NB Constant velocity ($a=0$)

Chapter Three

FORCE AND NEWTON'S LAWS OF MOTION

Force

Is defined as anything that causes motion or exerts pressure. Force is defined fully according to Newton's laws of motion as discussed below.

First Law

A body will remain at rest or will move with constant velocity unless external forces act on it otherwise

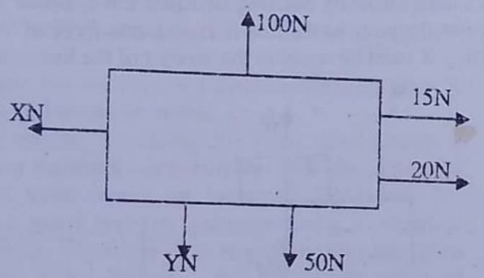
Case 1

A body at rest.

If forces act on a body and it does not move, then the opposite forces must be equal i.e. If a number of forces act on a body and it remains at rest, the resultant force in any direction must be equal to zero.

Illustration

Consider a body at rest subjected to forces as shown below.



Find X and Y

Solution

Since the system is in equilibrium;

The horizontal forces in the two directions must be equal and opposite.

i.e. $X = 15 + 20$

$X = 35\text{N}$

And the vertical forces also in the two directions must be equal and opposite.

I.e. $100 = Y + 50$

$Y = 100 - 50$

$Y = 50\text{N}$

$\therefore$ $X = 35\text{N}$ and $Y = 50\text{N}$

Case II

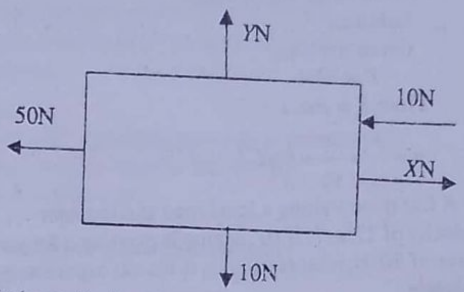
A body in motion;

A body can only change its velocity if and only if an external force acts on it.

Or, If a body is moving with constant velocity, then there will be no resultant force acting on it.

Illustration

1. A body moves horizontally at constant speed of 10ms^{-1} subjected to forces as shown below. Find the forces X and Y



Horizontally there is motion

Resultant force = Total force in the direction of motion – force opposing motion

$\Rightarrow$ $Ma = X - (50 + 10)$

$Ma = X - 60$

But with constant velocity, $a = 0$

$\Rightarrow$ $0 = X - 60$

$X = 60\text{N}$

Vertically, there is no motion

So $Y = 10\text{N}$

$\therefore$ $X = 60\text{N}$ and $Y = 10\text{N}$.

Second law

When a force acts on a body and it is said to move, then the acceleration caused varies directly as the force applied and inversely proportional to the mass of the body. So if the force is F, the mass is m and the acceleration is a ms^{-2}

Mathematically:

$$a \propto F \quad and \quad a \propto m^{-1}$$

By combining the two expressions using joint variations, we have;

$\Rightarrow$ $a \propto Fm^{-1}$

$$\frac{F}{m} \propto a$$

$$F \propto ma$$

Introducing a constant of proportionality K, this relationship becomes

$$F = Kma$$

Now if $m = 1$ and $a = 1$, then $F = K$, so the amount of force needed to give 1kg an acceleration of 1ms^{-2} is equal to K. If we choose this amount of force to be the unit of force then K=1 and the relationship above becomes, $F = ma$.

The unit force is therefore defined as that force which gives a mass of 1kg an acceleration of 1ms^{-2} and this unit of force is called a Newton (N)

Note: For computational purposes, we shall use the equation $F = ma$ from time to time even when the force and acceleration are both expressed in vector form

Examples.

1. A body of mass 10kg is acted upon by a force of 30N. Find the acceleration.

Solution

Given m=10kg

$F = 30N$

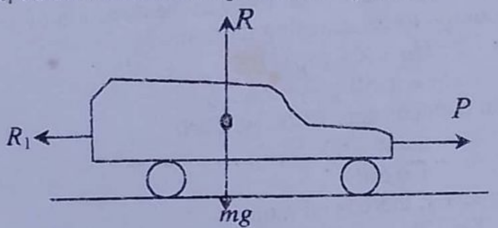
From $F = ma$

$$a = \frac{F}{m} = \frac{30}{10} = 3ms^{-2}$$

2. A Car moves along a level road at a constant velocity of $22ms^{-1}$. If its engine is exerting a forward force of 500N, what resistance is the car experiencing.

solution

Let R_1 be the resistance experienced by the car



Resultant force $= P - R_1$

$Ma = P - R1$

But at constant velocity, $a = 0$

$0 = P - R_1$

$0 = 500 - R_1$

$\therefore R_1 = 500$

Hence the resistance experienced = 500N

3. Find in vector form, the acceleration produced in a body of mass 500g when forces of $(5\mathbf{i} + 3\mathbf{j})$ N, $(6\mathbf{i} + 4\mathbf{j})$N and $(-7\mathbf{i} - 7\mathbf{j})$ N act on the body

Solution

Let R = resultant of forces

$$\textbf{OR} = \begin{pmatrix}5\\3\end{pmatrix} + \begin{pmatrix}6\\4\end{pmatrix} + \begin{pmatrix}-7\\-7\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} \quad \begin{pmatrix}5\\3\end{pmatrix} + \begin{pmatrix}6\\4\end{pmatrix} + \begin{pmatrix}-7\\-7\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix}$$

From $F = Ma$

$F = R,\ M = 500g = 0.5kg$

$$\mathbf{a} = \frac{R}{M} = \frac{1}{0.5}\begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}8\\0\end{pmatrix}$$

Hence acceleration is $8\mathbf{i}\ ms^{-2}$

4. Forces of $(10\mathbf{i} + 2\mathbf{j})$ N and $(a\mathbf{i} + b\mathbf{j})$ N act on a body of mass 500g causing it to accelerate at $(24\mathbf{i} + 3\mathbf{j})\ ms^{-2}$ Find the constants a and b

Solution

Resultant of the forces,

$$\textbf{OR} = \begin{pmatrix}10\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}10+a\\2+b\end{pmatrix}$$

$M = 500g = 0.5kg$

$a = 24\mathbf{i}+3\mathbf{j}$

From $F = Ma$

$$\begin{pmatrix}10+a\\2+b\end{pmatrix} = 0.5\begin{pmatrix}24\\3\end{pmatrix}$$

Equating the corresponding unit vectors,

$\Rightarrow\ 10 + a = 12$

$a = 2$

and $2 + b = \frac{3}{2}$

$b = -\frac{1}{2}$

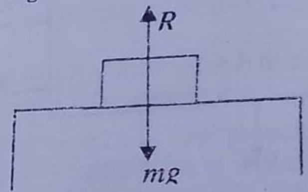
Hence $a = 2$ and $b = -\frac{1}{2}$

Third law

To every action there is an equal and opposite reaction. **OR;** Action and reaction are equal and opposite.

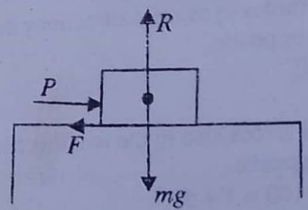
Illustration

Consider a box of mass m kg resting on a horizontal table .The box exerts a force on a table and a table reacts by exerting an equal and opposite force on the box. As the box is at rest, this force of reaction R must be equal to the weight of the box i.e. $R = mg$

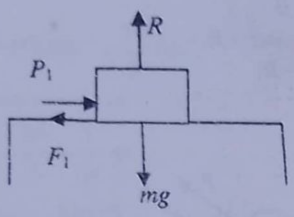


Cases involving friction

Let a heavy block be placed on a rough horizontal floor and let a small push P be applied on it



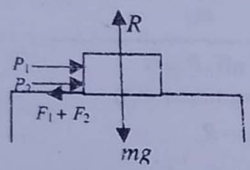
Since the block doesn't sink into the floor, there must be an equal and opposite reaction provided by the floor to the weight mg. This is called the normal reaction, R. It will be observed that the small push will not set the block into motion. This seems to disobey the first and the second laws of Newton. But what occurs in reality is that to the small push the surface of the rough floor provides an equal and opposite reaction and nullifies the push. The force provided by the rough floor surface, which opposes motion, is called frictional force.



In equilibrium position,

$F_1 = P_1$

Suppose we increase the push by just a small amount of P_2, still motion doesn't occur.



This means that the surface must have increased its frictional force by an equal and opposite amount $F_2 = P_2$, so that again the total push is nullified and motion doesn't occur.

This means that the frictional force on the surface is not constant but can be varied.

If we continued increasing the push, there comes a limit when the block starts moving. This means that the frictional force cannot be increased indefinitely i.e. forever, it has a limit or maximum value to which it can build up. When the push is greater than maximum value, the block moves, this maximum value of friction is called limiting static friction denoted by F. So for push P greater than F the block moves but for P $\le$ F the block doesn't move. If blocks of various weights are placed on the floor, we see that the greater the weight, the greater the normal reaction and the minimum force required to move the block. i.e. the greater the limiting force.

$\Rightarrow \quad F \propto R$

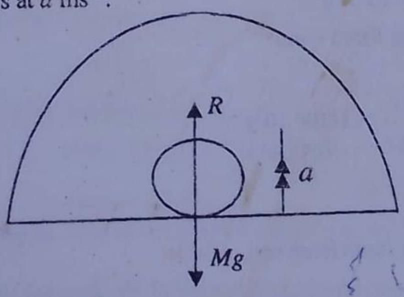
$F = \mu R$ where μ is a constant of proportionality called coefficient of friction.

Note: We shall see more about friction in chapter 11

Apparent Weights

Apparent Weight of a Body accelerating Up.

Let a man of mass M kg stand in a lift accelerating upwards at a ms^{-2}.



Resultant force = $R - Mg$

$Ma = R - Mg$

$R = M(a + g)$

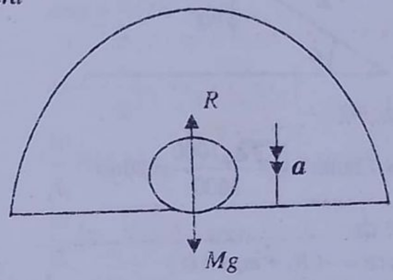
Since the man remains on the floor of the lift,

$\Rightarrow$ to this pull there must be an equal and opposite reaction down.

Conclusion

For a man in a lift ascending upwards at acceleration a, he exerts a weight equal to reaction R, where $R = M(a + g)$

Apparent Weight of a Body accelerating Downward



Resultant force = $Mg - R$

$Ma = Mg - R$

$R = M(g - a)$

Since the man stays on the floor of the lift, his apparent weight is $R = M(g - a)$.

When $a = g$, the apparent weight = 0,

$\Rightarrow$The man feels weightless and exerts no reaction on the floor and in this case the man falls freely under gravity.

If the lift accelerates at $a > g$, the man is left behind the lift as it descends. This is suicidal as this may cause death.

However, for safe acceleration in a lift down wards, $a < g$

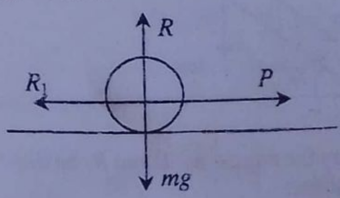
General Examples

1. The resistance to the motion of the train due to friction, etc is equal to $\frac{1}{160}$ of the weight of the train. If the train is travelling on a level road at 72kmh^{-1} and comes to the foot of an incline of 1 in 150 and steam is then turned off, how far will the train go up the incline before it comes to rest.

Solution.

Let the mass of the train be m kg

On the level, we have:



Note. R = Normal reaction, R_l = Total resistance and
P = Tractive force

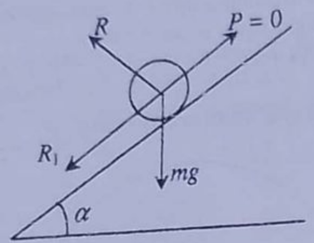
Resultant force $= P - R_1$

$ma = P - R_1$

With constant speed, $a = 0$

$\Rightarrow P = R_1 = \frac{1}{160}mg$

On an incline, we have;



Given $R_1 = \frac{1}{160}mg$

$$u = 72\text{kmh}^{-1} = \frac{72\times1000}{3600} = 20\text{ms}^{-1}$$

$$\sin\alpha = \frac{1}{150}$$

Resultant force $= -(R_1 + mg\sin\alpha)$

$$ma = -\left(\frac{1}{160}mg + \frac{1}{150}mg\right)$$

$$a = -\left(\frac{1}{160}g + \frac{1}{150}g\right) = \frac{-310g}{24000}$$

When the train comes to rest, $v = 0$

From $v^2 = u^2 + 2as$

$$0 = 20^2 - 2g\left(\frac{310}{24000}\right)S$$

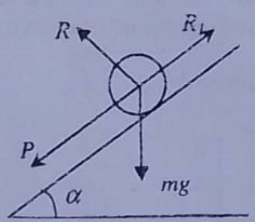
$$S = \left(\frac{400\times24000}{2\times9.8\times310}\right) = 1579.987\text{m} = 1.58\text{km}$$

Hence the train goes up 1.58km before stopping

2. A truck is found to travel with uniform speed down a slope which falls 1m vertically for every 112m length of the slope. If the truck starts from the bottom of the slope with a speed of 18kmh^{-1}, how far up will it travel before coming to rest

Solution

When going down



Given $\sin\alpha = \frac{1}{112}$

Let force applied by the engine be P and R_l be the resistance to the motion

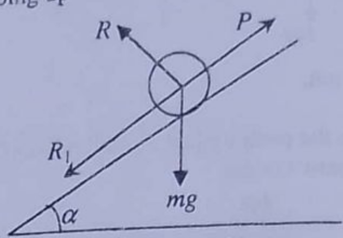
With uniform speed, $a = 0$

Resultant force $= P + mg\sin\alpha - R_1$

$$0 = P + mg\sin\alpha - R_1$$

$$R_1 = P + \frac{1}{112}mg$$

When going up



When steam is turned off, $P = 0$

Resultant force $= P - (mg\sin\alpha + R_1))$

$$ma = P - \frac{1}{112}mg - R_1$$

$$ma = P - \frac{1}{112}mg - P - \frac{1}{112}mg$$

$$ma = -\frac{2}{112}mg$$

$$a = -\frac{1}{56}g$$

$$u = 18\text{kmh}^{-1} = 5\text{ms}^{-1}$$

$v = 0$ since it comes to rest.

From $v^2 = u^2 + 2as$

$$0 = 5^2 - \frac{2gS}{56}$$

$$S = \frac{25\times28}{9.8} = 71.42857$$

Hence the truck goes 71.429m before it comes to rest

3. A train of mass $160Mg$ starts from a station ,the engine exerting a tractive force of $\frac{1}{64}$ of the weight of the train in excess of the resistances until a speed of 60km/h is attained .This speed continues constant until the brakes causing a retardation of 0.75m/s^2,bring the train to rest in 8km away. Find the time taken

(i) During acceleration

(ii) During retardation

(iii) Altogether

Solution

(i)Let P = tractive force

Given $m = 160Mg = 160\times10^3$ kg

$P = \frac{1}{64}\times160\times10^3\times g$

From Resultant force $= ma$

$\Rightarrow \quad ma = P$

$$160\times10^3 a = \frac{1}{64}\times160\times10^3 g$$

$$a = \frac{g}{64}$$

Since the train starts from rest, $u = 0$

$v = 60\text{kmh}^{-1} = \frac{50}{3}\text{ ms}^{-1}$

From $v = u + at$

$$\frac{50}{3} = \frac{1}{64}gt$$

$$t_1 = \frac{50 \times 64}{3 \times 9.8} = 108.8s$$

Hence the time taken during acceleration is 108.8s.

(ii) For retardation

$v = 0$

$u = \frac{50}{3}\text{ ms}^{-1}$

$a = -0.75\text{ms}^{-2}$

From $v = u + at$;

$$0 = \frac{50}{3} - 0.75t_3$$

$$\frac{50}{3} = 0.75t_3$$

$$t_3 = \frac{50}{3 \times 0.75} = 22.2s.$$

Hence the time taken during retardation is 22.2s.

(iii). But the total distance covered is 8km,

During acceleration,

From $v^2 = u^2 + 2aS$

$$S_1 = \frac{v^2}{2a}$$

Since $u = 0$

$$S_1 = \frac{2500 \times 64}{2 \times 9 \times 9.8} = 907m$$

During the deceleration,

From $v^2 = u^2 + 2as$

$$0 = u^2 - 2as_2$$

$$s_2 = \frac{u^2}{2a}$$

$$S_2 = \frac{2500}{2 \times 9 \times 0.75} = 185.185\text{ m}$$

At constant speed, $s = vt_2$

$$s_3 = \frac{50}{3}t_2$$

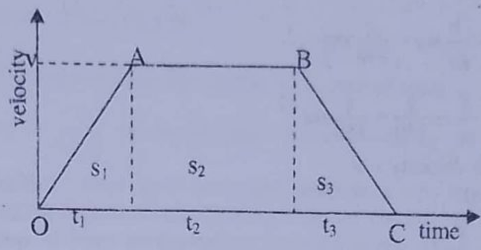
But total distance $= s_1 + s_2 + s_3$

$$907 + 185.185 + \frac{50}{3}t_2 = 8000$$

$t_2 = 414.4689\text{m}$

The total time taken $= 108.8 + 22.2 + 414.5 = 545.5\text{s}$

***Alternatively*: By using graphical approach,**



After obtaining t_1 and t_3 as shown above, we may find the value of t_2 by simply finding the area of the trapezium and equating it to 8000m

$$Area = \frac{v}{2}(OC + AB)$$

$$Area = \frac{v}{2}(t_1 + t_2 + t_3 + t_2)$$

$$\frac{50}{6}(108.8 + t_2 + 22.2 + t_2) = 8000$$

$$\frac{25}{3}(2t_2 + 131) = 8000$$

$$2t_2 = \frac{24000}{25} - 131$$

$t_2 = 414.5s$

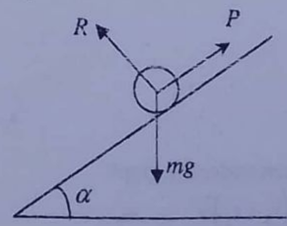
Total time $, T = 108.8 + 414.5 + 22.2 = 545.5s$

4. The pull exerted by an engine is $\frac{1}{80}$ of the weight of the whole train and the maximum brake force which can be exerted is $\frac{1}{30}$ of the weight of the train. Find the time in which the train travels from rest up a slope of 1 in 240 and 4.8km long, the brakes being applied when steam is shut off.

Solution.

Given pull, $P = \frac{1}{80}mg$

Braking force $= \frac{1}{30}mg$



During acceleration:

Resultant force $= \frac{1}{80}mg - mg\sin\alpha$

$$ma = \frac{1}{80}mg - \frac{1}{240}mg$$

$$a = \frac{g}{80} - \frac{g}{240} = \frac{g}{120}\ \text{ms}^{-2}$$

Let maximum velocity = v

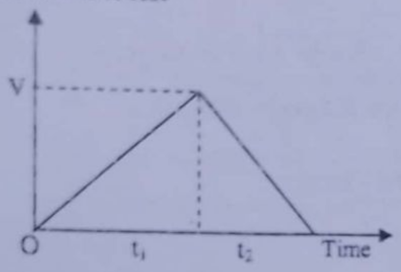
From $v = u + at$

$$v = \frac{g}{120}t_1$$

$$t_1 = \frac{120V}{g} \ \text{.........................} \ \text{(i)}$$

where t_1 is the time taken during acceleration.

But the train must accelerate until it is only necessary to decelerate to rest



During deceleration,

Resultant force = brake force + $mg\sin\alpha$

$$-ma = \frac{1}{30}mg + \frac{1}{240}mg$$

$$a = -\left(\frac{1}{30}g + \frac{1}{240}g\right) = \frac{-3g}{80}\ \text{ms}^{-2}$$

From $v = u + at$

$$0 = v - \frac{3g}{80}t_2$$

$$t_2 = \frac{80v}{3g} \ \text{.........................} \text{(ii)}$$

where t_2 is the time taken during deceleration

Total time, $T = t_1 + t_2$

$$T = \frac{120v}{g} + \frac{80v}{3g}$$

$$T = \frac{440v}{3g}$$

$$v = \frac{3gT}{440}$$

But distance covered = area under the graph

$$S = \frac{t_1 v}{2} + \frac{t_2 v}{2} = \frac{(t_1 + t_2)v}{2} = \frac{Tv}{2}$$

Substituting for S and v,

$$4800 = \frac{3gT^2}{880}$$

$$T = \sqrt{\frac{4800 \times 880}{3 \times 9.8}} = 379s$$

Hence the total time taken is 379 seconds

5. In a lift accelerated upwards at a certain rate, a spring balance indicates a weight to have a mass of 10kg. When the lift is accelerated downwards at twice the rate, the mass appears to be 7kg. Find the upward acceleration of the lift and the actual mass.

Solution

Let acceleration up = a ms^{-2}

For upward motion;

$$R - mg = ma$$

$$R = m(g + a)$$

$$\Rightarrow m(g + a) = 10g \text{....................(i)}$$

For downward motion;

$$mg - R = ma_1$$

$$R = mg - ma_1$$

$$= m(g - a_1)$$

But $a_1 = 2a$

$$\Rightarrow m(g - 2a) = 7g \text{..................(ii)}$$

Eqn (i) ÷ Eqn (ii)

$$\frac{g + a}{g - 2a} = \frac{10}{7}$$

$$7(g + a) = 10(g - 2a)$$

$$27a = 3g$$

$$a = \frac{g}{9} = 1.1s$$

Hence the acceleration of the lift is 1.1s.

Substituting for a into Eqn (i)

$$mg + \frac{g}{9} = 10g$$

$$\Rightarrow \frac{10gm}{9} = 10g$$

$$m = \frac{910g}{10g} = 9$$

Hence the actual mass is 9kg

Examination Questions

1. A particle of mass 3kg is moving on the curve described by $\mathbf{r} = 4\sin 3t\mathbf{i} + 8\cos 3t\mathbf{j}$ where $\mathbf{r}$ is the position vector of the particle at time t.
   (i) Determine the position and velocity of the particle at the time $t = 0$.
   (ii) Show that the force acting on the particle is $-27r$. **(1993 N0 5)**

   i) $v = 12\mathbf{i}$

2. a) A bullet travelling at 150 ms^{-1} will penetrate 8 cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4 cm of the block.

b) A particle of mass 2 kg, initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix} N$

Find i) its acceleration at time t
   ii) Its velocity after 3 seconds.
   iii) The distance the particle has travelled after 3 seconds. **(1994 No 5)**

Answers: i) 140.625 ms^{-2}

ii) $v = \frac{9}{2}\mathbf{i} + \frac{9}{4}\mathbf{j} + \frac{27}{4}\mathbf{k}$ iii) 8.4m

3. A carton of mass 0.4 kg is thrown across a table with a velocity of 25 ms^{-1}. The resistance of the table to its motion is 50N. How far will it travel before coming to rest? What must be the resistance if it travels only 2 meters **(March 1998 N0 2)**

Answer: 2.5m; 62.5 N

4. The resistance to the motion of a lorry of mass m kg is $\frac{1}{200}$ of its weight. When travelling at 108km^{-1} on a level road and ascends a hill, its engine fails to work. Find how far up the hill (in km) the lorry moves before it comes to rest. Give your answer close to one decimal place. **(2002 No. 6)**

Answer: $s = \frac{90000}{g(1+200\sin\alpha)}$

Note. *We leave the answer in terms of α as there is no provision for it.*

5. A vehicle of mass 2.5 metric tonnes is drawn up on a slope of 1 in 10 from rest with an acceleration of 1.2 ms^{-2} against a constant frictional resistance of $\frac{1}{100}$ of the weight of the vehicle, using a cable. Find the tension in the cable. **(2006 No. 6)**

Answer: 5695 N

6. The engine of a train exerts a force of 35, 000 N on a train of mass 240 tonnes and draws it up a slope of 1 in 120 against resistance totalling to 60N/tonne. Find the acceleration of the train. **(2008 No 6)**

Answer: 0.004167 ms^{-2}

Application of Newton's Laws to Motion of Connected Particles

Simple(single) connections

When two particles are connected by a light inextensible string passing over a smooth fixed pulley and allowed to move, then, as long as the string is taut (tight), the following must obey

(i). The velocity of one body = the velocity of the other. And also the acceleration of one body in general direction of motion = acceleration of the other

(ii). The particles will move like a rigid body.

(i) The resultant force for every particle = total force acting in direction of motion – total force opposing notion = ma

(ii) The tension, T in the string is constant

Cases considered.

Case 1

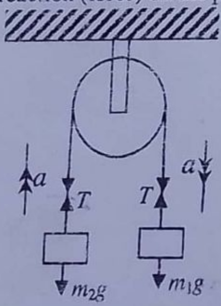
Consider two particles of mass m_1 and m_2 connected by a light inelastic string passing over a smooth fixed pulley as shown,

Suppose $m_1 > m_2$

Find (i) The acceleration of the particle

(ii) The tension in the string

(iii) The reaction (force) on the pulley



Considering forces on each particle.

For $m_1$kg mass, Resultant force = m_1g - T

$$m_1a = m_1g - T \quad \text{.....................(i)}$$

For $m_2$kg mass, Resultant force = $T - m_2g$

$$m_2a = T - m_2g \quad \text{.....................(ii)}$$

Eqn (i) + Eqn (ii)

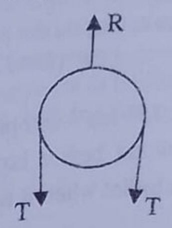
$a(m_1 + m_2) = g(m_1 - m_2)$

$$a = \frac{g(m_1 - m_2)}{m_1 + m_2}$$

Substituting for a into either Eqns and by simplifying, we have,

$$T = \frac{2m_1m_2g}{m_1 + m_2}$$

Reaction on the pulley.



Resolving forces vertically,

$$R = 2T$$

$$R = \frac{4m_1m_2g}{m_1 + m_2}$$

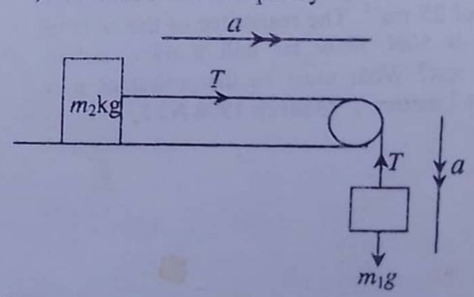
Case 2

(a) Consider a mass $m_2$kg placed on smooth horizontal table and connected by a light inextensible string passing over a small smooth pulley at the edge to a mass $m_1$kg hanging freely. Find

i) The acceleration of motion

ii) The tension in the string

iii) The reaction on the pulley



(i) For $m_1$kg mass, Resultant force $= m_1g - T$

$$m_1a = m_1g - T \quad \text{.....................(i)}$$

For $m_2$kg mass, Resultant force =T

$$m_2a = T \quad \text{.....................(ii)}$$

Eqn (i) + Eqn(ii)

$a(m_1 + m_2) = m_1g$

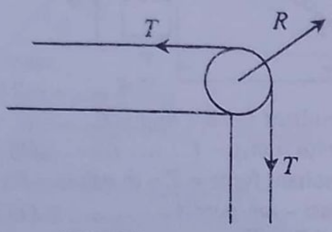
$$a = \frac{m_1g}{m_1 + m_2}$$

(ii) Substituting for a into Eqn (ii)

$$T = \frac{m_2(m_1g)}{(m_1+m_2)}$$

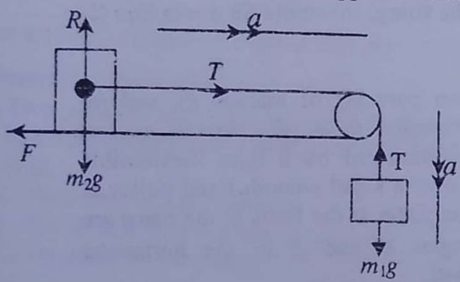
$$= \frac{m_1m_2g}{(m_1+m_2)}$$

(iii) Reaction on the pulley



$$R = \sqrt{T^2+T^2} = T\sqrt{2} = \frac{m_1m_2g\sqrt{2}}{m_1+m_2}$$

(b). In case of rough horizontal table. Here there will be frictional force that will act in opposite direction



For $m_1$kg mass, Resultant force $= m_1g - T$

$m_1a = m_1g - T$.....................(i)

For $m_2$gkg mass, Resultant $= T - F$

Where, F = (Frictional force)

$m_2a = T - F$.......................(ii)

Eqn (i) + Eqn (ii)

$a(m_1+m_2) = m_1g - F$

$$a = \frac{m_1g - F}{m_1+m_2}$$

But $F = \mu R = \mu m_2g$

By substitution,

$$a = \frac{m_1g - \mu R}{m_1+m_2} = \frac{m_1g - \mu m_2g}{m_1+m_2}$$

From Eqn (i)

$T = m_1g - m_1a$

$$= m_1g - m_1\left(\frac{m_1g - \mu m_2g}{m_1+m_2}\right)$$

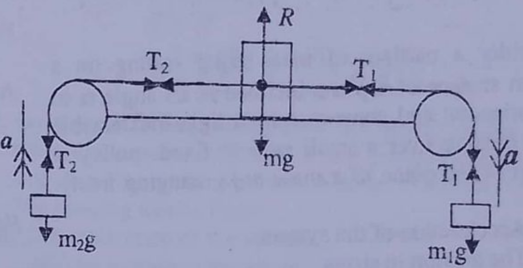
$$T = \frac{m_1^2g + -m_1m_2g - m_1^2 + m_1m_2g}{m_1+m_2}$$

$$T = \frac{m_1m_2g(1+\mu)}{m_1+m_2}$$

Cases 3

(a). Let a mass of m kg rest on a smooth horizontal table and attached by two inelastic strings to masses m_1 and m_2 kg ($m_1>m_2$),which hang over smooth pulleys at opposite edges of the table. Find

i. The acceleration of the system
ii. The tension in the strings
iii. The reactions on the pulleys



For $m_1$kg mass, Resultant force $= m_1g - T_1$

$m_1a = m_1$kg $- T_1$...................(1)

For $m_2$kg mass, Resultant force $= T_2 - m_2g$

$m_2a = T_2 -$
m_2g.....................(2)

For $m$kg mass, Resultant force $= T_1 - T_2$..........(3)

Adding equations (1), (2) and (3)

$a(m_1+m_2+m) = m_1g - m_2g$

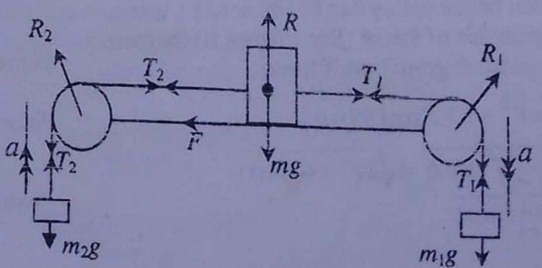
$$a = \frac{(m_1-m_2)g}{(m_1+m_2+m)}$$

For tensions T_1 and T_2, substitute for a into Eqns (1) and (2)

Reactions on pulleys can be solved out as in case (II) above.

i.e. $R_1 = T_1\sqrt{2}$ and
$R_2 = T_2\sqrt{2}$

(b). Considering a rough plane, Here, frictional force, F acts in opposite direction



For $m_1$kg mass, Resultant force $= m_1g - T_1$

$m_1a = m_1g - T_1$...........................(i)

For $m_2$kg mass;

Resultant force $= T_2 - m_2g$

$m_2a = T_2 - m_2g$..............................(ii)

For $m$kg mass;

Resultant force $= T_1 - T_2 - F$

$ma = T_1 - T_2 - \mu mg$..........................(iii)

Adding Eqns (i), (ii), and (iii)

$a(m_1 + m_2 + m) = (m_1 - m_2 - m\mu)g$

$$a = \frac{(m_1 - m_2 - m\mu)g}{(m_1 + m_2 + m)}$$

For tensions T_1 and T_2 substitute for a into Eqns (i) and (ii) respectively.

For reactions like in the case above;

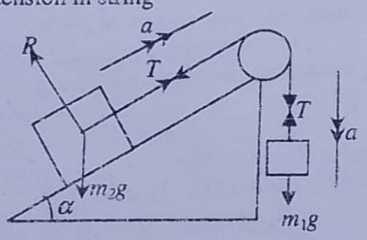
$R_1 = T_1\sqrt{2}$ and $R_2 = T_2\sqrt{2}$.

Case 4

a) Consider a particle of mass $m_2$kg resting on a smooth surface of a plane inclined at an angle α to the horizontal and connected by a light inextensible string passing over a small smooth fixed pulley at the top of the plane to a mass $m_1$kg hanging freely. Find

i) Acceleration of the system.

ii) The tension in string



For $m_1$kg mass, Resultant force $= m_1g - T$

$m_1a = m_1g - T$...............(1)

For $m_2$kg mass, Resultant force $= T - m_2g\sin\alpha$

$m_2a = T - m_2g\sin\alpha$...............(2)

Adding equations (1) and (2)

$a(m_1 + m_2) = (m_1 - m_2\sin\alpha)g$

$$a = \frac{(m_1 - m_2\sin\alpha)g}{(m_1 + m_2)}$$

For tension, substitute for a into either eqn.

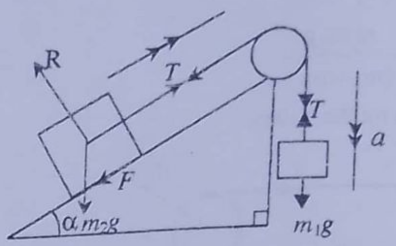
And reaction on the pulley can be obtained by using parallelogram law of forces (See chapter 10 for more details on parallelogram law of forces)

$$R^2 = T^2 + T^2 + 2.T.T\cos(90 - \alpha)$$

$$R = \sqrt{2T^2 + 2T^2\sin\alpha} = \sqrt{2T^2(1 + \sin\alpha)}$$

$$= T\sqrt{2(1 + \sin\alpha)}$$

(b) ***Considering a rough inclined plane.***



For $m_1$kg mass, Resultant force $= m_1g - T$

$m_1a = m_1g - T$..................(1)

For $m_2$kg mass, Resultant force $= T - m_2g\sin\alpha - F$

$m_2a = T - m_2g\sin\alpha - \mu m_2g\cos\alpha$.................(2)

Adding Eqns (1) and Eqn (2)

$a(m_1+m_2) = (m_1 - m_2\sin\alpha - \mu m_2g\cos\alpha)g$

$$a = \frac{(m_1 - m_2\sin\alpha - \mu m_2\cos\alpha)g}{(m_1 + m_2)}$$

For tension in the string, substitute for a into Eqn (1)

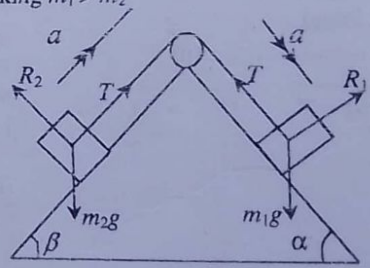
Case 5

(a) Consider two particles of masses m_1 and m_2 resting on the ***smooth faces*** of a double inclined plane and are connected by a light inextensible string passing over a small smooth fixed pulley at the vertex of the plane. If the faces of the plane are inclined at angles α and β to the horizontal respectively. Find

(i) Acceleration of the system

(ii) The tension in the string.

Taking $m_1 > m_2$



For $m_1$kg mass, Resultant force $= m_1g\sin\alpha - T$

$m_1a = m_1g\sin - T$.........................(1)

For $m_2$kg mass, Resultant force $= T - m_2g\sin\beta$

$m_2a = T - m_2g\sin\beta$.............................(2)

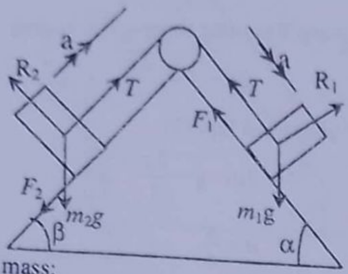
Adding equations (1) and (2)

$a(m_1+m_2) = (m_1g\sin\alpha - m_2g\sin\beta)$

$$a = \frac{m_1g\sin\alpha - m_2g\sin\beta}{m_1 + m_2}$$

For tension, substitute for a into Eqn (1)

(b). **For rough plane.**



For $m_1$kg mass;

Resultant force $= m_1g\sin\alpha - T - F_1$

$$m_1a = m_1g\sin\alpha - T - m_1g\cos\alpha\mu \ldots\ldots\ldots\ldots(1)$$

For $m_2$kg mass;

Resultant force $= T - m_2g\sin\beta - F_2$

$$m_2a = T - m_2g\sin\beta - m_2g\cos\beta\mu \ldots\ldots\ldots\ldots\ldots(2)$$

Adding equations (1) and (2)

$$a(m_1+m_2) = m_1g\sin\alpha - m_2g\sin\beta - m_1g\cos\alpha\mu - m_2g\cos\beta\mu$$

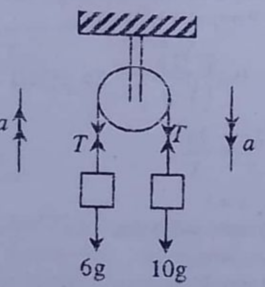
$$a = \frac{m_1g\sin\alpha - m_2g\sin\beta - m_1g\cos\alpha\mu - m_2g\cos\beta\mu}{m_1 + m_2}$$

For tension, substitute for a into Eqn (1)

Illustrative Examples.

1. Two particles of masses 6 and 10kg, are connected by a light string passing over a smooth fixed pulley. Find:
   i. Their common acceleration
   ii. The tension in the string
   iii. The force on the pulley

Solution



(i) For the 10kg mass,

Resultant force $= 10g - T$

$$10a = 10g - T \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

For the 6kg mass;

Resultant force $= T - 6g$

$$6a = T - 6g \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Eqn (i) + Eqn (ii);

$$16a = 4g$$

$$a = \frac{4g}{16} = 2.45\text{ms}^{-1}$$

(ii) Substituting for a into Eqn (i);

$$T = 10g - 10\left(\frac{4g}{16}\right)$$

$$= 10 - \frac{40g}{16} = 73.5\text{N}$$

(iii) Let R = force on the pulley;

Resolving forces vertically;

$$R = 2T$$

$$= \frac{2\times120g}{16} = \frac{240g}{16} = 147\text{N}$$

2. (a) A mass of 9kg resting on a smooth horizontal table is connected by a light string ,passing over a smooth pulley at the edge of the table, to a mass of 7kg hanging freely. Find:

(a)(i).Acceleration of the system

(ii)The tension in the string

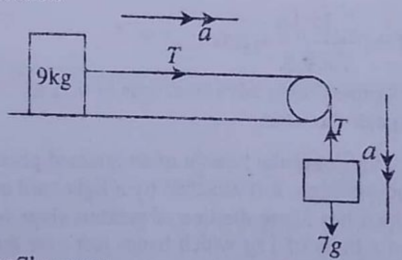
(iii)The reaction on a pulley

(b). If in (a) above, the 7kg mass starts from the level of edge, which is 2.1m above the ground and the string, which is 4.2m long, is taut and perpendicular to the edge. Find

(i) How long the 7kg mass takes to reach the ground

(ii) How long after that the 9kg mass takes to reach the edge of the table

Solution



For 7kg mass;

Resultant force $= 7g - T$

$$7a = 7g - T \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For 9kg mass, Resultant force $= T$

$$9a = T \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Adding Eqn (1) and Eqn (2)

$$16a = 7g$$

$$a = \frac{7g}{16} = 4.2875\text{ms}^{-2}$$ 4·2919

Hence the common acceleration is 4.2875ms^{-2}

ii) From equation (2)

$$T = 9a$$

$= \frac{63g}{16} N = 38.5875N$

iii) Reaction on the pulley is given by

$R = T\sqrt{2} = \frac{63g\sqrt{2}}{16} = 54.57N$

b)(i) Since the 7kg mass starts from rest,

$u = 0,\ a = \frac{7g}{16}$, Now $S = 2.1m$

From $s = ut + \frac{1}{2}at^2$

$2.1 = \frac{1}{2} \times \frac{7g}{16} t^2$

$t = \sqrt{\frac{2.1 \times 32}{7 \times 9.8}} = .9874 = 1s$

Hence the time taken for 7kg mass to reach the ground is roughly 1 second

(ii). When the 7kg mass reaches the ground, the 9kg mass will move with constant velocity at that instant as acceleration becomes zero.

Velocity gained in 1 second is

$v = u + at$

$v = \frac{7g}{16}$

So when the 7kg mass reaches the ground, it covers 2.1m, the remaining distance is $4.2 - 2 = 2.1m$.

With this remaining distance of 2.1m, it will be covered with constant speed

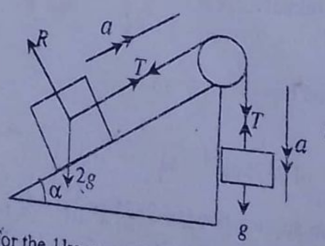
From $s = v \cdot t$

$2.1 = \frac{2.1 \times 16}{7 \times 9.8} = 0.5s$

Hence the 9kg mass takes additional time of 0.5s to reach the edge of the table.

3. A mass of 2 kg lies at the bottom of an inclined plane 9m long and 3m high .it is attached by a light cord of 9m long which lies along the line of greatest slope of the plane, to a mass of 1kg which hangs just over the top of the plane. The system is allowed to move. Assuming that the hanging mass comes to rest when it reaches the ground, find the distance that the mass of 2 kg will travel before it first comes to rest.

Solution



$\sin\alpha = \frac{1}{9} = \frac{1}{3}$

For the 1kg mass, Resultant force $= g - T$

$a = g - T$.............................(1)

For 2kg mass, Resultant force $= T - 2g\sin\alpha$

$2a = T - \frac{2g}{3}$ ....(

Adding Eqn (1) and Eqn (2)

$3a = g - \frac{2g}{3}$

$a = \frac{g}{9}$

Since the 1kg mass falls from rest, $u = 0$. $S = 3$

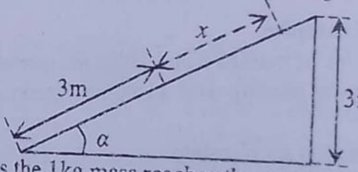
From $v^2 = u^2 + 2aS$, we need to find the velocity gained

$v^2 = 2 \times \frac{g}{9} \times 3$

$v^2 = \frac{2g}{3} \Rightarrow v = \sqrt{\frac{2g}{3}}$

Where v = velocity gained in falling through 3 metres

After the 1kg mass has hit the ground, the 2kg mass decelerates to rest under retardation = $-g\sin\alpha$



Now as the 1kg mass reaches the ground, the 2kg mass decelerates upwards, for a short while before coming to rest.

If x = distance covered after the 1kg mass reaches the ground, then from $v^2 = u^2 + 2aS$;

$0 = \left(\sqrt{\frac{2g}{3}}\right)^2 - 2 \times g\sin\alpha$

$\frac{2g}{3} = \frac{2g}{3}x$

$x = 1$

Total distance covered $= 3 + 1m$

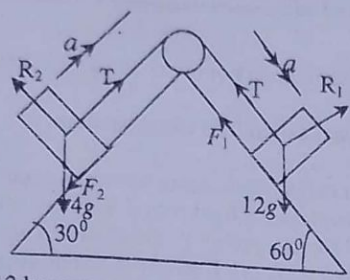
$= 4m$

So all together, it would have moved 4 m.

4. Two rough planes inclined at 30^0 and 60^0 to the horizontal and of the same height are placed back to back. Masses of 4kg and 12kg are placed on the faces and connected by a light string passing over smooth pulley at the top of the planes. If the coefficient of friction is ½ on both faces. Find the acceleration of the system.

Solution

Let F_1 and F_2 be the frictional forces to motion of the 12kg and 4kg masses respectively



For the 12 kg mass,

Resultant force $= 12g\sin60 - T - F_1$

$12a = 6g\sqrt{3} - 12g\cos60\mu$

$12a = 6g\sqrt{3} - 12g\times\frac{1}{2}\times\frac{1}{2}$

$12a = 6g\sqrt{3} - T - 3g$..........................(1)

For 4 kg mass,

Resultant force $= T - 4g\sin30 - 4g\cos30\mu$

$4a = T - 4g\times\frac{1}{2} - 4g\times\frac{\sqrt{3}}{2}\times\frac{1}{2}$

$4a = T - 2g - g\sqrt{3}$ ....(2)

Adding equations (1) and (2)

$16a = 5g\sqrt{3} - 5g$

$16a = 5g\left(\sqrt{3}-1\right)$

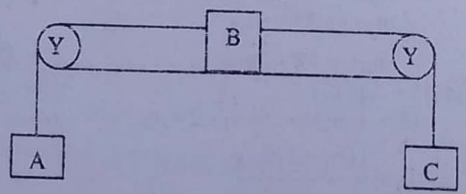
$$a = \frac{5g\left(\sqrt{3}-1\right)}{16}$$

$a = 2.2ms^{-2}$

Hence the acceleration of the system is $2.2ms^{-2}$

Examination Questions

1

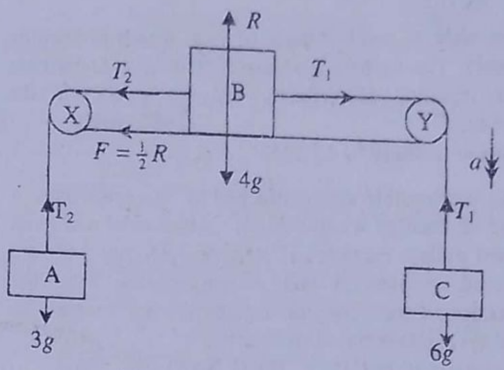


The diagram above shows particles A, B and C of masses 3, 4, and 6kg respectively connected by light inextensible strings which pass over smooth pulleys X and Y. Mass B rests on a horizontal rough table, the coefficient of friction between the table and mass B being ½. The system is released from rest. Determine

i. The acceleration of the masses.
ii. The tension in the strings.
iii. After mass C has dropped through a distance of 2m, the string connecting it to mass B snaps, determine the velocity and the time at which it occurs.

1992 No 7

Solution



Note: $F = \frac{1}{2}R = \frac{1}{2}\times 4g = 2g$

For 6kg mass, Resultant force $= 6g - T_1$

$6a = 6g - T_1$..(1)

For 3kg mass, Resultant force $= T_2 - 3g$

$3a = T_2 - 3g$..(2)

For 4kg mass, Resultant force $= T_1 - T_2 - 2g$

$4a = T_1 - T_2 - 2g$......................................(3)

Adding equations (1) + (2) + (3)

$13a = 6g - 3g - 2g$

$$a = \frac{g}{13} = 0.754ms^{-1}$$

Hence the acceleration of the masses is $0.754ms^{-2}$

(ii) From Eqn (1) $T_1 = 6g - 6a$

$$T_1 = 6g - \frac{6g}{13} = \frac{72g}{13}N = 54.277N$$

From Eqn (2) $T_2 = 3g + 3a$

$$= 3g + \frac{3g}{13} = \frac{42g}{13}N = 31.662N$$

(iii) When the string snaps, distance fallen = 2m

$$a = \frac{g}{13}$$

The initial velocity, $u = 0$

From $s = ut + \frac{1}{2}at^2$

$$2 = 0 + \frac{1}{2}\times\frac{g}{13}t^2$$

$$t = \sqrt{\frac{52}{9.8}} = 2.3s$$

From $v = u + at$

$$v = \frac{g}{13}\times\sqrt{\frac{52}{9.8}} = 1.736ms^{-1}$$

Hence the time taken is 2.3s and velocity acquired $1.736\ ms^{-1}$

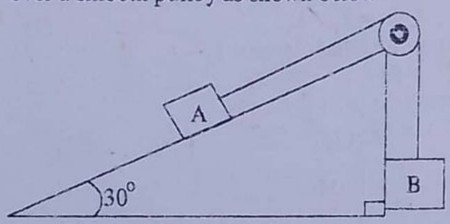
6. A mass of 3kg is at rest on a smooth horizontal table. It is attached by a light inextensible string passing over a smooth fixed pulley at the edge of

the table to another mass of 2kg, which is hanging freely. The system is released from rest. Determine the resulting acceleration and the tension in the string. **(1997 No 8)**

Answer: $1.96ms^{-2}$, 11.75N

7. An inextensible string attached to two scale pans A and B, each of weight 20gm, passes over a smooth fixed pulley. Particles of weights 3.8N and 5.8N are placed in pans A and B respectively. Find the reaction of the scale pan holding the 3.8N weight, if the system is released from rest.
   [Take $g = 10\ m/s^2$] **(2001 No 8)**
   Answers: 4.56N.

8. Blocks A and B of masses 2 and 3 kg respectively are connected by a light inextensible string passing over a smooth pulley as shown below



Block A is resting on a rough plane inclined at at 30° to the horizontal while block B hangs freely. When the system is released from rest, block B travels a distance of 0.75m before it attains a speed of $2.25ms^{-1}$.
Calculate the

(i) acceleration of the blocks

(ii) coefficient of friction between the plane and block A

(iii) reaction of the pulley on the string

(2003 No 11)

Answer: (i) $3.375\ ms^{-2}$, (ii) 0.16 (iii) 33.385 N

Multiple Connections

In multiple connections, the following hold:

- The tension in each uninterrupted portion of the string is uniform and acts in all portions.
- The tension in two different strings are likely to be different unless the strings are similar.
- The particles will not have the same acceleration and velocity like for single connections.
- For each particle, Resultant force = total force in the direction of motion - total force opposing motion.
- The acceleration of a particle moving between two portions of the strings is equal to a half the net acceleration of the particles attached to the end points of the string.
- So the particles do not move like a rigid body.

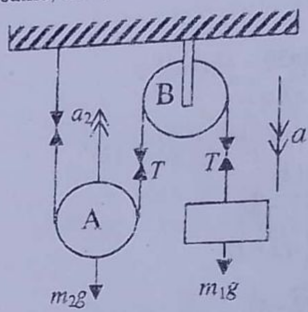
Cases taken into consideration

Case 1

Consider an inextensible string whose one end is fixed and runs under a light movable pulley A then over a fixed smooth pulley B, the other end being connected to a mass m_1. If pulley A has mass m_2 attached to it. Find the accelerations of m_1 and m_2 and the tension in the string.

Solution

Since the string passing over the two pulleys is the same, then, the tension in it is constant, T.



Note: If m_1 kg mass moves down with acceleration $= a_1$, Then m_2 kg mass moves upwards with acceleration $a_2 = \frac{1}{2}a_1$

For $m_1$kg mass, Resultant force $= m_1g - T$

$$m_1a_1 = m_1g - T \quad \text{.................(1)}$$

For $m_2$kg mass, Resultant force $= 2T - m_2g$

$$\tfrac{1}{2}m_2a_1 = 2T - m_2g$$

$$m_2a_1 = 4T - 2m_2g \quad \text{.........................(2)}$$

4(Eqn (1) + Eqn (2)

$$(4m_1 + m_2)a_1 = (4m_1 - 2m_2)g$$

$$a_1 = \frac{(4m_1 - 2m_2)g}{4m_1 + m_2}$$

Substituting for a_1 into Eqn (1) and by simplifying,

$$T = \frac{3m_1m_2g}{4m_1 + m_2}$$

Case 2

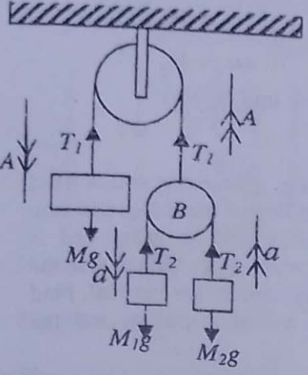
Let to one end of a light string passing over a fixed pulley be attached a particle of mass M kg and to the other end a light smooth pulley B. Over this pulley passes a light string attached to two particles of masses M_1 and M_2kg respectively. Find

(i) The acceleration of M kg mass and M_1kg mass.

(ii) The tensions in the strings

Solution

Let acceleration of M and pulley B be A and that of M_1 and M_2 be a and let the tensions be T_1 and T_2 respectively.



For M kg mass, Resultant force = Mg - T_1

$MA = Mg - T_1$ ....(1)

For pulley B, Resultant force = $T_1 - 2T_2$

$0 = T_1 - 2T_2$ since mass of pulley = 0

$T_1 = 2T_2$ ....(2)

For M_2kg mass, Resultant force = $T_2 - M_2g$

$M_2(A + a) = T_2 - M_2g$ ....(3)

For M_1kg mass, Resultant force $= T_2 - M_1g$

(since M_1 also finally moves up.)

$M_1(A - a) = T_2 - M_1g$ ....(4)

Eqn(3) ÷ M_2 $\Rightarrow$ $A + a = \frac{T_2}{M_2} - g$

Eqn. (4) ÷ M_1 $\Rightarrow$ $A - a = \frac{T_2}{M_1} - g$

Adding the equations $\Rightarrow 2A = \frac{T_2}{M_2} - g + \frac{T_2}{M_1} - g$

$$A = \frac{T_2}{2}\left(\frac{M_1 + M_2}{M_1M_2}\right) - g \text{ .... (5)}$$

Substituting Eqn (2) into Eqn (1)

$MA = Mg - 2T_2$

$$T_2 = \frac{Mg}{2} - \frac{MA}{2}$$

Substituting for T_2 into Eqn (5)

$$A = \left(\frac{Mg}{4} - \frac{MA}{4}\right)\left(\frac{M_1 + M_2}{M_1 + M_2}\right) - g$$

$$A = \frac{Mg}{4}\left(\frac{M_1 + M_2}{M_1M_2}\right) - \frac{MA}{4}\left(\frac{M_1 + M_2}{M_1M_2}\right) - g$$

$$A\left[1 + \frac{M}{4}\frac{(M_1 + M_2)}{M_1 + M_2}\right] = \frac{Mg}{4}\frac{(M_1 + M_2)}{M_1 + M_2} - g$$

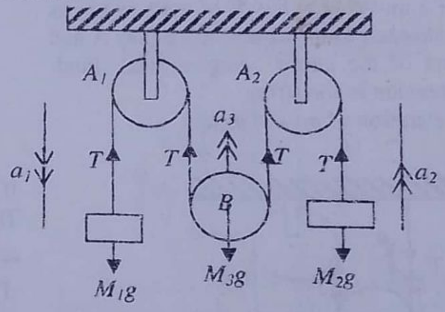
$$A = \frac{MM_1g + MM_2g - 4M_1M_2g}{MM_1 + MM_2 + 4M_1M_2}$$

Solve for T_1 from Eqn. (1)
and for T_2 from Eqn. (2)
Solve for a either using Eqn (3) or Eqn (4)

Case 3

Consider a light inextensible string whose one end carries a particle of m_1 kg and passes over a smooth fixed pulley A_1 then under a movable pulley B of mass m_3 kg, then over a smooth fixed pulley A_2 and its other end is attached to the particle of mass m_2 kg. if the portion of the string not in contact are vertical,

Solution



Taking $M_1 > M_2$.

Since B moves in between the portions of the string, then $a_3 = \frac{1}{2}(a_1 - a_2)$

For M_2kg mass, Resultant force = $T - M_2g$

$M_2a_2 = T - M_2g$ ....(1)

For M_1kg mass, Resultant force = $M_1g - T$

$M_1a_1 = M_1g - T$..........................(2)

For $m_3$kg mass, Resultant force = $2T - M_3g$

$\frac{1}{2}M_3(a_1 - a_2) = 2T - M_3g$.................(3)

From the Eqn. (1), $a_2 = \frac{T}{M_2} - g$ ....(4)

From Eqn. (2), $a_1 = g - \frac{T}{M_1}$ ....(5)

Eqn(5) – Eqn (4)

$$a_1 - a_2 = \left(g - \frac{T}{M_1}\right) - \left(\frac{T}{M_2} - g\right)$$

$$a_1 - a_2 = 2g - \left(\frac{M_2 + M_1}{M_1M_2}\right)T \text{ ....(6)}$$

From Eqn. (3), $a_1 - a_2 = \frac{4T}{M_3} - 2g$ ....(7)

Equating Eqn (6) to Eqn(7)

$$2g-\left(\frac{M_2+M_1}{M_1M_2}\right)T=\frac{4T}{M_3}-2g$$

$$\left(\frac{M_2+M_1}{M_1M_2}\right)T+\frac{4T}{M_3}=4g$$

$$T=\frac{4M_1M_2M_3g}{4M_1M_2+M_1M_3+M_2M_3}$$

For a_1, substitute for T into Eqn (5)

For a_2, Substitute for T into Eqn (4)

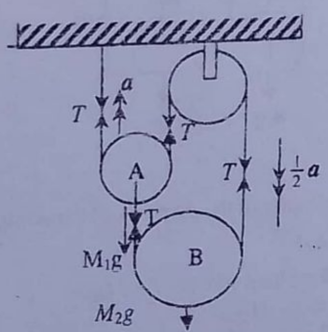
For a_3, We use $a_3=\frac{1}{2}(a_1-a_2)$

Case 4

Let a string whose one end is fixed pass under a movable pulley A of mass $m_1$kg, then over a fixed pulley and under a movable pulley B of mass $m_2$kg, its other end being attached to the axle of the pulley A and the hanging parts of the string being vertical. Find:

(i) The tension in the string.

(ii) Acceleration of m_1 and $m_2$kg.



When one pulley is connected to the axle of the other pulley by a string, its acceleration is a half that one of the other pulley.

If acceleration of $A=a$

Then acceleration of $B=\frac{1}{2}a$

For M_1kg mass, Resultant force $=2T-T-M_1g$

$$M_1a=T-M_1g\text{.....................(1)}$$

For M_2kg mass, Resultant force $=M_2g-2T$

$$\frac{1}{2}M_2a=2M_2g-4T\text{...............(2)}$$

4Eqn (1) + Eqn (2)

$$4M_1a+M_2a=2M_2g-4M_1g$$

$$a(4M_1+M_2)=(2M_2-4M_1)g$$

$$a=\frac{(2M_2-4M_1)}{4M_1+M_2}$$

$\therefore$ Acceleration of A is $\dfrac{(2M_2-4M_1)}{4M_1+M_2}$

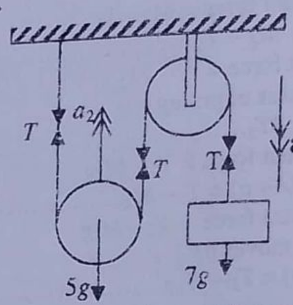
Acceleration of $B=\frac{1}{2}a=\dfrac{(M_2-2M_1)}{4M_1+M_2}$

For tension, substitute for a into Eqn(1)

Illustrative Examples.

1. A light inextensible string whose one end is fixed passes under a movable pulley of mass 5kg and then over a smooth fixed pulley and its other end is attached to a particle of mass 7kg. Given that the portions of the string not in contact are vertical. Find the accelerations of the movable pulley and the particle

Solution.



If the 7 kg mass moves down with acceleration = a_1

Then the 5kg mass moves upwards with acceleration $a_2=\frac{1}{2}a_1$

For 7kg mass, Resultant force = 7g − T

$$7a_1=7g-T\text{..........................(1)}$$

For 5kg mass, Resultant force = $2T-5g$

$$\frac{5}{2}a_1=2T-5g$$

$$5a_1=4T-10g\text{.......................(2)}$$

4(Eqn.(1)+ Eqn.(2)

$$33a_1=18g$$

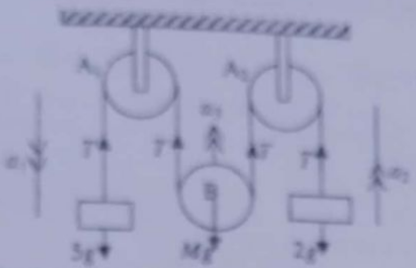
$$a_1=\frac{18g}{33}=5.345ms^{-2}$$

$$a_2=\frac{9g}{33}=2.673ms^{-2}$$

Hence the acceleration of the pulley and the particle are 2.673 ms^{-2} and 5.145 ms^{-2} respectively

2. Masses of 5kg and 2kg are suspended from the ends of a string, which passes over 2 fixed pulleys and under a movable pulley whose mass is Mkg, the portions of the string not in contact with the movable pulley being vertical. Find the value of M in order that when the system is released, the movable pulley remains at rest and find in this case the acceleration of other masses and the tension of the string.

Solution



For 5kg mass, Resultant force $= 5g - T$

$5a_1 = 5g - T$ ....(1)

For 2kg mass, Resultant force $= T - 2g$

$2a_2 = T - 2g$ ....(2)

For Mkg mass, Resultant force $= 2T - Mg$

$Ma_3 = 2T - Mg$

Since the pulley remains at rest, $a_3 = 0$

$2T - Mg = 0$ ....(3)

But $a_3 = \frac{1}{2}(a_1 - a_2)$

$a_2 = a_1$

Eqn (1) + Eqn (2)

$5a_1 + 2a_2 = 3g$

$7a_1 = 3g$ Since $a_1 = a_2$

$a_1 = \frac{3g}{7}$

Similarly $a_2 = \frac{3g}{7}$

From Eqn (2)

$T = 2a_1 + 2g$

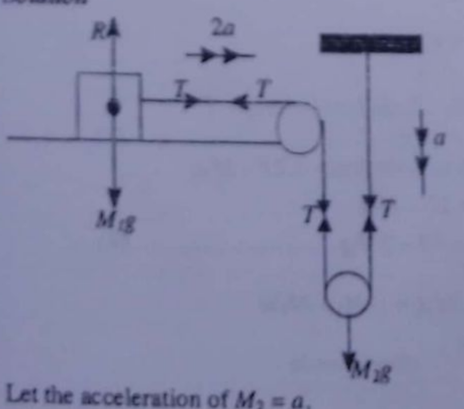
$T = \frac{6g}{7} + 2g = \frac{20g}{7}$

From Eqn (3)

$M = \frac{2T}{g} = \frac{40}{7}kg$

3. A particle of mass M_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass M_2 its other end being fixed so that the parts of the string beyond the table are vertical. Show that M_2 descends with acceleration $\frac{M_2 g}{4M_1 + M_2}$

Solution



Let the acceleration of $M_2 = a$,

Then acceleration of $M_1 = 2a$

For M_2kg mass, Resultant force $= M_2g - 2T$

$M_2a = M_2g - 2T$ ....(1)

For $m_1$kg mass, Resultant force $= T$

$2M_1a = T$ ....(2)

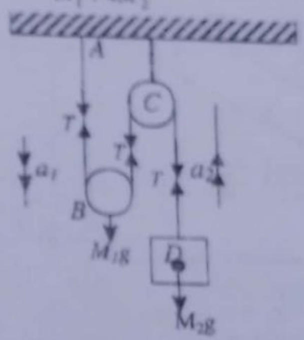
Eqn (2) + 2Eqn (1)

$M_2a + 4M_1a = M_2g$

$a(M_2 + 4M_1) = M_2g$

$$a = \frac{M_2 g}{M_2 + 4M_1}$$

4. A light string ABCD has one end fixed at A and passing under a movable pulley of mass M_1 at B and over a fixed pulley at C, carries a mass M_2 at D. the portions not in contact of the string are supposed to be vertical . Show that M_1 descends with acceleration $\frac{(M_1 - 2M_2)g}{M_1 + 4M_2}$



For that type of system

For M_1kg, Resultant force $= M_1g - 2T$

$M_1a_1 = M_1g - 2T$

But $a_1 = \frac{1}{2}a_2$

$\frac{1}{2}M_1a_2 = M_1g - 2T$

$M_1a_2 = 2M_1g - 4T$ ....(1)

For M_2kg mass, Resultant force $= T - M_2g$

$M_2a_2 = T - M_2g$ ....(2)

Eqn (1) + 4Eqn(2)

$a_2(M1 + 4M_2) = 2M_1g - 4M_2g$

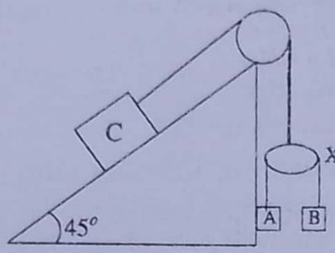
$$a_2 = \frac{2M_1g - 4M_2g}{M_1 + 4M_2}$$

$$a_1 = \frac{1}{2}a_2$$

$$a_1 = \frac{M_1g - 2M_2g}{M_1 + 4M_2}$$

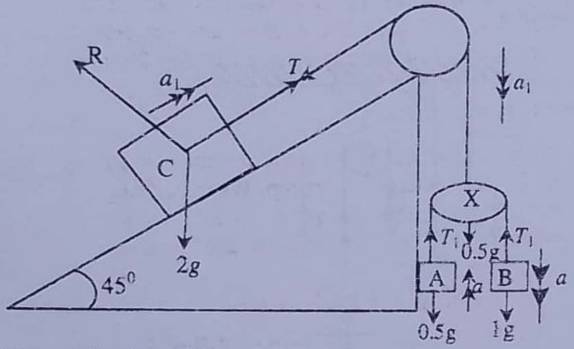
5. The diagram below shows two masses A and B of 0.5kg and 1kg respectively connected by a light inextensible string passing over a smooth pulley X of mass 0.5kg. Pulley X is connected to a mass C of 2kg lying on a smooth plane inclined at an angle 45^0 to the horizontal by a light

inextensible string passing over a fixed pulley . Find
(i)The acceleration of masses C and B
(ii)The tension in the strings when the system is released



Solution

Let acceleration of C and X be a_1 and that of masses A and B be a



Net acceleration of $B = a_1 + a$
Net acceleration of $A = a_1 - a$
For 2kg mass, Resultant force $= T - 2g\sin 45$

$2a_1 = T - \sqrt{2}g$.......................(1)

For pulley X, Resultant force $= 2T_1 + 0.5g - T$

$0.5\, a_1 = 2T_1 + 0.5g - T$

$\frac{1}{2}a_1 = 2T_1 + \frac{1}{2}g - T$

$a_1 = 4T_1 + g - 2T$ .... (2)

For 1kg mass, Resultant force $= g - T_1$

$1(a_1 + a) = g - T_1$

$a_1 + a = g - T_1$..........................(3)

For 0.5kg mass, Resultant force $= \frac{1}{2}g - T_1$

$A\text{-}a = \frac{1}{2}g - T_1$.......................(4)

Eqn (3) + Eqn(4)

$2a_1 = \frac{3g}{2} - 2T_1$ ....(5)

2Eqn (1)+ Eqn (2)

$5A = 4T_1 + (1 - 2\sqrt{2})g$................(6)

2Eqn (5) + Eqn. (6)

$9A = 3g + g - 2\sqrt{2}g = (4 - 2\sqrt{2})g$

$A = 1.2757\ \text{ms}^{-2}$

Hence acceleration of C is 1.28ms^{-2}

From (1), $T = 2\, a_1 + \sqrt{2}g$

$= 2 \times 1.2757 + \sqrt{2}g = 16.4107\text{N}$

From (5), $2T_1 = \frac{3g}{2} - 2a_1$

$= 14.7 - 2.5514 = 12.1486$

$T_1 = 6.0743$ N

From Eqn (3)

$a = g - T_1 - A$

$= 9.8 - 6.0743 - 1.2757 = 2.45$

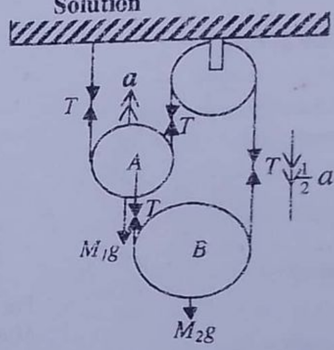
But acceleration of $B = a + A$

$= 2.45 + 1.2757$

$= 3.73\ \text{ms}^{-2}$

5 .A string with one end fixed passes under a pulley A of mass M_1 then over a fixed pulley ,then under a pulley B of mass M_2 and its other end is attached to the axle of A .The string is taut and its hanging parts are vertical .Find the ratio of the velocities of A and B when the system is in motion and show that the acceleration of A is $\frac{(4M_1 - 2M_2)g}{4M_1 + M_2}$

Solution



If acceleration of $A = a\ \text{ms}^{-2}$

Then acceleration of $B = \frac{1}{2}a\ \text{ms}^{-2}$

Since the system starts to move from rest,

$U = 0$

From $V = U + at$

$V_A = 0 + at$

$V_A = at$.......................................(1)

Also $V_B = 0 + \frac{1}{2}at$

$V_A : V_B = at : \frac{1}{2}at$

$= 2 : 1$

For M_1kg mass, Resultant force $= M_1g - T$

$M_1a = M_1g - T$...............................(3)

For M_2kg mass, Resultant force $= 2T - M_2g$

$\frac{1}{2}M_2a = 2T - M_2g$

$M_2a = 4T - 2M_2g$........................ (4)

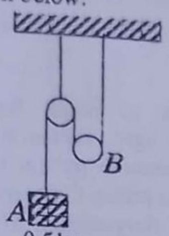
4Eqn(3) + Eqn.(4)

$4M_1g - 2M_2g = (4M_1 + M_2)a$

$a = \frac{(4M_1 - 2M_2)g}{4M_1 + M_2}$ down wards

Examination Questions

1. The diagram shows particle A of mass 0.5kg attached to one end of a light inextensible string passing over a fixed light pulley and under a movable light pulley B. The other end of the string is fixed as shown below.



0.5kg

(i) What mass should be attached at B for the system to be in equilibrium.

(ii) If B is 0.8kg what are the accelerations of particle A and pulley B?

(1997 No. 13)

Answers: (i) 1 kg (ii) $a_2 = \frac{g}{14}\text{ms}^{-1}$

7. To one end of a light inelastic string is attached a mass of 1kg which rests on a smooth wedge of inclination 30^0. The string passes over a smooth fixed pulley at the edge of the wedge, under a second smooth moveable pulley of mass 2kg and over a third smooth fixed pulley, and has a mass of 2kg attached to the other end. Find the accelerations of the masses and the moveable pulley and the tension in the string. (Assume the portions of the string lie in the vertical plane).

(1998 NOV/DEC No 13)

Answer: 4.9ms^{-2}, 0ms^{-2}, 9.8N

EXERCISE THREE

1. Forces of $(a\mathbf{i} + b\mathbf{i} + c\mathbf{k})$ N and $(2\mathbf{i} - 3\mathbf{i} + \mathbf{k})$ N acting on a body of mass 2kg causes it to accelerate at $(4\mathbf{i} + \mathbf{k})\text{ms}^{-2}$. Find the constants a, b and c.

2. Find the constant force necessary to accelerate a car of mass 1000kg from 15ms^{-1} to 20ms^{-1} in 10s against a resistance totalling 270N.

3. A train of mass 60×10^3 kg is travelling at 40ms^{-1} when the brakes are applied. If the resultant braking force is 40KN, find the distance the train travels before coming to rest.

4. A train of mass 100×10^3 kg starts from rest at station A and accelerates uniformly at 1ms^{-2} until it attains a speed of 30ms^{-1}. It maintains this speed for a further 90s and then the brakes are applied, producing a resultant braking force of 50KN. If the train comes to rest at station B, find the distance between the two stations.

5. A lift of mass 600kg is raised or lowered by means of a cable attached to its top. When carrying passengers, whose total mass is 400kg, the lift accelerates uniformly from rest to 2ms^{-1} over a distance of 5m. Find
   i) the magnitude of the acceleration
   ii) the tension in the cable if the motion takes place vertically upwards.
   iii) the tension in the cable if the motion takes place vertically downwards.

6. A stone of mass 50g is dropped into some liquid and falls vertically through it with an acceleration of 5.8ms^{-2}. Find the force of resistance acting on the stone.

7. A cage of mass 420kg contains three persons of total mass 280kg. The cage is lowered from rest by a cable. For the first 10 seconds the cage accelerates uniformly and descends a distance of 75m. Find the force in the cable during the first 10seconds

8. A bucket has a mass of 5kg when empty and 15kg when full of water. The empty bucket is lowered into a well, at a constant acceleration of 5ms^{-2}, by means of a rope. When full of water the bucket is raised at a constant velocity of 2ms^{-1}. Neglecting the weight of the rope, find the force in the rope:

i) When lowering the empty bucket

ii) When raising the full bucket.

9. A particle of mass 8kg slides down a rough plane which is inclined at arc sin1/6 to the horizontal .if the acceleration of the particle is g. Find the coefficient of friction between the particle and the plane.

10. A bullet of mass 0.02kg is fired into a wall with a velocity of $400ms^{-1}$. If the bullet penetrates the wall to a depth of 0.1m. Find the resistance of the wall assuming it to be uniform.

11. A lift of mass 500kg is drawn up by a cable .it makes an ascent in three stages: it is brought from rest to its maximum speed by a constant acceleration of ½ g, it then moves with its maximum speed for an interval of time and is then brought to rest by a acceleration of g. Find the tension in the cable in each of the three stages.

12. A car of mass 300kg is brought to rest in a time of 4 seconds from a speed of $20ms^{-1}$ .If there is no resistance to motion. Find the force exerted by the brakes assuming it to be constant

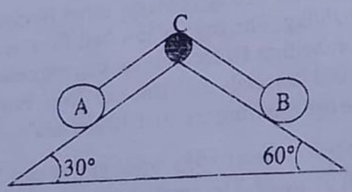
13. A car of mass 500kg is capable of braking with a deceleration of g. If the resistance to motion is constant and equal to 50N, find the braking force assuming this to be constant.

14. Two particles of mass 9kg and 10kg are connected by a light inelastic string, which passes over a fixed Pulley. Find the acceleration of the system and the tension in the string.

15. A particle of mass 4kg rests on a smooth plane which is inclined at 60^0 to the horizontal . The particle is connected by a light inelastic string passing over a smooth pulley at the top of a plane to a particle of mass 2kg which is hanging freely. Find the acceleration of the system and the tension of the string

16. A particle of mass 5kg rests on a rough horizontal table. It is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a particle of mass 6kg which is hanging freely. The coefficient of friction between the mass5kg and the table is 1/3. Find the acceleration of the system and the tension in the string.

17.



Two particles A and B rest on an inclined face of a fixed triangular wedge as shown in the diagram above. A and B are connected by a light inextensible string which passes over a light smooth pulley at C. The faces of the wedge are smooth and A and B are both of mass 7kg. Find the force exerted by the string on the pulley at C when the system is moving freely with both particles in contact with the wedge.

18. A particle of mass 10 kg lies on a rough horizontal table and connected by a light inextensible string passing over a smooth light pulley at the edge of the table to a particle of mass 8kg hanging freely. The coefficient of friction between the 10kg mass and the table is ¼. The system is released from rest with the 10kg mass at a distance of 1.5m from the edge of the table. Find

a) the acceleration of the system.
b) the resultant force on the edge of the table.
c) the speed of the 10kg mass as it reaches the pulley.

19. Two particles of mass 3kg and 5kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a light smooth moveable pulley C which carries a particle D of mass 6kg , the positions of the string not in contact are vertical, if the system is released from rest , find

a) the acceleration of particle A
b) the acceleration of pulley C.
c) the tension in the string.

20. A particle of mass 6kg is connected by a light inextensible string passing over a fixed smooth pulley to a light smooth moveable pulley B. Two particles C and D of mass 2kg and 1kg are connected to a light inextensible string passing over the pulley. When the system is moving freely, find the acceleration of the 1kg mass and the tension in the string.

21. A particle A of mass $2m$ is initially at rest on a smooth plane inclined at an angle α to the horizontal. It is supported by a light inextensible string which passes over a smooth light pulley P at the top edge of the plane. The other end of the string supports a particle B of mass m, which hangs freely.

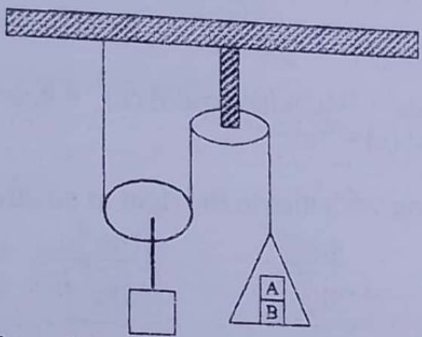
(a) Given that the system is in equilibrium, find α and the magnitude and direction of the resultant force exerted by the string on the pulley.
(b) A further particle of mass m is now attached to B and the system is released. Find , for the ensuing motion:
  (i) the tension in the string,
  (ii) the acceleration of B,
  (iii) the magnitude and the direction of the resultant force exerted by the string on the pulley.

d) One end of the light inextensible string is attached to a ceiling. The string passes under a smooth light pulley carrying a weight C and then over fixed smooth light pulley. To the free end of the string is attached a light scale pan in which two weights

A and B are placed with
The portions of the strin
pulley are vertical.

Each of the weights A a
C has a mass of kM. If t

A and B are placed with A on top of B as shown. The portions of the string not in contact with the pulley are vertical.



Each of the weights A and B has a mass M and weight C has a mass of kM. If the system is released from rest. Find the acceleration of the moveable pulley and the scale pan and show that the scale pan will ascend if $K > 4$. When the system is moving freely, find

a) the tension in the string
b) reaction between the weights A and B.

23. A body of mass 5kg moves in a straight line across the horizontal surface against a constant resistance of magnitude 10N. The body passes through point A and then comes to rest at a point B, 9 m from A. Find the speed of the body when it is at A.