PROPOSED MTC 2 UNEB GUIDE 2024.

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No. 1

$P(A) = 0.4$ $P(B) = 0.7$ $P(A \cap B) = 0.35$

$P(A'/B') = \dfrac{P(A' \cap B')}{P(B')}$. M_1

From the Contigency table.

$P(A' \cap B') = P(B') - P(A \cap B')$

$= P(B') - P(A) + P(A \cap B)$.

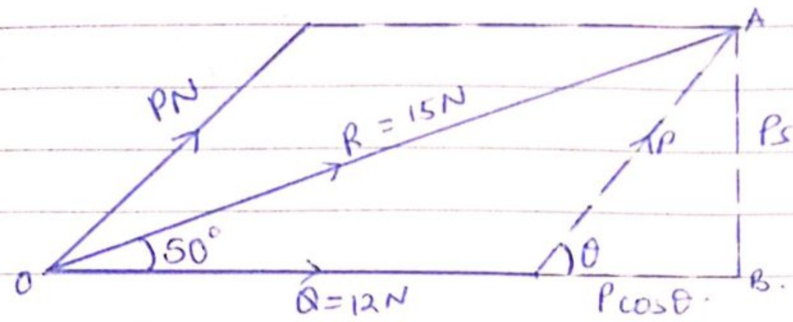
$= 0.3 - 0.4 + 0.35$ M_1

$= 0.25$ B_1

$P(A'/B') = \dfrac{0.25}{0.3}$ M_1

$= \frac{5}{6}$ or 0.8333. A_1

No.2.



Consider triangle OAB.

$$\cos 50^\circ = \frac{12 + P\cos\theta}{15}$$

$$P\cos\theta = 15\cos 50^\circ - 12 \quad \text{—— ①}$$ M1

Also ~~Psin~~ $\sin 50^\circ = \frac{P\sin\theta}{15}$

$$P\sin\theta = 15\sin 50^\circ \quad \text{—— ②}$$ M1

Squaring and adding ① and ②.

$$P^2(\sin^2\theta + \cos^2\theta) = (15\sin 50^\circ)^2 + (15\cos 50^\circ - 12)^2$$ M1

$$P = \sqrt{137.5964}$$

$$P = 11.73015\,N$$ A1

$$P\sin\theta = 15\sin 50^\circ$$

$$\sin\theta = \frac{15\sin 50^\circ}{11.73015}$$

$$\theta = 78.4^\circ$$ A1

B1

No.3.

(a).

t(s)	1	1.5	2
$V(ms^{-1})$	2	V	7.

$$\frac{V-2}{1.5-1} = \frac{7-2}{2-1}$$ M1

$$\frac{V-2}{0.5} = \frac{5}{1}$$

$$V-2 = 5 \times 0.5$$

$$V = 2 + 2.5$$

$$V = 4.5 \ ms^{-1}$$ A1

(b)

t(s)	3	4	t
$V(ms^{-1})$	8	10	13

$$\frac{t-4}{13-10} = \frac{4-3}{10-8}$$ M1

$$\frac{t-4}{3} = \frac{1}{2}$$ B1

$$t-4 = 3 \times 0.5$$

$$t = 4 + 1.5$$

$$t = 5.5 \text{ seconds}$$ A1

No. 4

$$\text{Weighted Price index} = \frac{\sum\left(\frac{P_{2010}}{P_{2005}} \times W\right)}{\sum W} \times 100$$

$$= \frac{\left(\frac{3500}{2500} \times 25\right) + \left(\frac{7000}{5000} \times 10\right) + \left(\frac{2000}{1500} \times 50\right) + \left(\frac{8000}{5000} \times 5\right) + \left(\frac{1200 \times 50}{800}\right)}{25 + 10 + 50 + 5 + 50} \times 100$$ M_1

$$= \frac{35 + 14 + \frac{200}{3} + 8 + 75}{140} \times 100$$ M_1

$$= \frac{596}{3 \times 140} \times 100$$ M_1

$$= 141.905.$$ A_1

Comment. There is an increase of 41.91% in the prices between 2005 and 2010. A_1

No. 5

$r = (t^3 \underset{\sim}{i} + \sin t \underset{\sim}{j})$ m.

Acceleration $a = \dfrac{d^2 r}{dt^2}$

$$a = \frac{d^2}{dt^2}\begin{pmatrix} t^3 \\ \sin t \end{pmatrix}.$$ M1

$$= \frac{d}{dt}\begin{pmatrix} 3t^2 \\ \cos t \end{pmatrix}.$$ M1

$$a = \begin{pmatrix} 6t \\ -\sin t \end{pmatrix}.$$

$$a = 6t \underset{\sim}{i} - \sin t \underset{\sim}{j} \ \ ms^{-2}$$

when $t = \pi/3$.

$$a = 6(\pi/3)\underset{\sim}{i} - \sin(\pi/3)\underset{\sim}{j}$$

$$= 2\pi \underset{\sim}{i} - \frac{\sqrt{3}}{2}\underset{\sim}{j}.$$

$$a = \frac{1}{2}\left(4\pi \underset{\sim}{i} - \sqrt{3}\underset{\sim}{j}\right) \ ms^{-2}$$ B1

From $F = Ma$.

$$F = 4\left(\frac{1}{2}\left(4\pi \underset{\sim}{i} - \sqrt{3}\underset{\sim}{j}\right)\right).$$ M1

$$F = 8\pi \underset{\sim}{i} - 2\sqrt{3}\underset{\sim}{j} \ N.$$ A1

No. 6.

$l = 2.7m$ $\quad w = 4.80m$ $\quad h = 3.281m.$

$\Delta l = 0.05$ $\quad \Delta w = 0.005$ $\quad \Delta h = 0.0005.$ B1

$V = l \times w \times h.$

$V_{max} = l_{max} \cdot w_{max} \cdot h_{max}.$

$= (l+\Delta l)(w+\Delta w)(h+\Delta h).$ 1

$= (2.7+0.05) \cdot (4.80+0.005)(3.281+0.0005).$ M1

$= 2.75 \times 4.805 \times 3.2815.$

$= 43.36092.$

≈ 43.361 (3dp. A1

$V_{min} = l_{min} \cdot w_{min} \cdot h_{min}$

$(l-\Delta l)(w-\Delta w)(h-\Delta h)$

$= (2.7-0.05)(4.80-0.005)(3.281-0.0005)$ M1

$= 2.65 \times 4.795 \times 3.2805$

$= 41.68449$

≈ 41.684 (3.dp). A1

No. 7.

$$P(X=x) = \begin{cases} \frac{1}{10}x & ; \ 1, 2 --- n \\ 0 & : \text{otherwise} \end{cases}$$

From $\sum_{\text{all } x}^{n} P(X=x) = 1$

$\frac{1}{10}(1+2+3+---n) = 1$ M

$1+\cancel{2}+3+---n = 10$

$\frac{n(n+1)}{2} = 10.$

$n(n+1) = 20$ ——— ①. M

$E(X) = \sum_{\text{all } x}^{n} x\, P(X=x)$

$3 = \frac{1}{10}\left(1+2^2+3^2+---n^2\right)$

$1+2^2+3^2+---n = 30.$

$\frac{n(n+1)(2n+1)}{6} = 30.$

$n(n+1)(2n+1) = 180$ ——— ②. M

Sub ① in ②.

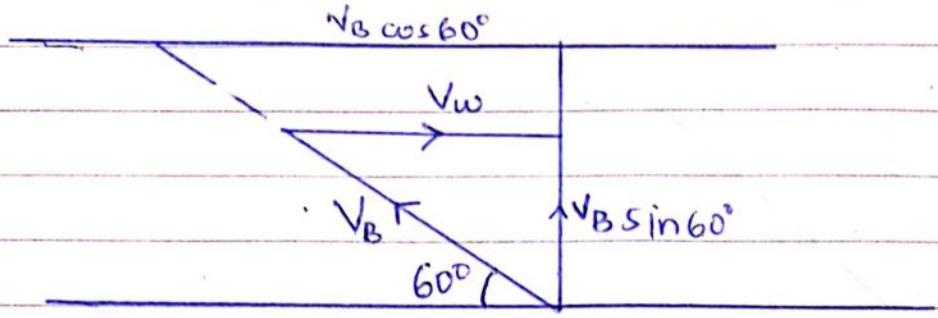
$20(2n+1) = 180.$ M

$2n+1 = \cancel{90}\ 9$

$2n = 8$

$n = 4.$ A

No. 8



B_1

Time taken $= \dfrac{AB}{V_B \sin 60°}$ M_1

$= \dfrac{50}{2.5 \sin 60°}$

$= 23.09$ seconds A_1

Resultant velocity $V = V_B \sin 60°$ M_1

$= 2.5 \sin 60°$

$= 2.1651 \text{ ms}^{-1}$ A_1

No. 9

Time (min)	f	c	x	fx	f.d
0 − 10	20	10	5.0	100	2
10 − 15	18	5	12.5	225	3.6
15 − 30	60	15	22.5	1350	4
30 − 45	45	15	37.5	1687.5	3
45 − 55	50	10	50.0	2500	5
55 − 60	30	5	57.5	1725	6
60 − 80	60	20	70.0	4200	3
80 − 90	10	10	85.0	850	1
	$\Sigma f = 293$			$\Sigma fx = 12{,}637.5$	

B_1 Σf

$B_1 B_1$ Midpoint x

B_1 Σfx

B_1 frequency density

(a) Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{12{,}637.5}{293}$ M_1

$= 43.1314$ minutes. A_1

(b) (i) Using a graph paper.

(ii) Mode $= 55 + (1.5 \times 1)$ M_1

$= 55 + 1.5$

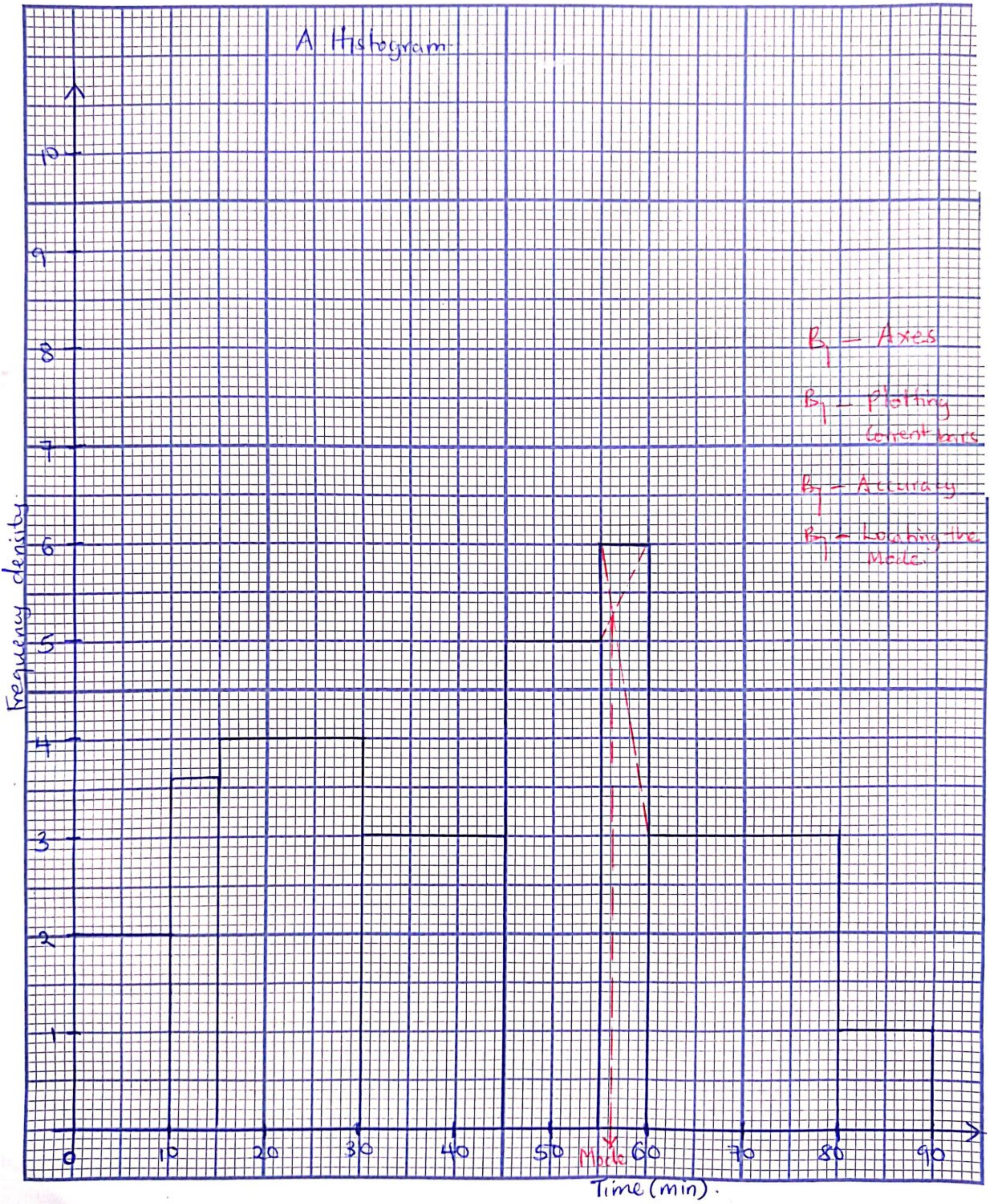
$= 56.5$ minutes A_1

Candidate's Name ....

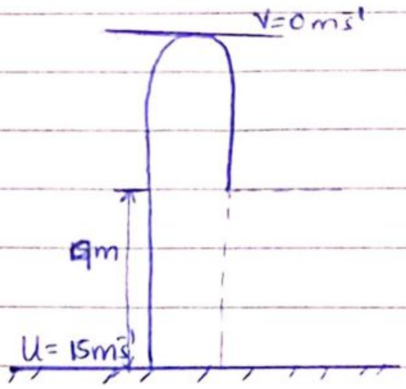
Signature ....

Subject .... Paper code ..../....

Random No.	
Personal Number	



10(a).



From $S = Ut - \frac{1}{2}gt^2$.

$9 = 15t - \frac{1}{2}(9.8)t^2$ M1

$9 = 15t - 4.9t^2$.

$4.9t^2 - 15t + 9 = 0$.

$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(4.9)(9)}}{2 \times 4.9}$ M1

$t = \frac{15 \pm \sqrt{48.6}}{9.8}$

$t_1 = 2.242$, $t_2 = 0.8192$. B1

Time taken before the ball was caught is 2.242 seconds. A1

At max point, $v = 0$.

$0 = 15 - 9.8t$.

$t = 1.532s$ B1

$V = U + gt$

$V = 0 + 9.8(2.242 - 1.532)$. M1

$V = 6.972\ ms^{-1}$ A1

CamScanner

(b)

$a = \frac{1}{5}(2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k})\ ms^{-2}$.

$u = 11\underset{\sim}{i} - 8\underset{\sim}{j} + 3\underset{\sim}{k}$

$r_0 = (-2\underset{\sim}{i} + \underset{\sim}{j})m$

$r_{(t=t)} = r_0 + ut + \frac{1}{2}at^2$.

$$r_{(t=5)} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 11 \\ -8 \\ 3 \end{pmatrix}(5) + \frac{1}{2}\begin{pmatrix} 2/5 \\ 3/5 \\ -4/5 \end{pmatrix}(5^2) \quad M_1$$

$$= \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 55 \\ -40 \\ 15 \end{pmatrix} + \begin{pmatrix} 5 \\ 7.5 \\ -10 \end{pmatrix} \quad M_1$$

$$= \begin{pmatrix} 58 \\ -31.5 \\ 5 \end{pmatrix}$$

$r_{(t=5)} = 58\underset{\sim}{i} - 31.5\underset{\sim}{j} + 5\underset{\sim}{k}$. A_1

Distance $= |r_{(t=5)}|$

$= \sqrt{58^2 + (-31.5)^2 + 5^2}$ M_1

$= \sqrt{3364 + 992.25 + 25}$

$= \sqrt{4381.25}$

$\approx 66.2\ m$

The distance covered is 66.2m. A_1

No. 11

Let $y_n = x_n e^{(x_n^2+1)}$.

$h = \frac{1-0}{6-1} = \frac{1}{5} = 0.2$. B1

n	x_n	y_0, y_5	$y_1, ----, y_4$
0	0.0	0.0000	
1	0.2		0.5658
2	0.4		1.2760
3	0.6		2.3377
4	0.8		4.1241
5	1.0	7.3891	
Total		7.3891	8.3038

B1 B1 B1 A1 A1

$$\int_0^1 x e^{(x^2+1)}\,dx \approx \frac{0.2}{2}\left[7.3891 + 2(8.3038)\right]$$ M1

$$\approx 2.3996$$

$$\approx 2.400 \text{ (3 dp)}.$$ A1

$$\text{Percentage error} = \frac{|\text{actual value} - \text{Approximate}|}{\text{actual value}} \times 100$$

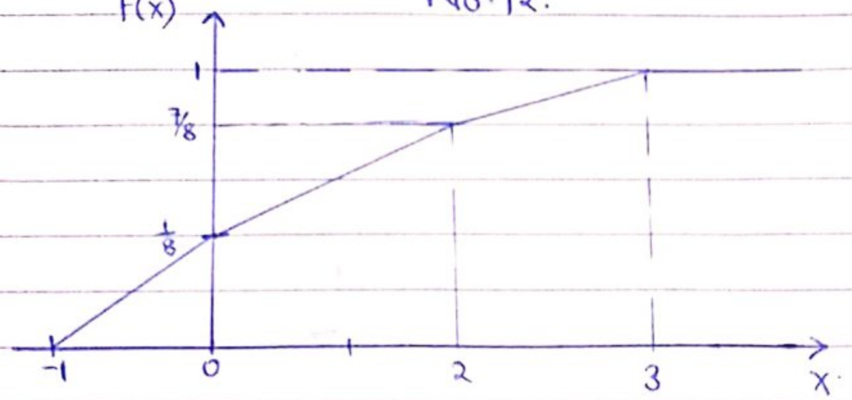
$$= \frac{|2.335 - 2.400|}{2.335} \times 100$$ M1

$$= \frac{0.065}{2.335} \times 100$$ M1

$$= 2.770$$ A1

(c). By increasing on the number of ordinates with in the given interval. A1

No. 12.



Consider interval $-1 \leq x < 0$

$(-1, 0)$ $(0, \frac{1}{8})$

$$\cancel{0-(-1)} \quad \frac{\frac{1}{8} - 0}{0 - -1} = \frac{F(x) - 0}{x - -1}$$

$$\frac{1}{8} = \frac{F(x)}{x+1}$$

$$F(x) = \frac{1}{8}(x+1).$$

Mh

For interval. $0 \leq x < 2$.

$(0, \frac{1}{8})$ $(2, \frac{7}{8})$.

$$\frac{\frac{7}{8} - \frac{1}{8}}{2 - 0} = \frac{F(x) - \frac{1}{8}}{x - 0}.$$

$$\frac{3}{8} = \frac{F(x) - \frac{1}{8}}{x}.$$

$$F(x) = \frac{3}{8}x + \frac{1}{8}$$

$$F(x) = \frac{1}{8}(3x + 1).$$

Mh

For interval $2 \leq x < 3$.

$(2, \frac{7}{8})$ $(3, 1)$

$$\frac{1 - \frac{7}{8}}{3-2} = \frac{F(x) - 1}{x - 3}.$$

$$\frac{1}{8} = \frac{F(x) - 1}{x - 3}$$

$$F(x) - 1 = \frac{1}{8}(x - 3).$$

$$F(x) = \frac{1}{8}x - \frac{3}{8} + 1$$

$$F(x) = \frac{1}{8}(x + 5)$$

Mh

For interval $x \geqslant 3$

$F(x) = 1$ M1

$$F(x) = \begin{cases} 0 & ; \quad x < -1 \\ \frac{1}{8}(x+1) & ; \quad -1 \leq x < 0 \\ \frac{1}{8}(3x+1) & ; \quad 0 \leq x < 2 \\ \frac{1}{8}(x+5) & ; \quad 2 \leq x < 3 \\ 1 & ; \quad x \geqslant 3. \end{cases}$$ A1

ii) $P(1 < x < 2.5) = F(2.5) - F(1).$

$$= \frac{1}{8}(2.5+5) - \frac{1}{8}(3(1)+1).$$ M1

$$= \frac{7.5}{8} - \frac{4}{8}$$

$$= \frac{7}{16} \text{ or } 0.4375.$$ A1

b) (i) p.d.f, f(x).

$$f(x) = \frac{d}{dx} F(x).$$

For $-1 \le x < 0$. $F(x) = \frac{1}{8}(x+1)$

$$f(x) = \frac{d}{dx}\left(\frac{1}{8}(x+1)\right).$$

$$f(x) = \frac{1}{8}$$

For $0 \le x \le 2$. $F(x) = \frac{1}{8}(3x+1)$

$$f(x) = \frac{d}{dx}\left(\frac{1}{8}(3x+1)\right)$$

$$f(x) = \frac{3}{8}$$ B1

For $2 \le x < 3$

$$F(x) = \tfrac{1}{8}(x+5).$$

$$f(x) = \frac{d}{dx}\left(\tfrac{1}{8}(x+5)\right).$$

$$f(x) = \tfrac{1}{8}$$

$$f(x) = \begin{cases} \frac{1}{8} ; & -1 < x < 0 \\ \frac{3}{8} ; & 0 \le x < 2 \\ \frac{1}{8} ; & 2 \le x < 3 \\ 0 ; & \text{elsewhere} \end{cases}$$ A1

$$E(x) = \int_{\text{all } x} x f(x)\, dx.$$

$$= \int_{-1}^{0} \frac{1}{8} x \, dx + \int_{0}^{2} \frac{3}{8} x \, dx + \int_{2}^{3} \frac{1}{8} x \, dx. \quad M_1$$

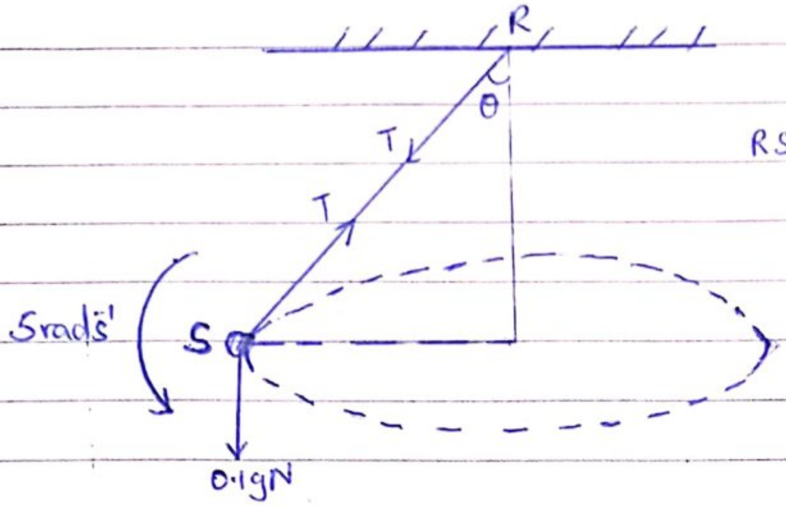
$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_{-1}^{0} + \frac{3}{8}\left[\frac{x^2}{2}\right]_{0}^{2} + \frac{1}{8}\left[\frac{x^2}{2}\right]_{2}^{3} \quad M_1$$

$$= \tfrac{1}{8}\left(0-(\tfrac{1}{2})\right) + \tfrac{3}{8}(2-0) + \tfrac{1}{8}\left(\tfrac{9}{2}-2\right)$$

$$= -\frac{1}{16} + \frac{3}{4} + \frac{5}{16}$$

$$E(x) = 1 \quad A_1$$

No.13.



$RS = 80cm$
$= 0.8m$

B4 Correct diagram

Applying Newton's law horizontally gives

$T\sin\theta = mr\omega^2$ M1

$T\sin\theta = m \times l\sin\theta \times \omega^2$ B1

$T = ml\omega^2$.

$T = 0.1 \times 0.8 \times 5^2$ M1

$T = 2N$. A1 Answer

(b)

Resolving vertically.

$T\cos\theta = mg$. M1 Resol

$2\cos\theta = 0.1(9.8)$

$\cos\theta = \frac{0.1(9.8)}{2}$. M1 Subs

$\theta = 60.66°$ A1 Corre

From $r = l\sin\theta$.

$r = 0.8\sin 60.66$ A1

$r = 0.6974m$.

The radius of the horizontal circle $= 69.74$ cm. A1 Corre

No. 14.

$2x^3 - 4x + 3 = 0$.

$\therefore y = 2x^3 - 4x + 3$.

x	-2.0	-1.75	-1.50	-1.25	-1.0
y	-5	-0.72	2.25	1.91	5.0

B1 — minimum of 5 points with 1 or 2 dp

From the graph, Root $x_0 \approx -1.68$

(b) $f(x) = 2x^3 - 4x + 3$.

$f'(x) = 6x^2 - 4$ B1 — Derivative

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

$$= X_n - \frac{(2X_n^3 - 4X_n + 3)}{6X_n^2 - 4}.$$ M1 — Substitution

$$= \frac{6X_n^3 - 4X_n - 2X_n^3 + 4X_n - 3}{6X_n^2 - 4}$$

$$X_{n+1} = \frac{4X_n^3 - 3}{6X_n^2 - 4}$$

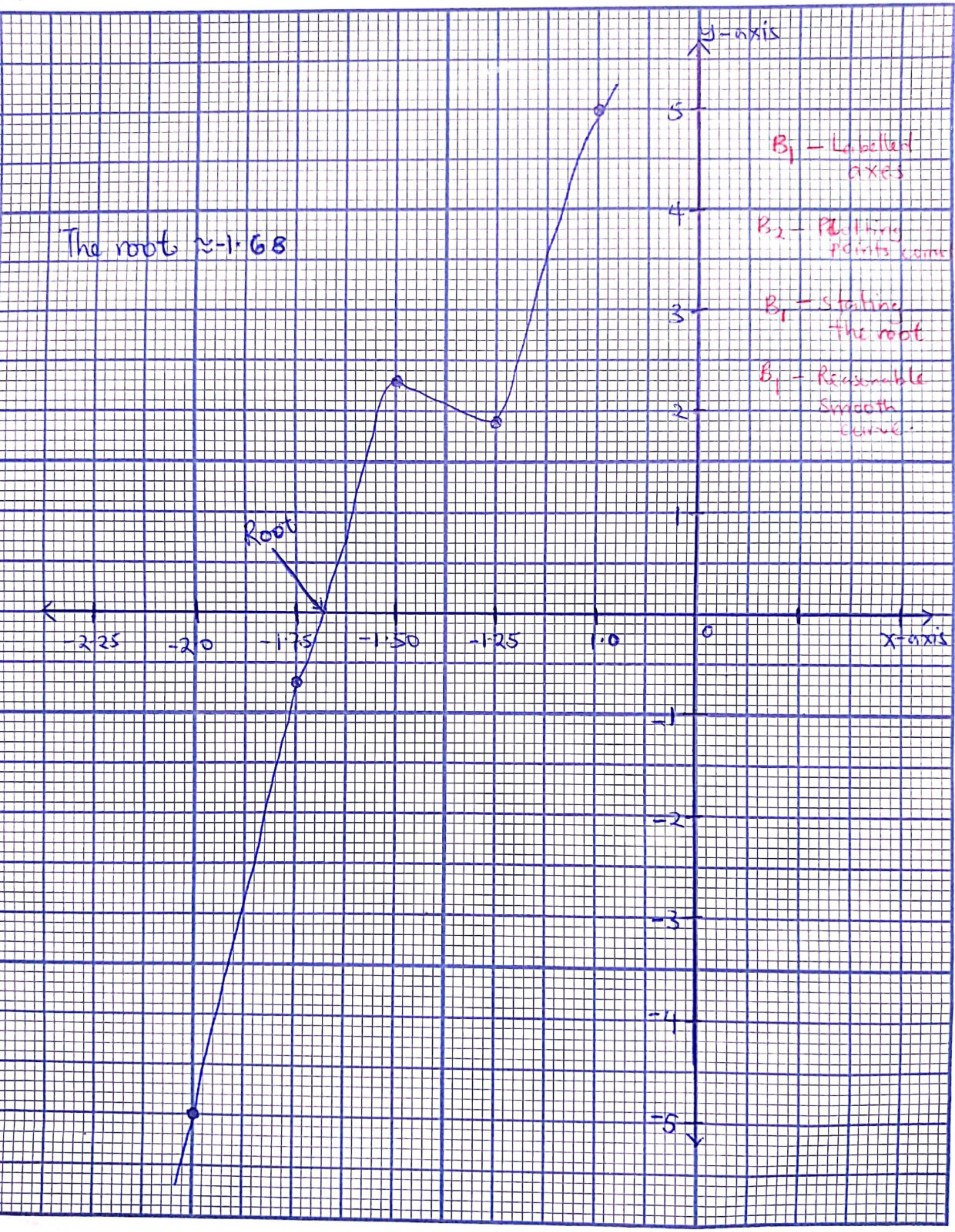
$X_0 = -1.68$ M1, B1 — Substitution and value of X_1

$$X_1 = \frac{4(1.68)^3 - 3}{6(1.68)^2 - 4} = -1.6983$$

$$X_2 = \frac{4(-1.6983)^3 - 3}{6(-1.6983)^2 - 4} = -1.6981$$ B1 — Value of X_2

$|X_2 - X_1| = |-1.6981 - -1.6983| = 0.0003$

$\therefore$ The root is -1.698 (3.d.p). B1 — Required root to 3 dp


y-axis
x-axis
The root ≈ -1·68
Root
-2·25
-2·0
-1·75
-1·50
-1·25
1·0
0
5
4
3
2
1
-1
-2
-3
-4
-5
B1 – Labelled axes
B2 – Plotting points
B1 – Stating the root
B1 – Reasonable smooth curve.

No. 15

$n = 120 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

$\mu = np$

$= 120 \times \frac{1}{4}$

$\mu = 30$ — B_1 Mean and Standard deviation

$\delta = \sqrt{npq}$

$= \sqrt{120 \times 0.25 \times 0.75}$

$= \sqrt{\frac{45}{2}}$

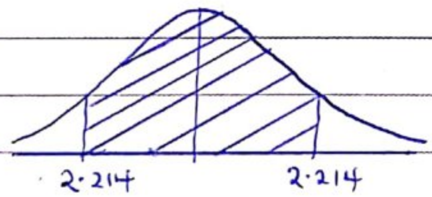
$= 4.7434.$

$X \sim$ number of correct options.

$P(20 \le X \le 40) = P(19.5 < X < 40.5).$ — M_1 Continuity correction &

$= P\left(\frac{19.5 - 30}{\sqrt{45/2}} < Z < \frac{40.5 - 30}{\sqrt{45/2}}\right)$ — M_1 Standardising

$= P(-2.214 < Z < 2.214)$



$P(-2.214 < Z < 2.214) = \phi(2.214) + \phi(+2.214)$ — M_1 Addition

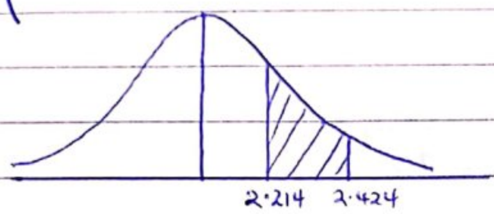
$= 0.4867 + 0.4867$ — B_1 Table value

$= 0.9734$ — A_1

$P(x=41) = P(40.5 < X < 41.5)$ M1 Continuity

$$= P\left(\frac{40.5-30}{\sqrt{22.5}} < Z < \frac{41.5-30}{\sqrt{22.5}}\right)$$

$$= P(2.214 < Z < 2.424)$$



$P(2.214 < Z < 2.424) = \phi(2.424) - \phi(2.214)$ M1 Subtraction

$= 0.4924 - 0.4867$

$= 0.0057$ A1 Output.

(b)

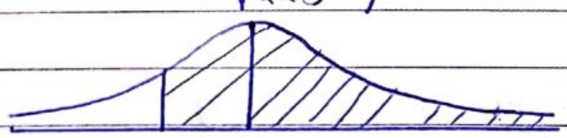
$P(X \geq X_1) = 0.8$

$P(X > X_1 - 0.5) = 0.8$ M1 Continuity

$$P\left(\cancel{Z} > \cancel{>} \frac{X_1 - 0.5 - 30}{\sqrt{22.5}}\right) = 0.8$$

M1 Standardisation

$$P\left(Z > \frac{X_1 - 30.5}{\sqrt{22.5}}\right) = 0.8$$



$$\phi\left(\frac{X_1 - 30.5}{\sqrt{22.5}}\right) = 0.3$$

M1 Table value

$$\frac{X_1 - 30.5}{\sqrt{22.5}} = -0.842$$

M1 Subtraction

$X_1 = 30.5 + \sqrt{22.5}(-0.842) = 26.506.$

The passmark is 26.5 A1 Output

CamScanner

No. 16

Let W be the weight per unit Area.

Lamina	Area	Weight	Distance of C.O.G from AH	Distance of C.O.G from AB	
ABCH	$6\,m^2$	6 W	2.5	0.6	M_1
GDFE	$3\,m^2$	3 W	2.5	2.7	M_1
Composite	$9\,m^2$	9 W	$\bar{x}$	$\bar{y}$	M_1

C.O.G from AH: $9W\bar{x} = 2.5(3W) + 2.5(6W)$ B_1

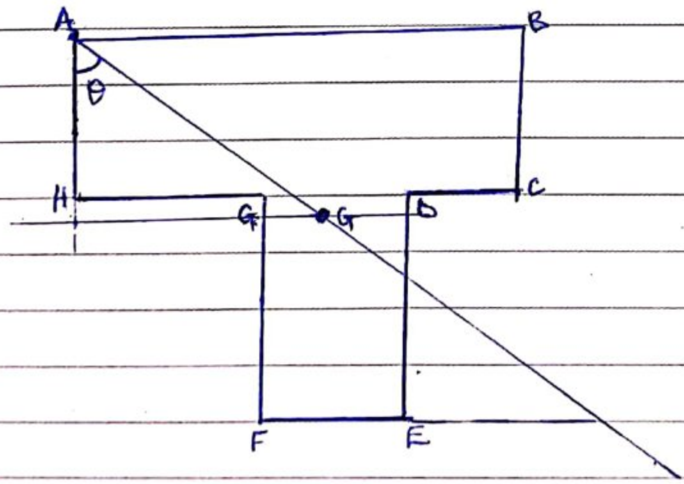
$9\bar{x} = \cancel{11.7}\ 7.5 + 15$ M_1

$\bar{x} = 2.5\ m$ A_1

Centre of gravity from AB: $9W\bar{y} = 2.7(3W) + 0.6(6W)$ M_1

$9\bar{y} = 11.7$

$\bar{y} = 1.3\,m$ A_1



$\tan\theta = \dfrac{2.5}{1.3}$ M_1

M_1

$\theta = \tan\left(\frac{2.5}{1.3}\right) = 62.5^\circ$ A_1

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The solutions in this guide are according to my opinion. I accept to own any mistake detected.