1. Solve the simultaneous equations; $\frac{1}{2y} + \frac{1}{x} = 4$; $\frac{3}{x} - \frac{1}{y} = 7$. (5marks)

2. Prove that; $\frac{log_2 x - log_2 x^2}{log_4 x^3} + \frac{5}{3} = log\,10$. (5marks)

3. Given the parabola $y^2 = 8x$,
a) Express a point T on the parabola in parametric form using t as the parameter. (2marks)
b) If parameter r gives point R, show that the gradient of chord TR is $\frac{2}{t+r}$. (3marks)

4. Find $\int x^3 e^{x^2} dx$. (5marks)

5. The line $r = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ a \\ b \end{pmatrix}$ meets a plane P perpendicularly at the point $(3,1,2)$.
Find the vector equation of the plane. (5marks)

6. Solve $sin(120^0 + 3x) = cos(90^0 - x)\ for\ 0^0 \leq x \leq 90^0$. (5marks)

7. A roll of fencing material $152m$ long is used to enclose a rectangular area using two existing perpendicular walls. Find the maximum area enclosed. (5marks)

8. Solve the differential equation $\frac{dy}{dx} x - x = y$ given that $y = e$ when $x = e$. (5marks)

SECTION B : (60MARKS)

9. a) Prove that; $^{n+1}_{r+1}C + {}^{n+1}_{r+2}C = {}^{n+2}_{n-r}C$. (6marks)

b) Two blue, three red and four black beads are to be arranged on a circular ring made of a wire so that the red are separated. Find the number of different arrangements. (6marks)

10. Given that; $f(x) = \frac{1+2x}{1-x}$
a) Find Maclaurin's expansion of $f(x)$ upto the term in x^3. (8marks)
b) Hence, find the value of $\frac{1.02}{0.99}$ to four significant figures. (4marks)

11. a) Given that; $y\sin x + x\cos y = \frac{\pi}{2}$, find $\frac{dy}{dx}$. (4marks)

b) A square prism is always three times the width in length. If the volume increases at a constant rate of $4cm^3 s^{-1}$, find the rate of change of the cross-sectional area when the width is $12cm$. (8marks)
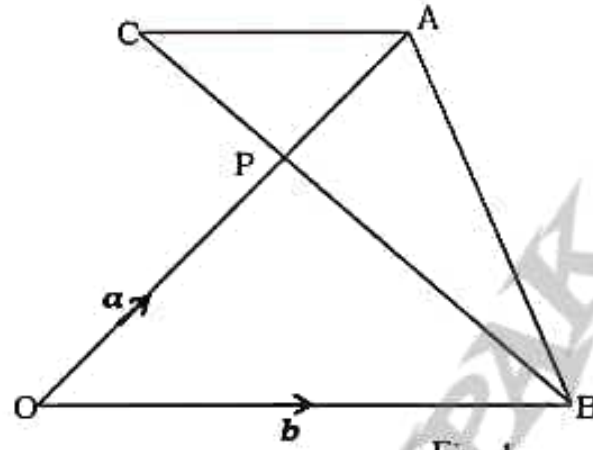
12.

Fig. 1

Figure 1 shows points A and B with position vectors a and b respectively. $3AC = BO$.
a) Express each of the following in terms of vectors a and b.
i) BA (2marks)
ii) BC (3marks)

b) Find the ration $BP:PC$ (7marks)

13. a) Prove that $\cos(\tan^{-1}x) = (x^2+1)^{-\frac{1}{2}}$. (4marks)
b) i) Prove that $\frac{\cos^2 4x + \cos 4x + \sin^2 4x}{\cos^2 4x - \cos 4x + si\ ^2 4x} = 3$ for $0 \le x \le \pi$. (4marks)

14. The lines L_1 and L_2 are perpendicular and intersect at $P(0,5)$. Line L_1 meets the x-axis in the first quadrant at Q such that $PQ = 13$ units. If L_2 meets the x-axis at R, without graphical construction, find the area of the triangle PQR. (12marks)

15. Given that $Z1 = 2 - 3i, Z_2 = 1 + 2i$ and $Z_3 = 3 - 4i$.
a) Express $\frac{Z_1 + Z_2}{Z_1 Z_2}$ in the form $a + bi$ where a and b are real numbers. (6marks)
b) Find a polynomial $p(x)$ of degree four where the roots of $p(x) = 0$ are Z_2 and Z_3. (6marks)

16. Evaluate; $\int_2^3 \frac{x^4 - x^3 - x^2 + 4x - 1}{(x-1)(x^2+1)} dx$. (12marks)

END