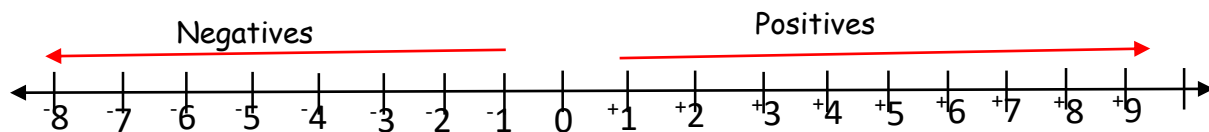


## TOPIC: INTEGERS

### CONTENT: Introduction and ordering integers

- An integer is a positive (+) or negative (-) number and zero.
- Examples of integers are {...-4,-3,-2,-1,0,+1,+2,+3,+4.....}
- Zero (0) is neither a negative (-) nor a positive integer.
- Integers can be represented on a number line.
- On a number line, positive (+) integers are put on the right hand side while negative (-) integers are put on the left as shown below



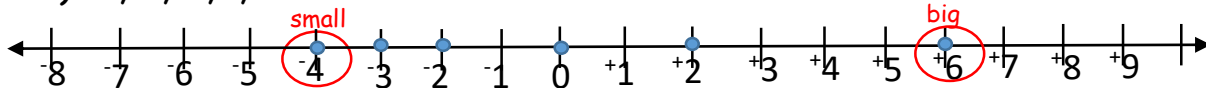
### Ordering integers

Integers can be ordered in either ascending or descending order

#### Examples

1. Arrange the following integers in ascending order

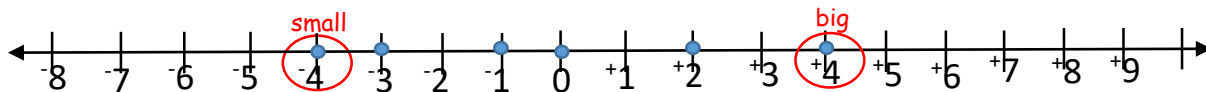
a) +6, -4, +2, 0, -2 and -3



Ascending order = {-4, -3, -2, 0, +2, +6}

2. Arrange the following integers in descending order

a) +4, -4, +2, 0, -1 and -3



Descending order = {+4, +2, 0, -2, -3, -4}

## Activity

1. Arrange the following integers in ascending order

a)  $+5, -3, +3, 0, -1$  and  $-4$

b)  $-5, -2, +4, 0, +1$  and  $-6$

c)  $+6, -7, +5, 0, -4$  and  $+3$

d)  $+4, -4, +2, 0, -1$  and  $-3$

2. Arrange the following integers in descending order

a)  $-5, -3, +3, 0, -1$  and  $+4$

b)  $+7, -3, +3, 0, -1$  and  $-4$

c)  $+4, -2, +3, 0, +1$  and  $-1$

d)  $+6, -3, +3, 0, -1$  and  $+4$

---

## Comparing integers using $<$ or $>$ or $=$

Examples

1. Compare the following integers using  $<$  or  $>$  or  $=$

a)  $-100 < 0$

b)  $+67 > -67$

c)  $-10 > -50$

## Activity

1. Compare the following integers using  $<$  or  $>$  or  $=$

a)  $-10 \dots \dots \dots 0$

b)  $+60 \dots \dots \dots -60$

c)  $-1 \dots \dots \dots -5$

d)  $-24 \dots \dots \dots +24$

e)  $+6 \dots \dots \dots -30$

c)  $-15 \dots \dots \dots -5$

---

## Additive inverse

### Points to note

- An additive inverse is an integer which when added to another integer gives zero.
- The inverse property states that "any number added to its inverse or opposite, gives zero".
- For example; a)  $+4 + -4 = 0$                       b)  $-9 + +9 = 0$

## Finding additive integers

### EXAMPLES

1. Find the additive inverse of +7

Let the inverse be k

$$k + +7 = 0$$

$$k + 7 - 7 = 0 - 7$$

$$k = -7$$

b) Find the additive inverse of -10

Let the inverse be r

$$r + -10 = 0$$

$$r - 10 + 10 = 0 + 10$$

$$r = +10$$

### Activity

a) Find the additive inverse of +6

b) Find the additive inverse of -4

c) Find the additive inverse of +9

d) Find the additive inverse of -5

e) Find the additive inverse of -12

f) Find the additive inverse of +8

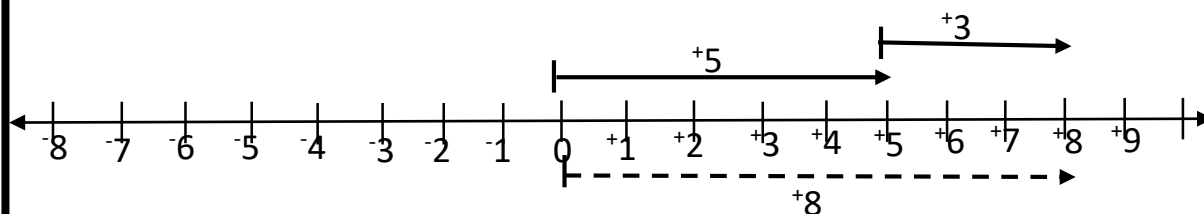
g) Find the additive inverse of -1

h) Find the additive inverse of +11

## Addition of integers using a number line

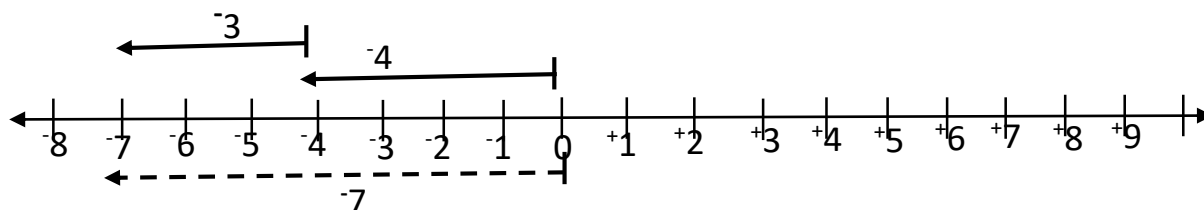
### Examples

1. Add +5 + +3 using a number line



$$+5 + +3 = +8$$

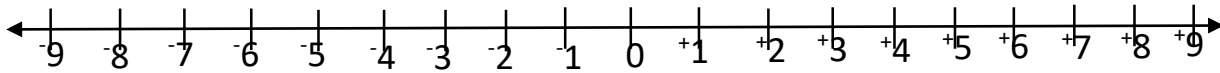
2. Add -4 + -3 using a number line



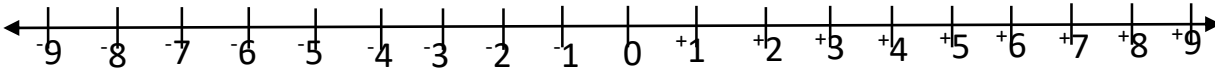
$$-4 + -3 = -7$$

## Activity

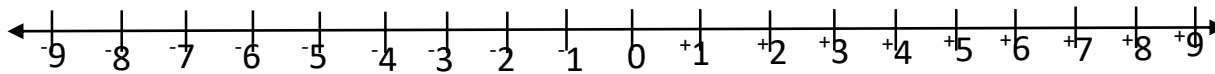
1. Add  $-5 + -4$  using a number line



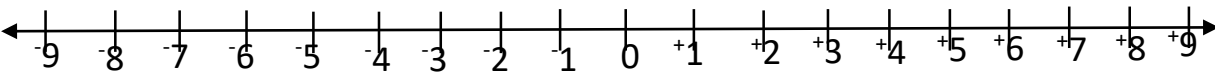
2. Add  $+6 + +3$  using a number line



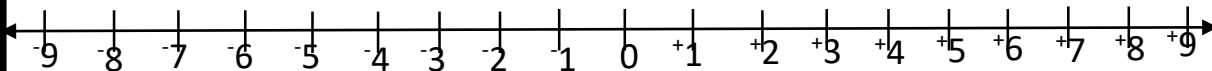
3. Add  $-6 + -2$  using a number line



4. Add  $-3 + -5$  using a number line



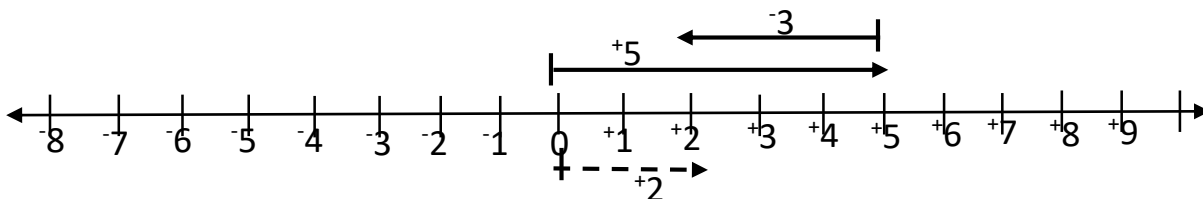
5. Add  $+4 + +2$  using a number line



## Addition of negative and positive integers using a number line

### Examples

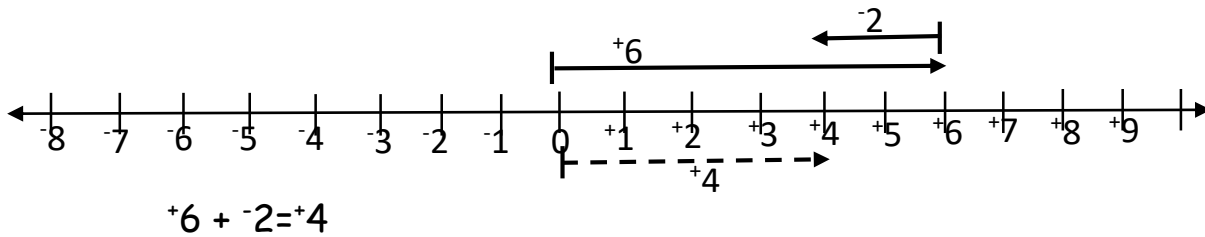
1. Add  $+5 + -3$  using a number line



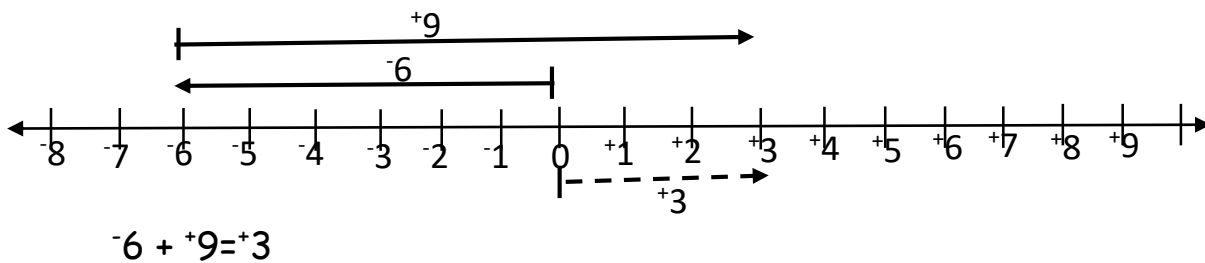
$$+5 + -3 = +2$$

---

2. Add  $+6 + -2$  using a number line

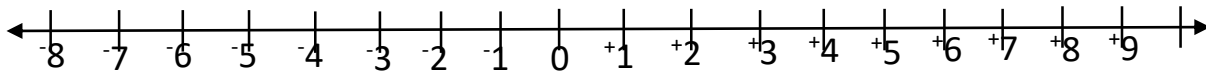


3. Add  $-6 + +9$  using a number line

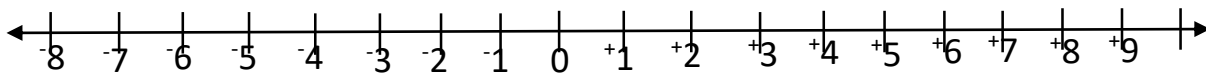


### Activity

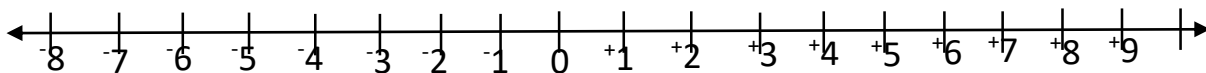
1. Add  $+7 + -3$  using a number line



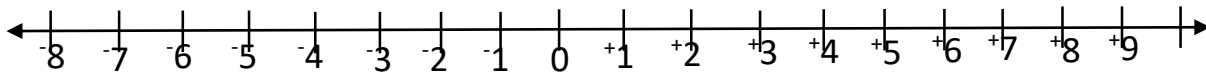
2. Add  $+9 + -5$  using a number line



3. Add  $-8 + +3$  using a number line



4. Add  $-4 + +9$  using a number line



## Addition of integers without using a number line

Points to note

➤  $+(+) = +$

➤  $+(-) = -$

➤  $-(-) = +$

➤  $-(+) = -$

Examples

1.Simplify:  $+6 + -2$

$$+6 + (-2)$$

$$+6 - 2$$

$$+4$$

2.Simplify:  $-7 + +2$

$$-7 + (+2)$$

$$-7 + 2$$

$$= -5$$

3.Simplify:  $-7 + -2$

$$-7 + (-2)$$

$$-7 - 2$$

$$-9$$

Activity

1.Simplify:  $+9 + -5$

2.Simplify:  $+7 + -4$

3.Simplify:  $+5 + -2$

4.Simplify:  $-6 + -2$

5.Simplify:  $-9 + -6$

6.Simplify:  $+11 + -4$

7.Simplify:  $+8 + -5$

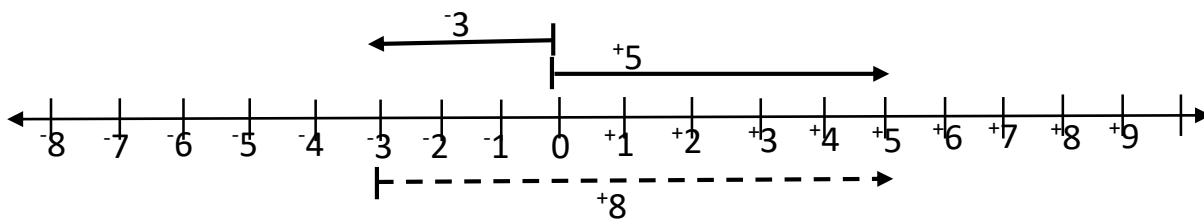
8.Simplify:  $-7 + -5$

9.Simplify:  $+5 + -3$

## Subtraction of integers using a number line

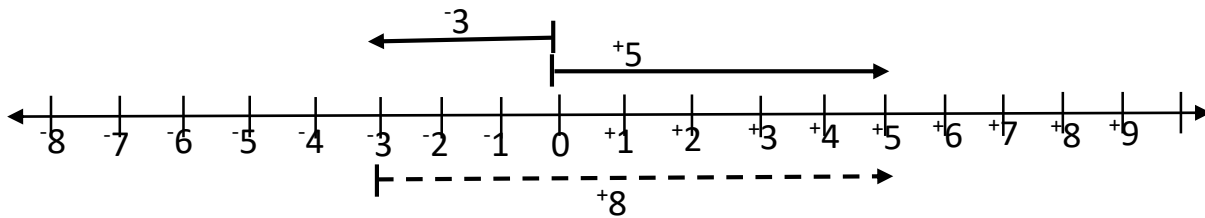
Examples

1.Subtract  $+5 - -3$  using a number line



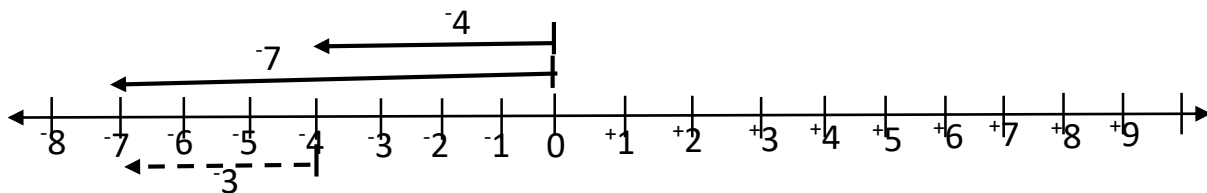
$$+5 - -3 = +8$$

2. Subtract  $+8 - -4$  using a number line



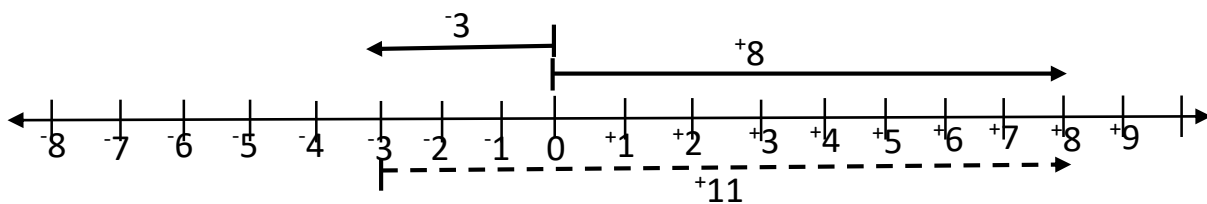
$$+5 - -3 = 8$$

3. Subtract  $-7 - -4$  using a number line



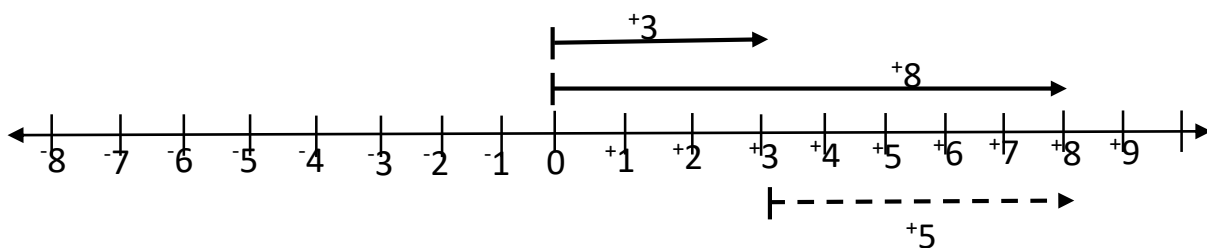
$$-7 - -4 = -3$$

4. Subtract  $+8 - -3$  using a number line



$$+8 - -3 = +11$$

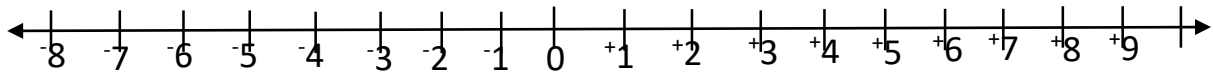
5. Subtract  $+8 - +3$  using a number line



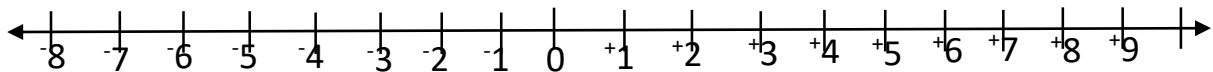
$$+8 - +3 = +5$$

## Activity

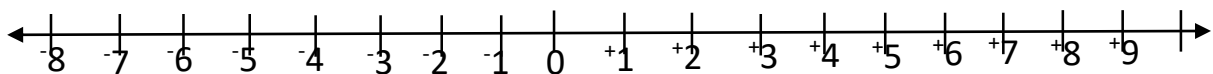
1. Subtract  $+5 - -2$  using a number line



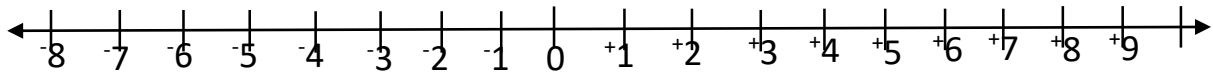
2. Subtract  $+7 - -4$  using a number line



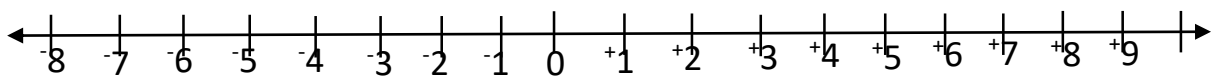
3. Subtract  $+5 - +2$  using a number line



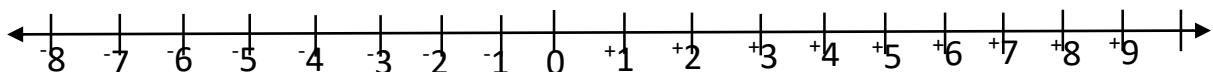
4. Subtract  $+7 - +5$  using a number line



5. Subtract  $-6 - -2$  using a number line



6. Subtract  $-8 - -5$  using a number line





## Subtraction of integers without using a number line

### Points to note

➤  $+(+)= +$

➤  $+(-)= -$

➤  $-(-)= +$

➤  $-(+)= -$

---

### Examples

1.Simplify:  $+6 - ^{-}2$

$$+6 - (^{-}2)$$

$$+6 + 2$$

$$+8$$

2.Simplify:  $^{-}7 - +2$

$$^{-}7 - (+2)$$

$$^{-}7 - 2$$

$$= ^{-}9$$

3.Simplify:  $^{-}6 - ^{-}2$

$$^{-}6 - (^{-}2)$$

$$^{-}6 + 2$$

$$^{-}4$$

---

### Activity

1.Simplify:  $+9 - ^{-}5$

2.Simplify:  $+7 - +4$

3.Simplify:  $+5 - ^{-}2$

4.Simplify:  $^{-}6 - +2$

5.Simplify:  $^{-}9 - ^{-}6$

6.Simplify:  $+11 - +4$

7.Simplify:  $+8 - ^{-}5$

8.Simplify:  $^{-}7 - +5$

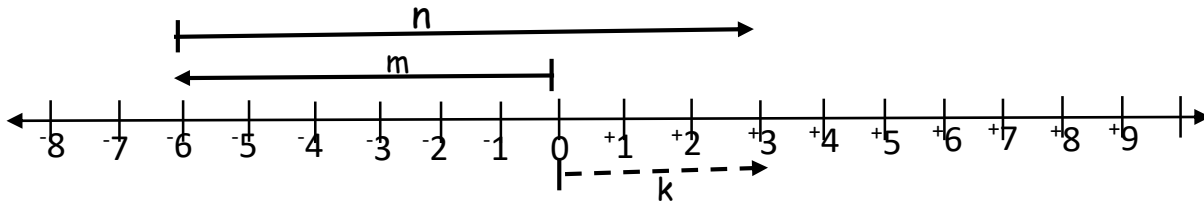
9.Simplify:  $+5 - ^{-}3$

---

## Forming addition mathematical statements from a number

### EXAMPLES

Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows  $m$ ,  $n$  and  $k$

$$m = -6$$

$$n = +9$$

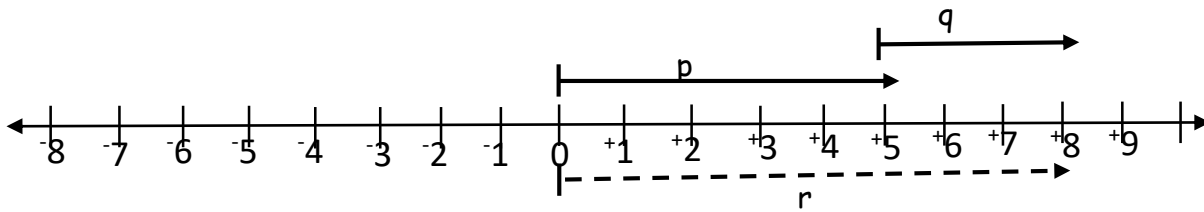
$$k = +3$$

b) Write down the addition mathematical statement shown on the above number line

$$m + n = k$$

$$-6 + +9 = +3$$

Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows  $p$ ,  $q$  and  $r$

$$p = +5$$

$$q = +3$$

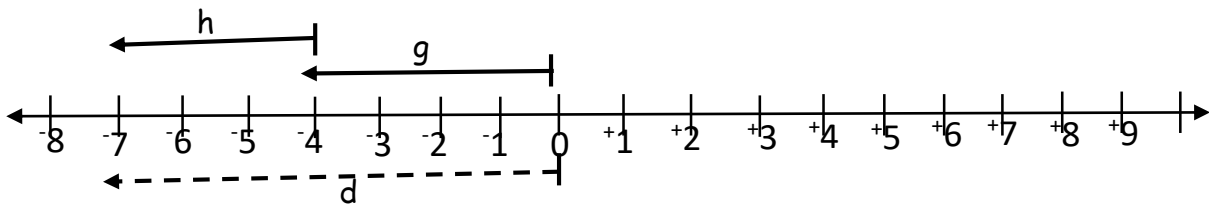
$$r = +8$$

b) Write down the addition mathematical statement shown on the above number line

$$p + q = r$$

$$+5 + +3 = +8$$

3. Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows  $g$ ,  $h$  and  $d$

$$g = -4$$

$$h = -3$$

$$d = -7$$

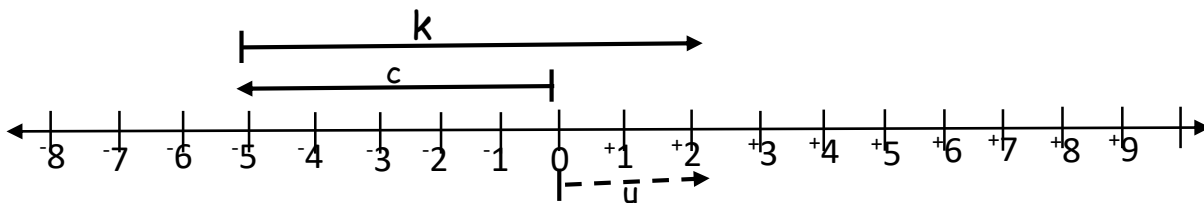
b) Write down the addition mathematical statement shown on the above number line

$$g + h = d$$

$$-4 + -3 = -7$$

### Activity

1. Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows  $c$ ,  $k$  and  $u$

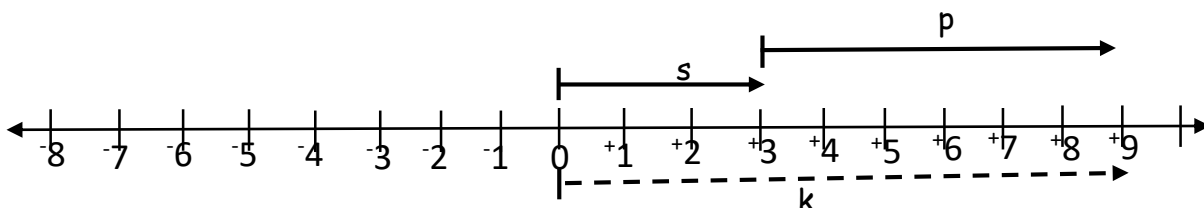
$$c = \dots\dots\dots$$

$$k = \dots\dots\dots$$

$$u = \dots\dots\dots$$

b) Write down the addition mathematical statement shown on the above number line

Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows p, q and r

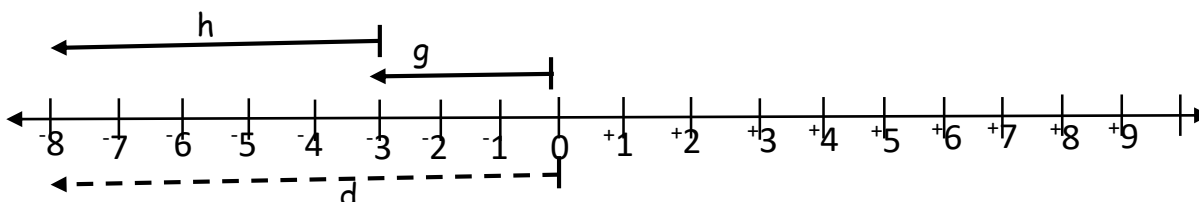
s=.....

p=.....

K=.....

b) Write down the addition mathematical statement shown on the above number line

3. Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows g, h and d

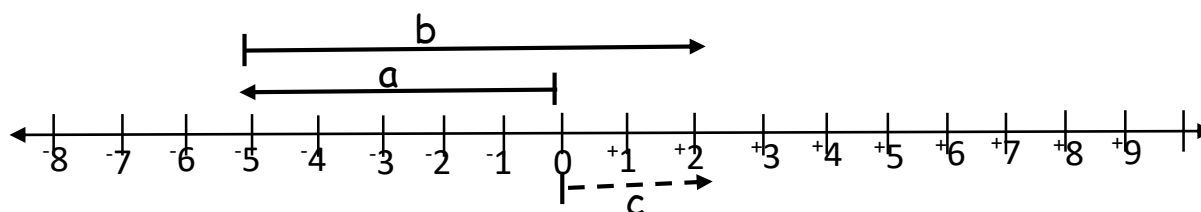
g=.....

h=.....

D=.....

b) Write down the addition mathematical statement shown on the above number line.

4. Use the number line below to answer the questions that follow



a) Write down the integer represented by the arrows a, b and c

a=.....

b=.....

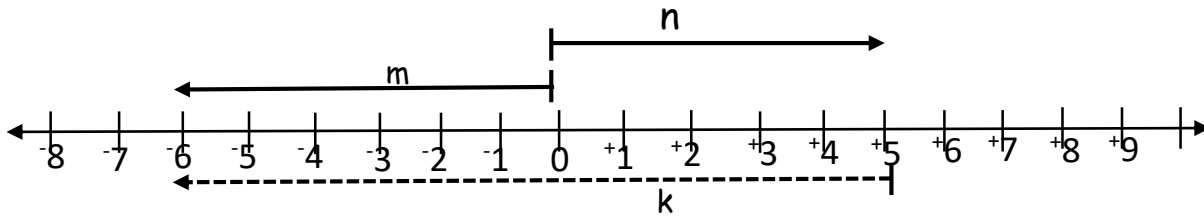
c=.....

b) Write down the addition mathematical statement shown on the above number line

## Forming subtraction mathematical statements from a number

### EXAMPLES

Use the number line below to answer the questions that follow



b) Write down the integer represented by the arrows m, n and k

$$m = -6$$

$$n = +5$$

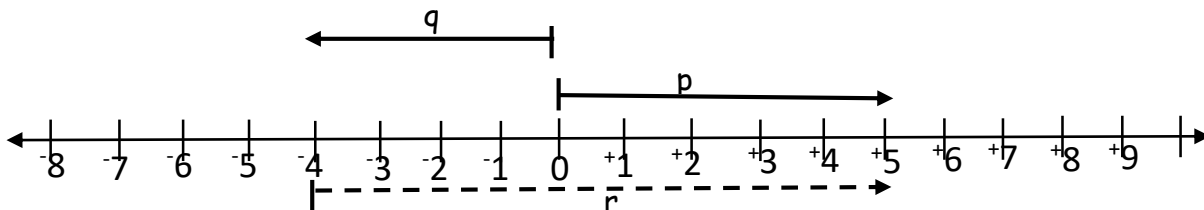
$$k = -11$$

b) Write down the subtraction mathematical statement shown on the above number line

$$m - n = k$$

$$-6 - +5 = -11$$

Use the number line below to answer the questions that follow



c) Write down the integer represented by the arrows p, q and r

$$p = +5$$

$$q = -4$$

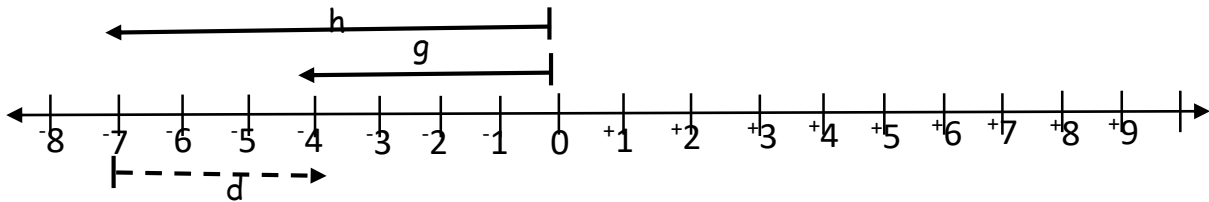
$$r = -9$$

d) Write down the subtraction mathematical statement shown on the above number line

$$p - q = r$$

$$+5 - -4 = +9$$

3. Use the number line below to answer the questions that follow



c) Write down the integer represented by the arrows  $g$ ,  $h$  and  $d$

$$g = -4$$

$$h = -7$$

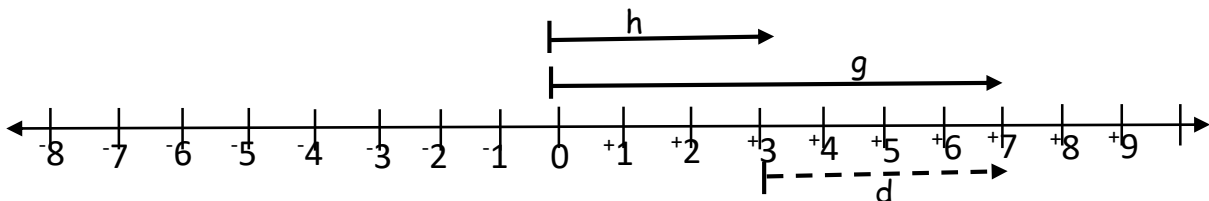
$$d = +3$$

b) Write down the subtraction mathematical statement shown on the above number line

$$g - h = d$$

$$-4 - -7 = +3$$

3. Use the number line below to answer the questions that follow



d) Write down the integer represented by the arrows  $g$ ,  $h$  and  $d$

$$g = +7$$

$$h = +3$$

$$d = +4$$

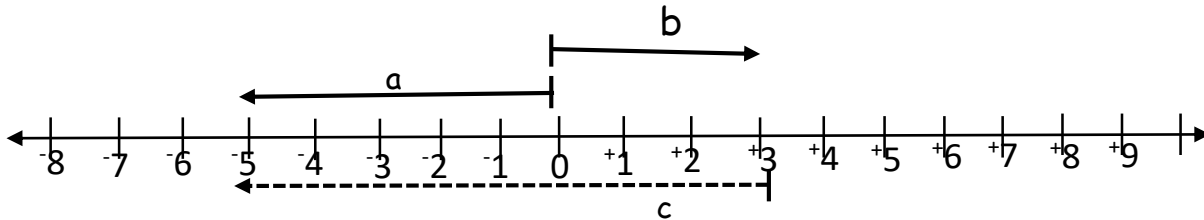
b) Write down the subtraction mathematical statement shown on the above number line

$$g - h = d$$

$$+7 - +3 = +4$$

## Activity

1. Use the number line below to answer the questions that follow



c) Write down the integer represented by the arrows a, b and c

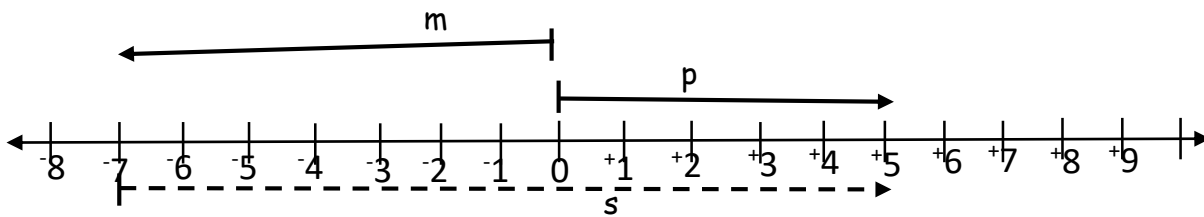
a=-----

b=-----

c=-----

b) Write down the subtraction mathematical statement shown on the above number line

Use the number line below to answer the questions that follow



e) Write down the integer represented by the arrows m, p and s

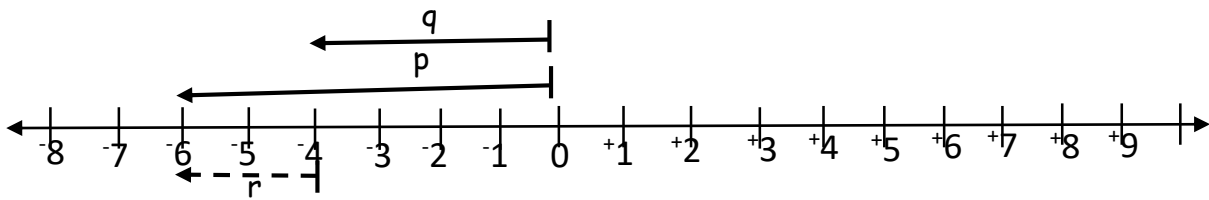
m=-----

p=-----

s=-----

f) Write down the subtraction mathematical statement shown on the above number line

3. Use the number line below to answer the questions that follow



e) Write down the integer represented by the arrows p, q and r

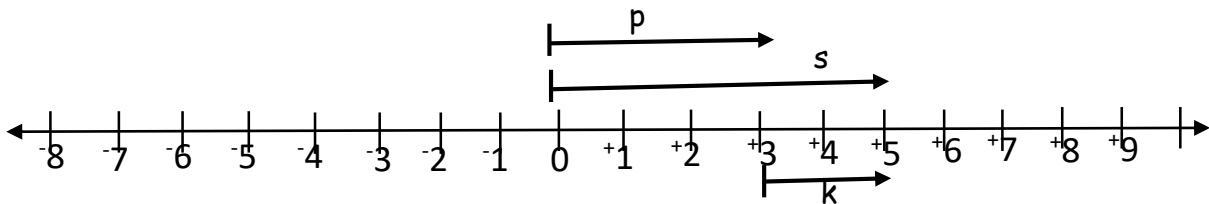
p=-----

q=-----

r=-----

b) Write down the subtraction mathematical statement shown on the above number line

3. Use the number line below to answer the questions that follow



f) Write down the integer represented by the arrows p, s and k

p=-----

s=-----

k=-----

b) Write down the subtraction mathematical statement shown on the above number line



## Multiplication of integers without using a number line

### Points to note

- $+ \times +$  or  $+(+)= +$
- $+ \times -$  or  $+(-)= -$
- $- \times -$  or  $-(-)= +$
- $- \times +$  or  $-(+)= -$

---

### Examples

1.Simplify:  $+6 \times -2$

$$+6 \times -2$$

$$-12$$

2.Simplify:  $-7 \times +2$

$$-7 \times +2$$

$$-14$$

3.Simplify:  $-6 \times -2$

$$-6 \times -2$$

$$+12$$

---

### Activity

1.Simplify:  $+9 \times -5$

2.Simplify:  $+7 \times +4$

3.Simplify:  $+5 \times -2$

4.Simplify:  $-6 \times +2$

5.Simplify:  $-9 \times -6$

6.Simplify:  $+11 \times +4$

7.Simplify:  $+8 \times -5$

8.Simplify:  $-7 \times +5$

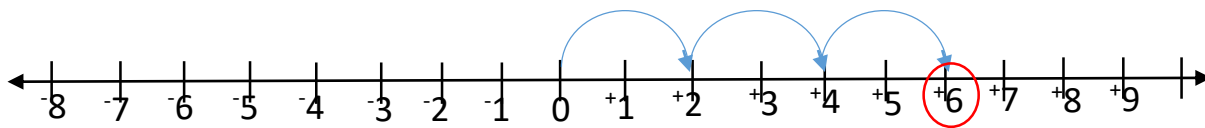
9.Simplify:  $-5 \times -3$

---

## Multiplication of integers using a number line

### Examples

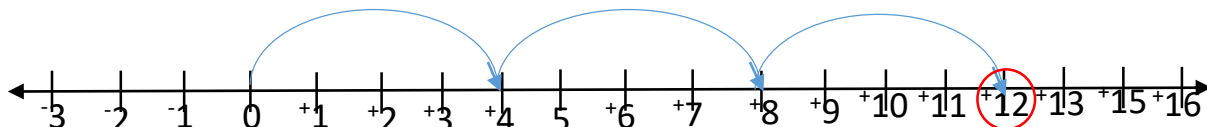
1. Workout:  $3 \times +2$  using a number line  
( $+3 \times +2$  means three groups of twos)



$$3 \times +2 = +6$$

---

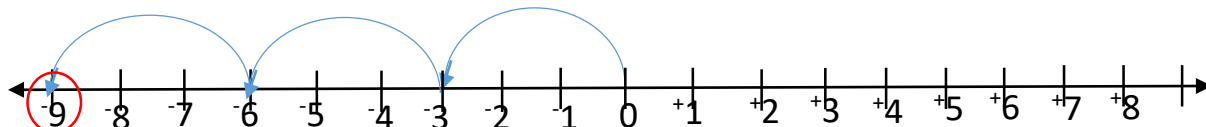
2. Workout:  $3 \times +4$  using a number line



$$3 \times +4 = +12$$

---

3. Workout:  $3 \times -3$  using a number line

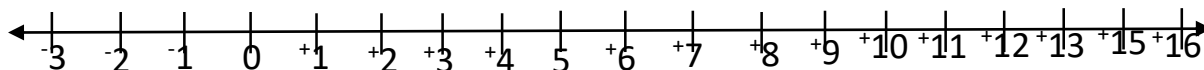


$$3 \times -3 = -9$$

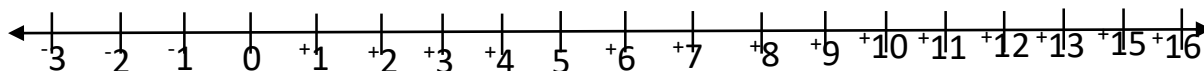
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### Activity

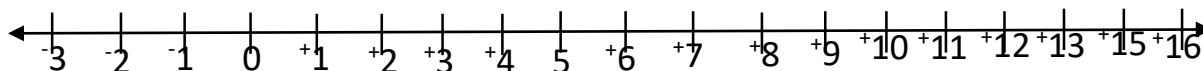
1. Workout:  $2 \times +3$  using a number line



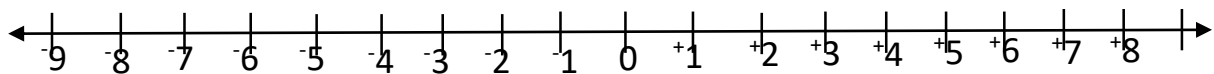
2. Workout:  $3 \times +3$  using a number line



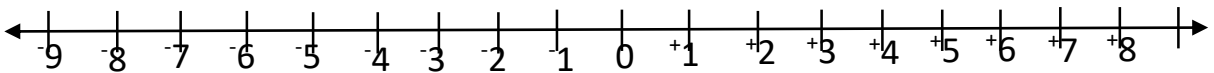
3. Workout:  $4 \times +2$  using a number line



4.Workout:  $3 \times -2$  using a number line



5.Workout:  $4 \times -2$  using a number line



## Division of integers without using a number line

Points to note

➤  $+\div+=+$

➤  $+\div=-$

➤  $-\div=+$

➤  $-\div=-$

---

Examples

1.Simplify:  $+6 \div -2$       2.Simplify:  $-32 \div +8$       3.Simplify:  $-16 \div -4$

$+6 \div -2$

$-32 \div +8$

$-16 \div -4$

$-3$

$-4$

$+12$

---

Activity

1.Simplify:  $+9 \div -3$

2.Simplify:  $+36 \div +4$

3.Simplify:  $+24 \div -4$

4.Simplify:  $-6 \div +2$

5.Simplify:  $-42 \div -6$

6.Simplify:  $+18 \div +2$

7.Simplify:  $-28 \div -4$

8.Simplify:  $-35 \div +5$

9.Simplify:  $-15 \div -3$

---

## Application of integers

Terms used and their meanings

- Before-----negative
- After-----positive
- Dropped-----negative
- Rise/Rose-----positive
- Loss-----positive
- Gain-----positive
- Increased-----positive
- Decreased -----negative
- Lower-----negative

### Examples

1. The temperature of a place was  $20^{\circ}\text{F}$  in the morning and later it dropped by  $23^{\circ}\text{F}$ . Find the new temperature of the place.

$$20^{\circ}\text{F} - 23^{\circ}\text{F} = -3^{\circ}\text{F}$$

2. The temperature on the top of a mountain was  $-14^{\circ}\text{C}$  at night but by morning, it had increased to  $10^{\circ}\text{C}$ . Find the rise in the temperature.

$$\begin{aligned} &= 10^{\circ}\text{C} - (-14^{\circ}\text{C}) \\ &= 10^{\circ}\text{C} - (-14^{\circ}\text{C}) \\ &= 10^{\circ}\text{C} + 14^{\circ}\text{C} \\ &= +24^{\circ}\text{C} \end{aligned}$$

3. The temperature of a pawpaw dropped by  $-5^{\circ}\text{C}$ . If the new temperature is  $15^{\circ}\text{C}$ . What was the initial temperature?

Let the initial temperature be  $h$

$$\begin{aligned} h - 5^{\circ}\text{C} &= 15^{\circ}\text{C} \\ h - (-5^{\circ}\text{C}) &= 15^{\circ}\text{C} \\ h + 5^{\circ}\text{C} - 5^{\circ}\text{C} &= 15^{\circ}\text{C} - 5^{\circ}\text{C} \\ h &= 10^{\circ}\text{C} \end{aligned}$$

Alternatively,

$$\begin{aligned} \text{Initial temp}^{\circ} &= \text{New} + \text{Drop} \\ \text{Initial temp}^{\circ} &= 15^{\circ}\text{C} + (-5^{\circ}\text{C}) \\ \text{Initial temp}^{\circ} &= 15^{\circ}\text{C} - 5^{\circ}\text{C} \\ \text{Initial temp}^{\circ} &= 10^{\circ}\text{C} \end{aligned}$$

## Activity

1. The temperature of a place was  $30^{\circ}\text{F}$  in the morning and later it dropped by  $33^{\circ}\text{F}$ . Find the new temperature of the place.
2. The temperature on the top of a mountain was  $-14^{\circ}\text{C}$  at night but by morning, it had increased to  $10^{\circ}\text{C}$ . Find the rise in the temperature.
3. The temperature of a pawpaw dropped by  $-3^{\circ}\text{C}$ . If the new temperature is  $12^{\circ}\text{C}$ . What was the initial temperature?
4. A girl moved 4 metres backwards and 4 more metres backwards. Write the final position of the girl.
5. John climbed 9 steps from his flat upwards and later descended 16 steps to the ground floor. What was John's final position from the flat?

## TOPIC: INTEGERS

### SUBTOPIC: THE FINITE SYSTEM / CLOCK ARITHMETIC

#### Definition:

- Finite system is a way of counting numerals in a specified group of number numerals.
- Finite system may also be called the modular (mod). In a special way, finite system sticks only on the remainders after grouping and regrouping a certain numeral.

#### Expressing numbers in finite system

#### Examples:

1. Express 23 in finite 5

$$23 \div 5 = 4 \text{ remainder } 3$$

$$23 = 4(\text{finite } 5)$$

2. Express 45 in finite 7

$$45 \div 7 = 6 \text{ remainder } 3$$

$$45 = 3(\text{finite } 7)$$

#### Activity

1. Express 24 in finite 3

2. Express 59 in finite 5

3. Express 54 in finite 6

4. Express 46 in finite 5

5. Express 34 in finite 7

6. Express 73 in finite 5

#### Addition and subtraction of numbers in finite system

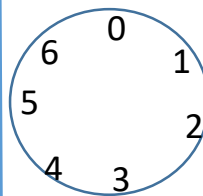
1. Work out:  $2 + 5 = \text{-----}$  (finite 7)

$$(2+5) \div 7 \text{ (finite 7)}$$

$$7 \div 7 = 1 \text{ remainder } 0$$

$$2 + 5 = 0 \text{ (finite 7)}$$

Alternatively, you can use a dial



2. Work out:  $3 + 4 = \text{-----}$  (finite 5)

$$(3+4) \div 5 \text{ (finite 5)}$$

$$7 \div 5 = 1 \text{ remainder } 2$$

$$3 + 4 = 2 \text{ (finite 5)}$$

3 .Work out:  $2 - 4 = \text{-----}$  (finite 5)

$$(2 + 5) - 4 = 3 \quad (\text{finite } 5)$$

$$7 - 4 = 3 \quad (\text{finite } 5)$$

$$2 - 4 = 3 \quad (\text{finite } 5)$$

4. Work out:  $0 - 5 = \text{-----}$  (mod 7)

$$(0 + 7) - 5 = 2 \pmod{7}$$

$$7 - 5 = 2 \pmod{7}$$

$$0 - 5 = 2 \pmod{7}$$

**Activity:**

1. Work out the following finite systems.

a)  $4 + 3 = \text{-----}$  (Finite 5)

b)  $3 + 3 + 2 = \text{-----}$  (Finite 5)

c)  $2 + 2 + 2 = \text{-----}$  (Finite 3)

d)  $4 + 4 + 4 = \text{-----}$  (finite 7)

e)  $6 + 4 + 5 = \text{-----}$  (Finite 7)

f)  $3 + 3 + 3 = \text{-----}$  (Finite 4)

Complete the table below in finite 5

+	2	3	4
1			
2			1
3		1	
4			3

2. Work out these finite systems.

a)  $1 - 4 = \text{-----}$  (mod 5)

b)  $3 - 5 = \text{-----}$  (mod 7)

c)  $4 - 3 = \text{-----}$  (mod 5)

d)  $5 - 2 = \text{-----}$  (mod 7)

e)  $0 - 4 = \text{-----}$  (finite 7)

f)  $2 - 3 = \text{-----}$  (finite 5)

## **SUBTOPIC:** Multiplication of numbers in finite system

### **Examples**

1)  $2 \times 3 = \text{-----} \pmod{5}$

$(2 \times 3) \div 5 \pmod{5}$

$6 \div 5 = 1 \text{ remainder } 1$

$2 \times 3 = 1 \pmod{5}$

2)  $3 \times 2^2 \pmod{5}$

$3 \times 2 \times 2 \pmod{5}$

$12 \pmod{5}$

$12 \div 5 = 2 \text{ remainder } 2 \pmod{5}$

$2 \pmod{5}$

### **ACTIVITY:**

#### **Multiply:**

a)  $4 \times 5 = \text{-----} \pmod{7}$

b)  $2 \times 4^2 = \text{-----} \pmod{9}$

c)  $23 \times 3 = \text{-----} \pmod{7}$

d)  $2 \times 32 = \text{-----} \pmod{6}$

f)  $4 (5 \times 2) = \text{-----} \pmod{6}$

Complete the table below in finite 5

x	2	4	5
1		4	
2			
3			0
4		1	

Ref: MK Mathematics pupils bk 7 pg 332.

## **SUBTOPIC:** Division of numbers in finite system

1. Workout  $2 \div 3 = \text{-----} \pmod{5}$

$(2+5) \div 3 = \text{-----} \pmod{5}$

$(7+5) \div 3 = \text{-----} \pmod{5}$

$12 \div 3 = 4 \pmod{5}$

$= 4 \pmod{5}$

2. Workout  $5 \div 3 = \text{-----} \pmod{7}$

$(5+7) \div 3 = \text{-----} \pmod{7}$

$12 \div 3 = \text{-----} \pmod{7}$

$12 \div 3 = 4 \pmod{7}$

$= 4 \pmod{7}$

### **Activity**

1. Workout the following

1.  $3 \div 2 = \text{-----} \pmod{5}$

2.  $3 \div 4 = \text{-----} \pmod{5}$

3.  $5 \div 4 = \text{-----} \pmod{7}$

4.  $1 \div 4 = \text{-----} \pmod{5}$

5.  $3 \div 2 = \text{-----} \pmod{7}$

6.  $3 \div 5 = \text{-----} \pmod{12}$



## Application of finite 7

### Examples

1. If today is Wednesday, what day of the week after 25 days?

Sun	Mon	Tue	Wed	Thur	Fri	Sat
0	1	2	3	4	5	6

$$3+25=---- \text{ (finite 7)}$$

$$(3+25 \div 7)=4 \text{ rem } 0$$

Since 0 represents Sunday on the table, the day will be Sunday

- If today is Friday, what day of the week was it 24 days ago?

Sun	Mon	Tue	Wed	Thurs.	Fri	Sat
0	1	2	3	4	5	6

$$5-24=---- \text{ (finite 7)}$$

$$(5+7)-24=(\text{finite } 7)$$

$$(12+7)-24=(\text{finite } 7)$$

$$(19+7)-24=(\text{finite } 7)$$

$$26-24=2 \text{ (finite } 7)$$

Since 2 represents Tuesday on the table, the day was Tuesday

### Activity

- 1.If today is Thursday, what day of the week will it be after 27days?
- 2.If today is Wednesday, what day of the week will it be after 52days?
- 3.If yesterday was Friday, what day of the week will it be after 20days from today?
- 4.If today is Monday, what day of the week was it 38 days ago?
- 5.If today is Tuesday, what day of the week was it 20 days ago?
- 6.If today is Friday, what day of the week was it 22 days ago?