

INTRODUCTION

This is a topical trial questions book designed to tackle major topics in the current syllabus.

The questions are graded in each topic and the main objective is to present the material in a manner for easy comprehension and understanding.

The book consists of Review sections in a number of topics which help a learner to recall several formulars where need be and there exists examples in some topics where need be in order to bring out the meaning clearly.

The main reason for the designing of this book is to provide enough questions for the students' practice in order to perfect in the subject and also provide confidence in the students after being exposed to a number of questions in this book.

It is hoped that the book will be found usefull for students' revision questions at O'level.

Any suggestions for improvement of this book are most welcomed.

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LCM, BASES, HCF

TRIAL QUESTIONS

1. Muvule trees are planted in Busoga on both sides of the road at 30 m apart on one side of the road and 45 m apart on the other side. At one end of the road, two trees are directly opposite each other. After what distance, measured from that end would a pair of trees be directly opposite each other? [Ans: 90 m]
2. (a) Find the L.C.M of 18, 45 and 42
[Ans: 630]
(b) Find the LCM of 42, 48 and 36
[Ans: 1008]
3. Which value of n gives $304_n = 117_{\text{eight}}$
[Ans: $n = 5$]
(b) Given that $202_n = 57_{\text{nine}}$ find the base that n represents
[Ans: $n = 5$]
4. (a) Find the LCM of 9, 10, 12 and 15
[Ans: 180]
(b)Find the L.C.M of $3a^2b^2c$ and $6ab^2c^3$
[Ans: $6ab^2c^3$ s]
5. Given that $72_x = 213_{\text{five}}$ find the value of x .
[Ans: $x = 8$]
6. Given that $3t5_{\text{eight}} = 245_{\text{ten}}$. Find the value of t .
[Ans: $t = 6$]
7. (a) Convert 200_{five} to actal base. [Ans: 50_{ten}]
(b) Find the value of base x in the equation; $305_x + 45_x = 353_x$
[Ans: $x = 7$]
8. Find the LCM of the following numbers:
 - (i) 88, 96, and 126
[Ans: 22176]
 - (ii) 56 and 84
[Ans: 168]
 - (iii) 112, 152, 160, 172
[Ans: 915040]
 - (iv) 144, 72, 60 and 54
[Ans: 2160]
9. The bell for changing lessons for O'level at Luwero high school rings after 40 minutes while that for A'level rings after 60 minutes. Lessons for both 'O' and 'A' level begin at 7:20 am with the sounding of the bells. Find the time when the two bells ring together again.
[Ans: 9:20 am]

10. Three bells ring at intervals of 20, 30, 40 seconds. If they begin by ringing together, after what length of time will they ring together again?
[Ans: 120 seconds]
11. Makanga, Zahara and Jjinggo are running a 10,000 metres race. Makanga completes his first lap after 120 seconds, Zahara completes her first lap after 130 seconds while Jjinggo completes his after 142 seconds. When will they next be all at the starting point together?
[Ans: 110760 seconds]
12. A family of Mr & Mrs.Kirya and their daughter was promised some money from a their family friend. Find the smallest sum of money which can be given to a family so that a child, mother and father can get shs 7500, shs 9000 and shs12000 respectively. [Ans: 180,000]
13. A certain number of oranges can be divided into equal trips each containing 10, 15 or 25 oranges. Find the smallest number of oranges for which this is possible. [Ans: 150]
14. Square tiles are used to cover an area measuring 16.5 m by 12.75 m. If the tiles are all alike and only whole ones are used, what is the greatest size they can be? How many are there?
[Ans: 0.5625 m², 374 litres]
15. (a) Find the H.C.F of 18, 45 and 42
[Ans: 3]
(b) Find the HCF of 42,48 and 36
[Ans: 6]
16. Equal squares, as large as possible, are ruled off on a rectangular board 54 by 78 cm.
 - (i) Find the size of each square.
[Ans: 6 × 6 cm or 36 cm²]
 - (ii) How many squares are there? [Ans: 117]
17. If the product of two numbers is 1260 and their HCF is 6, find their LCM [Ans: 210]
18. Find the length of the shortest piece of string that can be cut into equal lengths, each of either 28 cm, 35 cm or 42 cm.

19. A rectangular piece of room measuring 10 m long by 8 m wide is to be covered with square tiles of side $x\text{ cm}$ each.

- (a) Find the value of x [Ans: $x = 2\text{ m}$]
- (b) Find how many tiles are needed if only whole ones are used? [Ans: 20 tiles]

20. Find the HCF of;

- (i) 136, 142, and 156
- (ii) 112, 144 and 210
- (iii) 72, 108, 180 and 28

21. The sitting room of a house measures 1440 cm by 1640 cm . It's floor is to be tiled using square tiles which are as large as possible.

- (a) Find the size of these square tiles.
[Ans: 40×40 or 1600 cm^2]
- (b) How many square tiles are required for this purpose?. [Ans: 1476 tiles]

22. Three pieces of timber of length 164 m , 286 m and 358 are to be cut into smaller pieces of the same length. Find the greatest possible length which can be cut without leaving any remainder.
[Ans: 2 cm]

23. (a) Find the HCF of a^3b^4c , $a^2b^3c^2$ and $a^4b^2c^3$.
[Ans: a^2b^2c]

(b) The L.C.M of 24, 36 and 60 is x and their G.C.D is y , find the ratio $y : x$. [Ans: 1:60]

24. Evaluate;

- a) $110_{\text{two}} + 101_{\text{two}}$ (Give your answer in base 2)
- b) $111_{\text{two}} - 100_{\text{two}}$ (Give your answer in base 4)
- c) $123_{\text{four}} \times 103_{\text{four}}$ (Give your answer in base 6)
- d) $143_{\text{five}} + 104_{\text{five}}$ (give your answer in base 10)

25. Solve for n in the equation $11_n + 21_{3n} = 20_{4n}$.
[Ans: $n = 2$]

26. Given that $463x$ is divisible by 3, Find the possible values of x . [Ans: 2, 5 and 8]

27. Find the next term in each of the following sequences;

- i) 1, 6, 13, 22, 33, _____, _____, _____
- ii) 49, 31, 17, 7, 1, _____, _____, _____

28. Solve for n in the following equations;

- i) $213_n = 39_{\text{ten}}$ [Ans: $n = 4$]
- ii) $12_{\text{five}} = 21_n$ [Ans: $n = 3$]

iii) $23_{\text{seven}} = 32_n$ [Ans: $n = 5$]

iv) $154_n = 1000110_{\text{two}}$ [Ans: $n = 6$]

29. Convert the following to decimal

- i) 431.24_{five} [Ans: 116.56]
- ii) 1.6_{eight} [Ans: 1.75]
- iii) 110.01_{two} [Ans: 6.25]
- iv) 230.12_{four} [Ans: 44.375]
- v) 3.52_{six}

30. Five small containers of capacity 16, 72, 12, 24 and 56 litres are to be used to fill a bigger container. What is the capacity of the bigger container which can be filled by each of the above containers exactly without remainder when used separately? [Ans: 8,064 litres]

31. Musa, John and David start at the same time, position and direction to run round a circular field. Musa takes 180 seconds, John takes 480 seconds and David takes 720 seconds to complete one circuit. If they start running at 3:00 pm, at what time will they all be at the same position?
[Ans: 3:24 pm]

32. Find the value of n in each of the following if;

- i) $79_{13} = 144_n$ [Ans: $n = 8$]
- ii) $124_n = 52_{\text{ten}}$ [Ans: $n = 6$]

33. Express $\frac{108}{28}$ as a product of it's prime factors.
[Ans: $3^3 \times 7^{-1}$]

34. A room measures 540cm by 420cm. Find the length of the largest square tiles that can be used to cover the floor without requiring any cutting.
[Ans: 60 cm]

35. Traffic lights at three different junctions show green light at intervals of 10 seconds, 12 seconds and 15 seconds. They all show green at 1:00 pm. At what time will they all again show green together? [Ans: 1:01 pm]

36. In a large school, it is possible to divide the pupils into groups of equal numbers of 24, 30 or 32 and have no pupils left over. Find the least number of pupils in the school that makes this possible. [Ans: 960 pupils]

37. Find the shortest length that can be cut into exactly equal length of 4 cm or 7 cm or 18 cm.
[Ans: 252 cm]

FRACTIONS AND OPERATIONS

RECALL

- ◊ A fraction is referred to as part of a whole quantity. It is represented as $\frac{\text{numerator}}{\text{denominator}}$.
- ◊ **Proper fraction** is a fraction whose numerator is less than the denominator.
- ◊ **Improper fraction** is a fraction whose numerator is greater than the denominator.
- ◊ **Mixed fraction** is a fraction containing a whole number and a proper fraction.
- ◊ **Operations on fractions;** This involves addition, subtraction, division and multiplication. But with addition and subtraction, if the denominators are different, we calculate their **LCM** and then proceed normally. On the other hand, if the denominators are the same, we add the numerators directly.
- ◊ When dividing two fractions, we find the reciprocal of the second fraction and then multiply them.
- ◊ When multiplying two or more fractions, we multiply the numerators together and multiply the denominators also together.
- ◊ Compound operations on fraction; here we consider cases of simplifying fractions in which brackets and other arithmetic operations are involved. This process is governed by law called **BODMAS**
- ◊ Converting recurring decimals to fractions eg $0.66666666\ldots\ldots$ to fractions

Operations

- ◊ Operations as a topic is represented by several symbols connecting an equation. The operations may include; \downarrow , \uparrow , $*$, $@$, Δ , ∇ , \oslash , ϕ , etc. for example; If $a*b = a^2 - b^2$. Find $3*2$. This implies that 3 stands for a and 2 stand for b . Thereafter substitute the values into the equation;

$$3*2 = 3^2 - 2^2$$

$$9 - 4 = 5$$

Therefore, $3*2 = 5$

TRIAL QUESTIONS

1. Convert $5.2727\ldots\ldots$ to fractions.
[Ans: $\frac{58}{11}$]
2. Simplify $\frac{2}{3} + \frac{1}{5}$ of $\frac{1}{2} \div 1.25$ [Ans: $\frac{56}{75}$]
3. Solve for a in $\frac{a+8}{2} - \frac{3(a+10)}{5} = 1$
[Ans: $a = -30$]
4. A bottle contained $6\frac{2}{5}$ litres of milk. Berochan took $\frac{3}{8}$ of the milk and Oola took $\frac{3}{4}$ of the remainder. How much milk remained in the bottle. [Ans: 1 litre]
5. Express $0.245\ldots\ldots$ in the simplified form $\frac{a}{b}$.
Hence find $b-a$ [Ans: $\frac{245}{999}$, $b-a=754$]
6. Given that $X * Y = X^2 + Y$. Find $4*3$
[Ans: 19]
7. Express 3.36 as an improper fraction in its simplest form. [Ans: $\frac{84}{25}$]
8. If $a * b = \frac{a}{b} + \frac{b}{a}$. evaluate $\frac{1}{2} * \frac{3}{2}$ [Ans: $\frac{10}{3}$]
9. Work out $(4\frac{1}{2} - 3\frac{2}{3}) \div 1\frac{2}{3}$ [Ans: $\frac{1}{2}$]
10. Simplify $5\frac{1}{6} - 3\frac{2}{3} + 6\frac{7}{12}$ [Ans: $\frac{61}{12}$]
11. Write $0.4545\ldots$ in the form $\frac{a}{b}$ in its simplest form, hence find the value of $a-b$
[Ans: $\frac{5}{11}$, $a-b=-6$]
12. (a) Simplify : $\frac{9}{16} \times 2\frac{2}{3} - \frac{9}{6} \div \frac{1}{8}$ [Ans: $\frac{-21}{2}$]
(b) Evaluate $2\frac{10}{11} \times 4\frac{1}{6} \div 1\frac{1}{5}$ [Ans: $10\frac{10}{99}$]
(c) Given that $m * n = 2n - 3m$ find the value of $(2 * -1) + (-3 * 2)$ [Ans: 5]
13. Simplify without using a calculator.
$$\frac{-3 - (-12) \times 4 - (-20)}{-6 \times 6 \div 3 + (-6)}$$
 [Ans: $\frac{-65}{18}$]
14. Solve for x if, $\frac{x-2}{3} = \frac{x-3}{3} + \frac{x+3}{8}$.
[Ans: $x = \frac{1}{3}$]
15. Simplify; $\frac{1\frac{1}{2} - [8\frac{1}{3} \div 2\frac{1}{2}]}{1\frac{1}{5} \text{ of } [1\frac{1}{4} + 1\frac{2}{3}]}$ [Ans: $\frac{11}{21}$]

Fractions and Operations

16. Given that $x * y = \frac{x+y}{x-y}$, find the value of $(5 * 3) * -2$. [Ans: $\frac{1}{3}$]

17. (a) Simplify; $\frac{\frac{4}{5} \text{ of } \frac{25}{18} \div 1\frac{2}{3} \times 24}{2\frac{1}{3} - \frac{1}{4} \text{ of } 12 \div \frac{5}{3}}$ [Ans: $\frac{135}{2}$ or $67\frac{1}{2}$]

(b) Solve for x in the equation; $\frac{(x+3)}{4} - \frac{3(4-2x)}{4} = 3\frac{7}{10}$ [Ans: $x = 3.4$]

18. (a) Express $0.83333\dots$ as a fraction in its simplest form. [Ans: $\frac{5}{6}$]

(b) Given that $x * y = \frac{x^2 + y^2}{10y}$, evaluate $7 * (4 * -8)$. [Ans: -5]

(c) Evaluate; $6\frac{1}{4} \div \frac{8}{13} \times \frac{8}{13} \text{ of } \frac{26}{52}$ [Ans: $\frac{25}{8}$]

(d) Express the recurring decimal as a fraction $0.1515\dots$ [Ans: $\frac{5}{33}$]

19. (a) Given that $a * b = a^2 - b^2$, find the value of x in $x * \sqrt{3} = 7 * 4$. [Ans: $x = 6$]

(b) Evaluate; $\frac{\frac{2}{4}^3 - 1\frac{7}{8}}{\frac{2}{4}^3 + 1\frac{5}{8}}$ [Ans: $\frac{1}{5}$]

(c) Evaluate; $\frac{7\frac{1}{8} \div 1\frac{2}{3}}{\frac{1}{4} + 9\frac{1}{2}}$ [Ans: $\frac{57}{130}$]

(d) Simplify; $\frac{3 - \frac{1}{2} \text{ of } \frac{1}{3}}{7\frac{1}{3} \div 3\frac{2}{3}}$ [Ans: $\frac{17}{12}$]

20. (a) Solve the equation; $\frac{3x-1}{5} - \frac{9-2x}{3} = 1$ [Ans: $x = \frac{63}{19}$]

(b) Evaluate; $3 * (1 * 4)$ if $a * b = a^2 + \sqrt{ab}$. [Ans: 12]

(c) Convert $0.891891891\dots$ into a fraction in its simplest form. [Ans: $\frac{33}{37}$]

21. (a) Evaluate; $\left(\frac{\frac{1}{2} + 3\frac{1}{6}}{4\frac{1}{3} - 3\frac{2}{5}} \right) \div 1\frac{2}{3}$ [Ans: 3]

(b) If $P * R = \frac{P^2 - R^2}{PR}$, find $(4 * 2) * 1.5$ [Ans: 0]

(c) The operation $a \wedge b$ is defined as $\frac{a(8+b-3a)}{a+b}$.

Find; (i) $2 \wedge 1$ [Ans: 2]

(ii) $3 \wedge (2 \wedge 1)$ [Ans: $\frac{3}{5}$]

d) Evaluate; $\frac{\frac{3}{4} + 1\frac{5}{7} \div 1\frac{4}{7} \text{ of } 2\frac{1}{3}}{\left[1\frac{3}{7} - \frac{5}{8} \right] \times \frac{2}{3}}$ [Ans: $2\frac{3}{11}$]

22. (a) Solve for t in the equation; $\frac{t}{t-9} = 13 + \frac{t}{t-9}$. [Ans: $t = 9$]

(b) Given that $a * b = \frac{a^2 + b}{10b}$, find

i) $4 * -8$ [Ans: -0.1]

ii) $7 * (4 * -8)$ [Ans: -48.9]

23. (a) Express $0.277777\dots$ as a fraction in its simplest form. [Ans: $\frac{5}{18}$]

(b) Evaluate; $\frac{3\frac{1}{2} - 1\frac{5}{6} \times \frac{3}{11}}{1\frac{3}{4} + 7\frac{2}{3} \div 3\frac{5}{6}}$ [Ans: $\frac{4}{5}$]

(c) Solve the equation; $\frac{(x+1)}{3} - \frac{x}{6} = \frac{(x-2)}{4}$ [Ans: $x = 10$]

24. (a) Evaluate; $2\frac{1}{2} \div \frac{4\frac{1}{3} - 2\frac{1}{4}}{4\frac{1}{6}}$ [Ans: 5]

(b) Evaluate; $\frac{1\frac{1}{6} - 3\frac{1}{2}}{4\frac{1}{3} - 1} \div \frac{1}{5}$ [Ans: $-\frac{7}{2}$]

25. (a) Evaluate; $\frac{1\frac{1}{5} + 4\frac{1}{2} \div 1\frac{1}{2}}{3\frac{3}{5} - 2\frac{2}{5} \times 1\frac{1}{4}}$ [Ans: 7]

(b) If $a \Delta b$ means $2a + 3b$ and $a * b$ means $5a + 4b$ evaluate the expression $2\Delta(4 * 3)$. [Ans: 100]

(c) A ball is dropped from a height of 81cm. After each bounce it rebound two-thirds of the distance it fell. How far does the ball fall in the fifth fall? [Ans: 270 cm]

(d) Given that, $x \Delta y = x^2 - 6y^2$, evaluate $(8\Delta - 3)\Delta 4$. [Ans: 4]

26. (a) The operations * and ^ are defined as $m * n = am + n$ and $m^{\wedge}n = m^2 - n$.

Find the value of a if $-4^{\wedge}(2 * 3) = 9$ [Ans: $a = 2$]

(b) Simplify $\frac{\left(\frac{1}{2}\right) \div \left(\frac{2}{3}\right) \times 8 - 4\frac{1}{2}}{\frac{3}{4} - \left(2\frac{3}{4}\right) \div \frac{11}{8}}$ [Ans: $-\frac{6}{5}$]

(c) Simplify: $\frac{5\frac{1}{4} \div 4\frac{1}{5} + 3\frac{1}{4}}{2\frac{1}{2}}$ [Ans: $\frac{15}{4}$]

(d) Given that $a \Psi b = \frac{b^2 - a^2}{a^2 + b^2}$, find the values of

(i) $1 \Psi^{-1}$ [Ans: 0]

(ii) $(1 \Psi^{-1}) \Psi^{-4}$ [Ans: 1]

(e) Without using a calculator, simplify $\frac{14}{3a}$ of $1\frac{1}{6}$
 $\div \frac{4}{9a^2} \times 1\frac{4}{5}$ [Ans: $14.7a$]

(f) If $p * q = p^2 - 3q$ find

(i) $5 * 3$ [Ans: 16]

(ii) x such that $[(x * 2) * 3] = 7$
[Ans: $x = \sqrt{10}$ or $x = \sqrt{2}$]

27. A car consumes one litre of petrol for every $\frac{2}{5}$ km of distance covered. How many litres of fuel are needed to cover a distance of 55 km by the same car? [Ans: 137.5 litres]

28. A boy ate one fifth of a pineapple and the sister ate three quarters of what was left.

(a) What portion of the pineapple was eaten all together?. [Ans: $\frac{4}{5}$]

(b) What fraction of the pineapple was left?
[Ans: $\frac{1}{5}$]

29. Namulondo spent $\frac{2}{5}$ of her money on sugar, $\frac{3}{8}$ on pens and $\frac{1}{5}$ on pencils. She has shs3000 left in her pocket. How much money did she have before she bought the items? [Ans: shs 120,000]

30. One day Jesca decided to travel from Kampala to Tororo. He covered $\frac{3}{5}$ of the distance on foot and the remaining distance by taxi. He was left with 30 km to complete the journey. Find the total distance between Kampala and Tororo.
[Ans: 75 km]

31. In a refrigerator, $\frac{8}{5}$ of the items stored are drinks. Items are categorized into; drinks and eats. If all the eats as well as $\frac{3}{4}$ of the drinks are bad, what fraction of the items are good? [Ans: $\frac{2}{5}$].

32. (a) If $\frac{7}{15}$ of a number is 210, What is the number? [Ans: 450]

(b) After spending $\frac{3}{8}$ of his salary a man has shs. 2000 left. Find the man's salary. [Ans: 3200]

(c) Jessica ate $\frac{2}{5}$ of a loaf of bread. The following morning at breakfast she ate $\frac{4}{7}$ of the remaining bread. What fraction of the original loaf was left after breakfast? [Ans: $\frac{9}{35}$]

33. There are 420 people in a village, $\frac{3}{5}$ of whom are children. Of the children $\frac{1}{7}$ can read and write

while $\frac{1}{28}$ of the adults cannot read or write. How many people in the village are literate?

[Ans: 198 people]

34. If 20% of a number is 740, Find the number.

[Ans: 3700]

35. A number is increased by 80%. The new number is then increased by 60% giving a final result of 144. Find the original number. [Ans: 50]

36. What is 10% of 60% of 500? [Ans: 30]

37. In a certain year the number of books in a public library was increased by 5%. The following year the new number of books was increased by 4%. What was the overall percentage increase in the two - year period?

38. In a mixed primary school 2.5% of the pupils are prefects. Of the prefects 56% are boys. What percentage of the school are the girls prefects?
[Ans: 1.1%]

39. A sum of shs 40,000 was slashed by 99%. The new amount was then reduced by 30%. Find,

i) the final amount [Ans: 280]

ii) the overall percentage reduction.

40. 60% of the passengers on a bus are females and $16\frac{2}{3}\%$ of the females are students. Also $12\frac{1}{2}\%$ of the males are students. If there are four male students on the bus, find the number of;

i) passengers [Ans: 80 passengers]

ii) students [Ans: 12 students]

41. Kaddu, Mwanje and Nalubega got $\frac{5}{6}$, $\frac{7}{12}$, $\frac{9}{10}$ respectively of the maximum mark possible in an examination.

a) Determine who of the three candidates got the highest mark in the exam.

b) If Kaddu scored 50 marks out of the maximum possible mark, determine;

i) the maximum mark that can be scored by a candidate. [Ans: 60]

ii) the mark scored by the remaining.

[Ans: Mwanje scored 35 and Nalubega scored 54]

42. Mr. Wasswa has 4 sons; John, Peter, David and Moses. On a certain day, Mr. Wasswa was paid his gratuity which distributed to his sons as follows. John received $\frac{1}{4}$ of the whole gratuity,

Peter was given $\frac{1}{2}$ of what was left after John had received his share. David was given $\frac{1}{4}$ of what had been left after both John and Peter had received their shares. The remainder after John, Peter and David had received their shares was given to Moses. If Moses received 225,000/=, determine how much money;

i) Mr. Wasswa had received as his gratuity.

[Ans: 800,000/=]

ii) John, Peter, and David received.

[Ans: John = 200,000/= , Peter = 300,000/= , David = 75,000/=]

43. At the end of the first week of a term $\frac{2}{3}$ of the students had paid fees. If the following week one hundred more students paid fees, the fraction of paid up students rose from $\frac{2}{3}$ to $\frac{3}{4}$, determine the total number of students in the school.

[Ans: Number of students is 1200]

44. When $\frac{6}{7}$ of a sack of sugar has been sold, 36kg remain. Determine the quantity of sugar that was

i) originally available in the sack [Ans: 252 kg]

ii) sold [Ans: 216 kg]

45. A poultry farmer used $\frac{1}{3}$ of his birds for a party. After selling a quarter of what was left after the party, he was left with 200 birds. Determine the number of birds the farmer originally had.
[Ans: 400 birds]

46. Three quarters of a number is the same as a quarter of the number plus 100. What is the number? **[Ans: 200]**

47. Two - thirds of Wasswa's books in number exceeds three - quarters Makubuya's books by 50. If Wasswa had 300 books, how many books has Makubuya? **[Ans: 200]**

48. Madam Prossy used $\frac{3}{8}$ of her retirement package to pay school fees. She then used a fifth of the remaining to pay worker's salaries and was left with shs. 300,000. Determine the total value of Madam Prossy's retirement package.

[Ans: shs. 600,000]

49. At a certain school, $\frac{4}{9}$ of the pupils are girls. Given that the number of boys at this school exceeds the number of girls by 200, Determine the;

i) total number of pupils at the school. **[Ans: 1800]**

ii) number of girls at the school. **[Ans 800]**

THE SET THEORY

RECALL

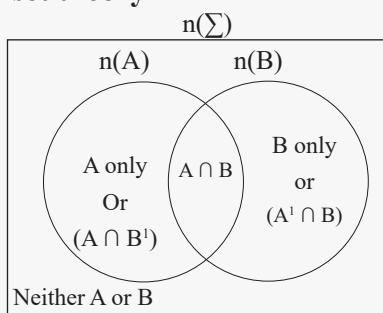
Set notation

- ◊ These provide a description of symbols used in set language.
- ◊ The language in set should be mastered and interpreted well.

The set theory may be categorised into two;

- (a) Two set theory. (Two sets are involved)
- (b) Three set theory (Three sets are involved)

Two set theory



$\Sigma \rightarrow$ Universal set (includes members of the union set and those outside it.
 $'\cap'$ → Intersection of sets. (This is a set consisting of all common members in the given sets.)

$n(A) \rightarrow$ number of elements in set A

$n(B) \rightarrow$ Number of elements in set B

$A \cup B \rightarrow$ all elements which belong to either A or B or both A and B without repetition.

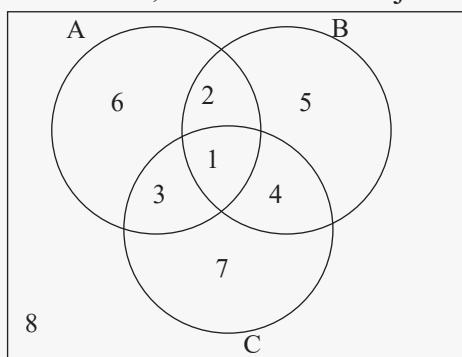
$$A \cup B = A \cap B + A \cap B^1 + A^1 \cap B \quad A \cup B = A \text{ only} + B \text{ only} + A \cap B$$

$A \cap B$ means elements in A but are also in B

(b) Three set theory

This involves large number of elements and it involves several methods of solving unknowns where need be.

Consider A, B and C to be subjects attended by students.



The regions 1, 2, 3 8 are given by the following sets.

(i) $A \cap B \cap C = 1$ (intersection of all sets)

(ii) $A \cap B \cap C^1 = 2$ ($A \cap B$) only

(iii) $A \cap B^1 \cap C$ or $(A \cap C)$ only = 3

(iv) $A^1 \cap B \cap C$ ($B \cap C$) only = 4

(v) $A^1 \cap B \cap C^1$ (B) only = 5

(vi) $A \cap B^1 \cap C^1$ (A) only = 6

(vii) $A^1 \cap B^1 \cap C$ (C only) = 7

(viii) $(A \cup B \cup C)^1 = 8$

(ix) $A \text{ and } B \text{ or } A \cap B = (A \cap B) \text{ only} + (A \cap B \cap C)$
= 2 + 1 = 3

(x) $A \text{ and } C \text{ or } A \cap C = (A \cap C) \text{ only} + (A \cap B \cap C)$
= 3 + 1 = 4

(xi) $B \text{ and } C \text{ or } B \cap C = (B \cap C) \text{ only} + (A \cap B \cap C)$
= 2 + 1 = 3

Set Theory

Note; atmost two subjects from the language , we consider 2 and below including these who do not offer any subject. $2 + 3 + 4 + 6 + 5 + 7 + 8 = 35$ (atmost two subjects but $2 + 3 + 4 + 1 = 10$ (students who offer atleast two subjects). For only one subject, consider 6, 7 and 5 and add all of them together.

N.B: Consider the language used in the sets and solve the challenges under this topic.

LANGUAGE

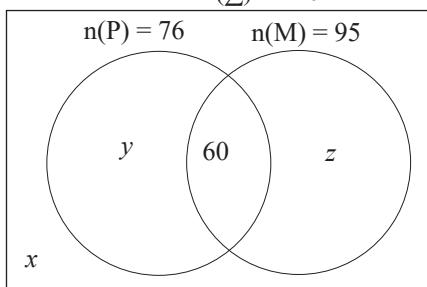
- ◊ “OR” → means Union, ie $A \cup B$
- ◊ “And” → means intersection of sets i.e A and B means $A \cap B$.
- ◊ “Neither - nor” - neither A nor B means members who do not belong to the union of the sets in context.
- ◊ Compliment of a set: Is a set consisting of all elements in the universal set that are not in the given set. e.g A^1 , means members who do not belong to A . But it should be noted that members of A^1 also include members outside the union set.
- ◊ Atmost: Means that number and below with zero inclusive (members outside the union inclusive)
- ◊ Atleast: means that number and above with the intersection inclusive.
- ◊ More than: Above that member in context. i.e Find the number of students doing more than one subject. Therefore look for students who do two and above subjects.

TRIAL QUESTIONS

1. In S4, there 30students, 18 are doing math(M), 15 are doing physics(P) and 13 chemistry(C). The number of students who are doing all the three subjects equals the number of those students who are not doing any of these subjects. Ten Students do M and C, 8 do M and P and 3 do only C and P. Determine;
 - (i) the number of students doing all the three subjects [Ans: 5]
 - (ii) the number of students who are doing only one subject [Ans: 9]
 - (iii) the probability that a student selected at random from the class does at least two of these subjects.
[Ans $\frac{8}{15}$]

2. The venn diagram shows girls who offer physics (P) and Maths (M).

$$n(\sum) = 120$$



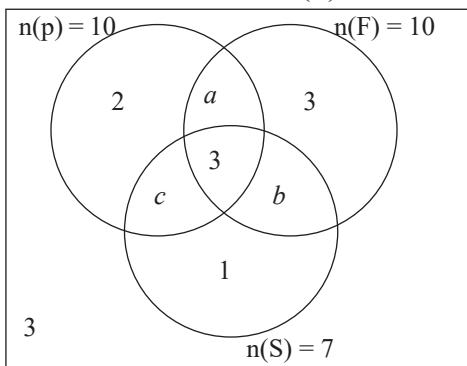
Find the;

- (i) Value of x, y and z . [Ans: $y = 16, z = 35, x = 9$]
- (ii) $n(P \cup M)$ [Ans: $n(P \cup M) = 111$]

3. In a group of 16 peoples, 7 ate Matooke (M), 8 ate Rice (R) and 3 ate neither Matooke nor Rice. Represent the information on a Venn diagram and hence find $n(M \cap R)$ [Ans: $n(M \cap R) = 2$ people]
4. Given that $M = \{ \text{All prime numbers less than } 20 \}$ and $N = \{ \text{All triangle numbers less than } 20 \}$, find $n(M \cap N)$ [Ans: $n(M \cap N) = 1$]
5. If set $T = \{ \text{All triangle numbers less than } 20 \}$, and $F = \{ \text{All factors of } 12 \}$ Find the members of $F \cap T$. Hence find $n(F \cap T)$. [Ans: $F \cap T = \{1, 3, 6\}, n(F \cap T) = 3$ members]
6. Given that { all odd numbers less than 20} and { all multiples of three less than 20}, find $n(D \cap M)$ [Ans: 3 members]
7. Given that $F = \{ \text{All factors of } 24 \}$ and $G = \{ \text{all factors of } 30 \}$. Find $n(F^1 \cap G)$ where F^1 is the compliment of F. [Ans: 4 members]
8. In a class of 80 pupils, 45 like English(E) and 50 like Maths(M).
 - (i) Represent this information on a venn diagram
 - (ii) Find the number of pupils who like both E and M. [Ans: 15 pupils]

9. Given that $n(A) = 7$, $n(B) = 9$, $n(A \cup B) = 2$ and $n(\sum) = 15$. Use a venn diagram to find;
- $n(A \cap B)$ [Ans: 3]
 - $n(A \cap B^1)$ [Ans: 4]
10. In a class of 70 girls, 30 like Math(M), 40 like Physics(P) and 20 like English(E). If 10 like both Math and Physics, 8 like English and Physics, 9 like both Math and English and 4 don't like any of the three subjects.
- Represent the information on a venn diagram
 - Use the venn diagram to find the number of girls that;
- like all the three subjects [Ans: 3 girls]
 - don't like Math [Ans: 40 girls]
- (c) If a student is picked at random, what is the probability that she likes only one subject. [Ans: $\frac{9}{14}$]

11. The venn diagram below shows the representation of members of a community council to three different committees of Finance (F), Production(P) and Security(S).



- Determine the values of a, b and c.
- Find the;

 - total number of members that make up the community council
 - number of members who belong to at least two committees.

- Given that a member is chosen at random from the council, what is the probability that the member belongs to;

 - only one committee?
 - atmost two committees?

[Ans: (a) $a = 3, b = 1, c = 2$ (b)(i) 18, (ii) 9, (c)(i) $\frac{1}{3}$, (ii) $\frac{5}{6}$]

12. In S.4 there are 50 students who offer three subjects namely Luganda(L), Computer (C) and Agriculture(A). 24 offer Agriculture, 29 offer computer and the number offering Luganda is 3 less than the number offering Computer. 9 offer both Agriculture and Computer, 13 offer both Luganda and Computer, 11 offer Agriculture and Luganda. Each of these students offer at least one of the three subjects.

- (a) Represent the above information on a Venn diagram

- (b) Find the number of students that offer

- all the three subjects [Ans: 4 students]
- only one subject [Ans: 25 students]

- (c) Find the probability that a student selected at random offers

- Luganda only [Ans: $\frac{3}{25}$]
- two of the subjects [Ans: $\frac{21}{50}$]

13. In a certain school there are 50 students who play three games, namely; Chess, Tennis and Volleyball. 24 play Chess, 26 play Tennis and 29 play Volleyball, 9 play both Chess and Volleyball, while 13 play both Tennis and Volleyball, 11 play both Chess and Tennis. Each of these students plays atleast one of the three games.

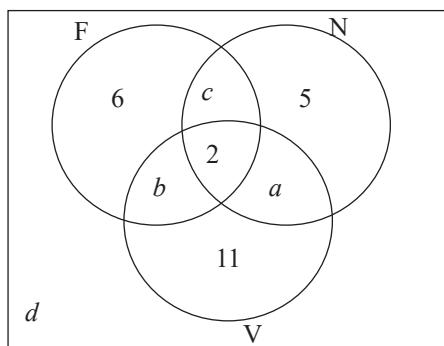
- (a) Represent the above information on a venn diagram.

- (b) Find;

- how many students play all the three games [Ans: 4 students]
- the number of students who play atmost two games [Ans: 46 students]
- the number of students who play atleast two games [Ans: 25 students]
- the probability that a student selected at random does not play Tennis [Ans: $\frac{12}{25}$]

Set Theory

14. The diagram below shows 35 students who participated in Football (F), Netball (N) and Volleyball (V) tournament.



If $n(V) = 20$, $n(N) = n(F) = 12$,

- (a) Find the values of a , b , c and d [Ans: $a = 4$, $b = 3$, $c = 1$, $d = 3$]
- (b) Determine; (i) $n(F \cap N \cap V^c)$ [Ans: 1]
(ii) $n(F^c \cap V \cap N^c)$ [Ans: 11]
- (c) Find the probability that a student picked at random plays neither game. [Ans: $\frac{3}{35}$]

15. In a certain town, 92 tourists were found to have visited other countries. 28 had visited Zambia(Z), 36 had visited Kenya(K) and 50 had visited Tanzania(T); 8 had visited Zambia and Kenya, 14 had visited Kenya and Tanzania. The number of those who had visited all the three countries was equal to those that had visited Zambia and Tanzania only. 6 people did not visit any of the three countries.

(a) Represent this information in a venn diagram.

(b) Find the number of tourists who had visited

- (i) all the three countries [Ans: 6]
- (ii) atleast two countries [Ans: 22]
- (iii) Zambia and Tanzania [Ans: 12]

16. In a certain school, there are 87 students in S.3. Of these, 43 play hockey (H), 42 play football (F) and 47 play Volleyball (V). 15 play hockey and volleyball, 17 play volleyball and football and 21 play hockey and football. Each student plays atleast one of the three games.

(a) Using a venn diagram, represent the information given above.

(b) Determine the number of students who play all the three games [Ans: 8 students]

(c) How many students play only two of these games. [Ans: 29 students]

(d) If a student is selected at random from the class, find the probability that the student plays only one of the games. [Ans: $\frac{50}{87}$]

17. In Ridar hotel, thirty nine guests ordered either Matooke, Cassava or rice. 24 ordered for Matooke(M), 16 ordered for Rice(R) and 17 ordered for Cassava(C). Those who ordered for Matooke and Cassava were more than those that ordered for both Cassava and Rice by one person. 9 ordered for Matooke and rice. 2 ordered for all the three.

(a) Represent the information on a venn diagram.

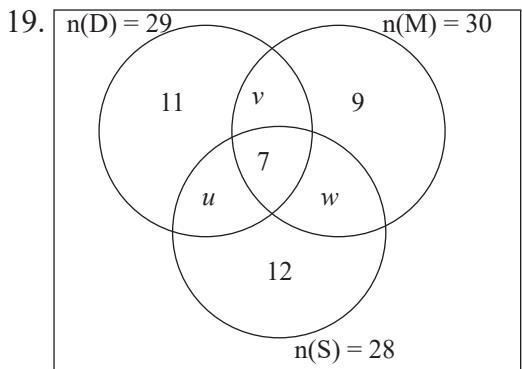
(b) Find how many guests that ordered for both cassava and Rice. [Ans: 5]

(c) If a guest is picked at random from the hotel, what is the probability that a guest did not order for Matooke? [Ans: $\frac{5}{13}$]

18. In a training college, students must take atleast one of the following subjects. Fine Art(A), Physical Education(P) and Music (M). Of the 70 stdents in the first year, none takes P and M. 26 take P only. Of the 35 students taking A, 20 have specialized in this subject alone. The number of students taking P and A is three more than the number taking M and A.

Using a Venn diagram, find the number of students taking

- (i) A and M [Ans: 6 students]
- (ii) M only [Ans: 9 students]
- (iii) Two subjects only [Ans: 15 students]
- (iv) Only one subject [Ans: 55 students]



The diagram above shows the representation of students of a S. 4 class who belong to the clubs of debating (D) Mathematics (M) or Science (S) which are in their school.

- Determine the values of u , v and w . [Ans: $u = 3$, $v = 8$, $w = 6$]
- Find the total number of students in the S.4 class. [Ans: 56 students]
- If a student is selected at random from the class, find the probability that the student belongs to atmost two clubs? [Ans: $\frac{7}{8}$]

20. At noon on Tuesday, a group of news paper vendors were asked whether they still had News papers from the Monitor(M), the New Vision(N) and the Red paper(R). All the vendors had atleast one of the three papers. 28 had the monitor, 29 had the New Vision and 30 had the Red Paper. 15 had the Monitor and the New Vision, 9 had New Vision and Red Paper, 8 had the Monitor and the Red Paper while 8 had Monitor only.

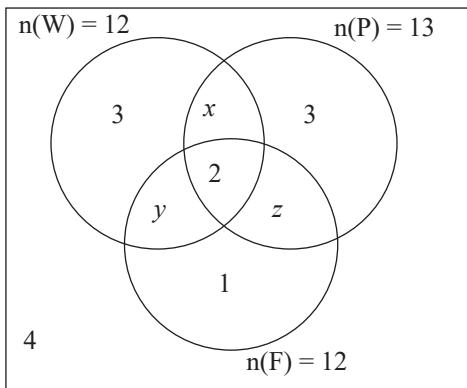
- Represent the information above on a Venn diagram.
- Find the number of Vendors with;
 - all the three types of papers. [Ans: 3 vendors]
 - only two types of papers left [Ans: 23 vendors]
 - atleast two types of papers [Ans: 26 vendors]
- Find the probability that a vendor picked at random from the group still had the Monitor and Red Paper News papers only. [Ans: $\frac{5}{58}$]

21. In a certain class, 20 students like German, 10 like both French and Germany, 9 like both French and Kiswahili, 7 like Kiswahili and Germany only and 2 like none of these languages. 22 like atleast two of the languages, 12 like one language only and 11 like either Kiswahili or German or both but not French.

- Repeat this information on a suitable Venn Diagram
- Find the number of students who like
 - all the three languages [Ans: 2 students]
 - Kiswahili only [Ans: 1 student]
 - French only. [Ans: 6 student]
- find the number of students in the class. [Ans: 36 students]
- What is the probability that a student selected at random likes neither Kiswahili nor Germany?
[Ans: $\frac{2}{9}$]

22. The venn diagram below shows the members of a district council who sit on three different committees of works (W), production (P) and Finance (F) .

Set Theory



- (a) Determine the values of x , y and z [Ans: $x = 3, y = 4, z = 5$]
 (b) Find the total number of members who;
 (i) make up the district council. [Ans: 25 members]
 (ii) belong to more than one committee. [Ans: 14 members]
 (c) Given that a member is selected at random from the council, find the probability that the member belongs to only two committees. [Ans: $\frac{12}{25}$]

23. In a class of 75 students, 38 students like Matooke (M), 44 students like Posho(P) while 33 students like Rice (R). Of these students 17 like both Matooke and Posho, 16 like both Matooke and Rice while 20 like both Posho and Rice. Only 6 of the students do not like any of the foods.

- (a) Represent the above information on a Venn diagram.
 (b) Determine the number of students who like all the three foods. [Ans: 7 students]
 (c) How many students like only two of these foods? [Ans: 32 student]
 (d) If a student is selected at random from the class, find the probability that the student likes only one of the foods. [Ans: $\frac{2}{5}$]

24. In a class of 39 students, 24 offer Divinity(D), 16 offer History (H) and 17 offer Commerce (C). Those who offer Divinity and Commerce are more than those who offer Commerce and History by one student. 9 offer both Divinity and History. 2 offer all the three.

- (a) Represent the information on a Venn diagram.
 (b) Find how many students offer both Commerce and History. [Ans: 5]
 (c) If a student is picked at random from the class, what is the probability that the student does not offer Divinity? [Ans: $\frac{5}{13}$]

25. There are 100 students in A– Level taking various subject combinations. Of these, 60 students take Maths(M) , 45 take Physics(P) and 40 take Chemistry (C). Out of these, 60 students taking M, 16 do neither P nor C. of the 45 taking P, 8 do neither M nor C. Of those taking C, 5 take neither M nor P. 7 take both M and C but not P.

- (a) Find the number who take
 (i) M and P [Ans: 37]
 (ii) P and C but not M [Ans: 0(zero)]
 (iii) all the three subjects [Ans: 28]
 (iv) none of these subjects [Ans: 27]

- (b) Find the probability that a student picked at random offers only one subject [Ans: $\frac{29}{100}$]

26. In an examination, 93 candidates attempted at least Qn1, Qn2 and Qn3. Qn 1 was attempted by 67 candidates, Qn2 by 46 candidates and Qn3 by 40 candidates , 28 attempted Qn1 and Qn2, 26 attempted Qn1 and Qn3 and those who attempted all the three questions were four less than those who attempted Qn2 and Qn3 only.

- (a) Represent the above information on a venn diagram.
 (b) How many candidates attempted Qn2 and Qn3 but not Qn1 [Ans: 6]
 (c) If a candidate is picked at random form the examination, what is the probability that the candidate attempted atleast two questions. [Ans: $\frac{58}{93}$]

27. In watching Premier football team games, 56 members watched Arsenal (A), 48 watched Liverpool (L) and 36 watched Chelsea (C). All the members watched one or more teams except 6 members who did not have entry fee. 32 watched Arsenal only, 10 watched Liverpool only, 8 watched Liverpool and Arsenal but not Chelsea. 39 watched more than one team.

Use a Venn diagram to find how many members watched;

- (a) the teams [Ans: 86 members]
- (b) Chelsea only [Ans: 5 members]
- (c) more than two teams. [Ans: 15 members]

28. On a wedding ceremony 71 guests were asked which flavours of Mirinda (M), Novida (N) and Fanta (F) they each prefer. It was found out that an equal number of guests preferred M and N. 10 guests preferred M and F, 11 guests prefer F and N while 6 preferred M and N only. 26 preferred F and 5 preferred M only. The number of guests who preferred F only doubles those who preferred N only.

- (a) Represent the above information on a venn diagram
- (b) Find the number of guests who; (i) Preferred N only [Ans: 4 guests]
- (ii) Preferred all the flavours [Ans: 3]
- (iii) did not like any of the three [Ans: 30]

- (c) If a guest is chosen at random from the group, find the probability that he/she preferred at most two drinks. [Ans: $\frac{68}{71}$]

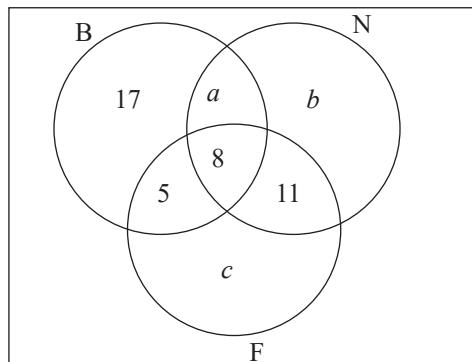
29. In Luwero High School 32 students were asked about their favourite games and the responses were as follows:

- 17 play football (F)
- 20 play hockey (H)
- 18 play volleyball (V)
- 11 play F and H
- 2 play V and H but not F.
- 5 play F and V but not H.

Determine :

- (a) the number of students who play all the 3 games. [Ans: 5 students]
- (b) the number of students who play only one game. [Ans: 14 students]
- (c) what percentage of the students play at least 2 games [Ans: 56.3%]

30. In a class of 94 S.3 students, each student is to play at least one of the three games; Badminton, Football and Netball as represented in the Venn diagram below.



If $n(B) = 40$ and $n(N) = 55$. Find the values of a, b and c.

- (a) What percentage of students play football?
- (b) Find the probability that a student selected at random plays at most 2 games.

[Ans: (a) 43.6% (b) $\frac{43}{47}$]

31. A school receives three National Newspapers everyday, namely The New Vision (V), The Monitor (M) and Red Pepper (R). In the same school there is an ‘A’ level class of S.6 science consisting of 90 students. Of these students 13 read both V and M, 18 read both V and R, 16 read both M and R. 40 read V, 37 read M and 46 read R. The number of students in that same class who read all the three newspapers is equal to the number who do not read any of them.

Set Theory

- (a) Represent the above information on a Venn diagram
- (b) Determine the number of students who read all the three newspapers. [Ans: 7 students]
- (c) How many students read only two of the newspapers? [Ans: 26 students]
- (d) If a student is selected at random from the class, find the probability that the student reads only one of the newspapers. [Ans: $\frac{5}{9}$]
32. In a certain school there are students who play football (F), Tennis (T) or volleyball (V). 24 play football, 25 play Tennis and 29 play volleyball. 11 play both F and T, 10 play both T and V while 13 play both F and V. The number of students who play tennis or volleyball but not football is equal to twice those who play neither of the three games. If those who play neither of the games are 12,
- (a) Represent the above information on a venn diagram.
- (b) Find the;
- (i) total number of students in the school. [Ans: 60 students]
- (ii) number of students who play only two games. [Ans: 22 students]
- (c) Find the probability that a student chosen at random plays not more than one game. [Ans: $\frac{17}{30}$]
33. A school has a teaching staff of 22 teachers 8 of them teach mathematics, 7 teach physics and 4 teach chemistry. Three teach both mathematics and physics and one teaches mathematics and chemistry. No teacher teaches all the three subjects. The number of teachers who teach physics and chemistry is equal to that of those who teach chemistry but not physics
- (a) Represent the above information on a venn diagram
- (b) Find the number of teachers who teach
- (i) mathematics only [Ans: 4 teachers]
- (ii) none of the three subjects [Ans: 9 teachers]
34. Seventy five candidates sat for a paper containing 3 sections A, B and C. Forty-five attempted section C, 26 attempted section A but not C, 10 attempted all the three sections. Twenty-eight attempted section A and C, 23 section B and C and 4 could not attempt any of the sections. If 20 attempted section A and B:
- (a) Represent this information in a Venn Diagram.
- (b) Determine how many candidates attempted (i) section B [Ans: 33 candidates]
- (ii) only one section. [Ans: 20 candidates]
- (c) A candidate was selected at random, what is the probability that he/she attempted at least 2 sections.
[Ans: $\frac{17}{25}$]
35. A tourist office looks at 20 East African national parks to see which of them have elephants(E), lions(L) and rhinoceros(R). They find that; $n(E \cap L^1 \cap R^1) = 2$, $n(L \cap E^1 \cap R^1) = 1$, $n(R \cap L^1 \cap E^1) = 0$, $n(E \cup L \cup R)^1 = 3$, $n(E \cap L \cap R^1) = 8.5$ parks have elephants and rhinoceros and 4 parks have lions and rhinoceros.
- (a) Draw a venn diagram to show this information
- (b) How many parks had all three of these animals? [Ans: 3 parks]
- (c) What is the probability that a park chosen at random has atleast two of these animals? [Ans: $\frac{7}{10}$]
37. In a mathematics class the teacher told students to bring a pen (P), a graph book (G) and a ruler (R) for use. During the next lesson it was found out that only 16 students brought all the items. 5 students did not have any of the items. 13 did not have a pen, 14 students did not have a graph book and 20 did not have a ruler. One student only had a pen, 2 students had only a graph book and no student had only a ruler.
- (a) Represent the above information on a venn diagram.
- (b) How many students;
- (i) were in the class? [Ans: 50]
- (ii) had a pen and a ruler only? [Ans: 8]

- (c) If a student is selected from this class at random, find the probability that he had;
- at least 2 items. [Ans: $\frac{21}{25}$]
 - only one item. [Ans: $\frac{3}{50}$]
38. A group of students play either football (F) Volleyball (V) or Tennis(T), 21 play F, 17 play V and 18 play T. 7 play F and V, 11 play T and F while 3 play only T and V. Those playing V only exceeds those playing the three games by 2.
- Draw a venn diagram representing the given information.
 - How many students play Tennis only? [Ans: 4]
 - How many students are in a group? [Ans: 35]
 - Find the probability of picking on a student playing atleast two games. [Ans: $\frac{16}{35}$]
39. In a random sample of 1000 men, the following facts emerged. 271 of the men were bald (B), 248 of the men wore spectacles (S), 251 of the men had false teeth (T), 64 men were both bald and wore spectacles, 97 men were both bald and had false teeth, 59 wore spectacles and had false teeth. 434 were not bald, did not wear spectacles and had all their own teeth.
- Represent the information on a venn diagram.
 - How many were bald, wore spectacles and had false teeth. [Ans: 16]
 - How many men were bald but neither wore spectacles nor have false teeth? [Ans: 126]
 - If a man is chosen at random, what is the probability that he wears spectacles and has false teeth?
[Ans: $\frac{59}{100}$]
40. The number of people who play football (F) or basket ball (B) is twice the number of people who play F and B. If $n(F)= 9$ and $n(B) = 6$, how many play both games?
41. There were 92 people in cinema hall of these 47 had put on hats, 33 had put on jackets and 27 had neither put on hats nor jackets.
- Represent this information on the venn-diagram.
 - From the venn diagram, determine the number of people in the cinema hall who had put on

(i) a hat and a jacket	[Ans: 13 people]	(iii) a hat but not a jacket	[Ans: 34 people]
(ii) either a hat or a jacket	[Ans: 67 people]	(iv) a jacket but not a hat.	[Ans: 20 people]
42. At a birth day party, there were 217 guests served with 3 major soft drinks. The major drinks were mineral water, soda and milk. A survey conducted during the party revealed that; 14 of the guests were served with mineral water and soda but not with milk. The total number of guests served with exactly two of the three major drinks was 42. The number of guests served with soda and milk but not with mineral water exceeded the number served with mineral water and milk but not soda by 2. The number of guest served with soda and milk was 45. The number served with mineral water was 112. 127 of the guests were served with only one of the drinks. The number served with milk alone exceeded those served with soda alone by 8.
- Represent this information on a venn diagram.
 - Using the venn - diagram, find the number of gusets who were served with;

(i) atleast one of the drinks	[Ans: 199 guests]	(iii) all the drinks	[Ans: 30 guests]
(ii) none of the drinks	[Ans: 18 guests]	(iv) soda	[Ans: 91 guests]
(v) milk	[Ans: 98 guests]		

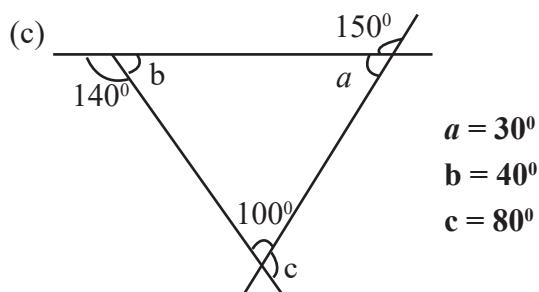
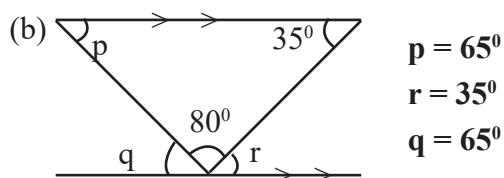
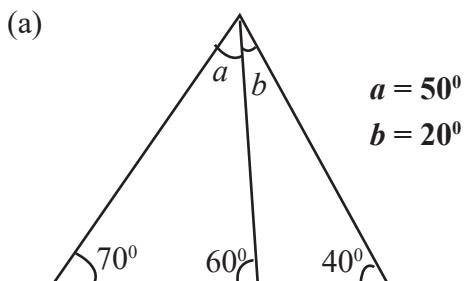
ANGLES AND POLYGONS

TRIAL QUESTIONS

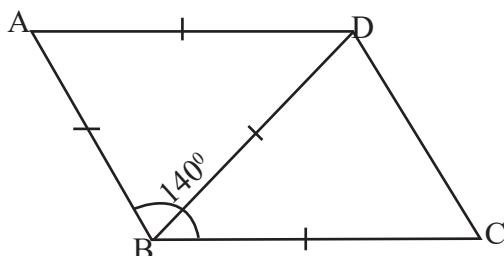
1. A regular polygon has 18 sides. Find the size of each exterior angle and hence determine the angle sum of interior angles of this polygon.
[Ans: Exterior angle = 20° , Angle sum of interior angles = 2880°]

2. A seven sided polygon has two of its angles 140° and 160° , and the remaining angles are all equal. Find the size of the equal angles. [Ans: 120°]

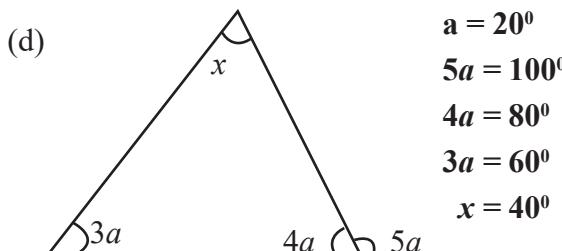
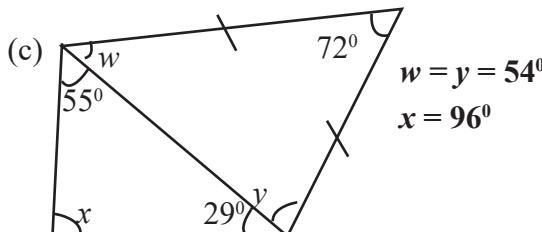
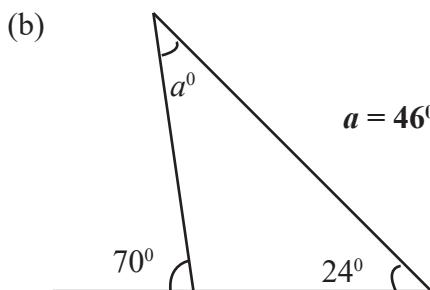
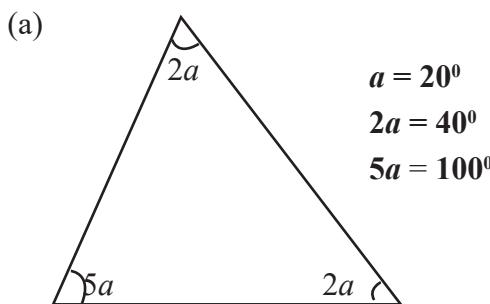
3. Find the angles represented by small letters in the diagrams below;



4. In the diagram ABD is an equilateral triangle, BC = BD and angle ABC = 140° . Find angle BCD.



5. Calculate the unknown angles in the following figures.



6. Find the interior angle of a regular decagon.
[Ans: 144°]

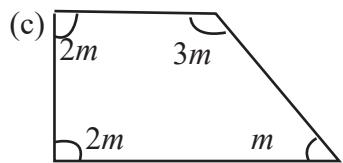
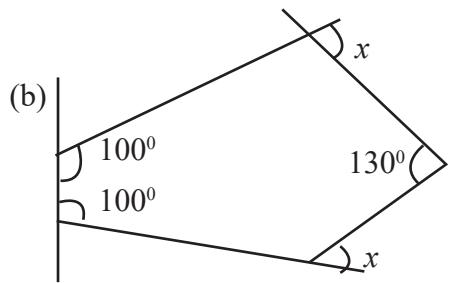
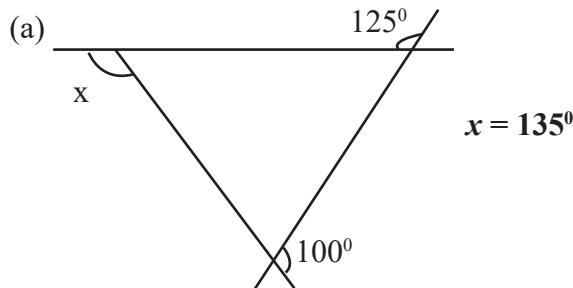
7. The interior angle of a regular polygon is 168° . How many sides has the polygon?
[Ans: 30 sides]

8. The interior angles of a regular polygon add up to 2340° . Find the size of the interior angle.
[Ans: 150°]

9. Find the interior angle of a 20 sided regular polygon. [Ans: 162°]

10. The exterior angles of a quadrilateral are z° , $2z^{\circ}$, $2z^{\circ}$ and 80° . Find the value of z . [Ans: $z = 56^{\circ}$].

11. Find the size of the angles marked with letters.

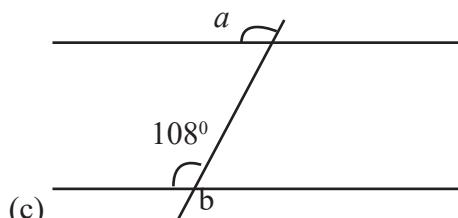
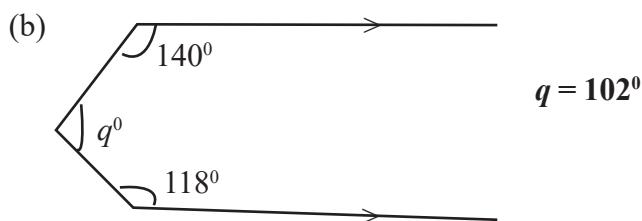
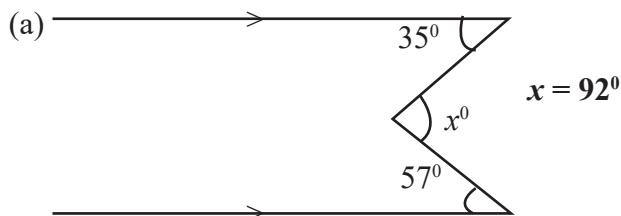


12. A regular polygon has 15 sides. If its interior angle is $(x + 30)^{\circ}$, calculate the value of x .

[Ans: $x = 120^{\circ}$]

13. An irregular pentagon has the following four interior angles 110° , 108° , 135° and 152° . Determine the value of the exterior angle adjacent to the fifth interior angle. [Ans: 145°]

14. Find the unknown angles;



CONSTRUCTION

RECALL

Before attempting any question in construction, you must;

- ◊ Have a pair of compasses, a set, and a ruler
 - ◊ Know how to construct relevant angles
 - ◊ Be able to bisect angles, lines and know the relevance of the respective processes.
 - ◊ Be able to construct triangles etc
 - ◊ Be able to drop a perpendicular from a point which is not on the line onto the line.
 - ◊ Be able to inscribe and circumscribe a triangle and any figure where necessary
 - ◊ Recall the formula for finding the area of a triangle and a circle.

TRIAL QUESTIONS

- and measure its radius

(ii) Find the area of the circle.

5. Using a ruler and a pair of compasses only

(a) Construct a triangle ABC in which $AB = 6\text{ cm}$, $BC = 8\text{ cm}$ and angle $ABC = 60^\circ$.

(b) Inscribe a circle in the triangle ABC and measure its radius

- (c) Construct a perpendicular from vertex A to meet BC at R. Measure AR.

(d) Calculate the area outside the circle but inside the triangle correct to 3 significant figures.

6. (a) Using a ruler and a pair of compasses only, construct a rhombus ABCD in which $AB = BC = 5 \text{ cm}$ and angle $ABC = 120^\circ$.

(b) Measure angle DAB.

(c) Construct perpendicular bisectors to sides AB and BC to meet at O.

(d) Construct a circle that passes through vertices A, B and C.

7. (a) Using a ruler and a pair of compasses only construct a trapezium ABCD in which AB is parallel to DC and $AB = 6 \text{ cm}$, $AD = 4 \text{ cm}$ $DC = 3 \text{ cm}$ and angle $DAB = 60^\circ$.

(b) Drop a perpendicular from D to meet AB at M. Measure DM.

(c) Calculate the area and perimeter of the trapezium ABCD.

8. Using a ruler, pencil and a pair of compasses only;

(a) Construct a triangle ABC, where $AB = 8.0 \text{ cm}$ and C is 5.4 cm from A and 6.5 cm from B.

(b) D is a point on AC produced such that $AD = 7.4 \text{ cm}$ and E is 6.2 cm from D on the same side of AD as B, such that angle $ADE = 135^\circ$

(c) Draw a circle to circumscribe points A, B and E. Measure

- (i) \overline{AE}
(ii) The radius of the circle
9. (a) using a ruler and a pair of compasses only, construct a parallelogram ABCD such that $AB = 9 \text{ cm}$, $BC = 8 \text{ cm}$, angle $BAC = 45^\circ$ and angle $ABC = 135^\circ$.
(b) Bisect angles ABC and BAD and let the bisectors meet at X. Measure AX.
(c) Construct a circle that passes through points A, B and X. Measure the radius of the circle and hence calculate its circumference.
10. (a) Using a ruler and compass only, construct $\triangle PQR$ such that $PQ = 8 \text{ cm}$, $QR = 6 \text{ cm}$ and $\angle QPR = 45^\circ$.
(b) Measure \overline{PR} and $\angle PQR$
(c) Draw a circle that passes through P, Q and R.
(d) Calculate the area of;
(i) the circle
(ii) the triangle PQR
11. Using a ruler, pencil and pair of compasses ONLY.
(a) Construct the quadrilateral ABCD in which $AB = 6 \text{ cm}$, $BC = 5 \text{ cm}$, $CD = 7 \text{ cm}$, angle $ABC = 105^\circ$ and angle $BCD = 120^\circ$.
(b) Construct an in circle of triangle ACD with centre O.
(c) Measure; (i) the length OA
(ii) the radius of the circle.
12. Construct triangle ABC such that $\overline{AB} = 4 \text{ cm}$, $\overline{BC} = 3.6 \text{ cm}$ and $\overline{AC} = 3.8 \text{ cm}$. Construct perpendicular bisectors of lines \overline{AB} and \overline{AC} such that they intersect at point P. Measure line AP. Draw a circumcircle of triangle ABC.
13. Using a ruler and a pair of compasses only, construct a quadrilateral ABCD in which $AB = 5 \text{ cm}$, $BC = 6 \text{ cm}$, $CD = 4 \text{ cm}$ and angle $ABC = 75^\circ$ and angle $BCD = 120^\circ$. Construct a circle passing through points A, C and D.
(a) Measure
(i) AD
(ii) the distance from the centre of the circle to point B
(b) Find the area of the circle
14. Using a ruler, pencil and a pair of compasses only, construct a quadrilateral PQRS such that $PQ = 5 \text{ cm}$, $QR = 7 \text{ cm}$, angle $SPQ = \text{angle } PQR = 90^\circ$ and angle $QRS = 60^\circ$.
(b) Construct a circle circumscribing triangle SQR. Hence measure;
(i) length RS and PS.
(ii) the angle PSR
(iii) the distance from the centre, O, of the circle to R.
(c) Calculate the area of the circle.
15. Using a ruler and pair of compass only. Construct triangle ABC such that $\overline{BC} = 4.8 \text{ cm}$ and angles $BAC = 30^\circ$, and $ABC = 90^\circ$. D is a point on \overline{BC} produced 2.7 cm away from \overline{AB} . Construct angle $BDE = 45^\circ$ with $\overline{DE} = 10.1 \text{ cm}$. Join points A to D and B to E. Construct a circle circumscribing triangle ACD such that it also passes through the point E. Measure;
(i) length AB and BE
(ii) angle ADC
(iii).the radius of the circle
16. (a) On a squared paper, plot points P(2.0, 3.5), Q(7.0, 3.5) and R(4.5, 6.5). Use a scale of 1 cm to represent 0.5 units on both axes. Join the points to form triangle PQR.
(b) Using a ruler and a pair of compasses only construct a circle circumscribing triangle PQR. State the coordinates of the centre, C, of the circle.
(c) Measure its radius and angle PCQ. Hence calculate area of the minor sector PCQ.
17. A circle passes through the points P(0.5, 3), Q(2, 4.7) and R(4.5, 1).
i) Plot and join the points P, Q and R on a graph paper (Use 2 cm to represent one unit on either scales).
ii) By construction, determine the centre and radius of the circle.
iii) Calculate the area of the minor segment cut off by the chord PQ.
18. Using a ruler and pair of compass only, construct a triangle ABC such that $AC = 9.6 \text{ cm}$, $BC = 4.8 \text{ cm}$ and the angles $BAC = 30^\circ$ and $ABC = 90^\circ$. D is a point on BC produced 2.7 cm away from

Construction

- AB. Construct and $BDE = 45^\circ$ with $De = 10.1$ cm. Join points A to D and B to E. Construct a circle circumscribing triangle ACD such that it also passes through the point E. Measure;
- lengths AB and BE
 - angle ADC
 - the radius of the circle
19. Using a ruler, pencil and a pair of compasses only,
- Construct a triangle ABC, measure AB and angle ACB.
 - draw a circle circumscribing triangle ABC, state the radius of the circle.
20. Using a ruler and pair of compasses only, construct
- a quadrilateral PQRS such that $QRS = 45^\circ$, $QR = 4.5$ cm, $RS = 6.0$ cm, $SP = 7.5$ cm and $PQ = 10.5$ cm.
 - Point T on RQ produced such that $PT = ST$. Join the point P, S, and T. Measure length PT and angle PTS.
 - a circle passing through the points P, T and R. Measure the radius of the circle.
21. Using a ruler, pencil and pair of compasses only,
- Construct a triangle PQR, where angle QPR = 135° , $PQ = 8.4$ cm and $QR = 12.5$ cm. State the length of PR.
 - S and T are points such that TS bisects QR, where T is on QR and S on the same side as PQ. Draw a circle to circumscribe the points P, Q, R and S. Measure and state the;
- length ST
 - radius of the circle
22. Using a pair of compasses and ruler only,
- Construct triangle ABC such that $Bc = 10.6$ cm and angles $ACD = 75^\circ$ and $ABC = 60^\circ$.
 - Construct a circum circle of triangle ABC with O as its centre.
 - Measure length AB and AC and the radius of the circle.
23. Using a ruler, pencil and pair of compasses only,
- Construct a triangle ABC such that $AB = 8.7$ cm, $AC = 10.6$ cm and angle $BAC = 60^\circ$.
 - Inscribe a circle on the triangle ABC.
 - Construct a perpendicular from B onto AC to meet it at point D.
 - Measure length BC and the radius of the circle.
 - Measure BD and calculate the area of triangle ABC.
24. Using a pencil, a ruler and a pair of compasses only, construct a triangle ABC in which $AB = 9.2$ cm, angle $CAB = 45^\circ$ and angle $ABC = 75^\circ$.
- Measure the length BC
 - Draw a circumscribing circle through the points A, B and C
 - Measure the radius of the circle.

BUSINESS MATHEMATICS

RECALL

- ◊ Profit = selling price – cost price
- ◊ Percentage profit = $\frac{\text{profit}}{\text{cost price}} \times 100$
- ◊ Loss = cost price – selling price
- ◊ Taxable income = Gross income – total allowances
- ◊ Income tax is obtained basing on taxable income using given rates as per question. Rates are given in percentage. The taxable income gives the limit for taxation on the individuals income.
- ◊ Expressing income tax as percentage of gross income; $\frac{\text{income tax}}{\text{gross income}} \times 100$
- ◊ Simple interest (I) = $\frac{PRT}{100}$, but also, interest(I) = Amount(A) – Principle(P)
- ◊ Compound interest: $A = P(1 + \frac{r}{100})^n$, $A \rightarrow$ amount, $P \rightarrow$ principle, $r \rightarrow$ rate, $n \rightarrow$ number of years.
- ◊ Total hire purchase = Deposit + Total installment.
- ◊ Taxable income = Gross income – Total allowances
- ◊ Gross income = taxable income + total allowances.
- ◊ Net income = Gross income – income tax.

TRIAL QUESTIONS

1. (a) If the exchange rate for shillings to dollars is \$1 = 1675/=, how many dollars does Mukasa get if he has 418, 750/=? [Ans: \$250]
(b) Find the principal which gains shs 30,000 as simple interest in 12 months at a rate of 15% per half a year. [Ans: shs.100,000]
(c) A British traveller takes £500 (pounds) to Uganda. How many shillings can this be exchanged for if £1 = shs 3000 give your answer in words [Ans: One million five hundred thousand shillings]
2. (a) Find the amount of £ 85.20 in 8 years at $7\frac{1}{2}\%$ per annum simple interests. [Ans: £136.32]
(b) What sum of money is invested for 5 years at $7\frac{1}{2}\%$ per annum yields shs. 150,000 simple interest. What is the amount at the end of this period? [Ans: shs.550,000]
(c) Find how long it takes shs 96,000 invested at $8\frac{1}{3}\%$ simple interest to amount to sh 102,000. [Ans: 9 months (0.75 years)]
(d) Find the interest earned when sh 450,000 is invested at 8% p.a. compound interest for 4 years. [Ans: 162,220]
3. In a certain country, the monthly gross income has certain allowances deducted from it before it is subjected to taxation. The allowances are:

married man	Shs 25,000
unmarried	shs 15000
transport	shs 3,000 per day
insurance	shs 15000
electricity	shs 18000
medical	shs 480,000 per annum

Family allowance for 4 children: Shs 9000 for each child above 18 years, shs 12000 for each child below 18 but above 10 years and 15000 for each child below 10 years of age .Mukasa is a married man with 5 children 3 of them below 10, one aged 14 and the elder one 20 years old.

- (a) Find Mukasa's taxable income and the income tax he pays under the tax rates below given that he earns shs 960,000 monthly. [Ans: taxable income = 715,000; Income tax = 90,475]

Taxable income	Rate %
0 - 200000	2%
200001 - 400000	10%
400001 - 600000	18%
600001 - 800000	26.5%
above 800000	35%

(b) Express his income tax is a percentage of his gross monthly income. [Ans: 9.2%]

4. The table below shows the tax structure on taxable income of workers of National water and Sewerage co-operations.

Income (sh) per month	Tax rate
0 - 60,000	12%
60,001 - 150,000	17%
150,001 - 250,000	22.5%
250,001 - 350,000	34%
350,001 - 500,000	48%
above 500,000	50%

An employee of National water and Sewerage co-operations earns a gross salary of sh. 890,000 per month. The employee's allowances include the following:

Marriage allowance	one tenth of his gross monthly income
Electricity allowance	Sh. 35,000 per month
Insurance allowance	shs 360,000 per annum
Transport allowance	shs 25,000 per month
Medical allowance	shs 20,000 per month
Housing allowance	shs 60,000 per month

Family allowance for four children only. For children in the age bracket of 0 to 11 years shs 25,500 per child per month; between 11 and 18 years shs. 15,500 per child per month; and over 18 years shs 5,500 per child per month.

- (a) Calculate the employees' taxable income and the income tax paid, given that the employee is a married person with three children who are 5 years, 9 years and 13 years old. [Ans: 564,500, 183,250]
 (b) What percentage of the employee's gross income goes to tax? [Ans: 20.59%]

5. Mr. Okello is a secondary school gatekeeper whose monthly gross income is subjected to taxation after the following allowances have been deducted.

Marriage allowance	$\frac{1}{20}$ th of the gross income
Housing and electricity	shs 480,000 per annum
Head of security department	shs 60,000 per annum
Lunch and breakfast	shs 15,000 per annum
Air time	shs 60,000 per annum
Medical	shs 240,000 per annum

If Mr. Okello earns a gross monthly income of shs 435,000

- (a) Calculate the taxable income and income tax he pays under the tax structure below.

Income (shs) Rate(%)

0 – 12,000	6.25
12,001 – 45,000	9.0
45,001 – 87,000	13.5

87,001 – 145,000	18.0
145,001 – 217,000	26.0
Above 217,000	37.5

[Ans: 342,000 85,425]

- (b) Determine the percentage of his gross monthly income paid in tax. [Ans: 19.64%]
6. (a) Kato bought a car at shs. 4,800,000 and sold it to Tom at shs. 3,600,000. Calculate the percentage loss.
[Ans: 25%]
- (b) Otieno deposited some money in a bank that paid simple interest at 5% p.a, for a period of 5 years. How much money did he deposit if he withdrew a total amount of Kshs 15,000 from the bank?
[Ans: 12,000]
7. A saleswoman earns a basic salary of shs 120,000 and a commission of 8% of the month's total sales. If the month's total sales were shs 1,350,000. find her income for that month. [Ans: 228,000]
8. (a) A forex bureau buys one US Dollar at Ugshs 1,900 and sells one Pound Sterling at Ugshs 3,450. Atim wants to exchange 3,000 US Dollars to Pound Sterling. How many pound Sterling will she get?
[Ans: £1652.17]
- (b) The price of an article is shs 24,000. If a discount of 12% is given, find the selling price of the article. [Ans: shs.21120]
- (c) The selling price of an article is shs 6250. if the seller makes a profit of 25%, what is the cost price?
[Ans: 5000]

9. A book was sold for shs.2160 at a loss of 10%. What did it cost? [Ans: 2400]

10. (a) A certain amount of money was invested at compound interest at a rate of 10% for 5 years. Given that at the end of the period the owner received sh. 500,000. Find the amount originally deposited.
[Ans: 310,460.66]

(b) The income tax rates of a certain country are as follows.

Income (shs)	Rates(%)
0 – 394,000	Tax free
394,001 – 694,000	30
694001 and above	36

- (i) Find the income of an employee who paid shs. 385,200 of tax. [Ans: 1,908,000]
(ii) Express the income tax paid as a percentage of her income. [Ans: 20.19%]

11. In a certain country, income tax is levied as follows.

Taxable income (shs)	Rates(%)
0 – 10,000	10
10,001 – 20,000	25
20,001 - 30,000	30
30,001 - 40,000	45
Above 40,000	50

A person's monthly gross income has certain allowances deducted from it before it is subjected to taxation (This includes family relief and insurance).

The allowances are as follows:-

Married man	Sh. 12000
Unmarried man	Sh. 8500
Each child below 11 years	Sh.9000
Above 11 but below 18 years	Sh.7500
Insurance premium	Sh. 5000

- (i) Sally earns sh 90,000. He is married with children aged 5,7, 12, 14 and 18 years. Given that he is insured and has claimed transport allowance of sh.1700, calculate the income tax he pays under the income tax rates above. [Ans: 10,235]
- (ii) After two years, Sally's salary is increased but the allowances remained the same. If he is found to pay a tax amounting to sh 27,650, find his gross income and the Net income.
[Ans: Gross income = 125,000, Net income = 97,350]
12. (a) Nambuya deposited shs 270,000 in a bank that offers 12.5% compound interest per annum. After n years, her money had accumulated to shs 384,433.60. Calculate the value of n . [Ans: 3 years]
- (b) Under hire-purchase, an item was bought by paying a deposit of shs 85,000 followed by six monthly installments of shs 120,000 each. If the hire-purchase price was higher than the cash price by 15%, calculate the cash price of the article. [Ans: 700,000]
13. (a) Pauline deposits shs. 200,000 in her Bank account at a simple interest rate of 10% per annum for 5years. Calculate the amount of money she received at the end of the period. [Ans: 300,000]
- (b) (i) Tom bought a car at shs 4,500,000 and sold it at a profit of 7%. Calculate the selling price of the car. [Ans: 4,815,000]
- (ii) A radio costs shs 300,000. Peter is given a 15% discount when he pays cash. How much does he pay for it? [Ans: 225,000]
- (iii) Apio sells 125 shirts each at shs 9,000 and gets a 12% commission of the value of the shirts sold. Calculate her commission. [Ans: 135,000]
14. Below is an advertisement for a brand new laptop.
- | | | | |
|---|---------------|---------------|---------------|
| LAPTOP | LAPTOP | LAPTOP | LAPTOP |
| HURRY WHILE STOCK LASTS! | | | |
| PRICE: SHS 1,200,000 | | | |
| CASH PAYMENT: GET A DISCOUNT OF 15% | | | |
| HIRE PURCHASE: DEPOSIT 20% OF THE PRICE AND EITHER PAY SHS.390,000 MONTHLY FOR 3 MONTHS OR SHS.100,000 WEEKLY FOR THIRTEEN WEEKS | | | |
- (a) Calculate the cost of purchasing a laptop using;
- the weekly hire purchase terms. [Ans: 1,540,000]
 - the monthly hire purchase [Ans: 1,410,000]
- (b) Find the savings one would make by paying cash rather than using weekly hire purchase terms.
[Ans: 520,000]

15. Income tax for all income earned is charged at rates shown below.

Income per annum	Rates(%)
0 - 1980	2
1981 - 3960	3
3961 - 5940	5
5941 - 7920	7
7921 - 9900	9
Over 9900	10

Mr. Mukasa earns a monthly gross income of shs 26,200. He also has a house allowance of shs 3200 per month and a family relief of shs 3500 per month.

- Calculate Mr. Mukasa's annual taxable income [Ans: 234,000]
- How much tax does he pay per month? [Ans: 1,474.8]
- What is his net salary per month? [Ans: 24,725.2]

16. A certain country's tax structure is such that a person's monthly income has certain allowances deducted from it before it is subjected to taxation. The allowances spelt out are as follows;
- marriage allowances = $\frac{1}{10}$ of gross monthly income.
 - water and electricity = shs. 12,500 per month
 - housing allowance = shs. 40,000 per month
 - medical allowance = shs. 240,000 per annum

Taxable income (shs)	Rates(%)
00001 - 50,000	0
50,001 - 100,000	10
100,001 - 200,000	15
200,001 - 350,000	20
350,001 - 450,000	30
450,001 and above	35

- (a) Given that an employee is married and earns shs. 540,000 per month, calculate the tax she pays under the above income tax rates. [Ans: 69,050/=]
- (b) What percentage of her income is paid as tax? [Ans: 12.79%]

17. A bank manger earns a gross salary of sh. 1,600,000 per month which includes an allowance of shs. 400,000 tax free. The rest of his income is subjected to an income tax which is calculated as follows;
- 8.5% on the first sh. 500,000.
 14.5% on the next shs. 300,000
 30% on the next shs. 200,000
 45% on the next shs. 160,000
 65% on the remainder.

Calculate the;

- (a) taxable income. [Ans: 1,200,000]
 (b) monthly tax paid by the bank manager. [Ans: 244,000]
 (c) percentage of the bank manager's monthly gross salary that goes to tax. [Ans: 15.25%]

18. A telecommunication company provides some benefits and allowances to its employees. The allowances given are:

- | | |
|-------------------------------------|------------------------------|
| Medical | - shs. 80,000/= per month |
| Electricity | - shs. 52,000/= per month |
| Marriage | - shs. 70,000/= per month |
| Telephone bills | - shs. 696,000/= per annum |
| Transport | - shs. 1,500/= per day |
| Security | - shs. 50,000/= per month |
| Insurance | - shs. 1,800,000/= per annum |
| A child above 10 years | - shs. 5,000/= |
| A child between 5 years and 10years | - shs. 8,000/= |
| A child below 5 years | - shs. 10,000/= |

The company gives family allowance for 2 children only.

Drake is married with 4 children, one child is 13years old, another child is 8 years old and has twins of 3 years old. His monthly salary is shs. 1,180,000/=

The country's tax structure is as shown below.

Tax income (shs)	Rates(%)
0 - 40,000	8
40,001 - 90,000	15
90,001 - 160,000	20
160,001 - 310,000	28
310,001 - 500,000	30
Above 500,000	40

Determine;

- (a) the taxable income Drake paid in June 2010, with 30days. [Ans: 655,000]
 (b) The income tax he paid as a percentage of his monthly salary. [Ans: 15.74%]

19. (a) Exchange rates in a Forex Bureau are that a US dollar to Uganda shillings is 1\$ = 2785 and a pound sterling to US dollar is £1 = \$ 2.59. How much pound sterling will a Ugandan trader get from shs. 4,570,185/=.

[Ans: £633.59]

(b) The tax rates in a country for Government employees are as follows;

Taxable income (shs)	Rates(%)
0 - 80,000	2.0
80,001 - 190,000	5.0
190,001 - 280,000	7.5
280,001 - 380,000	12.0
380,001 - 490,000	15.0
Above 490,000	20.0

Every employee is entitled to the following allowances

Medical - 720,000/= per annum

Electricity - 40,000/= per month

Transport - 2,500/= per day

Housing - 90% of his monthly medical income

If the employee paid a monthly income tax of shs. 125,350/= in June, 2013. Calculate;

- (i) the employee's taxable income [Ans: 905,000]
 (ii) the employee's monthly gross income [Ans: 1,134,000]

20. An Askari at Mandela S.S in Hoima earns gross annual salary of shs 1,068,000 which includes the following allowances. Transport shs. 2,500 per month, water shs 1,050 per month, medical shs.60,000 per annum and children allowance of $\frac{1}{5}$ of his gross monthly earnings. The following tax structure is applicable on the taxable income of his earnings per month.

Taxable income (shs)	Rates(%)
00001 - 5,000	9.0
5001 - 10,000	15.5
10,001 - 20,000	20.0
20,001 - 40,000	30.0
40,001 and above	45

Calculate;

- (a) the taxable income for the askari
 (b) the income tax paid by the askari
 (c) his/her net income
 (d) the tax paid as a percentage of the gross income earned.

21. (a) Tom bought a car at shs 3,800,000 and sold it to Patrick at a loss of 15%. How much did Patrick pay for it? [Ans: 3,230,000]
- (b) Mr. Opio gets 7% commission on every fan he sells. In one week, he sells 4 large fans for shs 25,000 each and 9 small fans for shs 22,500 each. Calculate his total commission. [Ans: shs 21,175]
- (c) A television set costs shs 300,000 cash. Ms. Apio buys it on hire purchase terms by paying a 15% deposit of the cash price and 12 equal monthly installments of shs 26,250 each. How much does she pay for it? [Ans: 360,000]
22. A private school's income tax structure is such that a teacher's monthly income for the month of October has certain allowances deducted from it before its subjected to taxation. The allowances are spelt out as follows:

Marriage allowance is one-tenth of the gross monthly income. Family relief is shs. 240,000 per annum. Housing shs. 35,000 per month. Medical shs. 120,000 per annum. Transport is shs 1000 per day. Departmental allowance shs. 15,000 per month. Administrators allowance shs. 50,000 per month. Family allowance for 3 children only at the following rates.

Class level	Allowances (shs)
A - level	6,800
O- level	7,200
Primary	9,000

Mr. Opio is married with 4 children and 2 of them are in primary, one in O-level and the other at A-level and he is not a head of department. Given that he earns a gross monthly income of shs. 680,000 calculate:

- (a) The taxable income and the income tax he pays under the tax rates shown in the table below.
[Ans: taxable income = 485,000, income tax = 156,840]

Taxable income (shs)	Percentage tax rate
0 - 15,000	8
15,001 - 84,000	16
84,001 - 170,000	25
170,001 - 285,000	34
285,001 - 435,000	40
Above 435,000	48

- (b). Determine the percentage of his gross monthly income paid in tax. [Ans: 23.06%]

23. Income tax in a certain country is calculated after deducting the following allowances. Marriage Sh 91,000 which is one – tenth of the gross monthly income, transport ; Sh 60,000 per month, insurance; Sh 372,000 per annum, medical;sh28,000 per month, housing; Sh 42,000, electricity ;Sh 15, 000 and water ;Sh 500 per day. Family allowance for only 3 children at the following rates; Sh21, 500 for each child below 10 years,Sh 20, 000 for each child between10 years and 15 years;Sh 15,000 for each child above 15 years. The tax rates are as shown below

Taxable income (shs)	Rates(%)
0 - 80,000	10
80,001 - 180,000	12
180,001 - 280,000	15
280,001 - 380,000	17.5
380,001 - 500,000	18
Above 500,000	20

Mr. Okiror is married and has 4 children aged 7, 9, 14 and 20 years.

(a) Calculate Mr.Okiror's taxable income. [Ans: shs 565,000]

(b) What percentage of Mr.Okiror's gross income goes to tax? [Ans: 9.57%]

24. Juma deposited shs. 1,600,000 in a bank which paid compound interest at the rate of 12% per annum. At the end of 5 years, he withdrew all his money. Determine how much money he withdrew.
[Ans: 2,819,746.693]

25. (a) Peter deposited shs.2,500,000 in a bank which offers a compound interest of 15% per annum. How much money did he have in the bank at the end of two years? [Ans: 3,306,250]

(b) The cash price of a radio is shs 720,000. It can be bought on hire purchase terms by making a deposit of 30% of the cash price and then paying 8 monthly instalments of shs 85,000 each.

(i) Find the cost of the radio on hire purchase terms [Ans: 896,000]

(ii).How much more does one pay on hire purchase rather than on cash terms? [Ans: 176,000]

26. The tax structure in a certain country on the taxable income is as follows:

Income(shs) per month	Tax rates (%)
01 - 130,000	Tax free
130,001 - 280,000	9.0
280,001 - 480,000	12.5
480,001 - 880,000	20.0
880,001 and more	30.0

Mr. Mugalu's gross income is shs978,000. The allowances include;

Allowances	Amount (shs)
Transport	75,000 per months
Family	A tenth of gross pay
Medical	150,000 per annum
Insurance	15,000 per month
Housing	80,000 per months

Determine Mr.Mugalu's;

- (a) Total monthly allowances [Ans: 280,300] (b) Taxable income [Ans: 697,700]
(c) Tax paid annually [Ans: 984,480] (d) Net pay. [Ans: 895,960]

27. A vendor receives commission of 5% on the first total sales up to Shs 100,000 and 8% on the total sales exceeding Shs100,000. Determine the total sales of the vendor who received Shs 16,200 commission.
[Ans: 240,000]

28. A worker is entitled to the following allowances;

Insurance Shs.1,524,000 per annum, Housing Shs.120,000 per months, Fuel of 5 litres per day, Electricity Shs.60,000 per month, Medical Shs.972,000 per year., His income is further subject to the tax structure below;

Taxable income (shs)	Rates(%)
01 - 150,000	8
150,0001 - 330,000	10
330,001 - 530,000	12
530,001 - 750,000	15
750,001 - 1,030,000	20
1,030,001 and above	25

N.B: 1 month = 30 days.

If the worker paid an income tax of shs.173,000 and the price per litre of fuel is shs.3,800. Calculate his;

- (a) taxable income [Ans: 1,150,000] (b) monthly net income [Ans: 1,935,000]

29. The tax structure of a certain country is as below.

Taxable income (shs)	Rates(%)
01 - 100,000	Tax free
100,001 - 200,000	12.5
200,001 - 300,000	18
300,001 - 400,000	25
400,001 - 500,000	28
500,001 - 600,000	30
Above 600,0000	45

The following allowances are offered to employees: transport shs50,000 per month, housing/rent shs192,000 per annum, marriage shs24,000 per month, insurance shs720,000 per annum. Water shs1000 per day, electricity shs40,000 and every child 20 years and above shs3500.

Mr.Ssenyange is married with five children aged 3,7,11,18 and 24 years. He earns a gross monthly income of shs800,000. Calculate

- (a) Total monthly allowances [Ans: 201,900] (b) Monthly taxable income [Ans: 598,100]
 (c) Income tax as a percentage of his gross income [Ans: 14.12%] (d) Net monthly income [Ans: 687,070/=]

30. Below is an advertisement for a brand new dinning set.

BRAND NEW DINNING SET BRAND NEW DINNING SET

HURRY WHILE STOCK LASTS!

PRICE: SHS 2MILLION

CASH: DISCOUNT OF 5%

HIRE PURCHASE: DEPOSIT 30% OF THE PRICE AND PAY EITHER SHS 120,000 MONTHLY FOR 13 MONTH OR SHS 50,000 WEEKLY FOR 30 WEEKS.

(a) Calculate the amount one would save by paying cash rather than using the monthly hire purchase term. [Ans: 260,000]

(b) If the whole sale price of the dinning set was 10% below the price. Calculate the percentage profit the seller gets on the weekly hire purchase terms. [Ans: 16.67%]

31. Babirye is a sales lady in a biscuit company. She receives a salary of shs4,000 per month and a commission of 10% of the total sales for the first 1000 biscuits sold and further discount of 12% of the sales for biscuits in excess of 1,000. The biscuits are sold at shs2,000 each. For June she sold 1500 biscuits.

(i) How much did she receive at the end of the months? [Ans: 324,000]

(ii) For July the biscuits sold increased by 10%. How much did she receive at the end of July?
 [Ans: 360,000]

(iii) For a months of August she received shs420,000. How many biscuits did Babirye sell during the month of August? [Ans: 1900 biscuits]

32. (a) Mr. Zziwa borrowed shs9,000,000 for 3 years at a compound interest of 5% per annum. How much money does he repay by the end of 3 years? [Ans: 10,418,625]

(b) John invested Shs 3 million at a 25% simple interest rate per annum. How much money does she receive by the end of five years? [Ans: 6,750,000]

(c) Mr. Lubuulwa bought 10 sacks of sugar at shs50,000 each, 10 bags of beans at shs30,000 each, 10 sacks of rice at shs100,000 each and 10 sacks of groundnuts at shs120,000 each. These items were offered 5%, 10%, 6% and 4% discounts respectively.

(i) how much money did he spend? [Ans: 2,837,00]

(ii) what was his total discount? [Ans: 163,000]

33. (a) Okello bought a car at 30 millions. If his car depreciates at a rate of 25% p.a, determine the amount he will sell his car after 4 years. [Ans: 9,492,187.5/=]
- (b) a car costs 9600 dollars and depreciates at a rate of 15% per annum. What is the value of the car after 3 years? [Ans: 5,895.6 dollars]
34. The value of a certain excavator increased by 20% in the first year, depreciated by 10% in the second year and again increased by 10% in the third year. If the value of the excavator at the end of the third year is 297,000,000 million, find its original value. [Ans: 250,000,000]
35. Below is the advertisement for a Refrigerator.

REFREGERATOR FOR SALE!
TERMS: CASH shs800,000 LESS 5% DISCOUNT
OR
FOR MONTHLY HIRE PURCHASE: DEPOSIT 300,000 AND THEN 4 MONTHLY INSTALLMENTS OF shs150,000 EACH OR 8 WEEKLY INSTALLMENTS OF 90,000 EACH

Calculate

- (a) The cash price [Ans: 760,000] (b) The monthly hire purchase [Ans: 900,000]
- (c) Weekly hire purchase [Ans: 1,020,000]
- (d) The savings a customer would make by buying a radio on cash basis rather than
- (i) Weekly hire purchase. [Ans: 260,000] (ii) Monthly hire purchase. [Ans: 140,000]
36. In a certain country, the monthly gross income has certain allowances deducted from it before it is subjected to taxation. The allowances are as follows;

Allowance	Amount (shs)
Marriage	18,000
single	12,000
Each child below 11 yrs	5,000
Each child above 11 yrs and below 18 yrs	7,000
Insurance premium	12,000

The income tax rates are as indicated in the table below.

Taxable income	Rates(%)
0 - 50,000	10
50,001 - 100,000	15
100,001 - 200,000	20
200,001 - 300,000	25
300,001 - 400,000	30
400,001 - 500,000	35
500,001 - above	40

Thomas is married and a father of John, Mary and Fred of ages 9, 10 and 13 years respectively. Given that he is insured and pays 654,000 as income tax per month, calculate;

- (i) His gross monthly income [Ans: 1,875,750]
- (ii) His taxable income [Ans: 1,828,750]
- (iii) The percentage of his monthly income he pays as income tax. [Ans: 34.87%]
37. A vehicle has a market price of shs12 million. A customer can buy either by cash and get a discount of 15% or by a 55% deposit and 12 monthly installments 550,000. Find how much he would pay by hire purchase than paying cash. [Ans: 600,000]

38. James is employed with an electricity installation company that pays him a gross salary of shs6,600,000 per year. He is married with 4 children, two of whom are aged 17 years and 19 years respectively while the other two are aged 11 years and 14 years respectively. The company pays allowance for only 3 children for every employee. A summary of allowance is as follows: Marriage; $\frac{1}{10}$ of gross monthly income, Medical ; 300,000 per year, Transport; 2000 per day, Children above 15years but below 29 years of age; 1000 per month, Child above 10 years but below 15 years of age; 6000 per months.

(a) Calculate James's taxable income for the month of june [Ans: 397,000]

(b) (i) calculate the income tax John pays in June using the tax rate below; [Ans: 75,750]

Income (shs) per month	Tax rate
0 - 100,000	10
100,001 - 150,000	15
150,000 - 220,000	20
220,001 and above	25

(ii) Calculate the income tax he pays as a percentage of his taxable income. [Ans: 13.77%]

39. In a certain organization the income tax system operates as follows;

Rates on taxable income:

30% on the first \$4000

40% on the next \$2000

50% on the next \$2000

60% on the next \$2000

65% on any amount in excess of the above

Allowances:

$\frac{2}{9}$ of earned income is tax free

Single \$400, married \$750

Children; one child(\$200), 2 children (\$450), 3 children (\$700)

Insurances and other expenses.

A doctor is married with two children, has an annual salary of \$21600. He is entitled to \$300 for life insurance premium.

Calculate;

(i) How much income tax would he pay? [Ans: £7645]

(ii) What percentage of his annual salary he would pay as income tax? [Ans: 35.39%]

40. Peter deposited shs 7500 in a bank which paid compound interest of 14% per annum. Calculate;

a) the interest earned at the end of one year. [Ans: shs. 1,050]

b) the amount after two years. [Ans: shs. 9,747]

41. A printer bought a machine at shs 2,100,000. If the depreciation is 15% per annum, Calculate the value of the machine after 3 years. [Ans: shs. 1,289,622.5]

42. The out fitter for the school was asked to stock three different types of school jumper. He decided to purchase 40 V - neck jumpers, 30 pull-neck jumpers and 30 crew-neck jumpers.

a) He bought the V-neck jumpers for shs 137.50 each and sold them at a profit of 40% on the cost price. Calculate the selling price of each V-neck jumper. [Ans: 19,250/=]

b) He bought the polo-neck jumpers at shs 157.50 each and sold them at shs 210 each. Express the profit as a percentage of the cost price. [Ans: $33\frac{1}{3}\%$]

c) He sold the crew-neck jumpers at shs 180, thereby losing 10% on the cost price. Calculate the cost price of each crew-neck jumper. [Ans: shs. 20,000]

- d) Calculate the total profit he made on the sale of all the jumpers and express this profit as a percentage of his initial out-lay. [Ans: profit = 317,500, 19.6%]
44. (a) By selling a radio at shs 80,000 each, a profit of 25% would be made on the cost of manufacture. Calculate the cost of manufacture of a radio. [Ans: shs. 64,000]
- b) 2000 of these radios were manufactured. 23 of them were found to be imperfect and were sold for shs 24,000 while the perfect ones were still sold for shs 80,000. Express the total profit as a percentage of the total cost of manufacture assuming that all 2000 radios were sold. [Ans: 24%]
45. Below is an advertisement for a radio.

RADIO	RADIO
TERMS: CASH 700,000/= LESS 15% DISCOUNT,	
OR MONTHLY HIRE PURCHASE: DEPOSIT 200,000/= AND THEN 4 MONTHLY INSTALLMENTS OF SHS 150,000/= EACH.	
OR WEEKLY HIRE PURCHASE: DEPOSIT 100,000/= AND THEN 8 WEEKLY INSTALLMENTS OF 100,000/= EACH	

- Calculate the (a) Cash price. [Ans: shs. 595,000]
- (b) Monthly hire purchase price. [Ans: shs. 800,000]
- (c) Weekly hire purchase price. [Ans: shs. 900,000]
- (d) saving a customer would make by buying the radio on cash basis rather than weekly hire purchase. [Ans: shs. 305,000]
- (e) saving a customer would make by buying the radio on cash basis rather than monthly hire purchase. [Ans: shs. 205,000]

46. (a) A customer deposited shs 2,800,000 in a bank at compound interest rate of 8% per annum. If his interest amounted to shs 727,193.60, how long did he invest this money? [Ans: 3 years]

(b) Advertisement

GET YOURSELF A COMPUTER CHEAPLY WHILE STOCK LAST !!
TERMS: CASH AT UG SHS 1.96 MILLION
HIRE PURCHASE: DEPOSIT 30% OF THE CASH PRICE AND PAY EITHER UG SHS 85,000 WEEKLY FOR 20 WEEKS
OR UG SHS 245,000 MONTHLY FOR 10 MONTHS

Calculate the:

- i) savings a customer would make by buying the computer on cash basis rather than on weekly hire purchase. [Ans: shs. 328,000]
- ii) Which mode of hire purchase payment would be cheaper? [Ans: weekly hire purchase]

BEARING

RECALL

Remember the following key issues before attempting any question

- ◊ All drawings in bearing must be drawn on a graph paper.
- ◊ All angles in bearing are measured starting from the North in a clockwise direction.
- ◊ You must know the compass and all the different directions described on it.
- ◊ A sketch diagram is a must as any answer without a sketch diagram is invalid. It should be drawn using a free hand(you don't need to measure).
- ◊ Be in position to convert from one metric unit to another. Eg km to m or cm etc.
- ◊ Never attempt any question on bearing without a protractor, a ruler, squared paper and a pencil.
- ◊ Neatness is a must requirement that earns you a bonus mark.
- ◊ You must know how to calculate speed, distance and time.
- ◊ The formula for distance = speed \times time = $S \times T$. This formula has to be mastered.
- ◊ Read the question at least two times before starting to attempt any bearing numbers.

TRIAL QUESTIONS

1. A plane flew due west from airstrip P at a speed of 280 kmhr^{-1} for $\frac{3}{4}$ hours before reaching airstrip Q. It then altered its course and flew North-West to airstrip R at 220 km/hr . From there it flew on a bearing of 060° to airstrip S at 240 km/hr for $1\frac{1}{2}$ hours. The total time of flight between the four airstrips was $4\frac{1}{2}$ hours.

- (i) By scale drawing, determine the distance and bearing of P from S. Use a scale of 1 cm to represent 50 km.
- (ii) Determine the total distance of flight from P to S and hence the average speed for the journey.

- (iii) If the plane flew directly back to P at a speed of 200 km/h , determine how long it took to fly back to P.

2. A plane moves from airport A to B, 610 km away on a bearing of 010° , there after it changes its course on a bearing of 217° and covers a distance of 500 km to C. It then moves 500 km eastwards to airport D.
 - (a) Using a scale of 1cm to represent 50km, draw an accurate diagram for the whole journey
 - (b) Find the bearing and distance of A from D.
 - (c) If the plane was using a speed of 400 km/h , calculate the time taken for the plane to move from A to D.
3. A plane flew due west from airstrip P at a speed of 280 kmhr^{-1} for $\frac{3}{4}$ hours before reaching airstrip Q. It then altered its course and flew North-West to airstrip R at 220 km/hr . From there it flew on a bearing of 060° to airstrip S at 240 km/hr for $1\frac{1}{2}$ hours. The total time of flight between the four airstrips was $4\frac{1}{2}$ hours.
 - (i) By scale drawing, determine the distance and bearing of P from S. Use a scale of 1 cm to represent 50 km.
 - (ii) Determine the total distance of flight from P to S and hence the average speed for the journey.
 - (iii) If the plane flew directly back to P at a speed of 200 km/h , determine how long it took to fly back to P.
4. A place flies from airport A to airport B, 610 km away on a bearing of 010° . There after, it changes its course and flies on a bearing of 217° for a distance of 500 km to airport C. It then flies eastwards to airport D, also 500 km away from C.
 - (a) Using a scale of 1cm represent 50 km, draw an accurate diagram for the whole journey.
 - (b) Find the bearing and distance of A from D.
 - (c) If the plane flies at a speed of 400 km/hr , calculate the time it will take to fly directly from d back to A.

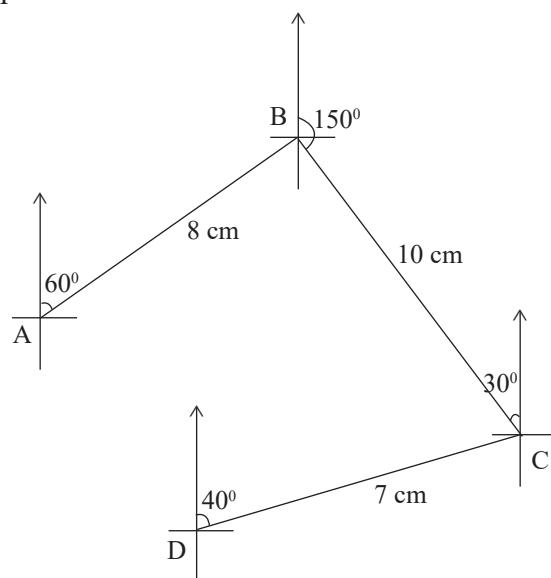
Bearing

5. The bearings of towns A and B from town P are 240° and 150° respectively. If B is 720 km from A on the bearing of 120° ,
 - (a) Draw a sketch to show the relative positions of the three towns.
 - (b) Using your sketch in (a) above calculate distance; i) AP (ii) BP
 - (c) Find the bearing of P from B.
 - (d) If a plane flies from P to B via A at a speed of 450 km/h, find how long it takes.
6. (a) A, B and C are three points on the same level ground. The bearing of C from A is 150° and B from A is 060° . From A, the angle of elevation of the top of a tower at C is 30° , and from B the angle of elevation of the top of the same tower is 17° . If the distance BC is 2 km. Calculate the;
 - (i) the height of the tower
 - (ii) distance AC
 - (iii) the bearing of B from C.
- (b) Two ships Q and R are sailing towards P. At 1144 hours, ship Q is exactly 120 km on bearing of 030° from P and ship R is 50km on a bearing of 300° from P. At this instant, ship R develops engine troubles and cannot continue with the journey. Ship Q receives a signal from ship R and has to change course and steam straight towards ship R at 50 km/h. With out using scale drawing, calculate the time of the day ship Q reaches ship R.
7. Two aeroplanes P and Q leave an airport at the same time. P flies on a bearing of at $900\text{ km}/\text{h}$ while Q flies due East at $750\text{ km}/\text{h}$.
 - (a) Using a scale of 1 cm to represent 100 km, draw an accurate diagram showing the positions of the two aeroplanes after 1hour 20 minutes.
 - (b) From the diagram in (a) above determine;
 - (i) the distance between the two aeroplanes
 - (ii) bearing of P from Q
 - (iii) the bearing of Q from P.
8. Town Q is 400 km on a bearing of 080° from town P. Town R is 350 km on a bearing of 200° from town Q and town S is 250 km on a bearing of 210° from P.
 - (a) Using a scale of 1 cm to 50 km, draw an accurate diagram to show the relative position of the four towns.
 - (b) Find the distance and bearing of S from R.
 - (c) A helicopter flies from one round the four towns. Find it's average speed if it takes 4 hours 30 minutes all together.
9. A helicopter flies from Moroto due south for 300 km. it then flies on a bearing of 255° for 350 km. from there it flies on a bearing of 02° for 400 km.
 - (i) Draw an accurate diagram showing the journey of the helicopter using a scale of 1 cm: 50 km.
 - (ii) Use your diagram to find the distance and the bearing of Moroto from the final position of the helicopter.
 - (iii) Given that the helicopter flies at a steady speed of $200\text{ km}/\text{hr}$, find how long the whole journey took
10. A plane leaves an airport A for B which is 300 km away on a bearing of 315° . At B, the plane changes it's course and flies due south to air port C which is 400 km from B. At C, the plane flies on a bearing of 180° to airport D 500 km from C.
 - (a) Using a scale of 1cm to represent 50km, draw an accurate diagram to represent the helicopter's journey.
 - (b) Use the diagram to determine;
 - (i) The bearing of airport A from airport D.
 - (ii) The distance of airport A from airport D in km.
 - (c) If the plane was flying at an average speed of $1500\text{ km}/\text{hr}$. Find the time it took to reach D given no stop over on the way.
11. A ship leaves a port and sails for 120 km on the bearing of 062° . It then changes direction to a bearing of 160° and sails for 200 km to an island. Using a scale drawing with 1cm representing 20 km, find;
 - (a) the distance of the island from the port.
 - (b) the bearing of the port from the island.
 - (c) How long would it take a ship to sail directly back to the port at a speed f $20\text{ km}/\text{hr}$.
12. A ship leaves a port and sails for 130 km on a bearing of 65° . It then changes direction on a bearing of 170° and sails for 180 km to an island.

- Use a scale drawing with 1 cm representing 20 km find;
- the distance of the island from the port
 - the bearing of the port from the island
 - how long would it take the ship to sail directly back to the port at speed of 20 km/hr.
13. Town B is 300 km due west of town A. Town C is 450 km from town B in the direction N60° W and D is 600 km from C in the direction S30°W. plane k starts from town A and then flies to town B, C and then D at an average speed of 300 km/hr. plane M flies directly from A to D at an average speed of 200 km/hr.
- find the time taken by each plane.
 - what is the bearing of town A from town D?
14. A boat sailed from P on a bearing of 060° for 40 km to Q, then sailed on a bearing of 110° to point R then finally on a bearing of 210°. If the boat sailed at a speed of 20 kmh⁻¹ and from Q to R it took $2\frac{1}{2}$ hours and then R to S it was $1\frac{1}{2}$ hours.
- Draw a sketch showing the points P,Q,R, and S
 - Using a scale of 1 cm to 10 km, draw a scale drawing showing the route of the boat.
 - what is the; (i) distance (ii) bearing of S from P
 - Find the total time taken for the boat to sail from P to S
15. Leocardia travels from home to a trading centre at a bearing of 030°, a distance of 300 km. from the trading centre, she then travels 600 km to the school which is at a bearing of 130° from the trading centre. Assuming that she travels at a constant speed of 20 km/hr and that 1 cm represents 50 km.
- draw a sketch to show her journey to school.
 - Draw an accurate diagram for the journey
 - what is the bearing of; (i) Home from school (ii) school from home
 - measure the distance between home and school
 - calculate the total time spent on her journey
 - if she uses the shortest route to school, calculate the time she takes if she moves at the same speed.
16. Jesca has to travel from town A to town D but because of a very big island between towns A and D, he travels from town A at a bearing of 150° a distance of 200 km to town B. from town B, he moves at a bearing of 090° a distance of 300 km to town C. from C he moves to town D which is 400km from C at a bearing of 020°. Using a scale of 1 cm to represent 45 km.
- Draw a diagram to show her journey
 - Measure,
 - Distance of A from D
 - Bearing of D from A
 - If he took a total time of five hours to travel and assuming that he travelled at a constant speed. Calculate his speed.
17. From the airstrip at Johannesburg, an aeroplane moves at a bearing of 090° to Botswana a distance of 200 km. it then moves at a bearing of 020° a distance of 300 km to town A. at town A, it rests for two hours and then moves at a distance of 170° to Kigali which is 500 km from town A. (use scale 1cm: 50 km)
- draw a diagram to show the journey of the plane
 - distance between Johannesburg and Kigali
 - Bearing of Johannesburg from Kigali.
 - Time spent, if the plane travelled at a constant speed of 80 km/hr.
18. A cargo ship sails from island A to island B at a bearing of 250° and a distance of 250 km from island B. It moves N30°W a distance of 550 km to island C. If 1cm represents 60 km.
- Draw a diagram to illustrate the ship's journey.
 - Distance between island C and A
 - If the shop took 2 hours to travel from island A to B and $3\frac{1}{2}$ hours to travel from B to C. calculate the respective speeds.
 - Bearing of;
 - Island C from A
 - Island A from C

Bearing

19. In the last year's motorcar sport competitions, motorcars started from town P and moved at a bearing of 060° for a distance of 120 km to town Q. They then moved in a direction described as $\text{N}80^{\circ}\text{E}$ a distance of 150 km to town R. Then they moved at a bearing of 230° for 300 km to town S. If 1 cm represents 20 km ;
- Draw a diagram to illustrate their journey
 - Bearing of P from S
 - If a car was to take a short cut from Q to S, what distance did it go through.
 - What is the bearing of Q from S.
20. Dorcus started running from station L at a bearing of 330° for a distance of 10 km to station M. she then moved in the Easter direction at a speed of 5 km/hr for 2 hours to junction K. she then turned through a bearing of 072° and moved for 15 km to station N. she then moved to station P which is 18 km south of station L. If 1 cm represents 2 km ,
- Draw a diagram to show her journey
 - distance and bearing of L from P
 - distance and bearing of L from N
 - Total time spent if she was running at a speed of 2 km/hr .
21. Points P, Q, R and S are oriented on the play ground such that point B is at a bearing of 000° from point P, point S is at a bearing of 340° from point P, point R is at a bearing of 070° from point Q. if distance PS is 100 m , PQ is 40 m and QR is 30 m and that 1 cm represents 7 m .
- Draw an accurate diagram to show the relationship between the points.
 - bearing of S from Q and R.
22. The figure below shows the shape of Rebecca's farm formed by poles A, B, C and D and 1 cm represents 100 m



If the owner wants to add a pole between B and C such that $BK : KC = 1 : 3$ where K is the pole point.

- Draw an accurate diagram including point K to represent the farm.
- Measure distance AK and KD in metres
- Determine the bearing of;
 - K from A
 - A from K
 - K from D.
- Determine distance KD in metres and bearing of A from D

FACTORIZATION

RECALL

- ◊ This is the process of creating a list of factors. When these are multiplied together, will produce a desired quantity.
- ◊ In general, it is a process of breaking up an expression into its separate factors.
- ◊ The easiest type of factor to find is a single term factor common to all terms of the expression and this is usually obvious.

For example;

Factorise $2p - 2q$.

Solution

- ◊ Realise that 2 is common through out, then factorise it out and introduce brackets, i.e; $2(p - q)$
Note: Factors of $2p - 2q$ are 2 and $(p - q)$.
- ◊ Therefore when factorizing an expression, first see if there is a common factor of each term.
- ◊ Group together all terms which have a common factor and then factorize each term before finding a bracket which is one of the factors of the original expression.

Example 1

Factorize $4x - 4y + 7x - 7y$

Solution

Collect together the terms which have a common factor.

$$4x - 4y + 7x - 7y$$

$$4(x - y) + 7(x - y)$$

$$(x - y)(4 + 7)$$

$$= 11(x - y)$$

OR

Collect like terms

$$4x + 7x - 4y - 7y$$

$$11x - 11y$$

$$11(x - y)$$

Example 2

Factorize the following expressions

(a) $33km - 44kr$

Find the H.C.F

$$\text{HCF} = 11k$$

$$11k \times 3m - 11k \times 4r$$

factor out $11k$ because its common

$$\underline{11k(3m - 4r)}$$

(b) $7k^2 - k$

HCF is k

$$k \times 7k - k \times 1$$

k is common

$$\underline{k(7k - 1)}$$

(c) $au + av - u - v$

solution

collect like terms

$$au - u + av - v$$

$$u(a - 1) + v(a - 1)$$

but $(a - 1)$ is common, so factor it out

$$(a - 1)(u + v)$$

(d) $fm + fn + gm + gn$

Solution

collect like terms

$$fm + gm + fn + gn$$

$$m(f + g) + n(f + g)$$

$(f + g)$ is common, so

$$(f + g)(m + n)$$

Example 3

Factorize the following

$$\begin{aligned}
 & \text{(a) } a^2p + a^2p + a^2q, a^2 \text{ is common} \\
 & \quad pq^2 \text{ and } p^2q, pq \text{ is common} \\
 & \quad a^2p + a^2q - pq^2 - p^2q \\
 & \quad a^2(p+q) - pq(q+p) \\
 & \quad \text{but } (p+q) \text{ is common} \\
 & \quad = (p+q)(a^2-pq)
 \end{aligned}$$

$$(b) 1 + y^2 + xy(1 + y^2)$$

Realise that $(1+y^2)$ is common then factor out
 $= (1+y^2)(1+xy)$

$$\begin{aligned}
 & (c) xp - 4xp^2 - z + 4zp \\
 & \quad xp(1-4p) - z(1-4p) \\
 & \quad = (1-4p)(xp-z)
 \end{aligned}$$

DIFFERENCE BETWEEN TWO SQUARES

- ◊ The product of the sum and the difference of two numbers is equal to the difference of their squares.
- ◊ $a^2 - b^2 = (a+b)(a-b)$. This is a fundamental identity at o'level.

Example 1

Factorize the following

$$(i) x^2 - 9$$

solution

$$(x)^2 - (3)^2$$

$$\text{But } (a^2 - b^2) = (a+b)(a-b)$$

$$(x)^2 - (3)^2 = (x+3)(x-3)$$

$$(ii) p^2 - 36$$

solution

$$(p)^2 - (6)^2$$

$$\text{But } a^2 - b^2 = (a+b)(a-b)$$

$$p^2 - 6^2 = (p+6)(p-6)$$

$$(iii) p^2 - 4q^2$$

solution

$$p^2 - 2^2q$$

$$p^2 - (2q)^2$$

$$\text{But } a^2 - b^2 = (a+b)(a-b)$$

$$p^2 - (2q)^2 = (p+2q)(p-2q)$$

$$(iv) 4y^2 - 9a^2b^2$$

solution

$$4y^2 - 9a^2b^2$$

$$2^2y^2 - 3^2a^2b^2$$

$$(2y)^2 - (3ab)^2$$

$$\text{but } a^2 - b^2 = (a+b)(a-b)$$

$$(2y)^2 - (3ab)^2 = (2y+3ab)(2y-3ab)$$

$$(v) (a+b)^2 - c^2$$

solution

$$\text{But } a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^2 - c^2 = (a+b+c)(a+b-c)$$

$$(vi) 8x^2 - 18$$

solution

$$2(4x^2 - 9)$$

$$2(2^2x^2 - 3^2)$$

$$2((2x)^2 - 3^2)$$

$$\text{But } (2x)^2 - 3^2 = (2x+3)(2x-3)$$

$$2[(2x+3)(2x-3)]$$

$$2(2x+3)(2x-3)$$

Example 2

Factorise the following

$$(a) 27a^2 - 12$$

solution

$$3(9a^2 - 4)$$

$$3(3^2a^2 - 2^2)$$

$$3(3a)^2 - 2^2)$$

$$3[(3a+2)(3a-2)]$$

$$3(3a+2)(3a-2)$$

$$(b) a^2b^2c^2 = 1$$

solution

$$(abc)^2 - 1^2$$

$$= (abc+1)(abc-1)$$

$$(c) 48a^3b^2 - 27ab^4$$

solution

$$3ab^2(16a^2 - 9b^2)$$

$$3ab^2(16a^2 - 9b^2)$$

$$\text{But } 3ab^2(4^2a^2 - 3^2b^2)$$

$$3ab^2[(4a)^2 - (3a)^2]$$

$$3ab^2[(4a+3b)(4a-3b)]$$

$$3ab^2(4a+3b)(4a-3b)$$

Example 3

Without using calculators or tables, evaluate;

$$(i) 7.46^2 - 2.54^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$7.46^2 - 2.54^2 = (7.46 + 2.54)(7.46 - 2.54)$$

$$= (10)(4.92)$$

$$= 49.2$$

$$(ii) 5.2 \times (3.75^2 - 1.25^2)$$

solution

$$a^2 - b^2 = (a + b)(a - b)$$

$$3.75^2 - 1.25^2 = (3.75 + 1.25)(3.75 - 1.25)$$

$$5.2 \times (3.75 + 1.25)(3.75 - 1.25)$$

$$5.2 \times (5)(2.5)$$

$$\frac{52}{10} \times \frac{5}{1} \times \frac{25}{10}$$

$$= 65$$

$$(iii) \frac{6.51^2 - 2.81^2}{0.932 \times 4}$$

solution

$$\frac{(6.51 + 2.81)(6.51 - 2.81)}{0.932 \times 4}$$

$$\frac{(9.32)(3.70)}{\frac{932}{1000} \times 4} = \frac{\frac{932}{100} \frac{37}{10}}{\frac{932 \times 4}{1000}}$$

$$= \frac{932 \times 37}{1000} \div \frac{932 \times 4}{1000}$$

$$= \frac{932 \times 37}{1000} \times \frac{1000}{932 \times 4}$$

$$= \frac{37}{4}$$

FACTORIZATION OF A QUADRATIC EXPRESSION.

- ◊ Remember a quadratic expression is of form $ax^2 + bx + c$ but when we equate the expression to zero ie; $ax^2 + bx + c = 0$, it becomes an equation because of the equal sign.
- ◊ When factorizing the above quadratic expression, we find two numbers such that their sum is b and their product is ac

$$\begin{array}{c} ax^2 + bx + c \\ \diagup \quad \diagdown \\ ac \end{array}$$

Example 1

Factorise $x^2 - 4x + 3$

Look for numbers such that their sum is -4 and their product is 3 .

$$\begin{array}{c} x^2 - 4x + 3 \\ \diagup \quad \diagdown \\ ac = 1 \times 3 = 3 \end{array}$$

The numbers are -1 and -3

$$x^2 - x - 3x + 3$$

$$x(x - 1) - 3(x - 1)$$

$$(x - 1)(x - 3)$$

$$x^2 - 4x + 3 = (x - 1)(x - 3)$$

Example 2

$$2y^2 + y - 6$$

solution

$$\begin{array}{c} 2y^2 + y - 6 \\ \diagup \quad \diagdown \\ 2 \times 6 = -12 \end{array}$$

look for two numbers that when multiplied give -12 but when added, give 1 . the numbers are 4 and -3

$$2y^2 + 4y - 3y - 6$$

$$2y(y + 2) - 3(y + 2)$$

$$(y + 2)(2y - 3)$$

$$2y^2 + y - 6 = (y + 2)(2y - 3)$$

Example 3: Factorize: $(p + 2q)^2 - 4(p + 2q) + 3$

Let $p + 2q$ be y (becomes $p + 2q$ is common)

$$y^2 - 4y + 3. \text{ Then,}$$

$$\begin{array}{c} y^2 - 4y + 3 \\ \diagup \quad \diagdown \\ 3 \end{array} \text{ factors are } -1, -3$$

$$y^2 - y - 3y + 3$$

$$y(y - 1) - 3(y - 1)$$

$$(y - 1)(y - 3) \text{ but } y = p + 2q$$

$$(p + 2q - 1)(p + 2q - 3)$$

Example 4: Factorize $20x^2y^2 + xy - 1$

Solution

$$20x^2y^2 + xy - 1$$

$$20(xy)^2 + xy - 1$$

$$\text{let } xy = n$$

$$\begin{array}{c} 20n^2 + n - 1 \\ \diagup \quad \diagdown \\ -20 \end{array} \text{ factors are } 5 \text{ and } -4$$

$$20n^2 + 5n - 4n - 1$$

$$5n(4n + 1) - (4n + 1)$$

$$(4n + 1)(5n - 1)$$

$$\text{but } n = xy$$

$$(4xy + 1)(5xy - 1)$$

TRIAL QUESTIONS

Without using tables or calculators (Use factorization)

1. Find the values of;

(a) $0.1 \times 45 + 0.1 \times 55$

[Ans: 10]

(b) $500 \times 4 + 4 \times 600$

[Ans: 4400]

(c) $(21.73 \times 14.6) + (2173 \times 5.4)$

[Ans: 434.6]

(d) $(617 \times 793) + (786 \times 793) + (597 \times 793)$

[Ans: 1586000]

(e) $(75 \times 130 - 75 \times 30)$

[Ans: 7500]

(d) $\frac{(98 \times 154 - 98 \times 54)}{(10 \times 6.3) + 10 \times 3.7}$

[Ans: 98]

2. Factorise the following expressions;

(a) $8xy + 4xy^2$

[Ans: $4xy(2 + y)$]

(b) $8x^2yz + 4xy^2z + 12xyz^2$

[Ans: $4xyz(2x + y + 3z)$]

(c) $P + \frac{PRT}{100}$

[Ans: $P\left(1 + \frac{RT}{100}\right)$]

(d) $\pi r^2 h + \frac{2}{3} \pi r^3$

[Ans: $\pi r^2(h + \frac{2}{3}r)$]

3. Factorize the following

(a) $(a - b)^2 - 2(a - b)$

[Ans: $(a - b)(a - b + 2)$]

(b) $p(x - 1) + (x - 1)$

[Ans: $(x - 1)(p + 1)$]

(c) $2a^2(a - x) + ax(a - x)$

[Ans: $a(a - x)(2a + x)$]

(d) $a(a - b) - 3(a - b)$

[Ans: $(a - b)(a - 3)$]

(e) $x(p^2 + 2pq + q^2) - 3y(p + q)^2$

[Ans: $(p + q)^2(x - 3y)$]

(f) $(x - 1)^2 - yx + y$

[Ans: $(x - 1)(x - y - 1)$]

(g) $x(p + w + z) + y(p + w + z)$

[Ans: $(p + w + z)(x + y)$]

4. Factorize the following expressions

(a) $8y^2 - 32$

[Ans: $8(y + 2)(y - 2)$]

(b) $144p^2 - 25$

[Ans: $(12p + 5)(12p - 5)$]

(c) $2x^2 - 18$

[Ans: $2(x + 3)(x - 3)$]

(d) $5x^2 - 125$

[Ans: $5(x + 5)(x - 5)$]

(e) $9q^2 - 9$

[Ans: $9(x + 1)(x - 1)$]

(f) $3x^4 - 3$

[Ans: $3(x + 1)(x - 1)(x^2 + 1)$]

(g) $25p^2 - 36$

[Ans: $(5p + 6)(5p - 6)$]

(h) $x^2 - 16$

[Ans: $(x + 4)(x - 4)$]

(i) $9 - 16p^2q^2$

[Ans: $(3 + 4pq)(3 - 4pq)$]

(j) $16x^2 - 64$

[Ans: $(16(x + 2)(x - 2)$]

(k) $25p^2 - 49q^2$

[Ans: $(5p + 7q)(5p - 7q)$]

(l) $m^4 - n^4$

[Ans: $(m^2 + n^2)(m + n)(m - n)$]

(m) $45 - 20k^2$

[Ans: $5(3 + 2k)(3 - 2k)$]

(n) $8x^2 - 18y^2$

[Ans: $2(2x + 3y)(2x - 3y)$]

(o) $q^4 - 625$

[Ans: $(q^2 + 25)(q + 5)(q - 5)$]

(p) $3(x - 4)^2 - 12$

[Ans: $(3(x - 2)(x - 6)$]

(q) $3p^3 - 12pq^2$

[Ans: $3p(p + 2q)(p - 2q)$]

5. Use factorization to find the values of

(a) $416^2 - 284^2$

[Ans: 92400]

(b) $90^2 - 10^2$

[Ans: 8000]

(c) $\frac{6.51^2 - 2.81^2}{0.932 \times 74}$

[Ans: $\frac{1}{2}$]

6. Factorize the following;

(a) $x^2 - x - 156$

[Ans: $(x + 12)(x - 13)$]

(b) $x^2 - 3x$

[Ans: $x(x - 3)$]

(c) $x^2 + 9x - 36$

[Ans: $(x - 3)(x + 12)$]

(d) $x^2 - x - 12$

[Ans: $(x + 3)(x - 4)$]

(e) $2x^2 + 9x - 11$

[Ans: $(x - 1)(2x + 11)$]

(f) $2t^2 + t + 6$

[Ans: $(2t + 3)(2 - t)$]

(g) $9n^2 + 18n - 7$

[Ans: $(3n - 1)(3n + 7)$]

(h) $6x^2 - x - 12$

[Ans: $(3x + 4)(2x - 3)$]

(i) $36 - (x^2 + 2xy + y^2)$

[Ans: $(6 + x + y)(6 - x - y)$]

j) $2 + 2x^2 + xy + x^3y$

[Ans: $(1 + x^2)(2 + xy)$]

(k) $9(x + 2)^2 - 12(x + 2)$

[Ans: $3(x + 2)(3x + 2)$]

ALGEBRA OF FRACTIONS AND EQUATIONS

RECALL

- ◊ This involves dividing the numerator by the denominator.
- ◊ This involves creating factors of a denominator and the numerator where need be and then cancel the factors of the same kind.
- ◊ When operations (adding, subtraction) are involved, then find the LCM of the denominators and proceed normally to simplify. (review operations on fractions).

Example: Simplify the following fractions;

$$(a) \frac{x+1}{x^2-1}$$

Solution

$\frac{x+1}{x^2-1}$, But $\frac{x^2+1}{x^2-1^2} = (x+1)(x-1)$
(creating factors of the denominator)

$\frac{(x+1)}{(x+1)(x-1)}$ (canceling the same factors)

$$= \frac{1}{x-1}$$

$$(b) \frac{x}{1-x} - \frac{x}{1+x} + \frac{2x}{1-x^2}$$

solution

$$1-x^2 = (1-x)(1+x)$$

$$\frac{x(1+x)-x(1-x)+2x}{(1-x)(1+x)}$$

$$\frac{x+x^2-x+x^2+2x}{(1-x)(1+x)}$$

$$\frac{2x^2+2x}{(1-x)(1+x)} = \frac{2x(x+1)}{(1-x)(1+x)} = \frac{2x}{1-x}$$

- ◊ But when an expression is equated to zero, then it becomes an equation which requires to be solved.

Equations

- ◊ An equation is a statement in algebra which states that the two given expressions are the same. In between the two expressions there exists an equal sign.

e.g, $x+9=10$, $\frac{x^2-x+2}{4(x-2)^2}=0$, But equations with one unknown of first degree are called linear equations
e.g $x+3=5$.

Example: Solve for x : $\frac{5}{x} + \frac{1}{3} = \frac{8}{9}$

Solution

LCM of x , 3 and 9 is $9x$, Then multiply $9x$ throughout on both sides.

$$\frac{5}{x} \times 9x + \frac{1}{3} \times 9x = \frac{8}{9} \times 9x$$

$$45 + 3x = 8x$$

$$45 = 5x$$

$$\frac{45}{5} = \frac{5x}{5}$$

$$x = 9$$

TRIAL QUESTIONS

1. $\frac{2x+5}{x-1} - \frac{x+4}{x+3} = 1$

[Ans: $x = \frac{-11}{3}$]

2. $\frac{4}{3x} - 4 + \frac{2}{x} = \frac{4}{5}$

[Ans: $x = \frac{25}{36}$]

3. $\frac{3}{x+3} - \frac{1}{5x-2} = 0$

[Ans: $x = \frac{9}{14}$]

4. $\frac{2m-5}{5} + \frac{m-4}{3} = \frac{m+14}{30}$

[Ans: $m = 4$]

5. $\frac{p-5}{2} + \frac{p+2}{3} = \frac{4p-1}{3}$

[Ans: $p = -3$]

6. Mr. Zziwa is three times as old as his daughter. 15 years ago, the father was eighteen times as old as his daughter. Find the present age of;

(i) the daughter [Ans: 17 years]

(ii) Mr. Zziwa [Ans: 51 years]

7. Solve the following equations

(a) $\frac{4x-1}{2} + \frac{x+3}{3} + \frac{x+7}{6} = 0$ [Ans: $\frac{2}{3}$]

(b) $\frac{1}{x} + \frac{3}{2x-7} = 0$ [Ans: $\frac{7}{5}$]

(c) $\frac{4}{5}(k+1) - \frac{k}{3} = 2\frac{2}{3}$ [Ans: $k = 4$]

(d) $\frac{y+3}{2\frac{1}{2}} + \frac{y-2}{3\frac{1}{3}} = \frac{3}{5}$ [Ans: $y = 0$]

(e) $\frac{1}{6}(x+2) - \frac{1}{9}(2x-3) = \frac{1}{12}(1-3x)$
[Ans: $x = -3$]

(f) $\frac{3}{4}(2p-5) + \frac{1}{6}(p+7) = \frac{2}{3}(p-2)$
[Ans: $p = \frac{5}{4}$]

(g) $\frac{x}{1-x} - \frac{x}{1+x} + \frac{2x}{1-x^2} = 1$ [Ans: $x = \frac{1}{3}$]

8. Muzungu is three times as old as his son. In 12 years time, he will be twice as old as his son.

(i) How old is his son now? [Ans: 12 years]

(ii) How old is the father now? [Ans: 36 years]

9. Rebecca is five years younger than her sister Muwanguzi and Annet is twice as old as Rebecca. The sum of their ages is 49. Find;

(i) Muwanguzi's age [Ans: 16 years]

(ii) Rebecca's age [Ans: 11 years]

(iii) Annet's age [Ans: 22 years]

10. Solve the following equations

(a) $\frac{3y}{2} - \frac{y}{3} = \frac{5(y-4)}{6}$ [Ans: $y = -10$]

(b) $\frac{1}{3}(t-2) = \frac{1}{4}(t+1)$ [Ans: $t = 11$]

(c) $\frac{q+3}{5} = 8 - \frac{q-1}{4}$ [Ans: $q = 17$]

(d) $\frac{x}{2} - \frac{x+1}{3} = \frac{1}{3}$

[Ans: $x = 4$]

11. After marking a test, Recheal got twice as many marks as Nakato and Christine got 8 marks more than Recheal. The total of all their marks was 223. What is;

(i) Nakato's marks? [Ans: 43 years]

(ii) Recheal's marks? [Ans: 86 years]

(iii) Christine's marks? [Ans: 94 years]

12. Nyangoma travels 6 km to work. He walks parts of the way at 5 km/hr and gets a bus the rest of the way at 40 km/hr. The whole journey takes 30 minutes. How far does he walk? [Ans: 2 km]

13. Winniefred has twice as much money as her sister. Together we have shs 60,000. How much money does Winniefred's sister have?

[Ans: 20,000]

14. Kaguta is twice as old as his son Kasolo. Ten years ago he was three times as old as his son. How old is the son at present? [Ans: 20 years]

15. Angle A of triangle ABC is double the size of angle B which is three times the size of angle C. Find the size of the angles A, B, C.
[Ans: A = 108°, B = 54°, C = 18°]

16. In a certain shop, a 75W bulb costs shs 200 less than a 100W bulb. Joseph bought two 75W bulbs and five 100W bulbs and altogether paid shs 6,600. Determine the cost of one 75W bulb.
[Ans: shs. 800]

17. A man pays an electricity bill of shs. 250,000 partly in twenty thousand shilling notes and partly in ten thousand shilling notes. Altogether fifteen notes were handed over to the Uganda Electricity Board cashier. Determine the number of notes of each denomination.
[Ans: twenty thousand shilling notes = 10, ten thousand shilling notes = 5]

18. John is now 10 years. If in 17 years John's father will be twice as old as John, how old is John's father now? [Ans: 37 years]

19. When 5 is added to a certain number and the sum is doubled, the result is 22. Find the original number. [Ans: 6]

20. When 7 is added to a certain number and the sum is doubled, the result is 10 more than 4 times the original number. [Ans: 2]

21. When 11 is added to a certain number, and the sum tripled the result is 12 less than 8 times the original number. Find the original number.
[Ans: 11]

22. John is 5 times as old as his son. Fifteen years from now, the son will be half as old as his father. Find the present age of the son.
[Ans: 5 years]

23. Susan is 7 years older than Ibrah. In five years time, Suzan will be twice as old as Ibrah. What will be their ages in ten years time?
[Ans: Suzan = 19 years, Ibrah = 12 years]

24. A transport company uses two types of vehicles to transport bricks to a site. The vehicles used are a lorry and a tipper. The profit earned by the company when it transports bricks to the site using a lorry is shs 10,000 above the cost of using a tipper. When 10 lorries and 15 tippers are used to deliver the bricks to the site, the company altogether earns a profit of shs. 600,000. determine the profit earned when;

i) One tipper is used to transport the bricks.

[Ans: 20,000/=]

ii) 20 lorries and 25 tippers are used to transport the bricks. [Ans: shs.1,100,000]

25. Mr. Samuel paid his house rent of shs.300,000 using five thousand and ten-thousand shilling notes. If the number of five thousand shilling notes exceeds the number of ten-thousand shilling notes by 30, determine the number of

i) five- thousand shillings notes. [Ans: 40]

ii) ten-thousand shilling notes. [Ans: 10]

26. Kakoola a secondary school which is partly day and boarding charges a student according to whether a student is a day scholar or a boarding. A boarder is charged shs.100,000 more than a day scholar. In term two of year 2005, the school had 200 boarders and 300 day scholars. The total charge collected during the term after all the 500 students had fully cleared school dues was shs.95,000,000. Determine the full amount paid by a;

i) boarding student. [Ans: shs.250,000]

ii) day scholar [Ans: 150,000]

iii) Parent of 2 boarding students and 5 day scholar students. [Ans: 1,250,000]

27. Mr. Opio goes for shopping with a twenty -thousand shilling note. After buying twelve kilograms of cassava flour, he was given a balance of shs. 4,400. Determine the price of one kilogram os cassava flour. [Ans: shs.1,300]

28. The price of a TV set is shs.10,000 more than the price of 5 radio-cassettes. Mr. Kato bought 4 TV sets and 5 radio cassettes and altogether paid shs.890,000. Determine the cost of;

i) a TV set [Ans: shs. 180,000]

ii) a radio cassette [Ans: shs. 34,000]

iii) 6 Tv sets and 10 radio cassettes.

[Ans shs. 1,420,000]

29. Okidi uses five thousand, ten-thousand shilling and twenty-thousand shillings notes to pay a bill of shs.300,000. Given that the total value of the bill paid in the five-thousand, ten-thousand and twenty-thousand shilling notes are in the ratio 2 : 3 : 10, determine the number of the;

i) five-thousand shilling notes [Ans: 8]

ii) ten-thousand shilling notes [Ans: 6]

iii) twenty-thousand shilling notes.[Ans: 10],
Okidi used to in order to pay his telephone bill.

30. When 40 is added to a number and the sum is divided by five, the result exceeds a quarter, the original number by 2. What is the original number? [Ans: 120]

31. Namirimu traveled from Kampala to Seeta at an average speed of X km/hr. The distance between Kampala and Seeta is 10 km. He then traveled to Lugazi and an average speed of $(X + 6)$ km/hr. Given that Seeta and Lugazi are 25 km apart and that Namirimu takes the same time to travel from Kampala to Seeta as she takes to travel from Seeta to Lugazi, determine the value of X .
[Ans: $X = 4$ km/hr]

SIMULTANEOUS EQUATIONS

RECALL

Methods that are majorly used to solve simultaneous equations.

- ◊ Elimination method
- ◊ Substitution method
- ◊ Inverse method
- ◊ Matrix method

TRIAL QUESTIONS

1. (a) Solve the simultaneous equations below:

$$\begin{aligned} 5x + 2y &= 5 \\ 3x - 0.2y &= 10 \end{aligned} \quad [\text{Ans: } x = \frac{9}{5}, y = 7]$$

- (b) In the market Ndgire buys 4 kg of tomatoes and 2 kg of potatoes for shs 3,800 = Kiwanuka buys 5 kg of tomatoes and 3 kg of potatoes for shs 5,000. Find

- (i) the cost of 1 kg of tomatoes and 1 kg of potatoes **[Ans: 700, 500]**
(ii) if Claudia wants to buy 10 kg of tomatoes and 15 kg of potatoes find how much she will spend. **[Ans: 14,500]**

2. Solve the simultaneous equations

- (i) $2a + b = 10$ and $a - 5b = -6$ **[Ans: $a = 4, b = 2$]**
(ii) $2x + y = -5$ and $y - 4x = 13$ **[Ans: $x = -3, y = 1$]**
(iii) $y + 5x = 7$ and $2x - 4y = 7$ **[Ans: $x = \frac{35}{22}, y = \frac{-21}{22}$]**
(iv) $2x + 5y = 12$ and $-5 + x = -2y$
 [Ans: $x = \frac{-1}{9}, y = \frac{22}{9}$]
(v) $2a + 3b = 13$ and $3a - 4b = 11$
 [Ans: $a = 5, b = 1$]

3. (a) Use matrix method to solve the following pair of simultaneous equations.

$$\begin{aligned} 3a + 4b &= 8 \\ a + 2b - 3 &= 0 \end{aligned} \quad [\text{Ans: } a = 2, b = \frac{1}{2}]$$

- (b) A grocery sells two kinds of meat products A and B . Okot bought 4 kg of A and 6 kg of B paying a total of shs. 5280. Namusisi bought 5 kg of A and 3 kg of B at a total cost of shs. 4440.

- (i) Write down two equations to describe okot's and Namusisi's purchases.

- (ii) By combining the two equations in matrix form, determine the cost of a kg of each meat product.

- (iii) How much would Mugisha pay for 6 kg of A and 5 kg of B? **[Hint: Refer to No.1(b)]**

- (c) Use matrix method to solve the equations

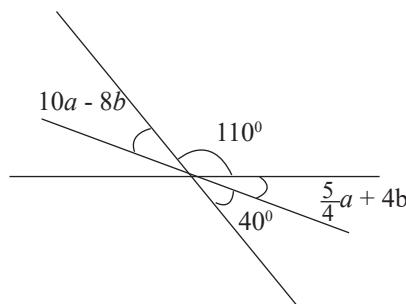
$$\begin{aligned} 3a - 5b + 9 &= 0 \\ 5a + 2b &= 16 \end{aligned} \quad [\text{Ans: } a = 2, b = 3]$$

4. Three kilograms of sugar and four kilograms of rice cost shs 22,400 while two kilograms of rice and five kilograms of sugar cost shs 21,000.

- (i) Find the cost of a kilogram of sugar and a kilogram of rice.

- (ii) How much will Mr. Musoke spend if he buys six kilograms of rice and nine kilograms of sugar?

5. Solve for a and b in the diagram below.



$$[\text{Ans: } a = 12, b = 10]$$

6. $p + q = 21$

$$p - q = 3 \quad [\text{Ans: } p = 12, q = 9]$$

(7) $3x + y = 7$

$$x + y = 3 \quad [\text{Ans: } x = 2, y = 1]$$

8. $x + y = 10$

$$x - y = -4 \quad [\text{Ans: } x = 3, y = 7]$$

(9) $2p - q = 5$

$$3p + q = 5 \quad [\text{Ans: } p = 2, q = -1]$$

10. $3x + y = 1$

$$4x - y = 6 \quad [\text{Ans: } x = 1, y = -2]$$

(11) $5x - 2y = 7$

$$3x - 2y = 1 \quad [\text{Ans: } x = 3, y = 4]$$

Simultaneous equations

12. $4x - 3y = 23$

$$4x + 3y = 17 \quad [\text{Ans: } x = 5, y = -1]$$

(13) $s - 3r = 1$

$$s - 5r = -1 \quad [\text{Ans: } s = 4, r = 1]$$

14. $x - 3y = 0$

$$2x - y = 15 \quad [\text{Ans: } x = 9, y = 3]$$

(15) $3x + 3y = 7$

$$6x - 6y = 7 \quad [\text{Ans: } x = \frac{7}{4}, y = \frac{7}{12}]$$

16. $5x - 3y = 4$

$$3x + y = 1 \quad [\text{Ans: } x = \frac{1}{2}, y = \frac{1}{2}]$$

(17) $a - b + 1 = 0$

$$3a = 2b \quad [\text{Ans: } a = 2, b = 3]$$

18. $2a + 9b = 3$

$$a + 2b = 5$$

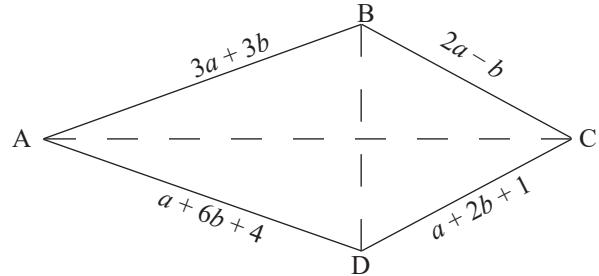
(19) $5x + 5y = 3$

$$3y - x = 0.2 \quad [\text{Ans: } x = \frac{2}{5}, y = \frac{1}{5}]$$

20. $7y - 2x = 0$

$$3y - 2x = 1 \quad [\text{Ans: } x = \frac{-7}{8}, y = \frac{1}{4}]$$

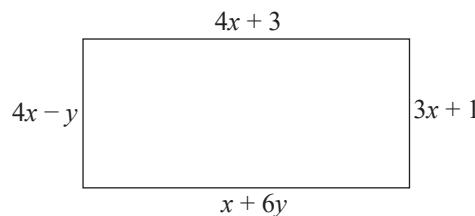
21. The figure below shows a kite ABCD.



Find the values of a and b

$$[\text{Ans: } a = 3, b = \frac{2}{3}]$$

22. The figure below shows a rectangle whose dimensions are given in cm.



Find the value of x and y and the area of the rectangle.

$$[\text{Ans: } x = 3, y = 2]$$

23. Solve the simultaneous equations;

$$\begin{aligned} y &= x + 1 & [\text{Ans: } x = \frac{8}{5}, y = \frac{-3}{5}] \\ x^2 + 4y^2 &= 4 \end{aligned}$$

QUADRATIC EQUATIONS

RECALL

- ◊ There are three methods of solving quadratic equations.

Factorization method;

- ◊ A quadratic equation is of form $ax^2 + bx + c = 0$. The left hand side of equation can be broken into two factors. A quadratic equation has two roots. The roots may be equal or unequal or one of these root may be zero. (Refer to factorization of quadratic expressions).

Example; Solve the equation $x^2 + 9x - 36 = 0$

Solution;

$$\begin{aligned}x^2 + 9x - 36 &= 0 \\ \text{factors are } -3 \text{ and } 12 \\ x^2 + 12x - 3x - 36 &= 0 \\ x(x + 12) - 3(x + 12) &= 0 \\ (x + 12)(x - 3) &= 0\end{aligned}$$

$$\begin{aligned}\text{either } x + 12 &= 0 \\ x &= -12\end{aligned}$$

$$\begin{aligned}\text{or } x - 3 &= 0 \\ x &= 3\end{aligned}$$

when two roots are given then
the general equation becomes;
 $x^2 - x(\text{sum of roots}) + \text{product of roots} = 0$ where α and β are the roots.

Completing squares method

- ◊ Under this method, a quadratic equation $ax^2 + bx + c = 0$ is first expressed in the form; $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
- ◊ Then divide the coefficient of x by 2. $x^2 + \frac{b}{2a}x + \frac{c}{a} = 0$
- ◊ We add the coefficient of x after dividing by two. $x + \frac{b}{2a}$ and then square the sum $(x + \frac{b}{2a})^2$
- ◊ Subtract the square of the coefficient of x after dividing by two from the result.

$$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2}$$

From that we add $\frac{c}{a}$ and equate to zero. $(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$. Generally $(x + p)^2 + q = 0$

Example: Use completing squares to solve $x^2 + 3x + 2 = 0$

Solution

$$\begin{aligned}x^2 + 3x + 2 &= 0 \\ (x^2 + 3x) + 2 &= 0 \\ (x + \frac{3}{2})^2 - (\frac{3}{2})^2 + 2 &= 0 \\ (x + \frac{3}{2})^2 - \frac{9}{4} + 2 &= 0 \\ (x + \frac{3}{2})^2 &= \frac{9}{4} - 2 \\ (x + \frac{3}{2})^2 &= \frac{1}{4} \\ \text{take square root on both sides} \\ \sqrt{(x + \frac{3}{2})^2} &= \sqrt{\frac{1}{4}} \\ x + \frac{3}{2} &= \pm \frac{1}{2} \\ \text{either } x + \frac{3}{2} &= \frac{1}{2} \\ x + \frac{3}{2} &= -\frac{1}{2} \\ x &= \frac{-3}{2} - \frac{1}{2} \\ x &= -2 \\ x &= -1, x = -2\end{aligned}$$

Method of quadratic formula

- ◊ Equation of form $ax^2 + bx + c = 0$ can be solved using the formula; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- ◊ Whenever b is negative, then the negative on b disappears. $x = \frac{-(-b) \pm \sqrt{b^2 - 4ac}}{2a}$
$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

TRIAL QUESTIONS

Use completing squares to solve the following.

1. $x^2 - 5x + 6 = 0$ [Ans: $x = 3, x = 2$]
2. $3x^2 + 8x + 4 = 0$ [Ans: $x = -2, x = -\frac{2}{3}$]
3. $2x^2 - 5x + 2 = 0$ [Ans: $x = \frac{1}{2}, x = 2$]
4. $16x^2 - 8x + 1 = 0$ [Ans: $x = \frac{1}{4}, x = \frac{1}{4}$]
5. $x^2 + 4x - 5 = 0$ [Ans: $x = -5, x = 1$]
6. $x^2 - 24 = 2x$ [Ans: $x = 6, x = -4$]
7. $x^2 - 11x + 28 = 0$ [Ans: $x = 4, x = 7$]
8. $2x^2 - 3x - 2 = 0$ [Ans: $x = 2, x = -\frac{1}{2}$]
9. $4x^2 + 1 = 4x$ [Ans: $x = \frac{1}{2}, x = \frac{1}{2}$]
10. $4x^2 - 4x - 35 = 0$ [Ans: $x = \frac{7}{2}, x = -\frac{5}{2}$]

Solve the following quadratic equations;

11. $x^2 + 2x - 63 = 0$ [Ans: $x = 7, x = -9$]
12. $x(x - 2) = 3(x - 2)$ [Ans: $x = 2, x = 3$]
13. $x^2 - 5x = 50$ [Ans: $x = 10, x = -5$]
14. $3x^2 + x = 2$ [Ans: $x = \frac{2}{3}, x = -1$]
15. $2x^2 - 3x - 5 = 0$ [Ans: $x = \frac{5}{2}, x = -1$]
16. $(p - 4)^2 = (p - 4)(2p - 1)$ [Ans: $p = 4, p = -3$]
17. $q^4 - 10q^2 + 24 = 0$
[Ans: $q = 2, q = -2, q = \sqrt{6}, q = -\sqrt{6}$]
18. $(2 + x)^2 + (x - 1)^2 = 5$ [Ans: $x = 0, x = -1$]
19. $(x - 1)(x + 2) = 4$ [Ans: $x = 2, x = -3$]
20. $2x^2 + 7x - 4 = 0$ [Ans: $x = \frac{1}{2}, x = -4$]

Solve the following quadratic equations.

21. $(2x - 7)^2 = 1$ [Ans: $x = 3, x = 4$]
22. $6x^2 - 23x - 4 = 0$ [Ans: $x = 4, x = -\frac{1}{6}$]
23. $3x^2 - 14x - 5 = 0$ [Ans: $x = 5, x = -\frac{1}{3}$]
24. $4x^2 - 3x - 10 = 0$ [Ans: $x = 2, x = -\frac{5}{4}$]
25. $10x^2 - 3x - 1 = 0$ [Ans: $x = \frac{1}{2}, x = \frac{1}{5}$]
26. $6k^2 + 7k + 1 = 0$ [Ans: $k = -1, k = -\frac{1}{6}$]
27. The length of a rectangle is $2 m$ more than the width and the rectangle is $63 m^2$. Find the dimensions of the rectangle.
[Ans: width = 7 m, length = 9 m]

28. The sides of a right angled triangle are $(2x - 3)m$, $(2x + 1)m$ and $(2x + 5)m$. Find the value of x and hence find the dimensions of the triangle.
[Ans: $x = 7.5 m$, opposite = 12 m, adjacent = 16 m, hypotenuse = 20 m]

29. The length of a rectangular room exceeds the breadth by $5 m$ and the area of the floor is $50 m^2$. Find the breadth of the room. [Ans: 5 m]

30. The hypotenuse of a right angled triangle is $17 cm$ long and one of the sides containing the right angle is $7 cm$ more than the other.

- (i) Find the length of the shortest side of the triangle. [Ans: 8 cm]
- (ii) Find the area of the triangle. [Ans: 60 cm²]

31. The numbers add up to 20. The sum of their squares is 20 less than 3 times of their product. Find the numbers. [Ans: 6 and 14]

32. The co-ordinates of the points P and Q are P(5, 4) and Q(-3, k). Find the possible values of k such that $|PQ| = \sqrt{65}$. [Ans: $k = 3, k = 5$]

33. If $x = 2$ is a solution of the equation $x^2 + kx - 10 = 0$. Find the value of k and the second value of x . [Ans: $k = 3, x = -5$]

34. Find the quadratic equations corresponding to the following solution sets.

- (a) $\{-3, 4\}$
- (b) $\{-4, 6\}$
- (c) $\{3.5, -2.5\}$
- (d) $\{\frac{-6}{7}, 1\}$
- (e) $\{\frac{-4}{3}, \frac{3}{2}\}$
- (f) $\{-2, -1\}$
- (g) $\{\frac{-3}{2}, 5\}$

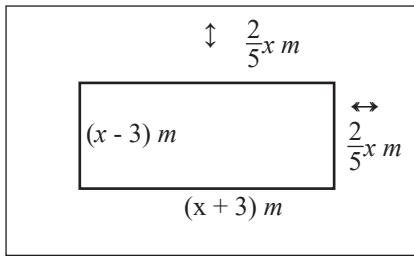
35. (a) The difference between the squares of two numbers is 40. If the larger number exceeds the smaller by 4, find the two numbers.

[Ans: 7 and 3]

- (b) A man is now four times as old as his son. Eight years ago, the product of their ages was 220 years. Find the son's present age.
[Ans: 13 years]

36. Jane is three years older than Peace. In two years time, the product of their ages will be 238. How old is Jane now? [Ans: 15 years]

37. The figure below represents the floor of a dining hall with a carpet margin all around of $\frac{2}{5}x$ wide m wide leaving a dancing space of $(x + 3)$ m long by $(x - 3)$ m wide.

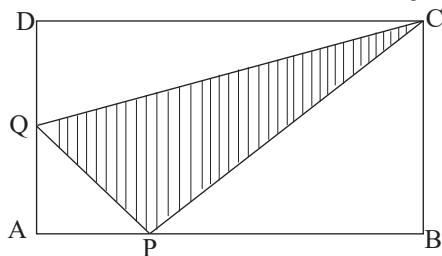


- (a) If the total area of the entire room is 315 m^2 , calculate the value of x . [Ans: $x = 10 \text{ m}$]
- (b) Hence calculate the area of the carpeted margin. [Ans: 224 m^2]
- (c) If the carpet costs shs 750 per m^2 , calculate the total cost of the carpeted margin.
[Ans: shs 168,000]

38. The number of diagonals in a polygon $= \frac{n(n-3)}{2}$, where n is the number of sides. How many sides are there if the number of diagonals is;

- (a) 54 [Ans: 12]
(b) 135 [Ans: $n = 18$]

39. In the given figure, ABCD is a rectangle with $AB = 2x \text{ cm}$ and $BC = x \text{ cm}$. P and Q are points on AB and AD such that $AP = AQ = 3 \text{ cm}$.



- (i) Show that the sum of the areas of triangles PBC and QDC $= \left(2x^2 - \frac{9x}{2}\right) \text{ cm}^2$
- (ii) If the area of triangle PCQ $= 22.5 \text{ cm}^2$, find x . [Ans: $x = 6 \text{ cm}$]

40. The square of a number is 3 more than half the number. Find the number.
[Ans: $x = 2$ and $x = -\frac{3}{2}$]

41. Find two consecutive numbers such that the sum of their squares is equal to 145.
[Ans: 8, 9 or -9, -8]

42. The perimeter of a rectangle is 26 cm. If the area of the rectangle is 36 cm^2 , calculate the length of the sides. [Ans: 9 cm and 4 cm]

43. The area of a triangle is 44 cm^2 , and the height is 3 cm more than the base. Calculate the length of the base. [Ans: 8 cm]

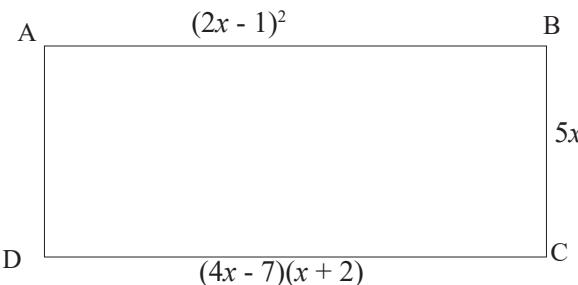
44. The sum of the first n natural numbers $(1 + 2 + 3 + \dots) = \frac{1}{6}n(n+1)$. How many consecutive natural numbers, starting with 1, has a sum of 630?
[Ans: 35]

45. A square is changed to a rectangle by doubling two of its sides and by increasing two of its sides by 2 cm. The area of the rectangle so formed is 70 cm^2 . Find the area of the original square.
[Ans: 25 cm^2]

46. The length of a reactangle exceeds it's breadth by 4 cm. If the length were halved and the breadth increased by 5 cm, the area would be decreased by 35 cm^2 . Find the dimensions of the rectangle. [Ans: $14 \text{ cm} \times 10 \text{ cm}$]

47. A piece of wire which is 18 cm long is cut into two parts. The first part is bent to form the four sides of a square; the second part is bent to form the four sides of a rectangle having length = X cm, and breadth = 1 cm. If the sum of the area of the square and the rectangle is $A \text{ cm}^2$, then show that $A = 16 - 3x + \frac{x^2}{4}$. If $A = 8 \text{ cm}^2$, solve the equation and obtain the length of the sides of the square.
[Ans: 2 cm]

48. In the figure below, ABCD is a rectangle with sides $AB = (2x - 1)^2 \text{ cm}$, $CD = (4x - 7)(x + 2) \text{ cm}$, $BC = 5x \text{ cm}$. Form an equation in x and solve it to find the length of the sides.



[Ans: $x = 3$, length = 25 cm, width = 15 cm, AD = BC = 15 cm, AB = DC = 25 cm]

QUADRATIC CURVES

TRIAL QUESTIONS

- Draw a graph of $y = 2x^2 - 4x - 10$ for values of x from -2 to 4 . Use a scale of $1\text{ cm} : 1\text{ unit}$ on the vertical axis and $2\text{ cm} : 1\text{ unit}$ on the horizontal axis. Use your graph to solve $2x^2 - 4x - 10 = 0$
- (a) Copy and complete the table below for the function $y = 2 + 3x - 2x^2$ for values of x from -4 to $+5$.

x	-4	-3	-2	-1	0	1	2	3	4	5
x^2										
$3x$										
$-2x^2$										
2										
y										

- (b) Draw the graph of $y = 2 + 3x - 2x^2$ using a scale of $2\text{ cm} : 1\text{ unit}$ on the x -axis and $2\text{ cm} : 5\text{ units}$ on the y -axis.
- (c) From your graph read the roots of:
- $2 + 3x - 2x^2 = 0$
 - $7 + 3x - 2x^2 = 0$
 - $6 + 9x - 6x^2 = 5x - 15$
- (a) Draw a graph of $y = (x + 1)(x - 3)$ for the domain $-2 \leq x \leq 4$. Using a scale of $2\text{ cm} : 1\text{ unit}$ on both axes
 (b) using your graph find the roots of the equations;
 (i) $x^2 - 2x - 3 = 0$ (ii) $x^2 - 3x - 1 = 0$ by drawing
 - (a) Copy and complete the following table if $y = 6 - x - 2x^2$ for $-3 \leq x \leq 3$

x	-3	-2.5	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0	2.5	3.0
6	6	6	6	6	6	6	6	6	6	6	6	6	6
$-x$													
$-2x^2$													
y													

- (b) Draw the graph of $y = 6 - x - 2x^2$ for $-3 \leq x \leq 3$ using your values in (a)
 (c) Use your graph to find the roots of;
 (i) $6 - x = 2x^2 = 0$ (ii) $1 + x - 2x^2 = 0$
- Copy and complete the table below in which $y = x^2 - 6x + 2$

x	-1	0	1	2	3	4	5	6	7
x^2		0			9			36	49
$-6x$	6	0	-6			-24			-42
$+2$		$+2$		$+2$					$+2$
y		2		-6			-3		9

- (a) Draw the graph of $y = x^2 - 6x + 2$ for $-1 \leq x \leq 7$
 (b) Use your graph to estimate the roots of the equation $x^2 - 6x + 2 = 0$
 (c) Using the same axes, draw the line $y = 4$
 (d) Use your graphs to estimate the roots of the equation $x^2 - 6x - 2 = 0$.
- (a) Using a scale of 1 cm to represent 1 unit on the x -axis and 1 cm to represent 2 cm on the y -axis, draw the graph of $y = 2x^2 + 3x - 11$ for the range of values of $x = -4$ to $x = 3$.
 (b) On the same axes draw the graph of $y = 2x + 1$.
 (c) Use your graph to solve the quadratic equations; (i) $2x^2 + 3x - 11 = 0$ (ii) $2x^2 + x - 12 = 0$

x	-4	-3	-2	-1	0	1	2
$2x^2$					0		
$3x - 1$					-1		
y					-1		

- (b) From the above table draw the graph of the function $y = 2x^2 + 3x - 1$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation;

(i) $2x^2 + 3x - 1 = 0$ (ii) $2x^2 + x - 3 = 0$

Quadratic Curves

16. (a) Plot the graph of $y = 3x^2 + 2x - 16$ for values of x , $-3 \leq x \leq 3$.
 (b) Use your graph to solve the equation; $3x^2 + 2x - 8 = 0$
17. (a) (i) Draw a table for values of y and x for the curve $y = 10 - x^2$. Use values of x from -4 to $+4$.
 (ii) Use your table to draw a graph of $y = 10 - x^2$.
 (b) On the same axes, draw the graph of the line $y = 2x + 3$
 (c) Use your graphs to solve the equation $x^2 + 2x - 7 = 0$.
18. (a) Draw the graph of the curve $y + 2x + 3 = x^2$ for values of x from -3 to 5 . Using a scale of 1 cm to represent 1 unit on both axes.
 (b) Use your graph to find the roots of the equations
 (i) $x^2 - 2x = 3$ (ii) $x^2 - x - 6 = 0$ (iii) $2x^2 - 9x + 4 = 0$
19. (a) Draw the graph of $y = 3x^2 + 4x - 4$ for $-4 \leq x \leq 4$
 (b) Using your graph solve the equations;
 (i) $3x^2 + 4x - 4 = 0$ (ii) $3x^2 + 4x - 6 = 0$ (iii) $3x^2 + 2x - 5 = 0$
20. (a) Draw a graph of $y = 12 - 2x - 2x^2$ for $-5 \leq x \leq 4$. Using a scale of $1\text{ cm} : 2$ units for vertical axis and $2\text{ cm} : 1$ unit for horizontal axis.
 (b) Using your graph solve the equations;
 (i) $12 - 2x - 2x^2 = 0$ (ii) $10 - 8x - 2x^2 = 0$
 (c) State the equation of a line of symmetry of this curve $y = 12 - 2x - 2x^2$.
21. (a) Draw a graph of $y = 6 - x - x^2$ for values of x from -4 to $+3$ using a scale of $2\text{ cm} : 1$ unit on both axes.
 (b) Use your graph to solve: (i) $6 - x - x^2 = 0$ (ii) $2 - x - x^2 = 0$
 (c) State the range of values of x for which y is positive.
22. (a) Draw the graph of $y = 2x^2 + 3x - 1$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation $2x^2 + 3x - 1$ to 1 decimal place. Use a scale of 2 cm for 1 unit on the x -axis and 1 cm for 2 units on the y -axis.
 (b) By drawing a suitable straight line graph on the plotted function in (a) above, estimate the roots of the equation $2x^2 + x - 3 = 0$.
23. The table below shows values for $y = 5 + 8x - 2x^2$ for $-1.5 \leq x \leq 6$
- | x | -1.5 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|----------|------|----|---|----|---|----|---|----|-----|
| $8x + 5$ | | | | | | 29 | | | |
| $-2x^2$ | | -2 | | | | | | | -72 |
| y | | | | 11 | | | | -5 | |
- (a) Copy and complete the table above.
 (b) Draw the graph of $y = 5 + 8x - 2x^2$ for $-1.5 \leq x \leq 6$
 (c) Use your graph to solve the equation.
 (i) $5 + 8x - 2x^2 = 0$ (ii) $-2x^2 + 5x + 3 = 0$
 (d) State the coordinates of the turning point of the curve $y = 5 + 8x - 2x^2$
24. Copy and complete the table below.
- | x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|------------|----|----|----|----|---|---|---|---|---|
| $x^2 - 2$ | | | | | | | | | |
| $6x - x^2$ | | | | | | | | | |
- (a) Plot on the same axes the graphs of $y = x^2 - 2$ and $y = 6 - x^2$ for $-4 \leq x \leq 4$
 (b) Use your graphs to solve the equations $x^2 - 2 = 6 - x^2$
 (c) Find the equation of the line through the point of intersection of the two curves.
25. (a) On the same axis draw graphs of $y = x^2 + 4x - 16$ and $y = -x^2 - 5x + 8$ for $x: -4 \leq x \leq 5$
 (b) Use your graph in (a) above to solve the equation $2x^2 + 9x - 24 = 0$
 (c) Find the equation of the line joining the points of intersection of the two curves.

VARIATION AND PROPORTIONALITY

RECALL

Variation means a change in form, position, state, quantity or quality of a thing.

Variation is categorise as follows;

- ◊ Direct variation/ direct proportionality.
- ◊ Inverse variation/ inverse proportionality.
- ◊ Joint variation
- ◊ Partial variation.

Under direct and inverse variation/ proportionality, one variable depends on one another.

Direct proportionality.

Circumference of a circle is proportional to its diameter. This is usually expressed in the form of an equation.

$$C = \pi d$$

We say C varries as its diameter d.

$$C \propto d$$

\propto - proportionality sign.

This means that increase in diameter increases the circumference. To form an equation, we substitute the proportionality sign with an equal sign and introduce the constant of proportionality.

$$C = kd$$

k → constant of proportionality

But with this case $k = \pi$

$$C = \pi d$$

NB: The word “varies as” is the same as direct proportionality.

Example

Given that y varies directly as the square of x . when $y = 8$, $x = 4$. Find the value of y when $x = 2$.

Solution;

$$y \propto x^2$$

$$y = kx^2$$

$$8 = k \times 4^2$$

$$8 = 16k$$

$$k = \frac{8}{16}$$

$$k = \frac{1}{2}$$

$$y = \frac{1}{2}x^2$$

$$\text{when } x = 2, y = ?$$

$$y = \frac{1}{2}(2)^2$$

$$y = \frac{1}{2} \times 4$$

$$y = 2$$

INVERSE PROPORTIONALITY

One variable increases with decrease in the other and vice versa.

Example

Given that p varies inversely as the square of q and that $p = 9$ when $q = 4$, find the value of p when $q = 8$

Solution;

$$p \propto \frac{1}{q^2}$$

$$p = \frac{k}{q^2}$$

when $p = 9$, $q = 4$

$$9 = \frac{k}{4^2}$$

$$k = 9 \times 4^2$$

$$k = 144$$

$$p = \frac{144}{q^2}$$

when $q = 8$, $p = ?$

$$p = \frac{144}{8^2}$$

$$p = \frac{144}{64}$$

$$p = 2.25$$

JOINT VARIATION

One variable depends on two or more others eg volume (V) of a right circular cylinder is given in terms of its radius r and height (h) by formula $V = \pi r^2 h$.

$$\begin{aligned} V &\propto r^2 & V &\propto h \\ V &\propto r^2 h \\ V &= kr^2 h \text{ but } k = \pi \\ V &= \pi r^2 h \end{aligned}$$

But under this section, both inverse and directly proportionality can be combined together.

Example

Given that y varies directly as the square of x and inversely as s . When $y = 8$, $x = 4$ and $s = 6$. Find y when $x = 6$ and $s = 13.5$.

Solution;

$$\begin{aligned} y &\propto x^2 \\ y &\propto \frac{1}{s} \\ y &\propto \frac{x^2}{s} \end{aligned}$$

$$y = \frac{kx^2}{s}$$

NB: The first values ie $y = 8$, $x = 4$, $s = 6$ are used to get the constant

$$\begin{aligned} y &= \frac{kx^2}{s} \\ 8 &= \frac{k \times 4^2}{6} \\ 8 \times 6 &= 16k \end{aligned}$$

$$k = \frac{8 \times 6}{16}$$

$$k = 3$$

$$y = \frac{3x^2}{5}$$

$$y = ?, x = 6, s = 13.5$$

$$y = \frac{3 \times 6^2}{13.5}$$

$$y = 8.$$

PARTIAL VARIATION

This is the type of variation that involves two constants and these are found by forming two simultaneous equations and solving them using a suitable method. ie elimination, substitution and matrices to find the value of constants.

N.B: Each variable has its own constant.

Example

P varies partly as the square of V and partly as the cube of V . When $V = 2$, $P = -20$ and when $V = -3$, $P = 135$. Find the relationship between P and V . Find the value of P when $V = -1$.

Solution;

$$P \propto V^2 + V^3$$

$$P = k_1 V^2 + k_2 V^3$$

$$\text{when } V = 2, P = -20$$

$$-20 = k_1(2)^2 + k_2(2)^3$$

$$4k_1 + 8k_2 = -20$$

$$\text{reduce by 4}$$

$$k_1 + 2k_2 = -5 \quad \text{(i)}$$

$$\text{when } V = -3, P = 135$$

$$P = k_1 V^2 + k_2 V^3$$

$$135 = k_1(-3)^2 + k_2(-3)^3$$

$$135 = 9k_1 + k_2(-27)$$

$$9k_1 - 27k_2 = 135$$

reduce by 9

$$k_1 - 3k_2 = 15 \quad \text{(ii)}$$

Solve(i) and (ii) simultaneously

$$k_1 + 2k_2 = -5 \quad \text{(i)}$$

$$k_1 - 3k_2 = 15 \quad \text{(ii)}$$

$$(i) - (ii)$$

$$k_1 + 2k_2 = -5$$

$$k_1 - 3k_2 = 15$$

$$0 + 2k_2 - (-3k_2) = -5 - 15$$

$$5k_2 = -20$$

$$k_2 = \frac{-20}{5}$$

$$k_2 = -4$$

Substitute k_2 into

$$k_1 - 3k_2 = 15$$

$$k_1 - 3(-4) = 15$$

$$k_1 + 12 = 15$$

$$k_1 = 15 - 12$$

$$k_1 = 3$$

$$k_1 = 3, k_2 = -4$$

$$P = 3V^2 - 4V^3$$

$$P = ?, V = -1$$

$$P = 3(-1)^2 - 4(-1)^3$$

$$P = 3 - 4(-1)$$

$$P = 3 + 4$$

$$P = 7$$

TRIAL QUESTIONS

- (1) The volume v of a cylinder varies directly as its height H . If the volume $V = 50 \text{ cm}^3$ when $H = 5 \text{ cm}$. Find the volume of the cylinder when $H = 20 \text{ cm}$, [Ans: $V = 200 \text{ cm}^3$]
- (2) Given that y is directly proportional to x^2 . If $y = 16$ when $x = 1$, find x when $y = 1$. [Ans: $x = \frac{1}{4}$]
- (3) The force (F) which acts between two magnetic poles is inversely proportional to the square of the distance (d) between them. If $F = 18$ when $d = 4$, find F when $d = 3$. [Ans: 32]

- (4) Given that V is inversely proportional to t and V = 25 when $t = 2$. Find the value of V when $t = 5$.
[Ans: V = 10]

- (5) Given that y is directly proportional to x^3 and that y is 250 when $x = 10$. Find the equation connecting x and y, hence find the value of y when $x = 4$.
[Ans: y = 16]

- (6) (a) If y varies directly as the cube root of x, and that when $x = -8$, $y = -4$. Find x when $y = -8$.

- (b) Given that w is inversely proportional to the square of h and that $w = h = -2$. Find h when $w = \frac{-1}{2}$
[Ans: h = 4]

- (c) The volume V of water oozing out of a narrow water pipe varies as the square of its diameter D and inversely as the reciprocal of the length L of the pipe.

- (i) Write down a formula connecting V, D, L and the constant of proportionality k.

- (ii) When $D = 10$ cm, $L = 0.3$ m and $V = 300$ cm^3 . Find k and determine the value of D when $V = 1000$ cm^3 and $L = 40$ m.
[Ans: k = 0.1, D = 50 cm]

- (7) A quantity T is partly constant and partly varies as the square of q. When $q = 2$, $T = 40$. when $q = 3$, $T = 65$.

- (a) Form an equation relating T and q.
[Ans: T = 20 + 5q²]

- (b) Determine the values of q when T = 100.
[Ans: q = 4]

- (8) A quantity M is partly constant and partly varies as the square of N. When $N = 4$, $M = 95$. When $N = 3$, $M = 60$.

- (a) Form an equation relating M and N.
[Ans: M = 15 + 5N²]

- (b) Determine the values of N when M = 140.
[Ans: N = 5]

- (9) The expenditures (E) of Luwero high school is directly proportional to income (Y) collected in form of school fees and inversely proportional to square number of children (n). When E = 540,000/=, Y = 1,800,000/= and n = 30. Find E when Y = 3,000,000/= and n = 50.
[Ans: 324,000/=]

- (10) x varies directly as the square of y and inversely as z. Given that $x = 6$, $y = 4$, and $z = 2$. Find z when $x = 8$ and $y = 6$.
[Ans: $\frac{27}{8}$]

- (11) x is inversely proportional to y. $x = 4$ when $y = 5$. Find

- (a) x when $y = 2.5$.
[Ans: x = 8]

- (b) y when $x = 1.25$.
[Ans: y = 16]

- (12) A quantity B varies as the square of V and is inversely proportional to R. When B = 27, V = 6 and R = 4. Determine the value of B when V = 8 and R = 16.
[Ans: B = 12]

- (13) The length (L) of a simple pendulum varies as the square of the period (T). A pendulum with length 0.994 m long has a period of approximately 2 s. Find

- (a) an equation connecting L and T.
[Ans: L = 0.2485T²]

- (b) the length of a pendulum whose period is 3 s.
[Ans: L = 2.237 m]

- (14) The distance of the horizon D km varies as the square root of the height H m of the observer above sea level. An observer at a height of 100 m above sea level sees the horizon at a distance 35.7 km, Find;

- (a) an equation connecting D and H.
[Ans: D = 3.57\sqrt{H}]

- (b) Find the distance of the horizon from an observer 70 m above sea level.
[Ans: D = 29.87 km]

- (15) The cost (C) of printing a circular on octavo paper is partly constant and partly varies as the number of copies (n) printed. If 100 and 500 copies cost £8.25 and £14.25 respectively.

- (a) Find an equation connecting C and n.
[Ans: C = 6.75 + 0.015n]

- (b) Find how much will 200 copies?
[Ans: £9.75]

- (c) Find how many copies produced at a cost of £12.99.
[Ans: 416 copies]

- (16) (a) Given that x varies directly as y and inversely as the square of z and that $x = y$ when $z = 3$. Calculate the value of x when $y = 5$ and $z = 2$.
[Ans: x = 11.25]

- (b) The formula $d = kV + mV^2$ gives the distance d metres travelled by a certain car being brought to rest from a speed $V \text{ ms}^{-1}$ by the application of breaks. If $d = 62$ m when $V = 40 \text{ ms}^{-1}$ and $d = 117$ m when $V = 60 \text{ ms}^{-1}$.

- (i) Find the values of the constants k and m .
[Ans: $k = 0.75$, $m = 0.02$]
- (ii) If the breaks are applied when the car's speed is 50 ms^{-1} , Find the distance it travels before coming to rest. **[Ans: $d = 87.5 \text{ m}$]**
- (17) The annual cost (C) of running a certain car is partly constant and partly varies as the distance (D) run by the car in the year. In one year the car ran 6000 km at a total cost of £900. In the next year, it ran 7200 km at a total cost of £950.
- (a) Find an equation connecting cost (C) and distance (D). **[Ans: $C = 648 + 0.042D$]**
- (b) How much would it cost to run the car in a year during which it ran 12000 km ?
[Ans: £1152]
- (18) Given that x varies directly as y and inversely as the square of Z and that $x = 2$, $y = 2$ and $Z = 3$. Calculate the value of x when $y = 5$ and $Z = 2$.
- (b) Given that d partly varies as V and partly varies as the square of V and that $d = 62$ when $V = 40$ and $d = 117$ when $V = 60$, find the value of d when $V = 50$.
- (19) The cost (C) of printing a copy of a newspaper is partly constant and is also inversely proportional to the number (n) of copies printed. When 200 copies are printed the cost per copy is shs 850. When 300 copies are printed the cost per copy is shs 750.
- (a) Form an equation relating C and n .
- (b) Calculate the cost per copy when 150 copies are printed.
- (20) Time t hours taken to dig a pit latrine partly varies as the depth (d - metres) of the latrine and partly as the square of the depth. If $t = 80$, $d = 20$ and when $t = 150$, $d = 30$.
- (a) Write down an expression connecting t and d . **[Ans: $t = 2d + 0.1d^2$]**
- (b) Find t when $d = 40$. **[Ans: $t = 240$]**
- (21) (a) Given that $(3x - y) : (2x + 3y) = 3 : 4$. Find the value of $x : y$. **[Ans: $13 : 6$]**
- (b) If $(x + y) : (x - y) = 8 : 3$, Find the ratio of x to y . **[Ans: $11 : 5$]**
- (22) Three quantities a , b , and c are given in the ratios $a : b = 3 : 4$ and $b : c = 5 : 6$. Determine the ratio $a : b : c$. **[Ans: $15 : 20 : 24$]**
- (23) The ratio of boys to girls in a math club is $4 : 5$, when 5 boys join the club and 2 girls leave, the ratio of boys to girls becomes $15 : 16$. Find the original number of boys and girls in the club.
[Ans: 40 boys and 50 girls]
- (24) In the manufacture of a radio set costing shs..495,000 the cost of labour, materials and overheads are in the ratio $7 : 5 : 3$. Find the difference between the costs of labour and overheads. **[Ans: 132,000]**
- (25) (a) If $a : b = 5 : 7$, find the ratio $(5a - 3b) : (2a + 3b)$. **[Ans: 4 : 31]**
- (b) Given that $x : y = 2 : 3$ and $y : z = 9 : 11$, find $x : z$ **[Ans: 6 : 11]**
- (26) Two numbers are in the ratio $4 : 3$. if the larger number is 120, find the smaller number.
[Ans: 90]
- (27) The ages of two men are in the ratio $7 : 8$. If the younger is 42 years old, What is the age of the older man? **[Ans: 48 years]**
- (28) (a) Increase 280 in the ratio $15 : 14$.
[Ans: 300]
- (b) Increase shs 1800 in the ratio $23 : 12$.
[Ans: 3450]
- (c) Decrease 60 metres in the ratio $1 : 25$, giving the new length in centimetres.
[Ans: 240 cm]
- (29) Three children share 30 biscuits in the ratio $1 : 2 : 2$. Afterwards the child with the least number of biscuits is given two more biscuits by each of the two children. Find the final ratio in which the 30 biscuits are shared. **[Ans: 1 : 1 : 1]**
- (30) Apolo, Ben, David and James contributed shs.5000, shs 6000, shs 5000 and shs4000, respectively in order to buy a bull. Later they sell the bull for shs 24,000. Divide the profit proportionally among them.
[Ans: Apolo = 1000, Ben = 1200, David = 1000, James = 800]
- (31) Given that $3A - B = 2A + B$, determine the ratio
- (i) $A : B$ **[Ans: 2 : 1]**
- (ii) $2A + B : A + 2B$ **[Ans: 5 : 4]**

SURDS, INDICES AND LOGARITHM

TRIAL QUESTIONS

1. Simplify ; $\frac{12m^2 \times 2m}{3mn}$ [Ans: $\frac{8m^2}{n}$]
2. Without using tables or calculator, find the value of x : $\log(3x + 8) - 3 \log 2 = \log(x - 4)$.
[Ans: $x = 8$]
3. Without using calculators and math tables, evaluate $\frac{64 \times 4^3 \times 128}{2^4 \times 32^3}$ [Ans: 16]
4. Without using calculators or tables simplify $\frac{1}{2} \log_{10} 16 + 2 \log_{10} \left(\frac{a}{5}\right) + \log_{10} a^2$
[Ans: $2 \log_{10} \left(\frac{2a^2}{5}\right)$]
5. (a) Use math tables to find the log of 0.0234
[Ans: 2.3692]
(b) What is the antilog of 4.098? [Ans: 12530]
(c) Evaluate without using tables or calculators:
(i) $\log 16 - \log 96 + \log 6000$ [Ans: 3]
(ii). $\log 27^3$ [Ans: $\log 3$]
6. Simplify $\frac{3^3 \times 9^2 \times 125}{9^3}$ [Ans: 375]
7. Given that $16^x = (\frac{1}{2})^{x-5}$ find the value of x .
[Ans: $x = 1$]
8. Find the value of a , if $\log_a 2 + \log_a 4 = 3$
[Ans: $a = 2$]
9. Without using tables and calculators find the value of $\log_{10} 4000 + \log_{10} 500 - \log_{10} 20$
[Ans: 5]
10. (a) Without using mathematical tables or calculator simplify $\left[\frac{27}{8}\right]^{\frac{2}{3}} \times \left[\frac{1}{81}\right]^{\frac{1}{4}}$ [Ans: $\frac{4}{27}$]
(b) Evaluate using logarithms to find the cube root of 0.0271 correct to three decimal places.
[Ans: 0.300]
(c) Solve $[8]^{(x+1)} = [2]^{(x+9)}$ [Ans: $x = 3$]
11. (a) Find x if: $81^x = \left[\frac{1}{3}\right]^{x-5}$ [Ans: $x = 1$]

12. Without using tables or calculators work out $\left(\frac{27}{8}\right)^{\frac{2}{3}} \times \left(\frac{16}{81}\right)^{-\frac{1}{2}}$ [Ans: $\frac{81}{16}$]
13. Given that $\frac{3}{2} \log a^3 - \log a^{\frac{1}{2}} - 2 \log a = 4$, find a
[Ans: $a = 100$]
14. Express $\frac{13}{3\sqrt{2} - \sqrt{5}}$ in the form $a\sqrt{m} + b\sqrt{n}$ where a, b, n and m are Integers [Ans: $3\sqrt{2} + \sqrt{5}$]
15. (a) Solve for x in the equation $3^x = \frac{1}{81}$
[Ans: $x = -4$]
(b) Use log tables to find the value of $\frac{218 \times 0.024}{95.43}$
[Ans: 0.0548]
(c) If $\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}} = a + \sqrt{b}$, find the values of a and b
[Ans: $-2 + \sqrt{6}, a = -2, b = 6$]
(d) Evaluate without using tables or calculators $\frac{4}{5} \log 32 + \log 50 - 3 \log 2$ [Ans: 2]
16. Express $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ in the form $a + b\sqrt{c}$ where a, b and c are integers. Hence state the values of a, b and c . [Ans: $5 + 2\sqrt{6}, a = 5, b = 2, c = 6$]
17. Express $\frac{3}{3\sqrt{2} - 2\sqrt{5}} - \frac{1}{2\sqrt{2} + 2\sqrt{5}}$ in the form $a\sqrt{m} + b\sqrt{n}$ where a, b, m and n are integers.
[Ans: $-\frac{19}{6}\sqrt{5} - \frac{13}{3}\sqrt{2}$]
18. (a) Without using tables or calculator find the value of x if $\log_5(x-2) + \log_5(2x) - \log_5(6x) = 1$
[Ans: $x = 17$]
(b) Solve for x in $3^x \div \frac{1}{81^{(1-x)}} = 1$ [Ans: $x = \frac{4}{3}$]
19. If $\frac{4\sqrt{3}}{\sqrt{5} - \sqrt{3}} = a + b\sqrt{c}$. Find the values of a, b and c . [Ans: $6 + 2\sqrt{5}, a = 6, b = 2, c = 5$]
20. (a) If $\frac{\sqrt{5} - 1}{2\sqrt{5} - 4} = a + b\sqrt{5}$, find the values of a and b where a and b are constants.
[Ans: $\frac{3}{2} + \frac{1}{2}\sqrt{5}, a = \frac{3}{2}, b = \frac{1}{2}, c = 5$]
(b) Find the value of x , if, $\log(2x - 11) - \log 2 = \log 3 - \log x$. [Ans: $x = 6$]

surds, indicies and logarithm

- (c) If $x^3 = 3.375$, use tables to find the value of x , correct to three significant figures.

[Ans: 1.50]

- (d) Given that $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$, find without using tables or calculator, the value of;

(i) $\log_{10}72$ [Ans: 1.8572]

21. (a) Given that $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$ and $\log_{10}5 = 0.6990$, evaluate $\log_{10}150$
[Ans: 2.1761]

- (b) Use logarithms tables to evaluate;
 $\sqrt[3]{\frac{0.3215 \times 1.439}{0.00485}}$ [Ans: 4.568]

22. (a) Without using mathematical tables or calculator, simplify, $\frac{\frac{1}{64^{\frac{1}{3}}}}{27^{\frac{1}{3}}}$ [Ans: $\frac{3}{4}$]

- b) Given that; $\sqrt{15} = 3.873$, $\sqrt{6} = 2.450$ and $\sqrt{10} = 3.162$, without using a calculator, evaluate to two significant figures; $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$
[Ans: 0.387]

23. (a) Without using tables or calculators, solve the equation; $\log(10x + 5) - \log(x - 4) = \log 100$
[Ans: 4.5]

- (b) Use logarithms to find the square root of 0.0576. [Ans: 0.2400]

- (c) Given that $8^{2n} \times 3^m = 36$, find the values of n and m . [Ans: $n = \frac{1}{3}$, $m = 2$]

- (d) Using mathematical tables, evaluate;
 $\frac{87.5 \times 0.0243}{0.003142}$ [Ans: 676.7]

24. (a) Solve for x given that $\log_{125}x + \log_{125}5x = 1$.
[Ans: $x = 5$]

- (b) If $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = p + q\sqrt{r}$, state the values p , q and r . [Ans: $p = 7$, $q = 4$, $r = 3$]

25. (a) Without using tables or calculator, find the value of $2\log_{10}50 + \log_{10}80 - \log_{10}2$

- (b) (i) Express 150 as a product of its prime factors.

- (ii) Using your result in (i) above, find $\log_{10}150$ given that $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$ and $\log_{10}5 = 0.6990$. [Refer to number 21 above]

- (c) Without using tables or calculator, evaluate $4\log_{10}2 - \log_{10}48 + \log_{10}30$. [Ans: 1]

- (d) Solve the equation ; $2\log x = \log(x + 6)$
[Ans: $x = 3$ and $x = -2$]

26. (a) Express $\sqrt{20} + \sqrt{125} - \sqrt{45}$ in the form $a\sqrt{b}$ where a and b are constants. [Ans: $4\sqrt{5}$]

- (b) Express $\frac{2}{2 - \sqrt{3}}$ in the form $a + b\sqrt{c}$, where a , b and c are constants. [Ans: $4 + 2\sqrt{3}$]

27. (a) Without using tables or calculator, evaluate $2\log_{10}5 + \log_{10}80 - \log_{10}20$. [Ans: 2]

- (b) Use the fact that $\log_{10}3 = 0.4771$ and $\log_{10}5 = 0.6990$ to evaluate $\log_{10}3375$.
[Ans: 3.5283]

28. (a) Solve for x in the equation $2^{(3x-1)} \times 8^{(x-1)} = 256$. [Ans: $x = 2$]

- (b) Given that $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$ and $\log_{10}5 = 0.6990$ find the value of $\log_{10}4320$
[Ans: 3.6353]

- (c) With out using tables or calculators simplify
 $\left[\frac{8}{125}\right]^{\frac{-2}{3}} \left[\frac{5}{8^{\frac{1}{2}}}\right]^{-2}$ [Ans: $\frac{1}{32}$]

- (d) Simplify, $\frac{4^2 \times 2^2 \times 16^{\frac{1}{2}}}{8^3}$ [Ans: 1]

29. (a) Use log table to evaluate; $\frac{\sqrt[5]{3.15 \times 0.047}}{9.24 \times 26.1}$
[Ans: 0.2278]

- (b) Find the value of x in $\log(x + 3) = \log(5 - x)$.
[Ans: $x = 1$]

- (c) Without using tables or a calculator, evaluate $4\log 2 - \log 48 + \log 30$ [Ans: 1]

- (d) Without using tables or calculator, evaluate $\frac{5^{-6} \times 27^4 \times 15^8}{81^5}$ [Ans: 25]

- (e) Use tables to evaluate; $(0.0367)^3 \times 47.9$
[Ans: 0.002368]

- (f) Simplify: $\frac{2^x \times 8^{(x-1)}}{16^{(x-1)}}$. [Ans: 2]

30. (a) Simplify; $\frac{24^2 \times 27^3}{2^6 \times 81}$ [Ans: 2187]

- (b) Given that $128^a = 16^b$, find $\frac{a}{b}$. [Ans: $\frac{a}{b} = \frac{4}{7}$]

- (c) Solve for x in $\frac{x^2}{7} = \frac{49}{x}$ [Ans: $x = 7$]

31. (a) Without using mathematical tables, solve for x in the equation; $9^{(2x-\frac{1}{4})} \times 27^{(x-\frac{1}{2})} = 729^{(x+\frac{1}{3})}$
[Ans: $x = 4$]

- (b) Given that $9^x = 243$, find the value of x .
[Ans: $x = 2.5$]

- (c) Evaluate without using tables or a calculator $\log_{10}120 - \log_{10}36 + \log_{10}3$. [Ans: 1]

32. (a) Solve the equation for x ; $\sqrt[3]{\left(\frac{2}{5}\right)^{2-x}} = \frac{8}{125}$
[Ans: $x = -7$]

(b) Evaluate without using tables or calculator
 $4\log_{10}2 + \log_{10}5 - 3\log_{10}2.$ **[Ans: 1]**

(c) Given that $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$ evaluate without using tables or a calculator $\log_{10}0.6.$ **[Ans: -0.2219]**

33. (a) Evaluate; $\left[\frac{27}{125}\right] \times \left[\frac{243}{3125}\right]^{\frac{2}{5}}$ **[Ans: $\frac{3}{5}$]**

(b) Given that $\frac{6}{2\sqrt{5}+\sqrt{2}} = a\sqrt{5} + b\sqrt{2}$, determine the values of a and $b.$ **[Ans: $a = \frac{2}{3}, b = -\frac{1}{3}$]**

(c) Express $\frac{\sqrt{3}-2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ in the form $a + b\sqrt{c}$, where a, b and c are real numbers.
[Ans: $7 - 3\sqrt{6}$]

(e) Solve the equation $\log_{10}(7x+2) - \log_{10}(x-1) = 1.$ **[Ans: $x = 4$]**

34. (a) It is given that $\log_{10}3 = 0.4771$ and $\log_{10}2 = 0.3010.$ Find the value of $\log_{10}\left[\frac{108}{64}\right].$ Without using a calculator or tables. **[Ans: 0.2273]**

(b) Solve the following equations simultaneously

$$\log(x-1) + \log y = 2\log 2$$

$$\log x + \log y = \log 6 \quad \text{[Ans: } x = 3, y = 2\text{]}$$

(c) Without using tables or a calculator, simplify
 $\frac{1}{2}\log 64 + 2\log 3 - \log 12 \quad \text{[Ans: } 2\log 2\text{]}$

35. (a) Solve for x in the equation; $3^{2x+1} = 243$
[Ans: $x = 2$]

(b) Find the value of x in the equation.

(i) $\log_x 625 = 4 \quad \text{[Ans: } x = 4\text{]}$

(ii) $\log_3 729 = x \quad \text{[Ans: } x = 6\text{]}$

36. (a) Solve for x in the equation; $32^x \times 4^{(x+3)} = 2^{27}$
[Ans: $x = 3$]

(b) If $\log_{10}5 = 0.6990$ and $\log_{10}2 = 0.3010,$ find the value of $\log_{10}500 \quad \text{[Ans: } 2.699\text{]}$

(c) Use logarithms tables to evaluate; $\frac{685.3}{29.86}.$
[Ans: 22.95]

(d) Simplify; $x^2 + (\sqrt{3})x$ if $x = (2 - \sqrt{3}).$ **[Ans: $2(2 - \sqrt{3}) \text{ or } (4 - 2\sqrt{3})$]**

37. (a) Simplify; $\frac{2^{-2} \times 3^6}{2^{-4} \times 9 \times 54}.$ **[Ans: 6]**

(b) Evaluate without using tables or calculators;
 $2\log_{10}6 + 2\log_{10}5 - 2\log_{10}3 \quad \text{[Ans: 2]}$

(d) Use tables to evaluate; $\sqrt[3]{18.95 \times 4.2}$
[Ans: 4.301]

38. (a) Given that; $\frac{4\sqrt{5} + 3\sqrt{2}}{2\sqrt{2} + \sqrt{5}} = p + q\sqrt{r},$ find the values of p, q and $r.$
[Ans: $p = \frac{8}{3}, q = \frac{5}{3}, r = 10$]

(b) Use logarithm tables to evaluate;
 $(1.58 \times 37.25)^{\frac{1}{3}} \quad \text{[Ans: 3.890]}$

(c) Find the value of x in the equation $2\log_{10}x + \log_{10}6 - \log_{10}15 = 1.$ **[Ans: $x = 5$]**

d) Given that $\log_{10}a = 1.699$ and $\log_{10}b = 1.913,$ evaluate $\log_{10}a^2b^{\frac{1}{3}} \quad \text{[Ans: 9.137]}$

(e) Use logarithms tables to evaluate;
 $\sqrt[4]{\frac{4.562 \times 0.38}{0.82}} \quad \text{[Ans: 1.206]}$

39. (a) Find the value of y if $\frac{243 \times 3^{2y}}{729 \times 3^y \times 3^{(2y-1)}} = 81$
[Ans: $y = -4$]

(b) Without using tables or calculators, evaluate the following

(i) $3\log_{10}2 + \log_{10}200 - \log_{10}1.6 \quad \text{[Ans: 3]}$

(ii) $32^m \times \frac{1}{8} \times 4^{(m-3)} = 2^{12} \quad \text{[Ans: } m = 3\text{]}$

(iii) Using mathematical tables evaluate:
 $\frac{35.6}{1.47 \times 12.6} \quad \text{[Ans: 1.922]}$

40. (a) Simplify: $\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}} \quad \text{[Ans: } \frac{6}{7}\text{]}$

(b) Without using tables or calculator, solve the equation: $\log_{10}(10x+5) - \log_{10}(x-4) = 2$
[Ans: $x = 4.5$]

(c) Use logarithm tables to evaluate;
 $\sqrt[3]{\frac{0.3216 \times 62.58}{41.57}} \text{ to 3 significant figures.} \quad \text{[Ans: 0.785]}$

41. (a) Simplify; $\frac{4m^6 \times 3m^3 \times 2m^2}{8m^2 \times 3m^7} \quad \text{[Ans: } m^2\text{]}$

(b) Simplify the following;

(i) $\log_5 x^2 - \log_5 2x = 2 \quad \text{[Ans: } x = 50\text{]}$

(ii) $\log_{10}4a + \log_{10}3a \quad \text{[Ans: } 2\log a + \log_{10}12\text{]}$

(iii) $\log_2 y^2 + \log_2 y \quad \text{[Ans: } 3\log_2 y\text{]}$

(iv) $5\log_{10}2 + \log_{10}\left[\frac{1}{32}\right] \quad \text{[Ans: 0]}$

(v) $\log_3 3^{\frac{1}{2}} + \frac{1}{2}\log_3 3 \quad \text{[Ans: 1]}$

(vi) $\log_4 x^2 + \log_4 x^3 + \log_4 x^4 \quad \text{[Ans: } 9\log_4 x\text{]}$

(vii) $\log_{10}z^{10} - \log_{10}z^7 + \log_{10}z^6 \quad \text{[Ans: } 9\log_{10}z\text{]}$

(viii) $3\log_8 x + \log_8 x^3 - \log_8 x^{\frac{1}{2}} \quad \text{[Ans: } \frac{11}{2}\log_8 x\text{]}$

(c) Without using tables or calculators evaluate;
 $\sqrt[3]{0.002406} \text{ to 3dp.} \quad \text{[Ans: 0.134]}$

surds, indicies and logarithm

(d) Given that $\log_{125}x + \log_{125}5x = \frac{1}{3}$, find the values of x . [Ans: $x = \pm 1$]

(e) Given that $\log_{10}7 = 0.845$ and $\log_{10}2 = 0.301$, find $\log_{10}\left[\frac{49}{64}\right]$ [Ans: -0.116]

42. Use logarithm tables to evaluate the following;

(a) $\sqrt[3]{72.4 \times 0.569}$ [Ans: 3.45]

(b) 8.05×23.5^2 [Ans: 4450]

(c) $19.7 \div (8.27)^2$ [Ans: 0.288]

(d) $\frac{3.74 \times 0.582}{0.0507}$ [Ans: 42.9]

(e) $3\sqrt{11 \times 5.132}$ [Ans: 58.5]

(f) $\sqrt[3]{3.94 \times 0.632}$ [Ans: 1.36]

(g) $\frac{0.078 \times 34.3}{4.8}$ [Ans: 0.557]

43. The formula for the radius of a circle is $r = \sqrt{\frac{A}{\pi}}$, when A is its area. Using logarithm tables, find r when A = 16.9 and $\pi = 3.14$. [Ans: 2.32]

44. Given that $\log_{10}2 = m$, $\log_{10}7 = n$, find the value of x if;

(a) $\log_{10}x = 2m + n$ [Ans: $x = 28$]

(b) $\log_{10}x = 3m - n + 1$ [Ans: $x = \frac{80}{7}$]

CO-ODINATE GEOMETRY

RECALL

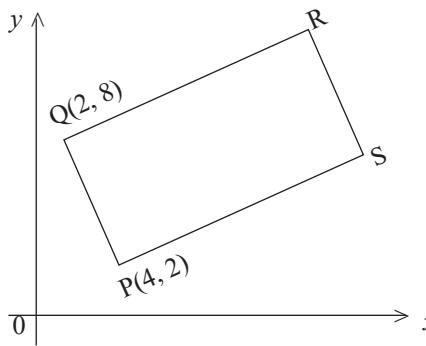
- ◊ $y = mx + c$ is the general form of an equation of line, where m is the gradient of a line and C , the intercept.
- ◊ Parallel lines have the same gradient $M_1 = M_2$
- ◊ For perpendicular lines, the product of their gradient is -1
- ◊ $M_1 M_2 = -1$
- ◊ To find the point of intersection of two lines, we solve the two equations simultaneously to find the values of x and y .
- ◊ Mid point of a line $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- ◊ Gradient of a line, $m = \frac{y_2 - y_1}{x_2 - x_1}$
- ◊ Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

TRIAL QUESTIONS

1. Find the equation of a line which passes through point $(4, 3)$ and parallel to the line $y = 2x + 4$
[Ans: $y = 2x - 5$]
2. Find the equation of a line that passes through point $(2, 5)$ and is parallel to the line $y = 3x - 15$.
[Ans: $y = 3x - 1$]
3. (a) Determine the equation of line that passes through points $(2, 1)$ and $(3, 3)$ [Ans: $y = 2x - 3$]
(b) Two lines L_A and L_B are parallel. Line L_A passes through points $(4, 2)$ and $(-3, 1)$. Determine the equation of line L_B that passes through point $(7, 0)$. [Ans: $y = \frac{1}{7}x - 1$]
4. The line that passes through point $A(m, 4)$ and $B(4, m)$ is parallel to the line whose equation is $y = 2x - 4$. Find the value of m . [Ans: $m = 4$]
5. A line through $(2, 3)$ and $(-1, 2)$ crosses at right angle, a line $ax + by = 11$ at point $(3, 2)$. Find the values of a and b . [Ans: $a = 3, b = 1$]
6. Determine the equation of the straight line passing through the point $(2, 5)$ and perpendicular to the line joining the point $A(-3, 3)$ to point $B(6, 9)$
[Ans: $y = \frac{-3}{2}x + 8$]

7. (a) Find the equation of the line passing through $(5, 4)$ and perpendicular to the line $2y = 3x - 7$
[Ans: $3y = -2x + 22$]
(b) Find the equation of a straight line which passes through the point $(0, 5)$ and is perpendicular to the line $y + 3x = 7$.
[Ans: $3y = -x + 5$]
8. (a) Find the equation of a straight line that goes through $(1, 7)$ and $(4, 9)$. [Ans: $3y = 2x + 19$]
(b) Find the equation of a straight line whose gradient is $\frac{-1}{2}$ and passes through the point $(4, 5)$. [Ans: $y = \frac{-1}{2}x + 7$]
(c) Find the equation of a straight line which passes through $(1, 3)$ and is perpendicular to the line $2y - 3x = 7$.
[Ans: $3y = -2x + 1$]
9. (a) Find the equation of a line that passes through the point $A(4, 0)$ and is parallel to the line whose equation is $y = 2x + 4$. [Ans: $y = 2x - 8$]
(b) If $A(2, 9)$ and $B(-1, 5)$ are end points of a line segment \overline{AB} , find;
(i) the coordinates of M , the midpoint of line segment \overline{AB} . [Ans: $(\frac{1}{2}, 7)$]
(ii) the distance of line \overline{AB} . [Ans: 5 units]
(c) Find the coordinates of the y -intercept and x -intercept of the line whose equation is $y - 4x - 6 = 0$.
[Ans: y intercept $(0, 6)$, x -intercept $(\frac{-3}{2}, 0)$]
10. (a) Find the equation of a line that joins $A(1, 5)$ and $B(-2, -1)$. [Ans: $y = 2x + 3$]
(b) Determine the equation of a straight line which passes through the point $(3, 4)$ and is parallel to the line $y + 3x - 5 = 0$.
[Ans: $y = -3x + 13$]
11. (a) For the equation $3y - 4x + 9 = 0$, find;
(i) the gradient of the equation. [Ans: $\frac{4}{3}$]
(ii) the coordinates of the point where the line in
(i) cuts the y -axis. [Ans: $(0, -3)$]
(b) Find the equation of a straight line passing through the point $(2, 3)$ and is perpendicular to the line $2y - 4x - 7 = 0$. [Ans: $2y = -x + 4$]

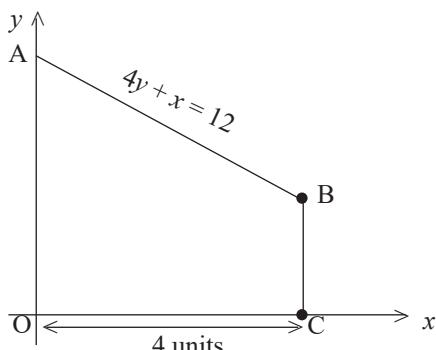
12. (a) The line through points A(a , 2) and B(3, 6) is parallel to the line $y = 4x - 5$. Find the value of a . [Ans: $a = 2$]
- (b) Find the equation of a line passing through a point A(2, 0) and perpendicular to the line joining the points B(-10, 3) and C(6, -9).
[Ans: $3y - 4x + 8 = 0$]
13. The points (-1, q) and (r , 2) lie on the line $y = 2 - x$. Find the values of q and r . [Ans: $q = 3, r = 0$]
14. A line is given by the equation $45 - 15x + 3y = 0$. Find the coordinates of the x -intercept.
[Ans: (3, 0)]
15. Two points P(5, 2) and Q(2, 4) are in a plane. Find;
- (a) Coordinates of M the mid point of \overline{PQ}
[Ans: $\frac{7}{2}, 3$]
- (b) $|\overline{OM}|$ where O is the origin. [Ans: 4.61 units]
16. Given the curve $y = 2x^2 + 3x$ and the line $y = 5 + 4$, determine the coordinates of the points of intersection of the curve and the line. [Ans: (-1, -1), (2, 14)]
17. (a) Find the equation of a line which is a perpendicular bisector of the line passing through the points P(-2, 3) and Q(6, 1).
[Ans: $y = 4x - 6$]
- (b) Find the equation of a straight line which passes through A(-2, 6) and B(-5, 4) and hence determined the coordinates of the point Q where this line cuts the x -axis.
[Ans: $3y = -2x + 22$, Q (11, 0)]
18. (a) Find the equation of a straight line through (2, 3) and is parallel to the line $y = \frac{1}{2}x - 7$.
[Ans: $y = \frac{1}{2}x + 2$]
- (b) Find the equation of a line through the origin and perpendicular to the line joining the points P(6, 8) and R(3, 5). [Ans: $y = -x$]
19. (a) Find the equation of a line, in the form $y = mx + c$, parallel to $4y + 2x = 5$ and passing through the point(1, -3). [Ans: $2y + x - 7 = 0$]
- (b) The line $3x - 5y - 6 = 0$ passes through point $(p, 0)$. Determine it's
- (i) gradient [Ans: $\frac{3}{5}$]
(ii) value of p [Ans: $p = 2$]
20. Line M passes through points (8, -1) and (-2, 12). Line N is $2y - x = 2$. Determine;
- (a) the equation of line M.
[Ans: $10y + 13x - 94 = 0$]
- (b) the co-ordinates of point of intersection for lines M and N. [Ans: $(\frac{14}{3}, \frac{10}{3})$]
- (c) the x - intercepts for the lines M and N.
[Ans: $(\frac{94}{13}, 0)$ and $(-2, 0)$]
- (d) the area enclosed between lines M and N and the x - axis.
21. Find the equation of a line whose y - intercept is 5 and perpendicular to the line whose equation is $y - 2x = 3$.
[Ans: $y = \frac{-1}{2}x + 5$]
22. (a) Find the equation of a perpendicular bisector of a line passing through (2, 4) and (2, 6)
- (b) find the equation of a line whose x - intercept is -4 and y - intercept is 5, hence state it's gradient. [Ans: $y = \frac{-5}{4}x + 5$, Gradient = $\frac{-5}{4}$]
23. Find the lengths of the line joining the following pair of points.
- (a) (-4, -1) and (4, 1) (c) (-2, 5) and (-2, 7)
(b) (-1, -3) and (-9, -9) [Ans: (a) $\sqrt{68}$ units
(b) 10 units (c) 2 units]
24. (a) Find the distance of the point A(-7, 24) from the origin.
(b) A(4, 7) and B(x, y) are the end-points of the straight line AB and C(5, 6) is its mid-point. Find the values of x and y .
[Ans: $x = 6, y = 5$]
25. (a) A(0, 0), B(5, 1), C(8, 4) and D(3, 3) are the vertices of a parallelogram. Find the length of the diagonal AC. [Ans: $\sqrt{80}$ units]
- (b) Show that the co-ordinates of the mid-points of the diagonal AC and BD are the same.
26. P is (2,5) and O is the origin. The line $y = mx + c$ is parallel to OP. What is m ? [Ans: $m = \frac{5}{2}$]
27. In the figure PQRS is a rectangle and P and Q are the points (4, 2) and (2, 8) respectively. Given that the equation of line PR is $y = x - 2$, find;



- (i) the equation of QR. [Ans: $3y - x = 22$]
 (ii) the coordinates of R [Ans: 14, 12]
 (iii) the co-ordinates of S. [Ans: 16, 6]
 (iv) the area of the rectangle PQRS
 [Ans: 80 sq.units]

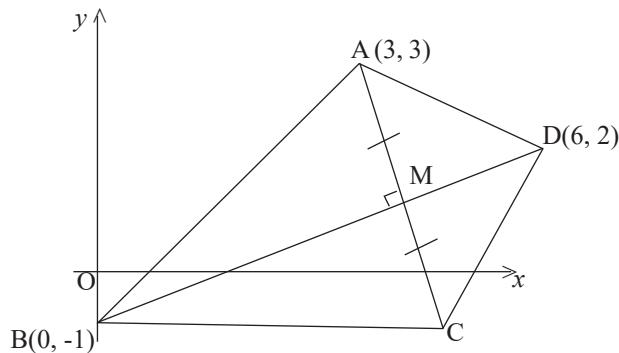
28 (a) The points $(-1, q)$ and $(r, 2)$ lie on the line $y = 2 - x$. Find the values of q and r .
 [Ans: $q = 3, r = 0$]

(b) In the figure below, C is 4 units from O, the equation of the line AB is $4y + x = 12$. Find the area of OABC.



[Ans: 10 sq.units]

29. In the quadrilateral ABCD, the point A, B and D are at $(3, 3)$, $(0, -1)$ and $(6, 2)$ respectively. The line BD bisects the line AC at right angles at the point M.



- a) Find the equation of BD and of AC.
 [Ans: BD: $y = \frac{1}{2}x - 1$, AC: $y = -2x + 9$]
 b) Calculate the coordinates of M and C.
 [Ans: M(4, 1), C(5, -1)]

30. Given points A(4, 3) and B(8, 13). Find

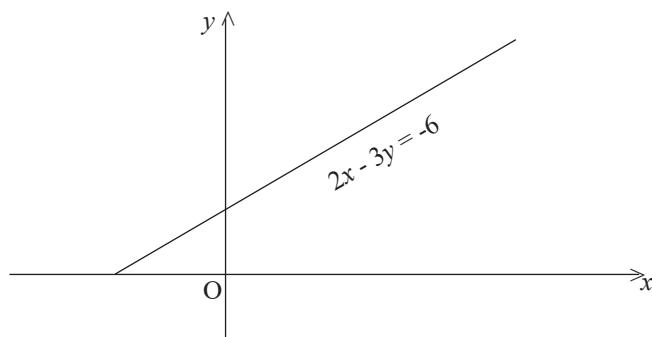
- a) the gradient of the line AB. [Ans: $\frac{5}{2}$]
 b) the co-ordinates of the mid - point of AB.
 [Ans: (6, 8)]
 c) the length of the line segement AB.
 [Ans: 10.77 units]

31. (a) i) Find the point of intersection of the lines $y = 2x - 3$ and $y = -x - 3$. [Ans: (0, -3)]

- ii) Calculate the area of triangle enclosed between the two lines and the x-axis.
 [Ans: 6.75 sq. units]

(b) Find the equation of the line which is a perpendicular bisector of the line passing through points A(5, 4) and B(3, 8)
 [Ans: $y = \frac{1}{2}x + 4$]

32. Find the gradient and the intercepts on the axes of the line shown in the diagram.



[Ans: grad = $\frac{2}{3}$, x- intercept = -3, y- intercept = 2]

FUNCTIONS

RECALL

This includes the following;

- ◊ Domain(object) and Range (image)
- ◊ Solving functions.
- ◊ Undefined functions/meaningless functions / infinite functions. (When denominator of a function is zero, then equate denominator to zero.)
- When a function is equal to zero, $f(x) = 0$, (equate the numerator to zero)
- ◊ Composite functions, This is a combination of two or more functions. e.g $fg(x)$, $hf(x)$
- ◊ Inverse of a function: $f^{-1}(x)$, $(gf)^{-1}(x)$ etc.

Example 1	Example 2	Example 3
<p>Given that $f(x) = \frac{x+1}{2x-4}$, find the value of x for which</p> <ul style="list-style-type: none"> (i) $f(x)$ is undefined (ii) $f(x)$ is equal to zero. <p>Solution:</p> <ul style="list-style-type: none"> (i) for undefined function, the denominator of $f(x)$ is equal to zero. $f(x) = \frac{x+1}{2x-4}, \text{ then } 2x-4=0$ $2x=4$ $\frac{2x}{2} = \frac{4}{2}$ $x=2$ <ul style="list-style-type: none"> (ii) $f(x)$ is equal to zero $f(x) = \frac{x+1}{2x-4}, f(x)=0$ $0 = \frac{x+1}{2x-4}$ $0(2x-4) = (x+1)$ $x+1=0$ $x=-1$	<p>If $g(x) = \frac{4x+9}{x+4}$, find the inverse of $g(x)$ [$g^{-1}(x)$]</p> <p>Solution</p> <p>Let any letter be equal to the given function.</p> $y = \frac{4x+9}{x+4}, \text{ then make } x \text{ the subject of the formula}$ $\frac{y}{1} = \frac{4x+9}{x+4}$ $y(x+4) = 1(4x+9)$ $yx+4y = 4x+9$ <p>collect like terms</p> $yx-4x = 9-4y$ $x(y-4) = 9-4y$ $x = \frac{9-4y}{y-4}$ <p>Then substitute the value selected back to x</p> $f^{-1}(x) = \frac{9-4x}{x-4}$	<p>If $f(x) = 2x+3$ and $g(x) = 6x$, find (i) $fg(x)$ (ii) $gf(x)$</p> <p>Solution</p> <ul style="list-style-type: none"> (i) $f(x) = 2x+3$, $g(x) = 6x$ <p>$fg(x)$ means that where there is x in $f(x)$, substitute $g(x)$</p> $fg(x) = 2(g(x)) + 3$ <p>but $g(x) = 6x$</p> $fg(x) = 2(6x) + 3$ $fg(x) = 12x + 3$ <ul style="list-style-type: none"> (ii) $gf(x)$: This means that, where there is x in $g(x)$, substitute $f(x)$ $gf(x) = 6(f(x))$ $= 6(2x+3)$ $gf(x) = 12x + 18$

TRIAL QUESTIONS

(1) Given that $g(x) = x^3 + 1$, find the values of

- (a) $g(0)$, [Ans: 1]
- (b) $g(5)$ [Ans: 126]
- (c) $g\left(\frac{3}{4}\right)$, [Ans: $\frac{91}{64}$]
- (d) $g(-2)$ [Ans: -7]

(2) The domain of the function $f(x) = 5x + 1$, is $\{0, 1, 2, 3, 4, 5\}$. Find its range.
[Ans, {1, 6, 11, 16, 21, 26}]

(3) If $f(x) = 7x$ and $g(x) = x + 3$ and $fg : x_1 \rightarrow y$, express as simply as possible the rule which maps x onto y . [Ans: $7x + 21$]

Find the values of p , q , r such that; (a) $fg: 5 \rightarrow p$
(b) $fg: 10 \rightarrow q$, (c) $fg: r \rightarrow 35$.
[Ans: $p = 56$, $q = 91$, $r = 2$]

Find also the function (F) which reverses the function fg , that is it maps y onto (back to) x .

$$[Ans: \frac{x-21}{7}]$$

(4) If $f(x) = ax + b$ and $f(2) = 10$, while $f(-3) = 5$.
Find

(i) value of a and b [Ans: $a = 1, b = 8$]

(ii) $f^{-1}(x)$ [Ans: $x - 8$]

(5) (a) If $h(x) = nx + 7$ and $h(5) = 22$, find;

(i) the value of n [Ans: $n = 3$]

(ii) $h(4)$ [Ans: 19]

(b) Given that $f(x) = 8x^2 + 5$, find

(i) $f(-3)$ [Ans: 77]

(ii) $f(2)$ [Ans: 37]

(iii) The value of x for which $f(x) = 293$

[Ans: $x = \pm 6$]

(6) If $f^{-1}(x) = 2x^2 - 3$, find

(i) an expression for $f(x)$ [Ans: $f(x) = \sqrt{\frac{x+3}{2}}$]

(ii) $f(15)$ [Ans: 3]

(7) If $h(x) = 4 - x^2$ and $g(x) = 2x + 1$,

Find (i) $gh(x)$ [Ans: $9 - 2x^2$]

(ii) $gh(-2)$ [Ans: 1]

(8) (a) Given that $g(x) = px + q$, $g(2) = 6$ and $g(-4) = 3$. Find the values of p and q .

[Ans: $p = \frac{1}{2}, q = 6$]

(b) If $f(x) = \frac{1}{x}$, find $\frac{1}{f^2(x)}$ for $x = 1, 2, 3$, and 4
[Ans: 1, 4, 9, 16]

(9) (a) Given that $f(x) = 3x - 2$, find $f^{-1}(x)$ and $f^{-1}(10)$. [Ans: $\frac{x+2}{3}, 4$]

(b) If $h(x) = rx^2 + t$, $h(1) = 7$ and $h(0) = 2$, find the values of r and t . [Ans: $t = 2, r = 5$]

(10) (a) Given that $f(x) = \frac{1-x^2}{4x^2-1}$, find the values of x for which

(i) $f(x) = 0$ [Ans: $x = \pm 1$]

(ii) $f(x)$ is undefined [Ans: $\pm \frac{1}{2}$]

(b) If $f(x) = \frac{3x}{2x+1}$, find $f^{-1}(x)$ Hence calculate $f^{-1}(2)$ [Ans: $\frac{x}{3-2x}, -2$]

(11) (a) Given that $f(x) = px - q$, $f(2) = 10$ and $f(1) = 7$, find the values of p and q .

[Ans: $p = 3, q = -4$]

(b) If $g(x) = \frac{2x-1}{x^2-9}$, find the values of x for which the function $g(x)$ is undefined. [Ans: $x = \pm 3$]

(c) Given that $f(x) = x^2 + 7$ and $g(x) = x - 2$, find the value of x for which $fg(x) = \frac{38-x^2}{2}$. [Ans: $x = 4$ or $x = -\frac{4}{3}$]

(12) (a) Given that $f(x) = 3x + 5$ and $g(x) = \frac{2}{x-5}$, find

(i) $gf(x)$ [Ans: $\frac{2}{3x}$]

(ii) $gf\left(\frac{1}{2}\right)$ [Ans: $\frac{4}{3}$]

(b) If $h(x) = px + 3$ and that $h(4) = 23$, find the value of

(i) p [Ans: $p = 5$]

(ii) $h(0)$ [Ans: 3]

(iii) $h(-5)$ [Ans: -22]

(c) Determine;

(i) $h^{-1}(x)$ [Ans: $\frac{x-3}{5}$]

(ii) $h^{-1}(13)$ [Ans: 2]

(13) Given that the function $f(x) = \frac{x}{x-2}$ and $g(x) = \frac{1}{2x-3}$

(a) Find the value of x for which $f(x) - g(x) = \frac{8}{15}$, [Ans: $x = -1, x = \frac{9}{7}$]

(b) The functions f and g are defined by $f(x) = \frac{x}{x-5}$ and $g(x) = x + 4$. Find;

(i) $g(-10)$ [Ans: -6]

(ii) $f^{-1}(x)$ and hence $f^{-1}(6)$. [Ans: $\frac{5x}{x-1}, 6$]

(iii) the value of x for which $gf(x) + fg(x) = 0$

[Ans: $x = \frac{13}{3}$ or $x = 0$]

(14) (a) Given that $f(x) = 2x + 6$ and $g(x) = \frac{x}{3x-10}$
Find;

(i) $gf(x)$ [Ans: $\frac{x+3}{3x+4}$]

(ii) $gf\left(\frac{1}{3}\right)$ [Ans: $\frac{2}{3}$]

(b) A function h is defined by $h(x) = \frac{x+4}{3x+2}$, find

(i) $h^{-1}(x)$ [Ans: $\frac{4-2x}{3x-1}$]

(ii) The value of x for which $h^{-1}(x)$ is meaningless.

[Ans: $x = \frac{1}{3}$]

(15) (a) Given that $g(x) = (x+2)(x-3)$ find

(i) $g(5)$ [Ans: 14]

(ii) the values of x for which $g(x) = 0$

[Ans: $x = -2, x = 3$]

(iii) the range of values of x for which $f(x) < 0$

[Ans: $f: x \rightarrow x \leq -4$]

- (b) Given that $f(x) = 2x + 9$ and $g(x) = 2 - x$, write an expression for $gf(x)$. Hence find the value of $gf(3)$. [Ans: $-7 - 2x, -13$]
- (16) (a) Given that $g(x) = \frac{3x}{x+4}$ and $h(x) = x^2 - 2$. Find
 (i) $g(-5)$ [Ans: 15]
 (ii) $gh(2)$ [Ans: 1]
- (b) Find the value of a , given that $f(x) = \frac{2x+a}{4-x}$ and $f(0) = 1$. [Ans: $a = 1$]
- (17) (a) A function $f(x) = x - m$ and $f^{-1}(x) = 2xk - 7$ where m and k are constants. Calculate the values of m and k . [Ans: $k = \frac{1}{2}, m = -7$]
 (b) Given that $h(x) = px^2 - qx + 1$, $h(2) = 11$ and $g(1) = 2$. Find the values of p and q . [Ans: $p = 4, q = 3$]
 (c) Given that $f(x) = \frac{x+5}{6}$ and $fg(x) = \frac{7-x}{2}$. Find;
 (i) $f(-17)$ [Ans: -2]
 (ii) an expression for $g(x)$ and hence evaluate $g(4)$ [Ans: $g(x) = 16 - 3x, g(4) = 4$]
- (18) (a) Determine the range corresponding to the domain $\{-3, -2, 0, 1, 3, 4\}$ for the mapping $x \rightarrow x^2 + 1$. [Ans: {10, 5, 1, 2, 10, 17}]
 (ii) Represent the mapping in (i) above on an arrow diagram.
 (iii) State the mapping in (ii) above.
 [Ans: Many to one]
- (b) Given that the functions $h(x) = x + 2$, $g(x) = x^2$ and $f(x) = -x$, find the values of x for which $gh(x) = f(x)$. [Ans: $x = -1$ or $x = -4$]
- (19) (a) The function h is defined as $h(y) = my^2 - 2$. Given that $h(3) = 16$, find;
 (i) the value of m [Ans: $m = 2$]
 (ii) $h^{-1}(6)$ [Ans: ± 2]
 (b) Given that $f(x) = x + 4$, $g(x) = 3x^2$, find the value of x for which $gf(x) = fg(x)$. [Ans: $-\frac{11}{6}$]
- (20) (a) Given that $f(x) = \frac{1}{2x-6}$, find
 (i) $f^{-1}(x)$ [Ans: $\frac{1+6x}{2x}$]
 (ii) $f^{-1}(-1)$ [Ans: $\frac{5}{2}$]
 (b) Given that $f(x) = x - 5$, $g(x) = x^2$ and $h(x) = \frac{x+2}{6x^2+x-2}$, find $f(-6)$. [Ans: -11]
- (c) Find the value of x for which
 (i) $gf(x) = fg(x)$ [Ans: $x = 3$]
 (ii) $h(x)$ is not defined. [Ans: $x = \frac{1}{2}$ or $x = -\frac{2}{3}$]
- (21) (a) Given that function $f(x) = \frac{9+24x+8x^2}{10}$ and $g(x) = \frac{8-x}{5}$, find the values of x for which $f(x) - g(x) = 0$. [Ans: $x = -\frac{7}{2}, x = \frac{1}{4}$]
 (b) If the functions $f(x) = \frac{x+3}{2}$ and $g(x) = \frac{1-2x}{5}$, determine the value of x for which $fg(x) + gf(x) = 0$. [Ans: $x = 3$]
- (22) (a) Draw an arrow diagram for the function $f(x) = 2x^2 + 2$ given that $-2 \leq x \leq 2$.
 (b) Given that $f(x) = \frac{(x+2)(x-3)}{(x-5)}$, find;
 (i) $f(-5)$ [Ans: -2.4]
 (ii) the values of x for which $f(x) = 0$.
 [Ans: $x = -2, x = 3$]
 (iii) The values of x for which $f(x)$ is meaningless. [Ans: $x = 5$]
 (c) Given that $f(x) = x^2 + 1$ and $h(x) = x - 3$, find the value of x for which $fh(x) = hf(x)$. [Ans: $x = 2$]
 (d) The function $h(x) = \frac{4}{5x^2 - 12x + 4}$, find the value of x for which $h(x)$ is meaningless.
 [Ans: $x = \frac{2}{5}, x = 2$]
- (23) (a) Given that $f(x) = ax + b$, $f(4) = 4$ and $f(-1) = -6$. Find the values of a and b .
 [Ans: $a = 2, b = -4$]
 (b) Given that $g(x) = \frac{2}{x+2} + \frac{3x+4}{x^2-4}$, express $g(x)$ in the form $\frac{px}{x^2+q}$, where p and q are integers. State the values of x for which $g(x)$ is not defined.
 [Ans: $x = \pm 2$]
 (c) The function $h(x) = \frac{4}{5x+1}$ find;
 (i) $h^{-1}(x)$ [Ans: $\frac{4-x}{5x}$]
 (ii) $h^{-1}(2)$. [Ans: $\frac{1}{5}$]
 (24) Given that $f(x) = \frac{x+5}{2}$ and $g(x) = \frac{1-3x}{3}$, determine the values of x for which $fg(x) = \frac{x^2+2x-20}{6}$. [Ans: $x = 4$ and $x = -9$]
 (25) (a) Given f is a function defined by $f: x \rightarrow 2x^2 - 1$, find $f(x+3)$ and hence $f(-2)$.
 [Ans: $f(x+3) = 2x^2 + 12x + 17, f(-2) = 7$]
 (b) Given a function $f(x) = \frac{3-x}{x^2-4}$. Find the value of x for which

(i) $f(x) = 0$ [Ans: $x = 3$]

(ii) $f(x)$ is undefined. [Ans: $x = 2, x = -2$ or $x = \pm 2$]

(26) Given $g(x) = 2x - 1$, write an expression for $gg(x)$ hence determine the value of x for which $gg(x) = 17$.

[Ans: $gg(x) = 4x - 3, x = 5$]

(27) Given $f(x) = 2 - x$ and $g(x) = \frac{3}{x+1}$

(a) Write expressions for

(i) $gf(x)$ [Ans: $\frac{3}{3-x}$] (ii) $fg(x)$ [Ans: $\frac{2x-1}{x+1}$]

(b) Determine the value of x for which

(i) $fg(x) + gf(x) = 0$ [Ans: $x = 0$ or $x = 5$]

(ii) $gf(x)$ is meaningless [Ans: $x = 3$]

(28) (a) Given that $f(x) = \frac{x^2 - 1}{5}$, find;

(i) $f^{-1}(x)$ [Ans: $\sqrt{5x+1}$]

(ii) $f^{-1}(16)$ [Ans: 9]

(b) If $g(x) = x^2 - 2x - 1$ and $h(x) = x + 3$, find the value of x for which $hg(x) - gh(x) = 2$ [Ans: $\frac{-1}{3}$]

STATISTICS

TRIAL QUESTIONS

(1) The following table shows the marks scored by 36 students in a mathematics test.

Marks	Frequency
30 - 39	4
40 - 49	6
50 - 59	3
60 - 69	12
70 - 79	2
80 - 89	5
90 - 99	4

- (a) Calculate to 2 decimal places;
- (i) Mean mark
 - (ii) Median mark
- (b) Find the probability that a student picked at random scored below 50.

(2) The table below shows the marks obtained by 40 students in a mathematics test.

Marks	Number of students
10 - 14	1
15 - 19	4
20 - 24	5
25 - 29	8
30 - 34	12
35 - 39	7
40 - 44	2
45 - 49	1

- (a) (i) State the modal class
(ii) Calculate the mode
- (b) Draw a histogram to show the marks obtained.

(3) The following are heights of seedling in a nursery bed rounded to the decimal point.

4.7	2.7	2.3	4.6	3.7	2.8	2.9	3.6
4.9	3.9	4.5	3.4	4.2	3.5	1.7	1.1
2.0	3.7	3.3	3.8	3.8	1.8	3.1	3.6
4.1	1.4	1.6	2.1	2.8	2.6	3.3	4.0
3.2	4.3	3.5	2.4	4.4	4.1	2.9	3.2

- (a) Draw a frequency distribution table starting with 1.0 - 1.4
- (b) (i) State the class interval
(ii) State the modal class.
- (c) Calculate:
- (i) mean height.
 - (ii) the median height.

(4) The table below shows marks scored by S.3 students in a Chemistry test in a certain school.

Score	fx
41 - 48	222.5
49 - 56	787.5
57 - 64	1452
65 - 72	753.5
73 - 80	918
81 - 88	253.5

- (a) State the modal class.
- (b) Calculate;
- (i) the mean score
 - (ii) the median score of the information.

(5) The table below show the marks scored by 75 students in a test.

Mark	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44
Number of students	3	9	14	23	16	8	2

(a) Calculate the; (i) Mean

(ii) Mode

(b) Represent the given data on an Ogive and use it to estimate the median.

(6) The table below shows the mark scored by 50 students in a Biology test.

Mark	Number of students
20 - 29	3
30 - 39	7
40 - 49	16
50 - 59	14
60 - 69	6
70 - 79	3
80 - 89	1

(a) Represent the information on a histogram.

(b) Find the;

(i) Median

(ii) Modal mark

(iii) Mean

(7) Below are marks scored by S.4 students of Luwero High School in the Chemistry mock examinations 2013.

19	20	31	72	40	39	35	32
45	21	13	51	77	15	19	64
10	50	62	40	19	20	27	37
20	53	39	15	42	56	31	12
12	38	31	26	39	63	44	22
58	78	64	18	36	49	54	35
40	38	70	16	29	39	20	21

(a) (i) Form a frequency distribution table consisting of classes of equal width starting with 10 – 19 as the first class.

(ii) Using 44.5 as your assumed mean, calculate the mean mark of students.

(b) Draw the cummulative frequency curve and use it to estimate the median mark.

(8) Draw a cummulative frequency curve for the following data which shows the marks obtained by 80 form 2 pupils in a school.

Marks	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Frequence	3	5	5	9	11	15	14	8	6	4

(a) Draw a cummulative frequency curve and use it to find the following.

(i) the median

(ii) the interquartile range

(iii) the mean.

(9) The table below shows the marks scored by 80 students in a mathematics test.

Marks	Number of students
10 - 19	2
20 - 29	5
30 - 39	8
40 - 49	12
50 - 59	20
60 - 69	12
70 - 79	10
80 - 89	7
90 - 99	4

(a) Using an assumed mean of 54.5, calculate the mean mark.

(b)(i) Draw a histogram for the data.

(ii) Use the histogram to estimate the modal mark.

(10) Below are the heights to the nearest cm of 40 students.

150	170	152	155	169	167	157	158	157
167	164	165	164	163	162	163	158	158
160	160	159	161	161	161	160	160	160
159	162	160	159	160	161	161	156	150

- (a) Make a frequency distribution table starting with a class interval 150 - 152
- (b) Draw an ogive and use it to estimate the median, interquartile range and 20th percentile height.

(11) The numbers of the eggs collected from a poultry farm for 40 consecutive days were as follows.

138	145	145	157	150	142	154	140
146	135	128	149	164	147	152	138
168	142	135	125	158	135	148	176
146	150	165	144	126	153	136	163
161	156	144	132	176	140	147	130

- (a) Construct a frequency distribution table with classes of equal interval width 5, starting from 125 - 129.
- (b) Draw a cumulative frequency curve (ogive) and use it to estimate the;
 - (i) interquartile range
 - (ii) Median number of eggs.

(12) The table below shows the masses of 40 students to the nearest kg.

Mass (kg)	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Number of students	2	m	9	$4m$	8	1

- (a) Find the value of m .
- (b) Using 50 as the working mean, calculate the mean mass.
- (c) Calculate the standard deviation.

(13) The table below shows the marks of 36 candidates in oral examination.

30	31	55	49	56	47
36	41	39	45	39	50
42	43	44	39	46	56
30	48	53	38	50	63
40	54	61	46	56	44
53	60	56	50	62	52

- (i) Construct a frequency distribution table having an interval of 6 marks starting with the 30 - 35 class group.
- (ii) Draw a cumulative frequency curve and use it to estimate the median mark.
- (iii) Calculate the mean.

(14) The table below shows the frequency distribution of marks of 800 candidates who sat a national examination.

Marks (%)	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Frequency	30	50	100	150	150	130	90	60	30	10

- (a) (i) Construct a cumulative frequency distribution for the data.
- (ii) Draw a cumulative frequency curve for the distribution.
- (b) Use your graph to estimate;

- (i) Median mark
(ii) Percentage number of candidates that failed if the pass mark was 50%.
(iii) Inter quartile range
(c) Calculate the mean mark.

(15)

Marks (%)	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79
Number of students	1	2	4	5	6	9	5	1	5	2

The table above shows the marks obtained in a mathematics examination.

- (a) (i) Draw a histogram to represent this data.
(ii) Estimate the mode from the histogram.

- (16) Study the table below,

Marks	40	42	44	46	48	50
Frequency	1	3	x	7	3	1

- (a) Find the value of x if the mean mark is 45.1.
(b) State the modal Frequency.

(17)

65	48	53	53	60	54	60	59	52
55	54	57	57	56	57	62	56	49
55	56	52	54	54	52	58	61	54

- (a) Starting with the class of 48 - 50 and using equal class sizes, draw a frequency distribution table.
(b) State the modal class.
(c) Estimate the mean mark using your table in (a) above.
(d) What percentage of the class scored above the mean mark?

- (18) The table below shows the ages in years of 80 people who were allowed to enter a cinema hall.

Ages (years)	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
Number of people	6	5	8	12	21	18	10

- (a) Calculate the mean age.
(b) (i) Draw a cumulative frequency curve (ogive) for the data.
(ii) Use the ogive to estimate the median.

- (19) The weight of pupils bags at happy hours primary school in kilograms are given below.

3.3	6.0	4.0	2.5	5.4	5.2	4.0	4.3
3.6	5.2	5.5	7.3	4.9	4.4	4.8	5.4
4.5	4.4	2.2	4.9	3.8	4.4	2.5	3.8
3.0	4.8	5.4	3.0	6.1	6.4	4.5	6.0
6.2	3.8	6.5	3.5	4.5	3.0	4.7	4.5

- (i) Starting with a class of 2.0 - 2.4, 2.5 - 2.9 and using equal class intervals, form a frequency distribution table for this data.
(ii) Plot a cumulative frequency curve (ogive) for the data. Use your data to estimate the median weight.
(iii) Calculate the mean weight using an assumed mean of 4.7 correct your answer to one decimal place.

Statistics

- (20) The following table shows the distribution of marks obtained by 50 students.

Marks	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79
Students	3	9	13	15	5	4	1

Calculate; (i) The mean (ii) The mode (iii) The median

- (21) The cumulative frequency table below shows the marks obtained by 70 candidates in a mathematics mock exam.

Marks	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89
Cumulative frequency	8	18	38	52	64	70

Use the information in the table above to;

(a) Draw an ogive and use it to estimate

(i) The median.

(ii) Number of students who scored below 50%.

(b) Make a frequency table and find the mean mark using an assumed mean of 54.

- (22) In a certain tertiary institution, the marks obtained in mathematics test were as follows.

77	54	32	66	88	66	45	84	64
64	25	76	87	67	49	31	58	41
52	69	54	91	75	71	42	74	70
63	50	65	46	56	58	47	54	68
22	64	5	52	57	77	45	49	51

(a) (i) Construct a grouped frequency distribution table starting with 1 - 10, 11 - 20 etc.

(ii) Calculate the mean mark using 50.5 as assumed mean

(b) Construct a cumulative frequency curve and use the C.F curve to estimate the number of students who scored between 40 and 75 marks.

- (23) Copy and complete the table below

Marks	Frequency	Mid-mark(x)	fx
20 - 29	24.5	24.5
30 - 39	2	34.5
40 - 49	8
50 - 59	14	54.5
60 - 69
70 - 79	6	74.5
80 - 89	2	169.0
	$\sum f = 40$		$\sum fx =$

(b) Use the table to determine the

(i) mean mark

(ii) the median mark.

(24) The mean of the numbers 13, 8, 6, 0, 3, 12, x , 11 and 5 is 7.

Determine;

- (i) the value of x
- (ii) the median

(25) In a class of 50 students, there are 30 boys and the rest are girls. Given that the mean mark for the whole class in a test was 64 marks and that for the boys was 60 marks. Find the mean mark for the girls.

(26) The figures below show the weight in kg, of 30 people in a street.

60	50	45	57	51	41	80	77	48	108	49
58	53	46	36	63	61	32	74	93	78	66
42	75	38	55							
58	66	60	83							

- (a) Draw the grouped frequency table taking weight groups 20 - 39, 40 - 59 etc.
- (b) Draw a histogram representing this data.
- (c) Draw a pie - chart for the data, marking each central angle.
- (d) State the modal class.

THE PROBABILITY THEORY

RECALL

- ◊ The term probability arose from games chances. For example, tossing a coin, rolling a die etc.
- ◊ Probability is a mathematical measurement of how something is likely to happen.
- ◊ Probability ranges from **0(zero)** to **1(one)**. Zero means that the event is impossible to happen but **1** means that the event is absolutely certain.
- ◊ Sample space (S); is a set of all possible outcomes of an experiment. E.g when a coin is tossed, a head (H) or a tail (T) is expected. Therefore $S = \{H, T\}$.but each out in the sample space is called a **sample point**
- ◊ Probability = $\frac{\text{number of events (E)}}{\text{number of sample space (S)}} = \frac{n(E)}{n(S)}$
- ◊ “**And**” means intersection
- ◊ “**Or**” means Union
- ◊ Mutually exclusive events; are events which cannot occur at the same time.e.g when a coin is tossed ,events head and tail are mutually exclusive. Meaning that if you get a head, you cannot get a tail. this implies that there is no intersection of two events.

Therefore $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$
 $P(A \cap B) = 0$

- ◊ Independent events; these are events which occur without being affected by each other. For example, a student offering French and history. French is independent from history.

For two independent events A and B,

$$P(A \cap B) = P(A) \times P(B)$$

$$\text{NOTE: } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

TRIAL QUESTIONS

1. A fair die is tossed only once and the number which appears on its top face noted. What is the probability of showing
 - (i) a number greater than 3 [Ans: $\frac{1}{2}$]
 - (ii) an odd number or triangle number. [Ans: $\frac{2}{3}$]
2. A bag contains 3 black balls 4 blue balls and 2 yellow balls.
 - (a) If two balls are picked at random without replacement find the probability that both are of the same colour. [Ans: $\frac{5}{18}$]
 - (b) How many black balls must be added to the bag so that probability of drawing a black ball should be $\frac{1}{2}$. [Ans: **3 black balls**]
 - (c) How many yellow balls must be added to the bag so that the probability of drawing a black ball should be $\frac{1}{5}$? [Ans: **6 yellow balls**]
3. The probability that a pupil passes a physics examination is 0.8 and the probability that she will pass in a chemistry examination is 0.6, Find the probability that the pupil will pass in both examinations. [Ans: $\frac{12}{25}$]
4. A bag contains four red balls and six black balls. A ball is picked at random from the bag and not replaced. A second ball is then picked. Find the probability that
 - (i) both balls are black [Ans: $\frac{1}{3}$]
 - (ii) both balls are red [Ans: $\frac{2}{15}$]
 - (iii) the balls are of different colours [Ans: $\frac{8}{15}$]
5. There are 5 red and 4 white balls in a bag. Two balls are picked from the bag at random one after another without replacement. Find the probability that:
 - (a) both are white [Ans: $\frac{1}{6}$]
 - (b) they are of different colours. [Ans: $\frac{5}{9}$]
6. The numerals 1, 2 and 3 are written at random to form two and three digit numbers.

- (i) Write down all the possible two and three digit numbers that can be formed from the given numerals if a numeral may be used only once in a number to be formed.
- (ii) What is the probability that the number formed is a prime number? [Ans: $\frac{1}{4}$]
7. A number is packed at random from the set below; 1, 3, 5, 7, 9, 11, 13, 15. What is the probability that a number packed is;
- (i) a prime number. [Ans: $\frac{5}{8}$]
(ii) divisible by 3 [Ans: $\frac{3}{8}$]
8. A box contains 4 black beads and 3 white beads. A bead was drawn and then replaced. Then a second was drawn and put aside, and then another was drawn. What is the probability that all 3 were white? [Ans: $\frac{3}{49}$]
9. A two digit number is formed using the numerals 4, 5 and 6 without repeating any numeral.
- (a) Write down the possibility space.
(b) Find the probability that the two digit number is a multiple of three. [Ans: $\frac{1}{3}$]
10. The numerals 2.3 and 4 are written at random to form one, two and three digit numbers.
- (a) If a numeral is used only once in a number to be formed, write down all the possible one, two and three digit numbers that can be formed from the given numerals.
(b) What is the probability that the number formed is divisible by three. [Ans: $\frac{3}{5}$]
11. A bag contains 2 red and 4 blue pens. If a pen is picked at random, find the probability that it is;
- (i) Red [Ans: $\frac{1}{3}$]
(ii) Red or blue [Ans: 1]
12. A coin is tossed twice. Find the probability of getting
- (a) a head followed by a tail, [Ans: $\frac{1}{4}$]
(b) either two heads or two tails. [Ans: $\frac{1}{2}$]
13. A die is thrown. Find the probability that the upper face is showing
- (a) a number 4 [Ans: $\frac{1}{6}$]
(b) a prime number [Ans: $\frac{1}{2}$]
- (c) neither a 5 nor [Ans: $\frac{2}{3}$]
14. A family of Ibrah and Becky wishes to have three children
- (a) Write down the possible outcome for the children
(b) Hence find the probability that a family of three children will consist of 2 boys and one girl
- (i) in that order, [Ans: $\frac{1}{8}$]
(ii) in any order. [Ans: $\frac{3}{8}$]
15. A box contains 3 red balls, 7 yellow balls and 2 green balls. A ball is taken at random. What is the probability of it being;
- (a) red [Ans: $\frac{1}{4}$]
(b) yellow [Ans: $\frac{7}{12}$]
(c) not red [Ans: $\frac{3}{4}$]
(d) neither red nor green [Ans: $\frac{7}{12}$]
16. (a) Calculate the probability that when a die is thrown, the number obtained on the upper face will be a prime number. [Ans: $\frac{1}{2}$]
(b) A coin is tossed and a die is thrown. Calculate the probability of obtaining a head on the coin and a prime number on the die.
[Ans: $\frac{1}{2}$]
17. A bag contains 14 marbles of which 10 are red and 4 are green. Two marbles are picked at random one at a time without replacement. Find the probability of picking marbles of different colours. [Ans: $\frac{40}{91}$]
18. Two dice are thrown once. Find the probability that
- (i) both dice show an odd number. [Ans: $\frac{1}{4}$]
(ii) the sum of the numbers on the dice is nine.
[Ans: $\frac{1}{9}$]
19. A box contains n red balls and $(n - 8)$ blue ones. The probability of drawing a red ball is $\frac{3}{4}$. Find the number of balls in the box. [Ans: 16 balls]
20. In a game, a player throws two fair dice, each with faces numbered 1 to 6. When the two dice show different numbers, the player scores the sum of the two numbers. When the two dice show the same numbers, the player scores twice

Probability Theory

the sum of the two numbers. Calculate the probability that a player;

(a) Scores 6 in one throw of the two dice.

$$[\text{Ans: } \frac{1}{9}]$$

(b) Scores a multiple of 4 in one throw of the two dice. $[\text{Ans: } \frac{1}{3}]$

(c) Scores 3 in each of two successive throws of the two dice $[\text{Ans: } \frac{1}{324}]$

21. A number is selected at random from the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find the probability that the number selected is:

(i) A triangular number $[\text{Ans: } \frac{2}{5}]$

(ii) A factor of 72 $[\text{Ans: } \frac{7}{10}]$

22. (a) From a group of five children, consisting of three girls and two boys, one child is chosen at random. Write down the probability that the child chosen is a girl. $[\text{Ans: } \frac{3}{5}]$

(b) Two children are chosen from this group of three girls and two boys on another occasion. Calculate the probability that;

(i) They are both boys $[\text{Ans: } \frac{4}{25}]$

(ii) They are both girls. $[\text{Ans: } \frac{9}{25}]$

(iii) They are of different sexes. $[\text{Ans: } \frac{4}{25}]$

23. The table shows the means of transport 180 students used to report back to school.

	Boys	Girls
Car	15	18
Train	5	8
Walk	50	44
Bus	30	10

If a student is chosen at random in the school, what is the probability that;

(a) The student is a boy? $[\text{Ans: } \frac{5}{9}]$

(b) The student is a girl who travels to school by car? $[\text{Ans: } \frac{1}{10}]$

(c) The student is a boy who walks to school?
 $[\text{Ans: } \frac{5}{18}]$

24. (a) A bag contains white balls and black balls. The probability of choosing a black ball is $\frac{3}{8}$. If the bag contains 32 balls, find;

(i) the number of white balls in the bag.

$$[\text{Ans: } 20]$$

(ii) the probability of picking a white ball

$$[\text{Ans: } \frac{5}{8}]$$

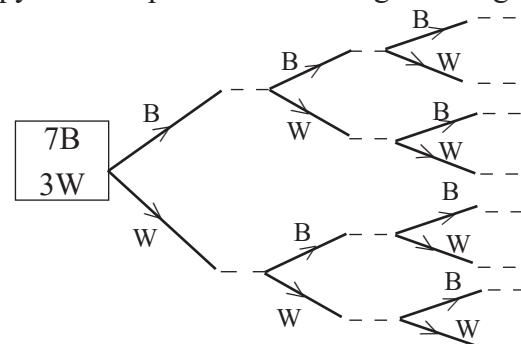
(b) A basket contains 4 Green and 3 Red apples. Joan picked two apples without replacement, find the probability that they are of the same colour. $[\text{Ans: } \frac{3}{7}]$

25. A bag contains 8 black marbles and a number of white marbles. The probability of drawing one of the black marbles is $\frac{1}{6}$. How many white marbles are there in the bag and find the probability of picking a white marble.

$$[\text{Ans: } n(W) = 40 \text{ mables}, \quad P(W) = \frac{5}{6}]$$

26. A bag contains 7 black and 3 white marbles. Three marbles are chosen at random and in succession, each marble being replaced after it has been taken out of the bag.

Copy and complete the following tree diagram



(b) a multiple of 3 [Ans: $\frac{3}{10}$]

29. A coin is tossed 3 times. Find the probability of obtaining;

(i) atmost two Heads [Ans: $\frac{7}{8}$]

(ii) two Heads and two Tails [Ans: $\frac{9}{64}$]

(iii) atmost one Head or one Tail. [Ans: $\frac{3}{4}$]

30. A bag contains 4 red and 3 blue marbles. Three marbles are selected at random one after the other without replacement.

(a) Draw a probability tree to find the possible outcomes.

(b) Find the probability that:

(i) all are blue.

(ii) the first two are red and last is blue.

(iii) all are of the same colour.

MATRICES

TRIAL QUESTIONS

- Given the matrices $A = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -4 \\ -1 & 2 \end{pmatrix}$. Determine
 - $AB + C$
 - The determinant of $(AB + C)$
- Given that $\begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & y \\ x & 2 \end{pmatrix} = \begin{pmatrix} 7 & 7 \end{pmatrix}$. Find the values of x and y
- Use matrices to solve the pair of simultaneous equations

$(a) x - 3y = 9$	$(b) 2y - 4x + 2 = 0,$	$(c) 3x - 5y + 9 = 0$
$7x + 2y = 17$	$3x - 2y = 5$	$2y - 16 = -5x$
- (a) Given that $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Find A^2
 (b) Given that $A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ and $C = AB$, find C^{-1}
- Given that; $\begin{pmatrix} 3 & a \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 4 & b \\ c & 1 \end{pmatrix} = \begin{pmatrix} d & -4 \\ 12 & 2 \end{pmatrix}$ Find the values of a , b , c and d .
- A news paper vendor supplied copies of “The New Vision”, “Red Pepper”, “Etop”, and “The Monitor” to 4 offices A, B, C , and D once a week as shown in the table below.

	A	B	C	D
Etop	4	2	0	1
New Vision	2	0	1	3
Monitor	3	1	5	1
Red paper	2	1	1	3

The prices per copy of The New Vision, Red Pepper, Monitor and Etop are sh. 1,000, 1,200 , 1,000 and 800 respectively,

- Write down a 4×4 matrix (s) for numbers of copies supplied per week.
 - Write down a 1×4 cost matrix (C).
 - If the vendor supplied the same numbers of copies for 3 weeks , write down a 4×4 matrix (T) for the copies supplied per office.
 - Find by suitably multiplying any of the two matrices above, how much each office pays the vendor after the 3 weeks.
- (a). Given that $A = \begin{pmatrix} 6 & -3 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ 4 & 8 \end{pmatrix}$.Find $(3A^{-1})B$
 - Four secondary schools football teams of Kibuli S.S, Lubiri S.S Mvara S.S and Nagaalama S.S qualified for a football tournament, which was played in two round with other teams.

In the first round:

- Kibuli S.S won two matches, drew three and lost one match.
- Lubiri S.S won one match, drew two and lost three matches.
- Mvara S.S won four matches, drew one and lost one match.
- Nagaalama S.S won three matches, drew two and lost one match.

In the second round:

- Kibuli S.S won three matches and lost three matches
- Lubiri S.S won two matches drew one and lost three matches.
- Nvara S.S won three matches drew two and lost one match.
- Naggaalama S.S won one match, drew one and lost four matches.

- Write down;
 - a 4×3 matrix to show the performance of the four teams in each of the two rounds

- (ii) a 4×3 matrix which shows the overall performance of the teams in the two rounds
- (b) If three points are awarded for a win, one point for a draw and no point for a loss, use matrix multiplication to determine the winner of the tournament.
- (d) Given that MTN donated sh. 2,070,000 to be shared by the four teams according to the ratio of their points scored in the tournament, find how much money each team got.
9. (a) Given that $P = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 5 \\ 2 & -3 \end{pmatrix}$ and $R = \begin{pmatrix} 4 & 3 \\ 1 & -2 \end{pmatrix}$, find
 (i) QR - P (ii) the determinant of QR - P
 (b) If $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, determine the values of x and y .
10. Three men John, peter and David went for shopping. John bought 2 kg of sugar, 4 kg of tealeaves and one loaf of bread, peter bought 10 kg of sugar, 3 kg of tealeaves and 2 loaves of bread; David bought 5 kg of sugar, 5 kg of tea leaves and 3 loaves of bread. The prices are; sugar shs. 2000 per kg, tealeaves shs. 1000 per kg and shs. 1800 for one loaf of bread. Use a suitable matrix multiplication to find the total amount of money spent by each of John, peter and David.
11. (a) Given that $P = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$ and that $R = PQ$, find the inverse of R.
 (b) Three men, Ali, Ben and Charles went to a supermarket to buy groceries. Ali bought 4 kg of rice, 2 kg of sugar and 3 kg of Posho. Ben bought 4 kg of Posho and 3 kg of Sugar. Charles bought 6 kg of sugar, 2 kg of rice and 3 kg of Posho
 (i) Write down a 3×3 matrix for the purchases.
 (ii) If a kg of rice costs shs.2500, a kg of sugar shs3,000 and a kg of Posho shs 2,000, write down a 3×1 matrix for the costs.
 (c) By matrix multiplication, determine the amount of money paid by each customer
 (d) Find the supermarket's total earning from the three customers.
12. Three ladies; Peace, Queen and Ritah went to a certain supermarket. Peace bought 2 kg of sugar, 1 kg of tea leaves and 2 loaves of bread. Queen bought 10 kg of sugar and 2 loaves of bread while Ritah bought 5 kg of sugar, 2 kg of tea leaves ad 1 loaf of bread. The cost of a kilogram of sugar was shs 1800, a kilogram of tea leaves was shs 900 and a loaf of bread shs 1400.
 (i) Write down a 3×3 matrix for the items bought.
 (ii) Write down a 3×1 cost matrix.
 (iii) By matrix multiplication, determine the total expenditure by the three ladies.
13. (a) Given that matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 6 \\ 10 & 15 \end{pmatrix}$, find matrix N such that $A2 + 3B - C$.
 (b) Use the matrix method to solve the simultaneous equations. $a - 2b = 3$

$$2a = 11 - b$$
14. (a) Given the matrices; $A = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$. Find a matrix M such that $M = 2AB + 3C^2$.
 (b) A boarding school uses 15 bags of maize, 8 bags of beans, 16 bags of maize flour, and 4 bags of rice in first term. The prices are shs. 1000, shs 1200, shs 1400 and shs 1400 respectively. In second term, the school uses 16 bags of maize flour and 5 bags of rice at shs.1400, shs.2600, shs.1600 and shs.1500 respectively. In third term, the school uses 12 bags of maize, 5 bags of beans, 12 bags of maize flour and 3 bags of rice at shs.1800, shs. 2200, shs. 2000, and shs. 1500 respectively. Use matrices to find the total cost of food stuff that year.
15. (a) If $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$, find; (i) AB (ii) $(AB)^{-1}$
16. Alpha made an order of the items in Pallisa supermarket as follows;

	$\frac{1}{2}$ kg packet	1 kg packet	2 kg packet	5 kg packet
Kaboloi Millet	0	4	2	0
Agole millet	4	2	5	2
Kumi millet	0	3	1	4

The table below shows the cost of each size of the packet.

$\frac{1}{2} \text{ kg}$ packet	800
1 kg packet	1500
2 kg packet	2800
5 kg packet	6500

- (i) Find the number of packets ordered
 - (ii) Write down 4 by 3 matrix for the order
 - (iii) Write down 1 by 4 matrix for the costs
 - (iv) Given that he had to pay 10% discount for the order made of the millet, use matrices to find the total expenditure by Alpha.

17. Find the values of x given that the matrix $\begin{pmatrix} x & 3x+4 \\ 1 & x+3 \end{pmatrix}$ is a singular matrix.

18. Shell petroleum company owns two pump stations, one in Mukono town and the other in Seeta Town. Over two consecutive days, the sales were recorded as follows (in litres).

First day;

Town	Petrol	Diesel	Kerosene
Mukono	1300	850	750
Seeta	1200	950	700

Second day;

Town	Petrol	Diesel	Kerosene
Mukono	1350	925	650
Seeta	1250	850	850

- (a) (i) Represent the above information on 2×3 matrices showing the sales of each day.
(ii) Write down a matrix representing the total sales over the two days.

(b) The cost of fuel at each station of given by; Petrol shs 2300 per litre, Diesel shs 2100 per litre and Kerosene shs 1800 per litre.

(i) State a 3×1 cost matrix.
(ii) Using a suitable matrix multiplication, find the total income from each station.
(iii) Find the total income from both stations for the two days.

19. (a) If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$ are matrices, find;

20. (a) Using matrix method find the values of x and y which satisfy the equations:

$$\begin{aligned}2x - 3y &= 12 \\ x + 2y + 1 &= 0\end{aligned}$$

(b) Given that matrix $A = \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix}$, find a matrix B such that $AB = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$. Hence or otherwise find the inverse of A.

21. A supplier of food stuffs to school per week summarised his information for supplies made to Kibuli SS, Seeta HS and Buddo SS for 2 weeks as follows:

First week:

Kibuli SS: 2bags of posho,1 bag of rice and 3bags of potatoes
Seeta HS: 2bags of posho and 2 bags of rice
Buddo SS: 1 bag of posho, 1bag of rice and 2bags of potatoes.

Second week:

Kibuli SS: 3 bags of posho and 2 bags of potatoes
Seeta HS: 1 bag of posho, 2 bags of rice and 1 bag of potatoes
Buddo SS: 3 bags of posho and 1 bag of potatoes.

The cost of posho, rice and potatoes is shs 20,000, shs 30,000 and shs 10,000 per bag respectively.

(a) Write down 3×3 matrices for the supplies made per week for two weeks.

(b) Write down a 3×1 cost matrix

(c) Which school spent most in the two weeks?

22. (a) Solve the following simultaneous equations using matrix method, $2x - y = -4$, $x + 5y = 9$

(b) Given that $(x \ 1-x) \begin{pmatrix} 2 & 18 \\ 2 & 6 \end{pmatrix} = 4(1-x \ y)$, find the values of x and y .

(c) Find x and y if $\begin{pmatrix} 3 & -1 \\ 20 & y \end{pmatrix} \begin{pmatrix} x \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 20 \end{pmatrix}$

(d) Use matrix method to solve the pair of simultaneous equations; $m+2n=2$
 $4n+1=2m$

(e) Given that $\begin{pmatrix} x-1 & x+1 \\ 3x & x \end{pmatrix}$ is a singular matrix. Find the possible values of x .

23. Students of S.1 to S.3 in Rock High school were distributed in 3 streams; South, North and East as follows: S.1; 65 in South, 60 in North and 70 in East. S.2; 50 in South, 62 in North and 54 in East while S.3 had 54 in South, 60 in North and 42 in East. Each student per steam had to pay subscription for club membership as shown in the table below.

	South	North	East
Subscription fee	5,000	4,000	3,000

(a) Write down (i) a 3×3 matrix for the student population in the school.

(ii) a 1×3 matrix for subscription fees.

(b) By multiplying the two matrices in part (a) above, calculate how much money is collected per class.

(c) How much money will go for charity work if 12.5% of the S.2 subscription fee is allocated for charity work?

24. (a) Given that $\begin{pmatrix} 3-a & -3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} -3 \\ x \end{pmatrix} = \begin{pmatrix} -3 \\ x \end{pmatrix}$. find the values of a and x .

(b) Given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. Show that $A^2 - 4A = I$ where I is a 2×2 identity.

25. (a) Given that matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$, find the values of the scalar k for which $A - kI$ is singular.

(b) If $\begin{pmatrix} 0 & 4 \\ 3 & -1 \end{pmatrix}$ is pre multiplied by the column matrix $\begin{pmatrix} x \\ y \end{pmatrix}$ to give $\begin{pmatrix} -8 \\ x \end{pmatrix}$, find the values of x and y .

26. Find the possible values x can take on given $A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$ and $AB = BA$.

27. Given that $D = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and I is a 2×2 identity matrix , obtain the values of p and q such that $D^2 = pD + qI$.

28. (a) Given that matrix $P = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$. If $PR = Q$, determine:

(i) the order of matrix R . (ii) matrix R .

(b) Given that $\det \begin{pmatrix} x+2 & 2 \\ y-1 & 3 \end{pmatrix} = 4$ and $\det \begin{pmatrix} 2 & x+2 \\ 3 & 1-y \end{pmatrix} = 0$. Determine the values of x and y , hence the two matrices.

29. Given that matrix $Q = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, find the values of k such that $Q^2 - 5Q + kI = 0$.

30. (a) Given that matrix $P = \begin{pmatrix} 2m & m \\ m-1 & m-2 \end{pmatrix}$ has no inverse, Find the values of m .

(b) Given that $M = \begin{pmatrix} 1 & -4 \\ 2 & 0 \end{pmatrix}$ and $N = \begin{pmatrix} 3 & -1 \\ 2 & -5 \end{pmatrix}$ Find $(MN)^{-1}$

31. (a) Use matrix method to solve the following pair of simultaneous equations.

$$\begin{aligned} -5x + 19y &= 24 \\ 6x - 2y &= -8 \end{aligned}$$

(b) Mr. Ddumba went to market and bought 4 kg of sugar, 4 kg of maize flour and 5 kg of millet. Kisakye bought 10 kg of sugar, 6 kg of maize flour and 7 kg of millet flour. The cost of sugar is 2,600/= per kg, 2,000/= per kg of maize flour and 2,500/= per kg of millet.

- i) Express the above information as a 2×3 matrix for the items and a 3×1 matrix for the price.
ii) Find Mr. Ddumba and Kisakye's expenditure by matrix method.

Answers

1. (i) $\begin{pmatrix} 3 & 2 \\ 3 & 1 \end{pmatrix}$

(ii) -3

2. $x = 1, y = -1$

3. (a) $x = 3, y = -2$

(b) $x = -3, y = -7$

(c) $x = 2, y = 3$

4. (a) $\begin{pmatrix} 1 & 8 \\ 0 & 9 \end{pmatrix}$

(b) $\begin{pmatrix} \frac{1}{25} & \frac{3}{25} \\ \frac{6}{25} & \frac{-7}{25} \end{pmatrix}$

5. $a = 2, b = -2, c = 2, d = 16$

6. (c) $T = \begin{pmatrix} A & B & C & D \\ 12 & 6 & 0 & 3 \\ 6 & 0 & 3 & 9 \\ 9 & 3 & 15 & 3 \\ 6 & 3 & 3 & 9 \end{pmatrix}$

(d) $\begin{pmatrix} A & B & C & D \\ 31,800 & 11,400 & 21,600 & 25,200 \end{pmatrix}$

7. $\begin{pmatrix} \frac{10}{7} & \frac{27}{7} \\ \frac{34}{7} & \frac{33}{7} \end{pmatrix}$

W D L
5 3 4
3 3 6
7 3 2
4 3 5

(b) Winner : Mvara
Points : 24

(d) Kibuli : shs 540,000
Lubiri : shs 360,000
Mvara : shs 720,000
Nagalama : shs 450,000

9. (a)(i) $\begin{pmatrix} 7 & -6 \\ 2 & 14 \end{pmatrix}$

(ii) 110

(b) $x = 2, x = -6$
 $y = -4$ or $y = 28$

10. John $\begin{pmatrix} 9800 \\ 26,600 \end{pmatrix}$

Peter
David $\begin{pmatrix} 20,400 \end{pmatrix}$

11. (a) $R = \begin{pmatrix} 4 & -9 \\ -2 & 7 \end{pmatrix}$ Inverse of $R = \frac{1}{10} \begin{pmatrix} 7 & 9 \\ 2 & 4 \end{pmatrix}$

(c) Ali $\begin{pmatrix} 22,000 \\ 17,000 \end{pmatrix}$
Ben
Charles $\begin{pmatrix} 29,000 \end{pmatrix}$

(d) shs.68,000

12. (iii) shs 40,300

13. (a) $N = \begin{pmatrix} 2 & 9 \\ 23 & 24 \end{pmatrix}$

(b) $a = 5$
 $b = 1$

14. $M = \begin{pmatrix} 22 & -6 \\ 9 & 23 \end{pmatrix}$

(b) shs 136,300

15. (i) $AB = \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix}$

(ii) $\begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix}$

16. (iv) shs 70,290

(17) $x = \pm 2$

18.(a)(ii) $\begin{pmatrix} 2650 & 1775 & 1400 \\ 2450 & 1800 & 1550 \end{pmatrix}$ (b)(ii) Mukono $\begin{pmatrix} 12,342,500 \\ 12,205,000 \end{pmatrix}$ (iii) shs 24,547,500

19. (a)(i) $AB = \begin{pmatrix} 5 & -4 \\ 5 & -8 \end{pmatrix}$

(ii) -20

(b) $\begin{array}{l} a = 21 \\ b = 16 \end{array}$ $\begin{array}{l} c = 19 \\ d = 22 \end{array}$

20.(a) $\begin{array}{l} x = 3 \\ y = -2 \end{array}$

(b) $B = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$

Inverse of $A = \frac{1}{7} \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$

21.(c) Seeta = 190,000/=

22.(a) $\begin{array}{l} x = -1 \\ y = 2 \end{array}$ (b) $\begin{array}{l} x = \frac{1}{2} \\ y = 20 \end{array}$

(c) $\begin{array}{l} x = 2 \\ y = 20 \end{array}$

(d) $m = 1.25, n = 0.375$

(e) $x = 0$ or $x = -2$

23.(b) $\begin{array}{l} S.1 \\ S.2 \\ S.3 \end{array} \begin{bmatrix} 775,000 \\ 660,000 \\ 636,000 \end{bmatrix}$

(c) shs 82,500

(24) $a = 1, x = 1$

25.(a) $k = 4$ or $k = -1$

(b) $x = -1, y = -2$

26. $x = 3, x = \frac{-1}{2}$

(27) $(p = 2, q = -1)$

28. (a)(i) 2 x 1 matrix

(ii) $R = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

(b) $x = , y = 0$

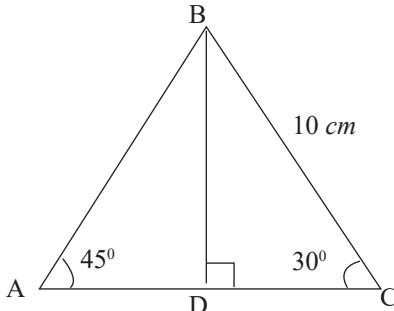
(29) $k = 6$

TRIGONOMETRY

TRIAL QUESTIONS

1. (a) The height of a right cone is 12 cm and the angle at the vertex of the cone is 30° . Find the volume of the cone correct to 2 decimal places. (Take $\pi = \frac{22}{7}$) [Ans: 41.47 cm^3]

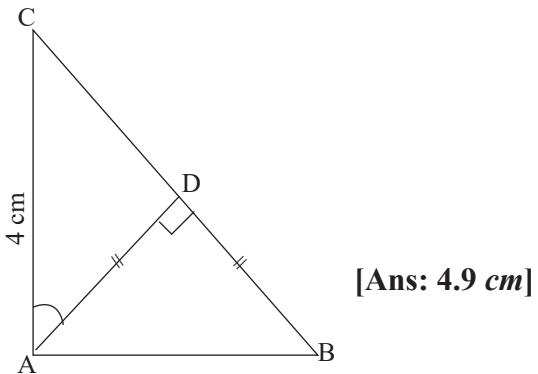
(b)



The figure shows triangle ABC in which $\overline{BC} = 10 \text{ cm}$, $\angle BCA = 30^\circ$ and $\angle BAC = 45^\circ$. Line \overline{BD} is perpendicular to \overline{AC} . Find;

- (i) \overline{BD} [Ans: 5 cm] (ii) \overline{DC} [Ans: 8.66 cm]
- (iii) Area of the triangle ABC [Ans: 34.15 cm^2]

2. If $\sin \theta = \frac{3}{5}$ and $0^\circ \leq \theta \leq 360^\circ$, find the values of $\cos \theta + \tan \theta$ without using tables or calculator. [Ans: $\frac{31}{20}, -\frac{31}{20}$]
3. Given that $\tan \theta = \frac{12}{5}$ and $0^\circ \leq \theta \leq 180^\circ$, find without using tables or calculators the value of $\sin \theta + \cos \theta$. [Ans: $\frac{7}{13}$]
4. The angle of depression of a boat from the top of a vertical cliff. 50 m high, is 10° . Find the distance of the boat from the foot of the cliff. [Ans: 283.56 m]
5. (a) Given that $\tan \theta = \frac{5}{12}$, find without using tables or a calculator the value of $4 \cos \theta - 3 \sin \theta$ [Ans: $\frac{33}{13}$]
 (b) Given that $\tan \theta = \frac{3}{4}$ and that $0^\circ \leq \theta \leq 360^\circ$, find the values of $\sin \theta + \cos \theta$. [Ans: $\frac{1}{5}, -\frac{7}{5}$]
6. A man of height 1.7 m tall is standing 12 m away from the foot of a tree. When he looks up at the top of the tree, the angle of elevation is 48° . Determine the height of the tree, correct to two decimal places. [Ans: 15.03 m]
7. (a) Building A is 40 m high. The angle of depression of the top of building B from the top of A is 26° . If the two buildings are 10 m apart, find the height of building B. [Ans: 35.12 m]
 (b) Given that $\tan \theta = \frac{12}{5}$ and $0^\circ < \theta < 180^\circ$, find without using tables or calculator the value of $3\sin \theta + 2\cos \theta$. [Ans: 2]
 (c) Given that $\sin \alpha = \frac{8}{17}$ and that $0^\circ < \alpha < 90^\circ$, find without using tables or calculator the value of; $2\cos(90^\circ - \alpha) - \sin \alpha$. [Ans: $\frac{8}{17}$]
8. (a) If $\sin \theta = \frac{4}{5}$ and $\sin \alpha = \frac{12}{13}$ where $0^\circ < \theta < 90^\circ$ and $90^\circ < \alpha < 180^\circ$, find the value of $\sin \theta \cos \alpha + \cos \theta \sin \alpha$. [Ans: $\frac{16}{65}$]
 (b) Given that $\tan \theta = -\frac{3}{4}$ and $0^\circ \leq \theta \leq 360^\circ$, find the possible values of $\cos \theta - \sin \theta$. [Ans: $\frac{7}{5}, -\frac{7}{5}$]
 (c) A student stands 16 m away from a vertical flag mast 10.5 m high. Find the angle of elevation of the top of the mast from the foot of the student. [Ans: 33.3°]
9. In the figure below AD is perpendicular to BC, AD = DB. AC = 4 cm and angle CAD = 30° . Find the length AB.



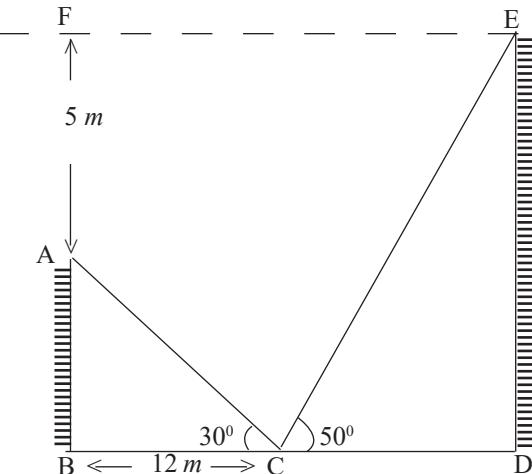
19. At a certain point on the ground, the angle of elevation of the top of the tower T is 28° . At another point 100 m away from the first point, the angle of elevation is 35° . Find the two expressions for the height of the tower and hence find the height. [Ans: 221 m]

20. The angles of depression of the two boats from the top of the cliff are 39° and 60° . The boats are in a straight line from the point of observation. How far apart are the two boats if the cliff is 400 m high? [Ans: 263.02m]

21. (a) Draw the graph of the curve $y = \cos 3x$ for $0 \leq x \leq 150^\circ$.

(b) Using your graph, determine the values of x for which $4 \cos 3x + 3 = 0$.

22. In the diagram, two buildings AB and DE are shown. DE is 5 m higher than AB. The angles of elevation of the tops A and E from a fixed point C are 30° and 50° respectively. BC is 12 m from C,



Find:

- (i) Height of AB [Ans: 6.93 m]
- (ii) Height of DE. [Ans: 11.93 m]
- (iii) The distance BD between the two buildings [Ans: 22 m]

23. The angle of elevation of the top of a cliff from a point A due East of it and 90 m away from its base is 30° . From another point B due West of the cliff, the angle of elevation of the top of the cliff is 60° .

(a) calculate height of the cliff. [Ans: 51.96 m]

(b) Calculate how far B is from the base of the cliff. [Ans: 30.0 m]

(c) How far is point B from the top of the cliff. [Ans: 60.0 m]

24. Two men 4 m and 3 m tall stand on two opposite sides of a mast, which is 30 m tall. The angle of elevation of the top of the mast from each of the men is 45° . Calculate:

(i) how far apart the two men are on the ground. [Ans: 60 m]

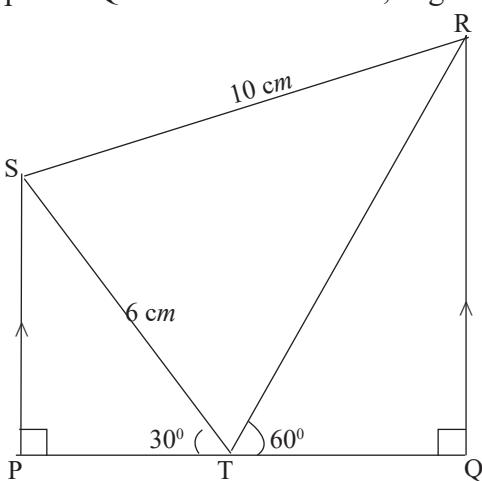
(ii) the angle of depression of the shorter man from the taller man. [Ans: 3.8°]

25. (a) At a certain point on the level ground the angle of elevation of the top of a tower, T, is 30° . At another point 100 m away from the first point, the angle of elevation is 35° .

(i) Find two expressions for the height of the tower

(ii) Hence find the height of the tower to the nearest metre. [Ans: 329 m]

26. The figure below shows a trapezium in which PS is parallel to QR, $SR = 10\text{ cm}$ and angle PQR = 90° . T is a point PQ such that $ST = 6\text{ cm}$, angle PTS = 30° and angle QTR = 60° .



(a) Calculate; leaving your answers in surd form where applicable the lengths:

(i) TR. [Ans: 8] (ii) QR. [Ans: $4\sqrt{3}$]

(iii) QT [Ans: 4] (iv) PS [Ans: 3]

(v) PT [Ans: $3\sqrt{3}$]

(b) Hence show that the area of the trapezium
 $= \frac{1}{2} (48 + 25\sqrt{3}) \text{ cm}^2$

Trigonometry

27. (a) Draw a graph of $y = \sin\theta$ for values of θ from 0 to 420° using a scale of $2\text{ cm} : 300$ on the x -axis and $4\text{ cm} : 1$ unit on the y -axis.

(b) Use your graph to solve (i) $\sin\theta = 0.7$ (ii) $\sin\theta = 0.5$

(c) Given that $\cos\theta = \frac{12}{13}$ for $270^\circ \leq \theta \leq 360^\circ$, find without using tables or calculators the values of:
 (a) $\sin\theta$ (b) $\tan\theta$

28. A ladder 10 m long leans against a wall and makes an angle of 63° with the horizontal.

(i) How high up the wall does the ladder reach? [Ans: 8.91 m]

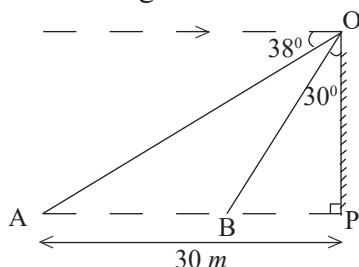
(ii) How far from the wall is the foot of the ladder? [Ans: 4.54 m]

29. A vertical post casts a shadow 6 m long on the ground. The height of the post is 15 m . Find the angle of elevation of the sun, correct to the nearest degree. [Ans: 68°]

30. Nairobi fire Brigade has a 10m ladder. The greatest angle at which it can be held against a vertical wall is 75° with the horizontal. What is the maximum vertical height to which it can reach? How far from the wall will it be placed? [Ans: $9.66\text{ m}, 2.59\text{ m}$]

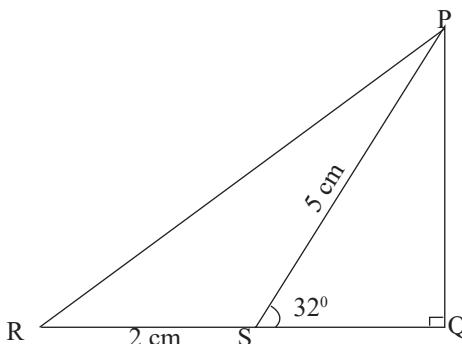
31. From the top of a cliff PO, a man observes two boats A and B.

(i) What is the angle of elevation of the top O from boat A, boat B? [Ans: 38°]



(ii) What is the angle of depression from O of the nearer boat B and of the farther boat A? [Ans: $60^\circ, 38^\circ$]

32. In the figure below, PQR is a right angled triangle. The point S on RQ is such that $PS = 5\text{ cm}$, $RS = 2\text{ cm}$ and angle $PSQ = 32^\circ$.



Calculate;

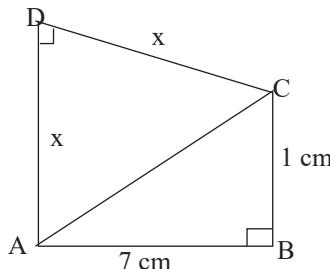
(a) PQ [Ans: 2.65 cm]

(b) SQ [Ans: 4.24 cm]

(c) angle PRQ [Ans: 23°]

33. In the quadrilateral ABCD, $\angle B = \angle D = 90^\circ$, $BC = 1\text{ cm}$, $AB = 7\text{ cm}$, $AD = DC = x\text{ cm}$. Calculate x.

[Ans: $x = 5$]

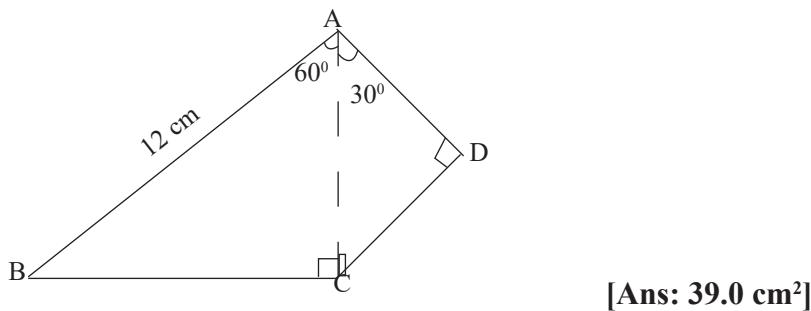


34. A ladder with the bottom 3 m from a vertical wall reaches 4 m up the wall.

(a) Find the angle of inclination of the ladder to the horizontal and the length of the ladder.

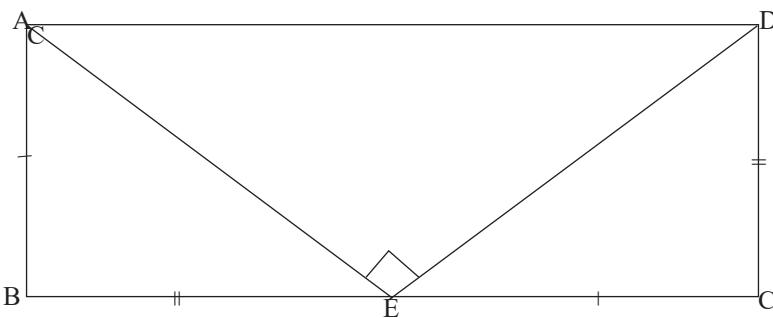
[Ans: 53.1°] [length = 5m]

35. Calculate the area of the quadrilateral ABCD having AB = 12 cm, $\angle ACB = \angle ADC = 90^\circ$, $\angle BAC = 60^\circ$, $\angle CAD = 30^\circ$.



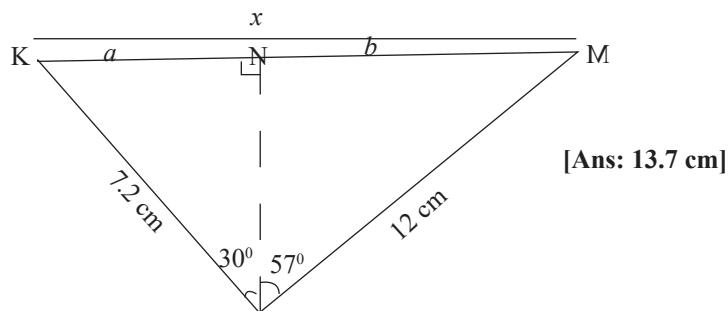
[Ans: 39.0 cm^2]

36. In the figure AB = EC = 3 cm, BE = DC = 7 cm and $\angle AED = 90^\circ$. Calculate the length of AD.



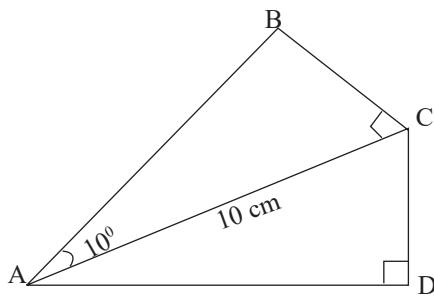
[Ans: AD = 10.8 cm]

37. Find the length of KM in this figure.



[Ans: 13.7 cm]

38. In a quadrilateral ABCD, $\angle BAC = 10^\circ$, $\angle BAD = 70^\circ$ and $\angle C = \angle D = 90^\circ$



Calculate the;

- (a) length of AB (correct to 3 dp) [Ans: 10.154 cm]
- (b) length of AD [Ans: 5 cm]
- (c) area of the quadrilateral ABCD. (correct to 3 dp) [Ans: 30.467]

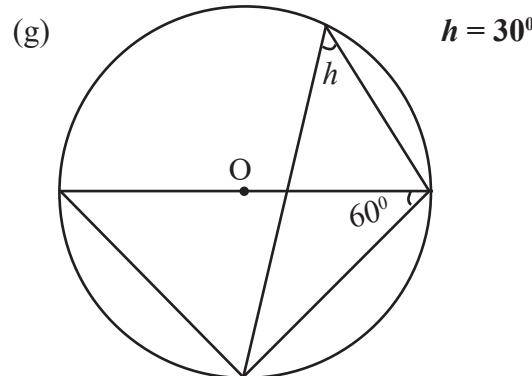
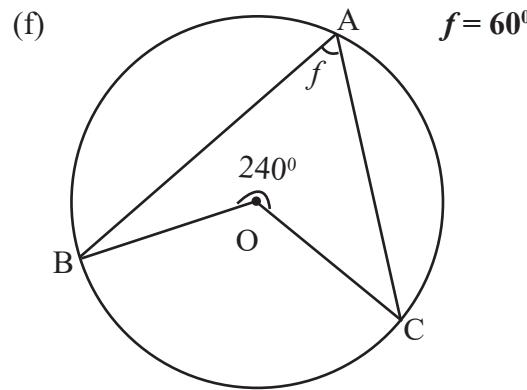
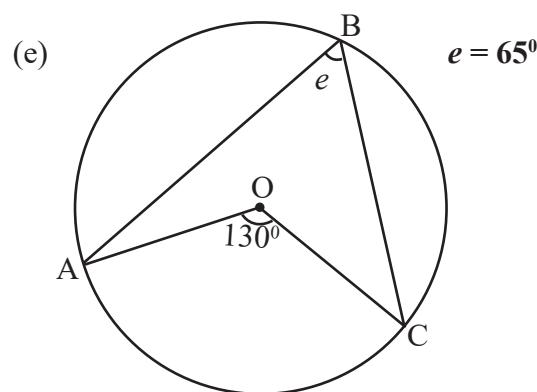
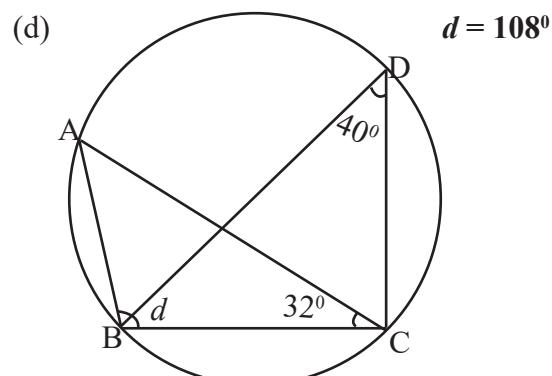
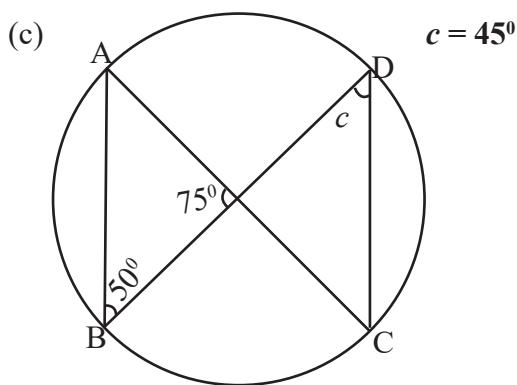
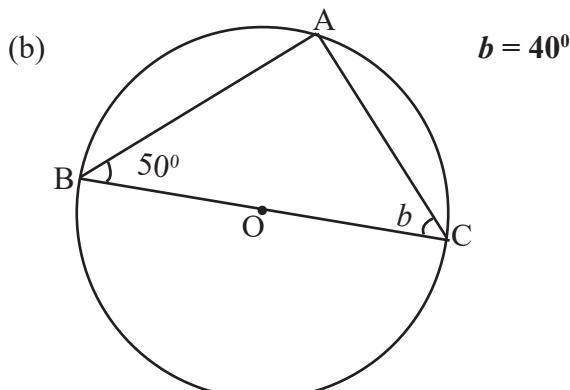
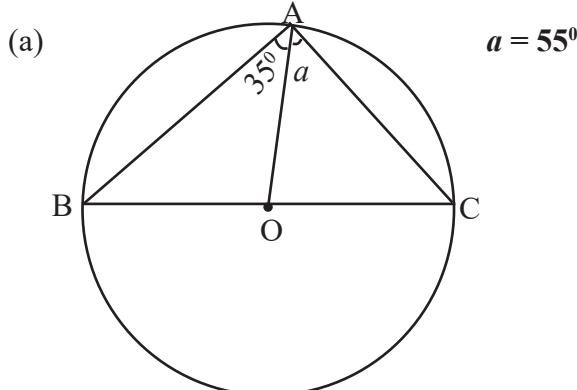
39. The legs of a pair of dividers are 9 cm long each. They are opened so that the points are 6 cm apart. Find the angle between the legs. [Ans: 38.9°]

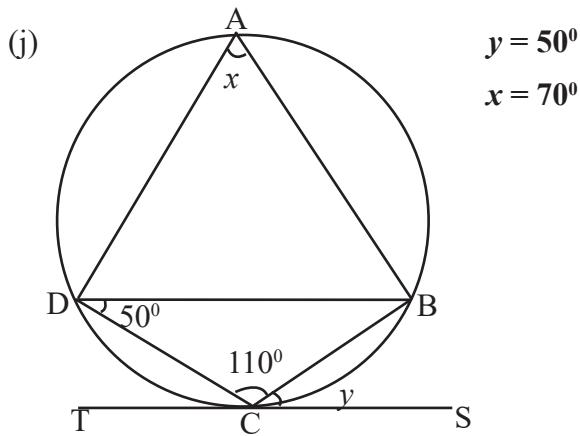
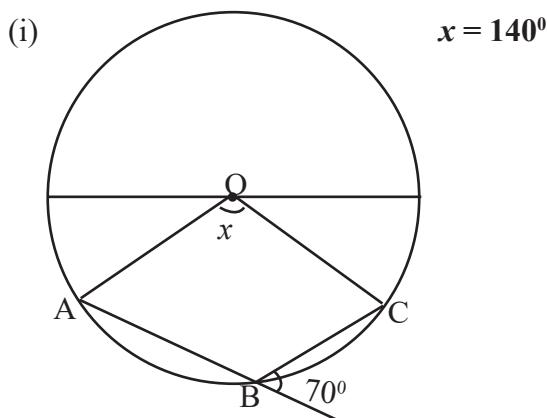
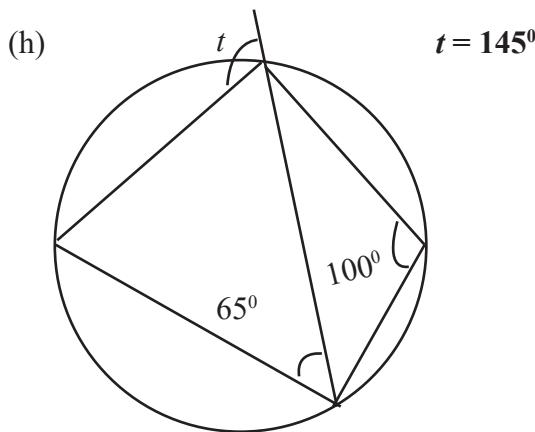
CIRCLE PROPERTIES

- ◊ NB: Before you encounter any of those questions, make sure that you recall all the properties governing the circles.
- ◊ Recall the formulas as well as where calculations are needed e.g;
- ◊ Area of a circle = πr^2
- ◊ Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$
- ◊ length of an arc = $\frac{\theta}{360^\circ} \times 2\pi r$
- ◊ Circumference of the circle = $2\pi r$ or πd

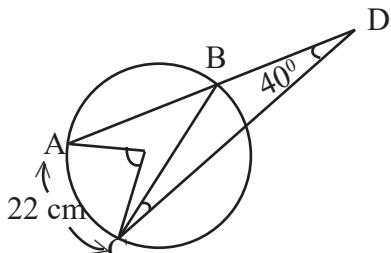
TRIAL QUESTIONS

1. Calculate the angles marked with small letters. O is the centre of circle wherever given.





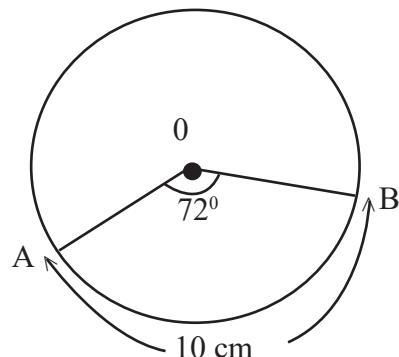
2. In the figure, O is the centre of a circle with radius 10.5 cm. Angle ADC = 40° . Minor arc AC = 22 cm. Calculate angle BCD.



[Ans: $\angle BCD = 20^\circ$]

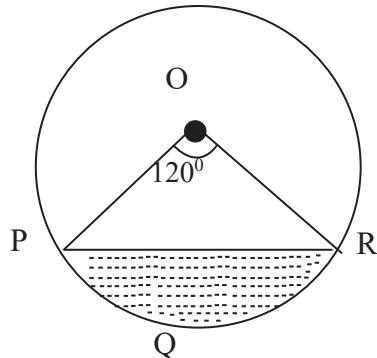
3. In the figure below, length of the arc AB = 10 cm and angle AOB = 72° . Find;

- (a) the radius of the circle in terms of π . [Ans: $r = \frac{25}{\pi}$]
 (b) the circumference of the circle in terms of π .



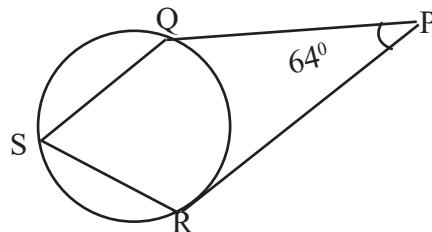
[Ans: 50 cm]

4. A circular cylinder with closed ends has diameter 21 cm and length 0.56 m. It contains some water and is lying horizontally. The figure represents a cross-section of this cylinder. O is the centre of the circle and PR represents the surface of the water. Given that angle POR = 120° , calculate;



- (i) the area of the sector OPQR. [Ans: 115.5 cm^2]
 (ii) the volume of water in the cylinder.
 [Ans: $3,794.56 \text{ cm}^3$]

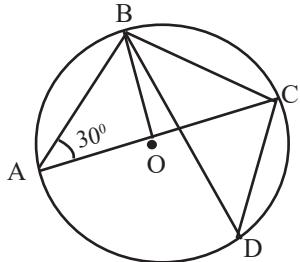
5. In the figure, PQ and PR are tangents to the circle at Q and R respectively, and QS = RS. If the angle QPR = 64° , Calculate angle QRS.
 [Ans: 61°]



[Ans: 61°]

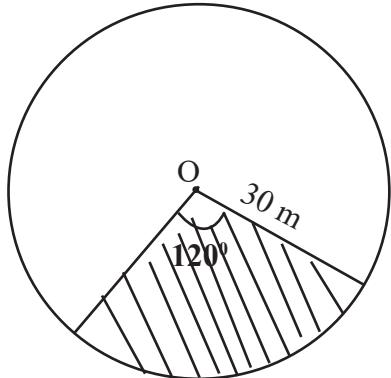
Circle properties

6. O is the centre of the circle ABCD and AOC is a straight line. Given that $\angle BAC = 30^\circ$, Calculate



- (a) $\angle ACB$ [Ans: 60°]
 (b) $\angle BDC$ [Ans: 30°]
 (c) $\angle BOC$ [Ans: 60°]

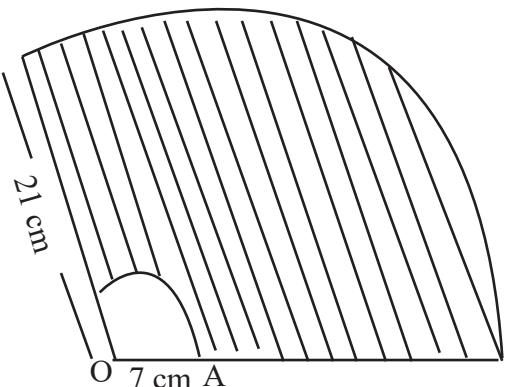
7. The curved surface of a cone is formed from the shaded sector of circle as shown in the diagram below.



Find the radius of the base of the cone.

[Ans: Radius = 10 m]

8. The wind screen wiper of a car sweeps through an angle of 120° . The shaded region in the figure below represents the area swept clean by the blade AB. If OA = 7 cm and OB = 21 cm, find the area of the glass swept clean.

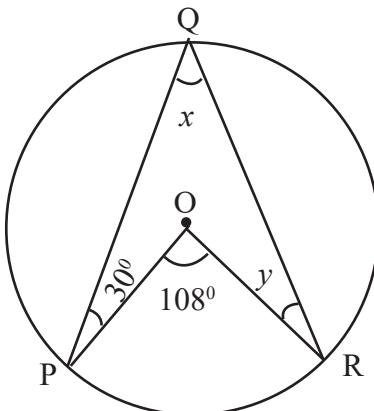


[Ans: $410\frac{2}{3}$ or 410.67 cm^2]

9. In a circle of radius 6 cm, an arc AB has a length of 16 cm.

- (a) Find the angle subtended by it at the centre O. [Ans: 152.7°]
 (b) Calculate the area of the sector AOB. [Ans: 48 cm^2]

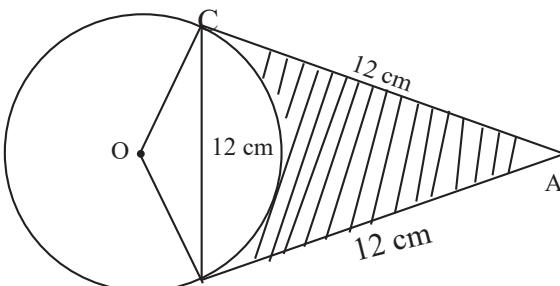
- 10.



In the figure above O is the centre of the circle. Angle POR and OPQ are 108° and 30° respectively. Find the values of x and y.

[Ans: x = 54° , y = 24°]

- 11.

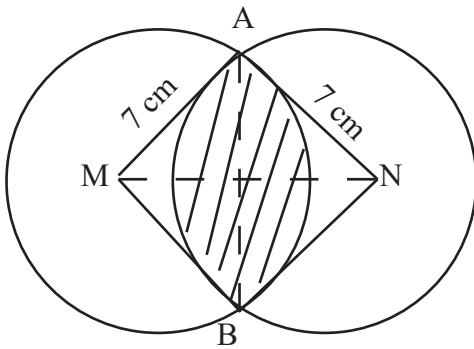


In the diagram above, AB and AC are tangents to the circle at point B and C respectively. O is the centre. Given that AB = AC = BC = 12 cm. Determine;

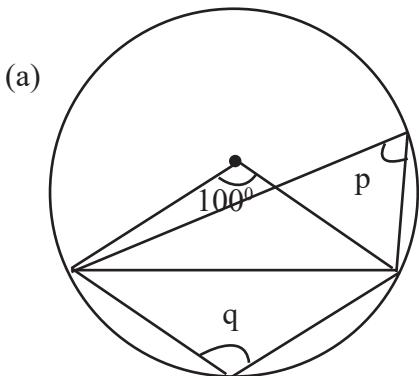
- (a) the angle BOC [Ans: 120°]
 (b) Area of the shaded region. [Ans: 32.8544 cm^2]

12. Two circles having the same radius of 7 cm, intersect at points A and B. If their centres are 10 cm apart, Calculate;

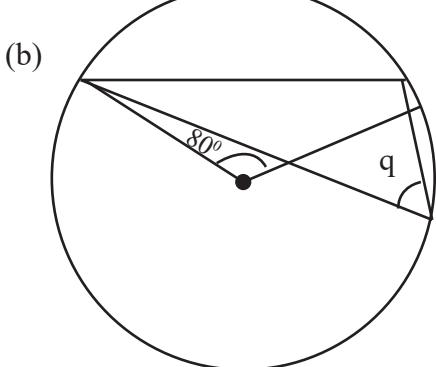
- (i) the length of a common chord AB. [Ans: $7\sqrt{2}$ or 9.9 cm]
 (ii) the area common to both circles. [Ans: 28 cm^2]



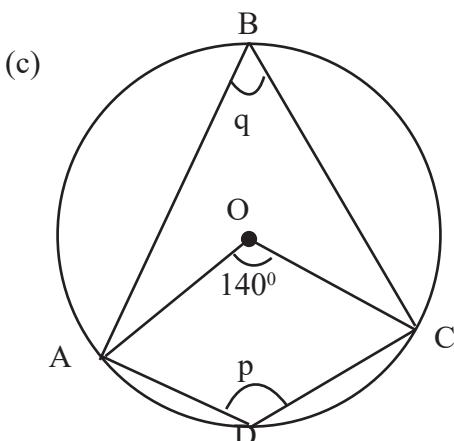
13. The circles below have their centre at O. Calculate angles P and Q in each case using circle properties.



[Ans: $p = 50^\circ$, $q = 130^\circ$]

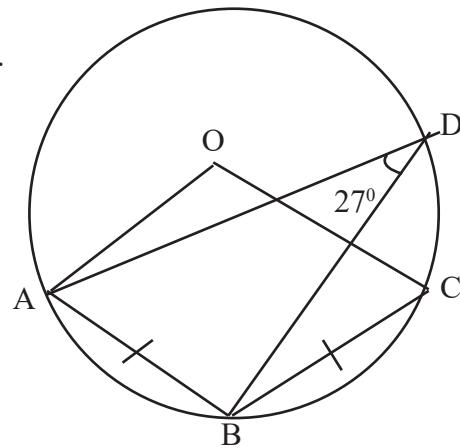


[Ans: $q = 40^\circ$]



[Ans: $q = 70^\circ$, $p = 110^\circ$]

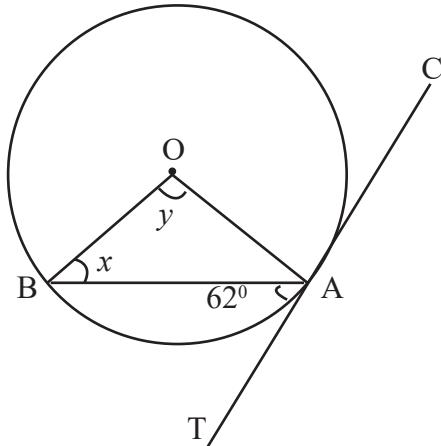
15.



In the figure given, O is the centre of the circle, $AB = BC$ and angle $ADB = 27^\circ$. Calculate;

- (i) the obtuse angle AOC [Ans: 108°]
- (ii) Angle ABC . [Ans: 126°]

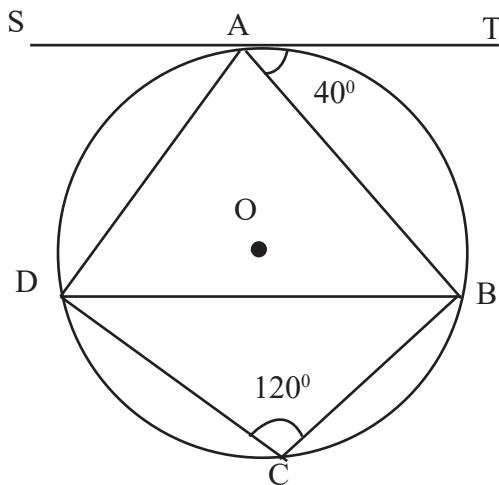
16. The figure shows a circle centre O with TAC as a tangent to the circle at point A. If angle $BAT = 62^\circ$, Find the size of x and y .



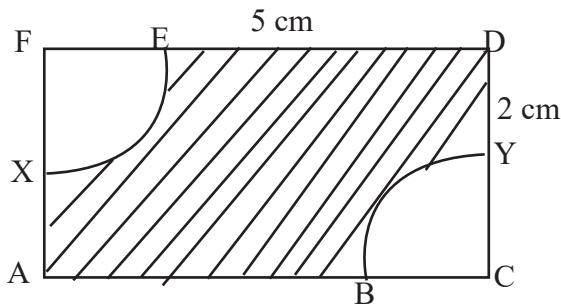
[Ans: $x = 28^\circ$, $y = 124^\circ$]

17. In the figure below, SAT is a tangent to the circle at T, $\angle BAT = 40^\circ$, $\angle BCD = 120^\circ$. Find;

- (a) $\angle DAB$ [60°]
- (b) $\angle ABD$ [80°]

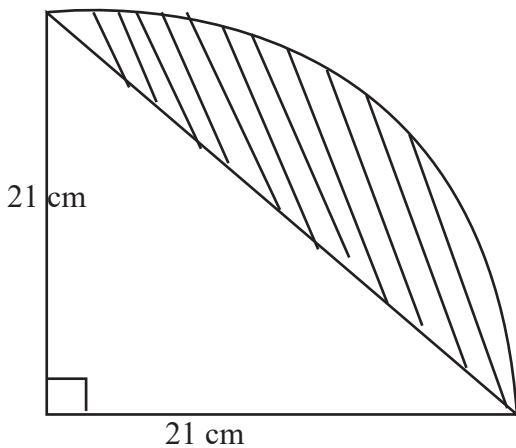


18. In the rectangle ABCD, $AC = 8 \text{ cm}$, $CD = 7 \text{ cm}$, $ED = 5 \text{ cm}$ and $DY = 2 \text{ cm}$.



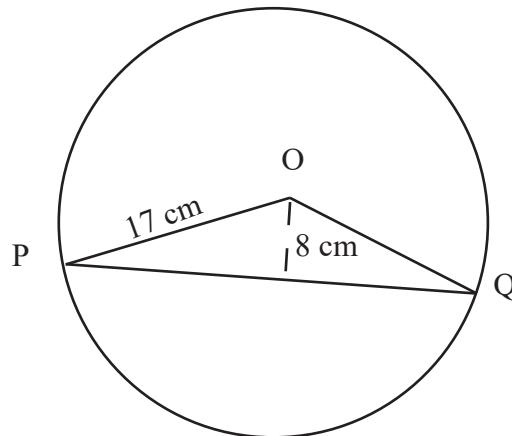
FEX and BCY are quadrants with centres F and C respectively. Find the area of the shaded region. [Ans: 29.2857 cm^2]

19. The diagram below shows a sector of a circle of radius 21 cm. Calculate the area of the shaded part (take $\pi = 22/7$)



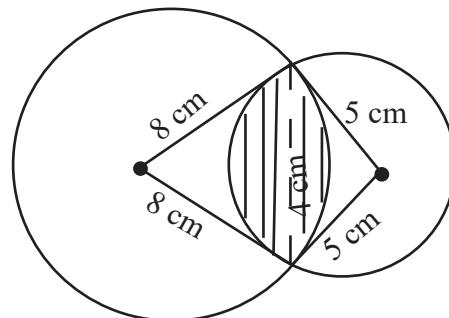
[Ans: 126 cm^2]

20. In the figure below, the chord is 8 cm away from the centre of the circle of radius 17 cm. Find the length of the chord PQ.



[Ans: $PQ = 30 \text{ cm}$]

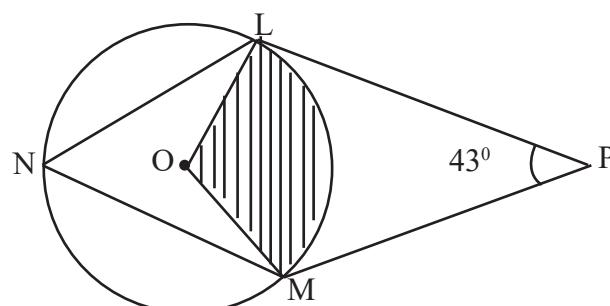
21. The diagram below shows two intersecting circles of radii 5 cm and 8 cm with a common chord of 4 cm.



Find the area of the shaded part.

[Ans: 1.8193 cm^2]

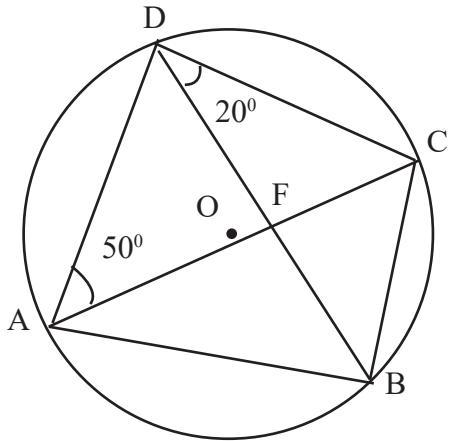
22. PL and PM are tangents to the circle with centre O and radius 5 cm.



(a) Find the obtuse angle LOM. [Ans: 137°]

(b) Find the shaded area. [Ans: 29.9 cm^2]

23. The figure shows a circle centre O and a circle quadrilateral ABCD whose diagonals AC and BD intersect at F. Angle $DAC = 50^\circ$, Angle $BDC = 20^\circ$ and AOC is a straight line.



(a) Giving reason for your answer, find the size of;

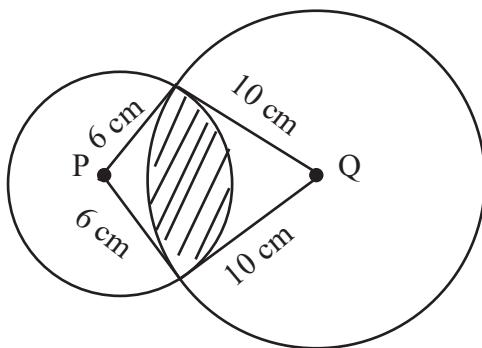
- (i) $\angle ADB$ [Ans: 70°]
- (ii) $\angle ABD$ [Ans: 40°]
- (iii) $\angle BAC$ [Ans: 20°]

(b) Show that;

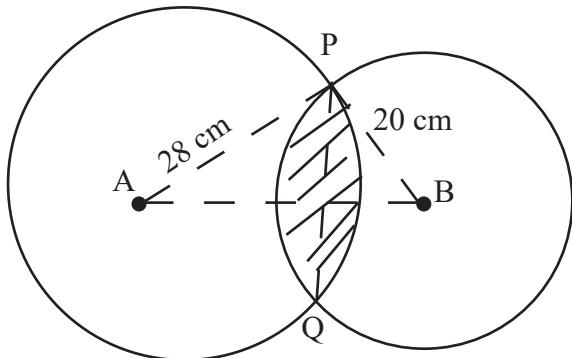
- (i) $\angle BCD = 110^\circ$
- (ii) $\angle DFC = 120^\circ$

24. A circle with centre P intersects a circle with centre Q at A and B. The radius of circle with centre P is 6 cm and the radius of a circle with centre Q is 10 cm. The length of the common chord is 8 cm. Calculate;

- (a) the distance between the centres P and Q. [Ans: 11.7 cm]
- (b) Area of the shaded region. [Ans: 12.9063 cm²]



25. Given that A and B are centres of the circles, lines PA and PB are tangents to the circles respectively. PQ = 30 cm while AB is a perpendicular bisector of PQ.

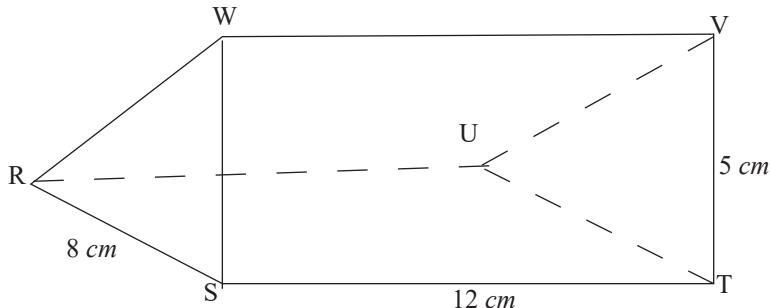


Find the area of the shaded part which is common to the two circles. (Take $\pi = 3.142$)
[Ans: 229.6199 cm²]

MEASUREMENT

TRIAL QUESTIONS

1. The figure below represents a triangular prism with isosceles ends. Given that $RW = WS = VU = VT = 5 \text{ cm}$, $RS = 8 \text{ cm}$ and $ST = 12 \text{ cm}$.

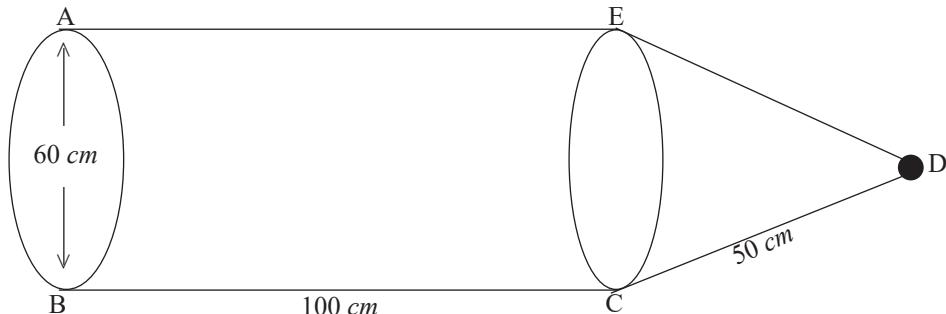


Calculate;

- (a) the volume of the prism
- (b) the total surface area of the prism
- (c) the cost of repainting the prism if a painter charges shs 500 per m^2 .

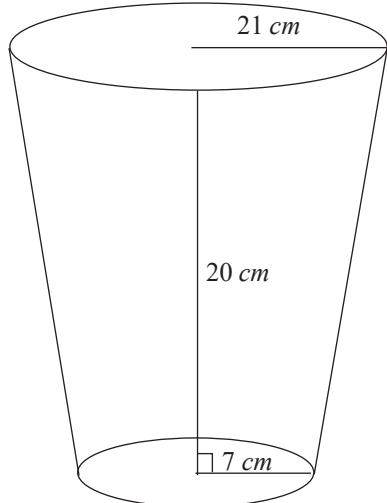
2. A wooden box is 2 m long, 0.5 m wide and 3 m high. Find the total surface area of the box.

3. The figure below shows a cylindrical piece of solid metal with a conical end.



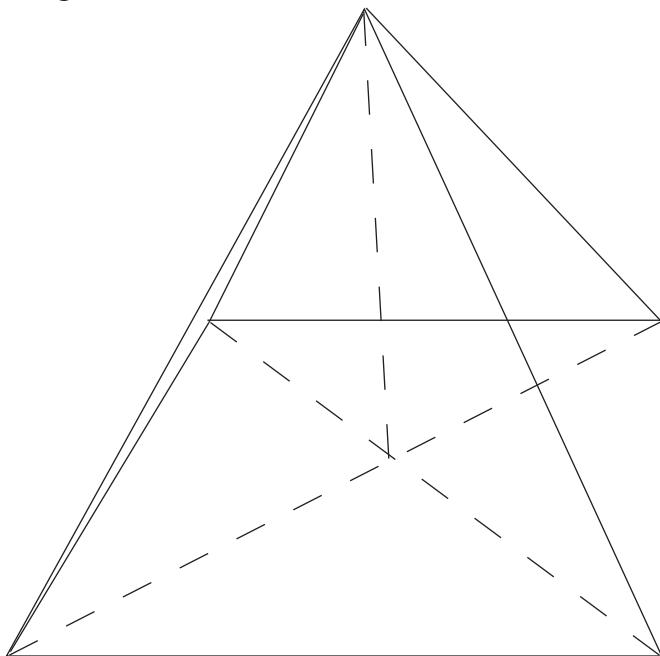
Given that the base diameter $AB = 60 \text{ cm}$, height of the cylindrical part $BC = 100 \text{ cm}$ and the slant height of the conical part $CD = 50 \text{ cm}$, calculate:

- (a) the volume of the solid
 - (b) the total surface area of the solid. (Use $\pi = 3.14$)
4. (a) A student had his rectangular photograph of dimensions 30 cm by 20 cm framed with a uniform border. If the area of the border is 216 cm^2 , how wide is the border?
- (b) A cone has a radius of 7 cm and a vertical height of 30 cm . Find
- (i) Its volume, ($\text{Use } \pi = \frac{22}{7}$).
 - (ii) the volume of another similar bigger cone which has a linear scale factor of 2.
5. A bucket in form of a frustum of a cone has a height of 20 cm and radii of 12 cm at the top and 8 cm at the bottom. If the bucket is open at the top,
- (a) Calculate its volume
 - (b) Surface area of the bucket
6. The figure below shows a wooden frustum of a cone. The radius of the base is 7 cm and that of the top is 21 cm . The frustum is 20 cm high.



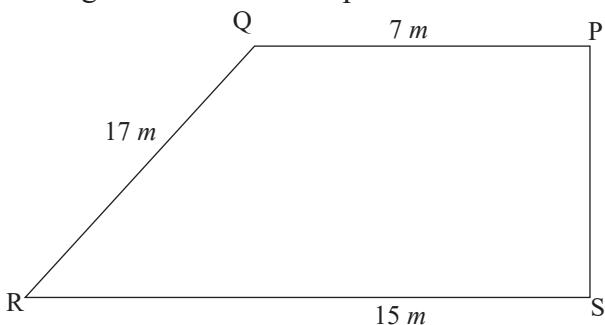
- (a) Calculate the volume of the bucket, in cm^3 .
 (b) If all the water in the bucket is emptied into a cylinder of base radius 14 cm, calculate the height of the cylinder, correct to 4 decimal places.

7. The figure below shows a wooden pyramid ABCDV on a square base ABCD of side 40 cm. The pyramid is 21 cm high as shown.



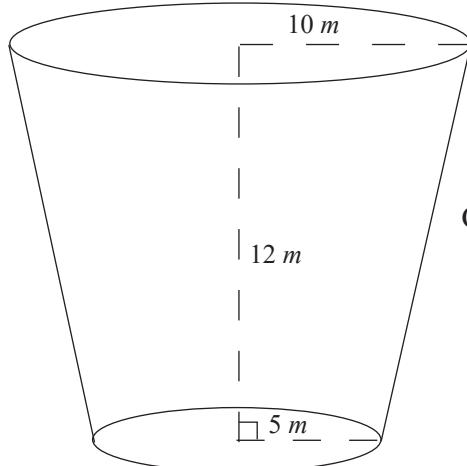
Calculate its;
 (i).volume in cm^3 .
 (ii).total surface area in m^2 .

8. (a) The figure below shows a plot of land in form of a trapezium, PQRS.



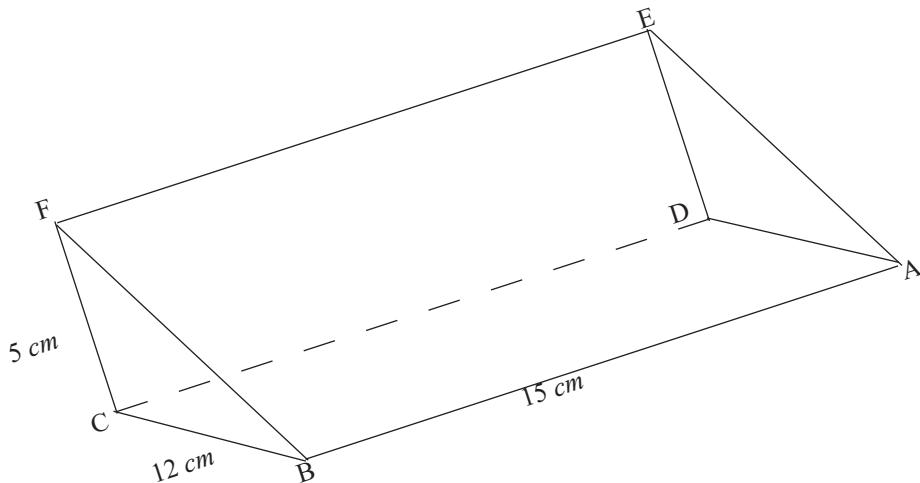
Calculate; (i) length hence find its perimeter
 (ii) area of the plot in m^2 .

- (b) The figure below shows a bucket in form of a frustum of a cone with top radius 10 m and bottom radius 5 m. The bucket is 12 m high



Calculate its volume in m^3 . (Take $\pi = \frac{22}{7}$)

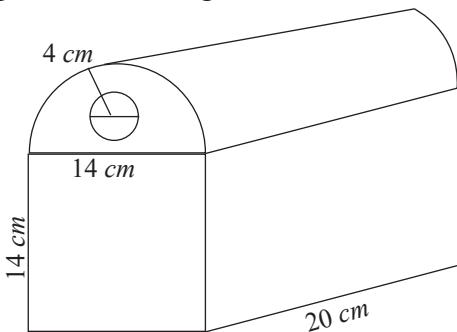
9. A solid in the shape of a frustum of a cone has radii of 7 cm and 14 cm. if the original cone had a height of 30 cm. calculate;
 - (a) Capacity of the solid.
 - (b) Its surface area when both ends are closed.
10. The diagram below shows a triangular prism in which AB = 15 cm, BC = 12 cm, CF = 5 cm and angle BCF = 90° .



Calculate the;

- (a) Volume and capacity of the prism.
- (b) Total surface area of the prism

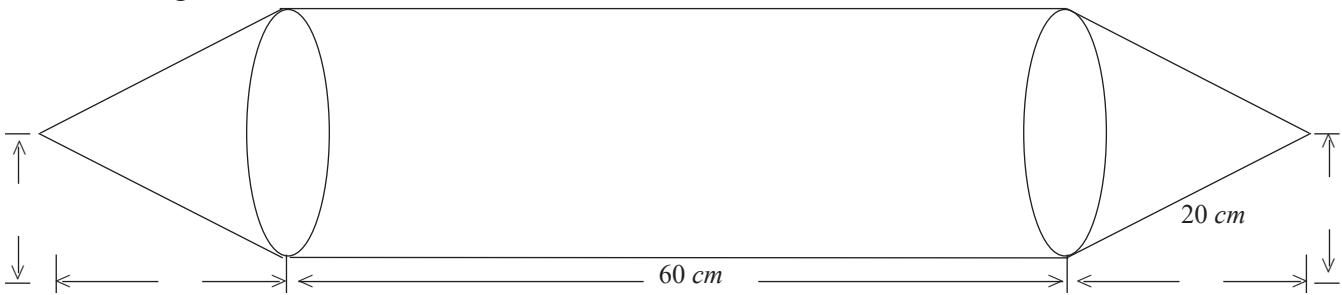
11. The diagram below represents a metal block with a uniform cross section. The uniform cross-section consists of a semi-circle of diameter 14 cm mounted on a square. A circular hole of diameter 4 cm runs through the entire length of the block. The length of the block is 20 cm.



Calculate:

- (a) the area in cm^2 of the cross-section
- (b) the volume in m^3 of the metal in the block.

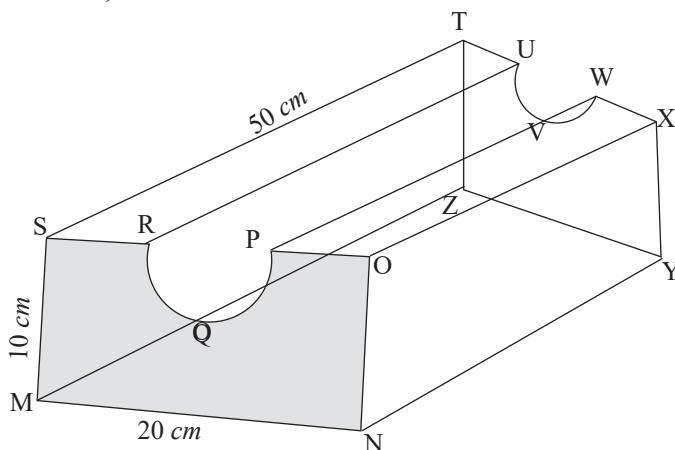
12. The figure below is of a hollow cylinder whose ends are covered by similar cones of radius 14 cm and vertical height 20 cm.



Calculate;

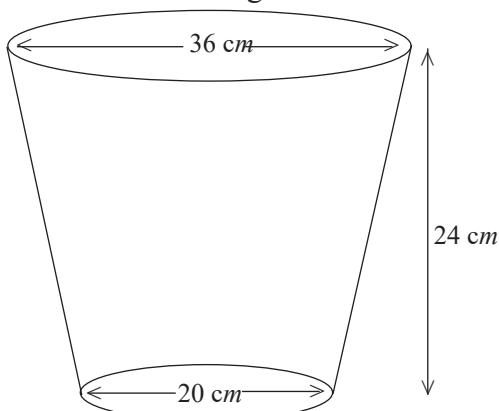
- the total surface area of the figure
- the volume of the figure

13. The diagram below shows a solid piece of metal made by drilling a semicircular groove of uniform cross-section PQR in a rectangular block. The radius of the semicircular groove is 5 cm. ST = 50 cm, SM = 10 cm, MN = 20 cm, OP = R = 5 cm.



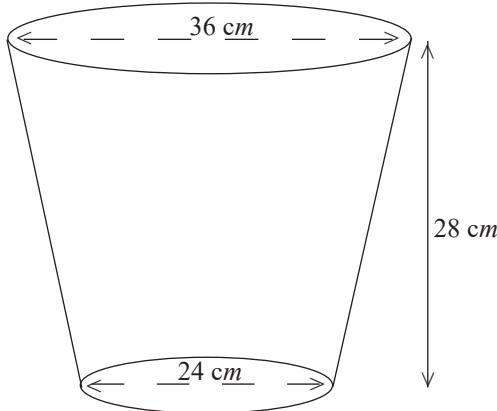
Calculate correct to 3 significant figures the;

- area of the shaded cross-section of the piece of metal.
 - volume of the metal.
14. (a) The diagram below shows a pail formed from a cone with a top diameter of 36 cm, bottom diameter of 20 cm and is 24 cm high.



- Calculate the volume of the pail.
- If this pail is filled with water and the water is later poured to full capacity into a cylinder of base radius 10 cm, determine the height of the cylinder. (Take $\pi = 3.142$).

- (b) The diagram below shows a bucket in form of a frustum of a cone with diameters 36 cm and 24 cm respectively. The bucket is full of water.

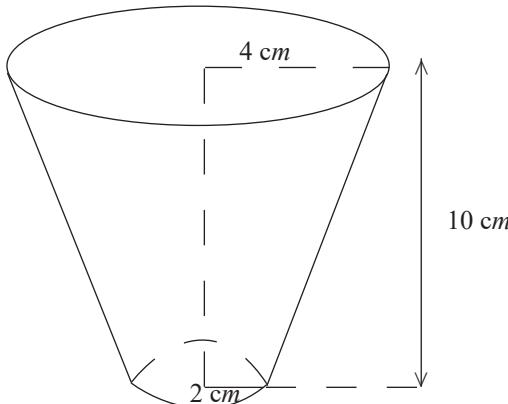


- (i) Determine the volume of the bucket, in cm^3
 (ii) If the water is poured into a hemisphere, it fills it.
 Find the radius of the hemisphere. (Use $\pi = \frac{22}{7}$)

15. (a) A hollow plastic cone has a base radius of 4 cm and a vertical height of 20 cm . Calculate:

- (i) the slant height.
 (ii) the volume of air inside the cone. [volume = $\frac{1}{3}\pi r^2 h$, $\pi = 3.14$]

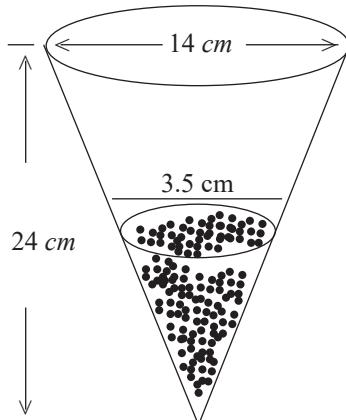
(b) The cone described in part (a) above is cut half-way between the top and the base (the cut is parallel to the base). The smaller circular end of the larger piece is then closed to make a drinking cup as below.



Calculate;

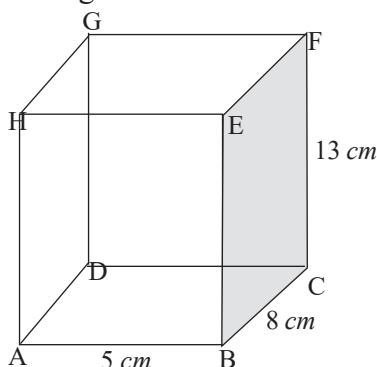
- (i) the area of the curved surface of the cup
 (ii) the capacity of the cup in millilitres.

16. The cone in the figure below is exactly full of water by volume.



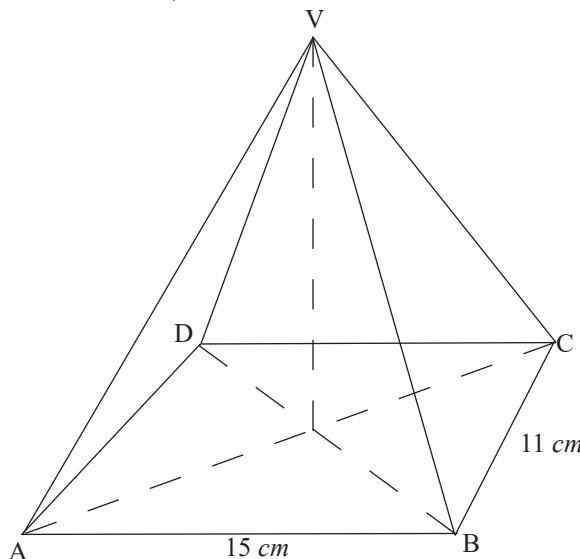
- (a) How deep is the water in the cone?
 (b) If the water in the cone is drawn out at a rate of 10 cm^3 per second, find the time taken to empty the cone? (Take $\pi = 3.14$)

17. The figure below shows a cuboid ABCDEFGH with the dimensions as shown.



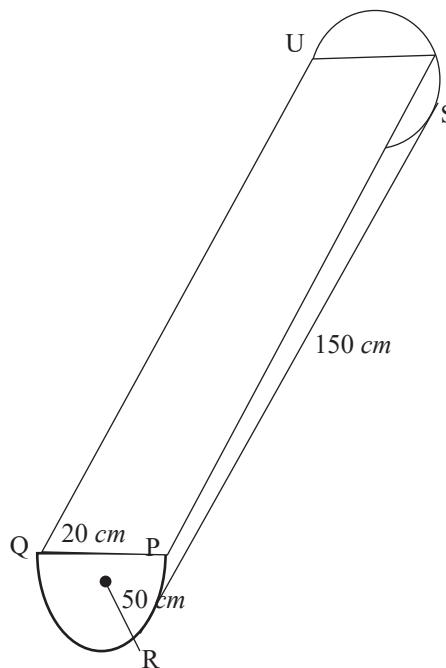
- Calculate the;
 (i) length BF and BG.
 (ii) angle between the line BG and the base ABCD.
 (iii) angle between the plane BDF and the base ABCD.

18. The figure below represents a right pyramid with vertex, V and a rectangular base ABCD. $VA = VB = VC = VD = 20\text{ cm}$, $AB = 15\text{ cm}$ and $BC = 11\text{ cm}$. M is the mid point of BC. Find the;



- (a) height V is above the base
- (b) angle of elevation of V from A
- (c) angle between planes VBC and ABCD.

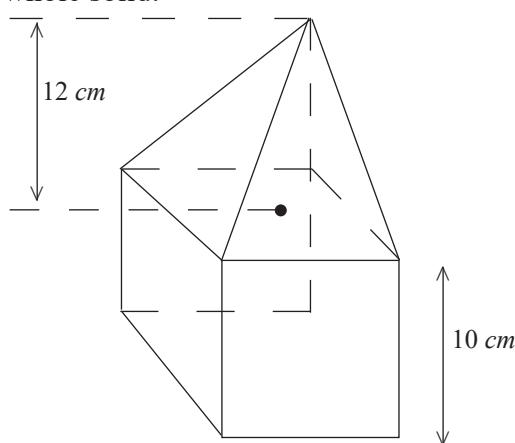
19. Diagram below shows a hollow right cylinder PQRSTV of negligible thickness. Part of which has been cut off as shown below. If the radius of the circular end is 50 cm $RS = 150\text{ cm}$ and $PQ = TU = 20\text{ cm}$.



Find;

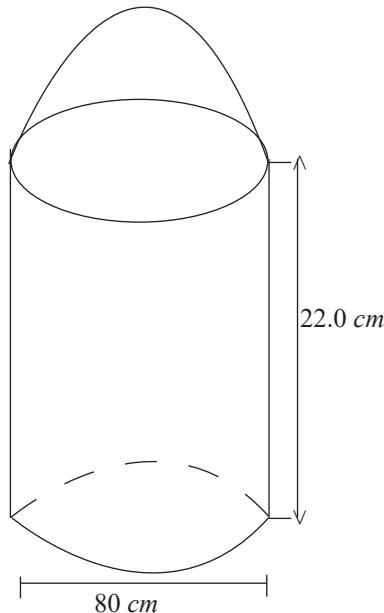
- (i) the area of the cross section PQR
- (ii) How much water (in litres) would fill this container
(Use $\pi = 3.14$)

20. The figure below shows a cube of side 10 cm surmounted by a pyramid of vertical height 12 cm . Find the volume of the whole solid.



Measurement

21. A flask is made from a cylinder of diameter 80 cm and height 22.0 cm topped by a hemisphere.

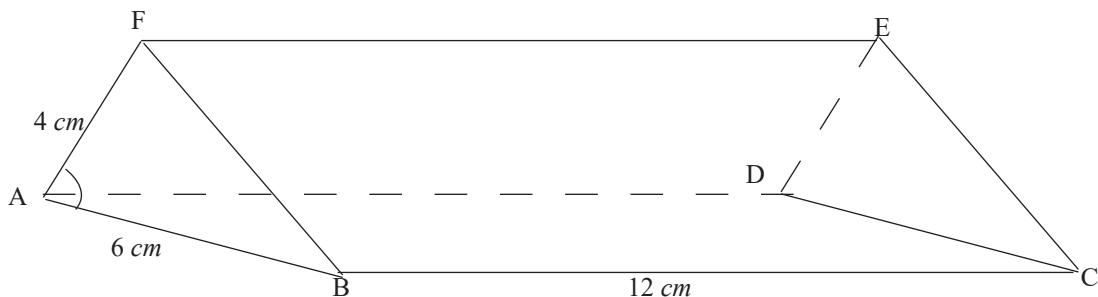


Find the total volume of the flask

22. VABCD is a pyramid with a square base ABCD of side 5 cm. V is its vertex vertically above the base such that VA = VB = VC = VD = 8 cm and O is the centre of the base. Calculate the;

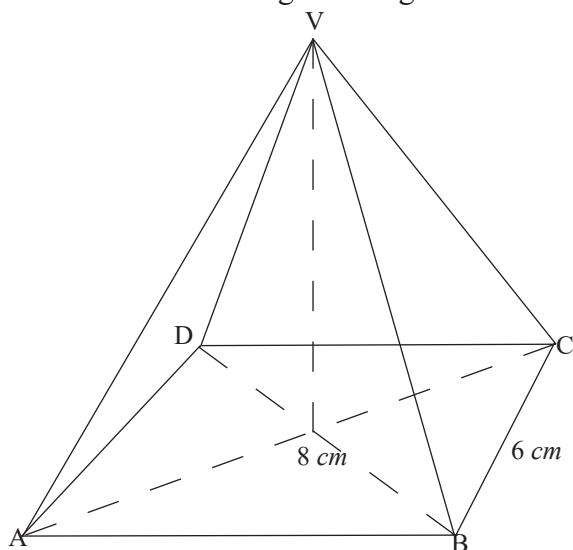
- (a) height OV of the pyramid (Correct to 2d.p)
- (b) angle VA makes with the base ABCD
- (c) angle between plane VDC and the base ABCD
- (d) volume of the pyramid (correct to four significant figures).

23. (a) The figure below shows a triangular prism of uniform cross-section ABF in which AF = 4 cm, AB = 6 cm and BC = 12 cm.



If angle BAF = 30°, calculate the volume of the prism.

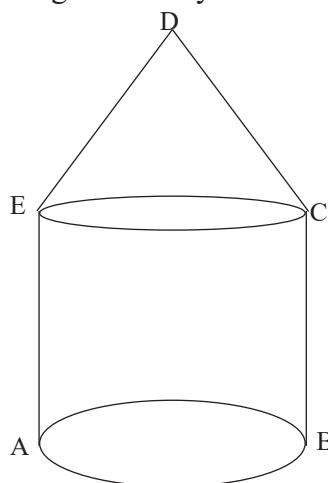
24. The figure below shows a right pyramid with vertex V and the edges VA = VB = VC = VD = 13 cm. The base ABCD is a rectangle of length 8 cm and width 6 cm.



Calculate;

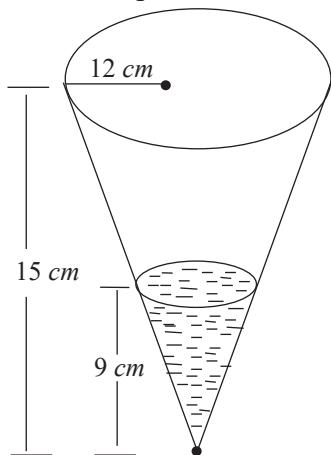
- (i) the height of the pyramid.
- (ii) the volume of the pyramid in cm^3 .
- (iii) the total surface area of the pyramid.

25. The diagram below shows a giant water tank ABCDE which is made in the shape of a right cone mounted on a cylinder. The radius of the cylinder is 6 m. The slant length of the cone ED = CD = 7.5 m and the height of the cylinder BC = AE = 10 m.



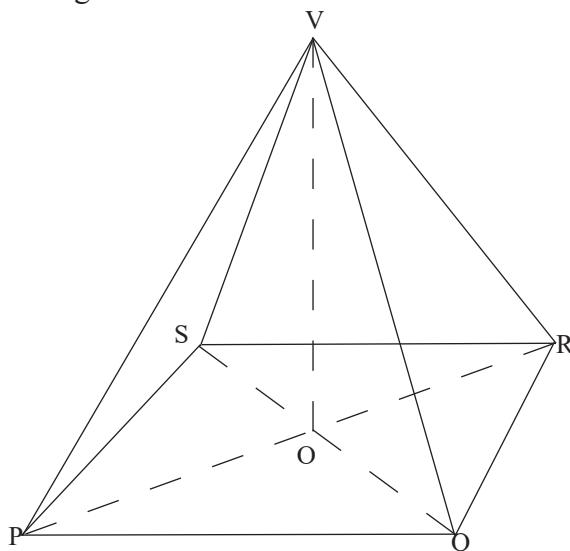
- (a) If the water tank has to be painted on the outer surface, calculate the surface area to be painted in m^2 . (Take $\pi = 3.142$).
 (b) Calculate the volume of water needed to fill the tank in m^3 . (Take $\pi = 3.142$).

26. The diagram below shows a right cone – shaped flask of base radius 12 cm and a depth of 15cm containing water to a depth of 9 cm.



- (a) Calculate the radius of the surface of water
 (b) If the water is poured in a cylinder of radius 4cm and it fills the cylinder, determine the height of the cylinder.
 (c).If the water is poured in a cube to the brim, find the length of the side of the cube. (take $\pi = 3.14$ and volume of cone = $\frac{1}{3}\pi r^2 h$)

27. VPQRS is a right pyramid with a rectangular base. PQ = 48 cm, QR = 36 cm and each slant edge is 50 cm long.

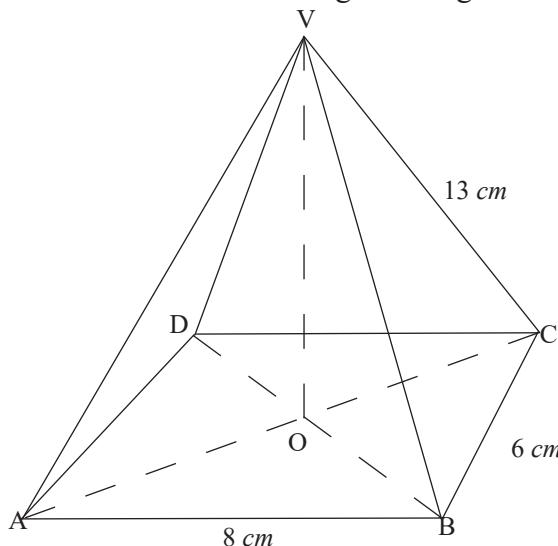


Calculate the;

- (a) the height of the pyramid.
 (b) angle between the slanting edge and the base
 (c) angle between the plane VQR and the base.
 (d) angle between planes VQR and VPS

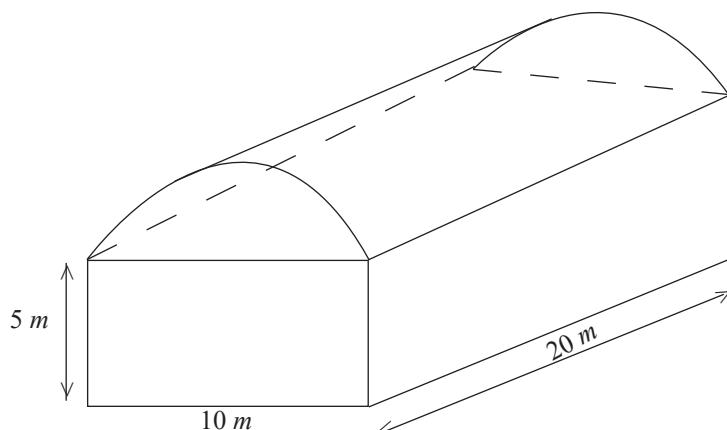
Measurement

28. The diagram below shows a right – pyramid with vertex V and the edges $VA = VB = VC = VD = 13 \text{ cm}$.
The base ABCD is a rectangle of length 8 cm and width 6 cm .



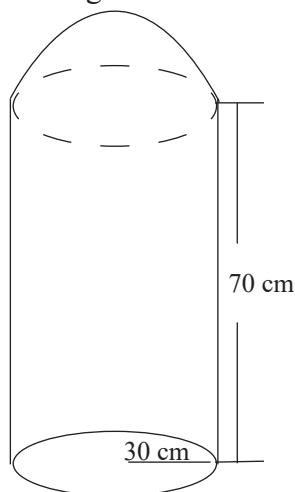
Calculate the;
 (a) vertical height OV
 (b) volume of the pyramid
 (c) total surface area of the pyramid

29. The figure below shows a building which consists of a cuboid 20 m long, 10 m wide and 5 m high surmounted by half a cylinder, diameter 10 m and length 20 m .



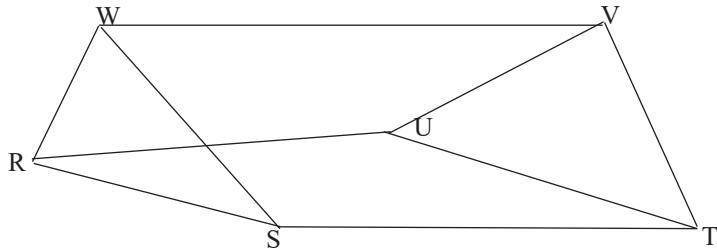
Calculate
 (i) the volume of the building
 (ii) the total area of the end, walls and roof

30. Figure below shows a hot water tank made by joining a hemisphere of radius 30 cm to an open cylinder of height 70 cm .



- (a) Calculate the total surface area of the tank.
 (b) The tank is full of water
 (i) Calculate the capacity of the tank in litres
 (ii) The water drains from the tank at a rate of 3 litres per second. Calculate the time, in minutes taken to empty the tank.
 (surface area of a sphere = $4\pi r^2$, volume of a sphere $\frac{4}{3} \pi r^3$)

31. The figure below is a triangular tent with isosceles ends. Given that $RW = WS = VU = VT = 5 \text{ m}$, $RS = 8 \text{ m}$ and $ST = 12 \text{ m}$.

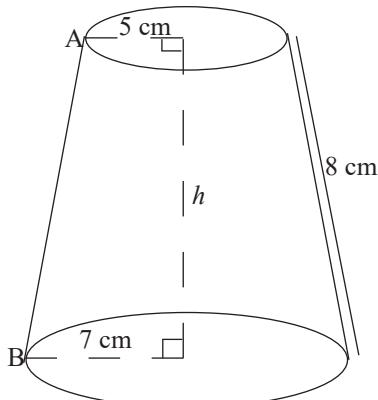


Calculate: (a) the volume of the tent.

(b) the total surface area of the tent.

(c) the cost of repainting this tent if a painter charge sh 500 per m^2 .

32. The figure below shows a lampshade in the form of a conical frustum.

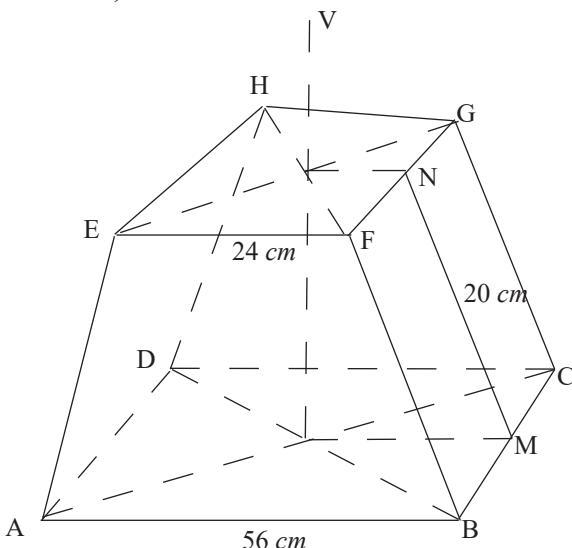


The top and bottom radii are 5 cm and 7 cm respectively. The slant height AB is 8 cm.

(a) Calculate the volume of the lamp shade to 4 significant figures.

(b) Find the vertical height of the lampshade. (Take $\pi = 3.142$)

33. The diagram below shows a solid block of wood ABCDEFGH which is part of a right pyramid on a square base ABCD with its vertex at V. Given that M and N are the mid-points of BC and FG respectively, AB = 56 cm, EF = 24 cm and MN = 20 cm. Calculate the:

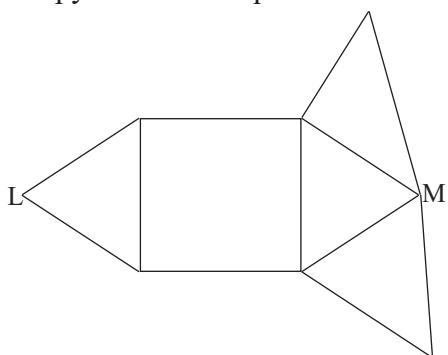


(a) height ST of the block of wood

(b) total surface area of the block of wood

(c) volume of the block of wood

34. The figure shows a square of side 6 cm and four congruent isosceles triangles. It represents the net of a pyramid on a square base. The distance LM is 20 cm. Calculate;



(a) The total surface area of the pyramid

(b) The perpendicular height of the pyramid when the net is folded.

(c) The angle of inclination of a triangular face to the base of the pyramid.

TRANSFORMATION, TRANSLATION, REFLECTION & ENLARGEMENT

REVIEW

Reflection

- ◊ An object and mirror line are required such that an image is found after reflection by the mirror. Or, when we are given the object and it's image, we can find the position of the mirror line.
- ◊ The object and the image are oppositely congruent.
- ◊ The image is of the same size as the object.
- ◊ The line joining the object to it's corresponding image meets the mirror line at 90° .

Transformation

- ◊ Transformation \times Object = Image
- ◊ $T \times O = I$

Transformation and area scale factor

- ◊ Area scale factor = determinant of a matrix of transformation
- ◊ Area scale factor = $\frac{\text{Area of the image}}{\text{Area of the object}}$
- ◊ Area scale factor = (Linear scale factor)²

Combined or Successive Transformations

- ◊ Suppose that the triangle P undergoes transformation T_1 to produce P' followed by a transformation T_2 to produce P'' then the single transformation T that maps P to P'' is $T_2 \times T_1$.
- ◊ $T = T_1 + T_2$
- ◊ A single matrix that maps P'' back to P is the inverse matrix of the successive transformation (T^{-1})

Matrix Transformations

Transformations	Matrix of transformation
Reflection in the x - axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Reflection in the y - axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
Reflection in the line $y = x$ or $y - x = 0$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Reflection in the line $y + x = 0$ or $y = -x$	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
Positive quarter (90°) turn about $(0, 0)$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
Negative quarter (-90°) turn about $(0, 0)$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
Half turn ($+180^{\circ}$) about $(0, 0)$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
Identity	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- ◊ General matrix for rotation; $T = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$
Where θ is the given angle.

TRIAL QUESTIONS

- On graph papers, plot the points A(1, 3) B (3, 3) and C (3, 1). Join the points to form triangle ABC. Find the co-ordinates of A', B' and C' after an enlargement of
 - Scale factor 2 with centre of enlargement (0, 0)
 - Scale factor -1 with centre of enlargement at (0, 0)
 - Scale factor $-2\frac{1}{2}$ with centre of enlargement at (2, 2)
- (a) The vertices of triangle RPQ are (0, 3), (1, 5) and (3, 3) respectively. Find the image $R_1P_1Q_1$ of triangle RPQ under a reflection in the line $x + y = 0$. [Ans: $R_1(-3, 3)$ $P_1(-5, -1)$ $Q_1(-3, -1)$]

(b) The image $R_1P_1Q_1$ is then mapped onto $R_2P_2Q_2$ under a rotation about the origin through 270° . Find the coordinates of the vertices of triangle $R_2P_2Q_2$. [Ans: $R_2(3, 3)$ $P_2(-1, 5)$ $Q_2(-1, 3)$]

(c) Find a single transformation matrix that will map triangle $R_2P_2Q_2$ back to RPQ

(d) Describe the transformation in (c) above fully.
- (a) Plot the points A(1, 0), B(0, -2), C(2, 0) on a graph using a scale of 2 cm : 1 unit on both axes.

(b) Graphically, find the image
 - triangle $A_1B_1C_1$ of triangle ABC after an enlargement by scale factor +2 about the origin.
 - triangle $A_2B_2C_2$ of triangle ABC after an enlargement by scale factor -2 about the origin.
(c) If triangle $A_1B_1C_1$ is the image of triangle $A_2B_2C_2$ under an enlargement, find:
 - the centre of enlargement
 - the scale factor.
- Plot on the same axes the
 - vertices P(2, 0), Q (0, 2), R (-2, 0) and S(0, -2)
 - image $P'Q'R'S'$ of PQRS under the matrix transformation $\begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$
 - Image $P''Q''R''S''$ of $P'Q'R'S'$ under a reflection in the x -axis. Determine the matrix of this transformation

- Find the matrix M of the transformation which maps PQRS onto $P''Q''R''S''$

[Ans: $\begin{pmatrix} 2 & 0 \\ -4 & -2 \end{pmatrix}$]
- (a) The image of quadrilateral P(2, -1) Q(0, -3) R(2, -4) S(4, -2) is $P_1(1, 2)$, $Q_1(3, 0)$, $R_1(4, 2)$ and $S_1(2, 4)$ under a certain transformation T_1
 - Find the matrix of transformation for T_1

[Ans: $T_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$]
 - Describe the transformation T_1 fully. [Ans: **A matrix of reflection in positive quarter turn**]
- (b) Given that quadrilateral $P_1Q_1R_1S_1$ is then mapped onto quadrilateral $P_2Q_2R_2S_2$ after a reflection in the line $y + x = 0$. Find the coordinates of the vertices of quadrilateral $P_2Q_2R_2S_2$ [Ans: $P_2(-2, -1)$ $Q_2(0, -3)$ $R_2(-2, -4)$ $S_2(-4, -2)$]

(c) Determine a single matrix of transformation that maps quadrilateral PQRS directly onto $P_2Q_2R_2S_2$.

[Ans: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$]
- (a) The image of triangle P with vertices (0, 0), (0, -2), (2, 0) is P' with vertices (0, 0), (0, -4), (4, 0) under transformation M. P' is then given a transformation N = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to give P''
 - Find the transformation M

[Ans: $M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$]
 - Describe M fully [Ans: **M is a matrix of enlargement at (0, 0) with scale factor 2**]
- (b) Find the co-ordinates of P'' [Ans: (0, 0) (-4, 0) (0, 4)]

(c) Give a single matrix transformation which maps P onto P''

[Ans: $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$]
- (a) A triangle PQR with vertices P (-2, 1), Q(3, 1) and R(0, 3) is mapped onto triangle $P_1Q_1R_1$ whose vertices are $P_1(-6, -1)$, $Q_1(9, 4)$ and $R_1(0, 3)$ by a transformation described by matrix M. Triangle $P_1Q_1R_1$ is further transformed by a matrix N = $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$ to triangle P_2, Q_2, R_2

Find;
 - matrix M [Ans: $M = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$]
 - the co-ordinates of P_2, Q_2, R_2 [Ans: $P_2(-6, 3)$ $Q_2(9, 3)$ $R_2(0, 9)$]

- (b) Find the single matrix that would map PQR onto $P_2Q_2R_2$. Describe this transformation fully. [Ans: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$] Matrix of enlargement at (0, 0) with scale factor 3]
- (c) Find the area of;
- (i) PQR [Ans: 3 Sq.Units]
(ii) $P_2Q_2R_2$ [Ans: 27 Sq.Units]
8. A transformation represented by the matrix $\begin{pmatrix} 4 & -2 \\ 3 & -2 \end{pmatrix}$ maps the vertices of triangle ABC onto its image vertices A' (2, 1) B' (10, 7) and C' (2, 0) respectively. The image of triangle ABC further undergoes another transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ to be mapped onto triangle A'' B'' C''. Find the:
- (a) co-ordinates of the vertices A, B and C
[Ans: A(1, 1) B(3, 1) C(2, 3)]
- (b) co-ordinates of A'', B'' and C'' single matrix of the transformation which would map triangle A'' B'' C'' back onto triangle ABC
[Ans: A''(4, 2) B''(20, 14) C''(4, 0); $\frac{1}{4} \begin{pmatrix} +2 & -2 \\ +3 & -4 \end{pmatrix}$]
Matrix of reflection in the y-axis.]
9. Triangle ABC was transformed to its image triangle A' B' C' by transformation $T = \begin{pmatrix} 6 & 3 \\ 4 & 3 \end{pmatrix}$. Find the area of triangle A' B' C' if the area of triangle ABC is 12 cm^2 . [Ans: 72 cm²]
10. The points P(0, 2), Q(1, 4) and R(2, 2) are vertices of triangles PQR. The images of P, Q, and R under a reflection in the line $x - y = 0$ are P', Q' and R' respectively. The points P', Q' and R' are then mapped onto points P'', Q'' and R'' respectively under an enlargement with centre (0, 0) and scale factor -2.
- (a) Write down the matrix for;
- (i) reflection [Ans: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$]
(ii) enlargement [Ans: $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$]
- (b) Hence determine the coordinates of the points
- (i) P', Q' and R'. [Ans: P'(2, 0) Q'(4, 1) R'(2, 2)]
(ii) P'', Q'' and R'' [Ans: P''(-4, 0) Q''(-8, -2) R''(-4, -4)]
11. The point A(5, 2) undergoes the transformation represented by $M = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ followed by a transformation T = $\begin{pmatrix} -6 & 0 \\ 11 & 0 \end{pmatrix}$. Determine the coordinates of the final image of A. [Ans: A(13, 16)]
12. (a) ABCD is a square with A(0, 5), B(0, 3), C(2, 3) and D(2, 5). It is rotated through 270° about the origin. Find the coordinates of A', B', C' and D', the images of A, B, C and D respectively.
[Ans: A'(5, 0) B'(3, 0) C'(3, -2) D'(5, -2)]
- (b) Given that A' B' C' D' is then reflected onto A'' B'' C'' D'' in the line $x - y = 0$ find the coordinates of A'', B'', C'' and D''. [Ans: A''(0, 5), B''(0, 3), C''(-2, 3) D''(-2, 5)]
- (c) Determine the matrix of a single transformation which would map A'' B'' C'' D'' back onto ABCD. Describe the matrix fully. [Ans: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$]
Matrix of reflection in the y-axis.]
13. (a) Point B(-2, 3) was mapped onto its image point B' under a reflection in $y = x$. Find the coordinates of B'. [Ans: B'(3, -2)]
- (b) The transformation described by the matrix $\begin{pmatrix} 3 & x \\ y & 3 \end{pmatrix}$ maps point A(3, 5) onto point A'(4, 9). Find the values of x and y.
[Ans: x = -1, y = -2]
14. (a) Triangle ABC with A(7, 8), B(1, 9) and C(3, 5) is reflected in the line $x - y = 0$ to give its image triangle A' B' C'. Triangle A' B' C' is then given a positive quarter turn to form its image triangle A'' B'' C'' about centre O(0, 0). Find;
- (i) A', B' and C' [Ans: A'(8, 7) B'(9, 1) C'(5, 3)]
(ii) A'', B'' and C'' [Ans: A''(-7, 8) B''(-1, 9) C''(-3, 5)]
- (b) Show triangle ABC and its images on the same axes.
15. Triangle PQR with area 15 cm^2 was transformed to its image triangle P' Q' R' by a transformation matrix N = $\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$. Find the area of triangle P' Q' R'. [Ans: 105 cm²]
16. The vertices of triangle PQR, P(0, 1), Q(2, 1) and R(2, 5) are mapped onto the triangle P' Q' R' by the transformation whose matrix is $\begin{pmatrix} 4 & 0 \\ -2 & 1 \end{pmatrix}$.
- (a) Find the;
- (i) Coordinates of the vertices of the image triangle P' Q' R' [Ans: P'(0, 1) Q'(8, -3) R'(8, 1)]

- (ii) Ratio of the area of triangle PQR to area of triangle P' Q' R' [Ans: PQR : P'Q'R' = 1 : 4]

- (b) Plot on the same axes triangle PQR and its image P' Q' R'

- (c) Determine the matrix of transformation which maps P' Q' R' back onto PQR. [Ans: $\frac{1}{4} \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$]

17. The vertices of triangle P are (1, 1), (2, 3) and (4, 3). The matrix $R = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$ maps P onto P' and the matrix $E = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ maps P' onto P".

- (a) Draw the three triangles P, P' and P" on the same diagram

- (b) What is the image of P" under the transformation described by RE⁻¹?

$$[\text{Ans: } \left(\frac{1}{4}, \frac{1}{4} \right) \left(\frac{3}{2}, \frac{3}{4} \right) \left(1, \frac{3}{4} \right)]$$

- (c) What is the image of P under the transformation described by ER⁻¹? [Ans: (2, 2) (6, 4) (6, 8)]

18. A transformation represented by the matrix $N = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ maps the points A(0, 0), B(2, 0), C(2, 3) and D(0, 3) of quadrilateral ABCD onto A' B' C' D'.

- (a) Draw the quadrilateral ABCD and its image A' B' C' D'.

- (b) Hence or otherwise determine the area of A' B' C' D'. [Ans: 18 Sq.units]

- (c) A transformation represented by matrix $M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps A' B' C' D' onto A" B" C" D". Find the coordinates A", B", C" and D". Hence draw A" B" C" D" on the same graph.

$$[\text{Ans: } A''(0, 0) B''(-2, -4) C''(-8, -7) D''(-6, -3)]$$

- (d) Determine the single matrix which maps A" B" C" D" back to ABCD.

$$[\text{Ans: } \frac{1}{4} \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}]$$

19. A transformation T_1 is defined by a reflection in the line $y = x$ followed by a half turn about the origin. Find the matrix of transformation T_1 .

$$[\text{Ans: } T_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}]$$

20. The points A(-6, 5), B(3, 2) and C(3, -1) are vertices of triangle ABC. Triangle ABC undergoes a transformation represented by

matrix $M = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ to form image triangle A'B'C'. Triangle A'B'C' is then reflected in the x-axis to form the image triangle A"B"C".

- (a) Find the coordinates of;

- (i) A', B' and C'.

$$[\text{Ans: } A'(-6, 3) B'(3, 12) C'(3, 3)]$$

- (ii) A", B" and C"

$$[\text{Ans: } A''(-6, -3) B''(3, -12) C''(3, -3)]$$

- (b) On the same graph, show triangle ABC and its images.

- (c) Calculate the area of triangle A'B'C', the image of ABC and hence calculate the area of triangle ABC. [Ans: Area A'B'C' = 40.5 sq.units. Area ABC = 13.5 sq.units]

21. Under an enlargement scale factor 3, a point P is mapped onto its image P'(18, -15), find the coordinates of P. [Ans: P(6, -5)]

22. The transformations T and S are defined as follows; T = reflection in the line $y = x$. S = positive quarter turn about the origin. The points A(3, 7), B(3, 4) and C(-1, 4) are the vertices of triangle ABC whose image under T is triangle A'B'C'. Triangle A"B"C" is the image of triangle A'B'C' under S.

- (a) Find the coordinates of;

- (i) A', B' and C'

$$[\text{Ans: } A'(7, 3) B'(4, 3) C'(4, -1)]$$

- (ii) A", B" and C"

$$[\text{Ans: } A''(-3, 7) B''(-3, 4) C''(1, 4)]$$

- (b) Show triangle ABC and its images on the same graph.

- (c) Transformation T followed by S can be replaced by a single transformation P. Find the matrix P. [Ans: $P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$]

23. A transformation represented by the matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ maps the points A(0, 0), B(2, 0), C(2, 3) and D(0, 3) of the quadrilateral ABCD onto A'B'C'D'. Another transformation $N = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps A'B'C'D' onto A" B" C" D".

- (a) State the coordinates of A', B', C' and D' the images of A, B, C and D and the coordinates of A", B", C" and D" the images of A', B', C' and D' respectively. [Ans: A'(0, 0) B'(4, 2) C'(7, -4) D'(6, -3); A''(0, 0) B''(-2, -4) C''(4, -7) D''(6, -3)]

- (b) Draw the quadrilateral ABCD and its images on the same graph.
- (c) Hence or otherwise determine the area of A' B' C' D' [Ans: 18 sq.units]
- (d) Determine the single matrix which maps A''B''C''D'' back to ABCD.
[Ans: $\frac{1}{5} \begin{pmatrix} -1 & -2 \\ 2 & -1 \end{pmatrix}$]

24. (a) Triangle A'B'C' is the image of triangle ABC under transformation $M = \begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}$. Find the area of ABC if the area of A'B'C' is 28 cm^2 .

[Ans: 3.5 cm²]

- (b) Point A(0, 3) is reflected in the line $y + x = 0$. Find the coordinates of its image A'.
[Ans: (-3, 0)]

25. (a) A unit square whose vertices O(0, 0), A(1, 0), B(0, 1) and C(1, 1) is transformed by rotating through a positive quarter turn about the origin. Find the matrix for this transformation.

[Ans: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$]

(b) Given $T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $M = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, find;

- (i) the images of the points A(0, 3) and B(5, 3) under the transformation TM.
[Ans: A'(3, 0) B'(3, -5)]

- (ii) the matrix of transformation which will map the images of A and B back to their original positions.
[Ans: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$]

26. (a) A transformation represented by the matrix $M = \begin{pmatrix} 1 & -1 \\ 4 & 8 \end{pmatrix}$ maps triangle ABC whose area is 24.5 cm^2 onto A'B'C'. Find the area of triangle A'B'C'. **[Ans: 294 cm²]**

27. Rectangle OABC is transformed into a parallelogram O'A'B'C' by the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ where O'(0, 0), A'(6, 3), B'(8, 7), C'(2, 4). Find the;

- (a) matrix which maps the parallelogram O'A'B'C' back onto the rectangle OABC.
[Ans: $\frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$]

- (b) Coordinates of O, A, B and C.
[Ans: O(0, 0) A(3, 0) B(3, 2) C(0, 2)]

- (c) Area of the rectangle OABC and hence the area of the parallelogram O'A'B'C'.
[OABC = 6 sq.units; O'A'B'C' = 18 sq.units]

28. The points A(1, 0), B(3, 0), C(3, 1), D(1, 1) are vertices of a rectangle ABCD. The images of A, B, C and D under a transformation $T = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ are A', B', C' and D' respectively. The images A', B', C' and D' are then mapped onto the points A'', B'', C'' and D'' respectively under a transformation $M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

- (a) Determine the coordinates of the points

(i) A', B', C' and D' **[Ans: A'(-2, 0) B'(-6, 0) C'(-6, -2) D'(-2, -2)]**

(ii) A'', B'', C'' and D'' **[Ans: A''(0, 2) B''(0, 6) C''(2, 6) D''(2, 2)]**

(b) Find a single matrix of transformation which would map rectangle A''B''C''D'' back onto ABCD. **[Ans: $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$]**

29. The points A(-2, 1), B(1, 4) and C(1, 1) are vertices of a triangle ABC. The points A', B' and C' are the images of A, B and C under a transformation described by matrix $M = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$. The points A'', B'' and C'' are the images of A', B' and C' under a transformation described by matrix $N = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$.

- (a) Find the coordinates of;

(i) A', B' and C'.
[Ans: A'(-6, 5) B'(3, 2) C'(3, -1)]

(ii) A'', B'' and C'' **[Ans: A''(-6, 3) B''(3, 12) C''(3, 3)]**

(b) Find the ratio of areas of triangles ABC to A''B''C''. **[Ans: ABC: A''B''C'' = 1 : 9]**

(c) Describe a single matrix that maps triangle ABC onto A''B''C''

[Ans: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ A matrix of enlargement at the centre (0, 0) with scale factor 3]

30. (a) The vertices of a triangle RPQ are (1, 3) (1, 4) and (3, 3) respectively. Find the coordinates of the image triangle R₁P₁Q₁ under the reflection in the line $x - y = 0$.

[Ans: R₁(3, 1) P₁(4, 1) Q₁(3, 3)]

(b) The image R₁P₁Q₁ is then mapped onto R₂P₂Q₂ by a positive $\frac{3}{4}$ turn about the origin. Find the coordinates of triangle R₂P₂Q₂.
[Ans: R₂(1, -3) P₂(1, -4) Q₂(3, -3)]

Transformation

39. (a) Using a scale of 1 cm : 1 unit, plot a triangle PQR where P(-1, 2), Q(4, 2), R(1, 7) on a graph paper.
- (b) Reflect triangle PQR using $y = -x$ as the reflection mirror to form P'Q'R', starting clearly the co-ordinates of P'Q'R'.
- (c) Rotate P'Q'R' using (-1, 1) as the centre and angle of 80° to form P"Q"R". State the co-ordinates of P"Q"R".
40. (a) Find the image of point (2, 4) after a reflection in the line $y = x - 4$ [Ans: (8, -2)]
- (b) Triangle A(1, 2), B(2, 5), C(3, 3) is mapped onto A'(3, 0), B'(6, 1) and C'(4, 2) by a reflection. Draw both triangles on the same graph and find the equation of the mirror line. [Ans: $y = x - 1$]
41. The end points of line AB whose co-ordinates are A(3, -1) and B(4, -3) undergo a rotational transformation to give the image line A'B' with A'(1, 3) and B'(3, 4) respectively.
- (a) Plot the line AB and its image on the same set of axes using a scale of 2 cm to 1 unit.
- (b) Determine the centre and angle of rotation of the line.
42. (a) What are the co-ordinates of the image of A(-1, 2) after enlargement, centre (0, 0) and scale factor -2?
- (b) Under an enlargement, centre (0, 3) and scale factor 2, a point P(2, 5) is mapped onto the point P'(a, b). State the values of a and b.
43. Draw triangle ABC such that A(2, 1), B(1, 1) and C(1, 3) are its vertices. Draw its reflection in the line $x = -2$ and state the co-ordinates of the image points
44. A point P(2, 3) is reflected in the x-axis to a point P₁. P₁ is reflected in the line $x = y$ to a point P₂.
45. (a) Find the equation of the line passing through A(-3, 3), B(-2, 2).
- (b) A point P(5, 8) is reflected in the above line. What are the co-ordinates of the image point P'?
46. Triangle ABC with vertices, A(2, 1) B(4, 4) and C(2, 4) is reflected in the line $y = 0$ to get triangle A'B'C'. Triangle A'B'C' is then given a negative quarter turn about the origin to get triangle A"B"C".
- (a)(i) Draw the three triangles on the same graph paper.
- (ii) Write down the co-ordinates of A'B'C' and A"B"C". [Ans: A'(4, -4) B'(2, -1) C'(2, -4) and A"(-1, -2) B"(-4, -4) C"(-4, -2)]
- (b) Use your graph to describe fully the transformation which maps A"B"C" onto ABC.
47. A triangle with vertices A(-1, 1), B(-1, 3) and C(-2, 3) is mapped onto triangle A'B'C' by translation $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Triangle A'B'C' is then rotated through 90° about the point (3, 2) to triangle A"B"C".
- (a) Plot the points A, B and C on a graph paper. Join the points to form triangle ABC.
- (b) Find the co-ordinates of:
- (i) A', B' and C' [Ans: A'(1, 2), B'(1, 4), C'(0, 4)]
- (ii) A", B", and C" [Ans: A"(3, 0), B"(1, 0), C"(1, -1)]
- (c) Find the centre and angle of the rotation which maps A"B"C" back onto ABC. [Ans: centre (1.5, 2.5), Angle (90°)]
48. Line $y + x = 2$ is reflected in the y-axis. Find the equation of the image of this line after this transformation.

KINEMATICS

RECALL

Remember the following key issues before attempting any question

- ◊ All drawings in kinematics must be drawn on a graph paper
- ◊ Be in position to convert from one metric unit to another. Eg km to m or cm etc.
- ◊ Neatness is a must requirement that earns you a bonus mark.
- ◊ You must know how to calculate speed, distance and time.
- ◊ The formula for distance = speed \times time = $S \times T$. This formula has to be mastered.
- ◊ Read the question at least two times before starting to attempt any kinematics numbers.

TRIAL QUESTIONS

1. Sylvester left UMSS at Noon and cycled towards Jinja 100 km away at a steady speed of 16 kmh^{-1} . After cycling for 180 minutes he rested for one hour at Lugazi. Then continued his journey with the same speed up to Jinja. Forty five minutes after the departure of Sylvester, Michael also started walking from UMSS towards Jinja at 6 kmh^{-1} for 1 hour and reached Kireka. He waited for 45 minutes at Kireka and later boarded a bus travelling at governed speed 32 kmh^{-1} . It travelled for one hour and stopped at Namawojolo for another one hour; then resumed its journey at 42 kmh^{-1} non stop up to Jinja.
 - (a) Using a scale of 2 cm: 1 hour and 2 cm: 10 km draw their distance time graphs on the same axes.
 - (b).From your graph determine;
 - (i).the time and distance from UMSS the bus overtook the cyclist
 - (ii).the distance between Namawojolo and Lugazi.
 - (iii).How long did Michael wait before Sylvester joined him at Jinja sailing club.
2. Towns A and B are 300 km apart by road. At 7.00 a.m. Ojok leaves town A for town B driving

a pick-up at a steady speed of 80 kmh^{-1} . After driving for 2 hours, the pick-up breaks down and he is forced to stop for half an hour after which he continues towards B at a reduced speed of 60 kmh^{-1} up to town B. One and a half hours after Ojok left town B, Kase leaves town B for town A driving at a constant speed non-stop all the way to A. Using a scale of 1 cm to 1 hour on the horizontal axis and 1 cm to 50 km on the vertical axis, draw the graph of both men's journeys on the same pair of axes, given that both reached their respective destinations at exactly the same time. Use your graph to find

- (a) Kase's speed in kmh^{-1}
 - (b) When they met and how far this was from town A
 - (c).Their time of arrival.
3. Gulu is 360 km away from Kampala. At 7:30 a.m. a taxi mini-bus leaves Gulu for Kampala travelling at a steady non stop speed of 50 kmh^{-1} . Two hours later a Gateway bus travelling at a steady non stop speed of 80 kmh^{-1} left Kampala for Gulu.
 - (a) Using a scale of 2 cm to represent 1 hour and 2 cm to represent 50 km, draw on the same axes graphs showing the journeys of the two vehicles
 - (b).Using your graphs determine the:
 - (i).distance from Gulu where the two vehicles met
 - (ii).time when the two vehicles met
 - (iii).times when the vehicles arrive at their stations
 - (iv).difference in the times of arrivals of the vehicles at their respective stations.

4. A taxi started its journey from Kampala at 0620 hours traveling at constant speed of 80 km/hr to Masindi 200 km away. After traveling for $1\frac{1}{2}$ hours it broke down, and put right for 1 hour then resumed its journey using an average speed of 30 kmh^{-1} to Masindi. At 0720 hrs the express bus left Masindi for Kampala traveling at a constant non stop speed of 50 kmh^{-1} . using a

- scale of 2 cm : 20 km 2 cm : 1 hr . Draw a distance time graph for the two vehicles on the same axes. From your graph find
- when and where from Masindi the bus meet the taxi
 - the difference in the time of arrival of the two vehicles
 - the average speed of taxi for the whole journey.
5. Arua town is 540 km from Kampala city. A pajero left Kampala for Arua at $7:00\text{ am}$ traveling at a steady speed of 100 kmh^{-1} . After traveling for 2 hours, the pajero had a mechanical breakdown which took exactly 2 hours to repair after which it continued non-stop to Arua at a steady speed of 110 kmh^{-1} . One hour after the pajero left Kampala, a bus left Arua for Kampala traveling at a steady speed of 80 kmh^{-1} but had a stop of 30 minutes at Karuma which is 240 km from Arua town. The bus then continued non-stop to Kampala at a steady speed of 20 kmh^{-1} more than its original speed.
- Using a scale of 2 cm to represent 1 hour on the horizontal axis and 2 cm to represent 50 km on the vertical axis, draw on the same axes a distance-time graph for the two vehicles.
 - Using your graphs, determine the;
 - time at which the two vehicles met.
 - distance form Kampala at which the two vehicles met.
 - time of arrival of the pajero in Arua town.
 - time of arrival of the bus in Kampala city.
 - difference in the times of arrival of the two vehicles at their respective stations.
6. Mbale and Kampala are 165 km apart. A bus leaves Mbale at $8:00\text{ am}$ traveling at 40 km/hr . 45 minutes later it breaks down and stops for an hour. It resumes its journey and reaches Kampala at $11:30\text{ am}$. A tax leaves Kampala at $9:30\text{ am}$ and travels at a steady speed of 60 km/hr to Mbale.
- On the same pair of axes draw both journeys described above.
 - When did the taxi reach Mbale?
 - What was the speed of the bus on the final leg?
7. Soroti is 340 km from Kampala. At $7:00\text{ am}$ a car traveling at a steady non-stop speed of 60 kmh^{-1} leaves Kampala for Soroti. One hour and 20 minutes later a bus leaves Kampala for Soroti traveling at a constant speed; After an hour of traveling the bus gets a puncture at a place 100 km from Kampala. It takes 40 minutes to fix the tyre and then resumes traveling at a steady non-stop speed so that it reaches Soroti at mid-day.
- Using scales of 2 cm to represent 40 km on the vertical axis and 3 cm to represent 1 hour on the horizontal axis, draw on the same axes graphs of the journeys of the two vehicles.
 - Using your graphs, estimate the;
 - time at which the bus overtakes the car
 - distance from Soroti where the bus overtakes the car
 - time when the car arrives in Soroti
 - time taken by the bus to travel from the point of fixing the tyres to Soroti
 - time taken by the bus driver to wait for the driver of the car in Soroti.
8. Towns A and B are 45 km apart. James starts from A at $8:00\text{ am}$ and travels towards B at 20 km/hr . When he is 15 km from A, he changes speed so that he arrives at B at $10:30\text{ am}$. David leaves town B at $8:30\text{ am}$ and travels to A at a speed of 28.4 km/hr . Taking 2 cm to represent 5 km on the vertical axis and 1 cm on to represent 15 minutes on the horizontal axis, draw graphs on the same axes to find,
- the time James and David met
 - the speed of James in the last part of the journey
 - the time when David reaches A.
9. A car travelling at a steady speed of 50 km/hr leaves town A at $8:00\text{ am}$ for town B, 180 km away. Two hours later it breaks down. It is repaired for $1\frac{1}{2}$ hours before resuming the journey at the same speed. At $10:00\text{ am}$ a bus leaves town B for town A non-stop at a steady speed of 40 km/hr .
- Using a scale of 2 cm representing 1 hour on the time axis and 2 cm representing 20 km

- on the distance axis, draw distance – time graphs of the two vehicles on the same axes.
- (b) Use your graphs to determine;
- When and where from town B the two vehicles bypass each other.
 - The times of arrival of the two vehicles at their respective destinations.
10. Charles moved from town A to town B, 31.5 km away at a speed of 10.5 kmh^{-1} . He covered the remaining distance to town C at a speed of 6 kmh^{-1} . If the total time taken was $5\frac{1}{4}\text{ hours}$,
- find the distance between B and C.
 - the average speed for the journey
11. Peter starts cycling from his home at exactly 8 am , at a steady sped of 16 kmh^{-1} to attend a meeting at the Gombolola headquarters, 20 km away. At the same time, Opela is sent from the Gombolola headquarters and comes at a steady speed of 6 kmhr^{-1} to inform Peter that the meeting was postponed. Unfortunately, at exactly $8:30\text{ am}$, Peter gets a puncture and works on it for 20 minutes before proceeding his journey, cycling carefully at 8 kmh^{-1} . Using a scale of 1cm to 1 km on vertical axis and 1 cm to 10 minutes on horizontal axis.
- draw the distance – time graph on the same axes for both Peter and Opela.
 - Use your graphs to determine;
 - the time Peter met Opela
 - the time when they were 3 km apart
 - the distance from the point where they met to the Gombolola headquarters.
12. Two towns, S and T, are connected by a road. The distance between S and T is 11 km . John leaves town S at $10:00$ hours and cycles along this road at a constant speed of 12 km/h . After 30 minutes he stops and rests for 10 minutes. He then continues his journey to T at a constant speed, arriving at $11:00$ hours.
- (i) Draw the distance – time graph for John's journey, using a scale of $2\text{ cm} : 10\text{ minutes}$ for the horizontal axis and $2\text{ cm} : 1\text{ unit}$ for the vertical axis.
 - At what speed does John cycle after the rest.
- Michael leaves town T at $10:00$ hours. He walks along the same road towards S for one hour at a constant speed of 3 km/hr .
- On the same graph, draw the distance-time graph for Michael's journey.
 - Write down the time at which John and Michael meet and the distance from town T.
13. A man leaves his home at 8.00 a.m cycling to a market 21 km away at a speed of 20 km/h . At the same time his wife leaves the market; riding back home at a speed of 8 km/hr .
- Calculate (i) how far from home they will meet each other.
 - at what time they will meet.
 - If the man's bicycle develops a fault immediately after meeting his wife hence forcing him to reduce the speed to 8 km/hr , at what time will he reach the market?
14. A lorry set off from Tororo at $07:30$ hours at a steady speed of 40 km/hr to Kampala, a distance of 180 km away. After travelling for 2 hours it stopped and rested for $1\frac{1}{2}\text{ hours}$, then continued at a steady speed of 50 km/hr for the rest of its journey. A car also set off from Kampala to Tororo at the same time as the lorry at a steady speed of 60 km/hr but suddenly reduced its speed after 2 hours to 15 km/hr due to some mechanical fault for the remaining journey. Using scales of 1 cm to 10 km and 1 cm to 30 minutes on the vertical and horizontal axes respectively;
- Draw distance time graphs showing the routes of the two vehicles
 - Using your graphs determine the
 - (i) distance between the two vehicles after 2 hours
 - difference in time of arrival at respective.
15. A car leaves town A at $9:30\text{ a.m}$ for town B, 100 km away at a steady speed of 60 km/hr . After reaching town B the car rests for 20 minutes before resuming its journey back to town A at a constant speed thus reaching A at $12:40\text{ p.m}$.
- Using a scale of $2\text{ cm}:30\text{ minutes}$ on the x-axis and $2\text{ cm}: 10\text{ km}$ on the y-axis, draw the graph of this motion.
 - Use your graph to determine
 - the time at which the car reached town B.

Kinematics

- (ii) the speed at which the car travelled back to town A.
16. Towns A and B are 300 km apart. At 9:30 a.m., Mukasa was 60 km away from town A moving towards town B on a Mate motor bike, when Byaruhanga set off from A on a boxer moving non-stop at 50 km/hr towards B. At 2:42 p.m., Byaruhanga overtook Mukasa and they continued with their journey. By using calculation, determine;
- the distance from A when Byaruhanga overtook Mukasa.
 - Mukasa's average speed (to 1 decimal point).
 - Times when Byaruhanga and Mukasa arrived at B.
 - Difference in times of arrival
17. The distance between Nakasongola and Gulu is 130 km. car x starts from Nakasongola at 8.00 am and travels at 40 km/hr towards Gulu. A second car y starts travelling from Gulu at 8.15 am towards Nakasongola at an average speed of 35 km/. At what time will the two cars meet?.
18. A car leaves town A at 9.30 am for town B, 100 km away, at a steady speed of 60 km/hr. After reaching town B, the car rests for 20 minutes, before resuming its journey back to town A at constant speed thus reaching A at 12.40 p.m
- Using a scale of 3 cm : 30 min on the time -axis and 2 cm: 10 km on the Distance-axis; draw the graph for this motion.
 - Use your graph to determine;
 - the speed at which the car travelled back to A
 - the time at which the car reached town B
19. A bus sets off from town A, at 7:30 am at a steady speed of 60 km/hr to town B, 240 km away. After travelling for 90 minutes it stopped at a service centre for 30 minutes after which it increased its speed by 15 km/hr until its final destination. 30 minutes later after the departure of the bus, a taxi sets off from town B towards town A at a steady speed and arrives 15 minutes before the bus.
- Using scales of 2 cm to represent 30 minutes and 2 cm to represent 30 km, draw on the same axes distance – time graphs showing the journeys for the two vehicles.
 - Using your graph(s) in (a) above, find the time;
- (i) when the vehicles by pass each other
(ii) taken by the taxi to cover the journey
(c) Calculate the average speed of the journey.
20. The distance from Jinja to Kampala is 73 km, Kampala to Gayaza is 17 km and Mwiri to Jinja is 20 km. This term, S4 students from Gayaza went for a geography field work to Mwiri. The bus left Mwiri to Gayaza at 5:00 pm and travelled at an average speed of 60 km/hr. It stopped at Namawojolo for 15 minutes so that students can buy roasted chicken. Namawojolo is about 40km from Jinja. It then proceeded non-stop to Gayaza at an average speed of 40 km/hr because of traffic jam. At 5:30 pm a teacher left Gayaza high school to Jinja in his car at an average speed of 80 km/hr non-stop. Assume the teacher and the school bus used the same route.
- on the same axes draw graphs showing the students and the teacher's journey.
 - Use your graph to determine the following
 - the time the bus arrived in Gayaza.
 - the time and distance from Kampala when the teacher met the students.
 - the difference in times of arrival of the teacher and the students. [use a scale of 1 cm for 10 km and 4 cm to represent 1 hour]
21. A cyclist P leaves town B at 1.06 pm for village A riding non-stop at a steady speed of 15 km/hr and arrives in village A at 3.06 pm. Another cyclist Q leaves village A at noon for town B. From town A cyclist Q rides at a steady speed of 20 km/hr for 45 minutes. He then rests for 30 minutes and then continues with a steady speed of 15 km/hr and reaches town B at 2.15 pm.
- Represent the motion of cyclists P and Q on a distance time graph. (Use a scale of 1 cm : 15 minutes on the x-axis, 2 cm : 5 km on the y-axis)
 - Use your graph to find;
 - when did the two cyclists pass each other and how far from B were they at this time?
 - how far apart were the two cyclists at 2:00 pm?

22. At 7:00 am a cyclist left town A for town B 120 km away, at an average speed of 20 km/hr. after cycling for 3 hours he rested for an hour. He continued to town B at the same speed. At 7:30 am, a motorist left town B for town A at an average speed of 60 km/hr. He then stayed in town A 1 hour before returning to town B. the return journey to town B took him $2\frac{1}{4}$ hours.
- (a) On the same axes, using a scale of 1 cm to represent 10 km and 2 cm to represent 1 hour, draw a distance time graph for cyclist and motorist.
- (b) From your graph, determine;
- the times and distance from town A the motorist by passed the cyclist on his way to and from town A.
 - how long had the motorist been in town B before cyclist arrived
23. The distance between A and B is 432 km. A lorry travelling at steady speed of 72 km/hr leaves A at 6:45 am for town B. $1\frac{1}{2}$ hours later, a minibus leaves from A at a steady non – stop speed of 108 km/hr heading from town B.
- (a) On the same axes, show the journeys of the two vehicles. (**use scales of 2 cm to represent 40 km and 2 cm to represent 1 hour**)
- (b) use your graph to estimate the;
- time and distance from A when the min-bus overtakes the lorry.
 - the times when the two vehicles arrive in town B.
- (iii) the difference in time of arrival of the two vehicles.
24. The distance from Gulu to Kampala is about 380km. otada bus leaves Gulu at 7:30 am and travels nonstop to Kampala at 60 km/hr. at 8:50 am the headteacher of Gulu college leaves Kampala in his car and travels towards Gulu at steady speed of 120 km/hr.
- (a) Using the same axws, draw a distance-time graph showing the journeys of both vehicles, hence find when and at what distance from Kampala they meet.
- (b) If the bus increases it's speed by 10 km/hr,
- Calculate the time at which the bus arrived in Kampala.
 - Determine the difference in times of arrival of the vehicles. (**use 2 cm : 50 km and 2 cm:1 hr**)
25. A bus travels from a town P to another town Q and then back to P. For the outward journey the speed is 5 km/h faster than the speed for the return journey and the return journey takes half an hour longer. If the towns are 120 km apart and km/h is the return speed;
- Show that $x^2 + 5x - 1200 = 0$
 - Find the value of to 3 significant figures.
 - Use the value of in (ii) to find the time taken to reach Q from P to the nearest minute.

VECTORS

RECALL

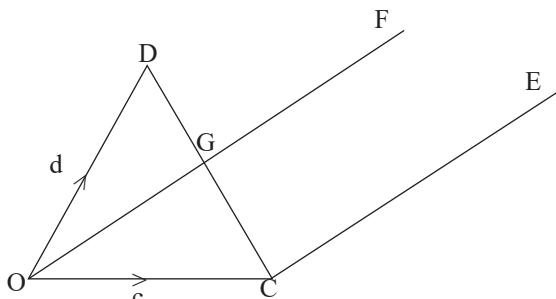
- ◊ A vector can be represented as a column vector, $\begin{pmatrix} x \\ y \end{pmatrix}$ where x denotes the horizontal movement and y denotes vertical movement.
- ◊ To add or subtract vectors, we add or subtract corresponding components.
For example;

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1+3 \\ -1+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$
- ◊ To multiply a vector by a scalar, multiply each component by a scalar. For example;

$$2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \times 2 \\ 2 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$
- ◊ Length of a vector is called a magnitude and is calculated as: $|p| = \sqrt{x^2 + y^2}$
- ◊ Reverse direction in vectors is denoted with a negative i.e $OA = -AO$
- ◊ If one vector is a scalar multiple of the other, then the two vectors are parallel ie $\mathbf{PQ} = \gamma \mathbf{DR}$
- ◊ Collinear points are points that lie on a straight line. They have a common point as parallel.
- ◊ Consider points A, B and C are on a straight line, then $\frac{AB}{BC} = \mu$ or $AB = \mu BC$ where μ is a scalar

TRIAL QUESTIONS

1. Two points P(5, 2) and Q (2, 4) are in a plane. Find the
 - (a) Co-ordinates of M the midpoint of PQ
 - (b) $|OM|$, where O is the origin
2. (a) If $AB = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $OB = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, find the coordinates of point A.
 (b) The coordinates of R and S are (2, 7) and (14, 2) respectively. Find:
 (i) RS (ii) the magnitude of RS.
3. The position vectors of points R and T are $r = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $t = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ respectively. Find the magnitude of RT.
4. The co-ordinates of A and B are (-6, 15) and (4, 5) respectively, x is a point on AB such that $AX:XB = 1:4$.
 Find (i) AB (ii) OX
5. Given that QεPR such that $PQ:QR = 3 : 2$, find the co-ordinates of Q if P is (2,5) and R is (7, -5)
- 6.

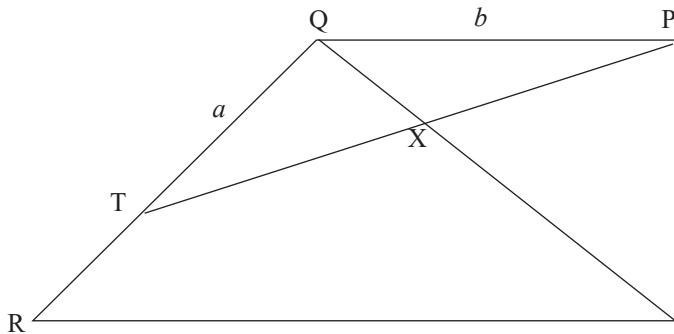


In the diagram above $Oc = c$, $OD = d$, $7DG = 4DC$ and $OG:GF = 5:2$

- (a) Find in terms of c and d
 - (i) DG
 - (ii) OG
 - (iii) OF
- (b) If $OE = \frac{1}{5}(12c - d)$ show that D, F and E are collinear.

7. Given that $OQ = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$ and $OP = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$
 - (a) Determine the column vector for PQ
 - (b) Hence find the length of vector PQ

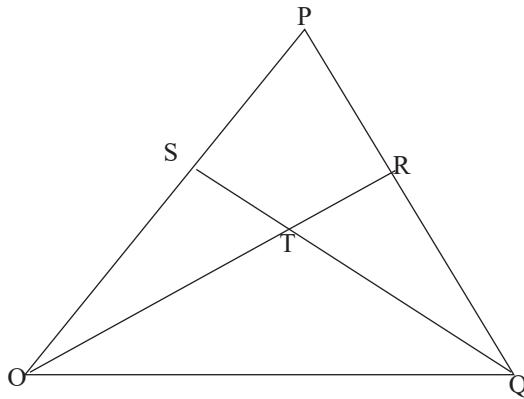
8.



In the figure above vectors $QT = a$, $QP = b$, $QR = 3a$, and $RS = 2b$.

- Express the following vectors in terms of a and b .
 - \vec{PT}
 - \vec{QS}
- If $PX = kPT$, express \vec{QX} in terms of a , b and k where k is a scalar.
- If $\vec{QX} = t\vec{QS}$, find the values of k and t

9.



In the figure OPQ is a triangle in which $OS = \frac{3}{4} OP$ and $PR : RQ = 2 : 1$. Lines OR and SQ meet at T .

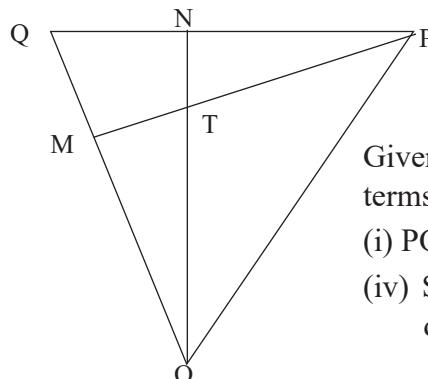
- Given that $OP = p$ and $OQ = q$, express the following vectors in terms of p and q ;
 - \vec{PQ}
 - \vec{OR}
 - \vec{SQ}
- If $ST = mSQ$ and $OT = nOR$, determine the values of m and n .

10. (a) Given that $a = \begin{pmatrix} -2 \\ -9 \end{pmatrix}$, $b = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ and $m = a + 2b$ find the magnitude of m .

(b) Given that $S(-2, 6)$ and $T(3, 3)$ are two points, find the coordinates of R if $\vec{OR} = 4\vec{OS} + \frac{1}{3}\vec{OT}$ and O is the origin.

11. (a) Given that $\vec{OA} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate the length of AB .

(b) The diagram shows triangle OPQ in which $QN : NP = 1 : 2$, $OT : TN = 3 : 2$ and M is the mid point of \vec{OQ} .



Given that $OP = p$ and $OQ = q$ express the following vectors in terms of p and q .

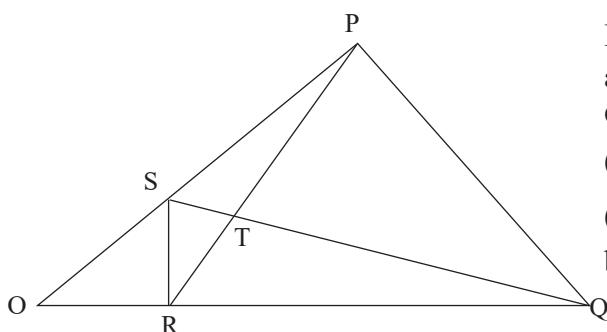
- \vec{PQ}
- \vec{ON}
- \vec{PT}
- Show that the point P , T and M are collinear and hence determine the ratio $MT : TP$.

12. (a) Given that $a = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Calculate; $|a + b|$.

(b) Given that $a = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, $b = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $c = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$, find the magnitude of $a + 2b + 3c$.

(c) Given that $a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ and $c = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Evaluate $|2a - 2b + \frac{3}{8}c|$

13.



In the above figure OPQ in a triangle in which $OS = \frac{1}{2} OP$ and $OR = \frac{1}{3} OQ$. T is a point on QS such that $QT = \frac{3}{4} QS$. Given that $OP = p$ and $OQ = q$.

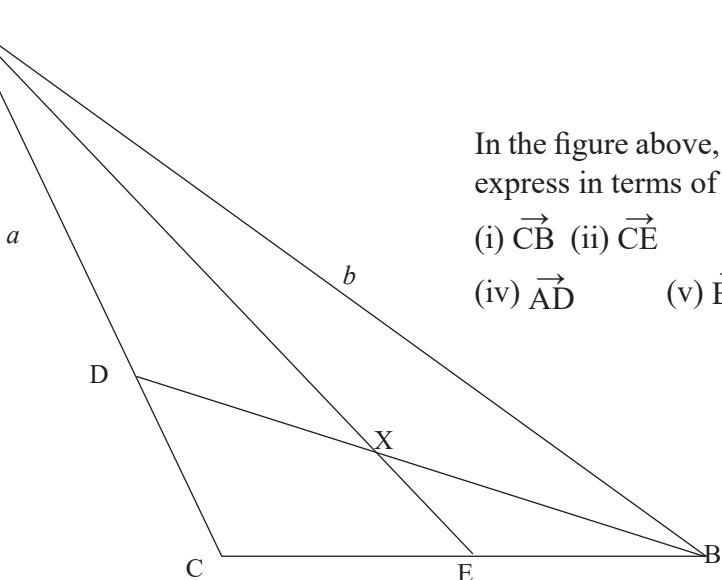
- Express the following vectors in terms of p and q
 - \vec{SR}
 - \vec{QS}
 - \vec{PT}
 - \vec{TR}
- Find the ratio of PR to TR .

14. (a) Given the vectors $a = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and $c = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$, find the values of the constants p and q such that $c = pa + qb$ without using tables or calculator.

(b) M is the point (-6, 8). If $OM = a - 2b$ and $a = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, find;

(i) the value of vector b (ii) the magnitude of b

15.



In the figure above, $\vec{AB} = b$, $\vec{AC} = c$, $2AD = 3DC$, $3CE = CB$, express in terms of b and c .

- (i) \vec{CB} (ii) \vec{CE} (iii) \vec{AE}
 (iv) \vec{AD} (v) \vec{BD} (vi) \vec{DE}

16. Given that $\vec{OL} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\vec{OM} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$ are position vectors of points L and M respectively. Find the length of \vec{LM} .

The diagram above shows a trapezium OABC. $\vec{OA} = a$, $\vec{OC} = c$ and $\vec{CB} = 3a$. X and Y are points on \vec{AC} such that $AX : XC = 1 : 2$ and $AY : YC = 1 : 3$.

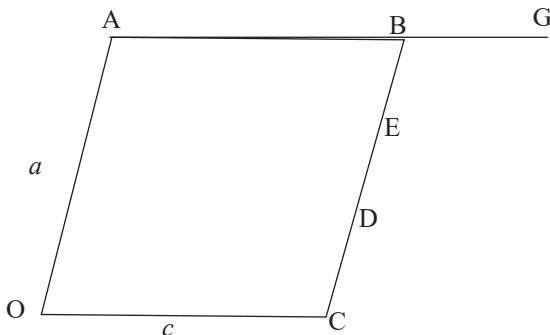
- (a) Express the following vectors in terms of a and c .

 - (i) \vec{AC}
 - (ii) \vec{OY}
 - (iii) \vec{OX}
 - (iv) \vec{AB}
 - (v) \vec{AY}

(b) Hence, show that O, Y and B are collinear.

(c) OX is produced and cuts CB at Z. Find the ratio $OX : XZ$

18. In the diagram below, OABC is a parallelogram with $OA = a$, $OC = c$, $CD = DE$, $DE : CB = 1:3$ and $AB : BG = 2 : 1$



- (a) Find in terms of a and c the vectors \overrightarrow{DA} and \overrightarrow{DG} .
 - (b) Show that the points O , E and G are collinear.
 - (c) Find the ratio $OE : OG$.

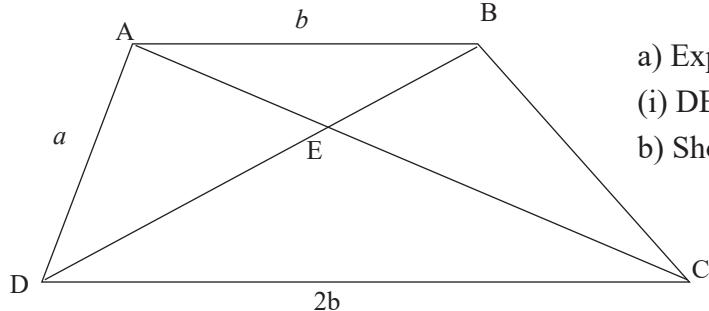
19. In a triangle ABC, L and M are points on AB and BC respectively such that $AL = \frac{2}{3}AB$ and $BM = \frac{1}{3}BC$. X is a point on AM such that $AX = \frac{3}{4}AM$. Given that $AB = b$ and $AC = c$, express the following vectors in terms of b and c .

- (i) AM (ii) CL (iii) CX

(b) Show that C, X and L are collinear points.

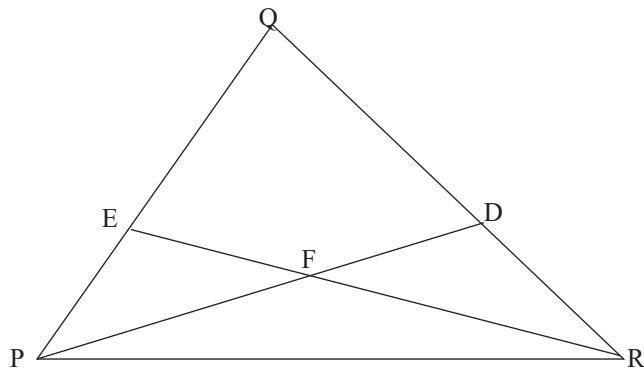
(c) State the ratio CL: CX.

20. The diagram below shows a quadrilateral ABCD with $DE = 2EB$, $\vec{AB} = b$, $\vec{DC} = 2b$ and $\vec{DA} = a$.



- a) Express in terms of a and b the vectors;
- (i) \vec{DB} (ii) \vec{BE} (iii) \vec{AE} (iv) \vec{AC}
- b) Show that the points C, E and A are collinear.

21. In triangle PQR, $PR = p$ and $PQ = q$. E divides PQ in the ratio 1: 3 and D divides QR in the ratio 5 : 2. DP meets RE at F.



- (a) Express the following vectors in terms of p and q .
- (i) \vec{PD} (ii) \vec{RE}
- (b) If $\vec{PF} = h\vec{PD}$ and $\vec{RF} = k\vec{PE}$ where h and k are scalars, by expressing in two different ways find the values of h and k .
- (c) Hence find the ratio $EF : FR$.

22. (a) Given the vectors $a = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $c = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$, find the magnitude of $3a - 4b + \frac{1}{2}c$.

- In the figure above, vectors $QT = a$ and $QP = b$.
- (i) Express vector \vec{PT} in terms of a and b .
 - (ii) If $\vec{PX} = k\vec{PT}$, express \vec{QX} in terms of a , b and k where k is a scalar.
 - (iii) If $\vec{QR} = 3a$ and $\vec{RS} = 2b$, express \vec{QS} in terms of a and b .
 - iv) If $\vec{QX} = h\vec{QS}$, find the values of k and h .

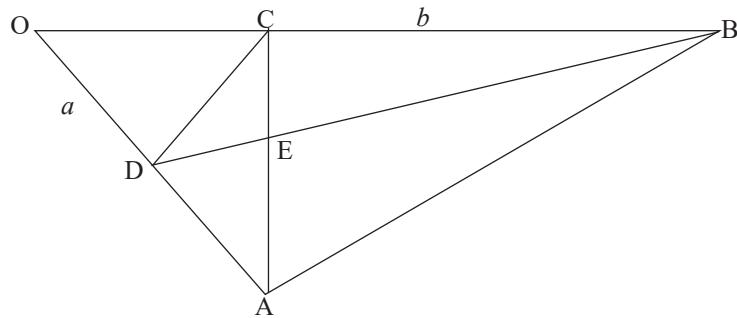
23. Given that; $\vec{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $\vec{AB} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$; find; (i) \vec{OB} (ii) $|\vec{OB}|$

24. OABC is a quadrilateral. P, Q, R are points on OA, OB, OC respectively such that $\vec{OA} = 3\vec{OP}$, $\vec{OB} = 5\vec{OQ}$ and $\vec{OC} = 2\vec{OR}$.

- (a) Given that $OP = p$, $OQ = q$ and $OR = r$, Find in terms of p , q and r , the vectors:
- (i) \vec{AB} (ii) \vec{BC} (iii) \vec{CA}
- b) If OABC is a parallelogram, show that;
- $3p - 5q + 2r$ hence express \vec{PQ} and \vec{QR} in terms of p and q only.
- (c) Show that P, Q and R are collinear
- (d) Find the ratio $PQ : QR$

25. In the figure below, OAB is a triangle such that $3OD = OA$, $3OC = OB$. E is a point on BD such that $4BE = 3BD$.

Vectors



(a) Given that $OA = a$, $OB = b$, express the following vectors in terms of a and b .

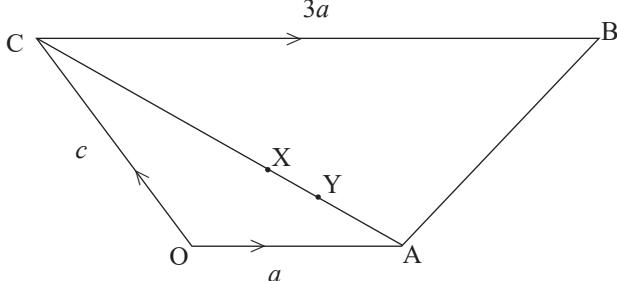
- (i) DC (ii) BD (iii) AE (iv) EC

(b) Show that the points A, E and C are in a straight line.

26. The diagram below shows a trapezium OABC. $OA = a$, $OC = c$ and $CB = 3a$. X and Y are points on AC such that $AX : XC = 1 : 2$ and $AY : YC = 1 : 3$

Give the following vectors in terms of a and c

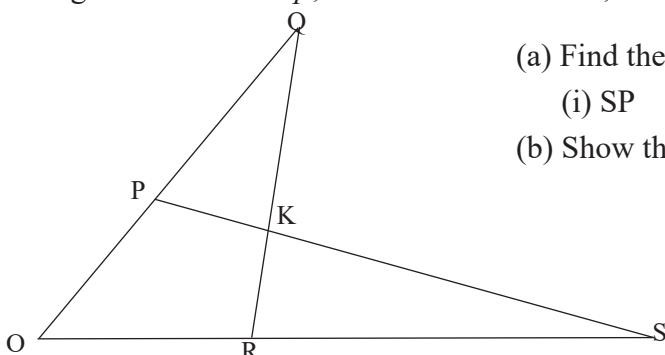
- (i) AC (ii) OY (iii) OX (iv) AB (v) AY



Hence, show that O, Y and B are collinear.

OX is produced and cuts CB at Z . Find the ratio $OX : XZ$

27. In the figure below $OP = p$; $OR = r$. $OR:RS = 1:1$, $OP:OQ = 2:3$ and $2QK = QR$

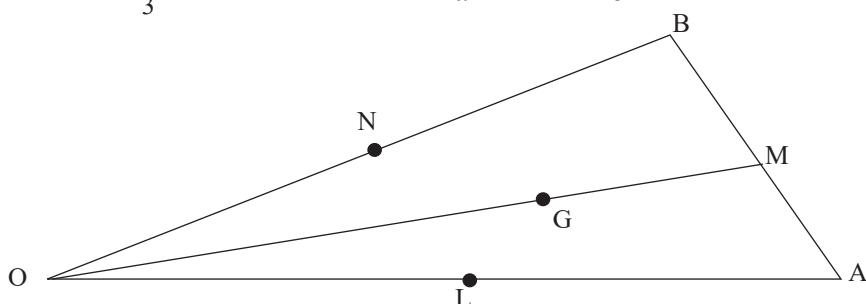


(a) Find the following vectors in terms of p and r .

- (i) SP (ii) OK (iii) PK

(b) Show that the points P, K and S are collinear.

28. In the figure below L, M and N are the mid points of OA, AB and OB respectively. G is a point on OM such that $OG = \frac{2}{3} OM$. Given that $OA = a$ and $OB = b$.

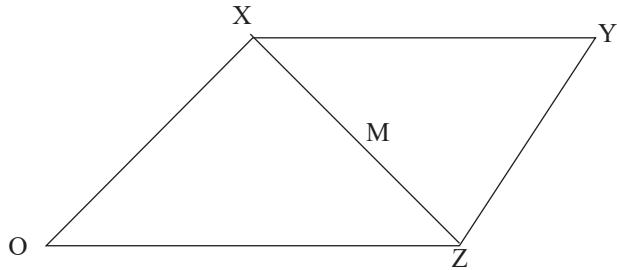


(a) Express in terms of a and b the vectors (i) OM (ii) OG (iii) BG (iv) GL

(b) Show that points B, G and L are collinear

29. Point A has coordinates $(3, 2)$ and $AB = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Find the magnitude of $2OA + OB$.

30. In the figure below, M is the mid point of XZ. $\vec{OX} = p + 2q$, $\vec{OZ} = 7p + 2q$ and $\vec{OY} = 3kq + np$ where k and n are constants and p and q are vectors.



(a) Express as simply as possible in terms of p and q each of the following vectors.

- (i) \vec{XZ} (ii) \vec{XM} (iii) \vec{OM}

(b) Express in terms of p, q, k and n as simply as possible.

- (c) If Y lies on OM produced and $OY:YM = 3:2$, find the values of k and n hence express \vec{OY} in terms of p and q only.

LINEAR PROGRAMMING AND INEQUALITY

RECALL

Graphs for inequalities

- ◊ If the inequality is \geq or \leq , i.e the points on the boundary line are included in the region representing the inequality. The boundary line is drawn as a continuous line, _____
- ◊ If the inequality is $>$ or $<$, the points on the boundary line are not included in the region representing the inequality. The boundary line is drawn as a broken line (.....)
- ◊ Find the unwanted region out of the two by choosing a test point on the side of the boundary line. The co-ordinates of the test point are substituted in the inequality a test is performed to find out whether the point satisfies the inequality.
- ◊ Shade the unwanted region.

Linear programming

- ◊ The following are some of the words and their corresponding symbols as used in linear programming.

Doesn't exceed \leq

Must transport \geq

Exceeds $>$

A maximum of \leq

Wishes to transport \geq

More than $>$

At least \geq

Contracted \geq

Less than $<$.

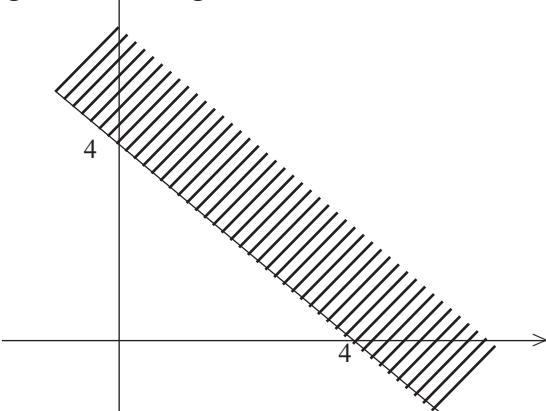
TRIAL QUESTIONS

1. By shading the unwanted region on the same axes show the region satisfying the inequalities; $y \leq 4x + 12$, $3y + x > -3$ and $x \leq 2$
2. By shading the unwanted regions show the region which satisfies the inequalities.

$$x + y \leq 3, \quad y \geq x - 4, \quad y + 7x > -4$$

Find the area of the wanted region

3. Find the inequality that represents the unshaded region in the diagram below.



4. (a) By shading the unwanted regions show on a graph using a scale of 2 cm: 1 unit on both axes the region R which satisfies all the 3 inequalities below:
 - (i) $3x + 5y \geq 15$
 - (ii) $x < y$
 - (iii) $y \leq 5$

(b) Study the graph below and use it to complete the inequalities below which satisfy the unshaded region R.

- 5. The manager of a cinema wishes to divide the seats available into two classes A and B. There are not more than 120 seats available. There must be at least twice as many B class as there are A class seats. Class A seats are priced at 1500 shs each and class B at 1000 shs each. At least 100,000/= should be collected at each show to meet the expenses. Taking x as the number

of class A seats sold and y as the number of class B seats sold, write down the inequalities representing the above information and plot them on a graph. From the graph;

- (a) (i) Find the number of seats of each kind which must be sold to give the maximum profit
 (ii) State the maximum profit.
- (b) Find the least number of seats that must be sold in order to incur no loss.
6. Solve the inequality and show the solution on a number line $-8 < \frac{3}{4}x - 2 \leq x - 3$
7. A certain school wishes to take its S4 students for a picnic at a national park. The school has hired a bus and a mini-bus to take the students. Each trip of the bus costs sh 80,000 and that of the mini-bus shs 50,000. The bus has a capacity of 57 students and the mini-bus 19 students. All the 171 students in S4 contributed a total of shs 400,000 for hiring the vehicles and must all go for the picnic. The mini-bus must make more trips than the bus because it travels faster. If x and y represent the number of trips made by the bus and mini-bus respectively;
 - (a) Write down five inequalities representing the above information
 - (b) Plot these inequalities on the same axes.
 - (c) By shading the unwanted region, show the region satisfying all the above inequalities
 - (d) Find the number of trips each vehicle should make so as to spend the least amount of money. Hence find the amount of money saved.
8. A manufacturer makes two types of drinks A and B. The following conditions apply to daily production.
 - (i) Each of type A costs shs 3,000 and each of type B costs shs 5,000 and the manufacturer has a maximum of shs 450,000 available.
 - (ii) Due to labour shortage, the production of type A plus four times that of B should not exceed 160.
 - (iii) A market study recommended that the number of type B produced should not exceed twice the number of type A produced.

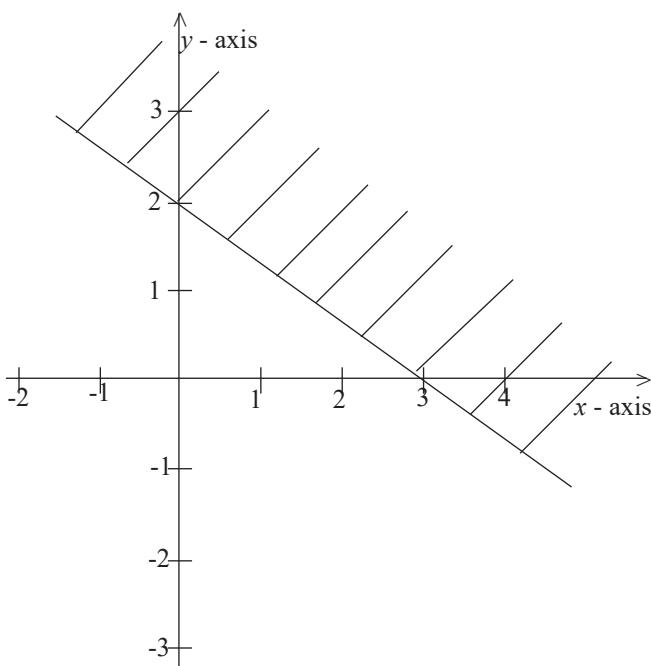
- (a) Given that x drinks of type A and y drinks of type B are made, write down three inequalities apart from $x \geq 0, y \geq 0$ satisfying the above conditions.
- (b) Show graphically the region containing the points satisfying the above conditions.
- (c) Taking $x + 2y$ as a suitable expression for the manufacturer's profit, find the number of each type of drink that should be made to obtain the greatest profit.
9. (a) On the same axes, show the region satisfying the following inequalities by shading out the unwanted regions; $y \leq 4, y + 1 \geq 0, x + y \leq 6$ and $x - y + 8 \geq 0$.
 (b) Calculate the area of the region so formed..
10. Given that $P = \{(x, y); 2x - 3y \leq 6\}$ and $Q = \{(x, y); x + y < 6\}$ show by shading the unwanted regions, the region representing $P \cap Q$
11. Karibu Hotel has 7 roasters of 200 kg oven capacity and 6 roasters of 400 kg oven capacity. The 200 kg oven capacity roaster can be used 5 times a day. The 400 kg oven capacity roaster can be used only 2 times a day. Each roaster must be operated by only one chef. On a given Saturday the Hotel is contracted to roast 9000 kg of meat for guests at a wedding ceremony. On that day, only 11 chefs were available. The 200 kg oven capacity roasters each needs shs, 12,000 per day to run and the 400 kg oven capacity roasters each needs shs. 20,000 per day to run. If x and y represent the number of 200 kg oven capacity roasters and 400 kg oven capacity roasters to be used respectively by the Hotel.
 - (a) Write down six inequalities representing the above information.
 - (b) Plot on the same axes, graphs for the inequalities, shading out the unwanted region.
 - (c) Use your graph to find the number of each type of roaster the Hotel should use so as to minimise costs.
12. The profits for two types of chicken, broilers x and layers y , produced by NICO poultry farm are shs 100,000 and shs 150,000 respectively. Broilers require 40 kg of starter meal and 25 kg of smash meal. Layers require 20 kg of starter

Linear programming and inequality

meal and 30 kg of smash meal. Given that there are 120 kg of starter meal and 150 kg of smash meal available.

- Use inequalities to represent this information
- Represent the inequalities above on a graph and use it to find the possible number of chicken of each type that can be kept.
- Calculate the maximum profit.

13. State the inequality that satisfies the unwanted region shown below.



14. Solve the inequality $3x - 2 < 10 + x < 2 + 5x$. Show the solution on a number line.

15. A workshop makes two types of chairs: Rockers and Swivels. Two operations A and B are used. Operation A is limited to 20 hrs a day. Operation B is limited to 15 hrs. a day. The table below shows the time each operation takes for a chair and the profit made.

Chair	Operation A	Operation B	Profit
Rocker	2 hrs	3 hrs	\$12
Swivel	4 hrs	1 hr	\$10

- Write down all the inequalities that satisfy these information.
- Represent these inequalities on a graph.
- Find the maximum profit.
- Find how many chairs of each type should be made to maximize the profit.

16. A school intends to transport 900 students to a field trip using a school bus and van. The bus can carry a maximum of 60 students while a van can carry a maximum of 45 students. The van has to make more than twice the number of trips made by the bus. The total number of trips has to be less than 20. The cost per trip is shs. 100,000 and shs. 80,000 per bus and a van respectively.

- Taking x and y to represent the number of trips by a bus and a van respectively. Write down five inequalities representing above information.
- Use a scale of 4 cm: 5 units on both axes. Graph these inequalities and shade out the unwanted regions.
- List down all the possible trips made by the vehicles.
- Determine the maximum expenditure incurred by the school

17. (a) Using a number line, illustrate the solution set for the inequality $-4 \leq x < 1$

- (b) Dot service wants to transport 400 workers to a working centre using x small vans of capacity 40 persons each and y large buses of capacity 80 persons each. The transport department has a maximum of 89 drivers on duty and the largest number of buses is 4.

- Write down inequalities representing the restrictions on the values of x and y .
- Plot on the same axes the inequalities, shading out the unwanted region.
- If the expenses of running each small van are shs.200,000 and each large bus are shs.300,000. Determine the maximum cost.

18. A farmer has x goats and y cows. The food cost for each goat is shs.800 and for each cow is shs.1600 per day. Only shs.14,400 is available for animal food. There is no room for more than 14 animals. There must be at least 9 goats and at least 3 cows.

- Write down all the inequalities that represent the above information.
- Represent the inequalities on a graph by shading the unwanted regions.

- (c) From your graph, find the number of goats and cows that must be kept so that the food cost is to be minimum.
19. A tailor has shs.180,000 to spend on materials for making shirts and trousers in a week. The material for a shirt cost him shs.12,000 and for a pair of trousers shs.15,000, the time taken to make a shirt is $2\frac{1}{2}$ hours and a pair of trousers 5 hours but he works at most 40 hours per week. He needs to make at least as many shirts as pairs of trousers. He makes a profit of shs.2000 on each shirt and shs.5000 on each pair of trousers. If x and y represent shirts and pair of trousers respectively,
- Write down all the inequalities for the above information
 - Plot a graph of the inequalities, shading out the unwanted regions.
 - How many of each should he make in a week to maximize his profit?
20. A shop keeper orders bags of rice. The cost price of a large bag is shs.180,000 and that of small of a small bag is shs.80,000. She is prepared to spend up to shs.4,000,000 altogether and needs twice as many large bags as small bags, with a minimum of 10 large and 20 small bags.
- What is the greatest number of bags she can buy?
21. A school has 70 hectares of land available for growing maize and beans. The cost per hectare for growing maize and beans is shs.90,000 and shs.60,000 respectively, and it has only shs.5,400,000 available. The labor per hectare is 2 man days for the maize and 4 man days for the beans, and a total of 240 man days of labor are available. If x and y represent the number of hectares to be used for maize and beans respectively, write down in their simplest form the inequalities which x and y must satisfy.
- Using a scale of 1 cm: 5 units on both axes, draw the appropriate straight lines to find the region in which the point (x, y) must lie if the inequalities are to be satisfied.
22. Kampala city council has an area of $40,000 \text{ m}^2$ set aside for a modern car taxi park. The average area for parking a car is 800 m^2 while the area for parking a minibus is 2000 m^2 . Not more than 30 vehicles can be accommodated. The parking charge for the car is shs.1500 and the charge for a minibus is shs.2500.
- How many of each should be parked for maximum income.
 - State the maximum income.