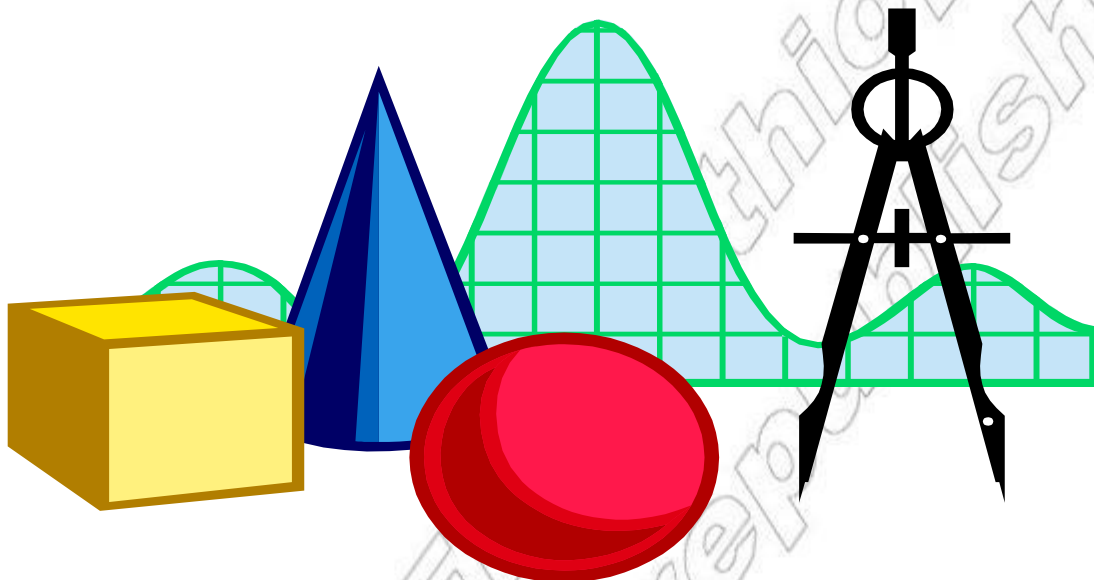


UNIT

5

GEOMETRIC CONSTRUCTION



Learning Competencies:

Up on completion of this unit you should be able to:

- ✓ Identify different types of plane geometry and their basic elements;
- ✓ Construct different types of geometrical figures;
- ✓ Apply methods and rules of construction for different types of geometrical shapes.

5.1 Introduction

- When we say geometric construction it implies that there are things to be constructed, what do you think are the elements to be constructed? List them.

The geometric construction forms the basis for all technical drawings. The purpose of the geometric construction is to accurately develop plane geometric shapes ranging from squares, triangles, and three-dimensional cylinders to complex irregular curves and ellipses. These constructions are normally produced without the aid of a scale, but rather with simple drafting tools.

Engineers, designers, and drafters regularly perform the task of producing geometric construction in their work, applying the principles of plane geometry. The process involved in the production of geometric constructions requires a basic understanding of plane geometry. Geometric construction skills can be acquired and demand precision and the correct use of drafting instruments. In developing geometric constructions it is important that drafting tools be in good condition. 4H to 6H leads are normally used for constructions that produce very lightweight lines. These lines need not necessarily be erased when the construction is completed. A small error or inaccuracy in the solving of a geometric problem could result in a serious error in the final construction.

There are a number of basic geometric constructions with which the student should be familiar. At the beginning the student should follow the basic sequence in each illustration that follows the learner how to develop the various geometric forms.

5.2 Construction of Point, Line and Angle

1. Define in your own words:
- Point - Line - Angle
2. Try to illustrate them with sketch.

5.2.1 Points

The construction of geometric figures begins from representation of points on drawings. A *point* represents an exact location in space which has neither width, height nor depth. Points are commonly represented by intersection of two lines, a short cross bar on a line or by small cross (Fig.5.1).



Fig. 5.1 Representation of points

5.2.2 Lines

A line can be defined as:

- A path between two points
- A moving point
- Geometric figure that has only one dimension: length.

Line may be straight, curved, or a combination but the shortest distance between two points is called straight line and it is commonly referred simply as a "line".

If the length of line is indefinite or without fixed endpoint, its length is any distance you select. If the line has fixed end points mark them with small mechanically drawn cross hairs.

A straight line drawn from left to right relative to a horizontal reference line is called a *horizontal line* and when drawn from top to bottom perpendicular to the horizontal is

called *vertical line*. If two lines are equidistant throughout their length and will never meet or cross are called *parallel lines*. The symbol for parallel line is //. Two or more lines crossing each other are called *intersecting lines*. The exact location where two lines intersect is called *point of intersection*. Lines that intersect or cross and form a 90° or right angle are called *perpendicular lines*. The symbol for perpendicularity is \perp . Fig. 5.2 shows the types of lines mentioned above.

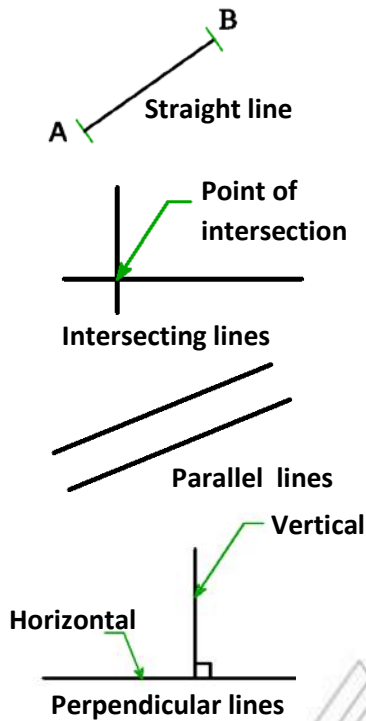


Fig. 5.2 Types of lines

Drawing Parallel Lines

To draw a line parallel to a given line through a given distance

Method 1 Fig. 5.3

1. Draw a line AB at any angle.

2. Locate point E near A and point F near B on line AB.
3. E and F as centers and radius CD, draw two arcs.
4. Draw a line (GH) tangent to the arcs using any straight edge.

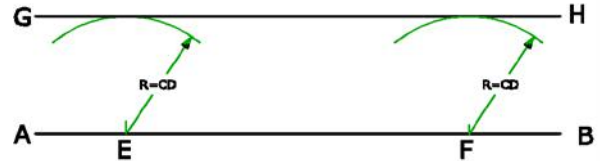


Fig. 5.3 Drawing parallel lines

Drawing Parallel Lines

Method 2 Fig. 5.4 (preferred method)

1. Draw line AB at any angle.
2. Open your compass to radius = CD.
3. With any point E on AB as a center, and radius = CD, strike an arc.
4. Align your set square with line AB with the T-square or another set square as a support.
5. Slide the set square until it is tangent to the arc.
6. Draw line GH using the edge of your set square.

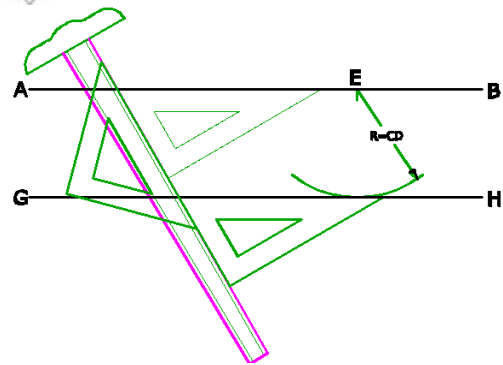


Fig. 5.4 Drawing parallel lines

Checkpoint 5.1

Draw a 120mm long line AB at an angle of 35° from the horizontal. Then, draw a line parallel to AB which is 25 mm away from AB.

Drawing Perpendicular Lines

Constructing perpendicular to a line through a point on the line (Fig. 5.5)

1. Draw line AB and locate point P on it.
2. With arbitrary radius r and p as a center, strike arcs to intersect AB at C and D.
3. With radius greater than $\frac{1}{2} CD$, and centers at C and D, draw arcs to intersect at E. Use straight edge to draw line EP. $EP \perp AB$.

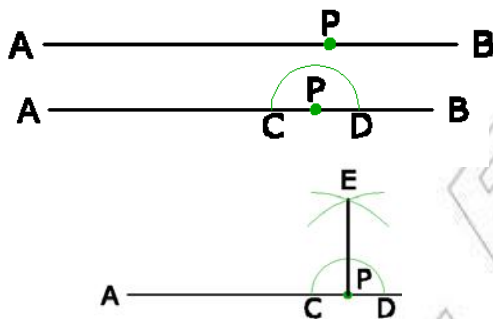
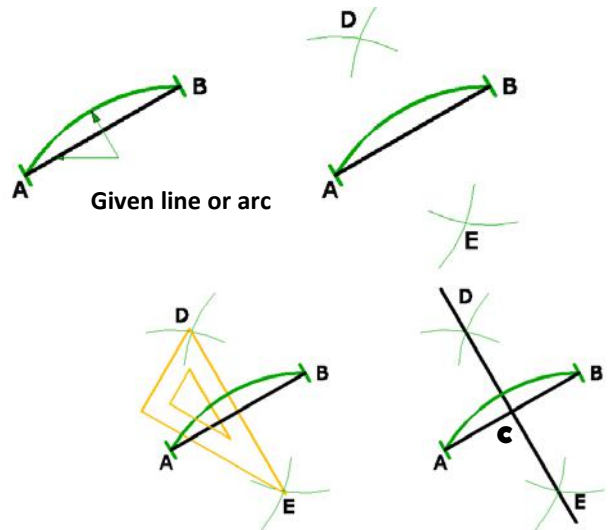


Fig. 5.5 Drawing perpendicular line through a point on the line

Constructing a perpendicular bisector of a given line or arc (Fig. 5.6)

1. Draw line or arc AB.
2. Adjust your compass to a radius greater than $\frac{1}{2} AB$.
3. With centers at A and B, draw intersecting arcs at D and E.
4. Draw a line DE. $DE \perp AB$ and $AC = CB$.



Constructing a perpendicular to a line from a point outside the line

Method 1 (Fig. 5.7)

1. Draw line AB and locate point P anywhere but not on AB.
2. P as a center and with arbitrary radius R , strike arcs on AB to get points C and D on the left and right side of P.
3. Adjusting the compass radius r to greater than $\frac{1}{2} CD$, and C and D as a center, strike two arcs intersecting at E.
4. Use straight edge to draw line $PE \perp AB$.

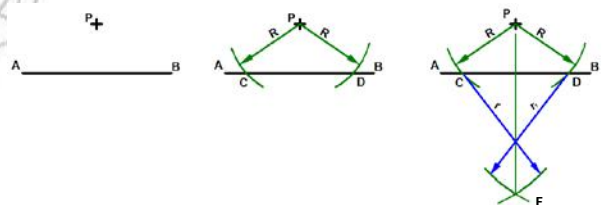


Fig. 5.7 Drawing perpendicular line to a given line through a point not on the line

Method 2 (Fig. 5.8)

1. Draw line AB at any angle and locate point P.
2. Draw a convenient inclined line from P to intersect AB at point C.
3. Locate the midpoint O of line PC as described in previous discussion.
4. With O as center and radius OP, draw an arc that intersects line AB at E.
5. Connect P and E. $PE \perp AB$.

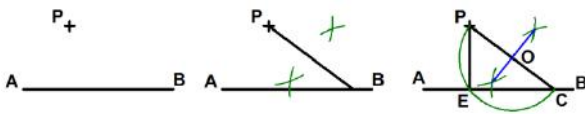


Fig. 5.8 Drawing perpendicular to a line through a point not on the line

Checkpoint 5.2

1. Draw line AB 80mm long. Draw perpendicular line to AB at a point 35mm from A on AB.
2. Draw an arc having a radius of 35 mm. Draw the perpendicular bisector of the arc.

Trisecting a Straight Line

Using 30-60° set square. (Fig. 5.9)

1. Draw a straight line AB.
2. Draw 30° lines from points A and B.
3. Extend them to intersect at C.
4. Draw 60° lines with the horizontal from point C in both directions and extend them to intersect AB at D and E. Check that $AD = DE = EB$.

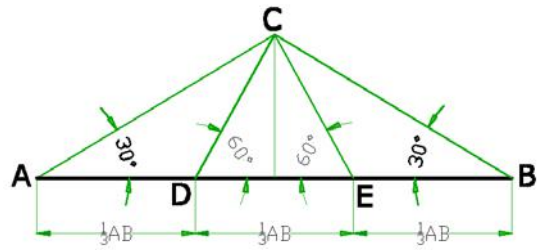


Fig. 5.9 Trisecting a straight line

Dividing Lines into n Number of Congruent Line Segments (Equal parts)

In this example, $n=5$. (Fig. 5.10)

1. Draw a line segment AB.
2. Either from A or B, draw a line at any convenient acute angle to AB. Here from B and label its one end as C.
3. From the intersection point of the lines (B) with compass or scale, step off as many equal divisions as needed, in this case five equal parts.
4. Draw a line from the last (fifth) interval to A.
5. Through each of the other points on line BC, draw lines parallel to line AC intersecting AB. Now line AB is divided into five equal parts. Use a triangle and T-square to draw the parallel lines.

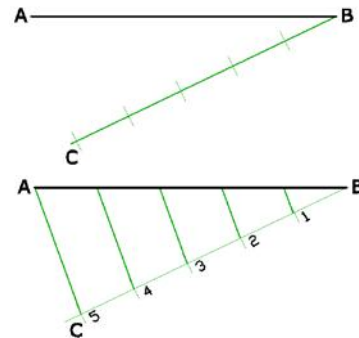


Fig. 5.10 Dividing a line into any number of equal parts

5.2.3 Angles

- Do you have any idea on classification of angles? If you do, on what specification are angles classified.
- What do you think is the relation of verticality and horizontality in terms of angles?

Angles are formed by the intersection of two lines. The point of intersection is called the *vertex*. Angles are measured in degrees ($^{\circ}$), minutes ($'$), and seconds ($''$). There are 360° in a full circle, 60 minutes in a degree, and 60 seconds in a minute. $26^{\circ} 43' 36''$ is read as 26 degrees, 43 minutes, 36 seconds. The concept of dividing angle by minutes and seconds is based on the fact that, the earth completes its rotation (360°) in 24 hours.

A straight angle is an angle of 180° and appears as a straight line. Obtuse angles are angles less than 180° but more than 90° . An angle of 90° is referred to as a right angle because of the relationship between the two intersecting lines. Acute angles are angles less than 90° . When two angles are combined to total 90° , they are referred to as complimentary angles. Supplementary angles form when two angles combine to total 180° . You may draw angles at any degree of angularity using triangles or a protractor. Fig. 5.11 illustrates the different degrees of standard angularity.

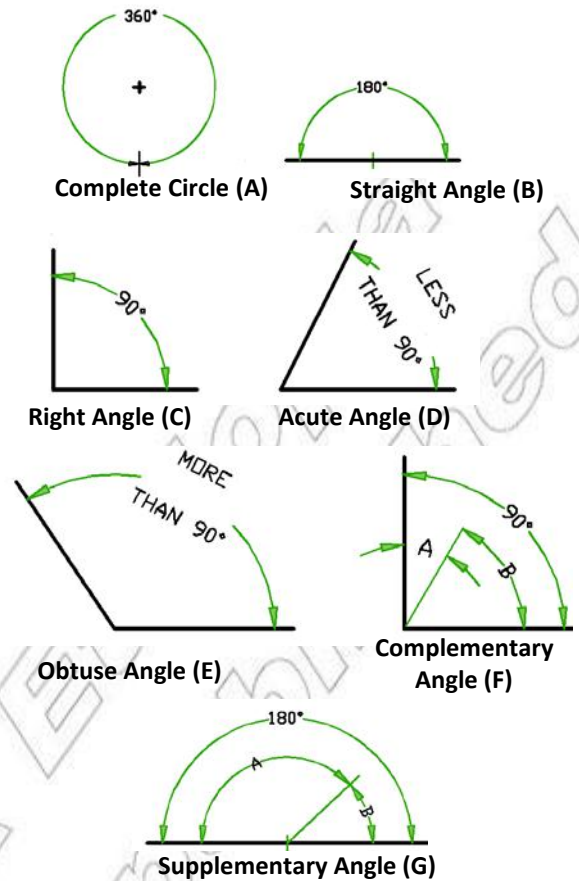


Fig. 5.11 Types of angles

Drawing Angles

Tangent Method (Fig. 5.12)

Both the tangent and sine methods are based on mathematical solutions and require a basic knowledge of trigonometry. To construct an angle assume a convenient value for x . (preferably multiple of 10).

1. Draw line AB whose length is x at any convenient angle.
2. Draw a line perpendicular to AB at A.
3. Find the tangent of angle θ in a table of natural tangents.
4. Calculate $y = x \tan \theta = R$.

5. With A as a center and radius $R=y$, draw an arc to intersect the perpendicular at C.
6. Connect B to C. Angle ABC is the required angle.

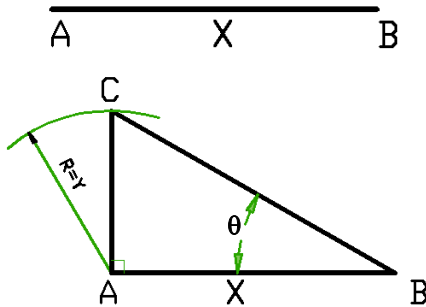


Fig. 5.12 Tangent method

The Sine Method (Fig. 5.13)

1. Draw line AB whose length is x.
2. Find the sine of angle θ in a table of natural sine.
3. Calculate $y=R=x \sin \theta$.
4. Strike an arc with radius R and center B.
5. Draw a tangent line AC through A to the arc which is the other side of the angle.

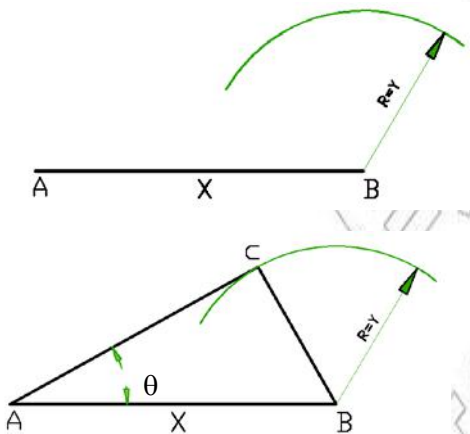


Fig. 5.13 Sine method

The Chord Method (Fig. 5.14)

1. Draw line AB whose length is x.
2. With center at B and radius $R=x$, draw an arc.
3. Find the chord length C for a given angle θ :
 $C=2 \times AB \times (\sin(1/2 \times \theta))$
4. With center A and radius $r=C$, draw an arc to intersect the arc drawn in Step 2 at D.
5. Connect D to B. Angle $ABD=\theta$.

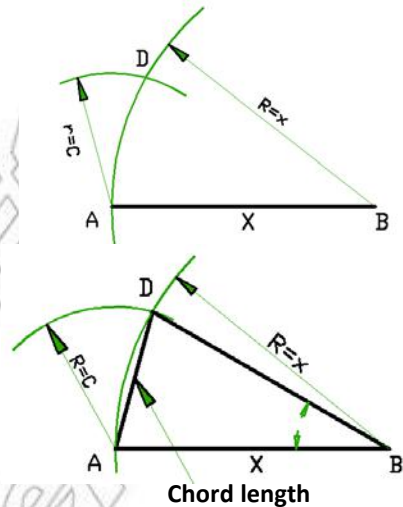


Fig. 5.14 The chord method

Dividing Angles

To bisect an angle means to divide it into half.

To trisect an angle means to divide it into three equal parts.

Bisecting an Angle (Fig. 5.15)

1. Draw the angle CAB.
2. Set the compass to any radius R.

3. Mark off two points E and F from A equal to the radius R on AC and AB respectively.
4. With centers at E and F, strike intersecting arcs of equal radius r at point D.
5. Draw line AD. This line bisects the angle CAB.

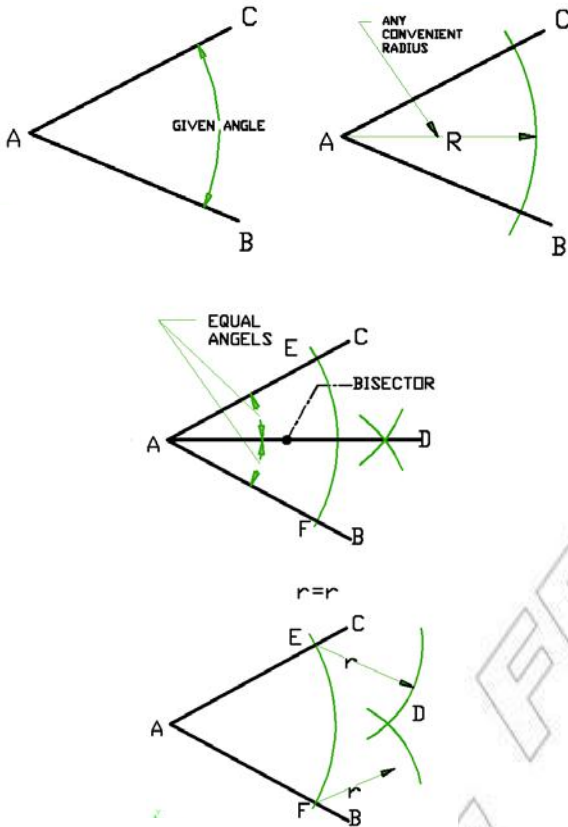


Fig. 5.15 Bisecting an angle

Dividing angles into any number of equal parts (Fig. 5.16)

1. Draw an angle AOB.
2. With O as center, draw an arc with any radius R to intersect line OA at C and line OB at D.

3. Divide the arc CD to the required number of parts (say 5) by trial and error using your divider or compass.
4. Connect the vertex O and the division points on the arc to get the angle divided.

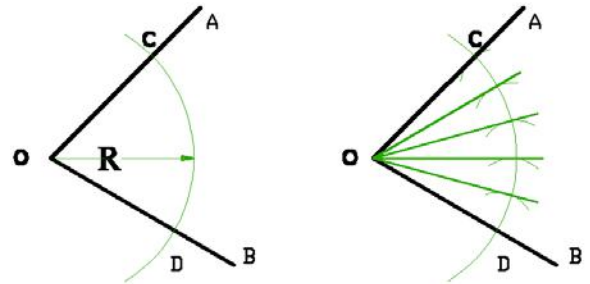


Fig. 5.16 Dividing an angle

Constructing an Angle Congruent to a Given Angle (Copying an Angle)

1. Draw angle CAB at any angle.
2. Draw line A'B' at any desired direction.
3. With A as center and any convenient radius R, draw an arc that intersects the given angle at E and F.
4. With the same radius $R=R'$ and using A' as center draw an arc that intersect line A'B' at E'.
5. Adjust your compass to EF on the given angle and draw an arc with E' as center to intersect the previous arc at F'.
6. Connect point A' to F' and check that $CAB = C'A'B'$. Note that $AF=AE=A'F'=A'E'$.

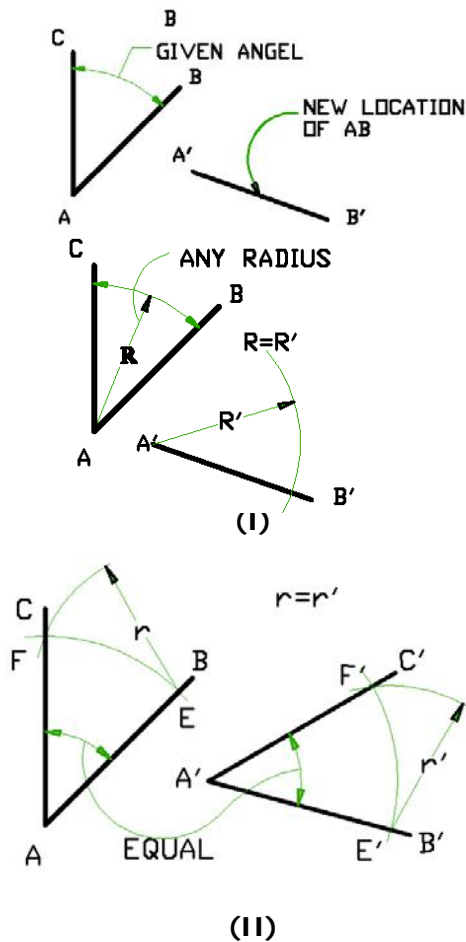


Fig. 5.17 Copying an angle

Checkpoint 5.3

1. Draw two lines 75mm long and making an angle of 65° with each other, using (a) the tangent method (b) the sine method (c) the chord method.
2. Bisect the angle drawn under 1 (a) and transfer one half in a new position.
3. Divide the angle drawn in exercise 1 (b) into eight equal angles.

5.3 Polygons

1. What do you understand from the word polygon?
2. Can you give examples of polygons?

A polygon is a plane, having boundary defined by three or more sides that are all straight. Polygons can be mainly classified as regular and irregular.

Irregular polygons are planes which either their sides or angles are of different sizes.

Regular polygons are planes which their sides are kept in a regular manner, such as equal length, equal angles and so on (including equilateral triangles and squares) and can be constructed by inscribing in or circumscribing around a circle, a technique covered later in this chapter.

The following list shows how the names of the regular polygon change with the number of sides:

Sides	Name
3	Triangle
4	Square
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon

The figure below illustrates some examples on regular and irregular polygons.

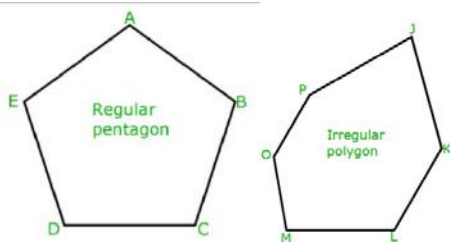


Fig. 5.18 Regular and irregular polygons

Activity 5.1

- What do you think is the difference between regular and irregular polygons?

5.3.1 Triangles

A Polygon having three sides is called a triangle. The sides of a triangle make three interior angles and the sum of these angles is always 180° .

When all sides and all interior angles (60°) are equal, the triangle is referred to as an equilateral triangle. When two sides and two angles are equal, the triangle is an isosceles triangle. A scalene triangle does not have any equal sides or angles. A right triangle has one angle equal to 90° and the long side opposing that angle is called the hypotenuse.

A triangle is classified either by its side or with its angle. (Fig.5.19 and 5.20)

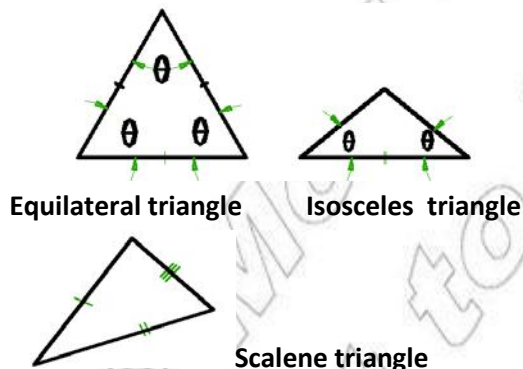


Fig. 5.19 Classified based on sides

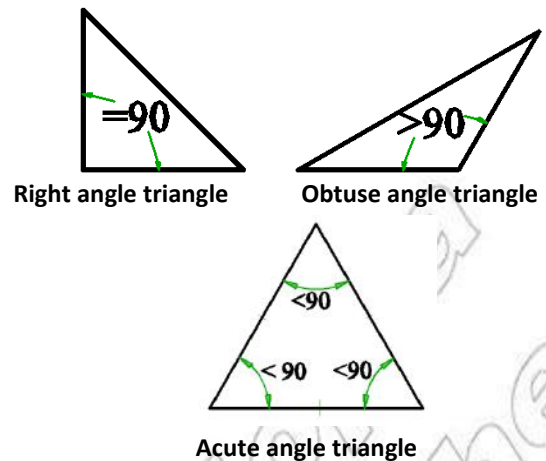


Fig. 5.20 Classified based on angles

Construction Methods: The construction of a triangle can be done in several methods. Some of the methods are described as follows.

Constructing any triangles given three sides AB, AC, and BC. (Fig. 5.19)

1. Draw line AB.
2. With center at A and radius AC, draw an arc.
3. With center at B and radius BC, draw another arc that cut the previous arc at C.
4. Connect point C to A and B.

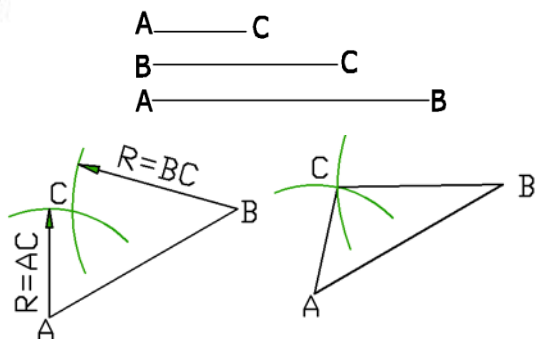


Fig. 5.21 Constructing triangle given three sides

Constructing a triangle given two sides and an included angle (Fig. 5.22)

Let the given angle CAB be θ , sides given be AB and AC .

1. Draw the base line AB .
2. Draw line AD at an angle of θ from AB , using any proper instrument.
3. A as a center and AC as a radius, strike an arc on AD to get point C .
4. Connect C to B . ABC is the required triangle.

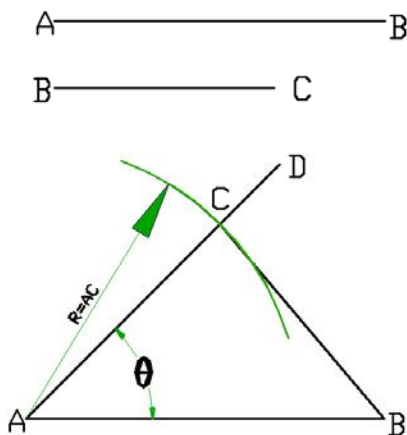


Fig. 5.22 Constructing triangle given two sides and included angle

Constructing a triangle given one side and two included angles (Fig. 5.23)

Let the given side be AB and the angles be θ and α .

1. Draw the base line AB .
2. Draw angle θ using set squares, or protractor from point A .
3. Draw angle α using set squares or protractor from point B .
4. Extend lines drawn at Steps 2 and 3 to intersect each other at point C .

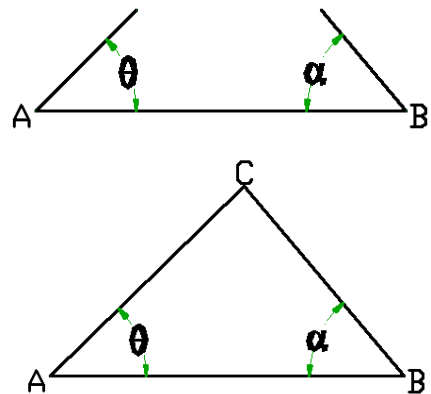


Fig. 5.23 Constructing a triangle given one side and two included angles

Checkpoint 5.4

1. Draw a triangle whose sides are 76mm, 85mm, and 65mm.
2. Draw a triangle whose two sides are 45 mm and 60mm and included angle is 75° .
3. Draw a triangle whose side is 60mm and included angles are 60° and 75° .

5.3.2 Quadrilaterals

1. What do you think the word quadrilateral stands for?
2. Can you give examples of quadrilaterals based on your experience of mathematics earlier classes?

A quadrilateral is a four-sided polygon. The sum of the interior angles of a quadrilateral is 360° .

Quadrilaterals may be subdivided into:

1. **Parallelogram** is a quadrilateral whose opposite sides are parallel, e.g. square, rectangle, and rhombus.
2. **Square** is a quadrilateral with all the four sides and all the angles are equal.
3. **Rectangle** is a quadrilateral, with all four angles are right angles and opposite sides are equal and parallel.
4. **Rhombus** is a quadrilateral, with all four sides have the same length, and equal opposite angles.
5. **Trapezium** is a quadrilateral having two parallel sides.

Some of the construction methods for quadrilaterals are discussed below.

Constructing a square given length of side AB (Fig. 5.24)

1. Draw a horizontal line AB.
2. With a T-square and a 45° triangle draw diagonals from A and B at 45° .
3. Draw perpendicular from A and B intersecting the diagonals at C and D.
4. Connect CD the points of intersection.

Note: The same method can be used to draw a rectangle.

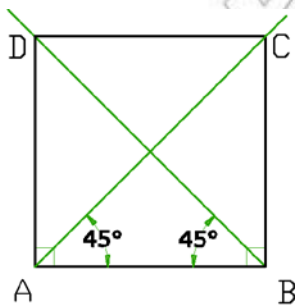


Fig. 5.24 Constructing a square given length

Constructing a square given length of diagonal AB (Fig. 5.25)

1. Draw horizontal line AB.
2. Locate O at the midpoint of AB.
3. Draw CD through O, perpendicular to and slightly longer than AB.
4. With T-square and a 45° triangle, draw AF and BE at 45° to AB intersecting CD at E and F.
5. Connect AE and FB.

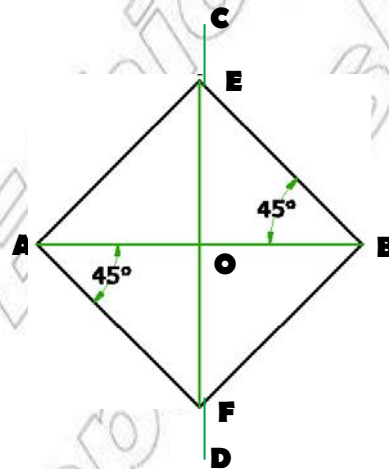


Fig. 5.25 Constructing a square given length of diagonals

Constructing a rhombus given side AB and an included angle θ . (Fig. 5.26)

1. Draw a horizontal line AB.
2. Draw lines from points A and B at θ .
3. Lay off distances AD and BC equal to AB.
4. Connect C and D.
ABCD is the required rhombus.

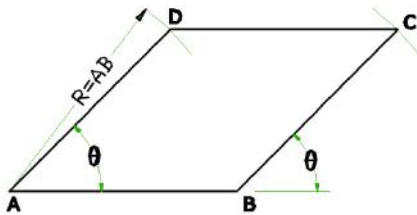


Fig. 5.26 Constructing a rhombus given side and included angle

Constructing a trapezium given sides and two interior angles (Fig. 5.27)

Let the sides of the trapezium be AC, AB and BD and the two interior angles be θ and α each.

1. Draw line AB at any convenient angle.
2. Draw two lines through, A and B at α and θ from line AB respectively.
3. Using B as center and radius BD, draw an arc to locate D.
4. Using A as center and radius AC, strike an arc to locate point C.
5. Connect C to D. ABCD is the required trapezium.

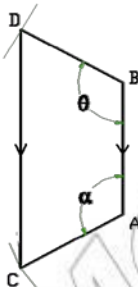


Fig.5.27 Constructing a trapezium given sides and two interior angles

Checkpoint 5.5

1. Draw a square whose sides are 55 mm long by all methods discussed.
2. Draw a rhombus whose sides are 40mm and an interior angle is 50° .
3. Draw a trapezium ABCD such that $AB = 60\text{mm}$, $BC = 48\text{mm}$, $AD = 50\text{mm}$, $CBA=105^\circ$, and $BAD =120^\circ$.

5.3.3 Construction of Regular Polygons

Most of the construction methods for regular polygons are simple and memorable. This is because they can be inscribed in or circumscribed around a circle. Under this section, first we will discuss special construction methods for particular regular polygons and then common construction methods for any regular polygons.

Constructing a regular pentagon in a given circumscribing circle (Fig. 5.28)

1. Draw horizontal diameter AB and vertical diameter CD intersecting at O.
2. Draw the circle with radius equal to half of AB.
3. Construct the midpoint of OB and mark as E.

4. With E as a center and radius equal to CE, strike the arc CF to intersect AB at F.
5. With C as a center and CF as a radius, strike the arc FG to intersect the circle at G.

Note: A line from G to C is one side of the pentagon.

6. Set a compass to GC and lay off this interval from C around the circle.
7. Connect the points of intersection

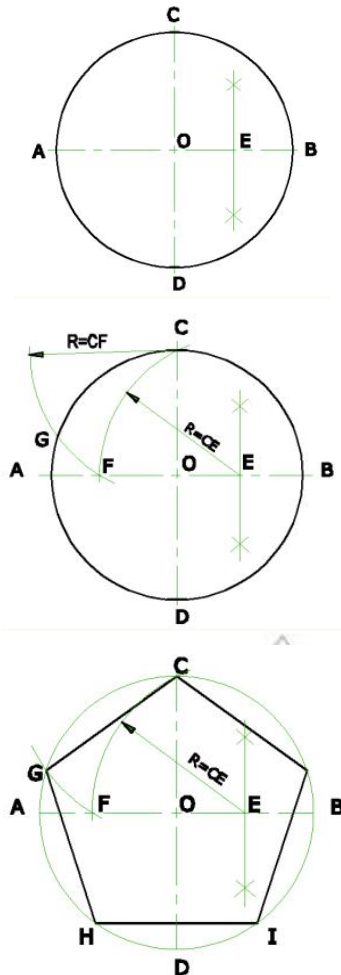


Fig. 5.28 Constructing a regular pentagon in a given circumscribing circle

Constructing a regular hexagon in a given circumscribing circle (Fig. 5.29)

1. Draw diameters AB and CD intersecting at O.
2. OB as radius, draw the circle.
3. Starting from C and with radius OB, lay off points around the circle.
4. Connect the points.

Note: The diameter of the circumscribing circle is equal to the long diameter (across corner) of the hexagon. The radius of the circumscribing circle (which is equal to one half the long diameter of the hexagon) is equal to the length of side of the regular hexagon.

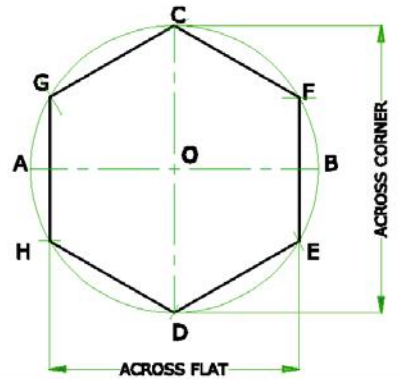


Fig. 5.29 Constructing a regular hexagon in a given circumscribing circle

Constructing a regular hexagon, given the distance across corners (Fig. 5.30)

1. Draw a circle with AB as a diameter.
2. With A and B as centers and radius $\frac{1}{2}$ AB, draw arcs to intersect the circle at C, D, E and F.

3. Connect the A, C, E, B, F and D to complete the hexagon.

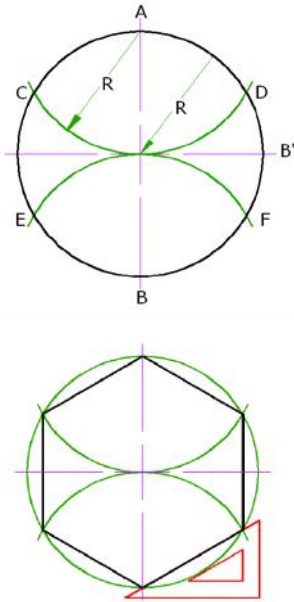


Fig. 5.30 Constructing a regular hexagon, given the distance across corners

Constructing a regular heptagon given the circumscribing circle

(Fig. 5.31)

1. Draw vertical line AB whose length is equal to the diameter of the given circle.
2. Locate the midpoint O of line AB and draw the given circle.
3. With B as center and OB as radius, draw an arc to intersect the given circle at C and D.
4. Connect C to D to intersect AB at M.
5. With radius $MC=MD$, strike arcs around the circle to locate points E, F, G, H, I and J starting from A.

6. Connect points A, E, F, G, H, I, and J to complete the regular heptagon.

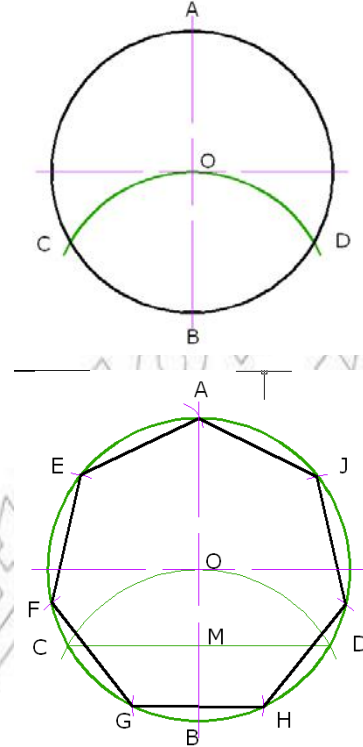


Fig. 5.31 Constructing a regular heptagon given the circumscribing circle

Checkpoint 5.6

1. Draw a regular pentagon whose circumscribed circle is 60mm diameter.
2. Inscribe a regular hexagon in a circle whose diameter is 60mm.
3. Draw a regular hexagon whose across corner is 90mm.
4. Inscribe a regular heptagon in a circle whose radius is 37mm.

Constructing any regular polygon with a given length of side (approximate method) (Fig. 5.32)

1. Draw the given side AB.
2. With radius equal to AB and centers at A and B draw arcs that intersect at point 6.
3. Draw a perpendicular to line AB through point 6 and extend it upward beyond point 6.
4. Draw 45° line from either A or B to intersect the perpendicular line at point 4.
5. Bisect the distance 4-6 to locate point 5.
6. Step up (lay off) distance equal to 5-6 beyond point 6 to get points 7, 8, 9....
7. With center at 8 and radius 8-A or 8-B, draw a circle that will inscribe the polygon.
8. Adjust your compass to AB and divide the circle into eight equal parts and connect the division points to complete the construction of the regular polygon (in this case octagon).

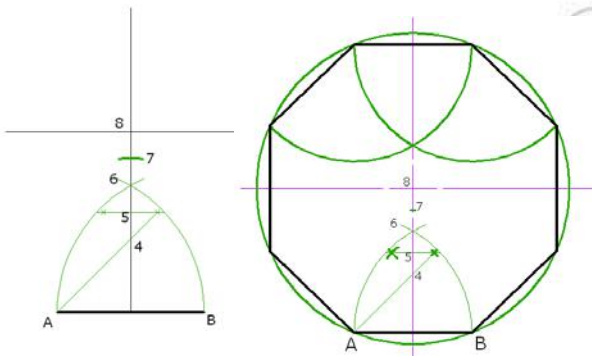


Fig. 5.32 Constructing a regular polygon with a given length of side

Note: If the circle is drawn using 4 as center and radius 4-A, you can construct a square whose side are equal to AB. If the circle is drawn using 6 as center and radius 6-A, you can construct a regular hexagon, and so on.

5.4 Circles and Tangents

A **circle** is a closed curve created by a set of points in a plane that are the same distance from a fixed point called the *center*. These set of points from the perimeter of the circle. Part of a circle is called an arc. Definitions of terms related to circles are given below.

The distance from the center point to any point along the circle edge is called a radius (RAD or R).

The distance from one side of the circle through the center point to the opposing side of the circle is the circle *diameter* (DIA).

Half of the distance around a circle is called a *semicircle*.

Circumference refers to the total distance around the circle. Calculate the circumference of a circle by multiplying the diameter of the circle by 3.1416 or π (pronounced pi).

A *chord* is a straight line joining two points on a curve.

A *segment* is the section of the curve cut off by the line or chord.

Quadrants result from the intersection of two radii at 90° including the portion of the circle between the radii.

Sectors are the part of the circle bound by two radii at other than right angles including the bound portion of the circle.

Angles are formed by the intersection of radii but do not include the bound portion of the circle.

An *arc* is a segment of the curved portion of the circle bound by the intersection of two radii but does not include the radii.

A straight line that intersects and passes through two points on the circle is called a *secant*.

Straight lines that touch but do not intersect at one point on a circle are said to be *tangent*.

Multiple circles sharing a common center point are called *concentric circles*.

Multiple circles that do not share a common center point are referred to as *eccentric circles*. Fig.5.33 illustrates circle terminology.

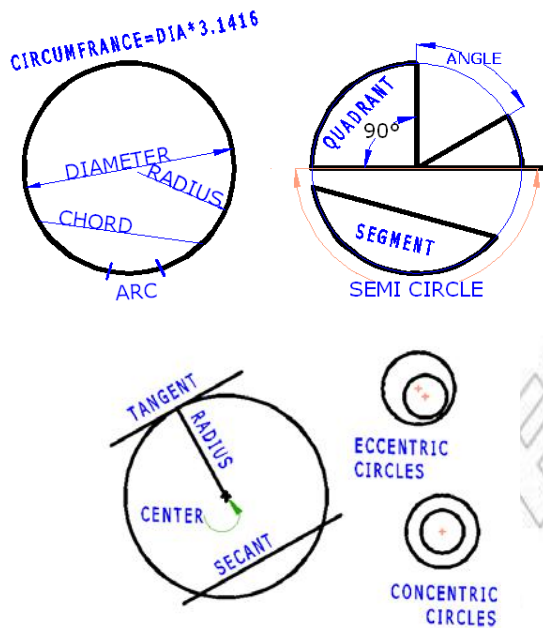


Fig. 5.33 Circle terminology

Activity 5.2

- What do you understand from the word *tangent*? And what relation circle and tangent have?

To draw a circle through three non-collinear points A, B and C (Fig. 5.34)

1. Mark any three points and connect them.
2. Draw the perpendicular bisectors of both AB and BC to intersect at O.
3. With O as centers and radius OA, draw the required circle.

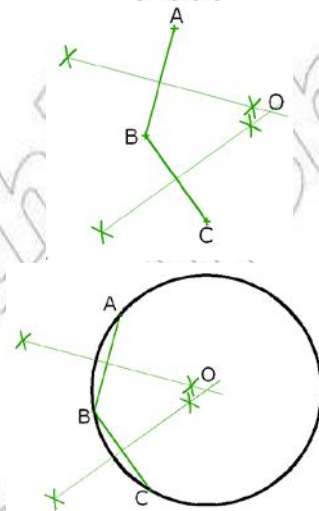


Fig. 5.34 Drawing a circle through three points

Constructing a line tangent to a circle at given point on the circle (Fig. 5.35)

1. Draw the given circle and locate point P on it.
2. Align one side of your set square to pass through P and the center of the circle keeping the T-square at the base.
3. Slide the aligned set square until the other side passes through point P and draw the required tangent line.

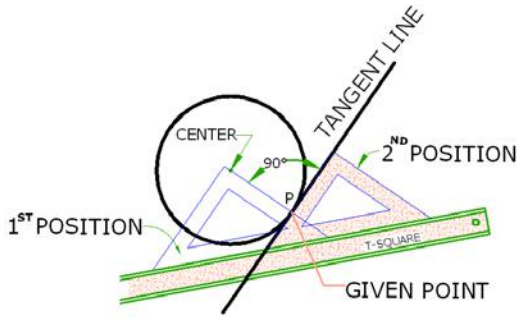


Fig. 5.35 Constructing a line tangent to a circle at given point on the circle

Constructing a line tangent to a circle through an external point (Fig. 5.36)

1. Draw a circle and locate given point P.
2. Construct the center O of the circle.
3. Connect OP.
4. Locate point M by constructing the midpoint of OP.
5. With M as a center and radius OM, draw an arc to intersect the circle at point of tangency T1 and T2.
6. Connect T1 to P and T2 to P.

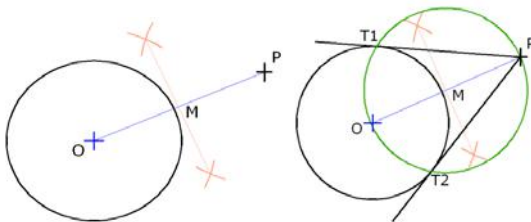


Fig. 5.36 Constructing a line tangent to a circle through an external point

Constructing tangent line to two circles of different size (Fig. 5.37)

Suppose the radii of the two circles are r and R , where $R > r$.

1. Draw two given circles with center points O and O'.

2. Connect the center point of the circles with line OO'.
3. Construct midpoint C of line OO'.
4. With C as a center and radius OC, draw a semicircle through OO'.
5. With O' as a center and radii $R-r$ and $R+r$, strike arcs that intersect the semicircle on OO' at D and E respectively.
6. Draw O'D to intersect circle O' at F and similarly draw O'E to intersect circle O' at G.
7. Through center O, draw OH parallel to O'F and OK parallel to O'G.

Note: Lines FH and GK are tangent to both circles. GK intersects the line OO'.

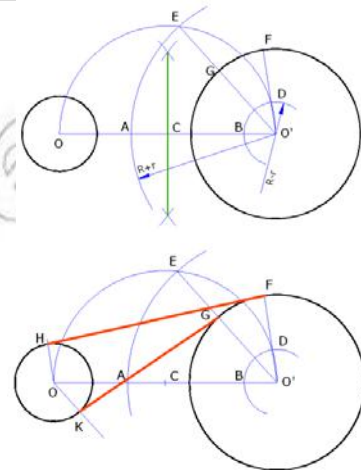


Fig. 5.37 Constructing tangent line to two circles of different size

Checkpoint 5.7

1. Draw 53 mm diameter circle. Mark a point P on the right side of the circle between the vertical and horizontal diameter. Draw a line tangent to the circle at P.
2. Locate a point T to the right of the circle drawn in the above exercise at 45mm from the center of the circle. Draw two tangents to the circle through T.

Constructing a tangent arc of radius R to two lines at right angle

(Fig. 5.38)

1. Draw perpendicular lines intersecting at O.
2. With O as a center and radius R, strike an arc intersecting the lines at A and B.
3. With same radius R and centers A and B, strike arcs intersecting at O' as shown.
4. With O' as a center and radius R, draw the tangent arc.

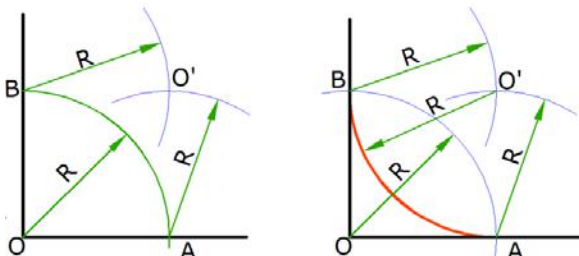


Fig. 5.38 Constructing a tangent arc to two lines

Constructing tangent arc of radius R to two lines that form any angle
(Fig. 5.39)

1. Draw lines AB and CD at any angle.
2. Draw EF parallel to AB and JH parallel to CD at a distances equal to the given radius of the tangent arc
3. Locate the intersection of EF and JH as O.
4. With O as center and radius R, draw the tangent arc.

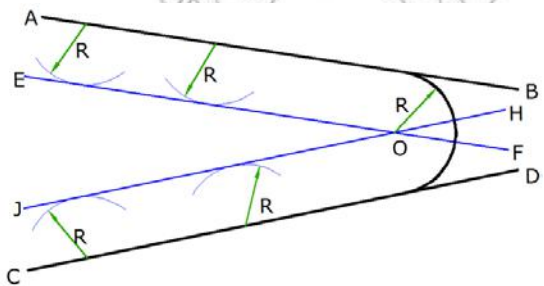
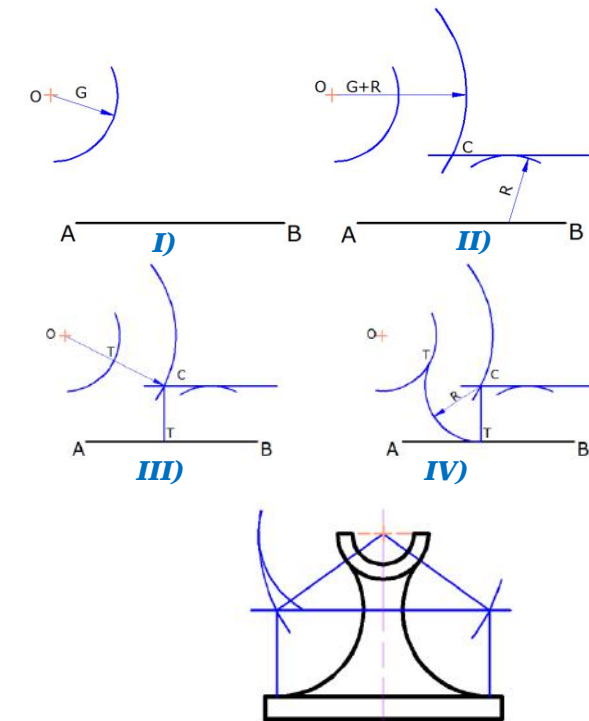
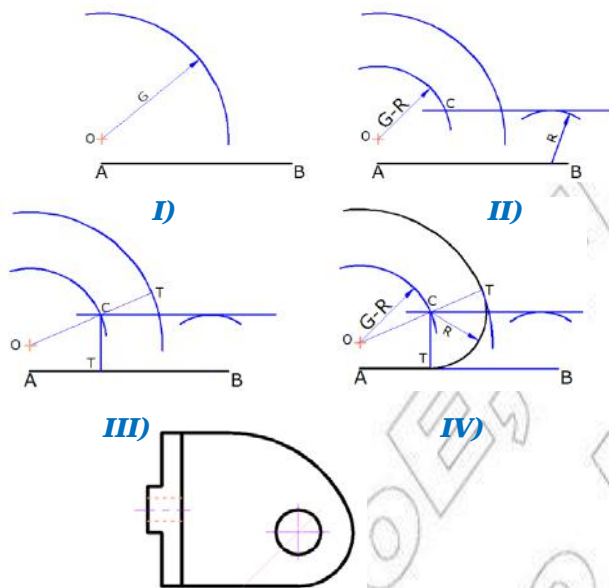


Fig. 5.39 Constructing tangent arc to two lines that form any angle

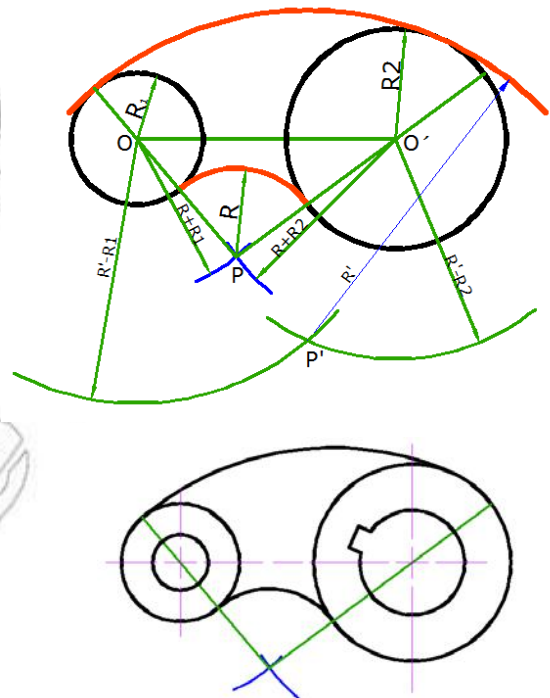
Constructing a tangent arc of radius R to a circle and a straight line
(Fig. 5.40)**Suppose the radius of the required arc is R .**

1. Draw the given arc and the given line.
2. Draw a parallel line to the given line AB at a distance of R.
3. Draw a concentric arc to the given arc with radius $G+R$ (or $G-R$ depending on the way the tangent arc is to be drawn), intersecting the line drawn in step 2 at point C. Point C is the center of the required arc.

**Example****Example****Fig. 5.40** Constructing a tangent arc to a circle and a straight line

Constructing internal and enclosing tangent arcs of radii R and R' respectively to two circles of radii R_1 and R_2 (Fig. 5.41)

1. Draw two given circles with radii R_1 and R_2 and centers O and O' respectively.
2. With O as a center and radii $R+R_1$ and $R'-R_1$, strike two arcs as shown in the figure.
3. With O' as a center and radii $R+R_2$ and $R'-R_2$, strike two arcs intersecting the previous arcs at points P and P' . P and P' are the centers of the tangent arcs.
4. With P and P' as centers and corresponding radii R and R' respectively, draw the tangents.

**Example****Fig. 5.41** Constructing internal and enclosing tangent arcs to two circles

Constructing tangent arc of radius R to two circles of radii R_1 and R_2 enclosing one circle (R_1) but not enclosing the other (Fig. 5.42)

1. Draw two given circles with radii R_1 and R_2 and centers O and O' respectively.
2. With O as a center and radius $R-R_1$, strike an arc.
3. With O' as a center and radius $R+R_2$, strike an arc to intersect the previous arc at P .
4. With P as a center and radius R , draw the tangent arc. T_1 and T_2 are points of tangency.

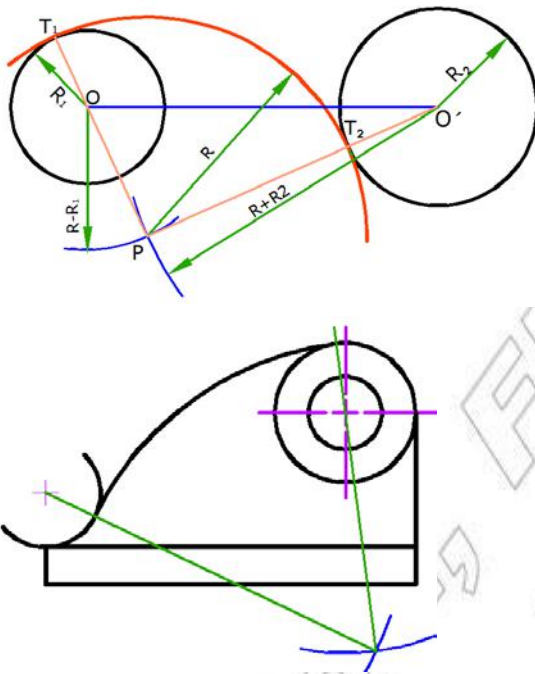


Fig. 5.42 Constructing tangent arc to two circles enclosing one but not the other

Checkpoint 5.8

1. Draw two circles whose radii are 19mm and 27mm. The distance between the centers of the two circles is 100mm. Draw a line tangent to both circles.
2. Redraw the circles under exercise 1. Draw two tangent lines to the circles which cross the line connecting the centers of the circles.
3. Draw an angle of 75° whose sides are 45mm long. Draw a tangent arc to both sides whose radius is 12mm.
4. Draw a 120° angle whose sides are 55mm. Draw a tangent arc whose radius is 20mm.
5. Draw a tangent arc whose radius is 70mm to the circles drawn under exercise 1.
6. Redraw circles whose data are given in exercise 1. Draw a tangent arc whose radius is 75mm externally to both circles.

An ogee or reverse curve is composed of two consecutive tangent circular arcs that curve in opposite directions. It is used in road design, decoration of buildings and on many real objects.

To draw an ogee curve that connects parallel lines (Fig. 5.43)

1. Draw two parallel lines AN and BM and connect A and B.
2. Locate point T on line AB at any convenient distance from either A or B.
3. Draw perpendiculars to lines AN and BM at points A and B.
4. Draw the perpendicular bisectors of AT and TB to intersect the lines drawn in step 3 at points F and C. F and C are the centers of the required tangent arcs.
5. With center at F and radius AF, draw an arc between points A and T.
6. With center at C and radius CB, draw another arc between points B and T to complete the ogee curve.

Note : Point T is the common point of tangency.

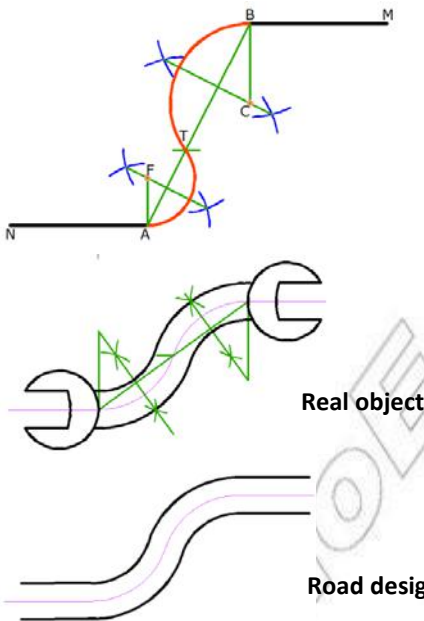


Fig. 5.43 Drawing ogee curve that connect lines

Checkpoint 5.9

Draw an ogee curve connecting two parallel lines spaced at 50mm

- a) Using two arcs of equal radii.
- b) Using two arcs of radii 20 and 30mm.

5.5 Construction of an Ellipse

Activity 5.3

1. What is ellipse? Use a sketch to define it.
2. Compare and contrast circle and ellipse based on their forms.

The ellipse is the locus of all coplanar points, the sum of whose distances from two fixed points (foci) is constant. This constant distance is the major axis. The longer axis of an ellipse is called major axis (diameter) and the shorter axis is called minor axis (diameter). The major and minor diameters are perpendicular to each other. The foci of an ellipse (F_1 and F_2) are located on the major axis and are obtained by striking an arc with radius equal to half the major axis and center at the end points of the minor axis as shown in Fig. 5.44.

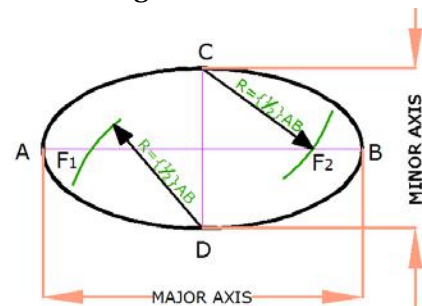


Fig. 5.44 Construction of an ellipse

An ellipse is constructed in several methods, few of which are

1. Four-center method (an approximate method)
2. Concentric circles method
3. Parallelogram or conjugate diameter method.

Constructing ellipse by four-center method (Fig. 5.45)

1. Draw the major axis AB and the minor axis CD intersecting at O.
2. Connect the end points of the two axes AD.
3. With radius OA and center O, strike an arc to intersect the extension of minor axis at E.
4. With radius DE and Center on D, mark off point F. Note $AF=AO - DO$ and $OA=OD+DE$.
5. Draw perpendicular bisector of AF and extend it to intersect the major axis at K and the minor axis at H.
6. With a divider or compass mark off OM equal to OK, and OL equal to OH.
7. With H as a center and radius equal to HD, draw the bottom arc between TT.
8. With L as a center and same radius HD, draw the top arc between TT.
9. With M as a center and radius equal to MB, draw the right side end arc between TT.
10. With K as a center and radius MB, draw the left side end arc between TT.

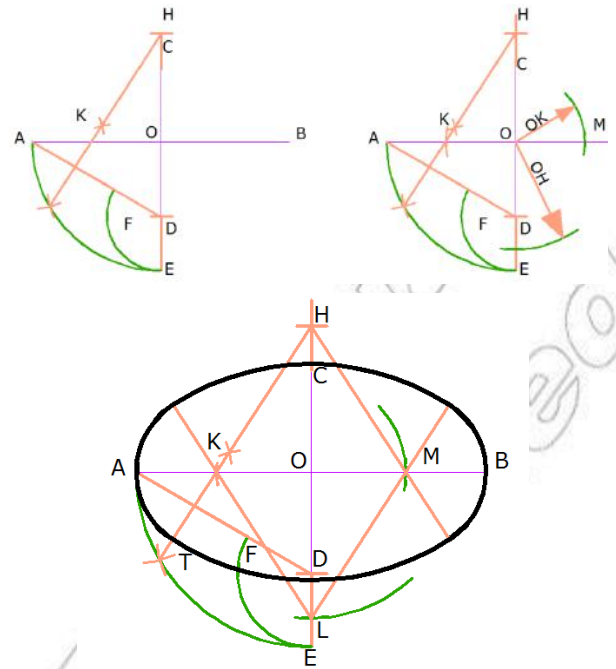


Fig. 5.45 Constructing an ellipse by four centered method

Constructing ellipse by concentric circles method (Fig. 5.46)

1. Draw the major axis AB and minor axis CD intersecting at O.
2. With center at O and radii half of AB and half of CD, draw concentric circles.
3. Draw a number of diameters equal to AB at any angle.
4. From each intersection point of the diameters with the larger circle, draw vertical lines.
5. From each intersection point of the diameters with the smaller circle, draw horizontal lines intersecting the nearest vertical lines drawn in step 4.
6. With a French curve, draw a smooth arc through all points to complete the ellipse.

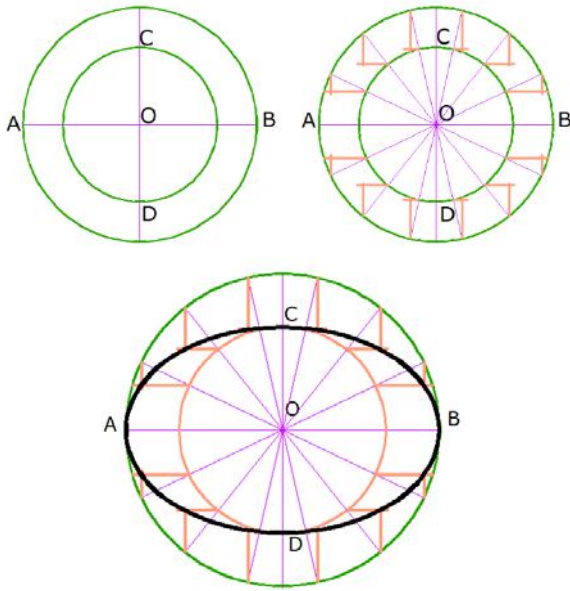


Fig. 5.46 Constructing an ellipse by concentric circles method

Constructing an ellipse by parallelogram method (Fig. 5.47)

1. Draw a parallelogram (JKLM) whose sides are equal to the major diameter AB and minor diameter CD of the ellipse.
2. Divide OA and OB into any appropriate number of equal parts (say 5) and number them as shown in Fig.5.45. Similarly, divide AJ, BK, BL, and AM into the same number of equal parts and number them as shown in the figure.
3. Connect point C to points 1, 2, 3, 4 on line AJ and BK. Similarly, connect D to points on lines AM and BL.
4. Connect point D to points 1, 2, 3, 4 on line OA and OB and extend them intersecting the corresponding lines drawn in step 3. Similarly, connect C to points 1, 2, 3, 4 on line OA and OB and

extend them to intersect the corresponding lines drawn in step 3.

5. Draw a smooth curve through the intersection points found in step 4 using French curves to complete the ellipse.

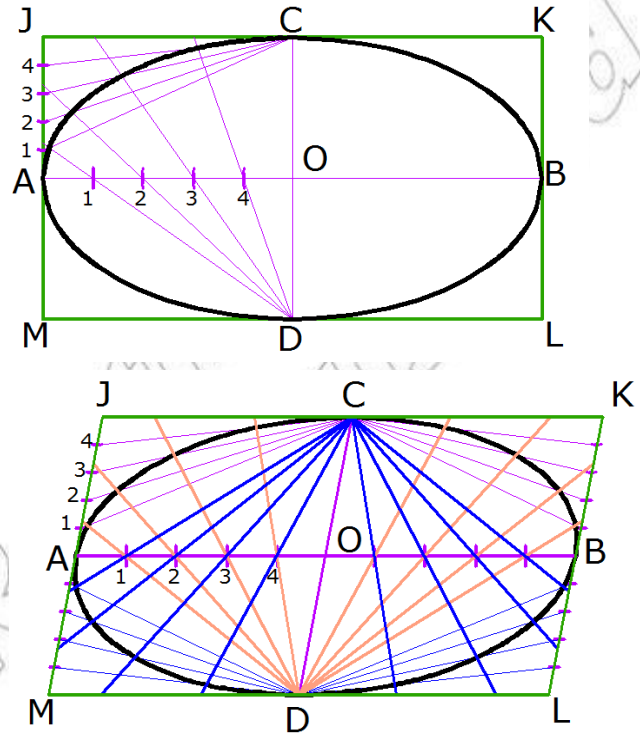


Fig. 5.47 Constructing an ellipse by parallelogram method

Checkpoint 5.10

1. Draw an ellipse whose axis is 102mm long (horizontally) and whose minor axis is 64mm long, using:-
 - a) The four-center method.
 - b) The concentric circle method.

UNIT SUMMARY

Familiarity with the step-by-step methods used for constructing geometric figures and knowing related definition of terms help you understand the practical applications of geometric construction to problem solving. Simplified or preferred methods of geometric construction, as well as alternate methods, are valuable knowledge factors when used with drafting instruments to create accurate drawings.

Geometric nomenclature primarily of the following concepts: points in space, line, angle, triangle, polygon, quadrilateral, circle, polyhedron, prism, pyramid, cone, and sphere; however the concepts from polyhedron to sphere are not considered in this unit.

Elements and polygon construction principles consist primarily of how to do the following: bisect a line; bisect an angle; draw an arc or circle through three points; draw a line parallel to a straight line at a given distance; draw a line perpendicular to a line at a point; draw a line parallel to a curved line at a given distance; draw a perpendicular to a line from a point not on the line; divide a line into equal parts; divide a line into proportional parts; transfer an angle; transfer an odd shape; transfer complex shapes; proportionately enlarge or reduce a shape; draw a triangle with sides of known length; draw a square; draw a pentagon; and draw a hexagon.

Circular construction consists primarily of knowing how to do the following: locate the centre of a given circle; construct an arc tangent a right angle, an acute angle, an obtuse angle, a straight line, a curve, and two radii or diameters; draw an ogee curve; draw an ellipse using the four centre method, concentric circle method and parallelogram method

Exercise I: Lines

1. Draw an 80 mm long line at an angle of 30° from the horizontal and draw a line parallel to it at a distance of 30mm.
2. Draw a 75mm long line at 15° from the vertical and draw a perpendicular bisector of it.
3. Draw a 95mm long vertical line and divide it into three proportional parts to 3,5, and 9.

Exercise II: Angles

4. Draw 120° using straight edge and a compass and bisect it.
5. Draw a 165° using straight edge and a compass and divide it into seven equal angles.
6. Draw a 60° angle using straight edge and a compass. Transfer and bisect it.

Exercise III: Triangles

7. Draw a triangle whose sides are 70mm, 60 mm and 80 mm long.
8. Draw a triangle whose one side is 50 mm and the included angles from this line are 38° and 65° .
9. Construct a triangle if the included angle is 65° and two of the sides are 80mm and 56 mm.

Exercise IV: Polygons

10. Circumscribe and inscribe a regular pentagon in a circle of diameter of 70mm long.
11. Draw a regular hexagon,
 - (a) If the distance across flat is 60mm.
 - (b) If the distance across corner is 64mm.
 - (c) Whose sides are 50mm.
12. Draw a regular heptagon given circumscribing circle whose diameter is 70mm.
13. Draw a square with 60mm sides, and inscribe an octagon.

Exercise V: Circles, tangent and arcs.

14. Locate three non-collinear points, and draw a circle through the points.
15. Draw two circles whose radii are 30mm and 20 mm with centers 54 mm apart.
 - (a) Draw a tangent line to the lower sides of both circles which doesn't cross the line connecting the center points.
 - (b) Draw a tangent line to both circles which intersects the line connecting the centers.
 - (c) Redraw the circles and draw arc of 70mm radius tangent to upper sides of circles and enclose them.

Exercise VI: Ellipse

16. Use the four centered method to draw an ellipse whose major and minor diameters are 76mm and 54mm, respectively.
17. Draw an ellipse by the concentric circles method given major axis as 75mm and 47 mm of minor axis.
18. Draw an ellipse whose major axis is 80mm and minor axis 50mm by parallelogram method. (Use 70°).