$P425/1$

PURE MATHEMATICS

Paper 1

August, 2023

3 hours

Uganda Advanced Certificate of Education

POST MOCK EXAMINATIONS- 2023

PURE MATHEMATICS

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3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt ***all*** *the* ***eight*** *questions in* ***Section A*** *and* ***Not*** *more than* ***five*** *from* ***Section B.***

Any additional question(s) will not be marked.

All working must be shown clearly.

Silent non-programmable calculators and mathematical tables with a list of formulae may be used.

Graph papers are provided.

SECTION A (40 MARKS)

Attempt **all** questions in this section

1. The first term of an Arithmetic Progression ($\boldsymbol{A.P}$) is equal to the first term of a Geometric Progression ($\boldsymbol{G.P}$) whose common ratio is $\frac{1}{3}$ and sum to infinity is $\mathbf{9}$. If the common difference of he A.P is $\mathbf{2}$, find the sum of the first ten terms of the $\boldsymbol{A.P}$.

(05marks)

2. Find the equation of a line through the point $(\mathbf{5}, \mathbf{3})$ and perpendicular to the line $\mathbf{2}\boldsymbol{x} - \boldsymbol{y} + \mathbf{4} = \mathbf{0}$ *(05marks)*

3. Solve for $\boldsymbol{x}$ in: $\log_a(x+3) + \frac{1}{\log_x a} = 2\log_a 2.$ *(05marks)*

4. Given that $\boldsymbol{D}(\mathbf{7}, \mathbf{1}, \mathbf{2}), \boldsymbol{E}(\mathbf{3}, -\mathbf{1}, \mathbf{4})$ and $\boldsymbol{F}(\mathbf{4}, -\mathbf{2}, \mathbf{5})$ are points on a plane, show that $\boldsymbol{ED}$ is perpendicular to $\boldsymbol{EF}$. *(05marks)*

5. In a triangle $\boldsymbol{ABC}$ all angles are acute. Angle $\boldsymbol{ABC} = \mathbf{50^o}, \boldsymbol{a} = \mathbf{10}\boldsymbol{cm}$ and $\boldsymbol{b} = \mathbf{9}\boldsymbol{cm}$. Solve the triangle *(05marks)*

6. Differentiate $e^{-x^2}x^3 \sin x$ with respect to $\boldsymbol{x}$. *(05marks)*

7. The region enclosed by the curve $\boldsymbol{y} = \boldsymbol{x}^2$ the $\boldsymbol{x} - \boldsymbol{axis}$ and the line $\boldsymbol{x} = \mathbf{2}$ is rotated through revolution about $\boldsymbol{x} - \boldsymbol{axis}$. Find the volume of the solid generated.

(05marks)

8. Solve $\frac{dy}{dx} = e^{x+y}$ given that $\boldsymbol{y} = \mathbf{2}$ when $\boldsymbol{x} = \mathbf{0}.$ *(05marks)*

SECTION B (60 MARKS)

Attempt **FIVE** questions in this section

9. (a) Given $\boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x} - \boldsymbol{a})^2\boldsymbol{g}(\boldsymbol{x})$, show that $\boldsymbol{f}'(\boldsymbol{x})$ is divisible by $(\boldsymbol{x} - \boldsymbol{a})$.

(03marks)

(b) A polynomial $\boldsymbol{P}(\boldsymbol{x}) = \boldsymbol{x}^3 + \mathbf{4}\boldsymbol{a}\boldsymbol{x}^2 + \boldsymbol{b}\boldsymbol{x} + \mathbf{3}$ is divisible by $(\boldsymbol{x} - \mathbf{1})^2$ Use the result in $(\boldsymbol{a})$ above to find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$. Hence solve the equation $\boldsymbol{P}(\boldsymbol{x}) = 0$ *(09 marks)*

10. Sketch on the same co−ordinate axes the graphs of the curve $y = 2 + x - x^2$ and $y = x + 1$. Hence determine the area of the region enclosed between the curve and the line. *(12 marks)*

11. (a) Solve $Z\overline{Z} - 5iZ = 5(9 - 7i)$ where $\overline{Z}$ is the complex conjugate of Z. *(06 marks)*

(b) (i) Find the Cartesian equation of the curve given as $|Z + 2 - 3i| = 2|Z - 2 + i|$.

(ii) Show that it represents a circle. Find the centre and radius of the circle. *(06 marks)*

12. (a) Simplify $\frac{\cos 3\theta + \cos 5\theta}{\sin 5\theta - \sin 3\theta}$ *(03 marks)*

(b) Show that $\cot 2\theta = \frac{1 - tan^2\theta}{2\tan\theta}$. Hence solve the equation $\cot 2\theta = 4 - tan\,\theta$ for values of θ between 0^o and 360^o *(09 marks)*

13. Express $\frac{1}{x^2(x-1)}$ as partial fractions. Hence evaluate $\int_2^3 \frac{1}{x^2(x-1)}$ correct to 3 decimal places. *(12 marks)*

14. (a) Show that lines $a = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ intersect. *(06 marks)*

(b) Find the

(i) point of intersection, P, of the two lines in (a) above. *(02 marks)*

(ii) Cartesian equation of the plane which contains a and b. *(04 marks)*

15. The tangents at the points $P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ on the rectangular hyperbola $xy = c^2$ intersect at R. given that R lies on the curve $xy = \frac{c^2}{2}$, show that the locus of the midpoint of PQ is given by $xy = 2c^2$. *(12 marks)*

16. The rate of increase of a population of a certain bird species is proportional to the number in the population present at the time. Initially the number in the population was $32,000$. After $70\ years$ the population was $48,000$. Find the:

(a) Number of birds in the population after $82\ years$

(b) Time when the population doubles the initial number. *(12 marks)*

END