Angles

In geometry, an angle can be defined as the figure formed by two straight lines meeting at a common end point.

An angle is represented by the symbol ∠. Here, the angle below is ∠AOB.

Parts of an Angle:

Arms: The two rays joining to form an angle are called arms of an angle. Here, OA and OB are the arms of the ∠AOB.

Vertex: The common end point at which the two rays meet to form an angle is called the **vertex**. Here, the point O is the vertex of ∠AOB.
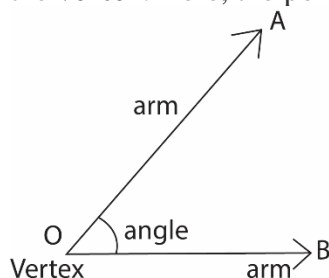
Types of angle

Angles can be classified on the basis of their measurements as

An **acute Angle** is an angle that measures less than 90 degrees

A right Angle is an angle which is equal to 90°,

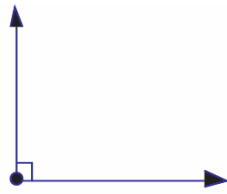**An obtuse Angle** is an angle has a measurement greater than 90 degrees but less than 180 degrees.

A straight Angles is an angle is 180 degrees

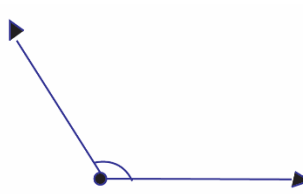**A reflex Angles** is an angle that is more than 180deg but less than 360deg

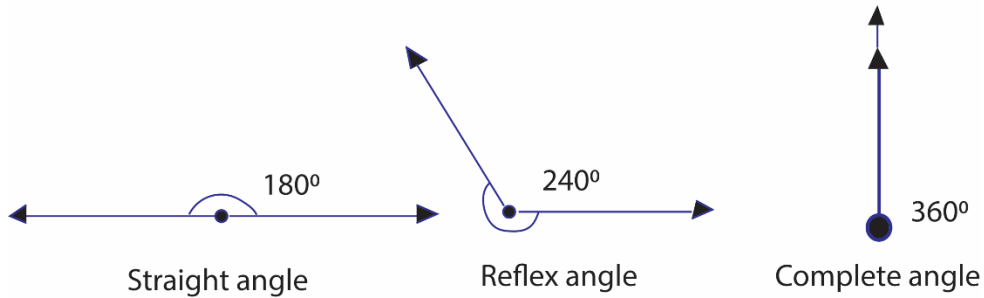A **complete Angle** is a straight line makes an angle of 360∘
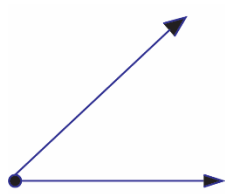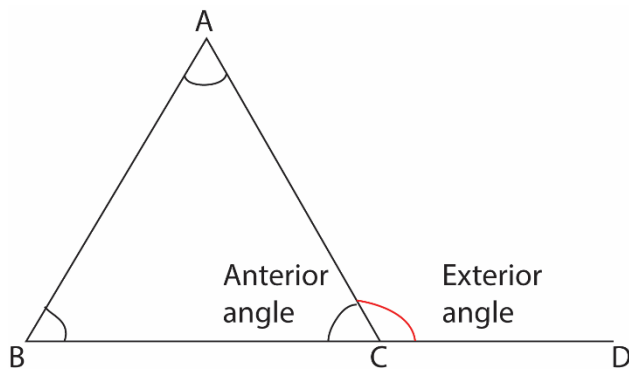
Acute angle   Right angle   Obtuse angle

180⁰   240⁰   360⁰

Straight angle   Reflex angle   Complete angle

Interior and Exterior Angles:

A

Anterior angle   Exterior angle

B   C   D

Interior angle

- An interior angle of a polygon is an angle inside the polygon at one of its vertices.

- The sum of the measures of the *interior angles* of a polygon with n sides is $(n – 2)180$.

- The measure of *each interior angle* of an equiangular n-gon is

$$\frac{(n-2)180}{n} \text{ or } 180 - \frac{360}{n} \text{ (the supplement of an exterior angle).}$$

Exterior angle

- An exterior angle of a polygon is an angle outside the polygon formed by one of its sides and the extension of an adjacent side.

- If you count one exterior angle at each vertex, the *sum* of the measures of the *exterior angles* of a polygon is always 360°.

The measure of each exterior angle of an equiangular *n*-gon is $\dfrac{360}{n}$.
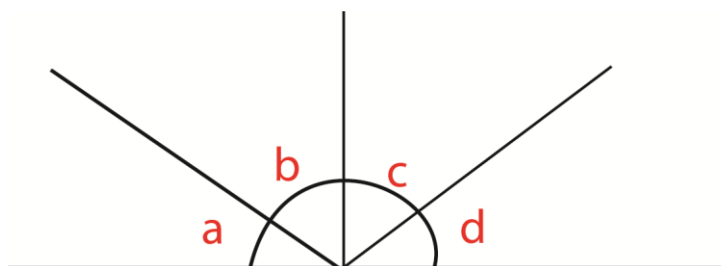
Terms related to sum of angles

Complementary angles add up to 90^0
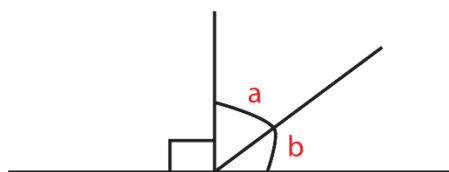
$a + b = 90^0$
thus, a and b are complementary angles

Supplementary angles add up to 180^0

$a + b + c + d = 180^0$

thus, a, b, c, and d are supplementary angles

Polygons

Polygons are usually defined by the number of sides that they have.

(a) A triangle
Is a three-sided polygon. Triangles can be defined by the length of the sides.
  (i)   **Equilateral triangle** has all sides equal and all its internal angles are 60^0.
  (ii) **Isosceles triangle** has two equal sides, with the third one a different length. Two of the internal angles are equal
  (iii) **Scalene triangle** has all the three sides and all the three internal angles different
  (iv) **Right angled** triangle has one of the angle = 90^0
  (v)   All internal angles of triangles add up to 180^0.

Types of triangle

Equilateral triangle | Isosceles triangle | Scalene triangle | Right angled triangle
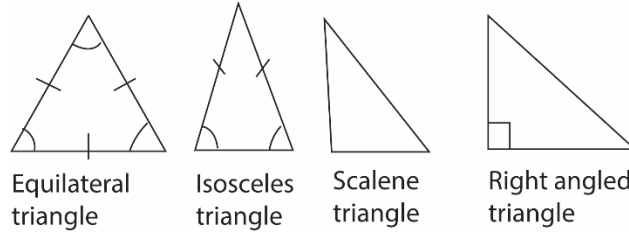
Quadrilaterals

This are four-sided polygons

The internal angles of all quadrilaterals add up to 360^0.

They include the square, rectangle, rhombus, parallelograms, trapezium and kite

- (i)   Square: all its sides are equal and each of its internal angles is equal to 900
- (ii) Rectangle: has four internal angles each equal to 900 and equal opposite sides
- (iii) Parallelogram has equal opposite sides and angles
- (iv) Rhombus has all sides equal and equal opposite angles
- (v)   Trapezium: has two parallel sides and unequal angle
- (vi) Kite has two pairs of equal adjacent sides, the shape has an axis of symmetry
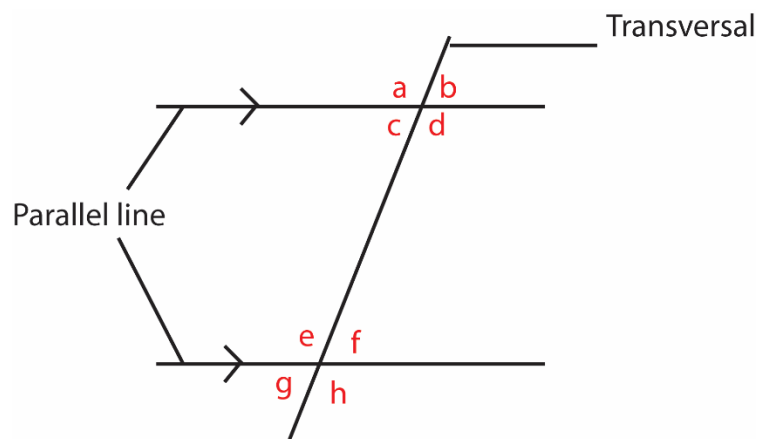

Quadrilaterals

Square | Rectangle | Parallelogram | Rhombus

Trapezium | Kite
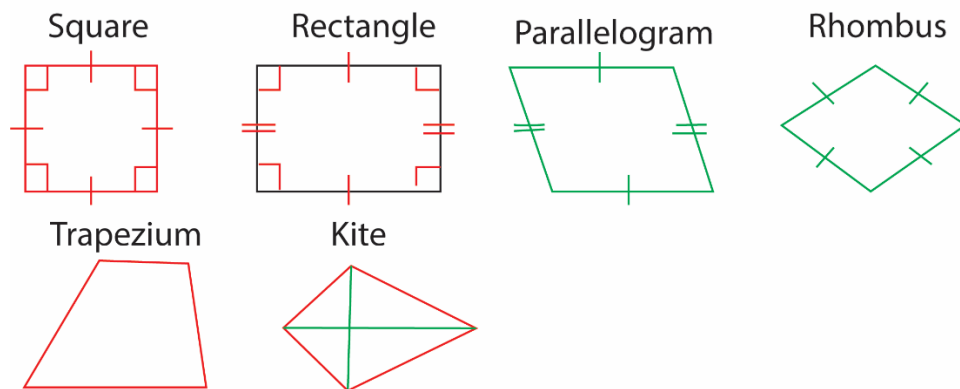
Parallel lines

These are lines that are always same distance apart and will never meet.

Transversal line is a line that cuts through two or more parallel lines

The following about parallel lines are equal

Transversal
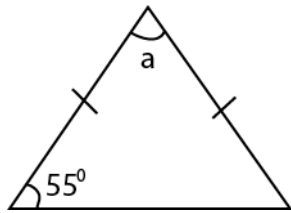
a b
c d

Parallel line

e f
g h

Equal angles
Alternative angles are: d-e, c-f, b-g, a-h
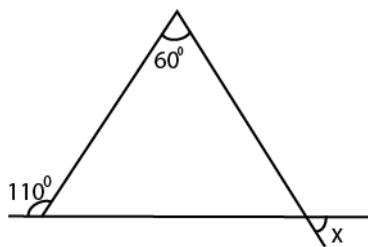corresponding angles: a-e, b-f, c-g, d-h
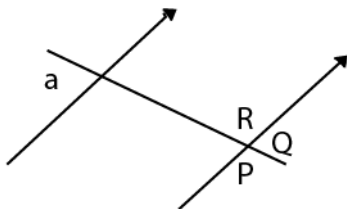Verticle angles: a-d, b-c, e-h, f -g

Revision questions

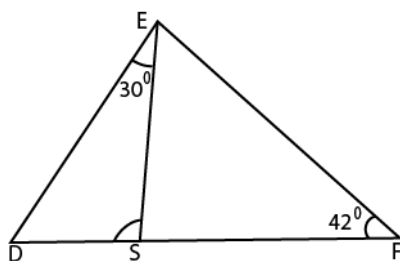1. In the diagram below, what is angle a?
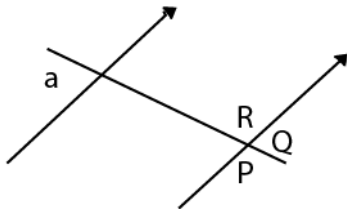
2. In the diagram below, find the size of angle *x*

3. In the diagram below, which of the angles, Q, R is
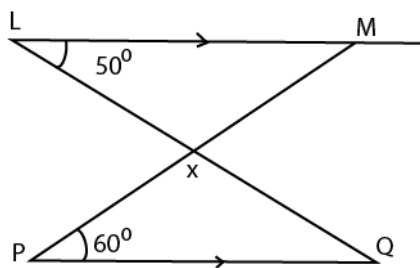
   the same as the angle marked a?

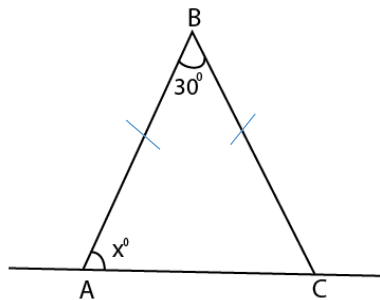4. In the triangle DBF, angle DEF = 60° and angle DFE = 42°. Find the size of angle DSE.

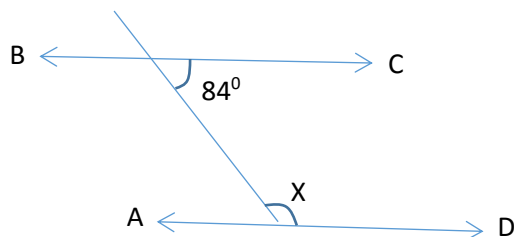5. In the diagram below, which of the angles, Q, R is the same as the angle marked a?

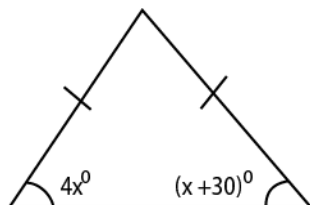6. In the diagram below, LM is parallel to PQ. Find the Value of x.
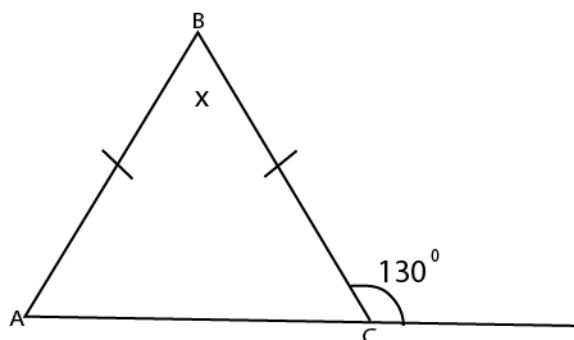

7. In the figure below, find the value of angle x

8. Find the size of the angle marked x in the figure below. BC is parallel to AD.
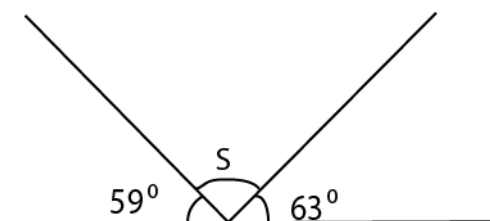
9. The figure below is an isosceles triangle, find the value of x.
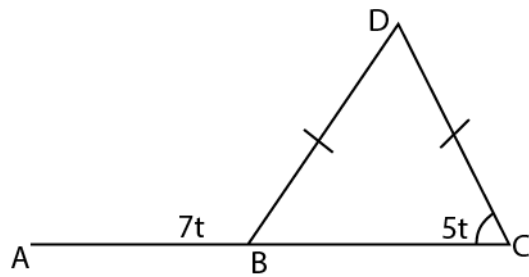

4x⁰   (x+30)⁰

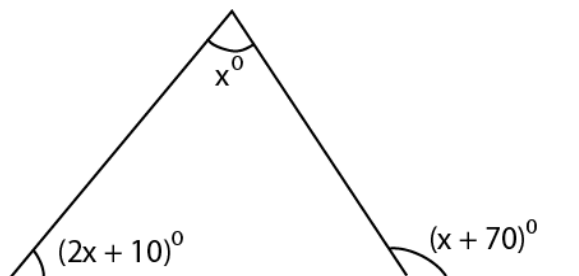10. In the figure below, find the size of angle marked x.
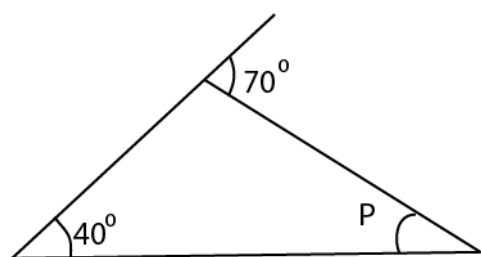

11. Find the size of the angle marked S in the figure below:

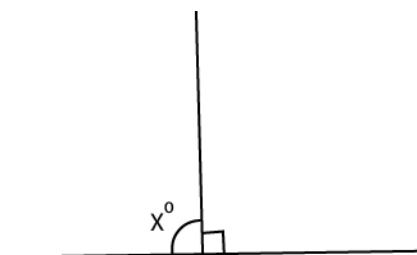12. In the figure below, BCD is a triangle and ABC is a straight line; Find the value of t.

13. Find the value of the angle marked (2x +10)° in the figure below:
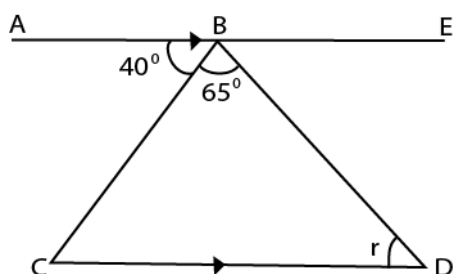
14. Find the value of angle P in the figure below:
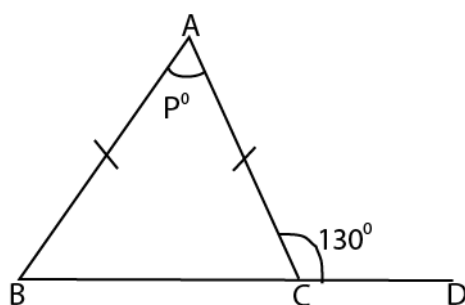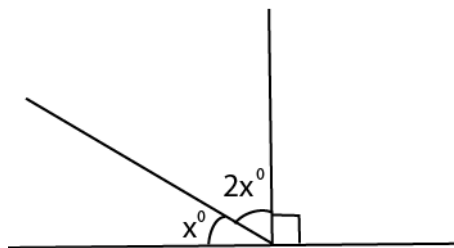

15. Find the value of x in the figure below.

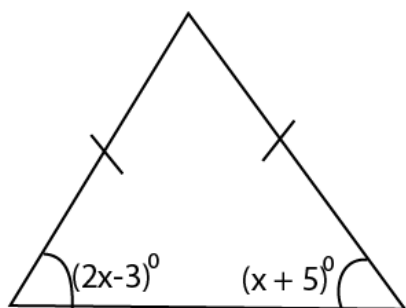16. In the diagram below AE is parallel to CD. Find the size of angle r.

17. In the diagram below, ABC is an isosceles triangle. Find the value of P at A
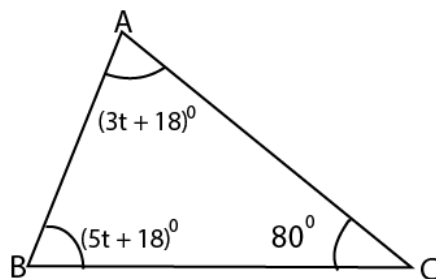
18. Find the value of x in the figure below.

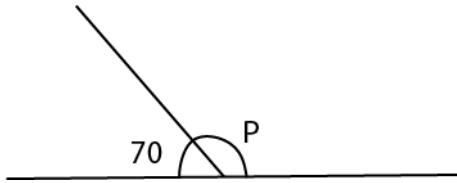19. Find the value of *x* in the figure below

20. Study the triangle below and use it to answer the questions that follow. A
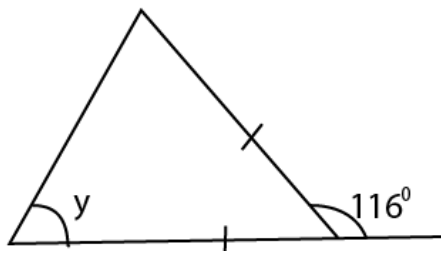

i)   Find the value of t.

ii)   Find the size of angle ABC.

21. Find the value of the angle marked p in the figure below.
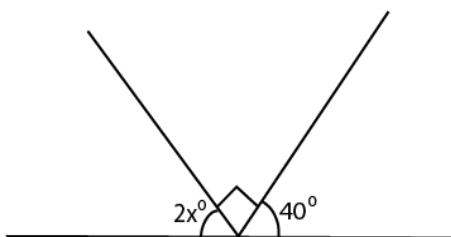

70   P

22. Find the complement of an angle of 70°.

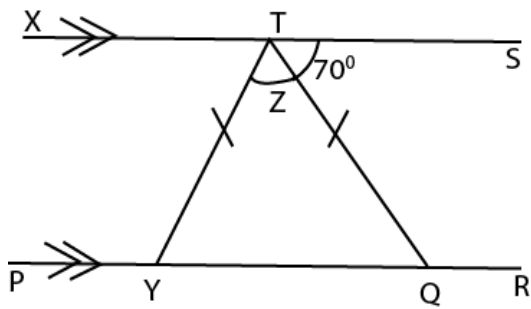23. Find the size of angle v in the figure below.

y   116⁰

24. Two angles are supplementary. If one of them is 28°, find the other angle.

25. Find the value of x in the figure below.
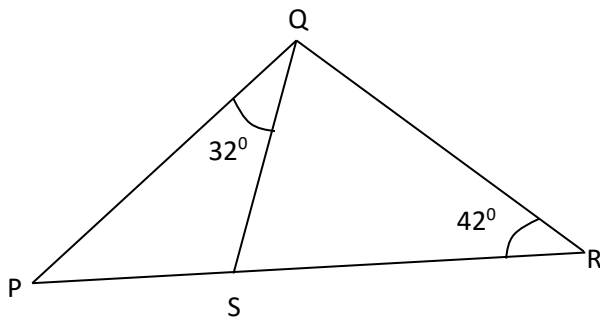

2x⁰   40⁰

26. In the diagram below, XTS is parallel to PYQR. Use it to find the size of angle Z.
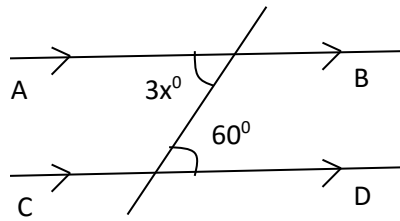

27. Find the complement of the angle of 25^0

28. In the triangle *PQR*, angle *PQR* = 78^0, angle *PRQ*=42^0 and angle *PQS = 32^0*. Find the size of angle *PSQ*.
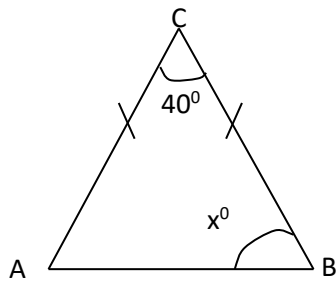

29. Find the size of one of the exterior angles of a regular octagon.
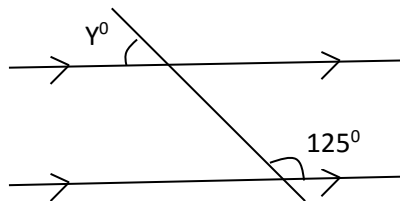
30. In the figure below, *AB* is parallel to *CD, find* the value of *x*

31. The figure below is an isosceles triangle. Find the size of angle *x*

32. Find the valve of Y in the figure below.

33. Find the value of x in the figure below.
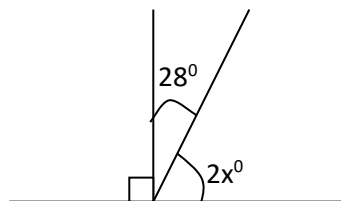
34. In the diagram below, PTUV is a straight line, angle TSU =40^0, angle *SUV*=150^0 and angle *PQT*=30^0.use the given information to find the value o the angle marked k and n.

(04 marks)
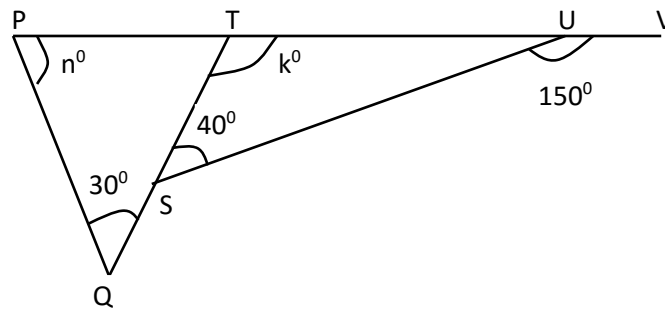
35. Find the value w in the figure below

36. In the diagram below, *CAB* is a triangle and *DCA* a triangle line. Study it and answer the question below(3marks)

(a) What is the valve of *Y?*
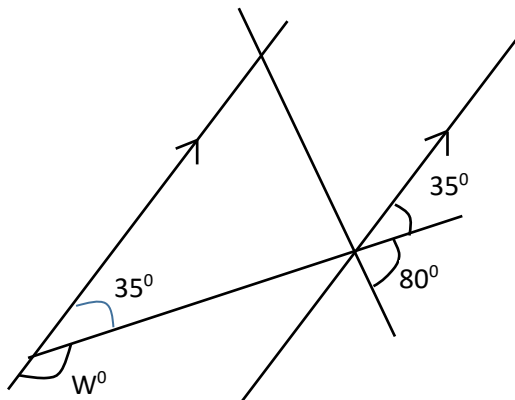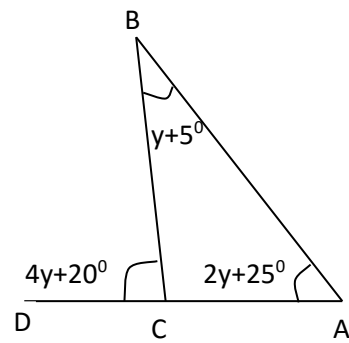

(b) What the size of angle ACB? (3mark)

37. In the figure below, find the value of n in degrees.
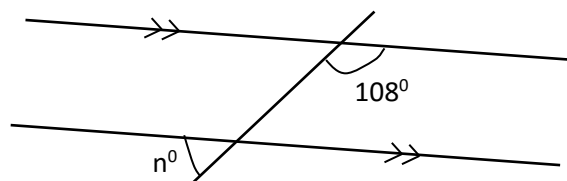
38. In the triangle below, find the size of angle g in degrees.
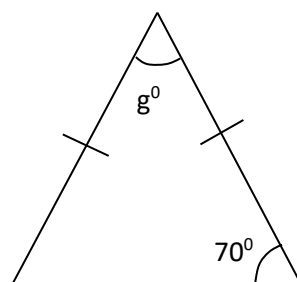
39. In the diagram below, OP is parallel to QR, HKL is triangle, angle $HLR=110^0$ and angle $SHP=50^o$

Study it and answer the questions that follow.
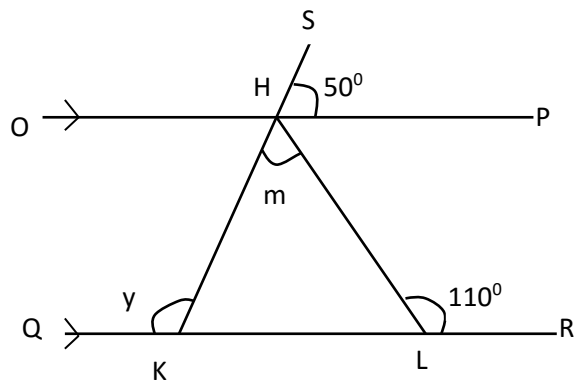
Find the size of

(a) Angle y: (2marks)

(b) angle M   (3mark)

40. In the diagram below, find the valve of x.

41. In the figure below, find the bearing of town B from town A.

42. 36 In the diagram below, line *AB* is parallel to *CD* and *BCD* is an isosceles triangle. Study it carefully and use it to answer the questions that follow.
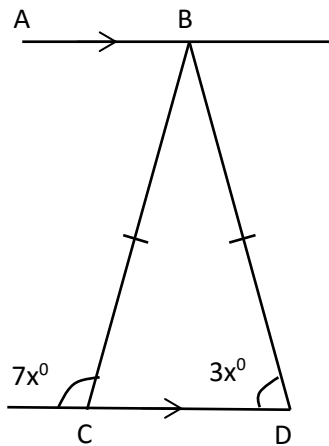
(a) Find the value of *X*   (3marks)

(b) Work out the size of angle *ABC*   (2marks)
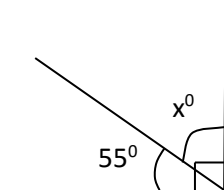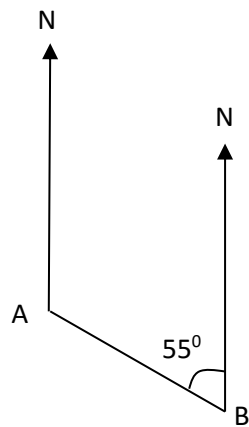
43. In the diagram below, find the valves of K
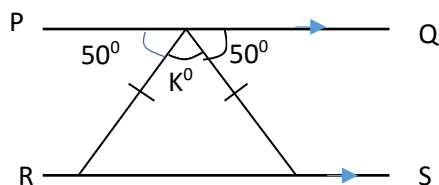
. (a) The interior angle of a regular polygon is 36 more than its exterior angle. What is the size of each exterior angle?   (02marks)

(b) In the figure below DE is parallel to FG, angle ODE =127^0 and angle OFG=136^0. Calculate the size of angle x.

(02marks)

44. Find the value of X in the diagram below.

45. In the figure below, BCD is a straight line. Line BX bisects angle ABC. Line AB is parallel to line XC.

Angle BCE = 50^0 and angle BAC = 70^0
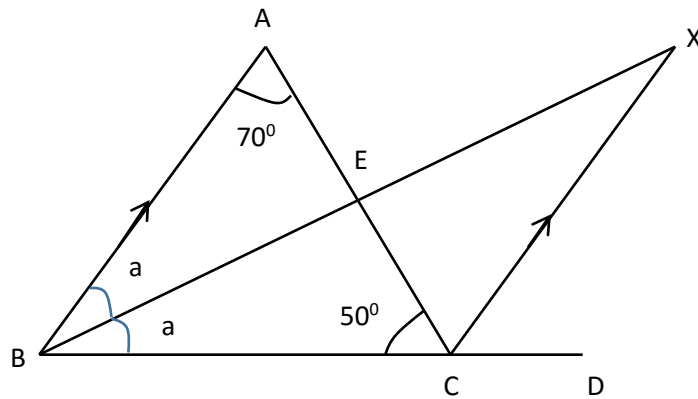
Find the size of the angles:

(i)   CEX   (04 marks)

(ii)   DCX   (01 marks)

46. Find the size of angle *y* the figure below.

47. In the figure below line AB is parallel to CD, Angle CTV = 44 and angle TQR=56. **Study use**

    it to answer the questions that follow:
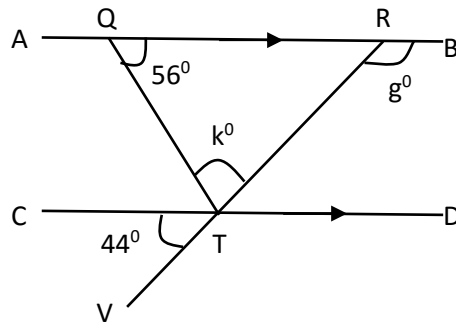
Find the size of

    (a) Angle k (02mark)

    (b) Angle g   (02mark)
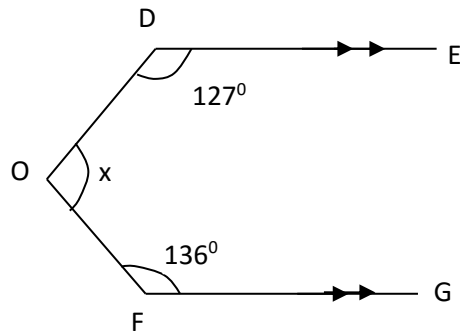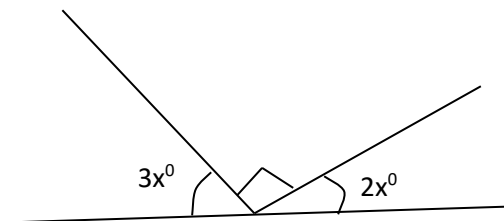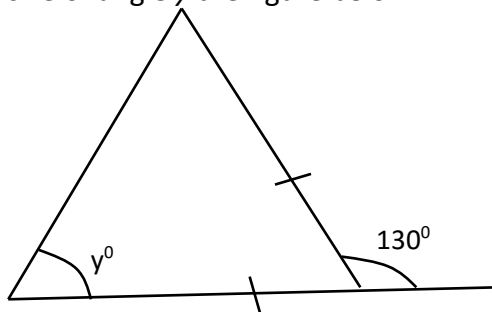
48. In the triangle below, find the value of d in degrees

49. Study the figure below and use it to answer the questions that follow.

(a) Find the value of g (03marks)

(b) Calculate the size of angle RST

50. Find the value of p in degrees in the diagram below

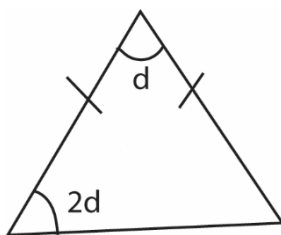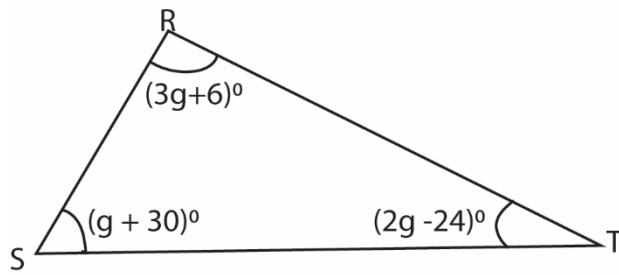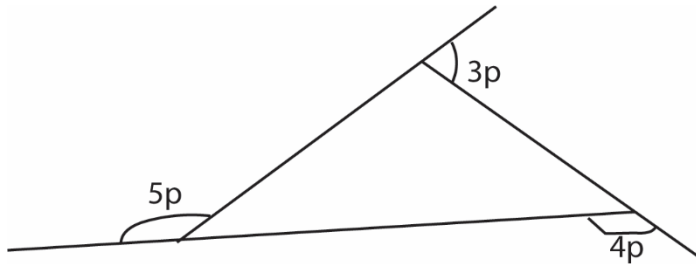51. (a) The interior angle of a regular polygon is 108 more than the exterior angle. How many sides has the polygon? (03marks)

(b) Find size of angle z in the figure below
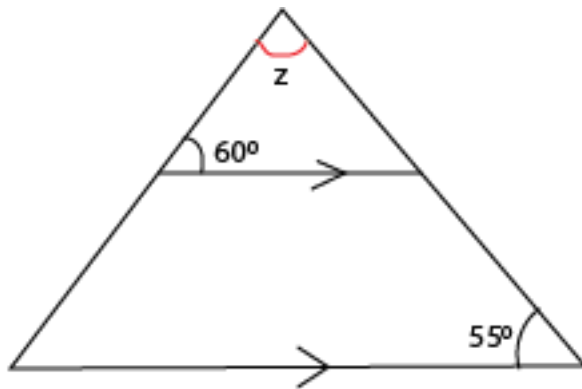
52. In the diagram below, line DH is parallel to FE. Angle ACB = 75^0 and angle CBE = 135^0. Angle FAD is twice angle DAC. Study the diagram and use it to answer the questions that follow.
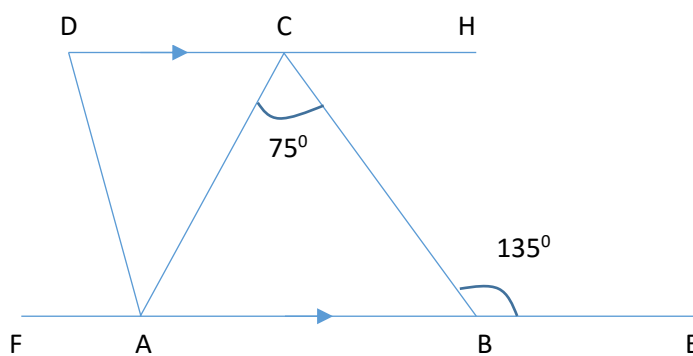
(a) Calculate the size of angle DAC   (03marks)

(b) Find the size of angle A

Suggested solution

1. In the diagram below, what is angle a?
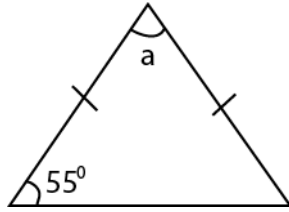
$55° + 55^0 + a = 180°$ (angle sum of triangle)

$110^0 + a = 180^0$

$110^0 - 110^0 + a = 180^0 - 110^0$

$a = 70^0$

2. In the diagram below, find the size of angle *x*

Solution

$60^0 + x = 110^0$

$60^0 - 60^0 + x = 110^0 - 60^0$
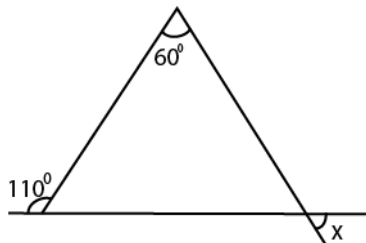
$x = 50^0$

3. In the diagram below, which of the angles, Q, R is the same as the angle marked a?
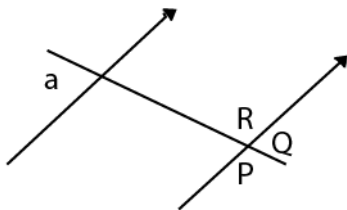
Solution

R (vertical angles are equal)

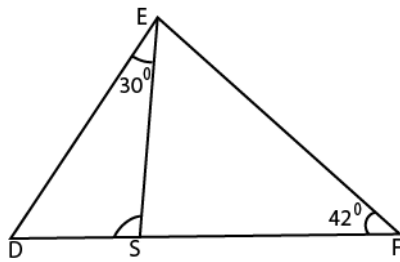4. In the triangle DBF, angle DEF = 60° and angle DFE = 42°. Find the size of angle DSE.

Finding angle EDF first,

<EDF + 42⁰ + 60⁰ = 180⁰

$<EDF + 42^0 + 60^0 = 180^0$

$<EDF + 102^0 = 180^0$

$<EDF + 102^0 - 102^0 = 180^0 - 102^0$

$<EDF = 78^0$

Now $<DSE + 78^0 + 30^0 = 180^0$

$<DSE + 108^0 = 180^0$

$<DSE + 108^0 - 108^0 = 180^0 - 108^0$

$<DSE = 72^0$

5. In the diagram below, which of the angles, Q, R is

the same as the angle marked a?
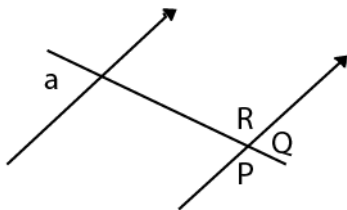
Solution

Q (vertical angles are equal)

6. In the diagram below, LM is parallel to PQ. Find the Value of x.

$< PQL = < QLM = 50^0$

⇨ $60^0 + 50^0 + x = 180^0$ (angle sum of triangle)

⇨ $x = 180^0 - (110^0)$

⇨ $x = 70^0$

7. In the figure below, find the value of angle x
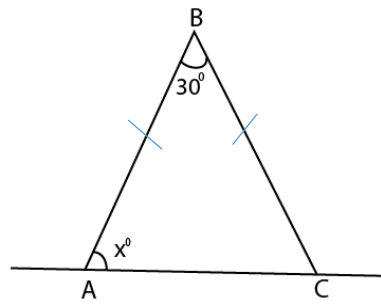
Angle BAC = angle ACB

$30^0 + x + x = 180^0$

$30^0 + 2x = 180^0$

$2x = 180^0 - 30^0$

$x = 75^0$

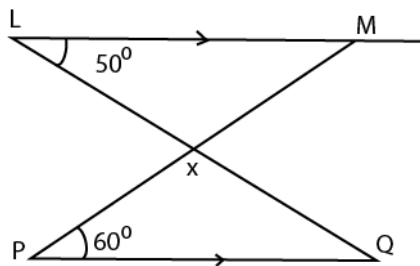8. Find the size of the angle marked x in the figure below. BC is parallel to AD.

$84 + x = 180$

$x = 180 - 84$

$= 96^0$
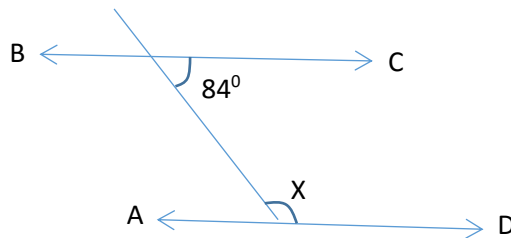
9. The figure below is an isosceles triangle, find the value of x.
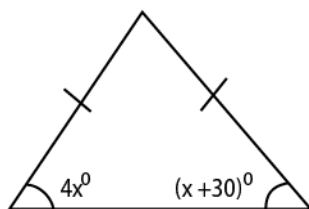
$4x = x + 30$

$4x - x = 30$

$3x = 30$

$x = 10^0$

10. In the figure below, find the size of angle marked *x*.
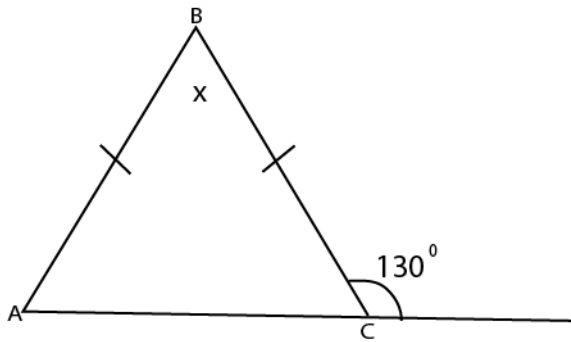
$< BAC =< ACB = 180 - 130 = 50^0$

$< BAC + < ACB + x = 180^0$ (angle sum

of triangle)

$50^0 + 50^0 + x = 180^0$

$x = 80^0$

11. Find the size of the angle marked S in the figure below:
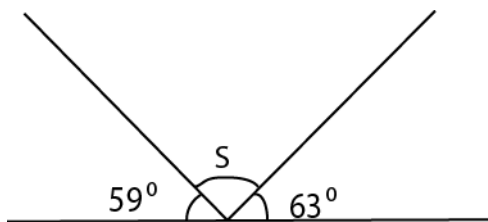
59 + S + 63 = 180⁰ (angle sum of straight line

122 + S = 180⁰

S = 58⁰

12. In the figure below, BCD is a triangle and ABC is a straight line; Find the value of t.
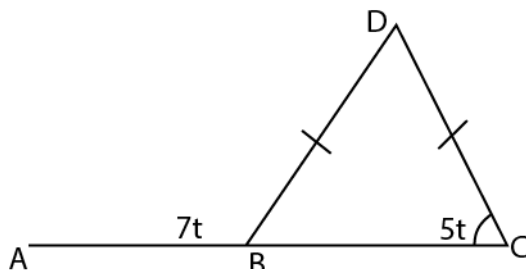
Angle DBC = angle DCB = 5t

7t + 5t = 180⁰ (angle sum of a straight line)

12 t = 180⁰

t = 15⁰
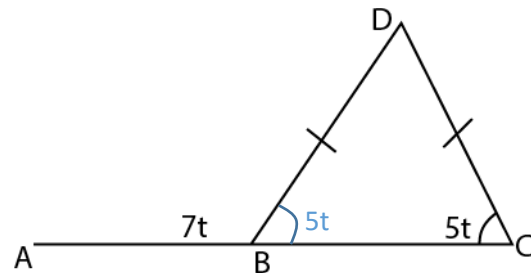
13. Find the value of the angle marked $(2x + 10)°$ in the figure below:
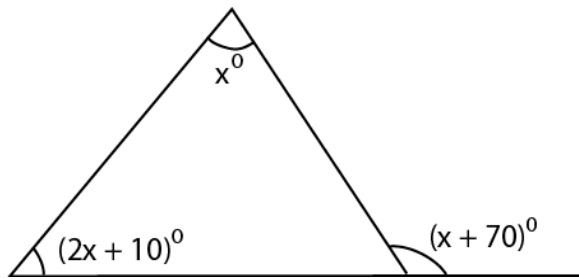
The sum of two opposite anterior angles= exterior angles

$(2x + 10) + x = x + 70$

Remove brackets

$2x + 10 + x = x + 70$

Collect like terms

$2x = 60^0$

$X = 30$

The required angle $= 2x + 10 = 2 \times 30 + 10 = 70^0$

14. Find the value of angle P in the figure below:
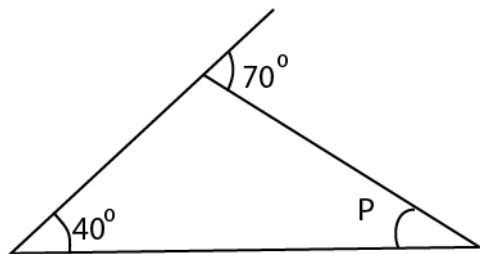
Two opposite interior angles = exterior angle

$40 + P = 70$

$P = 30^0$

15. Find the value of x in the figure below.
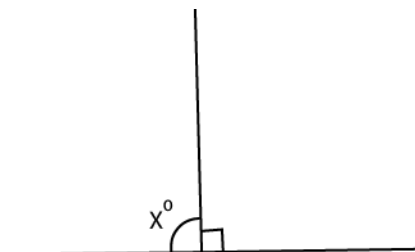
$x + 90 = 180^0$ (angle sum on a straight line)

$x = 90^0$

16. In the diagram below AE is parallel to CD. Find the size of angle r.
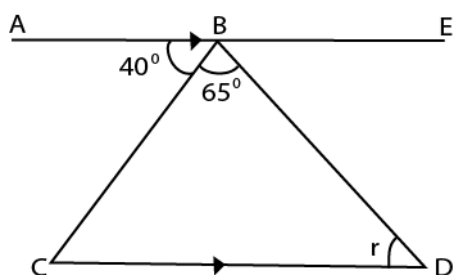
$40^0 + 65^0 + <EBD = 180^0$ (angle sum of

straight line)

Angle EBD $= 180^0 - (40 + 65)^0$

$= 75^0$

r = angle EBD = 75^0 (alternative angles)

17. In the diagram below, ABC is an isosceles triangle. Find the value of P at A
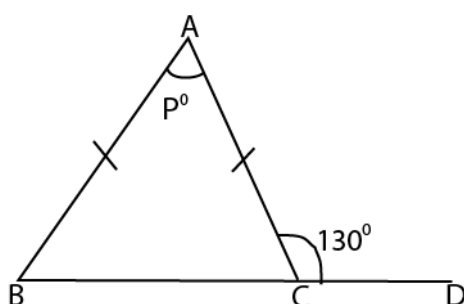
$<ACB + 130 = 180$ (angle sum on straight line)

Angle ACB = $180 - 130 = 50^0$

$<ABC = <ACB = 50^0$

$P + <ABC + <ACB = 180^0$

$P + 50 + 50 = 180^0$

$P = 80^0$

18. Find the value of x in the figure below.

$x + 2x = 90^0$

$3x = 90^0$
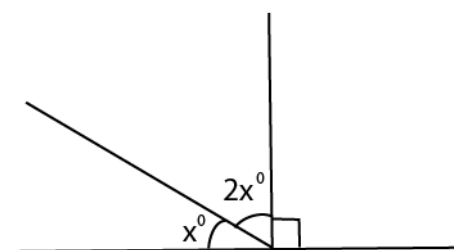
$x = 30^0$

19. Find the value of x in the figure below
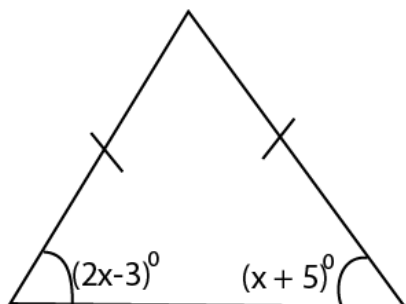
$(2x - 3) = (x + 5)$

Remove brackets

$2x - 3 = x + 5$

$x = 8$

20. Study the triangle below and use it to answer the questions that follow. A
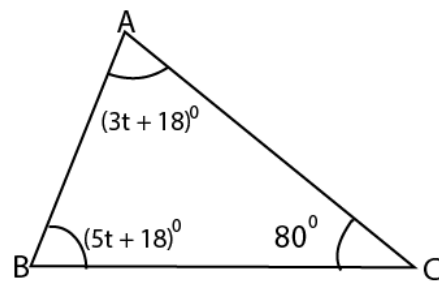
iii)   Find the value of t.

(3t + 18) + (5t + 18) + 80^0 = 180^0 (angle sum of triangle)

Remove brackets

3t + 18 + 5t + 18 + 80 = 180

8t + 116 = 180

8t = 64

t = 8^0

iv)   Find the size of angle ABC.

Angle ABC = 5 x 8 + 18 = 58^0

21. Find the value of the angle marked p in the figure below.
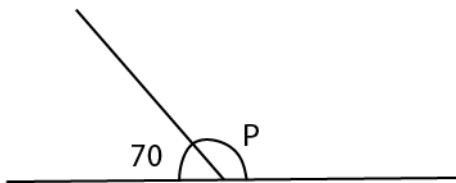
70 + P = 180^0 (angle sum of a triangle)

P = 110^0

22. Find the complement of an angle of 70°.

70^0 + complement = 90^0

Complement = 20^0

23. Find the size of angle *y* in the figure below.

$2y = 116$

$y = 58^0$
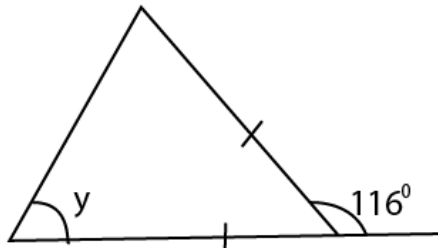
24. Two angles are supplementary. If one of them is 28°, find the other angle.

Supplementary angles add up to 180^0

∴ the supplement of $280 = (180 - 28)^0 = 152^0$

25. Find the value of *x* in the figure below.
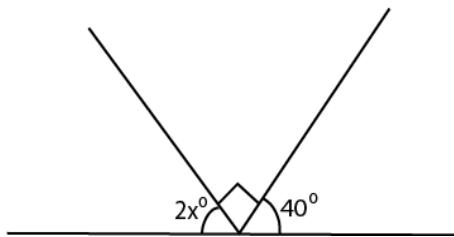
$2x + 90 + 40 = 180$ (angle sum on a straight line)

$2x + 130 = 180$

$2x = 180 - 130$

$2x = 50$

$x = 25$

26. In the diagram below, XTS is parallel to PYQR. Use it to find the size of angle Z.
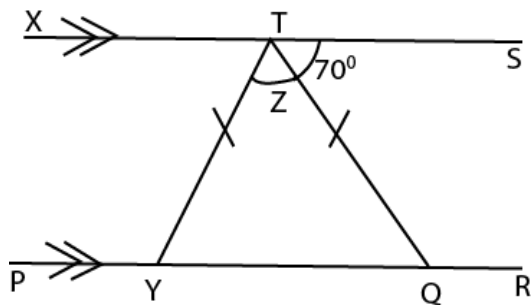
<TYQ = <TQY = 70^0 (alternative angle)

<TYQ + <TQY + Z = 180^0 (angle sum of triangle)

$70^0 + 70^0 + Z = 180^0$

$140^0 + Z = 180^0$

$Z = 40^0$

27. Find the complement of the angle of 25^0

Complementary angles add up to 90^0

Let the angle be y

$y + 250 = 90^0$

$y = 65^0$

28. In the triangle *PQR*, angle *PQR* = 78⁰, angle *PRQ*=42⁰ and angle *PQS* = 32⁰. Find the size of angle *PSQ*.
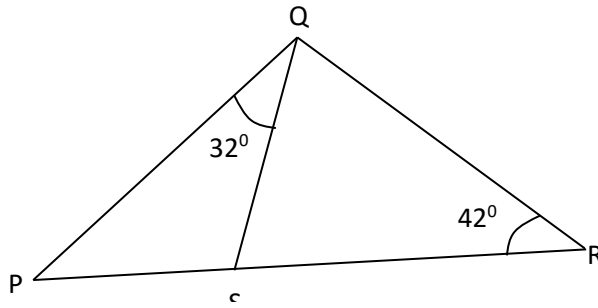
Angle QPR + angle PQR + angle QRP = 180⁰ (angle sum of a triangle)

Angle QPR + 78⁰ + 42⁰ = 180⁰

Angle QPR = 60⁰

Angle PSQ + angle QPS + angle PQS = 180⁰ (angle sum of a triangle)

Angle PSQ + 60⁰ + 32⁰ = 180⁰

Angle PSQ = 88⁰

29. Find the size of one of the exterior angles of a regular octagon.

Exterior angle $= \frac{360}{number\ of\ sides} = \frac{360}{8} = 45^0$

30. In the figure below, *AB* is parallel to *CD, find* the value of *x*
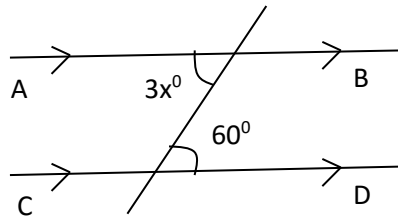
3x = 60⁰ (alternative angle)

$\frac{3x}{3} = \frac{60^0}{2}$

x = 20⁰

31. The figure below is an isosceles triangle. Find the size of angle *x*
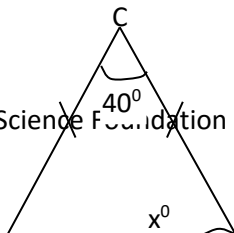
Angle CAB = angle ABC = x

It implies that: x + x + 40 = 180⁰ (angle sum of a triangle)

$$2x + 40 = 180$$

$$2x = 140$$

$$x = 70^0$$

32. Find the valve of Y in the figure below.

Interior angle + exterior angle = 180⁰

$$Y + 125 = 180^0$$

$$Y = 55^0$$
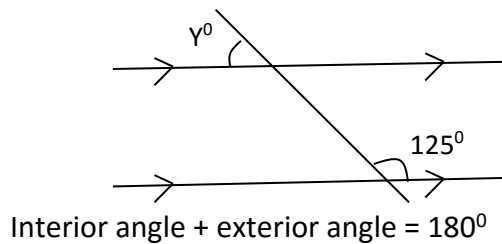
33. Find the value of x in the figure below.
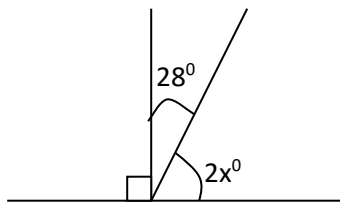
Angle sum on a straight line add up to 180⁰.

$$90 + 28 + 2x = 180$$

$$118 + 2x = 180$$

$$2x = 180 - 118$$

$$= 62^0$$

$$x = 31^0$$

34. In the diagram below, PTUV is a straight line, angle TSU =40⁰, angle *SUV*=150⁰ and angle *PQT*=30⁰.use the given information to find the value o the angle maerked k and n.

P   T   U   V

n^0 k^0

150^0

40^0

30^0

S

Q

K + 40 = 150

K = 110^0
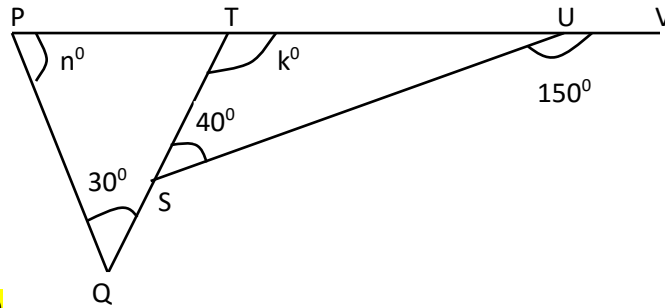
n + 30 = 110

n = 80^0

35. Find the value w in the figure below
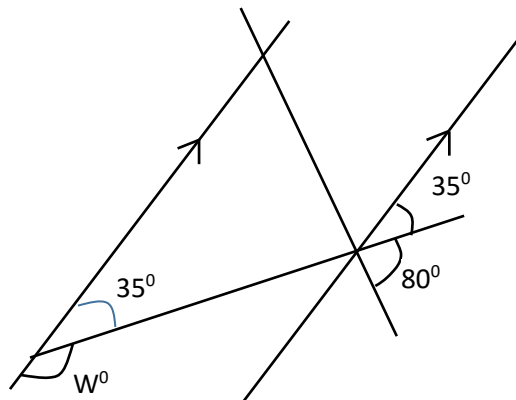
35^0

35^0 80^0

w^0

W + 35 = 180

W = 180 – 35

= 145^0

36. In the diagram below, *CAB* is a triangle and *DCA* a triangle line. Study it and answer the question below(3marks)

(a) What is the valve of *Y*

B

y+5⁰

4y+20⁰   2y+25⁰
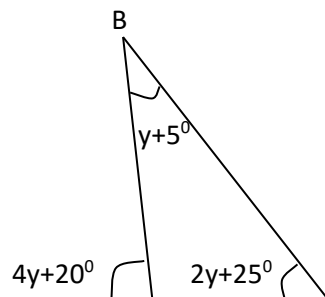
$4y + 20 = y +5 + 2y + 25$

$y =10^0$

(c) What the size of angle ACB? (3mark)

$angle\ ACB = 180^0 - (4y + 20)$

$= 180^0 - (4 \times 10 + 20)$

$= 120^0$

37. In the figure below, find the value of n in degrees.

108 + n =180

n = 72^0

38. In the triangle below, find the size of angle g in degrees.
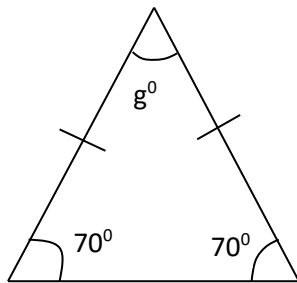
$70^0 + 70^0 + g0 = 180^0$(angle sum of a triangle)

$g^0 = 40^0$

39. In the diagram below, OP is parallel to QR, HKL is triangle, angle HLR=110^0 and angle SHP= 50°

Study it and answer the questions that follow.
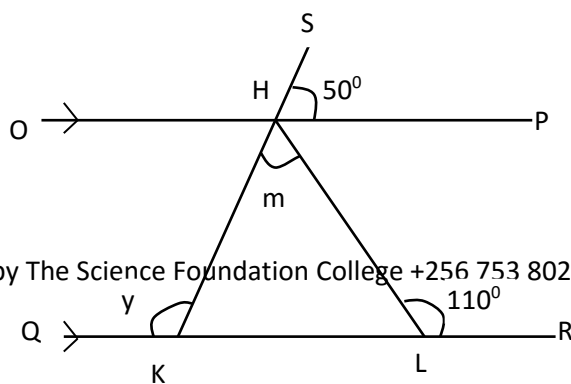
Find the size of

(c) Angle y: (2marks)

Angle SKI = angle SHP = 50^0 (corresponding angle)
Angle y + angle SKI = 180^0 (angle sum on a straight line)
$$y + 50^0 = 180^0$$
$$y = 130^0$$

(d) angle M   (3mark)

Angle HIK + 110^0 = 180^0 (angle sum on a straight line)
Angle HIK = 70^0
Angle HIK + angle HKI + m = 180^0 (angle sum of triangle)
$$70^0 + 50^0 + m = 180^0$$
$$m = 60^0$$

40. In the diagram below, find the valve of x.

$$55 + x = 90^0$$

x^0

$$x = 35^0$$

55^0

41. In the figure below, find the bearing of town B from town A.

N

N
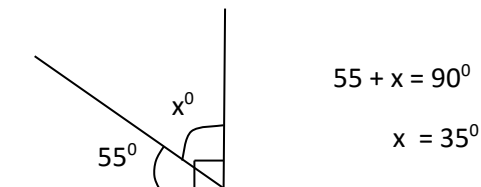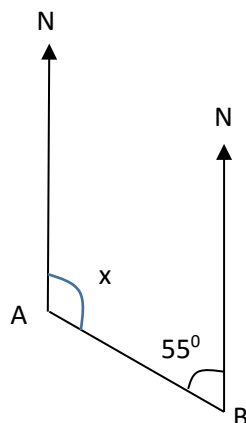
Let the bearing be x

$$X + 55^0 = 180^0$$

$$X = 180^0 - 55^0$$

x

$$X = 125^0$$

A

55^0

The bearing of B from A = 125^0

B

42. 36 In the diagram below, line *AB* is parallel to *CD* and *BCD* is an isosceles triangle. Study it carefully and use it to answer the questions that follow.
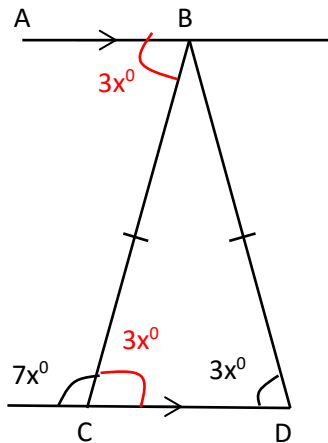
(c) Find the value of *X* (3marks)

Angle BCD = Angle CDB = $3x^0$ (corresponding angles)

$$7x^0 + 3x^0 = 180 \text{ (angle sum of a straight line)}$$
$$X = 18^0$$

(d) Work out the size of angle *ABC* (2marks)

Angle ABC = angle BCD (alternate angles) = $3x^0 = 18 \times 3 = 54^0$

43. In the diagram below, find the valves of K
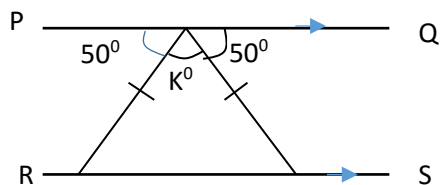
$50^0 + k + 50^0 = 180^0$ (angle sum of a straight line)

$$k = 80^0$$

. (a) The interior angle of a regular polygon is 36 more than its exterior angle. What is the size of each exterior angle? (02marks)

Let the exterior angle be x

The anterior angle = x + 36

Then, x + x + 36 = 180

X = 72

(b) In the figure below DE is parallel to FG, angle ODE =127^0 and angle OFG=136^0. Calculate the size of angle x.

(02marks)
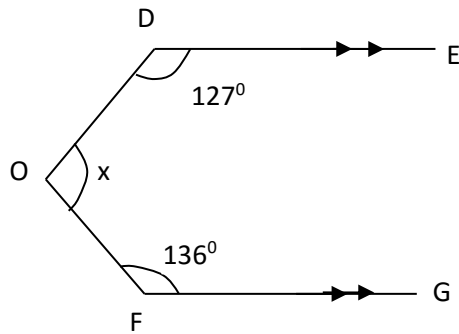
$$X + 127 + 136 = 360$$
$$X = 97$$

44. Find the value of X in the diagram below.
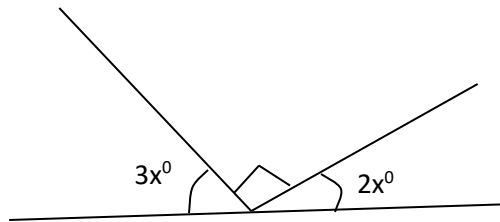
$$3x + 90^0 + 2x = 180^0 \text{ (angle sum of straight line)}$$

$$x = 18^0$$

45. In the figure below, BCD is a straight line. Line BX bisects angle ABC. Line AB is parallel to line XC.

Angle BCE = 50⁰ and angle BAC =70⁰
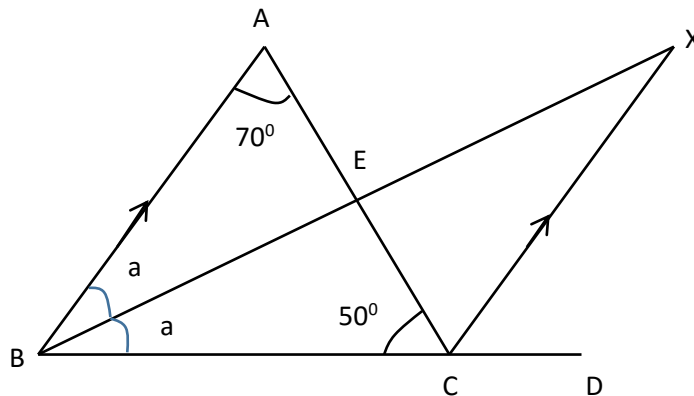
Find the size of the angles:

(iii)   CEX   (04 marks)

Considering triangle ABC   Using triangle BAE

$a + a + 50^0 + 70^0 = 180^0$ (angle sum of a triangle) $a + \text{Angle BEA} + 70^0 = 180^0$

$2a = 60^0$ $30^0 + \text{angle BEA} + 70^0 = 180^0$

$A = 30^0$ Angle BEA $= 80^0$

                                              But, angle BEA = angle CEK

                                              ∴ Angle CEK $= 80^0$

(iv)   DCX

Angel XCA = angle BAC = 70^0 (alternative angles)

Angle BCA + angle ACX + angle XCD = 180^0 (angle sum on a straight line)

$50^0 + 70^0 + \text{angle XCD} = 180^0$

                              Angle XCD $= 60^0$

46. Find the size of angle *y* the figure below.
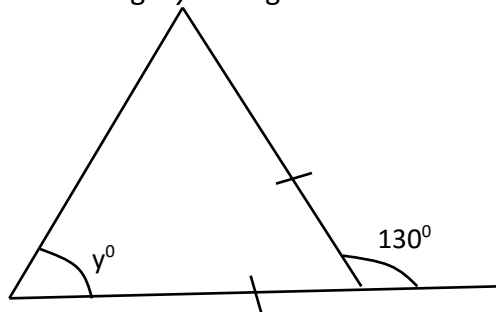
2y = 130⁰

y = 65⁰

47. In the figure below line AB is parallel to CD, Angle CTV = 44 and angle TQR=56. **Study use it to answer the questions that follow:**

Find the size of

(c) Angle *k*   (02mark)
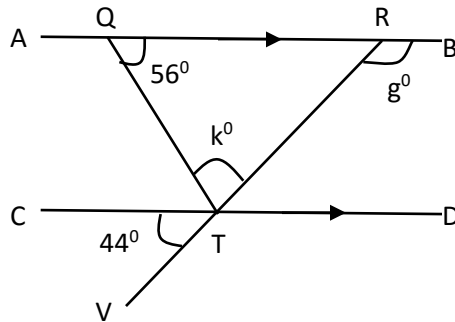
        Angle RTD = 44⁰ (corresponding angles)

        Angle RTD + k + 56 = 180

                k = 180 − (44 + 56) = 80⁰

(d) Angle g   (02mark)
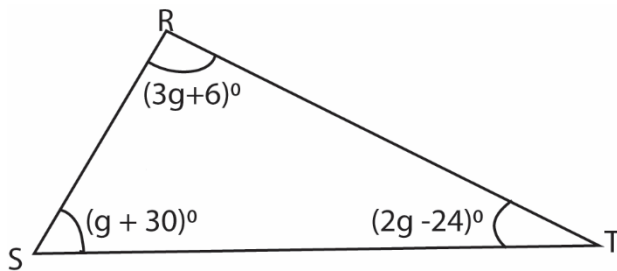
      g + angle RTD = 180⁰

        g = 180 -44 =136⁰

48. In the triangle below, find the value of d in degrees
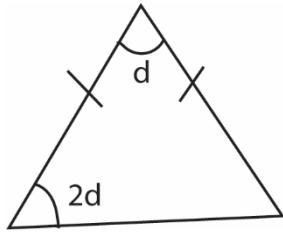
5d = 180 (angle sum of a triangle)

d= 36^0

49. Study the figure below and use it to answer the questions that follow.

(c) Find the value of g (03marks)

(3g + 6) + (g + 30) + (2g − 24) = 180^0 (angle sum of a triangle)

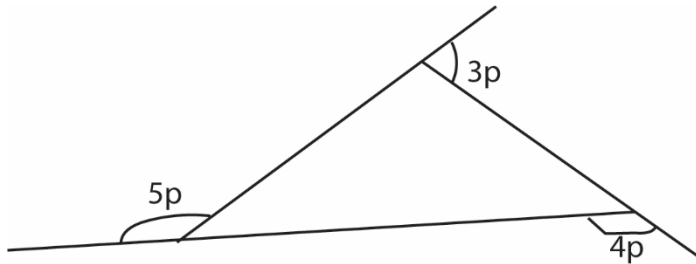3g + 6 + g + 30 + 2g − 24 = 180^0

6g +12 = 180^0

6g = 192^0

g = 32^0

(d) Calculate the size of angle RST

Angle RST = (g + 30)^0

= 32 + 30

= 62^0

50. Find the value of p in degrees in the diagram below

$$180 - 5p + 180 - 3p + 180 - 4p = 180 \text{ (angle sum of triangle)}$$

$$540 - 12p = 180$$

$$12p = 360 \text{ (sum of exterior angle)}$$

$$p = 30^0$$

51. (a) The interior angle of a regular polygon is 108 more than the exterior angle. How many sides has the polygon? (03marks)
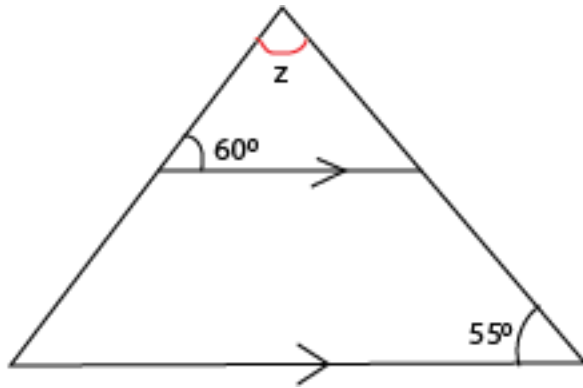
Let exterior angle be x

$$x + x + 108^0 = 180^0 \text{ (angle sum of straight line)}$$
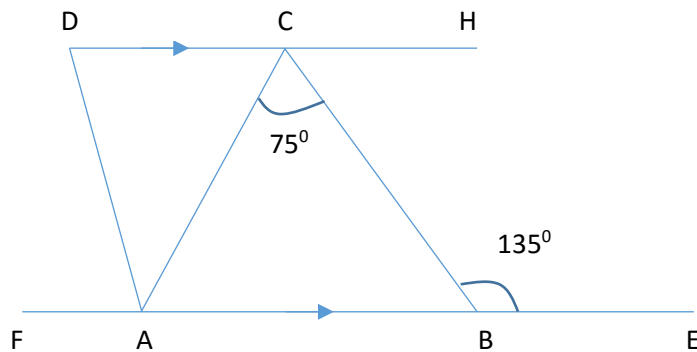
$$x = 36^0$$

Number of sides = 360/36 = 10sides

(b) Find size of angle z in the figure below

$55^0 + 60^0 + z = 180^0$ (angle sum of a triangle)

$z = 65^0$

52. In the diagram below, line DH is parallel to FE. Angle ACB = 75^0 and angle CBE = 135^0. Angle FAD is twice angle DAC. Study the diagram and use it to answer the questions that follow.

(c) Calculate the size of angle DAC   (03marks)

Angle CBA + angle CBE = 180^0 (angle sum of a straight line)

Angle CBA = $(180 - 135)^0$

= 45^0

Then angle ACB + angle CAB + angle ABC = 180^0 (angle sum of a triangle)

It implies that: angle CAB = $(180 - (45 + 75))^0$

= 60^0

(d) Find the size of angle DAC

Angle DAC = angle FAD = 2X = 2 x 40 = 80^0 (corresponding angles)

Thank you