Probability

25

Contents:

Opening problem

Jenaro and Marisa are playing a game at the local fair. They are given a bag containing an equal number of red balls and blue balls, and must each draw one ball from the bag at the same time. Before doing so, they must try to guess whether the balls they select will be the same colour or different colours.

Jenaro thinks it is more than likely that the balls will be the same colour. Marisa thinks it is more likely that the balls will be different colours. Their friend Pia thinks that both outcomes are equally likely.

Who is correct?

A INTRODUCTION TO PROBABILITY [10.1]

Consider these statements:

"The Wildcats will probably beat the Tigers on Saturday."
"It is unlikely that it will rain today."
"I will probably make the team."
"It is almost certain that I will understand this chapter."

Each of these statements indicates a **likelihood** or **chance** of a particular event happening.

We can indicate the likelihood of an event happening in the future by using a percentage.

0%	indicates we believe the event **will not occur**.
100%	indicates we believe the event **is certain to occur**.

All events can therefore be assigned a percentage between 0% and 100% (inclusive).

A number close to 0% indicates the event is **unlikely** to occur, whereas a number close to 100% means that it is **highly likely** to occur.

In mathematics, we usually write probabilities as either decimals or fractions rather than percentages. However, as $100\% = 1$, comparisons or conversions from percentages to fractions or decimals are very simple.

> An **impossible** event which has 0% chance of happening is assigned a probability of 0.
>
> A **certain** event which has 100% chance of happening is assigned a probability of 1.
>
> All other events can be assigned a probability between 0 and 1.

For example, when tossing a coin the probability that it falls 'heads' is 50% or $\frac{1}{2}$ or 0.5.

We can write $\text{P(head)} = \frac{1}{2}$ or $\text{P}(H) = \frac{1}{2}$, both of which read 'the probability of getting a head is one half'.

So, a **probability value** is a measure of the chance of a particular event happening.

The assigning of probabilities is usually based on either:

- observing past data or the results of an experiment (experimental probability), or
- using arguments of symmetry (theoretical probability).

> If A is an event with probability $\text{P}(A)$ then $0 \leqslant \text{P}(A) \leqslant 1$.

The probability of an event cannot be **negative**, or **greater than 1**. It does not make sense to be "less than impossible" or "more than certain".

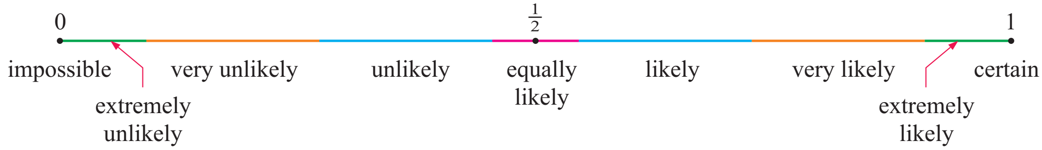
If $\text{P}(A) = 0$, the event cannot occur.

If $\text{P}(A) = 1$, the event is certain to occur.

If $\text{P}(A)$ is very close to 1, it is highly likely that the event will occur.

If $\text{P}(A)$ is very close to 0, it is highly unlikely that the event will occur.

The probability line below shows words which can be used to describe the chance of an event occurring.



EXERCISE 25A

1 Assign suitable words or phrases to these probability calculations:

a 0 **b** 0.51 **c** $\frac{1}{1000}$ **d** 0.23 **e** 1 **f** $\frac{1}{2}\%$

g 0.77 **h** 0.999 **i** $\frac{15}{26}$ **j** $\frac{500}{1999}$ **k** 0.002 **l** $\frac{17}{20}$

2 Suppose that $\text{P}(A) = \frac{1}{3}$, $\text{P}(B) = 60\%$ and $\text{P}(C) = 0.54$.
Which event is: **a** most **b** least likely?

3 Use words to describe the probability that:

a the maximum temperature in London tomorrow will be negative
b you will sleep in the next 48 hours
c Manchester United will win its next football match
d you will be eaten by a dinosaur
e it will rain in Singapore some time this week.

B ESTIMATING PROBABILITY [10.2, 10.6]

Sometimes the only way of finding the probability of a particular event occurring is by experimentation or using data that has been collected over time.

In a probability experiment:

- The **number of trials** is the total number of times the experiment is repeated.
- The **outcomes** are the different results possible for one trial of the experiment.
- The **frequency** of a particular outcome is the number of times that this outcome is observed.
- The **relative frequency** of an outcome is the frequency of that outcome divided by the total number of trials.

$$\textbf{relative frequency} = \frac{\textbf{frequency}}{\textbf{number of trials}}$$

For example, when tossing a tin can in the air 250 times, it comes to rest on an end 37 times. We say:

- the number of trials is 250
- the outcomes are *ends* and *sides*
- the frequency of *ends* is 37 and *sides* is 213
- the relative frequency of *ends* $= \frac{37}{250} \approx 0.148$
- the relative frequency of *sides* $= \frac{213}{250} \approx 0.852$.

The **relative frequency** of an event is an estimate of its **probability**.

We write estimated P(end) ≈ 0.148 and estimated P(side) ≈ 0.852.

Suppose in one year an insurance company receives 9573 claims from its 213 829 clients. The probability of a client making a claim in the next year can be predicted by the relative frequency:

$$\frac{9537}{213\,829} \approx 0.0446 \approx 4.46\%.$$

Knowing this result will help the company calculate its charges or premiums for the following year.

Activity — Rolling a pair of dice

In this experiment you will roll a pair of dice and add the numbers on the uppermost faces. When this is repeated many times the sums can be recorded in a table like this one:

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency											
Relative Frequency											

What to do:

1. Roll two dice 100 times and record the results in a table.
2. Calculate the relative frequency for each possible outcome.
3. Combine the results of everyone in your class. Calculate the overall relative frequency for each outcome.
4. Discuss your results.

The larger the number of trials, the more confident we are that the estimated probability obtained is accurate.

Example 1 — Self Tutor

Estimate the probability of:

a tossing a head with one toss of a coin if it falls heads 96 times in 200 tosses

b rolling a *six* with a die given that when it was rolled 300 times, a *six* occurred 54 times.

a Estimated P(getting a head)
$= $ relative frequency of getting a head
$= \frac{96}{200}$
$= 0.48$

b Estimated P(rolling a *six*)
$= $ relative frequency of rolling a *six*
$= \frac{54}{300}$
$= 0.18$

Example 2 — Self Tutor

A marketing company surveys 80 randomly selected people to discover what brand of shoe cleaner they use. The results are shown in the table alongside:

Brand	*Frequency*
Shine	27
Brite	22
Cleano	20
No scuff	11

a Based on these results, estimate the probability of a community member using: **i** Brite **ii** Cleano.

b Would you classify the estimate of **a** to be very good, good, or poor? Why?

a We start by calculating the relative frequency for each brand.

i Experimental P(Brite) $= 0.275$

ii Experimental P(Cleano) $= 0.250$

b Poor, as the sample size is very small.

Brand	*Frequency*	*Relative Frequency*
Shine	27	0.3375
Brite	22	0.2750
Cleano	20	0.2500
No scuff	11	0.1375

EXERCISE 25B

1 Estimate the probability of rolling *an odd number* with a die if *an odd number* occurred 33 times when the die was rolled 60 times.

2 Clem fired 200 arrows at a target and hit the target 168 times. Estimate the probability of Clem hitting the target.

3 Ivy has free-range hens. Out of the first 123 eggs that they laid she found that 11 had double-yolks. Estimate the probability of getting a double-yolk egg from her hens.

4 Jackson leaves for work at the same time each day. Over a period of 227 working days, on his way to work he had to wait for a train at the railway crossing on 58 days. Estimate the probability that Jackson has to wait for a train on his way to work.

5 Ravi has a circular spinner marked P, Q and R on 3 equal sectors. Estimate the probability of getting a Q if the spinner was twirled 417 times and finished on Q on 138 occasions.

6 Each time Claude shuffled a pack of cards before a game, he recorded the suit of the top card of the pack. His results for 140 games were 34 hearts, 36 diamonds, 38 spades and 32 clubs. Estimate the probability that the top card of a shuffled pack is:

a a heart **b** a club or diamond.

7 Estimate probabilities from these observations:

a Our team has won 17 of its last 31 games.

b There are 23 two-child families in our street of 64 families.

8 A marketing company was commissioned to investigate brands of products usually found in the bathroom. The results of a soap survey are shown alongside:

a How many people were randomly selected in this survey?

b Calculate the relative frequency of use of each brand of soap, correct to 3 significant figures.

Brand	*Freq*	*Rel Freq*
Silktouch	125	
Super	107	
Just Soap	93	
Indulgence	82	
Total		

c Using the results obtained by the marketing company, estimate the probability that the soap used by a randomly selected person is:
  i Just Soap **ii** Indulgence **iii** Silktouch?

9 Two coins were tossed 489 times and the *number of heads* occurring at each toss was recorded. The results are shown opposite:

Outcome	*Freq*	*Rel Freq*
0 heads	121	
1 head		
2 heads	109	
Total		

a Copy and complete the table given.
b Estimate the chance of the following events occurring:
  i 0 heads **ii** 1 head **iii** 2 heads.

10 At the Annual Show the toffee apple vendor estimated that three times as many people preferred red toffee apples to green toffee apples.

Colour	*Freq*	*Rel Freq*
Green	361	
Red		
Total		

a If 361 people wanted green toffee apples, estimate how many wanted red.
b Copy and complete the table given.
c Estimate the probability that the next customer will ask for:
  i a green toffee apple **ii** a red toffee apple.

11 The tickets sold for a tennis match were recorded as people entered the stadium. The results are shown:

Ticket Type	*Freq*	*Rel Freq*
Adult	3762	
Concession	1084	
Child	389	
Total		

a How many tickets were sold in total?
b Copy and complete the table given.
c If a person in the stadium is selected at random, estimate the probability that the person bought a Concession ticket.

12 The results of a local Council election are shown in the table. It is known that 6000 people voted in the election.

Councillor	*Freq*	*Rel Freq*
Mr Tony Trimboli	2167	
Mrs Andrea Sims	724	
Mrs Sara Chong	2389	
Mr John Henry		
Total		

a Copy and complete the table given.
b Estimate the chance that a randomly selected person from this electorate voted for a female councillor.

C PROBABILITIES FROM TWO-WAY TABLES [10.2, 10.6]

Two-way tables are tables which compare two categorical variables. They usually result from a survey.

For example, the year 10 students in a small school were tested to determine their ability in mathematics. The results are summarised in the two-way table shown:

	Boy	*Girl*
Good at maths	17	19
Not good at maths	8	12

there are 12 girls who are not good at maths.

In this case the variables are *ability in maths* and *gender*.
We can use these tables to estimate probabilities.

Example 3

Self Tutor

To investigate the breakfast habits of teenagers, a survey was conducted amongst the students of a high school. The results were:

	Male	*Female*
Regularly eats breakfast	87	53
Does not regularly eat breakfast	68	92

Use this table to estimate the probability that a randomly selected student from the school:

a is male

b is male *and* regularly eats breakfast

c is female *or* regularly eats breakfast

d is male, given that the student regularly eats breakfast

e regularly eats breakfast, given that the student is female.

We extend the table to include totals:

	Male	*Female*	*Total*
Regularly eats breakfast	87	53	140
Does not regularly eat breakfast	68	92	160
Total	155	145	300

a There are 155 males out of the 300 students surveyed.

$\therefore$ P(male) $= \frac{155}{300} \approx 0.517$

b 87 of the 300 students are male and regularly eat breakfast.

$\therefore$ P(male *and* regularly eats breakfast) $= \frac{87}{300} \approx 0.29$

c $53 + 92 + 87 = 232$ out of the 300 are female or regularly eat breakfast.

$\therefore$ P(female *or* regularly eats breakfast) $= \frac{232}{300} \approx 0.773$

d Of the 140 students who regularly eat breakfast, 87 are male.

$\therefore$ P(male given that regularly eats breakfast) $= \frac{87}{140} \approx 0.621$

e Of the 145 females, 53 regularly eat breakfast

$\therefore$ P(regularly eats breakfast, given female) $= \frac{53}{145} \approx 0.366$

EXERCISE 25C

1 Adult workers were surveyed and asked if they had a problem with the issue of wage levels for men and women doing the same job. The results are summarised in the two-way table shown.

	Problem	*No Problem*
Men	146	175
Women	188	134

Assuming that the results are representative of the whole community, estimate the probability that the next randomly chosen adult worker:

a is a woman

b has a problem with the issue

c is a male with no problem with the issue

d is a female, given the person has a problem with the issue

e has no problem with the issue, given that the person is female.

2 310 students at a high school were surveyed on the question "Do you like watching basketball being played on TV?". The results are shown in the two-way table alongside.

	Like	*Dislike*
Junior students	87	38
Senior students	129	56

a Copy and complete the table to include 'totals'.

b Estimate the probability that a randomly selected student:

i likes watching basketball on TV and is a junior student

ii likes watching basketball on TV and is a senior student

iii likes watching basketball on TV, given that the student is a senior

iv is a senior, given that the student likes watching basketball on TV.

3 The two-way table shows the students who can and cannot swim in three different year groups at a school.

	Can swim	*Cannot swim*
Year 4	215	85
Year 7	269	31
Year 10	293	7

If a student is randomly selected from these year groups, estimate the probability that:

a the student can swim

b the student cannot swim

c the student is from year 7

d the student is from year 7 and cannot swim

e the student is from year 7 or cannot swim

f the student cannot swim, given that the student is from year 7 or 10

g the student is from year 4, given that the student cannot swim.

D EXPECTATION [10.3]

The probability of an event can be used to predict the number of times the event will occur in a number of trials.

For example, when rolling an ordinary die, the probability of rolling a '4' is $\frac{1}{6}$.
If we roll the die 120 times, we expect $120 \times \frac{1}{6} = 20$ of the outcomes to be '4's.

Suppose the probability of an event occurring is p. If the trial is repeated n times, the **expectation** of the event, or the number of times we expect it to occur, is np.

Example 4 — Self Tutor

In one week, 79 out of 511 trains were late to the station at Keswick. In the next month, 2369 trains are scheduled to pass through the station. How many of these would you expect to be late?

We estimate the probability of a train being late to be $p = \frac{79}{511}$.

We expect $2369 \times \frac{79}{511} \approx 366$ trains to be late.

EXERCISE 25D

1. In a particular region in Africa, the probability that it will rain on any one day is 0.177. On how many days of the year would you expect it to rain?

2. At practice, Tony kicked 53 out of 74 goals from the penalty goal spot. If he performs as well through the season and has 18 attempts to kick penalty goals, how many is he expected to score?

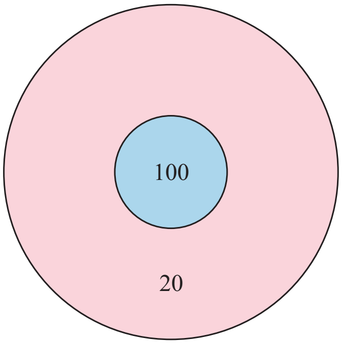
3. A certain type of drawing pin, when tossed 400 times, landed on its back 144 times.
   a. Estimate the probability that it will land on its back if it is tossed once.
   b. If the drawing pin is tossed 72 times, how many "backs" would you expect?

4. A bag contains 5 red and 3 blue discs. A disc is chosen at random and then replaced. This is repeated 200 times. How many times would you expect a red disc to be chosen?

5. A die has the numbers 0, 1, 2, 2, 3 and 4 on its faces. The die is rolled 600 times. How many times might we expect a result of:
   a. 0 b. 2 c. 1, 2 or 3 d. not a 4?

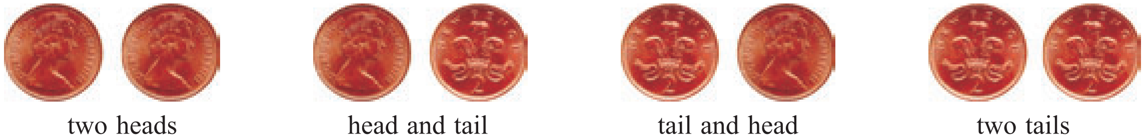
6. a. If 2 coins are tossed, what is the chance that they both fall heads?
   b. If the 2 coins are tossed 300 times, on how many occasions would you expect them to both fall heads?

7. On the last occasion Annette threw darts at the target shown, she hit the inner circle 17% of the time and the outer circle 72% of the time.
   a. Estimate the probability of Annette missing the target with her next throw.
   b. Suppose Annette throws the dart 100 times at the target. She receives 100 points if she hits the inner circle and 20 points if she hits the outer circle. Find:
      i. the total number of points you would expect her to get
      ii. the mean number of points you would expect per throw.



E REPRESENTING COMBINED EVENTS [10.4, 10.6]

The possible outcomes for tossing two coins are listed below:



These results are the **combination** of two events: tossing coin 1 and tossing coin 2.

If H represents a 'head' and T a 'tail', the sample space of possible outcomes is HH, HT, TH and TT.

A **sample space** is the set of all possible outcomes of an experiment.

Possible ways of representing sample spaces are:

- listing them
- using a 2-dimensional grid
- using a tree diagram
- using a Venn diagram.

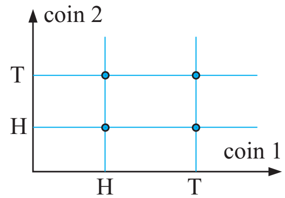
Example 5 Self Tutor

Represent the sample space for tossing two coins using:

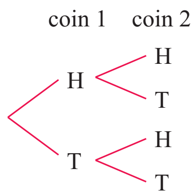
a a list **b** a 2-D grid **c** a tree diagram

a {HH, HT, TH, TT}

b



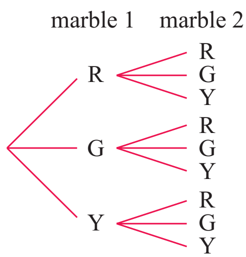
c



Example 6 Self Tutor

Illustrate, using a tree diagram, the possible outcomes when drawing two marbles from a bag containing several marbles of each of the colours red, green and yellow.

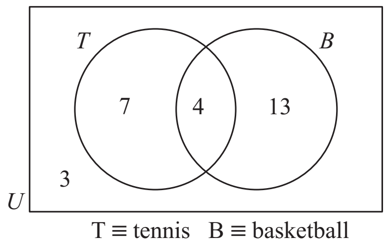
Let R be the event of getting a red
G be the event of getting a green
Y be the event of getting a yellow.



We have already seen Venn diagrams in **Chapter 2**.

If two events have common outcomes, a Venn diagram may be a suitable way to display the sample space.

For example, the Venn diagram opposite shows that of the 27 students in a class, 11 play tennis, 17 play basketball, and 3 play neither of these sports.



T ≡ tennis B ≡ basketball

EXERCISE 25E

1 List the sample space for the following:

a twirling a square spinner labelled A, B, C, D

b the sexes of a 2-child family

c the order in which 4 blocks A, B, C and D can be lined up

d the 8 different 3-child families.

e spinning a coin **i** twice **ii** three times **iii** four times.

2 Illustrate on a 2-dimensional grid the sample space for:

- **a** rolling a die and tossing a coin simultaneously
- **b** rolling two dice
- **c** rolling a die and spinning a spinner with sides A, B, C, D
- **d** twirling two square spinners: one labelled A, B, C, D and the other 1, 2, 3, 4.

3 Illustrate on a tree diagram the sample space for:

- **a** tossing a 5-cent and 10-cent coin simultaneously
- **b** tossing a coin and twirling an equilateral triangular spinner labelled A, B and C
- **c** twirling two equilateral triangular spinners labelled 1, 2 and 3 and X, Y and Z
- **d** drawing two tickets from a hat containing a number of pink, blue and white tickets.
- **e** drawing two beads from a bag containing 3 red and 4 blue beads.

4 Draw a Venn diagram to show a class of 20 students where 10 study History, 15 study Geography, and 2 study neither subject.

F THEORETICAL PROBABILITY [10.4, 10.6]

From the methods of showing sample spaces in the previous section, we can find the probabilities of combined events.

These are theoretical probabilities which are calculated using

$$\text{P(event happens)} = \frac{\text{number of ways the event can happen}}{\text{total number of possible outcomes}}.$$

Example 7 Self Tutor

Three coins are tossed. Write down a list of all possible outcomes.
Find the probability of getting:

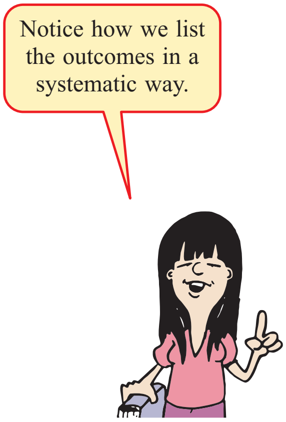
- **a** 3 heads
- **b** at least one head
- **c** 3 heads if it is known that there is at least one head.

The sample space is: HHH HHT TTH TTT
HTH THT
THH HTT

a P(3 heads) $= \frac{1}{8}$

b P(at least one H) $= \frac{7}{8}$ {all except TTT}

c P(HHH knowing at least one H) $= \frac{1}{7}$
{The sample space now excludes TTT}

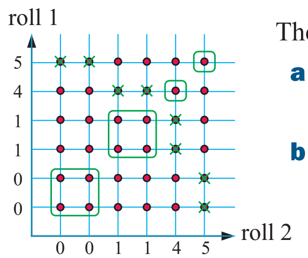


Example 8 — Self Tutor

A die has the numbers 0, 0, 1, 1, 4 and 5. It is rolled *twice*. Illustrate the sample space using a 2-D grid. Hence find the probability of getting:

a a total of 5 **b** two numbers which are the same.

2-D grid



There are $6 \times 6 = 36$ possible outcomes.

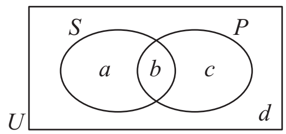
a P(total of 5)
$= \frac{8}{36}$ {those with a $\times$}

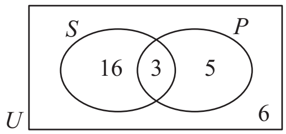
b P(same numbers)
$= \frac{10}{36}$ {those circled}

Example 9 — Self Tutor

In a class of 30 students, 19 play sport, 8 play the piano, and 3 both play sport and the piano. Display this information on a Venn diagram and hence determine the probability that a randomly selected class member plays:

a both sport and the piano **b** at least one of sport and the piano
c sport, but not the piano **d** exactly one of sport and the piano
e neither sport nor the piano **f** the piano if it is known that the student plays sport.





Let S represent the event of 'playing sport',
and P represent the event of 'playing the piano'.

Now $a + b = 19$ {as 19 play sport}
$b + c = 8$ {as 8 play the piano}
$b = 3$ {as 3 play both}
$a + b + c + d = 30$ {as there are 30 in the class}

$\therefore$ $b = 3$, $a = 16$, $c = 5$, $d = 6$.

a P(S and P)
$= \frac{3}{30}$ or $\frac{1}{10}$

b P(at least one of S and P)
$= \frac{16+3+5}{30}$
$= \frac{24}{30}$ (or $\frac{4}{5}$)

c P(S but not P)
$= \frac{16}{30}$
$= \frac{8}{15}$

d P(exactly one of S and P)
$= \frac{16+5}{30}$
$= \frac{7}{10}$

e P(neither S nor P)
$= \frac{6}{30}$
$= \frac{1}{5}$

f P(P given S)
$= \frac{3}{16+3}$
$= \frac{3}{19}$

In **f**, since we know that the student plays sport, we look only at the sport set S.

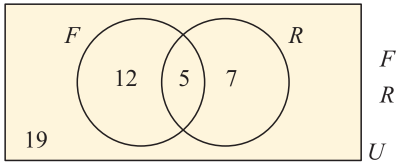
EXERCISE 25F

1 **a** List all possible orderings of the letters O, D and G.

b If these three letters are placed at random in a row, what is the probability of:

i spelling DOG
ii O appearing first
iii O not appearing first
iv spelling DOG or GOD?

2 The Venn diagram shows the sports played by boys at the local high school.
A student is chosen at random.
Find the probability that he:



$F \equiv$ football
$R \equiv$ rugby

a plays football
b plays both codes
c plays football or rugby
d plays exactly one of these sports
e plays neither of these sports
f plays football, given that he is in at least one team
g plays rugby, given that he plays football.

3 Draw the grid of the sample space when a 10-cent and a 50-cent coin are tossed simultaneously. Hence determine the probability of getting:

a two heads
b two tails
c exactly one head
d at least one head.

4 A coin and a pentagonal spinner with sectors 1, 2, 3, 4 and 5 are tossed and spun respectively.

a Draw a grid to illustrate the sample space of possible outcomes.

b How many outcomes are possible?

c Use your grid to determine the chance of getting:

i a head and a 4
ii a tail and an odd number
iii an even number
iv a tail or a 3.

5 List the six different orders in which Alex, Bodi and Kek may sit in a row. If the three of them sit randomly in a row, determine the probability that:

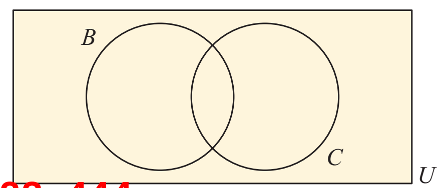
a Alex sits in the middle
b Alex sits at the left end
c Alex sits at the right end
d Bodi and Kek are seated together.

6 **a** List the 8 possible 3-child families, according to the gender of the children.
For example, BGB means *"the first is a boy, the second is a girl, and the third is a boy"*.

b Assuming that each of these is equally likely to occur, determine the probability that a randomly selected 3-child family consists of:

i all boys
ii all girls
iii boy, then girl, then girl
iv two girls and a boy
v a girl for the eldest
vi at least one boy.

7 In a class of 24 students, 10 take Biology, 12 take Chemistry, and 5 take neither Biology nor Chemistry. Find the probability that a student picked at random from the class takes:



a Chemistry but not Biology

b Chemistry or Biology.

8 **a** List, in systematic order, the 24 different orders in which four people P, Q, R and S may sit in a row.

b Hence, determine the probability that when the four people sit at random in a row:

i P sits on one end

ii Q sits on one of the two middle seats

iii P and Q are seated together

iv P, Q and R are seated together, not necessarily in that order.

9 A pair of dice is rolled.

a Show that there are 36 members in the sample space of possible outcomes by displaying them on a grid.

b Hence, determine the probability of a result with:

i one die showing a 4 and the other a 5 **ii** both dice showing the same result

iii at least one die showing a result of 3 **iv** either a 4 or 6 being displayed

v both dice showing even numbers **vi** the sum of the values being 7.

10 60 married men were asked whether they gave their wife flowers or chocolates for their last birthday. The results were: 26 gave chocolates, 21 gave flowers, and 5 gave both chocolates and flowers. If one of the married men was chosen at random, determine the probability that he gave his wife:

a flowers but not chocolates **b** neither chocolates nor flowers

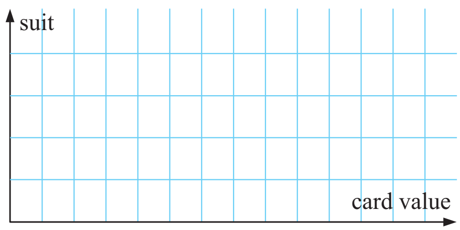
c chocolates or flowers.

11 List the possible outcomes when four coins are tossed simultaneously. Hence determine the probability of getting:

a all heads **b** two heads and two tails **c** more tails than heads

d at least one tail **e** exactly one head.

12 **a** Copy and complete the grid alongside for the sample space of drawing one card from an ordinary pack.



b Use your grid to determine the probability of getting:

i a Queen **ii** the Jack of hearts **iii** a spade

iv a picture card **v** a red 7 **vi** a diamond or a club

vii a King or a heart **viii** a Queen and a 3.

13 The medical records for a class of 28 children show whether they had previously had measles or mumps. The records show 22 have had measles, 13 have had measles and mumps, and 27 have had measles or mumps. If one child from the class is selected at random, determine the probability that he or she has had:

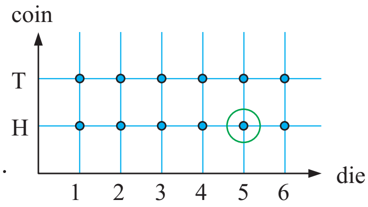
a measles **b** mumps but not measles **c** neither mumps nor measles.

G COMPOUND EVENTS [10.4]

We have previously used two-dimensional grids to represent sample spaces and hence find answers to certain probability problems.

Consider again a simple example of tossing a coin and rolling a die simultaneously.

To determine the probability of getting a head and a '5', we can illustrate the sample space on the two-dimensional grid shown. We can see that there are 12 possible outcomes but only one with the property that we want, so the answer is $\frac{1}{12}$.



However, notice that P(a head) $= \frac{1}{2}$, P(a '5') $= \frac{1}{6}$ and $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

This suggests that P(a head **and** a '5') = P(a head) $\times$ P(a '5'),
i.e., we multiply the separate probabilities.

INDEPENDENT EVENTS

It seems that if A and B are two events for which the occurrence of each one does not affect the occurrence of the other, then P(A and B) = P(A) $\times$ P(B).

The two events 'getting a head' and 'rolling a 5' are events with this property, as the occurrence or non-occurrence of either one of them cannot affect the occurrence of the other. We say they are **independent**.

If two events A and B are **independent** then P(A **and** B) = P(A) $\times$ P(B).

Example 10 Self Tutor

A coin is tossed and a die rolled simultaneously. Find the probability that a tail and a '2' result.

'Getting a tail' and 'rolling a 2' are independent events.

$\therefore$ P(a tail **and** a '2') = P(a tail) $\times$ P(a '2')

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

COMPLEMENTARY EVENTS

Two events are **complementary** if exactly one of them *must* occur.

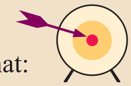
The probabilities of complementary events sum to 1.

The **complement** of event E is denoted E'. It is the event when E fails to occur.

For any event E with **complementary** event E',
$P(E) + P(E') = 1$ or $P(E') = 1 - P(E)$.

Example 11 **Self Tutor**

Sunil has probability $\frac{4}{5}$ of hitting a target and Monika has probability $\frac{5}{6}$.



If they both fire simultaneously at the target, determine the probability that:

a they both hit it **b** they both miss it.

Let S be the event of Sunil hitting and M be the event of Monika hitting.

a P(both hit)
$= \text{P}(S \text{ and } M \text{ hits})$
$= \text{P}(S) \times \text{P}(M)$
$= \frac{4}{5} \times \frac{5}{6}$
$= \frac{2}{3}$

b P(both miss)
$= \text{P}(S' \text{ and } M')$
$= \text{P}(S') \times \text{P}(M')$
$= \frac{1}{5} \times \frac{1}{6}$
$= \frac{1}{30}$

EXERCISE 25G.1

1 A coin and a pentagonal spinner with edges marked A, B, C, D and E are tossed and twirled simultaneously. Find the probabilities of getting:

a a head and a D **b** a tail and either an A or a D.

2 A spinner with 6 equal sides has 3 red, 2 blue and 1 yellow edge. A second spinner with 7 equal sides has 4 purple and 3 green edges. Both spinners are twirled simultaneously. Find the probability of getting:

a a red and a green **b** a blue and a purple.

3 Janice and Lee take set shots at a netball goal from 3 m. From past experience, Janice throws a goal on average 2 times in every 3 shots, whereas Lee throws a goal 4 times in every 7. If they both shoot for goals, determine the probability that:

a both score a goal **b** both miss

c Janice scores a goal but Lee misses.

4 When a nut was tossed 400 times it finished on its edge 84 times and on its side for the rest. Use this information to estimate the probability that when two identical nuts are tossed:


edge side

a they both fall on their edges **b** they both fall on their sides.

5 Tei has probability $\frac{1}{3}$ of hitting a target with an arrow, while See has probability $\frac{2}{5}$. If they both fire at the target, determine the probability that:

a both hit the target **b** both miss the target

c Tei hits the target and See misses **d** Tei misses the target and See hits.

6 A certain brand of drawing pin was tossed into the air 600 times. It landed on its back  243 times and on its side  for the remainder. Use this information to estimate the probability that:

a one drawing pin, when tossed, will fall on its **i** back **ii** side

b two drawing pins, when tossed, will both fall on their backs

c two drawing pins, when tossed, will both fall on their sides.

DEPENDENT EVENTS

Suppose a cup contains 4 red and 2 green marbles. One marble is randomly chosen, its colour is noted, and it is then put aside. A second marble is then randomly selected. What is the chance that it is red?

If the first marble was red, $\quad$ P(second is red) $= \frac{3}{5}$ ← 3 reds remaining / 5 to choose from

If the first marble was green, $\quad$ P(second is red) $= \frac{4}{5}$ ← 4 reds remaining / 5 to choose from

So, the probability of the second marble being red **depends** on what colour the first marble was. We therefore have **dependent events**.

Two or more events are **dependent** if they are **not independent**.

Dependent events are events for which the occurrence of one of the events *does affect* the occurrence of the other event.

For compound events which are dependent, a similar product rule applies as to that for independent events:

If A and B are dependent events then $\quad$ P(A **then** B) = P(A) $\times$ P(B given that A has occurred).

Example 12 — Self Tutor

A box contains 4 blue and 3 yellow buttons of the same size. Two buttons are randomly selected from the box without replacement. Find the probability that:

a both are yellow **b** the first is yellow and the second is blue.

a P(both are yellow)

= P(first is yellow *and* second is yellow)

= P(first is yellow) $\times$ P(second is yellow given that the first is yellow)

$= \frac{3}{7} \times \frac{2}{6}$ ← 2 yellows remaining / 6 to choose from

$= \frac{1}{7}$

b P(first is Y and second is B)

= P(first is Y) $\times$ P(second is B given that the first is Y)

$= \frac{3}{7} \times \frac{4}{6}$ ← 4 blues remaining / 6 to choose from

$= \frac{2}{7}$

EXERCISE 25G.2

1 A packet contains 8 identically shaped jelly beans. 5 are green and 3 are yellow. Two jelly beans are randomly selected without replacing the first before the second is drawn.

a Determine the probability of getting:

i two greens **ii** a green then a yellow

iii a yellow then a green **iv** two yellows.

b Why do your answers in **a** add up to 1?

2 A pocket in a golf bag contains 6 white and 4 yellow golf balls. Two of them are selected at random without replacement.

a Determine the probability that:

i both are white **ii** the first is white and the second is yellow

iii one of each colour is selected.

b Why do your answers in **a** not add up to 1?

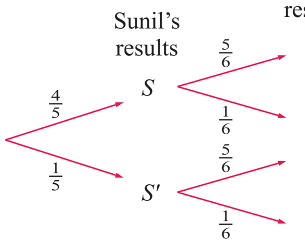
3 A container has 4 purple, 3 blue and 1 gold ticket. Three tickets are selected without replacement. Find the probability that:

a all are purple **b** all are blue **c** the first two are purple and the third is gold.

H USING TREE DIAGRAMS [10.5]

Tree diagrams can be used to illustrate sample spaces, provided that the alternatives are not too numerous. Once the sample space is illustrated, the tree diagram can be used for determining probabilities.
Consider **Example 11** again. The tree diagram for this information is:

S means Sunil hits
M means Monika hits



outcome	probability
S and M	$\frac{4}{5} \times \frac{5}{6} = \frac{20}{30}$
S and M'	$\frac{4}{5} \times \frac{1}{6} = \frac{4}{30}$
S' and M	$\frac{1}{5} \times \frac{5}{6} = \frac{5}{30}$
S' and M'	$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$
	total 1

Notice that:

- The probabilities for hitting and missing are marked on the branches.
- There are *four* alternative paths and each path shows a particular outcome.
- All outcomes are represented and the probabilities of each of the outcomes are obtained by **multiplying** the probabilities along that path.

Example 13 Self Tutor

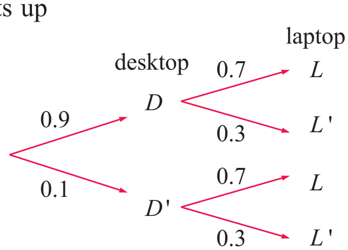
Stephano is having problems. His desktop computer will only boot up 90% of the time and his laptop will only boot up 70% of the time.

a Draw a tree diagram to illustrate this situation.

b Use the tree diagram to determine the chance that:

i both will boot up **ii** Stephano has no choice but to use his desktop computer.

a D = desktop computer boots up
L = laptop boots up



outcome	probability
D and L	$0.9 \times 0.7 = 0.63$
D and L'	$0.9 \times 0.3 = 0.27$
D' and L	$0.1 \times 0.7 = 0.07$
D' and L'	$0.1 \times 0.3 = 0.03$
total	1.00

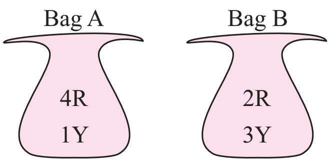
b **i** P(both boot up)
$= \text{P}(D \text{ and } L)$
$= 0.9 \times 0.7$
$= 0.63$

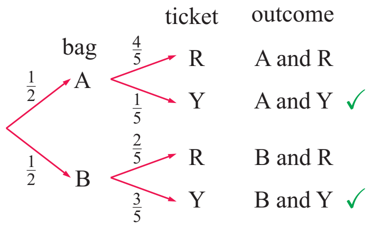
ii P(desktop boots up but laptop does not)
$= \text{P}(D \text{ and } L')$
$= 0.9 \times 0.3$
$= 0.27$

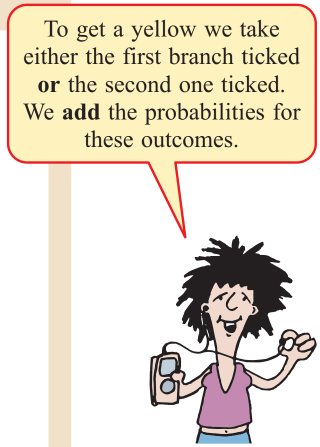
Example 14

Self Tutor

Bag A contains 4 red jelly beans and 1 yellow jelly bean. Bag B contains 2 red and 3 yellow jelly beans. A bag is randomly selected by tossing a coin, and one jelly bean is removed from it. Determine the probability that it is yellow.







P(yellow) = P(A and Y) + P(B and Y)

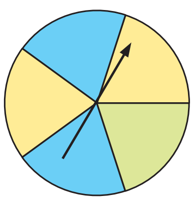
$= \frac{1}{2} \times \frac{1}{5} + \frac{1}{2} \times \frac{3}{5}$ {branches marked ✓}

$= \frac{4}{10}$

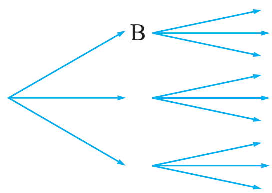
$= \frac{2}{5}$

EXERCISE 25H

1 Suppose this spinner is spun twice:



a Copy and complete the branches on the tree diagram shown.



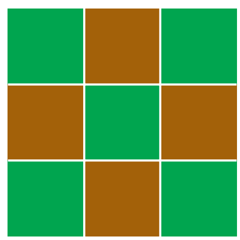
b What is the probability that blue appears on both spins?

c What is the probability that green appears on both spins?

d What is the probability that different colours appear on both spins?

e What is the probability that blue appears on *either* spin?

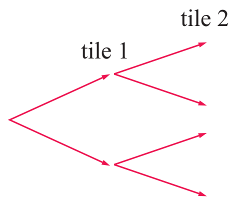
2 In a particular board game there are nine tiles: five are green and the remainder are brown. The tiles start face down on the table so they all look the same.



a If a player is required to pick a tile at random, determine the probability that it is:

i green **ii** brown.

b Suppose a player has to pick two tiles in a row, replacing the first and shuffling them before the second is selected. Copy and complete the tree diagram illustrating the possible outcomes.



c Using **b**, determine the probability that:

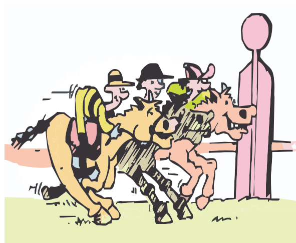
i both tiles are green

ii both tiles are brown

iii tile 1 is brown and tile 2 is green

iv one tile is brown and the other is green.

3 The probability of the race track being muddy next week is estimated to be $\frac{1}{4}$. If it is muddy, Rising Tide will start favourite with probability $\frac{2}{5}$ of winning. If it is dry he has a $\frac{1}{20}$ chance of winning.



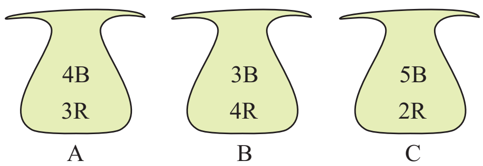
a Display the sample space of possible results on a tree diagram.

b Determine the probability that Rising Tide will win next week.

4 Machine A cans 60% of the fruit at a factory. Machine B cans the rest. Machine A spoils 3% of its product, while Machine B spoils 4%. Determine the probability that the next can inspected at this factory will be spoiled.

5 Box A contains 2 blue and 3 red blocks and Box B contains 5 blue and 1 red block. A box is chosen at random (by the flip of a coin) and one block is taken at random from it. Determine the probability that the block is red.

6



Three bags contain different numbers of blue and red tickets. A bag is selected using a die which has three A faces, two B faces, and one C face.
One ticket is selected randomly from the chosen bag. Determine the probability that it is: **a** blue **b** red.

I SAMPLING WITH AND WITHOUT REPLACEMENT [10.5]

Sampling is the process of selecting one object from a large group and inspecting it for some particular feature. The object is then either **put back** (sampling **with replacement**) or **put to one side** (sampling **without replacement**).

Sometimes the inspection process makes it impossible to return the object to the large group.
Such processes include:

- Is the chocolate hard- or soft-centred? Bite it or squeeze it to see.
- Does the egg contain one or two yolks? Break it open and see.
- Is the object correctly made? Pull it apart to see.

The sampling process is used for quality control in industrial processes.

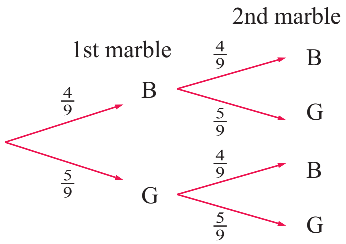
Example 15 — Self Tutor

A bin contains 4 blue and 5 green marbles. A marble is selected from this bin and its colour is noted. It is then *replaced*. A second marble is then drawn and its colour is noted. Determine the probability that:

a both are blue **b** the first is blue and the second is green

c there is one of each colour.

Tree diagram:
B = blue
G = green



outcome	probability
B and B	$\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$
B and G	$\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$
G and B	$\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$
G and G	$\frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$
	total 1

a P(both blue)
$= \frac{4}{9} \times \frac{4}{9}$
$= \frac{16}{81}$

b P(first is B and second is G)
$= \frac{4}{9} \times \frac{5}{9}$
$= \frac{20}{81}$

c P(one of each colour)
= P(B then G or G then B)
= P(B then G) + P(G then B)
$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9}$
$= \frac{40}{81}$

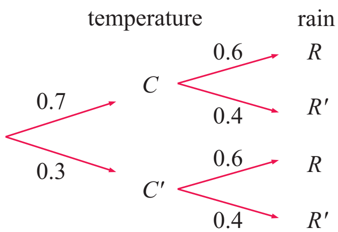
Example 16 — Self Tutor

Sylke has bad luck with the weather when she takes her summer holidays.
She estimates that it rains 60% of the time and it is cold 70% of the time.

a Draw a tree diagram to illustrate this situation.

b Use the tree diagram to determine the chance that for Sylke's holidays:
  i it is cold and raining **ii** it is fine and cold.

a C = the weather is cold R = it is raining



outcome	probability
C and R	$0.7 \times 0.6 = 0.42$
C and R'	$0.7 \times 0.4 = 0.28$
C' and R	$0.3 \times 0.6 = 0.18$
C' and R'	$0.3 \times 0.4 = 0.12$
	total 1.00

b **i** P(it is cold and raining)
= P(C and R)
$= 0.7 \times 0.6$
$= 0.42$

ii P(it is fine and cold)
= P(R' and C)
$= 0.4 \times 0.7$
$= 0.28$

EXERCISE 25I

1 A box contains 6 red and 3 yellow tickets. Two tickets are drawn at random (the first being *replaced* before the second is drawn). Draw a tree diagram to represent the sample space and use it to determine the probability that:

a both are red
b both are yellow
c the first is red and the second is yellow
d one is red and the other is yellow.

2 7 tickets numbered 1, 2, 3, 4, 5, 6 and 7 are placed in a hat. Two of the tickets are taken from the hat at random *without replacement*. Determine the probability that:

a both are odd
b both are even
c the first is even and the second is odd
d one is even and the other is odd.

3 Jessica has a bag of 9 acid drops which are all identical in shape. 5 are raspberry flavoured and 4 are orange flavoured. She selects one acid drop at random, eats it, and then takes another, also at random. Determine the probability that:

a both acid drops were orange flavoured
b both acid drops were raspberry flavoured
c the first was raspberry and the second was orange
d the first was orange and the second was raspberry.

Add your answers to **a**, **b**, **c** and **d**. Explain why this sum is 1.

4 A cook selects an egg at random from a carton containing 7 ordinary eggs and 5 double-yolk eggs. She cracks the egg into a bowl and sees whether it has two yolks or not. She then selects another egg at random from the carton and checks it.
Let S represent "a single yolk egg" and D represent "a double yolk egg".

a Draw a tree diagram to illustrate this sampling process.
b What is the probability that both eggs had two yolks?
c What is the probability that both eggs had only one yolk?

5 

Freda selects a chocolate at random from a box containing 8 hard-centred and 11 soft-centred chocolates. She bites it to see whether it is hard-centred or not. She then selects another chocolate at random from the box and checks it.
Let H represent "a hard-centred chocolate" and S represent "a soft-centred chocolate".

a Draw a tree diagram to illustrate this sampling process.
b What is the probability that both chocolates have hard centres?
c What is the probability that both chocolates have soft centres?

6 A sporting club runs a raffle in which 200 tickets are sold. There are two winning tickets which are drawn at random, in succession, without replacement. If Adam bought 8 tickets in the raffle, determine the probability that he:

a wins first prize
b does not win first prize
c wins both prizes
d wins neither prize
e wins second prize *given that* he did not win first prize.

J MUTUALLY EXCLUSIVE AND NON-MUTUALLY EXCLUSIVE EVENTS [10.5]

Suppose we select a card at random from a normal pack of 52 playing cards. Consider carefully these events:

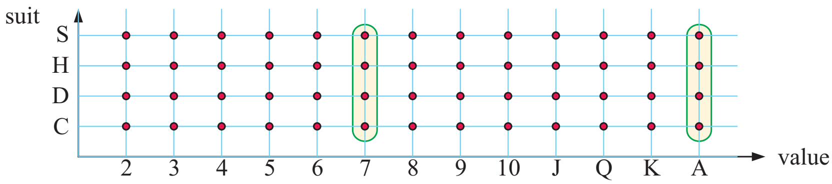
Event X: the card is a heart *Event Y:* the card is an ace *Event Z:* the card is a 7

Notice that:

- X and Y have a common outcome: the Ace of hearts
- X and Z have a common outcome: the 7 of hearts
- Y and Z do not have a common outcome.

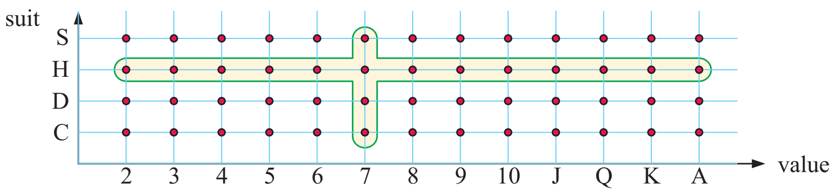
When considering a situation like this:

- if two events have no common outcomes we say they are **mutually exclusive** or **disjoint**
- if two events have common outcomes they are **not mutually exclusive**.



Notice that: P(ace **or** seven) $= \frac{8}{52}$ and P(ace) + P(seven) $= \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

If two events A and B are **mutually exclusive** then P(A **or** B) = P(A) + P(B)



Notice that: P(heart **or** seven) $= \frac{16}{52}$ and P(heart) + P(seven) $= \frac{13}{52} + \frac{4}{52} = \frac{17}{52}$.

Actually, P(heart **or** seven) = P(heart) + P(seven) − P(heart **and** seven).

If two events A and B are **not mutually exclusive** then P(A **or** B) = P(A) + P(B) − P(A **and** B).

EXERCISE 25J

1 An ordinary die with faces 1, 2, 3, 4, 5 and 6 is rolled once. Consider these events:

A: getting a 1 B: getting a 3 C: getting an odd number
D: getting an even number E: getting a prime number F: getting a result greater than 3.

a List all possible pairs of events which are mutually exclusive.

b Find: **i** P(B or D) **ii** P(D or E) **iii** P(A or E)
iv P(B or E) **v** P(C or D) **vi** P(A or B or F).

2 A committee consists of 4 accountants, 2 managers, 5 lawyers, and 6 engineers. A chairperson is randomly selected. Find the probability that the chairperson is:

a a lawyer **b** a manager or an engineer **c** an accountant or a manager

3 A jar contains 3 red balls, 2 green balls, and 1 yellow ball. Two balls are selected at random from the jar without replacement. Find the probability that the balls are either both red or both green.

4 A coin and an ordinary die are tossed simultaneously.

a Draw a grid showing the 12 possible outcomes.

b Find the probability of getting: **i** a head and a 5 **ii** a head or a 5.

c Check that: P(H or 5) = P(H) + P(5) − P(H **and** 5).

5 Two ordinary dice are rolled.

a Draw a grid showing the 36 possible outcomes.

b Find the probability of getting: **i** a 3 and a 4 **ii** a 3 or a 4.

c Check that: P(3 or 4) = P(3) + P(4) − P(3 **and** 4).

K MISCELLANEOUS PROBABILITY QUESTIONS [10.4 - 10.6]

In this section you will encounter a variety of probability questions. You will need to select the appropriate technique for each problem, and are encouraged to use tools such as tree and Venn diagrams.

EXERCISE 25K

1 50 students went on a 'thrill seekers' holiday. 40 went white-water rafting, 21 went paragliding, and each student did at least one of these activities.

a From a Venn diagram, find how many students did both activities.

b If a student from this group is randomly selected, find the probability that he or she:

i went white-water rafting but not paragliding

ii went paragliding given that he or she went white-water rafting.

2 A bag contains 7 red and 3 blue balls. Two balls are randomly selected without replacement. Find the probability that:

a the first is red and the second is blue **b** the balls are different in colour.

3 In a class of 25 students, 19 have fair hair, 15 have blue eyes, and 22 have fair hair, blue eyes or both. A child is selected at random. Determine the probability that the child has:

a fair hair and blue eyes **b** neither fair hair nor blue eyes

c fair hair but not blue eyes **d** blue eyes given that the child has fair hair.

4 Abdul cycles to school and must pass through a set of traffic lights. The probability that the lights are red is $\frac{1}{4}$. When they are red, the probability that Abdul is late for school is $\frac{1}{10}$. When they are not red the probability is $\frac{1}{50}$.

a Calculate the probability that Abdul is late for school.

b There are 200 days in the school year. How many days in the school year would you expect Abdul to be late?

5 

28 students go tramping. 23 get sunburn, 8 get blisters, and 5 get both sunburn and blisters. Determine the probability that a randomly selected student:

a did not get blisters
b either got blisters or sunburn
c neither got blisters nor sunburn
d got blisters, given that the student was sunburnt
e was sunburnt, given that the student did not get blisters.

6 An examination in French has two parts: aural and written. When 30 students sit for the examination, 25 pass aural, 26 pass written, and 3 fail both parts. Determine the probability that a student who:

a passed aural also passed written **b** passed aural, failed written.

7 Three coins are tossed. Find the probability that:

a all of them are tails **b** two are heads and the other is a tail.

8 Marius has 2 bags of peaches. Bag A has 4 ripe and 2 unripe peaches, and bag B has 5 ripe and 1 unripe peaches. Ingrid selects a bag by tossing a coin, and takes a peach from that bag.

a Determine the probability that the peach is ripe.
b Given that the peach is ripe, what is the probability it came from B?

9 Two coins are tossed and a die is rolled.

a Illustrate the sample space on a grid.
b Find the probability of getting:
  i two heads and a '6' **ii** two tails and an odd number
  iii a head and a tail, or a '6'.

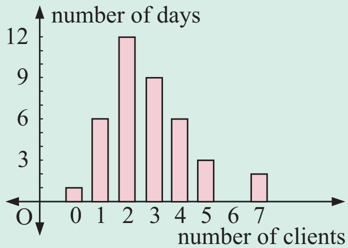
10 In a country town there are 3 supermarkets: P, Q and R. 60% of the population shop at P, 36% shop at Q, 34% shop at R, 18% shop at P and Q, 15% shop at P and R, 4% shop at Q and R, and 2% shop at all 3 supermarkets. A person is selected at random.
Determine the probability that the person shops at:

a none of the supermarkets
b at least one of the supermarkets
c exactly one of the supermarkets
d either P or Q
e P, given that the person shops at at least one supermarket
f R, given that the person shops at either P or Q or both.

11 On a given day, Claude's car has an 80% chance of starting first time and André's car has a 70% chance of the same. Given that at least one of the cars has started first time, what is the chance that André's car started first time?

Review set 25A

1 Donna kept records of the number of clients she interviewed over a period of consecutive days.



- **a** For how many days did the survey last?
- **b** Estimate Donna's chances of interviewing:
  - **i** no clients on a day
  - **ii** four or more clients on a day
  - **iii** less than three clients on a day.

2 Illustrate on a 2-dimensional grid the possible outcomes when a coin and a pentagonal spinner with sides labelled A, B, C, D and E are tossed and spun simultaneously.

3 University students were surveyed to find who owns a motor vehicle (MV) and who owns a computer. The results are shown in the two-way table.

	MV	no MV
computer	124	168
no computer	16	22

Estimate the probability that a randomly selected university student has:

- **a** a computer
- **b** a motor vehicle
- **c** a computer and a motor vehicle
- **d** a motor vehicle given that the student does not have a computer.

4 What is meant by saying that two events are "independent"?

5 Use a tree diagram to illustrate the sample space for the possible four-child families. Hence determine the probability that a randomly chosen four-child family:

- **a** is all boys
- **b** has exactly two boys
- **c** has more girls than boys.

6 In a shooting competition, Louise has 80% chance of hitting her target and Kayo has 90% chance of hitting her target. If they both have a single shot, determine the probability that:

- **a** both hit their targets
- **b** neither hits her target
- **c** at least one hits her target
- **d** only Kayo hits her target.

7 Two fair six-sided dice are rolled simultaneously. Determine the probability that the result is a 'double', i.e., both dice show the same number.

8 A bag contains 4 green and 3 red marbles. Two marbles are randomly selected from the bag without replacement. Determine the probability that:

- **a** both are green
- **b** they are different in colour.

9 A circle is divided into 5 sectors with equal angles at the centre. It is made into a spinner, and the sectors are numbered 1, 2, 3, 4, and 5. A coin is tossed and the spinner is spun.

- **a** Use a 2-dimensional grid to show the sample space.
- **b** What is the chance of getting: **i** a head and a 5 **ii** a head or a 5?

10 Bag X contains three white and two red marbles. Bag Y contains one white and three red marbles. A bag is randomly chosen and two marbles are drawn from it. Illustrate the given information on a tree diagram and hence determine the probability of drawing two marbles of the same colour.

11 At a local girls school, 65% of the students play netball, 60% play tennis, and 20% play neither sport. Display this information on a Venn diagram, and hence determine the likelihood that a randomly chosen student plays:

- **a** netball
- **b** netball but not tennis
- **c** at least one of these two sports
- **d** exactly one of these two sports
- **e** tennis, given that she plays netball.

Review set 25B

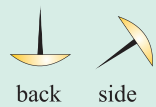
1 Pierre conducted a survey to determine the ages of people walking through a shopping mall. The results are shown in the table alongside. Estimate, to 3 decimal places, the probability that the next person Pierre meets in the shopping mall is:

Age	*Frequency*
0 - 19	22
20 - 39	43
40 - 59	39
60+	14

a between 20 and 39 years of age
b less than 40 years of age **c** at least 20 years of age.

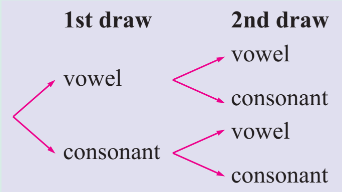
2 **a** List the sample space of possible results when a tetrahedral die with four faces labelled A, B, C and D is rolled and a 20-cent coin is tossed simultaneously.
b Use a tree diagram to illustrate the sample spaces for the following:
i Bags A, B and C contain green or yellow tickets. A bag is selected and then a ticket taken from it.
ii Martina and Justine play tennis. The first to win three sets wins the match.

3 When a box of drawing pins was dropped onto the floor, it was observed that 49 pins landed on their backs and 32 landed on their sides. Estimate, to 2 decimal places, the probability of a drawing pin landing:



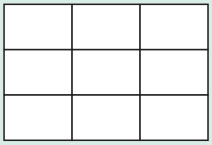
a on its back **b** on its side.

4 The letters A, B, C, D, ..., N are put in a hat.
a Determine the probability of drawing a vowel (A, E, I, O or U) if one of the letters is chosen at random.
b If two letters are drawn without replacement, copy and complete the following tree diagram including all probabilities:



c Use your tree diagram to determine the probability of drawing:
i a vowel and a consonant
ii at least one vowel.

5 A farmer fences his rectangular property into 9 rectangular paddocks as shown alongside.
If a paddock is selected at random, what is the probability that it has:



a no fences on the boundary of the property
b one fence on the boundary of the property
c two fences on the boundary of the property?

6 Bag X contains 3 black and 2 red marbles. Bag Y contains 4 black and 1 red marble. A bag is selected at random and then two marbles are selected without replacement. Determine the probability that:
a both marbles are red **b** two black marbles are picked from Bag Y.

7 Two dice are rolled simultaneously. Illustrate this information on a 2-dimensional grid. Determine the probability of getting:
a a double 5 **b** at least one 4
c a sum greater than 9 **d** a sum of 7 or 11.

8 A class consists of 25 students. 15 have blue eyes, 9 have fair hair, and 3 have both blue eyes and fair hair. Represent this information on a Venn diagram.
Hence find the probability that a randomly selected student from the class:

a has neither blue eyes nor fair hair

b has blue eyes, but not fair hair

c has fair hair given that he or she has blue eyes

d does not have fair hair given that he or she does not have blue eyes.

9 The two-way table alongside shows the results from asking the question "Do you like the school uniform?".

	Likes	*Dislikes*
Year 8	129	21
Year 9	108	42
Year 10	81	69

If a student is randomly selected from these year groups, estimate the probability that the student:

a likes the school uniform

b dislikes the school uniform

c is in year 8 and dislikes the uniform

d is in year 9 given the student likes the uniform

e likes the uniform given the student is in year 10.

10 The probability of a delayed flight on a foggy day is $\frac{9}{10}$. When it is not foggy the probability of a delayed flight is $\frac{1}{12}$. If the probability of a foggy day is $\frac{1}{20}$, find the probability of:

a a foggy day and a delayed flight

b a delayed flight

c a flight which is not delayed.

d Comment on your answers to **b** and **c**.