MT ST HENRY'S HIGH SCHOOL MUKONO

S.4 ANNUAL MATHEMATICS SEMINAR QUESTIONS 2023

HELD ON 17TH JUNE 2023

STIPULATION OF PAPER CONTENT

PAPER 1 (456/1)	PAPER 2 (456/2)
1. Algebraic symbols/expressions a) Substitutions b) Linear equations c) Operations and numbers d) Change of subject of a formula e) Expansion and factorization **2. Matrices** a) Operations with matrices b) Order of matrix c) Zero matrix and identity matrix d) Determinant and inverse of a 2X2 matrix e) Matrices and simultaneous equations f) Applications **3. Equations and inequalities** a) Coordinates, linear equations and gradients b) Simultaneous equations c) Quadratic equations (non – linear equations) d) Graphic methods of solving equations e) Inequalities and regions **4. Li near Programming** a) Wanted and unwanted regions b) Regions c) Maximizing and minimizing **5. Two-Dimensional Geometry** a) Angle properties of a triangle, quadrilaterals and polygons b) Trigonometry and its applications c) Circle and angle properties d) Construction and Bearings e) Perimeter, area of shapes and lengths **6. Statistics and Probability** a) Data presentation b) Measures of central tendency (mean, median, mode) Grouped data (mode using Histogram, calculation and median using and Ogive) c) Experimental probabilities d) Mutually exclusive and independent events e) Random selection and Tree diagrams **7. Transformation geometry** a) Congruency and symmetry b) Translations, reflections and rotations c) Enlargement d) Combined transformations e) Matrices of transformations	**1. Numerical concepts** a) Numbers, Number bases, Factors, multiples, primes and integers b) Sequences and number patterns c) Fractions, decimals and percentages d) Indices, logarithms e) Surds (simplifying and rationalizing the denominator) f) Ratios (increasing or decreasing, representative fraction, multiple ratios) g) Proportion (direct, inverse, joint partial variation) **2. Graphs** a) Graphs of lines, curves and waves b) Coordinates and lines c) Cartesian graphs and equations d) Kinematics (Distance, speed and time) e) Graphs of motion (distance – time and speed – time graphs) **3. Relations, Mappings and Functions** a) Relations and mappings b) Domain and range c) Arrow diagrams and papygram d) Function notation e) Inverse functions f) Composite functions g) Undefined, meaningless or infinite functions 4. **Set theory and logic** a) Elements of a set b) Universal set and subsets c) Intersections, Unions and compliments of two sets d) Three set problems **5. Vectors** a) Column and position vectors b) Adding and subtraction of vectors c) Mid – point of a vector d) Magnitude of vector e) Parallel vectors and collinear points f) Vector geometry **6. Business mathematics** a) Profit, Loss, Discounts and commissions b) Interest, Appreciation and depreciation c) Hire purchase, Exchange rates and insurance d) Taxation **7. 3-D Dimensional geometry** a) Areas of plane figures b) Similarity and enlargement (scale factors) c) Surface areas and volumes of solid shapes (mensuration) d) Angle between a slanting edge(line) and a plane e) Angle between two planes

ALGEBRA

1. Given that $a * b = \frac{(2a^2-b)^2}{ab}$, evaluate $(3 * 6) * 4$
2. Find the value of b if $37_{\text{b}} = 114_{\text{five}}$
3. Factorize the following completely
   (i) $27x^2 - 12y^2$
   (ii) $2x^4 - 32$
   (iii) $x^2 - 2xy - 5x + 2y + 4$
   (iv) $a^2 - b^2$ hence evaluate $\frac{76.45^2 - 23.55^2}{5.29}$ without using tables or a calculator.
4. Make t the subject of the formula $P = \frac{n}{2m}\sqrt{\frac{f}{k-t}}$

5. Two numbers differ by 4. Their sum is 14. Find the two numbers.

EQUATIONS

6. Solve the equations $\frac{5}{2}(y-1) - \frac{2}{5}(5-3y) = 21$
7. a) solve the pair of simultaneous equations below;
   $m = 4 - n$
   $n^2 + 2m^2 = 67$
   b) Wasike was given Shs 6,200 which is exactly enough to buy 3 loaves of bread and 2 kg of salt. However, he made a mistake and purchased 2 loaves of bread and 3 kg of salt. He then had a balance of Shs 400. Calculate the cost of
   (i) a loaf of bread
   (ii) a kilogram of salt
8. a) Form a quadratic equation with roots $\left\{-4, \quad {}^{5}/_{3}\right\}$
   b) Express the expression $2x^2 - 5x + 2$ in the form $a(x-b)^2 + c$. State the values of a, b and c. Hence solve the equation $2x^2 - 5x + 2 = 0$
9. Mubiru walks 10 kilometers to a waterfall at an average speed of x kilometers per hour.
   a) Write down in terms of x, the time taken in hours.
   b) Mubiru returns from the water falls but this time he walks the 10 kilometers at an average speed of $(x+1)$ kilometers per hour. The time of the return journey is 30 minutes less than the time of the first journey. Write down an equation in x and show that it simplifies to $x^2 + x - 20 = 0$.
   c) Solve the equation $x^2 + x - 20 = 0$.
   d) Find the time Mubiru takes to walk to the water falls.

10. To buy a plot of land in a certain town, 7.2 million shillings had to be raised by some members. Each member was to contribute some equal amount. Before the agreed date, five members withdrew. This means that the remaining members had to pay more to meet the target.

a) If n stands for the number of members in the group originally, show that the increase per person was Shs. $\frac{36,000,000}{n(n-5)}$.
b) If the increase per person was Shs. 120,000, find the original number of members in this venture.
c) How much could each have contributed if the five hadn't withdrawn?
d) Calculate the percentage increase in contribution per member caused by the withdrawals to one decimal place.

MATRICES

11.Given that $A = \begin{pmatrix} 3x & x-6 \\ -6 & x+2 \end{pmatrix}$, find the values of x for which the matrix A is singular.

12.Given the matrices $A = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & -2 \end{pmatrix}$ find;

a) $2A + 3B$
b) $4B - A$
c) B^2

13.Calculate the inverse of matrix $T = \begin{pmatrix} -3 & -2 \\ 1 & 1 \end{pmatrix}$

14. a) If $\begin{pmatrix} 3 & x \\ 5 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ y \end{pmatrix}$, find the values of x and y.

b) Use matrix method to solve the pair of simultaneous equations;

$x - 4 + 2y = 0$
$x + y = 3$

15. a) The matrix $A = \begin{pmatrix} a & 14 \\ 1 & b \end{pmatrix}$ and its inverse $A^{-1} = \begin{pmatrix} 1 & -7 \\ -\frac{1}{2} & 4 \end{pmatrix}$. $AA^{-1} = I$ where I is a 2×2 identity matrix. Find the values of a and b.

b) Given that $A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & -3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 3 & -1 \\ 1 & -4 \end{pmatrix}$ and $C = AB$, determine the;

i) Order of C
ii) Matrix C
iii) C^{-1}

16. A trader orders for bubs in different colours and wattage as shown in the table below.

	40W	60W	70W	100W
Green bulbs	400	200	0	400
Yellow bulbs	200	300	100	600
Blue bulbs	300	600	200	0

The cost for bulbs was as follows; Shs.800 @ 40W, Shs. 900 @ 60W, Shs. 950 @70W and Shs. 1000 @100W.
a) Write down for bulbs a 4X3 matrix.
(iii) Write down the cost as a 4X1matrix.

b) If the trader paid 10 % on top of the expenditure in form of taxes, find his total expenditure.
c) Given that the bulbs were sold as follows; Shs 900 @40W, Shs.1000 @60W, Shs. 1050 @70W and shs.1100 @100W.Find the percentage profit the trader makes.

STATISTICS AND PROBABILITY

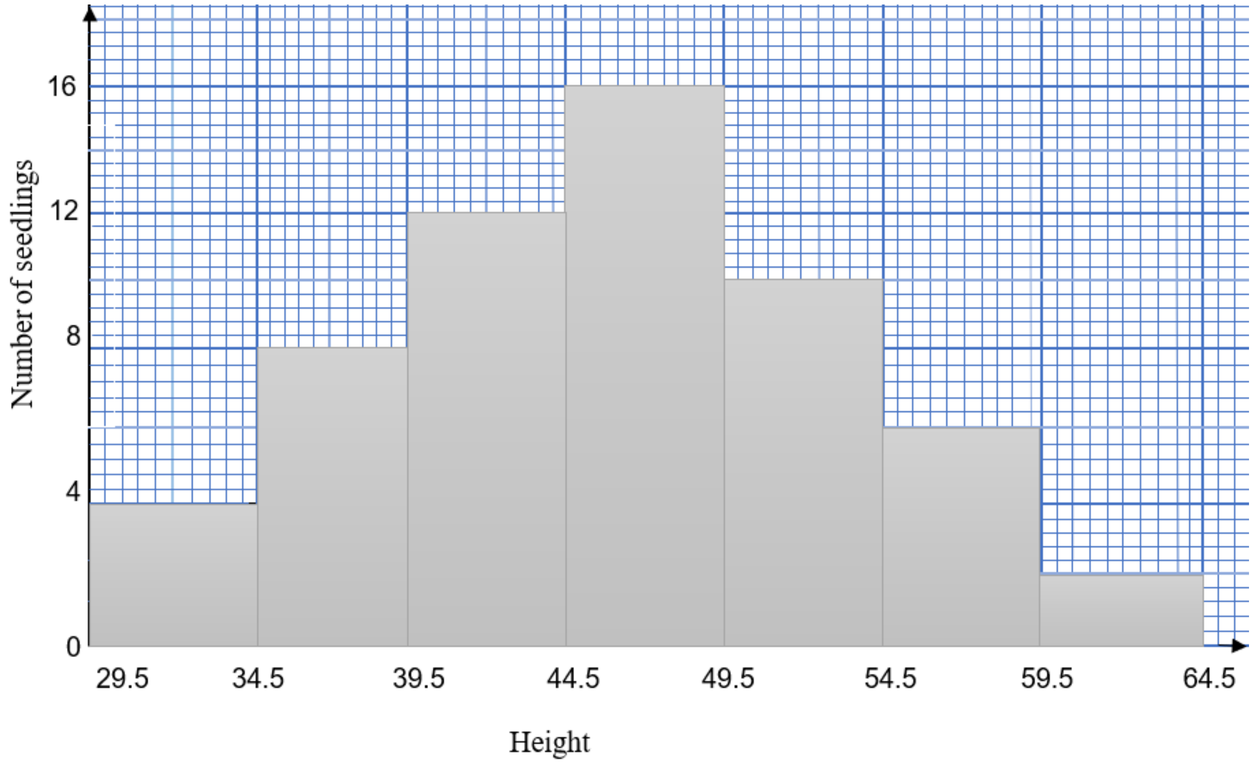
17. A bag contains 4 green marbles, 6 red marbles and some yellow marbles. If the probability of picking a yellow marble from the bag is $^1/_6$. Find the total number of marbles in the bag.
18. The probability that Paul comes to work on Monday is $\frac{3}{5}$. The probability that Joan works on the same day is $\frac{2}{3}$. Determine the probability that Paul will come to work on Monday and Joan will not come.
19. A die has faces numbered 7, 8, 9, 10, 11 and 12. A second die has faces numbered 1, 2, 3, 4, 5 and 6. During a game, the two die are tossed. The difference between the numbers on the first and second die are recorded as scores.
    a) Construct a possibility space for the scores.
    b) Find the probability of obtaining a score that is a multiple of 2.
    c) Find the probability of obtaining a score of 3 or more.
20. A bag contains 4 white and 5 black balls. Two balls are picked from the bag one after the other without replacement.
    a) Draw a probability tree to illustrate the information.
    b) Calculate the probability that the balls picked are of;
        i) the same colour
        ii) different colours
21. A box contains 4 black, 3 green and 5 red pens. If two pens are picked at random with replacement. Find the probability that;
    (i) The two pens are red.
    (ii) Both pens are of the same colour
    (iii) At least a black pen is picked
22. The frequency table below shows the performance of 100 students in a class.

Height (cm)	Number of students
0 – 9	10
10 – 19	x
20 – 29	25
30 – 39	30
40 – 49	y
50 – 59	10

    a) Given that the median mark is 30.5, find the values of x and y.
    b) Hence use your table to calculate;
        (i) The mean
        (ii) The modal mark.

23. The graph is a histogram showing the heights in millimeters of seedlings in a nursery bed.

a) Use it to draw a frequency distribution table. How many seedlings were considered?



b) Using your table;

i) Calculate the mean height using the working mean of 44.5 millimeters

ii) Find the modal height

Construct an Ogive and use it to estimate the median height of the seedlings.

GEOMETRY

24. Express the following ratios as trigonometric ratios of acute angles

a) $\sin 660^0$

b) $\cos 1080^0$

A regular polygon has an exterior angle of 24^0. Determine;

(i) the number of sides of the polygon

(ii) the angle sum of the polygon

25. a) Given that $\sin \theta = \frac{5}{13}$ and that θ for $90^0 < \theta < 180^0$. Find the possible value(s) of θ.

b) Find $\cos \theta$ and $\tan \theta$

c) Copy and complete the table below;

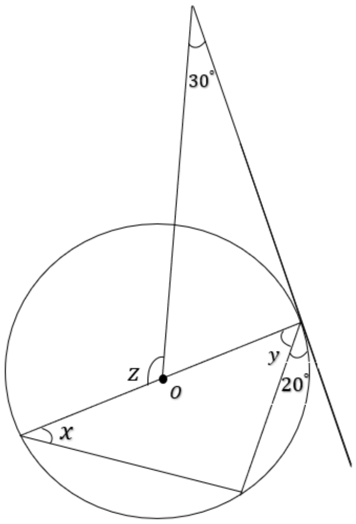
Θ	30^0	40^0	50^0	60^0	70^0	80^0	90^0	100^0	110^0	120^0	130^0	140^0	150^0
sin θ			0.77			0.98				0.87		0.64	
y = 2 sin θ			1.54			1.96				1.74		1.28	

(i) Use your completed table to draw a graph of $y = 2 \sin \theta$

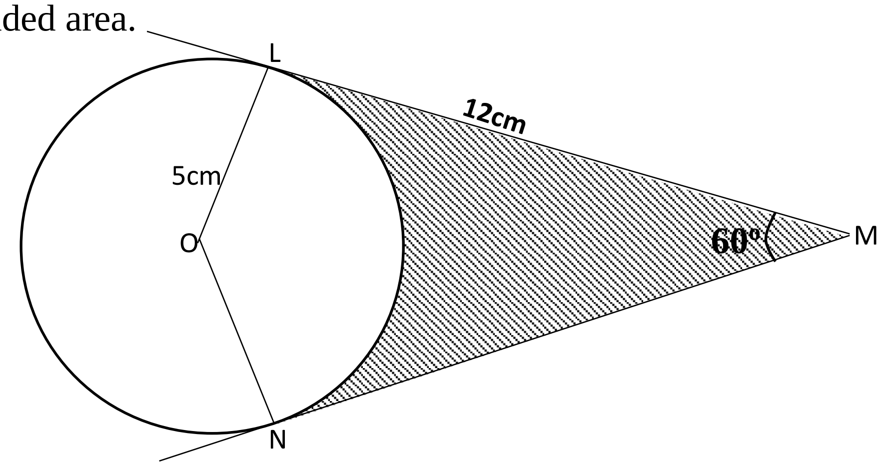
(ii) On the same axes, draw a line $y = 1.2$, hence obtain values of θ when $y = 1.2$

(iii) Use the graph to find the value of y when $\theta = 56^0$. Hence determine $\sin 56^0$

26. In the figure below, $\boldsymbol{O}$ is the center of the circle. Find the values of x, y and z.

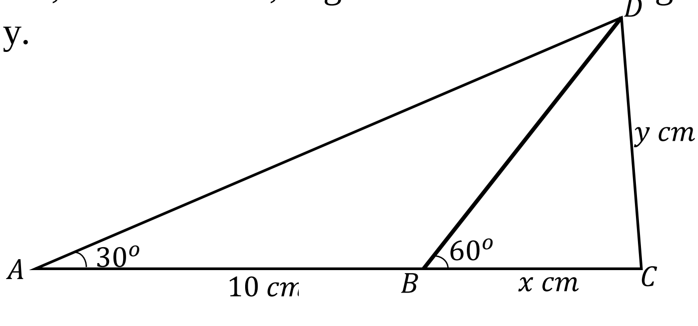


27. In the figure below, O is the center and LM and MN are tangents to the circle. Calculate the shaded area.



28. Using a ruler and a pair of compasses only,
   a) Construct a triangle PQR in which QR $= 9.2$ cm, PR $= 8.5$ cm and angle PQR $= 60^0$. Measure length QP.
   b) Bisect the sides PQ and PR. Produce the line bisectors to meet at M.
   c) Using M as the center, draw a circle to circumscribe triangle PQR, measure the radius of the circle and hence calculate the area of the circle.
   d) Find the area of the triangle PQR.

29. In the figure below, AB $=$ 10 cm, angle DAB $= 30^0$ and angle DBC $= 60^0$. Calculate the values of x and y.



30. Kiprotich started running from station P at a bearing of 330^0 for a distance of 10km to station T. He then moved in the Eastern direction at a speed of 5km/hr for 2hours to

junction M. He then turned through a bearing of 072^0 and moved for 15km to station N. He then moved to station K which is 18km south of station P. If 1cm represents 2km,

a) Draw a diagram to show his journey. Find;
b) Distance and bearing of P from K.
c) Distance and bearing of P from N.
d) Total time spent if he was running at an average speed of 2km/hr.
e) Distance and bearing of K from N.

31. The angle of depression from the top of a cliff to a boat moving towards the cliff at a speed of 10 Kmh^{-1} is 45^0. After 2 hours, the angle of depression of the boat from the top of the cliff is 60^0. Calculate the height of the cliff.

TRANSFORMATONS

32. Given that triangle PQR with area 25 cm^2 is mapped onto its image whose area is 125 cm^2 by a transformation matrix N $\begin{pmatrix} x & 4 \\ 1 & 3 \end{pmatrix}$. Find x.

33. Triangle ABC with vertices A (1, 2), B (2, 6) and C (4, 2) is mapped onto triangle A'B'C' by reflection in the line $x + y = 0$. Triangle A'B'C' is then mapped onto triangle A"B"C" by a transformation whose matrix is $\begin{pmatrix} 2 & 5 \\ -4 & -5 \end{pmatrix}$.

a) Use I (1, 0) and J (0, 1) to find the matrix of reflection in the line $y + x = 0$
b) Find the;
(i) co – ordinates of A', B' and C'
(ii) co – ordinates of A", B" and C".
c) Determine a matrix for the single transformation which maps A"B"C" back onto ABC

34. A triangle ABC has vertices $\text{A}(0, 0)$, $\text{B}(0, -2)$ and $\text{C}(2, 0)$. Its image under a transformation matrix M has vertices $\text{A}'(0, 0)$, $\text{B}'(0, -4)$ and $\text{C}'(4, 0)$.

a) Find matrix M and describe fully the transformation.
b) $\text{A}'\text{B}'\text{C}'$ is then further transformed by matrix $\text{N} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ to form image A"B"C". Find the coordinates of A", B" and C".
c) Determine a single transformation that maps A"B"C" directly back onto ABC.

35. A triangle ABC with vertices A(1, 3), B(3, 3) and C(3, 1) is enlarged with scale factor – 4 about (2, 2) to form triangle A'B'C'. Triangle A'B'C' is then rotated through a positive quarter turn about (0, - 4) to form a triangle A"B"C".

a) Draw on the same axes the triangles ABC, A'B'C' and A"B"C"
b) Write the coordinates of
(i) A', B' and C'
(ii) A", B" and C"

QUADRATIC CURVES

36. a) Draw a graph of $\text{y} = 3 + 2\text{x} - \text{x}^2$ for $-4 \leq \text{x} \leq 4$; using a scale 2 cm:1 unit on the $\text{x} -$ axis and $1\text{cm}: 1$ unit on the y – axis.

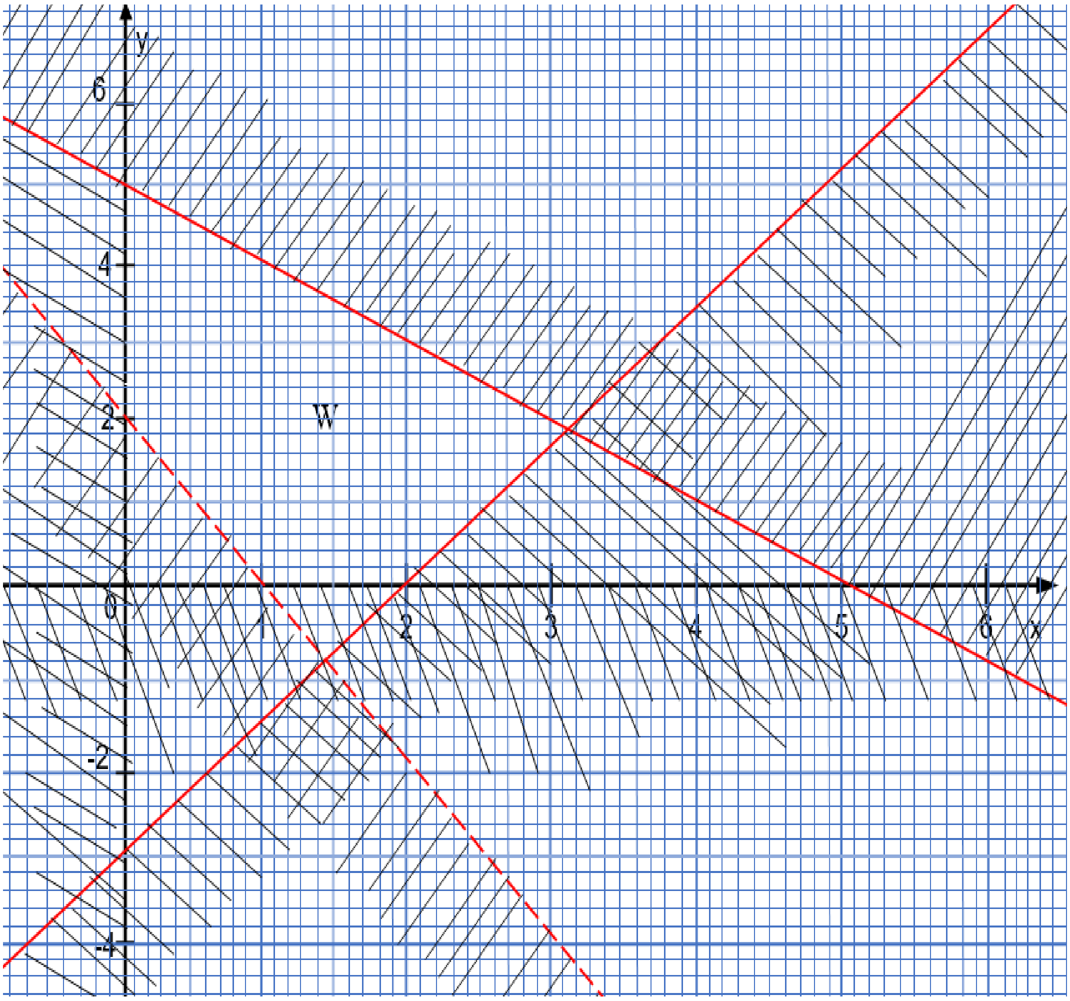
b) State the; (i) turning point of the curve.
(ii) range of values for which $y \leq 4$
c) From your graph, solve; (i) $3 + 2\text{x} - \text{x}^2 = 0$ (ii) $6 + x - x^2 = 0$

INEQUALITIES AND LINEAR PROGRAMMING

37. Solve the inequalities and represent them on a number line

(i) $3x + 7 \leq 5x + 1$ (ii) $3x + 1 \geq 2x - 2 < 6$

38. Study the graphical diagram below and answer the questions that follow;



a) Determine the five inequalities satisfied by the unshaded region, W, shown in the diagram above.
b) State the integral coordinate point of the feasible region that gives the maximum value of the expression $2x + y$.

39. A certain hotel has seven roasters of 200kg oven capacity and six roasters of 400kg oven capacity. The 200kg oven capacity roaster can be used five times a day. The 400kg oven capacity roaster can be used 2 times a day. Each roaster must be operated by one chef. On a given Saturday, the hotel was contracted to roast 9000 kg of meat for guests at a wedding ceremony. On that day, 11 chefs were available, the 200kg oven capacity roaster needs shs. 12,000 per day to run and the 400kg oven capacity roaster each needs shs.20,000 per day to run. If x and y represent the number of 200kg and 400kg oven capacity roasters to be used by the hotel respectively.

a) Write down six inequalities representing the above information.
b) Plot, on the same axes, graphs for the inequalities in (a) above shading out the unwanted regions.
c) Use your graph to find the number of each type of roaster the hotel used to keep its cost as minimal as possible.

NUMERICAL CONCEPTS

40. Find the **LCM** and **HCF** of 54, 72 and 144

41. Use logarithm tables to evaluate $\frac{0.6327\times\sqrt[3]{2.834}}{2.918}$.

42. Simplify $\frac{1\frac{1}{2}-\left(8\frac{1}{3}\div 2\frac{1}{2}\right)}{1\frac{1}{5}\text{ of }\left(1\frac{1}{4}+1\frac{2}{3}\right)}$

43. a) Evaluate $\left(\frac{16}{81}\right)^{-\frac{3}{4}}\times\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

b) Simplify $\sqrt{72}+\sqrt{50}-\sqrt{98}$ ii) $\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{20}}$ in the form $\frac{a}{b}\sqrt{c}$

c) Express the following in the form $\mathrm{a}+\mathrm{b}\sqrt{\mathrm{c}}$, hence state the values of a, b and c.

(i) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ (ii) $\frac{\sqrt{7}}{\sqrt{7}+\sqrt{5}}$

d) Solve the equations (i) $\frac{1}{2^{x}}\times 8^{x+1}=\frac{1}{32}$ (ii) $\log_5 625 = x$

e) Solve the simultaneous equations

$8^{x}=4^{2y-1}$ and $27^{2x}=9^{y-3}$

44. a) Work out $2\log_3 9-\frac{1}{3}\log_3 729+\frac{3}{4}\log_3 81+2$

b) Given that $\log x = 2.3412$ and $\log y = \bar{3}.1212$, find the value of;

(i) $\log\sqrt{x^2y}$ (ii) $\log\frac{x}{y^2}$

45. Convert the following to recurring decimals

(i) 0.777 ... . (ii) $0.\dot{3}\dot{8}$

RATIOS AND PROPORTIONS

46. An aircraft carries fuel in three tanks whose capacities are in the ratio $3:4:5$. The capacity of the smallest tank is 720 litres. Calculate;
a) the capacity of the largest tank.
b) The total capacity of the three tanks.

47. The scale of a map is $\frac{1}{200{,}000}$. Calculate the area, in cm^2, on the map of a forest reserve which covers 84 km^2 on the actual ground.

48. A machine used to paint white lines on a road uses 250 liters of paint for each 8 km of road marked. Calculate;
(i) how many liters of paint would be needed for 200 km of road
(ii) what length of road could be marked with 4000 liters of paint?

49. Two jugs are similar. The smaller jug has a surface area of 800 cm^2 while the larger jug has a surface area of 1250 cm^2. The smaller jug has a volume of 1280 cm^3. Find the volume of the larger jug.

50. The volume of wood in a tree (V) varies directly as the height (h) and inversely as the square of the girth (g). If the volume of a tree is 144 m^3 when the height is 20 m and the girth is 1.5 m, what is the height of a tree with a volume 1000 m^3 and girth of 2 m.

51. Six people can dig a trench in 8 hours. How long would it take;
   (i) 4 people
   (ii) 12 people to dig the same piece of land.

52. A food Aid agency carried out a survey to ascertain the average monthly expenditure on food by a family in Seeta. The expenses on food were found to be in two parts; a constant expenditure and another part varying as the square of the number of children in the family. A family of 3 children needed shs 190,000 while that of 5 children needed shs 310,000.
   a) Write down an expression for the total expenditure on food spent per month by a family with n children.
   b) What is the monthly expenditure for;
      (i) a childless family?
      (ii) a family with four children?
   c) How many children are in a family which needs an average expenditure of Shs. 392,500?

COORDINATE GEOMETRY

53. A line passes through the points A (-1, 7) and B (5, 3). Determine the;
   a) midpoint of the line AB.
   b) length of the line AB.
   c) gradient of the line AB.
   d) equation of the line joining points A and B.

54. A line has a gradient $-\frac{2}{3}$ and its x – intercept is 6. It cuts the y – axis at point P. Determine the;
   a) equation of the line
   b) coordinates of P.

55. a) Find the equation of a line passing through (5, 5) and is parallel to the line $5 - y = -3x$.
   b) Find the equation of the line passing through $(2, 1)$ and perpendicular to the line $4y + 7 + 3x = 0$

56. A line L_1 passes through the points $(-2, 5)$ and $(5, 3)$.
   a) Determine the equation of the line L_1
   b) Another line L_2 is perpendicular to L_1 and also passes through the point $(-2, 5)$. Find the equation of L_2.

RELATIONS, MAPPINGS AND FUNCTIONS

57. Given that $T = \{2, 5, 6, 8, 9, 10, 12, 13\}$. Illustrate on papygrams the relations; (i) "greater than by 3" (ii) "is a factor of"

58. If $h(x) = \frac{x^2-1}{x+1}$, find; (i) $h(0)$ (ii) $h(2)$ (iii) $h(-4)$

59. Given that $f(x) = \frac{2x-3}{x^2-3x-10}$, find;

(a) $f(0)$

(b) the value of x for which $f(x)$ is undefined.

60.(a) Given that $f(x, y) = ax^2 + by$, $f(-2, 1) = 10$ and $f(3, 2) = 16$

i) find the values of a and b

b) Given that $g(x) = \frac{2}{x^2-1}$; find (i) $g^{-1}(x)$ (ii) $g^{-1}\left(\frac{2}{15}\right)$

c) Given that $f(x) = 8x + 5$ and $g(x) = 3x - 5$, find

(i) $fg(x)$ (ii) $fg(2)$

d) If $g(x) = 1 + 5x$ and $gf(x) = 2x^2 + 4$, find $f(2)$

SETS AND LOGIC

61. a) Given that $n(A) = 9$, $n(A \cap B) = 4$, $n(A \cup B) = 12$ and $n(\varepsilon) = 18$. Find the;

(i) $n(A')$ (ii) $n(A' \cap B)$ (iii) $n(A \cup B)'$

b) Given the sets;

$A = \{\text{all composite numbers less than } 30\}$

$B = \{\text{all triangle numbers between 1 and } 30\}$

Find; i) $n(A \cap B')$ (ii) $n(A' \cap B)$

62. In the S.3 class, 7 like football and basketball, 6 like basketball and volleyball, 5 like football and volleyball, 8 like none of the three games, 12 like volleyball, 16 like either football or basketball but not volleyball. The number for basketball exceeds the number for football by 2. 18 students like only one type of the games. Taking the number of students that like football to be y, represent the above information on a Venn-diagram. Find the probability that a student a student picked at random likes at least two games.

63. In MSHHSM, S.4 students were asked which of the following drinks they liked, Coke (C), Pepsi (P) and Fanta (F). The following information was obtained.
$n(C \cap P) = 4$, $n(C \cap P' \cap F') = 4$, $n(F \cap P) = 7$, $n(F \cup C \cup P)' = 2$, $n(C \cap F) = 3$, $n(F) = 12$, those who do not like Coke are 20 and those who like Pepsi only are three times those who like Fanta only.

a) Draw a venn diagram to represent the above information and use it to find;

(i) $n(F \cup C \cup P)$

(ii) $n(C \cap F)$

(iii) $n(\varepsilon)$

b) If a student is picked at random, what is the probability that he plays atmost one game.

KINEMATICS

64. Mbale is about 255 Km away from Kotido. A bus leaves Kotido for Mbale at 6:45 am travelling at a steady speed of 50 kmh^{-1}. A taxi leaves Kotido an hour later at a speed of 70 kmh^{-1} but gets a flat tire after travelling for 1 hr 30 min. The mechanical problem was fixed after 30 minutes and then the taxi driver decided to increase the speed by 15 kmh^{-1}. Draw on the same axes, the distance – time graphs showing the journey of the bus and taxi.(Use scales of 2 cm: 30 km and 2 cm : 1 hr). Determine;

a) the time and distance from Mbale when the taxi overtakes the bus.

b) state the times when the two vehicles arrive in Mbale.
c) differences in the times of arrival of the two vehicles.

VECTORS

65. a) Given the vectors $\mathbf{OA} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, $\mathbf{OB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.Find

(i) **BA**

(ii) **|BA|**

b) If $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{s} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, find;

i) $\mathbf{r} + \mathbf{s}$ iii) $|2\mathbf{r}|$

ii) $\mathbf{s} - \mathbf{t}$ iv) $|3\mathbf{s} + 2\mathbf{t}|$

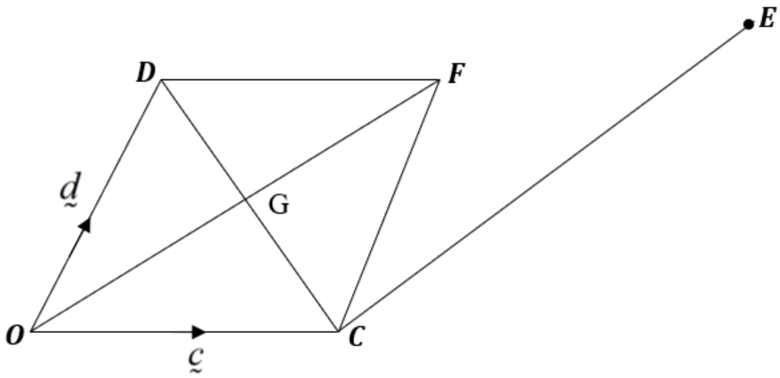
c) If $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$, find numbers m and n such that $\text{m}\mathbf{a} + \text{n}\mathbf{b} = \mathbf{c}$

66.a) The position vectors of D and E are $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -11 \end{pmatrix}$ respectively. M is on DE such that $\text{DM}: \text{DE} = 2:3$, find

(i) $\overline{DE}$ (ii) $\overline{DM}$ (iii) the position vector of M

b) Show that the points A(1, 1), B(2, 3) and C(5, 9) lie on a straight line.

67.



In the diagram above $\overrightarrow{\text{OC}} = \mathbf{c}$, $\overrightarrow{\text{OD}} = \mathbf{d}$, $7\overrightarrow{\text{DG}} = 4\overrightarrow{\text{DC}}$ and $\overrightarrow{\text{OG}}:\overrightarrow{\text{GF}} = 5:2$. ODFC is not a parallelogram, In terms of c and d only,

a) Find (i) $\overrightarrow{\text{DG}}$ (ii) $\overrightarrow{\text{OG}}$ (iii) $\overrightarrow{\text{OF}}$

b) Show that DFE are collinear given that $\overrightarrow{\text{OE}} = \frac{1}{5}(12\text{c} - \text{d})$

68.The position vectors of the points A and B are **a** and **b** respectively. The point P lies on OA produced such that $\overline{OP} = 3\overline{OA}$. Point Q lies on OB such that $\overline{OQ} = \frac{1}{3}\overline{OB}$. The lines AB and PQ meet at C.

a) Express PQ in terms of vectors **a** and **b**

(i) If also $PC = \mu PQ$, express OC in terms of $\mu, \boldsymbol{a}$ and $\boldsymbol{b}$

(ii) If also $AC = \lambda AB$, express OC in terms of $\lambda, \boldsymbol{a}$ and $\boldsymbol{b}$**.** Hence find the value of λ and μ

b) Show that $\overline{AQ}$ is parallel to $\overline{PB}$

BUSINESS MATHEMATICS

69. A house was bought at Shs 1,500,000. In the first year, its value appreciated by 25%, in the second year by 10% but dropped by 20% in the third year. Find its value after 3 years.

70. a) A cash discount of 15% is allowed on a colour TV set with a marked price of Shs 900,000. Hire purchase terms are; a deposit of 30% of the marked price and 12 monthly instalments of Shs. 65,000. Find the percentage profit made on the hire purchase scheme over the cash payment.

b) Mr Bogere deposited UGX 2,000,000 on a fixed deposit account at a compound interest of 20% per annum for 4 years. Calculate the;

i) amount on his fixed deposit account after 4 years

ii) interest he earned after the 4 years

c) A smart phone from M-KOPA was bought on hire purchase. A deposit of 100,000 was paid and 15 monthly instalments of UGX 55,000 was required.

i) Calculate the total amount paid on hire purchase.

ii) If the hire purchase price is 20% higher than the cash price, find the cash price.

d) A black book was sold at UGX 19,000 making a loss of 5%. At what price should it be sold to gain 5% profit.

71. The table below shows the tax structure on taxable income of public servants working in a certain country.

Income per annum (shs)	Tax rate (%)
0 – 1,200,000	12.5
1,200,001 – 2,400,000	30.0
2,400,001 – 3,600,000	36.5
3,600,001 and above	45.0

A man's gross annual income is Shs 6,460,000. His allowances are;

Housing – Shs 125,000 per month

Marriage - $\frac{1}{10}$ of his gross annual income.

Medical – Shs 354,000 per annum

Family allowances per annum for only 3 children are as follows.

Shs 25,000 for each child between 10 and 18 years

Shs 32,000 for each child below 9 years

He has to pay insurance premium of Shs 48,900 per annum

He has four children with two of them below eight years, one is 16 years and the oldest is 20 years.

Calculate:

a) his taxable income
b) income tax paid annually
c) net income

72. In a certain organization, the following allowances are not taxed. For any employee,

Transport shs. 35,000 per month

Housing shs. 80,000 per month
Water and electricity shs. 25,000 per month
Medical care shs. 240,000 per annum.
His remaining allowance is subjected to the tax structure below.

Taxable income	Rate (%)
1 – 150,000	10
150,001 – 350,000	15
350,001 – 600,000	20
Above 600,000	25

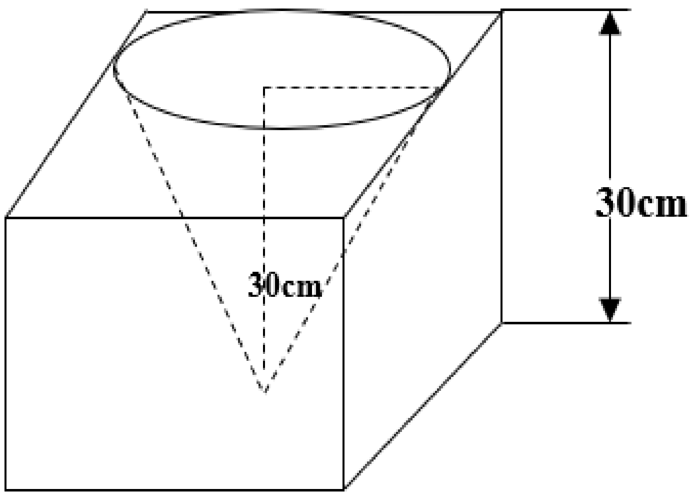
Given that an employee paid shs. 113,750 as tax, calculate his;

(i) total monthly allowances
(ii) taxable income
(iii) gross income
(iv) net income

MENSURATIONS

73. A rectangular swimming pool is constructed such that when the pool is completely full, the shallow end is 1 metre deep and the deep end is 4 metres deep. The pool is 25 metres long from the shallow end to the deep end and 20 metres wide.
(a) Calculate the
(i) inclination of the floor of the swimming pool to the horizontal
(ii) volume of the water (in m^3) that can fill the pool.
(b) Starting with the pool empty, a tap which delivers water at a rate of 400 litres per minute is used to fill the pool. How long (to nearest hour) will the pool take to fill?

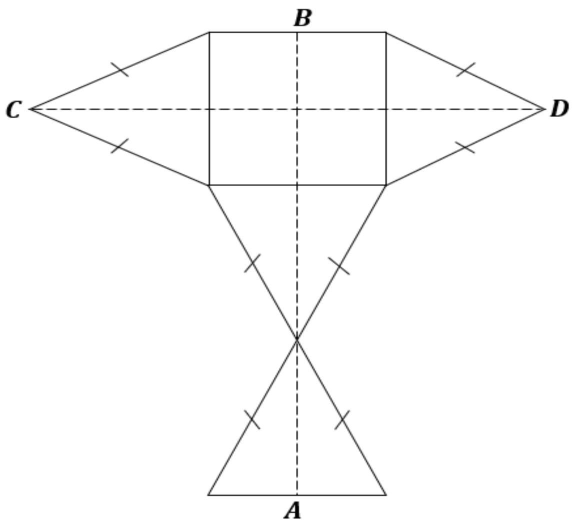
74. The figure below shows a metal cube of side 30 cm. A mortice is cut in form of a cone of height 30 cm.



Find the; (i) volume of the cut out piece
(ii) surface area of the remaining piece
(iii) cost of painting the remaining piece at Shs. 5/= per 1 cm^2.

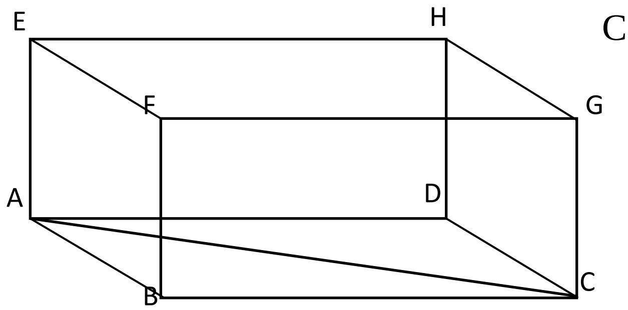
(Take $\pi = 3.142$, volume of a cone = $\frac{1}{3}\pi r^2 h$ and curved surface area of a cone = $\pi r l$)

75. The diagram below shows a square of side 12 cm and four congruent isosceles triangles, representing the net of a pyramid on a square base.
Given that $AB = CD = 40\ cm$, calculate the;
a) (i) height of the vertex of the pyramid from the square base,
(ii) angle between a triangular face and the base of the pyramid
(iii) volume of the pyramid



(b) If the pyramid is cut horizontally at a vertical height of 2.6 cm from the square base, and the upper part of the pyramid containing the vertex is thrown away, what volume remains?

76. The figure below is cuboid with square faces ABFE and DCGH. Where $\mathrm{BC} = 12$ cm and $\mathrm{AC} = 13$ cm



Calculate; (a) the lengths BF and BH
(b) the angle between BH and ABCD
(c) the angle between ABGH and EFGH

MATHEMATICAL FORMULAE

Quantity	Formula
Midpoint (M) between $A(x_1, y_1)$ and $B(x_2, y_2)$	$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
Length/distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Gradient (m) of $A(x_1, y_1)$ and $B(x_2, y_2)$	$m = \frac{y_2-y_1}{x_2-x_1}$
Equation of line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$	$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$
Equation of a line	$y = mx + c$, m = gradient, c = y – intercept
For two parallel lines $y_1 = m_1x + c$ and $y_2 = m_2x + c$	$m_1 = m_2$
For two perpendicular lines $y_1 = m_1x + c$ and $y_2 = m_2x + c$	$m_1.m_2 = -1$

Addition of logarithms	$\log_a x + \log_a y = \log_a xy$
Subtraction of logarithms	$\log_a x - \log_a y = \log_a \frac{x}{y}$
Power law	$\log_a x^m = m \log_a x$
Same base	$\log_a a = 1$ e.g. $\log_{10} 10 = 1$
Mean	$\bar{x} = \frac{\sum fx}{\sum f}$
Mean with assumed mean (working mean)	$\bar{x} = A + \frac{\sum fd}{\sum f}$; $d = x - A$
Mode	$M = L_1 + \left(\frac{d_1}{d_1 + d_2}\right) i$
Median	$M = L_0 + \left(\frac{\frac{N}{2} - Cfb}{fm}\right) i$
Modulus/magnitude of a vector $\boldsymbol{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$	$\lvert \mathbf{OA} \rvert = \sqrt{x^2 + y^2}$
Formula for compound interest	$A = P\left(1 + \frac{r}{100}\right)^n$
Formula for appreciation	$A = P\left(1 + \frac{r}{100}\right)^n$
Formula for depreciation	$A = P\left(1 - \frac{r}{100}\right)^n$
Hire purchase	$H.P = \text{Deposit} + \text{total instalments}$
Taxable Income	T.I = Gross Income – Tax free income
Circumference of a circle	$C = 2\pi r$ or $C = \pi d$
Area of a circle	$A = \pi r^2$ or $A = \frac{\pi d^2}{4}$
Length of an arc	$L = \frac{\theta}{360^0} \times 2\pi r$
Area of a sector	$A = \frac{\theta}{360^0} \times \pi r^2$
Total surface area of a cuboid	$T.S.A = 2(lh + lw + wh)$
Surface area of a cube	$S.A = 6l^2$
Surface area of an open cone	$S.A = \pi rl$
Surface area of a closed cone	$S.A = \pi r(r + l)$
Surface area of a sphere	$S.A = 4\pi r^2$
Surface area of a hemisphere	$S.A = 3\pi r^2$
Volume of a cylinder	$V = \pi r^2 h$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Volume of the sphere	$V = \frac{4}{3}\pi r^3$
Total angle sum of a polygon	$T.A.S = (n - 2)180^0$, n=number of sides
Exterior Angle of a polygon	$\text{Exterior angle of a polygon} = \frac{360^0}{\text{number of sides}}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

END