U.C.E Mathematics 1

(For S.1)

CONTENTS

1. NUMBER SYSTEMS

1.1 NUMBER BASES

Definition: Number bases are different ways of using the same number. We use a system called base 10, or decimal, for our arithmetic, but there are as many number bases as there are numbers.

These common bases also have proper names, shown in parentheses:

- base 2 (binary)
- base 8 (octal)
- base 10 (decimal)
- base 12 (duodecimal)
- base 16 (hexadecimal)

We are accustomed to writing numbers in base ten, using the symbols for 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, 75 means 7 tens and five ones. However numbers can be written in any number base.

If we use base 8 instead of base ten, then 75 is written as 113 which denotes one sixty four (8^2), one eight (8^1) and 3 units (instead of hundreds, tens and units).

Base 2 is particularly useful as it only requires two symbols, for zero and one, and it is the way numbers are represented in computers.

Just as, in base ten, the columns represent powers of 10 and have 'place value' 1, 10, 10^2, 10^3 etc. (reading from right to left), so in base 2, the columns represent powers of 2. Hence the number 1001011 denotes (reading from right to left):

1 unit (2^0), 1 two (2^1), no fours (2^2), 1 eight (2^3), no six teens (2^4), no thirty twos (2^5), 1 sixty four (2^6).

The number 1001011 in base 2 is the same as the number 75 in base ten.

As another example, we use the symbols 0, 1, 2, 3 and 4 to represent numbers in base 5. The columns in base 5 have 'place value' 1, 5, 25, 125, 625 etc reading from right to left. The number 75 in base ten is the same as the number 300 in base five, that is 3 twenty fives, no fives and no units.

The number 4102 in base 5 denotes 2 units, no fives, 1 twenty five and 4 one hundred and twenty fives making altogether 527 in base ten.

Writing the number 75 in base six we get 203, which represents 2 thirty sixes, no sixes and 3 units.

We have seen that 75 (base10), 1001011 (base 2), 300 (base 5), 113 (base 8), and 203 (base 6) all represent the same number.

Similarly, we can write 75 in any base we choose, and we can write all numbers in any base.

To write numbers between 0 and 1, we use negative powers of the base. For example, in base 2 we use halves, quarters, eighths, sixteenths etc instead of the tenths, hundredths, thousandths etc. which we use in base ten.

So if we write 11.11 in base 2 this denotes $2^1 + 2^0 + 2^{-1} + 2^{-2}$. The equivalent in base 10 is $2 + 1 + {}^1/_2 + {}^1/_4$, that is, 3.75 in base 10.

Converting from Decimal to any other base

Example 1.

Express 5213_{ten} with a base 8.

To express 5213_{ten} with a base 8 we divide repeatedly by 8, and the remainders in reverse order give the required number.

8	5213	
8	651	rem 5
8	81	rem 3
8	10	rem 1
8	1	rem 2
	0	rem 1

$\therefore 5213_{ten} = 12135_{eight}$

Example 2.

Express 21_{ten} with a base 2.

2	21	
2	10 rem	1
2	5 rem	0
2	2 rem	1
2	1 rem	0
	0 rem	1

$\therefore 21_{ten} = 10101_{two}$

Converting to decimal from any other base.

One method is to write out the number in full:

$432_{five} = (4 \times 5^2) + (3 \times 5) + (2 \times 1) = 100 + 15 + 2 = 117_{ten}$

$10101_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2) + (1 \times 1)$

$= 16 + 0 + 4 + 0 + 1 = 21_{ten.}$

$526_{eight} = (5 \times 8^2) + (2 \times 8) + (6 \times 1) = 320 + 16 + 6 = 342_{ten.}$

A quicker method can often be done mentally. Thus, for 526_{eight}, we say:
5×8 is 40, plus 2 makes 42; 42×8 is 336, plus 6 makes 342.
$\therefore 526_{eight} = 342_{ten}$.

Conversion of others

Whenever we want to change one base to another base we must convert it to base ten first.
For example,

(i) Change $8et1_{twelve}$ to base 7.
(ii) Find n if $100001_{two} = 45_{n.}$

Solutions

(i) Changing $8et1_{twelve}$ to base ten,
$8et1_{twelve} = (8 \times 12^3) + (11 \times 12^2) + (10 \times 12^1) + (1 \times 12^0)$
$= 13824 + 1584 + 120 + 1$
$= 15529$
Converting 15529 to base seven we have

7	15529
7	2218 rem 3
7	316 rem6
7	45 rem 1
7	6 rem 3

Hence, $8et1_{\text{twelve}} = 63163_{\text{seven}}$

(ii) Converting 100001 two to base ten we have

$(1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1\times 2^0)$

$1\times 32 + 0 \times 16 + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1$

$= 32 + 0 + 0 + 0 + 0 + 1$

$= 33$

Converting 45_n to base ten we have $(4 \times n^1) + (5 \times n^0)$

$= 4n + 5$

$\Rightarrow 33 = 4n + 5$

$\Rightarrow 4n = 33 - 5 = 28$

$\Rightarrow n = 7$

Calculations with any base other than 10

When performing the ordinary operations of arithmetic in any base other than ten, we must remember that the digits no longer denote successive powers of ten, but of the base which is being used. This means that the carrying figures are found by dividing by the base, not by ten.

Example 3.

Add 232_{five} to $344_{\text{five.}}$

We say 2 + 4 = six = (1×5) + 1. Put down 1
And carry 1; 3+4+1 = eight= (1×5) +3;
Put down 3 and carry 1; 2+3+1=six
= (1×5) +1; put down 1 and carry 1 into the fourth place from the right.

$$\begin{array}{r} 232 \\ +\underline{344} \\ \underline{1131} \end{array}$$

Ans. 1131_{five}

Example 4.

Subtract 43_{eight} from $72_{\text{eight.}}$

We cannot subtract 3 from 2, so we borrow one from the eights column and call it eight in the ones column, making 2+8, or ten; 3 from 10 leaves 7.
In the eights column we can either add 1 to 4 and subtract 5 from 7, or take 1 from the 7, leaving 6, and then subtract 4 from 6.
In either case it leaves 2.

$$\begin{array}{r} 72 \\ \underline{-43} \\ \underline{27} \end{array}$$

Ans.$27_{\text{eight.}}$

Example 5

Multiply 6734_{eight} by27_{eight}.

$$\begin{array}{r} 6734 \\ \underline{27} \\ 15670 \\ \underline{60404} \\ \underline{237304} \end{array}$$

When multiplying by the 2, we say 2×4 = eight = $(1\times8) + 0$; put 0 and carry 1; $(2\times3) +1$= seven, so we put seven; 2×7 = fourteen= $(1\times8) + 6$, so we put 6 and carry 1; $(2\times6) +1$ =thirteen = $(1\times8) +5$, so we put 5 and carry 1.

When multiplying by the 7 we say $7\times4 = 28$. **Ans. 237304_{eight}**
= $(3\times8) +4$; put 4 and carry 3; $(7\times3) +3$ = twenty four = $(3\times8) + 0$; put 0 and carry 3; $(7\times7)+3$ = fifty two = $(6\times8) +4$; put 4 and carry 6; $(7\times6) + 6$ = forty eight = $(6\times8) +0$; put 0 and carry 6. The addition is then done as in example 3.

Example 6

Divide 110111_{two} by 101_{two}

Solution

$$\begin{array}{l} 101\,]\,110111\,[\,1011 \\ \quad\;\;\underline{101} \\ \quad\quad\;\;111 \\ \quad\quad\;\;\underline{101} \\ \quad\quad\quad\;\underline{101} \\ \quad\quad\quad\;\underline{...} \end{array}$$

Ans. 1011_{two}

NOTE: To solve problems with number bases, it is easiest to write everything in base ten and then finally express the answer in the required base. However, you can use whichever method you prefer.

EXERCISE 1.1

1. Convert to base ten: 43_{eight}, 2314_{eight}, 7_{eight}, 27_{eight}, 127_{eight}.

2. Express with base 8: 47_{ten}, 252_{ten}, 725_{ten}, 866_{ten}, 9_{ten}, 39_{ten}.

3. Write 5432_{ten} with base: (i) five (ii) three (iii) six.

In numbers 4 to 10, carry out the calculations and give the answers in the base indicated.

4. $2102_{three} + 2202_{three}$

5. $11011_{two} + 1011_{two}$

6. $72_{eight} + 41_{eight} + 267_{eight}$

7. $201_{three} - 122_{three}$

8. $11011_{two} - 1101_{two}$

9. $152_{eight} \times 43_{eight}$

10. $124_{five}\times\; 32_{five}$

7. $201_{three} - 122_{three}$

11. $11111_{two} \times 1001_{two}$

12. $11001_{two} \times 111_{two}$

13. $3256_{eight} \div 46_{eight}$

14. $3212_{five} \div 14_{five}$

15. $10010_{two} \div 110_{two}$

Point notation

Example 7. Express 0.75_{ten} in binary.

Solution:

In binary, place values after the point are as shown below:

0. ___ ___ ___ ___ etc.

$\left(\frac{1}{2}\right)_{ten}$ $\left(\frac{1}{4}\right)_{ten}$ $\left(\frac{1}{8}\right)_{ten}$ $\left(\frac{1}{16}\right)_{ten}$

But $0.75_{ten} = \left(\frac{3}{4}\right)_{ten} = \left(\frac{1}{2}+\frac{1}{4}\right)_{ten}$.

Therefore, $0.75_{ten} = 0.11_{two}$.

Example 8. Express 0.12_{six} as a fraction in base ten.

Solution:

$0.12_{six} = \left(1\times 6^{-1} + 2\times 6^{-2}\right)_{ten} = \left(\frac{8}{36}\right)_{ten} = \left(\frac{2}{9}\right)_{ten}$.

Example 9. Work out: (i) $28.57_{nine} + 6.34_{nine}$

(ii) $30.241_{five} - 14.143_{five}$.

Solution:

(i)
$$\begin{array}{r} 28.57_{nine} \\ +\ \underline{6.34}_{nine} \\ \underline{36.02}_{Nine} \end{array}$$

(ii)
$$\begin{array}{r} 30.241_{five} \\ -\ \underline{14.143}_{five} \\ \underline{11.043}_{five} \end{array}$$

EXERCISE 1.2

1. Use point notation to express $\left(\frac{5}{12}\right)_{ten}$ in base eight.
2. Express 0.6_{eight} in base ten in the form $\frac{a}{b}$.
3. Express $\left(\frac{3}{8}\right)_{ten}$ in binary in point notation.
4. Express 1001.01_{two} in base ten in point notation.
5. Use point notation to express 45.3_{six} in base ten.
6. Express $\left(\frac{2}{10}\right)_{seven}$ in base ten.
7. Using point notations express 6.41_{eight} in base four.

Work out the following:

8. $0.111_{two} + 0.011_{two}$

9. $11.1_{two} - 0.111_{two}$

10. $2.123_{four} + 0.313_{four}$

11. Find n: (i) $45_n = 41_{ten.}$ (ii) $103_n + 26_n = 131_n$

1.2 Numbers

1.21. Natural numbers (counting numbers).

These are numbers used in counting.
Thus, $\{Natural\ numbers\} = \{Counting\ numbers\} = \{1,2,3,4,5,...\}$.

1.22. Whole numbers.

These are counting numbers including zero. i.e.

$\{Whole\ numbers\} = \{0,1,2,3,4,5...\}$

1.23. Integers:

This is a set of whole numbers together with negative whole numbers. We can write:

$\{Integers\} = \{...^{-}4,\ ^{-}3,\ ^{-}2,\ ^{-}1, 0, 1, 2, 3, 4, ...\}$

1.24 Factors and Multiples.

A whole number which divides exactly into another is said to be a factor of that number. Thus, if one number goes exactly into another number, the first number is called a **factor** of the second and the second number is called a **multiple** of the first.
Thus $3\times4=12$, $\therefore$ 3 and 4 are factors of 12. Again $2\times 6=12$, 2 and 6 are also factors of 12. Also 12 is a multiple of any of the numbers 2, 3, 4, or 6, since each of these numbers goes exactly into 12.
A **prime** number is a number which has only two factors, itself and 1. Thus 2, 3, 5, 7, 11, 13, 17, 19, 23, …are all prime numbers. A factor which is a prime number is called a **prime factor**.
Thus 3 is a prime factor of 12 and so is 2.

Note:

(a) Every number is a factor of itself.
(b) 1 has only itself as a factor and therefore it is not a prime number.
(c) Numbers that have more than two factors are called **composite numbers. Composite numbers** can be expressed as products of their prime factors. We obtain prime factors of a number

by successive division of the number starting with the least possible prime factor.

Example 10: Express 36 in terms of its prime factors.

Solution:

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

So $36 = 2\times 2 \times 3\times 3 = 2^2\times3^2$

Example 11. Express 4312 in prime factors.
Solution:

$$\begin{array}{r|l} 2 & 4312 \\ \hline 2 & 2156 \\ \hline 2 & 1078 \\ \hline 7 & 539 \\ \hline 7 & 77 \\ \hline 11 & 11 \\ \hline & 1 \\ \hline \end{array}$$

Hence, $4312 = 2^3 \times 7^2 \times 11$.

Note: The above process of writing numbers in terms of their prime factors is called **prime decomposition** or **prime factorization.**

Exercise

1. List all the factors of each of the following numbers:

(a) 8 (b) 12 (c) 20
(d) 11 (e) 45 (f) 32

2. Express the following numbers as products of their prime factors.

(a) 8 (b) 42 (c) 90

(d) 240 (e) 72 (f) 1024

(g) 360 (h) 800 (i) 625

(j) 280 (k) 102 (l) 3465

(m) 1000 (n) 1764 (o) 3969

(p) 1728 (q) 9000 (r) 2744

(s) 9520 (t) 6936 (u) 1440

H.C.F and L.C.M

The **Highest Common Factor** (usually written as H.C.F) of two or more numbers is the greatest number which is a factor of each of them.
The **Lowest Common Multiple** (L.C.M) of two or more numbers is the least number which is a multiple of each of them.

To find the HCF of two or more numbers:

(i) Express each of the numbers as a product of prime factors,
(ii) Pick out the least power of each common factor. The product of these gives the HCF or GCF

To find the LCM we proceed as follows:

(i) If 2 divides any of the numbers, divide through by 2 as many times as possible. Write down, as it is, any number not divisible by 2.
(ii) Go to 3 and repeat the process. Then go to 5, 7, 11, … If a number does not divide any of the numbers, skip it.
(iii) Proceed as in (ii) until you arrive at a row of 1s.
(iv) Pick the highest power of each of the prime factors that appear. The factors need not be common. Multiply them.

Example 12

Find the H.C.F and the L.C.M of 84, 1386 and 210.

Solution

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \\ \hline \end{array} \qquad \begin{array}{r|l} 2 & 1386 \\ \hline 3 & 693 \\ \hline 7 & 231 \\ \hline 7 & 77 \\ \hline 11 & 11 \\ \hline & 1 \\ \hline \end{array} \qquad \begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \\ \hline \end{array}$$

Hence $84 = 2^2 \times 3 \times 7$.

$1386 = 2 \times 3^2 \times 7 \times 11$

$210 = 2 \times 3 \times 5 \times 7$

The common factors available are 2, 3 and 7. The ones with least power are 2^1, 3^1 and 7^1.
Therefore, HCF = $2^1 \times 3^1 \times 7^1 = 42$.

The prime factors available are 2, 3, 5, 7 and 11. Their highest powers are 2^2, 3^2, 5^1, 7^1 and 11^1.
Therefore, LCM = $2^2 \times 3^2 \times 5 \times 7 \times 11 = 13860$.

Example 13

Find the LCM of 56, 70 and 98.

$56 = 2^3 \times 7$
$70 = 2 \times 5 \times 7$
$98 = 2 \times 7^2$
The LCM = $2^3 \times 5 \times 7^2 = 1960$.

Example 14

Find the LCM and HCF of 630 and 150

Solution

$630 = 2 \times 3^2 \times 5 \times 7$
$150 = 2 \times 3 \times 5^2$.
The common prime factors with the least power are 2, 3 and 5.

Therefore, the HCF = $2 \times 3 \times 5$

To find the LCM;
$630 = 2 \times 3^2 \times 5 \times 7$
$150 = 2 \times 3 \times 5^2$
The prime factors available are 2, 3, 5 and 7. Their highest powers are 2^1, 3^2, 5^2 and 7^1. Multiplying them gives the LCM. So LCM is $2 \times 3^2 \times 5^2 \times 7 = 3150$.

Alternative method for finding the LCM:

Example 15 Find the LCM of 16, 12 and 24.

	16	12	24
2	8	6	12
2	4	3	6
2	2	3	3
2	1	3	3
3	1	1	1

Therefore, LCM of 16, 12 and 24 = $2^4 \times 3$ = 48

Alternative method for finding the HCF:

Example 16

Find the HCF of 12 and 15.

Solution

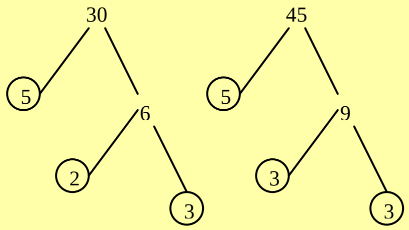
$F_{12} = \{1, 2, 3, 4, 6, 12\}$ and $F_{15} = \{1, 3, 5, 15\}$

The common factors are $\{1, 3\}$. The highest of these is 3. Therefore, the HCF of 12 and 15 is 3.

Example 17

Find the H.C.F of 30 and 45.

Solution



$30 = 2 \times 3 \times 5$
$45 = 3 \times 3 \times 5$
Both 30 and 45 have 3×5 in common; thus the H.C.F of 30 and 45 is 15.

EXERCISE 1.3

State the LCM and HCF of

1. 2×3, $2^2 \times 3 \times 5$
2. 3×7, $2^2 \times 7$
3. 2×11^2, 3×11
4. $2^2 \times 7$, $2^3 \times 5^2 \times 7$, $2^2 \times 7 \times 13$
5. 3^4, $32 \times 5^{2.}$

Find the HCF and the LCM, leaving the LCM in prime factors.

6. 72, 162
7. 126, 198
8. 210, 336, 294.

9. 455, 286.

10. 616, 2156.

11 Five small containers of capacity 16, 72, 12, 24 and 56 litres are to be used to fill a bigger container. What is the capacity of the bigger container which can be filled by each of the above containers exactly without remainder when used separately?

12 Musa, John and David start at the same time, position and direction to run round a circular field. Musa takes 180 seconds, John takes480 seconds and David takes 720 seconds to complete one circuit. If they start running at 3.00 pm, at what time will they all be at the same position?

13 Find the value of n if $79_{13} = 144_n$.

14 Find the value of n if $124_n = 52_{ten}$

15 Express $\frac{108}{28}$ as a product of its prime factors.

16 A room measures 540 cm by 420 cm. Find the length of the largest square tiles that can be used to cover the floor without requiring any cutting.

17 Traffic lights at three different junctions show green light at intervals of 10 seconds, 12 seconds and 15 seconds. They all show green at 1.00 p.m. At what time will they all again show green together?

18 In a large school, it is possible to divide the pupils into groups of equal numbers of 24, 30 or 32 and have no pupils left over. Find the least number of pupils in the school that makes this possible.

19 Find the shortest length that can be cut into exactly equal lengths of 4 cm or 7 cm or 18 cm.

20 Find the LCM of the following groups of numbers. Leave the answers in power form.

(a) 3, 9, 15
(b) 4, 5, 6
(c) 28, 36
(d) 48, 90, 125
(e) 200, 350
(f) 18, 42, 84
(g) 7, 75, 150
(h) 30, 56, 72

1.25 Number patterns

A **sequence** is a set of numbers following one another in order according to a special rule.

Examples

Write down the next two numbers in each of these sequences:

(a) 7, 14, 21, 28, 35, …………

(b) 2, 3, 5, 7, 11, 13, …………

(c) 1, 3, 6, 10, 15……………

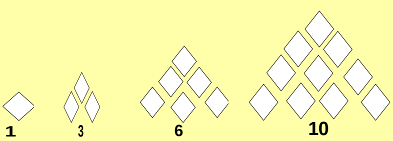
(d) 2, 4, 6, 8, 16, 32,…………

(e) 1, 4, 9, 16, 25, ………….

(f) 1, 1, 2, 3, 5, 8,……………

Triangle Numbers

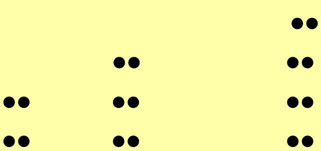
Are 1 and any natural number which can be represented by a triangular pattern. E.g. 1, 3, 6, 10 etc.



etc

Rectangle numbers

These are numbers which can give a rectangular pattern of dots. They must have at least two different factors apart from 1 and itself. Hence all even numbers except 2 are rectangular numbers.



4 6 8 etc

Square numbers

A square number or a perfect square is 1 and any natural number which can be represented by a square pattern. E.g. 1, 4, 9, 16, 25 etc

1.26 Divisibility test

(i) A number is divisible by 2 if its last digit is even e.g. 136, 1760.

(ii) A number is divisible by 3 if the sum of its digits is divisible by3, e.g. 4713 since 4 + 7 + 1 + 3 = 15 which is divisible by 3.

(iii) A number is divisible by 4 if the number formed by the last two digits is divisible by 4, e.g. 144; 128.

(iv) A number is divisible by 5 if its last digit is 5 or 0.

(v) A number is divisible by 6 if it is divisible by 2 and 3.

(vi) A number is divisible by 8 if the number formed by the last 3 digits is divisible by 8. e.g. 29848; 1048.

(vii) A number is divisible by 9 if the sum of its digits is divisible by 9 e.g. 447129 since 4+4+7+1+2+9 = 27 which is divisible by 9.

(viii) A number is divisible by 11 if the difference between the sum of the digits in even places and the sum of digits in odd places is 0 or divisible by 11. e.g., 733689

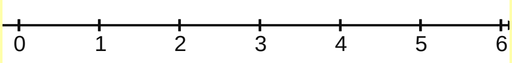
Digits in even places (3+6+9 = 18)
Digits in odd places (7+3+8 = 18).
The difference is 0. Hence 733689 is divisible by 11.
What about 80927?
Digits in even places are (0+2 = 2)
Digits in odd places are (8+9+7 = 24)
The difference is 22 which is divisible by 11. Hence 80927 is divisible by 11.

1.3 Integers

This is a set of whole numbers together with negative whole numbers. We can write
$\{Integers\} = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, \ldots.\}$.
When numbers are used with symbols + or - in front of them, they are called **directed** numbers. E.g. +1, +2, +3, …-1, -2, -3, …

The number line

Natural numbers can be represented on a number line as shown in the figure below



On the number line, numbers to the left of 0 are less than 0 and are called **negative numbers**. Those to the right of 0 are greater than 0 and are called **positive numbers**.

Addition of integers

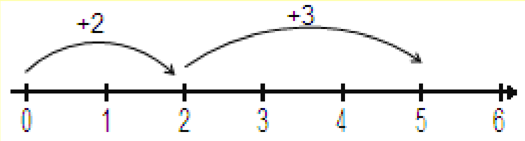
Example

Use the number line to find the value of: (a) $^{+}2 + ^{+}3$ (b) (-3)+(-4)

(c) 7+(-5)

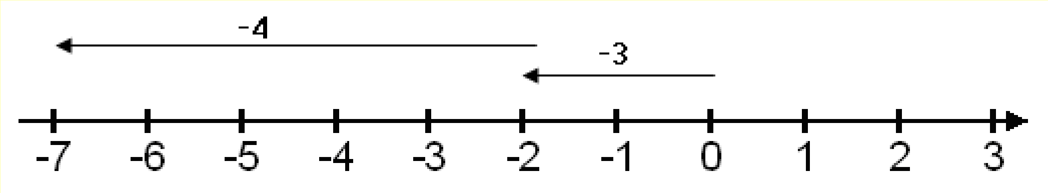
Solution:

(a) On the number line we move 2 steps from 0 to +2 and then from this point we move a further 3 steps to the right to represent +3. We end up at +5. See figure below.



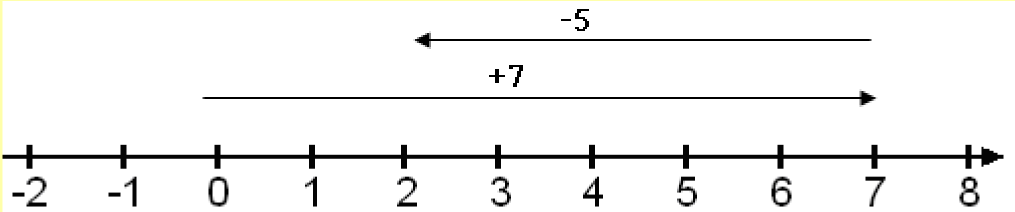
This shows that $^{+}2 + ^{+}3 = ^{+}5$.

(b)



(-3) + (-4) = -7 or -3 - 4 = -7

(c)



7+ (-5) = +2 or 7+ (-5) = 7 - 5 = 2
On the number line, move 7 steps to the right of 0 to represent +7 and then 5 steps to the left of +7 to represent -5. The end point is +2.

Subtracting integers

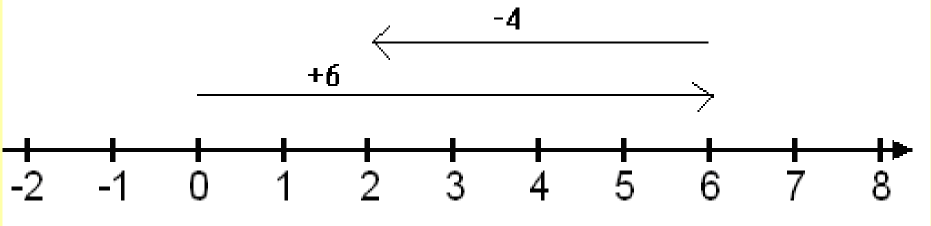
Example

Use a number line to find the value of:

(a) +6 - (+4) (b) -3 - (+5) (c) 4 - (-3)

Solution

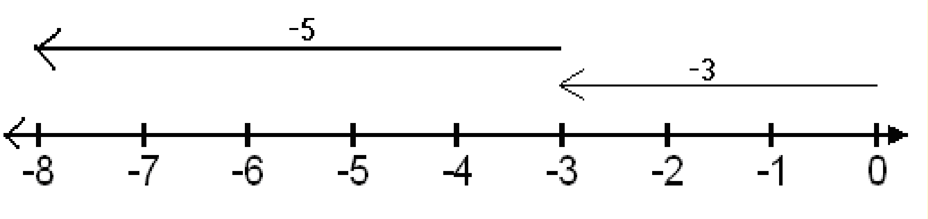
(a) In the figure below, we move 6 steps to the right of 0 to represent +6, and then from this point we move 4 steps to the left. This is because we are taking away 4 from 6.



We see that 6 - 4 = 2. Notice that (+6) - (+4) = (+6) + (-4) = 2. This means that subtracting a positive number is the same as adding a negative number.

(b) In the figure below we move 3 steps to the left of 0 to represent -3 and then from this point we move another 5 steps to the left.

Thus, -3 - (+5) = -8. Again, note that, -3 - (+5) = (-3) + (-5) = -(3 + 5) = -8.



(c) In the figure below, to subtract -3 from 4 we look for a number which when added to -3 gives 4. Therefore, we look for the number of steps between 4 and -3. Since (+4) = (+7) + (-3), it follows that (+4) - (-3) = (+7). Thus, 4 - (-3) = 4 + 3 = 7. This means that subtracting a negative number is the same as adding the positive number. Therefore, in general, if a and b are numbers, then,

(i) a - (+b) = a - b
(ii) a - (-b) = a + b

EXERCISE 1.4

Give the value of:

1. (+6) – (+2)

2. (-2) – (+5)

3. +3 + (-6)

4. (-2) - (-7)

5. 6 + (-2)

6. (-9) + (-1)

7. (-6) + 6

8. 0 – 4

9. 300 – 500

10. 0 – (-6)

11. 18 – (-20)

12. -43 - (-43)

13. (+2) – (-3)

14. 0 - 58

Simplify:

15. (+5a) – (+a)

16. (-2b) – (-3b)

Multiplying integers

When the same number is added to itself a number of times, the result obtained is the same as multiplying the number by the number of times it is added. For example:

(i) $3 + 3 + 3 + 3 + 3 = 15$

$3 \times 5 = 15$

(ii) $(-4) + (-4) + (-4) = -12$

$\therefore -4 \times 3 = -12$

However, this relationship between addition and multiplication does not apply in cases where two negative numbers are multiplied. Consider the following multiplication table.

$4 \times -2 = -8$

$3 \times -2 = -6$

$2 \times -2 = -4$

$1 \times -2 = -2$

$0 \times -2 = 0$

$-1 \times -2 = +2$

$-2 \times -2 = +4$

$-3 \times -2 = +6$

$-6 \times -2 = +8$

You will notice that the numbers in the first column decrease by 1 as you go down the table whist the corresponding products increase by 2. The numbers in the first column are all multiplied by the same number (-2).

Whenever two negative numbers are multiplied, the product is a positive number.

In general the **rules of multiplication** of are:

- Two numbers with like signs have a positive product.
- Two numbers with unlike signs have a negative product.
- Multiplication of any number by zero always gives zero.

Dividing integers

When a number is divided by another number, the result obtained is called a **quotient**. For example, the quotient of 12 ÷ 6 is 2. The rules of dividing integers are the same as those of multiplying except that we do not divide numbers by zero. It is meaningless to write 5 ÷ 0 because the quotient does not exist. However, zero divided by any number is always zero, for example,

$0 \div (-42) = 0 \div 1000 = 0$

Exercise 1.5

Find the values of:

1. $(-2) \times (+3)$

2. $(-10) \div (-2)$

3. $(-5) \times (-4)$

4. $(+6) \times (-5)$

5. $(-4) \times (-2)$

6. $(-6) \div (+3)$

7. $(+24) \div (+6)$

8. $0 \times (-3)$

9. $(+3) \times (+4)$

10. $(-18) \div (-6)$

11. $(-7) \div (+7)$

12. $(-3) \times (-3)$

13. $\frac{(-9)}{(-1)}$

14. $\frac{(-14)}{(+7)}$

15. $\frac{(-3)}{(-3)}$

Simplify:

16. $(-4x) \div (+x)$

17. $(-3x) \times (-2x)$

18. $(-6y) \div (-2)$

19. $(+ab) \div (-b)$

20. $(-xy) \div (-x)$

21. $(+18x) \div (-3x)$

22. $(-2) \times (+3y)$

23. $(-5) \times (-2d)$

24. $\frac{(-a)\times(-b)}{(-c)}$

Combined operations

When a question involves more than one operation, then there is a sequence of operations which has to be followed. The sequence is: Brackets, Of, Division, Multiplication, Addition, and Subtraction. Whenever there are brackets, the operations in the brackets must be performed first followed by the others according to the above sequence. Division and multiplication have no priority over each other so they can be performed in the order they appear in the expression.

When addition and subtraction are the only operations in an expression, they can be carried out in the order in which they appear in the expression. For example:

$10 + 5 - 3 = (10 + 5) - 3 = 15 - 3 = 12.$

Or $10 + 5 - 3 = 10 + (5 - 3)$

$= 10 + 2 = 12.$

$6 - 4 + 1 = (6 + 1) - 4 = 7 - 4 = 3$

Or $6 - 4 + 1 = (6 - 4) + 1$

$= 2 + 1 = 3$

Example

Find the values of:

(a) $4 + 12 \div 6$

(b) $3 + 20 \times 2 - 12$

(c) $39 \div (16 - 3) + 4 \times 5 - 8$

(d) $6 \times 5 - 33 \div 3 + 40$

Solutions

(a) $4 + 12 \div 6 = 4 + (12 \div 6)$

$= 4 + 2$

$= 6$

(b) $3 + 20 \times 2 - 12 = 3 + (20 \times 2) - 12$

$= 3 + 40 - 12$

$= 43 - 12 = 31$

(c) $39 \div (16 - 3) + 4 \times 5 - 8 = 39 \div 13 + 4 \times 5 - 8$

$= (39 \div 13) + (4 \times 5) - 8$

$= 3 + 20 - 8$

$= 23 - 8$

$= 15$

(d) $6 \times 5 - 33 \div 3 + 40 = 30 - 11 + 40$

$= (30 + 40) - 11$

$= 70 - 11 = 59$

Remember that we use brackets when carrying out any operation involving negative numbers. For example, we write $8 \times (-4)$ and not 8×-4 or $5 + (-2)$ and not $5 + -2$.
However, note that it is appropriate to leave out brackets when you use the raised negative sign. Thus we can write the above expressions as, $8 \times {}^{-}4$ or
$5 + {}^{-}2$.

Exercise 1.6:

Work out:

1. $16 + 2 - 5$
2. $13 - 4 + 8$
2. $7 + 1 - 3 + 2$
4. $11 + 3 \times 3$
5. $5 \times 3 - 5$
6. $4 \times 8 - 8 \times 3$
7. $18 \div 6 - 3 \times 4 - 2$
8. $18 \div (6 - 3) \times 4 - 2$
9. $30 \div (-6) - 4$
10. $5 - (-12) \times 3 - 24 \div 6$
11. $63 \div 3 - 42 \div (-7)$
12. $-3 \times 23 + (-5) \times (-1) - 8 \times (-4)$
13. $6 + 36 \div 9 + 14 \div 2 \times 5$
14. $\{9 + (-2) \times (-15)\} \times (-2 + 7) \div 3$

Evaluate:

15. $\dfrac{-8 \times {}^{-}3}{-4}$
16. $\dfrac{9 \times {}^{-}2}{-4-(-2)}$
17. $\dfrac{-6+(-5)+8 \times {}^{-}2}{-4+(-2)}$
18. $\dfrac{-2 \times {}^{-}5+14}{-3 \times 4}$

2. SETS AND OPERATIONS

Definition: A set is a collection of objects, ideas etc. Sets are named by

1. Describing them, or
2. Listing their members

Examples:

(a) The set of people in this room
(b) The set of exercise books in S.1.
(c) The set whose members are 1, 2, 3,4,5,6.
(d) The set whose members are C, A, N, T, O, R.

2.1 Symbols:

Below are some of the symbols used in sets:

(a) **Curly brackets:** {} this means 'the set of'
E.g. {people in this room} means 'the set of people in this room'

(b) **Members (or elements) of sets.**

{Vowels} = {a, e, i, o, u}. So a is a member of {a, e, i, o, u}.
The symbol used is $\in$ which means 'is a member of'. Is r a member of
{a, e, i, o, u}?
No r is not a member of {a,e,i,o,u}. Here we write: r$\notin${a, e, i, o, u}.

(c) **Capital letters**

Capital letters are often used for sets. For example, we could write **A** = {a, e, i, o, u}. 'A' here is read as 'the set A'. Using capital letters to represent sets enables us to write statements about sets in a very brief way. E.g.: a $\in$A; z $\notin$A etc.

(d) **Number of members**

Consider A = {a, e, i, o, u}. A has 5 members. We write: $\cap$(A) = 5.
We can also write: $\cap${a, e, i, o, u} = 5.

(e) **The empty set**

Let T = {people with three eyes}. There are no members in this set. So we call it an empty set. The symbol used for empty set is ϕ or{}.
Thus for the above set we write T = {} or ϕ.

2.2 Relationship between sets.

1. Disjoint sets

These are sets that have no members common to both sets.

Example:

If A = {a, e, i, o, u} and B = {1, 2, 3, 4, 5}. Since the sets A and B have no common members found in both sets, they are said to be disjoint.

2. Equal sets

Two or more sets are equal if every member in one set is also a member in the other set. For example: A = {l, m, n, o, p}, C = {l, m, n, o, p}. So A = C.

3. Intersection of sets

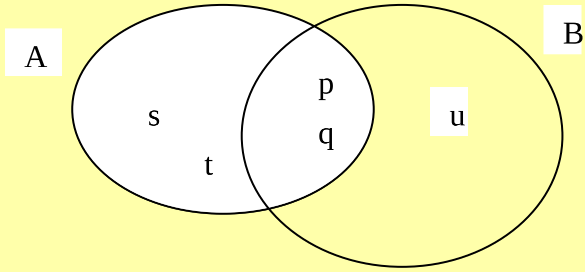
Suppose we have two sets A and B. Then the intersection of A and B, written as $A \cap B$, is a set consisting of members that are found in both sets.

Example

It is given that A = {p, t, s, q, r} and B = {p, q, u}.
$A \cap B$ = {p, q}.

NOTE:

We can use a Venn diagram to represent the above information:



From above $\cap(A \cap B) = 2$.

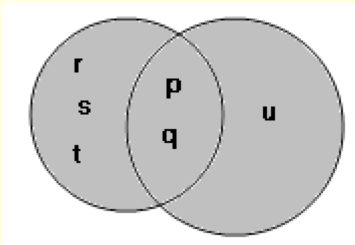
EXERCISE 2.1

Draw Venn diagrams to illustrate the relationship between the pairs of sets below.

1. A = {6, 8, 5}, B = {5,6,3,2}.
2. M = {odd numbers less than 10};
   N = {the even numbers less than 11}

4. **Union of sets**

Consider the sets A = {p, t, s, q, r}; B = {p, q, u}
The single set consisting of the members of A and the members of B, written as $A \cup B$, is called the union of A and B. The shaded region represents the union of A and B.



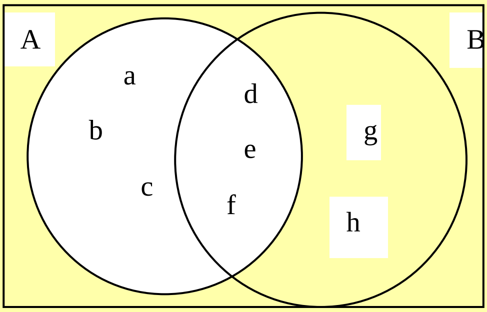
The union of A and B contains:

- the members of A only
- the members of $A \cap B$
- the members of B only.

Example

It is given that A = {a, b, c, d, e, f} and B = {d, e, f, g, h}.
(a)List the members of $A \cup B$
(b) State $\cap(A \cup B)$.

Solution



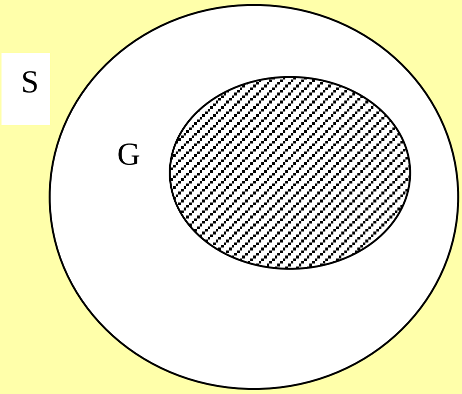
(a) $A \cup B$ = {a, b, c, d, e, f, g, h,}
(b) $\cap(A \cup B) = 8$.

Note

The members d, e, and f which are common to A and B are listed only once in the union of A and B.

5. **Subsets**

Suppose Form 1E contains boys and girls.
Let S = {students in form 1E}; and G = {girls in Form 1E}. This Venn diagram shows the relationship between G and S.



All the members of G are also members of S. we say G is a **subset** of S. The symbol for 'is a subset of' is $\subset$. G is a subset of S is written G $\subset$ S. The symbol $\not\subset$ means
'is not a subset of'.

EXERCISE 2.2

Draw Venn diagrams to illustrate the relationship between the pairs of sets below:

1. C = {Games}; D = {Chess, mweso, ludo}

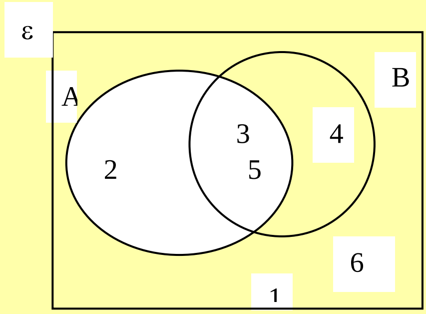
2. A = {countries in Africa}; B = {countries in Europe}

3. G = {a,b,c,d,e}; H = {b,g,h,e,m,n}.

6. Universal set (ε)

This is a set containing all members in a described set. For example, ε = {counting numbers less than 10} etc
Suppose A = {1,2,3,4}; and B = {3,4,5,6,7,8,9}. Then both A and B are subsets of ε.
We can define the compliment of a set as that set containing members that are in the universal set but are not found in the set under consideration. For example, the compliment of A, written as A′, is given by
A′ = {5, 6,7,8,9} and B′ = {1, 2}

Example
It is given that ε = {1,2,3,4,5,6}; A= {2,3,5}; B = {3,4,5}. List the members of:
(i) A′, (ii) B′, (iii) A′$\cap$B, (iv) A$\cup$B′, (v) (A$\cap$B)′, (vi) A′$\cap$B′



From the above figure

(i) $A' = \{1, 4, 6\}$

(ii) $B' = \{1, 2, 6\}$

(iii) $A' \cap B = \{4\}$

(iv) $A \cup B' = \{1, 2, 3, 5, 6\}$

(v) $(A \cap B)' = \{1, 2, 4, 6\}$

(vi) $A' \cap B' = \{1, 6\}$

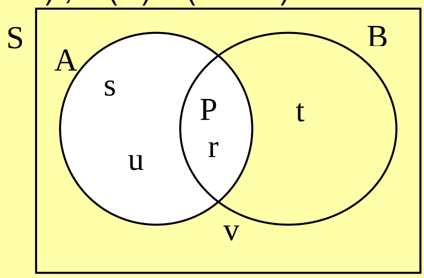
Exercise 2.3

1. The pupils of senior one class were asked about the sports they play. Seventeen of them play football. Fourteen play tennis. Five of them play both football and tennis. There are thirty pupils in the class. Draw a Venn diagram to show this information.
   (a) How many play football but not tennis?
   (b) How many play neither football nor tennis?

2. In a survey of a certain senior two class, the following information was discovered. Five pupils liked Coca cola but not Fanta. Nine pupils liked Fanta but not Coca cola. Three pupils liked neither cola nor Fanta. There are 20 pupils in the class. How many pupils liked both cola and Fanta?

3. Using the Venn diagram below, determine:

   (i) $A \cap B$; (ii) $A \cap B'$;
   (iii) $(A \cup B)'$; (iv) $\cap(A' \cap B')$.



4. Given A = {1, 2, 3, 4, 5}, B = {2, 4, 6} and C = {1, 3, 5, 7, 9}, answer the following questions about the sets A, B and C.
   (a) List the set $A \cap B$
   (b) write down n(A)
   (c) List the set $A \cup B$
   (d) List the set $A \cup B \cup C$.
   (e) Using a universal set ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, list the set C'.
5. A survey was carried out in a shop to find out how many customers bought bread, or milk or both or neither. Out of a total of 79 customers for the day, 52 bought milk, 32 bought bread and 15 bought neither. Draw a Venn diagram to show this information and use it to find out
   (a) how many bought bread and milk
   (b) how many bought bread only,
   (c) how many bought milk only.

6. In a certain group of children, all of them study French of German or both languages. 15 study French but not Germany, 12 study German of whom 5 study both languages. Draw a Venn diagram to show this information and use it to calculate how many children there are in the group.

7. In a class of 30 pupils, all pupils are required to take part in at least two sports chosen from football, gymnastics and tennis. 9 do football and gymnastics; 19 do football and tennis; 6 do all three sports. Draw a Venn diagram to show this information. Use your diagram to help calculate how many pupils do gymnastics and tennis but not football.

Operations

Any rule for combining two elements from a set is called an operation on that
set. For example, + is an operation on the set of real numbers.

Example

Given A = $\{1, 2, 3, 4\}$ and $a * b = a^2 + b^2$, for all elements of A, calculate

(a) $1 * 2$ (b) $3 * 4$

(c) $4 * 3$ (d) $2 * 2$

Solution

(a) $1 * 2 = 1^2 + 2^2$
$= 1 + 4$
$= 5$

(b) $3 * 4 = 3^2 + 4^2$
$= 9 + 16$
$= 25$

(c) $4 * 3 = 4^2 + 3^2$
$= 16 + 9$
$= 25$

(d) $2 * 2 = 2^2 + 2^2$
$= 4 + 4$
$= 8$

Example

Given A = 0, 1, 2 and a * b = remainder after a + b is divided by 3 for all a, b in A, calculate

(a) 0 * 1 (b) 2 * 0 (c) 1 * 2

Solution

(a) $0 + 1 = 1$ (b) $2 + 0 = 2$ (c) $1 + 2 = 3$

$1 \div 3 = 0$ rem. 1 $2 \div 3 = 0$ rem. 2. $3 \div 3 = 1$ rem. 0

So 0 * 1 = 1 so 2 * 0 = 2 so 1 * 2 = 0

Example

Given that $a * b = a^b$ calculate

(a) (0 * 1) * 2 (b) (2 * 1) * 2

Solution

(a) Remember we must evaluate the bracket first.

$(0 * 1) * 2 = (0^1) * 2$
$= 0 * 2$
$= 0^2 = 0.$

(b) $(2 * 1) * 2 = (2^1) * 2$
$= 2 * 2$
$= 2^2 = 4.$

Exercise

1. Given the operation $a * b = \frac{a+b}{2}$, calculate

   (a) 2 * 3 (b) -2 * 1 (c) (-2 * 1) * 3

2. Given p * q = p + q, calculate
   (a) 2 * 3 (b) 3 * 5 (c) (8 * 10) * 2

3. Given $a * b = \frac{a}{b} + \frac{b}{a}$, calculate

   (a) 1 * 2 (b) 2 * 3 (c) 4 * (1 * 2)

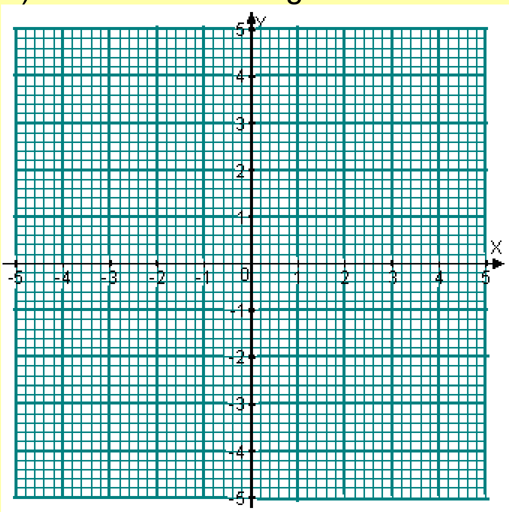
4. The operation * is defined as $x * y = x^2 - y^2$, calculate

   (a) (3 * 2) * 1 (b) $4 * {}^{-}2$

3 COORDINATES

The Cartesian plane

The position of a point on a plane surface is found by using two number lines that are perpendicular to each other. These two lines are the frame of reference and they meet at a fixed point O, called the **origin**. One of these lines is drawn horizontally and is called the **x-axis**. The other line is drawn vertically and is called the **y-axis.** For each axis, a suitable scale is chosen and then marked at equal intervals (distances) as shown in the figure below.

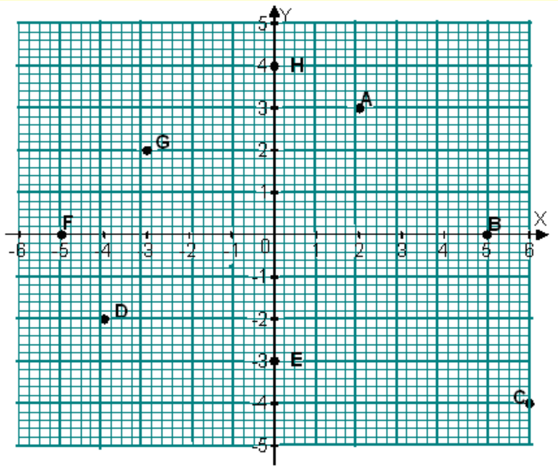


The measurements on the axes are called **coordinates**. The coordinates are given as an ordered pair (x, y) with the x-coordinate first and the y-coordinate second. This order is mandatory. The two coordinates are separated by a comma within a pair of brackets. The plane on which the points lie is called the **Cartesian plane**.

Cartesian coordinates are pairs of numbers which describe the positions of points.

Example

Write down the coordinates of points A, B, C, D, E, f, G and H in the figure below.
Figure 3.1



Solution

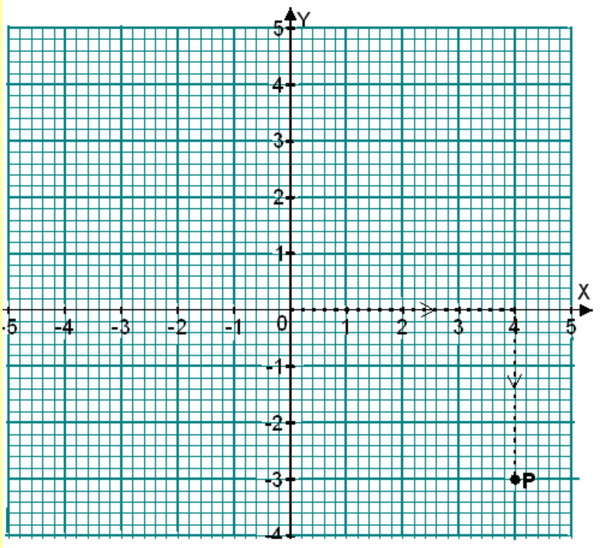
The coordinates are:

A (2, 3) **B** (5, 0) **C** (6, -4)
D (-4, -2) **E** (0, -3) **F** (-5, 0)
G (-3, 2) **H** (0, 4)

Plotting points

To plot a given point requires you to locate and mark the point on the Cartesian plane. For example, to plot point P (4, -3), we proceed as follows (see figure below):
The dotted line with arrows show the movements made to locate point
P (4, -3). Start at the origin and move 4 units parallel to the x-axis. Then move 3 units down parallel to the y-axis. Mark point P with a dot.

Figure 3.2



Example

The vertices of a quadrilateral ABCD have coordinates A (-10, 0), B (-10, 25), C (15, 25) and D (25, -10).

(a) Using the scale 1 cm represents 5 units on both axes, plot the points A, B, C and D.

(b) Join the points and name the quadrilateral ABCD.

(c) What are the coordinates of the point of intersection of the diagonals?

Solutions

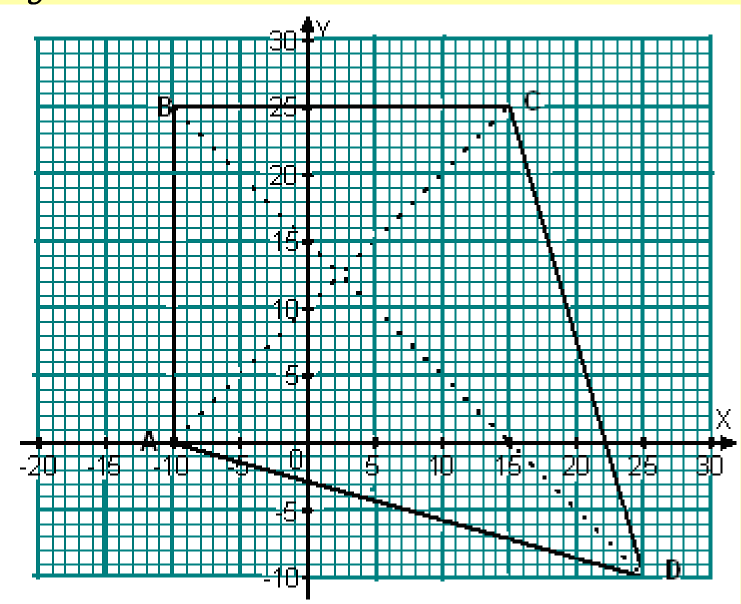
(a)

(b) The quadrilateral is a kite.

(c) The diagonals intersect at (2.5, 12.5).

The coordinates can be fractions or decimals. Sometimes a scale is not given. In such cases choose a scale that allows you to include all the required points. It is also common to use the same scale on both axes but this need not always be the case.

Figure 3.3



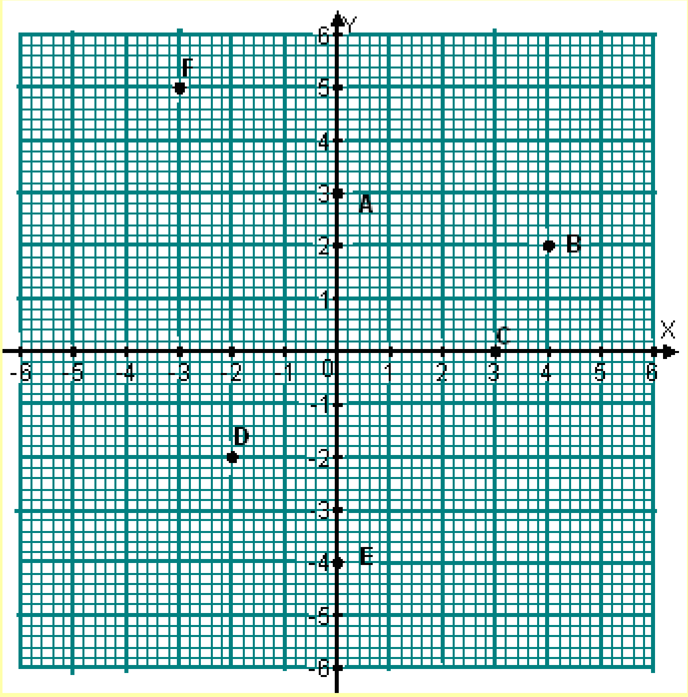
Exercise 3.1

1. Draw the axes and plot the points: A(2,3), B(3,4), C(4,0); D(5,3); E(4,2).

2. Plot the points, A (1, 3) and B (7, 3) on graph paper. Find the coordinates of the mid-point of AB.

3. Write down the coordinates of the points A, B, C, D, D, E, F shown below

Figure 3.4



4. Plot the points J (0, 7); K (6, 1); L (3, 8) and M (1, 2) on graph paper. Join the points J and K. Join the points L and M. Find the coordinates where the two lines meet.

5. P (2, 5); Q (2, 2) and R (8, 2) are three of the corners of a rectangle. Draw the rectangle on squared paper. Give the coordinates of the fourth corner. How long is the side from Q to R, of the rectangle?

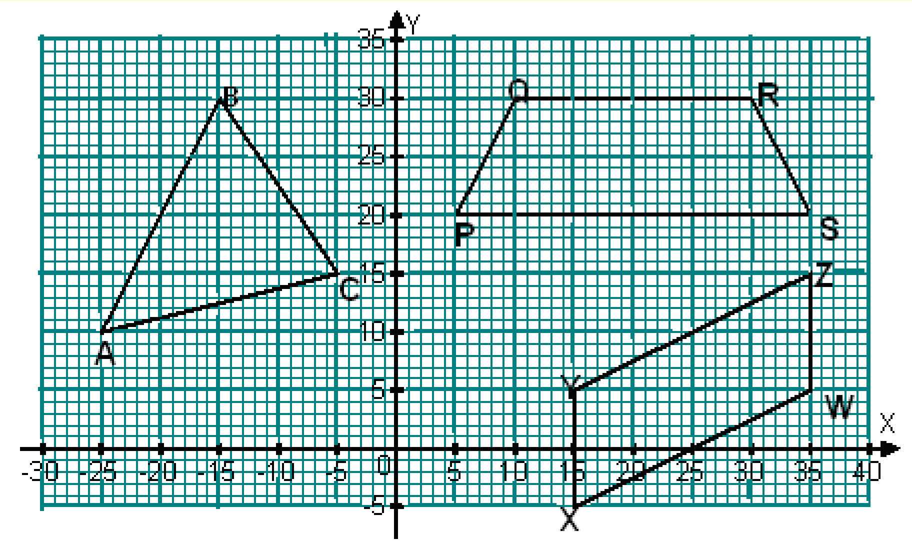
6. On a pair of axes, plot the points represented by the coordinates given below.
(3,5); (1,3); (4,6); (0,2); (2,4); (5,7). What do you notice about these points?

7. Plot the points (2,4); (2,2); (2,0); (4,0); (6,0); (4,2) on squared paper. What shape do the points you have plotted suggest?

8. The point (3, 4) is the center of a square. Two of the corners of the square are (1, 2) and (1, 6). Draw the square. Write down the coordinates of the third and fourth corners.

9. (a) Write down the coordinates of the vertices of the shapes in the figure 3.5 below.
   (b) Write down the coordinates of the mid-point of line AB in triangle ABC.

Figure 3.5



10. Plot the points given by these coordinates: M (-3,-1); N (-1,-1); O (5,-1);

    P (3,-1); Q (7,-1); R (1,-1).

11. P (-3, 4); Q (-3,-2); and R (1, 4) are the corners of a triangle. Draw the triangle on squared paper.

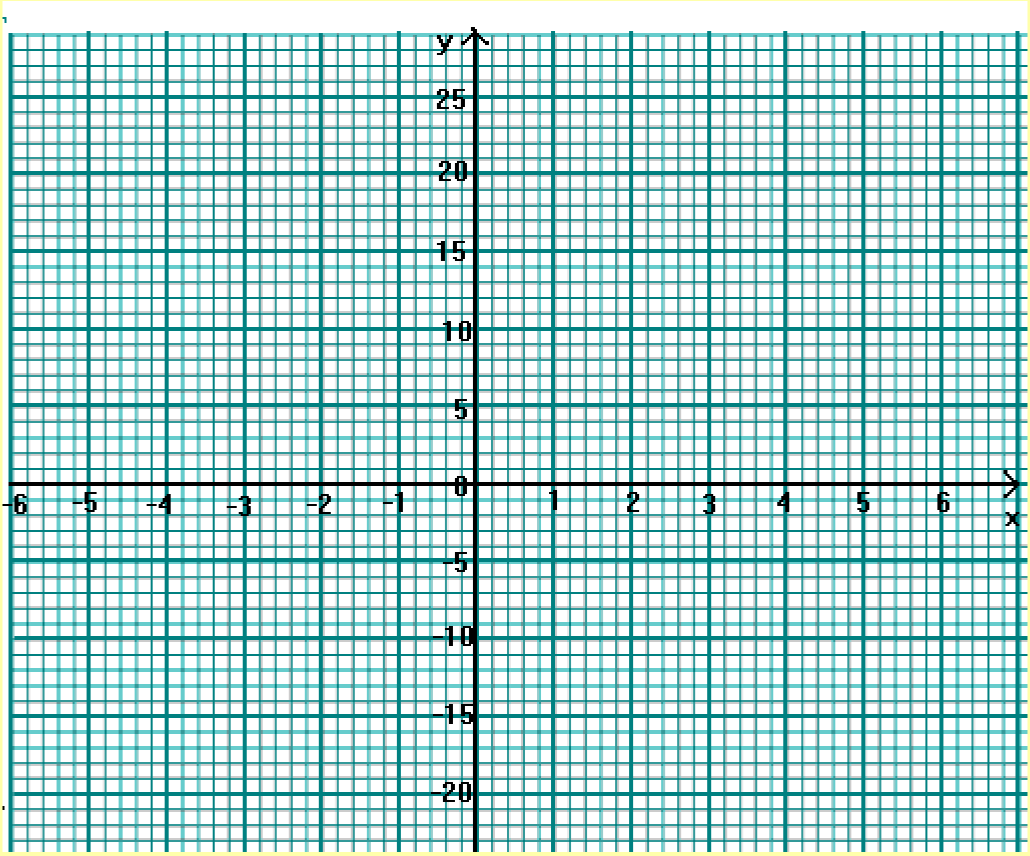
(a) Find the coordinates of the point half way between the points P and Q.

(b) Find the coordinates of the point half way between the points P and R.

3.1 Scales

How would you plot the points (5, 25); (3, 20); (7, 35)?

Using one sheet of graph paper, draw a pair of axes. Mark the horizontal axis: 0, 1, 2, 3,…
Mark the vertical axis: 0, 5, 10, 15, 25,…



The scale:

(a) on the horizontal axis is 1 unit for each space of 1cm
(b) on the vertical axis is 5 units for each space.

Plot the points on your axes.

Draw three more pairs of axes. Choose scales to plot each of the following sets of points.

(a) (4, 30); (7, 90); (2, 70)

(b) (10, 15); (50, 20); (90, 5)

(c) (1,150); (5,450); (7,200).

Note:

On each axis:

(i) the space between the numbers must always be the same.
(ii) each space must always represent the same amount.

3.3 Lines

A line may be thought of as a set of points. In math, a line is assumed to extend for ever in both directions. A line may be described by giving its equation. For example, $x = 4$, etc.
The equation simply describes the relationship between the coordinates of the points that lie on it. Thus the line $x = 4$ is a set of all points (x, y) for which $x = 4$.

Example

Write down the equation on which each of the following sets of points lie.
(a) (-2, 2), (-1, 2), (0, 2), (1, 2), (5, 2)
(b) (-1, -1), (0, -1), (3, -1), (5, -1), (7, -1).
(c) (-2, 0), (-2, 3), (-2, 5), (-2, -3).

Solution

(a) Every point has the y-coordinate equal to 2. Therefore, the equation of the line is $y = 2$.
(b) Each point has its y-coordinate equal to -1. The equation of the line is
$y = -1$.
(c) Each point has the x-coordinate equal to -2. The equation of the line is
$x = -2$.

Exercise 3.2

1. Write down the equations of the lines on which the following sets of points lie.
   (a) (5, 1.5), (8, 1.5), (10, 1.5), (-5, 1.5).
   (b) (0, -1), (0, 0), (0, 1), (0, 2), (0, 6).
   (c) (5, 1), (5, -2), (5, 5), (5, -6), (5, 7).

2. Give the coordinates of two points that belong to each of the following lines:
   (a) $x = 0$ (b) $y = -1$
   (c) $y = 8$ (d) $x = -6$

3. The point (4, -3) lies on the lines $x = 4$ and $y = -3$. On which lines do the following points lie?
   (a) (0, 5) (b) (6, 3)
   (c) (-1, -3) (d) (4, 4)

4. State the coordinates of the point at which the following pairs of lines intersect.
   (a) $x = 4, y = 0$ (b) $x = -2, y = 6$

(c) x = 0, y = -3 (d) x = -1, y = -1.

Plotting linear graphs

Consider the relation, y = x + 3. This is an equation which relates x to y, and it means that for every value of x there is a corresponding value of y. Since x and y can take any values, depending on the equation, they are called **variables**. For example, when x = 1, then y = 1 + 3 = 4. Thus, (1, 4) is a point on the line
y = x + 3. Similarly, when x = 2, then y = 2 + 3 = 5. Therefore, point (2, 5) lies on the line y = x + 3. Other points are found in a similar way and tabulated as shown in the table below.

Table 3.1

x	1	2	3	4	5
y = x + 3	4	5	6	7	8

Example

Draw the graph of y = 3 – x for values of x from -3 to +3.

Solution

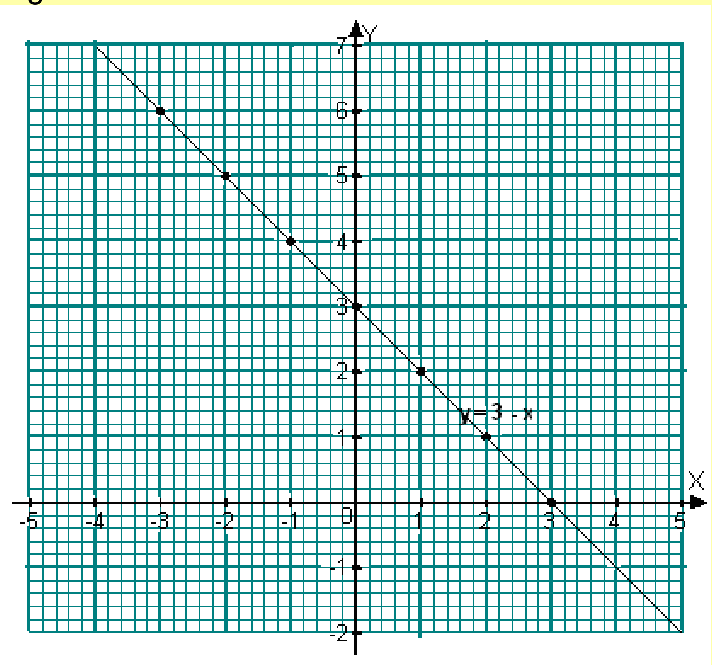
We have been given the range of the values of x. We need to find the corresponding values of y as shown in the table below:

Table 3.2

x	-3	-2	-1	0	1	2	3
y	6	5	4	3	2	1	0

These ordered pairs of values can now be plotted on a Cartesian plane as shown in Figure below. Two points are enough to draw any graph of a straight line, since other points satisfying the equation lie on the line.

Figure 3.6



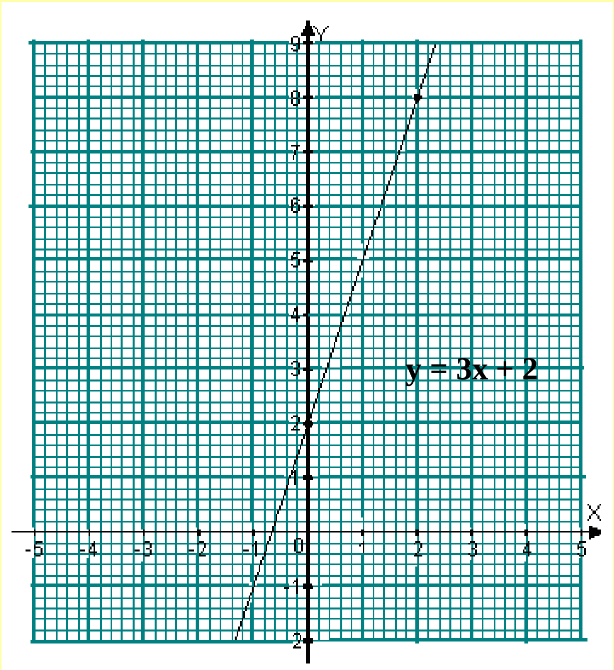
Example

Draw the graph of y = 3x + 2.

Solution

Since the relation is linear, we need only two points to draw the graph. Thus, when x = 0, then y = 3×0 + 2 = 2. (0, 2) is one of the required points. When
x = 2, y = 3×2 + 2 = 8. (2, 8) is another point. These two points are plotted on a Cartesian plane as shown in Figure 3.7.

Figure 3.7



Example

Give the coordinates of three points which lie on the line with equation y = 3.
Plot the three points on squared and draw the line y = 3. Give the coordinates of the point where the line crosses the y-axis.

Solution

The equation y = 3 means a set of points (x, y) such that the y-coordinates are equal to 3. Thus, x can take any value. Three such points are given in Table 3.3 below:

Table 3.3

x	-2	1	4
y	3	3	3

Figure 3.8

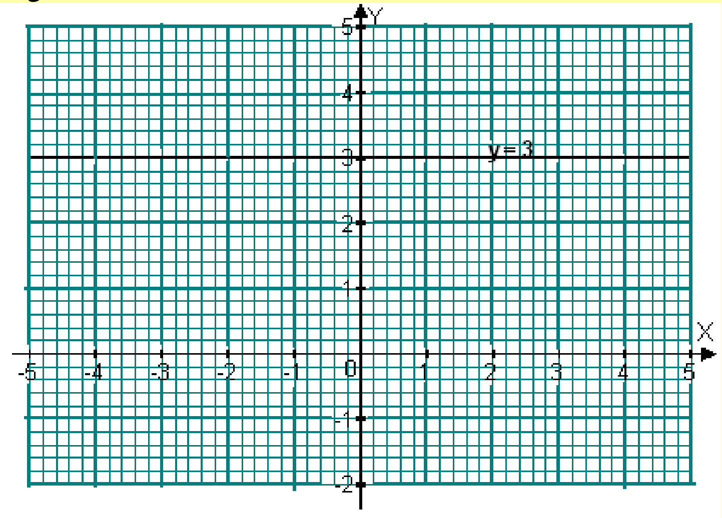


Table 3.3 gives the points: (-2, 3); (1, 3); (4, 3). The line crosses the y axis at the point (0, 3). When lines meet we say they **cut** or they **intersect**.

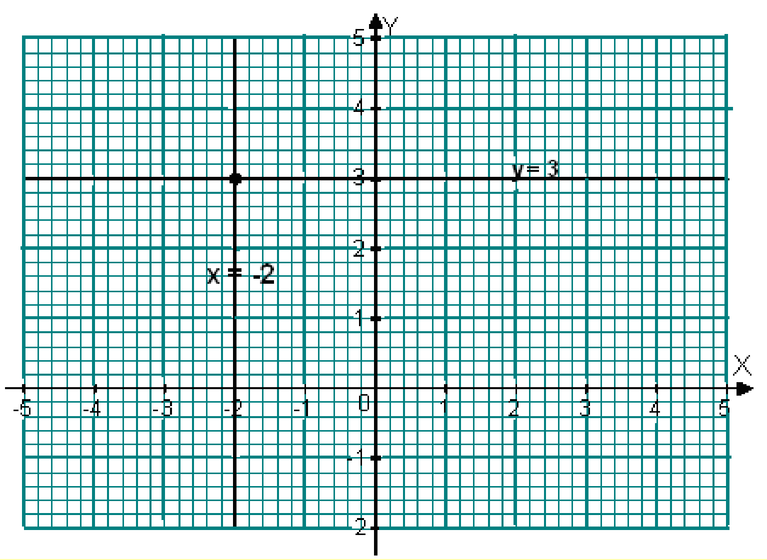
Example

Find the point of intersection of the lines y = 3 and x = -2.

Solution

Draw the line y = 3. Draw the line x = -2. The lines meet at the point (-2, 3).
The point of intersection is (-2, 3).

Figure 3.9



Exercise 3.3

1. For each of the parts (a) to (d) write down the equation of the line on which the points lie.
   (a) (3, 2); (3, 0); (3, 7); (3, 6).

   (b) (1, 4); (2, 4); (5, 4).

   (c) (0,9); (0,8); (0,6)

   (d) (-2, 7); (-2, 1); (-2,-5).

2. Dr aw the lines given below on the same pair of axes.

   (a) $x = 4$

   (b) $y = -2$

   (c) $x = -9$

   (d) $y = 3$.

3. Give the coordinates of the point at which each of the following lines cuts the y axis:
   (a) $y = 5$

   (b) $y = 7$

   (c) $y = -4$

   (d) $y = -12$

4. Give the coordinates of the point at which each of the

   following lines cuts the x axis:

   (a) $x = 5$; (b) $x = 9$; (c) $x = -6$; (d) $x = 0$.

5. Copy and complete each table for the given equation.

   (a) $y = x$

x	-1	0	1	2
y		0		

   (b) $y = -x$

x	-3	-1	2	5
y	3			

   (c) $y = 1 - 3x$

x		0	2	
y	7			-11

(d) $y = 2x + 3$

x	-5			5
y		3	7	

6. Draw a table of values for the equation $y = 5 - 2x$. Plot the points on a pair of axes and draw the graph. From your graph, find:
   (a) the y-coordinate when $x = 4$.
   (b) the x-coordinate when $y = 13$.
   (c) State the coordinates of the points where the line crosses the axes.

7. For each of the following equations, find three points and draw the graphs.
   (a) $y = 2x - 1$ (b) $y = \frac{1}{2}x + 3$
   (c) $y = 3 - 2x$ (d) $y = 3x$

8. Give the coordinates of three points that lie on:
   (a) $x + y = 5$ (b) $y + x = 0$
   (c) $y = 4x + 3$ (d) $y + x = 2$

9. Plot on a pair of axes the points (8, 0) and (0, 8). Draw the line joining them and give the coordinates of three other points on this line.

10. Draw the lines joining:
    (a) (-6, 0) and (8, 7)
    (b) (7, -1) and (-2, 8)
    State the coordinates of the point at which the two lines meet.

3.32 Equations of lines

Equations of straight lines give the relationship between the x and y coordinates of points on the line. For example, the points (3, 3); (5, 5); (-2,-2) lie on the line $y = x$ (the x-coordinate is always equal to the y-coordinate)
The points (3, 4); (4, 5); (7, 8); etc lie on the line $y = x + 1$.
(The y-coordinate is equal to the x coordinate plus one).

Alternatively, the relationship can be determined by finding the difference between successive x-coordinates on the one hand and y-coordinates on the other. When the differences are equal, addition or subtraction is then tried. If the addition or subtraction of the paired x- and y-coordinates gives a constant value, the relationship is generalized.

Example 1

Find the equation of a straight line passing through points (3, 2), (4, 3), (5, 4), (6, 5), (7, 6).

Solution

The differences in the x and y coordinates are found as shown below.

x: $3 \xrightarrow{1} 4 \xrightarrow{1} 5 \xrightarrow{1} 6 \xrightarrow{1} 7$

y: $2 \xrightarrow{1} 3 \xrightarrow{1} 4 \xrightarrow{1} 5 \xrightarrow{1} 6$

Subtraction in this case gives a constant value and addition but addition does not give a constant value. Thus, x – y = 1 is the equation of the line.

Example 2

Find the equation of a straight line passing through the points

(-1, -3), (0, -1), (1, 1), (2, 3), (3, 5), (4, 7).

Solution

x: $-1 \xrightarrow{1} 0 \xrightarrow{1} 1 \xrightarrow{1} 2 \xrightarrow{1} 3 \xrightarrow{1} 4$

y: $-3 \xrightarrow{2} -1 \xrightarrow{2} 1 \xrightarrow{2} 3 \xrightarrow{2} 5 \xrightarrow{2} 7$

Since the differences in the x-coordinates are not equal to the differences in the y-coordinates addition or subtraction cannot be applied. The x-coordinates should be doubled to make the differences equal.

2x: $-2 \xrightarrow{2} 0 \xrightarrow{2} 2 \xrightarrow{2} 4 \xrightarrow{2} 6 \xrightarrow{2} 8$

y: $-3 \xrightarrow{2} -1 \xrightarrow{2} 1 \xrightarrow{2} 3 \xrightarrow{2} 5 \xrightarrow{2} 7$

2x – y = 1 is the equation of the line.

Example 3

Find the equation of a straight line passing through points (-3, 10), (0, 8), (3, 6), (6, 4), (9, 2).

Solution

x: -3 $\xrightarrow{3}$ 0 $\xrightarrow{3}$ 3 $\xrightarrow{3}$ 6 $\xrightarrow{3}$ 9

y: 10 $\xrightarrow{-2}$ 8 $\xrightarrow{-2}$ 6 $\xrightarrow{-2}$ 4 $\xrightarrow{-2}$ 2

Multiply the x and y coordinates by 2 and 3 respectively to make the differences the same.

2x: -6 $\xrightarrow{6}$ 0 $\xrightarrow{6}$ 6 $\xrightarrow{6}$ 12 $\xrightarrow{6}$ 19

3y: 30 $\xrightarrow{-6}$ 24 $\xrightarrow{-6}$ 18 $\xrightarrow{-6}$ 12 $\xrightarrow{-6}$ 6

Thus, 2x + 3x = 24 is the equation of the line.

When given at least two points that lie on a straight line, say,
A (-1, 3) and B (0, -1);

- Subtract the y-coordinate of A from the y-coordinate of B, i.e -1 - 3 = -4.
- Subtract the x-coordinate of A from the x-coordinate of B, i.e. 0 - (-1) = 0 + 1 = 1.
- Write a fraction with the result of subtracting the y-coordinates of B and A as the numerator and the result of subtracting the x-coordinates of B and A as the denominator, i.e. $\frac{-1-(3)}{0-(-1)} = \frac{-4}{1}$.

The fraction or integer obtained is called the **gradient**. This gradient should be the same irrespective of the points you use. Which means, if we take a general point C(x, y) and point B(0, -1) then,

$$\frac{y-(-1)}{x-0} = \frac{-4}{1}$$

Therefore, $\frac{y+1}{x-0} = \frac{-4}{1}$

1(y + 1) = -4(x - 0)

y + 1 = -4x

y + 4x = -1 or y = -4x - 1.

Example 4

Find the equation of a straight line passing through points A (3, 2); B (4, 3); C (6, 5).

Solution

Take any two points, e.g. A and B

$$\frac{3-2}{4-3} = \frac{1}{1}$$

Take a general point E(x, y) and C (6, 5).

Therefore, $\frac{y-5}{x-6} = \frac{1}{1}$

y - 5 = x - 6
y - 5 + 5 = x - 6 + 5
y = x -1 or x = y + 1

Exercise 3.4

1. Plot the points (2, 2); (4, 3); (8, 5) on graph paper. Draw a straight line through the points. Find the coordinates of the point at which the line cuts the
   (a) x-axis (b) y-axis.

2. Find the coordinates of three points which lie on the line y = x-1. Find the coordinates of the point of intersection of the line y = x-1 and the *x-axis*.

3. Find the equations of the lines on which the following points lie:
   (a) (0,0); (5,5); (8,8); (-1,-1)
   (b) (-4, 4); (6,-6); (7,-7); (3,-3).
   (c) (3, 2); (8, 7): (10, 9); (11, 10).

4. On separate diagrams plot the points in question 3 above and draw the three lines. Write the equation of the line alongside each line.

5. Find the equations that are satisfied by the following sets of points.
   (a) (0, 5), (2, 11), (3, 14), (4, 17), (5, 20)
   (b) (-2, 9), (0, 6), (4, 0), (6, -3)
   (c) (0, -4), (3, 0), (6, 4), (9, 8), (12, 12)

6. The following coordinates are vertices of polygons. Plot them and name the figures formed.
   (a) A (-3, -2), B (1, -5), C (4, -1)
   (b) E (0, 0), F (4, 1), G (2, -3), H (-2, -4)

7. Three vertices of a rhombus have the following coordinates: (1, -1), (3, -6) and (3, 4). Find the coordinates of the fourth vertex.

8. State the equations of the lines on which the following points lie.
   (a) (-1, -2), (-1, 0), (-1, -5), (-1, 4)
   (b) (2, -1), (3, -1), (4, -1), (-2, -1)

9. State the coordinates of the points at which the following pairs of lines intersect.

(a) $x = -1, y = -2$ (b) $x = 5, y = 6$

(c) $x = 2, y = -4$ (d) $x = 9, y = 11$

10. Draw the graphs of:

(a) $2y = 5x + 3$ (b) $y = -2x + 1$

11. Find the equations of the straight lines passing through:

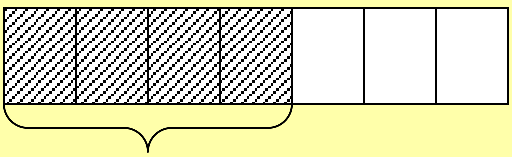
(a) (-2, -5), (0, -2), (2, 1), (4, 4), (8, 10)

(b) (-3, 6), (0, 5), (3, 4), (6, 3), (9, 2)

FRACTIONS

The meaning of a fraction

Fractions are used to describe parts of a whole item or quantity. For example, if a pineapple is cut into 4 equal parts, each part is one fourth ($\frac{1}{4}$) of the whole pineapple. Two parts of the same pineapple are two quarters ($\frac{2}{4}$) of the pineapple.
In the figure below, the rectangle is divided into 7 equal parts. Four sevenths ($\frac{4}{7}$) of the rectangle is shaded. What part of the rectangle is not shaded?



$$\frac{4}{7}$$

A number such as $\frac{4}{7}$ is called a fraction. The number 7 is called the **denominator** of the fraction; the number 4 is called the **numerator** of the fraction.

Equivalent fractions

Two or more fractions are equivalent if they have the same value. A fraction will not change its value if its numerator and denominator are multiplied or divided by the same number at the same time. For example,
$\frac{2}{5}=\frac{2\times 4}{5\times 4}=\frac{8}{20}$ and $\frac{6}{8}=\frac{6\div 2}{8\div 2}=\frac{3}{4}$. $\frac{2}{5}$ is equivalent to $\frac{8}{20}$ and $\frac{6}{8}$ is equivalent to $\frac{3}{4}$.
When the numerator and denominator have no common factors, other than one, the fraction is said to be in its lowest term. For example, $\frac{3}{4}$ and $\frac{7}{10}$ are in their lowest terms. Fractions that are not in their lowest terms can be reduced or simplified by dividing the numerator and the denominator by their highest common factors.

Reduction of fractions

The value of a fraction is unaltered if we divide the denominator and numerator by the same number. The process of dividing the denominator and numerator by the same number is called '**reducing** the fraction to its lowest terms' or canceling.

Example 1

Reduce $\frac{144}{180}$ to its lowest terms.

Solution

$\frac{144}{180} = \frac{36}{45}$ (dividing numerator and denominator by 4)

$= \frac{4}{5}$ (dividing numerator and denominator by 9).

Example 2

Express $\frac{7}{16}$ with a denominator 48.

Solution

The new denominator 48 is three times the original one (16).
Therefore, the new numerator must be 3 × 7 or 21.
Therefore,

$\frac{7}{16} = \frac{21}{48}$.

Comparing fractions

The idea of equivalent fractions can be used to compare fractions.

Example 3

Arrange in order of size beginning with the smallest, the fractions

$\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{7}{9}$.

Solution

$4 = 2^2$; $8 = 2^3$; $12 = 2^2 \times 3$; $9 = 3^2$.
Therefore the LCM of the denominators is $2^3 \times 3^2 = 72$.
Expressing the fractions as equivalent fractions with the same denominator 72, the fractions are:

$\frac{54}{72}, \frac{45}{72}, \frac{42}{72}, \frac{56}{72}$.

The fraction with the least numerator has the least value. In this case, $\frac{42}{72}$ is the least and $\frac{56}{72}$ is the greatest.

Therefore, the order is $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{7}{9}$.

Exercise 4.1

Reduce the following fractions to their lowest terms:

1. $\frac{9}{12}$ (2) $\frac{14}{18}$ (3). $\frac{60}{75}$ (4). $\frac{45}{72}$ (5). $\frac{12}{12}$.

6. Express with denominator 12 the fraction: $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}$.

7. If $\frac{7}{12}$ has the same value as the fraction $\frac{x}{60}$, say what number you think x stands for.

Arrange in order of size, beginning with the smallest, the fractions:

8. $\frac{1}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{12}$. 9. $\frac{3}{10}, \frac{1}{4}, \frac{3}{14}, \frac{2}{7}$.

4.2 Kinds of fractions

A fraction such as $\frac{3}{8}$, in which the numerator is less than the denominator, is called a **proper** fraction.

A fraction such as $\frac{11}{8}$, in which the numerator is greater than the denominator, is called an **improper** fraction.

The integers or whole numbers 1, 2, 3, 4, 5 … may be replaced by the fractions $\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}$.....

Notice that $\frac{11}{8} = \frac{8}{8} + \frac{3}{8} = 1 + \frac{3}{8}$, *and this is written simply* $1\frac{3}{8}$.

A number such as this is called a **mixed** fraction and it contains a whole number and a proper fraction.

Improper fractions can be expressed as mixed numbers and vice versa.

Example 4

Write $\frac{241}{72}$ as a mixed number.

First divide 241 by 72.

$$\begin{array}{r|r|l} 72 & 241 & 3 \\ & \underline{216} & \\ & 25 & \end{array}$$

The answer is $3\frac{25}{72}$.

Example 5

Express as improper fractions:

$(i)\ 3\frac{6}{7}\quad (ii)5\frac{6}{13}.$

Solution

(i) $3\frac{6}{7}=3+\frac{6}{7}=\frac{21}{7}+\frac{6}{7}=\frac{27}{7}$. The mental work is to say (3 × 7) + 6 = 27, and then to put 27 over 7 to make the fraction.

(ii) $5\frac{6}{13}=\frac{5\times 13+6}{13}=\frac{65+6}{13}=\frac{71}{13}$.

Exercise 4.2

Express as mixed numbers (or as integers):

(1). $\frac{10}{3}$ (2). $\frac{21}{8}$ (3). $\frac{20}{5}$ (4). $\frac{13}{4}$ (5). $\frac{17}{6}$

Express as improper fractions:

(6) $5\frac{2}{3}$ (7) $1\frac{5}{16}$ (8) $9\frac{2}{11}$ (9) $3\frac{3}{5}$ (10). $2\frac{5}{8}$

11. Write an equivalent fraction with denominator 36 for each of the following fractions.

    (a) $\frac{1}{2}$ (b) $\frac{5}{6}$ (c) $\frac{4}{9}$

    (d) $\frac{11}{12}$ (e) $\frac{13}{18}$ (f) $1\frac{8}{9}$

12. Simplify each of the following fractions.

    (a) $\frac{6}{14}$ (b) $\frac{45}{261}$ (c) $1\frac{3}{6}$

13. Arrange each group of the following fractions in descending order.

    (a) $\frac{3}{4}, \frac{5}{6}, \frac{1}{2}, \frac{4}{5}$ (b) $\frac{4}{5}, \frac{1}{3}, \frac{5}{9}, \frac{11}{16}$

    (c) $\frac{9}{16}, \frac{1}{2}, \frac{7}{8}, \frac{21}{32}$ (d) $\frac{1}{4}, \frac{1}{3}, \frac{2}{7}$

14. Arrange the following numbers in ascending order

(a) $\frac{12}{5}$, $3\frac{1}{4}$, 2, $\frac{17}{9}$ (b) $\frac{13}{7}$, $\frac{5}{3}$, $\frac{16}{9}$, $2\frac{2}{3}$, $1\frac{2}{2}$

(c) $2\frac{7}{8}$, $\frac{14}{5}$, 3, $\frac{17}{9}$ (d) $\frac{4}{3}$, $\frac{11}{6}$, $\frac{8}{5}$, $\frac{43}{30}$

4.3 Addition of fractions

To add two or more fractions together, we must express them all with the same denominator. This common denominator should be the **LCM** of the denominators of the fractions which are to be added. The numerators are then added and the resulting fraction reduced to its lowest terms.

Example 6

Simplify: $\frac{2}{3}+\frac{1}{4}+\frac{1}{12}+\frac{3}{8}$.

Solution:

$$\begin{aligned}\frac{2}{3}+\frac{1}{4}+\frac{1}{12}+\frac{3}{8}&=\frac{16}{24}+\frac{6}{24}+\frac{2}{24}+\frac{9}{24}\\&=\frac{16+6+2+9}{24}\\&=\frac{33}{24}\\&=\frac{11}{8}=1\frac{3}{8}\end{aligned}$$

(24 is the LCM of 3, 4, 12 and 8)

When adding mixed numbers, any improper fraction should be put as a mixed number. Then the whole numbers are added together, and the fractions are written with a common denominator and added as in the last example. *Alternatively*, the mixed numbers are expressed as improper fractions first and then proceed.

Example 7

Simplify: $2\frac{1}{2}+\frac{13}{8}+3\frac{1}{5}$.

Solution:

$$\begin{aligned}2\frac{1}{2}+\frac{13}{8}+3\frac{1}{5}&=2\frac{1}{2}+1\frac{5}{8}+3\frac{1}{5}\\&=6+\frac{20}{40}+\frac{25}{40}+\frac{8}{40}\\&=6+\frac{53}{40}\\&=6+1\frac{13}{40}=7\frac{13}{40}.\end{aligned}$$

Example 8

Work out $5\frac{1}{4}+2\frac{1}{3}$.

Solution:

Express the mixed numbers as improper fractions first: i.e.

$5\frac{1}{4}=\frac{21}{4}$ and $2\frac{1}{3}=\frac{7}{3}$

$\therefore 5\frac{1}{4}+2\frac{1}{3} = \frac{21}{4} + \frac{7}{3}$

The LCM of 4 and 3 is 12.

$\frac{21}{4}=\frac{63}{12}$ and $\frac{7}{3}=\frac{28}{12}$

$$\therefore \frac{21}{4} + \frac{7}{3} = \frac{63}{12} + \frac{28}{12} = \frac{63+28}{12}$$

$$= \frac{91}{12}$$

$$= 7\frac{7}{12}$$

Alternatively, we can add the whole parts first and then the fractional parts.

$$5\frac{1}{4}+2\frac{1}{3} = 5 + 2 + \frac{1}{4}+\frac{1}{3}$$

$$= 7 + \frac{3+4}{12}$$

$$= 7 + \frac{7}{12}$$

$$= 7\frac{7}{12}$$

Exercise 4.3

Simplify

1. $\frac{7}{16}+\frac{5}{16}$ **2.** $\frac{5}{x}+\frac{4}{x}$ **3.** $\frac{1}{4}+\frac{3}{4}$

4. $\frac{5}{8}+\frac{7}{8}$.

5. $\frac{10}{21}+\frac{4}{21}$ 6. $\frac{1}{2}+\frac{3}{4}$ 7. $\frac{1}{3}+\frac{1}{5}$

8. $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$

9. $\frac{2}{5}+\frac{1}{10}$ 10. $\frac{b}{3}+\frac{2b}{7}$ 11. $\frac{2}{3x}+\frac{2}{x}$

12. $\frac{3}{5}+\frac{1}{10}+\frac{2}{25}$

13. $2\frac{1}{2}+3\frac{3}{8}$ 14. $\frac{10}{7}+2\frac{2}{21}+3\frac{1}{3}$ 15. $1\frac{5}{18}+2\frac{2}{9}$.

4.4 Subtractions of fractions

To subtract fractions that have the same denominator, we subtract the numerators. For example:

$$\frac{8}{13}-\frac{5}{13} = \frac{8-5}{13} = \frac{3}{13}$$

Fractions with different denominators must first be expressed as fractions with a common denominator before the numerators are subtracted.

To subtract one mixed number from another, subtract the whole parts first and then add the result to the difference of the fractional parts.

Example 8

Simplify: $3\frac{7}{12}-1\frac{1}{3}$.

Solution:

$$\begin{aligned} 3\frac{7}{12}-1\frac{1}{3} &= 3-1+\frac{7}{12}-\frac{4}{12} \\ &= 2\frac{7-4}{12} \\ &= 2+\frac{3}{12}=2\frac{1}{4} \end{aligned}$$

Example 9

Simplify: $8\frac{7}{12}-2\frac{5}{8}$.

Solution:

$$8\frac{7}{12}-2\frac{5}{8}=6\frac{14}{24}-\frac{15}{24}\left(\begin{array}{l}\textit{the fractional part of the second mixed number is greater than the fractional part of the}\\ \textit{first, and we have to take one from the whole number } 6 \textit{ and change it to } \frac{24}{24}\end{array}\right)$$

$$=6+\frac{14-15}{24}$$

$$=5+\frac{24}{24}+\frac{14}{24}-\frac{15}{24}$$

$$=5+\frac{24+14-15}{24}$$

$$=5+\frac{23}{24} \quad = \quad 5\frac{23}{24}.$$

Exercise 4.4

Simplify

1. $\frac{5}{8}-\frac{3}{8}$
2. $\frac{7}{12}-\frac{5}{12}$
3. $1-\frac{1}{4}$
4. $\frac{5}{x}-\frac{2}{x}$
5. $2-\frac{2}{7}$
6. $3-\frac{3}{4}$
7. $5-\frac{7}{10}$
8. $1\frac{3}{4}-\frac{1}{2}$
9. $\frac{5}{6}-\frac{1}{3}$
10. $\frac{5}{8}-\frac{1}{4}$
11. $3\frac{3}{4}-2\frac{1}{8}$
12. $6\frac{5}{12}-3\frac{3}{4}$
13. $4\frac{1}{5}-1\frac{3}{5}$
14. $5\frac{3}{10}-1\frac{9}{10}$
15. $5\frac{1}{6}-2\frac{4}{9}$
16. $6\frac{5}{12}-3\frac{3}{4}$
17. $4\frac{1}{5}-1\frac{3}{5}$
18. $\frac{35}{16}-\frac{19}{12}$.

4.5 Multiplication of fractions

We multiply numerators together to give the numerator of the answer; multiply the denominators together to give the denominator of the answer.

Thus $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$.

Example 10

Multiply: $\frac{3}{10}\,by\,\frac{5}{6}$

Solution: $\frac{3}{10}\times\frac{5}{6}=\frac{3\times5}{10\times6}=\frac{15}{60}=\frac{1}{4}$.

When mixed numbers are to be multiplied, they must first be expressed as improper fractions.

Example 11

Multiply $3\frac{8}{9}\,by\,3\frac{3}{5}$

Solution: $3\frac{8}{9}\times3\frac{3}{5}=\frac{35}{9}\times\frac{18}{5}=\frac{7\times2}{1\times1}=14$.

Notice that $3\frac{1}{2}\times4\frac{1}{4}\,is\ not\,(3\times4)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

Multiplying a fraction by a whole number

Multiplication is equivalent to adding a number to itself repeatedly. For example,
$2 + 2 + 2 = 2 \times 3$

Similarly, $\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{3}{5}\times4$

Or $\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{3+3+3+3}{5}=\frac{3\times4}{5}$

Thus, $\frac{3}{5}\times4=\frac{3\times4}{5}=\frac{12}{5}=2\frac{2}{5}$.

Note that the product of a fraction and a whole number is obtained by first multiplying the numerator by the whole number. A whole number is regarded as a fraction with 1 as the denominator.

Thus, $4=\frac{4}{1}$, so that, $\frac{3}{5}\times4=\frac{3}{5}\times\frac{4}{1}$

$$=\frac{3\times4}{5\times1}=\frac{12}{5}=2\frac{2}{5}$$

The use of '*of*' in operation

In problems involving fractions, the word 'of' is frequently used. For example, we know that ½ *of* 8 is 4 and $\frac{3}{4}$ *of* 100 is 75. These two expressions are the same as
½ × 8 = 4 and ¾ × 100 = 75.
Therefore, '**of**' means '**multiply by'**.

Thus: ½ of ¾ = ½ × ¾ = $\frac{3}{8}$

Simplifying fractions

Fractions that have common factors should be simplified by *cancellation* before multiplication.

For example, $\frac{2}{5}\times\frac{10}{13}$ = $\frac{2}{\cancel{5}_1}\times\frac{\cancel{10}^2}{13}=\frac{2}{1}\times\frac{2}{13}=\frac{4}{13}$

And $\frac{4}{25}\times\frac{15}{32}=\frac{{}^1\cancel{4}}{\cancel{25}_5}\times\frac{\cancel{15}^3}{\cancel{32}_8}=\frac{1}{5}\times\frac{3}{8}=\frac{3}{40}$

Exercise 4.5 A

Work out the following leaving your answers in their simplest form.

1. $\frac{1}{3}\times\frac{5}{6}$
2. $\frac{2}{7}\times\frac{3}{5}$
3. $\frac{6}{11}\times\frac{3}{4}$
4. $\frac{2}{5}\times 30$
5. $\frac{1}{3}\times 2$
6. $12\times\frac{2}{3}$
7. $\frac{3}{5}\times\frac{5}{6}$
8. $\frac{1}{7}\times\frac{14}{25}$
9. $\frac{8}{15}\times\frac{3}{16}\times\frac{1}{2}$
10. $\frac{1}{3}\times\frac{6}{7}\times\frac{35}{48}$
11. $\frac{2}{7}\times\frac{7}{12}\times\frac{4}{21}$
12. $\frac{9}{4}\times\frac{2}{3}\times\frac{4}{27}$
13. $2\frac{3}{4}\times\frac{4}{5}$
14. $1\frac{1}{3}\times 9$
15. $3\frac{1}{3}\times 4\frac{1}{2}$
16. $6\frac{1}{4}\times 1\frac{3}{5}$
17. $6\frac{3}{10}\times 1\frac{4}{21}$
18. $\frac{1}{2}\ of\ 20$
19. $\frac{1}{3}\ of\ 4\frac{1}{2}$
20. $\frac{5}{8}\ of\ 13\frac{1}{3}$
21. $\frac{1}{3}\ of\ 24\times\frac{1}{4}\ of\ 32$

Reciprocals of numbers

Two numbers whose product is 1 are said to be **reciprocals** of each other. For example,

$\frac{3}{4}\times\frac{4}{3}=1$ and $\frac{1}{5}\times\frac{5}{1}=1$

Thus, $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$ and $\frac{5}{1}$ is the reciprocal of $\frac{1}{5}$ and vice versa. Therefore, a reciprocal of a number is obtained by interchanging the numerator and the denominator. For example, the reciprocal of $\frac{7}{2}$ is $\frac{2}{7}$.

4.6 Division of fractions

In division, the number to be divided is called a **dividend** whereas the number dividing the dividend is called a **divisor**. In a fraction, the numerator is a dividend and the denominator is a divisor. When one fraction is divided by another, the dividend is multiplied by the reciprocal of the divisor.

Thus the reciprocal of 3 is $\frac{1}{3}$; that of $\frac{1}{3}$ is 3; that of $\frac{2}{3}$ is $\frac{3}{2}$.

Hence, dividing by a fraction is the same as multiplying by the reciprocal.

Obviously $\frac{3}{4}\div 5$ is the same as $\frac{1}{5}\,of\,\frac{3}{4}$ which is $\frac{1\times 3}{5\times 4}\,or\,\frac{3}{20}$.

Notice that 5 is $\frac{5}{1}$, and to divide by $\frac{5}{1}$ we multiply by $\frac{1}{5}$.

Example 12

Simplify: $1\frac{5}{7}\div 6$.

Solution:

$$1\frac{5}{7}\div 6=\frac{12}{7}\div\frac{6}{1}$$
$$=\frac{12}{7}\times\frac{1}{6}=\frac{2}{7}.$$

Example 13

Simplify: $3\frac{3}{5}\div 2\frac{7}{10}$.

Solution:

$$3\frac{3}{5}\div 2\frac{7}{10}=\frac{18}{5}\div\frac{27}{10}$$
$$=\frac{18}{5}\times\frac{10}{27}=\frac{4}{3}=1\frac{1}{3}$$

Combined operations

The order of operations used for fractions is the same as that used for whole numbers. That is, work out the **brackets** first, then **divide**,

multiply, **add** and **subtract**. *Of* is taken as multiplication but its operation is done after working out the brackets and before division.

Example

Evaluate each of the following:

(a) $\frac{5}{8} \div \left(2\frac{1}{4} \times 5\frac{1}{3}\right)$

(b) $\frac{4}{5} \times \left(\frac{3}{4} + \frac{5}{8}\right) \div \frac{3}{8}$

(c) $\frac{4}{9} + \frac{5}{6} \times \left(\frac{2}{9} - \frac{1}{3}\right)$

Solutions

(a) $\frac{5}{8} \div \left(2\frac{1}{4} \times 5\frac{1}{3}\right) = \frac{5}{8} \div \left(\frac{\cancel{9}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{\cancel{3}_{1}}\right)$

$= \frac{5}{8} \div (3 \times 4)$

$= \frac{5}{8} \div 12$

$= \frac{5}{8} \times \frac{1}{12}$

$= \frac{5}{96}$

(b) $\frac{4}{5} \times \left(\frac{3}{4} + \frac{5}{8}\right) \div \frac{3}{8} = \frac{4}{5} \times \left(\frac{6+5}{8}\right) \div \frac{3}{8}$

$= \frac{4}{5} \times \left(\frac{11}{8}\right) \div \frac{3}{8}$

$= \frac{4}{5} \times \frac{11}{\cancel{8}_{1}} \times \frac{\cancel{8}^{1}}{3}$

$= \frac{44}{15}$

$= 2\frac{14}{15}$

(c) $\frac{4}{9}+\frac{5}{6}\times\left(\frac{2}{9}-\frac{1}{3}\right) = \frac{4}{9}+\frac{5}{6}\times\left(\frac{-1}{9}\right)$

$= \frac{4}{9}-\frac{5}{54} = \frac{24-5}{54} = \frac{19}{54}$.

Exercise 4.5 B

Evaluate:

1. $2\frac{1}{7}\times1\frac{1}{3}$ **2.** $\frac{5}{27}\times\frac{18}{35}$ **3.** $1\frac{5}{9}\times\frac{3}{7}$ **4.** $1\frac{2}{7}\times9\frac{1}{3}$

5. $2\frac{5}{8}\times4\frac{4}{7}$ **6.** $3\frac{3}{5}\times3\frac{1}{3}$ **7.** $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$ **8.** $2\frac{1}{3}\times4\frac{2}{7}$

9. $3\frac{6}{13}\times1\frac{11}{15}$ **10.** $\frac{1}{3}\times2\frac{2}{5}\times8\frac{3}{4}$. **11.** $\frac{1}{3}+\frac{1}{6}\times\frac{3}{4}$

12. $\frac{6}{7}+\frac{8}{11}\div\frac{32}{33}$ **13.** $\frac{2}{3}\times\frac{5}{6}+\frac{1}{8}$ **14.** $\frac{1}{5}\div\frac{19}{20}+\frac{7}{8}$

15. $\frac{3}{5}+\frac{1}{10}\times1\frac{4}{9}$ **16.** $\frac{7}{9}\times\frac{3}{14}+\frac{9}{11}$ **17.** $\left(\frac{5}{8}+\frac{1}{2}\right)\times3\frac{1}{8}$

18. $\frac{4}{7}\div\left(\frac{1}{2}-\frac{6}{7}\right)$ **19.** $2\frac{2}{3}+1\frac{1}{5}\div5\frac{4}{5}$ **20.** $1\frac{2}{9}+\left(\frac{5}{6}-\frac{1}{4}\div4\frac{1}{2}\right)$

21. $1\frac{3}{13}-\frac{7}{8}\times\frac{6}{7}+\frac{1}{2}$ **22.** $\left(3\frac{2}{5}-\frac{9}{10}\right)\div\left(3\frac{1}{2}-5\frac{2}{3}\right)$

23. $2\frac{3}{4}\times\frac{5}{6}\div\frac{11}{24}-1\frac{1}{3}$ **24.** $\frac{1}{3}\,of\left(\frac{5}{6}-\frac{1}{4}\right)\div12$

25. $\frac{3}{4}\,of\left(\frac{2}{3}+\frac{1}{2}\right)\times\frac{2}{5}\,of\left(1\frac{4}{5}-1\frac{2}{3}\right)$

26. $\frac{5}{8}\,of\,\frac{4}{15}\div\frac{7}{10}\,of\,\frac{25}{28}$

Exercise 4.6

Simplify:

1. $3\frac{1}{2} \div 1\frac{3}{4}$

2. $1\frac{1}{9} \div 1\frac{2}{3}$

3. $5\frac{5}{12} \div 3\frac{1}{3}$

4. $1 \div 5\frac{3}{4}$

5. $1\frac{7}{8} \div 1\frac{7}{8}$

6. $6\frac{3}{7} \div \frac{9}{14}$

7. $2\frac{2}{11} \div \frac{8}{22}$

8. $2\frac{2}{15} \div 1\frac{19}{45}$

9. $\dfrac{3\frac{1}{3}}{\frac{5}{9}}$

10. $\dfrac{\frac{3}{4}}{2\frac{1}{7}}$.

11. $\dfrac{2\frac{1}{6} \times \frac{9}{28}}{6\frac{3}{7} - 3\frac{9}{14}}$

12. $\dfrac{1\frac{3}{4} \times 5\frac{1}{3}}{6\frac{11}{20} - 5\frac{3}{10}}$

13. $\dfrac{3\frac{1}{3} - 1\frac{5}{6}}{2\frac{3}{4} + 1\frac{1}{6} + \frac{1}{3}}$

14. $\dfrac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}}$

15. $\dfrac{1\frac{4}{5} + \frac{7}{13}}{1 + \left(1\frac{4}{5} \times \frac{7}{13}\right)}$

16. $\dfrac{6\frac{1}{7} - 3\frac{5}{14}}{3\frac{1}{2} + 4\frac{3}{4}}$

17. $\dfrac{2\frac{3}{8} + 6\frac{1}{4} - 1\frac{1}{12} + \frac{1}{3}}{3\frac{3}{4} \times 1\frac{1}{6}}$

18. $\dfrac{(3\frac{1}{7} \times 2\frac{1}{3}) - 2}{4\frac{1}{2} - 3\frac{1}{9}}$

4.7 Application of fractions

Example 14.

If $\frac{3}{5}$ of a school are boys, and there are 96 girls in the school, how many children are there altogether?

Solution

Since $\frac{3}{5}$ of the school are boys, $\frac{2}{5}$ of the school are girls.

Therefore, $\frac{2}{5}$ of the school = 96.

Therefore, $\frac{1}{5}$ of the school = $\frac{96}{2}$

" $\frac{5}{5}$ of the school = $\frac{96 \times 5}{2}$

= 48×5 = 240.

Therefore, the total number of children = 240

Example 15.

How many pieces of string $2\frac{1}{2}$ cm long can be cut from a length of 84 cm, and how much remains?

Solution

$$84 \div 2\frac{1}{2} = \frac{84}{1} \div \frac{5}{2}$$
$$= \frac{84}{1} \times \frac{2}{5}$$
$$= \frac{84 \times 2}{1 \times 5} = \frac{168}{5}$$
$$= 33\frac{3}{5}.$$

Therefore, 33 pieces can be cut with a remainder of $\frac{3}{5}$ of $2\frac{1}{2}$

i.e. $\frac{3}{5} \times \frac{5}{2} = \frac{3}{2}$ = $1\frac{1}{2}$ cm.

Example 16

In a class of 36, three quarters of the pupils are boys and $\frac{1}{3}$ of the girls wear glasses. How many girls do not wear glasses?

Solution

Number of boys = $\frac{3}{4} \times 36 = 27$

Number of girls = 36 - 27 = 9

Number of girls who wear glasses are

$\frac{1}{3} \times 9 = 3$

Number of girls who do not wear glasses are

9 - 3 = 6.

Alternative method

The number of pupils in the class is taken as one whole. Since the pupils are either boys or girls, the fraction of girls in the class is, $1 - \frac{3}{4} = \frac{1}{4}$.

Of these girls, $\frac{1}{3}$ wear glasses. Therefore, $\frac{2}{3}$ of the girls do not wear glasses. Thus, $\frac{2}{3}\,of\,\frac{1}{4}\,of\;36$ are the girls who do not wear glasses in the class. This is simplified to give us the expression,
$\frac{2}{3}\times\frac{1}{4}\times 36=6.$

Exercise 4.7

1 A can contains 12 litres of water. How much be poured out to leave $5\frac{3}{4}$ of litres?

1. A man left $\frac{1}{2}$ of his money to his wife, $\frac{1}{5}$ to each of his two sons, and the rest to his daughter. What fraction did the daughter receive?
2. How many jugs each holding $1\frac{1}{2}$ litres can be filled from a jerican holding 30 litres?
3. A boy bought 28 apples and found that $\frac{2}{7}$ of them were bad. How many apples were fit to eat?
4. A boy's stride is $\frac{7}{9}$ of a meter. Find how many strides he takes in walking a distance of 966 m.
5. After spending $\frac{5}{6}$ of his money, a boy finds that he has 2,400 shillings left. How much had he at first?
6. A man walks at $5\frac{2}{5}$ km per hour. How far does he walk in 1hr 20 min.?
7. After using $\frac{5}{8}$ of his fuel stock, a house holder had $10\frac{1}{2}$ tones left. How many tones had he at first?
8. The distance all round a rectangular field is 437 m and the width of the field is $92\frac{1}{4}m$. Calculate the length.
9. When a man has traveled $7\frac{1}{2}$ km he has gone $\frac{5}{9}$ of his journey. Find the total journey.
10. How many $2\frac{3}{4}$ kg are there in $16\frac{1}{2}$ kg?

11. In a school of 720 pupils, $\frac{2}{5}$ are athletes, $\frac{1}{8}$ play football and $\frac{2}{3}$ of the remaining pupils play hockey. How many pupils play hockey?

12. On a Monday, $\frac{2}{5}$ of the students in a class ate oranges, $\frac{9}{20}$ ate bananas and the rest ate pineapples. What fraction of the students ate pineapples?

13. The profits of a business are sh. 24,000. Three-eighths of the profits are invested in the business and the rest of the profit is shared between two partners. If one of them receives $\frac{7}{12}$ of the profits, how much money does the other partner receive?

14. Two equal pineapples and two-thirds of a similar pineapple are to be shared equally among 5 people. What fraction of a pineapple will each person get?

15. An athlete completes one lap in $\frac{2}{3}$ of a minute. If he runs at the same rate, how long will he take to complete $2\frac{1}{2}$ laps?

16. Musa, Kevin and Swale shared out money. Musa received $\frac{3}{5}$ of the money, Kevin received $\frac{1}{6}$ of the remainder and Swale received
sh. 36,500 which was the actual amount of money that remained after Musa and Kevin had received their shares.
    (a) How much money did each of them receive?
    (b) How much money was shared among the three?

17. In a ranch $\frac{1}{4}$ of the animals are goats and sheep and $\frac{4}{5}$ of these are sheep. Two-fifths of the remaining animals are cows and the rest are bulls. What fraction of the animals are:
    (a) goats? (b) sheep?
    (c) cows? (d) bulls?

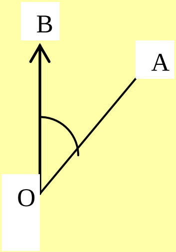
5 ANGLES AND BEARINGS

5.1 ANGLES

An angle is how much something has turned. The most common units for measuring angles is 'degrees'. We can measure angles using a protractor which is marked in degrees. The degree is further subdivided into 60 minutes and each minute into 60 seconds.
1 right angle = 90 degrees (90^0)
1 degree = 60 minutes (60')
1 minute = 60 seconds (60")
Notice that seconds and minutes of angle measurement resemble seconds and minutes of time.

(a) Naming angles.

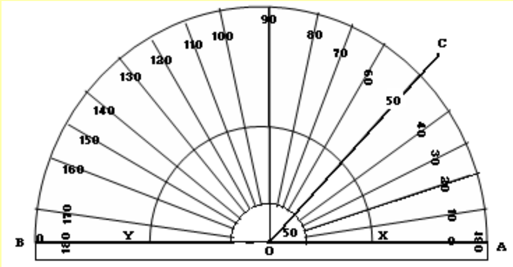
If you are standing at O and facing in the direction OA, the amount of turning which you have to make before you are facing in the direction OB is called the angle between OA and OB. O is called the **vertex** of the angle, OA and OB are called **arms** of the angle.



The angle above is named as angle AOB or BOA abbreviated as $\angle AOB$. We can also name <AOB as angle O if there is only one angle at O.
An angle is often denoted by a small letter placed in it, and the angle marked by a circular arc. E.g. <AOB = 62^0

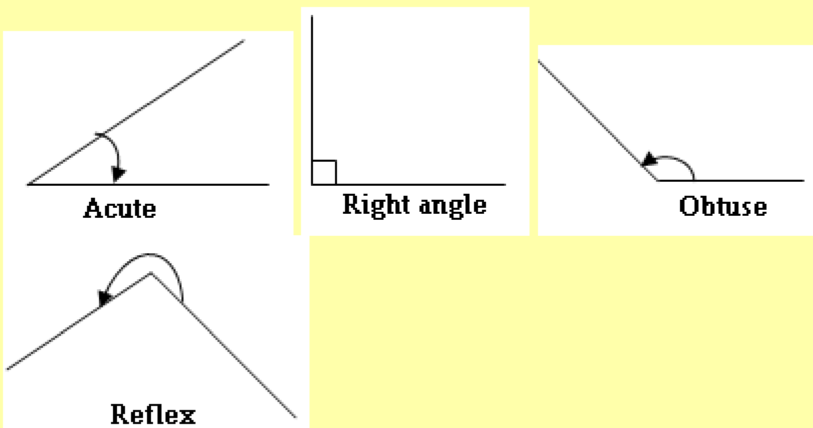
(b) Measuring angles

Angles are measured in degrees using a protractor. The figure below shows a protractor measuring an angle of 50^0. To measure an angle, place the protractor so that the centre O is at the vertex of the angle, and either OX or OY along one arm. Then seen under which graduation the other arm of the angle lies. In the figure below, the angle AOC is 50^0.



(c)Types of angles

1. An angle of 90^0 is called a right angle. The lines that form a right angle are said to be **perpendicular** to each other.
2. An angle less than a right angle is called an **acute** angle.
3. An angle bigger than a right angle but less than 180^0 is called an **obtuse** angle.
4. An angle bigger than 180^0 but less than 360^0 is called a **reflex** angles.



(d) Drawing angles

To make an angle of any given size at a point P in a line AB, place the protractor with the centre O at P, and the 0^0 - 180^0 line of the protractor along AB. Make a very small dot at the edge of the protractor against the necessary graduation, at Q, say. Join QP.

Exercise 5.1

1 Using a ruler, draw on your paper a fairly large triangle and measure the number of degrees in each angle. Make a list of the sizes of the angles, and add them together.

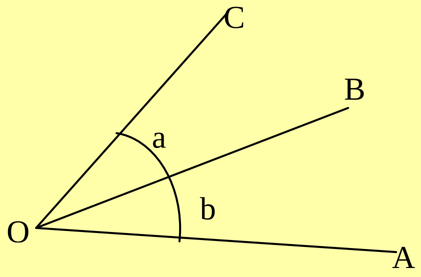
2 Draw the angle ABC which is 62^0. Make AB = 5 cm long and BC = 6.5 cm long. Draw AC. How long is it?

3. Draw angle DEF = 41^0 with DE = 38 mm and EF = 62 mm. Find the length of DF.

5.2 Definitions

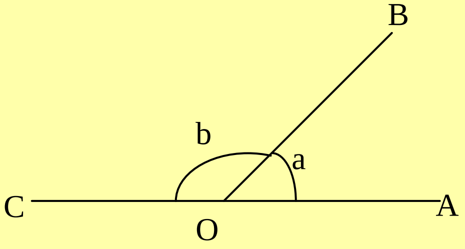
- **Adjacent angles.**

These are angles that lie next to one another. E.g. angles AOB and BOC.



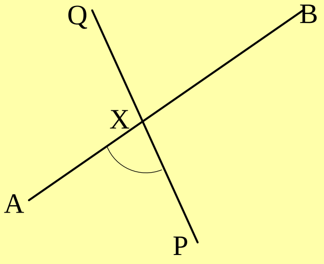
In the above figure, if the two angles AOB and BOC are equal, OB is said to bisect angle AOC, or to be the **bisector** of angle AOC

- Adjacent angles on a straight line are **supplementary** i.e. their sum is two right angles (180^0).



- If two straight lines intersect, four angles are formed. The two angles that are opposite each other are called **vertically opposite** angles. Vertically opposite angles are equal.

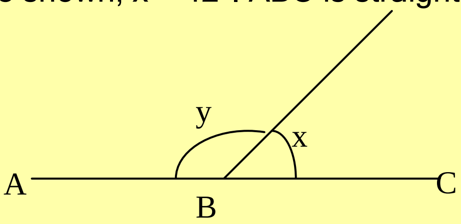
Angles AXP, BXQ are called vertically opposite angles.



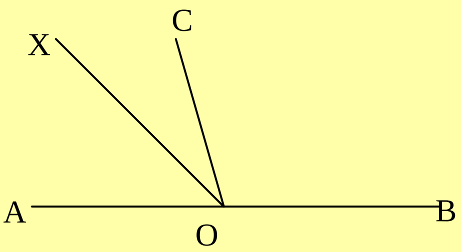
- Angles whose sum is 90 are called **complementary angles**.

Exercise 5.2

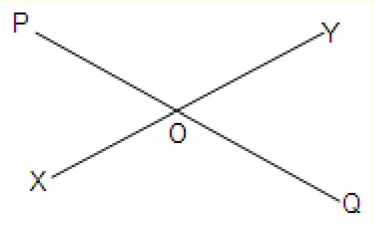
1. In the figure shown, $x = 42^0$. ABC is straight line. Find y.



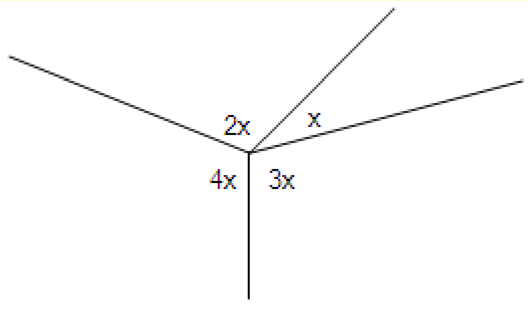
2. In the figure below AOB is a straight line and OX bisects angle AOC. If angle AOX = 62^0, calculate angle BOC.



3. In the figure below, OA is the bisector of angle POX and OB is the bisector of angle YOQ, and angle POX = 40^0, calculate angles POA, POY and YOB.



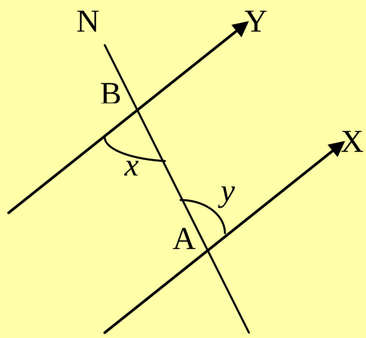
4. In the figure below, find x.



5.3 Properties of parallel lines

Meaning of Parallel lines:

These are lines in one plane which do not meet and they have the property that they are everywhere the same distance apart.

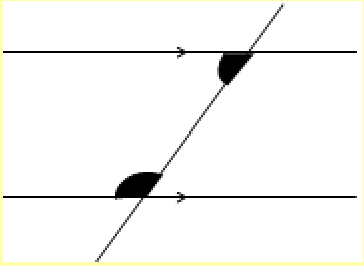


In the figure above, AX and BY are parallel lines. The line NBA cutting the parallel lines at A and B is called a **transversal**. Angles NBY, BAX are called **corresponding** angles.
The angles x and y are called **alternate** angles.

- corresponding angles are equal.
- Alternate angles are equal.

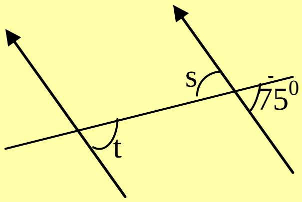
Interior angles

The figure below shows a pair of shaded angles on the same side of a transversal and lying between the parallel lines. This is a pair of **interior** angles. These interior angles are **supplementary** angles.



Example 1.

1. Find angles s and t in the diagram.

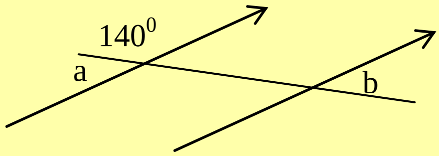


Solution

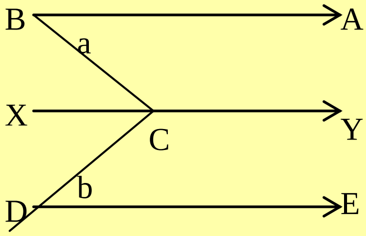
s and 75^0 are vertically opposite angles
so s = 75^0.
t and 75^0 are corresponding angles
so t = 75^0.

Exercise 5.3

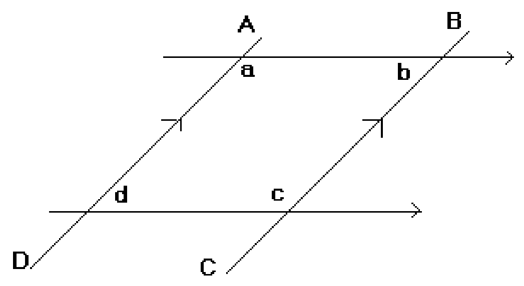
1. Find the angles marked *a* and *b* in the diagram.



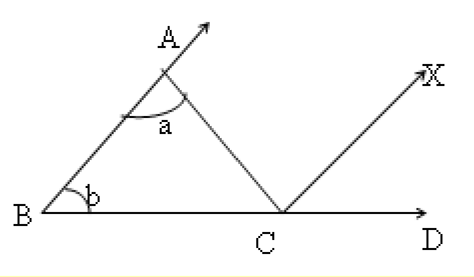
2. In the figure below if a = 27^0, and b= 56^0, calculate angle BCX, XCD and BCY.



3. In the diagram, if AB is parallel to DC and AD is parallel to BC, and b = 61^0. Calculate a, c and d.

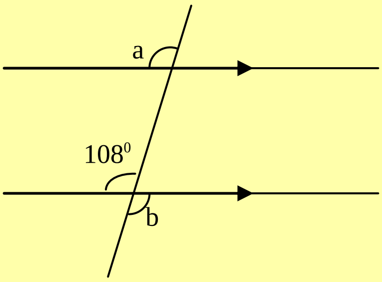


4. In the figure BCD is a straight line. a = 50^0 and b = 35^0. Calculate angles XCD, ACX and ACB.

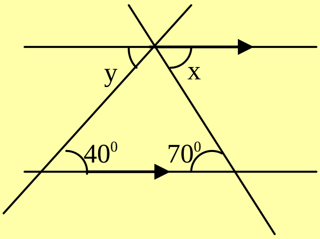


Find the size of the angles marked with letters.

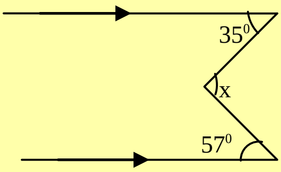
5.



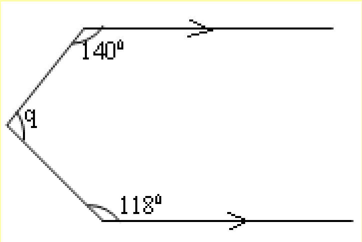
6.



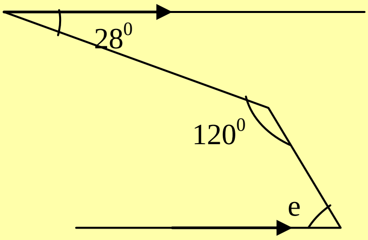
7.



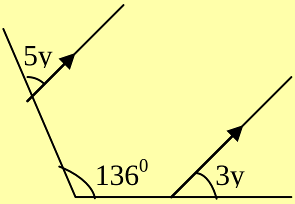
8.



9.

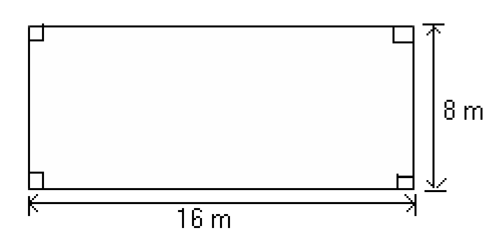


10.



Scale drawing

In representing real length of say, a plot whose dimensions are 16 m by 8 m, on paper, we use a **scale**. This means that a small length on paper represents a large length on ground. This representation is called a **scale drawing** of the real area. For example, the figure below is a scale drawing of a rectangular field whose measurements are 16 m by 8 m.



The measurements of the scale drawing are 8 cm by 4 cm. This means that
1 cm represents 2 m. This is the scale used to draw the plan of the field on paper.
When making an accurate scale drawing, state what it is. Do not write any thing on the accurate diagram except letters marking

points, as in your sketch. When asked for some particular length, the answer you give should be the actual length required and not the scaled length.

Indicating scale

There are 4 main ways of indicating the scale used.

1. **Statement scale**: *1 cm represents 100km* is a statement of the scale that means 1 cm on the scale drawing represents 100 km on the ground.

2. **Linear scale**: A line is drawn and then divided into equal divisions, for example, divisions of 1 cm. see figure below. Against the linear scale the real distance on the ground is indicated, in this case kilometers. This means 5 cm on the map represent 400 km on the ground, or 1 cm on the map represents $\frac{400}{5}$ = 80 km.

3. **Ratio form**: 1:100 means 1 cm represents 100 cm. The units in the scale in ratio form must be the same.

4. Representative fraction: (RF): A ratio scale such as 1:1000 may be written as a representative fraction as $\frac{1}{1000}$. The units in the scale must be the same.
   One type of scale can be converted to another.

Example

A scale is given in statement form as 1 cm represents 3 km. write this scale in ratio form.

Solution

Scale in ratio form must have the same units. We therefore convert 3 km to centimeters.
3 km = 3 $\times$ 1000 $\times$ 100 = 300 000 cm.
Therefore 1 cm represents 300 000 cm.
In ratio form, the scale is 1:300 000.

Example

A scale is given as $\frac{1}{2000}$ in representative fraction (RF) form. What is this scale in statement form?

Solution

The scale $\frac{1}{2000}$ means 1 cm represents 2000 cm in statement form.

This is the same as 1 cm represents $\frac{2000}{100}$ m = 20 m, i.e. 1 cm represents 20 m.

Example

On a map whose scale 1:20000, a school's rectangular farm measures 2 cm by 1 cm. Calculate the actual area of the farm in hectares. (1 hectare = 10 000 m^2).

Solution

The given scale 1:20 000 means 1 cm on paper represents 20 000 cm on the ground.

$\therefore$ 1 cm represents $\frac{20000}{100}$ m = 200 m.

$\therefore$ the length of the farm is $200 \times 2 = 400$ m. and the width is $200 \times 1 = 200$ m

The area of the farm is $400 \times 200 = 80\ 000\ m^2$.

1 hectare = 10 $000m^2$.

$\therefore 80\ 000\ m^2 = \frac{80000}{10000}$

= 8 ha.

Exercise 5.5

1. Find the R.F of a map drawn to scale of 1 cm to 500m.
2. If on a map the distance between two towns is 6.8cm and the scale of the map is 1cm to 2km, what is the actual distance between the towns? What would the distance be on a map whose scale was
   (i) 1cm to 5km, (ii) 1cm to 15km?
3. The scale of a map is 1cm to 5km.What is the actual distance between two towns which are actually 12km apart?
4. Find the scale in the following cases:
   (a) the length of a rectangular farm on the scale drawing is 4 cm while on the ground it is 500 m.
   (b) the length in a model of a car is 10 cm and the car is 5 m.
   (c) the length of a laboratory on scale drawing is 5 cm and the length of the laboratory is 25 m.

(d) the distance between two towns on a map is 4 cm while on the ground it is 100 km.

5. The floor of a school hall is rectangular and measures 40 m by 25 m. Draw a scale drawing using a scale of 1 cm to 5 m and then determine the length of the diagonal of the floor in metres.

6. The scale of a map is 1:100 000. Calculate the distances between two towns that are:
(a) 15 cm apart on the map.
(b) 8 cm apart on the map
(c) 17.5 cm apart on the map.

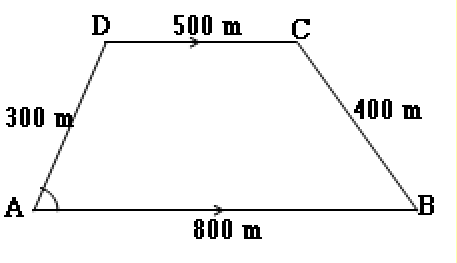
7. The scale of a map is 1 cm to 5 km. Calculate the distances on the map between points that are:
(a) 20 km apart on the ground.
(b) 37 km apart on the ground.
(c) 58.5 km apart on the ground.

8. The height of a building in a photograph is 10 cm. If the scale of the photograph is 1:350, calculate the height of the building.

9. The model of a van is 60 cm long and 2.4 cm wide and the scale of the model is 1:80.
(a) Find the actual length and width of the van.
(b) If the van is 1.8 m high, how high is the model?

10. The model of an aircraft is 25 cm long from the nose to the tail while the wingspan is 30 cm long. The wingspan of the actual aircraft is 27 m. How long is the aircraft?

11. A farm is in the shape of a trapezium ABCD.



AB is parallel to DC, AB = 800 m, BC = 400 m, CD = 500 m, DA = 300 m and $\angle DAB = 60^0$.
(a) Using a suitable scale, draw the plan of the farm.
(b) Find the area of the farm.

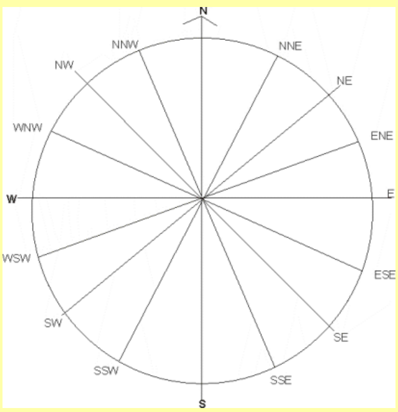
12. The area of a forest is 1 000 hectares. Find the area of the forest on a map whose scale is 1:50 000.

13. The scale of a map is 1:250 000. Calculate the area in m^2 of a game park on the map whose actual area is 25 km^2.

5.4 BEARINGS

Bearing gives the direction of a point with reference to another point. The bearing of a point is given by the angle turned from another given point. In addition, the distance between the two points may be given so as to fix their exact positions. There are two types of bearings: **compass** bearings and **true north** bearings. The direction of a point from another can be determined using a compass. A compass gives the direction of an object starting from the North.

5.41 The compass.

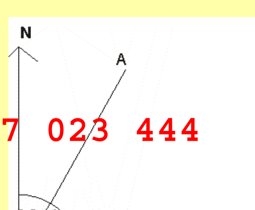


The four main directions of a compass are: North, South, East and West. The secondary bearings are North East (NE), South East (SE), North West (NW) and South West (SW).
NW lies exactly between North and West; NNW lies exactly between N and NW.
There are 16 points on the compass. Any other direction in between is stated using an angle in degrees. For example, N 20^0 W or S 40^0 E.
The angle between the main directions, for example, North and East, is 90^0. The angle between North and North East is 45^0.

There are two methods of giving a bearing:

(i) Compass bearings

On a compass, the bearings are given in terms of the angle turned from north or south. In the figure below the bearing of A from O is said to be N 30^0 E or 30^0 East of North.



(ii) True north bearings

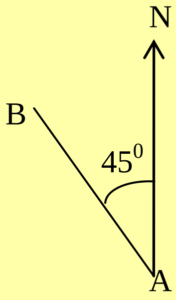
The bearing is given in terms of the angle, described clockwise, which the direction makes with the north line. For example, OA is on a bearing of 030^0. True north bearings are also called three-digit bearings because they are given in three digits. The bearing directly due north is is given as 000^0, the bearing due North East is 045^0, due South as 180^0.

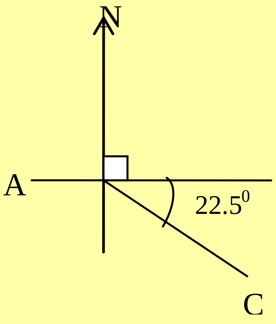
Example 1.

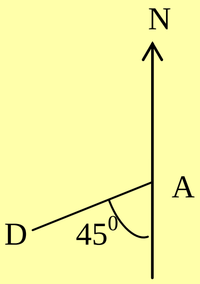
(a) The point B is in the direction North West (NW) from the point A.

(b) The direction of C from A is East South East (ESE)

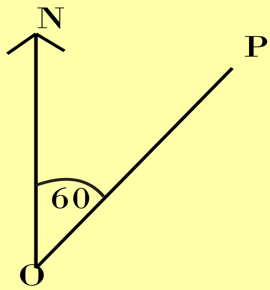
(c) The direction of D from A is South West (SW).

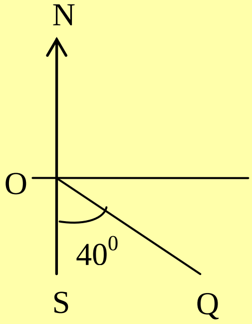






Example 2



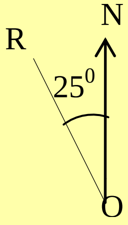


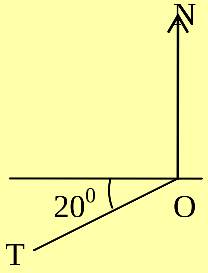
(a) The direction of P from O is N 60^0 E. (start from North turn 60^0 towards the East)

(b) The direction of Q from O is S 40^0 E (start from South, turn 40^0 towards the East)

Directions should always be measured eastwards or westwards from the north or from the south; never from the east or west.

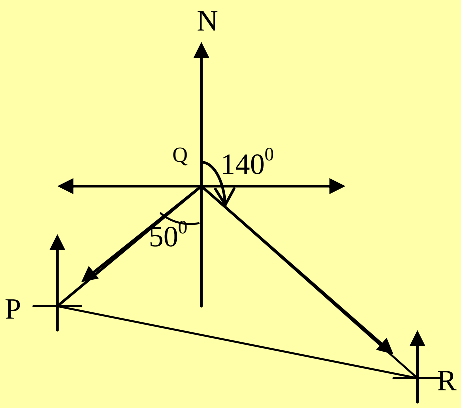
(c) The direction of R from O is N 25^0 W (start from North turn 25^0 towards the west).
(c) The direction of T from O is South 70^0 west.





Example 3

Consider points P, Q and R in the following figure.



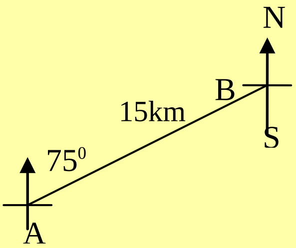
The bearing of each point from one another can be found by measuring the angles. The mark is inserted to assist in identifying the bearing or the required angle. The bearing of R from Q is 140^0. The bearing of P from Q is 230^0. The bearing of Q from P is 050^0. The bearing of Q from R is 320^0.

Example 4

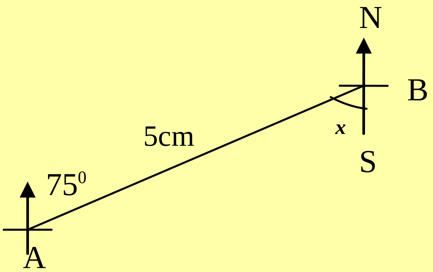
Village A and B are such that the bearing of B from A is 075^0. The distance between A and B is 15 km.
(a) Represent the above information in a scale drawing.
(b) Calculate the bearing of A from B.

Solution

(a) First we draw a sketch.



Then we choose a suitable scale to draw an accurate scale drawing. Use the scale of 1 cm represents 3 km. The accurate scale drawing is shown below.



(b) To calculate the bearing of A from B, consider line AB as a transversal of two parallel lines passing through A and B respectively. The bearing of A from B is greater than 180^0. The angle indicated as x is alternate to the angle marked at A, which is 75^0. Therefore $x = 75^0$.
Therefore the bearing of A from B is $180^0 + 75^0 = 255^0$.

Example 5

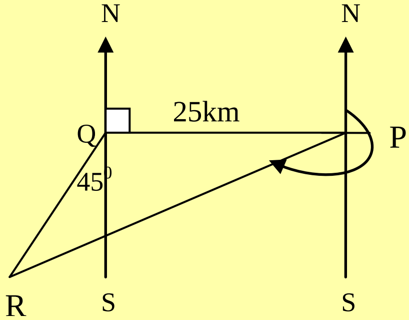
Village P is 25 km due east of village Q. Village R is 20 km from Q on a bearing of S 45^0 W.

(a) Find the distance between R and P.

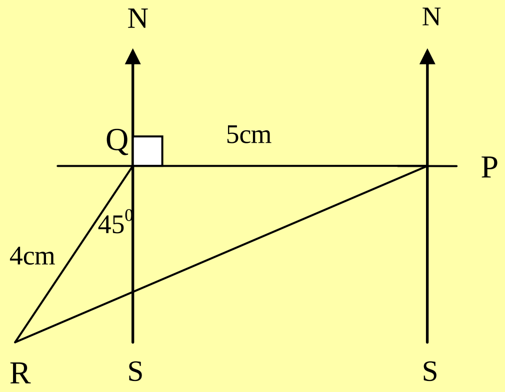
(b) Find the bearing of R from P.

Solution

We draw a sketch first.



We use a scale of 1 cm represents 5 km in making an accurate scale drawing.



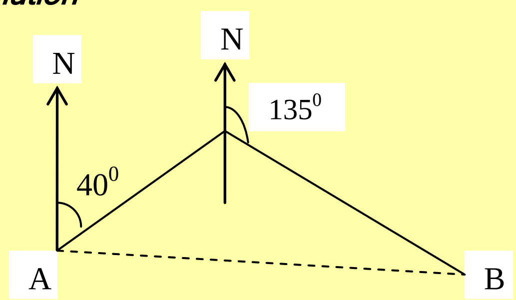
(a) Measuring from the scale drawing, the distance between R and P is about $8.3 \times 5 = 41.5$ km.

(b) To find the bearing of R from P, we draw a perpendicular through P and indicate North (N) and South (S). Then we measure the angle between PR and the line NS. This angle is about 70^0. Therefore, the bearing is S 70^0 W
or $180^0 + 70^0 = 250^0$.

Example 6

Starting from a point A, I walk 40m on a bearing of 040^0. Then I turn and walk a further 30m on a bearing of 135^0 to a point B.
(a) What is the bearing of B from A?
(b) How far is it from A to B?

Solution

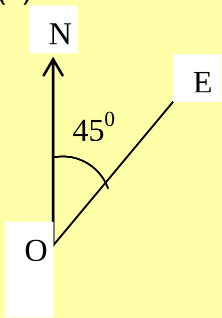


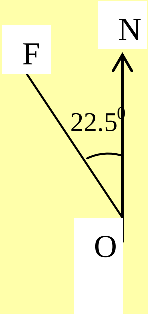
First we draw a rough sketch:
Choose a suitable scale: 1cm = 10m
- Draw the north line from the starting point.
- Measure the bearing from the north line. Draw the line 4cm = 40m
- Draw a new north line at the next turning point. It must be parallel to the first north line. Measure the turn to a bearing of 135^0 to the point B. Draw the line 3cm = 30m (use a ruler and set square to draw the parallel lines)

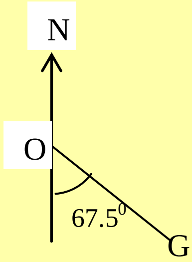
- We draw the line joining A to B. Measure angle NAB = 79^0. Therefore the bearing of B from A = 079^0.
- We measure AB = 4.8cm. But 1cm represents 10m. Therefore the distance from A to B = (4.8 ×10) m = 48m.
- **NB**: To measure the bearing of B from A, the north line must go through the point A.

Exercise 5.4

1. Using the compass, what is the direction of
   (a) E from O
   (b) F from O
   (c) G from O?







2. A cotton ginnery, G, is due East of a football field, F. A hospital, H, is also due east of the football field. What can you say about the direction of H from G? Illustrate your answer with sketch diagrams.

3. Find the angles between i) N and NNE (ii) ENE and ESE (iii) SE and WNW.

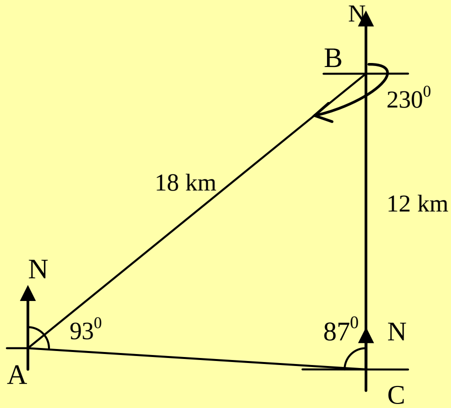
4. A ship sails from a port on a bearing of 120^0 for 75km. It then changes course and sails on a bearing of 015^0 for 80km. Finally the ship sails due east for 50km.
   Taking 1 cm to represent 10 km, make a scale drawing for the ship's three leg voyage. From your scale drawing find the bearing and distance of the ship's final position from the port.

5. A straight road runs east of a point P. A man at P finds that the bearing of two eucalyptus trees S and T are 050^0 and 145^0, respectively. After walking 40 m eastwards along the road, the man finds that he is due north of T. Use a scale drawing to determine
   (i) the shortest distance from T to the road;
   (ii) the bearing of T from S, given that S and T are 75m apart.

6. A helicopter flies from Gulu for 140km on a bearing of 205^0. It then flies for 170km on a bearing or 150^0. The helicopter then flies for 230km on a bearing of 240^0. Taking 1cm to 25km make a scale drawing for the helicopter's flight. From your scale drawing find:
   (i) how far from Gulu the helicopter is after the second leg of its journey.
   (ii) The bearing of Gulu from the helicopter's final position.

7. Three towns, X, Y and Z, are such that X is in a bearing of 120^0 and 20 km from Y. Town Z is in a bearing of 220^0 and 12 km from Y.
   (a) Using a suitable scale, draw the positions of X, Y and Z.
   (b) Find:
      (i) the distance between X and Z in km.
      (ii) the bearing of Z from X
      (iii) the bearing of X from Z
      (iv) the bearing of Y from Z
      (v) the bearing of Y from X.

8. The figure below represents the positions of three points.



   Draw the figure accurately using a suitable scale. Determine:
   (i) the bearing of B from A.
   (ii) the bearing of A from C.

9. School P is 040^0 and 30 km from school R. Dispensary D is 140^0 and 27 km from school R. Village Q is 030^0 and 40 km from D. The distance between P and Q is 20 km. Use a scale drawing to determine:
   (a) The distance between;
      (i) R and Q. (ii) P and D.
   (b) The bearing of Q from R.
   (c) The bearing of D from P

6. POLYGONS

- A polygon is any plane figure or flat shape which is enclosed by three or more straight lines.
- The point at which two sides of a polygon intersect is called a vertex. The plural is 'vertices'.
- A line segment joining two vertices which are not next to each other is called a diagonal.
- Polygons are named according to the number of angles or edges they have.

6.1 Types of polygons

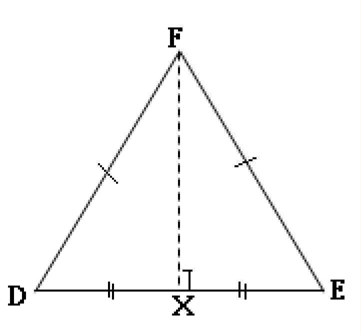
Name	Number of sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Dodecagon	12

6.2 Triangles

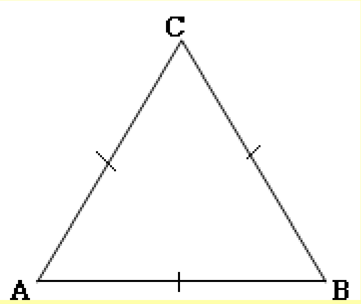
Types of triangles:

1. **Scalene triangle**: The sides of the triangle
   (i) are all of different length;
   (ii) angles are also of different sizes.

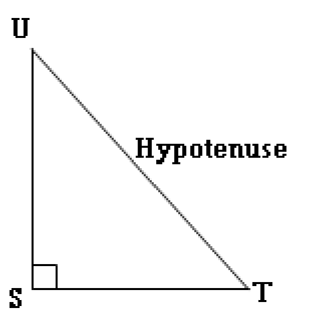
2. **Isosceles triangle**: (i) two sides equal; (ii) two angles equal. The third side is the base. The vertex (F) opposite the base is called the **vertex**. The angle at the vertex is called **Vertical** angle. The bisector of the vertical angle of an isosceles triangle bisects the base at right angles.
   (i.e. DX = XE).



3. **Equilateral triangle**: All the sides and angles are equal.

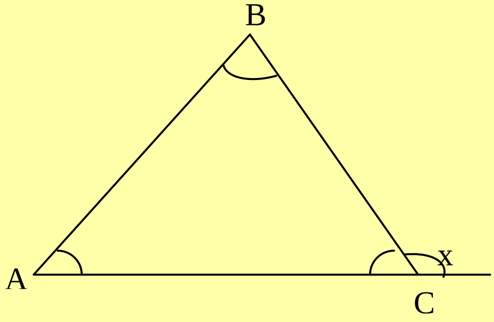


4. An **acute** angled triangle has all three angles acute.
5. An **obtuse**-angled triangle has one of its angles obtuse.
6. A **right angled** triangle has one of its angles a right angle.



Right angled triangles are either scalene or isosceles.

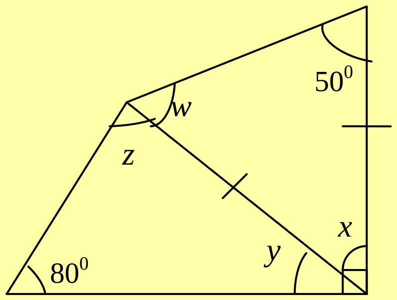
7. The side opposite the right angle is called the **hypotenuse**.
8. The angle sum of a triangle is 180^0
9. If ABC is a triangle, the angle A is said to be included between the sides AB and AC.
10. If one side of a triangle is produced, the exterior angle formed is equal to the sum of the two interior opposite angles.



x = A + B

Examples

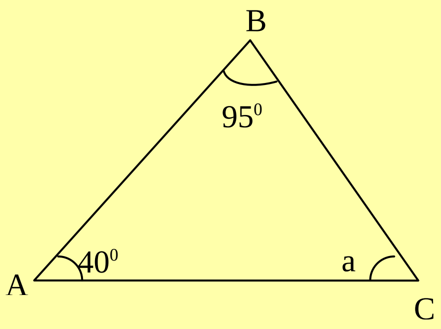
1. Find the angles represented by the letters w, x, y and z in the diagram.

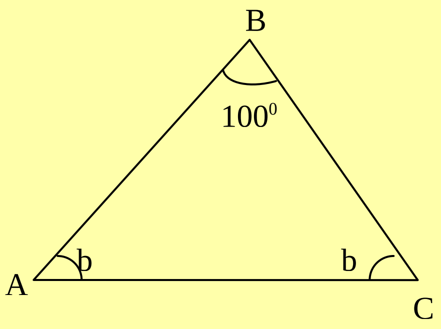


Solution

The top triangle is isosceles. So w = 50^0
50^0 +w +x = 180^0
50^0 + 50^0 + x = 180^0
So x = 80^0.
x + y = 90^0
80 + y = 90^0
So y = 10^0.
z+ y + 80^0 = 180^0
Z + 10^0 + 80^0 = 180^0
Z + 90^0 = 180^0
So z = 90^0.
The angles are: w = 50^0, x = 80^0, y = 10^0 and z = 90^0.

2. Calculate the angles marked by letters in the following figures:
(a) (b)





Solutions

(a) $40^0 + 95^0 + a = 180^0$ (angles of a triangle)

$a + 135^0 = 180^0$

$a = 180^0 - 135^0$

$a = 45^0$

(b) $b + b + 100^0 = 180^0$

$2b + 100^0 = 180^0$

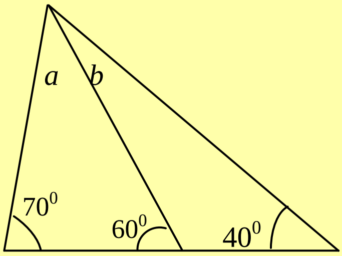
$2b = 180^0 - 100^0$

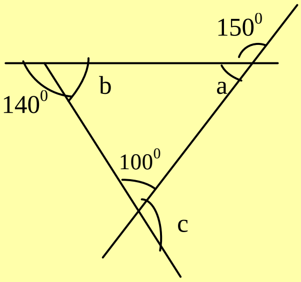
$2b = 80^0$

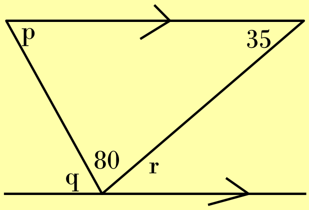
$b = 40^0$.

Exercise 6.1

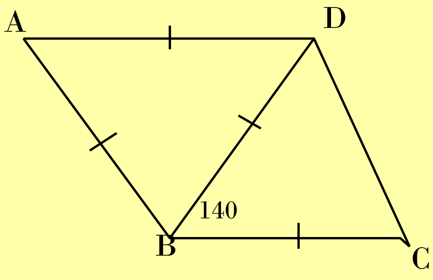
1. Find the angles represented by small letters in the diagrams below:





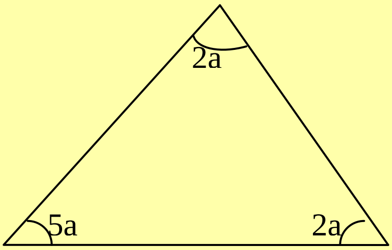


2. In the diagram ABD is an equilateral triangle, BC = BD and angle ABC is 140^0. Find angle BCD

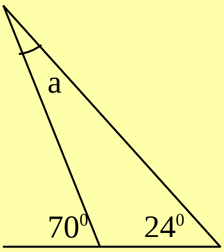


3. Calculate the unknown angles.

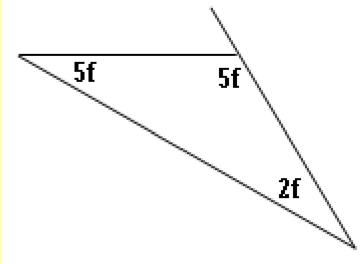
(a)



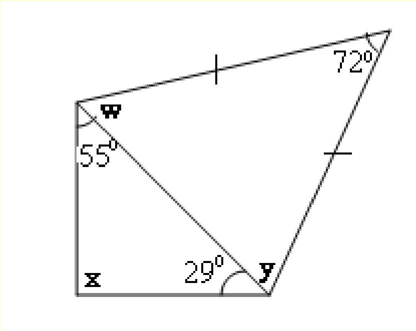
(b)



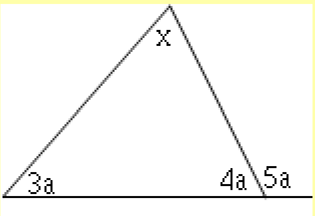
(c)



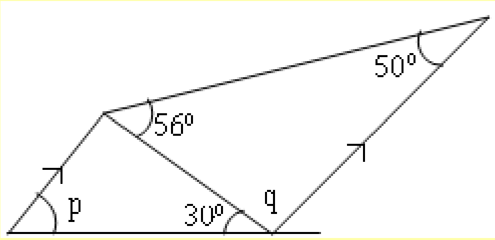
(d)



(d)



(e)



6.3 Quadrilaterals

These are flat figures enclosed by four line segments. (The diagrams below show the different kinds of quadrilaterals).

1. A kite is a quadrilateral with two adjacent sides equal and the other two adjacent also equal.

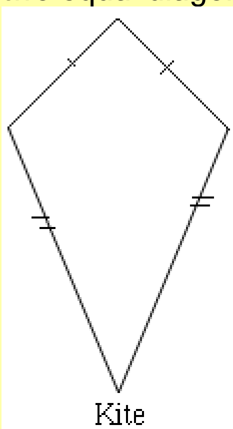
2. A trapezium is a quadrilateral with one pair of parallel sides.

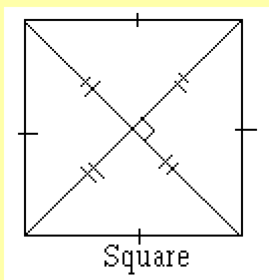
3. A parallelogram is a quadrilateral with both pairs of opposite sides parallel. The opposite sides are equal in length. Its diagonals are not of equal length and the sides are not perpendicular.

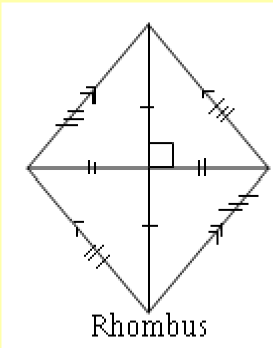
4. A rectangle is a parallelogram with four right angles. Its diagonals are equal in length but they do not meet at right angles.

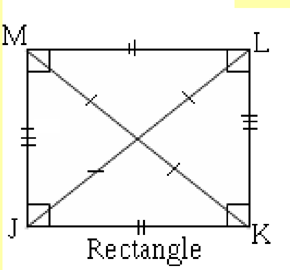
5. A rhombus is a parallelogram with all sides equal. The sides do not necessarily meet at right angles. While the diagonals of a rhombus meet at right angles, they are not necessarily equal in length.

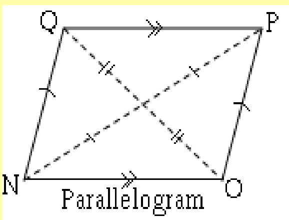
6. A square is a rhombus with four right angles. A square has two equal diagonals that meet at right angles

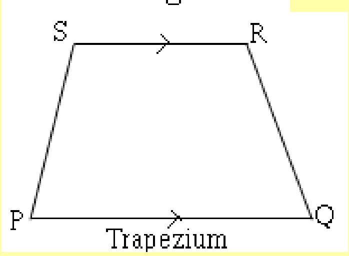








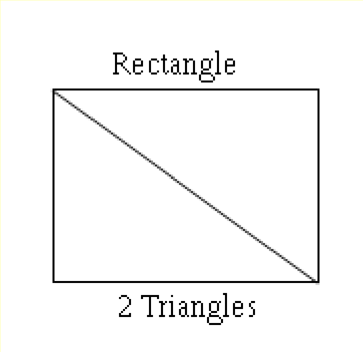


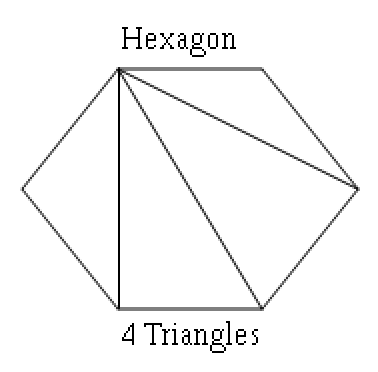


6.4 Other polygons

1. Sum of interior angles

A polygon may be regular or irregular. A regular polygon has equal sides and equal angles. The number of interior angles of a polygon is as many as the number of sides. If a polygon is divided into triangles by drawing in all the diagonals from one vertex, the number of triangles formed is given by n – 2, where n is the number of sides of the polygon.





Copy and complete the table below

No. of sides of polygon	No. of triangles	sum of interior angles
3	1	$1\times180^0 = 180^0$
4	2	$2\times180^0 = 360^0$
5	3	$3\times180^0 = 540^0$
6	4	
.	.	
.	.	
n	n-2	

From the above table we see that:
For a polygon with n sides:
the sum of the interior angles is = $(n - 2) \times 180^0$.

2. Sum of exterior angles of a convex polygon.

A **convex** polygon has all its sides pointing outwards. The sum of the exterior angles of a polygon is 360^0.
A **regular** polygon has all its sides equal and all angles equal.

Example 2

(i) Find the sum of the interior angles of a pentagon
(ii) The sum of the interior angles of a polygon is 900^0. How many sides has the polygon?

Solution

(i) The sum of the interior angles of a polygon with n sides is $(n - 2) \times 180^0$. A pentagon has 5 sides. So the sum of the interior angles of a pentagon is $(5 - 2) \times 180^0 = 3 \times 180^0 = 540^0$.

(ii) $900^0 = (n - 2) \times 180^0$

$\frac{900^0}{180^0} = n - 2 \quad \Rightarrow \quad 5 + 2 = n$

Therefore, n = 7. The polygon has 7 sides.

Example 3

Find the interior angle of a regular octagon.

Solution

Sum of interior angles of an octagon is
$(8 - 2) \times 180^0 = 6 \times 180^0$
$= 1080^0$.
Therefore, interior angle = $\frac{1080}{8} = 135^0$.
Alternative method:
Exterior angle of regular octagon = $\frac{360}{8} = 45^0$
Therefore, interior angle of regular octagon = $180^0 - 45^0 = 135^0$.

Example 4

Find the interior angle of a regular polygon with 9 sides.

Solution

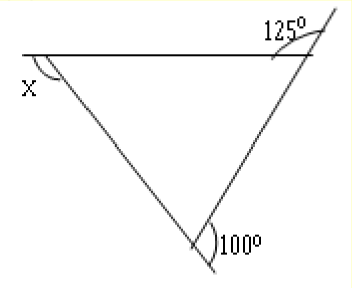
The sum of all the 9 exterior angles = 360^0

Therefore each exterior angle = $\frac{360}{9} = 40^0$
Therefore each interior angle = $180^0 - 40^0 = 140^0$.

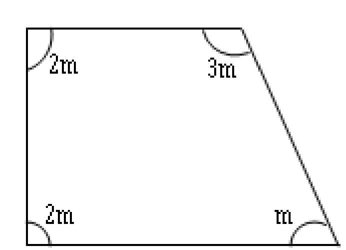
Exercise 6.2

1. Find the interior angle of a regular decagon.
2. The interior angle of a regular polygon is 168^0. How many sides has the polygon?
3. The interior angles of a regular polygon add up to 2340^0. Find the size of the interior angle.
4. Find the interior angle of a 20-sided regular polygon.
5. The exterior angles of a quadrilateral are z^0, $2z^0$, $2z^0$ and 80^0. Find the value of z.
6. Find the size of the angles marked with letters.

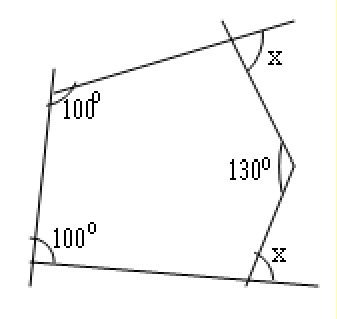
(a)



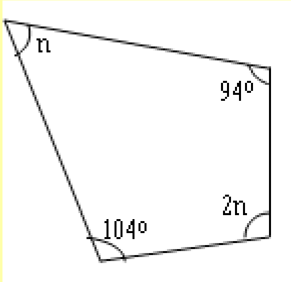
(b)



(c)



(d)



7. A square has two its sides given as (2x + 12) cm and (x + 20) cm. Calculate its length.

8. One of the large and one of the small angles of a parallelogram are marked 2x and x respectively. Calculate the value of the large angle.

9. Given that one of the angles of a rhombus is 120^0, calculate the smallest angle between one side and the diagonal.

10. The larger angle formed by the intersection of the diagonals of a rectangle is 150^0. Calculate the angle between the longer side and the diagonal.

11. The angles of a trapezium are x, 2x, 3x and 4x. Calculate the value of the second smallest angle.

12. The exterior angle of a rhombus is five times the size of the adjacent interior angle. Calculate the value of the smallest and largest angles of the rhombus.

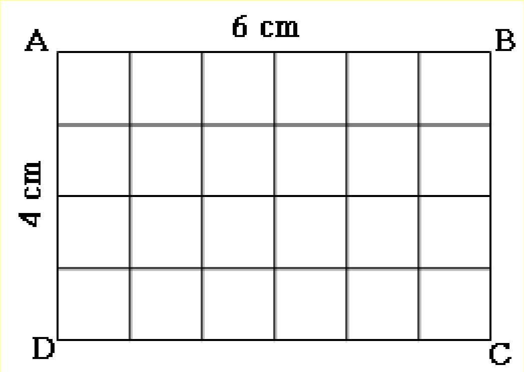
13. A regular polygon has 15 sides. If its interior angle is $(x + 36)^0$, calculate the value of x.

14. An irregular pentagon has the following four interior angles: 110^0, 108^0, 135^0, and 152^0. Determine the value of the exterior angle adjacent to the fifth interior angle.

7. AREA AND VOLUME

Area

The area of a plane figure is the amount of space covered by its surface.
Let us consider the rectangular frame below.



In the figure, AB, BC, CD, and DA are the boundaries. AB is 6 cm long and BC is 4 cm long. The rectangle is divided into 24 small squares. The side of each small square is 1 cm in length. The area of rectangle ABCD is
6 cm × 4 cm = 24 cm^2.

Area of a rectangle = Length × width.

Example 1

Find the area of a rectangle measuring $4\frac{1}{2}cm$ by $3\frac{1}{4}cm$.

Solution

Area of a rectangle = Length × width.

$$= 4\frac{1}{2} \times 3\frac{1}{4}$$

$$= \frac{9}{2} \times \frac{13}{4}$$

$$= \frac{117}{8}$$

$$= 14\frac{5}{8}cm^2 \text{ or } 14.625\text{ cm}^2$$

Example 2

A rectangular lawn 80m long and32m wide has a gravel path 4m wide all around it. Find the area of the path.

Solution

The area of the grass = $80 \times 32\text{m}^2$

$= 2560\text{m}^2$

Now the length of the whole rectangle formed by the lawn and path

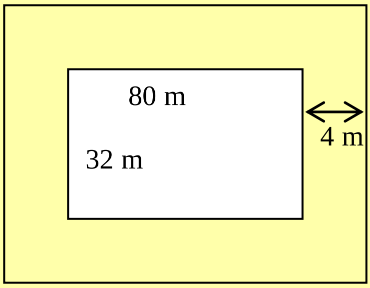
$= (80 + 4 + 4)\text{ m} = 88\text{m}$

The width $= (32 + 4 + 4)\text{ m}$

$= 40\text{m}$

The total area $= 88 \times 40\text{m}^2 = 3520\text{m}^2$

The area of the path $= (3520 - 2560)\text{ m}^2 = 960\text{m}^2$.



Method ii

The path can be divided up into two rectangles each 88m by 4m, together with two more rectangles, each 32m by 4m.

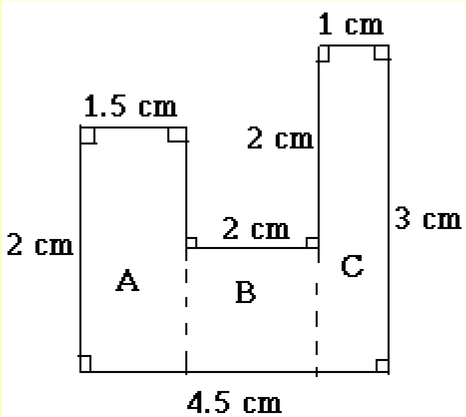
The area of the path $= (2 \times 88 \times 4 + 2 \times 32 \times 4)\text{ m}^2$

$= (704 + 256)\text{ m}^2$

$= 960\text{m}^2$.

Area of combined rectangles

When the corners of a plane figure are right angles, the area is found by dividing it into rectangles as shown below.



Area of rectangle A $= 2 \times 1.5 = 3\text{ cm}^2$

Area of rectangle B $= 2 \times 1 = 2\text{ cm}^2$

Area of rectangle C $= 3 \times 1 = 3\text{ cm}^2$

When working out problems involving areas, the following points should be notd.

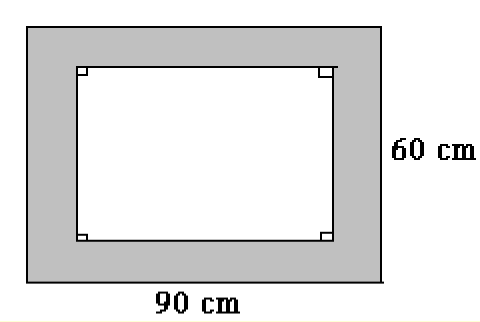
1. Measurements must be in the same units. For instance, if length is in centimeters, width must also be in centimeters.
2. Linear measure × linear measure = square measure.
3. Square measure ÷ linear measure = linear measure.

Converting units of area

The units of area are: millimeter squared (mm^2), centimeter squared (cm^2), decimeter squared (dm^2), metre squared (m^2), Dekametre squared (Dm^2), hectometer squared (hn^2), and kilometer squared (km^2).
The conversion flow chart on the next page can be used to change one to another. For instance, to change a smaller unit of area to a larger unit area, use the arrows on the right side.. Conversely, to change a larger unit of area to a smaller unit use the arrows on the left side. Thus an area of $1000\ cm^2 = (1000 \div 100)\ dm^2 = 10\ dm^2$ an
$1000\ cm^2 = (1000 \times 100)\ mm^2 = 100\ 000\ mm^2$.
Similarly an area of $1\ km^2 = (1 \times 100 \times 100 \times 100)\ m^2$.

Exercise 7.1

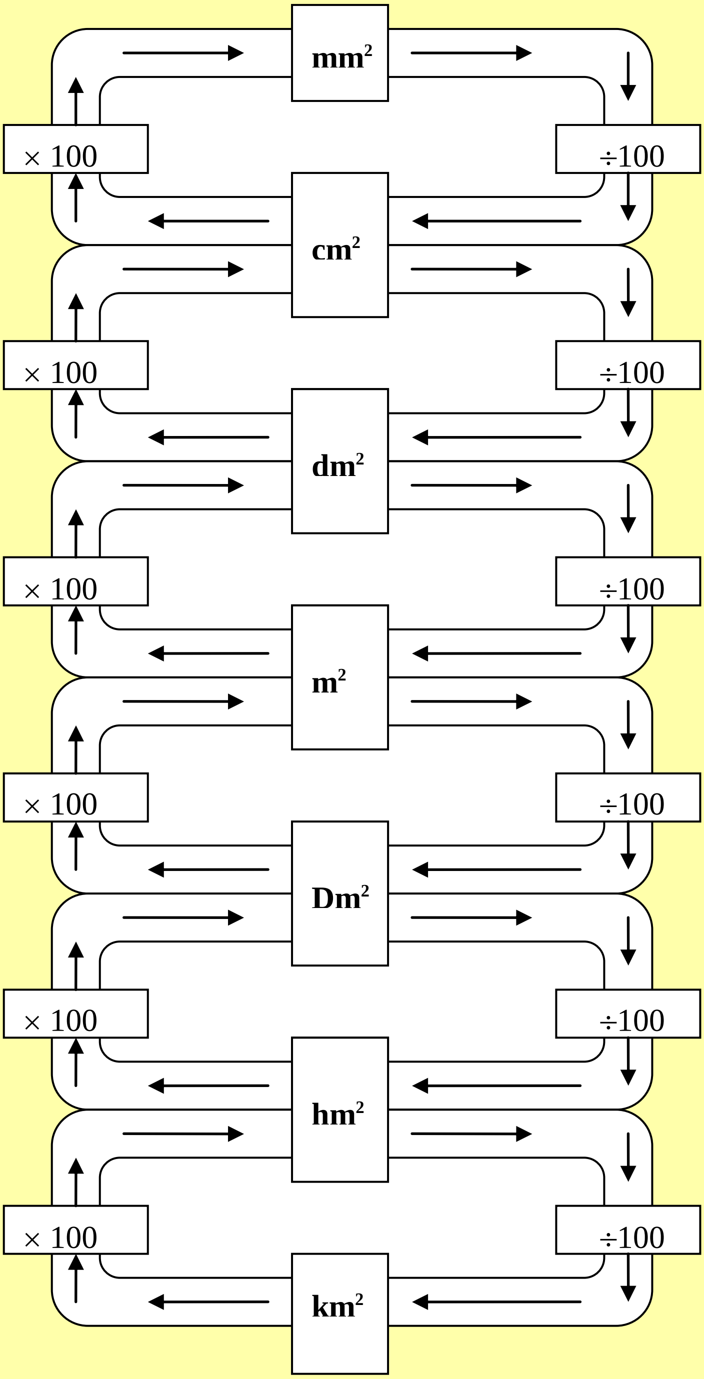
1. The figure below represents a rectangular table top. Find the area of the shaded part given that the width of the shaded part is 5 cm all round.



2. A rectangular piece of land measuring 100 m by 90 m has an area of 40 m by 80 m planted with beans and another area of 30 m by 30 m planted with simsim. The rest of the land has maize. Find the area under maize.

3. The diagonal of a square is 2.42 cm. Find the area of the square.

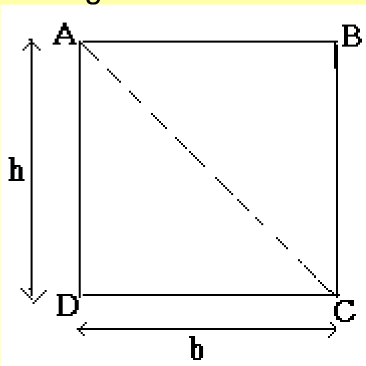
4. Convert each of the following to the units in the brackets.
(a) $45\ m^2$ (cm^2) (b) $15\ km^2$ (m^2) (c) $4650\ m^2$ (km^2)



mm²
× 100
÷100
cm²
× 100
÷100
dm²
× 100
÷100
m²
× 100
÷100
Dm²
× 100
÷100
hm²
× 100
÷100
km²

Area of a triangle

A triangle is a geometrical figure with three sides and three angles. The figure below shows rectangle ABCD divided into two equal triangles. Thus, area of triangle ADC = area of triangle ABC.



In triangle ADC, AD is the perpendicular height (h) and DC is the base (b). In triangle ABC, BC is the height an d AB is the base.

$2 \times$ area of triangle ABC = area of rectangle ABCD.

Area of rectangle ABCD = $h \times b$.

Therefore, area of triangle ABC

$= \frac{1}{2}$ area of rectangle ABCD.

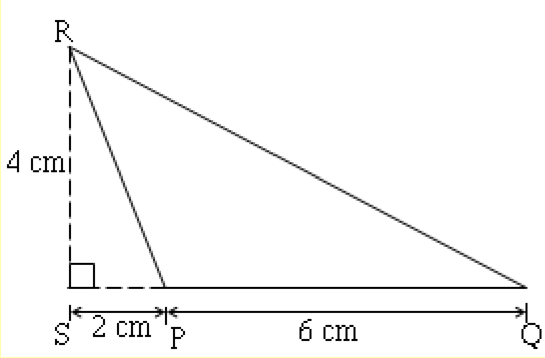
$= \frac{1}{2} \times h \times b$

$= \frac{1}{2}bh.$

$\therefore$ Area of any triangle = ½ × base × perpendicular height.

Example 1

Find the area of triangle PQR in the figure below.



Area of triangle PQR = Area of triangle QRS – Area of triangle PRS

$= \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 2 \times 4$

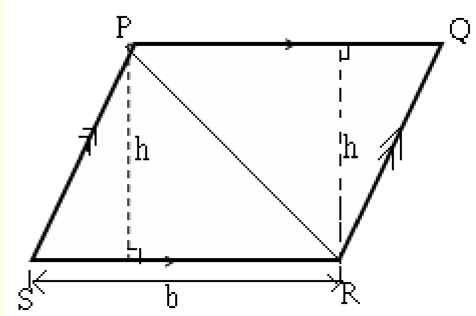
$= 16 - 4$

$= 12\ cm^2$.

Alternatively, the area of triangle PQR = $\frac{1}{2}$bh = $\frac{1}{2} \times 6 \times 4 = 12$ cm^2.

Area of a parallelogram

A parallelogram is a quadrilateral with opposite side that are parallel and equal in length. The figure below shows a parallelogram PQRS. PQ is parallel and equal to SR while PS is parallel and equal to QR.



In the figure, h is the height of triangle PQR and PRS. PQ is the base of triangle
PQR = SR = b (the base of the triangle PRS). The area of parallelogram PQRS = area of triangle PRS + area of triangle PQR.
Area of parallelogram PQRS

$= (\frac{1}{2} \times b \times h) + (\frac{1}{2} \times b \times h) = bh.$

Note that h is the perpendicular distance between the parallel sides.

Example 2

Find the area of a parallelogram whose base and height are 5 cm and 8 cm respectively.

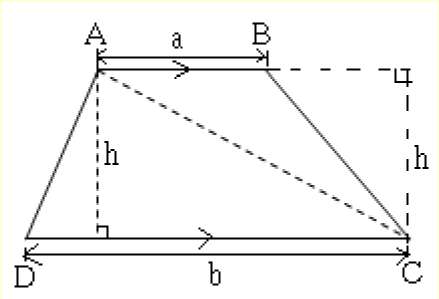
Solution:

Area of parallelogram = base $\times$ height
$= 5 \text{ cm} \times 8 \text{ cm}$
$= 40 \text{ cm}^2$.

The above formula is also used for finding the area of a rhombus, which is a special parallelogram with all its sides equal in length and diagonals meeting at right angles.

Area of a trapezium

A trapezium is a quadrilateral with only two sides that are parallel. The figure below shows a trapezium in which AB is parallel to DC and the lengths of AB and DC are *a* and *b* respectively. Let the perpendicular distance between the parallel sides be h. When line AC is drawn, trapezium ABCD is divided into two triangles ABC and ACD.



Area of trapezium ABCD

= Area of triangle ABC + Area of triangle ACD

$= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$

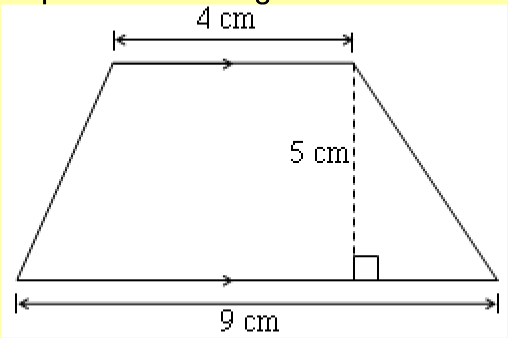
$= \frac{1}{2}ah + \frac{1}{2}bh$

$= \frac{1}{2}h(a + b)$.

Note that (a + b) is the sum of the lengths of the parallel sides. Thus, the area of a trapezium is equal to $\frac{1}{2}$(perpendicular distance between parallel sides) $\times$ sum of the parallel sides. Therefore, the area of a trapezium is $\frac{1}{2}h(a + b)$ where h = perpendicular height, a = length of shorter side and b = length of longer parallel side.

Example 3

Find the area of the trapezium in the figure below.



Solution:

Perpendicular height = 5 cm

Sum of parallel sides = (4 + 9) cm

= 13 cm

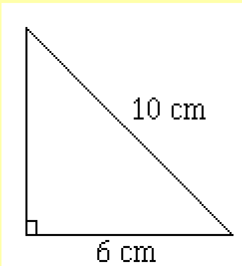
Area $= \frac{1}{2} \times 5 \times 13$

$= 32.5\ cm^2$

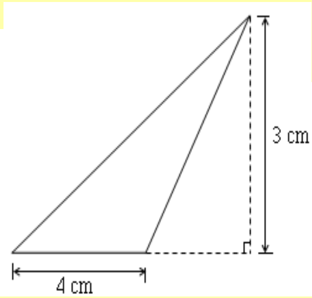
Exercise 7.2 A

1. Find the area of each of the following figures.

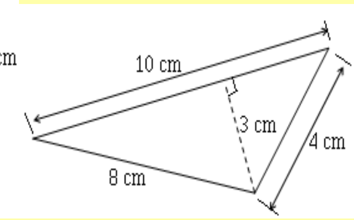
(a)



(b)



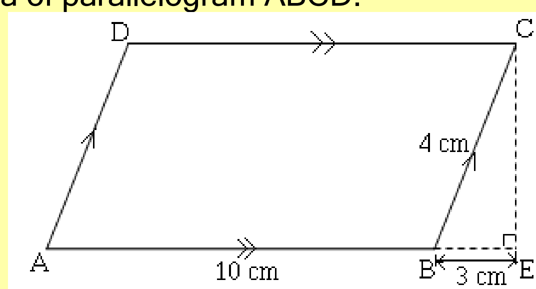
(c)



2. A right-angled triangular sheet of metal has its hypotenuse measuring 1.7 m and the base 1.5 m. Find the area of the sheet of metal.

3. A triangle has an area of 10.5 cm^2 and a perpendicular height of 3 cm. Find the length of the base.

4. Find the area of parallelogram ABCD.



5. The length of a rhombus is 13 cm and its longer diagonal is 24 cm. Find the area of the rhombus.

6. The area of a parallelogram is 15.4 cm^2. If its height is 7 cm,

   find its base.

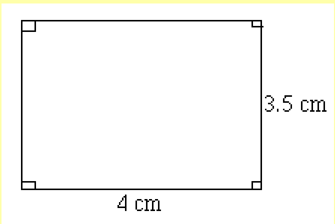
7. The parallel sides of a trapezium are 9.5 cm and 4.5 cm respectively. If the perpendicular distance between the parallel sides is 5 cm, find the area of the trapezium.

8. The area of a trapezium is 16.5 cm^2 and the parallel side are 6.5 cm and 4.5 cm respectively. Find the perpendicular height between the parallel sides.

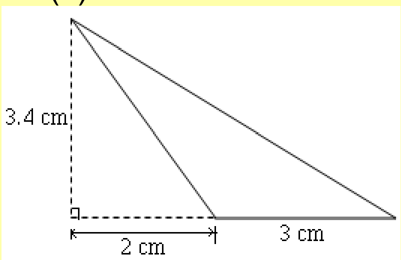
9. A trapezium has an area of 25.5 cm^2 and a perpendicular height of 6 cm. If the length of one of the parallel sides is 5 cm, find the length of the other parallel side.

10. Find the area of the shapes below.

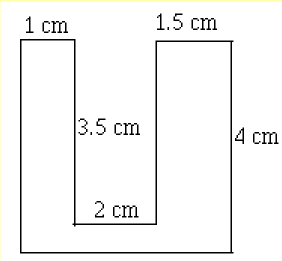
(a)



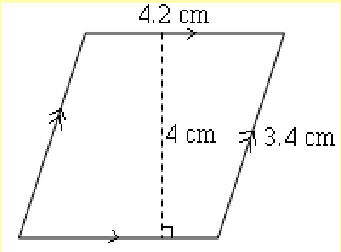
(b)



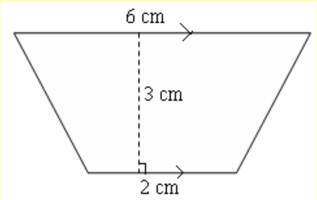
(c)



(d)



(e)



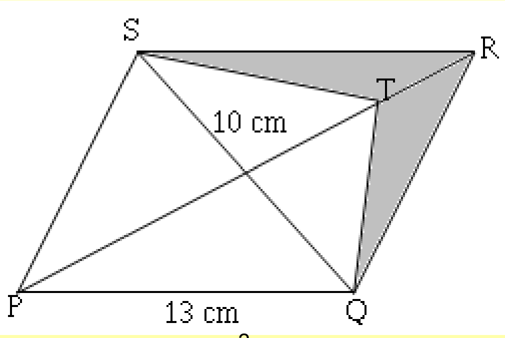
11. A rectangular table top measures 1.2 m by 80 cm. Find the area of the table top in square metres.

12. The length of a rectangular plot is 200 m. If the area of the plot is 4800 m^2, find the width of the plot.

13. A room 5.5 m long and 4.2 m wide has a carpet in the middle. If a margin of 20 cm wide is left all round, find the area of the floor that is not covered by the carpet.

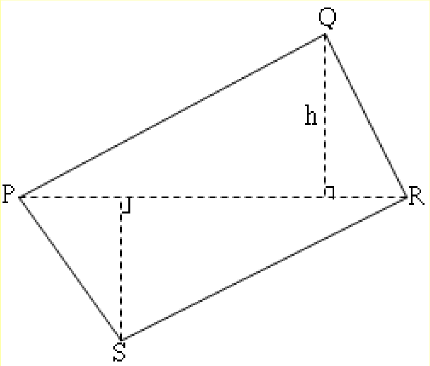
14. A photograph 20 cm wide and 30 cm high is mounted on a cardboard so that there is a margin 2 cm wide at the top and at the bottom. A margin of 1.5 cm is on both the left and the right sides. Calculate the area that is not covered by the photograph.

15. Find the area of the shaded part given that PQRS is a rhombus, PQ = 13 cm, TR = 2 cm and QS = 10 cm.



16. The area of a rhombus is 240 cm^2. If the length of one of the diagonals is 16 cm, find the length of the rhombus.

17. The area of quadrilateral PQRS is 54 cm^2. If the area of PRS is 20 cm^2, find the value of h if PR = 10 cm.



Area of a circle

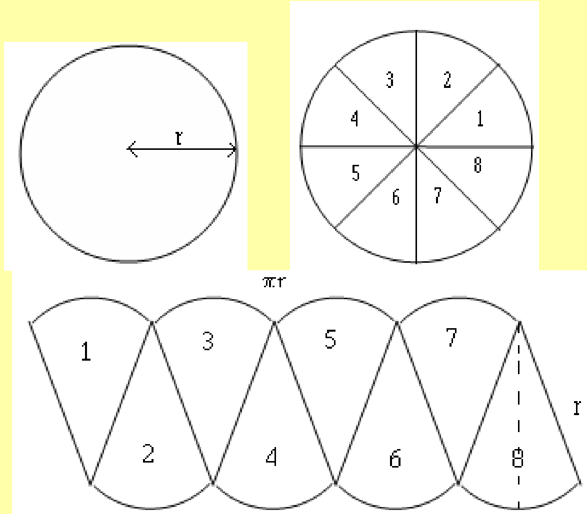
The figure below shows a circle of radius r. It is divided into 8 equal sectors. The sectors are cut out and then arranged to form a parallelogram. The curved lines of adjacent sectors are placed opposite each other as shown below. The dotted line shows the height of the parallelogram which is equal to the radius r. The base of the parallelogram is equal to πr, that is, half the circumference of the circle.

Circumference of a circle = $2\pi r$

Area of a parallelogram = base $\times$ height

$= \pi r \times r = \pi r^2$

Thus, the area of any circle = πr^2, where r is the radius and $\pi = \frac{22}{7}$ or 3.142 (correct to 3 d.p.)



Example 1.

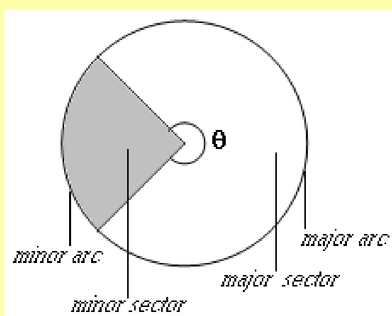
Find the area of a circle whose radius is 7 cm.

Solution

Area = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$.

Area of a sector of a circle

A sector is a part of a circle enclosed by two radii and an arc (minor or major). The figure below shows the major and minor sectors of a circle.



If the angle subtended by the major arc at the centre is θ, then the area of the major sector $= \frac{\theta}{360}\pi r^2$.

Example 2.

A circle has a radius of 18 cm. Find the area of a sector of the circle whose arc subtends an angle of 70^0 at the centre. (Take $\pi = \frac{22}{7}$).

Solution

Area of a sector = $\frac{\theta}{360}\pi r^2 = \left(\frac{70}{360} \times \frac{22}{7} \times 18 \times 18\right) cm^2$ = 198 cm^2.

Exercise 7.2 B

1. Calculate the areas of the circles of radii:

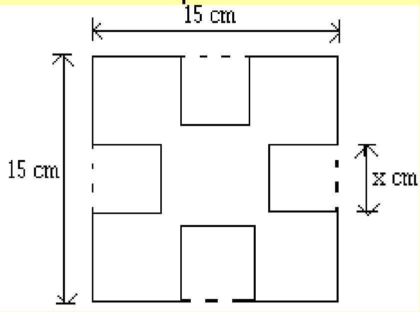
   (a) 1.4 m, (b) 4.2 cm. (Take $\pi = \frac{22}{7}$)

2. Calculate the area of the sector of a circle when:
   (a) radius = 14 cm, angle of sector = 50^0.
   (b) radius = 21 cm, angle of sector = 22.5^0.
   (Take π = 3.142).

3. Calculate the angles at the centre of the circle if the areas of the sectors and the radii are as follows:
   (a) area = 3.698 cm^2, radius = 2.8 cm
   (b) area = 28.14 cm^2, radius = 7 cm
   (c) area = 31.4 cm^2, radius = 6 cm

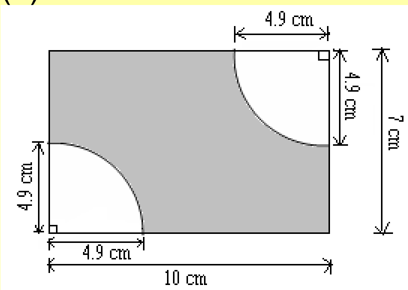
4. A circular metal sheet of radius 3.5 cm was cut out from a rectangular metal sheet measuring 8 cm by 9 cm. Find the area of the metal left.

5. The figure below shows a square plate whose side measures 15 cm. Four squares each of side x cm have been cut out. If the perimeter of the square plate after the small squares are cut out is 84 cm, find the area of the plate left.

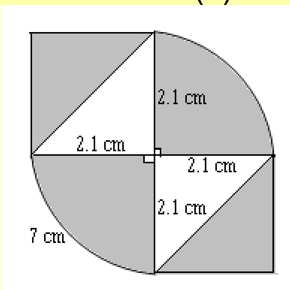


6. Calculate the area of the shaded region in the following shapes.

(a)



(b)



Area of irregular plane figures

So far we have learnt how to find areas of regular plane figures. In this section we will discuss how to find areas of irregular plane figures. The figures are drawn on a grid whose scale is known. However, the areas obtained are approximations. In order to find the area of any irregular plane figure, the following steps must be followed.

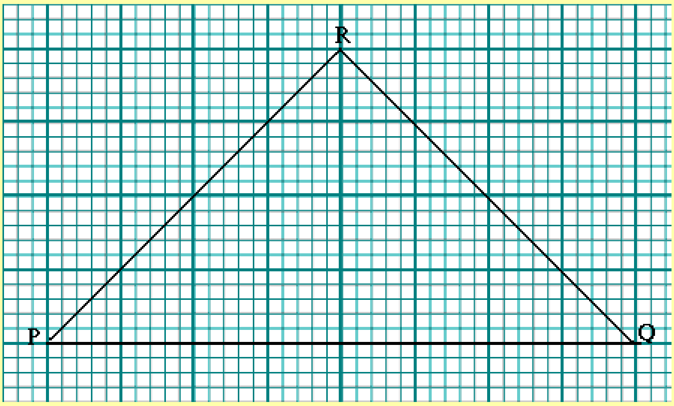
Step 1. Count the number of whole squares enclosed within the figure.

Step 2. Count all the incomplete squares covered by the figure and divide the total by two.

Step 3. Add the values obtained in steps 1 and 2 together and then multiply the total sum by the area of one square. The value obtained is the estimated area.

Example 1

Estimate the area of triangle PQR which is drawn on 1 cm squares.



Solution

Number of whole squares = 12
Number of incomplete squares = 8

Total number of squares $= 12 + \frac{8}{2}$

$= 16$

Area of triangle PQR $= 16 \times 1\ \text{cm}^2 = 16\ \text{cm}^2$

Note that triangle PQR is a regular plane figure and its area can be calculated using the following formula

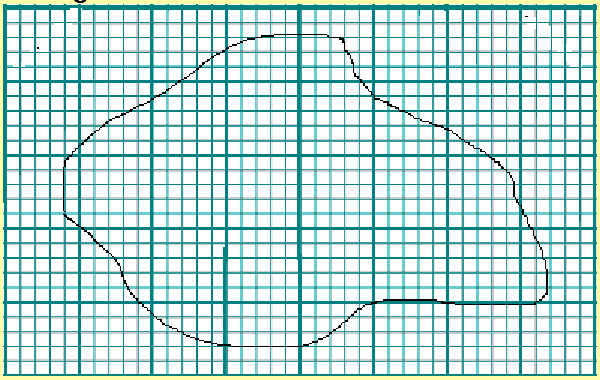
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

In this case,

$$\text{Area} = \frac{1}{2} \times 8 \times 4 = 16\ \text{cm}^2$$

Example 2

An outline of a swamp is drawn on a 1 cm square paper as shown in the figure below.



Estimate the area of the outline in cm^2.

Solution

Number of whole squares = 11
Number of incomplete squares = 16

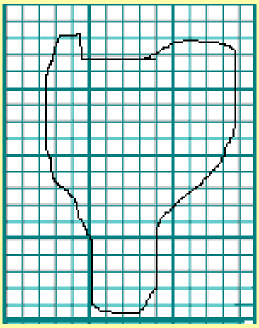
Total number of squares $= 11 + \frac{16}{2} = 19$

Area of outline $= 19 \times 1$ cm^2
$= 19$ cm^2.

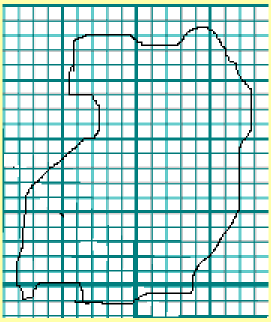
Exercise 7.2 C

Estimate the area of the following irregular figures drawn on 1 cm squares.

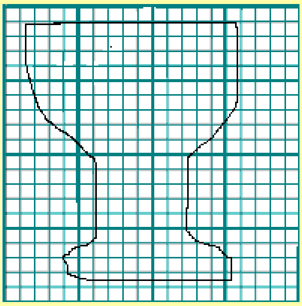
1.



2.



3.

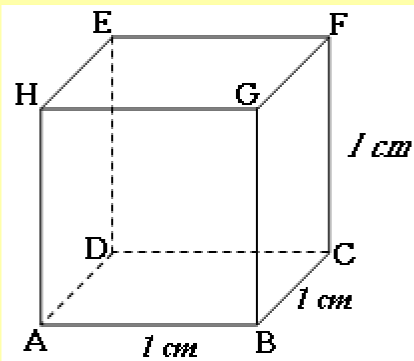


Surface area of solids

A solid is a shape formed in a 3-dimensional space. The most common of these are the cube, cuboid, cylinder, cone, pyramid, prism and sphere. The surface area of any solid is equal to the sum of the areas of all the faces.

Surface area of a cube

A cube has six equal faces. The area of each face $= l \times l$. The surface area of the whole cube $= 6(l \times l) = 6l^2$



Example 1

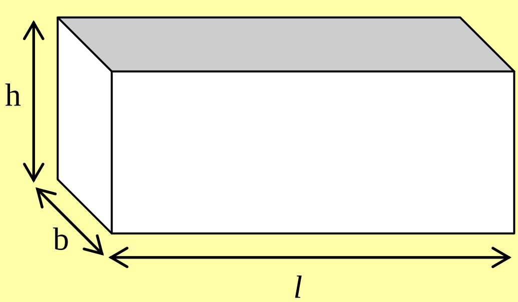
Calculate the surface area of a cube whose length is 6 cm.

Solution

Surface area $= 6l^2$ cm^2
$= 6 \times 6 \times 6$ cm^2
$= 216$ cm^2.

Surface area of a cuboid

The figure below shows a cuboid of length *l*, breadth *b* and height *h.*



A cuboid has 3 pairs of faces of different sizes. Therefore, surface area

$= 2(l \times b) + 2(l \times h) + 2(b \times h)$
$= 2(lb + lh + bh)$

Example

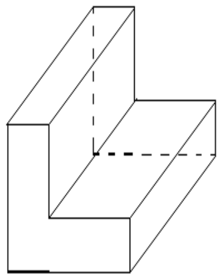
Find the surface area of a cuboid measuring 6 cm by 4 cm by 3 cm.

Solution

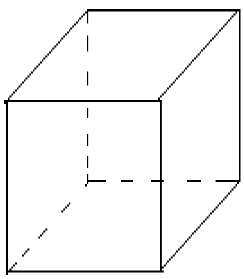
Surface area = $2(lb + lh + bh)$
$= 2(6 \times 4 + 6 \times 3 + 4 \times 3)$
$= 2 \times 54$
$= 108 \text{ cm}^2$.

Surface area of prisms

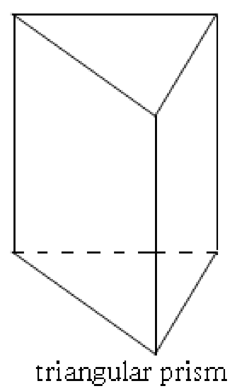
A prism is a geometric solid having 2 faces that are identical and parallel to each other. Any plane cut, made parallel to the ends, produces a cross-section of the same shape and size as the ends. Prisms are named after the shape of the cross-section, for example, a triangular prism has a triangular end and a hexagonal prism has a hexagonal cross-section.
The ends of a prism are parallel and are of the same size while the faces are parallelograms.
A prism is said to have a cross-section of uniform area. Different types of prisms are shown below.

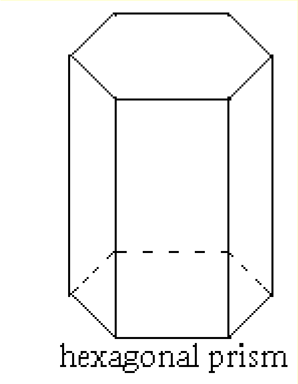


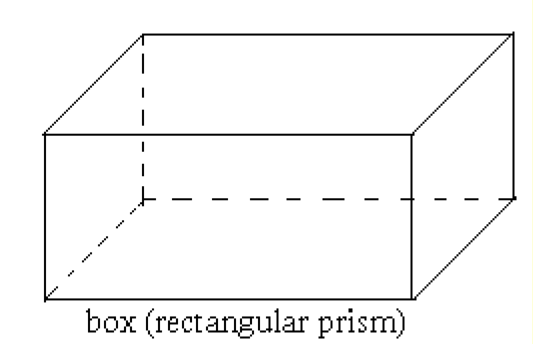
L-shaped prism



cube (square prism)





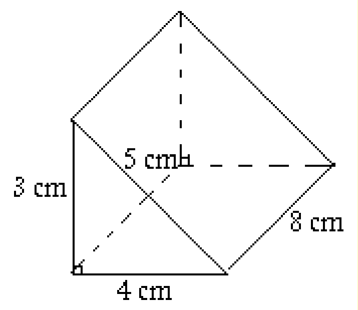


To find the surface area of a prism, the following steps should be followed.

1. Identify the shape of the face, for example, square, rectangle, triangle, parallelogram and trapezium.
2. Find the area of each face using a suitable formula.
3. Add the areas of all the faces together.

Example

Calculate the surface area of the prism below.



Solution

Area of triangular faces = $2\left(\frac{1}{2}\times 4\times 3\right)$ cm^2

= 12 cm^2

Area of rectangular faces

= $(4 \times 8) + (5 \times 8) + (3 \times 8)$

= 32 + 40 + 24

$= 96\ cm^2$

Total surface area $= (96 + 12)\ cm^2$

$= 108\ cm^2$

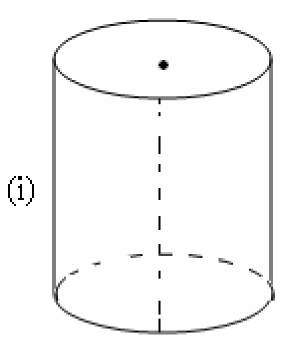
Surface area of a cylinder

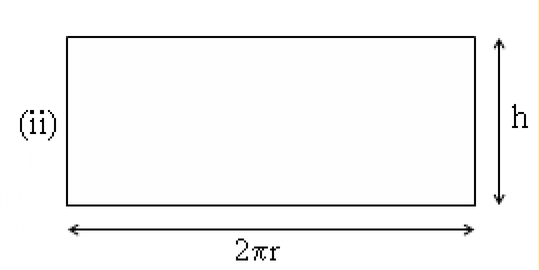
A cylinder is a prism with a circular cross-section. In the figure below we notice that a cylinder has one curved surface and two circular surfaces at the two ends. Assume the height and radius of the cylinder are *h* and *r* respectively.

When the two circular faces of a cylinder are removed and then the curved surface is cut along the vertical dotted line, see figure (i), a rectangular face is obtained and its measurements are shown in figure (ii). Note that the circumference of the circular edge becomes the length of the rectangular surface, hence our representation of the length as $2\pi r$, the formula for the circumference of a circle.

The surface area of the curved surface $= 2\pi r \times h$.

And the area of the circular ends $= 2 \times \pi r^2$





The surface area of a closed cylinder $= 2\pi rh + 2\pi r^2$

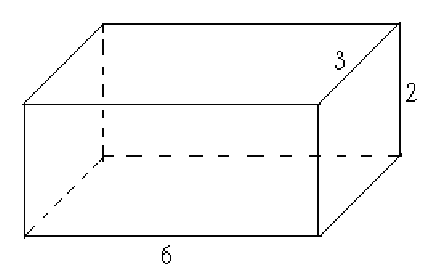
The surface area of an open cylinder (with one end closed)

$= 2\pi rh + \pi r^2$.

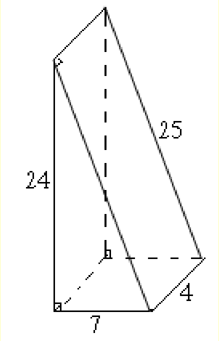
Exercise 7.2 D

1. Calculate the surface areas of each of the following shapes. (All measurements are in cm).

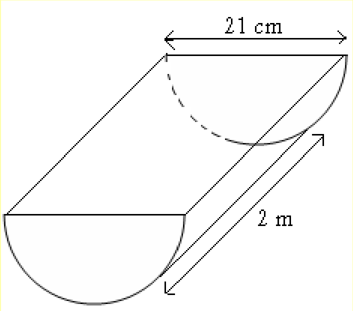
(a).



(b).



2. A water gutter was made from a thin sheet of metal in the shape shown below. Calculate the area of the sheet of metal used.



3. An open cylindrical tank has a diameter of 4.2 m and a height of 2 m. Calculate the surface area of the curved surface.

4. The height of a cylindrical tin is 15 cm. The area of the curved surface is 330 cm^2. Find the radius of the tin.

5. A large-scale farmer spent sh. 200,000 on land preparation per square kilometer. If the farmer planted wheat on 0.2 km^2, maize on 0.2 km^2 and barley on 0.1 km^2, calculate the amount of money spent by the farmer.
6. A rectangular block is 6cm long, 3cm wide and 2cm high. Find the total area of the six faces.

7. The width of a rectangle is 7cm and the perimeter is 38cm. Find the length of the rectangle and its area.

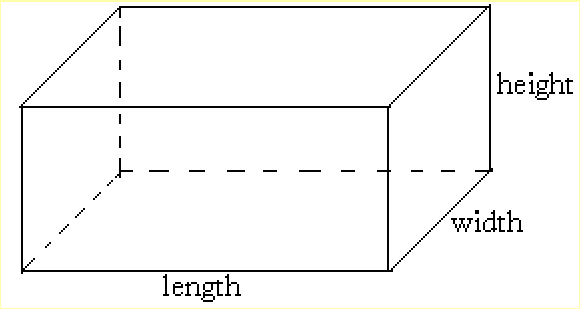
8. The perimeter of a square is 48m. Find its area.
9. A rectangle is 3x m long and 7m wide. Find its area and perimeter.
10. A photograph 10cm by 8cm is mounted on a piece of cardboard which is 16cm by 12cm. Find the area of the margin round the photograph.

Volume and capacity

Volume

Volume is the amount of space occupied by a shape or how much space is contained within the shape

Volume of a cuboid

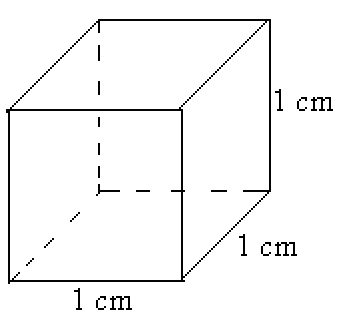


Volume = length × breadth × height.

Note that the units used for the three dimensions must be the same.

Converting units of volume

The units of volume are based on the units of length, that is mm, cm, m, and km.
Hence the corresponding units if volume are mm^3, cm^3, m^3, and km^3 respectively.
Note that a cube of side 1 cm has a volume of 1 cm^3, that is,



$$\text{Volume} = 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 1\text{ cm}^3$$

To convert cubic centimeters to cubic meters, remember that:

$$1\text{ cm} = 10\text{ mm}$$
$$1\text{ cm}^3 = 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$$
$$= 10\text{ mm} \times 10\text{mm} \times 10\text{mm} = (10 \times 10 \times 10)\text{ mm}^3$$
$$\therefore 1\text{cm}^3 = 1000\text{ mm}^3$$

Similarly, to convert cubic meters to cubic centimeters:

$$1\text{ m} = 100\text{ cm}$$
$$1\text{ m}^3 = 1\text{ m} \times 1\text{ m} \times 1\text{ m}$$
$$= 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}$$
$$= 1000\,000\text{ cm}^3.$$

Example 1

Convert the following measurements to cm^3.
(a) 20 mm^3 (b) 0.015 m^3

Solution

(a) $1000\text{ mm}^3 = 1\text{ cm}^3$
$20\text{ mm}^3 = \frac{20}{1000}cm^3$
$= 0.02\text{ cm}^3$

(b) $1\text{ m}^3 = 1000000\text{ cm}^3$

$0.015\text{ m}^3 = 0.015 \times 1000000\text{ cm}^3$
$= 15\,000\text{ cm}^3$

Remember: *To convert from larger units to smaller units we multiply, while to convert from smaller units to larger units we divide.*

Example 2

Find the volume of a cuboid 8 cm by 7 cm by 3 cm.
Volume $= 8 \times 7\times 3\text{cm}^3$
$= 168\text{ cm}^3$.

Example 3

Calculate the volume of a cuboid measuring 2 m by 50 cm by 110 mm. Give your answer in : (a) cm^3 (b) m^3

Solution

(a) Change all measurements to cm:
Length = 2 m = 2 × 100 cm = 200 cm
Width = 50 cm
Height = 110 mm = 11 cm
Volume $= 200 \times 50 \times 11\text{ cm}^3 = 110\,000\text{ cm}^3$

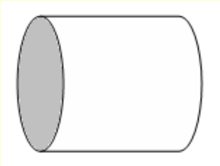
(b) Change all the measurements to metres:
Length = 2 m
Width $= 50 \div 100\text{ m} = 0.5\text{ m}$
Height $= 110 \div 1000\text{ m} = 0.11\text{ m}$
Volume $= 2 \times 0.5 \times 0.11\text{ m}^3 = 0.11\text{ m}^3$

Note that the answer in (b) can also be obtained from the answer in

(a) thus, $1\ m^3 = 1000\ 000\ cm^3$
$110\ 000\ cm^3 \div 1000\ 000 = 0.11\ m^3$

Volume of a cylinder

A cylinder is like a prism with a circular cross-section or it may simply betaken as a circular prism. In the figure below the cross-section of the cylinder is shaded.



Volume = area of cross-section × length.
= area of circular end × height.

However, cylinders usually stand on their base and therefore, we often use height rather than length, that is, the volume of a cylinder with a circular end, radius r and height h is given by the formula,
volume = area of circular end × height
$= \pi r^2 h$.

Example 4

Find the volume of a cylinder whose radius is 7 cm and height 5 cm.

Solution

Area of cross-section = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ cm^2$

Volume $= 154\ cm^2 \times 5\ cm = 770\ cm^3$

The volume of a pyramid

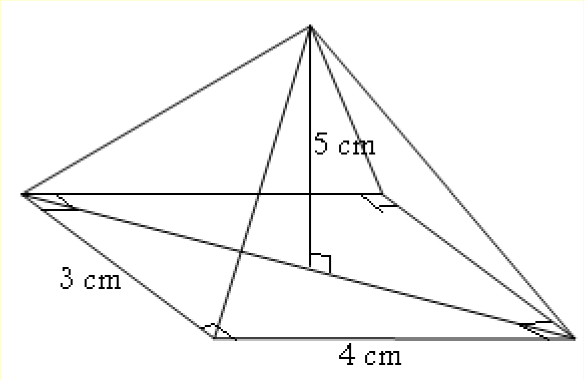
Since there are three pyramids that make up a cube, then the volume of each Pyramid is

$$V = \frac{1}{3} \times \text{ base area} \times \text{perpendicular height}$$

The same formula is used to find the volume of a pyramid with a base of any shape. *The height of the pyramid must be vertical from the vertex and perpendicular to the horizontal base.*

Example 5

Find the volume of the pyramid given below.



Solution
Area of the base = 4 × 3 = 12 cm^2. Perpendicular height = 5 cm

Volume = $\frac{1}{3}$ × 12 × 5 = 20 cm^2.

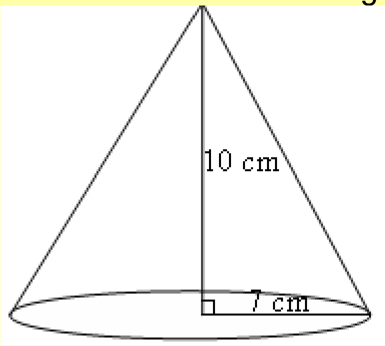
The volume of a cone

A solid cone is a pyramid with a curved surface and a circular base.

The volume of a cone = $\frac{1}{3}$ × base area × height = $\frac{1}{3}\pi r^2 h$.

Example

Find the volume of the cone in the figure below.



Solution

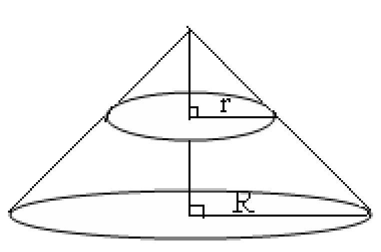
The base area = $\frac{22}{7}\times 7\times 7$ = 154 cm^2; height = 10 cm

Volume = 154 × 10 = 1540 cm^3

Note: The surface area of a solid cone = area of curved surface + area of circular base and is given by the formula:
S.A = $\pi rl + \pi r^2 = \pi r(l + r)$, where l is the slant height and r is the base radius.

The volume of a frustum

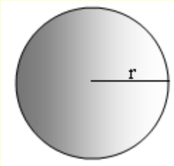
When a sector of a cone is cut off in such a way that the cut is parallel to the base, the remaining sector between the cut and the base is called a **frustum**.



The part cut off at the top is similar to the original cone. The volume of the frustum is found by subtracting the volume of the part cut off from the volume of the original cone.

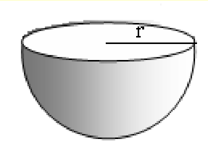
The volume of a sphere

The volume of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$



Volume of a hemisphere of radius, r, is given by the formula

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$



Capacity

The capacity of a container is the volume of liquid it can hold. There are special units for measuring capacity and the most common is the litre (l). Others include: centiliter, milliliter etc.

conversion of units of capacity.

10 millilitres (ml) = 1 centilitre (cl)
10 cl = 1 dl
10 dl = 1 litre (l)
1000 ml = 1 litre

Since capacity can be obtained from volume, there is a relationship between the units of measuring capacity and the units of measuring volume, thus:

1 litre = 1000 cm^3
1 millilitre = 1 cm^3
1000 litres = 1 m^3

Example 5

How many litres of milk will a tank measuring 3m long, 2 m wide and 1 m deep hold?
(Assuming the measurements given are internal measurements).

Solution

Volume = 6 m^3
1 m^3 = (100 × 100 × 100) cm^3
6 m^3 = 6 000 000 cm^3
1000 cm^3 = 1 litre
6 000 000 cm^3 = $\frac{6000000}{1000} \times 1l$ = 6 000 l

Exercise 7.2 E

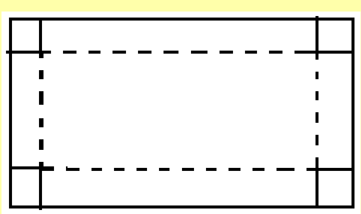
1. What is the height of a rectangular block which is 5m long, 3m wide and has a volume of 30 m^3?

2. A tank is 2.5 m long and 1.8 m wide. How much does the water level rise if 540 litres of water are poured in?

3. A rectangular tank 2 m long and 1.62 m wide contains water to a depth of 75 cm. The water is transferred to an empty tank which is 1.5 m long and 1.2 m wide. Find the depth of the water.

4. A classroom is 9.6 m long, 8.4 m wide and 3 m high. Find how many pupils can be accommodated in the room if each is allowed 0.5 m^3 of air space.

5. A sheet of cardboard is a rectangle 24 cm by 16 cm. Equal squares of side 4 cm are cut out of each corner and a tray is formed by folding along the dotted lines. Find in cm^3, the capacity of the tray.



6. A bottle contains 2 litres of water. How many cubic centimeters of water remain when 715 cm^3 is poured out?
7. A drum of uniform cross-section holds 240 litres of petrol. The area of the cross-section is 1200 cm^2. Calculate the height of the drum in metres.

8. A tank is 2m long, 1.5 m wide and 2 m deep. It contains water to a depth of 25 cm. How many litres of water should be added to make the tank half full?

9. A bottle contains exactly 1 litre of water. All the water is poured into a rectangular container that is 10 cm long, 8.4 cm wide and 9 cm high. Will any of the water spill over, and if so, how much?

10. A tank has a base area of 1.2 m^2. Water flows from a tap into the tank at the rate of 60 litres per minute. What will be the height of the water in the tank after 60 minutes?

11. One litre of milk is poured into a cylindrical jug of radius 6 cm. Calculate the depth of the milk in the jug.

12. Find the volume of the soil removed after digging a water tank that is 4 m deep, 3 m long and 2.5 m wide.

13. Find the volume of the plastic used to make a pipe 3 m long with internal and external radii of 7 cm and 8 cm respectively. Take $\pi = 3.14$.

14. A tin has a square base whose sides measure 20 cm. The vertical sides are 5 cm high. Cake mixture is poured into the tin and leveled off at the 3 cm mark above the base. Find the:
    (a) capacity of the tin.
    (b) volume of the cake mixture in the tin.

15. Find the height of a rectangular room that measures 4.7 m long and 3.2 m wide, if the space within it is 33.84 m^3.

8 LINEAR RELATIONS

8.1 Introduction

In chapter 3 we looked at equations like: $y = 5$, $y = x + 1$ etc. If we draw a graph of an equation and we get a straight line then the relation is called a **linear relation**.

A relation may be expressed in any one of the following four ways:

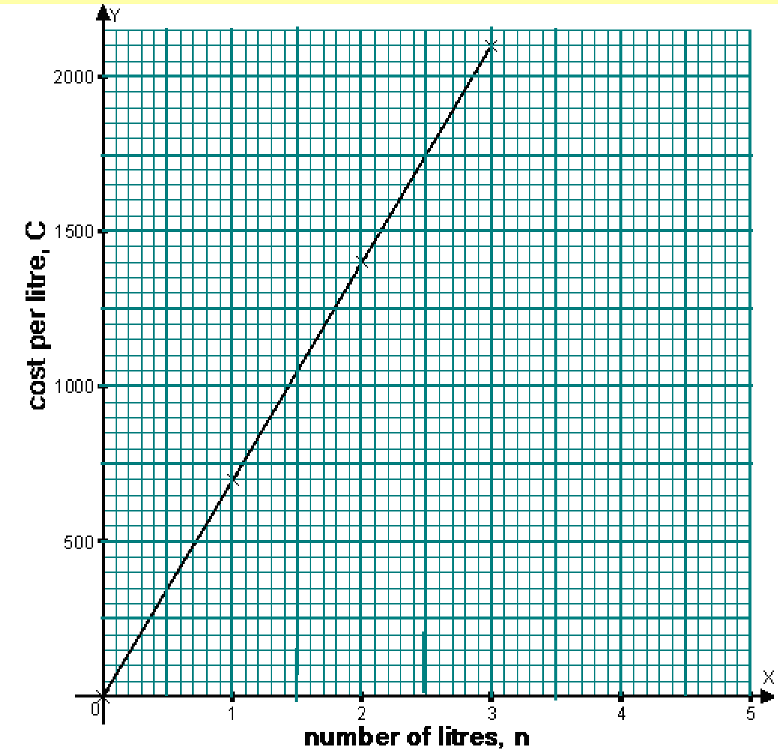
1. Using words. E.g.: the cost Sh C of n litres of paraffin at Sh 700 per litre equals seven hundred multiplied by n.

2. Using a formula or an equation: $C = 700n$.
   Note: symbols like sh., cm, kg etc are not included in a formula or equation.

3. Using a table of values
   e.g.

n	0	1	2	3
c	0	700	1400	2100

4. Using a graph. This is drawn by plotting the pairs of values in the table above as coordinates: (0, 0); (1,700); (2, 1400); (3, 2100). We plot these points on a pair of axes. This gives the graph representing the relation.



Exercise 8.1

1. Copy and complete the tables using the given relation
   (a) $p = 4n$

n	0	1	2	3	4
p	0	-	-	12	-

   (b) $m = 3d - 2$

d	0	1	2	3	4
m	-2	-	-	-	10

2. Use the values in the table to find the relation between the given quantities. Express the relation as an equation. Make a list of the coordinates given by the pairs of values in the table. Draw a graph representing the relation.
   (a)

a	0	1	2	3	4
b	0	3	6	9	12

   (b)

c	0	1	2	3	4
d	0	400	800	1200	1600

(c)

x	0	1	2	3	4
y	1	3	5	7	9

3. Express each relation using an equation:

(a) The distance traveled d (km) by a train equals the product of the average speed 40 km/h and the time t (hours).

(b) The temperature F degrees Fahrenheit equal nine – fifth of the temperature C degrees Celsius plus thirty two.

4. (a) A tank has 2000 liters of water in it. More water begins to flow in at a rate of 80 liters per minute. How much water is there in the tank after:

(i) 5 minutes

(ii) 10 minutes.

(b) Make an equation for the volume V liters of water in the tank when more has been flowing in for t minutes.

8.2 Graphs of linear relations

Consider the graph showing the cost of paraffin. Using the graph, find the cost of 4 liters of paraffin.

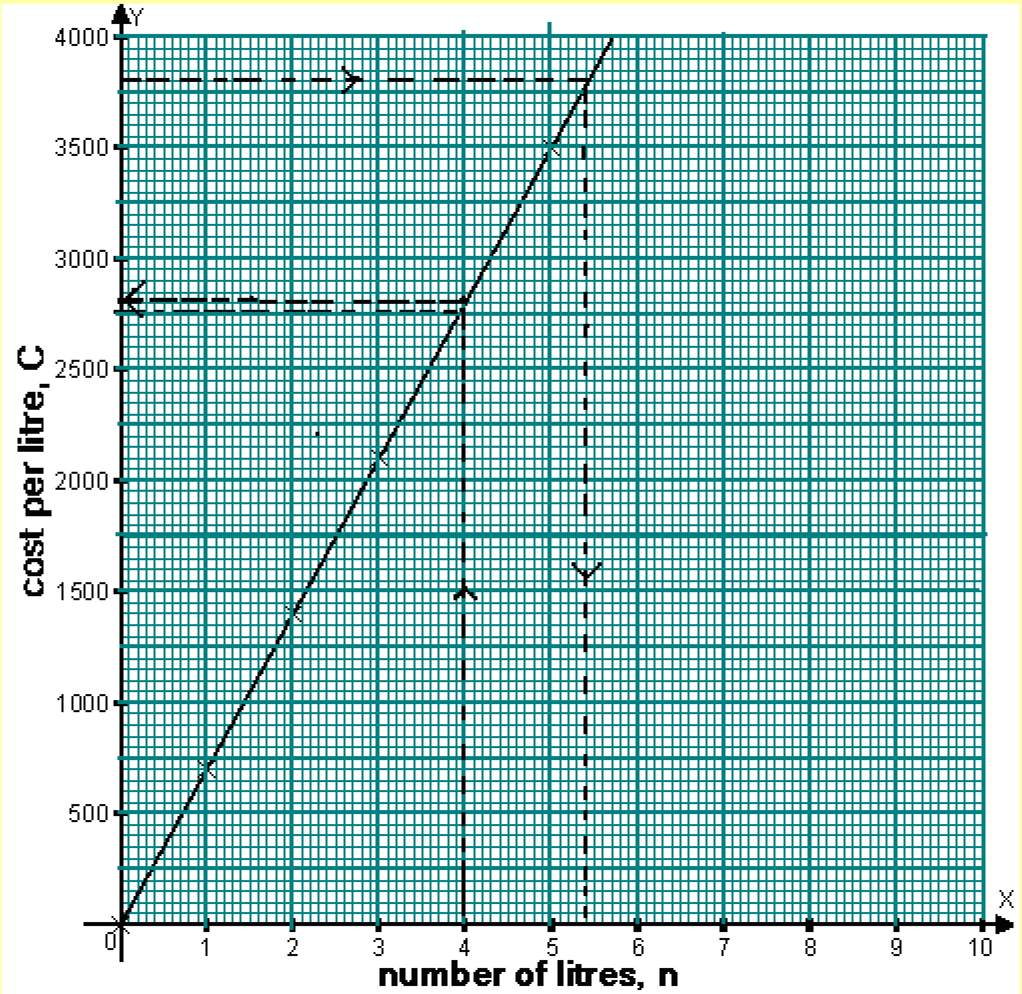
Solution

Start at 4 (the number of liters). Go (vertically) up to the line. Go horizontally across to the axis. Read the cost.
Check that you read sh. 2800.

Using the graph, find how many liters of paraffin you get for sh3, 800.

Solution

Start at 3,800 (the cost), go (horizontally) across to the line. Go (vertically) down to the axis. Read the number of liters. Check that you read 5.4 liters.



Exercise 8.2

1. Copy and complete the table below for the relation $q = 3p - 5$.

p	-5	0	5	10	15	20	25	30
q	-20	-	10	-	40	-	-	85

2. Draw a graph of $q = 3p - 5$ for values of p from -5 to 30. Take 1 cm to 5 units for each values of p and 1 cm to 10 units for the values of q. Use your graph to find
   (a) the value of q when p is: (i) 18 (ii) 23 (iii) -3.
   (b) the value of p when q is: (i) 34 (ii) 4 (iii) 76.

8.21 Distance - Time graphs

In a distance-time graph the distance is represented on the vertical axis and the time on the horizontal axis. However, before plotting any point, we must choose suitable scales for both axes. A suitable scale is the scale that will enable you to fit in all your points adequately. Also make sure that the intervals are uniform on each axis.

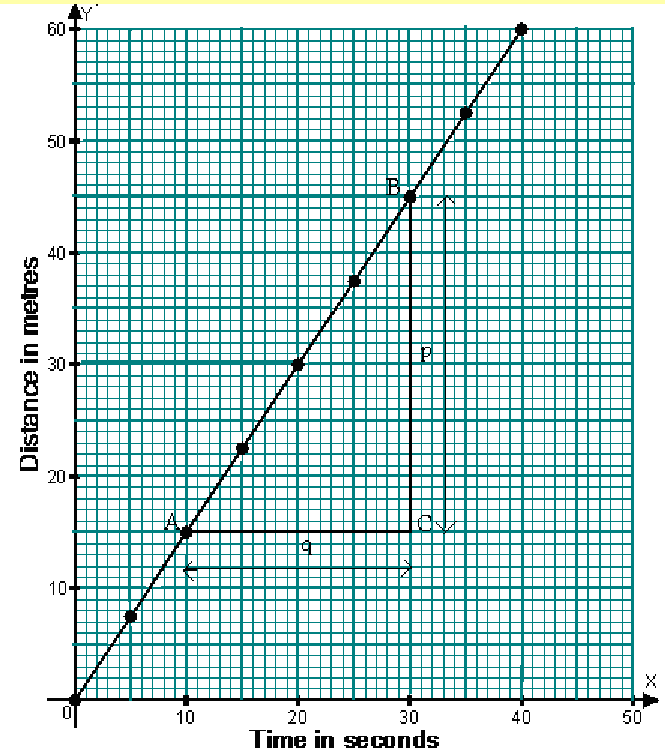
Example 1.

The table below shows the distance, in metres, walked along a road and the time taken, in seconds.

Distance (m)	Time (s)
0	0
7.5	5
15.0	10
22.5	15
30.0	20
37.5	25
45.0	30
52.5	35
60.0	40

(a) Draw a distance-time graph to represent this information.
(b) Determine the gradient of the graph.

Solution



(b) Choose two suitable points on the line. For example, A and B. Draw a line from point B parallel to the vertical axis and another line from point A parallel to the horizontal axis both to meet at point C. Label length BC and AC as p and q respectively.

Gradient = $\frac{p}{q}$

$= \frac{45-15}{30-10} = \frac{30}{20} = 1.5$ m/s.

(The gradient represents speed).

No matter which two points we take, the gradient is the same. This means that the speed is constant (uniform). Also if the vertical axis represents displacement, then the gradient would represent velocity.

Example 2

In a bicycle race, a cyclist covered 70 km as follows: 30 km in 30 minutes, 10 km in $1\frac{1}{2}$ hours and 30 km in 30 minutes.

(a) Draw a distance-time graph for the journey.

(b) From the graph, determine the average speed for the whole journey.

Solution

(a) In each stage of the journey, we need two points. That is the start and the end of each stage.

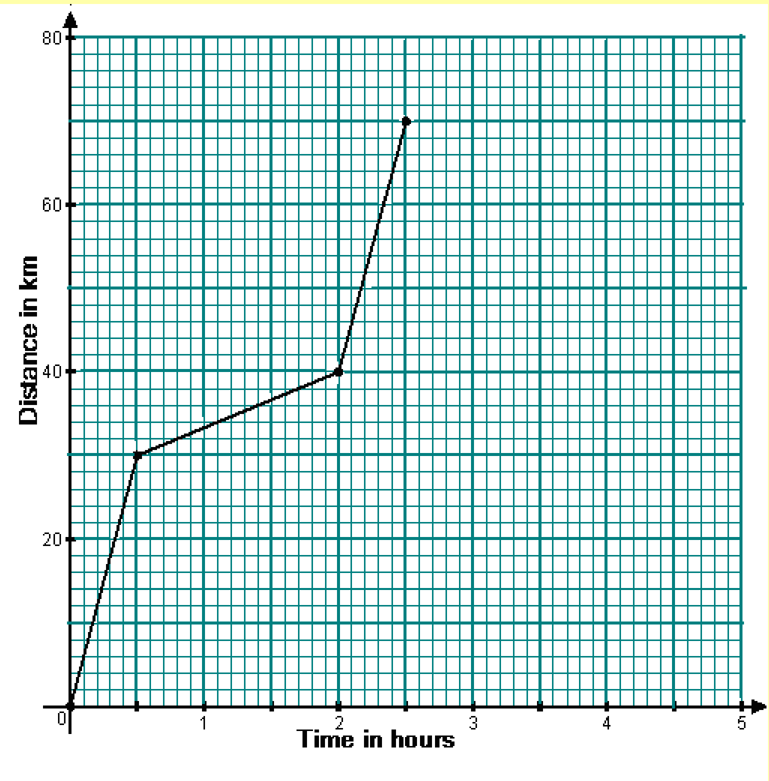
Stage I: (0, 0), $\left(\frac{1}{2}, 30\right)$

Stage II: $\left(\frac{1}{2}, 30\right)$, (2, 40)

Stage III: (2, 40) , ($2\frac{1}{2}$, 70)

Choose a suitable scale for each axis. In this case, in the vertical scale, 1 cm represents 20 km and in the horizontal scale, 1 cm represents 30 minutes. Draw the axes and plot the points.

Join the points in each stage with a straight line as shown in the figure below.



(b) The average speed is found by joining the first point of stage I to the last point of stage III. Then find the gradient of this line by choosing suitable points such as (0, 0) and $(1\frac{1}{4}, 35)$.

Gradient = (35 - 0) ÷ $(1\frac{1}{4} - 0)$ = 28 km/h.

Checking by calculation,

Average speed = $\frac{total\ distance}{total\ time}$ = 70 ÷ $\frac{5}{2}$ = 28 km/h.

Example 3

Two towns, X and Y, are 110 km apart. A cyclist, C_1, leaves Y at 9.00 a.m. and travels towards X at an average speed of 44 km/h. At the same time, another cyclist, C_2, leaves X and travels towards Y at an average speed of 25 km/h.

(a) On the same axes draw, distance-time graphs for each cyclist.

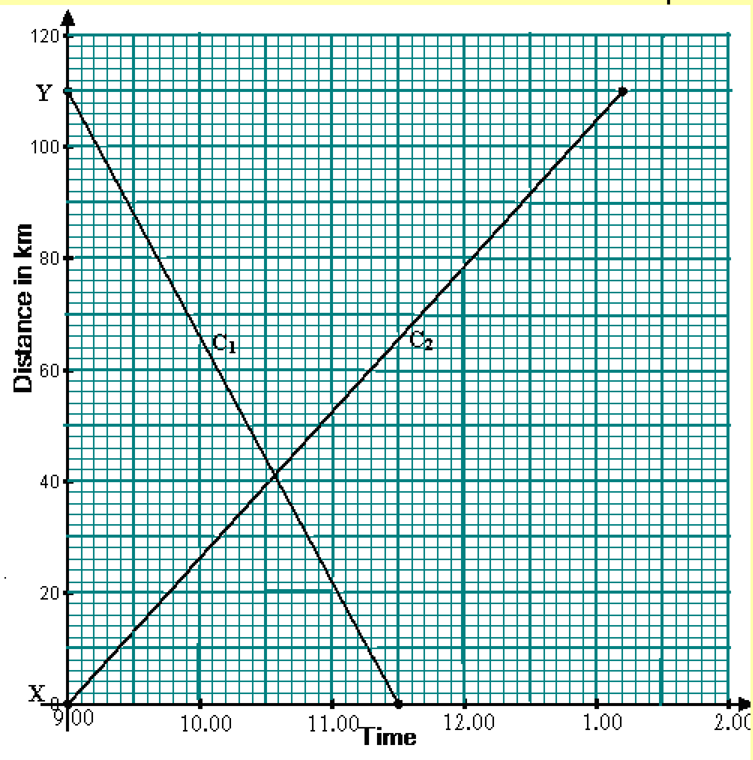
(b) From the graph determine:

(i) when the two cyclists met,

(ii) the distance cyclist C_1 had traveled before meeting cyclist C_2.

Solution

(a) Using suitable scales, mark on the vertical axis points X and Y at the correct distance. On the horizontal axis, mark the time starting from 9.00 a.m. then 10.00 a.m. etc. Since the speed for each motion is constant the graphs are straight lines. C_1 takes 2 hours and 30 minutes and arrives at X at 11.30 a.m. C_2 takes 4 hours and 12 minutes. She arrives at Y at 1.12 p.m.



(b) (i) They meet at 10.36 a.m.

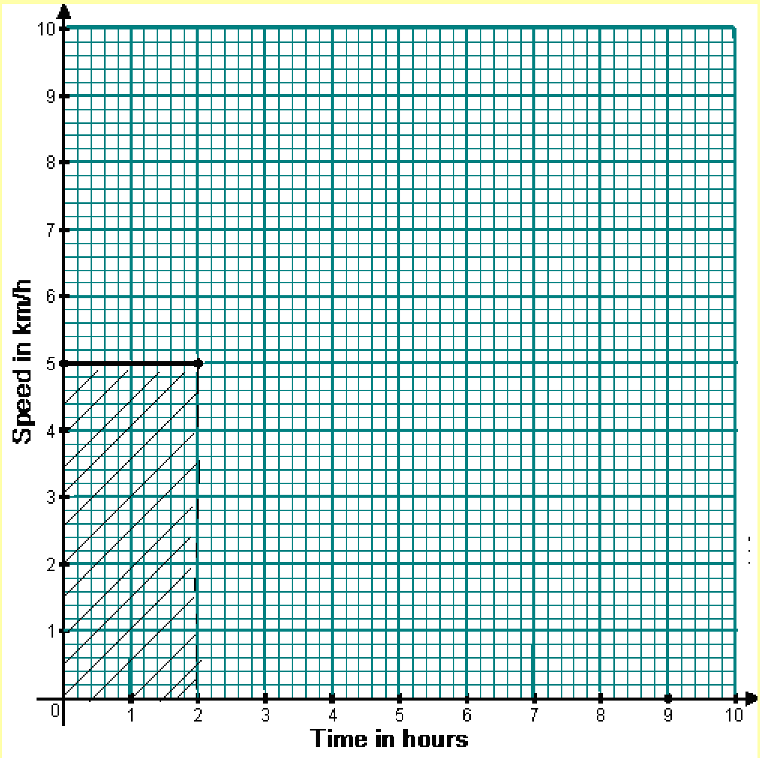
(ii) C_1 had covered 70 km from Y which is the intersection of graphs C_1 and C_2.

Speed-time graphs

In speed-time graphs, speed is represented on the vertical axis and time on the horizontal axis. If the motion is in a specific direction then velocity should be represented on the vertical axis.

Example 1

A man walking at a steady speed covers a distance of 10 km in 2 hours. We therefore say that his speed is 5 km/h.
Note: steady speed means that the speed stayed the same or constant.



The graph shows that speed remained constant. The graph is called a **speed - Time graph**.

Since the speed is constant, the distance traveled = speed $\times$ time

= $5 \times 2 = 10$ km.

If we shade the graph as in the figure above, we notice that the shaded area is equal to the distance traveled by the man in 2 hours.

Area of a rectangle = length $\times$ width

=2 h $\times$ 5 km/h

= 10 km/h.

In general, in speed-time graphs, the distance traveled is obtained by finding the area enclosed by the graph and the axes.

Exercise 8.3

1. An athlete sets off in a long distance race at a steady speed of 4 m/s. Draw a graph to show the distance *d* meters he has covered after *t* seconds, taking values of *t* from 0 to 25. Use scales of 2 cm to 5 seconds and 2 cm to 20 meters. From your graph find:

   (a) the distance covered after: (i) 5 sec. (ii) 22 sec. (iii) 10.5 sec.

   (b) the time taken to run: (i) 40 m (ii) 70 m (iii) 36 m

   (c) Give an equation connecting *d* and *t.*

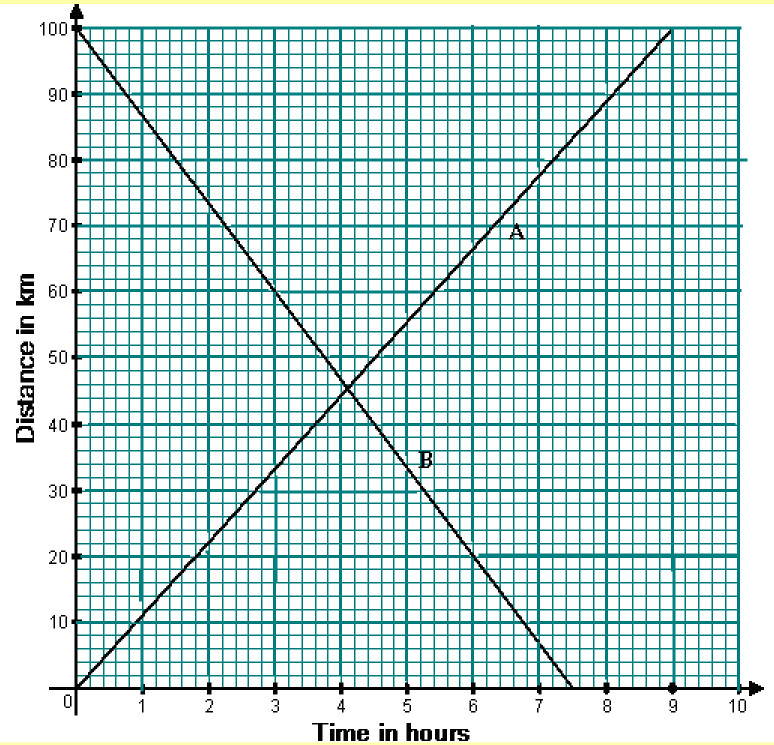
2. Dauda left school at 4.00 pm and walked home at 6 km/h.
   (a) How far from the school was he after
   (i) 30 minutes (ii) 20 minutes (iii) 10 minutes?
   (b) Draw a distance – time graph extending from 4.00 pm to 5.00 pm. Take 2 cm to 10 minutes and 2 cm to 1 km.
   (c) Use the graph to find:
   (i) the distance Dauda had walked by 4.40 pm.
   (ii) the time he reached home if he lived 4.5 km from the school.

3. From Kampala to Jinja is 80 km. At 08:15 hours a lorry leaves Kampala for Jinja at 60 km/h. Draw a graph showing the distance traveled with respect to time. Use scales of 1 cm to 5 minutes and 1 cm to 10 km. From your graph find:
   (a) how far from Kampala the lorry is when the time is:
   (i) 08:20 hours (ii) 08:50 hours (iii) 09:15 hours.
   (b) the time when the lorry reaches Jinja.

4. In a 100-m race, an athlete took 9.8 seconds.
   (a) Draw a distance-time graph to represent this information.
   (b) From the graph determine:
   (i) the average velocity correct to 1 decimal place,
   (ii) the distance traveled in three seconds,
   (iii) the time taken to cover 80 m.

5. A safari rally car travels at an average speed of 180 km/h for 5 hours between two towns.
   (a) Draw a distance-time graph to illustrate its motion.
   (b) From the graph determine:
   (i) the distance traveled in 36 minutes,
   (ii) the time it took to cover 290 km.

6. From the distance-time graph drawn below, calculate
   (a) the velocity, in km/h, of A and B respectively,
   (b) how
   (i) long they took before meeting
   (ii) far they had traveled before meeting.



7. Rashidah left her house at 8.00 am for Kampala city. After traveling at a constant speed for 1 hour, she arrived at Mukono town, 15 km from her home. She rested for 15 minutes and then proceeded at a constant speed towards Kampala 30 km from Mukono, she arrived in Kampala at 12.15 p.m.
   (a) Using a scale of 2 cm represents 1 hour and 1 cm represents 5 km draw a distance-time graph for the whole journey.
   (b) From the graph, determine Rashidah's average speed from:
      (i) her house to Mukono town,
      (ii) Mukono town to Kampala city.
   (c) Calculate her average speed for the whole journey.

8. Three towns, R, P and Q are such that Q is between P and R. From P to Q is 105 km and from Q to R is 15 km. A car leaves Q at 9.00 a.m. and travels towards P at an average speed of 60 km/h. At the same instant, a cyclist leaves P and travels towards Q at an average speed of 24 km/h. An ambulance leaves R at 9.30 a.m. and travels towards P via Q at an average speed of 160 km/h.
   (a) On the same axes, draw a distance-time graph for each vehicle.
   (b) From the graph determine:
      (i) when the ambulance caught up with the car,

(ii) when the ambulance and the cyclist met,

(iii) when the car and the cyclist met,

(iv) the distance the car had travelled before the ambulance caught up with it.

9 REFLECTION

9.1 Geometric transformations

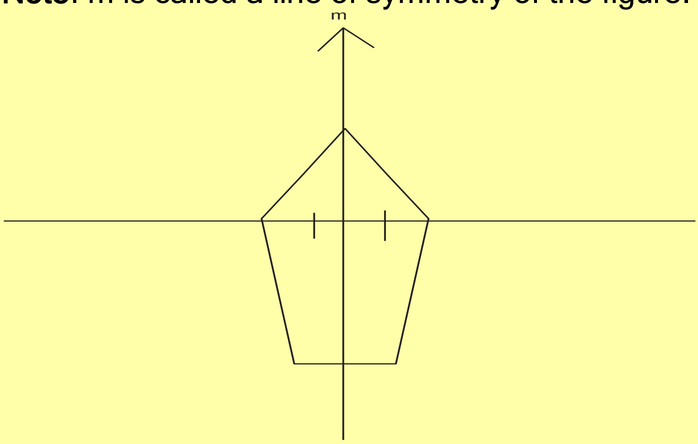
A geometric transformation is an operation that changes the position and appearance of a geometric figure.

Line symmetry.

A figure has line symmetry if:

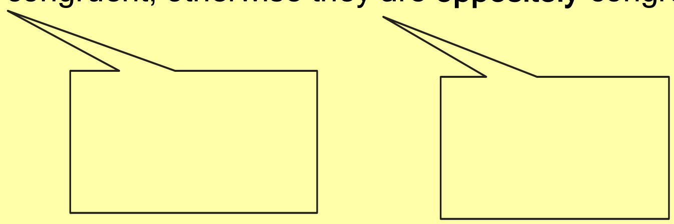
(i) both sides of the line are the same size and shape.
(ii) Any two opposite points are the same distance from the fold line.
(iii) The line joining any two points is perpendicular to the fold.

Note: m is called a line of symmetry of the figure.



Note (a) Two shapes are congruent if they are the same size and shape.

(b) If we can slide and fit one figure onto the other without turning over one of them, the figures are **directly** congruent; otherwise they are **oppositely** congruent.



9.2 Reflection in a plane mirror

- A **reflection** is a transformation such that any two corresponding points in the object and the image are both the same distance from a fixed line. Any line joining these points is perpendicular to the fixed line.
- A **plane mirror** is a normal flat mirror that gives an accurate reflection.

- The figure we are reflecting is called the **object** and its reflection the **image**.

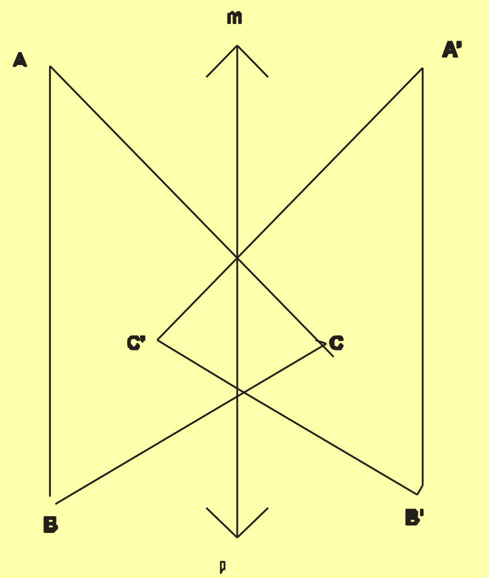
- The line along which the edge of the mirror rests is called the **mirror line**.

- Under reflection, objects and images are symmetric. The mirror line is the line of symmetry.

- An object point and its image are the same distance from the mirror line.

- Reflection alters direction except for lines parallel to the mirror line.

- Points which are on the mirror line are their own image.

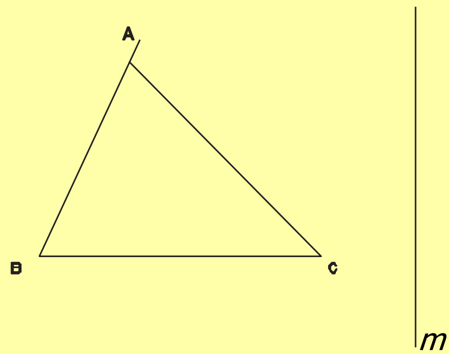


Constructing images of objects

In order to construct the image of an object under reflection, we use the properties of reflection.

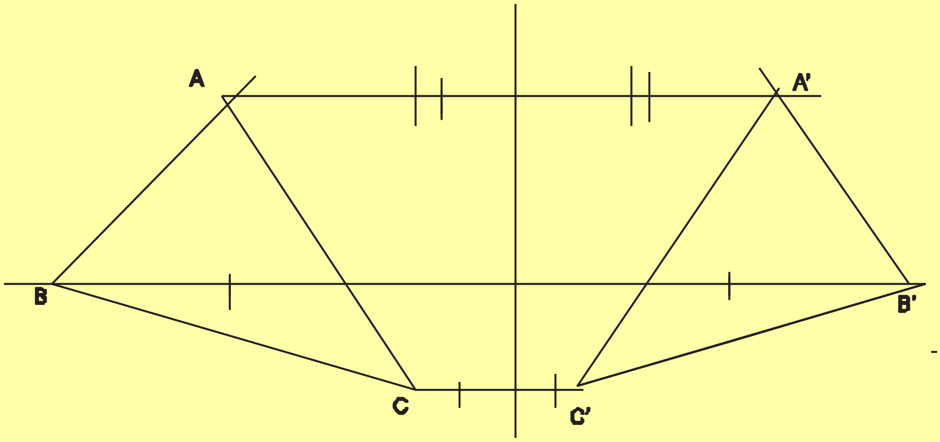
Example 1

Trace the diagram below and draw the image of triangle ABC after reflection in the given mirror line *m*.



Solution

Through the three vertices of the triangle ABC we construct lines perpendicular to *m*. Then across the mirror line we mark off equal distances on both sides of the mirror line. This will give us the positions of the image points A', B', and C'.



9.3 Reflecting coordinates

Example 2.

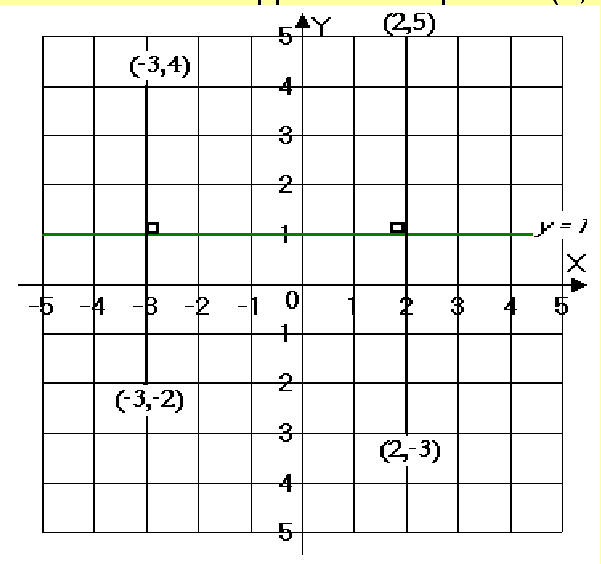
(a) Find the image of the point (-3, 4) under reflection in the y = 1.
(b) After reflection in the line y = 1, the image of a point is (2, 5). What was the point?

Solution

Draw a graph of the line y = 1.

(a) Plot the point (-3, 4). Draw and extend a perpendicular to the line from the point. Measure an equal distance on both sides of the line. The image point is (-3,-2)

(b) Plot the point (2, 5). This is the image, but the original point must also be opposite. The point is (2,-3).



Example 3.

Find the images of the following points under reflection in the line y = x + 1: (3, 1); (-1,-3); (0, 0); (3, 4).

Solution

Draw a graph of y = x + 1. For each point draw and extend a perpendicular. Measure an equal distance on both sides of the line.

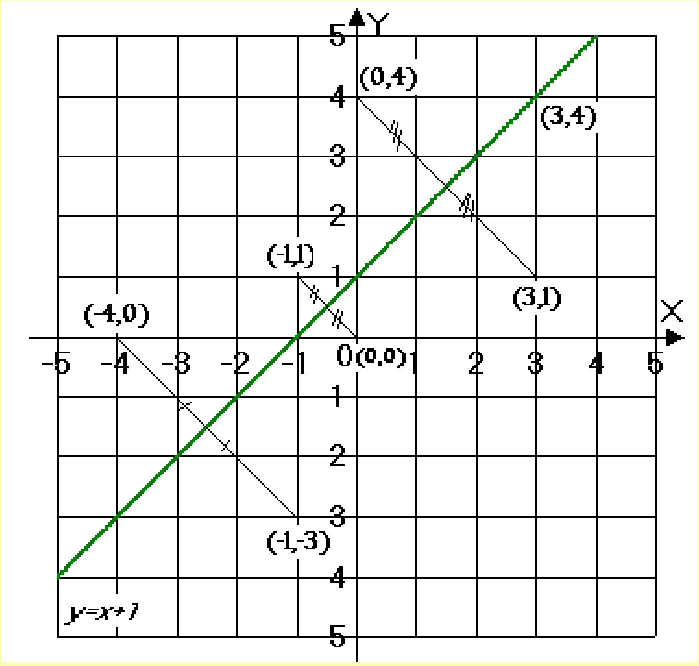
The image of (3, 1) is (0, 4)

The image of (-1,-3) is (-4, 0)

The image of (0, 0) is (-1, 1)

The image of (3, 4) is (3, 4).

Note: A point which lies on the mirror line will not move when it is reflected.



Exercise 9.1

1. Find the image of the point (5, 2) under reflection in the y axis.
2. Find the image of point (-1, 2) under reflection in the line x = 2.
3. After a point has been reflected in the x axis, its image is at (3, 2). Find the coordinates of the object point.
4. The point P (-2, 4) is reflected in the line x = 0. Find the coordinates for P' the image of P.
5. The points A (4, 2) and B (1, 3) are reflected in the line y = x. Find the coordinates of A' and B', the images of A and B.
6. A reflection maps the point (5, 5) onto the point (1, 5). Find the equation of the mirror line.
7. A (3, 3); B (3, 1); C (5, 1) and D (5, 3) are the vertices of a square ABCD. On the same axes draw ABCD and its image A'B'C'D' under reflection in the line x = 2. State the coordinates of A', B', C' and D'.
8. Find the image of each of the following points after a reflection in the lines:
   (a) y = x (b) y + x = 0
   (i) (4, 4) (ii) (3, 1) (iii) (-5, 5) (iv) (-4, 6)

9. Find the equation of the mirror line in each of the following:
   (a) P (4, 3) is mapped onto P_1(8, 3)
   (b) Q (6, -2) is mapped onto Q_1 (6, -2)
   (c) R (-9, 3) is mapped onto R_1 (-1, -3)
   (d) S (3, 2) is mapped onto S_1 (-3, 8).

10 BRACKETS

10.1 Use of brackets:

Brackets are used to show the order in which operations are intended to be done. Thus
(x + 3) means the number obtained by adding 3 to x.
6(5 + 2) means 6× (5 +2) or 6 × 7.
4 × (2 + 3) = 4 × 5 = 20.

When an expression in a bracket is multiplied by a number, in order to remove the bracket, every term inside must be multiplied by that number. Thus
P(x + y) means px + py
P(x – y) means px – py.

To see what rules we must follow when removing brackets, let us consider these illustrations:

(i) 9 + (5 + 3). This means: add 3 to 5, and add the result to 9. The same answer would be obtained if we added 5 to 9 and then added 3 to the sum.
Therefore 9 + (5 + 3) = 9 + 5 + 3.

(ii) 9 – (5 + 3). Means: add 3 to 5, and subtract the result from 9. This is the same as subtracting 5 from 9and then subtracting 3 from the difference. Therefore,
9 – (5 + 3) = 9 – 5 – 3.

(iii) 9 + (5 – 3). Means: subtract 3 from 5, and add the result to 9. This is the same as adding 5 to 9 and then subtracting 3 from the sum. Therefore, 9 + (5 – 3) = 9 + 5 – 3.

(iv) 9 – (5 – 3) means: subtract 3 from 5, and subtract the difference from 9. If 5 were subtracted from 9, we should be subtracting too much, and should require adding 3 to make up for it.
Therefore 9 – (5 – 3) = 9 – 5 + 3.

The rules for the removal of brackets are therefore:

1 If a bracket has a + sign in front of it, the signs of the terms inside the bracket remain unchanged.
2. If a bracket has a – sign in front of it, a + sign inside the bracket changes to – and – changes to +.

Note:

(i) In the absence of brackets, the operations of '*of*', '*multiplication*', and '*division*' should be performed before *addition* and *subtraction*. If an expression involves addition and subtraction only and no brackets are used, then the operations should be performed in the order in which they come, from left to right.**E.**g. 5 – 2 + 2 = 3 + 2 =5.

(ii) When we multiply an expression in brackets by a number, we usually omit the multiplication sign, i.e. $3 \times (15 + 6) = 3(15 + 6)$. These brackets can be removed by multiplying each term in the brackets by 3. This is called expansion.
Thus, $3(15 + 6) = 3 \times 15 + 3 \times 6$.
In general, $a(x + y) = ax + ay$
Similarly, $a(x - y) = ax - ay$.

(iii) Signs after the brackets do not affect the terms inside the brackets.

Example 1

(i) $2 - 2x - (x + 1) = 2 - 2x - x - 1$
$= 2 - 1 - 2x - x$
$= 1 - 3x$.

(ii) $2(5x - 1) - 4(2x - 3) = (10x - 2) - (8x - 12)$
$= 10x - 2 - 8x + 12$
$= 12 - 2 + 10x - 8x$
$= 10 + 2x$.

(iii) $a - (b - c + d) = a - b + c - d$.

Note:
When inserting brackets be careful when the brackets are inserted after a
'–' sign, as all the signs inside the bracket must be changed.

Example 2

$a - x - 2y + z = a - (x + 2y - z)$.
$2x - 4y - 3y = 2x - (4y + 3y) = 2x - 7y$.

Example 3

Simplify $2x(4x - 3) - 5(x + 1)$

Solution

$2x(4x - 3) - 5(x + 1) = (8x^2 - 6x) - (5x + 5)$.
$= 8x^2 - 6x - 5x - 5$.
$= 8x^2 - 11x - 5$.

You should multiply brackets by the number in front as the first step, and then remove brackets in a separate step to avoid making mistakes.

Example 4

Solve the equation $3(2x - 1) - 2(x + 1) = 4$.

Solution

$$\begin{aligned}3(2x - 1) - 2(x + 1) &= (6x - 3) - (2x + 2)\\ &= 6x - 3 - 2x - 2\\ &= 4x - 5.\end{aligned}$$

Therefore, $4x - 5 = 4$

$$\begin{aligned}4x &= 9\\ x &= \tfrac{9}{2}.\end{aligned}$$

Exercise 10.1

Remove brackets and simplify:

(i) $2(4x + 3y) - 3(2x + y)$

(ii) $4(a - b) + 3(b - a)$

(iii) $5(x - y) + 4(x + y)$

(iv) $-a(a + b)$

(v) $½ (2x - 6)$

(vi) $3(p + q) - 6(q - p)$

(vii) $\dfrac{3(x+1)-2(x-1)}{6}$

(viii) $\dfrac{a-(a-b)}{a}$

Solve the equations (No. ix – xiv):

(ix) $x - (6 - x) = 12$

(x) $3y = 4 - (1 + y)$

(xi) $2x + (3(x - 2) = 2x + 3$

(xii) $2(t + 2) + 3(t + 1) = t + 31$

(xiii) $3(y + 2) + 2(3y - 2) = 3y + 4$

(xiv) $x - 3[x - 4(x - 1) + 1]$. (HINT: Remove the inside bracket first.)

11 EQUATIONS AND INEQUALITIES

11.1 Equations

Mathematical statements with equal signs are called **equations**. For instance: "I'm thinking of a number, I add 3 and multiply by 5 to get 35. What is the number?"
Suppose the number I thought of is x.
"I add 3": $x + 3$
"I multiply by 5": $(x + 3) \times 5$ or $5(x + 3)$
"I get 35": $5(x + 3) = 35$.

Thus $5(x + 3) = 35$ is an equation. X is called the **unknown**. The process of finding what number x stands for is called **solving** the equation.
What we do when we solve an equation is to find what we call a root of the equation, i.e. a value of the unknown which makes the two sides of the equation the same.

With an equation we can do any of the following four things:

(i) Add the same number to each side
(ii) Subtract the same number from each side
(iii) Multiply both sides by the same number
(iv) Divide both sides by the same number.

For **example**, the equation obtained above was

$5(x + 3) = 35$

First divide by 5: $x + 3 = 35 \div 5$
$x + 3 = 7$
Then subtract 3: $x = 7 - 3$
$x = 4$.
So my number is 4.

Check: Add 3 to 4 gives 7. Multiply 7 by 5 gives 35, so 4 is the correct number.

NOTE: When solving a problem by algebra begin by taking a letter to stand for some unknown number, and state what units you are using.

Example

A man walks from his home to a neighboring town at 6 km per hour and then returns home by the same route at 5 km/h. He finds that the total journey takes 2 minutes longer than if he went there and back at 5 ½ km/h. Find the distance of the town from his home.

Solution

Let the total distance of the town from his home be x km.
The time to walk x km at 6 km/h = $\frac{x}{6}$ hours.

The time to walk x km at 5 km/h = $\frac{x}{5}$ hours.

The total time = $\left(\frac{x}{6}+\frac{x}{5}\right)$ hours.

The time to walk 2x km at 5 ½ km/h = $\frac{2x}{5\frac{1}{2}}h=\frac{4x}{11}h$. $\therefore \frac{x}{6}+\frac{x}{5}=\frac{4x}{11}+\frac{1}{30}$

(Notice that every term is a number of hours)
Multiply every term by 330, the L.C.M of 6, 5, 11, and 30.

55x + 66x = 120x + 11

121x = 120x + 11

Therefore x = 11. Therefore the distance of the town is 11 km.

Exercise 11.10

1. I'm thinking of a number. I subtract 6 and multiply by -4. The answer is -8. What is my number?
2. What is y if: (a) 3(y + 4) = 15. (b) 5(y + 3) = 55.
3. Find e if: (a) $\frac{(e+7)}{3}=4$ (b) $\frac{e-2}{4}=5$
4. Find g when: (a) 3(g – 1) = 21 (b) 12(g – 6) = 108
5. Find the value of x when: $\frac{x-8}{2}=19$
6. I think of a number and multiply it by 4; the result is the same as if I added 24 to the original number. Find the number.

7. Solve the equations: (a) 29 = 7p + 8 (b) ½ x + 5 = 13
   (c) 5 ½ = t + 1¼.

Solve the following problems by forming equations.

8. The sum of the angles of a polygon of n sides is (n – 2) 180^0. Find the number of sides if the sum is$1{,}260^0$.

9. A man, who is x years old now, has a son aged 5. In seven years the father will be 4 times as old as his son will be then. How old is the father now?

10. The perimeter of a rectangle is 44 cm. If the breadth is x cm and the length (x + 2) cm, find the length and the breadth.

11. A rod 30 m long is broken into two pieces, one of the length is x m and the other (x – 4) m. Find x.

12. I walked for x hours at a steady speed of 5 ½ km/h and then cycled for the same number of hours at a steady speed of 15 ½ km/h. I had then traveled 49 km altogether. Find x.

13. A father is 35 years old and his son is 11. In how many years will the father be twice as old as the son?

14. A boy cycles from P to Q at 20 km/h and returns at 16 km/h. The total journey takes 4 ½ hours. Find the distance from P to Q.

15. A boy starts out from a town A to cycle towards a town B, 90 km away at an average speed of 16 km/h. At the same moment a motorist leaves B and travels towards A at an average speed of 56 km/h. After how many hours do they meet?

16. A cyclist sets out along a certain road at an average speed of 16 km/h. Half an hour later a motorist starts from the same place to overtake him. If the motorist's average speed is 48 km/h, find how many kilometers he must go before he overtakes the cyclist.

NOTE:

A term can be moved from one side of the equation to the other side, provided the sign in front of it is altered.
If the equation contains brackets, these should be removed. If the equation contains fractions, multiply every term (on both sides) by the **LCM** of the denominators of the fractions, thus making the equation free from fractions.

Example 2

Solve the equation: 6(x + 4) – (2 – x) = 15.

Removing brackets, 6x + 24 – 2 + x = 15
Therefore 7x + 22 = 15

$$7x = 15 - 22$$
$$7x = -7$$
$$x = -1.$$

Example 3

Solve the equation: $\frac{3x}{2} - \frac{2(x-3)}{3} + 4 = 0$

Multiply each term by 6 (the L.C.M of 2 and 3);

$$\frac{6 \times 3x}{2} - \frac{6 \times 2(x-3)}{3} + 24 = 0$$

Therefore, 9x – 4(x – 3) + 24 = 0

$$
\begin{aligned}
9x - 4x + 12 + 24 &= 0 \\
5x + 36 &= 0 \\
5x &= -36 \\
x &= -7\tfrac{1}{5}
\end{aligned}
$$

Exercise 11.11

Solve the following equations.

1. $2x + 1 = 2(2x - 3) - 3(2x - 1)$
2. $2(3 - 2x) - (x + 3) = 2(x + 8) + 1$
3. $\frac{x+2}{2} + \frac{x}{3} = 1$
4. $3(x + 1) - 1 = x - 3$
5. $2(t - 7) = 6 + t/3$
6. $\frac{1}{2}(2z + 4) = \frac{1}{4}(z - 8)$
7. $\frac{y}{2} - \frac{y}{3} = \frac{1}{5}$
8. $\frac{(5x-3)}{8} + 1 = \frac{(4x-3)}{5}$
9. $\frac{x-1}{2} - \frac{x-3}{4} = \frac{1}{2}$
10. $4(x - 5) = 10 - 2(x + 3)$
11. $\frac{3x+2}{2} - \frac{3x-2}{3} = \frac{11}{4}$
12. $\frac{x-1}{3-2x} = \frac{3}{4}$

11.2 Inequalities

Statements with these signs: <, >, ≤ or ≥ are called inequalities.

< read: ‘is less than’

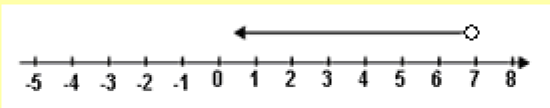
> read: ‘is greater than’

≤ read: ‘is less than or equal to’

≥ read: ‘is greater than or equal to’

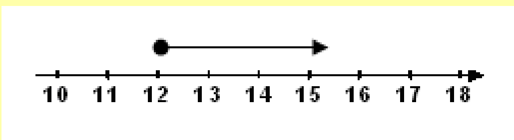
Example 1

I’m thinking of a number less than seven. If I call my number w, then $w < 7$.
This type of statement is called an inequality because the two sides are not equal. We can illustrate the inequality on a number line. 7 is not included in the set of numbers less than 7 so we put ‘○’ above 7.



Example 2

I’m thinking of a number greater than or equal to 12. If I call my number x, then $x \geq 12$.
On the number line: 12 is included, so we put ‘●’ to show “or equal to 12’ The arrow shows the numbers greater than 12.

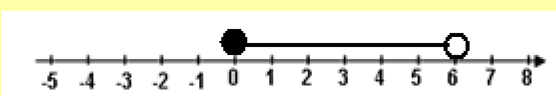


Note: ‘at least 15’ means the same as ‘15 or more’ and ‘at most 15’ means the same as ‘15 or less than 15’.

Example 3

Illustrate the following inequality on a number line.
$1 \leq x < 6$

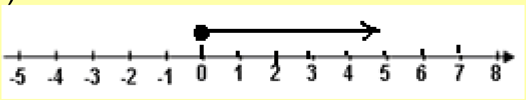
Solution



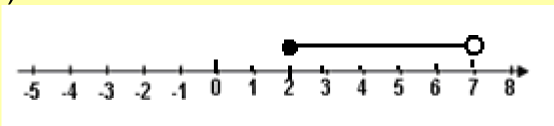
Exercise 11.20

1. Draw a number line to illustrate the following inequalities:
   (a) $a > 5$ (b) $b < 7$ (c) $c \leq 7$,
   (d) $d \geq -3$.
2. For each of the inequalities in question 1, write down three integers which make the inequality true.
3. Write down the inequality represented by each of the number lines below:

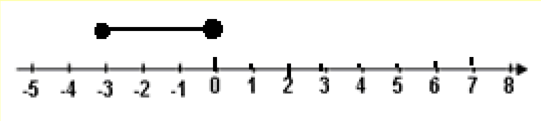
(a)



(b)



(c)



11.21 Solving linear inequalities

The methods of solving inequalities are similar to those used in solving equations. However, the solutions for inequalities are different from those of inequalities. While an equation has a particular value as a solution, an inequality has a range of values. When solving inequalities, the following **rules** must be followed:

(a) Addition and subtraction

Any quantity can be added or subtracted on both sides of an inequality. In general, for any three numbers a, b and c,

If $a < b$, then $a + c < b + c$ for all c

$a < b$, then $a - c < b - c$ for all c

$a > b$, then $a + c > b + c$ for all c

$a > b$, then $a - c > b - c$ for all c.

Note that the direction of the inequality remains unchanged.

(b) Multiplication and Division

The direction of the inequality is **reversed** if you multiply or divide by a **negative** number. When both sides are of a inequality are multiplied or divided by the same positive number, the inequality remains unchanged. For example, when both sides of an inequality $6 < 8$ are multiplied by 3, we get $18 < 24$ which is true. Also dividing both sides by 2 gives $3 < 4$, which is true.

Thus, if $a < b$, then

(i) $ac < bc$ for all positive c

(ii) $ac > bc$ for all negative c

(iii) $\frac{a}{c} < \frac{b}{c}$ for all positive c

(iv) $\frac{a}{c} > \frac{b}{c}$ for all negative c.

Example 4.

Solve the inequality: 3x – 4 < 5.

Adding 4 to both sides of the inequality, we have
3x – 4 + 4 < 5 + 4
3x < 9 ….dividing both sides by 3, we have
x < 3.

Example 5.

Solve for x: $\frac{3}{2}-\frac{5x}{3}>8+\frac{x}{2}$.

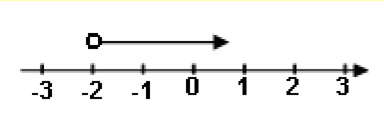
Multiplying both sides (each term) by 6 (the L.C.M of 2, 3)

$6\times\frac{3}{2}-6\times\frac{5x}{3}>6\times8+6\times\frac{x}{2}$
9 – 10x> 48 + 3x
Subtracting 3x from each side, we have
9 – 10x – 3x > 48 + 3x – 3x
9 – 13x > 48
Subtracting 9 from each side, we have
-13x > 39
Dividing both sides by -13 (reverses the direction of the inequality).
Therefore, x < -3.

Example 6.

Solve the inequality 5x + 3 > -11 – 2x and represent the solution on a number line.

5x + 3 > -11 - 2x
7x > -14
x > -2



Example 7.

Find the greatest integral value of y which satisfies the inequality
$2-\frac{3y}{2}>y+3$.

Solution:

$2-\frac{3y}{2}>y+3$….multiply each term by 2
4 – 3y > 2y + 6
- 5y > 2 …dividing by -5

$y < \frac{-2}{5}$ i.e. y < -0.4
The greatest integral value is -1.

Compound inequalities

Consider the value of x such that x > 2 and x < 6. This means that x lies between 2 and 6. We can combine the two inequalities and write 2 < x < 6. This is called a **compound inequality**.

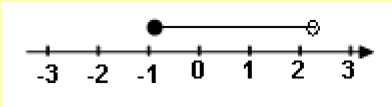
Example 8.

Find all integral values of x which satisfy
$$7 \geq 4 - 3x > -3.$$

Solution:

Take each inequality separately:
(i) $7 \geq 4 - 3x$
$3x \geq -3$
$x \geq -1$ or $-1 \leq x$

(ii) $4 - 3x > -3$
$-3x > -7$
Therefore $x < \frac{7}{3}$
Combining $-1 \leq x$ and $x < \frac{7}{3}$, we have $-1 \leq x < \frac{7}{3}$



Integral values of x are the integer values of x. Therefore the required values of x are; -1, 0, 1, 2.

Exercise 11.21

Solve the following inequalities:

1. 3x + 2 > -14

2. x - 4 < 5x + 3

3. 3x - 5 > -2

4. -4x < 8 - 2x

5. $4x + \frac{3}{2} < \frac{5}{2}$

6. $\frac{x}{5} - 2 < \frac{x}{3}$

7. $-5 - \frac{x}{4} < -6$

8. $\frac{y+2}{3} > 2 + \frac{y}{2}$

9. $4y + \frac{3}{2} < \frac{-3}{2}$

10. $\frac{2-4x}{3} > \frac{1}{2}$

11. 2(x - 1) > x - 1

12. 5(3 - 2x) < 3(4 - 3x).

Find the solutions of the following inequalities and illustrate the solution on a number line:

13. 4x + 1 > 7x - 5

14. 2(x + 3) ≥ 5(x - 4)

15. 7x - 6 ≥ 4 + 17(x - 5)

16. -7 < 3x + 2 < x + 5

17. $8 - x < 12 \leq 16 - 2x$

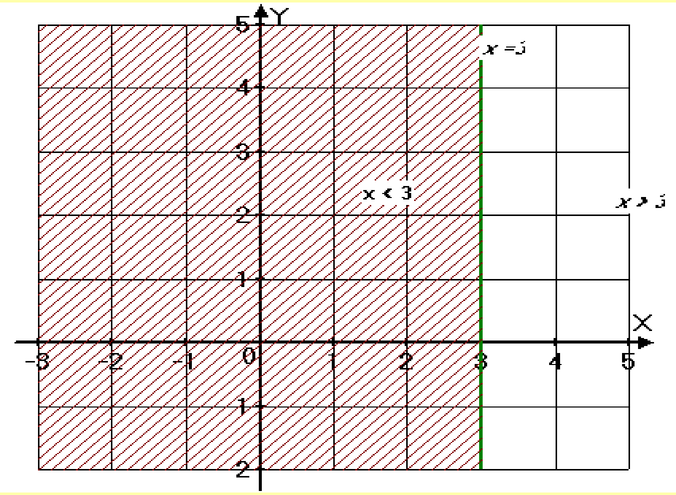
18. $11 \leq 3x + 5 \leq 17$

19. $-3 \leq -x + 2 < 0$

20. $-2x + 1 < x - 5 < 5 - x$.

11.22 Graphical representation of linear inequalities

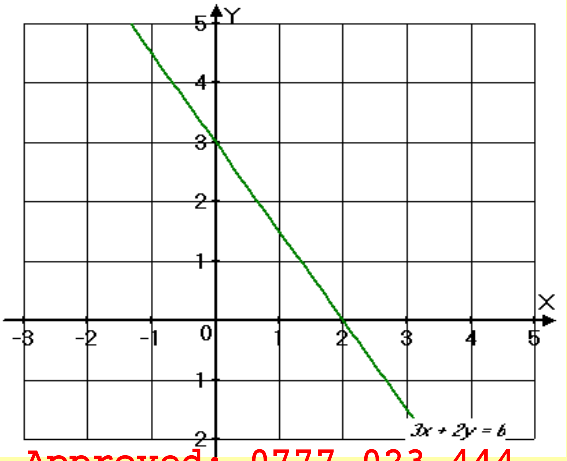
It is sometimes more useful to use *x* and *y* axes, rather than a number line. For example, to represent $x \geq 3$, we draw the line $x = 3$. The boundary line $x = 3$ represents all the points for which $x = 3$, and the region to the right contains all the points with x-coordinates greater than 3.
We use a continuous line for the boundary when it is included and shade the unwanted region.



In the inequality $x > 3$, x may not take the value 3. In this case we use a broken lie for the boundary.

Graphical representation of inequalities involving two variables (x, y).

Any straight line is determined by two points on it. Also any linear algebraic equation in x and y, such as $3x + 2y = 6$, can be represented by a straight line graph, with the x and y axes.



This straight line determines the graphical representation of four linear inequalities:
$3x + 2y > 6$, $3x + 2y < 6$, $3x + 2y \geq 6$, $3x + 2y \leq 6$.
Let us first consider the graph of $3x + 2y = 6$

x	0	2
y	3	0

We draw a straight line through these points. The line divides the plane into 3 sets of points:

(i). Those on the line, where 3x + 2y = 6.

(ii). Those on one side of the line (or other side of the origin) the half plane where 3x + 2y > 6, or 3x + 2y ≥ 6

(iii) Those on the other side of the line (unshaded region), where 3x + 2y < 6 or 3x + 2y ≤ 6 as the case may be.

Note: (i) You can show a particular region by shading the unwanted region.

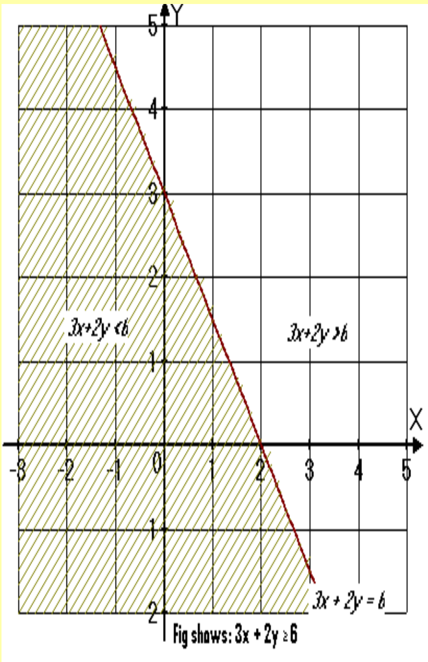
(ii) The boundary line is **solid** or **continuous** when the inequality is: **≤** or **≥**; otherwise it is a **dotted line** i.e. when the inequality is **<** or **>** (see figures below).

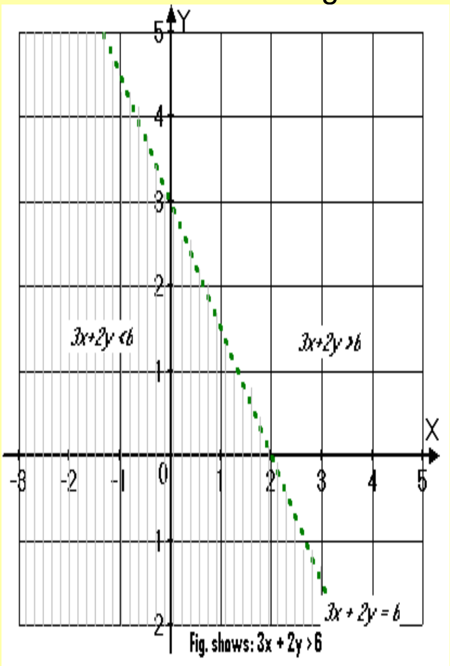
The following steps are taken when drawing the required region.

Step 1: Write the equation of the boundary line, and find two points that lie on the line,

Step 2: The sign **≥ or ≤** means that the values on given line are included. Plot the points and join them with a **solid** line.

Step 3: To decide which side of the boundary line is the required region, take a point on either side of the line and substitute in the inequality. If the point satisfies the inequality, then it lies in the *wanted* region, otherwise, it is in the *unwanted* region.

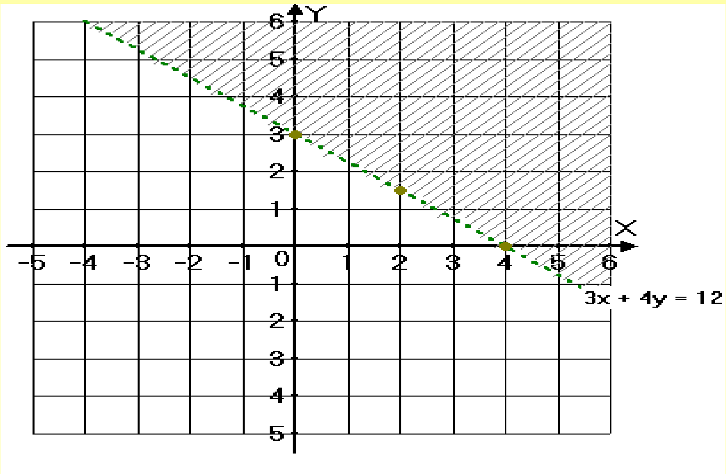




Example 7.

Show the region in which 3x + 4y < 12 by shading out the unwanted region.

The boundary line is 3x + 4y = 12.



First draw this line, taking 3 points on the line:

x	0	2	4
y	3	1.5	0

On the boundary line:
$3x + 4y = 12$;
On one side of the line,
$3x + 4y < 12$ and on the other side
$3x + 4y > 12$.
To find the required side of the line, take any point on either side of the line as a test point. If O (0, 0) is not on the boundary line, it is the simplest **test point**.
Hence using the origin O (0, 0):
$3x + 4y < 12$
$3(0) + 4(0) < 12$
$0 < 12$.

Thus (0, 0) satisfies the inequality. Therefore, (0, 0) lies in the required region. Hence we shade the other side of the line. (The shaded region represents $3x + 4y > 12$).

Graphical solution of simultaneous linear inequalities.

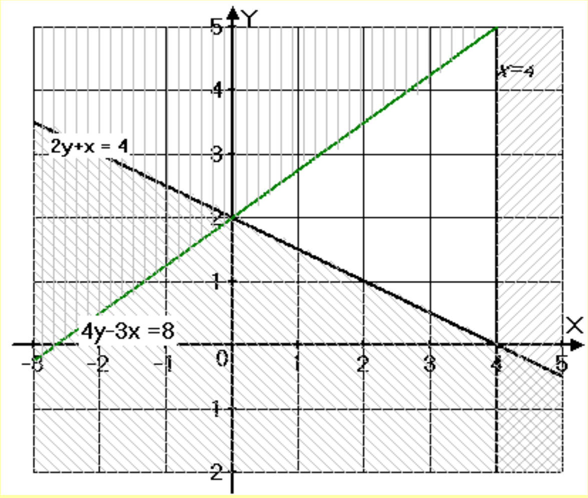
When the inequalities are drawn on a graph paper, the unshaded region R contains the points (x, y) which satisfy all the inequalities (constraints) given in a question. The set of points (x, y) in the region R are called the solutions of the simultaneous linear inequalities. In most solutions the x and y values taken are integers (whole numbers). In this case the solution, i.e. the set of points (x, y) where *x* and *y* are integers is called an **integral solution**.

Example 8

Find the region of the Cartesian plane which contains points whose coordinates satisfy the following inequalities: $x \leq 4$; $2y + x \geq 4$ and

$4y - 3x \leq 8$.
Find the integral solution of the simultaneous linear inequalities.

The solutions are:
(4,0), (4,1), (4,2), (4,3), (4,4), (4,5), (3,1), (3,2), (3,3), (3,4), (2,1), (2,2), (2,3), (1,2) and (0,2).



Exercise 11.22

Draw graphs to illustrate the following linear inequalities on Cartesian plane.
1. $x < 2$ **2**. $y < -2$ **3**. $x \leq 0$ **4**. $y > 0$ **5**. $x + y < 2$
6. $x + y \geq 2$ **7**. $3x - y \geq 0$ **8**. $x - y < 2$.

Show on a graph the region defined by the inequalities and hence write down the integral solution set.

9. $y < x$, $5y > x$, $x + y \leq 6$
10. $y \leq x + 1$, $x \leq 0$, $y + 2 > 0$
11. $x \geq 0$, $y > 0$, $x + y < 3$
12. $x \geq 4$, $y \geq 3$, $x + y < 13$, $6x + 5y \geq 60$
13. Shade, on a graph paper, the region R in which the following inequalities are simultaneously satisfied: $x \geq 2$, $y \geq 0$, $3x + 4y \leq 24$, $3x + 4y \geq 12$.

Interpretation of regions

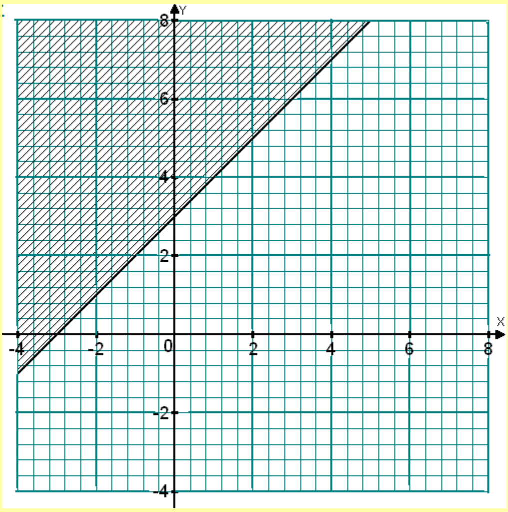
So far, we have been shading graphs to indicate the required regions defined by given inequalities. We can also define a required region already shown on a graph. We proceed as follows:

Step 1: Determine the equations of the boundary lines, if these have not been given on the graph.
Step 2: Choose a point in the required region and substitute in the equation of each line to determine the inequality.
Step 3: Check whether the boundary line is continuous or broken. If it is continuous, use ≤ or ≥ as appropriate. If it is broken, use < or > as appropriate.

Note: *Describe the required region completely. All the boundaries should be included.*

Example 1

Find the inequality that defines the unshaded region in the figure below.



Solution

Using the intercepts (-3, 0) and (0, 3), the gradient of the boundary line is

$$\frac{3-0}{0-(-3)} = \frac{3}{3} = 1.$$

Let (x, y) be any other point on the line, then

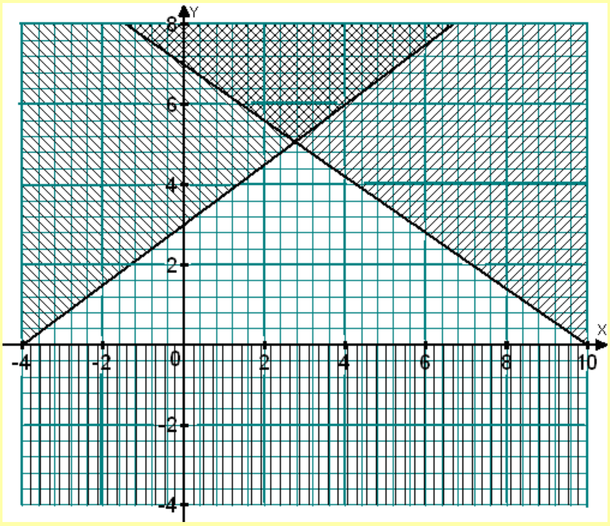
$$\frac{y-0}{x-(-3)} = 1$$

$\Rightarrow$ y = x + 3 is the equation of the boundary line.
The boundary line is continuous so it is included in the unshaded region. The point (0, 0) is in the required region.
At (0, 0), y = 0 and x + 3 = 0 + 3 = 3.

But, 0 < 3. So, the inequality is y ≤ x + 3.

Example 2

Give the inequalities that define the unshaded region shown below.



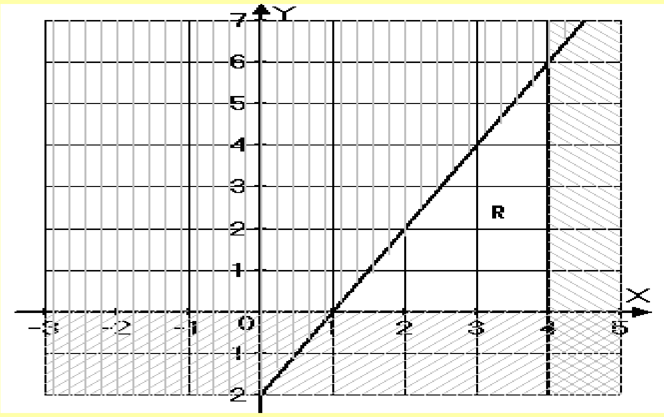
Solution

y = 0 is a boundary line and therefore, y ≥ 0 is one of the inequalities. The boundary line passing through (0, 7) and (10, 0) is given by $y = 7 - \frac{7}{10}x$ or 10y + 7x = 70.

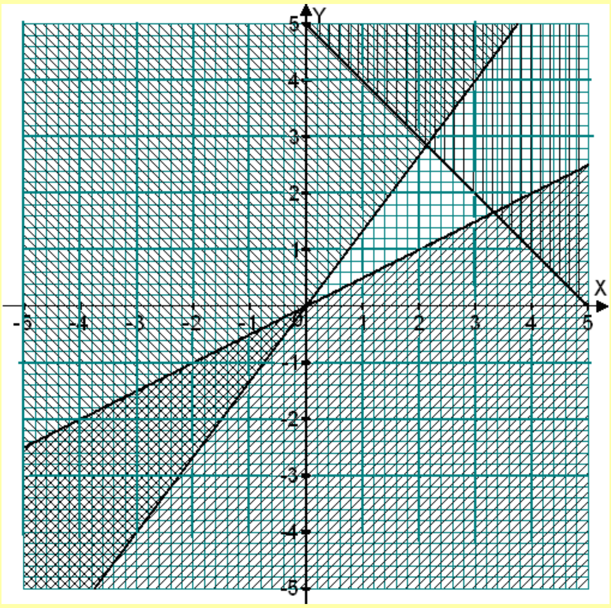
Point (4, 3) is in the required region. If x = 4 and y = 3, 10y + 7x = 58, but
58 < 70. Therefore, 10y + 7x ≤ 70 is another inequality.
The boundary line passing through (0, 3) and (4, 6) is given by
4y – 3x = 12.
Testing point (4, 3), when x = 4 and y = 3, 4x – 3y = 0 which is less than 12. The unshaded region is defined by y ≥ 0, 10y + 7x ≤ 70 and 4y – 3x ≤ 12.

Exercise

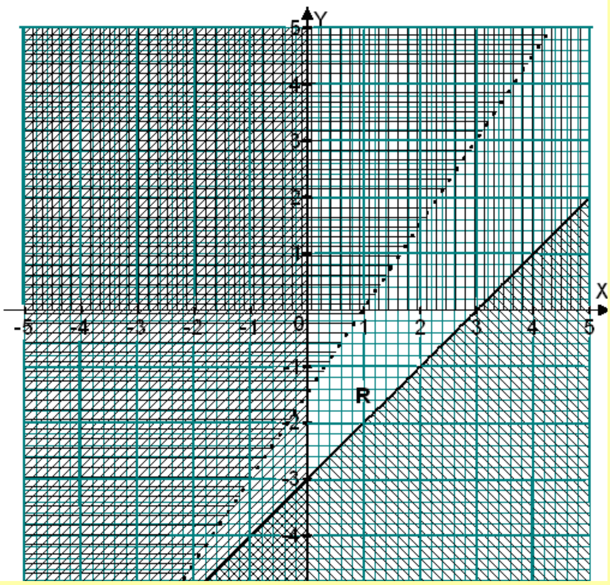
1. Give the inequalities that define the unshaded region R in each of the following.
   (a)


Y
X
R
7
6
5
4
3
2
1
0
-1
-2
-3
-2
-1
1
2
3
4
5

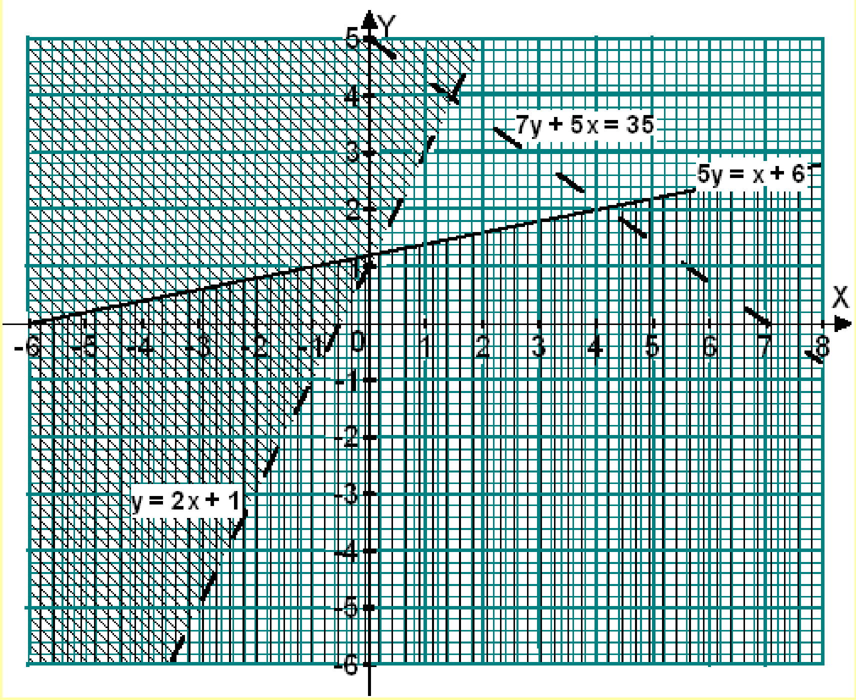
(b)


Y
X
5
4
3
2
1
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5

(c)

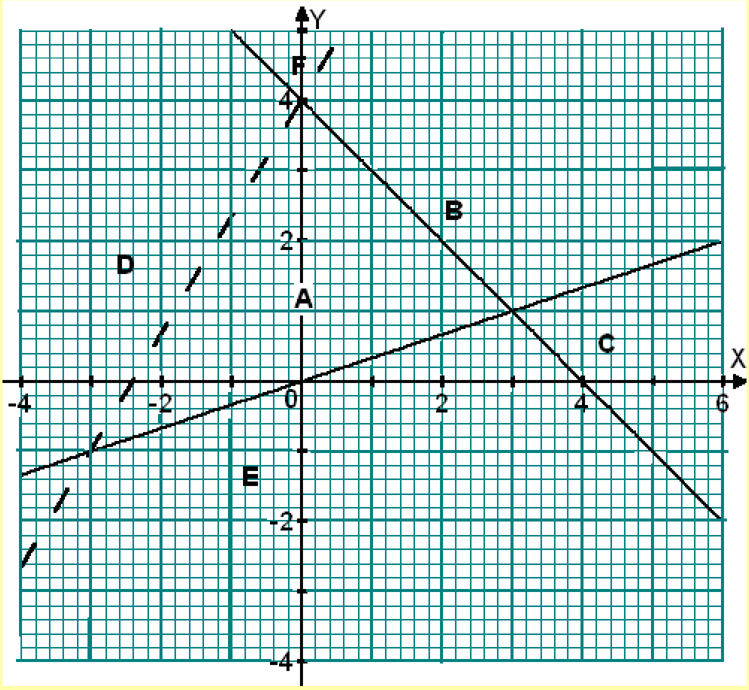

Y
X
R
5
4
3
2
1
0
-1
-2
-3
-4
-5
-4
-3
-2
-1
1
2
3
4
5

2. Give the points whose coordinates are integers and lie in the unshaded region.



3. Draw a graph and give the integral coordinates of the points that lie in the region defined by the inequalities:
$y > x - 2$, $2y < 3x + 6$, $x + y > -2$ and $x + y \leq 3$

4. The graph below shows regions A to F enclosed by lines
$x + y = 4$, $y = \frac{1}{3}x$ and $y = \frac{5}{3}x + 4$.



Use inequalities to describe region:
(a) A (b) B (c) C (d) D (e) E (f) F

12 APPROXIMATIONS

12.1 Rounding off

Examples

1. A boy counted 3641 beans in a bag. (read the number as: Three thousand, six hundred and forty one)

   How many is that to the nearest hundred?
   3641 is closer to 3600 than to 3700. So it is 3600 to the nearest hundred.

   How many is that to the nearest thousand?
   3641 is closer to 4000 than to 3000. So it is 4000 to the nearest thousand

2. A girl scored 95.7% in the math test.
   What is her score to the nearest whole percentage?
   95.7 is closer to 96 than to 95. So it is 96% to the nearest percent.

   What is her score to the nearest ten percent?
   95.7 is closer to 100 than to 90. So it is 100% to the nearest ten percent.

Exercise 12. 1

1. Write these quantities to the nearest hundred.
   (**a**) 346 kg (**b**) 780 m (**c**) 152 mm (**d**) 61 cm
   (**e**) 932 litres (**f**) 967 litres (**g**) 5412 g (**h**) 9650 kg.
2. Write these quantities to the nearest whole number.
   (**a**) 5.2% (**b**) 7.6 m (**c**) 9.8 mm (**d**) 5.36
   (**e**) 25.7 g (**f**) 90.13 kg (**g**) 146.713 m (**h**) 389.92 cm
3. The area of Gulu district is 11,560 square kilometers. What is that to the nearest thousand?
4. In Arua district the animal population in 1992 was as follows:
   Cattle: 100,746 Goats: 188,798 Sheep: 38,651
   (a) Write each of these to the nearest thousand.
   (b) Add up the total population and write that to the nearest thousand.

Decimal places

A number may have many decimal places as possible. The number of decimal places a number has is counted from a digit immediately after the decimal point toward the right. For example, 4.2341706 has 7 decimal places; 0.10051 has five decimal places.
If rounding to wanted number of decimal places, you check the digit in the next decimal place. If this digit is less than five, the last digit in the require decimal places remains unchanged but if it is 5 up to 9, the last digit in the decimal places increases by 1.

Examples

1. Write 5.762 to 2 decimal places

   5.762 becomes 5.76
   The answer is 5.76 (to 2 dp).
   It is helpful to write dp instead of 'decimal places'

2. Write 354.392 to 2 decimal places

   The answer is 354.40 (to 2 dp)

3. Write 56.9097 to 3 decimal places

   The answer is 56.910 (to 3 dp)

Exercise 12.2

1. Write the following quantities to 1 dp
   (a) 5.76
   (b) 12.34
   (c) 8.473
   (d) 14.96

2. Write the following quantities to 2 dp.
   (a) 3.926
   (b) 52.3921
   (c) 4.696
   (d) 21.9951

3. Write 36.1986345 to:
   (a) 3 dp
   (b) 6 dp

4. Write 5.3890962 to
   (a) 2 dp
   (b) 5 dp.

5. 1 mile = 1.609 kilometres. Write 5 miles in kilometers correct to 2 dp.

Significant figures (sf)

Significant figures (also called significant digits) of a number are those digits that carry meaning contributing to its accuracy.

Rules for counting significant figures are summarized below:

1. All non-zero digits are considered significant. E.g 123.45 has five significant figures: 1, 2, 3, 4, 5.

2. Zeros appearing anywhere between two non-zero digits are significant. E.g. 101.12 has five significant figures: 1, 0, 1, 1, and 2.

3. Leading zeros are not significant. For example, 0.00012 has two significant figures: 1 and 2.

4. Trailing zeros in a number containing a decimal point are significant. For example, 12.2300 has six significant figures: 1, 2, 2, 3, 0 and 0. The number 0.00122300 still has only six significant figures (the zeros before the 1 are not significant). In addition, 120.00 has five significant figures.

5. Trailing zeros in a number not containing a decimal point are not significant. For example 470,000 has two significant figures.

Examples

Rounding to 2 s.f:

1. 12300 becomes 12000
2. 13 stays as 13.
3. 0.00123 becomes 0.0012
4. 0.1 becomes 0.10, the trailing zero indicates that we are rounding to 2 s.f
5. 0.0125 becomes 0.013
6. 19800 becomes 20000.

Exercise 12.3

1. Write 3426 to 3 significant figure.

2. Write 5294379 to 4 significant figure.

3. Write 398.123 to 2 significant figures.

4. Round off the following numbers correct to: (i) 2 SF, (ii) 3 SF, (iii) 4 SF

(a) 39.6582, (b) 2.61925

(c) 304.9258 (d) 5.7984,

(e) 0.0025498 (f) 76839

(g) 81.9814, (h) 526.789

(i) 3000.8

ANSWERS

Exercise 1.2

1. $\left(\frac{5}{14}\right)_{eight}$

2. $\left(\frac{3}{4}\right)_{ten}$

3. 0.011_{two}

4. 925_{ten}

5. 29.5_{ten}

6. $\left(\frac{2}{7}\right)_{ten}$

7. 12.201_{four}

8. 1.01_{two}

9. 10.101_{two}

10. 3.102_{four}

11. (i) n=9 (ii) n=8

Exercise 1.3

1. 2×3; $2^2 \times 3 \times 5$

2. 7; $2^2 \times 3 \times 7$

3. 11; $2 \times 3 \times 11^2$

4. $2^2 \times 7$; $2^3 \times 5^2 \times 7 \times 13$

5. 3^2; $3^4 \times 5^2$

6. 18; $2^3 \times 3^4$

7. 18; $2 \times 3^2 \times 7 \times 11$

8. 42; $2^4 \times 3 \times 5 \times 7^2$

9. 13; $2 \times 5 \times 7 \times 11 \times 13$

10. 308; $23 \times 7^2 \times 11$.

11. 1008 litres

12. 9.24 pm

13. n = 8

14. n = 6

15. $3^3 \times 7^{-1}$

Exercise 4.3

1. $\frac{3}{4}$ **2.** $\frac{9}{x}$ **3.** 1 **4.** $1\frac{1}{2}$ **5** . $\frac{2}{3}$ **6.** $1\frac{1}{4}$ **7.** $\frac{8}{15}$ **8.** $\frac{3}{4}$

9. $\frac{1}{2}$ **10** . $\frac{136}{21}$ **11.** $\frac{8}{3x}$ **12.** $\frac{39}{50}$ **13.** $5\frac{7}{8}$ **14.** $6\frac{6}{7}$ **15.** $3\frac{1}{2}$

Exercise 4.4

1. $\frac{1}{4}$ **2.** $\frac{1}{6}$ **3.** $\frac{3}{4}$ **4.** $\frac{3}{x}$ **5.** $1\frac{5}{7}$ **6.** $2\frac{1}{4}$ **7.** $4\frac{3}{10}$ **8.** $1\frac{1}{4}$ **9.** $\frac{1}{2}$ **10.** $\frac{3}{8}$ **11.** $\frac{1}{2}$ **12.** $\frac{5}{29}$

13. $1\frac{3}{8}$ **14.** $2\frac{1}{3}$ **15.** $1\frac{5}{8}$ **16.** $2\frac{2}{3}$ **17.** $2\frac{3}{5}$ **18.** $\frac{29}{48}$.

Exercise 4.5

1. $2\frac{6}{7}$ **2.** $\frac{2}{11}$ **3.** $\frac{2}{3}$ **4.** 12 **5.** 12 **6.** 12

7. $\frac{2}{5}$ **8.** 10 **9.** 6 **10.** 7

Exercise 4.6

1. 2 **2.** $\frac{2}{3}$ **3.** $1\frac{5}{8}$ **4.** $\frac{4}{23}$ **5.** 1 **6.** 10 **7.** 6 **8.** $1\frac{1}{2}$ **9.** 6 **10.** $\frac{7}{20}$

11. $\frac{1}{4}$ **12.** $7\frac{7}{15}$ **13.** $\frac{6}{17}$ **14.** 18 **15.** $1\frac{3}{16}$ **16.** $\frac{26}{77}$ **17.** $1\frac{4}{5}$ **18.** $3\frac{21}{25}$

Exercise 4.7

1. $6\frac{1}{4}\,litres$ **2.** $\frac{1}{10}$ **3.** 20 **4.** 20

5. 1242 **6.** Sh.14, 400 **7.** $7\frac{1}{5}\,km$ **8.** 28

9. $126\frac{1}{4}\,m$ **10**. $13\frac{1}{2}\,m.$

Exercise 5.5

1 $\frac{1}{50000}$ **2**. 13.6km,
3. 9km, 2.4cm (i) 2.7cm (ii) 0.9cm
4. 073^0; 142km
5. (i) 58m (ii) 162^0
6. (i) 275km; (ii) 024^0

Exercise 6.2

1. 144^0 **2**. 30 sides **3**. 156^0 **4**. 162^0. **5**. $z = 56^0$

Exercise 7.1

1. $72cm^2$
2. 12cm; $84cm^2$
3. $144m^2$
4. $21x\ m^2$; (6x+14) m
5. $112cm^2$.

Exercise 7.2

2. 12 cm **3**. 135 cm **4**. 48 **5**. 512 cm^3

Exercise 11.12

1. 13 **2**. 40 km **3**. $1\frac{1}{4}$ **4**. 12 km.

Exercise 11.21

1. $x > -2$
2. $x > -1\frac{3}{4}$
3. $x > 1$
4. $x > -4$
5. $x < \frac{1}{4}$
6. $x > -15$
7. $x > 4$
8. $y < -8$
9. $y < -\frac{3}{4}$
10. $x < \frac{1}{8}$
11. $x > 1$
12 $x > 3$
13. $x < 2$
14 $x \le 8\frac{2}{3}$
15 $x < 7.5$
16. $-3 < x < 1\frac{1}{2}$
17. $-4 < x \le 2$
18. $2 \le x \le 4$
19. $2 < x \le 5$
20. $2 < x < 5$