P425/1

PURE MATHEMATICS

Paper 1

3rd November 2023

3 hours

HOLY CROSS LAKE VIEW SSS WANYANGE

Post Mocks

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt **all** the **eight** questions in **Section A** and **five** questions from Section **B.**

Any additional questions answered will not be marked.

All working **must** be shown clearly.

Mathematical tables with a list of formulae may be used.

Silent, non-programmable calculators may be used.

Begin each answer on a fresh page.

State the degree of accuracy. Indicate **CAL** for calculator and **TAB** for tables.

SECTION A: (40 MARKS)

*Answer **all** questions in this section.*

1. In a geometrical progression, the third term is 32 and the sixth term is 4. Find the sum of the first eight terms. *(05 marks)*

2. Solve the equation $\sin(x + 15^0)\cos(x - 15^0) = 0.5$ for $0^0 \leq x \leq 360^0$ *(05 marks)*

3. Solve the equation $2^{2x+8} - 2^{x+5} + 1 = 0$ *(05 marks)*

4. Find the acute angle between the line $\frac{x-1}{6} = \frac{y+3}{-4} = \frac{z-2}{5}$ and the plane $2x + 3y - z = 9$

   (05 marks)

5. Point P moves on the curve $y^2 = 4x$ and A is the point (1,0). Find the locus of the mid-point of AP. *(05 marks)*

6. Find the equation of the normal to the curve $x^2 - 3yx + 2y^2 - 2x = 4$ at the point (1, -1).

   (05 marks)

7. Show that $\int_0^{\frac{2\pi}{3}} 2x\sin\frac{1}{2}x\,dx = 4\sqrt{3} - \frac{4\pi}{3}$ *(05 marks)*

8. A hemispherical pot of internal radius 13 cm contains water to a maximum depth of 8 cm. Find, to four significant figures, the volume of water in the pot. *(05 marks)*

SECTION B: (40 MARKS)

*Answer any **five** questions in this section*

9. (a) Solve $Z\bar{Z} - 5iZ = 5(9 - 7i)$ where $\bar{Z}$ is the complex conjugate of Z *(06 marks)*

   (b) if $z_1 = \frac{2i}{1+3i}$ and $z_2 = \frac{3+2i}{5}$, find $|z_1 - z_2|$ *(06 marks)*

10. (a) The first three terms of the expansion $(9 + ax)^n$ are $27 - 9x + \frac{1}{2}x^2$. Find n and a.

(06 marks)

(b) find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^{10}$ *(06 marks)*

11. (a) The function $f(x) = x^3 + px^2 - 5x + q$ has a factor (x-2) and has a remainder of 5 when divided by x + 3. Find the values of p and q. *(04 marks)*

(b) The roots of the equation $ax^2 + bx + c = 0$ are α and β. Find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ *(04 marks)*

(c) prove by mathematical induction that $9^n + 7$ is divisible by 8 *(04 marks)*

12. (a) Find the maximum value of $24\sin\theta - 7\cos\theta$ and the smallest value of θ which gives this maximum value to 4 significant figures.
(06 marks)

(b) Prove that $\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta} = \tan 2\theta$ *(06 marks)*

13. (a) Given that $r = 3\cos\theta$ is an equation of a circle, find its cartesian equation. State its centre, radius and sketch the circle. *(06 marks)*

(b) (i) Find the equation of the locus of the point which moves such that its distance from the point (3,4) is a half its distance from the line y = 8. *(06 marks)*

14. Given that $y = \frac{4x-10}{x^2-4}$

(a) find the range of values where the curve doesn't exist.

(b) Hence

(i) determine the stationary points of the curve

(ii) state the equations of the three asymptotes

(iii) sketch the curve.

15. (a) Four points have coordinates A(3, 4, 7), B(13, 9, 2), C(1, 2, 3) andD(10, κ, 6). The lines AB and CD intersect at P. Determine the

(i) vector equation of the lines AB and CD *(06 marks)*

(ii) value of κ *(04 marks)*

(iii) coordinates of P *(02 marks)*

16. (a) Find the general solution of the equation $\frac{dy}{dx} = (xy)^{\frac{1}{2}} lnx$,
given that y = 1 when x = 1. Hence find the value of y when x = 4 *(06 marks)*

(b) The rate at which the population of a country increases is proportional to the number of people P. Given that the population in 2002 was 25m,

(i) form a differential equation for the population growth and solve it.

(ii) given that the population in 2012 was 35m, find the year in which the population will be double that of 2002. *(06 marks)*

END