UCE MATHEMATICS PAPER 1 2013 guide

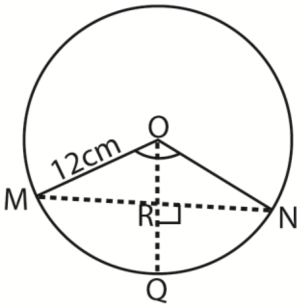
SECTION A (40 marks)

Answer all questions in this section

1. Factorize $a^2 - b^2$ completely. Hence evaluate $3.14^2 - 0.14^2$. (04marks)
2. Solve for x in the equation $2(x^2 + 4x + 4) = 5 + x$. (04marks)
3. Given that cos θ = $\frac{-12}{13}$ and$0^0 \leq \theta \leq\ 180^0$, find tanθ. (04 marks)
4. The following marks were obtained by 30 students in a mathematics test

45	41	41	52	48
58	39	38	26	59
70	61	46	39	65
29	49	63	69	60
40	42	64	41	46
41	32	49	56	49

   (a) Construct a frequency distribution table starting with the class 21- 30
   (b) State the modal class (04 marks)
5. Simplify:$3^{2x+y} x\ 3^{x-y} x\ 9^{-x}$. (04marks)
6. Solve$\frac{n-1}{2} - \frac{n-3}{4} = \frac{1}{2}$. (04marks)
7. If A = $\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 6 & -2 \\ 7 & 3 \end{pmatrix}$, calculate 2A – B. (04marks)
8. A translation $\binom{2}{3}$ followed by a translation $\binom{x}{y}$gives the translation$\binom{5}{-2}$. Find the values of x and y, (04marks)
9. In the diagram below, O is of the circle. R is the mid-point of NM and the angle MON = 150^0. Find the length of RQ. (04marks)



10. A number is chosen at random from integers 1 to 8. Find the probability that the number chosen is either a multiple of 4 or a prime number. (04marks)

SECTION B (60 MARKS)

Answer any ***five*** questions from this section. All questions carry equal marks

11. (a) (i) Make y the subject of $\frac{1}{y} + B = \frac{x}{2n}$.

    (ii) Find the value y when n = 10, x = 65 and B = 3 (06marks)

    (b) Betty is 9 years younger than David. John is three times as old as Betty. The sum of all their ages is 49 years. Find

    (i) John's age

    (ii) David's age (06marks)

12. (a) by shading the unwanted regions, show on a graph the region satisfying the inequalities below;

    $x \geq 0$

    $y \geq 0$

    $x + y \leq 5$

    $x + 3y \leq 9$ (08marks)

    (b) Use your graph to find the values of x and y which give the maximum values for both x + y and x + 3y. (04marks)

13. A ship leaves a port and sails for 120km on a bearing of 062^0. It then changes direction toa bearing of 160^0 and sails for 200km to an island. Using a scale drawing with 1cm representing 20km, find

    (a) the distance of the island from the port

    (b) the bearing of the port from the island

    (c) how long it would take the ship to sail directly back to the port at a speed of 20km/h. (12marks)

14. The table below shows the marks scored by 75 students in a test.

Marks	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44
Number of students	3	9	14	23	16	8	2

(a) Draw the given data on a histogram. (04marks)

(b) Use your histogram to estimate the modal mark. (04marks)

(c) Calculate the mean mark using a working mean of 27. (04marks)

15. (a) Draw a graph of $y = (x - 2)(x - 3)$ for the domain $0 \leq x \leq 5$. (06marks)

    Use 2cm to represent 1 unit on both axes

    (b) From your graph, find the values of x for which $x^2 - 5x + 6 = 0$

    (c) By drawing an appropriate line on the graph, find the values of x for which $x^2 - 5x + 2 = 0$ (04marks)

16. (a) given that P= $\begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}$, Q = $\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$ and R = PQ

    (i) Determine the order of R

    (ii) Find the matrix R (06marks)

    (b) Solve the following simultaneous equation using the matrix method

    $5x + 3y = 7$

    $2x - 4y = 3$ (06marks)

17. Triangle ABC with vertices A(1, 3), B(3, 3) and C (3, 1) is enlarged with a scale factor -4 about (2, 2) to triangle A'B'C'. Triangle A'B'C' is then rotated through a positive quarter turnabout (0, -4) to triangle A''B''C''.

(a) Draw on the same axes the triangles ABC, A'B'C' and A''B''C''. (10marks)

(b) Write the coordinates of

   (i) A', B', and C'

   (ii) A'', B'', and C'' (02marks)

Solutions

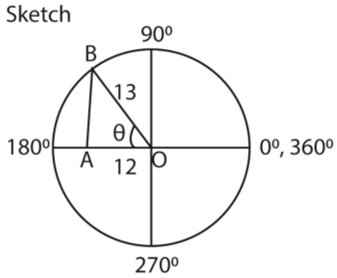
SECTION A (40 marks)

Answer all questions in this section

1. Factorize $a^2 - b^2$ completely. Hence evaluate $3.14^2 - 0.14^2$. (04marks)
   $a^2 - b^2 = (a+)(a-b)$
   Comparing $3.14^2 - 0.14^2$ with $a^2 - b^2$
   we have
   $3.14^2 - 0.14^2 = (3.14 + 0.14)(3.14 - 0.14)$
   $= 3.28 \times 3$
   $= 9.84$
2. Solve for x in the equation $2(x^2 + 4x + 4) = 5 + x$. (04marks)
   Method I; completing squares
   $2(x^2 + 4x + 4) = 5 + x$.
   $2x^2 + 8x + 8 = 5 + x$
   $2x^2 + 7x + 3 = 0$
   $(2x + 1)(x + 3) = 0$
   Either
   $2x + 1 = 0$
   $x = \frac{-1}{2}$
   Or
   $x + 3 = 0$
   $x = -3$
   Hence $x = \frac{-1}{2}$ and $x = -3$
   Method II using quadratic formula
   The root of a general quadratic equation of the form $ax^2 + bx + c = 0$; where a, b, c are real numbers are given by
   $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   Comparing $2x^2 + 7x + 3 = 0$ with $ax^2 + bx + c = 0$
   $X = \frac{-7\pm\sqrt{7^2-4(2)(3)}}{2(2)} = \frac{-7\pm\sqrt{49-24}}{4} = \frac{-7\pm\sqrt{25}}{4} = \frac{-7\pm5}{4}$
   Either
   $x = \frac{-2}{4} = \frac{-1}{2}$ Or $x = \frac{-12}{4} = -3$
   Hence $x = \frac{-1}{2}$ and $x = -3$

3. Given that $\cos\theta = \frac{-12}{13}$ and $0^0 \leq \theta \leq 180^0$, find $\tan\theta$. (04 marks)
   Cosine of an angle is negative in the second quadrant ($90^0 \leq \theta \leq 180^0$) and in the third quadrant ($180^0 \leq \theta \leq 270^0$)
   Since the question restricts the range of θ as $0^0 \leq \theta \leq 180^0$, the angle required is in the second quadrant.

Sketch



Note cosθ = $\frac{OA}{OB} = \frac{-12}{13}$

Using Pythagoras theory to find AB

$AB^2 = 13^2 - 12^2 = 169 - 144 = 25$

AB =$\sqrt{25}$ = 5

tanθ = $\frac{AB}{OA} = \frac{5}{-12} = \frac{-5}{12}$

4. The following marks were obtained by 30 students in a mathematics test

45	41	41	52	48
58	39	38	26	59
70	61	46	39	65
29	49	63	69	60
40	42	64	41	46
41	32	49	56	49

(c) Construct a frequency distribution table starting with the class 21- 30

Classes	Tally	Frequency
21 – 30	//	2
31 – 39	~~////~~	5
41 – 49	~~////~~ ~~////~~ //	12
51 – 59	~~////~~	5
61 – 69	~~////~~ 1	6

(d) State the modal class (04 marks)

Model class = 41 – 49 (i.e. class with highest frequency)

5. Simplify; $3^{2x+y} x\ 3^{x-y} x\ 9^{-x}$. (04marks)

$3^{2x+y} x\ 3^{x-y} x\ 9^{-x} = 3^{2x+y} x\ 3^{x-y} x\ 3^{-2x}$

$= 3^{2x+y+x-y-2x}$

$= 3^{x}$

6. Solve$\frac{n-1}{2} - \frac{n-3}{4} = \frac{1}{2}$. (04marks)

$\frac{n-1}{2} - \frac{n-3}{4} = \frac{1}{2}$

Multiplying through by 4

2(n – 1) – (n – 3) = 2

2n – 2 – n + 3 = 2

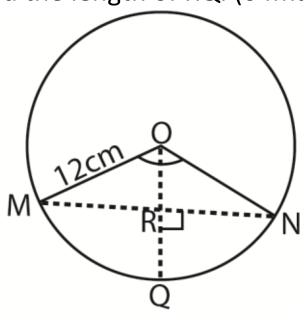
n = 1

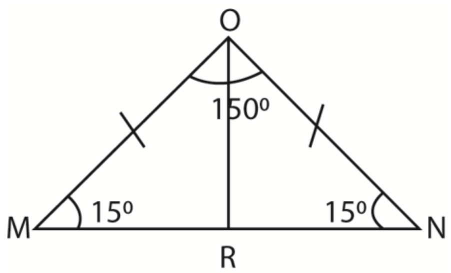
7. If A = $\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 6 & -2 \\ 7 & 3 \end{pmatrix}$, calculate 2A – B. (04marks)

2A – B = $2\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 6 & -2 \\ 7 & 3 \end{pmatrix}$

$= \begin{pmatrix} 4 & 6 \\ 8 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -2 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 8 \\ 1 & 1 \end{pmatrix}$

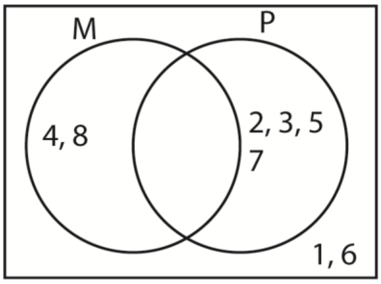
8. A translation $\begin{pmatrix}2\\3\end{pmatrix}$ followed by a translation $\begin{pmatrix}x\\y\end{pmatrix}$gives the translation$\begin{pmatrix}5\\-2\end{pmatrix}$. Find the values of x and y, (04marks)
Resultant translation is obtained by adding the individual translations
⇨ $\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}5\\-2\end{pmatrix}$
   2 + x = 5; x = 3
   3 + y = - 2, y = -5
   Hence x = 3 and y = -5
9. In the diagram below, O is of the circle. R is the mid-point of NM and the angle MON = 150^0. Find the length of RQ. (04marks)





Considering triangle OMN
$\frac{\overline{OR}}{\overline{OM}} = \sin 15^0$
OR = 12sin 15^0
RQ = 12 – OR = 12 - 12sin 15^0 = 8.89cm

10. A number is chosen at random from integers 1 to 8. Find the probability that the number chosen is either a multiple of 4 or a prime number. (04marks)
Let S= sample space
   M = multiple of 4
   P = prime number
S = {1, 2, 3, 4, 5, 6, 7, 8}
M = {4, 8}
P = {2, 3, 5, 7}
P(M$\cup P$) = P(M) + P(P)
$= \frac{n(M)}{n(S)} + \frac{n(P)}{n(S)}$
$= \frac{2}{8} + \frac{4}{8} = \frac{6}{8} = 0.75$
   Or
By using set theory



P(M∪P) = $\frac{n(M\cup P)}{n(E)} = \frac{6}{8} = \frac{3}{4} = 0.75$

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

11. (a) (i) Make y the subject of $\frac{1}{y} + B = \frac{x}{2n}$.

$\frac{1}{y} + B = \frac{x}{2n}$.

$\frac{1}{y} = \frac{x}{2n} - B = \frac{x-2Bn}{2n}$

$y = \frac{2n}{x-2Bn}$

(ii) Find the value y when n = 10, x = 65 and B = 3 (06marks)

Substituting the values

$y = \frac{2(10)}{65-2(3)(10)} = \frac{20}{65-60} = \frac{20}{5} = 4$

(b) Betty is 9 years younger than David. John is three times as old as Betty. The sum of all their ages is 49 years. Find

(iii) John's age

(iv) David's age (06marks)

Let Betty's age = x

David's age = x + 9 while John's age = 3x

Total = x + x+ 9 + 3x = 49

5x = 40

x = 8

(i) John's age = 3 x 8 = 24 years

(ii) David's age = 8 + 9 = 17years

12. (a) by shading the unwanted regions, show on a graph the region satisfying the inequalities below;

$x \geq 0$

$y \geq 0$

$x + y \leq 5$

$x + 3y \leq 9$ (08marks)

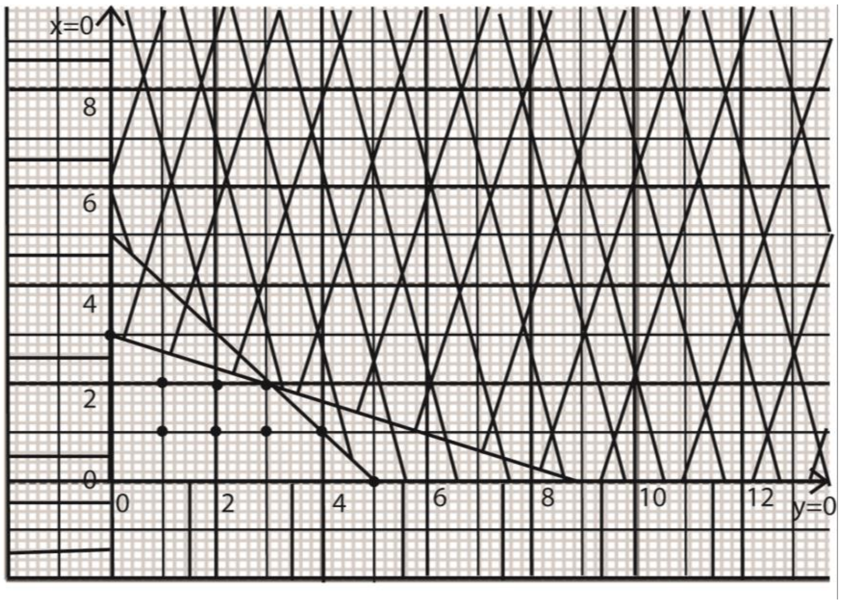
For $x + y \leq 5$, the boundary line is $x + y = 5$

x	0	5
y	5	0

Testing for wanted region using point (0, 0)
LHS = 0 + 0 = 0 RHS = 5
LHS < RHS, hence the region containing point (0, 0) is the wanted region
For x + 3y $\leq$ 9, the boundary line is x+ 3y = 9

x	0	9
y	3	0

Testing for wanted region using point (0, 0)
LHS = 0 + 0 = 0 RHS = 9
LHS < RHS, hence the region containing point (0, 0) is the wanted region



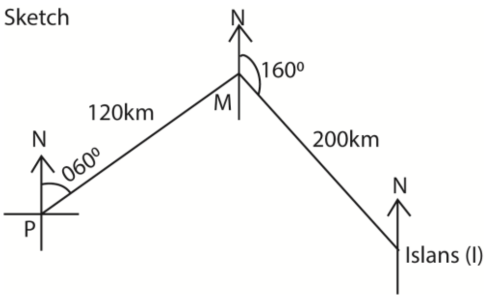
(b) Use your graph to find the values of x and y which give the maximum values for both x + y and x + 3y. (04marks)
Possible lattice points of (x, y) are that give maximum value are the extreme ones which are (5, 0), (4, 1), (3, 2), (0, 3)

(x, y)	x + y	X + 3y
(5, 0)	5	5
(4, 1)	5	7
(3, 2)	5	9
(0, 3)	3	9

Hence the values of x and y which gives the maximum values of both x + y and x + 3y are (x, y) = (3, 2)

13. A ship leaves a port and sails for 120km on a bearing of 062^0. It then changes direction to a bearing of 160^0 and sails for 200km to an island. Using a scale drawing with 1cm representing 20km, find

Sketch

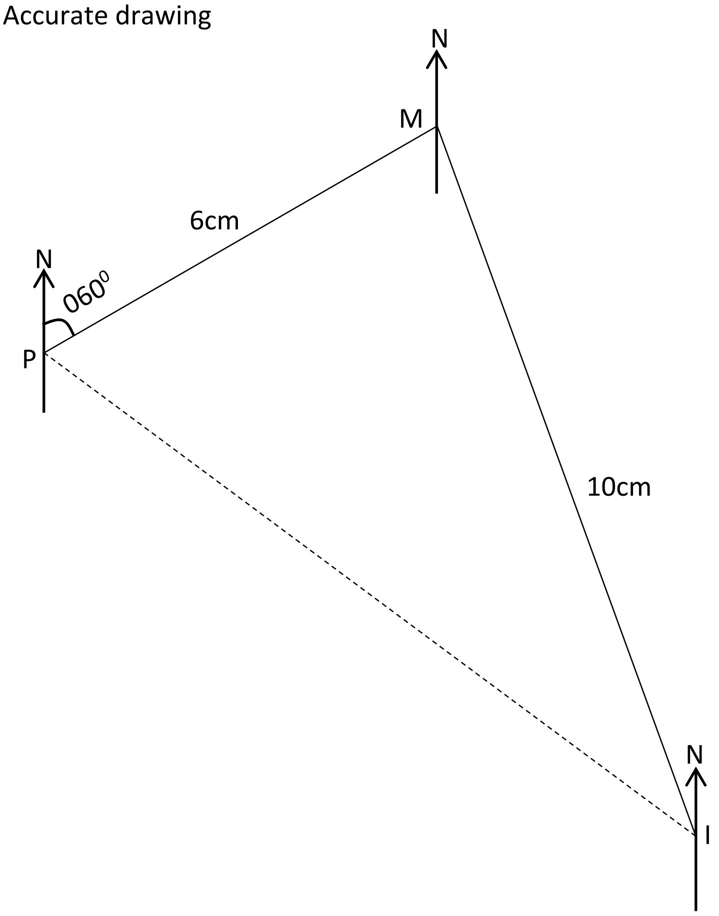


Drawing to scale

$\overline{PM}$ = 120km = $\frac{120}{20} = 6cm$

$\overline{MI}$ = 200km = $\frac{200}{20} = 10cm$

Accurate drawing



(a) the distance of the island from the port

$\overline{PI}$ = 10. 7cm

= 10.7 x 20 = 214cm

The distance of the Island from the port = 214cm

(b) the bearing of the port from the island

306^0

(c) how long it would take the ship to sail directly back to the port at a speed of 20km/h. (12marks)

time taken = $\frac{Distance\ \overline{PI}}{Speed} = \frac{214}{20} = 10.7hours$

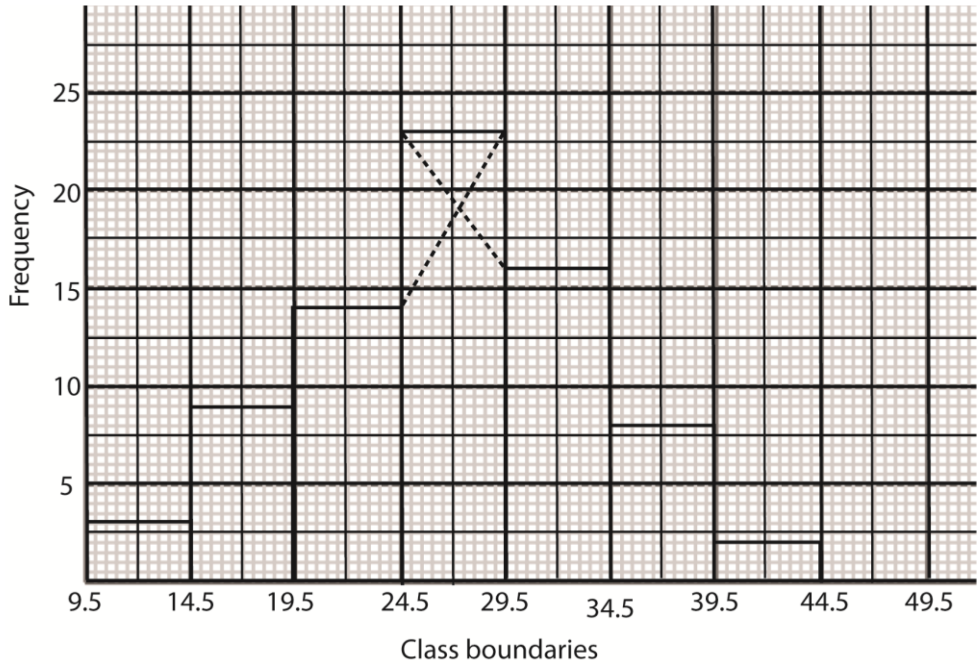
14. The table below shows the marks scored by 75 students in a test.

Marks	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44
Number of students	3	9	14	23	16	8	2

(a) Draw the given data on a histogram. (04marks)

Table of results

Marks	x	f	x – A = d	fd
10 – 14	12	3	-15	-45
15 – 19	17	9	-10	-90
20 – 24	22	14	-5	-70
25 – 29	27	23	0	0
30 – 34	32	16	5	80
35 – 39	37	8	10	80
40 – 44	42	2	15	30
		$\sum f = 75$		$\sum fd = -15$



(b) Use your histogram to estimate the modal mark. (04marks)

24.5 + 0.6 x 5 = 27 .5

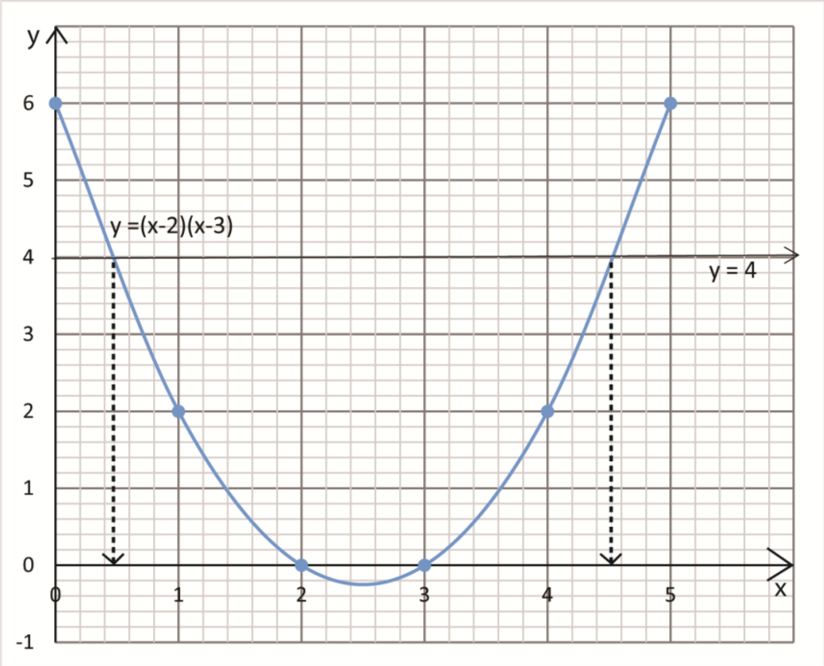
(c) Calculate the mean mark using a working mean of 27. (04marks)

Mean = A + $\frac{\sum fd}{\sum f}$ = 27 + $\frac{-15}{75}$ = 26.8

15. (a) Draw a graph of y = (x – 2)(x – 3) for the domain 0≤ x ≤5. (06marks)

Use 2cm to represent 1 unit on both axes

x	0	1	2	3	4	5
x – 2	-2	-1	0	1	2	2
x – 3	-3	-2	-1	0	1	2
y = (x – 2)(x – 3	6	2	0	0	2	6



(b) From your graph, find the values of x for which $x^2 - 5x + 6 = 0$

The curve meets the x-axis at (2, 0) and (3, 0)

Hence the values of x are x = 2 and x = 3

(c) By drawing an appropriate line on the graph, find the values of x for which

$x^2 - 5x + 2 = 0$ (04marks)

Comparing

$x^2 - 5x + 2 = 0$ ……….. (i)

with

$X^2 - 5x + 6 = 0$ ……… (ii)

Eqn. (ii) – eqn. (i) = 4

So we draw a line y = 4 on the same axes; the value of x at the points of intersection are x = 0.5 and x = 4.5

16. (a) given that P= $\begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}$, Q = $\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$ and R = PQ

(i) Determine the order of R

R = PQ

$= \begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$

a 2 x 3: 3 x 2 = a 2 x 2

Hence order of R is a 2 x 2

(ii) Find the matrix R (06marks)

$$R = \begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3x1 + 1x0 + 1x5 & 3x - 1 + 1x3 + 7x2 \\ -1\,x1 + 3x0 + 2x5 & -1\,x - 1 + 3x3 + 2x2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 + 0 + 5 & -3 + 3 + 4 \\ -1 + 0 + 0 & 1 + 9 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 14 \\ 9 & 14 \end{pmatrix}$$

(b) Solve the following simultaneous equation using the matrix method

5x + 3y = 7

2x -4y = 3 (06marks)

Expressing the equation in matrix form

$$\begin{pmatrix} 5 & 3 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

We can solve the above matrix equation either by pre-multiplying both sides by adjunct matrix inverse matrix of given matrix.

However, solving using inverse matrix is recommended.

Case 1: considering adjunct matrix

Adjunct matrix = $\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}$

Pre-multiplying both sides by adjunct matrix

$$\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -4\,x5 + -3x2 & -4x3 + -3\,x - 4 \\ -2x5 + -3x2 & -2x3 + 5x - 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4x7 + -3x3 \\ -2\,x7 + 5x3 \end{pmatrix}$$

$$\begin{pmatrix} -20 - 6 & -12 + 12 \\ -10 + 10 & -6 - 20 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -28 - 9 \\ -14 + 15 \end{pmatrix}$$

$$\begin{pmatrix} -26 & 0 \\ 0 & -26 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -37 \\ 1 \end{pmatrix}$$

-26x =-37

$x = \frac{-37}{-26} = 1.03$

-26y = 1

y = $\frac{-1}{26}$

case II: considering inverse matrix

Let M = $\begin{pmatrix} 5 & 3 \\ 2 & -4 \end{pmatrix}$

Adjunct M = $\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}$

Det(M) = 5 x -4 -3 x2 = -26

$M^{-1} = \frac{adjunct\ M}{\det(M)} = -\frac{1}{26}\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}$

Pre-multiplying both sides by M-1

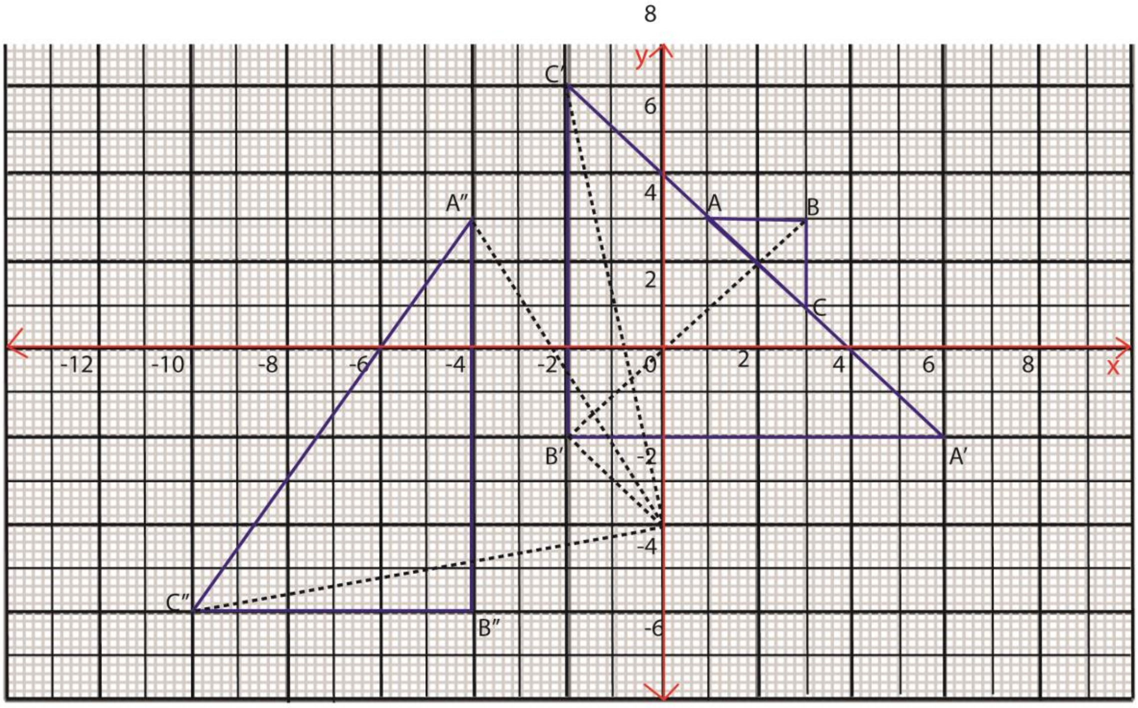
$$-\frac{1}{26}\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{26}\begin{pmatrix} -4 & -3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{26}\begin{pmatrix} -4x7 + -3x3 \\ -2\,x7 + 5x3 \end{pmatrix} = -\frac{1}{26}\begin{pmatrix} -37 \\ 1 \end{pmatrix}$$

$x = \frac{-37}{-26} = 1.03$

y = $\frac{-1}{26}$

17. Triangle ABC with vertices A(1, 3), B(3, 3) and C (3, 1) is enlarged with a scale factor -4 about (2, 2) to triangle A'B'C'. Triangle A'B'C' is then rotated through a positive quarter turnabout (0, -4) to triangle A''B''C''.
    (a) Draw on the same axes the triangles ABC, A'B'C' and A''B''C''. (10marks)



    (b) Write the coordinates of
    (i) A', B', and C' =A'(6, -2), B'(-2, -2), C(-2, 6)
    (ii) A'', B'', and C'' = A''(-2, 3), B''(-2, -6), c''(-10, -6)(02marks)

Thank you

Dr. Bbosa Science