

Kabojja Junior School
P.6/7 Maths. Lesson Notes

TOPIC : SETS

1. What is a set?

A set is a collection of well-defined objects.

Examples of sets

A sets of 5 books.

A set of 2 chairs.

A set of 3 cups.

A set of 6 girls.

2. Types of sets

- (a). An empty set
- (b). Subset
- (c). Equivalent sets
- (d). Equal sets
- (e). Union sets
- (f). Intersection sets
- (g). Disjoint sets
- (h). Universal sets
- (i). Complement of sets
- (j). None equivalent sets

Universal Sets:

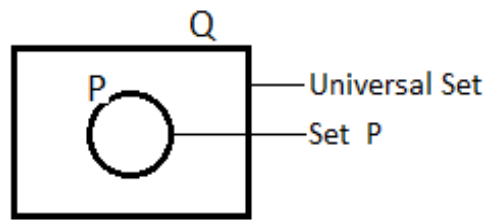
- 1. A Universal set is the set which contains all the things that we are considering.
- 2. A Universal set is a mother set from which we can get other sets.
- 3. The symbol of the Universal set is
 ξ or \mathcal{E} or Ξ
- 4. The universal set is shown using a rectangle.



EXAMPLE 1

Given that $Q = \{ \text{all pupils in a class} \}$ and $P = \{ \text{all girls in a class} \}$

Represent this information on a Venn diagram



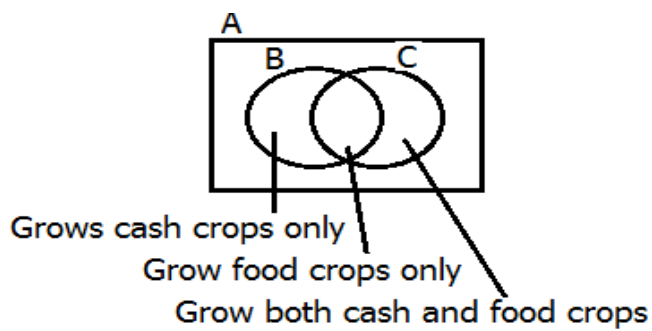
EXAMPLE 2

Given that $A = \{ \text{all farmers in ojwin village} \}$

$B = \{ \text{farmers who grow cash crops} \}$

$C = \{ \text{farmers who grow food crops} \}$

Representing this on a Venn diagram.

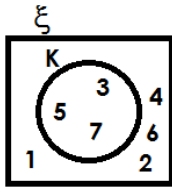


5. Examples of Universal Sets are:

- (a). $\{ \text{all pupils in P.6} \}$
- (b). $\{ \text{all pupils in Kabojja Junior School} \}$
- (c). $\{ \text{all countries in Africa} \}$
- (d). $\{ \text{all days of the week} \}$
- (e). $\{ \text{all months of the year} \}$
- (f). $\{ \text{all districts in Uganda} \}$
- (g). $\{ \text{all schools in Uganda} \}$
- (h). $\{ \text{all odd numbers less than 30} \}$
- (i). $\{ \text{all quadrilaterals} \}$
- (j). $\{ \text{all letters of the alphabet} \}$
- (k). $\{ \text{all animals with four legs} \}$

Complement of a set.

Consider the Venn diagram below.



- (a). Write down the members of set K.

$$K = \{3, 5, 7\}$$

- (b). Write down the members of ξ
 $\xi = \{1, 2, 3, 4, 5, 6, 7\}$

- (c). Write down the members which are not found in set K.
 $\{1, 2, 6, 4\}$

These members form the complement of set K.

1. The complement of set K is the set of elements in the universal set but not in set K.
i.e Elements outside set K form the complement of set K.

NB: The complement of a set is the set of elements found in the universal set but not in the given set.

2. The complement of K is written as K'
3. K' is read as set k complement.
4. $K \cap K' = \{ \}$ and $K \cup K' = \xi$

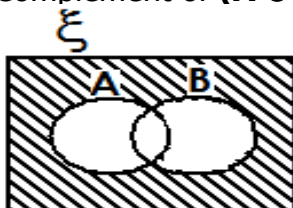
SHADING SET REGIONS

1. Complement of set K or K'



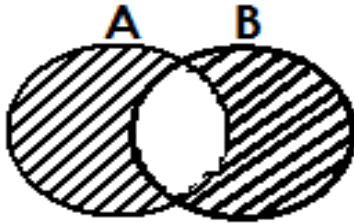
i.e shade the region outside set K.

2. Complement of $(A \cup B)$ or $(A \cup B)'$



i.e shade the region outside $(A \cup B)$.

3. Complement of $(A \cap B)$ or $(A \cap B)'$



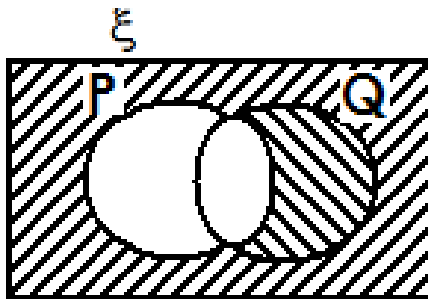
i.e shade the region outside $(A \cap B)$.

OR

(b).

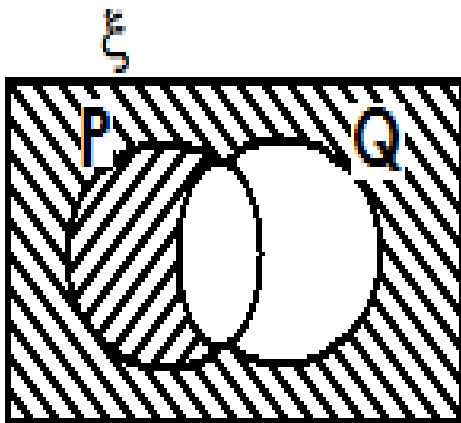


4. Complement of P or P'



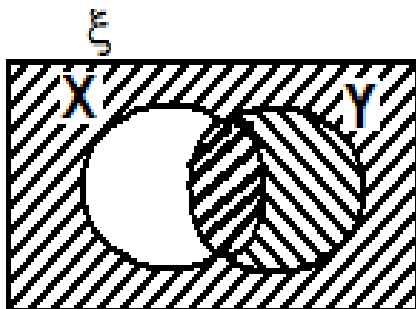
i.e shade the region outside set P.

5. In the Venn diagram below, shade set Q' .



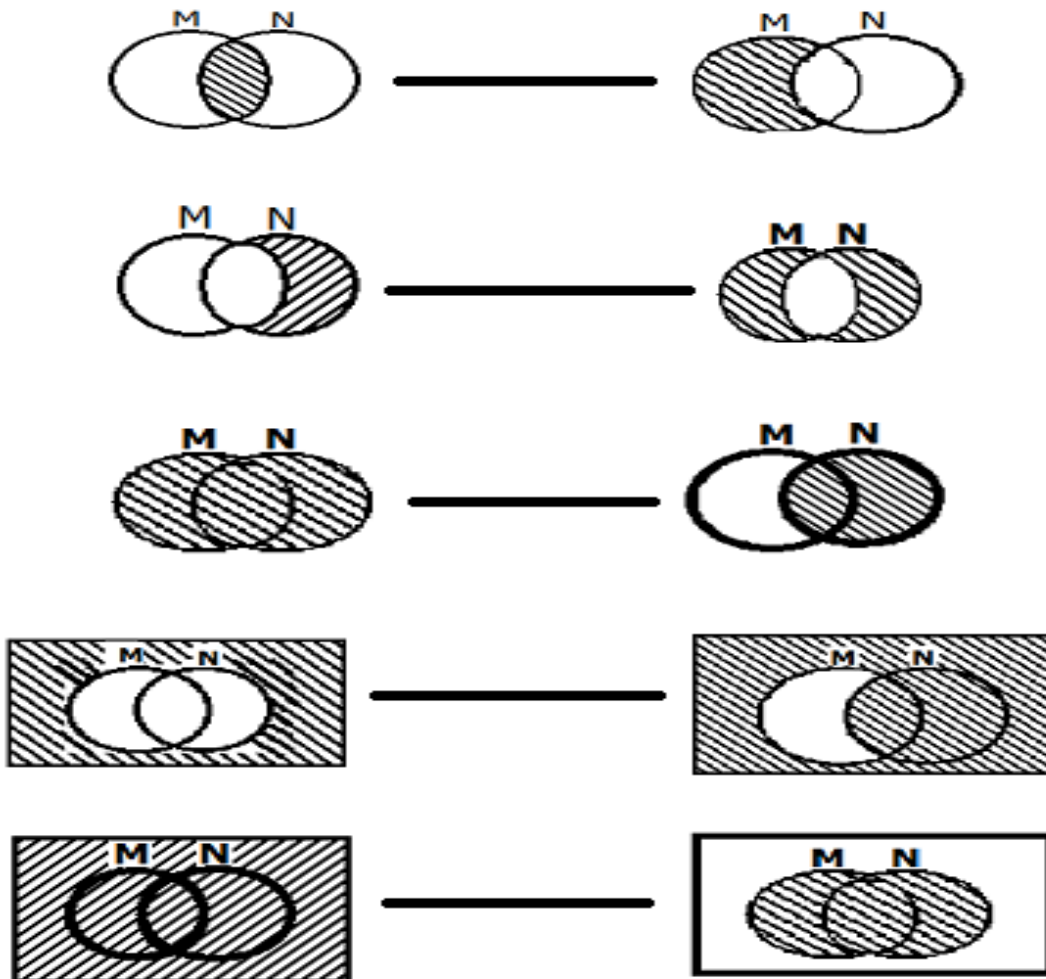
i.e shade outside set Q.

6. In the Venn diagram below shade $(X - Y)'$.



i.e shade outside $X - Y$.

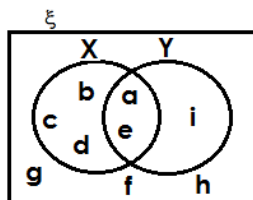
Describe the shaded set region



INTERPRETING INFORMATION GIVEN ON VENN DIAGRAM.

Examples:

1. Use the Venn diagram below to answer the questions that follow.



Write down the members of:

- (a). $X = \{b, c, d, a, e\}$
- (b). $Y = \{i, a, e\}$
- (c). $X \cap Y$

$$X \cap Y = \{a, e\}$$

(d). $X \cup Y$

$$X \cup Y = \{b, c, d, a, e, i\}$$

(e). $\xi = \{b, c, d, a, e, i, h, f, g\}$

(f). $X' = \{i, h, f, g\}$

(g). $Y' = \{b, c, d, h, f, g\}$

2. Find $n(X \cup Y)'$

$$(X \cup Y)' = \{h, f, g\}$$

$$n(X \cup Y)' = \underline{3}$$

3. Find $n(X \cap Y)'$

$$(X \cap Y)' = \{b, c, d, i, h, f, g\}$$

$$n(X \cap Y)' = \underline{7}$$

4. Find $n(X - Y)'$

$$(X - Y)' = \{a, e, i, h, f, g\}$$

$$n(X - Y)' = \underline{6}$$

5. Find $n(Y - X)'$

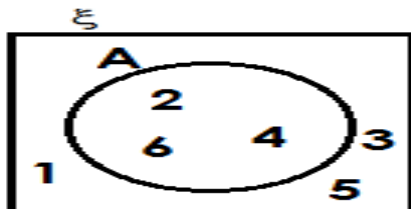
$$(Y - X)' = \{a, e, b, c, d, f, g, h\}$$

$$n(Y - X)' = \underline{8}$$

Exercise:

Use the Venn diagrams below to answer the questions that follow.

1.

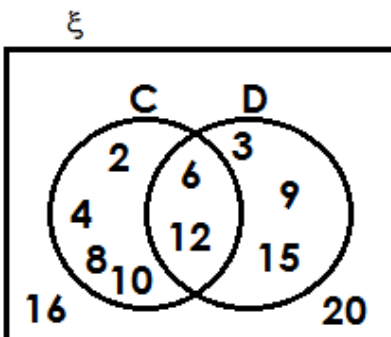


Write down the members of:

(a). A (b). A'

(c). ξ

2.



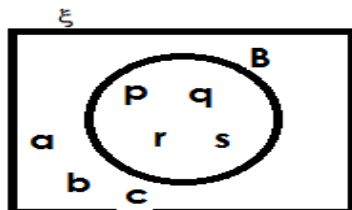
- (a). Write down the elements of;
- C
 - D
 - $C \cap D$
 - $C \cup D$
 - ξ
 - C'
 - D'
- (b). Find $n(C \cup D)'$
- (c). Find $n(C \cap D)'$
- (d). Find $n(C - D)'$
- (e). Find $n(D - C)'$

Representing information on Venn diagrams.

Examples:

1. If $\xi = \{a, b, c, p, q, r, s\}$ and $B = \{p, q, r, s\}$

- (a). Show the above information on a Venn diagram.



$$\xi = \{a, b, c, p, q, r, s\}$$

$$B = \{p, q, r, s\}$$

- (b). Find $n(B)'$

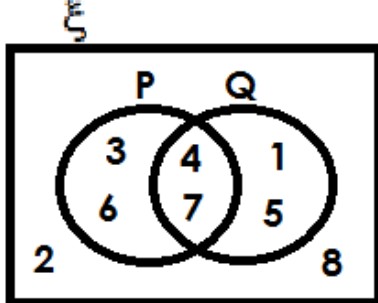
$$B' = \{a, b, c\}$$

$$n(B') = 3$$

2. Given that $\xi = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$P = \{3, 4, 6, 7\} \text{ and } Q = \{1, 4, 5, 7\}.$$

- (a). Show the above information on a Venn diagram.



$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$P = \{3, 4, 6, 7\}$$

$$Q = \{1, 4, 5, 7\}$$

$$P \cap Q = \{4, 7\}$$

- (b). Find $n(P \cap Q)'$

$$(P \cap Q)' = \{3, 6, 1, 5, 2, 8\}$$

$$n(P \cap Q)' = \underline{6}$$

Exercise:

1. If $\xi = \{1,2,3,4,5\}$ and $A = \{2,4\}$.
 - (a). Show the above information on a Venn diagram.
 - (b). Find $n(A')$
2. Given that $\xi = \{a,b,c,d,e,f,g,h,i,j\}$ and $C = \{d,e,f,g,h\}$.
 - (a). Show the information above on a Venn diagram.
 - (b). Find $n(B \cap C)'$
3. Given that $\xi = \{1,2,3,4,5,6,7\}$,
 $M = \{1,2,6,7\}$ and $N = \{2,4,6\}$.
 - (a). Show the above information on a Venn diagram.
 - (b). Find $n(M - N)'$

SUBSETS

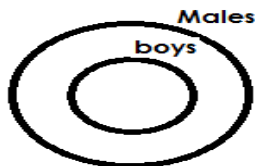
1. A subset is a small set from a big set (**universal set**).
2. The symbol for "is a subset of" is \subset
3. A subset is a part of the set.
4. Set A is a subset of set B. If each member of A also a member of B.
5. The symbol \in means "is a member of".

Examples:

1. Draw a Venn diagram to show that all cows (C) are domestic animals (A).

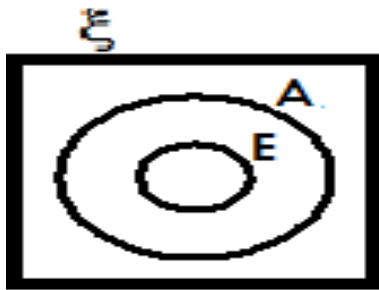


2. Draw a Venn diagram to show that all boys are males.

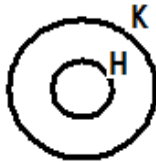


3. Given that $\xi = \{\text{all countries in the world}\}$, $A = \{\text{all countries in Africa}\}$ and $E = \{\text{countries in East Africa}\}$.

Draw a Venn diagram to represent this information.



4. In the Venn diagram below, what is the relationship between set H and set K?



Set H is a subset of set K or $H \subset K$.

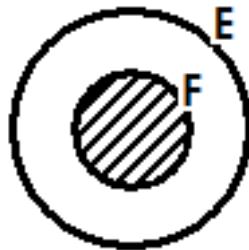
5. E and F are sets. Draw Venn diagrams to show the following relations.

- (a). $E \cap F = F$

$$E = \{1,2,3,4,5,6\}$$

$$F = \{2,4,6\}$$

$$E \cap F = \{2,4,6\}$$



- (b). $E \cup F = E$

$$E = \{1,2,3,4,5,6\}$$

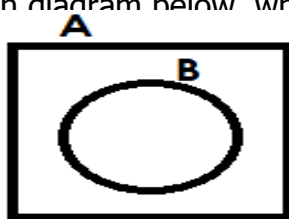
$$F = \{2,4,6\}$$

$$E \cap F = \{1,2,3,4,5,6\}$$



Exercise:

1. Draw a Venn diagram to show that all girls (G) are females.
2. Draw a Venn diagram to show that all cats are animals.
3. Show the relationship between the following sets using a Venn diagram.
4. In the Venn diagram below, what is the relationship between set A and set B?



5. Given that $\xi = \{\text{all days of the week}\}$ and $A = \{\text{Saturday, Sunday}\}$, what is the relationship between ξ and A?

Forming Subsets.

NOTE:

1. An empty set is a subset of every set.
2. Every set is a subset of itself.

Examples:

1. If $Y = \{0\}$
 - (a). List all the possible subsets of Y.
 $\{ \} , \{ 0 , 1 \}$
 - (b). How many subsets does set Y have?
Set Y has 2 subsets.
2. Given that $K = \{1,2,3,4,5,6\}$ and
 $N = \{3,5,7\}$.
Find $K \cap N$.
 $K = \{1,2,3,4,5,6\}$
 $N = \{3,5,7\}$
Subsets of $K \cap N$ are:
 $\{ \} , \{3\} , \{5\} , \{3,5\}$
 $K \cap N$ has **4 subsets**.
3. Given that set $M = \{1,b,c,d\}$, how many subsets does set M have?
Subsets of M are:
 $\{ \} , \{a\} , \{b\} , \{c\} , \{d\} , \{a,b\} , \{a,c\} , \{a,d\} , \{b,c\} , \{b,d\} , \{c,d\} , \{a,b,c\} ,$
 $\{a,b,d\} , \{a,c,d\} , \{b,c,d\} , \{a,b,c,d\}$
Set M has 16 subsets.

Exercise:

1. Given that $K = \{9\}$
 - (a). List all the subsets of K.
 $\{ \} , \{a\}$
 - (b). How many subsets does set K have?
Set K has 2 subsets.
2. If $A = \{x,y\}$, how many subsets does set A have?
Subsets of A are:

$\{ \}, \{x\}, \{y\}, \{x,y\}$

No of subsets in A.

Set A has 4 subsets.

3. How many subsets does set $X = \{ \}$ have?

Subsets of x are: $\{ \}$

No. of subsets in A are:

Set A has 1 subset.

4. Given that $S = \{1,2,3\}$.

- (a). Write down all the possible subsets.

Subsets of S are:

$\{ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

- (b). How many subsets does set S have?

Set S has 8 subsets.

5. If $B = \{2,3,5,7\}$, how many subsets does set B have?

Subset in set B are:

$\{ \}, \{2\}, \{3\}, \{5\}, \{7\}, \{2,3\}, \{2,5\}, \{2,7\}, \{3,5\}, \{3,7\}, \{5,7\}, \{2,3,5\}, \{2,3,7\}, \{2,3,5,7\}, \{3,5,7\}, \{2,5,7\}$

Set B has 16 subsets.

Formula for finding the number of subsets.

Consider the table below.

No. of elements	The set	No. of subsets	No. of subsets in expanded form	No. of subsets in power form
0	$\{ \}$	1	1	2^0
1	$\{a\}$	2	2	2^1
2	$\{a,b\}$	4	2×2	2^2
3	$\{a,b,c\}$	8	$2 \times 2 \times 2$	2^3
n			$2 \times 2 \times 2 \dots n \text{ times}$	2^n

From the table, we see that:

- Number of subsets = 2^n

where **n** is the number of elements in a given set.

Examples:

1. If $E = \{2,3,5,7,11,13\}$, find the number of subsets in E.

$$n(E) = 6$$

$$\text{No. of subsets} = 2^n$$

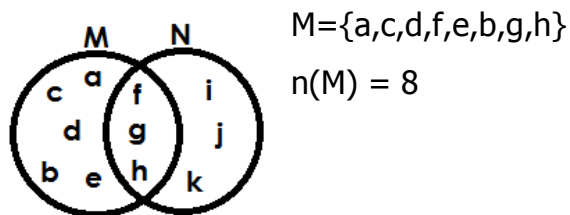
$$= 2^6$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 8 \times 8$$

$$= \underline{\underline{64}}$$

2. In the Venn diagram below, find the number of subsets in set M.



$$\text{No. of subsets} = 2^n$$

$$(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2)$$

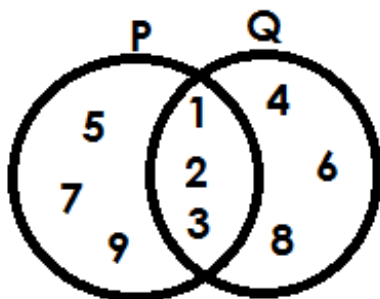
$$8 \times 8 \times 4$$

$$= 64 \times 4$$

$$= \underline{\underline{256}}$$

Exercise:

1. If $Q = \{g, h\}$, find the number of subsets in Q.
2. Given that set $B = \{p, q, r\}$, how many subsets does set B have?
3. If $P = \{0, 2, 4, 6, 8\}$, how many subsets does P have?
4. Given that $A = \{a, b, c, d\}$, find the number of subsets in A.
5. Given that $n(K) = 7$, find the number of subsets in set K.
6. In the Venn diagram below, find the number of subsets in $P \cup Q$.



Finding the number of elements in a set.

Examples:

1. There are 256 subsets in set X. How many elements does set X have?

$$2^n = \text{No. of subsets}$$

$$2^n = 256$$

2	256	
2	128	
2	64	
2	32	$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
2	16	
2	8	$= 2^8$
2	4	$= 2^n = 2^8$
2	2	$n = \underline{8 \text{ elements}}$
	1	

2. The number of subsets in B is 128. How many members are in set B?

$$2^n = \text{No. of subsets}$$

$$2^n = 128$$

2	128	
2	64	$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
2	32	$= 2^7$
2	16	$= 2^n = 2^7$
2	8	
2	4	$n = \underline{7 \text{ members}}$
2	2	
	1	

Exercise:

- Set T has 2 subsets. How many members are in set T?
- The number of subsets in set A is 16. How many elements are in set A?
- Set P has 8 subsets, find the number of elements in set P.
- The number of subsets in Q is 32. How many members are in set Q?
- Set K has 4 subsets. Find the number of elements in set K.
- The number of subsets in set E is 64. Find the number of members in set E.
- Set M has 1 subset. How many members are in set M?

Proper Subsets

1. A proper subset is a subset which does not contain all the members of the given set.
2. A proper subset is a subset that is not equal to the given set.

Improper Subset:

1. An improper subset is a subset which contains every elements of the original (given) set.

Examples:

1. If $A = \{1,3,5\}$, write down all the proper subsets of set A.

- (a). Proper sets of A are:

$\{ \} , \{1\} , \{3\} , \{5\} , \{1,3\} , \{1,5\} \{3,5\}$

- (b). How many proper subsets are in set A?

Set A has 7 proper subsets.

The number of proper subsets is one less than the number of subsets in a set.

No. of proper subsets = $2^n - 1$

2. If $K = \{c,d,e,f,g\}$, find the number of proper subsets in set K.

$$n(K) = 5$$

No. of proper subsets = $2^n - 1$

$$= 2^5 - 1$$

$$= (2 \times 2 \times 2 \times 2 \times 2) - 1$$

$$= 32 - 1$$

$$= 31 \text{ proper subsets}$$

Exercise:

1. If $A = \{p,q\}$, write down all the proper subsets of A.
2. How many proper subsets are in a set with 4 elements?
3. Given that $P = \{2,4,5\}$, find the number of proper subsets in set P.
4. If set $M = \{a,b,c,d,e,f\}$, how many proper subsets are in set M?
5. Given that set $Z = \{6,7,8,9\}$, write down all the proper subsets of Z.

Finding the number of elements in a set.

Examples:

1. The number of proper sets in set G is 127. How many elements are in set G?

$$\begin{aligned}2^n - 1 &= \text{No. of proper subsets} \\2^n - 1 &= 127 \\2^n - 1 + 1 &= 127 + 1 \\2^n &= 128\end{aligned}$$

2	128	$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ $= 2^7$ $2^n = 2^7$ $n = 7$
2	64	
2	32	
2	16	
2	8	
2	4	
2	2	
2	1	

Exercise:

- The number of proper subsets in set A is 3. Find the number of elements in set A.
- Set H has 15 proper subsets. How many members are in set H?
- There are 7 proper subsets in asset K. How many elements are in set K?
- Set P has 31 proper subsets. Find the number of members in set P.
- The number of proper subsets in set W is 63. How many elements are in set W?

DIFFERENCE IN SETS.

3. This is a set of elements in one set but not in the other. Difference of sets are donated by $A - B$.

$A - B$ means elements in set A which are not in set B. (Set A only)

$B - A$ means elements in set B which are not in set A (Set B only)

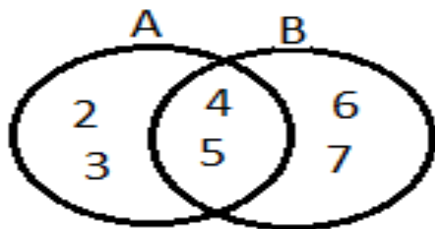
Example I:

- (a). Find $A - B$

$$A - B = \{2, 3\}$$

- (b). $B - A$

$$B - A = \{6, 7\}$$



Example II:

Given that $A = \{a, b, c, d, e\}$ and $B = \{a, e, t, q, u\}$

Find; i. $A - B$

ii. Find $B - A$

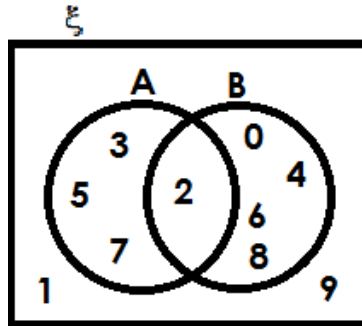
SUBTOPIC: SHOWING NUMBER OF MEMBERS

Examples:

1. If $\xi = \{\text{all numbers less than 10}\}$

$A = \{\text{all prime numbers less than 10}\}$ and $B = \{\text{all even numbers less than 10}\}$.

(a). Show the above information in the Venn diagram.

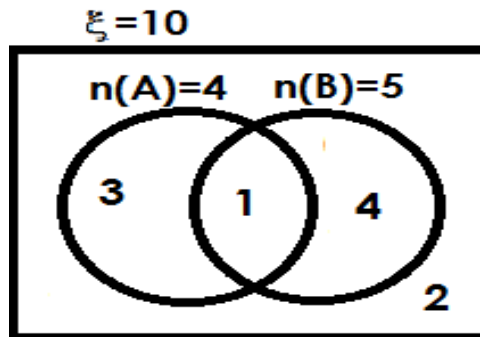


(b). Draw a Venn diagram to show the number of elements in the above sets.

$$n(A) = 4, n(B) = 5, n(\xi) = 10,$$

$$n(A \cap B) = 1, n(A - B) = 3,$$

$$n(B - A) = 4, n(A \cup B)' = 2$$



(c). Use the Venn diagram to find:

i. $n(A \cup B)$

$$n(A \cup B) = n(A - B) + n(A \cap B) +$$

$$n(B - A)$$

$$3 + 1 + 4 = \underline{8}$$

ii. $n(A)$

$$n(A) = n(B) + n(A \cap B)$$

$$3 + 1 = \underline{4}$$

iii. $n(B)$

$$n(B) = n(B - A) + n(A \cap B)$$

$$4 + 1 = \underline{5}$$

iv. $n(\xi)$

$$n(\xi) = n(\xi - B) + n(A \cap B) +$$

$$n(B - A) + n(A \cup B)'$$

$$= 3 + 1 + 4 + 2 = \underline{10}$$

v. $n(A \cap B)'$

$$n(A \cap B)' = n(A - B) + n(B - A) +$$

$$n(A \cup B)'$$

$$= 3 + 4 + 2 = \underline{9}$$

EXERCISE.

Show the number of elements of the following sets in the Venn diagram.

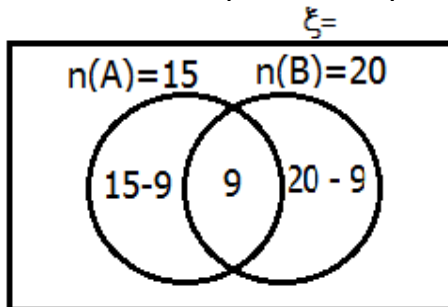
1. $G = [1, 2, 3, 4, 5, 6]$
 $H = [0, 2, 4, 7, 9]$
2. Set $M = [a, e, I, o, u]$
Set $N = [a, d, u, w, f]$
3. Set $L = [1, 2, 3, 4, 5, 6]$
Set $M = [2, 4, 9, 11]$
4. Set $P = [a, e, I, o, u]$
Set $Q = [a, b, c, d, e, f, g]$
5. Set $V = [Jane, Sarah, Andrew, Henry, Marvin]$
Set $D = [Amos, Josehp, Deo, Henry, Andrew]$

SUBTOPIC : DRAWING AND REPRESENTING THE INFORMATION ON A VENN DIAGRAM

Example 1

Given that $n(A) = 15$, $n(B) = 20$ and $n(A \cap B) = 9$

Draw the Venn diagram and represent the information



Find $n(A - B)$

Find $n(B - A)$

TRY

The number of pupils who do Maths.(M) = 24 and the number of pupils who do English (E) = 30 .

If there are 16 pupils who do both.

- i. Draw a venn diagram and find out how many pupils do one subject.
- ii. Find $n(M - E)$
- iii. Find $n(E - M)$
- iv. How many pupils like one subject?
- v. How many pupils are in the class?

EXERCISE

1. Draw the Venn diagram for these sets $n(P) = 16$, $n(Q) = 27$ and $(P \cap Q) = 8$
 - i. Find $(P - Q)$
 - ii. $n(Q - P)$
 - iii. $n(P \cup Q)$
2. Given that $n(K) = 32$, $n(L) = 27$ and $n(K \cap L) = 19$
 - i. Draw the Venn diagram for these sets
 - ii. Find $n(K - L)$
 - iii. Find $n(L - K)$
 - iv. Find $n(L \cup K)$
3. Given that $n(Q) = 17$, $n(P) = 21$ and $n(P \cap Q) = 12$
 - i. Draw a Venn diagram for these sets
 - ii. Find $n(Q - P)$
 - iii. Find $n(R - Q)$
 - iv. Find $n(P \cup Q)$
4. Given that $n(M) = 15$, $n(N) = 20$ and $n(M \cap N) = 8$
 - i. Draw a Venn diagram to show the sets.
 - ii. Find $n(M - N)$
 - iii. Find $n(N - M)$
 - iv. Find $n(M \cup N)$
5. Given that $n(A - B) = 4$, $n(B - A) = 6$ and $n(A \cup B) = 15$
 - i. Represent the above information on the Venn diagram.
 - ii. Find $n(A \cap B)$
 - iii. Find $n(A)$
 - iv. Find $n(B)$
6. Given that $n(X \cup P) = 28$, $n(X) = 17$, $n(P - X) = 11$ and $n(P) = 16$
 - i. Represent the above information on the Venn diagram.
 - ii. Find $n(X \cap P)$
 - iii. Find $n(X - P)$
7. Given that $n(J)$ only = 7, $n(W)$ only = 8 and $n(J \cup W) = 25$
 - i. Represent the above information on the Venn diagram.
 - ii. Find $n(J \cap W)$
 - iii. Find $n(J)$
 - iv. Find $n(W)$
8. Given that $n(W) = 20$, $n(M) = 32$ and $n(W \cap M) = 7$.
 - i. Represent the above information on the Venn diagram.
 - ii. Find $n(W \cup M)$
 - iii. Find $n(W - M)$
 - iv. Find $n(M - W)$
9. Given that $n(A - B) = 2$, $n(B - A) = 2$, $n(A \cup B) = 7$ and $n(A \cup B)' = 1$.
 - (a). Represent the above information on a Venn diagram.
 - (b). Find $n(A \cap B)$, $n(\xi)$, $n(A \cap B)^1$, $n(B - A)^1$

SUBTOPIC:

APPLICATION OF SETS AND PROBABILITY.

PROBABILITY (Chances)

1. What's probability?

Probability is the measure of the likelihood that an event will happen/ occur.

2. An event(E) is something that happens.
3. Likelihood is the chance of something happening.
4. Chance is the possibility of something happening.
5. If you throw an ordinary dice, there are six possible outcomes namely: 1, 2, 3, 4, 5 and 6
6. The act of throwing a dice is called an **experiment**.
7. The score that you get is called an **out come or event**.
8. **Sample space** is the set of all possible outcomes of an experiment.

PROBABILITY SCALE

1. An event that has a probability of 1 is a **Certain event**.

Ie $P(\text{certainty})=1$

Eg the probability that one day each of us will die is 1.

2. An event that has a probability of 0 is an **impossible event**.

Ie $P(\text{impossibility})=0$

Eg the probability of a goat flying is 0

3. All probabilities must therefore have a value between 0 and 1.
4. Probability can be expressed as a fraction, decimal or percentage.
5. The probability of an event is the ratio of the number of favorable outcomes to the number of all possible outcomes.

Ie $P(E)=\frac{\text{number of favourable outcomes}}{\text{Total number of possible outcomes.}}$

$P(E)=\frac{n(E)}{n(S)}$

TOTAL PROBABILITY

$P(\text{favourable outcomes}) + P(\text{unfavourable outcomes}) = 1$

ROLLING OR THROWING A DICE

EXAPMLE 1

A dice is rolled once on a table, what's the probability that a 6 will show up?

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(s) = 6$$

$$E = \{6\}$$

$$n(E) = 1$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$n(S)$$

$$P(6) = \frac{1}{6}$$

$$6$$

Example 2

A dice is rolled once on a table, what's the probability that a number less than 5 will appear on top?

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(s) = 6$$

$$E = \{1, 2, 3, 4\}$$

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$n(S)$$

$$P(\text{No} < 5) = \frac{4}{6}$$

$$6$$

EXERCISE

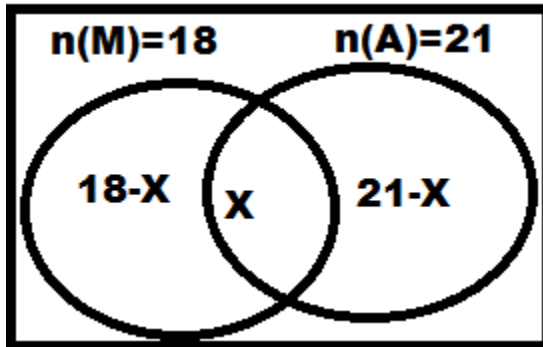
1. A dice is tossed once, what is the probability that a 4 will appear?
2. A dice is tossed once, what is the probability that an odd number appears?
3. A dice is rolled once, what is the probability that a number greater than 4 appears on top?
4. A cube whose faces are numbered from 1 to 6 is rolled once, what is the probability that an even number appears?

APPLICATION OF SETS

Examples

1. In a class of 30 pupils 18 like music (M), 21 like Art (A) and x like both subjects.
(a) Represent the above information on a Venn diagram.

$$n(\Sigma) = 30$$



- (b) How many pupils like both subjects.

Solution

$$18 - x + x + 21 - x = 30$$

$$18 + 21 - x = 30$$

$$39 - x = 30$$

$$39 - 39 - x = 30 - 39$$

$$-x = -9$$

$$\frac{-x}{-1} = \frac{-9}{-1}$$

$$\underline{\underline{X = 9}}$$

\therefore 9 Pupils like both subjects

- b) If a pupil is picked at random, what is the probability that the pupil picked took only one subject.

n(M) only + n(A) only

$$18 - x + 21 - x$$

$$18 - 9 + 21 - 9$$

$$9 + 12$$

$$11 \text{ pupils}$$

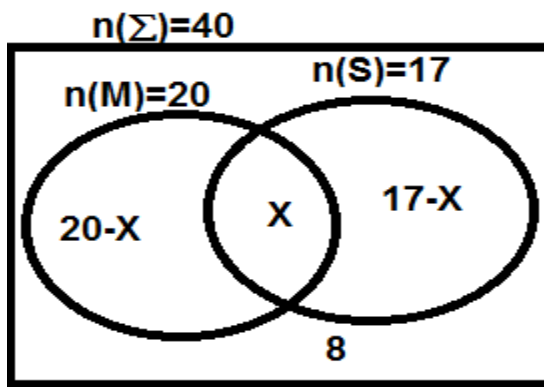
$$n(E) = 11 \text{ and } n(S) = 30$$

$$\text{hence } P(\text{only one subject}) = \frac{n(E)}{n(S)}$$

$$= \frac{11}{30}$$

2. In a class of 40 pupils, 20 like mathematics (M), 17 like science (S), 'x' like both subjects while 8 do not like any of the subjects.
(a) Represent the above information on a Venn diagram.

Solution



(b) How many pupils like both subjects?

Solution

$$8 + 20 - x + x + 17 - x = 40$$

$$28 + 17 - x = 40$$

$$45 - x = 40$$

$$45 - 45 - x = 40 - 45$$

$$-x = -5$$

$$\frac{-x}{-1} = \frac{-5}{-1}$$

$$-1 = -1$$

$$\underline{X = 5}$$

$\therefore 5$ Pupils like both subjects

(c) What is the probability of selecting a pupil who likes only one subject?

Solution

$$(20 - x) + (17 - x)$$

$$(20 - 5) + (17 - 5)$$

$$15 + 12$$

$$= \underline{27}$$

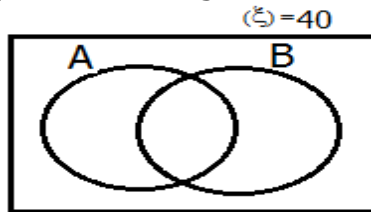
$$n(E)=27, n(S)=40$$

$$P(\text{only one subject}) = \frac{n(E)}{N(S)}$$

$$= \frac{27}{40}$$

EXERCISE

1. Given that, in a class of 30 pupils, 18 like Music (M), 21 like Art (A). If x pupils like both music and Art.
 - i. draw the Venn diagram and find the value of x
 - ii. how many pupils like music only?
 - iii. how many pupils like Art only?
 - iv. how many pupils like only one subject?
 - v. What is the probability of picking a pupil who likes only Art?
 - vi. What is the probability of picking a child who likes Art?
2. Study the Venn diagram. Given that $n(\xi) = 40$, $n(A - B) = 15$ and $n(B - A) = 20$



- i. Find $n(A \cap B)$
- ii. Find $n(A)$
- iii. Find $n(B)$

3. There are 24 boys in the field. 12 like football (F) 16 like hockey (H). x like both.
 - i. Draw the Venn diagram to show this information
 - ii. How many boys like football only?
 - iii. How many boys like only one game?
 - iv. What is the probability of picking a boy who likes only one game?
 - v. What is the probability of picking a boy who likes football only?
4. In a class of 42 pupils, 6 like Maths. Only. 10 like English only and 12 like neither.
 - i. Draw the Venn diagram and show the information.
 - ii. How many pupils like all the three subjects?
 - iii. How many pupils like only one subject?
 - iv. How many pupils like at least one subject?
 - v. What is the probability of selecting at random a pupil who likes Maths.?

Give more examples involving three Venn diagrams. Reference Bk 7(MK) Pg 11-19

MORE ABOUT PROBABILITIES 1

Examples:

1. A school bag contains 6 yellow and 5 green. What is the prob. Of picking a yellow pencil at random?
2. In a class of 36 pupils, 12 are girls and the rest are boys. Find the prob. of picking at random who is a boy?

Exercise

1. A bag contain 7 blue and 6 red pens. What is the prob. of picking a red pen at random from the bag?
2. A basket contains 3 rotten eggs and 6 good eggs. If the eggs in a basket are mixed, what is the prob. of picking a rotten egg from the basket?
3. A tin contain 12 mangoes, 7 of them are ripe. Find the prob. of picking an unripe mangoes from the tin.
4. In class, there are 30 boys and 40girls. What is the prob. that pupil picked to be a class monitor is a girl?

MORE ABOUT PROBABILITIES 2

Examples

1. A class will have a party next week. What is the prob. that the party will take place on a day starting with letter T.
2. Kimati's mother is pregnant. What is the prob. that she will give birth to a baby girl?
3. A coin is tossed once, what is the prob. that it will show a tail?

Exercise

1. Primary six pupils will have a party next week. Find the prob. that the party will take place on day starting with letter S.
3. A football team can win, draw or lose a match. What is the probability that it will win a match?
4. Musa's mother is pregnant. What is the prob. that she will give birth to a baby boy?
5. If a coin is tossed up, what is the probability of a head showing up?

MORE ABOUT PROBABILITIES 3

Examples

1. The Prob. that an event will take place is $\frac{3}{4}$. Find the prob. that it will not take place.

$$\begin{aligned}P(\text{event not take place}) &= 1 - P(\text{event take place}) \\ &= 1 - \frac{3}{4}\end{aligned}$$

2. In a car park, there is a prob. that a car picked at random is $\frac{3}{7}$ made from Britain. There are 175 cars in the car park. How many cars are not British made?

$$\begin{aligned}P(\text{not British made}) &= 1 - P(\text{British made}) \\ &= 1 - \frac{3}{7} \\ &= \frac{7}{7} - \frac{3}{7} \\ &= \frac{4}{7}\end{aligned}$$

$$\text{NO. of cars not made in Britain} = \frac{4}{7} \times 175 = 100 \text{ cars}$$

Method II

No. made in British

$$\frac{3}{7} \times 175 = 75 \text{ cars}$$

No. not made in British

$$175 \text{ cars} - 75 \text{ cars} = 100 \text{ cars}$$

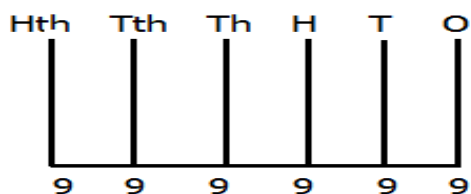
Exercise

1. The prob. that Sarah will pass her exams is $\frac{3}{5}$. What is the prob. that she will fail.
2. The prob. that Chelsea FC will win a match is 0.8. Find the prob. that it lose the match?
3. The prob. that a football team

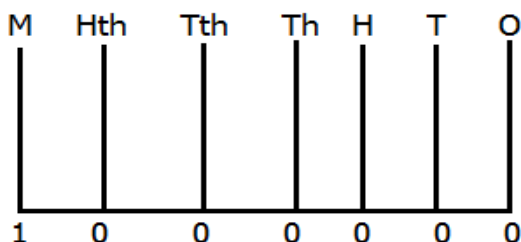
***TWO*: WHOLE NUMBERS**

Making abaci with place values up to 9 million

a). Show 999999 on an abacus below.



b). Show 1,000,000 on an abacus below.



The new number has six zeros. It is called one million.

Reading place values as indicated on the abaci

Identify the place value of each digit.

a). 7277

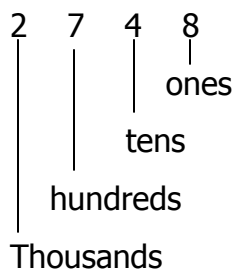
b). 201481

c). 100020

d). 4138294

Finding the value of each digit.

Example 1: 2748



The value of 2 = $2 \times 1000 = 2000$

The value of 7 = $7 \times 100 = 700$

The value of 4 = $4 \times 10 = 40$

The value of 8 = $8 \times 1 = 8$

1. 2 5 6 3

 | | | |

 | | | 3 x 1 = 3

 | | 6 x 10 = 60

 | 5 x 100 = 500

 2 x 1000 = 2000 0 + 60 + 3

+ 0 + 2000 + 60 + 3

2.

8	4	0	7	9
				$9 \times 1 = 9$
			$7 \times 10 = 60$	
		$0 \times 100 = 0$		
	$4 \times 1000 = 4000$			
$8 \times 10000 = 80000$				

Expand the following numbers using values.

- Find the value of each underlined digit in the following:**

1. Find the sum of the value of the underlined digits in the following.

2. Find the difference of the value of the underlined digits.

3. Find the product of the value of the underlined digits.

- i. 7238 ii. 9645 iii. 693427

Expressing numbers in words.

Example: Write in words 2, 045, 300

M	Thou	Unit
2	045	300

Two million, forty-five thousand, three hundred

Activity:

Express the following in words.

1. 3, 542, 125
2. 760,000
3. 3003
4. 101,740
5. 70,006
6. 530,540
7. 1,001,001
8. 99,099,099

Expressing in figures.

Example: Seven million, three hundred twenty six thousand eight hundred fifty seven.

Seven million = 7,000,000

Three hundred Twenty six thousand = 326,000

Eight hundred fifty seven = 857

7,326,857

Express the following in figures.

1. Three million, forty three,
2. Two million, eight hundred thousand,
3. One million, two hundred thirty four thousand five hundred sixty eight.
4. Six million, three hundred nineteen.
5. Seven million, three hundred fifty two thousand.
6. Nine million, forty seven thousand thirty six.
7. **More work on page 32 (Understanding Mtcs Bk 6).**

ROMAN / HINDU ARABIC NUMERALS.

Hindu Arabic	1	5	10	50	100	500	1000
Roman	I	V	X	L	C	D	M

1. The following are repeated numerals.

I, X, C and M e.g

2 = II, 20 = XX, 300 = CCC, 2000 = MM

Maximum 3 times.

2. The following are not repeated; V, L and D

3. Numbers with 6, 7 and 8 are additional Roman numerals.

6 = 5 + 1 = VI 8 = 5 + 3 = VIII 60 = 50 + 10 = LX

7 = 5 + 2 = VII 600 = 500 + 100 = DC 800 = 500 + 300 = DCCC

Expressing Hindu Arabic Numerals in Roman numerals.

Example 1:

75 = 70 + 5

+ 5

= LXX + V

+ V

= LXXV

Example 2:

555 = 500 + 50 + 5

D + L + V

DLV

Example 3:

445 = 400 + 40

CD + XL

CDXLV

Express the following in Roman numerals.

1. 68 2. 489 3. 572 4. 72 5. 445

6. 141 7. 392 8. 458 9. 764 10. 868

Express the following to Hindu Arabic numerals.

1. XIX 2. XCV 3. XXI 4. XXIV 5. CXIX

6. CX 7. CIV 8. XLVIII 9. CL 10. LXXV

11. XC 12. CD 13. MCDL 14. MCMXX 15. MM

16. MMDV 17. MDCLIX 18. MCCXXII

Word problems involving Roman Numerals Pg. 50 Mk 6.

SUBTOPIC : BASES

1. Counting in groups is referred to as bases.
There are two ways of grouping
 - i. Decimal system. This is counting in groups of ten
 - ii. Non decimal system. This is counting in other groups other than ten.
2. Special names for different bases

Base Two - binary

Base Three – Ternary

Base four - quaternary

Base five - quinary

Base six - Senary

Base seven - septenary

Base eight – Octal / Octonary

Base nine – nonary

Base ten – decimal / denary / whole number

4. Numerals used in each base.

Base two= 0,1

Base three = 0,1,2

Base four = 0,1,2,3

Base five = 0,1,2,3,4

Base six = 0,1,2,3,4,5

Base seven = 0,1,2,3,4,5,6

Base eight = 0,1,2,3,4,5,6,7

Base nine = 0,1,2,3,4,5,6,7,8

Base ten = 0,1,2,3,4,5,6,7,8,9

5. Each number base has a different place value.

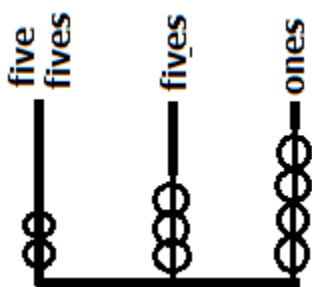
Example 1:

4 3 2 five
| | |
| | ones
fives
twenty fives (five fives)

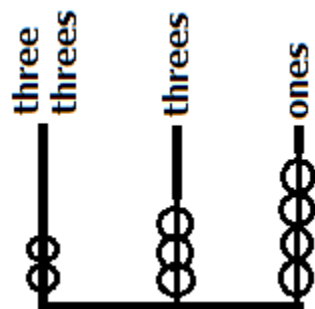
1 **0** **1** two
| | |
ones
twos
two twos

Representing Bases on Abacii

(a). $2\ 3\ 4_{\text{five}}$



(b). $1\ 2\ 0_{\text{three}}$



EXERCISE

Give the place value of the following.

1. 23_{five}

2. 43_{five}

3. 1001_{two}

4.

372_{eight}

5. 111_{two}

6. 101_{two}

7. 24_{five}

8.

231_{five}

9. 314_{five}

NOTE: To get the next place value from ones, multiply the previous one by the given base.

SUBTOPIC: READING AND WRITING BASES

Example 1

$1111_{\text{two}} = \text{one, one, one, one base two}$

Example 2

$123_{\text{four}} = \text{one, two, three base four}$

EXERCISE

1. 102_{five}

2. 223_{five}

3. 101_{two}

4. 423_{five}

5. 212_{three}


6. 1011_{two}

SUBTOPIC : CHANGING FROM BASE TEN TO OTHER BASES

When we are changing from base ten to other bases, we divide by the given base.

Example 1

Change 25_{ten} to base seven

B	No.	Rem.
7	25	
	3	4

$= 25_{\text{ten}} = 34_{\text{seven}}$

EXERCISE

Change to base two

1. 19_{ten}

2. 31_{ten}

3. 26_{ten}

Change to base five

1. 19_{ten}

2. 31_{ten}

3. 26_{ten}

Change to base two

1. 14_{ten}

2. 11_{ten}

3. 21_{ten}

Change to base two

1. 8_{ten}

2. 12_{ten}

3. 22_{ten}

SUBTOPIC: CHANGING FROM OTHER BASES TO BASE TEN

When we are changing from other bases to base ten we expand.

Example 1

Change 204_{five} to base ten

$$\begin{aligned} 204_{\text{five}} &= (2 \times 5^2) + (0 \times 5^1) + (4 \times 5^0) \\ &= (2 \times 5 \times 5) + (0 \times 5) + (4 \times 1) \\ &= 50 + 0 + 4 \\ &= 54 \end{aligned}$$

EXERCISE

1. 122_{five}

2. 1101_{two}

3. 1011_{two}

4. 141_{five}

5. 232_{five}

6. 1021_{four}

7. 111_{two}

8. 13_{five}

9. 123_{four}

SUBTOPIC : CHANGING FROM ONE BASE TO ANOTHER.

When we are changing from one base to another, we first change to base ten then divide by the base you are changing to.

Example 1

Change 101_{two} to base three

$$\begin{aligned} 101_{\text{two}} &= (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= (1 \times 2 \times 2) + (0 \times 2) + (1 \times 1) \\ &= 4 + 0 + 1 \\ &= 5_{\text{ten}} \end{aligned}$$

$$101_{\text{two}} = 12_{\text{three}}$$

B	No.	Rem
3	5	2
3	1	1
	0	

EXERCISE

1. Change 21_{three} to base two
2. Change 123_{five} to base two
3. Change 234_{five} to base two
4. Change 101_{two} to base five
5. Change 1001_{two} to base five
6. Change 222_{four} to base five
7. Change 111_{two} to base five
8. Change 13_{seven} to base two

SUBTOPIC : ADDITION OF BASES

Example 1

1. Add: $111_{\text{two}} + 110_{\text{two}}$
2. 110_{two}
3. $1011_{\text{two}} +$
4. 101_{two}
 $+ 110_{\text{two}}$

5. $111_{\text{two}} + 11_{\text{two}}$
6. $1011_{\text{two}} + 111_{\text{two}}$
7. 101_{two}
 $+ 11_{\text{two}}$

EXERCISE

1. $122_{\text{five}} + 322_{\text{five}}$
1. $444_{\text{five}} + 213_{\text{five}}$
3. $2211_{\text{three}} + 1122_{\text{three}}$
4. $321_{\text{five}} + 123_{\text{five}}$
5. $142_{\text{five}} + 231_{\text{five}}$
6. $32_{\text{five}} + 21_{\text{five}}$
7. $401_{\text{five}} + 42_{\text{five}}$

SUBTRACTION OF BASES

Example 1

1. $231_{\text{five}} - 120_{\text{five}}$
 1011_{two}

4. $344_{\text{five}} - 134_{\text{five}}$
 10_{two}

7. $111_{\text{two}} - 101_{\text{two}}$
 112_{five}

10. $464_{\text{eight}} - 237_{\text{eight}}$

2. $101_{\text{two}} - 11_{\text{two}}$

5. $322_{\text{five}} - 132_{\text{five}}$

8. $100_{\text{two}} - 11_{\text{two}}$

3. $1101_{\text{two}} -$

6. $111_{\text{two}} -$

9. $203_{\text{five}} -$

MULTIPLICATION OF BASES

1. 110_{two}
 $\times 10_{\text{two}}$

2. 423_{five}
 $\times 14_{\text{five}}$

3. $11_{\text{two}} \times 11_{\text{two}}$

4. $101_{\text{two}} \times 11_{\text{two}}$

5. 244_{five}
 0_{two}
 $\times 22_{\text{five}}$
 1_{two}

6. 1010_{two}
 $\times 101_{\text{two}}$

7. 333_{five}
 $\times 12_{\text{five}}$

8. 1
 $\times 1$

9. 101_{two}
 $\times 10_{\text{two}}$

10. 44_{five}
 $\times 21_{\text{five}}$

SUBTOPIC: SOLVING FOR THE UNKNOWN BASES

Example 1

If $17_x = 15_{\text{ten}}$. Find x

$$(1 \times x^1) + (7 \times x^0) = 15$$

$$x + 7 = 15$$

$$x + 7 - 7 = 15 - 7$$

$$x + 0 = 8$$

$$x = 8$$

NOTE: Expand if it is in any base apart from base ten. ie if its in base ten leave it as it is.

EXERCISE

1. $23_x = 11_{\text{ten}}$

2. $24_x = 42_{\text{five}}$

3. $77_y = 63_{\text{ten}}$

4. $45_x = 32_{\text{nine}}$

5. $100_n = 213_{\text{six}}$

6. $p^2 = 54_{\text{nine}}$

7. $33_p = 15_{\text{ten}}$

8. $42_x = 34_{\text{ten}}$

9. $13_x = 11_{\text{ten}}$

10. $31_x = 41_{\text{six}}$

11. $16_{\text{seven}} = 15_x$

12. $23_x = 33_{\text{five}}$

13. $203_k = 38_{\text{nine}}$

THREE: OPERATION ON WHOLE NUMBERS**Addition (up to 7 digits)**

Example 1:

M	HTh	TTh	Th	H	T	O
1	2	3	4	6	7	8
+	2	1	4	2	1	0
1	4	4	8	8	8	8

Work out:

1.
$$\begin{array}{r} 1\ 1\ 3\ 4\ 5 \\ +\ 1\ 6\ 7\ 8 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 3\ 3\ 2\ 4\ 5 \\ +\ 7\ 2\ 4\ 5 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 2\ 4\ 3\ 2\ 1 \\ +\ 6\ 7\ 4\ 2 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 8\ 2\ 4\ 5\ 3\ 6 \\ +\ 6\ 7\ 8\ 9 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 2\ 3\ 4\ 5\ 6 \\ +\ 3\ 1\ 4\ 5 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 6\ 3\ 4\ 5\ 8\ 2 \\ +\ 9\ 8\ 6\ 7 \\ \hline \end{array}$$

7. WORD PROBLEMS ON ADDITION. (I)

1. In a school, there are 287 boys and 319 girls. how many children are there in the school?
2. The number of trees on a district farm were 846839. If 146785 more trees were planted. How many trees were there altogether?
3. Class VI has 23 English books. If there are 18 more Mathematics books than English books; (a) how many Mathematics books are there?
(b) Find the total number of books.

4. During the presidential elections, a candidate received 10211 votes from county A, 11124 from B, 8221 from C 7001 from D and 15001 from E. how many votes did he get altogether?
5. A parent paid sh 338500 for his child in P.1, sh. 90500 for the one in P.7 and sh. 335900 for the one in S.I. How much money did he pay as school fees that term?

Subtraction

Example 1:

$$\begin{array}{r} 1\ 2\ 0\ 1\ 8\ 6 \\ -\ 2\ 0\ 1\ 2\ 3 \\ \hline 1\ 0\ 0\ 0\ 6\ 3 \end{array}$$

Example 2:

$$\begin{array}{r} 5\ 2\ 3\ 3\ 1\ 8\ 6 \\ -1\ 3\ 4\ 5\ 1\ 0\ 2 \\ \hline 3\ 8\ 8\ 8\ 0\ 8\ 4 \end{array}$$

Work out:

$$\begin{array}{r} 1.\quad 2\ 4\ 5\ 1\ 6\ 3 \\ -\ 4\ 3\ 1\ 7\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 2.\quad 7\ 5\ 8\ 3\ 6\ 1\ 4 \\ -\ 5\ 8\ 9\ 3\ 1\ 3\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 3.\quad 5\ 4\ 4 \\ -\ 2\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4.\quad 2\ 1\ 8\ 4\ 1\ 4\ 9 \\ -\ 4\ 3\ 6\ 2\ 4\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5.\quad 3\ 4\ 5\ 2\ 4\ 8 \\ -\ 2\ 3\ 1\ 3\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6.\quad 6\ 7\ 5 \\ -\ 1\ 5\ 3 \\ \hline \end{array}$$

WORD PROBLEMS ON SUBTRACTION (II)

1. What must be added to sh. 153000 to make sh. 250000?
2. The reading on a water meter was 00400702 units and at the end of the month it was 0049611 units. How much water was used during the month?
3. In one week, a factory packed 9877 kg of tea. In the following week it packed 7988 kg of tea. How many more kg were packed during the first week?
4. Mutebi wants to buy a radio which will cost him sh. 164000. He has saved sh. 98500. How much more money will he have to save?
5. In a class of 53 children, 25 are boys. How many girls are there in the class?

COMBINED OPERATIONS

(ADDITION AND SUBTRACTION) (III)

Examples

$$1.\quad 17 - 9 + 3$$

$$2.\quad 4 - 11 + 12$$

$$3.\quad 29 - (11 + 10)$$

$8 + 3$

$\underline{\underline{11}}$

$(4 + 12) - 11$

$16 - 11$

$\underline{\underline{5}}$

$29 - 21$

$\underline{\underline{8}}$

4. A pupil got a total of 375 marks in 5 subjects. She got 83 in Maths, 78 in Science, 59 in S.ST and 70 in RE. J=how many marks did he get in English?

Total for 4 subjects

marks for English

83

375

78

$\underline{- 290}$

59

$\underline{\underline{85}}$

$\underline{+70}$

$\underline{\underline{290}}$

OR: Marks for English

$375 - (83 + 78 + 59 + 70)$

$375 - 290$

$\underline{\underline{85}}$

Activity

1. Work out

a. $(176 + 29 + 407) - 456$

b. $19 - 10 + 7$

c. $4 - 18 + 16$

d. $45 - (13 + 25).$

2. A city has 1200013 people. If the number of children is 631176 and women is 253059, how many men are in the city?
3. A printing press mad 39678 exercise books in 3 days. On the first day, it made 13657 and on the second day 12890. how many books did it make on the third day?

Multiplication (A 3 digit number by a 2 digit number).

Example 1:

$$\begin{array}{r} 143 \\ \times 18 \\ \hline 1144 \\ + 1430 \\ \hline 2574 \end{array}$$

Example 2:

$$\begin{array}{r} 132 \\ \times 13 \\ \hline 396 \\ + 1320 \\ \hline 1716 \end{array}$$

Work out.

1a) 345

3

$$\begin{array}{r} \\ \times 12 \\ \hline \end{array}$$

7

b) 445

$$\begin{array}{r} \\ \times 35 \\ \hline \end{array}$$

c) 675

$$\begin{array}{r} \\ \times 263 \\ \hline \end{array}$$

d) 45

$$\begin{array}{r} \\ \times 52 \\ \hline \end{array}$$

e) 438

4

$$\begin{array}{r} \\ \times 13 \\ \hline \end{array}$$

6

f) 463

$$\begin{array}{r} \\ \times 83 \\ \hline \end{array}$$

g) 456

$$\begin{array}{r} \\ \times 14 \\ \hline \end{array}$$

h) 63

$$\begin{array}{r} \\ \times 5 \\ \hline \end{array}$$

MULTIPLICATION (WORD PROBLEMS) IV

Examples

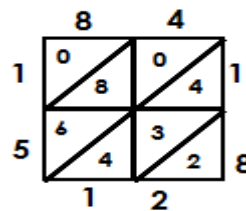
1. A bus carries 84 passengers each trip. How many passengers will it carry if it makes 18 trips?

MI

$$\begin{array}{r} 84 \\ \times 18 \\ \hline 672 \\ + 840 \\ \hline \end{array}$$

1512 passengers.

MII lattice.

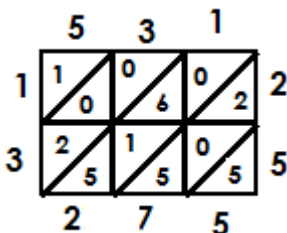


1512 passengers

2. 531 farmers sold 25 bags of rice each. Each bag weighed 80 kg. What was the total mass of the rice sold?

Total number of bags

$$531 \times 25$$



Number of kg sold

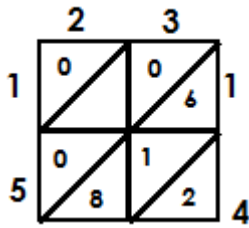
$$\begin{array}{r} 13275 \\ \times 80 \\ \hline 00000 \\ 106200 \\ \hline 1062000 \text{ kg} \end{array}$$

13275 bags.

3. Airnugisha sold 23 cartons of fat in a day. Each carton had 24 tins of fat.

(a) How many tins did he sell in a day?

(b) Find the number of tins he sold in a week



5 5 2 tins

x 7

3864 tins

552 tins

c. If he sold each tin at sh. 4500, how much money did he get in a day?

552 x sh 4500

sh. 4500

x 552

9000

225000

+ 2250000

sh. 2484000

Activity

1. In a school, there are 28 classrooms, with 45 pupils in each class. How many pupils are in the school?
2. On a coffee farm, there are 125 rows with 80 trees in each row. How many trees are there altogether?
3. A train has 8 second – class coaches each with seats for 36 passengers and 4 first – class coaches each with seats for 12 passengers. How many passengers can it carry?
4. A dairy farm had 75 cows each giving 13 litres of milk everyday.
 - a. How many litres does it get per day?
 - b. Find the number of litres sold in a week.
 - c. If each litre is sold at sh. 1600, how much does it get per day?

Division

Example 1:

$$\begin{array}{r}
 1976 \div 13 \\
 \begin{array}{r}
 13 \overline{) 1976} \\
 \underline{- 13} \\
 67 \\
 \underline{- 65} \\
 26 \\
 \underline{- 26} \\
 0
 \end{array} \\
 = \underline{152}
 \end{array}$$

1×13
 5×13
 2×13

Table 13

$1 \times 13 = 13$
$2 \times 13 = 26$
$3 \times 13 = 39$
$4 \times 13 = 52$
$5 \times 13 = 65$

Example 2:

$$\begin{array}{r}
 6360 \div 120 \\
 \begin{array}{r}
 120 \overline{) 6360} \\
 \underline{- 600} \\
 360 \\
 \underline{- 360} \\
 000
 \end{array} \\
 = \underline{53}
 \end{array}$$

5×120
 3×120

DIVISION (WORD PROBLEMS)

Examples

- 1767 building stones were piled 19 piles each with the same number of stones. How many stones were in each pile?

$$\begin{array}{r}
 19 \overline{) 1767} \\
 \underline{9 \times 19} \\
 57 \\
 \underline{3 \times 19} \\
 0
 \end{array}$$

93 stones.

- 95 petrol stations were supplied with 389500 litres of petrol. If the petrol was shared equally, how many litres did each petrol station get?

$$\begin{array}{r}
 95 \overline{) 389500} \\
 \underline{95 \times 4} \\
 95 \\
 \underline{95 \times 1} \\
 0
 \end{array}$$

4100 litres.

3. Bottle of soda are packed in crates each containing 24 bottles. How many crates are needed to pack 1344 bottle?

$$\begin{array}{r} 56 \\ 24 \overline{) 1344} \\ \underline{24 \times 5 = 120} \\ 144 \\ \underline{24 \times 6 = 144} \\ 0 \end{array}$$

56 crates.

Activity

1. There are 1127 children in a primary school. If there are 23 classrooms in the school, what is the average number of children per class?
2. A women's group has 1800 iron sheets to distribute to its members. If 72 sheets are required to complete each house, how many houses can the group complete?
3. A taxi can carry 29 passengers when full. in one day, the taxi carried 696 passengers. If it carried a full load on each journey, how many journeys did the taxi make?
4. Sodas are sold in crates containing 24 bottles. A head teacher wants to buy 840 bottles of soda for a party. How many crates of sodas must he buy?
5. A safari rally car traveled a total distance of 4214 km in 43 hours. What was his average speed in Km/hr?
6. A shopkeeper packed 400 tins of Kimbo in cartons each containing 24 tins. How many more tins does he need in order to fill the last carton?

MIXED OPERATIONS

Examples

1. 25560 bags of coffee each weighing 90 kg were repacked into 50 kg bags. How many 50 kg bags were obtained?

Either

No of kg of coffee
 25560 x 90 kg
 25560
 x 9
2300400kg

no of 50 kg bags
2300400kg
 50kg

46008 bags

or

no of 50 kg bags in
 25560 kg
25560kg
 50 kg
2556 bags
 5

total no of bags
2556 x 90
 5
 2556 x 18 bags

46008bag

2. A factory produced 50400 bottles of soda. If the factory makes 360 bottles per hour and works for 10 hours a day, how many days did it take to produce these bottles of soda?

No of bottles per day

360 x 10 bottles
3600 bottles

no of days

50400 days
 3600

14 days.

3. At a sports meetings, there were 200 men. The number of girls was four times that of men and 169 more than that of women. The number of boys was 40 more than that of men. What was the total number of people at the meeting?

No of girls

200 x 4
800

No of women
 800
 - 160
640

no of boys

200
 +40
240

total

800
 640
 200
 +240
1880 people

Activity

1. A factory produced 20822 packets of cooking fat out of these, 8860 were 100g packets, 2724 were 250g packets, 1731 were 500g packets, 1 kg packets were 800 fewer than 250g packets. If the rest were 2 kg packets, how many 2 kg packets were there?
2. An oil tanker can carry 6045 litres of diesel. 15 tankers delivered diesel to a filling station, where the diesel was stored in barrels each of which held 195 litres of diesel. How many barrels were filled with diesel?
3. A poultry farmer collected 56 trays of eggs. Each tray contained 30 eggs. On the way to the market 36 eggs broke. How many eggs were left for sale?
4. A poultry farmer collected 56 trays of eggs. Each tray contained 30 eggs. On the way to the market 36 eggs broke. How many eggs were left for sale?
5. Six truck are used to transport bags of cement. If each truck makes 12 trips carrying 316 bags per trips;
 - a. how many bags will they transport altogether?
 - b. how many more trucks would be needed if the same number of bags is to be transported in four trips only?
6. A factory packs soap in sachets which are packed in cartons each carton holds 80 sachets. The factory packed 120 such cartons. 12 cartons were destroyed by rain, the remaining soap was shared equally among 8 dealers. How many sachets did each dealer get?

Addition and subtraction without brackets.

Using all operations

Work out: I

- | | | |
|----------------------------------|----------------------------------|------------------------------|
| 1. $3 + 8 \times 4$ | 2. $15 + 4 \times 9$ | 3. $18 \times 7 + 12$ |
| 4. $6 \times 7 + 8 + 9 \times 3$ | 5. $2 \times 3 + 4 + 5 \times 6$ | 6. $13 \times 9 + 7 + 3 + 9$ |
| 7. $3 \times 7 + 8 \times 9$ | 8. $12 + 13 \times 8$ | 9. $5 \times 9 + 6$ |

Work to do: II

- | | | |
|---|--------------------------------|-----------------------------------|
| 1. $15 \times 3 + 10 \div 2 - 5$ | 2. $90 - 50 \div 25 \times 5$ | 3. $(5 \times 3) + 10 \div 2 - 5$ |
| 4. $300 \div 15 \times 2$ | 5. $(35 \div 7) - (18 \div 6)$ | 6. $50 \div 10 + 40 \div 4$ |
| 7. $(24 \div 2) \times (3 \times 6) \div (18 \div 2)$ | 8. $30 \times 11 + 105 \div 5$ | 9. $(25 - 7) \div 3$ |

PROPERTIES OF NUMBERS

Commutative property

a). Addition

$$a + b = b + a$$

$$7 + 4 = 4 + 7 \text{ (check)}$$

$$11 = 11$$

b). Multiplication

$$a \times b = b \times a$$

$$7 \times 4 = 4 \times 7$$

$$28 = 28$$

Using commutative property to complete the following statements.

- | | | |
|---|---|--------------------------------|
| 1. $4 + 5 = 5 + \underline{\hspace{2cm}}$ | 2. $6 \times 3 = \underline{\hspace{1cm}} \times 6$ | 3. $7 \times 10 = 10 \times n$ |
| 4. $y + 5 = 5 + 7$ | | |

Associative property of:

a). Addition

$$3 + (8 + 9) = (3 + 8) + 9$$

$$3 + 17 = 11 + 9$$

$$20 = 20$$

b). Multiplication

$$(4 \times 6) \times 5 = 4 \times (6 \times 5)$$

$$24 \times 5 = 4 \times 30$$

$$120 = 120$$

Complete the statements below using associative property.

1. $5 + (6 + 4) = (5 + 6) + p$

2. $25 \times 4 \times 9 = (n \times 4) \times 9$

3. $9 + 11 + 10 = (x + 9) + 10$

4. $4 \times 5 \times 10 = (4 \times 5) \times m$

Distributive PropertyExample: $(4 \times 5) + (4 \times 6)$ **Put 4 outside the baskets (It's a common factor)**

$$4 (5 + 6)$$

$$4 \times 11 = \underline{\underline{44}}$$

Using distributive property, work out the following.

1. $(2 \times 3) + (4 \times 3)$

2. $(5 \times 7) + (7 \times 4)$

3. $(63 \times 17) - (17 \times$

53)

4. $(8 \times 5) + (6 \times 8)$

5. $(3 \times 9) + (9 \times 4)$

6. $(78 \times 33) - (78 \times$

13)

7. $(46 \times 0.7) + (54 \times 0.7)$

8. $(26 \times 2.5) - (2.5 \times 16)$

POWERS / INDICES / EXPONENTS

Singular = Index

Write the following products in short form (index form).

- $5 \times 5 = 5^2$

- $2 \times 2 \times 2 \times 2 = 2^4$

- $m \times m \times m \times m \times m = m^5$

* In a term 5^2 , **5** is called the **base** and **2** is called an **index / power / exponent**.

* Similarly m^5 , **m** is called the **base** and **5** is called an **index / power / exponent**.

NOTE: The power shows the number of times the base is multiplied by itself.

Reading Indices.

$5^2 = 5$ squared or 5 to the power of 2.

$a^3 = a$ cubed or a to the power of 3.

$m^5 = m$ to the power of 5.

Write the following products in index form.

i. $2 \times 2 \times 2 = 2^3$

ii. $3 \times 3 \times 3 \times 3 = 3^4$

iii. $19 \times 19 = 19^2$

iv. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

v. $y \times y \times y = y^3$

vi. $n \times n \times n \times n \times n \times n \times n = n^7$

Finding Values of numbers written in Index Form.

* Evaluate the following:

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243 \text{ (**243 is the fifth power of 3**)}$$

$$\begin{aligned} 5^2 \times 6^3 &= (5 \times 5) \times (6 \times 6 \times 6) \\ &= 25 \times 216 \\ &= 5400 \end{aligned}$$

Exercise:

Evaluate the following.

i. $2^3 = 2 \times 2 \times 2 = 8$

ii. $5^2 = 5 \times 5 = 25$

iii. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

iv. $10^5 = 10 \times 10 \times 10 \times 10 \times 10 =$

100000

v. $4^4 = 4 \times 4 \times 4 \times 4 = 256$

$$\begin{aligned} \text{vi. } 2^2 \times 3^3 &= (2 \times 2) \times (3 \times 3 \times 3) \\ &= 4 \times 27 \\ &= \underline{\underline{108}} \end{aligned}$$

vii. $3^2 \times 5^1 = 3 \times 3 \times 5 = 135$

viii. $4^3 \times 10^2 = (4 \times 4 \times 4) \times (10 \times 10) =$

6400

Multiplying numbers written in Index Form.

NOTE: Simplify is different from evaluate.

(In simplify the answer remains in power form while in evaluate you multiply and write one value).

Simplify the following by expanding.

i. $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
 $= 2^5$

ii. $7^2 \times 7^3 \times 7$
 $(7 \times 7) \times (7 \times 7 \times 7) \times (7)$
 7^6 (The base must have an index which is equal to the sum of the given indices).

Exercise

Simplify the following by fully expanding the terms.

i. $C^3 \times C^4$

ii. $10^4 \times 10^2$

iii. $P^2 \times P$

iv. $a^2 \times a^3 \times a^4$

v. $8^2 \times 8^3$

LAW (Rule) OF INDICES ON MULTIPLICATION

NOTE: To multiply the same bases raised to different powers, add the powers.

ie $a^m \times a^n = a^{m+n}$

Compare the following.

1. $2^3 \times 2^2$

$2^3 \times 2^2$

$2 \times 2 \times 2 \times 2 \times 2$

2^{3+2}

2^5

2^5

2. $y \times y^3$
raised to

Note: If a base is written without a power, know that it is the power one.

$y^1 \times y^3$

y^{1+3}

y^4

$$\begin{aligned}
 3. \quad & m^5 \times m^6 \times m^{-4} \\
 & m^{(5+6+-4)} \\
 & m^{(11-4)} \\
 & \underline{\mathbf{m^7}}
 \end{aligned}$$

Exercise

Simplify the following (using law of indices).

- | | | |
|------------------------|------------------------------|------------------------|
| 1. $m^3 \times m^5$ | 2. $a^6 \times a^4$ | 3. $4^2 \times 4^7$ |
| 4. $k^7 \times k$ | 5. $9^3 \times 9^5 \times 9$ | 6. $m^{-4} \times m^5$ |
| 7. $b^7 \times b^{-3}$ | | |

More work on multiplication of indices.

In the term $6p^2$, **6** is called the **co-efficient** of p^2 .

Examples:

$4a^2 \times 3a^4$	or	$4a^2 \times 3a^4$
$4 \times 3 \times a^2 \times a^4$		$4 \times 3 \times a^2 \times a^4$
$12a^{(2+4)}$		$12 \times (a \times a) \times (a \times a \times a \times a)$
<u>$12a^6$</u>		<u>$12a^6$</u>

Exercise

Simplify the following numbers.

- | | | |
|------------------------------------|--|---------------------|
| 1. $3p^2 \times 2p^2$ | 2. $8n^4 \times 9n^3$ | 3. $3x^2 \times 4x$ |
| 4. $2a^2 \times 3ab$ | 5. $5t^3 \times 3t^5$ | 6. $xy \times xy$ |
| 7. $3a^2b \times 4ab^3$ | 8. $2mn^2 \times 5m^3n^4$ | 9. $3xy^2 \times$ |
| $6y^4x^3$ | | |
| 10. $6a^3 \times \underline{2a^2}$ | 11. $\underline{5}x^2y^3 \times \underline{3}m^2x^3$ | 12. $5m^2n \times$ |
| $3mn^5 \times mn$ | 9 5 | |
| 3 | | |

Dividing numbers written in Index Form.

Method I: (By expanding terms)

Simplify the following.

$$1. \quad 2^3 \div 2^2$$

$$\begin{array}{r} 2^3 \\ 2^2 \\ \hline \cancel{2} \times \cancel{2} \times 2 \\ \cancel{2} \times \cancel{2} \\ \hline 2 \end{array}$$

$$2. \quad b^7 \div b^4$$

$$\begin{array}{r} b^7 \\ b^4 \\ \hline b \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times b \times b \\ \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \\ \hline b^3 \end{array}$$

Exercise:

$$1. \quad a^5 \div a^3$$

$$2. \quad p^7 \div p^3$$

$$3. \quad 8^8 \div 8^3$$

$$4. \quad 10^4 \div 10$$

$$5. \quad k^8 \div k^5$$

$$6. \quad 3^5 \div 3^4$$

$$7. \quad 5^6 \div 5^2$$

$$8. \quad n^4 \div n^2$$

$$9. \quad g^3 \div g$$

Law of indices on Division.

When dividing powers of the same base, subtract the powers.

Compare the following.

$$2^3 \div 2^2$$

or

$$2^3 \div 2^2$$

$$2^{3-2}$$

$$2^1$$

$$\underline{2}$$

$$\frac{2^3}{2^2} = \frac{\cancel{2} \times \cancel{2} \times 2}{\cancel{2} \times \cancel{2}} = 2$$

$$= \underline{2}$$

Exercise

$$1. \quad 6^{12} \div 6^5$$

$$2. \quad x^5 \div x^2$$

$$3. \quad 5^4 \div 5^3$$

$$4. \quad n^9 \div n^4$$

$$5. \quad m^{10} \div m^6$$

$$6. \quad 2^6 \div 2^4$$

$$7. \quad C^7 \div C$$

$$8. \quad y^7 \div y^2$$

$$9. \quad 13^3 \div 13^2$$

$$10. \quad 7^9 \div 7^5$$

Note: $3^2 \div 3^2 = \frac{3^2}{3^2} = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = 1$

and $3^2 \div 3^2 = 3^{2-2} = \underline{3^0}$

From the above result, we see that $3^0 = 1$. Any number to the power one it is equal to that number.

NEGATIVE INDICES

Simplify $10^2 \div 10^5$

(a). By expanding each term.

(b). By subtracting indices.

$$10^2 \div 10^5 = 10^{2-5} = \frac{10 \times 10}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{10^3}$$

$$(b). \quad 10^2 \div 10^5 = 10^{2-5} \\ = \underline{10^{-3}}$$

From the above results of parts (a) and (b), we see that $\frac{1}{10^3} = 10^{-3}$

In general $x^{-a} = \frac{1}{x^a}$

Exercise:

1. $a^4 \div a^6$

2. $4^2 \div 4^7$

3. $10^5 \div 10^8$

4. $r^7 \div r^9$

5. $a^2 \div a^7$

6. $x^2 \div x^{-7}$

7. $y^5 \div y^{-2}$

8. $p^{-2} \div p^{-7}$

9. $5^6 \div 10^{10}$

MORE WORK ON DIVISION OF INDICES

Method I (by expanding)

$$12a^7 \div 3a^2 = \frac{\overset{4}{\cancel{12}} \times a \times a \times a \times a \times a \times a \times a}{\underset{1}{\cancel{3}} \times a \times a} \\ = \underline{4a^5}$$

Method II (by subtracting indices)

$$12a^7 \div 3a^2 = \frac{12a^7}{3a^2} = \frac{12}{3a} a^{7-2} \\ = \underline{4a^5}$$

Exercise:

Simplify the following.

1. $4a^5 \div 2a^3$

2. $6m^3 \div 3m$

3. $10y^7 \div 5y^2$

4. $6k^8 \div 3k^5$

5. $18m^{12} \div 6m^5$

6. $6xy^6 \div 2x^2y^3$

7. $10a^2b^4 \div 2ab$

8. $36p^2q^3 \div 9pq^2$

9. $15b^9 \div 3b^4$

10. $12b^7 \div 3b^5$

Multiplication and Division (Mixed).

$$1. \quad \frac{n^{10} \times n^4}{n^8}$$

$$2. \quad \frac{p^5 \times p^9}{p^3}$$

$$3. \quad \frac{y^8 \times y^{10}}{y^6}$$

$$4. \quad \frac{2n^2 \times 3n^2}{8n^4}$$

$$5. \quad \frac{7^8 \times 7}{7^6}$$

$$6. \quad \frac{5^3 \times 5^4}{5^2}$$

$$7. \quad \frac{2^2 \times 2^3}{2^6}$$

$$8. \quad \frac{m^6 \times m}{m^4}$$

$$9. \quad \frac{3^5 \times 3^4}{3^4}$$

$$10. \quad \frac{a^7 \times a^3}{a^6}$$

More work on Multiplication and Division

Evaluate the following.

$$1. \quad \frac{3^3 \times 3^4}{3^6}$$

$$2. \quad \frac{5^3 \times 5^4}{5^5}$$

$$3. \quad \frac{2^6 \times 2^3}{2^4}$$

$$4. \quad \frac{2n^2 \times 3n^2}{8n^4}$$

$$5. \quad \frac{4^3 \times 6^3}{3^2 \times 2^7}$$

$$6. \quad \frac{2^7 \times 2^3}{4^2 \times 8}$$

$$7. \quad \frac{3^3 \times 4^3}{6^2 \times 2^4}$$

$$8. \quad 10^4 \div 10$$

$$9. \quad 6^7 \div 6^4$$

$$10. \quad 8^5 \div 8^3$$

ADDITION AND SUBTRACTION OF INDICES

Find the value of the following.

$$1. \quad 2^2 + 2$$

$$2. \quad 3^2 + 3^0$$

$$3. \quad 2^2 - 2^0$$

$$4. \quad 3^2 -$$

$$2^3$$

$$5. \quad 4^3 + 5$$

$$6. \quad 2^2 + 3^2 + 4^0$$

$$7. \quad 5^2 - 4^2$$

$$8. \quad 1^4 +$$

$$4^2 - 3^2$$

$$9. \quad 11^2 + 2^5$$

$$10. \quad 3^3 - 4^2$$

$$11. \quad 4^2 - 2^4$$

$$12. \quad 10^2 -$$

$$7^2$$

$$13. \quad 3^4 + 10^0 - 8^2$$

$$14. \quad 2^6 + 9^1 + 100^0$$

Solving equations involving indices.

Given that;

$$1. \quad 2a \div 2^2 = 2^5$$

$$2^{a-2} = 2^5$$

$$a - 2 = 5$$

$$a - 2 + 2 = 5 + 2$$

$$\underline{\mathbf{a = 7}}$$

$$2. \quad \text{If } 3^p \times 3 = 3^6, \text{ find } p.$$

$$3^{p+1} = 3^6$$

$$p + 1 = 6$$

$$p + 1 - 1 = 6 - 1$$

$$\underline{\mathbf{p = 5}}$$

$$3. \quad 3^k \times 3^2 = 243$$

prime factorise 243

side work

3	243
3	81
3	27
3	9
3	3
	1

$$\mathbf{243 = 3^5}$$

$$3k \times 3^2 = 3^5$$

$$3k+2 = 3^5$$

$$k + 2 = 5$$

$$k + 2 - 2 = 5 - 2$$

$$\underline{\mathbf{k = 3}}$$

$$4. \quad 5^{3y} \times 25 = 5^8$$

factorise 25

5	25
5	5
	1

$$= \underline{\mathbf{5^2}}$$

$$5^{3y} \times 5^2 = 5^8$$

$$3y + 2 = 8$$

$$3y + 2 - 2 = 8 - 2$$

$$3y = 6$$

$$\frac{3y}{3} = \frac{6}{3}$$

$$y = \underline{2}$$

$$5. \quad 8^{2m} \div 64 = 8^4$$

factorise 64

$$\begin{array}{r|l} 8 & 64 \\ \hline 8 & 8 \\ \hline & 1 = \underline{8^2} \end{array}$$

$$8^{2m} \times 8^2 = 8^4$$

$$2m - 2 = 4$$

$$2m - 2 + 2 = 4 + 2$$

$$2m = 6$$

$$\frac{2m}{2} = \frac{6}{2}$$

$$m = \underline{3}$$

$$6. \quad 2^6 \div 2^x = 1$$

If a number is 1, it means you raise the base to power 0 therefore $1 = 2^0$ in this case.

$$2^6 \div 2^x = 2^0$$

$$2^{6-x} = 2^0$$

$$6 - x = 0$$

$$6 - 6 - x = 0 - 6$$

$$+x = +6$$

$$x = \underline{6}$$

STANDARD FORM

Standard form is a way of writing down very large or very small numbers easily.

Standard form is also known as:

- scientific notation

- standard index form
- exponential form

Standard form is commonly used in calculators by scientists like, Mathematicians, engineers, Astronomers, biologists and other professionals who encounter very big or very small numbers.

Formula for writing a number (big/small) in standard form / scientific notation.

Formula = $a \times 10^n$ where;

- * **a** is the first digit in a number written in standard form called **Mantissa**. This number must be between **0 and 10**.
- * **n** is an integer got from the number of steps . places a decimal point moves either to the right (negative movement) or the left (positive movement).

Writing the following standard form / scientific notation.

(a). 900	(b). 35714	(c). 49000	(d). 3.2
9×10^n	$a \times 10^n$	$a \times 10^n$	$a \times$
10^n			
9×10^2	3.5714×10^4	4.9×10^4	$3.2 \times$
10^0			

(e). 76.21	(f). 562.19×10^4
$a \times 10^n$	$5.6219 \times 10^2 \times 10^4$
7.621×10^1	5.6219×10^6
	or 562.19×10^4
	<u>56219</u> x 10000
	100
	= 5621900
	= 5.6219×10^6

Activity:

Express the following in Standard form.

- | | | | |
|--------|---------|----------|----|
| 1. 600 | 2. 9000 | 3. 12000 | 4. |
| 64790 | | | |

5. 240030 6. 836.4 7. 114.01 8. 5.72
9. 960.43×10^3

Writing small numbers in Standard form / scientific notation.

- * When a given number is less than one (1) the decimal point has to move to the right so the power of ten will be a negative.

Example:

- (a). 0.055 standard form

$$\overbrace{0.0055}$$

$$a \times 10^n$$

$$5.5 \times 10^{-3}$$

- (b). $\overbrace{0.000803}$

$$a \times 10^n$$

$$8.03 \times 10^{-4}$$

Express the following in scientific notation.

1. 0.04 2. 0.856 3. 0.0062 4. 0.9
5. 0.04789 6. 0.00621

Writing a number from standard form / scientific notation as ordinary number.

- (a). 2.43×10^2

$$\frac{243}{100} \times 100$$

$$\mathbf{243}$$

- (b). 7.01×10^4

$$\frac{701}{10000} \times 10000$$

$$\mathbf{70100}$$

- (c). 9.6×10^1

$$\frac{96}{10} \times 10$$

$$\mathbf{96}$$

- (d). 2.4×10^{-2}

$$\frac{24}{10} \times \frac{1}{100}$$

$$\frac{24}{1000}$$

$$\mathbf{0.024}$$

- (e). 8.3×10^0

$$\frac{83}{10} \times 1$$

$$\mathbf{8.3}$$

Activity

Which number has been written in standard form / scientific notation?

1. 2.1×10^2 2. 7.5×10^6 3. 8×10^5

4. 1.01×10^1

5. 4.73×10^0

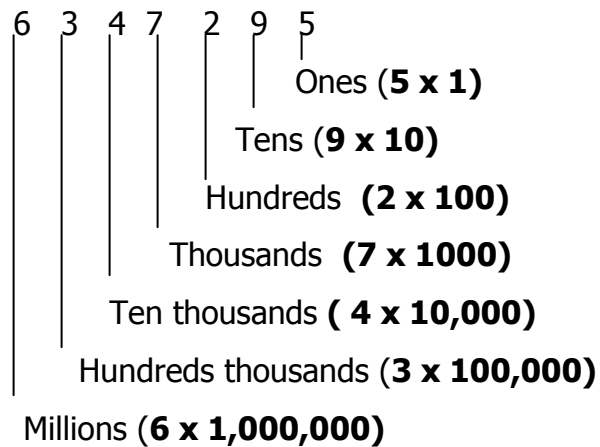
6. 8.3×10^{-1}

7. 4.004×10^{-2}

8. 6.4×10^5

EXPANDING NUMBERS NOT EXCEEDING 8 DIGITS

Example 1: Expand 6347295 using values.



$$6\ 3\ 4\ 7\ 2\ 9\ 5 =$$

$$6000000 + 300000 + 40000 + 7000 + 200 + 90 + 5$$

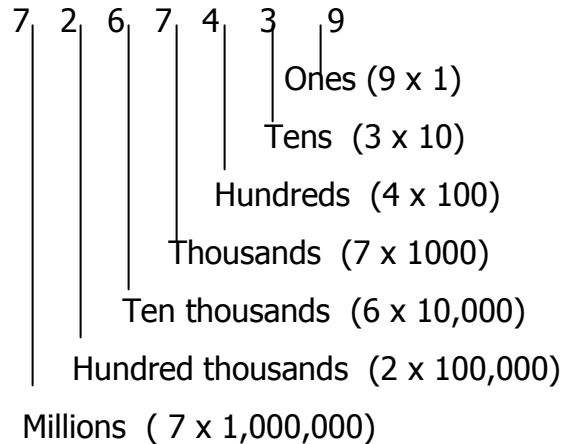
Exercise:

Expand the following using values.

- a). 5,119,023 b). 7,654,321 c). 108,450 d). 712
e). 9,536,008 f). 800,004

Expand using powers.

Example: 7,267,439



7267439 =

$$(7 \times 10^6) + (2 \times 10^5) + (6 \times 10^4) + (7 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (9 \times 10^0)$$

Expand using powers.

1. 935 2. 354212 3. 7277 4. 238
5. 4773468

Expand using powers.

1. 49.5 2. 127.4 3. 24.15

Writing numbers in standard form (Scientific notation)

Example:

1. $254 = 2.52 \times 10^2$ 2. $4576 = 4.576 \times 10^3$ 3. $0.0054 = 5.4 \times 10^{-3}$

Exercise.

1. 4620 2. 321 3. 0.0021 4.
0.347
5. 73.45 6. 253.7 8. 0.063 9. 8573

Writing numbers in standard form as single numbers

$$1. \quad 3.4 \times 10^2 = \frac{34}{10} \times 100 = 340$$

$$2. \quad 5.4 \times 10^{-3} = \frac{54}{10} \times \frac{1}{1000} = \frac{54}{10000} = 0.0054$$

$$3. \quad 2.13 \times 10^3 \times 10^3$$

$$4. \quad 6 \times 10^{-3}$$

$$5. \quad 4.5 \times 10^{-2}$$

$$6. \quad 7.35$$

$$7. \quad 8.248 \times 10^3 \times 10^{-2}$$

$$8. \quad 3.2 \times 10^5$$

$$9. \quad 7 \times 10^{-3}$$

$$10. \quad 4.3 \times 10^{-2}$$

A REVIEW ON FACTORS.

Factors are numbers that divide exactly. They don't leave any remainder.

Example: List all the factors of 10. (**Look for numbers that divide 10 equally**)

$$10 \div 1 = 10$$

$$10 \div 2 = 5$$

$$10 \div 5 = 2$$

$$10 \div 10 = 1$$

Factors of 10 are: 1, 2, 5, 10.

Example: What are the factors of 24?

$$24 \div 1 = 24$$

$$24 \div 2 = 12$$

$$24 \div 3 = 8$$

$$24 \div 4 = 6$$

$$24 \div 6 = 4$$

$$24 \div 8 = 3$$

$$24 \div 12 = 2$$

$$24 \div 24 = 1$$

$$F_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}.$$

Exercise: List all factors of the following:

$$1) \quad 6 \quad 2) \quad 8 \quad 3) \quad 12 \quad 4) \quad 15 \quad 5) \quad 18$$

$$6) \quad 20 \quad 7) \quad 32 \quad 8) \quad 30 \quad 9) \quad 36 \quad 10) \quad 48$$

Find the common factors of:

$$1. \quad 15 \text{ and } 12 \quad 2. \quad 18 \text{ and } 20 \quad 3. \quad 12 \text{ and } 8 \quad 4. \quad 20 \text{ and } 24$$

$$5. \quad 30 \text{ and } 36 \quad 6. \quad 8 \text{ and } 28 \quad 7. \quad 12 \text{ and } 54$$

PRIME FACTORISATION

These are factors, which are prime numbers. Prime numbers = {2,3,5,7,11,13,17,19,23,}

Example 1: Find the prime factors of 54.

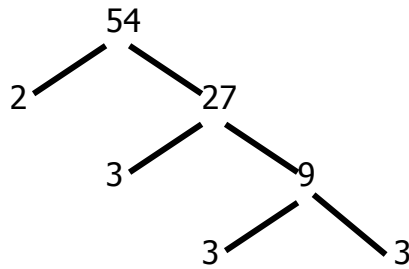
A list of prime factors/numbers = {2,3,5,7,11,.....}.

Ladder Method

2	54
3	27
3	9
3	3
	1

$PF_{54} = \{2_1, 3_1, 3_2, 3_3\}$ (Set notation)

Factor tree method



or $\{2^1 \times 3^3\}$ (Power form)

Set notation/subscript form or Power form/multiplication form.

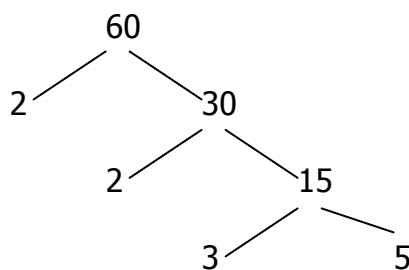
Example 2: Prime factorise 60.

Ladder Method

2	60
2	30
3	15
5	5
	1

$PF_{60} = \{2_1, 2_2, 3_1, 5_1\}$

Factor tree method



or $\{2^2 \times 3^1 \times 5^1\}$

Exercise:

Prime factorise the following, write your answer in set notation.

1. 18 2. 30 3. 24 4. 36 5.
40

Prime factorise the following and give your answer in multiplication form.

6. 45 7. 54 8. 60 9. 70 10.
84

More practice work in page 82 MK 6.

FINDING THE PRIME FACTORISED NUMBER.

Example 1: Find the number which is prime factorised to get:- $\{2_1, 2_2, 2_3, 3_1\}$

$$\text{Number} = 2 \times 2 \times 2 \times 3 = \mathbf{24}$$

Example 2: Find the number whose factorization is $\{2_2 \times 3_2 \times 5_1\}$.

$$\text{No.} = 2 \times 2 \times 3 \times 3 \times 5$$

$$= 4 \times 9 \times 5$$

$$= 20 \times 9$$

$$\mathbf{180}$$

Exercise:

Find the numbers whose prime factorization are given below.

1. $\{2_1, 2_2, 2_3\}$ 2. $\{3_1, 5_1, 7_1\}$ 3. $\{2^1 \times 3^2 \times 5^2\}$ 4. $\{2_1, 2_2, 3_1\}$
5. $\{2_1, 3_1, 3_2\}$ 6. $\{2_1, 2_2, 3_1, 3_2\}$ 7. $\{2^2 \times 5^1 \times 7^1\}$ 8. $\{2_2, 5_1, 7_1\}$

Finding the unknown prime factor.

Example: The prime factors of 60 are:- $2 \times 2 \times p \times 5$. Find p

$$2 \times 2 \times p \times 5 = 60$$

$$\begin{array}{r} 20p = 60 \\ 20 \quad 20 \\ p = \underline{\underline{3}} \end{array}$$

2	60
2	30
3	15
5	5
	1

Find the missing prime factors.

1. If $PF_{30} = 2 \times w \times 5$, find x. 2. $PF_{36} = 2 \times 2 \times 3 \times r$, find r.
3. $PF_{70} = 2 \times 5 \times n$, find n. 4. $PF_{90} = p \times 3 \times 3 \times 5$, find p.
5. $PF_{100} = 2 \times 2 \times 5 \times k$, find k.
6. The prime factorization of 120 is $2 \times 2 \times 2 \times m \times n$. Find the value of m and n.
7. The prime factorization of 144 is $a^4 \times b^2$; find a and b.

EXPRESSING A NUMBER AS A PRODUCT OF ANOTHER.

Example 1: Write 32 in powers of 2.

2	32
2	16
2	8
2	4
2	2
	1

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

Write 64 in powers of 4

4	64
4	16
4	4
	1

$$64 = 4 \times 4 \times 4 = 4^3$$

Exercise:

Express:

1. 64 in powers of 2. 2. 49 in powers of 7. 3. 256 in powers of 4.
4. 343 in powers of 7. 5. 216 in powers of 6. 6. 729 in powers of 3.
7. 8 in powers of 2. 8. 169 in powers of 13.

Finding the unknown, say $7^x = 49$.

REPRESENTING PRIME FACTORS ON VENN DIAGRAMS.

Use a Venn diagram to show prime factors of 36 and 30.

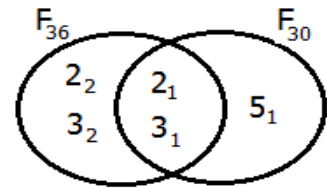
2	36
2	18
3	9
3	3
	1

$$F_{36} = \{2_1, 2_2, 3_1, 3_2\}$$

Common factors = $\{2_1, 3_1\}$

2	30
3	15
5	5
	1

$$F_{30} = \{2_1, 3_1, 5_1\}$$



Represent the prime factors of the following pairs of numbers on Venn diagram.

1. 24 and 30
2. 30 and 48
3. 48 and 60
4. 18 and 40
5. 15 and 20
6. 36 and 54.

FINDING THE GCF AND LCM.

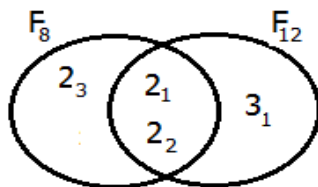
Example: Find the GCF and LCM of 8 and 12 using a Venn diagram.

2	8
2	4
2	2
	1

$$F_8 = \{2_1, 2_2, 2_3\}$$

2	12
2	6
3	3
	1

$$F_{12} = \{2_1, 2_2, 3_1\}$$



a). $GCF = 2 \times 2 = 4$

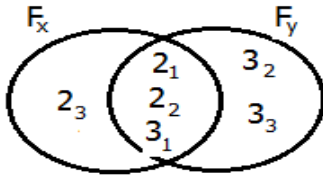
b). $LCM = 2 \times 2 \times 2 \times 3 = 24$

Refer to pages 88-900f-A new MK Primary Mathematics 2000 Pupils book 6.

FINDING THE UNKNOWN IN VENN DIAGAMS.

Example 1: Find the value of x and y, GCF and LCM.

a). $F_x = \{2_1, 2_2, 2_3, 3_1\}$ a).



$F_x = \{2_1, 2_2, 2_3, 3_1\}$

$x = 2 \times 2 \times 2 \times 3 =$

$x = 8 \times 3 =$

$x = \mathbf{24}$

b). $F_y = \{2_1, 2_2, 3_1, 3_2, 3_3\}$

$y = 2 \times 2 \times 3 \times 3 \times 3$

$y = 4 \times 27$

$y = \mathbf{108}$

c). $GCF = 2 \times 2 \times 3$

$= 4 \times 3$

$= \mathbf{12}$

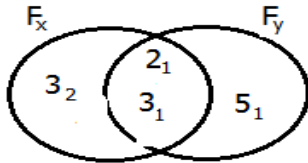
d). $LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

8×27

$= \mathbf{216}$

Exercise: Study the Venn diagrams and answer the questions that follow.

1.

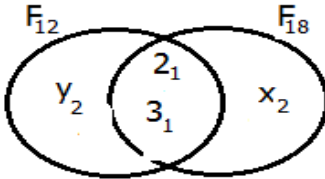


a). Find the value of; i. x ii. y

b). Find the GCF of x and y.

c). Find the LCM of x and y.

2.

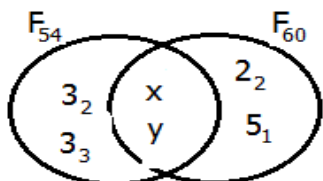


a). Find the value of; i. x ii. y

b). Find the GCF of 12 and 18.

c). Find the LCM of 12 and 18.

3.

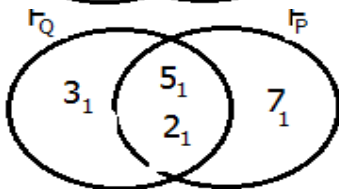


a). Find the value of; i. x ii. y

b). Find the GCF of 54 and 60.

c). Find the LCM of 54 and 60.

4.



a). Find the value of; i. Q ii. P

b). Find the GCF of q and p.

c). Find the LCM of q and p.

5. Given that $PF_{24} = 2^3 \times 3^1$ and $PF_{36} = 2^2 \times 3^2$. Find the GCF and LCM of 24 and 36.

More practice exercise on page 89 MK 6.

APPLICATION OF LCM AND GCF.

- The LCM of two numbers is 60 and their GCF is 4. One of the numbers is 12. Find the second number.
- The LCM of two numbers is 36 and GCF is 3. Find the value of Q if P is 9.
- The LCM of two numbers is 60 and their GCF is 5, find the second number given that the first number is 20.
- The LCM of two numbers is 180. Their GCF is 4. If one of the numbers is 20, find the second number.
- The LCM of two numbers is 18. Their GCF is 9, find the second number if one the numbers is 9.
- Two bells ring at intervals of 30 minutes and 40 minutes respectively. After how long do the two bells ring together?
- Two bells ring at intervals of 1 hour and 40 minutes respectively. If they ring together at 8:00am, at what time will they ring together?
- A bus leaves Kampala to Mbale every after 30 minutes and another leaves Kampala to Mbarara every after 45 minutes. If both buses leave Kampala at 6:00am, at what time will the two buses leave the park at the same time?

DIVISIBILITY BY 2, 3, 4 up to 11.

1. Divide the following numbers by 2: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Any number ending with an even digit or ending with 0,2,4,6,8 is divisible by 2.

Exercise:

Choose numbers divisible by 2 from the following.

- | | | | | |
|---------|---------|--------|--------|--------|
| 1. 10 | 2. 310 | 3. 11 | 4. 314 | 5. 36 |
| 6. 196 | 7. 22 | 8. 313 | 9. 907 | 10. 23 |
| 11. 105 | 12. 998 | | | |

2. Divisibility by 3: **Any number is exactly divisible by three if the sum of its digits**

is divisible by 3 or if the sum of its digits is a multiple of 3.

Example: Is 144 divisible by 3?

Sum of digits $1 + 4 + 4 = 9$ ($9 \div 3 = 3$)

List only those numbers which are exactly divisible by 3.

Exercise:

- | | | | | |
|---------|-------|-------|--------|--------|
| 1. 0 | 2. 10 | 3. 91 | 4. 15 | 5. 11 |
| 6. 93 | 7. 21 | 8. 13 | 9. 155 | 10. 90 |
| 11. 768 | | | | |

3. Divisibility by 4:

A number is divisible by 4 if its last two digits are zero or divisible by 4.

Find only those numbers that are exactly divisible by 4.

- | | | | | |
|--------|-----------|-------|--------|-------|
| 1. 0 | 2. 6 | 3. 36 | 4. 1 | 5. 7 |
| 6. 356 | 7. 2 | 8. 18 | 9. 244 | 10. 3 |
| 11. 19 | 12. 10000 | | | |

4. **Divisibility test by 5:**

Divisibility test by 6. The number must be an even number divisible by 3

A number is divisible by 5 if it ends with 0 or 5.

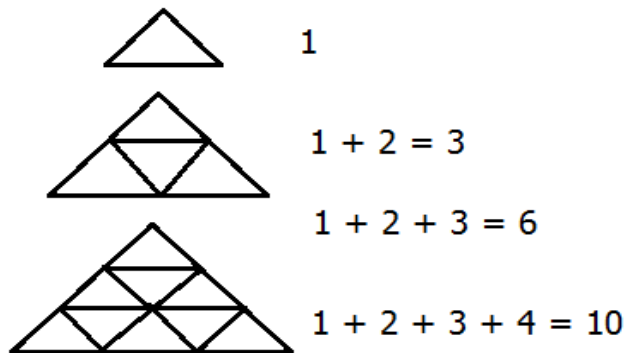
a). Write down multiples of 5 less than 60. $M_5 = \{ \quad \quad \quad \}$

NOTE: Other divisibility tests refer to MK 2000 pupil Bk 7 Pg 59-62 and Fountain Primary MTC pupils Bk 7 page 59-61.

b). Underline only those numbers that are divisible by 5:- 142, 345, 700, 1196, 752, 850, 1190

c). List the missing multiples of 5:- {170, __, 180, __, 190, __, 200, __, 210, __, 220}

TRIANGULAR NUMBERS - TRIANGULAR PATTERNS



Using triangular patterns given the next 3 triangular numbers.

When you add consecutive natural numbers from 1, the sum is always a triangular number.

Triangular numbers = {1,3,6,10,15,21,28,36,}

Example:

What is the sum of the first 7 counting numbers?

List of numbers.

Sum = $1 + 2 + 3 + 4 + 5 + 6 + 7$

= $6 + 9 + 13$

= $15 + 13$

= **28.**

The sum can also be obtained by using a short method: $\frac{n(n+1)}{2}$

$$\begin{aligned}\text{So } n(n+1) &= \frac{7(7+1)}{2} \\ &= \frac{7 \times 8}{2} = \frac{56}{2} \\ &= \underline{28} \text{ (is the sum)}\end{aligned}$$





Exercise:

1. List all triangular numbers less than 30.
2. What is the sum of the first 10 triangular numbers.
3. Fill in the missing numbers – {1, 3, 6, 10, _____, _____ }
4. What is the sum of the third and sixth triangular numbers.
5. Use the formular $\frac{n(n+1)}{2}$ to get;
 - i. the 30th triangular number
 - ii. the sum of all numbers from 1 to 50

RECTANGULAR NUMBERS.

These are counting numbers which make a pattern of a rectangle by using dots or squares.

1. Rectangular numbers can be arranged to make a rectangle.

<u>Rectangle</u>	<u>No. of squares</u>
	2
	6
	8
	10

Arrange squares to form the next four rectangular numbers.

Rectangular numbers are = {2,6,8,10,12,14,15,20}

How to obtain rectangular numbers.

Exercise:

Study the rectangular patterns above then draw and write rectangular numbers for each of these.

- | | | | |
|-----------|-----------|-----------|-----------|
| 1. 2 by 3 | 2. 3 by 6 | 3. 4 by 6 | 4. 4 by 7 |
| 5. 3 by 7 | 6. 6 by 7 | 7. 3 by 5 | 8. 4 by 9 |

Square numbers: A number whose dot pattern make a square.

These are numbers obtained by adding consecutive odd numbers or by squaring counting numbers.

Method I:

$1 \times 1 = \mathbf{1}$	$2 \times 2 = \mathbf{4}$	$3 \times 3 = \mathbf{9}$	$4 \times 4 = \mathbf{16}$	$5 \times 5 = \mathbf{25}$
$6 \times 6 = \mathbf{36}$	$7 \times 7 = \mathbf{49}$	$8 \times 8 = \mathbf{64}$	$9 \times 9 = \mathbf{81}$	$10 \times 10 = \mathbf{100}$
$11 \times 11 = \mathbf{121}$	$12 \times 12 = \mathbf{144}$			

Set of square numbers = $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \dots\}$

What is the square of:

- | | | | | |
|-------|-------|-------|--------|--------|
| 1. 13 | 2. 16 | 3. 49 | 4. 100 | 5. 81 |
| 6. 14 | 7. 21 | 8. 19 | 9. 17 | 10. 15 |

Note: The shape formed by triangular number is a triangle.

The shape formed by square number is a square.

Example:

1×1	2×2	3×3	4×4
--------------	--------------	--------------	--------------

How is the next number obtained?

Method II:

$$\begin{array}{rcl}
 1 + 3 & = & 4 \\
 4 + 5 & = & 9 \\
 9 + 7 & = & 16 \\
 16 + 9 & = & 25 \\
 25 + 11 & = & 36
 \end{array}$$

Square numbers are: $1, 4, 9, 16, 25, 36, \dots\}$

Method 2:

$$\begin{array}{rcl}
1 & & = 1 \\
1 + 3 & & = 4 \\
1 + 3 + 5 & & = 9 \\
1 + 3 + 5 + 7 & & = 16 \\
1 + 3 + 5 + 7 + 9 & & = 25 \\
1 + 3 + 5 + 7 + 9 + 11 & & = 36
\end{array}$$

Obtain the next four square numbers using the same method.

Method 3:

1×1	2×2	3×3	4×4	5×5
1^2	2^2	3^2	4^2	5^2

Exercise:

1. Find the value of the unknown.

$1 \times 1 = a$	$2 \times 2 = k$	$4 \times k = 16$	$y \times y = 25$
$z = 7 \times 7$	$9 = p \times p$	$11 \times 11 = f$	$13 \times b =$

139

2. Work out the following.

a). $10t = 100$	b). $169 = k^2$	c). $20a = 400$	d). $12n$
$= 144$			

3. What is the square of:

a). 11	b). 17	c). 14	d). 16	e). 13
f). 19	g). 12	h). 18	i). 15	

WHOLE NUMBER AND COUNTING.

1. whole numbers = $\{0,1,2,3,4,5,6,\dots\}$

Note: a). whole numbers are all positive numbers.

b). 0 is not a counting number.

Counting Number:- $\{1,2,3,4,5,6,7,8,9,\dots\}$

Exercise:

1. Give a set of counting numbers between 5 and 11.
2. Give a set of the first five whole number.
3. Write elements in a set of counting numbers greater than 15 but less than 24.
4. List elements in a set of counting numbers less than 30 which are divisible by 3.

Practice work on page 73 MK 6.

EVEN NUMBERS / ODD NUMBERS.

0×2	1×2	2×2	3×2	4×2	5×2
0	2	4	6	8	10

Even numbers are = $\{0,2,4,6,8,10,\dots\}$ ($2 \times n = 2n$)

Odd numbers are = $\{1,3,5,7,9,11,13,15,17,\dots\}$ ($2n + 1$)

Note: If n is a whole number.

A whole number $\times 2 = 2n$ (even number)

A whole number $\times 2$ plus 1 = $2n + 1$ = odd number.

Exercise:

1. List elements in a set of even numbers below 20.
2. List elements in a set of even numbers between 8 and 30.
3. What is the first even number?
4. List down members in a set of even numbers divisible by 3 less than 50.
5. List down elements in a set of odd numbers greater than 4 but less than 20.

More practice work on page 74 MK 6.

FINDING CONSECUTIVE NUMBERS.

Example: The sum of three consecutive counting numbers is 36. What are these numbers?

Let them be n , $(n+1)$, $(n+2)$.

$$n + n + n + 1 + 2 = 36$$

$$3n + 3 = 36$$

$$3n + 3 - 3 = 36 - 3$$

$$\frac{3n}{3} = \frac{33}{3}$$

$$n = 11$$

$$\text{The } 1^{\text{st}} n = 11$$

$$\text{The } 2^{\text{nd}} n + 1 = 11 + 1 = 12$$

$$\text{The } 3^{\text{rd}} n + 2 = 11 + 2 = 13$$

Exercise:

1. The sum of 3 consecutive counting numbers is 21. What are these numbers?
2. The sum of 3 consecutive counting numbers is 39. Find these numbers.
3. Find the consecutive counting numbers whose total is 51.
4. Find 4 consecutive counting numbers whose sum is 86.
5. List down 3 consecutive counting numbers whose total is 72.

More practice work on page 76 MK 6.

Consecutive Even/Odd Numbers.

Example 1: The sum of 3 consecutive even numbers is 24. List down the three numbers.

Let the 1^{st} number be: (x)

2^{nd} number be: $(x+2)$

3^{rd} number be: $(x+4)$

Form an equation and solve for x :

$$x + (x + 2) + (x + 4) = 24$$

$$3x + 6 = 24$$

$$3x + 6 - 6 = 24 - 6$$

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

$$x = 6$$

$$x + 2 = 6 + 2 = 8$$

$$x + 4 = 6 + 4 = 10$$

The numbers are: 6, 8 and 10

Example 2: The sum of 4 consecutive odd numbers is 32. What are the numbers?

Let the 1st number be: p

2nd number be: $p + 2$

3rd number be: $p + 4$

4th number be: $p + 6$

$$p + p + 2 + p + 4 + p + 6$$

$$4p + 12 = 32$$

$$4p + 12 - 12 = 32 - 12$$

$$4p = 20$$

$$4 \quad 4$$

$$p = 5$$

$$p = 5$$

$$(p + 2) = (5 + 2) = 7$$

$$(p + 4) = (5 + 4) = 9$$

$$(p + 6) = (5 + 6) = 11$$

The numbers are: 5, 7, 9 and 11.

Exercise:

1. Find the three consecutive even numbers whose total is 42.
2. The sum of 3 consecutive odd numbers is 45. Find the numbers.
3. The sum of 3 consecutive even numbers is 36. Find the third if two of them are 12 and 14.
4. The sum of 4 consecutive even numbers is 52. List all the numbers.
5. Find the four consecutive odd numbers whose total is 88.

More practice work on page 76 MK 6.

PRIME NUMBERS.

A prime number is a number with only two factors that is, "one and itself".

Examples of prime numbers:

2,3,5,7,11,13,17,19,23,29,31,41,43,47,53,59,61,67,71,73,79,83,89,97

Exercise:

1. Give a set of prime numbers between 1 and 10.
2. Write elements in a set of prime numbers between 10 and 30.
3. List members in a set of prime numbers between 30 and 50.
4. How many prime numbers are there between 50 and 60?
5. How many prime numbers are there between 70 and 80?
6. How many prime numbers are there between 90 and 100?
7. What is the sum of the 3rd and seventh prime number?
8. What is the sum of prime numbers between 80 and 100?
9. How many even prime numbers are there between 1 and 100?

COMPARING PRIME NUMBERS AND COMPOSITE NUMBERS:

No.	Set of facts	No. of facts	Type of No.
0	0	1	Not prime
1	1	1	Not prime
2	1,2	2	Prime number
3	1,3	2	Prime number
4	1,2,4	3	Composite no.
5	1,5	2	Prime number
6	1,2,3,6	4	Composite no.
7	1,7	2	Prime number
8	1,2,4,8	4	Composite no.

COMPOSITE NUMBERS

Composite numbers are numbers with more than two factors. The first composite number is 4.

Examples of composite numbers.

<u>Number</u>	<u>Factors</u>
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10

- What is the next composite numbers after 10?
- Write a set of the first 5 composite numbers.
- Find the sum of the first 4 composite numbers.
- Find the average of composite numbers between 22 and 27.

SQUARE NUMBERS

These are numbers obtained after squaring an integer. These are numbers got after multiplying a number by itself. Square numbers can also be got by adding consecutive odd numbers.

Example

$$1 \times 1 = 1^2 = 1$$

$$\text{set } P = \{1, 4, 9, 16, \dots\}$$

$$2 \times 2 = 2^2 = 4$$

Therefore set P is a set of Square numbers.

$$3 \times 3 = 3^2 = 9$$

$$4 \times 4 = 4^2 = 16$$

$$5 \times 5 = 5^2 = 25$$

$$\begin{array}{ccc} & \text{NB} & \\ & 3 \times 3 = 9 & \\ \downarrow & & \downarrow \\ \text{square} & & \text{square} \\ \text{root} & & \text{number} \end{array}$$

Exercise

Find the square of the following numbers.

- a. 4 b. 5 c. 2 d. 8 e. 10
f. 12 g. 20

SQUARE OF FRACTIONS

a. $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b. $\frac{3}{5} = 3 \times 3 = \frac{9}{25}$
75

c. $1\frac{1}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

d. $\frac{3}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

Exercise

Find the square of the following fractions

a. $\frac{1}{4}$

b. $\frac{3}{4}$

c. $\frac{7}{5}$

d. $2\frac{1}{3}$

e. $\frac{2}{5}$

f. $\frac{6}{4}$

g. $3\frac{1}{5}$

h. $4\frac{1}{2}$

Square of Decimals

$0.5 = 0.5 \times 0.5 = 0.25$

$0.4 = 0.4 \times 0.4 = 0.16$

$0.2 = 0.2 \times 0.2 = 0.04$

$0.12 = 0.12 \times 0.12 = 0.0144$

$0.6 = 0.6 \times 0.6 = 0.36$

$0.25 = 0.25 \times 0.25 = 0.0625$

Exercise

Find the square of the following decimals.

a. 0.1

b. 0.7

c. 0.13

d. 0.14

e. 0.8

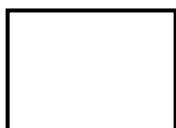
f. 0.15

g. 4.3

h. 2.5

Exercise b word problems.

- Find the square of 0.17
- One side of a square garden is 0.9cm. Find the area of the garden.
- Find the area of the square whose side is 0.8m
- The area of the square is 0.64 cm². Find one side of the square.
- Below is a square whose area is 0.25m². use it to answer that questions that follow.



a. Find the length of the one side.

b. Calculate the perimeter of that square.

SQUARE ROOT

Square root of whole numbers.

Find the square root of 36.

2	36
2	18
3	9
3	3
	1

$$36 = \sqrt{(2 \times 2) \times (3 \times 3)} = \sqrt{2 \times 3} = 6$$

Exercise

Find the square root of the following numbers.

a. 4

b. 16

c. 25

d. 100

e. 144

f. 49

g. 225

Square root of fractions

1. Find the square root of $\frac{1}{9}$

$$\sqrt{\frac{1}{9}} = \sqrt{\frac{1}{3} \times \frac{1}{3}} = \frac{1}{3}$$

2. Find the square root of $\frac{36}{81}$

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ \hline & 1 \end{array} \quad \sqrt{36} = 6 \quad \sqrt{\frac{36}{81}} = \frac{6}{9}$$

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ \hline & 1 \end{array} \quad \sqrt{81} = 9 \quad \text{Therefore } \sqrt{\frac{36}{81}} = \frac{6}{9}$$

3. Find the square root of $6\frac{1}{4}$

$$\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}}$$

$$\sqrt{25} = \frac{5}{5} \quad \frac{5}{5} \quad \frac{1}{1}$$

$$\sqrt{5 \times 5} = \frac{2}{2} \quad \frac{4}{2} \quad \frac{1}{1}$$

$$\sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$\text{Therefore } \sqrt{6\frac{1}{4}} = \frac{5}{2} = 2\frac{1}{2}$$

4. Find the square root of $3\frac{1}{16}$

$$\sqrt{3\frac{1}{16}} = \sqrt{\frac{49}{16}}$$

$$\sqrt{49} = \frac{7}{7} \quad \frac{7}{7} \quad \frac{1}{1}$$

$$\sqrt{\frac{49}{16}} = \frac{7}{4}$$

$$\text{Therefore } \sqrt{3\frac{1}{16}} = \frac{7}{4} = 1\frac{3}{4}$$

CUBIC NUMBERS

Cubic numbers are numbers obtained by multiplying a counting number by itself three times.

i.e

$$1 \times 1 \times 1 = 1$$

$$2 \times 2 \times 2 = 8$$

$$3 \times 3 \times 3 = 27$$

$$4 \times 4 \times 4 = 64$$

$$5 \times 5 \times 5 = 125$$

Therefore, 1, 8, 27, 64, 125 are examples of cubic numbers.

What are the next two cubic numbers after 125?

Give the next number in the sequence: 64 , 27 , 8, ____

Find the value of;

- a). 10^3 b). 5^3 c). 12^3 d). $(\frac{1}{3})^3$
e). $(\frac{1}{9})^3$ f). $(\frac{1}{2})^3$

ALGEBRA

It is a branch of mathematics in which symbols and letters are used to represent numbers.

Letters are called terms. Examples: 3a, 5y, 2p etc

The terms with the same letters are called like terms while terms with different letters are called unlike terms.

Examples: $7p+8w$. They cannot be simplified any further.

In mathematics 4×2 can be written as $2+2+2+2$

Similarly in algebra $5a$ can be written as $a+a+a+a+a = 5a$

HOW TO SIMPLIFY EXPRESSIONS WITH MANY TERMS

Example 1: Simplify: $3a - 8a + 5a + 9a - 2a$

Solution: First group all the terms with positive signs

$$3a + 5a + 9a - 8a - 2a.$$

$$= 17a - 10a$$

$$= \underline{7a}$$

This method is called grouping positives and negative terms

Example a)

$$\begin{array}{r} ab^2 - 5ab^2 + 3ab^2 \\ ab^2 + 3ab^2 - 5ab^2 \\ 4ab^2 - 5ab^2 \\ \underline{-ab^2} \end{array}$$

Example b)

$$\begin{array}{r} 6ab - 2ab - 3ab \\ 6ab - 5ab \\ \underline{ab} \end{array}$$

MK2000 new edition pupils book 6 page 377, Fountain book 6 page 187

Fountain book 7 page 215

Collection of like terms

NB A term without a sign is a positive term. A sign before the term is the **term for that term**

Examples:

a. with positives only e.g. $7a + 6 + 3a + 5$

$$\Rightarrow 7a + 3a + 6 + 5$$
$$\Rightarrow \underline{\underline{10a + 11}}$$

b. with both negatives and positives.
e.g. $8y - 3x - 5y + 9x$.

N.B: a term moves with its sign.

$$\begin{array}{r} 8y - 5y + 9x - 3x \\ \underline{\underline{3y + 6x.}} \end{array}$$

$$\begin{aligned} \text{i} \quad & -m + 2p + 5m - 8p - m \\ & -m + -m + 5m - 8p + 2p \\ & -2m + 5m - 6p \\ & \underline{\mathbf{3m - 6p}} \end{aligned}$$

$$\begin{aligned} \text{ii} \quad & 3xy - 5ac + 4xy + 6ac \\ & 3xy + 4xy - 5ac + 6ac \\ & 7xy + 6ac - 5ac \\ & \underline{\mathbf{7xy + ac}} \end{aligned}$$

$$\begin{aligned} \text{iii} \quad & 4a + 6b - 9a + 2b \\ & 4a - 9a + 6b + 2b \\ & \underline{\mathbf{-5a + 8b}} \end{aligned}$$

$$\begin{aligned} \text{iv} \quad & 8w - 5k - 11w + 4k \\ & 8w - 11w - 5k + 4k \\ & \underline{\mathbf{-3w - k}} \end{aligned}$$

Removing brackets Expressions which involve brackets may have terms inside the ***Brackets simplified first, then collect the like terms***

$$\begin{aligned} \text{Example} \quad & \mathbf{3(2x + 4x)} \\ & = \mathbf{6x + 12x} \\ & = \underline{\mathbf{18x}} \end{aligned}$$

If the terms inside the brackets are unlike then you have to use the terms outside the brackets in order to remove the brackets

$$\begin{aligned} \text{Example:} \quad & 2(3y + 6p) \\ & \underline{\mathbf{6y + 12p}} \end{aligned}$$

Involving brackets

i. (positive only)

$$\begin{aligned} \text{e.g.} \quad & 4(y + 3) + 2(y + 2) \\ & 4y + 12 + 2y + 4 \\ & 4y + 2y + 12 + 4 \\ & \underline{\mathbf{6y + 16}} \end{aligned}$$

ii. (both positives and negatives)

$$\text{eg. } 5(m - 3) - 3(m - 6)$$

NB. i. **a negative sign before the brackets changes the signs inside the brackets.**

$$\begin{aligned} & 5m - 15 - 3m + 18 \\ & 5m - 3m + 18 - 15 \\ & \underline{\mathbf{2m + 3}} \end{aligned}$$

$$\begin{aligned} 2. \quad & 8(p + 2q) - 6(p + q) \\ & 8p + 16q - 6p - 6q \\ & 8p - 6p - 16q - 6q \\ & \underline{\mathbf{2p + 10q}} \end{aligned}$$

positive sign before the brackets

From the above examples it is clear that any positive sign (term) outside the brackets does not change the sign of the term inside the brackets. On the other hand a negative sign outside the brackets changes the sign inside the brackets

$$\begin{aligned} \text{Examples} \quad & 5(3m + 2a) \\ & \underline{\mathbf{15m + 10a}} \end{aligned}$$

$$(b) \quad -(5-n) \\ \underline{-5 + n}$$

$$(c) \quad -3(4a - 6y) \\ \underline{-12a + 18y}$$

$$(b) \quad -x(-2x + 3y) \\ \underline{2x^2 - 3xy}$$

$$(e) \quad -13(x+4) - 21(1-x) \\ \underline{-13x - 52 - 21 + 21x} \\ -13x + 21x - 52 - 21 \\ \underline{8x - 73}$$

Subtraction of expressions

I Before subtracting any expression from another, the terms must be put into brackets
 ii) Start writing the terms which come immediately after the word from, insert the subtraction sign.

Example i Subtract $12x$ from $-8x$ ii Subtract: $2m-3w$ from $4m + w$

$$\begin{array}{r} (-8x) - (12x) \\ -8x - 12x \\ - \quad \underline{20x} \end{array} \qquad \begin{array}{r} (4m+w) - (2m-3w) \\ 4m+w - 2m + 3w \\ \underline{4m - 2m + w + 3w} \\ \underline{2m+4w} \end{array}$$

iii Subtract: $2x + y$ from $3x + 2y$ Subtract: $2(x+3)$ from $3(x+1)$

$$\begin{array}{r} (3x+2y) - (2x+y) \\ 3x + 2y - 2x + y \\ 3x - 2x + 2y + y \\ \underline{x + 3y} \end{array} \qquad \begin{array}{r} 3(x+1) - 2(x+3) \\ 3x + 3 - 2x - 6 \\ 3x - 2x + 3 - 6 \\ \underline{x - 3} \end{array}$$

Thrice the difference between x and 7 is written as $3(x-7)$

1. Let the term to be subtracted from be given any unknown, which is not in the terms mentioned.

2 Put the terms in brackets

2 Let the terms you have given be left alone on one side by making them positive.
 Example: I) What must be subtracted from $3x+2y$ to give $x + 3y$?

Solution

Let the number to be subtracted be w

$$\begin{aligned} (3x+2y) - (w) &= (x+3y) \\ (3x + 2y) - w + w &= (x+3y) + w \\ (3x+2y) - (x + 3y) &= w \\ 3x + 2y - x - 3y &= w \\ 3x - x + 2y - 3y &= w \\ \underline{2x - y} &= w \end{aligned}$$

ii) What must be subtracted from $4a + m$ to get $2a + 4m$

Let the number be n

$$\begin{aligned} (4a + m) - (n) &= (2a + 4m) \\ (4a + m) - n + n &= (2a+4m) + n \\ (4a + m) - (2a + 4m) &= n \\ 4a + m - 2a - 4m &= n \\ 4a - 2a + m - 4m &= n \\ \underline{2a - 3m} &= n \end{aligned}$$

iii) What must be added to $4a+b$ to make $6a-3b$?

Let it be m

$$\begin{aligned}(4a+b) + m &= (6a-3b) \\ (4a+b) - (4a+b) + m &= (6a-3b) - (4a+b) \\ m &= 6a-3b-4a-b \\ m &= 6a-4a-3b-b \\ \underline{m} &= \underline{2a-4b}\end{aligned}$$

iv) What must be added to $\frac{1}{2}$ to get $\frac{3}{4}$

Solution Let the number added be p

$$\begin{aligned}p + \frac{1}{2} &= \frac{3}{4} \\ p + \frac{1}{2} - \frac{1}{2} &= \frac{3}{4} - \frac{1}{2} \\ p &= \frac{6-4}{8} \\ p &= \underline{\underline{\frac{1}{4}}}\end{aligned}$$

Exercise: Let pupils do Exercise below

1: What must be subtracted from $\frac{3}{4}$ to get $\frac{1}{3}$?

2 What must be subtracted from $3x+y$ to get $x+y$?

3 What must be added to x to get $2x-5$?

4 What must be added to $-m$ to get $3m-6$?

4 What must be added to $2p+2k$ to get $k-4p$?

Substitution:

It means to replace (put in place). Usually each letter is given a representation

Examples: Given that $a=3$, $b=4$ and $c=5$

Evaluate: $3(5+4)$

$$3(7)$$

$$3 \times 7$$

$$\underline{\underline{21 \text{ Ans}}}$$

$$a(b^2 - c)$$

$$3(4^2 - 5)$$

$$3(4 \times 4 - 5)$$

$$3(16 - 5)$$

$$3 \times 11$$

$$\underline{\underline{33 \text{ Ans}}}$$

2: If $a=5$, $b=10$, $c=6$, $d=\frac{1}{2}$ and $e=\frac{1}{5}$

Work out the following :

$$\begin{aligned}(a) \quad &4(a+b) \\ &4(5+10) \\ &4 \times 15\end{aligned}$$

$$\begin{aligned}(b) \quad &-3(a+3c) \\ &-3(5+3 \times 6) \\ &-3(5+18)\end{aligned}$$

$$= \underline{\underline{60 \text{ Ans}}}$$

$$-3 \times 23 \\ = \underline{\underline{-69 \text{ Ans}}}$$

$$\begin{aligned} \text{(c)} \quad & d(b- 6) \\ & \frac{1}{2} (10 -6) \\ & \frac{1}{2} (4) \\ & \frac{1}{2} \times 4 \\ & = \underline{\underline{2 \text{ Ans}}} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & ad(b - c) \\ & 5 \times \frac{1}{2}(10 - 6) \\ & 5 \times \frac{1}{2} \times 4 \\ & 5 \times 2 \\ & = \underline{\underline{10 \text{ Ans}}} \end{aligned}$$

If $x = a + 2b$ and $y = 2a - b$. Express $3x - y$ in terms of a and b

$$\begin{aligned} \text{Solution} \quad 3x - y &= 3(a+2b) - (2a-b) & (2x+y) &= 2(a+2b) + (2a - b) \\ & 3a + 6b - 2a + b & &= 2a + 4b + 2a - b \\ & 3a - 2a + 6b + b & &= 2a + 2a + 4b - b \\ & \underline{\underline{a + 7b}} & &= \underline{\underline{4a + 3b}} \end{aligned}$$

If $a=3x$, $b=6y$ $c= 2z$ Work out the following:

$$\begin{aligned} \text{a)} \quad & 3(b-2c) \\ & 3(6y-2 \times 2z) \\ & 3(6y -4z) \\ & \underline{\underline{18y - 12z}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & -a(b+c) \\ & -3x(6y+2z) \\ & (-3x \times 6y) + (-3x \times 2z) \\ & \underline{\underline{-18xy - 6xy}} \end{aligned}$$

Complex

i. If $m = n$, $n= 8$ and $p = -5$, evaluate:

$$\begin{aligned} & m^2 - np \\ & (mxm) - (n \times p) \\ & (8 \times 8) - (8 \times -5) \\ & 64 - -40 \\ & 64 + 40 \\ & \underline{\underline{104}} \end{aligned}$$

Exercise:

Let pupils do the following exercise.

Given that $a = -2$, $b=3$, Find the value of : 1) $a-b^2$ ii $b^2 -a$

2) If $x= -3$, $y=2$, and $p = -1$, Evaluate

$$\text{i) } x + y + p \quad \text{ii) } xyp \quad \text{iii) } 2y + x - y \quad \text{iv) } 2x - p \quad \text{v) } 3xy + px$$

3) If $a= (x-y)$ and $b= (x+y)$ Write the expression for :

$$\text{i). } a+b \quad \text{ii) } b-a \quad \text{iii) } a - b \quad \text{in terms of } x \text{ and } y$$

Removing brackets involving fractions:

$$\text{i) } \frac{1}{3}(3a+9b)$$

$$\begin{aligned} &= \frac{1}{3} \times 3a + \frac{1}{3} \times 9b \\ &= a + 3b \end{aligned}$$

$$\text{ii} \quad \frac{2}{7}(42b -14a)$$

$$\frac{2 \times 42b - 2 \times 14a}{7}$$

$$= \underline{\underline{12b - 4a}}$$

FRACTIONAL TERMS:

In fractional terms, any term without a denominator is assumed to have the denominator as 1.

Example: $a + \frac{a}{5}$ ii $\frac{x}{2} + \frac{x}{3}$ LCM = 6

$$\frac{5xa + a/\cancel{5} \times 5}{5a + a} \qquad \frac{\cancel{6} \times x}{2} + \frac{x \times \cancel{6}}{3}$$

$$= \underline{\underline{6a}} \qquad \frac{3x + 2x}{6}$$

$$\underline{\underline{5x}}$$

$$\underline{\underline{6}}$$

ii) $p + \frac{p}{3}$

$$3Xp + 3x \frac{p}{3} \quad \text{LCM} = 3$$

$$= \frac{3p + p}{3}$$

$$= \underline{\underline{\frac{4p}{3}}}$$

iv) $\frac{b}{4} - \frac{b}{3}$ LCM = 12

$$\frac{\cancel{b}/\cancel{4} \times 12}{12} - \frac{b/\cancel{3} \times 12}{12}$$

$$\frac{3b - 4b}{12}$$

$$\underline{\underline{\frac{-b}{12}}}$$

v) $\frac{x+1}{2} + \frac{x-2}{3}$ Lcm = 6

$$\frac{\cancel{6} \times (x+1)}{2} + \frac{6 \times (x-2)}{\cancel{3}}$$

$$\frac{3x + 2x + 3 - 4}{6}$$

$$\underline{\underline{\frac{5x - 1}{6}}}$$

Equations:

This is a mathematical statement which shows that the two sides are equal.

a. Adding the same number both sides.

e.g i. $a - 2 = 5$
 $a - 2 + 2 = 5 + 2$
 $\underline{\underline{a = 7}}$

b. Subtracting the same number both sides.

e.g ii. $p + 5 = 17$
 $p + 5 - 5 = 17 - 5$
 $\underline{\underline{p = 12}}$

c. Dividing the same number both sides.

e.g iii. $4n = 12$
 $\underline{\underline{4n = 12}}$

$$\frac{4}{n} = \frac{4}{3}$$

$$\underline{n = 3}$$

d. Multiplying the same number both sides.

e.g iv. $\frac{1}{4}x = 9$

$$4 \times \frac{1}{4}x = 9 \times 4$$

$$x = 36$$

v. $\frac{3}{5}y = 12$

$$\frac{5}{3} \times \frac{3}{5}y = 12 \times \frac{5}{3}$$

$$\underline{y = 20}$$

e.g With unknowns on both sides

e.g vi. $8y + 6 = 5y + 18$

$$8y - 5y + 6 = 5y - 5y + 18$$

$$3y + 6 = 18$$

$$3y + 6 - 6 = 18 - 6$$

$$\underline{3y = 12}$$

$$\underline{y = 4}$$

or: $8y - 5y = 18 - 6$

NB. Transfer of terms ie. A negative becomes a positive and a positive becomes a negative. When a term crosses the equal signs, its sign changes.

$$3y = 12$$

$$\underline{3y = 12}$$

$$\underline{y = 4.}$$

f. With brackets on one side: e.g. vii. $3(x + 2) = 18$.

NB. A whole number before the brackets should be multiplied by each term inside the brackets.

$$3x + 6 = 18$$

$$3x + 6 - 6 = 18 - 6$$

$$3x = 12$$

$$\underline{3x = 12}$$

$$\underline{x = 4}$$

viii. $5(y - 2) = -20$

$$5y - 10 = -20$$

$$5y - 10 + 10 = -20 + 10$$

$$5y = -10$$

$$\underline{y = -2}$$

g. With brackets on both sides: i. Positives

e.g ix. $3(2m + 4) = 4(m + 5)$

$$6m + 12 = 4m + 20$$

$$6m + 12 - 12 = 4m + 20 - 12$$

$$6m = 4m + 8$$

$$6m - 4m = 4m - 4m + 8$$

$$\underline{2m = 8}$$

$$\underline{m = 4}$$

Or: $6m + 12 = 4m + 20$

$$6m - 4m = 20 - 12$$

$$2m = 8$$

$$2m = 8$$

$$\underline{2m = 8}$$

$$\underline{m = 4}$$

ii. Negatives: e.g $x + 7(m - 2) = 2(2 - m)$

Either: $7m - 14 = 4 - 2m$

$$7m + 2m - 14 = 4 - 2m + 2m$$

$$9m - 14 = 4$$

$$9m - 14 + 14 = 4 + 14$$

$$\frac{9m}{9} = \frac{18}{9}$$

$$\underline{m = 2}$$

h. cross multiplication

xi $\frac{x+2}{3} = \frac{x+6}{5}$

$$5(x+2) = 3(x+6)$$

$$5x + 10 = 3x + 18$$

$$5x - 3x = 18 - 10$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

$$\underline{\underline{x = 4}}$$

i. With two brackets on one side.

13. (Positives) $5(d+2) + 3(d+3) = 35$

$$5d + 10 + 3d + 9 = 35$$

$$5d + 3d + 10 + 9 = 35$$

$$8d + 19 = 35$$

$$8d + 19 - 19 = 35 - 19$$

$$\frac{8d}{8} = \frac{16}{8}$$

$$\underline{\underline{d = 2}}$$

Or: $7m - 14 = 4 - 2m$

$$7m + 2m - 14 = 4 + 14$$

$$9m = 18$$

$$\frac{9m}{9} = \frac{18}{9}$$

$$\underline{m = 2}$$

$$\frac{8}{6-y} = \frac{2}{y-1}$$

$$8(y-1) = 2(6-y)$$

$$8y - 8 = 12 - 2y$$

$$8y + 2y = 12 + 8$$

$$10y = 20$$

$$\frac{10y}{10} = \frac{20}{10}$$

$$\underline{\underline{y = 2}}$$

14. (Negatives) $7(x+3) - 4(x+4) = 20$

$$7x + 21 - 4x - 16 = 20$$

$$7x - 4x + 21 - 16 = 20$$

$$3x + 5 = 20$$

$$3x + 5 - 5 = 20 - 5$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$\underline{\underline{x = 5}}$$

j. With fractional co-efficient.

e.g (15). $\frac{3x}{4} + 9 = 12$

$$\frac{3x}{4} + 9 - 9 = 12 - 9$$

$$\frac{3x}{4} = 3$$

$$\frac{4}{3} \left(\frac{3x}{4} \right) = 3 \times \frac{4}{3}$$

$$\underline{\underline{x = 4}}$$

16. $\frac{1}{8}y - \frac{1}{6} = \frac{11}{24}$

$$24\left(\frac{1}{8}y\right) - 24\left(\frac{1}{6}\right) = \left(\frac{11}{24}\right) 24$$

$$3y - 4 = 11$$

$$3y - 4 + 4 = 11 + 4$$

$$3y = 15$$

$$\frac{3y}{3} = \frac{15}{3}$$

$$\underline{\underline{y = 5}}$$

h. With square root and coefficient

E.g 17 $y^2 - 3 = 33$

$$y^2 - 3 + 3 = 33 + 3$$

$$y^2 = 36$$

18. $3m^2 + 5 = 80$

$$3m^2 + 5 - 5 = 80 - 5$$

$$3m^2 = 75$$

EQUATIONS WITH FRACTIONAL COEFFICIENTS:

Examples: I) $\frac{1}{2}t = 6$
 $2 \times \frac{1}{2}t = 6 \times 2$
 $t = 12$

ii) $4\frac{1}{3}p + 2 = 15$
 $3 \times \frac{13p}{3} + 2 - 2 = (15 - 2) \times 3$
 $13p = 13 \times 3$
 $p = 3$

iii) $0.4m + 0.5 = 2.1$
 $\frac{4m}{10} + \frac{5}{10} = \frac{21}{10}$
 $4m + 5 - 5 = 21 - 5$
 $\frac{4m}{4} = \frac{16}{4}$
 $m = 4$

iv) $p - \frac{2}{3}p = 7$
 $3 \times p - \frac{2}{3} \times 3p = 7 \times 3$
 $3p - 2p = 21$
 $p = 21$

APPLICATION OF ALGEBRA :

1) Think of a number add 4 to it the result is 10 find the number.

Solution: Let the number be p

$$p + 4 = 10$$

$$p + 4 - 4 = 10 - 4$$

$$p = 6$$

2) Think of a number, multiply it by 2 then divide the result by 3, the answer is 10 What is the number?

Solution: Let the number be y

$$\frac{y \times 2}{3} = 10$$

$$\frac{3 \times 2y}{3} = 10 \times 3$$

$$\begin{array}{rcl} \frac{2y}{2} & = & \frac{30}{2} \\ \hline y & = & 15 \end{array}$$

3) A sheep costs 6000/= more a goat. If their total cost is 70,000/= Find the cost of each animal.

Solution: Let the cost of a goat be p:

Goat	Sheep
P	p + 6000/=
P + p + 6000	= 70,000=
2p + 6000 - 6000	= 70,000 - 6000
$\frac{2p}{2}$	= $\frac{64,000}{2}$
p	= 32000=

Goat	= 32000/=
Sheep p + 6000	
32 000 + 6000	= <u>38,000/=</u>

5) Alice is 4 years younger than Abbo, If their total age is 24years .Find their ages.

Solution: Let the age of Abbo be y years

Abbo	Alice
Y	y - 4
Y + y - 4	= 24
2y - 4 + 4	= 24 + 4
$\frac{2y}{2}$	= $\frac{28}{2}$
Y = 14	

Abbo = 14 years

Alice = y - 4
= 14 - 4

= 10 years

6) Peter is 5 years older than Moses. If their total age is 49 years How old is Moses?

Solution: Peter Moses
Y + 5 y

Y + y + 5 = 49

2y + 5 - 5 = 49 - 5 $\frac{2y}{2} = 44$

y = 22

Moses is 22 years

7) A ball and a pair of boots cost 150,000/= If boots cost twice as much as a ball Find the cost of each item. **Solution:** Let the cost of the ball be p then boots be 2p

P + 2p	= sh 150.000
$\frac{3p}{3}$	= $\frac{150,000}{3}$

Ball costs sh 50,000

Boots cost 2x 50,000
= sh **100,000**

P = sh 50000

- 8) A mother bought 8 exercise books at shs.(x-150) each and two mathematical sets at (x+100) each .If she spent shs.5300 altogether, how much did she spend on:
a) books? b) sets.

Solution:

$ \begin{array}{rcl} 8(x-150) + 2(x + 100) & = & 5300 \\ 8x - 120 + 2x + 200 & = & 5300 \\ 8x + 2x - 1200 + 200 & = & 5300 \\ 10x - 1000 + 1000 & = & 5300 + 1000. \\ \cancel{10x} - \cancel{1000} + \cancel{1000} & = & \cancel{6300} \\ -10 & & -10 \\ \hline x & = & \underline{630} \end{array} $	<p>Books $8(630 - 150)$ 8×480 <u>3840/=</u></p> <p>Sets $2(630 + 100)$ 2×730 <u>1460/=</u></p>
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9: **Solve:** $\frac{3y+3}{4} + 2 = \frac{2y+12}{3}$ Get L.C.M of 4 and 3

Solution:

$$\begin{array}{rcl}
 \frac{12(3y+3)}{4} + 2 \times 12 & = & \frac{12(2y+12)}{3} \\
 9y + 9 + 24 & = & 8y + 48 \\
 9y + 33 & = & 8y + 48 - 33 \\
 9y - 8y & = & 15 \\
 \underline{y} & = & \underline{15}
 \end{array}$$

- 10: Betty, Joyce and Alice shared shs.72000 such that Betty got 3 times as much as Alice
 Alice got twice as much as Joyce. Calculate their shares.

Solution:

Joyce P	Alice 2p	Betty 3x2p	total 72000/=
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Equation: $p + 2p + 6p = 72000$

$$\begin{array}{rcl}
 9p & = & 72000 \\
 \frac{9p}{9} & = & \frac{72000}{9} \\
 p & = & 8000
 \end{array}$$

<u>Joyce</u> Shs p Shs 8000	<u>Alice</u> Shs 2x 8000 Shs 16000	<u>Betty</u> Shs 6x 8000 Shs 48000
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EXERCISE

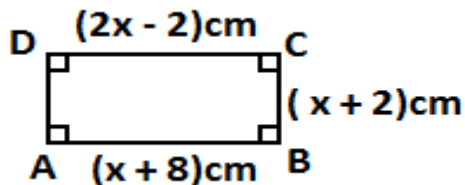
- 1.The number of boys in a school is less than the number of girls by 80. If there 300 pupils in the school how many boys are in the school?
- 2:Kato was told to share Shs 45000 with Nakato .If Kato got twice as much as Nakato. Find their shares.
- 3:A shirt and a dress cost Shs 14400 If a shirt costs Shs6400less than a dress. What are their costs?
- 4: John bought 2kg of sugar at Shs 3p and 1 kg of salt at Shs (p + 200) .Work out the value p, if John spent Shs.37000 .

Let pupils do exercise 3:nos 1—9 on page 31 primary maths revision and practice

Application 2

a. Perimeter and Area.

e.g The figure below is a rectangle ABCD



i. Work out the value of x

$$(2x - 2) \text{ cm} = (x + 8) \text{ cm}$$

(opposite sides equal)

$$2x - 2 = x + 8$$

$$2x - x = 8 + 2$$

$$\underline{x = 10}$$

ii. Find its length and width.

length

width

$$(x + 8) \text{ cm}$$

$$(x + 2) \text{ cm}$$

$$(10 + 8) \text{ cm}$$

$$(10 + 2) \text{ cm}$$

$$\underline{18 \text{ cm}}$$

$$\underline{12 \text{ cm}}$$

iii. Calculate its area and perimeter.

$$\text{Area} = \text{Length} \times \text{Width}$$

$$= 18 \text{ cm} \times 12 \text{ cm}$$

$$\underline{\text{Area} = 216 \text{ cm}^2}$$

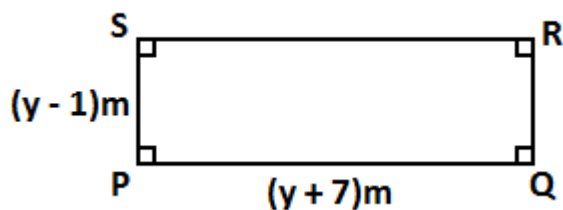
$$\text{Perimeter} = 2(\text{length} + \text{Width})$$

$$2(18 \text{ cm} + 12 \text{ cm})$$

$$2 \times 30$$

$$\underline{\text{Perimeter} = 60 \text{ cm}}$$

2. The perimeter of the rectangle below is 44m



a. Work out the value of y

$$2(\text{length} + \text{width}) = \text{perimeter}$$

$$2(y + 7 + y - 1) \text{ m} = 44$$

$$\frac{2(y + y + 7 - 1) \text{ m}}{2 \text{ m}} = \frac{44}{2 \text{ m}}$$

$$2y + 6 = 22$$

$$2y + 6 - 6 = 22 - 6$$

$$\frac{2y}{2} = \frac{16}{2}$$

$$\underline{y = 8}$$

b. Calculate its area

Length

Width

Area

$$\begin{array}{l} (y + 7) \text{ m} \\ (8 + 7) \text{ m} \end{array}$$

$$\begin{array}{l} (y - 1) \text{ m} \\ (8 - 1) \text{ m} \end{array}$$

$$\begin{array}{l} \text{Length} \times \text{width} \\ 15 \text{ m} \times 7 \text{ m} \end{array}$$

$$\underline{15 \text{ m}}$$

$$\underline{7 \text{ m}}$$

$$\underline{105 \text{ m}^2}$$

3. The length of rectangle is 5cm greater than the width. If its perimeter is 58cm, calculate its area.

Let the width be w cm.

The length is $(w + 5)$ cm

$$2(\text{length} + \text{width}) = \text{Perimeter}$$

$$2(w + 5 + w) \text{ cm} = 58 \text{ cm}$$

$$\frac{2(w + w + 5) \text{ cm}}{2 \text{ cm}} = \frac{58 \text{ cm}}{2 \text{ cm}}$$

$$2w + 5 = 29$$

$$2w + 5 - 5 = 29 - 5$$

$$\frac{2w}{2} = \frac{24}{2}$$

$$w = 12$$

Length

$$(w + 5)$$

$$(12 + 5) \text{ cm}$$

$$17 \text{ cm}$$

width

$$w \text{ cm}$$

$$12 \text{ cm}$$

Area = Length x width

$$= 17 \text{ cm} \times 12 \text{ cm}$$

$$\text{Area} = 204 \text{ cm}^2$$

Formation of equations about time to come.

- 1: 1. Brenda is 10 years older than Jane. If the total of their ages is 48 years, how old is each of them?

Let Jane's age be d yrs.

Jane	Brenda	Total
d	$d + 10$	48

$$d + d + 10 = 48$$

$$2d + 10 = 48$$

$$2d + 10 - 10 = 48 - 10$$

$$\frac{2d}{2} = \frac{38}{2}$$

$$d = 19$$

Jane is 19 years

Brenda is $(19 + 10)$

$= 29$ years.

2. Abdul is twice as old as Fred. The total of their ages is 45 years. How old is each of them?

Let Fred's age be m yrs.

Fred	Abdul	Total
m	$m \times 2 = 2m$	45

$$m + 2m = 45$$

$$3m = 45$$

$$\frac{3m}{3} = \frac{45}{3}$$

$$m = 15$$

Fred is 15 years

Abdul is 2×15 yrs

30 years

3. Shafick is 8 years younger than Amiina. In 6 years' time, the total of ages is 54 years. How old is each of them now?

	Shafick	Amiina	Total
now	R	R + 8	
After 6 yrs	R + 6	R + 8 + 6	54

$$R + 6 + R + 14 = 54$$

$$R + R + 6 + 14 = 54$$

$$2R + 20 = 54$$

$$2R + 20 - 20 = 54 - 20$$

$$2R = 34$$

$$R = 17$$

Shafick's age 17 years

Aminna's age $17 + 8$

$\Rightarrow 25$ years

4. Fredda is thrice as old as Peter. In 9 years' time the total of their ages is 66 years. How old is each of them now?

Let Peter's age be y

	Peter	Fredda	Total
Now	Y	$3 \times y = 3y$	
9 yrs time	y + 9	3y + 9	66

$$y + 9 + 3y + 9 = 66$$

$$y + 3y + 9 + 9 = 66$$

$$4y + 18 = 66$$

$$4y + 18 - 18 = 66 - 18$$

$$4y = 48$$

$$4 \quad 4$$

$$y = 12$$

Peter is 12 years

Fredda is 12×3

36 years.

Exercise One

1. Nathan is 7 years older than Anne. If the total of their ages is 89 years, how old is each of them?

2. Abdul is thrice as old as Joanna. The total of their ages 36. how old is each of them?
3. Mwebe is 14 years younger than Namuli. After 8 years, the total of the their ages is 56 years. How old is each of them now?
4. Ashaba is twice as old as Mugabi. In 10 years' time, the total of their ages will be 83 years. Find each one's age now.
5. Jeremy is twice as old as Jane's Martha is 4 years older than Jeremy. If the total of their ages is 44 years, how old is each of them?

CASE TWO

1. Moses is 24 years older than Jane. In 7 years' time, he will be twice as old as Jane. How old is each of them now?

Let Jane's age be m years.

Moses is $(m + 24)$ years

In 7 years' time,

Jane will be $(m+7)$ years

Moses will be $(m + 24 + 7)$ years

But

$2(\text{Jane's age}) = \text{Moses' age}.$

$$2(m+ 7) = m+31$$

$$2m + 14 = m + 31$$

$$2m - m = 31 - 14$$

$$\underline{m = 17.}$$

Jane is 17 years old.

Moses is $(17 + 24)$ years = 41 years old.

2. Abdu is 7 years old while Jamila is 43 years old. In how many years' time will Jamila be thrice as old as Abdu?

Let the time be a years

After a years

Abdu will be $(a + 7)$ years

Jamilla will be $(a + 43)$ years

But.

$3(\text{Abdul's age}) = \text{Jamila's age}$

$$3(a + 7) \text{ years} = (a + 43) \text{ years} \quad \underline{2a = 22}$$

$$3a + 21 = a + 43. \quad \quad \quad \underline{2} \quad \quad \underline{2}$$

$$3a - a = 43 - 21$$

$$a = 11$$

$$2a = 22$$

after 11 years.

- 2: Anne is 15 years younger than Peter. In 5 years time, Anne's age will be half the age of Peter. Find their ages now.

Solution:

	Anne	Peter	
Now	$m - 15$	m	Peter is 25 years old now.
5 years time	$(m - 15 + 5)$	$= 2 \times \frac{1}{2}(m + 5)$	
	$2(m - 10)$	$= m + 5$	
	$2m - 20 + 20$	$= m + 5 + 20$	Anne is $(m - 15)$ years
	$2m - m$	$= 25$	$= (25 - 15)$ years
	$m = 25$		$= 10$ years

**What will be their ages then? Anne 10+5
15 years**

**Peter = $m + 5$
 $25 + 5$
30 years**

- 3) A son is 20 years younger than the mother. In 10 years time the son will be half the age of the mother. Calculate their present ages. Solution: Let the mother's age be y

	Son	mother
now	$y - 20$	y
10 yrs time	$(y - 20 + 10)$	$= \frac{1}{2}(y + 10)$
	$2(y - 10)$	$= 2 \times \frac{1}{2}(y + 10)$
	$2y - 20 + 20$	$= y + 10 + 20$
	$2y - y$	$= 30$
	$y = 30$	

mother is 30 years old

Son is $(y - 20)$

$$= (30 - 20) \text{ years}$$

$$= 10 \text{ years}$$

- a) What will their ages be then?

**Solution Mother $y + 10$
 $30 + 10$
40 years old**

**Son $10 + 10$
20 years**

EXERCISE

- Annet is 20 years younger than Musa now. 10 years ago Annet was $\frac{1}{2}$ the age of Musa. Work out their present ages.
- A father is 20 years older his son. In 10 years time a father will be twice the age of his son. a) Calculate their ages now.
- Peter is 20 years older than John now. 10 years ago Peter was twice as old as John. How old are they now?
- A father is 3 times as old as his son. In 10 years time the son will be half the age of the father. Calculate their present ages.
- Mary is 10 years old and Aisha is 30 years old. In how many years time will Mary be half the age of Aisha?**

6. A daughter is 3 years old. A mother is 21 years old. In how many years' time will the mother be 3 times the age of the daughter. What will be their ages then?
7. A mother 14 years older than her daughter. In 8 years time a mother will be twice the age of the daughter. Calculate their ages now.
8. Jane is 3 years old. Betty is 7 years old. At what time will Jane's age be half the age of Betty?
9. Moses is 26 years old and George is 4 years old. In how many years' time will Moses be 6 times as old as George?
10. Paul is 14 years old and Sarah is 2 years old. At time will Sarah be $\frac{1}{4}$ the age of Paul?
11. A father is 28 years old and a son is 6 years. In how many years' time will the son be $\frac{1}{3}$ of the father's age.
12. Kato is 3 times as old as Jojo. The difference in their ages is 36 years. Find their ages.