

UGANDA ADVANCED CERTIFICATE OF EDUCATION
POST - MOCK EXAMINATIONS September 2023
PURE Mathematics
Paper One
3 Hours

INSTRUCTIONS:

Answer *all* questions in *section A* and *any five* questions in *section B*

SECTION A (40 MARKS)

1. Solve the equations;
$$\begin{aligned}2x - y + z &= 5 \\ x + 2z - 3y &= 2 \\ 2x + 4z + y + 3 &= 0\end{aligned}$$

(5 marks)
2. Find the acute angle between the line $\frac{x-6}{5} = 1 - y = 1 + z$ and the plane $7x - y + 5z + 5 = 0$ giving your answer to the nearest degree.

(5 marks)
3. Show that $x + 1$ is a factor of the expression; $x^3 - 5x^2 + 2x + 8$ hence solve the equation $x^3 - 5x^2 + 2x + 8 = 0$

(5 marks)
4. Show that the line $3x - y + 5 = 0$ is a chord to the circle $x^2 + y^2 - 2x - 6y + 5 = 0$ and hence find the length of this chord.

(5 marks)
5. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{3 + 5\cos \theta} d\theta$

(5 marks)
6. Differentiate $\frac{5^{\cos x}}{e^{\sqrt{x}}}$ with respect to x

(5 marks)
7. Prove that: $\frac{\cos 2\theta \cos \theta - \sin 3\theta \sin 4\theta}{\sin 8\theta \cos \theta - \sin 6\theta \cos 3\theta} = \cot 2\theta$

(5 marks)
8. Lecturer deposited shs800,000 in Centenary bank at the beginning of 2012. The bank offers a compound interest at a rate of 15% per annum. He deposits another shs800,000 at the beginning of every year and makes no withdraws. How much money will he receive by the end of 2023?

(5 marks)

SECTION B

Attempt any five (5) questions from this section.

9. (a) If $Z = x + iy$ is a complex number that satisfies the equation
- $$\left| \frac{Z - 1}{Z + 2 - 3i} \right| = 2$$
- (i) Show that the locus of Z is a circle centred at $-3 + 4i$ hence find the radius of the circle.
- (ii) Represent Z on an Argand diagram (6 marks)
- (b) Find the fourth root of the complex number $1 - \sqrt{3}i$ (6 marks)
10. (a) Solve the equation; $\cos^{-1}x + \cos^{-1}(x\sqrt{3}) = \frac{\pi}{2}$ for $0 \leq x \leq \pi$ Verify your answer. (5 marks)
- (b) Prove that in any triangle ABC;
- $$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot\left(\frac{C}{2}\right)$$
- Hence solve the triangle for $a = 7 \text{ cm}$, $b = 5 \text{ cm}$ and the angle included 48° (7 marks)
11. (a) Evaluate; $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx$ (6 marks)
- (b) Given that $y = a e^{-2x} \sin(x + \beta)$, where a and β are constants. Show that;
- $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$
- (6 marks)
12. Given the curve;
- $$y = \frac{2x+5}{(x^2-4)}$$
- (a) Prove that y cannot take any value in the range $-1 < y < -\frac{1}{4}$
- (b) Hence or otherwise determine the turning points of the curve.
- (c) Sketch the curve. (12 marks)
13. (a) Given that $(\alpha + \beta) = 3$ and $\alpha\beta = -10$ Find the value of $\alpha^2 + \beta^2$ Hence find the values α and β . (6 marks)
- (b) When expanded in ascending powers of x , the Maclaurin's expansion of $\ln(e^{2x} + e^{-2x}) = a + bx^2 + cx^4$ Determine the values of a , b and c . (6 marks)

14. The line L has its vector equation $\mathbf{r} = i - j + 3k + \lambda (i + 2j - 2k)$.
A plane P has its Scalar product equation $\mathbf{r} \cdot (6i - 2j + k) = -3$
- Show that the line L is parallel to plane P.
 - Determine the Cartesian equation of another plane M that is parallel to plane P and contains the line L.
 - Find the perpendicular distance from the line L to the plane P
15. Given the hyperbolic curve $xy = c^2$
- Determine the equation of the tangent to the curve $xy = c^2$ at the point $A\left(ct_1, \frac{c}{t_1}\right)$.
(4 marks)
 - Determine the equation of the normal to the curve $xy = c^2$ at the point $B\left(ct_2, \frac{c}{t_2}\right)$.
(4 marks)
 - If the tangent at A above meets the normal at B above intersecting on the y - axis;
Show that; $2t_2 = t_1(1 - t_1^4)$ (4 marks)
16. A body loses mass at a rate that is proportional to the mass M of the body present at any time t. If initially, the body's mass was 400kg and 20 days later, the body's mass had reduced to 100kg;
- Form a differential equation relating the body's mass present and time t.
 - Solve the differential equation.
 - Use the solved differential equation in (b) above to find the;
 - Mass of the body present after five weeks
 - Time taken for 250kg of the body to decay.

END

October 2023