Based on the

National Curriculum

NEW FOR 2024

Primary Six

MATHEMATICS

First Edition

TekArt Learning

P.6 MATHEMATICS NOTES

2024

TOPIC   **: SET CONCEPTS**

SUBTOPIC   **: EQUAL AND EQUIVALENT SETS**

Equal Sets

Equal sets are sets with the same number of elements of the same type. The symbol = is used to denote equal sets.

Example:

1. If set R = {r, a, t} and set P = {t, a, r}

    N(R) = 3 members, n(S) = 3 members.

    Members of R and S are similar

    Sets R and P are therefore equal sets.

    So we write;   **R = P**

    N.B: The arrangement of members does not matter provided they are exactly the same.

Equivalent Sets

Equivalent sets are sets with the same number of elements. The members may be different or the same. The symbol for equivalent is ⟷ .

Examples:

Set B = {4, 5, 6, 7, 8} and Set C = {a, b, c, d, e}

     n (B) = 5 members   n (C) = 5 members

     Therefore Sets B and C are equivalent since they both have 5 members each.

     They can be written as, **B ⟷ C**

ACTIVITY

   a) Define equal sets.

     _____

   b) What are equivalent sets?

     _____

   c) Given the sets below;

Set A = {0, 2, 4, 6, 8}

Set B = {2, 4, 6, 8, 10, 12, 14}

Set C = {s, n, a, i, l}

Set D = {4, 6, 8, 0, 2}

Set E = {n, a, i, l, s}

Set F is of even numbers between 1 and 15.

Use '**equal'** or '**equivalent**'

a) Set A and Set D

_____

b) Sets A and C

_____

c) Sets B and F

_____

d) Sets E and C

_____

e) Sets D and E

_____

d) Mr. Mulindwa has goats, cows and sheep on his farm and Mr. Muwonge has sheep, cows and pigs on

 his farm. Write the sets of the two farms and state either they are equal of equivalent.
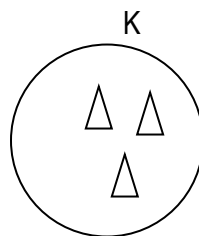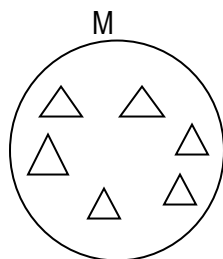

SUBTOPIC: UNEQUAL SETS

Unequal sets are the sets with different members or different number of members.

N.B: Unequal means not equal.

The symbol for Unequal sets is =
Examples:

1.   M   K

2

Set M has 6 members and Set K has 3 members.

Therefore Sets M and K are Unequal sets

$$M \neq K$$

2. Set T = {4, 5, 6, 7} and Set R = {a, p, q, k}

   Set T is a set of 4 numbers and Set R is a set of 4 letters.

   Sets T and R are unequal sets because their members are different though they have the same number of elements.

   $$T \neq R$$

ACTIVITY:

Given the sets below, write equal or unequal.

1. Set P = {0, 2, 4, 6, 8} and   Set Q = {8, 2, 4, 6}

   Sets P and Q are _____ sets

2. Set B = {man, woman, boy}

   Set C = {man, woman, girl}

   Sets B and C are _____ sets.

3. Set D is a set of all the months of the year that start with letter J

   Set E = {January, June, July}

   Sets E and D are _____ sets

4. Given that sets   F = { , , } and   G=◯ ,▢ ,△   }

   Sets F and G are_____sets

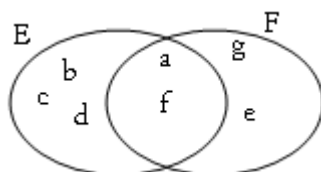# INTERSECTION AND UNION SETS

Intersection set

This is a set of common members of given sets.

Union set:

A set of all members in the given sets altogether.

Examples:

Given the venn diagram below,

3

1. Find E∩F

    <u>E∩F = {a, f}</u>

b) Find n(E∩F) = 2 members

    E∩F = {a, f}

    <u>Hence   n(E∩F) = 2 members</u>

c) What is E ∪ F?

    <u>E∪F= {a, f, b, c, d, g, e}</u>

2. Given that   Set Z ={1, 2, 3, 4, 5} and W={0, 2, 4, 6, 8}.
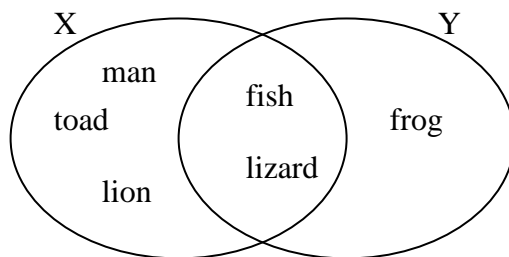
a) Represent the sets on a Venn diagram.

2. Find n(Z∪W)

    Z ∪ W = {0, 1, 2, 3, 4, 5, 6, 8}

    <u>n(Z ∪ W) = 8 members</u>

ACTIVITY:

a) Use the Venn diagram to answer the questions.

a) Find X∩Y.

b) Find n(X∩Y).

c) Find Y∪X.

d) What is n(X∪Y)?

2. Given that Set K is a set of all factors of 12 and Set L is a set of all factors of 30.

i. Find K∩L.

ii. Find the union set of K and L.

iii. How many elements are in L∪K?
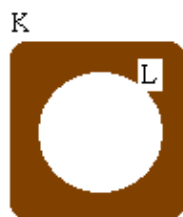
iv. Find L∪K

DIFFERENCE OF SETS

Examples:

i. Shade N - M on the Venn diagram below.

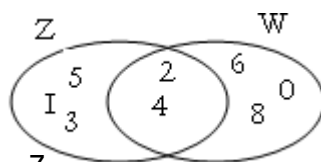**NB** N – M refers to the region for M only.

It also means: N - (M∩N)

ii. Shade K- L in the sets.

3. Given the Venn diagram below:-

1. Find W – Z

W – Z = {0, 6, 8}

b. Find n(X - W)

$Z - W = \{1, 3, 5\}$

<u>$n(Z - W) = 3$ members.</u>

4. Given that Set R is a set of all vowel letters in the word "chair" and Set K is a set of all vowel letters in the word "education".

1. Find K – R

2. Find n(R - K)

<u>solutions</u>

Set $R = \{a, i\}$

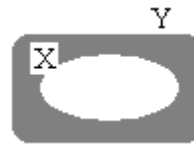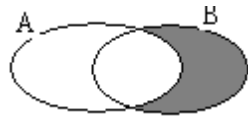Set $K = \{a, e, i, o, u\}$

a) <u>$K - R = \{e, o, u\}$</u>

b) $n(R - K)$

$R - K = \{ \ \}$
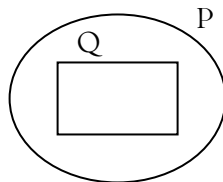
<u>$n(R - K) = 0$</u>

ACTIVITY:

1. Describe the shaded regions
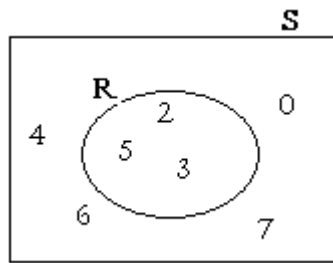

2. Shade P - Q

3. Set $B = \{a, h, k, r, s\}$ Set $H = \{b, h, t, r, v\}$.

   i. Find i) H – B

   ii. n(B - H)

4. Study the diagram and answer the questions

i. List the members of set R

ii. Find n(S - R)

5. Set T is a set of all multiples of 4 less than 19.   Set M is a set of all factors of 24.

   a) Find T – M

   b) Find n(M - T)

COMPLEMENT OF SETS

Complement of a set refers to the region or members with in the union of the given sets but do not belong to that given set.

We use the apostrophe sign to write the complement of a set e.g. the complement of a set B is written as B'.

Complement of a Set

The complement of set A, denoted by A' , is the set of all elements in the universal set that are not in A.

Example.

1. Shade F' in the sets.

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2. Given the sets below, find R'

$R^I = \{k, s, x, y\}$

3. Set Z = {p, q, ꜰ, ꜱ, ꞇ} Set Y = {ꜰ, ꜱ, ꞇ, u, v}

   Find n(Z')

   Z' = {u, v}

   <u>n(Z') = 2 members.</u>

ACTIVITY:

1. Shade the M – K on the Venn diagram below

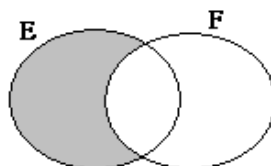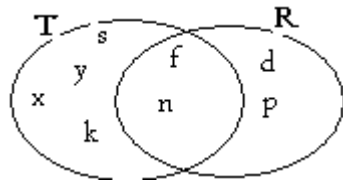2. Describe the shaded region in terms of complement

3. Set W is a set of all composite numbers less than 10 and set X is a set of all even numbers less than 16. Find n(X')

4. Let set Y be the set of all prime numbers less than 15, and set Z be the set of all multiples of 3 less than 20. Find $n(Y')$, the number of elements in the complement of set Y.

5. Consider set A as the set of all odd numbers less than 12, and set B as the set of all square numbers less than 25. Determine $n(A')$, the number of elements in the complement of set A.

6. Let set C be the set of all multiples of 5 less than 30, and set D be the set of all numbers with a unit digit of 2 less than 25. Calculate $n(C')$, the number of elements in the complement of set C.

7. Consider set E as the set of all prime numbers less than 20, and set F as the set of all multiples of 4 less than 25. Find $n(E')$, the number of elements in the complement of set E.

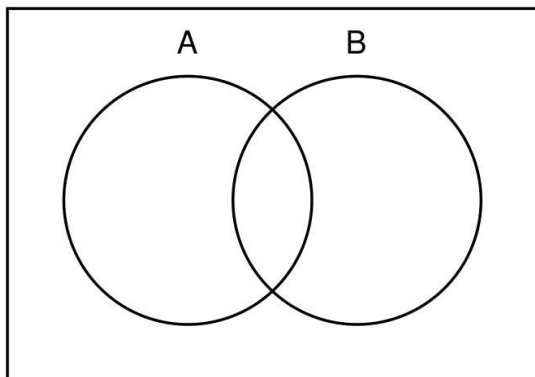8. Let set G be the set of all square numbers less than 30, and set H be the set of all numbers divisible by 7 less than 35. Determine $n(G')$, the number of elements in the complement of set G.
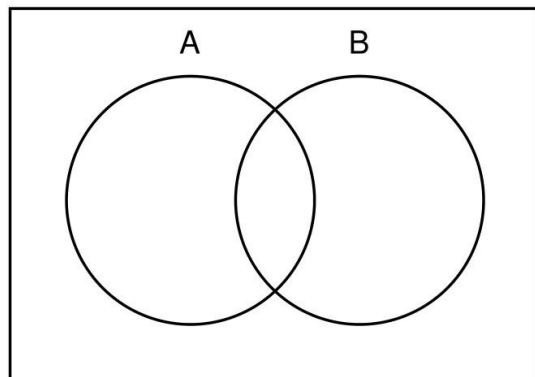
9. Consider set I as the set of all multiples of 6 less than 40, and set J as the set of all numbers with a unit digit of 5 less than 30. Calculate $n(I')$, the number of elements in the complement of set I.

Shade the Venn Diagrams

1) Shade _____A_____

2) Shade _____B_____

3) Shade ___A ∩ B___

4) Shade ___B - A___

5) Shade ___A ∪ B___

6) Shade ___A - B___

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Solve the Problems Using the Venn Diagrams

1)

K   L

10

15

8   13

2

19   11

7   16

1   5   3

K' - L'	= _____
K' - L	= _____
K	= _____
L' - K	= _____

2)

G   H

25   40

55

45

50   80

90

95   85

30   10   35

G U H'	= _____
(G U H)'	= _____
G	= _____
G U H	= _____

3)

B   C

b

x   y

g   z

s   v

w

a

d   q   r

(B - C)'	= _____
(C - B)'	= _____
(B U C)'	= _____
C' - B'	= _____

Math-Aids.Com
Venn Diagrams

SUBTOPIC   : UINVERSAL AND SUBSETS

CONTENT   :

Universal set

This is the mother set or the main/ bigger set. For example, if set Q is a set of all children in Victorious Primary School and set R is a set of all children in P.6 class of Victorious, then, set Q is a universal set

Note; Set R is just part of Set Q.

The symbol for universal set is **ε**

The sets Q and R can be represented as

Subset:

A subset is the smaller set which can be obtained from any given set. For example set R above is a subset of set Q since it is just part of Q. The symbol $\subset$ is used to imply 'is a subset of'

The sets above can be written as; $R \subset Q$

Proper subsets

Proper subsets are subsets with the exception of the main set itself.

Number of proper subsets is got by using $(2^n) - 1$

Consider

Given that,

Set M is a set of all farmers in Masiku Village.

Set P is a set of farmers who grow food crops.

Set Q is a set of farmers who rear animals.

Represent the sets on a venn diagram

N.B: - All farmers (M) is the universal set.

All farmers who grow food crops (P) is a subset of M, thus $P \subset M$

All farmers who rear animals (Q) is subset of M, thus $Q \subset M$

Farmers who rear animals and grow food crops is a subset of M, thus $(P \cup Q) \subset M$

Farmers who grow other crops is a subset of M, thus $(PUQ)' \subset M$

ACTIVITY:

1. Write the relationship between the sets in the Venn diagram below

2. Draw a Venn diagram to show that all animals (A) are Living things (L).

3. It is true that Kampala (K) is found in Uganda (U) which is in Africa (A).

    Represent this statement on a venn diagram.

LISTING AND FINDING NUMBER OF SUBSETS

Subsets are smaller sets obtained from a given set.

<u>Listing subsets:</u>

Example

1. Set B = {2, 4, 6}. List all the subsets in set B.

    { }, {2}, {4}, {6}, {2,4}, {2,6}, {4,6}, {2,4,6}.

    Note:

    The empty set is also a subset of any given set. The set itself is also a subset of itself.

Finding Number of Subsets

This can be done in either of two ways:

> 5   First listing the subsets then count them and finally state the number of subsets formed   or

> 6   Using the formula; thus,

Number of Subsets = 2^n

Where n stands fro number of elements in that given set.

Examples

1. Set P = {a, b, c, d}. Find the number of subsets in set P

n(P) = 4 members

No. of subsets = 2^n

= 2^4

= 2 X 2 X 2 X 2

= 16 Subsets.

<u>P has 16 subsets.</u>

2. Given that n(K) = 6. Find the number of subset in set K.

n(K) = 6

No. of subsets = 2^n

= 2^6

= 2 X 2 X 2 X 2 X 2 X 2

= 64 Subsets.

<u>K has 64 subsets.</u>

ACTIVITY:

1. List all the subsets in each of the given sets

a) Set B = {2,3,4}

b) Set M = {a, b, c, d}

2. Find the number of subsets in each set by first listing them.

a) Set Z = {p, q, r}

b) Set R = {6}

3. Using the formula, calculate the number of subsets the sets below have:

c) Set W = {0, 3, 6}

d) Set T = { ◌ }

e) Set X is a set of 3 blue cows on Mr. Muwonge's farm. Calculate the number of subsets in set X.

APPLICATION OF SETS

The Venn diagram below shows how all P.6 children prefer two clubs i.e Maths club and Science club.

a) How many pupils prefer Maths club?

Maths club = n(M) only + n(M∩ N)

=   30   +   10

= 40 pupils.

b) How many pupils prefer both clubs?

10 pupils prefer both clubs

c) How many pupils prefer only one club?

n(M) only + n(S) only

=   30   +   25

= 55 pupils

d) How many pupils are in P. 6 class?

$n(\varepsilon) = n(M)$ only $+ n(M\cap N) + n(S)$ only

    30   +   10   +   25

        = 65 pupils

ACTIVITY:

The venn diagram below shows how a school football team some use left leg (L), others use the right leg(R) and few use both legs

a) How many members are in the school team?

b) Find the number of players who use left leg.

c) How many players use only one leg?

d) How many players use either left or right leg?

e) How many players use at least on leg?

TOPIC: SET CONCEPTS

SUBTOPIC: APPLICATION OF SETS

CONTENT:

Representing information on a Venn diagram

1. In a p.6 class, 30 pupils prefer fish (F), 15 prefer meat (M) only, 10 prefer both fish and meat and 6 don't like any of the two.

a) Draw a venn diagram to show the information.

$n(F)=30$ $n(M)$only $= 15$ $n(F\cap M)= 10$ $n(F\cup M)^1=6$, $n(\varepsilon)=$

b) How many pupils prefer fish only?

$n(F)$only $= n(F) - n(F\cap M)$

$= 30 - 10$

$\underline{= 20 \text{ pupils}}$

c) What is the population of this class?

$n(\varepsilon) = n(F) + n(F\cap M)+ n(M)$only $+ n(F\cup M)^1$

$(30 - 10) + 15 + 10 + 6$

$= 20 + 25 + 6$

$= 51 \text{ pupils}$

d) Find the probability of picking at random a member likes fish only

$DC = 30$ $TC= 51$

$P (Fish) = \dfrac{DC}{TC}$

$= \dfrac{30}{51}$

ACTIVITY:

1. In a family, 12 members use English (E), 8 use Luganda (L), 4 use both English and Luganda and 3 use neither of the two languages.

a) Draw a venn diagram to represent the information

b) B) How many members use only one language?

c) If each member in this family was given sh. 10,000 for weekend, how much money was given to this family?

d) What is the probability of getting a member who uses English only?

2. In a group of 30 students, 18 students play football (F), 12 students play basketball (B), and 8 students play both sports. If 5 students do not play any sport, answer the following questions:
   a) Draw a Venn diagram to represent the information.

   b) How many students play only one sport?

   c) If each student received a snack for participating, and each snack costs shs. 2000, how much money was spent?

   d) What is the probability of randomly selecting a student who plays basketball only?

3. A group of 40 people participated in a survey. 25 people like to read books (B), 18 people enjoy watching movies (M), and 10 people do both

activities. If 8 people do not engage in either activity, answer the following:

    a) Draw a Venn diagram to represent the information.

    b) How many people like only one of the activities?

    c) If each participant received a gift, and each gift costs Shs. 15000, how much money was spent?

    d) What is the probability of randomly selecting a person who enjoys watching movies only?

SUBTOPIC   : APPLICATION OF SETS

a) In a school of 20 teachers, 12 teachers prefer posho (P), 15 prefer rice (R), some prefer both posho and rice and 2 prefer neither of the two kinds of food.

1. Represent the information on a venn diagram

Let $n(P \cap R)$ be X

$n(P)=12$ $n(R)= 15$ $n(P \cap R)= X$ $n(P \cup R)^1 =2$, $n(\varepsilon)= 20$

2. Find the number of teachers who prefer both posho and rice.

Note; for both, it is represented by X

Method I:

$$X + 12 - X + 15 - X + 2 = 20$$

Method III:

$$15 + 12 - X + 2 = 20$$

$$27 - X + Y + 2 = 20 + X$$

$$27 + 2 = 20 + X$$

$$X - X - X + 12 + 15 + 2 = 20$$

$$^-X + 29 = 20$$

$$^-X + 29 - 29 = 20 - 29$$

$$^-X = ^-9$$

$$X = 9$$

<u>Therefore 9 teachers prefer both posho and rice.</u>

Method II:

$$12 + 15 - X + 2 = 20$$

$$12 + 15 - X + X + 2 = 20 + X$$

$$27 + 2 = 20 + X$$

$$29 - 20 = 20 - 20 + X$$

$$9 = X$$

<u>X = 9</u>

Method IV:

$$X = (15 + 12 + 2) - 20$$

$$X = 29 - 20$$

<u>X = 9</u>

3. Find the number of teachers who prefer only on type of food.

$$(12 - X) + (15 - X)$$

$$(12 - 9) + (15 - 9)$$

$$3 + 6$$

<u>= 9 teachers</u>

ACTIVITY:

1. In a class of 40, 25 pupils prefer Math (M), 20 prefer English (E), p prefer both Math and English and 5 prefer neither Math nor English.

   a) Represent the information on a venn diagram

   b) Find the number of pupils who like both Math and English

   c) How many pupils prefer only one subject?

20

d) What is the probability of picki8ng a pupil at random who prefers Math only to be the class monitor?

2. In a class of 35 students, 20 students like geography (G), 15 like biology (B), t like both geography and biology, and 6 students dislike both subjects. Answer the following:
   a) Draw a Venn diagram to represent the information.

   b) Find the value of t, the number of students who like both geography and biology.

   c) How many students like only one subject?

   d) What is the probability of randomly selecting a student who likes geography only to be the class captain?

   More Set problems

   Problem 1:
   In a class of 40 students, 25 pupils prefer Math (M), 20 prefer English (E), p prefer both Math and English, and 5 prefer neither Math nor English. Answer the following questions:
   a) Draw a Venn diagram to represent the information.
   b) Find the value of p, the number of pupils who like both Math and English.
   c) How many pupils prefer only one subject?
   d) What is the probability of randomly selecting a pupil who prefers Math only to be the class monitor?

   Problem 2:
   In another class of 50 students, 30 students like Science (S), 25 like History (H), q like both Science and History, and 8 students dislike both subjects. Answer the following:
   a) Draw a Venn diagram to represent the information.
   b) Find the value of q, the number of students who like both Science and History.

c) How many students like only one subject?
d) What is the probability of randomly selecting a student who likes Science only to be the class treasurer?

Problem 3:
In a school of 60 students, 35 students participate in sports (S), 30 participate in art (A), r participate in both sports and art, and 10 students do not participate in either. Answer the following:
a) Draw a Venn diagram to represent the information.
b) Find the value of r, the number of students who participate in both sports and art.
c) How many students participate in only one activity?
d) What is the probability of randomly selecting a student who participates in sports only to be the class secretary?

Problem 4:
In a group of 45 students, 25 students enjoy reading (R), 20 enjoy playing music (M), s enjoy both reading and playing music, and 7 students do not enjoy either. Answer the following:
a) Draw a Venn diagram to represent the information.
b) Find the value of s, the number of students who enjoy both reading and playing music.
c) How many students enjoy only one activity?
d) What is the probability of randomly selecting a student who enjoys reading only to be the class president?

REVISION WORK

1. Draw a set symbol for an empty set.

2. Given that set P = {a, b, c}. List down all the proper subsets in set P.

3. Given that set A = {vowel letters} and set B = {a, b, c, d, e}. Find n(A∩B).

4. Amina formed two sets as shown below.

K = $\left\{ \square, \triangle, \bigcirc \right\}$

L = $\left\{ \triangle, \square, \bigcirc \right\}$

What relationship do the two sets have?

5. Shade the complement of set R in the diagram below.

6. Describe the shaded region in the Venn diagram below.

7. Given that set X has 5 elements. How many subsets are in set X?

8. Set G has 64 subsets. How many elements are there in set G**?**

9. M = {1, 2, 3, 4} and
N = {a, b, c, d}

Set M are N are _____

10. The Venn diagram below shows how pupils in a class like Matooke (M) and rice (R).

How many pupils like only one type of food**?**

11. Given that;

P = {Even numbers less than 10} and set Q = (All factors of 12}.

(a) List the members of set P.

(b) List the members of set Q.

(c) Draw a Venn diagram to show set P and Q.

(d) Find (i) n(PUQ)

(b) n(Q – P)

(c) n(Q')

12. Study the Venn diagram below and answer questions that follow.

(a) List down all the elements of;

(i) Set Y

(ii) Set X

(ii) The universal set.

(b) Find; (i) n(X′)

(c) n(Y)

13. There are 25 pupils in a class. 15 like English (E), 12 like Science (S) while 5 like neither of the two subjects.

(a) Represent the above information on a Venn diagram below.

$$n(\Sigma) = 25$$

n(E) = 15   n(S) = 12

_____ (y) _____

_____

(b) Find how many pupils like both subjects.

(c) Find the probability of pupils how like English only.

TOPIC: NUMERATION SYSTEMS AND PLACE VALUE

SUBTOPIC: PLACE VALUES AND VALUES

Examples:

Find the place value of each digit in 6 2 4 9 8 7

2 Find the value of each digit in 8 6 4 2 7.

T Th	Th	H	T	O
8	6	4	2	7

Value of a digit = digit x place value

Value of:

$7 = 7 \times 1 = 7$ $6 = 6 \times 1000 = 6000$

$2 = 2 \times 10 = 20$ $8 = 8 \times 10000 = 80000$

$4 = 4 \times 100 = 400$

3 State the place value of each digit in 6 3 4. 7 8

H	T	O		Tth	Hth
6	3	4 .		7	8

27

Place value of: 6 = Hundreds

3 = Tens

4 = Ones

7 = Tenth

8 = Hundredths

4   Find the value of each digit in 7 2. 9 2 9

T	O			Tth	Hth	THth
7	2	.		9	2	9

Value = digit x place value

7x10 = 70

2x1 = 2

$9x \, 1/10$ $= 9/10$ = 0.9

$2x \, 1/100$ $= 2/100$ = 0.02

$9x \, 1/1000$ $= 9/1000$ = 0.009

ACTIVITY:

a)   Write the place value of each digit

1. 3 6 9 8 5 3

2. 1 9 0 0 6 2 4

3. 7 8 3. 3 6

4. 8 9. 8 6 6 3

1.   Find the value of each of the digits.

1. 6 6. 4 2

2. 1 9 8 6. 7 9 7

3. 6 1 6. 7 8 9

b)   Find the sum of the place value of 7 and the value of 2 in 2 0 0 7 6 3

c) What is the product of the value of 8 and place value of 4 in 8 6 3. 4 7?

EXPANDING NUMBERS

a) Expand 4 9 6 3 in place value form

TH H   T   O
4   9   6   3

= 4 X 1000 + 9 X 100 + 6 X 10 + 3 X 1

b) Expand 6 8. 6 0 4 in place value form

T   O   T   H   TH
6   8.   6   0   4

= (6 X 10) + (8 X 1) + (6 X 0.1) + (0 X 0.01) + (4 X 0.001)

= (6 X 10 + (8 X1) + (6 X 0.1) + (4 X 0.001)

OR

= (6 X 10) + (8 X 1) + (6 X $^1/_{10}$) + (4 X $^1/_{1000}$)

c) Expand 6 8. 6 0 4 in value form.

T   O   Tth   Hth   THth
6   8.   6   0   4

= (6 X 10) + (8 X 1) + (6 X $^1/_{10}$) + (4 X $^1/_{1000}$)

= (6 X 10) + (8 X 1) + (6 X 0.1) + (0 X 0.01) + (4 X 0.001)

= (6 X 10) + (8 X1) + (6 X 0.1) + (4 X 0.001)

= 60 + 8 + 0.6 + 0.00 + 0.004

d) Expand 8 5. 7 6 4 in power form

Note: In expanding using powers/exponents, the whole numbers take positive powers while the decimal places tale powers. These exponents/powers are of ten.

ACTIVITY:

a) Expand the following using powers of ten.

    a) 6 8 8 4 9

    b) 2. 6 6 5

    c) 1 9 6 3. 3 0 4

b) Expand the following in place value form

    1. 1 7 1 7

    2. 6 3 4. 5 7 8

    3. 4 9. 8 5 7

c) Expand the following in value form

   1. 5 4 3 2 1

   2. 7 8 . 9 0 2

   3. 89.128

WRITING EXPANDED NUMBER IN SINGLE NUMBER

1. What number has been expanded below?

   60000 + 20 + 500 + 3

   60000

      500

       20

   +   3
   _____

   60523
   _____

2. Find the number that has been expanded

   $(7 \times 1000) + (6 \times 0.1) + (5 \times 10)$

   $= 7000 + 0.6 + 50$

   7000.0

     50.0

   +   0.6
   _____

   7050.6
   _____

3. Namuli expanded a certain number and got ,

   $(6 \times 10^4) + (5 \times 10^0) + (3 \times 10^1) + (7 \times 10^3)$

What number did she expand?

ACTIVITY:

Find the numbers which have been expanded below

a) 6000 + 20 + 7

b) (7 X 1000) + (8 X 10) + (9 X 100) + (7 X 1)

c) 400+60+5

d) (5×100)+(2×10)+(6×1)

e) 9000+300+20+1

f) (4×1000)+(2×100)+(7×1)

g) 800+70+6

WRITING NUMBERS IN WORDS

1. Write 6 2 4 9 1 4 in words

Thousands			Units		
H	T	O	H	T	O
6	2	4	9	1	4

624,914 = Six hundred twenty four thousand nine hundred fourteen

2. Write 1 9 0 0 3 0 0 4 7 in words

Millions			Thousands			units		
H	T	O	H	T	O	H	T	O
1	9	0	0	3	0	0	4	7

190,030,047 = One hundred ninety million thirty thousand forty seven

3. Write 2 4. 6 3 in words

T	O		Tth	Hth
2	4	.	6	3

24.63 = Twenty four and sixty three hundredths

ACTIVITY:

Write the following in words

1. 62,493

2. 171717

3. 9009009

4. 666666666

6. 14.14

7. 272.009

8. 4634.665

9. 0.0004

10. 6.789

5. 100100100

WRITING numbers from words to figures

a) Write in figures

Seventy four million, six hundred ninety two thousand, five hundred eleven

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
0	7	4	6	9	2	5	1	1

<div align="center">= 74,692,511</div>

b) Seventy nine point four five six.

Seventy nine point four five six = 79.456

c) Our hundred nine and forty six hundredths

One hundred nine = 109

Forty six hundredths = $^{46}/_{100}$ = 0.46

<div align="center">

109.00

0.46

109.46

</div>

ACTIVITY:

1. Write the following in figures

i) Seventeen million, seven thousand, seventeen

ii) Two hundred thousand, three hundred sixty four

iii) Sixty six point seventy six million, five hundred forty three thousand, two hundred ten.

iv)   Ninety and nine thousandths

2. Write the number represented on the abacus

ROUNDING OFF WHOLE AND DECIMAL NUMBERS

Rounding off means – correcting to the nearest values

Other terms;

- Rounding up

- Rounding down

Examples:

1. Round off 4965 to the nearest,

    1. Tens

        Note: When the next number to the right of the required place is 5 and above, we round up (add one to the digit in the required place) and when it is less than 5, we round (do not add one to the digit) in the required place.

2. Round off 96329 to the nearest thounsands.

$$\begin{array}{c|ccc} \text{Tth} & \text{Th} & \text{H} & \text{T} & \text{O} \\ + \quad 9 & 6 & \cancel{3} & \cancel{2} & \cancel{9} \\ & 0 & & & \\ \hline 9 & 6 & 0 & 0 & 0 \end{array}$$

3. Round off 728.36 to the nearest whole number

4. Note: Rounding off to the nearest whole number means to the nearest ones.

5. Round off 68.964 to the nearest tenths.

ACTIVITY:

1. Round off 666 to the nearest tens

2. Round off 19634 to the nearest THOUSANDS

3. Round off 45.36 to the nearest tenths

4. WRITE 689.99 to the nearest whole number.

5. WRITE 999.9999 to the nearest thousandths

6. Round off 123121 to the nearest ten thousands.

HINDU ARABIC TO ROMAN NUMERALS

Letters used in Roman Numerals;

I – 1	C - 100
V – 5	D – 500
X – 10	M – 1000
L -50	

a) Write 462 in Roman Numerals

462 = 400 + 60 + 2

= CD + LX + II

= CDLXII

b) Write 1629 in Roman Numerals

1629 = 1000 + 600 + 20 + 9

= M + DCCC + XL + IX

= MDCCCXLIX

Note: We use capital letter form when writing in Roman Numerals

ACTIVITY

1. Write the following in Roman Numerals

     1.   49   d) 2424

     2.   176   e) 964

     3.   3332   f) 1234

2. Lukule was given 3965 books. Express his number in Roman Numerals.

3. How would a Roman girl write 5260?

ROMAN NUMERALS TO HINDU ARABIC

1. Write CDLX in Hindu Arabic Numerals

    Note: When a letter of less value comes before that of a greater value, it means subtraction

    CDLX = CD + LX

    = 400 + 60

    = 460

2. Express CMLXVIII in Hindu Arabic Numerals

    CMLXVIII = CM + LX + VIII

    = 900 + 60 + 8

    = 968

3. Wasswa wrote MMMDXIX on a card. What number is this in Hindu Arabic Numerals?

    MMMDXIX = MMM + D + XIX

    = 3000 + 500 + 19

    = 3519

Activity

Write the following in Hindu Arabic Numerals

1. CCX

6. MMLXXIV

2. CCCIX

7. CCCIII

3. DCCCLXXXVIII

8. CDVII

4. CDXCII

9. XIX

5. LIX

10. CXIX

ADDITION OF NUMBERS

a)　Add: 469046 + 63942

　　　469046

　　+63942

　　　532988

b)　There are four million sixty thousand people in Eastern Uganda, six million forty thousand six hundred in Western Uganda and three million fifty thousand in Northern part. Find the total population of the three regions.

Eastern　- 4 0 6 0 0 0 0

Northern - 3 0 5 0 0 0 0

Tota　　- 7 1 1 0 0 0 0

The total population in the regions is 7,110,000 people.

ACTIVITY

1. Add the following numbers

   1. 96114 + 3224

   2. 630004 + 99963

   3. 17171717 + 222222

   4. 10000 + 100000 + 1000

2. There are 46920 female and 32690 male in Kamuli District. Find the population of the district.

3. Uganda's population is approximately 33 million and that of South Africa is 66.5million. Find the approximate total population

Subtraction of large numbers

a) Subtract 85604 – 64503

   - Arrange the numbers vertically according to the place values of the given digits the subtract.

b) Subtract 2896475 from 8331843

   Interpretation of the '*from*' operation, thus
   8331843 – 2896475
   - arrange vertically and subtract

c) There are 49625 text books in Victorius Library. 16240 are maths books and the rest are other subjects. How many books are for other subjects?

Total no. of books -   4 9 6 2 5

Number of math   - <u>1 6 2 4 0</u>

Other subjects   -   <u>3 3 3 8 5</u>

ACTIVITY:

1. Subtract the following

    1. 40000 – 3000

    2. 562003 – 49999

    3. 634963241 – 100100100

2. Subtract 99 from 10000000

3. What is the difference between 3694 and 76300?

4. How far is 50,000 metres away from 19500m?

5. In a country of 36 million people, 2,563,200 are adults and the rest are child. Find the number of children in this country.

OPERATION ON NUMBERS

MULTIPLICATION

1. Multiply 242 X 12

$$
\begin{array}{r}
2\,4\,2 \\
\times 1\,2 \\
\hline
4\,8\,4 \\
+\,2\,4\,2\,0 \\
\hline
\mathbf{2\,9\,0\,4}
\end{array}
$$

2. A school bus carries 68 passengers when full. If it makes 42 trips, how many passengers will be carried altogether?

In one trip, it carries 68 passengers

In 48 trips it carries (42 x 68) passengers

$$
\begin{array}{r}
4\,2 \\
\times 6\,8 \\
\hline
2\,5\,2\,0 \\
+\ \ \ 3\,3\,6 \\
\hline
2\,8\,5\,6
\end{array}
$$

The bus carries 2856 passengers in the 42 trips

ACTIVITY:

Multiply the following

1. 66 X 424   4. 9103 X 133

2. 117 X 24   5. 817 X 1313

3.   6636 X 36   6. 312 X 495

The average number of children in 136 schools in Mukono district is 1250. Find the total population in all the schools.

Find the product of 396 and 3298.

Word Application in division

EXAMPLES.

a.   Primary six has 225 pupils and 5 streams. If the pupils are distributed equally in all streams. How many pupils are in each stream?

   5   streams = 225pupils.

   1 stream = $(^{225}/_5)$ pupils

```
          0 4 5
        ┌─────────
      5 │ 2 2 5
5 x 0   0
      ─────
5 x 4   2 2
          2 0↓
        ─────
        0 2 5
5 x 5   2 5
        ─────
          0 0
```

Each stream has 45 pupils

Activity:

1. share 4848 microscopes among 12 schools.

42

2. 3 districts have to share 2451 text books from the ministry of education. How many text books will each district take?

3. What is the quotient of 1515 and 15?

4. Jophy bought 144 books. How many dozens did she buy?

5. A total of 4848 microscopes are to be shared among 12 schools. How many microscopes will each school receive?

6. The Ministry of Education is distributing 2451 textbooks among 3 districts. How many textbooks will each district receive?

2. A gardener planted 360 flowers in his garden. If he wants to arrange them equally in rows with each row having 12 flowers, how many rows will he have?

DIVISION OF NUMBERS

1. Divide 7620 by 20

```
              0 3 8 1
          _____
      20 )7 6 2 0
          - 0
          7 6
        - 6 0
          1 6 2
        - 1 6 0
          .  . 2 0
            - 2 0
```

2. Divide 76050 ÷ 234

Children. be encouraged to divide using multiples if the divisor (see P.5 notes)

ACTIVITY:

Divide the following numbers:

2 1256 ÷ 13

3 25610 ÷ 132

4 Divide 5600 by 250

NUMBER PATTERNS AND SEQUENCES

DIVISIBILITY TESTS

Divisibility test by 2

A number is divisible by 2 when it ends with, 0, 2, 4, 6, 8

e.g. 66, 200, 7204, 98, 24, 62

Divisibility test by 3

A number is divisible by 3 when the sum of its digits is divisible by 3.

e.g. i) 291

291   2+9+1 = 12

Therefore 291 is divisible by 3

ii) State whether 12631 is divisible by 3 or not.

12631 – 1+2+6+3+1

= 13

Therefore 12631 is divisible by 3

Divisibility test by 5

A number is divisible by 5 when it ends with either 0 or 5. E.g. 500, 25, 2795, 35090, 33000

Divisibility test by 4

A number is divisible by 4 when its last two digits are multiple of 4 i.e 00, 04, 08, 12, 16, 20, 24, 28, 32…………….

Examples:

a) Check whether 224 is divisible by 4

224 – The last two digits make 24 and 24 is divisible by 4

Therefore 224 is divisible by 4

ACTIVITY:

a) Which of the following numbers is divisible by 2

i) 37

ii) 9990

iii) 179

b) Test for divisibility by 3 and state whether the number is divisible by 3 or not

    i) 63

    ii) 29631

    iii) 178

c) Is 694 divisible by 4 or not?

d) Check whether 3595 is divisible by 5

e) Complete the table by using YES or NO

Number	By 2	By 3	By 4	By 5
3334	YES			
69250		NO		YES
1304	NO			
630001				
8896			YES	

GEOMETRICAL SEQUENCES

Solving using distributive property

Distributive property is used for numbers with a common factor.

EXAMPLES:

1. **Solve (7 x 19) + (7 x 11) using distributive property**

   (7 x 19) – (7 x 11)

      7 (19 + 11)   3 0

        7 (30)   x 7

        7 x 30   2 1 0

        = 210

2. **Solve (6 x 8) – (4 x 8) using distributive property.**

   (6 x 8) – (4 x 8)

      8 (6 – 4)

       8 (2)

       8 x 2

       = 16

ACTIVITY:

Using distributive property, solve the following.

1. (2 x 9) + (2 x 6)

2. (8 x 14) – (8 x 10)

3. (7 x 10) + (7 x 4)

4. (3 x 14) + (8 x 14)

5. (9 x 12) + (3 x 9)

NUMBER PATTERNS AND SEQUENCES

GEOMETRICAL SEQUENCES

a) Square numbers

A square number is obtained or got by multiplying a number by itself.

e.g. $1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$5^2 = 5 \times 5 = 25$

$6^2 = 6 \times 6 = 36$

$7^2 = 7 \times 7 = 49$

$8^2 = 8 \times 8 = 64$

Therefore 1, 4, 9, 16, 25, 36, 49, 64,…………………… are square numbers

b) Triangular numbers

Triangular numbers are obtained by adding consecutive counting numbers. They can be represented as the pattern below.

$1 = 1$

$1+2 = 3$

$1+2+3 = 6$

$1+2+3+4 = 10$

The sequence;

1, 3, 6, 10, 15, 21, 28, 36…………………………

c) Cubic numbers.

These are numbers got by multiplying the same number three times

$1^3 = 1 \times 1 \times 1 = 1$

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$

$4^3 = 4 \times 4 \times 4 = 64$

$5^3 = 5 \times 5 \times 5 = 125$

ACTIVITY

1. Write down all the square numbers between 10 and 65

2. Find the sum of the second and fifth triangular numbers.

3. Find the first four cubic numbers

4. Find the value of 6^3

5. What is the square number of 99?

NUMBER PATTERNS AND SEQUENCES

ARITHMETIC PROGRESSION

1. **Even numbers**

   Numbers which are divisible by 2 with no remainder e.g. 0, 2, 4, 6, 8, 10, 12, 14, 16……

2. **Odd numbers**

   Numbers you divide by two and get a remainder as 1 e.g. 1, 3, 5, 7, 9, 11, 13, 15……….

   Note; the pattern fro a even and odd number is by adding 2

3. **Counting numbers**

   These are numbers from 1 up to no end.

   They are also called National numbers e.g. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,……

4. **Prime numbers**

A prime number is a number with only two factors, which is 1 and itself, e.g 2, 3, 5, 7, 11, 13, 17, 19, 23, 29…………..

5. Composite numbers

A composite number is a number with more than two factors. E.g. 4, 6, 8, 9, 10, 12, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30…………………

ACTIVITY

1. Find the sum of the first four even numbers.

2. What is the product of the second and sixth odd number?

3. Divide the tenth counting number by the first prime number.

4. What is the difference between the sixth composite number and the third prime number?

5. Write all prime numbers between 20 and 36

6. If the sum of three consecutive integers is 72, find the integers.

7. A rectangle has a length of 15 units and a width of 8 units. Find its perimeter.

8. The average of five numbers is 21. If four of the numbers are 18, 25, 20, and 22, what is the fifth number?

9. If the square of a number is 81, what is the number?

10. A train travels 240 miles in 4 hours. What is its average speed?

NUMBER PATTERNS AND SEQUENCES

CONSECUTIVE NUMBERS

i) The sum of three consecutive counting numbers is 15. Find the numbers.

Let the first number be Y

The pattern of counting numbers is by adding

1st be y; 2nd = y+1 3rd = y+2

Sum = 15

y+y+1+y+2 = 15

y+y+y+1+2 = 15

3y+3 = 15

3y+3-3 = 15 – 3

$3y/3 = 12/3$

y = 4

$$y = 4$$
$$y+1 = 4+1 = 5$$
$$y+2 = 4+2 = 6$$
The numbers are 4, 5, 6

ii) Find the sum of four consecutive even numbers when the smallest number is 6

6, 8, 10, 12

= 6+8+10+12 = 36

ACTIVITY

1. Musa wrote three consecutive counting numbers on the chalkboard. If the second number was 9, find the sum of the numbers he wrote.

2. The median of three consecutive odd numbers is 21. Find the numbers.

3. Find the four consecutive counting numbers if their sum is 86

4. The total of three consecutive even numbers is 60. Find their range

5. The product of three consecutive even numbers is 192. Find the numbers.

6. The sum of three consecutive multiples of 5 is 75. Find the numbers.

7. The average of three consecutive integers is 14. Find the integers.

8. The sum of four consecutive odd numbers is 92. Find the numbers.

9. A father is three times as old as his son. If the sum of their ages is 48, find their ages.

10. The perimeter of a rectangle is 26 units. If the length is three more than twice the width, find the dimensions of the rectangle.

11. The sum of three consecutive odd integers is 45. Find the integers.

12. The product of three consecutive numbers is 210. Find the numbers.

13. The sum of five consecutive even integers is 130. Find the integers.

SUBTOPIC: PRIME FACTORISATION

CONTENT:

- Prime factorization can be done in two methods.

1. Using ladder method

2. Using factor tree method

- Prime factorization can also be in power form or subscript form.

Examples

a) Prime factorize 36 and give your answer in a power form.

F_{36} F_{36}

2	36
2	18
2	9
3	3
3	1

= (2 X 2) X (3 X 3)   = (2 X 2) X (3 X 3)

= 2^2 X 3^2 = 2^2 X 3^2

b) Prime factorize 48 and give your answer in subscript form

$$F_{48} = 2 \times 2 \times 2 \times 2 \times 3$$

$$= \{2_1, 2_2, 2_3, 2_4, 3_1\}$$

NOTE: Subscript form is also called set form.

ACTIVITY

a) Prime factorize the following and give the answer in a power form

1. 12   d) 100

2. 24   e) 125

3. 72   f) 18

b) Prime factorize and give the answer in subscript

1. 90   b) 32   c) 15   d) 120

FINDING PRIME FACTORISED NUMBERS

1. What number has been expanded below?

$2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 2) \times (3 \times 3) \times 3$

$= 4 \times 9 \times 3$

$= 36 \times 3$

$= 108$

2. Find the prime factorized number to get $2^3 \times 3^2$

$2^3 \times 3^2 = (2 \times 2 \times 2) \times (3 \times 3)$

$= 8 \times 9$

$= 72$

3. Find the number that has been expanded below;

$\{2_1, 2_2, 2_3, 3_1, 5_1\}$

$= 2_1 \textbf{X} 2_2 \textbf{X} 2_3 \times 3_1 \times 5_1$

$= (2 \times 2 \times 2) \times (3 \times 5)$

$= 8 \times 15$

$= 120$

ACTIVITY

Find the numbers which have been prime factorized;

1. $2 \times 3 \times 5$ 6. $\{2_1, 3_1, 5_1\}$

2. $2 \times 3 \times 3 \times 5$ 7. $\{7_1, 11_1\}$

3. $2^3 \textbf{X} 3^1$ 8. $7^1 \textbf{X} 11^1 \textbf{X} 13^1$

4. $5^2 \textbf{X} 7^1$

PRIME FACTORS ON VENN DIAGRAMS

1. Prime factorize 18 and 12 and represent their prime factors on a venn diagram

Note: On a venn diagram, we put subscripts. So we prime factorize in subscript from.

```
2 |18          2 |12
3 |9           2 |6
3 |3           3 |3
  |1
```

$18 = 2_1 \; 2_2 \; 3_1 \; 3_2$ $12 = 2_1 \; 2_2 \; 2_3 \; 3_1$

Common prime factors $\{2_1, 3_1\}$

Find the G.CF of F_{18} and F_{12}

G.C.F   = Product of factors in F18 $\cap$ F12 .

          = $2_1 \times 3_1$

          = 2 X 3

          = 6

2. Work out the L.C.M of F_{18} and F_{12}

L.CM = Product of factors F18 $\cup$ F12 .

L.C.M = $2_1 \; X3_1 \; X \; 2_2 \; X3_2$

L.CM = 2 X 3 X 2 X 3

      = 6 X 6

L.C.M = 36.

ACTIVITY

a) Prime factorize 40 and 15 and represent their prime factors on a venn diagram

b) Find their L.C.M

c) Calculate their G.C.F

INTERPRETING VENN DIAGRAMS

1. The venn diagram below shows the prime factors of Y and X

a) Find the value of Y

$Y = 2_1 \times 3_1 \times 3_2$

$Y = 2 \times 3 \times 3$

$Y = 6 \times 3$

$Y = 18.$

b) Find the value of X

$X = 2_1 \times 2_2 \times X3_1 \times 5_1$

$X = 2 \times 2 \times X3 \times 5$

$X = 12 \times 5$

$X = 60.$

c) Find the L.C.M of F_Y and F_X

L.CM = Product of factors $F_Y \cup F_X$.

$L.C.M = 2_1 \times 2_2 \times 5_1 \times 3_1 \times 3_2$

$= (2 \times 2 \times 5) \times (3 \times 3)$

$= 20 \times 9$

L.C.M = 180

2. a) Find the value of P and K in the figure

$$p \times 2_1 \times 3 = 12 \qquad\qquad K = 2_1 \times 3_1 \times 3_2$$

$$p \times 2 \times 3 = 12 \qquad\qquad K = 2 \times 3 \times 3$$

$$\frac{6P}{6} = \frac{12}{6} \qquad\qquad K = 6 \times 3$$
$$\qquad\qquad\qquad\qquad \underline{K = 18}$$

$$\underline{P = 2.}$$

a) Find the G.C.F of F_{12} and F_K

      G.C.F = Product of Intersection

$$= 2_1 \times 3_1$$

$$= 2 \times 3$$

$$\underline{G.C.F = 6}$$

ACTIVITY

    i)   Given the venn diagram below

1. Find the value of

1. m   ii) n

2. Work out the L.C.M of F_m and F_{20}

3. Calculate the G.C.F of F_m and F_{20}

SUBTOPIC : **SQUARES AND SQUARE ROOTS OF NUMBERS**

CONTENT :

Squares:

1. Find the square of 5

   5^2 = 5 X 5 = 25

2. What is the square of 16

   16^2 = 16 X 16 = 256

Square Roots:

a) Find the square root of 9

$^2\sqrt{9}$ =

3	9
3	3
	1

= 3 X 3
= 3^2

$^2\sqrt{9}$ = $^2\sqrt{3^2}$ = 3

Therefore $\sqrt{9}$ = 3.

b) Work out the square root of 64

$\sqrt{64}$ =

= 2 X 2 X 2 X 2 X 2 X 2

= 2^6

So, $^2\sqrt{64}$ = $^2\sqrt{2^6}$ = 2^3 = 2 X 2 X 2 = 8

Therefore, $\sqrt{64}$ = 8.

Square roots can also be got by using odd numbers

c) Find the square root of 6

16 – 1 = 15

15 – 3 = 12

12 – 5 = 7

7 – 7 = 0

Therefore, √16 = 4.

Note: We subtract odd numbers in their order sequence until we get 0 and count the number of odd numbers used.

ACTIVITY:

a) Find the square root of each

1. 4   d) 121

2. 25   e) 196

3. 100   f) 255

b) Find the square of each

1. 6   c) 100

2. 10   d) 25

SUBTOPIC : SQUARE ROOTS OF FRACTIONS

CONTENT :

a) Find the square root of $^4/_9$

$$\frac{\sqrt{4}}{\sqrt{9}} = \frac{\sqrt{4}}{\sqrt{9}} = 4 \qquad\qquad = {}^2\sqrt{2^2} = 2.$$

$$\sqrt{9} = 9$$

$$= 3 \times 3 = 3^2 = {}^2\sqrt{3^2} = 3.$$

Hence $\underline{\sqrt{(^4/_9)} = {}^2/_3}$

b) Work out the square root of $2^7/_9$

$$\sqrt{2^7/_9} = \sqrt{25/_9} = \frac{\sqrt{25}}{\sqrt{9}}$$

5	25
5	5
	1

3	9
3	3
	1

$$= {}^2\sqrt{5 \times 5} \qquad\qquad = {}^2\sqrt{3 \times 3}$$

$$= {}^2\sqrt{5^2} \qquad\qquad = {}^2\sqrt{3^2}$$

$$= 5. \qquad\qquad\qquad \underline{= 3.}$$

SO $\sqrt{25/_9} = {}^5/_3$

$$= \sqrt{2^7/_9} = \underline{1^2/_3.}$$

ACTIVITY

a) Work out the square roots of the following

1. $^{100}/_{1000}$ c) $^9/_{16}$ e) $6^1/_4$ g) $1^7/_9$

61

2. $^1/_4$ d) $1^{19}/_{81}$ f) $^{81}/_{100}$

SUBTOPIC   : SQUARES ROOTS OF DECIMALS

1. Find the square root of 0.49

$$^2\sqrt{\dfrac{0.49}{100}} = {}^2\sqrt{\dfrac{49}{100}} = \dfrac{7}{10} = 0.7$$

$$\sqrt{\dfrac{49}{100}} \qquad \begin{array}{c|c} 7 & 49 \\ 7 & 7 \\ \hline & 1 \end{array} \qquad = {}^2\sqrt{7 \times 7} = {}^2\sqrt{7^2} = 7$$

$$\sqrt{100} \qquad \begin{array}{c|c} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \qquad = {}^2\sqrt{2 \times 2 \times 5 \times 5} = {}^2\sqrt{2^2 \times 5^5} = 2 \times 5 = 10$$

Note: Square roots of decimals, we change the decimal into a common fraction first and after the square root of each part, we take it back to decimal form.

2. Work out the square root of 0.0081

$$\sqrt{0.0081} = \dfrac{\sqrt{81}}{\sqrt{10000}} = \dfrac{9}{100} = 0.09$$

ACTIVITY

Find the square root of each decimal;

1. 0.36   5) 2.25

2. 0.81   6) 1.21

3. 1.44   7) 0.0004

4. 1.96   8) 0.0064

MORE ON SQUARES ROOTS

1. The area of a square is 81 cm². Find the length of each side of the square.

Area   = S²

81 Cm²   = S²

²√ 81 Cm² = ²√S²

S   = **9 cm**

$$\begin{array}{c|c} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline \end{array}$$

√81 = √(3 X 3 X 3 X 3 X 3)

    = ²√3⁴

    = 3 X 3

    = 9

Therefore, Side = 9 Cm

2. Solve, K² = 0.0004

√K² = √0.0004

√K² = √ ⁴/₁₀₀₀₀

K = 2
    —
    100

K = 0.02

Solve 2Y² = 50

2y² = 50

Dividing each term by the coefficient

2y² = 50
—   —
2   2

Y² = 25

√25 =   5 25

         2   1

√25 = 5x5

   = ²√5²

   = 5

Taking the square root on both sides

$$\sqrt{y^2} = \sqrt{25}$$

$$\underline{y \quad = \quad 5}$$

ACTIVITY

a. Mulindwa's square garden covers an area of 100 m². Calculate the length of each side of the garden.

b. The base area of a cube is 16 cm². Find its height.

c. A farmer used a barbed wire to fence his square garden that covers an area of 196 m². Find the length of the barbed wire used.

d. Solve $P^2 = 1$

e. Solve $900 = m^2$

f. Find the value of $3r^2 = 12$

e)   2   3 four

   - 2   2 four
   _____

f) 6 5 seven

   - 4 6 seven
   _____

g) 464 eight - 237 eight

h) 463 nine - 155 nine

i) 354 six - 245 six

LESSON TWO

CHANGING DECIMAL TO NON-DECIMAL BASE

Changing from decimal to non-decimal bases

1. Change 25 to base seven.

Base	No.	Rem
7	25	4
	3	

Therefore: $25 = 34_{seven}$

2. Change 38 to base eight.

Base	No.	Rem
8	38	6
	4	

Therefore: $38 = 46_{eight}$

MULTIPLICATION OF BASES

Example:

1. Multiply: 232_{five} x 3

    $\overset{①\ ①\ ①}{2\ 3\ 2}_{five}$ | $6 \div 5 = 1$ rem ①
    $\underline{x\quad 3}$ | $10 \div 5 = 2$ rem ⓪
    $\underline{1201}_{five}$ | $7 \div 5 = 1$ rem ②

2. Multiply: 214_{four} x 3

    $\overset{①\ ①\ ③}{2\ 1\ 4}_{four}$ | $12 \div 4 = 3$ rem 0
    $\underline{x\quad 3}$ | $6 \div 4 = \ 1$ rem 2
    $\underline{1320}_{four}$ | $7 \div 4 = \ 1$ rem 3

Activity

Multiply the following bases

1. 214_{five} by 3
2. 432_{four} x 4
3. 320_{five} x 4
4. 354_{six} x 5
5. 122_{three} x 3
6. 464_{eight} x 5

ACTIVITY

1. Change the following to base three.

    a) 19_{ten} b) 31_{ten} c) 26_{ten}

2. Convert the following to base four.

    a) 34_{ten} b) 42_{ten} c) 75_{ten}

3. Change the following to base six.

    a) 31_{ten} b) 46_{ten} c) 94_{ten}

4. Convert the following to base seven.

    a) 96_{ten} b) 68_{ten} c) 536_{ten}

5. Change the following to base eight.

    a) 73_{ten} b) 26_{ten} c) 431_{ten}

LESSON FOUR

CHANGING FROM A NON-DECIMAL BASE TEN.

1. Change 234six to base ten.
   2 3 4six

   $(2 \times 6^2) + (3 \times 6^1) + (4 \times 6^0)$
   $2 \times 6 \times 6 + 3 \times 6 + 4 \times 1$
   $72 \quad + 18 \quad + 4$
   $= \quad \underline{94_{\text{ten}}}$

2. Change 41_{five} to base ten
   $\begin{matrix} 1 & 0 \end{matrix}$
   $4 \; 5 \;_{\text{five}}$
   $4 \; 1 \;_{\text{five}}$

   $41 \text{ five} = (4 \times 51) + (1 \times 50)$

   $= \quad (4 \times 5) + (1 \times 1)$

   $= \quad 20 + 1$

   $= \quad 21_{\text{ten}}$

Change the following to base ten.

a) 23_{four} f) 214_{six}

b) 131_{four} g) 63_{seven}

c) 55_{eight} h) 62_{seven}

d) 413_{five} i) 1011_{two}

e) 122_{three} j) 144_{five}

LESSON FIVE

CHANGING FROM NON DECIMAL TO A NON DECIMAL BASE
Examples

1. Change 413five to base two
   $\begin{matrix} 2 & 1 & 0 \end{matrix}$
   $\begin{matrix} 5 & 5 & 5 \end{matrix}$
   $4 \; 1 \; 3 \text{ five}$
   $(4 \times 5^2) + (1 \times 5^1) + (3 \times 5^0)$
   $4 \times 5 \times 5 + (1 \times 5) + (3 \times 1)$
   $(4 \times 25) + 5 + 3$
   $100 + 8$
   $108 \;_{\text{ten}}$

B	N	R
2	108	0
2	54	0
2	27	1
2	13	1
2	6	1
2	3	1
2	1	1
	0	

1101100

2. Change 43_{five} to base six

B	N	R
6	23	5
6	3	3
	0	

= 35_{six}

    4 3 five
    5 5

    $(4 \times 5^1) + (3 \times 5^0)$

    $4 \times 5 + 3 \times 1$

    20 + 3

    Therefore $43_{\text{five}} = 35_{\text{six}}$

ACTIVITY

Change the following as instructed.

a) 413_{five} to base three
b) 101_{two} to base five
c) 203_{five} to base six
d) 144_{five} to base two
e) 341_{six} to base five
f) 1110_{two} to base four

LESSON SIX

DETERMINING THE UNKNOWN BASES

1. If $17y = 15\text{ten}$, find base y

    $17y$ = 15ten

    $(1 \times y^1) + (7 \times y^0) + 15$

    $1 \times y + 7 \times 1 = 15$

    y + 7 = 15

    y + 7-7 = 15-7

    y   = 8

  = y is eight

2. If $24_z = 42_{\text{five}}$

    Find base z

    24z   =   42five

    $(2 \times z^1) + (4 \times z^0)$ = $(4 \times 51) + (2 \times 50)$

    2xz + 4x1   = 4x5 + 2x1

    2z + 4   = 20 + 2

    2z + 4   = 22

    2z+4 − 4   = 22 - 4

    2z   = 18

    $\dfrac{2z}{2}$ = $\dfrac{\cancel{18}}{2}$

    z   =   9

LESSON SEVEN

Addition of bases in table from

Completion of bases in table form.

Base five

+	1	2	3	4	10
1	2	3	4	10	11
2	3	4	10	11	12
3	4	10	11	12	13
4	10	11	12	13	14
10	11	12	13	14	20

1 + 1 = 2   3 + 4 = 7

1 + 2 = 3   7÷5 = 1 rem 2

1 + 3 = 4

1 + 4 = 5   4 ÷ 4 = 8

5÷5 = 1 rem 0   8÷5 = 1 rem 3

Activity:

1. Complete the base six additions in the table below.

+	1	2	3	4	5	10
1						
2						
3						
4						
5						
10						

MULTIPLICATION OF BASES IN TABLE FORM

x	1	2	3	4	10
1	1	2	3	4	10
2	2	4	11	13	20
3	3	11	14	22	30
4	4	13	22	31	40
10	10	20	30	40	100

TOPIC: OPERATIONS ON WHOLE NUMBERS

Examples

1.   Add   1 2 3 4 6 7 8

          +   2 9 7 8 6 8

            1 5 3 2 5 4 6

2.   Add   1 8 4 8 6 9 4
               + 3 3 0 2 5 2 0
                 5 1 5 1 2 1 4

3.   Subtract $\overset{4\ 11\ 12\ 13}{\cancel{5}\ \cancel{2}\ \cancel{3}\ 3}$ 1 8 6
                 - 1 3 4 5 1 0 2
                   3 8 8 8 0 8 4

4.   Subtract   8 4 0 0 0 7
                 -    3 4 6 5 2
                   8 0 5 3 5 5

Solving word problems involving addition and subtraction of whole

Examples

1.   Find the sum of 67802 and 14007
      Soln
        6 7 8 0 2
      + 1 4 0 0 7
        8 1 8 0 9

2.   A diary processed 6500 litres of milk, if 5650 litres of milk of milk were sold, how many litres remained?

            $\overset{5\ 14\ 10}{\cancel{6}\ \cancel{5}\ \cancel{0}}$ 0 litres
           - 5 6 5 0 litres
               8 5 0 litres of milk

      Note:
      Sum   is the result of adding any given numbers.
      Difference: Is the result got after subtracting numbers

3.   Multiplication of whole numbers
      Examples
      1. Multiply   3 2 4
                    x 1 8

      2. Multiply 465 by 472

70

3. Use lattice method of multiplication to work out

    9 4 7

    x 6 5

    60608

Division of whole numbers using long division

Examples

1.   Divide

Activity:

Ref to Mk bk6 pg 62

Ref to fountain bk6 pg 37

<u>Solving word problems involving division</u>

Examples

1. A petrol station manager bought 22,000 litres of petrol. If she put equal amount of oil in 440 drums. How many litres of oil were in each drum?

    <u>Soln</u>:

    22,000 ÷ 440 = 50

    There were 50 litres of oil in each drum.

Mixed operations (BODMAS)

Examples
1. Simplify 3x4+5
2. Workout 5-2+6
3. Work out 3+9÷3-1
4. Workout 4x7+9x3

TOPIC: **PATTERNS AND SEQUENCE**

LESSON I

Sub topic: Divisibility tests

Content: Divisibility tests of 2, 3 and 5

Examples

(a) **By 2**

A number is divisible by 2 when it is an even number e.g 2, 4, 6, 8, 10, 12, 14

b) **By 3**

A number is divisible by 3 when the sum of its digits are a multiple of 3 e.g. 612

$$= 6+1+2$$
$$= 9 \div 3 = 3$$

Therefore 612 is divisible by 3

c) **By 5**

A number is divisible by 5 when the last digit is either 0 or 5. e.g 10, 15, 20, 770, 405 etc

LESSON II

Sub topic: Developing number patterns

Content: Odd numbers, even numbers, Triangular numbers, Square, numbers, cube numbers, composite numbers, counting numbers, whole numbers

Examples

i)   List down the following
   a) Counting numbers/natural numbers less than 15
   b) Whole numbers lessthan 10
   c) Even numbers between 10 and 20
   d) Odd numbers lessthan 20
   e) Triangular numbers lessthan 36
   f) Square numbers lessthan 49

NB:
   ➢ Triangular numbers are obtained by adding consecutive counting numbers.
   ➢ Square numbers are obtained by adding the consecutive odd numbers starting with one as the first square number
   Or by squaring natural numbers

LESSON III

Subtopic: Prime numbers and composite
Content: -   List prime numbers
            -   List composite numbers
Examples
1.   What is the sum of the 3rd and the 7th prime numbers?
     2, 3, ⑦, 11, 13, ⑰, 19, 23
     sum = 5 + 17
         = 22

2.   Workout the sum of the first five composite numbers.
     4, 6, 8, 9, 10, 12, 14, 15
     sum = 4+6+8+9+10
         =   37

LESSON IV
Subtopic: Consecutive counting numbers, odd and even numbers
Content: Finding consecutive counting, odd and even numbers
Examples:
The sum of 3 consecutive whole numbers is 36. What are these numbers?
Let the 1st number be n.

2nd number n+1	1st number = n
3rd number n+2	
$n+n+1+n+2 = 36$	2nd number = n+1
$3n + 3 = 36$	= n+1
$3n + 3 -3 = 36$	= 12
$\dfrac{3n}{3} = \dfrac{\overset{11}{\cancel{33}}}{3}$	3rd number = n+2
	= 11+2
n = 11	= 13

Even and odd numbers increase in intervals of 2
The sum of three consecutive even numbers is 24.
List down the 3 numbers.

Let the 1st number be x	1st number = x
2nd number x+2	= 6
3rd number x+4	2nd number = x+2
x+x+2+x4 = 24	= 6+2
3x+6 = 24	= 8
3x+6-6 = 24	3rd number = x+4
3x = 24-6	= 6+4
$\dfrac{3x}{3} = \dfrac{\cancel{18}}{3}$	= 10
x = 6	

LESSON V
Subtopic: Prime factorization
Content: Methods used in prime factorization

- Multiplication, Subscript method/set notation
- Powers of ten/exponents

Examples

a) Find the prime factors of 60

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 1 \end{array}$$

$= 2\times2\times3\times5$ (multiplication)

$= 2^2\times3^1\times5^1$ (powers/exponents)

$= 2_1, 2_2, 3_1, 5_1$ (subscript form)

b)

LESSON VI

Content: - Finding prime factorized number
- Finding the missing prime factors

Examples:

i) What number has been prime factorized

ii) Prime factorise and find missing factors

The prime factorization of 30 is 2xyx5, find y

$$2xyx5_1 = 30$$
$$\frac{10y}{10} = \frac{30}{10}$$
$$y = 3_1$$

If $144 = a^4 \times b^2$ find "a" and "b"

$$\begin{array}{r|l} 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$= 2^4 \times 3^2$

$= a^4 \times b^2$

$\div a = 2$

$b = 3$

Given that $2^{2x} \times 2 = 32$ find the value of x

$$2^{2x}x2 = 2^5$$
$$2x+1 = 5$$
$$2x+1-1 = 5-1$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 1 \end{array}$$

LESSON VII

Content: Finding L.C.M and G.C.F using venn diagram

Examples

Show the prime factors of 30 and 36 on the venn diagram

30
2 15 = $\{2, 3, 5\}$
3   5
  5   1

36
2   18 $\{2_1, 2_2, 3_1, 3_2\}$
2   9
  3   3
    3   1

GCF = F30 ∩ F36
  = $\{2_1, 3_1\}$
  = 6

L.C.M = F30 U F36
  = $\{2_1, 2_2, 3_1, 3_2, 5_1\}$
  = 2X2X3X3X5
  = 12X3X5
  = 180

LESSON VIII

Subtopic: Unknown values/prin

Content: 1) Find the missing numbers
          2) Find the unknown factors
          3) Workout the H.C.F and L.C.M

Example:

Find x and y

F_x F_y

F_x = $\{2_1, 2_2, 2_3, 3_1\}$
  = 2x2x2x3
  = 24
F_y = $\{2_1, 2_2, 3_1, 3_2, 3_3\}$
  = 2x2x3x3x3
  = 108

G.C.F = F_x ∩ F_y
  = $\{2_1, 2_2, 3_1\}$
  = 2X2X3
  = 12

L.C.M = $\{2_1, 2_2, 2_3, 3_1, 3_2, 3_3\}$
  = 2x2x2x3x3x3
  = 216

Find the unknowns

F_{20} F_{30}

i) $\{2_1, 5_1 x\}$ = F_{20}
  = x+2x5 = 20
  $\overline{10}$x = $\overline{20}$ 2
  $\overline{10}$ $\overline{10}$ 1
  X = 2_2

75

ii) $\quad = \quad$ 21, 51, y = F30

$\quad\quad = \quad$ 2x5xy = 30

$\quad\quad = \quad \cancel{10}$y $\quad = \quad \dfrac{\cancel{30}}{10}$

$\quad\quad\quad\quad \dfrac{}{10}$

$\quad\quad\quad\quad$ y = 3_1

LESSON IX

Sub-topic: Application of G.C.F/L.C.M

Content: - Relationship between G.C.F and L.C.M

$\quad\quad\quad\quad$ - Other problem related to H.C.F/G.C.F

Examples

1. The LCM of two numbers is 144 their GCF is 12 and one of these numbers is 48. Find the other number.

Solution

2nd no. $= \dfrac{\text{LCM x GCF}}{1^{st} \text{ no.}}$

$\quad\quad = \dfrac{\overset{36}{\cancel{144}} \times \cancel{12}^{1}}{\underset{\cancel{4}_1}{48}}$

What is the largest possible divisor of 24 and 36?

2	24	36
2	12	18
2	6	9
3	3	9
3	1	3
	1	1

$= \quad$ 2x2x3

$= \quad$ 12

LESSON X

Subtopic: Application of LCM

Content: Find the smallest number which when divided by 9 and 12 leaves

a) no remainder?

b) Remainder of 1?

c) Remainder of 5?

2	9	12
2	9	6
3	9	3
3	3	1
	1	1

Solution

a) $=$ 2x2x3x3

$\quad = $ 36

b) (2x2x3x3)+1

$\quad\quad$ 36+1

$\quad\quad$ 37

c) (2x2x3x3)+5

$\quad\quad$ 36+5

$\quad\quad$ 41

ii) Kelvin has an article of 40cm and his father has a stride of 60cm. What is the width of the narrowest path and they both cross in a whole number of strides.

M_{40} = {40, 80, 120, 160 …}

M_{60} = { 60, 120, 180, …}

L.C.M = 120

Therefore: The width is 120cm

LESSON XI

Subtopic: <u>Working with powers of whole number</u>

Content: - Find a number from powers
   - Express numbers as a product of powers of a given numbers
   - Operation on powers

Example   i) What is 7^3

= 7x7x7

= 343

ii) Express 64 using powers of 4.

```
4 | 64
4 | 16     Therefore 64 = 4x4x4
4 |  4                 = 4³
   |  1
```

Therefore 64 = 4x4x4 $= 4^3$

LESSON XII
Subtopic **=**   Square numbers and square roots
   - Square of :-
   a) Whole numbers
   b) Fractions
   c) Mixed fractions
   d) Decimals

<u>Example</u>

1. What is the square of 12?
   12^2 = 12x12
       = 144

2. Workout the square of ¾
   $(¾)^2$ = ¾ x ¾
   = $\frac{9}{16}$

3. What is the square of 0.6?

$(0.6)^2 = \dfrac{0.6}{10} \times \dfrac{6}{10} \quad / \quad \dfrac{36}{100} = 0.36$

4. Find the square root of?

a) 144

$\sqrt{144}$

$\sqrt{144 = 2 \times 2 \times 3}$

$\sqrt{144 = 12}$

$2\begin{cases} 2 \\ 2 \end{cases} \begin{array}{|l} 144 \\ 72 \end{array}$

$2\begin{cases} 2 \\ 2 \end{cases} \begin{array}{|l} 36 \\ 18 \end{array}$

$3\begin{cases} 3 \\ 3 \end{cases} \begin{array}{|l} 9 \\ 3 \\ 1 \end{array}$

b) $\sqrt{\dfrac{9}{16}}$

$\begin{array}{c|c} 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$

$\sqrt{\dfrac{9}{16}} = \dfrac{3}{2 \times 2}$

$\sqrt{\dfrac{9}{16}} = \dfrac{3}{4}$

$2\begin{cases} 2 \\ 2 \end{cases} \begin{array}{|l} 16 \\ 8 \end{array}$

$2\begin{cases} 2 \\ 2 \end{cases} \begin{array}{|l} 4 \\ 2 \\ 1 \end{array}$

LESSON XIII

Content: Finding cube numbers and cube roots

Examples

1.   What are the cubes of the following?

a) $2 = 2^3$

= 2x2x2

= 8

b) $\dfrac{2}{3} = \left(\dfrac{2}{3}\right)^3$

= $\dfrac{2 \times 2 \times 2}{3 \; 3 \; 3}$

= $\dfrac{8}{27}$

Find the cube root of 8.

```
2| 8|
2| 4
2| 2
   1
```

$$\sqrt[3]{8} = \sqrt[3]{(2 \times 2 \times 2)}$$

$$\sqrt[3]{8} = \sqrt[3]{2^3}$$

$$\sqrt[3]{8} = 2$$

THEME: **NUMERACY**
TOPIC: **FRACTIONS**
Review on addition and subtraction of fractions.

Examples

1. Add: $^3/_5$ + $^1/_3$
    ### *Solution*
    $^3/_5$ + $^1/_3$ = $\dfrac{(3 \times 3) + (5 \times 1)}{15}$

    $= \dfrac{9 + 5}{15}$

    $= \dfrac{14}{15}$

2. Subtract $^5/_7 - ^1/_4$

    ### *Solution*
    $^5/_7 - ^1/_4$ = $\dfrac{(4 \times 5) - (7 \times 1)}{28}$

    $= \dfrac{20 - 7}{28}$

    $= \dfrac{13}{28}$

    Activity 1:1
1. Add the following
    a) ¾ + ½   b) $^1/_3$ + $^1/_5$ c) $^2/_3$ + $^3/_5$

    d) 1 ½ + 2 ¼   e) 3 $^1/_5$ + $^1/_3$

2. A boy was given $^5/_{12}$ of the bread in the morning and ½ of the remainder in the evening. What fraction of the bread did he get altogether?

3. Mugisha divided his land as follows:

    ¼ occupied by cows $^1/_3$ occupied by crops and $^1/_8$ is occupied by goats while the rest of the land is occupied by cash crops.

    Find the piece of land occupied by animals.

Activity 1:2

1. Subtract the following:

   a) $\frac{1}{2} - \frac{1}{3}$ b) $\frac{1}{4} - \frac{1}{6}$ c) $\frac{1}{3} - \frac{1}{5}$ d) $\frac{5}{7} - \frac{2}{5}$

2. Natasha had $\frac{7}{10}$ of the bread. She gave out $\frac{2}{5}$, what fraction of the bread remained?

3. I ate ¾ of the chapatti, what fraction of the chapatti remained?
4. The teacher instructed pupils to read $\frac{4}{20}$ pages of the book. What fraction of pages remained un-read?
5. A bus covered $\frac{7}{15}$ of the journey before breaking down. Find the part of the journey it had remained with.

Multiplication of fractions:

<u>Finding reciprocal</u>:

Reciprocal is also known as multiplicative inverse. This is because, a number multiplied by its reciprocal, the result is 1.

Example 1

Find the reciprocal of 3.
Number x reciprocal = 1
3 x reciprocal = 1
$\frac{3 \times r}{3} = \frac{1}{3}$
∴ Reciprocal of 3 = $\frac{1}{3}$

Example 2

Find the reciprocal of 2/5
Number x Recip = 1
$\frac{2}{5}$ x Recip = 1
5 x $\frac{2}{5}$ x R = 1 x 5 $\frac{2 \times R}{2} = \frac{5}{2}$
∴ Recip of $\frac{2}{5} = \frac{5}{2}$

Activity 1:3

Find the reciprocal or multiplicative inverse of the following:

a) $\frac{3}{3}$ d) 10   g) 2 ½   j) 0.2

b) ¼   e) 5   h) 1 $\frac{1}{3}$ k) 4.5

c) $\frac{4}{5}$ f) 4   i) 3 ¼   l) 3.2

<u>Multiplying fractions</u>

It should be noted that, it is only in multiplication of fractions where L.C.M is not applied.

<u>Examples;</u>

1. $\dfrac{2}{3} \times \dfrac{1}{4}$

   $= \dfrac{2 \times 1}{3 \times 4}$

   $= \dfrac{2}{12}$ (by reducing)

   $\dfrac{1}{6}$

2. $1\dfrac{1}{2} \times \dfrac{1}{3}$

   $\dfrac{3}{2} \times \dfrac{1}{3}$

   $\dfrac{3 \times 1}{2 \times 3}$

   $\dfrac{3}{6}$ (by reducing)

   $= \dfrac{1}{2}$

Activity 1:4

Simplify the following.

1. $\dfrac{1}{3} \times \dfrac{1}{2}$

2. $\dfrac{1}{3} \times \dfrac{1}{4}$

3. $\dfrac{2}{3} \times \dfrac{1}{4}$

4. $\dfrac{2}{5} \times \dfrac{3}{4}$

5. $\dfrac{3}{8} \times \dfrac{1}{3}$

6. $\dfrac{1}{10} \times \dfrac{2}{5}$

7. $\dfrac{2}{9} \times \dfrac{4}{10}$

8. $\dfrac{3}{7} \times \dfrac{4}{8}$

9. $\dfrac{3}{8} \times \dfrac{4}{5}$

Simplify:

a) $1\dfrac{1}{5} \times \dfrac{1}{3}$

b) $1\dfrac{1}{4} \times \dfrac{1}{5}$

c) $2\dfrac{1}{3} \times \dfrac{1}{14}$

d) $2\dfrac{1}{10} \times \dfrac{3}{9}$

e) $4\dfrac{1}{2} \times \dfrac{4}{9}$

f) $9\dfrac{1}{7} \times 10\dfrac{1}{5}$

<u>Division of fractions:</u>

Division is always done in two ways;

i) Use of reciprocal

ii) Use of L.C.M

Examples;

1. Workout

   $8 \div 2$

   $^8/_1 \times \frac{1}{2}$

   $\dfrac{8 \times 1}{1 \times 2}$

   $^8/_2$

   $= 4$

2. Simplify

   $^2/_3 \div 2$

   $^2/_3 \div ^2/_1$

   $^2/_3 \times \frac{1}{2}$

   $^2/_6$

   $= ^1/_3$

3. Divide

   $\frac{1}{4} \div ^2/_3$ (len =12)

   $^3\cancel{12} \times \frac{1}{4} \div ^2/_3 \times \cancel{12}^4$

   $(3 \times 1) \div (2 \times 4)$

   $3 \div 8$

   $^3/_8$

4. How many small bottles of ¼ litre can be obtained from a 20litre jerrican?

Soln
we divide 20 ÷ ¼

$$20 ÷ ¼$$

$$20 \times \frac{4}{1}$$

80 quarter litre bottle

Activity 1:5

1. Workout the following.

   a) $20 ÷ 4$ b) $^3/_{10} ÷ 6$ c) $¼ ÷ ^2/_3$ d) $9\,^1/_5 ÷ ^2/_{10}$

2. Simplify

   a) $^2/_3 ÷ ^1/_{12}$ b) $^4/_9 ÷ 3\,¾$ c) $^5/_9 ÷ 3\,^1/_3$

3. Divide the following

   a) $¾ ÷ 7/8$ b) $^2/_3 ÷ ^7/_9$ c) $2\,^4/_5 ÷ 1\,^1/_3$ d) $^8/_9 ÷ ^2/_3$

Activity 1:6

1. How many ½ liter cups of water can be got from a 5 litre container?
2. How many small spoons of 1 ½ ltr can be obtained from 2 ¼ ml?
3. By what fraction can 6 ½ be divided to get 2 ½?
4. How many pieces of ¾ m of cloth can be cut from a long piece of 9m?
5. A bag contains 5 ¼ kg of posho. How many ¼ kg packets can be got from the bag?

Mixed operations of fractions:

Examples;

$^2/_3 \times ^4/_9 ÷ ^1/_3$ $^1/_3 \times ¼ + ½$ $1/12 + ½$

$^2/_3 \times ^4/_9 \times ^3/_1$ Using Bodmas, Multiplication $\dfrac{(1 \times 1) + (6 \times 1)}{12}$

$\dfrac{2 \times 4 \times 1}{3 \times 3 \times 1}$ comes first. $\dfrac{1 + 6}{12}$

                $(^1/_3 \times ¼) + ½$ 12

$^8/_9$ $\dfrac{1 \times 1}{12} + \dfrac{1}{2}$ $^7/_{12}$

Activity 1:7

Workout the following:

$\frac{1}{2}$ x $^3/_5$ ÷ $\frac{3}{4}$	$1\,^2/_3$ x $3\,\frac{1}{2}$ ÷ $^1/_5$	$^5/_6$ ÷ $(^5/_4$ of 3)
$^4/_9$ x $^2/_3$ ÷ $^1/_3$	$(^1/_3$ x $\frac{1}{4})$ + $\frac{1}{2}$	$^1/_3$ x $^1/_8$ + $\frac{1}{4}$ ÷ $^1/_7$
$^3/_8$ x $^1/_9$ ÷ $^2/_3$	$\frac{3}{4}$ x $^2/_3$ – $\frac{1}{2}$	$^1/_3$ + $\frac{1}{2}$ of $^1/_7$ x $^1/_5$

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<u>Converting fractions to decimals</u>

<u>Rational fractions</u>

<u>Examples;</u>

Convert $^2/_5$ to a decimal

$$
\begin{array}{r}
0.4 \\
^2/_5 = 5\overline{\smash)2} \\
\cdot0 \\
20 \\
^-20 \\
\bullet\bullet
\end{array}
$$

$^2/_5 = \underline{\ 0.4\ }$

Change ¼ to a decimal

$$
\begin{array}{r}
0.25 \\
\frac{1}{4} = 4\overline{\smash)1} \\
\cdot0 \\
10 \\
^-8 \\
20 \\
^-20 \\
\bullet\bullet
\end{array}
$$

$\frac{1}{4} = \underline{0.25\ }$

Activity 1:8

Convert the following fractions to decimals.

a) $\frac{1}{2}$	b) $^1/_5$	c) $^4/_5$	d) $^2/_8$
e) $\frac{1}{4}$	f) $^2/_5$	g) $^3/_5$	h) $\frac{3}{4}$

<u>Operation on decimals</u>

<u>Addition and subtraction</u>

Addition and subtraction is done after arranging digits vertically in their correct place values.

Examples:

a) Add: 0.23 + 2.3

   Soln

$$\begin{array}{r} 2.3 \\ +\ 0.23 \\ \hline 2.53 \end{array}$$

b) Subtract: 7 – 0.34

   Soln

$$\begin{array}{r} 7.00 \\ -\ 0.34 \\ \hline 6.66 \end{array}$$

Activity 1:9

1. Add the following.
   a) 8.24 + 0.16
   b) 0.25 + 2.5
   c) 8 + 2.3 + 1.54
   d) 0.11 + 1.1 + 11
   e) 17.04 + 1.3

2. Subtract the following.
   a) 8 – 0.94
   b) 7.00 – 2.34
   c) 0.23 – 0.13
   d) 2.5 – 0.25
   e) 48.6 – 0.057

Activity 1 :10

Simplify the following.

a) 1.64 + 5 – 4.42
b) 9 – (2.31 + 1.69)
c) 2.71 – 1.88 + 7.24
d) 3.82 – 7.34 + 9.68
e) 2.78 – 6.06 + 10.77

<u>Multiplication of decimals</u>

<u>Note:</u> The product of decimals must reflect the number of decimal places in the question.

a) 0.4 x 0.3

$$\begin{array}{r} =\ 0.4 \\ x\ 0.3 \\ \hline 1.2 \\ +\ 00 \\ \hline 0.12 \end{array}$$

= 0.12

OR 0.4 x 0.3

$^4/_{10}$ x $^3/_{10}$

= $^{12}/_{100}$

= 0.12

Activity 1 : 11

Work out the following.

a) 0 . 17 x 0.3   b) 9.6 x 0.2

c) 0.36 x 0.4   d) 7.5 x 0.5

e) 1.2 x 0.5   f) 4.5 x 3.2

g) 3.21 x 0.4   h) 0.45 x 0.12

Division of decimals

(Decimals by whole numbers and whole numbers by decimals)

Examples:

a) Divide: $8 \div 0.2$

$$\frac{8}{1} \div \frac{2}{10}$$

$^4\frac{\cancel{8}}{1}$ x $\frac{10}{\cancel{2}_1}$

$\frac{4 \times 10}{}$

= 40

b) Work out: $0.32 \div 8$

$$\frac{32}{100} \div \frac{8}{1}$$

$^4\frac{\cancel{32}}{100}$ x $\frac{1}{\cancel{8}_1}$

$\frac{4 \times 1}{100}$

100

$= \, {}^4/_{100}$

= 0.04

Activity 1:12

Work out the following.

a) $10 \div 0.2$ b) $0.24 \div 8$ c) $5.6 \div 0.07$
d) $56 \div 0.7$ e) $0.3 \div 9$ f) $8.1 \div 0.027$

g) $27 \div 0.3$ h) $1.2 \div 48$ i) $9.6 \div 0.08$

j) $36 \div 0.12$ k) $0.06 \div 36$ l) $1.44 \div 0.12$

m) $55 \div 0.11$ n) $1.2 \div 0.6$ o) $19.6 \div 0.07$

Division and multiplication
Examples:

a) Simplify: $\frac{0.24 \times 0.2}{0.08}$

Soln

$\frac{0.24 \times 0.2}{0.08}$

$^{24}/_{100}$ x $^{2}/_{10}$ ÷ $^{8}/_{100}$

$^{24}/_{100}$ x $^{2}/_{10}$ x $^{100}/_{8}$

$\frac{24 \times 2}{10}$

$\frac{3 \times 2}{10}$

= 0.6

b) Work out: $\frac{1.44 \times 3.6}{0.12 \ \times 0.4}$

Soln

$\frac{1.44 \times 3.6}{0.12 \times 0.4}$

$\frac{1.44 \times 3.6 \times 1000}{0.12 \times 0.4 \times 1000}$

$\frac{144 \times 36}{12 \times 4}$

12 x 9

= 108

Activity 1:13

Work out the following.

a) $\frac{0.9 \times 0.8}{0.3}$ b) $\frac{0.72 \times 0.2}{0.036}$ c) $\frac{0.09 \times 0.6}{0.18}$

d) $\frac{0.72 \times 0.96}{0.014}$ e) $\frac{0.36 \times 0.4}{0.018}$ f) $\frac{4.5 \times 1.6}{4.8 \times 1.5}$

g) $\frac{9.6 \times 1.25}{4.8 \times 0.5}$ h) $\frac{2.4 \times 0.54}{0.36}$

Rounding off decimals
To round off is to estimate a number to the nearest value.
Note: In rounding off decimals, the decimal digits cancelled are not replaces with zeros.
Examples:

a) Round off 5.72 to the nearest whole number.

```
  O    Tth   Hth
  5  .  7̶    2̶
+ 1 ←┘
  6  .
```

Therefore; 5.72 ≈ 6

b) Round off 29 . 97 to the nearest tenths.

```
       T    O     Tth   Hth
       2    9  .  9    7̶
   +              1  ←⌐
       3    0  .  0
```

Therefore; 29.97 = 30.0

Zero after the decimal point represents the tenths place value required.

Activity 1:14

1. Round off the following to the nearest tenths.

   a) 1.32   b) 6.85   c) 2.41

   d) 7.96   e) 3.93   f) 5.49

   g) 8.54   h) 8.985

2. Round off to the nearest hundredths.

   a) 12.623   b) 20.841   c) 6.829

   d) 8.728   e) 7.936   f) 0.483

   g) 12.998   h) 3.452

3. Round off the following to the nearest whole number,

   a) 36.7   b) 0.736   c) 9.39

   d) 6.94   e) 142.83   f) 11.52

   g) 68.77   h) 68.259   i) 4.930

Square and square root of fractions and decimals

Square and square roots of fractions.

Examples:

1. Find the square of;

   a) $^2/_9$ b) $1\,^1/_5$

   Square of $^2/_9$ $^6/_5$

   $= \,^2/_9 \times \,^2/_9$ square $= \,^6/_5 \times \,^6/_5$

   $= \,^4/_{81}$ $= \,^{36}/_{25}$

2. Find the square root of $^1/_9$

   $\sqrt{^1/_9}$

   $= \sqrt{\dfrac{1 \times 1}{3 \times 3}}$

   $= \dfrac{1}{3}$

$\sqrt{36}/81$

= $\dfrac{\sqrt{36}}{\sqrt{81}}$

= $\dfrac{6}{9}$

3	81
3	27
3	9
3	3
	1

3 x 3 x 3 x 3

3 x 3

9

2	36
2	18
3	9
3	3
	1

2 x 2 x 3 x 3

2 x 3

6

Activity 1:15

1. Find the square of the following.
   - a) $^1/_6$
   - b) ¾
   - c) $^3/_8$
   - d) 1 ½
   - e) ½
   - f) 4 ½
   - g) $^2/_3$
   - h) $^4/_5$
   - i) $^1/_5$
   - j) 1 $^2/_3$
   - k) 3 $^2/_3$
   - l) 5 $^2/_3$

2. Find the square root of the following.
   - a) ¼
   - b) $^9/_{16}$
   - c) $^4/_{25}$
   - d) 6 ¼
   - e) 7 $^1/_9$
   - f) 5 $^1/_{16}$
   - g) $^4/_9$
   - h) $^9/_{25}$
   - i) $^{36}/_{49}$
   - j) 1 $^9/_{16}$
   - k) 1 $^{11}/_{25}$
   - l) 5 $^4/_9$

3. Find the area of the figure.

3 ½ cm

4. Find the area if a square land whose side is 4 ¼ km.
5. The area of a square is 144cm². Find one side of that square.
6. A square land has an area of 3 $^6/_{25}$ m². How long is one side of that land?

Square and square roots of decimals

Examples

1. Find the square of 0.3

   Soln

   Sq. of 0.3 = $(0.3)^2$

   = 0.3 x 0.3

   = $^3/_{10}$ x $^3/_{10}$

   = $^9/_{100}$

   = 0.09

2. Find the square root of 0.25

   Soln

   $\sqrt{0.25}$ = $\sqrt{\dfrac{25}{100}}$

   $\dfrac{\sqrt{25}}{100}$

   = $^5/_{10}$

   = 0.5

90

Activity 1 :16

1. Find the square of the following.
   a) 01
   b) 1.10
   c) 0.18
   d) 0.72
   e) 0.5
   f) 0.12
   g) 0.34
   h) 1.2
   i) 0.7
   j) 0.16
   k) 0.42
   l) 2.5

2. Calculate the square root of the following.
   a) 0.09
   b) 0.81
   c) 2.25
   d) 0.0025
   e) 0.49
   f) 1.44
   g) 0.04
   h) 0.0064
   i) 0.64
   j) 3.24
   k) 0.0016
   l) 0.0169

Converting fractions to recurring decimals

Note: Fractions are easily changed to decimals by use of long division.

Examples:

1. Change $^5/_9$ to a decimal.
   Soln:

$$
\begin{array}{r}
0.55 \\
9\,\overline{|\,5} \\
-0 \\
\overline{50} \\
-\,45 \\
\overline{50} \\
-\,45 \\
\overline{5}
\end{array}
$$

   Therefore; $^5/_9$ = 0.55 …

2. Convert $^1/_6$ to a decimal

$$
\begin{array}{r}
0.166 \\
6\,\overline{|\,1} \\
-0 \\
\overline{10} \\
-6 \\
\overline{40} \\
-36 \\
\overline{40} \\
-36 \\
\overline{4}
\end{array}
$$

   Therefore $\dfrac{1}{6}$ = 0.166 ….

Activity 1:17

Express each of the following fractions as a decimal.

a) $1/3$ b) $3/11$ c) $4/11$ d) $5/7$

e) $2/3$ f) $2/9$ g) $1/11$ h) $1/7$

i) $1/9$ j) $5/11$

Converting recurring decimals to fractions

Note:

- Recurring decimals are of two types.
- Pure recurring decimals whose digits all repeat. E.g:0.22...
- Mixed recurring decimals whose digits don't all repeat. E.g: 0.122...

Examples:

a) 0.77...

Let p be the fraction be p

p = 0.77...

10p = 7.7...

We subtract

10p = 7.7

- p = 0.7

9p   = 7.0

$\cancel{9}$p   = 7

$\cancel{9}$ 9

p   = $7/9$

b) 0.1515...

Let the fraction be y

y = 0.1515...

10y = 1.515...

100y   = 15.15

100y   = 15.$\cancel{15}$

- y   = 0.$\cancel{15}$

99y   = 15

$\cancel{99}$y   = 15

$\cancel{99}$ 99

y   = $5/33$

c) 0.233...

Let the fraction

x   = 0.233... ___1

10x = 2.33... ___2

100x = 23.3... ___$\cancel{3}$

100x = 23.$\cancel{3}$

- 10x   2.3

90x   = 21

$\cancel{90}$x   = 2$\cancel{1}$⁷

$\cancel{90}$ $\cancel{90}$³⁰

x = $7/30$

Activity 1:18

Convert the following fractions as common fractions

a) 0.33...   b) 0.1212...   c) 0.7272...

d) 0.153153...   e) 0.55...   f) 0.2424...

g) 0.3636...   h) 0.133...   i) 0.255...

j) 0.66...   i) 0.1010...   j) 0.123123...

k) 0.255...   l) 0.11...   m) 0.2121...

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Application of fractions

1. ¾ of my father's age is 36 years. How old is my father?

   Let the father's age be y

   ¾ of y $\qquad$ = 36 yrs

   ¾ x y $\qquad$ = 36yrs

   4 x $\dfrac{3y}{4}$ $\qquad$ = 36 yrs x 4

   $\dfrac{\cancel{3}y}{\cancel{3}y}$ $\quad$ = $\dfrac{^{12}\cancel{36}\text{ yrs x 4}}{3}$

   y $\qquad$ = 12yrs x 4

   y $\qquad$ = 48yrs

   Therefore father's age is 48years

2. $^1/_3$ of my age is equal to ½ of John's age. If John is 24 yrs old. How old am I?

   Let my age be m.

   $^1/_3$ of m = ½ of 24

   $^1/_3$m $\qquad$ = (½ x 24)

   $^1/_3$m $\qquad$ = 12

   $\cancel{3}$ x $^1/_{\cancel{3}}$ m $\quad$ = 12 x 3

   m $\qquad$ = 12 x 3

   m = 36yrs

3. On Kahima's farm, ½ of the animals are cows, $^1/_3$ of the remaining animals are goats. If the rest 12 animals are pigs, how many animals are on the farm?

   Soln

   Whole farm $\qquad\qquad$ = 1

   Cows $\qquad\qquad\qquad$ = ½

   Remaining animals $\quad$ = 1 – ½

   $\qquad\qquad\qquad\qquad$ = ½

   Goats $\qquad\qquad\qquad$ = $^1/_3$ of rem

   $\qquad\qquad\qquad\qquad$ = $^1/_3$ of ½

   $\qquad\qquad\qquad\qquad$ = $^1/_3$ x ½

   $\qquad\qquad\qquad\qquad$ = $^1/_6$

   Total (cows + goats)

   ½ + $^1/_6$

   $\dfrac{3 + 1}{6}$

$^4/_6$

Fraction for pigs = $^6/_6 - ^4/_6$

                    = $^2/_6$

              Or $^1/_3$

$^1/_3$ is equivalent to 12 pigs

Let the total be y.

$^1/_3$ of y   = 12 animals

$\cancel{3}$ x $^1/_{\cancel{3}}$ y   = (12 x 3) animals

y   = (12 x 3) animals

y   = 36 animals

Therefore; there are 36 animals on Kahima's farm.

Activity 1:19

a) $^2/_3$ of my salary is 6,000/=. What is my salary?

b) ¼ of my grandmother's age is 20yrs. How old is my grandmother?

c) ¾ of the Askari's salary is 3,000/=. How much does he get?

d) $^2/_5$ of Kadodi's weight is 10kgs. Find Kadodi's weight?

e) $^1/_3$ of the distance from Iganga to Jinja is equal to ¼ of the distance from Jinja to Kampala. If it is 80km from Jinja to Kampala, how far is Iganga to Jinja?

f) $^1/_3$ of my salary is equal to $^5/_6$ of Musa's salary. If my salary is 12000/=, what is Musa's salary?

g) $^1/_6$ of the area of a triangle is $^7/_{12}$ that of the area of the rectangle. If the area of the triangle is 420m². Find the area of the rectangle.

h) John spends ¼ of the salary on food and ¼ of the remainder on rent and is left with 600/=. What is the salary?

i) A teacher spends ½ of the salary on rent and ¼ of the remainder on fees and is left with sh. 8000. Calculate his salary.

j) ½ of the pupils in a school like maths and $^2/_5$ of the remainder like English. If there remains 180pupils who like other subjects, how many pupils are there in the school?

Ratios and proportions

- Ratios are comparisons between two or more quantities by division.

- Ratios are always expressed in the simplest form.

Forming ratios

Examples

In a class, there are 24 girls and 18 boys. Express the number of boys as a ratio of girls.

Soln:

Ratio   Boys to Girls

         18 : 24 (divided by 6)

      3~~18~~ : ~~24~~ 4

       ~~6~~   ~~6~~

      3 :   **4**

      Girls to Boys

      24 :   18

      <u>24</u> :   <u>18</u>

      6   6

      4 :   **3**

Activity 1:20

1. In a music club, there are 12 singers and 8 instrumentalists.
   a) What is the ratio of singers to instrumentalists?
   b) Find the ratio of instrumentalists to singers.
2. There are 120 pupils in a school. If 40 are girls;
   a) Find the ratio of girls to boys.
   b) Find the ratio of boys to girls.
   c) Express the number of boys as a ratio of the whole class.
3. Express 500g as a ratio of 1 hour.
4. Express 500g as a ratio of 1 kg.
5. In a village, 40 farmers grow beans, 30 grow maize and 60 grow cabbages.
   a) Find the ratio of farmers who grow maize to those who grow cabbages.
   b) Find the ratio of those who grow maize to the total number of farmers.
   c) What is the ratio of those who grow maize to those who grow beans and those who growing cabbages?

Increasing and decreasing in ratios

Increasing

For ratio increase, the new amount is bigger than the old amount.

Example

Increase 300/= in the ratio 5 : 2

New   old

5   2

?   300/=

2 parts   = 300/=

1 part   = $\frac{300}{2}$

3 parts   = $(^{300}/_2 \times 5)$/=

               150 x 5

             750/=

OR: $\underline{New}$ x Amount
     Old

    $\frac{5}{\cancel{2}}$ x $\cancel{300}$/=

    5 x 150/=

    750/=

Decreasing

For ratio decrease, the new amount is smaller.

Example I

Decrease 400kg in the ratio 3:4

New   Old

3   4

?   400kg

4 parts   = 400kg

1 part   = $\frac{400kg}{4}$

3 parts   = $(^{400}/_4 \times 3)$kg

             = (100 x 3)kg

             300kg

OR: $\underline{New}$ x Amount
      Old

$$\frac{3}{4} \text{ x } \cancel{400}\text{kg}$$

3 x 100kg

300kg

Example II

A class had 60 pupils, the number reduced to 50 pupils. In what ratio did it reduce?

Old number = 60

New number = 50

New : Old

50   :   60

5   :   6

Activity 1:21

1. Increase 300 in the ratio 2:1
2. Increase 600 in the ratio 3:2
3. Increase 4800/= in the ratio 5:4
4. Increase 360 in a ratio 5:4
5. The school fees was 18,000/- and increase in the ratio 11:10. What is the new amount of school fees?
6. The school had 800 pupils last year. This year they have 1000 pupils. In what ratio has the number increased?
7. What is the ratio increase from 800 to 960?
8. Decrease the following in their respective ratios.
    a) 600 in ratio 2:3
    b) 400kg in the ratio 2:5
    c) 1000/= in the ratio 3:5
    d) 800litres in the ratio5:8
    e) 700bags in the ratio 7:10
9. A class had 72 pupils; the number reduced in the ratio 7:8. What is the new number of pupils?
10. The marked price of a radio is 90,000/=. The man bought a radio at a reduced price of the ratio of 2:3. How much did he buy the radio?

Sharing in ratios

Examples:

1. Divide 4200kgs of sugar in the ratio of 2:5

   Ratio = 2:5

   Total = 2 +5

   = 7

   1st share = $\frac{2}{7} \times \overset{600}{\cancel{4200}}$kg   2nd share = $\frac{5}{7} \times \overset{600}{\cancel{4200}}$kg

   = 2 x 600kg   = 5 x 600kg

   = 1200kg   = 3000kg

2. Share 200/= in the ratio 2:3

   All parts = 2 + 3

   = 5 parts

   5 parts = 200/=

   1 part = 200

   2 parts = $\frac{(200 \times 2)}{5}$/=

   = 40 x 2

   80/=

   3 parts = $\frac{(200 \times 3)}{5}$/=

   = (40 x 3)/=

   120/=

   Therefore, the shares are 80/= and 120/=

Activity 1:22

1. Share 360 in the ratio 2:3
2. Divide 72 in the ratio of 5:3
3. Divide 4500/= in the ratio 7:8
4. Share 90kg of sugar between two people in the ratio of 7:3
5. John and Diana shared 3000/= in the ratio 2:3. How much did each get?
6. A man and his wife had 200kg of coffee. They decided to share it in the ratio 7:3 respectively.
   a) How many kilograms did the man get?
   b) How many kilograms did the wife get?

7. Amos, Andrew and Allan shared 24,000/= in the ratio 1:2:3 respectively. How much did each get?
8. A B and C contributed money for a business in the ratio 3:4:5. If C contributed 10,000/=. How much did they contribute altogether?
9. Share 480 in the ratio 4:5
10. Dan and Mike shared money in ratio 3:5 respectively. If Mike got 3000/=,
     how much did Dan get?

Proportions

There are three types of proportions.
- Simple direct proportions
- Inverse proportions
- Constant proportions

Simple proportions

In simple proportions, the more the number of items, the more the amount.

Examples

1. 4 girls have 8 breasts. How many breasts have 7 girls?

   Soln:

4 girls	=	8 breasts
1 girl	=	($8/4$) breasts
7 girls	=	($8/4$ x 7) breasts
	=	(2 x 7) breasts
	=	**14 breast**

2. 4 pen costs 2000/=. Find the cost of 9 pens.

   Soln:

4 pens	=	2000/=
1 pen	=	($2000/4$)/=
9 pens	=	($2000/4$ x 9)/=
9 pens	=	500 x 9
	=	**4500/=**

Activity 1:23

1. One book costs 600/=. Find the cost of 5 books.
2. 2 bags weigh 70kg. What is the weight of 5 bags?
3. 1200/= can buy 2kg of maize flour. How many kgs can you get from 3600/=

4. A bag of coffee weighs 65kg. How many kilograms will 12 bags weigh?
5. 3 dresses can be made from 6 metres of cloth. How many metres of cloth can I use for 9 dresses?
6. 5 pieces of timber are used to make 2 tables. How many tables can you make from 15 pieces of timber?
7. The bus fare for 3 people is 25000/=. What is the fare for 2 people?
8. 5 jerrycans cost 75000/=. How many jerrycans can one buy with 105,000/=?

Inverse proportions

In this type of proportion, the more the number employed to do work, the less the time they will take and vice versa.

Examples

1. 3 men can do a piece of work in 6 days. How long will 9 men take to do the same piece of work?
   Soln:

   3 men   =   6days
   1 man   =   (3 x 6) days
   9 men   = $\dfrac{(3 \times 6)}{9}$ days

   9 men   = $\dfrac{(18)}{9}$ days

   9 men   **=   2 days**

   Therefore; 9 men can do the same piece of work in 2 days.

Activity 1: 24

1. It takes 4 days for 12 women to dig a shamba. How long will it take 8 women to do the same job?
2. 25 girls can construct a road in 8 days. How many girls will construct a road in 10 days?
3. A carpenter takes 2 hrs to make a chair. How many chairs will be made in 6 hours?
4. 12 technicians can paint a school building in 10 days. How long will 15 technicians take?
5. 6 porters can dig a piece of land in 5 days.
   a) How many porters can do the same work in 10 days?
   b) How many days will 15 porters take?
6. 5 children take 4 days to slash the school compound. How many days will 10 children take?

Constant proportions

This is a type of proportion whereby time taken to complete a task remains the same though the number of parties change.

Examples:

1. 6 men can sing a song in 10 minutes. How long will 10 men take to sing the same song?

2. 5 shirts take 20 minutes to dry. How long will 20 shirts take to dry?

Percentages

- Percent means out of 100.
- The symbol for percent is %

That is;

$6\% = {}^6/_{100}$

$20\% = {}^{20}/_{100}$

Changing percentages to fractions and vice versa

Examples

1. Express 25% as a fraction in its lowest terms.

$25\% = \dfrac{25}{100}$

$= \dfrac{\cancel{5}^1 \times \cancel{5}^1}{2 \times 2 \times \cancel{5}_1 \times \cancel{5}_1}$

$= \dfrac{1}{4}$

2. Express 12 ½ % as a fraction.

$12\frac{1}{2} = \dfrac{25\%}{2}$

$= \dfrac{{}^{25}/_2}{100}$

$= \dfrac{25}{2} \div \dfrac{100}{1}$

$= \dfrac{25}{2} \times \dfrac{1}{100}$

$= \dfrac{25}{200}$

$= {}^1/_8$

3. Express ⁴/₅ as a %

${}^4/_5$

101

$\dfrac{4}{5}$ x $\cancel{100}^{20}$

$= (4 \times 20)\%$

= 80%

4. Convert 0.25 as a percentage.

0.25

$= \dfrac{25}{100}$

$= 25 \times \dfrac{\cancel{100}^{1}}{\cancel{100}_{1}}$

$= 25 \times 1$

= 25%

5. Change 20% as a decimal.

20%

$= \dfrac{20}{100}$

$= \dfrac{2}{10}$

= 0.2

Activity 1:25

1. Convert the following percentages to fractions in their lowest terms.

   a) 50%   b) 30%   c) 21%   d) 33 ½ %
   e) 12%   f) 40%   g) 65%   h) 90%
   i) 62 ½%   j) 16 $^2/_3$%   k) 37 ½%

2. Change the following fractions to percentages.

   a) ½   b) $^7/_{20}$ c) $^3/_{20}$ d) $^3/_8$
   e) ¼   f) $^{12}/_{25}$ g) $^2/_{25}$ h) $^5/_8$
   i) $^9/_{20}$ j) $^4/_{10}$ k) $^{13}/_{50}$ l) $^2/_3$

3. Change the decimals to percentages.

   a) 0.7   b) 0.45   c) 0.003   d) 0.9
   e) 0.001   f) 0.12   g) 0.75   h) 0.025
   i) 0.067

4. Change these percentages to decimals.

   a) 10%   b) 32%   c) 2.5%   d) 75%

   e) 25%   f) 82%   g) 1.2%   h) 62%

   i)   30%   j) 6 ¼%   k) 12 ¼%

Expressing percentages as ratios
Examples

1. 2%

$$= \frac{\cancel{2}^{1}}{\cancel{100}_{50}}$$

$$= \frac{1}{50}$$

= 1:50

2. $33\,^{1}/_{3}\%$

$$= \frac{100\%}{3}$$

$$= \frac{100}{3} \div \frac{100}{1}$$

$$= \frac{\cancel{100}^{1}}{3} \times \frac{1}{\cancel{100}_{1}}$$

$$= \frac{1}{3}$$

= 1:3

Express as percentages
Examples

1. 3:10

$$= \frac{3}{\cancel{10}_{1}} \times \cancel{100}^{10}$$

$$= 3 \times 10$$

$$= 30\%$$

2. $^{1}/_{5} : \,^{1}/_{3}$

$$= \frac{1}{5} \div \frac{1}{3}$$

$$= \frac{1}{5} \times \frac{3}{1}$$

$$= \frac{3}{5}$$

$= \underline{3} \ \times \ \cancel{100}\!-\!20$
$\quad \cancel{5}\ 1$

$= 3 \times 20$

$= 60\%$

Activity 1:26

1. Express the following percentages as ratios

   a) 10% b) 56% c) 125%
   d) 25% e) 76% f) 80%
   g) 50% h) 98% i) 144%

2. Express the following ratios as percentages.

   a) 1:2 b) 3:5 c) 5:16
   d) 2:3 e) 3:8 f) 3:4
   g) 2:5 h) $^1/_5 : \frac{1}{4}$ i) $^1/_8 : ^1/_5$
   j) $^1/_{20} : ^1/_7$ k) $^2/_5 : \frac{1}{2}$ l) $\frac{3}{4} : ^1/_5$

Activity 1:27

1) A child paid 55% of the school fees. What percentage is left for him to pay?

2) A buyer paid 85% of the cost of a radio. What percentage is left for him to pay?

3) 30% of the people in Uganda are male, 50% are female and the rest are children. What percentage is for children?

4) Henry had 40 cows, he sold 15.
   i)   What percentage was sold?
   ii)   What percentage of the cows was not sold?

5. A boy got 8 marks out of 20. What percentage is this?

6. If 40 out of 120 pupils in a class passed their exams.
   i)   What percentage of the pupils passed?
   ii)   What percentage of the pupils failed?

7. Write 40 as a percentage of 200.

8. Express 200gm as a percentage of 1 kg.

9. What percentage of 1 hour is 150 minutes?

10. Express 60 as a percentage of 80.

Percentage of numbers

Examples

1. What is 20% of 200/=?

$$\frac{20}{\cancel{100}_{\ 1}} \times \cancel{200}^{\ 2}/=$$

$(20 \times 2)/=$

40/=

2. What is 12 ½ of 800 people?

25 % of 800 people

$$\left[\frac{25}{2} \div \frac{100}{1} \right] \text{of } 800$$

$$\frac{25}{\cancel{2}_{\ 1}} \times \frac{1}{\cancel{100}_{\ 1}} \times \cancel{800}^{\ 4}$$

25×4

$= 100$ people

Activity 1:28

What is;

    a) 15% of 240 books?

    b) 20% of 400 kgs?

    c) 11% of 8000 books?

    d) 60% of 2400 cows?

    e) 25% of 1200/=?

    f) 12 ½ of 3200 kg?

    g) 90% of 360cm?

    h) 80% of 1500?

<u>Application of percentages</u>

Activity 1:29

1. In a class 10% of the pupils are absent. If there are 60 pupils.

    a) How many pupils are absent?

    b) How many are present?

2. There are 280 animals in the zoo. 10% are birds, 50% are primates and 40% consists of others.

    How many animals of each type named are in the zoo?

3. A school has 360 books. 30% are English books, 20% are science books and the rest are maths books

    How many books of each category are in the school?

4. 20 % of the pupils in a school are girls. If there are 35 girls in the school. How many pupils are in the school?

5. 10% of a number is 40. What is the number?
6. 25% of a number is 80. What is the number?

Percentage increase and decrease

Examples

1. Increase 200/= by 20%

    Get 20% of 200/=

    $\dfrac{20 \times \cancel{200}^{2}}{\cancel{100}_{1}}$/=

    (20×2)/=

    40/=

    New amount = 200/=

    $\begin{array}{r} + 40/= \\ \hline \mathbf{240/=} \\ \hline \end{array}$

    OR

    100%   = 200/=

    1%   = $\dfrac{200/=}{100}$

    100% + 20%   = $\left[\dfrac{\cancel{200}^{2} \times 120}{\cancel{100}_{1}}\right]$/=

                   = 2 x 120/=

                          240/=

2. Decrease 300 by 10%

    $\dfrac{10 \times \cancel{300}^{3}}{\cancel{100}_{1}}$

    10 x 3

    30

    New amount   = 300

    $\begin{array}{r} - 30 \\ \hline \mathbf{270} \\ \hline \end{array}$

    OR

    100% - 10 = 90%

    $\dfrac{90 \times \cancel{300}^{3}}{\cancel{100}_{1}}$

    90 x 3

    270

Activity 1:30

1. Increase 80 by 10%
2. Increase 240/= by 20%
3. Increase 400 by 15%
4. Increase 15000 by 30%
5. Okidi's pay was 1200 dollars. It was increased by 30%. What is his new pay?
6. The number of pupils in a class were 50 but they increased by 10%. What is the new number?
7. A shirt was priced at 9000/= last year. This year its price increased by 40%. What is the new price of the shirt?
8. Last year, there were 30,000 cars in Uganda. This year there are 20% more cars imported. How many cars are in Uganda this year?
9. Decrease 400 by 20%
10. Decrease 500kg by 10%
11. Reduce 80kg by 20%

Profit and Loss

- Profit is realized when the selling price of a commodity is more than the buying price.
- Loss is suffered when one sells a commodity at a lower price than it was bought.

Formulae

Profit   = Selling Price (S.P) – Buying Price (B.P)

Loss   = Buying Price (B.P) – Selling Price (S.P)

Examples

1. Nakajjumbe bought a phone at 80,000/= and sold it at 110,000/=. What profit did he make?

   Soln:

   Buying price   = 80,000/=

   Selling price   = 110,000/=

   Profit   = S.P – B.P

                   = 110,000/= - 80,000/=

                   = 30,000/=

2. Makonde bought a pair of trouser at 19,000/= and later sold it at 15,500/=. What loss did he suffer?

Buying price   = 19,000/=

Selling price   = 15,500/=

Loss   = Buying price – Selling price

= 19,000/= - 15,500/=

= 3,500/=

Activity
1. A cattle dealer bought a cow at 135,000/= and sold it at 147,000/=. Calculate the profit he made.
2. A land agent bought a piece of land at 750,000/= and sold it at 870,000/=. Find the profit made.
3. A trader bought 50kg of maize flour at 600/= per kg and sold it at 800/= per kg.
   a) Find the profit per kilogram.
   b) Find the total amount of money used to buy all the maize flour.
   c) Calculate the total amount earned as profit.
4. A soda agent bought 50 crates of soda at 23,250 each and sold at sh. 24,250 each.
   a) Calculate the profit made on each crate.
   b) Calculate the total profit made.
5. Akram bought a radio at 75,000/= and later sold it at 69,000/=. Calculate the loss he made.
6. A business lady bought a box of cosmetics at 240,000/= and collected 226,000/=. Calculate the loss.

Percentage profit and loss
Formulars

Percentage profit   = $\underline{\text{Selling price – Buying price}}$ x 100
                                        Buying price

OR: $\underline{\text{Profit}}$ x 100
              Buying price

Percentage loss   = $\underline{\text{Buying price – Selling price}}$ x 100

OR: $\dfrac{\text{Loss}}{\text{Buying price}}$ x 100

Examples

1. A boy bought a bicycle at 120,000/= and sold it at 130,000/=.
    a) Express the profit as a fraction of the buying price.

    Fraction   = $\dfrac{\text{Profit}}{\text{Buying price}}$

                      = $\dfrac{130,000/= - 120,000 /=}{120,000}$

                      = $\dfrac{10,000/=}{120,000/=}$

                      = ¹/₁₂

    b) Calculate the percentage profit.

    %age profit   = $\dfrac{\text{profit}}{\text{B.P}}$ x 100

                    = $\dfrac{10,000/=}{120,000/=}$ x ~~100~~

                    = $\dfrac{\cancel{100}\ ^{25}}{\cancel{12}\ _3}$

                    = $\dfrac{25}{3}$

                    = 8 ¹/₃%

2. Obiina bought a bicycle at sh. 70,000. Two years later, he sold it at a loss of 15%.
    a) Calculate the loss.

    %age loss   = $\dfrac{\text{loss}}{\text{B.P}}$ x 100

    $\dfrac{15}{1}$ = $\dfrac{\text{Loss}}{70000}$ x 100

    $\dfrac{\cancel{100}\,\text{loss}}{\cancel{100}}$ = $\dfrac{15 \times 700\cancel{00}}{\cancel{100}}$

    Loss   = 15 x 700

                  = 10500/=

Activity

1. Namuwaya bought a dress at 10,000/= and sold it at 12,000/=. Calculate the percentage profit.
2. Okello bought a blanket at 30,000/= and sold it at 350,000/=.
   a) Express his profit as a fraction of the cost price.
   b) Find his percentage profit.
3. A business man bought a 50 kg of G-nuts at 800/= per kilogram. He paid 5000/= as transport. If he sold each kg at 1000/=.
   a) Find the total amount collected from the sales.
   b) Find his profit.
   c) Calculate the percentage profit.
4. A passenger bought an air ticket at 500,000/=, he later sold it at 550,000/=.
   a) Express the profit as a fraction of the cost price.
   b) Calculate the passenger's percentage profit.
5. A marial vendor bought 20 bunches of matooke at sh. 8000/=. She sold them at a loss of 500/= per bunch.
   a) Find the amount got from the sales.
   b) Find the total loss.
   c) Calculate the percentage loss.
6. After selling a bed at 60,000/=. Mulonde made a percentage profit of 20%.
   a) Find the price at which Mulonde sold the bed.
   b) Find Mulonde's profit.

Simple Interest
In banking
- The money banked, borrowed or lent is called **Principal (P)**
- The percentage used to calculate interest is called **Rate (R).**
- The period that the principal is invested is called **Time** given in years or months.
- The additional amount paid back is **Interest**
- Total Amount = Principal + Interest
  Simple interest = P X R X T

Examples

1. My father deposited sh. 120,000 in the bank that offers an interest rate at 10% per year.
   a) Calculate the interest got after 2 years.

   $S I = P \times T \times R$

   $= 120000 \times 2 \times 10/100$

   $= 1200 \times 10 \times 2$

   = 24,000/=

   b) Calculate the amount collected after 2 years.

   Amount = principal + Interest

   $= 120,000/= + 24,000/=$

   = 144,000/=

2. A trader borrowed 400,000 from a bank at an interest rate of 5% per annum for 6 months.
   a) Calculate the simple interest.

   $S I = P \times T \times R$

   $= 40,000 \times {}^6/_{12} \times {}^5/_{100}$

   $= (100 \times 2 \times 5)/=$

   = 10,000/=

   b) What amount will the trader pay altogether?

   Amount = Principal + Interest

   $= 400,000 + 10,000$

   = 410,000/=

Activity

1. Calculate the simple interest on 50,000/= at a rate of 15% per year for 2 years.
2. Calculate the simple interest on 150,000/+ at 5% per annum for 3 years.
3. What interest is paid on a loan of 70,000/= at a rate of 20% per annum for 2 years?
4. A school kept 800,000/= in a bank at a rate of 15% per year for 1 ½ years.
   Calculate the simple interest.

Graphs

1. The Headteacher gave out balls to the classes to make practices for football.

   Scale: (⊕) = 10 balls

P.4	⊕ ⊕ ⊕
P.5	⊕ ⊕ ⊕ ⊕ ⊕
P.6	⊕ ⊕
P.7	⊕ ⊕ ⊕ ⊕

   a) Which class got the least number of balls?
   b) How many less balls did P.6 get than P.7?
   c) Find the total number of balls given to all the classes.
   d) If each ball was costing sh. 20,000, how much money did the H/M pay for the balls of P.6.

2. In Maya village, five men planted three in order to trap strong winds.

Name	Number of trees
Moses	300
Peter	500
Paul	450
Chris	150

   a) Who planted the highest number of trees?
   b) How many trees did Paul and Chris plant altogether?
   c) Construct a pictograph for the information above.
   d) If each seedling costs sh. 300, how much did Chris spend?

Bar graphs

Definition:
   - These are graphs constructed using bar like structures.
   - The bars can be vertical or horizontal depending on the information of wish of the drawer.

Scales

- It has the vertical and horizontal scales. The scales used depends on the broadness of the quality.

Bars
- All bars should be proportional and bars are always shaded.

Example
The table below shows the number of pupils who like different food types.

Type of food	Rice	Matooke	Yams	Millet	Cassava
Number of pupils	8	6	8	4	10

We can put the data above in a bar graph.

Exercise

1. The bar graph below shows the number of pupils in different classes at Mother Mony p/s.

a) Which class has the highest number of pupils?
b) What is the total number of pupils in the classes?
c) Which two classes have the same number of pupils?
d) Find the average number of pupils in the school/per class.

2. Below is a bar graph showing the number of vehicles that park at different hotels per day.

Number of vehicles

i)   Find the total number of vehicles that park at Speke hotel and Mando Bado.

ii) If each vehicle pays sh. 2000 for parking, how much money is collected at Equatorial hotel?

iii) Which Hotel is the least busy?

iv) How many more vehicles park at Africana that Speke hotel?

3. The table below shows the number of time each type of food is prepared in a term at a school.

Food	Matooke	Posho	Rice	Cassava	Millet
Number of times	10	20	30	20	25

a) Which food type is prepared 30 times?

b) Which two food types are prepared the same times?

c) Find the total cost matooke takes in a term if each time the Headteacher spends 400,000/=

d) Draw a bar graph to show the above information.

Line graph

- This type of graph is interpreted using a straight line drawn on the graph. The line can also be zig-zag

- The graph has two scales i.e the horizontal and the vertical.

It is mostly used for;

- Distance against time used.

- Quantity of goods sold against cost.

- Temperature change against time.

Example:

A motorist covers 40km in 2 hrs.

Distance 20, 40 on vertical axis; 1, 2 Hours on horizontal axis.

Exercise

1. The graph below shows the number of litres of petrol consumed by a car through a distance.

a) How many kilometres does the car travel on 1 litre?
b) What distance can the car cover on 6 litres of petrol?
c) How many litres does the car need to cover 45km?

2. The weight of Nderema's children is shown on the graph below.

a) Who is the heaviest child?
b) Which of the children weighs 30kgs?
c) Which two children have the same weight?
d) Find the average weight of all the children.

3. The graph below shows the cost of units of different items in a supermarket.

a) What is the cost of 1kg of rice?
b) Find the cost of 2 ½ kg of maize flour.
c) How many kg of maize flour can I buy with 3000/=
d) What is the cost of ½ kg of Nido?
e) Find the total cost of 1 kg of Nido, 4 kg of rice and 3 kg of maize flour.

Pie – chart
- Pie - charts are also called circle graph.
- The chart is circular in form.

It constitutes one of the following;
a) 1 whole
b) 10%
c) 360^0

½ or 50% or 180^0

¼ or 25% 0r 90^0

Exercise

1. A man spends his salary as shown by the pie chart below. He earns 180,000/= per month.

a) Find the value of x.

$$x + 90^0 + 150^0 \qquad = 360^0$$
$$x + 240^0 \qquad\qquad = 360^0$$
$$x + 240^0 - 240^0 \qquad = 360^0 - 240^0$$
$$x \qquad\qquad\qquad = 120^0$$

b) How much does he spend on fees?

$$\frac{120^0}{360^0} \times 180,000$$
$$= 1 \times 60,000$$
$$= 60,000/=$$

c) Express the expenditure on food as a fraction of the total.

$$\frac{150^0}{360^0}$$
$$= \frac{5}{12}$$

2. The pie chart below shows the types of books in the school library of 1440 books

a) Find the value of y.
b) How many books are there for MTC?
c) How many more books are for English than Science?
d) What fraction of the book represents SST books?

3. Mrs Bogoyo spends her salary of sh. 72,000 as follows;

a) Find the percentage for rent.
b) How much does she spend on medical?
c) How much more does she spend on fees than on food?

Constructing pie charts

Steps:
- Change the amount given (in the sectors to degrees)
- Check and see if the degrees got add up to 360^0.
- Draw a circle (with a reasonable radius)
- Use the outer scale reading of the protractor and count the degrees clockwise.

Example
1. Alupo spends his money as follows.
¼ on food, $^1/_3$ on rent and $^5/_{12}$ on fees. Construct a pie chart of radius 3cm.
Changing fraction parts to degrees.

Food = ¼ x 36^0
      = **90^0**

Rent = $^1/_3$ x 360^0
      = **120^0**

Fees = $^5/_{12}$ x 360^0
      = **150^0**

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<u>Pie chart (3cm radius)</u>

<u>Exercise</u>

1. On a farm, $^1/_{10}$ of the animals are goats, $^1/_5$ are sheep, $^2/_5$ are cattle and $^3/_{10}$ are chicken. Use the information to draw a pie chart.
2. In a village meeting, 35% of the people were women, 40% men and the rest children.
    a) What percentage represented children.
    b) Construct a pie chart.
3. A man got sh. 60000 from the sales of beans, sh. 80000 from peas, sh. 50000 from tomatoes and sh. 60000 from others. Use the above information to draw a pie chart.
4. Mwebe's land is divided into plots as shown below.

Land for;	Sheep	Goats	Cows
Percentage	30%	50%	20%

If he had 270 hectares;

i)   How many hectares are for sheep?

ii)   Use the information to draw a pie chart.

<u>STATISTICS</u> (Mean, Median, Mode and Range)

Definition:

Mean: Is the total/sum of scores divided by the number of scores.

Median: Is the midway mark between the highest and lowest.

Mode: Is the score which appears more than others.

Range: the difference between the highest and lowest marks.

Frequency: The number of times an event occurs.

<u>Example:</u>

Given 9 , 2 , 6 , 3 and 4, Work out;

a) The mean

   Mean   = <u>Total of scores</u>

                     Number of scores

                = <u>9 + 2 + 6 + 5 + 6 + 3 + 4</u>

                              7

                = <u>35</u>

                     7

   Mean   = 5

b) The mode

Scores	freq
9	1
2	1
6	2
5	1
3	1
4	1

6 is the mode

c) The range

Range   = Highest – Lowest

                = 9 – 2

Range   = 7

d) Median

Median   = ②,③,④, 5 ,⑥,⑥,⑨

Median   = 5

Activity

1. Given that 18 , 12 , 6 , 24 , and 30. Find the;
   - i)   Mean
   - ii)   Range
2. Work out the median of 6 , 7 , 4 , 9 and 8.
3. Given 20, 4 , 8 , 8 , 4 , 7 and 14, find;
   - i)   The mode
   - ii)   The median
4. Okello scored the following marks in a series of maths tests; 55% , 60% , 40% 60% , 55% , 80% and 60%.
   - i) Find his modal mark.
   - ii)   Work out the average

5. Calculate the mean of 40 and 42.

MORE ABOUT STATISTICS

Examples

The table below shows a pupil's marks in a science test.

Scores	80	70	60	12	30	40
Frequency	1	2	3	1	2	5

Find;

a) The modal mark

   Modal mark was 40

b) The modal frequency

   Modal frequency was 5

c) The mean

$$\text{Mean} = \frac{80 + (70\times2) + (60\times3) + (30\times2) + (40\times5)}{14}$$

$$= \frac{80 + 140 + 180 + 60 + 12 + 200}{14}$$

$$= \frac{672}{14}$$

$$= 48$$

Activity

1. A group of boys was given a test. They scored as follows;

Scores	20	80	60	70
Number of boys				

   i) How many boys did the test?

   ii)   Find the modal mark

   iii)   Calculate the mean mark.

2. The following are the marks in their raw form.

   20, 15 , 10 , 35 , 40, 10 , 30 , 30 , 20 , 40 , 30 , and 40.

   i) Make a table to show the scores, tallies and frequency.

   ii)   What is the modal frequency?

   iii)   What is the mode?

   iv)   Find the range of scores.

MORE ABOUT AVERAGE

Examples

1. The mean age of 5 pupils is 14 yrs. The age of 4 of the pupils are 16 , 12 , 13 and 15.

   a) What is the age of the fifth pupil?

Total age of 5 pupils	= 5 x 14
	= 70 yrs

Total age of the 4 pupils	= 12 + 16 + 13 + 15
	= 56 yrs
Age of the fifth pupil	= 70 – 56
	= 14 years

   b) Find the median.

   Ages in order   = 12 , 13 , ⑭ , 15 , 16

   Median age   = 14 years

2. The average weight of 4 men is 55kg. If one of the men weighs 70kg, what is the average weight of the other 3 men?

Total weight of all (4 x 55)	= 220kg
Weight of one man	- 70kg
Weight of 3 remaining men	= 150kg
Average weight for the 3 men	= 150
	3
	= 50 kg

Exercise

1. The mean age of 4 girls is 16 years. If three of them are aged 17, 12 and 15, find the age of the 4th girl.
2. The average number of books for 4 classes is 20. If one class has 23 books, what is the average number of books does the other class have?
3. The average of 3 numbers is 15. Of which one of them is 21, find the average of the other 2 numbers.
4. The average length of 8 sticks is 9cm. If three sticks of them measure 6cm, 10cm and 6cm, find the average length of the remaining 5 sticks.

PROBABILITY

Probability is the measure of chances for an even to occur.

Some things happen   -   certain
 Some things don't ever happen   -   impossible
Some things may happen sometimes   likely

The probability scale

0%   50%   75%
            100%

|————————————————|————————————————||————————————————————|————————————————|

0 very unlikely   ¼ unlikely   ½ likely   ¾
very
 (impossible)
likely(certain)

If something can't happen then the probability is 0.
If something can happen certainly, then the probability is 1.
Therefore: $0 \leq p(x) \leq 1$
Probability = Desired chance
               Sample space

Example

1. Amos has five cards numbered; {3 , 4 , 5 , 6 , 7 }
    If the cards are mixed and put in a box, what is the probability that he randomly chooses;
    a) a card marked 5?
        Prob (5)   = $1/5$

    b) a card of an even number?
        Even numbers {4 , 6 }
        Prob (even) = $2/5$

    c) a counting number?
        Counting numbers = {3 , 4 , 5 , 6 , 7}
        Prob (counting numbers) = $5/5$ = 1

Exercise

1. A die is tossed once. What is the probability that;
   a) An odd number appears on top.
   b) A number less than 4 appears on top.

2. Okello will visit his mother next week. What is the probability that he will visit her on;
   a) a day that begins with letter "T"
   b) Sunday
   c) on a day that ends with letter "Y"

3. 4 red pens and 6 blue pens are mixed and put in a box. If a pen is picked at random, what is the probability that it is;
   a) a red pen
   b) blue pen
   c) black pen

4. Letter card are as below.

$$\text{F} \quad \text{L} \quad \text{O} \quad \text{W} \quad \text{E} \quad \text{R}$$

   If the cards are mixed and put in a bucket, what is the probability of picking at random;
   a) a vowel
   b) a consonant

MEASURES (TIME)

Note: 1 hr   = 60mins

1 min   = 60sec

1 hr   = (60 x 60) sec

= 3600 seconds

Changing hours to minutes

Example

Change 4 hrs to minutes.

1 hr   = 60 mins

4 hrs   = (4 x 60) mins

= 240mins

= 240 minutes

__Activity__

Change the following hours to minutes.

a) 3 hrs   b) 11 hrs   c) 4 ½ hrs

d) 6 hrs   e) 1 ½ hrs   f) 20 hrs

<u>Changing minutes to seconds</u>

Example

Change 5 minutes to seconds

1 min   = 60 seconds

5 mins = (5 x 60) seconds

        = <u>300 seconds</u>

__Activity__

Change the following minutes to seconds

a) 20 minutes   b) 30 minutes   c) 15 minutes

d) 72 minutes   e) 60 minutes   f) 25 minutes

g) 10 minutes   h) 3 minutes   i) 50 minutes

<u>Changing hours to seconds</u>

__Example__

How many seconds are there in 1 hr.

1 hr   = 60 mins

1 min   = 60 seconds

Therefore: 60 mins   = (60 x 60)

                        = 3600 seconds

__Activity__

Change the following hours to seconds.

a) ½ hrs   b) 4 hrs   c) 6 hrs

d) 5 hrs   e) 20 hrs   f) ¼ hrs

g) 3 ½ hrs   h) ¾ hrs   i) $^1/_{10}$ hrs

MEASURES (DISTANCE, TIME & SPEED)

Distance, Time and speed go can be realized concurrently.

ITEM	UNITS
Distance	km, metres
Time	hours, minutes , seconds
Speed (the rate of moving)	m/sec , km/hr

Examples

1. A car moves 40km every hour. Find its average speed.

   Its speed = 40km/hr
2. A tax covers 40km in 30 mins

   Therefore; 40km take = 30m

                80km take = (30 +30) min

   Therefore its speed = 80km/hr

Finding distance

Distance = speed x time

Example

Find the distance covered by a cyclist at 14km/hr for 3 hrs.

Distance = speed x time

        = $\frac{14km}{hr}$ x 3 hrs

        = (14 x 3)

        = 42 km

Activity

1. A bus moved at a speed of 60km/hr for 4 hrs. What distance did it cover?
2. The speed of a train is 20km/hr. What distance does it cover in 5 hrs?
3. Mayanja drove his car for 45km at a speed of 40km/hr. What distance did he cover?
4. A motorist makes a journey from 8:30am to 10:50am at 70km/hr. What distance does he cover?
5. What distance is covered by a lorry that moves for 6 hrs at a speed of 35km/hr.

MORE ABOUT DISTANCE (Given time points)

1. Moving at 60km/hr, a bus completed the journey from 10:30am to 1:20pm. How long was the journey?
2. At a speed of 54km/hr a cyclist left Katonga at 9:00am and arrived at Kamapala at 12:30pm. How far is Kampala from Katonga?
3. A bus moves at an average speed of 90km/hr from 8:15am to 11:15am. What distance does it have?

Finding Time

Example:

How long will it take a car to cover a distance of 120km at a speed of 40km/hr?

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Time = $\dfrac{\text{Distance}}{\text{Speed}}$

= $100km ÷ \dfrac{40km}{hr}$

= $120km × \dfrac{hr}{40km}$

Time = **3 hrs**

Activity

1. How long will it take a car to cover a distance of 80km at a speed of 20km/hr?
2. A bus travels at a distance of 200km at a speed of 40km/hr. Find the time it takes.
3. The speed of a cyclist is 70km/hr. How long will he take to cover a distance of 350km?
4. Moving at a sped of 70km/hr, Lule covered a distance of 490km on his bicycle. How many hours did he take?
5. It is 220km from Masaka to Kampala. How long will a car take to cover that distance at a speed of 40km/hr?

Finding speed

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Example

A car travels for 2 hrs to cover a distance of 210km. Find its average speed.

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

= $\dfrac{210\cancel{hr}}{3\cancel{hr}}$

= $\underline{70km/hr}$

Activity

1. A bus travelled for 2 hrs to cover a distance of 160km. At what speed was the bus travelling?
2. Muguma took 10 minutes to run a distance of 100m. What was his speed (m/min).
3. Find the speed at which a driver should drive to cover a distance of 240km in 5 hrs.
4. Find the average speed of a train that covered 144km in 4hrs.
5. It is 150m from Kampala to Malaba. At what speed should a taxi run to cover the journey in 2 ½ hrs.

Changing km/hr to m/sec

Example

Change 180km/hr to m/sec.

1 km = 1000m

1 hr = 3600 sec

Therefore; $\dfrac{\text{Distance in metres}}{\text{Time in seconds}}$

= $\dfrac{180 \times 1000}{(3600 \times 1) \text{ sec}}$

= **50m/sec**

Exercise

Express the speed below in m/second.

a) 36km/hr   b) 72km/hr   c) 216 km/hr

d) 162km/hr   e) 144km/hr   f) 360km/hr

Changing m/sec to km/hr

Example

Convert 20m/sec to km/hr.

1000m = 1km

Therefore; 1m = $\frac{1}{1000}$ km

20m = $\left(\frac{20}{1000}\right)$ km

= $^{20}/_{1000}$ ÷ ($^1/_{3600}$) hr

= $\frac{20\ km}{1000}$ x $\frac{3600}{1hr}$

= 2 x 36

= 72km/hr

Activity

Convert to km/hr.

a) 40m/sec	b) 30m/sec	c) 90m/sec
d) 100m/sec	e) 25m/sec	f) 70m/sec
g) 60m/sec	h) 150m/sec	i) 50m/sec

Finding Average speed

Example

A car takes 3 hrs to cover a certain journey at 60km/hr but it takes only 2 hrs to return through the same distance. Calculate the average speed for the whole journey.

Distance   = S x T

          = $\frac{60km}{hr}$ x 3 hr

          = 180km

Total distance = (180 x 2)

Total time = 3 + 2

         = 5

Average speed   = $\frac{180\ x\ 2}{5}$

$$= 72km/hr$$

Activity

1. A car takes 2 hrs to cover a certain distance at 60km/hr but it returns in 3 hrs. Calculate its average speed for the whole journey.
2. A lorry takes 4 hrs to travel from Kampala to Lyantonde at 45km/hr but returns in 6 hrs. Find its average speed for the whole journey.
3. Ali took 4 hrs to cover a journey at 60km/hr but it takes only 2 hrs to return though the same distance. Calculate its average speed for the whole journey.
4. Bosco ran for 3 hrs at a speed of 6km/hr and another 2 hrs at a speed of 5km/hr. Find the average speed for the whole journey.
5. A bus takes 6 hrs to cover a distance of 80km/hr but it returns in only 4 hrs. Calculate its average speed for the whole journey.

TRAVEL GRAPHS

1. Town A and C are 100km apart. A motorist travelled from A to B for 2 hrs at a speed of 30km/hr, rested for 30mins and continued to C in only 1 ½ hr.

i) What is the distance from A to B.
ii) How far is C from B?
iii) At what speed did he more from B to C.
iv) What time did he take from A to B.
v) Find the average of the motorist for the whole journey.

TERM III
MEASURES
CIRCUMFERENCE
Finding Radius

Radius = $\underline{\text{Diameter}}$
            2

Example

1. Find the radius of a circle of diameter 9cm.

   Radius   = $\underline{\text{Diameter}}$
                        2

               = $\underline{9}$cm
                  2

               = **4 ½ cm   or 4.5cm**

Activity

Find the radius of a circle whose diameter is;

a) 6cm   b) 12cm   c) 24cm
d) 20cm   e) 45cm   f) 100cm
g) 17cm

Finding diameter
Example

Find the diameter of a circle whose radius is 15cm.

Diameter   = Radius x 2
            = 15cm x 2
            = **30cm**

Activity

Find the diameter of a circle whose radius is;

a) 4cm   b) 10cm   c) 16cm   d) 6cm
e) 10 ½ cm   f) 11cm

Calculating the circumference of a circle
Example

1. Find the circumference of a circle whose diameter is 10cm. (Use π = 3.14)

   C   = π D
          = 3.14 x 10cm
          = **31.4cm**

2. Calculate the circumference of a circle whose radius is 3 ½ .(Use π = $^{22}/_7$)

C   = π D or π r

       = 2 x <u>22</u> x 3 ½ cm
              7

       = 2̶ x 2̶2̶ x 7̶ cm
         1   7̶   2̶

C   = **22cm**

Activity

1. Find the circumference of a circle whose diameter is 5cm. (Use π = 3.14)
2. A circular bottom of a mug has a radius of 50mm. Find the circumference.
   (Use π = 3.14)
3. Find the circumference of a circle whose radius is 7cm. (Use π = $^{22}/_7$)

Area of squares
Examples

1. Find the area of a square whose side is 6cm.

6cm

Area = side x side

      = 6cm x 6cm

      = **36cm²**

2. Find the area of a square whose side is pcm.

Pcm

Area = side x side

      = pcm x pcm

      = p² cm²

Activity
  1. Find the area of the following squares.

  a)

  b)   10cm

  7cm

  2. The side of a square is 8cm. Find its area.

Finding the side of the square

Example
  1. The area of a square is 64cm2. Find the length of each side of the square.

  Let each side be y.

   ycm

  y x y = Area

  $\sqrt{y^2}$ = $\sqrt{64}$

  $\sqrt{y \times y}$ = $\sqrt{2 \times 2 \times 2 \times 2}$

  y = 8

  Therefore each length = 8cm

Activity
  1. Find the length of each side of the square whose area is;

  a) 25cm^2 b) 36cm^2 c) 81cm^2

  2. Find the area of a square piece of paper whose side is 12cm. Find the value of x^2.

  3. If 3x^2 = 27. Find the value of x.

  4. The area of a square is 900cm2. Find the length of each side of the square.

Area of rectangle

Example
A rectangle is 10cm long and 5cm wide. Find the area of the rectangle.

5cm

10cm

Area of a rectangle   = Length x Width

                                  = 10cm x 5cm

                                  = 50cm²

Activity
1. The length of the field is 700m long and 600m wide. Find its area.
2. Find the area of a rectangle whose length is 40cm and width 30cm.
3. A rectangle measures 25m by 20m. Find its area.
4. The length of a rectangle field is 120m by 80m. Find the area of the field.

Finding the side of a rectangle when area is given
Example
The area of a rectangle is 56cm2. The length is 8cm. Find the width of the rectangle.

Area = 56cm² w

8cm

Length x width   = Area

8cm x w   = 56cm²

~~8cm~~ x w   = 56~~cm²~~

~~8cm~~   8~~cm~~

Width   = 7cm

Activity
1. A rectangular piece of paper is 4800mm², its width is 60mm. Find its length.
2. The area of a rectangular field is 96cm², its width is 8cm. Find the length.
3. The area of a rectangle is 42cm2. The length is 7cm. Find its width.
4. A rectangular garden is 50m², its width is 5cm. Find its length.

Finding area when given perimeter
Example
1. The perimeter of the rectangle is 24cm and the width is 5cm.
    a) Find length
    b) Find area

$$P = 24cm \quad w (5cm$$
Length

L + W + L + W	= Perimeter
L + L + W + W	= p
2L + 5cm + 5cm	= 24cm
2L + ~~10cm~~ ~~10cm~~	= 24cm – 10cm
2L	= 14cm
2	2
L	= 7cm
The length	**= 7cm**

Area = Length x Width
= 7cm x 5cm
= 35cm²

Activity
1. The perimeter of a rectangle is 40cm and the width of the rectangle is 8cm.
    a) Find the length of the rectangle.
    b) Find the area of the rectangle.
2. The perimeter of a rectangle is 36cm, its length is 10cm.
    a) Find its width.
    b) Find its area.
3. The perimeter of a rectangular garden is 80cm, its width is 18cm.
    a) Find its length.
    b) Find its area.

Finding sides, area & perimeter
Example
ABCD is a rectangle.

A   (2x – 5)cm   B

(x – 1)cm

D   (x + 3)cm   C

a) Find the value of x.

$2x - 5 = x + 3$

$2x - 5 + 5 = x + 3 + 5$

$2x - x = 8$

x = 8

b) Find the width and the length.

Width	Length
= (x + 3)cm	= (x - 1)cm
= (8 + 3)cm	= (8 - 1)cm
= 11cm	**= 7cm**

c) Find the area and perimeter.

Area	Perimeter
A = L x W	P = 2(L x W)
A = 11cm x 7cm	P = 2(11 + 7)
A = 77cm²	**P = 36cm**

Activity

1. Work out the following.

(2x)cm

5cm

(x + 6)cm

The diagram is a rectangle.

a) Find the value of x.

b) Find the length.

c) Find the area.

d) Work out the perimeter.

2.   x + 9cm

$\frac{x}{2}$

2x + 1cm

a) Find the value of x.

b) Find the length and width of the rectangle.

c) Find the perimeter of the rectangle.

d) Find the area of the rectangle.

3.

(2y + 5)cm

(y+1)cm

(4y + 3)cm

a) Find the value of y.
b) Find the width and length of the rectangle.
c) Work out the area
d) Find the perimeter of the rectangle.

Area if trapezium

Example

Find the area of the trapezium below.

8cm

7cm

10cm

Area of trapezium   = ½ h (a + b)
            = ½ x 7cm (8 + 10)
            = ½ x 7 x 18 cm
            = ½ x 7cm x 9cm
            = 7cm x 9cm
            = **63cm²**

Activity

1. Find the area of the shapes below.
a)

5cm

6cm

8cm

b)

6cm

8cm

10cm

138

c)

4cm

9cm

8cm

2. Find the area of a trapezium of height 10cm and the parallel sides are 12cm and 18cm.

3. If A = ½ h (a + b). Find A if h = 8cm, a = 14cm and b = 15cm.

Finding one side of a trapezium
Example
The area of a trapezium is 60cm2, height is 4cm and one of the parallel sides id 10cm. Find the length of the second parallel side.

10cm

4cm

a

½ h(a + b) = Area

½ x 4 (a + b) A

2(a + 10) = 60

2a + 20 = 60

2a + 20 – 20 = 60 – 20

$\frac{2a}{2} = \frac{40}{2}$

a = 20cm

Activity
1. Find the length of the second parallel side of a trapezium if the area is 56cm^2, the height 8cm and one of the parallel sides is 4cm.
2. A = ½ h(a + b). Find the value of A, if b = 6cm, h = 9cm and a = 10cm.
3. The parallel sides of a trapezium are 10cm and 12cm. Find the height of the trapezium if the area is 77cm^2.

Area of parallelogram
Example
Find the area of the parallelogram below.

15cm

9cm   10cm

Area of parallelogram = b x h

= 15cm x 9cm

= 135cm²

Activity
Find the area of the shapes below

1.

7cm   8cm

12cm

2.

11cm   14cm

18cm

3.   A   B

7cm   8cm

E   D   13cm   C

Area of triangle
Examples
1. Find the area of the △

8cm

6cm

A   = ½ b h

= ½ x 6cm x 8cm

$= 3cm \times 8cm$

$= \mathbf{24cm^2}$

2. Workout the area of the triangle below.

$A = \frac{1}{2} \times b \times h$

$A = \frac{1}{2} \times 7cm \times 8cm$

$A = 7cm \times 4cm$

$\mathbf{A = 28cm^2}$

Activity

Find the area of the triangle below.

1.

2.

3.

Find the area of the shaded triangle

4.

10cm
8cm   6cm
5cm   3cm

5.

14cm
8cm
11cm
8cm   4cm

Finding one side of a triangle when area is given

Example

Find the base of a triangle whose area is 60cm² and height is 12cm.

12cm

base

½ x base x height = Area

½ x b x 12cm   = 60cm²

b x 6cm   = 60cm2

$\underline{6}$b cm   = 60cm x cm

6 cm   6cm

b   = 10cm

Note: Base   = $\underline{2 \text{ x Area}}$
                    Height

    Height   = $\underline{2 \text{ x Area}}$
                      base

Activity

1. Find the height of a triangle whose area is 36cm² and its base is 12cm.
2. Find the base of a triangle whose area is 20cm2 and height 8cm.

3. The area of a triangle is 40cm2. Find the height if the base is 10cm.
4. The height of a triangle is 9cm and its area is 36cm². Find its base.

Finding base or height by comparing area
Example
ABD is a triangle. AC and BE are heights of the same triangle. BD = 12cm AC = 10cm BE = 8cm. Find length AD.

½ x AD x 8cm   = ½ x 12cm x 10cm

AD x 4cm   = 6cm x 10cm

4ADcm   = 60cm

~~4ADcm~~   = 60cm x ~~cm~~
~~4cm~~   4~~cm~~

AD   = 15cm

Activity
Find the value of the unknown in the figure below.

1.

2.

143

3.

Finding sum of area
Example
Find the total area of each figure below.

a)

Area of fig. A
= ½ x b x h
= ½ x 4 x 8
= 2 x 8
= 16cm²

Area A + Area B
= (16 + 21)
= 37cm²

Area of fig. B
= ½ x b x h
= ½ x 7 x 6
= 7 x 3
= 21cm²

b)

144

Area of fig. A	Area of fig. B
= ½ x b x h	= L x W
= ½ x 3 x 8	= 8 x 7
= 12cm²	**= 56cm²**

Total area = (12 + 56)cm²

= 68cm²

Exercise

Find the area of the shapes below.

1.

2.

Finding differences of Area
Examples

1. Find the area of the shaded part.

Area of outer rectangle = L x W

= 11cm x 8cm

= 88cm²

Area of inner rectangle = L x W

= 9cm x 4cm

= 36cm2

Area of shaded part = 88cm² – 36cm²

= 52cm²

2. Study the diagram below carefully and find the area of the shaded part.

16cm

2cm

2cm   3cm   10cm

1cm

Finding length of inner figure.

= 16 – (2 + 3)

= 16 – 5

= 11cm

Finding width of inner figure.

= 10 - (2 + 1)

= 10 – 3

= 7cm

Area of outer fig. = L x W

= 16 x 10

= 160cm²

Area of inner fig. = 11cm x 7cm

= 77cm²

Exercise

Find the area of the shaded parts.

1.

5cm

12cm   10cm

7cm

2.   15cm

9cm

8cm   6cm

3.

14cm

10cm

Finding total surface area of cubes and cuboids
Examples

1. Find the Total Surface Area of the cube below.

5cm

$$\text{T.S.A} = \left[(S \times S) + (S \times S) + S \times S\right]$$
$$= 2 (S^2 + S^2 + S^2)$$
$$= 2 \times 3S^2$$
$$= 6s2$$
$$= 6 \times 5^2$$
$$= 6 \times 25m$$
$$\mathbf{= 150m^2}$$

2. A cuboid measure 8cm long 4cm wide and 5cm high. Find its volume.

5cm

4cm

8cm

$$\text{T.S.A} = 2 (lw + wh + lh)$$
$$= 2(8x4 + 4 \times 5 + 8 x5)$$
$$= 2 (32 + 20 + 40)$$
$$= 2 \times 92$$
$$\mathbf{= 184cm^3}$$

Find the area of the figures below.

a)

2cm

3cm

4cm

b)

8cm

4cm

2cm

c)

3cm

d)

16cm

Finding the measures of each side of a cube given its total surface area

Examples

The total surface area of a cube is 384cm³

T.S.A = 6s2

$6S^2$ = 384

$6S^2$ = $\underline{384}$
 6   6

S^2 = 64

$\sqrt{S^2}$ = $\sqrt{64}$

S   = 8cm

Each side measures 8cm

Activity

Find the measure of each side of a cube whose total surface area is;

a) 96cm²   b) 150cm²   c) 726cm²

d) 600m²   e) 2646m²   f) 2904m²

g) 486cm²   h) 6m²

Finding volume of cubes and cuboids

Volume is the amount of space occupied by an object.

Units can be m³ , cm³ , mm³

1. Find the volume of cuboid below.

3cm

2cm

3cm

Volume = l x w x h
        = 3 x 2 x 3
        = 6 x 3
        = **18cm³**

2. A cube measures 4cm a side. Find its volume.

Volume = S x S x S
        = S³
        = 4 x 4 x 4
        = 16 x 4
        = **64cm³**

4cm

Activity

1. Find the volume of the figures below.

5cm

4cm

6cm

8cm

2cm

5cm

   2. Calculate the volume of a cuboidal tank whose length is 11m, width 4m and height 6m.

3. A cubical milk tank measures 6m a side. Find the volume of the tank.

4. A soap box measures 40cm by 10cm by 5cm. What is its volume?

Finding one unknown side of a cuboid given the volume and the other sides.

Examples

1. Find the height of the rectangular prism below whose volume is 180cm³.

h

4cm

9cm

Volume = l x w x h

$$9 \times 4 \times h = 180$$
$$36h = 180$$
$$\frac{36\,h}{36} = \frac{180}{36}$$
h = 5cm
height = 5cm

2. Find the width of a rectangular prism whose volume is 480cm3, length 15cm and height 8cm.

8cm

wcm

15cm

Vol = l x w x h

$$15 \times 8 \times w = 480cm^3$$
$$120cm\,w = 480cm^3$$
$$\frac{120cm^2 w}{120\,cm^2} = \frac{480cm^3}{120cm^3}$$
w = 4cm

Exercise

1. Find the side marked by the letter.

3cm

wcm

6cm

Volume = 36cm³

h

5cm

6cm

The volume is 120cm³

3. The volume of a rectangular prism is 135m3. If the height and length are

3m and 5m respectively, what is the width?

3. The square base area of a cuboidal prism is 36cm2, and the volume is 360cm2. Find the height of the tank.

Finding volume of rectangular prism/cubes in litres
Examples
   1.   A rectangular tank is 30cm by 60cm by 90cm. Find its volume in litres.

   Vol. of the tank = l x w x h

   = (30 x 60 x 90

   But 1 litre = 1000cm3

   No of litres in the tank = (30 x 60 x 90)cm³

   1000 cm³

   = 3 x 6 x 9

   = 18 x 9

   = 162 litres

   2.   A tank in form of a cube measures 50cm a side. Find the number of litres at full capacity.

   Vol = S x S x S

   = (50 x 50 x 50)cm³

   But 1 litre = 1000cm3

   Number of litres = 50 x 50 x 50

   1000

   = 5 x 5 x 5

   = 25 x 5

   = 125 litres

Activity
   1.   Calculate the volume in litres of a rectangular tank 80cm by 70cm by 20cm.
   2. Below is a tank. Find its volume in litres.

   3.   How many litres of petrol are in a rectangular tank measuring 50m by 100m by 20m?
   4. The bottom area of a rectangular tank is 1480cm2. Find its volume in litres if the height is 300cm.

MORE ABOUT VOLUME IN LITRES

Examples

1. The tank below is holding 72 litres of water.

i)   Calculate the volume of h.

Volume of tank = 72litres

But 1 litre = 1000cm

Therefore;   72 hrs = 72 x 1000

40 x 60 x h = 72000

h   = $\underline{72000}$

                   40 x 60

h   = $\underline{120}$

                        4

h   = 30cm

ii)   How many litres are needed to fill the tank?

Height needed = (80 – 30)

                        = 50cm

Volume needed   = 40 x 60 x 50

                        = 2400 x 50

                        = $120000cm^3$

Number of litres   = $\underline{120000}$

                          1000

= 120 litres

2. Container has a volume of 108000cm3.

i)   Find the capacity of the container.

ii)   If the container is ¾ full of water. How many litres are needed to fill it?

Changing litres to millitres

Conversion table

L	dl	cl	ml
1	0	0	0
	1	0	0
		1	0

1 litre = 1000ml

Example

1. Change 7 litres to millitres

L to ml
1 l   = 100ml
7l   = (7 x 1000)ml
        = 7000ml

2. How many ml are in 4 ½ l?
    L to ml
    1 l   = 1000ml
    9l   = 9 x 1000 ml
    2   2
            = **4500ml**

Exercise
1. Change the following litres to millitres
a) 5 litres   b) 6 ¼ litres   c) 0.8 litres
d) 7 litres 300 lillilitres

More about litres and millitres
2. A cow gives 14 litres of milk daily. Express its daily milk production in millitres.
3. Aber fetched 15 litres of water and Mugura 18 litres of water. How many millitres of water did the two fetch?

Changing millitres (ml) to litres (l)
Examples
1. Change 3500 millitres to litres
    Ml   to l
    1000ml   = 1l
    1ml   = $\frac{1}{1000}$ l

    3500ml   = ($^{1}/_{1000}$ x 3500)l
    = $^{1}/_{1000}$ x 3500
    = **3 ½ litres**
    OR = **3.5 litres**

Exercise
1. Express the following millitres as litres.
a) 2000ml   b) 6000ml   c) 12000ml
d) 8500ml   e) 870ml   f) 5600ml

2. A baby takes 250 millitres of milk every feeding.
   i) How many litres does it take in 1 feeding.
   ii) How many litres does it take in 4 feedings.

GEOMETRY

Construction of regular polygons
- Equilateral triangle
- Square
- Pentagon
- Septagon/heptagon
- Octagon
- Nonagon

Construction of parallel lines

Angle proportion parallel lines

A line which intersects a set of parallel lines is called transversal line

Transversal line

When a transversal line intersects a pair of parallel lines, 8 angles are formed.

Examples

<a + <b1 = 180^0

<m + <n = 180^0

<x + <y = 180^0

<p + <q = 180^0

Finding the unknown angles

Examples

1.

X^0

m^o

c + 111^0 = 180^0 (co-interior <)

x + 111^0 – 111^0 = 180^0 – 111^0

x = 69^0

2.

75^0

p +60^0

$p + 60^0 + 75^0 = 180^0$ (co-ext <)

$p + 135^0 - 135^0 = 180^0 - 135^0$

$p = 45^0$

Activity

1.

x

100^0

2.

30^0

p

3.

50^0

p

Alternate angles

1.

x

x_1

$<x = <x_1$ (alternate <s)

2.

b

$b1$

$<b = <b1$ (alternate <s)

3.

60^0

p

$<p = 600$ (alternate <s)

4.

2x = 100⁰ (alternate <s)

$$\frac{2x}{2} = \frac{100}{2}$$

x = 50⁰

Activity

1. Find the size of the marked angles below.

Finding angles formed by parallel lines
Examples

1. Find the values of the unknowns below.

3p + 60⁰ = 180⁰

3p + 60⁰ – 60⁰ = 180⁰ – 60⁰

3p = 120

$$\frac{3p}{3} = \frac{120}{3}$$

p = 40⁰

2.

$a + 100^0 = 180^0$

$a + 100^0 - 100^0 = 180^0 - 100^0$

$a = 80^0$

$b + 110^0 = 180^0$

$b + 110^0 - 110^0 = 180^0 - 110^0$

$b = 70^0$

$a + x + b = 180^0$

$8^{00} + x + 70^0 = 180^0$

$150^0 + x = 180^0$

$150^0 - 150^0 + x = 180^0 - 150^0$

$x = 30^0$

1. Find the value of the unknowns

2.

3.

CONSTRUCTION

- Constructing perpendicular lines on a given point.
- Constructing perpendicular bisectors on a given line.
- Constructing perpendicular bisectors from a given point.
- Constructing of angles (30^0, 60^0, 90^0, 45^0 ....) using a pencil, ruler and a pair of compasses.

- Constructing triangles using;
  S.S.S
  S.A.S
  A.S.A
- Constructing quadrilaterals
  Rectangles
  Squares

Vertically opposite angles

Lines TR and PQ have a common point O.
O is the point of intersection
<a and <c are vertically opposite angles and are equal.
<b and <d are vertical <s and they are equal.

Activity

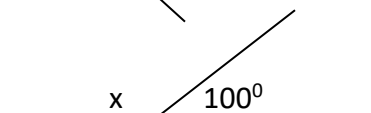1. Find the size of the angles marked by letters giving reasons.

    i)   <a = _____
    ii)   <b + 300 = _____
    iii)   <b = _____
    iv)   <a + <b = _____
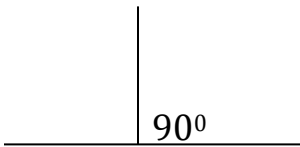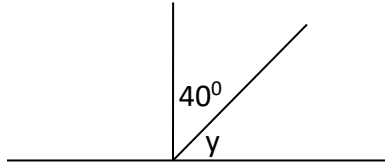   v)   <a ++ <c = _____

2.

3.

4.

5.

An angle with 90^0 is called a right angle.

Any two angles that add up to 90^0 are complementary angles.

Complementary angles add up to 90^0.

40^0 and 50^0 are complementary angles because $40^0 + 50^0 = 90^0$

40^0 is a complement of 50^0.

Finding complementary angles

What is the complement of 30^0?

Let the complement be y.

$y + 30^0 = 90^0$

$y + 30^0 - 30^0 = 90^0 - 30^0$

$y = 60^0$

Activity

What is the complement of each of the following?

a) 15^0 b) 89^0 c) 75^0

d) 50^0 e) 27^0 f) 25^0

  g) $(x + 20^0)$ h) $(x - 20^0)$

Angle descriptions

-   Acute angles
- Obtuse angles
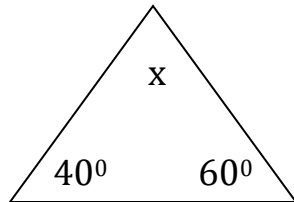- Right angles
- Angles on a straight line
- Angles at a point

Interior and exterior angles of a triangle
- Right angled triangle
- Equilateral triangle
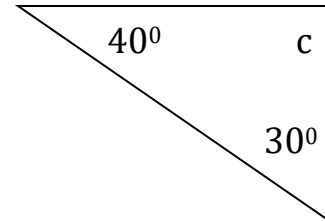- Scalene triangles
- Isosceles triangles

Angles in a triangle

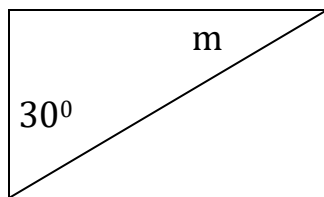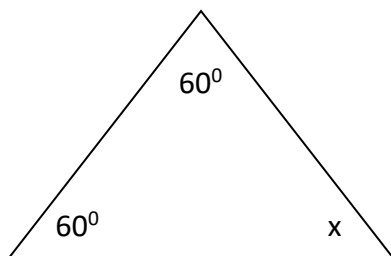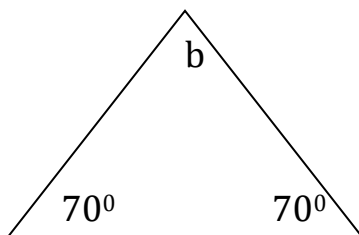Find the size of the angles marked by letters.

1.

x

40^0 60^0

2.

40^0 c

30^0

3.

m

30^0

4.

60^0

60^0 x

Angles of Isosceles triangles

b

Two base <s of an isosceles triangle are equal

70^0 70^0

b + 70^0 + 70^0 = 180^0

b + 140^0 – 140^0 = 180^0 – 140^0

b = 40^0

a)

40   d

x

b)

m   p

c)

e

m   45°

MORE ABOUT INTERIOUR AND EXTERIOR ANGLES

a

50°

b   60°   70°

c

Using angles on a straight line

<a = 180° – 50°

= 130°

<b = 180° – 60°

= 120°

<c = 180° – 70°

= 110°

130°

60°   70°

<a = 60° + 70°

= 130°

Finding centre angles given sides

1. Find the size of each centre angle of a regular polygon of 3 sides.
   All centre angles = 360^0
   Each centre angle = $\underline{360^0}$
   3
   $= \mathbf{120^0}$

2. A regular polygon has 12 sides. What is the size of each centre angle?
   No of sides = 12
   All centre angles = 3600
   Each centre angle = $\underline{\text{All centre}}$
   No of sides
   $= \underline{360^0}$
   12

   Centre angle = 30^0

Activity

Find the size of each angle of the following regular polygons whose number of sides are;

a) 4 sides	b) 8 sides	c) 6 sides	d) 10 sides
e) 5 sides	f) 12 sides	g) 7 sides	h) 20 sides

Finding the number of sides when the centre angle or exterior angle is given

1. Find the number of sides of a regular polygon whose centre angle is 60^0.
   Number of sides = $\underline{\text{All centre <s}}$
   Each centre <s
   $= \underline{360^0}$
   60^0
   $= \mathbf{6\ sides}$

2. Find the number of sides of a regular polygon whose exterior angle is 72^0.
   Number of sides = All centre <s
   Each Ex <z
   $= \underline{360^0}$
   72^0
   $= \mathbf{5\ sides}$

Activity

Find the number of sides of regular polygons whose centre are?

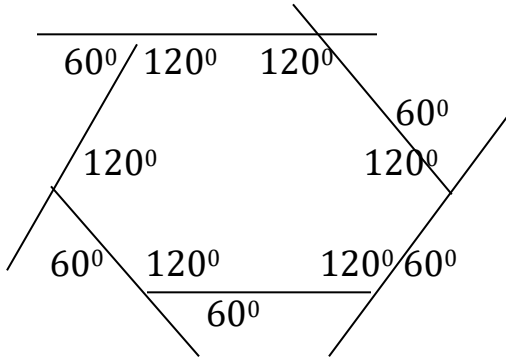a) 10^0 b) 60^0 c) 30^0 d) 90^0

e) 40^0 f) 120^0 g) 45^0 h) 36^0

Exterior angles and interior angles of a regular polygon

Find the size of each interior angle of a regular polygon whose exterior angle is 120^0.

Let the interior be y.

$y + \text{ext} < \ = 180^0$

$y + 120^0 = 180^0$

$y + 120^0 - 120^0 = 180^0 - 120^0$

$y = 60^0$

Find the size of each interior angle of a regular polygon whose exterior angle is;

a) 10^0 b) 80^0 c) 30^0 d) 70^0

e) 40^0 f) 20^0 g) 50^0 h) 60^0

Find the size of each exterior angle of a regular polygon whose interior angle is;

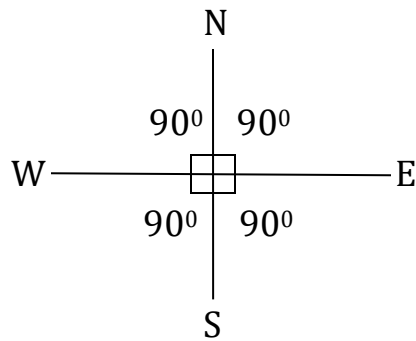a) 120^0 b) 160^0 c) 108^0 d)

130^0

e) 150^0 f) 100^0 g) 170_0 h)

140^0

Angles on a compass

1. What is the larger angle between N and E?
2. What is the smaller angle between W and S?
3. What is the larger angle between W and S?
4. What is the angle between N and W?

Turns

Right hand turn clockwise   Left hand turn ant-clockwise

1. Through what angle do I turn if I turn from North to SE clockwise?
2. Through what angle do I turn if I turns clockwise;
   a) From NE to South?
   b) From West to SE
   c) From SN to SE
3. Through what angle do I turn if I turned anti-clock wise.
   a) From NE to West.
   b) From West to East.

PYTHOGRAS THEOREM

Short Side — Hypotenuse — Short side

Finding the longest side (hypotenuse) of a right angled triangle.

1.

4cm   hcm

3cm

Find the value of h.

$a^2 + b^2 = c^2$

$3^2 + 4^2 = h^2$

$(3 \times 3) + (4 \times 4) = h^2$

$9 + 16 = h^2$

$\sqrt{25} = \sqrt{h^2}$

$\sqrt{5 \times 5} = \sqrt{h \times h}$

$5 = h$

Therefore h = 5cm

2.

12cm   hcm

Find the value of h

$a^2 + b^2 = c^2$

$5^2 + 12^2 = h^2$

$(5 \times 5) + (12 \times 12) = h^2$

$25 + 144 = h^2$

$\sqrt{169} = \sqrt{h^2}$

$\sqrt{12 \times 13} = \sqrt{h \times h}$

$13cm = h$

Therefore h = 13cm

Pythogram theorem

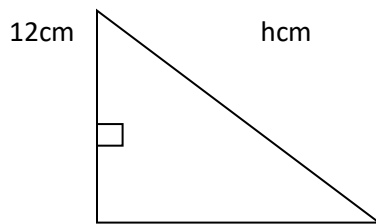1. Given that PS = PQ = 10cm, PR = 6cm and bisect <P

P

10cm

6cm

Q   R   S

i)   Find the length of QS.

PR = 6cm

PS = 10cm

$RS^2 + 6^2 = 10^2$

$RS^2 + 6 \times 6 = 10 \times 10$

$RS^2 + 36 = 100$

$RS^2 + 36 - 36 = 100 - 36$

$\sqrt{RS^2} = \sqrt{65}$

$\sqrt{RS \times RS} = \sqrt{8 \times 8}$

$RS = 8cm$

The length of QS = 8cm x 2

= 16cm

ii)   Calculate the perimeter of the figure.

P = sum of all sides

= 16cm + 10cm + 10cm

= 26cm + 10cm

= 36cm

iii)   Calculate the area of the figure.

base = 16cm

height = 6cm

Area = ½ bh

= ½ x 16cm x 6cm

= 8cm x 6cm

= 48cm²

iv)   Find the area of the triangle PRS

$$= \tfrac{1}{2}\,bh$$

$$= \tfrac{1}{2}\ 8cm \times 6cm$$

$$= 4cm \times 6cm$$

= 24cm²

2. PQS is an isosceles triangle. PQ = 13cm

i)   Calculate the value of x.

20pp sides equal

$(2x + 1)cm$	= 13
$2x + 1$	= 13
$2x + 1 - 1$	= 13 – 1
$2x$	= 12
$\underline{2x}$	= $\underline{12}$
2	2
x	**= 6**

ii)   Find the length of QS using pythogras theorem

$RS^2 + 12^2$ = 13^2

$RS^2 + (12 \times 12)$ = 13 x 13

$RS^2 + 144$ = 169

$RS^2 + 144 - 144$ = 169 – 144

$\sqrt{RS^2}$ = $\sqrt{25}$

$\sqrt{RS \times RS}$ = $\sqrt{5 \times 5}$

RS   = 5cm

The length of QS = RS + RS

                       = 5cm + 5cm

                       = 10cm

iii)   Find the perimeter of the figure

        P = sum of all sides

          = 13cm + 13cm + 10cm

          = 26cm +10cm

          = 36cm

iv)   Calculate the area of the triangle.

        A = ½ bh

           = ½ x 10cm x 12cm

           = 5cm x 12cm

           = 60cm²

Calculate the height, the perimeter and the area of each of the figures below

a)

h   10cm

12cm

b)

7cm 13cm

Q   R

10cm

c)

A   16cm   C

h

10cm

B

d)

Find x, height of lines, the area and the perimeter

P

2cm   (3x + 2) cm

Q   32cm   R

25cm   (3x+1)cm

48cm

Lines of folding symmetry

- Equilateral triangle
- Isosceles triangle
- A rectangle
- A kite
- A square
- A trapezium
- An isosceles triangle
- A circle
- A regular pentagon
- Letters of alphabet

Length

Converting km to metres.

1. Change 3km to metres

   1km = 100m

   3km = 3 x 1000m

       = 3000m

2. Change 0.4km to metres

   1km = 1000m

   0.4km = 0.4 x 1000m

$$= \left(\frac{4}{10} \times 1000 \right) m$$

      = 4 x 100 m

      = 400m

Activity

Change cm to km.

a) 40000cm	b) 110,000cm	c) 160,000cm
d) 48,000cm	e) 32,000cm	f) 490,000cm

Changing metres to cm

1. Change 4m to cm

   1m = 100cm

   4m = 4 x100cm

      = 400cm

2. Express 0.9 m to cm

    1m = 100cm

    0.9m = 0.9 x 100cm

        = $\underline{9}$ x 100cm
          10

        = 9 x 10 cm

        = 90cm

Activity

Change the following m to cm.

a) 5m   b) 1.2m   c) 25m   d) 9.6

e) 36m   f) 0.18m

Changing cm to metres

1. Change 40cm to metres.

    100cm = 1m

    40cm = $\left(\dfrac{40}{100}\right)$ m

        = $\left(\dfrac{4}{10}\right)$ m

    = 0.4m

2. Change 600cm to metres.

    100cm = 1m

    600cm = $\left(\dfrac{600}{100}\right)$ m

    = 6 cm

Activity
Change the following cm to metres.

a) 120cm   b) 18cm   c) 360cm

d) 80cm   e) 700cm   f) 90cm

Activity

Change the km to metres.

a) 5km   b) 0.6km   c) 7km   d) 24km

e) 8km   f) 36cm   g) 9km   h) 0.93km

i) 13km   j) 11km

Changing metres to km

1. Change 9000m to km.
    Since 1000m   = 1km
    9000m   = $\left(\dfrac{9000}{1000}\right)$ km
                    = **9km**

2. **Change 800m to km**
    1000m   = 1km
    800m   = $\left(\dfrac{800}{1000}\right)$km
                        $\dfrac{= \underline{8} \text{ km}}{10}$
                    = **0.8km**

Activity

Change the following metres to km.
    a) 6000m   b) 700m   c) 8000m   d)
        80m
    e) 900m   f) 130m

Changing km to cm

1. Change 7km to centimeters
    1km = 10000cm
    7km = 7 x 10000cm
         = **70000cm**

2. Change 0.4km 10 cm
    1km = 100000cm
         = 0.4 x 100000 cm
         = $\dfrac{4 \text{ x } 100000\text{cm}}{10}$
         = 4 x 10000cm
         = **40000cm**

Change km to cm
    a) 4km   b) 0.06km   c) 11km   d) 48km
    e) 14km   f) 69km   g) 18km   h) 53km

Change cm to km

Since 100,000cm   = 1km
    800,000cm = $\dfrac{800000}{100000}$
                = **8km**

Changing square metres to cm²

1. Change 2m² to cm²

   1cm   = 100cm

   1m x 1m   = 100cm x 100cm

   1m²   = 100,000 cm²

   2m²   = (2 x 10000)cm²

   = 20,000cm²

2. Express 1.2m2 to cm2.

   1m   = 100cm

   1m x 1m   = 100cm x 100cm

   1m²   = 10,000cm²

   1.2m²   = 1.2 x 10,000cm²

   $= \left(\dfrac{12 \ \times \ 10{,}000)}{10}\right)$ cm²

   = (12 x 1000)cm²

   = 12000 cm²

Activity

Change the following to square centimeters. (cm²)

a) 3m²   b) 8.2m²   c) 5m²   d) 10.5m²

e) 4m²   f) 20m²   g) 6m²   h) 12m²

Expressing km² to m²

1. Express 4km2 as m2

   1km = 1000m

   1km x 1km = 1000m x 1000m

   1km²   = 1000,000m²

   4km   = (4x 1,000,000)m²

   = 4,000,000m²

2. Change 2.5km² to m²

   1km²   = 1,000,000m²

   2.5km²   = (2.5 X 1,000,000) m²

   $= \left(\dfrac{25}{10} \ X \ 1{,}000{,}000\right)$ m²

   = 2,500,000m²

Activity

Change the following to square metres.
- a) 2km²
- b) 0.02km²
- c) 4km²
- d) 0.03km²
- e) 0.25km²
- f) 3.6km²
- g) 0.03km²
- h) 8km²

Finding perimeter of geometry shapes.

1. Find the perimeter of the figure below.

10cm

5cm   13cm

15cm

P   = 15cm + 13cm + 10cm + 5cm

     = 28cm + 15cm

     = 43cm

Activity

Find the perimeter of the following figures.

8cm   10cm   12cm

6cm

10cm

TOPIC 5: <u>INTEGERS</u>

Intergers are positive and negative numbers including zero plotted equidistantly on a number line.

Negative   Positive

$^+$7

A set of positive integers = {1 , 2 , 3 , 4 , 5 , 6 , ...}

A set of negative integers = { ... -4 , -3 , -2 , -1 }

N.B $_1$: Zero in neither positive nor negative.

N.B $_2$: Any integer without a sign is a positive integer.

Review the following;

- Inverse
- Integers on a number line/arrows on a number line.
- Ordering integers

<u>Addition and subtraction of integers</u>

Addition of integers

<u>*Examples*</u>

1. Add: +3 + +2

   $^+$3 + 2

   Therefore $^+$3 + $^+$ 2 = $^+$5

2. Add: +2 + +6

   +2 + 6

   $^+$2 + $^+$6 = $^+$8

3. Work out: -2 + -5

   $^-$2 – 5

   $^-$2 + $^-$5 = $^-$7

4. Add: -6 + -1

   -6 – 1

   Therefore $^-$6 + $^-$1 = $^-$7

Subtraction of integers
Examples
1. Subtract: +3 - +2

   +3 – 2

   Therefore +3 - +3 = +1

2. Work out: +5 - +4

   +5 – 4

   Therefore; +5 - +4 = +1

Activity
1. Add the following integers

   a) +2 + +3   b) +5 + +1   c) -6 + -2

   d) -3 + +2   e) +6 + +6   f) -3 + -3

   g) +7 + -2   h) -2 + +6

2. Subtract the following integers.

   a) +3 - +2   b) +4 - +6   c) -4 - -4

   d) -7 - +8   e) 2 – 4   f) -1 -1

   g) +3 - -3   h) +5 – 3

Integers on a number line
Addition and subtraction of integers on a number line.
Examples
1. Add: +3 + +2

   Therefore: +3 + +2 = +5
2. Simplify: +2 + -6

   Therefore; +2 + -6 = -4

3. Subtract: -2 - -3

- (-3)

-2

+
-7   -6   -5   -4   -3   -2   -1   0   +1   +2   +3   +4   +5
    +6   +7

− +1

So; -2 - -3 = +1

Activity
1. Simplify the following using a number line.
   a) +4 + +2   b) -3 + -4   c) -2 – 5
   d) -3 - -3   e) 4 – 6   f) +5 + -3
   g) -4 + 6   h) 3 - -6   i) 4 - +3
2. Show +3 + -6 on a number line.

Mathematical statements (Addition and subtraction statement) and algebraic statement
Examples
Study the number line below and answer questions that follow.

b

a

+   +
-7   -6   -5   -4   -3   -2   -1   0   +1   +2   +3   +4   +5
    +6   +7

c

a) Write an algebraic statement for the above number line.
   An algebraic statement = (+a) + (-b) = -c
b) Write the mathematical statement for the above number line.
   Mathematical statement consists of either addition or subtraction statement.
   i)   Addition statement
          a = +3
          b = + (-10)
          c = -7
          Addition statement = +3 + -14 = -7

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ii)   Subtraction statement
        a = +3
        b = - (+10)
        c = -7
        Subtraction statement = +3 - +10 = -7

Activity

1. Write the subtraction statement for the number line below.

-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
	+	+										
	+6	+7										

2. Write the addition statement for the given number line below.

-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
	+	+										
	+6	+7										

3. Study the number line below and answer questions that follows.

y

x

-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
	+	+										
	+6	+7										

Z

a) What integer is respected by letter x , y and z?
   i)   X _____   ii) Y _____   iii) z _____
b) Write the mathematical statement for the above number line.

APPLICATION OF INTEGERS
Questions (some)
   i)   Kato borrowed shs. 5,000. If he paid back shs. 3,000. Find Kato's
            financial status.

ii)   The temperature was 20⁰F but has dropped by 23⁰F. What is the temperature now?

ii)   The temperature was 20^0F but has dropped by 23^0F. What is the temperature now?

iii)   Peter was born 20AD and died 15BC. How old was he when he died?

iv)   A motorist moved 100m forward and reversed 150km. How far is she from the starting point?

v)   A patient's temperature dropped by 20C and by another 30c, find the patient's present temperature.

SOLUTION SETS AND INEQUALITIES

Inequality symbols are $>$, $<$, $\geq$, $\leq$

Solution set is a set of all possible values of unknown letter from the given inequality like;

x > 2

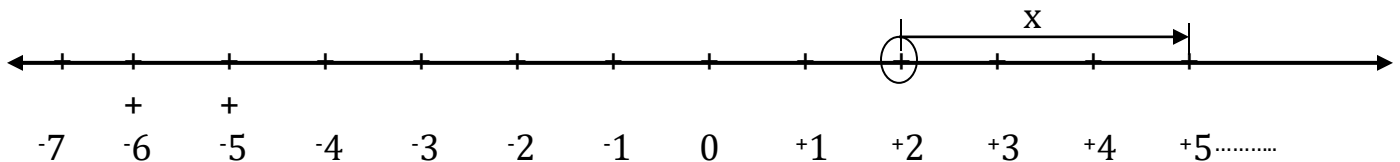## Finding solution set from the given inequality.

Examples

1. Given that; x > 2. Find all possible values of x if x is a positive integers.
    X > 2 means all positive integers greater than 2

    +   +
-7   -6   -5   -4   -3   -2   -1   0   +1   +2   +3   +4   +5··········
Solution set for x = {3 , 4 , 5 , ...}

2. If x ≥ 2. Find the solution set for x (x is a positive integer)
    x ≥ 2 means all positive integers greater or equal to two.

    +   +
-7   -6   -5   -4   -3   -2   -1   0   +1   +2   +3   +4   +5········

        The solution set for x = { 2 , 3 , 4 , 5 ....}
N.B: The circle is shaded because of equal sign

3. Given that; p ≥ -5, find the solution set for p if p is a negative integers.
    P > -5 means p is a set of negative integers greater that or equal to -5

    +   +
-7   -6   (-5)   -4   -3   -2   -1   0   +1   +2   +3   +4   +5
    +6   +7

The solution set for p = { -5, -4 , -3 , -2 , -1 }

NB: Zero is neither negative nor positive integer, so all negative integers greater or equal to -5 stops in -1. Hence no dot after negative one (-1)

Activity
1. Find the solution set for x < 5 if x is a positive integer.
2. Given that p ≥ - 6. Find the possible values of p is a negative integer.
3. Find the solution set for x ≤ -2.
4. Given that; -4 < x ≤ 4
5. Find the solution set for -3 ≤ p < 6

TOPIC: ALGEBRA
Writing phrases for algebraic expressions.
Examples
Add b to a   = a + b

Subtract b from a = a – b

Multiply b by a   = ab

Divide b by a   = $\underline{b}$
                    a

2x + 3 = multiply x by 2, then add 3 to the result

2(x +3) = Add 3 to x then multiply the results by 2

4y – 7 = multiply y by 4, then subtract 7 from the result

4(y – 7) = subtract 7 from y then multiply the result by 4

x + 5 = Divide x by 4, then add 5 to the result
4

$\underline{x + 5}$ Add 5 to x then divide the result by 4
  4

Exercise
Write phrases for the following.

a) x + y	b) x y	c) x + 2	d) x – y
e) $\underline{x}$	f) 3n +5	g) 10x + 4	h) 12a + 10
y			
i) 2a – 1	j) 6p – 6	k) (m + 10)	l) $\underline{t + 3}$
			2
m) $\underline{7}$	n) $\underline{b}$ + 20		
x – 1	5		

Expressing phrases as algebraic expressions

2 more than p   = p + 2

2 less than x = x – 2
Twice x = 2x
Three times of q = 3q
Half a = $\frac{a}{2}$

5 years younger than x = x – 5
7 years older than p = p + 7
Twice as old as n = 2n
Four times k's age = 4k
Average of a and b = $\frac{a + b}{2}$

Square p = p^2
Multiply the square of a by 3 = 3a2
Multiply n by 3 and square the result = $(3n)^2$
Multiply the sum of m and 9 by 7 = 7(m +9)

Exercise
Express the following as algebraic statements.
  a) a more than 5
  b) b divide by 4
  c) m times 3
  d) 13 more than d
  e) X subtracted from 8
  f) Thrice the sum of x and y
  g) A third the difference between 9 and m
  h) Half the sum of 2b , 3b , 8t and 6

Meaning of algebraic term
Example
2p means 2 x p or p x p
3qp means 3 x q x p
4q2 means 4 x q x q
$(4q)^2$ means 4q x 4q
Exercise
Expand the following.
  a) 3x   b) 3mn   c) $2y^2$ d)
      $(5x)^2$
  e) n^3 f) 6y   g) 7ab   h) $5x^3$
  i) $(ab)^2$ j) $(5x)^3$ k) xy^2 l) a^5
  m) $2a^5$ n) $4ab^2$ o) $(4ab)^2$

<u>Substitution</u>

<u>Examples</u>

1. Given b = 6

   Find b + 8

   6 + 8

   = 14

2. If = = 8, q = 6, what is pq9

   p x q x 9

   8 x 6 x 2

   = 96

3. Given b = 6 , c = -3 , a = 2

   Find <u>b c</u>

        9

   = b x c

      9

   = 3 x -3

   = -9

<u>Exercise</u>

If p = 8, q = 6 , r = 4 , a = 2, b = 6

Find the value of the following;

a) 3p   b) p + 3   c) qr   d) 2p

e) ¼ pc   f) pqr   g) q – r   h) 2q + r

i) 6ac   j) $4c^2$ k) abc   l) ab

m) $a^b c$

<u>Collecting like terms</u>

<u>Examples</u>

1. Simplify: x + y + 2x + 4y

               x + 2x + y + 4y

           3x + 5y

2. 3x + 6y – x – 2y

   3x + 6y – x – 2y

   3x – x + 6y – 2y

   2x + 4y

Exercise

Simplify;

a) p + q + p

b) 8x + 3 + 4y – x

c) x + y + 2x + 3y

d) 6t + 5 – 2t + 5

e) m + p + p + m

f) 3x + 4 + 4x + 5x

g) q + 4p + 3q + 2p

h) 2b + 4 – b

i) m + 2b + 3m + 5

j) b + 3k – 4b – k

Removing brackets

+ (2x + 3) = 2x + 3

- (4x + 6) = -4x - 6

Simplify; ½ (8a + 4b)

   ½ x 8a + ½ x 4b

   1 x 4a + 1 x 2b

   4a + 2b

Exercise

a) Remove the brackets

i)   2 (a + 3)

ii)   2 (6 + b)

iii)   5 (4b – 2)

iv)   4 (a – 2b)

v)   3 (8x + 5y)

b) Substitute if a = 4 , b = 1 and c = 3

i)   3b – c

ii)   2 (a + b)

iii)   2 (c – b)

iv)   5 (a – b)

v)   7 (b + c)

Change of positive and negative signs involving brackets.

i)   -2 (x – 2y)

ii) -2 (3x + 5)

iii) +4 (x + 1)

iv) +3x (y – 1)
v) +5 (x – 6)

Simplify

a) ½ (2a + 4b)
b) $^1/_3$ (6x – 9y)
c) $\underline{2a + 4b}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ 2

d) $^1/_3$ (+2xy – 15x)
e) $^1/_5$ (15x – 20y)

More about removing brackets
Examples
1. Remove the brackets
a) 3 (2 + x) + 2(x + 4)
b) 3x 2 + 3x y + 2 x y + 2 x 4
c) 6 + 3x + 2x + 8
d) 3x + 2x + 8 + 6
e) 5x + 14

2.
a) 3 (x + 3) – 2(x – 1)
b) 3 x y + 3 x 3 – 2 x y – 2x – 1
c) 3x + 9 – 2x + 2
d) x + 11

Exercise
Remove the brackets and simplify.
a) (x + 1) + (2x + 3)
b) (2x + 3) + (4x + 4)
c) (9x – 4) – (x – 1)
d) 5 (q + 3) + 3 (q – 1)
e) 4 (m + 3) + 3 (m + 1)
f) 5 (q + 3) – 3 (q – 1)

Finding the products of powers with the same base

Simplify

a) m x m

    m x m

    m^2

b) $y^2 \times y^3$

    $y^2 \times y^3$

    y x y x y x y x y

    $= y^5$

OR

$m^1 \times m^1$

$= m^{1+1}$

$= m^2$

$x^2 \times x^3$

$= x^{2+3}$

$= x^5$

Simplify; $4y^2 \times 3y^4$

$= 4y^2 \times 3y^4$

$= (4 \times 3)\ (y^2 \times y^4)$

$= 12(y2 + 4$

$= 12\ y^6$

Exercise

Simplify the following.

a) m x 3 x m

d) $w^3 \times w \times w^2$

g) $8m^2 \times 3m^6$

b) q x q x q x q

e) $5y^1 \times 4y^5$

h) $7m^2 \times 6m^3$

c) $y^2 \times y^3$

f) $2x^2 \times 4x^3$

Dividing powers of the same base
Examples

1. Simplify: $p^5 \div p^3$

    $P^5 \div p^3$ $= \dfrac{p \times p \times p \times p \times p}{P \times p \times p}$

          $= p^2$

2. Simplify: $y4 \div y3$

          $\dfrac{y \times y \times y \times y}{y \times y \times y}$

          $= y$

3. Simplify: $12\,t^4 \div 3t^2$

$$= \frac{12 \times t \times t \times t \times t}{3 \times t \times t}$$

$$= 4 \times t \times t$$

$$= 4t^2$$

Exercise

a) $y^4 \div y^3$ b) $m^7 \div m^2$ c) $d^9 \div d^5$

d) $q^7 \div q^4$ e) $t^7 \div t^4$ f) $10m9 \div 2m5$

g) $18p^3 \div 9p2$ h) $48x^5 \div 16x^2$ i) $36m^6 \div 6m^4$

Finding unknown

Examples

1. Solve: $p + 4 = 12$

$p + 4 = 12$

$p + 4 - 4 = 12 - 4$

$p = 12 - 4$

$p = 8$

2. $h - 15 = 12$

$h - 15 + 15 = 12 + 15$

$h = 12 + 15$

$h = 27$

3. Solve $2x = 8$

$$\frac{2x}{2} = \frac{8}{2}$$

$x = 4$

Exercise

Find the value of the unknown

a) $y + 3 = 7$ b) $a - 2 = 7$ c) $m + 11 = 24$

d) $p - 11 = 12$ e) $x + 51 = 84$ f) $n - 57 = 63$

g) $r + 49 = 75$ h) $q - 43 = 41$ i) $p + 24 = 42$

j) $x - 28 = 36$ k) $3a = 21$ l) $8w = 72$

m) $13m = 260$ n) $21h = 168$ o) $20p = 400$

Word problems in algebra
Example
Katamba bought some eggs. On his way home 4 eggs broke and he was left with 8 eggs. How many eggs did he buy?

Let the eggs bought be n.

Eggs bought n

Eggs broken 4

Therefore n – 4 = 8

n – ~~4 + 4~~ = 8 + 4

n = 12

Exercise
1. A farmer had some cows. She paid 8 cows as dowery. Altogether she had 15 cows. How many cows had she before?
2. I think of a number, add 7 to it, my answer is 12. What is the number?
3. A tube brewed y litres of local beer. He sold 17 litres and was left with 4 litres. How many litres did he brew?
4. Bulya had y mathematics numbers to work out. She worked out 23 and was left with 2. How many numbers was she given altogether?

5. A number multiplied by 13 gives 52. Find the number.

Solving equations
Example
1. Solve: m +4m = 20

$$\frac{5m}{5} = \frac{20}{5}$$

$$m = 4$$

2. Solve: 3g + g + 2g = 30

$$\frac{6g}{6} = \frac{30}{6}$$

$$g = 5$$

Activity
Solve the equations
a) 2y + y = 12   b) 2x + 2x + x = 25

c) t + 4t = 45   d) p + 5p + 2p = 40

e) n + 7n = 32

f) A mother is 4 times as old as her daughter. Their total age is 30 years.
Find the daughter's age.

g) A father is 3 times as old as his daughter. Their total age is 48 years.
How old is the daughter?

Finding the unknown involving fractions

Examples

1. Solve: $a = 4$
3

$= 3 \times \dfrac{a}{3} = 4 \times 3$

a = 12

2. Find the number of oranges that can be divided among 5 boys so that
each gets 6 oranges.

Let the number of oranges be p.

So; $\dfrac{p}{5} = 6$

$5 \times \dfrac{p}{5} = 6 \times 5$

p = 30

Exercise

Find the value of the unkown.

a) $\dfrac{a}{2} = 13$　　　　　　　　b) $\dfrac{m}{4} = 8$　　　　　　　　c) $\dfrac{x}{7} = 5$

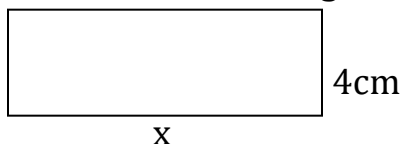d) $\dfrac{k}{9} = 8$　　　　　　　　e) $\dfrac{d}{4} = 20$

f) 4 pupils shared x books equally. Each pupil received 16 books. How
many books were there?

g) 8 homes shared p litres of paraffin equally. Each home received 64
litres. How much paraffin was shared?

Forming and solving equations

Example

The perimeter of a rectangle is 24cm. Find x.

```
┌─────────────────────┐
│                     │ 4cm
└─────────────────────┘
          x
```

l x w x l x w = perimeter

x + 4 + x + 4 = 24

x + x + 4 + 4 = 24

2x + 8   = 24

2x + ~~8 - 8~~   = 24 – 8

$\underline{2}$x   = $\underline{16}$

2   2
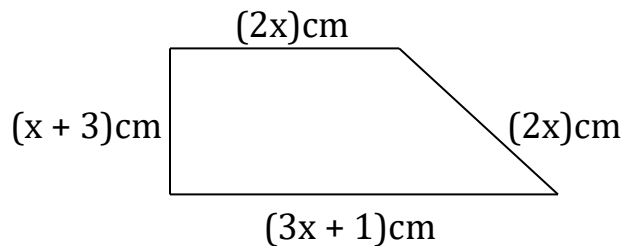
x   = **8cm**

Exercise

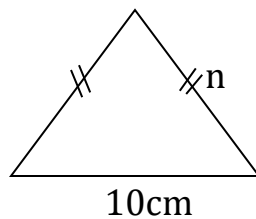Find the values of the unknown letter.

1. The perimeter of the triangle below is 60cm. Find x.

2. The perimeter of the trapezium below is 44 cm. Find x.

3. The perimeter of an isosceles triangle below is 36cm. Find n.

Solving equations involving brackets
Examples

1. Solve: 3 (y +4)   = 21

   3 x y + 3 x 4   = 21

   3y + 12   = 21

   3y + ~~12 – 12~~   = 21 – 12

   $\underline{3}$y   = $\underline{9}$

   3   3

$$y = 3$$

2. $5(y +1) -3(y - 1)$

$5y + 5 - 3y + 3$	$= 14$
$5y - 3y + 5 + 3$	$= 14$
$2y + 8$	$= 14$
$2y + 8 - 8$	$= 14 - 8$
$2y$	$= 6$
2	2
y	$= 3$

Exercise

Solve;

a) $2(x + 2) = 10$

b) $3 (x -2) + 2(x - 1) = 12$

c) $5(x + 1) = 15$

d) $4 (x +2) + 3(x - 1) = 12$

e) $7(x - 3) = 7$

f) $7(n + 3) - (2n - 4) = 35$

g) $6(p - 4) = 30$

h) $(p - 2) + (p - 4) = 0$

i) $6(x + 3) = 30$

j) $5(t - 1) - 3(t - 7) = 0$

More of equations
Examples

1. Solve: $4x - 3 = x + 6$

$4x - x$	$= 6 + 3$
$3x$	$= 9$
3	3
x	$= 3$

Exercise

a) $2x + 4 = x + 11$

b) $2x - 7 = x + 1$

c) $5x + 1 = 4x + 4$

d) $11x + 3 = x + 33$

e) $7x - 4 = 3x + 8$

f) $5(x - 2) = 2(x - 2)$

g) $6(x + 4) = 3 (x - 2)$

h) $(x - 1) = 4 (x - 12)$

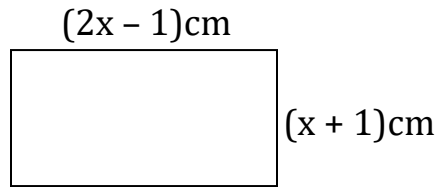i) $3(x - 2) = 2(x - 1)$

j) $7(x -2) = x + 10$

Forming equations and finding the unknown
Examples

1. Find the value of x in the figure.

(2x – 1)cm

(x + 1)cm

Opposite sides of a rectangle are equal

Therefore; 2x – 1 = x + 3

2x – x = 3 + 1

x   = 4

Exercise

Form equations and find x.

a)

2x   8cm

3x

b)

2x   5x

10x

c)

3x cm   15cm

2x + 5cm

d)

x   8cm

6cm

12cm

e)
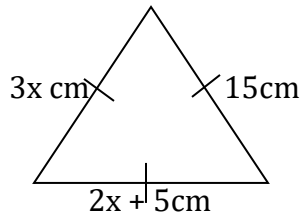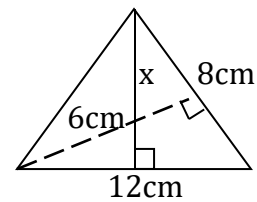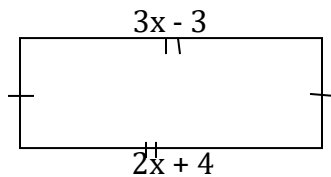

3x - 3

2x + 4

Basic algebraic symbols

=   equal to

≠   not equal

>   greater than

<   less than

≤   less than or equal to

≥   greater than or equal to

Examples

190

a) n = 4   means n is equal to 4

b) n ≠ 4   means n is not equal to 4

c) p > 5   means p is greater than 5

Exercise

1. Write in words.

a) x = 3

b) x ≠ 3

c) p ≤ 7

d) p < 7

e) y > 6

f) y ≥ 6

g) y < 10

h) t ≤ 10

i) x = 30

j) a > 13

2. State each of these using symbols.

a) m is less than 9

b) w is equal to 4

c) 8 is greater than x

d) h is less than or equal to 11

e) ½ and $^2/_4$ are equal

Finding the solution set for the inequality

Examples

1. Given the solution set for; x < 5

    (If x is a whole number)

    x is a whole number less than 5

    members of x = {0 , 1 , 2 , 3 , 4}

2. Given the solution set for; x > -5

    (if x is a negative integer)

    x > -5 means

    x is a negative integer than -5

    members of set x = { -4 , -3 , -2 , -2}

Find the solution set for the inequalities
1. a) x < 3 (If x is a counting number)
    b) x ≤ 3

2. n > -4 (If n is a negative integer)
    b) n ≥ -4
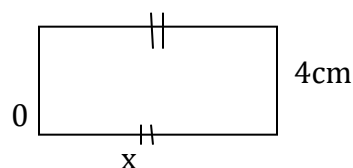
3. a) p < 8 (If p is a whole number)
    b) p ≤ 8

4. m > 7 (If m is a counting number)
    m ≥ 7

Forming and solving equations
Example
The perimeter of a rectangle is 24cm. Find x.

0

x

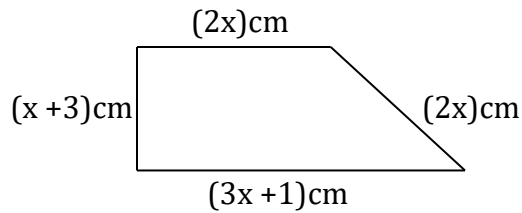|	
L + W + L + W	= Perimeter
x + 4 + x + 4	= 24
x + x + 4 + 4	= 24
2x + 8	= 24
2x + ~~8 - 8~~	= 24 – 8
2x	= 16
2	2
x	**= 8cm**

Exercise
Find the values of the unknown letter.
1. The perimeter of the triangle below is 60cm. Find x.

3x
25cm
4x

2. The perimeter of the trapezium below is 44cm. Find x.

(2x)cm

(x +3)cm

(2x)cm

(3x +1)cm

3. The perimeter of an isosceles triangle below is 36cm. Find n

Finding the solution set for the inequalities

Give sets of numbers and number lines for the following.

Examples

1. -1 < x < 3

   x represents integers between -1 and 3

The solution set for x = {0 , 1 , 2}

Find the solution sets and their number lines

a) -2 < y < 3

b) -2 $\leq$ y $\leq$ 3

c) -2 $\leq$ y < 3

d) -1 < p < 4

e) -1 $\leq$ p $\leq$ 4

f) -5 < q < 2

g) -5 < q $\leq$ 2

h) -3 $\leq$ x $\leq$ 4

Solving inequalities and finding solution set.

Example

Solve the inequality and find the solution set for x.

When 3 is added to x, the result is greater than 5

x + 3 > 5

x + 3 -3 > 5 – 3

x > 2

If x is a counting number less than 10

Therefore; x > 2 = a set of number greater than 2 but less than 10

= { 3 , 4 , 5 , 6 , 7 , 8 , 9 }

<u>***Exercise***</u>

Find the solution set for the inequalities below.

a) y + 2 > 4

b) p + 6 > 10

c) x + 3 < 9

d) n + 5 < 12

e) y + 7 > 11

f) When 4 is added to an integer (x) the result is less than 5. Find all possible values of n.

Measures (Money)

Uganda currency

The Uganda currency consists of the following;

Coins, bank notes

a) Coins are usually or a smaller value.

b) Bank notes are usually or bigger denomination than coins but also differ.

Each bank note has a value and an identifying number which is different from that on any other bank note.

The central bank writes new coins, and prints bank notes. The bank notes in each bundle are numbered.

<u>***Example***</u>

1. If bank notes are numbered consecutively from AP 003782 to AP 003881. How many notes are there?

First note   AP003782

Last note   AP003881

Number of notes

   AP   = 003881

- AP   =003782

               99

This means there are 99 notes without the last note. Therefore, the notes are (99 + 1) = 100 notes

Subtract and later add 1 note because during subtraction one note is left out. (99 + 1 = 100)

2. Amos has bank notes numbered from AP 004300 to AP 004399

a) How many bank notes does Amos have?

Number of notes

$$\begin{array}{r} \text{AP} \quad 004399 \\ - \underline{\text{AP} \quad 004300} \\ 99 + 1 \end{array}$$

100 notes

b) If each note is worth 1,000shillings in value, how much money does Amos have?

Amount of money in the bundle = 100notes x 1000/=

= sh. 100,000

Activity

1. Find how much money is in a bundle of sh. 1000 bank notes if they are numbered from UH 627400 to UH 627499.
2. A cashier is paying salaries to workers. How many 1000 shilling notes will give to a worker who gets a salary of sh. 90,000?
3. How many 500 shilling coins are equivalent to one ten thousand shilling notes?
4. Ali deposited some money. He has bank notes numbered from AZ 00360 to AZ 00389. The cashier told him that bank notes from AZ 000372 to AZ 00382 are counterfeit (forged). If the bank notes were of 10,000 shillings denomination;
   a) How much money did he deposit in the bank?
   b) How many bank notes were forged?

Uganda and other currencies

Different countries use different types of money. The money used by a country is its currency.

Example

Country	Currency
Uganda	Uganda shilling (Ush)
Kenya	Kenya shilling (Ksh)
Rwanda	Rwanda Francs (RWF)
South Africa	Rand (ZAB)
Zambia	Kwacha (Kch)
USA	US Dollars ($)
Britain	Pound sterling (€)
Japan	Japanese Ven (¥)
European union	Euro (Euro)
German	Deutsh mark (DM)

CURRENCY	BUYING	SELLING
1 pound sterling (€)	3730	3780
1 US Dollar (US $)	2355	2370
Kenya shilling	28	29
1 Rwanda Francs	2	2.4
1 Tanzania shilling (TZ)	1.57	1.58

Activity
1. Convert Ush. 34,000 to Kenya shillings.
2. I have Ush. 860,000. Find how much money I have in US Dollars.
3. A Tanzanian trader arrived in Uganda with TZ sh. 40,000. Find how much money she had in Uganda shillings if 1 T.Z sh. = 1.