

PERCENTAGES

Summary:

1. (i) *Percentage is a fraction whose denominator is 100.*

(ii) *The symbol for percentage is written as %.*

2. (i) *To change a percentage into a fraction or decimal divide by 100. Thus*

$$75\% = \frac{75}{100} = \frac{3}{4} = 0.75$$

(ii) *To change a fraction or decimal into a percentage multiply by 100. Thus*

$$\frac{3}{4} = \frac{3}{4} \times 100 = 75\% \text{ and } 0.75 = 0.75 \times 100 = 75\%$$

EXAMPLES:

1. *Express each percentage as a fraction in its simplest form*

(i) 20% **(ii)** 25% **(iii)** 24% **(iv)** 12.5% **(v)** $33\frac{1}{3}\%$

2. *Express each percentage as a decimal*

(i) 20% **(ii)** 25% **(iii)** 45% **(iv)** 48.5% **(v)** $12\frac{1}{4}\%$

3. *Express each fraction as a percentage*

(i) $\frac{3}{4}$ **(ii)** $\frac{1}{5}$ **(iii)** $\frac{1}{3}$ **(iv)** $\frac{3}{5}$ **(v)** $\frac{2}{3}$

4. *Express each decimal as a percentage*

(i) 0.2 **(ii)** 0.25 **(iii)** 0.45 **(iv)** 0.125 **(v)** 0.4825

5. (i) *Find 35% of 80*

(ii) *Find 25% of 120*

6. *Express 20 as a percentage of 80*

7. *In a bag of 120 eggs 30 are rotten. Find the percentage of rotten eggs*

8. In a test marked out of 60, a student obtained 48 marks. Find his percentage mark
9. In a class, 90 students are boys and 25% are girls. Find the number of girls in the class
10. In a school with a population of 4,000 students, 60% are boys. 80% of the boys and 25% of the girls are day scholars. Find the:
- (i) male students in the school
 - (ii) female boarding students
 - (iii) ratio of male day scholars to female day scholars
 - (iv) number of boarders in the school
11. A man spent 40% of his salary on food, 30% of the remainder on fees and 25% of the remainder on rent. If he was left with Shs 252,000, find:
- (i) the man's salary
 - (ii) how much did he spend on rent
12. The population in a village with three zones A, B and C is in the ratio 9:8:4. 80% of the people in zone A are literate, 40% of the people in zone B are illiterate and 90% of the people in zone C are literate. Find the percentage of literacy in the village.

Soln:

$$A : B : C = 9:8:4 = 9y: 8y:4y$$

$$\Rightarrow \text{Population in: } A = 9y, B = 8y \text{ and } C = 4y$$

$$\text{Village population} = 9y + 8y + 4y = 20y$$

$$\text{Village literate population} = \left(\frac{80}{100} \times 9y \right) + \left(\frac{60}{100} \times 8y \right) + \left(\frac{90}{100} \times 4y \right) = 15.6y$$

$$\text{Required percentage} = \frac{15.6y}{20y} \times 100 = 78$$

EER:

- 1. Calculate 25% of 1600**
- 2. If 20% of a number is 80, find 30% of that number**
- 3. In a class of 45 students 18 are girls. Find the percentage of boys in the class**
- 4. A farmer sold 40% of the eggs and still has 420 eggs left. Find how many eggs were sold**
- 5. Candidates P, Q and R contested in an election and received 1,200, 7,600 and 11,200 votes respectively. Find the percentage of the total votes each of them got**
- 6. In a farm, 40% of the cows died and 1200 were left. Find how many cows died**
- 7. 7500 voters polled in an election between two candidates P and Q. 20% of the votes were declared invalid and 55% of the valid votes were for P. Find the number of valid votes obtained by Q**
- 8. A farmer vaccinated 540 of his 720 cows against a disease. A few days later, 5% of the vaccinated and 80% of the unvaccinated cows contracted the disease. Find how many chicken contracted the disease.**
- 9. In a class, 60% of the students passed French and 50% of the students passed Arabic. All the students passed at least one subject. Find the percentage of students who passed both subjects**
- 10. In a class, 80% of the students passed French, 70% passed Arabic while 15% failed both subjects. If 325 students passed both subjects, find the number of students in the class.**
- 11. The gross monthly salary of Tom is Shs 800,000. He contributes 5% of his salary to a retirement account and then pays 25% of the remainder in taxes. Finally he pays a monthly health insurance of Shs 40,000. Find Tom's monthly take home pay**

12. In a French test one student scored **9** marks more than the other and his marks was **56%** of the sum of their marks. Find the marks obtained by each of them

13. In a school with a population of **4,000** students, **75%** are boys. **80%** of the boys and **60%** of the girls are day scholars. Find the:

(i) male students in the school

(ii) female boarding students

(iii) ratio of male day scholars to female day scholars

(iv) number of boarders in the school

14. A man spent **35%** of his salary on food, **20%** of the remainder on fees and **25%** of the remainder on rent. If he was left with **Shs 351,000**, find:

(i) the man's salary

(ii) how much did he spend on fees

15. In a country with a population of **14 million** people, **55%** are females. **75%** of the males and **40%** of the females are employed. Find the:

(i) male population in the country

(ii) female population unemployed

(iii) ratio of male population employed to female population employed

(iv) number of people employed in the country

PERCENTAGE CHANGE

Summary:

1. (i) **Percentage change** = $\frac{\text{Change in value}}{\text{Original value}} \times 100$

(ii) **Change in value** = $|\text{New value} - \text{Old value}|$

3. (i) **Percentage increase** = $\frac{\text{Increase in value}}{\text{Original value}} \times 100$

(ii) An increase of **20%** means the new value is **120%** of the old value

4. (i) **Percentage decrease** = $\frac{\text{Decrease in value}}{\text{Original value}} \times 100$

(ii) A decrease of **20%** means the new value is **80%** of the old value

5. (i) **Percentage error** = $\frac{\text{error}}{\text{Exact value}} \times 100$

(ii) **Error in value** = $|\text{Exact value} - \text{Inexact value}|$

EXAMPLES:

1. The price of an item increased from **Shs 5,000** to **Shs 6,000**. Find the percentage increase in the price of the item

Soln:

$$\text{Percentage increase} = \frac{\text{Increase}}{\text{Original}} \times 100$$

$$\text{Percentage increase} = \frac{(6,000 - 5,000)}{5,000} \times 100 = 20$$

2. The price of an item reduced from **Shs 8,000** to **Shs 6,000**. Find the percentage decrease in the price of the item

Soln:

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{Original}} \times 100$$

$$\text{Percentage increase} = \frac{(8,000 - 6,000)}{8,000} \times 100 = 25$$

3. The depth of a river was measured as **1.8m** instead of **1.6m**. Find the percentage error in the measurement

Soln:

$$\text{Percentage error} = \frac{\text{error}}{\text{Exact}} \times 100$$

$$\text{Percentage error} = \frac{(1.8 - 1.6)}{1.6} \times 100 = 12.5$$

4. Find the percentage error made in evaluating $\sqrt{xy - 14}$ instead of $\sqrt{xy + 14}$ when $x = 5$ and $y = 10$

Soln:

$$\text{Exact value} = \sqrt{5(10) + 14} = 8$$

$$\text{Inexact value} = \sqrt{5(10) - 14} = 6$$

$$\text{Percentage error} = \frac{\text{error}}{\text{Exact value}} \times 100$$

$$\text{Percentage error} = \frac{(8 - 6)}{8} \times 100 = 25$$

5. (i) Increase **500** by **20%**

(ii) Decrease **400** by **10%**

Soln:

$$(i) \text{ New value} = \frac{120}{100} \times 500 = 600$$

$$(ii) \text{ New value} = \frac{90}{100} \times 400 = 360$$

6. An item costing **Shs 2,400** is increased by **25%**. Find its new cost price

Soln:

$$\text{New cost price} = \frac{125}{100} \times 2,400 = \mathbf{3,000}$$

7. An item costing **Shs 4,000** is reduced by **20%**. Find its new cost price

Soln:

$$\text{New cost price} = \frac{80}{100} \times 4,000 = \mathbf{3,200}$$

8. The cost of an item is **Shs 4,600** after increasing it by **15%**. Find its original cost price

Soln:

$$\frac{115}{100} \times CP = 4,600$$

$$\therefore CP = \mathbf{4,000}$$

9. The cost of an item is **Shs 4,500** after a reduction of **10%**. Find its original cost price

Soln:

$$\frac{90}{100} \times CP = 4,500$$

$$\therefore CP = \mathbf{5,000}$$

10. A trader sells **30 pens** at **Shs 75,000**. If the cost of a pen is increased by **20%**, find how many pens can be bought with the same amount of money after the increase

Soln:

$$\text{Original cost price of a pen} = \frac{7,500}{30} = \mathbf{250}$$

$$\text{New cost price} = \frac{120}{100} \times 250 = \mathbf{300}$$

$$\therefore \text{Number of pens bought} = \frac{7,500}{300} = 25$$

11. A salary of **Shs 50,000** was increased by **8%** and then by a further **20%**. Find the final salary after the increments

Soln:

$$\text{New salary} = \frac{108}{100} \times 50,000 = \mathbf{54,000}$$

$$\therefore \text{Final salary} = \frac{120}{100} \times 54,000 = \mathbf{64,800}$$

12. An item costing **Shs 80,000** was reduced by **10%** and then by a further **5%**. Find its final cost price after the reductions

Soln:

$$\text{New cost price} = \frac{90}{100} \times 80,000 = \mathbf{72,000}$$

$$\therefore \text{Final cost price} = \frac{95}{100} \times 72,000 = \mathbf{68,400}$$

13. An item costing **Shs 25,000** was increased by **12%** and then later reduced by **20%**. Find its final cost price after the changes

Soln:

$$\text{New cost price} = \frac{112}{100} \times 25,000 = \mathbf{28,000}$$

$$\therefore \text{Final cost price} = \frac{80}{100} \times 28,000 = \mathbf{22,400}$$

14. The length and width of a rectangle were increased and reduced by **20%** and **5%** respectively. Find the percentage change in the area of the rectangle

Soln:

$$\text{Original area} = lw$$

$$\text{New area} = \left(\frac{120}{100}l\right) \times \left(\frac{95}{100}w\right) = \mathbf{1.14lw}$$

$$\text{Percentage change} = \frac{\text{Change}}{\text{Original}} \times 100$$

$$\text{Percentage change} = \frac{(1.14lw - lw)}{lw} \times 100 = 14$$

15. Give that y varies directly as x^3 , find the percentage change in y when x increases by 10%

Soln:

$$\text{Original value of } y = kx^3$$

$$\text{New value of } y = k\left(\frac{110}{100}x\right)^3 = 1.331kx^3$$

$$\text{percentage change} = \frac{\text{change}}{\text{Original}} \times 100$$

$$\text{Percentage change} = \frac{(1.331kx^3 - kx^3)}{kx^3} \times 100 = 33.1$$

16. The volume v of a cylinder varies jointly as its height h and the square of its radius r . find the percentage change in v when r increases by 20% and h decreases by 15%

Soln:

$$\text{Original value of } v = kr^2h$$

$$\text{New value of } v = k\left(\frac{85}{100}h\right)\left(\frac{120}{100}r\right)^2 = 1.224hr^2$$

$$\text{percentage change} = \frac{\text{change}}{\text{Original}} \times 100$$

$$\text{Percentage change} = \frac{(1.224hr^2 - khr^2)}{khr^2} \times 100 = 22.4$$

17. The salaries of A, B and C are in the ratio **2:3:5**. If their salaries are increased by **15%, 10% and 20%** respectively, find the new ratio of their salaries.

Soln:

$$A : B : C = 2:3:5 = 2y : 3y : 5y$$

\Rightarrow **Salary for:** $A = 2y$, $B = 3y$ and $C = 5y$

$$\text{New ratio} = \left(\frac{115}{100} \times 2y\right) : \left(\frac{110}{100} \times 3y\right) : \left(\frac{120}{100} \times 5y\right)$$

$$\text{Required ratio} = \frac{23y}{10} : \frac{33y}{10} : 6y = \mathbf{23:33:60}$$

18. The length, width and height of a cuboid are in the ratio **1:2:3**. If the dimensions of the cuboid are increased by **10%, 20% and 25%** respectively, find the percentage change in the volume of the cuboid

Soln:

$$L : W : H = 1:2:3 = y : 2y : 3y$$

\Rightarrow **Dimension for:** $L = y$, $W = 2y$ and $H = 3y$

$$\text{Original volume} = y(2y)(3y) = \mathbf{6y^3}$$

$$\text{New cost price} = \left(\frac{110}{100} \times y\right) \times \left(\frac{120}{100} \times 2y\right) \times \left(\frac{125}{100} \times 3y\right) = \mathbf{9.9y^3}$$

$$\text{Percentage change} = \frac{\text{Change}}{\text{Original}} \times 100$$

$$\text{Percentage change} = \frac{(9.9y^3 - 6y^3)}{6y^3} \times 100 = \mathbf{65}$$

EER:

1. (i) Increase 800 by 25%

(ii) Decrease 600 by 15%

2. The population of a school increased from 1,200 to 1,500 students. Find the percentage increase in the population of the school

3. The price of an item reduced from Shs 4,000 to Shs 3,400. Find the percentage decrease in the price of the item

4. A student wrote $(p + q)^2$ as $p^2 + q^2$. Find the percentage error the student made in evaluating $(p + q)^2$ when $p = 5$ and $q = 7$

5. A man's salary was increased by 10 % and then by a further 20 %. Find the:

(i) single percentage representing the two increments.

(ii) original salary if the man's final salary is Sh.660,000.

6. Given that $\sqrt{1 + x} \approx 1 + \frac{x}{2}$, find the percentage error made in approximating $\sqrt{1 + x}$ when $x = 1.56$

7. A student multiplied a number by $\frac{3}{5}$ instead of $\frac{5}{3}$. Find the percentage error in the calculation

8. A trader sells 5 cakes at Shs 7,000. If the cost of a cake is increased by 25%, find how many cakes can be bought with the same amount of money after the increase

9. A trader sells 9 books at Shs 4,500. If the cost of a book is reduced by 40%, find how many books can be bought with the same amount of money after the reduction

10. A trader sells 12 pens at Shs 3,000. If the cost of a pen is increased by 50%, find how many pens can be bought with the same amount of money after the increase

- 11.** Give that y varies directly as x^2 , find the percentage change in y when x increases by **10%**
- 12.** The length, width and height of a cuboid were increased by **10%, 20% and 15%** respectively. Find the percentage change in the volume of the cuboid
- 13.** The area of a triangle is given by $A = \frac{1}{2}bh$. Find the percentage change in A when b increases by **40%** and h decreases by **10%**
- 14.** Give that z varies directly as x and inversely as the square of y , find the percentage change in z when x increases by **25%** and y decreases by **20%**
- 15.** Give that p varies directly as the square of x and inversely as the square root of y , find the percentage change in p when x increases by **40%** and y decreases by **36%**
- 16.** In a stock of drinks, beer, wine and soda are in the ratio **5:7:8**. If this stock is increased by **40%, 50% and 75%** respectively, find the new ratio of the drinks.
- 17.** The ratio of boys to girls in a class is **7:8**. If the boys and girls are increased by **20% and 10%** respectively, find the new ratio of boys to girls in the class.
- 18.** An item costing **Shs 8,000** is increased by **15%**. Find its new cost price
- 19.** An item costing **Shs 5,000** is reduced by **10%**. Find its new cost price
- 20.** The cost of an item is **Shs 7,200** after increasing it by **20%**. Find its original cost price
- 21.** The cost of an item is **Shs 3,000** after a reduction of **25%**. Find its original cost price
- 22.** A salary of **Shs 80,000** was increased by **5%** and then by a further **15%**. Find the final salary after the increments
- 23.** An item costing **Shs 12,000** was reduced by **10%** and then by a further **5%**. Find its final cost price after the reductions

- 24.** An item costing **Shs 60,000** was increased by **8%** and then later reduced by **15%**. Find its final cost price after the changes
- 25.** The population of a school is **800** students. If it annually increases by **20%**, find the population of the school after two years
- 26.** The population of a school is **1000** students. If it annually decreases by **20%**, find the population of the school after two years
- 27.** A cow and a calf cost **Shs 500,000** and **Shs 200,000** respectively. If the price of the cow and that of the calf is increased by **24%** and **15%** respectively, find the cost of **one** dozen cows and **2** dozen calves