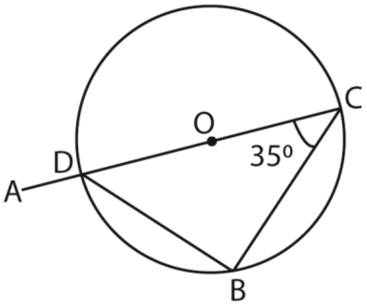
UACE MATHEMATICS PAPER 1 2013 guide

SECTION A (40 marks)

Answer all questions in this section

1. Solve the quadratic equation: $p^2 - 7p + 12 = 0$(04marks)
2. The length of eight trousers in centimetres are 90, 115, 98, 103, 108, 105, 101 and 98. Find the:
   (i) modal length
   (ii) median length (04marks)
3. Given $\tan\theta = \frac{-5}{12}$ and $270^0 \leq \theta \leq 360^0$, determine the value of $\cos\theta$. (04marks)
4. Factorise completely the following expression
   (a) $(a + 1)^2 - 3(a + 1)$ (02marks)
   (b) $47 - (x - 4)^2$ (02marks)
5. A square of area $36cm^2$ is transformed to an image using matrix $\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}$.
   Determine the area of the image (04marks)
6. Matovu is twice a od as Nankya. After four year, the sum of their ages will be 26 years. Find Nankya's age. (04marks)
7. The figure below shows a circle with centre O and BCD = 35^0.



   Calculate
   (a) angle CDB
   (b) angle ADB (04marks)
8. solve the simultaneous equation(04marks)
   $2y - 3x = 13$
   $3y + x = 3$
9. the table below shows the ages in years of 40 teachers in a school

Ages (years)	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69
Number of teachers	2	4	8	10	7	5	3	1

Draw a cumulative frequency curve (orgive) for the dat. (04marks)

10. Given that $\begin{pmatrix} x & 3 \\ 4 & y \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 18 \end{pmatrix}$, find the value of x and y (04marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

11. Mukisa stays 6km away from the factory where he works. One day, he started on his journey at 6.42 am and arrived at 7.30am. he walked part of the journey at 5km/h. Realising he would be late, he ran the rest of the journey at 10km/h.
    (a) What distance did he ran (07marks)
    (b) The factory closes its gate to its workers at 7.45 am. Determine the number of minutes by which Mukisa would have been late had he not run part of the journey. (05marks)
12. (a) Given the matrices B = $\begin{pmatrix} 2 & 8 \\ 16 & -4 \end{pmatrix}$ and $C = \begin{pmatrix} 6 & -4 \\ -12 & 8 \end{pmatrix}$
    find the inverse of matrix (B + C) (05 marks)
    (b) Mayo sells shirts of sizes small (S), Medium (M) and extra large (XL). The table below shows his sales for 3days

Size	Day		
	Mon	Tues	Wed
S	2	2	1
M	7	4	1
XL	3	5	3

    He sells each shirt at shs. 40,000 for S, shs. 50,000 for M and shs. 60,000 for XL
    (i) Write down a:
        - 3 x 3 matrix for the sale
        - 1 x 3 matrix for the prices of the shirt
    (ii) Use the matrices to calculate his total income for the shirts (07marks)
13. On a farm there are four houses P Q, R and S. P is 800m on a bearing of 020^0 from Q. R is 500m on bearing of 160^0 from Q. S is 1200m on a bearing of 045^0 from R.
    (a) Use a scale of 1cm to represent 100m to construct a scale diagram showing the positions of the four houses (09marks)
    (b) Find th distance and bearing of S from P. (03marks)
14. (a) A basket contains red balls and whit balls. The probability of picking a white ball is $\frac{1}{8}$. If there 24 balls in the bag, find the number of red balls. (04marks)
    (b) A basket contains 30 bananas. Ten of them are ripe and the rest unripe. Two bananas ar selected at random from the basket with replacement. Find the probability that:
    (i) are both ripe
    (ii) one is ripe and one is unripe (08marks)
15. The height y metres of a wave on a certain day is given by y = 5 + cos $(30x)^0$ where x is the number of hours after midnight.
    (a) Use x at intervals of one hour from 0 to 6hours to find the corresponding values of y. put the values of x and y in a table. (04marks)
    (b) Use the table to draw a graph of y against x (06marks)
    (c) From the graph, find
        (i) Height of the wave at 3.30 am
        (ii) Time when the height of the wave is 5.2m (02marks)
16. The triangle ABC with vertices A(-4, 2), B(-5, 5) and C(-1 4) is mapped onto triangle A'B'C' by a transformation matrix T = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

The triangle A'b'C' is mapped onto triangle A''B''c'' by another matrix M = $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.

(a) Determine the coordinates of the vertices of
   (i) A', B' and C'
   (ii) A'', B'' and c'' (04marks)

(b) On the same axes draw triangles ABC, A'B'C' and A''B''C''. (04marks)

(c) Describe fully the transformation represented by
   (i) T
   (ii) M (04marks)

17. A school has organized a Geography study tour for 90 students. Two types of vehicles are needed; taxis and costa buses. The maximum capacity of the taxi is 15 passengers while that of costa bus is 30 passengers. The number of taxis will be greater than the number of costa buses. The number of taxis will be less than five. The cost of hiring a taxis is shs. 60, 000 while that of costa bus is shs. 100,000. There is only shs. 600,000 available

(a) If x represents the number of taxis and y the number of costa buses, write six inequalities for the given information. (05marks)

(b) Represent the inequalities on graph paper by shading the unwanted regions. (Use the scale of 2cm to 1 unit on both axes)(04marks)

(c) Find from your graph the number of taxis and costa buses which are full to capacity that must be ordered so that the students are transported. (03marks)

Solutions

1. Solve the quadratic equation: $p^2 - 7p + 12 = 0$(04marks)

Method I: by factorization

$p^2 - 7p + 12 = 0$

(P-3)(P-4) = 0

Either P -3 = 0

P = 3

Or

(P-4) = 0

P = 4

Method 2: using quadratic equation

$p = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)} = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm \sqrt{1}}{2} = \frac{7 \pm 1}{2}$

Either: $p = \frac{8}{2} = 4$

Or: $p = \frac{6}{2} = 3$

2. The length of eight trousers in centimetres are 90, 115, 98, 103, 108, 105, 101 and 98. Find the:

By arranging the values in ascending order

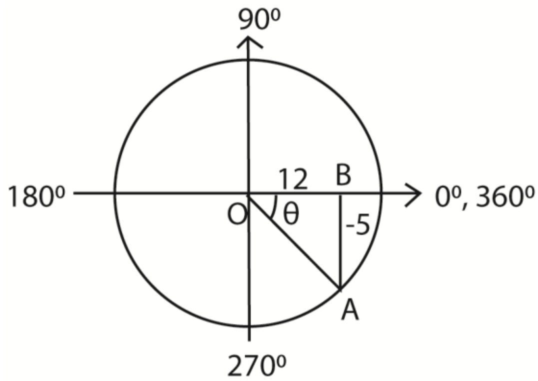
90, 98, 98, 101, 103, 105, 108, 115

(i) modal length: 98

(ii) median length (04marks)

$= \frac{101+103}{2} = \frac{204}{2} = 102$

3. Given tanθ = $\frac{-5}{12}$ and $270^0 \leq \theta \leq 360^0$, determine the value of cosθ. (04marks)



$\overline{OA}^2 = 12^2 + (-5)^2 = 144 + 25 = 169$

$\overline{OA} = \sqrt{169} = 13$

$\cos\theta = \frac{OB}{OA} = \frac{12}{13}$

4. Factorise completely the following expression
   (a) $(a + 1)^2 - 3(a + 1)$ (02marks)
   $(a + 1)(a + 1 - 3)$
   $(a + 1)(a - 2)$
   (b) $49 - (x - 4)^2$ (02marks)
   $(7 + (x-4))(7-(x - 4))$
   $(7 + x - 4)(7 - x + 4)$
   $(3 + x)(11 - x)$
5. A square of area $36cm^2$ is transformed to an image using matrix $\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}$.
   Determine the area of the image (04marks)
   Area of the image = area of the object x area scale factor
   But area scale factor = determinant of the matrix given
   Hence area scale factor = (3 x 5) –(1 x 5)
   = 15 – 5 = 10
   Area of the image = $36\ cm^2 \times 10 = 360cm^2$.
6. Matovu is twice a od as Nankya. After four year, the sum of their ages will be 26 years. Find Nankya's age. (04marks)
   Let x = the current age of Nankya

	Nankya	Matovu
Current age	x	2x
After four years	x +4	2x + 4

Sum after four years = 26years

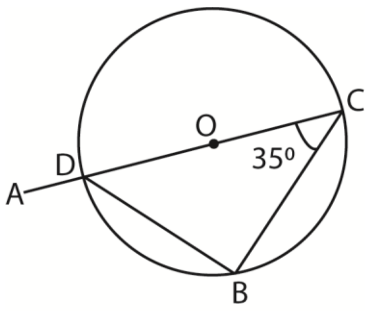
$x + 4 + 2x + 4 = 26$

$3x + 8 = 26$

$x = 6$

hence Nankya's age is 6years.

7. The figure below shows a circle with centre O and BCD = 35^0.



Calculate

(a) angle CDB

the angle subtended by an arc of a circle at the centre is twice the angle subtended by the same arc at any other point on the circumference of the circle

Hence <CBD = $\frac{1}{2}(180^0) = 90^0$

<CBD + <CDB + 35^0 = 180^0

<CBD + 90^0 + 35^0 = 180^0

<CBD = 180 -125 = 55^0

Or

Angle subtended at the circumference of circle by the diameter is 90^0.

⇨ <CBD = 90^0

<CDB + 35^0 = 90^0

<CDB = 90^0 – 35^0 = 55^0

(b) angle ADB (04marks)

<ADC + <CDB = 180^0

<ADC + 55^0 = 180^0

<ADC = 180^0 - 55^0 = 125^0

Or

35^0 + <CBD =<ADB

<ADB = 35^0 + 90^0 = 125^0

8. solve the simultaneous equation(04marks)

2y – 3x = 13

3y + x = 3

Method I: Using elimination method

2y – 3x = 13 ……………. (i)

3y + x = 3……………….. (ii)

(i) + 3(ii)

$$\begin{array}{r} 2y - 3x = 13 \\ +\ 9y + 3x = 9 \\ \hline 11y + 0 = 22 \end{array}$$

y = $\frac{22}{11} = 2$

From (i)

3(2) + x = 3

6 + x = 3

x = -3

Hence x = -3 and y = 2

Method II: using substitution method.

2y – 3x = 13 ……………. (i)

3y + x = 3……………….. (ii)

From (ii)

x = 3 – 3y

Substituting x in (i)

2y –3(3 -3y) = 13

2y – 9 + 9y = 13

11y = 22

y = $\frac{22}{11} = 2$

x = (3 – 3(2)) = 3 – 6 = -3

Hence x = -3 and y = 2

Method III: using Matrix method

2y – 3x = 13

3y + x = 3

$\begin{pmatrix} 2 & -3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \end{pmatrix}$

Pre-multiplying both sides by adjunct matrix,

$\begin{pmatrix} 1 & 3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 13 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 2+9 & -3+3 \\ -6+6 & 9+2 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 13+9 \\ -39+6 \end{pmatrix}$

$\begin{pmatrix} 11 & 0 \\ 0 & 11 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 22 \\ -33 \end{pmatrix}$

11y = 22

y = $\frac{22}{11} = 2$

11x = -33

x = $\frac{-33}{11} = -3$

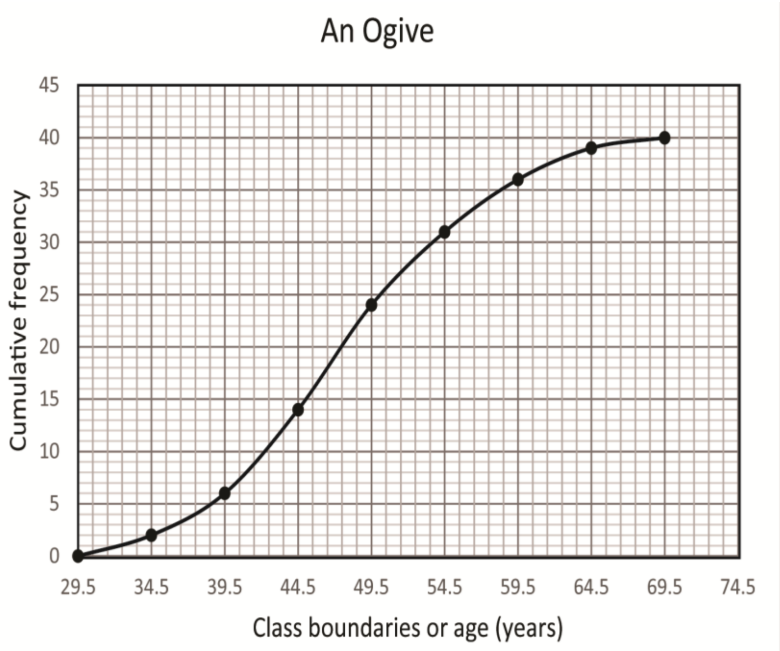
9. The table below shows the ages in years of 40 teachers in a school

Ages (years)	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69
Number of teachers	2	4	8	10	7	5	3	1

Draw a cumulative frequency curve (orgive) for the dat. (04marks)

Table of results

Age (years)	Class boundaries	Frequency	Cumulative frequency
30 – 34	29.5 – 34.5	2	2
35 – 39	34.5 – 39.5	4	6
40 – 44	39.5 – 44.5	8	14
45 – 49	44.5 – 49.5	10	24
50 – 54	49.5 – 54.5	7	31
55 – 59	54.5 – 59.5	5	36
60 – 64	59.5 – 64.5	3	39
65 – 69	64.5 – 69.5	1	40



10. Given that $\begin{pmatrix} x & 3 \\ 4 & y \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 18 \end{pmatrix}$, find the value of x and y (04marks)

$\begin{pmatrix} x & 3 \\ 4 & y \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 18 \end{pmatrix}$

2x + 15 = -1

2x = -16

x = $\frac{-16}{2} = -8$

8 + 5y = 18

5y = 10

y = $\frac{10}{5} = 2$

hence x = -8 and y = 2

SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

11. Mukisa stays 6km away from the factory where he works. One day, he started on his journey at 6.42 am and arrived at 7.30am. he walked part of the journey at 5km/h. Realising he would be late, he ran the rest of the journey at 10km/h.

(a) What distance did he ran (07marks)

Let x be the distance walked

Distance he ran = (6- x)

Time taken to move from home to work = 7.30am – 6.42am = 48min = $\frac{48}{60} = \frac{4}{5}hour$

From distance = speed x time

x = 5t

t = $\frac{x}{5}$

6-x = 10($\frac{4}{5} - t$)……. (ii)

Substituting for t in eqn. (ii)

6 – x = 10$\left(\frac{4}{5} - \frac{x}{5}\right)$

6 – x = 8 – 2x

x = 2

Distance he ran = 6 -2 = 4km

(b) The factory closes its gate to its workers at 7.45 am. Determine the number of minutes by which Mukisa would have been late had he not run part of the journey. (05marks)

t = $\frac{D}{s} = \frac{6}{5} = 1.2hours = 1h12min$

Time of arrival =6:42am + 1h 12min = 7:54 am

Time for being late = 7:54am – 7:45am = 9min

12. (a) Given the matrices B = $\begin{pmatrix} 2 & 8 \\ 16 & -4 \end{pmatrix}$ and $C = \begin{pmatrix} 6 & -4 \\ -12 & 8 \end{pmatrix}$ find the inverse of matrix (B + C) (05 marks)

B + C= $\begin{pmatrix} 2 & 8 \\ 16 & -4 \end{pmatrix} + \begin{pmatrix} 6 & -4 \\ -12 & 8 \end{pmatrix} = \begin{pmatrix} 2+6 & 8-4 \\ 16-12 & -4+8 \end{pmatrix}$= $\begin{pmatrix} 8 & 4 \\ 4 & 4 \end{pmatrix}$

Determinant of (B + C)= 8 x 4 – 4 x 4 = 32 – 16 = 16

Adjunct of (B+C) = $\begin{pmatrix} 4 & -4 \\ -4 & 8 \end{pmatrix}$

Inverse of (B + C) = $\frac{1}{16}\begin{pmatrix} 4 & -4 \\ -4 & 8 \end{pmatrix}$ =$\begin{pmatrix} \frac{4}{16} & \frac{-4}{16} \\ \frac{-4}{16} & \frac{8}{16} \end{pmatrix}$ =$\begin{pmatrix} \frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{2} \end{pmatrix}$

(b) Mayo sells shirts of sizes small (S), Medium (M) and extra large (XL). The table below shows his sales for 3days

Size	Day		
	Mon	Tues	Wed
S	2	2	1
M	7	4	1
XL	3	5	3

He sells each shirt at shs. 40,000 for S, shs. 50,000 for M and shs. 60,000 for XL

(i) Write down a:

- 3 x 3 matrix for the sale

$$\begin{pmatrix} 2 & 2 & 1 \\ 7 & 4 & 1 \\ 3 & 5 & 3 \end{pmatrix}$$

- 1 x 3 matrix for the prices of the shirt

$$(40.000 \quad 50,000 \quad 60,000)$$

(ii) Use the matrices to calculate his total income for the shirts (07marks)

$$(40.000 \quad 50,000 \quad 60,000)\begin{pmatrix} 2 & 2 & 1 \\ 7 & 4 & 1 \\ 3 & 5 & 3 \end{pmatrix}$$

= 40,000 x 2 + 50000 x 7 + 60000 x 3 + 40,000 x 2 + 50000 x 4 + 60000 x 5 + 40,000 x 1 + 50000 x 1 + 60000 x 3

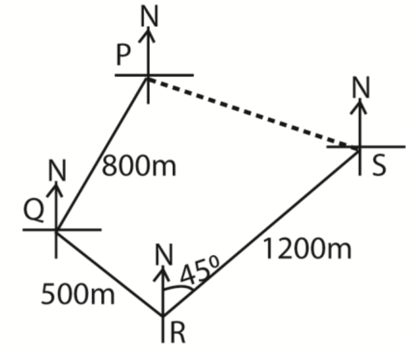
= 80,000 + 350000 + 180,000 + 80,000 + 200,000 + 300,000 + 40,000 + 50,000 + 180,000

= 1,460,000

13. On a farm there are four houses P Q, R and S. P is 800m on a bearing of 020^0 from Q. R is 500m on bearing of 160^0 from Q. S is 1200m on a bearing of 045^0 from R.

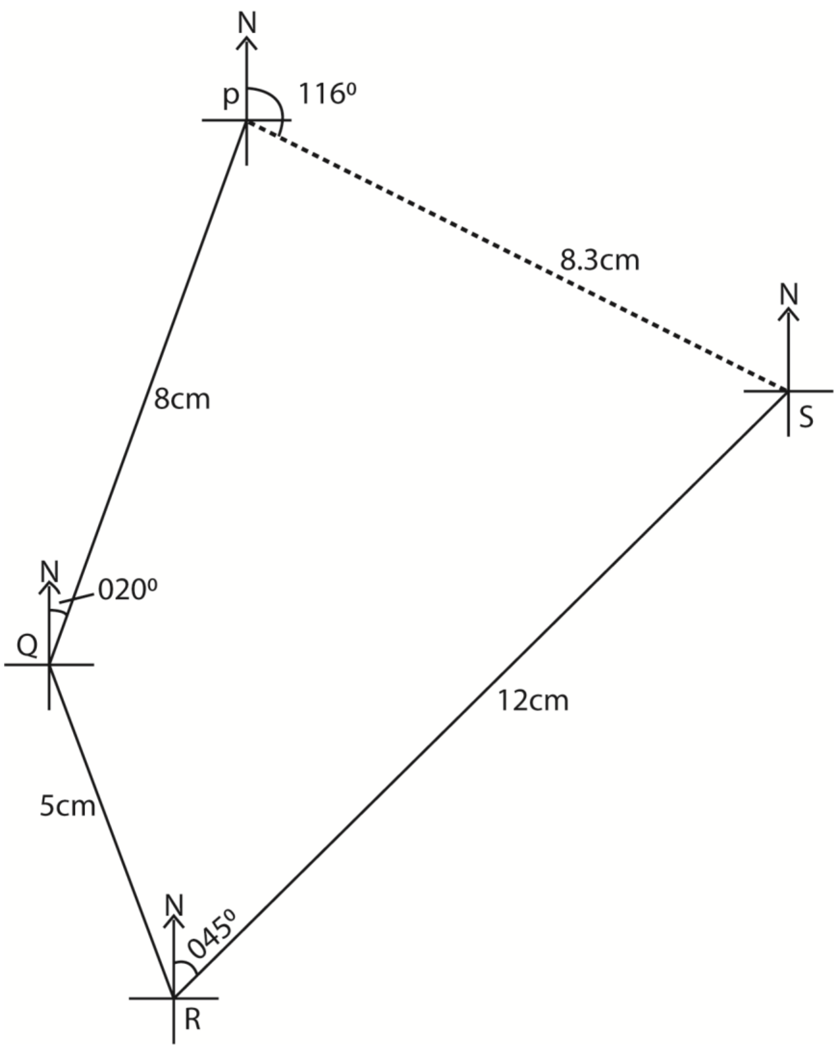
(a) Use a scale of 1cm to represent 100m to construct a scale diagram showing the positions of the four houses (09marks)

Sketch work



Scale drawing

Houses	Distance in m	Bearing	Distance in cm
P from Q	800	020^0	$\frac{800}{100} = 8$
R from Q	500	160^0	$\frac{500}{100} = 5$
S from R	1200	045	$\frac{1200}{100} = 12$



(b) Find the distance and bearing of S from P. (03marks)

Distance of S from P, = 8.3cm

In metres

$\overline{PS} = 8.3 \: x \: 100 = 830m$

Bearing of S from P = 116^0

14. (a) A basket contains red balls and whit balls. The probability of picking a white ball is $\frac{1}{8}$. If there 24 balls in the bag, find the number of red balls. (04marks)

Let x be the number of white balls

Probability of picking white ball = $\frac{x}{24}$

=> $\frac{x}{24} = \frac{1}{8}$; x =3

Number of red balls = 24 – 3 = 21

Or

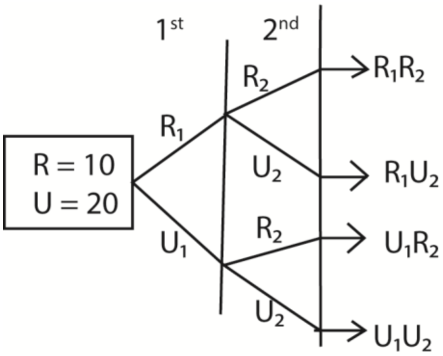
Probability of picking red ball = 1 - $\frac{1}{8} = \frac{7}{8}$

Let y = number of red balls

Probability of picking red ball = $\frac{y}{24} = \frac{7}{8}$; y = 21

(b) A basket contains 30 bananas. Ten of them are ripe and the rest unripe. Two bananas ar selected at random from the basket with replacement. Find the probability that:

(i) are both ripe



Probability of ripe banana, P(R) = $\frac{10}{30} = \frac{1}{3}$

Probability of unripe bananas = 1 - $\frac{1}{3} = \frac{2}{3}$

Probability that both are ripe =P($R_1 \cap R_2$) = $\frac{1}{3} \: x \frac{1}{3} = \frac{1}{9}$

(ii) one is ripe and one is unripe (08marks)

Probability that one is ripe and the other is unripe

P($R_1 \cap U_2$) + P($U_1 \cap R_2$)

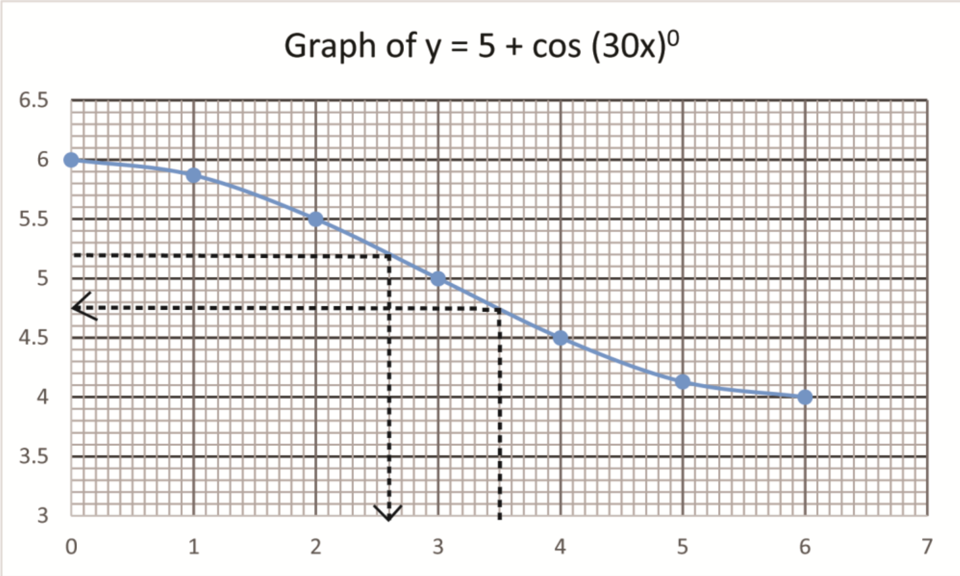
$\frac{1}{3} \: x \frac{2}{3} + \frac{2}{3} \: x \frac{1}{3} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

15. The height y metres of a wave on a certain day is given by y = 5 + cos $(30x)^0$ where x is the number of hours after midnight.

(a) Use x at intervals of one hour from 0 to 6hours to find the corresponding values of y. Put the values of x and y in a table. (04marks)

x	0	1	2	3	4	5	6
y = 5 + cos $(30x)^0$	6	5.87	5.50	5.00	4.50	4.13	4.00

(b) Use the table to draw a graph of y against x (06marks)



(c) From the graph, find

(i) Height of the wave at 3.30 am : 4.75

(ii) Time when the height of the wave is 5.2m: 2:36a.m (02marks)

16. The triangle ABC with vertices A(-4, 2), B(-5, 5) and C(-1 4) is mapped onto triangle A'B'C' by a transformation matrix T = $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$.

The triangle A'b'C' is mapped onto triangle A''B''c'' by another matrix M = $\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$.

(a) Determine the coordinates of the vertices of

(i) A', B' and C'

$$\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{array}{c}\begin{array}{ccc}A&B&C\end{array}\\\begin{pmatrix}-4&-5&-1\\2&5&4\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A'&B'&C'\end{array}\\\begin{pmatrix}0+2&0+5&0+4\\4+0&5+0&1+0\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A'&B'&C'\end{array}\\\begin{pmatrix}2&5&4\\4&5&1\end{pmatrix}\end{array}$$

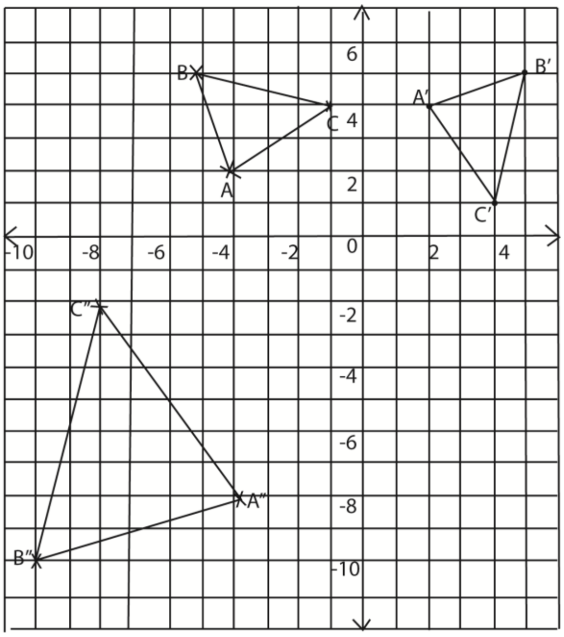
Hence A'(2, 4), B'(5, 5) and C' (4, 1)

(ii) A'', B'' and c'' (04marks)

$$\begin{pmatrix}-2&0\\0&-2\end{pmatrix}\begin{array}{c}\begin{array}{ccc}A'&B'&C'\end{array}\\\begin{pmatrix}2&5&4\\4&5&1\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A''&B''&C''\end{array}\\\begin{pmatrix}-4+0&-10+0&-8+0\\0-8&0-10&0-2\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A''&B''&C''\end{array}\\\begin{pmatrix}-4&-10&-8\\-8&-10&-2\end{pmatrix}\end{array}$$

Hence A''(-4, -8), B''(-10, -10) and C'' (-8, -2)

(b) On the same axes draw triangles ABC, A'B'C' and A''B''C''. (04marks)



(c) Describe fully the transformation represented by

(i) T: rotation through +900 about the origin or positive quarter turnabout (0,0)

(ii) M: Enlargement of scale -2about the origin. (04marks)

17. A school has organized a Geography study tour for 90 students. Two types of vehicles are needed; taxis and costa buses. The maximum capacity of the taxi is 15 passengers while that of costa bus is 30 passengers. The number of taxis will be greater than the number of costa buses. The number of taxis will be less than five. The cost of hiring a taxis is shs. 60, 000 while that of costa bus is shs. 100,000. There is only shs. 600,000 available

(a) If x represents the number of taxis and y the number of costa buses, write six inequalities for the given information. (05marks)

All students must go for the tour.

Hence $15x + 30y \geq 90$

Number of trips made $x > y$

Number of trips made by taxis; $x < 5$

Cost incurred: $60,000x + 100,000y \leq 600,000$

Some trips must be made by both, hence

$x \geq 0 \text{ and } y \geq 0$

Therefore the six inequalities are

$15x + 30y \geq 90$ ………………………. (i)

$x > y$ …………………………………………(ii)

$x < 5$ ……………………………………… (iii)

$60,000x + 100,000y \leq 600,000$ ….(iv)

$x \geq 0$ …………………………………………(v)

$y \geq 0$ ……………………………………… (vi)

(b) Represent the inequalities on graph paper by shading the unwanted regions. (Use the scale of 2cm to 1 unit on both axes)(04marks)

For $15x + 30y \geq 90$, the boundary line s$15x + 30y = 90$

$x + 2y = 6$

x	0	6
y	3	0

For x> y

The boundary line is x = y

x	0	5
y	0	5

For x < 5

The boundary line is x = 5

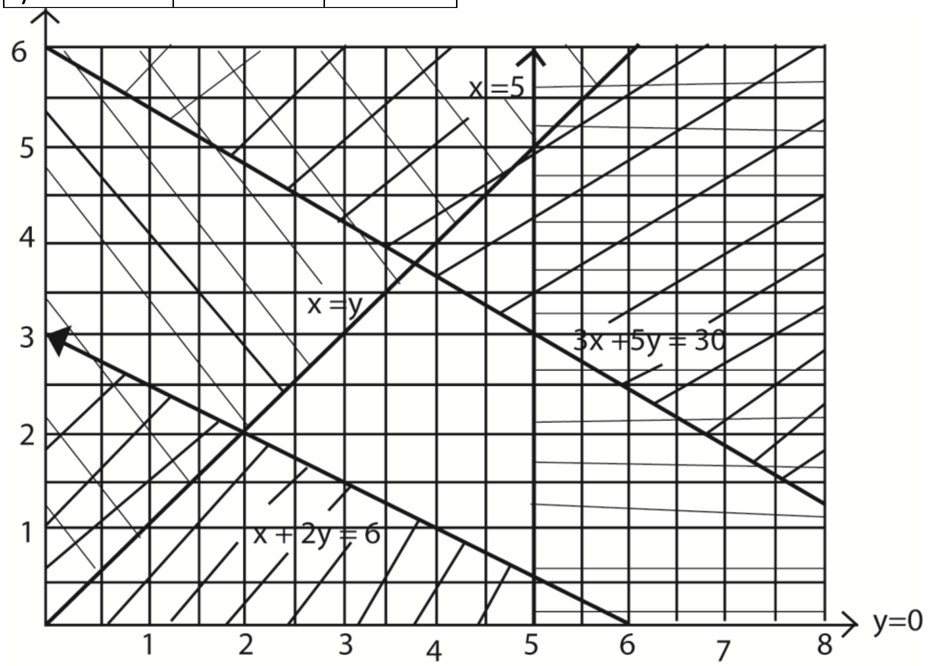
For $60{,}000x + 100{,}000y \leq 600{,}000$

The boundary line

$60{,}000x + 100{,}000y = 600{,}000$ or

$3x + 5y = 30$

x	0	5
y	6	3



(c) Find from your graph the number of taxis and costa buses which are full to capacity that must be ordered so that the students are transported. (03marks)
The combination are (x,y) = (2, 2) and (4, 1)
Hence either two taxis and two costa buses or four taxis and one costa bus should be ordered so that all students are transported.

NB: we pick points on the transportation equation which is the equation x + 2y = 6

Thank you
Dr. Bbosa Science