Course Name : Statistical Inference 1
Course code : MGS1204
Course Level : 1
Course Credit : 3
Contact Hours : 45

COURSE DESCRIPTION: This is an introductory course of statistical inference that concentrates on estimation of all parameters.

COURSE OBJECTIVES: The objective of the course is to equip students with the knowledge of estimation of population parameters from sample observations.

LEARNING OUTCOMES: By the end of the course unit the students should be able to use point and interval estimation techniques to generalize about the population parameters using the sample statistics.

COURSE OUTLINE

Topic	**Sub Topics**	**Hours**
Introduction to statistical inference	• Overview of statistical inference • Basic concepts used in Statistical inference which include among others, parameter, statistic, population, sample, Dependent and independent samples.	2
Sampling distributions, Central limit theorem and the law of large numbers	• Overview of sampling distributions • Sampling distribution of the sample mean, proof of the mean and variance of the sampling distribution of the mean. • Standard error of the mean • Central limit theorem and the law of large numbers. • Sampling distribution of the proportion. • T-distribution	12
Introduction to Estimation	• Overview of estimation • Estimator and Estimate • Point estimation (mean, proportion and variance) • Methods used in point estimation ✓ Maximum Likelihood Estimation ✓ Method of moments	28

	• Properties of estimators; Unbiasedness, consistency, efficiency and sufficiency and proof of the properties • Interval estimation ✓ Single mean (population standard deviation unknown and population standard deviation known) ✓ Single proportion ✓ Single variance ✓ Difference between two means. (Population variances known and Population variances unknown) ✓ Mean difference ✓ Ratio of two variances ✓ Differences between two proportions ✓ Simple linear regression parameters (intercept and slope) ✓ Correlation coefficient. (Pearson and Spearman)	
Sample size determination/calculation	• When the population standard deviation is known • When the population proportion is known and standard deviation is unknown • When the population proportion is unknown and standard deviation is unknown	3

Assessment

The course is assessed by assignments, coursework and final examinations whose contributions are shown below.

Coursework	30%
Final course examination	70%
Total	100%

Reference:

Casella, G., Berger, R.L. (2001). *Statistical Inference*. Duxbury Press. ISBN 0-534-24312-6

Cox, D. R. (2006). *Principles of Statistical Inference*, Cambridge University Press. ISBN 0-521-68567-2.

Held L., Bové D.S. (2014). *Applied Statistical Inference—Likelihood and Bayes* (Springer).

Kiefer, J. C. (1987). *Introduction to Statistical Inference.* New York: Springer-Verlag.

Rahlf, Thomas (2014). *"Statistical Inference*", in Claude Diebolt, and Michael Haupert (eds.), "Handbook of Cliometrics (Springer Reference Series)", Berlin/Heidelberg: Springer.http://www.springerreference.com/docs/html/chapterdbid/372458.html

Rao, C. R. (1973). *Linear Statistical Inference and Its Application.* New York: J. Wiley.

Ronald E.Walpole, 1998, "*Probability and Statistics for Engineers and Scientists*" Prentice Hall-Gale

Young, R. L. Smith, 2005, "*Essentials of Statistical Inference*" University of North Carolina, Chapel Hill, Cambridge University Press.

STATISTICAL INFERENCE 1 LECTURE NOTES.

Facilitators: Nuwamanya Wilber & Mpagi Julius

Recall that the purpose of descriptive statistics is to make the collected data more easily comprehensible and understandable. Some tools we examined in descriptive statistics include frequency distributions, measures of central tendency, and measures of dispersion, among others.

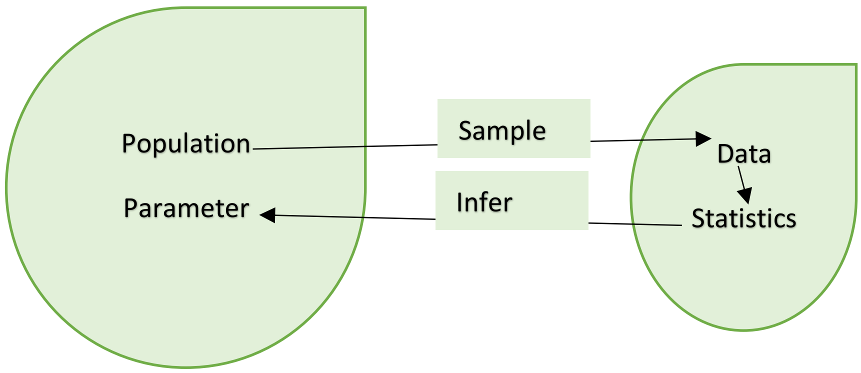
Because it is not always possible to address every member of the population, we take samples. The statistical question that needs to be answered is whether or not the characteristics observed in the sample are likely to reflect the true characteristics of the larger population from which the sample was taken. Inferential statistics provide us with the tools we need to answer this question.

Inference refers to reaching a conclusion based on available information/ knowledge or facts.

Statistical inference refers to the act of **generalizing** from a sample to a population with calculated degree of certainty. The aim of **statistical inference** is to make certain determinations with regard to the unknown **constants** known as parameter(s) in the underlying distribution. **In other words, Statistical inference** aims at learning characteristics of the population from a sample.

Illustration

We want to learn about population ***parameters*** but we can only calculate sample ***statistics***. *What do we do?*



Statistical Inference is divided into **estimation** and **hypothesis testing**. **Estimation** is carried out using numerous procedures that are used to estimate the population parameters using sample data while **Hypothesis testing** involves determining whether the population parameters estimated are realistic or not.

BASIC CONCEPTS IN INFERENTIAL STATISTICS:

1. **Population**: refers to all elements of interest. It is the totality of observations with which a statistician is concerned.
2. **Census**: This is when every member or unit in the population is surveyed.
3. **Sample:** is the subset of the population.
4. **Sampling Units**: These are people /items to be sampled.
5. **Sampling frame**: This is a list of sampling units.
6. **Parameter**: This is a numerical value describing the characteristic of the population. It is usually assumed to be fixed but unknown. Examples of parameters include the population mean (μ) and the population standard deviation (σ).
7. **Statistic**: This is a numerical value describing the characteristic of a sample. A statistic estimates a parameter and it changes with each new sample. **Examples of sample statistics**: sample mean ($\bar{x}$) and sample variance (s^2).

Symbols used to denote parameters and statistics.

Population parameter	**Sample statistic**
N: Number of observations in the population	n: Number of observations in the sample
N_i: Number of observations in population *i*	n_i: Number of observations in sample *i*
Π or P: Proportion of successes in population	p: Proportion of successes in sample
μ: Population mean	$\bar{x}$: Sample mean
σ: Population standard deviation	s: Sample standard deviation
σ^2: Population Variance	s^2: Sample variance

8. **Variable**: This refers to the characteristic being measured and can be described as either qualitative/ quantitative.
9. **Sampling:** This is the process of obtaining a sample from a population. A sample is usually taken to make useful inferences about the population. The sample must be representative of the whole population. However, there are errors involved, i.e. sampling errors and non-sampling errors. **Sampling errors** are errors that are introduced in the parameter by studying a sample rather than a population e.g. selecting a wrong sample/biased sample. A biased sample is a sample which consistently over/under estimates some/all of the characteristics of the population. Other sampling errors include wrong choices of a sampling unit, lack of a good sampling frame etc. **Non- sampling errors** are errors that are introduced in the parameter due to incorrect information like communication errors, transcription errors at data entry, ignorance on the part of respondents, deliberate false responses, etc.
10. **Sampling methods**: Once a sampling frame has been established, you can choose a method of sampling. There are two categories of this; namely- Random/Probability sampling methods and Non-random/ Non-probability sampling methods.
11. **Dependent samples**: These are samples in which the values in one sample affect the values in another sample. Such samples are paired/matched measurements for one set of items.
12. **Independent samples**: These are samples in which the values in one sample do not affect the values in another sample. I.e. the occurrence of one sample doesn't affect the occurrence of another sample.

SAMPLING DISTRIBUTIONS

Sampling distributions are probability distributions of statistics. In general, the sampling distribution of a given statistic is the probability distribution of the values taken by the statistic in all possible samples of the same size from the same population.

In other words, if we repeatedly collect samples of the same sample size from the population, compute the statistics (mean, standard deviation, proportion), and then draw a graph (histogram)/frequency distribution table of those statistics, the distribution of that histogram/table is called the sampling distribution of the statistics (mean, standard deviation, proportion).

Steps in generating a sampling distribution.

- Choose a population and sample for this experiment.
- Select a sample randomly out of the given population.
- Calculate the sample statistic.
- Follow the above steps for obtaining a number of similar statistics out of the same population.
- Generate a frequency distribution: Plot the statistics on a graph or tabulate the data. The final graph or table will represent your sampling distribution.

Significance of Sampling Distributions

The primary purpose of Sampling Distribution is to establish representative results of small samples of a comparatively larger population. This helps researchers and analysts to dig deep into the population, get a closer look into small groups of the population, and create generalized results based on the sample. The significance of sampling distribution is immense in the field of statistics.

- Firstly, the concept of sampling distribution provides accuracy. For any population being studied, it is important for a researcher to collect all possible samples to generate an inclusive and effective result. Sampling Distribution allows one to do that by collecting all possible samples and developing the sample statistics to give the best possible result.

- Secondly, the repeated collection of samples from the same set of subjects leads to consistency. What's more, the standard error also allows a researcher to reflect on the deviation and thus identify the unbiased nature of the sampling distribution altogether.

- Thirdly, the variability of the sampling distribution is immensely significant as it reflects the inclusion of numerous samples from the same set of subjects. This leads to an almost symmetric graph. The variability also ensures that all possible samples are collected from the population.

Sampling distribution of the sample mean

The sampling distribution of the sample mean focuses on calculating the means of all possible samples of sample size n which are then arranged to form a probability distribution of the sample mean. When the average of every sample is put together, the mean and variance of the sampling distribution is calculated which reflects the nature of the whole population.
Illustration

Imagine a population with 3 members. Let us select 6 random samples (with replacement) each of sample size 2 and note down the height for each member of the sample. The sampling distribution of the sample mean height can be obtained as below,

Step 1. Draw the 6 samples, each of size 2 and calculate the mean height for each.

Sample	Members of the sample	Sample average height
1	6.2ft,5.2ft	5.7ft
2	5.5ft,6.2ft	5.85ft
3	6.2ft,6.2ft	6.2ft
4	5.5ft,5.5ft	5.5ft
5	5.2ft,6.2ft	5.7ft
6	5.2ft,6.2ft	5.7ft

Step 2: Construct the sampling distribution of the sample mean

Sample average height (ft)	5.7	5.85	5.5	6.2
Frequency	3	1	1	1
Probability	3/6	1/6	1/6	1/6

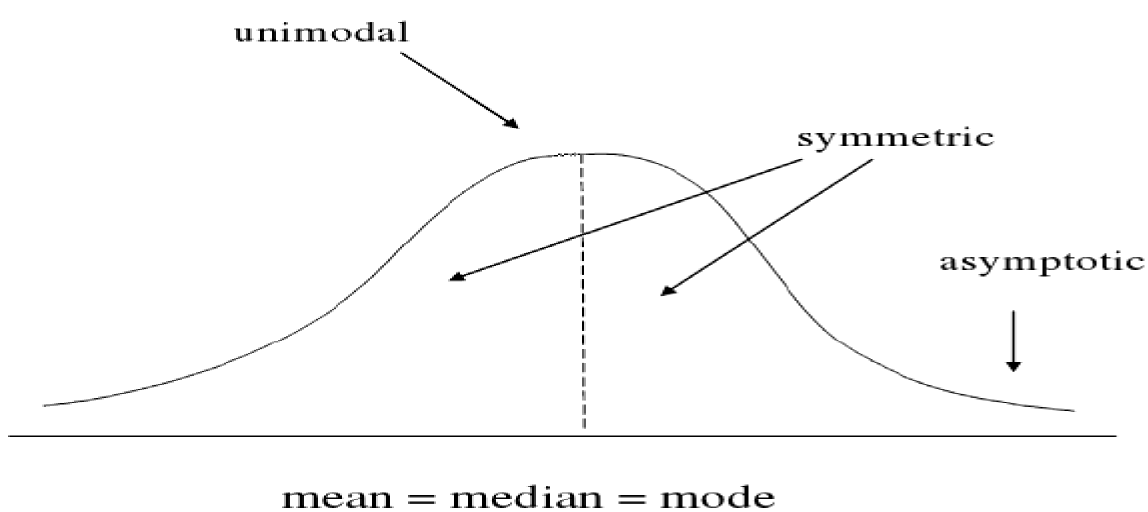
Sampling Distribution of Sample Means from a Normal Population

A normal distribution is one in which the values are evenly distributed both above and below the mean. A population has a precisely normal distribution if the mean, mode, and median are all equal. For example, the population of 3,4,5,5,5,6,7, the mean, mode, and median are all 5 hence being normally distributed.

A normal distribution can also be defined as a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean. In graph form, normal distribution will appear as a bell curve.

Properties of the normal distribution.

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.
- Unimodal (one mode)
- Asymptotic to the x-axis. i.e. as the distance from the mean increases the curve approaches to the base line more and more closely.



Mean and Variance of the Sampling distribution of the sample mean from a normal population.

If the population is normal, then the sample mean also has a normal distribution, regardless of the sample size. For samples of any size drawn from a normally distributed population, the sample mean is normally distributed, with mean $\mu_{\bar{X}}$ = μ and variance $\frac{\sigma^2}{n}$, where n is the sample size. This implies that for a normally distributed population, the mean of the sampling distribution of the sample mean is the mean of the population from which the scores were sampled.

Proof

Suppose you draw n random independent observations $x_1, x_2, x_3, \dots, x_n$ from a normally distributed population with mean μ and variance σ^2.

Mean is given as,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$E(\bar{x}) = \mathrm{E}\left[\frac{x_1+x_2+x_3+\cdots+x_n}{n}\right]$$

But $E(ax) = aE(x)$

$$E(\bar{x}) = \frac{1}{n}\mathrm{E}[x_1] + \frac{1}{n}\mathrm{E}[x_2] + \frac{1}{n}\mathrm{E}[x_3] + \cdots + \frac{1}{n}\mathrm{E}[x_n]$$

But $E(x) = \mu = \mathrm{E}[x_1] = \mathrm{E}[x_2] = \mathrm{E}[x_3] = \cdots = \mathrm{E}[x_n]$

$$E(\bar{x}) = \frac{1}{n}(\mu + \mu + \mu + \cdots + \mu)$$

$$E(\bar{x}) = \frac{1}{n}(n\mu)$$

$E(\bar{x}) = \mu,$ which is the mean of the sampling distribution of the sample mean.

Variance is given as,

$$\mathrm{Var}(\bar{x}) = Var(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n})$$

$$Var(\bar{x}) = \mathrm{Var}\left(\frac{x_1}{n}\right) + \mathrm{Var}\left(\frac{x_2}{n}\right) + \mathrm{Var}\left(\frac{x_3}{n}\right) + \cdots + \mathrm{Var}\left(\frac{x_n}{n}\right)$$

But $Var(ax) = a^2 Var(x)$

$Var(\bar{x}) = \frac{1}{n^2}\mathrm{Var}(x_1) + \frac{1}{n^2}\mathrm{Var}(x_2) + \frac{1}{n^2}\mathrm{Var}(x_3) + \cdots + \frac{1}{n^2}\mathrm{Var}(x_n)$

But $Var(x) = \sigma^2 = \mathrm{Var}(x_1) = \mathrm{Var}(x_2) = \mathrm{Var}(x_3) = \cdots = \mathrm{Var}(x_n)$

$Var(\bar{x}) = \frac{\sigma^2}{n^2} + \frac{\sigma^2}{n^2} + \frac{\sigma^2}{n^2} + \cdots + \frac{\sigma^2}{n^2}$

$Var(\bar{x}) = n(\frac{\sigma^2}{n^2})$

$Var(\bar{x}) = \frac{\sigma^2}{n}$, which is the variance of the sampling distribution of the sample mean.

And, the **standard error of the mean** (***SEM***) = $\sqrt{(Var(\bar{x}))} = \frac{\sigma}{\sqrt{n}}$

Generally, if $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the sample mean of a random sample of size n drawn from a normal population having mean μ and standard deviation σ, then $\bar{x}$ follows an exact normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. That is, $x_i \sim$ N (μ, σ) =⇒ $\bar{x} \sim$ N (μ, $\frac{\sigma}{\sqrt{n}}$).

CENTRAL LIMIT THEOREM AND THE LAW OF LARGE NUMBERS

Central limit theorem states that;

If a random sample of size n is selected from any population with mean μ and standard deviation σ, then $\bar{x}$ is approximately N (μ, $\frac{\sigma}{\sqrt{n}}$) when n is sufficiently large.

i.e.

If $\bar{x}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ ^{2}, then $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \longrightarrow$ N (z; 0,1) as n ⟶ ∞. Where Z is a standard normal distribution.

NOTE. The Central Limit Theorem is important because, for reasonably large sample size, it allows us to make an approximate probability statement concerning the sample mean, without knowledge of the shape of the population distribution.

- Again, one of the essential assumptions is a random sample.
- The distribution of X has the approximately normal distribution if the random sample is from a population other than normal.
- How large a sample size? Usually, it would be safe to apply the CLT if n ≥ 30. It also depends on the population distribution, however. More observations are required if the population distribution is far from normal.

The Law of Large numbers states that;

As sample size increases, its mean gets closer to the average of the whole population

Example 1:

The average male drinks 2 L of water when active outdoors with a standard deviation of 0.7 L. You are planning a full day nature trip for 50 men and will bring 110 L of water.

Required;

i) What is the mean and standard deviation of the sampling distribution of sample mean?
ii) State the sampling distribution of the sample mean.
iii) What is the probability that you will run out of water?

Example 2.

An auto-maker does quality control tests on the paint thickness at different points on its car parts since there is some variability in the painting process. A certain part has a target thickness of 2mm. The distribution of thicknesses on this part is skewed to the right with a mean of 2mm and a standard deviation of 0.5mm. A quality control check on this part involves taking a random sample of 100 points and calculating the mean thickness of those points.

Required;

i) What is the shape of the sampling distribution of the sample mean thickness?
ii) Find the mean and standard deviation of the sampling distribution of the sample mean.
iii) Assuming the stated mean and standard deviation of the thicknesses are correct, what is the probability that the mean thickness in the sample of 100 points is within 0.1mm of the target value?

Student's t distribution/t distribution.

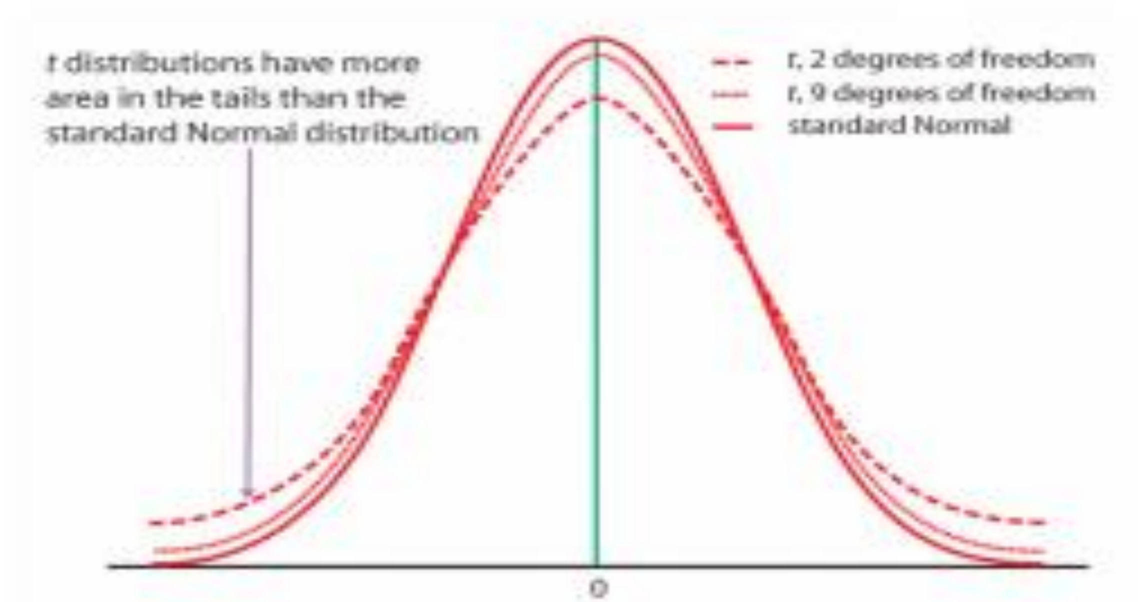
We have learnt that $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ (exactly or approximately) follows the standard normal distribution, where the data are from a random sample of size n from the population with mean μ and standard deviation σ. And, it is very likely that both μ and σ are unknown parameters. In practice, it suffices that the distribution is symmetric and single-peaked unless the sample is very small. Since most of the simple work in statistical inference focuses on the unknown population mean μ, we will need to deal with the unknown σ especially when n is not large **(n<30)**. It is quite intuitive and natural to estimate the unknown population standard deviation σ using the sample standard deviation, s.

We have another statistic $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ $\boldsymbol{instead\ of}$ $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

Let $x_1, x_2, x_3, \ldots, x_n$ be independent random variables that are all normal with mean μ and standard deviation σ. Let $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$. Then the random variable; $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ has a t-distribution with ν = n−1 degrees of freedom.

NOTE. When n is very large, s is a very good estimate of σ, and the corresponding t distributions are very close to the normal distribution. The t distributions become wider for smaller sample sizes, reflecting the lack of precision in estimating σ from s.

In statistics, **the number of degrees of freedom** is the number of values in the final calculation of a statistic that are free to vary.



It can be seen that the t-distributions have slightly greater variability than the standard normal distribution. Also, as degrees of freedom increase, the t-distribution curve gets closer to the standard normal curve.

Properties of the Student t distribution

• The t distribution is different for different sample sizes, or different degrees of freedom.

• The t distribution has the same general symmetric bell shape as the standard normal distribution, but it reflects the greater variability (with wider distributions) that is expected with small samples.

• The t distribution has a mean of 0.

• The standard deviation of the t distribution varies with the sample size, but it is greater than 1.

• As the sample size n gets larger, the t distribution gets closer to the standard normal distribution.

Sampling distribution of sample proportions (p-hat)

The probability distribution of the values of the sample proportions (p-hat) in repeated **samples** of the same size is called the **sampling distribution of p-hat**.
If the population is normally distributed/approximately normally distributed with a proportion of p, then random samples of the same size drawn from the population will have sample proportions close to p. More specifically, the sampling distribution of sample proportions will have a mean of p.

Consider a sample proportion $\hat{p} = \frac{X}{n}$ where x is the number of subjects in the sample with the characteristic of interest and n is the sample size. x is a binomial random variable with parameters n and p.

The binomial random variable x has;

- Mean = np
- Variance = np (1 – p)
- Approximately normal distribution for large sample sizes.

The sampling distribution of sample proportion $(\hat{p})$ has mean p and variance $\frac{p(1-p)}{n}$.

The standard deviation of all sample proportions $(\hat{p})$ is exactly $\sqrt{\frac{p(1-p)}{n}}$

Proof;

Mean

$$E(\hat{p}) = E(\frac{X}{n})$$

$$E(\hat{p}) = \frac{1}{n}E(X)$$

$$E(\hat{p}) = \frac{1}{n}(np)$$

$$E(\hat{p}) = p$$

Variance

$$Var(\hat{p}) = Var(\frac{X}{n})$$

$$Var(\hat{p}) = \frac{1}{n^2}Var(X)$$

$$Var(\hat{p}) = \frac{1}{n^2}(np(1-p)$$

$$Var(\hat{p}) = \frac{p(1-p)}{n}$$

Standard deviation = $\sqrt{Var(\hat{p})} = \sqrt{\frac{p(1-p)}{n}}$

Note:

- The sample size required to achieve approximate normality depends on the value of p, i.e.; if p is close to 0.5, the sample size doesn't need to be very large. Whereas if p is close to 0 or 1, a much larger sample size is required.

- Since the sample size n appears in the denominator of the square root, the standard deviation does decrease as sample size increases. Finally, the shape of the distribution of p-hat will be approximately normal as long as the sample size n is large enough. The convention is to require both **np** and **n (1 – p)** to be at least **5.**

- We standardize using $Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

Example.

A random sample of 100 students is taken from the population of all part-time students in the United States, for which the overall proportion of females is 0.6.

(a) There is a 95% chance that the sample proportion (p-hat) falls between what two values?
First note that the distribution of p-hat has mean p = 0.6, standard deviation

(b) What is the probability that sample proportion p-hat is less than or equal to 0.56?

ESTIMATION

Estimation refers to the process of using numerous procedures to estimate the population parameters using sample data. The formula/rule/procedure that is used to calculate an estimate of a population parameter is called an **Estimator** while the numerical value that is used to estimate the population parameter is called an **Estimate**.

An estimate of a population parameter may be expressed in two ways:

1.**Point estimate**. A point estimate of a population parameter is a single value used to estimate the population parameter. For example, the sample mean $\bar{x}$ is a point estimate of the population mean μ. Similarly, the sample proportion $\hat{p}$ is a point estimate of the population proportion p.

2.Interval estimate. An interval estimate refers to the range of values within which a population parameter is said to lie. For example, $a < \mu < b$ is an interval estimate of the population mean μ.

POINT ESTIMATION

This is when a single sample statistic is taken as the estimate of the unknown population parameter.

Example:

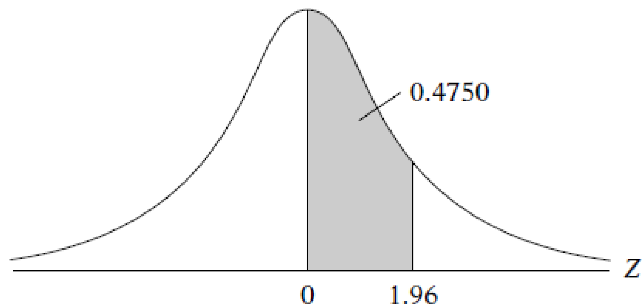
A random sample of 10 students was drawn from a statistics class of 100 students and their ages were found to be as follows: 21,20,21,22,23,25,25,25,22,22. Find the point estimates for the population mean and variance of the students' ages in the class.

AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION

Example

$\Pr(0 \le Z \le 1.96) = 0.4750$

$\Pr(Z \ge 1.96) = 0.5 - 0.4750 = 0.025$



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., $Z \ge 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $P(-1.96 \le Z \le 0) = 0.4750$. Therefore, $P(-1.96 \le Z \le 1.96) = 2(0.4750) = 0.95$.