

P425/1
PURE
MATHEMATICS
Paper 1
July /Aug. 2024
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all questions in section A and any five from section B.

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

*Any extra question(s) attempted in section B will **not** be marked.*

SECTION A (40 MARKS)

Answer ALL questions in this section

1. Solve the equation $\sin 3\theta = \cos \theta$ for $0^\circ \leq \theta \leq 180^\circ$. (05 marks)
2. Given that $x^3 = (y - 3x)^2$ show that; $2x \frac{dy}{dx} = 3y - 3x$. (05 marks)
3. $A(3, 5)$ and $B(-5, -1)$ are points on a line. Find the coordinates of point C that divides \overline{AB} in the ratio 3:1.
(a) Internally
(b) Externally (05 marks)
4. Find the equation of the tangent to the curve $y = 1 + 2\sin x$ at $x = \frac{\pi}{4}$. (05 marks)
5. Given that $z = 1 + 3i$ find the real numbers, x and y such that $xz + y\bar{z} = 7 + 3i$. (05 marks)
6. $O(0, 0)$ and $Q(4, 0)$ are fixed points. $P(x, y)$ is a variable point. Given that $\angle OPQ = 45^\circ$, find the locus of $P(x, y)$. (05 marks)
7. When a polynomial $P(x)$ is divided by $x^2 - 4$, the remainder is $3x + 7$. find the remainder when $P(x)$ is divided by;
(a) $x - 2$
(b) $x + 2$ (05 marks)
8. Evaluate; $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3 - x^2}}$ without using tables. (05 marks)

SECTION B (60 MARKS)

9. (a) Express $\sqrt{3} \sin \theta + \cos \theta$ in the form $R \sin(\theta + \alpha)$. (03 marks)
Hence solve the equation $\sqrt{3} \sin \theta + \cos \theta = \sqrt{2}$,
for $0^\circ \leq \theta \leq 360^\circ$. (03 marks)

- (b) Prove the Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ for any triangle ABC. Hence solve the triangle in which $b = 5\text{cm}$, $c = 8\text{cm}$ and $A = 60^\circ$. (06 marks)

10. (a) Use the mathematics of small changes to evaluate $\sin 29.5^\circ$ to 4 dpls. (05 marks)

- (b) A right circular cone has a slant length of $9\sqrt{3}\text{cm}$. Calculate the maximum volume of the cone; and state the corresponding values of the height and the radius in this case. (07 marks)

11. (a) Find the term in x^{-3} in the expansion of;
 $\left(x^2 + \frac{1}{2x}\right)^9$ (04 marks)

- (b) Expand $\sqrt{1 - \frac{1}{4}x}$ up to the term in x^3 ; and use the expansion to evaluate;

(i) $\sqrt{15}$ to 3dps

(ii) $\sqrt{7}$ to 4dps

(08 marks)

12. Express $\frac{x^6 + 64}{x^4 - 16}$ in partial fractions; hence evaluate;

$$\int_3^4 \frac{x^6 + 64}{x^4 - 16} dx \text{ to 4dps}$$

(12 marks)

13. (a) Find the coordinates of the point of intersection of the lines;

$$\mathbf{r}_1 = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \text{ and } \mathbf{r}_2 = \begin{pmatrix} 10 \\ 1 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix};$$

and compute the acute angle between the lines. (09 marks)

- (b) Write down the vector equation of the plane containing the two lines in (a) above. (03 marks)

14. Given the curve $y = \frac{2x - 5}{x^2 - 4}$,

- (a) Find its stationary points; hence state the region within which the curve does not lie. (06 marks)

- (b) Sketch the curve, and deduce the solution to the inequality $\frac{2x - 5}{x^2 - 4} \geq 0$ (06 marks)

15. (a) Prove that the equations of the tangent to the parabola $y^2 = 4ax$ At the variable point $(at^2, 2at)$ is $x - ty + at^2 = 0$. (03 marks)

- (b) Deduce the equations of the tangents to the parabola $y^2 = 4ax$ from the external point $A(-6a, a)$; (04 marks)

Hence (i) find the coordinates of the points of contact of the tangents with the parabola. (02 marks)

- (ii) show that the tangents make 45° with each other. (03 marks)

16. The temperature, $\theta^\circ\text{C}$, at a height h metres risen above the foot of a 1000 m high mountain, decreases at a rate which is directly proportional to the height risen.

- (a) A tourist notices that the temperature of water drops from 16°C , at the foot of the mountain, to -9°C at the peak of the mountain. Set up a differential equation for this problem and solve it. (06 marks)

- (b) Calculate the;

(i) height at which water starts to freeze. (03 marks)

(ii) temperature the water should have at the foot of the mountain if it just freezes at the top of the mountain. (03 marks)

END