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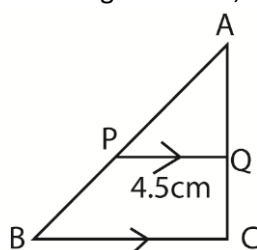


## UCE MATHEMATICS PAPER 1 2014 guide

### SECTION A (40 marks)

Answer all questions in this section

- Solve for x in the inequality  $\frac{x+3}{4} - \frac{x-2}{3} < 2$  (04marks)
- Given that  $2^{2y} = \frac{1}{8}$ , find the value of y. (04marks)
- In the figure below, PQ is parallel to BC. AP: PB = 3:4 and PQ = 4.5cm.



Calculate the length of BC (04marks)

- The average of 8 numbers is 30 while that of a different set of 7 numbers is 15. Find the average of all the numbers. (04marks)
- Simplify  $(4x^2 - 9) \div (2x + 3)$ . (04marks)
- If  $A = \begin{pmatrix} x & -7 \\ 4 & 6y \end{pmatrix}$  and  $B = \begin{pmatrix} 17-y & -21 \\ 12 & 36 \end{pmatrix}$ ; find the value of x and y given that  $3A = B$ . (04marks)
- Factorize the expression  $3x^2 - 10x + 3$   
Hence find values of x when  $3x^2 - 10x + 3 = 0$
- Given that  $\sin A = \frac{3}{5}$  and A is obtuse, find, without using mathematical tables or a calculator, the value of  $\tan A$ . (04marks)
- An object whose area is 4cm<sup>2</sup> is transformed by a matrix  $\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$  to its image P'. Find the area of P'. (04marks)
- A number is chosen from numbers 1 to 9. Find the probability that the number chosen is a triangle number. (04marks)

### SECTION B (60 MARKS)

Answer any **five** questions from this section. All numbers carry equal marks

- The table below shows ages of 60 university students, to the nearest years.

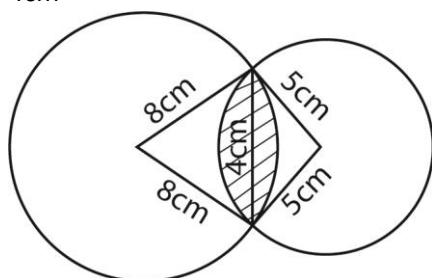
| Ages               | 17 – 19 | 20 – 22 | 23 – 25 | 26 – 28 | 29 – 31 |
|--------------------|---------|---------|---------|---------|---------|
| Number of students | 3       | 7       | 13      | 25      | 12      |

- Calculate the students' mean age (05marks)
- (i) Draw a cumulative frequency curve (Ogive) for the data

- (ii) Use the Ogive to find the median age. (07marks)
12. A group of members had to raise shs. 3,600,000 to buy a plot of land. Each member was to contribute the same amount of money.
- Write down expressions for each member's contribution before and after the 10 members dropped out
  - After the 10 members dropped out, each member had to pay shs. 60,000 more. Find
    - The original number of members in the group
    - How much each member contributed. (09marks)
13. (a) Copy and complete the following table for the curve  $2x^2 - 3x - 5 = 0$  for values of  $x$  from -3 to 4.

| x      | -3 | -2 | -1 | 0  | 1  | 2  | 3  | 4  |
|--------|----|----|----|----|----|----|----|----|
| $2x^2$ |    |    |    |    |    | 8  |    |    |
| $3x$   |    |    |    |    |    | -6 |    |    |
| -5     | -5 | -5 | -5 | -5 | -5 | -5 | -5 | -5 |
| y      |    |    |    |    |    | -3 |    |    |

- Using a scale of 2cm for 1 unit on the x-axis and 2cm for 5 units on the y-axis, draw the graph  $2x^2 - 3x - 5 = 0$  for  $-3 \leq x \leq 4$ . (04 marks)
  - Using your graph, solve the equation  $2x^2 - 3x - 5 = 0$  (04marks)
14. (a) The matrix  $A = \begin{pmatrix} a & 14 \\ 1 & b \end{pmatrix}$  and its inverse  $A^{-1} = \begin{pmatrix} 1 & -7 \\ -\frac{1}{2} & 4 \end{pmatrix}$ .  $AA^{-1} = I$ , where  $I$  is a  $2 \times 2$ . Identify matrix. Find the values of  $a$  and  $b$ . (05marks)
- A housewife buys the following items in three weeks. Week one she buys 2packetsof tea, 2tins of margarine, 2k of sugar and 3packets of biscuits. Week two she buys 1 packet of tea, 2kg of sugar and 2 packets of biscuit.
    - Write this information in a  $3 \times 4$  matrix.
    - A packet of tea costs shs. 1000, a tin of margarine costs shs. Shs. 1,500, a kilogram of sugar costs shs. 2,800 and a packet of biscuit costs shs. 500. Write a column matrix for the cost of items.
    - Find her expenditure in the three weeks
15. A triangle with vertices  $A(-1, 1)$ ,  $B(-1, 3)$  and  $C(-2, 3)$  is mapped onto triangle  $A'B'C'$  by a translation  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ . Triangle  $A'B'C'$  is rotated through  $+90^\circ$  about the point  $(3, 2)$  to triangle  $A''B''C''$ .
- Plot the points  $A$ ,  $B$  and  $C$  on a graph paper. Join the points to form triangle  $ABC$ . (02marks)
  - Find coordinates of
    - $A'$ ,  $B'$  and  $C'$
    - $A''$ ,  $B''$  and  $C''$
  - Find the centre and angle of the rotation which maps  $A''$ ,  $B''$  and  $C''$  back onto  $ABC$ . (04marks)
16. The diagram below show two intersecting circles of radii 5cm and 8cm with a common chord of 4cm



Find the area of the shaded part.

17. An export company is to transport 300 tonnes of pineapple. Two cargo planes are available. A Boeing which carry 30 tonnes per flight an Airbus which can carry 20 tonnes of pineapple per flight. The Airbus has to make more flights than Boeing. The Boeing has to make at least 3 flights. The company has 150,000 US dollars for transport costs. The cost per flight is 12,000 dollars for Boeing and 9,000 dollars for Airbus. If  $x$  is the number of flights made by the Boeing and  $y$  is the number of flights made by the airbus.
- Write down four inequalities satisfying the given conditions. (04marks)
  - Plot graphs of the inequalities you formed on the same axes and shade the unwanted regions. (06 marks)
  - Find the number of flights each plane should make if the cost of transport is to be minimum. (02marks)

## Solutions

### SECTION A (40 marks)

Answer all questions in this section

1. Solve for  $x$  in the inequality  $\frac{x+3}{4} - \frac{x-2}{3} < 2$  (04marks)

$$\frac{x+3}{4} - \frac{x-2}{3} < 2$$

Multiplying through by 12

$$3(x+3) - 4(x-2) < 2 \times 12$$

$$3x + 9 - 4x + 8 < 24$$

$$-x + 17 < 24$$

$$-7 < x$$

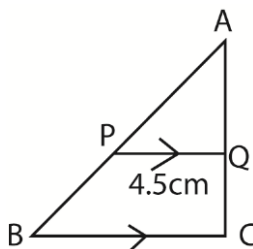
2. Given that  $2^{2y} = \frac{1}{8}$ ; find the value of  $y$ . (04marks)

$$2^{2y} = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

$$2y = -3$$

$$y = \frac{-3}{2} = -1.5$$

3. In the figure below, PQ is parallel to BC. AP: PB = 3:4 and PQ = 4.5cm.



Calculate the length of BC (04marks)

$$\frac{BC}{PQ} = \frac{AB}{AP}$$

$$\frac{BC}{4.5} = \frac{7}{3}$$

$$BC = \frac{7}{3} \times 4.5 = 10.5\text{cm}$$

4. The average of 8 numbers is 30 while that of a different set of 7 numbers is 15. Find the average of all the numbers. (04marks)

$$\text{Average of 15} = \frac{(8 \times 30) + (7 \times 15)}{15} = \frac{240 + 105}{15} = \frac{345}{15} = 23$$

5. Simplify  $(4x^2 - 9) \div (2x + 3)$ . (04marks)

$$(4x^2 - 9) \div (2x + 3) = \frac{(2x-3)(2x+3)}{(2x+3)} = (2x - 3)$$

6. If  $A = \begin{pmatrix} x & -7 \\ 4 & 6y \end{pmatrix}$  and  $B = \begin{pmatrix} 17-y & -21 \\ 12 & 36 \end{pmatrix}$ ; find the value of x and y given that  $3A = B$ . (04marks)

$$3A = 3 \begin{pmatrix} x & -7 \\ 4 & 4y \end{pmatrix} = \begin{pmatrix} 3x & -21 \\ 12 & 18y \end{pmatrix}$$

$$\text{But } 3A = B$$

$$\begin{pmatrix} 3x & -21 \\ 12 & 18y \end{pmatrix} = \begin{pmatrix} 17-y & -21 \\ 12 & 36 \end{pmatrix}$$

$$18y = 36; y = 2$$

$$3x = 17 - 2 = 15$$

$$x = 5$$

7. Factorize the expression  $3x^2 - 10x + 3$

$$3x^2 - 10x + 3 = 3x^2 - x - 9x + 3$$

$$= x(3x-1) - 3(3x-1)$$

$$= (3x-1)(x-3)$$

Hence find values of x when  $3x^2 - 10x + 3 = 0$

$$3x^2 - 10x + 3 = 0$$

$$(3x-1)(x-3) = 0$$

$$3x-1 = 0$$

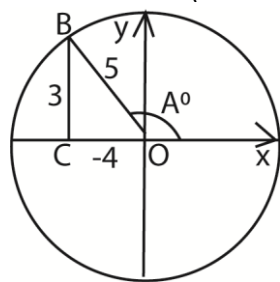
$$x = \frac{1}{3}$$

$$\text{and } x-3 = 0$$

$$x = 3$$

$$\therefore x = \frac{1}{3} \text{ or } x = 3$$

8. Given that  $\sin A = \frac{3}{5}$  and A is obtuse, find, without using mathematical tables or a calculator, the value of  $\tan A$ . (04marks)



$$OC = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$$

$$\tan A = \frac{-3}{4}$$

9. An object whose area is  $4\text{cm}^2$  is transformed by a matrix  $\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$  to its image  $P'$ . Find the area of  $P'$ . (04marks)

$$\text{Det of matrix} = (1 \times 1) - (-2 \times 1) = 3$$

$$\text{Area of } P' = 4 \times 3 = 12\text{cm}^2$$

10. A number is chosen from numbers 1 to 9. Find the probability that the number chosen is a triangle number. (04marks)

Let T = triangle numbers

$$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$T = \{1, 3, 6\}$$

$$P(T) = \frac{n(T)}{n(\varepsilon)} = \frac{3}{9} = \frac{1}{3}$$

### SECTION B (60 MARKS)

Answer any **five** questions from this section. All numbers carry equal marks

11. The table below shows ages of 60 university students, to the nearest years.

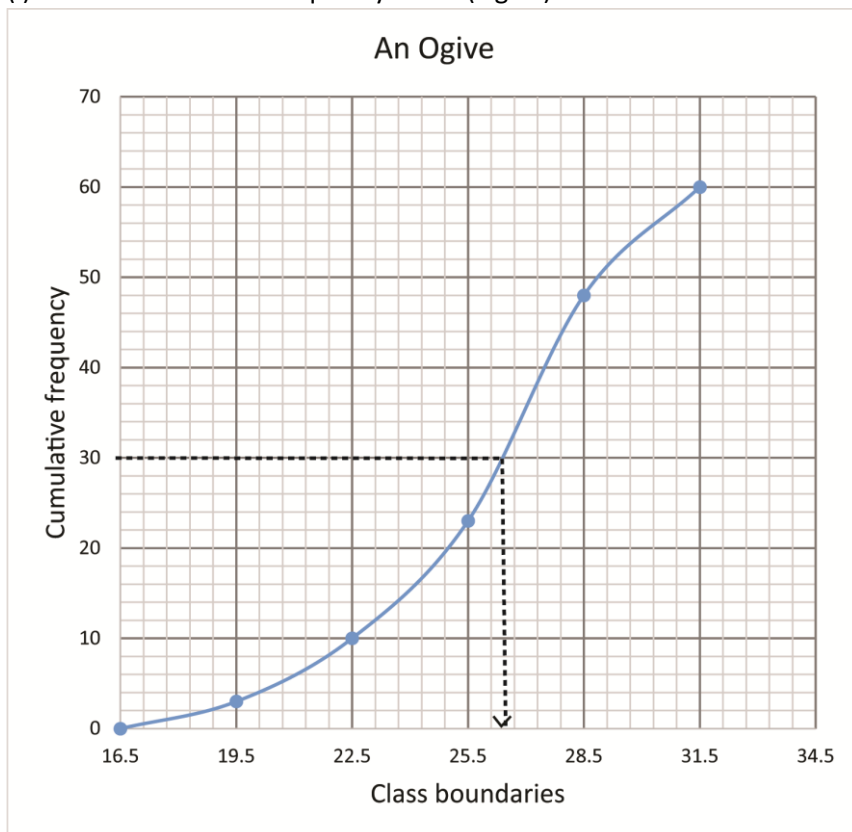
| Ages               | 17 – 19 | 20 – 22 | 23 – 25 | 26 – 28 | 29 – 31 |
|--------------------|---------|---------|---------|---------|---------|
| Number of students | 3       | 7       | 13      | 25      | 12      |

- (a) Calculate the students' mean age (05marks)

| Age(years) | f             | x  | fx               | cf |
|------------|---------------|----|------------------|----|
| 17 – 19    | 3             | 18 | 54               | 3  |
| 20 – 22    | 7             | 21 | 147              | 10 |
| 23 – 25    | 13            | 24 | 312              | 23 |
| 26 – 28    | 25            | 27 | 675              | 48 |
| 29 – 31    | 12            | 30 | 360              | 60 |
|            | $\sum f = 60$ |    | $\sum fx = 1548$ |    |

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{1548}{60} = 25.8$$

- (b) (i) Draw a cumulative frequency curve (Ogive) for the data



- (ii) Use the Ogive to find the median age. (07marks)

The median is the 30<sup>th</sup> value

$$\text{From the graph} = 25.5 + \frac{1.5}{5} \times 3 = 26.4$$

12. A group of members had to raise shs. 3,600,000 to buy a plot of land. Each member was to contribute the same amount of money.

- (a) Write down expressions for each member's contribution before and after the 10 members dropped out

Let the number of members be  $x$

Initial contribution by each member before others drop out =  $\frac{3,600,000}{x}$

After 10 members drop out, the remaining members are  $x - 10$

So each contribute =  $\frac{3,600,000}{x-10}$

- (b) After the 10 members dropped out, each member had to pay shs. 60,000 more. Find

- (iii) The original number of members in the group

$$\frac{3,600,000}{x} + 60,000 = \frac{3,600,000}{x-10}$$

*multiplying LCM =  $x(x - 10)$*

$$3,600,000(x - 10) + 60,000x(x - 10) = 3,600,000x$$

$$3,600,000x - 36,000,000 + 60,000x^2 - 600,000x = 3,600,000x$$

$$60,000x^2 - 600,000x - 36,000,000 = 0$$

$$x^2 - 10x - 600 = 0$$

$$(x - 30)(x + 20) = 0$$

$$\text{Either } x - 30 = 0; x = 30$$

$$x + 20 = 0; x = -20$$

Hence the original number = 30

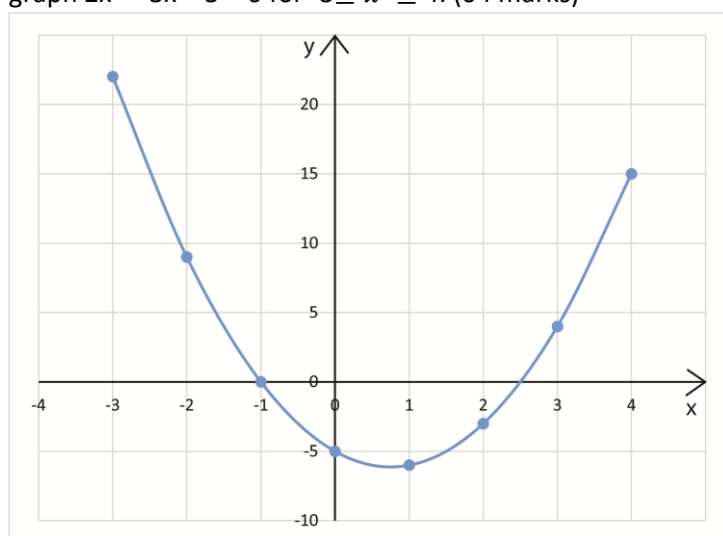
- (iv) How much each member contributed. (09marks)

$$\text{Each member contributed} = \frac{3,600,000}{20} = \text{shs. } 180,000$$

13. (a) Copy and complete the following table for the curve  $2x^2 - 3x - 5 = 0$  for values of  $x$  from -3 to 4.

|        |    |    |    |    |    |    |    |     |
|--------|----|----|----|----|----|----|----|-----|
| $x$    | -3 | -2 | -1 | 0  | 1  | 2  | 3  | 4   |
| $2x^2$ | 18 | 8  | 2  | 0  | 2  | 8  | 18 | 32  |
| $3x$   | 9  | 6  | 3  | 0  | -3 | -6 | -9 | -12 |
| $-5$   | -5 | -5 | -5 | -5 | -5 | -5 | -5 | -5  |
| $y$    | 22 | 9  | 0  | -5 | -6 | -3 | 4  | 15  |

- (a) Using a scale of 2cm for 1 unit on the  $x$ -axis and 2cm for 5 units on the  $y$ -axis, draw the graph  $2x^2 - 3x - 5 = 0$  for  $-3 \leq x \leq 4$ . (04 marks)



- (b) Using your graph, solve the equation  $2x^2 - 3x - 5 = 0$  (04marks)  
The solutions is  $x = -1$  and  $x = 2.5$

14. (a) The matrix  $A = \begin{pmatrix} a & 14 \\ 1 & b \end{pmatrix}$  and its inverse  $A^{-1} = \begin{pmatrix} 1 & -7 \\ -\frac{1}{2} & 4 \end{pmatrix}$ .  $AA^{-1} = I$ , where  $I$  is a  $2 \times 2$ .

Identify matrix. Find the values of  $a$  and  $b$ . (05marks)

$$\begin{pmatrix} a & 14 \\ 1 & b \end{pmatrix} \begin{pmatrix} 1 & -7 \\ -\frac{1}{2} & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding terms

$$a \times 1 + 14 \left(-\frac{1}{2}\right) = 1$$

$$a - 7 = 1$$

$$a = 8$$

$$1 \times 1 + b \left(-\frac{1}{2}\right) = 0$$

$$1 - \frac{1}{2}b = 0$$

$$b = 2$$

(b) A housewife buys the following items in three weeks. Week one she buys 2 packets of tea, 2 tins of margarine, 2kg of sugar and 3 packets of biscuits. Week two she buys 2 tins of margarine, 3kg of sugar and 4 packets of biscuit. And week 3 she buys 1 packet of tea, 2kg of sugar and 2 packets of biscuit.

(i) Write this information in a  $3 \times 4$  matrix.

$$\begin{matrix} \text{week 1} \\ \text{week 2} \\ \text{week 3} \end{matrix} \begin{pmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 3 & 4 \\ 1 & 0 & 2 & 2 \end{pmatrix} \text{ Hence a } 3 \times 4 \text{ matrix is } \begin{pmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 3 & 4 \\ 1 & 0 & 2 & 2 \end{pmatrix}$$

(ii) A packet of tea costs shs. 1000, a tin of margarine costs shs. Shs. 1,500, a kilogram of sugar costs shs. 2,800 and a packet of biscuit costs shs. 500. Write a column matrix for the cost of items.

The cost matrix

$$\begin{matrix} \text{tea} \\ \text{margarine} \\ \text{sugar} \\ \text{biscuit} \end{matrix} \begin{pmatrix} 1,000 \\ 1,500 \\ 2,800 \\ 500 \end{pmatrix} \text{ Hence the cost matrix is } \begin{pmatrix} 1,000 \\ 1,500 \\ 2,800 \\ 500 \end{pmatrix}$$

(iii) Find her expenditure in the three weeks

$$\begin{aligned} \text{Total expenditure} &= \begin{pmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 3 & 4 \\ 1 & 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1,000 \\ 1,500 \\ 2,800 \\ 500 \end{pmatrix} \\ &= \begin{pmatrix} 2 \times 1,000 + 2 \times 1,500 + 2 \times 2,800 + 2 \times 500 \\ 0 \times 1,000 + 2 \times 1,500 + 3 \times 2,800 + 4 \times 500 \\ 1 \times 1,000 + 0 \times 1,500 + 2 \times 2,800 + 2 \times 500 \end{pmatrix} \\ &= \begin{pmatrix} 12,000 \\ 13,400 \\ 7,600 \end{pmatrix} \\ &= 12,000 + 13,000 + 7,600 \\ &= \text{shs. } 33, 100 \end{aligned}$$

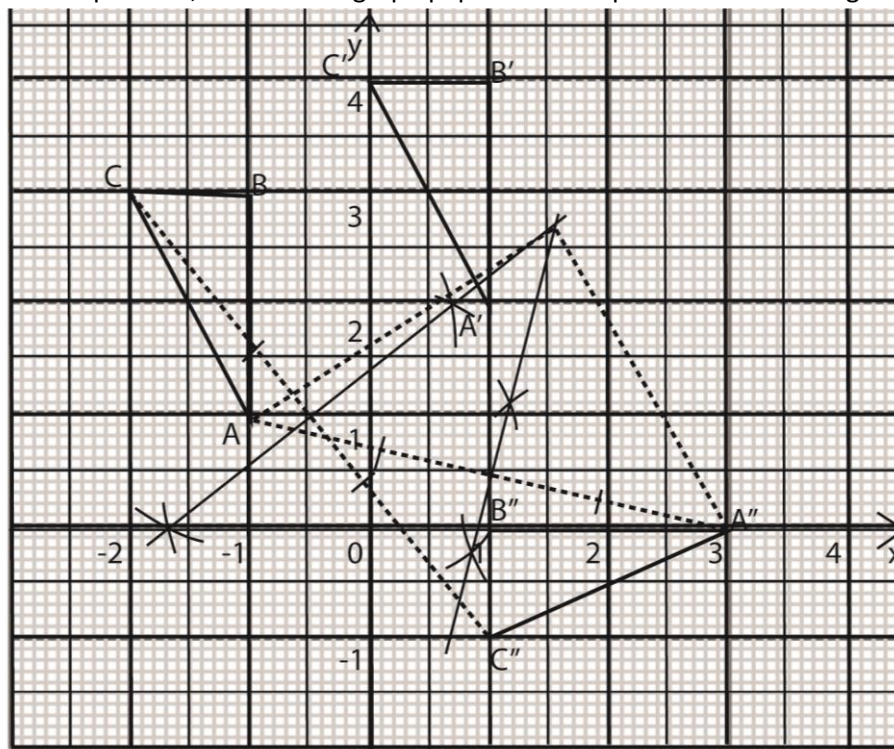
Or

By adding the units bought in each category

$$\begin{aligned} \text{Total expenditure} &= (3 \ 4 \ 7 \ 9) \begin{pmatrix} 1,000 \\ 1,500 \\ 2,800 \\ 500 \end{pmatrix} \\ &= (3 \times 1000 + 4 \times 1,500 + 7 \times 2,800 + 9 \times 500) \\ &= 3000 + 6000 + 19600 + 4500 \\ &= \text{shs. } 33,100 \end{aligned}$$

15. A triangle with vertices A(-1, 1), B(-1, 3) and C(-2, 3) is mapped onto triangle A'B'C' by a translation  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ . Triangle A'B'C' is rotated through +90° about the point (3, 2) to triangle A''B''C''.

(a) Plot the points A, B and C on a graph paper. Join the points to form triangle ABC. (02marks)



(b) Find coordinates of

(iii) A', B' and C'

Joining the points A', B', and C' to form triangle A'B'C'

A'(1, 2), B'(1, 4), C'(0, 4)

(iv) A'', B'' and C''

Joining the points A'', B'' and C'' to form triangle A''B''C''

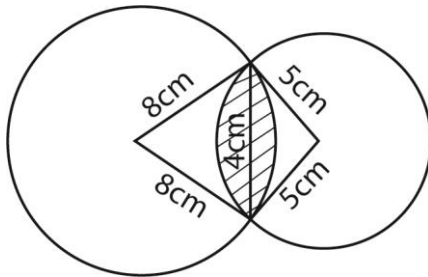
A''(3, 0), B''(1, 0), C''(1, -1)

(c) Find the centre and angle of the rotation which maps A'', B'' and C'' back onto ABC. (04marks)

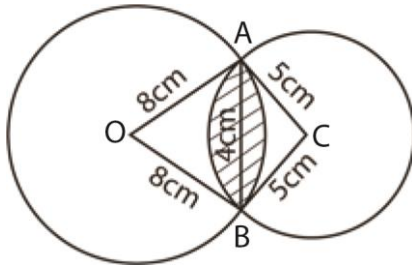
By drawing any two mediators of  $\overline{AA''}$ ,  $\overline{BB''}$  or  $\overline{CC''}$ . Centre of rotation is where the perpendicular bisectors of the mediators meet (1.6, 2.6). the angle of rotation is -90° or 90° clockwise



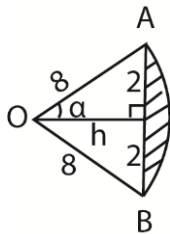
16. The diagram below show two intersecting circles of radii 5cm and 8cm with a common chord of 4cm



Find the area of the shaded part.



Considering sector OAB



Find h and angle  $\alpha$

$$h^2 = 8^2 - 2^2 = 64 - 4 = 60$$

$$h = \sqrt{60}$$

$$\sin \alpha = \frac{2}{8} = 0.25$$

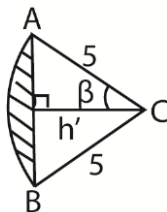
$$\alpha = \sin^{-1} 0.25 = 14.5^\circ$$

$$2\alpha = 29^\circ$$

$$\text{Area of the sector OAB} = \frac{2\alpha}{360} \pi r^2 = \frac{29}{360} \times \frac{22}{7} \times 8 \times 8 = 16.20 \text{ cm}^2$$

$$\text{Area of triangle AOB} = \frac{1}{2} \times 4 \times \sqrt{60} = 15.49 \text{ cm}^2$$

$$\text{Area of the shaded part} = 16.20 - 15.49 = 0.71 \text{ cm}^2$$



Find h and angle  $\beta$

$$h^2 = 5^2 - 2^2 = 25 - 4 = 21$$

$$h = \sqrt{21}$$

$$\sin \alpha = \frac{2}{5} = 0.4$$

$$\beta = \sin^{-1} 0.4 = 23.6^\circ$$

$$2\beta = 47.2^\circ$$

$$\text{Area of the sector ABC} = \frac{2\beta}{360} \pi r^2 = \frac{47.2}{360} \times \frac{22}{7} \times 5 \times 5 = 10.30 \text{ cm}^2$$

$$\text{Area of triangle ABC} = \frac{1}{2} \times 4 \times \sqrt{21} = 9.17 \text{ cm}^2$$

$$\text{Area of the shaded part} = 10.30 - 9.17 = 1.13 \text{ cm}^2$$

$$\text{Total area of shaded part} = 0.71 + 1.13 = 1.83 \text{ cm}^2$$

17. An export company is to transport 300 tonnes of pineapple. Two cargo planes are available. A Boeing which carry 30 tonnes per flight an Airbus which can carry 20 tonnes of pineapple per flight. The Airbus has to make more flights than Boeing. The Boeing has to make at least 3 flights. The company has 150,000 US dollars for transport costs. The cost per flight is 12,000 dollars for Boeing and 9,000 dollars for Airbus. If x is the number of flights made by the Boeing and y is the number of flights made by the airbus.

(a) Write down four inequalities satisfying the given conditions. (04marks)

1 flight of a Boeing = 30tonnes

X flight = 30x

1flight of Airbus = 20 tonnes

y flight = 20y

$$\Rightarrow 30x + 20y \geq 300$$

$$\text{Or } 3x + 2y \geq 30 \dots\dots\dots(i)$$

$$y > x \dots\dots\dots(ii)$$

$$x \geq 3 \dots\dots\dots(iii)$$

1 flight of Boeing = 12,000 US dollars

X flight of Boeing = 12,000x US dollars

1 flight of Airbus = 9,000 US dollars

Y flights of airbus = 9,000 US dollars

$$\Rightarrow 12,000x + 9,000y \leq 15,000$$

On reduction

$$4x + 3y \leq 50 \dots\dots\dots(iv)$$

(b) Plot graphs of the inequalities you formed on the same axes and shade the unwanted regions. (06 marks)

For equation (i) the boundary line is

$$3x + 2y = 30$$

|   |    |   |    |
|---|----|---|----|
| X | 0  | 4 | 10 |
| y | 15 | 9 | 0  |

For equation (ii)

$$y = x$$

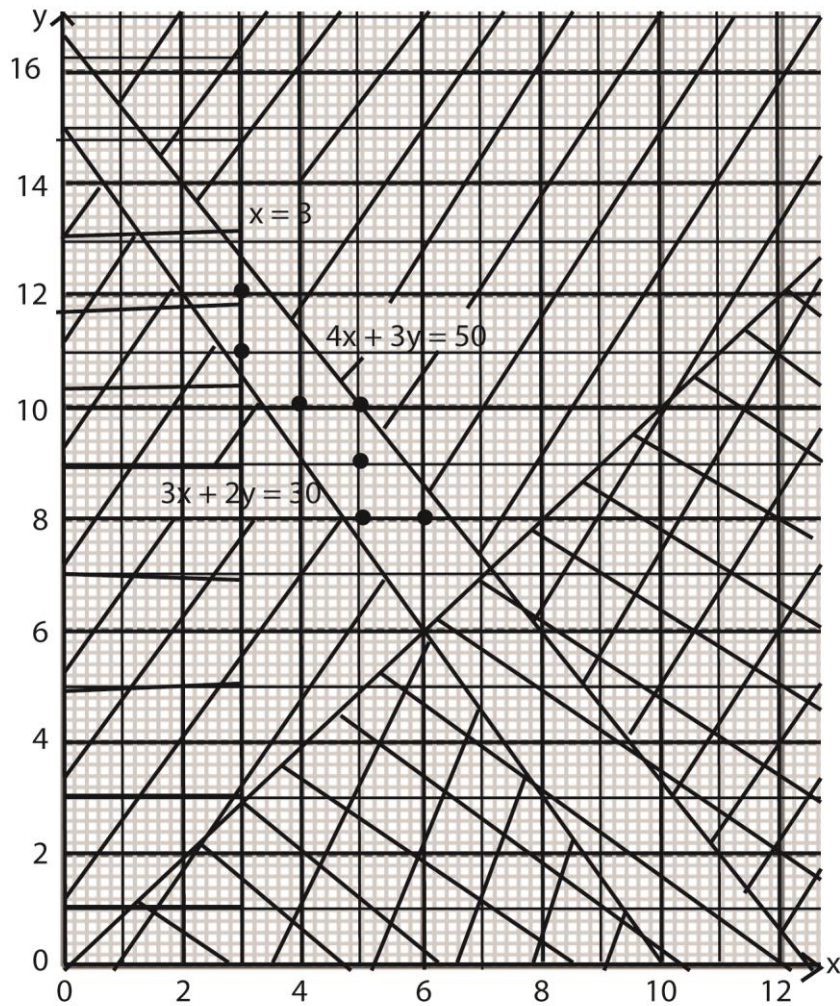
For equation (iii)

$$x = 3$$

For equation (iv)

$$4x + 3y = 50$$

|   |    |    |    |
|---|----|----|----|
| X | 2  | 5  | 11 |
| y | 14 | 10 | 2  |



- (c) Find the number of flights each plane should make if the cost of transport is to be minimum. (02marks)

The possible lattice points in the feasible region are (6, 7), (6, 8), (5, 8), (4, 9), (5, 9), (4, 10), (5, 10), (3, 11), (4, 11), (3, 12). However for minimization of the cost, we consider the lattice points on the line or near to line on the left hand side.

Now considering points (3, 11), (4, 9) (5, 8), (6, 7)

Cost function =  $12,000x + 9,000y$

| Points  | Cost  |
|---------|---|
| (3, 11) | $12,000 \times 3 + 9,000 \times 11 = \$135,000$ |
| (4, 9)  | $12,000 \times 4 + 9,000 \times 9 = \$129,000$  |
| (5, 8)  | $12,000 \times 5 + 9,000 \times 8 = \$ 132,000$ |
| (6, 7)  | $12,000 \times 6 + 9,000 \times 7 = \$ 135,000$ |

$\therefore$  Boeing makes 4 flight and Airbus 9 flights

Thank you

Dr. Bbosa Science