***VECTORS***

***Summary:***

***1.*** *A vector has both magnitude and direction.*

***2.***  *is the position vector of point* 

***3.*** *The magnitude* ***or*** *length* ***or*** *modulus of vector* ***OP*** *is denoted by*



***4.*** *To add two vectors we add the corresponding numbers*

***5.*** *To subtract two vectors we subtract the corresponding numbers*

***6.*** *A scalar* ***k*** *multiplied by vector* *is treated as follows****:***



***7.*** *A displacement vector* ***AB*** *is represented by a directed line segment* ***AB*** *as*

*shown****:***

***A***

***B***

*The vectors* ***AB*** *and* ***BA*** *are equal in length but opposite in direction*

***∴ BA = −AB***

***8.*** *In the triangle* ***OAB,*** *the displacement* ***OA*** *followed by* ***AB*** *is equal to a single*

*displacement* ***OB.***

***O***

***A***

***B***

***OB = OA + AB***

***∴ AB = OB − OA “****The vector triangle equation”*

***9.*** *If vector* ***AB*** *is parallel to* ***CD,*** *then* 

***10.*** *If* ***ABCD*** *is a parallelogram, then the two opposite sides are parallel and*

*also equal in length* ***(AB = DC*** *and* ***AD = BC).***

***11.*** *If* ***AB*** *is parallel to* ***BC*** *with a common point* ***B,*** *then the points* ***A, B*** *and* ***C***

*are collinear* ***(  )***

***EXAMPLES:***

***1.*** *Given the points* ***A(4, 1)*** *and* ***B(12, 16),*** *find the****:***

***(i)*** *column vector* ***AB***

***(ii)*** *length of* ***AB***

***2.*** *The position vectors of* ***P*** *and* ***Q*** *are*  *and* *respectively****,*** *find the*

*magnitude of* ***PQ***

***3.*** *Find the distance between the points* ***P(−8, 2)*** *and* ***Q(4, 7 )***

***4.*** *Given that*  *and*  *find the****:***

***(i)*** *position vector of* ***B***

***(ii)*** 

***5.*** *Given that*  *and*  *find the****:***

***(i)*** *coordinates of* ***A***

***(ii)*** *modulus of* ***OA***

***6.*** *Given the points* ***P(−2, 3)*** *and* ***Q(3, 6),*** *find the coordinates of* ***R,*** *if*



***7.*** *Given the points* ***A(3, 4)*** *and* ***B(9, 2),*** *find the coordinates of* ***T,*** *if*



***8.*** *Given that*   *and*  *find the magnitude of* ***m.***

***9.*** *Given the vectors*   *and*  *find the length of*



***10.*** *Given the vectors* *and*  *find the****:***

***(i)*** *column**vector* ***AC***

***(ii)*** *modulus of* ***AC***

***11.*** *Given the vectors* *and*  *find****:***

***(i)*** *vector* ***PR***

***(ii)*** *the length**of* ***PR***

***12.*** *Given the vectors* *and*   *find the magnitude of* ***BC.***

***13.*** *If*  *and*  *find the values of* ***a*** *and* ***b*** *such that*



***14.*** *If*  *and*  *find the values of* ***x*** *and* ***y*** *such that*



***15.*** *If*  *and*  *find the values of* ***x*** *and* ***y*** *such that*



***15. ABCD*** *is a parallelogram with* ***A(−2, −2), B(6, −2)*** *and* ***C(2, 1).*** *Find the*

*coordinates of* ***D.***

***16. ABCD*** *is a parallelogram with* ***A(2, 1), B(3, 4)*** *and* ***C(−1, 2).*** *Find the*

*coordinates of* ***D***

***17. PQRS*** *is a parallelogram with* ***P(1, 1), Q(5, 3)*** *and* ***R(7, 7).*** *Find the****:***

***(i)****column vector* ***PS***

***(ii)*** *coordinates of* ***S.***

***18. ABCD*** *is a quadrilateral with* ***A(4, 1), B(2, −2,******C(−2, 0)*** *and* ***D(0, 3).*** *Show*

*that* ***ABCD*** *is a parallelogram*

***19.*** *The vectors*  *and*  *are parallel to each other****.*** *Find the*

*value of* 

***20.*** *The vectors*  *and*  *are parallel to each other****.*** *Find the*

*value of* 

***21.*** *Show that the points* ***A(−1, 3), B(2, 1)*** *and* ***C(8, −3)*** *are collinear****.***

***22.*** *Show that the points* ***P(−1, −5), Q(0, −2)*** *and* ***R(2, 4)*** *are collinear****.***

***23.*** *Given the points* ***A(−2, −1), B(1, 5)*** *and* ***C(2, 7),*** *find the value of* ***k*** *such that*

****** *hence state the ratio*******

***24.*** *In the vector triangle* ***OAB, M*** *is a point on* ***AB*** *such that* ***AM: AB = 2:5.***

*Express****:***

***(i) AM*** *in terms of* ***AB***

***(ii) MB*** *in terms of* ***AB***

***(iii) AB*** *in terms of* ***AM***

***(iv) AB*** *in terms of* ***MB***

***(v) OM*** *in terms of* ***OA*** *and* ***AB***

***(vi) OM*** *in terms of* ***OB*** *and* ***AB***

***25.*** *In the vector triangle* ***OAB, K*** *is a point on* ***AB*** *such that* ***3AK = 2KB.***

*Express****:***

***(i) AK*** *in terms of* ***AB***

***(ii) KB*** *in terms of* ***AB***

***(iii) AK*** *in terms of* ***KB***

***(iv) KB*** *in terms of* ***AK***

***(v) OK*** *in terms of* ***OA*** *and* ***AB***

***(vi) OK*** *in terms of* ***OB*** *and* ***AB***

***26.*** *In the vector triangle* ***OAB, N*** *is the midpoint of* ***AB.*** *Express****:***

***(i) ON*** *in terms of* ***OA*** *and* ***AB***

***(ii) ON*** *in terms of* ***OB*** *and* ***AB***

***27.***  *The position vectors of the points* ***A*** *and* ***B*** *are and *

*respectively****.*** *Point* ***M*** *is on* ***AB*** *such that* ***AM: AB =******2 : 3,*** *find the:*

***(i)*** *column vector* ***AB***

***(ii)*** *column vector* ***AM***

***(iii)*** *position vector of* ***M.***

***28.*** *Given that*  *and* ***M*** *is a point on* ***AB*** *such that*

***3AM = 2MB,*** *find the****:***

***(i)*** *coordinates of* ***M***

***(ii)*** *magnitude of* ***OM***

***29.*** *Given that*   *and* ***M*** *is the midpoint of* ***AB,*** *find the****:***

***(i)*** *column vector* ***AB***

***(ii)*** *position vector of* ***M***

***30.*** *Given that   and point* ***E*** *divides* ***AB*** *in the ratio*

***1 : 3,*** *find the position vector of* ***E.***

***31.*** *Given that*   *and* ***M*** *is the midpoint of* ***AB,***

***(a)*** *Draw a vector diagram showing vector* ***AB***

***(b)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) AM***

***(iii) OM***

***32.*** *In a triangle* ***OAB,***  *and point* ***K*** *divides* ***AB*** *in the ratio* ***1:2,***

*Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) AK***

***(iii) OK***

***33.*** *In a triangle* ***OAB,***  *and* ***N*** *is a point on* ***AB*** *such that*

***2AB = 3NB.*** *Express vector* ***ON*** *in terms of* ***a*** *and* ***b.***

***34.*** *In a triangle* ***OAB,***  *point* ***C*** *divides* ***AB*** *in the ratio* ***2:3*** *and*

***D*** *is the midpoint of* ***OC.***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) OC***

***(iii) BD***

***(b)*** *Taking* ***O*** *as the origin****,*** *point* ***A(−15, 20)*** *and* ***B(10, 0),***  *find the****:***

***(i)*** *position vector of* ***C*** *in* ***(a)(i)*** *above****.***

***(ii)*** *coordinates of* ***C.***

***(iii)*** *length of* ***OC.***

***35.*** *In a triangle* ***OAB,******M*** *and* ***N*** *are midpoints of* ***OA*** *and* ***OB*** *respectively****.***

  *and* ***P*** *is a point on* ***AB*** *such that* ***4AP = 3AB.***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) OP***

***(iii) MB***

***(iv) NP***

***(b)*** *Show that* ***AB*** *is parallel to* ***MN.***

***36.*** *In a triangle* ***OAB,******M*** *and* ***N*** *are midpoints of* ***AB*** *and* ***OB*** *respectively****.***

  *and* ***P*** *is a point on* ***OM*** *such that* ***3OP = 2OM.***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) OM***

***(iii) PB***

***(iv) AP***

***(b) (i)*** *Show that the points* ***A, P*** *and* ***N*** *are collinear****.***

***(ii)*** *Find the ratio in which* ***P*** *divides* ***AN.***

***37.*** *In a triangle* ***OAB,***  ***P*** *and* ***Q*** *are points on* ***OA*** *and* ***AB***

*respectively such that* ***3OP = PA, AQ = 2QB*** *and* ***N*** *is the midpoint of* ***OQ.***

***ANM*** *is a straight line which is such that* ***AN = 5NM.*** *Given also that*

*where* ***h*** *is a scalar****.***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) OQ***

***(ii) AN***

***(iii) PN***

***(iv) NB***

***(b)*** *Show that the points* ***P, N*** *and* ***B*** *are collinear*

***(c)*** *Find the value of* ***h.***

***38.*** *In the figure below****,******P*** *is a point on* ***AD*** *such that* ***PD : AP = 1 : 2,*** 

 ***3OB = 2BD*** *and* ***OC = 3CE = 3AP.***

**•**

**•**

**•**

***a***

***b***

***A***

***O***

***D***

***C***

***E***

***P***

***B***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AD***

***(ii) BP***

***(iii) DC***

***(b)*** *Show that* ***AD : OE = 3 : 8***

***39.*** *In the figure below****,***    *and*



***P***

***T***

***Q***

***R***

***V***

***S***

***3a***

***3b***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) RS***

***(ii) PV***

***(iii) RQ***

***(b)*** *Find the ratio of* ***RV*** *to* ***RQ.***

***37.*** *In the figure below****,***  ***F*** *and* ***G*** *are points on* ***AC*** *such that*

***AF : AB = 3 : 4*** *and* ***AG : AC = 2 : 3.*** *Point* ***D*** *is on* ***OA*** *such that*

***OD : DA = FB : BG = 1 : 2.***

**•**

**•**

**•**

**•**

***O***

***A***

***D***

***B***

***C***

***F***

***G***

***a***

***b***

***(a)*** *Express* ***AG*** *and* ***AC*** *in terms of* ***AB.*** *Hence find the following vectors in*

*terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) AC***

***(iii) DG***

***(iv) OF***

***(b)*** *Find the ratio* ***DG : OC***

***39.*** *In the figure below****,***   *and* ***PR : RQ = 2 : 1***

***q***

***p***

***S***

***R***

***P***

***Q***

***O***

***T***

***(a)*** *Express the following vectors in terms of* ***p*** *and* ***q:***

***(i) PQ***

***(ii) OR***

***(iii) SQ***

***(b)*** *Line* ***OR*** *and* ***SQ*** *meet at point* ***T*** *such that* ***OT = hOR*** *and* ***ST = kSQ.***

***(i)*** *By expressing* ***OT*** *in two different ways****,*** *find the values of* ***h*** *and* ***k***

***(ii)*** *Determine the ratio in which* ***T*** *divides* ***SQ***

***40.*** *In the figure below****,******OABC*** *is a parallelogram where* ***OA = a*** *and*

***AB = b.*** *Point* ***N*** *is on* ***OA*** *such that* ***ON:NA = 1:2.***

***C***

***O***

***N***

***A***

***X***

***B***

•

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AC***

***(ii) BN***

***(b)*** *Line* ***AC*** *and* ***BN*** *meet at point* ***X*** *such that* ***AX = hAC*** *and* ***BX = kBN***

***(i)*** *By expressing* ***OX*** *in two different ways****,*** *find the values of* ***h*** *and* ***k***

***(ii)*** *Determine the ratio in which* ***X*** *divides* ***AC***

***35.*** *In a triangle* ***OAB,***  ***N*** *and* ***M*** *are points of* ***AB*** *and* ***OB***

*respectively****.*** *Line* ***ON*** *and* ***AM*** *meet at point* ***T*** *such that* ***AT = TM*** *and*

Given that ***OM = xOB*** and ***AN = yAB,*** *Express the vectors****:***

***(i) AM*** *and* ***OT*** *in terms of* ***a, b*** *and* ***x.***

***(ii) ON*** *and* ***OT*** *in terms of* ***a, b*** *and* ***y,*** *hence find the values of* ***x*** *and* ***y.***

***EER:***

***1.*** *Given the points* ***A(3, 4)*** *and* ***B(9, 1),*** *find the coordinates of* ***P,*** *if*



***2.*** *Given the vectors*   *and*  *find the values of*

***m*** *and* ***n*** *for which *

***3.*** *Given the vectors   and  find the values of*

***a*** *and* ***b*** *for which *

***4.*** *Given that vector  and  find the magnitude of*

*vector *

***5.*** *Given that vector*  *find the possible values of* ***x*** *for which* 

***6.*** *The position vectors of the points* ***A*** *and* ***B*** *are and *

*respectively****.*** *If point* ***E*** *divides* ***AB*** *in the ratio* ***1 : 3,*** *find the position*

*vector of* ***E.***

***7.*** *Find the distance between the points* ***P(−8, 2)*** *and* ***Q(4, 7 )***

***8.*** *Given the points* ***A(−1, 2), B(2, 8), C(−2, −5)*** *and* ***D(4, y),*** *find the value of* ***y*** *for which* ***AB*** *is parallel to* ***CD.***

***9.*** *Given the vectors   and  find the values of*

***a*** *and* ***b*** *for which *

***10.*** *Given the vectors  and find the****:***

***(i)*** *vector* ***AC***

***(ii)*** *magnitude of* ***AC***

***11. ABCD*** *is a parallelogram with  and point* ***C*** *is*

***(−5, 2).*** *Find the****:***

***(a)*** *coordinates of****:***

***(i) B (ii) D (iii) A***

***(b)*** *length of the diagonal* ***AC***

***(c)*** *point of intersection of the diagonal* ***AC*** *and* ***BD***

***12****. The vectors ,  and *

***(a)*** *Find the****:***

***(i)*** *position vector of* ***N (02 marks)***

***(ii)*** *length of* ***ON (02 marks)***

***(iii)*** *coordinates of point* ***E,*** *where* ***E*** *divides* ***PQ*** *in the ratio* ***1:3. (03 marks)***

***(b)*** *Use the vector method to show that* ***N*** *lies on* ***PQ.*** *Hence state the*

*ratio* ***PN : PQ.******(05 marks)***

***13.*** *Given that* ***OA = a, OB = b*** *and* ***C*** *is the midpoint of* ***AB,***

***(a)*** *Draw a vector diagram showing vector* ***AB****.* ***(01 mark)***

***(b)****Express in terms of* ***a*** *and* ***b*** *the vectors****:***

***(i) AB (01 mark)***

***(ii) OC (02 marks)***

***14.*** *In the triangle* ***OAB,***  ***C*** *is a point on* ***AB*** *such that*

***AC:AB = 1:3*** *and* ***D*** *is the midpoint of* ***OB.***

**O**

**B**

**C**

**D**

***a***

***X***

***A***

***b***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) AB***

***(ii) OC***

***(iii) AD***

***(b)******X*** *is a point on* ***AD*** *such that* ***AX : AD = 1 : 2.*** *Find in terms of* ***a*** *and* ***b*** *the*

*vectors****:***

***(i) AX***

***(ii) OX***

***(c)*** *Find in simplest form the ratio* ***OX : OC.***

***15.*** *In the triangle* ***OAB,***  ***D*** *is a point on* ***OB*** *such that*

***OD:OB = 2:5*** *and* ***E*** *is the midpoint of* ***AB.***

**O**

**B**

**E**

**D**

***a***

***FE***

***A***

***b***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) OE***

***(ii) AD***

***(b)*** *Given that* ***AF = tAD*** *and* ***OF = hOE,*** *find the values of* ***t*** *and* ***h***

***(c)*** *Show that the points* ***O, F*** *and* ***E*** *are collinear*

***16.*** *In the triangle* ***ABC,***  ***D*** *is a point on* ***AC*** *such that*

***AD:DC = 3:2*** *and* ***E*** *is the midpoint of* ***BC.***

**A**

**C**

**E**

**D**

***b***

***FE***

***B***

***c***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) BD***

***(ii) AE***

***(b)*** *Given that* ***BF = tBD*** *and* ***AF = nAE,*** *find the values of* ***t*** *and* ***n***

***(c)*** *State the ratio of* ***BD*** *to* ***BF***

***17.*** *In the triangle* ***OAB,***  *Point* ***R*** *divides* ***AB*** *in the ratio* ***2:5***

*and point* ***T*** *divides* ***OB*** *in the ratio* ***1:3.***

**O**

**B**

**R**

**T**

***a***

***D***

***A***

***b***

***(a)*** *Express the following vectors in terms of* ***a*** *and* ***b:***

***(i) BT***

***(ii) OR***

***(iii) AT***

***(b)*** *Given that* ***AD = kAT*** *and* ***RD = hRO,*** *find the values of* ***k*** *and* ***h.*** *Hence*

*express vector* ***AD*** *in terms of* ***a*** *and* ***b***

***18.*** *In the triangle* ***OPQ,***   *and point* ***T*** *is*

*on* ***QS*** *such that* 

***q***

***p***

***T***

***R***

***Q***

***P***

***O***

***S***

***(a)*** *Express the following vectors in terms of* ***p*** *and* ***q:***

***(i) SR*** ***(ii) QS (iii) PT (iv) TR***

***(b)*** *Show that the points* ***P, T*** *and* ***R*** *are collinear*

***TRANSLATION***

***Summary:***

***1.*** *Translation deals with movement of an object to a new position*

***2.*** *A translation  means that an object is moved a distance* ***a*** *in the* ***x−****direction and a distance* ***b*** *in the* ***y−****direction*

***3.*** *A translation  moves point* ***P(x, y)*** *to a new position*  *Thus ,* ***Translation + object = image***

***EXAMPLES:***

***1.*** *A translation  maps the points* ***P(3, 7)*** *and* ***Q(6, 1)*** *onto**the points* *and*  *respectively. Find the coordinates of* *and* 

***2.*** *A triangle with vertices* ***A(2, 1)******B(2, 3)*** *and* ***C(4, 1)*** *is mapped onto its image by a translation  Find the coordinates of the image of the triangle* ***ABC.***

***3.*** *A translation  maps point* ***P*** *onto* *Find the coordinates of point* ***P***

***4.*** *Find the translation that maps point A****(2, 6)***  *onto*

***5.*** *A translation* ***T*** *maps point* ***P(2, 5)*** *onto* *Find the image of* ***Q(5, 7)*** *under translation* ***T***

***6.*** *A triangle with vertices* ***A(1, 2)******B(3, 4)*** *and* ***C(5, 2)*** *is mapped onto its image by a translation  followed by a translation*  *Find****:*** ***(i)*** *a single translation representing the two successive translations*

***(ii)*** *the coordinates of the image of the triangle* ***ABC.***

***7.*** *A triangle with vertices* ***A(2, 0)******B(1, −3)*** *and* ***C(−2, 1)*** *under goes a translation  to give triangle  Triangle  is then mapped onto triangle  by a translation *

***(a)*** *Find the coordinates of the vertices of****:***

***(i)*** *triangle *

***(ii)*** *triangle *

***(b)*** *Plot triangle* ***ABC*** *and its images on the same axes****.***

***8.*** *A translation  maps the line* ***y = 2x + 1*** *onto its image. Find the equation of the image line*