§ 9.5 Using Permutations and Combinations in Probability

Fundamental Counting Principle

Remember back - if two events are independent, then

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This is known as the Multiplication Rule. We have a similar rule for counting problems.

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If A can occur in a ways and B can occur in b ways, then $A \cap B$ can occur in ab ways. This is called the Fundamental Counting Principle.

Examples of FCT

Example

You are hungry and head to a deli. You have 4 choices for the type of bread, 5 choices for meat and 3 choices for condiments. How many different sandwiches can we make if we can only use one type of each per sandwich?

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Example

You play the Daily Numbers, which involves picking 4 numbers between 0 and 9. How many different numbers could you choose?

Since the choice of the numbers is independent, we can apply the multiplication rule. There are $10^4 = 10000$ different numbers we could play.

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$$\underline{59} \times \underline{58} \times \underline{57} \times \underline{56} \times \underline{55} \times {\color{red}\underline{35} = 21{,}026{,}821{,}200}$$

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So, the probability of winning Powerball, if it was fair, would be $\frac{1}{21{,}026{,}821{,}200}$

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Are these independent?

$$4 \cdot 3 \cdot 2 \cdot 1 = 24 = 4!$$

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Is there a formula?

$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

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$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

What do we *not* want?

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7!$$

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Since this is multiplication, we can use the inverse operation to rid ourselves of what we don't want.

$$\frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 10 \cdot 9 \cdot 8 = \frac{10!}{7!}$$

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The number of ways to select r objects from a set of n objects where repetition is not allowed and order matters is

$$\frac{n!}{(n-r)!} =_n P_r$$

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How many ways are there to arrange none of 5 objects?

If we use the same formula, we'd get

$$_5P_0 = \frac{5!}{(5-0)!} = \frac{5!}{5!} = 1$$

Why does this make sense?

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Is this a problem?

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Is this a problem?

$$\frac{5!}{0!} = \frac{5!}{1} = 120$$

Example

If we have the numbers 1-4, we already saw that there were $4! = 24$ different arrangements where order mattered. How many when order does not matter?

Combinations

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We can expect the number of combinations to be smaller. But, how much smaller?

Back to the Track

Example

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How many horses are we ordering?
How many ways can we arrange those horses where order matters?

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How many horses are we ordering?
How many ways can we arrange those horses where order matters?
So if we divide our number of permutations by 3!, we will destroy the order.

$$\frac{720}{3!} = \frac{\frac{10!}{(10-3)!}}{3!} = \frac{10!}{(10-3)!3!}$$

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Notice that this 3 comes back here. If we write this in general, we'd have our formula for combinations.

Combinations

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If we wanted to find the number of arrangements of r objects taken from a set of n objects where repetition is not allowed and order does not matter is

$$\frac{n!}{r!(n-r)!} =_n C_r = \left(\begin{array}{c} \text{n} \\ \text{r} \end{array} \right)$$

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Since it doesn't, we sometimes ask how many ways we can *choose* the participants.

$$_{20}C_5 = \frac{20!}{(20-5)!5!} = \frac{20!}{15!5!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

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We can simplify this further

$$ \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{\cancel{20} \cdot 19 \cdot \cancel{18}^{3} \cdot 17 \cdot 16}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 19 \cdot 3 \cdot 17 \cdot 16 = 15,504 $$

Relationship Between Permutations and Combinations

Notice how we found the formula - a combination is really a permutation where order is not considered.

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The whole thing with permutations and combinations is that the calculations are not difficult since it is just multiplication and division. What is hard about them is deciding whether or not the order matters, telling us which formula we need to use.

One More Example

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2. 3 people?
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How many words can we form from the letters ‘stats’?

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Formula

If we want to arrange n objects, where there are k types of objects with r_i of type i, then there are

$$\frac{n!}{r_1! \cdot r_2 \cdots r_k!}$$

arrangements.

Repetition Examples

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How many arrangements are there of the letters in the word ‘statistics’?

Repetition Examples

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How many arrangements are there of the letters in the word 'statistics'?

$$\frac{10!}{3! \cdot 3! \cdot 2! \cdot 1! \cdot 1!} = 50,400$$

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If we continued, we'd end up with $6 + 5 + 4 + 3 + 2 + 1 + 0$ handshakes, which is a total of 21 handshakes.

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Now consider this: How many people shake hands at a time?

$$_7C_2 = \frac{7!}{2! \cdot 5!} = 21$$

This is known as the handshake lemma.

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If we added the probability for each number made, we would get 1, or 100%.

Back to the FCT

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Straight ahead- what do we need?
What is the alternative?

$$P(\textit{at least one jack}) = 1 - P(\overline{\textit{at least one jack}}) = 1 - P(\textit{no jacks})$$

Back to the FCT

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Now,

$$P(\textit{no jack}) = \frac{48 \times 47 \times 46}{52 \times 51 \times 50} \approx .7826$$

So, the probability of at least one jack would be $1 - .7826 = .2174$.

Example

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Back to the FTC

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$$.1275 \times_3 C_2 = .3825$$

Back to the FCT

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Probability of winning?

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Probability of winning? $\frac{1}{6}$.
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Now, we subtract the price to play from this and get

$$\$4.17 - \$5.00 = -\$.83$$

So, the game is unfair.