# TOPIC: INTEGERS

CONTENT: <u>Introduction and ordering integers</u>

- ➢ An integer is a positive (+) or negative (-) number and zero.
- ➢ Examples of integers are {..........$^-4,^-3,^-2,^-1,0,^+1,^+2,^+3,^+4$..........}
- ➢ Zero (0) is neither a negative (-) nor a positive integer.
- ➢ Integers can be represented on a number line.
- ➢ On a number line, positive (+) integers are put on the right hand side while negative (-) integers are put on the left as shown below
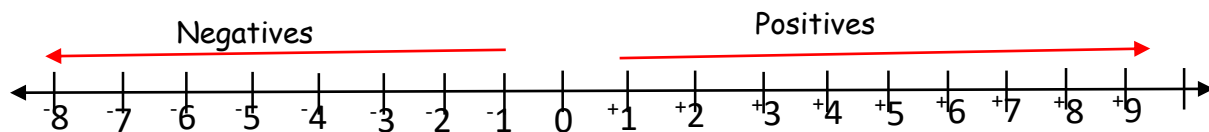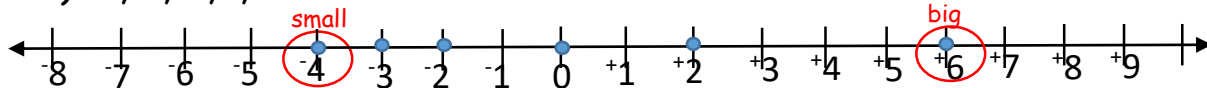

<u>Ordering integers</u>

Integers can be ordered in either ascending or descending order

Examples

1.Arrange the following integers in ascending order
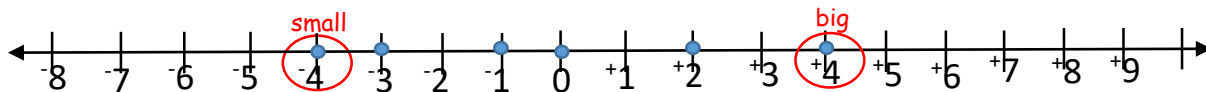
a) $^+6,^-4,^+2,0,^-2$ and $^-3$

Ascending order={$^-4,^-3,^-2,0,^+2,^+6$}

2.Arrange the following integers in descending order

a) $^+4,^-4,^+2,0,^-1$ and $^-3$

Descending order={$^+4,^+2,0,^-2,^-3,^-4$}

1. Arrange the following integers in ascending order

    a) $^{+}5, ^{-}3, ^{+}3, 0, ^{-}1$ and $^{-}4$ b) $^{-}5, ^{-}2, ^{+}4, 0, ^{+}1$ and $^{-}6$

    c) $^{+}6, ^{-}7, ^{+}5, 0, ^{-}4$ and $^{+}3$ d) $^{+}4, ^{-}4, ^{+}2, 0, ^{-}1$ and $^{-}3$

2. Arrange the following integers in descending order

    a) $^{-}5, ^{-}3, ^{+}3, 0, ^{-}1$ and $^{+}4$ b) $^{+}7, ^{-}3, ^{+}3, 0, ^{-}1$ and $^{-}4$

    c) $^{+}4, ^{-}2, ^{+}3, 0, ^{+}1$ and $^{-}1$ d) $^{+}6, ^{-}3, ^{+}3, 0, ^{-}1$ and $^{+}4$

Comparing integers using $<$ or $>$ or $=$

Examples

1. Compare the following integers using $<$ or $>$ or $=$

a) $^{-}100 < 0$ b) $^{+}67 > ^{-}67$ c) $^{-}10 > ^{-}50$

Activity

1. Compare the following integers using $<$ or $>$ or $=$

a) $^{-}10 … … …0$ b) $^{+}60 … … ….^{-}60$ c) $^{-}1 … … … ^{-}5$

d) $^{-}24 … … …^{+} 24$ e) $^{+}6 … … ….^{-}30$ c) $^{-}15 … … … ^{-}5$

Additive inverse

Points to note

➢ An additive inverse is an integer which when added to another integer gives zero.

➢ The inverse property states that "any number added to its inverse or opposite, gives zero".

➢ For example;   a) $^{+}4 + ^{-}4 = 0$ b) $^{-}9 + ^{+}9 = 0$

Finding additive integers

EXAMPLES

1. Find the additive inverse of $^+7$
   Let the inverse be k
   $K+ ^+7=0$
   $K+7-7=0-7$
   $K=^-7$

b) Find the additive inverse of $^-10$
   Let the inverse be r
   $r+ ^-10=0$
   $r-10+10=0+10$
   $r=^+10$

Activity

a) Find the additive inverse of $^+6$

b) Find the additive inverse of $^-4$

c) Find the additive inverse of $^+9$

d) Find the additive inverse of $^-5$

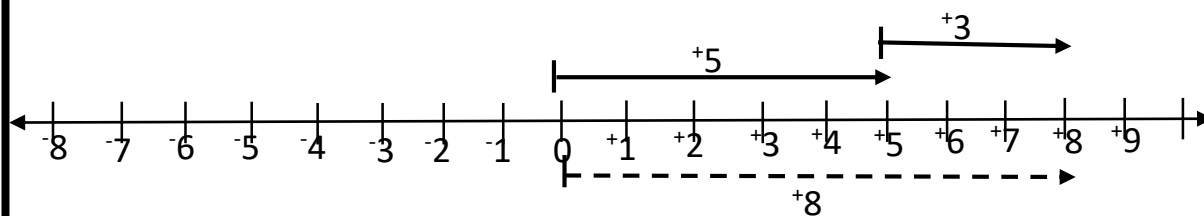e) Find the additive inverse of $^-12$

f) Find the additive inverse of $^+8$

g) Find the additive inverse of $^-1$

h) Find the additive inverse of $^+11$
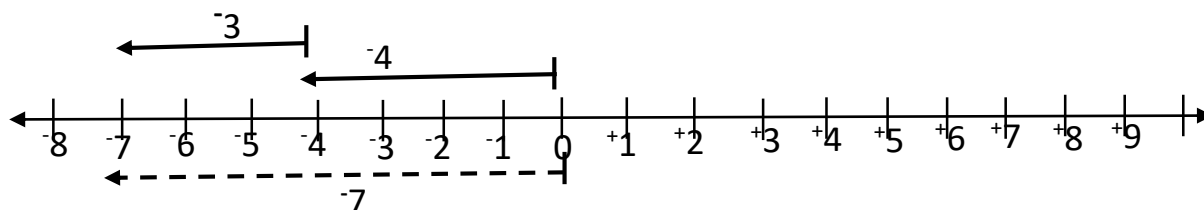
Addition of integers using a number line
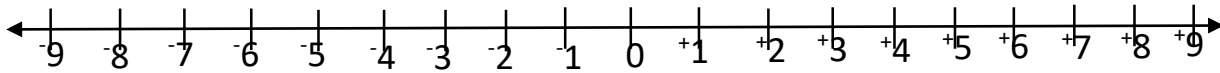
Examples

1. Add $^+5 + ^+3$ using a number line
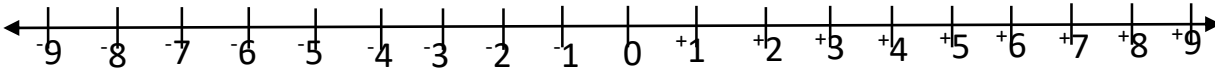

$^+5 + ^+3 =^+8$

2. Add $^-4 + ^-3$ using a number line

$^-4 + ^-3 =^-7$

Activity

1. Add ⁻5 + ⁻4 using a number line

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9

2. Add ⁺6 + ⁺3 using a number line

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9

3. Add ⁻6 + ⁻2 using a number line

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9
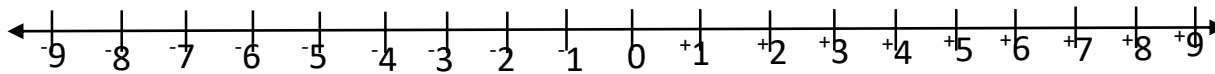
4. Add ⁻3 + ⁻5 using a number line

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9
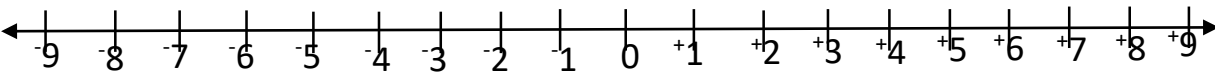
5. Add ⁺4 + ⁺2 using a number line

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9
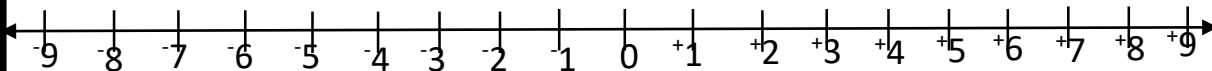
Addition of negative and positive integers using a number line

Examples
1. Add ⁺5 + ⁻3 using a number line

⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8 ⁺9
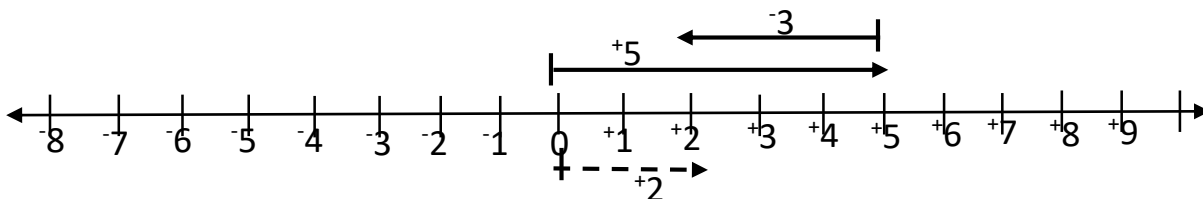
⁺5 + ⁻3=⁺2

2.Add ⁺6 + ⁻2 using a number line

⁺6 + ⁻2=⁺4
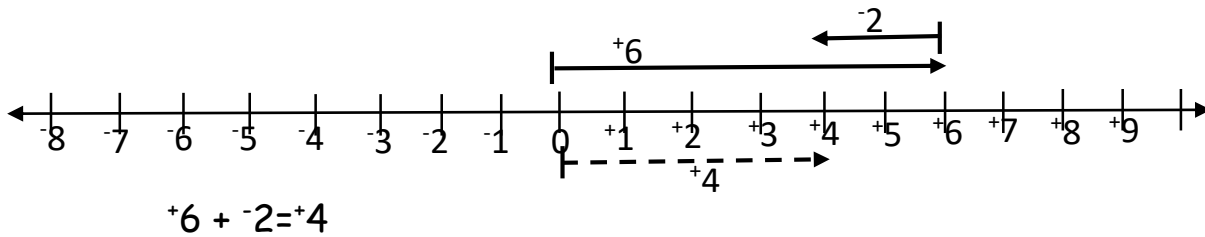
3.Add ⁻6 + ⁺9 using a number line

⁻6 + ⁺9=⁺3
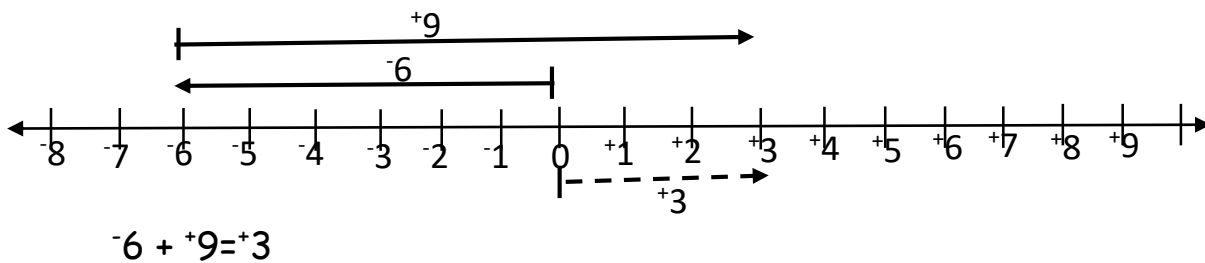
Activity

1.Add ⁺7 + ⁻3 using a number line
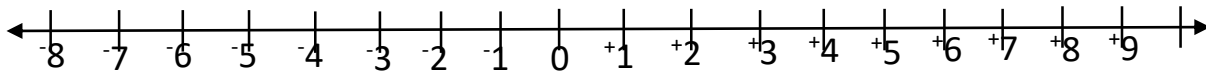
2.Add ⁺9 + ⁻5 using a number line

3.Add ⁻8 + ⁺3 using a number line
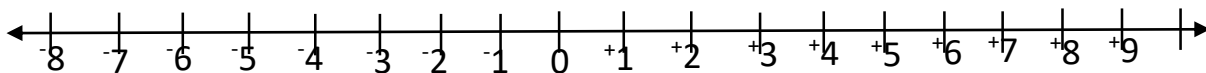
4.Add ⁻4 + ⁺9 using a number line

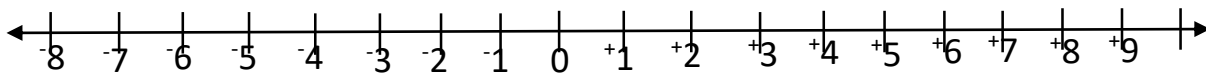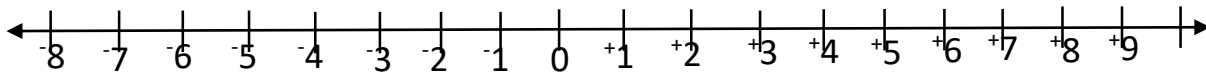# Addition of integers without using a number line

Points to note

- +(+)= +
- +(-)= -
- -(-)=+
- -(+)=-

Examples

1.Simplify: $^+6 + {}^-2$

$^+6 +(^-2)$

$^+6 - 2$

$^+4$

2.Simplify: $^-7 + {}^+2$

$^-7 + (^+2)$

$^-7 + 2$

$= {}^-5$

3.Simplify: $^-7 + {}^-2$

$^-7 + (^-2)$

$^-7 - 2$

$^-9$

Activity

1.Simplify: $^+9 + {}^-5$

2.Simplify: $^+7 + {}^-4$

3.Simplify: $^+5 + {}^-2$

4.Simplify: $^-6 + {}^-2$

5.Simplify: $^-9 + {}^-6$

6.Simplify: $^+11 + {}^-4$

7.Simplify: $^+8 + {}^-5$

8.Simplify: $^-7 + {}^-5$

9.Simplify: $^+5 + {}^-3$

Subtraction of integers using a number line
Examples

1.Subtract $^+5 - {}^-3$ using a number line

$^+5 - {}^-3 = {}^+8$

2. Subtract ⁺8 - ⁻4 using a number line

$^-3$

$^+5$

$^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad ^+1 \quad ^+2 \quad ^+3 \quad ^+4 \quad ^+5 \quad ^+6 \quad ^+7 \quad ^+8 \quad ^+9$

$^+8$

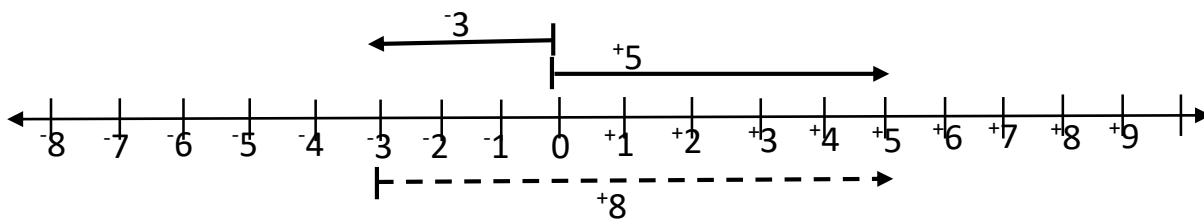$^+5-\ ^-3=8$

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3. Subtract ⁻7 - ⁻4 using a number line

$^-4$

$^-7$

$^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad ^+1 \quad ^+2 \quad ^+3 \quad ^+4 \quad ^+5 \quad ^+6 \quad ^+7 \quad ^+8 \quad ^+9$

$^-3$

$^-7-\ ^-4=\ ^-3$

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4. Subtract ⁺8 - ⁻3 using a number line

$^-3$

$^+8$

$^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad ^+1 \quad ^+2 \quad ^+3 \quad ^+4 \quad ^+5 \quad ^+6 \quad ^+7 \quad ^+8 \quad ^+9$

$^+11$
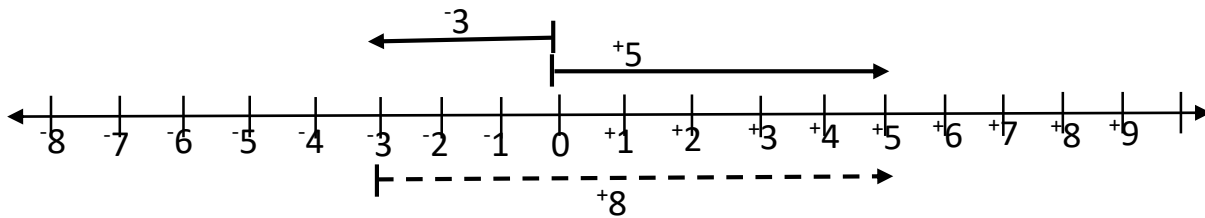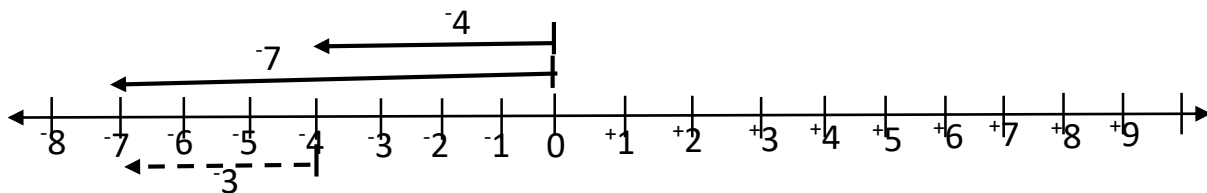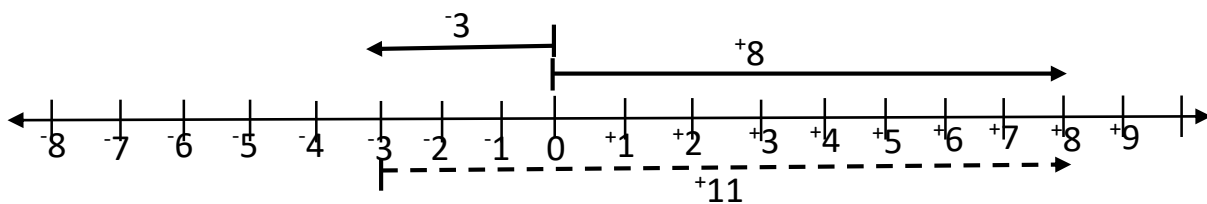
$^+8-\ ^-3=\ ^+11$

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5. Subtract ⁺8 - ⁺3 using a number line

$^+3$

$^+8$

$^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad ^+1 \quad ^+2 \quad ^+3 \quad ^+4 \quad ^+5 \quad ^+6 \quad ^+7 \quad ^+8 \quad ^+9$

$^+5$

$^+8-\ ^+3=\ ^+5$

1.Subtract ⁺5 - ⁻2 using a number line

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◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```

2.Subtract ⁺7 - ⁻4 using a number line

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```
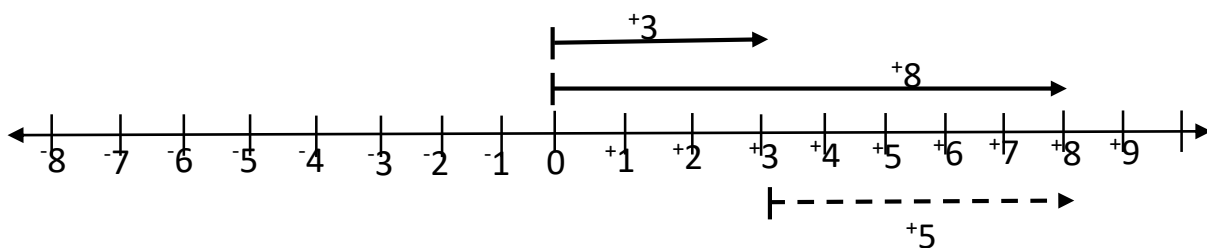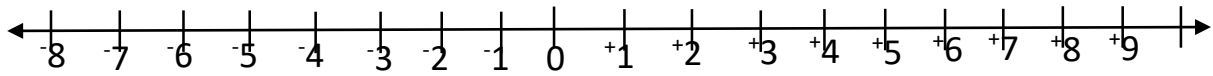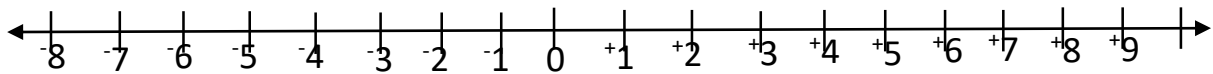
3.Subtract ⁺5 - ⁺2 using a number line

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```
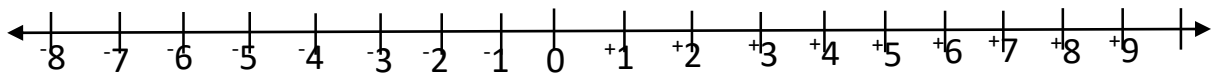
4.Subtract ⁺7 - ⁺5 using a number line

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```
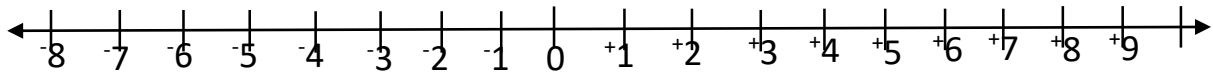
5.Subtract ⁻6 - ⁻2 using a number line

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```
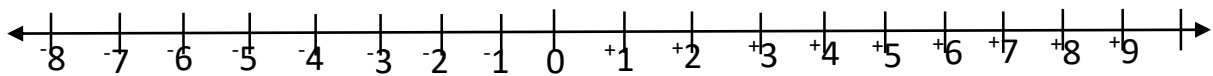
6.Subtract ⁻8 - ⁻5 using a number line

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──►
  -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9
```
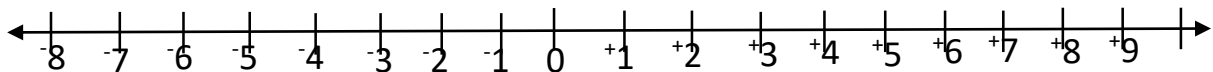
Subtraction of integers without using a number line

Points to note

- +(+)= +
- +(-)= -
- -(-)=+
- -(+)=-

Examples

1. Simplify: $^+6 - ^-2$

$^+6 -(^-2)$

$^+6 + 2$

$^+8$

2. Simplify: $^-7 - ^+2$

$^-7 - (^+2)$

$^-7 - 2$

$= ^-9$

3. Simplify: $^-6 - ^-2$

$^-6 - (^-2)$

$^-6 + 2$

$^-4$

Activity

1. Simplify: $^+9 - ^-5$

2. Simplify: $^+7 - ^+4$

3. Simplify: $^+5 - ^-2$

4. Simplify: $^-6 - ^+2$

5. Simplify: $^-9 - ^-6$

6. Simplify: $^+11 - ^+4$

7. Simplify: $^+8 - ^-5$

8. Simplify: $^-7 - ^+5$

9. Simplify: $^+5 - ^-3$

Forming addition mathematical statements from a number

EXAMPLES

Use the number line below to answer the questions that follow

a) Write down the integer represented by the arrows m,n and k

    M= ⁻6   n= ⁺9   k=⁺3

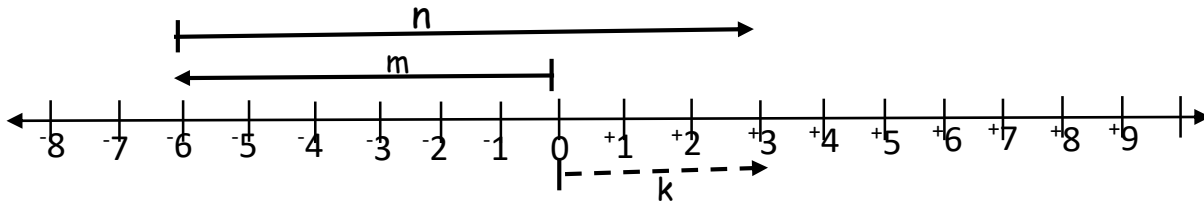- -

b) Write down the addition mathematical statement shown on the above number line

      m + n = k

      ⁻6 + ⁺9=⁺3

- -

Use the number line below to answer the questions that follow

a) Write down the integer represented by the arrows p, q and r

     p= ⁺5   q= ⁺3   r=⁺8

- -

b) Write down the addition mathematical statement shown on the above number line

      p + q = r

      ⁺5 + ⁺3=⁺8

- -

3.Use the number line below to answer the questions that follow

a) Write down the integer represented by the arrows g, h and d

g= ⁻4   h= ⁻3   d=⁻7

b)Write down the addition mathematical statement shown on the above number line

      g + h = d

      ⁻4 + ⁻3=⁻7

Activity

1. Use the number line below to answer the questions that follow

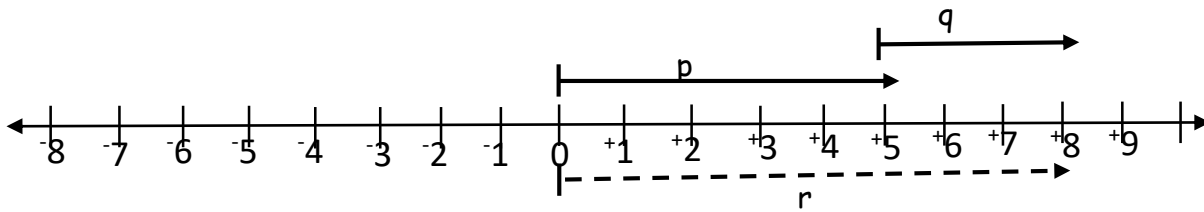a) Write down the integer represented by the arrows c, k and u

    C=………………   k=……………………….   u=……………………………

b) Write down the addition mathematical statement shown on the above number line

2.   Use the number line below to answer the questions that follow

a) Write down the integer represented by the arrows p, q and r

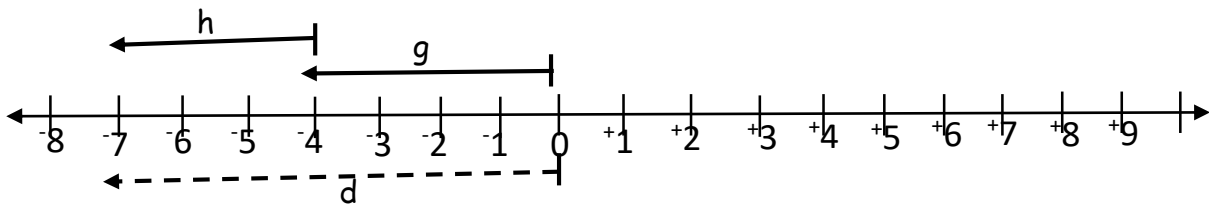s=…………….   p=……………………   K=………………………………………

b) Write down the addition mathematical statement shown on the above number line

3. Use the number line below to answer the questions that follow

a) Write down the integer represented by the arrows g, h and d

g=…………………….   h=…………………….   D=…………………………………….

b) Write down the addition mathematical statement shown on the above number line.

4. Use the number line below to answer the questions that follow

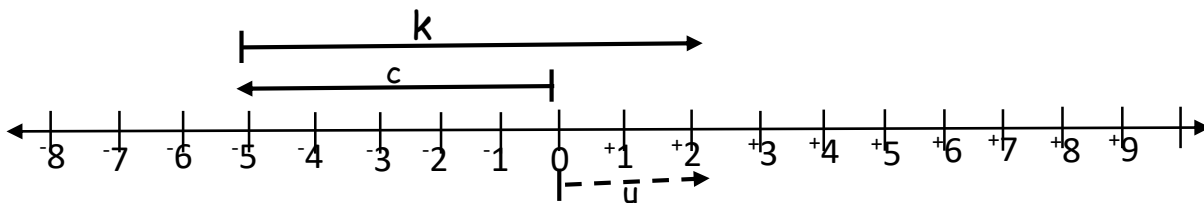a) Write down the integer represented by the arrows a, b and c

a=……………..   b=…………………….   c=……………………………

b) Write down the addition mathematical statement shown on the above number line

Forming subtraction mathematical statements from a number

EXAMPLES

Use the number line below to answer the questions that follow

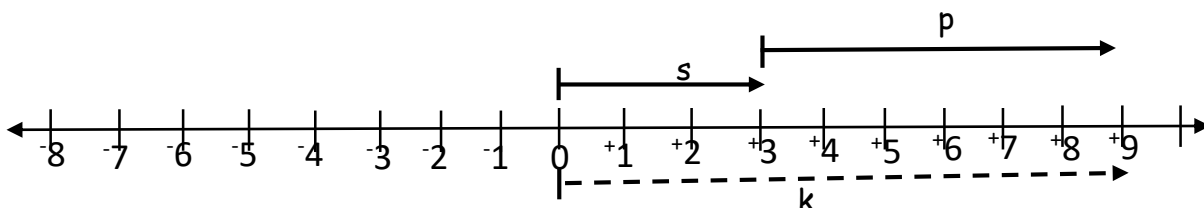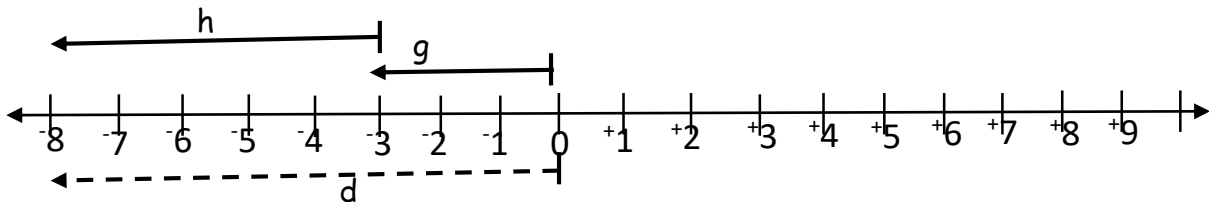b) Write down the integer represented by the arrows m,n and k

$M = ^-6$ $\qquad$ $n = ^+5$ $\qquad$ $k = ^-11$

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b) Write down the subtraction mathematical statement shown on the above number line

$m - n = k$

$^-6 - ^+5 = ^-11$

Use the number line below to answer the questions that follow

c) Write down the integer represented by the arrows p, q and r

$p = ^+5$ $\qquad$ $q = ^-4$ $\qquad$ $r = ^+9$

---

d) Write down the subtraction mathematical statement shown on the above number line

$p - q = r$

$^+5 - ^-4 = ^+9$

3.Use the number line below to answer the questions that follow

c) Write down the integer represented by the arrows g, h and d

g= -4   h= -7   d=+3

- -

b)Write down the subtraction mathematical statement shown on the above number line

g - h = d

-4 - -7=+3

- -

3.Use the number line below to answer the questions that follow

d) Write down the integer represented by the arrows g, h and d

g= +7   h= +3   d=+4

- -

b)Write down the subtraction mathematical statement shown on the above number line

g - h = d

+7 - +3=+4

Activity

1.Use the number line below to answer the questions that follow
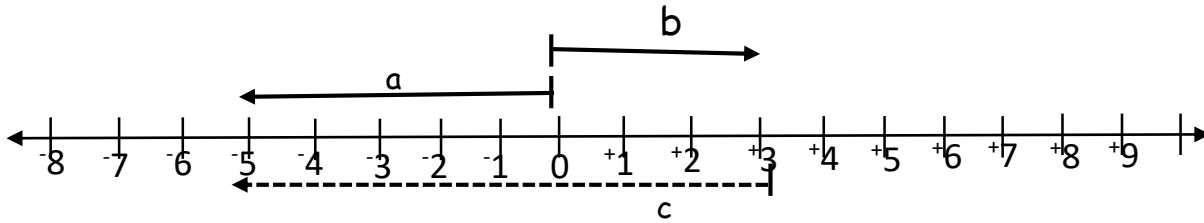
c) Write down the integer represented by the arrows a,b and c

a=----------------   b=----------------------   c=------------------------

b)Write down the subtraction mathematical statement shown on the above number line

Use the number line below to answer the questions that follow

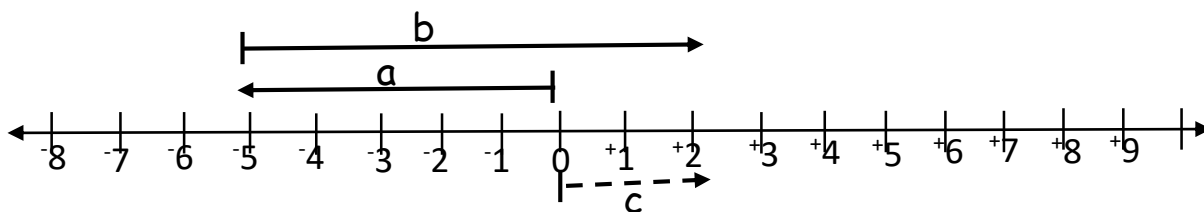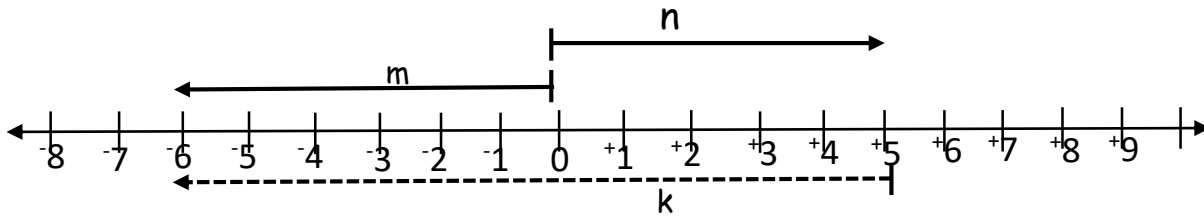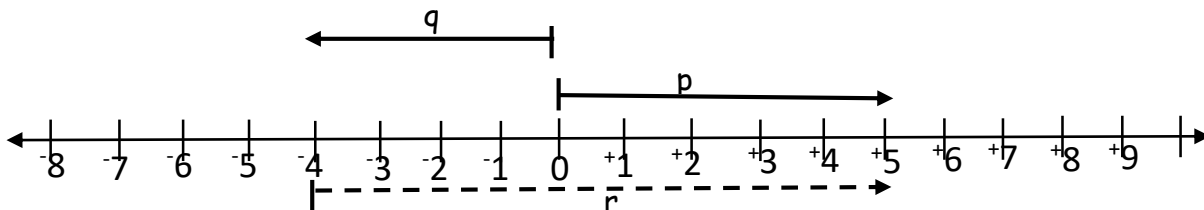e) Write down the integer represented by the arrows m, p and s

m=----------------   p=-----------------   s=--------------------

f) Write down the subtraction mathematical statement shown on the above number line

3.Use the number line below to answer the questions that follow
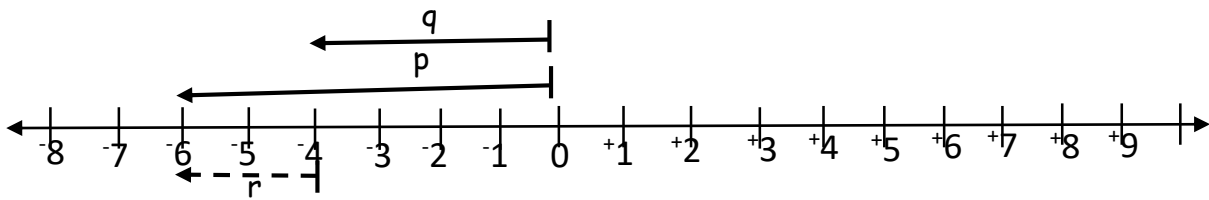
e) Write down the integer represented by the arrows p, q and r

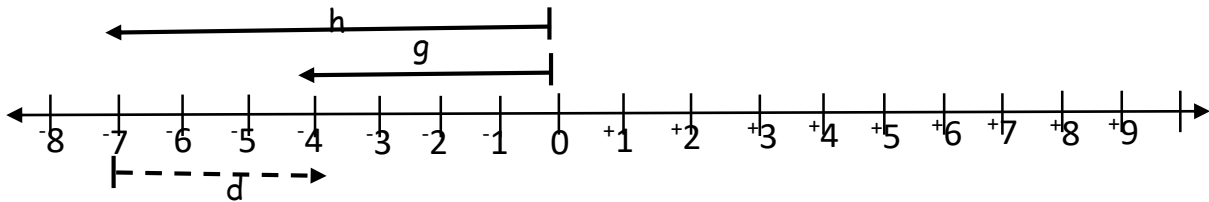p=----------------   q=------------------   r=-----------------

b)Write down the subtraction mathematical statement shown on the above number line

3.Use the number line below to answer the questions that follow

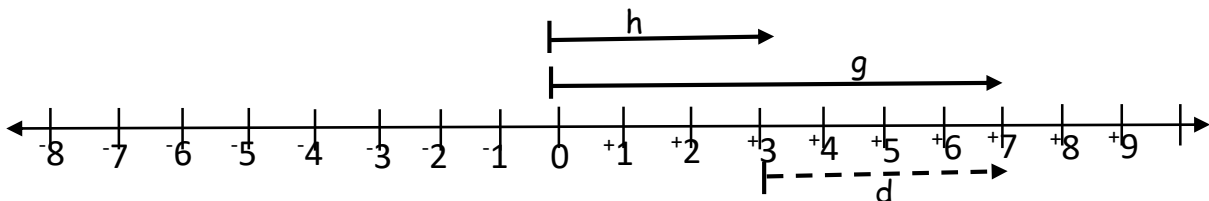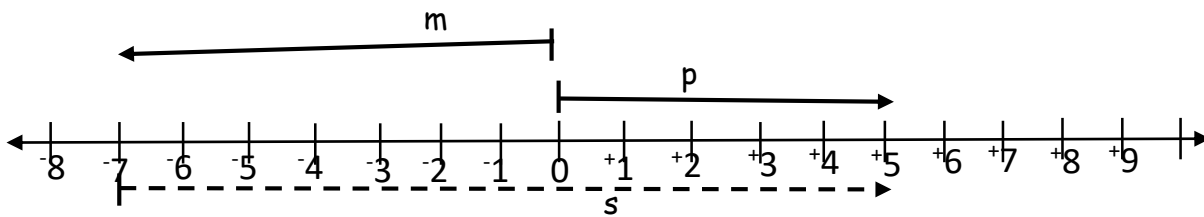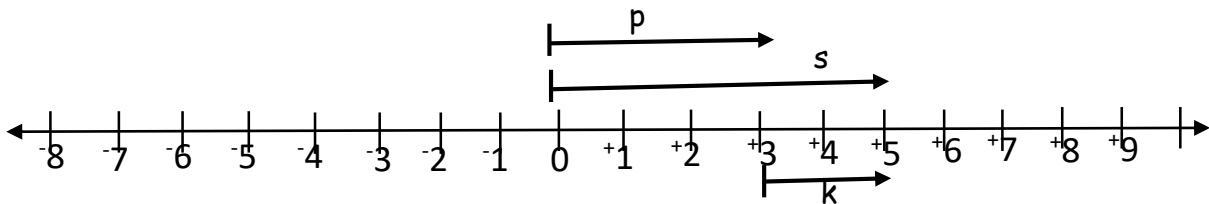f) Write down the integer represented by the arrows p, s and k

p=----------------   s=---------------   k=--------------------

b)Write down the subtraction mathematical statement shown on the above number line

Multiplication of integers without using a number line

Points to note

➢ + x + or +(+)= +

➢ + x – or +(-)= -

➢ - x – or -(-)=+

➢ - x + or -(+)=-

Examples

1.Simplify:$^+6$ x $^-2$ 2.Simplify: $^-7$ x $^+2$ 3.Simplify: $^-6$ x $^-2$

$^+6$ x $^-2$ $^-7$x $^+2$ $^-6$ x$^-2$

$^-12$ $^-14$ $^+12$

Activity

1.Simplify:$^+9$ x $^-5$ 2.Simplify:$^+7$ x $^+4$ 3.Simplify:$^+5$ x $^-2$

4.Simplify:$^-6$ x $^+2$ 5.Simplify:$^-9$ x $^-6$ 6.Simplify:$^+11$ x $^+4$

7.Simplify:$^+8$ x $^-5$ 8.Simplify:$^-7$ x $^+5$ 9.Simplify:$^-5$ x $^-3$

Multiplication of integers using a number line

Examples

1. Workout: 3 x $^{+}$2 using a number line
   ($^{+}$3 x $^{+}$2 means three groups of twos)
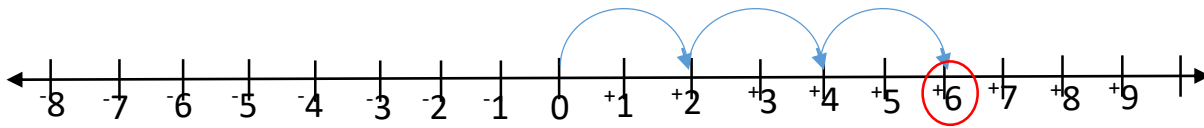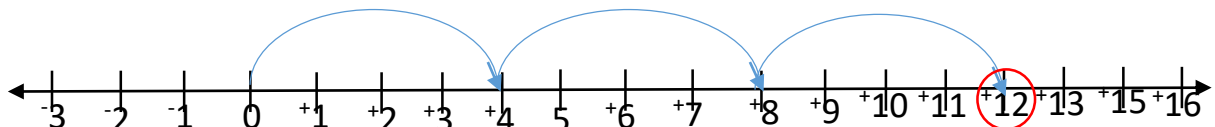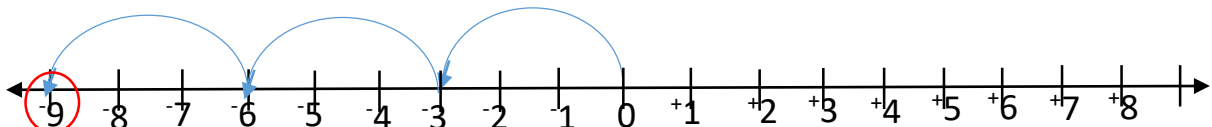
-8 -7 -6 -5 -4 -3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 $^{+}$5 $^{+}$6 $^{+}$7 $^{+}$8 $^{+}$9

   3 x $^{+}$2= $^{+}$6

2. Workout: 3 x $^{+}$4 using a number line

-3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 5 $^{+}$6 $^{+}$7 $^{+}$8 $^{+}$9 $^{+}$10 $^{+}$11 $^{+}$12 $^{+}$13 $^{+}$15 $^{+}$16

   3 x $^{+}$4= $^{+}$12

3. Workout: 3 x $^{-}$3 using a number line

-9 -8 -7 -6 -5 -4 -3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 $^{+}$5 $^{+}$6 $^{+}$7 $^{+}$8

   3 x $^{-}$3= $^{-}$9

   Activity

   1. Workout: 2 x $^{+}$3 using a number line

-3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 5 $^{+}$6 $^{+}$7 $^{+}$8 $^{+}$9 $^{+}$10 $^{+}$11 $^{+}$12 $^{+}$13 $^{+}$15 $^{+}$16

   2. Workout: 3 x $^{+}$3 using a number line

-3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 5 $^{+}$6 $^{+}$7 $^{+}$8 $^{+}$9 $^{+}$10 $^{+}$11 $^{+}$12 $^{+}$13 $^{+}$15 $^{+}$16

   3. Workout: 4 x $^{+}$2 using a number line

-3 -2 -1 0 $^{+}$1 $^{+}$2 $^{+}$3 $^{+}$4 5 $^{+}$6 $^{+}$7 $^{+}$8 $^{+}$9 $^{+}$10 $^{+}$11 $^{+}$12 $^{+}$13 $^{+}$15 $^{+}$16

4.Workout: $3 \times {}^-2$ using a number line

$\xleftarrow{\quad} \overset{-9}{|} \; \overset{-8}{|} \; \overset{-7}{|} \; \overset{-6}{|} \; \overset{-5}{|} \; \overset{-4}{|} \; \overset{-3}{|} \; \overset{-2}{|} \; \overset{-1}{|} \; \overset{0}{|} \; \overset{+1}{|} \; \overset{+2}{|} \; \overset{+3}{|} \; \overset{+4}{|} \; \overset{+5}{|} \; \overset{+6}{|} \; \overset{+7}{|} \; \overset{+8}{|} \xrightarrow{\quad}$
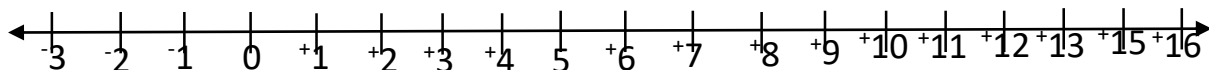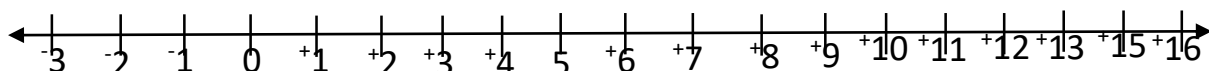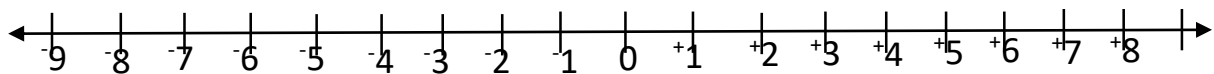
5.Workout: $4 \times {}^-2$ using a number line

$\xleftarrow{\quad} \overset{-9}{|} \; \overset{-8}{|} \; \overset{-7}{|} \; \overset{-6}{|} \; \overset{-5}{|} \; \overset{-4}{|} \; \overset{-3}{|} \; \overset{-2}{|} \; \overset{-1}{|} \; \overset{0}{|} \; \overset{+1}{|} \; \overset{+2}{|} \; \overset{+3}{|} \; \overset{+4}{|} \; \overset{+5}{|} \; \overset{+6}{|} \; \overset{+7}{|} \; \overset{+8}{|} \xrightarrow{\quad}$

Division of integers without using a number line

Points to note

➢ + ÷ + = +

➢ + ÷ − = −

➢ − ÷ − = +

➢ − ÷ + = −

Examples

1.Simplify: ${}^+6 \div {}^-2$

$\quad {}^+6 \div {}^-2$

$\quad {}^-3$

2.Simplify: ${}^-32 \div {}^+8$

$\quad {}^-32 \div {}^+8$

$\quad {}^-4$

3.Simplify: ${}^-16 \div {}^-4$

$\quad {}^-16 \div {}^-4$

$\quad {}^+12$

Activity

1.Simplify: ${}^+9 \div {}^-3$ 2.Simplify: ${}^+36 \div {}^+4$ 3.Simplify: ${}^+24 \div {}^-4$

4.Simplify: ${}^-6 \div {}^+2$ 5.Simplify: ${}^-42 \div {}^-6$ 6.Simplify: ${}^+18 \div {}^+2$

7.Simplify: ${}^-28 \div {}^-4$ 8.Simplify: ${}^-35 \div {}^+5$ 9.Simplify: ${}^-15 \div {}^-3$

Application of integers

Terms used and their meanings

- Before-----------negative
- After-------------positive
- Dropped------------negative
- Rise/Rose-----------positive
- Loss---------------positive
- Gain----------------positive
- Increased------------positive
- Decreased ----------negative
- Lower---------------negative

Examples

1. The temperature of a place was 20^0F in the morning and later it dropped by 23^0F. Find the new temperature of the place.

$20^0F - 23^0F = {}^-3^0F$

2. The temperature on the top of a mountain was $^-14^0C$ at might but by morning, it had increased to 10^0C. Find the rise in the temperature.

$= 10^0C - {}^-12^0C$

$= 10^0C - ({}^-12^0C)$

$= 10^0C + 12^0C$

$= {}^+12^0C$

3. The temperature of a pawpaw dropped by $^-5^0C$. If the new temperature is 15^0C. What was the initial temperature?

Let the initial temperature be h

$h - {}^-5^0C = 15^0C$

$h - ({}^-5^0C) = 15^0C$

$h + 5^0C - 5^0C = 15^0C - 5^0C$

$h = 10^0C$

Alternatively,

Initial temp^0 = New + Drop

Initial temp$^0 = 15^0c + {}^-5^0c$

Initial temp$^0 = 15^0c + ({}^-5^0c)$

Initial temp$^0 = 15^0c - 5^0c$

Initial temp$^0 = 10^0c$

4.Kintu was born in 12BC and died in 23AD.At what age was he when he died?

$=12-{}^-23$

$=12-({}^-23)$

$=12+23$

$=35$years.

5.When marking a test, a teacher a warded2 marks for every correct and subtracted a mark for every wrong answer. The test had 20 questions.

a) If a pupil got 18 correct questions, how many marks did he score?

Number of questions he failed=20-18

$=2$questions

Marks scored$=(18\times2)-(2\times1)$

$=36-2$

$=34$marks

b) A pupil got 25marks.How many correct questions did he get?

Let the number of questions answered correctly be m.

Number of questions he failed=(20-m)

$(m\times2) - (20-m) = 25$

$2m - 20 + m = 25$

$2m + m - 20 = 25$

$3m - 20 \quad = 25$

$3m - 20 + 20 = 25 + 20$

$3m = 45$

$\dfrac{3m}{3} = \dfrac{45}{3}$

$M=15$

He passed 15questions

c) How many question did he fail?

$=20-m$

$=20-15$

$=5$questions

1. The temperature of a place was 30^0F in the morning and later it dropped by 33^0F. Find the new temperature of the place.

2. The temperature on the top of a mountain was $^-14^0$C at might but by morning, it had increased to 10^0C. Find the rise in the temperature.

3. The temperature of a pawpaw dropped by $^-3^0$C. If the new temperature is 12^0C. What was the initial temperature?

4. A girl moved 4metres backwards and 4 more metres backwards. Write the final position of the girl.

5. John climbed 9 steps from his flat upwards and later descended 16 steps to the ground floor. What was John's final position from the flat?

6. In an interview, two marks are a warded for every correct answer and one mark is deducted for every wrong answer. What is the score for a candidate who got;

a) three correct questions and two wrong questions?

b) five correct questions and 3 wrong questions?

7. When marking a test, a teacher a wards 4 marks for every correct answer and subtracts a mark for every wrong answer. The test had 20 questions.

a) How many marks did a pupil who passed 18 questions score?

b) A pupil scored 45 marks. How many questions did he answer correctly?

8. In a test containing 30 questions, a teacher a warded 3 marks for every correct answer but deducted a mark for every wrong answer .

a) If a pupil passed 25 questions, how many marks did she score?

b) Sarah scored 50 marks, how many questions did he fail?

9. When marking a test, a teacher awarded 4 marks for every correct answer and subtracted 2 marks for every wrong answer. The test had 20 questions.

a) A pupil answered 12 questions correctly. How many marks did he score?

b) Joan scored 50 marks. How many questions did he answer correctly?

TOPIC: INTEGERS

SUBTOPIC: THE FINITE SYSTEM / CLOCK ARITHEMATIC

Definition:

➢ Finite system is a way of counting numerals in a specified group of number numerals.

➢ Finite system may also be called the modular (mod). In a special way, finite system sticks only on the remainders after grouping and regrouping a certain numeral.

Expressing numbers in finite system

Examples:

1. Express 23 in finite 5

$23 \div 5 = 4$ remainder 3

23 = 4(finite5)

2. Express 45 in finite 7

$45 \div 7 = 6$ remainder 3

45 = 3(finite7)

Activity

1. Express 24 in finite 3

2. Express 59 in finite 5

3. Express 54 in finite 6

4. Express 46 in finite 5

5. Express 34 in finite 7

6. Express 73 in finite 5

Addition and subtraction of numbers in finite system

1. Work out: 2 + 5 =------ (finite 7)
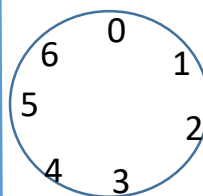
$(2+5) \div 7$ (finite 7)

$7 \div 7 = 1$ remainder 0

2 + 5 = 0 (finite 7)

Alternatively, you can use a dial

2. Work out: 3 + 4 = ------ (finite 5)

$(3+4) \div 5$ (finite 5)

$7 \div 5 = 1$ remainder 2

3 + 4 = 2 (finite 7)

3 .Work out: 2 – 4 = -------- (finite 5)

(2 + 5) – 4 = 3   (finite 5)

7 – 4 = 3   (finite 5)

2 – 4 = 3 (finite 5)

4. Work out: 0 – 5 = -------- (mod 7)

(0 + 7) – 5 = 2(mod 7)

7 – 5 = 2 (mod 7)

0 – 5 = 2 (mod 7)

Activity:

1. Work out the following finite systems.

a) 4 + 3 = _____ (Finite 5)

b) 3 + 3 + 2 = _____ (Finite 5)

c) 2 + 2 + 2 = _____ (Finite 3)

d) 4 + 4 + 4 = _____ (finite 7)

e) 6 + 4 + 5 = _____ (Finite 7)

f) 3 + 3 + 3 = _____ (Finite 4)

Complete the table below in finite 5

+	2	3	4
1			
2			1
3		1	
4			3

2. Work out these finite systems.

a) 1 – 4 = _____ (mod 5)

b) 3 – 5 = _____ (mod 7)

c) 4 – 3 = _____ (mod 5)

d) 5 – 2 = _____ (mod 7)

e) 0 – 4 = _____ (finite 7)

f) 2 – 3 = _____ (finite 5)

SUBTOPIC: <u>Multiplication of numbers in finite system</u>

Examples

1) 2×3 = ------- (mod 5)

   $(2 \times 3) \div 5$ (mod 5)

   $6 \div 5 = 1$ remainder 1

   $2 \times 3 = 1$ (mod 5)

2) 3×2^2 (finite 5)

   $3 \times 2 \times 2$ (finite 5)

   12 (finite 5)
   $12 \div 5 = 2$ remainder 2 (finite 5)

   2 (finite 5)

ACTIVITY:

Multiply:

a) $4 \times 5 =$ _____ (finite 7)

b) $2 \times 4^2 =$ _____ (mod 9)

c) $23 \times 3 =$ _____ (finite 7)

d) $2 \times 32 =$ _____ (finite 6)

f) $4 (5 \times 2) =$ _____ (mod 6)

Ref: MK Mathematics pupils bk 7 pg 332. Exercise: 17:2

Complete the table below in finite 5

x	2	4	5
1		4	
2			
3			0
4		1	

SUBTOPIC: <u>Division of numbers in finite system</u>

1. Workout $2 \div 3 =$ ------(finite 5)
   $(2+5) \div 3 =$ ------(finite 5)
   $(7+5) \div 3 =$ ------(finite 5)
   $12 \div 3 = 4$(finite 5)
     $= 4$(finite 5)

2. Workout $5 \div 3 =$ ------(finite 7)
   $(5+7) \div 3 =$ ------(finite 7)
   $12 \div 3 =$ ------(finite 7)
   $12 \div 3 = 4$(finite 7)
     $= 4$(finite 7)

Activity

1. Workout the following

   1. $3 \div 2 = - - - - (finite\ 5)$

   2. $3 \div 4 = - - - - (finite\ 5)$

3. $5 \div 4 = - - - - (finite\ 7)$

4. $1 \div 4 = - - - - (mod\ 5)$

5. $3 \div 2 = - - - - (mod\ 7)$

6. $3 \div 5 = - - - - (finite\ 12)$

SUBTOPIC: <u>Application of finite 7</u>

Examples

1. If today is Wednesday, what day of the week after 25 days?

Sun	Mon	Tue	Wed	Thur	Fri	Sat
0	1	2	3	4	5	6

3+25=---- (finite 7)

(28÷7)=4 rem 0

Since 0 represents Sunday on the table, the day will be Sunday

If today is Friday, what day of the week was it 24 days ago?

Sun	Mon	Tue	Wed	Thurs.	Fri	Sat
0	1	2	3	4	5	6

5-24=---- (finite 7)

(5+7)-24=(finite 7)

(12+7)-24=(finite7)

(19+7)-24=(finite 7)

26-24=2 (finite 7)

Since 2 represents Tuesday on the table, the day was Tuesday

3. If today is Saturday 12th May, what day of the week will it be on 23rd July the same year?

May	June	July
31-12=19days	30days	23days

Total number of days=19+30+23

=72days

Sun	Mon	Tue	Wed	Thur	Fri	Sat
0	1	2	3	4	5	6

The day will be Monday

6 + 72=------(finite 7)

78=------(finite 7)

(78÷ 7)=11 rem 1 (finite 7)

Activity

1.If today is Thursday, what day of the week will it be after 27days?

2.If today is Wednesday, what day of the week will it be after 52days?

3.If yesterday was Friday, what day of the week will it be after 20days from today?

4.If today is Monday, what day of the week was it 38 days ago?

5.If today is Tuesday, what day of the week was it 20 days ago?

6.If today is Thursday 19th March, what day of the week will it be on 20rd May the same year?

7.If today is Monday 18th April, what day of the week will it be on 25th July the same year?

Ref: Fountain Primary mathematics Pupils bk 7 pg 339

Application of finite 12

Examples

1.If it is November now, what month of the year will it be after 27 months?

J	F	M	A	M	J	J	A	S	O	N	D
1	2	3	4	5	6	7	8	9	10	11	0

2 + 27=--- (finite12)

(2+27)÷12(finite 12)

29÷12=2 rem 5 (finite 12)

The month will be May

1.If it is April now, what month of the year was it 30 months ago?

J	F	M	A	M	J	J	A	S	O	N	D
1	2	3	4	5	6	7	8	9	10	11	0

4 - 30=--- (finite12)

(4+12) - 30=--- (finite12)

(16+12) - 30=--- (finite12)

(28+12) - 30=--- (finite12)

40 - 30=10 (finite12)

The month was October

Activity

1. If it is September now, what month of the year will it be after 26 months?

2. If it is April now, what month of the year will it be after 42 months?

3. If it is June now, what month of the year will it be after 35 months?

4. If it is July now, what month of the year was it 32 months ago?

5. If it is August now, what month of the year was it 25 months ago?

More about application of finite system

Examples

1. Find the least (smallest) number which when divided by 5, 2 remains and when divided by 8, 3 remains.

2 (finite 5) = 7, 12, 17, 22, ㉗, 32, 37………………….

3 (finite 8) = 11, 19, ㉗, 35, 43, 51………………….

The number is 27

2. Martha had playing cards. When he grouped them in threes, 2 cards remained, when grouped them in fours, 3 cards remained and when he grouped, them in fives, 3 remained. Find the least number of playing cards Martha had?

2 (finite 3) = 5, 8, 11, 14, 17, 20, ㉓, 26, 29………

3 (finite 4) = 7, 11, 15, 19, ㉓, 27, 31, 35……..

3 (finite 5) = 8, 13, 18, ㉓, 28, 33, 38,…….

Martha had 23 playing cards

ACTIVITY:

1. What is the least number of cows such that when shared by 4 people,3 cows remain but when divided by 8 people, 7 cows remain?

2. Find the least number which when divided by seven, 3 remains, when divided by 4, 2 remains but when divided by eight, 6 remains.

3. Find the least number of sweets that can be shared by 8 pupils or 5 pupils leaving remainder 4

4. Find the smallest number which when divided by 5 leaves remainder 2 but when divided by 4 leaves remainder 3.

5. A man bought some mangoes. When he put them in groups of 8, five mangoes remained but when he put them in groups of 9, eight mangoes remained. How many mangoes did he buy?

6. Alex had a certain number of oranges. When he put them in groups of 5, four oranges remained but when he put them in groups of 6, three oranges remained. How many mangoes did he have?

7. Nakintu had visitors to be given seats. When she sat them in fours, one visitor remained, when she sat them in nines, six visitors remained but when she sat them in fives, three visitors. How many visitors did she have?

8. Find the least number of pens which when grouped in fours, three pens remain but when grouped in fives, two pens remain.

9. Find the smallest number which when divided by 7, gives remainder 6, when divided by 4, gives remainder 2 and when divided by 8, gives remainder6.

Ref: MK primary mathematics pupils bk 7 pg 340, 341. Exercise 17:9.

Finding the unknowns of finite system.

Examples

1. Find the value of P if 3P – 3 = 4 (finite 7)

3P – 3 = 4 (finite 7)

3P – 3 + 3 = 4 + 3 (finite 7)

3P = 7 (finite 7)

3P = 7+7 (finite 7)

3P = 14+7 (finite 7)

3P = 14+7 (finite 7)

$$\frac{^1\cancel{3}P}{\cancel{3}_1} = \frac{\overset{7}{\cancel{21}}}{\cancel{3}_1} \text{ (finite 7)}$$

P = 7 (finite 7)

P = 0 (finite 7)

2. Find w. If 2w – 4 = 2 (mod 5)

2w – 4 = 2 (mod 5)

2w – 4 + 4 = 2 + 4 (mod 5)

$\frac{\overset{1}{\cancel{2}}w}{\underset{1}{\cancel{2}}} = \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{2}}}$ (mod 5)

w = 3 (mode 5)

3. Find n. If 3n + 4 = 6 (finite 7)

3n + 4-4 = 6-4 (finite 7)

3n = 2 (finite 7)

3n = 2+7 (finite

$\frac{\cancel{3}n}{\cancel{3}} = \frac{\overset{3}{\cancel{9}}}{\cancel{3}}$ (finite 7)

n = 3 (finite 7)

ACTIVITY:

1. Find the value of m. If 5m = 3 (finite 9)

2. Work out: 4k – 4 = 3 (finite 5)

3. Find the value of q if 3q – 3 = 4 (finite 7)

4. Evaluate 2x – 1 = 4 (finite 5)

5. Work out: 3Y – 4 = 5 (finite 6)

6. Find the value of g if 3g = 3 (finite 4)

7. Find the value of r if 3r – 6 = 2 (mod 8)

8. Work out: 4r – 2 = 5 (finite 7)

9. Solve for r: 3 + r =0 (finite 5)

10. Solve for p: 4 + p =1 (mod 5)

Ref: MK primary Mathematics pupils bk 7 pg 333,