KINEMATICS

Summary:

1. *Kinematics deals with speed, distance and time*

2. *The following terms are computed as follows:*

(i) Distance $= speed \times time$ ***(ii) Time*** $= \frac{distance}{speed}$

(iii) Speed $= \frac{distance}{time}$ ***(iv) Average speed*** $= \frac{total\ distance}{total\ time}$

3. *Time in minutes can be converted into hours and vice versa as follows:*

(i) 15 minutes $= \frac{15}{60} =$ ***0·25 hours***

(ii) 0·25 hours = $0{\cdot}25 \times 60$ ***= 15 minutes***

4. *Speed in* $\boldsymbol{kmh^{-1}}$ *can be converted into* $\boldsymbol{ms^{-1}}$ *as follows:*

$$\boldsymbol{90kmh^{-1}} = \frac{90 \times 1000}{3600} = \boldsymbol{25ms^{-1}}$$

5. *If a car travels at a constant speed of* $\boldsymbol{60kmh^{-1}}$***,*** *it means that the car covers* ***60km*** *after every one hour*

EXAMPLES:

1. *A car travels with a constant speed of* $\boldsymbol{40kmh^{-1}}$***.*** *How far can it travel in* ***21*** *minutes****?***

Soln:

$$\boldsymbol{Distance} = s \times t = 40 \times \frac{21}{60} = \boldsymbol{14km}$$

2. *A car travels* ***75km*** *at a constant speed of* $\boldsymbol{50kmh^{-1}}$***.*** *How long does the journey take****?***

Soln:

$$\textbf{Time} = \frac{d}{s} = \frac{75}{50} = \textbf{1·5h}$$

3. *A car travels* ***36km*** *in* ***45****minutes****. Find its average speed*

Soln:

$$\textbf{Speed} = \frac{d}{t} = 36 \div \frac{45}{60} = \textbf{48kmh}^{-1}$$

4. *A car travels for* ***5*** *hours with a constant speed of* $\textbf{85kmh}^{-1}$ *and then travels for* ***3*** *hours with a constant speed of* $\textbf{69kmh}^{-1}$***.*** *Find its average speed*

Soln:

$$\textbf{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Distance ***I*** $= 85 \times 5 =$ ***425km***

Distance ***II*** $= 69 \times 3 =$ ***207km***

$$\therefore \quad \textbf{Average speed} = \frac{425 + 207}{5 + 3} = \textbf{79kmh}^{-1}$$

5. *A car travels* ***97·5km*** *with a constant speed of* $\textbf{65kmh}^{-1}$ *and then travels* ***60km*** *with a constant speed of* $\textbf{80kmh}^{-1}$***.*** *Find its average speed*

Soln:

$$\textbf{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{Time } \textbf{I} = \frac{97 \cdot 5}{65} = \textbf{1·5h} \qquad \text{Time } \textbf{II} = \frac{60}{80} = \textbf{0·75h}$$

$$\therefore \quad \textbf{Average speed} = \frac{97 \cdot 5 + 60}{1 \cdot 5 + 0 \cdot 75} = \textbf{70kmh}^{-1}$$

6. *A car travels for* ***5*** *hours at an average speed of* $\boldsymbol{36kmh^{-1}}$ *for the entire journey.* *For the first two hours its steady speed is* $\boldsymbol{30kmh^{-1}}$***.*** *Find its steady speed for the last three hours*

Soln:

$$\textbf{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Distance ***I*** $= 30 \times 2 =$ ***60km***

Distance ***II*** $= v \times 3 =$ ***3v***

$$\Rightarrow 36 = \frac{60 + 3v}{5}$$

$\therefore$ $\boldsymbol{v = 40kmh^{-1}}$

7. *A man covers a distance of* ***15km*** *in* ***3 hours,*** *partly by walking and partly by running.* *If he walks at* $\boldsymbol{3kmh^{-1}}$ *and runs at* $\boldsymbol{9kmh^{-1}}$***,*** *find the distance he covers by running*

Soln:

If ***x*** *= distance walked,* ***y*** *= distance ran*

$\Rightarrow x + y = 15$ ----------*(i)*

Also : $\frac{x}{3} + \frac{y}{9} = 3$

$\Rightarrow 3x + y = 27$ ----------*(ii)*

On solving, $\boldsymbol{y = 9km}$

8. *Tom arrives early to school by* ***10*** *minutes when he rides from home at a steady speed of* $\boldsymbol{9kmh^{-1}}$. *When he rides at a steady speed of* $\boldsymbol{7\cdot 5kmh^{-1}}$, *he arrives late by* ***6*** *minutes. Calculate:*

(i) *how far the school is from his home*

(ii) *the speed that enables him to be punctual*

Soln:

(i) *let* $\boldsymbol{d}$ = *required distance,* $\boldsymbol{t}$ = *time required to be punctual*

If difference in time = $\frac{10}{60}$

$\Rightarrow \quad t - \frac{d}{9} = \frac{1}{6}$ ***" less time is used"***

$\therefore\ 54t - 6d = 9$ ----------*(i)*

Also: If difference in time = $\frac{6}{60}$

$\Rightarrow \quad \frac{d}{7\cdot 5} - t = \frac{1}{10}$ ***" more time is used"***

$\therefore\ 10d - 75t = 7\cdot 5$ ----------*(ii)*

On solving, $\boldsymbol{d = 12km}$, $\boldsymbol{t = 1\cdot 5h}$

(ii) Speed $= \frac{d}{t} = \frac{12}{1\cdot 5} = \boldsymbol{8kmh^{-1}}$

9. *A train takes* ***15*** *minutes less for a journey of* ***156km*** *if its speed is increased by* $\boldsymbol{4kmh^{-1}}$ *from its normal speed. Find its normal speed*

Soln:

let $\boldsymbol{v}$ = *required speed* *If difference in time =* $\frac{15}{60}$

$\Rightarrow \quad \frac{156}{v} - \frac{156}{v+4} = \frac{1}{4}$ ***" more time is for slow speed"***

$\therefore\ v^2 + 4v - 2496 = 0$

$$v = \frac{-4 \pm \sqrt{16 + 9984}}{2}$$

$$v = 48 \text{ or } -52$$

$$\therefore \boldsymbol{v = 48kmh^{-1}}$$

10. *Two cyclists* $\boldsymbol{C_1}$ *and* $\boldsymbol{C_2}$ *left town* ***P*** *for town* ***Q, 18km*** *away at the same time.* $\boldsymbol{C_1}$ *travelled at a steady speed of* $\boldsymbol{15kmh^{-1}}$ *faster than* $\boldsymbol{C_2}$. *When* $\boldsymbol{C_1}$ *had covered half the distance, he delayed for half an hour, after which he travelled at a speed* ***20%*** *less his original speed and arrived in town* ***Q 15*** *minutes earlier than* $\boldsymbol{C_2}$. *Determine the original speeds of the two cyclists* $\boldsymbol{C_1}$ *and* $\boldsymbol{C_2}$

Soln:

let $\boldsymbol{v}$ = *speed of cyclist* $\boldsymbol{C_2}$

Time to reach by $\boldsymbol{C_2} = \frac{18}{v}$

Time to reach by $\boldsymbol{C_1} = \frac{9}{v+15} + \frac{1}{2} + \frac{9}{0 \cdot 8(v+15)} = \frac{2v+111}{4(v+15)}$

If difference in time $= \frac{15}{60}$

$$\Rightarrow \frac{18}{v} - \frac{2v+111}{4(v+15)} = \frac{1}{4} \quad \textbf{"more time is for slow speed"}$$

$$\therefore v^2 + 18v - 360 = 0$$

$$v = \frac{-18 \pm \sqrt{324 + 1440}}{2}$$

$$v = 12 \text{ or } -30$$

$$\therefore \boldsymbol{v = 12kmh^{-1}}$$

$\Rightarrow$ *Speed of cyclist* $\boldsymbol{C_1}$ = $v + 15 = 12 + 15 = \boldsymbol{27kmh^{-1}}$

EER:

1. *A car travels for* ***40*** *minutes with a constant speed of* $84kmh^{-1}$**.** *Find the speed of another car which takes* ***48*** *minutes to travel the same distance*

$$[\ Ans:\ 70kmh^{-1}\ ]$$

2. *A car travels* $97 \cdot 5km$ *with a constant speed of* $65kmh^{-1}$ *and then travels for* ***45*** *minutes with a constant speed of* $80kmh^{-1}$**.** *Find its average speed*

$$[\ Ans:\ 70kmh^{-1}\ ]$$

3. *Tom walking at* $6kmh^{-1}$ *from home to school takes* ***20*** *minutes less when he returns at* $10kmh^{-1}$**.** *Calculate how far the school is from his home*

$$[\ Ans:\ 5km\ ]$$

4. *Tom walks to work at* $6kmh^{-1}$ *and returns home at* $5kmh^{-1}$**.** *If the entire journey takes him* ***1*** *hour* ***39*** *minutes, calculate how far the place of work is from his home*

$$[\ Ans:\ 4 \cdot 5km\ ]$$

5. *A car takes* ***15*** *minutes less for a journey of* ***70km*** *if its speed is increased by* $5kmh^{-1}$ *from its normal speed. Find its normal speed*

$$[\ Ans:\ 35kmh^{-1}\ ]$$

6. *A man covers a distance of* ***9·5km*** *in* ***2 hours,*** *partly by walking and partly by running. If he walks at* $4kmh^{-1}$ *and runs at* $6kmh^{-1}$**,** *find the distance he covers by running*

$$[\ Ans:\ 4 \cdot 5km\ ]$$

7. *A train takes two hours less for a journey of* ***300km*** *if its speed is increased by* $5kmh^{-1}$ *from its normal speed. Find its normal speed*

$$[\text{Ans: } 25kmh^{-1}]$$

8. *Towns* ***P*** *and* ***Q*** *are* ***156km*** *apart. A car left* ***P*** *for* ***Q*** *at a steady speed of* $Vkmh^{-1}$. *On the return journey, it increased the speed by* $4kmh^{-1}$ *and took* ***15*** *minutes less. Calculate the value of* ***V***

$$[\text{Ans: } 48kmh^{-1}]$$

9. *It takes* ***3*** *hours to travel between two successive distances at respective speeds of* $50kmh^{-1}$ *and* $60kmh^{-1}$. *When the speeds are interchanged, the journey takes* ***8*** *minutes less. Calculate the distance of the entire journey*

$$[\text{Ans: } 160km]$$

10. *Kampala and Jinja are* ***300km*** *apart. A car moves from Kampala to Jinja and back. Its average speed on the return journey is* $30kmh^{-1}$ *greater than that on the outward journey and it takes* ***50*** *minutes less. Find the average speed of the outward journey*

$$[\text{Ans: } 90kmh^{-1}]$$

11. *Tom arrives early to school by* ***10*** *minutes when he rides from home at a steady speed of* $5\cdot4kmh^{-1}$. *When he rides at a steady speed of* $3\cdot6kmh^{-1}$, *he arrives late by* ***15*** *minutes. Calculate:*

(i) *how far the school is from his home*

(ii) *the speed that enables him to be punctual*

$$[\text{Ans: (i) } 4\cdot5km \quad \text{(ii) } 4\cdot5kmh^{-1}]$$

12. *Two cyclists* C_1 *and* C_2 *left town* **P** *for town* **Q, 24km** *away at the same time.* C_1 *travelled at a steady speed of* $10kmh^{-1}$ *faster than* C_2. *When* C_1 *had covered half the distance, he delayed for three quarters of an hour, after which he travelled at a speed* **25%** *less his original speed and arrived in town* **Q 15** *minutes earlier than* C_2. *Determine the original speeds of the two cyclists* C_1 *and* C_2

$$[\text{Ans: (a) } 20kmh^{-1},\ 10kmh^{-1}\ \text{ (b)(i) } 16km\ \text{ (ii) } 0{\cdot}8h]$$

13. *Towns* **P** *and* **Q** *are* **130km** *apart. At* **9:00am,** *a car left* **P** *for* **Q** *at a speed of* $60kmh^{-1}$ *and stopped at a petrol station for* **10** *minutes. It resumed its journey at a speed of* $75kmh^{-1}$ *until it reached* **Q** *at* **11:00am.** *Calculate:*

(i) *how far the petrol station is from town* **P**

(ii) *the average speed for the entire journey*

$$[\text{Ans: (i) } 30km\ \text{ (ii) } 65kmh^{-1}\ ]$$

14. *A motorist travelled 8km up a hill at a steady speed of* $x\ kmh^{-1}$. *On the return journey down the hill, his speed was* $(x+4)\ kmh^{-1}$. *The difference in time between the uphill and downhill journeys was* **10** *minutes.*

(a) *Write down an expression for the time taken for the:*

(i) *uphill journey*

(ii) *downhill journey*

(b) (i) *Form a quadratic equation for the difference in time for the two journeys*

(ii) *Solve the quadratic equation*

(c) *Find his average speed for the uphill and downhill journeys*

$$[\text{Ans: (b)(i) } x^2 + 4x - 192 = 0\ \text{ (ii) } 12kmh^{-1}\ \text{ (c) } 13{\cdot}7143kmh^{-1}\ ]$$

MOTION WITH CATCH UP OR MEETING

Summary:

*In case of two bodies catching up or meeting during motion**:***

(i) Time to catch up $= \dfrac{\text{relative distance}}{\text{relative speed}}$ *"Motion is in the same direction"*

(ii) Time to meet $= \dfrac{\text{relative distance}}{\text{total speed}}$ *"Motion is in opposite direction"*

(iii) *Relative speed is the same as speed difference*

(iv) *Relative distance is the same as distance apart at the start of timing*

(v) *Timing in this case starts with the latter rather than the former*

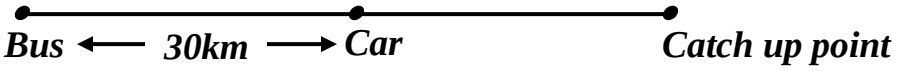
EXAMPLES:

1. *A car is moving at* 40kmh^{-1} *and a bus* ***30km*** *behind it is moving in the same direction at* 60kmh^{-1}**.** *Calculate the**:***

(i) *time taken by the bus to catch up with the car*

(ii) *distance travelled by the bus to catch up with the car*

Soln:



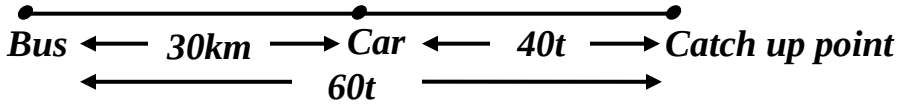
(i) $\text{Time to catch up} = \dfrac{\text{relative distance}}{\text{relative speed}} = \dfrac{30}{60-40} = 1\cdot 5\text{h}$

(ii) *Distance to catch up* $= s \times t = 60 \times 1\cdot 5 =$ ***90km***

Or *Distance to catch up* $= 30 + (40 \times 1\cdot 5) =$ ***90km***

METHOD 2

(i) *Let* t *= time taken by the bus to catch up*



Bus's total distance = car's total distance

$60t = 30 + 40t$

$\therefore t = 1{\cdot}5h$

(ii) *Distance to catch up* $= s \times t = 60 \times 1{\cdot}5 =$ ***90km***

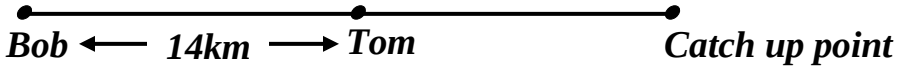
Or *Distance to catch up* $= 30 + (40 \times 1{\cdot}5) =$ ***90km***

2. *Tom left home riding steadily at* $\mathbf{7kmh^{-1}}$**.** *Two hours later Bob left the same home riding steadily along the same road at* $\mathbf{15kmh^{-1}}$**.** *Calculate**:***

(i) *how long will it take Bob to catch up with Tom*

(ii) *how far will Bob travel to catch up with Tom*

Soln:



(i) *Tom's distance in* $\mathbf{2h} = 7 \times 2 =$ ***14km***

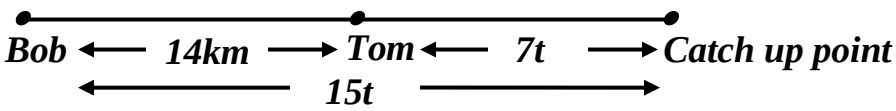
$$\text{Time to catch up} = \frac{\text{relative distance}}{\text{relative speed}} = \frac{14}{15-7} = \mathbf{1{\cdot}75h}$$

(ii) *Distance to catch up* $= s \times t = 15 \times 1{\cdot}75 =$ ***26·25km***

Or *Distance to catch up* $= 14 + (7 \times 1{\cdot}75) =$ ***26·25km***

METHOD 2

(i) *Let* $\mathbf{t}$ *= time taken by the Bob to catch up*



Bob's total distance = Tom's total distance

$15t = (7 \times 2) + 7t$

$\therefore t = 1{\cdot}75h$

(ii) *Distance to catch up* $= s \times t = 15 \times 1{\cdot}75 =$ ***26·25km***

Or *Distance to catch up* $= 14 + (7 \times 1{\cdot}75) =$ ***26·25km***

3. *Bob and Tom have to go to church* ***12·5km*** *away from their home****.*** *When Bob had covered* ***3·2km,*** *riding steadily at* $\mathbf{3kmh^{-1}}$***,*** *Tom left the same home riding steadily along the same road at* $\mathbf{5kmh^{-1}}$***.***

(a) *Calculate****:***

(i) *how long will it take Tom to catch up with Bob*

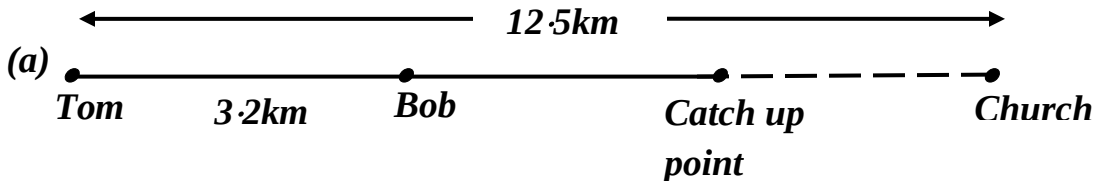
(ii) *how far will Tom travel to catch up with Bob*

(b) *Immediately Tom caught up with Bob****,*** *he then reduced his speed and arrived* ***0·35*** *hours later than if he had maintained the* $\mathbf{5kmh^{-1}}$ *speed.*

(i) *Calculate by how much he reduced his speed*

(ii) *For how long was he in church before Bob joined him*

Soln:



(i) $\text{Time to catch up} = \dfrac{\text{relative distance}}{\text{relative speed}} = \dfrac{3 \cdot 2}{5-3} = \mathbf{1 \cdot 6h}$

(ii) *Distance to catch up* $= s \times t = 5 \times 1{\cdot}6 =$ ***8km***

(b) *let* $\mathbf{v}$ *= new speed* *Remaining distance* $= 12{\cdot}5 - 8 =$ ***4·5km***

If difference in time $= 0{\cdot}35$

$\Rightarrow \quad \dfrac{4 \cdot 5}{v} - \dfrac{4 \cdot 5}{5} = 0 \cdot 35$ ***" more time is for slow speed"***

$\therefore \; \mathbf{v = 3 \cdot 6kmh^{-1}}$

$\Rightarrow$ *Reduction in speed* $= 5 - 3 \cdot 6 = \mathbf{1 \cdot 4kmh^{-1}}$

(ii) *Waiting time* $= \frac{4 \cdot 5}{3} - \frac{4 \cdot 5}{3 \cdot 6} = \mathbf{0 \cdot 25h}$ ***" more time is for slow speed"***

4. *Towns* ***P*** *and* ***Q*** *are* ***500km*** *apart. A car left* ***P*** *for* ***Q*** *at an average speed of* $\mathbf{60kmh^{-1}}$. *After* $\mathbf{2\frac{1}{2}}$ *hours, a bus left* ***P*** *left for* ***Q*** *and travelled along the same road at an average speed of* $\mathbf{100kmh^{-1}}$.

(a) *Calculate the:*

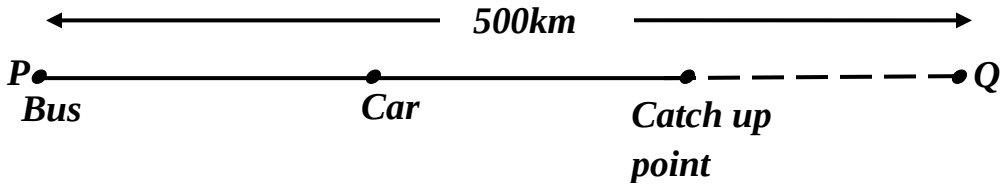
(i) *distance of the car from* ***Q*** *when the bus took off*

(ii) *distance from* ***P*** *to where the bus caught up with the car*

(b) *Immediately the bus caught up with the car, the bus stopped for* ***25*** *minutes. Find the new average speed at which the bus travelled in order to reach* ***Q*** *at the same time as the car*

Soln:

(a)



(i) *Car's distance in* $\mathbf{2\frac{1}{2}h} = 60 \times 2 \cdot 5 = \mathbf{150km}$

$\therefore$ *Distance from* ***Q*** $= 500 - 150 = \mathbf{350km}$

(ii) *Time to catch up* $= \frac{\text{relative distance}}{\text{relative speed}} = \frac{150}{100 - 60} = \mathbf{3 \cdot 75h}$

$\therefore$ *Distance to catch up* $= s \times t = 100 \times 3 \cdot 75 = \mathbf{375km}$

(b) *let* $\mathbf{v}$ = *new speed* *Remaining distance* $= 500 - 375 = \mathbf{125km}$

If the total time to reach is the same

$\Rightarrow \frac{25}{60} + \frac{125}{v} = \frac{125}{60}$

$\therefore \mathbf{v = 75kmh^{-1}}$

5. *A car and a bus left town* ***P*** *for town* ***Q 240km*** *away at* ***8:00 am*** *traveling at* $\boldsymbol{90kmh^{-1}}$ *and* $\boldsymbol{120kmh^{-1}}$ *respectively. After* ***20*** *minutes the bus stopped for* ***30*** *minutes and then resumed its journey at the same speed.*

(a) *Calculate the:*

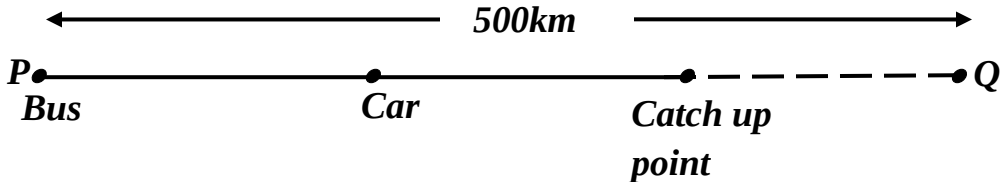
(i) *time when the bus caught up with the car*

(ii) *distance from* ***P*** *to where the bus caught up with the car*

(iii) *time of arrival of the car to town* ***Q***

Soln:

(a)



(i) *Bus's distance in* ***20*** *minutes* $= 120 \times \frac{20}{60} =$ ***40km***

Car's distance in ***50*** *minutes* $= 90 \times \frac{50}{60} =$ ***75km***

$$\text{Time to catch up} = \frac{\text{relative distance}}{\text{relative speed}} = \frac{75-40}{120-90} = \frac{7}{6}\boldsymbol{h} = \boldsymbol{1h\ 10\ minutes}$$

$\Rightarrow$ *Required time = 8:50 + 1h 10 minutes* $=$ ***10:00 am***

(ii) *Distance from* ***P*** $= 40 + \left(120 \times \frac{7}{6}\right) =$ ***180km***

(iii) *Car's travel time* $= \frac{240}{90} = \frac{8}{3}\boldsymbol{h} = \boldsymbol{2h\ 40\ minutes}$

$\Rightarrow$ *Arrival time = 8:00 + 2h 40 minutes* $=$ ***10:40 am***

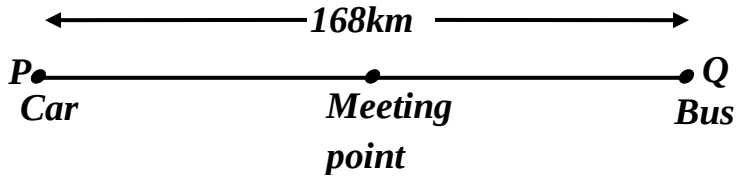
6. *Towns* ***P*** *and* ***Q*** *are* ***168km*** *apart. A car left* ***P*** *for* ***Q*** *at an average speed of* $\boldsymbol{60kmh^{-1}}$. *At the same time a bus left* ***Q*** *for* ***P*** *at an average speed of* $\boldsymbol{80kmh^{-1}}$.

(i) *After how long will the two vehicles meet?*

(ii) *How far is the meeting point from town* ***P?***

(iii) *Just as they met, the car increased its speed by* $\boldsymbol{16 \cdot 8 km h^{-1}}$**.** *Find the difference in the times of arrival of the two vehicles*

Soln:



(i) *Time to meet* $= \dfrac{\text{relative distance}}{\text{total speed}} = \dfrac{168}{60+80} = \boldsymbol{1 \cdot 2h}$

(ii) *Distance from* **P** $= s \times t = 60 \times 1 \cdot 2 =$ **72km**

(iii) *Bus's distance to* **P** $=$ **72km**

$\therefore$ *Bus's time to* **P** $= \dfrac{72}{80} = \boldsymbol{0 \cdot 9h}$

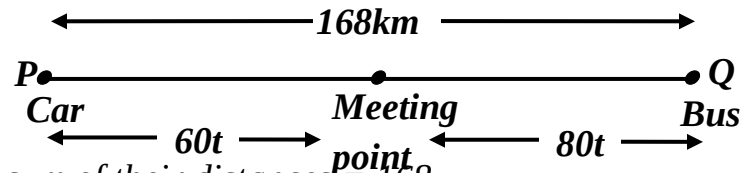
Car's distance to **Q** $= 168 - 72 =$ **96km** *New speed* $= \boldsymbol{76 \cdot 8 km h^{-1}}$

$\therefore$ *Car's time to* **Q** $= \dfrac{96}{76 \cdot 8} = \boldsymbol{1 \cdot 25h}$

$\Rightarrow$ *Arrival time difference* $= 1 \cdot 25 - 0 \cdot 9 =$ **0·35h**

METHOD 2

(i) *Let* **t** $=$ *time taken by the Bus to meet*



If the sum of their distances $= 168$

$\Rightarrow$ $60t + 80t = 168$

$\therefore$ $\boldsymbol{t = 1 \cdot 2h}$

(ii) *Distance from* **P** $= s \times t = 60 \times 1 \cdot 2 =$ **72km**

17. *Bob and Tom live* ***62km*** *apart.* *At* ***7:00 am,*** *Bob left his home cycling towards Tom's home at* $\boldsymbol{20kmh^{-1}}$. *At* ***7:21 am,*** *Tom left his home cycling towards Bob's home at* $\boldsymbol{24kmh^{-1}}$.

(a) *Calculate the:*

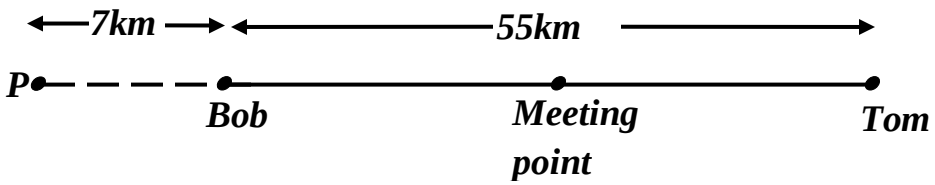
(i) *time when the two men met*

(ii) *distance from Bob's house to where the two men met*

(b) *The two took* ***12*** *minutes at the meeting point and then travelled to Tom's house at an average speed of* $\boldsymbol{20kmh^{-1}}$. *Find the time they arrived at Tom's house*

Soln:

(a) (i)



Bob's distance in ***21*** *minutes* $= 20 \times \frac{21}{60} =$ ***7km***

$\therefore$ *Time to meet* $= \frac{relative\ distance}{total\ speed} = \frac{55}{20+24} =$ $\boldsymbol{1 \cdot 25h = 1h\ 15minutes}$

$\Rightarrow$ *Required time = 7:21 + 1h 15 minutes* **= 8:36 am**

(ii) *Distance from Bob's house* $= 7 + (20 \times 1 \cdot 25)$ **= 32km**

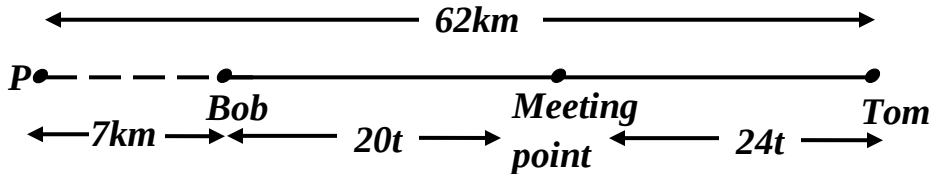
(b) *Distance to Tom's house* $= 62 - 32$ **= 30km**

$\therefore$ *Their time to Tom's house* $= \frac{30}{20} =$ $\boldsymbol{1 \cdot 5h}$

$\Rightarrow$ *Arrival time = 8:36 + 12 + 1h 30 minutes* **= 10:18 am**

METHOD 2

(i) *Let* ***t*** *= time taken by Tom to meet*



Bob's distance in ***21*** *minutes =* $20 \times \frac{21}{60}$ *=* ***7km***

If the sum of their distances = 62

$\Rightarrow$ *(7 + 20t)+ 24t = 62*

$\therefore$ ***t = 1·25h = 1h 15 minutes***

$\Rightarrow$ *Required time = 7:21 + 1h 15 minutes* ***= 8:36 am***

(ii) *Distance from Bob's house = 7 + (20 × 1·25)* ***= 32km***

EER:

1. *Bob and Tom have to go to school* ***9km*** *away from their home****.*** *When Bob had covered* ***3·6km,*** *walking steadily at* $\mathbf{2 \cdot 5 km h^{-1}}$***,*** *Tom left the same home running steadily along the same road at* $\mathbf{4 km h^{-1}}$***.***

(a) *Calculate****:***

(i) *how long will it take Tom to catch up with Bob*

(ii) *how far will Tom travel to catch up with Bob*

(iii) *how long was Tom at school before Bob joined him*

[Ans: (a)(i) 1·5h (ii) 6km (iii) 0·45h]

2. *Bob and Tom live* ***62km*** *apart.* *At* ***7:00 am,*** *Bob left his home cycling towards Tom's home at* $\boldsymbol{20kmh^{-1}}$. *At* ***8:00 am,*** *Tom left his home cycling towards Bob's home at* $\boldsymbol{8kmh^{-1}}$.

(a) *Calculate the:*

(i) *time when the two men met*

(ii) *distance from Bob's house to where the two men met*

[Ans: (a)(i) 9:30am (ii) 50km]

3. *Towns* ***P*** *and* ***Q*** *are* ***100km*** *apart.* *At* ***4:00 am*** *a cyclist left* ***P*** *for* ***Q*** *at a steady speed of* $\boldsymbol{20kmh^{-1}}$. *At* ***7:30 am,*** *a motorist left* ***P*** *for* ***Q*** *along the same road at a steady speed of* $\boldsymbol{100kmh^{-1}}$.

(a) *Calculate the:*

(i) *time when the motorist overtook the cyclist*

(ii) *distance from* ***P*** *to where the motorist overtook the cyclist*

(iii) *time of arrival of the cyclist*

[Ans: (a)(i) 8:22 am (ii) 87·5km (iii) 9:00 am]

4. *Towns* ***P*** *and* ***Q*** *are* ***170km*** *apart.* *At* ***8:25am*** *car left* ***P*** *for* ***Q*** *at an average speed of* $\boldsymbol{40kmh^{-1}}$. *At* ***8:55am*** *bus left* ***Q*** *for* ***P*** *and travelled along the same road at an average speed of* $\boldsymbol{80kmh^{-1}}$.

(a) *Calculate the:*

(i) *time when the two vehicles met*

(ii) *distance from* ***P*** *to where the two vehicles met*

(b) *Just as they met, the car increased its speed by* $\boldsymbol{10kmh^{-1}}$. *Find the difference in their times of arrival at their destinations*

[Ans: (a)(i) 10:10am (ii) 70km (b) 1·125h]

5. *Bob and Tom have to go to church* ***30·8km*** *away from their home****.*** *When Bob had covered* ***9km,*** *riding steadily at* $\boldsymbol{4kmh^{-1}}$***,*** *Tom left the same home riding steadily along the same road at* $\boldsymbol{7kmh^{-1}}$***.***

(a) *Calculate****:***

(i) *how long will it take Tom to catch up with Bob*

(ii) *how far will Tom travel to catch up with Bob*

(b) *Immediately Tom caught up with Bob****,*** *he then reduced his speed and arrived* ***0·6*** *hours later than if he had maintained the* $\boldsymbol{7kmh^{-1}}$ *speed.*

(i) *Calculate by how much he reduced his speed*

(ii) *For how long was he in church before Bob joined him*

[Ans: (a)(i) 3h (ii) 21km (b)(i) $\boldsymbol{2\cdot1kmh^{-1}}$ ***(ii) 0·45h]***

6. *Bob and Tom have to go for a burial* ***138km*** *away from their home****.*** *When Bob had covered* ***18km,*** *riding steadily at* $\boldsymbol{24kmh^{-1}}$***,*** *Tom left the same home riding steadily along the same road at* $\boldsymbol{30kmh^{-1}}$***.***

(a) *Calculate****:***

(i) *how long will it take Tom to catch up with Bob*

(ii) *how far will Tom travel to catch up with Bob*

(iii) *how long Tom will take waiting for Bob at the burial*

(b) *If Bob increased his speed immediately he was overtaken such that they both arrive at the burial at the same time****,*** *calculate by how much he increased his speed*

[Ans: (a)(i) 3h (ii) 90km (b)(i) $\boldsymbol{2\cdot1kmh^{-1}}$ ***(ii) 0·45h]***

7. *Bob and Tom live* ***190km*** *apart.* *At* ***7:00 am,*** *Bob left his home cycling towards Tom's home at* $\mathbf{30kmh^{-1}}$. *At* ***7:30 am,*** *Tom left his home cycling towards Bob's home at* $\mathbf{40kmh^{-1}}$.

(a) *Calculate the:*

(i) *time when the two men met*

(ii) *distance from Bob's house to where the two men met*

(b) *The two took* ***15*** *minutes at the meeting point and then travelled to Tom's house at an average speed of* $\mathbf{20kmh^{-1}}$. *Find the time they arrived at Tom's house*

[Ans: (a)(i) 10:00am (ii) 90km (b) 3:15pm]

8. *Two cyclists* $\boldsymbol{C_1}$ *and* $\boldsymbol{C_2}$ *left town* ***P*** *for town* ***Q, 24km*** *away at the same time.* $\boldsymbol{C_1}$ *travelled at a steady speed of* $\mathbf{10kmh^{-1}}$ *faster than* $\boldsymbol{C_2}$. *When* $\boldsymbol{C_1}$ *had covered half the distance, he delayed for three quarters of an hour, after which he travelled at a speed* ***25%*** *less his original speed and arrived in town* ***Q 15*** *minutes earlier than* $\boldsymbol{C_2}$.

(a) *Determine the original speeds of the two cyclists* $\boldsymbol{C_1}$ *and* $\boldsymbol{C_2}$

(b) *If cyclist* $\boldsymbol{C_1}$ *started from town* ***P*** *while* $\boldsymbol{C_2}$ *at the same time started from town* ***Q*** *and both travelled non–stop,*

(i) *find how far from* ***P*** *the two cyclists will meet*

(ii) *After how long will they meet?*

[Ans: (a) $\mathbf{20kmh^{-1}}$, $\mathbf{10kmh^{-1}}$ ***(b)(i) 16km (ii) 0·8h]***

DISTANCE –TIME GRAPHS

EXAMPLES:

1. *Towns* ***P*** *and* ***Q*** *are* ***500km*** *apart.* *At* ***8:15 am*** *a car left* ***P*** *for* ***Q*** *traveling at a steady speed of* $\boldsymbol{60kmh^{-1}}$. *Two and a half hours later, a bus left* ***P*** *for* ***Q*** *along the same road at a steady speed of* $\boldsymbol{100kmh^{-1}}$.

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***50km*** *and* ***2cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *distance of the car from* ***Q*** *when the bus took off*

(ii) *time and distance from* ***P*** *where the bus overtook the car*

(iii) *difference in the times of arrival of the two vehicles*

Soln:

Table for the car (P *to* ***Q)***

Distance moved	***0***	***60***	***120***
Time of the day	***8:15***	***9:15***	***10:15***

Table for the bus (P *to* ***Q)***

Distance moved	***0***	***100***	***200***
Time of the day	***10:45***	***11:45***	***12:45***

2. *Towns* ***P*** *and* ***Q*** *are* ***360km*** *apart.* *At* ***7:30 am*** *a car left* ***P*** *for* ***Q*** *traveling at a steady speed of* $\boldsymbol{80kmh^{-1}}$. *At the same time a bus left* ***Q*** *for* ***P*** *at an average speed of* $\boldsymbol{100kmh^{-1}}$.

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***50km*** *and* ***2cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *time when the two vehicles met*

(ii) *distance from* ***Q*** *to where the two vehicles met*

(iii) *difference in the times of arrival of the two vehicles*

Soln:

Table for the car (P *to* ***Q)***

Distance moved	***0***	***80***	***160***
Time of the day	***7:30***	***8:30***	***9:30***

Table for the bus (Q *to* ***P)***

Distance moved	***0***	***100***	***200***
Time of the day	***7:30***	***8:30***	***9:30***

3. *Towns* ***P*** *and* ***Q*** *are* ***180km*** *apart.* *At* ***0730*** *hours a car left* ***P*** *for* ***Q*** *traveling at a steady speed of* $\boldsymbol{40kmh^{-1}}$. *After* ***2*** *hours the car stopped for* $\boldsymbol{1\frac{1}{2}}$ *hours and then proceeded with its journey at a speed of* $\boldsymbol{50kmh^{-1}}$. *A bus left* ***Q*** *for* ***P*** *at the same time as the car at a steady speed of* $\boldsymbol{60kmh^{-1}}$ *but suddenly reduced its speed after* ***2*** *hours to* $\boldsymbol{15kmh^{-1}}$ *for the rest of its journey*

(a) *On the same axes show the journeys of the two vehicles*

[*Use a scale of* ***2cm*** *to represent* ***20km*** *and* ***2cm*** *to represent* ***1*** *hour* ***]***

(b) *Use your graphs to find the****:***

(i) *time and distance from* ***Q*** *where the two vehicles met*

(ii) *distance between the two vehicles at* ***0930*** *hours*

(iii) *times of arrival of the two vehicles at their destinations*

(iv) *difference in the times of arrival at the respective towns*

Soln:

Table for the car (P** to **Q)

Distance moved	***0***	***40***	***80***	***80***	***130***	***180***
Time of the day	***0730***	***0830***	***0930***	***1100***	***1200***	***1300***

Table for the bus (Q** to **P)

Distance moved	***0***	***60***	***120***	***135***	***150***	***165***
Time of the day	***0730***	***0830***	***0930***	***1030***	***1130***	***1230***

***4.** Towns **P** and **Q** are **150km** apart. At **1100** hours a car left **P** for **Q** traveling at a steady speed of $\mathbf{50kmh^{-1}}$. After half an hour a bus left **P** for **Q** at a steady speed of $\mathbf{120kmh^{-1}}$ but after traveling **30km,** it stopped for **15** minutes and then resumed its journey at a speed of $\mathbf{120kmh^{-1}}$. The bus arrived at **Q** and rested for **30** minutes before returning to **P** by the same road where it arrived at **1442** hours*

***(a)** On the same axes show the journeys of the two vehicles*

*[Use a scale of **2cm** to represent **25km** and **4cm** to represent **1** hour]*

***(b)** Use your graphs to find the:*

(i)** time and distance from **Q** where the bus overtook the car on its way to **Q

(ii)** time and distance from **Q** where the bus met the car on its way back to **P

***(iii)** average speed of the bus for the outward journey*

***(iv)** average speed of the bus for the return journey*

***(v)** average speed of the bus for the entire journey*

Soln:

Table for the car (P** to **Q)

Distance moved	***0***	***50***	***100***
Time of the day	***1100***	***1200***	***1300***

***Table for the bus (P** to **Q** and back)*

Distance moved	***0***	***30***	***30***	***150***	***150***	***0***
Time of the day	***1130***	***1145***	***1200***	***1300***	***1330***	***1442***

$Time\ to\ cover\ \mathbf{30km} = \frac{30}{120} = \mathbf{0 \cdot 25h = 15\ minutes}$

EER:

1. *Towns* ***P*** *and* ***Q*** *are* ***360km*** *apart.* *At* ***8:15 am*** *a car left* ***P*** *for* ***Q*** *traveling at a steady speed of* $\mathbf{90kmh^{-1}}$***.*** *After* $\mathbf{1\frac{2}{5}\ hours,}$ *a bus left* ***P*** *for* ***Q*** *along the same road at a steady speed of* $\mathbf{120kmh^{-1}}$***.***

(a) *On the same axes show the journeys of the two vehicles*

[*Use a scale of* ***2cm*** *to represent* ***50km*** *and* ***2cm*** *to represent* ***1*** *hour* ***]***

(b) *Use your graphs to find the****:***

(i) *distance of the car from* ***Q*** *when the bus took off*

(ii) *time and distance from* ***P*** *to where the bus overtook the car*

(iii) *difference in the times of arrival of the two vehicles*

2. *Towns* ***P*** *and* ***Q*** *are* ***100km*** *apart.* *At* ***5:00am,*** *a car left* ***P*** *and travelled for one hour at a speed of* $\mathbf{30kmh^{-1}}$***.*** *It then increased its speed to* $\mathbf{100kmh^{-1}}$ *until it reached* ***Q.*** *At* ***5:30am,*** *a bus left* ***Q*** *for* ***P*** *and travelled at a steady speed of* $\mathbf{60kmh^{-1}}$ *until it broke down* $\mathbf{1\frac{1}{2}\ hours}$ *later****.***

(a) *On the same axes show the journeys of the two vehicles*

[*Use a scale of* ***2cm*** *to represent* ***10km*** *and* ***2cm*** *to represent* ***30*** *minutes* ***]***

(b) *Use your graphs to find the****:***

(i) *time and distance from* ***P*** *when the two vehicles met*

(ii) *distance from* ***Q*** *to where the bus broke down*

(iii) *time the car reached town* ***Q***

3. *Towns* ***P*** *and* ***Q*** *are* ***200km*** *apart.* *At noon,* *a car left* ***P*** *and travelled for one hour at a speed of* $\boldsymbol{50kmh^{-1}}$. *It stopped for* ***30*** *minutes then continued to* ***Q*** *at a speed of* $\boldsymbol{60kmh^{-1}}$. *At* ***12:30pm,*** *a bus left* ***Q*** *and travelled for one hour at a speed of* $\boldsymbol{40kmh^{-1}}$. *It then changed and travelled at a speed of* $\boldsymbol{V\ kmh^{-1}}$ *and arrived at* ***4:30pm*** *at town* ***P***

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***20km*** *and* ***4cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *time when the two vehicles met*

(ii) *distance from* ***P*** *to where the two vehicles met*

(iii) *time of arrival of the car*

(iv) *speed* ***V*** *of the bus*

4. *Towns* ***P*** *and* ***Q*** *are* ***360km*** *apart.* *At* ***7:00am,*** *a car left* ***P*** *and travelled for two hours at a speed of* $\boldsymbol{50kmh^{-1}}$. *It stopped for* ***1*** *hour then continued to* ***Q*** *at a steady speed for* ***4*** *hours.* *At* ***8:00am,*** *a bus left* ***Q*** *for* ***P*** *and travelled non–stop for* $\boldsymbol{4\frac{1}{2}}$ ***hours.***

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***40km*** *and* ***2cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *time when the two vehicles met*

(ii) *distance from* ***P*** *when the two vehicles met*

(iii) *average speed of the bus*

5. *Towns* ***P*** *and* ***Q*** *are* ***45km*** *apart.* *At* ***0815*** *hours,* *Bob left* ***P*** *for* ***Q*** *riding at a speed of* $\boldsymbol{15kmh^{-1}}$. *His bicycle broke down at* ***0915*** *hours and was delayed for* ***45*** *minutes.* *He then walked back to* ***P*** *and arrived at* ***1230*** *hours.* *At* ***0915*** *hours Tom left* ***P*** *for* ***Q*** *riding at a steady speed and arrived at* ***1200*** *hours.*

(a) *On the same axes show the journeys of the two men*

[Use a scale of ***2cm*** *to represent* ***20km*** *and* ***4cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *distance from* ***P*** *when Bob's bicycle broke down*

(ii) *speed at which Bob walked back to* ***P***

(iii) *average speed of Tom*

(iv) *time when the two men met*

(iv) *distance from* ***P*** *when the two men met*

6. *Towns* ***P*** *and* ***Q*** *are* ***90km*** *apart.* *At* ***7:00 am,*** *a car left* ***P*** *for* ***Q*** *traveling at a steady speed of* $\boldsymbol{24kmh^{-1}}$. ***45*** *minutes later,* *a bus left* ***P*** *for* ***Q*** *at a steady speed of* $\boldsymbol{60kmh^{-1}}$ *but after traveling* ***15km,*** *it stopped for half an hour and then resumed its journey at a speed of* $\boldsymbol{60kmh^{-1}}$. *The bus arrived at* ***Q*** *and rested for* ***15*** *minutes before returning to* ***P*** *by the same road where it arrived at* ***11:15 am.***

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***10km*** *and* ***4cm*** *to represent* ***1*** *hour]*

(b) *Use your graphs to find the:*

(i) *time and distance from* ***P*** *where the bus overtook the car on its way to* ***Q***

(ii) *time and distance from* ***Q*** *where the bus met the car on its way back to* ***P***

(iii) *average speed of the bus for the return journey*

7. *Towns* ***P*** *and* ***Q*** *are* ***450km*** *apart. At* ***7:42am,*** *a Van and a Bus left* ***P*** *for* ***Q*** *travelling at* $90kmh^{-1}$ *and* $150kmh^{-1}$ *respectively. After* ***30*** *minutes, the bus had a puncture which took* ***1·8*** *hours to mend before resuming the journey at the same speed.*

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***50km*** *and* ***2cm*** *to represent* ***1*** *hour]*

(b) *Use your graph, to find the:*

(i) *distance from* ***P*** *to where the two vehicles met for the first time.*

(ii) *time and distance from* ***Q*** *to where the two vehicles met for the second time*

(iii) *difference in the times of arrival of the two vehicles*

(iv) *average speed of the bus for the entire journey*

8. *Town* ***P*** *is* ***300km*** *from town* ***Q.*** *A lorry left town* ***P*** *for* ***Q*** *at* ***7:30am*** *and travelled at a steady speed of* $80kmh^{-1}$. *At the same time, a bus left town* ***Q*** *for town P and travelled at a steady speed of* $120kmh^{-1}$.

(a) *On the same axes show the journeys of the two vehicles*

[Use a scale of ***2cm*** *to represent* ***50km*** *and* ***2cm*** *to represent* ***1*** *hour]*

(b) *Use your graph, to find the time and distance from* ***Q*** *to where the two vehicles met*

(c) *Just as they met, the lorry and the bus were then driven at speeds of* $100kmh^{-1}$ *and* $75kmh^{-1}$ *respectively, calculate the difference in their times of arrival at their destinations.*