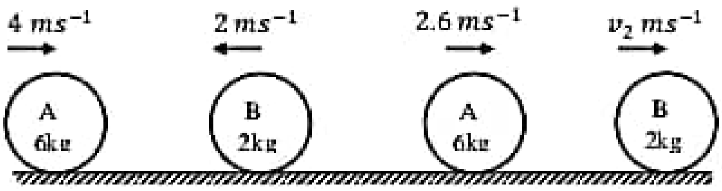
PROPOSED
MARKING GUIDE
APPLIED MATHEMATICS
P425/2 2023

NO	SOLUTION	MKS	COMMENT
1	Let X be the number of tails obtained $P(H) = 2P(T)$ $P(H) + P(T) = 1$ $2P(T) + P(T) = 1$ $3P(T) = 1$ $P(T) = \frac{1}{3}, P(H) = \frac{2}{3}$ $\Rightarrow n = 7, p = \frac{1}{3}, q = \frac{2}{3}$ $P(X = 2) = {}^7C_2 \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^5$ $= \frac{224}{729}$ or 0.3073		
		05	
2	 a) From $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $6 \times 4 - (2 \times 2) = 6 \times 2.6 + 2 \times v_2$ $24 - 4 = 15.6 + 2v_2$ $2v_2 = 4.4$ $v_2 = 2.2\ ms^{-1}$ b) Loss in kinetic energy $= K.E_{After} - K.E_{Before}$ $K.E_{Before} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ $= \frac{1}{2} \times 6 \times 4^2 - \frac{1}{2} \times 2 \times 2^2$ $= 44\ J$ $K.E_{After} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ $= \frac{1}{2} \times 6 \times 2.6^2 + \frac{1}{2} \times 2 \times 2.2^2$		

$= 15.44\ \text{J}$

$\therefore$ Loss in K. E $= 44 - 15.44 = 28.56\ \text{J}$

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3

$h = \frac{1}{2} - 0 = 0.5$

x	$f(x)$	
0	0.1003	
0.5		0.0391
1		0.0801
1.5		0.0602
2		0.0649
2.5		0.0380
3	0.0327	
Total	**0.1330**	**0.2823**

$$\int_0^3 f(x)\,dx \approx \frac{1}{2} \times \frac{1}{2}[0.1330 + 2(0.2823)]$$

$$\approx 0.1744$$

$$\approx 0.174\ \text{(3dps)}$$

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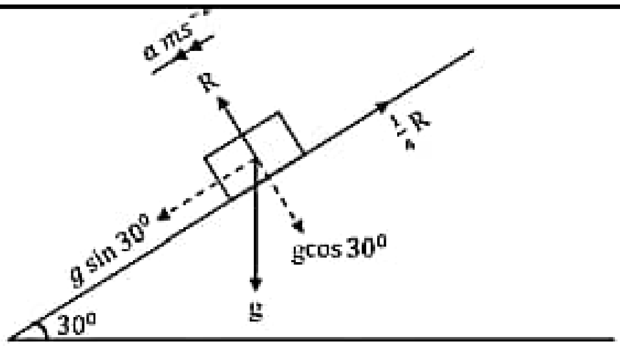

Perpendicular to the plane;

$R = g\cos 30^0$ ....(i)

Parallel to the plane;

$g\sin 30^0 - \frac{1}{4}R = a$ ....(ii)

Putting (i) into (ii);

$a = g\sin 30^0 - \frac{1}{4}(g\cos 30^0)$

$a = 9.8 \times \frac{1}{2} - \frac{1}{4} \times 9.8 \times \frac{\sqrt{3}}{2}$

$a = 2.7782\ ms^{-2}$

From $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2 \times 2.7782 \times 4$$

$$v^2 = 22.2256$$

$$\therefore v = 4.7144\ ms^{-1}$$

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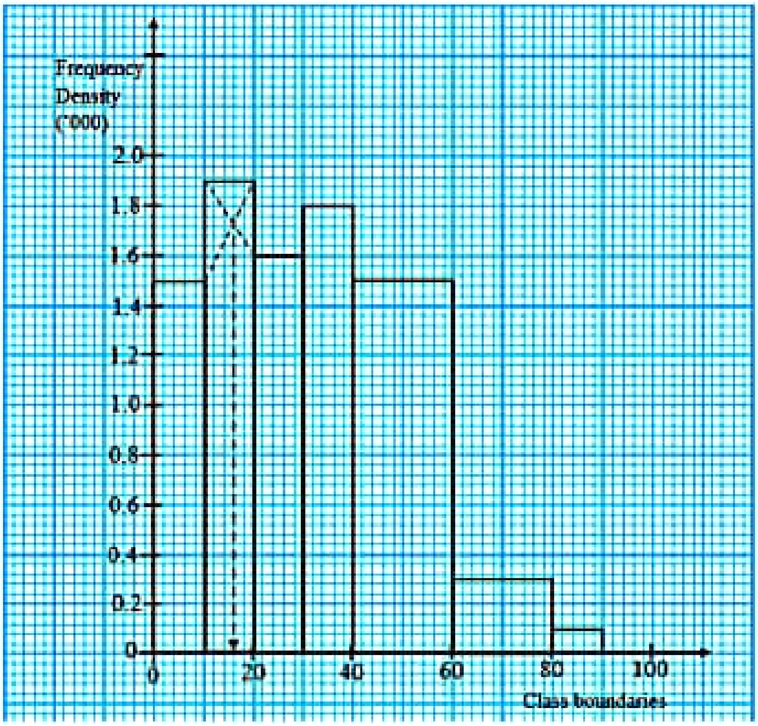
5

Age (years)	f ('000)	c	f.d
0 – 10	15	10	1.5
10 – 20	19	10	1.9
20 – 30	16	10	1.6
30 – 40	18	10	1.8
40 – 60	30	20	1.5
60 – 80	6	20	0.3
80 – 90	1	10	0.1

a)



b) Modal age = 16 years

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6

$x = 6.45, y = 0.00215, z = 2.7$

$e_x = 0.005, e_y = 0.000005, e_z = 0.05$

$$w_{min} = \frac{(x+z^3)_{min}}{(\sqrt{y})_{max}}$$

$$= \frac{6.445+(2.65)^3}{\sqrt{0.002155}}$$

$= 539.7147$

$$w_{max} = \frac{(x+z^3)_{max}}{(\sqrt{y})_{min}}$$

$$= \frac{6.455+(2.75)^3}{\sqrt{0.002145}}$$

$= 588.4137$

$\therefore$ Internal $= 539.7147 \leq w \leq 588.4137$

05

7 a) From $\mathrm{P}(S' \cap R') = \mathrm{P}(S') \cdot \mathrm{P}(R')$

$$\mathrm{P}(S) = [1 - P(S)] \cdot \mathrm{P}(R')$$

$$\mathrm{P}(S) = \frac{1}{4}(1 - P(S))$$

$$\mathrm{P}(S) = \frac{1}{4} - \frac{1}{4}\mathrm{P}(S)$$

$$\frac{5}{4}\mathrm{P}(S) = \frac{1}{4}$$

$\therefore \mathrm{P}(S) = \frac{1}{5}$ or 0.2

b) $\mathrm{P}(S' \cap R) = \mathrm{P}(S') \cdot \mathrm{P}(R)$

$$= \frac{4}{5} \times \frac{3}{4}$$

$= \frac{3}{5}$ or 0.6

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8 Let K = weight per unit are, W = KA

Area of square $= 60 \times 60 = 3600\ cm^2$

Area of circle $= \pi r^2 = \pi \times 20^2 = 400\pi\ cm^2$

Remainder $= 3600 - 400\pi$

Figure	Area	Weight	Distance of C.O.G from AD
Square	3600	3600K	30
Circle	400π	400πK	40
Remainder	$3600-400\pi$	$(3600-400\pi)$K	$\bar{x}$

Taking moments about AD

$$(3600 - 400\pi)K \times \bar{x} = 3600K \times 30 - 400\pi K \times 40$$

$$(3600 - 400\pi)\bar{x} = 108{,}000 - 16{,}000\pi$$

$$\bar{x} = \frac{108{,}000 - 16{,}000\pi}{3600 - 400\pi}$$

$\bar{x} = 24.6375\ cm$ from AD

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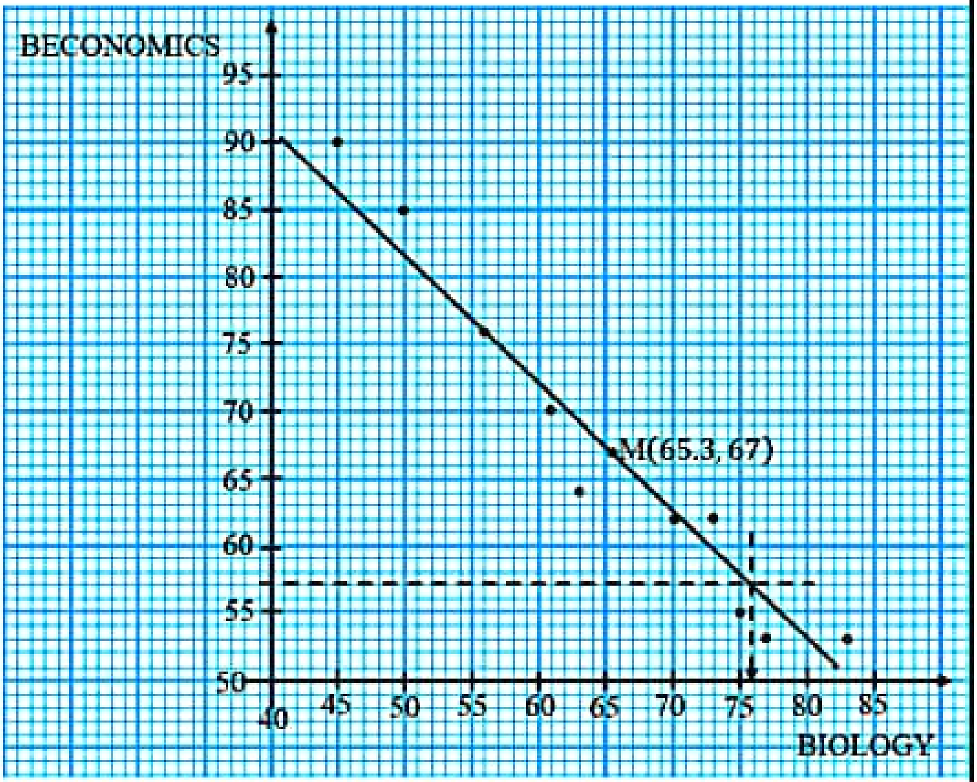
9 a) $\mathrm{M}(\bar{x}, \bar{y})$

$\bar{x} = \frac{\sum x}{n} = \frac{653}{10} = 65.3$

$\bar{y} = \frac{\sum y}{n} = \frac{670}{10} = 67$

$\therefore M(\bar{x}, \bar{y}) = M(65.3, 67)$



b) Let B = BIOLOGY

E = ECONOMICS

B	E	R_B	R_E	d	d^2
45	90	10	1	9	81
63	64	6	5	1	1
56	76	8	3	5	25
61	70	7	4	3	9
75	55	3	8	-5	25
83	53	1	9.5	-8.5	72.225
73	62	4	6.5	-2.5	6.25
50	85	9	2	7	49
77	53	2	9.5	-7.5	56.25
70	62	5	6.5	-1.5	2.225
653	670				$\sum d^2 = 327$

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$

$= 1 - \frac{6(327)}{10(10^2-1)}$

$= -0.9818$

Comment: *It is significant at 5%*

Or It is significant at 1%

Or It is very high negative correlation

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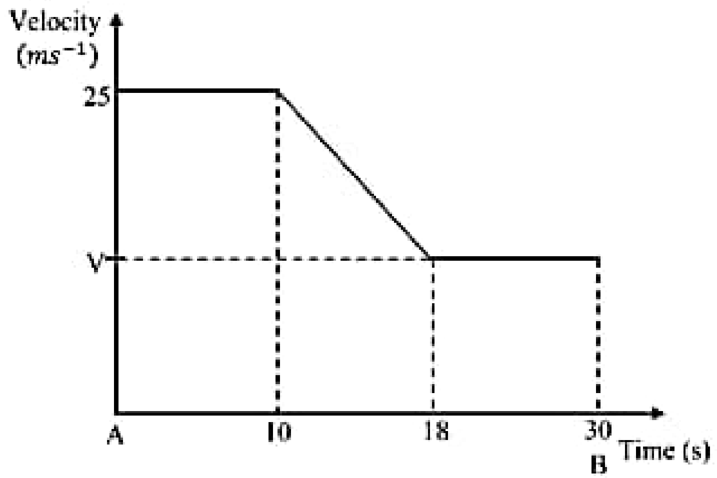
10 a)



b)

(i) $526 = (25 \times 10) + \frac{1}{2} \times 8(25 + V) + (V \times 12)$

$526 = 250 + 100 + 4V + 12V$

$526 = 350 + 16V$

$16V = 176$

$V = 11\ ms^{-1}$

(ii) From $v = u + at$

$11 = 25 + 8a$

$8a = -14$

$a = -1.75$

$\therefore$ the deceleration is $1.75\ ms^{-2}$

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11 a)

(i) $f(1) = 1e^1 + 5(1) - 10 = -2.2817$

$f(2) = 2e^2 - 5(2) - 10 = 14.7781$

(ii) Since $f(1) \cdot f(2) < 0$, thus $1 < \text{root} < 2$

b)

1	x_0	2
-2.2817	0	14.7781

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$\frac{x_0-1}{0+2.2817}=\frac{2-1}{14.7781+2.2817}$
$x_0=1.1337$
$f(1.13337)=1.13337e^{1.1337}+5(1.1337)-10$
$=-0.8089$
<table>
<tr><td>1.1337</td><td>x_1</td><td>2</td></tr>
<tr><td>-0.8089</td><td>0</td><td>14.7781</td></tr>
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$\frac{x_1-1.1337}{0+0.8089}=\frac{2-1.13337}{14.7781+0.8089}$
$x_1=1.1787$
$\therefore$ Root ≈ 1.179 (3dps)
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a) Testing
For $-1\le x\le 0$;
$=F(0)-F(-1)$
$=\frac{1}{6}-0$
$=\frac{1}{6}<\frac{1}{2}$

For $0\le x\le 2$;
$=F(2)-F(0)$
$=\frac{5}{6}-\frac{1}{6}$
$=\frac{2}{3}>\frac{1}{2}$
Let m =median
$F(m)=\frac{1}{2}$
$\frac{1+2m}{6}=\frac{1}{2}$
$1+2m=3$
$2m=2$
$\therefore m=1$

b) For $-1\le x\le 0$; $f(x)=\frac{d}{dx}\left(\frac{1+x}{6}\right)=\frac{1}{6}$
For $0\le x\le 2$; $f(x)=\frac{d}{dx}\left(\frac{1+2x}{6}\right)=\frac{1}{3}$
For $2\le x\le\frac{8}{3}$; $f(x)=\frac{d}{dx}\left(\frac{4+3x}{12}\right)=\frac{1}{4}$
For $x\ge\frac{8}{3}$; $f(x)=\frac{d}{dx}(1)=0$
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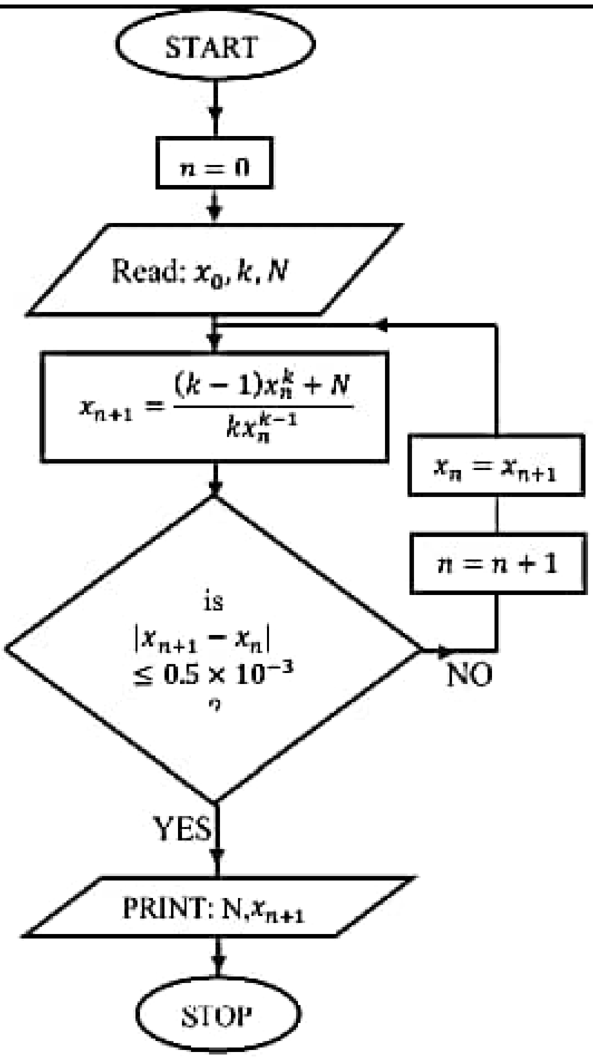
	$\therefore f(x)=\begin{cases}\frac{1}{6}, -1\le x\le 0\\ \frac{1}{3}, 0\le x\le 2\\ \frac{1}{4}, 2\le x\le \frac{8}{3}\\ 0, elsewhere\end{cases}$ c) $P(1\le X\le 2.5)=\int_1^2 \frac{1}{3}dx+\int_2^{2.5}\frac{1}{4}dx$ $=\left[\frac{1}{3}x\right]_1^2+\left[\frac{1}{4}x\right]_2^{2.5}$ $=\frac{1}{3}(2-1)+\frac{1}{4}(2.5-2)$ $=\frac{1}{3}+\frac{1}{8}$ $=\frac{11}{24}$ or 0.4583 d) $E(X)=\int_{-1}^0\frac{1}{6}x\,dx+\int_0^2\frac{1}{3}x\,dx+\int_2^{\frac{8}{3}}\frac{1}{4}x\,dx$ $=\frac{1}{6}\left[\frac{x^2}{2}\right]_{-1}^0+\frac{1}{3}\left[\frac{x^2}{2}\right]_0^2+\frac{1}{4}\left[\frac{x^2}{2}\right]_2^{\frac{8}{3}}$ $=\frac{1}{12}(0-1)+\frac{1}{6}(4-0)+\frac{1}{8}\left(\frac{64}{9}-4\right)$ $=-\frac{1}{12}+\frac{2}{3}+\frac{7}{18}$ $=\frac{35}{36}$ or 0.97222		
		12	
13	a) Let F_R be the resultant force $F_R=F_1+F_2+F_3$ $=\begin{pmatrix}2\\-3\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}-2\\-11\end{pmatrix}$ $=\begin{pmatrix}5\\-12\end{pmatrix}$ N $\therefore\ \lvert F_R\rvert=\sqrt{5^2+(-12)^2}=13$ N b) About the origin; $G=\begin{vmatrix}2&3\\2&-3\end{vmatrix}+\begin{vmatrix}-2&3\\5&2\end{vmatrix}+\begin{vmatrix}3&-2\\-2&-11\end{vmatrix}$ $=(-6-6)+(-4-15)+(-33-4)$ $=-12-19-37$ $=-68\ Nm$ Using $G-xF_y+yF_x=0$ $-68+12x+5y=0$ $\therefore 12x+5y-68=0$ c) It cuts the x-axis when $y=0$;		

	$12x - 68 = 0$ $12x = 68$ $x = 5.6667\ m$ from the origin. d) Let $(a\boldsymbol{i} + b\boldsymbol{j})$ be the force to be added to form a couple For a couple the $\boldsymbol{F}_R = \begin{pmatrix}0\\0\end{pmatrix}$ $\Rightarrow \begin{pmatrix}5\\-12\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ $5 + a = 0 \qquad \therefore a = -5$ $-12 + b = 0 \qquad \therefore b = 12$ $\therefore (-5\boldsymbol{i} + 12\boldsymbol{j})$ N		
		12	
14	a) Let $x = N^{\frac{1}{k}}$ $x^k = N$ $x^k - N = 0$ Let $f(x) = x^k - N$ $f'(x) = kx^{k-1}$ $x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}}$ $x_{n+1} = \frac{kx_n^k - x_n^k + N}{kx_n^{k-1}}$ $= \frac{(k-1)x_n^k + N}{kx_n^{k-1}}$ b)		



c) Dry run

$N = 13, k = 4, x_0 = 1.6$

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.6	1.9935	0.3935
1	1.9935	1.9054	0.0881
2	1.9054	1.8989	0.0065
3	1.8989	1.8989	0.0000

$\therefore$ Root $= 1.899, N = 13$

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15 a) $P(A) = 0.3, P(A') = 0.7$

$P(B) = 0.5,\ P(B') = 0.5$

$P(C) = 0.7,\ P(C') = 0.3$

(i) P(only one) $= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= (0.3 \times 0.5 \times 0.3) + (0.7 \times 0.5 \times 0.3) + (0.7 \times 0.5 \times 0.7)$

$= 0.045 + 0.105 + 0.245$

	$= 0.395$ (ii) P (two or more) = P (only two) + P(all) P(only two) $= P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A \cap B \cap C')$ $= (0.7 \times 0.5 \times 0.7) + (0.3 \times 0.5 \times 0.7) + (0.3 \times 0.5 \times 0.3)$ $= 0.245 + 0.105 + 0.045$ $= 0.395$ P (All) $= P(A \cap B \cap C)$ $= 0.3 \times 0.5 \times 0.7$ $= 0.105$ $\therefore$ P (two or more) $= 0.395 + 0.105 = 0.5$ **ALT:** P (two or more) $= 1-[P(none) + P(only\ one)]$ P (none) $= P(A' \cap B' \cap C')$ $= 0.7 \times 0.5 \times 0.3$ $= 0.105$ $\therefore$ P (two or more) $= 1-[0.105 + 0.395]$ $= 1 - 0.5$ $= 0.5$ b) $P(P) = 0.6, P(L/P) = \frac{2}{3}$ $P(Q) = 0.4, P(L/Q) = \frac{1}{3}$ (i) $P(L) = \left(0.6 \times \frac{2}{3}\right) + \left(0.4 \times \frac{1}{3}\right)$ $= \frac{8}{15}$ or 0.5333 (ii) $P(P/L') = \frac{P(P \cap L')}{P(L')}$ $= \frac{\left(0.6 \times \frac{1}{3}\right)}{1 - \frac{8}{15}}$ $= \frac{3}{7}$ or 0.4286		
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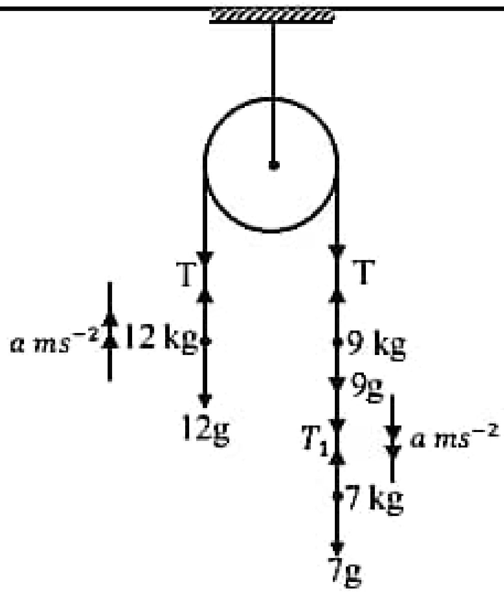
16	Let a be the acceleration of the system, T and T_1 be the tensions in the strings		



a)

(i) For 12 kg mass:

$T-12g = 12a$ ……………………..(i)

For 9 kg mass:

$T_1 + 9g - T = 9a$ ………………..(ii)

For 7 kg mass:

$7g - T_1 = 7a$ ……………………..(iii)

(i)+(ii)+(iii);

$4g = 28a$

$a = \frac{4g}{28} = \frac{g}{7} = \frac{9.8}{7} = 1.4\ ms^{-2}$

(ii) From (i);

$T = 12a + 12g$

$T = 12 \times 1.4 + 12 \times 9.8$

$T = 16.8 + 117.6$

$= 134.4$ N

From (iii);

$T_1 = 7g - 7a$

$= 7 \times 9.8 - 7 \times 1.4$

$= 68.6 - 9.8$

$= 58.8$ N

b) Let the velocity be v

	Using $v = u + at$ $v = 0 + 1.4 \times 1.5$ $v = 2.1\ ms^{-1}$		