SENIOR THREE SELF STUDY MATHEMATICS

SET THEORY

DAY 1

Summary:

1. A set is a collection of well-defined objects**.**

2. An empty set is a set with no elements. It is denoted by curly brackets with nothing inside {} **or ϕ.**

3. A subset is a set that is part of a larger set

4. A universal set is a set of all other elements under consideration

5.A Venn diagram is a set diagram that shows relations between different sets.

6.For any two sets **A** and **B:**

(i) n(ε)is read as number of members in the universal set. This means number of members in all the regions of the Venn diagram

(ii) n(A) is read as number of members in set **A**.

(iii) $n(\overline{A})$ or $n(A')$ is read as number of members in set **A** complement. This means number of members of the universal set that are not in set **A.**

(iv) $A \cup B$ is read as **A** union **B.** This means members of eitherset **A** or **B** **(**the entire region covering the two sets**)**.

(v) $A \cap B$ is read as **A** intersection **B.** This means the region common to the two sets.

(vi) $A \cap \overline{B}$ is read as **A** intersection **B** complement**.** This means members of **A** only**.**

(vii) $\overline{A} \cap B$ is read as **A** complement intersection **B.** This means members of **B** only**.**

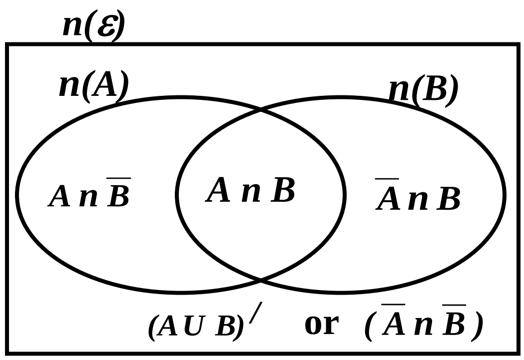
(ix) $\overline{A} \cap \overline{B}$ is read as **A** complement intersection **B** complement**.** This means neither **A** nor **B.** $\overline{A} \cap \overline{B}$ is the same as $(A \cup B)'$

(x) $A \cup \overline{B}$ is read as **A** union **B** complement**.** This means of members of the universal set that are not in set **B** only**.**

(xi) $\overline{A}\ u\ B$ is read as **A** complement union **B.** This means of members of the universal set that are not in set **A** only**.**

(xii) $\overline{A}\ u\ \overline{B}$ is read as **A** complement union **B** complement**.** This means of members of the universal set that are not in the intersection**.**

(xiii) The Venn diagrams illustrating the regions relating any two sets**A** and **B**is as follows**:**



EXAMPLES:

1. Given the sets**ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 4, 5, 8}**and

B = {1, 3, 5, 7, 8, 9}, where **ε**is the universal set**,** find**:**

(i) $A\ n\ B$

A = {2, 3, 4, 5, 8}

B = {1, 3, 5, 7, 8, 9}

$A\ n\ B$ = { 3, 5, 8 }

(ii) $A\ u\ B$

A = {2, 3, 4, 5, 8}

B = {1, 3, 5, 7, 8, 9}

$A\ u\ B$ **={1, 2, 3, 4, 5, 6, 7, 8, 9}**

(iii) $A \cap \overline{B}$

$A \cap \overline{B}$ **= {2, 4}**

(iv) $\overline{A} \cap B$

$\overline{A} \cap B$ **= {1, 7, 9}**

ACTIVITY 1

1. In a class of **53** students**, 36**drink tea**, 18**drink coffee while **10**drink neither tea nor coffee. Find how many students drink both tea and coffee

2. In a class of **29** boys**, 22** liked rice and **18** liked matooke. All the boys liked at least one of the foods. Find how many liked both.

(b) Given the sets **P =** {All factors of **90**}and **Q =** { All factors of **60**}**,** find**:**

(i) n(P n Q)(ii) $n(\overline{P} \cap Q)$ **(iii) n(P U Q)**

3. Given the sets **P = {**All triangular numbers less than **40**}and

Q = { All factors of **60**}**,** find**:**

(i) n(P n Q) **(ii) n(P U Q)** **(iii)** $n(P \cap \overline{Q})$ **(iv)** $n(\overline{P} \cap Q)$

4. If **ε = { x: 0 < x < 13}, A = { x: 1 < x <9}**and **B = { x: 4< x < 11},** where **x** is an integer and **ε**is the universal set**,** find**:**

(i) n(A n B) **(ii) n(A U B)** **(iii)** $n(A \cap \overline{B})$ **(iii)** $n(A \cup B)'$

5. If **ε = { x: 1 < x < 14}, A = { x:** $2 \le x \le 9$ **}**and **B = { x:** $6 \le x \le 11$**},** where **x** is an integer and**ε**is the universal set**,** find**:**

(i) n(A n B) **(ii) n(A U B)** **(iii)** $n(A \cap \overline{B})$ **(iii)** $n(A \cup B)'$

6. Given the sets **ε = {6, 7, 8, 9, 15, 16, 17, 18, 20, 21},**

M = { x: x are multiples of **3**}and **N = { x: x** are odd numbers}**,** where **ε**is the universal set**,** find**:**

(i) n(M n N) **(ii) n(M U N)** **(iii)** $n(M \ n \ \overline{N}\ )$

(iv) $n(\ \overline{M}\ n\ \overline{N}\ )$ **(v)** $n(M\ U\ \overline{N}\ )$ **(vi)** $n(\ \overline{M}\ )$

Hint: The elements of **M** and N have to be chosen from the universal set

7. Sets **A** and **B** are such that **n(ε) = 30, n(A) = 18, n(B) = 14** and $n(A\ U\ B)^{/} = 5.$ find**:**

(i) n(A n B) **(ii) n(A U B)(iii)** $n(A\ n\ \overline{B}\ )$ **(iv)** $n(A\ U\ \overline{B}\ )$

8. Sets **P** and **Q** are such that **n(P) = 12, n(Q) = 8, n(P U Q) = 15** and $n(P\ U\ Q)^{/} = 2.$ find**:**

(i) n(P n Q)

(ii) n(ε), where **ε**is the universal set

9. Sets **A** and **B** are such that **n(ε) = 40, n(A) = 25,** $n(\ \overline{A}\ n\ B) = 10$ and $n(A\ n\ B) = n(A\ U\ B)^{/}$ **,** where **ε**is the universal set**.** Use a Venn diagram to find:

(i) n(A n B) **(ii) n(A U B)**

10. Sets **M** and **N** are such that **n(ε) = 19, n(M) = 8** and $n(\ \overline{N}\ ) = n(\ \overline{M}\ n\ N) = 7,$ where **ε**is the universal set**.** Use a Venn diagram to find**:**

(i) n(M n N) **(ii)** $n(M\ U\ \overline{N}\ )$

ACTIVITY 2

1. Sets **A** and **B** are such that **n(ε) = 28, n(A) = 10, n(B) = 17**and **n(A U B) = 22,** where **ε**is the universal set**.** Use a Venn diagram to find**:**

(i) $n(A \cap B)$ **(ii)** $n(A \cap \overline{B})$ **(iii)** $n(\overline{A} \cap B)$ **(iv)** $n(\overline{A} \cap \overline{B})$

(v) $n(A \cap B)^{\prime}$ **(vi)** $n(A \cup \overline{B})$ **(vii)** $n(\overline{A})$

2. Given the sets **M =** {All multiples of **6** less than **72**}and

N = {All multiples of **4** less than **50**} **,** find**:**

(i) $n(M \cap N)$ **(ii)** $n(M \cup N)$ **(iii)** $n(M \cap \overline{N})$ **(iv)** $n(\overline{M} \cap N)$

3. Sets **A** and **B** are such that **n(ε) = 23, n(A n B) = 8, n(B) = 14**and $n(\overline{A})$ **= 10,** where **ε**is the universal set**.** Use a Venn diagram to find**:**

(i) $n(A)$ **(ii)** $n(\overline{A} \cap \overline{B})$ **(iii)** $n(\overline{A} \cap B)$ **(iv)** $n(A \cup \overline{B})$

4. In a class of **30** students**, 18** play volley ball**, 14** play hockey while **5** play neither. Find how many students play**:**

(i) both games

(ii) only one game

5. Given the sets **M =** {The first **10** rectangle numbers}and

N = {The first **5** square numbers} **,** find**:**

(i) $n(M \cap N)$ **(ii)** $n(M \cup N)$ **(iii)** $n(M \cap \overline{N})$ **(iv)** $n(\overline{M} \cap N)$

6. In a class of **28** boys**, 13** passed history and **25** passed physics. All the boys passed at least one subject. Find how many passed both subjects.

7. Sets **A** and **B** are such that **n(ε) = 35, n(A n B) = 8, n(A) = 17**and $n(A \, U \, B)' = 5$**,** where **ε**is the universal set**.** Use a Venn diagram to find**:**

(i) $n(\overline{A} \, n \, B)$ **(ii)** $n(\overline{A})$

8. Given the sets **M =** {All multiples of **3** less than **20**}and

N = {All odd numbers less than **20**} **,** find **n(M n N)**

9. In a class of **50** boys**, 23** passed history**, 35** passed physics and **2** passed neither subject**.** Find how many passed**:**

(i) both subjects.

(ii) only one subject.

10. Given the sets **M =** {All integers greater than **4** but less than **10**}and

N = {All multiples of **3** between **1** and **20**}**,** find**:**

(i) n(M n N) **(ii) n(M U N)** **(iii)** $n(M \, n \, \overline{N})$

11. Given the sets **T =** {All triangle numbers less than **20**}and

F = {All factors of **12**} **,** find the members of **T n F.** Hence find **n(T n F)**

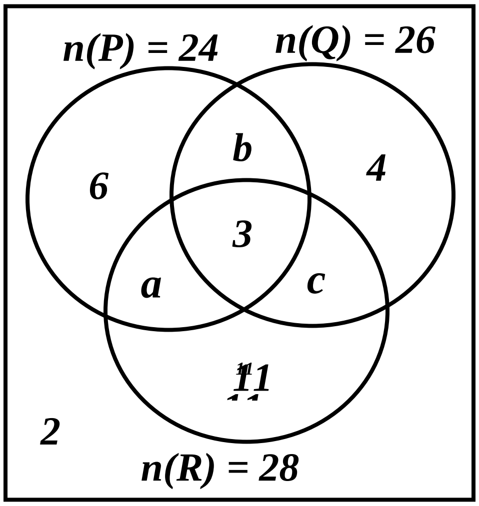
12. Given the sets **P =** {All factors of **24**}and **Q =** { All factors of **30**}**,** find

$n(\overline{P} \, n \, Q)$

DAY TWO

FURTHER MORE ON SETS

Study the Venn diagram below**:**



Find**:**

(i) the values of **a, b** and **c**

from *P*

24=6+b+3+a

15=b+a

a=15-b………………………………. Eqn1

From Q

26=b+4+3+c

19=b+c

b=19-c……..…………………………Eqn2

From R

28=a+3+c+11

14=a+c

c=14-a …………………………………Eqn3

sub **b** in **a**

a=15-(19-c)

a=c-4…………………………………...Eqn4

sub **a** in **c**

c=14-(c-4)

c=18-c

2c=18

c=9
sub **c** in Eqn 2
b=19-c
b=19-9
b=10
sub **b** in Eqn 1
a=15-b
a=15-10
a=5
a=5, **b**=10, **c**=9

(ii) n(ε), where **ε**is the universal set

n(ε)= 6+10+9+3+5+11+4+2

n(ε)=50

ACTIVITY 3

1. In a class of **30** students**, 18** play Tennis**, 15** play Golf and **13** play Hockey**.** The number of students who play all the three games are equal to those who play neither game**. 10** play both Tennis and Hockey**, 8** play Tennis and Golf**,3** play only Golf and Hockey**.**

(a) Represent the information on Venn diagram

(b) Find the number of the students who play**:**

(i) all the three games

(ii)at most one game

(c) Find the probability that a student chosen at random plays at least two games

2. In a class of **56** students**, 28** play Tennis**, 24** play Chess and **32** play Hockey**.10** play both Tennis and Chess**, 6** play both Chess and Hockey**, 4** play all the three games**.**

(a) Represent the information on a Venn diagram

(b) Find the number of the students who playboth Tennis and Hockey only

(c) Find the probability that a student chosen at random plays**:**

(i)at least two games

(ii) only one game

3. Sets **A, B** and **C** are such that **n(ε) = 100, n(A) = 46, n(B) = 40, n(C) = 49, n(A n B) = 14, n(A n C) = 17, n(B n C) = 15**and $\boldsymbol{n(\overline{A}\ n\ \overline{B}\ n\ \overline{C}\ ) = 6}$**.** Use a Venn diagram to find**:**

(i) n(A n B n C)

(ii) n(A U B U C)

(iii) $\boldsymbol{n(\overline{A}\ n\ B\ n\ C)}$

(iv) $\boldsymbol{n(A\ n\ \overline{B}\ n\ C)}$

(v) $\boldsymbol{n(A\ n\ B\ n\ \overline{C}\ )}$

4. In a class of **53** students**,30** study Art**,20** study French and **15** study Computer**. 6** study both Art and French**, 4** study Art and Computer**,5** study French and Computer**.** Each student studies at least one of the three subjects

(a) Represent the information on Venn diagram

(b) Find the number of the students who study**:**

(i)all the three subjects

(ii)at least two subjects.

(c) Find the probability that a student chosen at random studies**:**

(i) only one subject

(ii) French only

(iii) French but not Computer

5. In a class of **100** students**,15** take Art only**,12** take French only and **8** take Computer only**. 10** take both Art and French**, 40** take Art and Computer**,20** take French and Computer**, 65** take Computer**.**

(a) Represent the information on Venn diagram

(b) Find the number of the students who take**:**

(i) all the three subjects

(ii) Art

(iii) French

(c) Find the probability that a student chosen at random takes neither subject

6. In a class of **52** students**,** an equal number of students visited Arua and Kasese. **24** visited Mbale**, 11** visited both Mbale and Arua**, 12** visited Arua and Kasese**, 13** visited Mbale and Kasese**. 8** visited all the three towns and **4** visited neither town**.**

(a) Represent the information on Venn diagram

(b) Find the number of the students who**:**

(i) visited Kasese

(ii)did not visit Arua

(c) Find the probability that a student chosen at random visited at least two towns

7. In a class of **72** students**,** each student must at least take of the subjects Art **(A),** Computer **(C)** and French **(F).** None of the students takes **F** and **C,26** take **F** only**.** Of the **35** students taking **A, 20** take the subject alone. The number of students taking **F** and **A** is three more than those taking **A** and **C.**

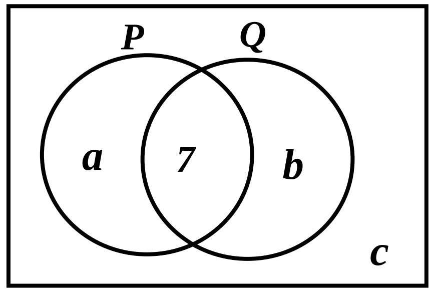
(a) Represent the information on Venn diagram

(b) Use the Venn diagram to find the number of the students who take;

(i) Art and Computer
(ii) Computer.

(c) What is the probability that a student chosen at random takes only one subject.

15. In the Venn diagram below, sets **P** and **Q** are such that **n(PUQ) = 16,**
n(P') = 7, and **n(Q') = 6.**



Find the values of **a,b** and **c.** Hence obtain **n(ε)**

7. Sets **A, B** and **C** are such that **n(A) = 23, n(B) = 24, n(C) = 25, n(A n B n C) = 5,**
$n(A \text{ n } \overline{B} \text{ n } \overline{C}) = 8$, $n(\overline{A} \text{ n } B \text{ n } \overline{C}) = 12$, $n(\overline{A} \text{ n } \overline{B} \text{ n } C) = 9$ and $n(\overline{A} \text{ n } \overline{B} \text{ n } \overline{C}) = 2$.
Use a Venn diagram to find**:**

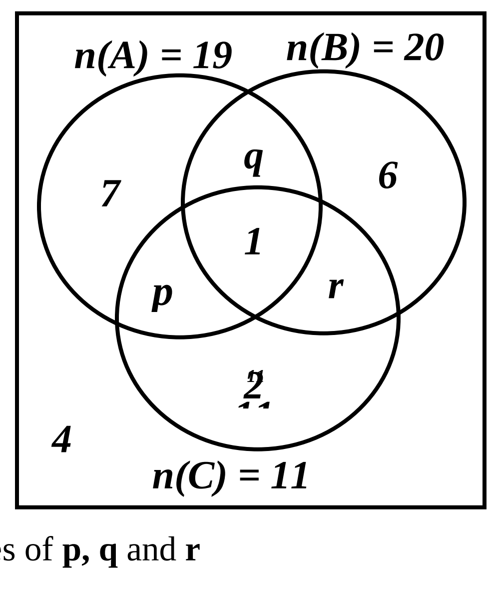
(i) $n(\overline{A} \text{ n } B \text{ n } C)$

(ii) $n(A \text{ n } \overline{B} \text{ n } C)$

(iii) $n(A \text{ n } B \text{ n } \overline{C})$

(iv) n(ε) , where **ε**is the universal set**.**

6. Study the Venn diagram below**:**



Find**:**

(i) the values of **p, q** and **r**

(ii) n(ε), where **ε**is the universal set

8. In a class of **42** students**,15** like Chemistry **(C),19** like Physics **(P),** and **28** like Mathematics **(M). 6** students like both Physics and Chemistry**,10** students like both Mathematics and Chemistry and **8** like Physics and Mathematics but not Chemistry**.** Given that the number of students who like all the three subjects is equal to those who do not like any of the subjects**.**

(a) Represent the above information on a Venn diagram**.**

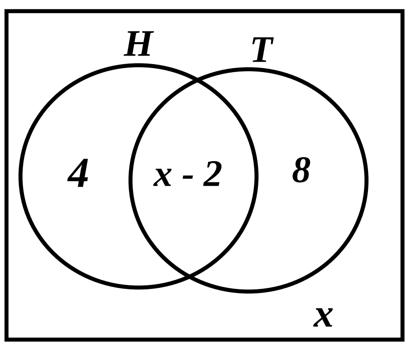
(b) Find the number of students who like**:**

(i) all the three subjects**.**

(ii) at least two of the subjects**.**

(c) Find the probability that a student selected at randomlikes other subjects**.**

9. The Venn diagram below shows a group of students playing Hockey **(H)** or Tennis **(T)**

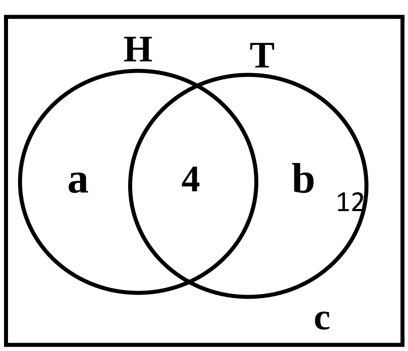


If the probability that a student picked at random from the group plays both or none of the games is **0·4,** find the**:**

(i) value of **x.**

(ii) number of students playing both of the games.

10. In the Venn diagram below**, 20** students play either Hockey **(H)** or Tennis **(T)**



If **14** and **12** students do not play Hockey and Tennis respectively**,** find the**:**

(i) values of **a, b** and **c.**

(ii) probability that a student picked at random plays neither of the games

11. In a sports club **19** members play Hockey **(H),18** play Rugby **(R),17** play Tennis **(T)** and **5** play neither of the games**.** **10** play both **H** and **R, 6** play both **H** and **T, 7** play both **R** and **T, 20** play only one game**.**

(a) Represent the above information on a Venn diagram**.**

(b) Find the number of members**:**

(i) who play all the three games in the club**.**

(ii) who play at most one game**.**

(c) Find the probabilitythat a member picked at random plays at least two games

12. In a class of **42** students**,15** like Chemistry **(C),19** like Physics **(P),** and **28** like Mathematics **(M). 6** students like both Physics and Chemistry**,10** students like both Mathematics and Chemistry and **8** like Physics and Mathematics but not Chemistry**.** Given that the number of students who like all the three subjects is equal to those who do not like any of the subjects**.**

(a) Represent the above information on a Venn diagram**.**

(b) Find the number of students **:**

(i) who like all the three subjects**.**

(ii)at least two of the subjects**.**

(c) Find the probability that a student selected at random from the class likes other subjects**.**

DAY 3

MATRICES

Summary:

1. *A matrix is a bracket with numbers in rows and columns. Thus* $\begin{pmatrix} \boldsymbol{1} & \boldsymbol{2} \\ \boldsymbol{3} & \boldsymbol{4} \end{pmatrix}$ *and* $\begin{pmatrix} \boldsymbol{1} & \boldsymbol{2} & \boldsymbol{0} \\ \boldsymbol{-2} & \boldsymbol{3} & \boldsymbol{5} \end{pmatrix}$ *are matrices.*

2. *The order of a matrix with* $\boldsymbol{m}$ *rows and* $\boldsymbol{n}$ *columns is written as* $\boldsymbol{m} \times \boldsymbol{n}$ *and is called an* $\boldsymbol{m} \times \boldsymbol{n}$ *matrix.*

3. *The numbers in a matrix are called its elements or entries.*

4. (i) *To add and subtract matrices of the same order, add and subtract corresponding elements*

(ii) *Two matrices are equal if their corresponding elements are equal*

(iii) *A scalar* $\boldsymbol{k}$ *multiplied by a matrix* $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$ *is treated as follows:*

$$\boldsymbol{kA} = \boldsymbol{k}\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \boldsymbol{ka} & \boldsymbol{kb} \\ \boldsymbol{kc} & \boldsymbol{kd} \end{pmatrix}$$

(iv) *Matrix multiplication is treated as follows:*

$$\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}\begin{pmatrix} \boldsymbol{w} & \boldsymbol{x} \\ \boldsymbol{y} & \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} \boldsymbol{aw+by} & \boldsymbol{ax+bz} \\ \boldsymbol{cw+dy} & \boldsymbol{cx+dz} \end{pmatrix}$$

Matrix product $\boldsymbol{AB} \neq \boldsymbol{BA}$ *.*

Matrix product $\boldsymbol{AB}$ *can be done if the number of columns in* $\boldsymbol{A}$ *is equal to the number of row in* **B.**
If a $\boldsymbol{2 \times 5}$ *matrix is multiplied by a* $\boldsymbol{5 \times 3}$ *matrix, then the resulting matrix has theouter dimensions* ***(****The new matrix is of order* $\boldsymbol{2 \times 3}$***)5.*** *An identity matrix* $\boldsymbol{I}$ *is a*

matrix with ones along the major diagonal and zeros elsewhere. Thus a 2×2 *identity matrix is given by* $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

6. *If matrix* $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$*, , then:*

(i)Determinant of $A(Det\ A)=ad-cb$

(ii) *Adjoint of matrix* $A=\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

(iii) *The inverse of* $A,(A^{-1})=\dfrac{1}{detA}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

(iv) *The product* $AA^{-1}=I$

(v) *A matrix multiplied by an identity matrix remains unchanged*

7. *A singular matrix is the one whose determinant is zero and thus has no inverse.*

EXAMPLES:

1. *If matrix* $A=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$,

(a) *State the order of matrix* ***A***

It's a 2x2

(b) *Determine the:*

(i) *determinant of* ***A***

Det A =3*4 - 2*1

=12 - 2

=10

(ii) inverse of ***A***

$$(A^{-1}) = \frac{1}{detA}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\frac{1}{10}\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$$

$$=\begin{pmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{-1}{10} & \frac{3}{10} \end{pmatrix}$$

ACTIVITY 4

2. *Given that matrix* $P = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, *find:*

(i) $P + Q$ ***(ii)*** $Q - R$ ***(iii)*** $3P - 2Q + R$ ***(iv)*** PQ ***(v)*** QP ***(vi)*** QRP

(vii) P^2 ***(viii)*** Q^2 ***(ix)*** $(P+Q)^2$ ***(x)*** $3P - 2I$ *where* I *is a* 2×2 *identity matrix*

3. *Given that matrix* $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $B = \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix}$, *find* ***det(AB)***

4. *Given that matrix* $P = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $R = P^2Q$, *find* R^{-1}

5. *Find the order of the resulting matrix when a* 3×4 *matrix is multiplied bya* 4×5 *matrix*

6. *If matrix* $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$,

(i) *determine the order of matrix* ***AB***

(ii) *find matrix* ***AB***

7. *If matrix* $\boldsymbol{P} = \begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}$, $\boldsymbol{Q} = \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$ *and* $\boldsymbol{R} = \boldsymbol{PQ}$,

(i) *determine the order of matrix* ***R***

(ii) *find matrix* ***R***

8. *Given the matrix equation* ***AY = B,*** *use matrix inversion method to find:*

(i) *matrix* ***Y*** ***(ii)*** *matrix* ***A***

9. *If matrix* $\boldsymbol{A} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find matrix* ***B*** *such that* $\boldsymbol{AB} = \begin{pmatrix} 17 & 13 \\ 21 & 14 \end{pmatrix}$

10. *If matrix* $\boldsymbol{B} = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$, *find matrix* ***A*** *such that* $\boldsymbol{AB} = \begin{pmatrix} 10 & 4 \\ -5 & 9 \end{pmatrix}$

11. *If matrix* $\boldsymbol{P} = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$, $\boldsymbol{Q} = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ *and* $\boldsymbol{PR} = \boldsymbol{Q}$, *determine:*

(i) *the order of matrix* ***R***

(ii) *matrix* ***R***

12. *If matrix* $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$, *find matrix* ***A*** *such that* $\boldsymbol{AP} = \boldsymbol{I}$, *where* ***I*** *is a* $\boldsymbol{2 \times 2}$ *identity matrix.*

13. *If matrix* $\boldsymbol{A} = \begin{pmatrix} x & -7 \\ 4 & 6y \end{pmatrix}$ *and* $\boldsymbol{B} = \begin{pmatrix} 17-y & -21 \\ 12 & 36 \end{pmatrix}$, *find the values* ***x*** *and* ***y*** *such that* ***3A = B***

14. *Find the values of* ***a*** *and* ***b*** *such that* $\begin{pmatrix} 3 & b \\ 4 & a \end{pmatrix}\begin{pmatrix} 7a \\ 2 \end{pmatrix} = \begin{pmatrix} 43 \\ 30 \end{pmatrix}$

15. *Find the values of* k *and* n *such that* $\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix} + k\begin{pmatrix} 3 & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$

16. *Find the values of* x *and* y *such that* $(1 \quad 3 \quad 2)\begin{pmatrix} 4 & 3 \\ x & 2 \\ 10 & y \end{pmatrix} = (39 \quad 25)$

17. *Find the values of* x *and* y *such that* $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

18. *Given that matrix* $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, *find the value of* λ *such that* $A^2 + \lambda I = 5A$, *where* I *is a* 2×2 *identity matrix*

19. *Given that matrix* $A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ *and* $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$, *find the possible values of* x *such that* $AB = BA$

20. *Find the values of* x *for which the matrix* $\begin{pmatrix} x & 6 \\ 8 & 3x \end{pmatrix}$ *has no inverse*

21. *Find the values of* x *for which the matrix* $\begin{pmatrix} x & 3 \\ 4 & x-4 \end{pmatrix}$ *is singular*

22. *Find the values of* x *for which the matrix* $\begin{pmatrix} 2x & 3x \\ 2 & x \end{pmatrix}$ *is singular*

23. *Given that matrix* $M = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find the values of* λ *such that the matrix* $(M - \lambda I)$ *is singular, where* I *is a* 2×2 *identity matrix*

DAY 4

SOLUTION TO SIMULTANEOUS EQUATIONS BY MATRIX METHOD

Summary:

The following steps apply in solving simultaneous equation using matrix method:

(i) *Write the equations in matrix form*

(ii) *Find the inverse of the* $\boldsymbol{2\times 2}$ *matrix*

(iii) *Pre multiply both sides of the matrix equation by the inverse matrix*

activity

1. *Use matrix method to solve the following simultaneous equations:*

(i) $x - y = 5$, $3x + 2y = 5$ **(ii)** $2x - 5y + 14 = 0$, $4x + 3y - 11 = 0$ **(iii)** $4x + 3y = 24$, $2y - 3y = -1$

(iv) $x + y = 15$

$$\frac{x}{3} + \frac{y}{9} = 3$$

2. *Find the inverse of* $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$*, hence solve the simultaneous equations*

$3x + 2y = 12$

$4x + 5y = 23$

3. *Tom bought* **3** *pens and* **2** *books at* **Shs 4,800.** *Bob bought* **5** *pens and* **4** *books from the same shop at* **Shs 9,000.**

(i) *Form two equations to represent the above information*

(ii) *Use matrix method to find the cost of each pen and that of each book*

(iii) *How much would Ben pay for* **10** *pens and* **6** *books*

4. *Shs* **4000** *can buy* **10** *bans and* **5***cakes or* **4***bans and* **10***cakes.*

(i) *Form two equations to represent the above information*

(ii) *Find by matrix method the cost of each ban and that of each cake.*

DAY 5

MATRIX WORD PROBLEMS

activity

1. *Tom, Bob and Ben went to a supermarket for shopping **.***
*Tom bought **3** pens and **5** books and **4** rulers*
*Bob bought **4** pens and **3** books and **2** rulers*
*Ben bought **6** pens and **3** rulers*
*The cost of a pen is **Shs 500,** a book is **Shs 800** and a ruler is **Shs 1500.***

(a) *Write down:*

(i) *a 3×3 matrix for the items bought by the three boys.*

(ii) *a 3×1 costmatrix for each item*

(b)*Use matrix multiplication to find the amount of money spent by each boy*

2. *In a swimming competition, **7** points were awarded for each first-place finish,**4** points for second and **2** points for third.Senior one had **4** first place finishes,**7** second place finishes and **3** third place finishes.*

*Senior two had **8** first place finishes,**9** second place finishes and **1** third place finish.*

*Senior three had **10** first place finishes,**5** second place finishes and **3** third place finishes.*

*Senior four had **3** first place finishes,**3** second place finishes and **6** third place finishes.*

(a) *Write down:*

__(i)__ a $\boldsymbol{4 \times 3}$ matrix for the number of finishes each class had.

__(ii)__ a $\boldsymbol{3 \times 1}$ matrix for the points awarded for each finish

__(b)__Use matrix multiplication to determine the winner of the competition

3. *Shops **A, B, C,** and **D**ordered for balls, bats and gloves as follows**:***

	Balls	***Bats***	***Gloves***
Shop A	***70***	***30***	***50***
Shop B	***60***	***20***	***25***
Shop C	***40***	***15***	***10***
Shop D	***50***	***40***	***30***

The balls cost **Shs 5,000***each, bats* **Shs 3,000***each and gloves* **Shs 2,000***each*

*__(a)__Write down**:***

__(i)__ a $\boldsymbol{4 \times 3}$ matrix for theitems ordered by each shop.

__(ii)__ a $\boldsymbol{3 \times 1}$ cost matrix for each item

*__(b)__By matrix multiplication**,** find the total cost of the items for each shop*

__(c)__ If the supplier had to pay a tax of __20%__ of the cost of the items sold__,__ find his expenditure on the order__.__

More exercises

1. *Given that **I** is an identity matrix of order* **2 × 2** *and matrix* $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{2} & \boldsymbol{-1} \\ \boldsymbol{-2} & \boldsymbol{-5} \end{pmatrix}$**,**

find matrix ***B = A+2I***

2. *Find the inverse of matrix* $\boldsymbol{P} = \begin{pmatrix} \boldsymbol{-2} & \boldsymbol{3} \\ \boldsymbol{-4} & \boldsymbol{5} \end{pmatrix}$

3. *Use matrix method to solve the simultaneous equations**:***

$$\frac{x}{2} + \frac{y}{3} = 5$$
$$\frac{x}{3} + \frac{y}{4} = 1$$

8. *A hotel rents double rooms at* ***Shs 40,000*** *per day and single rooms at* ***Shs25,000*** *per day**.** If* ***14*** *rooms were rented one day for a total of* ***Shs 470,000***

(i) *Form two equations to represent the above information*

(ii) *Find by matrix method how many rooms of each kind were rented**.***

4. *In the morning**,5*** *breads and* ***8*** *cakes were bought.*

*In the afternoon**, 7*** *breads and* ***6*** *cakes were bought.*

The cost of a bread is ***Shs 4000*** *and a cake is* ***Shs 1200***

(a) *Write down**:***

(i) *a* $\boldsymbol{2 \times 2}$*matrix for the bought items*

(ii)*a* $\boldsymbol{2 \times 1}$*cost matrix for each item*

(b) *Use matrix multiplication to find the expenditure in each case.*

17. *Given that matrix* $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ x & y \end{pmatrix}$, *find the values of* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *such that* $\boldsymbol{A^2} = \boldsymbol{I}$,

where $\boldsymbol{I}$ *is a* $\boldsymbol{2 \times 2}$ *identity matrix*

4. *Given that* $\boldsymbol{P} = \begin{pmatrix} 6 & -4 \\ 2 & -1 \end{pmatrix}$ *and* $\boldsymbol{PQ} = \begin{pmatrix} 16 & -18 \\ 6 & -5 \end{pmatrix}$, *find**:***

(i) *the inverse of* $\boldsymbol{P}$.

(ii) *matrix*$\boldsymbol{Q} = \boldsymbol{P^{-1}}[\,\boldsymbol{PQ}\,]$.

5. *Find the values of* $\boldsymbol{x}$ *for which the matrix* $\begin{pmatrix} x & x+9 \\ 2 & x+5 \end{pmatrix}$ *has no inverse*

19. *Find the values of* $\boldsymbol{x}$ *for which the matrix* $\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$ *is singular*

19. *Find the values of* $\boldsymbol{x}$ *for which the matrix* $\begin{pmatrix} x & 2x \\ x-1 & x+1 \end{pmatrix}$ *is singular*

19. *Find the values of* $\boldsymbol{x}$ *for which the matrix* $\begin{pmatrix} x-5 & 3 \\ -2 & x \end{pmatrix}$ *is singular*

19. *Find the values of* $\boldsymbol{x}$ *for which the matrix* $\begin{pmatrix} x & 4 \\ 1 & x-3 \end{pmatrix}$ *is singular*

21. *Given that matrix* $M = \begin{pmatrix} 2 & -1 \\ -6 & 1 \end{pmatrix}$, *find the values of* $\boldsymbol{k}$ *such that the matrix* $(kI - M)$ *is singular, where* $\boldsymbol{I}$ *is a* 2×2 *identity matrix*

21. *Given that matrix* $A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$, *find the values of* λ *such that the matrix* $(A - \lambda I)$ *is singular, where* $\boldsymbol{I}$ *is a* 2×2 *identity matrix*

16. *Find the values of* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *such that* $(1 \quad 3)\begin{pmatrix} 4 & y \\ x & 2 \end{pmatrix} = (7 \quad 7)$

2. *Given that matrix* $P = \begin{pmatrix} x+7 & x \\ 3 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} x-1 & 0 \\ 2 & 2 \end{pmatrix}$ *and* $R = P + Q$, *find the value of* $\boldsymbol{x}$ *for which the determinant of* $\boldsymbol{R}$ *is* **2**

21. *Given that matrix* $P = \begin{pmatrix} 4x+1 & 3x \\ 2x+1 & 2x \end{pmatrix}$, *find the values of* $\boldsymbol{x}$ *for which the determinant of* $\boldsymbol{P}$ *is* **6**

6. Shs 244,000 *can buy* **5** *bans and* **6***cakes***,** *while* **Shs356000** *can buy* **7** *bans* *and* **9***cakes.* *Find by matrix method the cost of each ban and that of a cake.*

7. *Find the values of* y *for which the matrix* $\begin{pmatrix} 2y & 5 \\ 4 & y+\frac{1}{y} \end{pmatrix}$ *is singular*

8. *Given that matrix* $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, *find the value of* λ *such that* $A^2 + \lambda I = 5A,$ *where* I *is a* 2×2 *identity matrix*

9. *Given the matrix* $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$,, *find the values of* x *and* y *such that*

$xA + yI = A^2$, *where* I *is a* 2×2 *identity matrix.*

9. *Given the matrix* $A = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$,, *find the values of* x *and* y *such that*

$A^2 = \begin{pmatrix} x & -6 \\ -2 & y \end{pmatrix}$

10. *Bob and Ben went to a supermarket for shopping.*
Bob bought **2kg** *of sugar*, **4** *bars of soap*, **5** *counter books and one bottle of cooking oil.*
Ben bought **5kg** *of sugar*, **3** *bars of soap and a dozen of counter books.*
The cost of sugar per kg was **Shs1,500,** *a bar of soap was* **Shs1,000,** *a counter book was* **Shs 3,000** *and a bottle of cooking oil was* **Shs 2,000.**

(a)*Write down:*

(i) *a* 2×4 *matrix for the items bought by the two people.*

(ii) *a* $\boldsymbol{4 \times 1}$ *cost matrix for each item*

(b) *Calculate the**:***

(i)*expenditure of each person by matrix multiplication*

(ii)*total expenditure of both Bob and Ben*

(c) *How much did Ben spend than Bob*

3. *A charity organization donated Ball pens, exercise books, graph books and table books to senior four, three and two Classes of a school as below;*

Senior four students got ***2*** *ball pens,* ***12*** *exercise books,* ***3*** *graph books and* ***1*** *table book each.*

Senior three students got ***2*** *ball pens,* ***8*** *exercise books,* ***1*** *graph books and* ***1*** *table book each.*

Senior two students got ***1*** *ball pens**,*** ***6*** *exercise books and* ***1*** *table book each*

There are ***100*** *students in senior four,* ***120*** *in senior three and* ***130*** *students in senior two.*

*The organization bought the items at the following rates**:***

Ball pens at ***Shs500*** *each, Exercise books at* ***Shs1500*** *each, graph book at* ***Shs 2000*** *each and table books at* ***Sh.6000*** *each.*

(a) *Write down*

(i) $\boldsymbol{1 \times 3}$ *matrix for the number of students.*

(ii) $\boldsymbol{3 \times 4}$ *matrix for the items*

(iii) $\boldsymbol{4 \times 1}$ *cost matrix.*

(b)*By matrix multiplication, determine the*

(i) *number of items of each type distributed.*

(ii) *total amount spent by the organization in acquiring the items.*

(c) *If the organization had to pay **5%VAT** on the items bought, determine the total amount spent.*

10.*If matrix* $\boldsymbol{B} = \begin{pmatrix} \boldsymbol{1} & \boldsymbol{2} \\ \boldsymbol{5} & \boldsymbol{3} \end{pmatrix}$*, find matrix* ***A****such that* $\boldsymbol{AB} = \begin{pmatrix} \boldsymbol{17} & \boldsymbol{13} \\ \boldsymbol{21} & \boldsymbol{14} \end{pmatrix}$

9.*If matrix* $\boldsymbol{P} = \begin{pmatrix} \boldsymbol{3} & \boldsymbol{-4} \\ \boldsymbol{-2} & \boldsymbol{3} \end{pmatrix}$*, find matrix* ***A****such that* $\boldsymbol{AP} = \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{1} & \boldsymbol{0} \end{pmatrix}$

15. If $\begin{pmatrix} x & -2 \\ -1 & y \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 18 & 0 \\ 0 & 18 \end{pmatrix}$*, find x and y*

16. Given that $\begin{pmatrix} 3 & x \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 1 & y \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 13 & 7 \end{pmatrix}$*, find x and y.*

1. *Given that* $A = \begin{pmatrix} 2 & -7 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 8 & 3 \\ 0 & 2 \end{pmatrix}$ and $C = BA$, find;
2. *i)* $C + 3B$
3. *ii)* C^{-1}

Give that $P = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 4 & 6 \\ 10 & 15 \end{pmatrix}$*, find matrix T such*

$T = P^2 + 3Q - R$

4. *If* $M = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$;

a) Determine ; i) M^2 *ii)* M^3

b) identify matrix M^2

12. Four Secondary schools football teams of Ntare H.S, LayibiCollege, Mvara S.S and Kitende S.S qualified for a football tournament , which was played in two rounds with other teams.

First round

Ntare H.S won three matches, drew one and lost one match.

Layibi college won two matches, drew one and lost two matches

Mvara S.S won one month , drew three and lost one match.

Kitende S.S won four matches , drew one and lost no match.

Second round:

Ntare H.S won three matches , drew two and lost no match.

Layibi college won two matches, drew two and lost one match.

Mvara S.S won no match, drew three and lost two matches.

Kitende S.S won three matches, drew two and lost no match.

a) Write down:

(i) a 4 x 3 matrix to show the performance of the four teams in each of the two rounds. (02 marks)

(ii) a 4 x 3 matrix which shows the overall performance of the teams in both rounds. (02 marks)

b) If three points are awarded for a win, one point for a draw and no point for a loss, use matrix multiplication to determine which school won the tournament.

(03 marks)

c) Given that MTN donated sh. 3, 450,000 to be shared by the four teams according to the ratio of their points scored in the tournament, find how much money each team got. (05 marks)

DAY 6

STATISTICS

Summary:

1. *For a set of* **n** *values***:**

$$\textbf{(i)}\ \textit{mean}\ \textbf{(Average)} = \frac{\textbf{sum of values}}{\textbf{number of values}} = \frac{\textbf{sum}}{\textbf{count}}$$

(ii) *Median is the middle value when the given data is listed in order of magnitude. If the number of items is even, the average of the middle two is used.*

(iii) *Mode is the value that occurs most frequently.*

There can be more than one mode in a given data.

(iv) *Range is the difference between the largest and smallest values*

EXAMPLES:

1. *Find the mean, mode and median of the following numbers**: 7, 8, 10, 12** and **8.***

2. *Find the mean, mode and median of the following numbers**: 31, 28, 30, 33, 25** and **30.***

3. *The mean of **3, 7, 10, 8** and **x** is **6.** Find **x***

4. *The marks scored by a boy in four tests were **45, 70, 35** and **40.** When he does a fifth test the mean mark of the five tests is **50.** Find his scored mark in the fifth test*

5. *If the mean of **6** numbers is **30,** find the sum of these numbers*

6. *The mean marks for a French test in a class of **30** boys and **20** girls are **60** and **70** respectively. Find the mean mark for the whole class*

Soln:

Required mean $= \dfrac{(30\times 60) + (20\times 70)}{30+20} = \mathbf{64}$

7. *In a class of boys and girls, the average age is* $\mathbf{15\frac{1}{2}}$ *years. The class has* ***12*** *boys whose average age is* $\mathbf{16\frac{3}{4}}$ *years. Find the size of the class, if the average age of the girls is* ***15*** *years.*

Soln:

If n = number of girls

$$\Rightarrow \quad \frac{(12\times 16\cdot 75) + (15\times n)}{12+n} = 15\cdot 5$$

$\mathbf{n = 30}$

$\therefore$ *Class size = 12 + 30 =* ***42***

8. *The mean age of a class of* ***30*** *students is* ***16*** *years* ***3*** *months. If* ***12*** *students whose mean age is* ***14*** *years* ***6*** *months left the class, find the mean age of those who remained.*

Soln:

Required mean $= \dfrac{(30\times 16\cdot 25) - (12\times 14\cdot 5)}{30-12} = \mathbf{17\cdot 4167}$

9. *The mean weight of a class of* ***30*** *boys is* ***x kg.*** *When two boys with a total weight of* ***150kg*** *are absent, the mean weight of those present is* ***2kg*** *less than the mean weight of the whole class. Find the value of* ***x.***

Soln:

If mean for those present = $x - 2$

$$\Rightarrow \quad \frac{30x - 150}{30-2} = x - 2$$

$\therefore\ \mathbf{x = 47}$

ACTVITY 7

1. *The mean of* ***n*** *numbers is* ***5.*** *If the number* ***13*** *is included with the* ***n*** *numbers, the new mean is* ***6.*** *Find the value of* ***n.***

2. *In a set of* ***10*** *numbers****,*** *the mean of* ***6*** *numbers is* ***64.5*** *and that of the* ***10*** *numbers is* ***68.*** *Find the mean of the other four numbers.*

3. The mean of ***3, 7, 3, x, 8, 10*** *and* ***x*** *is* ***7.*** *Find* ***x***

4. *The mean heights of* ***20*** *boys and* ***15*** *girls are* ***1·60m*** *and* ***1·52m*** *respectively. Find the mean height of the* ***35*** *boys and girls.*

5. *The average age of* ***6*** *men is* ***45*** *and* ***5*** *of the men are* ***47, 40, 38, 46*** *and* ***43*** *years old. Find the age of the sixth man.*

6. *The average age of* ***6*** *men is* ***37*** *and one of them is* ***42*** *years old. Find the average age of the other five men.*

7. *The average age of a class of* ***30*** *boys is* ***14*** *years* ***4*** *months. If five boys whose average age is* ***15*** *years* ***2*** *months leave the class, find the average age of the* ***25*** *remaining boys****.***

8. *In a class of* ***30*** *students, there are* ***20*** *boys whose average age is* ***19*** *years* ***7*** *months and the rest are girls****.*** *Given that the mean age for the whole class is* ***18*** *years* ***4*** *months****,*** *find the mean age of the girls in the class****.***

9. *A class of* ***15*** *boys took an examination in which* ***7*** *boys got an average mark of* ***40*** *and* ***7*** *others got an average mark of* ***50.*** *The average mark for the whole class was* ***46.*** *How many marks did the other boy get****?***

10. *The mean of four numbers is* ***20.*** *If two other numbers* ***(x + 3)*** *and* ***(x + 2)*** *are added****,*** *the new mean is* ***30.*** *Find the value of* ***x.***

DAY 7

FREQUENCY DISTRIBUTION TABLES

Summary:

1. *A frequency table shows a summary of values and their frequency*

2. (i) *Data that is listed is called ungrouped data*

(ii) *Data that is grouped together in classes is called grouped data*

3. *The following terms may be needed***:**

(i) *Class boundaries are class groups in continuous form*

(ii) *Class width = upper class boundary – lower class boundary*

(iii) *Cumulative frequency is obtained by adding frequencies as you go along*

4. *In a frequency distribution table***,** *mean can be computed as follows***:**

(i) *mean* $= \dfrac{\sum fx}{\sum f}$, *where*

f = *frequency* (*number of times of occurrence*)

x = *class mid values*

(ii) *mean* $= A + \dfrac{\sum fd}{\sum f}$, *where*

A = *assumed mean or working mean*

$d = x - A$ *(deviation)*

5. (i) *The class which contains the mode is called the modal class*

(ii) *The modal class is the one with the highest frequency*

6. *For grouped data, mode is calculated as follows:*

(i) *Determine the modal class*

(ii) $\textbf{mode} = L + \left(\dfrac{D_1}{D_1 + D_2} \right) c$, *where*

L = *lower boundary of the modal class*

D_1 = *modal frequency* $-$ *premodal frequency*

D_2 = *modal frequency* $-$ *post modal frequency*

c = *modal class width*

7. (i) *The class which contains the median is called the median class*

(ii) *The median class corresponds to a cumulative frequency of* $\frac{1}{2}\sum f$

8. *For grouped data, median is calculated as follows:*

(i) *Determine the median class*

(ii) $\textbf{median} = L + \left(\dfrac{\frac{1}{2}\sum f - Cf_b}{f_w} \right) c$, *where*

L = *lower boundary of the median class*

$\sum f$ = *total frequency*

Cf_b = *cumulative frequency before the median class*

f_w = *frequency within the median class*

c = *median class width*

9 (i) *The cumulative frequency curve* **or** *an ogive is a curve where cumulative frequencies are plotted against the upper class boundaries. It can estimate the median*

(ii) *A histogram consists of bars with frequency as the vertical and class boundaries as the horizontal. It can estimate the mode*

(iii) *A frequency polygon is a line graph drawn by plotting frequency against class mid values.*

NOTE:

(a) *The points are joined by straight lines.*

(b) *The polygon extends to the next lower and higher classes with zero frequencies*

activity

1. *The marks of students in a test were as follows:*

Marks	**4**	**5**	**6**	**7**	**8**
No of students	**2**	**6**	**4**	**5**	**3**

(a) *State the modal mark*

(b)*Find the:*

(i) *mean mark*

(ii) *median mark*

2. *The marks of students in a test were as follows:*

Marks	**3**	**4**	**5**
No of students	**3**	***x***	**4**

Given that the mean mark is **4·1,** *find* **x**

3. *The ages in years of **40** students were as follows**:***

12	***13***	***14***	***12***	***15***	***14***	***13***	***16***	***14***	***15***
13	***14***	***16***	***15***	***14***	***12***	***13***	***14***	***15***	***13***
15	***16***	***15***	***14***	***15***	***12***	***15***	***13***	***12***	***15***
13	***15***	***12***	***15***	***16***	***14***	***15***	***14***	***16***	***14***

(a) *Form an ungrouped frequency distribution table for the data*

(b) *State the modal mark*

(c) *Find the**:***

(i) *mean mark*

(ii) *median mark*

4. *The marks of students in a test were as follows**:***

Marks	***5***	***8***	***10***	***14***	***18***	***20***
No of students	***2***	***5***	***12***	***3***	***11***	***7***

*Calculate the mean mark using an assumed mean of **10,***

5. *The age distribution of **40** adults were as follows**:***

Age	***Frequency***	***Cumulative frequency***
20 – 29	***4***	***4***
30 – 39	***12***	***16***
40 – 49	***8***	…………
50 – 59	***9***	…………
60 – 69	***7***	…………

(a) *Copy and complete the cumulative frequency column*

(b) *State the**:***

(i) *class width*

(ii) *modal class*

(c) *Determine the median class*

(d) *Calculate the**:***

(i) *mean*

(ii) *mode*

(iii) *median*

(c) *Display the data on a histogram and use it to estimate the mode.*

(d) *Draw an ogive for the data and use it to estimate the median*

(e) *Display the data on a frequency polygon*

6. *The marks of **40** students were as follows**:***

26	11	10	12	14	16	20	25
21	22	13	17	18	27	30	32
27	35	40	44	39	28	37	26
44	37	36	39	28	46	32	15
16	19	34	43	26	38	48	40

(a) *Form a frequency distribution table with a lower class of* $\mathbf{10 - 14}$**.**

(b) *Calculate the**:***

(i) *mean*

(ii) *mode*

(iii) median

(c) *Display the data on a histogram and use it to estimate the mode.*

(d) *Plot an ogive for the data and use it to estimate the median*

7. *The age distribution of* ***40*** *adults were as follows:*

Ages	$2{\cdot}0-2{\cdot}4$	$2{\cdot}5-2{\cdot}9$	$3{\cdot}0-3{\cdot}4$	$3{\cdot}5-3{\cdot}9$	$4{\cdot}0-4{\cdot}4$	$4{\cdot}5-4{\cdot}9$
Frequency	8	9	10	6	12	5

(a) *State the:*

(i) *class width*

(ii) *modal class*

(b) *Determine the median class*

(c) *Calculate the:*

(i) *mean*

(ii) *mode*

(iii) *median*

(d) *Display the data on a histogram and use it to estimate the mode.*

(e) *Draw an ogive for the data and use it to estimate the median*

8. *The cumulative distribution table shows the marks scored by* ***50*** *students.*

Marks	Cumulative frequency
$30-39$	5
$40-49$	13
$50-59$	23
$60-69$	39
$70-79$	46
$80-89$	50

(a) *Draw an ogive for the above data and use it to estimate the:*

(i) *median mark*

(ii) *pass mark of the test if* ***39*** *students passed*

(iii) *number of students who scored* ***75*** *marks and above*

(b) *Form a frequency distribution table for the above data to calculate the mean mark*

9. *The weights in kg of* ***40*** *students were as follows:*

Weights	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	1	7	9	8	10	5

Calculate the mean weight using an assumed mean of ***54·5***

10. *The weights of* ***40*** *students were as follows:*

50	51	50	52	54	56	60	65
61	62	53	57	58	64	70	72
67	75	67	70	56	66	65	69
72	77	76	57	66	68	62	55
56	59	74	73	78	66	67	74

(a) *Form a frequency distribution table with class width of* ***5*** *starting with class of* ***50 – 54***

(b) (i) *Display the data on a histogram and use it to estimate the mode*

(c) *Calculate the:*

(i) *mean using a working mean of* ***62***

(ii) *mode*

(iii) *median*

activity

***1.** The weights in kg of **50** babies in a maternity ward were as follows:*

Age	***Frequency***	***Cumulative frequency***
2·0 – 2·4	***8***	…………
2·5 – 2·9	***9***	…………
3·0 – 3·4	***10***	…………
3·5 – 3·9	***6***	…………
4·0 – 4·4	***12***	…………
4·5 – 4·9	***5***	…………

***(a)** Copy and complete the cumulative frequency column*

(b)** State the**:

***(i)** class width*

***(ii)** modal class*

***(c)** Determine the median class*

(b)** Calculate the**:

***(i)** mean*

***(ii)** mode*

***(iii)** median*

***(c)** Display the data on a histogram and use it to estimate the mode.*

***(d)** Plot an ogive for the data and use it to estimate the median*

***(e)** Display the data on a frequency polygon*

2. *The weights in kg of* **50** *students were as follows***:**

Weights	***20 − 24***	***25 − 29***	***30 − 34***	***35 − 39***	***40 − 44***	***45 − 49***
Frequency	***8***	***9***	***10***	***6***	***12***	***5***

(a) *Calculate the***:**

(i) *mean*

(ii) *mode*

(iii) *median*

(b) *Display the data on a histogram and use it to estimate the mode*

(c) *Display the data on an ogive and use it to estimate the median*

3. *The age distribution of* **40** *adults were as follows***:**

Age	***F***	***x***	***fx***	***Cumulative frequency***
20 − 29	***4***	***24·5***	***98***	***4***
30 − 39	***12***	…….....	…………	***16***
40 − 49	…….......	***44·5***	***356***	…………
50 − 59	***9***	…….....	…………	…………
60 − 69	…….......	***64·5***	***451·5***	…………
	∑f = ……		***∑fx =……...***	

(a) *Copy and complete the frequency distribution table above*

(b) *State the***:**

(i) *class width*

(ii) *modal class*

(c) (i) *Determine the median class*

(ii) *Calculate the mean age*

4. *The cumulative distribution table shows the marks scored by* ***50*** *students.*

Marks	***Cumulative frequency***
30 – 39	***5***
40 – 49	***13***
50 – 59	***23***
60 – 69	***39***
70 – 79	***46***
80 – 89	***50***

(a) *Draw an ogive for the above data and use it to estimate the****:***

(i) *median mark*

(ii) *pass mark of the test if* ***39*** *students passed*

(iii) *number of students who scored* ***75*** *marks and above*

(b) *Form a frequency distribution table for the above data to calculate the mean mark*

5. *The ages in years of* ***40*** *students were as follows****:***

12	***13***	***14***	***12***	***15***	***14***	***13***	***16***	***14***	***15***
13	***14***	***16***	***15***	***14***	***12***	***13***	***14***	***15***	***13***
15	***16***	***15***	***14***	***15***	***12***	***15***	***13***	***12***	***15***
13	***15***	***12***	***15***	***16***	***14***	***15***	***14***	***16***	***14***

(a) *Form an ungrouped frequency distribution table for the data*

(b) *Find the:*

(i) mode

(ii) median

6. *The weights in kg of* **50** *babies in a maternity ward were as follows:*

$4{\cdot}2$	$3{\cdot}1$	$2{\cdot}8$	$4{\cdot}0$	$2{\cdot}3$	$3{\cdot}7$	$3{\cdot}3$	$4{\cdot}4$	$2{\cdot}5$	$3{\cdot}0$
$3{\cdot}6$	$4{\cdot}3$	$3{\cdot}2$	$2{\cdot}4$	$4{\cdot}1$	$3{\cdot}4$	$2{\cdot}7$	$4{\cdot}2$	$4{\cdot}8$	$2{\cdot}6$
$2{\cdot}2$	$3{\cdot}0$	$4{\cdot}1$	$4{\cdot}6$	$3{\cdot}7$	$2{\cdot}9$	$4{\cdot}3$	$2{\cdot}0$	$3{\cdot}2$	$4{\cdot}0$
$4{\cdot}7$	$2{\cdot}6$	$3{\cdot}8$	$2{\cdot}3$	$4{\cdot}0$	$3{\cdot}3$	$2{\cdot}7$	$4{\cdot}5$	$2{\cdot}4$	$3{\cdot}6$
$2{\cdot}0$	$3{\cdot}5$	$2{\cdot}7$	$3{\cdot}2$	$2{\cdot}1$	$4{\cdot}2$	$3{\cdot}0$	$4{\cdot}1$	$2{\cdot}8$	$4{\cdot}7$

(a) *Form a frequency distribution table with class width* $0{\cdot}5$ *starting from* $2{\cdot}0 - 2{\cdot}4$

(b) *Calculate the:*

(i) mean using an assumed mean of $3{\cdot}2$

(ii) mode

(iv) median

7. *The marks of* **40** *students were as follows:*

11	17	35	34	42	45	28	66
16	21	14	36	41	31	49	37
20	33	37	38	18	38	39	27
26	28	40	33	43	32	29	47
29	32	41	24	44	35	36	23

(a) *Form a frequency distribution table for the data starting with a class of* $10 - 14$

(b) *State the:*

(i) *class width*

(ii) *modal class*

(c) *Determine the**:***

(i) *mean mark*

(ii) *median class*

(d) *Display the data on a histogram and use it to estimate the mode*

(e) *Draw an ogive for the data and use it to estimate the median*

8. *The heights in* **cm** *of plants in a garden were as follows**:***

10.3	***9.7***	***10.2***	***9.8***	***10.1***
9.9	***10.1***	***9.9***	***10.1***	***10.2***
10.3	***10.0***	***10.2***	***10.1***	***9.8***
9.9	***10.1***	***10.0***	***10.1***	***9.9***
10.1	***10.1***	***10.1***	***10.1***	***9.9***
9.8	***9.8***	***10.0***	***9.9***	***10.2***

(a) *Copy and complete the frequency distribution table below**:***

Time ***(x)***	*Frequency**(f)***	*Cumulative frequency*	***fx***
9.7	…..	***1***	…..
9.8	***4***	***5***	…..
9.9	…..	…..	…..
10.0	***3***	…..	…..
10.1	…..	…..	…..
10.2	…..	…..	…..
10.3	…..	…..	…..
	$\sum f$ = ……		$\sum fx$ = ……..

(b) *Use the table to;*

(i) *State the modal height*

(ii) *Calculate the mean and median height*