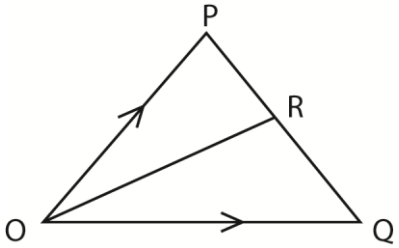
UCE MATHEMATICS PAPER 2 2017 guide

SECTION A (40 marks)

Answer all questions in this section

1. An increase of 15% in salaries makes the monthly expenditure on salaries for a factory to be shs. 22,425,000. Find the expenditure before the increase. (04marks)
2. The sets M and P are such that n(M) = 50, n(P) = 25 and n(M∪ P) = 60.
   Calculate n(P∩ M). (04marks)
3. Find the equation of the line joining the points (3, 5) and (-2, 10). (04marks)
4. A metallic cylindrical pipe of uniform cross-sectional area has an outer radius of 14cm and inner radius of 6.5cm. it has a length of 4.2metres. Calculate the volume in cm3, of the metal used to make the pipe. (04marks)
5. Express $\frac{3}{1-\sqrt{2}}$ in the form a + b$\sqrt{2}$. (04marks)
6. Given that f(x) = $\frac{3x+16}{4}$, find the value of f'(1). (04marks)
7. The coordinates of the mid-point of a line PQ are (8, -1). The coordinates of P are (5, -5). Determine the coordinates of Q. (04marks)
8. Two water tanks of the same shape. The larger tank is 80cm high with a capacity of 500litres. What is the capacity of the smaller tank whose height is 48cm? (04marks).
9. Fauza bought one dozen of shirt at shs. 40,000 per shirt. She sold them at a profit of 20% How much money did she earn as profit from the shirt sales? (04marks)
10. In the figure below, OP = a, OQ= b and PR = $\frac{1}{2}PQ$.



Express OR in terms of a and b. (04marks)

SECTION B (60MARKS)

Answer any five questions from this section. All questions carry equal marks

11. (a) Simplify $\frac{\left(3\frac{5}{6}\div 2\frac{2}{15}\right)x\frac{3}{23}}{5\frac{1}{3}-2\frac{7}{12}}$. (06marks)

(b) A forest covering an area of 807.5km^2 is represented on a map by a green area of 32.3cm^2. Determine the scale on the map. (06marks)

12. In a survey carried out in the department od languages at a certain University, the following data was collected, 20 students spoke German, 10spoke French and German, 9 spoke French and Kiswahili. 7 spoke Kiswahili and German only. 2 could not speak any of the three languages. 22 could speak at least two of the languages. 12 could speak only one languages. 11 could either speak Kiswahili or German but not French.
    (a) Use Venn diagram to represent the given information. (03 marks)
    (b) Find the number of students that could speak
        (i) all the three languages
        (ii) Kiswahili only
        (iii) French only (05marks)
    (c) What is the probability that a student picked at random from the group could speak neither Kiswahili nor German. (04marks)
13. A cyclist covered a journey of 48km from station A to station B in 5½ hours. A cyclist rode at 12km/hr for the first 1½ hours and changed speed for the remaining part of the journey.
    (a) (i) Determine the speed for the remaining part of the journey. (06marks)
        (ii) Represent the cyclist journey on a distance –time graph. (04 marks)
    (b) Calculate the average speed of the cyclist from a to B. (02marks)
14. (a) Given the set (2, 4, 6) draw a papygram to show the relation
    " is the smallest prime factor of" (03marks)
    (b) for the mapping x$\longrightarrow$ 4x + 5, fin the domain when the range is {1,13) (04marks)
    (c) The function f(x) = 2x^2 and g(x) = 5x -3. Find the value of x such that f(x) = g(x). (05mrks)
15. (a) If a = $\begin{pmatrix}-2\\4\end{pmatrix}, b = \begin{pmatrix}-5\\1\end{pmatrix}, c = \begin{pmatrix}-1\\3\end{pmatrix}$ and 2a + m + b = c. find
    (i) m
    (ii) $|m|$

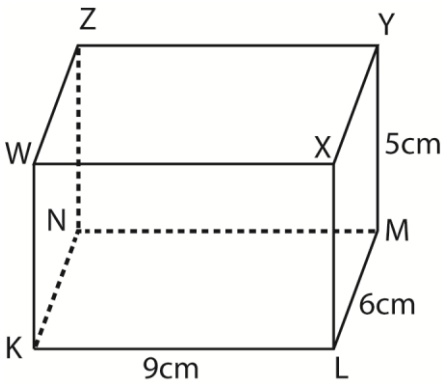
    (b) Using vectors show that the point P(-4, 1), Q(0,2) and R (8, 4) lie on straight line (06mrks)

16. The table below shows fares for some flights at an airport.

Destination	One way ticket (US dollars)	Return ticket (US dollars)
A	300	565
B	705	1295
C	380	714
	186	302

    - One way ticket means from airport to destination
    - Return ticket means from airport to destination and back to airport

    (a) Calculate the amount in Ug. Shs. For a one way ticket to B if the exchange rate is us is US 1 to Ug. Shs. 2,500/=
    (b) A family bout for return tickets for destination A at Ug. Shs. 5,737,000. Determine the exchange rate. (05marks)
    (c) A tourist bout a one way ticket to C at a rate of US 1 to Ug. Shs. 2,400. Another tourist bought a one way ticket to c at a rate of US 1 to Ug. Shs. 2,420, a week later. How much more in Ug. Shs. Did the second tourist pay? (05marks)
17. The diagram below shows a cuboid KLMNWXYZ in which KL= 9cm and MY = 5cm



(a) Calculate the length
  (i) KM
  (ii) KY

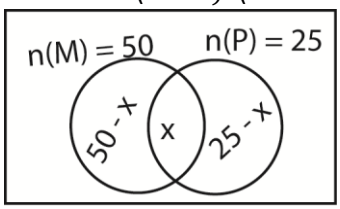
(b) Determine the angle between
  (i) Line KY and base KLMN
  (ii) Plane KZYL and plane WXY

SOLUTIONS

SECTION A (40 marks)

Answer all questions in this section

1. An increase of 15% in salaries makes the monthly expenditure on salaries for a factory to be shs. 22,425,000. Find the expenditure before the increase. (04marks)
   Let x = initial expenditure
   $\frac{115}{100}x = 22{,}425{,}000$
   $x = \frac{100}{115}\ x\ 22{,}425{,}000$ = 19,500,000
   $Hence\ initial\ cost = shs.\ 19,500{,}000$
2. The sets M and P are such that n(M) = 50, n(P) = 25 and n(M∪ P) = 60.
   Calculate n(P∩ M). (04marks)



   50 –x + x + 25 – x = 60
   75 – x = 60
   x = 75 – 60 = 15
   Hence n(P∩ M) = 15
3. Find the equation of the line joining the points (3, 5) and (-2, 10). (04marks)
   Method I; using general equation of the line
   y = mx + c
   Substituting for (x, y) = (3, 5)
   5 = 3m + c …… (i)
   Substituting for (x, y) = (-2, 10)
   10 = -2m + c …… (ii)

Eqn. (ii) – eqn. (i)

5 = -5m

m = $\frac{5}{-5} = -1$

Substituting for m = -1 in eqn. (i)

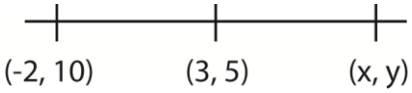
5 = -3 + c

c = 8

Hence the equation of the line is y = -x + 8

Method II

Let (x, y lie on the line



Gradients on the line are equal

$\frac{5-10}{3-2} = \frac{y-5}{x-3}$

-1 = $\frac{y-5}{x-3}$

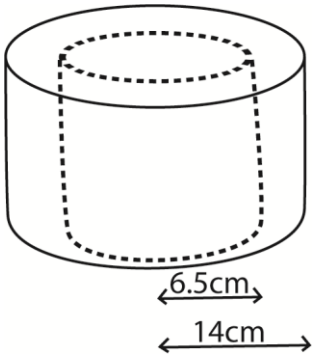
-1(x – 3) = y – 5

-x + 3 = y- 5

Y = -x + 8

Hence the equation of the line is y = -x + 8

4. A metallic cylindrical pipe of uniform cross-sectional area has an outer radius of 14cm and inner radius of 6.5cm. it has a length of 4.2metres. Calculate the volume in cm3, of the metal used to make the pipe. (04marks)



Volume of the metal used = volume of the metallic cylinder – volume of the hole

= $\frac{22}{7}\ x\ 14^2$x 420 - x $\frac{22}{7}\ x\ 6.5^2$x 420

= 258.720 – 55.770

= 202.950cm^3

5. Express $\frac{3}{1-\sqrt{2}}$ in the form a + b$\sqrt{2}$. (04marks)

$\frac{3}{1-\sqrt{2}} = \frac{3(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{3+3\sqrt{2}}{1^2-(\sqrt{2})^2} = \frac{3+3\sqrt{2}}{-1} = -3-3\sqrt{2}$

Hence a = -3 and b = -3

6. Given that f(x) = $\frac{3x+16}{4}$, find the value of f'(1). (04marks)

Let f(x) = y

⇨ y = $\frac{3x+16}{4}$

4y =3x + 16

3x =4y – 16

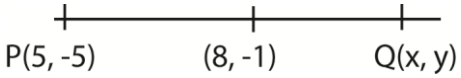
x = $\frac{4y-16}{3}$

f'(x) = $\frac{4y-16}{3}$

f'(1) = $\frac{4(1)-16}{3} = \frac{4-16}{3} = \frac{-12}{3} = -4$

7. The coordinates of the mid-point of a line PQ are (8, -1). The coordinates of P are (5, -5). Determine the coordinates of Q. (04marks)

Let Q(x,y)



$\frac{5+x}{2}$ = 8

$x + 5 = 16$

x = 11

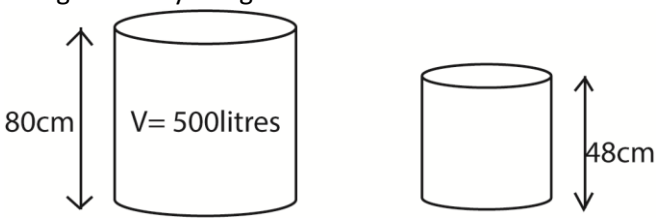
also

$\frac{y-5}{2}$ =-1; y = 3

$Hence\ Q(11, 3)$

8. Two water tanks of the same shape. The larger tank is 80cm high with a capacity of 500litres. What is the capacity of the smaller tank whose height is 48cm? (04marks).

Using similarity of figures



L.S.F = 80 ; 48

V.S.F = 80^2; 48^2

Let the volume of smaller tank be x

$\frac{500}{x} = \frac{80^2}{48^2}$

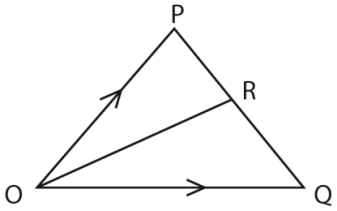
$x = \frac{500\ x\ 48^2}{80^2} = 108litres$

9. Fauza bought one dozen of shirt at shs. 40,000 per shirt. She sold them at a profit of 20% How much money did she earn as profit from the shirt sales? (04marks)

Cost of shirts = 12 x 40,000 = 480,000

Profit = $\frac{20}{100}\ x\ 480{,}000$ = shs. 96,000

10. In the figure below, OP = a, OQ= b and PR = $\frac{1}{2}PQ$.



Express OR in terms of a and b. (04marks)

OR = OP + PR

= a + $\frac{1}{3}(-a + b)$

= $\frac{3a-a+b}{3} = \frac{2a+b}{3}$

SECTION B (60MARKS)

Answer any five questions from this section. All questions carry equal marks

11. (a) Simplify $\frac{\left(3\frac{5}{6}\div 2\frac{2}{15}\right)x\frac{3}{23}}{5\frac{1}{3}-2\frac{7}{12}}$. (06marks)

$$\frac{\left(3\frac{5}{6}\div 2\frac{2}{15}\right)x\frac{3}{23}}{5\frac{1}{3}-2\frac{7}{12}} = \frac{\left(\frac{23}{6}\div\frac{32}{15}\right)x\frac{3}{23}}{\frac{16}{3}-\frac{31}{12}} = \frac{\left(\frac{23}{6}x\frac{15}{32}\right)x\frac{3}{23}}{\frac{64-31}{12}} = \frac{15}{64}\,x\frac{12}{33} = \frac{15}{176}$$

(b) A forest covering an area of 807.5km^2 is represented on a map by a green area of 32.3cm^2. Determine the scale on the map. (06marks)

32.3cm^2 represent 807.5km^2

1cm^{2}represents $\frac{807.5}{32.3} = 25km^2$

$\sqrt{1cm^2} = \sqrt{25km^2}$

1cm represent 5km = 5 x 100,000cm = 500,000cm

Scale is 1:500,000

12. In a survey carried out in the department od languages at a certain University, the following data was collected, 20 students spoke German, 10spoke French and German, 9 spoke French and Kiswahili. 7 spoke Kiswahili and German only. 2 could not speak any of the three languages. 22 could speak at least two of the languages. 12 could speak only one languages. 11 could either speak Kiswahili or German but not French.

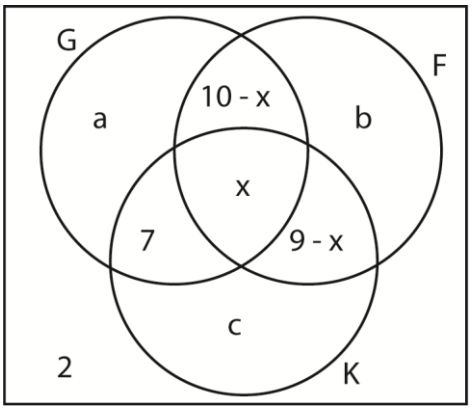
(a) Use Venn diagram to represent the given information. (03 marks)

Summary of information

n(G) = 20; n(F) = ?; n(K) =?; n(F∩ $G) = 10$; n(∩ $K) = 9$; $n(G \cup F \cup K)' = 2$;

n(K∪ $G)\ only = 11$.

Let $n(G \cap F \cap K) = x$



(b) Find the number of students that could speak

(i) all the three languages

10 – x + x + 7 + 9 – x = 22

26 –x = 22

x = 4

∴ the number of students who could speak all languages is 4

(ii) Kiswahili only

n(G) = 20

a + 6 + 4 + 7 = 20

a + 17 = 20

a = 3

n(KU G) $only = 11.$

3 + 7 + c = 11

c = 1

∴ the number of students who could speak Kiswahili only is 1

(iii) French only (05marks)

a + b + c = 12

3 + b + 1 = 12

b + 4 = 12

b = 8

∴ the number of students who could speak French only is 8

(c) What is the probability that a student picked at random from the group could speak neither Kiswahili nor German. (04marks)

P(KUG)' = $\frac{n(KUG)'}{n(\varepsilon)} = \frac{8+2}{20+8+5+1+2} = \frac{10}{36} = \frac{5}{18}$

13. A cyclist covered a journey of 48km from station A to station B in 5½ hours. A cyclist rode at 12km/hr for the first 1½ hours and changed speed for the remaining part of the journey.

(a) (i) Determine the speed for the remaining part of the journey. (06marks)

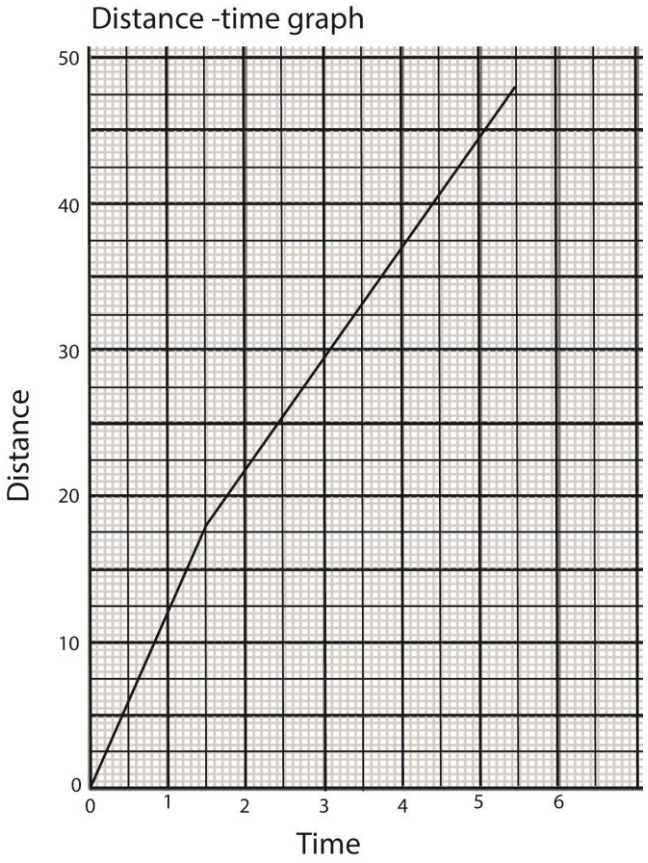
Distance covered before changing the speed = speed x time = 12 x $\frac{3}{2}$ = 18km

Remaining distance = 48 – 18 = 30km

Time taken = 5½ - 1½ = 3hours

Speed = $\frac{distance}{time} = \frac{30}{4}$ = 7.5km/hr

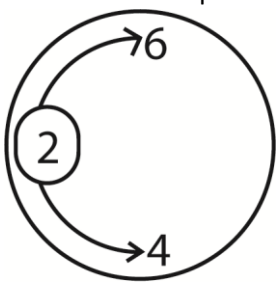
(ii) Represent the cyclist journey on a distance –time graph. (04 marks)



(b) Calculate the average speed of the cyclist from a to B. (02marks)

Average spee = $\frac{total\ distance}{total\ time} = \frac{48}{5.5} = 8.73km/h$

14. (a) Given the set (2, 4, 6) draw a papygram to show the relation
" is the smallest prime factor of" (03marks)



(b) for the mapping x⟶ 4x + 5, find the domain when the range is {1,13) (04marks)

Let y = 4x + 5

x = $\frac{y-5}{4}$

f'(x) =$\frac{x-5}{4}$

f'(1) =$\frac{1-5}{4} = -1$

f'(13) =$\frac{13-5}{4} = 2$

Hence the domain is {-1, 2}

(c) The function f(x) = $2x^2$ and g(x) = 5x -3. Find the value of x such that f(x) = g(x). (05mrks)

For f(x) = g(x)

$2x^2$ =5x -3

$2x^2$ -5x + 3 = 0

(x -1)(2x – 3) = 0

Either x -1 =0; x = 1

Or 2x -3 = 0; x =$\frac{3}{2}$

15. (a) If a = $\begin{pmatrix}-2\\4\end{pmatrix}, b = \begin{pmatrix}-5\\1\end{pmatrix}, c = \begin{pmatrix}-1\\3\end{pmatrix}$ and 2a + m + b = c. find

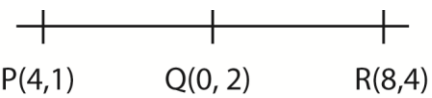
(i) m

$$2\begin{pmatrix}-2\\4\end{pmatrix} + m + \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$
$$\begin{pmatrix}-4\\8\end{pmatrix} + m + \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$
$$m + \begin{pmatrix}-9\\9\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$
$$m = \begin{pmatrix}-1\\3\end{pmatrix} - \begin{pmatrix}-9\\9\end{pmatrix} = \begin{pmatrix}8\\6\end{pmatrix}$$

(ii) $|m|$

$|m| = \sqrt{8^2 + 6^2}$ = $\sqrt{64 + 36} = 10$

(b) Using vectors show that the point P(-4, 1), Q(0,2) and R (8, 4) lie on straight line (06mrks)



PQ = OQ – OP

= $\begin{pmatrix}0\\2\end{pmatrix} - \begin{pmatrix}-4\\1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}$

QR = OR - OQ

$= \binom{8}{4} - \binom{0}{2} = \binom{8}{2} = 2\binom{4}{1}$

QR = 2PQ

Hence PR is parallel to PQ. But since QR and PQ share the same point Q, the PQR is a straight line

16. The table below shows fares for some flights at an airport.

Destination	One way ticket (US dollars)	Return ticket (US dollars)
A	300	565
B	705	1295
C	380	714
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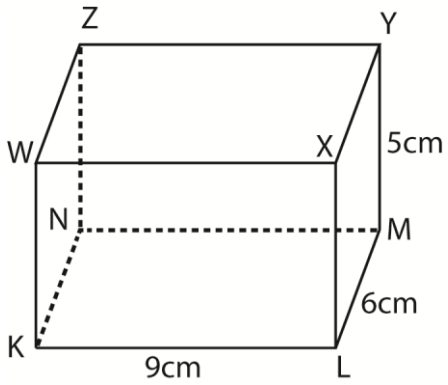
- One way ticket means from airport to destination
- Return ticket means from airport to destination and back to airport

(a) Calculate the amount in Ug. Shs. For a one way ticket to B if the exchange rate is us is US $ 1 to Ug. Shs. 2,500/=
US $ 1 = Ug. Shs. 2,500/=
US $ 705 = Ug. Shs. 2,500 x 705 = Ug. Shs. 1,762, 500

(b) A family bout for return tickets for destination A at Ug. Shs. 5,737,000. Determine the exchange rate. (05marks)
4 return tickets = Ug. Shs. 5,737,000
1return ticket = Ug. Shs. $\frac{5,737,000}{4}$ = Ug. Shs. 1,434,250
US $ 565 = Ug. Shs. 1,434,250
US $ 1 = Ug. Shs. $\frac{1,434,250}{565}$ = Ug. Shs. 2,538.50

(c) A tourist bout a one way ticket to C at a rate of US $ 1 to Ug. Shs. 2,400. Another tourist bought a one way ticket to c at a rate of US $1 to Ug. Shs. 2,420, a week later. How much more in Ug. Shs. Did the second tourist pay? (05marks)
First tourist
US $ 1 = Ug. Shs. 2,400
US $ 380 = Ug. Shs. 2,400 x 380 = Ug. Shs. 912,000
Second tourist
US $ 1 = Ug. Shs. 2,420
US $ 380 = Ug. Shs. 2,420 x 380 = Ug. Shs. 919,600
Difference = 919,600 – 912,000 = 7600
Hence the second tourist paid 7600 more than the first tourist.

17. The diagram below shows a cuboid KLMNWXYZ in which KL= 9cm and MY = 5cm



(a) Calculate the length

(i) KM

$\overline{KM}^2 = \overline{KL}^2 + LM^2$

$\overline{KM} = \sqrt{9^2 + 6^2} = \sqrt{117} = 10.82cm$

(ii) KY

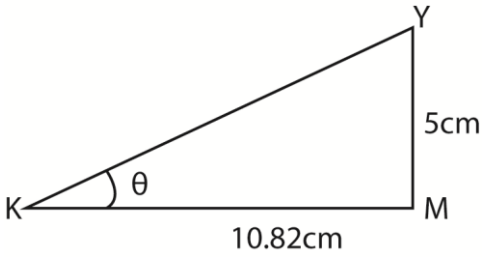
$\overline{KY}^2 = \overline{KM}^2 + MY^2$

$\overline{KY} = \sqrt{117 + 5^2} = \sqrt{142} = 11.92cm$

(b) Determine the angle between

(i) Line KY and base KLMN

Let the angle be θ

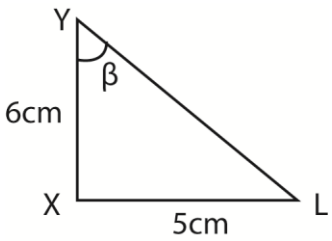


$\tan\theta = \frac{5}{10.82}$

$\theta = \tan^{-1}\left(\frac{5}{10.82}\right) = 24.8^0$

(ii) Plane KZYL and plane WXYZ

Let the angle be β



$\tan\beta = \frac{5}{6}$

$\beta = \tan^{-1}\left(\frac{5}{6}\right) = 39.8^0$

Thank you
Dr. Bbosa Science