THE VIRTUAL SCHOOL OF MATHEMATICS
PURE MATHEMATICS
3 HRS

SECTION A (40 MARKS)

1. Solve the in equality $\frac{x+3}{x-1} > 2$ for which interval its valid.

2. Given the equation $2x^2 - 3x - 8 = 0$ has roots $\propto and\ \beta$, find the equation whose roots are a and c such that $\propto = \frac{\beta}{a-1}$ and $\beta = \frac{\propto}{c-1}$

3. Expand $\sqrt{(x-2)}$ in descending powers of x as far as the 3rd term. Hence evaluate $\sqrt{2}$ to 5 significant figures.

4. Evaluate the integral of $\int_0^1 \frac{dx}{\sqrt{9-4x^2}}$

5. Use the method row – reduction to echelon form to solve,
$$2a + b + 3c = 11$$
$$4a + 3b + c = 15$$
$$a + 2b - 2c = 3$$

6. Given that $x = \sqrt{\left(\frac{y^2-2}{5}\right)}$, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$

7. Solve for x in $e^{2x} - 5e^x + 6 = 0$, hence or otherwise find the value of $\sqrt{e^5}$

8. Solve for θ in the range $0^0 \leq \theta \leq 720^0\ in\ cos4\theta + cos2\theta + cos6\theta = 0$.

SECTION B (60 MARKS)

9. Sketch the curve $y = \frac{x^2+2x-8}{x^2+x-2}$

10. (a) Prove that $\int_0^{½\pi} \frac{4cosx}{4-sin^2x}\ dx = 1n\sqrt{3}$

    (b) Use the expansion of $\sqrt{\left(\frac{1-x}{1+x}\right)}$ up to the third term to evaluate the integral of $\int_0^{\pi/6} \sqrt{\left(\frac{1-x}{1+x}\right)}\ dx$

11. (a) Find the coordinates of intersection of the circles
$x^2 + y^2 - 6x + 4y = 13$ and $x^2 + y^2 - 10x + 10y = 15$

    (b) (i) Find the focus and vertex of the parabola $y = 2x^2 + 3x - 5$

    (ii) Find the equations of the normal and tangent to the circle whose equation is given by $x^2 + y^2 - 10x - 6y + 24 = 0$ at (2,2)

12. (a) Find the derivatives with respect to X of the following functions;
    (i) $\frac{cos2x}{1+sin2x}$

    (ii) $in\ (secx + tanx)$

    (b) $\int_0^{\pi/2} x^2 sinx\ dx$

13. If $Z = \frac{(2-i)(5+12i)}{(1+2i)^2}$

    (a) Find the;
    (i) Modulus of Z
    (ii) Argument of Z.

(b) Represent Z on a complex plane

DRIVE WELL

(c) Write Z in the polar form.

14. (a) Sketch the curve $y = x(x + 2) and\ y = x(4 - x)$ on the same axes.

(b) Find the area enclosed by the two curves in (a)

(c) Determine the volume of the solid generated when the area enclosed by the two curves in (a) is rotated about the x – axis.

15. (a) (i) Show that the points (1, 2, 3), (3, 8, 1) and $(7, 20, -3)$ are collinear.
(ii) Coordinates A $(3,8,1) and\ B(7,20,-3)$ are externally bisected from point B by Point M, find coordinates of M.

(b) (i) Find the point of intersection of the lines, $\boldsymbol{r} = 2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + 5\boldsymbol{k} + \curlywedge (-\hat{\boldsymbol{\imath}} - 2\boldsymbol{k})$ and
$\frac{x-1}{1} = \frac{y-2}{0} = \frac{z-1}{3}$

(c) Find the shortest distance of $P(3, +1, -1)$ from the line
$\boldsymbol{r} = (1 - \curlywedge)\hat{\imath} + (2 \curlywedge - 1)\hat{\boldsymbol{\jmath}} + +(2 \curlywedge - 2)\hat{\boldsymbol{k}}$

16. At 3:00pm, the temperature of a hot metal was 80^0C and that of the surrounding 20^0C. At 3:03pm, the temperature of the metal had dropped to 42^0C. The rate of cooling of the metal was directly proportional to the difference between its temperature θ and that of the surroundings.

(a) (i) Write a differential equation to represent the rate of cooling of the metal.
(ii) Solve the differential equation using the given condition.

(b) Find the temperature of the metal at 3:05pm.