THE REPUBLIC OF UGANDA

MINISTRY OF EDUCATION AND SPORTS

REVISED PRIMARY TEACHER EDUCATION CURRICULUM

MATHEMATICS MODULE

MAY 2011

UNIT 1: RATIONALE AND LEARNING THEORIES IN MATHEMATICS

1.1 Introduction

You are most welcome to unit 1. In this unit you will be introduced to the rationale for teaching mathematics in the primary school. You will also discuss various learning theories advanced by psychologists and how they are applicable to teaching and learning mathematics in the primary school.

1.2 Content organization

Dear student, in this unit you are going to cover the following topics as indicated in the table below.

Topic	Subtopic
i. Rationale for teaching mathematics in primary schools	a) Importance of learning mathematics in primary school b) Objectives of teaching mathematics c) Mathematics in everyday life
ii. Teaching mathematics in primary school	a) Challenges of teaching and learning mathematics in the primary school b) Role of the teacher in teaching mathematics
c) Learning theories in mathematics	a) Piaget b) Ausubel c) Skemp d) Cagne e) Dienes f) Bruner

1.3 Learning outcome

Explain the rationale and apply various learning theories in teaching mathematics

1.4 Competences

a) Explain the rationale for teaching mathematics in primary school
b) Discuss the importance of mathematics in everyday life and all subject areas in the school curriculum
c) Identify challenges and roles of the teacher in teaching mathematics
d) Apply different learning theories to the teaching and learning of mathematics

1.5 Unit orientation

This unit will help to explain why mathematics should be taught to all pupils in the primary school. It will be interesting to discover the enormous uses of mathematics not only in all subject areas of the school curriculum but also in everyday life. The learning theories give you a basis on how the curriculum and teaching are designed.

1.6 Study requirements

To succeed in studying this unit, you will have to engage in plenty of discussions with your tutor and classmates. There will be group work activities too. You will need a pen, notebook, a copy of the Uganda Primary School Curriculum (1999) and some textbooks on learning theories from the library.

1.7 CONTENTS AND ACTIVITIES

1.7.1: RATIONALE FOR TEACHING MATHEMATICS IN PRIMARY SCHOOLS

a) Importance of learning mathematics in the primary school.

In your understanding, what is mathematics? Compare your views and answers with those of your colleagues. In mathematics we deal with numbers and computations. Some reasons why every child should learn mathematics are given below.

1. Mathematics provides learners with means of developing powers of logical thinking, spatial awareness and numerical skills.
2. Mathematics presents information in many ways through the use of numerals, tables, charts or graphs and other diagrams.
3. The figures and symbols used in mathematics can be manipulated and combined in systematic ways so that it is possible to deduce information about the situation to which the mathematics relates.
4. Mathematics is fundamental for the study of all subjects in the curriculum.

It is good that you work in groups and come up with more reasons for teaching mathematics in the primary school. Learners should be made aware of these reasons.

b) Objectives of teaching mathematics

Study the Uganda primary school curriculum, volume one 1999, starting from page 221. Write down the aims of teaching mathematics in the primary school. If you read further on, you will realize that there are set objectives for teaching each topic to each class. Write down a topic, say sets and list the objectives for teaching sets, for P.1 up to P7.

Below are some objectives which you should relate to a topic and possibly a class in the primary school.

1. To enable the learners to count, read and write numbers; and make simple calculations with numbers.
2. To enable learners workout problems involving money, distance, time, temperature and capacity.
3. To enable the learner apply the concepts of ratio, proportion and percentage in problems in daily life situations.
4. To enable the learner form patterns, recognize shapes and know their properties

The objectives above and those you listed on your own, if attained by all the learners at the end of their primary school, will lead them to having the vital components of mathematics as may be required of an ordinary citizen.

c) Mathematics in everyday life

What role does mathematics play in anyone's everyday life? You will agree with me that we all need to:

1. Be able to count and make simple calculations with numbers; note that calculations may be done with or without a calculator
2. Know about money, balance or change issues, profit and loss matters
3. Be able to measure mass, length, capacity, talk about time and temperature.
4. Be able to recognize shapes, patterns and know some of their properties.
5. Deal with ordinary fractions, decimals and percentages.
6. Read and understand charts and graphs in their various forms.
7. Use mathematics knowledge to solve specific problems in our everyday life.

List down a few more things you believe we should all be able to do as a result of the mathematics we learn. What mathematics will specifically be required by:

- A milk vendor
- A poultry farmer
- A taxi conductor
- A parent
- A nurse
- One with a telephone/food business
- A teacher can live without mathematics?

1.7.2: TEACHING MATHEMATICS IN THE PRIMARY SCHOOL

a) Challenges of teaching and learning mathematics in the primary school

One of the challenges generally faced by mathematics teachers and learners not only in Uganda but also in other parts of the world is according to cockcroft (1982, p67):
Mathematics is a difficult subject to teach and learn. This is because the ability of a learner to proceed to new work is very often dependent on a sufficient understanding of one or more pieces of work that have been done before. We may say that learning multiplication, for example is dependent on having learnt addition.

Some other challenges include the great differences in the rates of attainment between children of the same class and lack of spirit of hard work and practice.
In Uganda, specifically, we face challenges of:

- Lack of mathematics textbooks which can cater for the learners' different abilities
- Large classes, which are difficult to teach and have their written work marked.
- Teaching mathematics in English language, using technical words whose meanings in mathematics are different from their use in ordinary English language

Get into groups and discuss other challenges to teaching and learning mathematics in Uganda suggest strategies that will minimize these challenges.

b) Role of the teacher in teaching mathematics

As a primary school teacher, you are expected to play several roles as an individual and also as a member of staff or the profession at large. Here are some of the roles.

1. Make mathematics a reality in life by using methods and approaches to learning that are very practical and are based on the experience of the learners..
2. Integrate mathematics with other subjects in the curriculum by seeking opportunities out of a wide range of learner's activities.
3. develop in learners a positive attitude to mathematics, create awareness of its great power to communicate and provide explanations in matters of daily phenomena
4. demystify the subject and make it user friendly
5. constantly evaluate the teaching/learning process
6. Plan and manage your time. You have to plan your day, your week and entire year so that you can accomplish your work.

All these roles have to be undertaken along with your other commitments at home and in the community. Remember at school you are expected to take on the role of a parent. You ought to guide and counsel the learners.

1.7.3: LEARNING THEORIES IN MATHEMATICS

We shall learn about theories which are sets of ideas that attempt to explain the process of learning. These ideas have been advanced by different psychologists. We shall discuss how each one applies to teaching and learning mathematics.

a) Piaget

Piaget believes that a child's ability to learn develops in well defined stages that are related to "chronological" age. He believes in child-centered education. Piaget emphasizes discovery learning where a child is given chance to manipulate real objects.

As a teacher, this theory demands that you give children a chance to discover knowledge.
How will you guide children to discover the formulae for perimeter of a square or area of a rectangle? How can these be built onto what children already know? How about a sum like $5 \div \frac{1}{2}$ using pieces of paper, then using pictures or symbols and finally formulae?

All in all, always give pupils an opportunity to discover new ideas depending on what they already know.

b) Ausubel

Ausubel advocates for meaningful learning as opposed to rote learning. He stresses the importance of learners discovering formulae before they can practice using them. If this is done, the learner finds it easy to memorize the formulae. This of course requires the teacher to allow for plenty of practice for the learner to discover patterns that lead to formulae.

He criticizes rote learning (reception learning) in which the learner is presented with the entire content of what is to be learnt in its finished form. He says this usually involves multiple readings (cramming) of the material with little or no effort devoted to retention.

Teachers will impose on children formulae for area, arithmetic mean and angle sum of a polygon. Discuss with colleagues how these can be taught meaningfully with children building onto previous knowledge.
Remember Ausubel emphasizes that children should discover, practice, memorize then be in position to apply and relate.

c) Skemp

Skemp says we should teach mathematics from simple work to hard work. For example, you cannot teach addition of fractions before children have learnt addition of whole numbers.

How can you make use of what skemp advocates for when you are planning for your teaching?
In your scheme of work, topics which can be understood easily by the learners should appear first. Those which are hard should appear later. This explains why integers are not taught in lower primary classes.

When you are preparing a lesson, find out the simplest mathematical ideas in that topic and start with those. This requires that even before you write the scheme of work, you should read widely on a given topic then organize your work from simple to complex.

d) Gagne

According to Gagne learning will only be said to have occurred if there is a change in the learner's behavior or performance. He says that learning must be linked with the design of instructions. A design of instructions depends on the learning outcomes the teacher sets out to achieve. Gagne is concerned with how the teacher develops the learning outcomes or objectives.

He says that learning objectives should be formulated basing on skills, concepts and information; and attitudes and values to be gained by the learner. In the primary school, pupils are expected to acquire computational skills in mathematics. These skills involve how to add, subtract, multiply and divide.

The concepts or ideas to be learnt will include knowing ordering of numbers: 2 is less than 5, 6 exercise books are more than 3 exercise books.
Pupils should learn about measuring before learning area. Change in attitudes and values can be achieved by the teacher relating mathematics to the learner's experiences and environment. As a teacher, Gagne says you should observe a change in the learner's performance for you to have done some teaching.

e) Dienes

In his contribution to the teaching and learning of mathematics, Dienes uses cubes and cuboids to help pupils understand number notations and operations. He fitted the cubes or cuboids alongside one another to give comparisons of lengths and then reassembling again. Dienes says that in early years, a child realizes that 1 cube is for one unit and all the other numbers greater than 1 are represented by repetition.

For example 5=1+1+1+1+1

Later on children discover combinations that result into larger cubes.

Study the following:

Start with 1 cube = 1 cube
Arrange 3 cubes in a row = 3 cubes
Arrange 3 cubes in 3 rows = 3x3 cubes

As a child continues with the arrangement, they discover the rotation of $3^0, 3^1, 3^2, 3^3$,-- which we use when dealing with number bases and powers.

Dienes advocates for step understanding which is very important in learning mathematics. Study the Uganda Primary School Mathematics Curriculum and see its spiral nature. All the topics develop year by year from P.1 up to P.7.

f) Bruner

According to Bruner, the role of the teacher is to interpret the learner's environment then assist the learner to actively participate in learning through discovery activities. The teacher should design activities to suit the environment and the curriculum in order for learners to discover mathematical ideas.
How does Bruner's theory compare with the thematic curriculum?

He advances the following advantages of discovery learning.

- Independent thinking and problem solving is developed. The child uses techniques of discovery learning to find solutions to real problems outside the classroom. Think of some examples.

- The child is motivated to venture into new problems after discovering solutions on his or her own.

Bruner wants the teacher to teach the basics. After the basics, the teacher will pose a problem to the learner and encourage the learner to explore it, prompt the learner to use previous knowledge to solve the problem, give the learner a chance to demonstrate the new skills acquired then finally build on the learner's experience to give them the information they may not have discovered.

1.8: Unit summary

You have come to the end of unit1. In this unit you were introduced to the importance of learning mathematics in the primary school. You discussed how mathematics is used in daily life, the challenges faced in teaching and learning mathematics and what some psychologists say about the teaching and learning of mathematics

1.9: Glossary

Concept: an idea in mathematics e.g. addition.

← Discovery learning: learning in which facts and concepts are found out step by step

← Role learning: learning things by heart without understanding their meaning.
Or how they come about like $c = 2\pi r, \ v = \pi r^2 h$

Theory: a formal statement of ideas which are suggested to explain a fact.

1.10: Notes and answers to activities

a) Importance of learning mathematics in the primary school also it is because:

- Mathematics is a powerful means of communication, if **A** was born on 11/8/1960 and **B** was born on 8/11/1960, we can tell, who is older, by how many years.
- Mathematics can be used to predict the outcome of an event which is yet to come (probability especially).
- To arouse interest and appeal amongst children

b) Other objectives are for learners to be able to:

- Recognize shapes and know their properties
- Read and understand graphs in their various forms
- Use mathematics knowledge to solve problems in everyday life

c) People in their everyday life need to be able to:-
- Measure mass, length, capacity using various scales or containers
- Work with money by counting, adding, subtracting, dividing, multiplying often mentally
- Estimate time, distance, ingredients for cooking, doses of drugs etc

End of unit exercise

1. Give five examples of rote learning in mathematics in the primary schools. How can a teacher promote meaningful learning in mathematics
2. write down the procedure for an activity to help learners discover that:

i) $3 \div \frac{1}{2} = 6$

ii) The diameter of a circle $d = 2 \times radius$

d) Discuss the reasons why primary teachers may not give pupils chance to discover mathematical ideas

1.11: Self checking exercise

You have come to the end of unit 1. Listed below, are the learning outcomes. Please tick in the column that best reflects your learning.

Learning outcome	Not sure	Satisfactory
1. I can explain the reasons for teaching mathematics in the primary school		
2. I can discuss the importance of mathematics in everyday life and in other subjects		
3. I can identify the roles and challenges of the teacher in teaching mathematics.		
4. I can use different learning theories ← to → teach mathematics		

1.12: References for further reading

1. Alice Hansen et al (2005), Children's errors in mathematics: understanding common misconceptions in primary schools. Learning matters

2. Uganda primary school curriculum (1999). Volume one

UNIT STRUCTURE

UNIT 2: SET CONCEPTS

2.1 INTRODUCTION

You are most welcome to this **Unit 2**. This unit introduces you to the ideas of **set concept, operations on sets, solving problems in everyday life situations involving set concepts and how to teach set concepts in a primary school.**

2.2 CONTENT ORGANIZATION

Hello student, in this unit you are going to cover the following topics as indicated in the table below:-

Topic	Subtopic
1. Introduction to set concepts	a. Meaning of a set b. Formation of sets c. Types of sets
2. Operations	a. Intersection of sets b. Union of sets c. Difference of sets d. Compliment of sets
3. Application of sets	a. Use Venn diagrams b. Solving problems using set concept
4. Teaching set concept in primary school	a. Activities and instructional materials for teaching sets b. Planning lessons for teaching sets

2.3 LEARNING OUTCOME

At the end of this unit, you are expected to apply set concepts in solving problems in everyday life situations and teach set concepts in the primary school.

2.4 COMPETENCES

Dear student, now you know the expected learning outcome therefore as you study/ work through this unit, you will be able to:

a) Define a set
b) Give different examples of sets
c) Identify and name different types of sets
d) Carry out operations on sets
e) Draw Venn diagrams and use them to solve problems
f) Identify activities and instructional materials for teaching sets in primary schools
g) Apply knowledge of sets in solving problems in everyday life situations
h) Prepare lessons and demonstrate teaching set concept in primary schools

2.5 UNIT ORIENTATION

This unit is to get you exposed to plenty of materials to keep you actively learning by collecting, grouping objects, distinguishing between different groups formed and comparing them. This unit is very interesting because it starts with concrete objects then proceeds to pictures, numbers, letters being presented in Venn diagrams and finally application of set concepts in solving problems.

2.6 STUDY REQUIREMENTS

To be successful in studying this unit you are required to prepare plenty of materials collected from your local environment, read about pre-mathematical activities in the ECE module concerning sorting, identifying, matching and learning theories with respect to teaching mathematics in the previous unit. You also need to have primary mathematics course books, the mathematics primary school syllabus and secondary school mathematics book 3 for further practice.

2.7 CONTENTAND ACTIVITIES.

2.7.1: **Introduction to set concepts**

a) Meaning and formation of sets

What do you understand by a set?

Find out by doing the following:

Collect different items from your local environment, such as bottle tops, sticks, stones, etc.
Now you have collected them, may you group them according to colour, types, size and shape?

Can you name the groups formed?

For example a group of leaves, a group of stones etc. What do you call a group of objects formed?

Check this with this answer below:

These different groups formed are described as a collection of well defined objects called sets

Since you have understood the meaning of a set, you can now use the collected objects to list down five (5) examples of sets.
Thank you for listing down the sets. Share your answers with a colleague. Then, compare your answer with the following examples of sets:

i. A set of all chairs in my class room
ii. A set of all tutors in my college
iii. A set of red pens on the tutor's table
iv. A set of all vowel letters in the alphabet
v. A set of all factors of 12

Can you give 5 more examples of sets with well defined members?
Compare your answers with those of your colleagues. *You are doing well; let's continue.*
You can now read more about notations of sets

b) Set notations

Set notations: sets can be expressed in two forms namely;

1. Tabular form
2. Set builder form

A set can be named using any capital letter e.g. M={ all vowel letters in the alphabet} members of this set can be listed as M a,e,i,o,u}
When members of a set are listed as shown above, it is known as a tabular form. When a set is described in words, it is known set builder form.

All vowel letters listed are members of set M. A member / element of a set is shown by using small letters.
The symbol "ε" denotes that is a member of "G" is an element for example; 'a' is a member of set M is denoted as a ε M membe in a set are separated with comas.

Example 1:

Write the following set in a tabular form:

A = { all even numbers less than 10}
∴ A = {0,2,4,6,8}

Example 2.

Write the following is set builder form:

B = {January, June, July}
B is a set of all months of the year which begin with letter "J"

Using the acquired knowledge in above examples, try activity 2.1 in your exercise books

Activity 2.1

a) **Write the following sets in tabular form**

i. D = {all months of the year} (ii) E = {all days of the week} (iii) B = {all prime numbe less than 20}
(iv) G = {all English consonant letters}, (v) F= {all factors of 12}

b) **Write the following sets in a set builder form**

i. A={1,2,3,4,5,6,7,8,9,10
ii. C={w, x, y, z}
iii. K={2,3,5,7,11}
iv. L={red, yellow, blue}

i. n(D)
ii. n(E)
iii. n(F)
iv. n(G)
v. n(B)

(a) Types of sets

Equal and equivalent sets

(i) Equal sets

Look at the following examples:

Example 1:

A= {○ ,Ӝ,∆, □}
B= {□, ○ , Ӝ, ∆}

What have you noticed about set A and set B? You should have noticed that set A and set B have the same number of members which are exactly the same but with different arrangement. Therefore set A is equal to set B and is denoted as A=B. can you look at example 2:

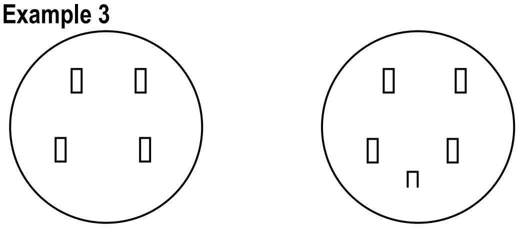
Example 2.

P= {2, 4, 6, 8}
Q= {2, 4, 6, 2, 8}

What can you say about example 2? In plenary Tutor guides you to share your answer with your colleagues. Then, read the following information. You need to note that:

> **i.** Using example 2, P=Q
> **ii.** A set is not changed if its members are repeated
> **iii.** The order in which the members of the set are written does not make any difference

Example 3



F G

Are the sets F and G equal? They are not equal. Why?

You must have found out that set F and G have different number of members therefore set F is not equal to set G. The symbol for 'not equal to' is ≠. We write for example: F ≠ G. using the above given information can you do activity 2.2.

Activity 2.2

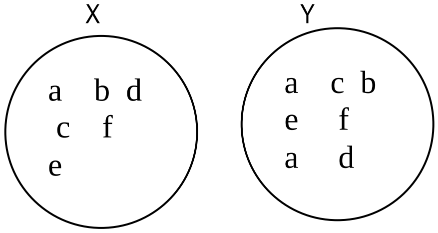
1. Which of the following pairs are equal and not equal sets?

   i. A={Uganda, Kenya, Tanzania, Rwanda}
   B= {all countries found in east Africa}

   ii. C= {1,2,3,4}
   D= {2, 1, 4, 3}

   iii. E={Δ,–,○}
   F= {□, Δ, ○}

   iv.
   X Y



   v. M= {whole numbers between 1 and 9}
   R= {3, 6, 8}

2. Write a set which is equal to each of the following sets.

   i. The set of the months of the year
   ii. The set of the letters in your name 'mukama'

Check your answers with the ones at the end of this unit.

- **Equivalent sets**

You are welcome to read about equivalent sets. Study example 4:

Example 4:

A={the letters in the word ‘mineral’}
B= {all days in a week}
The elements in the above sets will be:
A= {m, i, n, e, r, a, l}
B= {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

What do you say about set A and B?

Compare your observations with your colleague. Read about equivalent sets in the table below:

You will find that for example set A has the same number of members as set B but different members. Then set A and B are called equivalent sets. The symbol for ‘equivalent to’ is $\Leftrightarrow$ ≡. Therefore set A is equivalent to set B. this is written as $A \Leftrightarrow B$.

You can get more practice about equivalent sets by doing the following activity.

Activity 2.3

1. Which one of these are equivalent and not equivalent sets?

   i. K= {a, b, c, d, e}
      L= {a, e, i, o, u}

   ii. E= {a, b, c, a, b}
      F= {1, 2, 3, 1, 2}

   iii. P= {coat, book, shirt, dress, trousers}
      Q= {sheep, dog, elephant, lion}

2. Write five (5) examples of equivalent sets in your exercise book.

Share your answer with your colleague.

Note:

Using the above example part a (iii) in activity 2.3. Set P is not equivalent to set Q because they don’t have the same number of members. Therefore it is denoted as P $\not\Leftrightarrow$4?

Since you have read about equal and equivalent sets can you do the following activity individually in your exercise book?

Activity 2.4

(a) Use the following symbols to complete the given statement:
⟺Or =

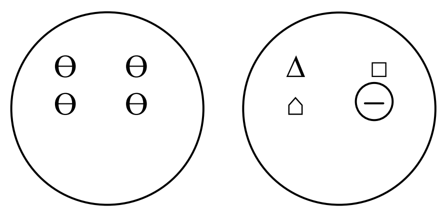
i. A= {muna, ntuza, gundi}
B= {gundi, muna, ntuza}

ii C= {all odd numbers from 1 to 9}
D= {1, 3, 5, 7, 9}

iii J= {yesterday, today, tomorrow}
K= {W, L, Y}

iv

L M



v N= {x, y, x, f, x}
1. P= {x, f, y}

(b) What have you observed?

Check your answers at the end of the unit.

For further practice on this competence read the exercises in primary mathematics course books P4, P5, and P6

Now you can take a break

Iii Empty sets.

Dear student, you are welcome from the break.

Let you read about the empty sets.

What is an empty set?

You can now answer this question, by; finding and listing the numbers of the following sets:-

i. A= {brothers and sisters who are 5 cm tall}
ii. B= {birds with teeth}
iii. C= {all students in my class with 3 hands}
iv. D= {all odd numbers between 10 and 30}
v. E= {the first three states in East African community}

Discuss your findings with your colleague then compare your findings with the following information:

You found that sets A, B, and C do not have members and these sets are examples of empty sets at times referred to as 'null sets'. The empty set is represented by the symbol { } or Ǿ. You should note that an empty set does not consist of any element/ member. The empty set { } is not the same as { 0 } or { 0 } or { Ǿ }

An empty set is one of the sub sets in all sets

Now you have known what an empty set is. Can you give 6 examples of empty sets.

Discuss your answers with your colleague.

I hope you are doing well.

(iv) Finite and infinite sets

With a colleague, can you discuss the following examples?

Example 1

i. The set of first five counting numbers
This is {1, 2 3, 4, 5}

Example 2

ii. The set of all the vowel English alphabet letters
This is {a, e, i, o, u}

Example 3

iii. The set of all composite numbers
This is {4, 6, 8, 9, 10, 12, 14, 16}

Example 4

iv. The set of positive integers
This is {+1, +2, +3, +5, +6, +7 …}

Example 5

v. The set of odd numbers less than 1000.
This is. {1, 3, 5, 9, ------------, 999}

Compare your findings with the text in the table below.

It is found that examples in parts i , ii, v have specific number of elements/ members or can be listed Up to the end. These are examples of finite sets. For part v a few members were written at the beginning, then series of dots before endings with a lot of members.

Can you list more examples of finite sets? Show your answers to your tutor.

You can also note that that:

Set with members that don't come to the end or with unlimited number of members are called Infinite sets e.g. part iii and iv.

If you have understood the difference between finite and infinite sets, can you do the following activity in your exercise book?

Activity 2.5

Use the listing method (tabular from) to write each of the following sets and state whether it is finite or infinite.

a) The set of names of months in a year
b) The set of names of government aided PTCs
c) The set of multiples 24
d) The set of triangular numbers
e) The set of all counting numbers
f) The set of multiples of 3 upto 27

You can check your answers at the end of this unit.

However you can also note that some are infinite in some special way as shown in the examples below:-

a) The set of all odd numbers
This is expressed as {1,2,3,4,5,6,7,8,9,10,…..}

b) The set of all multiples of 4.
This is expressed as {0,4, 8, 12, 16, 20, 24, ….}

c) The set of square numbers
This is expressed as {1, 4, 9, 16, 25 …}

v. Sub sets

Taking set to be a set of the letters that spell the word "cat".

P= {c, a, t}

Now write down the smaller sets from this set P.

How many such smaller sets have you obtained? You should have come up with the following sets:

{ c, a};{a, t};{c, t};{a};{t};{c};{} and {c, a, t}

Let you read the text in the table

The smaller sets obtained from the bigger set are called sub sets. You will note that the last set in the sub-sets listed above is set P itself. This is because every element of P is in P Any given set is a subset of itself and an empty set is also a subject of every set. If one set such as {a,t} from the list of sets obtained from set P is named Q, then the set Q is said to be a sub set of set This is denoted as $Q \subset P$. If a set say R={s,y} is not a subset of set P, it is denoted as $R \not\subset P$.

Let us now try to come up with a list of sub sets for the following sets:

i. {0,1}
ii. {□,∆,O}
iii. {a, b, c, d}

Compare the number of subsets obtained for each given set with the number of elements in those sets
What is the relationship between the number of elements in a set and the number of subsets formed?
You will note that the number of sub sets from a given set has some relationship with the number of elements in that set.

For example set P with 3 elements has 8 subsets. Now let us fill in this table using the lists of subsets obtained earlier.

Number of elements	Number of subsets	Relationship
2	4	4 = 2
3	8	8 = 2
4	………	….. = …..
5	………	….. = …..

From the table you will realize that the number of sub sets are obtained by using the formula 2n where n is the number of elements in that given set.

You can now try out the following activity:

i P= {1,8,5}
ii K= {all factors of 12}
iii A= {all vowels letters in the alphabet}
iv Q= {parrot, crow}
v T= {all days of the week}

b) Write down the relationship of the following sets;

A= {0,1, 2, 3,4, 5}, B= {0, 2, 4} and C= {7, 8}

Compare your answers with colleagues and then show your answers to your tutor.
Thank you for completing this topic.

2.7.2. OPERATIONS ON SETS

(a) INTERSECTION OF SETS

Since you know what a set is and listing down members in a given set from the previous topic, you can now look at these sets:
A= {a, e, i, o, u} and B= {c, h, a, i, r}

What do you notice about them? You must have noted that the two sets have some common members in them. You can now identify and list down the common members in the two sets.

The set obtained is {a,i} and is called the intersection set. It shows the common members in the two sets A and B. it is stated as set A intersection set B is {a, i}. The symbol for intersection of sets is ∩. so the above can be denoted as A∩B ={a,i}

You can now find out and write down the intersection of the following using the symbol '∩'

i C= {2, 4, 6, 8, 10} and D= {3, 6, 8, 9}
ii E= {mango, lemon, paw paw} and F= {pineapple, mango, apple}
iii K= {yellow, blue, green, red} and L= {purple, blue, orange, red}

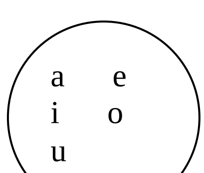
Compare your work with those of your colleague.

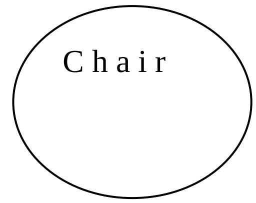
Did you know that sets can also be shown using Venn diagrams?

Look at these examples:

Example 1.

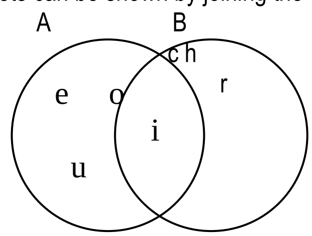
A is a set of all vowel letters in the alphabet and B is a set of letters that spell the word 'chair'.
These sets can be written as A= [a, e, i, o, u}



A B 

B={c, h, a, i, r}

The intersection of these two sets can be shown by joining the two sets as shown below.



With the knowledge gained in example1, can you now show the intersection of the following pairs of sets on the Venn diagram on a sheet of paper.

.

i. C= {2,4,6,8,10} D= {0,1,2,3,4,5,6,7,8,9}

ii. P= {∆,□,▯,○} Q= {□,○ ,⌂,◊,O}

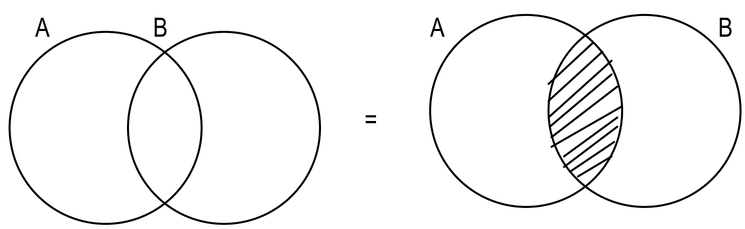
iii.

iv. R= { Dan, Tom, Anne, Mary} S= {Linda, Mary, Tom, John}

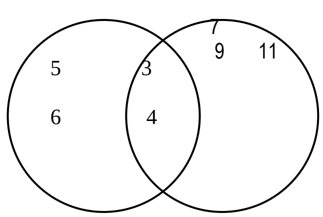
Compare your diagrams with those of your colleagues. Well done.

Example 2:-

Use the venn diagram to shade A ∩ B



Example 3



M ∩ Q **=** { 3,4}

With your colleague do the following activity on a sheet of paper;

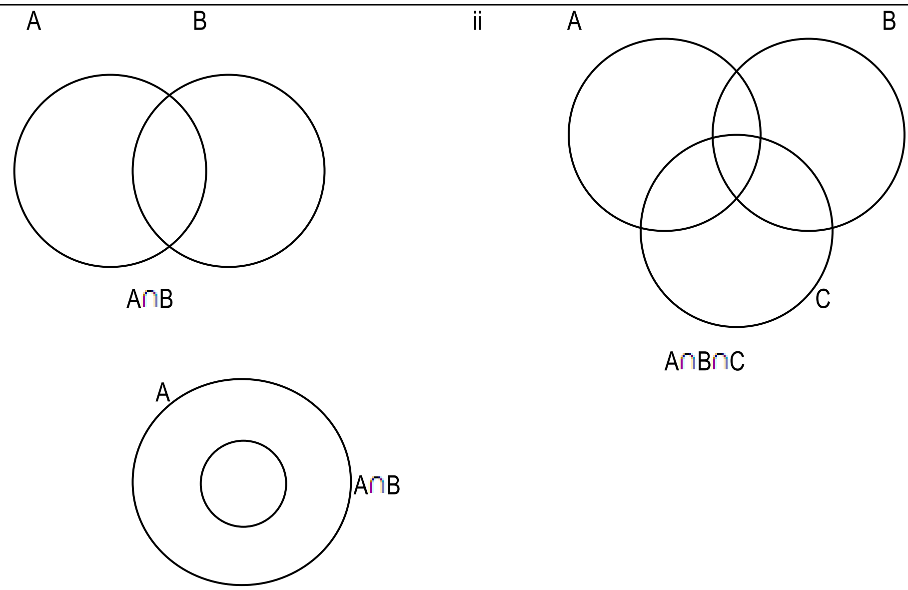
Activity 2.6

1. If M = { 1,2,3,4,5,} and N = { 3,4,5,6,7} find M ∩ N

2. (i) Using a venn diagram find A ∩ B. If A = { 1,3,5,}   B = { 2,3,4,5}

   (ii)Given that x = { 1,3,4,5},   y = { 2,3,4,6} and z = { 3,4,5,6} find A∩B∩C

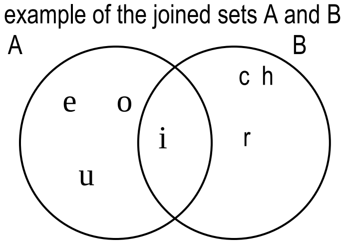
3. Shade the intersection region



Compare your answers with the ones the one at the end of unit

(b) Union of sets

Look at this example of the joined sets A and B from the previous work.



Can you list down all the elements in these joined sets as seen in the diagram? How many elements are there in this set?

The set you have written is called a union set. Union comes from the word ‘unite’ or combine. So the set of A union B is {a, e, i, o, u, c, h, r}. You will note that much as ‘a’ and ‘i’ are found in both sets; they are written once because they appear once in the intersection region. That means if you are writing down elements of a union set, you don’t repeat the common elements. The symbol ‘U’ is used to stand for union of sets. So AUB ={a, e, i, o, u, c, h, r}

Activity: 2-7

Can you now write down the union of the following sets using the symbol 'U'
i. P= {l, m, n, o, r} and Q= {m, r, s, t, n, u}
ii. R= {Paul, Anne, Mary, Dan} and S= {Dora, Anne, Peter}
iii. A={1,2,3,4,5,6,7}and B= {0,2,4,8,10}
iv. E= {dog, cat, cow, goat} and F= {cat, sheep, rabbit, cow}
v. K= {t, a, b, l, e} and L= {d, e, s, k}
Compare your work with that of your colleagues and show your answers to your tutor.

How are you feeling now?

Great

(C) Difference of sets
From the previous work of joining two sets as the example

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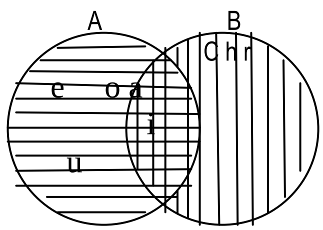


You will realize that there are 3 distinct regions on the Venn diagrams as shaded differently what does each of the region represent?

Read the information in the table about the difference of sets

You will realize that region shaded A epresents the intersection of set A and B. Region 'represents elements in set A that do belong to set B and region represents elements in set B that are not in set A. the two regions nd show the difference of the t sets A and B. The difference of the two sets is written as: A-B= {e, i, o, u} and B-A= {c, h, r}.

Study the examples below;

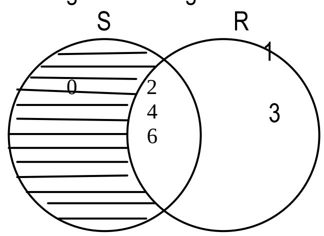
Example 1.

Given the following sets; S= {0,2,4,6,8}and R= {1,2,3,4,5,6}

You can now find the difference of the following:

i. S-R
ii. R-S
iii. Is S – R the same as R – S?

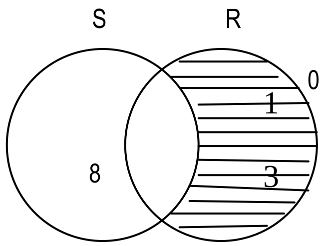
(i) Using a venn diagram:



S = {0,~~2~~,4,~~6~~,8}
R = {1,~~2~~,3,~~4~~,5,~~6~~}

∴S – R = {0,8}

(ii)

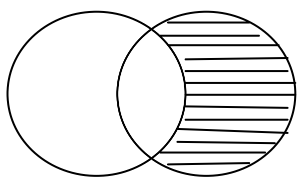


∴R – S = {1,3}

(iii) form the diagrams above S-R is not the same as

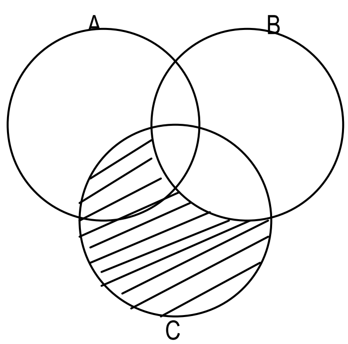
Example 2: **A** **B**

(i)



Shade B – A

(ii) What is the shaded region below?

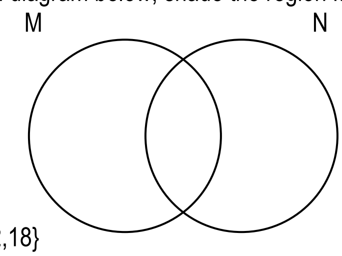


C – B

Now using the above examples can you do the following activity in your exercise book:

Activity 2.8

1. Given the venn diagram below, shade the region M - N

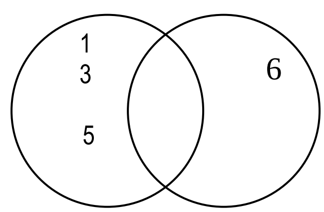


2. If D = {2,4,6,12,18}
C = {6,8,12,14}find

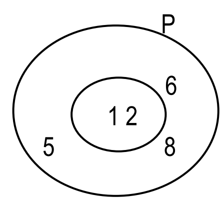
(i) D – C
(ii) C - D

3. Given information in the Venn diagram below:

(i)



(iii)



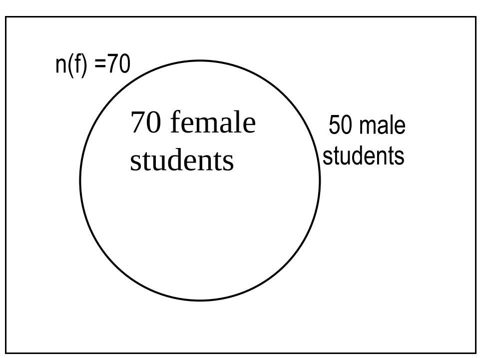
Q 7
Find P - Q

Compare your answers with the ones at the end of the unit.

(d) Compliment of sets

Before we look at this sub topic we shall need to learn about universal sets. Let us take all the students in the year one class to be a big set say of 120 students. The female and male students would be smaller sets within the big set. A set of 70 female students (F)can be shown on the venn diagram as:

$\varepsilon = 120$



The rectangle represents all students in the year one class and the circle (F) represents all female students within the class. The set of all students in year one class is called a universal set and is denoted by the symbol '∈' or 'U'.

Can you list down some examples of universal sets.

i. A set of fruits
ii. A set domestic animals
iii. A set of all students in the colleges
iv. A set of furniture
v. ξ = {1,2,3,4,5,6,7,8,9}

You can get other sets from a universal set. Try to write down other sets from the universal sets i - v above.

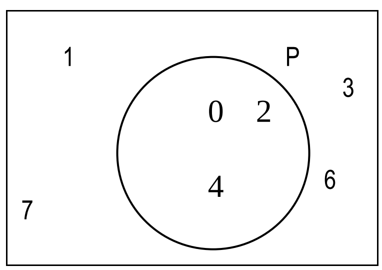
From the diagram above, you will note that the male students are within the universal set represented by the rectangle but outside the circle (F) which is the set of female students. In this case the male students are the compliment of the female students in the year one class.

The compliment of a set therefore is that set within the universal set outside a given set. As in the example above the compliment of male students are female students and vice versa the two sets are represented by letters "M" for males and "F" for females then the compliment Fis denoted as F^1 = {all male students in year one class} and the compliment of M^1 = { all female students in year one } the compliment of a set is represented as F^1 or M^1

You can now try out the following:

Activity: 2 - 9

a) Use the diagram to find the compliment of P



b) If A= {1,2,3,4,5,6,7,8,9,10} and Q= {1,3,5,7}

Find:

i. Q'
ii. R'
iii. (QUR)'

c) The ξ is a set of all days of the week.

If T= {Monday, Friday, Thursday, Sunday}

i. Show the information on the Venn diagram
ii. Find the compliment of T.

Compare your answers with the ones at the end of the unit.

2.7.3. APPLICATIONS OF SET CONCEPTS

You are thanked for reading through topic I and II. Now you are welcome to this topic.
You already know to use sets and Venn diagrams to solve problems.

(a) How to use a Venn diagram to solve problems

Dear student, the simplest way of solving problems is to draw a Venn diagram. Put all the numbers or elements in the appropriate places and then select the required region.

You study the examples below:

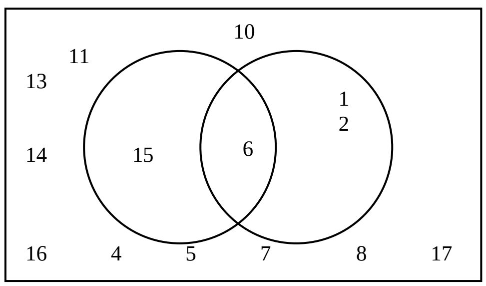
Given that ξ= {all integers from 1 to 18}, x = {all multiples of 3 from 8 to 15}, Y= {all factors of 18}

(a) List down the elements of (i) $X \cap Y$ (II) $X \cup Y$ (III) $(X \cup Y)'$

STEP I: write down all sets in tabular form.

ξ = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18}
X = {3,6,9,12,15}
Y= {1,2,3,6,9,12}

STEP II: Draw the Venn diagram



STEP III: Use the Venn diagram to get the answers:

(i) $X \cap Y$ = {3,6,9, 12}
(ii) $X \cup Y$ = {1,2,3,6,9,12,15,18}
(iii) $(X \cup Y)'$ = { 4,5,7,8,10,11,13,14, 16, 17}

I believe you have followed the examples. Can you do this problem in your exercise book for more practice?

Given that €= {all even numbers from 0 to 20}

A= {all multiples of 4 between 0 to 20}

B= {all square numbers less than 20}

Use the Venn diagram to list the members of the following sets

- **i.** $A \cap B$
- **ii.** A'
- **iii.** $(A \cap B)'$
- **iv.** $(A \cup B)'$
- **v.** $A' \cap B$
- **vi.** $A' \cap B'$

Share your answers with your colleague, then your tutor.

Are you finding it easy. Let you continue with solving problems.

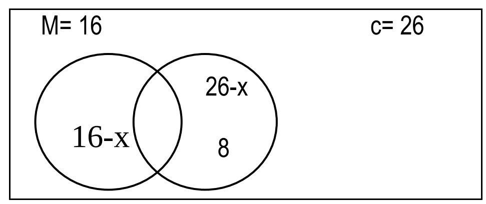
To begin with let you deal with two set problems.

Example 2:

At Kaliro PTC in a stream of 40 students 16 play mweso, 26 play chinese, and 8 play neither game. How many students play mweso and chinese. You study the working in the table below:

Step 1: you start by

i. Put the unknown value e.g. x in the appropriate place in the Venn diagram
ii. Let M= {mweso players}
   C= {chinese players}
iii. Use all the information given to compile the Venn diagrams
iv. Draw the Venn diagram as below



v. The total is 40; all sections add up to 40 thus

8+ (16-x) + x + (26-x) = 40
8+16 +26+ x –x –x = 40
50-x = 40
50-50-x= 40-50
-x/-1 = -10/-1
X= 10

vi. Substitute number (value for x) for letter since x =10

Then 16-x =16-10
=6 and
26-x = 26-10
16
Therefore the students who play both games are 10

Activity 2:10

a) Given that two sets N and M such that $\cap(M) = 12$, $\cap(N) = 13$, $\cap(N \cup M) = 20$ and $\xi = 24$, use a Venn diagram to find ou

$\cap(M \cap N)$
$\cap(M \cap N^1)$
$\cap(M \cup N)^1$

b) In a night club of 25 ladies, 12 support Arsenal, 10 support Manchester united and 11 support neither team. How ma ladies support both teams?

c) Peter bought a packet of assorted biscuits. 15 of them were circular, 11 of them were sugar coated while 6 of t circular biscuits were sugar coated.
Find how many biscuits were not circular; how many were only circular; how many were not sugar coated only?

d) 40 pupils were asked if they liked fish or meat. It was found out that 26 liked fish; 28 liked meat and 16 liked both fi and meat.

e) Show the above information on a Venn diagram; how many liked fish only; how many liked neither fish nor meat?

f) In a bus park, there are 30 buses. Ali noticed that 15 of them were white. He also noted that 12 had blue stripes. O 4 white buses had blue stripes.
Show this on a Venn diagram and how many buses had neither white nor blue stripes?

In plenary with the help of your tutor, share your answers with other groups. Check your answers with those at the end of this unit.

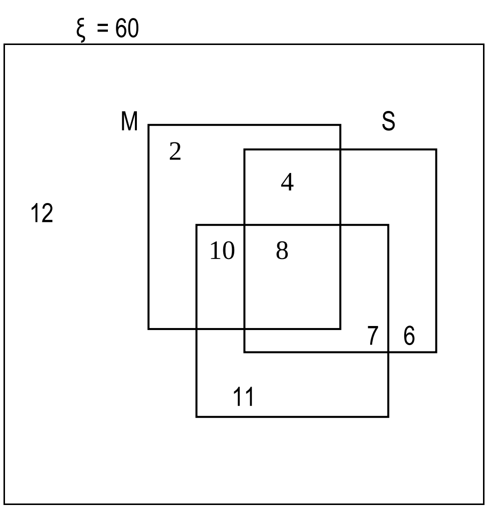
(b) Problems solving involving three sets

Dear student, welcome back from a walk. So far the only Venn diagrams you dealt with in the previous sub topic involved two sets. Let us now look at Venn diagrams involving three sets.

You study the following examples

Example 1:

Given the Venn diagram shown below



The diagram above shows the number of students studying Mathematics education (M), Science education (S), and Cultural Education (C) in a class.

i. How many students are studying science education?

ii. How many students are studying cultural and mathematics education only?
iii. How many students are studying all the three subjects?
iv. How many students are there altogether?
v. Write down $\cap(M'\cap S\cap C)$

You can now study the following working:

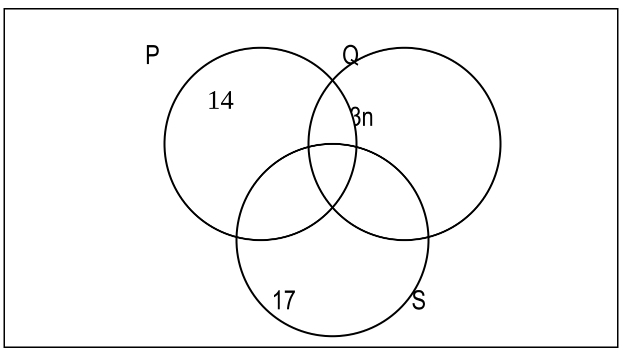
i. Students studying science education are 8+4+7+6= 25.
ii. Students studying cultural and mathematics education only are 10.
iii. Students who are studying all the 3 subjects are 8.
iv. Total students are 8+7+10+4+6+2+11+12= 60
v. $\cap(M'\cap S\cap C)$

M^1= {6, ~~7~~, 11, 12}
S= {4, 8, ~~7~~}
C= {8, ~~7~~, 10, 11}

$\therefore \cap(m^1\cap S\cap C) = 7$

Have you understood example 1. You can try example 2 with a colleague:

Example 2: Given that $\cap$ (P$\cup$ Q$\cup$ S) = 84 find n(PnQnS



Cross check your answers with the working shown in the table below:

Adding all the regions

14+ (n+1) +n+2n+17+ (n+4) +3n = 84

Collect like terms:

14+1+17+4+n+n+2n+n+3n=84
36+ 8n=84
36-36+8n=84-36
8n/8=48/8
n= 6

Can you do more problem solving examples like 1 or 2? If you have not understood any step share it with a colleague before you continue. Read the following text before you have some problems to solve. You need to understand the following expression in terms of
Set concepts.

'Both' means members which belong to the intersection set.

'All' means all members found in the common regions of more than two sets.

'Neither' means members which don't belong to particular sets.

'And' means members which belong to two sets.

Activity 2:11

At an inter house music festival at Nakaseke core PTC, there were 204 students. Students participated in different traditional dances as shown below:

110 danced bakisimba (ganda) (B)
96 danced kadodi (kigisu) (K)
77 danced naleyo (karamojong) (N)
20 danced both bakisimba and naleyo only
54 danced both bakisimba and kadodi only
18 danced both naleyo and kadodi only
17 didn't dance.

a. You represent the above information on a Venn diagram.
b. Using the above information answer the following questions
:
   i. How many students danced all types?
   ii. How many tried at least a dance?

May you revise your work before you could cross check with answers at the end of this unit.

Thank you for reading through this topic but you may have more numbers to practice.

Get National Curriculum Development Center Uganda (1997), Secondary School Mathematics Book 3, page 22 – 24. Exercise 1E numbers 1,3,6 and 10.

Share your work with colleagues.

Thank you for completing Topic 3

2.7.4. TEACHING SET CONCEPT IN PRIMARY SCHOOLS.

(a) Planning to teach sets:

In the previous topics you were taken through facts on basic concepts in sets.

Now imagine you are to teach sets in primary school. Think about how you will do it. As you think about how you will do it, you will be required to plan a lesson.

Planning to teach is a process that is intended to answer the following questions:

1. What am I going to teach?
2. How will I teach it?
3. Why am I teaching it?
4. How will I know if I have been successful?

As a teacher you are expected to plan for pupil's learning and provide a learning environment that will produce the desired learning outcome. It therefore requires that as you plan you identify the intended learning for the pupils by clearly knowing why particular experiences are beneficial for pupils' mathematics learning. This helps in coming up with specific learning competences and outcomes to be attained by the learners for a particular lesson in your plan. As you plan the lesson you need to consider the context in which you will teach in terms of school locality, class size, space, materials available, and background knowledge of the learners. You should remember that in your class you will have learners with different learning abilities and styles. You need to put this in consideration to ensure that all learners are fully helped and engaged.

The planned activities include revision, consolidation, and extension. Pupils make errors and misconceptions in learning different concepts in mathematics. You should have good knowledge of the common errors and misconceptions that occur in learning about sets as it is useful in preparing learning experiences that would correct them. You will need specific words related to sets and make sure they are clearly explained to the learners to avoid confusing them. Can you list specific words you may use in teaching sets? Now that you have some tips on what to take note of and the process of planning you can try out this activity.

Activity: 2.12

Identify a topic from the primary mathematics syllabus about sets in any class from P.4-P.7 and plan a lesson for micro teaching. Compare your lesson plan with those of your colleagues and correct where necessary.
Well done. You are ready to teach now.

(b) Activities and instructional materials for teaching sets:

You will note that children have numerous experiences right from home and in play situations where they form sets. Think of instances where children collect and group objects either at home or at school and list them down. So when teaching sets, you will build on these experiences children already have to make them understand the concepts better as you teach from known to unknown.

This therefore will require that you get various materials from within children's environment to use in teaching sets. It is very important to identify these materials well in advance as they guide you in planning activities for the lesson. Now let us stop and think about materials that could be used in teaching sets in primary school.

Can you list some of these materials? Your list will include materials like pupils themselves, books of different sizes and colours, coloured pencils, pens of different colours and types, cut out shapes of different colours , letter cards, number cards, leaves etc. As earlier noted materials help to determine the kind of activities you will structure for your lessons. Can you now come up with a list of activities the learners will do in the lessons using the materials listed above? You list will include activities like collecting objects, sorting, describing groups of objects, forming groups of objects, comparing groups of objects, counting objects in groups formed, joining groups of objects, forming smaller groups from big groups, portioning objects in a group, drawing Venn diagrams making rings around groups of objects and so on.

Now let's try out this activity:

Activity: 12 . 13

a. Pick the primary school mathematics syllabus and outline the content to be taught in classes P.4-P.7 in sets.
b. Suggest activities and instructional materials you would use in teaching each of the content outlined in (a) above.

You are doing well.

2.8 UNIT SUMMARY:

In this unit you have been introduced to sets concepts and learned about:

i. What a set is
ii. Formation of sets
iii. Types of sets
iv. Intersection of sets
v. Union of sets
vi. Difference of sets
vii. Complement of sets
viii. Subsets
ix. Solving problems using set concepts
x. Preparation for teaching sets in primary schools.

2.9 GLOSSARY

Set is a collection of well defined items.

Subset is a set which contains part of (or all of) another set.

Members/ elements is one of the items contained in a set

Null set is an empty set; one without any members.

Super set is a set from which other sets are contained.

Finite set is a set whose members can be counted and are terminate (end)

Infinite set is a set whose members cannot be counted and are non- terminate (do not end)

Universal set is a set defined by a list or rule which is general.

Complement of a set is all those members which are not in that set but are in the universal set originally given.

2.10 Notes and answers

Activity 2.1

a. (i) D ={Jan, Feb, Mar, April, May, Jun, July, Sept, Oct, Nov, Dec}

(ii) E = {Mon, Tue, Wed, Thur, Frid, Sat, Sun}

(iii) B = {2,3,5,7,11,13,17,19}

(iv) G ={b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z}

(v) F={1,2,3,4,6,12}

b. (i) A={all natural numbers up to ten}

(ii) B={all the last four (4) alphabet letters}
(iii) K={a set of all prime numbers between 1 and 12}
(iv) L={a set of all primary colours}

c. (i) n(D) = 12
(ii) n(E)=7)
(iii) n(F) = 6
(iv) n(G) = 21
(v) n(B) = 8

Activity 2.2

1(i) not equal
(ii) Equal
(iii) Not equal
(iv) Equal
(v) not equal

2. (i) {January, February, march, April…………December}
(ii) { m,u,k,a,m,a}

Activity 2.4

(i) A = B
(ii) C = D
(iii) $\Leftrightarrow$ J K
(iv) $\Leftrightarrow$ L M
(v) $N \neq P$

(b) It has been observed that all equal sets are equivalent set but not all equivalent sets are equal sets

Activity 2.5

a) {Jan, Feb, mar, april, may, jun, jul, aug, sept, oct, nov, dec}
(Infinite set)

b) {kaliro, nakaseke, kibuli, -------------, gulu}
(finite set)

c) {24, 48, 72, 96----------------}
(infinite set)
of

d) {1,3,6,10,15,21,28----------------}
(infinite set)

e) {1,2,3,4,5,6,7,8,9,-------------,}
(infinite set)

f) {3,6,9,12,15,21,24,27}
(Finite set)

Activity 2.9

a) P^1 = {1,3,6,7 }

b) (i) Q^1={2,4,6,8,9,10 }
(ii) A^1={ }
(iii) $(QuA)^1$ ={ }

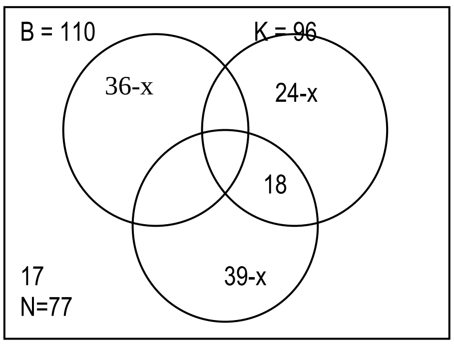
c) ε ={mon, tue,wed,thur,fri,sat,sun}

T={mon,fri,thur,sun}

(i) Diagram undrawn

(ii) T^1={tue,wed,sat }

Answers for activity 2.11
(a) ξ = 204



Let students who danced all be x.

Then B only = 110 – (54 + x + 20)
= 110 - 74 – x
= 36 – x

M only = 77 – (20 + 18 + x)
= 77 – 38 – x
= 39 -x

K 0nly = 96 – (54 + 18 + x)
= 96 – 72 - x
= 24 - x

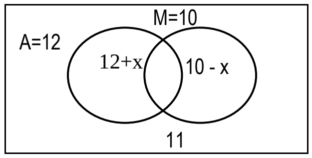
(b) is students who danced all

36 – x + 54 + x + 20 + 18 + 39 – x + 24 – x + 17 = 204
=36 + 54 + 20 + 18 + 39 +24 + 17 + x – x – x – x = 204
= 208 – 2x = 204
= 209 – 209 – 2x = 204 – 209
$\frac{-2x}{-2}$ = $\frac{-4}{-2}$
x= 2

(ii) Those who tried a dance = 204 - 17
= 187 students

Activity 2.10

(a) (i) n(M ∩ N) = 8
(ii) n(M ∩ N^1)
(iii) $(M \cup N)^1$= 24 – (4 + 8 + 12)
= 24 – 24
= 0
(b) ξ = 25



Let those who support both games be x

Then
= 12 – x + x + 10 – x + 11 = 25
12 + 10 + 11 + x – x – x = 25
33 – x = 25
33 – 33 – x = 25 – 33
$\frac{-x}{-1} = \frac{-8}{-1}$
X = 8

∴ Those who support both games are 8.

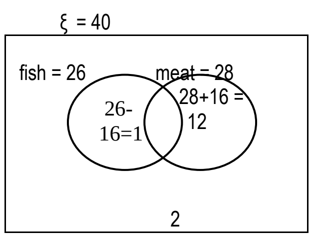
(c) (i) Biscuits not circular

11 – 6 = 5

(ii) Biscuits not sugar coated

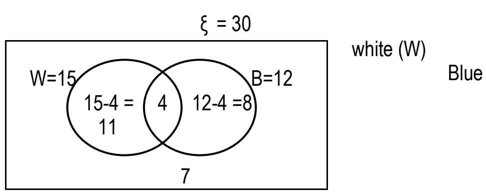
15 – 6 = 9

(d) (i)



(ii) Those who like fish only are 10
(iii) those who did not like fish and meat

40 – (10 + 16 + 12)
= 40 – 38
= 2 students



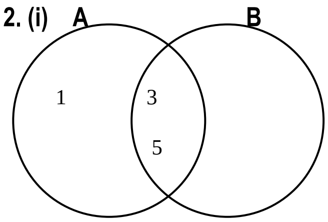
Those buses which were neither white or without blue stripe

30 – (11 + 4 + 8)
= 30 – 23
= 7 buses

Answers: (2 -6)

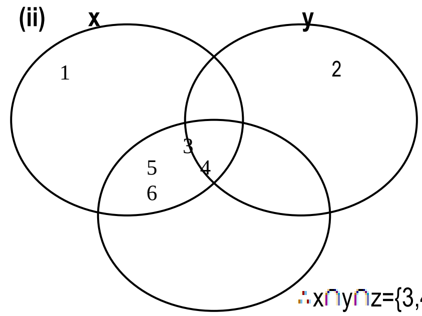
1. **M∩N = {3,4,5}**

2. (i)



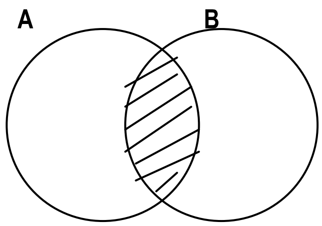
∴ A∩ B = {3,5}

(ii)



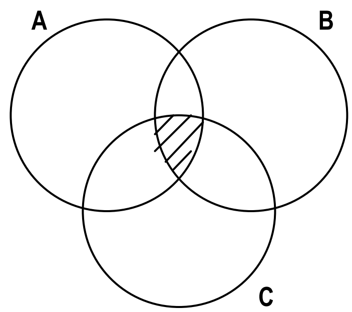
∴x∩y∩z={3,4}

3 (i)



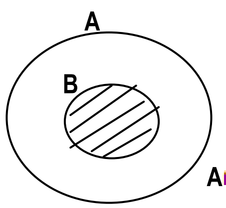
A ∩B

(ii)



A∩B∩C

(iii)



A∩B

Activity 2.8

2.11 END OF UNIT EXERCISE

1. Which of the following sets are equal or equivalent?

A= {1, 2, 3, 4}
B= {a, b, c, d}
C= {3, 4, 2, 1}
D= {a, e, i, o}
E= {b, c, d, a}

2. (a) List down the members of the following sets:

A is a set of all vowel letters of the alphabet.
B is a set of even numbers less than 20.
C is a set of the first five square numbers.
D is a set of the months of the year with 31 days.

(c) Find i.∩ (A) ii.∩ (B) iii.∩(C) iv.∩ (D) diagram undrawn

3. Use the Venn diagram to answer the following questions:

i. List down the members in set P and set Q.

ii. What is P∩Q

iii. What is PUQ

iv. Find P-Q and Q-P.

4. Draw Venn diagrams to show this information

€= {all letters of the alphabet}
B= {all vowel letters}
C= {the first 12 letters of the alphabet}
(c) Use the diagram to find

i. B∩C

ii. BUC

iii. (BUC)'

iv. ∩(BUC)'

5. Find the number of subsets in the following sets:

R= {l,m,n,o,p}
S= {all factors of 6}
T= {all months of the year}
V= {⌂,▯,∆,□,O}

6. Find the number of elements in the following sets given that

i. P has 32 subsets

ii. Q has 16 subsets

iii. R has 128 subsets

iv. T has 156 subsets

7. In their leisure time, 35 pupils out of class of 56 enjoy watching TV, 22 enjoy listening to music and 15 enjoy reading. 9 pupils enjoy doing all the three, 4 pupils enjoy listening to music and watching TV only while 2 enjoy listening to music and reading only. Draw a Venn diagram to help you answer the following questions.

i. How many pupils enjoy listening to music and reading?

ii. How many pupils enjoy reading only?

iii. How many pupils enjoy watching TV and reading only?

iv. How many pupils don't enjoy any of the three activities?

v. How many pupils enjoy listening to music only?

vi. Find the probability of a pupil selected at random who enjoys watching TV only?

Submit your work to your tutor.

Congratulation for completing unit 2

2.12 SELF CHECK/ASSESSMENT

No	Learning Outcome	Not sure	Satisfactory
1.	I can define a set		
2.	I identify and name different types of sets		
3.	I can carry out operations on sets		
4.	I can draw venn diagrams and use them to solve problems of sets		
5.	I can apply knowledge of sets in solving problems in everyday life situations		
6.	I can teach set concepts in Primary schools		

2.13 references for further reading

1. Primary School Mathematics text books.
2. School Mathematics of East Africa Books 1,2,3,4, Cambridge University Press
3. Okot- Uma, R, Kawooya M, et al (1995), Secondary School Mathematics; Macmillian Uganda.

UNIT 3: NUMERATION SYSTEMS AND PLACE VALUES

3.1 Introduction

You are welcome to this unit 3. This unit introduces you to the counting, writing of numbers in accordance to the idea of place values.

It also enables you to write numbers in both Roman numerals and Arabic, also how to teach numeration systems and place values in a primary school.

3.2 Content organization

Dear student, in this unit you are going to cover the following topics as indicated in the table below.

Topic	Sub topic
1. Introduction to numbers	a) Meaning of a number and a numeral b) Reading and Writing of numbers • Whole numbers • Natural numbers • Writing Hindu Arabic numerals in figures and vice versa.
2. Roman numerals and Hindu Arabic numerals	a) What are Roman and Hindu Arabic numerals b) Writing of Roman and Hindu Arabic numerals c) Advantages and disadvantages of Roman numerals d) Conversion of Roman numerals to Hindu Arabic numerals and vice versa e) Application of Roman and Hindu Arabic numerals
3. Place value concept	a) Place value charts b) Reading and writing in place values c) Decimal number fractions d) Application of place values
4. The counting system	a) Introduction to bases b) Counting and grouping numbers in (i) Binary system (ii) Base five system (iii) Base eight system (iv) Conversion of bases (v) Application of bases
5. Clock computations	a) Introduction to Clock computations b) The finite system c) Application of clock computations
6. Teaching of numeration systems and place values in a primary school	a) Activities and instructional materials for teaching numeration system and place value b) Planning lessons for teaching numeration systems and place values

3.3 Learning Outcome

You are expected to apply knowledge of numeration systems and place values to life situations and also to teach numeration systems and place values in the primary school.

3.4 Competences

Dear student, now you know the expected learning outcome, therefore as you study/work through this unit you will be able to

a) Properly use place values in writing and reading numbers
b) Write ordinary numbers in Roman and Hindu Arabic numerals and vice versa.
c) Express Hindu Arabic Numerals to roman numeral and vice versa.
d) Read, write and count in different groups of numbers other than base ten.
e) Convert a given base to another base and write given number of any base to its place value
f) Group numbers that have the same remainder in a given system
g) Carry out addition of finite system on clock faces and calculate the sum n finite system.
h) Identify activities and instructional materials for teaching numeration systems and place values in primary school.
i)Prepare a lesson and demonstrate teaching of numeration system and place values in primary school.

3.5 Unit orientation

This unit will make you exposed to variety of actively learning materials like grouping of pupils, sticks in fives, eight and tens. It will also need you to involve the learners so that the competences set can be achieved.

3.6 Study requirements

To be successful in studying this unit, you are required to collect plenty of materials from the environment like counting sticks, chalk board illustrations, wall clock with Roman figures and also plenary primary mathematics course books for further practice.

3.7 Content and activities

3.7. I. INTRODUCTION TO NUMBERS

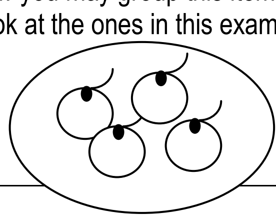
a) Meaning of a number and a numeral

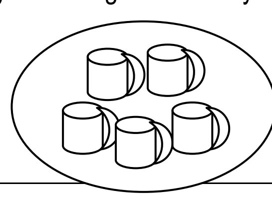
What do you think a number is?

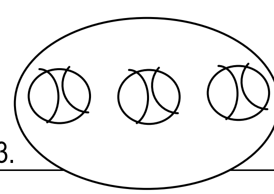
Find out by doing the following

Activity 3.1

Collect different items from the surrounding such as oranges, cups, balls etc
Now you may group this item differently according to what they are and form a set.
Look at the ones in this example

1. 

2. 

3. 

Compare your answers with a colleague.

Now compare your answers with the text below:

1) a) There are (4) oranges
   b) There are four oranges

2) a) There are (5) cups
   b) There are five cups

3) a) There are (3) balls
   b) There are three balls

Hullo student relax a little before you continue.

After comparing your answers with the text shown above

There is something you have to learn.

Then ask yourself! What is it?

This is what we shall discover

According to the tutors example in (1a) above,

1 a) There are (4) oranges "(4)" in the bracket is a digit which is the same as numeral. Now then ask yourself what is a number?
According to the tutor's example in the table above,

The word number or figure is used to express and record quantities of various objects e.g 4 oranges,2 cups,7.8 metres etc

Thank you very much. I hope you have now discovered the difference between a number and a numeral.

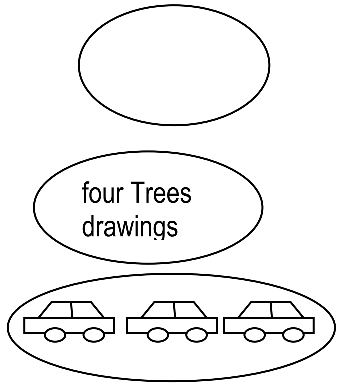
b Reading and Writing numbers.

(i) Whole numbers

Activity 3.2

What do you think is a whole number?

Find out by matching the following



1

3

4

What have you discovered?

Find out if your answer is the same as the one in the box below

What is in the sets of objects above is a whole. Therefore we can say

- There are three pencils in a set
- There are four tables in a set
- There is only one car in the set

We can also say, in the whole class room there is no pupil therefore a whole number indicates the number of items present in a given set and it begins from zero {0,1,2,3,4----}

Note: whole numbers are either natural numbers or counting numbers but natural numbers begin from one i.e. {1,2,3,4,5,----}

Well done

(ii) Reading and writing number in Hindu Arabic numeral system.

You are reminded that the most commonly used for counting numbers is the <u>decimal system reading numbers</u>.

When you were in primary six and seven, we considered the place values of the digits in a number using the following place value chart

Billions			**Millions**			**Thousands**			**Units**		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
				4	5	3	4	6	2	7	8

The chart shows 4 5 3 4 6 2 2 7 8 which is read as “Forty five million three hundred forty six thousand ,two hundred seventy eight.

Now, can you do this activity in your exercise book:

Activity 3.3

1. Using a place value chart, write these numbers in words:

(a) 321,858
(b) 957,781,612
(c) 19,463,100,342

2. Use the place value chart to write the following in figures.

(a) Seventy million, six thousand twenty three.
(b) Three billion, nine hundred and twenty one million for hundred thousand.
(c) A quarter of million.

Thank you for working out the above activity.

May you now show your answers to the tutor.

Thank you for completing topic 1. You can take a walk.

3.7.2: ROMAN NUMERALS AND HINDU ARABIC NUMERALS

a) What are Roman and Hindu Arabic numerals?

(i) Roman numerals

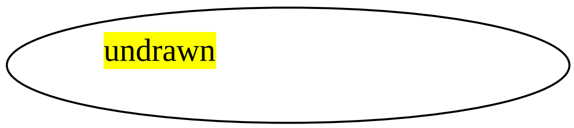
Dear student I hope that in the previous topic you understood what a numeral is.

This will help therefore be of help to us to understand how Romans, Hindu and Arabians used to write in numerals

Lets begin with the Roman numerals.

The Roman system developed from the idea of counting fingers on one hand and then groupings of ten on the counting of the fingers of the two hands.

Illustration



The Romans based there numerals on alternating groups of five and ten.

Lets see how this addition of symbols were used to indicate numbers.

Example 1

I	II	III	IV	V	VI	VII	VIII	IX	X
One	two	three	four	five	six	seven	eight	nine	ten

The used position to show whether a number was to be added to or taken from the next higher grouping when teaching reading or writing e.g.

They also expressed bigger numerals to the power of ten for example

Example 2

(a) VI means V + I i.e. 6
(b) LX means L + X i.e. 60
(c) DC means D + C i.e. 600
(d) IV means V – I i.e. 4
(e) XL means L – X i.e. 40
(f) CD means D – C i.e. 400

You are progressing well may you continue.

Now design other patterns to make learning Roman numerals interesting and enjoyable.

Compare your patterns with a colleague.

What have you learnt? Share it with a friend.

Example 3

X=10
C=10 $\times$ 10=100
M=10 $\times$ 10 $\times$ 10=1000

Apart from expressing in the powers of 10, they also expressed Roman notation for the fives

Example 4.

V=5
L=50
D=500

However, Romans also wrote, numerals beyond 4000. It is wrong to write 4000 as MMMM. Instead it is written as IV, where a bar is put above a group of Roman numerals. It means multiplying the group of Roman numerals by 1000.

Note that a numeral is a symbol used to represent a number and also we commonly call a numeral a figure/digit
Good. I hope you are catching up!

b) Now since you have known how Romans write their numerals, let's try and see how we can write a few whole numbers in Roman numerals.

Consider this example in the box

Qn. Write eleven in Roman numerals.

Note that eleven is built up as a result of addition of ten and one and it becomes eleven.
Therefore
Ten + one = eleven

In Roman numerals

Ten = X and one = 1
Ten + one = Eleven
Then,
X + I = XI

$\therefore$Eleven in Roman numerals is written as "XI"

Good. Can you smile?

Now, using the above example can you work out the following activity.

Activity 3.4

(a) Write the following numbers in Roman numerals
(i) 95
(ii) 132
(iii) 487
(iv) 7691
(v) 2253

You are doing well

Now that you have known how Roman numerals are written. Can you do this activity?

Activity 3.5
From the information above, can you now sight some advantages and disadvantages of writing numbers using Roman numerals? Discuss with a colleague

Well done. You have completed part one of the subtopic

(ii) Hindu Arabic numerals

Hello student, did you know Hindus and Arabians also have there own system of writing numerals

Note that: the numerals from 1-9 which we use today are said to have originated from India.

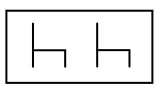
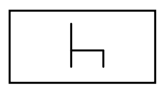
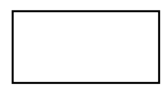
The system was developed and largely used by the Arabs. This is why it is referred to as the Hindu – Arabic system.

The most important difference with other systems is that this system begins with symbol for zero (0).

The symbol carried the idea of an empty set. This is very important because it made it easy for any set to be given a symbol using the numbers from 0-9.

This can be illustrated by the example.

How many are in the following sets.

(a) ________  (b)______  (c)______ 

Yes this is how it is

Therefore you study an example how Hindus and Arabians wrote their numerals.

Number in words	**Hindu – Arabic numeral**
(a) Eighteen (b) Two thousand (c) Five hundred sixty You can give more examples of your own.	18 2000 560

I hope since you often use the Hindu Arabic system you can easily change to Roman numerals.

c) The table below will help you express Hindu – Arabic numeral to Roman numerals

Example

Word (number)	**Hindu – Arabic numeral**	**Roman numeral**	**Meaning**
One	1	I	One
Two	2	II	two
Three	3	III	three
Four	4	IV	One less than five
Five	5	V	Five
Six	6	VI	One more than five
Seven	7	VII	Two more than five
Eight	8	VIII	Three more than five
Nine	9	IX	One less than ten
Ten	10	X	Ten
Twenty	20	XX	twenty
Thirty five	35	XXXV	thirty five

Well done you have completed the subtopic

Can you rest for a while.

d) Application of Roman and Hindu- Arabic numerals

Now using the above information, you could convert Hindu – Arabic numerals to Roman numerals.

Study the following examples:

Musa was born in the year one thousand nine hundred and ninety eight.

i) How can you write these words in both Hindu-Arabic and in Roman numerals?

Let's begin by writing in Hindu-Arabic numeral

One thousand	1000
Nine hundred	900
Ninety eight	+ 98
	1998

Note in writing in Hindu Arabic numeral, consider the idea of the place value concept then when totaling the figures/numeral you will arrive at the expected answer.

Good you are doing great!

ii) Let's proceed to writing the Hindu- Arabic numeral in Roman numeral.

Now you can see that the numerals to be written in Roman is relatively large numeral.

What then can you do?

You will have to follow the steps shown below to arrive at the final answer (break up the numeral into steps below)

Given that change 1990 in roman numeral

Step I: writing 1000 in Roman numeral is M

Step II: then 900 is got after subtracting 100 from 1000 i.e.

1000 – 100 = 900

M – C = CM

Note:

It is written as CM because 100 being a smaller number is subtracted from 1000 to get 900

Step III

Then 90 is got after subtracting 10 from 100 i.e

100 – 10 = 90

C – X = XC

Step iv

Now, add 1000 – M

1000 + 900 +90 = M + CM +XC

∴ 1990 = MCMXC

With the above information you can do the following

Activity 3. 6

1. Write the following using Roman numerals

   (a) 81 (b) 98 (c) 164 (d) 242 (e) 917 (f) 3729

2. Express these in Hindu Arabic

   (a) CDXL (b) CMXCIX (c) MDCLXXIII (e) CCCXVII (f) CCXI

May you cross check with the ones at the end of the unit.

Congratulations! You have come to the end of this subtopic.

3.7.3: PLACE VALUE CONCEPT

a) Place value chart

Dear student;

It is very important that you understand the concept of a place value.

Then what is a place value?

Study the place value chart below and write your observations..

Millions				**Thousands**			**Units**		
No.	Hundred million	Ten million	million	Hundred thousand	Ten thousand	thousand	hundreds	tens	ones
i							4	8	6
ii						3	2	5	1
iii					4	8	7	9	3
iv				8	5	2	6	4	9
v			7	2	6	4	1	3	0
vi		1	8	2	0	4	0	0	8

Can you compare the position (place) of the digits on the calculator with the one on the place value chart?

Now that you can do the activity below

Activity 3.7

Using the numbers shown on the chart, find the place values of the circled brackets

(i) 486

(ii) 3251

(iii) 48793

(iv) 852649

(v) 7264130

(vi) 18204008

Compare your answers with the one of your colleagues then give your answer to the tutor.

You are doing great!!

Let you continue

Dear student, we can therefore use the place value to:

i) Count a number
ii) Read a number
iii) Write a number

b) Decimal fraction

Did you also know that decimal numbers are decimal fraction and have got place values?

Let you study this example below:-

7.5643 has four digits after the decimal point. We say it has four decimal places.

Therefore you should be able to recognize the place values of the digits in a whole number.

How can you determine the place value?

The chart below will help you determine the place value of the digits in decimal fractions

Units			**Decimal fraction**			
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten hundredths
		0	6	1	5	
	6	8	1			
		7	8	2	3	5
		7	5	6	4	3
2	3	4	5	9	1	3
	3	4	0	0	4	4

What is the place value of the circled digits of the given numbers.

i) 0.6 1 5 = 6 is the tenths place value
ii) 6 8 1 = 8 is in the ones place value
iii) 7.8 2 3 = 3 is in the hundredths place value

With your colleague do iv, v and vi

Show your answers to the tutor

Therefore, a place value shows the position of the digit number in a whole number and a decimal number.

Note:

For you to understand the position of whole numbers and decimal numbers, you should have the picture of a place value chart in mind

c) Reading of a decimal fraction / decimal number

Do you know how to read decimal fraction?

You will discover how to read a decimal fraction from this table below.

How can 0.856 be read?

It is read as zero point eight five six.

Therefore the meaning of 0.856 is that there are eight tenths, five hundredths and six thousandths. This is so because the values become established in speech.

Thank you for knowing how to indicate decimal fraction on a place value chart, how to read decimal fractions.

Can you use all the information got above to try this activity?

Activity 3.8

a) Write the following in words
   - i 47.63
   - ii 208.029
   - iii 96.118

b) Writ the place values of the underlined digits
   - i 0.0<u>4</u>73
   - ii 395.66<u>9</u>2
   - iii 5<u>1</u>46.0136

c) Find the value of the circled digits.
   - i 7.56 43
   - ii 68.10 43

Please may you cross check with the answers at the end of this unit.

Great! You are doing well

You can now read more about decimal fractions under unit 6 ahead on fractions and Macmillian Uganda Primary Mathematics Pupils' Book 7.

3.7.4. The counting system

a) introduction to bases

Have you ever studied about base system?

A base is used in a counting system.

Ask yourself: What is a base and how is it used in the counting system?

Before you know what a base is, recall how to count numbers i.e whole numbers, natural numbers

You also reminded that we learned about bases in primary five, six, seven and senior one.

b) Counting in various bases

The base ten counting system is called the decimal/mother/denary system; it is a system used every day. All counting system which uses the idea of place value follow the same principal as are used in the decimal system. The difference between various base systems is the method of grouping used of each system.

Let you study the table below:-

No.	**Base (system)**	**Numbers written in different bases**									
i.	Decimal (10)	1	2	3	4	5	6	7	8	9	10
ii.	Quandary (5)	1	2	3	4	10	11	12	13	14	20
iii.	Binary (2)	1	10	11	100	101	110	111	1000	1001	1010
iv.	Octal (8)	1	2	3	4	5	6	7	10	11	12

With your collogue,

What have you observed?

Use the following questions as you study the table above

i How many symbols (digits) used in base; ten, eight, five and two?
ii What is the lowest and largest in each base?
iii What do you conclude about base systems?

Discuss your answers with your tutor

Read the following test before you continue.

There are three important principles to note about base systems;

i The number of symbols (digits) for the lowest for any base is always 0 and the largest symbol for any base is one less than the base itself.

ii Each base system has its own number symbols e.g.

Base ten = { 0.1.2.3.4.5.6.7.8.9 }

Base eight = { 0,1,2,3,4,5,6,7}

Base two = {0,1 }

iii Each system used place values and powers according to the value of each base

Activity 3.9

Now, you have read and understood the principles of base systems

With you group members copy the table about counting in various bases, on a piece of paper and continue it as far as 46 in bases ten, eight, five and two.

Give you work to your tutor.

(c) Convert number from base ten to any base;

Let you study the following example;

i Express 38_{ten} into base two. Use long division

2	38	remainder
2	19	1
2	9	1
2	4	0
2	2	0
2	1	1

0 $\therefore 38_{ten} = 100110_{two}$

ii convert 121_{ten} in base five

121_{ten} =

		remainder
5	121	1
5	24	4
5	4	4
	0	

$\therefore 121_{ten} = 441_{five}$

Now you have carefully studied the two examples.

You do the following problems in your exercise book

Activity 3.10

Express the following from base ten to other bases:-

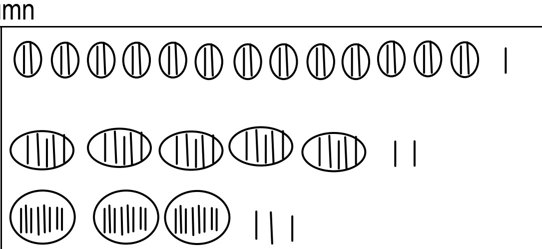
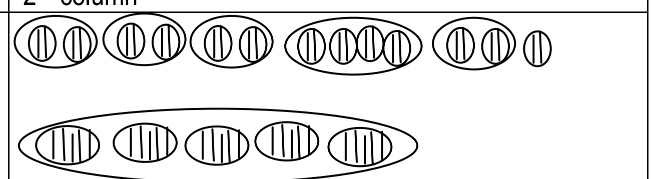
a) 14 in base five
b) 49 in binary system
c) 87 in octal system
d) 61 in base seven
e) 112 in duodecimal system

Compare your answers with those at the end of the unit.

Thank you for completing this exercise.

Now, you know how to change from decimal system to any base. Let you look at the vice vesra.

27 sticks:

1st column		2nd column
25 55 85		

Activity 3.8

Repeat the above example with different number of sticks and make a table of your results
Good! You can compare your answer with a colleague

Lets continue

Now what do you think is meant by the following?

a) i) 27 sticks (grouped in 2's)
ii) 27 sticks (grouped in 5's)
iii) 27 sticks (grouped in 8's)

b) i) 20 sticks (grouped in 2's)
ii) 20 sticks (grouped in 5's)
iii) 20 sticks (grouped in 8's)

Compare your answers with a friend

Lets continue

If you count/ group items (in this case the sticks) in the different groups of your choice, we call that grouping system a base.

Therefore 27 sticks (grouped in 2's) is

=11011 (base two)
$=11011_{two}$

Note the base is written below the digit(s) on the right as shown above. Counting in other groups is like counting in different bases known as the base system. This may include base five systems, base ten systems or the decimal system, base eight systems etc.

Dear student! In this way I believe you are now able to know what a base is and how it is used as a counting system.

Thanks you can now give more example of your own on different base system.

(a) Conversion of a given base to another base

Conversion of a given base to base ten

Have you known?

It is possible for you to convert any base number to base ten. The conversion of a given base to base ten is known as the decimal system.

Let you study the examples given below

Example 1:

i) Express 101_{two} in base ten

To begin, expand the base two number using powers of two
i.e. $101_{two} = 1^{23}0^{21}1^{20}$
$= (1x2^2)+(0+2^1)+(1x2^0)$
$= 4+ 0 + 1$
$\therefore 101_{two} = 5$

Let you study one more example on this.

Example 2:

ii) Express 2314_{five} in base ten

Remember the first procedure; expand the base five number using powers of five as shown below.

$2314_{five} = 2^{53}3^{52}1^{51}4^{50}$
Expand $= (2x5^3)+(3^{five}x5^2)+(1x5^1)+(4x5^0)$
$= (2x125)+(3x25)+5(1x5)+(4x1)$
$= 250 +75+5+4$
$\therefore 2314_{five}=334_{ten}$
Well done I hope these examples will help you practice more. Lets proceed

With your colleague do the following activity on a piece of paper.

Activity 3.10

a) Express each of the following to base ten:

(i) 111_{two} (ii) 312_{five}

b) Convert 94 to the following base.

(i) Two (ii) five (iii) eight

c) What base eight numeral is equal to 54?
d) Add the following and give your answer in decimal system;

(i) $10111_{two} + 111_{two}$
(ii) $44_{five} + 312_{five}$

You can check your answers at the end of the unit.

Well done you are doing fine.

Can you take a walk for a short time.

e) Application of bases

Welcome back from a walk

Since you know how to convert from base ten to any base or vice versa.

Let you study to apply the acquired concepts in bases:

Example 1:

Given that $102_{four} = 24_n$

Find the value of n.

Step 1:

First change to base ten

102four = 24n

$=1^{42}0^{41}2^{40}{}_4 = 2^{n1}4^{n0}{}_n$

$= (1 \times 4^2) + (0 \times 4^1) + (2 \times 4^0) = (2 \times n^1) + (4 \times n^0)$

$= (1 \times 16) + (0 \times 4) + (2 \times 1) = (2 \times n) + (4 \times 1)$

$= 16 + 0 + 2 = 2n + 4$

$= 18 – 4 = 2n + 4 – 4$

$= \frac{14}{2} = \frac{2n}{2}$

$= 7 = 7$

Example 2: solve for x, $4\ x^2 + x^2 = 112_{five}$

Finding to base ten

$x^2 + x^2 = 1^{52}1^{51}2^{50}$five

$= 2x^2 = (1 \times 5^2)+(1 \times 5^1)+(2 \times 5^0)$

$= 2x^2 = 25 + 5 + 2$

Find the value of x2

$= \frac{2x^2}{2} = \frac{32}{2}$

$= x^2 = 16$

Find the square root

$= \sqrt{x^2} = \sqrt{16}$

$\therefore x = 4$

Let you have more practice on this

Activity 3.11

a) Solve for the unknown base

i. $23_n = 19$ten

ii. $112_{three} = 22_t$

iii. $213_{six} = 100_q$

iv. $K_2 = 1304_{seven}$

v. $P_3 = 121_{seven}$

b) $132_{five} \div 12_{five}$

c) Express 168_{twelve} to grosses

Cross check with answers provided at the end of the unit.

Thank you for completing this topic.

3.7.5. Clock computations

a) Introduction to clock computation

Dear student, do you know that we live according to time?

If yes what helps you to keep time?

Do you see, touch or smell time?

Remember that there are certain things that happen everyday i.e. the sun rises in the morning, it sets and it gets dark every evening.

Also you wake up every day, every week, every month, every year until you die.

Hullo student

- What happens everyday when you get up in the morning?
- When do you go to sleep?
- Is it light (day) when you go to sleep or is it dark?
- What do you call this time of the day?
- What helps you to keep time in order for you to wakeup, go to school, go home and go to bed?

Now did you know that what helps you to tell time and keep time is a clock?

How does a clock look like? See how it looks like below

Remember you should know that a day is made up of 24 hours.

A day can be divided into two to make 2 half days, having 12 hours each.

Dear student for more information about reading time, read measures under unit 11 a head in this module.

Did you also know that?

You have special days in the week when special things happen.

Try and find out which days do these special things happen in a week.

Activity 3.11

i) When do Christians go to church?
ii) Which day follows after Christians have gone for prayers
iii) Which day do you participate in P.E
iv) Which is the middle day of the week?
v) Which day follows the middle day of the week?
vi) On which day do Muslims go to the mosque?
vii) On which day do the seventh day Adventists go to pray?
viii) Therefore, how many days are there in the week?

You can compare your answers with your friend

Well done. You are a great student! Let's continue

What have you discovered from the activity 3.11?

Read the text below.

- There are seven days which make up one week
- 4 weeks make up one month
- 12 months make up one year
- 10 years make up a decade
- 100 years make up a century
- 1000 years make up a millennium

Hullo student

Activity 3.11**.** exposed you to days of the week and some of the things that you do throughout the days.

Now ask yourself,

If days make up a week?

- What then makes up a year?
- How many months makeup a year?
- How many weeks makeup a months?
- How many days make up a month?

You can compare your answers with a colleague.

Well done

Have you discovered.

That different months in a year are arranged according to days and weeks? These are arranged together in what we call a calendar.

Let you study an example of a month in a calendar of a given year, 1992.

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Now using the calendar above you can discover the following.

If today is Sunday 2nd November 1992

- Next Sunday will be 9^{th} November
- The day of the week 25^{th} will be a Tuesday
- There are five Sundays in the month of November
- There are 30 days in the month of November

Now using the same calendar and the information above, do the following activity

Activity 3.12

Using the above calendar, if today is Monday 7^{th} November, 1992.

i) What date will be the following Saturday?
ii) In fifteen days time we shall have a mathematics test. What day will it be? What date will it be?
iii) When was eight days before today?

Now, compare your answers with a friend

"Thank you for completing the activity"

You are welcome to the next subtopic.

b) Finite system/ clock system

Dear student; you have been already introduced to the subtopic.

What if finite system?
Read the text below.

i Finite system is a way of finding remainders.
ii Finite is referred to clock arithmetic or modular (mod).

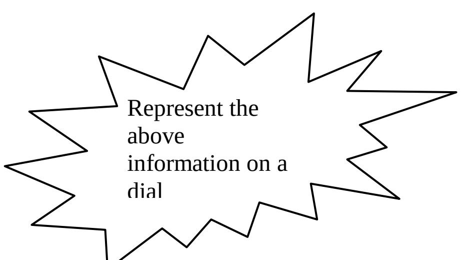
Now you have been reminded on the meaning finite system.

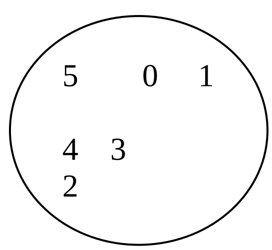
Let you study the following examples

Example 1:

Given that {0,1,2,3,4,5}

What is the finite system of the above set





By counting the whole number represented on the dial. It will help you to discover the finite of the given symbol.

∴ {0,1,2,3,4,5} = (finite 6)

With your classmate, try the following problems on a piece of paper.

a) What are the finite system for the following sets?

i {0,1} = -------
ii {0,1,2,3,4,5,6,7} = ------------
iii {all the English alphabet letters} = ----------------

b) What are the next two (2) equivalent whole numbers for

i 5 (finite 7) = 5,12,19,--, --
ii 4(finite 12) = 4,16,28,--,--

In plenary may you discuss answers.

c) Operations on the finite system.

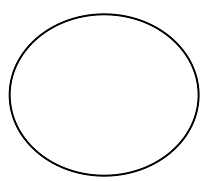
Let you study the following examples: On addition, subtraction, multiplication and division.

Example 1:

Add 6 + 5 (finite 9)

Method 1

Use a clock dial



∴ 6 + 5 = (finite 9) = 2 (finite 9)

method 2:

Use long division to find remainders

6 + 5 = (finite 9)

1 remainder 2

$$= 9\overline{)11}$$
$$\underline{-9}$$
$$2$$

Writing only remainder

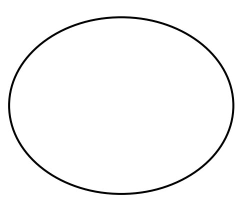
∴ 6 + 5 = (finite 9) = 2(finite 9)

Example 2:

Subtract: 3 – 2 (finite 5)

Method 1

Using a clock dial



Method 2

3 – 2 = (finite)
= 1(finites)

∴ 3 – 2 = (finite 5) = 1 (finite 5)

Working examples 3 and 4.

Can you try these problems? Choose any of the method you want.

Work out the following

i $6 + 2 =$ (finite 7)
ii $1 - 5 =$ (finite 6)

Share your answers with your groupmates.

Now, you can continue studying the following examples:-

Example 3

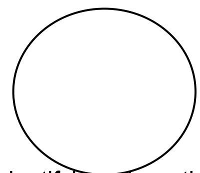
Multiply

Method 1

$5 \times 7 =$ ------ (finite 8)
$35 =$ ----- (finite 8)
$35 \div 8 = 3$ rem (finite 8)
$\therefore 5 \times 7 = 3$ (finite 8)

Method 2

Use a chat



show the product 35 on a dial

identifying where the last end:
$\therefore 5 \times 7 = 3$ (finite 8)

Example 4:

Divide

$2 \div 4 =$ ----(finite 6)

Method 1.

Using repeated subtraction

$2 \div 4 =$ ------ (finite 6)
$2 + 6 - 4 =$ (finite 6)
$8 - 4 = 4$ (finite 6) 1st subtraction

$4 - 4 = 0$ 2nd subtraction

$\therefore 2 \div 4 = 2$ (finite 6)

Method 2:

Using equivalent whole number of the dividend

$2 \div 4 =$ ------ (finite 6)

Look for numbers equivalent 2_2 (finite 6) are:-

{12,22,27,32,42,47,2,8,14,20}

Choose one of the numbers listed above which is exactly divisible by 4 and no remainder is 8 and 20.

But taking the least (8)

$\therefore$ 8 ÷ 4 = 2 (finite 6)

Method 3:

2 ÷ 4 = ------ (finite 6)
= (2 + 6) ÷ 4 =
= 8 ÷ 4 =2 (finite 6)

Since you have been able to go through all examples;

In your exercise book can you try this activity:

Activity 3.13

a) Add the following

i 2 + 3 = ---------(finite 5)
ii 4 + 5 = ---------(finite 7)
iii 3 + 7 + 8 + 2 = ------- (finite 11)
iv 2 + 3 + 1 + 2 = -------(finite 4)

b) Complete the table below

Finite 6

+	0	1	2	4
0				
1				
2				
3				
4				

c) Subtract the following

i 3 – 4 = ---------(finite 6)
ii 1 – 2 = ------ (mod 3)
iii n – 7 = 4 (mod 9)

d) Complete the following

i 3 + 2 – 7 = ------- (finite 12)
ii 5 – (3 + 3 + 2 + 4) = ------------- (finite 8)

e) Multiply

i 4 x 5 = ------- (finite 7)
ii 12 x 11 = ------ (finite 14)
iii 4^2 x 3^3 = ----- (mod 7)

f) Divide

i 2 ÷ 3 = ------ (finite 5)
ii 3 ÷ 9 = ------ (finite 12)

May you compare your answers at the end of the unit

You can now take a walk before you continue.

(d) Application of computations

Dear student,

(d) Application of clock computations

Dear student,

You have learnt about finite system, now you can discuss, how to solve problems in everyday life situation using finite system.

You have also already learnt how to use finite arithmetic's to solve problems related to days of the week and months of the year. Use it to solve problems in, to real life situations.

For instance

(a) Finite 7 can be associated to days of the week

Sun – 0
Mon – 1
Tue – 2
Wed – 3
Thur – 4
Fri – 5
Sat – 6

(b) Finite 12 can be associated to months of the year

Jan – 0
Feb – 1
Mar – 2
Apr – 3
May – 4
Jun – 5
Jul – 6
Aug – 7
Sept – 8
Oct – 9
Nov – 10
Dec – 11

Let you study.

Example

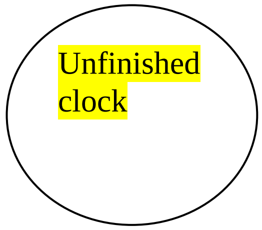
1. Today is Friday. What day of the week will it be in 15 days time?

Solution: working method 1.

Since Friday ⟶ 5, then added 15 to 5
5+15 = $\boxed{\ }$finite 7)
20 = $\boxed{\ }$ (finite 7) (20 ÷ 7 = 2 rem 6)
= 6 finite 7
6 ⇨Saturday, so the day will be a Saturday

Method 2

Use of dial



You will find that the arrow will begin from point 5 and end at point 6.

Example 2

Using repeater addition

Since Monday ⇨ 1, then subtract

1 – 20 = --------- (finite 7)

Find the equivalent of 20 symbol in finite 7.

20 ÷ 7 = 6 (finite 7)

Then 1 – 6 = ------(finite 7)

(1 + 7) – 6 = ------- finite 7

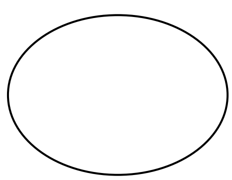
8 – 6 = 2 (finite 7)

Hence, 2 is associated to Tuesday:

∴ The day was Tuesday.

Method 2

Use of dial



A go refers to move 20 steps anti-clockwise from Monday ⇨ 1

The arrow has ended at point 2.

2 ⇨ Tuesday
∴ The day was Tuesday.

Example 3:

2x – 3 = 3 (finite 4), solve x
= 2 x – 3 + 3 = 3 + 3 (finite 4)
= 2x = 6 (finite 4)
= $\frac{2x}{2} = \frac{6}{2}$ (finite 4)
∴ x = 3 (finite 4)

In your exercise book do the following activity.

Activity 3.14

a) Today is Friday, what day of the week

i Will be 80 days time?
ii Will it be in 120 days time?
iii Was it 62 days ago?

b) February is the month now. What month of the year was it 72 months ago?

c)

i 3n = 3 (finite 4)
ii 4x – 3 = 4 (finite 7)
iii 6(q – 3) = 4 (finite 8)

For more practice do exercise 4:3 MK Pupils Primary Mathematics.

Compare your answers at the end of the unit.

3.7.6. Teaching numeration systems and place values in primary school.

Activities and instructional materials for teaching numeration systems and values.

Dear students,

In teaching, this unit, you have got a role and responsibility to mobilise a lot of learning materials and activities. Teaching and learning of this unit calls for participatory approach which will give your learners proper understanding for successful learning.

Activities like grouping numerals according to their values/places counting, reaching and expressing numbers in recommended base help learners build concepts of numeration system and place values and how to solve problems in our life situation. Use the environment effectively to encroach your teaching/learning process for better performance.

3.7,7. Unit Summary

In this unit you have been introduced to numeration systems and place values, learn them and put them into practice to help you solve problems in classroom situations and out of class.

i. Meaning of a number and numeral
ii. Writing and reading of numbers
iii. Roman numeral and Hindu Arabic numerals
iv. Reading ad writing in place values
v. Conversion of Roman numerals to Hindu Arabic numerals and vice versa.
vi. Application of Roman and Hindu Arabic numerals
vii. Introduction to bases.
viii. Counting and grouping numerals in different bases.
ix. The finite sets of days, weeks, months.
x. Clock computations
xi. Application of clock computations.
xii. Activities and instructional materials for teaching numeration systems and place value.
xiii. Planning lessons for teaching numeration systems and place values.

3.9. Glossary

Whole numbers are either natural numbers or counting numbers but natural numbers/counting numbers begin from one i.e. and this include zero (0).

Number system - set of defined symbols and the numbers they represent, together with rules forming larger numbers from those symbols.

Hindu- Arabic numbers - is the number system which is used for ordinary arithmetic whose symbols are 0,1,2,3,4,5,6,7,8,and 9

Base - is number/letter which gives a place value number system and controls the relationship between places.

Binary system - base system

Octal system - base eight

Natural numbers - the set of number 1,2,3,4,5,6---- as used in counting numbers

Decimal numbers are decimal fractions that have got place values and decimal point.

3.10. References:-

1. MK. Primary Mathematics 2000, Pupils Book 7..

3.11 END OF UNIT EXERCISE:

1. Find equivalent for these numbers:-

   a. 465
   b. 579
   c. 2009
   d. 4404

2. Find the Hindu – Arabic for these Roman numbers

   a) CMXXII
   b) MDCCC
   c) MMCL
   d) CCLIV

3. Write the following decimal fractions in words.

   a) 0.481
   b) 6.734
   c) 0.009
   d) 1.998

4. Express each of the following in the required number system.

   a) 112 in base five
   b) 10101_{two} in base ten
   c) 87 in base four

5. Complete the table below

x	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							

Finite 7x

6. Solve for

4(n – 2) = 2 (finite 9)

7. 27th June, 1997 was a Friday which of the week was 13th September, 1997.

Activity 3.9

1. (i) 7
   (ii) 82

2. (i) 1011110_{two}
   (ii) 334five
   (iii) 136eight

3. 66eight

3.10 Answers and notes

Activity 3.4

a) XCV
b) CXXXII
c) CDLXXVII
d) DCCXCI

Activity 3.6

1.
a) LXXXI
b) XCVIII
c) CLXIV
d) CCXLII
e) CMXVII
f) MMMDCCXXIX

2.
a) 440
b) 999
c) 1594
d) 1673
e) 317
f) 211

Activity 3.8

a)
i Forty seven point six three
ii Two hundred eight point zero, two, nine
iii Ninety six point one, one, eight

b)

i 4 is in hundredths
ii 9 is in thousandths
iii 1 is in hundreds

c)
i $6 \times 0.01 = 0.06$ or $\frac{6}{100}$
ii $0 \times 0.01 = 0$

3.13 References

Dear student you can get more information and work about this unit from the following reference books:-

a) D. Paling (2000) Teaching Mathematics.
b) E. William (1998) Primary Mathematics Today 4th edition.
c) Primary mathematics books 6 and 7 pupils. E.g. MK Publishers.

UNIT 4: OPERATION ON NUMBERS

4.1. Introduction

You are welcome to unit 4. This unit introduces you to the operations on: writing numbers in words and figures, additions, subtractions, multiplication, expansion of numbers, Laws of indices, square roots and how to teach operations of numbers in primary schools.

4.2. Content Organisation

Dear student, in this unit, you are going to cover the topics as listed in the table below.

Topic	Details
1. Addition of numbers	• Writing numbers in i). Words ii). Figures • Addition of numbers i). Horizontal addition ii). Vertical addition
2. Subtraction of numbers	• Subtraction of numbers i). Horizontal subtraction ii). Vertical subtraction
3. Multiplication of numbers	• Use of number lines • Multiplication of natural numbers • Multiplication with digits by: i). Two ii). Three
4. Division of numbers	• Introduction to division of numbers • Division of numbers by digits of: i). Two ii). Three
5. Expansion of numbers	• Expansion in base ten
6. Laws of Indices	• Relationship of exponents, powers and indices • Expansion of indices • Multiplication of numbers • Division of numbers involving indices
7. Square numbers	• Meaning of square numbers • Square roots of numbers and mixed fractions
8. Teaching operation of numbers in primary schools	• Introduction to counting, addition and multiplication of numbers in base five and ten. • Numbers on a number line • Operation on BODMAS • Planning lessons for teaching operation on numbers.

4.3. Learning Outcome

By the end of this unit, you are expected to apply operations on numbers on digits up to seven. Use brackets involving operations in additions, subtraction, multiplication and division of natural numbers; apply the laws of indices in multiplication and division of numbers and lastly apply the concept of operation on numbers to solve daily challenges in life like additions, subtractions, divisions and multiplication.

4.4. Competences

Dear students, now you know the expected learning outcome, therefore as you study/work through this unit, you will be able to;

a) add numbers up to seven digits.
b) Subtract numbers up to seven digits.
c) Multiply numbers up to seven digits by two and three digit numbers.
d) Divide numbers up to seven digits by two and three digit numbers.
e) Express numbers in expanded form.
f) Use (BODMAS) to work out problems.
g) Use laws of indices to carry out multiplication and division of numbers.
h) Find square numbers.
i) Find square roots of square numbers and mixed fractions.
j) Identify activities and instructional materials to use in teaching operation on numbers.
k) Demonstrate teaching operations on numbers in primary school class.

4.5 Unit Orientation

This unit will make you get exposed to plenty of learning materials which can help in studying the concepts of counting, additions, subtractions and multiplication. You will also be required to use this knowledge attained to solve other problems in operation on numbers.

4.6. Study Requirements

To study this unit, you will need to remind yourself on numeration system and place values, and have a place value chart. You will also need a quiet environment for study.

4.7. Content

Topic I: Addition of Numbers

1. Writing numbers

i). Writing numbers in words

Do you know that it's important to know how to write numbers in words?
How then can you write numbers in words?
To be able to write numbers in words, you need to remind yourself on the idea of place values. The place value table looks like this.

Millions	Thousands			Ones		
Millions	Hundred of thousand	Tens of thousand	Thousand	Hundred of ones	Tens of ones	Ones

How then can you use this table to write numbers up to seven digits in words?

Okay, let us try these example:

Example 1

Write 7650620 in words.
To be able to answer this, place each digit according to it's place value in the table shown above.
Therefore, this can be presented as follows

7 6 5 0 6 2 0

Ones (O)
Tens of Ones (T)
Hundreds of ones (H)
Thousand (T)
Tens of thousand (TT)
Hundred of Thousand (TH)
Millions (M)

Now this can be written as:

Seven millions six hundred fifty thousand six hundred twenty

Thank you for following!

We can also try another example.

Example 2.

Write 630578 in words.

We can use the same idea by placing the digit in place values such as:

6 3 0 5 7 8

Ones
Tens
Hundred
Thousand

Tens of thousand
Hundred of thousand

Therefore, this can be written as;

Six hundred thirty thousand five hundred seventy eight

Thank you for moving with to this point!

ii). Writing numbers in figures
Did you know that we can write the same words in 4.6.1.i) in figures?
Let us see how this can be done considering this example.

Example 3.

Write, nine million, one hundred nine thousand, nine hundred nineteen in figures. To be able to write this in figures, we need to split the words as follows.

Nine million	9,000,000
One hundred nine thousand	109,000
Nine hundred nineteen	+ 919
	9,109,919

Thank you for following!

We can work out another example.

Example 4.

Write, four million, two hundred eighty nine thousand six hundred eighty two in figures.

To be able to write this in figures, we need to split the words as follows;

Four million	4000000
Two hundred eighty nine thousand	289000
Six hundred eighty two	682
	4289682

Work through with a friend,

Write the following numbers in words

a) (i) 5123450
(ii) 2890435
(iii) 1111111

b) Write the following in figures

(i) Six million seven hundred twelve thousand nine hundred one
(ii) Four million four hundred thirty nine thousand two hundred fifty.

Share your answers with a friend

Let us continue to the next subtopic

2. Addition of numbers

i). Horizontal addition

Did you know that addition is the first and most obvious of the four operations on numbers, the one you most frequently use in your everyday life?

✍ **Activity 4.1**

Did you also know that the ability to add is a basic skill?

How then will you be able to add?

Let us try this example.

Example 1

Add;

7685600 + 528600

Note: For you to add this properly, you should consider the position of the place values.

$$
\begin{array}{rr}
= & 7685600 \\
+ & \underline{\ \ 528600} \\
 & \underline{8214200}
\end{array}
$$

Thank you for doing well!

Example 2

ii). Vertical addition

In a similar way, vertical addition is like horizontal addition where you need to keep in mind the position of the place value. This can be illustrated in this example.

The first step is arranging the figures in a vertical way considering the position of the place value.

	HT	TT	T	H	T	O
	7	6	0	0	0	0
	8	5	6	2	8	9
	9	2	0	0	0	0
+				5	2	0
	25	3	6	8	0	9

Well done!

Try out this activity.

Add the following numbers:

1. a) 768920 + 68590
   b) 8657890 + 5968920
   c) 9285680 + 4218

2. Add:

a)

	8396820
	89652
	560
+	20

b)

	5968902
	956890
+	7890

You are doing great!

You can compare your answer with a friend and with one at the end of this unit.

Let us move to the next topic

TOPIC II: SUBTRACTION OF NUMBERS

1. Horizontal subtraction of numbers

Hello student, for you to be able to subtract numbers correctly, just like in the previous subtopic, you need basically to have the idea of place value in your mind.

This can be illustrated using the example below:

Example 1.

Subtract:

4567890 – 82560

In order for such subtraction to take place, arrange the digit in order of place value as shown.

	M	HT	TT	T	H	T	O
	4	5	6	7	8	9	0
-			8	2	5	6	0
	4	4	6	5	3	3	0

Note: Keeping in mind the above order of arrangement, you will be able to subtract any figure. Always show the subtraction sign.

2. Vertical Subtraction of numbers

Hello student, you need to get the idea of subtraction clearly. The concept of place value should not be forgotten as is in the case of previous subtopics.

Let us try out this subtraction.

Example 2

	M	HT	TT	T	H	T	O
	5	2	8	9	3	4	5
-	0	2	6	0	9	3	4
	5	0	2	8	4	1	1

In the above subtraction, it is difficult to subtract a bigger digit from a smaller digit. Therefore student, you should not forget that we can borrow 1 from the next digit of another place value. Adding it to the present digit makes you subtract vertically easily.

Well done! I hope it is all clear.

Now test your own knowledge.

✍ **Activity 4.2**

1. Work out the following:

   a) 7896593 – 56254
   b) 59345 – 628
   c) 9859692 – 56930

2.

   a)
$$\begin{array}{r} 689352 \\ -\quad 4368 \\ \hline \\ \hline \end{array}$$

   b)
$$\begin{array}{r} 123345 \\ -\quad 45892 \\ \hline \\ \hline \end{array}$$

   c)
$$\begin{array}{r} 968210 \\ -\quad 45829 \\ \hline \\ \hline \end{array}$$

   d)
$$\begin{array}{r} 754692 \\ -\quad 1096 \\ \hline \\ \hline \end{array}$$

Great Job!

Cross check your answers at the back of the module

TOPIC III: MULTIPLICATION OF NUMBERS

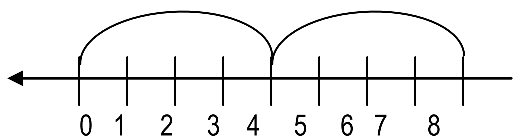
1. Use of number lines in multiplication

Did you know?

Jumps on stepping stones, number of tracks and number lines is a popular method to model multiplication. In these models, the first number refers to the number of jumps made and the second number is the size of every jump. Therefore, to get the total amount of jumps made, we need to add the total number of jumps made to the size of the jumps made.

This can be illustrated as;

Example 1: 2x4



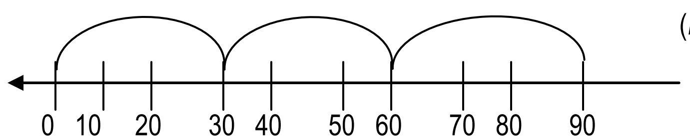
2 jumps of 4

Therefore, the 2 jumps made by the size of four gives us a total jump of 8.

I hope you are following along.

Example 2 3x30

You can also express multiplication on a number line as shown below.



(*Indicate a frog jump*)

3 jumps of 30

Therefore, 3 jumps of 30 give a total of 90.

This can be also found as 3×30 =90

Well done! Let us now move on to the next subtopic.

2. Multiplication of natural numbers

Hello student,

For you to be well equipped with multiplication of natural numbers, you should learn multiplication table. You should build a multiplication table in the order below, beginning with the easiest i.e. first, the 1-times table; second, the 2-times table; third, the 5-times table; fourth, the 10-times table.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2			5					10
2	2	4			10					20
3	3	6			15					30
4	4	8			20					40
5	5	10			25					50
6	6	12			30					60
7	7	14			35					70
8	8	16			40					80
9	9	18			45					90
10	10	20			50					100

Knowing the table in the mind, you are able to multiply numbers so easily.

Example: 3 (a) and (b)

a) Multiply

$$\begin{array}{r} \text{TT T H T O} \\ 5\ 6\ 8\ 2\ 5 \\ \times \qquad\quad 2 \\ \hline 11\ 3\ 6\ 5\ 0 \\ \hline \end{array}$$

Note: For you to be able to multiply, you should consider the order of the place values and the following steps in carrying out multiplication.

Step I: Multiply 5 Ones by 2 $= 5 \times 2 = 10$

You then write 0 under Ones and carry 1 to Tens.

Step II: Multiply 2 Tens by 2, and then add 1. $= 2 \times 2 + 1 = 4 + 1 = 5$ Tens

Step III: Multiply 8 Hundreds by 2 $= 8 \times 2 = 16$

Then write 6 under Hundreds and carry 1 to Thousand.

Step IV: Multiply 6 Thousands by 2, and then add 1 $= 6 \times 2 + 1 = 12 + 1 = 13$

Then write 3 in Thousands and carry one to the Tens of thousands.

Step V: Lastly, multiply 5 Tens of thousands by 2, and then add 1

$$= 5 \times 2 + 1 \qquad = 10 + 1 \quad = 11$$

Well done! This method will help you solve many problems involving bigger digits.

b) Multiplying a 5-digit number by 10

When multiplying a 5-digit number by 10, you can use repeated additions

Example:
Multiply 56892×10

56892+56892+56892+56892+56892+56892+56892+56892+56892+56892
= 568920

This can also be worked out by using the idea of place value as:

```
   TT T H T O                    TT T H T O
    5  6 8 9 2            or by   5  6 8 9 2
×            10                   ×        1
   0  0 0 0 0                    ___________
+ 5 6  8 9 2                        5 6 8 9 2
= 5 6 8 9 2 0
```

Any number multiplying one in that number

Well done!

c) Multiplication by two digits

Dear student, when multiplying digits, take your time to work through each step. This allows you to be more careful when arranging digits in order of their place values.

Note: Keep the columns so that each digit has its correct place value.

Example 5

32746×21

	3	2	7	4	6
$\times$				2	1
	3	2	7	4	6
6	5	4	9	2	0
6	**8**	**7**	**6**	**6**	**6**

$(\times 1)$
(x20) put down and then multiply by 2

Good work! Let us look at another example.

d) Multiplication by three digits

Did you know that we can also work out multiplication with bigger values?

How then can we do this?

Check this example.

Example 6

Multiply 7893462×562

Remember in the previous example, you need to keep the order of arrangement that is clear layout of the digits.

$$
\begin{array}{rl}
7893462 & \\
\times \quad 562 & \\
\hline
1\,5\,7\,8\,6\,9\,2\,4 & \rightarrow \quad (\times 2) \\
4\,7\,3\,6\,0\,7\,7\,2\,0 & \rightarrow \quad (\times 60) \text{ put down 0, and then multiply by 6} \\
+\,3\,9\,4\,7\,7\,3\,1\,0\,0\,0 & \rightarrow \quad (\times 500) \text{ put down two 0s, and then multiply by 5} \\
\hline
4\,4\,3\,7\,1\,2\,5\,6\,4\,4 & \\
\hline
\end{array}
$$

Well done!

Hoping that you have understood, work out these activities

✍ **Activity 4.3**

1.

a) $\begin{array}{r} 59726 \\ \times \quad 507 \\ \hline \\ \hline \end{array}$

b) $\begin{array}{r} 23456 \\ \times \quad 23 \\ \hline \\ \hline \end{array}$

c) $\begin{array}{r} 1387154 \\ \times \quad 122 \\ \hline \\ \hline \end{array}$

d) $\begin{array}{r} 3567891 \\ \times \quad 56 \\ \hline \\ \hline \end{array}$

2.

a) What is 45208 times 32?
b) Multiply 32958 by 420
c) Find the product of 9570 and 62

You can compare your answer with a friend and with one at the end of this unit

Thank you for completing this subtopic. You may now take a break.

Topic IV Division of numbers

Well come to division of numbers.

Dear students, do you know that.

1. Division of numbers has a relationship with multiplication i.e. 12 x $\square$ = 72

Example 1.

You ask yourself what number is multiplied by 12 to get 72. It is shown as:

72 ÷ 12 = $\square$

72 ÷ 12 = 6. Therefore 12 x 6 = 72 and 72 ÷ 6 = 12

Well done.

Let us continue, division of numbers helps solve problems in our ever day life e.g. sharing food, objects etc.

Example 2

Sharing 10 sticks between 2 people. Each will get 5 sticks.

Example 3.

Division is regarded as repeated subtraction. 42 ÷ 3, can a rise from. There are 42 children. You ask yourself. How many threes are there in forty two?

42 – 3 = 39
39 – 3 = 36
36 – 3 = 33
33 – 3 = 30
30 – 3 = 27
27 – 3 = 24
24 – 3 = 21
21 – 3 = 18
18 – 3 = 15
15 – 3 = 12
9 – 3 = 6
6 – 3 = 3
3 – 3 = 0

Then you ask yourself, how many times has been three (3) subtracted?
therefore 42 ÷ 3 = 14

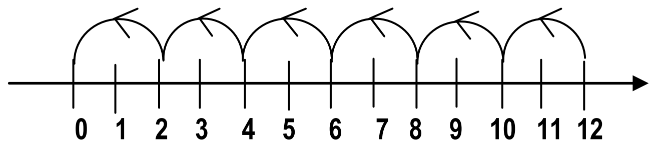
You are following

You can also use the number line

Example 4

12 ÷ 2

if you start at 12, how many jumps of 2 spaces at a time will make to reach 0?



The answer is 6 jumps of 2 which are in 12. This is written as 12 ÷ 2 = 6

Well done.

2. Dividing by single digit number

Example 5

675 ÷ 5

You write the working fully and show what is happening

```
  135
5√675
 -5     (1x5) this is really 100x5 because we are working in the hundreds column
 17
-15     (3x5) this is really 30 x 5 because we are working in the tens column
 25
 25      (5x5)
  - -
```

Thank you for following

Now putting in the idea of example 5 above you can divide six and seven digfit numbers by two and three digits numbers.

Example 6.

Divide 650244÷12

You arrange in long division.

```
   54187
12√650244
   -60
   50
  -48
     22
    -12
     104
     -94
       84
      -84
       - -
```

Example 7

Divide 6780600 ÷ 120

You also arrange this in long division

```
     56505
120√6780600
   - 600
     780
   - 720
      606
    - 600
        600
      - 600
       - -
```

Well done.

Hoping that you have understood work out these activities.

Activity 4.4

1. (a) Work out the following
   (i) 389083 ÷ 14
   (ii) 1230448 ÷ 212

(b) A shopkeeper bought 63658 arranges and he packed 14 oranges in such box. How many boxes did he make?

You can compare your answers with a friend.

Thank you for completing this sub topic.

You may now take a break.

Topic V: Expansion of numbers

Dear student to expand you have to know what base is used and place values.

Example 1:

Expand 134567 using addition. You have to know the place value of each

That is H TH TH TH H T O
1 3 4 5 6 7

100,000 + 30000 + 4000 + 500 + 60 + 7

Now you can try your own number.

Good.

Example 2:

Expand by using exponent.

10^2 is read as 10 squared or ten to the 2^{nd} power. 10 is the base and 2 is called the exponent. or
$1^5 3^4 4^3 5^2 6^1 7^0$ = $(1x10^5) + (3x10^4)+(4x10^3) +(5x10^2) + (6x10^1) + (7x10^0)$

Example 3:

Expand $7^6 6^5 7^4 5^3 9^2 2^1 4^0$

$(7x10^6)$ + $(6x10^5)$ + $(7x10^4)+(5x10^3)$ $+(9x10^2)$ + $(2x10^1)$ + $(4x10^0)$

Thank you for following

Welcome to the expanded notation involving decimal numbers.

Example 3

Expand 2065, 3456 using exponents. You also use the place values of the numbers like this:-

Place value									
Exponents	3	2	1	0		-1	-2	-3	-4
Number	**2**	**0**	**6**	**5**		**3**	**4**	**5**	**6**

This is where the point is

$(2x10^3) + (0x10^2)+(6x10^1) +(5x10^0) + (3x10-^1) + (4x10^{-2}) + (5x10^{-3})+(6x10^{-4})$
Yes, you have observed. Can you find the number that has been expanded?

Look here and see how you can.

Example 4

What number has been expanded?

(a) $(9x10^6) + (3x10^5)+(4x10^3) +(2x10^2) + (3x10^1) + (1x10^0)$
(b) $(2x10^4) + (3x10^3)+(5x10^2) +(0x10^1) + (1x10^0) + (7x10^{-1})+ (0x10^{-2}) + (3x10^{-3})$

To work out these numbers you keep on remembering the place values.

(a)	(b)
9000000	20000.000
300000	3000.000
4000	500.000
200	+ 1.703
30	23501.703
+ 1	
9304231	

Good

Hoping that you have understood workout the activity.

Activity 4.5

1. Expand the following numbers using exponents

- **(a) 4,008,545**
- **(b) 3551060**
- **(c) 395.0456**

2. What numbers have been expanded?

a) $(1x10^5) + (6x10^2)+(4x10^1) +(3x10^0)$
b) $(200000) + (2000)+(4x10^3) +(100) + (30) + (1)$
c) $(6x10^4) + (3x10^3)+(4x10^0) +(2x10^2) + (3x10^{-1}) + (3x10^{-2}) + (7x10^{-4})$

Great job!

Cross check your answers at the back of the unit.

Topic VI: Laws of Indices

1. Relationships of exponents, powers and indices

You can use powers or exponents when you wish to write a number in a short way.
Let us consider this example:

Example 1.

The area of a square is given by side by side e.g. $3cm \times 3cm = 9cm^2$
The other way of writing 3×3 is $3^2 = 9$
We can therefore say '3 squared = 9'.

In the above example i.e. 3^2, the number raised above 3 that is 2, is said to be a power or exponent or index and 3 is the base.
The exponent shows how many times the base occurs as a factor.
If 5 is the base and 3 is the exponent, we write it as 5^3, which means $5 \times 5 \times 5$
If 4 is the base and 2 is the exponent, we write it as 4^2, which means 4×4
Therefore, for any number a, $a^3 = a \times a \times a$.

Well done! I hope we are together.

2. **Expansion of indices in base 10**

Let us examine the following series.

What do you notice about the power of ten and the number of zeros in each base ten number?

Meaning of exponent with base ten	Base ten number	Number of zeros
$10^1 = 10$	10	1
$10^2 = 10 \times 10$	100	2
$10^3 = 10 \times 10 \times 10$	1000	3
$10^4 = 10 \times 10 \times 10 \times 10$	10000	4
$10^5 = 10 \times 10 \times 10 \times 10 \times 10$	100000	5
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$	1000000	6

Remember, in the previous subtopic, when we write 10^4, 10 is the base and 4 is the index / power / exponent.
Also, do not forget that a number raised to the power of zero is 1 e.g. $10^0 = 1$, or $3^0 = 1$.

Well done!

You can use the above information to do the activity below.

✍ **Activity 4.6**

Write each of the following in base ten number. The first 2 have already been done for you.

i). 2×10^{4} $= 2\times10\times10\times10\times10$ $= 20000$

ii). 6×10^{2} $= 6\times10\times10$ $= 600$

iii). 16×10^{3}

iv). 91×10^{0}

v). 28×10^{4}

vi). 5×10^{6}

vii). 10^{6}

viii). 8×10^{3}

ix). 7×10^{2}

x). 10×10^{4}

Great job!

You can compare your answers with a friend and with one at the back

3. Multiplication of numbers using indices

i). Multiplication of numbers with the same base

Did you know?

You can multiply numbers using powers.

When multiplying numbers with the same base, you first add the powers or exponent.

Consider the example below:

Example 1

Simplify: $2^{3}\times2^{3}$

Method: - Notice that the numbers have the same base that is 2.
- Then add the power and retain one of the bases as follows.

$2^{(3+2)}$ $= 2^{5}$

Well done! You have got it.

Let us proceed.

ii). Multiplying numbers with different bases

Ask yourself. How about multiplying numbers with different bases?

Note: When multiplying numbers with different bases, we first expand the numbers and then multiply by the final value.

Example 2

Simplify: $2^3 \times 3^2$

Step I: Expand the numbers first

$2^3 = 2 \times 2 \times 2 = 8$

$3^2 = 3 \times 3 \quad = 9$

$= 8 \times 9 = 72$

OR

Step II: After expanding them, group them as follows:

$2^3 \times 3^2 = (2 \times 2 \times 2) \times (3 \times 3)$

Step III: Then get the product of the grouping.

$= 8 \times 9$

$= 72$

In this way, you will be able to multiply numbers with different bases.

***Well done!* Let us proceed**

4. Division of numbers involving indices

a) Division of numbers with the same base

Also, did you know that when dividing numbers with the same base, the powers are subtracted? How?

Let us look at this.

$$a^m \div a^n = a^{m-n}$$

It's as simple as that.

Now we can illustrate this further with this example in the box.

Example 3

Simplify: $4^6 \div 4^3$

Retain the base and subtract the powers.

$= 4^{6-3}$

$= 4^3$

Well done!

b) Division of numbers with different bases

Hello student, is it possible to divide numbers of different bases?

How can this be achieved?

Let's try with this example.

Simplify: $5^2 \div 2^2$
Expand the numbers.
$5^2 \div 2^2 = (5\times5) \div(2\times2)$
$= 25\div4$
= 6 remainder 1

Therefore, note that when dividing numbers with different bases, we can expand the numbers and then divide to find the final value.

Great job!
Use the above information to carry out this activity.

✍ **Activity 4.7**
Simplify the following. The first two numbers have been simplified for you.

i). $m^3 \times m^0 \times m^2 = (m\times m\times m) \times (m\times m) = m^5$

ii). $3q^2 \times q^4 \times 2q^2 = 3\times(q\times q) \times (q\times q\times q\times q) \times 2\times(q\times q) = 3\times2\times q^8 = 6q^8$

iii). $10^4 \times 10^5$

iv). $p^4 \times p^2 \times p^{-1}$

v). $n^2 \times n^{-1}$

vi). $x^2 \times x^4 \times 5x^1$

vii). $2^5 \div 2^2$

viii). $x^7 \div x^2$

ix). $4y^4 \div 2y$

x). $p^3 \div p^7$

Well done!

Now cross check your answers with a friend and your tutor.

Dear student, there are also other laws of indices involving multiplication and subtraction.

These laws include:

i). A given number raised to power a number can be multiplied by a number outside the bracket, that is to say;

$(a^m)^n = a^{m\times n}$

This can be illustrated using this example.

Simplify: $(5^2)^3$

$= 5^{2\times 3}$

$= 5^6$

In the above example, the power 2 is multiplied by another power 3 outside the brackets.

ii). Another law states that any number raised to the power of zero is 1. That is to say;

$a^0 = 1$

Consider the examples;

$5^0 = 1$, $6^0 = 1$, $x^0 = 1$, $10000^0 = 1$.

iii). We can also find a root of a number raised to power another number.

Consider this example.

Work out the square root of a^4

$\sqrt{a^4} = (a^4)^{½}$

$= a^{4\times ½}$

$= a^2$

Note: The power half ($½$) is the root sign which is raised to the power a^4 .

This can then be simplified by the law (i) where, $(a^m)^n = a^{m\times n}$.

Let's also consider another example.

Find the cube root of 5^9

Cube root is expressed as $\sqrt[3]{x}$

Therefore, $\sqrt[3]{5^9}$

$= (5^9)^{⅓}$

$= 5^{9\times ⅓}$

$= 5^3$

iv). A fraction can be raised to a negative power which can later be expressed as a fraction with a positive power.

How then can this be done?

Look at this expression.

$(\frac{a}{b})^{-m} = (\frac{b}{a})^m$

$(\frac{a}{b})^{-m}$ can be written as $\frac{1}{(\frac{a}{b})^m} = \frac{1}{(\frac{a^m}{b^m})}$

Topic VII: Square Numbers

1. Meaning of square numbers

What do you understand by a square number?

Find out by doing the following.

Make a multiplication table like the one below.

$\times$	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

From the table, find the square numbers.

Good!

See if your answers are the same as these: 1, 4, 9, 16, 25, 36…

Therefore student, how can you define a square number?

Is your answer similar to the one above?

A square number is a number got after multiplying a number by it.

This can also help you to explain more of square numbers.

Try this:

Activity: 4.7

$1 \times 1 = \ldots$
$2 \times 2 = \ldots$
$3 \times 3 = \ldots$
$4 \times 4 = \ldots$
$5 \times 5 = \ldots$

Note: share your answers with a friend.

Thank you for going through this subtopic.

2. Square roots of numbers and mixed fractions.

Square root of numbers

Do you know what a square root of a number is?

If you do not, look at this:

Square root	If $x^2 = y$, then x is a square root of y

The symbol $\sqrt{\ }$, called a radical sign is used to indicate a principle square root sign.

For example

$\sqrt{25}$ = 5, this means that the square root of a number 25 is 5.

We say that 5 is a square root of 25, because $5^2 = 25$.

It is also true that $(^-5)^2 = 25$. This suggests that another square root of 25 is $^-5$.

Hello student, we therefore have a principle square root and negative square root;

$\sqrt{25} = 5$ is a principle square root.

$^-\sqrt{25} = {^-5}$ is a negative square root.

Thank you. Now that you have understood what a square root of a number is.

Activity

Find the square root of each of the following:

a)

$\sqrt{400}$ $\sqrt{225}$ $\sqrt{16}$ $\sqrt{36}$

$^-\sqrt{9}$ $^-\sqrt{100}$ $^-\sqrt{625}$ $^-\sqrt{125}$

b) Find two square roots of 225. Find which the principle square root is.

***Well done!* Compare your answers with a colleague and tutor.**

3. Find square root of numbers by use of its prime factors.

Dear student, did you that we can find square roots of numbers by use of its prime factors? Look at this example:

Example 1: To find the square root of 144, you need to factorise 144 as shown below:

2	144
2	72
2	36
2	18
2	9
3	3
	1

i.e. 144 $= 2\times2\times2\times2\times3\times3$

Therefore,

$$\sqrt{144} = \sqrt{2\times2\times2\times2\times3\times3}$$
$$= \sqrt{2^2\times2^2\times3^2}$$
$$= \sqrt{(2\times2\times3)^2}$$
$$= 2\times2\times3$$
$$= 12$$

Thank you for following this example.

You can use the above information to do the following activity.

Activity 4.9

What is the square root of?

a) 484
b) 441
c) 196
d) 576
e) 81
f) 2025

Now, can you share your answer with your colleague?

You did great!

(ii) Square root of fractions

Dear student, in a fraction you should identify the numerator and the denominator.

Look at this fraction:

$\frac{4}{9}$, 4 is the numerator while 9 is the denominator. In order to express a square root of the fraction, do the following.

Step I: Express the denominator as a prime factor.

Example 2:

2	4
2	2
	1

$\sqrt{4}$ = $\sqrt{2\times2}$ = $\sqrt{2^2}$ = 2

Express also the denominator as a prime factor

3	9
3	3
	1

$\sqrt{9}$ = $\sqrt{3\times3}$ = $\sqrt{3^2}$ = 3

Therefore, to combine the numerator and denominator in terms of square roots and prime factors, we shall say;

$\sqrt{4/9}$ = $\frac{\sqrt{4}}{\sqrt{9}}$ = $2/3$

Therefore, the square root of $4/9$ = $2/3$

Well done! Let's continue with the discussions.

In the second case where a fraction is a mixed fraction, you should convert the mixed fraction into an improper fraction. This can be done with the example below.

Work out the square root of $1\,7/9$.

Converting this mixed fraction into improper fraction, multiply 9 by the whole number 1 and the add 7 to it. Divide it by the previous denominator as shown below.

$1\,7/9$ = $\frac{(9\times1)+7}{9}$ = $\frac{9+7}{9}$ = $\frac{16}{9}$. This is an improper fraction.

You can now work out the square root of the fraction $16/9$

i). Square roots and decimal numbers

What do you think is the square root of?

a) 0.36

Examples 3

Dear student, in order to work this out, express the decimal as a fraction.

0.36 = $36/100$

Note: to express into a fraction, consider the position of the decimal place value. In this case, the position is in hundred of tens. Therefore, it is expresses as $36/100$.

From this, you can now work out the square root as below:

$^{36}/_{100}$ $= \sqrt{^{36}/_{100}}$ $= \frac{\sqrt{36}}{\sqrt{100}}$ $= {^6}/_{10}$ = 0.6

Now with the above example, can you try out the following with your partner?

i). 0.49
ii). 0.64
iii). 0.09
iv). 0.0169

Example 4

b) The square root of 2.25

This is also done using the same way, express 2.25 as a mixed fraction, then convert the mixed fraction into an improper fraction from which you can work out the square root.

This can be worked out as below:

$2.25 = 2\,{^{25}}/_{100}$ $= \frac{(2\times100)+25}{100}$ $= \frac{200+25}{100} = {^{225}}/_{100}$

Therefore,

225

5	225
5	45
3	9
3	3
	1

100

2	100
2	50
5	25
5	5
	1

$225 = 5\times5\times3\times3$ $\quad 100 = 2\times2\times5\times5$

$= 5^2 \times 3^2$ $\quad = 2^2 \times 5^2$

$\sqrt{2.25} = \sqrt{^{225}/_{100}}$ $= \frac{\sqrt{225}}{\sqrt{100}}$ $= \frac{\sqrt{5^2\times3^2}}{\sqrt{2^2\times5^2}}$ $= \frac{\sqrt{(5\times3)^2}}{\sqrt{(2\times5)^2}}$ $= {^{15}}/_{10}$

Simplify the fraction

${^{15}}/_{10}$ $= {^3}/_2$ $= 1\,{^1}/_2$

Activity 4.10

Find the square root of

(a) 0.49
(b) 5.76
(c) 0.0036

Yes, I believe you understand the concept well.

Topic VIII: Teaching Operation on numbers in Primary Schools

1. Introduction to writing, addition, and multiplication of numbers in base five and base ten.

i). Counting in base five and base ten

Counting is important because it helps us to establish a number of a given item. We can decide to count in various ways but for now, we shall learn how to count in base five and base ten, for example:

Dear student, how can you be able to count in base five or ten?

Let's see if this table will help you answer the question.

Base Five

5^2	5^1	5^0
5×5 (five fives)	5 (Five)	1 (Ones)
25 (Twenty five)	5 (Five)	1 (0nes)

Good! I hope you have understood.

What about base ten?

Base Ten

10^2	10^1	10^0
10×10 (Ten tens)	10 (Ten)	1 (Ones)
100 (Hundred)	10 (Ten)	1 (0nes)

With the tables above, what do you notice?
Find out, is your answer the same as this?
'The place values of numbers in different bases.'
Let us illustrate more in these examples.

a) 235_{five}

2 3 5

Ones…….5^0

Fives……………5^1

Twenty fives……………5^2

b) 780_{ten}

7 8 0

Ones…….10^0

Fives……………. 10^1

Hundreds ……………...10^2

Okay student, what next?

Let's see if we can now add numbers in base five and base ten.

ii). Addition in base five and base ten

Example 1.

Let's consider this example:

With base five,

Add $132_{five} + 221_{five}$

Arrange the digits in order of the place value.

	Twenty fives	Fives	Ones
	1	3	2_{five}
+	2	2	1_{five}
	4	0	3_{five}

Begin by adding the digits in ones i.e. (2+1) =3

3 therefore is less than the base five, so it is recorded as it is in place of Ones.
Moving on to the next place value, adding digits in place value of fives (3+2) =5
Since five is the same as the base.

Divide the product 5 with the base number.

5	5
1	0

1 remainder 0

Write 0 in place value of five and carry one. Transfer the group of one five to be added to the next place value of twenty five i.e
(1+1)+2 =4

Since 4 is less than the base, it's recorded as it is. In such steps, you would have added digits in base five.

Well done!

Now with your colleague, try out the next activity.

Example 2;

Add, $123_{five} + 410_{five}$

***Good!* Compare your answer with your colleague.**

Find out if your answer is the same as this in the box.

Add: $123_{five} + 410_{five}$

Twenty five	Fives	Ones
1	2	3_{five}
+ 4	1	0_{five}
1 0	3	3_{five}

Example iii). Addition of numbers in base ten

Hullo student, addition of base ten requires the same knowledge you have got in addition in base five. Remember, when you add the digits and it is greater than the required base, divide the digit by the base and you write the remainder and carry the quotient to be added to the next place value.

Let's try out this example.

Add: $986_{ten} + 758_{ten}$

$$\begin{array}{r} 986_{ten} \\ +\ 758_{ten} \\ \hline 1744_{ten} \\ \hline \end{array}$$

Thank you for working through this example. For more references incase you need more practice, visit Primary Mathematics Book 7. Page 34-35.

You can now relax as we prepare for the next subtopic.

(iv) Multiplication of numbers in base five and base ten.

Let us begin with;

- **Multiplication in base five and base ten**

Dear student, the idea of multiplying is different from what we have learnt in addition.

Remember that if you have a number higher than the base number, divide the number by the base, write the remainder and carry the quotient.

Let's have a look at these examples:

Example 4

Solve: $2 \times 43_{five}$

Solution:

$$\begin{array}{r} 43_{five} \\ \times\ 2 \\ \hline 141_{five} \\ \hline \end{array}$$

Work out: $569_{ten} \times 41_{ten}$

$$\begin{array}{r} 569_{ten} \\ \times\ 41_{ten} \\ \hline 569_{ten} \\ +\ 22760_{ten} \\ \hline 23329_{ten} \\ \hline \end{array}$$

2. Presenting numbers on a number line

- Representing inequalities on a number line

Did you know that you can show inequalities on a number line?

How? Remember the following ideas.

All the numbers to the left of a number are less than that number represented by a sign < or equal to the number represented by a combination of two signs $\leq$.

All the numbers to the right of a number are greater than that number represented by a sign > or the number may be equal to number represented as $\geq$.

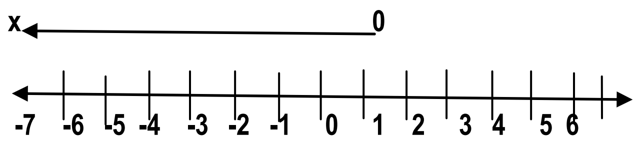
Therefore, the signs <, $\leq$, >, $\geq$ are called an inequality. You should therefore thoroughly understand the meaning of these signs. More explanations on these signs are in the table below.

Show these inequalities on a number line.

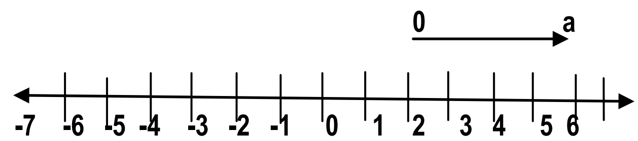
a) $X<1$
b) $a>2$
c) $y \geq 3$
d) $b \leq 0$

We draw the number line then draw an arrow above it.

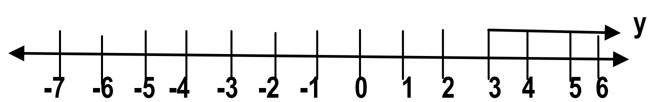
(a) x<



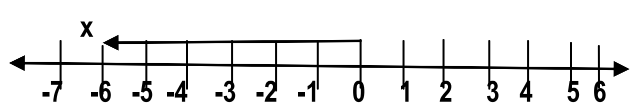
(b) a>2



(c) y≥3



(d) b ≤ 0



Well done

Teaching operation of numbers in primary schools

Activities and instructional materials to use in teaching of numbers in Primary schools.

Dear student, I believe after going through this unit, you have understood it thoroughly. Now let us see how this topic can be taught effectively in a primary school.

It is important that before your teaching as discussed previously, you need to plan.
What does planning involve?

Planning involves a lot of activities like scheming of the unit, preparing lesson plans, how to deliver the material to the learners, how to involve learners during the teaching and learning, and instructional materials like textbooks, books, pens, pencils. This will enable you program yourself on how to teach effectively to such a learner.

Remember as a teacher, you play a major role in the teaching learning process. It is there important that you know your learner(s) a little bit more in order to help you discover the challenges and weaknesses of your pupils.

Apart from knowing your pupils' capability, you should also consider another factor of class control. It is not so obvious that when you enter a class, pupils are always ready to learn, some may be taken up by the friends in class, or the teaching aid you may have brought in class. Therefore, as teacher, you are in full control of your class ensuring that discipline is maintained and that you provide a good environment for learning.

Now, let us see some of the activities you can carry out to demonstrate the teaching of numbers.

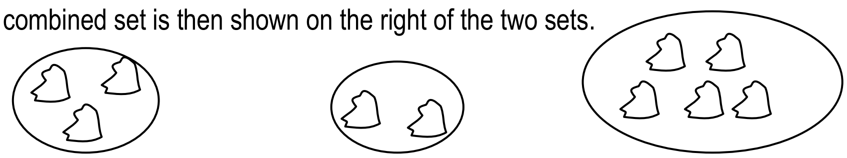
Activity 1: Addition

In carrying out addition, multiplication, subtraction, and division of numbers, use concrete objects like sticks, beans, straws, in order to bring out these ideas to the learners properly. Involve the learners during the learning of this operation.

Example:

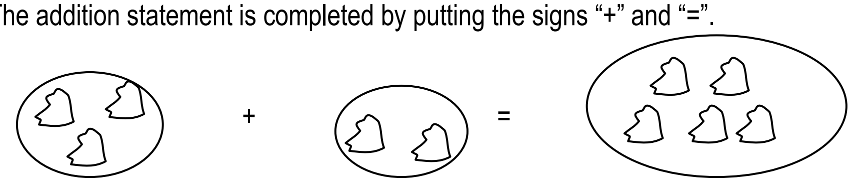
You may give two sets of objects, each of a number less than 5. they count each set and write down the two numbers. They then put the two sets together to form one set. This set is then counted and the number written down.
The two sets are first shown with their numbers.

The combined set is then shown on the right of the two sets.



3 2

The addition statement is completed by putting the signs “+” and “=”.



3 2 = 5

The pupils read the completed sentence as “Three plus two is equal to five.” Or “Three add two is equal to five.”

<u>**Note**</u>: The writing of the completed addition in a vertical form, as

$$\begin{array}{r} 3 \\ +\,2 \\ \hline 5 \\ \hline \end{array}$$

Should not be introduced at this stage. The vertical form becomes helpful when place value is first introduced. You should then build up more addition sentences for various sets to make sure the concept is grasped.

Activity 2: Subtraction

Select five children (3 girls and 2 boys) and stand in front of the class. The other pupils then count them (five). The boys then return to their seats. Then class count how many pupils left (three). You should now use a number sentence. The two boys come to the front of the class again. The pupils check that there are five children. You then put a numeral, 5 on the blackboard.

The two boys again return to their seats and you put a numeral, 2 on the blackboard. At this stage, then introduce the use of the subtraction sign to show the operation of 'taking away.'

You then ask the pupils how many children are left and you complete the sentence on the blackboard.

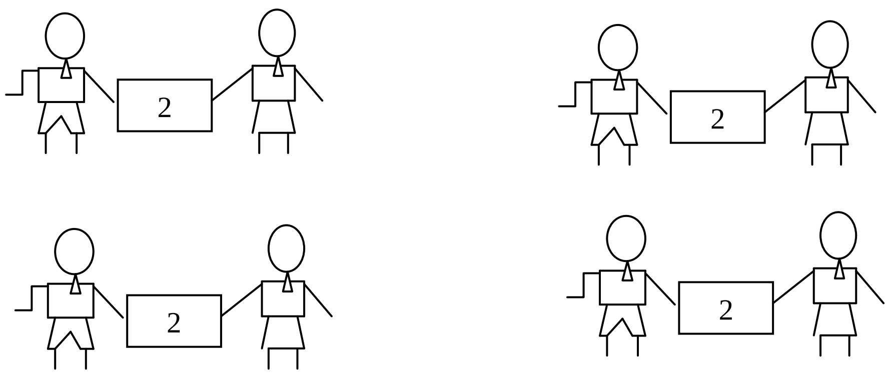
The pupils then read the sentence as, "five subtract two is equal to three" or "five take away two is equal to three."

You then repeat the other sets of boys and girls for various numbers so that they can get the idea of subtraction.

Activity 3: Multiplication

Dear student, before introduction to the idea of multiplication, make the students recall the idea of addition. You can do this demonstrating a few examples to make them recall.

Now select 8 children to come in front of the class (four boys and four girls). Pair the children to make four groups.



Make each pair hold up a large 2 numeral card. Then ask how many children are there in each group. You can now draw a large 2 numeral card on the blackboard and ask how many groups are there. Then show the numeral, 4 on the blackboard.
You then ask further, "how many children are there altogether?" Then write a big numeral, 8 on the blackboard.

You can then go ahead and explain how many times has the 2 appeared. Then introduce the word "times" and then tell the pupils in a sentence that "two times four is equal to eight."

You can now introduce the word and symbol of multiplication ($\times$) to mean times.
Therefore, substituting for the word times will be

$2\times4 = 8$

Operation on BODMAS

Mixed operation with numbers

Dear student when a problem involves different operations the order in which they operate should be followed properly as Brackets of Division Multiplication Addition Subtraction (BODMAS).

First work out "brackets" ()
Second workout "of" of
Third division ÷
Fourth multiplication x
Fifth addition +
And last subtraction -

Example: 1

Work out 20 of (96÷4)÷4+50 – 25
= 20 of (100) ÷ 4 + 50 – 25
= 2000 ÷ 4 + 50 – 25
= 500 + 50 – 25
= 550 – 25
= 525

Thank you for following.

Example 2

Work out 12500 ÷ 5 of (20 + 5) + 20 – 4
= 12500 ÷ 125 + 20 – 4
= 100 + 20 – 4
= 120 – 4
= 116

Now it is your duty to practice more numbers using BODMAS.

Activity

Work out:-

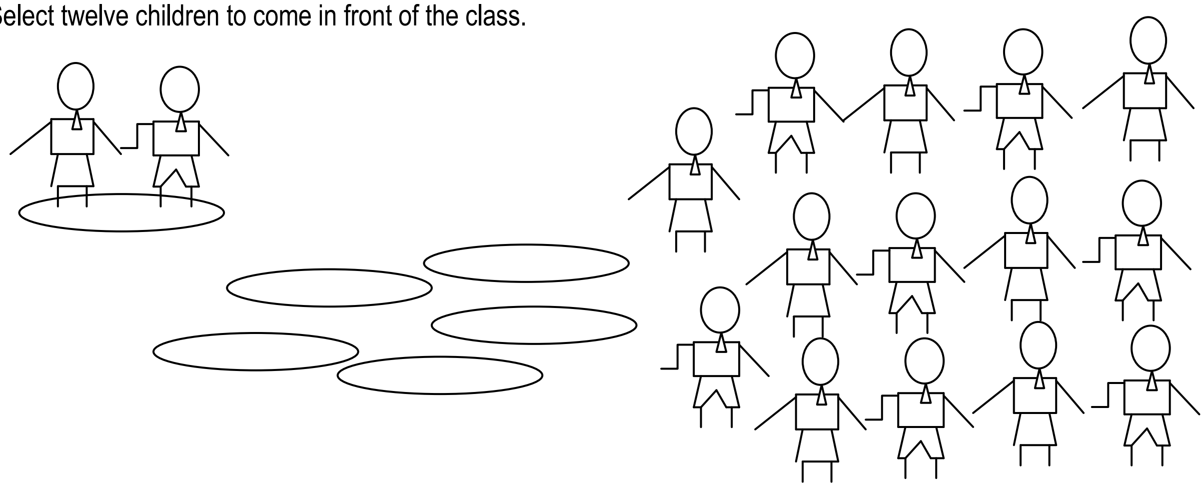
(a) 50+(26+74)÷10x6+129 – 36
(b) 20(165 – 65) ÷ 5 + 38 – 71

Repeat the activity to make sure the pupils understand the concept of multiplication. At a later stage, you can now tell the pupils that, repeated addition is multiplication.

Well done! You are becoming a great teacher

Activity 4: Division

Select twelve children to come in front of the class.



You can draw a set of small loops on the floor. The choose two of the children and stand in one of the loops. Then choose another two and make them stand in another loop. This goes on until all the loops are covered.Then ask the class, "How many groups of twos have I made?" The pupil will then count the twos and say, "there are six." Then say "I started with twelve children (at the same time write 12 on the blackboard), I the formed twos (write 2 on the blackboard) a little distance from the 12). I then found that I had six two (write 6 on the blackboard to the right of 2)."

Then go ahead to explain that to show this activity, we use a special symbol. It is called the 'division' symbol. Then write this on the blackboard between the 12 and the 2 and complete the statement as,

$12 \div 2 = 6$

Repeat this activity to make sure pupils get the idea of division.

4.8 unit summary

In this unit you have been introduced to operation on numbers and learned about:-

(i) Writing numbers in words and in figures.
(ii) Addition of numbers (horizontal addition and vertical subtraction).
(iii) Subtraction of numbers (horizontal subtraction and vertical addition).
(iv) Use of number lines in multiplication.
(v) Multiplication of natural numbers.
(vi) Multiplication of digits by a two and three digits numbers.
(vii) Introduction to division of numbers.
(viii) Division of numbers by digits of two and three.
(ix) Expansion in base ten.
(x) Relationship of exponents, powers and indices.
(xi) Expansion of indices.
(xii) Multiplication of numbers involving indices
(xiii) Division of numbers involving indices.
(xiv) Meaning of square numbers.
(xv) Square roots of numbers and mixed fractions.
(xvi) Introduction to counting, addition and multiplication of numbers in base five and ten.
(xvii) Numbers on a number line.
(xviii) Planning lessons for teaching operation on numbers.

4.9 Glossary

i. **Operation** is a rule for processing one or more objects.
ii. **Figure** may be either a digit or a number.
iii. **Addition** Is the operation of combining numbers, each of which represents a separate measure of quantity so as to produce a number representing the measure of all those qualities together.
iv. **Subtraction** is the operation of finding a number which gives a measure of the differences in size between two quantities or measures.
v. **Horizontal** is a cross the page
vi. **Vertica**l on the top to bottom direction of the page.
vii. **Number** line is a graduated straight line along which it is possible to mark all the real numbers.
viii. **Product** is the result given by the operation of multiplication. Multiplication is the operation which combines several equal measures of size giving the result as a single member.
ix. **Division** is an operation between two numbers which measure how many times bigger one number is than the other.

x. **Quantity** is the result given by the operation of division.
xi. Digits are the single symbols 0,1,2,3,4,5,6,7,8,9 as used in everyday life.
xii. BODMAS is an acronym that serves as a reminder of the order in which certain operations have to be carried out when working with operations and formulas brackets of division, multiplication, addition and subtraction.

4.10 Answers to activities

a.

i. Five million, one hundred twenty three thousand four hundred fifty.
ii. Two million, eight hundred ninety thousand four hundred thirty five.
iii. One million, one hundred eleven thousand one hundred eleven.

b.

(i) six million	6,000,000
Seven hundred twelve thousand	712,000
Nine hundred	+ 900
	6,712,900

(iii)	Four million	4,000,000	
	Four hundred thirty nine	439,000	
	Two hundred fifty	+	250
		4,439,250	

Activity 4.1

1. a. 837510
   b. 14626810
   c. 9289898

2.a. 8487052
  b. 5933682

Activity 4.2

1. a. 7840339
   b. 588717
   c. 9802762

2. a. 684984
   b. 77453
   c. 922381
   d. 753592

Activity 4.3

1. a. 30281082
   b. 539488

c. 169232788
d. 199801896

2. a. 1542656
b. 13716360
c. 593340

Activity 4.4

i. 16000
ii. 91
iii. 28000
iv. 5000000
v. 1000000
vi. 8600
vii. 700
viii. 100000

Activity 4.7

iii. 1000000000
iv. P^5
v. n
vi. $5x^7$
vii. 8
viii. x^5
ix. $2y^3$
x. 1^{-4}

Activity 4.8

a. (i) 20
(ii) 15
(iii) 4
(iv) 6
(iv) -3
(v) -10
(vi) -25
(vii) –

b. 15 or -15. 15 (a positive square root)

Activity

a. 22
b. 21
c. 18
d. 24

e. 9
f. 45

Activity

a) $(4x10^6) + (8x10^3)+(5x10^2) +(4x10^1) + (5x10^0)$
b) $(3x10^6) + (5x10^5)+(5x10^4) +(1x10^3) + (0x10^2) + (6x10^1)$
d) $(3x10^2) + (9x10^1)+(5x10^0) + (0x10^{-1}) + (4x10^{-2}) + (5x10^{-3}) (6x10^{-4})$

2.a. 100000
600
40
+ 3
100643

b) 200000
2000
100
30
+ 1
202131

c) 60000.0000
3000. 0000
4.0000
.0300
.0007
63004.3307

Activity 4.10

a. 0.7
b. 2.4
c. 0.06

4.11 End of Unit Exercise

1. A teacher made a roll call during night preps. He then discovered that there are 600 boys and 300 girls. How many students were there altogether?

2. What is the sum of 130470 and 4834?

3. A farmer had 1, 475 cows and 320 bulls. She later discovered that the following day, thieves stole her 20 cows and 6 bulls. How many cows and bulls remained altogether?

4. Mukasa borrowed sh.18,000 from a friend and he decided to give his three girls as pocket money. How much did each girl receive?

5. John borrowed 18,000 from a sister and is required to pay 6 times more. How much will her sister receive?
   Self Assessment

References

1. Primary Mathematics Today 4th Edition by Elizabeth Williams, pg 100
2. Primary Mathematics 2000, pg 41
3. Teaching Mathematics in Primary schools, by D. Paling, pg 65-79
4. Tapsan Frank (1999) The oxford mathematics Study Dictionary

UNIT 5 NUMBER PATTERNS AND SEQUENCES

5.1 INTRODUCTION

You are welcome to this unit 5. This unit will explore you to types of numbers, number sequence, factors, and multiples of numbers, solving numbers and how to teach number patterns and sequence in a primary school.

5.2 CONTENT ORGANIZATION

Hello student; in this unit you are going to cover the following topics.

Topic	Subtopic
1. Number patterns	a) Number sequences b) Types of numbers patterns (i) Even numbers (ii) Odd numbers (iii) Square numbers (iv) Triangular numbers (v) Cubic numbers (vi) Rectangular numbers
2. Test for divisibility	a) Divisibility tests for; (i) 6,9,10,11 Application of divisibility tests
3. Factors and multiple of numbers	a) Factors of numbers (i) Prime numbers (ii) Prime factors (iii) Expressing numbers in prime factors (prime factorization) (iv) Highest common factor (H.C.F) b) Multiple of numbers (i) Lowest common factors (L.C.M) (ii) Use of prime factors to find L.C.M and H.C.F, square roots and cube roots (iii)Application of L.C.M and H.C.F
4. Teaching number patterns and sequences in primary school	a) Analysis of number patterns in primary school syllabus. b) Planning lessons for teaching number patterns and sequences.

5.3 Learning outcome:

By the end of this unit, you should be able to demonstrate understanding of number patterns and sequence in solving problems and teach pupils how to form number patterns and sequence.

5.4 Competences

Hello student, now that you know the learning outcome, as you study through this unit, you should be able to;

a) Write down patterns of number
b) Distinguish between factors and multiples
c) Prime factorize numbers
d) Find square root of numbers using prime factors
e) Use prime factors to find LCM and HCF
f) Use the knowledge of number sequence and patterns to solve problems in everyday life.
g) Prepare lesson and demonstrate the teaching of number patterns and sequence in a primary school.

5.5 Subject orientation

This unit is to get you exposed to various of types number patterns and sequence. You will have to develop number patterns from natural numbers or counting numbers. You should therefore read the previous unit about whole numbers.

5.6 Study requirements

To be able to study this unit, you have to organize study materials like the counting chart plus mathematics tables, have knowledge on counting numbers from 0 to 100. You also need to have Primary Mathematics Course books, Primary Mathematics syllabus.

I wish you success in studying this unit!

5.7: Content and activities

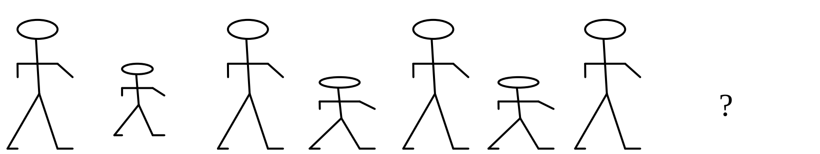
5.7.1: Number sequence.

Hello student! Welcome to this topic. This topic will require you to read about numbers under units 3 and 4.

Now, can you do the following?

- What is a sequence?

  Can you explain then what a number sequence is?
  You can find out by doing the following,



- What have you discovered from the arrangement of the pictures above.
- What do you think the next picture will be? Small or big

Lets use another example,

2, 4, 6, 8, 10, _?__

- What have you discovered from the arrangement of numbers.
- What do you think will be the next number in the arrangement?

Well done!

Note:

A sequence is an arrangement of two or more things in a successive order. Therefore, a number sequence is the arrangement of numbers in a successive order. This arrangement of numbers in a successive order is called the number pattern.

We can also decide to arrange a number pattern in ascending or descending order.

Read more about number pattern in ascending or descending order in Primary Mathematics Book 7. Page 70.

Lets proceed to discuss more number patterns.

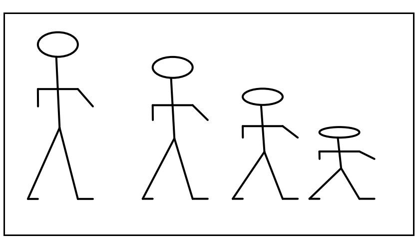
a) Number patterns

(i) Type of number patterns

What do you understand by pattern?

Can you now explain what a number pattern is?

Check this, a pattern is an order of arrangement. Look at this kind of arrangement.



What can you say about the arrangement of individuals in the bracket

Therefore, numbers can follow an order of arrangement and this order of arrangement is called the number pattern.

Thank you! Let’s continue

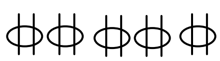
(i) Even and odd numbers

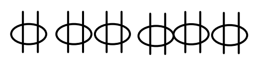
Did you know that there are different types of number patterns? Lets find out by doing the following. Now collect 10 sticks and 13 straws, pair the sticks and straws.

Sticks straws

10 sticks ////////// 13 straws //////////

Pairing up





a) (i) How many groups of sticks are formed
   (ii) How many sticks are left ungrouped?

b) (i) How many groups of straws are formed?
   (ii) How many straws are left ungrouped?

Good! You can compare your answer with a friend.

Note: 10 sticks grouped in twos is also known as 10 divided by 2 (10 ÷ 2) leaves no sticks left ungrouped. Whereas 13 straws grouped in twos is also known as 13 divided by 2 (13 ÷ 2) leaves one straw ungrouped. Therefore, when you divide any number by 2 and gives no remainder left this number will be called an <u>even number</u> i.e. 2,4,6,8,10, 12, etc. Also when you divide any number by 2 and leaves 1 as a reminder, this number will be called an <u>odd number</u> i.e. 3,5,7,9,11,13 etc

Well done! You have catched up!

You can now count sticks of any number and try to group in twos. Identify which number will leave a reminder and also find if the number is an even number or odd number.

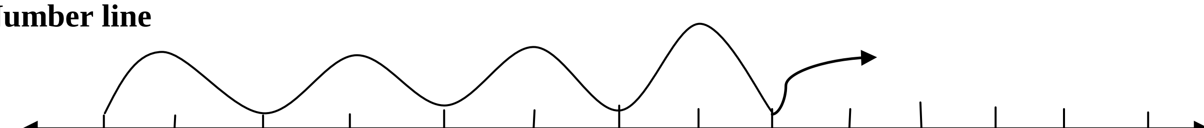
Let’s continue……………………………….

Did you know?

You can also use a number line to count numbers in two’s.

Let’s see how this works.

Number line



0 1 2 3 4 5 6 7 8 9 10 11
12 13

From the above number line, counting in two’s gives the numbers circled, these are known as <u>even numbers</u>

Good.

Activity 5.1

You can now continue to use number line to find out more of the even numbers up to 100 and also identify those numbers that form groups of two’s and leaves a remainder one.

Well done!

Let’s proceed………………………………………..

(ii) Square numbers.

Hallo student,

Do you know what a square is? Then can you draw a shape of a square? Can you now use dots to form a square?

Activity 5.2.

Now using dots laid out in rows and columns, make many square patterns.

Compare if your square patterns are the same as this below.

® ® ® ® ® ® ® ® ® ® ® ® ® ® ®
A ® ® ® ® ® ® ® ® ® ® ® ® ® ®
B ® ® ® ® ® ® ® ® ® ® ® ®
C ® ® ® ® ® ® ® ® ®
D ® ® ® ® ®
E

a) Join the outside dots to form a shape
b) Count and write down how many dots are used in shapes A,B,C.D and E.
c) What can you conclude about each of the numbers in A,B,C,D and E.

d) What do you call the pattern formed?

Well done you are doing great!

Note: The number of dots used to make each of the squares A,B,C,D and E are called square members. The arrangement of shapes formed follow a pattern, known as a square. You can now use the above information to form more square patterns.

(iii) Triangular numbers

Dear Student! With the same idea used to obtain a square pattern; you can now do the following

- What is a triangle? Draw a shape of triangle.
- Using dots make many triangular patterns.

Activity 5.3.

Now, can you use dots to make as many triangular patterns. Compare if your shapes look like the ones below.

® ® ® ® ®

® ® ® ® ® ® ® ®

A ® ® ® ® ® ® ® ® ®

B C ® ® ® ® ® ® ® ®

D ® ® ® ®

E

a) Join the outside dots to form a shape.
b) Count and write down how many dots are used to make up shapes A,B,C,D,and E
c) What can you conclude about each of the numbers in A,B,C,D and E
d) What do we call the patterns formed?

Well done

Note: The number of dots used to make each of the triangular numbers. The arrangement of the shapes formed follow a pattern, known as a triangular pattern

You can now give more examples of triangular patterns

Let's continue……………………………………………………………………

iv) Cubic numbers

Hallo student! Having known what square numbers and triangular numbers are, you can now find out this;

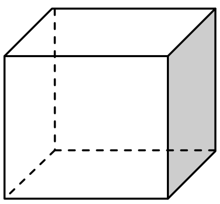
- Do you know how a cube looks like?
- Can you then draw the shapes of a cube?

Note, a cube is a four sided square.
Now carry the activity below.

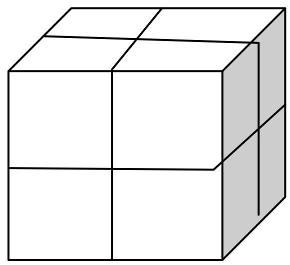
Activity 5.4

Collect small cubes and then use them to build bigger and larger cubes.

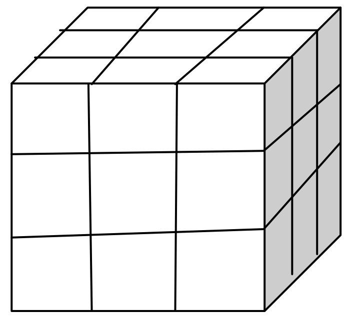
Compare if your drawing is the same as below.



A



B



C

- Count and write down how many cubes are in A,B,C.
- How do we call the numbers of cubes in A,B,C?
- What about the patterns formed by A,B,C.

Well done! You can cross check your answers at the end of the unit.

Note;

The numbers of cubes in A,B and C are called Cubic numbers.
The arrangement of the cubes A,B,C follow a pattern as cubic pattern.

Lets continue ……………………………………

v) Rectangular numbers.

Hallo student!

- What is a rectangle? Then draw its shape.
- Can you name any objects which have a shape of a rectangle?

Activity 5.5

Now, using dots form many shapes of rectangular pattern.

Compare if your shapes look like the ones below

® ® ® ® ® ® ® ® ® ® ®

A ® ® ® ® ® ® ® ® ® ® ®

B C ® ® ® ® ®

D

a) Join the outside dots to form a shape.
b) Count and write down the number of dots used in A,B,C and D respectively
c) Name the shapes formed above.
d) Make more shapes of similar pattern.

Well done

Note: the number of dots used make each of the shapes A,B,C and D are called rectangular numbers, the arrangement of the shapes formed follow a pattern known as rectangular pattern

5.7.2: TEST FOR DIVISIBILITY

Hallo student welcome to another topic of divisibility tests.

- What do you understand by something being divisible?
- How can you prove that something is divisible?

Find the answers by doing the following

Collect 10 mangoes and give to 2 girls,
How many mangoes does each girl get?
Get another 10 mangoes and give to 3 girls.
How many mangoes does each girl get?

Note; From the above examples, 10 mangoes to be given to two girls it's the same as 10 divided by 2, i.e. $10 \div 2$, the result is 5, we can see that 10 mangoes can be shared equally between the two girls. therefore 10 is divisible by 2.

On the other hand 10 mangoes cannot be exactly divided among three girls, therefore we can say 10 is not divisible by 3.

Therefore, to test, if a number is divided equally either by 2,3,4,5,6,7 etc is known as divisibility test.
Dear student,

Divisibility tests are useful to determine whether or not a number can be divided by another number before carrying out the division

Good

You have now understood what divisibility test is. Now carry out the following tests.

Activity

a) Make a set of numbers as shown below.

Set A

81, 29
2, 4
6,7,20,71,26,21, 69,
48, 49, 30, 15, 40, 45,

(i) From the set above choose numbers which can be divisible by 2.

i. Divisible by 2
ii. Divisible by 3
iii. Divisible by 4
iv. Divisible by 5
v. Divisible by 6
vi. Divisible by 7
vii. Divisible by 8
viii. Divisible by 9, 10 and 11

Did you know?

Note:

a) Divisibility test for 2.

For numbers to be divisible by 2, the digit or last digit of the number should end with either 0,2,4,6,8,etc. such numbers which are divisible by 2 are known as even numbers.

b) Divisibility test for 3.

For numbers to be divided by 3, the sum of the digits should be divisible by 3 or they should be multipliers of 3.

Consider this example

Number	**Sum of digits**	**Divisible ✓ or not divisible x**
12	1 + 2 = 3	✓
28	2 + 8 = 10	X
186	1 + 8 + 6 = 15	✓
487	4 + 8 + 7 = 19	X

Therefore, the numbers whose sums are divisible by 3 are 12 and 186.

c) Divisibility by 4

For a number to be divisible by 4, look at the last two digits of the number. If the two digits part at the end is divisible by 4, then the whole number will be.

Consider this example:

Which of these numbers is divisible by 4.

(i) 2564 (ii) 3663

Look at the last digits in the numbers

(i) 2564

We consider 64÷4=16, so 64 is divisible by 4
∴2564 is divisible by 4.

(ii) 3663

Look at 63. This is an odd number, so cannot be divisible by 2. Therefore, it cannot be possible to be divisible by 4.

(iii) Divisibility by 6

For the number to be divisible by 6, it must be divisible by both 2 and 3. Any even number (that is, a number ending in 0,2,4,6 or 8) is divisible by 2. Any number whose digits add up to a multiple of 3 is divisible by 6.
So, any number which is even and has digits which add up to a multiple of 3 is divisible by 6.

Consider the example below:-

Test the following numbers for divisibility by 6.

(i) 3204
(ii) 9406

(i) 3204 ends in 4 so it is an even number.
3+2+0+4=9 which is a multiple of 3?
∴3204 = is divisible by 6.

(ii) 9406 ends in 6 so it is an even number. 9+4+0+6 = 19 which is not a multiple of 3 so 9406 cannot be divisible by 6?

d) Divisibility by 7

Hello student, for a number to be divisible by 7, you double the last digit and subtract it from the remaining digits. If the answer is 0 or divisible by 7 leaving no reminder, then that number is divisible by 7.

For example

Test the following numbers for divisibility by 7.

For example

Test the following numbers for divisibility by 7.

(i) 84 (ii) 784 (iii) 1176

(i) 84 double 4x2=8
Subtract from 8 = 8 – 8 = 0
∴84 is divisible by 7

(ii) 784 double 4x2=8

Subtract from 78 = 78 – 8 = 70

70 ÷ 7 = 10

∴784 is divisible by 7

(iii) 1176

Take 6 and double it 6x2=12

Subtract from 117 = 117 – 12

= 105 ÷ 7

= 15

∴1176 is divisible by 7

e) Divisibility test for 5

For numbers to be divided by 5 their last digit should end with either 0 or 5 e.g. 5, 10,15,20,465,20850 etc

- Divisibility test for 8

For a number to be divided by 8, the number should be a multiple of 8 or the last three digits of the number should be divisible by 8.

Consider this example.

Which of these numbers is divisible by 8

h) 1047064 ii) 7829321

Consider the last three digits in the box.

i) 1047<u>064</u>

064 ÷ 8 = 8

∴ 1047064 is divisible by 8

321 ÷ 8 = 40 r

Let find out.

```
          130883
      8 ) 1047064
8x1 =      -8
```

ii) 7829321

Considering the last three digits

321 ÷ 8 = 40 r I.
∴ 7829321 is not divisible by 8.

- **Divisibility test for 9**

Dear student! To make sure that a number is divisible by 9, the sum of its digits should be divisible by 9. Lets consider this example;

Find out which number is divisible by 9.

i)	84652	ii) 40851
	number	sum of digit
	84652	8 + 4 + 6 + 5 + 2 = 25
	40851	4 + 0 + 8 + 5 + 1 = 18

In the above example, you will discover that 40851 is divisible by 9.

Well done. Let continue.

Divisibility by 11

Hello student, for a number to be divisible by 11 the difference between the sum of the digits in alternating position should be either 0 or divisible by 11.

Lets consider these examples.

Find out which of these numbers is divisible by 11.

i) 298276 ii) 458923

i) 298276

First alternate positions

2 9 8 2 7 6

Second alternate positions

Now, get the sum of the digits in the first alternate position i.e.

2 + 8 + 7 = 17

Then sum of the digits in the second alternate position is 9 + 2 + 6 = 17

You should then get the difference between the sums of the digits in alternate positions i.e. 17 – 17 = 0

Since the difference between the sums is 0, then the number is divisible by 11.

a) 4 5 8 9 2 3

Lets repeat the same procedure above

4 5 8 9 2 3

First alternate positions

Second alternate positions

Sum of digits in the first alternate position

4 + 8 + 2 = 14
Sum of digits in the second alternate position
5 + 9 + 3 = 17

Difference between the sum of digits in alternate position = 17 – 14 = 3
∴ Since the difference of 3 between the sums is not 0 or divisible by 11, this number is not divisible by 11

Well done

You may read more from functional Primary Mathematics for Uganda, book 7, page 77 – 86, Primary Mathematics Book 7 pages 68 - 70

Now with the above information, do the exercise below.

Exercise 5.2

Using divisibility tests, determine whether each of these numbers are divisible by 2, 3, 4,5,6,7,8,9,10,11

a) 58 b) 153 c) 881 d) 12345

Well done. You may compare your answers with a colleague

5.7.3: FACTORS AND MULTIPLIES

Welcome to this subtopic below

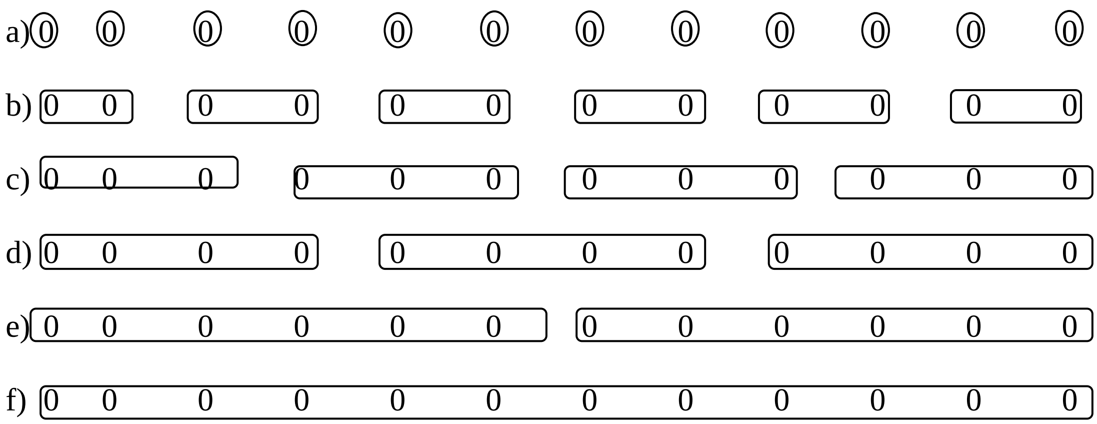
a) Factors of a number

Hello student! Do you knowabout factors?

What are factors?

Find out by doing the following

Using 12 as a particular number, find out the different multiplication which give that number. The different multiplication are formed in this diagram below.



List down the different multiplications in each of the letters a,b,c,d,e and f above.

Well done! Compare your answers with a colleague

Compare your answers also with the one in the box.

Different multiplication in the above diagram include	
a) 1 x 12 =12	d) 4 x 3 = 12
b) 2 x 6 = 12	e) 6 x 2 = 12
c) 3 x 4 = 12	f) 12 x 1 = 12

What have you discovered for the multiplication?

Note: When, 12 is divided by each of the numbers multiplied to give 12, there is no reminder.

Therefore the numbers which when multiplied to give 12 are said to be factors and one of the number is said to be a factor

Hope you have understood what factors are.

Let's continue

You can now list the factors of 12 as {1,2,3,4,6,12
}

In the above activity, when two factors are multiplied together they give a multiple (product). Therefore, in this case, 12 is said to be a multiple.

Consider this example.

18 = 3 x 6, 3 is a factor of 18 and also 6 is a factor of 18
Again 18 = 2 x 9, 2 and 9 are factors of 18.
∴ 18 is said to be a multiple of any of the numbers 2,3,6,or 9 since each of the number goes exactly into 18.

Let's continue

b) Multiple of a number.

Hallo student! From the information in (a) you are expected to answer this.
What are multiplies of a number?

Find out by doing the following.

List down whole numbers from 1 to 25 as shown below.

1 2 3 4 5 6 7
8 9 10 11 12
13
14 15 16 17 18 19
20 21 22 23 24
25

From the list circle numbers which can be formed by adding of 2's repeatedly.

Note; The numbers which can be formed by adding 2's repeatedly are called multiples of 2. In this case, they will be 2,4,6,8,10,12,14,16,18,20,22,24.

Well done.
You can now use the above information to do the following activity.

Activity 5.5

a) Write down the factors of the following numbers.

i) 1 ii) 9 iii) 16 iv) 24 v) 30

b) Write own the multiples of the following between 1 – 100.

i) Multiples of 5
ii) Multpies of 6
iii) Multiples of 10
iv) Multiples of 25

Well done relax.

c) Lowest Common Multiple and Highest Common Factor.

(i) Lowest common multiple (L.C.M)

Dear student! What are the lowest common multiples!
Can you find out by doing the following example;

List down multiples of 2,3 and 4 respectively.

Multiples of 2:	2	4	8	10	12	14	16	18	20	22	24	26
Multiples of 3:	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4:	4	8	12	16	20	24	28	32	36	40	44	48

- Circle the common multiple
- You will discover that 12 and 24 are common multiple of 2,3 and 4

∴ the lowest common multplie is 12.

Good you are doing great.

You may give more examples of your own.

Lets continue……………………………..

Highest common Factor (H.C.F)

Hallo student,

Use the information from factors of a number from the previous subtopic and the idea of lowest common multiple to carry out the activity.

Activity 5.6

a) List down factors of;

i) 20 and 30
ii) 12 and 18

b) List the common factors in a) I and ii above,
c) From b) determine the highest common factor in a) I and ii.

Well done! You can compare your answers with a friend and check at the end of the unit.

Activity 5.7;

Hullo student,

Did you know?

You can also use multiplication tables to find out multiples of a number and therefore finding lowest common multiplies of a number.

Now formula to a multiplication tables of number 1 – 10 as shown below.

x	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4					20					
5										
6		12								
7										
8										
9										
10										

a) Complete the table above
b) From the table list down the multiples of 3,4, and 5
c) What is the lowest number of the multiples you have listed?
d) What could we call this number?

Exercise 5.8

a) Find the H.C.F of;

i) 4, 6 ii) 8,12 (iii) 24, 36 (iv) 10,15,30

b) Find the L.C.M of :

i) 6,8 ii) 6, 8,12 (ii) 16, 20 (iv) 10, 25,4

Dear student,

More information about H.C.F and L.C.M will be learnt under prime factorization.

(iii) Prime factorization

- Prime numbers and composite numbers.

What do you understand by a prime number and composite number?
Hallo student! For you to understand the above, carry out the activity below.

Make a table of the first 1 – 100 numbers.
Circle 2 and then cross out all of its multiples.
Circle 3 and then cross out all its multiples.
Circle 5 and then cross out all its multiplies
Circle 7 and then cross out all its multiplies.

Eratosthenes sieve (hundreds grid)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- List the numbers which are left out?
- List down the factors of each number left out.
- What type of numbers are these?

Well done

Compare your answer with the one in the box.

a) It is seen that these numbers have only two factors, that is, one and itself, such numbers are called <u>prime numbers</u> therefore, besides these numbers, 2,3,5,and 7 are included.

b) From the above explanation, during factorization of any number, a factor which is a prime number is called <u>a prime factor</u> i.e.

Factors of 63 are, 3,7,9,12 thus 3 and 7 are prime factors but 9 and 21 although a factor of 63 are not prime factors.

c) Also, numbers with more than two factors are called <u>composite numbers</u>. i.e. 63 = 1 x 3 x 21

30 = 1 x 2 x 3 x 5

You can give more examples of you own on prime factors and composite numbers.

ii) Expressing numbers inform of prime factors

Dear student!

You can express numbers in form of <u>prime factors</u> using these methods below.

- The division known as ladder method.
- Factor tree.

Note, that expressing numbers in form of prime factors is known as <u>prime factorization</u> . you know what factors are; for example we said 2 x 5 = 10, 2, and 5 are factors of 10.

When we say prime factorization, we are after those factors of a number which are prime.

You may recall that a prime number is that number whose factors are 1 and itself.
Natural numbers can, therefore, be expressed as a product of prime numbers e.g. factors of 24 are 1,2,3,4,6,8,12,24.

Identifying the prime numbers, we have 2 and 3.

We can therefore use these prime numbers to divide 24, beginning with the smallest 2 number and then 3.

Let us carry out the division.

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\therefore 24 = 2x2x2x3$
$= 2^3 \text{ x } 3$

Lets consider another example

Prime factors of 30

Factor of 30 are 1,2,3,5,6,10,15,30

What do you think are the prime factor of 30?

Let us carry out the division,

Remember we begin by using the smallest prime factor.

2	30
3	15
5	5
	1

30 = 2 x 3 x 5

b. (ii) Using prime factors to find L.C.M

We can use prime factors to find L.C.M.

Example: Find the L.C.M of 16 and 20.

Lets find the prime factors of 16 and 20 using a factor tree method.

16
2 8
2 4
2 2
2 1

∴ factors of 16 = 2 x 2 x 2 x 2
$= 2_1 x 2_2 x 2_3 x 2_4$

20
2 10
2 5
5 1

∴ F_{20} = 2 x 2 x 5
$= 2_1 x 2_2 x 5_1$

We should now get the common factors between 16 and 20. Therefore, the L.C.M should have the common factors and the remaining factors in 16 and 20.

∴ L.C.M = $(2_1 x 2_2 x 2_3 x 2_4 x 5_1)$
= 2 x 2 x 2 x 2 x 5
= 80.

We ignore the suffices when finding the LCM

Let consider another example using the long division method.

16 and 20

2	16	20
2	8	10

2	4	5
2	2	5
5	1	5
	1	1

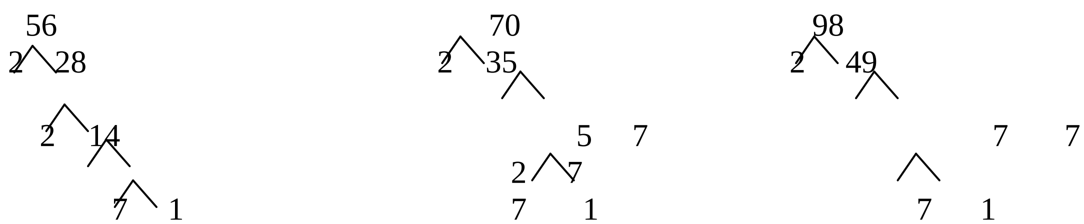
2 x 2 are common factors to both, then 2 x 2 are out, for the remaining factors in 16 and % for the remaining factor in 20. When you work out you find the L.C.M is 2 x 2 x 2 x 2 x 5 = 80. It means 80 is the L.C.M of 16and 20.

Well done

Let's consider another example
Find the L.C.M of 56, 70, and 98.

In this case, we can use another method called the factor tree method of factorization.

Therefore, factorizing 56, 70 and 98 respectively.



The prime factors of 56 are 2 x 2 x 2 x 7   2 x 2 x 2 x 7

70 are 2 x 5 x 7   2 x 5 x 7

98 are 2 x 7 x 7   2 x 7 x 7

Therefore, the common factor among 56,70,98, are the circled numbers which include 2 and 7. The remaining factors in 56 are 2 x 2, 70 is 5 and 98151.

Therefore, the L.C.M of 56, 70 and 98 must contain the common factors and the remaining factors in each of them i.e. 2 x 7 x 2 x 2 x 5 x 7 = 2^3 x 5 x 7^2

∴ L.C.M = 2^3 x 5 x 7^2

> Note, If the L.C.M is large, it should be left in prime factors i.e. 2^3 x 5 x 7^2

Congratulations! You have understood.

Let's continue!

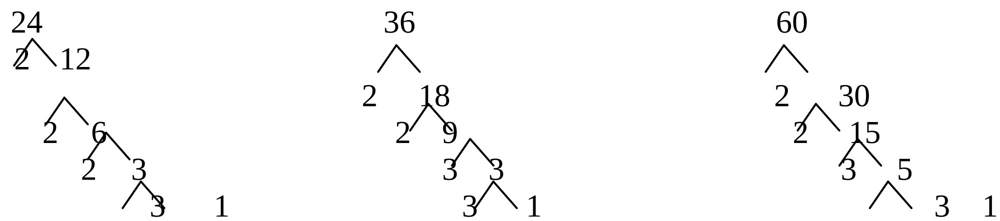
- **<u>Using the prime factors to find the highest common factors (H.C.F)</u>**

Dear student!

I believe you have understood how to use the prime factors to find. L.C.M and square roots of numbers but lets now see how we can find the H.C.F

Note. You can still use either the factor tree or the long division to find out the prime factors of the given numbers.

Let consider this example.
Find the H.C.F of 24, 36 and 60.
Using the factor tree, lets find the prime factors.



Prime factors of 24 = 2 x 2 x 2 x 3   2 x 2 x 2 x 3
36 = 2 x 2 x 3 x 3   2 x 2 x 3 x 3
60 = 2 x 2 x 3 x 5   2 x 2 x 3 x 5

Now select the common factors among the three numbers i.e., 24, 26,60.

From the above prime factors 2 x 2 x 3 are the common factors in each of them.

Therefore, to find the Highest common factor, multiply and get the product of the common factor i.e 2 x 2 x 23 = 12. Therefore, the H.C.F between 24, 36, 60, is 12.

- Using venn diagrams to find the HCF and the LCM. (check the attached sheet for the work)

Using the venn diagrams to find the lowest common multiples (LCM) and the Highest Common Factor (HCF).

Dear student,

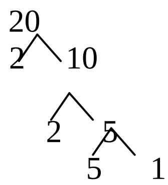
We can also find LCM and HCF using venn diagram.

Let's see it.

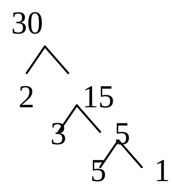
Example:-

Find the LCM and HCF of 20 and 30. You first find the prime factors of 20 and 30

F_{20} =



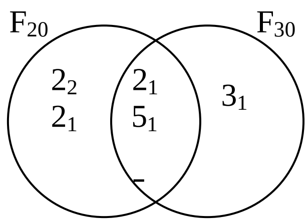
F_{30}



List the factors in a set form while putting suffice

$F_{20} = \{2_1, 2_2, 5_1\}$
$F_{30} = \{2_1, 3_1, 5_1\}$

Draw venn diagrams to represent the sets



LCM is got by taking the union of the two sets

$\therefore LCM = F_{20} \cup F_{30} = \{2_1, 2_2, 3_1, 5_1\}$
$= 2x2x3x5$
$=60$
$\therefore LCM = 60$

HCF is got by taking the intersection of the two sets.

$F_{20} \cap F_{30} = \{2_1, 5_1\}$
$= 2x5$
HCF = 10

Activity

By using venn diagrams, find the LCM and HCF of the following numbers.

i) 12 and 20 (ii) 15 and 18

In summary,

- The highest common factor (usually written H.C.F) of two or more numbers is the greatest number which is a factor of each of them; thus 12 is the H.C.F of 24, 36, and 60.

The lowest common multiple (L.C.M) of two or more numbers is the least number which is a multiple of each of them; thus 36 is the L.C.M of 12 and 18.

You can use the above information to work out.

Activity 5.8

1. State the H.C.F of
   a) 4, 6
   b) 8, 12
   c) 10,15,20
   d) 26,31
   e) 24,26
   f) 18,30,12,42

2. State the L.C.M of
   a) 6, 8
   b) 6, 8,12
   c) 16,20
   d) 9, 14,21

3. Find the smallest number which leaves a Reminder 2 when divided by 8. or 12 or 14

Well done! You may cross check your answer at the end of this unit.

- **Using prime factors to find square roots of numbers**

Remember, in the previous unit 4, under operations of numbers, we talked about using prime factors to find square roots of numbers.

But lets give an example to emphasize the concept.

Find the square root of 144.

To find such a square root, first express the number in form of a prime factor, you may decide to use a factor tree or long division method

Using factor tree method,

144
2 72
2 36
2 18
2 9
3 3
3 1 the prime factor of 144 are; 2 x 2 x 2 x 2 x 3 x 3

Then arrange/group the prime factors of 144 in pairs by putting them in brackets as show.

144= (2 x 2) x (2 x 2) x (3 x 3)

To form a square root, select one prime factor from each of the bracket.

The square root of 144 = 2 x 2 x 3
= 12

Well done! You are great.

Activity

Find the square roots of the following numbers

a) 81
b) 225
c) 1764
d) 2576.

5.7.4: Teaching number Patterns and Sequence in a primary school

a) **Analysis of the topic number patterns and sequence in a primary school syllabus.**

Hello student,

Now that you have gone through this unit, its important that you begin to analyze this unit, bearing in mind how you shall split the concept from the lower primary to the upper primary.

Let's begin with analysis in lower primary P1 – P4. In primary 1 -4, you have to introduce pupils to form number patterns using basic operations, identify even and odd numbers, find multiples using various methods.

Find the lowest common multiple (L.C.M) of numbers less than 10.

You should help pupils to distinguish between factors and multiples of numbers.

In upper primary P5 – 7, do more do more calculations on numbers using valid short cut methods. Use the knowledge of divisibility tests of numbers by 2, 3, 4, 5, and 10 in quick calculations.

Also develop and identify number patterns, include composite numbers, square and square roots of numbers.

For more analysis, refer to the primary school curriculum volume one.

Activity 5.9:

Using the information on unit 5, prepare schemes on the topic number patterns and sequence and select one lesson and prepare a lesson plan. Prepare three lessons from number patterns and sequence to show how you will teach

i) Divisibility concept
ii) Prime numbers and prime factors
iii) Finding Square roots using prime factorization method

5.8: Unit summary

In this unit, you have been introduced to sets and learned about;

i) What is a sequence and therefore number sequence?
ii) What is a number pattern?
iii) Identify the different types of number patterns.
iv) Find the factors and multiples of numbers.
v) What are L.C.M and H.C.F?
vi) Use divisibility test to carry out operation on subtraction.
vii) Solve problems using number pattern and sequence.
viii) Prepare a lesson for teaching number pattern and sequence.

5.9: Glossary

Concept **:**idea underlying a class of things

Digit **:** a symbol representing a number, it also shows a place value

Ascending order **:** an arrangement of a group of numbers in the order of their size, beginning with the smallest number e.g.1,2,5,9,20, etc

Descending order **:** an arrangement of a group of numbers in the order of their size, beginning with the biggest number e.g.250. 132, 9,6 etc

Number sequence **:** numbers that follow each other in a given order e.g. 1,3,4,5,etc

Odd numbers **:** numbers that are not completely divisible by two.

5.10: <u>NUMBER PATTERNS AND SEQUENCES</u>.
<u>ANSWERS</u>

<u>Exercise: 5.5</u> page 152

a) (i) {1} (ii) {1,3,9} (iii) {1,2,4,8,16} (iv) {1,2,3,4,6,8,12,24}
b) (i)M_5 ={5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95}
(ii) = {6,12,18,24,30,36,42,,48,54,60,66,72,78,84,90,96}
(iii) = {10,20,30,40,50,60,70,80,90}

(iv) = **{**25,50,75**}**

Activity 5.6 page 153

(a) (i) {1,2,4,5,10,20} and {1,2,4,5,10,20}
(ii) {1,2,3,4,6,12} and {1,3,6,9,18}
(b) (i) {1,2,5,10} (ii) {1,3,6}

Exercise 5.8 page 154
(a) (i) 2 (ii) 4 (iii) 12 (iv) 5
(b) (i) 24 (ii) 24 (iii) 80 (iv) 100

Activity page 159
i)

Activity 5.8 page 160
1. (a) 2 (b) 4 (c) 5 (d)----- (e) 2 (f) 6
2. (a) 24 (b) 24 (c) 80 (d) 126
3. 170

Activity- page 161
(a) 9 (b) 15 (c) 42 (d)-

5.11: End of unit exercise

This exercise is intended to help you consolidate what you have learnt about in this unit. You are, therefore, advised to read the whole unit before attempting to answer the question.

1. Give steps through which you can introduce number patterns and sequence in primary 4.

UNIT 6: FRACTIONS AND DECIMALS

6.1. Introduction

Dear students, you are welcome to Unit 6. This unit will enable you develop a full understanding of the concept of a fraction as a single number; and also to teach pupils how to use fractions and decimals to solve problems.

6.2. Content organization

Hello student, in this unit, you are going to study the following topics outlined in the table below:

Topic	Details
1. Introduction to fractions	a. Meaning of fractions b. Types of fractions c. Read, write of fractions d. Conversion of fractions (mixed to improper fractions and vice versa)
2. Equivalent fractions	a. Finding equivalent fractions b. Ordering fractions and decimals
3. Operation on fractions	a. Adding fractions b. Subtracting fractions c. Multiplying fractions d. Fractions of quantities e. Dividing fractions f. Application of fractions
4. Decimal fractions	a. Introduction to decimal fractions b. Converting decimal fractions to ordinary fractions (vice versa)
5. rational and irrational fractions	a. Conversion of ordinary fractions to recurring fractions pure repeating decimals mixed repeating decimals. b. Changing recurring decimals to common fractions c. Operation on decimal fractions
6. Teaching fractions and decimals in primary schools	a. Activities and instructional materials for teaching fractions and decimals b. Planning lesson for teaching fractions and decimals.

6.3. Learning Outcome

By the end of this unit, you are expected to teach students how to use fractions and decimals to solve problems.

6.4. Competences

Now that you have known the learning outcome, and you have studied through the unit outcome, you are expected to:

- i Identify equivalent fractions
- ii Put fractions with different denominators in order of size
- iii Change mixed numbers to improper fractions and vice versa
- iv Add fractions together
- v Solve problems requiring the multiplication and division of fractions
- vi Change decimals to fractions and vise versa

vii Add and subtract decimals
viii Multiply and divide decimals
ix Use decimals to solve problems.

6.5. Unit Orientation

This unit is quite long. It was thought that these topics should be thoroughly reviewed as they are so important to the development of later work in mathematics.

6.6. Study Requirements

When studying this unit, you will be required to carry out practical work as it is going to be illustrated. Do a lot of revision on what is going to be covered, allow yourself time to internalize the concepts, graph paper for teaching about mixed numbers, real objects and reasonable old box for illustration.

I wish you all the best while studying this unit!

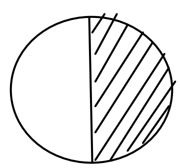
6.7. Content

6.7.1. Introduction to Fractions

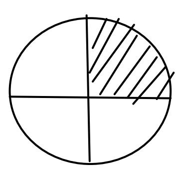
a. Meaning of a fraction

Hullo student! Do you know what a fraction is?

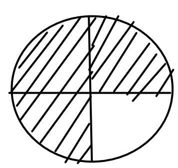
Find out the answer from the following shapes.



This shape has been divided into two equal parts. The shaded part is one-half of the circle. We write this as ½



A quarter (written as ¼) of this circle is shaded



Three quarters (written as ¾) of the circle is shaded

Note that; $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$ are all proper fractions.

The number of the bottom is called the **Denominator**, is the number of equal parts the whole circle has been divided into.

The number at the top is called the **numerator**, is the number of parts in the fraction.

Well done! Let's continue…

1. Types of fractions

i). Proper fractions

We can easily show that $\frac{3}{8}$ is the same as $3 \div 8$

Let you draw a line AB 3 cm. long, and mark a point C in it so that AC = 1 cm as the diagram below. If the whole line is divided into eighths of a cm, and AX is $\frac{3}{8}$ cm, AX is clearly one eighth of AB, for there are 24 small divisions (each $\frac{1}{8}$ cm). In AB, and AX contains three of these divisions, and 3 is one eighth of 24.

$\therefore \frac{3}{8} = 3 \div 8$

Hence $\frac{numerator}{denominator}$ = numerator ÷ denominator

A fraction such as $\frac{3}{8}$, in which numerator is less than denominator, is called a proper **fraction**

Diagram not drawn

ii). Improper fractions

A fraction such as $\frac{11}{8}$ in which the numerator is greater than the denominator, is called **improper fraction.**
The integers, or whole numbers; 1,2,3,4,5,…….. may be replaced by the fractions $\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}$ ……. Where convenient for $\frac{3}{1} = 3 \div 1 = 3$ etc

iii). Mixed fractions

You may discover that $\frac{11}{8} = \frac{8}{8} + \frac{3}{8} = 1 + \frac{3}{8}$ and this one is written simply $1\frac{3}{8}$.

Similar $\frac{23}{8} = \frac{16}{8} + \frac{7}{8} = 2 + \frac{7}{8} = 2\frac{7}{8}$

A number such as $2\frac{7}{8}$ is called a **mixed fraction**, as consists of a whole number 2 and proper fraction $\frac{7}{8}$

(d) Conversion of fractions.

Now you know the different types of fractions. Let you do something more practical.

Study the examples below:

Example 1

Write $\frac{25}{7}$ as a mixed fraction

First divide 25 by 7

$$\begin{array}{r} 3 \\ 7\,\big)\,25 \\ \underline{21} \\ 4 \end{array} \qquad \therefore \frac{25}{7} = 3\frac{4}{7}$$

May try this with your colleague on a sheet of paper: $\frac{241}{72}$

Let you look at example 2

Express $3\frac{6}{7}$ as an improper fraction

$3\frac{6}{7} = \frac{(3 x 7)+6}{7} = \frac{21+6}{7} = \frac{27}{7}$

$\therefore 3\frac{6}{7} = \frac{27}{7}$

In your exercise book do the following work

Activity 6.1

a. Express as mixed fractions

(i) $\frac{78}{16}$ (ii) $\frac{147}{18}$ (iii) $\frac{158}{100}$

b. Change mixed fractions to improper fractions

(i) $12\frac{4}{13}$ (ii) $27\frac{5}{12}$ (iii) $46\frac{3}{17}$

c. Write mixed fractions to improper fractions

(i) a $\frac{3}{4}$ (ii) $5\frac{a}{4}$

Can you compare your answers at the end of this unit.

How are you feeling?

Now let us change these numbers into improper fractions using another method.

Example 3

$5\, ^1\!/_3$

$= 5 + \, ^1\!/_3$

$= \, ^5\!/_1 + \, ^1\!/_3$ looking for the LCM of the denominator i.e. 3

$= \frac{15+1}{3}$ Divide the denominator by the LCM and then multiply each numerator by the result of the division, then add the numerators.

$= \frac{16}{3}$

We can also change improper fractions as mixed fractions. Let's consider this example.

Example 4

$\frac{13}{5}$

$= 13 \div 5 = 2$ remainder 3 / fifths)

$\frac{13}{5} = 2 + \frac{3}{5} = 2\frac{3}{5}$

Note that we are dividing the numerator by the denominator to find the whole number.

Well done!

Can you now use the above information to carry out the activity below?

Activity 6.2

a) What fraction is:

i 3 days of a week
ii 4 hours of a day
iii 30 minutes of an hour
iv 20 boys in a class of 160 students

b) express each of these as improper fractions

i $2\frac{1}{3}$ (ii) $5\frac{1}{3}$

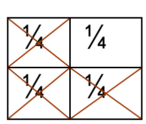
Compare your answers with those at the end of the unit.

6.7.2 Equivalent Fractions

(a) Finding equivalent fractions

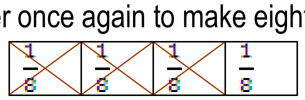
Can you demonstrate equivalent fractions by doing the following?

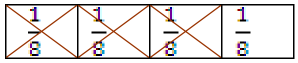
i 1Take a sheet of paper and fold it into four equal parts. Shade $\frac{3}{4}$ of the paper.



Shaded part is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

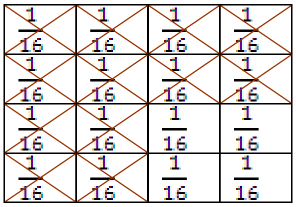
ii Fold the paper once again to make eight equal parts. Now shade $\frac{6}{8}$ of the paper.





Shaded part is $\frac{1}{8} x \frac{1}{8} x \frac{1}{8} x \frac{1}{8}$ x $\frac{1}{8} x \frac{1}{8} = \frac{6}{8}$

Fold further the paper once again to make 16 equal parts. Now shade $^{12}\!/_{16}$



Shaded part is $\frac{1}{16} x \frac{1}{16}\ x \frac{1}{16} x \frac{1}{16}$ x $\frac{1}{16} x \frac{1}{16}\ x \frac{1}{16} x \frac{1}{16}\ x \frac{1}{16} x \frac{1}{16}$ x $\frac{1}{16} x \frac{1}{16} = \frac{12}{16}$

What do you notice about the whole figure and the shaded parts?

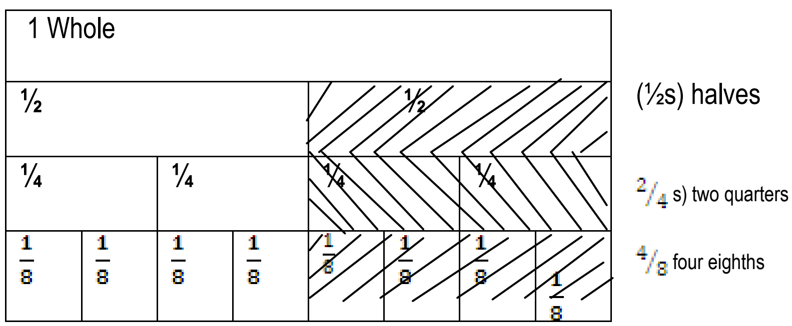
I hope, you will discover that

$\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$

Therefore the above illustrations clearly demonstrate that $^{3}/_{4} = {}^{6}/_{8} = {}^{12}/_{16}$

Therefore the above illustrations clearly demonstrate that $^{3}/_{4}, {}^{6}/_{8}$ and $^{12}/_{16}$ are examples of equivalent fractions.

Study the equivalent chart below



What do you notice about the shaded parts of the above diagram?

Discuss it with your colleague.

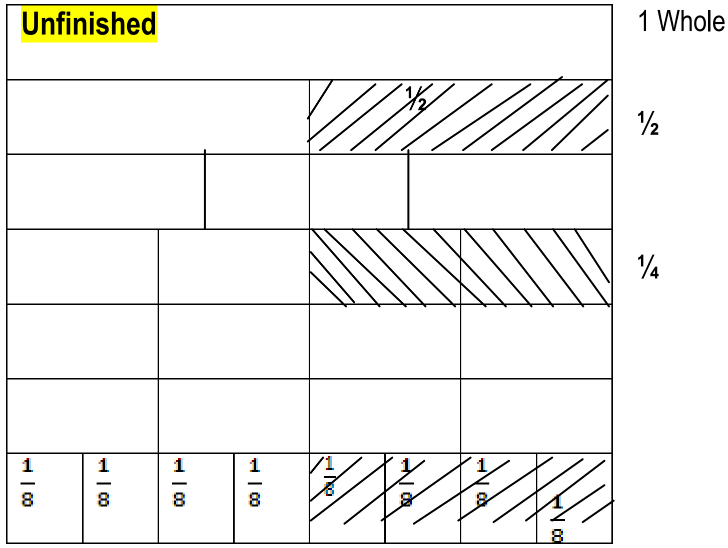
I believe you have found that:

$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$

Then $½ = \frac{2}{4} = \frac{4}{8}$

Still these are referred to as equivalent fractions.

From the above two examples, illustrated, the idea of equivalent fraction, can you use the diagram below to find out more about equivalent fractions.



Look at the above diagram, what have you observed?

Compare your answers with the text below

Note:
The area has remained the same, so ¾ is the same as $^6/_8$ which is also the same as $^{12}/_{16}$. this can be represented as ¾ = $^6/_8$ = $^{12}/_{16}$

The area shaded has remained the same, so $^3/_4$ is the same as $^6/_8$ which is also the same as $^{12}/_{16}$. This can be represented as $^3/_4$ = $^6/_8$

Dear student, it is important the above unaltered illustrations lead you that the value of a fraction is if we divide the numerator and the denominator by the same number or if we multiply the numerator and the dominator by the number.

Therefore, you can conclude that

$$\frac{a}{b} = \frac{a \, x \, n}{b x \, n} \text{ and } \frac{a}{b} = \frac{a \div n}{b \div n}$$

Do the following in your exercise book

Activity 6.3

Using the knowledge you have learnt from above, work out these problems

a. Write two equivalent fractions

i $\frac{4}{5}$ (ii) $\frac{7}{10}$

b. Find the equivalent fractions

i $\frac{5}{8} = \frac{?}{32}$ ii $\frac{18}{48} = \frac{3}{?}$

c. Reduce the fractions in their lowest terms i $\frac{9}{12}$ ii $\frac{39}{52}$ iii $\frac{210}{285}$

b) Ordering fractions

Did you know?

What do you think ordering fractions can be?

Let you consider these:

Five-eights is <u>greater than</u> three-eights. We can write this as $\frac{5}{8} > \frac{3}{8}$.
We can also write one-quarter is <u>less than</u> three-quarters as $\frac{1}{4} < \frac{3}{4}$
To compare fractions with the same denominators, we look to see which fraction has the greatest numerator.

To compare fractions with different denominators, we first of all find the equivalent fractions with the same denominator.

You can use the equivalent diagram as illustrated above to compare your fractions.

Study the examples:

Example 1

Which fraction is greater than the other: $\frac{2}{3}$ or $\frac{3}{4}$?

Finding equivalent fractions for $\frac{2}{3}$ and $\frac{3}{4}$ by using illustrations

i

ii

iii

iv

v

vi

vii

viii

unfinished

Now, present diagrams (iv) and (vii) on the same diagram (viii).

Find out which of the fractions is greater.

You will find out that

$\frac{9}{12} = \frac{3}{4}$ is greater than $\frac{8}{12} = \frac{2}{3}$

$\therefore \frac{3}{4} > \frac{2}{3}$

Alternatively the same fractions can be compared using a number line.

Study the diagram below:

Which of the fractions is greater? $\frac{3}{4}$ and $\frac{2}{3}$

Step 1. Find the LCM of the denominators 4 and 3. As 12

Step 2. Draw a number line and divide it into at least 12 cuts as shown below.

$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ and $\frac{2 \times 4}{4 \times 4} = \frac{8}{12}$

diagram

then, $\frac{9}{12}$ compared $\frac{8}{12}$

since $\frac{9}{12} = \frac{3}{4}$ and $\frac{8}{12} = \frac{2}{3}$

$\therefore \frac{9}{12} > \frac{8}{12} = \frac{3}{4} = \frac{2}{3}$

Activity 6.4

Can use the above illustration to compare the following fractions for more practice

a) Which fraction is greater than the other?

(i) ¼ and $\frac{2}{5}$ (ii) $\frac{5}{8}$ and $\frac{3}{4}$

b) Which fractions is less than the other?

⅓ and ¼

Show your answers to the tutor and also check at the end of the unit.

You can now take a rest.

Now let you study about how to arrange fractions in either ascending or descending order.

Let you study the following example;

Arrange in order of size, beginning with the smallest fractions $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{7}{9}$

Step 1: Finding LCM of denominators

2	4	8	12	9
2	2	4	6	9
2	1	2	3	9
3	1	1	1	9
3	1	1	1	3
	1	1	1	1

LCM = $2^3 \times 3^2$
= 72

Step 2: Expressed with dominator 72, the fractions are $\frac{54}{72}, \frac{45}{72}, \frac{42}{72}, \frac{56}{72}$

Step 3: Comparing the numerators

$$\frac{42, 45, 54, 56}{72}$$

Since $\frac{42}{72} = \frac{7}{12}$

$\frac{45}{72} = \frac{5}{8}$ =

$\frac{54}{72} = \frac{3}{4}$

$\frac{56}{72} = \frac{7}{9}$

∴ The order is $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{7}{9}$

With your colleagues the following discuss activity.

Activity 6.5

a) Copy the following and put > or < between each pair of fractions in order to make a true statement

i. $\frac{1}{4}$ $\frac{1}{2}$
ii. $\frac{3}{4}$ $\frac{2}{3}$
iii. $\frac{5}{6}$ $\frac{3}{7}$
iv. $\frac{2}{3}$ $\frac{3}{4}$
v. $\frac{10}{10}$ $\frac{5}{10}$

b) Express each of these as improper fractions.

i. $2\frac{1}{3}$
ii. $4\frac{1}{3}$
iii. $5\frac{1}{3}$

c) Arrange in order of size, beginning with the smallest fraction

i. $^{1}/_{2}, ^{5}/_{6}, ^{2}/_{3}, ^{7}/_{12},$

ii. $^{4}/_{15}, ^{7}/_{30}, ^{5}/_{21}, ^{3}/_{14},$

iii. $^{5}/_{9}, ^{7}/_{8}, ^{8}/_{9}, ^{3}/_{4},$

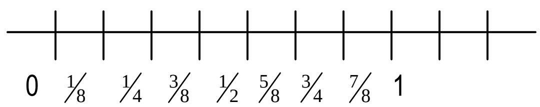
Check for the answers at the end of this unit.

You are doing well!

.

However, dear student remember that,

We can also show fractions on the number line when comparing them. The greater fractions are on the right

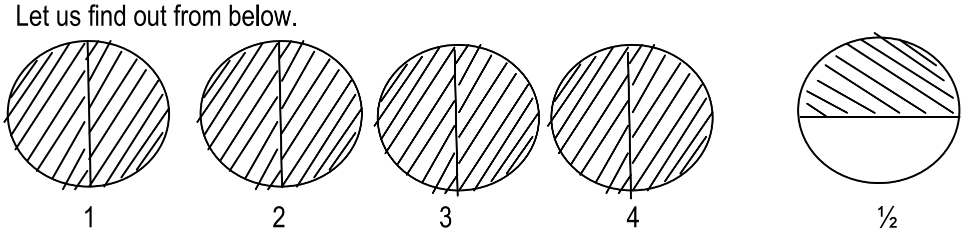


You can now write more fractions on the number line.

iv). Mixed numbers

Dear student, what do you think are mixed numbers?

Let us find out from below.



In the above picture, 4 wholes and $\frac{1}{2}$ are shaded. This can be written as $4\frac{1}{2}$. This means $4 + \frac{1}{2}$.
Also $3\frac{1}{2}$ means $3 + \frac{1}{2}$.

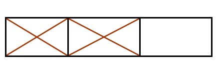
These are the mixed fractions. We can change these fractions into **improper fractions**, in which the numerator is greater than the denominator.

Note: Fractions with the numerator less than the denominator are called **proper fractions** e.g. $\frac{3}{5}$

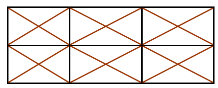
6.7.3. Operation on fractions

i). Addition of fractions

Dear student, how can you add fractions?
You can add fractions with the same denominator quite easily.
Let's find out by doing the following:

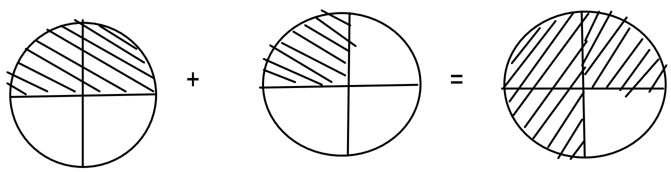


$\frac{4}{6} + \frac{2}{6}$

= 

$= \frac{4+2}{6} = \frac{6}{6}$

Also,



$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

Note: When fractions which are being added do not have the same denominator, you need to find the **common denominator.**

Example 1.

Add: $\frac{1}{3} + \frac{1}{4}$ .We need to find the LCM of 3 and 4 as the new denominator, then express each part of the addition as fractions with this new denominator. Therefore, the LCM of 3 and 4 is 12.

$\frac{1}{3} = \frac{4}{12}$

$\frac{1}{4} = \frac{3}{12}$

$\frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

Now let you practice, in your exercise book do the following activity

Activity 6.6

a) Add

i. $\frac{3}{5} + \frac{3}{5}$

ii. $\frac{2}{3} + \frac{3}{4}$

iii. $3 + \frac{3}{10}$

iv. $4\frac{2}{3} + 1\frac{5}{6}$

v. $\frac{1}{3} + \frac{1}{6} + \frac{2}{3}$

vi. $\frac{3}{20}+\frac{7}{10}+2\frac{1}{5}$

Compare your answers with your colleague.

b) Subtraction of fractions

Welcome to this subtopic!

Dear student, you can subtract fractions when the denominators are the same. We can therefore subtract one fraction from another quite simply. When the denominators are different, we again need to find the common denominator. Let you study the following examples;

i) Subtraction of fractions with the same denominator

Example 1

Method 1 $\frac{3}{5} - \frac{1}{5}$

$= \frac{3-1}{5} \quad = \frac{2}{5}$

Method 2. Number line undrawn

ii) Subtraction of fractions with different denominators.

$\frac{3}{5} - \frac{2}{3}$ The common denominator is 15. Divide the common denominator by each of the denominators and multiply the result by the numerator.

$\frac{3}{5} = \frac{9}{15}$

$\frac{2}{3} = \frac{8}{15}$

$= \frac{9}{15} - \frac{8}{15} \quad = \frac{9-8}{15} \quad = \frac{1}{15}$

iii) Subtraction of mixed fractions.

$2\frac{1}{3} - \frac{3}{5}$

When subtracting mixed numbers, change them to improper fractions

$2\frac{1}{3} \quad = \frac{(2\times3)+1}{3} \quad = \frac{6+1}{3} \quad = \frac{7}{3}$

$\frac{7}{3} - \frac{3}{5}$ The common denominator is 15.

Dividing the common denominator by each of the numerators

$15 \div 3 = 5$

Then $5 \times 7 = 21$, so $\frac{21}{15}$.

Also$15 \div 5 = 3$

Then $3 \times 3 = 9$, so $\frac{9}{15}$.

$^{7}/_{3}$ - $^{3}/_{5}$ = $^{21}/_{15}$ - $^{9}/_{15}$ = $\frac{21-9}{15}$ = $^{12}/_{15}$

You can reduce the fractions to its simplest by dividing the denominator and numerator by 3.

$^{12}/_{15} \div 3$ = $^{4}/_{5}$

Dear student, may you use the following books to practice what you have learnt.

i A New MK Primary Mathematics 2000, Book 7 page 78,
ii Macmillian Primary Mathematics Book 7, page 75 etc

Well done

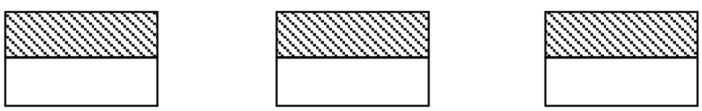
c) Multiplication of fractions

Hello student, in multiplication of fractions, we need to use the following sequence of activities to help the learners get concept..

i) Multiplication of a fraction by a whole number.

Example 1:

3x ½ means three groups of halves



The shaded part of each rectangle represents ½.

∴ The shaded parts make 1 ½.

Method 1

You can use represented addition.

3x ½ = ½ + ½ + ½ = $^{3}/_{2}$= 1 ½

Method 2.

We can get the answer by simply multiplying the numerators alone and the denominators alone in order to get the new fraction.

Thus:-

3x ½ = $\frac{3x1}{1x2}$ = $^{3}/_{2}$ = 1 ½

ii) Multiplication of a fraction by a whole number.

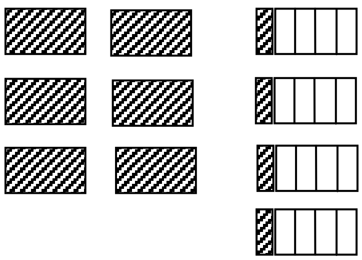
½ of 6 means what is a half of 6 objects.

= 3

(iii) Multiplication of a whole number by a mixed fraction.

4x 2 $^{1}/_{5}$ can be written as 2 $^{1}/_{5}$+ 2 $^{1}/_{5}$ + 2 $^{1}/_{5}$ = 8 $^{4}/_{5}$

The same problem can be illustrated using a teaching aid as shown below:-



= 8 $^{4}/_{5}$

You can as well use the quick way of multiplication.

4x 2 ½ = 4 x $^{11}/_{5}$

$= \frac{4x11}{5}$

$\frac{44}{5}$

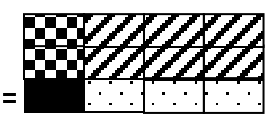
$= 8\frac{4}{5}$

iv) Multiplication of a fraction by a fraction.

Example 2:

Diagrammatic illustration.

(i) Multiply $^{2}/_{3} \times ^{1}/_{4}$ =



$^{2}/_{3} \times ^{1}/_{4} = ^{2}/_{12} = ^{1}/_{6}$

(ii) $^{2}/_{3} \times ^{1}/_{4}$

$= \frac{2\times1}{3\times4}$ $= ^{2}/_{12}$ simplifying the fraction, divide the denominator and numerator by 2.

$= ^{2}/_{12} \div 2 = ^{1}/_{6}$

Therefore, $^{2}/_{3} \times ^{1}/_{4} = ^{2}/_{12} = ^{1}/_{6}$

v) Multiplication of mixed fractions

a) $2\,^{1}/_{2} \times 3\,^{1}/_{3}$

Change the fractions into improper fractions

$2\frac{1}{2} \quad = \frac{(2\times2)+1}{2} \quad = \frac{4+1}{2} \quad = \frac{5}{2}$

$3\frac{1}{3} \quad = \frac{(3\times3)+1}{3} \quad = \frac{9+1}{3} \quad = \frac{10}{3}$

$\frac{5}{2} \times \frac{10}{3}$ We can cancel before we multiply.

$= \frac{5}{1} \times \frac{5}{3} \quad = \frac{5\times5}{1\times3} \quad = \frac{25}{3} \quad = 8\frac{1}{3}$

Let you continue looking at the next sub topic.

d) Fractions of quantities

Dear student, what are fractions of quantities?
Find out by doing this problem.

Share 6 oranges among five boys.
How many oranges does each boy get?

Compare your answer with the one below

$20 \div 5 = 4$

Therefore, each boy gets four oranges.
We can also write it as $6\times \frac{1}{5} = 4$

The other ways of representing fractions in quantities are:

Example:

a) What is $\frac{3}{5}$ of 10 balls?

$\frac{3}{5} \times 10$ cancel the denominator with the whole number.

$= \frac{3}{5} \times 10 \quad = 3\times2 \quad = 6$

b) $\frac{3}{4}$ of 6 Km

$\frac{3}{4} \times 20$ cancel denominator with the whole number.

$\frac{3}{4} \times 20 \quad = 3\times5 \quad = 15$Km

Let you have more practice using the above examples.

Activity 6.7

a) Simplify

i $\frac{3}{8} \text{ x } \frac{7}{10}$

ii $2\frac{1}{2} \text{ x } 1\frac{1}{8}$

iii $\frac{61}{27} \times \frac{27}{61}$

iv $(5 \times \frac{3}{5}) + (5 \times \frac{4}{5})$

v $\frac{2\frac{1}{8} \times 1\frac{4}{7} + \frac{1}{8}}{2\frac{1}{8} - \frac{1}{21}}$

b) Evaluate

i $\frac{3}{5}$ of $(1\frac{1}{5} \times 1\frac{1}{4})$

c) What is ¼ of $\frac{2}{3}$

Compare your answers with your classmates and submit your work to the tutor.

Thanks for doing the above exercise for a short time.

e) Division of fractions

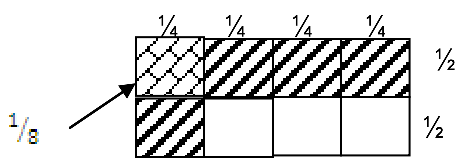
Dear student, division of fractions, can be illustrated diagrammatically as shown below.

The following directions may guide you to obtain the answer.

i Dividing of fraction by a whole number.

Example 1

¼ ÷ 2



Step 1: Divide the unit on one side into 4 parts so that each part is ¼
Step 2: Mark one ¼
Step 3: Divide the unit on the other side into two parts.
Step 4: Mark ½ by changing the markings.
Step 5: What fraction of the unit is doubly marked

Let us find out.

When you multiply 4 by ¼ ? The answer is 1. We say that 4 is the reciprocal of ¼ . We can also say that ¼ is the reciprocal of 4. When you multiply a number by its reciprocal, the answer is always 1.

Example 2:

a) Work out $10 \div 2$

$= 10 \times (\frac{1}{2})$, $\frac{1}{2}$ is the reciprocal of 2
Cross out 2 with 10, the answer is 5
$= 10 \times (\frac{1}{2}) = 5$

b) $8 \div 3$

$= 8 \times (\frac{1}{3})$, $\frac{1}{3}$ is the reciprocal of 3.
$= 8 \times \frac{1}{3} = \frac{8}{3} \quad = 3\frac{1}{3}$

2. You should be able to discover that division of a fraction by a whole number is the same as the multiplication of a fraction by the reciprocal of the whole number:-

$\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Example 3:

Divide $\frac{3}{4} \div 5$,

Obviously $\frac{3}{4} \div 5$ is the same as $\frac{1}{5}$ of $\frac{3}{4}$, which we have seen, is $\frac{1 \times 3}{5 \times 4} = \frac{3}{20}$

You can notice that 5 is $\frac{5}{1}$, and divide by $\frac{5}{1}$ we multiply by $\frac{1}{5}$
Then, let you consider $\frac{3}{4} \div \frac{5}{7}$ or $\frac{\frac{3}{4}}{\frac{5}{7}}$

This is $\frac{\frac{3}{4} \times 7 \times 4}{\frac{5}{7} \times 7 \times 4} = \frac{21}{20}$

But this is just the same answer as we should get if we multiply $\frac{3}{4}$ by $\frac{7}{5}$. Hence to divide by $\frac{5}{7}$, we have to multiply by $\frac{7}{5}$. This rule of division of two fractions is thus;

Turn the divisor the other way up and multiply as shown examples 1, 3 and 4.

Example 4.

Simplify $3\frac{1}{4} \div \frac{1}{6}$
$= \frac{13}{4} \div \frac{1}{6}$

$= \frac{13 \times \cancel{6}^{3}}{{}_{2}\cancel{4} \times 1}$

Can you try this activity in your exercise book?

✍ **Activity 6.8**

1. Add
   a. $\frac{2}{9} + \frac{4}{9}$
   b. $\frac{2}{3} + \frac{1}{6}$
   c. $2\frac{1}{4} + 3\frac{3}{4}$

2. Multiply
   a. $\frac{4}{3} \times \frac{1}{4}$
   b. $2\frac{1}{2} \times 3\frac{3}{4}$

3. Work out the following
   a. $\frac{1}{4}$ of 360
   b. $\frac{2}{3} \times 27$
   c. $\frac{1}{3}$ of 30

4. Divide the following fractions
   a. $6 \div \frac{1}{3}$
   b. $4\frac{1}{3} \div 1\frac{5}{8}$
   c. $3\frac{1}{2} \div 1\frac{1}{2}$

5. Simplify

   a. $\dfrac{3\frac{1}{3}}{\frac{5}{9}}$

   b. $\frac{8}{25} \div 8$

(6) if $\frac{3}{5}$ of a school are boys, and they are 96 girls in the school, how many children are there all together?

You can now compare your answer with the one given at the end of the unit.

Well done!

Before you continue, may you read the text below to take note;

The reciprocal of a number is 1 divided by that number. That is the reciprocal of 3 is $\frac{1}{3}$, that of $\frac{1}{3}$ is 3, that of $\frac{2}{3}$ is 1 ÷ $\frac{2}{3}$ or 1x $\frac{3}{2}$ which is $\frac{3}{2}$

We now see that dividing by fractions is the same as multiplying by the reciprocal.

Let you proceed.

(f) Application of fractions

Dear student, in solving numbers dealing with mixed operations with fractions, you need to use the knowledge of BODMAS which you acquired when dealing with operation on numbers.

Example1:

Simplify ($^1/_3$ + ½) x ¼
Step 1: Deal first with the ones in the brackets
Solution:- ($^1/_3$ + ½) x ¼

B	rackets	( )
O	f	of
D	ivision	÷
M	ultiplication	x
A	ddition	+
S	ubtraction	-

$\left(\frac{2+3}{6}\right)$ x 4

=$\left(\frac{2+3}{6}\right)$ x 4

= $^5/_6$ x ¼

= $^5/_{24}$

Example II.

$^5/_6$ - ¾ ÷ 1 ½

Rename the mixed fraction as an improper fraction.

= $^5/_6$ - ¾ ÷ $^3/_2$

= $^5/_6$ - ¾ x $^3/_2$ work at division

= $^5/_6$ - $^1/_2$

= $\frac{5-3}{6}$

= $^2/_6$ = $^1/_3$

Examples 3:

If $\frac{3}{5}$ of a school are boys and their 96 girls in the school, how many children are there altogether?

Since $\frac{3}{5}$ of the school are boys, $\frac{2}{5}$ of the school/1 are girls.

∴ $\frac{2}{5}$ of the school = 96

Then, $\frac{1}{5}$ of the school = $\frac{96}{2}$

$\therefore \frac{5}{5}$ of the school = $\frac{96}{2}$ x 5

The total number of = 240 children.

Now since you are well versed within the above example can you do the following activity?

Activity 6.9

a) Evaluate:

i $\frac{3}{5} + \frac{1}{3} \div \frac{2}{3}$

ii $\frac{7}{12} - \frac{1}{2}$ of $\frac{1}{3}$

iii $(\frac{2}{5} \times \frac{1}{7}) \div (\frac{1}{5} - \frac{1}{8})$

iv $\frac{2}{3} \times (\frac{1}{4} - \frac{1}{12}) \div \frac{1}{5}$

b) A plumber cut off 4 ¼ metres from a pipe 12 $\frac{3}{8}$ metres long. How much was left?

c) Simplify

i $\frac{1}{x} + \frac{1}{2x}$

ii $\frac{y}{y} + \frac{x}{q}$

d) Find the area of a rectangle $3\frac{2}{3}$ cm long and $2\frac{5}{6}$ wide.

Can you compare your answers with those of your colleagues?

Well do.

You can drink a bottle of water.

6.7.4. (a) Introduction to decimal fraction

Decimal Fractions

Dear student, you are welcome to this topic.
Do you remember in the counting system, the use of place values?
1 2 3 4 meant 1 thousand, 2 hundreds, 4 tens and 3 ones. The place of these digit numbers is important.

<u>Thousands</u> <u>Hundreds</u> <u>Tens</u> <u>Ones</u>

1	2	4	3
5	0	0	0 ⟶ five thousand
3	0	2	0 ⟶ three thousand and twenty

Also remember that place values are used for numbers less than 1.

Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousands
		4	•	8		
		4	•	8	5	
		4	•	8	5	6

The numbers on the left of the decimal point are whole numbers and those on the right of the decimal point are **decimal fractions.**

Therefore, 4.8 means 4+ $^8/_{10}$

4.85 means 4+ $^8/_{10}$ + $^5/_{100}$

4.856 means 4+ $^8/_{10}$ + $^5/_{100}$ + $^6/_{1000}$

Well done! Let's continue…

Decimal and fractions

Check this:

156.42 is a **decimal number,** 156 is a whole number, and the .42 is a decimal fraction.

Note: when the number has no whole number part, we usually put a 0 in front and write it as 0.42, 0.8 instead of .42, .8.

This is because the point can easily be unnoticed.

Dear student, you can change any decimal fraction into an ordinary fraction.

(b) Converting decimal fractions to ordinary fractions

Note this:

A number with one decimal place is a fraction with denominator10 e.g. 0.5 = $^5/_{10}$

A number with two decimal places is a fraction with denominator 100 e.g. 0.45 = $^{45}/_{100}$

Also a number with three decimal places is a fraction with denominator 1000

e.g. 0.623 = $^{623}/_{1000}$

0.008 = $^8/_{1000}$

The number of zeros is the same as the number of places of decimals e.g.

0.468293 = $^{468293}/_{1000000}$ (six decimal places).

6.7.5 RATIONAL AND IRRATIONAL FRACTIONS.

(a) Converting ordinary fractions to recurring decimal fractions

In order to understand the concept of recurring decimals you need to review the changing fractions to decimals.

In changing to decimals, you got:

$$\begin{array}{r} 0.8 \\ 5\overline{)40} \\ \underline{40} \\ \underline{--} \end{array}$$

$\therefore\ {}^{4}/_{5} = 0.8.$

The division carried out came to an end

By changing some fractions' to decimals, you can get a fraction whose decimal does not terminate.

Let you study:-

Example1:

Change ${}^{5}/_{9}$ to a decimal fraction.

$$\begin{array}{r} 0.55 \\ 9\overline{)50} \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

$\therefore\ \frac{5}{9} = 0.55$ ------thus 5 repeats without ending

Example II.

Change ${}^{3}/_{11}$ to a decimal fraction

$$\begin{array}{r} 0.2727 \\ 11\overline{)30} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 8 \end{array}$$

$\therefore\ \frac{3}{11}$ = 0.272……… (2 and 7 repeat without ending)

Repeating decimals are grouped into two groups:-

i **Pure repeating decimals**

Those have their digits recurring

Example 1:

0.33…….. (3 repeats)
0.2121 …….. (2 and 1 repeat)
0.123123……… (1,2 and 3 repeat)

ii Mixed repeating decimals

These have some digits which do not recur while other recurs.

Example 2:

0,122…… (only 2 repeats)
0.255……. (only five repeats)
0.1233…. (only 3 repeats)

Note: these are three different ways of representing recurring decimals

(i) 0.33….
0.2727…..
0.123…….

Using three dots means and so on

(ii) $0.\overline{3}$
$0.\overline{27}$
$0.\overline{123}$

a bar is put over all the recurring digits

(iii) $0.\dot{3}$
$0.\dot{2}\dot{7}$
$0.\dot{1}2\dot{3}$

a dot is put on the first and the last digit in the decimal

b) Changing recurring decimals to common fractions

Example 1:

Change 0.44…… to a common fraction.
Let x = 0.44----------

Then, 10x = 4.44-------
10x – x = 4.44------
-0. 44-----

9x = 4.00

$\frac{9x}{9} = \frac{4}{9}$

$= \frac{4}{9}$

$\therefore$ 0.44-------= $\frac{4}{9}$

With your colleague look at:

Example 2.

Express 6.73636------- into a fraction.

If r = 6.73636-----------

Then 100r = 673.636---------

100r – r = 673.636--------

- 6.736---------

99r = 666.900

$\frac{99r}{99} = \frac{666.9}{99}$

r = $\frac{666.9 \ x\ 10}{99.x\ 10}$

r = $\frac{666.9}{990}$

$\therefore$ 6.73636-------= $\frac{6669}{990}$

= $6\frac{81}{110}$

You can use the above information to answer the activity below.

Activity 6.10

1. Change these decimal fractions into ordinary fractions.

   a) 0.25
   b) 0.20
   c) 3.45

2. Change the recurring decimals to ordinary fractions

   a. 0.55…………..
   b. 0.1515……….
   c. 0.261261……
   d. 0.1666………

Compare your answers with your colleague and then check at the end of the unit.

For more practice:-

With your classmates, get school Mathematics of East Africa Book 2, page 175, Do exercise B or Primary School Mathematics Book 6 pages 22, Exercise 3.

Give your work to the tutor.

Bravo!

May you relax a bit.

c) Operation on decimal fractions

i). Adding decimal fractions

Dear student, adding decimal fractions is just like whole numbers, as long as you keep the decimal points lined up.

Example:

Add

$$\begin{array}{r} 234.921 \\ +\ 152.324 \\ \hline 387.245 \\ \hline \end{array}$$

Add 0.4 + 28 + 5.78

$$= \begin{array}{r} 0.40 \\ 28.00 \\ +\ 5.78 \\ \hline 34.18 \\ \hline \end{array}$$

e) Subtracting decimal fractions.

Example 1:

Subtract

$$\begin{array}{r} 567.923 \\ -\ 321.412 \\ \hline 246.511 \\ \hline \end{array}$$

Example 2:

Subtract 19.8 from 36.4

$$= \begin{array}{r} 36.4 \\ -\ 19.8 \\ \hline 16.6 \\ \hline \end{array}$$

> **Note:**
> It is important to keep the place value or the decimal points lined up.

With the above information, work out the activity.

✍ **Activity 6.11**

1. Work out the following:
    a) 0.8+0.3
    b) 0.04+2.3
    c) 0.5+25+2.45
    d) 4.25-0.45
    e) 66-12.8-15.9
    f) 23.8+0.24+1.09
    g) 100-28.6

2. 3.75m of cloth is cut from 2.2m long. How much cloth is left?

3. To make some bread, Musa needs 4.5kg of sugar and 2.5kg of flour. How much does Musa need altogether?

You may compare your answers with a friend and the one given at the end of this unit.

You are wished good luck.

ii). Multiplying Decimals

Hello student, you can multiply decimal numbers in the same way as multiplying whole numbers, but only take care of the decimal point.

(i) Multiplication by 10, 100 and 1000

To multiply by 10, you put down a zero, then shift all the other numbers one to the left. We can set it out like this.

$$\begin{array}{r} 35 \\ \times\ 10 \\ \hline 350 \\ \hline \end{array}$$

Let's also try: 2.4×10

$$= \begin{array}{r} 2.4 \\ \times\ 10 \\ \hline 24.0 \\ \hline \end{array}$$

We have kept the decimal point in the same place but shifted all the digits along by one place.

To multiply by 100, we put down the two zeros then shift all the other numbers two places to the left.

Example:

62×100

$$= \begin{array}{r} 62 \\ \times\ 100 \\ \hline 6200 \\ \hline \end{array}$$

Also 4.5×100

$$= \begin{array}{r} 4.5 \\ \times\ 100 \\ \hline 450.0 \\ \hline \end{array}$$

> **Note:**
> To multiply by 10 or by 100, we keep the decimal point still and move the digits one or two places to the left.

(ii) Multiplying two decimal numbers

Dear student, we can use the long multiplication process, ignoring the decimal points and then after counting back the positions of the decimal where it should be placed.

Check this out!

Multiply 34.20×6.82

$$= \begin{array}{r} 3420 \\ \times\ 682 \\ \hline 6840 \\ 273600 \\ +\ 652000 \\ \hline 2332440 \\ \hline \end{array}$$ *(Long multiplication process ignoring the decimal points)*

Considering the numbers to be multiplied, 34.20 has got two decimal places and also 6.82 has got two decimal places, so there are four decimal places altogether.

Now, you should count four digits from the right to the left and place the decimal point.

Therefore, $34.20 \times 6.82 = 233.2440$

Also multiply 1.5×8.2

$$= \begin{array}{r} 15 \\ \times\ 82 \\ \hline 30 \\ +1200 \\ \hline 1230 \\ \hline \end{array}$$ *(Using long multiplication process, ignore the decimal points)*

Therefore, 1.5 and 8.2 have got one decimal place each, making a total of two decimal places. Then counting back, indicate the positions of the decimal place from the right to the left.

$1.5 \times 8.2 = 12.30$

Well done! I hope you have understood.

You may now use the above information o workout the activity below.

✍ Activity 6.12

1.
   a) 8.32×5.2
   b) 4.62×58
   c) 0.28×5
   d) 9.8×6.75
   e) 960×1.2

2. A kilo of posho costs 10.6 cents; I want to buy 120kgs of posho. How much will I pay for it?

You can now cross check your answers with a friend and the one given at the end of the unit.

Thank you for the good work done.

(g) Dividing decimals

Dear student, study the example given below.

Example.

Divide 36.48 by 1.2

$36.48 = 36 + \frac{48}{100} \quad = \frac{3600+48}{100} \quad = \frac{3648}{100}$

$1.2 = 1 + \frac{2}{10} \quad = \frac{10+2}{10} \quad = \frac{10}{12}$

Therefore, $36.48 \div 1.2 \quad = \frac{3648}{100} \div \frac{12}{10}$

The reciprocal of $(\frac{12}{10}) = \frac{10}{12}$

$= \frac{3648}{100} \times \frac{10}{12} \quad = \frac{304}{100}$

$= 30.4$

Use the example studied above to discuss the activity below.

Activity 6.13

Now using the above example, work out the following:

i $45.5 \div 1.5$
ii $17.6 \div 1.6$
iii $219.2 \div 4$
iv $21.6 \div 9$

You may cross check your answer with the one at the end of the unit.

For further problems for practice you will find them in module two unit 16.

At the moment congratulations!

Let you look at the next topic of this unit.

6.7.5 Teaching fraction and decimal in primary school

i). Activities and instructional materials for teaching fractions and decimals.

Dear student, in teaching this unit, you have got an obligation to mobilize a lot of learning materials and activities. The teaching of this unit requires a practical approach which will give your learners proper understanding of this unit.

The learners should be involved in the learning process, asking them questions which require their previous experience will give them an upper hand in participating during the lesson.

Activities like finding out the number of students in the class, thereafter find the number of girls in the same class. The number of girls can be expressed as a fraction of the total number of students of that class. E.g. There are60 students, 150 students are girls and 50 students are boys. These numbers can be represented as fractions.

Fraction of girls will be $^{150}/_{200}$ = $^{3}/_{4}$ of the class
Then the boys will be $^{50}/_{200}$ = $^{1}/_{4}$.

In this way the students will be able to understand the concept of a fraction of a whole. Instructional materials like textbooks, writing books, pens or pencils should be available as this will enable the learners to practice more of the have learnt.

ii). Planning lesson of teaching fractions and decimals

Hello student, when you are to plan to teach this unit in ordinary school, it's always better to read and have enough knowledge about the unit outline. Consult various textbooks and other instructional materials to help you obtain an approach to proper teaching of fractions and decimals.

It is also observed that many pupils consider fractions and decimals not an easy going unit. It is therefore your responsibility to simplify the subject matter by teaching from known to unknown or teaching from simple to hard subtopics. Organization of the subtopics becomes very important in this case.

Also it is advisable to have enough time for yourself and the pupils to understand what you have taught. This can be done through vigorous exercises given to the learners so that they develop the concept that you intend to be achieved by the end of the lesson.

Class control should be maintained as learning takes place in an environment that is conducive.

6.8 UNIT SUMMARY

In this unit you have been introduced to fractions and decimals and learnt about:-

(i) Meaning of fractions
(ii) Types of fractions.
(iii) Adding fractions
(iv) Subtracting fractions
(v) Multiplying fractions
(vi) Fractions of quantities
(vii) Dividing fractions
(viii) Application of fractions
(ix) Meaning of a decimal fractions
(x) Operation on decimal fractions
(xi) Activity and instructional materials for teaching fractions.
(xii) Planning lessons for teaching fractions and decimals

6.9 GLOSSARY

Numerator - a part of equal parts taken from a whole

Denominator - shows the total number of equal parts a whole has been cut into. It gives the name of the fraction

Reciprocal - it is the value given by dividing 1 by that number, or dividing that number into 1.

Decimal fraction - a fraction expressed as a part of ten. It is written using a decimal point after the place of a unit

6.10 Answers to activities

Activity 6.1

a) (i) $4\frac{7}{8}$, (ii) $8\frac{1}{6}$ (ii) $1\frac{27}{50}$

b) (i) $\frac{160}{13}$ (ii) $\frac{329}{12}$ (iii) $\frac{785}{17}$

c) (i) $\frac{49+3}{4}$ (ii) $\frac{20+9}{4}$

Activity 2

a) (i) $\frac{3}{7}$ (ii) $\frac{1}{6}$ (iii) ½ (iv) $\frac{1}{8}$

b) (i) $\frac{7}{3}$ (ii) $\frac{16}{3}$

Activity 6.3

a) (i) $\frac{4}{5} = \frac{8}{10}, \frac{12}{15}$ (ii) $\frac{7}{10}, = \frac{14}{20}, \frac{21}{30}$
b) (i) $\frac{20}{32}$ (ii) $\frac{3}{8}$

c) (i) $\frac{3}{4}$ (ii) $\frac{3}{4}$ (iii) $\frac{42}{57}$

Activity 6.4

a) (i) $\frac{2}{5} > \frac{1}{4}$ (ii) $\frac{3}{4} > \frac{5}{8}$

b) $\frac{1}{4} < \frac{1}{3}$

Activity 5

a) (i) $\frac{1}{4} < \frac{1}{2}$ (ii) $\frac{2}{4} > \frac{3}{9}$ (iii) $\frac{5}{6} > \frac{3}{7}$ (iv) $\frac{2}{3} < \frac{3}{4}$ (v) $\frac{10}{10} > \frac{5}{10}$

b) (i) $\frac{7}{3}$ (ii) $\frac{13}{3}$ (iii) $\frac{16}{3}$

c) (i) $\frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{5}{6}$ (ii) $\frac{3}{14}, \frac{7}{30}, \frac{5}{21}, \frac{4}{15}$ (iii) $\frac{5}{9}, \frac{3}{4}, \frac{7}{8}, \frac{8}{9}$

Activity 6.8

a) (i) $\frac{2}{3}$ (ii) $\frac{5}{6}$ (iii) 6

b) (i) $\frac{1}{3}$ (ii) $9\frac{3}{8}$

c) (i) 90 (ii) 18 (iii) 10

d) (i) 18 (ii) 2 (iii) 2

e) (i) 6 (ii) $\frac{1}{25}$

f) 240 children

Activity 6.10

a) (i) $\frac{1}{4}$ (ii) $\frac{1}{5}$ (iii) $3\frac{9}{20}$

b) (i) $\frac{5}{9}$ (ii) $\frac{5}{33}$ (iii) $\frac{29}{111}$ (iv) $\frac{1}{6}$

Activity 6.11

a) (i) 1.1 (ii) 2.34 (iii) 27.95 (iv) 3.8 (v) 3.73 (vi) 27.95 (vii) 71.4

b) 1.55 metres left

c) 7 kg

Activity 6.12

1. a) 8.32 x 5.2 = 43.264
   b) 4.62 x 58 = 267.96
   c) 0.28 x 5 = 1.4
   d) 9.8 x 6.75 = 66.15
   e) 960 x 1.2 = 1152

2. 1 kg of posho costs 10.6 cents

∴ 120 kg of posho will cost 120 x 10.6 cents
= 1272 cents

Activity 6.13

a) 45.5 ÷ 1.5

$\frac{455}{10} \div \frac{15}{10}$

$= \frac{455}{10} \times \frac{10}{15}$

$= \frac{455}{15}$

$= 30\ ^{1}/_{3}$

b) 17.6 ÷ 1.6

$\frac{176}{10} \div \frac{1.6}{10}$

$\frac{176}{10} \times \frac{10}{16}$

$= \frac{176}{16}$

= 11

c) 219.2 ÷ 4

$= \frac{2192}{10} \div 4$

$= \frac{2192}{10} \times \frac{1}{4}$

= 54.8

d) 21.6 ÷ 9

$= \frac{216}{10} \div 9$

$= \frac{216}{10} \times \frac{1}{9}$

= 2.4

6.11 END OF UNIT EXERCISE

1. I have 3 $\frac{1}{3}$ meters of cloth. I give 2 $\frac{1}{2}$ to my brother. How much cloth is left?

2. John has got 25 pigs. He decides to sell a fifth of them.
    a) How many does he sell?
    b) How many has he left after selling them?

3. I can fill 5 cups of flour from a 2.5kg bag of flour. How much flour can go into each cup?

4. Alice and Peter share 6000. Alice gets $\frac{3}{5}$ of the money.
    a) What fraction does Peter get?
    b) How much does Peter get?

5. The price of sugar now is Shs. 2500 per kilogram. A year ago, the price was less by $\frac{1}{5}$ the current price.
    a) Find by how much more the price is this year.
    b) How much was the price of sugar last year?

6. The distance from Mukono to Kampala is 12km. from Mukono to Kireka is three thirds the distance from Mukono to Kampala.
    a) Find the distance from Mukono to Kireka.
    b) What is the distance left from Kireka to Kampala in km?

7. A water tank has a capacity of 1500litres. John decides to fill the tank $\frac{2}{3}$ full.
    a) Find the capacity in litres filled by John.
    b) What fraction of the tank is left unfilled?

8. A farmer earned Shs.650,000 from his harvest, he then distributed his earnings as follows: $\frac{1}{10}$ was given as tithe, $\frac{3}{10}$ was given as school fees and the rest was saved in his account.
    Find:
    a) How much did he give as:
        i). Tithe
        ii). School fees
    b) How much money did he save?
9. Mr. Dick drove for 380 kilometers. After traveling $\frac{2}{3}$ of this distance, he developed a mechanical fault. Find the distance he had covered before getting the fault.

10. I had a jerican of honey. I gave away one fifth of the honey, I remained with6kg. How much honey did I have at first?

6.13. REFERENCES FOR FURTHER READING

1. Anand N. and Peter P.: Mathematics Methods; A Resource Book for Primary School Teachers.

2. Cliff Green. Mathematics Revision and Practice for UCE

End of Unit .

UNIT 7. GRAPHS AND INTERPRETATION

7.1 INTRODUCTIONS

Hullo student you are most welcome to unit 7. This unit will introduce you to some basic strategies for organizing, presenting data graphically, interpreting data and use mathematics to communicate with others through different types of graphs.

7.2 CONTENT ORGANIZATION

Dear student, in this unit you will be covering the following topics as , indicated below;

Topic	Sub topic
1. Introduction to graphs	a) What are graphs? b) Types of graphs and their meanings (i) Pictographs (ii) Bar – graphs and column graphs (iii) Pie charts/pie graphs (iv) Line graphs- travel, temperature graphs (v) Time line graphs
2. Graphs and interpretation	a) Graphs representation b) Graphs interpretation - Scaling - Drawing graphs
3. Coordinate graphs	- Coordinate graphs

7.3 Learning out come

Dear student, at the end of this unit you should be able to;
Manipulate data using graphs and demonstrate teaching graphs in primary schools.

7.4 Competences

You should master the following competences;

(i) Describe graphs
(ii) Name the different types of graphs
(iii) Choose an appropriate scale for any graph
(iv) Draw a graph using a chosen scale
(v) Define data
(vi) Collect and organize data.
(vii) Represent organized data the pictographs, bar graphs/column graphs.
(viii) Interpret different graphs
(ix) Find mode, median and mean
(x) Find range, quartiles and percentiles
(xi) Draw coordinate graphs
(xii) Represent points on a coordinate plane.

(xiii) State the coordinates of a point on a coordinate plane
(xiv) Demonstrate how to teach graphs in a primary school.

7.5 unit orientation

In this unit you will be exposed to different data whereby every part of your life involves data. At college, on TV, on computer, magazine and newspapers, you are presented with data. You also learn how data is easily understood when it is organized and presented visually with a graph. For example you will see how graphs will be used to compare many things and how helpful in analyzing data in many walks of your life. Finally you will be exposed to graphing experiences which will enable you to experience in investigating using real data from your own life.

7.6 Study requirements

Hullo students in order to be successful in this unit you need to collect plenty of materials from your local environment e.g. colored pencils, mathematical sets, graph papers, magazines, tangible records, newspapers etc. At times you will be required to carry out a survey and organize the results geographically. You will be required to revise thoroughly about fractions to proportions and percentages in unit 6 in this module. It might be an added advantage to learn a computer for you being able to make bar, line, circle graphs. You will finally need to have primary mathematics course books and secondary course books 1,2,3 and 4 for further practice.

7.7. Content and activities

7.7.1. INTRODUCTIONTO GRAPHS

Dear student, you are welcome to this unit 7 and this topic in particular.

Imagine you have been studying graphs since primary one. You should revise and acquire more knowledge about it.

a) Meaning of graphs

Hullo student,

What are graphs?

Can you brain storm on it with your colleague?

Read the text below:

A Graph is a diagram showing either the relationship between some variables quantities or the connection that exists between set of points. They are means of expressing information or data in a form of picture and figures

But are you aware that a photograph e.g. your photograph is an example of a graph?

Discuss with your colleague

b) Types of graphs

What types of graphs do you know?

Can you list them on a piece of paper and their main use?

Read the text below:

Types of graphs and their uses

Types of graphs	**Major purpose**
1. Picture graph	Used to compare prices of things like postage, stamps etc
2. Time lines	Shows dates of several occurrences
3. Bar graphs i) Simple bar graph ii) Double/triple bar graph iii) Stacked bar graphs	Represent real life situations
4. Histogram bar graphs	To represent intervals
5. Line graphs	To show changes in two/three quantities
6. Pie charts	Cake graphs or circle graphs

Now you know some types of graphs. Are you able to teach about them?

If yes, that is good, if not study about them carefully.

(i) Picture graphs/Pictographs.

What are the other names for picture graphs? Can you guess?

Let you read the text below:

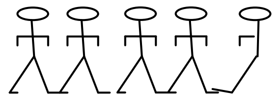
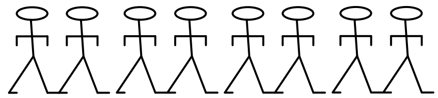
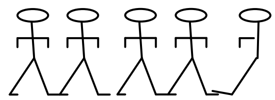
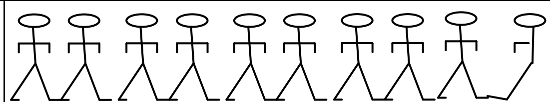
We can represent information which is in number form using pictures. Therefore picture graphs are also known as pictographs or pictograms.

Study the examples below: (1 and 2)

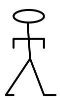
Example 1: Study the picture graph below.

At Kaliro primary teachers' college, during the games competition in 2009, points were scored students were selected from different streams to represent the college team;

Number of students selected to represented the college team.

Steam	Number of students selected
2K	
2 A	
2L	
2I	

Key:



- Stands for 20 students

Use the picture graph to answer the following questions;

a) Which stream had the highest number of students selected?
b) How many students were selected from 2I and 2A respectively?
c) Calculate the average number of students selected.
d) Find the difference between the number of students 2L and 2A.
e) In what ratio of the students were selected from 2K to 2A?

Before you answer any question work out the number of students selected for each stream as shown in the table below:

Study the table:

Stream	**Working**	**No. of students**
2 K	$4\frac{1}{2}$ stds x 20 students per symbol $= \frac{9}{2}$ x 20 = 90	90
2 A	8 stds x 20 students per symbol = 160	160
2 L	$4\frac{1}{2}$ stds x 20 students per symbol	90

	$= \frac{9}{2} \times 20 = 90$	
2 I	9 ½ stds x 20 students per symbol $= \frac{19}{2} \times 20 = 190$	190
4 streams	**90 + 160 + 90 + 190**	**530**

Use the working shown in the above table to derive the following answers;

a) The stream with the highest number of students selected 2I

b) 2I and 2A selected 190 and 90 students respectively.

c) Average of No. of students selected

$$\frac{90+160+90+90}{4}$$

= 132. 5 students

d) Method (1) using the table:

Difference number of students between 2L and 2A= 160 – 90 = 70 students.

Method 2: using symbols: (8 – 4 ½) x 20

$= (8 \times 20) - (\frac{9}{2} \times 20)$

$= 160 - 90$

∴ difference between 2L and 2I = 70 students

e) Ratio = 2K to 2A

= 4 ½: 8

= 4 ½ x 20 : 8 x 20

$= \frac{9}{2} \times 20 : 8 \times 20$

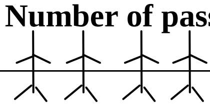
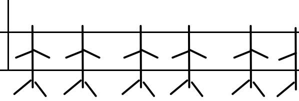
= 90 : 160

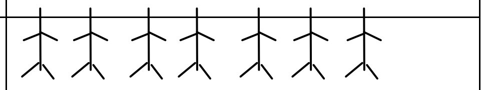
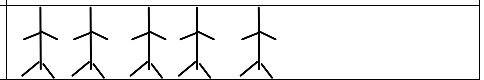
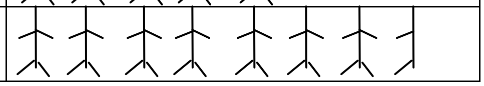
$= \frac{90}{160}$

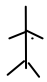
∴ ratio 2K to 2A= 9:16

Example 2: the graph shows the number of passengers that traveled on the five most heavily traveled airlines in 2001.

Top five airlines

Name of Airline	**Number of passengers in 2001**
North west	
USAir	

United	
Delta	
American	

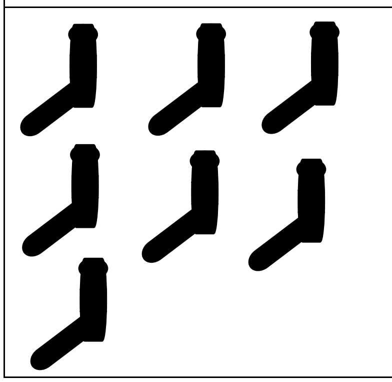
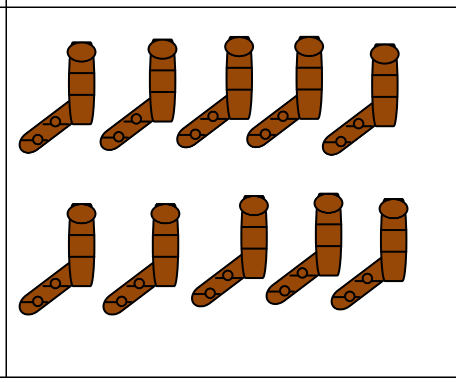
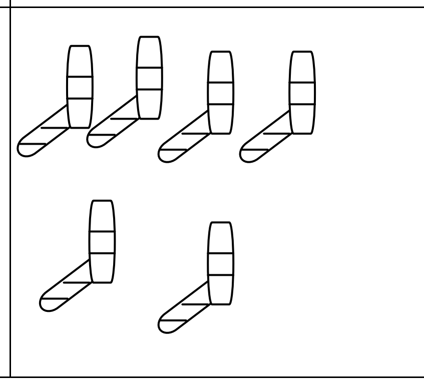
 = 10 million passengers

a) How many passengers does one air plane represent?
b) Estimate the number of passengers that travelled on united.
c) How many more passengers travelled on , American than USAir?
d) If one airplane represented 20 million passengers how would the picture graph change?

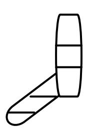
Study the activity and do the exercise in your book

Activity 7.1

1. The pupils in class 2 did a survey of shoes they were wearing. Some wore white shoes, some wore black shoes and some wore brown. They presented their results in the below pictogram.

Black shoes	**Brown shoes**	**White shoes**
		

Note:

 = represents 4 pupils

a) How many pupils wore the brown shoes?
b) How many wore white shoes?
c) How many pupils wore black shoes

2. Given that information was recorded from six schools in Luwero district as follows:

School	**No. of pupils (to nearest 100)**	**School**	**No. of pupils (to nearest 100)**

X	1550	V	1600
Y	1400	U	1200
Z	900	T	1350

Draw the picture graph (given hint: a symbol chosen represents 100 pupils)

In plenary discuss your answers with your colleagues in a plenary.

Read the text below to note some important points:-

1. The graph must have the title, key and the symbol or signs set out neatly in rows.
2. It is very easy to interpret a pictograph.
3. Pictograph is very simple way of showing information.
4. Be careful when to choose a suitable scale.
5. If you are dealing with large numbers you might use a scale of say, one symbol to represent 10, 50 or 100 or 1000 units etc

Do exercise 6A from David Nyakairu, Functional Primary Mathematics for Uganda Pupils book 7. Share your answers with your colleague.

Well done, can you take a walk?

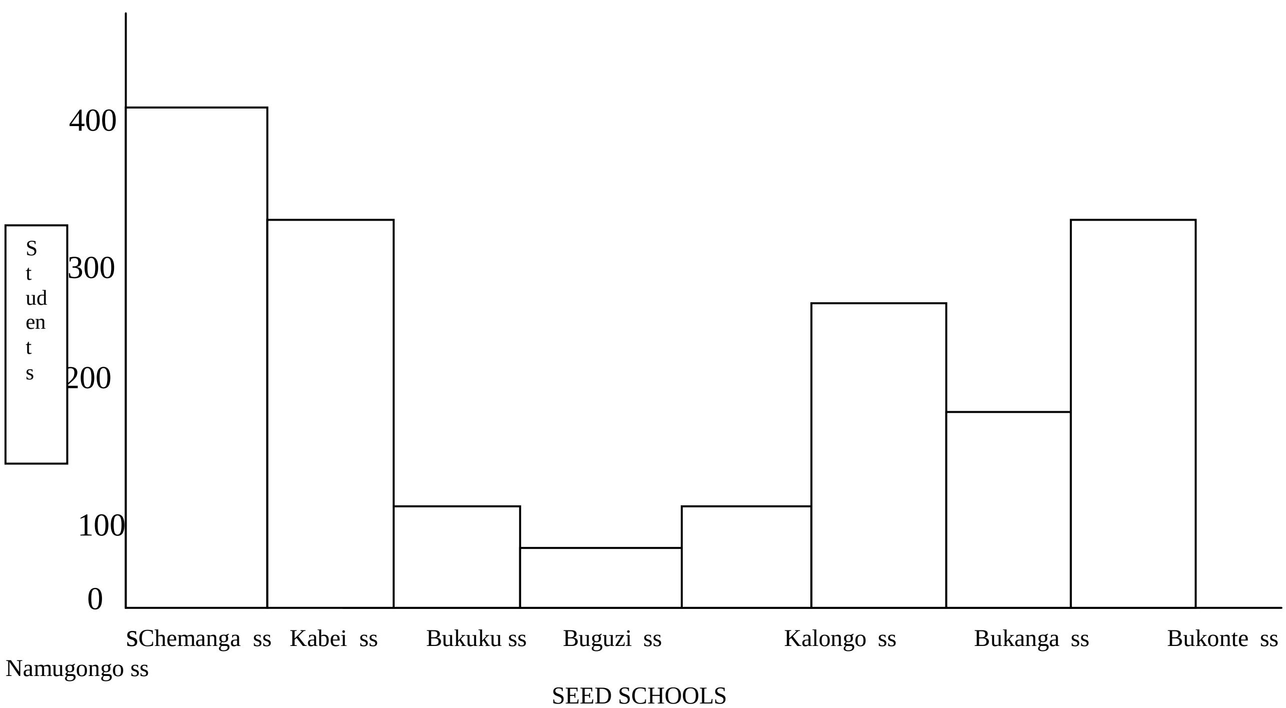
(ii) Bar graphs and column graphs

Before you read about bar graphs you need to have graph paper or graph book or squared papers and a pencil.

(1) Bar graph interpretation

With your colleague can you study the examples carefully.

Example 1: The bar graph showing the number of students attending eight SEED Schools



With your group mates discuss the following questions.

a) (i) Study the graph carefully,
(i) What observations have you made?

b) Which school has the most students?
c) Which school has the least students?

Study the text below, take note of important points and compare your answers

Step 1 (i) Find the scale used on horizontal and vertical axes
(ii) Find the number of students in each school and label on each bar using the original diagram

Step 2: begin to answer the provided questions

a) Observations are to be noted:

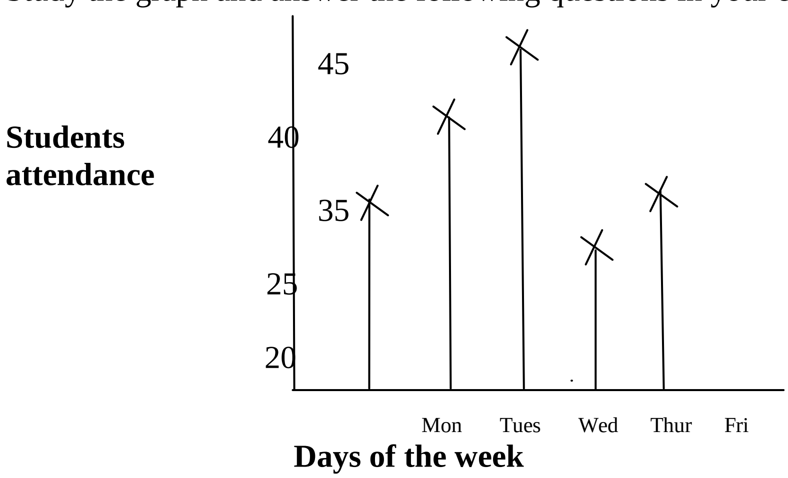
Note:
- Each graph must have a heading or title.
- writing along each axis what quantity is measured along it.
- marked the scales at regular intervals along both axis.

b) The school which has the most students is Chemanga ss
c) The school which has the least number of students is Buguzi ss
d) Schools which have less than 150 students are; Buguzi, Bukuku and Kalongo

Let me imagine you have understood the example 1. If yes, can you study example 2;

2) Column graph

Study the graph and answer the following questions in your exercise book.



Now answer the following questions.

a) What was the graph about?
b) What was the scale on the vertical axis?

c) On which day was the worst attendance?

d) On which day was the best and what could be the reasons?

e) What is the difference between the graph shown in example 1 and 2?

Compare your answers with your colleagues.

Study the steps to be taken and then compare your answers in the table below.

Step 1;

(i) Establish the scale represented on both vertical and horizontal axis.

(ii) Write the value on each column using the original diagram see the example.
Or represent the value of each column in a table.

Day	**Mon**	**Ttue**	**Wed**	**Thur**	**Frid**
No. stds	35	40	45	26	35

a) The graph shows the attendance for a week in a class

b) Scale on vertical each gap/space represents

- Take any two readings e.g. 20 and 25
- Get a difference 25 -20 = 5 students.

∴ scale on vert. 1 gap/space = 5 students

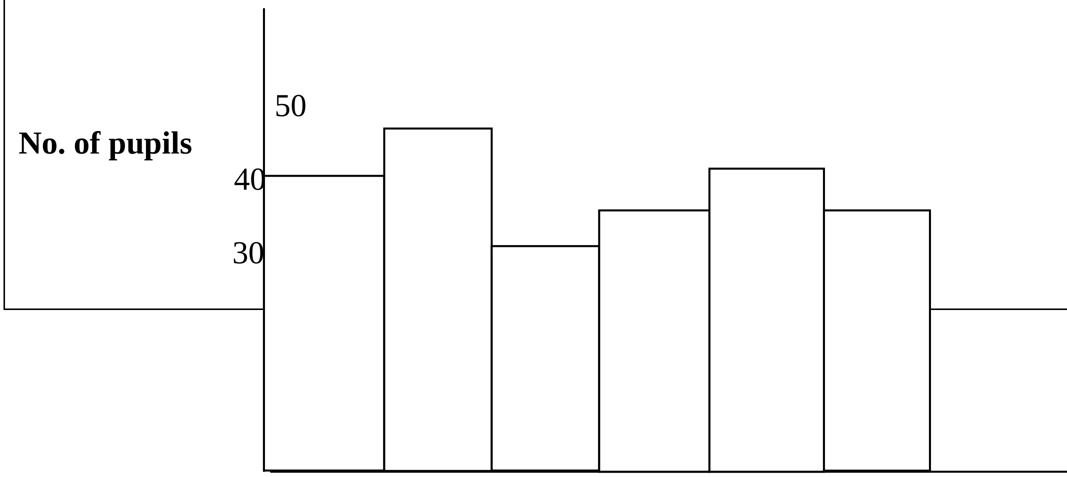
c) The worst day with the least attendance was Thursday (26 stds)

d) The best day with the highest attendance was Wednesday (45 stds)

Activity 7.2

1. Study the graphs and answer the following questions

a. Primary 7 intake in Apio's primary school



20

10

0

2005 2006 2007 2008 2009 2010

year

(i) In which year was the P.7 intake highest?
(ii) In which year was it lowest?
(iii) Make a table to show the yearly intake.

b. Mr. Okello poultry sales
graph undrawn

i. What does one square on the vertical axis represent?
ii. On which day did Okello sale most birds?
iii. On which day was the fewest birds sold?
iv. How many birds were sold altogether in the week?

Check your answers with the ones at the end of the unit.

ii) Drawing bar graph

Now you know the meaning of a bar graph and a column graph, you have also discussed how to interpret graphs.

Do you know how to draw a bar graph and a column graph?

Learn more about drawing of bar and column graphs.

Study example 1:

Given that a boy obtained the following marks as shown in the table below;

Day	**Mon**	**Ttue**	**Wed**	**Thur**	**Frid**
Marks	11	14	18	12	17

Choose your own scale and draw a bar graph

Can you follow those steps laid in the table below and take note;

Step ;

1. Measure your graph paper and its height and width
2. Choose a scale which is convenient for plotting the points and reading the values on the graph.

3. let the 5 days of the week be shown on the horizontal axis using a scale your have chosen e.g. 1 cm = 1 day or ½ cm = 1 day Or 10 small squares = 1 day or 20 small squares = 1 day
4. Let the marks be shown along the vertical axis. Consider the highest mark 13, 18 and lowest mark is 11. Remember to choose a scale which can be easily divided say by 2 e.g. 1 cm = 4 marks on 1 cm = 2 marks or 10 squares = 4 marks etc.
5. Draw the graph on your graph paper
6. Think of a title of the graph e.g. "Bar graph showing marks obtained in 5 days of the week by a boy".
7. Draw the horizontal and vertical axes and mark them.
8. Clearly label them e.g. horizontal axis – days in a week, vertical axis – marks obtained.

Now draw the bar graph on your graph paper.

I hope now you are capable of drawing either a bar graph or column graph using the guidelines given in the above table. With your colleagues, do the following activity: Compare your graph with the one below.

Graph undrawn

Activity 7.3

Choose your own scales and draw bar graphs for the following;

1. a) The PLE results of a school are shown in this table;

Year of exams	**2004**	**2005**	**2006**	**2007**	**2008**
Percentage results	40	46	50	56	48

a) The traffic accidents in a town are shown in this table;

Month	**Jan**	**Feb**	**Mar**	**April**
Number of accidents	36	24	30	35

2. Choose your own scales and draw column graphs for the following:

a) The imports of a country are

Items	**Motorcar**	**Petrol, oil**	**Machinery**	**Clothes**	**Electrical goods**	**Stationary**	**Others**
%	10	15	20	10	8	7	30

b) The number of people at college concert were;

Day	Wednesday	Thursday	Friday	Saturday

People	450	480	560	600

You can now cross check your answers with the ones at the end of the unit.

Before you move to another type of graph, read the information in the table below:

Note that:

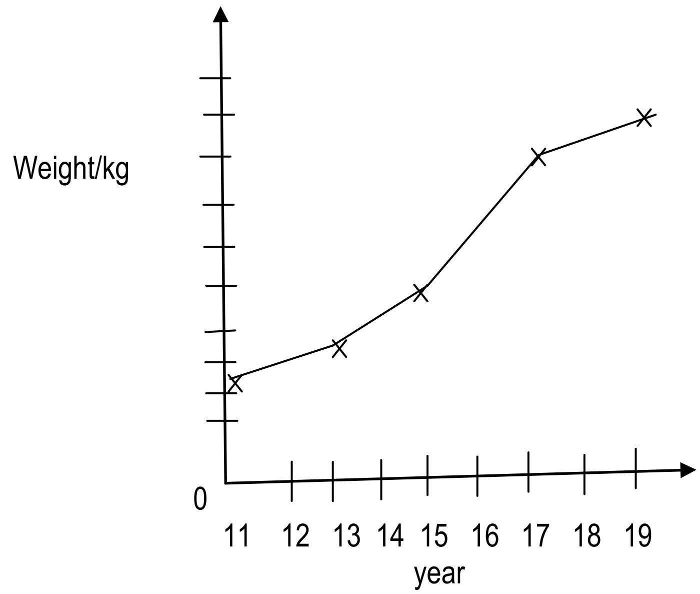
A bar graph at times is called a bar chart. A bar graph must have a heading (title), scale and the bars must have all equal in width

Thank you for going through pictographs, bar graphs and column graphs successfully
Can compare your with these in the table below.

1. 100 pupils 2. a) 19800 sq m, b) 6490 sq m, c) sh. 170,000/= 3. (a) Shs. 42500/= (b) shs. 25200/=

Activity 7.7

1. On line graphs
   a) 60 min
   b) 1^0 F
   c) At 8. Am 102^0F, at 11 am 98^0F, at 2 pm 101^0F, at 6pm 102^0F and 99^0F
   d) 99^0F



a) 40
b) 15 ½
c) 17 yrs

20.7.3 Graphical Representation of Data

(a) Pie charts

A pie chart displays portions of a whole as sectors of a circle (sector angles). The circle then represents the total of the component parts. Study the following example.

The table below shows the number of people employed on various jobs in a school.

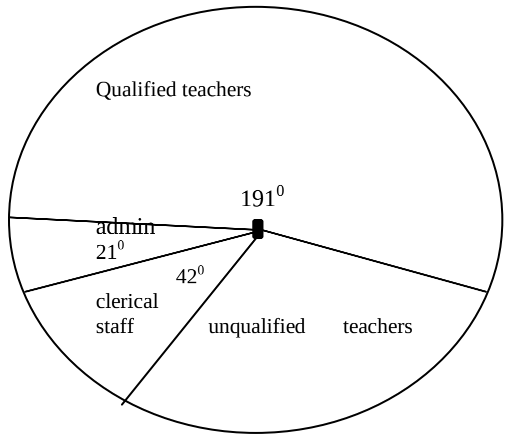
Type of Personnel	Number employed
Qualified teachers	45
Unqualified teachers	25
Clerical staff	10
Administrators	5
Total	**85**

Represent the data on a pie chart.
The first thing is to calculate the sector angles. Remember the angle at the centre is 360 degrees. Then calculate the sector angles as follows:

Type of Personnel	Sector Angle
Qualified teachers	$\frac{45}{85} \times 360^0 = 191^0$
Unqualified teachers	$\frac{25}{85} \times 360^0 = 106^0$
Clerical staff	$\frac{10}{85} \times 360^0 = 42^0$
Administrators	$\frac{5}{85} \times 360^0 = 21^0$

Using a protractor, the pie chart is now drawn.

A pie chart for types of people employed in a school



- Choose a radius.
- Mark a centre of a circle.
- Draw a starting line.
- Choose a scale on the protractor.
- Measure the sectors and label them.

(b) Bar Graph

In bar graphs, the information is represented by a series of bars of the same width. The height or length of each bar represents the frequency. Bar graphs are accurate, quick and easy to draw and can show a large number of component parts without confusion.

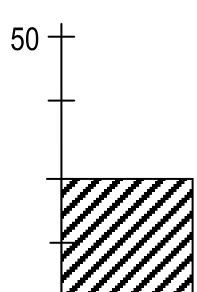
For the types of people employed in a school used to draw the pie chart in (b), we have the following bar graph

- Choose a scale
  Horizontal axis
  /space/gap represents/type of teachers vertical axis.

  /space/gap represents 5 teachers.

- Think of a title (heading)
- Then, draw the horizontal and vertical axes, label them
- Mark the graph

A bar graph showing the type of teachers employed in schools



45

40

35

Number of Teachers 30

25

20

15

10

5

0

Qualified Teachers | Unqualified Teachers | Clerical staff staff | Administrators

Type of teachers employed

(c) Line Graphs

Some information is collected and summarised as non-continuous or discrete values. Such data may be represented on a graph where points are joined using straight lines. This is common with temperature as in the following case.

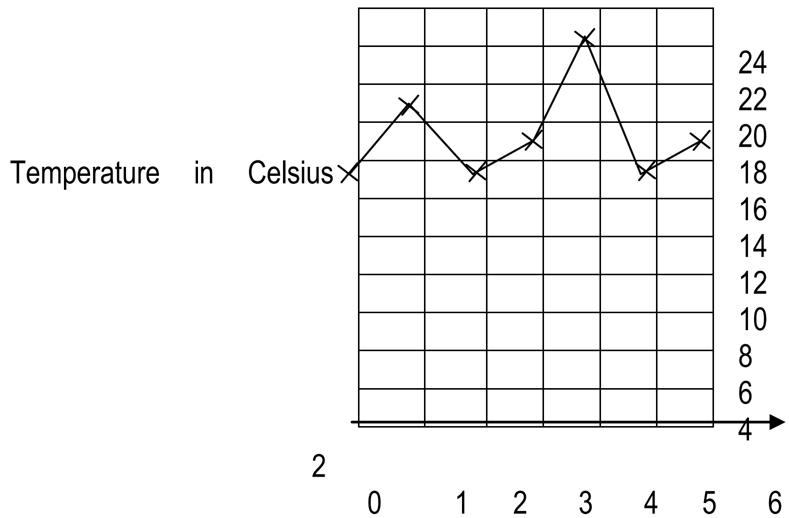
The table below gives the temperature at 4.00am on seven successive days. Plot a graph for the information.

Day	1	2	3	4	5	6	7
Temperature	16	20	16	18	24	16	18

- Choose a scale:

  Horizontal/axis:
  /space = 1 day
  Vertical axis
  /space = 2^0C
- Write a heading of the graph
- Draw the horizontal and vertical axes.
- Label the lines
- Plot the points

A line graph showing the temperature in Celsius a seven successive days.



Day

Use the above example to do the exercise in Glit Green Mathematics revision and practice for UCE page 94. Show your work to the tutor.

Activity 20.3

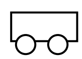
The figures below show the way in which commuters in a Kampala suburb travel to the city.

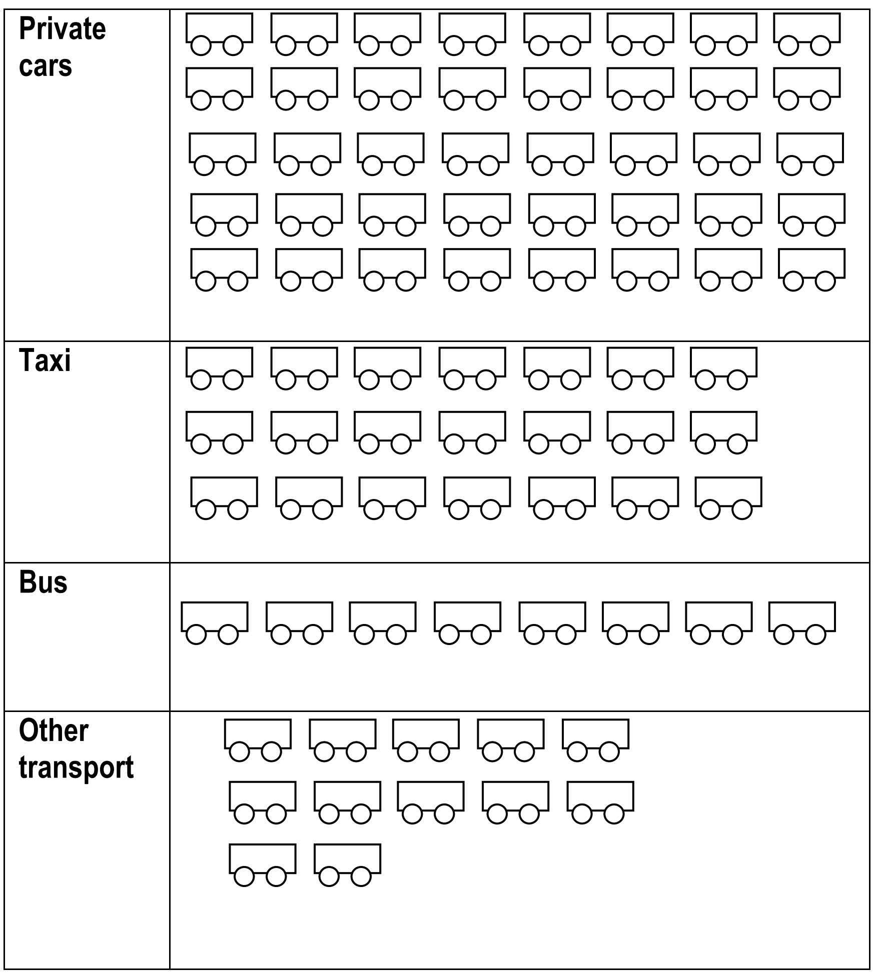
Type of transport	Type of transport
Private cars	1560
Taxi	840
Bus	320
Other transport	480

Represent this information on a:

i. Pictogram
ii. Pie chart
iii. Bar graph

Check your work with the answers in the table below.

(i) Choose a scale: e.g. represents 40 commuters. 



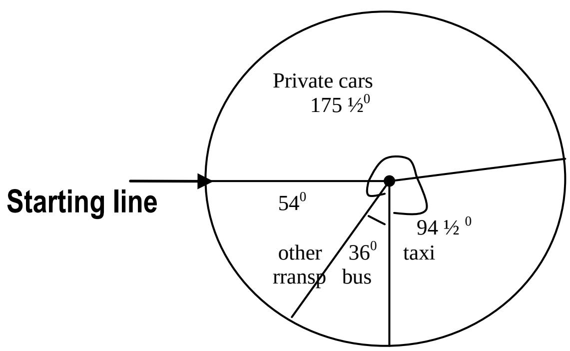
(ii)

- Establish the vehicles as. = 1560 + 840 + 320 + 480
- Find angles per sector.
- Use a table as shown:-

Section	Workings	Degrees
Private cars	$\frac{1560}{3200} \times 360$	175 ½ ^{0}
Taxi	$\frac{840}{3200} \times 360$	94 ½^0
Bus	$\frac{320}{3200} \times 360$	36^0
Other transport	$\frac{480}{3200} \times 360$	54^0

A pie chart showing commuters in Kampala suburban city travel, to the city.

- Chose a radius
- Mark a centre of circle
- Draw a radius as a starting point.
- Measure angles for each sector
- Label each sector.



A bar graph showing commuters in Kampala suburb travel to the city

Scale: horizontal axis
/space/gap/number of small squares represents: / type of transport

Vertical axis

/space represents 150 commuters.

For more practice, get cliff Green Mathematics Revision and practice for USE, pages 96 numbers; 1,2,3 and 5. Submit your work to the tutor.

(iv) Pie charts/pie graphs/circle graphs

(1) Pie chart interpretation

You are welcome to this sub topic.

Dear student, what is a pie chart? You can compare your answers with this explanation;

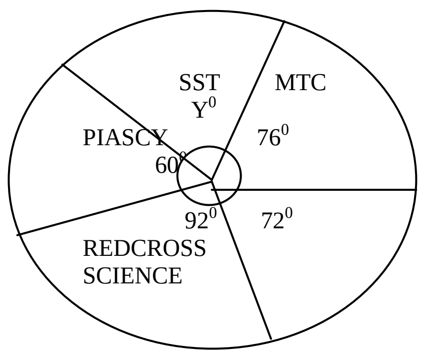
> You can also show data in a form of a pie-chart. A pie chart sometimes is called a circle graph or cake graph – why? A pie chart can be drawn by dividing the circumference of a circle into degrees. A pie chart displays portions of a whole as a sectors of a circle (sector angles). The circle then represents the total of the component parts.

Study the following examples.

Example 1: There are 450 students in a college. Each student belongs to one of the five clubs, namely maths, science, Red Cross, PIASCY and SST clubs.

Use the given pie chart to answer the following questions

A pie chart showing students in each club;



i How many students belong to SST?
ii If members in SST joined science how many members would they be altogether?
iii What would be the average number of students if all clubs had equal number of members?

Method 1.

Sector	**Angle in degrees**	**Working**	**Number of students**
MTC	76^0	$\frac{76^0}{360^0} \times 450$	95
SCIE	72^0	$\frac{72^0}{360^0} \times 450$	90
RED CROSS	92^0	$\frac{92^0}{360^0} \times 450$	115
PIASCY	$\underline{60^0}$	$\frac{60^0}{360^0} \times 450$	75
SST	y+76+72+92+60 $= 360^0 - 300^0$	$\frac{60^0}{360^0} \times 450$	75

	= 60^0		
	360^0		**450-**

Method II

360^0 = 450 students.

$1^0 = \frac{450}{360}$

MTC = $\frac{450}{360} \times 76^0 = 95$

Science = $\frac{450}{360} \times 74^0 = 90$

Red cross = $\frac{450}{360} \times 92^0 = 115$

PIASCY = $\frac{450}{360} \times 60^0 = 75$

SST = $\frac{450}{360} \times 60^0 = 75$

Use the table above to write correct answers.

i SST has 75 students .

ii SST and Science = 90 + 115

i. = 205 students

iii $\frac{95+90+115+75+75}{5}$

$\frac{\cancel{450}}{5}$

= 90 students

(2) Pie - charts drawing

Example 2:

Nabirye spends $\frac{3}{10}$ of her salary on food, $\frac{1}{10}$ on clothing and saves the rest. Show this on pie- chart.

Step 1: Change all fraction to degrees as shown below.

Item	Fraction of the sector	Working	Angle of the sector in degrees
Food	$\frac{3}{10}$	$\frac{3}{10} \times 360^0$	108^0
Rent	$\frac{1}{5}$	$\frac{1}{5} \times 360^0$	72^0

Clothing	$\frac{1}{10}$	$\frac{1}{10}$x 360^0	36^0
Saving	Find the unknown fraction supposing the fraction on saving = x Then, $X + \frac{3}{10!} + \frac{1}{5!} + \frac{1}{10} = 1$ $= X + \frac{3}{10!} + \frac{2}{10!} + \frac{1}{10} = \frac{10}{10}$ $= X + \frac{6}{10} - \frac{6}{10} = \frac{10}{10} - \frac{6}{10} = \frac{4}{10}$	$\frac{4}{10}$x 360^0	144^0
Total		$108^0+72^0+36^0+144^0$ =	360^0

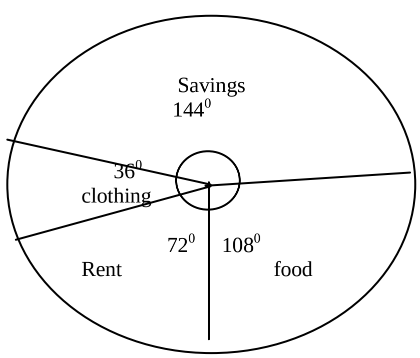
Step 2: To draw a pie – chart the degrees for each fraction will be represented within the pie chart e.g. the above table. Remember that a pie chart represents 360^0. Use a protractor to mark the degrees on the pie chart.

Step 3: since now you know degree by each sector, you are ready to draw the pie chart.

Procedures of drawing a pie chart

i. Choose an appropriate title.
ii. Choose a convenient radius.
iii. Mark a center e.g. center (**0**)
iv. Draw a starting line were you will start measuring the angles.
v. Measure each sector and label.

A pie chart showing Nabirye's salary expenditure

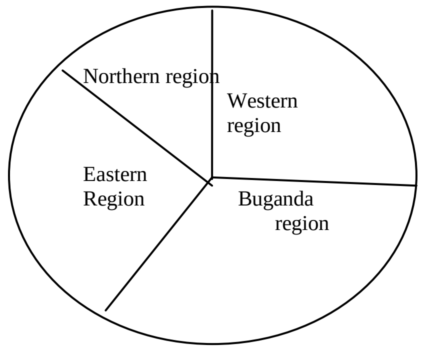


Note: The working for finding a sector angle and an accurate pie-chart will be more convenient to be on the same page.

Do the following activity in your exercise book

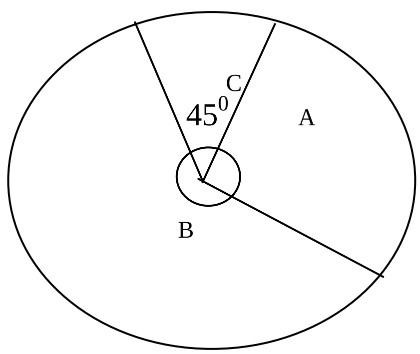
Activity 7.4

1. a) This pie chart shows the population of Uganda in 1969. Use the sizes of the sector to write down the regions in order starting with the smallest.



b) measure the angles and workout the population of each region given that the total population of the four region was 1,704,240

2. The pie – chart represents the candidates who passed P.L.E in a certain year in one particular country. Their grades were A,B and C. the total numbers of candidates who passed was 720.



a) How many candidates passed in grade /B?
b) What is the percentage of candidates who obtained grade A?

3. There are 10 goats, 11 sheep, 8 cows and 7 pigs in Okongo's home.

   a) Find the total number of animals in Okongo's home.
   b) Draw a pie chart to represent the animals in Okongo's home.

4. The exports of a country are: tea and coffee 25%, fruits 10%, milk products and meat 30%
, sisal 15%, others 20%. Show this on a pie chart accurately.

How are you getting on now?

I hope you are feeling well?

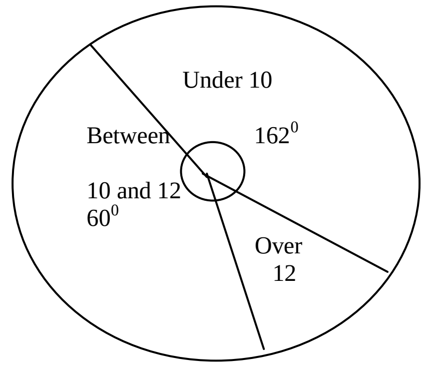
3) Application of pie chart

Let you look at example 3 critically.

Example 3:

This pie chart shows the ages of pupils of a school

There 200 pupils over 12 years of age



a) What is the total number of pupils in the school?
b) How many pupils are between the ages of 10 and 12 years?
c) How many more pupils are there under 10 years than there are over 12 years?

Study the working shown below:

Step 1: Find the angle sector of pupils between 10 and 12 in degrees.

Let the angle be x^0
Then, $x + 162^0 + 60^0 = 360^0$ (from an eqn)
$X + 222^0 = 360^0$
$X + 222^0 – 222^0 = 360^0 - 222^0$
$X = 138^0$

Step 2: Find the value of each sector of the pie – chart

Method 1: Using the known sector

Sector	Angle	Working	Number of students
Number of pupils over 12 years	60^0	$60^0 = 200$ pupils $1^0 = \frac{200}{60}$	200 pupils
Total number of pupils in a school.	360^0	$\therefore 360^0 = \frac{200}{60} \times 360 =$	1200 pupils
$\therefore$ number of pupils between 10 and 12 years	138^0	$60^0 = 200$ pupils $1^0 = \frac{200}{60}$ $\therefore 138^0 = \frac{200}{60} \times 138$	460 pupils

$\therefore$More pupils under 10 years than over 12 years:

$162^0 - 60^0 = 102^0$
If $60^0 = 200$ pupils
Then $1^0 = \frac{200}{60}$

$\therefore 102^0 = \frac{200}{60} \times 102$

$= 340$ pupils

Method 2:

Supposing the total number of pupils = y

a) Then the number of pupils over 12 years

$$= \frac{60}{360} \times y = 200$$

$$\frac{y}{6} \times 6 = 200 \times 6$$

$$\therefore y = 1200 \text{ pupils}$$

b) Then, the number of pupils between 10 and 12 years

$$= \frac{138}{360} \times 1200$$

= 460 pupils

c) More pupils between under 10 years and over 12 years

$$\left(\frac{162 \times 1200}{360} - \frac{60 \times 1200}{360}\right)$$

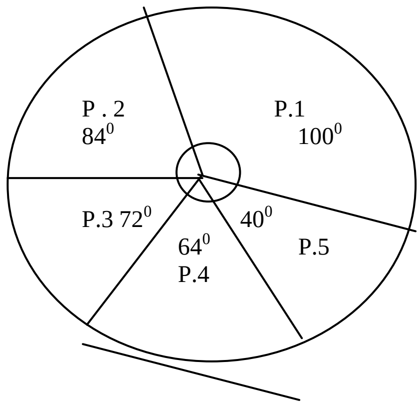
= 540 – 200

= 340 pupils.

Have more practice by doing these problems concerning pie charts in your exercise book.

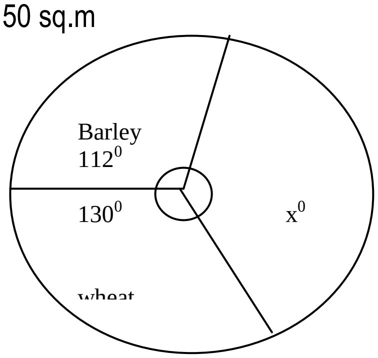
Activity 7.6

1. The pie chart below shows the number of pupils in a primary school in the first five years. If there were 900 pupils altogether in a school, how many were in primary five (5)?



2. This pie chart shows a field with maize, barley and wheat. Wheat is grown in an area of 7,150 sq.m

a)

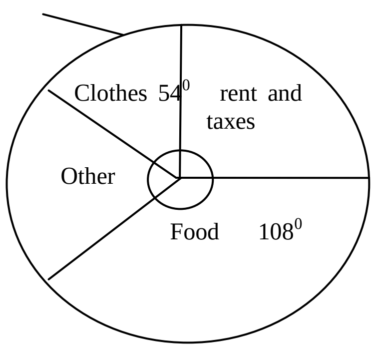


Answer the following questions.

a) Find the area of the field

b) What area is under maize

3.



This pie chart shows the expenses of a family

Use the pie chart above to answer the following questions.

a) If sh. 51,000/= is spent on food, what amount is spent on rent and taxes?
b) How much more money is spent on “other purposes” than on clothes?
c) What is the family’s total income?

Check the answers with the ones at the end of the unit.

Thank you for completing this subtopic.

(v) Line graphs

Dear student; having seen how data is presented on a bar graph, pictographs and a pie chart, you can also represent the same information on a line graph.

Note: a bar graph or histogram data is always changing as time goes on and these can be represented on a line graph, with the horizontal axis always showing the time.

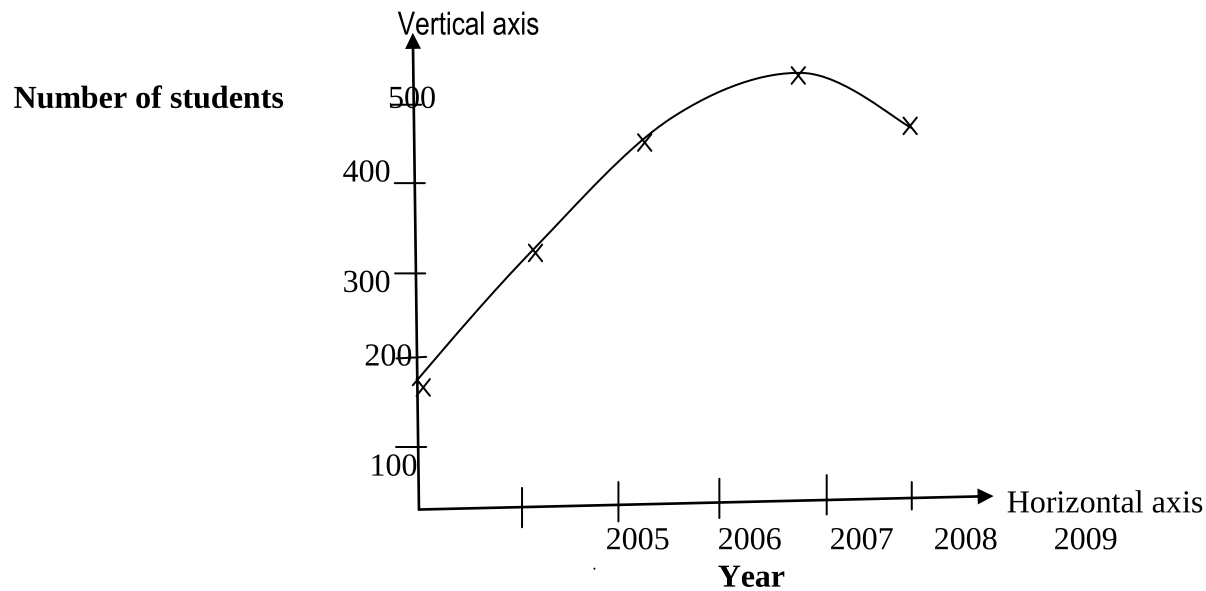
Let's consider this example.

The enrolment of students at Nakaseke PTC is shown in the table below.

Year	**Number of students**
2005	180
2006	314
2007	430
2008	510
2009	460

Show this information on a line graph

Number of students at Nakaseke PTC

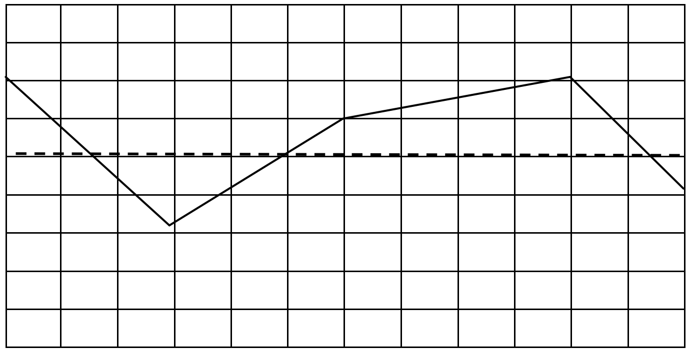


Well done!

Now use the above information to carry out the activity below.

Activity 7.7

1. The graph below shows the temperature of a sick person at different times of one particular day. The temperatures are taken 5 times during the day. When the points are joined, we get a line graph, which can be used to find certain information



8am 11am 2pm 5pm 8pm

time in hours

You can answer the following questions

a) What does one small square on the horizontal axis stand for in minutes?
b) What does on small square on the vertical axis stand for, in degrees?
c) At what time of the day were the temperatures taken? What were the temperatures?
d) What was the probable temperature at noon

2. The average weight of boys at different ages is given in the table below.

Age in years	11	13	15	17	19
Average weight	37	40	42	49	51

Plot a line graph using 2 kg to the cm on the vertical axis and 1 year to the on the horizontal axis.

From the graphs find

a) The average weight of boys of 14 years
b) The age of boys who have average weight of 44 kg
c) After what age is the growth slower.

Check your answers with the ones at the end of the unit.

Now, can you read the text below and take note if following points:

Note:

When deciding which type of graph to use, here are some guidelines

1. Use a bar graph when the data falls into district categories and you want to comare tatols
2. Use a line graph when you want to show the relationship between on consecutive amounts or data over time.
3. Use pictographs for information presentation in which you want a high visual appeal

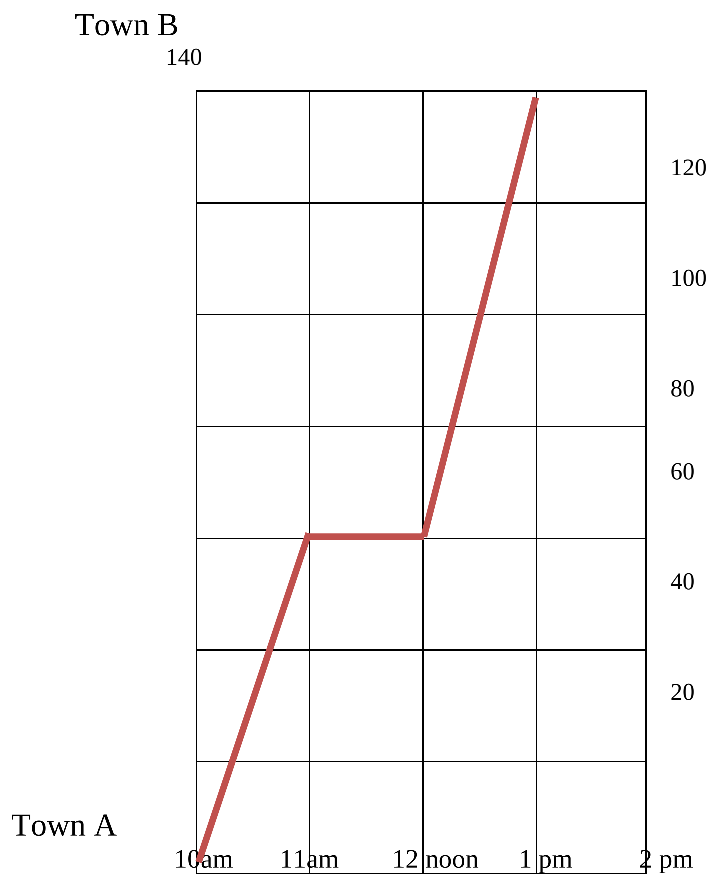
Well done! May you read about the next topic?

(vi) Travel graph

Interpreting travel graph

Example 1

Study the graph below and answer the questions that follow



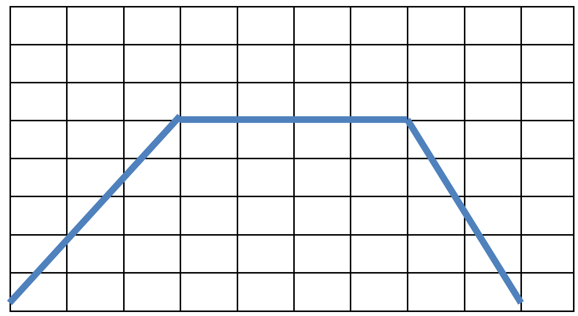
Number of hours

(i) What is the scale on vertical axis and the horizontal axis?
(ii) what is the motorist's speed for the 1^{st} part of the journey?
(iii) For how long did the motorist rest?
(iv) what was the speed of the motorist after resting?
(v) At what time did the motorist arrive at town B?
(vi) How long did the whole journey take from town A to town B?

Share your answers with your colleague.

Example II

Study the graph below and answer questions that follow.



(i) How long is the journey?
(ii) How long did the train rest?

(i) The length of the journey

(12 x 20) + (½ x 15 x 12) + (10 x 12 x ½)

240 + 90 + 60

= 390m

(b) Drawing travel graphs

Example III

A taxi driver left Kampala at 9. Am for Jinja and moved at 40km/ph for 1 hour. He then rested for 15min and started the journey at 80km/h for 30 minutes to Jinja.

a) Draw a graph showing the taxi driver's journey.
b) At what time did he reach Jinja?

b) He reached Jinja at 10.45am

Example (iv)

A train accelerates from rest to a speed of 15m/s and stays at that speed for 10 seconds. It again accelerates for 20m/s and stays at that speed and then steadily slows to rest in another 20 seconds.

a) Draw a speed time graph.
b) What is the distance travelled in the first 30 seconds.
c) The total distance travelled.

b) Distance = speed x times
= 15 x 20 (because the train was resting for 10 minutes).
= 300m

c) The total distance travelled

= (15 x 20) + (20 x 10) + (20 x 20)
300 + 200 + 400
(10 x 15) + ($\frac{10 \, x \, 5}{2}$)

150 25
= 300 + 200 + 400 + 150 + 25
= 1075m

Now that you have been able to go through distance time graphs, you can now do this activity.

Activity 7.8

1. Draw a distance time graph to illustrate each of the following

    i Kamugisha cycles for 1 hr at 15km/hr, stops for 1 hr and then continues in the same direction for 2 hr at 10km/hr.
    ii A pick up moves at 30 km/hr for 1½ hrs.

2. A cyclist leaves Soroti at 10.00 am and cycles at 15km/hr to Kapiri which is 25 km away
    i. At what time does he arrive?
    ii. Draw a distance- time graph for his journey.

7.7.3. CO-ORDINATE GRAPHS

Dear student;

Welcome to this subtopic, in order for you to effectively study through it, read about the number line as well as the negative and positive integers under unit 9.

Therefore; What do you think is a co-ordinate?

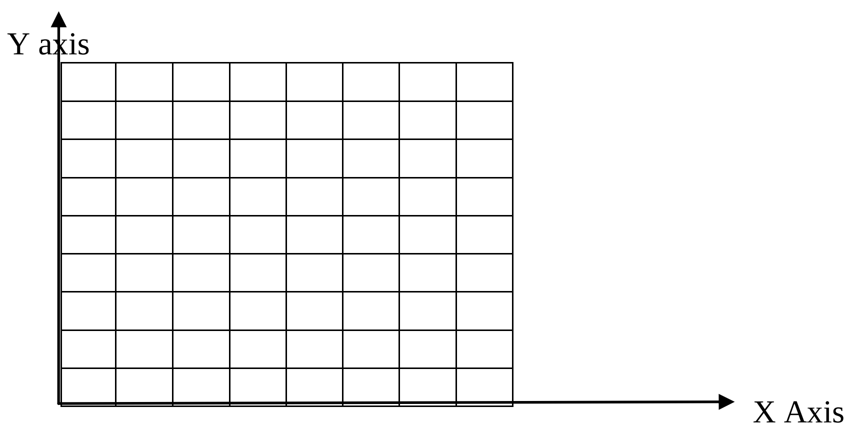
- Do you know how a coordinate graph look like?

Do the following activity.

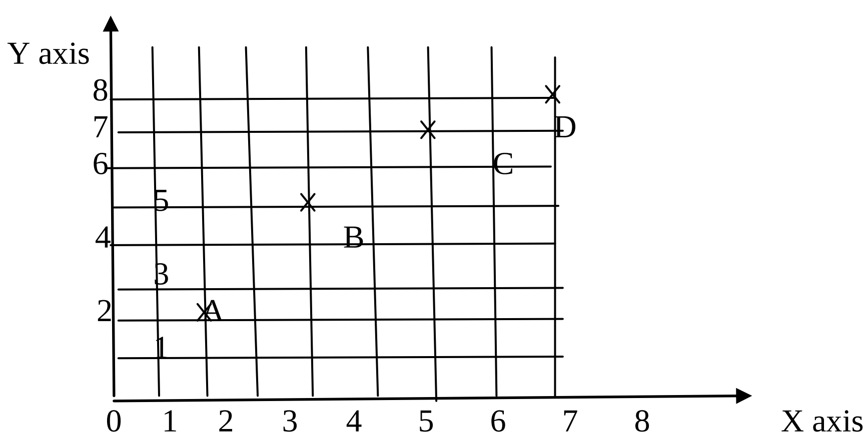
You can now follow the procedure below in order to answer the above questions.

Procedure to draw a coordinate graph.

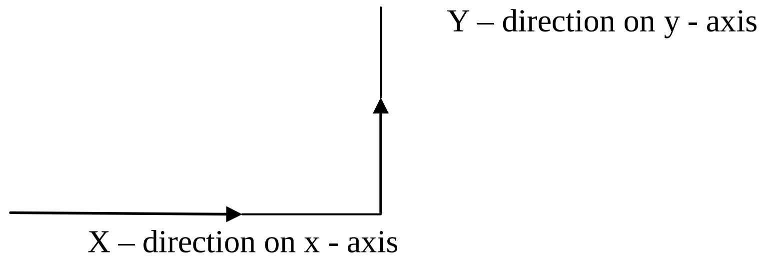
Draw a square grid as shown below



- Mark two lines on the extreme lines that is the horizontal line and vertical line as shown above.
- Label the two lines as axis i.e. the horizontal axis should be labeled as y axis.
- Indicate zero (0) where the two axis begin and this is known as the origin of the axis.
- Indicate the figures 0,,1,2,3,4,e.t.c. on y – axis



Now, using the y axis and x axis as a movement of steps, find the positions of A,B,C and D. note that the movement to their points begin from the origin 0 and more x – direction and then y – directions to find the positions of these letter A,B,C,D as shown above.



Considering point A, moving from origin 0 on x – axis, there are 2 steps and then 2 more steps of y – axis the new position A is then written down as (x,y) to which is (2,2).

Considering point D, you move 8 steps on x – direction and then 7 steps on y – direction. The directions are then represented as (x,y) = (8,7).

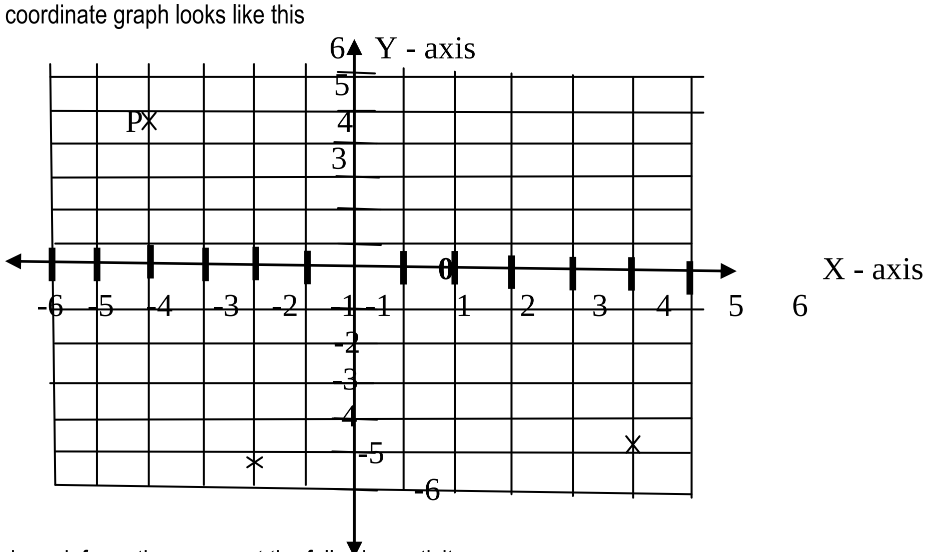
Can you then find the new positions of B and C.

Note! To locate the position of A.B,C,D, correspond to the number of steps along the x – axis and the y – axis represented as (x,y). in this way we are actually coordinating (linking) numbers on x – direction to members on y – direction and this is known as the coordinating systems. The coordinate on x – axis as (x,Y). These pairs up with the co-ordinate of y – axis as (x,y) . These pairs are known as the co-ordinate pairs.

Therefore, coordinate pairs are used to locate positions of places in the graph or grid, beginning to move from x – axis followed by y – axis.

Remember! In number line, the values of x ranges from negative to positive and their also applies to the values of y on the y – axis.

Therefore, the coordinate graph looks like this



Now, with the above information carry out the following activity.

Activity 7.8

Using the coordinate graph above, plot the points on x and y axe

1.

No.	Point	(x y) direction
i)	A	(3,2)
Ii)	B	(-3,3)
iii)	C	(0,6)
iv)	D	(7,0)
v)	E	(4, 5)
vi)	F	(6,3)

2. Read the position of the following points on the graph above in the (x,y) direction.

i) P
ii) Q

Share your answers with a colleague.

Well done! You can take a walk

7.7.4. TEACHING GRAPHS IN PRIMARY SCHOOLS

Dear student! Why do we draw graphs?

In everyday life and in mathematics, science, technology, business etc… people do not spend time off drawing a graph unless it serves some useful purpose when it is finished. You need to remember this in your teaching.

A graph can be useful in various ways e.g.

a) It can show information in a form which can be quickly and easily understood.
b) It can provide information which we previously did not have.
c) It can indicate relationship between the members of two sets.

Remember, a pupil can best build up and understand mathematical ideas and technique through their own activities and their own thinking.

So as a teacher, your job is to provide the activities and to encourage each child to think about what he or she is doing, to look for relationships which may emerge, and to build up a store of mathematical techniques.

You should also put into consideration that, most pupils understand a new mathematical idea with a start of a practical experience e.g. in graphs help children to come up with identification of squares. this normally they begin having an idea of square shirts. Make them draw bring in the idea of drawing two lines i.e. the horizontal line, the x – axis and the vertical line the y- axis. Make them know how to indicate and label the axis.

- Activities and instructional materials for teaching graphs.
- Make pupils draw a grid in their books and let them mark them clearly starting point zero and the directions from the starting point i.e. x and y axis. You may make them study the grid which has been located with points A,B,C.
- Divide the students to make them understand how to locate the position of these points. The activity is repeated so that the pupils get the ideas.

Congratulations

7.8 UNIT SUMMARY

In this unit you have been introduced to graphs. I now hope you have the following

i) What is a graph?
ii) Types of graphs
iii) Represent information on a graph
iv) Read and interpret a graph
v) What a co-ordinate graph is.
vi) Locate and read position on co-ordinate graphs.

7.9 ANSWERS TO ACTIVITIES

Activity 7.2

1. a) (i) In 2006
   (ii) in 2007

Year	2005	2006	2007	2008	2009	2010
No. of pupils	45	50	35	40	45	40

b) (i) 1 square represents 1 bird
(ii) on Saturday
(iii) on Thursday
(iv) Monday 26
Tuesday 24
Wednesday 24
Thursday 22
Friday 24
Saturday 28
148 birds

Graphs undrawn

(15 x 20) + (15 x 10) + (20 x 10) + (20 x 20) x ½ + $\frac{20 \times 10}{2}$ = 50

300 150 200 200 ½ 200

(15 x 10) (20 x 20) + (20 x 20) + ($\frac{\overset{5}{\cancel{10}} \times 5}{\cancel{2}}$)

150 400 400 25

$\frac{\overset{10}{\cancel{20}} \times 15}{\cancel{2}}$ = 150

= 380

(20 x 20) ½ = 200

20 x 20 = 400 600

$\frac{10 \times 5}{2}$ = 25 625

10 x 15 = 150 775

$\frac{20 \times 15}{2}$ = 300 1025

Activity 7.6

1. 100 pupils
2. a) 19800 sqm
   b) 6490 sqm
   c) Sh 170,000

3. a) Shs 42500
   b) Shs 25,200

7.9. GLOSSARY

Coordinate pairs (x,y) - Is a phrase to help remember the order of plotting the ordered pair (x,y) it means go along the x –value and up to the y – value

Axis – That are 2 fixed lines in a coordinate system i.e. the x – axis and y – axis and are placed at right angles to each other.

Grid – Is a set work of horizontal and vertical lines that are drawn on a map, charts in order to locate specific location.

7.10. REFERENCE BOOKS

1. Roland E Larson Passport to Algebra ad Geometry page 192 – 218
2. Mehta, Preparatory Mathematics, New edition pages 113 -123
3. Susan Sperry s, Early childhood mathematics 125 – 135

End of unit
Congratulations

UNIT 8: RELATIONS, MAPPINGS AND FUNCTIONS

8.1 Introductions

Hullo student, you are most welcome to unit 8. In this unit, you will be introduced to the idea of connecting people, places, objects, ideas etc belonging to the same group or two different groups.

8.2. Content Organization

Dear student, in this unit you will be covering the following topics as indicted below:-

No.	Topic	Sub-topic	
1.	**Graphical representation of relations**	a.	What relations are.
		b.	Relations between two sets
		c.	Arrow diagrams
		d.	Pap grams
2.	**Mappings**	a.	Mapping as a relation
		b.	A one – to- one mapping
		c.	A many – to-one-mapping
		d.	A one-to-many mapping
		e.	A many-to many mapping
		f.	Domain and range
		g.	Function as a special mapping
3.	**Cartesian graphs**		Plotting of functions on Cartesian graphs

8.3. Learning outcome

At in the end of this unit you are expected to relate the concepts in relations mappings and functions to everyday life situations.

8.4. Competences.

Dear student, now you know the expected learning outcome, therefore as you study/work through this unit, you will be able to:-

(i) Define relations
(ii) Explain if relations between two sets hold or not.
(iii) Draw arrow diagrams.
(iv) Draw papygrams.
(v) Draw arrows to represent different types of mappings.
(vi) Define domain and range.
(vii) Solve problems involving functions as a special mapping
(viii) Draw graphs of functions.

8.5 Unit orientation.

You have already realized in the foregone topics that mathematics is a language of its own which helps to bring out concepts clearly. In this unit, you need to have ideas about grouping objects in sets and determine how many there in each group. You also need to remind yourself on plotting and drawing Cartesian graphs which you did in unit 7.

8.6 Study requirements.

Hullo student in order to be successful in this unit you will need graph paper, pencil, pen, rubber and exercise book.

8.7. Content and activities.
8.7.1. Graphical representation of relations.

a) What relations are?

In order to state what relations are, we need to first understand how people are related within our communities. People are connected in the communities we live in different ways e.g. mother, father, brother, sister etc.

Would you think and list some more relations.

Bearing in mind that mathematics is often referred to as being a language of its own. Here are some mathematical statements we have already used.

(i) $Y = x$ (ii) $-4 < {}^{+}1$

You can read the above using ordinary English sentences as:-

(i) Y is equal to x
(ii) -4 is less than $^{+}$1

The statements is equal to and 'is less than' are known as 'relations'.
In the two examples given above, the relations were denoted by the mathematical symbols = and <. It may not always be possible to use a symbol.

For example: 8 is a factor of 24
PQ is the mediator of AA'

Write down some other examples of relations you have used in mathematics.
Relations are not confined to statements involving numbers, lines, sets and so on. In our everyday life we use such expressions as;

John sits next to Samuel.
Mary has not been to Nairobi.
Betty is taller than Martha.

Each of the phrases connecting the two people is a relation.
Now that we hope that we are together. Write down the answers to these questions.

1. What is the relation in this statement?
   Sarah is a sister of Sammy.
2. Think of and write three statements of relationships like the ones given above.
3. What is the relation in each of the statements given?

Great, you can now share your answers with your colleagues.

b) Relations between two sets.

We have now seen that a relation is a rule that connects two or more people, objects or ideas belonging we to the same set or two different sets. If there is a relationship between those sets, we say "relation holds". If there is no relation, then we say "relation does not hold".

Example.

Silas (11), Fatiya (9), Sarah (8) and Mathew (6) are four children of Joseph and Mary whose ages are as given in the bracket.

1. Can you identify small sets from the big set "the family of Joseph and Mary?
2. Name them.

Compare your answers with the following

ε = {Silas, Fatiya, Sarah, Mathew}
A= {Fatiya, Sarah}
B= {Silas, Mathew}
You can now clearly see the relationship between the sets and also between different members of the set.

(c) Arrow diagrams

Make a list of six boys or girls who sit near to you in the classroom and also list the games they play.

Your list should resemble like this one.

Set A = {Michael, John, Julius, Joseph, Samuel, Peter}
{Football, hockey, basketball, volleyball}

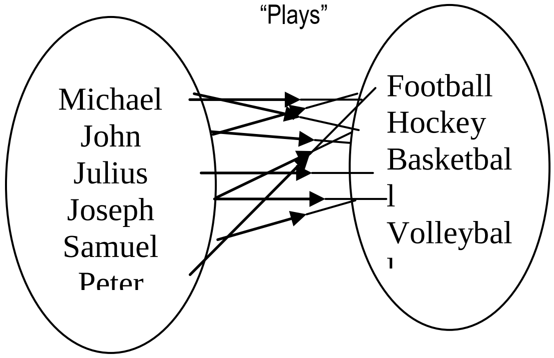
Suppose when you asked the boys what games they play and the answers come as follows:-

Michael: football and Hockey
John: football and basketball
Julius: basketball
Samuel: volleyball
Peter: football

1. What would the relation between set A and Set B be?
2. Do you agree that the relation is "plays"?

We can represent that relation diagrammatically as;

"Plays"



This is called an arrow diagram. Each arrow is drawn to connect members in the two sets between which there is a relation.

NB. The direction of the arrow indicates the relationship

Activity 8.1

1. Draw a diagram to represent the relation “is a factor of” between the members of {2,4,5,6,7} and {30,31,33,34,35,36}

Check your answer with the one at the end of the unit

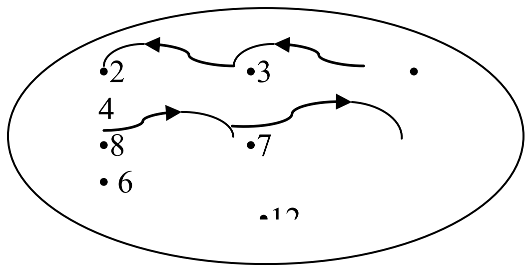
Great, you can now check your answers with those given out at the end of the unit.

d) Papycrams

You have now seen relations which exist between members of different sets. There are also relations which exist within a single set (same set). Such relations may• also be represented using a diagram as in example below.

Example:

The relation “is one more than” can be represented in the following numbers {2,3,4,6,7,8,12}



In each case the arrow begins from the number which is “one more than” and end in the number it is less than.

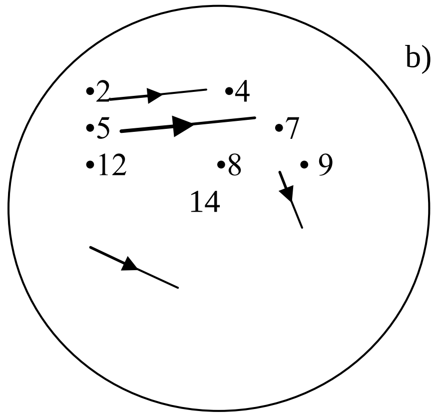
Diagrams like that are called papygrams. It represents the relationship between the members of the same set

In your exercise book, draw a papygram for your own family of the relation “is a sister of”. (Do not forget to include your brothers.)

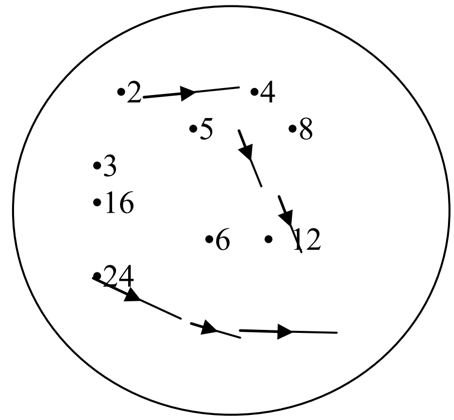
Activity 8.2

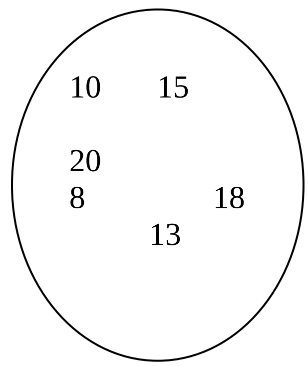
1. List the set and state the relation in each of the papygrams

a)



b)





2. Draw a diagram to illustrate the relation is a multiple of "between the sets E={12,13,14,15,16,17} and F={1,2,3,4,5}
3. Draw a diagram to illustrate the relation "is three more than" between the sets
   C={12,13,17,18} and D={9,10,14,15}

8.7.2 Mappings

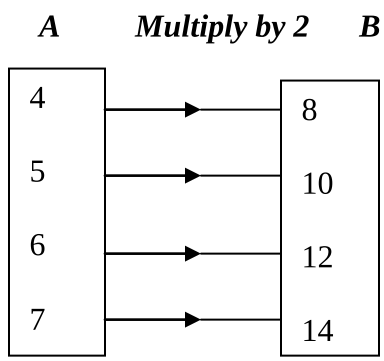
a) Mapping as a type of relation.

In the previous topic we discussed relation between two sets and relation between members within the same sets. In this sub topic we are going to look at special relations.

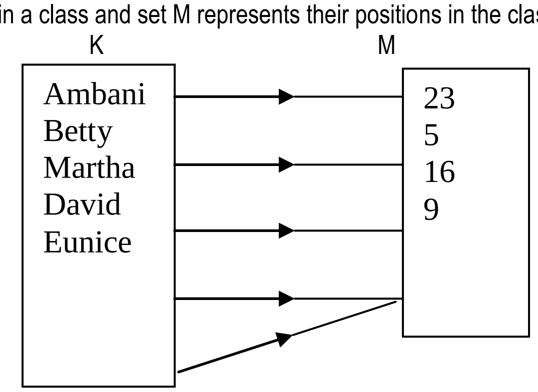
Example 1

Let set A = {4,5,6,7} and set B = {8,10,12,14}. What is the relation between these sets? Do you agree that the relation between the two sets is that the members of set A are multiplied by 2 to get members of set B.

∴ The operation of multiplying each member of A by 2 produces corresponding members of B.
We can represent it diagrammatically as:-



Example 2
Set K represents 5 children in a class and set M represents their positions in the class.



From the diagram, you can see that members of set K are related to the members of set M by the operation “position of”

Look at the two examples above.

What can you say about the direction of their arrows?
Among the answers you have written should be this:-

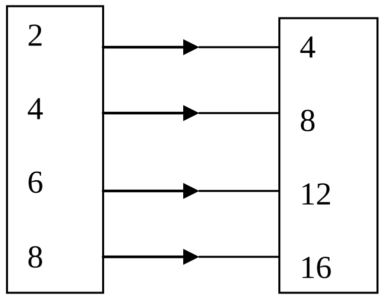
1. All the arrows face the same direction.
2. Only one arrow leaves each member of set K going to a member in set M.

That is great. We can now conclude that, a relation between two sets which connects each member of one set to one member of the other set is called a mapping.

I think we can now move to the types of mappings.
(b) A one - to – one mapping

Example 3.

The diagram shows the relation "is half of" between A = {2,4,6,8} and B= {4,8,12,16}



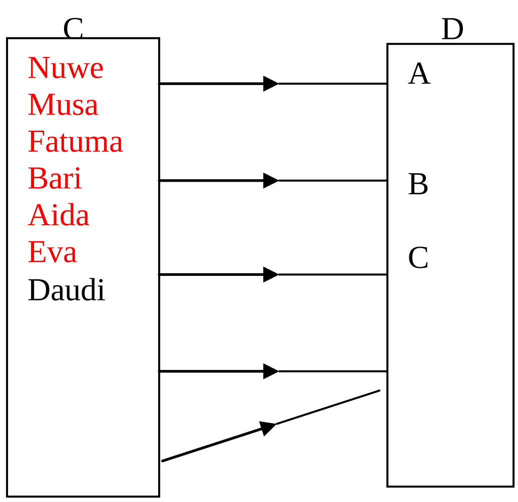
We can say that, 4 is the image of 2; 8 is the image of 4; 12 is the image of 6 and 16 is the image of 8;
Every member of A has one image in B. every member of B is the image of exactly one member of A. therefore this is a one-to-one mapping.

(c) Many to one mapping

Example 4.

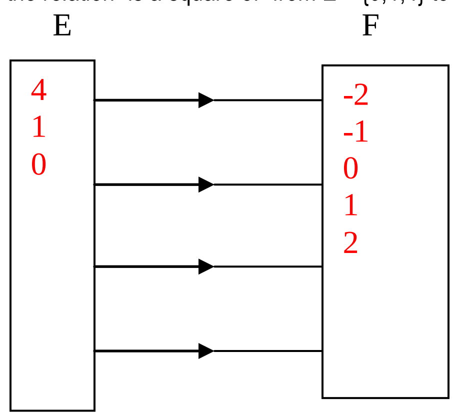
Pupils got the following grades in a test.

Muwe	B
Musa	B
Fatuma	B
Bari	C
Aida	A
Eva	C
Daudi	A



This is an example of many-to-one mapping. How many elements of set C have the same image in set D?
Both Bari and Eva for example, got the same grade, C. No elements of C have more than one image, because no pupil can get more than one grade.
(d) One-to-many mapping

The diagram below shows the relation "is a square of" from E = {0,1,4} to F={-2, -1,0,12}

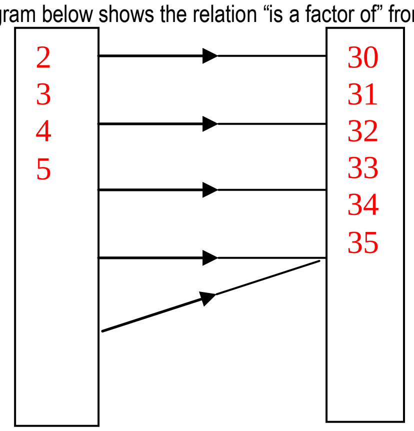


Study the diagram carefully. How many arrow starts from each member of E?
Share your answer with your colleague.

Note: This is an example of a one-to-many mapping. Some element of E have more than one image of F. no elements of F is the image of more than one element of E.

Many-to-many mapping

The diagram below shows the relation "is a factor of" from P={2,3,4,5} to S= {30,31,32,33,34,35}



Look at the diagram, what do you observe? You should be able to identify that same elements of P have more than one image of S. Some elements of S are images of many elements of P. But you notice that the 31 has no arrow going to it as its only factors are 1 and itself.
Therefore the example above is of many-to-many mapping.

(f) Domain ad range.

Looking at the diagrams we have drawn in the previous sub-topics, we realize that arrows begin from some sets and end at the other sets. When dealing with the relation the set where the arrows begin is called the domain end. The set where the arrows end is the range. The elements of the domain are called objects while the elements of the range are called images.

(g) Function as a special mapping

In the last topic we looked at a special relation called mapping. When we look back to example 1 in the last topic you realize that there was only one arrow starting from each member of the domain (left hand set). Therefore that mapping was special. That type of mapping is called a function.

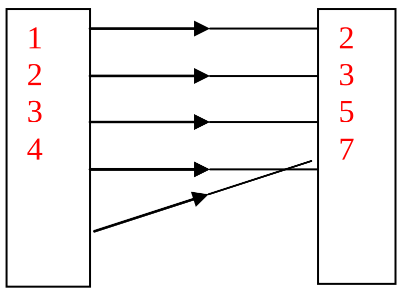
A function is defined to be a mapping in which each object has one image and why one image.

(h) Functional Notation

Functions as a special mapping has also special ways it may be written.

Set A = {1,2,3,4} Set B = {prime numbers taken in order} = {2,3,5,7}

Let the relations connecting members of set A to set B be “prime numbers taken in order”.



This relation is a function. We can represent that relation by letter f.

$A \xrightarrow{f}$ B to mean that all members of A are mapped onto members of B by the function f.

We can write it as

$1 \xrightarrow{f}$ 2,2 is the image of 1 f(1) = 4.

A function may be written using other letters like g,h,r

g(1) = 2

for any function f(x), f(3) is the value of the function when x = 3. The same applies for same number.

Example: 5

If f(x) = 5x – 3, find f(4)

Solution:

f(x) = 5x – 3

f(4) = (5 x 4)-3 subtracting x = 4

= 20 – 3

= 17

Example B

If $f(x) = x^2+bx-3$ and $f(3) = 15$, find b

Substituting x = 3 we find that:

$F(3) = 3^2 + 3b – 3$

$= 6 + 3b$

$6 + 3b = 15$

$6 – 6 + 3b = 15 – 6$

$3b = 9$

$b = 3$

Now that you have been able to solve numbers dealing with fractions, then you can do this activity.

8.7.3 Plotting of functions on Cartesian graphs.

Since you have already learnt about drawing coordinate graphs in Unit 7. In this topic you will be taken through plotting and drawing graphs of functions using two coordinate axes (that is, the Cartesian graph).When the graph of function is drawn, the domain is the x-axis and the range is the y-axis. The ordered pairs (x,y) are formed in such a way that the values of x are members in the domain (D) and the value of y are members of the range (R)

Example 1

Draw the graph of the function y= 2x – 1 using the values {0,1,2,3,4}

Let the relation between x and y be y = 2x -1. The mapping can be written as : x $\rightarrow$ y. Since y = 2x – 1, the relation becomes x $\rightarrow$ 2x-1. If we represent this relation by f, the function may be written as f:x $\rightarrow$ 2x-1. Another way of expressing this is: x $\rightarrow$ f(x). Since x $\rightarrow$ y and also x $\rightarrow$ f(x), we can write that y = f(x).

1. Plot the graph of the function for these values of x

x	0	1	2	3	4
y					

Using the values of x, we shall obtain the values of y corresponding to each value of x.

Solution

For x = 0, y = 2 x 0 – 1 = 0 – 1 = $^{-}1$
For x = 1, y = 2 x 1 – 1 = 2 – 1 = 0
For x = 2, y = 2 x 2 – 1 = 4 – 1 = 3
For x = 3, y = 2 x 3 – 1 = 6 – 1 = 5
For x = 4, y = 2 x 4 – 1 = 8– 1 = 7

x	0	1	2	3	4
y	$^{-}1$	1	3	5	7

You can now copy the ordered pairs and plot on a graph: (0,$^{-}1$)(1,1)(2,3)(4,7)
Graph drawings

Example 2

Find the set of ordered pairs {(x,y) if $y = x^2 – 4x + 3$ and D = { x/x is an integer $^{-}1 \leq x \geq 4$}

Here you have been asked to find the ordered pairs satisfying the equation $y = x^2 – 4x + 3$, when x = $^{-}1$,0,1,2,3,4.

You can now proceed to substitute x in the equation $y = x^2-4x + 3$ with values $^{-}1$,0,1,2,3,4.

Solution

Draw a table of values as shown. It helps to have separate rows for x^2 and 4x

x	$^-1$	0	1	2	3	4
x^2	1	0	1	4		
^-4x	4	0	$^-4$	$^-8$		
$^+3$	3	3	3	3	3	3
y	8	3	0	$^-1$		

Can you complete the table above and list down the ordered pairs.
Plot points at ($^-1$,8), (0,3), (1,0), (2,$^-1$) and soon.

Graph drawing

Note:

1. Be careful when working out tables with values of functions, particularly when negative numbers are involved.
2. If the points of a graph are not on a straight line, do not join them with a straight line but with smooth curve.

Activity 8.3

1 a) Given that f(x) = 2x + 1, find f(3)

b) Given that h(x) = x^2 + bx + 5, if h(2) = 13, find b

2. For each of the following functions, find the values of y when x is 0,1,2,3 and plot on the graph.

   a) Y = 2x + 1
   b) Y = 2 - x
3. Draw the graph of y = $2x^2 + 3x – 2$ for $^-3 \leq x \geq 2$

You can now check your answers with those given at the end of this unit.

8.8 UNIT SUMMARY

You have come to the end of unit 8. In this unit you have learnt about.

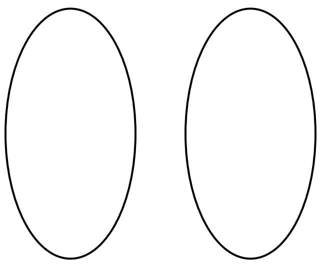
- Relations and how they are represented using papygrams.
- Arrows diagrams which represent relationships between members of two sets.
- Different types of mapping.
- How to draw Cartesian graphs for given functions.

8.9 GLOSSARY

Relations	:	A connection between corresponding members of two sets, or between members of the same set.
Mapping :		matching elements from one set to the elements of another set by use of a rule.
Function :		A relation between two sets where each member of the object set has one and only one image.
Papygram	:	a diagram representing relationship between members of the same set.

8.10 Answers to the activities.

Activity 8.1



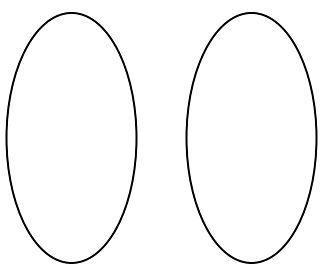
Unfinished drawing

Activity 8.2

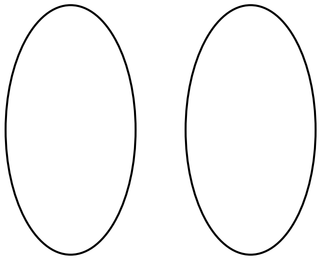
1. a) = {2,4,5,7,8,9,12,14,} relation – "is 2 less than"

   b) {2,3,4,5,6,8,12,16,24} is a factor of
   c) {8,10,13,15,18,20} is 5 less than

2.



3.



Activity 8.3

1. a) $f(x) = 2x + 1$

$f(3) = 2x3+1$
$= 6+1$
$= 7$

c) $h(x) = x^2+bx + 5$

$h(2) = 4 + 2b + 5 = 13$
$9 + 2b = 13$
$2b = 4$
$b = 2$

2 a) $y = 2x + 1$

x	0	1	2	3
y	1	3	5	7

(0,1), (1,3), (2,5), (3,7)

Graph undrawn

b) $y = 2 - x$

x	0	1	2	3
y	2	1	0	$^-1$

(0,2), (1,1), (2,0), (3, $^-1$)

Graph undrawn

3) $y = 2x^2 + 3x - 2$

x	$^-3$	$^-2$	$^-1$	0	1	2
$2x^2$	18	8	2	0	2	8
3x	$^-9$	$^-6$	$^-3$	0	3	6
$^-2$	$^-2$	$^-2$	$^-2$	$^-2$	$^-2$	$^-2$
y	7	0	$^-3$	$^-2$	3	12

($^-3$,7), ($^-2$,0), ($^-1$,$^-3$), (0,$^-2$), (1,3), (2,12)

Graph undrawn

8.11 end of unit exercise

This assignment is intended to help you consolidate what you have learnt in this unit. You are therefore advised to read the whole unit again before you attempt the following questions.

1. Draw a diagram to represent the relation "exceeds by more than 2" in {2,3,4,5,6,7,8}
2. Given f(x) = $\frac{x+2}{x-4}$, find f(4) and f(0)
3. Given that f(x) = ax^2 + bx, f(1) = 5 and f(2) = 14. Find the values of a and b.
4. Draw a graph of y = x^2 + 2 for the values of $^-4 \leq x \leq 4$

8.12 Self check/assessment

No.	Learning outcome	Not sure	Satisfactory
1.	I can represent relations by drawing arrow diagrams and papygrams		
2.	I can find images of elements using a given function		
3.	I can draw a Cartesian graph for a given function		

8.13: Reference for further reading

1. School Mathematics of East Africa, Book 2, Cambridge University Press.
2. Cliff Green (2000), Mathematics Revision and Practise for USE; Oxford University Press.
3. Okot – Uma, R, Kawooya M, etal (1995), Secondary School Mathematics; Macmillan, Uganda.

UNIT 9: GEOMETRY 1 (12 HOURS)

9.1 INTRODUCTION

You are very welcome to unit 9. In this unit, you will learn about plane figures Pythagoras, theoem, regular polygons solid figures and bearing and scale drawing.

9.2 CONTENT ORGANIZATION

Dear student, in this unit, you are going to cover the topics indicated in the table below.

Topic	Subtopic
1. Plane figures	a. Angles b. Parallel and perpendicular lines c. Drawing planes figures
2. Pythagoras' theorem	a. Introduction to Pythagoras' theorem b. Application of Pythagoras' theorem
3. Regular polygons	a. Properties of regular polygons b. Construction of regular polygons
4. Solid figures	a. Properties of solid figures b. Nets of solid figures c. Surface area of solid figures d. Volume of solid figures
5. Bearings and scale drawing	a. Bearings b. Scale drawing

9.3 LEARNING OUTCOME

Apply knowledge and skills in geometry for solving real life problems.

9.4 COMPETENCES

a) Apply Pythagoras theorem to solve problems.
b) Construct angles and regular polygons.
c) Make nets of solid figures.
d) Find surface area and volume of solids.

9.5 UNIT ORIENTATION

This unit will review the geometry you learnt in lower classes. It will also introduce you to the geometry of solids which are three dimensional (3 – D) objects.

9.6 STUDY REQUIREMENTS

In order to study this unit successfully, you need a mathematical set, paper, a chair and desk. You will also need small boxes, cylindrical containers, a pair of scissors or razorblade and glue.

9.7 CONTENT

9.7.1: PLANE FIGURES

a) Angles

What is an angle? What does it measure? Give three real life examples or chalk board of angles. Now look at your protractor. It is a tool for measuring angles drawn on paper. You can also use it to draw a given angle. Use your protractor to draw angles of 10^0, 30^0, 45^0, 90^0, 120^0 190^0

From your earlier knowledge about angles, classify the following angles as acute angle, right angle, obtuse angle and reflex angle. These are some of the examples; angles are; 15^0, 30^0, 45^0, 90^0, 105^0 135^0 165^0, 90^0, 120^0, 220^0,330^0,

We are now going to learn how to construct angles using a ruler and pair of compasses. By bisecting some angles we shall be able to divide angles into smaller equal parts.

Example: Let you construct an angle of 90^0,

Step 1: draw a line of about 9 cm and mark off point A

Step2: open a pair of compasses 6 cm wide against a ruler.

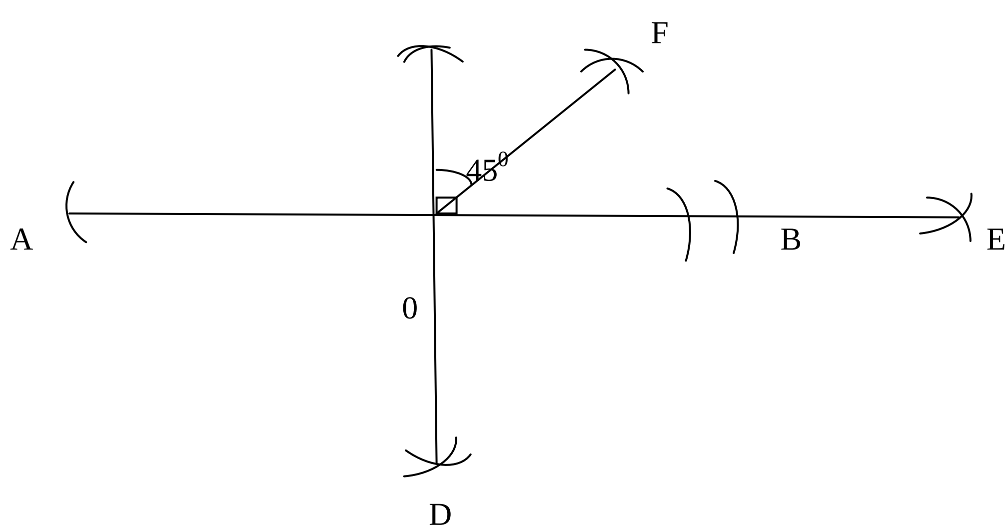
Step 3: put the compass needle at A and make an arc on the line with the pencil to get a point B.

Step 4: Take the pair of compass and open it again to about 5 cm wide.

Step 5: put the compass needle at A and make one arc below AB and another one above AB.

Step 6: without adjusting the compasses, repeat step 5 but with the compass needle at B. the arcs should cut each other.

Step 7: join the arcs with a straight line. Measure the angle formed by AB and the straight line joining the arcs. You should have a right angle as shown in the figure below



We are now going to construct an angle of 45^0 by bisecting the 90^0 .

Step 1: Place the needle of the compass at 0.
Step 2: With 0 as centre, using a radius of 4 cm, draw an arc to cut 0B at E and OC at C. keeping the same radius, place the compass needle at C and E to mark two arcs that intersect at F.
Step 3: Join OF and measure angle E0F with a protractor. What do you get?

Try this activity in your exercise book

Activity 9.1

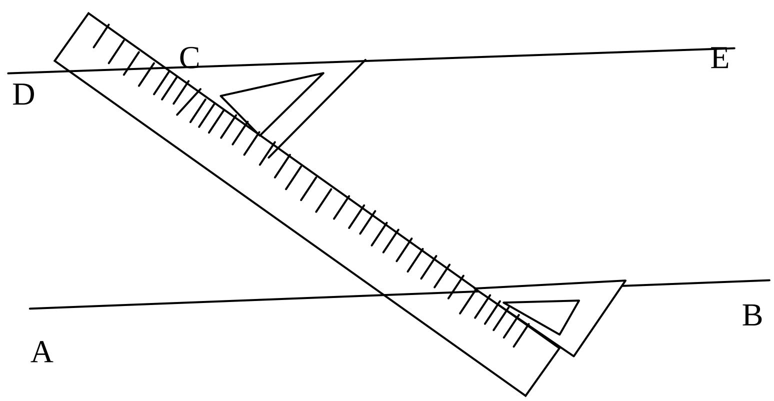
Take time to read primary school mathematics course books (book 5 – Book 7). Follow the procedures given in these books to construct angles of 60^0, 120^0, 30^0, How can you use bisection to construct angles of 15^0, 75^0, $22\ ½^0$, and 135^0 using a pair of compass, a ruler, and a pencil only?

b) Parallel and perpendicular lines

Carefully look at a door, exercise book or ruler near you. Identify any parallel lines. What is the angle between parallel lines? Also identify perpendicular lines. What angle is made by perpendicular lines?

Here is a method for drawing parallel lines.
You need a ruler and a set square.

Step 1: draw a line AB of about 10cm
Step 2: place a ruler and a set square as shown, along AB;
Step 3: slide the set square along the ruler so that it passes through C;
Step 4: Draw line DE along the other side of the set square to give a line parallel to AB.



Visit the library and read about drawing a perpendicular from a point to a line. Make notes and use them to construct a perpendicular form a point to a straight line. Share your finding with the tutor.

c) Drawing plane figures.

We shall use the knowledge of constructing angles, parallel lines and perpendicular lines to draw plane figures. List the plane figures you know. Let you discuss their properties and then draw them in your exercise book.

(i) Square

Measure the sides and angles of a square in a text book. What do you notice? Draw a square.

(ii) Repeat step (i) for a rectangle.

(iii) Using the procedure for constructing an angle of 60^0, construct an equilateral triangle.

(iv) Discuss with your classmates how you can draw a circle.

From the drawings and properties of the plane figures above, find how many times each one can be folded into two equal parts. What name do we give to the lines of folding? Determine the number of lines of symmetry for the plane figures in (i) to (iv) above.

Congratulations for completing topic 1.

May you take a walk?

9.7.2: PYTHAGORAS' THEOREM

Dear student you are welcome to topic 2.

a) Introduction to Pythagoras' theorem.

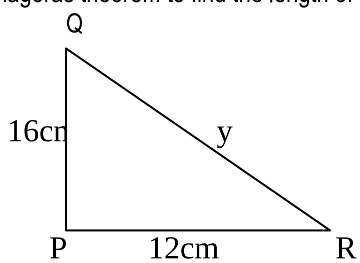
On graph paper draw a triangle ABC with AB=4cm, BC = 3cm and AC = 5 cm. you may construct ABC using a ruler and pair of compass. Measure angle ABC. What name is given to triangle ABC and the sides AC, AB, BC?

Draw a square on each of the sides AB, BC and AC. Write down the area of each of these squares. What relationship exists between the areas drawn on the sides of each side? Now suppose AB = b cm, BC = a cm and AC = c cm, write down a relationship between the areas. I hope you have found that;

$$a^2 + b^2 = c^2$$

Which is Pythagoras's theorem.

Use Pythagoras theorem to find the length of the unknown side in a triangle PQR below



Pythagoras' theorem states that $QR^2 = PQ^2 + PR^2$

hence, $y^2 = 16^2 + 12^2$

$= 256 + 144$

Then $y^2 = 400$

$\sqrt{y^2} = \sqrt{400}$

$\therefore y = 20$ cm

(b) Application of Pythagoras' theorem

How is Pythagoras theorem useful?
Pythagoras' theorem is useful in many activities. Some of them include;

Note.

- Finding direct distances between two given places.
- Measuring foundation of square or rectangular building.
- Finding heights of trees
- Constructing or ma mixing playgrounds.

Study the example below:

The following examples illustrate the application of Pythagoras' theorem.

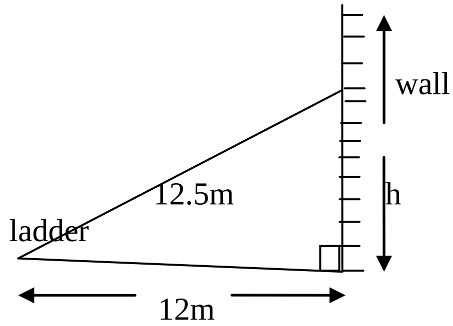
Example 1

Lutwama leans a ladder 12.5 m long against a vertical wall. If the foot of the ladder is 12 m from the wall, how far up the wall does it reach?

Solution

Let the ladder reach h metres

since:
$12.5^2 = 12^2 + h^2$
Then $12.5^2 - 12^2 = h^2$
$= 156.25 - 144 = h^2$
$\sqrt{12.25} = h^2$
$3.5 \text{ m} = h$
$\therefore h = 3.5\text{m}$

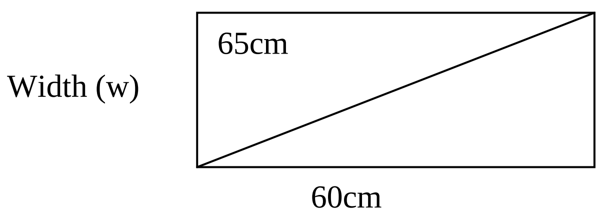


Example 2

A rectangular sheet of length 60 cm has a diagonal of length 65cm. determine its width.

Solution:

Step 1: representing the above information on the diagram.



Step 2: Let width be w cm length be Lcm, diagonal be d cm; then $d^2 = w^2 + L^2$

$65^2 = w^2 + 60^2$
$65^2 - 60^2 = w^2$
$625 = w^2$
$\therefore w = 25 \text{ cm}$

Activity 9.3

a. Budima is 12 km due west of Buwuma. Buwegnge is 9 km due south of Buwume. How far is Buwenge for Budima?

b. A boda boda cyclist leaves a road junction on a road running southwest at the same moment as a taxi crosses over the junction on a road going Northwest. The boda boda cyclist moves at an average speed of 35km/h. while the taxi moves at 94 km/h what is the distance between the two after

i. A quarter an hour?
ii. Half an hour?
iii. One hour?

c. The area of a square garden is 625^2m^2. Find the length of its diagonal.

Let you check for the correct answers with ones at the end of the unit.

Bravo for completing topic 2.

9.7.3. REGULAR POLYGONS

Now let you look at topic 3.

a) Properties of regular polygons.

What is a regular polygon? We know that a polygon is a many sided figure. The sides may be three or more. But what makes it regular? Indeed a regular polygon has equal sides and equal angles. A polygon can be divided up into triangles if one vertex of the polygon is joined to all the other vertices. Work in pairs and copy and complete the following table.

Name of polygon	**Number of sides**	**No. of triangles**	**Angle sum of interior angles**
Triangle	3	1	180^0
quadrilateral	4	2	-
Pentagon	5	-	-
Hexagon	6	-	-
Heptagon	7	-	-
Octagon	9	-	-
Nonagon	9	-	-
Decagon	10	-	-
Any polygon	n	-	-

I hope you have discovered the formula that gives the angle sum of the interior angles of a polygon with n sides. Can you now add a column to the table above to show the size of an interior angle of a regular polygon with sides from 3 to n? we shall use this information to construct regular polygons.

Activity 9.4

Copy and complete the following table

Polygon	Number of sides	Angle sum of interior angles	Size of interior angle	Size of exterior angle
Triangle	3	180^0	60^0	120^0
Square	-	360^0	-	90^0
Pentagon	-	-	-	-
Hexagon	-	-	-	-
Heptagon	-	-	-	-
Octagon	-	-	-	-
Nonagon	-	-	-	-
Decagon	-	-	-	-
In general	n	-	-	-

Compare your answers with the ones at the end of the unit.

a) Construction of regular polygons

You have already constructed an equilateral triangle when drawing plane figures. Revise the procedure and construct an equilateral triangle of side 5 cm. we shall construct regular polygons with more than three sides. First work in pairs.

The steps given below will work for any regular polygon but we shall use them specifically to construct a regular pentagon.

Step 1: find the size of the exterior angle of the regular polygon from the last sub topic, you found the

$$\text{Size of an interior angle} = \frac{\text{angle sum of interior angles}}{\text{Number of sides of a polygon}}$$

$$\text{Exterior angle} = 180^0 - \text{interior angle}$$

Step 2: Draw a line AB = 5cm

Step 3: Use the protractor to measure the inferior angle at point A. Draw the angle and mark off 5 cm at a point you will call.

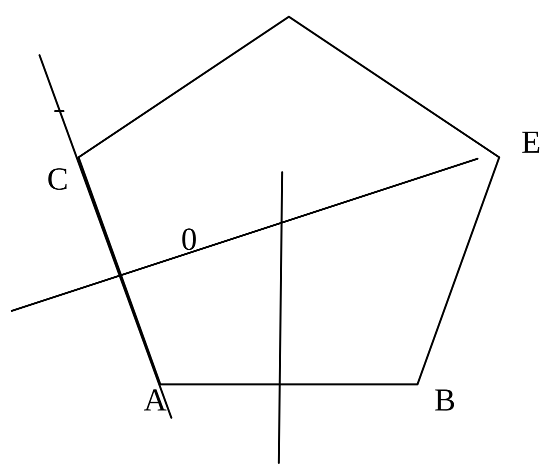
Step 4: Draw perpendicular bisectors of AB and AC. Do this by using radius of 3 cm, centre A, make arcs on either sides of AB. Repeat with centre B. the arcs must meet in. join the intersections with a straight lines. Then do the same using points A and C to bisect AC. Where the two perpendicular bisectors meet will be the centre of the regular polygon. Name the point 0

Step 5: With centre 0, open your compass to have the pencil point at C. Draw a circle to touch points C, A and B

Step 6: Mark off from B equal distances of 5 cm along the circumference and join them to form a regular polygon.

Below is the regular pentagon with interior angle 108^0

D



Now draw a square, regular hexagon and regular octagon. Follow through the steps shown above carefully. You may start by tabulating using columns of polygon, number of sides, size of exterior angle and size of interior angle.

Draw the following in your exercise book.

Activity 9.5

Construct a regular hexagon

Show your work to your tutor.

Thank you for completing Topic 3

Let you now move to the next topic

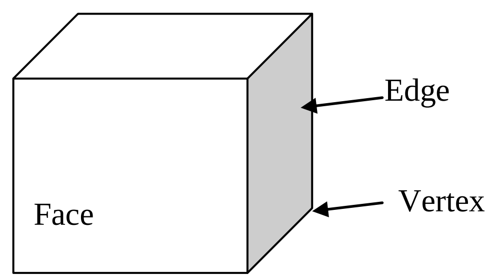
9.7. 4. SOLID FIGURES

Hello student you are welcome to topic 4. Let you study,

a) Properties of solid figures

The solid we wish to study are the cube, cuboid, cylinders, cone and pyramid. Get solids (boxes, cans, containers) that are in the shape of the solids above. We shall determine the properties of each solid then give it a name. Answer the following questions about each solid you have in front of you.

- How many faces (sides) has the solid got?
- What shape are the faces?
- How many edges have the solid?
- How many vertices have the solid?

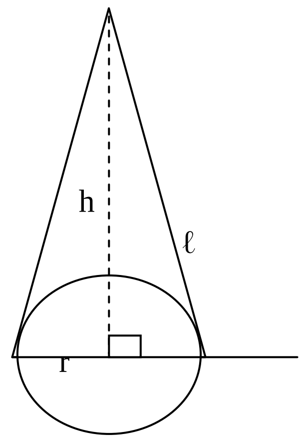


Parts of a solid

The names and properties of some of the solids are summarized in the table below.

Solid	**Number of faces**	**Number of edges**	**Number of vertices**
Cube	6 square faces	12	9
Cuboids	• 6 rectangular faces • 2 opposite faces are equal	12	9
Pyramid	5, one of them is base; square or rectangular	9	5

Why is the cone and the cylinder missing in the table? Describe their shapes in your own words. I hope you agree with me that a cylinder has circular top and bottom faces. They are not polygons. It will roll if placed with its curved face on a horizontal surface. Wheels are examples of a cylinder. A cone on the other hand has a circular bottom and a pointed top (or vice versa). We can best demonstrate it by drawing one here.

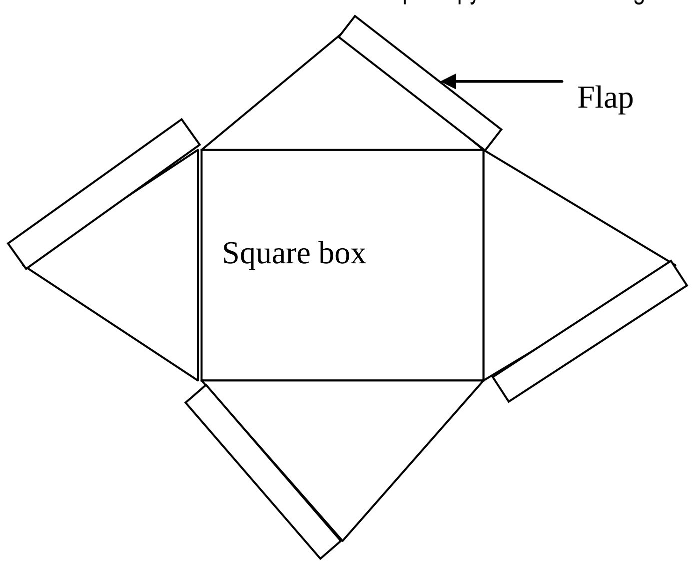


The cone has a height from the tip to the center of the base, h, a slant edge L and circular base has radius r.

Look out in your environment for objects that are shaped like the solids above. Collect as many as you can and use them to study the next subtopics.

b) Nets of school figures.

Get to your collection of solids. Open the edges of boxes or carefully cut open some containers along the edges of their shapes. When you draw figures similar to what you see, you have what we call "nets of solids". When you fold the net you ought to form the solid. The nets increase your understanding of the properties of the solids. They also make it easy for us to talk about the surface areas of the solids and their volumes. Here we show the net of a square pyramid. The triangular faces have isosceles triangles



Activity 9.6

Now draw the nets of a cube and cuboid, Tetrahedron. Fold the nets to form the solids. Keep the solids for use in the next subtopics.

Show your work to your tutor.

c) Total surface Area of solid figures

Surface area is the total external area of the faces that make up a solid.

(i) Cube

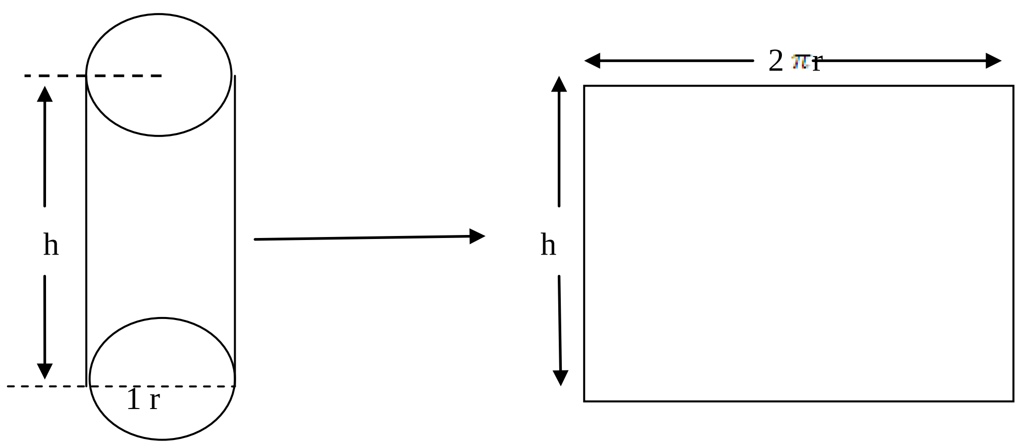
What do you know about the faces of a cube? Right, since all six faces are square, what is the surface area of a cube with side x cm? find the surface area of a cube whose sides are 4 cm.

(ii) Cuboid

Look at the net of a cuboid one more time. How can you find the surface area of a cuboid measuring L cm by w cm by h cm? we said two opposite faces are equal. This gives; S.A = 2(Lxw + Lxh + wxh) = 2(Lw+Lh+hw)cm^2. Find the surface area of a cuboid whose sides are a 4 cm by 7 cm by 10 cm.

(iii) Cylinder

Open out the curved surface of a paper cylinder. What shape do you get? What are the measurements of this shape? Study the figure below.



Cylinder with curved surfaces, two circular ends

Cylinder with its curved surface opened out. Why is the length = 2 π r?

From the figure find area of the curved surface and the area of each circular end.

Do you agree that surface area of a cylinder = $2\pi rh + 2\pi r^2$?

$= 2\pi r\,(h + r)$.

Now determine the total surface area of a cylinder whose height is 12 cm and radius 9 cm.

(iv) Cone

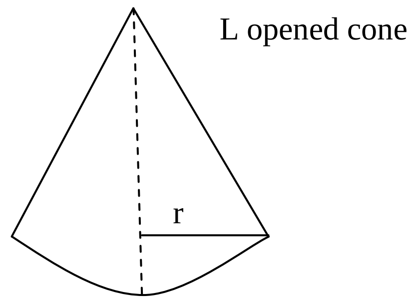
What will an opened out cone look like?

Area of cone = Area of circular base + area of curved surface.

Area of circular base = πr^2
Area of curved surface = $\pi r \times L$; remember r is radius of base and L is length of slant surface. See figure below.

$\therefore$ Area = $\pi r (r + L)$

Find the surface area of a cone whose radius is 4 cm and its height is 6 cm.

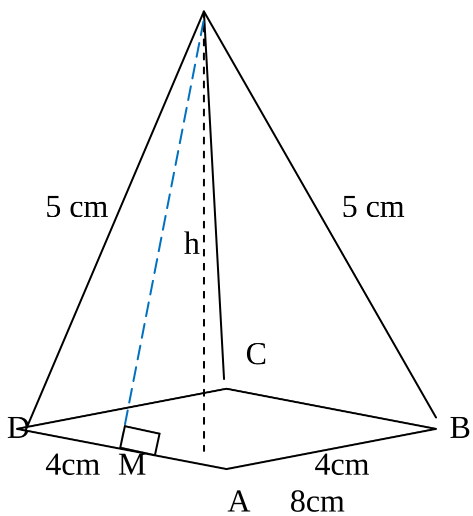


(v) Pyramid

From the net of a pyramid, how can you find total surface area? This is area of base plus area of the four triangles

The figure below shows a square based pyramid VABCD. Find the total surface area of the pyramid.

v



Area of base = $(8 \times 8)\ cm^2 = 64\ cm^2$
In triangle ADV, height $VM^2 + DM^2 = DV^2$

$VM^2 + 16 = 25$

$VM^2 = 9$

$\therefore VM = 3$ cm

Area of triangle ADV = $\frac{1}{2} \times 3 \times 9 = 12\ cm^2$
Area of 4 triangles = $(4 \times 12)\ cm^2 = 49\ cm^2$
$\therefore$ Area of pyramid = $(64 + 49)\ cm^2 = 112\ cm^2$

Try the following activity in your exercise book

Activity 9.7

1. Find the surface area of a cube whose sides are 14 cm.
2. The surface area of a cuboid is 240 cm^2.. If its length is 5cm and its width is 6 cm find its height.
3. A match box measures 64mm by 39mm by 16mm. fid its surface area.
4. A model in the shape of a cone have a radius of 35mm and a height of 45mm. determine the length of the slant side, hence find the surface area of the model.

Share your answers with your colleague

Volume of solid figures

(i) Cube

How many unit cubes will be required to fill a cube whose dimensions are x cm by x cm by x cm?
We shall fill x unit cubes along the length of the cube
x unit cubes along the width of the cube
And x unit cubes along the height of the cube
This gives us a total of x^3 (units) ^{3}.

If the cube has an edge of 7 cm, its volumes will be (7 x 7 x 7) cm^3 = 343 cm^3.

$$\therefore Side\ x\ side\ x\ side = (s^3)$$

(ii) Cuboid

Now suppose instead of a cube, we have a cuboid a cm by b cm by c cm; how many unit cubes will fill the cuboid? Volume of the cuboid is (axbxc) cm^3.

If we have a cuboid with volume V = 400 cm^3., length = 10 cm and width 9 cm, what is the height of the cuboid?

V = Lx w x h
400 = 10 x 9 x h
$\therefore$ h = $\frac{400}{9\cancel{0}}$ = 5 c$\cancel{m}$

(iii) Pyramid

Make three equal nets of a pyramid with square base 6 cm and having congruent equilateral triangles. Join the sides of each net so that you have three pyramids, then fit the three pyramids together to form a cube.

What is the length of the side of a cube?
What is the volume of the cube?
What is the volume of one pyramid?

In general, Volume of a pyramid c $^1/_3$ x base x perpendicular height..

Now find the volume of a pyramid whose base is 9 cm by 12 cm and height is 15 cm.

(iv) Cylinder

Just like for the pyramid, volume of cylinder
= base area x height
= πr^2 x h.

A cylinder of radius 7 cm and height 12 cm will have a volume V = $\frac{22}{7}$x 7 x 12 cm^3
= 1949 cm^3

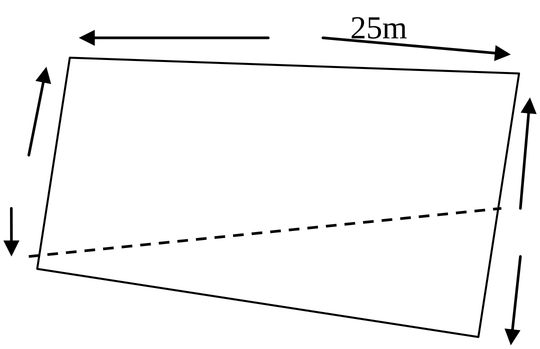
(v) cone

A cone may be considered as a pyramid with a circular base. We then have volume of cone = 1/3 x base area x height = $^1/_3\, \pi r^2 h$.
Try this problem with a friend.
A cone with base radius 9cm and height 14cm will have volume v=⅓x$\frac{22}{7}$x9x9x7
$\therefore$v = 594 cm^3

Activity 9.9

1. Find the volume of a cylinder with radius 24 mm and height 1 m.

2. The diagram on the right shows the cross section of a swimming pool 25 m long. The water is 1.2 m deep at the shallow end and 2.7m at the deep end. If the pool is 10m wide find the volume of water when the pool is full.

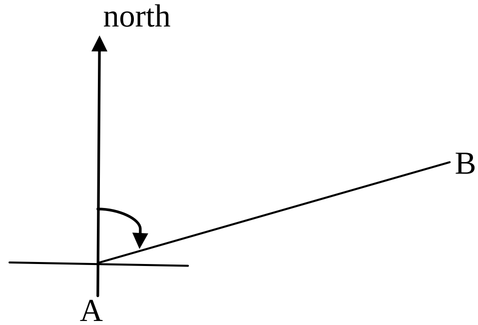


3. A paraffin funnel in the shape of a cone has a base radius 6 cm and height 19 cm find its volume

9.7.5: BEARINGS AND SCALE DRAWING

a) Bearings

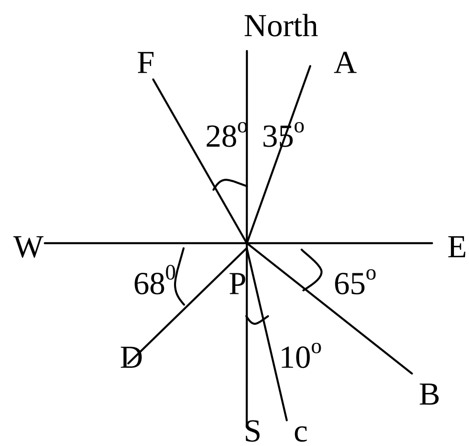
When we wish to find the direction of one place say B from another place A, we measure the angle between a north line drawn at A and the line along which B lies. See the example below.



The angle measured in a clockwise direction from the north gives what we call the bearing of B from A. We can say that the bearing of B from A is 090^0 or B is on a bearing of 090^0 from A. what will be the bearing of A from B? Find it by drawing a North line at B and using angle properties you already know. Is it 260^0? Discuss the result with your classmates.

In the diagram on the right find the bearing of
(i) A (ii) B (iii) C (iv) D

(v) F from P



b) Scale Drawing

Draw a line and measure AB 12 cm Suppose you now use 1 cm to represent 5 cm. what will be the real length of AB?

Scale Drawing: 1 Cm represents 5m
Drawn length = 12 cm
Actual length = 12 x 5 m
Actual length = 60m

As you realize, it would not have been possible for you to draw the actual length of 60m in your exercise book. That is why we use a scale, which uses smaller units, therefore smaller diagrams to represent reality. What is the difference between a scale drawing and a sketch?

Activity 9.9

1. A rectangle measuring 9.5 cm by 6 cm is drawn to represent a garden. The scale used is 1 cm to 5m. what is the actual;

   (i) Length
   (ii) Width
   (iii) Perimeter
   (iv) Area of the garden?

2. A patrolling aircraft sets out on a course of 125^0 for 20 km, then turns to a course of 290^0 for 30 km and finally follows a course of 210^0 for 10 km. Use scale drawing to find how far south and how for west the aircraft is from its starting point.

9.9 UNIT SUMMARY

In this unit you have learnt about angles, parallel lines, perpendicular lines, plane figures and lines of symmetry. You used this knowledge to identify and construct regular polygons, identify solids and find their area and volume.

9.9 GLOSSARY

Bisect :Divide a line or angle into two equal parts

Polygon : Many sided closed figure

Regular polygon :A polygon with all sides equal and all angles equal

Surface Area : The area of the faces or planes that form the solid

9.10 NOTES AND ANSWERS TO ACTIVITIES

Activity 9.1

When you construct an angle of 60^0, begin with AB = 5 cm say, using same radius with A as centre make an arc above AB at C, repeat with centre at B. this arc will interest the first one. Measure angle CAB. Using angles on a straight line; you also have the 120^0 drawn.

Bisect 60^0, what do you get? Bisect 30^0 what do you get? Can we have $75^0 = 45^0 + 30^0$?

Activity 9.2

Read primary 6 mathematics course books and review how to construct perpendicular and parallel lines. Practice the procedures until when you can work without reading the procedure.

Activity 9.3

1. 15 km
2. (a) 22.75 km (b) 45.5 km (c) 91 km
3. 35.4 km

Activity 9.4

Angle sum of interior angles = $(n – 2) \times 190^0$
Where n is the number of sides.
Size of interior angle = $\frac{\text{angle sum}}{n}$ since all angles are equal.
Size of exterior angle = $\frac{360}{n}$

Activity 9.5

Use table in activity 9.4 to get interior angle and exterior angle.

Activity 9.6

You may begin by opening up boxes in the shapes of a cube and cuboid.

Activity 9.7

1. 1176 cm^2
2. 9 cm
3. 9129 mm^2
4. 57 mm, area = 10120 mm^2

Activity 9.9

1. r = 0.024 m, V = 0.0019 m^3
2. Area = (25 x 1.2) + ½ x 25 x 1.5 = 49 .75m^2
   Volume = Area x width = 497. 5m^3
3. V = 1/3 πr^2 h = 679.24 cm^3

Activity 9.9

1. Length = 42.5m, width = 30m, perimeter = 145m, area = 1275m^2
2. 15 km south and 19 km west

9.11 END OF UNIT EXERCISE

1. Using a ruler and pair of compass draw aline AB = 9cm. construct a perpendicular at B.

2. The sum of the interior angles of a regular polygon is 2160^0, find;
   a) The number of side the polygon has
   b) The size of its interior and exterior angle. Construct the polygon.

3. Find the surface area of a cylindrical water tank whose diameter is 1m and height is 4.5m.
4. The volume of a cone is 1320 cm^3 and its height is 35cm. Find its base radius.

9.12 SELF – CHECKING EXERCISE

You have now completed unit 9. Demonstrate your competence by ticking the column of the listed learning outcomes that reflects your learning.

LEARNING OUTCOME	NOT SURE	SATISFACTION
1. I can use Pythagoras theorem to solve problems		
2. I can construct angles and regular polygons		
3. I can make nets of solids		
4. I can find surface area and		

volume of solids		

If you have placed a tick in the "NOT SURE" column, read the information in the unit again to reinforce your learning.

CONGRATULATIONS

9. 13 REFERENCES FOR FURTHER READING

1. Green, C. (1999), Mathematics Revision and Practice for UCE. Oxford University Press.
2. National Curriculum Development Centre (1997). Secondary School Mathematics Book 3 Macmillan.

<u>End of the unit</u>

UNIT 10: INTEGERS

10.1: INTRODUCTION

Dear student,

You are most welcome to this unit 10. This unit introduces you to related concepts and issues such as negative and positive numbers directed numbers, ordering integers, inverses, number line, inequalities, solution sets, solving problems in our daily situations and how to teach these concepts to primary school children.

10.2 CONTENT ORGANIZATION

Hello student, in this unit you are going to cover the following topics as indicated in the table below;

No.	Topic	Sub-topic
1.	**Introduction To integers**	a. Definition of integers b. Key concepts and issues in integers
2.	**Positive and negative integers**	a. Number line b. Marking out integers
3.	**Ordering integers**	a. Ordering
4.	**Comparing integers**	Inequality symbols a. Greater than b. Less than
5.	**Addition of integers**	a. Addition using a number line b. Addition using inverses
6.	**Subtraction of integers**	a. Subtraction using a number line b. Subtraction by adding the inverse
7.	**Multiplication of integers**	a. Multiplication of integers using a number line b. Multiplication of integers using a table
8.	**Divisions of integers**	a. Division of integers using a table
9.	**Properties of integers**	a. Properties of zero and one b. Closure property c. Commutative property d. Associative property e. Distributive property
10.	**Inequalities**	a. Inequality symbols b. Sets of numbers c. Representing inequalities on the number line.
11.	**Solution sets of inequalities**	a. Solving inequalities b. Solving and representing inequalities on a number line c. Representing inequalities on a graph

12.	**Application of integers in solving problems**	a. Using temperature scales b. Using number line c. Weather forecasts

10.3 LEARNING OUTCOME

The student will be able to;

i) Extend knowledge of number concepts to include integers.
ii) Apply the concept of integers in everyday life situations.
iii) Teach integers in the primary schools.

10.4 COMPETENCES

a. Define integers
b. Write positive and negative integers.
c. Order integers
d. Compare integers
e. Add integers
f. Subtract integers
g. Multiply integers
h. Divide integers
i. Find additive inverse of an integer
j. Give examples of inequalities.
k. Solve and represent solution sets of inequalities on number lines.
l. Apply integers in solving problems requiring the knowledge of integers.
m. Demonstrate how to teach integers in the primary schools.

10.5 SUBJECT ORIENTATION

This unit is meant to help the student to explore related concepts and issues in integers. There is conceptual leap for children in coming to terms with the concept of negative integers. As a symbol, a negative number may be met when using a calculator or seen in a television weather forecast, but there are no props (like fingers) to help pupils visualize a negative integer. The temperature scale would be an obvious model to use but children. Find it difficult to read scales or even to access them. Therefore to help children to conceptualize negative numbers, there is need to expose them to these appliances both at home and school.

10.6 STUDY REQUIREMENTS

To be successful in studying this unit you are required to have plenty of materials. These should include thermometer(s) , number line, graph paper, ruler, cards, calculator. You also need Primary School Maths Course books, Secondary school Maths Bk. 2 by E. Karuhije etc (2007). Learning to teach Number. A handbook for students and teachers in the primary school by Len Frobisher etl (1101010). In addition you need a pen and an exercise book.

10.7 CONTENT AND ACTIVITIES:

10.7.1: INTRODUCTION OF INTEGERS

Hullo student,

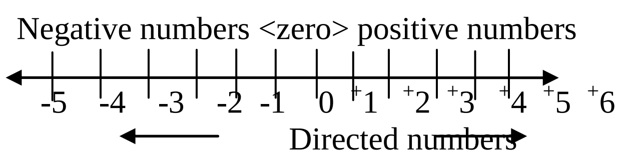
You are welcome to the 1st topic of integers.

Can you define integers?

a) **Definition of integers**

An integer is a ‘positive’ or ‘negative’ whole number together with zero.

See figure below;



Positive and negative integers as reflections in zero.

Activity 10.1

1. With help of a number line show a set of;

(i) Natural numbers less than the (10).

(ii) Directed numbers less than 10.

2. What do you notice about natural numbers and directed numbers?

Well Done!

10.7.2: POSITIVE AND NEGATIVE INTEGERS

Hello student,

Let's now explore more about integers by visualizing it on a number line.

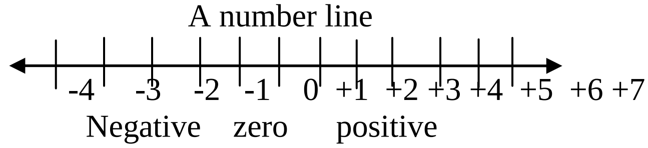
a) **Marking out integers**

The numbers to the right of zero are called **Positive integers** and so we refer to each of the number in the set {1,2,3,4,5,6,7…………..} as **positive** . The numbers to the left of zero are **negative integers** and so we refer to each of the numbers in the set {……….$^-7$, $^-6$,$^-5$,$^-4$,$^-3$,$^-2$,$^-1$} **negative**

The number zero has a special position between positive integers and negative integers. Zero is a whole member representing an empty set. Therefore zero is used to mark the point from where we measure out each unit lengths to the right and left of a number line

See figure below:

A number line



Can you read the set of integers represented in the number line above?

10.7.3: ORDERING INTEGERS

Hello student,

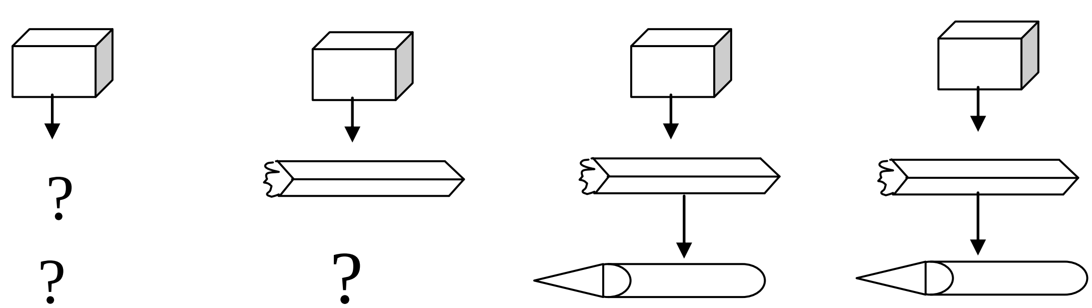
Do you still remember ordering concrete objects in your primary school?

Let's find out

Ordering numbers

Activity: 10.3

Get 4 boxes, 3 books and 2 pencils. Spread out the 4 boxes in a row. Match the books in front of the boxes, one to each box. One box will have no book. Match each book with a pencil. One book will not have a pencil. So the boxes are the greatest in number, the pencils the least.



1. Which group has the greatest number of objects?
2. Which group has the least number of objects?
3. Are there more books or pencils?
4. How many more books are there?

NB: you should be able to say:

There is one more book than pencils.
There is one less pencil than there are books.

Are there more boxes or books?

Activity 10.4

Identify pictures of 3 different objects in the classroom and match as in activity 10.3.

Remember to ensure that the sets are “one more” or “one less” the other.

Well done!

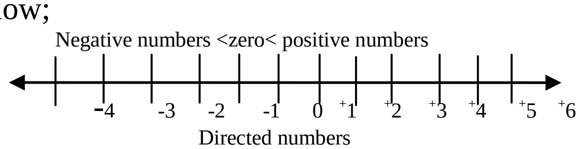
10.7.4: COMPARING INTEGERS

Hello student,

Let's compare integers with help of a number line.

Comparing integers

Study the figure below;



- **Inequality symbols;**

> **Read;**
> - Sentences involving phrases such as 'more than' and 'less than' can be written using mathematical symbols.
> - Their meanings are given below;
> - $>$ stands for more than or greater than
> - $<$ stands for less than

e.g. if there are more than 20 people in a room.
- if n is the number of people in the room, we write $n > 20$
- if he was driving at less than 60 km/hr

If his speed was S km/hr we write $S < 60$.

Activity 10.5

Rewrite the following using inequality signs;

a) T is less than 4
b) X is more than 17
c) F is less than -8
d) Y is greater than -3

Well done!

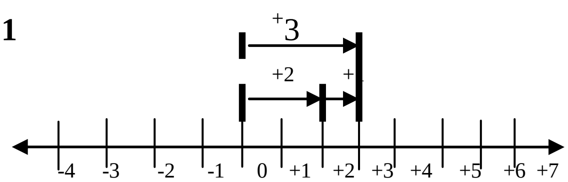
10.7.5: ADDITION OF INTEGERS

Can I add using a number line?

Let's find out

a) Addition using a number line.

Example 1

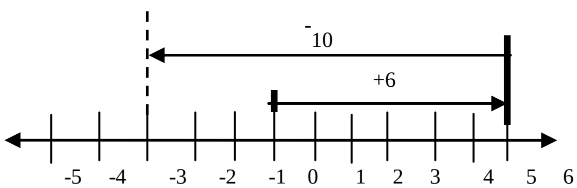


The operation shown on the numberline can be expressed as ${}^{+}2 + {}^{+}1 = {}^{+}3$

Example 2

What is +6 + -10?

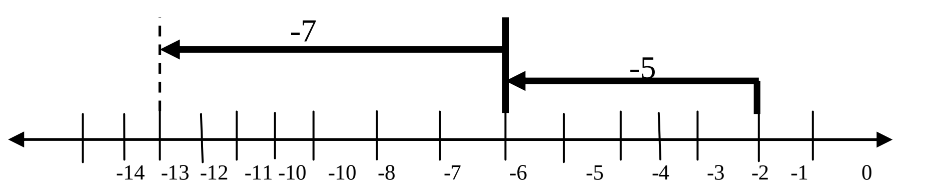
Solution:



${}^{+}6 + {}^{-}10 = {}^{-}4$.

Example 3:

What is ${}^{-}5 + {}^{-}7$?



-5 + -7 = -12

Activity: 10.6

Use number lines to work out the following:

1. $^{+}12 + {}^{+}8$
2. $+16 + {}^{+}11$
3. $^{+}13 + {}^{-}6$
4. $^{-}6 + {}^{-}10$
5. $^{-}11 + {}^{+}8$.
6. $^{-}8 + {}^{-}5$

b) Addition of integers without using a number line.

Adding a positive integers with another positive integer gives a positive integer,
E.g. $^{+}7 + {}^{+}10 = {}^{+}16$

Example:

$^{-}12 + {}^{-}5 = {}^{-}17$ (since both integers have a negative sign)

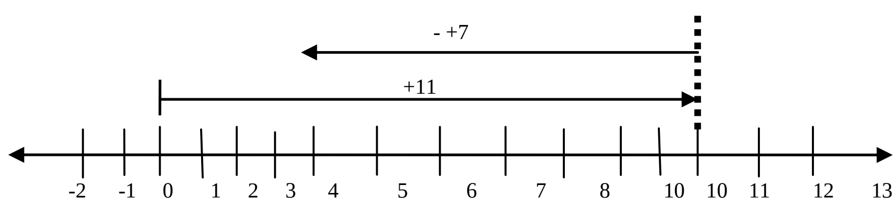
Add the following integers without using number lines

(i) $^{-}7 + {}^{+}11$
(ii) $^{-}12 + {}^{+}10$
(iii) $^{+}20 + {}^{-}60$

10.7.6: Subtraction if integers using a number line

Example:

$^{+}11 - {}^{+}7$



$^{+}11 - {}^{+}7 = {}^{+}11 - 7$
$= {}^{+}4$

Activity 10.7

1. Use number lines to work out the following:

   i) $^{+}14 - {}^{+}8$
   ii) $^{+}20 - {}^{+}12$
   iii) $^{+}24 - {}^{+}15$

2. Work out the following without using number lines

i) $^{+}16 - {}^{+}10$
ii) $^{+}26 - {}^{+}17$
iii) $^{+}47 - {}^{+}30$

c) Subtraction of integers using and without a number line.

Study the following examples:

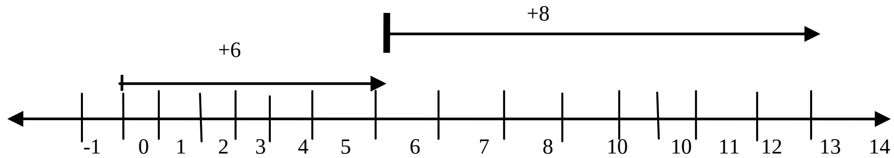
Example: 1;

Work out the following: $^{+}6$ - $^{-}8$

Solution:

Negative minus negative gives positive

Therefore $^{+}6$ - $^{-}$ 8 = $^{+}6$ + 8



From the numberline $^{+}6$ – ($^{-}8$) = $^{+}6$ + 8=$^{+}14$

Example 2:

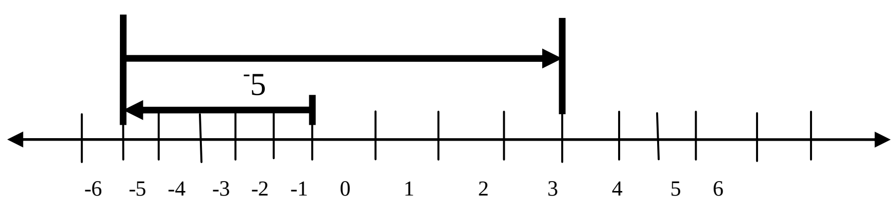
Work out the following -5 - -10

Solution 2:

Negative minus negative gives positive

Therefore -5 - -10 = -5 + 10

$^{+}10$



From the number line $^{-}5$ – ($^{-}10$)
= $^{-}5$ + 10
= $^{+}4$

Activity: 10.8

1. Use the number line to work out the following

i) $^{+}7 - ^{-}10$
ii) $^{+}10 - ^{-}6$
iii) $^{-}14 - ^{-}7$

2. Work out the following without using number lines;

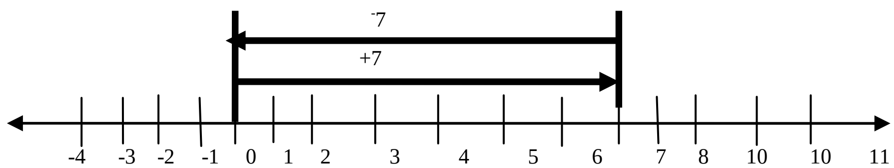
i) $^{+}8 - ^{-}4$
ii) $^{+}13 - ^{-}7$
iii) $^{-}12 - ^{-}24$
iv) $^{-}28 - 18$
v) $^{-}28 - ^{-}18$
vi) $^{-}17 - ^{-}10$

Additive inverses of integers.

Example 1:

What is +7 + $\boxed{}$ = 0?

Let us use is number line to find out

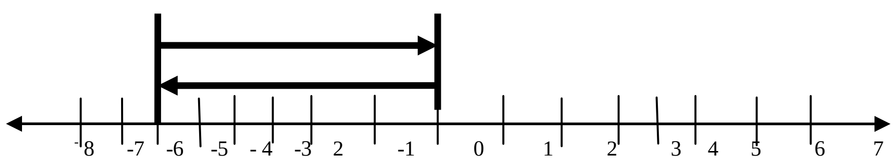


$\therefore\ ^{+}7 + ^{-}7 = 0$

-7 because we have gone 7 steps backwards before we could get to 0 from +7
Therefore the **Additive inverse** of +7 is -7.

Example 2

What is -6 +__ = 0?
We use a number line again to find out.



+6 because we have gone 6 steps forwards before we could get to 0 from -6
Therefore the **additive inverse** of -6 is +6

Activity 10.10

Use number lines to work out the following and then state the additive inverse of the given integer

1. $^{+}5 + \square = 0$
2. $^{+}7 + \square = 0$
3. $^{-}25 + \square = 0$
4. $^{-}21 + \square = 0$
5. $^{-}18 + \square = 0$

Write the Additive inverse of the following integers

6. $^{+}6$
7. $^{+}20$
8. $^{-}20$
9. $^{-}34$
10. $^{+}17$

10.7.7: MULTIPLICATION OF INTEGERS

Lets multiply integers

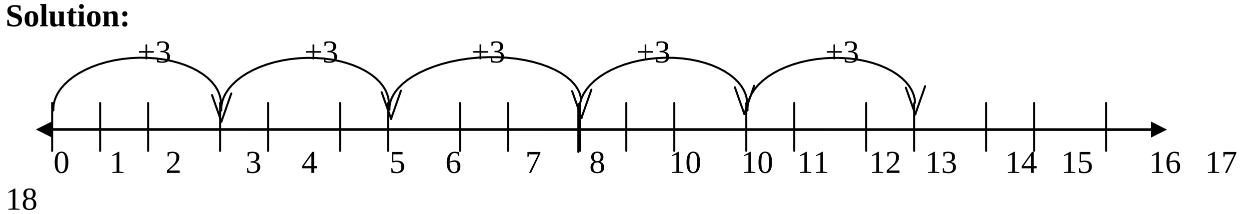
a) Multiplication of integers using a number line.

Multiplication of a positive integer by a positive integer.

Example 1:

Use a number line to find the value of $^{+}3 \times {}^{+}5$

Solution:



+3 x+5 = +15

> **Note:** Multiplication of a positive integer by a positive integer gives a positive integer i.e. +x+=+

Multiplication of a negative integer by a positive integer gives a negative integer

Example 2:

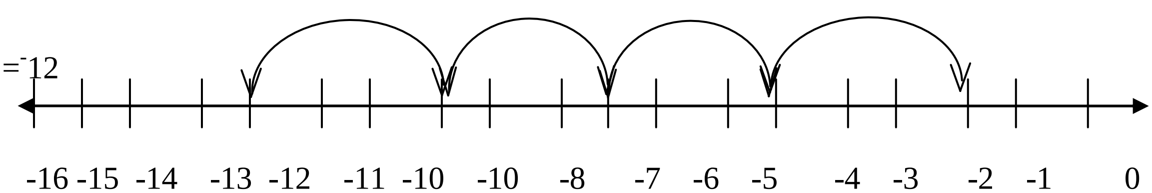
Use a number line to find the value of

$^{-}3$ x $^{+}4$

$^{-}3$ x $^{+}4$ = 3 x 4

$^{-}3$ x $^{+}4$

= $^{-}12$



Note: Multiplication of a negative integer by a positive integer gives a negative integer i.e. $^{-}$x + = -

Activity 10.10

Use number lines to find the values of the following products;

1) $^{+}2$ x $^{+}3$
2) $^{+}4$ x $^{+}2$
3) $^{-}5$ x $^{+}3$
4) $^{-}2$ x $^{+}6$

Use the rules of multiplication of integers to work out the following;

5) $^{+}6$ x$^{+}3$
6) $^{+}15$ x$^{+}5$
7) $^{-}7$ x$^{+}3$
8) $^{-}8$ x $^{+}5$
9) $^{-}13$ x$^{+}4$
10) $^{-}16$ x$^{+}6$

Check your answers with those given at the back of the unit.

Study the following multiplications table of integers

X	$^{-}3$	-2	-1	0	$^{+}1$	$^{+}2$	$^{+}3$
$^{+}3$	$^{-}10$	-6	-3	0	$^{+}3$	$^{+}6$	$^{+}10$
$^{+}2$	-6	-4	-2	0	+2	$^{+}4$	$^{+}6$
$^{+}1$	-3	-2	-1	0	$^{+}1$	$^{+}2$	
0	0	0	0	0	0	0	0
-1	$^{+}3$		$^{+}1$	0	-1	-2	

-2	$^{+}6$		$^{+}2$	0	-2	-4	
-3	$^{+}10$		$^{+}3$	0	-3	-6	

Activity 10.11

1. Copy and complete the table above.
2. Work out the following using the table above.

   i) $^{+}1 \times -2$
   ii) $^{+}2 \times -2$
   iii) $^{+}3 \times -2$
   iv) $^{+}2 \times -3$
   v) $^{+}3 \times -2$

Study all your answers. They should all be negative. This is because;

> Multiplication of <u>positive integer</u> by a <u>negative integer</u> gives a <u>negative integer</u> i.e. + x - = -

Use the table to work out the following;

i) -1×-1
ii) -2×-2
iii) -3×-2
iv) -2×-3
v) -3×-3

> Multiplication of a negative integer by negative integer gives a positive integer i.e. – x - = +

Use the rules of multiplication of integers to work out the following;

i) $^{+}8 \times -2$
ii) $^{+}8 \times -7$
iii) -7×-4
iv) -8×-2
v) -12×-5
vi) -16×-3
vii) -20×-4

Check your answers with those given at the end of the module

10.7.8: DIVISION OF INTEGERS

Let's study division of integers

Division of integers in relation to multiplication.

In the table below, study the multiplication sentences and the corresponding division sentences.

No.	**Multiplication sentences**	**Corresponding division sentences**
1.	$^{+}2$ x -5 = -10	-10 ÷$^{-}5$ = $^{+}2$
2.	-3 x $^{+}5$ = -15	-10 ÷$^{+}2$ = -5
3.	- 2 x -3 = $^{+}6$	-15 ÷$^{+}5$ = -3
		-15 ÷-3 = $^{+}5$
		$^{+}6$ ÷-3 = -2
		$^{+}6$ ÷-2 = -3

For each of the multiplication sentences there are two corresponding **division sentences.**

Activity 10.12

For each of the multiplication sentences, write the corresponding division sentences.

1) $^{+}4$ x -4 = -16
2) $^{+}10$ x -2 = -20
3) -4 x $^{+}5$ = -20
4) -2 x -8 = $^{+}16$
5) -8 x $^{+}6$ = -48
6) -7 x -4 = $^{+}28$

Study the following statements

1. -12 ÷$^{-}4$ = $^{+}3$
   because $^{+}3$ x -4 = -12
2. -12÷$^{+}3$ = -4
   because -4 x $^{+}3$ = -12

Activity 10.13

Write the missing integers to make the statement true

1. -6 ÷-2 = +3
   because +3 x ….. = -6
2. -14÷+7= -2
   because …….. x -7 = -14
3. -20 ÷-5 = +4
   because +4 x ……. = -20

4. -60÷+5 = -12

because ……..x ……. = -60

5. +48 ÷+ 6= +8

because +6 x ……. = +48

Check your answer with those at the end of the module.

10.7.9: PROPERTIES OF INTEGERS

Hello student,

Let's understand properties of integers.

a) Properties of zero and one.

Zero (0) is an identity element because if it is added to any whole number that whole number is unchanged

e.g.

8 +0 = 8 and 0 + 8 = 8
a + 0 = a and 0 + a = a

Does zero have the same property when a is any integer? Yes it has

e.g. -4 + 0 = -4 and 0 + -4 = -4

Therefore from the examples above, zero (0) is called the identity element for addition for integers.

One (1) has an identity element because if it is multiplied by any integer, the intger is unchanged.

e.g. 5 x 1 = 1 x 5 = 5
a x 1 = 1 x a = a

Does one (1) have the same property when a is any integer? Yes it has

e.g.

3 x1 = 1 x -3 = -3

So, a x 1 = 1 x a = a, where a is any integer. Therefore, 1 is called the identify element for multiplication of integers

b) Closure property

A set of whole numbers is closed under addition if the sum (result) of any two or more of the members is also a member of the set. Let us find out whether when you add integers, the same is always an integer.

e.g. +7 + +2 = $^{+}9$

$+6 + -2 = {}^{+}4$
$-8 + +2 = -6$

Therefore the set of integers is closed under addition because whenever you add integers, the sum is always an integer.

Is the set of integers also closed under multiplication?

Examples: ${}^{-}3\text{x-}4={}^{+}12$
${}^{+}2\text{x-}3= {}^{-}6$

The set of integers is also closed under the operation of multiplication, because the product is always an integer.

Since a + b is an integer if a and b are integers, the set of integers is closed under additions, since ax b is an integer, if a and b are integers, then the set of integers is closed under multiplication.

c) Commutative property

The commutative property means that the order in which two whole numbers are added will not affect their sum. That is a + b = b + a is true when a and b are whole numbers. Is this true when a and b are integers?

Examples 1

For whole numbers, 8 and 7

8 + 7 = 15 and 7 + 8 = 15
8 + 7 – 7 + 8
15 = 15

Examples 2

For integers +5 and -12

5 + -12 = -12 + 5 = -7
5 + -12 = -12 + 5
7 = 7

From these two examples we can conclude that a + b = b + a will also be true when a and b are integers. Hence, the order in which any two integers are added does not affect their sum. This is the commutative property of addition of integers.

Does the order in which any two integers multiplied affect their answer?

e.g. -6 x +5 = -30 and +5 x -6 = -30
- 4 x -2 = +8 and -2 x – 4 = +8

The answers above show that the order in which any two integers are multiplied does not affect their answers. That is a x b = b x a, where a and b are integers. This is the commutative property of multiplication of integers.

Associative property

Any three or more numbers can be grouped in any way you wish, while adding and the answer will remain the same;

Example:

(10 + 5) +3 and 10+(5+3)
14 + 3 = 10 + 8
17 = 17

Therefore for any set of whole number a,b, and c we can write
(a+b) + c = a+(b+c) because addition of integers is associative. This is also true with integers.

For example

$(^{+}4 + {}^{+}6) + {}^{-}2 = {}^{+}4 + ({}^{+}6 + -2)$
+10 + -2 = +4 + +4
+10 – 2 = +4 + +4
8 = 8

d) Distributive property

Addition

a x (b + c) = a x b + a x c is true for positive whole numbers. Is it true for integers

Example

-6 (-2 + +10) = (-6 x -2) + (-6 x +10)
= 12 + -54
= 12 – 54
= -42

Therefore the distributive property with respect to addition of integers holds.

Subtraction

a x (b – c) = ab – ac is true for subtraction of integers.

e.g.

Simplify -5(+15 - +6)
Two methods are possible

Method 1

-5 x (+15 - +6)
-5 x +15 - -5 x +6
- 75 +30
-45

Method 2

-5 (+15 - +6)
- 5 (+10)
- 45

Therefore the distributive property with respect to subtraction of integers holds.
Therefore a x(b – c) = (a x b) – (ax) ss is also true for the set of integers when a,b and c are integers.

10.10: ANSWERS TO ACTIVITIES
Activity 10.5 page 289

(a) $T < 4$
(b) $X > 17$
(c) $F < {}^{-}8$
(d) $Y > {}^{-}3$

Activity 10.6 page 290

Diagram missing

Activity 10.7 page 291

Diagram missing

2. (i) ${}^{+}6$ (ii) ${}^{+}9$ (iii) ${}^{+}17$

Activity 10.8 page 294

1. Diagram **missing**

2. (i) +4 (ii) +6 (iii) +12 (iv) -46 (v) -7

Activity 10.10 page 294

Diagram missing

(6) -6 (7) -20 (8)+20 (9) +34 (10) -17

Activity 10.10 page 296

Diagram missing

(5) +18 (6) +75 (7) -21 (8) -40 (9) -52 (10) -96

Activity 10.12

1) -16 ÷ +4 = -4
   - 16 ÷ -14 = +4

2) -20 ÷ -2 = +10
   -20 ÷ +10 = -2

3) -20 ÷ -4 = +5
   -20 ÷ +5 = --4

4) +16 ÷ -2 = -8
   +16 ÷ 8 = -2

5) -48 ÷ -8 = 16
   -48 ÷ 6 = -8

6) +28 ÷ -7 = -4
   + 28 ÷ -4 = -7

Activity 10.13

1) -2 2)+7 3) +5 4) -12 x +5 5) +8

TOPIC 10: INEQUALITIES

What do you understand by inequalities?

a) Inequality symbols

Sentences involving phrases such as ‘more than’ and ‘less then’ can be written using mathematical symbols.

Their meanings are given below;

>Stands for greater than.
< stands for less than
≥ stands for equal or greater than
≤ stands for equal than

b) Sets of numbers

We can write a set of numbers x, for which x ≤ 10, as ={x : ≤ 10} which is read as the set of numbers x such that x is less than or equal to 10.

{S:S < 7} is the set of numbers S such that S is less than 7. If S is a natural number we can list the possible values of S.

These are 1,2,3,4,5,and 6.

Example:

List the members of the following sets of whole numbers.

a. {x : x ξN, x < 7}
b. {e : e is even, 5 < e < 15}

Solution:

a) {x : x ξN, x < 7}

Therefore the natural numbers less than 7 are ={1,2,3,4,5,6}

b) e: e is even, 5<e<15

Therefore even numbers between 5 and 15 are: {6, 8,10,12,14}

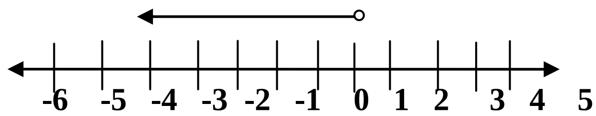
Representing inequalities on the number line.

We can show inequalities on a number line. Remember that all the numbers to the left of a number are less than that number; all the numbers to the right of the number are greater than that number.

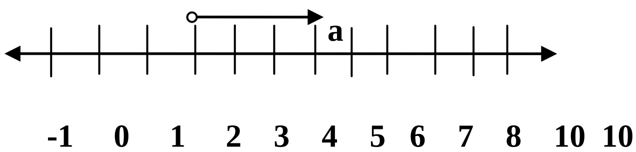
Example:

Show these inequalities on the number lines.

a) **X<1**

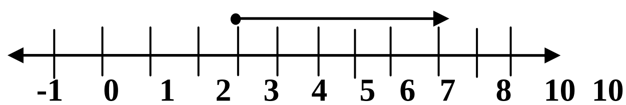


b) a>2

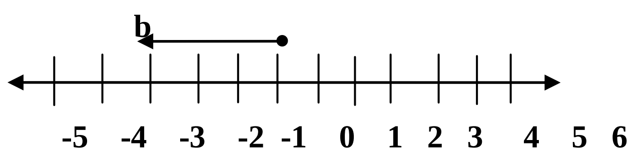


c) **.y ≥ 3**

y



d) **b ≤ 0**



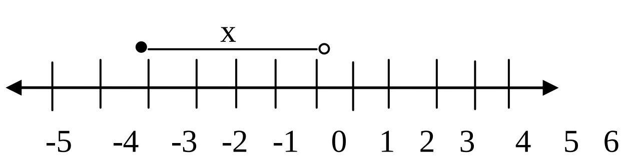
Example 2

Show these inequalities on a number line

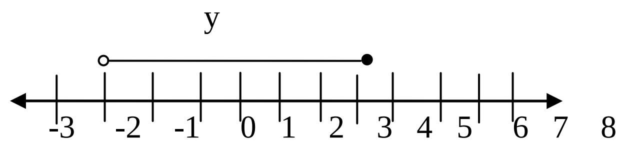
a) $-3 \leq x < 1$
b) $-2 < y \leq 4$

Solution

a) $-3 \leq x < 1$



b) $-2 < y \leq 4$



Activity 10.13

1. Show the following inequalities on the number line

   a) $X < 5$
   b) $Y > 4$
   c) $V > 5$
   d) $-4 \leq 10 < 0$
   e) $2 < b < 7$

Solving inequalities

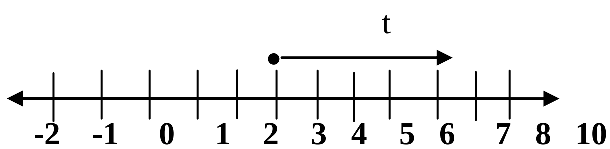
Solve the following inequalities and illustrate their solutions on number lines.

a) $5t - 3 \geq 12$

Solution

$5t - 3 \geq 12$
$5t - 3 + 3 \geq 12 + 3$
$5t/5 \geq 15/5$
$t \geq 3$
The solution set is $\{t: t \geq 3\}$

This can be represented on a number line as shown below:



Activity 10.14

1. Solve the following inequalities and show on number line

   a) $t - 5 \geq -3$
   b) $x + 6 \leq -2$
   c) $3x - 4 \geq -4$
   d) $8 - 2x < 16$

2. Find the solution set for the inequality $x > 4x$ and represent the solution on the number line.
3. List the members of the following sets and show them on a number line

a) {x: x is a natural number and 2x + 3 < 11}
b) {x:x is an even number and 3 < x ≤ 10}
c) {x: x is a prime number and 5 ≤ x < 17}

4. Solve the following inequalities and represent the solution set on a number line.

a) $5 - 4x \geq - x + 8$
b) $2 (2x + 1) < 3 - (2x - 1)$
c) $2 (8 - x) \leq 3(x + 2)$

Check your answers with those at the end of the module.

Congratulations!

References:

1. Secondary School Mathematics Book 2 by Karuhije etl.
2. Functional mathematics book 7 and Book 6 by Nyakairu etl
3. PTE Mathematics education module 2, ME/2
4. Mathematics methods. A resource Book for primary school teachers by Anand Nair and Peter pool Macmillian 110101

NOTES AND ANSWERS TO ACTIVITIES

Activity 10.1

1. (i) Natural numbers less than ten;

   {1,2,3,4,5,6,7,8,10}

   (iii) Directed numbers less than 10;
   {-------------$^{-}3$,$^{-}2$,$^{-}1$,0,1,2,3,4,5,6,7,8,10}
2. Directed numbers include both positive and negative integers whereas natural numbers comprise positive integers.

Activity 10.3

1. Boxes
2. Pencils
3. Books
4. There is one more book.

Activity 10.5

a) $t<4$
b) $x>17$
c) $f<{}^{-}8$
d) $y>{}^{-}3$

Activity 10.6

1. $^{+}20$
2. $^{+}27$
3. $^{+}7$
4. $^{-}16$
5. $^{-}3$
6. $^{-}13$

Activity 10.7

1.(i) $^{+}6$ (ii) $^{+}8$ (iii) $^{+}10$
2.(i) $^{+}7$ (ii) $^{+}10$ (iii) $^{+}17$

Activity 10.8

1. (i) $^{+}16$ (ii) $^{+}16$ (iii) $^{+}3$
2. (i) $^{+}12$ (ii) $^{+}20$ (iii) $^{+}12$ (iv) $^{-}10$ (v) $^{-}10$ (vi) -8

Activity 10.10

1. $^{+}6$
2. $^{+}8$
3. -15
4. -12
5. $^{+}18$
6. $^{+}75$
7. -21
8. -40
9. -52
10. -106

Activity 10.11

2(i) -2 (ii) -4 (iii) -6 (iv) -6 (v) -6

Activity 13

1) -2
2) -2
3) -5
4) $^{+}$5x -12
5) $^{+}$8

Activity 10.14

(1) (a) $t \geq 3$ (b) $x \leq 0$ (c) $x \geq 0$ (d) $x > 4$
(2) $X < 0$
(3) (a) x = {1,2,3,4,5} (b) x = {4,6,8,10} (c) x = {5,7,11,13}
(4) $x \leq 1$ (b) $x < {}^{1}/_{3}$

UNIT 11: MEASURES

11.1 Introduction

You are welcome to this unit 11. This unit introduces you ideas/concepts of length, mass, capacity, time, temperature, speed and money and how to effectively teach measures in primary schools.

11.2 Content Organisation

Hello student in this unit you are going to cover the following topics as indicated in the table below:-

Topic	Details
1. Money	• Definition of money • Denomination of money used in Uganda • Problems involving the use of money • Currencies used in other countries • Exchange rates • Comparison of exchange rates • Exchange of money
2. Length	• Definition of length • Non- standard units used in measuring length • Standard units and other metric units • Conversion of units of length • Addition using units of length • Perimeter • Subtraction using units of length • Multiplication using units of length • Division involving units of length • Problems involving length
3. Area	• Definition of area • Non- standard units in measuring area • Standard units and other metric units • Use metric units to find area
4. Volume	• Definition of volume • Non- standard units and metric units • Measurement of volume • Volume of cubes and cuboids
5. Capacity	• Measuring of capacity • Non-standard units of capacity • Standard units and metric units of capacity. • Volume and capacity • Addition using units of capacity • Subtraction using units of capacity • Using units of capacity in multiplication • Division using units of capacity
6. Mass	• Measuring of mass • Non standard units • Standard units and other metric units of

	measuring mass • Addition of mass • Subtraction of mass • Multiplication of mass • Division of mass • Solving problems involving mass
7. Time	• Meaning of time • Non standard units of time • Standard units of time • Addition involving time • Subtraction involving time • Multiplication of time • Division of time • Problems involving time
8. Temperature	• Meaning of temperature • Non-standard units used in measuring temperature • Standard units used in measuring temperature • Subtraction of temperature • Conversion of degrees in Celsius scale to Fahrenheit scale and vice versa • Solving problems involving temperature
9. Speed	• Meaning of speed • Words which imply speed in everyday life • Relationships between distance, time and speed. • Solving problems involving knowledge of speed • Demonstration of teaching measures in specified class in primary schools.

11.3 Learning outcome

The student is able to:

i. Apply concepts of measures in everyday life situations
ii. Demonstrate ability of teaching measures in primary schools

11.4 Competences

Dear students, as you are now aware of the expected learning outcome of this unit, as you study and do in-built activities through this unit, you will be able to;

i. Define money
ii. Discuss uses of money
iii. State different denominations of money used in Uganda
iv. Workout problems involving use of money
v. State currencies used in other countries
vi. Compare exchange rates
vii. Workout problems involving exchange rates

viii. Define length
ix. Use non-standard units to measure volume
x. Discuss standard units which are used to measure volume
xi. Use standard units to measure volume
xii. Find volume of cubes and cuboids using standard units
xiii. Define capacity
xiv. Use non-standard units of capacity to measure capacity of given a container.
xv. Use standard units of capacity to measure capacity of a given container.
xvi. Relate volume and capacity
xvii. Add using units of capacity
xviii. Subtract using units of capacity
xix. Use units of capacity in multiplication
xx. Solve problems involving capacity
xxi. Define mass
xxii. Use non-standard units to measure mass
xxiii. Use standard units to measure mass
xxiv. Add using units of mass
xxv. Subtract using units of mass
xxvi. Multiply mass
xxvii. Divide mass
xxviii. Solve problems involving mass
xxix. Define time
xxx. Use non-standard units to measure time
xxxi. Use standard units to measure time
xxxii. Add using units of time
xxxiii. Subtract using units of time
xxxiv. Multiply using units of time
xxxv. Divide using units of time
xxxvi. Solve problems involving time
xxxvii. Define temperature
xxxviii. Use non-standard units of measuring temperature
xxxix. Use standard units of measuring temperature
xl. Subtraction involving temperature problems
xli. Convert degrees in Celsius scale to Fahrenheit scale and vice versa
xlii. Solve problems involving temperature
xliii. Define speed
xliv. Discuss the relationship between distance, time and speed.
xlv. Solve problems requiring the knowledge of speed.
xlvi. Demonstrate how to teach measures in primary schools

11.5 Subject orientation

This unit exposes you to interesting and enjoyable study of non-standard and standard units used in measurement, like length, Area, mass, time, temperature, volume etc.

It helps you to carry out measurement of different things and apply the skills and knowledge in our daily lives.

11.6 Study requirements

In this unit you will require plenty of materials to effectively teach the concepts. These will include: a pair of scissors, a razor blade, squared paper, a meter ruler, a measuring stick, a pair of compasses, water, seeds, stones, sand, empty containers, thermometers, clock, time charts.

You will also need a pen, exercise book, Primary Maths Revision and Practice for Uganda, Primary School Curriculum Course books, UCE Essential Maths.

11.7 Content and Activities.

11.7.1 MONEY

a) Definition of money

What do you understand by money?

As you may recall that mathematics developed and grew out of man's practical needs. When society grew and became more organized there arose a need to exchange or trade what one had for what one needed. This came to be known as "barter trade". To date this is still a common practice in our society.

If for example a person had a goat and his neighbour had hens and if such a person got visitors whom he wanted to honour by slaughtering a chicken for them, he would approach his neighbours and offer a certain number of hens for the goat.

What do you think the neighbour would do?

The two would negotiate and agree on the number of hens worth the goat.
As society developed and progressed it was deemed necessary to find an easy way of transacting business. Instead of goods for other goods, man devised money for goods.

Money is a medium of exchange.

- What are the uses of money?
- List them down
- Now you agree that money enables you access other services.

b) Denomination of money used in Uganda

Can you state the different denominations of money used in Uganda?
Now compare with the facts below:
The unit of money (currency) in Uganda is the shillings (sh). The face value of a note is its denomination. At the moment (2010) we have in Uganda coins/notes of the following denominations.

Sh.50	Sh.100	Sh.200	Sh.500

Notes:

Sh.1000	Sh.2000	Sh.5,000	Sh.10, 000	Sh.20, 000	Sh.50, 000

c) **Problems involving the use of money:**

(i) Selling price, buying price or cost price profit, loss and percentage loss and profit.

Refer to the mathematics corner, identify things sold in our shop by examining the price list.
In the primary school, children are expected to solve simple problems on selling and buying from the shops or markets.

For example Mukasa bought 4 pens at 200/= each, how much money did he pay?
This is got by multiplying 4 by 200. the answer is 800/=.

Example 1

A shopkeeper sells pens at sh.800. if he bought them at sh.500.

i). What was his profit?
ii). What was the percentage profit?

Solution:

Selling price = sh800
Buying price = sh500
(i) Profit = selling price-buying price
(800-500)
=Sh. 300

(ii) The percentage profit is $\left(\frac{300}{500}\times 100\%\right)$

$\therefore$ Percentage profit = 60%

The percentage profit is calculated by getting the profit and dividing it by the buying price 1 cost price and multiplying it by 100%.

Example 2

Cherop sold an article at sh2400 and made a profit of 20%. What was the buying price?

Solution:

In the above example what is given is the selling price (s.p) and the profit.
They are asking for buying or cost price (CP).
We arrange the problem as follows:

$(100+20)$% = 2400 shillings
120% = 2400
1% = $\frac{2400}{120}$
100% = $\frac{2400}{120}\times 100$

= 2000

Therefore the cost price = sh.2000

Example 3

The cost price of a shirt is sh.15,000 and the percentage profit is 30%, find the selling price?

Solution:

100% = 15,000

1% = $\frac{15,000}{100}$

$\therefore$ 30%= $\frac{15000}{100} \times 30$

= 4500

$\therefore$ selling price = 15000+4500

Sh19500

Or

Another way would be:

100% = sh.15000

1% = $\frac{15000}{100}$

130% = $\frac{15000}{100} \times 130$

Selling price was sh 19500.

Example 4:

A woman bought a bag at sh.45000. She sold it at sh.40000. what was the percentage loss?
Solution:

The selling price = 40000
The cost price = 45000
Loss = 45000-40000
5000

Loss percentage = $\frac{5000}{45000} \times 100\%$

= 11 $^{1}/_{9}$%

ii). Discount and bills

A discount means you pay less. This discount is given to customers who have bought a bulk of goods. For example;
A shoe dealer is given a 10% discount when paying for a pair of shoes of sh25000. How much would he pay?

Solution:

100% = 25000

1% $= \frac{25000}{100}$

90% $= \frac{25000}{100} \times 90$

= 22500

∴ He would pay sh.22500

Tom went to the market and bought the following items;
4kg of sugar at 2200/= per kilogram
3kg of maize flour at 900/= per kilogram
500g of salt at 800/= per kilogram
750g of ghee at 2800/= per kilogram
1500g of meet at 5000/= per kilogram

i). Calculate his total bill
ii). Suppose he was given 15% discount, how much did he pay?

Solution:

One way the total bill can be presented is by having a simple table as shown below;

s/no	Particulars	Quantity	Rate	Amount
1.	Sugar	4	2200/=	Sh8800
2.	Salt	500g or $\left(\frac{1}{2}kg\right)$	800/=	Sh400
3.	Ghee	750g or $\left(\frac{3}{4}kg\right)$	2800/=	Sh2100
4.	Meat	1500g or $\left(1\frac{1}{2}kg\right)$	5000/=	Sh7500
Total				Sh18800

i). ∴The total bill paid is sh18800

ii). A discount of 15% = $\frac{18800 \times 15}{100}$

= 2820/=

∴He would pay (18800-2820)/=

= sh15980

Another way would be;
100% = sh18800

1% = sh $\frac{18800}{100} \times 85$

=sh15980

Activity 11.1

1. Sam got a 25% profit on a shirt sold at sh.70.000. What was the cost price of the shirt?
2. A farmer bought a chair for Shs.60000. He sold it at a profit of 20%. Calculate the selling price.
3. The cost price of a table is Shs.40, 000, and it was sold at Shs.44, 000. Calculate the percentage profit.
4. The profit after selling a bicycle for Shs.120, 000 is Shs.30, 000. Calculate the percentage profit.
5. A man got a 15% loss after selling his motocycle at Shs.700,000.
   a) How much was the loss?
   b) How much did he sell the motorcycle?
6. Masaba bought a radio at Shs.85, 000. If he was given 10% discount, what was the original price of the radio?
7. Kiprop bought the following items:
   - 5kg of meat at Shs.5000 per kg
   - 12 pineapples at Shs.800 each
   - 500g of salt at Shs.400 per kg
   - 4200g of maize flour at Shs.900 per kg
   - 3 chickens at Shs.8500 each.

Make out a bill, give 25% discount and determine how much money Kiprop paid.

Check your answers with those given at the end of this unit.

d) Currencies used in other countries

As we transact business with other countries, we get the currencies of those countries. However, it is necessary to get the money in the currency internationally accepted, for example, the American Dollar ($), the Euro, the Pound sterling, Japanese Yen, the Dutch mark (DM), Kenya Shilling (KS), Tanzania Shilling (Tz S) among others.

Can you state the currency used in the following countries?

- Rwanda
- Britain
- USA(United States of America)
- India
- South Africa
- Congo
- Germany
- Burundi
- DRC (Democratic Republic of Congo)

In Uganda, you can sell or buy international currency in Banks or Forex bureau.
Study the table below showing the exchange rates at the Crane Forex Bureau:

Currency	Buying	Selling
US $	2082	2100
Pound Sterling	3350	3450
Euro	2940	3100
KS	27	28

Source: Daily Monitor, August 18, 2008, pg 27

Note: The Buying rate for the Forex bureau is the selling rate for the customer, and the selling rate for the Forex bureau is the buying rate for the customer.

Now study the following example:

John wants to go for a holiday in New York. He needs dollars.

a) How many dollars will he get for Sh.900, 000 if the selling rate is Sh.2100 per dollar?
b) He returns with $85. How many shillings is this worth if the buying rate is Sh.2082 per dollar?

Solution:

a) 1$ = 2100/=
900, 000/= gives $ $\frac{900,000}{2100}$ = $428.57

John will get = $428.57

b) $1 = 2082/=
$85 = 2082 x 85
= 176,970/=John will get Sh.176,970

Activity 11.2

(Use the Crane Forex bureau rates in the table given)

1. How many dollars would Ayeko get from 2,650,000 Uganda Shillings?
2. How many Uganda Shillings would Sabila get from 368,500 Kenya Shillings?
3. Complete the following table showing the conversion of currency into another.

a)

U Shs	Pound Sterling
950,000	(i)
(ii)	275
135,750	(iii)
(iv)	960

b)

$	Ush
350	(i)
(ii)	850,000
665	(iii)
(iv)	1,050,600

Check your answers with those given at the end of the module.

Great Job! How are you feeling now!

11.7.2. LENGTH

a) Definition of Length

It refers to the distance from one end of an object to the other, normally the longest dimension.

b) Non standard Units used in measuring length

Can you name the non standard units used in measuring length?

- (The span or cubit , the cubit and the foot)

Thank you for trying.

Diagrams missing!

With your classmate, can you use these units to measure the length and height of the following; Table, chalkboard, desk, door, your bed, field etc. Record the results in your notebook. Compare your results with those of other colleagues, and discuss them.

It is possible that some of your observations may include the following:

- The methods and units of measure were not uniform.
- The length of objects not being exact multiples of the human body parts.

Dear student, you will agree that such challenges as those enlisted above prompted a need to come with a standard unit for measuring length.

c) Standard Units and other metric units

Hullo student, when people began forming groups to conduct business, industry construction and trade, there came a need for standard units of measuring with a common meaning. One such a unit was a 'Metre,' which was chosen to be the standard basic unit of measuring length.

Furthermore, on the continent of Europe, an attempt was made to get a unit of length which was convenient and tied to the physical features of the world. So originally, the metre was defined as one ten millionth of length from the North Pole to the Equator along the Meridian through Dunkirk and Barcelona. Based on this argument, parts and multiples of the basic unit (Metre) were defined.

The Basic Unit of Length is the METRE

Viz;

Milli = $\frac{1}{1000}$ Part of a unit (metre)

Centi = $\frac{1}{100}$ part of a unit (metre)

Deci = $\frac{1}{10}$ part of a unit (metre)
Metre = 1 unit (metre)
Deca = 10 units (metres)
Hecto = 100 units (metres)
Kilo = 1000 units (metre)

OR

It can be presented in a table form as shown below:

Km	HM	Dm	M	dm	cm	mm
1000	100	10	1 (Basic Unit)	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Dear student remember that when any measurement of length is needed, we first define a unit and arrange for a method by which the unit can be reproduced so that it can be fitted in.

Can you use the above information to do the following activity?

Activity 11.3

1. Name the first three non-standard units used to measure length.
2. List down two challenges encountered or which emerged as a result of using non standard units to measure length.
3. Name the basic unit for measuring length.
4. Identify and arrange in order units used in measuring length.

You can now check your answers with those given at the end of the module.

Now note the following:

If you measure something, we compare it with a uniform unit. This uniform unit is called a standard unit. A standard unit of measurement is more valuable if it is used all over the world. We use standard units based on metric system. this system is called the system international d' units (SI)
The SI unit of length is the Metre (M). The other standard units of length are based on metre Viz:

Kilo metre = 1000 m
Hecto metre = 100 m
Deca metre = 10 m
Deci metre = $\frac{1}{10}$ m
Centi metre = $\frac{1}{100}$ m
Milli metre = $\frac{1}{1000}$ m

Dear student, from the above arrangement, it should be noted that the prefixes for metric measurements are ordered by powers of 10. It is therefore very easy to change from one prefix to another for the same unit of measure as shown in the table below;

1000	100	10	1 (Basic Unit)	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Kilo	Hecto	Deca	Metre	Deci	Centi	Milli

You will notice that in the table above each prefix to the left of another is 10 times the one on the right. To change the prefix to any one on the left in the table you must divide the number by ten for each successive prefix.
For example, if you want to change 200mm to centimetres, you divide by 10 because a centimeter is one place to the left of the millimetre. Thus we can say, 200mm = 20cm.
Changing 20cm to dm, you divide by 10 because decimeter is one place to the left of centimeters. Viz; 20cm = 2dm

Therefore, to change from a larger prefix to a smaller one, we multiply the number by 10 for each place to the right. On the other hand, when changing a smaller prefix to a larger one, we divide the number by 10 for each place to the left.

Study the examples below:

Example 1:

Change 150,000mm to Km

Solution:

1Km = 1,000,000mm

Therefore, 150,000mm = $\frac{150{,}000}{1000{,}000}$

= 0.15Km

Example 2:

Change 5.684km to cm

Solution:
1km = 100,000cm
Therefore, 5.684km = 5.684 x 100,000
= 568400.000
= 568400cm

Example 3:

Add

$$\begin{array}{rr} 9\text{m} & 25\text{cm} \\ +\ 7\text{m} & 85\text{cm} \\ \hline 17\text{m} & 10\text{cm} \\ \hline \end{array}$$

Example 4:

Subtract

$$\begin{array}{lcc} & \text{m} & \text{cm} \\ & 6 & 70 \\ - & 3 & 85 \\ \hline & 2 & 85 \\ \hline \end{array}$$

Example 5:

A family of 8 people got 9m 75cm of cloth each. What was the total length of cloth got by the whole family?

Solution:

Each got 9m 75cm

The family of 8 got

= 9m 75cm

x 8

78m 00cm ⇨ 600 ÷ 100

= 6m 00cm

Example 6:

Divide 33km 80m by 8

Solution:

Km 4 **m 135**

8 33 80

32 +1000

1 1080

-8

28

1 km = 1000m 24

40

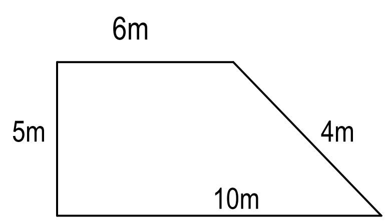
40

= 4Km 135m

•1 Km = 1000m

Example 7:

a) Find the distance round the diagram below:



Solution:

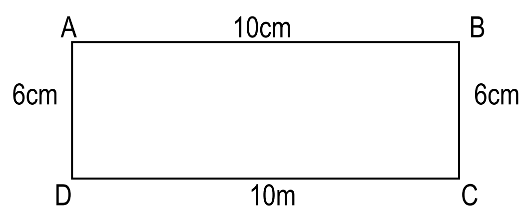
Distance round = (6+4+10+5)m

= 25m

b) Find the distance round the rectangle ABCD.

Hint: AB= Length (l)

BC = Width (w)



Solution:

The total distance round a figure is called its **perimeter** (p). But AB=DC and BC=AD
The distance round the rectangle = (10+6+10+6)
= 32cm
The perimeter (p) of the rectangle is 32cm.
This can be expressed as:
P = (l + w + l + w)
Or P= (2l +2w)
P= 2(l + w)

Activity 11.4

1. Express in millimeters;
a) 8cm b) 7.5cm c) 0.625cm

2. Change the following into metres
a) 2150cm b) 3500mm c) 0.079km

3. Express in kilometers
a) 7500m 450,000cm c)48500dm

4. Ayo had 4m 75cm of a tape, and Mukasa had 3m 65cm long. What is the total length of the tapes?

5. A rope measures 3m 10cm, if 1m 45cm is cut of, what will be the length of the rope?

11.7.3. AREA

Hello student,

Since you now know what perimeter is? Can you try to define area?

a) Definition of area

Amount/space covered by an object on the surface.

b) Non standard units

With your colleague identify non-standard method used in measuring area;

- Comparison of objects
- Use of grid
- Conversation of area

Form groups and find areas of the following objects using the above methods identified; table top, leaves, bottle tops, slates, etc

(1) Comparison of objects

Place squared paper or exercise book on a table top as shown and find how many of them fill the space (surface of the table)

Now compare your work with those of other groups and share your findings.

Note:

When we are finding area of plane figures by means of covering the figure with small squares and counting them. We usually choose a unit square shape of side 1cm or 1mm. the area should be expressed in square units i.e. cm^2 or mm^2

Activity 11.4.1

Use small squares of side 1cm to find the area of the following items;

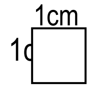
(i) Your desk top
(ii) Your text book top
(iii) Clock face

Thank you for your effort, now let's identify standard units used in measuring area.

(c) Standard units and other metric units in measuring area;

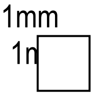
These are;

(i) Centimeter squared (cm^2) i.e.



A = 1cm x 1cm
= $1cm^2$

(ii) Millimeter squared (mm^2) i.e. mm

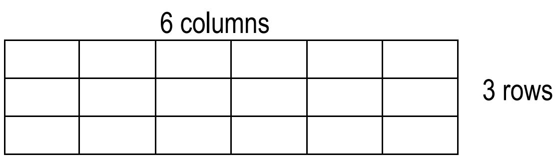


A = 1mm x 1mm
=$1mm^2$

Area of rectangles:

Suppose you are given a plane figure which is rectangular in shapes measuring 6 by 3.

It's area can be determined by placing $1cm^2$ on the plane shape and then counting to find out, how many of these units ($1cm^2$) fit the figure as shown below;



From the rectangle above, we observe that if we arrange square centimeters in rows and columns to cover the rectangular shape, the area in square units will be equal to the number of rows.
Area = 6x3=18 square units.

The area of the rectangle is 18 square units because 18 squares cover the surface of the rectangle. If we now measure the length and width (breadth). We are able to derive the formular for the area of the rectangle.
Hence area = 6cm x 3cm

Area = length x width or length x breadth
A= LxW or area = Lxb

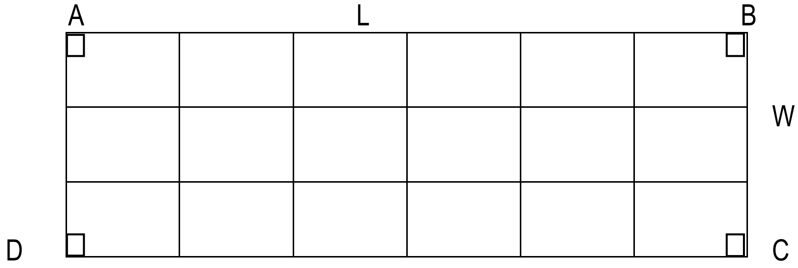
Example 1: use the formular above to find area of a rectangle which measures 3.6m by 2.1m

Solution;

Area = LxW
=$(3.6x2.1)m^2$
= $7.56m^2$
Area = $7.56m^2$

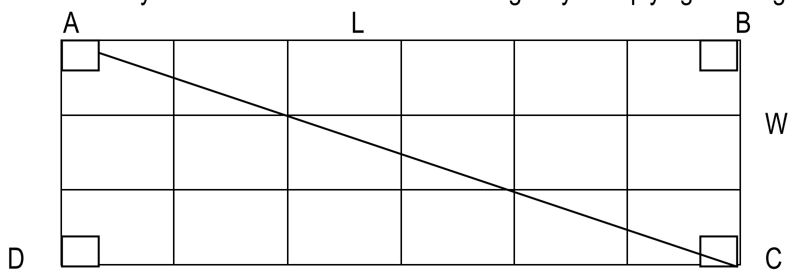
Area of triangles

When finding the area of triangle we use the idea of the area of a rectangle. See figure below;



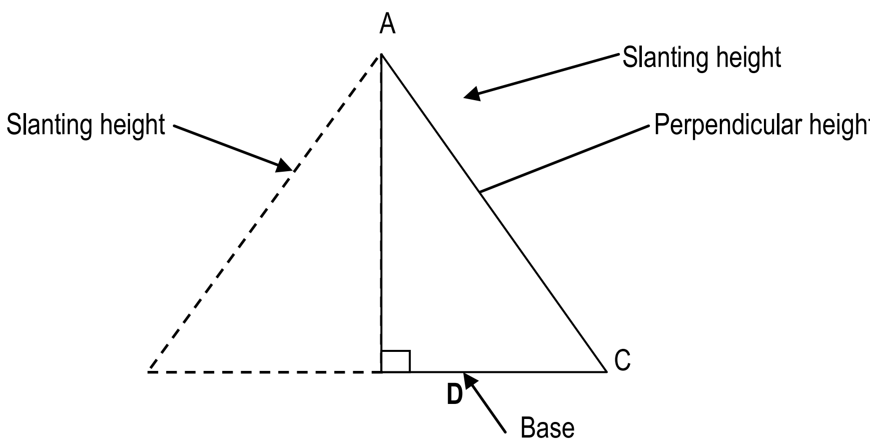
Do you still remember how to find the area of a triangle?

Remember that you can find the area of this rectangle by multiplying the length by width. Thus; area = LxW



You will note that the drawn diagonal AC has divided the rectangle into two equal parts forming two right angled triangles. Each of the triangles formed is exactly half the rectangle. This implies that if the area of the whole rectangle = LxW, then the area of half the rectangle = ½ x(LxW). But pairs ABC and ADC = ½ x (LxW). But for a triangle we have the base and the perpendicular height as shown below;

o



From the above figure, we can derive the formular of finding the area of the triangle as;

Area = ½ x base x perpendicular height.

Example 2:

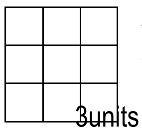
Find the area of a triangle whose base is 10cm and height 4cm.

Solution:

Area of triangle = ½ x base x height
A = (½ x 10 x 4) cm
A = 20 cm^2

Hello student now that you know how to find area of rectangle and triangle let us find out area of a square.

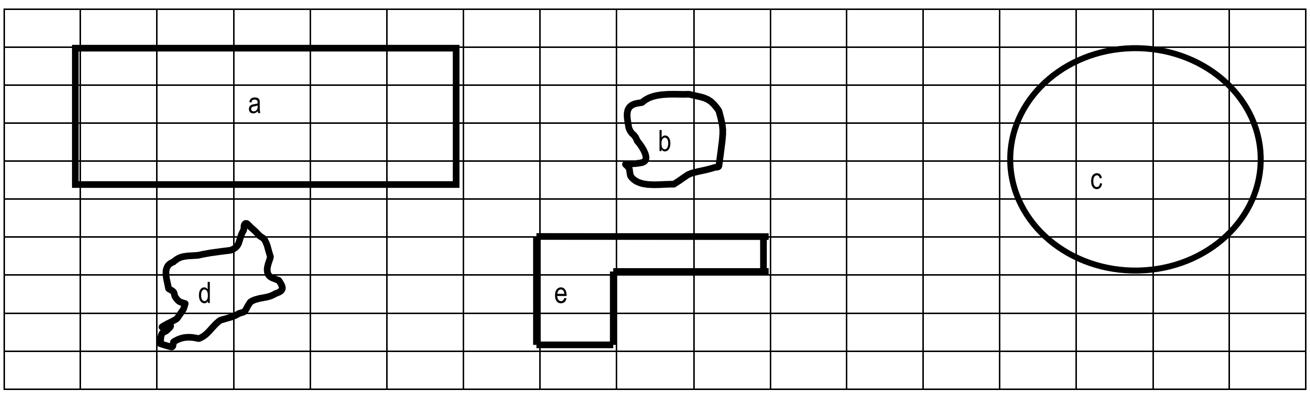
A square has all sides equal see figure below



Area = 9 square units since a square has all sides equal, then its area = side x side or length x length. Hence: area = (side)^2 or (length)^2 or A= S^2

(iii) Use of grid

Get a sheet of paper marked with one centimeter squares. Place irregular or regular objects on it such as leaves, seeds, rectangular objects etc and find their area by counting the number of small squares covered by the object as shown below;



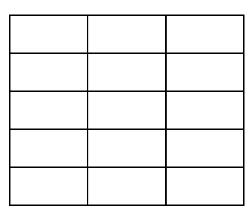
. Count and record the number of squares covered (fitted) by each figure.

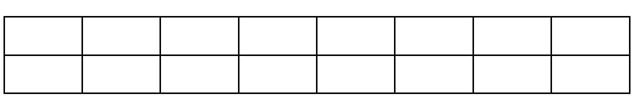
(iv) Conservation of area

Get one centimeter square sheet of paper/square bricks/triangular sheets of paper and make different shapes using the same number of squares e.g. 16 as shown below.

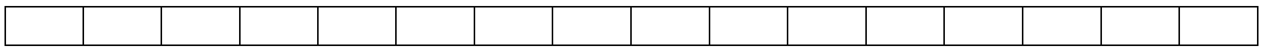
NB. It must be noted that the area covered by each shape is 16cm^2

(a)





(b)



(c)

Example:

Find the area of a square piece of land of length 175m?

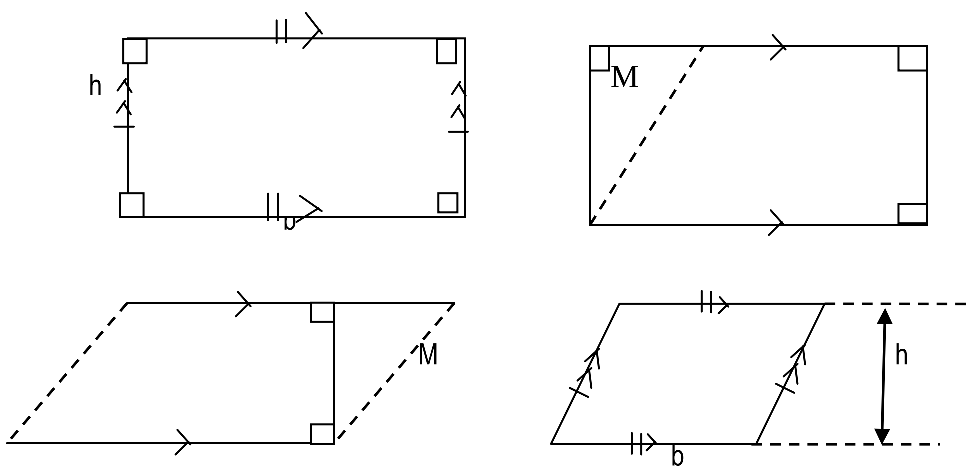
Solution;

Area = $(\text{side})^2$ or $(\text{length})^2$
Area = $(L\text{x}L)^2$
$= (175 \times 175)\ m^2$
$= 30{,}625\ m^2$

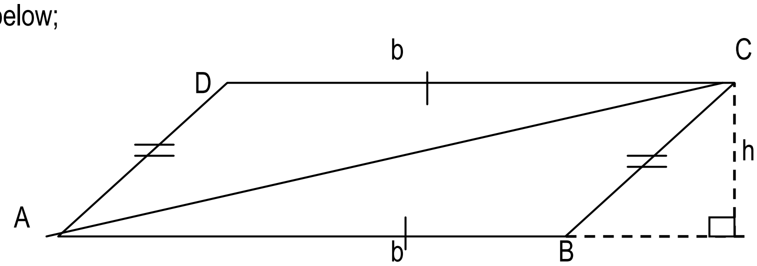
Let us go to look at another interesting area of a parallelogram

Area of a parallelogram

A parallelogram is formed out of a rectangle. It implies that the area of a parallelogram is related to the area of a rectangle. For the parallelogram, we have the base and the perpendicular height. A parallelogram may be formed from a (rectangle (such as a sheet of paper) by cutting off triangle M and placing it in position shown.



if we are to find the area of parallegram ABCD we should first of all draw a diagonal AC or DB in the parallelogram as shown below;



It can be observed that the diagonal AC has divided the parallelogram into two equal triangles sharing the same the same height (h).

Therefore the area of the parallelogram =

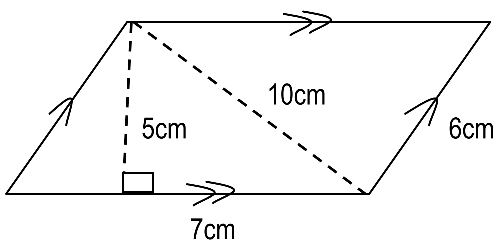
Area of triangle ABC + area of triangle ADC =
(½ x base x perpendicular height + ½ x base x perpendicular height). Since the base and perpendicular height of both triangles are given as b and h respectively, we substitute the formular above as follows;

Area of the whole parallelogram
$= \frac{1}{2} \times b \times h + \frac{1}{2} \times b \times h$
$= \frac{1}{2} bh + \frac{1}{2} bh$
$= \frac{bh + bh}{2}$
$= \frac{2bh}{2}$
= bh units

Therefore the area of a parallelogram can be defined as b x h (where b is base and h is height) A = bh units.

Example:

Find the area of the parallelogram below.

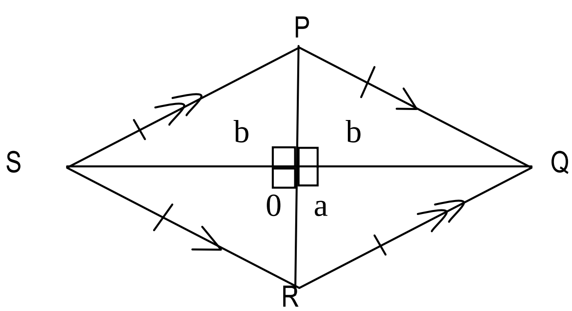


Solution:

Base = 7cm
Perpendicular height = 5
Area = base x perpendicular height
Area = 7 x 5
= $35cm^2$

Area of rhombus

A rhombus is a special parallelogram in which all the sides are equal in length and the opposite angles are equal. The diagonals of rhombus meet at 90^0 and they bisect each other forming four equal right angled triangles within a rhombus. See figure below;



To find the area of the rhombus SRQP, we need to follow two procedures;

Procedure 1:

Since diagonal SQ has divided a rhombus into two equal triangles, we can therefore; find the area of the rhombus through finding the area of the triangle SQR and triangle SQP and then adding the two areas together.

1. Area of triangle SQR = ½ x base x perpendicular height.

   = ½ x QS x OR
2. Area of triangle SQR = ½ x base x perpendicular height
   = ½ x SQ x OP
   = (½ x SQ x OR) + (½ x SQ x OP)
   = (½ SQ x OR + SQ x OP)
   = ½ SQ (OR + OP)

But OR + OP = PR

Therefore ½ SQ (OR + OP) = ½ SQ (PR)
= ½ (SQ x PR)

But SQ and PR are the two diagonals of the rhombus SRQP.

Therefore the area of a rhombus can be defined as Area = ½ (product of its diagonals)

Procedure 2:

The two diagonals drawn in the rhombus have divided it into four equal right angled triangles. Thus to find the area of the rhombus SRQP, we shall find the area of each of the four triangles and then add their areas together.

Solution:

1. Area of triangle OPQ = ½ x OP x OQ
2. Area of triangle OPQ = ½ x OP x OQ
3. Area of triangle ORS = ½ x OR x OS
4. Area of triangle OPS = ½ x OP x OS

But OR = OP = a
OQ = OS = b

Substituting a and b in 123 and 4 we should have the following;

1. Area of triangle ORQ = ½ x a x b
   = ½ ab
2. Area of triangle OPQ =½ x a x b
   = ½ ab
3. Area of triangle ORS =½ x a x b
   = ½ ab
4. Area of triangle OPS =½ x a x b
   = ½ ab

Total Area = ½ ab + ½ ab + ½ ab + ½ ab
= 4(½ ab)

But ½ ab is the area of one of the four right angled triangles formed in the rhombus. Area of a rhombus = 4(½ ab) = 4 (area of one of the triangles formed in a rhombus). In short using procedure 1 and procedure 2, we discover that either the area of a rhombus is defined as Area = ½ x (product of the diagonals of a rhombus). Or Area = 4 x (area of one of the triangles formed in a rhombus).

Example:

Calculate the area of a rhombus ABCD whose diagonals AC and BD are 12 cm and 28cm long respectively.

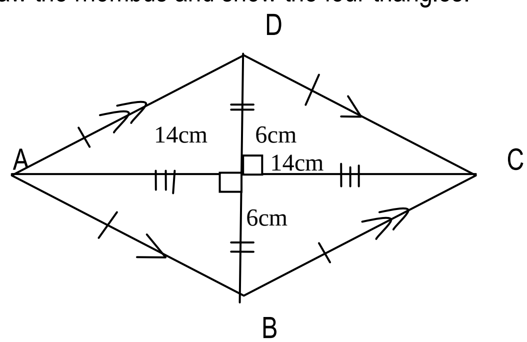
Solution:

Method;= ½ (product the diagonals)
= ½ (12 cm x 28 cm)
= ½ (12 x 28) cm^2
= ½ (336) cm^2
= 168 cm^2
Area of rhombus ABCD = 168 cm^2

Method 2:

Area of rhombus = 4(area of one of the four triangles formed in a rhombus).

Now we can draw the rhombus and show the four triangles.

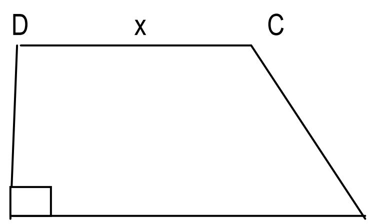


From the rhombus ABCD, we observe that each of the triangles formed in rhombus has a base measuring 6cm and a perpendicular height measuring 14cm.

Area of rhombus ABCD = 4(½ x 6 cm x 14 cm)
= 4(42) cm^2
= 168 cm^2

Hullo student, thank you for your continued concentration in studying areas of different shapes. Let us now study area of a trapezium.

Area of trapezium

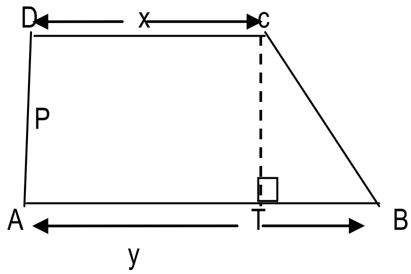


Find the area of the trapezium ABCD

A B

Can you suggest how we can find the area of the above trapezium?

One way to find the area above is to draw a perpendicular line from C to the base. AB as shown below.



The perpendicular line drawn form C to the base at point T has divided the trapezium into two parts, one part being rectangle and the other being a triangle. Here we can now find the area of rectangle ATCD and triangle TBC and then add their results together in order to get the area of the trapezium ABCD.

Solution:

Rectangular part ATCD length = DC = X
= width = AD = P
Triangular part TBC base = TB = y-x
Height = TC = P
Area of rectangular part ATCD = L X W
= PX x
= Px (i)
Area triangular part TBC = ½ P(y-x)
= ½ (Py – Px)
= ½ Py – ½ Px (ii)

Area of a trapezium ABCD = Area of rectangular part ATCD area of triangular Part TBC. Hence combining (i) and (ii) we obtain.

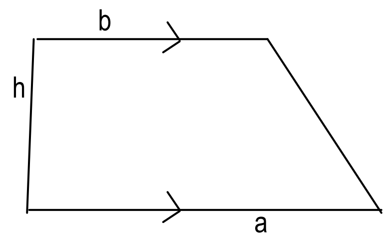
Area of trapezium: = Px + (1/2 Py – ½ Px)
= Px + ½ Py – ½ Px
½ Px + ½ Py

$$P\frac{(x+y)}{2}$$

Area of trapezium = ½ (x + y) x P

Therefore area of a trapezium = ½ x (sum of parallel sides) x perpendicular height.

Using the knowledge above, what is the area of the trapezium in the figure below;

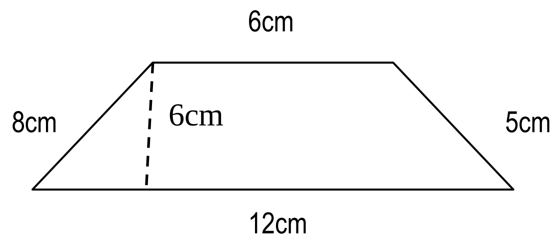


Solution:

Sum of parallel sides = (a + b)
Perpendicular height = h
Area of trapezium = ½ x (sum of parallel sides) x perpendicular height
½ x (a + b) x h

Example:

Find the area of the trapezium

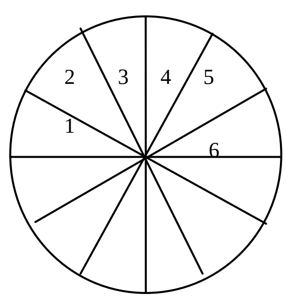


Solution:

Sum of parallel sides = (6 + 12) cm
Perpendicular height =6cm
Area = ½ (sum of parallel side x perpendicular height)
= ½ (18 x 6) cm^2
= ½ (18 x 16) cm^2
= (½ x 18 x 6) cm^2
= 54 cms
Therefore area of trapezium = 54 cm^2

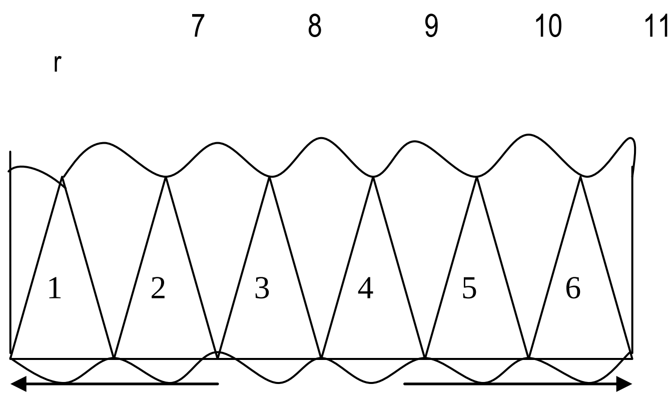
Area of a circle

Hullo students, you remember how to find the area of a triangle, we can use the same idea to find area of a circle. However, in finding area of a circle one can use the circumference and the radius to find the formula for getting the area. See figure below;



Procedure 0774182925 yoga

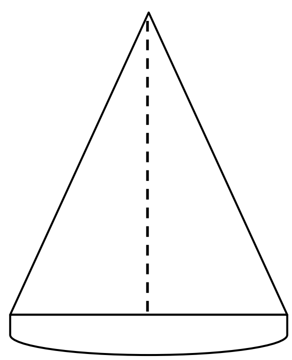
1. Draw a circle of any size as shown above.
2. Use ½ of radius to cut arcs.
3. Draw the diameter to produce 12 equal parts (sectors). Mark the 6 sectors 1-6.
4. Then cut out the circle into 12 piece as shown below;



½ of 2π or πr

The above figure has been rearranged to form a rectangle of length = half the circumference. Length is II r
Width (breadth) = r
Area = LxW. In this case it will be πr x r a circle = πr x r
= π r

Method 2:



The above figure is one part of a circle and it looks like a triangle.

Area of 1 part of a circle = ½ x $^1/_{12}$c x r
But c = circumference of a circle = 2 πr
Area of 1 part of a circle = ½ x 1/12 x 2π r x r x 12
= πr^2

Example:

Calculate the area of a circle whose circumference 88 cm, (Take π asw 22/7)

Solution:

Area of circle = πr^2
But circumference + 2 πr
C = 2 π r
Where C = 88 cm
88 = 2 πr
88 = 2 x 22/7 x r
88 = 44/7 r
44r = 88 x 7

R = 88 x7/44
R = 14 cm

Therefore Area = (22/7 x 14 x 14) cm^2
= 616 cm^2

With the knowledge gained do the following activity;

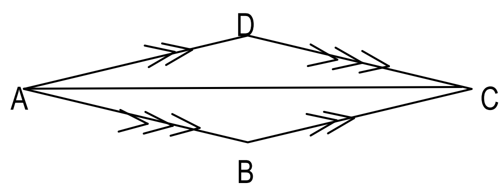
Activity 11.5

Hint; use specific formulars to find areas of each of the following;

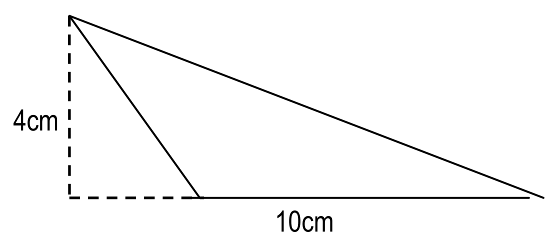
1. A rectangle of length 15 c m and width of 13cm.
2. A triangle of base 20 cm and perpendicular height of 13 cm.
3. A parallelogram having base 3 dm and perpendicular height of 12 cm.
4. A trapezium with parallel sides measuring 16 cm, 8 cm and perpendicular distance between them 15 cm
5. A rhombus whose two diagonals are 80cm and 60cm.

Find the areas of the figures below;

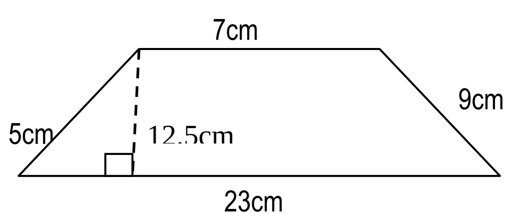
6 .Perpendicular distances between DC and AB is 86 cm



7.



8.



Check your answers with those at the end of module.

Congratulations for attempting this activity. You are now knowledgeable about area of two dimensional objects.
11.7.4 VOLUME

a) Definition of volume

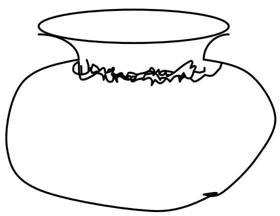
Hullo student can you define volume?

- Volume is the amount of space occupied by an object.

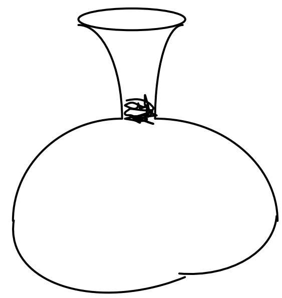
Have you ever tried to find out how people of long time ago used to measure volume of objects or substances?
Let us find out

b) Non standard units in measuring volume.\

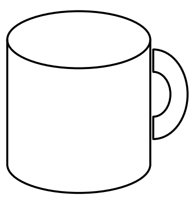
In most societies, capacity was measured using containers like; cups, bowls, gourds, calabashes, pots etc.
Today even in some communities, these containers are still being used.



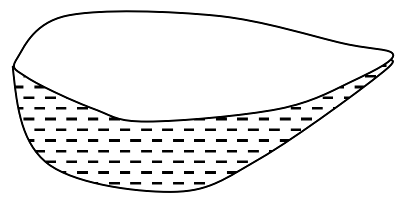
Pot



Gourd



Cup



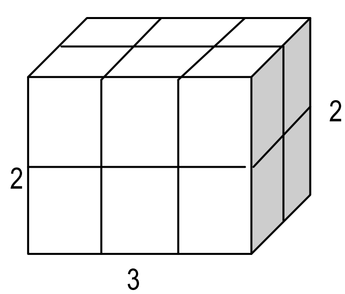
Calabash

It should be noted that different societies had different units of measure. It was difficult to establish a standard measure hence need for standard measure.

Standard units and metric units.

The cubic centimeter is the basic unit for volume.

Visit the mathematics corner sort out 3-dimensional objects which are rectangular or square in shape e,g match boxes, small and big empty boxes, bricks, plunks of wood, cubes. Get as many as possible of the same size.
Stack those of the same size and bind together. Find out what space is occupied?



- What space is occupied by the 1cm cubes shown above?
- Match boxes measuring 3x2x1cm are put in a big box measuring 9x6x3cm. Find the number of match boxes inserted in this box

i. Volume of regular objects

A solid is a substance which has size, shape and mass. A regular object or solid is one which has straight sides which can be measured. We also say the sides are uniform. E.g cubes, cylinders, tetrahedrons etc.

ii. Volume of irregular objects

Irregular solids are those solids whose length cannot be easily measured e.g broken stone, broken glass, broken wood.

Their volumes cannot be calculated using their lengths. The method used is to immerse the solid in a container full of liquid which does not dissolve the solid and measure the volume of liquid displaced by solid.

Activity 11.6

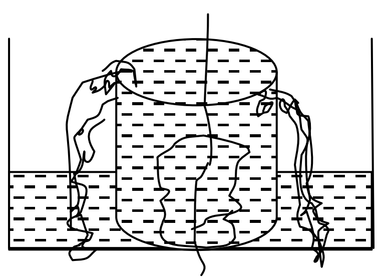
1) Get an irregular solid and tie a piece of string around it.
2) Get an empty container (tin) and another larger container.
3) Fill the empty tin to the brim with water (i.e until it overflows).

Carefully place it in the large container. Be careful not to spill any of the water.

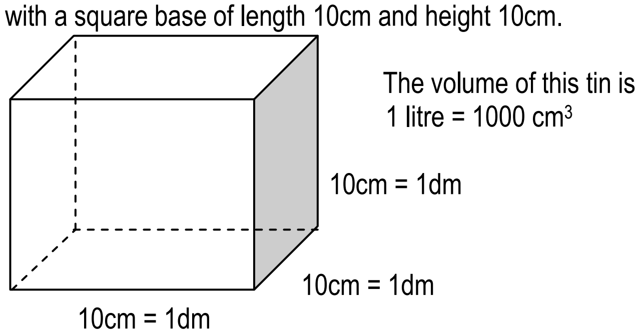
4) Lift the irregular solid by the end of the string and carefully lower it in the tin full of water.
    a) Explain what happens as you lower the solid into the tin
5) Lift the tin containing the stone out of the larger container, taking greater care not to pour any more water into the larger container.
6) Determine the volume of water which poured out of the tin due to displacement by the solid

Pour it into a measuring cylinder or any container of known capacity.
What meaning do you attach to the measurements you get?

The capacity of the displaced water is the volume of the solid. See diagram below:



Earlier we said the basic measure for capacity is the LITRE.
Suppose you have a tin with a square base of length 10cm and height 10cm.



When the above tin is filled with water. The volume will be 10cmx10cmx10cm=1000cm^3 =1litre
Hence 1ml=1cm^3

Volume of water = $1dm \times 1dm \times 1dm = 1dm^3$

$= 10cm \times 10cm \times 10cm = 1000cm^3$

= 1litre

To find volume of solid objects like cubes whose side is x units is x^3 cubic units.

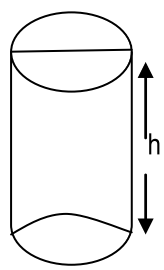
Example;

Find volume of a cube whose side is 3cm?

Solution;

Volume of cube = x^3 cubic units
But x=3cm
= 3^3 = $(3\times 3\times 3)cm^3$
= $27cm^3$

Volume of a cylinder



A cylinder

Volume = Area of base $\times$ *height*
Note: Earlier we learnt how to get area of a circle
Volume = $\prod r^2\ (h\ is\ the\ height)$
$\therefore$ Volume of a cylinder whose radius is r units and height is h units is $\prod\times r\times r\times h$ cubic units = $\prod r^2 h\,cubic\ units$

Example:

Find the volume of a cylinder whose height is 21cm and radius 5cm (take $\prod\ as = \frac{22}{7}$)

Solution

Volume = Area of base x height
= $\prod r^2 \times h$
But h=21, $\prod = \frac{22}{7}$, r = 5
Volume = $\prod\times r\times r\times h$
= $\frac{22}{7}\times 5\times 5\times 21$
= $1650cm^3$

Activity 11.7

1. Calculate the volume of a cube whose side 6.5cm
2. What is the volume of a cylinder of height 12cm and radius 7cm?
3. Calculate the volume of air in a room whose sides are 6m by 3.5m and height 5m.
4. A tank whose capacity is 150000 has a square base of side 6m. Find its height.

11.7.5. CAPACITY

Hullo student, you are welcome to capacity.

Let us find out what capacity is

a) **Definition of capacity**

Capacity is the amount of liquid a container can hold.
Capacity is a word that is commonly used in our daily life situations.
In villages, markets, people use empty tins, gourds, baskets etc to measure quantities of food stuffs such as beans, maize, millet, groundnuts, sorghum etc. When doing this, they are using capacity as measurement.

The word capacity features also in vehicles such as buses, taxis, Viz; "licensed to carry 12 passengers" or "licensed to carry 30 passengers". The numbers 12 and 30 represent the seating capacity of these vehicles.
The above examples give scenarios where capacity is used to mean something other than volume.

b) **Non standard units of capacity**

Refer to non standard measures of capacity.

Activity 11.8.0

1) Visit a nearby market on a market day with your colleague. Observe the way sellers measure out quantities of different foodstuffs (beans, cassava flour, maize flour, millet flour, sorghum flour etc) using tins of different sizes.

Note: That people use capacity as well as mass (weight) to measure out these quantities.

2) a) Collect empty tins, empty mineral water bottles and cups from shops and maths corner. In groups of your choice, measure liquids from given containers and record the results. Compare results of each used container.
   b) With your tutor discuss the rationale for using standard units.
   c) Standard units of capacity

The basic standard measure for capacity is the litre
However it should be noted that there are other standard measures in use such as:

The capacity of a beer bottle is 500 millilitres (500ml) or $\frac{1}{2} litre$.

The capacity of some plastic mugs is 500millilitres. The capacity of a soda bottle is 300millilitres. The capacity of some jerrycans of paraffin or cooking oil is 20litres.

Activity 11.8.1

Hullo student

In groups of your choice with your colleague collect empty containers from math corner or shop such as;
$\frac{1}{2}l\, beer\ bottles, 300ml\ and\ 1l\ soda bottles, 5l, 10l, and\ 20l\ jerrycans, \frac{1}{2}l\ mugs etc.$ use these empty containers to measure quantities of unknown capacities. Record results of each used container compare your results and discuss with other group members.
Other units of capacity:

Draw number line:

Kilolitre	hectolitre	dekalitre	litre	deciliter	centiliter	milliliter
1000	100	10	1L	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

1Kilolitre =1000litres
1Hectolitre =100litres
1Decalitre =10litres
1decilitre $= \frac{1}{10} litres = 0.1 litres$
1centimetre $= \frac{1}{100} litres = 0.01 litres$
1millilitre $= \frac{1}{1000} litres = 0.001 litres$

d) Relationship between volume and capacity

Hullo student do the activity below

Activity 11.

Get into groups with your colleagues; make cube measuring 10cm by 10cm by 10cm
Get a litre container of sand and pour sand in the made cube.

What do you discover?

The content just fills in exactly.
Similarly get rectangular and circular containers with same and different capacity and fill them with sand or soil or water
What do you discover?
Note:

1ml =1cm3
1litre =1000cm3
1litre =1000ml

Activity 11.9

1. A drum contains 86litres of juice, another 45litres is added. How many litres of juice does the drum now hold?
2. Kiplangat sold 98litres of milk on Monday and 73litres on Tuesday. How many litres of milk did he sell altogether?
3. $$\begin{array}{rll} & l & ml \\ & 4 & 350 \\ + & 3 & 725 \\ \hline & & \\ \hline \end{array}$$
4. A shopkeeper had 25litres 500millitres of paraffin in his shop. If he sell 7 litres 750millilitres of paraffin, how much paraffin is left?
5. A family uses 450litres and 600millilitres of water weekly. How much water does the family use in 4weeks?

6. Masaba has 6litres of milk. If he gives each child $\frac{1}{4} litre\ of\ milk.$ How many children does he have?

Check your answers with those given at the end of the module.

How are you feeling now?

Great !

11.7.6. MASS

Hullo student

Welcome to yet another interesting topic of mass in this unit of measures

Try to define mass

a) Definition of mass

Mass is the amount of matter in an object.

Activity 11. 10

Hullo student

1. With your colleague get some objects from the math corner such as stones, tins, sticks, fruits, books etc to find their mass or weights.

   Try to compare their heaviness by holding one in each hand.

2. In groups of three,

   Try to carry one of your friends on your back. Do this in turns. Compare your masses.

   Discuss your findings with your colleagues

In our daily situations we talk about weight of a substance when in fact we are trying to find the mass of an object i.e its heaviness.

Can you mention non standard, units used in measuring mass?

b) Non standard units of measuring mass

Can you recall in your local market where people sell foods like sorghum, cassava flour, rice, maize flour, beans etc using containers of known capacity and mass like empty tins of kimbo, blue band (margarine), cooking oil etc to measure quantities of these foods and sell them.

Our grandmothers used to do most of their measurements by estimation e.g when cooking they would estimate using their hands quantities of salt to put in food. They would estimate quantities of flour, curry powder, grain etc. In most rural areas this is still so. In modern kitchens, a lot of exact measurements are made and used. For example in order to bake a good cake, exact measurement of ingredients is done.

Some non standard units of measuring mass.

- Estimation
- Use of both hands to compare weights
- A simple scale
- A see-saw

c) Standard units of measuring mass

Now read this;

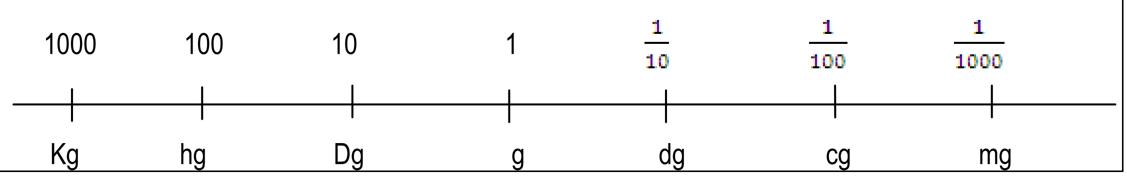
When babies are born in hospitals they are part in a basket on a weighing scale to determine their mass (weight) at birth. The weights are also taken during subsequent visits to the hospital and recorded in the baby's card. This is to help monitor increase in the baby's weight to ascertain good growth in the baby. Nowadays people in urban places measure their weights to ascertain whether they are overweight or under weight.

Activity 11.11

1. Collect objects with different weights e.g. stones, sand, bread, sugar, posho flour etc from college store.
2. Estimate their weights of the objects you have collected and record the estimated weight.
3. Weigh the objects you have collected using a weighing balance.
4. Compare estimated weight with weight got using a balance.

The basic measure for measuring mass (weight) is the Gram (g)
Other metric units of measuring mass

Draw a scale showing different weights



1 Kilogram=1000grams
1Hectogram=100grams
1Decagram = 10grams
1decigram = $\frac{1}{10} gram (0.1g)$
1centigram = $\frac{1}{100} gram (0.01g)$
1 milligram = $\frac{1}{1000} gram (0.001g)$
1tonne = $1000 kg$

Example

Express 35000g in kg

Solution

1000g =1kg

1g $= \frac{1}{1000} kg$

35000g $= \left(\frac{1}{1000} \times 35000\right) kg$

= 35kg

Example

Divide 52kg 320g by 5

Solution

kg	g
10	464
5 | 52	320
5	2000
2	2320
Change to grams	20
=2x1000	32
= 2000gms	30
∴add to	20
	20

= 10kg = 464g

∴ 52kg320g divided by 5 gives 10kg 464g

Activity 11.12

1. Change the following to kilogrammes

   (a) 6500g b) 12000g c) 75000g d) 250g

2. How many grams are in the following?

   a) 13kg b) 1.35kg c) 0.25kg d) 125kg

3. The weights of five bags of coffee are; 60.5kg, 58.2kg, 97.0kg, 68.01kg and 70.25kg. Find the total weight in
   (a) Kg (b) grams
4. Subtract 140kg300g from 620kg125g
5. The average weight of an eight month baby is 8kg400g. what is the total weight of 7 similar babies?
6. Divide 138kg400g by 2
7. The total weight of 12 children is 144kg60g. What is the average weight of each child?
8. Express the following in tones
   a) 25000kg b) 650kg

9. If 3kg of meat is enough for 10 students, how much meat is needed for 450 students?
10. Mukasa's family needs 2.5kg of sugar per week. About how much money will Mukasa spend on sugar in a month of February if the cost of sugar is sh2200 per kilogram?
11. A farmer has 25 bags of cotton. If the average weight of cotton is 42kg per bag, find out how much money he makes when he sells his cotton at 980/= per kg.

11.7.7: TIME

a) Definition of time

Time is a measure of duration for example:

- sunrise (dawn)
- morning
- mid-morning (break)
- lunch
- afternoon
- sunset (dusk)
- evening
- night

With your colleagues discuss the non-standard units of measuring time.

b) Non-standard units of measuring time.

- Using shadows
- Early morning
- Break
- Lunch
- Afternoon
- Evening
- Night

c) Standard units of telling time

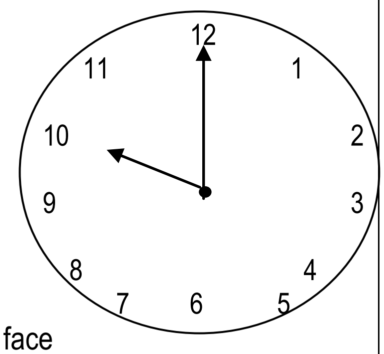
With your colleagues discuss the standard units of time; Viz;

- Second
- Minute
- Hour
- 24hour system
- 12hour system
- Week
- Fortnight
- Month
- Year
- Decade
- Century
- Millennium

Relation of time units:

60 seconds=1minute
60 minutes = 1hour
24hours = 1 day
7days = 1 week
4 weeks = 1 month
12months = 1year
10 years = 1 decade
10 decades or (100 years) = 1 century
10 centuries or 1000 years = 1 millennium

d) The 12 hour clock system



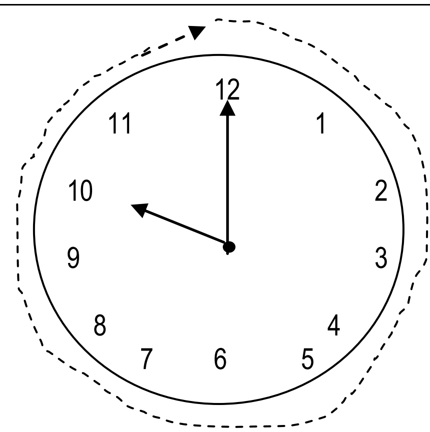
Clock face

Note:

1. Measuring of time on the clock begins at midnight
2. From midnight to noon the time is ante-meridian (a.m) (before mid-day)
3. From noon to mid-night the time is post meridian (p.m)
4. When we say one a.m we are saying it is one hour after midnight.

In the clock face above, it is ten (10) hours after midnight i.e. at ten in the morning (10.00am)

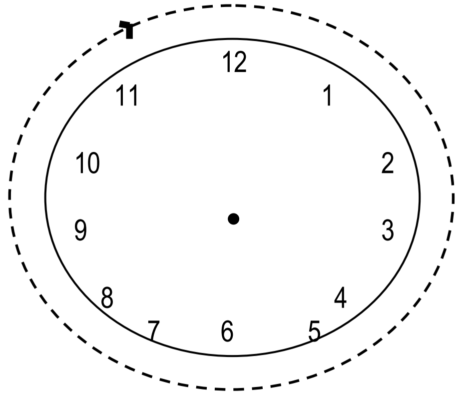
Study the time on the clock face and complete the tables below:



12.00 midnight to 12.00 noon

After midnight	1hr	2hrs	3hrs	4hrs	5hrs	6hrs	7hrs	8hrs	9hrs	10hrs	11hrs	12hrs
Time as a.m	1a.m	2a.m							9a.m			Noon or mid-day

12:00 noon to 12:00 midnight



Afternoon	1hr	2hrs	3hrs	4hrs	5hrs	6hrs	7hrs	8hrs	9hrs	10hrs	11hrs	12hrs
Time as p.m.	1p.m	2p.m	3p.m								11pm	Midnight

Read the time in full

i) 8.25a.m

Twenty five minutes past eight in the morning

ii) 10.45pm

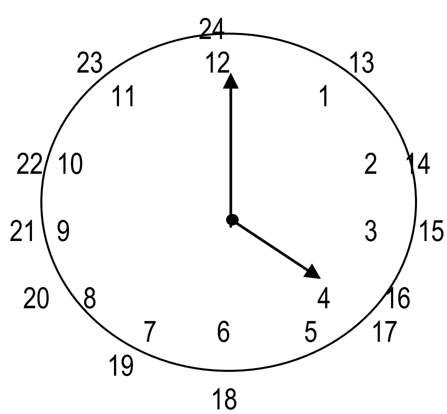
A quarter to eleven in the evening.

Activity 11.13

Write the time using words

3. 7.35a.m ii)9.20a.m iii) 1.40pm iv) 12.37p.m v) 12.10am

e) The 24 hour clock system.



clock face

The time shown on the clock face above is 4.00pm (4hours after midday or 16.00hours after midnight

Note:

When we write time using 24hr clock system, we write (4) four places like this 00.00. The first two places from the left are for hours and the next two places on the right are for minutes. So 10.30 a.m. is the same as 10.30hours. Even the reading of time in 24hour clock system is not the same as the 12hour clock system. For instance 10.30a.m. We read it as ten thirty hours. How will you read 8.00a.m? It would be eight hours and it is written as 08.00hours

1. Conversion of time from the 12-hour clock system to the 24-hour clock system

Time in a 12-hour system	Time in a 24-hour system
1.00a.m	0100hrs
2.00a.m	0200hrs
3.00a.m	----------
4.00a.m	----------
5.00a.m	----------
6.00a.m	----------
----------	0700hrs
----------	0800hrs
----------	0900hrs
10.00a.m	-----------
11.00a.m	-----------
12.00noon	-----------
1.00p.m	-----------
-------------	1400hrs
-------------	1500hrs
-------------	1600hrs
-------------	1700hrs
6.00p.m	-----------
-------------	1900hrs
-------------	2000hrs
9.00p.m	-----------
10.00p.m	-----------
--------------	2300hrs
--------------	2400hrs

Note: midnight to mid day time is a.m. (morning hours)
Mid day to midnight time is p.m (afternoon hours)

Example 1

Write 3.00a.m in the 24-hour clock system.

Note:

Changing a.m to 24-hour clock the hour remains unchanged because they are within the 12hours after midnight

3.00a.m=0400hrs

Example 2

Express 7.40a.m in the 24 hour clock

  7.40
+ 00.00
  07 40 = 07.40hrs

Example 3

Write 2.50p.m in the 24-hour system add 12 hours to the time given

To change pm to 24 hour system, add 12 hours to the time given.

= 2.50
+12 .00 = 1450 hrs
 14 50

Activity 11.14

Write the given time using the 24hour system

1) 2.00a.m 2) 4.00a.m 3)11.40p.m 4) 12.35p.m 5) 6.30p.m 6) 8.10a.m

7. The college staff meeting started at 10.00a.m and ended at 2.45p.m. Write the time the meeting started and ended in 24 hour system.

8. Nyondo PTC football team went for a return match to Kapchorwa PTC. The journey started at 11.30am and arrived at the venue at 1.45p.m

   i)How long did the journey take?
   ii) What time in the 24 hour system did they start and end the journey to kapchorwa?

f) Converting the 24-hour system to the 12-hour system

Example 1

Change 0900hours to the 12-hour clock system

Note:

i) When changing from a 24-hour clock to the 12 hour clock system, subtract 1200hrs for any time which is after 1200hrs. Write the time with p.m.
ii) Subtract 0000hrs for anytime which is before 1200hrs. Write the time in a.m.

Therefore to change 0950hrs, we have

  0950
- 0000
   950 = 9: 50 am

Example 2

Write 1542 hours to 12 hour clock system

1542

- 1200
  342 = 3.42pm

Example 3

Write 1255hrs using 12hour clock system

```
  1255
- 0000
  1255 = 12.55pm
```

Note: for 1255 the hours are not after 12.00 hours

Activity 11.15

1. (i)

Hrs	mins
1	30
+ 4	45

(ii)

Hrs	mins
9	12
- 5	35

2. (i)

Hrs	mins
15	28
X	5

(ii)

Hrs	mins
8 ⟌ 40	32

3. (i)

weeks	days
3	5
12	6

(ii)

weeks	days
24	2
- 15	5

4) Change 5 hours to minutes
5) Write a quarter to 6 in the morning figures
6) For each given time, change it to a 12-hour clock system

i) 0300hrs ii) 2140hrs iii) 1325hrs iv) 0035hr v) 2000hrs vi) 2400hrs

7) A taxi left Mbale in the morning and reached its destination at 1845 hours. What time is this in the 12hour clock system?

8) i) Change 0027hrs to the 12-hour clock system
ii) Change 0129hrs to the 12-hour clock system.

How are you feeling?

Encouraging!

1. Time tables

The bus timetable shows the departure and arrival times of the bus at different towns.

Town	Arrival time	Departure time
Jinja		8.00a.m
Iganga	8.30a.m	9.55a.m
Mbale	11.30a.m	12.15p.m

Tororo	12.45p.m	

Study the bus timetable and the examples

Example 1

At what time did the bus leave Jinja?
At 8.00a.m

Example 2

At what time did the bus arrive at Mbale?
At 11.30a.m

Example 3

How long did the bus stay in Iganga?
Departure time from Iganga-arrival time at Iganga

Example 4:

How long did the bus take to travel from Jinja to Tororo?
Arrival time at Tororo-departure time from Jinja

Activity 11.16

Study the train timetable and answer the questions that follow:

Station	**Kasese**	**Mityana**	**Kampala**	**Jinja**	**Tororo**
Arrival time		5.25pm	8.50pm	11.00pm	12.05am
Departure time	3.20pm	6.00pm	8.55pm	11.05pm	

1) At what time does the train:
i) Leave Kasese? ii) Reach Kampala? iii) Leave Jinja? iv) Arrive at Tororo?

2) i) How long does the train stay in Mityana?
ii) How long does the train stay in Jinja?
3) How long does the train take to travel from Kasese to Kampala?

Airline timetable

The timetable shows the arrival and departure times of an aero plane traveling from Kigali to Entebbe

Airport	Arrival time	Departure time
Kigali		1230hrs
Harare	1400hrs	1530hrs
Lusaka	1630hrs	2030hrs
Entebbe	2200hrs	

Example 1

How long does the plane stay in Harare?

Departure time from Harare – Arrival time at Harare

Dep. 1530hrs

Arri. 1400 hrs = 1 hrs 30 mins

1 30

The plane stays in Harare for 1hr30min

Example 2

How long does the plane take to travel from Kigali to Lusaka?
Arrival time at Lusaka – departure time from Kigali

Arri. 1630

Dep. 1230 = 4hrs

400

The plane takes 4hrs

Example 3

What time does the plane reach Entebbe?
(give your answer in 12hour clock system)
2200 hours to 12-hour clock system = 2200

- 1200

1000 = 10.00pm

The plane reaches Entebbe at 10.00pm

Study the marine timetable showing the departure times and arrival times of a ship

Port	**Arrival time**	**Departure time**
A		0630hrs
B	1030hrs	1045hrs
C	1130hrs	1150hrs
D	1340hrs	1400hrs
E	1700hrs	1700hrs
F	2030hrs	

Activity: 11.17

Use the tables above to answer the following questions:

1. How long does the plane stay at these airports?
i) Harare ii) Lusaka

2. How long does the plane take to travel from:
i) Kigali to Harare? ii) Harare to Lusaka? iii) Lusaka to Entebbe?
iv) Kigali to Entebbe?

3. What time in the 12hour clock system does the plane leave:

i) Kigali? ii) Harare? iii) Lusaka?

4. Which airport does the plane leave at 1530hrs
5. Which airport does the plane arrive at 2200hrs?
6. What time does the ship leave the following ports
i) A (in 12hour clock) ii) E (in 12hour clock) iii) F (in 12hour clock)
7. How long does the ship stay at the following ports?
i) D ii) E
8. How long does it take to travel between stations
i) A and B ii) B and E iii) A and F
9. Find the difference between the times given below:
i) 0400hrs and 0700hrs ii) 0040hrs and 0620hrs
10. A plane left Soroti flying school at 1328 and reached Entebbe at 1410hrs. How long did it take traveling from Soroti to Entebbe?

Congratulations!

11.7.8: TEMPERATURE

Hullo student

Can you define temperature?

a) Definition of temperature

Temperature is a degree of hotness or coldness

Now lets remember what our grand parents used to measure temperature.

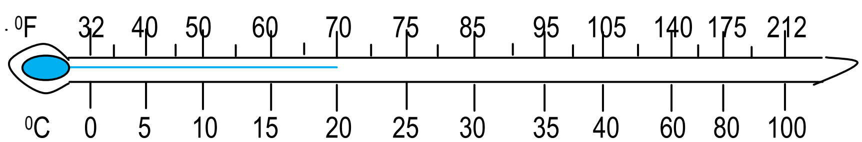
b) Non-standard units of measuring temperature

Feeling /touching the object or substance.

However as man developed, he invented an instrument for measuring temperature called thermometer.

c) Standard units of temperature

- Celsius scale
- Fahrenheit scale



A thermometer showing the two scales-Fahrenheit and Celsius

Read this;
In the diagram above the temperature scale on the Fahrenheit thermometer starts with 32, and ends with 212^0. whereas the temperature scale on the Celsius thermometer begins with 0^0 and ends with 100^0. The units of measure are called degrees. For these units (degrees) we use the symbol (^{0}) e.g. for 36 degrees we write 36^0. How would you write 65 degrees? It should be written as 65^0.

Since there are two ways of measuring temperature, it is important to show which scale you are using. We therefore include the letter F for Fahrenheit and C for Celsius. The complete reading therefore is 65^0F, meaning 65 degrees using the Fahrenheit scale. How would you read 15^0C?

It should be read "fifteen degrees Celsius" an indication that the Celsius scale was used to read the extent of heat of an object or place

Note;
On the Celsius scale, water at the sea-level freezes at 0 (zero) degrees and boils at 100^0. On the Fahrenheit scale, water freezes at 32^0 and boils at 212^0.

Activity 11.18

Either

1. Get thermometers from the maths corner or science department with the guidance of the tutor take readings of:
   i. The room temperature
   ii. Temperature of boiling water or ice
   iii. Boiling water/ hot tea

   Take recordings and discuss with your friend

   Or
2. Visit a nearby clinic/dispensary to see how a medical worker uses a thermometer to take temperature of patients. Request and study temperature records of patients.

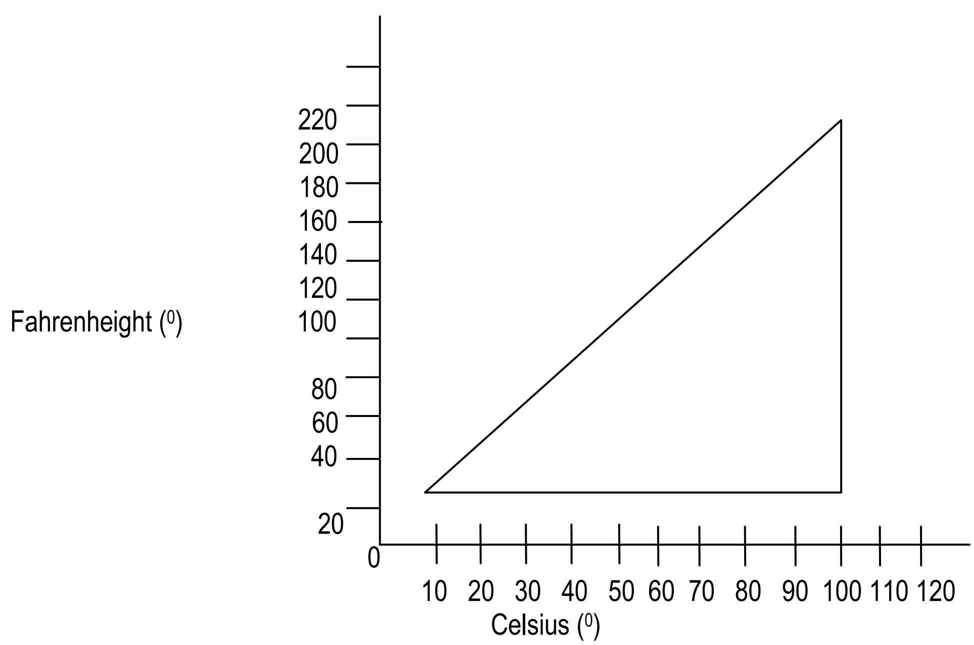
d) Conversion of units of temperature

Activity 11.19

1. Ask your tutor to get/ provide you a graph paper if you do not have one.\
2. Using knowledge of freezing and boiling points of water at sea level for the two scales Fahrenheit and Celsius, plot a graph, Fahrenheit against Celsius scale.
3. Identify the coordinates of the two points of the scales, freezing and boiling points viz; (0,32) and (100,212)
4. Choose a scale for the graph; Celsius axis as 1:10 and Fahrenheit axis as 1:20 see graph below

Scale: Celsius axis is 1:10
Fahrenheit axis is 1:20

Use the graph to complete the table below;



Fahrenheit scale against a Celsius scale.

Scale: Celsius axis is 1:10
Fahrenheit axis is 1:20

Use the graph to complete the table below;

^{0}C	10	20	30	40	50	60	70	80	90
^{0}F									

From the triangle drawn in the slope of the graph (212-32)=180 vertically as against 100 horizontally. When you compare the two quantities (remember what you did in ratios) you get;

$\frac{180}{100} reduce\ get \frac{9}{5}$

Inorder to change from one scale to another, we must use the equation of the line I.e ($y = mx + c)$

$y = mx + c)$ (where m is ratio, c is the constant)

F=$\frac{9}{5}c + 32$

We add 32 because on the Fahrenheit scale 32 ^{0} F corresponds to 0 ^{0} C.

When we want to make c our subject then we would use the knowledge of solving equations like this;

F=$\frac{9}{5}C + 32$ (subtract 32 from both sides)

F- 32=$\frac{9}{5}C + 32 - 32$

F- 32=$\frac{9}{5}C$ (multiply 5 by both sides)

$5(F - 32) = \frac{9C}{5} \times 5$

$\frac{5}{9}(F - 32) = \frac{9C}{9} Divide 9$

C = $\frac{5}{9}(F - 32)$

Therefore the two equations become our formula for changing from one unit to another.

a) Changing from Fahrenheit to Celsius

Example

Change 113 ^0F to Celsius

Procedure:

1. write the formula

C=$(F - 32) \times \frac{5}{9}$

2. substitute

C=$(113 - 32) \times \frac{5}{9}$

3. solve c=81$\times \frac{5}{9}$

=9$\times$5

$\therefore 113\,^{0}F = 45\,^{0}C$

b) Changing form Celsius to Fahrenheit

Example 1

Change $50\,^{0}C$ to Fahrenheit

Procedure:

$$F=\frac{9}{5}c+32$$

$$F=\left(\frac{9}{5}\times 50\right)+32$$

$=90+32$

$= 122$

$\therefore 50=122\,^{0}F$

Activity 11.20

1. Change from $^{0}F\ to\ ^{0}C$

a) $185\,^{0}F$ b) $208\,^{0}F$ c) $158\,^{0}F$ d) $194\,^{0}F$

2. Change from ^{0}C to ^{0}F

a) $15\,^{0}C$ b) $40\,^{0}C$ c) $65\,^{0}C$ d) $35\,^{0}C$

Good luck

11.7.9: speed

Hullo student

Let's look at the last item in this unit Viz; speed.

Can you explain what speed is?

Definition of speed:

Speed is a name given to distance you can cover in a certain length of time, usually an hour. Or speed is the rate of covering a distance with respect to time.

Changing units of speed.

Changing speed from kilometers per hour (km/hr) to metres per second (m/s)

Study the following examples

Example 1

Express 90km/hr in m/s

Distance is in km

Time is in hrs

We need to change km to m.

1km = 1000m
90km=(1000x90)m
=90000

We also have to change hours to seconds
1hour = 60 minutes
1minute = 60seconds
So 1 hour = 60x60 seconds

Speed(s) = $\frac{\text{distance in metres}}{\text{time in seconds}}$

= $\frac{90000m}{3600s}$

= 25m/s

Activity 11.21

Express the following speeds in m/s

i) 450km/hr ii 126km/hr iii 270km/hr
iv) 756km/hr v) 1080km/hr vi) 6120km/hr

Changing speed from metres per second (m/s) to kilometers per hour (km/hr)

Study the following examples

Example 1

1) Change 100m/s to km/hr

We need to change m to km

1000m=1km

1m = $\frac{1}{1000}km$

100m = $\left(\frac{1}{1000}\times 100\right)km$

= $\frac{1}{10}km$

Next we change seconds to hours 60x60 seconds = 1hour

1sec = $\frac{1}{60\times 60}hr$

S= $\frac{D}{T}=\frac{1}{10}km\div\frac{1}{60\times 60}\sec$

= $\frac{1}{10}km\times\frac{60\times 60}{1}$

= 360km/hr

Example 2

2) Express 40m/s in km/hr

1000m=1km

1m $= \frac{1}{1000}km$

40m $= \left(\frac{1}{1000} \times 40\right)km$

$= \frac{4}{100}km$

$60 \times 60\sec$ =1hr

1sec = $\frac{1}{60 \times 60}hr$

S= $\frac{D}{T} = \frac{4}{100}km \div \overline{60 \times 60}/hr$

$= \frac{4}{100} \times \frac{60 \times 60}{1hr}$

$= 4 \times 6 \times 6km/hr$

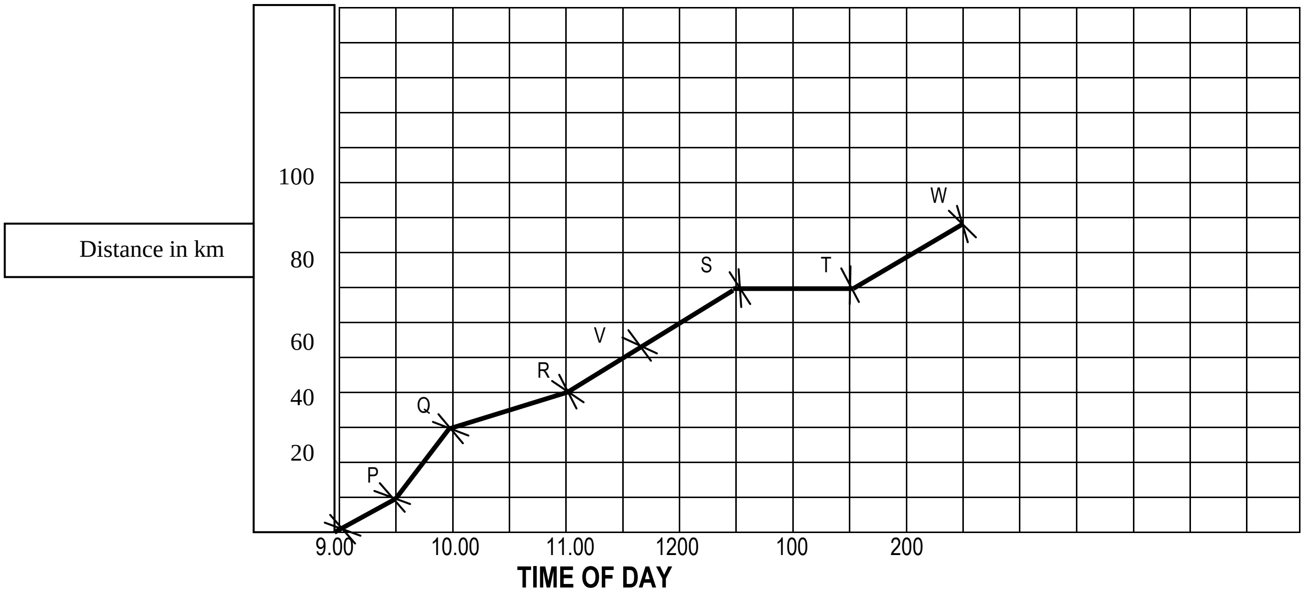
=144km/hr

Activity 11.22

1. Express the following speeds in km/hr

i)10m/s ii) 5m/s iii) 60m/s
iv)15m/s v)1000m/s vi)35m/s
vii)200m/s viii)75m/s

2. A boy ran 150metres in 15seconds. Calculate his speed in km/hr.
3. Express 25m/s in km/hr.
4. Express 270m/s in km/hr.



The graph shows the progress of a lorry from 9.00am to 2.00pm. The lorry leaves Soroti at 9.00am. It moves through places marked up to Q which is Bukedea.

Study the graph then answer the following questions;

Example 1:

What is the average speed of the lorry from Soroti to point R?

Solution:

Distance from Soroti to R=40km
Time taken from Soroti to R=2hrs

$\therefore$ speed = $\frac{40}{2} km/hr$

= 20km/hr

Example 2

What is the average speed from Soroti to Tororo?

Solution

Distance from Soroti to Tororo = 70km

Time taken = (12.00-9.00) +1
= (3+1) hr
=4hr

Remember S to T from 12.00noon to 1.00pm is resting time

S= $\frac{D}{T} km/hr$

$= \frac{70}{4} = 17\frac{1}{2} km/hr$

Activity 11.23

Use the graph to answer the following questions;

1. What was the average speed of the lorry from P to Q?
2. What was the distance from Q to R?
3. What was the average speed from R to S?
4. What was the distance from T to R?
5. What was the average speed of the lorry from Q to V?
6. A bus leaves Kampala for Arua at 8.00am. If it travelled at an average speed of 65km/hr. When did it arrive at Kigumba 208km away?

Check your answers

11.9 Unit summary

You have completed learning mathematical concepts in unit 10 on integers. In this unit, you were introduced to concepts of money, length, area, volume, capacity, mass, time, temperature and speed.

11.10 answers

Activity 11 .1

1. Sh 56000
2. Sh.50000
3. 10%
4. $33\frac{1}{3}\%$
5. Sh. 72250
6.
7. Sh. 48060

Activity 11.2

1. $ 1261.9
2. Ush. 9949500
3. (a) (i) 283.58 (ii) Shs. 921250 (iii) 34.34 (iv) shs. 3,216,000
   (b) (i) Ush. 728700 (ii) $. 404.76 (iii) Ush. 1,384,530 (iv) $ 500.28

Activity 11.3

Activity 11.4

1. (a) 80mm (b) 75mm (c) 6.25mm
2. (a) 29.5m (b) 3.5m (c) 79m
3. (a) 7.5km (b) 4.5km (c) 4.85km
4. 8m. 40cm
5. 1m. 65cm

Activity 11.5

1. 195cm^2
2. 130cm^2
3. 360cm^2
4. 180cm^2
5. 2400cm^2
6. --------
7. 20cm^2
8. 62.5cm^2

Activity 11.7

1. 274625cm^3

2. $1848cm^3$
3. $105cm^3$
4. $4167cm^3$(to whole number)

Activity 11.8.0

Activity 11.8.1

Activity 11.9

1. 131 litres
2. 171 litres
3. 8 litres 725 ml
4. 17l. 750ml
5. 1702l. 400ml
6. 24 children

Activity 11.12

1. (a) 6.5kg (b) 12 kg (c) 75kg (d) 0.25 kg
2. (a) 13000g (b) 1350g (c) 250g (d) 125000g
3. (a) 353.96kg (b) 353960g
4. 479kg. 825g
5. 58kg.800g
6. 69kg.200g
7. 12kg.05g
8. (a) 2.5 tonnes (b) 0.65 tonnes
9. 135kg.
10. Shs. 22000
11. Shs. 1,029,000

Activity 11.13

(i) Twenty five minutes to eight in the morning
(ii) Twenty minutes past nine in the morning.
(iii) Twenty minutes to two in the afternoon.

Activity 11.14

1. 0200 hrs.
2. 0400 hrs
3. 2300 hrs
4. 0035 hrs
5. 1830 hrs
6. 0810 hrs

7. Started at 1000hrs ended at 14.45 hrs
8. Start: 1130 hrs End: 13.45 hrs. The journey took 1 hour 15 min

Activity 11.15

1. (i) 6 hrs. 15min (ii) 3hrs. 37 mins
2. (i) 77 hrs. 20 mins (ii) 5 hrs. o4 mins
3. (i) 16 weeks. 4 days (ii) 8 weeks. 4 days
4. 300 min
5. 5.45 am
6. (i) 3.00 am (ii) 9.40 pm (iii) 1.25pm (iv) 12.00 (midnight)
7. 6.45pm
8. (i) 12.27 am (ii) 1.29 am.

Activity 11.16

1. (i) 3.20 pm (ii) 8.50pm (iii) 11.05pm (iv) 12.05am
2. (i) 35 min (ii) 5 min
3. 4 hrs. 30 min

Activity 11.17

1. (i) I hrs. 30 mins (ii) 4 hrs
2. (i) I hr. 30 mins (ii) 6 hrs. 30 mins (iii) 2 hrs. 30 mins
3. (i) 12.30 pm (ii) 3.30 pm (iii) 8.30 pm
4. Harare
5. Entebbe
6. (i) 10.45 am (ii) 5.00 pm (iii) ---------
7. (i) 20 min (ii) did not stop
8. (i) 5.00hr (ii) 6hrs. 15 mins (iii) 10 hrs
9. (i) 3 hrs (ii) 5 hrs. 40 mins
10. 42 mins

Activity 11.20

1. (a) 85^0c (b) 104^0 F (c) 149^0F (d) 95^0F

Activity 11.21

1. (i) 125mls (ii) 35mls (iii) 75mls (iv) 210mls (v) 300mls (vi) 1700mls.

Activity 11. 22

1.
   - i 36km/hr
   - ii 18km/hr
   - iii 216km/hr
   - iv 54km/hr

v 3600km/hr
vi 126km/hr
vii 720km/hr
viii 306km/hr

2. 36km/hr
3. 90km/hr
4. 97.2km/hr

Activity 11.23

1. 40km/hr
2. 10km/hr
3. 20km/hr
4. 30km
5. 14.12km/hr
6. 3hrs.12min

11.11 End of unit exercise

1. A hawker bought a leather belt at shs. 5000 and sold it at shs. 8000.

   (a) What was his profit?
   (b) Express his profit as a percentage.

2. The cost price of a car is Ug. Shs. 15 million and the percentage profit is 20% find the selling price.
3. Convert the following length to metres. (a) 2 km (b) 0.25Hm (c) 5827mm
4. Calculate the area of a trapezium with height 20m and the sum of the two parallel sides 28m.
5. (a) Calculate the radius of a cylinder with volume 5.5 litres and height 70cm
   (b) Find the surface area of the cylinder above

6. Change the following to grams. (a) 0.445kg (b) 2 ½ kg
7. Express the following speed in km/hr to m/sec (a) 56km/hr (b) 150km/hr
8. A sick man's body temperature by the time he reached hospital was 42^0c convert that temperature to ^{0}F.
9. Calculate the area of a rhombur with diagonals 12 cm and 18cm.
10. Find the side of a cube with volume $343m^3$.

11.12 self check/assessment
Congratulations upon completion of unit 11 of this module. Find here below the learning outcomes to help you tick in the appropriate column that best reflect your level of understanding.

Learning outcome	**Sure**	**Not sure**
I can solve problems involving money		
I can solve problems involving length		
I can calculate area of figures		
I can work out volumes of figures		
I can do problems involving capacity		
I can solve problems involving mass		

I can solve problems involving time		
I can solve problems involving temperature		
I can solve problems involving speed		

UNIT 12: ALGEBRA 1

12.1 Introduction

You are most welcome to this unit 12

This unit introduces you to the early algebra (algebra 1). Through this you will be introduced to ideas like algebraic expression, substitute a number for each variable and find the results, order of operations, exponential notations, simplifying algebraic expressions, solving simple equations and inequalities.

12.2 Content Organization

Dear student, in this unit you are going to cover the following topics as shown in the table below:-

Topic	**Subtopic**
1. Introduction to Algebra	a) Algebraic sentences (statement) b) Completion of Algebraic sentence c) Find value represented by a letter in an algebraic sentence d) Operation orders i.e. commutative, associative and distributive properties.
2. Simplify algebraic expressions	a) Like terms and unlike terms in algebraic expression b) Simplify of algebraic expressions c) Brackets d) Removing brackets
3. Substitution	a) Substitution of number for letters in algebraic expression b) Substitution of number for letters in formulae
4. Simple equation solving	a) Solving first degree equations b) Equations involving fraction and brackets c) Forming equations in one unknown d) Solve algebraic to solve wordy problems in mathematics
5. Demonstration lesson on algebra	Demonstration of how to teach algebraic concepts in primary schools.

12.3 Learning outcome

By the end of the unit, you are expected to:

i. Use algebra in solving problems
ii. Teach concepts in Algebra found in the primary school.

12.4 Competences

Dear student, now you know the learning outcomes therefore as you study this unit, you will be able to:

a) Give examples of algebraic sentences
b) Compete algebraic sentences

c) Find value represented by letter in an algebraic sentence.
d) Identify like and unlike term in an algebraic expression.
e) Collect like and unlike terms
f) Simplify algebraic expressions
g) Solve first degree equations involving fractions and brackets
h) Substitute numbers for letters in an algebraic expression
i) Substitute numbers for letters in formulae
j) Solve word problems in mathematics using algebra
k) Write inequalities in one variable
l) Solve inequalities in one variable
m) Demonstrate how to teach algebraic concepts in primary school

12.5 Unit orientation

Hullo student, many students in primary teachers' colleges find difficulties with the fundamental ideas of algebra and learning the skills of algebraic manipulation. This is because the reason lies in primary and secondary having little teaching of pre-algebraic ideas that prepare the ground work for more formal studies. This unit will expose you to a range of such ideas within the ability of covering the catch-ups of what you missed. It will not prepare you on how to teach children to manipulate symbols according to set of rules but it will expose you to different practical methods in different contexts and situations which will enable you understand what algebra is about.

12.6 Study requirements

Dear student in order to be successful in studying this unit you are required to:-

i. Prepare adequate instructional materials using collected materials from your local environment.
ii. Read/ revise about concepts of laws of indices in unit 4, how to add subtract, multiply and divide integers in unit9 and finding perimeters, areas, volumes, money, time, speed, temperature using formulas in unit 11 in this module.
iii. Have primary Mathematics course books, reference books like preparatory mathematics by B.J. Metha, H.e. par school Mathematics part1, algebra structure and methods book (Andrew M. Gleason) and mathematics education module ME/2

12.7 Content

12.7.1. Introduction to algebra

(a) Algebraic sentences

(i) Simple number sentences

In this subtopic you will review work covered in unit 4 on making additions, subtraction, multiplication and division statements true of false.

Let you look at the following carefully,

Example 1:

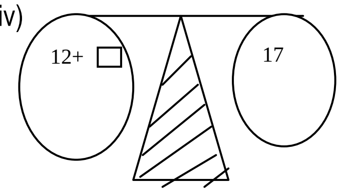
(a) Find the missing numbers in the following

(i) 7 + 4 = $\boxed{}$
(ii) $\boxed{}$ + 9 = 13

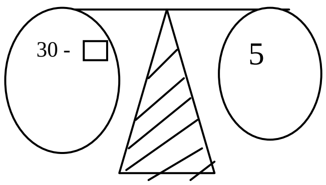
$\boxed{32}$

(iii) 6 - = $\square$

(iv)



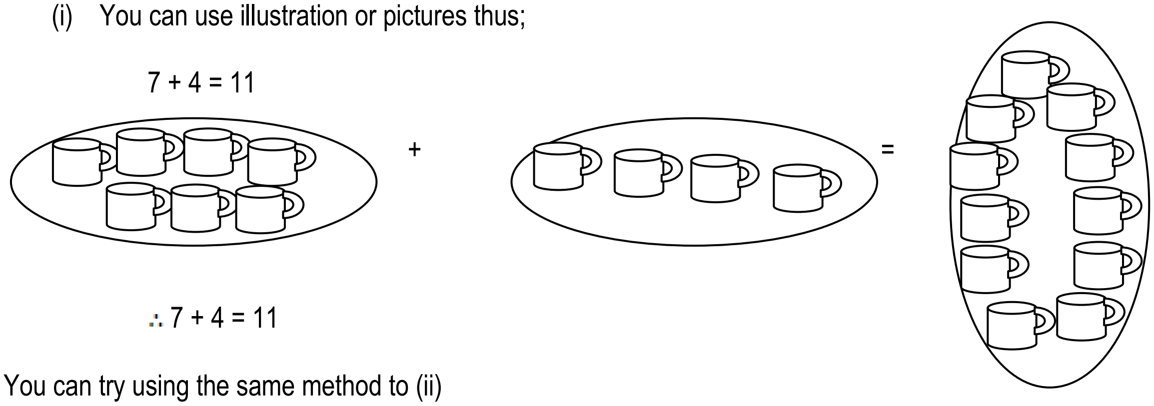
(v)



(b) Study how you can get solution/answers

(i) You can use illustration or pictures thus;

7 + 4 = 11

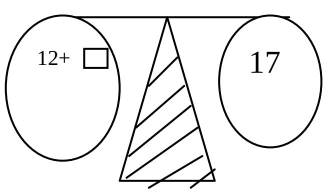


∴ 7 + 4 = 11

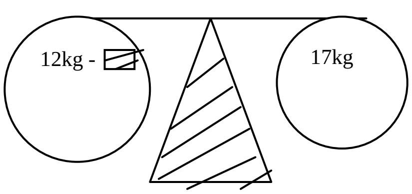
You can try using the same method to (ii)

Share your findings with your colleague.

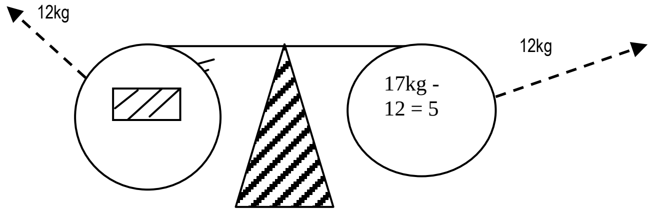
(c) Let you study the working below



Step 1: assume the weights are in kg.



Step 2: if you take away 12kg from each side you have

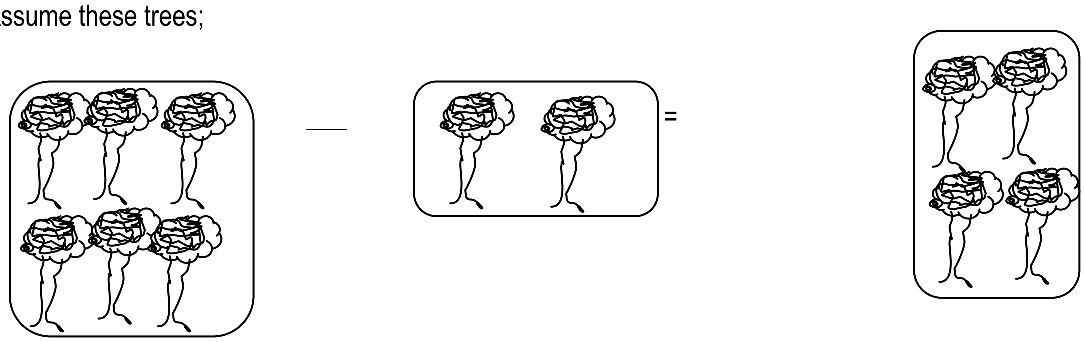


This shows that the stone weighs 5kg. When you take the same weight from each side, the two sides are balanced.

Get a colleague make weighing scale using local materials; do number (v) practically. Share your findings in a plenary. Note the general observation made by the class.
Study example 1 (iii)

6 – 2 = $\boxed{}$

Assume these trees;



∴ 6 - 2 = 4

From the illustration above you note that trees remaining in the set, after taking away two (2) are equal to 4.

Example 2.

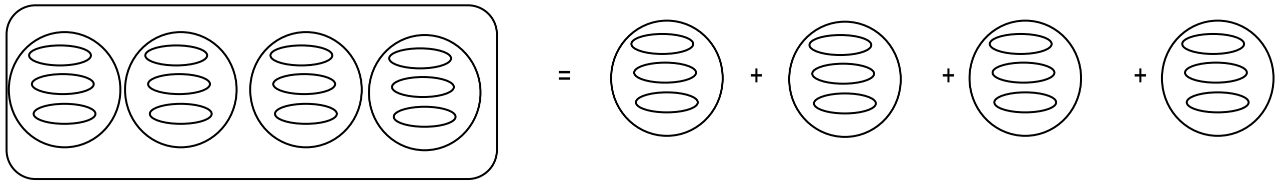
Let you also study example 2.

What are missing numbers below:

(i) 3 x = $\boxed{}$
(ii) 16 ÷ $\boxed{}$

Study example 2(i)

Assume there eggs;



12
12 eggs = 3 egg x 4 groups

$\therefore$ 3 x 4 = 12

To find the missing number in example 2(ii), you require to divide the answer thus 12 by the given number (factor) then, the missing number is equal to 12 ÷ 3 =4

In example 2(ii) the missing number (factor) is equal to 16 ÷ 4 = 4.

With colleague can you try this activity?

Activity 12.1

Using illustration find the missing numbers in the following number sentences

(a) 11 + 4 = $\square$

(b) 5 + $\square$ = 13

(c) $\square$ + 7 = 15

(d) 13 – 4 = $\square$

(e) $\square$ - 2 = $\boxed{5}$

(f) 17 - $\square$ = 15

(g) 4 x 2 = $\square$

(h) 6 x $\square$ = 24

(i) 18 ÷ $\square$ = 6

i) Can you discuss your answers with your colleagues?
ii) What observations have you made about the problems?
ii) What do you mean by Algebra?

Read the text below:

From the above explanation, you found that the simple number sentences had missing numbers. These missing numbers can be represented using letters to stand for missing numbers for example 3+$\square$=7 can be represented as 3+ $x = 7$, $\square$ - 2 = 15 as n – 2 = 15, 3 x $\square$= 21 as 3 x y = 21 and 14 ÷$\square$= 7 as 14 ÷ t = 7, and so on.

Once we use letters to represent missing numbers, this is called algebra. These letters used in expressing algebraic sentences are called variables e.g. x, y, t etc. A variable is a letter that is used to represent one or more numbers.

(iii) Variables in algebraic expressions

What is an algebraic expression

An algebraic expression is a collection of numbers, variables, operations and grouping symbols. Here are some examples

Example 1

2 times a, plus b times c. This can be represented as $2 \times a + b \times c = 2a + bc$

Example 2:

If n is a divisor of 12 without a remainder, find the value of n.

In this case n is an open statement.

The possible values are 1, 2, 3,4,5,6 and 12.

$\therefore$ The statement in the example above is true when n takes any of the values 1,2,3,4,5,6 and 12.

Look at example 3

Multiply the number P by 4, and then take 2 away; the result is equal to 34. This becomes simply 4p-2=34
Can you give us the number which p stands for?

Example 4:

State in words the operation represented by $\frac{3y-8}{5}$

$\therefore$ The operations are:-

Multiply y by 3, take 8 away and divide the result by 5.

Do the following in your exercise book

Activity 12.2

a) Write the following in algebraic expressions

i. Four times the square of x.
ii. Take x away from n.
iii. From 4 times x take away 5 times y.
iv. X multiplied by 3 equals 12.
v. Add 4 to twice Q; the answer is 20
vi. Add 4 times x to 6 and then divide the result by 2.
vii. Divide x by 5; the result is equal to 9.
viii. Add 2 to three times x and the result is 23; therefore x is equal to 7.
ix. 3 less than twice a number.

(b) State in words the operations represented by:

i) $\frac{1+7x}{4}$ ii) $\frac{x^2-3}{5}$ iii) $\frac{1}{2}x+3$ iv) $\frac{1}{2}(x+3)$

Check your answers with these in the table below:-

(a)	
i	4 x x^2 or $4x^2$
ii	n – x
iii	4x – 5y
iv	3x = 12
v	2Q + 4 = 20
vi	$\frac{6+4x}{2}$
vii	$\frac{x}{5}$ = 9
viii	$3x+2=23$
(b)	
i	Add 1 to seven times x divide the results by 4
ii	X squared take away 3 and divide the results by five
iii	3x + 2 = 23

Read to note the following text in the table below:-

An algebraic expression is a collection of numbers, variables, operations and grouping symbols. Here are some examples:-

Algebraic expression 5n	Meaning 5 times n
$4x^2$	4 times the square of x
2a+bc	2 times a, plus b times c

∴ The language of algebra. When we use letters to stand for numbers is known as algebra. Two symbols are often used. These are: = , meaning " is equal to" and ∴ meaning "therefore"

Well done for completing this lesson.

ii) Finding values represented by letters in algebraic sentences/ statement

Example 1:

5 is added to number x, the result is equal to 14. What is the number represented by x?

To find the value of x:-

Form a simple equation x + 5 = 14

Then x + 5 = 5 = 14 – 5

X = 5

Therefore, the number is 9.

Example 2:

Write down the values for **m**. In given algebraic sentence **m** is a multiple of 6, less than 24 using symbols:

M=M6 =

$$m = m_6 = 6 \times 1 = 6$$
$$= 6 \times 2 = 12$$
$$= 6 \times 3 = 18$$
$$= 6 \times 4 = 24$$
$$Then, m = \{6,12,18,24\}$$

$\therefore$ Possible values are 6 or 12 or 18

With your colleague can you have practice answering the following:-

Activity 12.3

a) Write the possible values/numbers represented in algebraic expressions:-

i Twice y is equal to 8
ii Add 3 to thrice k, the answer is 15
iii **x** multiplied by 3 equals 12
iv Twice **x** taken from 24 is equal to 6 times x.

b) What are the possible values for the unknown numbers in the following:-

i $q^2 = 16$
ii X is a division of 20
iii Y is a counting number less than 5.
iv n is multiple of 4 between 4 and 28
v Ten minus y is equal to six

Share your answer with your tutor in plenary.

I hope you now know the differences among the following terms if not discuss them with a colleague.

“Algebra”, a “variable”, “algebraic” sentence or statement and an “algebraic expression”

iii) Operation orders i.e.

Let you study about some of the operation orders:-

- **Commutative and identity properties**

In this lesson you will study number properties as they apply to algebraic expressions. I imagine you already know that the order in which you add two numbers does not affect the sum or answer.

- **Commutative property**

Example1:

Write an expression equivalent to y+5 using a commutative property of addition

Then, y+5=5+y

Example 2:

What is the expression of an expression equivalent to each other using commutative property:

i) xy is x .y = y.x

ii) 5 + ab is 5 + b = ab + 5

Another is 5 + ab = ba + 5

May you try the following:-

Try this;

Use a commutative property to write an equivalent expression

i) x + 9
ii) pq
iii) xy + t

Dear student thank you for learning this operation order, let you embark on another one,

(v) Identity properties

Example 1: Addition

a + 0 = a and 0 + a = a

∴(Adding 0 to any number gives that same number/letter/item)

Example 2: Multiplication

Given that

For any number a, 1.a=a
∴a.1=a (multiplying a number by 1 gives that some number/letter/item)

Example 3: dividing a number by itself:

For any number a, $a \neq 0, \frac{a}{a} = 1$

ii) $\frac{m+3}{m+1} = 1$

Can you do the following in your exercise book: while discussing the problems with your colleague.

Activity 12.4

a) Write an equivalent expression using a commutative property.

i) y + 8
ii) mn
iii) 9 + xy
iv) ab + c

b) Write an equivalent expression. Use the indicated name for 1.

i) $\frac{5}{6}$ use $\frac{8}{8}$ for 1

ii) $\frac{6}{7}$ use $\frac{100}{100}$ for 1

iii) $\frac{y}{10}$ use $\frac{2}{2}$ for 1

iv) $\frac{m}{3n}$ use $\frac{p}{p}$ for 1

c. Tell whether each pair of expression is equivalent

i) 3x + 5 and 5 + 3 x **x**
ii) 4x and x + 4
iii) bxy + b x and yxb + bx
iv) a + c + e + g and ca + cg

Thank you. You are now progressing well!

Associative property for addition and multiplication.

Let you study how to use associative property in algebraic expressions

Example 1

3 + (7 + 5) and (3 + 7) + 5 = 15
= 3 + 12 = 10 + 5
= 15 = 15

∴ 3 + (7 + 5) and 3 + (7 + 5) are equivalent

Note that for any numbers a, b and c

a + (b + c) = (a + b) + c

Number can be grouped in any order of addition.

Example 2: Multiplication

3 x (4 x 2) and (3 x 4) x 2
= 3 x 8 = 12 x 2

= 24 = 24

∴ 3 x (4 x 2)= 3 x (4 x 2)

Also note that for any numbers a, b and c whereby $a\times(b\times c)=(a\times b)\times c$ **(numbers can be grouped in any order for multiplication)**

Can you try the following in your exercise books?

Activity 12.5

a. Use a commutative property to write an equivalent expression

i) x + 9
ii) pq
iii) xy + t

b. use an associative property to write an equivalent expression

i) y + (2 + 3)
ii) (a + b) + 2
iii) 3 . (v . w)

c. Use the commutative and associative properties to write three equivalent expressions

i) 4 x (t x u)
ii) r + (2 + 5)

Check your answers with these at the end of the unit:-

Can you discuss the rest of the questions with your colleagues?

Using distributive property of multiplication over addition:

Example 1

$$\begin{aligned} &5\times(4+8) \quad or\ (5\times 4)+(5\times 8) \\ &=5\times 12 \qquad\qquad = 20+40=60 \\ &=60 \end{aligned}$$

Hence the expressions $5\times(4+8)$ and $(5\times 4)+(5\times 8)$ are equivalent. The distributive property over addition states that this will always be true.

Example 2: Use the distributive property to write an equivalent expression

$a(b+c)=(a\cdot b)+(a\cdot c)$

= $ab+ac$

Or $(b+c)a=(b\cdot a)+(c\cdot a)$

= $ba+ca$

In your group, discuss the following questions:-

Use the distributive property to write equivalent of the following expressions:

(i) 3 (x + 2)
(ii) (s + t + w)6

(iii) $6(x + 2y + 5)$

Check your answers with these in the table below:-

(i) $3x + 3.2 = 3x + 6$
(ii) $6x + 6.\,2y + 6 \times 5$
$= 6x + 12y + 30$
(iii) $S(6) + t(6) + w(6)$
$= 6s + 6t + 6w$

How are you feeling about algebra?

Be strong about this. Take a rest for a while.

12.7. 2. Simplify algebraic expressions.

a) Collecting like and unlike terms in algebraic expressions.

Let you study the examples shown below:

Example1: Collect like terms

(i) $X + x = 1.x + 1.x$ — Use the identity property

$= (1 + 1)x$ — use the distribution property

$= 2x$

(ii) $3x + 4x = (3 + 4)x$ — Use the distributive property

$= 7x$

(iii) $2x + 3y + 5x + 2y = 2x + 5x + 3y + 2y$ — use commutative property

$= (2 + 5)x + (3 + 2)y$ — use distributive property

$= 7x + 5y$

Try these problems in activity 12.6 your exercise book.

Activity 12.6

Collect like terms and simplify the following:

(a) $a + a + a$
(b) $3x + x + x + x - x$
(c) $3a + 2a + a$
(d) $f - f + f + f - f$
(e) $7x - 2x + 3x - x - 4x$
(f) $5r + 6r - r - 3r$
(g) $3a + 5b + 7a$
(h) $41a + 90 + 60a + 2$
(i) $8a + 8b + 3a + 3b$
(j) $100y + 2002 + 190.1g + 4002$
(k) $23 + 5t + 7y + t + y$
(l) $\frac{1}{2}b + \frac{1}{2}b$
(m) $\frac{1}{2}a + a + a + 5a$
(n) $X + 2x^2 + 3x^3 + 4x^2 + 5x$
(o) $X \times y \times x \times y$

Can you cross check your answers with the ones at the end of the unit.

Dear student I hope you are finding it easy now!

With your colleagues can you do the following exercise?

Exercise 2A nos.1,2 and 4 from secondary school mathematics Book2 page 18-19 (check in the library).

Note that: Read the text below and copy in your note book.

Thus 3a+2a contains the two terms 3a and 2a. These are called **like terms** because they can be collected together to make 5a. Again 5x-3y contains the two terms for 5x and 3y. These are called **unlike terms** because they cannot be collected together and replaced by a single term. 2a+3b-1 contains the three terms 2a, 3b and 1 all **unlike.** This expression cannot be simplified at all unless we know the values of the letters a and b.

2ab consists of only one term, a and b are **factors**, not separate terms.

You can therefore simplify an expression containing several terms by collecting together all like terms, putting those with the same sign together

If you have understood the above text, can you do more problems about collecting like terms:-

Activity 12.7

Collect like terms if possible and put them in shorter terms

1. $x + 2y + 3x – 2x – 2y$
2. $2a – b + 3b – a – b$
3. $a + b + c – a – b + c$
4. $1 + 2x$
5. $1 + 2x + 3a – x – a$
6. $ab + 2ab$
7. $7x + 5 – 2x – 2$
8. $4x – 2y + 2y – x – 3$
9. $5a + 3b – 2a + b + 2$
10. $3x – 3$
11. $3x – x$
12. $21x + 44x + 15y – 16x - 8y – 38xy + 2x + xy$
13. $a\{ 1 + b [1 + c (1 + d)]\}$. Hint: begin with $c(1 + d)$ and work outwards

Dear student, has this exercise been challenging?

Let you then continue working hard.

Can you discuss your answers in a plenary.

b) Simplify algebraic expression

Since you have learnt about collecting like terms, now it becomes easy for you to simplify algebraic expressions.

Let you look at the examples critically

Example 1: simplify

$X^2 + xy + 2x^2 + 2x^2 + xy + 6xy – 4 xy + 6xy$

Bring the like terms together and add: $= x^2 + 2x^2 + 2x^2 + xy + 6xy – 4 xy$

$= 5x^2 + 7xy – 4 xy$

$= 5x^2 + 3xy$

Example 2: multiplication

Simplify: (i) $a \times a \times a \times a$

Use of indices laws

$= a^1 \times a^1 \times a^1 \times a^1$

$= a^{1+1+1+1}$

$= a^4$

(i) $2xy \times 3x^2y$

Collecting like terms

$= 2 \times x \times x \times y \times 3 \times x \times x \times x \times y$

$= (2 \times 3) \times (x \times x \times x \times x) \times (y \times y)$

$= 6 \times x^3 \times y^2$

$= 6x^3y^2$

Example 3: Division

$$6m^3 \div 3m = \frac{6m^3}{3m}$$

$$= \frac{\overset{2}{\cancel{6}} \times \cancel{m} \times m \times m}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{m}}}$$

$$= 2m^2$$

Try the following numbers in your exercise book.

Activity 12.8

Simplify the following

1. $a + 3a + 2a$
2. $a^2 - 3a^2 + 6a^2$
3. $5p - p + 2p - 4p$
4. $3x - 2y + x + 3y$
5. $7pq - 6pq + pqr$
6. $a^3 \times a^2$
7. $m^2 \times m^2 \times m^2$
8. $2x^2y \times 4xy^2$
9. $5b^2c \times 3a^3$
10. $4a^2b \times \frac{3}{4} b^3$
11. $9p^2q \div 6q^2$
12. $5a^2 \div 5p$
13. $2a^3 \times 3a^2 \div 4a^4$
14. $\frac{15a^3}{12b^2} \times \frac{6m^2}{3a} \div \frac{5m}{4b^3}$

Thank you for completing this activity:

c) Brackets and removing brackets

Let you find out the effect of brackets:

Example 1

(i) $10 + (2 + 6) = 10 + 8 = 18$ or $10 + 2 + 6 = 18$

(ii) $10 - (2 + 6) = 10 - 8 = 2$ or $10 - 2 - 6 = 2$

(iii) $10 - (6 - 2) = 10 - 4 = 6$ or $10 - 6 + 2 = 6$

You should note carefully that the change in the signs of terms

Example 2

Thus, a + (b + c) = a + b + c
a – (b + c) = a – b – c
a – (b – c) = a – b + c

3 (a – 2) + 4 = 3a – 6 + 4 = 3a – 2
a – (2 – a) + 3 = a – 2 + a + 3 = 2a + 1

Example 3: Remove brackets from the following:

(i) 2 (4a – 3) + 3 (a – 1)
= 2 x 4a – 3 x 2 + 3 x a – 1 x 3
= 8a – 6 + 3a – 3
= 8a + 3a – 6 – 3
= 11a - 9

Remove brackets and simplify

(ii) $\frac{4}{5}$(10m + 15n) + ¾ (12m + 16 n)

Method 1:

Look for a common factor in each bracket

= $\frac{4}{5}$ x 5 (2m + 3n) + ¾ x 4 (3m + 4n)
= 4 (2m + 3n) + 3 (3m + 4n)
= 4 x 2m + 3n x 4 + 3 x 3 + 4n x 3
= 8m + 12n + 9m + 12n
= 8m + 9m + 12n + 12n
= 17m + 24n

Collect like terms

Method 2

$\frac{4}{5}$(10m + 15 n) + ¾ (12m + 16n)
= $\frac{4}{5}$ x 10m + 15m x $\frac{4}{5}$ x ¾ x 12m + 16^nx ¾
= 8m + 12n + 9m + 12n
= 8m + 9m + 12n + 12n
= 17m + 24n

Using the examples 1 – 4. Can you do the following exercise.

Activity 12.9

1. Remove brackets from the following
   a) 2(3x – y)
   b) 7 (2a – 3b)
   c) 3x (x – 1)
   d) 4 (x^2 – x + 1)
   e) a(2x – t)

f) $5a(2a - 4)$
g) $6(4p + 2q + 1)$
h) $2x (a + 3b)$
i) $5 (a + 4) + 2$
j) $a + b(c + d)$
k) $4x (1 - 8x)$
l) $\frac{1}{2} (4x + 8y)$
m) $\frac{1}{5}(10m - 15n$

2. Remove brackets and simplify

a) $3(a + 1) + 2$
b) $(q + 3)3 - 3q$
c) $2(a + 3) + 3 (a + 2)$
d) $2m (m - 4) + 5 (2m + 1)$
e) $k(k + 1) -2 (k + 1)$
f) $5(a + b + c) + 2 (2a - b - 3c)$
g) $a^2 (a + ab) + a (a^2 + 6)$

Are you doing well?

You can take a walk.

(e) More use of brackets.

Study the example below:-

Example 1:

Put phrases in brackets

Collect like terms

Subtract $2x - y$ from $3x + 2y$

$(3x + 2y) - (2x - y)$
$= 3x + 2y - 2x + y$
$= 3x - 2x + 2y + y$
$= x + 3y$

Example 2:

What must be added to $(4a + b)$ to make $(6a - 3b)$?

Think of the statement

$(4a + b) +$ --------$= (6a - 3b)$

To find the missing expression we subtract $(4a + b)$ from both the left hand side and the right hand side.

Then, the missing number is

$(6a - 3b) - (4a + b)$
$= 6a - 3b - 4a - b$
$= 6a - 4a - 3b - b$
$= 2a - 4b$

Check

$(4a + b) + (2a - 4b)$
$= 4a + b + 2a - 4b$
$= 4a + 2a + b - 4b$
$= 6a - 3b$

$\therefore$ Your answer of $2a - 4b$ is correct.

Using the above examples do the following problems.

Activity 12.10

1. Subtract $x + y$ from $2x - y$
2. Subtract $3a - 2b$ from $5a - 2b$
3. What must be added to $4x - 3y$ to make $3x + 2y$?
4. Subtract $4m - 3n$ from $8m - 5n$.
5. Peter had $(2x + y)$ hens. Later he lost $(x - y)$ of his hens. How many hens did he have left?
6. Mary is $5x$ years old. Betty is $2x$ years younger than Mary. How old is Betty?
7. Remove the brackets $\frac{4(15 + 5a + 5b}{5}$

Let you compare your answers with those in the table below;

If you have successfully completed this topic, then you will fin the following work very easy

You can now conclude that;

Remaining brackets:

First look at a numerical expression
$2 \times (13 + 5) = 2 \times 18 = 36$

Topic 3: substitution of numbers for letters in algebraic expression.

Dear student,

What is substitution for value?

Let us read the text below.

The process of replacing each variable i.e. letter/unknown in an expression by the numerical for a given value of the variable and simplifying the result is known as evaluating the expression or finding its value.

You will thoroughly understand by discussing the following examples;

If a = 2; b = 1, c = 3 find the value of

Example 1.

b) abc = a x b x c
= 2 x 1 x 3
= 6

Try this on own: and the check with the working below

ii

= (a x a) + (b x b) + (c x c)
= (2 x 2) + (1 x 1) + (3 x 3)
= 4 + 1 + 9
= 14.

Example 2: If the value of x = 6 and y = 2

Evaluate: $\frac{4x+3y}{x-2y}$

= (4 x 6) + (3 x 2)
= 6 – (2 x 2)

$= \frac{(4 \times x) + (3 \times y)}{6 - (y \times y)}$

$= \frac{24 + 6}{6 - 4} = \frac{30}{2} = 15$

Note that it is use ful short cut to write “if x = 6 for “ x if x has the value 6”

Here “u” means “has the value”.

Can you follow the examples to these numbers in the table below in your exercise book.

Activity 12.11

15. if a = 2; b = 1, c = 3 find the value of

a) a^2b
b) $a^3b + bc^2$
c) $a^3bc + 16 – a^2$
d) $3a^2 c – 2 abc$
e) $\frac{1}{3}C^2(a^2 – b^2$
f) $\frac{a^3c + a^2c}{a^2c^2}$

2. given that L= 4, b = 3, m = 2
π = 3.14; r = 5 what is the value of the following

a) Lb
b) πr^2
c) 2(L + b)
d) $\frac{4}{3}\pi r^2$

3. If x = 5, b = 3, c = 4, y = 2

a) $5b^2c – 3bc$

b) $x^y + x^b$

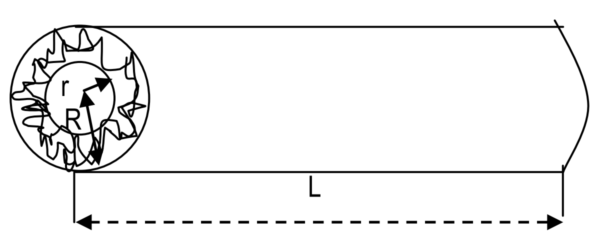
c) $\frac{2(2b+c}{x} - 5\ \frac{xc-xb}{xy}$

d) $\frac{(8x)^a - c^a}{C^2 – by}$

a) substitution of numbers for letters in formulae

Let you have more practice by studying through examples:

The diagram below represents a metallic pipe of mass Mkg.
Given that $M = \pi Ld(R^2 – F^2)$ where d is the density of the metal, find the mass of the pipe for L=5, d= 2.7 x 10^3, R = 0.08 and r = 0.05



Substituting in the formular:

$M = \pi l d\ (R^2 - r^2)$
$M = \pi\ x\ l\ x\ d$ [(RxR) – (r x r)]
= 3.14 x 5 x 2.7 x 10^3[(0.08 x 0.08) – (0.05 x 0.05)]
= 42390 [0.0064 – 0.0025]
∴ M = 165.321 kg
With your colleague try the following problems. Write your answers on a piece of paper.

Activity 12.12

1. If x = 2, y = -4 and 2 = 5, evaluate
   $\frac{(x+y)_2}{2(y-2}$
2. Given that h = 10t – 10 t –t^2, find h when t = s
3. In the formular v = u + a t
   v is the final velocity, u is the initial velocity, a, is the acceleration, t is the tine taken for the velocity to increase from u to v. find the acceleration, a, given that the initial velocity was 15m per second, the final velocity was 45 m per second and the time taken to increase the speed from 15 to 45 metres per second was 10 seconds
4. I = $\frac{PRT}{100}$, find the value of

I when, P = 250, R = 3, T = 6

Report your findings in a plenary. Then check your findings with these in the answers at the end of the unit.

Note that as you compare your answers, mind about the methods of working out the problems.

From practice about substitution in algebraic expressions with your group mates:

Do the revision exercise 2, secondary school mathematics Book 2, new edition page 25. Nos; 2,4,6(e), 8 and 12.

Read the notes given below;

1. A letter can be used as a variable or an known in an algebraic expression.
2. An algebraic expression involving an equal sign is an algebraic equation
3. Algebraic expression and algebraic equations can be simplified substitution involves replacing some variables by known values.
4. Formulae are algebraic equations.

Well done.

May you take a walk before you continue.

12.7.4. SIMPLE EQUATIONS SOLVING

a) Solving first degree equations.

Can you imagine: 5 + 4 = 9 is an example of an equation?

Discuss it with your colleague.

Study the following patterns:-

20 = 20
20 + 4 = 20 + 4 i.e. 24 = 24
20 – 8 = 20 – 8 i.e. 12 = 12
20 x 3 = 20 x 3 i.e. 60 = 60
20 ÷ 5 = 20 ÷ 5 i.e. 4 = 4

Note:

Whatever you do to one side of the equation, you must do the same thing to the other side of the equation. This is called the **balancing** of an equation.

Let you study more examples

Example 1

(i) Solve for x

a + 5 = 8
a + 5 – 5 = 8 – 5
∴ a = 3

(ii) Solve for n

2n = 12
= 2n ÷ 2 = 12 ÷ 2
∴ n = 6

(iii) 5 x + 6 = 21 rewrite original operation.

= 5x + 6 – 6 = 21 – 6 to isolate the x – term, subtraction 8 from each side
= 5x = 15 simplify
= $\frac{5x}{5} = \frac{15}{5}$ to isolate x, divide each side by 3
∴ x = 3 simplify

Example 2: find the value of K

(i) $\frac{5k}{8} = 5$ Rewrite original equation

= $\frac{5k}{8} \times 8 = 5 \times 8$ Multiply by 8 on both sides to get rid of a
= $\frac{5k}{5} = \frac{5x8}{5}$ Simplify

∴ K = 8

(ii) Solve 24 = ¼ (x – 8)

$24 = \frac{1}{4}x - \frac{1}{4} \times 8$

$24 = \frac{x}{4} - 2$

$24 + 2 = \frac{x}{4} - 2 + 2$

$26 \times 4 = \frac{x}{4} \times 4$

$104 = x$

$\therefore x = 104$

Can you have independent practice

Activity 12 .13 Use the examples above to work out the following	
Solve the equations. 1. $3a = 21$ 2. $2a = 12$ 3. $4p = 0$ 4. $n + 3 = 15$ 5. $x - 6 = 5$ 6. $2a + 3 = 15$ 7. $4p - 4 = 4$ 8. $18 = 26 + 6$ 9. $3p - 2s = 13$ 10. $5x + 8 = 9x$ 11. $2(a + 1) = 6$ 12. $2p + 3(1 + p) = 18$ 13. $7.49 = m - 5.86$	14. $n + 1.7 = 3.9$ 15. $x + 1024 = 9785$ 16. $y + \frac{3}{5} = 1$ 17. $\frac{1}{2}x = 17$ 18. $\frac{x}{3.2} = 8$ 19. $4h - 3h + 2h = 9$ 20. $\frac{3}{4}q = 3$ 21. $\frac{n}{3} - 5 = 20$ 22. $\frac{x}{-5} + 1 = 10$ 23. $14 = \frac{1}{4}(q - q)$ 24. $\frac{3}{4}x - \frac{2x}{4} + 12 = -8$ 25. $5 = 3m + 10 - 1\frac{1}{3}m$ 26. $0.5k + 0.8 + 0.2k = 3.6$

Compare your work with the answers at the end of the unit

Are you feeling well?

(b) More complicated equations

Let you continue with more complicated equations but feel at home!

Study:-

Example 1

$2x - 7 = x - 3$

$= 2x - x - 7 = x - x - 3$

$= x - 7 = -3$

$= x - 7 + 3 = -3 + 3$

$= x - 4 = 0$

$= x - 4 + 4 = 0 + 4$

$\therefore x = 0$

Try on your own:

(iii) $5x + 2 = 18 - 3x$

Compare your answers with this working

$= 5x + 3x + 2 = 18 - 3x + 3x$
$= 8x + 2 = 18$
$= 8x + 2 - 2 = 18 - 2$
$= 8x = 16$
$= \frac{8x}{8} = \frac{16}{8}$
$\therefore x = 2$

Check your answer by substituting the value of x = 2. In 5 x + 2 and 18 – 3x

5 x x + 2	18 – (3 x x)
5 x 2 + 2	18 – (3 x 2)
10 +2	18 – 6
12 =	12

With your colleague can you practice on a piece of paper;

Activity 12.14

Find the value of unknown

1. 6q – 6 10 + 2q
2. 5x – 3 = x +18
3. 2p – 2 = P+ 10
4. 7x – 17 = 3x – 7
5. 4x – 3 = 9 (x – 2)

Discuss your answers with colleagues.

I hope your finding it fine?

Let you look at another example;

Example 2;

Solve the equation;

(i) $\frac{2m+1}{3} = \frac{m-2}{2}$

$\frac{(2m+1)}{3} = \frac{m-2}{2} \times 6$ multiply both sides by LCM of all fractions in

$= 2(2m + 1) = 3(m - 2)$
$4m + 2 = 3m - 6$
$4m - 3m + 2 = 3m - 3m - 6$
$m = 2 - 2 = -6 - 2$
$\therefore m = -8$

(ii) Solve for t.

$\frac{(2t+3)}{4} + 2 = \frac{3t-8}{3}$

Multiply LCM on both sides but remember to multiply 2 by 12

$\frac{(2t+3)}{4} \times {}^{12} + 2 \times 12 = \frac{(3t-8)}{3} \times 12$

$= 3(2\ t + 3) + 24 = 4\ (3 + -8)$ Remove the brackets

$= 6t + 9 + 24 = 12t - 32$

$= 6t + 33 \quad = \quad 12t - 32$

$= 6t - 12t + 33 \quad = 12t - 12t - 32$

$= -6t + 33 - 33 \qquad = -32 - 33$

$= \frac{-6t}{-6} = \frac{-65}{-6}$

Remember dividing a negative by a negative number the answer is positive

$\therefore\ t = 10\frac{5}{6}$

Using examples 4 (i) and 4(ii) do the following problems in your exercise.

Activity 12.15

Solve the following;

a. $\frac{3}{4}x + 1 = 9$
b. $\frac{2}{3}x - 8 = 16$
c. $\frac{x-4)}{2} = \frac{x+2}{3}$
d. $\frac{x+1}{5} + = \frac{x-3}{2}$
e. $\frac{x+3}{2} + 4 = 6$
f. $\frac{2m+3)}{4} = \frac{3m-5}{3}$
g. $\frac{3}{4}\ (12x - 12) - \frac{2}{3}(9x - 18) = 0$
h. $\frac{t+1}{2} = \frac{(3t-1)}{11}$
i. $x - 5 = 8 - (2x + 4)$
j. $\frac{n+2}{5} = \frac{1}{2}$

Let you compare your answers with your colleagues; then, with these in the table below;

(c) Wordy problems involving using simple equations.

You are welcome to wordy problems;

Let you study the following examples **carefully**.

Example 1.

I think of a number, I then double it, and takes away, the result is 17. What is the number?

Suppose the number t thought of is x

Then, doubling it makes 2x

Taking away leaves 2x – 5

Forming and equation. 2x – 5 = 17

2x – 5 + 5 = 17 + 5

2x = 22

$\frac{2x}{2} = \frac{22}{2}$

x = 11

∴ the number I thought of must have been 11

Check twice 11 is 22, and when 5 is taken away the result is 17, so 11 is the correct answer.

Let you study more examples for more practice

Example 2;

I think of a number, divide by 3 and add 10. The result is 16. What was the number I thought of?

Let n be the number

Then $\frac{x}{3}$ + 10 = 16

Hence $\frac{26}{3}$ + 10 – 10 = $\frac{16}{6}$ - 10

$\frac{x}{3}$ = 6

$\frac{x}{3}$ x 3 = 6 x 3

x = 18

∴ the number was 18

Example 3:

Stephen is 4 years other than his sister. In 10 years' time the sum of their age will be 60 years. How old is each of them?

Let the sister's age be x years

Then, John's age will be (P+4) years.

After 10 years

Sister's age = (P+10) years
John's age = (P+4+10) years = (P+14).

Forming an equation

$= (P + 10) + (P + 14) = 60$
$= P + 10 + P + 14 = 60$
$= P + P + 10 + 14 = 60$
$= 2P + 24 = 60$
$= 2P + 24 – 24 = 60 – 24$
$= 2P = 36$
$= \frac{2P}{2} = \frac{36}{2}$
$P = 18$

∴ John's sister age is 18
John's age is 18 + 4 = 22 old

(d) Problem solving using formulae

Let you discuss problems solving formulae.

Now, dear student study;

Example 1

The length of a rectangle is 4 cm greater than its width. If the perimeter of the rectangle is 30cm, what are its length and width?

Suppose the width is w cm.

Then the length is (w + 4)cm.

Perimeter of the rectangle is given as 30 cm

Forming an equation:-

$2[(L + W)] = P$
$2[(w + 4) +W)] = 30$
$= 2(w + w + 4) = 30$
$= 2(2w + 4)] = 30$
$= 4w + 8 = 30$
$= 4w + 8 – 8 = 30 – 8$
$= 4w = 22$
$= \frac{4w}{4} = \frac{22}{4}$
W = 5 ½ or 5.5cm
∴ length w+4 = 5 ½ + 4
= 9 ½ cm
Width w = 5 ½ cm

Since you have studied the four(4) examples can you do the following numbers in your exercise book.

Activity 12.16

Use simple equations to solve the following problems;

a. Think of a number subtract 8 from it. The result is 21. Find the number.
b. I think of a number, double it and add 5. The result is 11. Find the number.
c. I think of a number, halve it and subtract 3. The answer is 7.
d. I think of a number, multiply it by 3, and add 6. The number is 30. Find the number.
e. I think of a number and multiply it by 4, the result is the same as if I added 24 to the original number. Fid the number.
f. I think of a number, x. how would you represent (i) the next integer above x, (ii) the next integer below x? these three are called "consecutive integers". Find three conservative integers whose sum is 96.
g. Nyende takes 2 parcels A and B to the Post Office. Parcel B is heavier than parcel A by 3kgs. Their total weight is 17kgs. What is the weigth of each parcel?
h. Andrew's father is three times as Andrew. The sum of their age is 44 years. What are their ages now? How old will of be 3 years from now?
i. Half of Reagan's age and a third Joshua's age add up to 11. Joshua is 3 years older than Reagan. What their ages now?
j. The length of a rectangle is 3 cm greater than its width. If the perimeter of the reactangle is 26cm, what are its length and width?
k. Aaron has saved some money. His sister has saved twice as much money. Altogether they have shs. 18,000/=. How much money has each saved?

Compare your answers with these at the end of this unit.

In a group of 4 students; do exercise 12 found in secondary Maths Book 2, page 179 – 180 Nos; 4,5,6 – 10 and (11) a,b and c.

With your colleague can you.

Summarize what you have learnt about solving simple equations?

Discuss your findings with other students in your class.

Then compare your answers with the summary indicated below:-

1. An equation is a statement that two quantities are equal or balancing e.g. $1 + 1 = 2$, $x + 4 = 6$ etc.
2. The sides of an equation are expression on the two sides of equality sign e.g. in $2y + 3 = 6$ the LHS is $2y + 3$ and the R.H.S is 6
3. Determining a number that a symbol in a equation represents if the equality is to hold is called a **solving the equation.**
4. A number that makes an equation on to true is called a **solution.**
5. In solving an equation we can add, subtract, multiply and divide the same number on both sides.
6. To check whether the solution of an equation is correct, you substitute the value obtained into equation, e.g. $4x + 6 = 0$

   $4x = -6$

   $x = -1.5$

   check

when $x = -1.5$ LHS = $4x - 1.5 + 6 = 0$ RHS = 0 LHS = RHS, so $x = -1.5$ is a solution.

Thank you for completing this topic.

Can you try another topic.

12.7.5. DEMONSTRATION OF HOW TO TEACH ALGEBRA CONCEPTS IN PLENARY SCHOOLS;

You are welcome to this practical part.

You are reminded that when you were reading different parts of this unit, most of the lessons were approached practically. However, you can read through this example of a demonstration lesson on how to find a value of unknown e.g. x

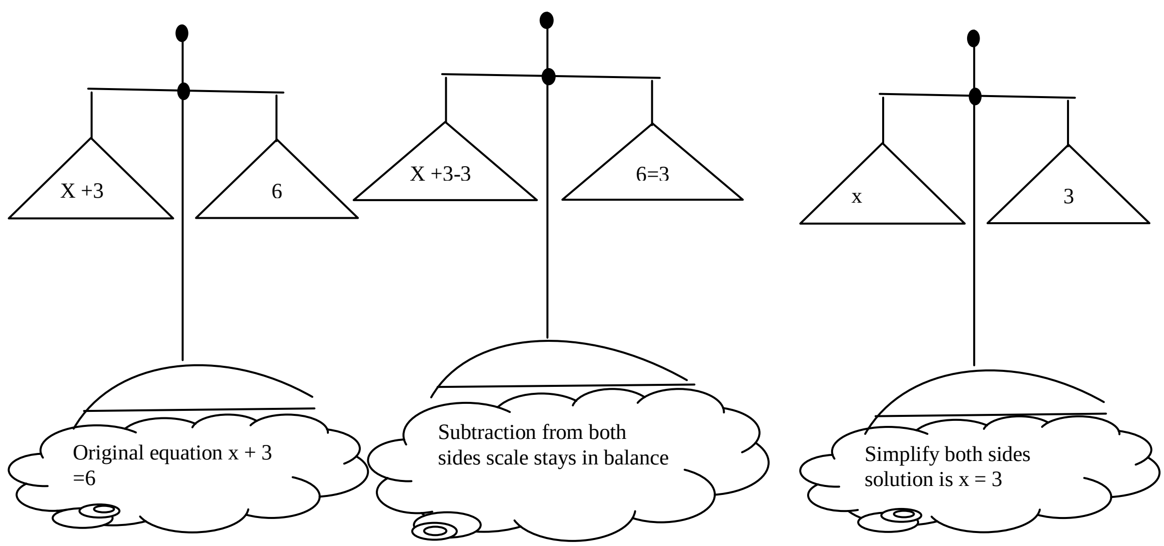
Demonstrate how you will teach to solve the equation for x

x + 3 = 6 in P.4 class

step 1 Remind yourself on the concepts of solving an equation
step2: Device a method e.g. make an improvised scale or already made.

Method 1.

Using a weighing scale

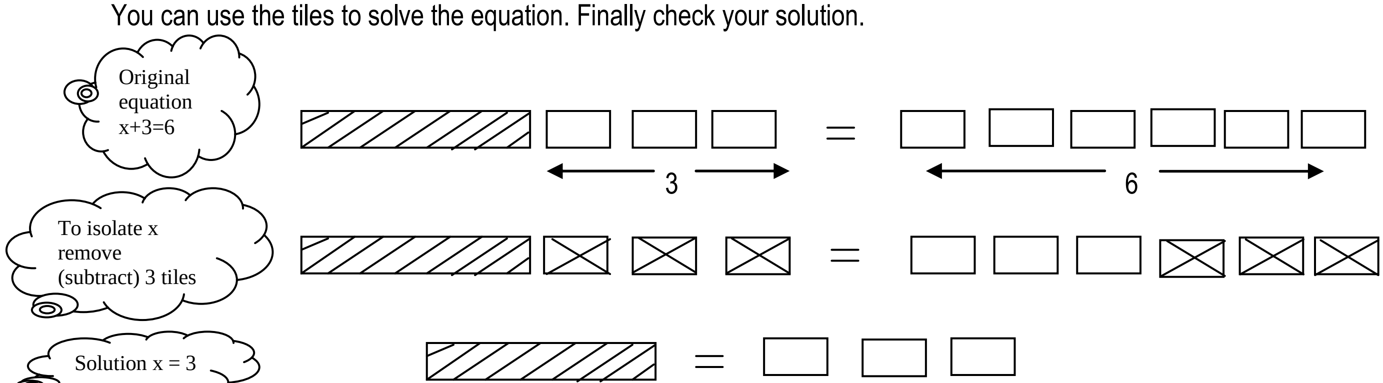


Remember you do to one side of the scale, you must also do to the other side so that the scale remains in balance.

Method 2

Using algebra tiles

You can use the tiles to solve the equation. Finally check your solution.



Hence, x + 3 - 3 = 6 – 3

∴ x = 3

You can now check that x = 3 is solution.

Note: checking solution is an important part of solving equations and inequalities. Throughout this course, you can learn to be better problems, solver if you develop the habit of always checking your solutions.

At your home can you practice using either the methods demonstrated above.

Activity 12.20

Solve the equations:-

(a) X + 4 – 7
(b) 5 = 1 + x

Dear student a tip to algebra

Problem solving: even though you could use mental mathematics to answer the questions. It is important to learn to use algebra to answer any question. Later you will encounter problems that are too complicated to be solved with mental mathematics. Then, your knowledge of algebra will really pay off!!

Congratulations for completing this unit 12

12.9 End of Unit exercise.

Do the following in your exercise book.

1. Write the algebraic expression:

   Square root x^4 and take away square of x

2. Simplify: ¼ a + a + a + ¾ a + a^2
3. Remove the brackets and simplify:- ⅓ (12x + 15y) – ½ (8x – 6y)
4. Given that m = $^-2$ and n = $^-1$ find the value of: $m^2 - n^2$
5. With help of illustrations, demonstrated how to teach P.5 that x + 8 = 13.

12.8 ANSWERS TO THE ACTIVITIES.

Activity

1.	(i) 8 + y	(ii) nm	(iii) xy + 9 or yx + 9	(iv) c + ab or c + ba
2.	(i) $\frac{5}{6}$	(ii) $\frac{6}{7}$	(iii) $\frac{m}{3n}$	(iv) $\frac{y}{10}$ (v) $\frac{m}{39}$
	(i) true	(ii) false	(iii) true	(iv) false

Activity

4x (t x u) = (4 x 1) x u
= 4 x (u x t)
4x (t x u)= (t x u) x u
4x (t x u)= tx (4 x u) etc

Activity

a) 3a
b) 5x
c) 6a
d) f
e) 3x
f) 10a + 5b
g) 101a + 92
h) 11a + 11b
i) 390y + 600z
j) 23 + 6t + 8y
k) b(m) 6 ½a
l) 6x + $6x^2$ + $3x^2$
m) X^2y^2

Activity

1. A
2. $4a^2$
3. 2p
4. 4x + y
5. Pq + pqr
6. a^5
7. m^6
8. $8x^3y^3$
9. $15a^3b^2c$
10. $3a^2b^4$
11. $\frac{2p^2}{q}$
12. $\frac{a^2}{p}$
13. $\frac{a}{2}$
14. 2ab

Activity

1. (a) 6x – 2y (b) 14a – 216 (c) $3x^2 - 3x$
(d) $4x^2 - 4x + 4$ (e) 2ax – a t (f) $10a^2 - 20a$
(g) 24p + 12q + 6 (h) 2ax + 6bx (i) 5a + 22
(j) a + bc + bd (k) $4x - 32x^2$ (l) 2x + 4y
(m) 2m – 3n

2. (a) 3a + 5 (b) 9 (c) $k^2 - k - 2$
(d) 5a + 12 (e) 9a + 3b – c (f) $2m^2 + 2m + 5$
(g) $2a^3 + a^3b + ab$

Activity 12.10

5. x – 2y
6. 2a
7. – x – 5y
8. 4m + 2n
9. X + 2y
10. 3x
11. 12 + 4a + 4b

1. a) 4 b) 17 c) 36 d) 24 e) 9 f) 1

2. a) 20 b) 78.5 c) 14 d) 104 ⅔ or 104.67

14. a) 144 b)150 c) 0

Activity 12.13

a. $\frac{2}{9}$2. h = 25

$a = \frac{45-15}{10}$

$= \frac{30}{10}$

b. ∴ V = 3 m per second

c. 45

Activity

1. a = 7	14. m = n = 2.2
2. P = 0	15. x = 8761
3. x = 1	16. $y = \frac{2}{5}$
4. P = 2	17. x = 34
5. P = 5	18. x = 25.6
6. a = 6	19. h = 3
7. n = 8	20. q = 4
8. a = 6	21. n = 75
9. b = 6	22. x = 45
10. x = 2	23. 65 = q
11. a = 2	24. x = -80
12. P = 3	25. k = 4
13. 13.35	

Activity 12.6

a. 29
b. 3
c. 20
d. 8
e. 8
f. 31, 32 and 33
g. A = 7, B = 10
h. Age now; Father = 33 years
   Andrew = 11 years
   After 3 years = 11 + 3 = 14 years

i. Reagan = 12 years =

Joshua = 15 years

j. length = 8cm
Width = 5cm

k. Aaron = 6000/=
Sister = 12000/=

12.9 GLOSSARY

Numerical expression:	Is a collection of numbers, operations and grouping symbols.
Factor an expression:	To write an equivalent expression as a product of factors.
Number:	Value of expression
Value of an expression:	Number
A variable:	It is letter such as a v,x,y etc represent one or more numbers
Substituting: Values of variable:	This is when we replace variables e.g. x,y, with numbers
Evaluate an algebraic: expression:	This is substitute a number for each variable and find the value
Simplifying:	it is writing an algebraic expression in shortest form or simplest form e.g. a x a x a x a = $a^{1+1+1+1}$ = a^4
Multiplicative identity:	when any number or algebraic expression is multiplied by 1 the product is that number or expression
Equivalent expression:	these are expressions such as 2+x and x+2, which always result in the same number when are substitute any value for their variables
Equation:	is mathematical sentence that uses an equal sign to state that two expressions represent the same number or are equivalent e.g. 3+2=5, x+15=12
Open sentence:	is that equation that contains at least one more variable e.g. x+6=13
Solution:	it is a replaced for a variable that makes an equation true or an answer.
Inverse operations:	is one of the methods of for solving equations. It is a way of undoing by using an opposite e.g. addition can be used undone by subtraction when solving equations of the type x+a =b
Formula:	is an equation that shows a relation between two or more variables e.g. P=2L + 2w
Evaluate:	when you perform the operations to get a single number or value. If to find all solutions a variable e.g. solve x+ 6 = 13

Solve an equation: this is where we can either add the same number to each side or subtract the same number from each side or multiply both sides by the same number or divide each side by the same number

e.g. $4x + 6 = 0$

$4x + 6 - 6 = 0 - 6$

$4x \quad = -6$

$\frac{4}{4} = \frac{-6}{4}$

$\therefore x = 1.5$ or $1\frac{1}{2}$

12.10 REFERENCE BOOKS.

Roland E. L Health passport to algebra and Geometry page 50-75

Karuhiji E. Secondary School Mathematics Book 2 (New edition), Pages 17-26.

Smith C Algebra

Mary P Dolciam: Algebra Structure and method

Ministry of Education, the second mathematics module ME/2, Primary mathematics

Republic of Uganda: course books from books 4 and book 7

End of algebra 1

UNIT 13: ORGANISING MATHEMATICS LESSONS

13.1 Introduction

Dear student, you are welcome to unit 13. This unit introduces you to organising mathematics lessons before starting to teach.

13.2 Content Organisation

Hello student, in this unit, you are going to cover the following topics as indicated in the table below.

Topic	Subtopic
1. Syllabus interpretation	• Topics covered in Primary School Mathematics Syllabus. • The nature of the Primary School Mathematics Curriculum.
2. Topic Analysis	• Forming subtopics • Developing teaching points
3. Learning difficulties in Mathematics	• Learner's learning difficulties in Mathematics
4. Mathematical terms and vocabulary	• Terms and vocabulary used in Mathematics
5. Preparing to teach	• Making a Mathematics scheme of work • Preparing a mathematics lesson plan • Identification and making teaching and learning aids. • Developing activities
6. Methods of teaching	• Different methods of teaching • Application of different methods in the teaching of Mathematics

13.3 Learning Outcome

At the end of this unit, you are expected to interpret the Primary Mathematics Syllabus and use in teaching.

13.4 Competences

1. Identify and name different topics in the Primary School Mathematics syllabus
2. Identify the main teaching points in a given Mathematics topic.
3. Identify the learning difficulties experienced by learners and strategies to overcome them.
4. Identify and explain terms and vocabulary used in a given topic in Mathematics.
5. Make Mathematics scheme of work
6. Make lesson plan for Mathematics and use it in teaching
7. Identify activities for teaching a given Mathematics concept.
8. Give different examples of methods used in teaching Mathematics
9. Prepare lessons and demonstrate teaching using the identified activities and methods for specific Mathematical concept.

13.5 Unit Orientation

This unit is to get you prepared for teaching Primary Mathematics. The unit is very interesting because it introduces you to a variety of activities and methods used in teaching Mathematics concepts.

13.6 Study Requirements

To be successful in studying this unit, you are required to prepare plenty of materials collected from your local environment, read about matching and learning theorists with respect to teaching Mathematics in Unit 1. You also need to have Primary Mathematics course books and the Mathematics Primary School Syllabus for further practice.

I hope you will enjoy studying this unit.

13.7 Content

Topic I: Syllabus Interpretation

1. Topics covered in the Primary School Mathematics Syllabus

What is a Mathematics syllabus?

Find out by doing the following.

Read the Primary Mathematics Syllabus included in the Uganda Primary School Curriculum (pgs 220-229)

Check your findings with the answers below:

a) An outline of topics to be covered in Mathematics

Since you have understood the meaning of the Mathematics syllabus, you can now list down 10 topics covered in the Primary mathematics.

Thank you for listing down the topics. Share yours answers with a colleague. Then compare the answers with the following outline.

- Set concepts
- Numeration systems and place values
- Operation on numbers
- Number patterns and sequences.
- Fractions
- Graphs and interpretation of information
- Geometry
- Integers
- Measures
- Algebra

Can you give one topic not taught in lower primary classes (P.1-P.4)?
Compare your answer with those of your colleagues. You are doing well. Let's continue.
You can now read more about the nature of the Primary Mathematics Curriculum.

b) The nature of the Primary Mathematics Curriculum

The syllabus is expected to be covered and completed by all classes. The Mathematics syllabus is itself structured so that any given topic is based on some earlier topic(s) that were successfully covered, and a successful coverage of a topic will lead on to more Mathematics.

Can you use the above information to do the following activity?

✍ **Activity 13.1**

a) What do you notice about the topics in the Primary Mathematics Syllabus?

b) Why do you think work on integers starts at Upper Primary level?

You can compare your answer with a friend and with one at the end of the module.

Well done; keep up!

Thank you for coming to the end of this topic.

Topic II: Topic Analysis

Analysis of a topic is an important aspect of preparation to teach. It is one which is omitted by many teachers and yet it is the most important. A number of things have to be considered under the following subtopic.

1. Forming Subtopics

Can you imagine yourself handling the whole topic within one lesson, is it possible? Definitely no!

Can you share with your colleague and give two possible reasons why it is not possible to handle the whole topic within one lesson. Compare your answers with those of other colleagues.

Thank you for giving the reasons.

Then compare your answer with the following possibilities:

- Few Mathematical concepts need to be covered at a time to avoid confusing the learners.
- Amount of activities given in a lesson should be related with the time available and the learners' ability to take in.
- To plan for the resources available for the topic.

Thank you for sharing with your colleague. Keep that spirit alive!

Since you have identified the topics covered in the Primary Mathematics syllabus in topic I, you can now use that knowledge to do the activity below:

✍ **Activity 13.2**

Using fractions as a topic to be taught in a P.4 class form the subtopics to be considered under this topic.

Read the following books to help you to do the activity:

- Primary Mathematics Syllabus
- Pupils' Mathematics textbook
- Teacher's Guidebook

You can compare your answer with a friend and with one at the end of this unit.

2. Developing Teaching points

Imagine yourself standing before the pupils in the classroom and taking them through a Mathematical problem that you had not done before.

A number of things have to be considered which include:

a) Study the topic from the textbooks and choose some of the more difficult examples within the topic and work them out.
b) While working out an example, choose a good layout of the solution.
c) Study the example you have chosen and solved. Study textbooks to discover what knowledge the pupil should possess before they start the topic (previous knowledge).
d) Study the example and textbooks again to discover what new ideas and techniques the pupils are going to learn.
e) Try to study the examples and discover what difficulties your pupil may have in this topic and state how you will over come these difficulties.

Examples of topic analysis

Class: P.3
Topic: Division
Subtopic: Division with a remainder
Competence: - Divide a 2 digit number by a unit numeral and leave a remainder
- Divide by a 3 digit number by a unit numeral and leave remainders.

Analysis
Divide:

a) $40 \div 3$
b) $209 \div 3$

Solution and layout

(a) 10+3
3)40
30 (10x3)
10
9 (3x3)
1

(b) 60+9
3) 209
- 180 (60 x 3)
29
27 (9x3)
2

$\therefore 209 \div 3 = (60+9)r^2$
$= 69\ r^2$

3. Previous Knowledge

a) Multiplication tables
b) Subtraction
c) Borrowing
d) Sharing
e) Symbols of division e.g. $\div$ and $^3/_2$

4. New knowledge

- Identify place values

- The quotient should be under the correct place value
- Decomposition of biggest place value to the next place value (borrowing)

5. Problem

- Divide equally
- Multiplication (quotient $\times$ divisior)
- Subtract
- Valuing zero

6. How to overcome the problem above

a) Use real objects to learn to divide equally e.g. sticks, stones etc.
b) Multiplication table : review these
c) Divide numbers without remainders (review)
d) Remind pupils of borrowing when dividing
e) Emphasize place values
f) Make the division problems realistic by setting them in everyday situation.

You can now tell why you are required to analyse a topic before you start teaching.

✍ **Activity 13.3**

Give five reasons why you need to analyse a topic before teaching it to the class.

Share your answer with a colleague and then compare your answer with a friend and with one at the end of this module.

Note: Analyzing a topic enables a teacher to develop manageable teaching points to be covered in a specific period of time.

You are doing well, keep it up!

Topic III: Learning difficulties in Mathematics

Dear student, this topic is a continuation of what you covered in the topic analysis under problems. However, the only difference is that learning difficulties in Mathematics cuts across all the topics in mathematics.

Did you know that your performance in Mathematics at the various levels you have covered may have been affected by the teachers' failure to identify the learning difficulties you were facing in learning mathematics?

Using your own experience, list down five learning difficulties you found in primary school and secondary school. Share with a colleague then compare your answers with the following examples:

- The place value concept
- The zero concept
- The concept of integers
- Multiplication table
- The concept of division
- The three dimensional concept
- The Cartesian coordinates concept

Can you give five more examples of learning difficulties you found in learning Mathematics?

- **Children's learning in Mathematics**

When planning to teach, teachers should have knowledge of the errors the children are likely to make and the possible misconceptions that can develop as seen in the topic analysis.

From the above information given the teacher should target:

- Ways children are likely to best understand and make sense of the conceptual or procedural knowledge in the content to be taught.
- Ways in which children will be able to use their owning thinking and reasoning powers.
- Children's ability to apply their knowledge in solving realistic problems.

Can you use the above information to do the following activity?

✍ **Activity 13.4**
With your colleagues, identify five learning difficulties and ways of overcoming them as shown below:

Topic	Learning difficulties	How to overcome them
1. Place value –addition of numbers	• Failure to understand the position of digits e.g. $\begin{array}{r} 56 \\ +\ 8 \\ \hline 514 \\ \hline \end{array}$	• Give children more practice on place values
2. Multiplication –multiplication of numbers	Carrying when multiplying $\begin{array}{r} 17 \\ \times\ 6 \\ \hline 62 \\ \hline \end{array}$	• Give children more practice on how to carry numbers when multiplying

Topic IV: Mathematical terms and vocabulary

Terms and vocabulary used in Mathematics

Consider the following situation where a teacher asks, "What is the difference between 7 and 10?" The response comes, '7 has got one figure in it while 10 has got 2 figures in it.

This is an example of one of the concerns of this subtopic: that words have different meanings and that this fact sometimes leads to a break down in communication between teacher and learner. It is therefore important for teachers to use technical terms correctly so that learners can acquire the meanings they have in mathematics. For example, the 'faces' of a cube should not be called 'sides' and the 'vertices' should not be called 'corners.'

✍ **Activity 13.5**

a). Using the 10 topics listed in the mathematics syllabus, identify the terms and vocabulary from each topic as in the table below.

Topic	**Terms and vocabulary**
Sets	Either/or, at least

b). Using these textbooks:

- Pupils textbooks
- The Oxford Mathematics Study Dictionary by Tapson F
- Improving the learning of Mathematics by Back House pgs 111-128
- Primary School Mathematics Curriculum

Explain the terms and vocabulary used in the topics.

Topic V: Preparing to teach

Dear student, you are once again welcome to yet another important huddle in our journey of organizing a mathematics lesson.

What you need to do as a teacher is to cover what factors affect your teaching. Before you teach your Mathematics lesson, you should find out the following about the school you are working in: school rules, timetable, syllabi (see unit7 in PES for the types), school records, class textbooks, exercise books, visual aids, materials, library, outdoor practical work, classroom supplies and class lists. These are fundamental factors which make your performance in the school easier.

1. Making Mathematics Scheme of work

Schemes of work are the individual teacher's plan for the terms work.
They are flexible guides which are for modification as the work progresses.
Some of the characteristics of effective schemes of work are listed below:

- They are made by those who are to use them.
- They are detailed, flexible and should not attempt to cover the impossible.
- They are tied to what has been taught before and should pave way for the future.
- They should contain statement of the learning outcome, competences to be developed, generic methods to be used, learning skills to be developed, learning aids to be used.

Below is a sample scheme of work for P.4

Learning Outcome: The learners are able to apply the set concepts in solving problems

PERIOD	THEME	TOPIC	S/TOPIC	L/SKILLS	COMPETENCES	CONTENT	METHOD	ACTIVITY	LEARNING AIDS	REFERNCE
1	SETS	Set concepts	Naming sets	- Logical thinking - Problem solving - effective communication	1. Forms sets 2. Name sets	- Forming set - Naming sets	- Guided discussion - Group work - Role Play	- Spelling games of words used as numbers - Forming sets - Naming sets	- Stones - Cups -Books	- Primary School Mathematics BK 4
2			Equivalent sets	- logical thinking - problem solving - effective communication	1. Identifies equivalent sets 2. Forms equivalent sets	- Identifying equivalent sets - Forming equivalent sets	- Guided reading - Think Pair	- Identifying equivalent sets - Forming equivalent sets	- cups - stones - books	Primary school mathematics Bk 4

Note:

When planning your scheme of work, you will need to use:

- The Mathematics Syllabus
- The Teacher's Guide
- The pupils' textbook

- Look at each topic for the theme, level of content and the competences to be covered
- Look at the number of lessons recommended for the topic
- Plan a variety of methods in your coverage of the topic. Too many lessons taught in the same way tend to be boring to the pupils
- Decide how and when you will ass your pupils
- The lay out of a scheme of work varies from person to person

With a colleague, can you list down the components of the scheme of work? Compare your list with the one below:

- Learning outcome
- Period
- Theme
- Topic
- Subtopic
- Learning skills
- Competences
- Content
- Method
- Activity
- Learning Aids
- Reference
- Remarks

If you have understood the components of the Mathematics scheme of work, can you do the following activity in your exercise books?

✍ **Activity 13.6**

What should you consider while making a Mathematics scheme of work?

Great job!
You can compare your answer with a friend and with one at the end of this unit.

2. Preparing a Mathematics Lesson plan

"Failing to prepare means preparing to fail."

Good planning is not simply a bureaucratic exercise. It is intended to help teachers become and remain professionals in their work.

If a teacher enters the classroom without a clear idea of what he/she is going to do during the lesson, the pupils will soon realise this and will not give the teacher the attention s/he deserves. If the teacher has spent time preparing for the lesson, then the lesson will be enjoyable and effective. You will feel that the time spent preparing the lesson was not wasted. You will work hard to pass on the ideas you have prepared most effectively. What the pupils feel about a subject is usually a reflection of what their teacher feels.

In order for you to reach such a high level of preparedness and confidence when teaching, you need to follow a good lesson plan. It will remind you to think about different parts of the lesson. The first thing a teacher must decide is what is going to be taught.

Here, you need to look at three things to help you choose the topic.

- The scheme of work which covers the whole topic of which the lesson is a part.
- Knowledge of the equipment the school
- Timing of the lesson

When you have decided what topic you are going to teach in a particular lesson, you should write down the title and then:

- Decide on the learning outcome of the lesson; the specific competences can only be written after you have studied the content thoroughly. Write the learning outcome in your subject notebook.
- Prepare the content of your lesson. This is split into two parts;

  - Content pupils must know
  - Enrichment materials to help them learn and be able to answer.

Make note of your content under the two sections in your notebook. You must know more than your pupils' textbooks. Remember you read widely when preparing the schemes of work.

- Decide the generic method you are going to use. This generic method has already been noted in your scheme of work. Now prepare to carry it out.

  - What motivation can you prepare? How can you capture pupils' interest and set their minds onto the lesson?
  - prepare your key questions
  - prepare your instructions very carefully. Pre-test them on yourself.
  - try your learning aids and study the room to find out where you are going to place them.

- Consolidation and testing: These two are often interrelated. Both the question and the expected answers must be prepared. You must prepare all the possible answers to the mathematics questions.
- Prepare the homework you will give to your pupils.
- Prepare the ending to your lessons. Be prepared to stop at some convenient point if you have too much material..
- At first, you have to prepare two or three places where you should stop if you run out of time.

Having done your preparations, you must discipline yourself and make your lesson plan.

Note:

A carefully planned lesson is always better than an unplanned one. Very few experienced teachers and hardly any inexperienced one can teach well without a lesson plan.

A good lesson plan includes the following features:

- Date, class, subject, time and number of pupils
- Competences

- A list of apparatus and materials which will be required by the teacher and/or pupils during the lesson.
- The sequence of planned activities. This usually includes an introduction, two or three main steps or stages (usually varied in methods, and conclusion.
- A time schedule
- A class board plan
- Space for the teacher's self evaluation should spell out the strengths, weaknesses and the way forward.

Note:

The advantage of a good format is that it makes the teacher aware of the pupils and what they are doing. If under the pupils' activity you find yourself writing again and again, 'sit and listen', then most probably your lesson plan is bad. Pupils are unable to listen for more than ten minutes continuously.

The Date: Is useful for reference. Pupils should be trained to date their work as well.

Subject: If the same book is used for lesson plans of other subjects, then it may be necessary to put the subject at the top of each lesson plan. If a section of the book is devoted to a subject, then the subject should only be put at the beginning of the section.

Time: Attempting to plan the timing of your lesson plan is very important. The best way to do this is to divide the lesson into short intervals of time.

For example, if the period is 40 minutes;

- Spend 3-4 minutes for introductory phase.
- Spend 10-15 minutes for experiencing phase
- Spend 8-10 minutes for open interaction (sharing experiences phase)
- Spend 8-10 minutes for evaluation activity and summing up.

Note:

One of the most common mistakes made by teachers is to plan an introduction to take 5 minutes, and then exceed this time so much that there is no hope of finishing the lesson. This half the lesson is taught in one period and the other half in the next period. Once this happens, it often goes on. Allow yourself 2-3 minutes of unplanned time.

Competences: This must state clearly what the child can do after s/he has understood the concept and has acquired clearly measurable skills. It should be expressed in a form that it emphasizes the transfer of learning.

Learning aids: This is a very important part of the lesson plan especially in practical lessons and those using visual aids. It is advisable to list the equipment that you will require and the quantities of each.

Method and language: this should be reflected in the teacher's and pupils' activities.

Introduction: This has several purposes:

- To stimulate interest
- To provide a link with the previous lesson on the subject
- To draw pupils' attention from the previous lesson they have just had to the lesson they are about to have.

Conclusion: A lesson can end in several ways:

- Recapitulation: going over the lesson in a different way e.g. writing notes, filling a prepared sheet

- Testing to see whether the pupils have understood the concept and have acquired clearly measurable skills –giving exercises and asking questions.
- Posing a problem to start pupils on thinking about the next day's lesson.
- Answering question posed at the beginning of the lesson.
- Giving homework

Class board plan: class board plan should help you organise your work on the chalk board.

Here is a sample lesson plan

Date	Class	Subject	Time	No. of pupils
27.8.2009	P.4	Mathematics	8.00-8.40am	45

Topic: Set concepts

Subtopic: Naming sets

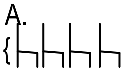
Competences:

1. Identifying members of the set
2. Naming sets
3. Counting the members of the set

Instructional materials: chairs, balls and cups

Methods: Think pair, group discussion, guided discovery

References: Teacher's Guide, Pupil's textbook P.4

Time	Phase	Teacher's activity	Pupil's activity
5min	Introduction	Teacher will review p.3 work on set. 1. what is a set?	Answering questions
15min	Experiencing	Teacher will guide the learners on how to identify members of set A. A= {  } Ask pupils to name and count the members. Teacher will draw set B on blackboard. B= {  } And ask the pupils to name it.	Pupils will answer: - set A has 4 members - the members in set A are chairs - Set A is a set of 4 chairs. Pupils will name the set. - set B has 3 members. - it is a set of cups
8min	Sharing experiences	- teacher will group learners - teacher will give them some questions to practice.	- form groups Answer questions
12min	Evaluation	- teacher will give an activity to the individual learners for set C, D and E	- learners will do the written exercise in their exercise books and answer as follows: This is a set of…

			It has…members It is set C.

Self Evaluation:

1. strengths
2. weakness
3. way forward

Thank you for reading through this subtopic. Are you finding it easy? If not go back, read through and share with your colleague.

Can you now try this activity with your colleague?

✍ **Activity 13.7**

1. How does a teacher know what s/he is going to teach in a particular lesson?
2. State what one should think about before teaching a Mathematics lesson.
3. Give two purposes of an introduction to a lesson.

You are doing great!
Compare your answer with a friend and with one at the end of this unit.

3. Identification and making teaching and learning Aids

Teaching aids are materials that a teacher can use to put across mathematical ideas to pupils. They help pupils in the process of understanding and learning Mathematical concepts/ideas.

According to Piaget, primary school pupils are at the concrete operation stage. At this stage, they can learn by working with physical objects. They benefit greatly from seeing, hearing and listening experiences. Teaching aids will therefore help them to understand Mathematical concepts such as basic operations like addition, subtraction, multiplication and division; and the geometrical properties of planes and solid shapes.

Teaching/learning aids also help to keep pupils interested, eager and ready to learn because they are tangible, colourful and varied.

Teaching aids can be categorised as visual and audio-visual materials. The visual aids are those that a pupil can and possibly touch, while the audio-visual are those that the pupils can see and listen to.

With the above background, can you list down five visual aids and five audio-visual aids? Compare your answer with the one below:

Visual aids

- Models, charts, diagrams, overhead projector
- Chalkboard, textbooks, graphs, e.t.c

Audio-visual

- Radio, tape recorder
- CD, Television
- Resource person e.t.c

Some teaching aids can be made by the teacher from locally available materials while others are manufactured commercially and can be bought from shops.

(i). Locally available teaching/learning aids

These can be made by the teacher and/or the pupils. They can be physical objects like sticks, stones, bottle tops, flowers and seeds, collected from the local environment. They can also be objects like cubes, boxes and cylinders made by the teacher using his own innovation and imagination (improvisation), from locally available materials.

You should involve the pupils in the collection and making of some of these objects. Diagrams drawn by the teacher or pupils, pictures cut out of magazines and newspapers can be used as good and attractive teaching/learning aids.

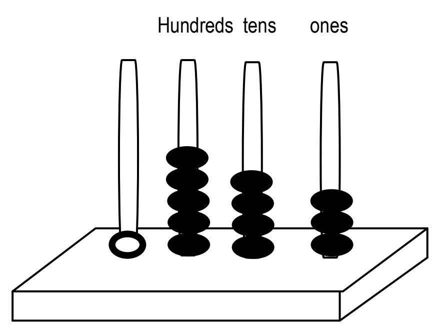
Examples of teaching aids made by the teacher/pupils include the following:

- The number pattern cards
- Square charts for numbers 1-100
- Clock face
- Geometrical models made out of manilla paper/or cardboard.
- Frameworks of plain shapes
- Hollow cubes
- Geoboard

(ii). Teaching/learning aids produced commercially

Examples of aids produced commercially include:

- Abacus –this is a device for teaching counting, the idea of place values and basic number operations of addition and subtraction of whole numbers. Here is an illustration of an abacus showing the number 543



- Tape measures
- Mathematical instruments e.g protractor and pair of compass
- Weighing balance
- Teaching aids for teaching capacity e.g. plastic containers and spoons of different sizes.
- Radio
- Film and slide
- Video tapes

(iii). Some special teaching aids

- The chalkboard

The chalkboard is the most commonly used aid in the teaching of Primary School Mathematics. This is mainly because most aspects of Mathematics can be clarified through writing and drawing.

When used properly, the chalkboard can greatly aid in teaching and learning of Mathematics because it:

- Permits point by point development and reference on the lesson progresses
- Allows emphasis on major points by outlining and summarizing.
- Provides a medium for participation of the pupils in the classroom activities
- Can be used for assignment, homework problems or discussion of questions.
- Combine visual and oral presentation of ideas

Chalkboard plan

Summary of main points	• **Topic** • **Development of the lesson**	**Illustrations**

Divide your chalkboard as shown in the plan above
Write clearly, neatly and correctly

Note:
Writing on a chalkboard should be accompanied by oral explanations, or vice versa unless you are giving straight notes.

- Draw simple accurate diagrams. If mathematical instruments are available, use them.
- Encourage pupils' participation in solving short appropriate problems on the chalkboard.
- Use coloured chalk to show key ideas.
- Check to see that what you have written on the chalkboard is visible and can be followed by all the pupils in the class.

- **The textbook**

The textbook is the most commonly used aid in teaching of Mathematics. Many times it is misused because most teachers use it blindly. Your duty as a teacher should be to help your pupils understand what is in the textbook rather than covering the content in the textbook.

- A good textbook gives you guidance while writing scheme of work and preparing daily lesson
- It provides you with a sequential approach to teaching.
- It gives you adequate exercise for pupils to master concepts and skills.

- It acts as support if you have little background knowledge in the content of mathematics.
- It is a dictionary and store of mathematics concepts, principles, formular and definitions.

.

- **Charts and pictures**

Mathematical pictures and charts make the classroom educative and conducive to learning. They save time in that you can prepare them well ahead of time. When it comes to teaching, you will just refer to them rather than spending time drawing them during the lesson. You can draw/use charts and pictures and charts for almost all the mathematical topics.

When drawing pictures and charts, try to use many colours and be artistic. Make them large enough to be seen from the furthest point in the classroom and inadequate numbers for them to go around the class. After use, they could be displayed in the classroom, stored away or placed in the Mathematics corner/area. In displaying charts, /pictures, you should keep in mind the pupil's eye level.

Since you have read about the making and the use of teaching/learning aids, can you do the following activity individually?

✍ **Activity 13.8**

a). Differentiate between visual aids and audio-visual aids

b). Give two ways in which the chalkboard can be used to clarify Mathematical concepts

You can compare your answer with a friend and with one at the end of this unit.

While you are concluding this topic, you note that:

The mere use of these materials however, does not guarantee effective communication, or effective teaching. It is their careful selection and skillful handling by the teacher that render them useful in facilitating learning.

4. Developing activities for teaching Mathematical concepts

In unit 1 of this module, you were introduced to the stage of pre-mathematical activities. The stages involved four stages that a pupil must go through before beginning number work.

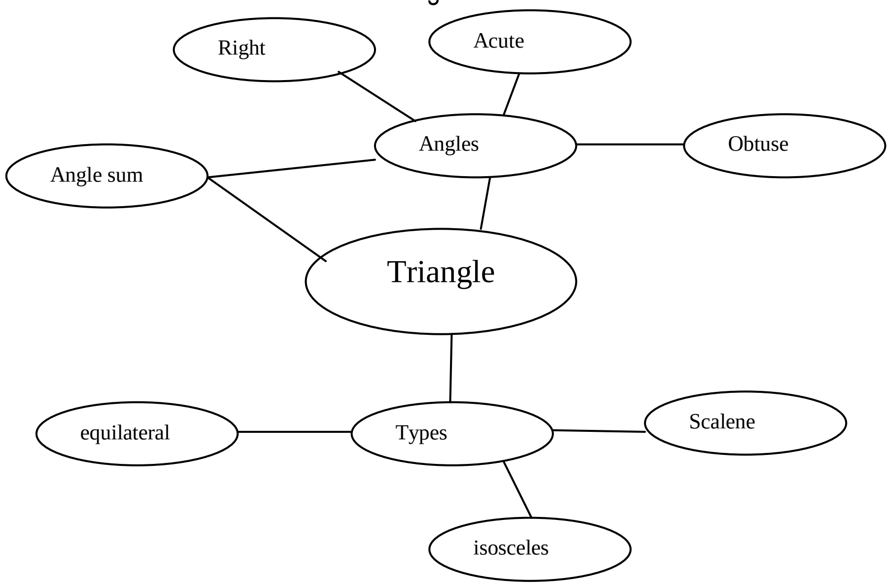
The stages are:

- Sorting and identifying groups
- Matching and comparing groups
- Beginning of number work
- Conserving numbers

These stages are important because they foundation for the learning of the mathematics concepts of integers.

Mathematics concepts

In mathematics, concepts are a group of related mathematics facts. For example, the concept of a triangle involves some of the mathematical facts that are shown in the figure.



The triangle concept

Adapted from Orton: Learning Mathematics

In the above figure, the concept of a triangle involves the knowledge of the:

- Acute angle
- Obtuse angle
- Equilateral triangle
- Scalene triangle
- Right angle
- Angle-sum of a triangle
- Isosceles triangle

Since a concept is a group of related facts, then what is a fact?

In mathematics, a fact is a true mathematical statement. Here are some of the mathematical facts.

- 7+4=11
- The sum of two odd numbers is an even number
- The angle sum of a triangle is 180^0
- The area of a rectangle is given by length $\times$ width
- The area of a circle is given by $\prod r^2$

The above listed mathematical facts can be categorised under the following concepts.

- Bullet 1 and 2 fall under the numerical concepts
- Bullet 3 falls under the triangle concept
- Bullet 4 falls under the rectangle concept
- Bullet 5 falls under the circle concept.

Note:

From the above example, it can be observed that one has to learn related Mathematical facts before the Mathematical concept is learned.

Learning mathematical concepts through the learning of Mathematical facts

Learning of mathematical facts is a process that leads to the learning of mathematical concepts. On the other hand, one must note that if one wants to learn Mathematics, it is useless to learn the facts in isolation. This means that the facts that are related must be learnt so as to help in the information of mathematical concepts. At this stage, we shall discuss how mathematical facts are learnt.

According to Bruner, one of the psychologists you learnt about in unit 1 of this module, the following are the stages of learning facts.

(i). Enactive stage –a stage where learners use concrete objects/equipment
(ii). Iconic stage –a stage where pictorial representation is used
(iii). Symbolic stage –a stage where pictorial representation is represented by symbols.

The above three stages should be regarded as a sequential approach to learning Mathematics. This means that the learner with the help of the teacher must be able to:

- Handle concrete objects first
- And then present concrete objects using pictures
- And lastly use symbols

The sequence outlined is mostly met in primary one and primary two of the Uganda Primary Schools. The lower primary mathematics teachers usually adopt the sequences (Concrete $\longrightarrow$ pictorial $\longrightarrow$ symbols) when:

- Playing with sets of concrete objects by making sets. Through matching sets, the meanings of equal to less than or more than are learnt.
- Learning the oral counting of concrete objects or pictures in a set
- Matching of concrete objects in a set with the counting set so that the number of elements in each set is defined.
- Matching of pictures in a set with the counting set so that the number of elements in each set is defined.
- Recognising and learning to write symbols for numbers 0 to 9, and using the symbols to record the number of concrete objects and pictures in any given set.

Although the learning of mathematical facts in the lower primary follows the sequential approach as outlined above, it is generally observed that the first stage is neglected by the teachers of both middle and upper primary. This leads to pupils learning mathematical concepts only in symbolic form. The result is that the mathematical concepts are learnt in an abstract manner. This is unfortunate because many pupils are introduced to symbolic ideas when they are not ready for them.

Why teach Mathematical concepts from concrete to abstract?

Mathematical concepts should be taught from concrete to abstract because in using different concrete objects, there are a variety of ways a concept can be represented, for example, three as a symbol may carry little meaning. The symbol 3 carries a lot of meaning when it is associated with say; 3 sticks, three oranges, three bananas, three houses and so on.

In this way, the learner becomes aware of the different aspects, a set of three things, three times as much can easily be appreciated if the learner is familiar with three in a variety of ways. On the other hand, when a teacher uses concrete objects, he will help the learners to see the use of Mathematics in everyday life situations.

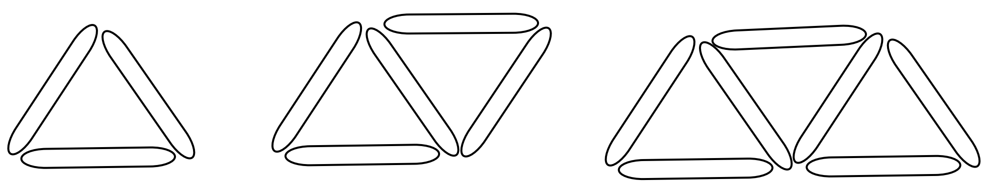
Most mathematical educators also argue that most pupils understand Mathematics concepts better if they work with concrete objects. This is because the pupils are able to:

- See concrete objects
- Touch or point at them
- Relate a set of concrete objects to another set of concrete objects.
- Arrange concrete objects in a required pattern.
- Manipulate them in a variety of ways.

The manipulation of objects, if well directed by the teacher leads to the development of abstract thinking amongst the learners.

Example:

In order to form a triangle, 3sticks are required; for two triangles, 5 sticks are required and for three triangles, 7 sticks are required as shown below.



Have you noticed the pattern formed by sticks in the figure above?
Then, how many sticks will be required for five triangles, 10 triangles and 30 triangles, if the above arrangement is to be maintained?

Then try out the example using the counters as suggested. After trying out the example, you should realise that you obtain a sequence of positive integers. Sequencing is an abstract concept in mathematics teaching.

Note:
It is worth noting that when one is teaching Mathematics, attempts should be made to use concrete objects. The concrete objects should be used as 'starting point' as illustrated in the example above involving the introduction of sequences. The concepts developed through using concrete objects have got to be eventually represented in a symbolic form. As Brunner points out "the ultimate approach to learning Mathematics is symbolic"
This is achieved through language and through other symbols of a specific mathematics nature.

I hope that you have found the above information enjoyable and enriching. *Good!*

Can you try the following activity?

✍ **Activity 13.9**

(i). What is a mathematical fact? Give two examples.

(ii). What is a mathematical concept?

(iii). State three stages that a teacher can use to introduce Mathematical facts to the pupils

You are doing great!

You can compare your answer with a friend and with one at the end of this unit.

For further practice, in a group of five, develop activities that can be used to introduce the concept of vertical addition to a P.1 class.

TOPIC V: METHOD OF TEACHING

Dear student, you are welcome to the last topic of this unit. The topic has been divided into two subtopics:

- Different methods of teaching
- Application of the different methods of teaching Mathematics

1. Different methods of teaching Mathematics

The primary school teacher is responsible for producing pupils who:

- Have well-formed basic mathematics concepts
- Are able to build on their concepts to increase and broaden their mathematical knowledge.
- Are well-motivated to study Mathematics
- Are able to apply their mathematical knowledge to real life situations

In order to achieve this, you must have a good understanding of the concepts to be aught and the way pupils learn Mathematics. You must be able to present these concepts to pupils in a way which will be easily understood and assimilated by them. You must show the pupils how to apply these concepts and subsequently how to use these concepts to develop other concepts.

When teachers have a very clear picture of the subject matter to be learned, they should then present how they will present the material to the pupils.

Different topics lend themselves to different methods of teaching.
There are an infinite number of ways in which a lesson can be presented.

Using the knowledge acquired from PES unit 6 on the different methods of teaching, list down the different methods used in teaching. Then compare both lists with the list below:

- Brain storming	- Ice breakers
- Guided discussion	- Role play
- Pair work	- Lecturing
- Jigsaw	- Guided discovery
- Question and answer	- Sorting
- Cooperative learning	- Team teaching
- Think pair share	- Project work
- Demonstration	- Gallery walk
- Experiment	- Exposition
- Problem solving approach	- Practical work

Can you devise any other methods in addition to the one listed?

As a teacher of Mathematics, you need to be given some idea of the wide range of teaching methods, let us now consider two of those methods listed above and see for ourselves the differences in their approach and the amount of pupils' involvement in each case.

(i). Demonstration

The teacher prepares a problem and demonstrates related experimental work. He then allows the pupils to draw their own summary. The teacher will have to find out whether each pupil has drawn the correct conclusions before giving them problems which require them to use the knowledge they have just gained.

(ii). Class discussion

The teacher prepares a problem which is then discussed by the entire class with the help of the teacher. They discuss what experiments or discussions should be undertaken and the teacher demonstrates the experiments. The pupils discuss the results and draw their own conclusions. The teacher checks the conclusions of each pupil. He then gives out work which requires them to use the knowledge they have just gained.

Activity

Using the above two examples and the knowledge of PES, in a group of 5, discuss how each method listed above can be used to teach a topic of your choice.

Good. Thank you for completing this subtopic!

2. Application of Different methods in the teaching of Mathematics

Dear student you are welcome to the last bit of unit 13.

Can you now go back to the first topic of this unit and revise on the topics covered in the Primary Mathematics syllabus?

Can you list the 10 topics covered in the Primary Mathematics syllabus?
If yes, then you are doing well.

Let us continue and do the following activity.

✍ **Activity 13.10**

In a group of 10 students, select one topic of your choice and prepare a lesson of 40 minutes using the methods of your choice and the learning aids.

Your group will present the lesson while the rest of the class listen and observe your presentation. There after, the students will comment and critique, e.g. what other methods could have been used to teach the topic?

Great job!

Have you prepared your lesson and presented it to the class? If so, then you are a brilliant student.

Note:

The criteria for choosing a teaching method depends on the content, class size, available resources and the competences to be developed.

Thank you for completing unit 13.

13.8. In this unit, you have been introduced to the organisation of the Mathematics lesson and learned about:

(i). Syllabus interpretation
(ii). Topic analysis
(iii). Learning difficulties in Mathematics
(iv). Mathematical terms and vocabulary
(v). Preparing to teach

13.9. Glossary

Mathematics facts: these are true Mathematics statements

Mathematics Concepts: relationship with a group of facts

Symbolic stage: a stage where concrete objects, and pictorials are represented by symbols.

13.10. Notes and answers to the activities

Activity 13.1

a. The topics are spiral in nature. That is to say:

- Mathematical knowledge keeps expanding from topic to another
- Mathematical concepts are built on the previous concepts learnt from the previous topics.

b. Work on the integers is abstract in nature for learners meeting number work for thefirst time.

Activity 13.2

(i). Equivalent fractions
(ii). Getting equivalent fractions
(iii). Reducing fraction to lowest term
(iv). Presenting fractions on a number line
(v). Addition of fractions with the same denominator
(vi). Addition of fraction involving word problem
(vii). Subtraction of fractions
(viii). Subtraction of fraction involving word problem
(ix). Changing improper fractions to mixed fractions
(x). Addition of mixed fraction
(xi). Subtraction of mixed fraction
(xii). Addition of fraction with different denominators
(xiii). Subtraction of fraction with different denominators

Activity 13.3

- To identify previous knowledge to be used in the lesson
- To identify new knowledge and skills to be acquired
- To identify problems pupils are likely to face during the lesson and remedies
- To avoid repetition and overlap of topics
- To avoid omission of concepts to ensure proper coverage of the topic

- To do proper resource allocation.

Activity 13.6

- Topics to be covered during a specified period of time say one week
- Availability of textbooks and teaching aids
- The method one need to use to teach the content in the topics appearing in the scheme of work

Activity 13.7

1. The teacher knows what s/he is going to teach in a particular lesson by looking at the scheme of work which covers the whole topic of which the lesson is part.

2. Which question he will ask the learners

- Which methods to apply.
- What teaching aids she will use.
- Learners in the class(ability and size)

3.

- To stimulate the interest of the learners
- To provide a link with a previous lesson with sub-skills of a lower order than the one you are going to teach.
- To draw the pupil's attention to the lesson

Activity 13.8

a) Visual aids refer to the materials that a pupil can see and touch while audio-visual aids can be listed too as well as seen

b)

- Permits point to point development and reference as the lesson progresses.
- Allows emphasis on major points
- Provides a medium for the participation of the pupils in classroom activities
- Presents assignments, homework problems or discussion questions
- Combines visual and oral presentation of ideas

Activity 13.9

i). A mathematical fact is a true statement in Mathematics e.g.
- $2+4=6$ and
- $27 \div 9 = 3$

ii). A mathematical concept is formed by a group of related mathematical facts

iii). Concrete $\longrightarrow$ pictorial $\longrightarrow$ symbols

13.11. End of unit exercise

This assignment is intended to help you consolidate what you have learnt in this unit. You are therefore advised to read the whole unit again before you attempt the following questions.

1). Identify one topic of your choice from the syllabus and prepare a P.4 scheme of work for a one week period.

2). State the difference between a teaching syllabus and an examination syllabus.
3). Identify one topic of your own from P.1 class and analyse it.

13.12. Self check / Assessment

Learning outcome	Not sure	Satisfactory
I am able to list down topics from the Primary School Syllabus		
I am able to analyse a Mathematics topic		
I am able to make a Mathematics scheme of work		
I can write competences and plan for teaching		

13.13. Reference for further reading

1. Primary School Mathematics Curriculum Volume 1
2. Errors in Mathematics by Hausen A.

UNIT 14: RATIONALE AND LEARNING THEORIES IN MATHEMATICS

1.1 Introduction

You are most welcome to unit 1. In this unit you will be introduced to tfor rationale fro teaching mathematics in the primary school. You will also discuss various learning theories advanced by psychologists and how they are applicable to teaching and learning mathematics in the primary school.

1.2 Content organization

Dear student, in this unit you are going to cover the following topics as indicated in the table below.

Topic	Subtopic
iii. Rationale for teaching mathematics in primary schools	d) Importance of learning mathematics in primary school e) Objectives of teaching mathematics f) Mathematics in everyday life
iv. Teaching mathematics in primary school	d) Challenges of teaching and learning mathematics in the primary school e) Role of the teacher in teaching mathematics
f) Learning theories in mathematics	g) Piaget h) Ausubel i) Skemp j) Cagne k) Dienes l) Bruner

1.3 Learning outcome

Explain the rationale and apply various learning theories in teaching mathematics

1.6 Competences

e) Explain the rationale for teaching mathematics in primary school
f) Discuss the importance of mathematics in everyday life and all subject areas in the school curriculum
g) Identify challenges and roles of the teacher in teaching mathematics
h) Apply different learning theories to the teaching and learning of mathematics

1.7 Unit orientation

This unit will help to explain why mathematics should be taught to all pupils in the primary school. It will be interesting to discover the enormous uses of mathematics not only in all subject areas of the school curriculum but also in everyday life. The learning theories give you a basis on how the curriculum and teaching are designed.

1.8 Study requirements

To succeed in studying this unit, you will have to engage in plenty of discussions with your tutor and classmates. There will be group work activities too. You will need a pen, notebook, a copy of the Uganda Primary School Curriculum (1999) and some textbooks on learning theories from the library.

1.9 Contents

TOPIC I: RATIONALE FOR TEACHING MATHEMATICS IN PRIMARY SCHOOLS

d) Importance of learning mathematics in the primary school.

In your understanding, what is mathematics? Compare your views and answers with those of your colleagues. In mathematics we deal with numbers and computations. Some reasons why every child should learn mathematics are given below.

5. Mathematics provides learners with means of developing powers of logical thinking, spatial awareness and numerical skills.
6. Mathematics presents information in many ways through the use of numerals, tables, charts or graphs and other diagrams.
7. The figures and symbols used in mathematics can be manipulated and combined in systematic ways so that it is possible to deduce information about the situation to which the mathematics relates
8. Mathematics is fundamental for the study of all subjects in the curriculum.

It is good that you work in groups and come up with more reasons for teaching mathematics in the primary school. Learners should be made aware of these reasons.

e) Objectives of teaching mathematics

Study the Uganda primary school curriculum, volume one 1999, starting from page 221. Write down the aims of teaching mathematics in the primary school. If you read further on, you will realize that there are set objectives for teaching each topic to each class. Write down a topic, say sets and list the objectives for teaching sets, for P.1 up to P7.

Below are some objectives which you should relate to a topic and possibly a class in the primary school.

5. To enable the learners to count, read and write numbers; and make simple calculations with numbers.
6. To enable learners workout problems involving money, distance, time, temperature and capacity.
7. To enable the learner apply the concepts of ratio, proportion and percentage in problems in daily life situations.
8. To enable the learner form patterns, recognize shapes and know their properties

The objectives above and those you listed on your own, if attained by all the learners at the end of their primary school, will lead them to having the vital components of mathematics as may be required of an ordinary citizen.

f) Mathematics in everyday life

What role does mathematics play in anyone's everyday life? You will agree with me that we all need to:

8. Be able to count and make simple calculations with numbers; note that calculations may be done with or without a calculator
9. Know about money, balance or change issues, profit and loss matters
10. Be able to measure mass, length, capacity, talk about time and temperature.
11. Be able to recognize shapes, patterns and know some of their properties.
12. Deal with ordinary fractions, decimals and percentages.
13. Read and understand charts and graphs in their various forms.
14. Use mathematics knowledge to solve specific problems in our everyday life.

List down a few more things you believe. We should all be able to do as a result of the mathematics we learn. What mathematics will specifically be required by:

- A milk vendor
- A poultry farmer
- A taxi conductor
- A parent
- A nurse
- One with a telephone/food business
- A teacher can live without mathematics?

TOPIC II: TEACHING MATHEMATICS IN THE PRIMARY SCHOOL

e) Challenges of teaching and learning mathematics in the primary school

One of the challenges generally faced by mathematics teachers and learners not only in Uganda but also in other parts of the world is according to cockcroft (1982, p67):
Mathematics is a difficult subject to teach and learn. This is because the ability of a learner to proceed to new work is very often dependent on a sufficient understanding of one or more pieces of work that have been done before. We may say that learning multiplication, for example is dependent on having learnt addition.

Some other challenges include the great differences in the rates of attainment between children of the same class and lack of spirit of hard work and practice.
In Uganda, specifically, we face challenges of:

- Lack of mathematics textbooks which can cater for the learners' different abilities
- Large classes, which are difficult to teach and have their written work marked.
- Teaching mathematics in English language, using technical words whose meanings in mathematics are different from their use in ordinary English language

Get into groups and discuss other challenges to teaching and learning mathematics in Uganda suggest strategies that will minimize these challenges.

f) Role of the teacher in teaching mathematics

As a primary school teacher, you are expected to play several roles as an individual and also as a member of staff or the profession at large. Here are some of the roles.

7. Make mathematics a reality in life by using methods and approaches to learning that are very practical and are based on the experience of the learners..
8. Integrate mathematics with other subjects in the curriculum by seeking opportunities out of a wide range of learner's activities.
9. develop in learners a positive attitude to mathematics, create awareness of its great power to communicate and provide explanations in matters of daily phenomena
10. demystify the subject and make it user friendly
11. constantly evaluate the teaching/learning process
12. Plan and manage your time. You have to plan your day, your week and entire year so that you can accomplish your work.

All these roles have to be undertaken along with your other commitments at home and in the community. Remember at school you are expected to take on the role of a parent. You ought to guide and counsel the learners.

TOPIC III: LEARNING THEORIES IN MATHEMATICS

We shall learn about theories which are sets of ideas that attempt to explain the process of learning. These ideas have been advanced by different psychologists. We shall discuss how each one applies to teaching and learning mathematics.

a) Piaget

Piaget believes that a child's ability to learn develops in well defined stages that are related to "chronological" age. He believes in child-centered education. Piaget emphasizes discovery learning where a child is given chance to manipulate real objects.

As a teacher, this theory demands that you give children a chance to discover knowledge.
How will you guide children to discover the formulae for perimeter of a square or area of a rectangle? How can these be built onto what children already know? How about a sum like $5 \div \frac{1}{2}$ using pieces of paper, then using pictures or symbols and finally formulae?

All in all, always give pupils an opportunity to discover new ideas depending on what they already know.

b) Ausubel

Ausubel advocates for meaningful learning as opposed to role learning. He stresses the importance of learners discovering formulae before they can practice using them. If this is done, the learner finds it easy to memorize the formulae. This of course requires the teacher to allow for plenty of practice for learner to discover patterns that lead to formulae.

He criticizes role learning (reception learning) in which the learner is presented with the entire content of what is to be learnt in its finished form. He says this usually involves multiple readings (cramming) of the material with little or no effort devoted to retention.

Teachers will impose on children formulae for area, arithmetic mean and angle sum of a polygon. Discuss with colleagues how these can be taught meaningfully with children building onto previous knowledge.
Remember Ausubel emphasizes that children should discover, practice, memorize then be in position to apply and relate.

g) Skemp

Skemp says we should teach mathematics from simple work to hard work. For example, you cannot teach addition of fractions before children have learnt addition of whole numbers.

How can you make use of what skemp advocates for when you are planning for your teaching?
In your scheme of work, topics which can be understood easily by the learners should appear first. Those which are hard should appear later. This explains why integers are not taught in lower primary classes.

When you are preparing a lesson, find out the simplest mathematical ideas in that topic and start with those. This requires that even before you write the scheme of work, you should read widely on a given topic then organize your work from simple to complex.

g) Gagne

According to Gagne learning will only be said to have occurred if there is a change in the learner's behavior or performance. He says that learning must be linked with the design of instructions. A design of instructions depends on the learning outcomes the teacher sets out to achieve. Gagne is concerned with how the teacher develops the learning outcomes or objectives.

He says that learning objectives should be formulated basing on skills, concepts and information; and attitudes and values to be gained by the learner. In the primary school, pupils are expected to acquire computational skills in mathematics. These skills involve how to add, subtract, multiply and divide.

The concepts or ideas to be learnt will include knowing ordering of numbers: 2 is less than 5, 6 exercise books are more than 3 exercise books.
Pupils should learn about measuring before learning area. Change in attitudes and values can be achieved by the teacher relating mathematics to the learner's experiences and environment. As a teacher, Gagne says you should observe a change in the learner's performance for you to have done some teaching.

h) Dienes

In his contribution to the teaching and learning of mathematics, Dienes uses cubes and cuboids to help pupils understand number notations and operations. He fitted the cubes or cuboids alongside one another to give comparisons of lengths and then reassembling again. Dienes says that in early years, a child realizes that 1 cube is for one unit and all the other numbers greater than 1 are represented by repetition.

For example 5=1+1+1+1+1

Later on children discover combinations that result into larger cubes.

Study the following:

Start with 1 cube = 1 cube
Arrange 3 cubes in a row = 3 cubes
Arrange 3 cubes in 3 rows = 3x3 cubes

As a child continues with the arrangement, they discover the rotation of $3^0,3^1,3^2,3^3$,-- which we use when dealing with number bases and powers.

Dienes advocates for step understanding which is very important in learning mathematics. Study the Uganda Primary School Mathematics Curriculum and see its spiral nature. All the topics develop year by year from P.1 up to P.7.

i) Bruner

According to Bruner, the role of the teacher is to interpret the learner's environment then assist the learner to actively participate in learning through discovery activities. The teacher should design activities to suit the environment and the curriculum in order for learners to discover mathematical ideas.
How does Bruner's theory compare with the thematic curriculum?

He advances the following advantages of discovery learning.

- Independent thinking and problem solving is developed. The child uses techniques of discovery learning to find solutions to real problems outside the classroom. Think of some examples.

- The child in motivated to venture into new problems after discovering solutions on his or her own.

Bruner wants the teacher to teach the basics. After the basics, the teacher will pose a problem to the learner and encourage the learner to explore it, prompt the learner to use previous knowledge to solve the problem, give the learner a chance to demonstrate the new skills acquired then finally build on the learner's experience to give them the information they may not have discovered.

1.10Unit summary

You have come to the end of unit1. In this unit you were introduced to the importance of learning mathematics in the primary school. You discussed how mathematics is used in daily life, the challenges faced in teaching and learning mathematics and what some psychologists say about the teaching and learning of mathematics

1.11Glossary

Concept: an idea in mathematics e.g. addition.

- Discovery learning: learning in which facts and concepts are found out step by step
- Role learning: learning things by heart without understanding their meaning.
- Or how they come about like $c = 2\ \pi r,\ \ v = \pi r^2 h$

Theory: a formal statement of ideas which are suggested to explain a fact.

11.12. Notes and answers to activities

a) Importance of learning mathematics in the primary school also because:

- Mathematics is a powerful means of communication, if **A** was born on 11/8/1960 and **B** was born on 8/11/1960, we can tell, who is older, by how much?
- Mathematics can be used to predict the outcome of an event which is yet to come (probability especially).
- To arouse interest and appeal amongst children

d) Other objectives are for learners to be able to:

- Recognize shapes and know their properties
- Read and understand graphs in their various forms
- Use mathematics knowledge to solve problems in everyday life

e) People in their everyday life need to be able to:-

- Measure mass, length, capacity using various scales or containers
- Work with money by counting, adding, subtracting, dividing, multiplying often mentally
- Estimate time, distance, ingredients for cooking, doses of drugs etc

End of unit exercise

3. Give five examples of role learning in mathematics in the primary schools. How can a teacher promote meaningful learning in mathematics
4. write down the procedure for an activity to help learners discover that:

iii) $3 \div \frac{1}{2} = 6$

iv) The diameter of a circle $d = 2 \times radius$

h) Discuss the reasons why primary teachers may not give pupils chance to discover mathematical ideas

1.12Self checking exercise

You have come to the end of unit 1. Listed below, are the learning outcomes. Please tick in the column that best reflects your learning.

Learning outcome	Not sure	Satisfactory
5. I can explain the reasons for teaching mathematics in the primary school		
6. I can discuss the importance of mathematics in everyday life and in other subjects		
7. I can identify the roles and challenges of the teacher in teaching mathematics.		
8. I can use different learning theories to teach mathematics		

1.13 References for further reading

3. Alice Hansen et al (2005), children's errors in mathematics: understanding common misconceptions in primary schools. Learning matters

4. Uganda primary school curriculum (1999). Volume one

UNIT 15: ORGANISING MATHEMATICS CLASSROOMS

15.1 Introduction

Dear student, you are welcome to unit15. This unit introduces you to the basic requirements for developing a mathematics classroom and how to organize the mathematics classroom to suit all types of learners in the mathematics class.

15.2 Content Organisation

Hullo student, in this unit, you are going to cover the following topics as indicated in the table below:

Topic	Subtopic
1. Grouping pupils in a Mathematics class.	• Concepts of mixed ability grouping • Types of learners in a Mathematics class • Learners with special needs • Helping learners with learning with learning needs in Mathematics
2. Mathematics learning areas	• Mathematics learning areas in lower and upper primary • Developing and maintaining a mathematics learning area • Importance of the mathematics learning areas.
3. Storage of instructional materials in a mathematics classroom	• Storage of instructional materials in a mathematics classroom
4. Mathematics class day	• Organisation of a mathematics class day

15.3 Learning outcome

At the end of this unit, you are expected to demonstrate the skills and knowledge of organizing a mathematics classroom and apply it in the primary school classroom situation.

15.4 Competences

Dear student, now you know the expected outcome, therefore as you study this unit, you will be able to:

1. Define mixed ability grouping
2. Name different ways of grouping learners of mixed abilities
3. Identify ways of helping each type of learners
4. Identify learners with special needs in mathematics
5. Identify ways of helping learners with special needs
6. Define mathematics learning area
7. Identify materials for upper and lower primary
8. Make storage facilities for the mathematics instructional materials
9. Organize a mathematics class day.

15.5 Unit Orientation

This unit is to get you exposed to the various skills and knowledge required for the organisation of the mathematics classroom. This unit is interesting because it starts with the very learners you are going to teach, their levels of abilities, and then proceeds to ways of grouping them, learners with special needs and how to help them, making instructional materials to be used for upper and lower classes and finally organizing a mathematics class day for stakeholders to come and view the work done in the mathematics department.

15.6 Study requirements

To be successful in studying this unit, you are required to prepare plenty of materials collected from your local environment, read about children's different levels of learning abilities and children with special needs in the ECE, concerning ways of grouping them according to ability levels and ways of helping children with special needs. You also need to have basic numeracy concept and teaching primary mathematics in primary school textbooks for further reading.

15.7 Content

Topic I: Grouping pupils in a Mathematics class

1. Concept of mixed ability groupings

What do you understand by a mixed ability grouping?
In order to answer this question, can you refer to (Paling pg 304-307) and (Mathematics Education Module pg 59-70). Compare your answers with your colleague and check your findings with the answer below.

A method of grouping learners after assessing them so that each group contains one of the brightest pupils, one of the weakest and two or three across the range in between brightest and weakest.

2. Types of learners in a mathematics mixed class

Types of learners in a mixed ability class include therefore among others children who are:

- Low achievers
- Average achievers
- High achievers

There are several ways of grouping pupils with mixed abilities. These include:

a). Banding

Banding occurs when all the pupils within an age range are assessed by reference to ability and are then grouped in bands so that each separate band:

- Contains pupils of mixed ability
- Generally contains enough pupils for two or three classes

b). Interest grouping

Interest grouping brings pupils together to work at a common interest. These groupings are most often used with investigational methods of teaching and are usually formed by a combination of the decisions made by pupils and teacher. For example, although the pupils are allowed to choose a mathematics topic which interests them, the teacher ensures that each pupil is following a progressive sequence of topics.

c). Streaming

Streaming extends over two or more years of school. Here pupils are assessed by reference to general ability. From this assessment, each pupil's name is placed on a list which ranks pupils from those with the highest ability down to those with the lowest ability. This list is then divided into several parts to assign the pupils to their respective working groups.

Now, are you able to name different ways of grouping and types of learners in a mathematics mixed class? If yes, good you are doing well.

✍ **Activity 15.1**
Can you compare the type of grouping mentioned above and the one in your college?

Share your answer with a colleague.

Note:
If you are to teach a mixed class satisfactory then you need to employ a variety of teaching methods (Revise unit 13 on methods)

How can you assist the individual learner in a mathematics class? To answer the above question, you can borrow a leaf from the six models of mixed ability teaching shown below as adopted from Dean (1982), Teaching and Learning Mathematics pg 51.

Model	Exercises	Exploratory tasks
X Whole class activity	All pupils study the same topic at the same time. Materials are in the form of work sheet, work cards or topic workbook	Pupils are involved in various levels of investigations of a teacher initiated problem/activity
Y Group work	Small groups in a class, work on different tasks from a textbook, topic book, worksheet or work card.	Small groups carryout different investigations or activities from a variety of sources.
Z Individualized scheme	Pupils work on materials from structured scheme which is designed to develop concepts. Task assignment is systematic or teacher directed.	The pupils work individually choosing their tasks, with guidance from the teacher, from a bank of investigation. Not necessarily structured.

✍ **Activity 15.2**
In a pair, discuss five advantages and disadvantages of grouping learners according to ability

Compare your answers with those at the end of this module.

d. Learners with special needs

While we have looked at learners of mixed abilities another category of learners are those with special needs.
A learner with special needs may be looked at as a learner who does not benefit from the general curriculum and only can benefit when modifications have been identified depending on the need. For instance, a blind child requires Braille and deaf child requires sign language.

✍ **Activity 15.3**
Visit a nearby primary school with a classmate and find out how learners with special needs in mathematics are identified,assisted and report to the class.

Compare your findings with those at the end of this unit.

d. Helping learners with learning needs in Mathematics

The topic you have just concluded assisted you in identifying learners with learning needs in Mathematics. It is now your duty again to identify ways these learners can be helped.

✍ **Activity 15.4**

Refer to SNE on numeracy and suggest five ways learners with special needs in mathematics can be helped following the table structured below:

Low ability	High ability
……………….. ……………….. ………………..	………………….. ………………….. …………………...

Compare your findings with your colleagues.

Thank you for coming to the end of this topic.

15.7.2 Mathematics learning areas

1. Mathematics learning areas in lower and upper primary

As a teacher of Mathematics, there are certain things you need to remember. For example, that the pupils you are teaching have different mental abilities. Also you should keep in mind that your pupils have different interests. These depend on various factors including the pupil's background, level of maturity and the nature of topic being taught.

It is important that as you teach, you see the needs of your learners. One way of doing this is to have a Mathematics learning area.
Using think-pair share, can you discuss what a mathematics learning area is?

Materials to be put in a Mathematics learning area vary between lower and upper primary.
Below are materials that should be kept in the mathematics learning area for upper primary:

- Work Cards
- Pictures
- Mathematical Instruments
- Scales
- Charts
- Clock Faces
- Wooden Cubes and Blocks
- Course Books and Other Textbooks
- Thermometer
- Children's Work Displayed
- Tape Measure

For lower primary, refer to the ECE module for items in the Mathematics learning area.

> ✍ **Activity 15.5**
> Can you make one instructional material for upper primary and one for lower primary for your mathematics learning area?

a. Developing and maintaining a Mathematics learning area. (Refer to ECE Syllabus)

b. Importance of the Mathematics learning area. (Refer to ECE Syllabus)

15.7.3: ORGANISING A MATHEMATICS CLASSROOM

1. Skills needed for a classroom organisation

a). Planning for essential requirements

For a start, you need to think ahead and plan for essential requirements to avoid panicking at the last moment. You should do this planning at the beginning of the year. You should categorise the essential teaching requirements under two headings:

Consumables and Durables

Under durable items, you would include items such as pairs of scissors, rulers, chalkboard, protractors, set square, cubes, scales, knives tins and so on.

Items such as chalk, pencils, rubbers, paper etc are consumables. For consumables, you have to consider the quantity you will require of each of the items. First estimate the number of children you are likely to have in the class, this will affect the total quantities of each item you will require. For example, if you have 30 pupils, you need to calculate the amount of materials you will require over a term and over the whole year, to provide for that number of pupils.

When it comes to durable items, find out whether they are available or not. If they are available, find the class. For example, find out whether there are enough textbooks for each pupil. If not, then find out the cost of each book and then work out the total cost.
This kind of planning will enable you to know the materials you need and the quantities you need for your teaching. You also need to be aware of the problems you have to overcome before getting the materials.

b). Ensuring class control (Discipline)

You need to ensure class discipline. Your class should have a sense of direction as to what they are expected to do or not to do. Class discipline is important if the class is to maintain order. For example, if you want materials to be used effectively and well kept, then it is only a disciplined class that can do this. Otherwise, things will be chaotic and likely to be destroyed or misused. As an example, it is a misuse if you found pupils using pairs of compasses and screw drivers on their desk.

In order to maintain class discipline, you need to have regular class meetings. Usually classes have class teachers. The purpose of having a class teacher is to meet the class and discuss the problems of the class. In such meetings, it is important to remind the class how to maintain materials in the class. During these meetings, different pupils are assigned duties. In this way, the whole class will feel responsible for keeping the class materials in good order.

Establish good communication between the pupils and yourself. If a pupil needs to borrow a book, s/he should not fear to approach you or the class librarian. Where communication is vague, materials will be misused and even lost, when such a situation comes about, books will be misplaced. With good communication and a class in which you have developed habits of responsibility, you can keep track of where things are.

You should not assume that you will carry out all these duties alone and with out the active participation and cooperation of your pupils. Pupils will appreciate your efforts and cooperate with you if when they understand why you are asking them to do things. It is therefore important to keep them informed, involved and active. You can do this by making them responsible for the materials they use in the classroom.

You have to select what work should be displayed. You might do this by categorizing the work using some set standards. Then you can decide to display the best together, the weak work and the average work together at the same time.

Even if the work is poor by the class standard, an individual pupil may feel a sense of success and achievement in seeing his work displayed. This feeling should not be neglected. What you need to do therefore is to have constructive comments on each of the displays. This is how pupils will be able to learn from another's work.

✍ **Activity 15.6**

1. Why is it important to anticipate the needs of your Mathematics class?
2. How would you establish class control?

You can compare your answer with a friend and with one at the end of this unit.

2. Making storage facilities

Lack of proper storage facilities may result into the loss of valuable materials. Some materials may be stolen and others destroyed by natural causes rain, termites and rats. It is therefore important to get storage space for materials in the Mathematics class. The space must be lockable and the materials arranged in an orderly way.
Such storage should be:

- Sufficient
- Secure
- Accessible
- Good enough to protect materials from damage

If you use a cupboard or a shelf to store books, it is important that you use the space wisely. The more books you have, the more space you should use. In order to put all the books of the same title together, choose the most suitable space available which will hold all the copies of these books. It is difficult to get all the books of the same title when they are not in one place. After you have identified the place to keep each item, the next thing is to arrange them in an orderly way. Just throwing things into the cupboard causes problems.

First, the books or materials might be damaged or destroyed. If they are books, pages might be torn. In case of some other materials, they can be misplaced.

.
Secondly, they are hard to get when they are needed. When you eventually get them, it might take a long time to organize them in order. This wastes time.

Third, it might be dangerous to get the materials from the cupboards particularly when some other item can fall and hurt the person as he is getting them out.

It is important to indicate where things can be found. You have to this by using labels. You have to use well written cards. An example could be 'MATHS'. If that is the label for mathematics, then a list of all mathematics textbooks should be under it. This helps you to find the books of this particular subject quickly. What should be emphasized to the class is that their items should be kept in permanent places. This simple reminder should be: *'PUT THE RIGHT THING IN THE RIGHT PLACE.'* Such a label can help you easily trace a misplaced book.

We have seen the importance of proper storage of the materials; therefore there is a need to make a storage facility for books.

✍ **Activity 15.7**
What materials from your local environment can be used for making racks, cupboard, stand and shelves?

Can you make one storage facility using materials from the local environment?

15.7.4 Mathematics class day

a. Organisation of a Mathematics class day

During the year, a lot of work is expected to have taken place in the Mathematics classes. The many activities involve classroom teaching, construction of materials and organisation of the mathematics learning areas. There is a need to account for the existence of the mathematics department in the school to the stakeholders.
We also need to popularise mathematics to the community, hence the need to organize a Mathematics day. The day to be successful requires you to draw a programme indicating the activities for the day.

✍ **Activity 15.8**
In a group of 10 students, can you draw the programme of the mathematics day and present it to the class for scrutiny and adjustment?

15.8 Unit Summary

In this unit, you have learnt about organizing a mathematics classroom. You have focused on:

- Grouping pupils in a mathematics class
- Mathematics learning area
- Storage of instructional materials
- Organisation of mathematics class day

15.9 Unit Glossary

Categorise : Create classes to which objects can be referred to.

Concrete : Solid

Store : A place where things are kept

Task : A piece of work requiring to be done

15.10 Notes and Answers to the activities

Activity 15.2

- Encourages competition in the learners.
- Promotes creative and critical thinking.

- Learners enjoy their successes as they learn,

- Some learners may be left and may not cope up with others.
- Demands detailed preparation and keeps the teacher occupies all the time.

Activity 15.3

- Through difficult in manipulating materials.
- Speech problems
- Withdraw from the rest of the learners activity.
- Irregular school attendance.

- Make learning learner friendly.
- Use concrete experiences.
- Motivate their learning.
- Use a variety of instructional materials.
- Attend to them individually.

Activity 15.6

1.
- It enables you to know ahead of time what you will need.
- It enables you to know the quantities of items needed
- You can tell whether it is possible to get these materials or not
- It helps you to know how much money is needed to purchase the materials

2. .
- Have regular meetings in order to establish rules and regulations to govern the activities of the class.
- Discuss the problems of the class particularly how best to use and maintain the materials and equipment.
- Distribute responsibilities among the pupils
- Have good communication channels within your class and between you and the pupils
- Keep pupils active when carrying out their activities
- Make pupils suggest how to use the materials

15.11 End of unit exercise

This assignment is intended to help you consolidate what you have learnt about in this unit. You are therefore advised to read the whole unit again before you attempt the following questions.

a. Prepare a 40 minutes lesson on a topic of your choice for a P.4 class of mixed abilities. Particular attention should be paid to the instructional materials and activities to cater for the varying abilities of the learners.

b. How would you distribute responsibilities to your pupils?

c. If a pupil's record shows that he cannot add two-digit numbers, how would you organize your work in order to assist him?

d. What are the most common requirements a class teacher should think of in organizing a mathematics class day?

e. How would you as a teacher use a Mathematics learning area?

15.12 Self Check/Assessment

You have now completed unit 15 of module1. The learning outcome is listed below; you are now expected to demonstrate your competence by ticking the column that reflects your learning.

Learning outcome	Not sure	Satisfactory
I can apply organizational skills in the mathematics classroom		
I have established a mathematics learning area in my classroom and I use it effectively		
I can identify learners with difficulties and I can plan for them effectively		

15.13 References for further reading

i. The Foundations of Mathematics in the Infant School by Taylor Joy
ii. Improving the learning of Mathematics by Backhouse John
iii. Books of Readings: MoES (2007)

End of unit

UNIT 16: MATHEMATICAL COMPUTATIONS

16.1: introduction

Hullo student are welcome to unit 16 which introduces you to easy methods of calculations. In this unit you will learn various methods of carrying out mathematical computations.

16.2 content organization

Dear student, in this unit you will be covering the following topics as indicated below:-

No.	Topic	Subtopic
1.	**Operations and properties of numbers**	- Mathematical operations - Commutative property - Associative property - Distributive property - Identity element
2.	**Estimation, approximation, rounding of, scientific form and significant figures.**	- Estimation - Approximation - Rounding of - Scientific form - Significant figures
3.	**Indices**	- Index notation - Laws of indices - Zero and negative indices - Fractional indices
4.	**Logarithms**	- Logarithms - Antilogarithms

16.3: Learning outcomes

At the end of this unit, you are expected to:-

(i) Apply quick computation skills to solve problems.
(ii) Develop skills of estimating within stated limits of accuracy
(iii) Teach computations to primary school pupils.

16.4 Competences

Now that you know the expected learning outcomes, as you work through this unit, you will be able to:-

(i) Identify mathematical operations.
(ii) Work out problems involving properties of numbers.
(iii) Use easy methods to carry out computations
(iv) Use the laws and powers of indices in carrying out mathematical computations.
(v) Define logarithms to base 10 and express numbers in standard form.
(vi) Use tables to find logarithms and antilogarithms of numbers.

16.5: Unit orientation

In order to study this unit, you need to remind yourself of unit 4 on operations on numbers and unit 10 on integers respectively. Work on mathematical computations is an extensive of the work you covered on those units.

16.6: Study requirements

During the study of this unit you will need logarithm tables.

16.7: content and activities

16.7.1: operations of properties of numbers

a) Operations of numbers

Dear student, you realize that when we are dealing with whole numbers we majorly have four operations thus:-

Additions, subtraction, multiplication other non common symbols are used that is star * and dot •; and in division a slash is used.
What do you think this would mean?
4*2

Share your answer with your colleague.

b) Properties of numbers.

(i) Commutative property

Look at each of the following pairs of sentences. In which pairs is the result of following the instructions the same?

(i) Add 15 to 7. Add 7 to 15.
(ii) Hoe the ground and then sow the maize. Sow the maize and then hoe the ground.
(iii) Turn right and then take one step forward. Take one step forward and then turn right.
(iv) Put on your socks and then put on your shoes. Put on your shoes and then put on your socks.
(v) Open the door and then open the window. Open the window and then open the door.
(vi) Subtract 9 from 15. Subtract 15 from 9.

Do you agree that only (i) and (v) give the same results?

The order in which we open the window and door does not make a difference to the result. In (i) when we added the two numbers, the order did not matter since 15 + 7 is the same as 7 + 15.

Now what can you conclude?

Do also have the same answer as this?

For example we conclude that a + b = b + a give the same answer. Hence, the order in which any two integers are added does not affect their sum.

The commutative property then means that the order in which two whole numbers are added will not affect their sum.

The commutative property then means that the order in which two whole numbers are added will not affect their sum.

b) Subtraction.

Taking the above example (vi) 15 – 9 is not the same as 9 – 15. That indicates that subtraction of numbers is non - commutative.

c) There are other operations which combine two numbers. Are both multiplication and division commutative? Explain your answers and share with your colleagues.

You can do the following multiplication problems to find out the answer.

5 x 7 = and 7 x 5 =
-6 x -11 = and -11 x -6 =
-9 x 8 = and 8 x -9 =

You will find that all the answers to those problems show that the order in which any integers are multiplied does not affect their answers. That is a x b = b x a. this is the commutative property of multiplication of integers.

Activity 16.1

1. In each sentence below find the value of n which makes the sentence true

    a) 28 + 17 = 17 + n
    b) 18 + n = 53 + 18
    c) 12 x 5 = 5 x n
    d) N x 9 = 9 x 7

2. Matovu planted 18 banana trees last week and 52 banana trees this week. In the same month last year he planted 52 banana trees and 18 banana trees respectively. In which year did he plant more banana trees?

    You can now compare your answers with the ones at the end of the unit

(ii) Associative property

Can you see two possible meanings for each of the following?

(a) Cold water seller
(b) Old car dealer

In each of the sentences the position of the punctuation matters. E.g. in (a) it could mean cold (water seller) or (cold water) seller.

Taking numbers:- 13 + 7 + 2 can be written as 13 + (7 + 2) is the same as (13 + 7) + 2. You realize that the two groupings all arrive at the same answer.

You now see that any three or more whole numbers can be grouped in any way you wish, while adding and the answer will remain the same.

Therefore, for any three numbers a,b,c (a + b) + c = a + (b + c). this is the associative property. Addition of numbers is associative.

Examples 1

a) 17 + 998 + 2
= (17 + 998) + 2
= 1015 + 2
= 1017

b) 17 + 998 + 2
= 17 + (998 + 2)
= 17 + 1000
= 1017

From the example you find that it's easier to add 998 and 2 first to give 1000, then add 17.
Let us consider the operation of multiplication;

a) 19 x 25 x 4
= (19 x 25) x 4
= 475 x 4
= 1900

b) 19 x 25 x 4
= 19 x (25 x 4)
= 19 x 100
= 1900

Note: always look at an addition or multiplication sum to see whether there is a quick way of doing it.

Now you can do this activity.

Activity 16.2

1. Do the following additions, grouping the numbers in any order you prefer.
   a) 39 + 46 + 54 (b) 813 + 87 + 65
2. Work out these multiplications in any order you choose.
   a) 25 x 4 x 20 b) 5 x 45 x 6

3. Check your answers with those given at the end of this unit

(iv) Distributive property

Look at the diagram below with crosses, how many black crosses are there? You probably found number by seeing that there are 3 rows and 5 columns, so the number is 3 x 5. What is the number of red crosses?

X X X X X X X X X
X X X X X X X X X
X X X X X X X X X

How many crosses are there altogether?

As there are 3 rows and (5 + 4) columns the total is 3 x (5 + 4) crosses.

But	total number of crosses	=	number of black crosses	+	number of red crosses
So	3 x (5 + 4)	=	3 x 5	+	3 x 4

With that knowledge, draw a rectangular pattern of crosses in two colours to illustrate.
2 x (4 + 3) = (2 x 4) + (2 x 3)

You realize that this property of numbers is so often used and it is called the distributive property and can be stated as :-
a x (b + c) = (a x b) + (a x c)

Work out 7 x 34 in your head. Did you say 7 x 34 = 7 x (30 + 4) = (7 x 4) = 210 + 28 = 238?

This is the easier way. Which part shows the distributive property? The property can sometimes be used to simplify calculations. For instance.
(17 x 3) + (17 x 7) = 17 x (3 + 7) = 170

Now that you have finished to prove that the distributive property holds with respect addition of numbers, you can try to find out what happens with subtraction of numbers.

Subtraction
Is b x (c – d) = (b x c) – (b x d)

Find out below

Example 3

Simplify 6(15 – 9)
= (6 x 15) – (6 x 9)
= 60 – 54
= 6

Now you should be able to work out numbers in activity below.

Activity 16.3

1. Draw a pattern of crosses in two colures to show that (1 + 6) x 4 = (1 x 4) + (6 x 4)
2. Use distributive property to solve these problems
   (a) 74 x 25
   (b) 66 x 22
   (c) 4 (51 – 16)
   (d) 5 (63 – 61)

Check your answers with those given at the end of the unit.

(v) Identify element

You have now learnt all the other properties. At this time try to find out what happens to 0 when added to a number.

Right, can you observe this?

Examples 4

5 + 0 = 5	and	0 + 5 = 5
8 + 0 = 8	and	0 + 8 = 8
x + 0 = x	and	0 + x = x

What can you see?

Is it true that you have seen that, "for any number x when 0 is added, the sum is that number. With that finding 0 is called an additive identity number.

Find out what happens to 1 when multiplied by any number in the example 2:-

Example 2

6 x 1 = 6	and	1 x 6 = 6
8 x 1 = 8	and	1 x 8 = 8
a x 1 = a	and	1 x a = a

You can now prove that, when any number is multiplied by 1, the product is that numbers we call 1 the multiplicative identity.

Can you move ahead.

Let look at division

Recall that the bar in expressions written as $\frac{a}{b}$ means to divide.

Using this idea, we see that the expressions $^{5}/_{5}$, $^{3}/_{3}$ and $\frac{26}{26}$ all name the number 1
With that you can conclude that, dividing a number by itself gives 1.

Now you can conclude that;

Addition:

For any number a, a + 0 = a and 0 + a = a. (adding 0 to any number gives that number).

Multiplication

For any number by 1 gives that number.

16.7.2 Estimation, approximation, rounding off, scientific form and significant figures

a) Rounding off numbers

Sometimes when we do calculations, an exact answer is not needed because an estimate answer is good enough, an approximate answer is easier to understand than an exact number or there is no exact answer.

To give an approximate answer, you can round to the nearest 10, 1000, and so on. In the process of rounding numbers, look at the digit in the ten places. Digits of 5,6,7,8,9 are rounded upwards.

Digits of 0,1,2,3,4 are rounded downwards.

(i) To round to the nearest 10

Look at the digit in the units column.

- If it is less than 5 round down.
- If its 5 or more round up.

Example 1

Round 687 to the nearest ten.

680, 681, 682, 683, 684, 685, 686, 687, 688, 689
680 and 690 are two nearest tens, but 687 is nearer to 690 than 680.
∴The number is 690.

You can now use the idea of looking at the digit in the unit column.
7 is greater than 580 we round it upwards and get the numbers 690.

(ii) To round to the nearest 100.

Look at the digit in the tens column

- If it is less than 5 round down.
- If it is 5 or more round up.

Example 2

Round 7650 to the nearest hundreds

7600 7700

7650

The 5 in the tens place is exactly five, so take it up to the next hundred 7700.

∴The hundred nearest to 7650 = 7700.

(iii)To round to the nearest 1000

Look at the digit in the hundreds column.

- If it is less than 5 round down.

If it is 5 or more round up.

Example 3.

Round 2446

4 is less than 5, therefore drop 446

So 2446 is nearest to 2000.

Rounding off decimals to the nearest tenths, thousandth, numbers with one, two, three etc digits after the decimal point can be rounded to the nearest tenth, hundredth, thousandths etc

For example 6.78 can be rounded to 6.8 as 6.78 is nearer to 6.8.

Remember of the digit being checked in any decimal is 5 and above, the decimal is rounded.

Activity 16.4

1. Round off these numbers to the nearest ten.
   (a) 189, (b) 1201 (c) 1885

2. Write the following in figures and round off to the nearest ten and then to the nearest hundred.
   a) One thousand six hundred eighty.
   b) Four hundred ninety.
   c) Sixty – two thousand and eight

Check your answers with the ones at the end of the unit.

(b) Estimation

When you use some aids to calculate, its important first to find a rough answer to the calculation.

Similarly in our everyday life situations, we are sometimes not certain when giving answers instead we give rough estimates.

For example when asked the number of guests who will come to attend a party, one can just say:- May be about 100 guests will come. Similarly when we are cooking we estimate the amount of food to cook, salt, water, etc.

Think of two occasions when you have to give rough estimate.

Share your answers with a colleague. The answer always to an estimation depends on the person who is giving it.

Note: when you estimate numbers, round numbers to the nearest whole number, then calculate without using a calculator. For very large or very small numbers, round to the nearest $\frac{1}{100}$, $\frac{1}{10}$, 10, 100 etc

Example 4

Estimate without using a calculator

(i) 3.08 x 9.8764

$\therefore 3.08 \sim 3$ (round down)

$9.8764 \sim 10$ (round up)

$\therefore 3.08 \times 9.8764 \sim 3 \times 10 = 30$

(ii) $\frac{14.963 - 2.8}{4.144}$

$14.963 \sim 15$ (round up)

$2.8 \sim 3$ (round up)

$4.144 \sim 4$ (round down)

$\therefore \frac{14.963 - 2.8}{4.144} \sim \frac{15-3}{4} = \frac{12}{4} = 3$

c) Approximation

The amounts given as an estimates are usually approximation of a quantity which has been decided by judgment rather than by carrying out the process needed e.g. measuring or doing the sum.
In other words an approximation is a stated value of a number that is close to (but not equal to) the true value of that number.

For example when finding the value of π are bound to have answers like 3,3.1, 3.141……… which are all approximations.

d) Significant figures

Think about these three conversations;

(i) What is your weight? 71.782 kg.

(ii) Thank you for the tickets. How far is it from Cairo to New York? 8891.74 km.

(iii)How much was your new car? Shs. 15371.45.

What is wrong with the answer in each case?
What would be more sensible answer?
You should see that, in each case, it is the first few figures only which are important.
In (i) 72 kg is reasonably accurate to 2 significant figures.
In (ii) 8900 km is probably quite accurate to 2 significant figures enough.
In (iii) Sh. 75,400 would be on accurate enough answer for most people to 3 significant figures.

So are significant figures used?
Have you also got this answer?

Significant figures (sf) are used to express the relative importance of the digits in a number. The most important being the first digit, starting from the left hand end of the number which is not zero. Starting with the non zero digit all digits are then counted as significant up to the last non-zero digit. After that, zeros may or may not be significant. It depends on the context.

Example 5

Write 3098.614 to

a) 5 significant figures = 3098.6
b) 3 significant figures.
c) 1 significant figure

Significant figures start from the 3.
∴ 3098.614 to

a) 5 significant figures = 3098.6
b) 3 significant figures = 3100 (the 8 makes the 9 into a 10)
c) 1 significant figure = 3000

Note.

- All the zeros to the left of the decimal point should be left in, otherwise you change the size of the number of factors of 10.

- Zeros on the right and left hand side of a decimal point may or may not be significant.

Activity 16.5
In each of the question find rough estimate for the calculation.

1. a) 217 x 0.29 b) 0.96 ÷ 101

2. $\frac{4.9 \times 81.3}{3.53}$

3. Express the following to a) 4 significant figures
b) 2 significant figures
(i) 409.887
(ii) 6.59432

Check your answers with the ones at the end of this unit

d) Standard form (scientific form)

When a number is in standard form, there is only one digit to the left of decimal point. The size of the number is given by a power of 10.

For instance

a) By what powers of ten must you multiply 1.3 to get
(i) 13; (ii) 130; (iii) 1300?

Copy and complete this table

$13 = 1.3 \times 10^1$
$130 = 1.3 \times 10^2$
$1300 = 1.3 \times 10^3$
$13000 = 1.3 \times$
$130000 =$
$1300000 =$

Numbers written in standards form are expressed as a number between 1 and 10, multiplied by an integral power of 10.
$A \times 10^n$ is in standard form $1 \leq A < 10$

Now find out if you can write numbers less than 1. For example 0.00013, in standard form.

Copy and complete this table.

13.0	=	$1.3 \times 10 = 1.3 \times 10^1$
1.3	=	$1.3 \times 1 = 1.3 \times 10^0$
0.13	=	$1.3 \times \frac{1}{10} = 1.3 \times 10^{-1}$
0.013	=	$1.3 \times \frac{1}{100} =$
0.0013	=	
0.00013	=	
0.000013	=	
0.0000013	=	

In standard form how many figures are there to the left of the decimal point?

Taking 173.2 in standard form is 1.732×10^2. What is the connection between the power of 10 moved to change 173.2 into 1.732?

Now write 0.000173 in standard form.

What is the connection between the number of places and the power of 10 this time?

Share your answers with your friend.

Do you realize that:-

1. When the decimal point moves any steps to the left, the powers of 10 takes the positive number of places the decimal point has moved.

   As in 1 7 3 . 2 = 1.732×10^{-2} = 1.732×10^{-2}

When the decimal point moves any steps to the right, the power of 10 takes the negative number of places the decimal point has moved.

As in 0. 000173 = 1.73×10^{-4}

Standard form is used for very large and very small numbers. Its use also makes it easier to obtain rough answers to calculations when very large or small numbers are used. It also helps us to estimate the position of the decimal point in a calculation.

Example 6

Find the rough estimate for 479 x 0.0032

$479 \times 0.0032 = 4.79 \times 10^{2} \times 3.2 \times 10^{-3}$

$\sim 5 \times 10^{2} \times 3 \times 10^{-3}$

$\sim 5 \times 3 \times 10^{2-3}$

$= 15 \times 10^{-1}$

$= 1.5$

Example 7

Find the rough estimate for 0.0634 ÷ 26.1

$\frac{0.0634}{26.1} = \frac{6.34 \times 10^{-2}}{2.61 \times 10^{-1}}$

$\sim \frac{6 \times 10^{-2}}{3 \times 10^{-1}}$

$= 2 \times 10^{-2-1}$

$= 2 \times 10^{-3}$

Activity 16.6

1. Express the following in standard form
   a) 0.05621 b) 980.52

2. Evaluate the following leaving your answer in standard form
   a) $3.6 \times 10^{8} \div 2 \times 10^{3}$
   b) $6.5 \times 10^{6} \div 5 \times 10^{-10}$

Check your answers with the ones at the end of this unit

16.7.3 indices

a) Index notations

A product in which the factors are the same is called a power. We can write 5 x 5 x 5 as 5^3. The number 3 is called **exponent** or **power** (index) and 5 is called the **base**. The exponent tells how many times the base is used as a factor.

You can write index number in a general way. For any number a, three lots of a multiplied together can be written;
$a \times a \times a = a^3$
And n lots of a multiplied together can be written:
$\underbrace{a \times a \times a - - - - - - - - - - - xa}_{\text{n lots of a}} = a^n$

When an expression is written with exponents we say the expression is written using experiential notation.
Power/exponent → P
Base → b

You can now do this work in your exercise book.

1. Write each of the following in your exercise book.
   b) 4 x 4 x 4 x 4 x 4 b) 7 x 7 x 7 x 7

2. Write each of the following in full
   a) 6^3 b) 5^5 c) 8^4

   After doing that exercise you can share your answers with your colleague.

c) Laws of indices

You are now going to look at the rules for solving numbers written in exponential form.

(i) First law

Example 1

Work out the following leaving your answer in exponential form. a) $5^3 \times 5^5$ b) $7^4 \times 7^5$

a) $5^3 \times 5^5 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5)$
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^8$

b) $7^4 \times 7^5 = (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7)$
$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
$= 7^9$

So
$5^3 \times 5^5 = 5^8$

$7^4 \times 7^5 = 7^9$

Do you notice a pattern in the indices?

What can you conclude?

You must have realized that, when you multiply numbers of the same base, we add their indices.

The above example now becomes

$5^3 \times 5^5 = 5^{3+5} = 5^8$

Therefore $a^n \times a^m = a^{n+m}$

(ii) Second law

What is a) $3^9 \div 3^6$ b) $8^5 \div 8^3$

a) $3^9 \div 3^6 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 3 \times 3 \times 3 = 3^3$

b) $8^5 \div 8^3 = \frac{8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8}$

$= 8 \times 8 = 8^2$

So $3^9 \div 3^6 = 3^3$ and

$8^5 \div 8^3 = 8^2$

Do you notice the pattern here?

What can you conclude?

You notice that, when you divide numbers of the same base, we subtract their indices.

So

a) $3^9 \div 3^6 = 3^9 - 6 = 3^3$

b) $8^5 \div 8^3 = 8^{5-3} = 8^2$

Therefore we conclude that

$a^n \div a^m = a^{n-m}$ or $\frac{a^n}{a^n} = a^{n-m}$

(iii) Third law

What is a) $(6^3)^2$ b) $(7^4)^2$

a) $(6^3)^2 = (6 \times 6 \times 6) \times (6 \times 6 \times 6)$

$= 6 \times 6 \times 6 \times 6 \times 6 \times 6$

$= 6^6$

b) $(7^4)^2 = (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7)$

$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$= 7^8$

Thus $(6^3)^2 = 6^{3 \times 2} = 6^6$ and

$(7^4)^2 = 7^4 \times 2 = 7^8$

Therefore you have found that to raise a number which is expressed as a power of a base to another power, just multiply the indices.

You can express the law as $(a^n)^m = a^{n \times m}$

Activity 16.7

Evaluate the following expressions

1. $5^2 \times 5^4$
2. $3^7 \div 3^5$
3. $6^9 \div 6^1$
4. $(7^5)^2$
5. Find x in the following expressions:-
   a) $3^x = 81$
   b) $2^{x+1} = 256$

Check your answers with those given at the end of the unit.

c) Zero and negative indices

In order to get the meaning of 3^0, you will have to write the index 0 as 3 – 3, 4 – 4, 5 – 5 or 6 – 6. Using 0 = 5 – 5, the statement becomes
$3^5 \div 3^5 = 3^{5-5} = 3^0$

But when you expand the expression $3^5 \div 3^5$ it becomes
$3^5 \div 3^5 = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$
$= 1$

Therefore 3^0 is equal to 1.

Try the same process for 4^0 and 7^0 what do you observe?

You should have noticed that any number raised to the power zero is always equal to one.

In general as a rule we write $a^0 = 1$ where a stands for any number.

(ii) Negative indices.

Sometimes numbers are raised to a negative power like 2^{-3}.

But for you to find the meaning of 2^{-3}, you have to first write the index -3 for example 3 – 6.

You will then write the number as:-
$2^{-3} = 2^{3-6}$
But $2^{3-6} = 2^3 \div 2^6 = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2^3}$

So $2^{-3} = \frac{1}{2^3}$

Try this number 4^{-2} using the same activity above.

What do you observe?

Have you got the same observation like this one that:- for a given counting number raised to a negative index, its value is equal to the reciprocal of that number (the base) raised to the additive inverse of the given index.

In general the expression is $a^{-m} = \frac{1}{a^m}$

Similarly $\frac{1}{a^{-m}} = a^m$ and $(\frac{a}{b})^{-m} = (\frac{b}{a})^m$

Example 1

Evaluate 3^{-5}

Solution.

First find the reciprocal of 3 and the additive inverse of 5.

Hence $3^{-5} = (\frac{1}{3})^5 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

Therefore $3^5 = (\frac{1}{3})^5 = \frac{1}{243}$

Example 2

Simplify $(\frac{1}{4})^{-3}$

Solution:-

Find the reciprocal of $\frac{1}{4}$ and the additive inverse of -3

Hence $(\frac{1}{4})$-3 = 4^3 = 4 x 4 x 4 = 64

Example 3

Evaluate $(\frac{3}{4})$-2

Solution:-

Find the reciprocal of $\frac{3}{4}$ raised to the value of the additive inverse of -2

$(\frac{3}{4})$-2 = $(\frac{4}{3})^2$ = $\frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

$\therefore (\frac{3}{4})$-2 = $\frac{16}{9}$

Activity 16.8
Evaluate the following expression

1. a) 4 -7 b) 5^{-3}
2. a) $(\frac{1}{3})$-2 b) $2 \times 3^2 \times 3^{-2}$
3. $5^n = \frac{1}{625}$

Check your answers with those given at the end of the unit

Fractional indices

Example 4

You know that 36^2 is 36 x 36

36-2 is $\frac{1}{36}{}^2 = \frac{1}{36} \times \frac{1}{36}$ and $36^0 = 1$

What can $36^{\frac{1}{2}}$ mean?

To give a meaning to a number raised to a fractional index, you use the laws of indices.

First you reduce the fractional index to 1
$36^{½} \times 36^{½} = 36^{½+½} = 36^1 = 36$
Since $36^{½} \times 36^{½} = 36$ and also 6 x 6 = 36
So $36^{½} = \sqrt{36} = 6$

Example 5.

Similarly taking $8^{\frac{1}{3}}$
$8^{⅓} \times 8^{⅓} \times 8^{⅓} = 8^{⅓+⅓+⅓}$
$= 8^1$
$= 8$
Since $8^{⅓} \times 8^{⅓} \times 8^{⅓} = 8 = 2 \times 2 \times 2 = 8$

Then it follows that $8^{⅓}$ is equivalent to the cube root of 8 which cam symbolically be expressed as $\sqrt[3]{8}$

Hence $8^{⅓} = \sqrt[3]{8} = 2$

Example 6.

Simplify $8^{\frac{4}{3}} = 8^{(⅓ \times 4)} = (8^{⅓})^4$
Since 8⅓ = $\sqrt[3]{8}$ then $(8^{⅓})^4$ can be expressed as $(\sqrt[3]{8})^4 = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

Therefore $8^{\frac{4}{3}} = 16$.

From the examples above, we note that when a number is raised to a fractional index, the denomination part of the index determines the root of the number, while the numerator of the same fractional index determines the power to which the calculated root should be raised.
Therefore, you can generally conclude that
$a^{\frac{1}{n}} = \sqrt[n]{a}$ which is read as the n^{th} root of a.

Similarly $a^{m/n}$ can be written as $a^{m/n} = (\sqrt[n]{a})^m$, which is read as the n^{th} root of a raised to the power m, where m and n are natural numbers.

Example 7: $(\frac{8}{27})^{-2/3}$

Solution

Step 1: Before simplifying the expression $(\frac{8}{27})^{-2}/_3$, first express the index as positive by getting the reciprocal of $^8/_{27}$ and the additive inverse of $^{-2}/_3$.
The reciprocal of
$^8/_{27} = {}^{27}/_8$ and additive inverse of $^{-2}/_3 = {}^{+2}/_3$

Step 11

Secondly you now expand the index such that the numerator part is reduced to 1.

$(\frac{27}{8})^{2/3} = (\frac{27}{8})^{1/3 \times 2} = [\ (\frac{27}{8})^{1/3}]^2$

$= (\ \frac{\sqrt[3]{27}}{8})^2$

$= (\frac{3}{2})^2$

$= \frac{9}{4}$

Activity 16.9
Evaluate the expressions
1. $25^{1/2}$
2. $64^{1/2}$
3. $27^{1/3}$
4. $16^{1/4}$
5. $16^{-\frac{3}{4}}$
6. $16^{1/4} \times 32^{2/5}$

Check your answers with those given at the end of this unit.

16.7.5 Logarithms and Antilogarithms

a) Logarithms

The study of logarithms is an extension of what you covered on indices. So you will have to keep referring to topic 4 of this unit; this is because inorder to understand the idea of logarithms, we need to see the relationship between exponents (indices) and logarithms.

You know that $10^2 = 100$
$10^3 = 1000$
$10^4 = 1000$

The logarithm of a number to base 10 is the power to which 10 must be raised to give the number. Logarithmic form we write the word logarithm in short **log**. We write it as follows log $10^{100 = 2}$. We read it as the logarithm of 100 to base 10 is two. If the base is b,N the number and P the power we can summarise the two forms as follows:-

Exponential form	**Logarithmic Form**
$b^p = N$	$\log_b N = P$
$5^3 = 125$	$\log_5 125 = 3$
$4^2 = 16$	$\log_4 16 = 2$
$10^2 = 100$	$\log_{10} 100 = 2$
$8^2 = 64$	___________
$4^3 = 64$	___________

Complete changing the numbers in exponential form to logarithmic form

Exponential form	**logarithmic form**
$\mathbf{b^{P=N}}$	**log** $\mathbf{b^{N=P}}$

Using that idea let's see how to find the following numbers.

Example 1

Find the following a) $\text{Log}_7 49$ b) $\text{Log}_3 81$

a) $\text{Log}_7 49 = x$

First equate it to the unknown and change the number to the exponential form.

$\text{Log}_7 49 = x$ becomes $7^x = 49$

Find prime factors for 49
$= 7^2$
$7^x = 7^2$

When dealing with numbers of the same bare you equate the powers.
So x = 2
Therefore $Log_7 49 = 2$

b) $Log_3 81 = x$
   $3^x = 81$
   $3^x = 3^4$
   X = 4

$\therefore Log_3 81 = 4$

Laws of logarithms

Laws No 1

Log of a number to its base is 1
$Log_b\ 10 = 1$

Law No 11

When we are multiplying logarithms we add them.
Log (ab) = Log a + log b

Example 2

Evaluate

a) $Log_{10}{}^{5} + log_{10}{}^{2}$
   $= log_{10}(5 \times 2)$
   $= log_{10}{}^{10}$
   = 1

Law No. III

When we are dividing the logarithms we subtract them.
Log ($^a/_b$) = Log a – log b

Example.

Evaluate $log_7 21$- log $_7 3$
$= Log_7(\frac{21}{3})$
$= Log_7 7$
= 1

Law No. IV
When we have logarithm of a number to the y^{th} power, we multiply the y^{th} power with the logarithm.

Example 3

Solve $8^x = 16$

Solution.

$8^x = 16$
Take logs on both sides.
Log 8^x = log 16
X log 8 = log 16
X = $\frac{\log 16}{\log 8}$ (taking 16 and 8 as powers of 2)
X = $\frac{\log 24}{\log 23}$
X = $\frac{4 \log 2}{3 \log 2}$
X = $\frac{4}{3}$ = 1 ⅓

Great, you have now completed the laws of logarithms. Can you now do this activity?

Activity 16.0

1. Find the following
   a) $\text{Log}_8 16$ b) Log_2(--------------)
2. Complete the following
   a) $\text{Log}_2{}^2 + \log_2 4 = \log_2$ (---------------)
   b) $\text{Log}_2{}^3 + \log_2 5 = \log_2$ (---------------)
   c) $\text{Log}_2 10 + \log_2 5 = \log_2$ (---------------)
   d) $\text{Log}_2 50 + \log_2 5 = \log_2$ (---------------)
   e) $\text{Log}_2 (6 \times 4)$ = $\log_2$ (---------------)+ $\log_2$ (---------------)
   f) $\text{Log}_2 (7.5 \div 6.5)$= $\log_2$ (---------------)- $\log_2$ (---------------)

Check your answers with those given at the end of each unit

(iii) Finding logarithms of numbers.

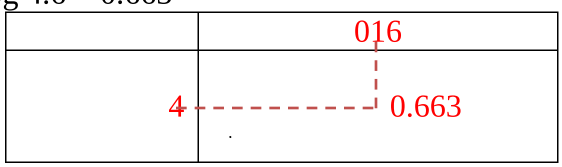
Logarithms of numbers are found from tables of logarithms. These give the logarithms of numbers between 1 and 10, correct to 3 significant figures (3sf). Get a mathematical textbook with tables at the end, get to the "logarithms of numbers" table.

The table below shows the value of common logarithms for numbers between 1 and 10.

Number	**0.0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1.	0.000	0.041	0.079	0.114	0.146	0.176	0.204	0.230	0.255	0.279
2.	0.301	0.322	0.342	0.362	0.380	0.398	0.415	0.431	0.447	0.462
3.	0.477	0.491	0.505	0.519	0.531	0.544	0.556	0.568	0.580	0.591
4.	0.602	0.613	0.623	0.633	0.643	0.653	0.663	0.672	0.681	0.690
5.	0.699	0.708	0.716	0.724	0.732	0.740	0.748	0.756	0.763	0.771
6.	0.778	0.785	0.792	0.799	0.806	0.813	0.822	0.826	0.833	0.839
7.	0.845	0.851	0.857	0.863	0.869	0.875	0.881	0.886	0.892	0.898
8.	0.903	0.908	0.914	0.919	0.924	0.929	0.934	0.939	0.944	0.949
9.	0.954	0.959	0.964	0.968	0.973	0.978	0.982	0.987	0.991	0.996

To find the log of 4.6 from tables for example, we look for 4 on the left and read along to the column under 0.6.

We read that log 4.6 = 0.663

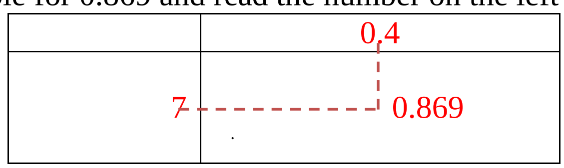


Saying log 4.6 is another way of saying
$10^{0.663} = 4.6$

Example 4

Use the logarithm table above to find
$10^{0.869}$

Look in the table for 0.869 and read the number on the left and the top.



So $0^{0.869} = 7.4$

Activity 16.11

1. Use common logarithm table to find these

   a) Log 3.4
   b) Log 9.6
   c) Log 6.9
   d) $10^{0.919}$
   e) $10^{0.806}$
   f) $10^{0.491}$

Check your answers with those given at the end of the unit

(iv) Multiplication using logarithms

Let us try and work out 2.4 x 3.5 using our logarithm table.

From the table, log 2.4 = 0.380

And log 3.5 = 0.544

This means $2.4 = 10^{0.380}$ and $3.5 = 10^{0.544}$

So $2.4 \times 3.5 = 10^{0.380} \times 10^{0.544}$

Using the law of multiplication of indices
$= 10^{0.380 + 0.544}$
$= 10^{0.924}$

To find out what $10^{0.924}$ is, we look in the table for 0.924

So $10^{0.924} = 8.4$

Therefore we found that 2.4 x 3.5 = 8.4. By using logarithms we have changed the problem from multiplication to addition.

You can now solve the number briefly using this procedure:-

Number	log
2.4	0.380
3.5	0.544
8.4 ←	0.924

(v) Division using logarithms

What is 8.5 ÷ 2.4

We use almost the same procedure as in multiplication using logarithms but taking care of the law of division of logarithms and indices.

Thus 8.5 ÷ 2.4 = log 8.5 ÷ log 2.4

Log 8.5 = 0.929 and log 2.4 = 0.380

$8.5 \div 2.4 = 10^{0.929} \div 10^{0.380}$
$= 10^{0.929\ -.380}$
$= 10^{0.549}$
∴ From the table 0.549 = 3.5
So 8.5 ÷ 2.4 = 3.5

When dividing by a number we change the problem to one where we subtract the logarithm of that number.

Activity 16.12

Work out these numbers using logarithms

1. a) 2.2×3.1 b) 5.8×1.5
2. a) $9.8 \div 3.5$ c) $7.3 \div 1.7$

Check your answers with those given at the end of the unit.

(iv) Logarithms of numbers more than 10

From our table, log 8 = 0.903

So $\log 80 = \log 8 \times 10$
$= \log 8 + \log 10$
$= 0.903 + 1$
$= 1.903$

$\text{Log } 800 = \log 8 \times 100$
$= \log 8 + \log 100$
$= 0.903 + 2$
$= 2.903$

Note:

The number before the decimal point is called the **characteristic** of the logarithm. The number after the decimal point is the **mantissa**.

In the forgone number, 2.903, 2 is the characteristic and .903 the mantissa.
When a number is between 1 and 10, the characteristic of its logarithm is 0. The simplest way to find the characteristic of a larger number is to write it in standard form.

Example 5

$\text{Log } 6400 = \log 6.4 \times 10^3$
$= \log 6.4 + \log 10^3$
$= \log 6.4 + 3 \log 10$
$= 0.806 + 3 \times 1$
$= 0.806 + 3$
$= 3.806$

Example 6

Find log 0.0048

Solution

0.0048 = 4.8 x 10^{-3}
Log 0.0048 = log 4.8 x log 10^{-3}
= 0.681 + $\bar{3}$
= 3.681

So log 0.048 = 3. 681

We read $\bar{3}$.681 as bar three point six eight one.

The characteristic is the same as the power of 10 in the standard form expression of the number. You can now work out products and quotients of larger numbers using logarithms.

It is good practice to obtain a rough estimate of the final result. This will help you to check that the answer is about the right size. To find the estimate, we approximate to one significant figures.

Example 7

Work out using logarithms
86.3 x 1750

Rough estimate
86.3 x 1750 = 90 x 2000 = 180,000

Number	**Standard form**	**Log**
0.37	3.0 x 10^{-1}	$\bar{1}$.477
0.22	2.2 x 10^{-1}	$\bar{1}$.342 +
0.065	10^{-2} x 6.5	← 2.819

Therefore 0.37 x 0.22 = 0.065

Activity 16.13

Evaluate using logarithms:-

1) a) 556 x 937 b) 821 x 124 c) 86.3 x 0.0175
2) a) 63.1 ÷ 8.66 b) 654 ÷ 210
3) given that log 2= 0.301 and log 3 = 0.477 find log 18
4) given that $\log_{10}x$ = 1.756 and $\log_{10}y$ = 3.681, find $\log_{10}(\frac{x}{y})$

check your answers with those at the end of the unit

b) Antilogarithms

Now you know how to find logarithms of numbers. If we want to find out the number whose logarithm is given, then you can use the table of logarithm as a reverse just as in example 2.

For example; find the number whose logarithm is 0.531

Method A

Using the logarithm table in reverse, look for the mantissa 0.531 in the table, it is in row 3 in column 4. Write 3.4. Use the characteristics to determine the size of the number. In this case the characteristic is 0 so you don't move the decimal point.

Write the number as antilog of 0.531 = $3.4 \times 10^0 = 3.4$

Method B

Using the antilogarithm tables, look up for a number corresponding to the mantissa 531. Get a row marked 53. Run along it until you reach column marked 1. Read the number at that place. This wil be 3.4. Use the characteristic is 0 so the number remains as 3.4.

Write the number as: the antilogarithm of 0.531 = $3.4 \times 10^0 = 3.4$ or antilog 0.531 = 3.4

***Note*:**
Both methods work but remember when using logarithm tables you are reversing the process. Also note that the antilogarithms are written as antilog in short.

Activity 16.14

1. Find the antilogarithms of the following

   (a) 0.976
   (b) 1.384
   (c) 2.905
   (d) 4.740
   (e) 2.196

Check your answers with those given at the end of each unit.

Thank you for completing unit 16.

16.8: Unit summary

In this unit you a have been introduced to:-

(i) Properties of numbers as commutative, distributive, associative and identity element.

(ii) Estimation and various ways of approximations were discussed, significant figures as a degree of giving accuracy was handled.
(iii) Indices and their laws were discussed.
(iv) Logarithms and antilogarithms were handled.

16.9 Glossary

Exponent : a number that shows how many times a quantity must be multiplied by itself.
Significant figure: a figure that is considered important in a given number.

16.10 answers to the activities

Activity 16.1

1. a) n = 28
   b) n = 53
   c) n = 12
   d) n = 7

2. Last week 18 + 52 bananas = 70 bananas
   Last year 52 + 18 bananas = 70 bananas
   He planted the same number of banana trees in the two years.

Activity 16.2

1. a) 39 + 46 + 54
   = 39 + (46 + 54)
   = 39 + 100
   = 139

   b) 813 + 87 + 65
   = (813 + 87) + 65
   = 900 + 65
   = 965

2. a) 25 x 4 x 20
   = (25 x 4) x 20
   = 100 x 20
   = 2000

   b) 5 x 45 x 6
   = 5 x (45 x 6)
   = 5 x 270
   = 1,350

Activity 16.3

1) **X X X X X X X**
X X X X X X X
X X X X X X X
X X X X X X X
(1 + 6) x 4

2) a) 74 x 25
= (70 + 4) x 25
= (25 x 70) + (25 x 4)
= 1750 + 100
1850

b) 66 x 22
= (60 + 6)22
= (60 + 22) + (6 + 22)
= 1320 + 132
= 1452

c) 4(51 – 16)
= (4 x 51) – (4 x 16)
= 204 – 64
= 140

d) 5(63 – 61)
= (5 x 63) – (5 x 61)
= 315 – 305
= 10

Activity 16.4

1. a) 200 b) 1000 c) 2000
2. a) 1680 – 2000
b) 490 – 500
c) 62008 – 60,000

Activity 16.5

1. a) 217 x 0.29
~ 200 x 0.3
=

b) 0.96 ÷ 101
~ 1 ÷ 100

$= \frac{1}{100}$

2. (i) a) 409.9 b) 410
   (ii) a) 6.594 b) 6.6

Activity 16.6

1. a) 5.621×10^{-2}
   b) 9.8052×10^{2}

2. $3.6 \times 10^{8} \div 2 \times 10^{3}$
   $\sim\frac{4 \times 10^{8}}{2 \times 10^{3}}$
   $= 2 \times 10^{8-3}$
   $= 2 \times 10^{5}$
   $= 200000$

Activity 16.7

1. $5^{2} \times 5^{4}$
   $= 5^{2+4}$
   $= 5^{7}$

2. $3^{7} \div 3^{5}$
   $= 3^{7-5}$
   $= 3^{2}$

3. $6^{9} \div 6^{1}$
   $= 6^{9-1}$
   $= 6^{8}$

4. $(7^{5})^{2}$
   $= 7^{10}$

5. a) $3^{x} = 18$
   $3^{x} = 3^{4}$
   $x = 4$

   b) $2^{x+1} = 2^{8}$
   $x+1 = 8$
   $x = 8 - 1$
   $x = 7$

Activity 16.8

1. a) $4^{-1} = (\frac{1}{4})^7 = \frac{1}{4^7}$ or $\frac{1}{16{,}384}$

   b) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

2. a $(\frac{1}{3})^{-2} = 3^2 = 9$

   b) $2 \times 3^2 \times 3^{-2} = 2 \times 3^{2+-2} = 2 \times 3^0$

   $= 2 \times 1 = 2$

3. $5^n = \frac{1}{625}$

   $5n = \frac{1}{5^4}$

   $5^n = 5^{-4}$

   $n = -4$

Activity 16.9

1. $25^{1/2} = \sqrt{25} = 5$
2. $64^{1/2} = \sqrt{64} = 8$
3. $27^{1/3} = \sqrt[3]{27} = 3$
4. $16^{1/4} = \sqrt[4]{16} = 2$
5. $16^{-3/4} = \frac{1}{16}^{3/4} = (\frac{\sqrt[4]{1}}{16})^3 = (\frac{1}{2})^3 = \frac{1}{8}$
6. $16^{1/4} \times 32^{2/5} = \sqrt[4]{16} \times (\sqrt[5]{32})^2$

   $= 2 \times 2^2$

   $= 8.$

Activity 16.10

1. $\text{Log}_8 16 = x$

   $8^x = 16$

   $(2^3)^x = 2^4$

   $3x = 4$

   $X = \frac{4}{3}$

   b) $\text{Log}_2 128 = x$

   $2^x = 128$

   $2^x = 2^7$

   $X = 7$

2. a) $\log_2 (2 \times 4) = \log_2 8$

   b) $\log_2 (3 \times 5) = \log_2 15$

   c) $\text{Log}_2 (\frac{10}{5})$

   d) $\text{Log}_2 \frac{50}{5}$

   e) $\text{Log}_2 6 + \text{Log}_2 4$

f) $\log_2 7.5 - \log_2 6.5$

Activity 16.11

a) 0.531
b) 0.982
c) 0.839
d) 8.3
e) 6.4
f) 3.1

Activity 16.12

1 a) 2.2 x 3.1 = log 2.2 x log 3.1

$2.2 \times 3.1 = 10^{0.342} \times 10^{0.491}$
$= 10^{0.341 + 0.491}$
$= 10^{0.832}$
$\therefore 0.832 \sim 6.8$
$2.2 \times 3.1 = 6.8$

) 5.8 x 1.5 = log 5.8 x log1.5
$= 10^{0.763 + 0.176}$
$= 10^{0.939}$
0.939 from the tables = 8.7

Therefore 5.8 x 1.5 = 8.7

Activity 16.11

a) 0.531
b) 0.982
c) 0.839
d) 8.3
e) 6.4
f) 3.1

Activity 16.12

1 a) 2.2 x 3.1 = log 2.2 x log 3.1

$2.2 \times 3.1 = 10^{0.342} \times 10^{0.491}$
$= 10^{0.341 + 0.491}$
$= 10^{0.832}$
$\therefore 0.832 \sim 6.8$
$2.2 \times 3.1 = 6.8$

b) 5.8 x 1.5 = log 5.8 x log 1.5
$= 10^{0.763} \times 10^{0.176}$

$= 10^{0.939}$

0.939 from the tables = 8.7

Therefore 5.8 x 1.5 = 8.7

2 a) 9.8 ÷ 3.5 = log 9.8 ÷ log 3.5

$= 10^{0.991} - 10^{0.544}$

$= 10^{0.991 - 0.544}$

$= 10^{0.447}$

0.447 from the tables = 2.8

Therefore 9.8 ÷3.5 = 2.8

b) 7.3 ÷ 1.7 = log 7.3 ÷ log 1.7

$= 10^{0.863 - 0.230}$

$= 10^{0.633}$

0.633 from the tables = 4.6

Therefore 7.3 ÷ 1.7 = 4.6

Activity 16.13

1 a) 556 x 937

No.	**Standard form**	**Log**
556	5.56×10^2	2.745
937	9.37×10^2	+2.972
5220000	$10^5 \times 52.2$ ←	5.717

Therefore 556 x 937 = 5220000

b) 821 x 124

No.	**Standard form**	**Log**
821	8.21×10^2	2.914
124	1.24×10^1	1.093
102000 ←	$10^4 \times 102$ ←	4.007

Therefore 821 x 124 = 102000

c) 86.3 x 0.0175

No.	**Standard form**	**Log**
86.3	8.63×10^1	1.936
0.0175	1.75×10^{-2}	+2.243
15.1	$10^0 \times 15.1$	0.179

Therefore 86.3 x 0.0175 = 15.1

2 a) 63.1 ÷ 8.66

No.	**Standard form**	**Log**

63.1	6.31×10^1	1.800
8.66	8.66×10^0	-0.978
	6.6 ←	0.822

Therefore 63.1 ÷ 8.66 = 6.6

b) 654 ÷ 210

No.	**Standard form**	**Log**
654	6.54×10^2	2.816
210	2.1×10^2	2.322
	3.1 ←	0.499

Therefore 6.54 ÷ 210 = 3.1

3. Log 18 = log 2 x 3^2

= log2 + 2 log 3

= 0.301 + 2(0.477)

= 0.301 + 0.954

= 1.255

4. $\text{Log}_{10}\frac{x}{y} = \log_{10}x - \log_{10}y$

= 1.756

3.681 –

2.075

Activity 16.14

a) Antilog 9.4
b) Antilog 24
c) Antilog 810
d) Antilog 55000
e) Antilog 0.016

16.11 End of unit exercise

This assignment is intended to help you consolidate what you have learnt in this unit.

1. Find a rough estimate for
   a) 479 x 0.0032
   b) 0.0634 ÷ 26.1

2. Workout these numbers leaving your answers in standard form.

   a) $5 \times 10^2 \times 4 \times 10^6$
   b) $7 \times 10^3 \times 3 \times 10^2$

3. Find the value of k in

   $3^{k+2} = 243$

4. Log 4 = 0.602, find Log 40

5. Use logarithms to carry out the following computations

   a) 2468 x 1357
   b) 1240 ÷ 278

16.12 Self check/assessment

Learning outcome	Not sure	Satisfactory
1. I am able to carry out computations using concepts of estimation indices and logarithms 2. I can teach more effectively mathematical operations on numbers using quick methods of calculations		

16 .13: Reference for further reading

1. Secondary school mathematics Book 2
2. Cliff Green (2000) Mathematics Revision and Practice for UCE, Ox.
3. School Mathematics of East Africa Book 1, Cambridge.

4. School Mathematics of East Africa Book 2; Cambridge.

UNIT 17: BUSINESS MATHEMATICS

17.1 INTRODUCTION:

Dear student, you are most welcome to unit 17.

This unit introduces you to business mathematics which is part of your life. Through this unit you will be introduced to basic knowledge and skills like finding percentages, quantities and costs, currency exchange , profit, loss, commission and discount; simple and compound interest; hire purchase and mortgage and types of taxes.

17.2 content organization

Hello student, in this unit, you are going to cover the following topics as shown below:-

Topic	Subtopic
1. Percentages	(a) Introduction to percentage (b) Fractions to percentages and vice – versa (c) Decimals to percentage and vice versa (d) Percentage increase and decrease.
2. Application of percentage	(a) Finding original and prices (cost price) and selling price (new price) (b) Profit/gain, loss, and commission. (c) Simple interest and compound interest. (d) Post price saving bank (e) Appreciation and depreciation
3. Currency exchange	(a) Introduction to currency exchange (b) Conversion of local currencies to others and vice versa.
4. Taxation and insurance	(a) Hire purchase (b) Income tax (c) Insurance (d) Others
5. teaching business mathematics in primary schools	(a) Activities and instructional materials for teaching business in mathematics (b) planning for a demonstration less of teaching any of subtopics under business mathematics

17.3 learning outcomes

Pay the end of the unit, you are expected to:

(i) Apply the acquired knowledge and skills of business mathematics in transacting day to day business.
(ii) Have ability to teach business mathematics in primary schools.

17.4 Competences:

Dear student, now you have learnt the learning outcomes of this unit therefore, as you study this unit you are required to;

(i) Define the term "percentage".
(ii) Convert percentages to ordinary fractions and decimal fractions and vice versa.
(iii) Express different quantities in percentage form.
(iv) Find percentage increase and decrease.
(v) Solve problems involving quantities sales and costs.
(vi) Profit, loss, discount compute and commission.
(vii) Compute simple and compound interest.
(viii) Convert local currencies to other currencies and vice versa.
(ix) Calculate different taxes used in business e.g. payment electricity, water bills, etc

Use the acquired, knowledge and skills to teach business mathematics in primary schools.

17.5 unit orientation

This unit has been designed to provide you with the basic knowledge and skills in business mathematics. This is meant to prepare you to be able to teach the basic concepts in this unit in primary schools without finding and difficulty secondly it will also enable you transact any business your daily life situation.

17.6 Study requirements

Hullo student in order to understand this unit very well, before you study it, you are required to:

(i) Visit the college library or the nearby coordinating centre and identify which of the reference books or text books having information about business mathematics for instance

- Heinemann Edexcel, GCSE Mathematics course page 228
- Karugaba J. UCE Study – book mathematics, a complete revision course pages 48-58
- A new MK, Primary Mathematics 2000 pupil's book 7 pages 119 – 164.
- Channon J.B, New General Mathematics book 2 pages 48-56 and others.

(ii) Acquaint yourself by visiting the nearest bank, post office bank, forex bureau and revenue tax offices to learn about some of taxes and their respective taxes and rates.

(iii) Revise thoroughly unit II on measures (money) and read about foreign exchange rates.

(iv) Get a variety of news papers to become well versed with current exchange rates.

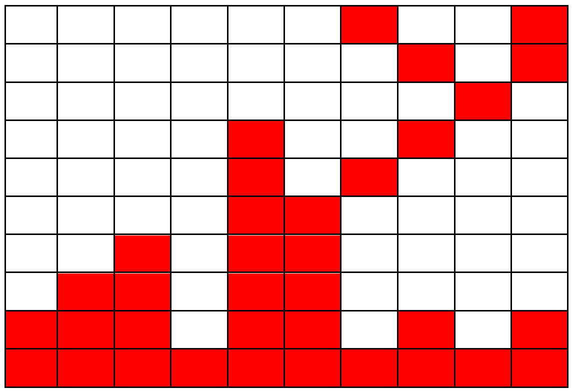
17.7 Content

17.7.1 Percentage

(a) Introduction to percentage

Dear student, what is the term “percentage”?

Let you tell the meaning of percentage by first looking at this large square below:-



This large square has been divided into 100 equal small squares.

With your colleague discuss the following questions.

(i) How many small squares have been shaded?
(ii) What fraction of small shaded squares are shaded out of the large square?
(iii) How else can you express the fraction of the shaded part?

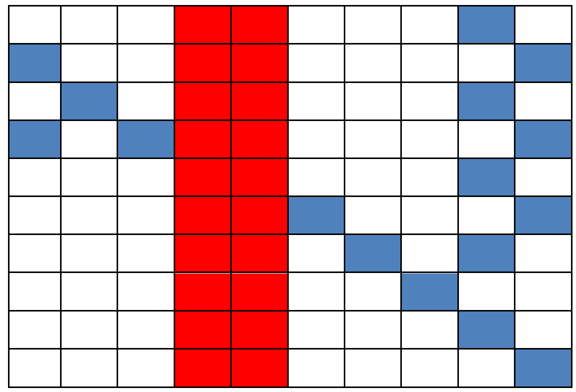
Let you compare your answers with the text below:

(i) There are 35 small squares shaded.
(ii) These make 35 out of 100 small squares shaded this expressed $\frac{35}{100}$

So it can be read as 35 percent. The term “percent” means per hundred $\therefore \frac{35}{100}$ can be read/ written as 35%.

Hence, number of parts per hundred is called percentage.
Percentage is symbolically written as % or PC.

Let you study this diagram below and the answer following question:



(a) What percentage of large square is shaded (i) Red? (ii) Blue?

(b) What percentage of the large square is unshaded?

Share your findings with you colleague.

I hope now, you can explain clearly what do we mean by the word percentage, “Well you can rest for short time”

(b) Percentage as a fraction and vice versa

(i) Percentage as a fraction

Study example below:-

Example 1. Write :

50% means $\frac{50}{100}$

Thus 50% = $\frac{50}{100}$

$\frac{\overset{1}{\cancel{50}}}{\cancel{100}}$ = ½

∴ 50% = ½

Can you do the following problems in your exercise book:

Activity 17.1

(a) Write the following as a fraction

(i) 20% (ii) 75% (iii) 2 ½ %

(b) Express 45% as a fraction

May you compare your answers with a colleague.

I hope it is becoming more interesting?

Let you continue looking at other areas

(ii) Changing fractions to percentage.

As you remember, in the introduction of this unit it was mentioned that when a number e.g. 35 is out 100 is written $\frac{35}{100}$ = 35%

Let you find out more.

Therefore study example 2:-

Express ¾ as a percentage:

$\frac{3}{4} = \frac{3}{4} \times 100\%$

$= \frac{3 \times 100}{4}$

$\therefore \frac{3}{4} = 75\%$

Example 3.

A boy get 23 out of 25 marks for a test what is as a percentage?

Form a fraction

23 out of 25 = $\frac{23}{25}$

$\therefore \frac{23}{25} = \frac{23}{25} \times 100\%$

$= \frac{23 \times 100\ \%}{25}$

= 92%

May use the above shown examples to the following activity in your exercise book.

Activity 17.3

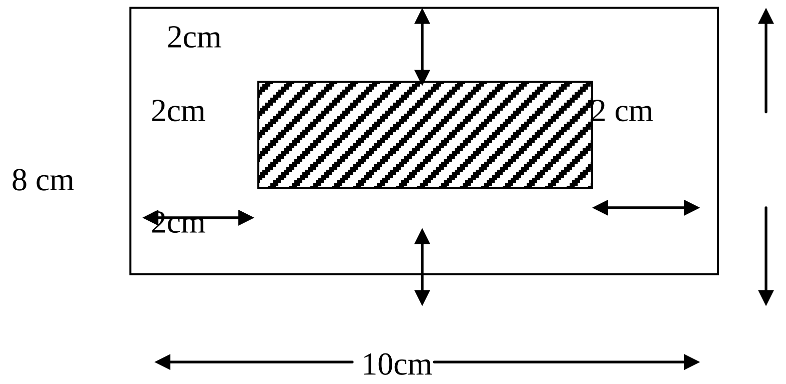
(a) Express these fractions as percentages

(i) $^{4}/_{5}$ (ii) $^{5}/_{8}$ (iii) ⅓ (iv) ⅔

(b) What percentage of the total area is shaded?

(c) Write the following as fractions:

(i) 4% (ii) 55% (iii) 22 ½ % (iv) 145%



(c) Write the following as fractions:

(i) 4% (ii) 55% (iii) 22 ½ (iv) 145%

Can you compare your answers with those ones at the end of this unit.

That is a good start.

You are now welcome the next subtopic:

Expressing decimals to percentage and vice versa

(c) Expressing decimals to percentages and vice versa

(i) Decimals to percentages.

Let you find out, how to change decimal numbers to fractions.

Look at the following examples.

Example 1:

Express 0.08 as a percentage.

Method 1

Change the decimal number to a fraction

$= \frac{08}{100}$

Then $\frac{08}{100}$ x 100%

∴ 0.8 = 80%

Method 2

0.8 = 0.8 x 100
= 080.0
∴ 0.8 = 80%

Multiplying the decimal number by 100

Can you try this problem on own?

Example 2:

Write 0.001 as a percentage. Share your findings with your tutor?

But I hope the answer 0.1%

Let you look at the following:-

(ii) Changing percentages to decimal numbers.

Example 3:

Write 25% as a decimal number

$25\% = \frac{25}{100}$

$= 0.25$

$7\% = \frac{7}{100}$

$$\begin{array}{r|l} & \underline{0.07} \\ = 100 & 700 \\ & \underline{700} \\ & \ldots \end{array}$$

∴ 7% = 0.07.

Use the above demonstrated examples and do the following activity in your book

Activity 17.4

(a) Convert the following to percentages.

(i) 0.03 (ii) 0.84 * (iii) 1.78 (iv) 2.5 * (v) 1

(b) Express the following to decimal numbers

(i) 40%	(ii) 2%	(iii) 72%	(iv) 175 %	* (v) 2 ½	*(vi) 4. 25%

(c) Express 4:5 as a percentage.

Compare your answers with group mates.

What did you notice with problems marked with *(stars).

Share your findings with the tutor.

Then, compare your answers the end of the unit.

Brave, you are doing well.

Let us move to another interesting subtopic:

(d) Percentage increase and decrease

Quantities in percentage.

You are most welcome this sub-topic, now

(i) Finding percentage of quantities:

Given example 1:

Find 10% of 36kg of maize

= 10% x 36kg

$= \frac{10}{100} x\ 36 = \frac{\cancel{3}60}{\cancel{1}00}$

= 3.6kg

Example 2.

What is 2 ½ of Shs. 30,000/=

= 2 ½% of 30,000

$= \frac{(2\frac{1}{2})}{100} \times 30000$

$= \frac{(\frac{5}{2})}{100} \times 30000/=$

$= (\frac{5}{2} \div \frac{100}{1}) \times 30000$

$= \frac{5}{2} \times \frac{1}{\cancel{100}} \times 300\cancel{00}$

= Shs. 750/=

With your colleague try the following on a piece of paper.

(a) Find 8% of 4500/=
(b) What is 15% of Shs. 72000/=
(c) Find 5% 0f 9700.
(d) Okurutu had 210 oranges in his basket and then sold 40% of them. How many oranges did he sell?
(e) What is 25% of 120 km?

May you share your answers with your tutor.

Let you look at another area much related to what you have been reading about:

(ii) Expressing one quantity as a percentage of another.

With your colleague study the following examples.

Example 1

Eriwala planted 50 beans. Only 25 came up. What percentage is that?

Step 1 Proportion of beans = $\frac{25}{50}$

Form a fraction

Step II $\therefore$ The percentage of beans came up = $\frac{25}{50} \times 100\%$

$= \frac{2500}{50}$

$= 50\%$

Let you discuss example 2:

There were 40 litres of milk in a big jerycan. Some milk was sold and now there are 35 litres of milk left. What percentage of the whole jerycan was that sold?

Step 1: Find the number of litres sold : 40 – 35

= 5

Step II: Find the fraction of milk sold = $\frac{5}{40}$

Step III: ∴ The percentage of milk sold $= \frac{5}{40} \times 100\%$

$= \frac{500}{40}$

= 12 ½ % or 12.5

Let you do the following in your exercise book.

Activity 17.5

a) In a basket of 80 oranges 12 are bad. What percentage is bad?
b) Between 6.00am and 12.00 noon a radio station broadcasts “News in “English” twice. Each broadcast lasts 15 minutes. What percentage of the total time is spent on “news in English”?
c) Peter had 7 letters to post; 5 to addresses in Uganda and 2 to foreign countries. What percentages were to foreign countries?
d) What is 12% of 40000/=?
e) A school bought 450 kg of rice at the beginning of the term if 72% was used. How many kg were used?
f) Three people A,B and C shared a certain amount of money.
   A got 60,000/=; B got 90,000/= and C got 40% of the money.

   (i) How much money was shared altogether?
   (ii) What percentage of the whole amount did A get?

Check for the correct answers at the end of the unit.

You are encouraged to have more work at home:

Do exercise 10:10 numbers; 1,2,5,10 (a,k,0). From A new MK Primary Mathematics 2000 Pupils book 7 page 124 – 125 and exercise 5b from Channon, New General Mathematics book 2, page 50- 56.

If you have finished may you thank each other.

(i) Percentage increase

Dear student, is it possible to have a percentage greater than 100%?

Can you share it with a colleague?

Then, let you study the table below:

This table shows the number of students in Kaliro PTC Year 1 from 1975 to 1978:

YEAR	NUMBER OF STUDENTS
1975	80
1976	120
1977	160
1978	240

Step 1: Look at 1975 and 1976

Finding the difference

1976	120
1975	-80
Increase =	40

The increase (40) is half the 1975 number (80) or the increase is $\frac{40}{80}$ x 100% = 50% of 1975 number

Step II: Then, look at 1975 and 1977

Finding the difference

1977	160
1975	- 80
Increase =	80

The increase (80) is the same as the 1975 number (80).
Then, the increase is $\frac{80}{80}$ x 100% = 100% of the 1975 number.

Step III: Finally, look at 1975 and 1978.

Finding the difference

1978	240
1975	- 80
Increase =	160

The increase (160) is twice the 1975 number (80)

Then, the increase is 200% of the 1978 number

∴ it is possible to have a percentage greater than 100%

Using the above information can you now look at percentage increase critically:

Example 1

The price of a plate is 6000. What will the new price be, if there is an increase of 5%?

Method

The increase in % = 5%

Then, the new price in % = (100% + 5%) of the old price

= (105%)

∴ The new price $= \frac{105}{100} \times 6000$

= Shs. 6300/=

Method

% increase = 5%

New price in % = 100% + 5%

= 105%

If the old price of plate in % = 100%
But, 100% = 6000/=
100% = 6000/=

$\frac{100\%}{100\%} = \frac{6000}{100}$

1% = 60/=

∴ The new price of plate = 105% x 60/=

= 105 x 60

= shs. 6300/=

Example 1.

Find the new price as percentage of the old price if there is an increase of 20%.

The old price is always = 100%

∴ The new price = 100% + 20%

= 120%

Before you do the following activity can you discuss the two examples with your colleague.

Now, using the above examples can you have more practice:

Activity 17.6

(a) Express the new price as a percentage of the old price if:

(i) There is an increase of 10%.
(ii) There is an increase of 40%.
(iii) There is an increase of 25%

(b) The price of the chair is increased by 10%. What is the new price, if then old price was Shs. 50,000/=

(c) School fees are increased by 7½%. What will the new fees be if the old fees were shs. 300,000/=

A man earned Shs. 150,000/= per month and was the given a 15% increase. What was his new wage?

Compare your answers at the end of the unit.

It has been quite challenging but well done.

To have mastery of the above work, you need to do more problems.
Let you do more work from A New Uganda Primary Mathematics Pupils Book 6 page 93-94 exercise 94.

Can you share your answers with classmates?

Let you look at example under:-

(i) Percentage decrease:

Example 1

Express the new price as a percentage of the old price if there is a decrease of 10%

Old price in % = 100%
Decrease % = 10%

∴ The new % after the decrease = 100% - 10%
= 90%

Are you ok?

Let you study

Example 2:

The price of a shirt is shs. 12000/=. What will the new price be, if there is a decrease of 25%?

Method

The old price in % = 100%

The decrease in % = 25%

Then, the new price in % = 100% - 25%

= 75%

∴ The new price of a shirt = 4000 x $\frac{75}{100}$

= shs. 9000

Method

Old price in percentage = 100%

Decrease % = 25%
The new price % = 100% - 25%
= 75%

Since 100% = 12000/=

$$\frac{100\%}{100} = \frac{12000}{100}$$

1% = 120/=

∴ The new price = 75% = 120 x 75
= 9000/=

May you try this activity in your exercise book.

Activity 17.7

Using the two examples demonstrated do the following problems.

(a) Express the new price as a percentage of the old if there is decrease of 33%.
(b) What is the new price, if there decrease of 40%?
(c) Find the new price, if there is a reduction of 22.5%.
(d) The price of a shirt was reduced from shs. 10,000/= to sh. 8500/= find the percentage decease.
(e) Prices fall by 8%. What is the new price if the old price was 6400/=?
(f) Decrease 80kg by 5%.
(g) Rice rises in price by 10%. If the old price was shs. 1000/=, what was the new price?

Please may check for correct answers at the end of the unit.

You are encouraged to continue revising the same problems with your colleagues.

Let you look at more challenging problems:-

Study:

Example 3:

If the price of a ruler decreased from shs. 500/= to shs. 450/=. What was the percentage decrease?

Method 1

Amount of money subtracted old price – new price

= 500/= - 450/=
= Shs. 50/=

∴ The percentage decrease = $\frac{50}{500}$ x 100
= 10%

Method 2

You can also use: Old price in % = 100%
Old price = 500/=
Now price = 450/=
Decrease in money = 500/= - 450/=
= 50/=

Hence, 500/= = 100%
1/= $\frac{100}{500}$

∴ 50/= $\frac{100}{500}$ x 50

= 10%

Dear student, you continue practicing solving problems using those above examples.

Please go to your library do the following exercise 5c numbers: 17, 21, 25, 31, 33 Channon J.B New General Mathematics book 2 page 55-56.

Submit your work to your tutor.

You are doing well.

Congratulation of finishing 17.7.1

In conclusion for increase is adding on the original/old price/number and for decrease we are subtracting from the original price/number. The words on enlisted below can give the same meaning as increase or decrease.

Increase (gain, profit, rise, raise, higher, upwards) etc and

Decrease (loss, fall, discount, reduction, deduction, lower) etc

Topic 17.7.2 application of percentages

(a) Finding original and new price (selling price) if the percentage gain or loss is known.

(i) Finding New Price (selling price)

Example 1: The cost price of 50kg sugar is shs. 110,000/=. At what price must he sell 50 kilograms in order to make a profit of 20%?

Cost price/original price in % = 100%
Selling price/new price in % = 100% + 20% = 120%
But cost price/original price = 110,000/=

$\therefore$ The selling of sugar = $\frac{120}{100}$ x 110,000

= shs. 132,000/=

Using the learnt example may you do the following activity:-

Activity 17.8

(a) The cost price of bag of rice is shs. 80,000. A what price must I sell the bag of rice in order to get a profit of 30%?

(b) A man bought a 72kg sack of Irish potatoes at shs. 40.000. He later sold the potatoes in kg making a loss of 10%.

(i) At what price was he selling each kg?
(ii) At what price did he sell the whole sack?
(iii) What would be his profit percentage if he had sold the whole sack at sh 45,000/=?

(c) Norah bought a goat at shs. 30,000/=, she sold it at a loss of 25%. How much did sell the goat?

Compare your answers with the ones at the end of the unit.

Well done! May you take a walk.

Welcome back from a walk.

Let you embark on the following subtopic.

(ii) Finding the original price (cost price)

Let you study the examples below.

Example 1

A man sells a motorcycle for shs. 7,200,000 and makes a gain/profit of 20%. What did he pay for the motorcycle?

Use method 1

Let the cost price of a motorcycle be x.
Then, his selling price in % = 100% + 20%
= 120%

Form an equation using two selling

The selling price of motorcycle = $\frac{120}{100}$ x x

But selling price $\frac{120}{100}$ x = 7,200,000

∴ The cost price of the motorcycle.

Solving the

$\frac{120x}{100}$ x 10 = 72000 x 10

$\frac{12x}{12} = \frac{7200000 \ x \ 10}{12}$

X = 6,000,000/=

Alternatively you can derive to the

Method 2: Original % = 100%
Selling price % = 100% + 20%
= 120%
But selling price = 7,200,000/=

Then 120% = 7,200,000/=

1% = $\frac{7,200,000}{120}$

∴ original price = $\frac{7,200,000}{120}$ x 100

= Shs. 6,000,000/=

You are free to study the two (2) methods and choose a better.

Look at example 2:-

A shop window has this notice “all prices have been reduced by 25%”. What was the original price if these are the new prices?

Method 1

Reduced % = 25%
Selling price in % = 100% - 25%
= 75%

Let the old price be = x
Then, selling price $\frac{75}{100}$ x x

Hence selling price of Teapot = shs. 6000

The original price $\frac{75x}{100}$ = 6000

$$\frac{75x}{100} \text{ x } 100 = 6000\text{x}100$$

$$\frac{75x}{75} = \frac{6000 \ x \ 100}{75}$$

X = 8000/=

∴ The cost price teapot was 8000/=

Method 2

Reduced % = 25%
Selling price in % = 100% - 25%
But selling price 75% = 6000/=
Then 1% = $\frac{6000}{75}$

∴ Original price 100% $\frac{6000}{75}$ x 100

= 8000/=

With your group mates do the following activity?

Activity 17.9

(a) A man now gets shs. 315,000 per year. This is 5% higher than he got last year. How much did he get last year?

(b) I bought a blanket and was given 20% discount . I paid sh 40000 what would be the price have been without discount?

(c) My grocer gives discount of 5% for cash .i pay him sh 104500/=.
What was the bill before discount was take off?

D] The new cost of oil will be 3000/= per litre . This is an increase of 7.2% what was old price per litre?

May you compare your answers with those at the end of the unit.

Let you have more work as an assignment at your time

Use: A New MK Primary Mathematics 2000
Pupils book 7 page 140-141
Exercise 11.3 Numbers 1-10

Thank you for trying all the work you can now relax

Profit, lose discount and commission

Your welcome to topic two [2]

Using the knowledge acquired from topic 17.7.1 you can comfortably study this topic.

Let you read about profit

Finding the percentage profit /gain or loss .

Study the following examples:-

Example1

A man bought a book for sh 100/= and sold it for sh 150/=.What was his percentage gain? Finding the money gained [profit] =New –old price

Gain =150-100

=50/=

Then, find the gain as a fraction $= \frac{gain/profit}{old\ price[orginal\ price)}$

$\therefore\ the\ percentage\ gain = \frac{50}{100}$ x 100%

= 50%

Example 2:

What is the percentage loss on a car which was bought for 6,500,000/= and sold 3,900,000/=.

Step 1: Find the amount of money list = old price of the car – new price/selling price of a car

= 6,500,000 – 3,900,000

= shs 2,600,000/=

Step 2: find the loss as a fraction:

$= \frac{loss}{original\ price} = \frac{2600000}{6500000}$

$\therefore\ the\ percentage\ loss\ \frac{2600000}{6500000}$ x 100%

= 25%

Let you practice in your exercise books following the above examples:

Activity 17.10

(a) A labourer's wage rose from 75000/= to 120,000/=. What was the percentage rise?

(b) Joshua bought eggs for sh. 250/= each. What percentage gain was this?

(c) Florence began work at €290 per year. After 4 years her salary had risen to €348. What percentage rise was this?

(d) This is an advertisement from a newspaper.

New mode just arrived
200cc
Motorcycles
New price shs 3,650,000/=
Old price shs 3,750000/=

What is the percentage fall in price?

(e) Grand clearance sale.

Item	**usual price per metre**	**sale price per metre**
Empress cotton	sh. 6000/=	shs 3500/=
Royal cotton	sh. 9000/=	shs. 5500/=
Mosquito net	sh. 5000/=	Shs. 3000/=

(i) What is the percentage reduction?
(iii)On which one is there greater reduction?

(f) Study the table and complete it,

Old price (original price)	**New price (selling price)**	**% gain/loss**
(i) £ 120	£ 120	
(ii) sh. 72000/=	-------------------	33 ⅓% less
(iii) -------------------	Shs. 499500	10% loss

May you discuss with the whole class then check at end of the unit.

Before you move to another sub-topic individually try this exercise at home/during prep time:

Exercise 9 numbers 1,3,5,7,9,11,13 and 15

Exercise 11.1 2, 4, 6, 8, and 7

All these work get from A New MK Primary Mathematics 2000 pages 136 – 141

Submit your work to the tutor.

Thank you good work so far covered.

You can read this text below:-

(iii) Discount

Dear student may you read the text below:

Discount is realized when a trader sells an article at a price less than the marked price. The amount a trader deducts is called **discount**. The price written on an article called the **<u>marked price</u>**.

Let you look the following examples to understand above text better.

Example 1:

The marked price of a dress is shs 20000. Mary paid shs. 16000/=

(i) How much was given as a discount?

(ii) What was the discount percentage?

(i) The discount = old price – new price

= 20000 - 16000

∴ discount = shs 4000

(ii) Using the answer in part (i)

Discount % = $\frac{discount}{old\ price}$ x 100%

Hence = $\frac{4000}{20000}$ x 100%

∴ Discount % = 20%

Now let you copy in your exercise book. Then study example 2:-

Tibatemwa found a marked bicycle price as 60,000/=. Then was given a discount of 15% for cash. How much money did he pay?

Method 1

Old price of price % = 100%

Discount given = 15%

New price paid in % = 100% - 15%

= 85%

$\therefore$ Tibatemwa paid = $\frac{85}{100}$ x 60,000

= sh. 51,000/=

Method 2

Discount as a % = 15%

$\therefore$ Discount as money = $\frac{15}{100}$ x 60000

= shs. 9000/=

$\therefore$ Tibatemwa paid = (60000 - 9000)

= Shs. 51,000/=

Can you come with other methods, well now you do the following activites:-

Activity 17.11

(a) A shopkeeper gives 5% of his sales for one day to the boy scouts. How much does he give, his sales are sh. 5000?

(b) A shop gives 10% discount on cash sales. What is the discount on sh. 23000/=?

(c) A book costs shs. 1050, it is sent away by post and the postage costs shs. 2000/=. What percentage of the total cost is the cost of the postage?

(d) A set of chairs is marked shs. 250000 if a customer is given a 20% discount,

(i) How much does the customer pay?

(ii) How much was the discount?

Share your answers in a plenary and copy the correction in your exercise book.

Well done you can now apply the acquired skills in your simple business.

However, may you discuss the following exercise with your colleague over the weekend:

NCDC, secondary school mathematics students' Book 3. Page 139-140 exercise 10A numbers 6 and 7.

Dear student let you discuss the new subtopic

Commission:

What do you think is a "commission"?

Share it with your colleague;

Let you compare your findings with the text below:-

Some people sell things for other people. Such people are called "**commission agents**" or **salesmen**. These are paid for that service. This payment is called a "**commission**"

Let you study the examples below:

Example 1.

Vuni was paid a salary of shs 10000 plus a commission of 20% of the value of goods he sold worth shs. 750000. How much money did he earn altogether?

Vuni's salary = shs 10,000
His commission = $\frac{20}{100}$ x 750000

= 20 x 7500
= shs. 150,000/=

∴ The total money earned = 10000 + 150000
= shs. 160,000/=

Example 2:

Yoro was given a commission of 6% of his sales. How much commission did he earn, if he sold 50 padlock at shs. 3500 each?

Total sales = 50 x 3500/=
= shs. 175,000/=

∴ The commission Yoro got = $\frac{6}{100}$ x 175000

= 6 x 1750

= shs. 10500/=

Example 3.

Birungi was receiving a wage of Shs. 21,000 a week and a commission of 8% on the goods she sold AF the end of the week she was given a total package of shs. 28000. Calculate the value of the goods she sold.

Method 1:

Birungi's wage = shs. 21,000
Total earning = shs. 28000 - 21000
= Shs 7000

Commission as % = 8%

Hence 8% = 7000/=

Then, 1% = $\frac{7000}{8}$ x 100

∴ 100% of the value = $\frac{7000}{8}$ x 100

= shs. 87,500

Now, dear student can you try the following activity in your exercise book.

Activity 17.12

(a) A Saleswoman is paid a commission of 3% of her sales. Calculate her commission if she sold goods worth shs. 24000.

(b) For every 50 copies of handwriting books sold by Leo, gets money worth 5 copies.

(i) Find the commission he gets after selling 6000 copies at 4000 each
(ii) Calculate his percentage commission

(c) Maliki employs 12 salesmen at 12% commission, if they sell goods worth shs. 120,000. How much does each salesman get?

(d) Get A New MK Primary Mathematics Pupils Book 7 page 147 – 149, Exercise 11.7 Numbers: 4,5,9,16

In plenary, this your answer, let the tutor look at your final work.

I hope now you can make a business man/woman.

Congratulations!

(c) Simple and compound interest

Let you begins by looking subtopic.

(i) Simple interest

Dear student,

What is simple interest?

Let you think about it.

Then, read the text below.

Here is a story a man wants to buy a car. The car costs shs. 6750000. The man finds that he has only sh 5750000. He therefore decides to borrow the extra sh. 1,000,000 from from a bank. The bank gives him the money, but he says that he must pay 5% interest per year. This means that for each year which he keeps the sh. 1,000,000 he must pay shs. 50000. Therefore, if he keeps it for 2 years, he must pay on 50,000 x 2 = shs. 100,000. The sum of money which he borrows is called the principal (p). In this case the principal is sh. 1000000. The extra sum of money which he has to pay the bank per you're called the simple interest (I). in case the interest is shs. 50000. The total sum of money which has to pay back to the Bank is called the Amount (A). NB. Interest is always expressed in percentage know as the rate (R%). Time (T) is the period which the money borrowed.

Now, dear student using the above read information study the following examples:-

Example 1:

How much interest must a man pay on a loan of shs. 6300000 borrowed for 5 years at 3 ½ per year ?

The interest on sh 100 for a year at 3⅓%

Shs. 3 ⅓ = $\frac{10}{3}$

∴ The interest on shs. 6300000 = $\frac{10}{3}$ x $\frac{6300000 \times 5}{100}$

Interest = shs. 1,050,000

Get a piece of paper and do this problem. Find the simple interest on £ 600 for 4 years at 4% per annum.

Compare your working with this example.

100 in 1 year earns = 4%

Then, £ 600 in 4 years earns = £ 4 x 6 x 4
= £96

∴ The simple interest is €96

Sample 2

Calculate the simple interest on shs. 32000 for 8 months at a simple rate of 12%.

Before you find the answer, remember. Time must be considered in years.

Therefore, 1 year = 12 month.

8 months = $\frac{8}{12}$ of a year

∴ Simple interest = 32000 x $\frac{8}{12}$ x $\frac{12}{100}$

$= \frac{320000 \ x \ 8 \ x \ 12}{12 \ x \ 100}$

= shs 2,560

Let you look at example 3:-

Bongo deposited sh. 50,000 for 3 ½ years at a simple rate of 2% per year. Find his simple interest and the total amount.

(i) Principal = 50000/=
Time = 3 ½
Interest = 2%

∴ Simple interest = 50000 x 3 ½ x2%
= 50000 x $\frac{7}{2}$ x $\frac{2}{100}$

$= \frac{50000 \ x \ 7 \ x \ 2}{2 \ x \ 100}$

= shs. 3500

(ii)

Method 1

Using the answer in (i)

Total amount = 50000 + 3500
= shs. 53500

Method 2

Total amount = (50000 x 3 ½ x 2%) + 50000
= (50000 x $\frac{7}{2}$ x $\frac{2}{100}$) + 50000

= ($\frac{50000 \ x \ 7 \ x \ 2}{2 \ x \ 100}$) + 50000

= 3500 + 50000
= Shs. 53500/=

Look at all the examples ths 1,2 and 3 what have you observed?

Let you compare your answer with text below:-

From the examples already done it has been observed that the simple interest is found by multiplying the principal by the Rate percent and by the number of time in years, and dividing the product by 100. Therefore if the symbol P is used for the principal, T for the number of years, R for the rate percent and I for the interest.

Then I = $\frac{PTR}{100}$

Hence if numerical values are given for any three of the letters the value of the fourth may be found.

Since $\frac{PRT}{100}$ = 1

PTR = 100 x 1

∴ P = $\frac{1001}{TR}$ or T = $\frac{1001}{PR}$ or R = $\frac{1001}{PT}$

Using the examples and information read. Let you try the following work.

Activity 17.13

(a) What is the interest on €600 borrowed for 3 years at 4% per year?
(b) Find the interest on Shs 550000 for 4 years at 2 ½% per year.
(c) Fin d the missing numbers in this table below:

Principal	**Interest**	**Am unt**
(i) Shs. 500000	5000	-------------
(ii) ------------------	£35	£750
(iii)Shs. 215000	------------------	Shs. 240000

(d) A man decides to buy a car. He has Shs. 5,000,000 of his own at 3% per year over 4 years.

(i) What must he pay back altogether?
(ii) What must he pay each month over 4 years?

Can you compare your answers at the end of the unit.
Tics Book pages 58

Brave ! Bravo!

Can you have more practice finding simple interest by doing exercises 6c, 6e and 6f from Channon New General Mathematics book pages 58 – 68. Or

Let you study above the next subtopic

(ii) Compound Interest

You study the following examples:

Example 1:

Calculate the compound interest on shs. 60000/= for 2 years at 10% per annum.

Step 1: Find the simple interest at end 1^{st} year.

$$= \frac{P \times R \times T}{100}$$

$$= \frac{600000 \times 10 \times 1}{100}$$

= Sh 6000.

Step 2: New amount in the bank at the beginning 2^{nd} year = P + I

= 60000 + 6000 per annum

= shs. 66000

Step 3: ∴ The interest for the second year

$$= \frac{P \times R \times T}{100}$$

$$= \frac{66000 \times 10 \times 1}{100}$$

= shs 6600

Dear student looking at the above example what have you noticed?

Let you compare you findings with this information below.

It means that the depositor will get an interest of shs. 6000 for the first year and shs 6600 for the second year, totaling to an interest of (6000 + 6600) = Shs. 12600 for 2 years.

The interest calculated for the first year also starts earning interest. This type of interest where interest is earned on the previous principal plus the interest is called compound interest.

The compound interest is usually much higher than the usual simple interest.

Note: In compound interest, interest also earns interest.

Since it is clearly understand about compound interest. Get your exercise book and do the following simple activity.

Activity 17.14

(a) Calculate the compound interest on shs. 25000/= for 2 years at 10% per annum.
(b) Obama had shs. 800000 of his own, he wanted to buy a car at shs. 2000000. He borrowed the rest of the money at a compound interest of 10% for 2 years.

(i) How much interest did he pay back?
(ii) How much money did he pay back?
(iii) How much did the car cost him?

(c) Jamada got a loan of shs. 90000 from a bank which charges a compound interest of 9% per annum. What was the total amount he returned to the bank?
(d) Betty borrowed sh,200,000 from the college association at a 15% simple interest for 2 years. After failing to pay back the money the association gave him and extra period of 2 years to pay the would be the amount of previus loan a compound interest of 10% per annum. Calculate the total amount she owes to the association.

Dear student may you share your answers with your classmates and submit all your work to the tutor.

I am sure now you well equipped with business Mathematics.

Let you look at something different.

A New MK Primary Mathematics 2000 Pupil's Book 7 exercise 11.8 and 11.9 pages 152 – 155.

Dear students let you find out about. Post office Bank in the next sub-topic :

(d) The Post Office Saving Bank

Hullo student,

Do you have a Post Office near your college> if yes what services does it render to your college.

Let you read the information below:

One way of savings money is to put it into the Post Office Savings Bank. While the money is in the Bank it gains interest.

Post Office interest is calculated at the end of December. The money is not sent to you, but is added to sum which you already have in the bank.

May you use the above information while doing the following examples:-

Example 1:

A man puts shs. 200,000 in the Post Office Savings Bank. How much interest will he get after 1 year?
Interest on shs. 100 after 1 year = Sh 2 ½ .

Interest on Shs. 200000 after 1 year = 2 ½ x $\frac{200000}{100}$ x 1

$= \frac{5 \times 200000 \times 1}{2 \times 100}$

∴ Interest Sh. 5000.

Example 2:

A man keeps shs. 16000 in the Post Office Savings Bank for 2 years. How much does he have in the Bank at the end of 2 years:

At the beginning of the first year.

Sum in the bank = sh 16000

At the end of the first year
Interest on shs. 100 for 1 year = shs. 2 ½

Interest on sh 1600 for 1 year = $\frac{2\frac{1}{2} x\ 16000}{100}$

∴ At the end of the 1st year the man

16000 + 400 = Shs. 16400

At the beginning of the 2nd year = shs. 16400
At the end of 2nd year
Interest on 100 for I year = Sh 2 ½

Then interest at the end of the 2nd year,

$2\frac{1}{2} \text{ x } \frac{16400\ x\ 1}{100}$

$= \frac{5\ x\ 16400\ x\ 1}{2\ x\ 100}$

= shs. 410.

∴ After 2 years the man has = 16400 + 410
= 16810

Let you try the following work in your exercise book.

Activity 17.

(a) Find the sum of money in the Post Office Savings Bank

(ii) Shs 800 for 1 year
(iii) Shs. 24000 for 20 months.
(iv) Shs. 48000 for 2 years.

(b) What was the Post Office interest for 1998 on Shs. 240000 which were put in the Bank on 21st February 1998?
(c) John begins his Post Office Savings with Shs. 40000 on 4th march. How much money will he have altogether at the end of the years?
(d) Shs. 12000 were put in the Post Office Savings Bank on 6th April 1993. How much money was in the bank at the end of 1994?

Check your answers with those ones at the end of the unit.

Dear student you and now lucky that you have some ideas about Post Office Saving Bank.

In groups of 10 classmates, do the following exercise 11.11 from A New MK Primary Mathematics 2000 Pupils' Book 7 pages 159.

Share your answers with your tutor.

(e) Appreciation and depreciation.

(i) Appreciation

Dear student,

What is appreciation and depreciation?

Let you study the examples before you explain.

Example 1:

Ali bought a piece of land 3 years ago at shs. 4000000 in Busalamu village. What is the cost of the same piece of land if its value appreciates at 10% every year?

The cost of land at end of 1^{st} year

$= (100\% + 10\%) \times 4000000$

$= \frac{110}{100} \times 4000000$

= sh. 4,400,000/=

The cost of land at end of 2^{nd} year

$= (100\% + 10\%) \times 4400000$

$= \frac{110}{100} \times 4400000$

= sh. 4,840,000/=

The cost of land at end of 3^{rd} year

$= (100\% + 10\%) \times 4840000$

$= \frac{110}{100} \times 4840000$

= sh. 5,324,000/=

Hullo students with a colleague may you try problem follow the above example:-

Problem for practice:

What will be the cost of that plot this year?

What have you discovered?

In our daily life there are communities whose value keeps on increasing/gaining every year by a certain percentage of the original value. This is called **appreciation** in value. For example land, houses, rental fees etc. can you give more examples.

Let you read about something the above example.

(ii)Depreciation

Study the example below:-

Muharo bought a new radio for 80,000/=. But it depreciates in value at 12% every year. What will be the cost of it after 3 years?

Solution.

The cost of the radio at end of first year

(100% - 12%) x 80000

$= \frac{88}{100} \times 80000$

= 70,400/=

The cost of the radio at the end of 2^{nd} year

(100% - 12%) x 70400

$= \frac{88}{100} \times 70400$

= shs. 61,952/=

The cost of the radio at the end of the 3^{rd} year

(100% - 12%) x 61952

$= \frac{88}{100} \times 61952$

= 54,517. 76/=

As you are calculating the example, what did you discover?

You might have noticed the same as the following text:-

There are certain commodities whose value keeps on reducing by a certain percentage, these include for instance radios, cars, bicycles, clothes, shoes etc. this is known as **depreciation** in value

Using the above example and information do:-

Activity 17.15

(a) Alice bought a piece of land 2 years ago at 2500000, if the land raises value at 20% every year, what will be the cost of this land this year?

(b) A set of chair depreciates value at 10% every year, what will be the cost of 3 years old set of chairs which was bought at sh. 400000 when new?

(c) A computer which was bought last year at sh. 3,000,000 depreciates value at 15% every year. What will be the cost of this computer after 2 years now?

Hullo student, well done, can you compare your answers at the end of this unit?

Can you have a short break?

Dear student, welcome back from your break.

Let now embark on another topic:

17.7.3 Currency Exchange

(a) Introduction to currency Exchange

You are welcome to this very important topic.

Why is it important to learn about?

Share your findings with a colleague?

Dear student what is currency?

Try to revise unit (11) about money. Then what is foreign exchange?

Let you read and discuss the following examples, and then you will understand better.

There are two types of currencies used in Uganda today while carrying out business and world market. These namely:-

Local currency known as Uganda shillings (Ush) and the foreign currencies.

Can you give some of the examples you know?

Let you compare your answers with those in the chart below:-

Currency	Symbol	Current exchange rate	Country
Us Dollar Pound sterling Euro Kenya shillings Tanzania shillings Rwanda France	US$ £ € K sh. Tz. Sh RF	2389 3835 3340 27.2 1	America England Europe Kenya Tanzania Rwanda

(b) Conversion of local currency to others and vice versa

(i) Conversion of local currency to others.

Let you carefully study the following examples:-

Example 1:

Convert US$ 500 to Uganda shillings (Ush(if US$ 1 = Ush 2389.

Given that US$ 1 = 2389

Then, US$ 500 = 2389 x 500

∴ US$ 500 = Ush 1,194,500

Let you look at example 2

Given that Jaffery Forex Exchange rate chart on date 26/05/2011

Forex exchange

Foreign currencies	**Uganda shillings**	
Currency	**Buy**	**Sell**
US dollar	2380	2390
Pound sterling	3830	3870
Euro	3325	3375
Kenya Shillings	27.3	28

Use the foreign exchange chart.

(i) Which foreign currency was strongest on 26/05/2011 in the currency.
(ii) Why is it more expensive when Forex bureau is selling foreign currency?

(iii) If a friend donated pound £ 12 to a girl friend in Uganda on 26/05/2011. How much was that in Uganda shillings?

Step choose a column from the chart

So – taking buying column
Pound £ 1 = Ush 33 25

Then Pound £5 = 5 x 3325

Then pound £5 = 5 x 3325

= Ush 16,625.

Why do you use the column of buying instead of selling shown in the chart? When working out the above example?

You require to be very careful when reading forex exchange chart?

On your own can try the following problems:

(a)	Change US \$ 1220 to Uganda shilling assuming the US\$ = 2390/=
(b)	Given that 2800 Euro to be exchanged with Uganda shillings on assumption that 1 euro = Ush 3375.

Can you discuss your answers with a colleague.

Share your answers with your tutor.

Now let you look at another subtopic.

Exchange of other currencies with Uganda currency.

Study the example demonstrated below:-

Taking £ 1 sterling = 3830. Find the amount in pound sterling that can be exchanged for Ush 7,670,000

Rate Ush 3830 = £1

Then, Ush 7,670,000 = $\frac{1}{3830}$ x 7670000

∴ Ush 7670,000 = £ 2000 sterlings

Now, try the following exercise for more practice:-

Activity 17.16

(a) Convert £ 750 sterling to Ush 3880
(b) Given that 1 japanese Yen = Ush 3550, convert 2800 Japanese Yen to Uganda shillings
(c) A lady in Nairobi rceieved Ush 500000 which she exchanged for Kenya shillings at the rate of Ksh 1 to Ush 28. How many Kenya shillings did she receive?
(d) The table below gives Economy class international airline fares for some flight from Entebbe airport (may 2011)

Destination	**Single ticket (US dollars)**	**Return ticket (US dollars)**
Accra	640	1280
Dar es Salaam	162	324
Cape town	434	868
London	789	694

(i) Find the equivalent amount in Ush for a single ticket to Accra, if the dollars are obtained from a "forex Bureau" at rate of US$1 to Ush 2388.

(ii) Which is cheap and by how many Ush.

- A single ticket to London for Us dollars bought at Ush 2378 to the dollar or
- A single ticket to Cape town for US dollars bought at Ush 3320 to the dollar.

Compare your answers with those at the end of the unit.

May you relax.

17.7.4 Taxation and insurance

Dear student, you are welcome to the topic.

Let you answer to this question. Why is taxation related to topic percentage?

You might have noticed that percentages are used very widely in money problems. This is because rates as expressed in percentages are usually of an easily understood size. However, you are reminded that is another way of writing a fraction. This means that tax rates are expressed in percentages.

Can you give some examples of taxes charged in Uganda/

Compare your list with the following:-

Some of the taxes charged are:- income tax, hire purchase, insurance, graduate tax, with holding tax, pay as you earn, mortgage, Value Added Tax (VAT).

Let you read about some of the above taxes:

(a) Hire purchase:

Study the example demonstrated below:

A set of chairs cost sh. 200,000. Peter makes 20 monthly payment of sh. 15000 to buy the set of chair on hire purchase. Calculate;

i. Total hire purchase cost,
ii. The extra, Peter paid to buy the chairs on hire purchase
iii. The extra cost as percentage of the cash price.

(i) 20 payments of sh 15000 = 20 x 15000
= 300000

(ii) The extra paid is = New price – old price
= 300000 - 200000
= Sh. 100,000

(iii) Percentage increase = $\frac{100000}{200000}$ x 100

= 50%

Have you ever bought commodities using hire purchase? If yes, what do you understand by term "hire purchase"?

Let you read the next below:-

Customers normally purchase good e.g. radios, cars, furniture, boats etc on credit. They are able to have the goods straight away. The payments each month of an amount they can afford. This method of purchase of goods is called hire purchase

Now you read about this sub topic. May you do the following work in your exercise book.

A radio costs sh 160000. The shopkeeper offers it on hire purchase for a monthly payments of Ush. 18000.

(a) Find total hire purchase cost.
(b) What was the extra cost as a percentage of the cash price?

May discuss your numbers with your classmates.

Well done

(b) Income tax

Dear student, you are welcome to discuss this subtopic.

But what is an income tax?

Let you study the example:

Example 1.

Find the amount of income tax that will pay on a taxable income of Ush 540000. If the rate of tax is 25%

Income tax as percentage = 25%

$\therefore$ Income tax to be paid = $\frac{25}{100}$ x 540000

= Ush 135,000/=

Can you copy this above example in your exercise book.

A tutor earns a monthly salary of Ush 569400. He is entitled to a personal allowance of Ush 130,000 and an earned income allowance of 10% of his remaining income. What amount of income tax due if the rest is taxed at 15%?

Tutor monthly salary = Ush. 569400

Subtracting personal allowance = 569400
- 130000
Ush = 439400

Earned income allowance = 10%
Tutor gets % = (100 - 10)%
= 90%

$\frac{90}{100}$ x 439400 = Ush. 395460

The income tax rate is 15%

$\therefore$ The income tax is $\frac{15}{100}$ x 395460

Ush 59319.

Using the above example do the following problems;

Mary earns sh. 961500 per year. She has personal allowance of shs. 58,000 and an earned income allowance of 15%. Income tax is charged at 25% on the first shs. 265000 and 30% on her remaining income. Find the amount of income tax due.
Therefore, dear student, one way in which the government collects this money is to take part of people's income. The money taken is called **income tax**.

Dear student thank you for going through some of the taxes, you are now asked to read about other taxes e.g. pay as you earn, withholding tax, and graduated tax.

Submit your findings to your tutor.

Finally let you look at

(d) Insurance

Dear student, you are welcome to this last subtopic.

Let you study the example:

Example 1:

Mr. Kato insures his house for Sh 300,000,000, it's content for shs. 100,000,000 and takes out all risk cover on items worth sh. 1500000. The insurance rate for the house is 2% for it's contents is 4% and for all risks is 8%. What is the annual premium he must pay?

Hullo, read the question three times before you study the working.

Step 1: For the house, the premium is 2% of shs 300,000,000 = $\frac{2}{100}$ x 300,000,000

= shs. 6,000,000

Step 2: for the house contents, the premium is 4% of Ush 100,000,000 = $\frac{4}{100}$ x 100,000,000

= 4,000,000

Step 3: For all risks, the premium

= $\frac{8}{100}$ x 15000000

= sh 120,000

Step 4:

∴ Kato's whole insurance bill

= 6,000,000
4,000,000

$$
\begin{array}{lr}
 & +\quad 120,000 \\
\text{Ush} & \underline{10120000} \\
\end{array}
$$

In groups use the above demonstrated example and do the following problem.

Matama has comprehensive insurance for her house and the content. The cover for the house is sh 840,000 and for contents is sh 960,000. The annual insurance rate for the house is 2 ½ and for the content is 5%. What is his annual premium?

Share your answers with your subject tutor.

Dear student congratulations for completing this topic.

17.7.5 (a) Teaching business Mathematics in Primary School

Dear student, in teaching this unit, you require to adequately prepare a lot of learning materials and activities. The teaching of this unit requires practical approaches e.g. visiting places like banks, post offices, shops and markets. This will enable your learners to have a proper understanding of this unit.

The learners should be actively involved in the learning/teaching process, asking them questions which require to use their own experiences.

Activities like finding out how much money is deducted for withholding tax on commodities bought worthy Ush 6,000,000 for a school? Thus requires a pupil to establish the rate of withhold as 6%.

Hence $\frac{6}{100}$ x 6000,000/=

= Ush. 360000.

In this way the pupils will be able to understand the concept percentages with its application in everyday life situation.

(c) Planning lesson of teaching percentages and its application.

Hullo student, when you are to plan to teach this unit in any primary school, it is always advisable revise the following units 6 (on fractions) and have enough knowledge about taxation by visiting banks, revenue offices and then the unit outline. Read a variety of books, consult many resources persons and other instructional materials to help you an approach to proper teaching of business Mathematics.

It is also observed that many teachers and pupils fear business Mathematics as a difficult unit. It is therefore your responsibility to make it practical and easy by leading the pupils from known to unknown.

Finally it is advisable to you to handle this unit from the elementary perspective whereby (use a lot of pupils' experience abstract idea.

17.8 Unit summary

In this unit you have been introduced to business mathematics and learnt about:-

(i) Meaning of percentages.
(ii) Conversion of fractions to percentages and vice versa.
(iii) Expressing decimals as percentages and vice versa
(iv) Percentage increase and decrease.
(v) Finding the percentage gain and loss.
(vi) Finding original (cost) price and selling price (new)
(vii) Percentage discount
(viii) Commission
(ix) Simple interest and compound interest
(x) The Post Office Saving Bank (POSB)
(xi) Appreciation and depreciation
(xii) Taxation and insurance
(xiii) Activity and instructional materials for teaching business mathematics
(xiv) Planning lesson for teaching any topic on business mathematics

17.9 Glossary

Income tax: - Money collected by government from what people get.
Personal allowance: is what people get without deduction

Tax free: - personal allowance
Insurance -
Percentage - a special kind of ratio of any given number to one hundred or a fraction whose denominator is hundred.

Increase - adding on the original number.
Decrease - subtracting from the original number
Discount - selling an article at a price less than a marked price. Amount of money deducted.
Marked price - price written on an article
Commission - is the payment paid for services of selling people articles
Compound interest the type of interest where interest is earned on the previous principal plus interest.
POSB - Post Office Savings Bank
Appreciation - increase in value by over certain percentage by a commodity
Depreciation - loss of value by a commodity by certain percentage
Premium - the amount of money the owner of proper agrees regularly to pay the insurance company. So that the company will pay out if the customer has a loss covered by the policy.
Policy - The agreement made between the owner of property being insured and company.
Personal allowance This is income that is free of tax
Taxable - Balance/difference after removing the personal allowance

Income tax - Money removed by the government form someone's income.
% - is a symbol for percent

Interest - The interest is the amount of extra money paid in return for having the use of someone's money

Depreciation - is the value of an object whose amount of money used to purchase has fallen.

Discount - is an amount of money which has been taken off.

17.10 answers and notes.

Activity 17.1

(a) (i) $\frac{1}{5}$ (ii) ¾ (iii) $\frac{1}{40}$
(b) $\frac{9}{20}$

Activity 17.3

(a) (i) 80% (ii) 62 ½ % (iii) $33\frac{1}{3}$ (iv) $66\frac{2}{3}$%
(b) 30%
(c) (i) $\frac{1}{25}$ (ii) $\frac{11}{20}$ (iii) $\frac{9}{40}$ (iv) $1\frac{9}{20}$

Activity 17.4

(a) (i) 3% (ii) 84% (iii) 178% (iv) 250% (v) 100%
(b) (i) 0.4 (ii) 0.02 (iii) 0.72 (iv) 1.75 (v) 0.025 (vi) 0.0425
(c) 80%

Activity 17.5

(a) 15% (b) $8\frac{1}{3}$ (c) 28.57% (d) 4800 (e) 324kg

Activity 17.6

(a) (i) 110% (ii) 140% (iii) 125%
(b) 55,000/= (c) 322500/= (d) 172,500/=

Activity 17.7

(a) 67% (b) 60% (c) 77.5% (d) 15% (e) 5888/= (f) 76
(g) 1100/=

Activity 17.8

(a) 104000/= (b) (i) 500/= (ii) 3600/= (iii) 25% (c) 22500/=

Activity 17.9

(a) 300000/= (b) 50000/= (c) 110,000/= (d) 2798.51/=

Activity 17.10

(a) 60% (b) 50% (c) 20% (d) $2\frac{2}{3}$ (e) (i) Empress cotton
- ($41\frac{2}{2}$ %)
- 40% (mosquito net)
- 27.78% (royal cotton)

(ii) Empress cotton

(e) (i) 7 ½ % (gain)
(ii) sh 48000
(iii)Ush 555000

Activity 17.13

(a) £ 72 (b) Sh 55000 (c) (i) 55000 (ii) £715 (iii) sh 25000
(d) (i) Sh 4,032,000 (ii) 1,008,000/=

Activity 17.15

(a) 3,600,000/= (b) 291,600/= (c) Sh 2167500

Activity 17.16

(a) Sh. 2,910,000 (b) sh 9,940,000/= (c) Ksh 20,000
(d) (i) Sh. 1,528,320
(ii) To Cape Town by sh. 435362

17.11 End of unit exercise

1. In a sale an article priced at Ush 24500 and another one priced at Ushs 12400 are each reduced by Ush 1000. What is the percentage reduction in each case.

2. A mobile phone was sold for Ush 150,000 at a loss of 2 ½ %. Find the cost price of the phone.
3. If 4 kg of bread cost Ush 440 a quarter, what should be the cost of 6kgs of bread when wheat is Ush 630 a quarter?
4. Find the simple interest on Ush 120000 for 8 months at $5\frac{1}{4}$% per annum.
5. A boy bought 417 stamps for Ush 8000 and sold them at Ush 250. If his profit was Ush 2000, find the value of n
6. Copy and complete this table

Cost price	**%profit/loss**	**Selling price**
(i) Sh. 3500	20% profit	------------
(ii) Sh 8000	---------------	sh 1500
(iii) sh 1200 per dozen	---------------	Sh 80

(iv)------------------- 20% sh 1200

7. Mary invests sh. 1000 at 10% per annum compound interest. How much money does she have after 3 years?

8. Goods worth sh 8400000 are insure at 5% per year for 6 years. What is the total premium paid in this period?

You can relax and look for more problems to work out.

17.12 references for further reading

Dear student listed below are references for further reading.

(a) Karuhije NCDC, Uganda Secondary Mathematics book 2
(b) Karuhije NCDC, Uganda Secondary Mathematics book 3 New Edition.
(c) Primary mathematics 2000 Pupil’s book 7 and 6 MK Publishers
(d) Karugaba J. Mathematics a complete Revision course.
(e) Channon J.B., Head HC, New General Mathematics.
(f) Okello Highway Primary Mathematics Book 7 and 8

End of Unit

UNIT 18: ALGEBRA II

18.1 introductions

You are most welcome to this unit on Algebra II.

This unit is a continuation of unit 12 on Algebra I where you introduced to concepts like algebraic expressing, solving equations and inequalities, order of operation and exponential notations among others.

18.2 Content Organization

This unit gives you an opportunity to learn mathematical ideas in the following topics;

No.	Topic	Subtopic	
1.	**Linear equations**	a.	Graphing linear equations
		b.	Solving linear equations
		c.	Formatting linear inequalities
		d.	Finding solution sets of
		e.	Representing solution sets of linear inequalities on a number line
2.	**Simultaneous equations**	a.	What are simultaneous equations?
		b.	Forming simultaneous equations
		C	Solving simultaneous equations using (i) Graphical method (ii) Elimination method (iii) substitution method
3.	**Subject of formulae**		Changing subject of a formula
4.	**Identities**		Diagrammatic illustration of identities
5.	**Quadratic equations**	a.	Forming quadratic equations
		b.	Solving quadratic equations

18.3 learning outcome

By the end of the unit you are expected to use the concept of algebra to solve problems.

18.4 Competences

i. Graph linear equations
ii. Form linear inequalities
iii. Solve linear inequalities
iv. Find solution sets of linear inequalities.
v. Represent solution sets of linear inequalities on a number line.
vi. Define simultaneous equation.
vii. Form simultaneous equations.
viii. Solve simultaneous equations.

ix. Charge subject of a formula.
x. Illustrate identities.
xi. Form quadratic equations.
xii. Solve quadratic equations.

18.5 unit orientation

In order to understand this topic well, you need to revise the concepts you earlier learnt in Algebra I module 1. Try to put more emphasis on solving algebraic equations and inequalities.

18.6 study requirements

The following materials are vital for you to successfully study this topic;

i. Prepare relevant instructional materials like graph book for effective learning of the topic.
ii. Use primary course books to study ideas on algebra.

18.7 Content and activities

18.7.1 Linear equations

(a) Graphing linear equations

Read and note the following;

A linear equation is an equation whose graph is a straight line. An equation is linear if the variables occur to the 1st power e.g.

$Y = 2x - 1$, $y - x = -3$, $3y = 7$, $x - 2y = 1$

From the examples above, you should have realized that there are no module of variables and no variable appears in a denominator. Examples of non-linear equations are;

$Y = x^2 - 1$, $x^2 + y^2 = 9$ $y = \frac{3}{x}$, $xy = -1$

Take note that a solution of an equation with two variables is an ordered pair of numbers. These pairs of numbers can be plotted on a coordinate plane. A **graph** of an equation is a drawing that represents its solutions.

Example 1

Study the example below

Graph the equation $y + x = 1$

Solving for y: y + x = 1
Y = 1 – x
X = -1, y = 1 - $^{-}1$ = 2
X = -0, y = 1 - 0 = 1
X = -1, y = 1 - 1 = 0
X = -1, y = 1 - 2 = $^{-}1$
X = -1, y = 1 - 3 = $^{-}2$

x	**y**
-1	2
0	1
1	0
2	-1
3	-2

The solution of the two variables has the following ordered pairs

($^{-}1$,2), (0,1), (1,0), (2,-1), (3,- 2).

In the example above it is interesting to note that two points are sufficient to draw the graph. We normally use the third point as a check.

Activity 18.1

(i) Determine whether the given point is a solution to the equation.

(a) (2,5), y + 1 = 3x (b) ($^{-}1$,4), y = 2 – 2x (c) (-2, -1), 3y + 2x = 7 (d) (10,-4), x + y = 4

Graphing using intercepts

Did you realize that the easiest way if graphing linear equations is plotting any two points that belong to the graph? In most cases, the earliest points to find are the points where the graph crosses the axes.

The line above shows the right line where the graph crosses the x – axis at (-3,0) and the y-axis at (0,2).

Note that we say the x-intercept is -3 and the y-intercept is 2 . The x-intercept of a line in the x-coordinate of the point where the line intercepts the x-axis. While the y-intercepts of a line is the y-coordinate of the point where the line intercepts the y-axis.

Activity 18.2

Graph the following equations using intercepts;

(a) $2y - 3x = {}^{-}6$ (b) $4x + 3y = 12$ (c) $3y = 2x - 7$ (d) $y = -4 - 4x$ (e) $5x + 7y = 35$

(f) $8x + 2y = 24$

(c) Linear inequalities

This is a question for you.

What is the difference between equations and inequalities?

Mathematical sentences containing = (equal to) are reffered to as equations on the other hand, inequalities are mathematical sentences containing <,>, ≤, or ≥

Note that the meanings of the symbols are as follows;

Symbol	Meaning
>	More than or greater than
<	Less than
≥	Equal to or greater than
≤	Equal to or less than

(d) Solutions of inequalities

Try to answer this question.

How do we determine the solution of an inequality?

Take note that a solution if on inequality is any number that makes the inequality true.

Given that $y > 4$. Think of possible values of y which will make the inequality true.

For the inequality $y > 4$, the possible values of y area the numbers less than 4 i.e. 3,2,1,0,⁻1…….

Activity 18.3

1. determine whether the given number is the solution if the inequality, true or false

(a) x > 7; (i) 2 (ii) 0 (iii) 8 (iv) $^{-}7$ (v) $^{-}10$
(b) x > $^{-}3$ (i) $^{-}5$ (ii) 0 (iii) 3 (iv) 10 (v) -11
(c) x ≤ 5 (i) $^{-}5$ (ii) $^{-}3$ (iii) 0 (iv) 5 (v) 7

(e) Graphing inequalities

We can represent inequalities on a number line. A graph if an inequality in one variable is a picture of all its solution on a number line.

Example

Show or graph these inequalities on number lines.

(a) X < 1

The solutions of x < 1 are all the numbers less than i.e. 0,-1,-2,-3…….. The solution can be shown on the number line as follows

Number line undrawn

NB: 1 is not a solution. We show this by an open circle.

(b) Solving inequalities using the addition principle.

We know that the inequality of 5 > 1 is true.

Adding the same number to both sides makes the inequalities still true

∴ 5>1
= 5 + 3 > 1 + 3
= 8 > 4

Note the addition principle for inequalities which states that;

For all rational numbers x, y,z

If $x < y$, then $x + z < y + z$
If $x > y$, then $x + z > y + z$

The addition principal for inequalities is used in the same way as solving equations.

Example

Solve the inequalities and graph the solution. $X + 5 > 6$

Using the additional principal

$X + 5 + (-5) > 4 + (-5)$
$\therefore x > {}^{-}1$

Unfinished numberline

Activity 18.4

Solve the following inequalities and graph their solutions.

(a) $x + 3 > 6$ (b) $x – 5 \leq 8$ (c) $x – 1 \geq 7$
(d) $x + 1 < 5$

(g) Solving the inequalities using the multiplication principle.

Note this true inequality

$2>5$

If we multiply both sides by 2, we get a true inequality.

$4 > 10$ True

If we multiply both sides by -2, we get a false inequality

${}^{-}8 > {}^{-}10$ false

If we reverse the inequalities symbol however, we get a true inequality.

${}^{-}8 < {}^{-}10$ false

Note that the multiplication principal for inequalities which states that;

For all rational numbers, x,y and z, where z is positive, where C is negative

If $x < y$, then $xz < bz$ if $x > y$, then $xz > bz$

If x > y, then xz > bz if x > b, then xz < bz

Similar statements are true for ≤ and ≥.

Example

(a) Solve and graph the solution

2x < 14

½ (x) < ½ (14) multiply both sides by ½

X < 7 since ½ is positive there is no need to change the sign inequality symbol.

Unfinished numberline

(b) Solve and graph the solution

$\frac{-1}{3}$ (-3y) < - $\frac{-1}{3}$ (21)Multiply both sides by $\frac{-1}{3}$ and reversing the inequality symbol

Y > $^{-}$7

Unfinished numberline

Activity 18.5

Solve and graph the solution

(i) x – 7 < 3 (ii) x – 10 > - 16 (iii) -5y + 6y – 8 < -9 (iv) x - $\frac{1}{3}$ > ¼ (v) -7x ≤ 21

(vi) -12 > 2x – 6x (vii) 4x – 9x > -12 (viii) 24 – 7x < 11x – 12 (ix) 3x < 30 + 2x (x) y + 3 ≥ 6(y-4) + 7

18.7.2 simultaneous equations

(a) Meaning of simultaneous equations.

A simultaneous equation consists of two equations with two unknowns, e.g. a+b = 1 and a – b = 2 are simultaneous equation.

A solution of two equation with two variables is an ordered pair that make both equations true. Therefore, since the solution satisfies both equations simultaneously, we refer to a system of simultaneous equations.

Examples

(i) Determine whether (0,1) is a solution of

X + y = 1
X – y = -1

X + y = 1 x – y = -1

1	1	
1	1	√

-1	-1	
-1	-1	√

(ii) Determine whether (-3,2) is a solution of the system.

X + y = -1
Y + 3x = 4

X + y = -1	
-3 + 2	-1
-1	-1

y + 3x = 4	
2 + 3 (-3)	4
2 – 9	4
-7	4 x

Since (-3,2) is not a solution of the y + 3x = 4, it is not a solution of the system.

Bear in mind that when we graph a system of two linear equations, any one of the following may happen;

(i) The two lines may have one point of intersection. The point of intersection is the ONLY solution of the system.

(ii) The two lines may be parallel. If this is so, then there is no point that satisfies the equation. In this case, the system has NO solution.

(iii) The two lines may coincide. In this case, the equations have the same graph and every solution if one equation is a solution of the other. Therefore there is an infinite number of solutions.

Activity 18.6

Solve the following simultaneous equations using the graphical method.

(i) x – 3 = 2y
X – 1 = y

(ii) 3x + y = 0
x – y = 2

(iii) y – 2x = 3
y – 2x = 5

(iv) 2x + 3y = 7
4x + 6y = 14

Which one has solution?

(ii) Which one has no solution? (iii) Which one has infinite numbers of solutions?

(b) Finding solution using the substitution method.

Have you realized that solving simultaneous equation by graphing is often not accurate if the solutions are not integers?

Substitution method is one of the most reliable ways of solving systems of equations.

If a variable in one equation is alone on one ride, you can easily substitute for the variable in the other equation.

Solving using the substitution method

$a + b = 6$ (i)
$a = b + 2$ (ii)

in (ii) above, a and b+2 are equivalent expressions. We can therefore substitute b+2 for a in (i);

$a + b = 6$

$\boxed{b} + 2 = 6$ substitute b + 2 in (i)

Solving for b and collecting like terms,

$2b + 2 = 6$
$2b = 4$
$\therefore b = 2$

Substituting 2 for b in any of the original equations,

$a + b = 6$
$a + 2 = 6$
$a = 4$

$\therefore b = 2$

Substituting 2 for b in any of the original equations,

$a + b = 6$
$a + 2 = 6$
$a = 4$

$\therefore$ The solution is (4,2).

Check: $a + b = 6$

$4 + 2$	6
6	$6\surd$

$a = b + 2$

4	$2 + 2$
4	$4 \quad \surd$

(c) Finding solution by graphing

Draw graphs of the following equation using any of the methods you have learnt.

X – y = 4 and x + 2y = 7

For you to draw the graph, you need to determine the coordinates of any two points on each line.

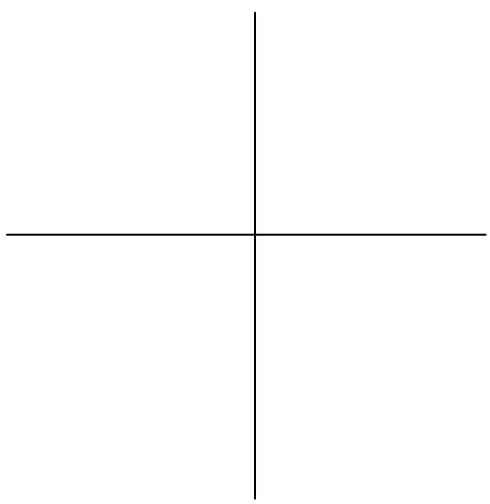
From x – y = 4, if x = 0, then y = -4
If y = 0, then x = 4

∴ The two points (0,-4) and (4,0) lie on the line.

From x + 2y = 7, if x = 0, then y = 3 ½
If y = 0, then x = 7

∴ The two points (0, 3 ½) and (7,0) lie on the line.

Use the points (0, 3 ½) and (7,0); and (0,-4) and (4,0) to draw the graphs.



Since point A (5,1) is the intersection, the solution is (5,1)
= x = 5 and y = 1

Activity 18.7

Solve using the substitution method

(a) (i) $x - 6 = 24$ $3x + 2y = 4$	(ii) $x - 2y = 8$ $2x + y = 8$	(iii) $3a + 4b = 2$ $2a - b = 5$	(iv) $2y - 3x = -16$ $y + x = 7$

(b) The sum of two numbers is 82. One number is twelve more than the other. find the larger number.
(c) Find two numbers whose sum is 66 and difference is 8.
(d) The perimeter of a rectangular shape is 350cm. the width is 15cm shorter than the length

(i) What is the length and the width of this rectangle.
(ii) Find the area of this rectangle.

(d) Finding solution using elimination method

You are going to be introduced to yet another method of solving simultaneous equations. In this method, it is useful when both equation are written in standard form i.e. $ax + by = c$

Example

Solve using elimination method

$$\begin{array}{l} X + y = 72 \\ \underline{X - y = 28} \\ 2x = 100 \\ \\ X = 50 \end{array}$$

Substituting 50 in x in either of the original equations.

$$\begin{array}{l} X + y = 72 \\ 50 + y = 72 \\ \quad Y = 22 \end{array}$$

= x = 50 and y = 22

Check:

$x + y = 72$
$50 + 22 = 72\surd$

$X - y = 28$
$50 - 22 = 28\surd$

18.7.3 Subject of formulae

All along you have been finding values of the unknown which is given in form of other letters e.g. $x = 2y - w$. in this case, x is the subject of the formula. We can easily find the value of x by substituting the values of y and w directly into the formula.

Suppose we want to find out the value of y which is not the subject of the formula. What will you do in this case? From x = 2y – w, the equation remains true so long as we did the same thing to both sides. Doing the same fling on both sides will keep both sides of the equation equal.

$\therefore x = 2y - w$

Adding w to both sides

$X + w = 2y - w + w$
$X + w = 2y$

$\Longrightarrow$ $2y = x + w$

$= y = \frac{1}{2}(x + w)$

Note that subject of the formula is normally on the left.
Example.

Make F the subject of the formula

$C = (F - 32) \times \frac{5}{9}$

Multiplying both sides by 9,

$9C = 5(F - 32)$

Dividing both sides by 5

$\frac{9}{5}C = F - 32$

$\therefore F - 32 = \frac{9}{5}C$

Adding both sides by 32

$F = \frac{9}{5}C + 32$

Activity 18.8

For each of the following, change the subject to the letter in brackets

(a) $A = \frac{1}{2}lb$ (b) (b) $V = \pi r^2 h$ (r) (c) $I = \frac{PTR}{100}$ (R) (d) $\tau = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ (a)

(e) V = u tat (a) (f) $\frac{7}{x} = 3y$ (y)

18.7.4 Identities

Find the solution set of the following;

(i) $2x - 1 = 5$ (ii) $(2y - 3) + 9 = y + 3$ (iii) $3(a + 1) = 3a + 3$

(iv) $2(b-3) + 7 = 2b + 1$ (v) $5y - 2(y + 1) = 3y - 2$ (vi) $2w + 7 - 3(w + 1) = 9$

Did you discover that (i), (ii), and (vi) are quite different from (iii), (iv) and (v)? What is the major difference between them?

Solving (ii),

$(2y - 3) + 9 = y + 3$

Remove brackets

$2y - 3 + 9 = y + 3$

$2y + 6 = y + 3$

$2y - y = 3 - 6$ group like terms

$\therefore y = -3$

Solving (v),

$5y - 2(y + 1) = 3y - 2$

$5y - 2y - 2 = 3y - 2$ remove brackets group like terms

$5y - 2y - 3y = -2 + 2$

$0 = 0$

What are the solutions for y then?

Substitute 1 for y in (v)

$5y - 2(y + 1) = 3y - 2$

$(5x1) - 2(1 + 1) = (3 \text{ x } 1) - 2$

$5 - (2 \text{ x } 2) = 3 - 2$

$5 - 4 = 3 - 2$

$1 = 1$

Substitute -3 for y in √

$5y - 2(y + 1) = 3y - 2$

$(5 \text{ x } -3) - 2(-3 + 1) = (3x -3) -2$

$-15 - (2 \text{ x } 2) = -9 -2$

$-15 + 4 = -11$

$-11 = -11$

NB: This is an example of an equation which is true for all values of the unknown. An equation of this nature is referred to as IDENTITY. Equations (iii), (iv) and (v) are identities.

Using the idea of expanding numbers that you learnt in Algebra 1 and since you have also to understand what an identity is you can now solve the three important identities.

What do you think $(a + 3)^2$ mean?
Have you realized that $(a + 3)^2$ is not the same as $a^2 + 3^2$?

When you consider expanding 3^2, it will become 3 x 3. Therefore, in the same way, we shall expand
$(a + 3)^2$ to become (a + 3) x (a + 3) which we usually write (a+3) (a + 3). At this point, you are ready to solve the identities.

1. $(x + y)^2 = (x + y) (x + y)$

$= x^2 + xy + xy + y^2$

$= x^2 + 2xy + y^2$

Congratulations!

Can you now solve the next two identities in your exercise book and share your answer with you colleagues.

2. $(x – y)^2$ 3. $(x + y) (x – y)$

In your answers you should have discovered that;

1. $(x + y)^2 = x^2 + 2 x y + y^2$
2. $(x - y)^2 = x^2 - 2 x y + y^2$
3. $(x + y) (x – y) = x^2 - y^2$

Note that these identities are very useful in solving mathematical problems. The first and second identities help us in solving numbers dealing with quadratic equations. While the third identity help us to solve numbers dealing with the difference of two sequences.

18.7.5 Quadratic equations

Equations which can be expressed in the form ax2 + bx + C = 0 where b or c may be zero, are called Quadratic Equations.

Assuming that the solution set of a quadratic equation is {-1,2}, can you find the equation?

Given solution set as {-1,2}

$\therefore$ x = -1 or x =2

$\Longrightarrow$ X + 1 = 0 or x – 2 = 0

Multiplying the two factors

$(x + 1)(x - 2) = 0$
$X^2 - 2x + x - 2 = 0$
$\therefore X^2 - x - 2 = 0$

Try to construct equation where solution sets are;

(a) {2,5}   (b) {1,-3}   (c) {0,3}

Solution by factors

Example

Solve (i) $(y - 5)(y + 7) = 0$
(ii) $a^2 = 36$
(iii) $a^2 - 3x = 0$

Solution.

(i) $(y – 5)(y + y) = 0$

$\Longrightarrow$ Either $y -5 = 0$ or $y + 7 = 0$
$Y = 5$ or $y = 7$

(ii) $a^2 = 36$

$\Longrightarrow$ $a^2 – 36 = 0$
Since $a^2 - 36$ is the difference between two sequences, $a^2 = a \times a$
$36 = 6 \times 6$

$\therefore$ $a^2 – 36 = 0$
$(a + b)(a – b) = 0$
$\therefore$ $a + b = 0$ or $a – b = 0$
$a = 6$ or $a = -6$

(i) $y^2 – 3y = 0$

Factorizing

$y^2 – 3y = 0$
$y(y – 3) = 0$

(ii) $m^2 + 5m + 4 = 0$

Look for two numbers whose

Sum = 5
Product = 4

$\therefore$ The two numbers are 1 and 4

Theme $(m + 1) (m + 4) = 0$
$M + 1 = 0$ or $m + 4 = 0$
$M = -1$ or $m = -4$

(v) $2a^2 – 6a = 80$
$2a^2 – 6a – 80 = 0$

Dividing by 2 all through

$a^2 – 3a – 40 = 0$

Two numbers whose sum = -3 and product = -40 are 8 and 5

$\therefore a^2 - 3a - 40 = (a - 8)(a + 5) = 0$

$a - 8 = 0$ or $a + 5 = 0$

$a = 8$ or $a = -5$

(vi) $7x^2 = 4 - 12x$

$\Longrightarrow 7x^2 + 12x - 4 = 0$

Two numbers whose sum is 12 and product (7 x -4) = 28 are -2 and 14

$\therefore 7x^2 + 12x - 4 = 7x^2 + 14x - 2x - 4 = 0$

$7x(x + 2) - 2(x + 2) = 0$

$\therefore (x + 2)(7x - 2) = 0$

$X + 2 = 0$ or $7x - 2 = 0$

$X = -2$ or $7x = 2$

$X = \frac{2}{7}$

(b) Completing the square

You have learnt that it is possible to solve quadratic equations which is the form

Perfect square = non negative constant

The method of perfect square transforms a quadratic equation which is not of tis form.

The main idea behind completing the square can be analyzed in the following

(i) $(x + b)^2 = x^2 + 12x + 36$

$(\frac{12}{2})^2 = 36$

(ii) $(x - 5)^2 = x^2 - 10x + 25$

$(\frac{-10}{2})^2 = 25$

(iii) $(x + a)^2 = x^2 + 2ax + a^2$

$(\frac{2a}{2})^2 = a^2$

Note that the constant term is the square of half the coefficient of x

You should have realized that the method of competing the square for x2 + bx + ______ follow these steps.

(i) Find half the coefficient 1k.
(ii) Square the result of step (i)
(iii) add the result of step (ii) to $x^2 + bx$

Example 1

Complete the square

(a) $x^2 - 14x +$ ______ 1
$X2 - 14x + 49 = (x - 7)^2$

$(\frac{-14}{2})^2 = 49$

(b) $x^2 + 3x +$ ___?_____

$X2 + 3x + \frac{9}{4} = (x + \frac{3}{2})^2$

$(\frac{3}{2})^2 = \frac{9}{4}$

Example 2

Solve $x^2 + 12x + 32 = 0$

$X^2 + 12x + 32 = 0$

$X^2 + 12x = -32$

$X^2 + 12\,x + (\frac{12}{2})^2 = -32 + (\frac{12}{2})^2$

$X^2 + 12x + 36 = -32 + 36$

$(x + 6)^2 = 4$

$X + 6 = \sqrt{4}$

$X = -6 \pm \sqrt{4} = 6 \pm 2$

$X = -4$ or $x = -8$

Example

Solve $2x^2 + 3x - 4 = 0$ by completing the square.

In order to complete the squares make the coefficient of x^2 to be ;

$\Longrightarrow$ $2x^2 + 3x - 4 = 0$

$\Longrightarrow$ $X^2 + \frac{3}{2}x - 2 = 0$

$X^2 + \frac{3}{2}x = 2$

$X^2 + \frac{3}{2}x + (\,½ \,.\, \frac{3}{2})^2 = 2 + (\,½ \,.\, \frac{3}{2})^2$

$X^2 + \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16}$

$(x + \frac{3}{4})^2 = \frac{4}{16}$

$X + \frac{3}{4} = \sqrt{\frac{41}{16}}$

$X = \frac{-3}{4} \pm \sqrt{\frac{41}{4}}$

$\therefore x = \frac{-3 \pm \sqrt{41}}{4}$

$X \approx 0.9$ or $x = -2.4$

(c) The quadratic formula

Earlier on you learnt how to solve quadratic equations which is of the form;. $ax^2 + bx + c = 0$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x = \frac{-c}{a}$

$x^2 + \frac{b}{a}x + (\frac{1}{2} \,.\, \frac{b}{a})^2 = \frac{-c}{a} + (\frac{1}{2} \,.\, \frac{b}{a})^2$

$x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = \frac{-c}{a} + (\frac{b}{2a})^2$

$(x + \frac{b}{2a})^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$

$X + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{4a^2}$

$X = \frac{-b}{24} \pm \frac{\sqrt{b^2 - \ 4ac}}{4a^2}$

$X = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - \ 4ac}}{2a}$

$\therefore x = \frac{^-b \pm \sqrt{b^2 - \ 4ac}}{2a}$

Note that the formula quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The formula gives the roots of $ax^2 + bx + c = 0$ in terms of the coefficients of a,b,c. In this case, the artembtions is that $a \pm 0$ and $b^2 - 4ac \geq 0$.

Also note the following ideas;

$b^2 - 4ac > 0 \Longrightarrow$ two solutions
$b^2 - 4\,ac = 0 \Longrightarrow$ one solution repeated
$b^2 - 4ac < 0 \Longrightarrow$ empty solution set

Example

Solve 3x2 – 7x + 1 = 0 using the formula.

Solution.

Then 3x2 – 7x + 1 = 0

a = 3, b = -7, c = 1

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 3 \times 1}}{2 \times 3}$

$= \frac{7 \pm \sqrt{49 - 12}}{6}$

6

$= \frac{7 \pm \sqrt{37}}{6}$ or $x = \frac{7 - \sqrt{37}}{6}$

Activity 18.9

(i) Solve the following equation using these methods;

a) Factorization
b) Competing the squares
c) The quadratic formula

Where applicable.

1. $X2 - 7x + 8 = 0$
2. $2x2 - 3x + 7 = 0$
3. $X2 + 3x - 6 = 0$
4. $3x2 + 6x - 9 = 0$
5. $4y2 + 5y - 21 = 0$

(ii) Find the equation whose roots are 14 and (1) $-1\frac{2}{3}$ (2) $\pm 1\frac{2}{3}$ (3) 2, -5 (4) 4,0

18.8 UNIT SUMMARY

You have come to the end of unit 18. In this unit, you were introduced to many concepts in the following topics;

- Linear equations
- Simultaneous equation
- Subjects of formulae identities and
- Quadratic equations

18.9 GLOSSARY

Linear equation : It is an equation whose graph is straight line

Simultaneous equation: it counts of two equations with two unknown. E.g. a+ b=1 and a– b = 2

Intercepts : the intercept of a graph is the point at which it cuts across an axis.

Graph : it is diagram showing either the relationship between some variables of the connections that exist between a set of points.

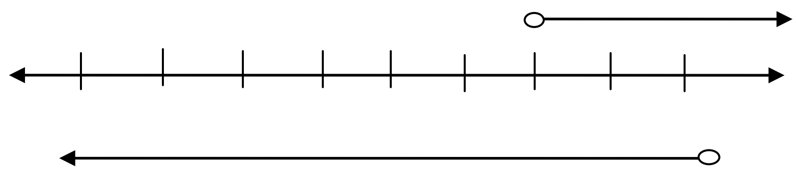
18.10 ANSWERS TO ACTIVITIES

Activity 18.3

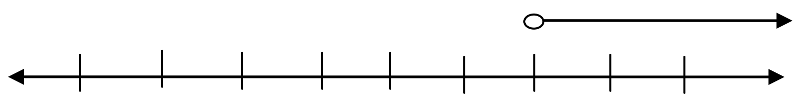
(a) (i) True (ii) True (iii) False (iv) True (v) true
(b) (i) False (ii) True (iii) True (iv) True (v) False
(c) (i) True (ii) True (iii) True (iv) True (v) False

Activity 18.4

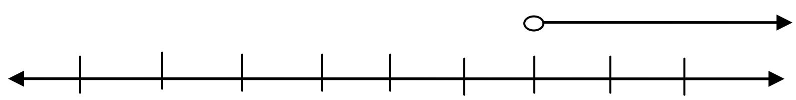
(a) $x > 3$



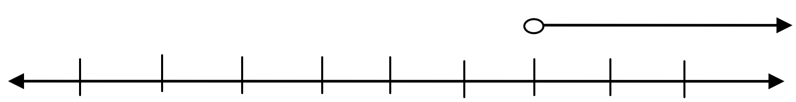
(b) $x \leq 13$



(c) $x \geq 8$



(d) $x < 4$



Activity 18.6

(i) $x = -1$, $y = -2$ (ii) $x = 0$, $y = 0$ (iii) No solution (iv) Infinite number of solution

Activity 18.7

(a) (i) $x = \frac{5}{2}$, $y = \frac{-7}{4}$ (ii) $x = \frac{24}{5}$, $y = \frac{-8}{5}$ (iii) $a = 2$, $b = -1$ (iv) $x = 6$, $y = 1$

(c) The two numbers are 35 and 47

$\therefore$ 47 is the larger number

Activity 18.8

(a) $b = \frac{2A}{L}$ (b) $r = \sqrt{\frac{v}{\pi h}}$ (c) $R = \frac{100\ I}{PT}$ (d) $L = \frac{T^2\ g}{4\ \pi^2}$ (e) $a = \frac{v-u}{t}$ (f) $y = \frac{7}{32}$

Activity 18.9

1) X = -1 0r x = 8
2) X = 4.1 or x = 2.6
3) X = 1.3 or x = -4.3
4) X = 1 or x = -3
5) Y = $\frac{7}{4}$ or y = 3

(ii) 1. $3x^2 - 12x - 20 = 0$ (2) $9x^2 - 15x + 25 = 0$ (3) $x^2 - 2x - 10 = 0$ (4) $x^2 - 4x = 0$

18.11 end of unit exercises

1. Draw the following lines on a graph

a) X – 5 = y
b) Y – 2x + 3 = 0
c) 3y – 2x – 4 = 0
d) X – y = 0

2. Factorize the following

a) $t^2 - 36$
b) $m^2 - 8m + 6$

3. Solve; $x(x - 4)(x + 6)^2 = 0$

4. Solve form in $mt^2 + mt + q = 0$. Using the method of completing the square.

18.12 self check/assessment

No.	Learning outcome	Not sure	Satisfactory
1.	Can now graph linear equations and inequalities		
2.	I can now solve simultaneous equations		
3.	I can now change subjects of the formulae		
4.	I can now solve equations using the three identities		
5.	I can now form and solve quadratic equations		

18.13 Reference for further reading

1. School mathematics of east Africa, book 1,2,3 and 4, Cambridge University press
2. Mathematics revision practice for UCE oxford university press.
3. New general mathematics

UNIT 19: GEOMETRY II

19.1 Introduction

You are most welcome to yet another unit 19.

The unit is a continuation of unit on geometry I which you introduced when you were in year !. in this unit you are expected to be introduced to concepts from the following topics; rectangular Cartesian coordinate and length of a straight line. Circle properties and skills of teaching geometry in primary school.

19.2 content organization

Topic	Sub topic
1. Rectangular Cartesian coordinate system.	(a) Identifying features on the Cartesian coordinate system (b) Plotting coordinates points on a rectangular Cartesian coordinate system (c) Drawing straight line (d) Locating mid-points on straight lines (e) Finding length of straight lines (f) Finding gradient of straight lines (g) Find equations of straight lines
2. Circle	(a) Describing different points of a circle (b) Drawing chords in a circle (c) Constructing tangents of a circle (d) Identifying angle properties in a circle (e) Drawing cyclic quadrilalards (f) Constructing circumscribed triangles in a circle (g) Constructing inscribed triangles of a circle
3. Teaching Geometry in the primary school	(a) Reviewing methods of leaching geometry in primary school

19.3 Learning outcome

By the end of the unit you are expected to use the knowledge of Cartesian graphs and circles to work out geometrical problems.

19.4 competences

(i) Identify features on the rectangular Cartesian coordinate system
(ii) Plot points on the rectangular Cartesian coordinates system.
(iii) Draw straight lines on the Cartesian coordinate system
(iv) Find length of straight lines.
(v) Find gradients of straight lines.

(vi) Relate mid-points on straight lines
(vii) Find equation of straight lines.
(viii) Describe different parts of a straight line.
(ix) Draw chord in a circle.
(x) Construct tangents of a circle.
(xi) Identify angle properties in a circle.
(x) Draw cyclic quadrilaterals.
(xi) Construct circumscribed and inscribed in angles in a circle.
(xii) Review methods of teaching geometry.

19.5 Unit orientation

This topic unit of geometry II is a continuation of geometry I. this requires you for you in fully understand the concept in this unit you need to revise the topics you learnt. More emphasis should be put on plane figures, polygons and Pythagoras' theorem.

19.6 Study requirement

In order to study this topic with ease, you need the following materials.

- A mathematical set with a ruler, a sharpener, paper and a desk/table with a flat surface.

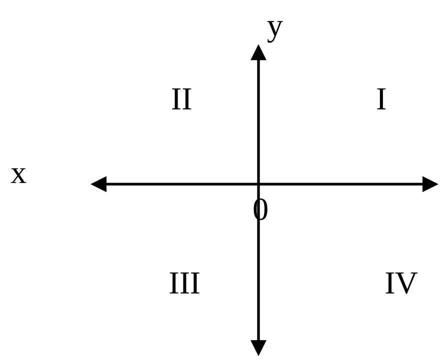
You also require glue, a pair of scissors, a sharpener.

19.7 Content and activities

19.7.1 Cartesian coordinate system

(a) What is a Cartesian graph?

Study the sketch diagram of the Cartesian graph below



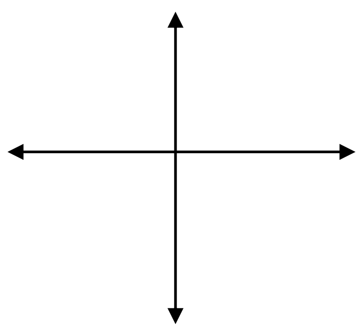
Take note that the vertical line labeled y is called the y-axis or the vertical axis. While the horizontal line labeled x is called x – axis or horizontal axis. The two axis are also called the coordinate axes. The intersection of the two axes divide the plane into four parts (section) called quadrants. These quadrants can be numbered I, II, III and IV in an anticlockwise direction as shown in the diagram above.

A plane in which the coordinate axis has been constructed is called Cartesian plane. The graph drawn on such a plane is a Cartesian plane. The Cartesian coordinate system is also called rectangular coordinate Cartesian system.
The word Cartesian is after the name of a French Mathematician Rene Decartes who first introduced the coordinate system.

(b) Plotting coordinates of points

A point in any quadrant is described by giving its two distances from the origin along the two axis. The distance (x) from the x = axis is always given first, followed by the distance (y) for the y-axis. Therefore, a point is always described as (x,y). it is called an ordered pair because the order of x and y matters.

The diagram below can help you plot coordinates of points.



In the diagram above point A is represented by the ordered pair (1,5), B is represented by (5,1), C is represented by (-4,2) and D is represented by (-2,-6).

Activity 19.1

Using Cartesian graph, plot the coordinates of the following points:

P(0,5), Q(-1,7), R(7,-1), S(-1,-7), T(-5,-1), U(1,7), V(7,1), W(1,-7), X(-7,1), Y(5,0) and Z(0,0)

(c) Drawing straight lines

You have learnt about plotting coordinates of points on the Cartesian graph.

In order to draw straight lines, you need to plot any two points and join them to form a line.

Activity 19.2
Draw straight lines using coordinates of these points on the same axes;

(a) A(0,6) and B(4,4),
(b) P(0,6) and Q(5,7)
(c) W(-3,-2) and X(5,-2)
(d) Y(0,3) and Z(4,-3)
(e) S(1,3) and T(-4,7)

(d) Locating midpoint and straight lines

Mid points of a straight lines are points which lie in the middle of the lines. They are points half way the line.

Example

(a) P(0,1) and Q(6,1)
(b) R (2,-3) and S(5,3)

Graph diagram undrawn

There are many ways of determining midpoints of a line.

One way is by drawing the line on the graph and locating the midpoint through estimation.

Another reliable method is as seen below;

(a) From P(0,1) and Q(0,1)

Midpoint of PQ = $(\frac{0+6}{2}, \frac{1+1}{2})$

$= (\frac{6}{2}, \frac{2}{2}) = (3,1)$

Generally the coordinates of Midpoint $=(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

(b) From R(2,-5) and 5(5,3)

Midpoint = $(\frac{x1+x2}{2}, \frac{y1+y2}{2}) = (\frac{2+5}{2}, \frac{-3+3}{2})$

$= =(\frac{7}{2}, \frac{0}{2}) = (3\ ½ , 0)$

The coordinate of the midpoint of the line RS is (3 ½ , 0) as reflected on the graph above.

Activity 19.3
Find the coordinates of the mid points of these lines;

(a) (2,5) and (0,5)
(b) (1,-2) and (9,2)
(c) (5,-3) and (2,0)
(d) (3,7) and (1, -3)
(e) (-5,1) and (2,-1)
(f) (-3,5) and (4,5)

(e) Finding lengths of straight lines

You have been introduced to finding the equation of a line passing through two points. We can now use Pythagoras' theorem to find the lengths between two points.

The diagram shows three points A (-3,2) B(5,4) and C (5,1)

Graph undrawn

The length of AC = 5—3 = 8 units and BC = 4 – 3 = 3 units

Triangle ABC is right – angled with hypotenuse AB.

Using Pythagoras' theorem,

$AB^2 = AC^2 + BC^2$

$= 3^2 + 3^2$

$= 64 + 9$

$\therefore AB = \sqrt{73} = 8.5$ units

From A(-3,2) and B(5,4)

$AB = \sqrt{(5 - 3)^2 + (4 - 1)^2}$

$= \sqrt{8^2 + 3^2}$

$= \sqrt{64 + 9} = \sqrt{73}$

$= 8.5$ units

This method can be used to find the length of any line whose end point are known.

In general, the length or distance between A(x,y) and B(x_2, y_2)

AB = $\sqrt{(x1 - x2)2 + (y1 - y2)2}$

Example

Find the length between

P= (-2,0) and Q(-5,2)
PQ = $\sqrt{(-2 - -5) + (0 - 2)^2}$

$= \sqrt{(-2 - 5) + (0 - 2)^2}$

$= \sqrt{(-2 + 5)^2 + (-2)^2}$

$= \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4}$

$= \sqrt{13} \cong 3.6$ units

Activity 19.4

(i) Find the length between each pair of these points

(a) (2,7),(4,5)
(b) (0,-3), (6,0)
(c) (4,2), (-8,-3)
(d) (-6,7), (1,-8)
(e) (-4,7),(2,7)
(f) (-5,-2),(1,-6)

(ii) A= (8,6), B= (3,1) and C (8,1). Show that triangle ABC in a right angled triangle

(f) Gradient of straight lines.

As you were drawing graphs of linear equations earlier on, have you noticed that some of them slant upwards from the left to right while others slant downwards. It is time that some graphs slant more steeply than others. The ratio that describes which way a line slants and how it is steep is called a gradient or slope.

A gradient of a straight line therefore, is a measure of its steepness.

The graph below shows line PQ.

Graph undrawn

In the graph above, a move from P to Q is the change in the x – coordinate i.e. 8-2 = 6 units. While the change in the y-coordinate is 4-1 = 3 units. In this case the line upward slopes. Note that the change in the x-coordinates is the **run** while that of the y-coordinates the **rise**

Therefore,the ratio for the line is 3 to 6 i.e.

Ratio = $\frac{rise}{run} = \frac{3}{6}$ = ½

Note the line slopes upward, which is positive gradient.
Since the ratio is the gradient,

$\therefore$ Gradient = $\frac{rise}{run} = \frac{change\ in\ y-coordinate}{change\ in\ x-coordinate}$

$= \frac{difference\ of\ y-coordinate}{difference\ in\ x-coordinate}$

Example

Graph the line containing points (-3,1) and (3,-4) and find the gradient.

Solution:

Not the points on the graph.

Graph undrawn

$\therefore$ Gradient = $\frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x^2 - x^1}$

$= \frac{{}^-4 - 1}{3 - {}^-3}$

$= \frac{-5}{6}$

$\therefore$ The gradient is $\frac{-5}{6}$

Note that the line slopes downward, hence negative gradient.

Activity 19.5

Find the gradient of the lines containing these points:

(a) (1,1) and (3,5) (b) (2,9) and (2,2) (c) (-2,3) and (2,1) (d) (-9,4) and (5,-11) (e) (-4,-6) and (3,-2) (f) (0,-3) and (-4,0) (g) (-10,3) and (-2,-1) (h) (-1,0) and (2,-3)

(g) Finding gradients of horizontal and vertical lines.

Example.

Find the gradient of the lines y=5 and x = -2 from the graph below.

Graph undrawn

From the above graph, the gradient of line y = 5 is

Gradient = $\frac{changing\ in\ y}{change\ in\ x}$

$= \frac{5-5}{3--4} = \frac{0}{7} = 0$

∴ the gradient is 0

For the line x = 0

Gradient = $\frac{change\ in\ y}{change\ in\ x}$

$= \frac{-2-4}{-2--2} = \frac{-6}{0}$ = undefined

∴ The line has no gradient.

Note that;

A horizontal line has gradient 0.
A vertical line has no slope.

(h) Finding Gradient from equations.

You should take note that it is possible to find the gradient from its equation. We begin by finding any two points from the line then apply the formula for the slope.

Example 1

Find the gradient of the line y=2x + 3. From the points (1,5) and (0,3)

Gradient = $\frac{change\ in\ y}{change\ in\ x}$

$= \frac{5-3}{1-0}$ Graph undrawn

$= \frac{2}{1}$

= 2

∴ The gradient = 2

Note that the gradient 2 is the coeffinent of the x – term in the equation y = 2x + 3.

When we have equation in the for y=Mx + C, the coeffinent of the x –term, m, is the gradient.

Example 2

Find the gradient of ½ x + 3y = 7

Solving for y

2x + 3y = 7
3y ÷ 7 – 2x
$\frac{3y}{3} = \frac{7}{3} - \frac{2x}{3}$

∴ y = $\frac{-2}{3}$ x + $\frac{7}{3}$

∴ The gradient is $\frac{-2}{3}$

Activity 19.6

Find the gradient of the following lines

(a) 3x + 8y = 9
(b) 5y + 4x = 7
(c) X – 7 = -5y
(d) 4y + 5x + 8 = 0

(i) Find the gradient and y-intercept

Given that;
(a) y + 4 – 3x = 0

(b) $2x = 6 - 3y$

Solution

(a) Rearranging the equation in the form y = mx + c

$Y + 4 - 3x = 0$

$Y = 3x - 4$

∴ Gradient = 3 and y
Y – Intercept = -4.

(b) $2x = 6 - 3y$
$6 - 3y = 2x$
$-3y = 2x - 6$

$\frac{-3y}{-3} = \frac{2}{-3}x - \frac{6}{-3}$

$= y = \frac{-2}{3}x + 2$

∴ Gradient = $\frac{-2}{3}$ and y – intercept = 2.

Note that in equation y = mx + c is called the gradient – intercept equation of a line. The gradient is in and the y – intercept is C.

Activity 19.7 Find the gradient and the y – intercept of the following; (a) $Y - 5x = 0$ (b) $\frac{3}{2}x + y + 6 = 0$ (c) $3x = 16 - 4y$ (d) $25 - 7x - 5y = 0$

(i) Finding equation of straight lines

You should take note that once we know the gradient of a straight line and the coordinates of one point on that line, it becomes easy to find the equation of the line.

Example.

If P=(2,1) and the gradient of a straight line through p is $\frac{1}{3}$, find the equation of the line

Let B = (x,y) be any point on the line.

∴ gradient of AB = $\frac{y-1}{x-2}$

Since gradient = $\frac{1}{3}$

$\Longrightarrow$ $\frac{y-1}{x-2} = \frac{1}{3}$

$\Longrightarrow$ 3(y – 1) = x -2

3y – 3 = x – 2

3y = x – 2 + 3

3y = x + 1

Y = $\frac{1}{3}$x + $\frac{1}{3}$

Note that the coeffinent of x is $\frac{1}{3}$ which is the gradient.

Example

Find the equation of the line and the coordinates of the points where the line cuts the axes with a line whose gradient is 2 and passes through point Q (-1,-3).

Let R(x,y) be any point on the line.

Gradient QR = $\frac{y- -3}{x- -2} = \frac{y+3}{x+2}$

Given that gradient ≅ 2

= $\frac{y+3}{x+1} = \frac{2}{1}$

$\Longrightarrow$ Y + 3 = 2 (x + 1)

Y + 3 = 2x + 2

Y = 2x -1

∴ The equation y y = 2x -1

(c) Note that the line cuts the x-axis where y = 0

From y = 2x – 1, when y = 0

$\Longrightarrow$ 2 x – 1 = 0

2x = 1

X = ½

∴ The point is (½ ,0)

Example

Find the equation of the straight line passing through S(1,1) and T(-2,2)

Solution:-

Given that S(1,1) and T(-2,2)

Gradient = $\frac{2-1}{-2-1}$ = $\frac{-1}{3}$

Let R (x,y) be a point on the line.

Gradient RT = $\frac{y-2}{k--2}$ = $\frac{y-2}{x+2}$

Since the gradient = $\frac{-1}{3}$

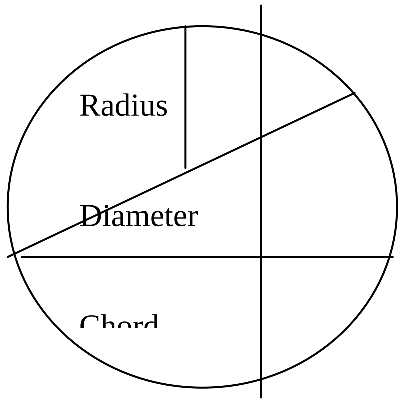
$\Longrightarrow$ $\frac{y-2}{x+2}$ = $\frac{-1}{3}$

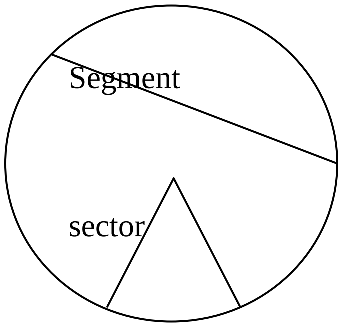
3(y – 2) = -(x + 2)

3y – 6 = -x -2
3y + x = -2 + 6
X + 3y = 4 or y = $\frac{-1}{3}$ x + $\frac{4}{3}$

19.7.2 Circle

(a) Different parts of a circle





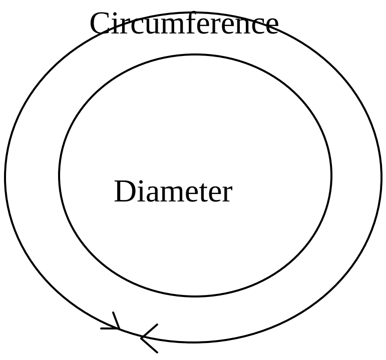
The above diagrams show the major parts of a circle.

What relation exists between the diameter and the radius? Do you still recall that the diameter is twice the radius? i.e. d = 2r.

(b) Circumference of a circle

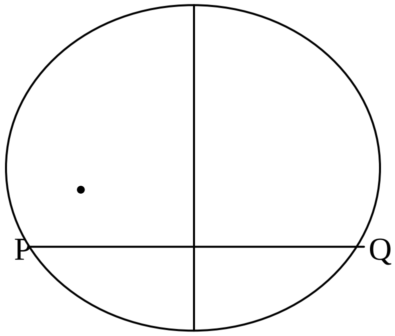
Circumference of a circle is the distance all around the circle. It is the perimeter of the circles.

The diagram below shows the circumference of the circle.



(c) Chords

A chord is a line joining two points in the circumference of a circle. A chord which passes through the centre is the diameter.



Using the diagram above we can establish the relation between the line of symmetry and the chord PQ as follows;

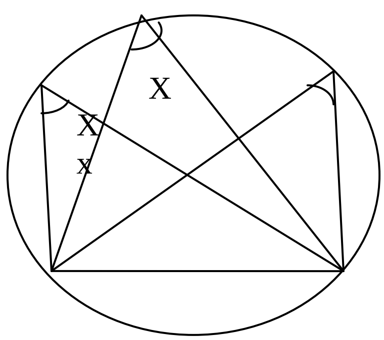
- The line of symmetry bisects the chord PQ.
- The line of symmetry is perpendicular to chord PQ.
- The line of symmetry goes through the centre of the circle O.

In general the perpendicular bisector of any chord of a circle passes through the centre of the circle.

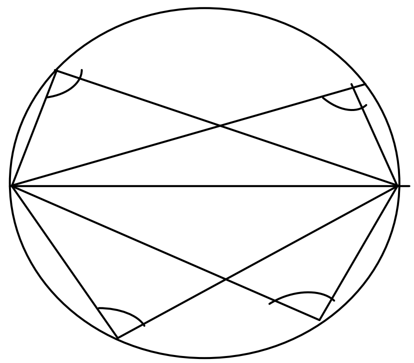
(d) Circle theorems

There are four major circle theorem you need to be familiar with;

(i) Angles subtended at the circumference by a common chord in the same segment are equal.

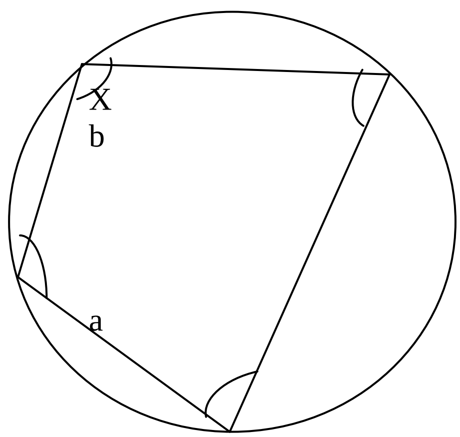


(ii) Angle subtended at the circumference from a diameter is 90^0

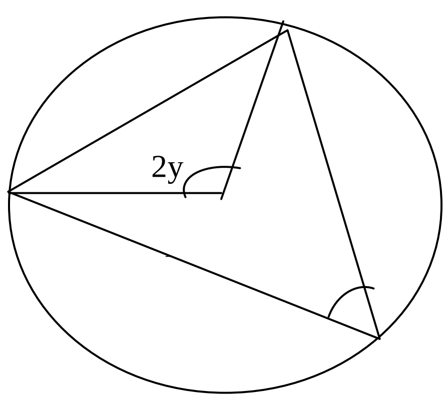


(iii) The sum of opposite angles in a cyclic quadrilateral is 180^0 i.e. from the diaphragm,

$a + b = 180^0$ and
$x + y = 18^0$



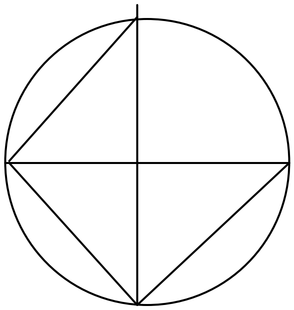
(ii) Angle subtend of the centre is true the angle subtended at the circumference from a common chord as shown in the diagram below.



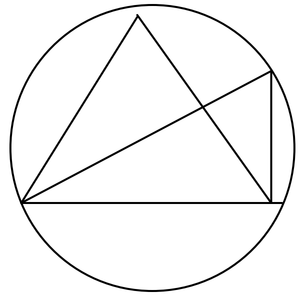
Activity 19.8

Find the missing angles in the diagram below

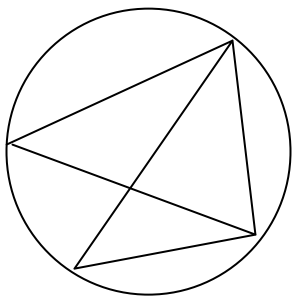
(i)



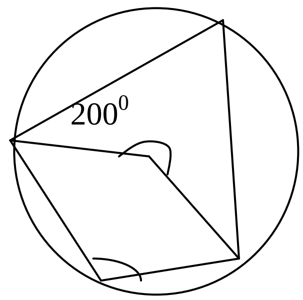
(ii)



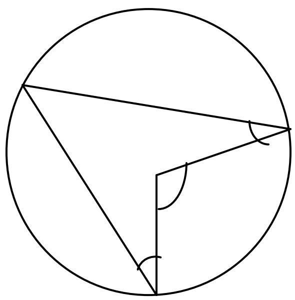
(iii)



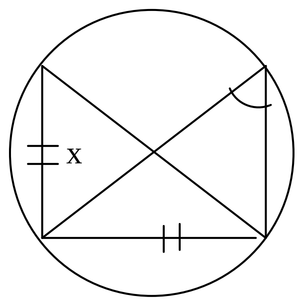
(iv)



(v)



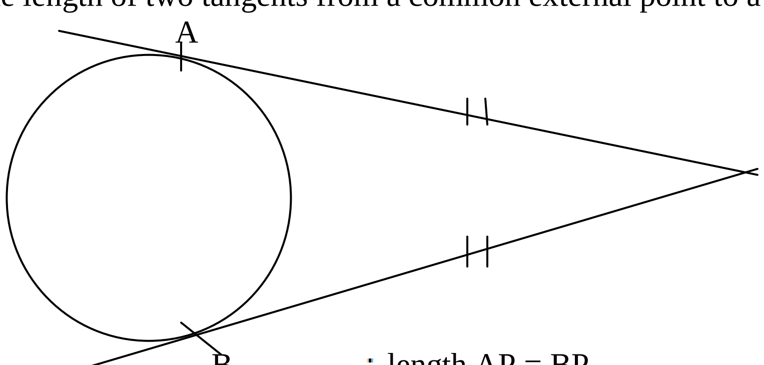
(vi)



(e) Tangents

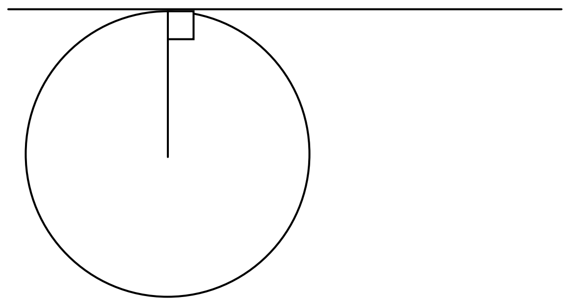
You need to know the three major rules for tangents

(i) The length of two tangents from a common external point to a circle are equal

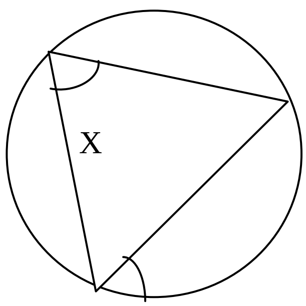


∴ length AP = BP

(ii) The angle between a tangent and a radius drawn from the point of contact is 90^0



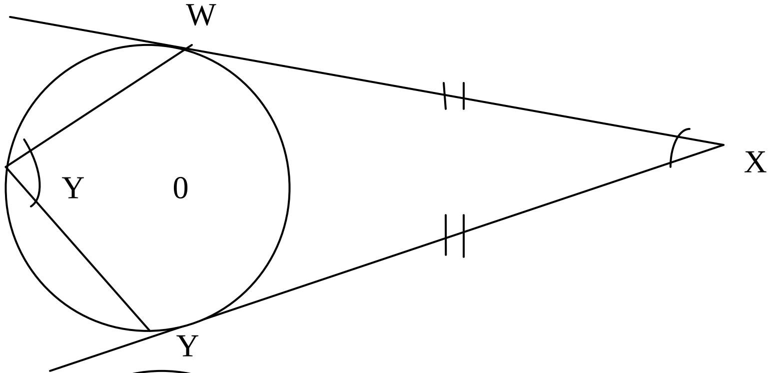
(iii) The alternate segment theorem states that angles marked x are equal



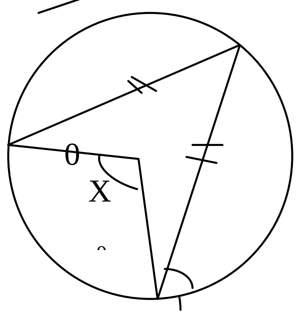
Activity 19.9

Find the value of the missing angles

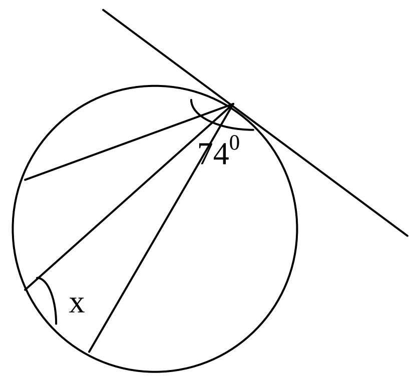
(i)



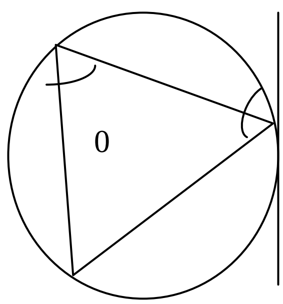
(ii)



(iii)



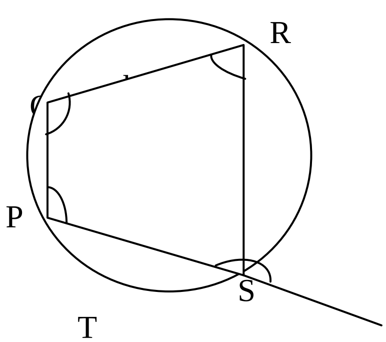
(vi)



(f) Cyclic quadrilaterals

Note that a cyclic quadrilateral is a quadrilateral which can be drawn so that all its corners lie on a circle.

e



Draw a circle radius of 4 cm as seen above.
Measure angles PQR and RSP. Find < PQR + RSP.

Measure angles QPS and QRS.

Find < QPS + QRS.

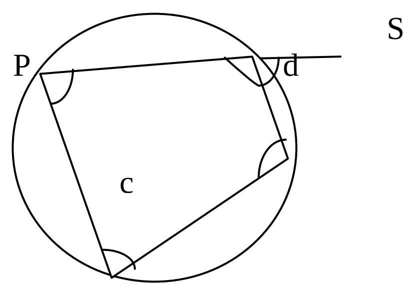
Have you discovered that the sum of the appoinrte angles of the cyclic quadrilateral is 180^0.

In general;

(i) Opposite angles of the cyclic quadrilateral are supplementary i.e. add up to 180^0.
(ii) If one side of the cyclic quadrilateral is produced the exterior angle found in equal to the interior opposite angle.

Activity 19.10

1. Using the figure below answer the questions that follow;



(a) d = 80^0. Give reasons
(b) 50^0 + C = 180^0. Give reasons
(c) 80^0 + W = 180^0. Give reasons
(d) Calculate (i) C (ii) W

2. PQRS is a cyclic quadrilateral and < QPS = 70^0.

(a) Given that PR bisects <QPS and < QRS, calculate these angles; (i) < QPR (ii) <PRQ (iii) < PQR (iv) <PSQ.

(b) Show that PR is the diameter of the cycle.

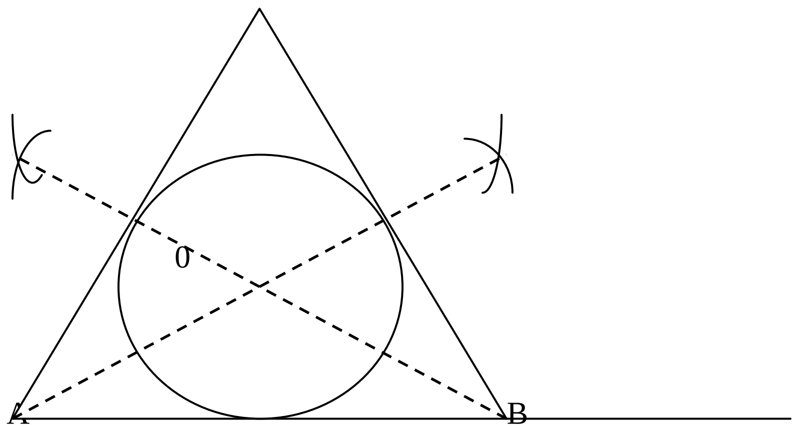
(g) Construction of a circle inscribed in a triangle.

Draw a triangle ABC with sides 6 cm each. Bisect any two angles and let the bisector meet at a point 0, the centre of the inscribed circle.

Draw the circle in the triangle as seen below.

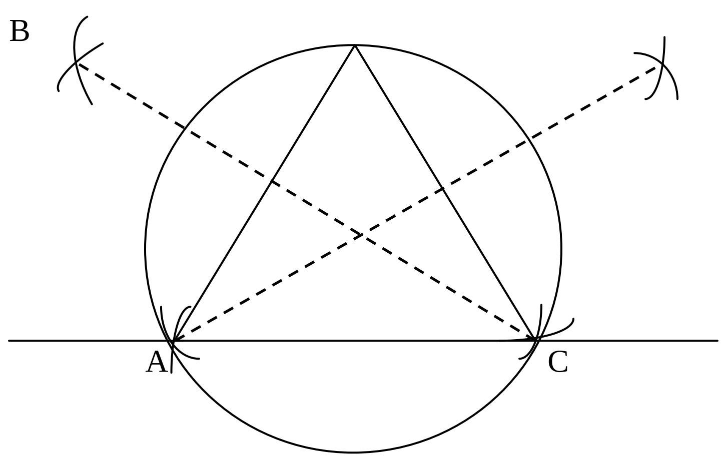
B

0



(h) Construction of a circle circumscribed on a triangle.

Draw a triangle ABC with sides 6cm each. Bisect any two sides of the triangle and let the bisects meet at a point 0; the centre of the circumscribed circle. Draw the circle passing through the point ABC as seen below.



Activity 19.11

Draw five triangles of different sizes and for each of them;

(a) Draw an inscribed circle.
(b) Draw a circumscribed circle.

19.7.3 Teaching Geometry in Primary school

Activity 19.12

(i) Pick a concept in one of the topic in geometry and prepare for a demonstration lesson.
(ii) Prepare a detail scheme of work for the lesson.
(iii) Prepare a detail lesson plan on how to learn the concept.
(iv) Ensure that you choose appropriate instructional materials.
(v) Select participatory methods and techniques for the lesson

Activity 19.4

(i) (a) 2.8 unit (b) 7.8 unit (c) 13 units (d) 10.6 unit (e) 10 units (f) 10 units

(ii) Graph undrawn

From the diagram above, from that $\triangle$ ABC is a right angled triangle, there is need to use pythagoras theorem.

i.e. AB = $AC^2 + BC^2$

AB = $\sqrt{5^2 + 5^2}$

AB = $\sqrt{50}$ = 7.07 unit

Length AB = $\sqrt{(3-8)^2 + (1-6)^2}$

= $\sqrt{-5^2 + -5^2}$

= $\sqrt{50}$ = 7.07 units

Since $AB^2 = AC^2 + BC^2 = 7.07$

$\triangle$ ABC is a right angled triangle

19.8 Unit summary

In this unit on geometry II, you have learnt concepts based on rectangular Cartesian coordinate system, circle and methods of teaching Geometry in primary school.
You need to consistently revise these concepts using as many problems as possible. Remember practice makes perfect.

19.9 Glossary

Cartesian Plan - It is a plane in which coordinate axes have been constructed.

Chord - It is a line joining two points on the circumference of a chord.

Cyclic Quadrilateral - It is a quadrilateral which can be drawn so that its corner lies on a circle.

Gradient - It is a measure of its steepness of a straight line.

Tangent - It is a line which no matter how far it is extended touches the circle at one point only.

Quadrants - These are sections within the Cartesian plane is divided into four quadrants.

19.10 Answers to the activities

Activity 19.1

Graph undrawn

Activity 19.2

Graph undrawn

Activity 19.3

(a) (1,5) (b) 50 (c) $(\frac{7}{2}, \frac{-3}{2})$ (d) (2,2) (e) $(\frac{-3}{2}, 0)$ (f) (½ , 2)

With angle C = 90^0

Activity 19.5

(a) $\frac{4}{3}$ (b) $\frac{7}{6}$ (c) $\frac{-1}{2}$ (d) $\frac{-8}{7}$ (e) $\frac{4}{7}$ (f) $\frac{3}{4}$ (g) 4 (h) -1

Activity 19.6

(a) $\frac{-3}{8}$ (b) $\frac{-4}{5}$ (c) $\frac{-1}{5}$ (d) $\frac{-4}{5}$

Activity 19.7

a) Gradient = 5, y - intercept = 0

b) Gradient = $\frac{-3}{2}$, y – intercept = -6

c) Gradient = $\frac{-3}{4}$, y intercepts = -4

d) Gradient = $\frac{-7}{5}$, y – intercepts = -5

Activity 19.8

(i) $y = 10^0$, $x = 10^0$ (ii) $x = 75^0$ (iii) $y = 50^0$ (iv) $x = 100^0$ (v) $x = 64^0$ (vi) $x = 45^0$

Activity 19.9

(i) $y = 62.5^0$ (ii) $x = 120^0$ (iii) $x = 106^0$ (iv) $x = 30^0$

19.11 End of unit exercise

1. Find the mid-point of the following lines,

(a) (-2,5) and (2,5) (b) (1,8) and (2,-5)

2. Find the length of these straight lines

(a) (0,4) and (0,-4)
(b) (9,1) and (-9, -1)

3. Find the gradient and state the y-intercepts for the following equations.

(a) $y – 4x = 2$ (b) $3x – 2 = y$ (c) $y – 4 + 7 = 0$ (d) $y + x = ½$

4. Find the equation of the lines joining the two points;

(a) (5,-3) and (2,0) (b) (-3,7) and (-1,-3) (c) (-5,1) and (2,-1) (d) (-3,0) and (2,-5)

5. Find the circumference, diaeter and area of a circle with radius 7 cm

19.12 Self check/Assessment

Learning outcome	**Not sure**	**Satisfactory**
1. I can draw Cartesian graphs		
2. I can plot coordinate of points		
3. I can draw straight lines		
4. I can locate midpoints of straight lines		
5. I can find lengths of straight lines		
6. I can find gradients and y-intercepts		
7. I can name different parts of a circle		
8. I can state circle theorems		
9. I can apply the rules for tangents		
10. I can find angles of the cycle quadrilateral using angle properties.		

19.3 Reference for further reading

1. School mathematics of East Africa Books 3 and 4
2. Secondary school mathematics Macmillan, Uganda Book 2
3. Mathematics Revision and Practice for UCE, Oxford University Press
4. Primary Mathematics Course Books.

UNIT 20: STATISTICS

20.1 Introduction

You are welcome to unit 20 which discusses different ways of collecting statistical data, summarizing it and interpreting it. It is hoped that you will be actively engaged in collecting data which you will use for this unit's activities.

20.2 Content Organisation

This unit is organized as shown in the table below:

Topic	Subtopics
3. Collecting statistical data	• Use of statistics in daily life • Different ways of collecting data
4. Formation of frequency tables	• Ungrouped data • Grouped data
5. Calculation of mode, median and Mean	• Mode of ungrouped data • Median of ungrouped data • Mean of ungrouped data • Mode for grouped data • Mean for grouped data
6. Graphical presentation of grouped data	• Frequency polygon • Commutative frequency curve
7. Measures of Spread	• Range • Interquartile range Histogram

20.3 Learning outcome

By the end of studying this unit, you should be able to collect, organize and analyse data.

20.4 Competences

- Outline the uses of statistics in daily life
- Organize data in tables
- Represent data in charts and graphs
- Analyse and interpret statistical information
- Teach statistics in the primary school

20.5 Subject orientation

You need to revise the topic of graphs and interpretation in the primary school. Be prepared to learn through your own activities in data collection and organisation.

20.6 Study Requirements

You are going to be practical and you will need a notebook, pen, ruler and a calculator. Also have a mathematical set and graph paper.

20.7 Content and activities

20.7.1. COLLECTING STATISTICAL DATA

1. Uses of statistics in daily life

What do you understand by the word statistics?
What does statistics deal with?
Compare your response with those of your classmates.
You will all agree that statistics deals with collecting, summarizing, analyzing and interpreting information. Most times, the information collected is numerical in form.
List five different types of information you may collect from your school and home environment.

Statistical data is usually collected in an attempt to understand or make predictions about a given situation. A teacher may wish to know what the learners have achieved and gives a test. A doctor may be interested in the number of new outbreaks of influenza at the beginning of the school term. Discuss with your classmates how statistics is used by different categories of people.

2. Different ways of collecting data

How do teachers, doctors, surveyors, governments and all institutions that use statistics gather information?
Some might use records already in existence while others carry out original data collection in the form of observations, interviews, censuses, questionnaires, and checklists. The data collected might be numerical (quantitative) or categorical (qualitative).

We shall discuss the nature of the different ways of collecting data.

(i) Observations

When you count the number of vehicles passing by your school at break time, lunch time, etc you are gathering data by direct observation.

Observation reduces the chances of collecting incorrect data. However, it may not always be possible to collect data this way.
Discuss with you classmates the advantages and disadvantages of collecting data by observation.

(ii) Interviews

Data may be collected through oral interviews. How will one keep data collected in this way? Conduct a simple oral interview to find out the age, number of sisters, number of brothers, best colour, favorite food and most liked subject for ten of classmates. Could you obtain this information through observation or do you find an oral interview a better tool?

(iii) Questionnaires

This form of collecting data involves handing out a set of written questions to respondents. The questions may be; simple Yes/No, rated on a 5 scale of; Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D) and Strongly Disagree (SD); or open-ended.

However, the questions should be simple, unambiguous, relevant and as short as possible.

(iv) Checklists

A checklist will have a list of items of interest which the person collecting data ticks as present, observed, seen, achieved or otherwise. As a teacher, you may wish to have a checklist to assess if a P.1 pupil is able (not able) to; count from one to five, write numerals from one to five, add using digits one to five or draw pictures to represent numbers from one to five. Now use the above ways to collect data that may be useful to a teacher.

20.7.2 FORMATION OF FREQUENCY TABLES

a). Ungrouped data

The data you gathered in topic one is known as raw data. It is not arranged or sorted in an order. One way of organising data is to form a frequency table. We shall do this using an example.

Study the examples below;

The scores of 20 pupils obtained in a test scored out of ten are shown below:

4, 6, 5, 4, 6, 6, 3, 5, 5, 3,
3, 5, 5, 4, 7, 6, 5, 8, 6, 2

To form a frequency table, we shall do the following:

i. Write down the smallest score which is 2.
ii. Write down the greatest score which is 8
iii. Write the scores from 2 to 8 in a column labelled score (item) or x.
iv. Take each score in the raw data in turn and place a tally mark (/) in the second column labelled tally, opposite the appropriate score. Note that the fifth tally mark for each score is made across (---), thereby tying the tally marks into bundles of five for easy translation into frequency values.
v. When the tally column is completed, count the tallies and record the numerical values in corresponding position in the column labelled 'frequency'. What is meant by frequency?

For the raw data above, we have:

Score (x)	**Tally**	**Frequency**
2	/	1
3	///	3
4	///	3
5	~~////~~ /	6
6	~~////~~	5
7	/	1
8	/	1
TOTAL		**20**

From the frequency table above, we can see that a score of 3 occurred three times and 5 occurred six times, 5 occurred six times, etc.

Here is an activity for you to do in your exercise book..

✍ **Activity 20.1**
The ages of 30 members of a youth club were recorded as below:

17 18 17 16 14 15
14 15 14 17 13 14
14 15 15 16 18 14
13 14 16 17 16 14
14 14 14 16 15 16

(i) Form a frequency table for the ages.
(ii) What is the mode?
(iii) Find the modal frequency.

b). Grouped data

When dealing with a large amount of data, it is helpful to group the information into classes. We then determine the number of items which belong to each class, thereby obtaining a class frequency. The number of classes is usually between 6 and 15, depending on the amount of data. Let us study the following data:

The time in seconds that it took 40 children to swim a certain length are shown below:

40, 42, 38, 30, 39, 36, 45, 32, 40, 42
46, 31, 35, 50, 47, 32, 46, 38, 39, 42
35, 44, 51, 52, 47, 48, 41, 44, 37, 43
49, 36, 48, 43, 41, 43, 41, 53, 43, 52

Show this data in a grouped frequency table using classes 30-34, 35-39, and so on. What is the biggest value? Our last class must contain this value.

A frequency table showing time in seconds used for swimming

Class	**Tally**	**Frequency (f)**
30-34	////	4
35-39	~~////~~ ////	9
40-44	~~////~~ ~~////~~ ////	14
45-49	~~////~~ ///	8
50-54	///	3
TOTAL		**40**

In the grouped frequency table above, the end numbers such as 30, 34, 35, and 39 are called class limits. When 30 and 35 are lower class limits, 34 and 39 are upper class limits. In addition, the times recorded are corrected to the nearest second. So the class interval 30-34, includes all the times between 29.5 and 34.5 seconds. These numbers are called the lower and upper class boundaries.

Using the above example can you try activity 20.2

✍ **Activity 20.2**

The figures below are measurements of the sound levels in decibels at 36 places of prayer

93, 87, 103, 98, 90, 91, 92, 105
98, 89, 88, 86, 88, 85, 102, 100
103, 95, 99, 92, 92, 86, 85, 96
89, 86, 98, 91, 82, 94, 100, 87
86, 85, 95, 88

Form a grouped frequency table for the data using classes 81-85, 86-90 and so on.

Draw a frequency table

20.7.3. Calculation of Mode, Median and Mean

(a) Mode of Ungrouped data

The mode of a set of data is the value that occurs most frequently. For example, the mode of 2, 3, 3, 4, 4, 5, 6, and 7 is 4, because it occurs three times which is more than any of the other numbers in the set.

(b) Median of Ungrouped data

If a set of values is arranged in increasing or decreasing order of size, the median is the value which lies in the middle. The set of values 2, 3, 3, 4, 4, 4, 5, 6 and 7 have median of 4. on the hand, the set of values 2, 3, 4, 5, 6 and 7 have median $\frac{4+5}{2}$ = 4.5.

In the case where the set has an even number of members, the median is the average of the middle values.

(c) Mean of Ungrouped data

The mean of a set of values is found by adding all the values in the set and then dividing the sum by the number of values.

The mean of 2, 3, 4, 5 and 6 is $\frac{2+3+4+5+6}{5} = \frac{20}{5} = 4$

✍ **Activity 20.3**

(a) Find the mean of:

i. 5, 3, 8, 6, 4, 2, 8
ii. 2, 4, 6, 5, 3, 1, 8, 9

(b) Determine the mode of the following marks obtained by 20 students: 40, 15, 43, 35, 35, 39, 27, 50, 17, 39, 17, 28, 18, 17, 14, 21, 30, 36, 12, 20.

(c) Find the mean of $3\frac{1}{2}$, $2\frac{1}{2}$, 2, 1, $3\frac{1}{2}$, 2, $3\frac{1}{2}$, 3, 3, 1, $1\frac{1}{2}$, $2\frac{1}{2}$ and $3\frac{1}{2}$

Check your answers with text below:-

Look at further calculation of mode, median and mean for grouped data.

(a) Mean of grouped data.

Using the following examples;

The masses of 80 young men.

Mass (kg)	40-44	45-49	50 -54	55-59	60-64	65-69	70-74	75-79
Number of men	1	6	14	21	18	12	6	2

Find the mean mass (kg)

Read the procedures in the table below:

You need to assume that all the men in the same class have the same age, which is the midpoint of the class, and proceed as before, e.g. the midpoint of the class 44-44 is the average of the end point referred to as lower and upper class limits respectively:- which is

$$\frac{40+44}{2} = 42$$

Therefore to find mean, develop a table as shown below:-

Mass (kg)	|Midpoint (x)	Number of men (f)	f.x
40-44	$\frac{40+44}{2} = 42$	1	42
45-49	$\frac{45+49}{2} = 47$	6	282
50-54	$\frac{54+54}{2} = 52$	14	728
55-59	$\frac{53+57}{2} = 57$	21	1197
60-64	$\frac{60+64}{2} = 62$	18	1096
65-69	$\frac{65+69}{2} = 67$	12	804
70-74	$\frac{70+74}{2} = 72$	6	432
75-79	$\frac{75+79}{2} = 77$	2	154
		ξf = 80	**ξfx = 4635**

∴ The mean $(\overline{x}) = \frac{\xi f x}{\xi f} = \frac{4635}{80}$

= 57.9 kg

If you have followed the above example read the text:

> The answer is an estimate. An accurate answer could be got by using all the ungrouped masses. But this takes much longer. It is the order of the size that is very important when finding mean of a large number of items. Mean is denoted as "x"

Activity 20.4

1. The score of 90 pupils in a test were recorded and data grouped. The results obtained were:-

Score	F
15 - 19	1
20 - 24	13
25 - 29	29
30 - 34	25
35 - 39	19
40 - 44	3

Use the grouped data to calculate the mean score.

2.

Volume in ml	94 - 120	110 – 114	115 – 119	120 - 124
Number of glasses	1	15	27	10

Find the mean volume in ml.

TOPIC 4: MEASURES OF CENTRAL TENDENCY

Hullo student, you are welcome to this topic.

- Do you know something on measures of central tendency?
- Can you list down these measures of central tendency.

Well, let you find out through the following.

(i) Mean

A group of five boys were given a mathematical assignment and each received the following scores after the assignment.

John	88
Peter	86
Paul	85
Eric	78
Isaac	88

Their teacher informed them that their mathematical assignment score would be the average of their groups that scores. Paul told his parent that he scored 86, peter told his dad that he scored 88 and Isaac told his mum he scored 85.

Now, how can all the three boys think they have told the correct scores.

> Note: there are three common ways to describe this set of data. These ways are called <u>measures of central tendency.</u> They are the mean, the mode and the medius

Lets see how the boys used this measures to active arrive at their scores.

i) Isaac those to add all the marks of the five boys and divided by the number of boys who did the assignment i.e.

John + Peter + Paul + Eric + Isaac

$$\frac{88 + 86 + 85 + 78 + 88}{5}$$

$$= \frac{425}{5} = \underline{85}$$

> Note the mean of data is got by adding up all the values and dividing by the number of values

(ii) Mode

Peter used the score that appears most often to describe the set of data. Lets see how he arrived at his score.

He had to arrange the scores from the least to the greatest score i.e. 78, 85, 86, 88, 88. He had to develop a table as shown.

Score	**Tally**	**No. of students**
78	I	1
85	I	1
86	I	1
88	II	2

$\xi = 5$

$\therefore$ Mode = 88

Note: the score which appears most often in the set of data is called the <u>mode</u> if there was another score of 86, then 86 would also be mode and the data would have two modes. A set of data in which no members appear more than once, has no mode.

(iii) Median

Then Paul used the median, he organized his scores from least to greatest and choose the middle number.

78, 85, $\boxed{86}$ 88, 88.

The middle score is 86, the median score is 86.

Note! If the number of data is even, the set has two middle numbers. In that case, the middle is the mean of the two numbers. E.g. m $\{11,12,15,22,26 \ and \ 29\}$, the middle numbers are 15 and 22.

The mean is $\frac{15+22}{2}$ or 185

Therefore the median is the number in the middle when the data are arranged in order. When there are two middle numbers, the median is their mean

(iv) Application of mean

Example I:

10 pupils had to sit for a maths paper and their mean mark was 64. Nine of the pupils got the following marks 80, 72, 63, 52, 45, 85, 58, 75, 50.

Determine the mark of the 10th student.

Remember to find mean = $\frac{\text{sum of scores}}{\text{number of scores}}$

Let the 10th mark be y

$$64 = \frac{80+72+63+52+45+85+58+75+50+y}{10}$$

$$64 = \frac{580+y}{10}$$

Then cross multiple by 10.

64 x 10 = (580 + y)
64 x 10 = 580 +y
640 = 580 + y
60 = y.

∴ The mark of the 10th pupil is 60.

Example iv

In a certain game, Bod scored the following points 3,12,2,8,0,3,5,7

Determine the median and the mode of the points scored in the game.

Median : to find the median arrange the points from the least to the greatest i.e. 0,2,3,3,5,7,8,12.

The median middle numbers and divide by

$$\text{Median} = \frac{3+5}{2}$$

$$= \frac{\cancel{8}}{\cancel{2}}$$

∴ The mode point is 4

ii) The mode; is got by finding the number which appears most often is 3

∴ The mode point is 3

Activity 7.9

Using the illustrated above examples. Do the following exercise 6B – numbers 1 1A numbers 1,2,3,5 pages 96, 101 and 102 from Secondary Mathematics book 3. In plenary share your answers with your colleagues.

Well done.

Please can you proceed with the next topic?

TOPIC 5: MEASURE OF DISPERSION

(i) Range

Hello student;

Can you tell what range is?

How is range of data determined?

Find out by doing the following.

Arrange the following scores in ascending order.

188, 180,156,174,145,136,135,170,125.

Find the difference between the highest score and the least score which 188 – 125 =

Compare your answer with a friend.

Note difference between the highest score and the least score is called the range.

Let you continue.

(ii) Interquartile range

The range of a data can be sub divided into three i.e. the lower quartile range, the median and the upper quartile range.

In a large set of data, such as thousand of college exam tests scores, it is helpful to separate the data into four parts called <u>quartiles</u> . quartiles are used in another measure of dispersion called <u>the inter quartile range.</u>

To find the inter quartile range, first find the middle half of the data.

Let's consider data given below

1, 2, 3, 3, 6,6, 7,9, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 14, 15, 17

In the 1st place, find the median of the data since the median separates the data into two halves

1, 2 ,3, 3, 6,6, 7,9, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 14, 15, 17

median

Then also find the median of the upper half. This number is called the upper quartile, indicated by (u Q)

1, 2, 3,3, 6,6, 7,9, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 14, 15, 17

Median UQ

Also find the median of the lower half. This number is called the quartile, indicated by LQ

1, 2, 3, 3, 6,6, 7, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 14, 15, 17

LQ median UQ

The middle half of the data goes from 7 to 13. Then subtract the lower quartile from the upper quartile is 13 – 7 = 6

The inter quartile range is 6.

Now with the above information carry out the activity below.

Activity 7.10
With your group mates do the following exercise on piece paper

1. Find the range of each set of data.
   a) 3,4,5,7,8,8,10
   b) 12,17,16,23,18
   c) 135,170,125,174,136,145,180,156,188

2. Find the mean, median, upper and lower quartiles of each set of data.

   a) 135,170,125,174,136,145,180,156,188
   b) 12,17,16,23,18,
   c) 3,4,5,7, 8,8,10

3. Find the inter quartile range of data in 2 (a)

Should your answers to your tutor.

Thank you for completing topic 4.

TOPIC V: HISTOGRAM, FREQUENCY POLYGON AND CUMULATIVE FREQUENCY (OGIVE)

Try to study the following example.

The following is frequency for the weights in kg of adult patients who visited a certain doctor in a certain week.

Weight (kg)	**50 - 54**	**55 - 59**	**60 – 64**	**65 - 69**	**70 - 74**	**75 - 79**
Number of patients (f)	3	5	8	11	21	19

(a) (i) Draw a histogram/bar graph
(ii) What is the modal class

(b) Construct a frequency polygon of the patients weight.

(c) Draw a cumulative frequency curve (ogive)

Histogram of grouped data

(i) Find the class boundaries of each class e.g. 50 – 54 becomes (50.0 – 0.5) – (54 .0 + 0.5)
(ii) Draw a new frequency table using class boundaries.

Class	**F**
49.5 – 54.5	3
54.5 – 59.5	5
59.5 – 64.5	8
64.5 – 69.5	11
69.5 – 74.5	21
74.5 – 79.5	19

Note that the class interval remains the same e.g. the class interval of 49.5 – 54.5 can be worked out as follows:

$$\begin{array}{r} 54.5 \\ -\ 49.5 \\ \hline 5.0 \\ \hline \end{array} \qquad \therefore \text{The class interval is } 5$$

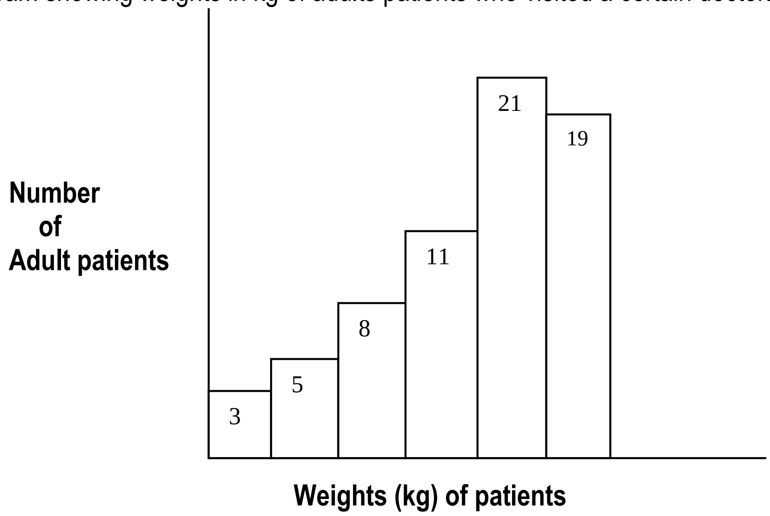
- And also the upper class boundary limit becomes the lower class boundary limit of the immediate following class.

Steps to draw a histogram

- Get a scale for the graph.
- Horizontal axis use the class boundaries to make the axis.
- Vertical axis. Use the following: get a range /space/gap represents: 2.

Think of a title for the graph.

(i) A histogram showing weights in kg of adults patients who visited a certain doctor.



Therefore you can easily get the modal class by looking at the histogram and the longest bar indicates the modal class.

∴ modal class 75 – 79.

Then the mode could be $\frac{75+79}{2}$

$= 77$

(b) A frequency polygon

To draw a frequency polygon. Follow the steps as shown below:-

- Create two (2) imaginary classes at the beginning and end of the given data and whose frequencies are "O". e.g. 45 – 49 and 80 – 84 respectively.
- Then find the mid points of the classes:

Study the new frequency table

	Class (kg)	f	Mid pt (x)
Created class	**45 - 49**	**0**	**47**
	50 - 54	3	52
	55 - 59	5	57
	60 - 64	8	62
	65 - 69	11	67
	70 - 74	21	72
	75 - 79	19	77
Created class	**80 - 84**	**0**	**82**

- Find the scale of the graph.

  Horizontal axis.
  Use the midpoint to mark the axis

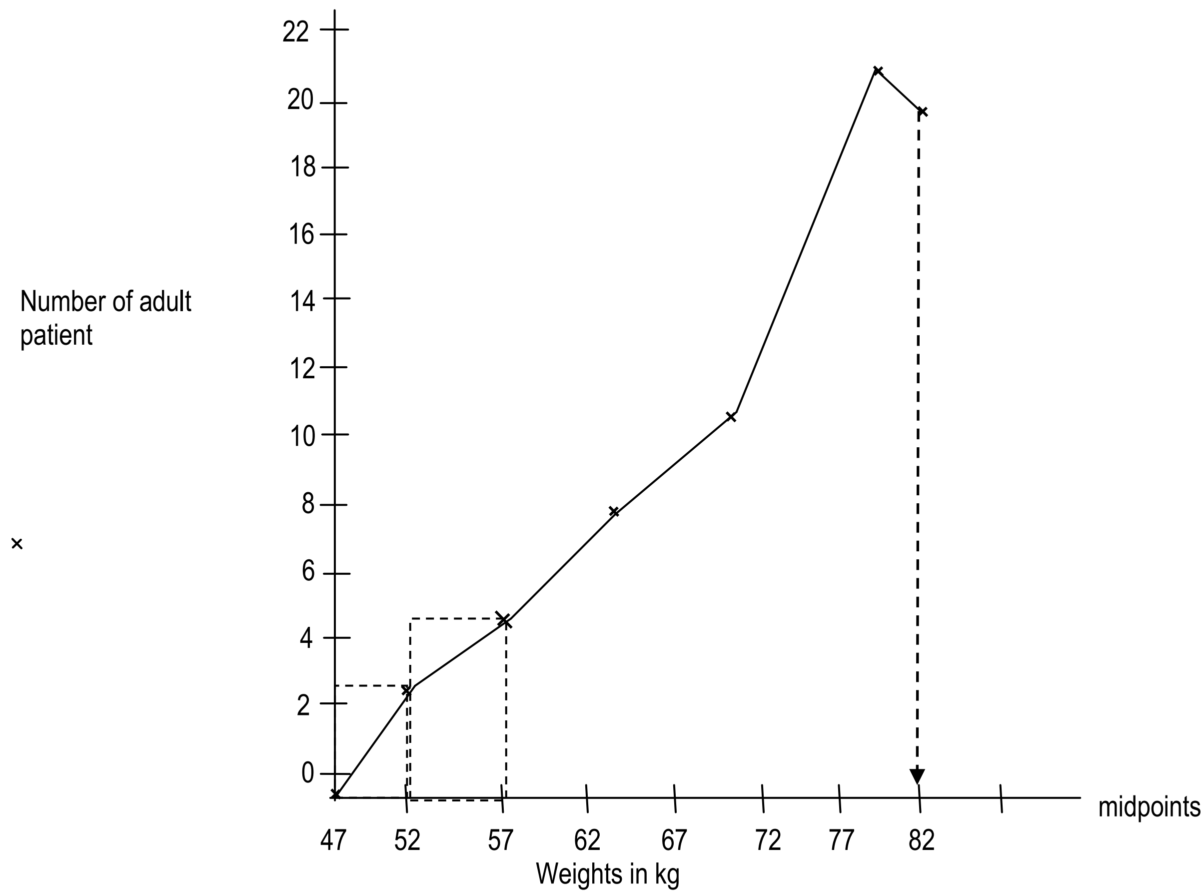
Then vertical axis

Use the frequency
/gap represents: 2

- Think of a title (heading) of the graph.
- Draw the axes perpendicular to one another.
- Label the axes.
- Plot the points.
- Join the point using a ruler.
- Observe and discuss with tutor/colleagues. E.g. why create the two imaginary classes? Why use a ruler while joining points?

See the graph below:-

A frequency polygon showing the weights in kg of adults patients who visited a certain doctor;



(c) Cumulative frequency (Ogive) and median of grouped data:

Read the text below:-

The median of grouped data may be determined by drawing a cumulative frequency curve. The median is the value of the variable corresponding to half the total frequency.

Study the example illustrated below:

Using the above example: to draw a cumulative frequency curve;

- Draw a frequency table as shown below.

Note: create an imaginary class at the beginning i.e. 45 – 49 whose frequency is 0.

class	f	Mid point (x) or less	Cumulative frequency (CF)
45 - 49	**0**	47	0
55 - 59	3	52	0+3=3
60 - 64	5	57	0+3+5=8
65 - 69	8	62	0+3+5+8=16
70 - 74	11	67	0+3+5+8+11=27
75 - 79	21	72	0+3+5+8+11+21=48
	$\underline{19}$ $\xi f = 67$	77	0+3+5+8+11+27+19=67

- With the guide of your tutor discuss some important observations

Compare your observation with the following:-

You will notice that cumulative frequency keeps on piling the figure until you get the sum which is equal the sun of frequency column.

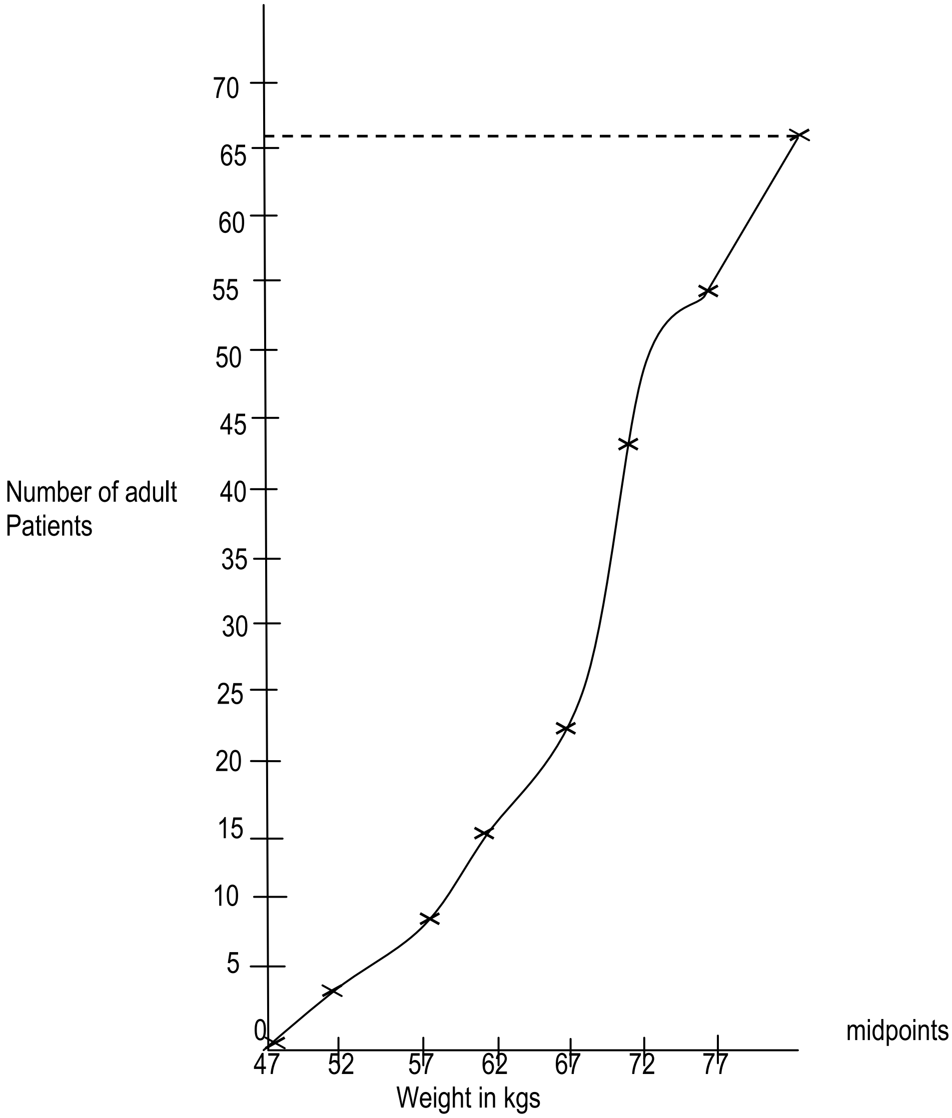
You need also to note that frequency is denoted by a small letter (f) were as cumulative frequency is denoted either by capital letter “F” or C.F.

Now to draw a cumulative curve graph.

- Think of a title.
- Use the midpoints (x) on the horizontal axis and Cumulative Frequency on the vertical axis.
- Get a scale.
- Plot the points, use free hand to join the points. (why?)

  See the example below:-

A cumulative frequency graph showing the weights of adult patients visiting a certain doctor



In groups, do the following work on sheets of paper.

Activity 20.5

Copy and complete the table below

Mass (kg)	**Midpoint (x)**	**f**	**f.x**
40-44	(40+44)/2 = 42	1	…
45-49	…	6	…
50-54	…	14	…
55-59	…	21	…
60-64	62	18	…
65-69	…	12	…
70-74	…	6	…
75-79	77	2	…
		$\sum f = \ldots$	$\sum f..x = \ldots$

(b) Find the mean mass.

(c) Draw the following graphs:

(i) A histogram
(ii) A frequency polygon
(iii) A cumulative frequency curve.

(d) Use the cumulative frequency curve to determine the median.

In plenary, present your findings and let the tutor summarize. Copy the work in your exercise book.

For further practice:

With your colleague get cliff Green Mathematics Revision and Practice for USE pages 100 – 110, do 22b UNEB questions numbers 1,3 and 5.

Compare your answers at the back of the reference book pages 138 – 139.

Thank you for completing this topic V.

TOPIC VI: MEASURES OF SPREAD

(a) Range

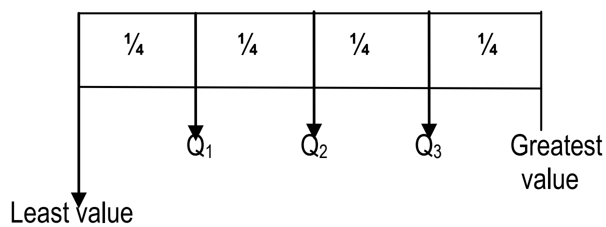
The range is the difference between the largest and the smallest values in a given data. It tells us how widely the values are spread out. The range e.g. 57, 62, 84, 68, 40 and 80 is 80 – 84 = 44. The range uses extreme values, but it is rarely used.

(b) Interquartile range

The interquartile range uses the upper and lower quartile values to measure spread. Each quartile divides data into four equal parts. The quartiles are denoted as Q_1, Q_2 and Q_3

This can be illustrated in the diagram below:

Data arranged in increasing order



What other name is given to Q_2 ? Q_1 is called the lower quartile while Q_3 is the upper quartile. The inter-quartile range then is: Q_3 - Q_1. It takes into account the middle half of the data. Values of Q_3, Q_2 and Q_1 can be read from the cumulative frequency curve. Find the interquartile range for the mass of the 80 young men in activity 20.6

20.8 Unit Summary

You have now completed unit 21 in which you learnt about statistics. In this unit, you collected data, summarised it in frequency tables and graphs and calculated modes, medians and mean. You also learnt about range and interquartile range as measures of spread.

20.9 Glossary

Checklist : a list of times that someone can tick off as present, realised or missing

Data: information

Pictogram: diagrams that use pictures to represent numerical information

Pie chart: a chart that displays information in parts (sectors) of a circle

Questionnaire: a set of written questions to be answered by several people to help collect data on a given topic.

Statistics: the collection, organisation and analysis of numerical facts

20.10 Answers and activities

Activity 20.1

A frequency distribution table showing ages of youth club members

(i)

Ages (x)	Tally	f
13	//	2
14	~~////~~ ~~////~~ /	11
15	~~////~~	5
16	~~////~~ /	6
17	////	4
18	//	2

(ii) The modal age ($\hat{x}$) = 14
(iii)The modal frequency 11

Activity 20.2

A frequency distribution showing measurements of the sound levels.

Class	Tally	f
81- 85	////	4
86 – 90	~~////~~ ~~////~~ /	11
91 – 95	~~////~~ ////	9
96 – 100	~~////~~ ///	8
101 - 105	////	4
Total		**36**

Activity 20.3

a) (i) Median = 5 (ii) Median = 4.5
b) Mode = 17
c) Mean = 2.5

Activity 20.4

Mean = 29.1 scores

21.11 End of Unit Exercise

3. Twenty five army cadets were given a blood test to determine their blood group and the results were as follows:

A	B	B	AB	O
O	O	B	AB	B
B	B	O	A	O
A	O	O	O	AB
AB	A	O	B	A

Construct a frequency table for this data. Which is the most common blood group?

4. The distribution of years of service for 75 employees of a certain bank is given in the following table.

Years of service	Number of employees
1-5	21
6-10	25
11-15	15
16-20	0
21-25	8
26-30	6

(a) Find the (i) Mean
(ii) modal class

(b) Draw (i) A histogram
(ii) A cumulative frequency curve

(c) What is the interquartile range?
(i) Compare your answers with a colleague
(ii) With the guide of your tutor, share your answers with a whole class.

20.12 Self Checking Exercise

You have now completed Unit 21 of this module. Below are the learning outcomes. Please tick in the column that best reflects your learning.

Learning Outcome	Sure	Not satisfactory
I can outline the different uses of statistics in daily life.		
I can collect data and organise it in frequency tables.		
I can represent data in charts and graphs		
I can analyse and interpret statistical data		
I can teach statistics in the primary school		

Congratulations!

21.13 References for further reading

Greer A. (1991). A First Course in Statistics. Stanley Thornes Ltd, London.

Green Cliff. (1998). Mathematics Revision and Practice for UCE. Oxford University Press.
Ogunnyi M. B. (1990). Education Measurement and Evaluation. Longman

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UNIT 21: PROBABILITY (21 HOURS)

21.1INTRODUCTION

You are most welcome to unit 21. This unit introduces you to the ideas of chance and simple probability, use of Cartesian and Venn diagrams to determine probability, mutually exclusive and independent events and how to teach probability in the primary school.

21.2 CONTENT ORGANIZATION

Dear student, in this unit you are going to cover the following topics as indicated in the table below.

Topic	Subtopic
I. Probability of an event	a. Idea of chance b. Possibility space c. Experimental probability d. Theoretical probability
II. Cartesian diagrams, Venn diagrams and probabilities of events	a. Cartesian diagrams for solving theoretical probability problems. b. Venn diagrams and probabilities of events
III. tables and their use in finding probability	a. Addition Tables b. Subtraction tables c. Multiplication tables
IV. Mutually exclusive and independent Events	a. Mutually Exclusive events b. Independent events
V. Probability Trees and their use in solving problems	a. Tree diagrams b. Using probability tree to solve problems
VI. Teaching probability in the primary school	a. Probability in the primary school curriculum b. teaching probability in the primary school

21.3 Learning outcome

- Discuss the usefulness of probability in everyday life.
- Design games of chance and use them to teach probability in the primary school.

21.4 Competences

a. Relate probability to chance
b. Construct different possibility spaces based on tossing a coin and dice.
c. Use different possibility spaces to find probability of different events.
d. Carry out simple probability experiment and record the outcomes.
e. Use Cartesian diagrams, Venn diagrams and tables to determine probability of an event.
f. Identify mutually exclusive and independent events and solve related problems.
g. Teach probability in the primary school

21.5 Unit orientation

This unit is to help you learn about probability using practical activities by observing your environment, playing games and recording outcomes and using diagrams to determine probabilities. This unit relates probability to set concepts in solving real life problems.

21.6 Study requirements

To be successful in studying this unit, you should interact very much, with your environment. You need coins, seeds, dice, pen and paper. You also need to have primary mathematics course books, the primary school mathematics syllabus and secondary school mathematics Books 3 and 4 for further practice.

21.7 CONTENT AND ACTIVITIES

21.7 1: PROBABILITY OF AN EVENT

a. idea of chance

When a farmer plants maize or bean seeds, is it certain that they will all germinate? Suppose that they all germinate, can the farmer be sure that they will all bear fruit? When your favorite football team gets into the pitch, must it win the game it plays? In these and many other situations in real life, we cannot say 'yes' or 'No' with certainty. We may use expressions like: perhaps my team will win, there is a fifty-fifty chance that it will rain, I have a chance of joining my first-choice secondary school many events in life are uncertain in nature.

There are also events that are certain. If today is Tuesday, could tomorrow be Monday by any chance? We can be sure that tomorrow will be Wednesday. On the other hand, we know that the chance that tomorrow will be Monday is non-existent. We can then say that in real life, certain events will not happen at all, some have a chance of happening yet others we can be sure they must happen. These ideas of chance are what probability is all about.

ACTIVITY 21.1

Write down five different things that;

1. Will surely not happen
2. May or may not happen
3. Will surely happen

b. Possibility space

When you select one number from the set of natural numbers, it may be either odd or even. Each of these possibilities is called an outcome. You may choose an odd or even number so all the possible outcomes are equally likely. We say that the probability of selecting an odd number is equal to the probability an even number. What is the value of this probability? Suppose we use E to represent selecting an Even number and D to represent selecting an Odd number. There are only two possible outcomes which form what we call the possibility space or sample space S. we wirte S={ D, E}. D and E are also referred to as events.

The probability that an event E happens may be defined as

$$P(E) = \frac{\text{The number of ways the event can occur}}{\text{The total number of possible outcomes}}$$

We shall illustrate this using an example.

The probability of certainty is 1

The probability of an impossibility is 0.

If the probability of an event is P and the probability that it does not happen is q, then P + q = 1

Example:

What is the probability of getting a prime number when a die is rolled?

Solution.

S={1,2,3,4,5,6}

Let Q be the set of prime numbers in a set S.
Then Q = {2,3,5}
We have n(Q) = 3; n(S) = 6

•

• • $P(Q) = \frac{n(Q)}{n(S)} = 3/6; \quad = ½$

P(prime number) = ½

Can you now find the P(even number), P(odd number), P(negative number) and P (number equal to or greater than 21)? Relate your answers to what we said about chance.

Remember it is always important that you first understand the information given to you whenever you have to draw a possibility space and use it to find probabilities.

Activity 21.2

1. Draw the possibility space for the outcomes of tossing two coins simultaneously. Use H for head T for tails.
2. A teacher wants each child in class to mention the (a) day of the week (b) month of the year when they were born. Draw the possibility space in each case.
3. A bag contains a blue and 6 red balls. One ball is removed. What is the probability that the ball is blue?
4. A number is selected at random from the set of counting numbers from 1 to 30 inclusive, find the probability that the number selected is

   (a) an even number
   (b) a composite number
   (c) the cube of 2
   (d) a multiple of 4 or 6
   (e) a factor of 28
   (f) a multiple of 3 and an odd number.

5. A card is picked at random from a pack of 52 playing cards, what is he probability that the card is (a) a queen (b) a spade (c) A of hearts (d) a card with a prime number on it.

Check your answers with the ones at the end of the unit.

c. Experimental probability

In situations where we can carry out a survey or an experiment then use the recorded outcomes to determine probabilities we shall refer to this as experimental probability. Some simple experiments are:

(i) Toss a coin 221 times, recording whether it lands with heads (H) or tails (T) up.
(ii) Roll a die 150 times and record how many times each score of 1,2,3,4,5 and 6 occurs.
(iii) Find the number of people in each car passing by your school.

From the above three experiments, answer the following questions.

Activity 21.3

1. When a coin is tossed 221 times, how many times will it land with heads (H) or tails (T) up? Write down P(H) and P(T)

2. For each of the scores of 1,2,3,4,5,and 6 when a die is rolled, 150 times, find the fraction; number of times the score occurs
   Number of roles of the die
3. what is the probability that a car passing by your school has
   (a) Exactly five people?
   (b) More than one person?
   (c) Less than the average number of people?
   (d) Theoretical probability

   Check your answers with the ones at the end of the unit

d. Theoretical probability

Many times it may not be practical to find experimental probability. We then estimate the "theoretical probability" of an event form the fraction:

$$\frac{\text{Number of ways the event can occur}}{\text{Number of all possible outcomes}}$$

We often do this when an activity involves equally likely outcomes. Indeed we shall generally assume that events are equally likely unless otherwise stated.

When we say that on tossing a coin P(H) = P(T) = ½ or when rolling a die P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6, we are assuming the events in each case to be equally likely. This means that the stated values of probability are "theoretical probabilities".

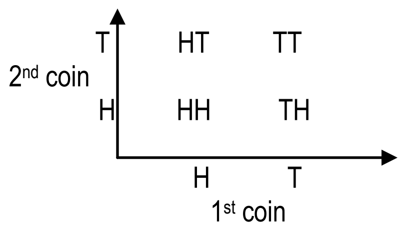
21.7.2: CARTESIAN DIAGRAMS, VENN DIAGRAMS AND PROBABILITIES OF EVENTS

a) Cartesian diagrams for solving theoretical probability problems

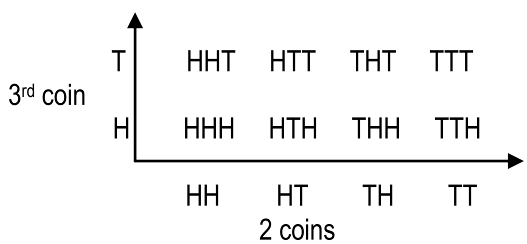
We can use a Cartesian diagram to represent the outcomes of an activity that involves probability. All you have to do is decide which event is put on the horizontal axis and which one is put on the vertical axis. Remember to label the axes very clearly. Let us look at some examples

Example1

When two coins are tossed, we can list the possibility space as S= {HH, HT, TH, TT}. We can also show this information in a Cartesian diagram:

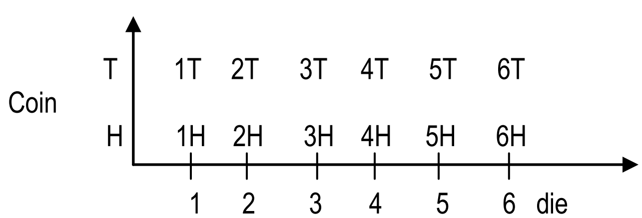


We can use the diagram above to get the possibility space for tossing three coins:



Example 2

Suppose we toss a coin and a die what will be the possibility space?
Study the diagram below.



From the above examples I hope you can now represent a possibility space on a Cartesian diagram. Try the following activity.

Activity 21.4

Four digits 2,3,4,6 are written down in random, order them to make

a) a two digit number
b) a three digit number

Show the possibility space in each case on a Cartesian diagram. State the total number of events in each possibility space
Check your answers with the ones at the end of the unit.

b) Venn diagrams and probabilities of events

There is a relationship between sets and probability that allows us to use venn diagrams to solve probability problems. Try to answer the following questions:

i) What is the relationship between a possibility space S and the universal set ξ?
ii) What is P(S)?
iii) What part of the venn diagram should represent S?
iv) Describe the event represented by each region on a venn diagram with two intersecting circles.

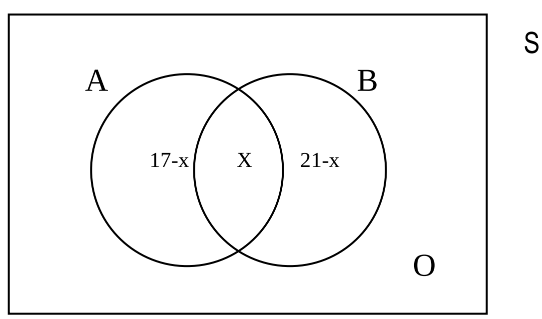
Discuss your answers with your colleagues. Now study the following example

Example 3.

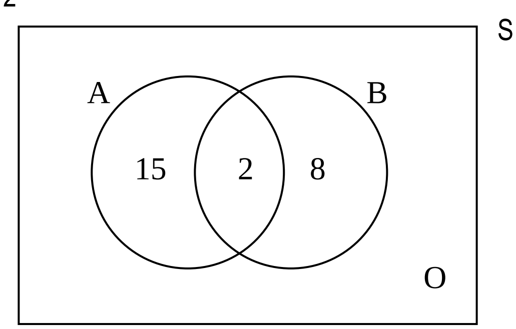
Two sets A and B are such that n(A)=17, n(B)=21 and n(AUB)=25. use a venn diagram to find the probability that a number picked at random from AUB belongs to (a) AnB b) A only c) B only

Solution:

We shall use the venn diagram to find the number of elements in each region. We first let n(AnB)=x



$17 - \cancel{x} + \cancel{x} + 21 - x = 25$
$27 - x = 25$
$X = 2$



(a) n(AnB) = 2 and P(AnB) = $\frac{2}{25}$
(b) n(A only) = 15 and P(A only) = 15/25 = 3/5
(c) n(B only) = 8 and P(B only) = 8/25

From the venn diagram, we also have n(AUB) = 25 so P(AUB) = 25/25 = 1

In general, we have P(S) = P(ξ) = 1.

Activity 21.5

Two events A and B are such that

P(A) = 0.29, P(B) = 0.48 and P(AnB) = 0.13

Use venn diagram to determine:

(1) P(AnB) 2. (A^1nB) (3) P(AuB). (4) P(A^1uB) (5) P(A^1nB) (6) P(A^1UB^1)

Check your answers with the ones at the end of the unit.

21.7.3. TABLES AND THEIR USE IN FINDING PROBABILITY

(d) Addition tables

Begin by revising the subtopic on Cartesian diagram. We shall transform these diagrams into addition tables whenever appropriate.

Example 1

A blue die and a red die are tossed together and the sum on the dice is recorded. Draw a possibility space for this activity and use it to find:

(i) P(a score of 8 or more)
(ii) P(same score on the two dice)
(iii) P(an odd score)
(iv) P(a score of 13)
(v) P(a score of between 2 and 12 inclusive)

Solution

Red die						
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
(+)	1	2	3	4	5	6

Blue die

Note that the table shows the outcome of adding. The outcome on the red die and the blue die;

(i) P(8 or more) = P (8 or 9 or 21 or 11 or 12)
$= P(8) + P(9) + P(21) + P(11) + P(12)$
$= \frac{5+4+3+2+1}{36} = \frac{15}{36} = \frac{5}{12}$

(ii) P(same score on the dice)
= P [(1,1) or (2,2) or (3,3) or (4,4) or (5,5) or (6,6)]
$= \frac{6}{36} = \frac{1}{6}$ (each occurs only once)

(iii) P(odd score) = P(3, or 5 or 7, or 9 or 11)
$= \frac{2+4+6+4+2}{36} = \frac{18}{36} = ½$

(iv) P(score of 13) = 0. this is not possible
(v) P(score between 2 and 12 inclusive) = 1
It is all the scores

(e) Subtraction tables

Suppose when we roll two dice we are interested in the difference between the two numbers on the dice. We shall consider the magnitude of the difference; in other words we shall take the positive difference. Our possibility space will now look like this.

	6	5	4	3	2	1	0
	5	4	3	2	1	0	1
Red die	4	3	2	1	0	1	2
	3	2	1	0	1	2	3
	2	1	0	1	2	3	4
	1	0	1	2	3	4	5
	(-) 1	2	3	4	5	6	
				Blue die			

We then have (a) P(odd score) = P(1 or 3 or 5)

$$= \frac{21 + 6 + 2}{36} = \frac{18}{36} = ½$$

(b) P(score less than 3)

$= P(0 \text{ or } 1 \text{ or } 2)$

$= \frac{6+21+8}{36} = \frac{24}{36} = \frac{2}{3}$

(c) Multiplication tables

If this time we get the product of the two numbers showing on the dice we have the following

				Die 1			
	X	1	2	3	4	5	6
	1	1	2	3	4	5	6
	2	2	4	6	8	21	12
	3	3	6	9	12	15	18
Die 2	4	4	8	12	16	20	24
	5	5	21	15	20	25	30
	6	6	12	18	24	30	36

Find: (a). P(odd score)
(b) P(score greater than 21)
(c) P(score which is a multiple of 2 or 3).

I hope now you can use tables to solve probability problems. Now have a go at the next activity.

Activitiy 21.6

1. A coin is tossed and a die is rolled. Use a Cartesian diagram to find the probability of;

(a) A head and a four
(b) A tail and a composite number.

2. Two dice are thrown and their scores are added together. Find the probability that the total will be

(a) 5
(b) More than 5
(c) Less than 5

Find the sum of the probabilities in (a), (b) and (c) and comment on your answer.

Check your answers with the ones at the end of the unit.

21.7.4. MUTUALLY EXCLUSIVE AND INDEPENDENT EVENTS

Study the following pairs of events

A1: Jane is attending school.
A2: Jane is weeding the flower garden at home.

B1: Today is Monday
B2: Today is Saturday

C1: The distance between town A and B is 8km
C2: The distance between town A and town B is 3000m.

What can you say about each pair of events?
Which pair of events can occur together?
If two events cannot occur together we say that they are mutually exclusive. If one event is A and the other is B then we have P(AnB) = 0 for A,B mutually exclusive. Write down five pairs of events A and B such that P(AnB) = 0.

Now study the following pairs of events.

A1: Mary's first born child is a boy.
A2: Mary's second born child is a boy.

B1: x is a counting number between 1 and 21.
B2: x is a multiple of 3.

C1: the teacher has not come to school.
C2: the teacher is sick

Is there a pair of events in which the probability that one event occurs depends on the other event? For example, if Mary's first born is a boy, does it affect the probability that her second child is a boy or it remains the same? What about the teacher not coming to school, does it depend on the teacher being sick?

If one event has no effect on subsequent events, the events are said to be independent. For example A1 and A2 above are independent events. Write down three more examples of independent events. If events A and B are independent P(AnB) = P(A), P(B)

Activity 21.7

1. The probability that a student will stay at home is defined as P(H) = 0.64, while the probability that she will go to a movie is P(M) = 0.21.

Find:
(a) P(H^1)
(b) P(HUM)
(c) P(HnM)

2. A coin is tossed three times. What is the probability of the coin showing heads all three times?
3. A box contains 2 blue and 3 red pens. A pen is drawn from the box and is not replaced. A second pen is then drawn. What is the probability that both are red pens?

Check your answers with the ones at the end of the unit.

21.7.5 Probability trees and their use in solving problems

In this topic you are going to learn about

- Tree diagrams
- Using probability trees to solve problems.

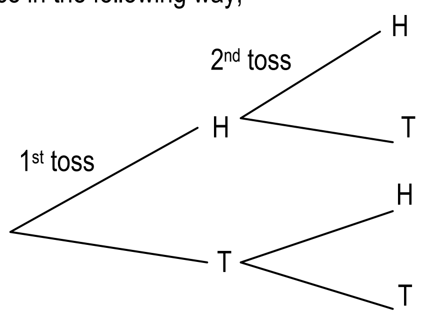
a) Tree diagrams.

You have already learnt how to list a sample space for fairly simple situations. You will now look at a different way in which you can arrange a sample space using a tree diagram.

A tree diagram can be defined as a sample space that has a starting point and at least two branches originating from this starting point. At the end of each branch you can have more branches.

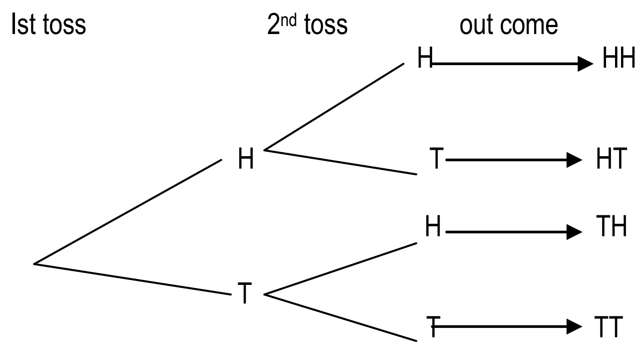
Example 1

When you toss a coin twice, you get this sample space 8 = { HH, HT, TH, TT}. Using a tree diagram, we can arrange this sample space in the following way;



We call this a tree diagram.

You can now draw this tree diagram more carefully



Therefore from the starting point you have two branches each representing a possible outcome of a single toss of a coin.

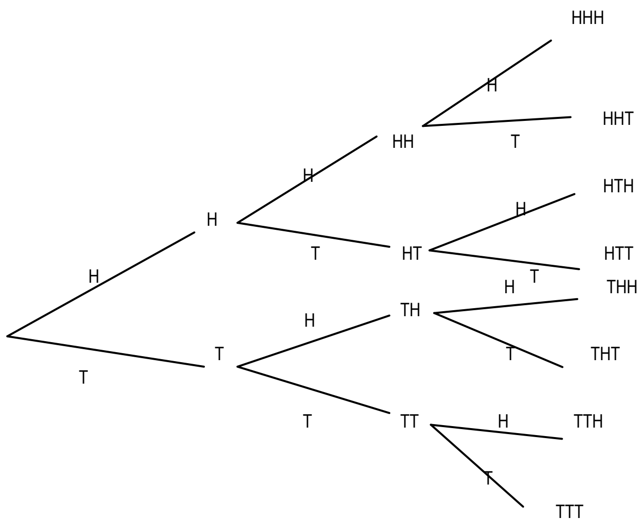
The outcome of tossing a coin a second time is independent of the outcomes of the second toss. Whatever the outcome of the first toss, therefore, you still expect Head (H) or Tail (T) on the second toss.

This is well represented in the region of the tree diagram under 2nd toss where for each outcome of the first toss, you have two branches each representing a possible outcome. In the end you can say that you have four complete branches, each representing a possible outcome of two tosses of a coin.

The top branch as you can see from the diagram is HH: that is both tosses show heads. In the same way, you can identify all the other outcomes that make up the sample space.

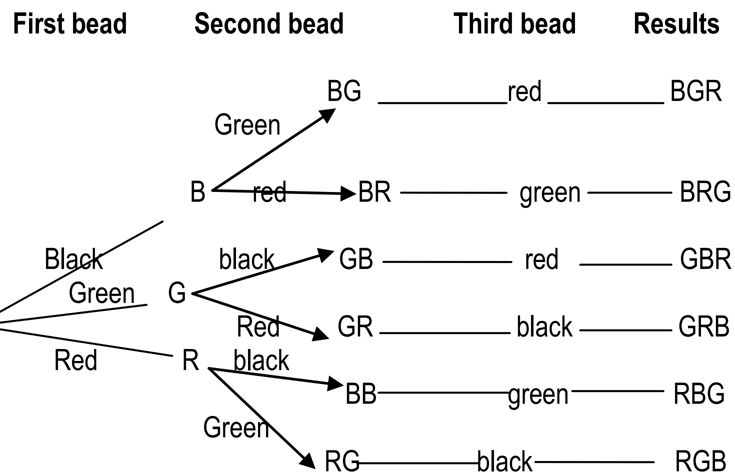
Example 2.

Use a tree diagram to show all possible results of throwing three coins.



Example 3

A bag contains 3 beads one black, one green and one red. The beads are removed one by one. Draw a tree diagram to show all possible ways in which they can be removed.



How does the second pick depend on the first pick? Remember there are three beads, so if the first pick was a black bead the second can only be a green or a red bead. If the first was green, the second black, the third has to be red. This is how the six possible outcomes can be got.

b) Using probability trees to solve problems.

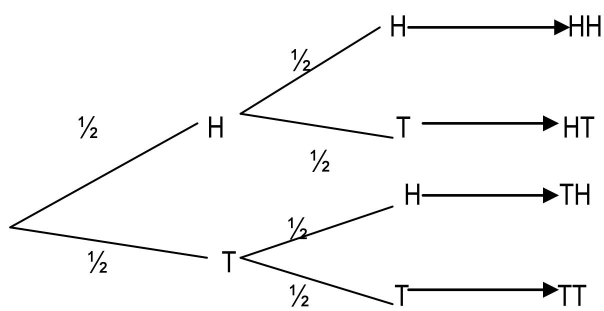
You have been drawing tree diagrams sharing possible outcomes of given trials without considering the probabilities with which those outcome occur. If these probabilities are indicated on their respective branches, the tree diagrams become probability trees? Once you can tell which outcome id represented by a certain route on the probability trees, then you would be in position to quickly solve any related problems. All you need to recall is that in probability we interpret the “and” as multiplication and the “or” as addition.

Example 4

Use a probability tree to find the probability that in two tosses of a coin:-

(i) A head occurs twice
(ii) A tail is obtained once.

Solution



(i) P (Head and Head) = P(HH)

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

(ii) P(Tail once = P(Head and Tail or Tail and Head)

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

Example 5

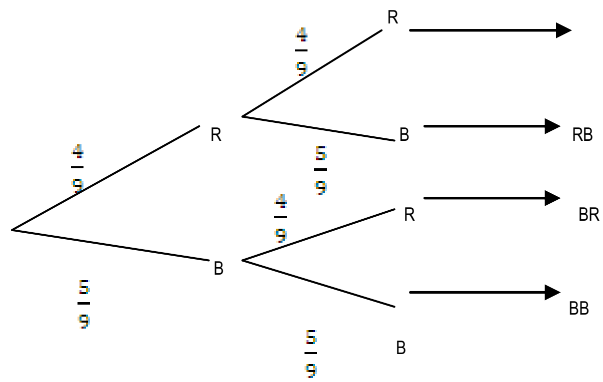
A bag contains 4 red (R) and 5 blue (B) balls. Akello picks a ball and goes out to play with it. When she is tired, John asks her for it. She refuses and brings it back to the bag. John then comes and picks a ball and goes out to play with it.

Find the probabilities that both Akello and John pick:

(i) Red balls
(ii) Blue balls

(iii) Balls of different colours
(iv) Balls of the same colours.

You can represent that information first on the probability tree for example:-



(i) P(both red balls) = P(RR)

$= \frac{4}{9} x \frac{4}{9}$

$= \frac{16}{81}$

(ii) P(both blue balls) = P(BB)

$= \frac{5}{9} x \frac{5}{9}$

$= \frac{25}{81}$

(iii) P(balls of different colours) = P (Red and Blue or Blue and Red)

$= \frac{4}{9} x \frac{5}{9} + \frac{5}{9} x \frac{4}{9}$

$= \frac{20}{81} + \frac{20}{81}$

$= \frac{40}{81}$

(iv) P(balls of the same colour)

= P (both Red or both Blue)

$= \frac{4}{9} x \frac{4}{9} + \frac{5}{9} + \frac{5}{9}$

$= \frac{16}{81} + \frac{25}{81}$

$= \frac{41}{81}$

Activity 21.8

Solving the following problems using probability trees.

1) A tin contains 7 reds and 5 blue beads. Two beads are picked at random. Find the probability of picking:-

(i) 1 red and 1 blue
(ii) Beads of the same colour assuming that the first bead is not replaced before the second one is picked.

2) Cards marked with the numbers 1,2,3,4,5 are put in a bag and two are drawn at random the first one being replaced before the second card is drawn. Find the probability that:-

   a) Both have odd numbers on them.
   b) Both have even numbers on them.
   c) Both have the same number.

3) A coin is tossed three times. Find the probability of getting:-

   a) All heads
   b) All tails
   c) At least one head and one tail

Check your answer with those at the end of this unit.

21.7.6. TEACHING PROBABILITY IN THE PRIMARY SCHOOL

(a) Probability in the Primary School Curriculum.

Study the Uganda Primary School Mathematics Curriculum. In which class is probability to be taught? What prior knowledge is expected of the pupil? It is important that children will have learnt set concepts, fractions and graphs and interpretation of information before probability is introduced?

Among the things children should already know are how to obtain number of elements in a given set, identifying described sets of numbers or objects and the meaning of fractions. Children should be able to represent information in graphical form and also to read and interpret graphs. These form a basis for learning probability when they get to P.7.

(b) Teaching probability in the primary school.

It is important that you engage pupils in a lot of practical activities. Children should toss coins, dice, and labeled tetrahedral, numbered pieces of paper and so on. They should be able to record the outcomes of their activities in tables.

The teacher should give plenty of guiding questions such as;

- How many times was the outcome a head? a tail, an even number? a prime number? a multiple of 4?
- In how many different ways can a coin, die, tetrahedron land?
- How many chances are there for the coin to land showing a head? for the die to land showing a 5? for the tetrahedron to land showing a 3?

The pupils' answers should lead you to introduce the vocabulary used in talking about probability. You can tell them what a possibility space, an outcome, an event and probability of an event mean in relation to their answers.

Activities 21.9

Play the following game with a partner. Both players copy the table shown below.

Guess H or T	Right (√)	Wrong (x)
Totals		

1. Give the coin to your partner. Guess whether your partner will toss a head (H) or a tail (T) and write your guess in the table. Say what it is.
2. Let your partner toss the coin. Was your guess right or wrong? Tick in the appropriate column in the table.
3. Now your partner guesses and you toss the coin.
4. Repeat until each one has tossed 20 times.
5. The winner is the player with the highest number of right guesses.

21.8 Unit summary

In this unit you have learnt how to make a possibility space of a probability activity. You also learnt how to use Cartesian diagrams, Venn diagrams and tables to find relevant probabilities. You have learnt that some events are mutually exclusive and others are independent.

21.9 Glossary

Independent event: An event whose probability of occurring does not affect and is not affected by the probability that another event can or will occur

Mutually exclusive events: Events that cannot occur simultaneously

Possibility space: The set of all possible outcomes of an activity or situation.

Probability: The chance or likelihood that an event will occur.

21.10 Notes and answers to activities

Activity 21.1

Things that will surely not happen have a probability of zero e.g. Uganda is a European country. Things that will surely happen have a probability of 1. e.g. if today is Monday, tomorrow will be Tuesday. Things that may or may not happen have a chance (probability) of happening between zero and one, e.g. it may rain this evening

Activity 21.2

1. S={HH, HT, TH, TT}
2. (a) S={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
   (b) S= {January, February, March, April, may, June, July, August, September, October, November, December}
3. $\frac{\text{number of blue balls}}{\text{Total number of balls}} = \frac{8}{14} = \frac{4}{7}$
4. (a) P(even) = ½ (b) P(composite) = 19/30, (c) P(cube of 2) = 1/30, (d) P(multiple of 4 and 6) = 1/3, (e) P(factor of 28) = 1/5, (f) P(multiple of 3 and odd) = 1/6
5. (a) 1/13, (b) ¼, (c) 1/52 (d) 16/36 = 4/9

Activity 21.3

1. You expect heads up = tails up = 50 so that P(H) = P(T) = ½
2. You expect fraction to be ≈ 1/6
3. Compare your values with those of your classmates who were at a different spot.

Activity 21.4

(a)

6	26	36	46	66
4	24	34	44	64
3	23	33	43	63
2	22	32	42	62
	2	3	4	6

n(S) = 16

(b)

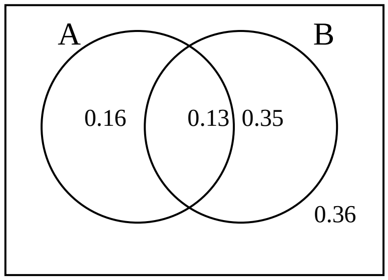
6	226	236	246	266	326	336	346	366	426	436	446	466	626	636	646	666
4	224	234	244	264	324	334	344	364	424	434	444	464	624	634	644	664
3	223	233	243	263	323	333	343	363	423	433	443	463	623	633	643	663
2	222	232	242	262	322	332	342	362	422	432	442	462	622	632	642	662
	22	**23**	**24**	**26**	**32**	**33**	**34**	**36**	**42**	**43**	**44**	**46**	**62**	**63**	**64**	**66**

n(S) = 64

Activity 21.5

Represent events A and B on Venn diagram. Fill in probabilities starting with P(AnB). Remember P(S) = 1 and S is representative by the rectangle.

S

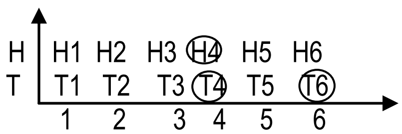


(1) P (AnB$^{1'}$) = 0.16
(2) P(A’nB) = 0.35
(3) P(AUB) = 0.64
(4) P(A’UB) = 0.84
(5) P(A’nB’) = 0.36
(6) P(A’UB’) = 0.87

Activity 21.6

1. coin

coin



die

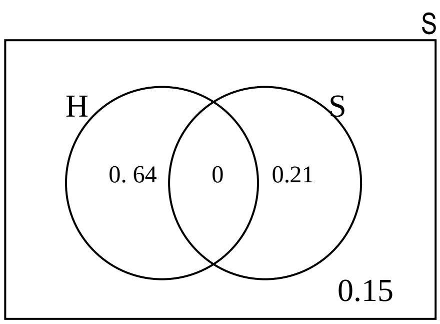
(a) P(Head and 4) = 1/12
(b) P(Tail and composite number) = 2/12 = 1/6

2. (a) 4/36 = 1/9 (b) 26/36 = 13/18, (c) 6/36 = 1/6

Probabilities add to 1; so the total can be either 5 less than 5 or more than 5. the three options cover the entire possibility space.

Activity 21.7

1. using venn diagrams



(a) P(H’) = 0.36 note that P(H’) = 1 – P(H).
(b) P(HUM) = 0.85; P(HUM) = P(H) + P(M)
(c) P(HnM) = 0; H and M are mutually exclusive

Since the student cannot be at home and also go to the movie!

2. P(HHH) = P(H)xP(H)xP(H) = ½ x ½ x ½ =1/8

The events of heads on any toss – first, second and third are independent, so P(H) remains at ½ each time the coin is tossed

3. P (first pen is red)=3/5
   P (second is also red) = 2/4

Note: That when the second pen is drawn, the total number of pens has reduced by 1 to4, also the red pens have reduced by 1 to 2

Therefore P(both are red) = 3/5 x 2/4½=3/21

The events of drawing one red pen followed by drawing a second red pen are dependent events because the probability of the second draw depends on what happened (what colour of the pen was drawn) in the first event.

In general if events A and B are dependent (i.e. not independent)

P(AnB) = P(A). P(B/A)
= P(B). P(A/B)

Where P(B/A) is red as probability of B given that A has occurred

Activity 21.8

1. (i) $\frac{35}{66}$ (ii) $\frac{31}{66}$
2. (i) $\frac{9}{25}$ (ii) $\frac{4}{25}$ (iii) $\frac{1}{5}$
3. (i) $\frac{1}{8}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{4}$

21.11 End of the unit exercise

1. Given the set S={3,7,21,13,16,19}, find the probability that a number chosen at random from S is (i) even (ii) odd (iii) prime (iv) an exact square.
2. A box contains 2 red balls, 3 green balls and 4 blue balls. A ball is picked from the box what is the probability that the ball is; (i) red, (ii) green (iii) blue
3. There are two black opens and one red pen. Two of the three pens are taken away one at a time. What is the probability that both will be black?
4. over a period of days, an institution recorded the following numbers of absentees;

Number of absentees	0	1	2	3
Number of days	1428	28	18	

What is the probability that on any one day there will be (a) no absentees? (b) one or 2 absentees? (c) less than 3 absentees? (d) more than 3 absentees

21.12 LEARNING OUTCOMES- SELF CHECKING EXERCISE

You have now completed unit 21. the learning outcomes are listed below. Demonstrate

Competence by ticking the column that best represents your learning.

LEARNING	NOT SURE	SATISFACTORY
I can construct possibility spaces based on tossing coins and dice		
I can use different possibility spaces to find probability of relevant events		
I can use Cartesian diagrams, Venn diagrams and tables to find probability of relevant events		
I can design games of chance and use them to teach probability in the primary school		

If you have placed a tick in the 'NOT SURE" column go back to the information in the unit to reinforce your learning.

21.13 References for further reading

1. Nyakairu D, Forman M. (1994), UCE, Essential Mathematics, Oxford University Press
2. M.E. Wardle, C.J. Weeks (1984) New mathematics, Revision and Practice; Oxford University Press.

Congratulations.