P425/1
PURE MATHEMATICS
PAPER ONE
AUGUST 2024
TIME: 3 HOURS

MBALE SECONDARY SCHOOL

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS - 2024

PURE MATHEMATICS

P425/1

PAPER ONE

3 HOURS

INSTRUCTIONS TO CANDIDATES

- **Answer all the eight questions in section A and any five from section B.**
- **Any additional question(s) answered will not be marked.**
- **All necessary working must be clearly shown.**
- **Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.**

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Show that

$$\left\{\left(\frac{(1+\sqrt{2})^2-(1-\sqrt{2})^2}{4(1+\sqrt{2})}\right)^2\right\}^2 = 2(3-2\sqrt{2})$$

(05 marks)

2. Solve $4\,Cos^2\theta - 3Sin2\,\theta + 4 = 0$ for $0^o \leq \theta \leq 90^o$. (05 marks)

3. Given that $y = \ln\left\{e^x\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right\}$ show that $\frac{dy}{dx} = 1 + cosecx$

(05 marks)

4. Find the point where line $\frac{x+5}{2} = \frac{y+4}{2} = \frac{z+9}{4}$ intersect the line $\underset{\sim}{r} = \begin{pmatrix}-1\\1\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$ (05 marks)

5. Evaluate: $\int_0^1 \frac{1}{5+x(x+2)}dx$ (05 marks)

6. A point$P\ (x,y)$ moves, with $A(2,0)$ and $B\ (1,4)$ such that $|PA|^2 + |AB|^2 = |PB|^2$. Describe the locus of point P. (05 marks)

7. Given that μ and λ are roots of a quadratic equation such that $\mu + \lambda = 1$ and $\mu\lambda = -2$ Find the value of $\frac{\mu}{2\lambda-\mu} + \frac{\lambda}{2\mu-\lambda}$ (05 marks)

8. Solve the differential equation: $\sin 2x\frac{dy}{dx} + 2ycos^2x = 2secx$ given that $y\left(\frac{\pi}{4}\right) = 2\sqrt{2}$

(05 marks)

SECTION B (60 MARKS)

Answer any FIVE questions from this section. All questions carry equal marks.

9. a) Find the perpendicular distance of the point $(1,1,4)$ from the line $\frac{x-1}{2} = y = \frac{z+1}{3}$ (04 marks)

b) Show that the angle θ, between two lines whose Cartesian equations are $\frac{x-2}{m_1} = \frac{y-1}{n_1}$ and $\frac{x-4}{m_2} = \frac{y+3}{n_2}$ is given by

$\theta = tan^{-1}\left(\frac{m_1n_2-n_1m_2}{m_1m_2+n_1n_1}\right)$, hence find the value of θ when

$m_1 = 1, m_2 = 2, n_1 = -1$ and $n_2 = 1$ (08 marks)

10. a) Find the equation of a parabola whose focus is $(3,2)$ with equation of the directrix at $x = -3$ (03 marks)

b) Given that $y = mx + c$ is a tangent to the parabola $\frac{x}{4} + \frac{y^2}{9} = 1$, show that $-c = \frac{81+576m^2}{144m}$. Hence find the equation of the tangents to the parabola $\frac{x}{4} + \frac{y^2}{9} = 1$ passing through point $(6,0)$ (09 marks)

11. a) Show that $Sin3\theta = 3Sin\theta - 4Sin^3\theta$ (04 marks)

b) Express $2Cos\theta + Sin\theta$ in the form $RCos(\theta - \beta)$ hence solve $2Cos\theta + Sin\theta - 2 = 0$ for $0^0 \leq \theta \leq 180^0$ and find the maximum value of $\frac{1}{2Cos\theta + Sin\theta - 2}$ (08 marks)

12. a) A team of 15 players is to be selected from 25 players of the school from which the first eleven is chosen and the Captain from the eleven is also chosen. Find in how many ways this can happen. (05 marks)

b) Use binomial theorem to expand $\sqrt[3]{\frac{(1+x)^2}{1-x}}$ up to a term in x^3, hence use the substitution $x = 0.2$ to find $\sqrt[3]{15}$ correct to 3 decimal places. (07 marks)

13. a) $\int_0^{\frac{\pi}{6}} Sin4\theta\ Cos2\theta\ d\theta$ (04 marks

b) $\int xSecx\ dx$ (04 marks)

c) $\int \frac{Sin^{-1}(x)}{\sqrt{1-x^2}} dx$ (04 marks)

14. a) Determine the turning points of the curve $y = x^3 - 5x^2 + 8x - 4$ and classify them, hence sketch the curve. (05 marks)

b) The area of the region between the curve in a) above, the line $y = 0, x = 1$ and $x = 2$ is rotated through four right-angles about the x-axis. Find the volume of the solid generated (Leave π in your answer) (07 marks)

15. a) Find the coordinates in the form (x, y) such that:
$x^3 - y^3 = -4$ and $x - y = 2$ (06 marks)

b) Prove by mathematical induction that when $k = 1,2,3,4,\ldots\ldots n$ and $a_k = k^2 - 2k + 1$, then
$a_1 + a_2 + a_3 + a_4 + \cdots + a_n = \frac{n}{6}(n-1)(2n-1)$
(06 marks)

16. a) Given a differential equation $\frac{dm}{dn} = e^{(n+m)}$ when $m(0) = 1$, show that $m = -\ln(e - e^n)$ for which $n < 1$ (05 marks)

b) A radio-active isotope decays at a rate directly proportional to the amount of isotope present at anytime (t). Given that the half life of the isotope is half a century.

i) For how long will it take the isotope to decay by 40%. (04 marks)

ii) Find the amount of isotope after 20 years given the the original amount of isotope was 5g. (03 marks)

END