51.0 Simple Mechanics

Definitions to know

- A **machine** is a device which allows the supply of energy at one point to do work at another point.
- **Work:** Work is the product of force applied on a particle and the displacement the particle makes.
- **Energy** is the ability to cause change; can change the speed, direction, shape, or temperature of an object.
- **Power** is the rate of doing work. i.e. $\text{Power} = \frac{\text{Force} \times \text{distance}}{\text{time}}$
- **Load** is the weight being lifted by the simple machine.
- **Effort** is the force placed on the simple machine to move the load. Also called applied force or input force.

51.1 Simple machines

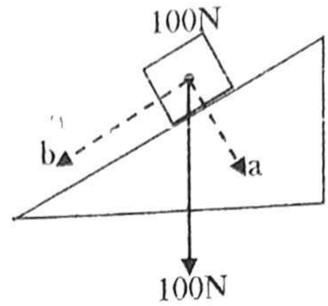
Simple machines are tools that make work easier. They have few or no moving parts and use energy to work. Make our work easier by letting us use less mechanical effort to move an object. They make work easier for us by allowing us to push or pullover increased distances. Simple machines give us an advantage by changing the amount, speed, or direction of forces. They allow us to use a smaller force to overcome a larger force.

The amount of effort saved when using machines is called **mechanical advantage** or MA.

Types of Simple Machines

1. **Inclined planes;** Ramp, Wedge, Screw.

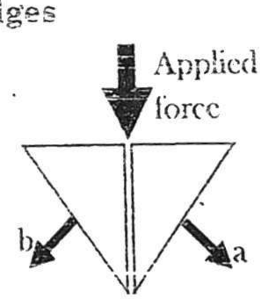
(i) Ramp



A flat surface that is higher on one end, slanting surface connecting a lower level to a higher level. You can use this machine to move an object to a lower or higher place. Inclined planes make the work of moving things easier, allows us to raise an object with less effort than if we lifted it directly upward. You would need less energy and force to move objects with an inclined plane.

Examples of Ramps; Slanted Road, Path up a Hill, Slide.

(ii) Wedges



A wedge is a simple machine used to push two objects apart. A wedge is usually made up of two inclined planes. These planes meet and form a sharp edge. This edge can split or push objects apart. A wedge is an inclined plane which moves. A wedge can also be used as a lifting device, by forcing it under an object. Most wedges (but not all) are combinations of two inclined planes. Can also be round, like the tip of a nail. The narrower the wedge (or the sharper the point of a wedge), the easier it is drive it in and push things apart.

Examples of Wedges on the farm:
Knife, Axe, Forks, Nails.

2. Levers

A lever is a rigid body capable of turning about a fixed point. This turning point is called the **fulcrum (Pivot)**. An object that a lever moves is called the **load**. The load is a force or object which must be overcome by the lever. The applied force or effort or input force is the force you use to move the lever and Lifts or moves loads. By changing the position of the fulcrum, you can gain extra power with less effort. The closer the object is to the fulcrum, the easier it is to move.

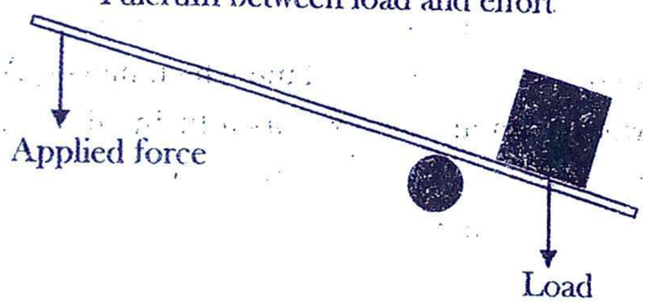
It is the same principle as the inclined plane, the greater the distance over which the force must be applied, the smaller the force required to do the work (lift the load).

There are three classes of levers:

- First class.
- Second class.
- Third class.

(i) First Class Levers

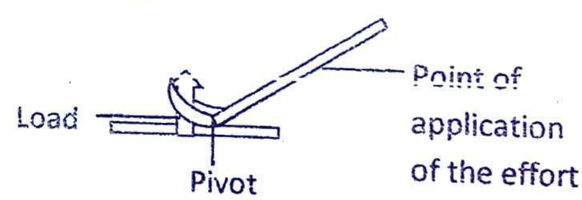
Fulcrum between load and effort



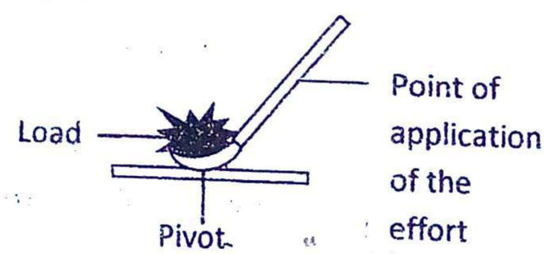
- Fulcrum (Pivot) in the center – between load & effort.
- The fulcrum is placed close to the load, and this will let you move the load with just a small applied force (effort).
- This type of lever system gives you a mechanical advantage, which means that the force you apply gets multiplied, so you can put a large force on the load.
- This type of lever system takes advantage of another property of some levers: they reverse the direction of the force. You can push in one direction, and the load moves the other way.
- If the fulcrum is nearer the applied force (effort), much more force than the force of the load itself must be applied.
- If you're lifting something, it will require much more force than would be needed if you were to just lift the load by yourself; this type lever system makes the work harder.

Examples of first Class Levers:

Claw hammer

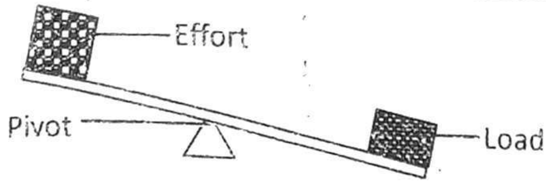


Crow bar

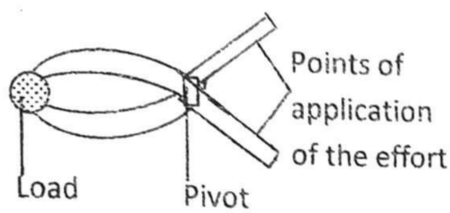


Sea saw

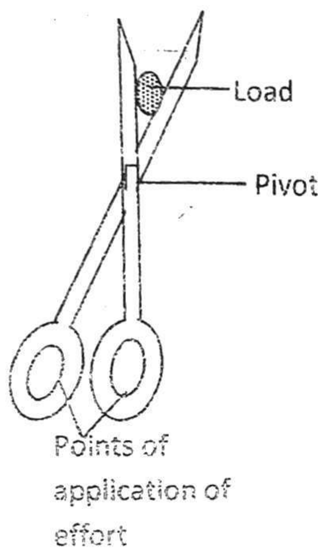
This is commonly used to determine weight of objects or human beings using standard weights. The sea saw operates on the principle of moments.



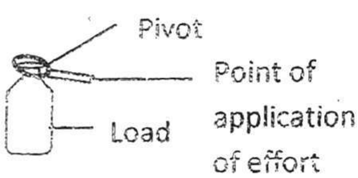
Scissors



Pair of pliers

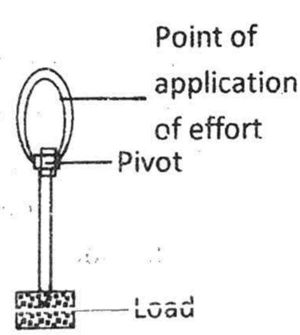


Bottle opener

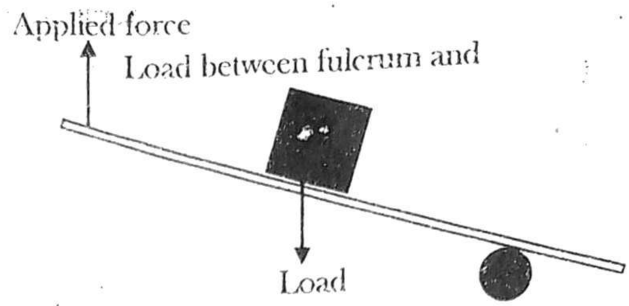


Steelyard

Commonly used to pick leaves or rubbish from compounds.



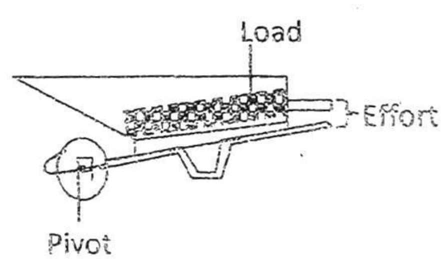
(ii) Second Class Levers



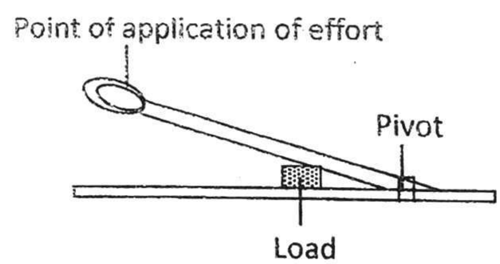
The load is in the center between the fulcrum and the applied force or effort. Causes the load to move in the same direction as the force you apply. When the load is nearer to the fulcrum, the effort needed to lift the load will be less. If you want to move a very large load with a small effort, you must put the load very close to the fulcrum.

Examples of 2nd Class Levers on the farm

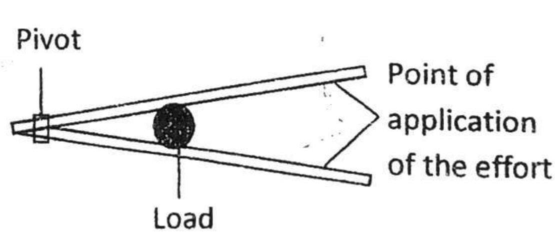
Wheel barrow



Paper cutter

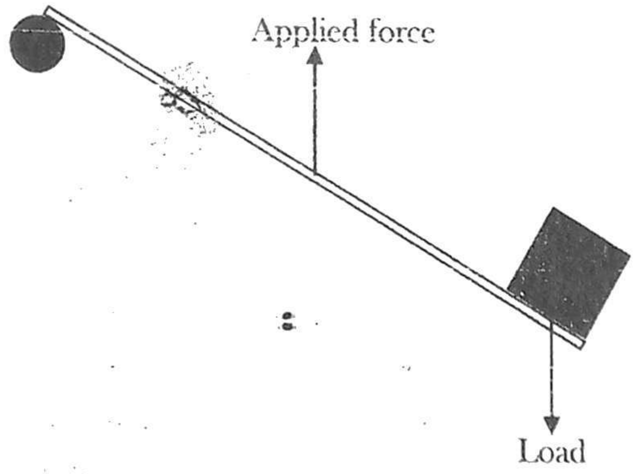


Nut cracker



(iii) Third Class Levers

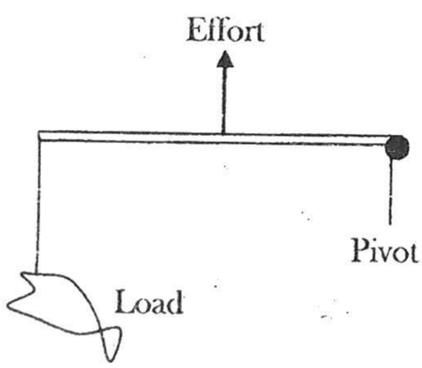
Effort between fulcrum and load



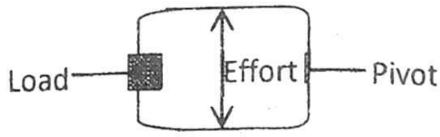
- The applied force or effort is in the center between the load and fulcrum
- This lever system does not give any mechanical advantage
- No matter where you apply the force, the force you apply must always be greater than the force of a load
- No matter how close or how far the load is from the fulcrum, the effort used to lift the load, has to be greater than the load!
- The load moves in the same direction as the force you apply. A motor is usually used with this lever system to lift loads at a distance

Examples of Third Class Levers on the farm

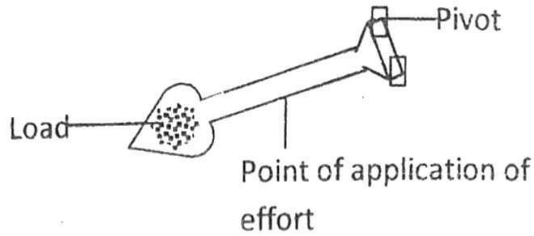
Fishing rod



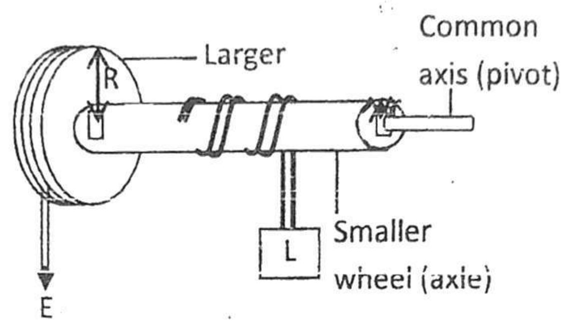
Pair of tongs



Spade

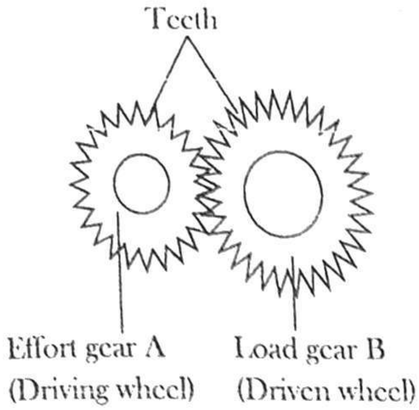


3. Wheel and Axle



- A wheel with a rod, called an axle, through its center lifts or moves loads.
- The axle is a rod that goes through the wheel, this lets the wheel turn.
- The wheel & axle can be used as a tool to multiply the force you apply or to multiply the distance traveled.
- A lever that is able to rotate through a complete circle (360°).
- The circle turned by the wheel is much larger than the circle turned by the axle.
- The increased distance over which the force is applied as the wheel turns results in a more powerful force on the axle, which moves a shorter distance.
- Trade-off: The larger the diameters of the wheel, the less effort you need to turn it, but you have to move the wheel a greater distance to get the same work done.

4. **Gears**



Gears are rigidly fixed on the axis and turned with their axis. If the effort is applied on a small gear, it drives a large gear which has a load attached to it.

5. **Screws**

The screw is used for the purpose of holding things together. The distance between successive threads of the screw is called the **pitch.**

6. **Pulleys**

This is a wheel with a grooved rim. There are three types of pulley systems;

- Single fixed pulley
- Single movable pulley
- Block and tackle pulley.

7. **The hydraulic press/lift**

40.2 Simple calculations in mechanics

These involve load, effort, pressure, work, force, energy, friction, mechanical advantage, efficiency and velocity ratio.

The ratio of load to effort is called the "**mechanical advantage**" i.e.

$$\text{Mechanical advantage} = \frac{load}{effort}$$

The ratio of distance moved by the effort to distance moved by the load is called the "**velocity ratio of the machine**" or "Speed ratio" i.e.

$$\text{Velocity ratio of a machine} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

The ratio of mechanical advantage to velocity ratio of a machine expressed as percentage is called the "**Efficiency of a machine**" i.e.

$$\text{Efficiency of a machine} = \frac{\text{mechanical advantage}}{\text{velocity ratio}} \times 100$$

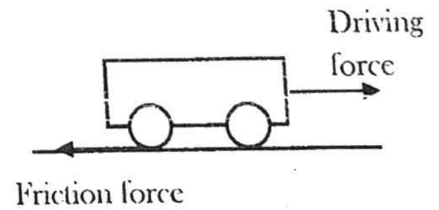
Efficiency of a machine may also be defined as the ratio of "work done on the load to work done by the effort" i.e.

$$\text{Efficiency of a machine} = \frac{\text{workdone on the load}}{\text{work done by the effort}}$$

$$\text{Work done on the load} = \text{Load} \times \text{Distance moved by the load}$$

$$\text{Work done by the effort} = \text{Effort} \times \text{Distance moved by the effort}$$

Friction is the force which opposes the relative sliding motion of two surfaces in contact with one another. This therefore means that friction always act in the direction opposite to that of actual motion since it opposes motion.



Friction force $= \mu R$, where μ is the coefficient of friction and **R** is the normal reaction.

Example

Find the frictional force on the car tyre whose mass is 2kg given that the coefficient of friction μ is 0.2.

Solution

$$Frictional\ force = \mu R$$

$$But \quad R = mg$$

Where m is the mass in **kg** and **g** is the acceleration due to gravity

$$\Rightarrow Frictional\ force = \mu mg$$
$$= 0.2 \times 2 \times 10$$
$$\therefore Frictional\ force = \mathbf{4N}$$

Examples on calculations of force

1. Find the gravitational force of a body of 2kg on earth.

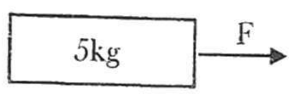
Solution

$$Gravitational\ force, F = mg$$

Where m is the mass in kg and **g** is the acceleration due to gravity

$$\Rightarrow \mathrm{F} = 2 \times 10$$
$$\therefore \mathbf{F = 20N}$$

2. A body of mass 5kg is acted on by a force in horizontal direction as shown below.



If the force causes an acceleration of $4m/s^2$, find the force F.

Solution

$$Force, F = ma$$

Where m is the mass in **kg** and a is the acceleration of the body

$$\Rightarrow Force, F = 5 \times 4$$
$$\therefore \mathbf{F = 20N}$$

3. A body of mass 5 kg is lifted through a distance of 6m. Calculate the work done.

Solution

$$Work\ done = Force \times distance\ moved$$

$$\text{But,} \quad Force, F = mg$$
$$\Rightarrow Force, F = 5 \times 10$$
$$\therefore \mathbf{F = 50N}$$

Now

$$Work\ done = 50 \times 6$$
$$\therefore Work\ done = \mathbf{300J}$$

4. Calculate the work done when a force of 30N moves through a distance of 9cm.

Solution

$$Work\ done = Force \times distance\ moved$$
$$= 30 \times \frac{9}{100}$$
$$\therefore Work\ done = \mathbf{2.7J}$$

KINETIC ENERGY

This is the energy possessed by a body in motion e.g. running water, moving bullet. Kinetic energy is given by:

$$k.e. = \frac{1}{2}mv^2$$

Where m is the mass of the body and v is the speed or velocity.

Example

Find the kinetic energy of a body mass 2kg moving with a speed of **4m/s.**

Solution

$$k.e. = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 2 \times 4^2$$
$$\therefore k.e. = \mathbf{16J}$$

PONTENTIAL ENERGY

This is the energy possessed by a body due to its position above the ground. It lifts a body to some height above the ground. Work is done against gravitational force and it is stored in the body as potential energy.

When the body is allowed to fall, its potential energy reduces as it approaches the ground.

$$Potential\ energy = Work\ done$$
$$= F\ x\ d$$
$$but\ F = mg\ and\ d = h$$
$$\therefore p.e. = mgh$$

Where $g = 10m/s^2$ and h is the height above the ground.

Example

A stone of mass 8kg is lifted through a height of 2metres. Find the potential energy the stone develops (Take $g = 10m/s^2$)

Solution

$$p.e. = mgh$$
$$= 8 \times 10 \times 2$$
$$\therefore p.e. = \mathbf{160J}$$

PRESSURE

Pressure (P) is the force per unit area.

$$P = \frac{Force\ (F)}{Area(A)}$$

Example

(a) A car piston exerts a force of 200N on a cross sectional area of $40cm^2$. Find the pressure exerted by the piston.

Solution

Given

$$Force = 200N$$
$$Area = 40cm^2 = \frac{40}{10000} = 0.004m^2$$

Now

$$P = \frac{Force\ (F)}{Area(A)}$$
$$= \frac{200}{0.004}$$
$$\therefore P = \mathbf{50000Nm^{-2}}$$

(b) The pressure exerted on a foot pedal of cross sectional area $5cm^2$ is $200Nm^{-2}$. Calculate the force.

Solution

Given

$$Pressure = 200Nm^{-2}$$
$$Area = 5cm^2 = \frac{5}{10000} = 0.0005m^2$$

Now

$$P = \frac{Force\ (F)}{Area(A)}$$

$$200 = \frac{F}{0.0005}$$
$$\Rightarrow F = 200 \times 0.0005$$
$$\therefore F = \mathbf{0.1N}$$

More worked examples under simple machines

1. A machine of effort 200N lifts a load of 800N through a distance of 5m. If the effort moves through a distance of 100m, Find;
   (i) The mechanical advantage of the machine
   (ii) The velocity ratio of machine
   (iii)

Solution

(i)

$$M.A. = \frac{load}{effort} = \frac{800}{200}$$
$$\therefore \mathbf{M.A. = 4}$$

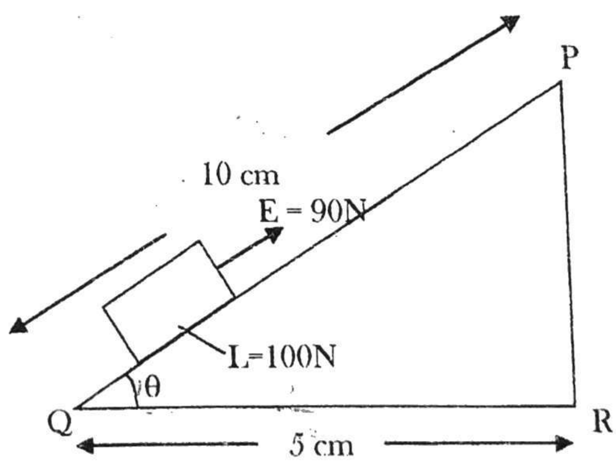
(ii)

$$V.R. = \frac{distance\ moved\ by\ effort}{distance\ moved\ by\ the\ load} = \frac{100}{5}$$
$$\therefore \mathbf{V.R. = 20}$$

(iii)

$$Efficiency = \frac{M.A.}{V.R.} \times 100$$
$$= \frac{4}{20} \times 100$$
$$\therefore Efficiency = 20\%$$

2. Below is an inclined plane used to lift a load from R to P as shown?



(a) Determine
   (i) mechanical advantage

(ii) angle of inclination
(iii) velocity ratio
(iv) Efficiency of the machine.

(b) What is meant by first class lever?

(c) By means of a lever, an effort of 50N moves a load of 200N through a distance of 3m. If the effort moves a distance of 16m. Calculate:

(i) The mechanical advantage
(ii) The efficiency.

Solution

(a) (i)

$$M.A. = \frac{load}{effort} = \frac{100}{90}$$

$$\therefore \mathbf{M.A. = 1.11}$$

(ii)

$$\cos\theta = \frac{Adjacent}{Hypotenuse} = \frac{QR}{QP}$$

$$\Rightarrow \cos\theta = \frac{5}{10} = 0.5$$

$$\therefore \theta = \cos^{-1}(0.5) = \mathbf{60^0}$$

(iii)

$$V.R. = \frac{length\ of\ incline}{height\ of\ incline} = \frac{QP}{PR}$$

$$= \frac{10}{10\sin 60^0} = \frac{1}{\sqrt{3}/2}$$

$$\therefore V.R. = \frac{2}{\sqrt{3}} = \mathbf{1.155}$$

(iv)

$$Efficiency = \frac{M.A.}{V.R.} \times 100$$

$$= \frac{1.11}{1.155} \times 100$$

$$\therefore Efficiency = 96.1\%$$

(b) This is where the pivot is in between the load and effort.

(i)

$$M.A. = \frac{load}{effort} = \frac{200}{50}$$

$$\therefore \mathbf{M.A. = 4}$$

(ii)

$$Efficiency = \frac{work\ done\ by\ the\ load}{work\ done\ on\ the\ effort} \times 100$$

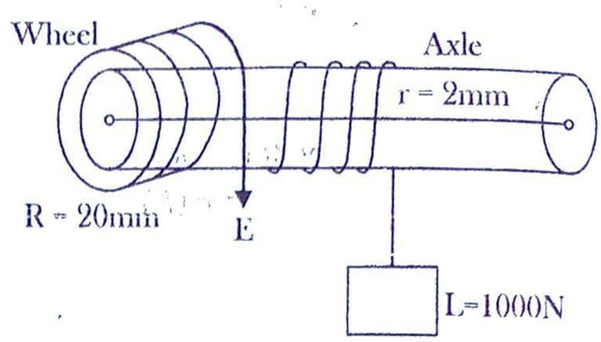
$$= \frac{L \times load\ distance}{E \times effort\ distance} \times 100$$

$$= \frac{200 \times 3}{50 \times 16} \times 100$$

$$= \frac{600}{800} \times 100$$

$$\therefore Efficiency = 75\%$$

Examples on Wheel and Axle



Assuming that the efficiency of the above system is 45%. Find;

(i) The effort required to raise the load.

(ii) The energy wasted when the effort moves through one turn.

Solution

(i)

After a complete turn, Effort (E) moves through a distance equal to $2\pi R$ and the load is raised through a distance $2\pi r$

Now

$$V.R. = \frac{distance\ moved\ by\ effort}{distance\ moved\ by\ load}$$

$$= \frac{2\pi R}{2\pi r} = \frac{R}{r} = \frac{20}{2}$$

$$\therefore \mathbf{V.R. = 10}$$

Also

$$M.A. = \frac{load}{effort} = \frac{1000}{E}$$

Given

$$Efficiency = 45\%$$

But

$$Efficiency = \frac{M.A.}{V.R.} \times 100$$
$$\frac{M.A.}{V.R.} \times 100 = 45$$
$$\frac{1000/E}{10} \times 100 = 45$$
$$\frac{10000}{E} = 45$$
$$\therefore \mathbf{E} = \frac{10000}{45} = \mathbf{222.2N}$$

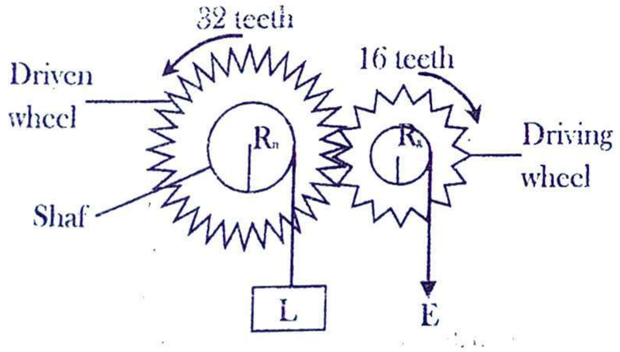
(ii)

$$work\ input = E \times effort\ distance$$
$$= E \times 2\pi R$$
$$= \frac{10000}{45} \times 2\pi \times \frac{20}{1000}$$
$$\therefore \boldsymbol{work\ input = 8.88\pi J}$$
$$work\ output = L \times load\ distance$$
$$= L \times 2\pi r$$
$$= 1000 \times 2\pi \times \frac{2}{1000}$$
$$\therefore \boldsymbol{work\ output = 4\pi J}$$

$Energy\ wasted$
$$= work\ input - work\ output$$
$$= 8.88\pi - 4\pi$$
$$= 4.88 \times \frac{22}{7}$$
$$\therefore \textbf{Energy wasted} = \mathbf{15.34J}$$

Gears

Gears are toothed wheels of different diameters like those of bicycles. Gears are used to change the velocity ratio and mechanical advantage of machines e.g. cars. This depends on which gears are engaged (grieved together).



With gears, the smaller wheel is called the "driving wheel" and the larger wheel is called the "driven wheel". This implies that the effort is applied on the smaller wheel (driving wheel) and the load is supported by the larger wheel (driven wheel). The two wheels rotate antagonistically.

Velocity ratio of gears is given by:

$$V.R. = \frac{number\ of\ teeth\ on\ driven\ wheel}{number\ of\ teeth\ on\ driving\ wheel}$$

For example, for the gears above:

$$V.R. = \frac{32}{16} = 2$$

Mechanical advantage of gears is obtained by conserving energy i.e.

$$Work\ done\ by\ the\ effort$$
$$= Work\ done\ on\ the\ load$$
$$effort \times \begin{matrix} distance\ moved\ by\ shaft \\ of\ driving\ wheel \end{matrix}$$
$$= load$$
$$\times \begin{matrix} distance\ moved\ by\ shaft \\ of\ driven\ wheel \end{matrix}$$
$$E \times 2\pi R_g = L \times 2\pi R_n$$
$$\frac{L}{E} = \frac{R_g}{R_n}$$
$$\therefore \mathbf{M.A.} = \frac{\boldsymbol{R_g}}{\boldsymbol{R_n}}$$

Example

A hydraulic machine has 120 teeth in the driven gears and 40 teeth in the driving gear. Calculate

(i) Its V.R.

(ii) Its M.A. (if the machine is 80% efficient)

<u>**Solution**</u>

(i)

$$V.R. = \frac{number\ of\ teeth\ on\ driven\ wheel}{number\ of\ teeth\ on\ driving\ wheel}$$
$$\therefore V.R. = \frac{120}{40} = 3$$

(ii)

$$Efficiency = \frac{M.A.}{V.R.} \times 100$$
$$\frac{M.A.}{3} \times 100 = 80$$
$$\Longrightarrow M.A. = \frac{80}{100} \times 3$$
$$\therefore \mathbf{M.A. = 2.4}$$