

P425/2
APPLIED
MATHEMATICS
Paper 2
July /Aug. 2024
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** questions in section A and any **five** from section B.*

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

*Any extra question(s) attempted in section B will **not** be marked.*

SECTION A (40 MARKS)

1. Given that $P(A) = 0.7$, $P(A' \cup B) = 0.6$, find
 (a) $P(A \cap B')$
 (b) $P(B/A)$ (05 marks)

2. A particle executing simple harmonic motion starts from rest and next comes to rest after 6 seconds, covering 10m in this time;
 Calculate the;
 (a) period of motion. (02 marks)
 (b) maximum speed of the particle. (03 marks)

3. Calculate the maximum error in the function $x \sin x$ at $x = 30^\circ \pm 0.5^\circ$ (05 marks)

4. Study the table below;

x	5	6	7	8	9	10
f	1	5	10	8	4	2

Using an assumed mean, $m = 7$, calculate the mean and variance of x .

(05 marks)

5. The position vectors of two bodies A and B after t seconds of motion are;
 $\mathbf{r}_A = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 8 \end{pmatrix}$ and $\mathbf{r}_B = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + t \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ metres. Calculate the shortest distance between the bodies, and state the value of t at which this occurs. (05 marks)

6. A box contains 3 red and 2 white balls of the same size. The balls are thoroughly mixed. Balls are picked one at a time with replacement. Ten balls are picked, find the probability that;
 (a) exactly 5,
 (b) over 8 balls are red. (05 marks)

7. The points in the table below were taken from the curve $y = x^2$

x	0.1	0.2
y	0.01	0.04

Calculate the absolute errors made in using linear interpolation or extrapolation to estimate;

- (a) y when $x = 0.18$
 (b) x when $y = 0.05$

(05 marks)

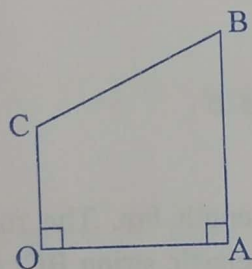
8. A particle is projected vertically upwards with a speed of 49ms^{-1} . Calculate the time interval within which the particle is at least 78.4 metres above the ground. (05 marks)

SECTION B (60 MARKS)

9. (a) Show that the equation $e^x = 2 - x$ has a root lying between 0 and 1. Hence use a graphical method to find the first approximate, x_0 , to the equation. (06 marks)
- (b) Use the Newton – Raphson method to find the root correct to 4 decimal places. (06 marks)

10. (a) ABCD is a uniform rectangular tray of mass 1kg, at rest on a horizontal table. Masses of 2, 4, 5 and 8kg are placed at the corners A, B, C and D respectively. Find the position of the centre of gravity of the system, with respect to the sides AB and AD. (04 marks)

(b)



OABC is a uniform lamina, in which $\overline{OA} = 6\text{cm}$, $\overline{AB} = 7\text{cm}$ and $\overline{OC} = 4\text{cm}$.

- (i) Find the coordinates of the centre of gravity of the lamina, taking \overline{OA} as the x – axis, and \overline{OC} as the y – axis. (04 marks)
- (ii) The lamina is suspended from point C, calculate the angle side OA makes with the vertical. (04 marks)

11. The table below shows the distribution of marks obtained by students in a paper 2 examination.

Marks (%)	0 - 10	10 - 20	20 - 40	40 - 45	45 - 60	60 - 100
Frequency density	0.8	1.0	1.5	4.4	2.8	0.4

- (a) Construct a histogram for this data, and use it to calculate the modal mark. (05 marks)
- (b) Plot a cumulative frequency curve, and use it to determine the;
- (i) Number of student who scored at least a 50%. (07 marks)
- (ii) Decile deviation.

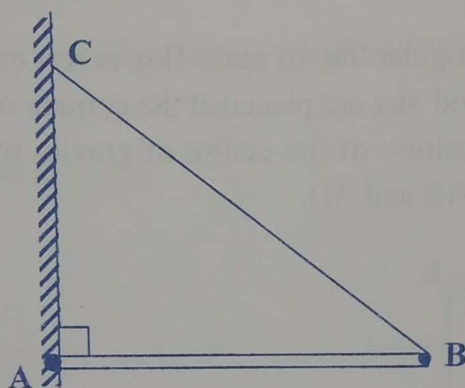
- ✓12. The heights of maize plants in a nursery bed are normally distributed with a mean of 16cm with a variance of 100cm^2 .

(a) Calculate the probability of obtaining a maize plant whose height is greater than 20cm. (04 marks)

(b) Twenty five plants are picked at random, find the probability that their mean height lies between 13cm and 19cm. (04 marks)

(c) How many plants should be picked at random before the probability that at least one of them has a height less than 16cm is greater than 0.9? (04 marks)

13. ✓



AB is a uniform rod of mass 12kg, and of length 8m. The rod is smoothly hinged, to a vertical wall, at point A. A light inelastic string BC, of length 10m, keeps the rod at rest, in a horizontal position, as shown in the diagram above.

(a) Calculate the;

(i) Tension, T in the string. (03 marks)

(ii) Reaction at the hinge A. (05 marks)

(b) A boy of mass 40kg starts to walk along the rod from point A towards B. Given that the tension in the string cannot be exceed $\frac{3}{2}T$, calculate the distance the boy will walk before the string snaps. (04 marks)

✓14. (a) Use the trapezium rule with 6 intervals to evaluate; (05 marks)

$$\int_0^2 x \sin x \, dx, \quad \text{correct to 4 decimal places.}$$

(b) Given that $x = 1.60 \pm 0.005$ and $y = 4.8 \pm 0.05$; calculate the maximum error in computing y/x ; hence or otherwise state interval within which the exact value of y/x lies. (07 marks)

15. (a) A car of mass 1000 kg, travels along a horizontal road against a constant resistance of 200 N. If the car develops a constant power of 30 kW, calculate the acceleration of the car at the instant when its speed is 30 ms^{-1} . (05 marks)

(b) A steamer is initially 60 km north of an observatory, O and travelling at 40 kmh^{-1} due $\text{N}60^\circ\text{E}$. At the same instant, a boat capable of a maximum speed of 64 kmh^{-1} and initially 80 km East of O, sets off to intercept the steamer.

Calculate the two possible courses the boat can take, and determine the shortest time of interception. (07 marks)

16. X is a continuous random variable whose probability density function is given as;

$$f(x) = \begin{cases} 2kx & ; 0 \leq x \leq 2 \\ k(6-x) & ; 4 \leq x \leq 6 \\ 0 & ; \text{elsewhere} \end{cases}$$

(a) Sketch the graph of $f(x)$ and use it to determine the value of the constant k . (04 marks)

(b) Find, $F(x)$, the cumulative distribution function of X, hence compute;

(i) $P(X > 5)$

(ii) the 80th percentile of X.

(08 marks)

END