P425/1

Pure Mathematics

Paper one

JULY/AUG 2024

3 HOURS

ASSHU ANKOLE JOINT MOCK EXAMINATIONS 2024

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER ONE

3 HOURS

INSTRUCTIONS TO CANDIDATES

- Answer **all** the **eight** questions in section **A** and any **five** questions in section **B**.
- Any additional question(s) answered will **not** be marked.
- All necessary working **must** be shown clearly.
- Begin each question on a fresh sheet of paper.
- Indicate the questions attempted
- Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

1. Solve the simultaneous equations
$2x + y - 3z = 7$
$4x - 2y + z = 15$
$3x + 3y + 2z = 1$ *(05 marks)*

2. Evaluate $\int_0^{\pi/2} \sin 3x \cos 5x \, dx$. *(05 marks)*

3. Solve the equation $4 \cos x + 3\cos \frac{x}{2} = 1$ for $0^0 \leq x \leq 360^0$ *(05 marks)*

4. Find the acute angle between the lines $\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z+3}{1}$ and $\underset{\sim}{r} = (2 + 2\lambda)\, \underset{\sim}{i} + (1 + 3\lambda)\, \underset{\sim}{j} + (6\lambda - 1)\, \underset{\sim}{k}$. *(05 marks)*

5. Differentiate $\log_2 \left(\frac{e^{\frac{2}{x^2}}}{\sin 2x} \right)$ with respect to x *(05 marks)*

6. Solve for x: $\log_4 x = \log_2(3 - 2x)$ *(05 marks)*

7. Points A(0, 2) and B(4, -2) lie on the circumference of a circle. Points C (-3, -3) and D(7, 2) lie outside the circle but the centre of the circle lies on line CD. Find the equation of the circle. *(05 marks)*

8. A curve is represented by parametric equations $x = \sqrt{t^2 + 3}$ and $y = 3t + 4$. Find the equation of the tangent to the circle at point (2 , 7). *(05 marks)*

SECTION B (60 marks)

Answer any five questions.

9. Show that
   i. $\int_0^1 \frac{3x+9}{x^2 + 5x + 4} dx = \ln 5$ *(06 marks)*
   ii. $\int_0^{2\pi/3} \frac{3dx}{5+4\cos x} = \pi/3$ *(06 marks)*

10. (a) Express $(-1 + i\sqrt{3})^8$ in the form $x + iy$ *(05 marks)*
(b) Find the Cartesian equation of the curve given as $|z - 2| = 2\,|z + 1 - 3i|$, show by leaving unshaded, the region $|z - 2| > 2\,|z + 1 - 3i|$ on the Argand diagram. *(07 marks)*

11. (a) Find, in vector form, the equation of a line passing through the point (1, 1, 3) and perpendicular to the plane $2x + 3y + 3z = 7$. *(03 marks)*
(b) Find the position vector of the point of intersection of the lines
$$\underset{\sim}{r_1} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} \text{ and } \underset{\sim}{r_2} = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}.$$
Write down the vector equation of the plane containing lines $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$ hence or

otherwise find the Cartesian equation of the plane containing lines r_1 and r_2.

(09 marks)

12. (a) If α and β are roots of the equation $2x^2 - 7x + 1 = 0$. Show that;

$$\left(\sqrt{\frac{\alpha}{\beta}} - \sqrt{\frac{\beta}{\alpha}}\right)^2 = \frac{41}{2}$$

(05 marks)

(b) Given that $(x-2)^2$ is a factor of the poly nomial $f(x) = x^4 + ax^3 + bx^2 + cx + 4$ and $f(x)$ leaves a remainder of 2 when divided by $(x-1)$. Find the values of a, b and c.

(07 marks)

13. (a) A is a cute angle and B is obtuse such that $\tan A = \frac{4}{3}$ and $\tan B = -2$, without using tables or a calculator. Find the values of;

i) $\sin(A - B)$

ii) $\cos(A + B)$

(06 marks)

(b) Prove that, in any triangle ABC, $\frac{a+b-c}{a+b+c} = \tan\frac{A}{2}\tan\frac{B}{2}$.

(06 marks)

14. (a) If $y = \frac{5x+3}{\sqrt{1-2x^2}}$. Find $\frac{dy}{dx}$

(05 marks)

(b) A cylindrical tin without a lid is made of a sheet metal. If 5 is the area of the sheet used, without waste, V is the volume of the tin and r is the radius of the cross-section, prove that $2V = Sr - \pi r^3$. If S is given, prove that the volume is maximum when the ratio of the height to diameter is 1 : 2.

(07 marks)

15. (a) A curve is represented by parametric equations $x = 4\cos\theta$, $y = 3\sin\theta$; show that the Cartesian equations of the curve represents the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(03 marks)

(b) The Normal to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at $P(4\cos\theta, 3\sin\theta)$ meets the x – and y-axes at A and B respectively. Find the equation of the normal. If M is the mid-point of AB, show that the locus of point M is also an ellipse.

(09 marks)

16. (a) Solve the differential equation $\frac{dy}{dx} = e^{2x+y}$.

(04 marks)

(b) Mbarara city's population is growing in a such way that at time t years, the rate at which the population is increasing is proportional to size, N, of the population at that time, t. If the population increases from 10,000 to 20,000 in five years. What will be the population in the next five years?

(08 marks)

END