NUMBER BASES

Summary:

1. *Number bases are different ways of writing down numbers.*

2. *The most common base system is base* **10.**

3. *The digits of a number in any base are less than the base itself*

4. *The digits* **10** *and* **11** *are represented by* ***t*** *and* ***e*** *respectively in number bases*

NOTE:

(i) *Base 10 is called* ***decimal base***

(ii) *Base 2 is called* ***binary base***

(iii) *Base 3 is called* ***trinary base***

(iv) *Base 8 is called* ***octal base***

EXAMPLES:

1. *Convert the following to base ten*

(i) 1011_{two} (ii) 346_{seven} (iii) 2210_{three}

(iv) $2et_{twelve}$ (v) $312\cdot 21_{four}$ (vi) $0\cdot 12_{six}$

solution

$$\begin{aligned}
\textbf{(i)}\quad 1011_{two} &= (1\times 2^3) + (0\times 2^2) + (1\times 2^1) + (1\times 2^0) \\
&= (1\times 8) + (0\times 4) + (1\times 2) + (1\times 1) \\
&= 11_{ten}
\end{aligned}$$

(iv) $312 \cdot 21_{four} = (3\times4^{2}) + (1\times4^{1}) + (2\times4^{0}) + (2\times4^{-1}) + (1\times4^{-2})$

$$= (3\times16) + (1\times4) + (2\times1) + (2\times\frac{1}{4}) + (2\times\frac{1}{16})$$

$$= 54 + \frac{1}{2} + \frac{1}{16}$$

$$= 54\frac{9}{16}_{ten} \quad \text{or} \quad 54\cdot5625_{ten}$$

CONVERTING FROM BASE TEN TO OTHER BASES

Summary:

(i) *Divide the number repeatedly by the required bases*

(ii) *The remainder in reverse order gives the required number*

EXAMPLES:

1. *Convert* 64_{ten} *to base three*

3	64	R
3	21	1
3	7	0
	2	1

$\therefore\ 64_{ten} = 2101_{three}$

2. *Convert* 246_{ten} *to base five*

3. *Convert* 2101_{three} *to base seven*

Hint: *First convert* 2101_{three} *to base ten*

$$2101_{three} = (2\times 3^3) + (1\times 3^2) + (0\times 3^1) + (1\times 3^0)$$

$$= 64_{ten}$$

7	64	R
7	9	1
	1	2

$$\therefore\ 2101_{three} = 121_{seven}$$

5. *Find the value of **n** in the following equations:*

(i) $45_n = 1112_{three}$ *(ii)* $21_n = 19_{ten}$ *(iii)* $303_n = 410_{six}$

(iv) $202_n = 37_{nine}$ *(v)* $112_n + 304_n = 421_n$

OPERATIONS WITH ANY BASE OTHER THAN 10

ADDITION:

If the sum of the digits exceeds the base, divide that sum by the base then write down the remainder and carry the whole number.

EXAMPLES:

1. *Workout the following leaving your answer in the base indicated*

(i) $136_{seven} + 254_{seven}$

(ii) $232_{five} + 344_{five}$

(iii) $28\cdot 57_{nine} + 6\cdot 34_{nine}$

Solution:

(i) $\begin{array}{r} 136_{seven} \\ +254_{seven} \\ \hline 423_{seven} \end{array}$

(ii) $\begin{array}{r} 232_{five} \\ +344_{five} \\ \hline 1131_{five} \end{array}$

(iii) $\begin{array}{r} 28\cdot 57_{nine} \\ +\ 6\cdot 34_{nine} \\ \hline 36\cdot 02_{nine} \end{array}$

2. *Workout* $122_{three} + 461_{seven}$ *giving your answer in base five*

Hint: *First convert* 122_{three} *and* 461_{seven} *to base ten and then finally express the answer in the required base*

$$122_{three} = (1\times 3^2) + (2\times 3^1) + (2\times 3^0) = 17_{ten}$$

$$461_{seven} = (4\times 7^2) + (6\times 7^1) + (1\times 7^0) = 239_{ten}$$

$$\Rightarrow \quad 122_{three} + 461_{seven} = 17_{ten} + 239_{ten} = 256_{ten}$$

5	256	R
5	51	1
5	10	1
	2	0

$$\therefore \quad 122_{three} + 461_{seven} = 2011_{five}$$

SUBTRACTION:

In case of borrowing the new value is the sum of the base and the digit which was small.

EXAMPLES:

1. *Workout the following leaving your answer in the base indicated*

(i) $72_{eight} - 43_{eight}$

(ii) $254_{eight} - 217_{eight}$

(iii) $30 \cdot 241_{five} - 14 \cdot 143_{five}$

Solution:

(i)
$$\begin{array}{r} 72_{eight} \\ -43_{eight} \\ \hline 27_{eight} \end{array}$$

(ii)
$$\begin{array}{r} 254_{eight} \\ -217_{eight} \\ \hline 35_{eight} \end{array}$$

(iii)
$$\begin{array}{r} 30 \cdot 241_{five} \\ +\ 14 \cdot 143_{five} \\ \hline 14 \cdot 043_{five} \end{array}$$

2. *Workout* $221_{three} - 101_{two}$ *giving your answer in base four*

Hint: *First convert* 221_{three} *and* 101_{two} *to base ten and then finally express the answer in the required base*

$$221_{three} = (2\times3^2) + (2\times3^1) + (1\times3^0) = 25_{ten}$$

$$101_{two} = (1\times2^2) + (0\times2^1) + (1\times2^0) = 5_{ten}$$

$$\Rightarrow \quad 221_{three} - 101_{two} = 25_{ten} - 5_{ten} = 20_{ten}$$

4	20	R
4	4	0
	1	1

$$\therefore \quad 221_{three} - 101_{two} = 110_{four}$$

MULTIPLICATION AND DIVISION

EXAMPLES:

1. *Workout the following leaving your answer in the base indicated*

(i) $152_{eight} \times 43_{eight}$

(ii) $et5_{twelve} \times 8t_{twelve}$

(iii) $124_{five} \times 32_{five}$

Solution:

(i) $\begin{array}{r} 152_{eight} \\ \times\ 43_{eight} \\ \hline 476 \\ +650 \\ \hline 7176_{eight} \end{array}$

(ii) $\begin{array}{r} et5_{twelve} \\ \times\ 8t_{twelve} \\ \hline 9t82 \\ +7te4 \\ \hline 88t02_{twelve} \end{array}$

(iii) $\begin{array}{r} 124_{five} \\ \times\ 32_{five} \\ \hline 303 \\ +432 \\ \hline 10123_{five} \end{array}$

2. *Workout* $\mathbf{1011_{two}} \times \mathbf{12_{three}}$ *giving your answer in binary base*

Hint: *First convert* $\mathbf{1011_{two}}$ *and* $\mathbf{12_{three}}$ *to base ten and then finally express the answer in the required base*

$$\mathbf{1011_{two}} = (1\times 2^3) + (0\times 2^2) + (1\times 2^1) + (1\times 2^0) = \mathbf{11_{ten}}$$

$$\mathbf{12_{three}} = (1\times 3^1) + (2\times 3^0) = \mathbf{5_{ten}}$$

$$\Rightarrow \mathbf{1011_{two}} \times \mathbf{12_{three}} = 11_{ten} \times 5_{ten} = \mathbf{55_{ten}}$$

2	**55**	**R**
2	**27**	**1**
2	**13**	**1**
2	**6**	**1**
2	**3**	**0**
	1	**1**

$$\therefore \mathbf{1011_{two}} \times \mathbf{12_{three}} = \mathbf{110111_{two}}$$

3. *Workout the following leaving your answer in the base indicated*

(i) $\mathbf{2001_{three}} \div \mathbf{12_{three}}$

(ii) $\mathbf{110111_{two}} \div \mathbf{101_{two}}$

Solution:

(i) Hint: *First convert* $\mathbf{2001_{three}}$ *and* $\mathbf{12_{three}}$ *to base ten and then finally express the answer in the required base*

$$\mathbf{2001_{three}} = (2\times 3^3) + (0\times 3^2) + (0\times 3^1) + (1\times 3^0) = \mathbf{55_{ten}}$$

$12_{three} = (1 \times 3^1) + (2 \times 3^0) = 5_{ten}$

$\Rightarrow \quad 2001_{three} \div 12_{three} = 55_{ten} \div 5_{ten} = 11_{ten}$

3	11	R
3	3	2
	1	0

$\therefore \quad 2001_{three} \div 12_{three} = 102_{three}$

(ii) $110111_{two} = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 55_{ten}$

$101_{two} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5_{ten}$

$\Rightarrow \quad 110111_{two} \div 101_{two} = 55_{ten} \div 5_{ten} = 11_{ten}$

2	11	R
2	5	1
2	2	1
	1	0

$\therefore \quad 110111_{two} \div 101_{two} = 1011_{two}$

EER:

1. *Convert the following to base ten*

(i) 2212_{three} (ii) 1011_{two} (iii) 234_{five}

2. *Express* $0 \cdot 24_{six}$ *as a fraction in base ten*

3. *Express* $45 \cdot 3_{six}$ *in base ten using point notation*

4. *Find the value of* **n** *if* $45_n = 100001_{two}$

5. *Find the value of* **n** *if* $103_n + 26_n = 131_n$

6. *Convert* 102_{three} *to binary base*

7. *Workout the following leaving your answer in the base indicated*

(i) $152_{eight} \times 43_{eight}$

(ii) $et5_{twelve} \times 8t_{twelve}$

(iii) $2212_{three} \div 21_{three}$

8. *Arrange the following numbers* 36_{eight}, 302_{four} *and* 202_{three} *in ascending order*