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## Lowest common multiple (LCM)

The Least Common Multiple (LCM) is also referred to as the Lowest Common Multiple (LCM) and Least Common Divisor (LCD). For two integers  $a$  and  $b$ , denoted  $\text{LCM}(a,b)$ , the LCM is the smallest positive integer that is evenly divisible by both  $a$  and  $b$ . For example,  $\text{LCM}(3,5) = 10$  and  $\text{LCM}(7,8) = 56$ .

How to find LCM

### 1 Listing Multiples

- (a) List the multiples of each number until at least one of the multiples appears on all lists
- (b) Find the smallest number that is on all of the lists
- (c) This number is the LCM

Example:  $\text{LCM}(6,7,21)$

- (i) Multiples of 6: 6, 12, 18, 24, 30, 36, **42**, 48, 54, 60
- (ii) Multiples of 7: 7, 14, 21, 28, 35, **42**, 56, 63
- (iii) Multiples of 21: 21, **42**, 63

Find the smallest number that is on all of the lists. We have it in bold red above.

So  $\text{LCM}(6, 7, 21)$  is 42

### 2 By prime Factorization

- (i) Create a set of prime factors of each given number.
- (ii) Create a union set for the factor.
- (iii) The products of union set of prime factors is the LCM

### Example 1

Find LCM of 9 and 12

A set A of prime factors of 12 = {2, 2, 3}

A set B of prime factors of 9 = {3, 3}

$$A \cup B = \{2, 2, 3, 3\}$$

$$\text{LCM of 9 and 12} = 2 \times 2 \times 3 \times 3 = 36$$

Or use a table to find the prime factors

2	9	12
2	9	6
3	3	3
3	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 3 = 36$$

### Example 2

There are three classes in a school with 4, 15 and 18 pupils respectively. Find the number of mangoes that each class can share out without any remainders.

### Solution

The questions requires to find the LCM of 4, 15 and 18

2	4	15	18
2	2	15	9
2	1	15	9
3	1	5	3
3	1	5	1
5	1	1	1

$$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5 = 180$$

## Greatest common Factor (GCF)

This is found by finding the products of product of the intersection set of prime factors of the numbers.

It can be obtained by

1. Listing Factors and identify the biggest common factor in the lists.

### Example 3

Find the greatest common factor, GCF, of 36 and 63.

Factors of 36 = {1, 3, 6, **9**, 18, 36}

Factors of 63 = {1, 3, 7, **9**, 21, 63}

The greatest common factor = 9 (in bold)

2. Using the product of the elements of intersection set of prime factors of the numbers

### Example 4

What is the greatest common factor (GCF) of 36 and 63?

Set A = prime factors of 36 or {2, 2, 3, 3}

Set B = prime factors of 63 or {3, 3, 7}

$A \cap B = \{3, 3\}$

Therefore,  $GCF = 3 \times 3 = 9$

### Exercise

1. (a) Find the LCM of 7 and 15.

(b) In Mugo primary School, there are two bells. One bell is for lower primary and the other is for upper primary. The bell for lower primary rings after every 30 minutes, that for upper primary rings after every 40 minutes. If both bells are rung at, 9: 10 a.m. when will they next be rung at the same time?

2. Find the greatest common factor of 18 and 30.

3. In Odokomit Primary School, two bells are rung at different intervals of 30 minutes and 40 minutes. If they are rung together at 10:00 a.m., at what time will they be rung together again?

4. Find the Lowest Common Multiple (L.C.M) of 6 and 9

5. What is the Greatest Common Factor (GCF) of 8 and 12?

6. Find the Greatest common factor (GCF) of 18 and 24
7. At Kampala Bus Park, buses travelling to Arua and Mbarara leave after very 40munites and 50 munities respectively. The first buses to the town leave together at 6:00am. At what time will buses to the two towns leave Kampala together again? (04marks)
8. Find the least number of sweets when divided among 8 boys or 6 girls equally, laves 2 sweets as a remainder.
9. The prime factors of 12 and 90 are given below:  
 $12 = 2^2 \times 3$   
 $90 = 2 \times 3^2 \times 5$   
Use the given prime factors above to find the Lowest Common Multip (LCM) of 12 and 90.

## Suggested answers

1. (a) Find the LCM of 7 and 15.

7	7	15
3	1	15
5	1	5
	1	1

LCM =  $7 \times 1 \times 5 = 105$

- (c) In Mugo primary School, there are two bells. One bell is for lower primary and the other is for upper primary. The bell for lower primary rings after every 30 minutes, that for upper primary rings after every 40 minutes. If both bells are rung at, 9: 10 a.m. when will they next be rung at the same time?

2	40	30
2	20	15
2	10	15
5	5	15
3	1	3
	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 5 \times 3 = 120 \text{ minute}$$

$$= 2 \text{ hour}$$

Time for ringing the belles together

9 : 10 am

+ 2

11:10 am

2. Find the greatest common factor of 18 and 30.

Common factor	18	30
2	9	15
3	3	5

$$\text{GCF} = 2 \times 3 = 6$$

3. In Odokomit Primary School, two bells are rung at different intervals of 30 minutes and 40 minutes. If they are rung together at 10:00 a.m., at what time will they be rung together again?

2	40	30
2	20	15
2	10	15
5	5	15
3	1	3
	1	1

The time taken before the bells ring again is

$$\text{LCM} = 2 \times 2 \times 2 \times 5 \times 3 = 120 \text{ minute}$$

$$= 2 \text{ hour}$$

Time for ringing the bells together

10 : 00 am

+ 2

12:00 noon

4. Find the Lowest Common Multiple (L.C.M) of 6 and 9

prime factors	9	6
3	3	2
3	1	2
2	1	1

$$\text{GCF} = 2 \times 3 \times 3 = 18$$



5. What is the Greatest Common Factor (GCF) of 8 and 12?

Common prime factors	9	6
3	3	2

GCF = 3

6. Find the Greatest common factor (GCF) of 18 and 24

Factors of 18 = (1, 2, 3, **6**, 9, 18)

Factors of 24 = (1, 2, 4, **6**, 8, 12, 24)

Greatest common factor = **6**

7. At Kampala Bus Park, buses travelling to Arua and Mbarara leave after very 40 minutes and 50 minutes respectively. The first buses to the town leave together at 6:00am. At what time will buses to the two towns leave Kampala together again? (04marks)

The lowest common multiple is the duration of time before the bus can leave together again. It is obtained by finding the prime factor

2	40	50
2	20	25
2	10	25
5	5	25
5	1	5
	1	1

The buses will leave together after 3hr 20min at

$$6.00 + 3.20 = 9.20 \text{ a.m.}$$

$$2 \times 2 \times 2 \times 5 \times 5 = 200 \text{ minutes or 3hrs 20mins}$$

8. Find the least number of sweets when divided among 8 boys or 6 girls equally, leaves 2 sweets as a remainder.

prime factors	9	6
3	3	2
3	1	2
2	1	1

$$\text{GCF} = 2 \times 3 \times 3 = 18$$

$$\text{The number} = 18 + 2 = 20$$

9. The prime factors of 12 and 90 are given below:

$$12 = 2^2 \times 3$$

$$90 = 2 \times 3^2 \times 5$$

Use the given prime factors above to find the Lowest Common Multip (LCM) of 12 and 90.

Union set of prime factors = {2, 2, 3, 3, 5}

$$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5$$

$$= 180$$