SECTION A

1. Solve for x given $y = x + \frac{1}{x}$, in the equation $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$ *(05 marks)*
2. Prove that $x = 3t^2 + 1$ and $2y = 3t + 1$ are parametric equation of a parabola. Find its vertex, focus and length of latus rectum. *(05 marks)*
3. Given A, B and C are angles of a triangle, prove that $\frac{a^2+b^2-c^2}{a^2-b^2+c^2} = \tan B \cot C$. *(05 marks)*
4. Differentiate from first principles $y = \frac{1}{x^2}$. *(05 marks)*
5. Find the square root of $14 + 6\sqrt{5}$. *(05 marks)*
6. Find $\int x \ln x \, dx$. *(05 marks)*
7. Using small changes, find the $\sqrt{627}$ to 4 significant figures. *(05 marks)*
8. Find the angle between the planes $4x + 3y + 12z = 10$ and $8x - 6y = 14$. *(05 marks)*

SECTION B

9. (a) When a polynomial $P(x)$ is divided by $x - 1$ the remainder is 3 and when divided by $x - 2$ the remainder is 1. Prove that when divided by $x^2 - 3x + 2$ the remainder is $5 - 2x$. *(06 marks)*

   (b) Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^{12}$. *(06 marks)*

10. (a) Find the region where the curve $y = \frac{3x+3}{x(3-x)}$ does not lie, hence determine the turning points and their nature. *(04 marks)*

    (b) State the asymptotes and intercept. *(03 marks)*

    (c) Sketch the curve. *(05 marks)*

11. (a) A man pays premium of 100 dollars at the beginning of every year to an insurance company on an understanding that at the end of 15 years they can receive back the premium he had paid with 5% compound interest. What did he receive? *(06 marks)*

    (b) A committee of six is to be formed from nine women and three men. In how many ways can this chosen so as to include at least one man. *(06 marks)*

12. Partialise $\frac{x^3-10x^2+26x+3}{(x+3)(x-1)^3}$. *(12 marks)*

13. (a) Solve the differential equation $x\frac{dy}{dx} - 3 = 2\left(y + \frac{dy}{dx}\right)$. *(04 marks)*

(b) The rate at which malaria spreads in the body is proportional to the number if infected cells in the body. If the number of infected cells in the body at any time is N. Given that after 1 month the number of cells infected is doubled and considering the initial number of cells infected to be N_0.

i) Show that $N = N_0 e^{t\ln 2}$.

ii) Show that five months later the number of the infected cells is $32N_0$. *(08 marks)*

14. (a) Prove that $\sin 3x + \sin 5x + \sin 7x + \sin x = 4\sin 4x \cos 2x \cos x$. *(06 marks)*

(b) Solve for x from 0^0 to 360^0. Given that $\sec x + 3 = \cos x + \tan x\,(2 + \sin x)$.

(06 marks)

15. (a) Given $y = e^{3x}\sin 4x$. Show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$. *(04 marks)*

(b) Differentiate and simplify $y = \sqrt{\frac{(x+2)}{x-1}}$. *(08 marks)*

16. (a) Find the vector equation of the line of intersection between the planes

$$\boldsymbol{r}\cdot\begin{pmatrix}1\\1\\-3\end{pmatrix} = 6 \quad \text{and} \quad \boldsymbol{r}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix} = 4$$ *(06 marks)*

(b) Using the dot product, find the equation of the plane containing points A$(0,1,1)$, B$(2,1,0)$ and C$(-2,0,3)$. *(06 marks)*

END

NB:

1. *Members of* **UMTA** *am sorry to edit this question paper without your consent but me that's how I saw the questions were supposed to be.*
2. *If I have corrected them wrongly am sorry.*
3. *Am expecting to hear from any members for the corrections.*
4. *Below is the proposed marking Guide.*

PROPOSED

MARKING GUIDE

UMTA P425/1 2023

NO	SOLUTION	MKS	Comment
1	Dividing through by x^2 $4x^2+17x+8+\frac{17}{x}+\frac{4}{x^2}=0$ $4\left(x^2+\frac{1}{x^2}\right)+17\left(x+\frac{1}{2}\right)+8=0$ From $y=x+\frac{1}{x}$ Squaring both sides $y^2=x^2+2+\frac{1}{x^2}$ $\Rightarrow x^2+\frac{1}{x^2}=y^2-2$ $4(y^2-2)+17y+8=0$ $4y^2-8+17y+8=0$ $4y^2+17y=0$ $y(4y+17)=0$ $y=0$ or $y=-\frac{17}{4}$ When $y=0$; $x+\frac{1}{x}=0$ $x^2+1=0$ $x^2=-1$, x is an defined When $y=-\frac{17}{4}$ $x+\frac{1}{x}=-\frac{17}{4}$ $4x^2+4=-17x$ $4x^2+17x+4=0$ $(4x+1)(x+4)=0$ $x=-\frac{1}{4}$ or $x=-4$ $\therefore$ values of x are $-4, -\frac{1}{4}$		
		05	
2	$x=3t^2+1, 2y=3t+1$		

	From $2y=3t+1$ $t=\frac{2y-1}{3}$ $\Rightarrow x=3\left(\frac{2y-1}{3}\right)^2+1$ $x=\frac{(2y-1)^2}{3}+1$ $(2y-1)^2=3x-3$ $4y^2-4y+1=3x-3$ $4y^2-4y=3x-4$ $y^2-y=\frac{3}{4}x-1$ $\left(y-\frac{1}{2}\right)^2=\frac{3}{4}x-1+\left(\frac{1}{2}\right)^2$ $\left(y-\frac{1}{2}\right)^2=\frac{3}{4}x-\frac{3}{4}$ $\left(y-\frac{1}{2}\right)^2=\frac{3}{4}(x-1)$ hence a parabola By comparing with $(y-k)^2=4a(x-h)$ $4a=\frac{3}{4}$ $\quad\therefore a=\frac{3}{16}, k=\frac{1}{2}, h=1$ Vertex, $(h,k)=\left(1,\frac{1}{2}\right)$ Focus, $s(h+a,k)=s\left(\frac{19}{16},\frac{1}{2}\right)$ Length of latus rectum $=2(2a+k)=2\left(\frac{7}{8}\right)=\frac{7}{4}$ units		
		05	
3	From sine rule; $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ $\Rightarrow a=2R\sin A, b=2R\sin B, c=2R\sin C$ L.H.S $=\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$ $=\frac{4R^2\sin^2A+4R^2\sin^2B-4R^2\sin^2C}{4R^2\sin^2A-4R^2\sin^2B+4R^2\sin^2C}$ $=\frac{\sin^2A+[(\sin B+\sin C)(\sin B-\sin C)]}{\sin^2A+[(\sin C+\sin B)(\sin C-\sin B)]}$ $=\frac{\sin^2A+2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\cdot 2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\sin^2A+2\sin\left(\frac{C+B}{2}\right)\cos\left(\frac{C-B}{2}\right)\cdot 2\cos\left(\frac{C+B}{2}\right)\sin\left(\frac{C-B}{2}\right)}$ $=\frac{\sin^2A+\sin(B+C)\sin(B-C)}{\sin^2A+\sin(C+B)\sin(C-B)}$ $=\frac{\sin^2A+\sin A\sin(B-C)}{\sin^2A+\sin A\sin(C-B)}$		

	$\sin A[\sin A+\sin(C-B)]$ $=\frac{\sin(B+C)+\sin(B-C)}{\sin(B+C)+\sin(C-B)}$ $=\frac{2\sin B\cos C}{2\sin C\cos B}$ $=\tan B\cot C$		
		05	
4	$y=\frac{1}{x^2}$ $y+\delta y=\frac{1}{(x+\delta x)^2}$ $\delta y=\frac{1}{(x+\delta x)^2}-\frac{1}{x^2}$ $\delta y=\frac{x^2-(x+\delta x)^2}{x^2(x+\delta x)^2}$ $\delta y=\frac{x^2-x^2-2x\delta x-(\delta x)^2}{x^2(x+\delta x)^2}$ $\delta y=\frac{-2x\delta x-(\delta x)^2}{x^2(x+\delta x)^2}$ $\frac{\delta y}{\delta x}=\frac{-2x-\delta x}{x^2(x+\delta x)^2}$ As $\delta x\rightarrow 0, \frac{\delta y}{\delta x}\rightarrow\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{-2x}{x^4}=\frac{-2}{x^3}$ $\therefore \frac{dy}{dx}=\frac{-2}{x^3}$		
		05	
5	Let $\sqrt{14+6\sqrt{5}}=\pm(\sqrt{a}+\sqrt{b})$ Squaring both sides; $14+6\sqrt{5}=a+2\sqrt{ab}+b$ Equating corresponding components; Surdic; $2\sqrt{ab}=6\sqrt{5}$ $ab=45; \quad a=\frac{45}{b}$ ....(i) Non –surdic; $a+b=14$ $\frac{45}{b}+b=14$ $45+b^2=14b$ $b^2-14b+45=0$ $(b-9)(b-5)=0$		

	When $b = 5$, $a = \frac{45}{5} = 9$ Taking $a = 5$ when $b = 9$ $\therefore \sqrt{14+6\sqrt{5}} = \pm(\sqrt{5}+\sqrt{9}) = \pm(3+\sqrt{5})$		
		05	
6	Let $u = \ln x, \frac{dv}{dx} = x$ $\frac{du}{dx} = \frac{1}{x}, v = \frac{x^2}{2}$ $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int\left(x^2 \cdot \frac{1}{x}\right)dx$ $= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$ $= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$		
		05	
7	Let $y = \sqrt{x}$, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ $y + \delta y = \sqrt{x+\delta x}$ Taking $x = 625, \delta x = 2$ $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $\delta y \approx \frac{dy}{dx} \cdot \delta x$ $\approx \frac{1}{2\sqrt{625}} \times 2$ ≈ 0.04 $\sqrt{627} = 25 + \delta y$ $\sqrt{627} \approx 25 + 0.04$ $\therefore \sqrt{627} \approx 25.04$(4sfs)		
		05	
8	Let $n_1 = \begin{pmatrix}4\\3\\12\end{pmatrix}, n_2 = \begin{pmatrix}8\\-6\\0\end{pmatrix}$ Using $n_1 \cdot n_2 = \|n_1\|\|n_2\|\cos\theta$ $\begin{pmatrix}4\\3\\12\end{pmatrix} \cdot \begin{pmatrix}8\\-6\\0\end{pmatrix} = \sqrt{4^2+3^2+12^2}\sqrt{8^2+(-6)^2}\cos\theta$		

	$32-18+0=\sqrt{169}\sqrt{100}\cos\theta$ $14=13\times 10\cos\theta$ $\cos\theta=\frac{7}{65}$ $\therefore\theta=\cos^{-1}\left(\frac{7}{65}\right)=83.82^0$		
		05	
9	(a) let $ax+b$ be the remainder; $P(x)=Q(x)(x^2-3x+2)+ax+b$ $P(x)=Q(x)(x-2)(x-1)+ax+b$ When $x=1$, $P(1)=3$ $P(1)=a+b=3$ $a+b=3$ ....(i) When $x=2$, $P(2)=1$ $P(2)=2a+b=1$ $2a+b=1$.................(ii) $(i)-(ii);\ -a=2$ $\therefore a=-2$ From (i); $-2+b=3$ $\therefore b=5$ $\therefore -2x+5=5-2x$ is the remainder (b) using $u_{r+1}={}^{n}C_{r}\,a^{n-r}b^{r}$ $u_{r+1}={}^{12}C_{r}\,(2x)^{12-r}\cdot\left(\frac{1}{x^2}\right)^{r}$ $={}^{12}C_{r}\cdot 2^{12-r}\cdot x^{12-r}\cdot x^{-2r}$ ⇨ $12-r-2r=0$ $3r=12$ $\therefore r=4$ $u_5={}^{12}C_4\cdot 2^8$ $=126720$ $\therefore 126720$ is the term independent of x		
		12	
10	(a) Region where the curve does not lie $y(3x-x^2)=3x+3$ $3xy-x^2y=3x+3$ $yx^2+(3-3y)x+3=0$		

For non –real values of x; $b^2 - 4ac \leq 0$

$(3-3y)^2 - 4 \times y \times 3 \leq 0$

$9 - 18y + 9y^2 - 12y \leq 0$

$9y^2 - 30y + 9 \leq 0$

$3y^2 - 10y + 3 \leq 0$

$(y-3)(3y-1) \leq 0$

Critical values of y

$y = 3, y = \frac{1}{3}$

y	$y < \frac{1}{3}$	$\frac{1}{3} < y < 3$	$y > 3$
$(3y-1)$	−	+	+
$(y-3)$	−	−	+
$(3y-1)(y-3)$	+	−	+

Hence the curve does not exist in the range $\frac{1}{3} \leq y \leq 3$

Turning points;

When $y = 3$;

$3x^2 - 6x + 3 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$, $\therefore (1,3)_{min}$

When $y = \frac{1}{3}$;

$\frac{1}{3}x^2 + 2x + 3 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x = -3$, $\therefore \left(-3, \frac{1}{3}\right)_{max}$

(b) intercepts and asymptotes

$x, y = 0$

$3x + 3 = 0$

$x = -1$, $(-1,0)$

$y, x = 0, y$ –undefined

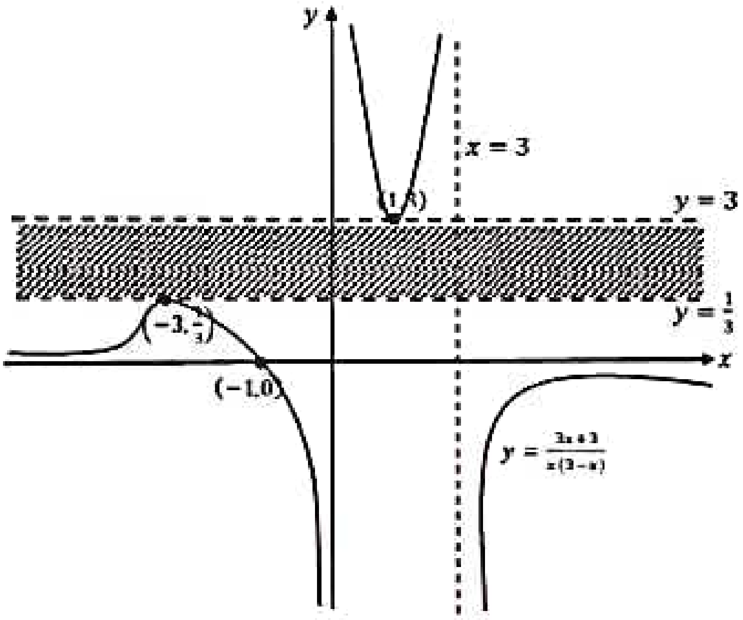
Vertical asymptote

$$y = \frac{\frac{3}{x}+\frac{3}{x^2}}{\frac{3}{x}-1}$$

As $x \to \pm\infty$; $y \to 0$

i.e $y = 0$

(c)



12

11 (a) From $A = P\left(1+\frac{r}{100}\right)^n$

Where P = \$100, $n = 15, r = 5\%$

Amount at the end of 1st year, $A_1 = P(1.05)$

Amount at the end of 2nd year, $A_2 = P(1.05)^2$

Amount at the end of 3rd year, $A_3 = P(1.05)^3$

.
.
.
.

Amount at the end of 15th year, $A_{15} = P(1.05)^{15}$

Total amount, A_T

$A_T = A_1 + A_2 + A_3 + \cdots + A_{15}$

$A_T = P[1.05 + 1.05^2 + 1.05^3 + \cdots + 1.05^{15}]$

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$A_T = P\left[\frac{a(r^n-1)}{r-1}\right]$ where $P = \$100$, $a = 1.05, r = 1.05$

$A_T = 100\left[\frac{1.05(1.05^{15}-1)}{1.05-1}\right]$

$A_T = \$\ 2265.749177$

(b)

Men (3)	Women (9)
1	5
2	4
3	3

$= \binom{3}{1}\times\binom{9}{5}+\binom{3}{2}\times\binom{9}{4}+\binom{3}{3}\times\binom{9}{3}$

$= 378 + 378 + 84$

$= 840$ ways

12

12

Let $\frac{x^3-10x^2+26x+3}{(x-3)(x-1)^3} \equiv \frac{A}{x-3}+\frac{B}{x-1}+\frac{C}{(x-1)^2}+\frac{D}{(x-1)^3}$

$x^3-10x^2+26x+3 \equiv A(x-1)^3+B(x-3)(x-1)^2+C(x-3)(x-1)+D(x-3)$

When $x = 1; 20 = -2D$ $\quad\therefore D = -10$

When $x = 3; 18 = 8A$ $\quad\therefore A = \frac{9}{4}$

When $x = 0; 3 = -A - 3B + 3C - 3D$

$3 = -\frac{9}{4} - 3B + 3C - 3(-10)$

$-\frac{99}{4} = -3B + 3C$

$-\frac{33}{4} = -B + C$ ……………………(i)

Comparing coefficients of;

$x^3; 1 = A + B$

$1 = \frac{9}{4} + B$ $\quad\therefore B = -\frac{5}{4}$

From (i); $-\frac{33}{4} = \frac{5}{4} + C$ $\quad\therefore C = -\frac{19}{2}$

$\therefore \frac{x^3-10x^2+26x+3}{(x-3)(x-1)^3} \equiv \frac{9}{4(x-3)} - \frac{5}{4(x-1)} - \frac{19}{2(x-1)^2} - \frac{10}{(x-1)^3}$

12

13

(a) $x\frac{dy}{dx} - 3 = 2y + 2\frac{dy}{dx}$

	$(x-2)\frac{dy}{dx}-2y=3$ $\frac{dy}{dx}-\frac{2}{x-2}y=\frac{3}{x-2}$ I.F $=e^{\int\frac{-2}{x-2}dx}$ $=-2\ln(x-2)$ $=\frac{1}{(x-2)^2}$ Multiplying through by $\frac{1}{(x-2)^2}$ $\frac{1}{(x-2)^2}\frac{dy}{dx}-\frac{2}{(x-2)^3}y=\frac{3}{(x-2)^3}$ $\int\left(\frac{y}{(x-2)^3}\right)dx=3\int(x-2)^{-3}dx$ $\frac{y}{(x-2)^3}=-\frac{3}{2(x-2)^2}+c$ $\therefore y=(x-2)^3\left[-\frac{3}{2(x-2)^2}+c\right]$ (b) $\frac{dN}{dt}\propto N$ $\frac{dN}{dt}=kN$ $\int\frac{dN}{N}=\int k\,dt$ $\ln N=kt+c$ $N=e^{kt+c}=e^{kt}\cdot e^{c}$ $N=Ae^{kt}$ When $t=0$, $N=N_0$ $N_0=Ae^0$ $\therefore A=N_0$ $N=N_0e^{kt}$ When $t=1$ month, $N=2N_0$ $2N_0=N_0e^k$ $e^k=2$ $\therefore k=\ln 2$ $\therefore N=N_0e^{t\ln 2}$ (ii) When $t=5$ months, $N=?$ $N=N_0e^{5\times\ln 2}$ $\therefore N=32N_0$		
		12	

14	(a) L.H.S$=\sin 5x+\sin 3x+\sin 7x+\sin x$ $=2\sin 4x\cos x+2\sin 4x\cos 3x$ $=2\sin 4x\,(\cos 3x+\cos x)$ $=2\sin 4x\cdot 2\cos 2x\cos x$ $=4\sin 4x\cos 2x\cos x$ =R.H.S (b) $\sec x+3=\cos x+\tan x\,(2+\sin x)$ $\frac{1}{\cos x}+3=\cos x+\frac{\sin x}{\cos x}(2+\sin x)$ Multiplying through by $\cos x$ $1+3\cos x=\cos^2 x+\sin x(2+\sin x)$ $1+3\cos x=\cos^2 x+2\sin x+\sin^2 x$ $1+3\cos x=1+2\sin x$ $\tan x=1.5$ $x=\tan^{-1}(1.5)$ $x=56.31^0, 236.31^0$		
		12	
15	(a) $y=e^{3x}\sin 4x$ $\frac{dy}{dx}=4e^{3x}\cos 4x+3e^{3x}\sin 4x$ $\frac{dy}{dx}=4e^{3x}\cos 4x+3y$ $\frac{d^2y}{dx^2}=4[-4e^{3x}\sin 4x+3e^{3x}\cos 4x]+3\frac{dy}{dx}$ $\frac{d^2y}{dx^2}=-16y+3(4e^{3x}\cos 4x)+3\frac{dy}{dx}$ $\frac{d^2y}{dx^2}=-16y+3\left(\frac{dy}{dx}-3y\right)+3\frac{dy}{dx}$ $\frac{d^2y}{dx^2}=-25y+6\frac{dy}{dx}$ $\therefore \frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=0$ (b) $y=\sqrt{\frac{(x+2)}{x-1}}$ $y^2=\frac{x+2}{x-1}$		

	$2y\frac{dy}{dx}=\frac{(x-1)\cdot1-(x+2)\cdot1}{(x-1)^2}$ $2y\frac{dy}{dx}=\frac{x-1-x-2}{(x-1)^2}$ $2y\frac{dy}{dx}=\frac{-3}{(x-1)^2}$ $\frac{dy}{dx}=\frac{-3}{(x-1)^2}\times\frac{(x-1)^{1/2}}{2(x+2)^{1/2}}$ $\frac{dy}{dx}=-\frac{3}{2\sqrt{(x-1)^3}\sqrt{x+2}}$		
		12	
16	(a) $\boldsymbol{r}\cdot\begin{pmatrix}1\\1\\-3\end{pmatrix}=6$ and $\boldsymbol{r}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix}=4$ Let $\boldsymbol{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ $x+y-3z=6$ ……………..(i) $2x-y+z=4$ ……………..(ii) (i) + (ii); $3x-2z=10$ Let $z=\mu$; $3x-2\mu=10$ $3x=10+2\mu$ $x=\frac{10}{3}+\frac{2}{3}\mu$ From (i): $\frac{10}{3}+\frac{2}{3}\mu+y-3\mu=6$ $y=\frac{8}{3}+\frac{7}{3}\mu$ $x=\frac{10}{3}+\frac{2}{3}\mu$ $y=\frac{8}{3}+\frac{7}{3}\mu$ $z=\mu$ $\boldsymbol{r}=\begin{pmatrix}\frac{10}{3}\\\frac{8}{3}\\0\end{pmatrix}+\mu\begin{pmatrix}\frac{2}{3}\\\frac{7}{3}\\1\end{pmatrix}$ or $\boldsymbol{r}=\begin{pmatrix}10\\8\\0\end{pmatrix}+\mu\begin{pmatrix}2\\7\\3\end{pmatrix}$		

(b) $n = AB \wedge AC$

$$AB = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$AC = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$$

$$n = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ -2 & -1 & 2 \end{vmatrix}$$

$$n = \begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix} i - \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} j + \begin{vmatrix} 2 & 0 \\ -2 & -1 \end{vmatrix} k$$

$$n = -i - 2j - 2k$$

Using $r \cdot n = a \cdot n$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$$

$$-x - 2y - 2z = 0 - 2 - 2$$

$$-x - 2y - 2z = -4$$

$$\therefore x + 2y + 2z = 4$$

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