P425/1

PURE

MATHEMATICS

Paper 1

July/Aug 2024

3 hours

RUKUNGIRI DISTRICT SECONDARY SCHOOLS

JOIN MOCK EXAMINATIONS 2024.

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours.

INSTRUCTIONS TO CANDIDATES

*Answer all questions in section A and any **five** questions from section **B**.*

Any additional question(s) answered will not be marked.

All necessary calculations must be done in the answer booklet provided. Therefore, no paper should be given for rough work.

Graph paper is provided

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

1. Given that $\propto$ *and* β are the roots of the equation $x^2 - px + q = 0$ obtain a quadratic equation whose roots are α^{-2} and β^{-2}. *(05marks)*

2. Find vector equation of a line of intersection of planes $2x + 3y + 4z = 1$ *and* $x + y + 3z = 0$ *(05marks)*

3. If $\sin 3\theta = p$ and $sin^2\theta = \frac{3}{4} - q$ prove that $p^2 + 16q^3 = 12q^2$ *(05marks)*

4. Show that $\int_0^{\frac{\pi}{2}} sin^2\left(\frac{x}{6}\right) dx = \frac{\pi-3}{4}$. *(05marks)*

5. Find 3 terms of an A.P such that their sum is 33 and sum of their product is 563. *(05marks)*

   Find co-ordinates of the vertex and focus of the parabola $y^2 + 8y = 4x - 12$. *(05marks)*

6. Expand using binomical theorem $\frac{1}{\sqrt{4+4x}}$ up to the third term and hence find $\frac{1}{\sqrt{6}}$. *(05marks)*

7. Given that $y = e^x cos3x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$ *(05marks)*

8. Find co-ordinates of the vertex and focus of the parabola $y^2 + 8y = 4x - 12$. *(05marks*

SECTION B

9. (a) If x is a real number , prove that the expression: $\frac{x^2+2x-11}{2(x-3)}$

   does not lie strictly between 2 and 6. *(07marks)*

   (b) If $a^x = b^y = c^z$ *and* $b^2 = ac$ prove that $y(x + z) = 2xz$. *(05marks)*

10. (a) Prove that $\frac{cos3\theta}{cos\theta} - \frac{cos6\theta}{cos2\theta} = 2\,(cos2\theta - cos4\theta)$ *(05marks)*

    (b) Solve for θ in the equation for $0^o \leq \theta \leq 360^o$ $sin\theta - sin2\theta = sin4\theta - sin3\theta$. *(07marks)*

11. (a) Evaluate $\int_0^1 \frac{4x^4}{\sqrt{1-x^6}} dx$ *(07marks)*

    (b) integrate $sin2xsin5x$. *(05marks)*

12.(a) Find the values of z in $z^3 - 8i = 0$. *(05marks)*

(b)The arguments of complex numbers $z - 2$ and $z - 2i$ differ by $\pi/2$.
Find the locus of point $P(x, y)$ which represent $z = x + yi$ and describe the locus. *(07marks)*

13.(a) Use small changes to find the value of $344^{\frac{1}{3}}$. *(05marks)*

(b)A rectangular sheet is 50cm by 40cm wide. A square of xcm by xcm is cut off from each corner. The remaining sheet is folded to form an open box.
Find the value of x which will give maximum volume of the box and state the maximum volume of the box. *(07marks)*

14.(a) Find vector equation of the line through point $(1,3,-2)$ perpendicular to the line $r = \begin{pmatrix}4\\1\\3\end{pmatrix} + \lambda \begin{pmatrix}-2\\3\\-1\end{pmatrix}$ and line through points $A(0,0,-8)$, $B(1,2,-3)$ *(05marks)*

(b)Give two lines parametrically given by

$$r_1 = (t-1)i + (2-2t)j + (1+3t)k$$

$$r_2 = (1-3\alpha)i + (m-\alpha)j + (7+\alpha)k$$

if lines intersect. Find;

(i) values of t, m and α *(05marks)*

(ii) position vector of the point of intersection. *(02marks)*

15.(a) Find equation of the locus of point $P(x, y)$ which moves so that its Distance from point $(5,0)$ is a half its distance from line x-8=0. *(05marks)*

(b)Find equation of a circle passing through points $(2,3)$ and $(4,5)$ having its centre on line. $\frac{y-3}{4} = x$. *(07marks)*

16.(a) Solve the differential equation $2x\frac{dy}{dx} = 1 - y^2$ given that $y(1) = 0$. *(05marks)*

(b)According to Newton's law, the rate of cooling of a body in air is proportion to the difference between temperature of the body and that temperature of the ai
If the air temperature is kept at 20^0C and the body cools from 100^oC to 60^oC in 20 minutes.
In what further time will the body cool to 30^oC. *(07marks)*

END