P425/1
PURE MATHEMATICS
Paper 1
July, 2023
3 hours

SHURE JOINT MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in section* **A** *and only five questions in section* **B**.

All necessary calculations **MUST** *be done on the same page as the rest of the answers.*

Any extra question(s) attempted in section **B** *will not be marked.*

Begin each other question on a fresh sheet of paper.

All working must be shown clearly.

Silent, non-programmable, scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Solve the simultaneous equations $x + siny = 1$

$$y + cos^{-1}x = \frac{\pi}{2}$$ *(05 marks)*

2. Find the coordinates of the point of intersection of the line.

$\frac{x}{3} = \frac{y-1}{-1} = \frac{z+1}{2}$ and the plane $x - 2y + 3z + 16 = 0$ *(05 marks)*

3. Evaluate $\int_0^1 3^{\sqrt{x}}\, dx$ giving your answer to three significant figures.

(05 marks)

4. Given that $cos45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that

$sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$ *(05 marks)*

5. Solve the differential equation; $\frac{dy}{dx} - ytanx = cosx$, given that $y(0) = \frac{\pi}{6}$

(05 marks)

6. Prove that if $\log_a\left(1+\frac{1}{8}\right) = l$, $\log_a\left(1+\frac{1}{15}\right) = m$ and $\log_a\left(1+\frac{1}{24}\right) = n$,

then $\log_a\left(1+\frac{1}{80}\right) = l - m - n$ *(05 marks)*

7. A point P is twice as far from the line $x + y = 5$ as from the point (3, 0). Find the locus of P.

(05 marks)

8. An inverted right circular cone of vertical angle 60^0 is collecting water from a tap at a steady rate of 20π cm^3/min. Find:-
   (i) The depth of the water after 10 minutes,
   (ii) The rate of increase of the depth at this instant. *(05 marks)*

SECTION B (60 MARKS)

Answer any five questions from this section.

9. (a) If $z = x + iy$, determine the Cartesian equation of the locus given by

$$\left|\frac{(z-1)}{(z+1-i)}\right| = \frac{2}{5}$$ *(6 marks)*

(b) Sketch the loci defined by the equations:

(i) $\arg(z+2) = \frac{-2\pi}{3}$ (ii) $\arg\left(\frac{z-3}{z-1}\right) = \frac{\pi}{4}$ *(6 marks)*

10. (a) Evaluate $\int_0^{\sqrt{3}} \frac{x^3}{(1+x^2)^{1/2}}\,dx$. *(05 marks)*

(b) Use the substitution $t = \tan {}^{x}/_{2}$ to evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{1-2\sin x+\cos x}\,dx.$

(07 marks)

11. (a) Use integration to show that the volume of the cone is

$V = \frac{1}{3}\pi r^2 h$ *(05 marks)*

(b) A student walks to school at a speed proportional to the square root of the distance he still has to cover. If the student covered 900m in 100 minutes and the school is 2500m from home, find how long he takes to get to school.

(07 marks)

12. (a) Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$. Hence or Otherwise solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ in the range $0^0 \le x \le 360^0$.

(b) Prove that, $\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$ hence or otherwise solve the equation $\tan(\theta - 45^0) = 6\tan\theta$ where $-180^0 \le \theta \le 180^0$ *(12 marks)*

13. (a) Use the substitution $y = x + \frac{1}{x}$ to solve the equation
$2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

(b) Prove that, if x is so small that its cube and higher powers can be neglected, $\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{x^2}{2}$ by taking $x = \frac{1}{9}$, prove that $\sqrt{5}$ is approximately equal to $\frac{181}{81}$. *(12marks)*

14.(a) Show that the points A, B and C with position vectors

$2\hat{\imath} + 3\hat{\jmath}$, $4\hat{\imath} + 5\hat{\jmath}$, $6\hat{\imath} + 9\hat{\jmath}$ respectively are the vertices of a triangle. Find the area of the triangle. *(5 marks)*

(b) Find a vector $\boldsymbol{r}$ perpendicular to the vectors $\boldsymbol{s} = 5\hat{\imath} + 3\hat{\jmath} + \boldsymbol{k}$ and $\boldsymbol{t} = -\hat{\imath} + 3\hat{\jmath} + 2\mathbf{k}$. Hence, find the equation of a plane passing through the point $A(5, -1, -2)$ and parallel to s and t. Find the angle between the plane and the line $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-2}{3}$. *(7marks)*

15.(a) A, B, C are the points (0, 0), (8, 0) and (4, 8) respectively.

(i) Show that the triangle ABC is isosceles.

(ii) Find the radius, and state the coordinates of the Centre of the circle through the Points A, B and C.

(iii) State the equation of the circle, and show that it is orthogonal to the circle $x^2 + y^2 + 2x + 18y = 62$. *(12 marks)*

16.(a) A watering can is in the shape of an inverted right pyramid with a square base 40cm x 40cm, and of height 80cm. Show that when the depth of the water in the container is h, the volume of the Water is $\frac{1}{12}h^3$.

(b) Given that the container is initially full, and the water starts to flow through the vertex at a constant rate such that when $h = 40$cm, the depth is falling at a rate of $\frac{1}{15}cms^{-1}$ find:-

(i) The rate at which the volume is decreasing

(ii) The time taken for the volume to change from $\frac{1}{2}v_0$ to $\frac{1}{8}v_0$ where v_0 is the original volume. *(12 marks)*

END