P425/1
PURE MATHEMATICS
Paper 1
JULY/AUGUST, 2024
3 Hrs

ASSHU BUSHENYI DISTRICT MOCK EXAMINATIONS 2024
UGANDA ADVANCED CERTIFICATE OF EDUCATION
PURE MATHEMATICS
Paper 1
3 hours

INSTRUCTIONS TO CANDIDATES.

- Attempt all the eight questions in section A and five questions from section B.
- Any additional question(s) answered will not be marked.
- Begin each answer on a fresh page.
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- All the necessary working must be clearly shown on the same page as the rest of the answer.

SECTION A (40 MARKS)

1. Solve the equation $x - 9\sqrt{x} + 20 = 0$. (05 marks)
2. Express $y = -x^2 + 10x - 21$ in the form $a(x+p)^2 + q$. Hence state and distinguish the turning point of y. (05 marks)
3. Find the co-ordinates of the point of intersection the line passing through the points A (2, -1, 5) and B (3, 1, -2) and the plane $7x + 2y + z = 19$. (05 marks)
4. Differentiate $e^{-x}(\sin x - \cos x)$ and hence evaluate $\int_0^{\frac{\pi}{2}} e^{-x} \cos x dx$. (05 marks)
5. A parallel line to the x-axis cuts the curve $y^2 = 4x$ at point M and the line $x = -2$ at point N. Find the equation of the locus of the midpoint of $\overline{MN}$. (05 marks)
6. Show that $\int_e^{e^3} \frac{dx}{x(1nx)^2} = \frac{2}{3}$. (05 marks)
7. Solve the equation $tan^{-1}(2x) + tan^{-1}(3x) = \frac{\pi}{4}$. (05 marks)
8. Find a particular solution of the equation $\frac{dy}{dx} = e^{2x}$-$3y$ given that $y(0) = 1$. (05 marks)

SECTION B (60 MARKS)

9(a) Solve the simultaneous equations. $x - 2y = 1, 3xy - y^2 = 8$. (05 marks)

(b) By using the substitution $p = x + \frac{1}{x}$,
Solve the equation $2x^4 + x^3 - 6x^2 + x + 2 = 0$. (07 marks)

10(a) Evaluate $\int_0^{\frac{\pi}{2}} \cos 3x \cos 2x dx$. (05 marks)

(b) Find $\int \frac{11x+12}{(2x+3)(x^2-x-6)}$dx. (07 marks)

11(a) Find the co-ordinates of the foot of the perpendicular from the point P (2, -1, 3) to the line $\frac{x-2}{-1} = \frac{y+1}{3} = \frac{z+4}{2}$. (05 marks)

(b) Find the Cartesian equation of the plane through the points A (1, 0, -2) and B (3, -1, 1) which is parallel to the line with vector equation
r $=3\boldsymbol{i} + (2\beta - 1)\boldsymbol{j} + (5 - \beta)\boldsymbol{k}$. Hence find the equation of the line of intersection of this plane with the plane $x - y + 3z = 5$. (07 marks)

12(a) Find the equation of the locus of z defined by arg $\left[\frac{z-1}{z+1}\right] = \frac{\pi}{4}$, where z is a complex number. (05 marks)

(b) Assuming that x is very small that terms in x^3 and higher powers can be neglected, find a quadratic approximation to $\sqrt{\frac{1-x}{1+2x}}$ and state the range of x- values for which the expansion is valid. (07 marks)

13(a) Show that $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$. (05 marks)

(b) Express $10 \sin x \cos x + 12 \cos 2x$ in the form $R\sin(2x + \alpha)$ where α is an acute angle and R is a positive constant. Hence state the minimum value of $\frac{1}{10 \sin x \cos x + 12 \cos 2x + 5}$ and the smallest value of x for which it occurs. (07 marks)

14(a) Differentiate $\frac{(x-1)e^{4x}}{(x+1)^3}$ with respect to x. (06 marks)

(b) Given that $y = \tan(log_e^x)$, prove that $X\frac{d^2y}{dx^2} + (1-2y)\frac{dy}{dx} = 0$. (06 marks)

15(a) Show that circles
$x^2 + y^2 + 4x - 2y - 11 = 0$ and $x^2 + y^2 - 4x - 8y + 11 = 0$ are orthogonal. (05 marks)

(b) A triangle ABC has vertices A (-3, 2), B (1, 4) and C (5, 2). Find the co-ordinates of the point of intersection of the perpendicular bisectors of sides AB and BC. Hence obtain the equation of the circle circumscribing triangle ABC. (07 marks)

16. The rate of increase of temperature, T of a liquid being heated in an oven is proportional to the excess temperature of the oven over that of the liquid. If the temperature of the liquid rises from 0°C to 120°C in five minutes and the temperature of the oven is maintained at 180°C, find the;

(a) temperature of the body after a further five minutes. (09 marks)

(b) time, to the nearest minute it takes for the temperature to rise to 140°C. (03 marks)