PATTERNS AND SEQUENCES

Divisibility test

Divisibility test is the quicker way of finding out whether a number is exactly divisible by another number or not.

Divisibility test for two and three.

i) Divisibility test for two.

A number is exactly disable by **two** when it is even i.e. when its last digit is 0, 2, 4, 6 or 8.

Examples

1. Without dividing, which of the two numbers is exactly divisible by two?

a) 345 and 654

345 is not an even number.

654 is an even number.

Therefore 654 is exactly divisible by two.

b) 1123 and 3112

1123 not even.

3112 is even. Therefore 3112 is exactly divisible by two.

ii) Divisibility test for three.

A number is exactly divisible by 3 when the **sum** of its digit is a multiple of three.

Example

Which of the following numbers 234 and 632 is exactly divisible by three?

234

Sum off digits = 2 + 3 + 4

= 9

9 is a multiple of three.

234 is exactly divisible by 3.

632

Sum of digits = 6 + 3 + 2

= 11

11 is not a multiple of three.

632 is not exactly divisible by three

Activity

1. Which of the following numbers are exactly divisible by two?

   a) 34 b) 51 c) 690, c) 755, d) 2348

2. Identify all the numbers which are exactly divisible by three in the list below.

   11, 33, 123, 1243, 873 and 4320

3. Without dividing identify the numbers which are exactly divisible by two and three in the list below. 237, 762, 651 and 4560.

Divisibility for 4, 5 and 10

Divisibility test for four.

A number is exactly divisible by four when the number formed by its last three digits is a multiple of 4.

Examples

Without dividing, find out whether the numbers below are exactly divisible by four.

a) 112.

Last two digits form 12

12 is a multiple of 4

112 exactly divisible by 4

b) 7,218

Last two digits form 18

18 is not a multiple of 4

7,218 is not exactly divisible by 4

Divisibility test for 5

The number is exactly divisible by five when its last digit is either **zero** or **five**.

Examples

Identify all the numbers that are exactly divisible by five from the list below. 24, 76, 55, 675, 3,453, 1,000, and 60,005

55, 675, 1,000 and 60,005 are exactly divisible by five.

Divisibility test for 10.

A number is exactly divisible by ten when its last digit is zero e.g. 10, 100,310, 5700 etc.

Activity.

1. Without dividing show whether the following numbers are exactly divisible by 4.

   a) 404 b) 5,402 c) 4,532, d) 65,911 e) 1,000,040

2. Identify all the numbers that are exactly divisible by five from the list below. 443, 655, 7, 695, 30,003 and 100,010.
3. Circle the numbers that are exactly divisible by ten in the list below. 601, 450, 10,001, 10,000, and 40,401.

Types of numbers.

i) Whole numbers

Whole numbers start with zero and keep increasing by 1.

0, 1, 2, 3, 4, 5,…

ii) Counting numbers/ Natural numbers

They start with one and keep on increasing by 1.

1, 2, 3, 4, 5,…

iii) Odd numbers:

These are numbers that give a remainder when divided by 2.

They start with one and keep on increasing by 2.

{1, 3, 5, 7,…}

iv) Even numbers:

These are numbers that are exactly divisible by two.

{0, 2, 4, 6, 8, 10,…}

v) Prime numbers:

These are numbers with only two factors.

The factors are one and the number itself.

{2, 3, 5, 7, 11, 13,…}

vi) Composite numbers:

These are numbers with more than 2 factors

{4, 6, 8, 9, 10, 12,…}

vii) Square numbers:

These are numbers got by multiplying a number by its self.

They can also be got by adding consecutive odd numbers.

{1, 4, 9, 16, 25,…}

viii) Cube numbers:

These are got by multiplying the same number twice.

{1, 8, 27, 64, …}

ix) Triangular numbers:

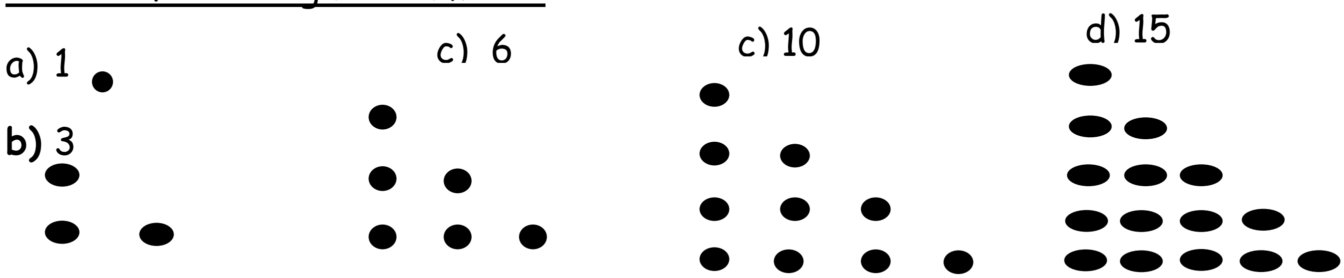
These are numbers got by adding consecutive counting numbers.

Their dot pattern forms a triangle.

{1, 3, 6, 10, 15, 21, …}

$^{+}2$ $^{+}3$ $^{+}4$ $^{+}5$ $^{+}6$

Pattern for triangular numbers



Activity

1. Find the next number in the series.

    a) 2, 3, 5, 7, ___.

b) 1, 3, 5, 7, ___.

c) 4, 6, 8, 9, 10, ___.

d) 49, 36, 25, 16, ___.

e) 625, 125, 25, ___.

2. What is the product of the 3rd and the 6th triangular numbers?
3. Write the seventh prime number in Roman numerals.
4. What is the difference between the 10th odd number and the 10th even number?
5. Work out the total of the first 8 triangular numbers.

Factors of whole numbers

A factor of a number is a number that exactly divides another number.

Examples

1. List all the factor of the following numbers.

a) 12

1 x 12 = 12

2 x 6 = 12

3 x 4 = 12

Factors of 12

= {1, 2, 3, 4, 6, 12}

Or

$12 \div \boxed{12} = 1$

$12 \div \boxed{6} = 2$

$12 \div \boxed{4} = 3$

$12 \div \boxed{3} = 4$

$12 \div \boxed{2} = 6$

$12 \div \boxed{1} = 12$

= {1, 2, 3, 4, 6, 12}

b) 36

1 x 36 = 36

2 x 18 = 36

3 x 12 = 36

4 x 9 = 36

6 x 6 = 36

Factors of 36

= {1, 2, 3, 4, 6, 9, 12, 18, 36}

Or

36 ÷ 1 = 36

36 ÷ 2 = 18

36 ÷ 3 = 12

36 ÷ 4 = 9

36 ÷ 6 = 6

36 ÷ 9 = 4

36 ÷ 12 = 3

36 ÷ 18 = 2

36 ÷ 36 = 1

= {1, 2, 3, 4, 6, 9, 12, 18, 36}

Greatest common factor (GCF)/Highest common factor (HCF)

Also known as;

- Greatest common divisor (GCD).
- Highest common divisor(HCD)

To find the greatest common factor of numbers;

- List all the factors of the numbers.
- Identify the common factors
- Identify the GCF from the common factors.

Examples

Find the GCF of the following numbers.

a) 6 and 8

Factors of 6

= {1, 2, 3, 6}

Factors of 8

{1, 2, 3, 6}

Common factors

{1, 2}

Greatest common factor

= 2

b) 12, 15 and 18

Factors of 12

= {1, 2, 3, 6, 12}

Factors of 15

= {1, 3, 5, 15}

Factors of 18

{1, 2,3, 6, 9,

Common factor

= {1, 3}

GCF = 3

ACTIVITY

1. List all the factors of the following numbers.

   a) 8
   b) 15
   c) 24
   d) 30
   e) 48
   f) 40
   g) 49

2. How many factors do the following numbers have?

   a) 7
   b) 14
   c) 17
   d) 23
   e) 27
   f) 39
   g) 47

3. Work out the GCF of the following numbers.

   a) 12 and 15
   b) 8 and 18
   c) 16 and 2
   d) 8, 9 and 12
   e) 24, 36 and 48
   f) 10, 25 and 30

Multiples of numbers

A multiple is a product of two factors.

Examples

i) List down the multiples of 6 less than 40.

M6 = {6, 12, 18, 24, 30, 36}

ii) List down the first 5 multiples of 12.

M12 = {12, 24, 36, 48, 60}

Finding lowest common multiple by listing down multiples

Example

1. Find the L.C.M of;

a) 6 and 8

M_6 = {6, 12, 18, 24, 30, 36, …}

M_8 = {8, 16, 24, 32, …}

L.C.M = 24

b) Find the LCM of 6 and 8.

M6 = {6, 12, 18, 24, 30, 36, 42, 48,…}

M9 ={9, 18, 27, 36, 45, 54, …}

CM = {18, 36}

LCM = 18

2. What least number of oranges can be shared equally among 6 or 8 pupils without living a remainder?

M6 = {6, 12, 18, 24, 30, 36, 42,…}

M8 = {8, 16, 24, 32, 40, 48,…}

LCM =24

Therefore 24 oranges were shared.

2. Find the least number of sweets when shared among 10 or 15 girls leaves five as a remainder.

M10 = {10, 20, 30, 40, 50, 60, 70, …}

M15 = {15, 30, 45, 60, 75, 90, …}

LCM = 30.

Number of sweets = 30 + 5

= 35 sweets

Activity

1. Find the LCM of the following.

   a) 12 and 18

   b) 14 and 21

   c) 20 and 30

   d) 30 and 40

2. What number can be divided by 16 and 12 without leaving a remainder?

3. Find the least number that can be divided by 20 and 25 leaving no remainder.

4. What least number of sweets can be shared equally among 7 or 8 children leaving 3 as a remainder?

5. The teacher had a certain number of sweets. When he shared them among 24 pupils or 36 pupils 13 sweets remained. How many sweets did the teacher have?

PRIME FACTORISATION OF NUMBERS

Prime factorizing means dividing a number by its prime factors.

We use prime factors when prime factorizing.

Prime factors are prime numbers.

They include 2, 3, 5, 7, 11, 13, …

The answer can be written in set notation, power form or product form

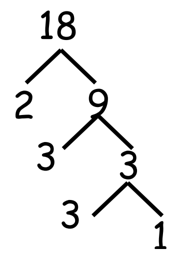
Example I

Prime factorize 18.

Ladder method

2	18
3	9
3	3
	1

Using a factor tree



Set notation / subscript form

F18 = $\{2_1, 3_1, 3_2\}$

Product form

F18 = 2 x 3 x 3

Power form

F18 = $2^1 \times 3^2$

Example II

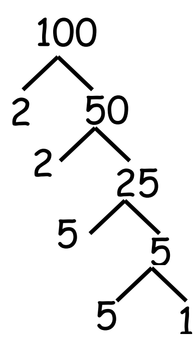
Prime factorize 100.

* Take note of factors of 100 that are prime numbers.

Ladder method

2	100
2	50
5	25
5	5
	1

Factor tree



Set notation

F100 = $\{2_1, 2_2, 5_1, 5_2\}$

Product form

F100 = 2 x 2 x 5 x 5

Power form

$2^2 \times 5^2$

ACTIVITY

1. Prime factorise the following numbers and write your answer in set notation.

a) 50

b) 36

c) 20

d) 30

2. Express the following numbers as a product of their prime factors.

a) 16

b) 24

c) 60

d) 72

3. Prime factorise the following numbers and write the answer in power form.

a) 15

b) 34

c) 48

d) 105

Finding prime factorised numbers.

Examples

What number has been prime factorised to give;

a) $\{2_1, 2_2, 3_1\}$

$= 2 \times 2 \times 3$

$= 4 \times 3$

$=12$

b) $2 \times 3 \times 3 \times 5$

$= 6 \times 15$

$= 90$

c) $2^2 \times 3^2 \times 5$

$= 2 \times 2 \times 3 \times 3 \times 5$

$= 4 \times 9 \times 5$

$= 36 \times 5$

$= 180$

Activity

1. Find the numbers whose prime factors are shown below.

a) $\{2_1, 2_2, 2_3, 5_1\}$

b) $\{3_1, 3_2, 7_1\}$

c) $2 \times 3 \times 3 \times 5$

d) $3_1, 5_1, 7_1$

e) $2^3 \times 3 \times 5$

f) $2^2 \times 3^2 \times 5$

2. Find the value of k.

a) $K = \{3 \times 5 \times 13\}$

b) $k = 3^2 \times 5^2 \times 7$

b) $k = \{2_1, 3_1, 7_1\}$

Finding L.C.M and GCF by prime factorisation

Example

Find the L.C.M of and GCF of 8 and 12 by prime factorization.

i) GCF

Product of common prime factors

2	8	12
2	4	6
	2	3

GCF = 2 x 2

= 4

ii) LCM

Product of all the prime factors

2	8	12
2	4	6
2	2	3
3	1	3
	1	1

LCM = 2 x 2 x 2 x 3

= 24

3. Work out the LCM and GCF of 18, 24 and 36

i) GCF

2	18	24	36
3	9	12	18
	3	4	6

GCF = 2 x 3

= 6

ii) LCM

2	18	24	36
2	9	12	18
2	9	6	9
3	9	3	9
3	3	1	1
	1	1	1

LCM = 2 x 2 x 2 x 3 x 3

= 8 x 9

= 72

Activity

1. Work out the GCF of the following numbers.

a) 12 and 15

b) 8 and 18

c) 8, 9 and 12

d) 24, 36 and 48

e) 10, 25 and 30

2. Find the LCM of the following.

a) 12 and 18

b) 14 and 21

c) 20 and 30

d) 30 and 40

Representing prime factors on Venn diagrams

Example I

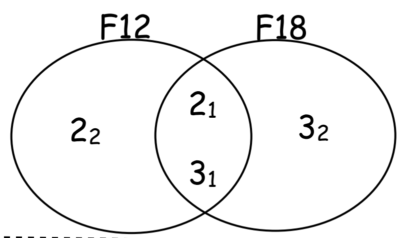
1. a) Represent the prime factors of 12 and 18 on a Venn diagram.

2	12
2	6
3	3
	1

2	18
3	9
3	3
	1

$F12 = \{2_1, 2_2, 3_1\}$

$F18 = \{2_1, 3_1, 3_2\}$



a) Find the GCF of 12 and 18.

G.C.F = product of $F_{12} \cap F_{18}$

$F_{12} \cap F_{18} = \{2_1, 3_1\}$

$\therefore$ G.C.F = 2×3

$= 6$

b) Work out the LCM of 12 and 18

Product of $F_{12} \cup F_{18}$

$F_{12} \cup F_{18} = \{2_1, 2_2, 3_1, 3_2\}$

$= 2 \times 2 \times 3 \times 3$

$= 4 \times 9$

$= 36$

Example 2

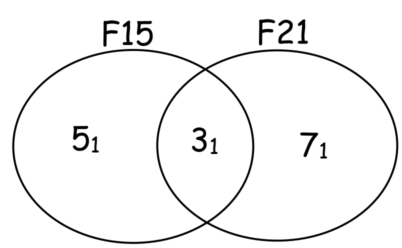
2. a) Prime factorise 15 and 20 and show the prime factors on the Venn diagram.

3	15
5	5
	1

3	21
7	7
	1

$F15 = \{3_1, 5_1\}$

$F21 = \{3_1, 7_1\}$



b) Find the GCF of 15 and 21.

$F_{15} \cap F_{21} = \{3_1\}$

GCF = 3

b) Work out the LCM of 15 and 21

$F_{15} \cup F_{21} = \{3_1, 5_1, 7_1\}$

$= 3 \times 5 \times 7$

$= 15 \times 7$

$= 105$

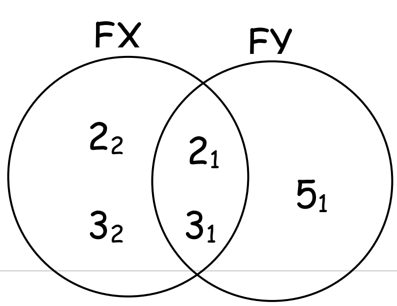
Activity

1. Prime factorise 10 and 20
   a) Show the prime factors of 10 and 20 on a Venn diagram.
   b) Calculate the GCF of 10 and 20
   c) Work out the LCM of 10 and 20.
2. Given the numerals 30 and 40.
   a) Show the prime factors of 30 and 70 on a Venn diagram.
   b) Work out the GCF of 30 and 70
   c) Calculate the LCM of 30 and 70.
3. Prime factorise 60 and 72.
   a) Represent the prime factors of 60 and 72 on a Venn diagram.
   b) Calculate the GCF of 60 and 72.
   c) Find the LCM of 60 and 72.

Finding the unknown number given prime factors on a Venn diagram

Examples

1. Study the Venn diagram below carefully and use it to answer questions that follow.



a) Find the value of x.

$FX = \{2_1, 2_2, 3_1\}$

$X = 2 \times 2 \times 3$

$X = 4 \times 3$

$X = 12$

b) Find the value of y

$Fy = \{2_1, 3_1, 5_1\}$

$Y = 2 \times 3 \times 5$

$y = 6 \times 5$

$y = 30$

c) Find the LCM of X and Y.

$LCM = 2 \times 2 \times 3 \times 5$

$= 4 \times 15$

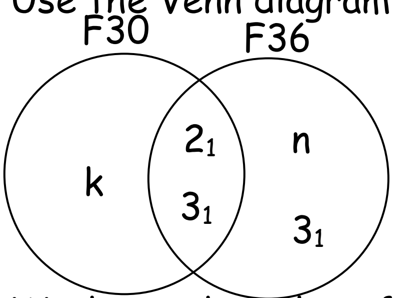
$= 60$

d) Work out the GCF of X and Y

$GCF = 2 \times 3$

$= 6$

2. Use the Venn diagram below to answer the questions that follow



a) Find the value of k.

$K \times 2 \times 3 = 30$

$6k = 30$

$$\frac{\cancel{6k}^{\,1}}{\cancel{6}_{\,1}} = \frac{\cancel{30}^{\,5}}{\cancel{6}_{\,1}}$$

$K = 5$

b) Find the LCM of 30 and 36

$LCM = 2 \times 3 \times 2 \times 3 \times 5$

$= 6 \times 6 \times 5$

$= 36 \times 5$

$= 180$

c) Work out the value of n.

$n \times 3 \times 3 \times 2 = 36$

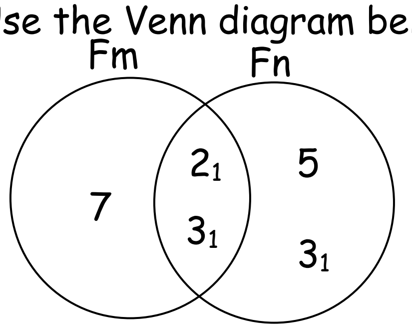
$18n = 36$

$$\frac{\cancel{18}^{\,1}n}{\cancel{18}_{\,1}} = \frac{\cancel{36}^{\,2}}{\cancel{18}_{\,1}}$$

$n = 2$

Activity

1. Use the Venn diagram below to answer the questions that follow.



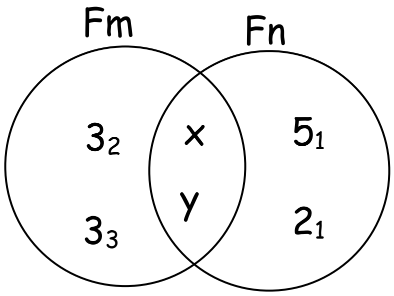
a) Find the value of;

i) m

ii) n

b) Work out the LCM and GCF of m and n.

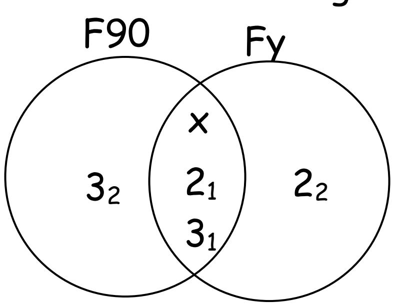
2. Study the Venn diagram below carefully and use it to answer the questions that follow.



a) Find the value of;
   iii) x
   iv) y

b) Work out the LCM and GCF of m and n.

3. Use the Venn diagram below to answer the questions that follow.



a) Find the value of;
   v) x
   vi) y

b) Work out the LCM and GCF of x and y.

4. The intersection set for two numbers A and B is $\{2_1, 3_1\}$. If A - B $=\{2_2, 3_2\}$ and B-A = $\{5_1\}$.

   a) Find the value of A

   b) Find the value of B

   c) Find the LCM of A and B

   d) Workout the GCF of A and B

Squares and square roots of whole numbers

<u>A square number</u> is got by multiplying a number by its self.

<u>A square root</u> is a number that is multiplied by its self to give a square number.

The symbol for square root is √

Example

1. Find the square of:

a) 7
$7^2 = 7 \times 7$
$= 49$

b) 13
$13^2 = 13 \times 13$
$= 169$

b) 18
$182 = 18 \times 18$
$=324$

2. Find the square root of;

a) 36

2	36
2	18
3	9
3	3
	1

$\sqrt{36} = \sqrt{(2x2)x(2x3)}$

$\sqrt{36} = 2 \times 3$
$= 6$

b) 196

2	196
2	98
7	49
7	7
	1

$\sqrt{196} = \sqrt{2x2x7x7}$
$= 2 \times 7$
$= 14$

3. Find the square root of 100

$\sqrt{100} = \sqrt{(2x2)x(5x5)}$

$= 2 \times 5$

$= 10$

2	100
2	50
5	25
5	5
	1

Activity

1. Work out the square of the following

a) 11 b) 15 c) 20 e) 47 f) 114

2. Calculate the square root of the following.

a) 64 b) 121 c) 144 d) 324 e) 625 f) 1600

3. Calculate the area of a square whose side measures ;

a) 14 cm c) 22m e) 41 dm

d) 21m f) 103 dm

Application of square roots

Examples

1. Find the value of p in $p^2 = 36$.

$P^2 = 36$

$\sqrt{P^2} = \sqrt{36}$

$P = 6$

2. The area of a square room is $400m^2$. Calculate the length of its side.

Side x side = area

$S^2 = 400m^2$

$\sqrt{S^2} = \sqrt{400}m^2$

$S = 20m$

Activity

1. Solve for the unknowns;

a) $r^2 = 196$ c) $q^2 = 361$

b) $K^2 = 256$ c) $n^2 = 441$

2. Calculate the length of the side of a square whose area is;

a) $64\ cm^2$ c) $289cm^2$

b) $900m^2$ d) $576dm^2$

3. Given that a = 3 and b= 4. Find the value of c if $c^2 = a^2 + b^2$.

4. Find the value of p if (p x p) = 81