

P425/1
PURE MATHEMATICS
Paper 1
July/August, 2023
3 hours



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GLORISO EXAMINATIONS BOARD (GEB)-KAMPALA
SECONDARY SCHOOLS JOINT MOCK EXAMINATIONS, 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES:

- ✓ Answer all the **EIGHT** questions in section **A** and any **FIVE** questions from section **B**. Any additional question(s) will **NOT** be marked.
- ✓ **All** workings **MUST** be shown clearly.
- ✓ Graph paper is provided.
- ✓ Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- ✓ State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- ✓ Clearly indicate the questions you have attempted on the answer scripts as illustrated, **DO NOT** hand in the question paper.

Question		Mark
Section A		
Section B		
Total		

SECTION A: (40 MARKS)

Attempt ALL questions in this section

1. Solve the simultaneous equations: $xy = 2$, $2\log(x - 1) = \log y$. (05 marks)
2. Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix and sketch. (05 marks)
3. Point $P(14 + 2t, 5 + 2t, 2 - t)$ lies on a fixed straight line for all values of t . Find the Cartesian equation of the line and find the cosine of the acute angle between the line and the plane $x = z$. (05 marks)
4. Prove that: $\sin(2\sin^{-1}x + \cos^{-1}x) = \sqrt{1 - x^2}$. (05 marks)
5. Find the common ratio of the geometric sequence $\sin 2x + -\sin 2x \cos 2x + \sin 2x \cos^2 2x + \dots$ and prove that the sum to infinity is $\tan x$. (05 marks)
6. Evaluate: $\int_0^{\frac{\pi}{2}} \sin 7x \cos 5x \, dx$. (05 marks)
7. Differentiate from first principles: $y = \sin x^2$. (05 marks)
8. Solve the d.e. $\frac{dy}{dx} + 2y = e^{-2x} \cos x$. (05 marks)

SECTION B: (60 MARKS)

Attempt ONLY five questions from this section.

9. Given that the first three terms in the expansion in ascending powers of x of $(1 + x + x^2)^n$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1-3ax}\right)^3$, find the value of a and n . (12 marks)

- 10a) Sketch the curve $y = \frac{4}{(5-x)(x-1)}$. (06 marks)

- b) Calculate the area of the region bounded by the curve, the x -axis, the line $x = 2$ and $x = 4$. (06 marks)

- 11a) Describe the locus given by: $|z + 2i| = |2zi - 1|$. (05 marks)

- b) Given that $z_1 = 6\left(\cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi\right)$ and $z_2 = 3\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)$, find z_1z_2 and $\frac{z_1}{z_2}$ in the form $x + yi$. (07 marks)

- 12a) Find the equation of a plane through $A(1, 2, -3)$, $B(3, 2, 1)$ and $C(0, -2, -1)$
i) in vector form ii) Cartesian form (06 marks)

- b) Find the equation of a plane through $(3, 2, 2)$ which is perpendicular to the line $\frac{x-1}{4} = \frac{y+2}{1} = \frac{z+3}{-3}$. (06 marks)

- c) Find the point of intersection of the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and the plane $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 10$. (12 marks)

- 13a) Prove that in a triangle ABC; $\cot\frac{(A-C)}{2} = \frac{a+c}{a-c} \tan\frac{B}{2}$. (06 marks)

- b) A point P lies on the line AC of a triangle ABC such that BCP is an equilateral triangle. Show that $AP^2 = a^2 + c^2 - ac \cos B - \sqrt{3}ac \sin B$. Deduce that $AP^2 = \frac{1}{2}(a^2 + b^2 + c^2) - 2\sqrt{3}\Delta$, where, Δ is the area of triangle ABC . (06 marks)

14. Find the equation of the tangent and the normal to the curve $xy = c^2$ at the point $P\left(ct, \frac{c}{t}\right)$. Given that the normal at P meets the curve again at Q , find the coordinates of Q . If the tangent at P meets the y -axis at R , find the equation of the locus of the midpoint M of PR . (12 marks)

- 15a) Differentiate w. r. t. x :

i) $y = 2x^{\cos x}$ ii) $y = \frac{e^{\sin x}}{\tan^{-1} x}$ (06 marks)

- b) If $y = e^{-x} \cos x$, determine the three values of x between 0 and 3π for which $\frac{dy}{dx} = 0$. “” (06 marks)

16. Evaluate: $\int_0^1 \frac{3-x}{(x+1)(x^2+1)} dx$ (12 marks)

“GOOD LUCK”

****THE END****