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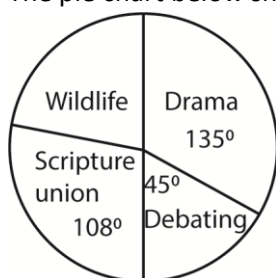


UCE MATHEMATICS PAPER 1 2016 guide

SECTION A (40 marks)

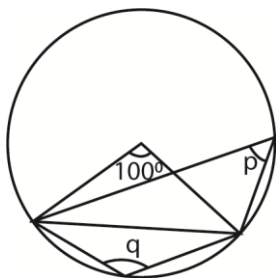
Answer all questions in this section

1. Make a the subject of the expression: $c = \frac{a^2}{(a-b)(a+b)}$.
2. The pie chart below shows the various clubs that 40 students belong to;



Determine the number of students in the wild life club.

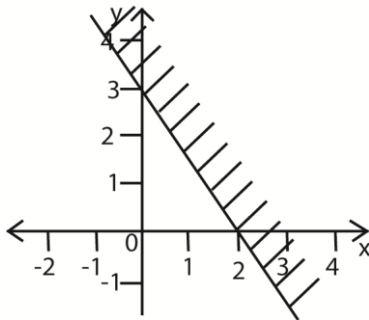
3. Given $a * b = a + b + ab$
 - (a) evaluate $3 * 5$.
 - (b) find the value of n , when $7 * n = 23$
4. The circle below has its centre at O



Calculate angle p and q .

5. An object at $(0, 0)$ undergoes a translation $A = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ then followed by translation $B = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$.
 - (a) Find a single translation equivalent to two translations A and B .
 - (b) How far is the object from $(0, 0)$?
6. Solve the equation; $\frac{5x+2}{3} - \frac{7x+2}{5} = 2$
7. Two fair coins are tossed
 - (a) Construct table showing all the possible outcomes.
 - (b) What is the probability of getting at least a tail?

8. Find the inverse of the matrix $A = \begin{pmatrix} 5 & 7 \\ -3 & -2 \end{pmatrix}$
9. Determine the inequality which is represented by the unshaded region on the graph below



10. A pilot in a plane at altitude of 500m above the ground sees a camp at an angle of 15° . Find the horizontal distance the pilot would have to fly so that the plane is directly above the camp.

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

11. A manager of a restaurant spends shs. 29,000 to purchase 4kg of rice and 7kg of Irish potatoes. Later he increases each of the above quantities by 1kg thus increasing his expenditure by shs. 5000.

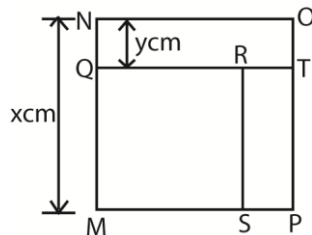
- Write down two equations that represent the manager's purchase
- Use your equations to find the cost of rice and Irish potatoes per kilogram
- How much would the manager pay for 10kg of rice and 15kg of Irish potatoes?

12. (a) Solve equation: $3 \begin{pmatrix} 1+x \\ y \end{pmatrix} - \begin{pmatrix} x \\ 1-2y \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$.

- (b) Given that $M = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

- Calculate N^2 and MN
- Find the value the scalar k if $N^2 + kN = MN$

13. In the figure below, MNOP and MQRS are squares, $MN = x$ and $QN = y$ cm



- If the area of rectangle QNOT is 1cm^2 less than area of MQRS, show that $y^2 - 3xy + x^2 = 1$
- Given that $y = 3$, find the appropriate value of x .
- Calculate the area of the rectangle PTRS

14. The following table shows the marks scored by 36 students in a mathematic test

Marks	Frequency
30-39	4
40-49	6
50-59	3
60-69	12
70-79	2
80-89	5
90-99	4

- (a) Calculate to 2 decimal places the
- Mean mark
 - Median mark
- (b) Find the probability that a student picked at random scored below 50.
15. (a) copy and complete the table for $y=(3x+1)(2x-5)$
- | | | | | | | |
|--------|----|---|-----|---|----|---|
| X | -1 | 0 | 1 | 2 | 3 | 4 |
| $3x+1$ | -2 | | 4 | | 10 | |
| $2x-5$ | -7 | | -3 | | 1 | |
| y | 14 | | -12 | | 10 | |
- (b) Use your completed table to draw a graph of $y = (3x+1)(2x - 5)$ with a scale of 2cm for 1unit on x-axis.
- (c) Draw on the same axes the line of $y=5$.
- (d) Use the two graphs in (b) and (c) to solve the equation $6x^2 -13x -10 = 0$.
16. (a) The image of P(6, 3) after reflection is P'(3, 6)
- Plot the points P and P' on the graph paper.
 - Construct the line of reflection. Hence find the equation of the line of reflection.
- (b) The image of ABCD under a matrix of transformation $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ is A'B'CD'; the coordinates of the image are A'(1, 0), B'(4, -6), C'(4, -4) and D'(1, 2). Determine the coordinated of A, B, c and D.
17. The manager of the cinema hall wishes to divide the seats available into two classes executive and ordinary. There are not more than 120 seats available. There must be at least twice as many ordinary seats as there are executive seats. Executive seats are priced at shs. 15,000 each. Ordinary seats are priced at shs. 10,000 each. At least shs. 1,000,0000 should be collected at each show to meet expenses.
- Taking x as the number of executive seats and y as the number of ordinary seats, write down five inequalities from the given information.
 - Represent the inequalities on a graph.
 - From the graph, find the number of seats of each kind which must be sold to give maximum profit.

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. Make a the subject of the expression: $c = \frac{a^2}{(a-b)(a+b)}$.

$$c = \frac{a^2}{(a-b)(a+b)} = \frac{a^2}{(a^2 - b^2)}$$

$$c(a^2 - b^2) = a^2$$

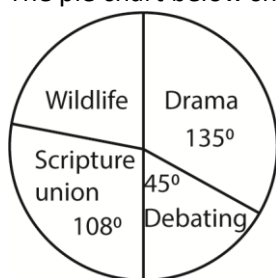
$$ca^2 - a^2 = cb^2$$

$$a^2(c - 1) = cb^2$$

$$a^2 = \frac{cb^2}{(c-1)}$$

$$a = \sqrt{\frac{cb^2}{(c-1)}}$$

2. The pie chart below shows the various clubs that 40 students belong to;



Determine the number of students in the wild life club.

$$\text{Degree of wildlife} = 360^\circ - (135^\circ + 108^\circ + 45^\circ) = 72^\circ$$

$$\text{Number of students in wildlife} = \frac{72}{360} \times 40 = 8 \text{ students}$$

3. Given $a * b = a + b + ab$

(c) evaluate $3 * 5$.

$$3 * 5 = 3 + 5 + 3 \times 5 = 8 + 15 = 23$$

(d) find the value of n , when $7 * n = 23$

$$7 * n = 23$$

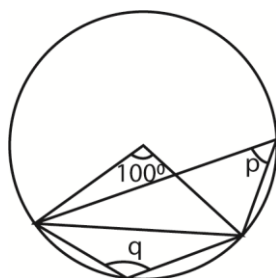
$$7 + n + 7n = 23$$

$$8n + 7 = 23$$

$$8n = 16$$

$$n = 2$$

4. The circle below has its centre at O



Calculate angle p and q.

$$p = \frac{1}{2}(100) = 50^\circ$$

Angle subtended at the centre is twice the angle subtended at any point on the circumference by the same arc of the circle

$$q = \frac{1}{2}(360^\circ - 100^\circ) = \frac{1}{2} \times 260^\circ = 130^\circ$$

5. An object at $(0, 0)$ undergoes a translation $A = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ then followed by translation $B = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$.

(a) Find a single translation equivalent to two translations A and B.

$$\text{Single translation} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

(b) How far is the object from $(0, 0)$?

$$\text{Distance from } (0, 0) = \left| \begin{pmatrix} 8 \\ -6 \end{pmatrix} \right| = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ units}$$

6. Solve the equation; $\frac{5x+2}{3} - \frac{7x+2}{5} = 2$

$$\frac{5x+2}{3} - \frac{7x+2}{5} = 2$$

Multiplying through by 15

$$5(5x + 2) - 3(7x + 2) = 2 \times 15$$

$$25x + 10 - 21x - 6 = 30$$

$$4x + 4 = 30$$

$$4x = 30 - 4 = 26$$

$$x = \frac{26}{4} = 6.5$$

7. Two fair coins are tossed

(a) Construct table showing all the possible outcomes.

		Coin I	
		T	H
Coin II	T	TT	TH
	H	TH	HH

(b) What is the probability of getting at least a tail?

Sample space, $S = \{TT, TH, HT, HH\}$

Possible outcome = $\{TH, HT, HH\}$

$$P(\text{getting at least a tail}) = \frac{3}{4} = 0.75$$

Or

$$P(\text{getting at least a tail}) = 1 - P(\text{getting no tail})$$

$$= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

8. Find the inverse of the matrix $A = \begin{pmatrix} 5 & 7 \\ -3 & -2 \end{pmatrix}$

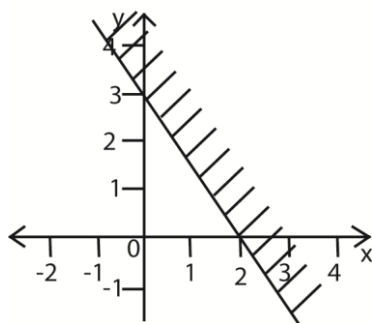
$$A^{-1} = \frac{\text{Adjunct of } A}{\det(A)}$$

$$\det(A) = 5 \times -2 - (7 \times -3) = -10 + 21 = 11$$

$$\text{Adjunct } A = \begin{pmatrix} -2 & -7 \\ 3 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{11} \begin{pmatrix} -2 & -7 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} \frac{-2}{11} & \frac{-7}{11} \\ \frac{3}{11} & \frac{5}{11} \end{pmatrix}$$

9. Determine the inequality which is represented by the unshaded region on the graph below



Finding boundary line

Method I: using difference approach

$$\begin{array}{c|c|c} & \xrightarrow{2} & \\ \hline x & 0 & 2 \\ \hline y & 3 & 0 \\ \hline & \xleftarrow{-3} & \end{array}$$

$$\begin{array}{r} -3x \quad 0 \quad -6 \\ -2y \quad 6 \quad 0 \\ \hline -6 \quad -6 \end{array}$$

$$-3x - 2y = -6 \text{ or } 3x + 2y = 6$$

Method II; using general equation of the line; $y = mx + c$

$$m = \frac{0-3}{2-0} = \frac{-3}{2}$$

$$y = \frac{-3}{2}x + c$$

using point (0, 3)

$$3 = 0 + c$$

By substitution

$$y = \frac{-3}{2}x + 3 \text{ or } 2y = -3x + 6 \text{ or } 2y + 3x = 6$$

Method II: using gradient approach

Let (x, y) line on the boundary line

$$\begin{array}{ccc} | & | & | \\ \hline (0, 3) & (2, 0) & (x, y) \end{array}$$

$$\frac{0-3}{2-0} = \frac{y-0}{x-0}$$

$$\frac{-3}{2} = \frac{y}{x-2}$$

$$-3(x-2) = 2y$$

$$-3x + 6 = 2y$$

$$3x + 2y = 6$$

Find the inequality of testing using point (0, 0)

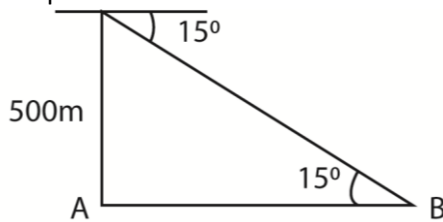
$$\text{L.H.S} = 3(0) + 2(0) = 0$$

$$\text{R.H.S} = 6$$

$$(\text{L.H.S} = 0) < (\text{R.H.S} = 6)$$

Hence the inequality is $3x + 2y \leq 6$

10. A pilot in a plane at altitude of 500m above the ground sees a camp at an angle of 15° . Find the horizontal distance the pilot would have to fly so that the plane is directly above the camp.



$$\tan 15^\circ = \frac{500}{AB}$$

$$AB = \frac{500}{\tan 15^\circ} = 1866\text{m}$$

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

11. A manager of a restaurant spends shs. 29,000 to purchase 4kg of rice and 7kg of Irish potatoes. Later he increases each of the above quantities by 1kg thus increasing his expenditure by shs. 5000.

(a) Write down two equations that represent the manager's purchase

Let x = price per 1kg of rice and y = price of 1kg of Irish potatoes

$$4x + 7y = 29,000$$

After increase

$$5x + 8y = 29,000 + 5,000 = 34,000$$

Hence equations are

$$4x + 7y = 29,000$$

$$5x + 8y = 34,000$$

(b) Use your equations to find the cost of rice and Irish potatoes per kilogram

In order to find x and y , we have to solve the two equations simultaneously

Method I; using elimination approach.

$$4x + 7y = 29,000 \text{ (i)}$$

$$5x + 8y = 34,000 \text{ (ii)}$$

$$5\text{eqn. (i)} - 4\text{eqn. (ii)}$$

$$20x + 35y = 145,000$$

$$-20x + 32y = 136,000$$

$$3y = 9,000$$

$$y = \frac{9,000}{3} = 3,000$$

By substituting for $y = 3000$ into eqn. (i)

$$4x + 7 \times 3000 = 29000$$

$$4x + 21000 = 29000$$

$$4x = 8000$$

$$x = \frac{8000}{4} = 2000$$

Hence the price of 1kg of rice is shs.2000 and a kg of Irish potatoes is shs. 3000.

Method II; using matrix method

$$\begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 29000 \\ 34000 \end{pmatrix}$$

Pre-multiplying both sides by adjunct matrix

$$\begin{pmatrix} 8 & -7 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 & -7 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 29000 \\ 34000 \end{pmatrix}$$

$$\begin{pmatrix} 32 - 35 & 56 - 56 \\ -20 + 20 & -35 + 32 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 232,000 - 23,8000 \\ 145,000 - 136,000 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6000 \\ -9000 \end{pmatrix}$$

$$\begin{pmatrix} -3x \\ -3y \end{pmatrix} = \begin{pmatrix} -6000 \\ -9000 \end{pmatrix}$$

$$-3x = -6000$$

$$x = \frac{-6000}{-3} = 2000$$

$$-3y = -9000$$

$$y = \frac{-9000}{-3} = 3000$$

Hence the price of 1kg of rice is shs.2000 and a kg of Irish potatoes is shs. 3000.

(c) How much would the manager pay for 10kg of rice and 15kg of Irish potatoes?

$$\text{Cost price} = 10x + 15y = 10 \times 2000 + 15 \times 3000 = 650000$$

12. (a) Solve equation: $3 \begin{pmatrix} 1+x \\ y \end{pmatrix} - \begin{pmatrix} x \\ 1-2y \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$.

$$3(1+x) - x = 5$$

$$3 + 3x - x = 5$$

$$3 + 2x = 5$$

$$2x = 5 - 3 = 2$$

$$x = 1$$

$$3y - (1 - 2y) = 9$$

$$3y - 1 + 2y = 9$$

$$5y - 1 = 9$$

$$5y = 10$$

$$y = \frac{10}{5} = 2$$

Hence $x = 1$ and $y = 2$

(c) Given that $M = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

(i) Calculate N^2 and MN

$$N^2 = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4+3 & 2+2 \\ 6+6 & 3+4 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 12 & 7 \end{pmatrix}$$

$$MN = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0+3 & 0+2 \\ 6+0 & 3+0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 6 & 3 \end{pmatrix}$$

(ii) Find the value the scalar k if $N^2 + kN = MN$

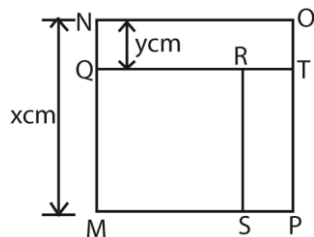
$$\begin{pmatrix} 7 & 4 \\ 12 & 7 \end{pmatrix} + k \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 6 & 3 \end{pmatrix}$$

$$7 + 2k = 3$$

$$2k = -4$$

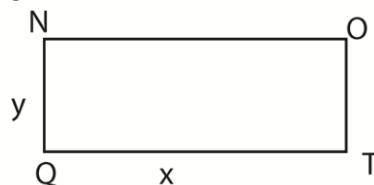
$$k = \frac{-4}{2} = -2$$

13. In the figure below, MNOP and MQRS are squares, $MN = x$ and $QN = y$ cm



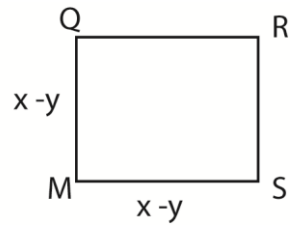
(a) If the area of rectangle QNOT is 1cm^2 less than area of MQRS, show that $y^2 - 3xy + x^2 = 1$

Considering QNOT



Area of QNOT = xy

Considering MQRS



$$\text{Area of MQRS} = (x-y)(x-y) = x^2 - 2xy + y^2$$

But area of MQRS - 1 = area of QNOT

$$\Rightarrow x^2 - 2xy + y^2 - 1 = xy$$

$$x^2 - xy + y^2 = 1$$

(b) Given that $y = 3$, find the appropriate value of x .

Substituting $y = 3$

$$x^2 - 3x + 3^2 = 1$$

$$x^2 - 3x + 8 = 0$$

$$(x-1)(x-8) = 0$$

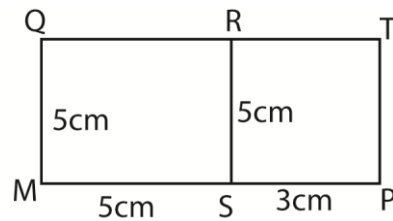
$$x-1 = 0; x = 1$$

Or

$$x-8 = 0; x = 8$$

since $x > y$; $x = 8$

(c) Calculate the area of the rectangle PTRS



$$\text{Area of PTRS} = 5 \times 3 = 15\text{cm}^2$$

14. The following table shows the marks scored by 36 students in a mathematic test

Marks	Frequency
30-39	4
40-49	6
50-59	3
60-69	12
70-79	2
80-89	5
90-99	4

(a) Calculate to 2 decimal places the

Marks	Mid-mark (x)	Frequency (f)	fx	cf
30-39	34.5	4	138	4
40-49	44.5	6	267	10
50-59	54.5	3	163.5	13
60-69	64.5	12	774	25
70-79	74.5	2	149	27
80-89	84.5	5	422.5	32
90-99	94.5	4	378	36
		$\Sigma f = 36$	$\Sigma fx = 2292$	

- (i) Mean mark

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{2292}{36} = 63.67$$

- (ii) Median mark

$$\text{Median mark} = Li + \frac{\left(\frac{N}{2} - cf_b\right)C}{fm}$$

$$\text{Median class} = 60 - 69$$

$$Li = 59.5, fm = 12, cf_b = 13, C = 10$$

$$\text{Median mark} = 59.5 + \frac{\left(\frac{36}{2} - 13\right)10}{12} = 59.5 + \frac{(18-13)10}{12} = 59.5 + \frac{50}{12} = 63.67$$

- (b) Find the probability that a student picked at random scored below 50.

$$\text{Number of students scored less than 50} = 6 + 4 = 10$$

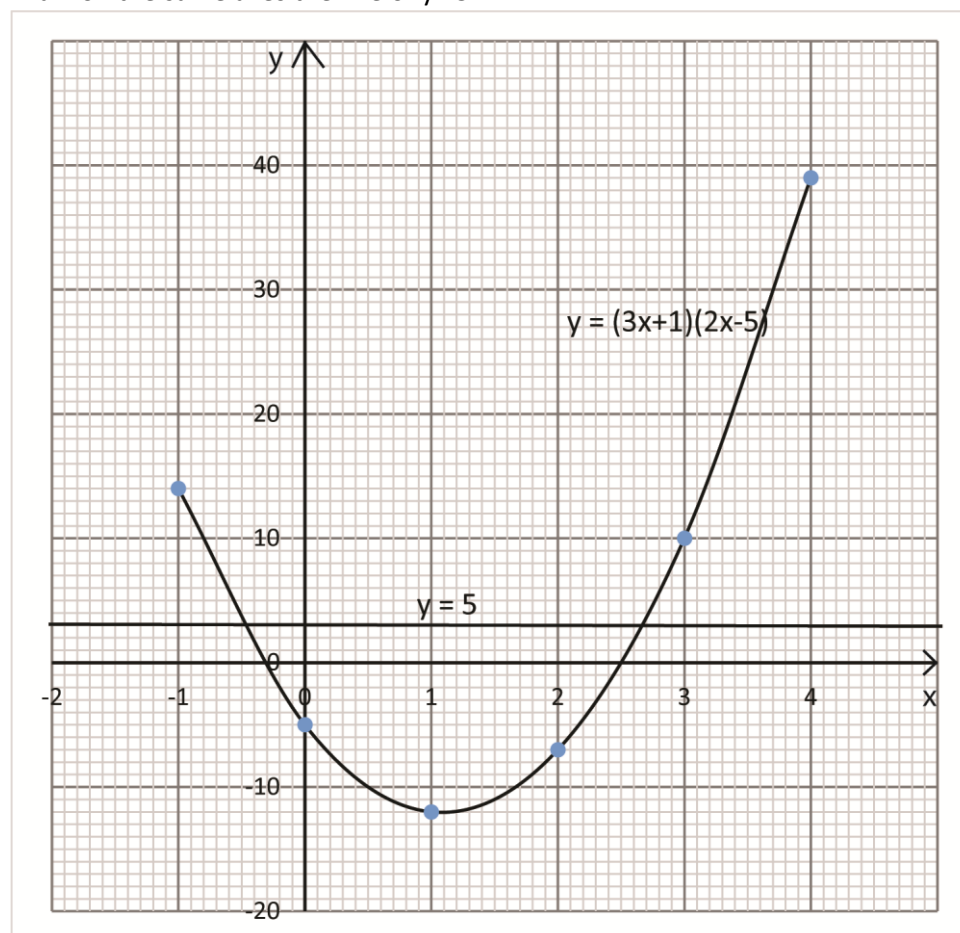
$$\text{Probability} = \frac{10}{36} = \frac{5}{18} = 0.278$$

15. (a) copy and complete the table for $y=(3x+1)(2x-5)$

X	-1	0	1	2	3	4
$3x+1$	-2	1	4	7	10	13
$2x-5$	-7	-5	-3	-1	1	3
y	14	-5	-12	-7	10	39

- (b) Use your completed table to draw a graph of $y = (3x+1)(2x-5)$ with a scale of 2cm for 1 unit on x-axis.

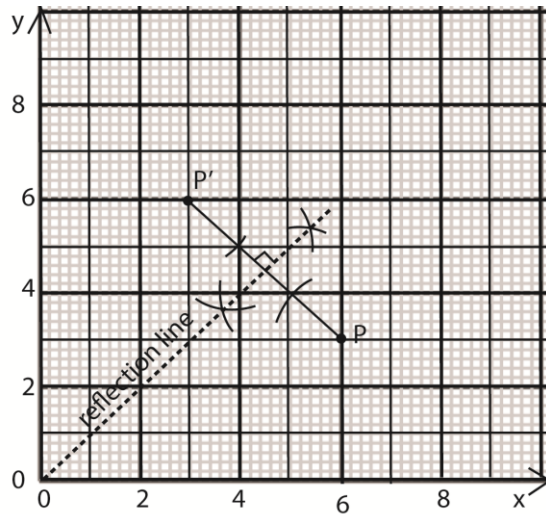
- (c) Draw on the same axes the line of $y=5$.



- (d) Use the two graphs in (b) and (c) to solve the equation $6x^2 - 13x - 10 = 0$.

Get points of intersection $x = 2.8$ or $x = -0.5$

16. (a) The image of $P(6, 3)$ after reflection is $P'(3, 6)$
- Plot the points P and P' on the graph paper.
 - Construct the line of reflection. Hence find the equation of the line of reflection.



Line of reflection is the perpendicular bisector of PP' . this line should be constructed
Possible points on the reflection line are $(0, 0)$, $(2, 2)$, $(4, 4)$
Hence the equation of the line of reflection is $x = y$.

- (b) The image of $ABCD$ under a matrix of transformation $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ is $A'B'CD'$; the coordinates of the image are $A'(1, 0)$, $B'(4, -6)$, $C'(4, -4)$ and $D'(1, 2)$. Determine the coordinates of A , B , C and D .

Object point = inverse matrix \times image

$$\text{Matrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$\text{Determinant} = 1 \times 1 - (0 \times -2) = 1 - 0 = 1$$

$$\text{Inverse matrix} = \frac{1}{\det} (\text{adjunct matrix})$$

$$= \frac{1}{1} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$\begin{matrix} & A' & B' & C' & D' & & A & B & C & D & & A & B & C & D \\ \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 4 \\ 0 & -6 \end{pmatrix} & \begin{pmatrix} 4 & 4 \\ 4 & 2 \end{pmatrix} & = & \begin{pmatrix} 1+0 & 4+0 & 4+0 & 1+0 \\ 2+0 & 8-6 & 8-4 & 2+2 \end{pmatrix} & = & \begin{pmatrix} 1 & 4 & 4 & 1 \\ 2 & 2 & 4 & 4 \end{pmatrix} \end{matrix}$$

Hence $A(1, 2)$, $B(4, 2)$, $C(4, 4)$, $D(1, 4)$

17. The manager of the cinema hall wishes to divide the seats available into two classes executive and ordinary. There are not more than 120 seats available. There must be at least twice as many ordinary seats as there are executive seats. Executive seats are priced at shs. 15,000 each. Ordinary seats are priced at shs. 10,000 each. At least shs. 1,000,000 should be collected at each show to meet expenses.

- (a) Taking x as the number of executive seats and y as the number of ordinary seats, write down five inequalities from the given information.

Number of seats

$$x + y \leq 120 \text{ (i)}$$

cost of seats

$$15,000x + 10,000y \geq 1,000,000 \text{ (ii)}$$

Ordinary seats are at least twice as executive

$$\Rightarrow y \geq 2x \text{ (iii)}$$

Hence five inequalities are

$$x + y \leq 120$$

$$15,000x + 10,000y \geq 1,000,000$$

$$y \geq 2x$$

$$y \geq 0$$

$$x \geq 0$$

- (b) Represent the inequalities on a graph.

For $x + y \leq 120$

The boundary line is $x + y = 120$

X	0	120
y	120	0

For $15,000x + 10,000y \geq 1,000,000$

The boundary line is $15,000x + 10,000y = 1,000,000$

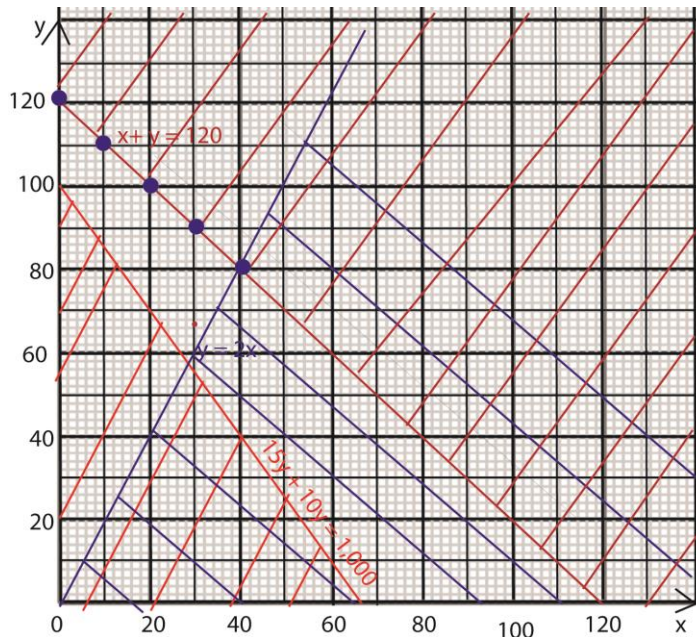
Or $15x + 10y = 1000$

X	0	60
y	100	10

For $y \geq 2x$

The boundary line is $y = 2x$

X	0	50
y	0	100



- (c) From the graph, find the number of seats of each kind which must be sold to give maximum profit.

For profit maximization, we pick extreme points on highest line

(40, 80), (30, 90), (20, 100), (10, 110), (0, 120)

The objective functions for profit maximization is

$15,000x + 10,000y$

Points	Function	Total
(40, 80)	$40 \times 15,000 + 80 \times 10,000$	1,400,000
(30, 90)	$30 \times 15,000 + 90 \times 10,000$	1,350,000
(20, 100)	$20 \times 15,000 + 100 \times 10,000$	1,300,000
(10, 110)	$10 \times 15,000 + 110 \times 10,000$	1,250,000
(0, 120)	$0 \times 15,000 + 120 \times 10,000$	1,200,000

Hence 40 executive and 80 ordinary seats should be sold for profit maximization (shs. 1,400,000)

Thank you

Dr. Bbosa Science