MATHEMATICS CONTEST PRACTICE WITH WORKED SOLUTIONS

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PART THREE

COMPILED BY; NUWAMANYA ERICKSON

Email: ericksonnuwamanya14@gmail.com

0782-605169/ 0751-256450

NAME:____

CLASS:____

An electric car is charged 3 times per week for 52 weeks. The cost to charge the car each time is \$0.78. What is the total cost to charge the car over these 52 weeks?

Since the car is charged 3 times per week for 52 weeks, it is charged $3 \times 52 = 156$ times. Since the cost per charge is \$0.78, then the total cost is $156 \times \$0.78 = \$121.68$.

Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga's and Julia's numbers were even. Karl's and Liu's numbers were prime numbers. Glenda's number was a perfect square. What was Ioana's number?

Of the given uniform numbers,

- 11 and 13 are prime numbers
- 16 is a perfect square
- 12, 14 and 16 are even

Since Karl's and Liu's numbers were prime numbers, then their numbers were 11 and 13 in some order.
Since Glenda's number was a perfect square, then her number was 16.
Since Helga's and Julia's numbers were even, then their numbers were 12 and 14 in some order. (The number 16 is already taken.)
Thus, Ioana's number is the remaining number, which is 15.

In the list $7, 9, 10, 11, 18$, which number is the average (mean) of the other four numbers?

The average of the numbers $7, 9, 10, 11$ is $\frac{7+9+10+11}{4} = \frac{37}{4} = 9.25$, which is not equal to 18, which is the fifth number.
The average of the numbers $7, 9, 10, 18$ is $\frac{7+9+10+18}{4} = \frac{44}{4} = 11$, which is equal to 11, the remaining fifth number.
We can check that the averages of the remaining three combinations of four numbers is not equal to the fifth number.
Therefore, the answer is 11.
(We note that in fact the average of the original five numbers is $\frac{7+9+10+11+18}{5} = \frac{55}{5} = 11$, and when we remove a number that is the average of a set, the average does not change. Can you see why?)

A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

We would like to find the first time after 4:56 where the digits are consecutive digits in increasing order.
It would make sense to try 5:67, but this is not a valid time.
Similarly, the time cannot start with 6, 7, 8 or 9.
No time starting with 10 or 11 starts with consecutive increasing digits.
Starting with 12, we obtain the time 12:34. This is the first such time.

We need to determine the length of time between 4:56 and 12:34.
From 4:56 to 11:56 is 7 hours, or $7 \times 60 = 420$ minutes.
From 11:56 to 12:00 is 4 minutes.
From 12:00 to 12:34 is 34 minutes.
Therefore, from 4:56 to 12:34 is $420 + 4 + 34 = 458$ minutes.

Reading from left to right, a sequence consists of 6 X's, followed by 24 Y's, followed by 96 X's. After the first n letters, reading from left to right, one letter has occurred twice as many times as the other letter. The sum of the four possible values of n is

irst, we note that we cannot have $n \leq 6$, since the first 6 letters are X's.
After 6 X's and 3 Y's, there are twice as many X's as Y's. In this case, $n = 6 + 3 = 9$.
After 6 X's and 12 Y's, there are twice as many Y's as X's. In this case, $n = 6 + 12 = 18$.
The next letters are all Y's (with 24 Y's in total), so there are no additional values of n with $n \leq 6 + 24 = 30$.
At this point, there are 6 X's and 24 Y's.
After 24 Y's and 12 X's (that is, 6 additional X's), there are twice as many Y's as X's. In this case, $n = 24 + 12 = 36$.
After 24 Y's and 48 X's (that is, 42 additional X's), there are twice as many X's as Y's. In this case, $n = 24 + 48 = 72$.
Since we are told that there are four values of n, then we have found them all, and their sum is $9 + 18 + 36 + 72 = 135$.

Suppose that p and q are two different prime numbers and that $n = p^2q^2$. The number of possible values of n with $n < 1000$ is

(A) 5 **(B)** 6 **(C)** 4 **(D)** 8 **(E)** 7

We note that $n = p^2q^2 = (pq)^2$.
Since $n < 1000$, then $(pq)^2 < 1000$ and so $pq < \sqrt{1000} \approx 31.6$.
Finding the number of possible values of n is thus equivalent to finding the number of positive integers m with $1 \leq m \leq 31 < \sqrt{1000}$ that are the product of two prime numbers.
The prime numbers that are at most 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.
The distinct products of pairs of these that are at most 31 are:

$$2 \times 3 = 6 \quad 2 \times 5 = 10 \quad 2 \times 7 = 14 \quad 2 \times 11 = 22 \quad 2 \times 13 = 26$$

$$3 \times 5 = 15 \quad 3 \times 7 = 21$$

Any other product either duplicates one that we have counted already, or is larger than 31.
Therefore, there are 7 such values of n.

On Monday, Mukesh travelled x km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. The value of x is

We recall that time $= \dfrac{\text{distance}}{\text{speed}}$. Travelling x km at 90 km/h takes $\dfrac{x}{90}$ hours.
Travelling x km at 120 km/h takes $\dfrac{x}{120}$ hours.
We are told that the difference between these lengths of time is 16 minutes.
Since there are 60 minutes in an hour, then 16 minutes is equivalent to $\dfrac{16}{60}$ hours.
Since the time at 120 km/h is 16 minutes less than the time at 90 km/h, then $\dfrac{x}{90} - \dfrac{x}{120} = \dfrac{16}{60}$.
Combining the fractions on the left side using a common denominator of $360 = 4 \times 90 = 3 \times 120$, we obtain $\dfrac{x}{90} - \dfrac{x}{120} = \dfrac{4x}{360} - \dfrac{3x}{360} = \dfrac{x}{360}$.
Thus, $\dfrac{x}{360} = \dfrac{16}{60}$.
Since $360 = 6 \times 60$, then $\dfrac{16}{60} = \dfrac{16 \times 6}{360} = \dfrac{96}{360}$. Thus, $\dfrac{x}{360} = \dfrac{96}{360}$ which means that $x = 96$.

Kamal turned his computer on at 2 p.m. on Friday. He left his computer on for exactly 30 consecutive hours. At what time did he turn his computer off?

We need to determine the time that is 30 hours after 2 p.m. on Friday.
The time that is 24 hours after 2 p.m. on Friday is 2 p.m. on Saturday.
The time that is 30 hours after 2 p.m. on Friday is an additional 6 hours later.
This time is 8 p.m. on Saturday.

Three integers from the list $1, 2, 4, 8, 16, 20$ have a product of 80. What is the sum of these three integers?

The three integers from the list whose product is 80 are 1, 4 and 20, since $1 \times 4 \times 20 = 80$.
The sum of these integers is $1 + 4 + 20 = 25$.
(Since 80 is a multiple of 5 and 20 is the only integer in the list that is a multiple of 5, then 20 must be included in the product. This leaves two integers to choose, and their product must be $\dfrac{80}{20} = 4$. From the given list, these integers must be 1 and 4.)

Wally makes a whole pizza and shares it with three friends. Jovin takes $\frac{1}{3}$ of the pizza, Anna takes $\frac{1}{6}$ of the pizza, and Olivia takes $\frac{1}{4}$ of the pizza. What fraction of the pizza is left for Wally?

Since Jovin, Anna and Olivia take $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{4}$ of the pizza, respectively, then the fraction of the pizza with which Wally is left is

$$1 - \tfrac{1}{3} - \tfrac{1}{6} - \tfrac{1}{4} = \tfrac{12}{12} - \tfrac{4}{12} - \tfrac{2}{12} - \tfrac{3}{12} = \tfrac{3}{12} = \tfrac{1}{4}\,.$$

Jeff and Ursula each run 30 km. Ursula runs at a constant speed of 10 km/h. Jeff also runs at a constant speed. If Jeff's time to complete the 30 km is 1 hour less than Ursula's time to complete the 30 km, at what speed does Jeff run?

When Ursula runs 30 km at 10 km/h, it takes her $\dfrac{30\text{ km}}{10\text{ km/h}} = 3$ h.
This means that Jeff completes the same distance in 3 h − 1 h = 2 h.
Therefore, Jeff's constant speed is $\dfrac{30\text{ km}}{2\text{ h}} = 15$ km/h.

Janet picked a number, added 7 to the number, multiplied the sum by 2, and then subtracted 4. If the final result was 28, what number did Janet pick?

Solution 1
We undo Janet's steps to find the initial number.
To do this, we start with 28, add 4 (to get 32), then divide the sum by 2 (to get 16), then subtract 7 (to get 9).
Thus, Janet's initial number was 9.

Solution 2
Let Janet's initial number be x.
When she added 7 to her initial number, she obtained $x + 7$.
When she multiplied this sum by 2, she obtained $2(x + 7)$ which equals $2x + 14$.
When she subtracted 4 from this result, she obtained $(2x + 14) - 4$ which equals $2x + 10$.
Since her final result was 28, then $2x + 10 = 28$ or $2x = 18$ and so $x = 9$.

Tobias downloads m apps. Each app costs \$2.00 plus 10% tax. He spends \$52.80 in total on these m apps. What is the value of m?

. Since the tax rate is 10%, then the tax on each \$2.00 app is $\$2.00 \times \frac{10}{100} = \$0.20$.
Therefore, including tax, each app costs $\$2.00 + \$0.20 = \$2.20$.
Since Tobias spends \$52.80 on apps, he downloads $\frac{\$52.80}{\$2.20} = 24$ apps.
Therefore, $m = 24$.

A solid cube is made of white plastic and has dimensions $n \times n \times n$, where n is a positive integer larger than 1. The six faces of the cube are completely covered with gold paint. This cube is then cut into n^3 cubes, each of which has dimensions $1 \times 1 \times 1$. Each of these $1 \times 1 \times 1$ cubes has 0, 1, 2, or 3 gold faces. The number of $1 \times 1 \times 1$ cubes with 0 gold faces is strictly greater than the number of $1 \times 1 \times 1$ cubes with exactly 1 gold face. What is the smallest possible value of n?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 4

.e call the $n \times n \times n$ cube the "large cube", and we call the $1 \times 1 \times 1$ cubes "unit cubes". The unit cubes that have exactly 0 gold faces are those unit cubes that are on the "inside" of the large cube.
In other words, these are the unit cubes none of whose faces form a part of any of the faces of the large cube.
These unit cubes form a cube that is $(n-2) \times (n-2) \times (n-2)$.
To see why this is true, imagine placing the original painted large cube on a table.
Each unit cube with at least one face that forms part of one of the outer faces (or outer layers) has paint on at least one face.
First, we remove the top and bottom layers of unit cubes. This creates a rectangular prism that is $n-2$ cubes high and still has a base that is $n \times n$.
Next, we can remove the left, right, front, and back faces.
This leaves a cube that is $(n-2) \times (n-2) \times (n-2)$.
Therefore, $(n-2)^3$ unit cubes have 0 gold faces.

The unit cubes that have exactly 1 gold face are those unit cubes that are on the outer faces of the large cube but do not touch the edges of the large cube.
Consider each of the six $n \times n$ faces of the large cube. Each is made up of n^2 unit cubes.
The unit cubes that have 1 gold face are those with at least one face that forms part of a face of the large cube, but do not share any edges with the edges of the large cube. Using a similar argument to above, we can see that these unit cubes form a $(n-2) \times (n-2)$ square.
There are thus $(n-2)^2$ cubes on each of the 6 faces that have 1 painted face, and so $6(n-2)^2$ cubes with 1 painted face.
We calculate the values of $(n-2)^3$ and $6(n-2)^2$ for each of the possible choices for n:

Choice	n	$(n-2)^3$	$6(n-2)^2$
(A)	7	125	150
(B)	8	216	216
(C)	9	343	294
(D)	10	512	384
(E)	4	8	24

From this information, the smallest possible value of n when $(n-2)^3$ is larger than $6(n-2)^2$ must be $n = 9$.

To see this in another way, we can ask the question "When is $(n-2)^3$ greater than $6(n-2)^2$?". Note that $(n-2)^3 = (n-2) \times (n-2)^2$ and $6(n-2)^2 = 6 \times (n-2)^2$, and so $(n-2)^3$ is greater than $6(n-2)^2$ when $(n-2)$ is greater than 6, which is when n is greater than 8.
The smallest positive integer value of n for which this is true is $n = 9$.

Sam thinks of a 5-digit number. Sam's friend Sally tries to guess his number. Sam writes the number of matching digits beside each of Sally's guesses. A digit is considered "matching" when it is the correct digit in the correct position.

Guess	Number of Matching Digits
51545	2
21531	1
71794	0
59135	1
58342	2
37348	2
71744	1

What is the sum of all of the possibilities for Sam's number?

We label the digits of the unknown number as $vwxyz$.
Since $vwxyz$ and 71794 have 0 matching digits, then $v \neq 7$ and $w \neq 1$ and $x \neq 7$ and $y \neq 9$ and $z \neq 4$.
Since $vwxyz$ and 71744 have 1 matching digit, then the preceding information tells us that $y = 4$.
Since $vwx4z$ and 51545 have 2 matching digits and $w \neq 1$, then $vwxyz$ is of one of the following three forms: $5wx4z$ or $vw54z$ or $vwx45$.

Case 1: $vwxyz = 5wx4z$
Since $5wx4z$ and 21531 have 1 matching digit and $w \neq 1$, then either $x = 5$ or $z = 1$.
If $x = 5$, then $5wx4z$ and 51545 would have 3 matching digits, which violates the given condition. Thus, $z = 1$.

Thus, $vwxyz = 5wx41$ and we know that $w \neq 1$ and $x \neq 5, 7$.
To this point, this form is consistent with the 1st, 2nd, 3rd and 7th rows of the table.
Since $5wx41$ and 59135 have 1 matching digit, this is taken care of by the fact that $v = 5$ and we note that $w \neq 9$ and $x \neq 1$.
Since $5wx41$ and 58342 have 2 matching digits, this is taken care of by the fact that $v = 5$ and $y = 4$, and we note that $w \neq 8$ and $x \neq 3$.
Since $5wx41$ and 37348 have 2 matching digits and $y = 4$, then either $w = 7$ or $x = 3$.
But we already know that $x \neq 3$, and so $w = 7$.
Therefore, $vwxyz = 57x41$ with the restrictions that $x \neq 1, 3, 5, 7$.
We note that the integers $57041, 57241, 57441, 57641, 57841, 57941$ satisfy the requirements, so are all possibilities for Sam's numbers.

Case 2: $vwxyz = vw54z$
Since $vw54z$ and 51545 have only 2 matching digits, so $v \neq 5$ and $z \neq 5$.
Since $vw54z$ and 21531 have 1 matching digit, then this is taken care of by the fact that $x = 5$, and we note that $v \neq 2$ and $z \neq 1$. (We already know that $w \neq 1$.)
Since $vw54z$ and 59135 have 1 matching digit, then $v = 5$ or $w = 9$ or $z = 5$.
This means that we must have $w = 9$.
Thus, $vwxyz = v954z$ and we know that $v \neq 2, 7, 5$ and $z \neq 1, 4, 5$.
To this point, this form is consistent with the 1st, 2nd, 3rd, 4th, and 7th rows of the table.
Since $v954z$ and 58342 have 2 matching digits and $v \neq 5$, then $z = 2$.
Since $v9542$ and 37348 have 2 matching digits, then $v = 3$.
In this case, the integer 39542 is the only possibility, and it satisfies all of the requirements.

Case 3: $vwxyz = vwx45$
Since $vwx45$ and 21531 have 1 matching digit and we know that $w \neq 1$, then $v = 2$ or $x = 5$.
But if $x = 5$, then $vw545$ and 51545 would have 3 matching digits, so $x \neq 5$ and $v = 2$.
Thus, $vwxyz = 2wx45$ and we know that $w \neq 1$ and $x \neq 5, 7$.
To this point, this form is consistent with the 1st, 2nd, 3rd and 7th rows of the table.
Since $2wx45$ and 59135 have 1 matching digit, this is taken care of by the fact that $z = 5$ and we note that $w \neq 9$ and $x \neq 1$.
Since $2wx45$ and 58342 have 2 matching digits, then $w = 8$ or $x = 3$, but not both.
Since $2wx45$ and 37348 have 2 matching digits, then $w = 7$ or $x = 3$, but not both.
If $w = 8$, then we have to have $x \neq 3$, and so neither $w = 7$ nor $x = 3$ is true.
Thus, it must be the case that $x = 3$ and $w \neq 7, 8$.
Therefore, $vwxyz = 2w345$ with the restrictions that $w \neq 1, 7, 8, 9$.
We note that the integers $20345, 22345, 23345, 24345, 25345, 26345$ satisfy the requirements, so are all possibilities for Sam's numbers.

Thus, there are 13 possibilities for Sam's numbers and the sum of these is 526758.

Which of the following is equal to 2 m plus 3 cm plus 5 mm?

(A) 2.035 m **(B)** 2.35 m **(C)** 2.0305 m **(D)** 2.53 m **(E)** 2.053 m

Since there are 100 cm in 1 m, then 1 cm is 0.01 m. Thus, 3 cm equals 0.03 m.
Since there are 1000 mm in 1 m, then 1 mm is 0.001 m. Thus, 5 mm equals 0.005 m.
Therefore, 2 m plus 3 cm plus 5 mm equals $2 + 0.03 + 0.005 = 2.035$ m.

ANSWER: (A)

Multiplying x by 10 gives the same result as adding 20 to x. The value of x is

From the given information, $10x = x + 20$.
Therefore, $9x = 20$ and so $x = \frac{20}{9}$.

When two positive integers p and q are multiplied together, their product is 75. The sum of all of the possible values of p is

Since $75 = 3 \times 5 \times 5$, we can factor 75 in three different ways:

$$75 = 1 \times 75 = 3 \times 25 = 5 \times 15$$

If $pq = 75$ with p and q integers, then the possible values of p are thus $1, 3, 5, 15, 25, 75$.
The sum of these values is $1 + 3 + 5 + 15 + 25 + 75 = 124$.

What is the tens digit of the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, and 15?

. Among the list 10, 11, 12, 13, 14, 15, the integers 11 and 13 are prime.
Also, $10 = 2 \times 5$ and $12 = 2 \times 2 \times 3$ and $14 = 2 \times 7$ and $15 = 3 \times 5$.
For an integer N to be divisible by each of these six integers, N must include at least two factors of 2 and one factor each of 3, 5, 7, 11, 13.
Note that $2^2 \times 3 \times 5 \times 7 \times 11 \times 13 = 60\,060$.
(This is the least common multiple of 10, 11, 12, 13, 14, 15.)
To find the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, 15, we can find the smallest six-digit positive integer that is a multiple of 60 060.
Note that $1 \times 60\,060 = 60\,060$ and that $2 \times 60\,060 = 120\,120$.
Therefore, the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, 15 is 120 120.
The tens digit of this number is 2.

Chris received a mark of 50% on a recent test. Chris answered 13 of the first 20 questions correctly. Chris also answered 25% of the remaining questions on the test correctly. If each question on the test was worth one mark, how many questions in total were on the test?

Suppose that there were n questions on the test.
Since Chris received a mark of 50% on the test, then he answered $\frac{1}{2}n$ of the questions correctly.
We know that Chris answered 13 of the first 20 questions correctly and then 25% of the remaining questions.
Since the test has n questions, then after the first 20 questions, there are $n - 20$ questions.
Since Chris answered 25% of these $n - 20$ questions correctly, then Chris answered $\frac{1}{4}(n - 20)$ of these questions correctly.
The total number of questions that Chris answered correctly can be expressed as $\frac{1}{2}n$ and also as $13 + \frac{1}{4}(n - 20)$.
Therefore, $\frac{1}{2}n = 13 + \frac{1}{4}(n - 20)$ and so $2n = 52 + (n - 20)$, which gives $n = 32$.
(We can check that if $n = 32$, then Chris answers 13 of the first 20 and 3 of the remaining 12 questions correctly, for a total of 16 correct out of 32.)

The average age of Andras, Frances and Gerta is 22 years.
What is Gerta's age?

Name	Age (Years)
Andras	23
Frances	24
Gerta	?

Since the average of the three ages is 22, then the sum of the three ages is $3 \cdot 22 = 66$.
Since Andras' age is 23 and Frances' age is 24, then Gerta's age is $66 - 23 - 24 = 19$.

If $n = 7$, which of the following expressions is equal to an even integer?

When $n = 7$, we have

$$9n = 63 \qquad n + 8 = 15 \qquad n^2 = 49 \qquad n(n-2) = 7(5) = 35 \qquad 8n = 56$$

Therefore, $8n$ is even.
We note that for every integer n, the expression $8n$ is equal to an even integer, since 8 is even and the product of an even integer with any integer is even.
If n were even, then in fact all five choices would be even. If n is odd, only $8n$ is even.

Jitka hiked a trail. After hiking 60% of the length of the trail, she had 8 km left to go. What is the length of the trail?

After Jitka hiked 60% of the trail, $100\% - 60\% = 40\%$ of the trail was left.
From the given information, 40% of the length of the trail corresponds to 8 km.
This means that 10% of the trail corresponds to one-quarter of 8 km, or 2 km.
Since 10% of the trail has length 2 km, then the total length of the trail is $10 \cdot 2 = 20$ km

What is the smallest positive integer that is a multiple of each of 3, 5, 7, and 9?
(A) 35 **(B)** 105 **(C)** 210 **(D)** 315 **(E)** 630

Solution 1
The sequence of symbols includes 5 $\heartsuit$'s and 2 $\spadesuit$'s.
This means that, each time the sequence is written, there are $5 - 2 = 3$ more $\heartsuit$'s written than $\spadesuit$'s.
When the sequence is written 50 times, in total there are $50 \cdot 3 = 150$ more $\heartsuit$'s written than $\spadesuit$'s.

Solution 2
The sequence of symbols includes 5 $\heartsuit$'s and 2 $\spadesuit$'s.
When the sequence is written 50 times, there will be a total of $50 \cdot 5 = 250$ $\heartsuit$'s written and a total of $50 \cdot 2 = 100$ $\spadesuit$'s written.
This means that there are $250 - 100 = 150$ more $\heartsuit$'s written than $\spadesuit$'s.

The operation $\otimes$ is defined by $a \otimes b = \dfrac{a}{b} + \dfrac{b}{a}$. What is the value of $4 \otimes 8$?

From the given definition,

$$4 \otimes 8 = \frac{4}{8} + \frac{8}{4} = \frac{1}{2} + 2 = \frac{5}{2}$$

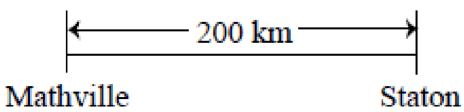
At the end of the year 2000, Steve had \$100 and Wayne had \$10 000. At the end of each following year, Steve had twice as much money as he did at the end of the previous year and Wayne had half as much money as he did at the end of the previous year. At the end of which year did Steve have more money than Wayne for the first time?

We make a table of the total amount of money that each of Steve and Wayne have at the end of each year. After the year 2000, each entry in Steve's column is found by doubling the previous entry and each entry in Wayne's column is found by dividing the previous entry by 2. We stop when the entry in Steve's column is larger than that in Wayne's column:

Year	Steve	Wayne
2000	\$100	\$10 000
2001	\$200	\$5000
2002	\$400	\$2500
2003	\$800	\$1250
2004	\$1600	\$625

Therefore, the end of 2004 is the first time at which Steve has more money than Wayne at the end of the year.

Anca and Bruce left Mathville at the same time. They drove along a straight highway towards Staton. Bruce drove at 50 km/h. Anca drove at 60 km/h, but stopped along the way to rest. They both arrived at Staton at the same time. For how long did Anca stop to rest?



Since Bruce drove 200 km at a speed of 50 km/h, this took him $\frac{200}{50} = 4$ hours.
Anca drove the same 200 km at a speed of 60 km/h with a stop somewhere along the way.
Since Anca drove 200 km at a speed of 60 km/h, the time that the driving portion of her trip took was $\frac{200}{60} = 3\frac{1}{3}$ hours.
The length of Anca's stop is the difference in driving times, or $4 - 3\frac{1}{3} = \frac{2}{3}$ hours.
Since $\frac{2}{3}$ hours equals 40 minutes, then Anca stopped for 40 minutes.

Krystyna has some raisins. She gives one-third of her raisins to Mike. She then eats 4 raisins, after which she gives one-half of her remaining raisins to Anna. If Krystyna then has 16 raisins left, how many raisins did she have to begin?

Solution 1
We work backwards from the last piece of information given.
Krystyna has 16 raisins left after giving one-half of her remaining raisins to Anna.
This means that she had $2 \cdot 16 = 32$ raisins immediately before giving raisins to Anna.
Immediately before giving raisins to Anna, she ate 4 raisins, which means that she had $32 + 4 = 36$ raisins immediately before eating 4 raisins.
Immediately before eating these raisins, she gave one-third of her raisins to Mike, which would have left her with two-thirds of her original amount.
Since two-thirds of her original amount equals 36 raisins, then one-third equals $\frac{36}{2} = 18$ raisins.
Thus, she gave 18 raisins to Mike and so started with $36 + 18 = 54$ raisins.

Solution 2
Suppose Krystyna starts with x raisins.
She gives $\frac{1}{3}x$ raisins to Mike, leaving her with $x - \frac{1}{3}x = \frac{2}{3}x$ raisins.
She then eats 4 raisins, leaving her with $\frac{2}{3}x - 4$ raisins.
Finally, she gives away one-half of what she has left to Anna, which means that she keeps one-half of what she has left, and so she keeps $\frac{1}{2}\left(\frac{2}{3}x - 4\right)$ raisins.
Simplifying this expression, we obtain $\frac{2}{6}x - \frac{4}{2} = \frac{1}{3}x - 2$ raisins.
Since she has 16 raisins left, then $\frac{1}{3}x - 2 = 16$ and so $\frac{1}{3}x = 18$ or $x = 54$.
Therefore, Krystyna began with 54 raisins.

The chart shown gives the cost of installing carpet in four rectangular rooms of various sizes. The cost per square metre of installing carpet is always the same.

Width (metres)

Length (metres)		10	y
	15	\$397.50	\$675.75
	x	\$742.00	\$$z$

What is the value of z?

Solution 1
Let $\$c$ be the cost per square metre of installing carpeting.
Then in each situation, the area of the room times the cost per square metre equals the total price.
From the top left entry in the table, $15 \cdot 10 \cdot \$c = \$397.50$.
From the top right entry in the table, $15 \cdot y \cdot \$c = \$675.75$.
From the bottom left entry in the table, $x \cdot 10 \cdot \$c = \$742.00$.
From the bottom right entry in the table, $x \cdot y \cdot \$c = \$z$.
Now,

$$z = x \cdot y \cdot c = x \cdot y \cdot c \cdot \frac{10 \cdot 15 \cdot c}{10 \cdot 15 \cdot c} = \frac{(x \cdot 10 \cdot c) \cdot (15 \cdot y \cdot c)}{15 \cdot 10 \cdot c} = \frac{(742.00) \cdot (675.75)}{397.50} = 1261.40$$

Therefore, $z = 1261.40$.

Solution 2
Let $\$c$ be the cost per square metre of installing carpeting.
Then in each situation, the area of the room times the cost per square metre equals the total price.
From the top left entry in the table, $15 \cdot 10 \cdot \$c = \$397.50$.
Thus, $c = \dfrac{397.50}{15 \cdot 10} = 2.65$.
From the top right entry in the table, $15 \cdot y \cdot \$c = \$675.75$.
Thus, $y = \dfrac{675.75}{15 \cdot 2.65} = 17$.
From the bottom left entry in the table, $x \cdot 10 \cdot \$c = \$742.00$.
Thus, $x = \dfrac{742.00}{10 \cdot 2.65} = 28$.
From the bottom right entry in the table, $x \cdot y \cdot \$c = \$z$.
Thus, $z = 28 \cdot 17 \cdot 2.65 = 1261.40$.
Therefore, $z = 1261.40$.

How many triples (a, b, c) of positive integers satisfy the conditions $6ab = c^2$ and $a < b < c \leq 35$?

ınce the left side of the given equation is a multiple of 6, then the right side, c^2, is also a multiple of 6.
Since c^2 is a multiple of 6, then c^2 is a multiple of 2 and a multiple of 3.
Since 2 and 3 are different prime numbers, then the positive integer c itself must be a multiple of 2 and a multiple of 3. This is because if c is not a multiple of 3, then c^2 cannot be a multiple of 3, and if c is not even, then c^2 cannot be even.
Therefore, c is a multiple of each of 2 and 3, and so is a multiple of 6.
Thus, there are five possible values for c in the given range: 6, 12, 18, 24, 30.
If $c = 6$, then $6ab = 36$ and so $ab = 6$.
Since $1 \leq a < b < 6$ (because $c = 6$), then $a = 2$ and $b = 3$.
If $c = 12$, then $6ab = 144$ and so $ab = 24$.
Since $1 \leq a < b < 12$, then $a = 3$ and $b = 8$ or $a = 4$ and $b = 6$.
(The divisor pairs of 24 are $24 = 1 \cdot 24 = 2 \cdot 12 = 3 \cdot 8 = 4 \cdot 6$. Only the pairs $24 = 3 \cdot 8 = 4 \cdot 6$ give solutions that obey the given restrictions, since in the other two pairs, the larger divisor does not satisfy the restriction of being less than 12.)
If $c = 18$, then $6ab = 324$ and so $ab = 54$.
Since $1 \leq a < b < 18$, then $a = 6$ and $b = 9$.
(The divisor pairs of 54 are $54 = 1 \cdot 54 = 2 \cdot 27 = 3 \cdot 18 = 6 \cdot 9$.)
If $c = 24$, then $6ab = 576$ and so $ab = 96$.
Since $1 \leq a < b < 24$, then $a = 6$ and $b = 16$ or $a = 8$ and $b = 12$.
(The divisor pairs of 96 are $96 = 1 \cdot 96 = 2 \cdot 48 = 3 \cdot 32 = 4 \cdot 24 = 6 \cdot 16 = 8 \cdot 12$.)
If $c = 30$, then $6ab = 900$ and so $ab = 150$.

If $c = 30$, then $6ab = 900$ and so $ab = 150$.
Since $1 \leq a < b < 30$, then $a = 6$ and $b = 25$ or $a = 10$ and $b = 15$.
(The divisor pairs of 150 are $150 = 1 \cdot 150 = 2 \cdot 75 = 3 \cdot 50 = 5 \cdot 30 = 6 \cdot 25 = 10 \cdot 150$.)
Therefore, the triples (a, b, c) of positive integers that are solutions to the equation $6ab = c^2$ and that satisfy $a < b < c \leq 35$ are

$$(a, b, c) = (2, 3, 6), (3, 8, 12), (4, 6, 12), (6, 9, 18), (6, 16, 24), (8, 12, 24), (6, 25, 30), (10, 15, 30)$$

There are 8 such triplets.

A sports team earns 2 points for each win, 0 points for each loss, and 1 point for each tie. How many points does the team earn for 9 wins, 3 losses and 4 ties?

. The team earns 2 points for each win, so 9 wins earns $2 \times 9 = 18$ points.
The team earns 0 points for each loss, so 3 losses earns 0 points.
The team earns 1 point for each tie, so 4 ties earns 4 points.
In total, the team earns $18 + 0 + 4 = 22$ points.

If $2 \times 2 \times 3 \times 3 \times 5 \times 6 = 5 \times 6 \times n \times n$, then n could equal

Solution 1
We rewrite the left side of the given equation as $5 \times 6 \times (2 \times 3) \times (2 \times 3)$.
Since $5 \times 6 \times (2 \times 3) \times (2 \times 3) = 5 \times 6 \times n \times n$, then a possible value of n is 2×3 or 6.

Solution 2
Since $2 \times 2 \times 3 \times 3 \times 5 \times 6 = 5 \times 6 \times n \times n$, then $1080 = 30n^2$ or $n^2 = 36$.
Thus, a possible value of n is 6. (The second possible value for n is -6.)

Solution 3
Dividing both sides by 5×6, we obtain $2 \times 2 \times 3 \times 3 = n \times n$, which is equivalent to $n^2 = 36$.
Thus, a possible value of n is 6. (The second possible value for n is -6.)

What number should go in the $\square$ to make the equation $\frac{3}{4} + \frac{4}{\square} = 1$ true?

. For $\frac{3}{4} + \frac{4}{\square} = 1$ to be true, we must have $\frac{4}{\square} = 1 - \frac{3}{4} = \frac{1}{4}$.
Since $\frac{1}{4} = \frac{4}{16}$, we rewrite the right side using the same numerator to obtain $\frac{4}{\square} = \frac{4}{16}$.
Therefore, $\square = 16$ makes the equation true.
(We can check that $\frac{3}{4} + \frac{4}{16} = 1$, as required.)

In the subtraction shown, X and Y are digits. What is the value of $X + Y$?

$$\begin{array}{r} 1\,X\,2 \\ -\quad 8\,Y \\ \hline 4\,5 \end{array}$$

We rearrange the given subtraction to create the addition statement $45 + 8Y = 1X2$.
Next, we consider the units digits.
From the statement, the sum $5 + Y$ has a units digit of 2. This means that $Y = 7$. This is the only possibility. Therefore, we have $45 + 87 = 1X2$.
But $45 + 87 = 132$, so $X = 3$.
Therefore, $X + Y = 3 + 7 = 10$.
(We can check that $132 - 87 = 45$, as required.)

If $x = 2y$ and $y \neq 0$, then $(x + 2y) - (2x + y)$ equals

. We simplify first, then substitute $x = 2y$:

$$(x + 2y) - (2x + y) = x + 2y - 2x - y = y - x = y - 2y = -y$$

Alternatively, we could substitute first, then simplify:

$$(x + 2y) - (2x + y) = (2y + 2y) - (2(2y) + y) = 4y - 5y = -y$$

How many pairs of positive integers (x, y) have the property that the ratio $x : 4$ equals the ratio $9 : y$?

The equality of the ratios $x : 4$ and $9 : y$ is equivalent to the equation $\frac{x}{4} = \frac{9}{y}$. (Note that x and y are both positive so we are not dividing by 0.)
This equation is equivalent to the equation $xy = 4(9) = 36$.
Thus, we want to determine the number of pairs of integers (x, y) for which $xy = 36$.
Since the positive divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36, then the desired pairs are

$$(x, y) = (1, 36), (2, 18), (3, 12), (4, 9), (6, 6), (9, 4), (12, 3), (18, 2), (36, 1)$$

There are 9 such pairs.

Nadia walks along a straight path that goes directly from her house (N) to her Grandmother's house (G). Some of this path is on flat ground, and some is downhill or uphill. Nadia walks on flat ground at 5 km/h, walks uphill at 4 km/h, and walks downhill at 6 km/h. It takes Nadia 1 hour and 36 minutes to walk from N to G and 1 hour and 39 minutes to walk from G to N. If 2.5 km of the path between N and G is on flat ground, the total distance from N to G is closest to

ıs Nadia walks from N to G, suppose that she walks x km uphill and y km downhill. We are told that she walks 2.5 km on flat ground.
This means that when she walks from G to N, she will walk x km downhill, y km uphill, and again 2.5 km on flat ground. This is because downhill portions become uphill portions on the return trip, while uphill portions become downhill portions on the return trip.
We are told that Nadia walks at 5 km/h on flat ground, 4 km/h uphill, and 6 km/h downhill.
Since speed $= \dfrac{\text{distance}}{\text{time}}$, then distance $=$ speed $\times$ time and time $= \dfrac{\text{distance}}{\text{speed}}$.
Thus, on her trip from N to G, her time walking uphill is $\dfrac{x}{4}$ hours, her time walking downhill is $\dfrac{y}{6}$ hours, and her time walking on flat ground is $\dfrac{2.5}{5}$ hours.
Since it takes her 1 hour and 36 minutes (which is 96 minutes or $\dfrac{96}{60}$ hours), then

$$\frac{x}{4} + \frac{y}{6} + \frac{2.5}{5} = \frac{96}{60}$$

A similar analysis of the return trip gives

$$\frac{x}{6} + \frac{y}{4} + \frac{2.5}{5} = \frac{99}{60}$$

We are asked for the total distance from N to G, which equals $x + y + 2.5$ km. Therefore, we need to determine $x + y$.
We add the two equations above and simplify to obtain

$$\begin{aligned} \frac{x}{4} + \frac{x}{6} + \frac{y}{6} + \frac{y}{4} + 1 &= \frac{195}{60} \\ x\left(\frac{1}{4} + \frac{1}{6}\right) + y\left(\frac{1}{4} + \frac{1}{6}\right) &= \frac{135}{60} \\ \frac{5}{12}x + \frac{5}{12}y &= \frac{9}{4} \\ x + y &= \frac{12}{5}\left(\frac{9}{4}\right) \end{aligned}$$

Thus, $x + y = \dfrac{108}{20} = \dfrac{27}{5} = 5.4$ km.
Finally, the distance from N to G is $5.4 + 2.5 = 7.9$ km.

Suppose that $\dfrac{2009}{2014}+\dfrac{2019}{n}=\dfrac{a}{b}$, where a, b and n are positive integers with $\dfrac{a}{b}$ in lowest terms. What is the sum of the digits of the smallest positive integer n for which a is a multiple of 1004?

irst, we simplify $\dfrac{2009}{2014}+\dfrac{2019}{n}$ to obtain $\dfrac{2009n+2014(2019)}{2014n}$ or $\dfrac{2009n+4\,066\,266}{2014n}$.

Since $\dfrac{2009n+4\,066\,266}{2014n}=\dfrac{a}{b}$ and $\dfrac{a}{b}$ is in lowest terms, then $2009n+4\,066\,266=ka$ and $2014n=kb$ for some positive integer k.

Since $2009n+4\,066\,266=ka$, then if a is a multiple of 1004, we must have that $2009n+4\,066\,266$ is a multiple of 1004 as well.

Therefore, we determine the values of n for which $2009n+4\,066\,266$ is divisible by 1004 and from this list find the smallest such n that makes a divisible by 1004. (Note that even if $2009n+4\,066\,266$ is divisible by 1004, it might not be the case that a is divisible by 1004, since reducing the fraction $\dfrac{2009n+4\,066\,266}{2014n}$ might eliminate some or all of the prime factors of 1004 in the numerator.)

We note that $2008=2\times 1004$ and $4\,066\,200=4050\times 1004$, so we write

$$2009n+4\,066\,266=(2008n+4\,066\,200)+(n+66)=1004(2n+4050)+(n+66)$$

(2008 and 4 066 200 are the largest multiples of 1004 less than 2009 and 4 066 266, respectively.)

Since $1004(2n+4050)$ is a multiple of 1004, then $2009n+4\,066\,266$ is a multiple of 1004 whenever $n+66$ is a multiple of 1004, say $1004m$ (that is, $n+66=1004m$).

Thus, $2009n+4\,066\,266$ is a multiple of 1004 whenever $n=1004m-66$ for some positive integer m.

.nese are the values of n for which the expression $2009n + 4\,066\,266$ is divisible by 1004. Now we need to determine the smallest of these n for which a is divisible by 1004.

When $m = 1$, we have $n = 1004 - 66 = 938$. In this case,

$$\frac{2009n + 4\,066\,266}{2014n} = \frac{2009(938) + 4\,066\,2006}{2014(938)} = \frac{5\,950\,708}{1\,889\,132} = \frac{1\,487\,677}{472\,283}$$

where in the last step we divided a common factor of 4 out of the numerator and denominator. Regardless of whether this last fraction is in lowest terms, its numerator is odd and so $\frac{a}{b}$ (the equivalent lowest terms fraction) will also have a odd, so a cannot be divisible by 1004. So, we try the next value of m.

When $m = 2$, we have $n = 2008 - 66 = 1942$. In this case,

$$\frac{2009n + 4\,066\,266}{2014n} = \frac{2009(1942) + 4\,066\,2006}{2014(1942)} = \frac{7\,967\,744}{3\,911\,188} = \frac{1\,991\,936}{977\,797}$$

where in the last step we divided a common factor of 4 out of the numerator and denominator. Now $1004 = 4 \times 251$ and 251 is a prime number. (251 is a prime number because it is not divisible by any of the primes 2, 3, 5, 7, 11, and 13, which are all of the primes less than $\sqrt{251}$.) Now $1\,991\,936 = 1984 \times 1004$ so is a multiple of 1004, and $977\,797$ is not divisible by 4 or by 251. This means that when $\frac{1\,991\,936}{977\,797}$ is written in lowest terms as $\frac{a}{b}$, then a will be divisible by 1004.

Therefore, the smallest value of n with the desired property is $n = 1942$, which has a sum of digits equal to $1 + 9 + 4 + 2 = 16$.

Owen spends \$1.20 per litre on gasoline. He uses an average of 1 L of gasoline to drive 12.5 km. How much will Owen spend on gasoline to drive 50 km?

Since Owen uses an average of 1 L to drive 12.5 km, then it costs Owen \$1.20 in gas to drive 12.5 km.

To drive 50 km, he drives 12.5 km a total of $50 \div 12.5 = 4$ times.

Therefore, it costs him $4 \times \$1.20 = \$4.80$ in gas to drive 50 km.

The time on a cell phone is 3:52. How many minutes will pass before the phone next shows a time using each of the digits 2, 3 and 5 exactly once?

There are six times that can be made using each of the digits 2, 3 and 5 exactly once: 2:35, 2:53, 3:25, 3:52, 5:23, and 5:32.
The first of these that occurs after 3:52 is 5:23.
From 3:52 to 4:00, 8 minutes pass.
From 4:00 to 5:00, 60 minutes pass.
From 5:00 to 5:23, 23 minutes pass.
Therefore, from 3:52 to 5:23, which is the next time that uses the digits 2, 3, and 5 each exactly once, a total of $8 + 60 + 23 = 91$ minutes pass.

The same sequence of four symbols repeats to form the following pattern:

$$\heartsuit \clubsuit \spadesuit \heartsuit \heartsuit \clubsuit \spadesuit \heartsuit \heartsuit \clubsuit \spadesuit \heartsuit \ \ldots$$

How many times does the symbol $\heartsuit$ occur within the first 53 symbols of the pattern?

Since the sequence repeats every 4 symbols and since $13 \times 4 = 52$, then the 52nd symbol is the last symbol in a sequence of 4 symbols.
Also, the first 52 symbols represent 13 sequences of these 4 symbols.
Each sequence of 4 symbols includes 2 $\heartsuit$s, so the first 52 symbols include $13 \times 2 = 26$ $\heartsuit$s.
Finally, the 53rd symbol in the pattern is the first of a sequence of 4, so is also $\heartsuit$.
Therefore, the first 53 symbols include $26 + 1 = 27$ $\heartsuit$s.

Which number from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ must be removed so that the mean (average) of the numbers remaining in the set is 6.1?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

The original set contains 11 elements whose sum is 66.
When one number is removed, there will be 10 elements in the set.
For the average of these elements to be 6.1, their sum must be $10 \times 6.1 = 61$.
Since the sum of the original 11 elements is 66 and the sum of the remaining 10 elements is 61, then the element that must be removed is $66 - 61 = 5$.

The integer 636 405 may be written as the product of three 2-digit positive integers. The sum of these three integers is

e begin by factoring the given integer into prime factors.
Since 636 405 ends in a 5, it is divisible by 5, so

$$636\,405 = 5 \times 127\,281$$

Since the sum of the digits of 127 281 is a multiple of 3, then it is a multiple of 3, so

$$636\,405 = 5 \times 3 \times 42\,427$$

The new quotient (42 427) is divisible by 7 (can you see this without using a calculator?), which gives

$$636\,405 = 5 \times 3 \times 7 \times 6061$$

We can proceed by systematic trial and error to see if 6061 is divisible by 11, 13, 17, 19, and so on. After some work, we can see that $6061 = 11 \times 551 = 11 \times 19 \times 29$.
Therefore, $636\,405 = 3 \times 5 \times 7 \times 11 \times 19 \times 29$.
We want to rewrite this as the product of three 2-digit numbers.
Since $3 \times 5 \times 7 = 105$ which has three digits, and the product of any three of the six prime factors of 636 405 is at least as large as this, then we cannot take the product of three of these prime factors to form a two-digit number.
Thus, we have to combine the six prime factors in pairs.
The prime factor 29 cannot be multiplied by any prime factor larger than 3, since $29 \times 3 = 87$ which has two digits, but $29 \times 5 = 145$, which has too many digits.
This gives us $636\,405 = 87 \times 5 \times 7 \times 11 \times 19$.
The prime factor 19 can be multiplied by 5 (since $19 \times 5 = 95$ which has two digits) but cannot be multiplied by any prime factor larger than 5, since $19 \times 7 = 133$, which has too many digits.
This gives us $636\,405 = 87 \times 95 \times 7 \times 11 = 87 \times 95 \times 77$.
The sum of these three 2-digit divisors is $87 + 95 + 77 = 259$.

Joshua chooses five distinct numbers. In how many different ways can he assign these numbers to the variables p, q, r, s, and t so that $p < s$, $q < s$, $r < t$, and $s < t$?

ppose that the five distinct numbers that Joshua chooses are V, W, X, Y, Z, and that $V < W < X < Y < Z$.
We want to assign these to p, q, r, s, t so that $p < s$ and $q < s$ and $r < t$ and $s < t$.
First, we note that t must be the largest of p, q, r, s, t. This is because $r < t$ and $s < t$, and because $p < s$ and $q < s$, we get $p < s < t$ and $q < s < t$, so $p < t$ and $q < t$.

Since t is the largest, then Z must be t.

Now neither p nor q can be the second largest of the numbers (which is Y), since p and q are both smaller than s and t.
Therefore, there are two cases: $Y = r$ or $Y = s$.

Case 1: $Y = r$
We have $Y = r$ and $Z = t$.
This leaves V, W, X (which satisfy $V < W < X$) to be assigned to p, q, s (which satisfy $p < s$ and $q < s$).
Since X is the largest of V, W, X and s is the largest of p, q, s, then $X = s$.
This leaves V, W to be assigned to p, q.
Since there is no known relationship between p and q, then there are 2 possibilities: either $V = p$ and $W = q$, or $V = q$ and $W = p$.
Therefore, if $Y = r$, there are 2 possible ways to assign the numbers.

Case 2: $Y = s$
We have $Y = s$ and $Z = t$.
This leaves V, W, X (which satisfy $V < W < X$) to be assigned to p, q, r.
There is no known relationship between p, q, r.
Therefore, there are 3 ways to assign one of V, W, X to p.
For each of these 3 ways, there are 2 ways of assigning one of the two remaining numbers to q.
For each of these 3×2 ways, there is only 1 choice for the number assigned to r.
Overall, this gives $3 \times 2 \times 1 = 6$ ways to do this assignment. (The 6 ways to assign V, W, X to p, q, r, respectively, are VWX, VXW, WVX, WXV, XVW, XWV.)
Therefore, if $Y = s$, there are 6 possible ways to assign the numbers.

Having examined the two possibilities, there are $2 + 6 = 8$ different ways to assign the numbers.

If 7:30 a.m. was 16 minutes ago, in how many minutes will it be 8:00 a.m.?

If 7:30 a.m. was 16 minutes ago, then it is currently $30 + 16 = 46$ minutes after 7:00 a.m., or 7:46 a.m.
Since 8:00 a.m. is 60 minutes after 7:00 a.m., then it will be 8:00 a.m. in $60 - 46 = 14$ minutes.

What is the difference between the largest and smallest of the numbers in the list 0.023, 0.302, 0.203, 0.320, 0.032?

We write the list in increasing order: $0.023, 0.032, 0.203, 0.302, 0.320$.
The difference between the largest and smallest of these numbers is $0.320 - 0.023 = 0.297$.

At the Lacsap Hospital, Emily is a doctor and Robert is a nurse. Not including Emily, there are five doctors and three nurses at the hospital. Not including Robert, there are d doctors and n nurses at the hospital. The product of d and n is

Since Emily is a doctor and there are 5 doctors and 3 nurses aside from Emily at the hospital, then there are 6 doctors and 3 nurses in total.
Since Robert is a nurse, then aside from Robert, there are 6 doctors and 2 nurses.
Therefore, $d = 6$ and $n = 2$, so $dn = 12$.

The operation ∇ is defined by $g \nabla h = g^2 - h^2$. For example, $2 \nabla 1 = 2^2 - 1^2 = 3$. If $g > 0$ and $g \nabla 6 = 45$, the value of g is

. Using the definition of the operation, $g \nabla 6 = 45$ gives $g^2 - 6^2 = 45$.
Thus, $g^2 = 45 + 36 = 81$.
Since $g > 0$, then $g = \sqrt{81} = 9$.

A hockey team has 6 more red helmets than blue helmets. The ratio of red helmets to blue helmets is $5 : 3$. The total number of red helmets and blue helmets is *olution 1*

Suppose that the team has r red helmets.

Since the team has 6 more red helmets than blue helmets, then the team has $r-6$ blue helmets.

Since the ratio of the number of red helmets to the number of blue helmets is $5:3$, then $\frac{r}{r-6}=\frac{5}{3}$ and so $3r=5(r-6)$ or $3r=5r-30$.

Therefore, $2r=30$ or $r=15$.

Thus, the team has 15 red helmets, 9 blue helmets, and $15+9=24$ helmets in total.

Solution 2

Since the ratio of the number of red helmets to the number of blue helmets equals $5:3$, then we can try multiplying both parts of this ratio by small numbers to see if we can obtain an equivalent ratio where the two parts differ by 6.

Multiplying by 2, we obtain $5:3=10:6$, which doesn't have the desired property.

Multiplying by 3, we obtain $5:3=15:9$. Since $15-9=6$, then we have found the correct number of red and blue helmets.

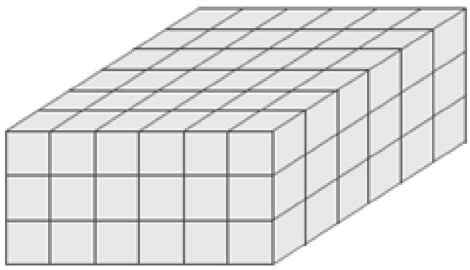
Therefore, the team has 15 red helmets, 9 blue helmets, and $15+9=24$ helmets in total.

(If we continue to multiply this ratio by larger numbers, the difference between the two parts gets bigger, so cannot equal 6 in a different case. In other words, the answer is unique.)

The entire exterior of a solid $6\times 6\times 3$ rectangular prism is painted. Then, the prism is cut into $1\times 1\times 1$ cubes. How many of these cubes have no painted faces?

We visualize the solid as a rectangular prism with length 6, width 6 and height 3. In other words, we can picture the solid as three 6×6 squares stacked on top of each other.

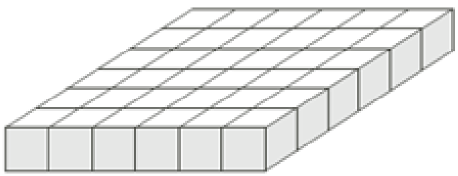
Since the entire exterior of the solid is painted, then each cube in the top layer and each cube in the bottom layer has paint on it, so we can remove these.



This leaves the middle 6×6 layer of cubes.

Each cube around the perimeter of this square has paint on it, so it is only the "middle" cubes from this layer that have no paint on them.

These middle cubes form a 4×4 square, and so there are 16 cubes with no paint on them.



When the three-digit positive integer N is divided by 10, 11 or 12, the remainder is 7. What is the sum of the digits of N?

Solution 1

When N is divided by 10, 11 or 12, the remainder is 7.
This means that $M = N - 7$ is divisible by each of 10, 11 and 12.
Since M is divisible by each of 10, 11 and 12, then M is divisible by the least common multiple of 10, 11 and 12.
Since $10 = 2 \times 5$, $12 = 2 \times 2 \times 3$, and 11 is prime, then the least common multiple of 10, 11 and 12 is $2 \times 2 \times 3 \times 5 \times 11 = 660$. (To find the least common multiple, we compute the product of the highest powers of each of the prime factors that occur in the given numbers.)
Since M is divisible by 660 and $N = M + 7$ is a three-digit positive integer, then M must equal 660. (The next largest multiple of 660 is 1320.)
Therefore, $N = M + 7 = 667$, and so the sum of the digits of N is $6 + 6 + 7 = 19$.

Solution 2

When N is divided by 10, 11 or 12, the remainder is 7.
This means that $M = N - 7$ is divisible by each of 10, 11 and 12.
Since M is divisible by each of 10 and 11, then M must be divisible by 110.
We test the first few multiples of 110 until we obtain one that is divisible by 12.
The integers 110, 220, 330, 440, and 550 are not divisible by 12, but 660 is.
Therefore, M could be 660. (This means that M must be 660.)
Finally, $N = M + 7 = 667$, and so the sum of the digits of N is $6 + 6 + 7 = 19$.

A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

Let L be the length of the string.
If x is the length of the shortest piece, then since each of the other pieces is twice the length of the next smaller piece, then the lengths of the remaining pieces are $2x$, $4x$, and $8x$.
Since these four pieces make up the full length of the string, then $x + 2x + 4x + 8x = L$ or $15x = L$ and so $x = \frac{1}{15}L$.
Thus, the longest piece has length $8x = \frac{8}{15}L$, which is $\frac{8}{15}$ of the length of the string.