

THEME: NUMERACY

TOPIC: OPERATION ON WHOLE NUMBERS

Consider; $4 + 5 = 9$

In the above mathematical problem, 4 and 5 are called addends while 9 is the sum.

Consider $20 - 14 = 6$

In the above mathematical problem, 20 is the minuend, 14 is the subtrahend while 6 is the difference.

REVISION ACTIVITY

1. Work out: $8975631 + 2867542$
2. Find the sum of 231048 and 524628
3. Increase 341678 by 987326
4. There were 351 272 books in a library and 189 242 more books were donated to the same library. How many books are there altogether?
5. Atiku paid sh.245,385 for a fridge and sh.437,498 for a television set. How much money did she spend altogether?
6. Work out: $6432578 - 2323451$
7. Find the difference of 678231 and 354986
8. What must be added to 237428 to get 885643?
9. Subtract 452 367 from 872 291
Decrease 845654 by 518349
10. A dairy processed 5300,450 litres of milk and sold 3450,833 litres. How many litres were left?
11. A lorry carried 750458 mangoes. If 678432 were ripe and the rest were raw, find the number of raw mangoes.

MULTIPLICATION OF WHOLE NUMBERS

Consider $3 \times 7 = 21$

In the above mathematical problem, 3 is the multiplier, 7 is the multiplicand while 21 is the product.

Examples

1. Work out: 357×9

$$\begin{array}{r} \textcolor{red}{5} \textcolor{red}{6} \\ 3 \ 5 \ 7 \\ \times \quad 9 \\ \hline 3 \ 2 \ 1 \ 3 \end{array}$$

Method II

$$\begin{array}{r} 3 \ 5 \ 7 \\ \times \quad 9 \\ \hline \end{array}$$

$$9 \times 7 = 63$$

$$9 \times 50 = 450$$

$$9 \times 300 = 2700$$

$$\begin{array}{r} \textcolor{red}{1} \ \textcolor{red}{1} \\ 2 \ 7 \ 0 \ 0 \\ \quad 4 \ 5 \ 0 \\ + \quad 6 \ 3 \\ \hline 3 \ 2 \ 1 \ 3 \end{array}$$

2. Find the product of 348 and 26.

$$\begin{array}{r} \textcolor{blue}{6} \\ \textcolor{red}{2} \textcolor{red}{4} \\ 3 \ 4 \ 8 \\ \times 2 \ 6 \\ \hline 2 \ 0 \ 8 \ 8 \\ + 6 \ 9 \ 6 \\ \hline 9 \ 0 \ 4 \ 8 \end{array}$$

Method II

$$\begin{array}{r} 348 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{rcl} 6 \times 8 & = & 48 \\ 6 \times 40 & = & 240 \\ 6 \times 300 & = & 1800 \\ 20 \times 8 & = & 160 \\ 20 \times 40 & = & 800 \\ 20 \times 300 & = & 6000 \end{array}$$

	1	1		
6	0	0	0	
1	8	0	0	
	8	0	0	
	2	4	0	
	1	6	0	
+		4	8	
<hr/>				
9	0	4	8	

3. Find the product of 4520 and 739.

$$\begin{array}{r}
 4520 \\
 \times 739 \\
 \hline
 40680 \\
 13560 \\
 + 31640 \\
 \hline
 3340280
 \end{array}$$

The product of 4520 and 739 is 3,340,280

4. There are 24 bottles of soda in a crate. How many bottles are in 297 crates?

(297 x 24) bottles

$$\begin{array}{r} 297 \\ \times 24 \\ \hline 1188 \\ + 5960 \\ \hline 7148 \end{array}$$

There are 7,148 bottles of soda in 297 crates

5. A carpenter needs 46 screws to fix a bed. How many screws does he need to fix 792 beds?

(792 x 46) screws

$$\begin{array}{r} 792 \\ \times 46 \\ \hline 4752 \\ + 3168 \\ \hline 36432 \end{array}$$

He needs 36,432 screws to make 792 beds

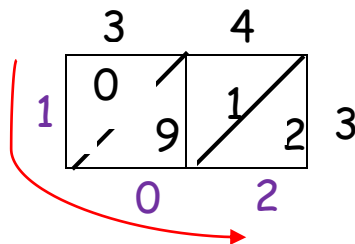
Activity

- Work out the product of the following numbers
 - 84 and 32
 - 637 and 18
 - 2508 and 16
- To plant a garden of bananas on a hectare of land, you need 538 suckers. How many suckers do you need to plant on 63 hectares of land?
- A bus carries 67 passengers per trip. How many people can be carried by the same bus in 98 trips?
- A school consumes 3568kg of rice per month. How many kilograms can the same school consume in 47 seven months?

MULTIPLICATION OF WHOLE NUMBERS USING LATTICE METHOD

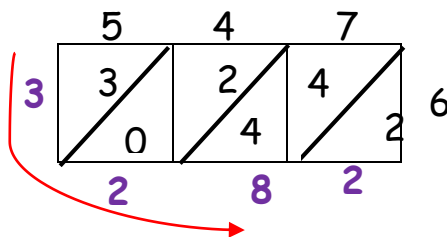
Examples

- Work out 34×3 using lattice method.



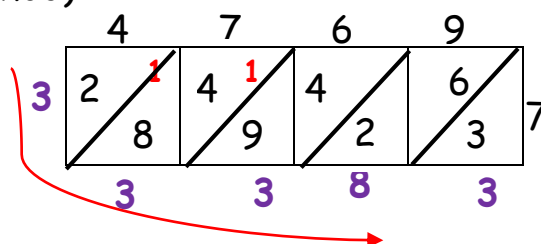
$$34 \times 3 = 102$$

- Work out 547×6 using lattice



$$547 \times 6 = 3,282$$

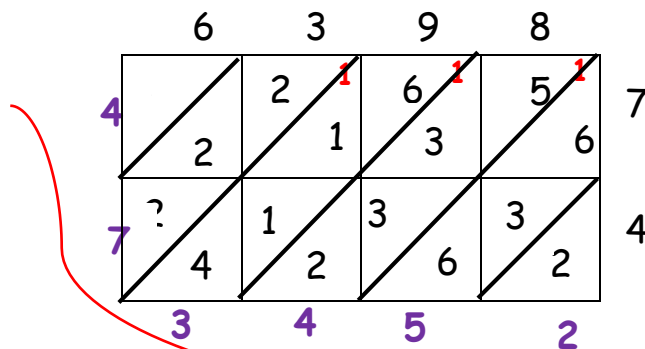
- A university uses 4769 litres of water daily. How much water does it use in a week? (Use lattice method)
(4769×7) litres



The university uses 33,383 litres of water in a week

4. Mulema collects 6398 eggs daily on his farm. How many eggs does he collect in 74 days?

(6398×74) eggs



He collected 473,452 eggs in 74 days

ACTIVITY

1. Work out the following using lattice method.

a) 82×4

b) 753×6

c) $2,3647 \times 5$

d) 438×69

2. There are 94 books in each bookshelf. If there are 75 bookshelves, how many books are there altogether?

3. A maize mill processes 873 bags of flour in a day. How many bags of flour are processed in 69 days?

DIVISION OF WHOLE NUMBERS

Consider; $10 \div 2 = 5$

In the above division problem, 10 is the dividend, 2 is the divisor while 5 is the quotient.

Examples:

1. Work out the following using long division.

a) $189 \div 3$

$$\begin{array}{r}
 63 \\
 3 \overline{) 189} \\
 \underline{3 \times 0 = 0} \\
 18 \\
 \underline{3 \times 6 = 18} \\
 009 \\
 \underline{3 \times 3 = 9} \\
 0
 \end{array}$$

$189 \div 3 = 63$

b) $81424 \div 7$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 1 & 1 & 6 & 3 & 2 \\
 7 & \overline{) 81424} \\
 7 \times 1 = & \underline{7} & & & & & \\
 & 1 & 1 & & & & \\
 7 \times 1 = & & \underline{7} & & & & \\
 & & 4 & 4 & & & \\
 7 \times 6 = & & & \underline{42} & & & \\
 & & & 2 & 2 & & \\
 7 \times 3 = & & & & \underline{21} & & \\
 & & & & 1 & 4 & \\
 7 \times 2 = & & & & & \underline{14} & \\
 & & & & & 0 & 0
 \end{array}
 \end{array}$$

$81424 \div 7 = 11,632$

c) $1845 \div 15$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 0 & 1 & 2 & 3 \\
 15 & \overline{) 1845} \\
 15 \times 0 = & - & 0 & & & & \\
 & & 1 & 8 & & & \\
 15 \times 1 = & & \underline{15} & & & & \\
 & & 0 & 3 & 4 & & \\
 15 \times 2 = & & & \underline{30} & & & \\
 & & & 4 & 5 & & \\
 15 \times 3 = & & & & \underline{45} & & \\
 & & & & 0 & 0
 \end{array}
 \end{array}$$

$1 \times 15 = 15$
 $2 \times 15 = 30$
 $3 \times 15 = 45$
 $4 \times 15 = 60$
 $5 \times 15 = 75$
 $6 \times 15 = 90$

$1845 \div 15 = 123$

2. The minister of health distributed 1485 cartons of masks to 20 sub-counties. How many cartons did each sub-county get?

$1485 \div 20 =$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 7 & 4 \\
 20 & \overline{) 1485} \\
 20 \times 7 = & - & 14 & 0 & & & \\
 20 \times 4 = & & \underline{00} & 8 & 5 & & \\
 & & & -8 & 0 & & \\
 & & & 0 & 5
 \end{array}
 \end{array}$$

$1 \times 20 = 20$
 $2 \times 20 = 40$
 $3 \times 20 = 60$
 $4 \times 20 = 80$
 $5 \times 20 = 100$
 $6 \times 20 = 120$
 $7 \times 20 = 140$
 $8 \times 20 = 160$

Each got 74 and 5 cartons remained

3. There are 24 bottles in a crate of soda. How many bottles can be obtained in 20,808 bottles?

$$\begin{array}{r}
 867 \\
 24 \overline{) 20808} \\
 \underline{-192} \\
 0160 \\
 \underline{-144} \\
 0168 \\
 \underline{-168} \\
 000
 \end{array}$$

$$\begin{aligned}
 1 \times 24 &= 24 \\
 2 \times 24 &= 48 \\
 3 \times 24 &= 72 \\
 4 \times 24 &= 96 \\
 5 \times 24 &= 120 \\
 6 \times 24 &= 144 \\
 7 \times 24 &= 168 \\
 8 \times 24 &= 192 \\
 9 \times 24 &= 216
 \end{aligned}$$

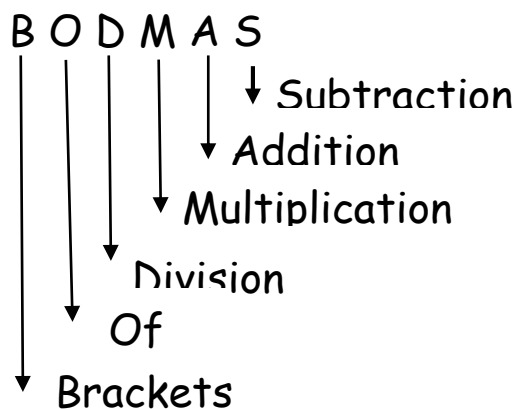
867 crates can be obtained in 20,808 bottles

ACTIVITY

- Work out the following using long division.
 - $8412 \div 4$
 - $18360 \div 6$
 - $7254 \div 18$
 - $16,384 \div 64$
- Share 594 carpets among 18 vendors.
- Find the quotient of 2880 and 36.
- Kapere the poultry farmer collected 6,500 eggs in 50 days. How many eggs did he collect daily?
- The Baby coach carried 3450 passengers in 50 trips. How passengers did it carry per trip?
- Jambi supplies 25kg of rice to each restaurant. If he has 527 kg, how many kilograms will remain?

COMBINED OPERATIONS

In case of more than one operation in a problem, we follow the order of **BODMAS**.



Examples

1. Work out the following correctly:

a) $17 - 9 + 23$
Brackets

$$\begin{array}{r} b) 8 \times 12 \div 4 \\ (12 \div 4) \times 8 \\ 3 \times 8 \\ 24 \end{array}$$

$$\begin{array}{r} c) 58 + (7 \times 6) \\ (7 \times 6) + 48 \\ 42 + 58 \\ 100 \end{array}$$

d) $\frac{1}{2}$ of $10 + 15 \div 3$

$$\frac{1}{2} \text{ of } 10 + (15 \div 3)$$

$$(\frac{1}{2} \text{ of } 10) + 5$$

$$\frac{1}{2} \times 10 + 5$$

$$(1 \times 2) + 5$$

$$2 + 5$$

$$7$$

e) $(8 - 5) - (3 \times 2) + (2 \times 2)$

$$(8 - 5) - (3 \times 2) + (2 \times 2)$$

$$3 - 6 + 4$$

$$(3 + 4) - 6$$

$$7 - 6$$

$$1$$

2. Janie had sh.3000 on her mobile money account. She received sh.2500 more from her mother and later used sh.1700 as airtime. How much was left on her account?

$$\text{Sh.}3000 + \text{sh.}2500 - \text{sh.}1700$$

$$(\text{sh.}3000 + \text{sh.}2500) - \text{sh.}1700$$

$$\text{Sh.}5500 - \text{sh.}1700$$

$$\text{Sh.}3800$$

She was left with sh. 3800 on her account

3. Kepondi had sh.81000 and used $\frac{1}{3}$ of it to pay electricity bills. She then sold her hen at sh.16000. How much money does she have now?

$$\text{sh.}81000 - (\frac{1}{3} \times \text{sh.}81000) + \text{sh.}16000$$

$$(\text{sh.}81000 + \text{sh.}16000) - \text{sh.}27000$$

$$\text{Sh.}81000 - (\frac{1}{3} \times \text{sh.}81000) + \text{sh.}16000$$

$$\text{Sh.}97000 - \text{sh.}27000$$

$$\text{Sh.}81000 - \text{sh.}27000 + \text{sh.}16000$$

$$\text{Sh.}70,000$$

She has sh.70,000 now

Activity

1. Work out the following correctly

a) $8 - 26 + 22$

b) $2.8 \div 4 \times 2$

c) $6 \div 6 + 2 - 3$

d) $42 \div (7 \times 6) + 2$

e) $(8 - 5) - (4 \times 3) + (5 \times 2)$

f) $\frac{3}{10}$ of 40 + $\frac{2}{5}$ of 25

g) $33 \times 2 + 12 \div 12$

h) $28 - 4 \times 5$

2. Kiku had 24 fruits and have away $\frac{1}{4}$ of them to a friend. Kiku later got 12 fruits more from his daddy. How many fruits did he remain with altogether?

3. Andema had sh.7500 and received sh.4500 more from a friend. He later equally shared the money with his elder brother. How much did each get?

BASES

- This is the counting and grouping of numbers.
- Grouping objects/numbers in twos forms base two, grouping in threes forms base three, grouping numbers in fours forms base four, etc.

Names of base and digits used

Base	Base name	Digits used
One	Unary	0
Two	Binary	0, 1
Three	Ternary	0, 1, 2
Four	Quaternary	0, 1, 2, 3
Five	Quinary	0, 1, 2, 3, 4
Six	Senary	0, 1, 2, 3, 4, 5,
Seven	Septenary	0, 1, 2, 3, 4, 5, 6
Eight	Octal	0, 1, 2, 3, 4, 5, 6, 7,
Nine	Nonary	0, 1, 2, 3, 4, 5, 6, 7, 8
Ten	Decimal/ denary	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

PLACE VALUES AND VALUES OF NON-DECIMAL BASES.

Examples

1. Write the place value of each digit in each number below.

a) 101_{two}

101_{two}

 Ones
 Twos
 Two twos

b) 2341_{five}

2341_{five}

 One
 Fives
 Five fives
 Five five fives

c) 637_{nine}

637_{nine}

 One
 Nines
 Nine nines

2. What is the value of each digit in the following numbers?

a) 212_{three}

212_{three}

 Ones (2×1) = 2
 Threes (1×3) = 3
 Three threes ($2 \times 3 \times 3$) = 18

b) 146_{seven}

146_{seven}

 Ones (6×1) = 6
 Sevens (4×7) = 28
 Seven sevens ($1 \times 7 \times 7$) = 49

3. Expand the following numbers as instructed in the brackets

a) 342_{five} (using place values)

342_{five}

 Ones
 Fives
 Five fives

$$(3 \times 5 \times 5) + (4 \times 5) + (2 \times 1)$$

b) 110_{two} (using values)

110_{two}

 Ones (0×1) = 0
 Twos (1×2) = 2
 Two twos ($1 \times 2 \times 2$) = 4

$$4 + 2 + 0$$

c) 432_{five} (using exponents)

5^2	5^1	5^0
4	3	2

$$(4 \times 5^2) + (3 \times 5^1) + (2 \times 5^0)$$

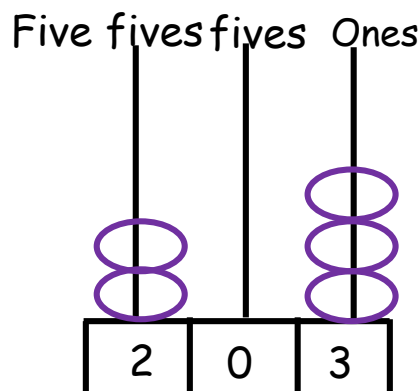
d) 1011_{two} (using exponents)

2^3	2^2	2^1	2^0
1	0	1	1

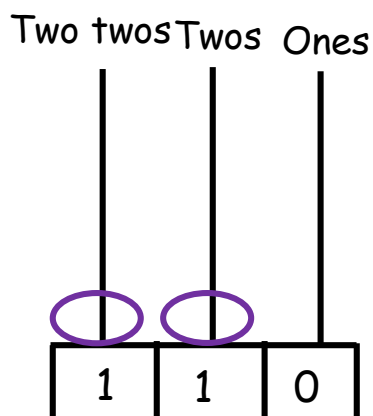
$$(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

Show the following numbers on the abacus.

a) 203_{five}



b) 110_{two}



Activity

1. Write the place value of the underlined digits.

a) 5 6 4_{eight}

b) 1 1 0 1 1_{two}

2. What is the value of 2 in the following numbers?

a) 3 4 2 6_{nine}

b) 1 2 3 0_{four}

3. Expand the following as instructed in the brackets

a) 3 2 2_{five}(Using values) b) 1 3 0 2_{four} (using exponents)

4. Show the following numbers on the abacus.

a) 2 2 1_{three}

b) 1 4 3 1_{six}

5. What base five number has been expanded to give $(2 \times 5^3) + (4 \times 5^2) + (3 \times 5^1) + (1 \times 5^0)$

CONVERTING FROM NON-DECIMAL SYSTEM TO DECIMA SYSTEM

Examples

1. Convert the following into base ten/denary base /decimal base.

a) 1101_{two}

$$\begin{array}{rcl}
 1 & 1 & 0 & 1_{\text{two}} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \text{Ones} & & & (1 \times 1) = 1 \\
 \text{Twos} & & & (0 \times 2) = 0 \\
 \text{Two twos} & & & (2 \times 2) = 4 \\
 \text{Two two twos} & & & (2 \times 2 \times 2) = 8 \\
 1 + 0 + 4 + 8 & & & \\
 13 & & & \\
 1101_{\text{two}} = 13_{\text{ten}} & & &
 \end{array}$$

2^3	2^2	2^1	2^0
1	1	0	1

$$\begin{aligned}
 & (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\
 & (1 \times 2 \times 2 \times 2) + (1 \times 2 \times 2) + (0 \times 2) + (1 \times 1) \\
 & 8 + 4 + 0 + 1 \\
 & 13
 \end{aligned}$$

$$1101_{\text{two}} = 13_{\text{ten}}$$

b) 1234_{five}

$$\begin{array}{rcl}
 1 & 2 & 3 & 4_{\text{five}} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \text{Ones} & & & (4 \times 1) = 4 \\
 \text{Fives} & & & (3 \times 5) = 15 \\
 \text{Five fives} & & & (2 \times 5 \times 5) = 50 \\
 \text{Five five fives} & & & (1 \times 5 \times 5 \times 5) = 125 \\
 125 + 50 + 15 + 4 & & & \\
 1234_{\text{Five}} = 194_{\text{ten}} & & &
 \end{array}$$

5^3	5^2	5^1	5^0
1	2	3	4

$$\begin{aligned}
 & (1 \times 5^3) + (2 \times 5^2) + (3 \times 5^1) + (4 \times 5^0) \\
 & (1 \times 5 \times 5 \times 5) + 2 \times 5 \times 5 + (3 \times 5) + (4 \times 1) \\
 & 125 + 50 + 15 + 4
 \end{aligned}$$

$$1234_{\text{Five}} = 194_{\text{ten}}$$

CONVERTING FROM DECIMAL SYSTEM TO NON-DECIMAL SYSTEM

EXAMPLES

1. Convert 63_{ten} into binary base.

BASE	NUMBER	REMAINDER
2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

$$63_{\text{ten}} = 111111_{\text{two}}$$

2. Express 100_{ten} a base five numeral.

BASE	NUMBER	REMAINDER
5	100	0
5	20	0
5	4	4
	0	

$$100_{\text{ten}} = 400_{\text{five}}$$

2. Express 43_{ten} to ternary base.

BASE	NUMBER	REMAINDER
3	43	1
3	14	2
3	4	1
3	1	1
	0	

$$23_{\text{ten}} = 1121_{\text{three}}$$

4. Convert 200_{ten} into nonary base

BASE	NUMBER	REMAINDER
9	200	2
9	22	4
9	2	2
	0	

$$200_{\text{ten}} = 242_{\text{Nine}}$$

ACTIVITY

1. Express the following in decimal system;

a) 1011_{two}

b) 302_{four}

c) 132_{eight}

d) 224_{six}

2. Convert the following as instructed in the brackets;

i) 48_{ten} (into quinary base)

ii) 51_{ten} (into binary base)

iii) 77_{ten} (into ternary base)

iv) 92_{ten} (into octal base)

CONVERTING NON DECIMAL BASE TO ANOTHER NON DECIMAL BASE

Examples

1. Convert 1011_{two} to base three.

2^3	2^2	2^1	2^0
1	0	1	1

$$\begin{aligned}
 &= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
 &= (1 \times 2 \times 2 \times 2) + (0 \times 2 \times 2) + (1 \times 2) + (1 \times 1) \\
 &= 8 + 0 + 2 + 1 \\
 &= 11_{\text{ten}}
 \end{aligned}$$

B	N	R
3	11	2
3	3	0
3	1	1
	0	

$$= 102_{\text{three}}$$



2. Express 132_{eight} to base six

8^2	8^1	8^0
1	3	2

$$= (1 \times 8^2) + (3 \times 8^1) + (2 \times 8^0)$$

$$= (1 \times 8 \times 8) + (3 \times 8) + (2 \times 1)$$

$$= 64 + 24 + 2$$

$$= 90_{\text{ten}}$$

B	N	R
6	90	0
6	15	3
6	2	2
	0	

$$= 230_{\text{six}}$$



3. Change 302_{four} to quinary base.

4^2	4^1	4^0
3	0	2

$$(3 \times 4^2) + (0 \times 4^1) + (2 \times 4^0)$$

$$(3 \times 4 \times 4) + (0 \times 4) + (2 \times 1)$$

$$48 + 0 + 2$$

$$50_{\text{ten}}$$

B	N	R
5	50	0
5	10	0
5	2	2
	0	

$$= 200_{\text{five}}$$



4. Express 224_{six} as an octal base.

6^2	6^1	6^0
2	2	4

$$(2 \times 6^2) + (2 \times 6^1) + (4 \times 6^0)$$

$$(2 \times 6 \times 6) + (2 \times 6) + (4 \times 1)$$

$$72 + 12 + 4$$

$$88_{\text{ten}}$$

B	N	R
8	88	0
8	11	3
8	1	1
	0	

$$= 130_{\text{eight}}$$



ACTIVITY

1. Convert 34_{five} to binary base.
2. Express 151_{six} to base eight.
3. Convert 1001_{two} to base five.
4. What is 26_{nine} as a base seven number?
5. Convert 130_{five} to four

OPERATION ON NON-DECIMAL NUMBERS

ADDING NUMBERS IN NON-DECIMAL SYSTEM

Examples

1. Work out: $110_{\text{two}} + 11_{\text{two}}$

$$\begin{array}{r} \overset{1}{1} \ 1 \ 0_{\text{two}} \\ + \ 1 \ 1_{\text{two}} \\ \hline 1 \ 0 \ 0 \ 1_{\text{two}} \end{array} \quad \begin{array}{l} 2 \div 2 = 1 \text{rem} 1 \\ 2 \div 2 = 1 \text{rem} 1 \end{array}$$

2. Find the sum of 124_{Five} and 42_{Five} .

$$\begin{array}{r} \overset{1}{1} \ \overset{1}{2} \ 4_{\text{Five}} \\ + \ 4 \ 2_{\text{Five}} \\ \hline 2 \ 2 \ 1_{\text{Five}} \end{array} \quad \begin{array}{l} 6 \div 5 = 1 \text{rem} 1 \\ 7 \div 5 = 1 \text{rem} 2 \end{array}$$

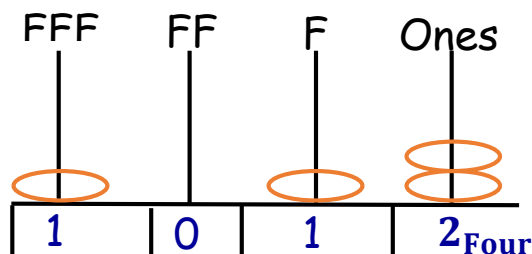
3. Work out: $256_{\text{Seven}} + 431_{\text{Seven}}$

$$\begin{array}{r} \overset{1}{2} \ \overset{1}{5} \ 6_{\text{Seven}} \\ + \ 4 \ 3 \ 1_{\text{Seven}} \\ \hline 1 \ 0 \ 2 \ 0_{\text{Seven}} \end{array} \quad \begin{array}{l} 7 \div 7 = 1 \text{rem} 0 \\ 9 \div 7 = 1 \text{rem} 2 \\ 7 \div 7 = 1 \text{rem} 0 \end{array}$$

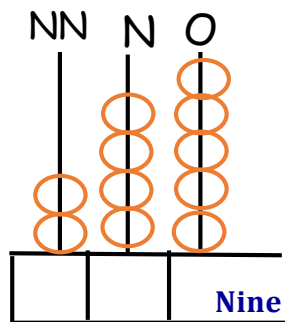
$$\begin{array}{r} \overset{1}{2} \ \overset{1}{1} \ 2_{\text{Three}} \\ + \ 2 \ 2_{\text{Three}} \\ \hline 1 \ 1 \ 1 \ 1_{\text{Five}} \end{array} \quad \begin{array}{l} 4 \div 3 = 1 \text{rem} 1 \\ 4 \div 3 = 1 \text{rem} 1 \\ 3 \div 3 = 1 \text{rem} 1 \end{array}$$

5. Represent the sum of 314_{four} and 32_{Four} on an abacus

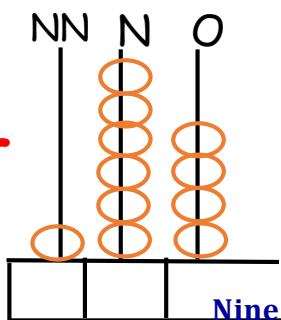
$$\begin{array}{r} \overset{1}{3} \ \overset{1}{1} \ 4_{\text{Four}} \\ + \ 3 \ 2_{\text{Four}} \\ \hline 1 \ 0 \ 1 \ 2_{\text{Four}} \end{array} \quad \begin{array}{l} 6 \div 4 = 1 \text{rem} 2 \\ 5 \div 4 = 1 \text{rem} 1 \\ 4 \div 4 = 1 \text{rem} 0 \end{array}$$



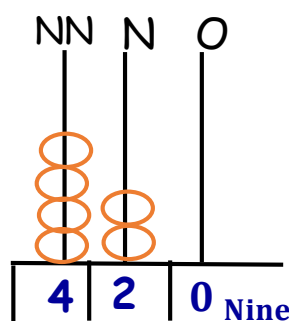
6. Work out and complete the abacus below correctly.



+



=



$$\begin{array}{r} \overset{1}{2} \ \overset{1}{4} \ 5_{\text{nine}} \\ + \ 1 \ 6 \ 4_{\text{nine}} \\ \hline 4 \ 2 \ 0_{\text{Nine}} \end{array}$$

$$\begin{array}{l} 9 \div 9 = 1 \text{em} 0 \\ 11 \div 9 = 1 \text{rem} 2 \end{array}$$

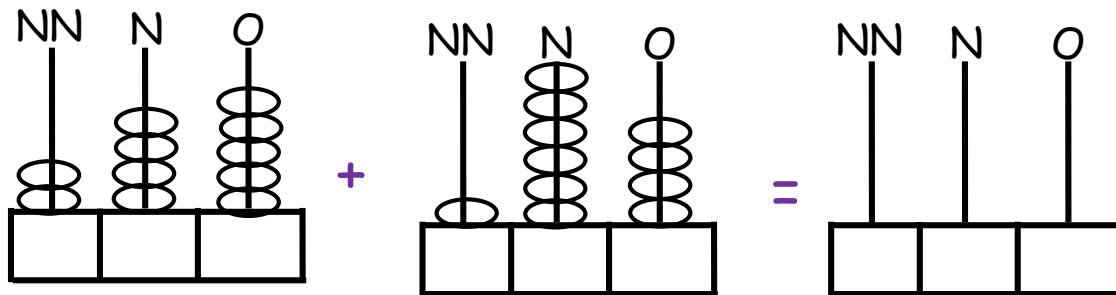
ACTIVITY

1. Work out the following.

a) $110_{\text{two}} + 11_{\text{two}}$ b) $102_{\text{three}} + 22_{\text{three}}$ c) $156_{\text{seven}} + 64_{\text{seven}}$

2. Represent the sum of 374_{eight} and 45_{eight} on the abacus

3. The abaci below show the addition problem of base nine. Work out and complete the abacus correctly.



SUBTRACTION IN NON-DECIMAL SYSTEM

EXAMPLES

1. Work out: $123_{\text{five}} - 31_{\text{five}}$

$$\begin{array}{r} 0 \quad 2+5 \\ \cancel{1} \cancel{2} 3_{\text{five}} \\ - \quad 31_{\text{five}} \\ \hline 042_{\text{five}} \end{array}$$

$$3 - 1 = 2$$

$$2 + 5 = 7 - 3 = 4$$

2. Find the difference between 342_{six} and 142_{six}

$$\begin{array}{r} 2 \quad 2+6 \\ \cancel{3} \cancel{2} 3_{\text{six}} \\ - 142_{\text{six}} \\ \hline 141_{\text{six}} \end{array}$$

$$3 - 2 = 1$$

$$2 - 1 = 1$$

$$2 + 6 = 8 - 4 = 4$$

3. Show the difference between 451_{nine} and 123_{nine} on the abacus.

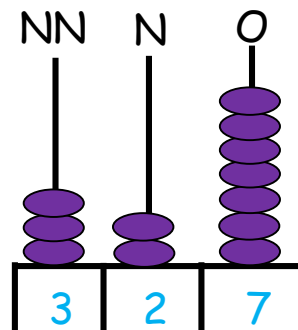
$$\begin{array}{r} 4 \quad 1+9 \\ 4 \cancel{5} \cancel{1}_{\text{nine}} \\ - 123_{\text{nine}} \\ \hline 327_{\text{nine}} \end{array}$$

$$(1 + 9) = 10 - 3$$

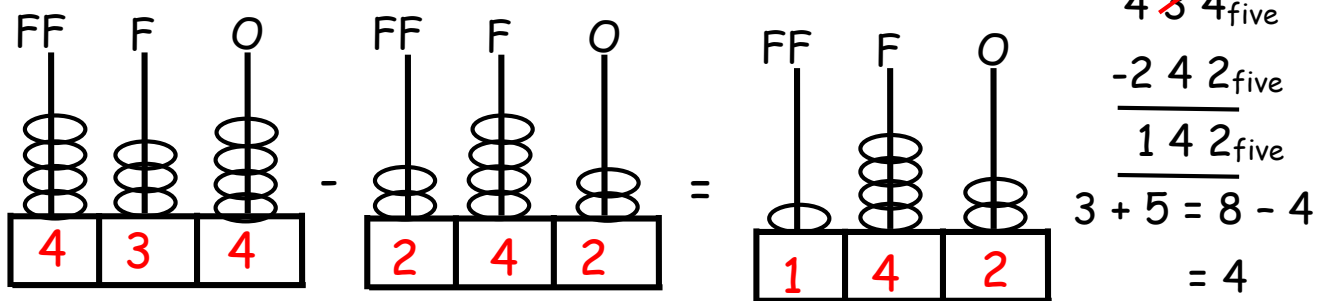
$$= 7$$

$$4 - 2 = 2$$

$$4 - 1 = 3$$



4. Complete the abaci below correctly.



Activity

1. Work out:

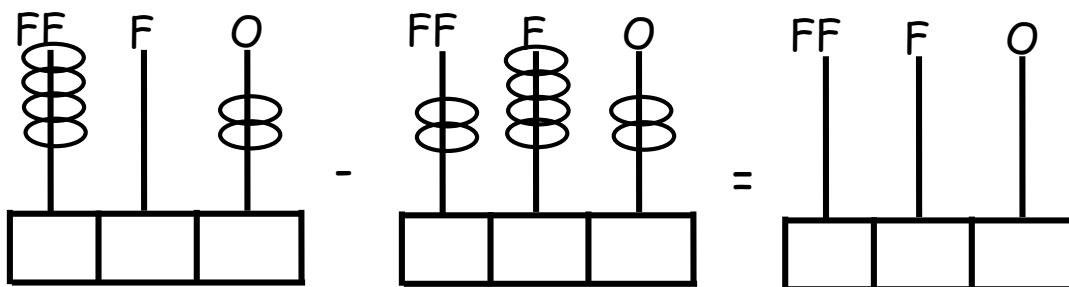
a)
$$\begin{array}{r} 110_{\text{two}} \\ -101_{\text{two}} \\ \hline \end{array}$$

b)
$$\begin{array}{r} 411_{\text{five}} \\ -123_{\text{five}} \\ \hline \end{array}$$

c)
$$\begin{array}{r} 212_{\text{three}} \\ -122_{\text{three}} \\ \hline \end{array}$$

2. Subtract 211_{four} from 301_{four} and show your answer on the abacus.

3. Complete the abaci below correctly.



MULTIPLICATION IN NON-DECIMAL SYSTEM

Examples

1. Work out the following

a) $1\ 1\ 1_{\text{two}} \times 1\ 0_{\text{two}}$

$$\begin{array}{r} 1\ 1\ 1_{\text{Five}} \\ \times 1\ 0_{\text{Five}} \\ \hline 0\ 0\ 0 \\ +1\ 1\ 1 \\ \hline 1\ 1\ 1\ 0_{\text{Five}} \end{array}$$

b) $1\ 0\ 1_{\text{two}} \times 1\ 1_{\text{two}}$

$$\begin{array}{r} 1\ 0\ 1_{\text{two}} \\ \times 1\ 1_{\text{two}} \\ \hline 0\ 0\ 0 \\ +1\ 1\ 1 \\ \hline 1\ 1\ 1\ 0_{\text{two}} \end{array}$$

1. Find the product of 43_{fives} and 123_{five}

$$\begin{array}{r} 1\ 2\ 3_{\text{Five}} \\ \times 4\ 3_{\text{Five}} \\ \hline 4\ 2\ 4 \\ +1\ 1\ 3\ 2 \\ \hline 2\ 2\ 3\ 0\ 4_{\text{five}} \end{array}$$

$$\begin{array}{l} 9 \div 5 = 1 \text{ rem } 4 \\ (6+1) \div 5 = 1 \text{ rem } 2 \\ (3+1) = 4 \\ (12 \div 5) = 2 \text{ rem } 2 \\ (8+2) \div 5 = 1 \text{ rem } 3 \\ (4+1) = 1 \text{ rem } 1 \end{array}$$

2. What is the product 424_{five} and 4?

$$\begin{array}{r} \textcolor{red}{3}\ \textcolor{red}{2}\ \textcolor{red}{2} \\ 4\ 2\ 3_{\text{Five}} \\ \times 4 \\ \hline 3\ 3\ 2\ 2_{\text{five}} \end{array}$$

$$\begin{array}{l} 12 \div 5 = 2 \text{ rem } 2 \\ (8+2) \div 5 = 2 \text{ rem } 0 \\ (16+2) \div 5 = 3 \text{ rem } 3 \end{array}$$

5) Work out : $1\ 2\ 1_{\text{three}} \times 1\ 2_{\text{three}}$

$$\begin{array}{r} 1\ 2\ 1_{\text{three}} \\ \times 1\ 2_{\text{three}} \\ \hline 1\ 0\ 1\ 2 \\ +1\ 2\ 1 \\ \hline 2\ 2\ 2\ 2_{\text{five}} \end{array}$$

$$\begin{array}{l} 4 \div 3 = 1 \text{ rem } 1 \\ 3 \div 3 = 1 \text{ rem } 0 \end{array}$$

ACTIVITY

Work out the following

1. $421_{\text{five}} \times 22_{\text{five}}$

2. $101_{\text{two}} \times 10_{\text{tw}}$

3. $34_{\text{three}} \times 43_{\text{three}}$

4. $121_{\text{four}} \times 22_{\text{four}}$

5. $53_{\text{six}} \times 45_{\text{six}}$

6. $203_{\text{seven}} \times 112_{\text{seven}}$

7. $132_{\text{eight}} \times 33_{\text{eight}}$

8. $33_{\text{nine}} \times 221_{\text{nine}}$

FINDING UNKNOWN BASES

Examples

w^1	w^0
5	2

Solve for the unknown value in the following

a) $54_n = 34_{\text{ten}}$

$$(5 \times p^1) + (4 \times p^0) = 34$$

$$(5 \times p) + (4 \times 1) = 34$$

$$5p + 4 = 34$$

$$5p + 4 - 4 = 34 - 4$$

$$\frac{\cancel{5}p}{\cancel{5}} = \frac{\cancel{30}6}{\cancel{5}}$$

$$p = 6$$

P is base six

b) $52_w = 17_{\text{ten}}$

$$(5 \times w^1) + (2 \times w^0) = 17$$

$$(5 \times w) + (2 \times 1) = 17$$

$$5w + 2 = 17$$

$$5w + 2 - 2 = 17 - 2$$

$$\frac{\cancel{5}w}{\cancel{5}} = \frac{\cancel{15}2}{\cancel{5}}$$

$$w = 3$$

P is base three

c) $43_m = 35_{\text{six}}$

m^1	m^0
4	3

=

6^1	6^0
3	5

$$(4 \times m^1) + (3 \times m^0) = (3 \times 6^1) + (5 \times 6^0)$$

$$(4 \times m) + (3 \times 1) = (3 \times 6) + (5 \times 1)$$

$$4m + 3 = 18 + 5$$

$$4m + 3 - 3 = 23 - 3$$

$$\frac{\cancel{4}m}{\cancel{4}} = \frac{\cancel{20}3}{\cancel{4}}$$

$$m = 5$$

P is base five

d) $32_k = 1011_{\text{two}}$

k^1	k^0
3	2

=

2^3	2^2	2^1	2^0
1	0	1	1

$$(3 \times k^1) + (2 \times k^0) = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$(3 \times k) + (2 \times 1) = (1 \times 2 \times 2 \times 2) + (0 \times 2 \times 2) + (1 \times 2) + (1 \times 1)$$

$$3k + 2 = 8 + 0 + 2 + 1$$

$$3k + 2 = 11$$

$$3k + 2 - 2 = 11 - 2$$

$$\frac{\cancel{3}k}{\cancel{3}} = \frac{\cancel{9}3}{\cancel{3}}$$

$$k = 3$$

k is base three

Activity

Work out the value of the unknown base in each of the following

1) $14_n = 7_{\text{ten}}$

2) $75_d = 26_{\text{ten}}$

3) $83_k = 35_{\text{ten}}$

4) $50_m = 50_{\text{six}}$

5) $46_q = 33_{\text{five}}$

6) $28_p = 10110_{\text{two}}$

7) $34_b = 31_{\text{nine}}$

8) $25_y = 102_{\text{three}}$

FINITE SYSTEM

NOTE

Finite system is the system of counting and grouping objects where remainders are considered.

This system is also called Remainder - arithmetic, clock arithmetic or modular system.

DIFFERENT FINITES AND DIGITS USED

Finite	Digits used
2	{0,1}
3	{0,1,2}
4	{0,1,2,3}
5	{0,1,2,3,4}
6	{0,1,2,3,4,5}

7	{0,1,2,3,4,5,6}
8	{0,1,2,3,4,5,6,7}
9	{0,1,2,3,4,5,6,7,8}
10	{0,1,2,3,4,5,6,7,8,9}
11	{0,1,2,3,4,5,6,7,8,9,10}
12	{0,1,2,3,4,5,6,7,8,9,10,11}

COUNTING AND GROUPING IN FINITE FIVE, SEVEN AND TWELVE

Examples

1. Count and group the following numbers in fives (finite five)

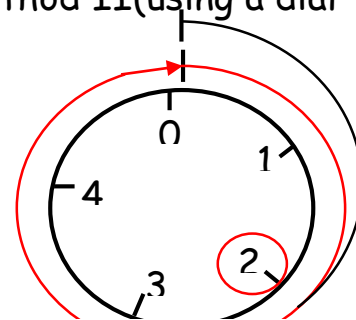
a) 7

Method I

$$(7 \div 5) = 1 \text{ rem } 2$$

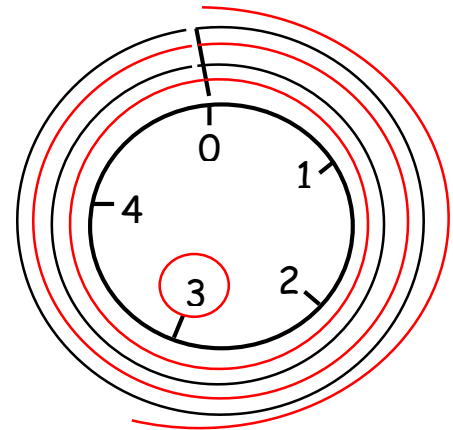
$$7 = 2(\text{finite } 5)$$

Method II(using a dial)



$$7 = 2(\text{finite } 5)$$

Method II (using a clock)



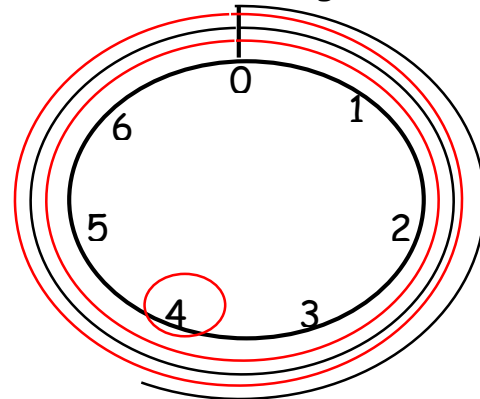
b) 23

Method I

$$23 \div 5 = 4 \text{ rem } 3$$

$$23 = 3(\text{finite } 5)$$

Method II (using a clock)



2. Count and group in sevens (finite 7)

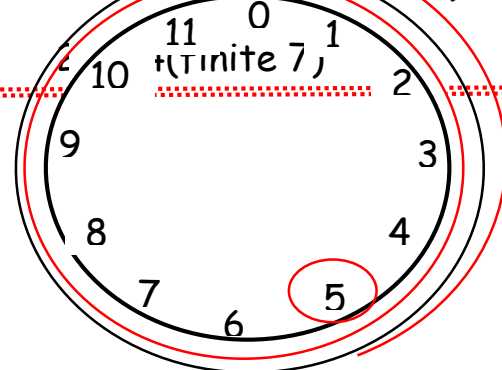
i) 25

Method I

$$25 \div 7 = 3 \text{ rem } 4$$

$$25 = 4(\text{finite } 7)$$

Method II (using a clock)



3. Count and group in twelves (finite 12)

i) 29

Method I

$$29 \div 12 = 2 \text{ rem } 5$$

$$29 = 5(\text{finite } 12)$$

$$29 = 5(\text{finite } 12)$$

3. Write down the equivalences of

a) 3(finite 7)

$$(3+7)=10$$

$$(10+7)=17$$

$$(17+7)=24$$

$$(24+7)=31$$

$$3(\text{finite } 7) = 10, 17, 24, 31, \dots$$

b) 0(finite 5)

$$(0+5) = 5$$

$$(5+5) = 10$$

$$(10+5) = 15$$

$$(15+5) = 20$$

$$0(\text{finite } 5) = 5, 10, 15, 20, \dots$$

c) 7(finite 12)

$$(7+12) = 19$$

$$(19+12) = 31$$

$$(31+12) = 43$$

$$(43+12) = 55$$

$$7(\text{finite } 12) = 19, 31, 43, 55, \dots$$

d) 1(finite 4)

$$(1+4) = 5$$

$$(5+4) = 9$$

$$(9+4) = 13$$

$$(13+4) = 17$$

$$1(\text{finite } 4) = 5, 9, 13, 17, \dots$$

ACTIVITY

1. Without using a dial, count and group the following as indicated in the brackets.

a) 5 (in fours)

2. Using a dial, count and group the following as indicated in the brackets.

(e) 18 (in eights)

(f) 12 (in sixes Finite 7

(b)

0	1	2	3	4	5	6

3. Complete the tables below correctly

Finite 5

0	1	2	3	4

a)

ADDITION IN FINITE SYSTEM

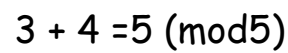
Example

1. Work out the following

a) $3 + 4 = \text{-----} (\text{mod } 5)$

Method II (using a dial)


$3 + 4 = \text{-----} (\text{mod } 5)$



Method II

$$6 + 4 = \text{----} \text{ (finite7)}$$

$$10 \div 7 = 1 \text{ rem } 3$$



$$4 + 5 + 2 + 1$$
$$6 + 4 = 3 \text{ (finite)}$$

A clock face diagram with numbers 0 through 11. A red circle highlights the number 0, and a red line connects it to the number 11.

$$4 + 5 + 2 + 1 = 12$$

$$12(\text{mod } 12)$$

$$12 \div 12 = 1 \text{ rem } 0$$

ACTIVITY

(1) Add the following using a dial or clock arithmetic

- (i) $2 + 3 = \underline{\hspace{1cm}}$ (finite 5)
- (ii) $9 + 7 = \underline{\hspace{1cm}}$ (finite 12)
- (iii) $3 + 6 = \underline{\hspace{1cm}}$ (mod 7)
- (iv) $5 + 3 = \underline{\hspace{1cm}}$ (mod 6)

(2) Workout the following without using a dial.

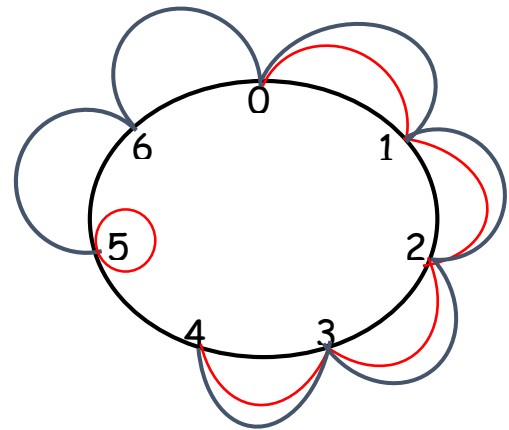
- (i) $6 + 7 = \underline{\hspace{1cm}}$ (mod 5)
- (ii) $4 + 5 = \underline{\hspace{1cm}}$ (mod 7)
- (iii) $6 + 5 + 1 = \underline{\hspace{1cm}}$ (finite 12)
- (iv) $2 + 1 + 2 = \underline{\hspace{1cm}}$ (finite 3)

SUBTRACTION IN FINITE SYSTEM WITH AND WITHOUT A DIAL

EXAMPLE

1. Workout: $4 - 6 = \underline{\hspace{1cm}}$ (mode 7)

Method I (using a dial)



Method II

$$\begin{aligned}
 4 - 6 &= \underline{\hspace{1cm}} \pmod{7} \\
 (4 + 7) - 6 &= \underline{\hspace{1cm}} \pmod{7} \\
 11 - 6 &= \underline{\hspace{1cm}} \pmod{7} \\
 &= (5 \pmod{7}) \\
 4 - 6 &= 5 \pmod{7}
 \end{aligned}$$

3. Work out: $4 - 3 = \underline{\hspace{1cm}}$ (mod 5)

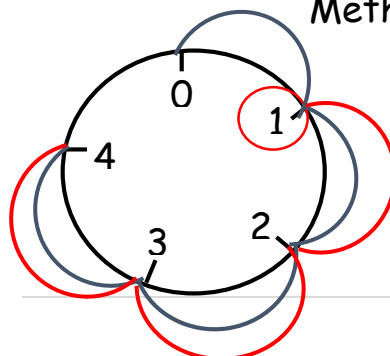
Method I

$$\begin{aligned}
 4 - 3 &= \underline{\hspace{1cm}} \pmod{5} \\
 4 - 3 &= 1 \pmod{5} \\
 4 - 3 &= 1 \pmod{5}
 \end{aligned}$$

Method II

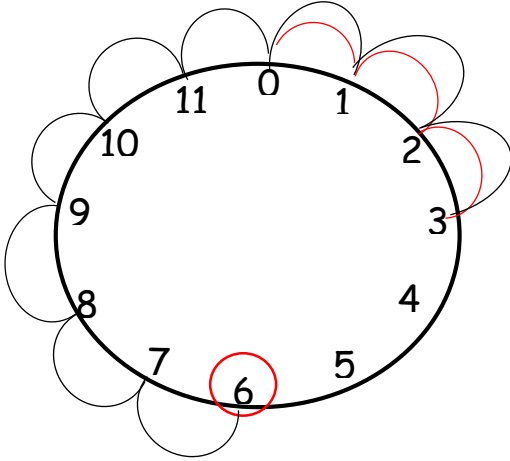
$$4 - 3 = \underline{\hspace{1cm}} \pmod{5}$$

$$4 - 3 = 1 \pmod{5}$$



Method I

$$3 - 9 = \underline{\hspace{1cm}} \pmod{12}$$



$$3 - 9 = 6 \pmod{12}$$

Method II

$$3 - 9 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$(3 + 12) - 9 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$15 - 9 = 6 \text{ finite 12}$$

ACTIVITY

(1) Workout the following using a dial.

- (i) $4 - 2 = \underline{\hspace{1cm}}$ (finite 5)
- (ii) $3 - 5 = \underline{\hspace{1cm}}$ (mode 7)
- (iii) $1 - 7 = \underline{\hspace{1cm}}$ (finite 8)
- (iv) $4 - 7 = \underline{\hspace{1cm}}$ (mode 12)

(2) Without using clock arithmetic, simplify the following:

- (i) $7 - 7 = \underline{\hspace{1cm}}$ (finite 12)
- (ii) $2 - 5 = \underline{\hspace{1cm}}$ (finite 7)
- (iii) $0 - 3 = \underline{\hspace{1cm}}$ (mode 4)
- (iv) $3 - 5 = \underline{\hspace{1cm}}$ (finite 6)

APPLICATION OF FINITE SYSTEM FINITE 7

Consider the table,

Sun	Mon	Tue	Wed	Thu	Fri	Sat
0	1	2	3	4	5	6

Examples

1. If today is Friday. What day of the week will it be after 23 days from now?

Method II

5 stands for Friday

$$\text{Day} + 23 = \underline{\hspace{1cm}} (\text{mode } 7)$$

$$5 + 23 = \underline{\hspace{1cm}} (\text{mode } 7)$$

$$28 = \underline{\hspace{1cm}} (\text{mode } 7)$$

$$= 28 \div 7$$

$$= 4 \text{ remainder } 0$$

0 stands for Sunday

Sun	Mon	Tue	Wed		Thu	Fri	Sat
					///		1
2	3	4	5		6	7	8
9	10	11	12		13	14	15
16	17	18	19		20	21	22
23							

Therefore the day will be Sunday

The day will be Sunday

2. Today is Tuesday. What day of the week was it 45 days ago?

Method II

Method I

2 stands for Tuesday

$$\text{Day} - 45 = \underline{\hspace{1cm}} (\text{finite } 7)$$

$$2 - 45 = \underline{\hspace{1cm}} (\text{finite } 7)$$

First express 45 in (finite 7)

$$6 \text{ rem } 3$$

$$7 \quad \underline{45}$$

$$- 42$$

$$01$$

$$45 = 3 (\text{finite } 7)$$

Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	1					
9	8	7	6	5	4	3
16	15	14	13	12	11	10
23	22	21	20	19	18	17
30	29	28	27	26	25	24
37	36	35	34	33	32	31
44	43	42	41	40	39	38
						45

$$2 - 3 = \underline{\quad} \text{ (finite 7)}$$

$$(2 + 7) - 3 = \underline{\quad} \text{ (finite 7)}$$

$$9 - 3 = 6 \text{ (finite 7)}$$

6 stands for Thursday

Therefore the day was Thursday

The day will be Saturday

ACTIVITY

1. The examination started at 11:00 and lasted for 3 hours. At what time did it end?
2. It is 3:00 a.m. now, what time will it be after 14 hours from now?
3. A train left Mombasa at 10:00 p.m., it reached Kasere after 42 hours. At what time did it arrive at Kasere?
4. A plane left London at 7:30 a.m. If it landed at Entebbe after 36 hours, at what time did it land?
5. A meeting started at 8:00 a.m. It lasted for 8 hours, at what time did it end?
6. Cissy travelled for 81 hours after leaving at 2:30 p.m. At what time did she arrive at the destination?

MONTHS OF THE YEAR.

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Equivalency in finite	1	2	3	4	5	6	7	8	9	10	11	0

Examples

1. This month is July. Which month of the year will it be after 32 months?
7 stands for July

$$7 + 32 = \underline{\quad} \text{ (mode 12)}$$

$$= 39 \text{ (mode 12)}$$

$$= 39 \div 12$$

$$= 3 \text{ remainder } 3$$

3 stands for March

So the month will be March.

2. It is April now, which month of the year was it 346 months ago?
4 stands for April

4 - 34 = ____ (finite 12)
Express 346 in finite 12)

$$\begin{array}{r} 28 \text{ rem } 10 \\ 12 \overline{) 346} \\ \underline{2 \times 12 - 24} \\ 106 \\ \underline{8 \times 12 = - 96} \\ - 10 \end{array}$$

= 28 remainder 10

10 stands for October

Therefore the month was October.

ACTIVITY

1. Kapere left school in August. He stayed home for 17 months. In which month of the year did he resume school?
2. If it is February now, which month of the year was it 124 months ago?
3. Kavuma was born 1341 months ago, in which month was he born if it is August now?
4. If this month is October, which month of the year will it be after:-
 - (i) 43 months
 - (ii) 92 months
 - (iii) 131 months?
5. My sister went to America in March. She will spend $2\frac{1}{2}$ years there. What month of the year will she come back?
6. Allan left Uganda in May and lived in Canada for $4\frac{2}{3}$ years. In which month did he return?

TOPICAL TEST

1. Workout: $5 - 9 + 4$
2. Find the product of 125 and 50
3. Use $>$, $<$, or $=$ to make this statement true.
 $\frac{1}{2}$ kg _____ 0.5kg
4. Kazooba sold the following trays of eggs in a week. 4, 8, 5, 0, 7, 9 and 2.
Find the average number of trays Kazooba sold each day.
5. A school bought 3435kgs of soya beans last month and 8237kgs this month.
If 1304kgs were stolen and bad soya beans how many kgs were good to eat?
6. A passenger train has 8 wagons each carrying 124 people. If the train Makes 2 trips each day, how many people does the train carry each day?
7. What bas eight numeral is equal to 124_{ten} ?
9. Express 321_{nine} as a denary system numeral.
10. List down 5 whole numbers which are equivalent to 3 (finite 7)
11. What base has been used in this operations $123 + 22 = 200$
12. Add: $24_{\text{five}} + 324_{\text{six}}$ (give the answer in base seven)
13. Telephone poles are 15metres apart. What is the distance from the 1st pole to the 10th pole?
14. If today is Monday. What day of the week will it be after 47 days?