PROPOSED
MARKING GUIDE
PURE MATHEMATICS
P425/1 2023

NO	SOLUTION	MKS	COMMENT
1	$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(2n+1)(n+1)$ **Solution** For $n = 1$; L.H.S $= 1^2 = 1$, R.H.S $= \frac{1}{6} \times 1 \times (3)(2) = 1$ It holds For $n = 2$; L.H.S $= 1^2 + 2^2 = 5$ R.H.S $= \frac{1}{6} \times 2 \times (5)(3) = 5$ It holds *Assume the result holds for* $n = k$ $1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{1}{6}k(2k+1)(k+1)$ For $n = k+1$; $1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2 = \frac{1}{6}k(2k+1)(k+1) + (k+1)^2$ R.H.S $= \frac{1}{6}k(2k+1)(k+1) + (k+1)^2$ $= \frac{k+1}{6}[2k^2 + k + 6k + 6]$ $= \frac{k+1}{6}[2k^2 + 7k + 6]$ $= \frac{1}{6}(k+1)(2k+3)(k+2)$ It holds for $n = k+1$		
		05	
2	If $y = mx + c$ is a tangent to $4x^2 + 3y^2 = 12$, then $4x^2 + 3(mx+c)^2 = 12$		

	$4x^2 + 3(m^2x^2 + 2mcx + c^2) = 12$ $4x^2 + 3m^2x^2 + 6mcx + 3c^2 = 12$ $(4 + 3m^2) + 6mcx + 3c^2 - 12 = 0$ For tangency, $b^2 = 4ac$ $(6mc)^2 = 4(4 + 3m^2)(3c^2 - 12)$ $36m^2c^2 = 4(12c^2 - 48 + 9m^2c^2 - 36m^2)$ $9m^2c^2 = 12c^2 - 48 + 9m^2c^2 - 36m^2$ $12c^2 = 48 + 36m^2$ $\therefore c^2 = 4 + 3m^2$		
		05	
3	$y = e^x \cos 3x$ $\frac{dy}{dx} = -3e^x \sin 3x + e^x \cos 3x$ $\frac{dy}{dx} = -3e^x \sin 3x + y$ $\frac{d^2y}{dx^2} = -3[3e^x \cos 3x + e^x \sin 3x] + \frac{dy}{dx}$ $= -9y - 3e^x \sin 3x + \frac{dy}{dx}$ $= -9y + \frac{dy}{dx} - y + \frac{dy}{dx}$ $= 2\frac{dy}{dx} - 10y$ $\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$		
		05	
4	Let $\boldsymbol{d} = 3\boldsymbol{i} + 12\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{n} = -\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ Let θ = required angle Using $\boldsymbol{d} \cdot \boldsymbol{n} = \lvert\boldsymbol{d}\rvert\lvert\boldsymbol{n}\rvert \sin\theta$ $\begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{3^2 + 12^2 + 4^2}\sqrt{(-1)^2 + 2^2 + 2^2} \sin\theta$ $-3 + 24 + 8 = \sqrt{169}\sqrt{9} \sin\theta$		

<table>
<tr><td></td><td>$29 = 13 \times 3 \sin\theta$
$\sin\theta = \frac{29}{39}$
$\theta = \sin^{-1}\left(\frac{29}{39}\right)$
$\theta = 48.04^0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>05</td><td></td></tr>
<tr><td>5</td><td>$\frac{7-2x}{(x+1)(x-2)} > 0$
Critical values
$x = -1, x = 2, x = \frac{7}{2}$
<table>
<tr><td>x</td><td>$x < -1$</td><td>$-1 < x < 2$</td><td>$2 < x < 3.5$</td><td>$x > 3.5$</td></tr>
<tr><td>$(7-2x)$</td><td>$+$</td><td>$+$</td><td>$+$</td><td>$-$</td></tr>
<tr><td>$(x+1)(x-2)$</td><td>$+$</td><td>$-$</td><td>$+$</td><td>$+$</td></tr>
<tr><td>$\frac{7-2x}{(x+1)(x-2)}$</td><td>$+$</td><td>$-$</td><td>$+$</td><td>$-$</td></tr>
</table>
$\therefore$ The range of values of x are: $x < -1, 2 < x < 3.5$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>05</td><td></td></tr>
<tr><td>6</td><td>$\int_0^{\pi/3}(1+\cos 3y)^2\,dy = \int_0^{\pi/3}(1 + 2\cos 3y + cos^2 3y)\,dy$
$= \int_0^{\pi/3}\left[1 + 2\cos 3y + \frac{1}{2}(\cos 6y + 1)\right]dy$
$= \left[y + \frac{2}{3}\sin 3y + \frac{1}{12}\sin 6y + \frac{1}{2}y\right]_0^{\pi/3}$
$= \left[\frac{3}{2}y + \frac{2}{3}\sin 3y + \frac{1}{12}\sin 6y\right]_0^{\pi/3}$
$= \left(\frac{\pi}{2} + \frac{2}{3}\sin\pi + \frac{1}{12}\sin 2\pi\right) - 0$
$= \frac{\pi}{2}$ or 1.5708</td><td></td><td></td></tr>
<tr><td></td><td></td><td>05</td><td></td></tr>
<tr><td>7</td><td>Let $2\sin\theta + 3\cos\theta \equiv R\sin(\theta + \alpha)$
$2\sin\theta + 3\cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$
$\equiv (R\cos\alpha)\sin\theta + (R\sin\alpha)\cos\theta$
Comparing coefficients of;
$\sin\theta$; $R\cos\alpha = 2$ ……………………(i)</td><td></td><td></td></tr>
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	$\cos\theta$; $R\sin\alpha = 3$ ....(ii) $(R\cos\alpha)^2 + (R\sin\alpha)^2 = 2^2 + 3^2$ $R^2(cos^2\alpha + sin^2\alpha) = 4 + 9 = 13$ $R^2 = 13$ $R = \sqrt{13}$ (ii)÷(i); $\tan\alpha = \frac{3}{2}$ $\alpha = \tan^{-1}(1.5)$ $\alpha = 56.31^0$ $\therefore 2\sin\theta + 3\cos\theta = \sqrt{13}\sin(\theta + 56.31^0)$		
		05	
8	Let $f(x) = \ln(2+x), f(0) = \ln 2$ $f'(x) = \frac{1}{2+x}, f'(0) = \frac{1}{2}$ $f''(x) = -(2+x)^{-2}\cdot 1 = \frac{-1}{(2+x)^2}, f''(0) = -\frac{1}{4}$ Using $f(x) = f(0) + xf'(0) + \frac{x^2f''(0)}{2!} + \frac{x^3f'''(0)}{3!} + \cdots$ $\therefore \ln(2+x) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$		
		05	
9	a) Let $f(z) = z^3 - 7z^2 + 19z - 13$ Putting $z = 1$ $f(1) = 1^3 - 7(1)^2 + 19(1) - 13$ $f(1) = 0$ $z = 1$ is a root and then $z - 1$ is a factor		

$$
\begin{array}{r|l}
 & z^2 - 6z + 13 \\
\hline
(z-1) & z^3 - 7z^2 + 19z - 13 \\
 & -\left(z^3 - z^2\right) \\
\hline
 & -6z^2 + 19z - 13 \\
 & -\left(-6z^2 + 6z\right) \\
\hline
 & 13z - 13 \\
 & -\left(13z - 13\right) \\
\hline
 & - \quad -
\end{array}
$$

$z^2 - 6z + 13 = 0$

$z = \frac{6 \pm \sqrt{(-6)^2 - 4\times1\times13}}{2\times1}$

$z = \frac{6 \pm \sqrt{-16}}{2}$

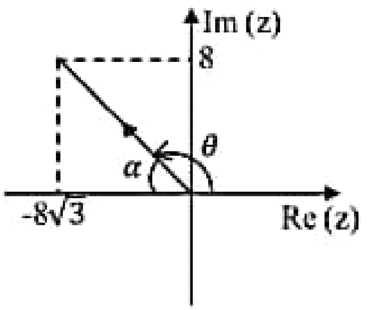
$z = \frac{6 \pm 4i}{2} = 3 \pm 2i$

The values of z are $1, 3 + 2i$ and $3 - 2i$

b) $8(-\sqrt{3} + i) = -8\sqrt{3} + 8i$

Let $z = -8\sqrt{3} + 8i$

$r = |z| = \sqrt{(-8\sqrt{3})^2 + 8^2} = 16$ units



$\arg(z) = \theta = 180^0 - \tan^{-1}\left(\frac{8}{8\sqrt{3}}\right) = 180^0 - 30^0 = 150^0 = \frac{5\pi}{6}$

Using $z = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$

$z = 16^{\frac{1}{4}}\left[\cos\left(\frac{\frac{5\pi}{6} + 2\pi k}{4}\right) + i\sin\left(\frac{\frac{5\pi}{6} + 2\pi k}{4}\right)\right]$

	$z = 2\left[\cos\left(\frac{5\pi+12\pi k}{24}\right) + i\sin\left(\frac{5\pi+12\pi k}{24}\right)\right]$ For $k = 0, z_1 = 2\left[\cos\left(\frac{5\pi}{24}\right) + i\sin\left(\frac{5\pi}{24}\right)\right]$ $= 2(0.7934 + 0.6088i)$ $= 1.5868 + 1.2176i$ For $k = 1, z_2 = 2\left[\cos\left(\frac{17\pi}{24}\right) + i\sin\left(\frac{17\pi}{24}\right)\right]$ $= 2(-0.6088 + 0.7934)$ $= -1.2176 + 1.5868i$ For $k = 2, z_3 = 2\left[\cos\left(\frac{29\pi}{24}\right) + i\sin\left(\frac{29\pi}{24}\right)\right]$ $= 2(-0.7934 - 0.6088i)$ $= -1.5868 - 1.2176i$ For $k = 3, z_4 = 2\left[\cos\left(\frac{41\pi}{24}\right) + i\sin\left(\frac{41\pi}{24}\right)\right]$ $= 2(0.6088 - 07934i)$ $= 1.2176 - 1.5868i$		
		12	
10	**Method I** Let $\frac{3x^3+2x^2-3x+1}{x(1-x)} \equiv Ax + B + \frac{C}{x} + \frac{D}{1-x}$ $3x^3 + 2x^2 - 3x + 1 \equiv x(Ax + B)(1 - x) + C(1 - x) + Dx$ Putting $x = 1; 3 = D$ $\quad\therefore D = 3$ Putting $x = 0; 1 = C$ $\quad\therefore C = 1$ Comparing coefficients of; $x^3; 3 = -A$ $\quad\therefore A = -3$ $x^2; 2 = A - B$ $2 = -3 - B$ $\quad\therefore B = -5$ $\therefore \frac{3x^3+2x^2-3x+1}{x(1-x)} \equiv -3x - 5 + \frac{1}{x} + \frac{3}{1-x}$ Hence;		

	$\int f(x)\,dx = \int(-3x-5)\,dx + \int \frac{1}{x}\,dx + \int \frac{3}{1-x}\,dx$ $= -\frac{3}{2}x^2 - 5x + \ln x - 3\ln(1-x) + c$ **Method II** $\frac{3x^3+2x^2-3x+1}{x(1-x)} = \frac{3x^3+2x^2-3x+1}{x-x^2}$ $\begin{array}{r l} & -3x-5 \\ (-x^2+x) & \overline{)\,3x^3+2x^2-3x+1} \\ & -\underline{(3x^3-3x^2)} \\ & 5x^2-3x+1 \\ & -\underline{(5x^2-5x)} \\ & 2x+1 \end{array}$ $\frac{3x^3+2x^2-3x+1}{x(1-x)} = -3x-5+\frac{2x+1}{x(1-x)}$ Let $\frac{2x+1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x}$ $2x+1 \equiv A(1-x) + Bx$ Putting $x = 1; 3 = B$ $\quad\quad \therefore B = 3$ Putting $x = 0; 1 = A$ $\quad\quad \therefore A = 1$ $\therefore \frac{3x^3+2x^2-3x+1}{x(1-x)} \equiv -3x-5+\frac{1}{x}+\frac{3}{1-x}$ Hence; $\int f(x)\,dx = \int(-3x-5)\,dx + \int \frac{1}{x}\,dx + \int \frac{3}{1-x}\,dx$ $= -\frac{3}{2}x^2 - 5x + \ln x - 3\ln(1-x) + c$		
		12	
11	a) Equation of a line through E$(2,0,-1)$ $\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ Let $\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$		

$x = 2 - 2\mu$
$y = \mu$
$z = -1 + 2\mu$

At point B;

$2 - 2\mu + 2\mu - 2(-1 + 2\mu) = 8$

$2 + 2 - 4\mu = 8$

$-4\mu = 4 \qquad \therefore \mu = -1$

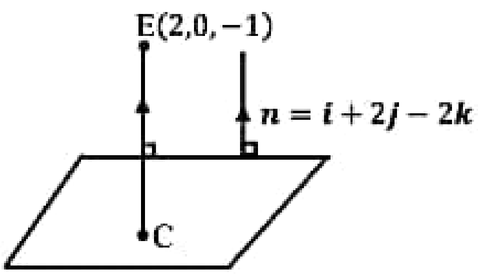
$\Rightarrow x = 2 - 2(-1) = 4$

$y = -1$

$z = -1 + 2(-1) = -3$

$\therefore B(4, -1, -3)$

b)



Equation of the perpendicular from E to the plane;

$$r = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

Let $r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$x = 2 + t$

$y = 2t$

$z = -1 - 2t$

At point C;

	$2 + t + 4t - 2(-1 - 2t) = 8$ $2 + 5t + 2 + 4t = 8$ $9t = 4$ $t = \frac{4}{9}$ $\Rightarrow x = 2 + \frac{4}{9} = \frac{22}{9}$ $y = 2\left(\frac{4}{9}\right) = \frac{8}{9}$ $z = -1 - 2\left(\frac{4}{9}\right) = -\frac{17}{9}$ $\therefore C\left(\frac{22}{9}, \frac{8}{9}, -\frac{17}{9}\right)$		
		12	
12	$a)\ No.\ of\ ways = 10! = 3{,}628{,}800\ ways$ $b)\ No.\ of\ ways = {}^9C_6 \times {}^7C_5 = 84 \times 21 = 1764\ ways$ $c)\ {}^{20}C_r = {}^{20}C_{r-2}$ $\frac{20!}{(20-r)!r!} = \frac{20!}{(20-(r-2))!(r-2)!}$ $(20 - r)!\,r! = (20 - (r - 2))!\,(r - 2)!$ $(20 - r)!\,r! = (22 - r)!\,(r - 2)!$ $(20 - r)!\,r(r - 1)(r - 2)! = (22 - r)(21 - r)(20 - r)!\,(r - 2)!$ $r(r - 1) = (22 - r)(21 - r)$ $r^2 - r = 462 - 22r - 21r + r^2$ $-r = 462 - 43r$ $42r = 462$ $r = 11$ ***Alternatively:*** If ${}^nC_x = {}^nC_y \Rightarrow x + y = n$ Then ${}^{20}C_r = {}^{20}C_{r-2}$ $r + r - 2 = 20$ $2r = 22$		

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<tr><td></td><td>$\therefore r = 11$</td><td></td><td></td></tr>
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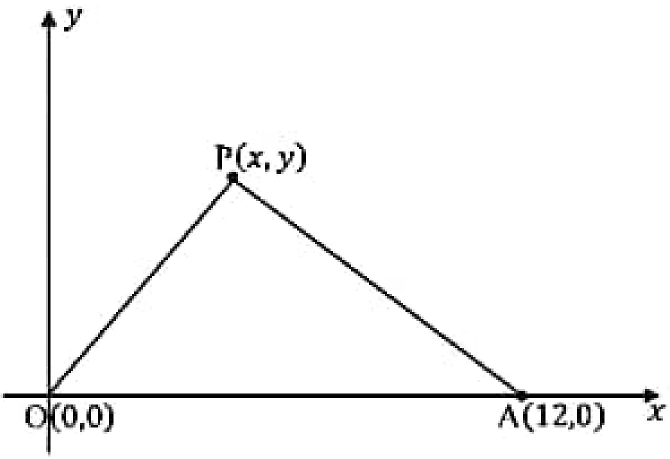
<tr><td>13</td><td>
a) $x = t^2 - 3, y = t(t^2 - 3)$
From $x = t^2 - 3$
$t = \sqrt{x+3}$
⇨ $y = \sqrt{x+3}(x)$
Squaring both sides gives
$y^2 = x^2(x+3)$
$\therefore y^2 = x^3 + 3x^2$ or $x^3 = y^2 - 3x^2$

Alternatively:
$y = tx \Rightarrow t = \frac{y}{x}$
Using $x = t^2 - 3$
⇨ $x = \frac{y^2}{x^2} - 3$
$\therefore x^3 = y^2 - 3x^2$ or $y^2 = x^3 + 3x^2$
b)
(i)


$\overline{OP} = 5\overline{PA}$
$\overline{OP}^2 = 25\overline{PA}^2$
$(x-0)^2 + (y-0)^2 = 25[(x-12)^2 + (y-0)^2]$
$x^2 + y^2 = 25(x^2 - 24x + 144 + y^2)$
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	$x^2 + y^2 = 25x^2 + 25y^2 - 600x + 3600$ $24x^2 + 24y^2 - 600x + 3600 = 0$ $x^2 + y^2 - 25x + 150 = 0$ hence a circle (ii) Completing squares $x^2 + y^2 - 25x = -150$ $\left(x - \frac{25}{2}\right)^2 + (y - 0)^2 = -150 + \left(\frac{25}{2}\right)^2$ $\left(x - \frac{25}{2}\right)^2 + (y - 0)^2 = \frac{25}{4}$ $\therefore$ Centre, $C\left(\frac{25}{2}, 0\right)$ and radius, $r = \sqrt{\left(\frac{25}{4}\right)} = \frac{5}{2} = 2.5$ units		
		12	
14	a) Turning points $\frac{dy}{dx} = \frac{(4x^2-1)\cdot 0 - 1\cdot 8x}{(4x^2-1)^2} = 0$ $8x = 0$ $x = 0$ When $x = 0, y = \frac{1}{0-1} = -1$ $\therefore (0, -1)$ Nature; $\frac{d^2y}{dx^2} = \frac{(4x^2-1)^2\cdot -8 + 8x\cdot 2(4x^2-1)\cdot 8x}{(4x^2-1)^4} = \frac{(4x^2-1)(96x+8)}{(4x^2-1)^4}$ When $x = 0, y = \frac{(0-1)(0+8)}{(0-1)^4} = -8 < 0$ $\therefore (0, -1)_{max}$ b) Asymptotes Vertical asymptote $4x^2 - 1 = 0$ $4x^2 = 1$ $x = \pm\frac{1}{2}$		

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$x = -\frac{1}{2}, x = \frac{1}{2}$

Horizontal asymptote

$y = \frac{\frac{1}{x^2}}{4 - \frac{1}{x^2}}$

As $x \to \pm\infty, y \to 0$

i.e $y = 0$

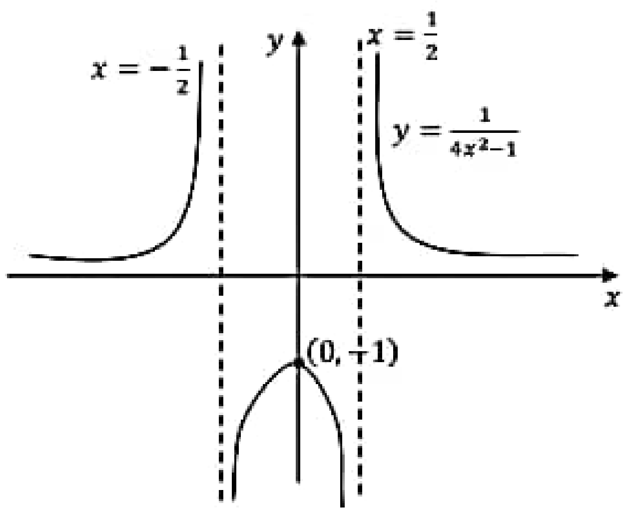
Intercepts

When $y = 0, x = ?$

$0 = 1, x$ is undefined

When $x = 0, y = ?$

$y = \frac{1}{0-1} = -1, (0, -1)$



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<tr><td>15</td><td>

a) $\tan 3\theta = \tan(2\theta + \theta)$

$= \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$

But $\tan 2\theta = \frac{2\tan\theta}{1 - tan^2\theta}$

$\Rightarrow \tan 3\theta = \frac{\frac{2\tan\theta}{1-tan^2\theta} + \tan\theta}{1 - \left(\frac{2\tan\theta}{1-tan^2\theta}\right)\cdot\tan\theta}$

$= \frac{2\tan\theta + \tan\theta - tan^3\theta}{1 - tan^2\theta - 2tan^2\theta}$

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$$= \frac{3\tan\theta - tan^3\theta}{1 - 3tan^2\theta}$$

$$\therefore \tan 3\theta = \frac{\tan\theta(3 - tan^2\theta)}{(1 - 3tan^2\theta)}$$

ALT:

From De Movoire's theorem;

$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$

$$= cos^3\theta + 3icos^2\theta\sin\theta - 3\cos\theta\, sin^2\theta - isin^3\theta$$

Equating components;

Real: $\cos 3\theta = cos^3\theta - 3\cos\theta\, sin^2\theta$ ....(i)

Imaginary; $\sin 3\theta = 3cos^2\theta\sin\theta - sin^3\theta$...............(ii)

(ii)÷(i); $\tan 3\theta = \frac{3cos^2\theta\sin\theta - sin^3\theta}{cos^3\theta \quad 3\cos\theta\, sin^2\theta}$

Dividing through the R.H.S by $cos^3\theta$

$\tan 3\theta = \frac{3\tan\theta - tan^3\theta}{1 - 3tan^2\theta}$

$\therefore \tan 3\theta = \frac{\tan\theta(3 - tan^2\theta)}{(1 - 3tan^2\theta)}$

b) $\cos 6x + \cos 2x + \cos 4x = 0$

$2\cos 4x\cos 2x + \cos 4x = 0$

$\cos 4x\,(2\cos 2x + 1) = 0$

Either $\cos 4x = 0$ or $2\cos 2x + 1 = 0$

For $\cos 4x = 0$

$4x = \cos^{-1}(0)$

$4x = 90^0, 270^0, 450^0, 630^0$

$x = 22.5^0, 67.5^0, 112.5^0, 157.5^0$

For $2\cos 2x + 1 = 0$

$\cos 2x = -\frac{1}{2}$

$2x = \cos^{-1}\left(\frac{1}{2}\right)$

$2x = 120^0, 240^0$

	$x = 60^0, 120^0$		
	$\therefore x = 22.5^0, 60^0, 67.5^0, 112.5^0, 120^0, 157.5^0$		
		12	
16	a) Let T be the body's temperature		
	$\frac{dT}{dt} \propto (T - 25)$		
	$\frac{dT}{dt} = -k(T - 25)$		
	$\int \frac{dT}{T-25} = -\int k\,dt$		
	$\ln(T - 25) = -kt + c$		
	$T - 25 = e^{-kt+c}$		
	$T - 25 = e^{-kt} \cdot e^{c}$		
	$T - 25 = Ae^{-kt}$, $A = e^{c}$		
	$T = 25 + Ae^{-kt}$		
	When $t = 0, T = 90^0$C		
	$90 = 25 + A$ $\therefore A = 65$		
	$T = 25 + 65e^{-kt}$		
	When $t = 6\,mins, T = 60^0$C		
	$60 = 25 + 65e^{-6k}$		
	$e^{-6k} = \frac{35}{65}$		
	$-6k = \ln\left(\frac{35}{65}\right)$		
	$k = \frac{1}{6}\ln\left(\frac{65}{35}\right)$		
	$\therefore T = 25 + 65e^{-\frac{1}{6}\ln\left(\frac{65}{35}\right)\cdot t}$		
	b) When $T = 40^0, t = ?$		
	$40 = 25 + 65e^{-\frac{1}{6}\ln\left(\frac{65}{35}\right)\cdot t_1}$		
	$-\frac{1}{6}\ln\left(\frac{65}{35}\right)\cdot t_1 = \ln\left(\frac{15}{65}\right)$		

	$t_1 = \frac{-6\ln\left(\frac{15}{65}\right)}{\ln\left(\frac{65}{35}\right)} = 14.2124$ minutes When $T = 30^0, t = ?$ $30 = 25 + 65e^{-\frac{1}{6}\ln\left(\frac{65}{35}\right)\cdot t_2}$ $-\frac{1}{6}\ln\left(\frac{65}{35}\right)\cdot t_2 = \ln\left(\frac{5}{65}\right)$ $t_2 = \frac{-6\ln\left(\frac{5}{65}\right)}{\ln\left(\frac{65}{35}\right)} = 24.8606$ minutes $\therefore$ Time taken $= 24.8606 - 14.2124$ $= 10.6482 \approx 11$ minutes *Alternatively:* Using $\ln(T - 25) = -kt + c$ Set $t = t_1$ at $T = 40 \Rightarrow \ln 15 = -kt_1 + c$ Set $t = t_2$ at $T = 30 \Rightarrow \ln 5 = -kt_2 + c$ Subtracting : $\ln 15 - \ln 5 = k(t_2 - t_1)$ The required time, $t_2 - t_1 = \frac{\ln 3}{k}$ $= \ln 3 \div \frac{1}{6}\ln\left(\frac{65}{35}\right)$ $= \frac{6\ln 3}{\ln\left(\frac{65}{35}\right)}$ $= 10.6482 \approx 11$ minutes		
		12	