P425/1

Pure Mathematics

Paper 1

Nov. 2023

Time: 3 Hours

WEST ANKOLE DIOCESE - CHURCH OF UGANDA

S.6 2024 ENTRANCE EXAMINATIONS

PURE MATHEMATICS

PAPER 1

TIME: 3 HOURS

INSTRUCTIONS:

*Answer **all** the **EIGHT** Questions in Section A and any **FIVE** from Section B.*

Any additional Question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40) MARKS

Answer all the Questions in this section.

1. Solve the simultaneous equations (05 marks)

$x + y + 2z = 3$

$2x + y - z = 5$

$3x + 2y + 5z = 8$

2. Solve the equation: (05 marks)

$\log_3 x + \log_x 9 = 3$

3. Solve $5\tan^2 A - 5\tan A = 2\sec^2 A$ for $0^0 \leq A \leq 360^0$ (05 marks)

4. Find the equation of the tangent to the curve.

$x^3 + 2y^3 + 3xy = 0$ at the point $(2, -1)$. (05 marks)

5. The 5th term of an arithmetic progression (AP) is 12 and the sum of the first 5 terms is 80. Determine the first term and common difference. (05 marks)

6. A line L passes through the point of intersection of the lines $x - 3y - 4 = 0$ and $y + 3x - 2 = 0$. If L is perpendicular to the line $4y + 3x = 0$, determine the equation of the line L (05 marks)

7. Determine the Binomial expansion.

$\left[1 + \frac{x}{2}\right]^4$

Hence evaluate $(2.1)^4$ to 2 decimal places. (05 marks)

8. The sum of the height and radius of a right circular cone is 9cm. Show that the maximum volume of the cone will be $36\pi cm^3$. (05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section.

9. (a) Given the geometrical Progression (G.P) 2, 6, 18, 54, …
Find the sum of the first ten terms of the G.P. (03 marks)

(b) In an Arithmetical Progression (A.P), the sum of the fifth and sixteenth terms is 44. The sum of the first 18 terms is three times the sum of the first ten terms.
Determine the:
(i) Value of the first term
(ii) Common difference of the A.P
(iii) Sum of the first 30 terms of the A.P (09 marks)

10. Express $8sin\theta - 15cos\theta$ in the form R sin $(\theta - \propto)$ where R and $\propto$ are constants. Hence find the maximum value of $8sin\theta - 15cos\theta$ and the smallest positive value θ at which the maximum value occurs. (12 marks)

11. (a) Differentiate with respect x.
(i) $\left(\frac{1+2x}{1+x}\right)^2$ (04 marks)

(ii) $\frac{x}{\sqrt{(1+x^2)}}$ (04 marks)

(b) Fine the gradient of the curve.
$x = \frac{t}{1+t}, y = \frac{t^3}{1+t}$ at the point $\left[\frac{1}{2}, \frac{1}{2}\right]$ (04 marks)

12. (a) Find the term independent of x in the expansion $\left(\frac{1}{x^2} - x\right)^{18}$ (04 marks)

(b) Show that, if x is small enough for its cube and higher powers neglected.

$$\sqrt{\left[\frac{1+x}{1-x}\right]} = 1 + x + \frac{1}{2}x^2$$

By putting $x = \frac{1}{7}$, show that $\sqrt{3} \approx \frac{196}{113}$ (08 marks)

3. When $f(x) = x^3 - ax + b$ is divided by $x + 1$, the remainder is 2 and $x + 2$ is a factor of $f(x)$. Find the values of a and b. (07 marks)

(b) If the roots of the equation $x^2 + 2x + 3 = 0$ are $\propto$ and β, form the equation whose roots are $\propto^2 - \beta$ and $\beta^2 - \propto$ (05 marks)

14. (a) Find $\frac{dy}{dx}$ and in terms of x, y when $x^2 + y^2 - 2xy + 3y - 2x = 7$ (02 marks)

(b) if $x = a(t^2 - 1), y = 2a(t + 1)$ Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t. (05 marks)

(c) An error of $2\frac{1}{2}\%$ is made in the measurement of the area of a circle. What percentage error results in:

(i) the radius (03 marks)

(ii) the circumference. (02 marks)

15. (a) Show that the tangents to the curve $y = 4 - 2x - 2x^2$ at points $(-1,4)$ and $(1/2, 2\,1/2)$ respectively, pass through the point (06 marks)

$(-1/4, 5\,1/2)$

(b) Calculate the area enclosed by the curve $y = 4 - 2x - 2x^2$ and the $x - axis$ (06 marks)

16. (a) Sketch the curvey $= 5x(2 - x)$ (08 marks)

(b) The area bounded by the curve and the $x - axis$ is rotated about the $x - axis$ through one revolution. Determine the volume of the solid generated. (04 marks)

END