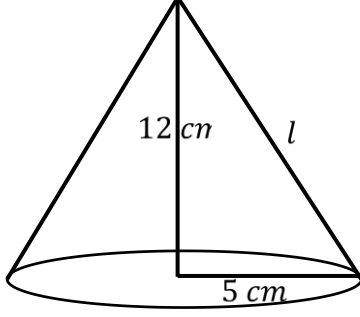
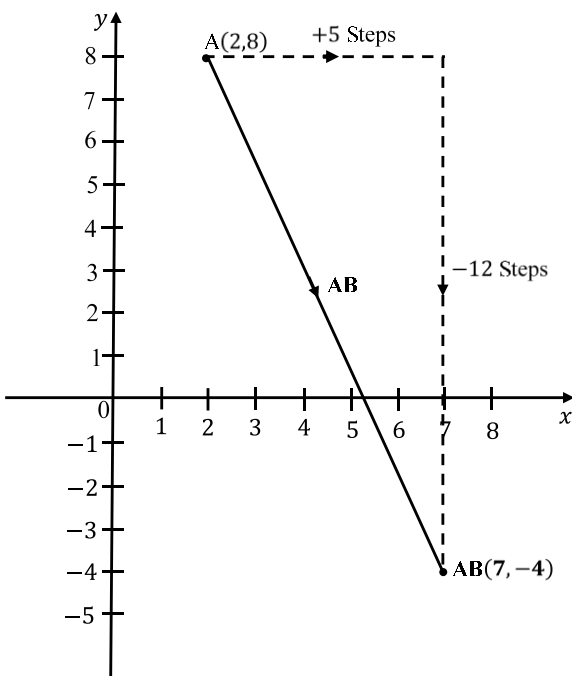


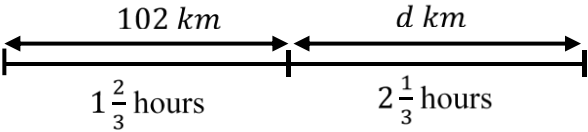
**PROPOSED
MARKING GUIDE
MATH 2 UMTA
O LEVEL 2023**

NO	SOLUTION	Mks	Comments
1	$= \left(\frac{13}{8} \div \frac{13}{4} \right) \div \left(\frac{11}{4} - \frac{15}{8} \right)$ $= \left(\frac{13}{8} \times \frac{4}{13} \right) \div \left(\frac{22-15}{8} \right)$ $= \frac{1}{2} \div \frac{7}{8}$ $= \frac{1}{2} \times \frac{8}{7}$ $= \frac{4}{7}$		
		04	
2	$P = \{3, 6, 9, 12\}$ $Q = \{1, 3, 6, 10, 15\}$ $P \cap Q = \{3, 6\}$ $\therefore n(P \cap Q) = 2 \text{ elements}$		
		04	
3	<p>For perpendicular lines; $m_1 \times m_2 = -1$</p> $-\frac{2}{3} \times m_2 = -1 \quad \therefore m_2 = 3/2$ $3/2 = \frac{h+4}{8-2}$ $2(h+4) = 3 \times 6$ $2h + 8 = 18$ $2h = 10 \quad \therefore h = 5$		
		04	
4	$\frac{4\sqrt{3}}{2\sqrt{3}-\sqrt{6}} = \frac{4\sqrt{3}(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3}-\sqrt{6})(2\sqrt{3}+\sqrt{6})}$ $= \frac{8 \times 3 + 4\sqrt{18}}{(2\sqrt{3})^2 - (\sqrt{6})^2}$		

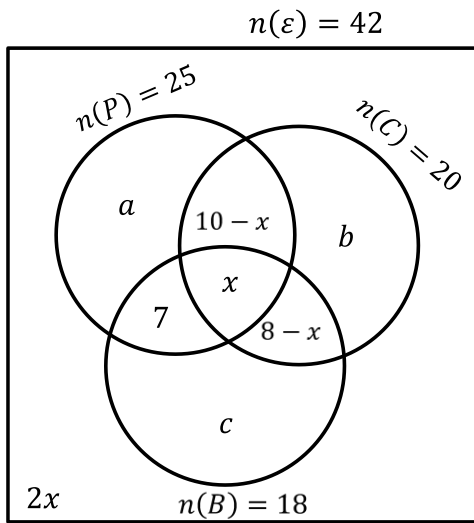
	$= \frac{24+4\sqrt{9 \times 2}}{4 \times 3 - 6}$ $= \frac{24+12\sqrt{2}}{6}$ $= 4 + 2\sqrt{3}$		
		04	
5	<p>From $A = P \left(1 + \frac{r}{100}\right)^n$</p> $2,850,000 = 2,420,000 \left(1 + \frac{805}{100}\right)^n$ $(1.085)^n = 1.17768565$ <p>Taking log to base 10 on both sides</p> $\log_{10}(1.085)^n = \log_{10} 1.17768595$ $n = \frac{\log_{10} 1.17768595}{\log_{10} 1.085}$ $n = 2 \text{ years}$		
		04	
6	$1 - \frac{1}{3} \log_{10} 64 + 2 \log_{10} 20$ $= \log_{10} 10 - \log_{10} 64^{1/3} + \log_{10} 20^2$ $= \log_{10} 10 - \log_{10} 4 + \log_{10} 400$ $= \log_{10}(10 \times 400) - \log_{10} 4$ $= \log_{10} 4000 - \log_{10} 4$ $= \log_{10} \left(\frac{4000}{4}\right)$ $= \log_{10} 1000$ $= \log_{10} 10^3$ $= 3 \log_{10} 10$ $= 3 \times 1$ $= 3$		
		04	

7	 <p> $l^2 = 5^2 + 12^2$ $l^2 = 25 + 144$ $l^2 = 169$ $\sqrt{l^2} = \sqrt{169}$ $l = 13 \text{ cm}$ $S.A = \pi r l$ $= 3.142 \times 5 \times 13$ $= 204.23 \text{ cm}^2$ </p>		
		04	
8	<p>From $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$</p> <p>$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$</p>		

	 <p> $OB = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ $\therefore (5,12)$ are the coordinates of B </p>		
		04	
9	$V.S.F = (L.S.F)^3$ $\frac{V_b}{V_s} = \left(\frac{H}{h}\right)^3$ $\frac{250}{54} = \left(\frac{10}{h}\right)^3$ $\frac{125}{27} = \left(\frac{10}{h}\right)^3$ $\left(\frac{10}{h}\right)^3 = \frac{5^3}{3^3}$ $\frac{10}{h} = \left(\frac{5^3}{3^3}\right)^{1/3}$ $\frac{10}{h} = \frac{5^{3 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}}}$ $\frac{10}{h} = \frac{5}{3}$ $5h = 30 \quad \therefore h = 6 \text{ cm}$		

		04	
10	 $d = 54 \times \frac{7}{3} = 126 \text{ km}$ <p>Average speed = $\frac{\text{Total distance}}{\text{total time taken}}$</p> $= \frac{102+126}{\frac{5}{3}+\frac{7}{3}}$ $= \frac{228}{4}$ $= 57 \text{ kmh}^{-1}$		
		04	
11	<p>(a) i) $f(-5) = \frac{5(-5)+1}{4}$</p> $= \frac{-25+1}{4}$ $= \frac{-24}{4}$ $= -6$ <p>ii) let $f(x) = y$</p> $y = \frac{5x+1}{4}$ $5x + 1 = 4y$ $5x = 4y - 1$ $x = \frac{4y-1}{5}$ $f^{-1}(x) = \frac{4x-1}{5}$ $\Rightarrow f^{-1}(4) = \frac{4(4)-1}{5}$ $= \frac{15}{5}$		

	$= 3$ <p>(b) $gh(x) = g[h(x)]$</p> $= \frac{4\left(\frac{1+2x}{8}\right)+3}{3}$ $= \frac{\frac{1+2x}{2}+3}{3}$ $= \frac{1+2x+6}{6}$ $= \frac{2x+7}{6}$ $\Rightarrow \frac{2x+7}{6} = \frac{6-x^2}{4}$ $8x + 28 = 36 - 6x^2$ $6x^2 + 8x - 8 = 0$ $3x^2 + 4x - 4 = 0$ $3x^2 - 2x + 6x - 4 = 0$ $x(3x - 2) + 2(3x - 2) = 0$ $(x + 2)(3x - 2) = 0$ <p>Either $x + 2 = 0$ or $3x - 2 = 0$</p> $x = -2 \text{ or } 3x = 2$ $x = \frac{2}{3}$		
		12	
12	(a)		



$n(P)$ only;

$$a + 7 + x + 10 - x = 25$$

$$a + 17 = 25$$

$$a = 25 - 17 = 8$$

$n(C)$ only;

$$b + 10 - x + x + 8 - x = 20$$

$$b + 18 - x = 20$$

$$b = 20 - 18 + x$$

$$b = 2 + x$$

$n(B)$ only;

$$7 + x + 8 - x + c + 18$$

$$c + 15 = 18$$

$$c = 18 - 15 = 3$$

(b) i) $8 + 7 + x + 10 - x + 2 + x + 8 - x + 3 + 2x = 42$

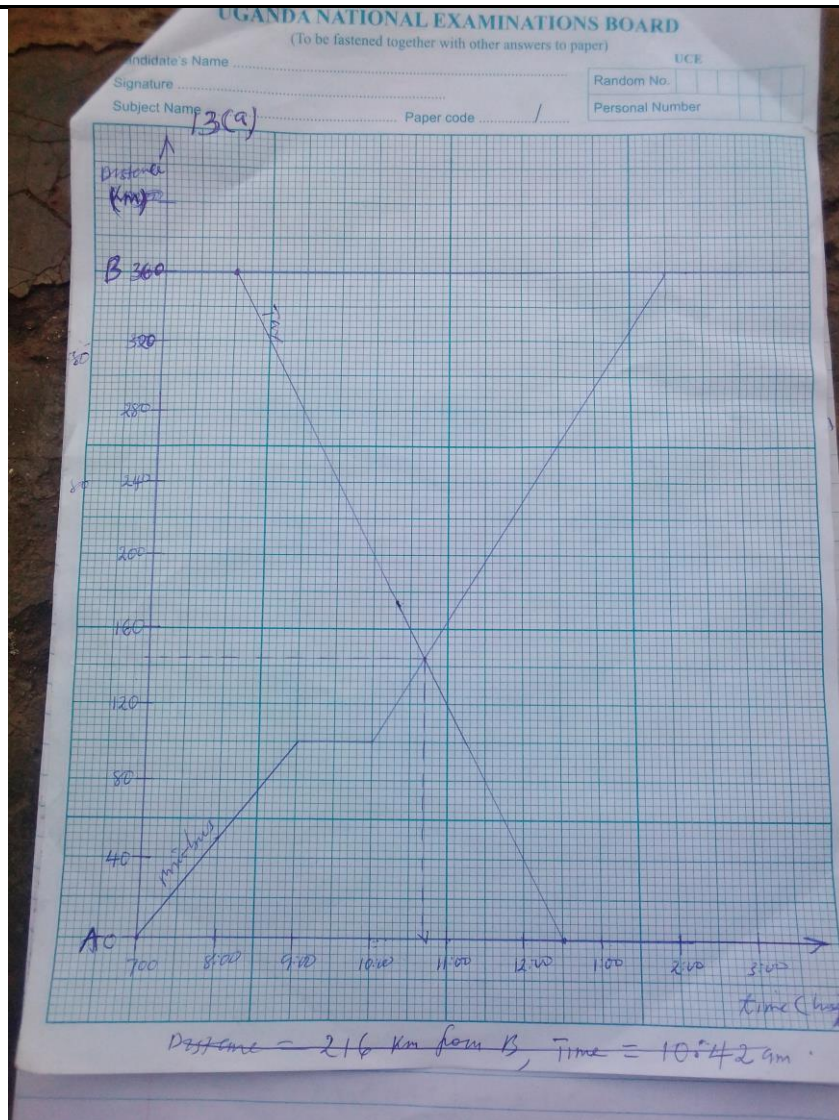
$$25 + 13 + 2x = 42$$

$$2x + 38 = 42$$

$$2x = 42 - 38$$

$$2x = 4 \quad \therefore x = 2$$

	<p>ii) At least two = $7 + (10 - 2) + (8 - 2) + 2$</p> <p style="text-align: center;">$= 7 + 8 + 6 + 2$</p> <p style="text-align: center;">$= 23$ students</p> <p>(c) Probability = $\frac{n(E)}{n(s)}$</p> <p style="text-align: center;">$= \frac{8+3+4}{42}$</p> <p style="text-align: center;">$= \frac{15}{42}$</p> <p style="text-align: center;">$= \frac{5}{14}$ or 0.3571</p>		
		12	
13	(a)		



(b) i) Time of arrival of:

Mini-bus = 2:00 pm

Taxi = 12:30 pm

ii) Time = 10:42 am

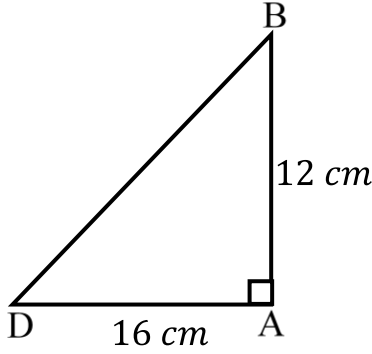
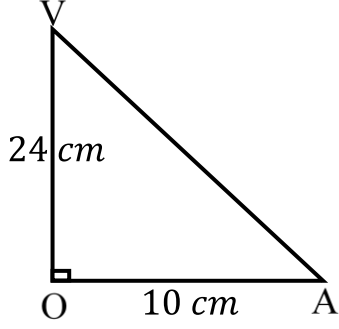
Distance = 216 km from B

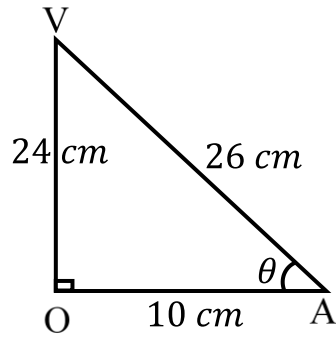
$$\begin{aligned}
 \text{(c) Average speed} &= \frac{\text{Total distance}}{\text{Total time taken}} \\
 &= \frac{260}{4} \\
 &= 65 \text{ km hr}^{-1}
 \end{aligned}$$

		12																						
14	<p>(a) Allowances</p> <p>Housing $= \frac{840,000}{12} = \text{Shs}70,000$</p> <p>Electricity $= \text{Shs}60,000$</p> <p>Medical $= \text{Shs}100,000$</p> <p>Total allowances $= \text{Shs}230,000$</p> <p>\Rightarrow Taxable income $= 950,000 - 230,000$</p> <p>$= \text{Shs}720,000$</p> <p>(b) income tax</p> <table><tr><th>Taxable income</th><th>Rate</th><th>Income tax</th></tr><tr><td>01-100,000</td><td>5</td><td>$\frac{5}{100} (100,000 - 0)$ $= \text{Shs}5,000$</td></tr><tr><td>100,001-300,000</td><td>7</td><td>$\frac{7}{100} (300,000 - 100,000)$ $= \text{Shs}14,000$</td></tr><tr><td>300,001-450,000</td><td>10</td><td>$\frac{10}{100} (450,000 - 300,000)$ $= \text{Shs}15,000$</td></tr><tr><td>450,001-600,000</td><td>14</td><td>$\frac{14}{100} (600,000 - 450,000)$ $= \text{Shs}21,000$</td></tr><tr><td>600,001-720,000</td><td>25</td><td>$\frac{25}{100} (720,000 - 600,000)$ $= \text{Shs}30,000$</td></tr><tr><td>Total</td><td></td><td>$= \text{Shs}85,000$</td></tr></table> <p>\thereforeIncome tax $= \text{Shs}85,000$</p> <p>(c) Net income = Gross -income tax</p> <p>$= 950,000 - 85,000$</p> <p>$= \text{Shs}865,000$</p>	Taxable income	Rate	Income tax	01-100,000	5	$\frac{5}{100} (100,000 - 0)$ $= \text{Shs}5,000$	100,001-300,000	7	$\frac{7}{100} (300,000 - 100,000)$ $= \text{Shs}14,000$	300,001-450,000	10	$\frac{10}{100} (450,000 - 300,000)$ $= \text{Shs}15,000$	450,001-600,000	14	$\frac{14}{100} (600,000 - 450,000)$ $= \text{Shs}21,000$	600,001-720,000	25	$\frac{25}{100} (720,000 - 600,000)$ $= \text{Shs}30,000$	Total		$= \text{Shs}85,000$		
Taxable income	Rate	Income tax																						
01-100,000	5	$\frac{5}{100} (100,000 - 0)$ $= \text{Shs}5,000$																						
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300,001-450,000	10	$\frac{10}{100} (450,000 - 300,000)$ $= \text{Shs}15,000$																						
450,001-600,000	14	$\frac{14}{100} (600,000 - 450,000)$ $= \text{Shs}21,000$																						
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Total		$= \text{Shs}85,000$																						
		12																						
15	<p>(a) i) $QR = \frac{2}{3}QT$</p> <p>but $QT = QO + OT$</p>																							

	$= -q + \frac{1}{2}p$ $= \frac{1}{2}(p - 2q)$ $\therefore QR = \frac{2}{3} \times \frac{1}{2}(p - 2q) = \frac{1}{3}(p - 2q)$ <p>ii) $OR = OQ + QR$</p> $= q + \frac{1}{3}(p - 2q)$ $= \frac{3q + p - 2q}{3}$ $= \frac{1}{3}(p + q)$ <p>iii) $QS = \frac{1}{2}QP$</p> <p>but $QP = OP - OQ$</p> $= p - q$ $\therefore QS = \frac{1}{2}(p - q)$ <p>iv) $TS = TQ + QS$</p> $= -\frac{1}{2}(p - 2q) + \frac{1}{2}(p - q)$ $= \frac{-p + 2q + p - q}{2}$ $= \frac{1}{2}q$ <p>(b) $OS = OQ + QS$</p> $= q + \frac{1}{2}(p - q)$ $= \frac{2q + p - q}{2}$ $= \frac{1}{2}(p + q)$		
--	--	--	--

	$\Rightarrow OS:OR = \frac{\frac{1}{2}(p+q)}{\frac{1}{3}(p+q)}$ $= \frac{1}{2} \times \frac{3}{2}$ $= 3/2$ <p>\therefore Since $2OS = 3OR$ and point O is common, then the points lie on a straight line.</p>		
		12	
16	<p>(a) $C = k_1 + \frac{k_2}{N}$</p> <p>When $N = 200, C = \text{shs}850$</p> $850 = k_1 + \frac{k_2}{200}$ $170,000 = 200k_1 + k_2 \dots\dots\dots(i)$ <p>When $N = 400, C = \text{shs}500$</p> $500 = k_1 + \frac{k_2}{400}$ $200,000 = 400k_1 + k_2 \dots\dots\dots(ii)$ <p>$(ii) - (i); 200k_1 = 30,000 \quad \therefore k_1 = 150$</p> <p>From (i); $200(150) + k_2 = 170,000$</p> $30,000 + k_2 = 170,000$ $k_2 = 170,000 - 30,000$ $k_2 = 140,000$ $\therefore C = 150 + \frac{140,000}{N}$ <p>(b) i) When $N = 560, C = ?$</p> $C = 150 + \frac{140,000}{560}$ $= \text{shs}400$ <p>ii) When $C = \text{shs}950, N = ?$</p>		

	$950 = 150 + \frac{140,000}{N}$ $800 = \frac{140,000}{N}$ $800N = 140,000$ $N = 175 \text{ copies}$		
		12	
17	<p>i) VA</p>  $\overline{DB} = \sqrt{16^2 + 12^2}$ $= \sqrt{400}$ $= 20 \text{ cm}$  $\overline{VA} = \sqrt{24^2 + 10^2}$ $= \sqrt{676}$ $= 26 \text{ cm}$ <p>ii)</p>		



$$\tan \theta = \frac{24}{10}$$

$$\theta = \tan^{-1}\left(\frac{24}{10}\right)$$

$$\theta = 67.38^\circ$$

$$\sin \theta = \frac{24}{26}$$

$$\theta = \sin^{-1}\left(\frac{24}{26}\right)$$

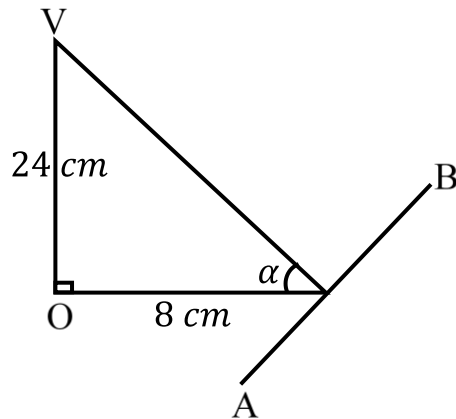
$$\theta = 67.38^\circ$$

$$\cos \theta = \frac{10}{26}$$

$$\theta = \cos^{-1}\left(\frac{10}{26}\right)$$

$$\theta = 67.38^\circ$$

iii)



$$\tan \alpha = \frac{24}{8}$$

$$\alpha = \tan^{-1}(3)$$

$$\alpha = 71.57^\circ$$

iv) volume = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$= \frac{1}{3} \times 16 \times 12 \times 24$$

$$= 1536 \text{ cm}^3$$