SERIES TEST I

TIME: 1HOUR 30MINUTES

INSTRUCTIONS;

Answer **ALL** Questions.

1. A geometric progression has the sum of the ~~sum of the~~ first and second terms equal to -4, if the sum of the fourth and the fifth terms is 108. Calculate the;
   (a) first term
   (b) common ratio of the progression. (5marks)

2. How many terms of the G.P $2 + 2\times(1.1) + 2\times(1.1)^2 + \ldots\ldots\ldots$ must be taken for the sum to exceed 100? (5marks)

3. The first, second and fourth terms of an arithmetic progression form a geometric progression. Find the common ratio of the G.P (5marks)

4. A man pays premium of 100 dollars at the beginning of every year to an insurance company on an understanding that at the end of 15years they can receive back the premium he had paid with 5% compound interest. what did he receive? (5marks)

5. .The n^{th} term of a series is 3^n+4n. Calculate the sum of the first 20 terms of the series. (5marks)

6. The first terms of the arithmetic progression (A.P) and geometric progression (G.P) are equal. The common ratio of a G.P is equal to the common difference of the A.P while the third term of the AP is also equal to the second term of the GP. If the fourth term of an AP is 10, find the two possible values of their first term. (7marks)

7. Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at the beginning of 2019 with shs 800,000 and continue to deposit the same amount at the beginning of every year. How much will she receive at the end of 2022 if she made no withdrawal within this period? (6marks)

8, a) The eighth term of an arithmetic progression is twice the third term and the sum of the first eight terms is 39. Find the first three terms of the progression and show that its sum to n terms is $\frac{3n}{8}$(n + 5). (6marks)

b) Find how many terms of the series $1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots\ldots.$ must be taken so that the sum will differ from the sum to infinity by less than 10^{-6}. (6marks)

END

MARKING GUIDE

No. 1

a - 1st term, r - common difference

$$a + ar = -4 \quad B_1$$

$$ar^3 + ar^4 = 108 \quad B_1$$

$$r^3(a + ar) = 108 \quad m_1$$

$$-4r^3 = 108$$

$$r = \sqrt[3]{-27} \quad A_1$$

$$= -3$$

from $a + ar = -4$

$$a = \frac{-4}{1+r}$$

$$= \frac{-4}{1-3} \quad A_1$$

$$= 2.$$

05

No. 2.

$$2 + 2\times(1.1) + 2\times(1.1)^2 + \ldots$$

$a = 2$, $r = 1.1$ m_1

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right) \geq 100 \quad m_1$$

$$\frac{2(1.1^n - 1)}{1.1 - 1} = 100$$

$$1.1^n = 6 \quad m_1$$

Logarithm on both sides

$$\log 1.1^n = \log 6 \quad m_1$$

$$n = \frac{\log 6}{\log 1.1}$$

$$n = 18.799 \quad A_1$$

19 terms must be taken

05

No. 3

Let a – 1st term, d – Common diff

G.P; $a + (a+d) + (a+3d) + \dots$

$$r = \frac{a+d}{a} = \frac{a+3d}{a+d}$$ m1

$$\Leftrightarrow a^2 + 3ad = a^2 + 2ad + d^2$$

$$d^2 - ad = 0$$ B1

$$d = 0 \text{ (ignore)}$$

$$\Rightarrow d = a$$ B1

$$\therefore r = \frac{a+a}{a}$$ m1

$$\frac{2a}{a}$$

Common ratio, $r = 2$. A1

05

No. 4

From: $A = P(1 + r/100)^n$

$P = \$100$, $n = 15$, $r = 5\%$ B1

Amount at the end of; 1st yr; $A_1 = P(1.05)$

2nd yr; $A_2 = P(1.05)^2$

⋮

15th yr; $A_{15} = P(1.05)^{15}$ B1

Total Amount $A_T = A_1 + A_2 + A_3 + \dots + A_{15}$

$$= P[1.05 + 1.05^2 + (1.05)^3 + \dots + (1.05)^{15}]$$

$$= P\left[\frac{a(r^n - 1)}{r-1}\right]$$ m1

$$= 100\left[\frac{1.05 \cdot (1.05^{15} - 1)}{1.05 - 1}\right]$$ B1

$$= \$\,2265.749177$$ A1

05

NO. 5

Series; $(3+4)+(9+8)+27+12)+\ldots$

$(3+9+27+\ldots)+(4+8+12+\ldots)$ M1

$\frac{a(r^n-1)}{r-1}+\frac{1}{2}n[2a+(n-1)d]$ B1

$=\frac{3(3^{20}-1)}{3-1}+\frac{1}{2}(20)[2(4)+(20-1)4]$ B1 B1

$=3,486,785,240$ A1

05

NO. 6

Let a - first term of G.P & A.P
$d=r=x$

AP	G.P
$u_1=a$	$u_1=a$
$u_3=a+2x$	$u_2=ax$
$u_4=a+3x$	

$\Leftrightarrow$ $a+3x=10$ ———— (i) B1

$a+2x=ax$ ———— (ii) B1

from $a=10-3x$ ———— *

* into (ii) M1

$10-3x+2x=ax(10-3x)$ M1

$3x^2-5x-6x+10=0$

$(x-2)(3x-5)=0$ A1

$x=2$ or $x=5/3$

$\therefore$ $a=10-3(2)$ or $a=10-3(5/3)$ B1 B1

$=4$ $=5$

07

NO. 7

$$\text{Amount } A = \frac{PR(R^n - 1)}{R - 1}$$

$P = 800,000$ B1

$R = (1 + r) = 1.05$ B1

$n = 4$ B1

$$\therefore A = 800,000(1.05)\frac{[1.05^4 - 1]}{1.05 - 1}$$ M1M1

$= 3,620,505$ /= A1

06

NO. 8

Let a - first term, d - common diff

nth term of AP $= a + (n-1)d$

$U_8 = 2U_3$

$a + 7d = 2(a + 2d)$ ———— (i)

Sum of nth term; $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_8 = 39 = \frac{8}{2}(2a + 7d)$ ———— (ii) M1

from (i) $a = 3d$ ———— *

* into (ii)

$\frac{8}{2}[2(3d) + 7d] = 39$

$52d = 39$ B1

$d = \frac{3}{4}$

$\therefore a = 3(\frac{3}{4})$

$= \frac{9}{4}$ B1

Terms; a, $a+d$, $a+2d$

$\frac{9}{4}$, 3, $\frac{15}{4}$, ... B1

Sum of n terms is given by;

$$S_n = \frac{n}{2}[2a + (n-1)d]$$ m1

$$= \frac{n}{2}\left[2(9/4) + (n-1)3/4\right]$$

$$= \frac{n}{2}\left[\frac{18}{4} + \frac{3}{4}n - 3/4\right]$$ A1

$$= n/2\left(\frac{15}{4} + 3/4 n\right)$$

$$\therefore S_n = 3/8 n(n+5) \quad \#$$

06

NO.8b

Series; $1 + 1/5 + \frac{1}{5^2} + \frac{1}{5^3} + \ldots$ is a G.P

$a = 1, \quad r = 1/5$

$$S_\infty = \frac{a}{1-r}$$ m1

$$= \frac{1}{1-1/5}$$

$$S_\infty = 5/4$$ A1

Let the no. of terms whose sum will differ from sum to ~~so~~ infinity by less than 10^{-6} be n.

$$S_n = \frac{a(1-r^n)}{1-r} \quad |r| < 1$$

$a = 1 \quad r = 1/5$ ~~M1~~

$$S_n = \frac{1(1-(1/5)^n)}{1-1/5}$$ B1

$$= 5/4\left[1-(1/5)^n\right]$$

But $S_\infty - S_n < 10^{-6}$

$$\frac{5}{4} - \frac{5}{4}\left[1-\left(\frac{1}{5}\right)^n\right] < 10^{-6}$$

$$\frac{5}{4}\left(\frac{1}{5}\right)^n \leq 10^{-6}$$

$$5\left(\frac{1}{5}\right)^n < 10^{-6} \times 4$$ B1

$$5^{1-n} < 4\times 10^{-6}$$

Introducing $\log_{10}$ &

$$\log 5^{1-n} < \log(4\times 10^{-6})$$ M1

$$1-n < \frac{\log(4\times 10^{-6})}{\log 5}$$

$$1-n < -7.7227$$

$$n > 1 + 7.7227$$

$$> 8.7227$$ A1

$\therefore$ $n = 9$ terms.

06