PROPOSE MARKING GUIDE
FOR WAKISSHA 2024
MATHEMATICS P425/1
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No 1.

$\frac{x}{y} + \frac{6y}{x} = 5$ — ①

$x - 2y - 2 = 0$ — ②.

from ②.

$x = 2(y+1)$ — ③.

Subst from ①.

$\frac{x^2+6y^2}{xy} = 5$

$x^2 + 6y^2 = 5xy.$ — (**)

Subst ③ into (**).

$[2(y+1)]^2 + 6y^2 = 5[2(y+1)y.$

$4(y+1)^2 + 6y^2 = 10y(y+1)$

$4(y^2+2y+1) + 6y^2 = 10y^2+10y$

$4y^2 + 8y + 4 + 6y^2 = 10y^2 + 10y.$

$10y^2 + 8y + 4 = 10y^2 + 10y.$

$8y + 4 = 10y$

$2y = 4$

$y = 2.$

Subst y into ③.

$x = 2(2+1)$

$x = 6.$

The co-ordinates are $(6, 2)$.

No 2.

$\int_0^1 \frac{x}{\sqrt{(1+x)}} dx.$

Let $\sqrt{1+x} = u$

$u^2 = 1 + x$

$x = u^2 - 1$

from $u^2 = 1 + x$

$2u\,du = dx.$

Changing limits.

x	u
0	1
1	$\sqrt{2}$

$$\Rightarrow \int_0^1 \frac{x}{\sqrt{(1+x)}}\,dx = \int_1^{\sqrt{2}} \frac{u^2-1}{u} \cdot 2u\,du.$$

$$= 2\int_1^{\sqrt{2}} (u^2-1)\,du.$$

$$= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$$

$$= \left[\frac{2u^3}{3} - 2u\right]_1^{\sqrt{2}}$$

$$= \left(\frac{2(\sqrt{2})^3}{3} - 2(\sqrt{2})\right) - \left(\frac{2(1)^3}{3} - 2(1)\right)$$

$$= -0.942809042 + 1.33333333$$

$$= 0.390524291$$

$$\int_0^1 \frac{x}{\sqrt{(1+x)}}\,dx = 0.3905 \ (4dps).$$

No 3.

$\sin x + \sin y = \beta_1$ ——— ①

$\cos x + \cos y = \beta_2$ ——— ②.

i, $\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2}$

Solution.

from ①.

$\sin x + \sin y = \beta_1$

$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_1$ ——— (*).

from ②.

$\cos x + \cos y = \beta_2$

$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_2$ ——— (**)

(*) ÷ (**)

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{\beta_1}{\beta_2}$$

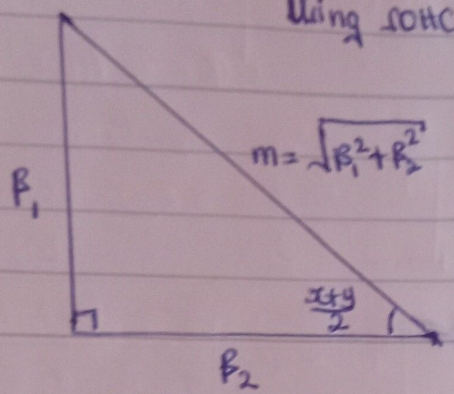
$\tan\left(\frac{x+y}{2}\right) = \beta_1/\beta_2$ As required.

ii,

from $\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2}$.

Consider the triangle below.

Using SOHCAHTOA.



From the triangle above;

$\sin\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\sqrt{\beta_1^2+\beta_2^2}}$

$\cos\left(\frac{x+y}{2}\right) = \frac{\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}$

Taking the angle to be $\left(\frac{x+y}{2}\right)$.

Using; $\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos\theta = 2\cos^2 \theta/_2 - 1$

Now taking θ to be $\left(\frac{x+y}{\cancel{2}}\right)$.

$\Rightarrow \cos(x+y) = 2\cos^2\left(\frac{x+y}{2}\right) - 1$

But $\cos(x+y) = 2\cos^2\left(\frac{x+y}{2}\right) - 1 \leftarrow$

$$\cos(x+y) = 2\left(\frac{\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}\right)^2 - 1$$

$$= \frac{2\beta_2^2}{\beta_1^2+\beta_2^2} - \frac{1}{1}$$

$$\cos(x+y) = \frac{2\beta_2^2 - (\beta_1^2+\beta_2^2)}{\beta_1^2+\beta_2^2}$$

$$\cos(x+y) = \frac{2\beta_2^2 - \beta_1^2 - \beta_2^2}{\beta_1^2+\beta_2^2}$$

$$\cos(x+y) = \frac{\beta_2^2 - \beta_1^2}{\beta_1^2+\beta_2^2}.$$

No 4.

$\sqrt[3]{27.15}$

Let $y = \sqrt[3]{x}$, $x = 27$, $\delta x = 0.15$

Since $y = \sqrt[3]{x} \Rightarrow y = \sqrt[3]{27} = 3$

$y = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$

$\frac{dy}{dx} = \frac{1}{3\sqrt[3]{x^2}}$

$\left.\frac{dy}{dx}\right|_{x=27} = \frac{1}{3\sqrt[3]{27^2}}$

$= \frac{1}{3 \times 9}$

$\frac{dy}{dx} = \frac{1}{27}$

In small changes, $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx \frac{dy}{dx} \cdot \delta x$

$\delta y \approx \frac{1}{27} \times 0.15$

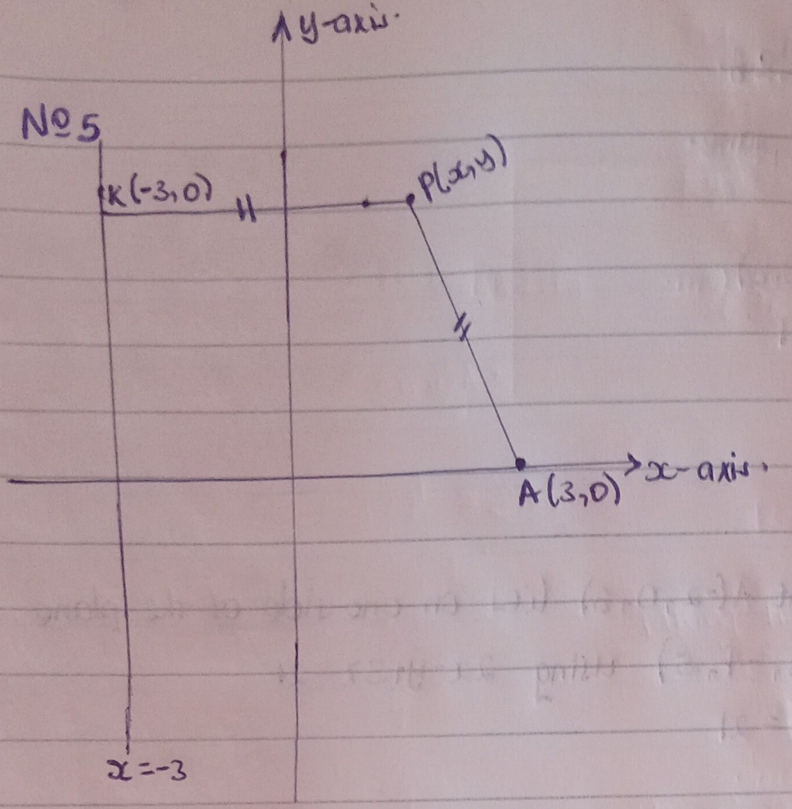
$\delta y \approx \frac{1}{81}$

$\sqrt[3]{27.15} = y + \delta y$

$= 3 + \frac{1}{81}$

$\sqrt[3]{27.15} = 3\frac{1}{81}$

No 5



Comparing $x=-3$ with the general equation of the line $ax+by+c=0$. $\Rightarrow x+3=0$

$a=1,\ b=0,\ c=3$

For the perpendicular distance KP

$$KP = \left| \frac{(1\times x)+(0\times y)+3}{\sqrt{1^2+0^2}} \right|$$

$$KP = x+3$$

But $|KP| = |PA|$

$$(x+3)^2 = \left(\sqrt{(x-3)^2+(y-0)^2}\right)^2$$

$$(x+3)^2 = (x-3)^2+(y-0)^2$$

$$x^2+6x+9 = x^2-6x+9+y^2$$

$$y^2-12x=0$$

$$y^2=12x$$

Which is a parabola with vertex, $V(0,0)$

Comparing with the genenal equation of a parabola, $y^2=4ax$.

$$4a=12$$

$$a=\frac{12}{4}$$

$$a=3$$

Focus, $F(3,0)$

Equation of the directrix.

$$x=-3$$

No 6.

Using the equation of the plane.

$2x - y + 3z = 21$

Testing with point $A(-2, 0, 6)$

$2(-2) - (0) + 3(6) = 21$

$-4 + 18 = 21$

$14 \neq 21$

$14 < 21$

This implies that the point $A(-2, 0, 6)$ lies on one side of the plane.

Testing with point $B(3, -4, 5)$ using $2x - y + 3z = 21$

$2(3) - (-4) + 3(5) = 21$

$6 + 4 + 15 = 21$

$25 \neq 21$

$25 > 21$

$\Rightarrow$ The point $B(3, -4, 5)$ lies on another side of the plane.

Since points A and B give opposite signs, the therefore lies on the opposite sides of the plane.

No 7.

G.P

$U_2 = 24$

$U_3 = 12(b+1)$

$b = ??$

$S_3 = 76.$

$$r = \frac{12(b+1)}{24}$$

$$r = \tfrac{1}{2}(b+1)$$

$$a = \frac{24}{\frac{1}{2}(b+1)}$$

$$a = \frac{48}{(b+1)}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$76 = \frac{\left(\frac{48}{b+1}\right)\left[\left(\frac{1}{2}(b+1)\right)^3 - 1\right]}{\frac{1}{2}(b+1) - \frac{1}{1}}$$

$$76 = \frac{\left(\frac{48}{b+1}\right)\left[\frac{(b+1)^3}{8} - 1\right]}{\frac{(b+1)-2}{2}}$$

$$76 = \frac{\left(\frac{48}{b+1}\right)\left[\frac{(b+1)^3 - 8}{8}\right]}{\frac{b-1}{2}}$$

$$76\left(\frac{b-1}{2}\right) = \left(\frac{48}{b+1}\right)\left[\frac{(b+1)^3 - 8}{8}\right]$$

$$38(b-1)(b+1) = 48\left[\frac{(b+1)^3 - 8}{8}\right]$$

$$38(b-1)(b+1) = 6\left[(b+1)^3 - 8\right]$$

$$19(b+1)(b-1) = 3\left[(b+1)^3 - 8\right]$$

$$19(b^2-1) = 3(b+1)^3 - 24$$

$$19b^2 - 19 = 3(b^3+3b^2+3b+1) - 24$$

$$19b^2 - 19 = 3b^3 + 9b^2 + 9b + 3 - 24$$

$$3b^3 - 10b^2 + 9b - 2 = 0$$

$b=1$ is a root of the equation

$\Rightarrow$ $(b-1)$ is a factor.

$$\begin{array}{r|l} & 3b^2 - 7b + 2 \\ \hline b-1 & 3b^3 - 10b^2 + 9b - 2 \\ - & 3b^3 - 3b^2 \\ \hline - & -7b^2 + 9b - 2 \\ & -7b^2 + 7b \\ \hline - & 2b - 2 \\ & 2b - 2 \\ \hline & 0 \end{array}$$

$$3b^2 - 7b + 2 = 0$$

$$3b^2 - 6b - b + 2 = 0$$

$$3b(b-2) - 1(b-2) = 0$$

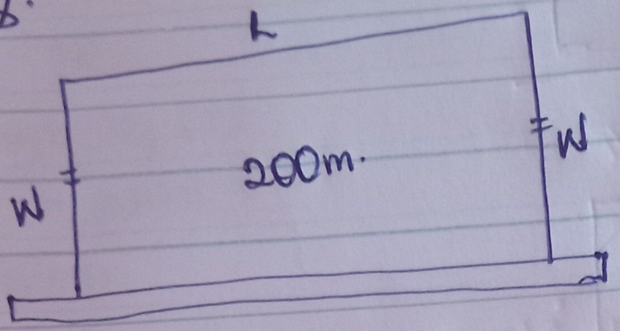
$$(3b-1)(b-2) = 0$$

Either $b = 1/3$

Or $b = 2$.

$\therefore$ $b = 1, 2, 1/3$.

No 8.



$P = L + W + W$

$200 = L + 2W$

$L = 200 - 2W$. —— ①

$A = L \times W$

$A = W(200 - 2W)$.

$A = 200W - 2W^2$

$\frac{dA}{dW} = 200 - 4W$.

For maximum area, $\frac{dA}{dW} = 0$.

$200 - 4W = 0$.

$4W = 200$

$W = 50m$.

from ①

$L = 200 - 2(50)$

$L = 100m$,

Area $= L \times W$.

$= 100 \times 50$

Area $= 5000m^2$.

SECTION B.

No 9a,

$$\frac{\cos(B+C)}{\operatorname{Cosec} B \operatorname{Cosec} C} = \frac{bc}{ab+ac}$$

Considering the L.H.S. Using Sine rule; $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$$\frac{bc}{ab+ac} = \frac{(2R\sin B)\cdot(2R\sin C)}{(2R\sin A\cdot 2R\sin B)+(2R\sin A\cdot 2R\sin C)}$$

$\Rightarrow a = 2R\sin A$.

$b = 2R\sin B$

$c = 2R\sin C$.

$$= \frac{4R^2\sin B\sin C}{4R^2\sin A\sin B + 4R^2\sin A\sin C}$$

$$= \frac{4R^2\sin B\sin C}{4R^2(\sin A\sin B+\sin A\sin C)}$$

$$= \frac{\sin B\sin C}{\sin A\sin B+\sin A\sin C}$$

Dividing $\sin B\sin C$ up and down.

$$\frac{bc}{ab+ac} = \frac{\frac{\sin B\sin C}{\sin B\sin C}}{\frac{\sin A\sin B}{\sin B\sin C}+\frac{\sin A\sin C}{\sin B\sin C}}$$

$$= \frac{1}{\frac{\sin A}{\sin C}+\frac{\sin A}{\sin B}}$$

$$= \frac{1}{\sin A\left(\frac{1}{\sin C}+\frac{1}{\sin B}\right)}$$

$$\frac{bc}{ab+ac} = \frac{1}{\sin A}\left[\frac{1}{\operatorname{Cosec} C+\operatorname{Cosec} B}\right]$$

But $A+B+C = 180°$

$A = 180° - (B+C)$

$\sin A = \sin[180° - (B+C)]$

$\sin A = \sin 180° \cos(B+C) - \sin(B+C)\cos(180°)$

$\sin A = \sin(B+C)$

$$\frac{bc}{ab+ac} = \frac{1}{\sin(B+C)}\left[\frac{1}{\operatorname{Cosec} B+\operatorname{Cosec} C}\right]$$

$$\frac{bc}{ab+ac} = \frac{\operatorname{Cosec}(B+C)}{\operatorname{Cosec} B+\operatorname{Cosec} C}$$

Thus the Proof had issues. Instead it would have been, $\frac{bc}{ab+ac} = \frac{\operatorname{Cosec}(B+C)}{\operatorname{Cosec} B+\operatorname{Cosec} C}$

No 9b,

$3\cot\theta + \operatorname{cosec}\theta = 2 \qquad 0^\circ \le \theta \le 360^\circ$

$\frac{3\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = 2$

Multiply through by $\sin\theta$.

$3\cos\theta + 1 = 2\sin\theta$.

Re-arranging.

$3\cos\theta - 2\sin\theta = -1$

Applying R-formula.

$3\cos\theta - 2\sin\theta \equiv R\cos(\theta+\alpha)$

$\Rightarrow R\cos(\theta+\alpha) = 3\cos\theta - 2\sin\theta$.

$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha = 3\cos\theta - 2\sin\theta$.

Comparing co-effecients;

$R\cos\alpha = 3$ —— ①.

$R\sin\alpha = 2$ —— ②,

② ÷ ①.

$\tan\alpha = 2/3$

$\alpha = \tan^{-1}(2/3)$

$\alpha = 33.7^\circ$

①^2 + ②^2 give;

$R^2\cos^2\alpha + R^2\sin^2\alpha = 3^2 + 2^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 13$

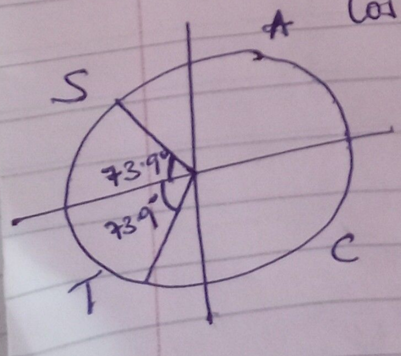
$R^2 = 13$

$R = \sqrt{13}$

$\Rightarrow R\cos(\theta+\alpha) = \sqrt{13}\cos(\theta+33.7^\circ)$.

$\Rightarrow \sqrt{13}\cos(\theta+33.7^\circ) = -1$

$\cos(\theta+33.7^\circ) = \frac{-1}{\sqrt{13}}$



$\theta + 33.7^\circ = \cos^{-1}\left(\frac{1}{\sqrt{13}}\right)$

$\theta + 33.7^\circ = 106.1^\circ, 253.9^\circ$

$\theta = \{72.4, 220.2\}$.

Nº 10.

a,

$$\frac{[\sqrt{3}(\cos\theta + i\sin\theta]^{8}}{[3\cos 2\theta + 3i\sin 2\theta]^{3}} = \frac{(\sqrt{3})^{8}(\cos\theta + i\sin\theta)^{8}}{(3)^{3}(\cos 2\theta + i\sin 2\theta)^{3}}$$

Using Demoivre's theorem, $(\cos\theta + i\sin\theta)^{n} = \cos n\theta + i\sin n\theta$.

$$\frac{[\sqrt{3}(\cos\theta + i\sin\theta)]^{8}}{[3\cos 2\theta + 3i\sin 2\theta]^{3}} = 3\left[\frac{(\cos\theta + i\sin\theta)^{8}}{(\cos\theta + i\sin\theta)^{2\times 3}}\right]$$

$$= 3\left[\cos\theta + i\sin\theta\right]^{8-6}$$

$$= 3(\cos\theta + i\sin\theta)^{2}$$

$$\frac{\sqrt{3}(\cos\theta + i\sin\theta)^{8}}{[3\cos 2\theta + 3i\sin 2\theta]^{3}} = 3\cos 2\theta + i\sin 2\theta.$$

b,

$(1+3i)z_1 = 5(1+i)$

$$z_1 = \frac{5+5i}{1+3i}$$

$$z_1 = \frac{(5+5i)(1-3i)}{(1+3i)(1-3i)}$$

$$z_1 = \frac{5-15i+5i+15}{1^2+9}$$

$$z_1 = \frac{20-10i}{10}$$

$$z_1 = 2-i$$

Let $z = x+iy$.

$|(x+iy)-(2-i)| = |2-i|$

$|(x-2)+(y+1)i| = |2-i|$

$\sqrt{(x-2)^2+(y+1)^2} = \sqrt{2^2+(-1)^2}$

Squaring both sides.

$(x-2)^2+(y+1)^2 = 5$

$x^2-4x+4+y^2+2y+1 = 5$

$x^2+y^2-4x+2y = 0$

Which is an equation of a circle of the form $x^2+y^2+2gx+2fy+c=0$.

$C(-g,-f)$

$2g = -4$ | $2f = 2$

$g = -2$ | $f = 1$

$c(2, -1)$

Radius, $r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-2)^2 + (1)^2 - 0}$

$r = \sqrt{5}$ units.

No 11a,

A.P

a

$d = 2$

$S_{10} = ??$

G.P.

a.

$r = 1/3$

$S_\infty = 9$

Solution.

from $S_\infty = \frac{a}{1-r}$

$9 = \frac{a}{1 - 1/3}$

$9 = \frac{a}{2/3}$

$a = 9 \times 2/3$

$a = 6$.

→ The first term of an A.P is 6.

$S_{10} = ??$, $n = 10$, $a = 6$, $d = 2$.

from $S_n = n/2 \left[2a + (n-1)d\right]$.

$S_{10} = 10/2 \left[(2 \times 6) + (10-1)2\right]$

$S_{10} = 5 (12 + 18]$

$S_{10} = 150$

No 11b,

DEFEATED.

3E, 2D, 1F, 1A, 1T

8 letters in the arrangement.

If they are to be separated, then there is no restriction made.

∴ The possible arrangement = $\begin{bmatrix} 8 \\ 3 \end{bmatrix}$

$$= {}^8C_3$$

$$= \frac{8!}{(8-3)!\,3!}$$

$$= \frac{8\times7\times6\times5\times4\times3!}{5!\,3!}$$

$$= \frac{8\times7\times6\times5\times4}{5\times4\times3\times2\times1}$$

$$= 56 \text{ ways.}$$

∴ There are 56 possible ways of arrangement.

No 12.a,

A(1, 1, 1) B(1, 0, 1) C(3, 2, -1).

$d_1 = AB$

$d_2 = AC$

for $d_1 = AB$

$= OB - OA$.

$= \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}$

$d_1 = \begin{pmatrix}0\\-1\\0\end{pmatrix}$

for $d_2 = AC$

$= OC - OA$.

$= \begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}$

$d_2 = \begin{pmatrix}2\\1\\-2\end{pmatrix}$

The normal to the plane, $n = |d_1 \times d_2|$

$$n = \begin{vmatrix} i & j & k \\ 0 & -1 & 0 \\ 2 & 1 & -2 \end{vmatrix}$$

$$n = i\begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} - j\begin{vmatrix} 0 & 0 \\ 2 & -2 \end{vmatrix} + k\begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix}$$

$$n = 2i - 0j + 2k$$

$$n = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$n = 2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

from $r \cdot n = a \cdot n$

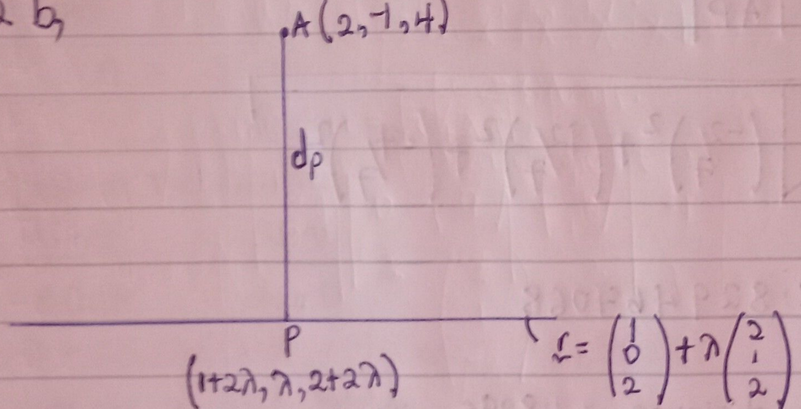
Let $r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$x + z = 1 + 0 + 1$$

$$x + z = 2.$$

12 b,



$$AP \cdot d = 0.$$

$$(OP - OA) \cdot d = 0$$

$$\left[\begin{pmatrix} 1+\lambda \\ \lambda \\ 2+2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right] \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1+\lambda \\ 1+\lambda \\ -2+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0.$$

$$2(-1+\lambda) + (1+\lambda) + 2(-2+2\lambda) = 0.$$

$$-2 + 2\lambda + 1 + \lambda - 4 + 4\lambda = 0.$$

$$7\lambda = 5$$

$$\lambda = 7/5$$

$$\lambda = 5/7$$

$$AP$$
$$\begin{pmatrix} -1+\lambda \\ 1+\lambda \\ -2+2\lambda \end{pmatrix}$$

But $\lambda = 5/7$.

$$\Rightarrow AP = \begin{pmatrix} -1+5/7 \\ 1+5/7 \\ -2+2(5/7) \end{pmatrix}$$

$$AP = \begin{pmatrix} -2/7 \\ 12/7 \\ -4/7 \end{pmatrix}$$

$$dp = |AP|$$

$$dp = \sqrt{\left(-\frac{2}{7}\right)^2 + \left(\frac{12}{7}\right)^2 + \left(-\frac{4}{7}\right)^2}$$

$$dp = 1.829464068$$

Perpendicular distance $= 1.8295$ units.

№13

Using the point $P(5\cos\theta, 4\sin\theta)$:

$x = 5\cos\theta$ $\qquad$ $y = 4\sin\theta$

$\frac{dx}{d\theta} = -5\sin\theta$ $\qquad$ $\frac{dy}{d\theta} = 4\cos\theta$

Using chain rule;

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = 4\cos\theta \cdot \frac{1}{-5\sin\theta}$$

$$\frac{dy}{dx} = -\frac{4\cos\theta}{5\sin\theta}$$

But the normal is perpendicular to the ellipse.

$$m \times n = -1$$

$$-\frac{4\cos\theta}{5\sin\theta} \cdot n = -1$$

$$n = \frac{5\sin\theta}{4\cos\theta}$$

For the equation the normal.

$$\frac{y - 4\sin\theta}{x - 5\cos\theta} = \frac{5\sin\theta}{4\cos\theta}$$

$$5x\sin\theta - 25\sin\theta\cos\theta = 4y\cos\theta - 16\sin\theta\cos\theta$$

Where it cuts the x-axis, at A, $y = 0$.

$\rightarrow$ $5x\sin\theta - 25\sin\theta\cos\theta = 4(0)\cos\theta - 16\sin\theta\cos\theta$

$$5x\sin\theta = 9\sin\theta\cos\theta$$

$$5x = 9\cos\theta$$

$$x = \frac{9}{5}\cos\theta$$ Thus $A(\frac{9}{5}\cos\theta, 0)$

Where it cuts the y-axis at B, $x = 0$.

$$5(0)\sin\theta - 25\sin\theta\cos\theta = 4y\cos\theta - 16\sin\theta\cos\theta$$

$$4y\cos\theta = -9\sin\theta\cos\theta$$

$$4y = -9\sin\theta$$

$$y = -\frac{9}{4}\sin\theta$$ Thus $B(0, -\frac{9}{4}\sin\theta)$

Mid-point of AB $= \left[\left(\frac{\frac{9}{5}\cos\theta + 0}{2}\right), \left(\frac{-\frac{9}{4}\sin\theta + 0}{2}\right)\right]$

Midpoint of AB $= \left(\frac{9}{10}\cos\theta, -\frac{9}{8}\sin\theta\right)$

No 13b,

from the equation $x^2+y^2-2ax+c^2=0$.

Compare with $x^2+y^2+2gx+2fy+c^2=0$.

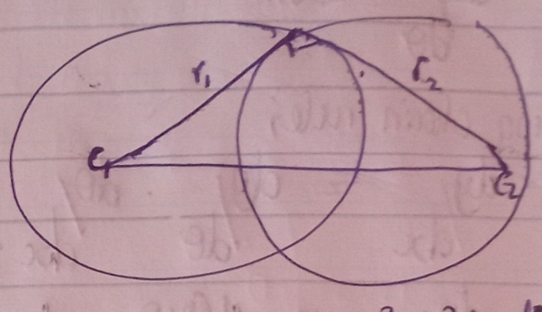
$2g_1=-2a$ | $2f_1=0$

$g_1=-a$ | $f_1=0$

$C_1(a,0)$

$r_1=\sqrt{a^2+0^2-c^2}$

$r_1=\sqrt{a^2-c^2}$



$r^2+r^2=d^2\ \overline{C_1C_2}^2$

from the equation, $x^2+y^2-2by-c^2=0$.

$2g_2=0$ | $2f_2=-2b$

$g_2=0$ | $f_2=-b$

$C_2(0,b)$

$r_2=\sqrt{0^2+b^2+c^2}$

$r_2=\sqrt{b^2+c^2}$

$C_1(a,0)$ $C_2(0,b)$

$\overline{C_1C_2}=\sqrt{(a-0)^2+(0-b)^2}$

$\overline{C_1C_2}=\sqrt{a^2+b^2}$

$r_1^2+r_2^2=(\sqrt{a^2-c^2})^2+(\sqrt{b^2+c^2})^2$

$r_1^2+r_2^2=a^2-c^2+b^2+c^2$

$r_1^2+r_2^2=a^2+b^2$

For circles to cut each other orthogonally, $r_1^2+r_2^2=\overline{C_1C_2}^2$

F $r_1^2+r_2^2=\overline{C_1C_2}^2$

$a^2+b^2=(\sqrt{a^2+b^2})^2$

$a^2+b^2=a^2+b^2$

$\therefore$ R.H.S $\equiv$ L.H.S. It therefore holds that these two circles cut each other orthogonally.

14a,

$$\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}\,dx = \int_1^3 \frac{x^2+1}{x(x^2+4x+3)}\,dx.$$

$$= \int_1^3 \frac{x^2+1}{x\left[x^2+x+3x+3\right]}\,dx.$$

$$= \int_1^3 \frac{x^2+1}{x\left[x(x+1)+3(x+1)\right]}\,dx.$$

$$= \int_1^3 \frac{x^2+1}{x(x+3)(x+1)}\,dx.$$

Consider $\dfrac{x^2+1}{x(x+3)(x+1)} \equiv \dfrac{A}{x} + \dfrac{B}{(x+3)} + \dfrac{C}{(x+1)}$

$x^2+1 = A(x+3)(x+1) + B(x+1)x + Cx(x+3)$

$x^2+1 = A(x+3)(x+1) + Bx(x+1) + Cx(x+3).$

Putting $x=0$.

$1 = A(3)(1) \Rightarrow A = \frac{1}{3}$

Putting $x=-1$

$2 = -2C \Rightarrow C = -1$

Putting $x = -3$

$10 = 6B \Rightarrow B = \frac{5}{3}$

$$\frac{x^2+1}{x(x+3)(x+1)} = \frac{1}{3x} + \frac{5}{3(x+3)} - \frac{1}{(x+1)}$$

$$\Rightarrow \int_1^3 \frac{x^2+1}{x(x+3)(x+1)}\,dx = \frac{1}{3}\int_1^3 \frac{1}{x}\,dx + \frac{5}{3}\int_1^3 \frac{1}{(x+3)}\,dx - \int_1^3 \frac{1}{(x+1)}\,dx.$$

$$= \frac{1}{3}\Big[\ln x\Big]_1^3 + \frac{5}{3}\Big[\ln(x+3)\Big]_1^3 - \Big[\ln(x+1)\Big]_1^3$$

$$= \frac{1}{3}(\ln 3 - \ln 1) + \frac{5}{3}(\ln 6 - \ln 4) - (\ln 4 - \ln 2)$$

$$= 0.366204 + 0.675775 - 0.6931471$$

$$= 0.3488319$$

$$\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}\,dx = 0.3488 \text{ (4dps)}.$$

No 14b,

$$\int \frac{1}{3x^2+5x+4}\,dx.$$

Consider $3x^2+5x+4$.

$$3x^2+5x+4 = 3\left(x^2+\tfrac{5}{3}x+\tfrac{4}{3}\right)$$

$$= 3\left[x^2+\tfrac{5}{3}x+\left(\tfrac{1}{2}\times\tfrac{5}{3}\right)^2-\left(\tfrac{1}{2}\times\tfrac{5}{3}\right)^2+\tfrac{4}{3}\right]$$

$$= 3\left[\left(x+\tfrac{5}{6}\right)^2+\tfrac{23}{36}\right]$$

$$3x^2+5x+4 = \tfrac{23}{12}+3\left(x+\tfrac{5}{6}\right)^2$$

$$\Rightarrow \int \frac{1}{3x^2+5x+4}\,dx = \int \frac{1}{\frac{23}{12}+\frac{3}{1}\left(x+\frac{5}{6}\right)^2}\,dx$$

$$= \int \frac{1}{\frac{23+36\left(x+\frac{5}{6}\right)^2}{12}}\,dx$$

$$= \int \frac{12}{23+36\left(x+\frac{5}{6}\right)^2}\,dx$$

Let $23+36\left(x+\tfrac{5}{6}\right)^2 = 23+23\tan^2 u$

$$= 23(1+\tan^2 u)$$

$$23+36\left(x+\tfrac{5}{6}\right)^2 = 23\sec^2 u$$

Also, from $23+36\left(x+\tfrac{5}{6}\right)^2 = 23+23\tan^2 u$

$$36\left(x+\tfrac{5}{6}\right)^2 = 23\tan^2 u$$

$$6\left(x+\tfrac{5}{6}\right) = \sqrt{23}\tan u$$

$$u = \tan^{-1}\left(\frac{6}{\sqrt{23}}\left(x+\frac{5}{6}\right)\right)$$

Also, from $6\left(x+\tfrac{5}{6}\right) = \sqrt{23}\tan u$

$$6\,dx = \sqrt{23}\sec^2 u\,du$$

$$\Rightarrow dx = \frac{\sqrt{23}\sec^2 u\,du}{6}$$

$$\Rightarrow \int \frac{12}{23+36\left(x+\frac{5}{6}\right)^2}\,dx = \int \frac{12}{23\sec^2 u} \cdot \frac{\sqrt{23}\sec^2 u \cdot du}{6}$$

$$= \frac{2\sqrt{23}}{23}\int (1)\,du$$

$$= \frac{2\sqrt{23}}{23}(u) + c$$

$$\int \frac{1}{3x^2+5x+4}\,dx = \frac{2\sqrt{23}}{23}\tan^{-1}\left[\frac{6}{\sqrt{23}}\left(x+\frac{5}{6}\right)\right]$$

№15

$$\frac{dN}{dt} \propto N-5$$

$$\frac{dN}{dt} = K(N-5)$$

$$\int \frac{dN}{(N-5)} = \int K\,dt$$

$$\int \frac{1}{(N-5)}\,dN = K\int (1)\,dt$$

$$\ln(N-5) = +Kt + C$$

At $t=0$, $N=120$

$$\ln(120-5) = K(0) + C$$

$$\Rightarrow c = \ln(115)$$

$$\Rightarrow \ln(N-5) = Kt + \ln(115)$$

At $t = 1$year, $N = 210$.

$$\ln(210-5) = K(1) + \ln(115)$$

$$\ln 205 = K + \ln 115$$

$$K = \ln 205 - \ln 115$$

$$K = \ln\left(\frac{205}{115}\right)$$

$$K = \ln \frac{41}{23}$$

$$\Rightarrow \ln(N-5) = \ln\frac{41}{23}\,t + \ln(115)$$

$$\ln(N-5) = t\ln\left(\frac{41}{23}\right) + \ln 115$$

$N = ??$, $t = 5$ years.

$\ln(N-5) = t\ln\left(\frac{41}{23}\right) + \ln 115$

$\ln(N-5) = 5\ln\left(\frac{41}{23}\right) + \ln 115$

$\ln(N-5) = \ln\left(\frac{41}{23}\right)^5 + \ln 115$

$\ln(N-5) = \ln\left[\left(\frac{41}{23}\right)^5 \cdot 115\right]$

$N-5 = \left(\frac{41}{23}\right)^5 \times 115$

$N = \left[\left(\frac{41}{23}\right)^5 \times 115\right] + 5$

$N = 2075.036217$

$N = 2075$ people

$\therefore$ The number of people after five years is 2075.

b,

$N = 37275$, $t = ??$

from $\ln(N-5) = t\ln\left(\frac{41}{23}\right) + \ln 115$

$\ln(37275-5) = t\ln\left(\frac{41}{23}\right) + \ln 115$

$\ln 37270 = t\ln\left(\frac{41}{23}\right) + \ln 115$

$t\ln\left(\frac{41}{23}\right) = \ln 37270 - \ln 115$

$t = \dfrac{\ln 37270 - \ln 115}{\ln\left(\frac{41}{23}\right)}$

$t = 10.00040368$

$t \approx 10.0$ years.

PROPOSED MARKING GUIDE FOR WAKISSHA JOINT EXAMINATION MOCK 2024

MATHEMATICS P425/1

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Thank You.