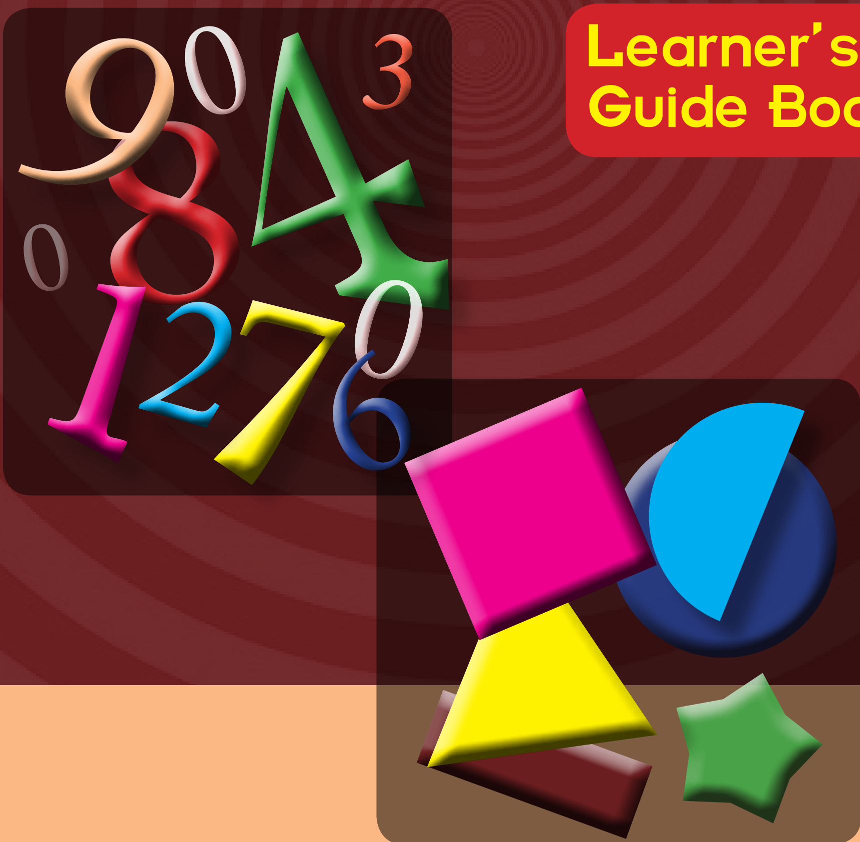
Mathematics

Integrated with Technology

for Selected Sub-Topics



Learner's Guide Book

NCDC
NATIONAL CURRICULUM DEVELOPMENT CENTRE

THE REPUBLIC OF UGANDA
Ministry of Education and Sports

Mathematics

Integrated with Technology

for Selected Sub-topics

Learner's Guide Book

Uganda Certificate of Education
(UCE)

National Curriculum Development Centre
P.O. Box 7002, Kampala
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Contents

Preface

The major reason for undertaking this new approach to teaching-learning Sciences and Mathematics at O level was to improve the academic achievements and interest of learners.

The idea of integrating technology into Sciences and mathematics at lower secondary education was conceived after realising that Science and mathematics teaching at this level hardly generated interest in the learners. In addition, these learners were lacking in practical skills and rarely applied Science and Mathematics to everyday life situations.

The National Curriculum Development Centre (NCDC) has developed this new approach of teaching Science and Mathematics by including practical activities that enable learners to come up with products or be able to offer services to the community leading to national sustainable development by generating their own income.

Acknowledgment

This book was developed and prepared in a series of writing and discussion workshops organised by the National Curriculum Development (NCDC) between 2007 and 2011.

I wish to acknowledge the contribution of the subject panel members, teachers and learners from various schools who participated at trial activity phase and other education stakeholders that provided input and direction of this material.

I am grateful to all those who worked behind the scenes for the commitment in ensuring the work is done and feedback from the field is incorporated.

I also thank the African Development Bank (ADB) Project Phase-III for the financial support.

Last but not least, I wish to recognise agencies, companies and websites for the reference materials and pictures used in this book.

Connie Kateeba
Director
National Curriculum Development Centre (NCDC)

Introduction

The idea of integrating Mathematics with Technology is a result of efforts to improve performance and developing an interest in the Mathematics teaching/ learning process. It involves applying Mathematical concepts into real life experiences.

The integration of Mathematics with technology encourages the application of theoretical knowledge to sustainable economic development. It aims at providing information and knowledge to learners that can be used to develop competences that can result in the provision of a service and/or a commercial product. Entrepreneurial skills are integrated with Mathematical concepts with a view to achieving academic excellence and economic development.

Rationale

The purpose of developing a learner's guide book which integrates Mathematics with Technology is to guide the teaching/ learning process towards academic achievement by applying the knowledge acquired in a classroom setting to real-life experience. You should develop the ability to use abstract knowledge or concepts to develop a commercial product or provide a service with a view to improving the standard of living and achieving sustainable development.

How to Use this Guide Book

This Guide Book is meant to be used together with the recommended Mathematics textbooks for S1 – S4. The Guide Book contains only sample sub-topics in which some activities are to be done. This means that not all topics/sub-topics are covered. Students/ teachers can select other activities to be done.

Assessment has not changed so you will be assessed as it has been that is daily exercises, weekly/monthly or end of term examinations. The final (summative) examination will be done at the end of S4.

Senior One

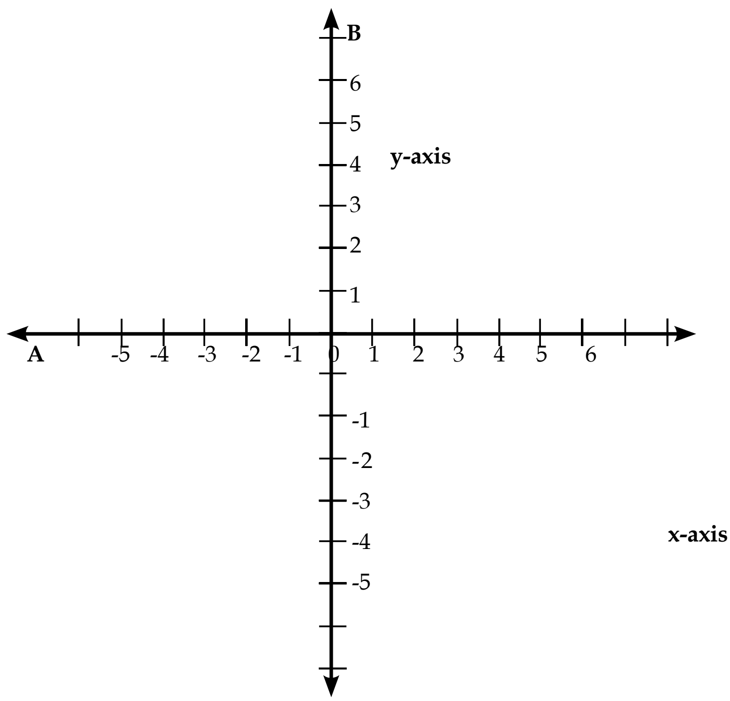
TOPIC 1 COORDINATES

Coordinates refer to a description of either a point or a place in two dimensions or three dimensions. It is an ordered pair of numbers which is expressed in terms of x-value and y-value for two dimension p(x, y) and in three dimension it is expressed as p(x, y, z).

Sub-Topic: Coordinates in Two Dimensions

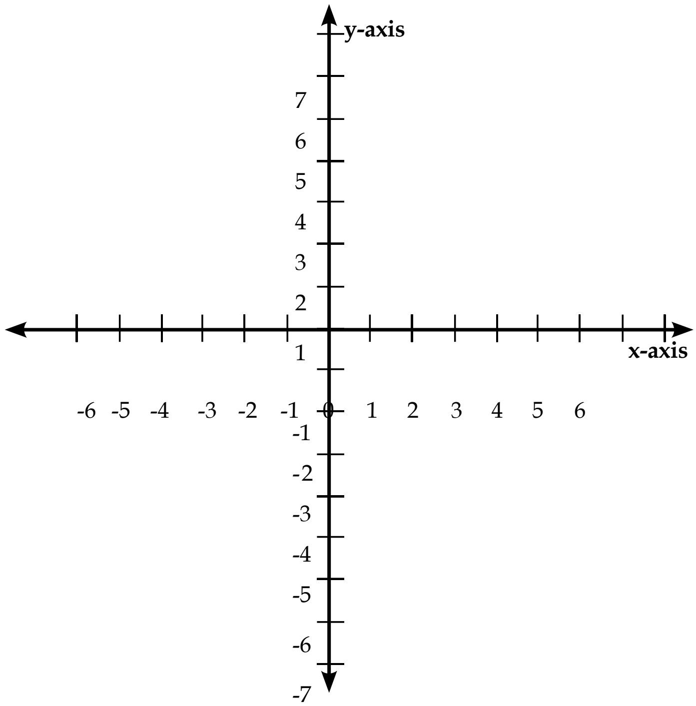
Coordinates

In coordinates the knowledge of a number line and integers is required. The horizontal number line is referred to as the X-axis and vertical one is the Y-axis. The combination of two number lines form a co-ordinate graph.



Hint: The two number lines must be perpendicular to each other.

Line A is the X-axis and line B is the Y-axis as illustrated in the figure below.



On each axis, the interval must be regular.

Coordinates are used in locating places/positions in a given space.

Exercise

Write down the coordinates of points indicated on Figure 1:

Answer

i) The coordinates for A are (2, 1)

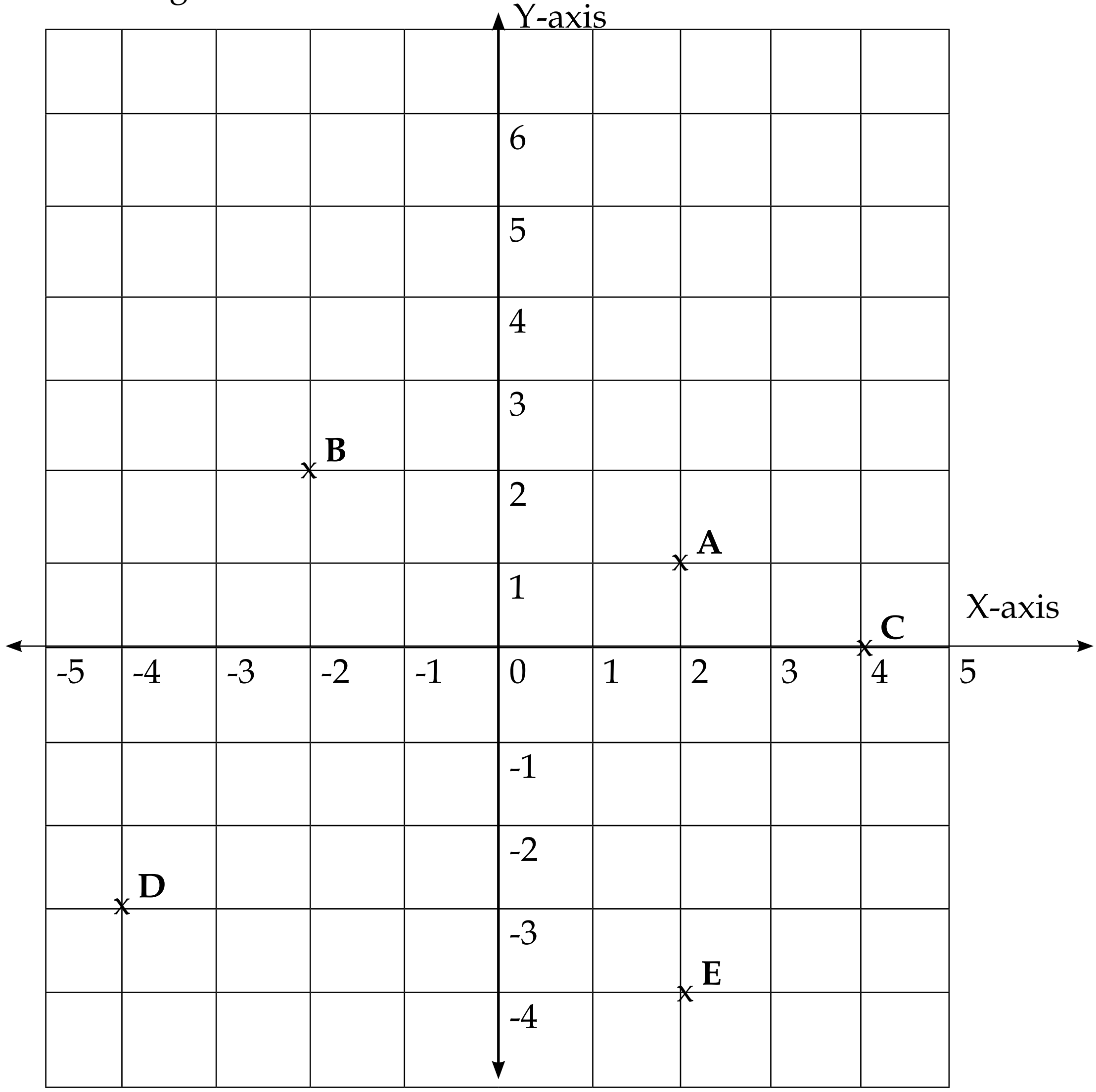
ii) The coordinates for B are (-2, 2)

iii) The coordinates for C are (4, 0)

iv) The coordinates for D are (-4, -3)

The coordinates for E are (2, -4)

Figure 1



Note: Points on the graph are described by the coordinates system i.e p(x,y). The x-coordinate is written first followed by y-coordinate.

Exercise 1

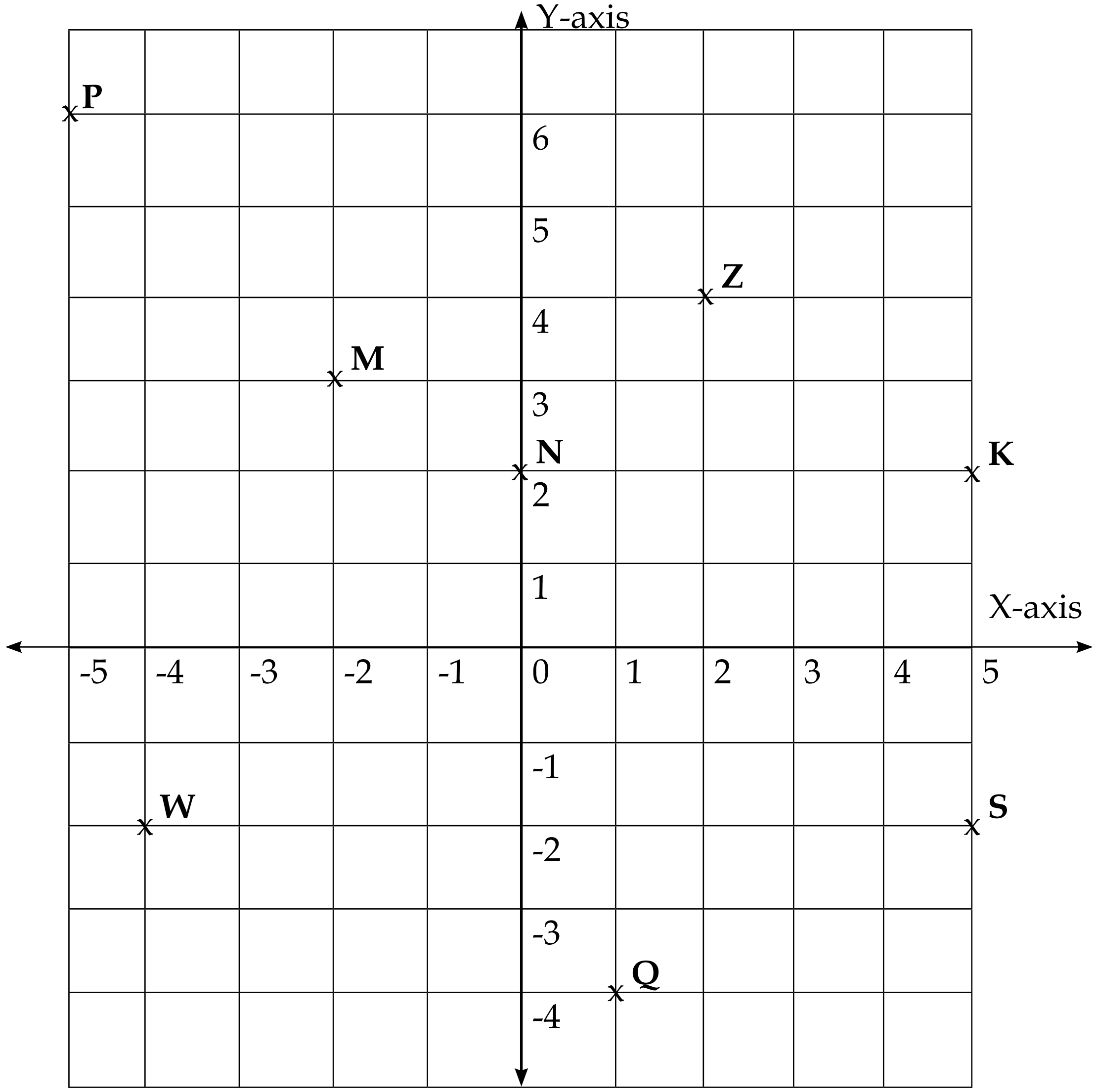
1. Plot the following points on the coordinates graph

   i) A(2, 3), ii) B(-3, 0) iii) C(4, 4) (iv) D(-2, -5)

   v) E(0, 7) vi) F(-4, -2)

2. Plot the following points on the coordinate system A(0, 3), B(1, 2), C(0, -1), D(-1, 2). Join A to B, B to C and C to D,0 What figure do you get?

3. State the coordinates of the points indicated on the figure below.



Generally, the coordinate of a given point on the Cartesian graph is described by recording its x-value, y-value i.e. p(x, y). The x-value is read from the x-axis and the y-value is read from the y-axis.

Think: How would one locate a particular house in a given housing estate?

Example 2

Imagine a meeting of 30 people seated in columns and rows as in the figure 3 below.

Figure 3

5						
4				Tr		
3	S					
2			VC			
1			CM			
	1	2	3	4	5	6

Key

S - Secretary VC - Vice Chairman

CM - Chairman Tr - Treasurer

Using the idea of columns and rows, there are 6 columns and 5 rows. We can describe the position of:

i) Vice Chairman as (3,2)

ii) Secretary as (1,3)

iii) Treasurer (4,4)

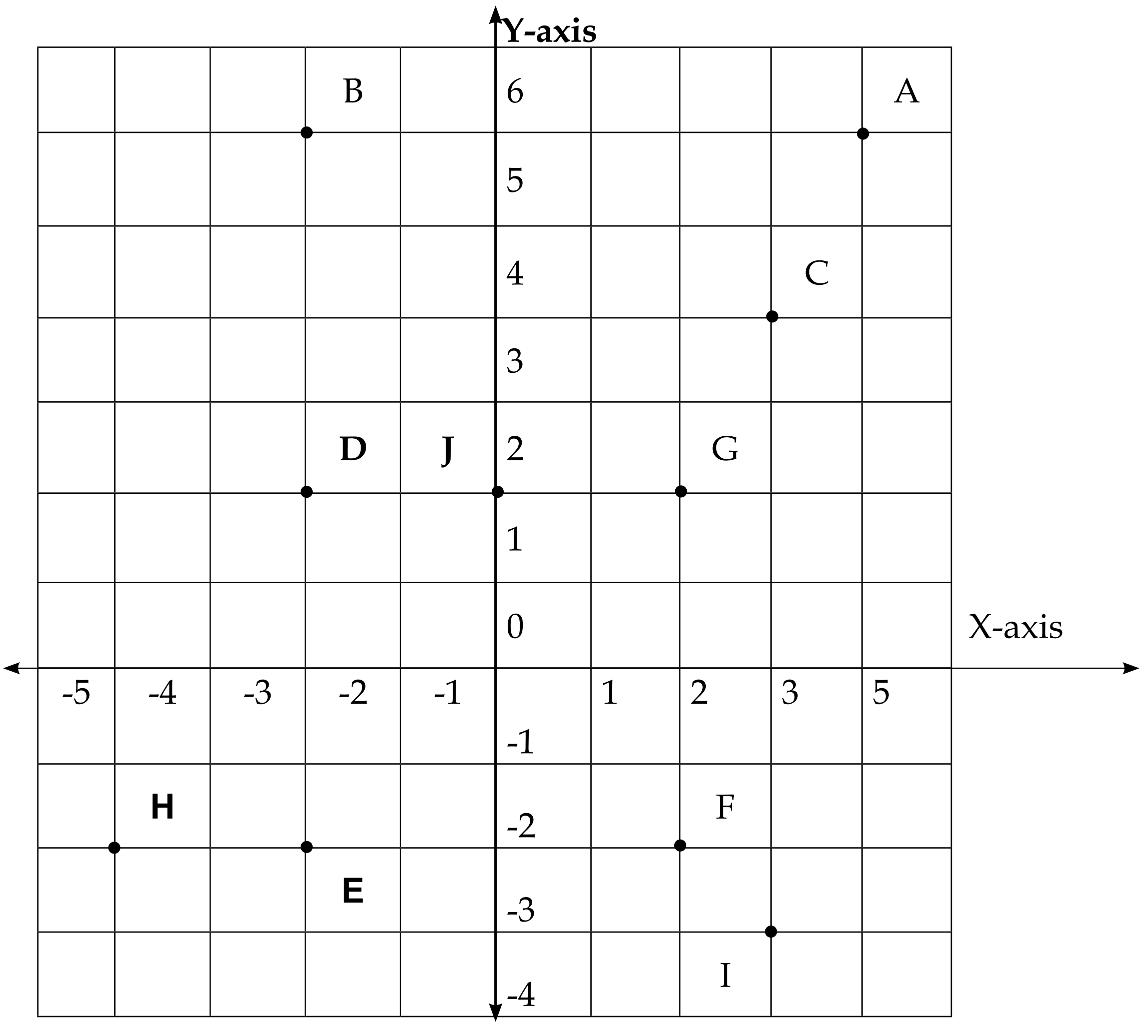
Example 3

Draw a Cartesian graph. Plot the following points:

A (2,0) B(0,3) C(4,3) D(-2,-1).

Example 4

State the coordinates of the points indicated with letters on the graph.



Exercise 2

1. On the same graph, plot the points A(0,4) B(-2,1) C(2,1) and D(0,-3). Name the figure formed.

Activity: Sitting classroom plan (Location of places in a classroom)

Introduction: It is the identification of points on the Cartesian graph.

Materials: Manila paper, markers, ruler, desks, cello tape/ glue, pair of scissors.

Procedure

i) Arrange the desks in columns and rows.

ii) Get manila papers and cut them into pieces of rectangles say 30cm by 15cm.

iii) Get one piece of the rectangle; write your name on it using a marker.

iv) Fix it on your desk using cello tape or glue for identification.

v) Label columns as X-lines and rows as Y-lines.

vi) Locate the origin among the arranged desks in the classroom.

vii) Locate your place by stating the number of column first, followed by the number of row.

viii) Write the number pair as (x, y).

Note: Write column followed by row for coordinates (x,y).

Observation: Columns represent x-values and rows represent y-values.

Interpretation: Sitting position in the class can be described by coordinates.

Application: Coordinates are used in the identification of places on the map, sitting arrangements in examination rooms, Geography of places and tickets in the theatre.

Follow up Activity: Identify where the knowledge of coordinates is applicable.

TOPIC 2 NUMBER BASES

Sub-Topic: Place Values

Introduction

In our daily lives, we usually use decimal (base 10) system in counting, writing and reading figures. In some cultures like Ateso, they use base 5 in counting and reading numbers. Other systems (bases) can also be used for example in computer technology, binary (base two) system is used.

Decimal system (Base Ten)

In base ten, the following numerals are used 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This means that when any mathematical operation is carried out, the answer is expressed/represented by the above numerals.

In decimal, place values are assigned from the right to left of the whole numbers in power of 10 i.e; ones, tens, hundreds, thousands, etc.

Operations in decimal system

Addition

Example 1

(i)
$$\begin{array}{rr} & 637_{ten} \\ + & \underline{524_{ten}} \\ & \underline{1161_{ten}} \end{array}$$

(ii)
$$\begin{array}{rr} & 316_{ten} \\ + & \underline{721}^{ten} \\ & \underline{1037}_{ten} \end{array}$$

Subtraction

Example 2

$$\begin{array}{r} 529_{ten} \\ -\ 63_{ten} \\ \hline 466_{ten} \\ \hline \end{array} \qquad \text{(ii)} \qquad \begin{array}{r} 716_{ten} \\ -\ 409_{ten} \\ \hline 307_{ten} \\ \hline \end{array}$$

Multiplication

Example 3

$$\begin{array}{r} 735_{ten} \\ \times\ 13_{ten} \\ \hline 2205_{ten} \\ +\quad 735_{ten} \\ \hline 9555_{ten} \\ \hline \end{array}$$

Division

Example 4

50,157 ÷ 9

$$\begin{array}{r} \mathbf{5573} \\ 9\,\big)\overline{50,157} \\ -\ 45 \\ \hline 51 \\ -\ 45 \\ \hline 65 \\ -\ 63 \\ \hline 27 \\ -\ 27 \\ \hline \end{array}$$

So 50,157 ÷ 9 = 5,573

Exercise 1

Perform the following operations:

1.
$$\begin{array}{r} 5\,3\,4_{ten} \\ +\ 5\,6\,9_{ten} \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{r} 8\,2\,4_{ten} \\ -\ 3\,1\,1_{ten} \\ \hline \\ \hline \end{array}$$

3. $3.534 \times 9_{ten}$

4 $6300_{ten} \div 21_{ten}$ = ____________ $x\ 9_{ten}$

5 Find the difference of 723_{ten} and 178_{ten}

6 Find the total sum of 1230_{ten} and $617_{ten.}$

7 Share 3015 cows amongst three children.

Other Bases

In addition to base ten, other bases can be used to represent numbers or things. The four mathematical operations can be done in these bases.

The following are numerals used in some of the common bases.

Binary system (base two) {0, 1}.

Base five {0, 1, 2, 3, 4} and base six {0, 1, 2, 3, 4, 5} as numerals to use.

Base twelve (duodecimal) has {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e}.

The place values at these bases are expressed in powers of their respective bases i.e in binary base place values include ones, twos fours, eights, etc,

Base five include; ones, fives, twenty fives, etc.

Base six include; ones, sixes, thirty sixes, etc.

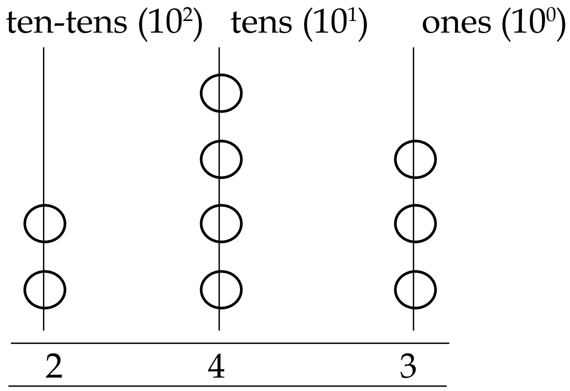
Base twelve include; ones, twelves, one hundred forty fours, etc.

For example when you are given 243_{ten}.

It means that the place value of three is ones but its value is 3, the place value of four is tens but its value is $4_{tens} = 40$.

The place value of 2 is ten-tens but its value is $2 \times 100 = 200$.

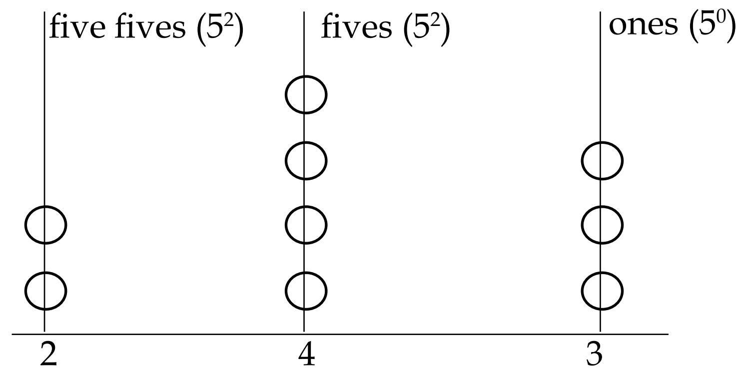
This can also be illustrated on an abacus.



Similarly, the same number can be put in base five $2\ 3\ 4_{five}$

This means the place value of three is ones but its value is 3, the place value of four is fives but its value is four fives $= 4 \times 5 = 20$, and the place value of two is five-fives its value is two-five-five $= 2 \times 5 \times 5 = 50$.

And on an abacus



Conversion of numbers from decimal to other bases

Note: Place value refers to the position of a digit and value is the quantity.

Example 1

Change 37_{ten} to base:

i) two ii) five iii) six iv) duodecimal

Answer

i) $37_{ten} = 100101_{two}$

2	37	
2	18	1
2	9	0
2	4	1
2	2	0
	1	0

ii) $37_{ten} = 122_{five}$

5	37	
5	7	2
	1	2

iii) $37_{ten} = 101_{six}$

6	37	
6	6	1
	1	0

iv) $37_{ten} = 31_{twelve}$

12	37	
	3	1

Note that when changing from decimal to any other base, you divide the given number by the base you are changing to until the quotient value is less than the base as illustrated in the examples above .

Example 2

Change the following to base ten

i) 144_{five} ii) 154_{six} iii) 20_{twelve} iv) 10110_{two}

Answer

i) $144_{five} = (1 \times 5^2) + (4 \times 5^1) + (4 \times 5^0)$

$= 1 \times 25 + 20 + 4 = 49_{ten}$

ii) $154_{six} = (1 \times 6^2) + (5 \times 6^1) + (4 \times 6^0)$

$= 36 + 30 + 4 = 70_{ten}$

iii) $20t_{twelve} = (2 \times 12^2) + (0 \times 12^1) + (t \times 12^0)$

$= (2 \times 144) + 0 + 10$

$= 288 + 10 = 298_{ten}$

iv) $10110_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= 16 + 0 + 4 + 2 + 0 = 22_{ten}$

Note that when you are changing from any base to decimal, you expand the given number.

Example 3

Change 135_{six} to the following bases:

i) decimal ii) duodecimal iii) five

Answer

i) $135_{six} = (1 \times 6^2) + (3 \times 6^1) + (5 \times 6^0)$

$= 36 + 18 + 5 = 59_{ten}$

ii) $135_{six} = 59_{ten}$

Duo decimal

12	59	
6	4	e

$= 4e_{twelve}$

iii) $135_{six} = 59_{ten}$

To base five

5	59	
5	11	4
	2	1

$= 214_{five}$

Note: When changing from a given base to any other base, change the number to decimal then to the required base.

Exercise 2

1. Change 104 to base:

   i) two ii) five iii) six iv) twelve

2. Change $1te_{twelve}$ to base:

   i) decimal ii) binary iii) six iv) five

Operations in various bases

Addition

In addition, we follow the idea of place values. If the sum of digits is equal or more than the base number, you divide by the base number, write the remainder and carry the quotient as illustrated below.

$$\begin{array}{r} 1 \\ 1011_{two} \\ +\ 11_{two} \\ \hline 1110 \\ \hline \end{array}$$

$2\,\underline{|2}$

1 r 0

$2\,\underline{|3}$

1 r 1

Example 1

1. Work out the following in the given bases.

a)
$$\begin{array}{r} 1\,0\,1\,1_{two} \\ +\quad 1\,1_{two} \\ \hline \end{array}$$

b)
$$\begin{array}{r} 1\,4\,3_{five} \\ +\,2\,1\,0_{five} \\ \hline \end{array}$$

2. Students activity work out using an abacus.

a)
$$\begin{array}{r} 1\,5\,1\,3_{six} \\ +\,2\,0\,3_{six} \\ \hline \end{array}$$

b)
$$\begin{array}{r} 1\,4\,6\,5_{twelve} \\ +\,3\,6\,5\,6_{twelve} \\ \hline \end{array}$$

Answer

1. a)
$$\begin{array}{r} 1\,0\,11_{two} \\ +\quad 11_{two} \\ \hline \underline{1\,1\,10} \end{array}$$

b)
$$\begin{array}{r} 1\,4\,3_{five} \\ +\,2\,1\,0_{five} \\ \hline \underline{4\;\;0\,3}_{five} \end{array}$$

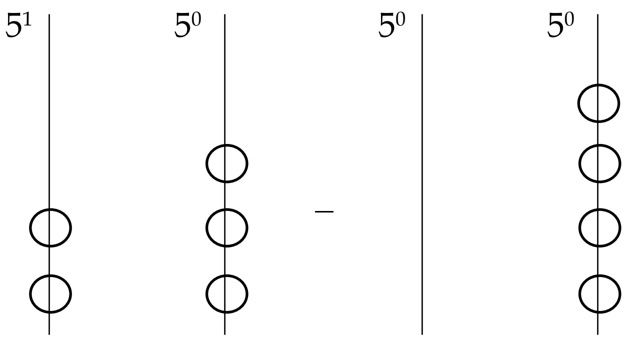
2. b)
$$\begin{array}{r} 1\,5\,1\,3_{six} \\ +\,2\,0\,3_{six} \\ \hline \underline{2\,1\,2\,0}_{six} \end{array}$$

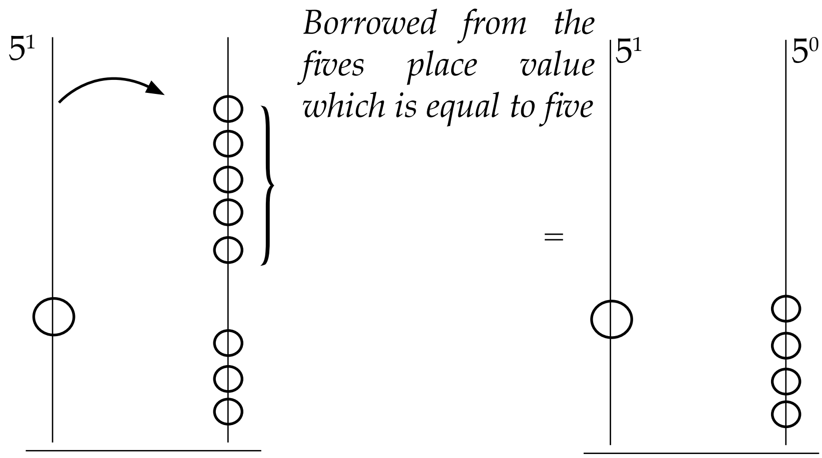
b)
$$\begin{array}{r} 1\,4\,6\,5_{twelve} \\ +\,3\,6\,5\,6_{twelve} \\ \hline \underline{4\;\;t\,e\,e}_{twelve} \end{array}$$

Subtraction

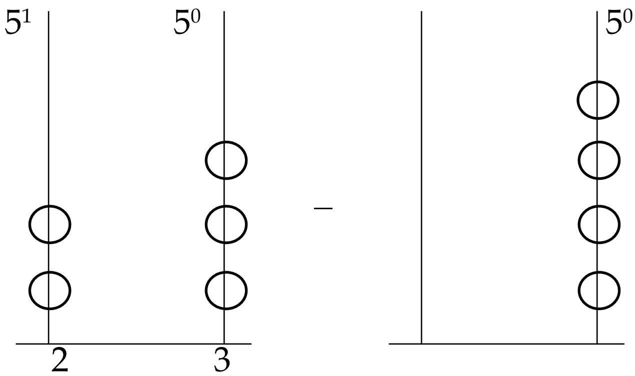
In subtraction the borrowed number is equivalent to the base as illustrated below.

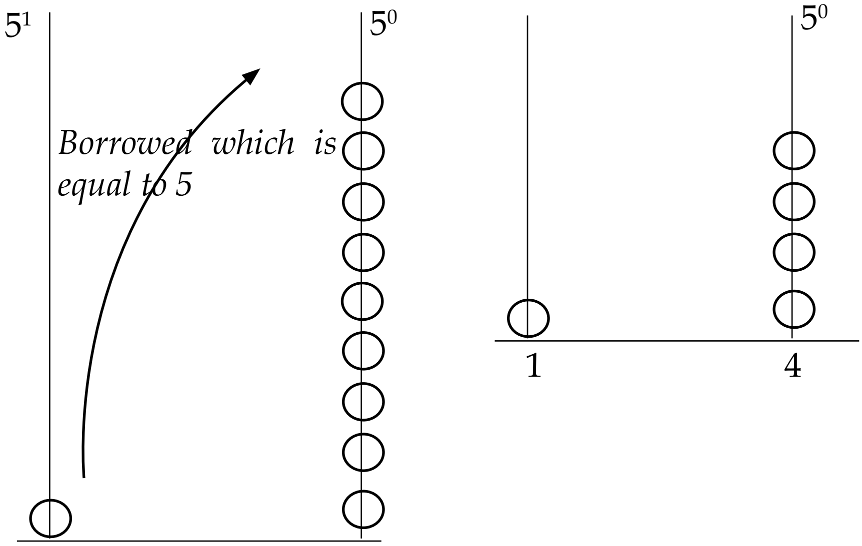
23_{five} – 4_{five}

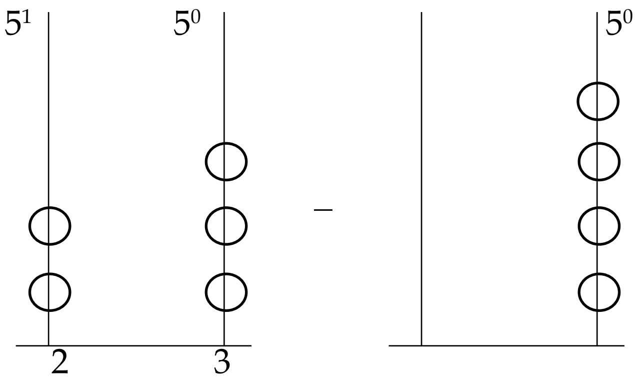


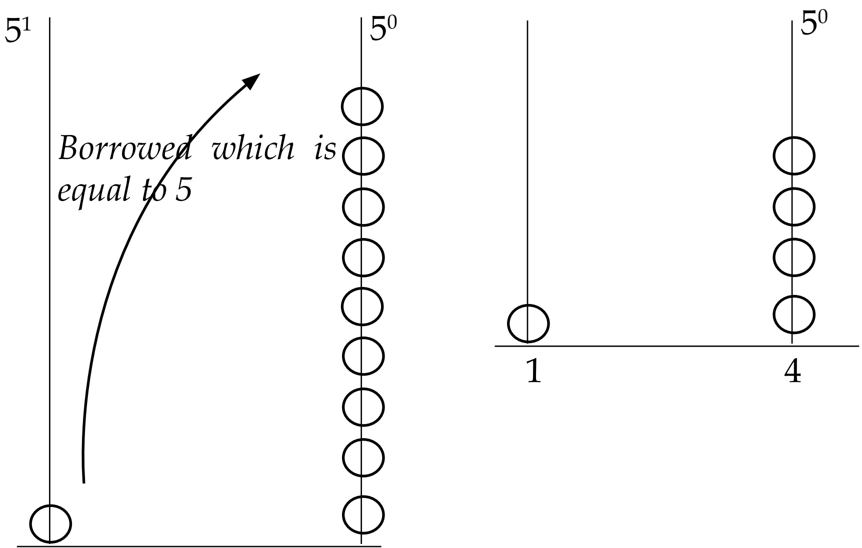


Unlike in base ten where the number borrowed is equal to 10. In base five the number borrowed is equal to the base(s) as illustrated on the abacus.




5^1
5^0
Borrowed which is equal to 5
5^0
1
4


5^1
5^0
2
3
–
5^0


5^1
5^0
Borrowed which is equal to 5
5^0
1
4

$23_{five} - 4_{five}$

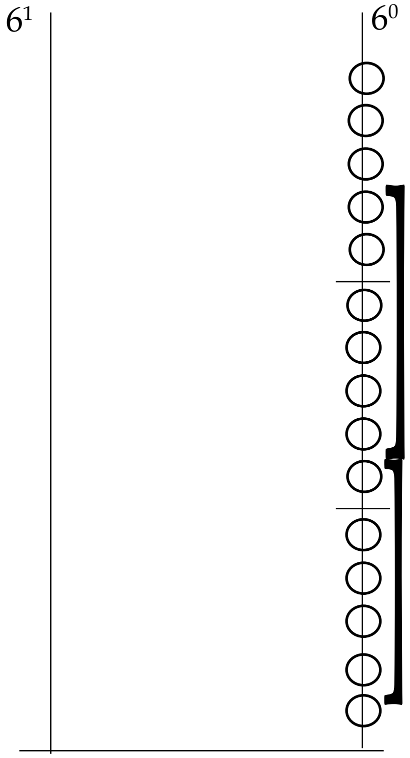
$= 14_{five}$

Exercise 3

a) $\begin{array}{r} 1\,10\,0_{two} \\ -\quad 11_{two} \\ \hline \\ \hline \end{array}$ b) $\begin{array}{r} 2\,1\,4_{five} \\ -\ 1\,2\,3_{five} \\ \hline \\ \hline \end{array}$ c) $\begin{array}{r} 1\,t\,31_{twelve} \\ -\ 6\,4\,5_{twelve} \\ \hline \\ \hline \end{array}$

Multiplication

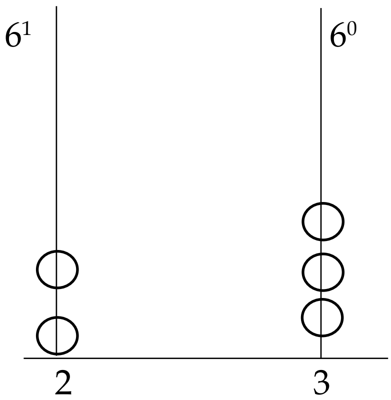
$5_{Six} \times 3_{six}$



It is 5 three times in base six, When using an abacus, we put all the five three times in the units place value.

Then we divide into sixes. This will give us 2 sixes and three ones.

23_{Six}



Division

Change numbers to decimal then carry out division after which change to the given base.

a) $11110_{\text{two}} = (1 \times 24) + 1 \times 23) + (1 \times 22) + (1 \times 21) + (0 \times 20)$

$= 16 + 8 + 4 + 2 + 0 \quad = 30$

$101_{\text{two}} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 4 + 0 + 1 \ = 5$

$\therefore 11110_{\text{two}} \div 101_{\text{two}} = \quad 30 \div 5 = 6$

2	6	
2	3	0
	1	1

110_{two}

$1110_{\text{two}} \div 101_{\text{two}} = 110_{\text{two}}$

b) $404_{six} \div 12_{six}$

$404_{six} = (4 \times 6^2) + (4 \times 0^0)$

$= 4 \times 36 = 144$

$12_{six} = (1 \times 6) + (2 \times 1)$

$= 6 + 2 = 8$

$404_{six} \div 12_{six}$

$144 \div 8 = 18$

6	18	
	3	0

$18_{ten} = 30_{six}$

$404_{six} \div 12_{six} = 30_{six}$

Exercise 4

Add

1. $\begin{array}{r} 1\,1\,0\,1\,0\,1_{two} \\ +\,1\,0\,1\,1\,1_{two} \\ \hline \end{array}$

2. $\begin{array}{r} 1\,0\,1\,0\,1_{two} \\ +\,1\,0\,0\,1_{two} \\ \hline \end{array}$

3. $\begin{array}{r} 4\,1\,3_{five} \\ +\,1\,1\,4_{five} \\ \hline \end{array}$

4. $\begin{array}{r} 1\,2\,0\,3_{five} \\ +\,4\,4\,3_{five} \\ \hline \end{array}$

5. $\begin{array}{r} 1\,2\,1\,3_{six} \\ +\,2\,2\,2_{six} \\ \hline \end{array}$

6. $\begin{array}{r} 4\,0\,1\,2_{six} \\ +\,2\,3\,0\,4_{six} \\ \hline \end{array}$

7. $\begin{array}{r} 1\,t\,e\,0_{twelve} \\ +\,2\,1\,5_{twelve} \\ \hline \end{array}$

8. $\begin{array}{r} t\,t\,e\,5_{twelve} \\ +1\,01\,6_{twelve} \\ \hline \end{array}$

Subtract

9. $\begin{array}{r} 1010_{two} \\ -\ \ 11_{two} \\ \hline \end{array}$

10. $\begin{array}{r} 424_{five} \\ -\ \ 31_{five} \\ \hline \end{array}$

11. $\begin{array}{r} 504_{six} \\ -120_{six} \\ \hline \end{array}$

12. $\begin{array}{r} 1te6_{twelve} \\ -\ 917_{twelve} \\ \hline \end{array}$

Multiply

13. $\begin{array}{r} t01_{twelve} \\ \times\ \ \ 9_{twelve} \\ \hline \end{array}$

14. $\begin{array}{r} 1423_{six} \\ \times\ 12_{six} \\ \hline \end{array}$

15. $\begin{array}{r} 444_{five} \\ \times\ 13_{five} \\ \hline \end{array}$

16. $\begin{array}{r} 1110_{two} \\ \times\ 111_{two} \\ \hline \end{array}$

Divide

17. $100011_{two} \div 111_{two}$

18. $220_{five} \div 30_{five}$

19. $240_{six} \div 12_{six}$

20. $200_{twelve} \div 20_{twelve}$

Activity: Making an Abacus

Materials

- Wooden frames, cardboards, wires/strings
- bottle tops/beads and nails.

Procedure

i) Get the wooden frames with holes or nails.
ii) Fix the strings/wires

iii) Indicate the place values on the wires or strings.

iv) Install the number of beads/bottle tops depending on the base.

Special precaution

The learner has to be careful with the place values of different bases.

Observation

i) Note down what happens when the number of the beads/bottle tops are equal or greater than the base.

ii) How do you transfer beads/bottle tops from one string to another (one place value to another place value)

Conclusion

i) The abacus helps to tell the difference between place values of different bases.

ii) It helps the learner to write numerals in words.

Application

The idea of abacus and place values is applied in:

i) The banking sector

ii) Business field

iii) Schools, etc.

Follow-up activity

i) Make different abaci for different bases.

ii) Write different numerals in words (e.g. filling in bank slips).

TOPIC 3 FRACTIONS AND PERCENTAGES

A fraction is a part of a whole which is expressed in terms of a numerator and a denominator i.e $\frac{a}{b}$ where a is the numerator and b the denominator.

Fractions are parts of whole numbers in which the numerator is over denominator. This includes basic concepts operations, decimals fraction and percentages.

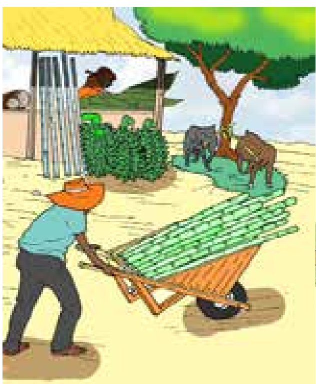
Sub-Topic 1: Basic Concepts

Types of Factions

i) Proper fraction

The numerator is less than the denominator and the fraction is less than one, e.g. $\frac{2}{5}, \frac{5}{6}, \frac{6}{7}$, ...

Figure 1



A man pushing whole fractions of sugarcane

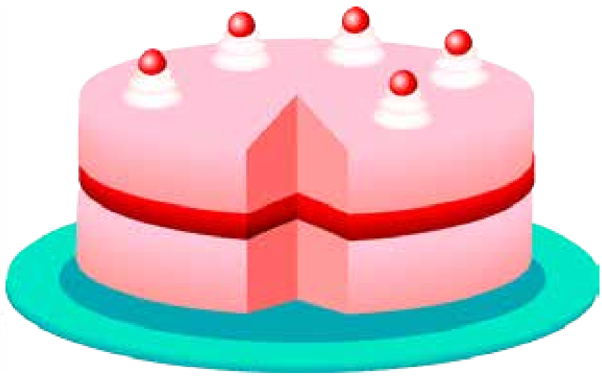
Girls eating different fractions of the fractions of sugarcane

ii) Improper fraction

The numerator is greater than the denominator, e.g $\frac{8}{5}, \frac{7}{6}, \frac{10}{6}$, etc.

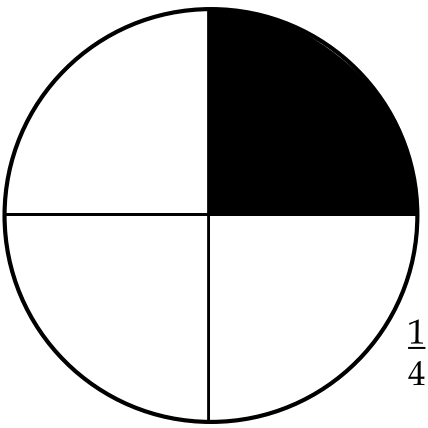
iii) Mixed fraction

It is formed by an integer and a proper fraction, e.g. $1^2/_3$, $5\frac{1}{10}$, etc.

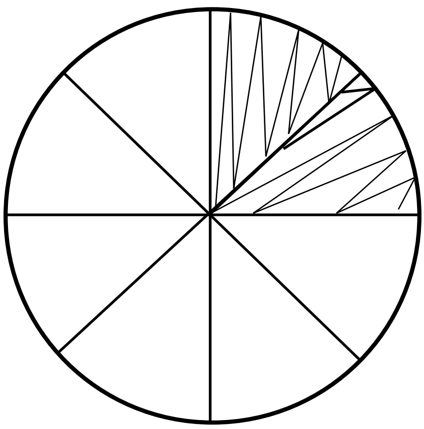


iv) Equivalent fraction

These are two or more fractions of the same size but expressed differently. They are formed by multiplying the same number by the numerator and denominator. See illustration below.



$\frac{1}{4}$



$\frac{2}{8}$

e.g. $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$, $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$, $\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$

$\therefore \frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$

$\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$ and $\frac{12}{16}$ are equivalent fractions.

Simplifying Fractions

This can be done if the numerator and denominator have the common factors and this leads to equivalent fractions.

Examples 1

1. Change the following mixed fractions into improper fractions.

Note: You multiply the denominator by an integer, add the numerator and divide by the denominator.

a) $1\frac{3}{4} = \frac{4 \times 1 + 3}{4} = \frac{4 + 3}{4} = \frac{7}{4}$

b) $3\frac{5}{6} = \frac{6 \times 3 + 5}{6} = \frac{18 + 5}{6} = \frac{23}{6}$

2. Express as mixed fractions:

Note: You divide the numerator by the denominator in order to get an integer and a proper fraction.

a) $\frac{17}{4} =$

$$\begin{array}{r} 4 \\ 4\overline{)\,17} \\ \underline{-16} \end{array}$$

$$4 + \frac{1}{4} = 4\frac{1}{4}$$

Alternatively: $\frac{17}{4} = \frac{16}{4} + \frac{1}{4} = 4\frac{1}{4}$

b) $4\frac{5}{9} =$

$$\begin{array}{r} 4 \\ 9\overline{)\,41} \\ \underline{-36} \\ 5 \end{array}$$

$$4 + \frac{5}{9} = 4\frac{5}{9}$$

Alternatively: $4\frac{1}{9} = \frac{36}{9} + \frac{5}{9}$

$$4 + \frac{5}{9} = 4\frac{5}{9}$$

3. Write down three fractions which are equivalent to $^3/_7$

$\frac{3}{7} = \frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$ $\qquad$ $\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$

$\frac{3}{7} \times \frac{4}{4} = \frac{12}{28}$ $\qquad$ $\frac{6}{14} = \frac{9}{21} = \frac{12}{28}$

Three equivalent fractions to 3/7 are:

$\therefore \frac{6}{14}, \frac{9}{21}$ and $\frac{12}{18}$

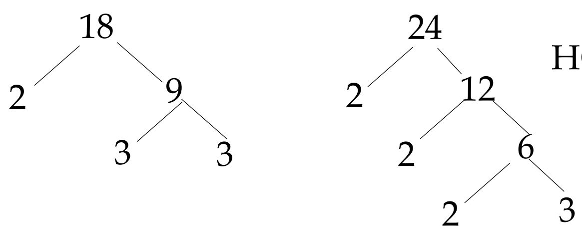
4. Express in lowest forms

i) $\frac{18}{24}$ $\qquad$ ii) $\frac{128}{144}$

Hint

i) Find Highest Common Factor of the numerator and denominator.

ii) Use it to cancel the numerator and denominator.



HCF is 2 x 3 = 6

$\therefore \frac{18}{24} \div \frac{6}{6} = \frac{3}{4}$

ii) $\frac{\overset{8}{\cancel{128}}}{\underset{9}{\cancel{144}}} \div \frac{16}{16} = \frac{8}{9}$ HCF is 16

$$= \frac{8}{9}$$

Exercise 1

1. Change into improper fractions:
   a) $4\frac{1}{5}$ b) $5\frac{5}{6}$
2. Express as mixed fractions
   a) $\frac{5}{2}$ b) $\frac{57}{7}$
3. Which of the following are equivalent fractions?

   $\frac{2}{3}, \frac{4}{7}, \frac{4}{8}, \frac{5}{10}, \frac{6}{9}, \frac{8}{12}, \frac{9}{15}$
4. Reduce to their lowest terms.
   a) $\frac{16}{40}$ b) $\frac{250}{350}$

Comparison of fractions

Two or more fractions are compared in order of size. This can be done by expressing the given fractions using the same denominator i.e Lowest Common Multiple

Example 2

Arrange $\frac{1}{2}, \frac{1}{4}$ and $\frac{2}{3}$ in order of size starting with the smallest.

Hint

i) Find the LCM of the denominators.

ii) The LCM of 2, 4 and 3 is illustrated in the table below.

2	2	4	3
2	1	2	3
3		1	3
			1

LCM is $2^2 \times 3 = 12$

$\frac{1}{2}, \frac{1}{4}, \frac{2}{3}$

$= \frac{6,\ 3,\ 2 \times 4}{12}$

$= \frac{6,\ 3,\ 8}{12}$

$\frac{1}{2} = \frac{6}{12}$

$\frac{1}{4} = \frac{3}{12}$

$\frac{2}{3} = \frac{8}{12}$

Comparing the numerator $\frac{3}{12}$ is the least and $\frac{8}{12}$ is the greatest.

$\therefore \frac{1}{4}, \frac{1}{2}, \frac{2}{3}$

Exercise 2

Arrange the fractions in order of size starting with the smallest.

i) $\frac{5}{6}, \frac{2}{3}, \frac{3}{4}$

ii) $\frac{4}{7}, \frac{9}{14}, \frac{1}{2}$

Sub-Topic 2: Operations of Fractions

These include addition, subtraction, multiplication and division.

a) Addition of Fractions

i) To add two or more fractions with the same denominator, add the numerators and denominator remains the same.

Example 4

$$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5}$$
$$= \frac{4}{5}$$

ii) If the denominators are not the same, find the L.C.M of the denominators.

Example 5

Work out $\frac{3}{4} + \frac{1}{3}$

$$\frac{3}{4} + \frac{1}{3} = \frac{3(3) + 4(1)}{}$$

12

2	4	3
2	2	3
3	1	3
		1

LCM is $2^2 \times 3 = 12$ $= \frac{9 + 4}{12} = \frac{13}{12} = 1\frac{1}{12}$

iii) Mixed fractions are changed into improper fractions in order to add.

Example 6

1. Work out $2\frac{1}{3} + 1\frac{1}{2}$

$$2\frac{1}{3} + 1\frac{1}{2} = \frac{(3 \times 2) + 1}{3} + \frac{(2 \times 1) + 1}{2}$$
$$= \frac{6 + 1}{3} + \frac{2 + 1}{2}$$
$$= \frac{7}{3} + \frac{3}{2}$$
$$= \frac{2(7) + 3(3)}{6}$$
$$= \frac{14 + 9}{6}$$
$$= \frac{23}{6} = 3\frac{5}{6}$$

2. I eat $\frac{1}{4}$ of a cake, my sister eats $\frac{1}{3}$ of the cake. How much have we eaten altogether?

$$= \frac{1}{4} + \frac{1}{3} = \frac{3 + 4}{12} = \frac{7}{12} \text{ of the cake}$$

b) Subtraction of fractions

i) To subtract fractions of the same denominator, we subtract directly the numerators and maintain the denominators.

Example 7

Work out $\frac{5}{9} - \frac{1}{9}$

$$= \frac{5-1}{9} = \frac{4}{9}$$

ii) When denominators are not the same, find their LCM of the denominators.

Examples 8

1. Find $\frac{5}{7} - \frac{2}{5}$

$$= \frac{5(5) - 7(2)}{35}$$

$$= \frac{25 - 14}{35}$$

$$= \frac{11}{35}$$

2. Work out $2\frac{1}{4} - 1\frac{1}{2}$

$$= \frac{(4 \times 2) + 1}{4} - \frac{(2 \times 1) + 1}{2}$$

$$= \frac{9}{4} - \frac{3}{2}$$

$$= \frac{9 - 2(3)}{4}$$

$$= \frac{9-6}{4} = \frac{3}{4}$$

3. Simplify $\frac{1}{21} + \frac{5}{6} - \frac{1}{14}$

LCM of

2	21	6	14
3	21	3	7
7	7	1	7
	1	1	1

LCM is 2 x 3 x 7 = 42

$$= \frac{2(1) + 7(5) - 3(1)}{42}$$

$$= \frac{2 + 35 - 3}{42}$$

$$= \frac{37 - 3}{42} = \frac{34}{42}$$

iii) I have $6\frac{1}{2}$ metres of cloth, $2\frac{5}{8}$ m was used to make a dress. How much cloth is left?

$$= 6\frac{1}{2} - 2\frac{5}{8}$$

$$= \frac{13}{2} - \frac{21}{8}$$

$$= \frac{4(13) - 1(21)}{8}$$

$$= \frac{52 - 21}{8}$$

$$= \frac{31}{8}$$

$$= 3\frac{7}{8} \text{ metres}$$

Exercise 2

Work out the following:

1. $\frac{3}{7} + \frac{2}{7}$
2. $\frac{4}{9} - \frac{2}{7}$
3. $6\frac{1}{3} - 4\frac{1}{2}$
4. $\frac{1}{6} + \frac{1}{3}$
5. $5\frac{1}{2} + 2\frac{1}{4}$
6. $\frac{3}{8} - \frac{1}{8}$
7. $3 - 2\frac{1}{3} + 2\frac{1}{2} - \frac{1}{4}$
8. John and James share shs 6,000. John gets $\frac{3}{5}$ of the money. What does James get?
9. Three pupils have ages $12\frac{1}{3}$, $13\frac{3}{4}$, and $14\frac{1}{4}$ years. What is the sum of their ages?
10. Jane eats $\frac{5}{8}$ of bread. Her sister eats $\frac{1}{4}$ of the bread. How much is left for the parents?

c) Multiplication of fractions

Note: i) "of" means multiplication.

ii) Multiply the numerators together and the denominators together.

Examples 9

Work out

1. $\frac{2}{9} \times \frac{3}{5} = \frac{2 \times \cancel{3}}{{}_{3}\cancel{9} \times 5}$

$= \frac{2}{15}$

2. $\frac{4}{7} \times \frac{2}{5} = \frac{4 \times 2}{7 \times 5}$

$= \frac{8}{35}$

3. $8\frac{6}{13}$ of $2\frac{4}{11} = 8\frac{6}{13} \times 2\frac{4}{11} = \frac{\overset{10}{\cancel{110}}}{\underset{1}{\cancel{13}}} \times \frac{\overset{2}{\cancel{26}}}{\underset{1}{\cancel{11}}} = 20$

Hint

i) Change the mixed numbers into improper fractions before proceeding.

ii) Write any integers as improper fractions with 1 as the denominator.

4. $2\frac{1}{2} \times 1\frac{1}{3} = \frac{5}{2} \times \frac{4}{3}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

5. Find $\frac{3}{4}$ of 20Km

$= \frac{3}{\cancel{4}} \times \cancel{20}^{5}$

= 15Km

6. What is $\frac{1}{2}$ of $4\frac{1}{2}$

$= \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{2}}$

= 3

7. If a dress needs $4\frac{1}{2}$ metres of cloth. How much cloth is needed for 6 dresses?

$= 4\frac{1}{2} \times 6$

$= \frac{9}{\cancel{2}} \times \frac{\cancel{6}^{3}}{1}$

= 27 metres.

d) Division of fractions

With division, the first fraction is maintained, the division sign changes to multiplication with the reciplical of the fraction..

Example 10

i) Work out the following:

1. $\frac{3}{7} \div \frac{1}{7}$

2. $1\frac{3}{5} \div \frac{6}{1}$

3. $2\frac{1}{2} \times 3\frac{2}{3} \div 5\frac{5}{6}$

Answer

1. $\frac{3}{7} \div \frac{1}{7} \quad = \quad \frac{3}{7} \times \frac{7}{1} = \quad 3$

2. $1\frac{3}{5} \div 6 \quad = \quad \frac{5 \times 1 + 3}{5} \div 6$

   $\frac{8}{5} \div 6 \quad = \quad \frac{8}{5} \times \frac{1}{6} = \frac{4}{15}$

3. $2\frac{1}{2} \times 3\frac{2}{3} \div 5\frac{5}{6}$

$$= \frac{(2 \times 2 + 1)}{2} \times \frac{(3 \times 3 + 2)}{3} \div \frac{(6 \times 5 + 5)}{6}$$

$$= \frac{5}{2} \times \frac{11}{3} \div \frac{35}{6}$$

$$= \frac{\overset{1}{5}}{\underset{1}{2}} \times \frac{11}{\underset{1}{3}} \times \frac{\overset{\cancel{2}\,1}{6}}{\underset{7}{35}} = \quad \frac{11}{7}$$

ii) A loaf of bread requires $3\frac{1}{3}$ cups of wheat. How many loaves of bread can be made from 30 cups of flour?

$$30 \div 3\frac{1}{3} = 30 \div \frac{10}{3}$$

$$= \cancel{30}^{3} \times \frac{3}{\cancel{10}}$$

$$= 9 \text{ loaves.}$$

Exercise 3

1. Work out the following:

(a) $\frac{3}{4} \times \frac{1}{2}$ (b) $\frac{2}{3} \times \frac{6}{7}$ (c) $3\frac{1}{3} \div \frac{3}{8}$

(d) $5\frac{1}{4}$ of $4\frac{4}{5}$ (e) $21 \div 1\frac{5}{9}$ (f) $3\frac{1}{2} \times \frac{2}{3} \div \frac{1}{3}$

2. A gardener has $4\frac{1}{2}$ rows of maize. In each row, there are 60 maize plants. How many maize plants are there altogether?

3. A sweater needs $8\frac{1}{2}$ balls of wool. How many sweaters can be made from 51 balls of wool?

4. What is i) $\frac{3}{4}$ of 2 hours?

ii) $\frac{2}{3}$ of 1km

e) Fractions with more than one operation i.e addition, subtraction, multiplication and division.

BODMAS is applied in simplification of fractions in which brackets and other signs occur.

Example 11

Simplify

1. $\frac{5}{7} \div \frac{2}{3}$ of $\frac{1}{18}$

$= \frac{5}{7} \div \left(\frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{18}_{9}}\right)$

$= \frac{5}{7} \div \frac{1}{27}$

$= \frac{5}{7} \times \frac{27}{1}$

$= \frac{135}{7}$

$= 19\frac{2}{7}$

2. $\frac{5}{7} \times \frac{2}{3} \div \frac{1}{18}$

$= \frac{5}{7} \times \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{18}^{6}}{1}$

$= \frac{60}{7} = 8\frac{4}{7}$

3. $3\frac{1}{2} \div \frac{2}{5} + \frac{1}{3} = \left(\frac{7}{2} \times \frac{5}{2}\right) + \frac{1}{3} = \frac{35}{4} + \frac{1}{3}$

$= \frac{3(35) + 4(1)}{12} = \frac{105 + 4}{12} = \frac{109}{12} = 9\frac{1}{12}$

$= 9\frac{5}{12}$

4.

$$\frac{5^1/_3 + 4^3/_8 - 1^5/_6}{7^7/_8} = \frac{16}{3} + \frac{35}{8} - \frac{11}{6} \Big/ \frac{63}{8}$$

$$= \frac{8(16) + 3(35) - 4(11)}{24} \Big/ \frac{63}{8} = \frac{128 + 105 - 44}{24} \Big/ \frac{63}{8}$$

$$= \frac{189}{24} \Big/ \frac{63}{8}$$

$$= \frac{\overset{3}{\cancel{\overset{21}{\cancel{189}}}}}{\underset{3}{\cancel{24}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{7}{\cancel{63}}} \quad = \quad 1$$

Example 12

Express the first quantity as a fraction of the second.

450g : 2Kg

1Kg = 1000g

$\overset{9}{\cancel{450g}}$: $\overset{20}{(2 \times \cancel{1000})\cancel{g}}$

9 : 40

= 9 : 40

Exercise 4

Simplify the following:

1. $3\frac{1}{2} \div \left(\frac{1}{2} + \frac{1}{2}\right)$

2. $\dfrac{3\frac{1}{2} + 1\frac{1}{40} + \frac{2}{5}}{1\frac{3}{20}}$

3. $\dfrac{3\frac{1}{2} - 2\frac{1}{7}}{3\frac{1}{4} + \frac{5}{6}}$

4. $4\frac{1}{2} - \frac{3}{5} \div \frac{1}{2}$

Decimals

Decimals help in identification of place values as illustrated in the examples below.

Examples 14

1. 2030

Thousands	Hundreds	Tens	Units
2	0	3	0

Means two thousands and thirty.

2.

Th	H	T	U
3	1	2	5

means Three thousands, one hundred and twenty five.

3. 3.6 means $3 + \frac{6}{10}$

3.65 means $3 + \frac{6}{10} + \frac{5}{100}$

3.652 means $3 + \frac{6}{10} + \frac{5}{100} + \frac{2}{1000}$

$\therefore$ Numbers on the left of a decimal point are called whole numbers and on the right are called decimal fractions.

Decimals and Fractions

13.65 is decimal fraction. 13 is a whole number and .65 is a decimal fraction.

Note: A number with no whole number part, a zero is put in the place of the whole number, e.g. .65 is written as 0.65.

i) Changing decimal to fractions

Note: A number with one decimal place is a fraction with denominator 10, 2 decimal places the denominator is 100 caps.

Example 15

Change the decimals into vulgar fractions in their lowest terms.

a) $0.7 = \frac{7}{10}$

b) $0.12 = \frac{\cancel{12}^{3}}{\cancel{100}_{4}} = \frac{3}{4}$

c) $2.25 = 2 + \frac{\cancel{25}^{1}}{\cancel{100}_{4}}$

$= 2\frac{1}{4}$

ii) Changing fractions to decimals

Some fractions can be expressed as exact decimals and are called terminating decimals.

Fractions can be changed into decimals by either finding an equivalent fraction with denominator 10, or 100 or 1000, or by dividing the numerator by the denominator.

Examples 16

Convert the following fractions into decimals.

1. $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$

2. $1\frac{1}{2} = 1 + \frac{1 \times 5}{2 \times 5} = 1 + \frac{5}{10}$

$= 1.5$

3. $\frac{3}{20} =$

$$\begin{array}{r|l} & 0.15 \\ \hline 20 & 30 \\ & 20 \\ & 100 \\ & \underline{100} \\ & \text{--} \end{array}$$

$= 0.\dot{1}\dot{5}$

4. $\frac{21}{25}$ =

$$\begin{array}{r|l} & 0.84 \\ \hline 25 & 210 \\ & \underline{200} \\ & 100 \\ & \underline{100} \\ & -- \end{array} \quad = \quad 0.84$$

iii) Recurring decimals

These are decimals in which one or more figures are continually repeated in the same order. They are formed by fractions which cannot be expressed as exact decimals.

Example 17

1/3	=	0.3333 …… (3 repeated)
5/11	=	0.70909….. (0 and 9 repeated)

Recurring decimals are written as follows by either placing a dot or a bar over the recurring period, i.e. 0.3333……is written as 0.3 or $0.\bar{3}$. 0.70909…... is written as 0.709 or $0.7\dot{0}\dot{9}$

Example 18

Express the following decimals into fractions.

1. $0.22\bar{2}$ …….

Name what you are given say T = $0.22\bar{2}$ …….

Find how figures are repeating themselves

Multiply (1) by 10 (because its only one digit repeating itself) and name it.

T x 10 = 0.222 ……x 10

10T = 2.222……

2 – 1 = 10T = 2.222….

$\underline{T}$ = $\underline{0.222….}$

9T = 2.0

Make T the subject T = $\frac{2}{9}$

$\therefore 0.22\bar{2}$ ……. = $\frac{2}{9}$

0.3737……..

Let y	=	0.3737 ....(i)	
100y	=	0.3737 ……x 100	
100y	=	37.3737…......................(ii)	
2 – 1	=	100y =	37.3737……

Equation (ii) − equation(i)

~~y~~	=	0.3737……
99y	=	37
y	=	$\frac{37}{99}$

$\therefore 0.3737\ldots.. = \frac{37}{99}$

0.1590

Let x	=	0.159090…….(i)
100x	=	0.159090……x 100
100x	=	15/90909……(ii)
100x	=	$15.909\bar{0}$
10000x	=	$1590.909\bar{0}$
− 100x	=	15.9090
9900x	=	1575
$\frac{9900x}{9900}$	=	$\frac{1575}{9900}$
x	=	$\frac{\cancel{1575}}{\cancel{9900}} = \frac{7}{44}$

Changing fractions to decimals

You divide the numerator by the denominator.

Example 19

Reduce the following to decimals and correct to 3 decimal places.

1. $\frac{2}{7}$

$$\begin{array}{r|l} & 0.2857 \\ 7 & 20 \\ & \underline{14} \\ & \ \ 60 \\ & \ \ \underline{56} \\ & \ \ \ \ 40 \\ & \ \ \ \ \underline{35} \\ & \ \ \ \ \ \ 50 \\ & \ \ \ \ \ \ \underline{49} \\ & \ \ \ \ \ \ \ \ 1 \end{array} = 0.286$$

Or

2. $\frac{8}{15}$

$$\begin{array}{r|l} & 0.5333...... \\ 15 & 80 \\ & \underline{75} \\ & \ 50 \\ & \ \underline{45} \\ & \ \ 30 \\ & \ \ \underline{45} \\ & \ \ 50 \\ & \ \ \ \underline{45} \\ & \ \ \ \ 50 \end{array}$$

$= \ 0.533\bar{3}$

$= \ 0.533$ (to 3 dp)

Exercise 5

1. Express the following fractions as decimals.

   a) $\frac{5}{8}$ b) $\frac{4}{5}$ c) $2\frac{1}{4}$

2. Convert the decimals below into vulgar fractions in their lowest form.

   a) 0.03 b) 4.05 c) 0.025

3. Reduce the fractions below to decimals, correct to 3 decimal places.

   a) $\frac{1}{9}$ b) $6\frac{3}{7}$

4. Express the following reccuring decimals as fractions.

a) $0.18\,18\,\overline{18}$ b) $3.\dot{6}\dot{9}$

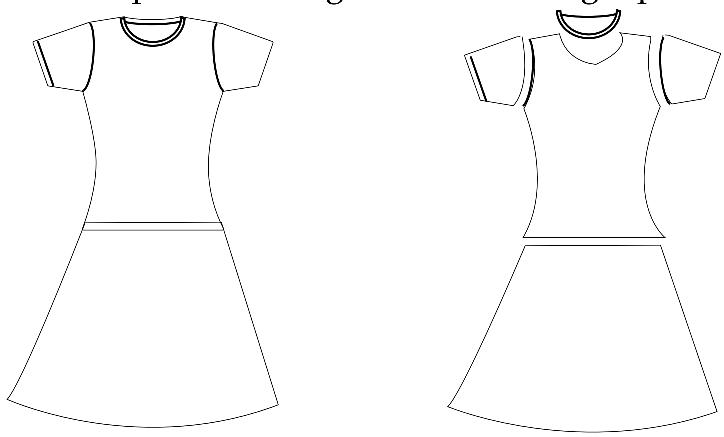
c) $0.148\,\overline{148}$

Activity: Making a dress

A fraction is a part of a whole number.

Materials

- Piece of cloth/paper, threads, needle, scissors, razor blade, tape measure, glue and masking tape.



Procedure

i) Get a piece of cloth material.

ii) Measure off the dimensions for the different parts of the dress.

iii) Cut off the pieces.

iv) Join the pieces together to form a dress.

v) Vary the dimensions of the parts for different sizes of the dresses.

Precaution

Take correct measurements for the parts.

Observation

i) Different fractions put together form a whole

ii) March the corresponding sizes (pieces)

Conclusion

Different fractions joined form a whole.

Application

i) Textile industries

ii) Tailors

iii) Construction field

Follow up activity

Make different sizes of clothes.

Sub-Topic 4: Percentage

Percentage is a fraction whose denominator is 100 e.g. 50% means $\frac{50}{100}$, 12% means $\frac{12}{100}$,etc. Changing fractions into percentages you multiply the fraction by 100%

Example 20

Convert the following to percentages

i) $\frac{3}{5}$ ii) $\frac{5}{8}$

Answers

i) $\frac{3}{5} \times 100\% = 60\%$ ii) $\frac{5}{8} \times 100\% = \frac{125}{2}\% = 62\frac{1}{2}\%$

Changing percentages into fractions

The percentage symbol is turned into a denominator as 100, then the fraction is reduced to its simplest form.

Example 21

Change the following percentages to fractions in lowest (form) term.

1. $45\% = \frac{\cancel{45}\ 9}{\cancel{100}\ 20} = \frac{9}{20}$
2. $0.95\% = \frac{0.95}{100} = \frac{95}{100} \times \frac{1}{100} = \frac{19}{2000}$
3. $2\frac{2}{5}\% = \frac{\frac{12}{5}}{100} = \frac{12}{5} \times \frac{1}{100} = \frac{3}{125}$

Note: To find the percentage of a given quantity, we use fractions.

Examples 22

Find the value of:

1. 25% of 1 day

   $\frac{25}{100}$ of 1 day

   $\frac{25}{100} \times 24$ hours

   $\frac{1}{4} \times 24$ hours $= 6$ hours

2. $\frac{\frac{12}{2}}{100}$ of 40 cm^2

   $\frac{25}{2} \times \frac{1}{100} \times 40 = 5\text{cm}^3$

Exercise 6

1. Express the following as fractions in their lowest terms:

   a) 15% b) 36% c) 87%

2. Express the following as a percentage:

   a) $\frac{1}{8}$ b) $\frac{17}{20}$ c) $\frac{12}{25}$

3. Find the value of:

   a) 4% of 50g

   b) 40% of 1 litre

   c) 10% of 1 hour

Percentage Change

a) An increase in percentage means that the number is increased by given percentage. The original number is taken to be 100.

New value: $\left(\frac{100 + \text{increase}}{100}\right)$ of old value.

Example 4

---

1. Increase shs 200 by 15%

$\left(\frac{100 + 15}{100}\right)$ of 200

Shs 230

2. Increase Shs 10,000 by 5%

$\left(\frac{100 + 5}{100}\right)$ of 10,000 = $\frac{105}{100} \times 10{,}000$

Shs. 105,000

b) A decrease in percentage means the number is decreased by a given percentage such that the new value becomes

$\left(\frac{100 - \text{decrease}}{100}\right)$ of old value

Example 24

i) Decrease 1800 by 15%

$\left(\frac{100-15}{100}\right)$ of 1800

$\frac{85}{100} \times 1800 \quad = 1530$

ii) Decrease 1200 by 20%

$= \left(\frac{100-20}{100}\right)$ of 1200

$= \frac{80}{100} \times 1200 \quad = 960$

Exercise 7

1. Increase 50 by 40%.
2. Increase Shs 8500 by 20%.
3. Decrease 350m by 505.
4. Decrease 2.5kg by 70%.

TOPIC 4 SET THEORY

A set is collection of well defined objects (members or elements). These elements can be people, animals, countries, cities and any other thing.

The elements of a set are always enclosed by curly brackets and separated by commas. Sets are denoted by capital letters, e.g. C, D, F…..

A set can also be described by words enclosed in curly brackets.

Sub-Topic 1: Symbols Used in Set Theory

Example 1

A set of students in a class whose surnames begin with letter B can be presented as {students whose surnames begin with letter B}.

Example 2

Use curly brackets to show:

i) some capital cities of East African countries.
ii) prime numbers less than 30.

Answer

i) {Nairobi, Kampala, Dar-es-Salaam, Kigali}
ii) {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

Note: The order of listing the members of a set does not matter.

Exercise 1

1. Write the following sets with curly brackets:
   i) Ten surnames of students in your class.
   ii) The numbers more than 10 but less than 30.
   iii) Even numbers less than 40.
   iv) The names of national parks in Uganda.
   v) The continents of the world.
2. Describe the following sets:
   i) C = {Kenya, Burundi, Rwanda, Tanzania, Uganda}
   ii) N = {1, 3, 5, 7, 9, ………..}
   iii) P = {Obote, Mutesa, Amin, Lule, Binaisa, Muwanga, Lutwa}
   iv) A = {2, 3, 5, 7, 11, 13, ………..}

1. Member of (€)

In the example above, part (ii) can be denoted by P such that;

P = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

We can also say that:

2 is a "Member of" Set P

3 is a "Member of" Set P and so on. The phrase "member of" can be I replaced by the symbol **€** (which means "member of"). In short, we can write as:

2 € P
3 € P
5 € P and so on

Is 7 a member of set P?
Is 15 a member of set P?

In the first question, the answer is yes. This can be written as $7 \in P$.

In the second question, the answer is no. This can be written as $15 \notin P$. So the symbol $\notin$ means "not a member of".

Example 3

S = {Capital city in Africa}. Use the symbols $\in$, $\notin$ correctly to complete the following statements:

i) Kampala………………….S

Accra……………………….S

London…………………….S

Paris……………………….S

Cairo……………………….S

2. Empty Set

If a set has no member/element, it is said to be an empty set. The symbol $\varnothing$ or $\{\ \}$ is used to indicate an empty set.

Example 4

A = {Birds which have 4 legs}
B = {Countries sharing a capital city}

The above two sets are empty. They can be written as

$A = \varnothing$ or $A = \{\ \}$

$B = \varnothing$ or $B = \{\ \}$

Note: That ∅ is not enclosed in curly brackets and if it is enclosed like

A = $\{\varnothing\}$ this means that Set A is not empty but it has one element Æ.

Exercise 2

For the given sets state using an appropriate symbol whether the given members after belong to or do not belong to the sets.

1. A = {2, 4, 6, 8}, 4
2. N = {National parks in Uganda}, Kidepo
3. D = {Districts in Uganda}, Nairobi
4. S = {Square numbers less than 1,000}, 289
5. M = {All subjects taught in O-level classes in Uganda}, Medicine

Sub-Topic 2: The Venn Diagram

3. Union and Intersection

Union refers to a collection of all members in the given sets to form one set.

Example 5

A = {11, 12, 13, 14, 15}
B = {2, 4, 8, 20, 23}

The new set formed by Set A and Set B is {2, 4, 8, 20, 23, 11, 12, 13, 14, 15}

The symbol U means the union of sets. So

A U B = {2, 4, 8, 20, 23, 11, 12, 13, 14, 15}.

Example 6

Find S U T if

S = {2, 4, 6, 8}

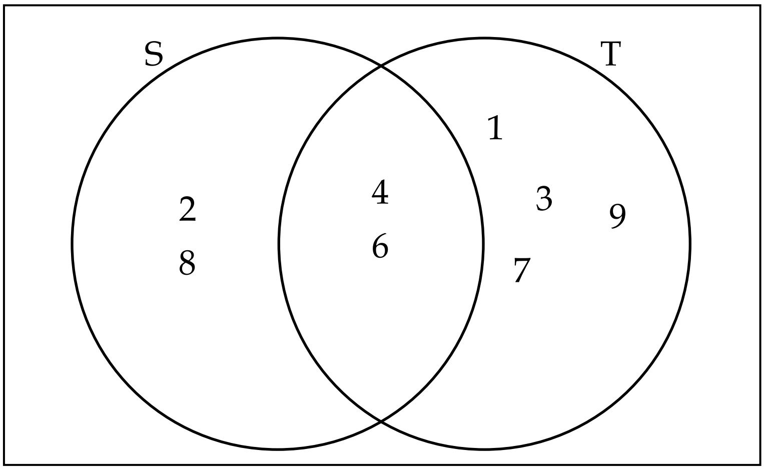
T = {1, 3, 4, 6, 7, 9}

S U T = {1, 2, 3, 4, 6, 7, 8, 9}

Note: (i) S U T has all members of S and T

(ii) 4 and 6 are found in the two sets, but when you write members of S U T, 4 and 6 are not repeated.

The above can be illustrated by use of a venn diagram as shown



4. Intersection of Sets

This refers to common members in given sets as illustrated in the venn diagram above. Element 4 and 6 are common to both sets S and T.

A symbol ∩ is used to mean intersection

Example 7

A = {Natural numbers less than 15}

B = {Even numbers less then 20}

Find A ∩ B.

Answer

A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

B = {2, 4, 6, 8, 10, 12, 14, 16, 18}

$A \cap B$ = {2, 4, 6, 8, 10, 12, 14}

5. Equivalent and Equal Sets

Look at the following sets

A = {2, 4, 6, 8, 10, 12}

B = {1, 3, 5, 7, 11, 13}

C = {12, 2, 6, 10, 4, 8}

How many members does each of the above sets have?

Set A has 6 members

Set B has 6 members

Set C has 6 members

When sets have the same number of members, they are said to be equivalent sets. Then $A \Leftrightarrow B$, $B \Leftrightarrow X$ and $A \Leftrightarrow C$ read as set A is equivalent to B, B is equivalent to C, e.t.c

Look at Set A and Set C, what do you notice?

Set A and C have the same number of members and the members are identical. Sets are said to be equal if they have the same number of members which are identical hence A = C.

Read as set A is equivalent to set B.

Note: Equal sets are equivalent sets but not all equivalent sets are equal sets.

7. Disjoint Sets

Look at the following sets:

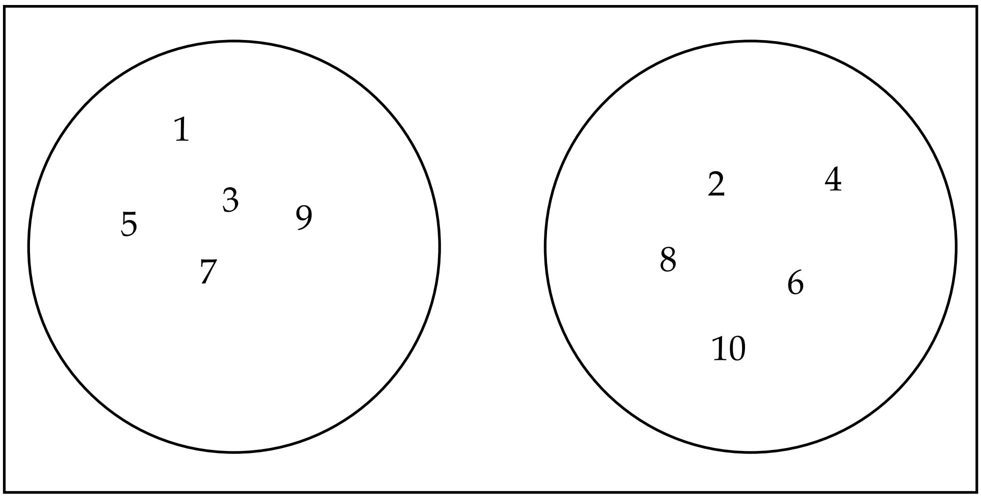
X = {1, 3, 5, 7, 9 }

Y = {2, 4, 6, 8, 10}

Which members are common to the two sets?

$X \cap Y = \varnothing$

Since there are no members which are common to the two sets, then Set X and Y are disjoint sets as illustrated in the diagram below.



Exercise 3

1. State whether the given pairs of sets are equal or equivalent.

i) P = {1, 2, 3} K = {Positive integers less than 4}

ii) S = {1, 2, 3} T = {a, b, c}

ii) M = {1, 2} N = {The set of all positive integers less than 3}

2. Let $A = \{2, 4, 6, 8, \ldots\ldots\}$ and $B = \{3, 6, 9, \ldots\ldots\}$, i.e. multiples of 2 and 3. Find

   i) $A \cap B$

   ii) What can you say about $\cap(A \cap B)$?

3. Let A = {Jesse, Roy, Ronney}

   B = {Roy, Dan, John}
   C = {Mary, John, Timothy}

Find (i) $A \cap B$, (ii) $B \cap C$, and $A \cap C$

8. Universal Set and Subsets

A universal set is a mother set / general set. It is a set that contains all members of the given set.

Example: $A = \{1, 2, 3, 4, 5, \ldots\ldots\}$

From Set A, we may get many other sets. For instance, Set B of even numbers or Set C of odd numbers can be obtained or drawn from Set A

$B = \{2, 4, 6, 8, 10 \ldots\ldots\}$

$C = \{1, 3, 5, 7, 9, 11 \ldots\ldots\}$

Form ten Sets from Set A. Set A is referred to as universal. The symbol representing universal set is ε.

Sets B and C which were formed from Set A are called sub-sets of A. The symbol for "subset of" is $\subset$ and is written as $B \subset A$ and $C \subset A$.

If all the members of the given set are not a subset, the symbol used is $\not\subset$.

Exercise 4

1. What is a subset?
2. ε = {Positive numbers less than 100}. Form 10 subsets from this universal set.
3. A = {1, 5, 7}. List all subsets formed from set A.
4. List all the subsets containing only two elements of the set {1, 2, 3, 4, 5, 6, 7, 8}
5. Given that Z = {Integers between eight and twenty}. Find the number of subsets of Z.
6. In a class of 20 boys, 15 like matooke and 10 like rice. How many like both?
7. In a team of 25 tourists, 10 visited Jinja, 17 visited Gulu and 7 visited both towns, With a help of a venn diagram, find;

   The number of tourists who visited neither towns.

   How many visited Jinja only

Activity: Sitting arrangement in a class

Materials: Metre rule, manila paper, pen a recorder, chairs, desks, classroom.

Procedure:

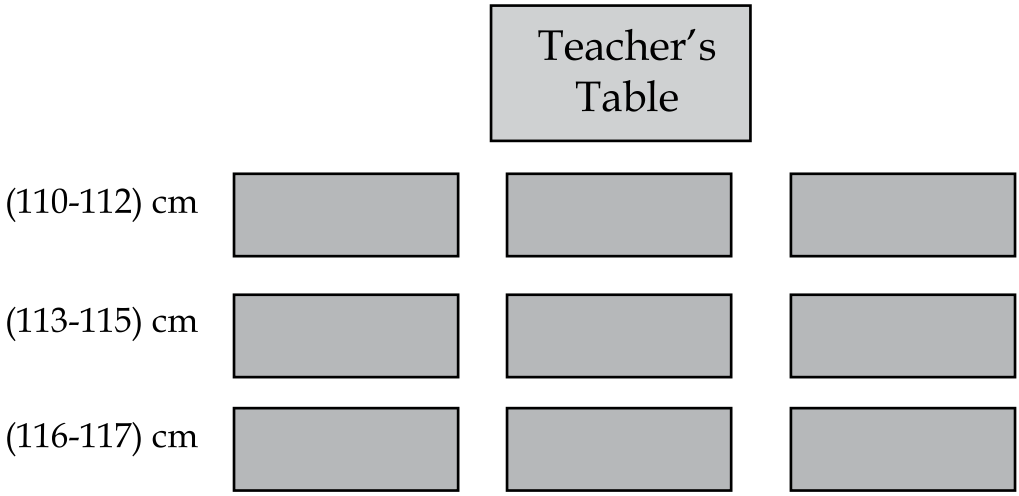
i) Take heights of the learners.

ii) Arrange heights (data) in sets, e.g. (110-112 cm) (113 – 115 cm)

iii) Arrange desks and chairs in columns and rows.

iv) Sit in rows of your respective sets of heights.

Classroom:



Observations: Identify/observe the neatness and arrangement of the classroom.

Interpretation: A disciplined class is formed.

Application: Sets are commonly used in arrangements of things in supermarkets, shops, textbooks in the library, markets, laboratories, hospitals, bedrooms, etc.

Follow up: Students should visit the school library and arrange books according to sets of subjects.

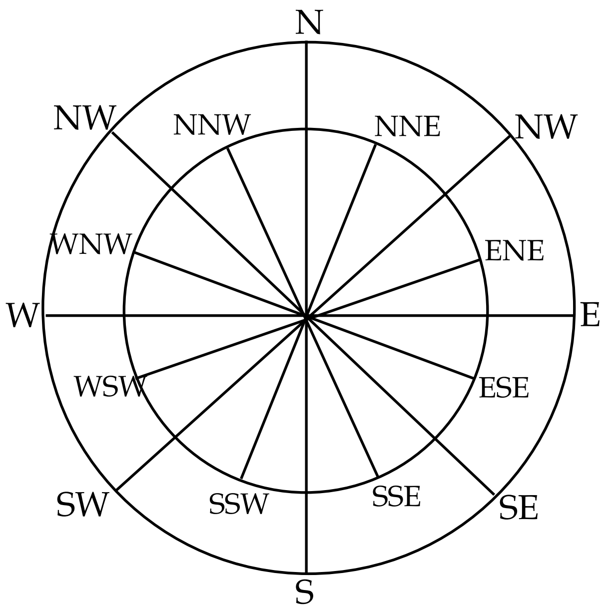
TOPIC 5 BEARING

Bearing refers to the location of a point or a place from a fixed point of reference i.e. the North direction of the point. It is expressed in terms of direction and distance.

Direction is expressed in degrees clockwise direction from the North direction.

Compass direction

The figure below shows the 16 points of the compass



The compass has sixteen points as shown above. The angle between directions that are next to each other is 360 ÷16 or 22 ½^{o}

Exercise 1

Give the angle turned through in a clockwise direction:

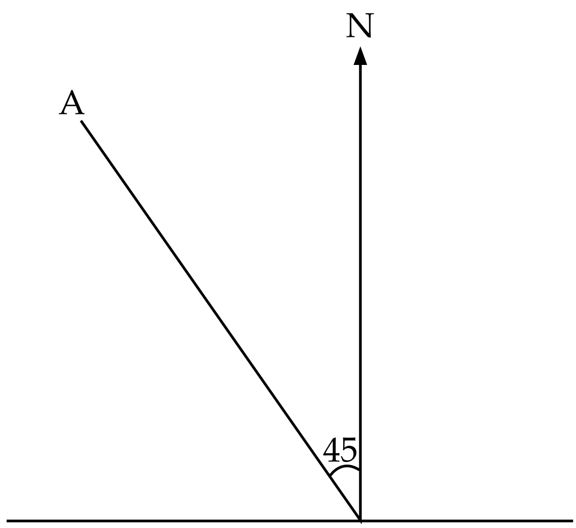
1.	N to SE	3.	NNE to ESE
2.	NE to SW	4.	E to NE

5.	S to E	8.	SSW to S
6.	SW to SSW	9.	WSW to ENE
7.	SSE to ENE	10.	NW to WSW

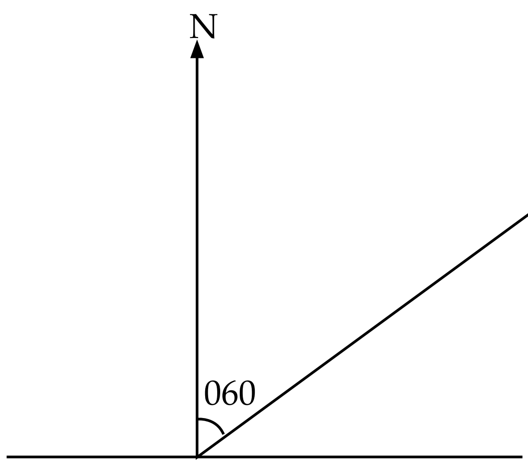
Example 1

O and A are two places. On seperate sketch diagrams, show the position of O and A given that the direction of A from O is:

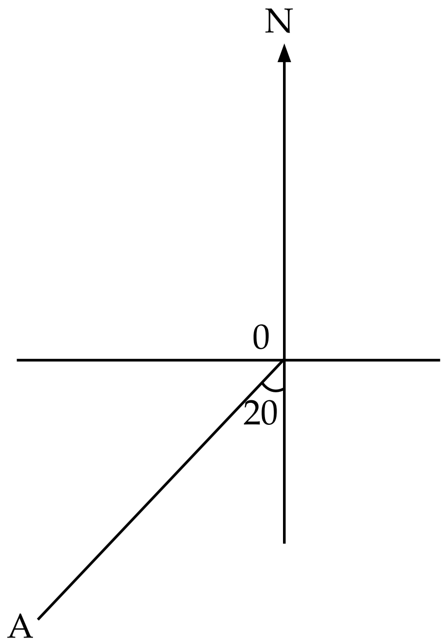
i) NW



ii) N60°E



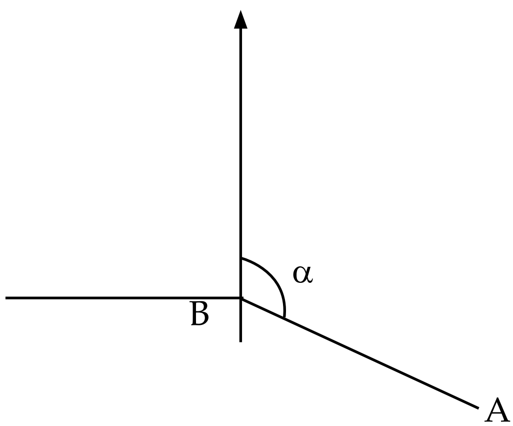
iii) 520°W



Answers

Note: Direction can be described by the use of the marines compass

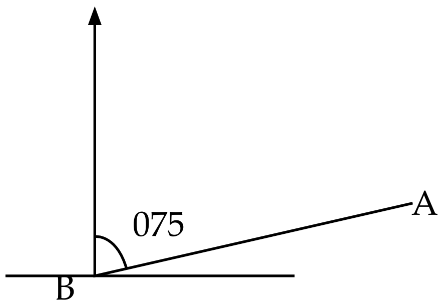
If there are two places A and B and you want to described the bearing of A from B, then stand at B, face in the north direction, turn clockwise until you see A. Diagramatically it can be illustrated as below:



Angle X represents the bearing of A from B if angle X is expressed in three figures. i.e. 75 = 0.75° 60= 060° etc.

In determining the bearing of a point we can also use the cardinal points of North or South. The starting point is either North or South. From either point, you can move West or East.

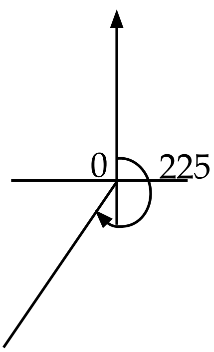
Figure 1



Example 2

From figure 1 the bearing of A from B can be expressed as either 075° or N 75°E

Figure 2



The bearing of P from O can be expressed as 225° or 545°W

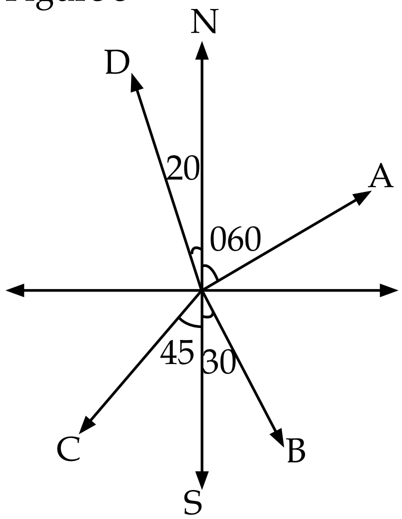
Exercise 2

Give the bearing corresponding to the following:

i)	N 40 E	ii)	S 35 W
iii)	S 80 E	iv)	N 52 W
v)	S44 S	v)	N 30 W

In the figure below, state the bearing of A B C and D from the point X

Figure 3



Scale drawing

In some cases, the size of the object cannot be represented on a piece of paper or the available space. Scale is used in describing the relation of the points or places on the paper and the actual points or places on the ground. For example, you cannot draw a line of 1 km length in your exercise book, but you can draw one centimetre in your exercise book. In this case, 11km can be represented by 1 cm i.e. 1cm ≡ 1km.

Note: ratio has no units and so the ratio 1cm to 1km can be written as 1:100,000.

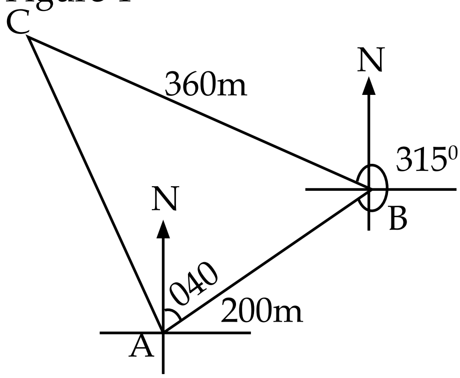
This means that 1cm on paper is equivalent to 100,000cm on the ground.

We can apply the knowledge of scale drawing in bearings.

Example 3

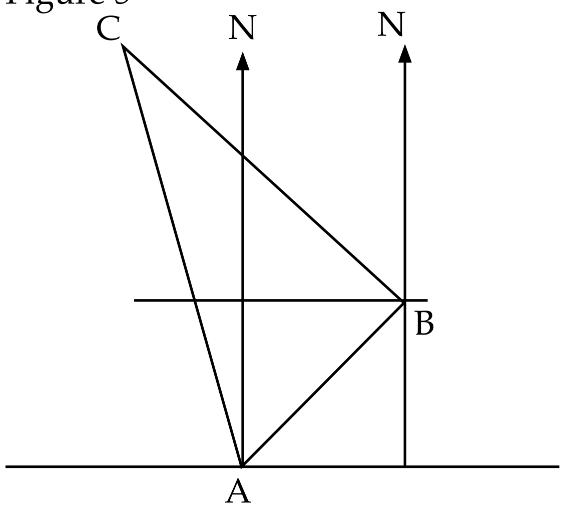
A boat leaves a setty and cruises for 200m on a bearing of 040^{o}.It then changes course to a bearing of 315^{o} for 360m. By means of a scale drawing, find the boat from the setty 50m.

Figure 4



Scale" 1cm represents 50

Figure 5



From figure 5 AC = 8.4cm. Therefore the distance of the boat from the setty is 8.4 x 50 = 420m

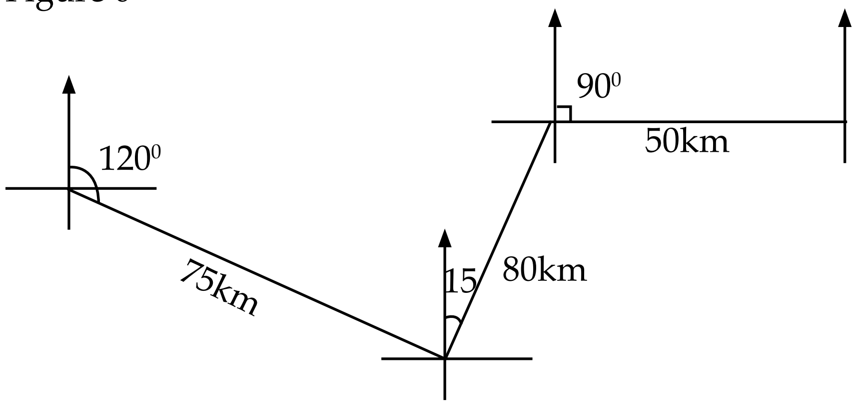
The bearing of the boat from the setty is $360^o - 16^o = 344^o$.

Example 4

A ship sails from a port on a bearing of 120 for 75km. It then changes course and sails on a bearing of 015 for 80km. Finally the ship sails due east for 50km.

Takiing 1cm to 10km, make scale drawing for the ship's three leg voyage. From your drawing, find the bearing and distance of the ship's final position from the port.

Figure 6



Learners draw accurate diagram using scale: 1cm = 10km

The bearing of ship's final position from the port is 073^o and distance is 14.2 x 10 = 142 km

Exercise 3

1. Mary's school is about 80m from her home if you take the N35°W direction, but to get there she first walks 48m North East to Mango town and then walks 52m in the direction of N50°W to her school.
   a) Use an appropriate scale to draw a scale diagram of Mary's journey to school.
   b) What is the bearing of Mango town from Mary's school?

2. A helicopter flies from Gulu for 140 km on a bearing of 205 °. It then flies for 170 km on a bearing of 150°. The helicopter then flies for 230°km on a bearing of 240°.

   Taking 1cm to 25km, make a scale drawing for the helicpter's flight.

   From your scale drawing, find:

   i) How far from Gulu the helicopter is after the second leg of its journey.

   ii) The bearing of Gulu from the helicpter's final position.

3. In a marathon, the runners set off on a bearing of 060^o for 3km then turn South East for 2.5km and finally turn in the direction N30^oE for 1.2km and the distance from the starting point use 1cm≡ 0.5km

4. The bearing of O and P from S are 200^o and 290^o respectively and the distance from S to O = 3.6km and from P to S = 2.7 km. Find

   i) The lengt in km of OP.

   ii) The bearing of O from P.

5. Town B is 100km away from town A on a bearing of 135^o. Town D is on a bearing of 90^o from town B, 124km apart. Town C is 160km away from town D on a bearing of 030^o from D.

   a) Using a scale of 1cm to represent 20km, make an accurate drawing to show the relative positions and distances of towns A,B,C and D.

   b) Determine the:

      i) shortest distance and bearing of town C from town A

      ii) Distance and bearing of town B from town C.

Activity: Construction of a mariner's compass

Objectives: i) To tell compass direction.
ii) To determine bearings of given directions.

Introduction: This involves location of particular places from afixed point of reference.

Materials: Manila paper/cardboard, straws, pins, glue, threads, B/board protector, pair of scissors.

Procedure

i) Sketch two circles on a manila paper with a common centre.

ii) Draw major axes, i.e. y and x axes in the drawn circles.

iii) Cut out the quadrants of inner circle leaving the axes.

iv) Cut the remaining part of the manila leaving the "annular."

v) Name the major directions, North, South, West and East.

vi) Sub-divide the quadrants with straws indicating the direction on the manila paper NE, SE, NW, SW.

Note

i) Major axes must meet at 90o.

ii) Quadrants are divided equally.

Observation

i) A sketch of a mariner compass is constructed.

ii) Major axes meet at 90 o.

Interpretation

i) Bearing is measured clockwise from North direction.

ii) Bearing is different from direction.

Application: It is applicable in navigation, air travel, surveying.

Follow up Activity

Identify bearing of different places on a map.

TOPIC 6 NUMERICAL CONCEPTS

Introduction

In this chapter, we shall consider different types of numbers such as directed, natural, odd, even, prime and composite numbers.

Natural numbers

In daily life, human beings are involved in counting things. Some people may use their fingers in counting or other things. The numerals used in counting are referred to as natural numbers.

The set of natural numbers is {1, 2, 3, 4,………..}. Numerals are number symbols. The number symbols we use today are Hindu Arabic numerals. There are other numerals which are used for example Roman numerals.

Sub-Topic 1: Directed Numbers

In real life situations, people are involved in measurements of quantities. For example, measuring heights, temperatures, lengths and many others. In all these measures, there is always a starting point.

In temperature, zero degree (0^o) is taken to be the starting point. Any reading below zero is negative and the reading above zero is positive, e.g. $^-15^o$, $^-8^o$, 7^o, 11^o, etc.

Example 1

The average weight of students in S1 is 45kg. Use directed numbers to write each of the following students' mass in relation to the average mass.
John = 55kg, Peter = 40kg, Sarah = 47kg, Mary = 50kg.

Answer

John's mass is 10kg above the average $^+10$ (55 – 45 = 10)

Peter's mass is 5kg below the average $^-5$ (40 – 45 = $^-5$)

Sarah's mass is 2kg above the average $^{+}2$ $(47 – 45 = {}^{+}2)$

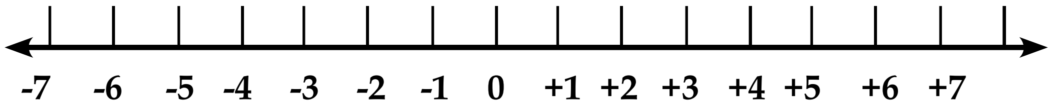
Mary's mass is 5kg above the average $^{+}5$ $(50 - 45 = {}^{+}5)$

Using the above examples $^{+}10$, $^{-}5$, $^{+}2$ and $^{+}5$ are referred to as directed numbers. They represent a value above or below the fixed measure.

Integers

These are directed numbers with no fractional parts.

$\{\ldots{}^{-}5,\ {}^{-}4,\ {}^{-}3,\ {}^{-}2, {}^{-}1,\ 0,\ {}^{+}1,\ {}^{+}2,\ {}^{+}3,\ {}^{+}4,\ {}^{+}5\ldots\}$. Integers can easily be illustrated using a number line.



On a number line, positive integers are marked to the right of O and negative integers on the left.

Example 2

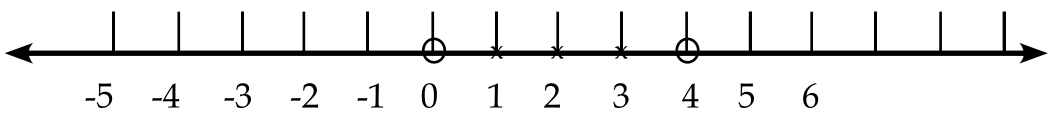
Illustrate the following sets of natural numbers on a number line

a) $\{N; N < 4\}$ b) $\{N; N > 2\}$

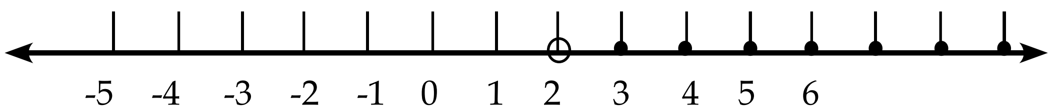
c) $\{N; N \leq 3\}$ d) $\{N: 2\ N \leq N \leq 8\}$

Answer

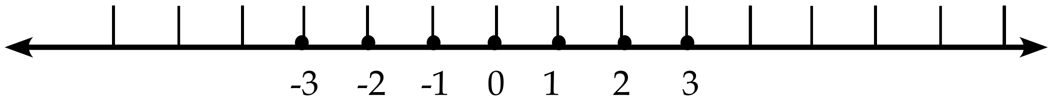
a)



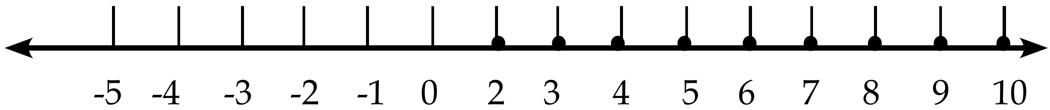
b)



c)



d)



In the example above, note the use of open circles and closed circles.

Activity

Explain when a closed or open circle is used.

Addition and subtraction of integers

This can be illustrated by use of a number line.

Example 3:

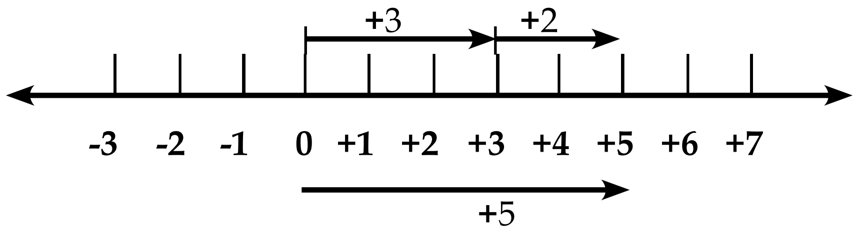
i) $+3 + +2$ ii) ${}^{+}2 + {}^{+}1 + {}^{+}4$

iii) $-3 + {}^{+}5$ iv) ${}^{+}7 + {}^{-}4$

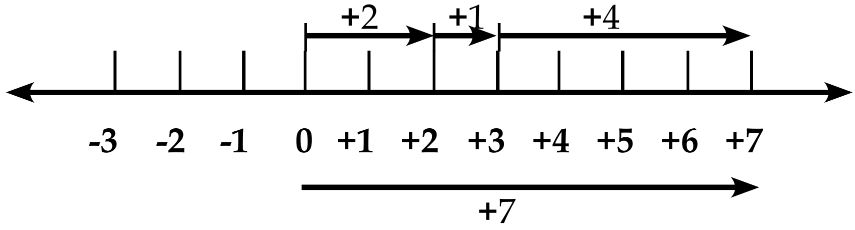
v) ${}^{-}4 - 2$

Work out using a number line.
Answer: using a number line.

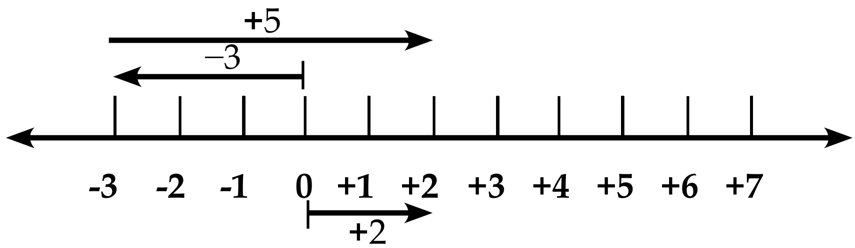
i)



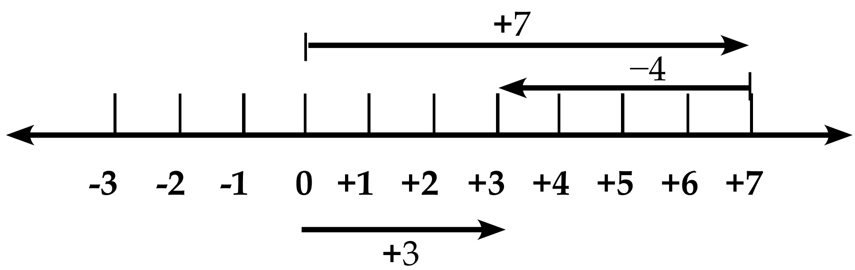
ii)



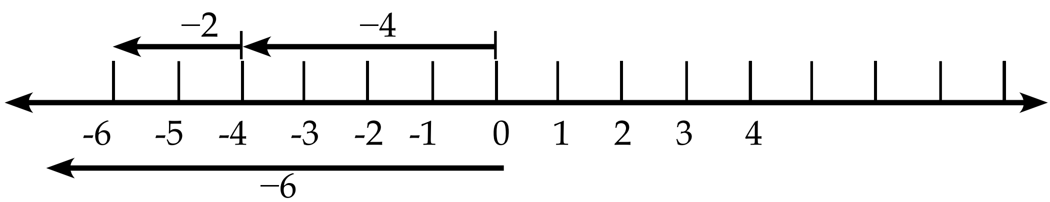
iii)



iv)

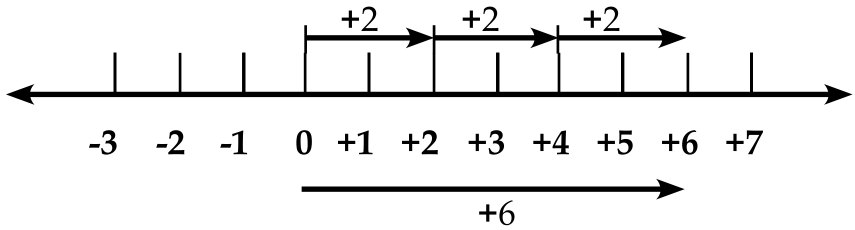


v)

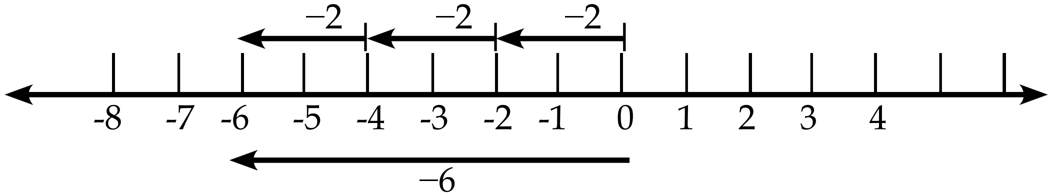


Multiplication and division of integers

Multiplication is repeated addition e.g $+2 \times 3 = 2 + 2 + 2 = 6$ as illustrated or the number line below;



$-2 \times 3 = -2 + -2 + -2 = -6$



Similarly

Negative x negative	=	positive
Positive x positive	=	positive
Negative x positive	=	negative

It also applies in division of integers.

Negative ÷ negative	=	positive
Negative ÷ positive	=	negative
Positive ÷ positive	=	positive
Positive ÷ negative	=	negative

Exercise 1

1. Work out the following

   a) $^{+}4 + ^{+}5 =$ b) $^{-}3 + ^{-}6 =$ (c) $^{+}15 + ^{+}20 =$

   d) $^{+}6 + ^{-}12 =$ e) $^{+}1 + ^{-}10 =$

   f) $^{+}6 + ^{-}5 =$

2. Find x

   a) $x + ^{+}3 = +9$ b) $^{-}9 - x = ^{+}3$ c) $^{-}2 + x = 0$

   d) $x + ^{-}5 = ^{-}2$ e) $x - ^{-}7 = 3$ f) $^{+}7 - ^{-}x = ^{-}3$

3. Work out the following

a) $^{+}2 \times {}^{+}3 =$ b) $^{-}2 \times {}^{-}5 =$

c) $^{-}2 \times {}^{-}4 \times +5 =$ d) $^{+}6 \div {}^{+}3 =$

e) $^{-}12 \div {}^{+}24 =$ f) $^{-}30 \div {}^{-}5 =$

4) Find the value of y in each of the following:

a) $1\frac{1}{2}y - 3 = {}^{+}7$ b) $y^2 = {}^{+}25$ c) $y^2 + 2 = {}^{+}18$

Even, odd, prime and composite numbers

In primary school, you studied the even, odd, prime and composite numbers.

Even numbers are whole numbers which are exactly divisible by two e.g {2, 4, 6, 8, 10,………..}.

Odd numbers are whole numbers which when divided by 2, there is always one as a remainder e.g {3, 5, 7, 9, ………..}.

Prime numbers are whole numbers with only two factors, i.e. one and itself e.g {2, 3, 5, 7, 11, 13, 17,………..}.

Composite numbers are whole numbers with more than two factors.

Exercise 2

1. List down the first 12

a) even numbers b) odd numbers c) prime numbers

2. Determine which of the following numbers are prime or composite.

a) 83 b) 107 c) 38

d) 77 e) 43 f) 97

3. Choose any odd number and an even number. Multiply them. Is the product an odd or even? Is it always odd or even?

4. Write down the 12^{th}, 15^{th}, 200^{th} and nth

   a) even number b) odd number

Sub-Topic 2: Factors and multiples

In multiplication, when two or more numbers are multiplied together, the product (answer) obtained is called a multiple of the numbers. The numbers multiplied together are called the factors.

Example 4

a) $2 \times 3 = 6$
b) $3 \times 7 \times 4 = 84$

In (a), 6 is a multiple of 2 and 3 and therefore 2 and 3 are some of the factors of 6.
In (b), 3, 7 and 4 are some of the factors of 84 and 84 is a multiple of 3, 4 and 7.

Exercise 3

1. List factors of:

   a) 45 b) 72 c) 144

   d) 79 e) 625

2. List the multiples of:

   a) 2 up to 100 b) 5 up to 300 c) 12 up to 276

3. In your group, suggest situations where the knowledge of multiples and factors you have gained can be applied/ used in daily life.

Lowest common multiple

Example 5

List down the multiples of 5 and 6.

Multiples of 5: {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65……….}.

Multiples of 6: {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66………..}.

From the above sets, you must have found out that the Common Multiples of 5 and 6 are {30, 60,….}.

Out of these common multiples of 5 and 6, the lowest is 30 and therefore the lowest common multiple (LCM) of 5 and 6 is 30.

Highest common factor (HCF)

List down the factors of 12 and 18
Factors of 12 (F_{12}) are {1, 2, 3, 4, 6, 12}.
(F_{18}) = {1, 2, 3, 6, 9, 18}.

Common factors of 12 and 18 ($F_{12} \cap F_{18}$) = {1, 2, 3, 6}.
From the common factors, 6 is the highest common factor HCF.

Activity: Find the highest common factor of 40 and 30

Highest common factor can be found by prime factorisation of numbers.

Example 6

Find the HCF of 16 and 24.

Answer

Find the prime factors of 16 and 24.

2	16	
2	8	
2	4	
2	2	
	1	

$\therefore \quad 16 = \quad 2^4$

2	24	
2	12	
2	6	
3	3	
	1	

$\therefore 24 \quad = \quad 2^3 \times 3$

The common factor of 16 and 24 is 2.

The HCF is given by the lowest power of the common prime factor.

So the HCF of 16 and 24 is $2^3 = 8$

Example 7

Find the HCF of 92, 96 and 120.

Answer

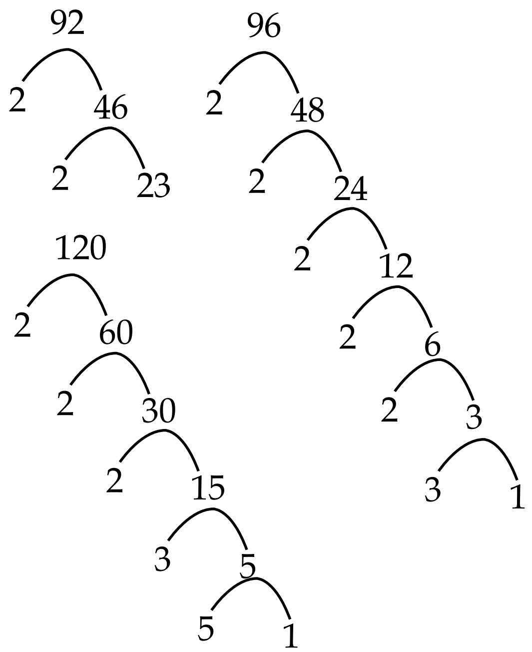
2	92	96	120
2	46	48	60
2	23	24	30
2	23	12	15
2	23	6	15
3	23	3	15
5	23	1	5
23	23	1	1
	1	1	1

$\therefore 92 \quad = \quad 2^2 \times 23$

$96 \quad = \quad 2^5 \times 3$

$120 \quad = \quad 2^3 \times 3 \times 5$

HCF of 92, 96 and 120 is $2^2 = 4$.



Alternative method is using a factor tree

$92 = 2 \times 2 \times 23$

$96 = 2 \times 2 \times 2 \times 2 \times 3$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

HCF = $2 \times 2 = 4$

Exercise 4

1. Find the HCF of:

   a) 15 and 42

   b) 36 and 72

   c) 12, 36 and 48

2. Find the HCF of:

   a) $2^2 \times 7^3 \times 17$ and $2 \times 3^2 \times 7 \times 17^2$

   b) $3^2 \times 5 \times 7^3$ and $2 \times 3^2 \times 5^2 \times 7^3$

   c) $2 \times 3 = 6$

   d) $3 \times 7 \times 4 = 84$

Disibility tests

Factors and multiples can be used to determine whether a given number can be divided or factorised. A number is divisible by:

i) 2 if the last digit of the number is 0 or even number, e.g. 106 or 50

ii) 3 if the sum of its digits is divisible by 3.

iii) 4 if the last two digits are divisible by 4 or are 00, e.g. 124 or 1200

iv) 5 if the last digit of the number is 0 or 5, e.g. 125 or 1020.

v) 6 if the number is even and is divisible by both 2 and 3, e.g. 96 or 144.

vi) 8 if the last three digits of the number make a sum divisible by 8 or they are 000, e.g. 12,000 or 1224.

vii) 9 if the sum of all the digits can be divisible by 9, e.g. 1233 or 207.

viii) 10 if the last digit of the number is 0, e.g. 110 or 100.

ix) 11 if the sum of the odd position digits and even position digits are equal or differ by a multiple of 11, then the number is divisible by 11, e.g. 132 or 506.

x) 12 if the number is divisible by both 3 and 4.

Exercise 5

1. Is 7936785 divisible by 11?

2. Which of the following is divisible by 9 ?

   a) 1926 b) 6163 c) 1378 d) 9774

3. Which of the following numbers are prime?

   a) 141 b) 221 c) 157 d) 113

4. In groups, find the importance of the divisibility tests.

Activity: Packing fruits

Objectives: To identify multiples of numbers and be able to apply them in daily life.

Introduction: A multiple is a product of two or more numbers which is divisible by each of them.

Materials; Paper bags/containers/boxes, fruits, cello tape/ glue.

Procedure

i) Group the fruits in multiples of 3.

ii) Pack them in paper bags/containers/boxes.

Precaution: Make sure there are in multiples of that particular number.

Observation: Multiples depict number patterns.

Interpretation: Multiples are products of numbers.

Application: Applied in supermarkets, when packing fruits, factories, and industries.

Follow up

i) Make multiplication tables.

ii) Identify areas where multiple numbers are applied.

TOPIC 7 ANGLES PROPERTIES OF GEOMETRICAL FIGURES

An angle is the amount a point or body turns; it is formed when two lines meet at a particular common point. An angle is measured by an instrument known as a protractor and it is expressed in degrees.

Angles can be clearly shown by the following illustrations.

Opening / closing a door or book.

Turning your head in any direction.

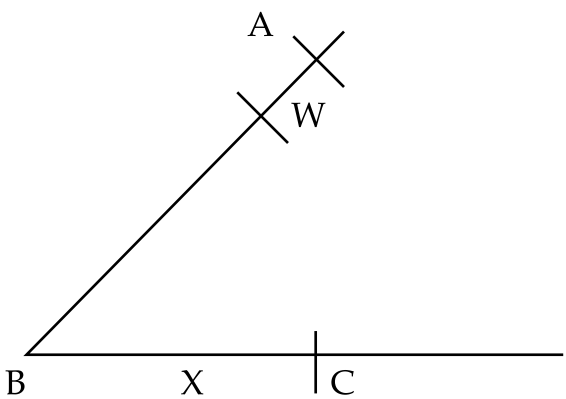
General angle properties of geometric figures

The amount of turn from one straight line to another at a common point (vertex) is termed as an angle.

Two lines that share the same end points form an angle. The point where the lines intersect is called the vertex of the angle.

An angle can be identified by using a point on each line and the vertex such as in the figure 1 below:

Figure 1



Angle ABC is equal to angle WBX because they share a vertex point which is always given in the middle.

Example 1

In the figures below, state the angles that are equal.
Figure 2 a

a)

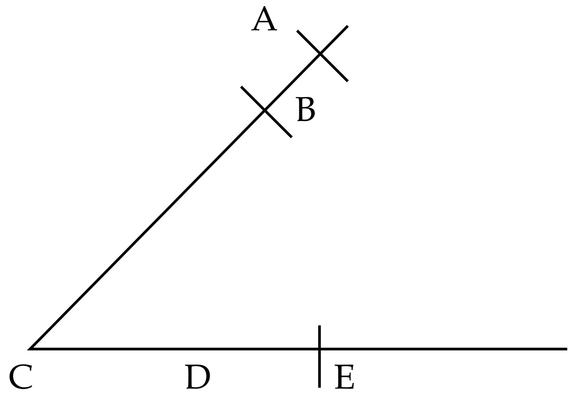
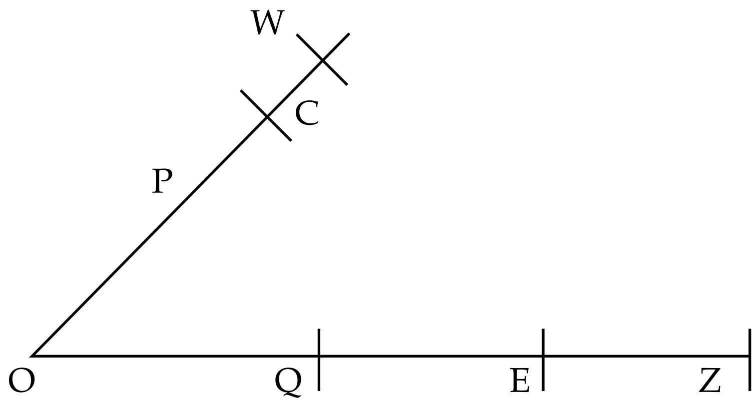


Figure 2b

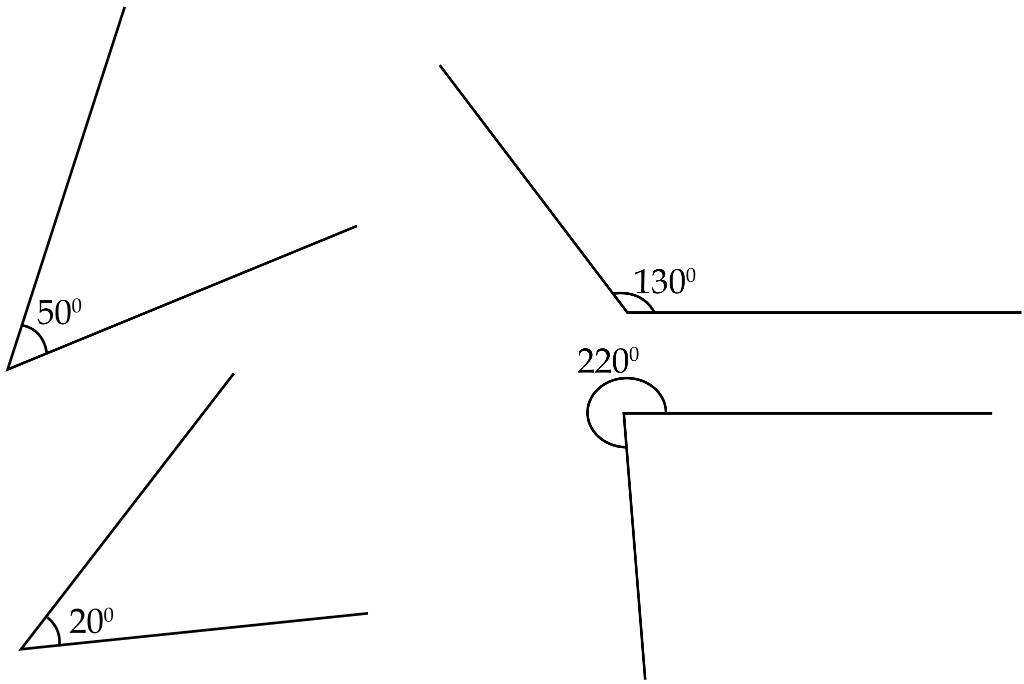
b)



Angles are measured in degrees with the help of a protractor.

Here are some examples of angles not drawn to scale.

Figure 3



Sub-Topic 1: Types of Angles

1. Acute angles: An acute angle is an angle measuring between 0^o and 90^o such as 15^o, 25^o, 85^o and 60^o.

Activity 1: List five more angles that are acute.

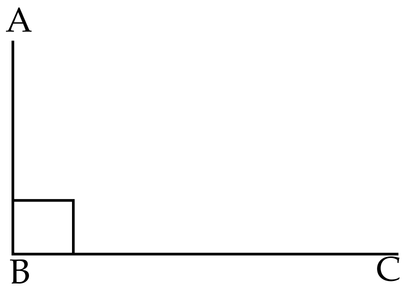
2. Obtuse Angles: These are angles measuring between 90^o and 180^o. These include 95^o, 110^o, 130^o, 149^o, etc.

Activity 2: List six angles that are obtuse.

3. Reflex Angles: These are angles between 180^o and 360^o. For example 190^o, 200^o and 290^o, etc.

4. Right Angles: These are angles measuring 90^0. Two lines or line segments that meet at right angles are said to be perpendicular as in the figure below:

Figure 3



Line AB is perpendicular to Line BC at point B.

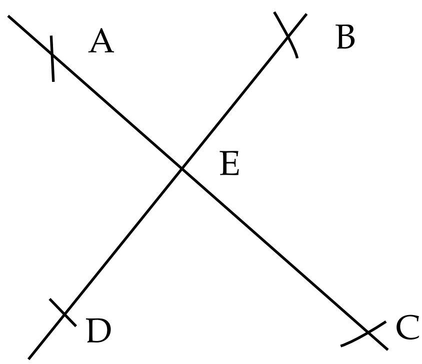
5. Complementary angles: Two angles are said to be complementary if the sum of their degree measurement equals 90^0. For example 40^0 and 50^0, 30^0 and 60^0, 20^0 and 70^o. List any 4 pairs of complementary angles.

6. Supplementary angles:

Activity 3

Two angles are supplementary if the sum of their degree measurements equals 180^o. For example 30^o and 150^o, 110^o and 70^o. List any five pairs of supplementary angles.

7. Vertically opposite ngles: For any two lines that meet such as in the figure below:

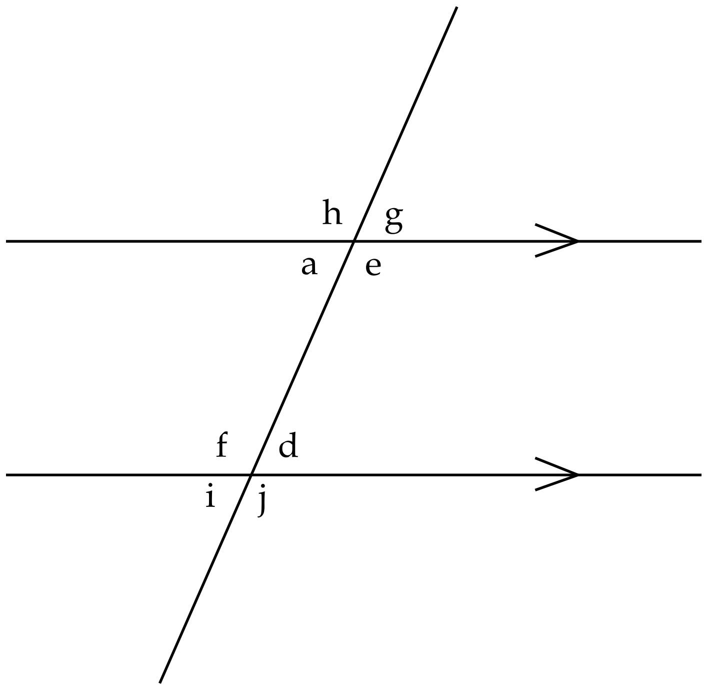


Activity 4

Angle AEB and DEC are vertically opposite and they have the same degree measurement.

State any other two angles on the figure that have the same degree measurement.

8. Alternate interior angles: These refer to any pair of parallel lines that are both intersected by a third line as in the figure below:



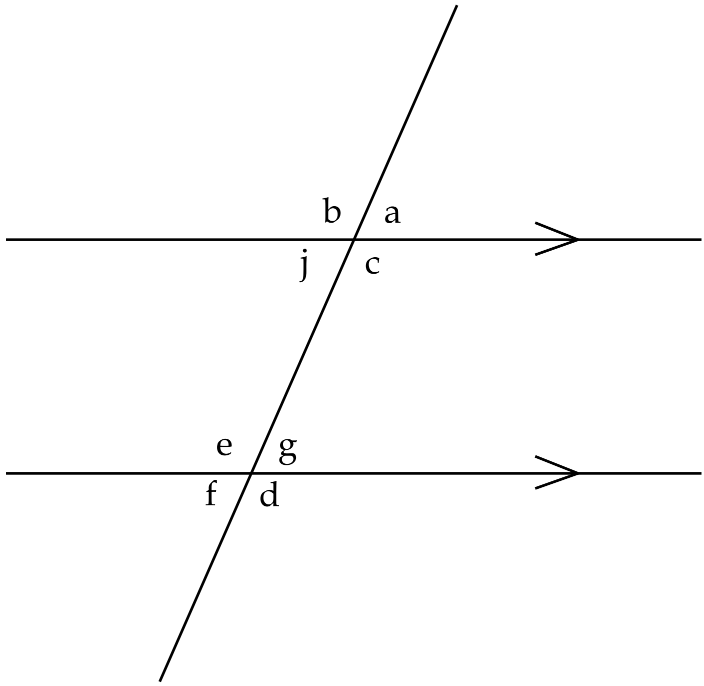
Angle a and d are called alternate interior angles. Alternate interior angles have the same degree measure.

a = b

Activity 5

State another pairs of alternate interior angles on the figure.

9. Alternate exterior angles: These refer to any pair of parallel lines that are both intersected by a third line as in the figure below:

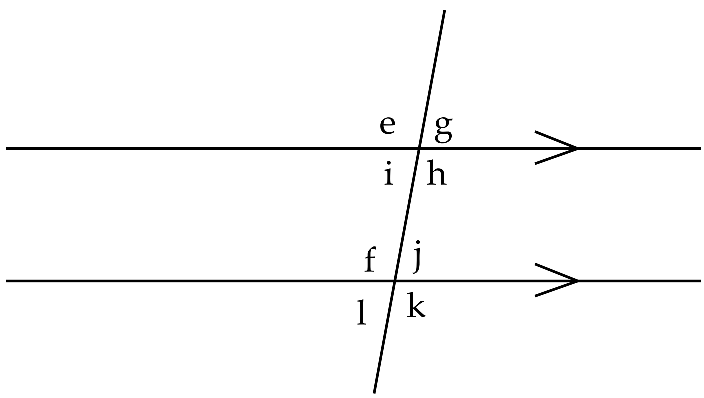


Angle b and d are alternate exterior angles. Alternate exterior angles have the same degree of measurement.

Activity 6

State a pair of angles in the figure that are alternate exterior.

10. Corresponding angles: These are formed when two parallel lines are intersected by a third line as in the figure below:

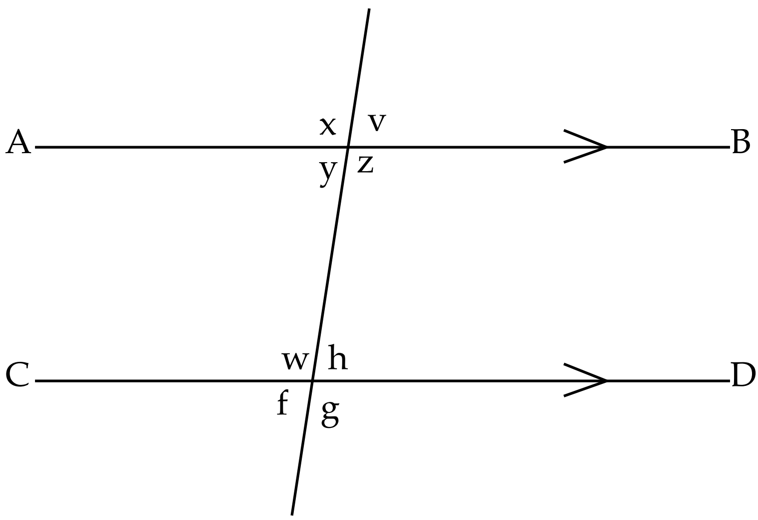


Angles e and f are corresponding angles and corresponding angles have the same degree of measurement.

Activity 7

State another pair of angles which are corresponding.

When the alternative interior angles, alternate exterior angles and corresponding angles are put together, they form a transversal as in the figure below.



If line AB is parallel to line CD then

$$\left.\begin{array}{l} x = w \\ y = f \\ v = h \\ z = g \end{array}\right\} \text{Sets of corresponding angles.}$$

$$\left.\begin{array}{l} y = h \\ z = w \end{array}\right\} \text{Sets of alternating angles.}$$

$$\left.\begin{array}{l} g = x \\ f = v \end{array}\right\} \text{Alternate exterior angles.}$$

$$\left.\begin{array}{l} z + v = 180 \\ v + x = 180 \\ z + y = 180 \\ h + w = 180 \end{array}\right\}\text{Supplementary angles}$$

$g + h = 180$

State the remaining supplementary angles.

Exercise 1

1. a) Find the supplements of the following angles:

   i) 130^o ii) 40^o iii) 80^o iv) 50^o

   b) Find the complements of the following angles:

   i) 35^o ii) 75^o iii) 15^o iv) 80^o

Sub-Topic 2: Angle Properties of Polygons

A polygon is a closed plane figure bounded by three or more line segments. There are two types of polygons: regular and irregular polygons. Regular polygons have all their sides and angles equal unlike the irregular polygons.

Triangles

These are polygons with three sides and three angles. The interior angle sum of a triangle is 180^0 or two right angles.

Example 2

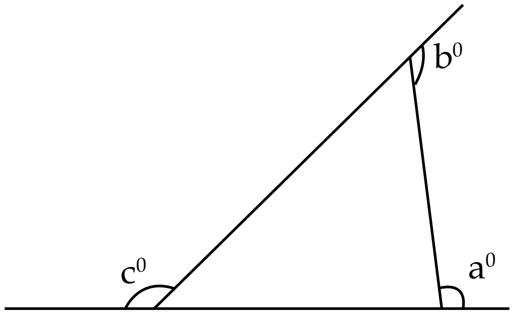
Given triangle ABC with BAC = 70^o, BCA = 50^o, and ABC = a^o

Find the value of a.

$$\begin{aligned} a + 50 + 70 &= 180 \\ a + 120 - 120 &= 180 - 120 \\ a &= 60 \end{aligned}$$

If one side of a triangle is produced, the interior angle is equal to the sum of two interior opposite angles.The exterior angle sum of a triangle is equal to 360^o.

Example 3



$a^o + b^o + c^o =$ 360

Using angle properties, the angles marked a^o, b^o and c^o can be found.

$a^o + b^o$ = 180 (angles on a straight line).

a = 180 – 60

a = 120

b + 70 = 180

b = 180 – 70

b = 110^o

c + 50 = 180

c = 180 – 50

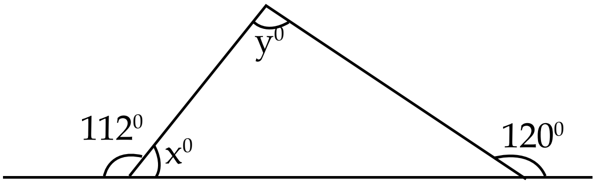
c = 130^o

Exterior angle sum

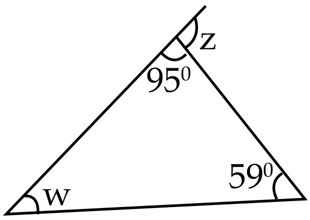
$120^o + 110 + 130^o =$ 360^o

Exercise 2

1. Find the values of the unknown in the following figures which are not drawn to scale.



2. Calculate the size of the angles given in the figures below:



Finding the angle of a regular polygon

If the polygon is regular, all its exterior angles are equal. Since at every vertex the interior plus external angle equals 180^o but the sum of all exterior angles equals 360^o.

We therefore find the exterior angle by dividing 360 by the number of sides of the polygon and the interior angle can be found by subtracting the exterior from 180.

Examples 4

1. Find the interior angle of a regular polygon with 9 sides.

Answer

The sum of all 9 exterior angles = 360^o

$\therefore$ Each exterior angle $= \frac{360}{9}$

$= 40^o$

$\therefore$ Each interior angle = $180 - 40$

$= 140^{o}$

2. Find the
   i) Exterior angle of a regular octagon.
   ii) Interior angle of a regular octagon.

Answer

The sum of all eight exterior angles $= 360^{o}$

Each exterior angle $= \frac{360^{o}}{8} = 45^{o}$

The interior angle $= 180 - 45$

$= 135^{o}$

Exercise 3

1. If each exterior angle of a regular polygon is 50^{o}, how many sides has the polygon.
2. Find the interior angle of a regular polygon with 20 sides.
3. Find the number of sides of a regular polygon whose interior angles are:
   i) 72^{0}
   ii) 108^{0}
   iii) 156^{0}
   iv) 60^{0}

Sum of the interior angles of a polygon

If all sides and all angles of a polygon are equal, then it is a regular polygon. A regular polygon of n sides has n angles.

Each exterior angle $= \frac{360}{n}$

Each interior angle $= 180 - \frac{360^o}{n}$

$= \frac{180n - 360}{n} = \frac{180(n-2)}{n}$

Total interior angle sum of the polygon will be:

$= \frac{180(n-2) \times n}{n}$

$= 180\,(n-2)$

Example 5

1. Find the sum of the interior angles of a 6 sided regular polygon.

$= 180\,(n - 2)$
$180\,(6 - 2)$
$180(4) = 180 \times 4 = 720^0$

Answer

2. An irregular pentagon has four of its angles as follows 120^o, 130^o, 40^o, and 110^o. Find the fifth angle:

Answer

Let the fifth angle be x

$x + 120 + 130 + 40 + 110 = 540$

$x + 400 = 540$

$x + 400 = 540 - 400$

$x = 140^o$

Exercise 4

1. Find the measure of each of the interior angles of a regular polygon with sides:

   i) 12 ii) 6 iii) 16

2. Find the number of sides of a regular polygon whose interior angles are:

   i) 90^{o} ii) 108^{o} iii) 135^{o}

   iv) 156^{o}

3. The angles of a pentagon are $x + 20$, $x - 30$, $2x + 10$ and

   $2x - 10$ degrees. Find the size of its largest angle.

4. A polygon has exterior angles of 61^{o}, 72^{o}, 53^{o}, and a^{o}. Find the value of a^{o}. State the interior angles of the polygon.

Dividing angles into quadrants/small portions or sectors.

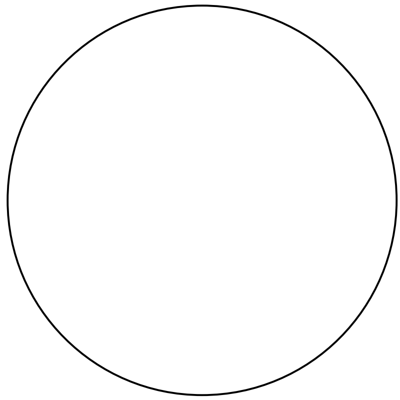
Activity 1: Making a wall clock

Materials

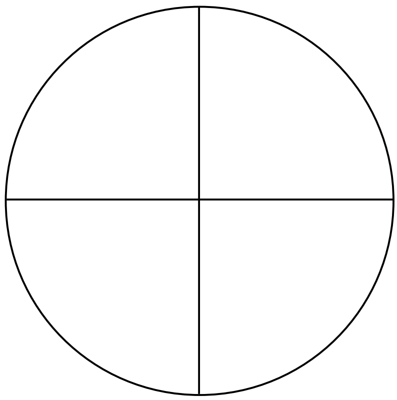
Manila paper, pencil, ruler, pair of compasses, cardboard, protractor, razor blade/pair of scissors, pins.

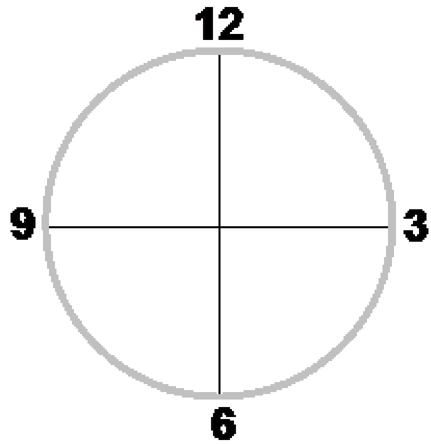
Procedure

i) Using a compass and convenient radius, draw a circle on a manila paper.



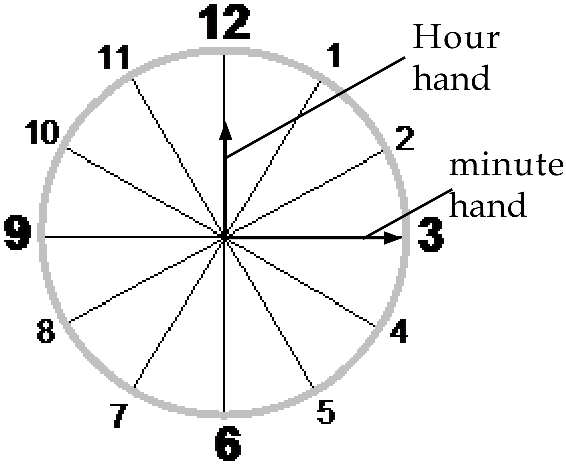
ii) Draw major axes to divide the circle into quadrants (4 equal parts).

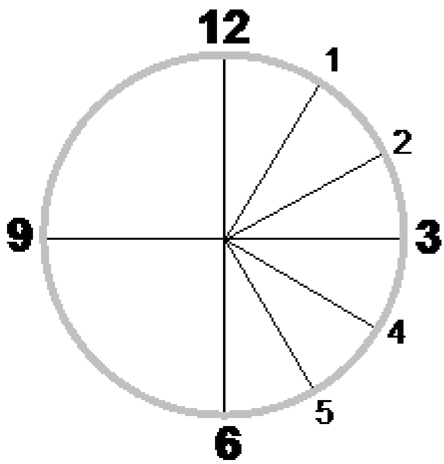




iii) From the Y-axis, sub-divide each quadrant of 90^0 into three equal sectors of 30^0 as illustrated below.

- Fill in the remaining numbers from 1 onwards.
- Cut two pieces of paper from the manila paper of not equal length to represent the hour and minute hand.
- Fix them together at the centre of the circle with a pin.





iv) Cut out the sketch and display on the notice board in the classroom.

Note: Angles must be accurately measured using a protractor.

Observations

Identify the time represented by each angle.

Interpretation

A wall clock is formed.

Application

Clocks are used in homes, classrooms, at places of work, shops, hospitals, etc.

Follow-up Activity: Make sketches of clocks using different shapes.

Activity 2: Making a door mat

Objective

Applying the knowledge of angles in designing a door mat.

Materials

Bottle tops, apiece of wood, nails, tape measure, hammer, saw/panga

Procedure

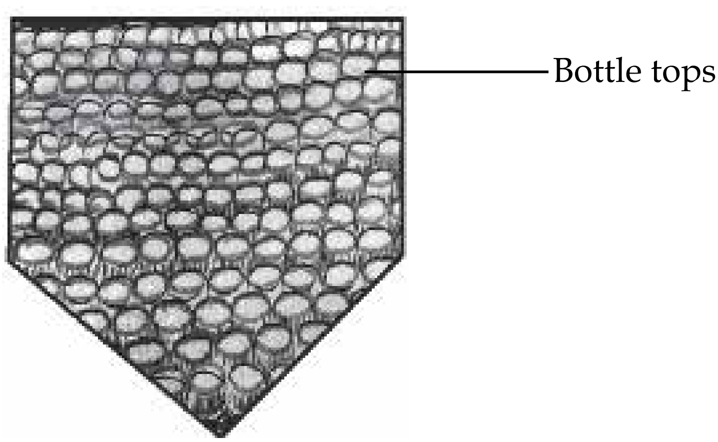
i) Cut the piece of wood of reasonable size of any shape i.e. rectangular, square and circular, etc.

ii) Draw lines on the piece of wood intersecting at the centre forming different angles.

iii) Align the bottle tops upside down.

iv) Nail the bottle tops onto the wood forming different patterns.

v) Flatten the bottle tops by hitting them to a rough surface as illustrated below.



Note:

i) Specific designs must be followed.

ii) Bottle tops must be nailed upside down.

Observation

Bottle tops are nailed at different angles.

Interpretation

Knowledge of angles is applicable in designing and construction.

Application

Doormats are used in homes, offices, hospitals, etc.

Follow-up Activity

Make many doormats in different shapes and sell.

Senior Two

TOPIC 8 RATIOS AND PROPORTIONS

Introduction

Ratios refers to sharing or dividing a quantity into different sizes. It can be expressed as a statement .i.e. 2:3, 1:2, 2:3:5, etc.

Proportion refers to the variations of two or more quantities in definite order (constant of proportional). Proportions include: Direct proportion, Inverse proportion, Joint proportion and Partial Proportions is denoted by the symbol (α)

The knowledge of fractions and decimals is a basic requirement.

Ratio and Proportion

This includes ratio as a fraction, proportion (direct and inverse) and representative fractions.

Sub-Topic 1: Ratio

1. **Ratio** is a type of fraction that enables to compare two or more quantities of the same kind and has no unit. The ratio of any two quantities say x and y is measured by the fraction $\frac{x}{y}$ and is expressed as x : y read as x to y. Ratios include statements, comparing quantities, multiple ratios, sharing and changing quantities in a given ratio.

   i) **Ratio as a Fraction**

Example 1

Express the following ratios as fractions in their simplest form.

a) 12:16 b) 10:15

$= \frac{12}{16} = \frac{3}{4}$ $\qquad$ $\frac{10}{15} = \frac{2}{3}$

$= \frac{3}{4}$ $\qquad$ $\frac{2}{3}$

Example 2

Find the value of 2a : 3b as ratio…. If a : b = 7:11

Steps

- First express the ratio as a fraction.
- Simplify the fraction.
- Express it as a ratio.

Answer

$= \frac{a}{b} = \frac{7}{11}$

a = 7 $\qquad$ b = 11

2a : 3b

$= \frac{2a}{3b} = \frac{2 \times 7}{3 \times 11} = \frac{14}{33}$

$\therefore$ 2a:3b = 14:33

Example 3

Express the following ratios in their simplest form.

a) 36 : 8

b) $\frac{1}{2}$: $\frac{3}{8}$

Answer

a) 36 : 8

$= \frac{36}{8} = \frac{9}{2}$

$\therefore$ 36:8 = 9:2

b) $\frac{1}{2} : \frac{3}{8}$

Answer

$= \frac{1}{2} / \frac{3}{8}$

$= \frac{1}{\cancel{2}_1} \times \frac{\cancel{8}^4}{3}$

$= \frac{4}{3}$

$= \frac{1}{2} : \frac{3}{8} = 4:3$

Example 4

Express the ratio 3 weeks:6 days in the simplest form.

Answer

1 week = 7 days

$\frac{3 \text{ weeks}}{6 \text{ days}} = \frac{(3 \times \cancel{7}^1) \text{ days}}{\cancel{6}_2 \text{ days}} = \frac{7}{2} = 7:2$

Exercise 1

1. Express the following ratios as fractions in their simplest form.

   a) 14:20 b) 72:81

2. Express the following ratios in their simplest form.

   a) 2m : 2cm b) $= \frac{3}{4} : \frac{5}{8}$ c) Shs.200 :Shs.1000

   d) 16m :1.5m If a : b = 3 : 4

3. Find the numerical values of:

i) $a^2:b^2$ ii) $\frac{1}{b} : \frac{1}{a}$

ii) Comparison representation and interpretation of quantities in ratio

Ratios are used to compare two or more quantities.

Example 5

In a school there are 250 boys and 150 girls. Find the ratio of the number of boys to the number of girls.

Answer

$$b:g = 250 : 150$$

$$= \frac{\cancel{250}^{5}}{\cancel{150}_{3}} = \frac{5}{3}$$

$$= 5:3$$

Example 6

Christine's age is quarter of her mother's age. Find the ratio of Christine's age to her mother's age.

Answer

Let her mother's age be x then Christine's will be $\frac{1x}{4}$

$$\frac{\text{Christine's age}}{\text{Mother's age}} = \frac{1x}{4} \times \frac{x}{4}$$

$$= \frac{1x}{4} \times \frac{1}{x} = \frac{1}{4}$$

$$= 1:4$$

Example 7

The price of bread was raised from Shs.1000 to Shs.1200. Find the:

i) Ratio of the new price to the old price.

ii) Ratio of the increase in price to the old price.

Answer

i) $\frac{\text{New price}}{\text{Old price}} = \frac{\cancel{1200}}{\cancel{1000}_{5}}$

$\therefore$ New price: Old price $= 6:5$

ii) Increase in price $= 1200 - 1000$

$= 200$

$= \frac{\text{Increase in price}}{\text{Old price}} = \frac{\overset{1}{\cancel{200}}}{\cancel{1000}_{5}}$

$= 1:5$

Example 8

Given that 100:1 = 8m : x. Find x

Answer

$= \frac{100}{1} = \frac{8}{x}$

$= 100x = 8$

$x = \frac{\overset{2}{\cancel{8}}}{\cancel{100}_{25}} = \frac{2}{25}$ or 0.08m

Exercise 2

1. Find x and y if 2:1:4 = 4:x:y
2. One day, 32 out of 480 pupils in a Senior Two Class were absent, what is the ratio of absentees to those who were present?
3. John earns Shs 1225 a day and Musa earns Shs 1350 a day. Find the ratio of John's earning to Musa's earnings.
4. In a school there are 90 students in S1, 80 students in S2 and 100 students in S3. Find the ratio of students in S1 to those in S2 to those in S3.
5. Given the ratio; 3:2000 = x:6Kg. Find x.

iii) Multiple ratios

This refers to a comparison of more than two quantities in their given rations.

Example 9

If A:B = 4:3 and B:C = 2:1. Find A:B:C

A:B = 4:3, B:C = 2:1

Answer

Since B is common to both ratios A:B and B:C, you combine the two ratios by making B's value in both ratios the same.

Multiply each ratio by B's value, i.e. multiply A:B = 4:3 by 2 and

B:C = 2:1 by 3

A:B = 2(4:3) = 8:6

B:C = 3(2:1) = 6:3

∴ A:B:C = 8:6:3

Example 10

If P:Q = 5:4 and R:Q = 2:3. Find P:R

Answer

P:Q = 5:4, R:Q = 2:3

P:Q = 5:4 R:Q = 4(2:3)

= 3(5:4) = 15:12

$\therefore$ P:Q:R = 15:12:8

$\therefore$ P:R = 15:8

Exercise 3

1. If A:B = 3:4 and B:C = 1:2. Find A:B:C
2. If X:Y = 3:5 and Y:Z = 4:7. Find X:Y:Z
3. If A:B:C = 1:2:3 and C:D:E = 4:3:2. Find A:B:C:D:E
4. Given that P:Q = 7:5 and M:Q = 3:2. Find P:M.

iv) Sharing

If a quantity is divided into two or more parts so that there are x units in the first part, and y units in the second part, then we say that the quantity is divided in the ratio x:y.

Example 11

James, Sarah and Mary shared 36 mangoes in the ratio 2:3:4. How many did each get?

Answer

Total shares = 2 + 3 + 4

= 9 shares

So the girls got $\frac{2}{9}$, $\frac{3}{9}$ and $\frac{4}{9}$ respectively of mangoes.

Jane got $\frac{2}{\cancel{9}_{1}} \times \cancel{36}^{4} = 8$ mangoes

Sarah got $\frac{3}{\cancel{9}_{1}} \times \cancel{36}^{4} = 12$ mangoes

Mary got $\frac{4}{\cancel{9}_{1}} \times \cancel{36}^{4} = 16$ mangoes

Example 12

Divide 23000 shillings between A, B and C if A's share to B's share is 2:3 and B:C is 5:7.

Answer

A:B	=	2:3	B:C	=	5:7
	=	5(2:3)		=	3(5:7)
	=	10:15		=	15:21
A:B:C	=	10:15:21			

Total shares = 10 + 15 + 21 = 46 shares

A's share $\frac{10}{\cancel{46}_{1}} \times \cancel{23{,}000}^{500} = 5{,}000/=$

B's share $\frac{15}{\cancel{46}_{1}} \times \cancel{23{,}000}^{500} = 7{,}500/=$

C's share $\frac{21}{\cancel{46}_{1}} \times \cancel{23{,}000}^{500} = 10{,}500/=$

Example 13

Divide 9,200 shillings among Sarah, Jane and Musa in the ratio $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$

Answer

Note: Change fractions to whole numbers by multiplying the denominators by their L.C.M.

2	2	3	4
2	1	3	2
3	1	3	1
	1	1	1

LCM $= 2^2 \times 3$
$= 12$

$\frac{1}{2} \times 12 = 6$, $\frac{2}{3} \times 12 = 8$, $\frac{3}{4} \times 12 = 9$

Sarah : Jane : Musa = 6:8:9

Total shares is $6 + 8 + 9 = 23$

Sarah's share $\frac{6}{\cancel{23}_{1}} \times \cancel{9,200}^{400} = 2,400/=$

Jane's share $\frac{8}{\cancel{23}_{1}} \times \cancel{9,200}^{400} = 3,200/=$

Musa's share $\frac{9}{23_{1}} \times \cancel{9,200}^{400} = 3,600/=$

Exercise 4

1. Divide 1800/= in the ratio 3:7
2. Four shareholders hold 480 shares, 350 shares, 525 shares and 645 shares. Divide Shs 150,000 among them in the ratio of their holding.
3. A line segment 12cm long is divided in the ratio 2:1 into two parts. What is the length of the longer part?
4. The ratio of boys to girls in a class is 9:11. If there are 40 pupils in the class, how many are boys and how many are girls?

v) Change in the given ratios

a) If the price of 1 kg of sugar has increased from Shs 2500 to Shs 2800. Then new price: old price

$= 28\cancel{00}:25\cancel{00}$
$= 28:25$

We say that the price has increased in the ratio 28:25. That is

New price $= \frac{28}{25}$ of original price

b) If the price of 1 kg of sugar has fallen from Shs 2800 to Shs 2500 then

New price : old price = 2500:2800
= 25:28

That is the price has increased in the ratio 25:28.

$\therefore$ New price $= \frac{25}{28}$ of original price

Example 14

1. Decrease 70 litres in the ratio 13:14

Answer

$$\text{New} = \frac{13}{14} \times 70 = 65 \text{ litres}$$

2. Increase 12 hours in the ratio 5:3

Answer

$$\text{New} = \frac{5}{\cancel{3}_{1}} \times \cancel{12}^{4} = 20 \text{ hours}$$

3. The price of coffee has gone up 15% in the last three years. In what ratio has it increased?

Answer

The new price is $\frac{100 + 15}{100} = 115\%$

Old price is 100%

$$\frac{\text{New price}}{\text{Old price}} = \frac{115\%}{100\%} = \frac{23}{20}$$

$\therefore$ The price has increased in the ratio 23:20

4. John is making some lemon squash for his family. He mixes lemon juice, sugar and water in the ratio 3:1:5 by the volume. John has 240ml of lemon juice.

    i) How much sugar and water will he need to make the squash?

    ii) How much lemon squash will he have altogether?

Answer

Lemon juice : sugar : water

i)

$$3 : 1 : 5$$

$$3 : 1 : 5 = 240 : s : w$$

$\frac{3}{240} = \frac{1}{s}$	$\frac{3}{240} = \frac{5}{w}$
$3s = 240 \times 1$	$3w = 5 \times 240$
$s = \frac{\cancel{240}^{80} \times 1}{\cancel{3}}$	$w = \frac{5 \times 240}{3}$
= 80mls of sugar	= 400mls of water.

ii) 240 + 80 + 400

= 320 + 400

= 720mls of lemon squash.

Exercise 5

1. Increase 42 litres in the ratio 7:6
2. Decrease 35 in the ratio:

   i) 5:7 ii) 3:5 iii) 4:9
3. A shopkeeper reduced all his prices by 10%. In what ratio have the prices been reduced?
4. The number of boys in a school has increased from 350 to 420. In what ratio has the number increased?
5. Last year Sarah sold 200 clusters of bananas. This year the number sold was reduced to the ratio 3:4. How many clusters did he sell this year?
6. The ratio of a man's weight to that of his wife is 4:3. The man weighs 84kg. How much does his wife weigh?

7. A shop is selling pads of (book) paper for Shs 1500 each. The price is increased in the ratio 7:5. What is the new price?

8. The shop is selling mathematical sets for 1500 shillings each. The price is decreased in the ratio 5:6. What is the new price?

Sub-Topic 2: Proportions

Introduction

Proportion is an increase or decrease of given quantities in a given ratio.

1. How much sugar is put in a tea cup?

2. How does a cook get the right amount of salt when preparing sauce?

At this level, we consider direct proportions and indirect (inverse) proportion. There are mainly two types of proportions namely direct and indirectly (inverse) proportion.

a) Direct Proportion

This is an increase or decrease in one item that causes an increase or decrease in the other in the same ratio.

Example 14

Price of notebooks bought is directly proportional to the number of notebooks.

Table I

Number of notebooks –	x	2	3	4	5	6
Price in shillings –	y	400	600	800	1000	1200

It is written as $y \propto x$.

$$\frac{\text{Number of notebooks}}{\text{Price in shillings}} = \frac{2}{400} = \frac{3}{600} = \frac{4}{800} = \frac{5}{1000} = \frac{6}{1200}$$

$$= \frac{1}{200} = 1:200$$

If the number of notebooks is doubled, the total price paid will also be doubled.

$\frac{y}{x}$ = constant (k)

$y = kx$

Using the table above, $x = 2$ when $y = 400$.

$k = \frac{400}{2} = 200$

$y = 200x$. This is called the linear function or equation.

Example 15

If y is directly proportional to x, copy and complete the table below:

x	8	--	48
y	--	42	84

Answer

$\frac{y}{x} = k \text{ - } y = kx$

$x = 48$
when $y = 84$

$k = \frac{84}{48} = \frac{7}{4}$

$y = \frac{7x}{4}$

$= \frac{7 \times \cancel{8}^{2}}{\cancel{4}_{1}}$

$= 14$

When $y = 42$

$y = \frac{7x}{4}$

$42 = \frac{7x}{4}$

$x = \frac{\cancel{42} \times 4}{\cancel{7}_{1}}$

$= 24$

Example 16

If y varies directly as the square root of x, copy and complete the table below:

x	1	--	4
y	--	12	20

Answer

$$y \propto \sqrt{x} \Rightarrow \frac{y}{\sqrt{x}} = k$$

$$y = k\sqrt{x}$$

$20 = k\sqrt{4}$

$20 = 2k$

$10 = k$

$x = 1, y = 10\sqrt{1}$

$y = 10$

$y = 12, \; x = ?$

$12 = 10\sqrt{x}$

By squaring both sides

$144 = 100x$

$x = \frac{\cancel{144}}{\cancel{100}}$

$k = 10$

$= \frac{36}{25}$

Example 17

Musa drives 65km in 50 minutes. How far will he have gone after 90 minutes at the same average speed?

Answer

Distance is directly proportional to time.

$D = kt$

Alternatively:

$D = 65\text{km},\ t = 50$ minutes

$65 = \overset{13}{k} \times 50$

$K = \dfrac{65}{50} = \dfrac{13}{10}$

$D = \dfrac{13t}{10}$

$t = 90$ minutes

$D = \dfrac{13 \times \cancel{90}^{9}}{\cancel{10}}$

$= 117\text{km}$

Alternative method use ratios.

Time has increased in the ratio $9\cancel{0}:5\cancel{0}$

$= 9 : 5$

Distance covered $= \dfrac{9 \times \cancel{65}}{\cancel{5}}$

$= 117$ km

Example 18

The men from our village clear a field of 10 acres in 4 days. How many acres could they clear in six days?

Answer

The days increase in the ratio 6:4

Number of acres $= \dfrac{{}^{3}\cancel{6} \times \cancel{10}^{2}}{\underset{2}{\cancel{4}}}$

$= 15$ acres

$6 = \left(\dfrac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}\right)$

$= 15$ acres

Alternatively:

$4 = 10$ acres

$1 = \dfrac{10}{4}$ acres

$6 = \dfrac{6 \times 10}{4}$ acres

$= 15$ acres

Exercise 6

1. If y is directly proportional to x, copy and complete the table below:

x	--	12	21
y	30	60	--

2. A car travels 94km on 14 litres of petrol. How far does it travel on 21 litres?

3. Fencing a house costs 240 per 2.6m length. How much would 6.5m cost?

4. A man earns Shs 4800 in 8 hours. How many hours does it take him to earn Shs 7200?

b) Inverse (indirect) proportion

This refers to an increase in one quality which leads to a decrease in another in a given proportion.

If y varies inversely as y and x are connected by the formula

$yx = k$ (constant)

$y \alpha \frac{1}{x}$

Examples 19

1. Given that v is inversely proportional to t^2 and v = 25 when t = 2, find v when t = 5.

Answer

$v \alpha \frac{1}{t^2}$

$vt^2 = k$

$v = 25; t = 2$

$k = 25 \times 2^2$

$= 25 \times 4$

$= 100$

$vt^2 = 100$

$t = 5, \ v = ?$

$v \times 5^2 \ 100$

$v = \frac{100}{5^2} \qquad = \frac{\cancel{100}^{20}}{\cancel{5} \times \cancel{5}}$

$v = 4$

2. If it takes 8 days for 9 men to weed a field of millet, how long will it take 6 men?

Answer

The men have decreased in the ratio 6:9, so the time will increase in the ratio 9:6

$\therefore$ Time $= \frac{\cancel{9}^{3}}{\cancel{6}_{2}} \times \cancel{8}^{4} = 12$ days

Alternatively

9 men take 8 days

1 man would take (9 x 8) days

6 men would take $\frac{(\cancel{9}^{3} \times \cancel{8}^{4})}{\cancel{6}_{2}}$ days

$= 12$ days

3. I have bought enough 10kg of meat at Shs 6000/kg. How much can I buy if the price is reduced to Shs 5000/kg?

Answer

The price decrease in the ratio 50 :60 = 5:6, but the weight increase in the ratio 6:5

$\therefore$ The weight $= \frac{6}{\cancel{5}} \times \cancel{10}^{2} =$ 12 Kg

Alternatively

$6000 \times 10 = 5000 \times x$

$x = \frac{\cancel{6000} \times \cancel{10}^{2}}{\cancel{5000}} = 12\text{kg}.$

4. Three machines can harvest a farm in 7 days if used for 10 hours a day. How many days would two machines take if used for 12 hours a day?

Answer

The work done by the machines increase in the ratio 3:2, and the time decrease in the ratio 10:12 = 5 : 6

$$\therefore \frac{\overset{1}{\cancel{3}}}{\cancel{2}} \times \frac{\overset{5}{\cancel{10}} \times 7}{\underset{4}{\cancel{12}}} = \frac{35}{\cancel{4}} \text{ days}$$

$$= 8\tfrac{3}{4}$$

Alternatively:

3 machines take 10 hours → 7 days

1 machines would take 10 hours → (7 x 3) days

1 machines would take 1 hour → (7 x 3 x 10) days

1 machines 12 hours → $\frac{(7 \times 3 \times 10)}{12}$ days

2 men 12 hours → $\frac{(7 \times \overset{1}{\cancel{3}} \times \overset{5}{\cancel{10}})}{\cancel{12} \times \underset{6}{\cancel{2}}}$ days

$= \frac{35}{4}$ days

$= 8\tfrac{3}{4}$ days

5. Otti takes 6 days to plough a certain piece of land. Musa takes 12 days to plough the same piece of land. Assuming that both work at the same rate, how long would the two take to plough the same piece of land?

Answer

Let the time taken by both be x days. Rate taken by Otti is $\frac{x}{6}$ and Musa $\frac{x}{12}$

Rate taken by both $\frac{x}{x}$

$$\therefore \quad \frac{x}{6} + \frac{x}{12} = \frac{x}{x}$$

$$\frac{2x + x}{12} = 1$$

$$\frac{3x}{12} = 1$$

$$x = \frac{\overset{4}{\cancel{12}} \times 1}{\cancel{3}}$$

$$= 4 \text{ days}$$

Exercise 7

1. Given that P is inversely proportional to q, copy and complete the table below:

p	1	--	7
q	--	14	10

2. John can drive to Masaka in 2 hours at $60km/h^{-1}$. How long will he take averagely at $50km/h^{-1}$?

3. There is enough food to feed 300 pupils for 21 days. Find how many more pupils would be needed for the same amount of food to last for 15 days?

4. A water tap takes 2 hours to fill a tank while another takes 3 hours to fill the same tank. How long would it take to fill the tank if both taps are running together?

5 Men working 8 hours a day can build a wall in 10 days. How long will it take 10 men to build the same wall if they work for only 6 hours a day?

Representative scale (RF)

This represents the length on the map to the actual length or distance on the ground.

Example 20

If 1cm on the map corresponds to 625000cm on the actual ground, then the scale of the map becomes 1:625000, and the representative fraction (RF) of the map is written as 1/625000. It is written as 1/n.

Examples 21

1. On a map of Uganda with 1/350,000, the distance from Masaka to Mbarara is 9.5cm. How far apart are these two towns, to the nearest km?

Answer

I cm on the map = 350,000cm on the actual ground
1km = 350,000cm

9.5cm = (350,000 x 9.5)cm

But 1km = 100,000cm

$$= \frac{(350,000 \times 9.5)\text{cm}}{100,000}$$

= 128.25km
= 128km.

2. The distance between Kitgum and Lira is 5cm on the map. The actual distance is 125km. What is the RF of the map?

Answer

5cm on the map represents 125km on the actual ground.
5cm = (125 x 100,000)cm

$$RF = \frac{5}{125 \times 100,000}$$

$\therefore$ RF = 1/2,500,000

3. The distance between Rubaga Hospital and Mengo Hospital is 1.5km. How far apart are the two hospitals on a map scale 1:16,000?

Answer

1cm on the map = 16,000cm on the actual ground.

16,000cm = 1cm

1cm = $\frac{1\text{cm}}{16000}$

$$1.5\text{km} = (1.5 \times 100{,}000)\text{cm} = \left(\frac{1}{\cancel{16{,}000}_{4}} \times 1.5 \times \cancel{100{,}000}^{25}\right)\text{cm}$$

$$= \frac{1}{4} \times \frac{\cancel{15}^{3}}{\cancel{10}_{2}} \times 25$$

$$= \frac{75}{8} = 9^3/_8\ \text{cm}$$

$$= 9\frac{3}{8}\ \text{cm}$$

Exercise 8

1. On a map of scale 1:250,000, the distance between two towns A and B is 16cm. Find the actual distance of AB in km.

2. The distance of 3.4cm on map represents 0.68m on the ground, find the representative fraction (RF).

3. On a map of scale 1:420,000, the distance between Murchison Falls and Karuma Falls is 15cm. How far apart are the two falls?

4. The length of Kazinga Channel is about 36km. How long is the channel on a map with representative fraction 1/360,000?

5. On a map the distance from Kasese to Fort Portal is 3.7cm. The actual distance between these two towns is 74km. What is the scale of the map?

Actvity : Making a cake

Objective: To get the right proportion of quantities in a given substance / expression

Introduction: It is considered to be a part share or number in relation to a given whole.

Materials: Baking powder, baking flour, blue-band, sugar, eggs, milk.

Procedure: Mix the ingredients below to form dough.

- Get 1 kg of wheat flour.
- ½ kg of sugar
- ½ kg of blue band
- 6 eggs
- 3 level tea spoons of baking powder.

Precaution: Take correct proportions of the ingredients. Use adjusted amount of heat.

Observation: Ingredients are in different proportions.

Interpretation: Different types of cakes require different proportions of the ingredients.

Application: Proportion is applied in the baking industry, catering services and homes.

Follow up activity: Make cakes of different sizes.

TOPIC 9 TRANSFORMATION

Sub-Topic 1: Similarity and Enlargement

Introduction

Two or more figures are similar when they have corresponding lengths in proportion and the corresponding angles are equal.

Enlargement is the decrease or increase in size of the image as compared to the object and image that are similar in shape.

Similarity And Enlargement

Any two geometric figures are said to be similar if they have the same shape but not necessarily the same size.

Comparing a small photograph and an enlarged copy shows that they have complete similarity between them. All the details of the small photograph are represented in the enlarged copy. The dimensions of the features are increased proportionally. Such figures are called similar figures.

Figure a

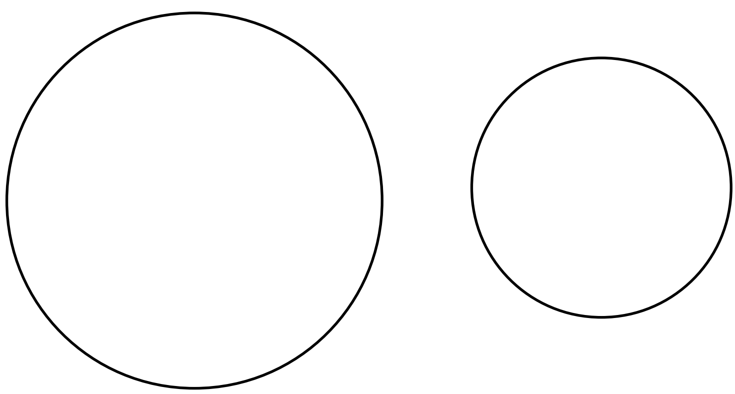


Figure b

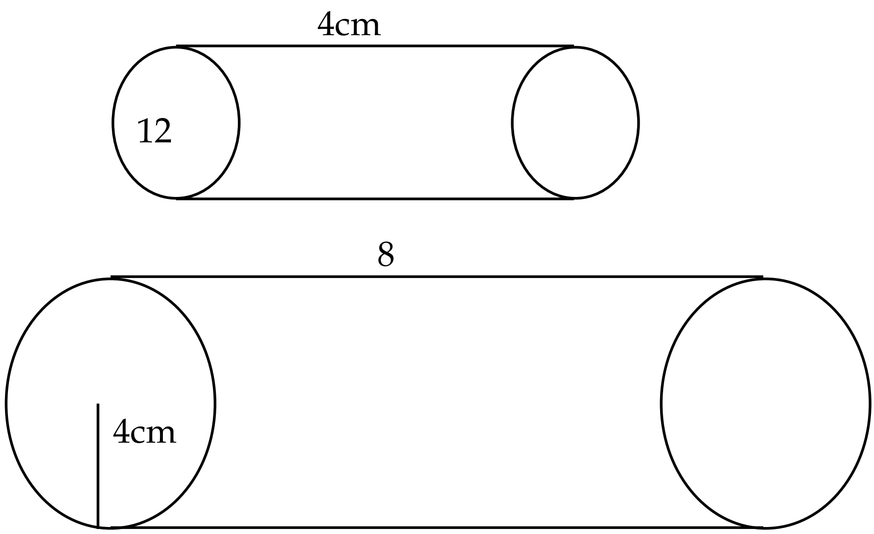


Figure c

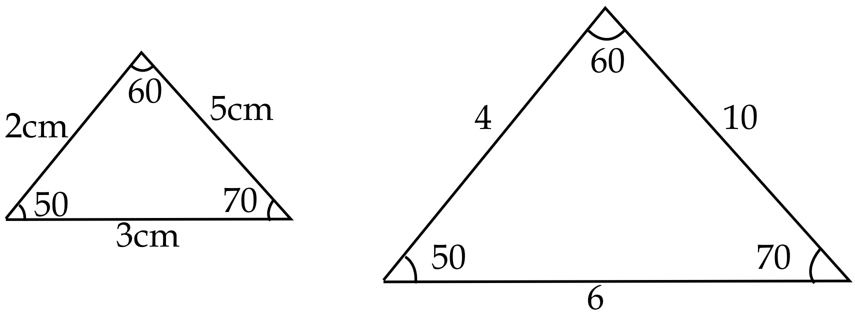
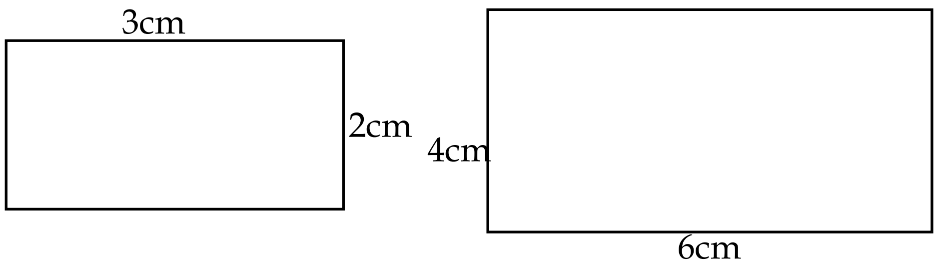


Figure d



The diagrams in figures a, b, c and d show examples of similar figures

Any two circles, squares, cylinders are similar as illustrated in the above figures a, b, and c.

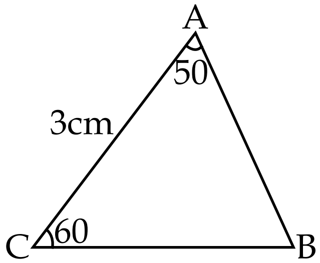
The following constructions and examples will illustrate more about proportionality.

i) Construction of similar figures

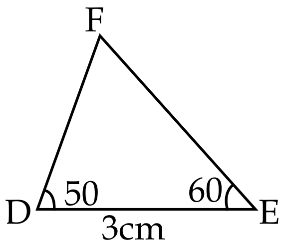
Constructions

Draw triangle ABC, DEF and PQR with the following measurements

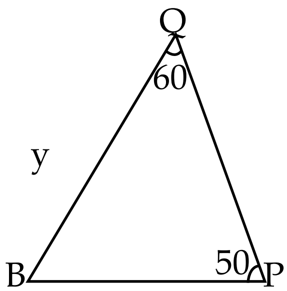
a) ΔABC having $\angle A = 50$, $\angle B = 60$ and $\overline{AB} = 2cm$



b) ΔADF having $\angle D = 50$, $\angle E = 60$ and $\overline{DE} = 3cm$



c) ΔPQR having $\angle P = 50$, $\angle Q = 60$ and $\overline{PQ} = 4cm$



Note: In the figures, sides AB, DE and PQ are the sides opposite to equal angles. These sides are called corresponding sides. State the other corresponding sides. Measure the remaining sides of the triangles and complete the following table:

i) $\frac{DE}{AB} = \frac{3}{2}$

ii) $\frac{PQ}{AB} = \frac{4}{2}$

iii) $\frac{PQ}{DE} = \frac{4}{3}$

iv) $\frac{EF}{BC}$ = ……………

v) $\frac{QR}{BC}$ = ……………

vi) $\frac{QR}{EF}$ = ……………

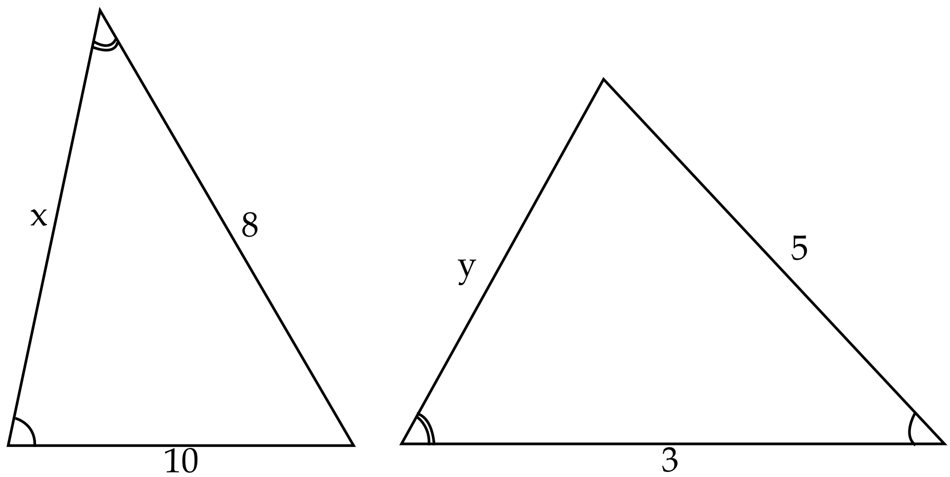
In general, two plane figures are similar if:

a) their corresponding angles are equal.

b) their corresponding sides are in the same ratio.

Example 1

1. The two triangles given below are similar. Find the numerical value of x and y.



Answer

Since the two triangles are similar, their corresponding sides are proportional

$$\frac{x}{3} = \frac{10}{5} = \frac{8}{y}$$

$$5x = 30$$

$$x = 6$$

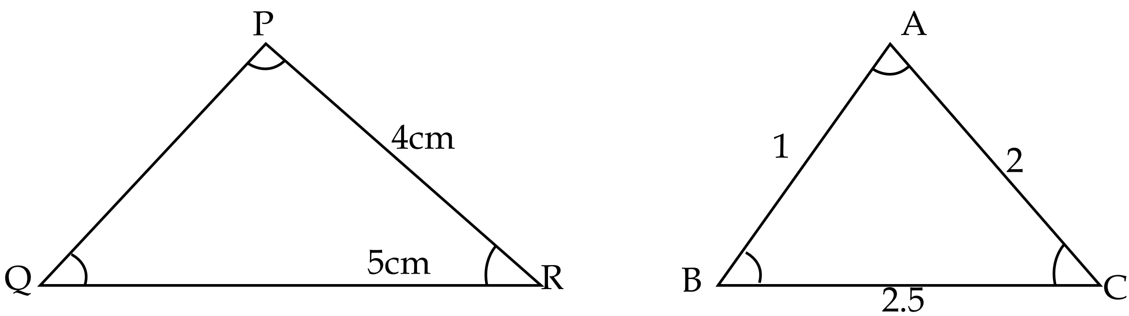
$$10y = 40$$

$$y = 4$$

Note: When taking the ratio of the sides involved in calculations, sides of the same triangle should appear either in the numerator or in the denominator.

Example 2

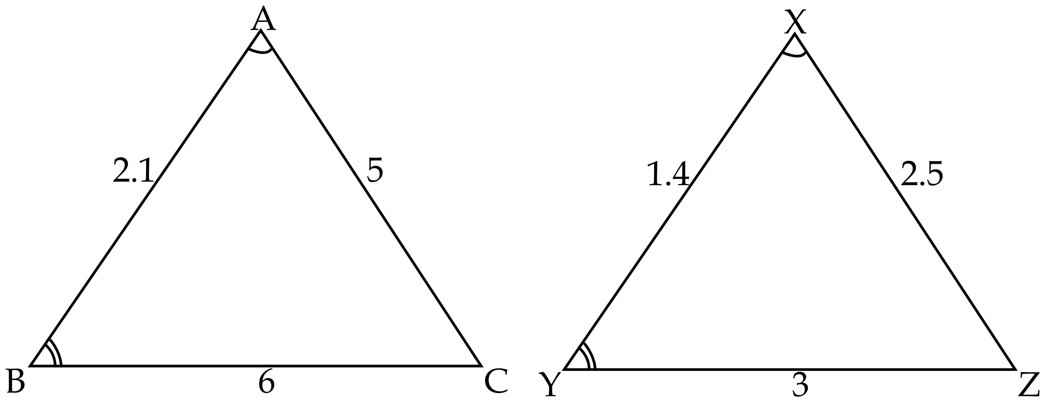
To find whether the given triangles are similar



Answer

i) $\frac{PQ}{AB} = \frac{2}{1}$ $\frac{QR}{BC} = \frac{5}{2.5} = \frac{2}{1}$ $\frac{PR}{AC} = \frac{\Delta}{2} = \frac{2}{1}$

The corresponding sides are proportional and hence the triangles are similar.

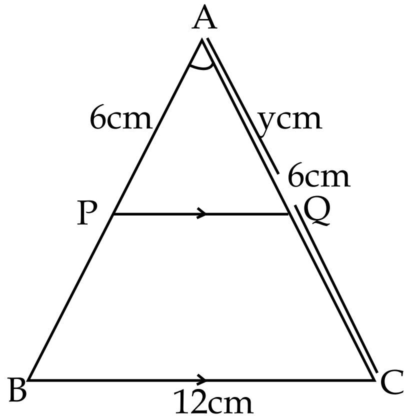


ii) $\frac{BC}{YZ} = \frac{6}{3} = \frac{2}{1}$ $\frac{AB}{XY} = \frac{2.1}{1.4} = \frac{3}{2}$ $\frac{AC}{XZ} = \frac{5}{2.5} = \frac{2}{1}$

The ratios of the corresponding sides are not equal hence the triangles are not similar.

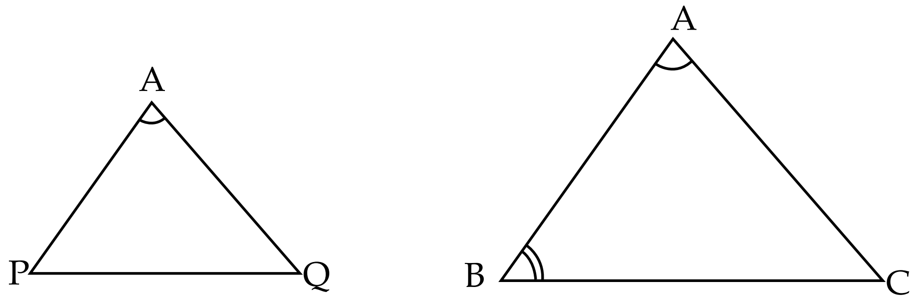
Example 3

In the figure below, find the length of the sides AB and AQ. PQ is parallel to BC and the dimensions are in centimeters.



Answer

Extracting similar triangles.



In Fig. 1

$\angle ABC = \angle APQ$ and $\angle ACB = \angle AQP$ corresponding angles.

$\therefore \Delta$ ABC is similar to Δ APQ

$\frac{AB}{6} = \frac{12}{9}$ and $\frac{12}{9} = \frac{6}{AQ}$

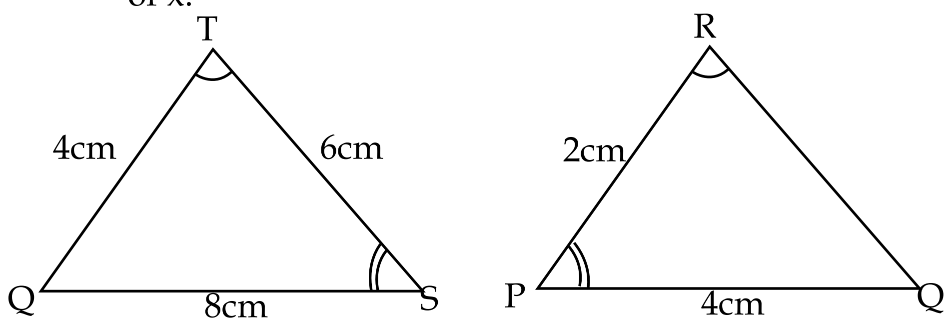
$AB = \frac{12 \times 6}{9}$ $\quad$ $12AQ = 9 \times 6$

$= 8cm$ $\quad$ $AQ = \frac{9 \times 6}{12}$

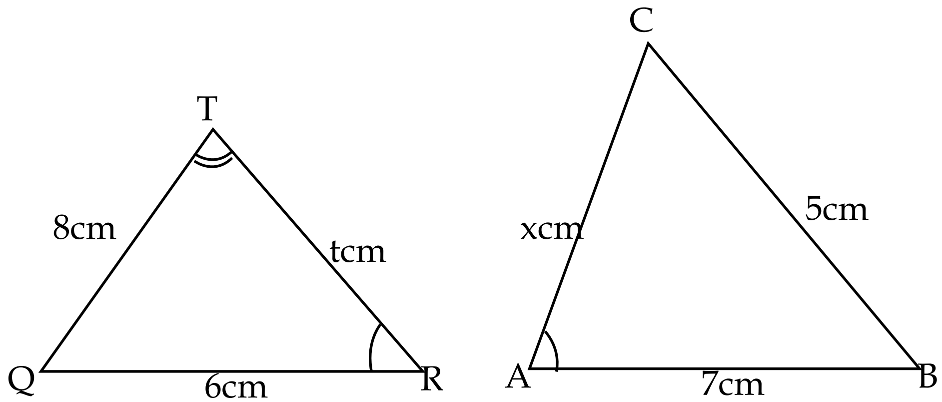
$= 4.5cm$

Exercise 1

1. State why triangles PQR and TQS below are similar. Write the ratios of the corresponding sides. Hence find the value of x.

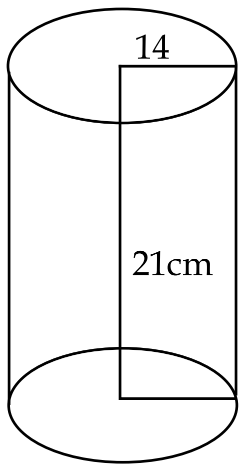


2. Calculate the length of the sides marked t and x.

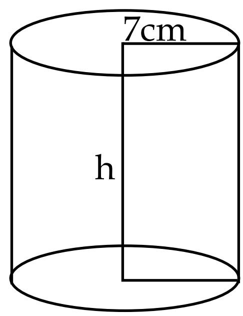


3. The figures below are two similar cylinders. Calculate:

i) The value of h



ii) The ratio of the volume of the bigger cylinder to that of the smaller cylinder.



Enlargement

This is a kind of transformation which is precisely specified by:

i) stating the centre of enlargement.

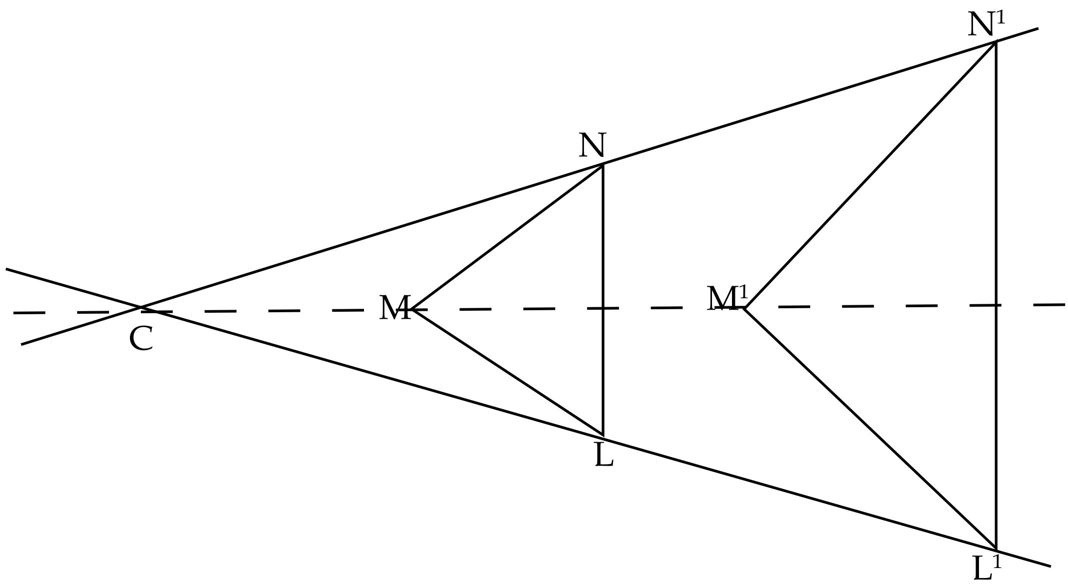
ii) the linear scale factor.

Here all the angles are preserved and the corresponding sides are proportional. Under enlargement, an object and its image are similar figures.

The centre of enlargement is the point through which every line joining an object point to its corresponding image will pass.

The linear scale factor is the ratio of the distance of the image from the centre of enlargement to the distance of the object from the centre of enlargement.

Figure 2



In the figure above, Δ LMN is mapped out triangle. $L^1M^1N^1$ under an enlargement. Given that LMN respectively maps into $L^1M^1N^1$, the centre of enlargement is found by joining L to L^1 M to M^1 and N to N^1. The point of intersection of the lines L^1L, MM^1, and N to N^1 is the centre of the enlargement. Thus C is the centre of the enlargement mapping LMN to $L^1M^1N^1$.

The linear scale factor (Lsf) is defined as

$$Lsf = \frac{CM^1}{CM} = \frac{CL^1}{CL} = \frac{CN^1}{CN}$$

$$\text{Or} \quad \frac{L^1M^1}{LMN} = \frac{M^1N^1}{MN} = \frac{L^1N^1}{LN}$$

Other useful scale factors are area factor and volume scale factor which are defined as:

$$Asf = \frac{\text{Area of Image}}{\text{Area of Object}}$$

$$Vsf = \frac{\text{Volume of Image}}{\text{Volume of Object}}$$

$\therefore$ Asf and Vsf are related to the linear scale factor (Lsf) by the equations:

$$Asf = (Lsf)^2$$

$$Vsf = (Lsf)^3$$

If the object and the image are on both sides of the centre of enlargement, the linear scale factor is positive.

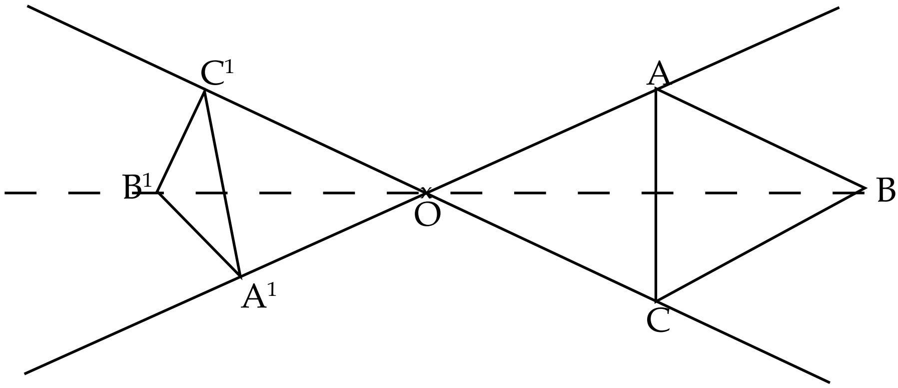
Negative scale factor of enlargement

Consider Figure 3 $\Delta A^1B^1C^1$ is the image of ΔABC, O is the centre of enlargement as A^1A, B^1B and C^1C intersect at O.

A and it's image A^1 are on opposite sides of O and the image is upside down. The object and image are still similar, corresponding sides are proportional and corresponding angles are equal.

Taking AOA^1 as a number line, if A corresponds to some positive number, O corresponds to zero then A^1 corresponds to a negative number, we refer to this as a negative enlargement and the scale factor K is a negative number.

Figure 3



$$\frac{OA^1}{OA} = \frac{-2}{4} = \frac{-1}{2}$$

$\therefore$ the scale factor k = $-\frac{1}{2}$

Note: Under enlargement where k = +1, the figures are directly congruent and where K = -1, the figures are oppositely congruent.

Enlargement in the Cartesian plane

Consider using Cartesian coordinates where the centre of enlargement is the origin.

A point A(x, y) or an object which is enlarged by LSF (i) 3 (ii) -1 with origin as centre will be:

Answer

i)	A(0,2)	3(0, 2)	=	$A^1(0, 6)$
ii)	A(0, 2)	-1(0, 2)	=	$A^1(0 – 2)$

Example

The square A(1,1) B(2, 1) C(2, 2) and D(1, 2) is enlarged with LSF:

i) 2

ii) $^-1$, with origin as centre. What are the coordinates of the corresponding images at points A^1 B^1 C^1 and D^1 in each case?

Answer

Doubling the coordinates we have:

i) $A^1(2, 2)$ B^1 $(4, 2)$ C^1 $(4, 4)$ and D^1 $(2, 4)$.

ii) Multiplying the coordinates by $^-1$

$A^1(^-1, {^-1})$ B^1 $(^-2, {^-1})$ C^1 $(^-2, {^-2})$ and D^1 $(^-1, {^-2})$.

Exercise 2

1. Two similar cups are 3cm and 5cm high respectively. What is the ratio of their capacities?

2. A figure has vertices P(4,0), Q(4,3), R(0,3) and S(3,-2). Plot these points on square paper. Draw the images of P, Q, R, S under an enlargement scale factor 3 with origin as the centre.

3. If ΔABC has vertices A(6,0), B(6,3), C(4,3) and $A^1(1,4,5)$, $B^1(1,3)$, $C^1(2,3)$ are the images under an enlargement. Find the centre and the scale factor of the enlargement.

4. PQ is an enlargement of AB with centre (0,1). The coordinates of A, B and P are (3,1), (5,1) and (9,1) respectively. Find the coordinates of Q.

5. Plot A(2,3), B(3,1) and C(2,2). If O, the origin, is the centre of an enlargement, draw the images of ΔABC under the enlargement:

a) $\Delta A_1B_1C_1$, scale factor 1½.

b) $\Delta A_2B_2C_2$ scale factor -1½.

If $\Delta A_2B_2C_2$, is an image of $\Delta A_1B_1C_1$ under an enlargement, find the centre and scale factor of the enlargement.

6. The image (0, 2) under an enlargement scale factor 3, is (4, 6). What is the centre of enlargement?

Activity: Making paper bags

Materials: Plain papers, old newspapers, ruler, pencil, glue, razor blades and pair of scissors

Procedure

i) Draw rectangles of same dimensions on the materials provided.

ii) Cut out the rectangles.

iii) Join them to form nets of cuboids/cubes.

iv) Fold the rectangles along the joints.

v) Stick them together using glue/masking tape forming paper bags of same size.

vi) Draw more rectangles using different scales.

vii) Go through the same process (forming many paper bags of different sizes.)

Precaution: Make sure that the ratios of corresponding parts are constant and angles are the same.

Observation

- Identify the type of polygons used to make the bags.
- Identify different sizes of rectangles.
- Identify the different sizes of paper bags made.

Interpretation

Properties of enlargement can be applied on polygons to make different sizes of paper bags.

Application

Similarity and enlargement is applied in all factories that make containers of different sizes, for instance paper bag factories, textile industries when making clothes of similar design but different sizes and in photography.

Follow up activity

i) Make different sizes of paper bags.

ii) Identify where they can be used.

TOPIC 10 APPROXIMATION AND ESTIMATION

Estimating and approximating are vital skills that need to be developed through the education of every individual. As children grow, their scope of understanding broadens to include situations beyond their grasp and experience. In order to sort out all these complications, genuine competence is needed in mathematical processes.

Estimating involves making a judgement based on general considerations in contrast to finding the quantity by an exact mathematical procedure. An estimation is an educated guess. We estimate quantities whose exact values are not known or not easily accessible, for example, the cost of a building, the size of the crowd and the amount of money to perform a task. These can be grouped into three categories:

i) Estimating the number of objects in a collection.
ii) Estimating the result of numerical computation.
iii) Estimating the measure of an object.

In estimates, it is difficult to assess the magnitude of the error involved in the estimate.

Approximating is an attempt to come near a target value which can be reached as closely as desired although sometimes not reached. Approximation is a result that is nearly but not exactly correct yet it is accurate enough for a specific purpose. Judging the accuracy of the approximation needed in a given situation is an essential part of approximation.

Sub-Topic 1: Approximation

In approximations and estimation, we will look at the following:

1. Rough answers (estimates) for calculations.
2. Significant figures.
3. Decimal places.

Rough answers (estimates):

This is a method that is used to obtain rough answers to calculations as stated in the examples.

Example 1: 36×31

$\approx 40 \times 30$

≈ 1200

Example 2: 310×7.8

$\approx 300 \times 8$

≈ 2400

Example 3: Find a rough answer to:

$$\frac{411.8 \times 18.42}{79.82}$$

$$\approx \frac{400 \times 20}{80}$$

≈ 100

Exercise 1

1. Obtain the rough answers (estimates) to the following calculations:

i) 330×21.3 ii) 3.17×630 iii) 0.136×7.47

iv) 53.6×1.87 v) 108×8.9 vi) $1.8^2 \times 2.4^2$

vii) $\frac{17.8 \times 0.723}{0.018}$ viii) $\frac{66.4 \times 12.5}{23.3}$

2. If a minibus is 3m by 33cm wide, roughly how many can be parked side by side in a garage 87m by 21cm wide.

3. A farmer has a rectangular field measuring 204m by 197m. He wishes to plant mango trees and thinks he needs to have about 1.5m between each tree and 1.75m between rows. Roughly how many rows will there be? How many mango plants roughly will he have?

Significant figures: These are digits of a number beginning with the first digit on the left which is not zero and ending with the last digit a non-zero.

They are figures in a number apart from any zeros which occur either before the first or after the last digit to show where the decimal comes as stated in the examples below.

Example 4

Write the following numbers correcting to two significant figures:

a) 6.089

= 6.1

b) 4.857

= 4.9

Example 5

State the number of significant figures in the following numbers:

a) 0.098

Two significant figures

b) 0.908

Three significant figures

c) 20.03

Four significant figures

Exercise 2

1. Approximate the following numbers to:

   a) 8294 b) 50135 c) 3.8775 d) 0.07279

   i) 1 significant figure
   ii) 2 significant figures
   iii) 3 significant figures

2. State the number of significant figures in the following:

   a) 4.269 b) 4003 c) 0.990 d) 403660

3. The length and breadth of a rectangle are measured each to two significant figures and found to be 7.8cm and 4.2cm. What are the greatest and least possible values of the area in cm^2.

Decimal places

Decimal places are counted from the decimal point to the right. Zeros after a decimal point are counted as in the examples below:

Example 6

2.0042 = This number is approximated to four decimal places.

Example 7

Approximate the following numbers to two decimal places:

i) 0.0375 = 0.04

ii) 67.1243 = 67.12

Example 9

Approximate the following to 3 decimal places:

a) 28.4467 = 28.447

b) 393.00462 = 393.005

c) 3.78496 = 3.795

Exercise 3

1. Approximate the following to the nearest

   i) whole number ii) tenth

   iii) hundredth

   a) 3.617 b) 0.947 c) 8.2967

2. Approximate the following to the nearest:

   i) Tens ii) Hundreds

   iii) Thousands iv) Whole number

   a) 56, 236.78 b) 41829.23 c) 16.467

   d) 3604.

3. Approximate the following numbers to;

   i) 1 significant figure ii) 2 significant figures

   iii) 3 significant figures

   a) 509.24 b) 0.09845 c) 824301

   d) 0.002367 e) 512632 f) 28.243

Approximation and **estimation** is a branch of applied arithmetic which involves counting and measuring. Sometimes measurements cannot be made easily. E.g. the amount of food one is to eat at a given lunch time, time taken by a student from home to school, the country's population at a given time.

In **approximation** and **estimation,** we make reasonable judgment to the likely value or quantity with numerals. It includes rounding off to whole numbers, decimal numbers.

Significant figures and standard form

Measurement can be expressed in terms of length, weight and capacity.

Accuracy in measurement is considered by what accuracy is necessary or possible with the method of measurement used.

Sub-Topic 2: Estimation

This involves judgment about the value or quantity required. It can be expressed in

i) Capacity (water, oil, milk)

ii) Weight (sugar, flour, rice)

iii) Length (length of a stick, desk radius).

Estimation in daily life is applied in homes:

When preparing food for the family, putting salt in source, sugar in tea, etc.

Example 9

Obtain a rough answer to the following Calculation.

i) $48 \times 21 \sim 50 \times 20 = 1000$

ii) $38 \times 11 \sim 40 \times 10 = 400$

iii) $407 \times 203 \sim 400 \times 200 = 80000$

Exercise 4

1 Give a rough answer to the following calculations.

i) $5005 + 101$

ii) 138×11

iii) $38 \div 11$

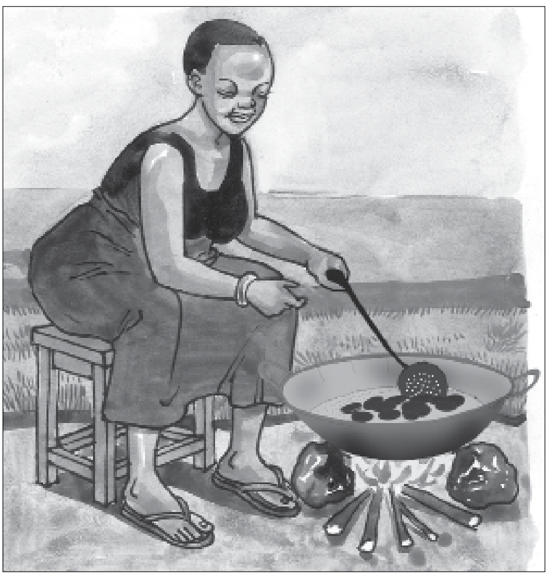
iv) $607 \div 209$

v) $523 + 149$

Activity: Making pancakes (Kabalagala)

Objective: To get the right quantities to use.

Materials: Cassava flour, ripe bananas, cooking oil, rolling stick/ bottle flat top, round cup, frying pan / saucepan and source of fire.



Procedure

i) Get the ripe bananas, peel and put them in the source pan.

ii) Estimate the right amount of cassava flour, mix the cassava flour and the bananas to form the dough.

iii) Roll the dough on a flat surface to form the right thickness of the pancake required.

iv) Cut the required size of pancake using a cup or any circular container; heat the cooking oil in a saurcepan to a boiling point.

v) Drop the cut pans in the boiling cooking oil.

vi) Estimate that they are ready and remove.

Note

i) Bananas to be used must be ripe.

ii) The cassava flour must be dust free.

iii) Heating saucepan must be moderate in size and clean.

Observation: Correct estimates of ingredients gives better products.

Follow up: Make as many pancakes as possible for sale.

Activity: Making tea

Objectives: To estimate required values/quantities.

Introduction: Involves rough idea on quantifiable values/ items.

Materials: Water, sugar, tea leaves, table spoon, cups of different sizes, heating source, kettle/container/ heater.

Procedure

i) Prepare boiled water.

ii) Pour the water in cups of different sizes.

iii) Estimate different quantities of sugar and tea leaves for each cup.

iv) Stir using a spoon.

v) Taste the mixture/solution.

Note: Water must reach the boiling point

Mix ingredients before the water cools.

Observation: Different sizes of cups need different estimates of ingredients.

Interpretation: Estimation is required in daily life.

Application: Tea is served/taken in homes, schools, offices, hotels, etc.

Follow up activity: Estimates of different sizes of children's wear, sugar, soap.

TOPIC 11 STATISTICS

Statistics is a branch of science which is concerned with the collection, organisation, interpretation and analysis of numerical information.

The numerical information is called data. This data can be presented in any of the following: frequency distribution table, bar charts, histogram, O-give curve, frequency polygon, pictograms and pie-charts.

Sub-topic 1: Collection and Presentation of Data

Statistical data refers to information in numerical form (figures). This can either be ungrouped or grouped. The frequency distribution table is one of the methods of data presentation which shows the distribution of data.

Frequency means the number of times an item appears or occurs in a given data which is denoted by (f).

Think

Figure 1

Data can be collected in three ways;

i) **Census surveying:** This is where every unit in the group is considered e.g. collecting the heights of all the students in the class.

ii) **Sample surveying:** This is where part of the total population is taken e.g. data can be collected from only 20 out of 70 students in a given class.

iii) **Administrative surveying:** This is collected as a result of organisation's day to day operations. For example data on birth, death, etc.

Data can be in one of the following forms;

i) **Discrete data:** It is recorded in whole figures e.g. number of people, cars, trees, etc.

ii) **Continuous data:** This refers to data that can be collected by a continuous scale e.g. heights of and weights of students, etc.

Example 1

An English teacher gave an exercise to his students in a class. The results were graded from A to F as follows:

E	E	A	D	C
A	F	C	B	C
D	A	A	B	D
B	F	A	E	D

a) Construct a frequency distribution table for the above grades.

b) State the number of students who did the exercise.

c) Find which grade was got by most students?

Answer

a) The frequency table

Grade	Tally	f
A	~~IIII~~	5
B	III	3
C	III	3
D	IIII	4
E	III	3
F	II	2
		Ef: 20

b) There are 20 students who did the exercise.

c) Grade A was got by most students (It has the highest frequency).

Exercise 1

1. A maths teacher gave a test to her students in Form 2 which was marked out of 5, the marks were as follows:

4	2	0	2	5
3	1	1	3	3
5	1	2	2	2
5	3	2	1	3
0	4	5	4	2

a) Construct a frequency table for the above marks.

b) How many students did the test?

c) Which mark was got by most students?

2. The following are students' marks obtained in a history test marked out of 100.

90	70	60	60
60	80	60	70
70	90	70	70
90	90	80	80
70	50	50	90

a) Construct a frequency table for the above marks.

b) Find the mark scored by most students?

c) Find the lowest mark using the frequency table?

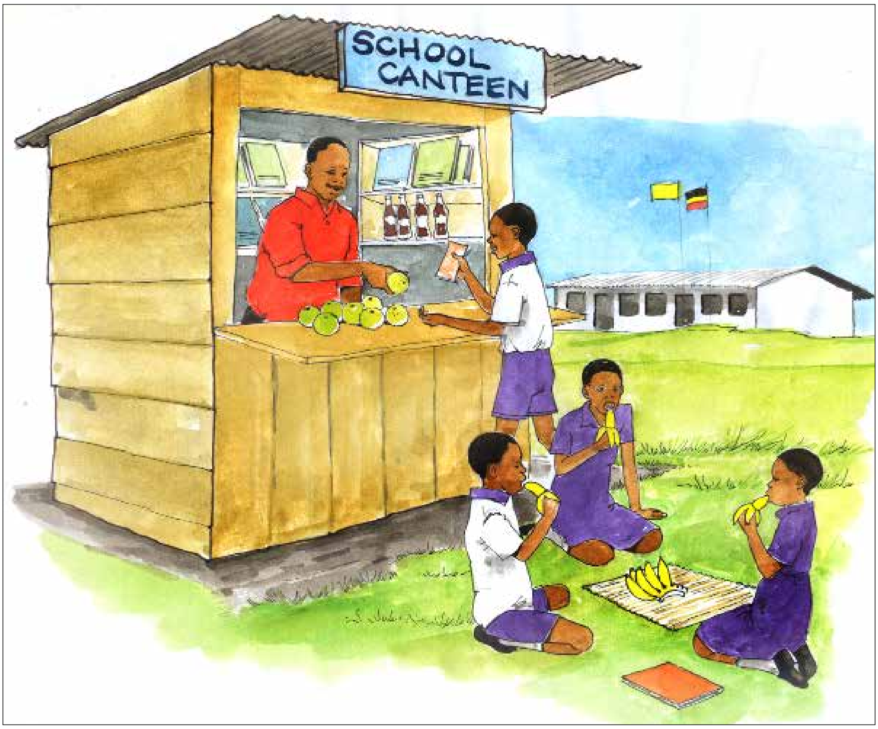
Activity: To find the most bought item in the school canteen

Objective

To collect required data and present it.

To interpret collected data.

Figure 2



In figure 2 above, identify the most consumed item in the school canteen.

Sub-Topic 2: Measure of Central Tendecy

This includes mode, median and radius.

i) Mode

Mode is an average, which gives us the most common member of a group. For example, if 10 students in S2 scored the following marks in an exercise marked out of 10: 5, 7, 6, 8, 5, 9, 6, 6, 7, 6, then the most common mark scored by students is 6. This is because 6 was scored by four students.

Mode can easily be determined if the data is arranged in either ascending or descending order. For example, if a die is thrown/ tossed 15 times with the following outcomes: 2, 3, 1, 2, 4, 6, 5, 2, 3, 3, 4, 1, 2, 6, 2, then these outcomes can be arranged in ascending order like this 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 5, 6, 6. The mode (the most common number) is 2. Note that it is possible to have more than one mode.

Frequency distribution tables may be used in determining the mode of a group of numbers or items if the data is large.

If a die is thrown 30 times giving the following results:

4, 6, 5, 1, 6, 2, 1, 3, 1, 3, 5, 5, 2, 5, 4,4, 3, 2, 6, 1, 2, 5, 6, 1, 2, 5, 2, 5, 4, 3, then a frequency table can be used to determine the mode.

A table showing the outcome of 30 die throws.

Result	Tally	Frequency
1	IIII	5
2	~~IIII~~ I	6
3	IIII	4
4	IIII	4
5	~~IIII~~ II	7
6	IIII	4

From the above table, the mode is 5 and it is appearing 7 times.

Exercise 2

1. The following were marks scored by S2 students in a test marked out of 25.

   22, 20, 17, 24, 22, 18, 17, 21, 21, 18

   18, 19, 20, 17, 24, 23, 23, 21, 20, 17

   22, 21, 20, 19, 19, 18, 20, 24, 23, 21

   18, 19, 17, 24, 23, 19, 20, 18, 23, 23.

   Find the mode of this class.

2. The heights (in cm) of 27 students at Koboko Secondary schools are given below:

158	152	163	165	152	166	153	164	165
158	162	153	154	160	154	153	162	163
151	157	153	159	156	160	161	153	155

   a) Present the data in a frequency distribution table.

   b) What is the mode?

ii) Median

Median is also an average which is given by the middle mark/ score when the data is arranged in ascending or descending order.

Example 2

Find the median of these numbers:

a) 40, 42, 41, 46, 48, 54, 43

b) 68, 73, 70, 71, 73, 72

Answer

a) Arrange in ascending order: 40, 41, 42, 43, 45, 46, 48
The middle number is 43. That means the median is 43.

b) Arrange in ascending order 68, 70, 71, 72, 73, 73

In this case, the middle numbers are two, i.e. 71 and 72.Add 71 to 72 and divide by 2 in order to get the median.

$$\frac{71 + 72}{2} = \frac{143}{2} = 71.5$$

So 71.5 is the median.

iii) Mean

This is an average which is the most likely or expected outcome because it represents all the members in the group.

The mean is obtained by finding the sum of all the group members and then dividing this sum by the number of members in the group.

Example 3

Find the mean of the following scores:

5, 5, 6, 7, 5, 6, 6, 6, 8, 8, 8, 7, 6, 6, 9, 8, 8, 6, 6, 8

Answer

$$\frac{5+5+6+7+5+6+6+6+8+8+8+7+6+6+9+8+8+6+6+8}{20}$$

$$= \frac{134}{20} = 6.7$$

If the data is large, then frequency distribution tables can be used.

Example 4

The following were marks scored by S2 students in a Maths test:

5	7	7	7	7	7	7	7	10	10
11	11	12	12	13	13	13	14	14	14
15	15	15	15	15	15	16	16	16	16
17	17	17	17	17	18	18	18	18	18
19	19	19	19	19	20	20	20	20	20
21	21	22	22	22	22	22	23	23	23
23	24	24	24	25	25	25	26	26	29

Calculate the mean score.

Answer

Score (x)	Tally	Frequency (f)	x x f
5	I	1	5
7	~~IIII~~ II	7	49
10	II	2	20
11	II	2	22
12	II	2	24
13	III	3	39
14	III	3	42
15	~~IIII~~ I	6	90
16	IIII	4	64
17	~~IIII~~	5	85
18	~~IIII~~	5	90
19	~~IIII~~	5	95
20	~~IIII~~	5	100
21	II	2	42
22	IIII	5	110
23	IIII	4	92
24	III	3	72
25	III	3	75
26	II	2	52
29	I	1	29
	Total	70	1197

Mean **=** $\dfrac{\text{Total of } \mathbf{x} \text{ x } \mathbf{f}}{\text{Total of f}}$ = $\dfrac{1197}{70}$ 17.1

Exercise 3

1. Find the mean for the values below:

   a) 4, 3, 2, 6, 8 b) -1, 1, 8, 6 c) 5, 1, 6, 8

   d) 6, 6, 4, 15 e) 2, 3, 4, 2, 3

2. Use the table below to calculate the mean.

Marks	10	13	20	25	30
Frequency	3	2	3	3	1

3. Use the table below to find x if the mean is $13^{1}/_{4}$

Score	10	12	15	20
Number of times	2	x	2	1

4. The mean of five numbers is 24. Four of the numbers are 18, 23, 25, 31. What is the fifth number?

5. The mean of five scores is 16. Their median is 17 and mode is 18. The two smallest scores differ by 1. What are the five scores?

6. The mean mass of 100 children is 56kg. The mean mass of the 40 girls is 53kg. What is the mean mass of the boys?

Activity: Displaying items sold in the canteen

Objective: To be able to collect data and analyse it.

Introduction: This is a science of collecting, presenting, interpreting and analysing data.

Materials: Items sold in the canteen.

Procedure

i) Record data for five days.

ii) Display the data in a frequency table.

iii) Represent the data in a bar graph/pie chart.

Precautions: Collaborate with the canteen sales people. Make sure the items are recorded in their categories.

Observation: Identify which items are bought most and those bought least.

Interpretation: Statistical knowledge is helpful in any field.

Conclusion: Facts can be drawn from statistical data.

Application: This is applied in industries, factories, population census, business world.

Follow-up activity: Collect data from any place of your convenience. Display the collected data in any statistical form.

Senior Three

TOPIC 12 THREE DIMENSIONAL GEOMETRY

The dimensional geometry involves

i) Length of lines.

ii) Angle between lines and planes.

iii) Angle between planes.

iv) Volumes and total surface area of 3-D geometrical figures.

Three dimensional geometry refers to solid geometrical figures with three different dimensions, i.e. length, breadth and height. It could be a cube, cuboid, pyramid, prisms, etc.

Sub-Topic 1: Points, Lines and Planes in Three Dimensional Geometry

Point: A point can be described in two dimensions as p(x, y) and in three dimensions as p(x, y, z) on the Cartesian plane. It can be anywhere on the line, plane or space.

Lines: A line can be uniquely determined either by two points or by two intersecting planes.

Planes: Aplane is determined by any:

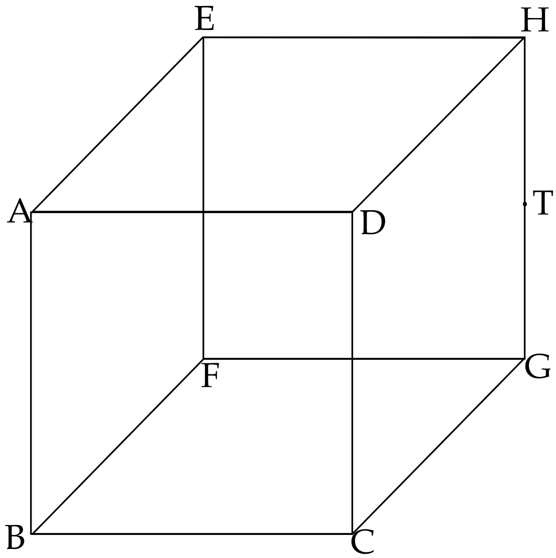
i) Three non-collinear points.

ii) One line and a point not on the line

iii) Two intersecting lines.

iv) Two parallel lines.

Figure 1



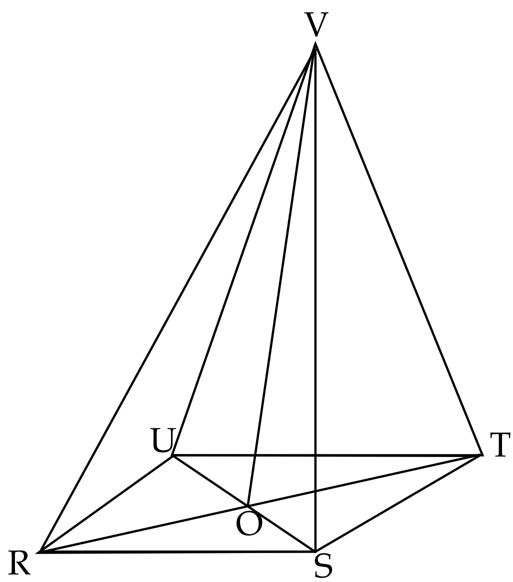
Examples of points from the figure above are E, D, B, C, G, A, FH, etc.

Lines: AB, BC, DG, HD, BT, etc.

Planes: ABCD, EFBA, HGAB, etc.

VOS, VST. RVS are examples of planes in figure 2.

Figure 2



Nature of lines

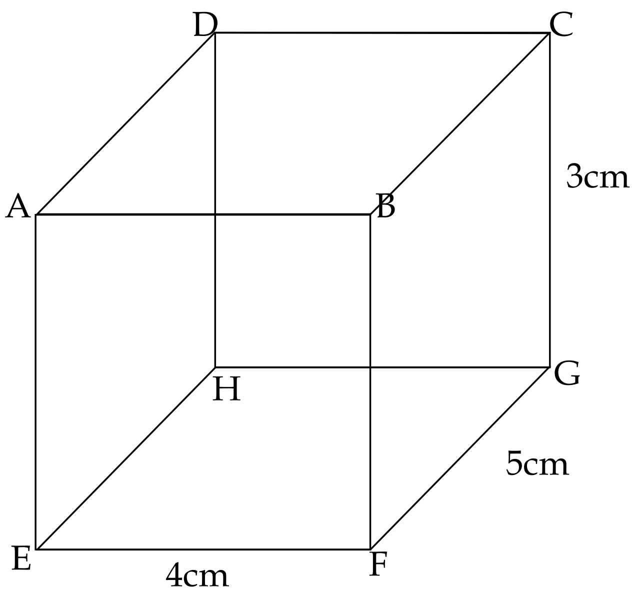
i) **Parallel lines:** These lines have no point of intersection, e.g. AB and DC, AD and BC figure 1

ii) **Skew lines:** These lines do not intersect and yet they are not parallel e.g. EF and GC, BC and HD Figure 1.

3 Points which are in a straight line are said to be collinear points.

Length of lines

Figure 3



Find the length of the following lines:

i) AF

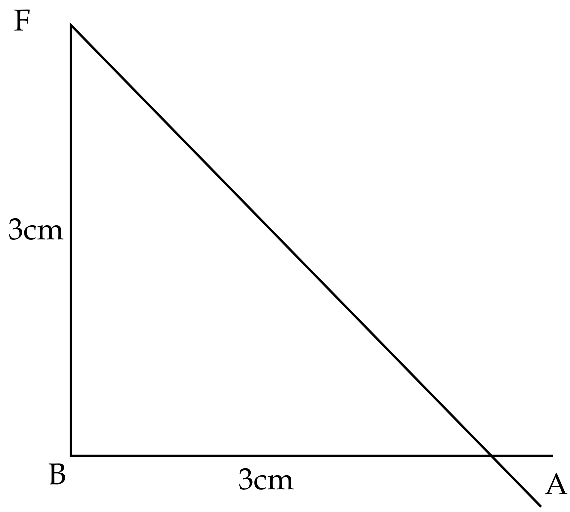
ii) BG

iii) HB

Answer

Here we extract right angled triangles and employ the Pythagoras theorem.

Length AF

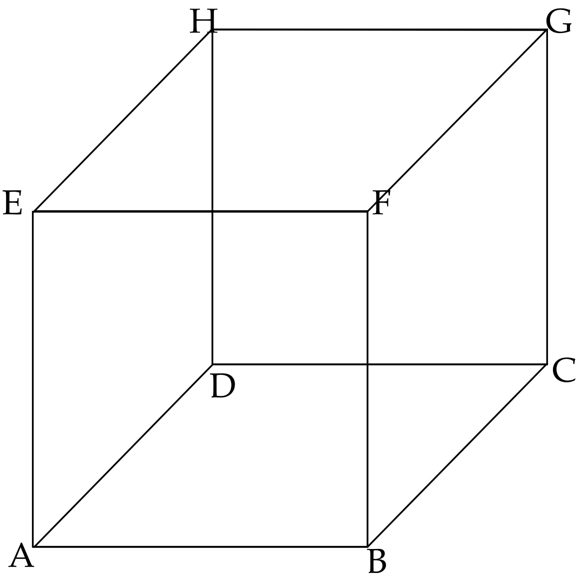


AF^2	=	$BA^2 + BF^2$
	=	$4^2 + 3^2$
	=	25
AF	=	$\sqrt{25}$
	=	5cm
BG^2	=	$GC^2 + CB^2$
	=	$3^2 + 5^2$
	=	34
BG	=	$\sqrt{34}$
DB^2	=	$\sqrt{4^2 + 5^2}$
	=	$\sqrt{41}$
HB^2	=	$HD^2 + DB^2$
	=	$3^2 + (\sqrt{41})\approx$
	=	9 + 41
HB	=	$\sqrt{50}$

Exercise 1

Use the figure 4 below to answer 1-3 the following questions

Figure 4



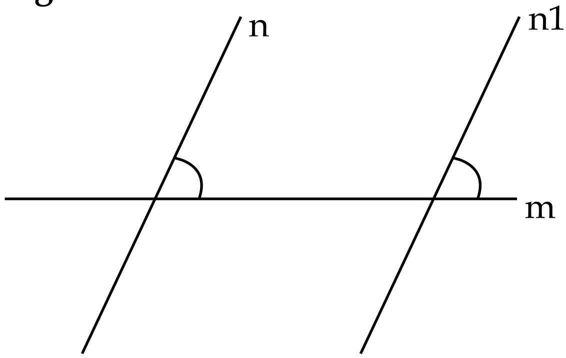
1. State 4 pairs of skew lines from figure 4.
2. Give lines that are parallel to:

   i) AB ii) DA iii) GC iv) DG
3. Calculate the length of;

   i) HA ii) AC iii) AG

Sub-Topic 2: Angles in Three D Geometry

i) Angles between two skew lines

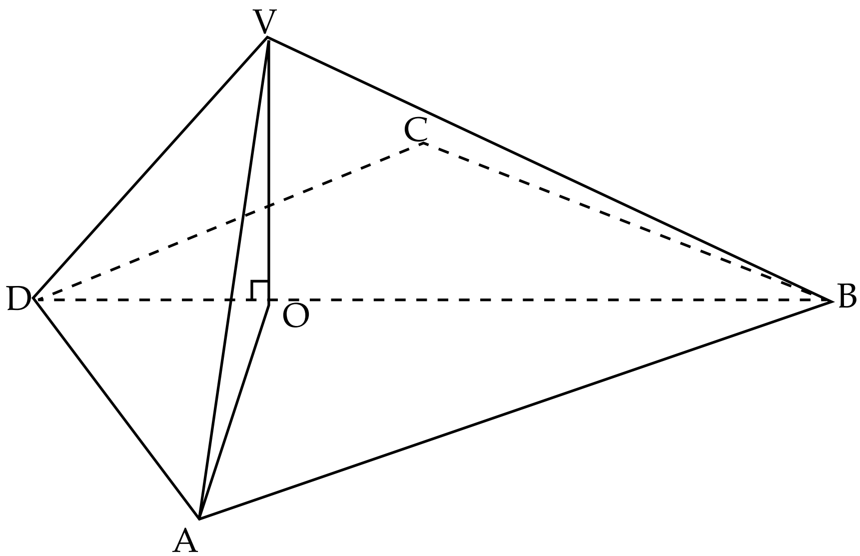


Angles are unchanged by translation this defines the angle between two skew lines. From Figure 2, the angle between lines HF and AB is got by either shifting line AB to line EF forming angle EFH or shifting line HF to DB forming angle ABD.

ii) Angles between a line and a plane

Suppose we have a plane ABCD and a line AV which is not in the plane. If a perpendicular to the plane is dropped from V to O, so that VOA is a right angle, then the angle VAO is the angle between the line and the plane. We say that AO is the projection of AV onto the plane ABCD. Notice that the three points V, A and O define a new plane.

Figure 5

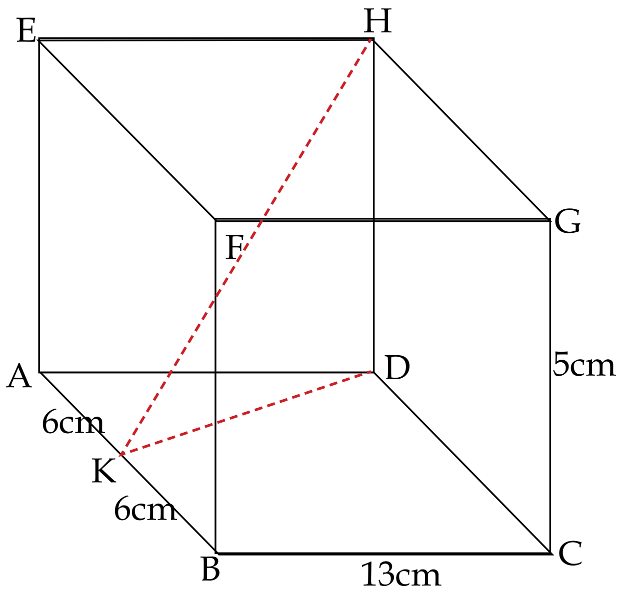


The angle between a line and a plane is defined as the angle between the line and its projection on the plane.

Example 1

The figure below shows a cuboid in which AB = 12cm, BC = 13 and CG = 5cm K is the mid point of AB.

Figure 6



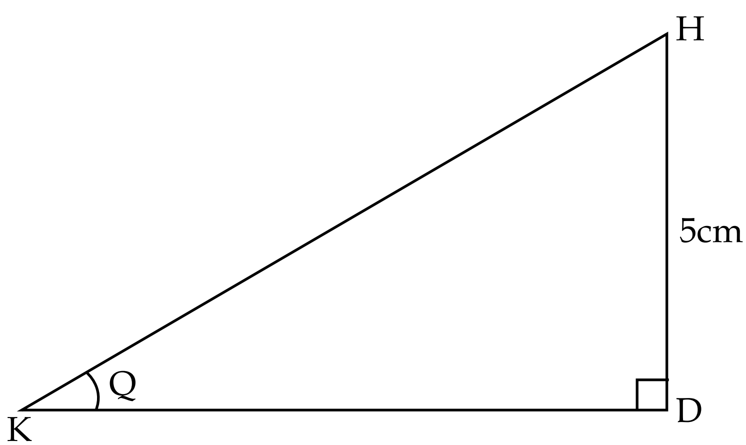
i) Calculate the angle between HK and the plane ABCD.

ii) Calculate the angle between HK and plane ADHE.

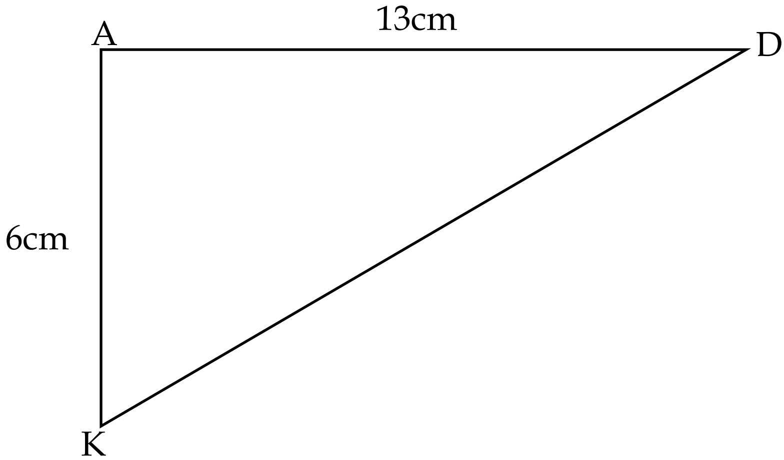
Answer

---

i) The projection of the line HK on the plane ABCD is the line KD. Therefore, the angle between the line HK and the plane ABCD is the angle HKD.



Getting the length of KD we use a triangle.



By Pythogra's theorem ΔK^2 $= AK^2 + AD^2$

$= 6cm^2 + 13cm^2$

$= 36 + 169$

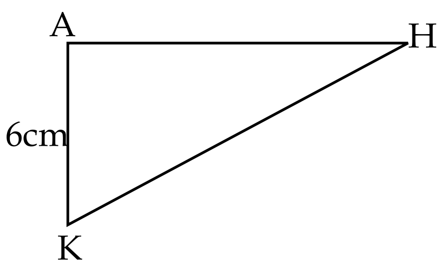
$=205$

$DK = 14.3cm$

In triangle KHD HD = 5cm $\angle HDK = 90^0$

then, $\angle HKD = \frac{HD}{KD} = \frac{5}{14.3}$

(b) AH is the projection of HK on ADHE. The angle we need is AHK. In triangle AHK, AK = 6cm KAH = 90^o



By Pythogra's theorem,

$AH^2 = EH^2 + AC^2$

$= 13^2 + 5^2$

$= 169 + 25$

$=194$

$AH = 13.9cm$

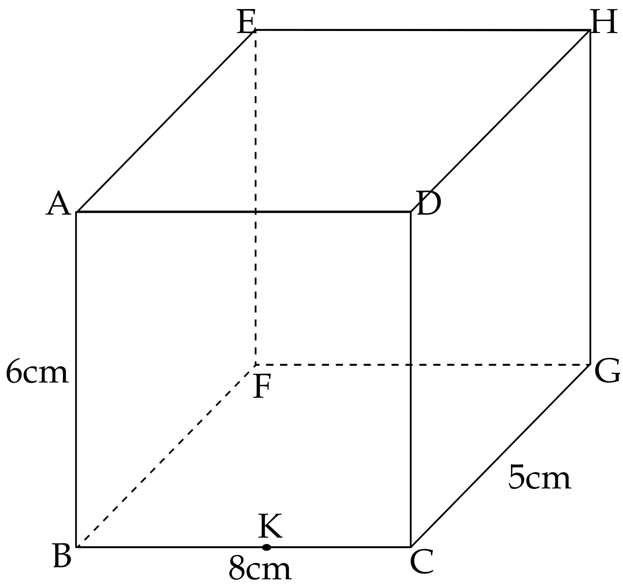
$$\tan \angle AHK = \frac{AK}{AH} = \frac{6}{13.9}$$

$$\angle AHK = 23.3 \text{ as required angle.}$$

Exercise 2

In the figure below shows a cuboid in which AB = 6cm BC = 8cm and CG = 5cm. Use it to answer the following questions.

Figure 7



K is the mid point of BC. Calculate:

a) The angle between GK and plane ABCD.

b) The angle between GK and plane DCGH.

c) The angle between Gk and plane EFGH.

Calculate the angle between these lines and planes

i) HB and EFGH.

ii) AF and ABCD.

iii) CD and EFGH.

iv) CH and EFGH.

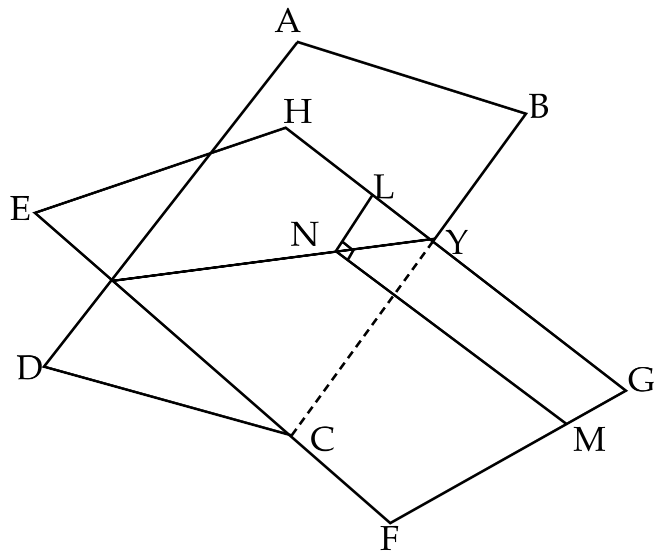
v) AB and ADHE.

iii) Angle between two planes

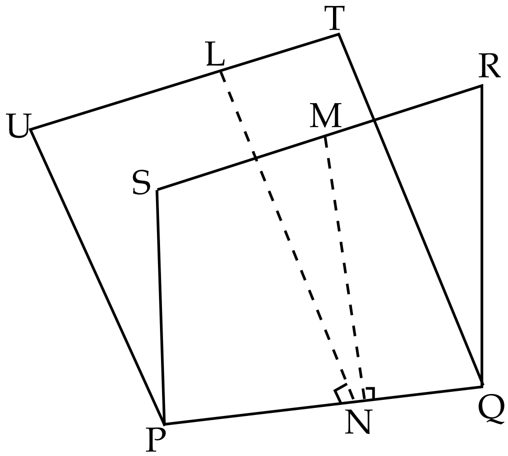
The angle between two planes is defined as the angle between two lines, one in one of the planes and the second line in the other plane, each meeting the common line (i.e. the line of intersection).

In the figure below, the angle LNM is the angle between planes, ABCD and EFGH. Both MN and LN are perpendicular to XY.

Figure 8



Note: That LNM is the angle between planes PQRS and PQTU. Both MN and LN are perpendicular to PQ.



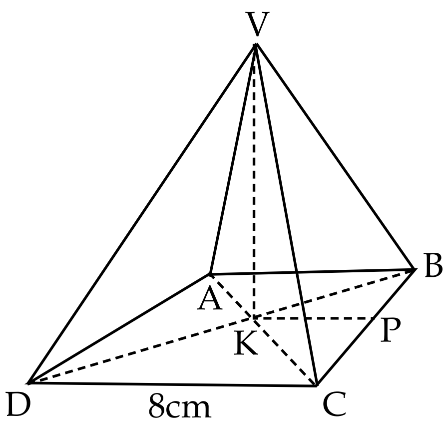
Example

VABCD is a right pyramid on a square base ABCD of side 8cm. The height of the pyramid is 6cm calculate:

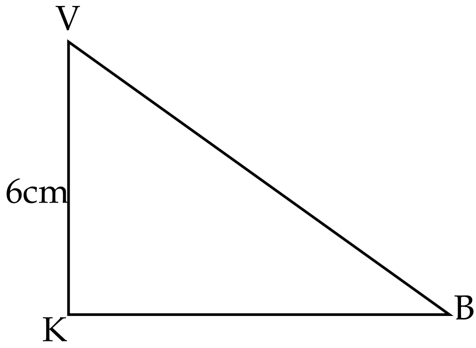
a) The angle which each side of the slant edge makes with the horizontal plane ABCD.

b) The angle between the plane VBC and the horizontal plane ABCD.

Figure 9



VBK is the angle which the slant edge VB makes with the horizontal plane ABCD. We first find the length of KB.



Now BD = DC + CB

= 82 + 82

= 64 + 64

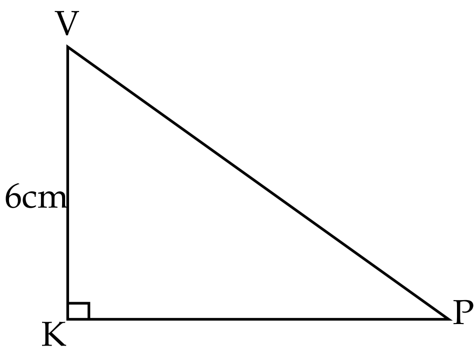
= 128

BD = 11.8cm

ICB = BD = 5.65

tan VBK = VBK = 46.7

The angle which the slant edge makes with the horizontal ABCD is 46.7.



The angle between the plane VBC and ABCD is the angle VPK

Now. In the right-angled triangle VKP

VP = ½DC (mid point theorem)

KP = ½ x 8 = 4cm

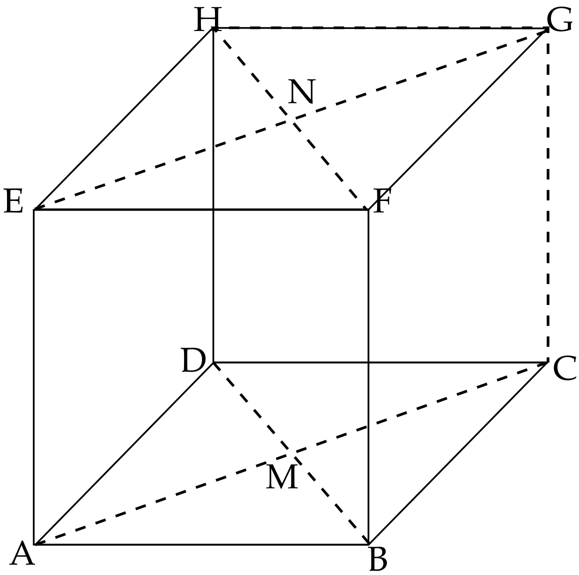
tan ∟VPK = 1.5

∟VPK = 56.3

the angle between the plane VBC and plane ABCD is 56.3

Exercise 3

Figure 10



1. Using the figure 10 of a cuboid above state the lines of intersection of the plane:

a) ABCD and BCGE

b) ABCD and BCHF

c) BDHF and ACGE

d) ADHE and EFGH

2. Give the size of the angle between each pair of planes in question 1.

3. VABCD is a right pyramid on a square base ABCD of side 10cm. The height of the pyramid is 8cm. Using the following figure, calculate:

a) The angle which each side of the slant edge makes with the horizontal ABCD.

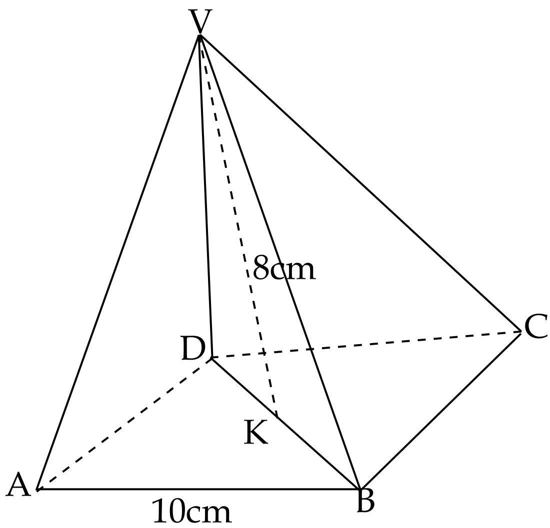
b) The angle between the plane VBC and the horizontal ABCD.

c) The angle between BDV and ABCD.

d) The angle which VD makes with VAB.

e) The angle which DV makes with VB.

Figure 11

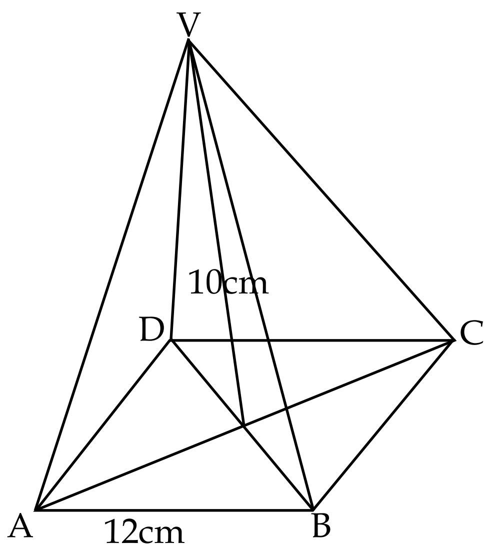


4. In the figure below determine the angle between:

   i) The plane VCD and ABC.

   ii) The plane BCD and VCD.

Figure 12



Activity: A sketch of a piggy bank

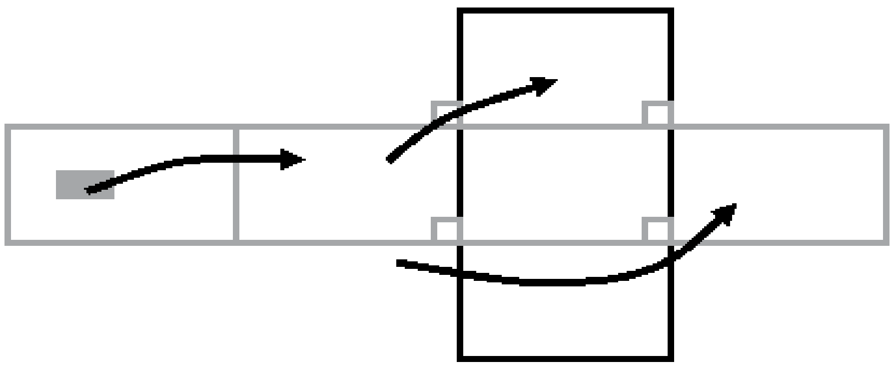
Objective: Tell the difference between plane figures and solid figures.

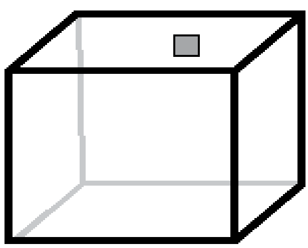
Introduction: Three dimensional figure (solid figures are made up of regular polygons (planes).

Materials: Manila paper, wood ruler, pins, masking tape, scissors/razor blade, glue, cardboard

Procedure

i) Draw a net on a manila paper/cardboard as shown below:





ii) Cut off the flaps and fold the polygons along the drawn lines, the sides and top may be laid out flat.

iii) Bind the six faces to form a cube or cuboid.

Note: The drawn lines of the net must meet at 90^{o} (perpendicular) to each other. Specific dimensions must be considered for different nets.

Observation/Results: The figure is made up of six faces. Identify the cube and cuboid.

Interpretation: Solid figures are formed by joining plane figures.

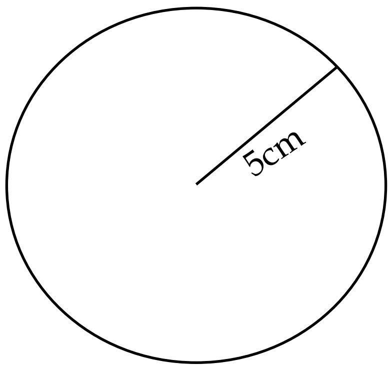
Application: 3D is commonly used by Architects, designers in industries and factories.

Follow up activity: Make different solid figures (3D) and identify where they are applied or used.

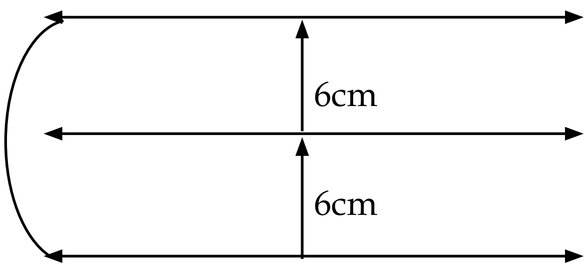
TOPIC 13 GEOMETRICAL CONCEPTS AND LOCUS

Locus and geometrical construction

A locus in mathematics is a path traced out by a moving object. It is a set of positions traced out by a point which moves according to the same law. For example, the locus of a point which moves so that it is 5cm from a fixed point is a circle with radius 5cm.

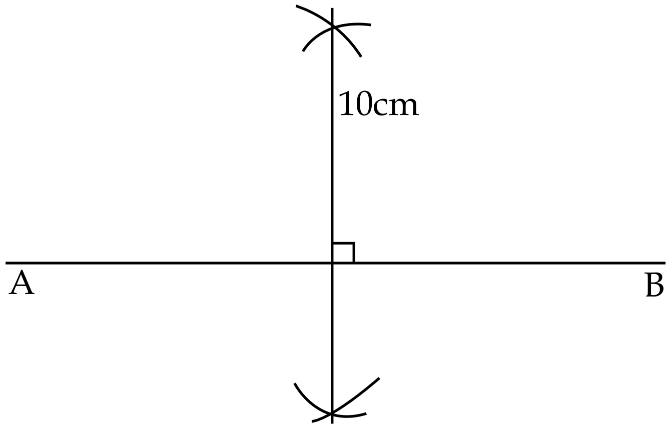


A locus of a point that is 6cm from a given line gives two lines which are parallel to the given line and at a distance of 6cm as in the figure below:

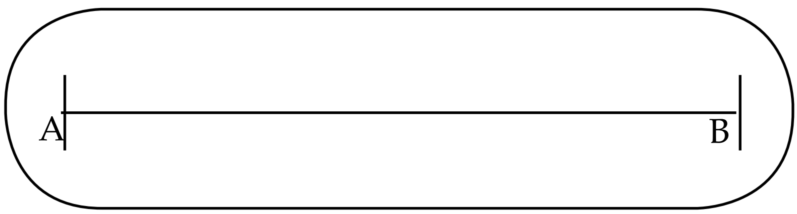


Example

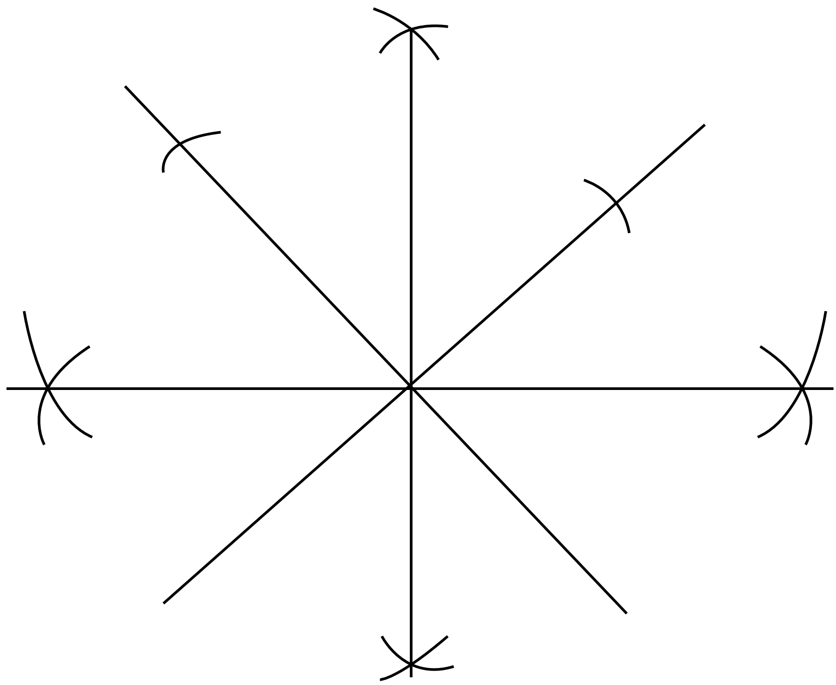
Locus of a point from two fixed points gives a perpendicular bisector of the two points joined as shown in the following figure.



The locus of a point from a fixed line segment is as illustrated below:



The locus of two intersecting lines gives the angle bisector of the lines as in the figure below:



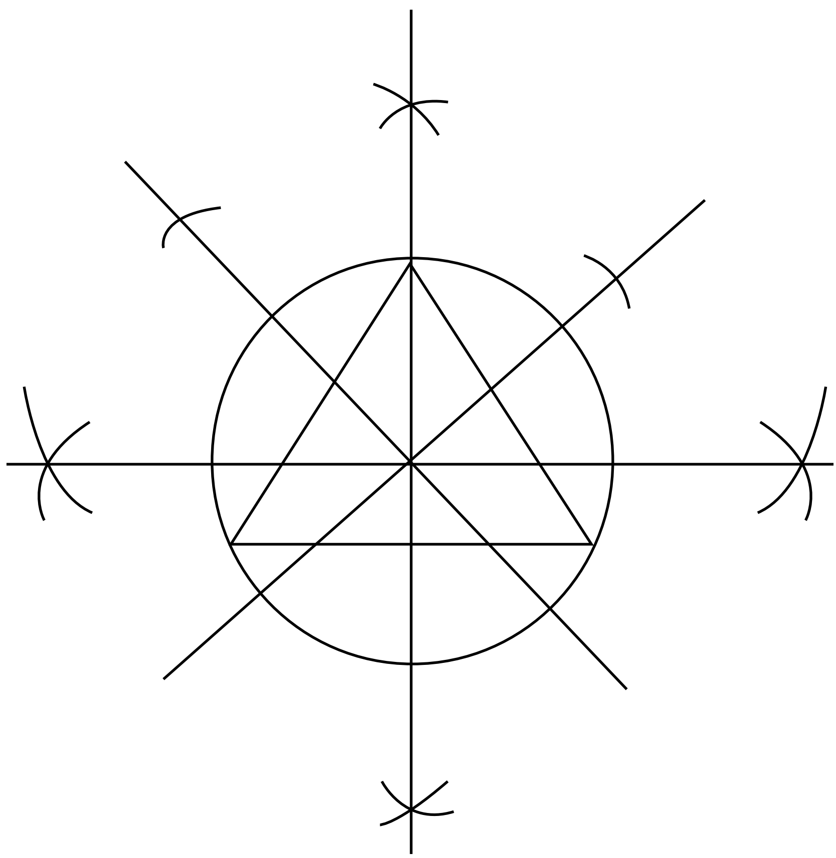
Intersecting loc 1

Frequently two pieces of information are given about the position of a point. Each of the two loci will determine the required position of the point.

Example 1

Find the locus of a point that is equidistant from all the sides of the given triangle. This locus is the centre of the circumcircle of the triangle as it is illustrated in the figure 1 below:

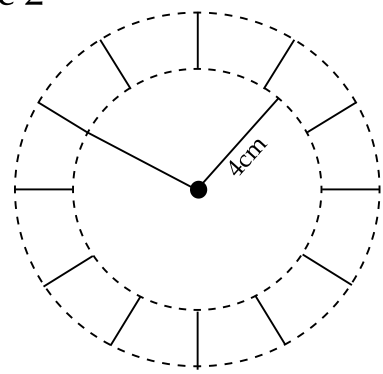
Figure 1



Example 2

Find the locus of a point which is more than 4cm from a fixed point but less than 6cm. This is illustrated in the figure 2:

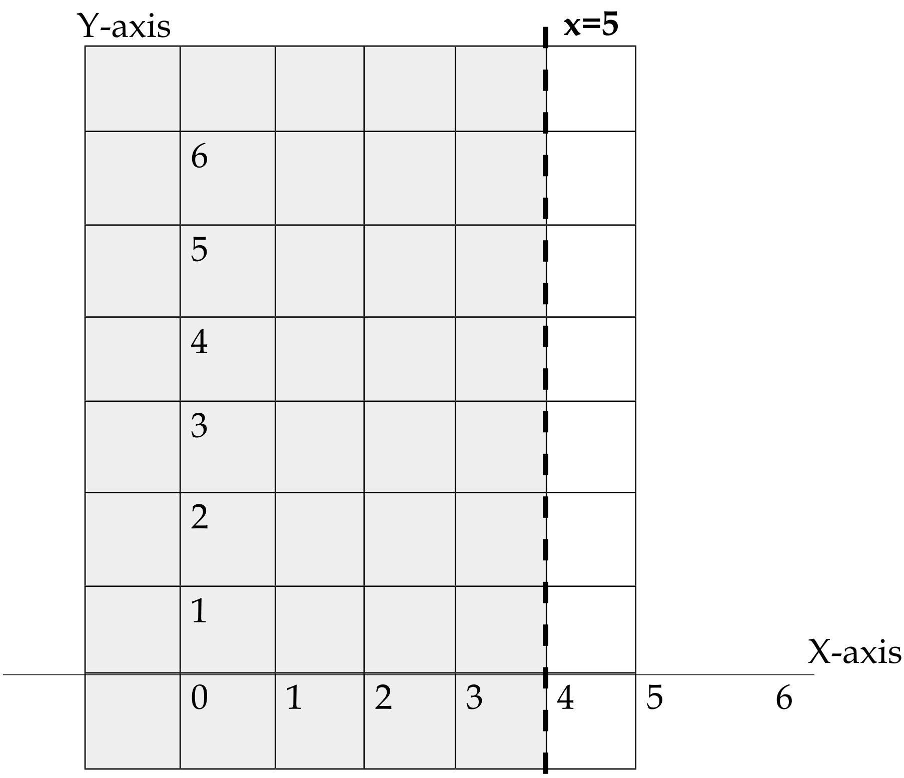
Figure 2



Example 3

Locus can be referred to as a region as illustrated in the following figure 3:

Figure 3



Find the possible positions of a point x in a set {(x,y) : x > 5}.

Possible area where x can be found (shaded region).

X = 4

Exercise 1

1. Describe fully the locus of a point that is 7cm from a fixed point.
2. Two trees M and Z are 20m apart. Treasure is buried 10m from Z and 15m from M. Draw a diagram to show the possible positions of the treasure.
3. A treasure is buried inside PQR, a triangular plot of the ground. PQ = 40m = QR and PR = 45m. W is a point on QR such that QW = 20m. The treasure lies at least 20m from QR and less than 24m from W. Make a scale drawing and show by shading the set of possible positions in which the treasure is hidden.
4. Construct triangle ABC with AB = 6cm, BC = 7cm, AC = 8cm. Find the locus of a point that is equidistant from all the vertices of the triangle. Taking the point as the centre (k) and KB as the radius, draw a circle. Find the area of the circle.
5. Using a ruler, pencil and pair of compasses only, construct a triangle XYZ where angle YXZ 135^0, XY = 8.4cm and YZ = 12.5cm. State the Length of XZ. S and T are points such that TS bisects YZ where T is on YZ and S on the same side as XY. Draw a circle to circumscribe the points X, Y, Z and S. Measure and state the length TS and the radius of the circle.

Activity: Making a bedcover

Objectives: i) To identify the different types of polygons.
ii) To construct different patterns of polygons.

Introduction: Triangles are three sided polygons.

Materials: Pencil, ruler, a pair of compasses, manila paper/ cloth, razor blade or pair of scissors, cardboard, pin, glue, threads

Procedure

i) Get five pieces of one metre manila paper of different colours.

ii) Construct equilateral triangles of the same dimension on each piece.

iii) Think of the design you require.

iv) Join the triangles according to the design of your choice.

v) Join them with buttons for cloth and glue for manila.

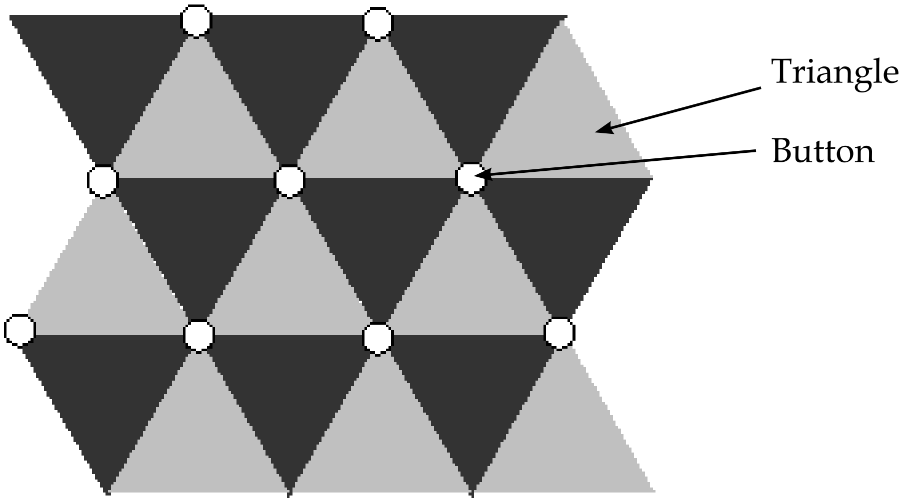
Note: i) The triangles must be equilateral.
ii) Join triangles in a sequence.

Observation: Identify the patterns formed.

Interpretation: Designs can be made using different polygons.

Application: Used in making tiles, bedcovers, floors of buildings, etc.

Follow-up activity: Make different shapes of polygons and identify where they are applied.


Triangle
Button

TOPIC 14 MATRICES

A matrix is a store of information. It is an array of numbers arranged in a particular order i.e. row by column. The members of a matrix are called elements or entries. The order of a matrix refers to the size of the matrix which is denoted by the number of rows by columns. Matrices are represented by capital letters.

Observation : The knowledge of Arithmetic is necessary.

Sub-Topic: Matrix Multiplication

Why do sellers in markets display their items according to their prices?

Matrix multiplication is done row by column.

The columns of the first matrix must be the same as the number of rows in the second matrix to be multiplied.

Commutative as illustrated below.

Example 1

Give $A = (2, 3)$ and $B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

The order of matrix

A is 1×2

B is 2×1

Therefore $AB = (2, 3)\begin{pmatrix} 4 \\ 1 \end{pmatrix} = (2 \times 4 + 3 \times 1)$

$= (8 + 3)$

$= \quad (11)$

But BA is not possible.

$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ (2, 3) because the two matrices are not compatible.

Example 2

Given P $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ Q $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$

Find P Q

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 \times 1 + 1 \times 0 & 2 \times -1 + 1 \times 2 \\ 1 \times 1 + 3 \times 0 & 1 \times -1 + 3 \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$$

Example 3

Jane went to the market and bought 5 oranges, 3 mangoes, 2 papaws and 4 pineapples.

1. How much money did Jane spend on the fruits?
2. Make a row matrix for the fruits and a column matrix for the cost then combine the row and column matrix together.

Exercise

1. Two sisters Alice and Sarah presented their shopping list to their father: Alice's list included 6 black books, 1 dozen of pencils 2 dozen of pens a mathematical set and a tin of biscuits. Sarah's list included 8 black books, 2 dozens of pencils, 1 dozen of pens and 2 tins of biscuits.

   The cost of the items were as follows:

Black books cost	Shs 2500 each,
A dozen of pencils	Shs 1000
A dozen of pens	Shs 2400
A mathematical set	Shs 200
A tin of biscuits	Shs 5000

   a) Using matrix multiplications, find how much money is spend on:
      i) Alice
      ii) Sarah

   b) Find the total expenditure on both daughters.

2. At a handover ceremony for the Maths club, the following items were required:

   A cake costing shs 100,000

   4 crates of soda at shs 11,000 each

   100 packets of snacks at shs 1500 each

   Using a matrix multiplication, find how much money was needed for the ceremony.

3. Given matrix

$$A = \begin{pmatrix} 4 & 2 & 4 \\ 3 & 1 & 2 \\ 3 & 2 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 4 \\ 3 & 1 \\ 3 & 2 \end{pmatrix}$$

Find A B

Activity: Making a week's budget

Objectives:

To be able to:

(i) state and use properties of multiplication.
(ii) apply matrix multiplication in daily use.

Introduction: Matrix multiplication involves combining rows by columns.
The number of columns of the first matrix must be equal to the number of rows of the 2nd matrix.

Materials Items sold in the school canteen and their respective prices, manila paper, markers

Procedure

i) List down the items you need from Monday to Friday in a table form (matrix).

ii) List their respective prices also in a matrix form.

iii) Combine the two formed matrices together to get the product (expenditure).

Note: Make sure the matrices are compatible.

Observation: Matrix multiplication gives a matrix product.

Conclusion: Matrix multiplication simplifies arithmetic work and saves time.

Application: Matrices are applied in industries, factories, business field, stores, schools, etc.

Follow up activity: Make a budget of the requirements you need at the beginning of the term.
Identify where else matrices can be applied.

Senior Four

TOPIC 15 PROBABILITY

Probability is the study of non deterministic experiments (study of chance). It is an area of mathematics that deals with the formal study of laws of chance and uncertainty.

$$P(E) = n(E) / n(S)$$

Where $O \leq P(E) \leq 1$

Observation: The knowledge of fractions and decimals is a requirement.

Sub–topic: Equally Likely Outcomes

This refers to events whose outcomes have equal chances of occurrence as illustrated below.

Tossing a coin.
Sample S = (H, T)

Out come	H	T
Probability	$\frac{1}{2}$	$\frac{1}{2}$

Tossing a Die

Sample space S = (1 , 2, 3, 4, 5, 6)

Out Come	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Example 1

A student picks an orange from a basket of 5 oranges of which 2 were ripe and 3 unripe.

i) Find the probability that the student picked.

A ripe orange.

An unripe orange.

Answer

$n(s) = 5$

$n(ripe) = 2$

$P(R) = n(R) / n(S)$

$P(R) = 2/5$

$n(unripe) = 3$

$P(u) = n(u) / n(s)$

$= 3/5$

Exercise

1. In a mixed class of 20 girls and 10 boys, find the probability of selecting randomly.

   i) a girl.

   ii) a boy

2. Find the probability of getting a prime number when a die is tossed once.

3. In a class of 60 students. If the probability of getting a boy at random is 2/3. Find the numbers of,

   i) Boys in the class.

   ii) Girls in the class.

Activity: Making "Luddo" (Probability game)

Materials: Manila paper, clay, rubber coke, blue tack, wood, dried cassava, paint/colours, ruler, pens, pencils, buttons of different colours, empty film container

Procedure

i) Draw the format of Luddo board on the manila paper.

ii) Cut the cube and make the six faces with dots (1-6) respectively.

iii) Cut 16 circular pieces from the manila paper. In sets of fours, paint each set differently.

Note: i) The die must be a cube.

ii) Buttons must be in four different colours.

Observation: The Luddo game formed is played by a maximum of four participants.
Scores are recorded to determine the winner.

Interpretation: Probability is applied in the game.
Each side of the die has equal opportunity of occurrence.

Conclusion: It enhances arithmetic (addition).

Application: It is a leisure game.
It is applied in the weather forecast (Meteorology).

Follow up activity: Design more games basing on probability.

TOPIC 16 ALGEBRA

Sub-Topic 1: Algebraic Expression

This refers to the combination of letters with numbers or figures. We use terms like 4x , 3y, etc. 4x means 4 times x, i.e. x + x + x + x = 4x: this is written in short form as 4x.

Note: in arithmetic 68 means

6 tens + 8 ones

(6 x 10) + (8 x 1)

60 + 8

= 68 (Not 6 x 8)

But in algebra, 4x always means 4 times x. 4 is called the coefficient of x and it tells how many x's are added together.

3y means 3y's are added together

3y = y + y + y = 3 x y

2b + 3b = b + b + b + b + b = 5b

5b – 3b + 2b = 4b

Expressions in algebra may be written in shorter form. This is termed as collecting like terms e.g. 5x + 3y – 3x + 4y

= 5x – 3x +3y + 4y

= 2x + 7y.

Example 1

I think of a number, add 5 to it and the result is 7.

Answer

Let the number be x

$x + 5 = 7$
$x = 7 - 5$
$x = 2$

Example 2

John bought 5 books at x shillings each and 3 pens at y shillings each. Find how much he spent on both items.

Answer

The total cost of 5 books	=	5 times x	=	Shs 5x.
The total cost of 3 pens	=	3 x y	=	Shs 3y
Total cost of 5 books and 3 pens			=	5x + 3y

Simplification of algebra

Daniel bought 4t books on one day and 7t books on the next day. In two days, he bought:

$4t + 7t = 11t$ books

James bought 3p pens on one day and again he bought 5a books for his shop. You cannot add pens to books and express as one quantity:

i.e. $3p + 5a$.

$3p + 5a$ are unlike terms and their sum is $3p+5a$.

Example 3

Simplify

i) $12a + 7b - 10a + 4a + 8b - 10b$

$12a - 10a + 4a + 7b + 8b - 10b$

$16a - 10a + 15b - 10b$

$6a + 5b.$

ii) $7x + 3 - 5x - 7 + 3x + 6$

$7x - 5x + 3x + 3 - 7 + 6$

$7x + 3x - 5x + 3 + 6 - 7$

$10x - 5x + 9 - 7$

$5x + 2$

Exercise 1

Write in short form:

1. $5b + 3b + b$
2. $5x + 3y - 2x - 7y$
3. $4x - 6x + 5x + x - 3x$
4. $5e - 2e - 1\frac{1}{2}e$
5. $a - 2a - 4a + 5a$
6. $6b - 3a - 4b + 7a$
7. $7t + 5x - 4t - 8x$
8. $3y + y + 4y$
9. $6m + 5n - 3m - 7n$
10. $5d + 3d + d$

Exercise 2

Write the following statement in algebraic expression:

1. Add 7 to twice x.
2. Take away 5 from three times y.
3. How many cents are there in x shillings and y cents?
4. John was x years old 10 years ago. How old is he now?
5. Sarah has t books, Mary has 3 times as many books as Sarah. Write an expression for the number of books they both have.
6. A rectangle is "a" metres long and "b" metres wide. Find its perimeter in centimeters.

7. For a party, David bough x large prizes costing Shs 500 each and y small prizes costing Shs 200 each. Write the total cost c, as an algebraic equation.

Brackets

In arithmetic, we make use of brackets. All the terms in a bracket form one quantity, e.g. $(2 + 3)$ means 5.

Similarly $(a + b)$ means the result of adding b to a.

Brackets help us to know in what order we should undertake addition, subtraction, multiplication and division, e.g.

$7 + 3\ (7-3)$	$= 7 + 3\ (4)$	$= 7 + 12$	$= 19$
$(7 + 3)\ (7 - 3)$	$= 10 \times 4$	$= 40$	
$(7 + 3)\ 7 - 3$	$= 10 \times 7 - 3$	$= 70 - 3$	$= 67$

The distributive property

When the whole of an expression within brackets is multiplied by a number, then if the bracket are removed, each term within the brackets must be multiplied by the number, e.g.

i) $t\ (x - y) = tx - ty$

ii) $x\ (a + b) = xa + xb$

iii) $p(p - 2q) = p^2 - 2pq$

iv) $5\ (x + 2y - 3z = 5x + 10y - 15z$

v) $\frac{1}{3}(a + b) = \frac{a}{3} + \frac{b}{3}$

Exercise 3

Remove the brackets and simplify

1. $5(x-y) - 3(x + y)$
2. $3 + 7\ (2a -5)$
3. $a + 3 - (2a)$
4. $(p - q) - (2p + 4q)$

5. $(a + b + c) + (a - b - c)$

6. $4(-a + b) - 2 (b - 5a)$

7. $1 - (x - 1)$

8. $6x - (4 - x) - 3$

9. $(a - b) - (b - a)$

10. $2(3x2 + y) - 3x (x -2)$

Factorisation

The reverse of multiplication of an algebraic expression that is breaking up an expression into its separate parts is what is termed as factorisation.

Each separate part is called a factor.

Consider a multiple 12

$$12 \to \left.\begin{matrix} 1 \times 12 \\ 2 \times 6 \\ 3 \times 4 \end{matrix}\right\} \text{Factors}$$

$\downarrow$

Multiple

12 is a multiple of 1, 12, 2, 6, 3, 4 and these numbers are factors of 12.

There are three types of factorisation.

1. Common factor
2. Difference of two squares
3. Quadratic expressions (polynomials)

1. Common factor

Factorise the following:

$ab + ay$ a is common to both terms ab and ay.

$a(b + y)$.

i) $4x + 4y \quad = \quad 4 (x + y)$

ii) $ab^2 - ab = ab(b - 1)$

iii) $x^2y^3 + xy^2 = y^2x(xy + 1)$

$6xy + 4xp = 2x(3y + 2p)$

Exercise 4

Factorise the following:

1. $3a^2 - 9a$
2. $4xy + 10xp$
3. $x^2y^3 + x^3y^2$
4. $12xy \quad + 6yt$
5. $m^3n^2 - m^2n^3 + 2m^2x$
6. $abc - bcd$
7. $7x + 21t$
8. $3xp - 2xt + xpt$
9. $4pq - 2px + 6py$
10. $a^5b^3 - b^3a^7$

2. Difference of two squares

Use of the identity

$X^2 - y^2 = (x + y)(x - y)$ $X^2 - y^2$ is a difference of two squares because X^2 and y^2 are squares.

Why is x^2 and y^2 squares?

Example 4

Factorise: i) $p^2 - q^2$ ii) $4x^2 - 9$ iii) $t^2 - 1$
iv) $2x^2 - 18$ v) $p^2 - 1$

Answer

i) $p^2 - q^2 = (p + q)(p - q)$ ii) $4x^2 - 9 = (2x + 3)(2x - 3)$

iii) $t^2 - 1 = (t + 1)(t - 1)$ iv) $2x^2 - 18 = 2(x^2 - 9)$

v) $p2 - 1 = (p + 1)(p - 1)$ vi) $p^2 - 1(p + 1)(p - 1)$

Exercise 5

Factorise the following:

1.	$8x^2 - 18$	6.	$p^2 - 81$
2.	$a^2 – 49d^2$	7.	$x^4 - 1$
3.	$4x^2 - 64$	8.	$25m^2 – 9n^2$
4.	$3p^2 – 12d^2$	9.	$z^2 - \frac{1}{4}$
5.	$x^2 – 4y^2$	10.	$18p^3 – 2p$

Quadratic expressions and equations

Factorisation of quadratic expressions. Quadratic expressions are of two forms:

i) $x^2 + bx + c$

ii) $ax^2 + bx + c$

When the above quadratic expressions are equated to zero then they become quadratic equations which can be solved by any of the following methods:

1. Factors method
2. Completing squares
3. Graphical method.

Factor method of quadratic functions:

1. $x^2 + bx + c$

   Find two integers whose product is c and whose sum is b.

Example 5

Factorise the following:

1. $x^2 – 15x + 44$

Find two integers whose product is $^+44$ and whose sum is $^-15$

We have $(^-11) \times (^-4) = 44$

And $(^-11) + (^-4) = {}^-15$,

$x^2 - 15x + 44$

$x^2 - 11x - 4x + 44$

$x(x - 11) - 4(x - 11)$

$(x - 4)(x - 11)$

2. $p^2 + 5p + 6$

Factors of 6 whose sum is 5 are 2, 3

$\therefore$ $p^2 + 5p + 6$

$p^2 + 2p + 3p + 6$

$p(p + 2) + 3(p + 2)$

$(p + 3)(p + 2)$

3. $y^2 - y - 20$

Factors of -20 whose sum is -1 are -5 and 4.

$y^2 - y - 20$

$y^2 - 5y + 4y - 20$

$y(y-5) + 4(y-5)$

$(y + 4)(y - 5)$

Exercise 6

Factorise the following:

1.	$u^2 + 7u + 12$	6.	$z^2 + 10z + 25$
2.	$p^2 + 8p + 15$	7.	$t^2 - 2t - 15$
3.	$x^2 + 9x + 14$	8.	$x^2 - x - 6$
4.	$x^2 - x - 2$	9.	$t^2 + 4t + 4$
5.	$y^2 - 3y - 10$	10.	$x^2 + 3x - 18$

$ax^2 + bx + c$

Here find two integers whose product is ac (a x c) and whose sum is b.

Example 6

Factorise $2b^2 + 5b - 3$

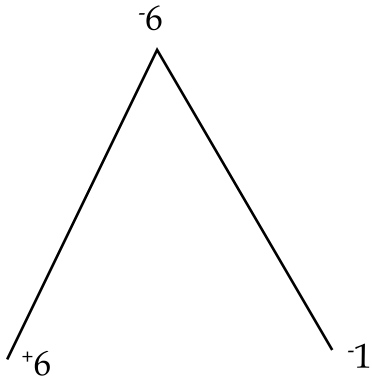
Find two integers whose product is $^{-}6$ (2x -3) and whose sum is 5.

$\therefore\ 2b^2 + 5b - 3$

$2b^2 + 6b - b - 3$

$2b(b + 3) - 1\,(b + 3)$

$(b + 3)\,(2b - 1)$



Example 7

Factorise $6y^2 - 17y + 5$

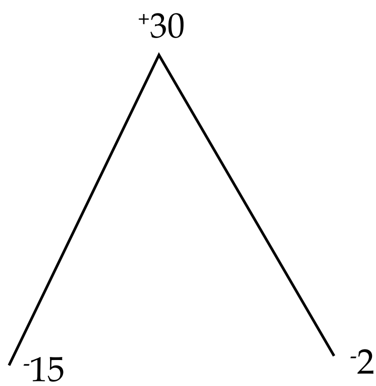
Find two integers whose product is $(6 \times 5) = 30$ and whose sum is $^{-}17$.

$\therefore\ 6y^2 - 17y + 5$

$6y^2 - 15y - 2y + 5$

$3y(2y - 5) - 1(2y - 5)$

$(2y - 5)\,(3y - 1)$



Factorise

$2x^2 - 5x + 2$

$2x^2 - 4x - x + 2$

$2x(x - 2) - (x - 2)$

$(2x - 1)(x - 2)$

Exercise 7

Factorise the following expressions:

1. $2x^2 + 5x - 3$
2. $6z^2 - 17z + 5$
3. $5c^2 - 16c - 12$
4. $4x^2 + 38x - 15$
5. $3x^2 - 7x - 6$

Sub-topic 2: Solutions of Quadratic Expressions

Solution of quadratic equations by factors method

An equation of the form $ax^2 + bx + c = 0$

Where a, b and c may be 0 is called an equation of second degree or a quadratic equation, i.e. $x^2 + b + c = 0$

$ax^2 + bx + c = 0$

e.g. $x^2 + 3x + 4 = 0$, $5x^2 + 2x + 7 = 0$

After factorising the expression, we use the zero property

if $ab = 0$

then $a = 0$ or $b = 0$ or both a and b may be zero.

1.	$p(p - 3) = 0$	then $p = 0$	or	$p - 3 = 0$	$p = 3$
2.	$(x - a)(x - b) = 0$	then $x - a = 0$	or	$x - b = 0$	$x = a$ or $x = b$
3.	$(2x - 3)(x - 5) = 0$	then $2x - 3 = 0$	or	$x - 5 = 0$	
		$2x = 3$	or	$x = 5$	
		$x = 3/2$			

Exercise 8

Solve the following equations:

1.	$y^2 - 7y = 18$	6.	$27 - 6y = y^2$
2.	$x^2 + 6x + 5 = 0$	7.	$9x^2 - 12x - 5 = 0$
3.	$x^2 + 10x + 9 = 0$	8.	$p^2 = 7p$
4.	$16x^2 + 9 = 24x$	9.	$4x^2 - 20x - 56 = 0$
5.	$x + 1 = 6/x$	10.	$3x^2 + 1 = -4x$

Perfect squares

The quadratic expression $x^2 + 2ax + a^2 = 0$. If it is written in the form $(x + a)^2$ then this expression is called a perfect square and it is the square of $(x + a)$.

The standard form of perfect squares are:

1. $(a + b)^2 \quad = a^2 + 2ab + b^2$
2. $(a - b)^2 \quad = a^2 - 2ab + b^2$
3. $(a + b)(a - b) \quad = a^2 - b^2$
4. $(a + b)^3 \quad = a^3 + b^3 + 3a^2b + 3ab^2$
5. $(a - b)^3 \quad = a^3 - b^3 - 3a^2b + 3ab^2$

A quadratic expression is a perfect square if:

a) The first and last terms are exact squares and positive.

b) The middle term is twice the product of the square root of the first and last term (the middle term may be positive or negative).

$$(A \pm B)^2 = +A^2 \quad \pm 2AB \quad +B^2$$

1st term — Middle term — Last term

Middle term (M.T) = 2√first term x √last term

$M.T = 2\sqrt{A^2} \times \sqrt{B^2}$

$= \pm 2AB$

Or $MT^2 = 4FT$

$F.T = \frac{(2AB)^2}{4 \times B^2}$

$= \frac{4A^2B^2}{4B^2}$

$= A^2$

$L.T = \frac{(2AB)^2}{4 \times A^2}$

$= \frac{4A^2B^2}{4A^2}$

$= B^2$

Exercise 9

1. Which of the following expressions are perfect squares?
   i) $4a^2 - 12ab + 9b^2$
   ii) $9x^2 – 15x + 25$
   iii) $9 + 6x + 2x^2$

2. If the following expressions are perfect squares, find the missing terms
   i) $16a^2$ - …………………. + 9
   ii) ………………….. + 4a + ¼
   iii) $a^2b^2 + 8ab$ + ………………

Completing the square

When the first two terms of quadratic expressions are given, we have to add third term (last term) to complete the square.

- If the quadratic expression has an x^2 coefficient of 1, we add the square of half the coefficient of x in order to get the last term. The expression will then be a complete square.
- For expressions whose x^2 coefficient is not 1, we use the formula:

$$\text{Last term} = \frac{(\text{middle term})^2}{4\text{x first term}}$$

In general, the process of expressing $x^2 + bx + c$ in the form $(x + p)^2 + q$ is called completing the square.

$$x^2 + bx + c = (x + p)^2 + q \text{ where}$$

$$p = \frac{b}{2} \text{ and } q = c - p^2$$

$$\text{i.e. } x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right)$$

$$\text{e.g. } x^2 + bx + c = x^2 + 2px + p^2 + q$$

So comparing corresponding terms

$b = 2p \quad \therefore p = \frac{b}{2}$

And $p^2 + q = c$

$q = c - p^2$

What must be added to:

a) $x^2 + 8x$

b) $y^2 - 13y$

to make the expression perfect squares? What expression in each is the square of?

c) If $4x^2 - 10x + k$ is a perfect square, find K.

Quadratic formula

When the coefficient a of x^2 in a quadratic equation $ax^{2+} bx + c = 0$ is not 1, and when the coefficient of x is an odd number the process of completing the square to solve the equation becomes rather difficult. Hence we use the method of completing square to solve the equation as follows:

General $ax^{2+} bx + c = 0$

$$ax^{2+} bx = {}^{-}c$$

$$x^{2+} \frac{bx}{a} = \frac{-c}{a}$$

Complete a square

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{4a}\right)^2 = \frac{c}{a}$$

$$\left(x^{+} \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$\therefore \quad x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{4a^2}$$

$$= \frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$\sqrt{x} = \frac{-b}{2a} + -\frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$x = \frac{-b +-\sqrt{b2 - 4ac}}{2a}$$

Exercise 10

Solve the following quadratic equations by completing squares.

1. $x^2 + 4x - 5 = 0$
2. $2x^2 + 8x - 10 = 0$
3. $x^2 - 2x = 2$
4. $4x^2 + 10x = 5$
5. $3x^2 = 7x + 9$
6. $2x^2 - 7x = 9$
7. $x^2 + x = 20$

Forming quadratic equations given the solution

Example 8

1. Form a quadratic equation in x whose solutions are:

a) $\{2, {}^{-}3\}$ b) $\{1, 2\}$ c) $\{\frac{2}{3}, 4\}$

Answer

a) $\{2, {}^{-}3\}$

This means that $x = 2$ or $x = -3$

If $x = 2$ and $x = -3$

Then $x - 2 = 0$ or $x + 3 = 0$

$(x - 2)(x + 3) = 0$

$x2 + x - 6 = 0$

b) $\{1\ 2\}$

$x = 1$ or $x = 2$

$x - 1 = 0$ or $x - 2 = 0$

$(x - 1)(x - 2) = 0$

$x^2 - 3x + 2 = 0$

c) $\{\frac{2}{3}\ 4\}$

$x = \frac{2}{3}$ or $x = 4$

$x - \frac{2}{3} = 0$ or $x - 4 = 0$

$(3x - 2)(x - 4) = 0$

$3x^2 - 14x + 8 = 0$

Exercise 11

1. Find a quadratic equation in y whose solution set are:

a) $\{^-3,\ 1\}$

b) $\{\frac{2}{3}, -\frac{1}{2}\}$

c) $\{^-2\ ^-3\}$

d) $\{\frac{1}{2},\ ^-3\}$

e) $\{4,\ ^-5\}$

Activity: Patterns of algebraic expression

Objective: To expand and factorise algebraic expressions

Introduction: It involves algebraic letters incorporated in number system to form expressions which can either be expanded or factorised

Materials: Manilla paper, markers, pair of scissors, ruler and pencil.

Procedure

i) Get a ruler marked from 1-10 and a piece of plain paper.

ii) Draw the polygon patterns.

iii) Formulate the algebraic expressions.

iv) Write them in their respective positions on the polygon.

v) Cut out the polygons

vi) Re–assemble the cut out polygon to the required pattern / design.

Note: The cut polygons must be regular.

Observation

i) Different polygons can be used in the illustration of expansion and factorisation.

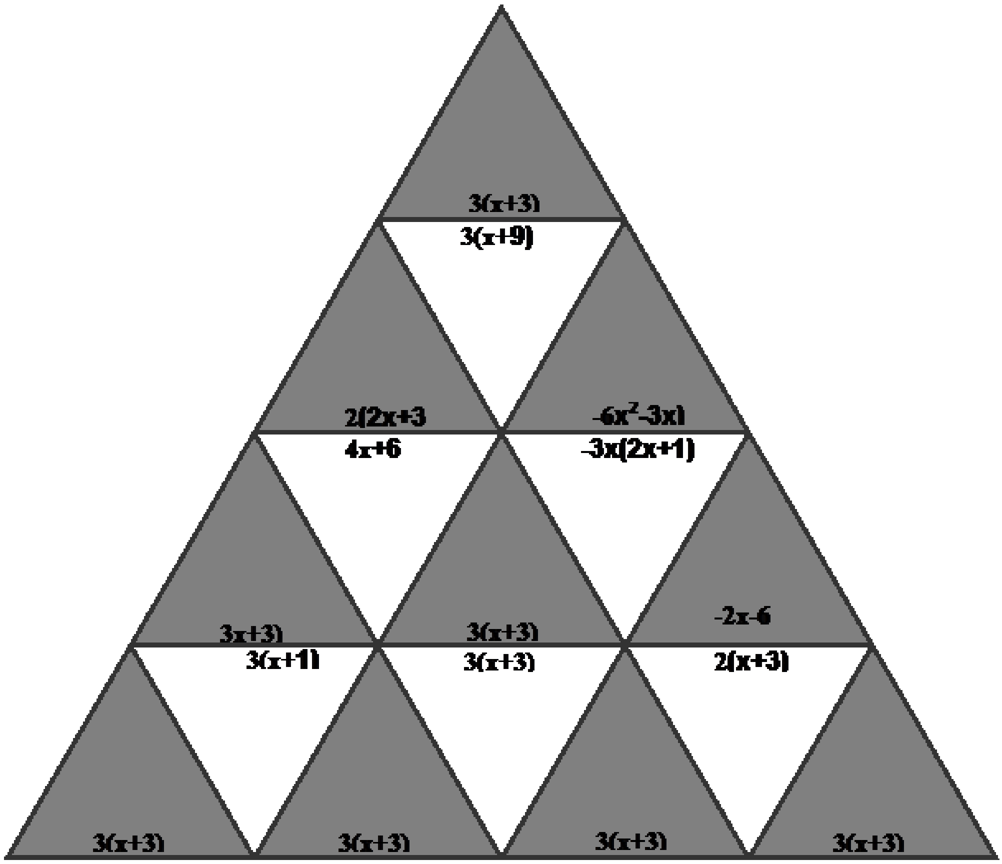
ii) The general polygon drawn should not have any algebraic expression along its perimeter.

Interpretation: Factorisation is the reverse of expansion.

Application: It is applied in Art and Design, construction.

Follow-up activity: Form different expressions to come up with different polygons.

Sketch of the general polygon



TOPIC 17 INEQUALITIES AND REGIONS

Inequality is an open mathematical sentence with the relation < or > and involves one or more variables. It includes inequality symbols, simple inequalities and regions.

Sub topic 1: Inequality

Inequalities can be represented by one or more inequality symbols namely

i) < which means "less than"

ii) > means "greater than" or "more than".

iii) ≤ means "less than or equal to" (at most).

iv) ≥ means "greater than or equal to" (at least).

Example 1

Rewrite the following using the inequality symbols:

i) x is less than 7
$\Rightarrow\ x < 7$

ii) Jane (J) is at least 7 years.
$J \geq 7$

iii) P is more than 10
$P > 10$

iv) t is less than or equal to 8
$t \leq 8$

v) w is greater than or equal to 11
$w \geq 11$.

Exercise 1

1. Rewrite using inequality signs.
   a) x is greater or equal to 7.
   b) x is more than 17.
   c) y is greater than 5.
2. Write the following inequalities in words.
   i) $x < 13$ ii) $r \leq 11$ iii) $p \geq 7$ iv) $2 > y$
3. The balance (b) of my account, is at least Shs. 20,000. Write this sentence as an inequality.
   i) Inequalities can be written as a set of numbers.

Example 2

1. The set of natural numbers N such that N is less than or equal to 15. This can be written as $\{N: N \leq 15\}$. We can list the possible values of N as $\{N: N \leq 15\} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$
2. List the members of the set of natural numbers less thans. $\{x: x \in N, x < 5\}$

Answer

The natural numbers less than 5 are 1,2,3 and 4.

Exercise 2

List the members of the following sets:

i) $\{x: x \in N, x < 6\}$

ii) $\{y: y \text{ is multiple of } 3, 5 \leq y \leq 15\}$

iii) $\{p: p \text{ is primes}, 4 < p < 19\}$

iv) $\{E: E \text{ is even}, E < 10\}$

Some inequalities can be found using more than one type of inequality sign.

Example 3

1. My Aunt has between 2 and 5 children. If C is the number of Children then C>2, or 2<C and C<5. So C is greater than 2 and C is less than 5. This can be combined as 2<C<5 and it can be written as {C: 2<C<5}

2. If P is a prime number between 3 and 10, rewrite it using inequality sign.

Answer

{P: 3<P<10}

Y is greater than or equal to -2 and less than or equal to 8.

$\{ -2 \leq y \leq 8\}$

3. List the members of a set of whole numbers.

   (i) $\{x: x \in N, 3 < x < 8\}$

Answer

The members are (4, 5,6,7)

   (ii) $\{y: y \text{ is an odd}, 5 < y < 10\}$

Answer

y = (6, 7, 8, 9)

4. Rewrite using inequality signs.

   (i) x is greater than or equal to 5 and less than 10.

Answer

$5 \leq x < 10$

   ii) y is greater than or equal to -2 and less than or equal to 8.

Answer

$-2 \leq y \leq 8$.

Exercise 3

1. Rewrite using inequality signs.
   i) x is greater than 5 and less than 13.
   ii) y is greater than or equal to 5 and less than 12.
   iii) y is less than 3 and greater than or equal to -4.
2. Write the following inequalities in words.
   i) $2 \leq y \leq 14$ iii) $5 < m < 21$
   ii) $6 < t \leq 10$ iv) $-3 \leq p < 5$.
3. "The number of absent (a) from class was between 10 and 20". Write the above sentence as an inequality.

b) Simple inequalities

Representing inequalities on a number line.

Note:

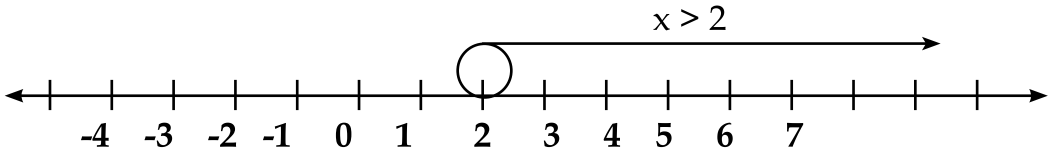
1. All numbers on the left of a number on a number line are less than that number.
2. All numbers on the right of a number are greater than that number.

Example 4

1. Show the inequality of the following on number line:
   a) $x > 2$.

Answer

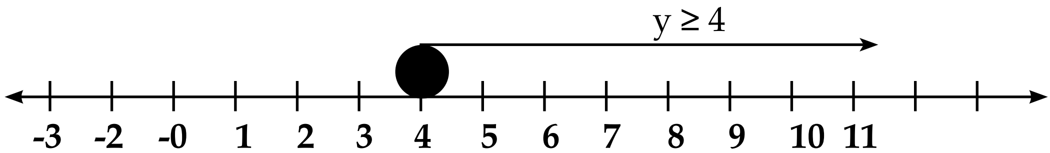
You draw a number line, then draw an arrow above it. Open circle means the number (integer) is not a member of the inequality.



b) y ≥ 4.

Answer

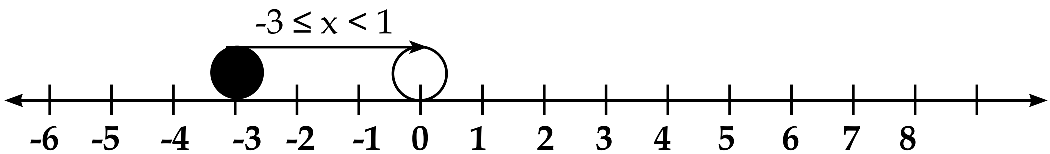
As number 4 is included, this time you draw a closed circle on the number line.



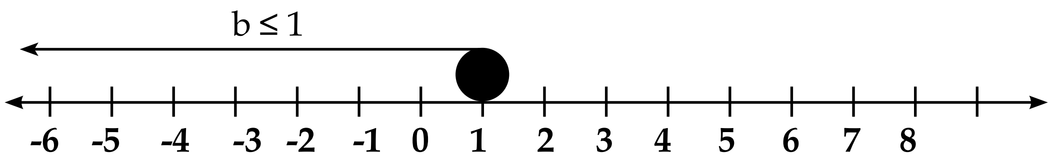
c) -3 ≤ x < 1

Answer

-3 is included but 1 is not included.



d) b ≤ 1



Exercise 4

Show the following on number lines

a) $x < 7$ b) $2 < h \leq 6$ c) $y \geq 8$

d) $-2 \leq t < 2$ e) $-2 \leq p \leq 3$

c) Solving linear inequalities in one unknown

Inequalities are solved in the same way as solving equations.

Note:

1. Never change an inequality sign to equal sign.
2. Multiplying or dividing by a negative number, the sign of the inequality ***must*** be reversed.

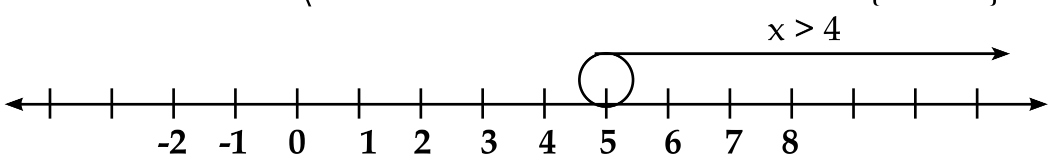
Example 5

1. Solve the inequality and illustrate it on a number line.

a) $x + 2 > 6$

Answer

$x + 2 - 2 > 6 - 2$ \The solution set is $x > 4$ $\{x : x > 4\}$

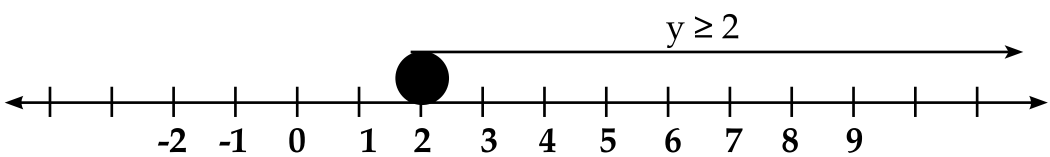


(b) $y - 4 \geq -2$

Answer

$y - 4 + 4 \geq -2 + 4$ ∴ The solution set is

$y \geq +2$ $\{y : y \geq 2\}$



c) $15 - 4x \geq 7$

Answer

$15 - 4x \geq 7$

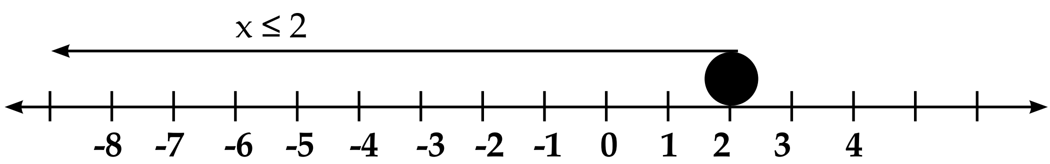
$15 - 15 - 4x \geq 7 - 15$

$-4x \geq -8$

$\frac{-4x}{-4} \leq \frac{-8}{-4}$ (change of the sign)

$x \leq 2$

$\therefore$ The solution set is $\{x : x \leq 2\}$



d) $2 < a + 8 < 4$

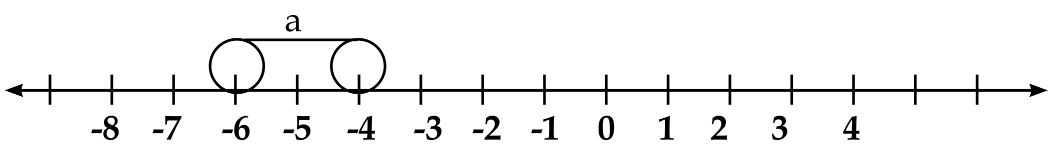
Answer

$2 < a + 8 < 4$

$2 - 8 < a + 8 - 8 < 4 - 8$

$-6 < a < -4$

$\therefore$ The solution set is $\{a : -6 < a < -4\}$



e) $-6 < 6 - 3r \leq 12$

Answer

$-6 < 6 - 3r \leq 12$

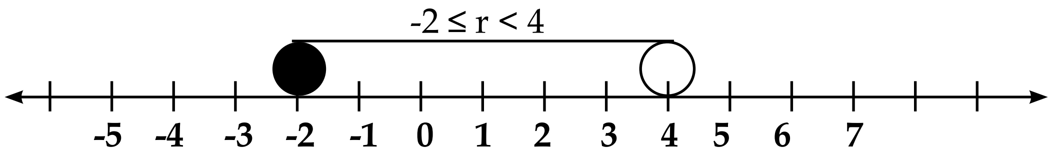
$-6 - 6 < 6 - 6 - 3r \leq 12 - 6$

$-12 < -3r \leq 6$

$\frac{-12}{-3} > \frac{-3r}{-3} \geq \frac{6}{-3}$

$4 > r \geq -2$

$\therefore$ The solution set is {r:-2 < r < 4}



2. Solve the following inequalities:

a) $1 - 4p < 9$

Answer

$1 - 1 - 4p < 9 - 1$

$$-4p < 8$$

$$\frac{-4p}{-4} > \frac{8}{-4}$$

$$p > -2$$

$\therefore$ The solution set is {p:p > -2}

b) $6p - 4(p - 1) > 5$

Answer

$6p - 4(p - 1) > 5$

$$6p - 4p + 4 > 5$$

$$2p + 4 > 5$$

$$2p + 4 - 4 > 5 - 4$$

$$2p > 1$$

$$P > \frac{1}{2}$$

$\therefore$ The solution set is {p:p > ½ }

c) $-3 \le 2x - 7 \le 2$

Answer

$-3 + 7 \le 2x - 7 + 7 \le 2 + 7$

$4 \le 2x \le 9$

$\frac{4}{2} \le \frac{2x}{2} \le \frac{9}{2}$

$2 \le x \le \frac{9}{2}$

$\therefore$ The solution set is $\{x : 2 \le x \le 4\frac{1}{2}\}$

3. i) Find the integral value of x which satisfy
$7 \ge 4 - 3x > -3$

Answer

$7 \ge 4 - 3x > -3$

$\Leftrightarrow 7 - 4 \ge 4 - 4 - 3x > -3 - 4$

$\Leftrightarrow 3 \ge -3x > -7$

$\Leftrightarrow \frac{3}{^-3} \le \frac{-3x}{^-3} < \frac{-7}{^-3}$

$\Leftrightarrow -1 \le x < \frac{7}{3}$

-1 and $2\frac{1}{3}$ are the range values of x

$\therefore$ The required integral values of x are $\{-1, 0, 1, 2\}$.

ii) Find the greatest integral value of x which satisfies the inequality

$x - \frac{1}{2} \underline{<} \frac{x}{6}$

Answer

$$x - \frac{1}{2} < \frac{x}{6}$$

$$\Leftrightarrow \quad x \times 6 - \frac{1}{\cancel{2}} \times \cancel{6}^{3} < \frac{x}{\cancel{6}} \times \cancel{6}$$

$$\Leftrightarrow \quad 6x - 3 < x$$

$$\Leftrightarrow \quad 6x - 3 + 3 < x + 3$$

$$\Leftrightarrow \quad 6x < x + 3$$

$$\Leftrightarrow \quad 6x - x < x - x + 3$$

$$\Leftrightarrow \quad 5x < 3$$

$$\Leftrightarrow \quad x < \frac{3}{5}$$

$\therefore$ The greatest integral value of x is 0.

iii) Find the greatest integral value of y which satisfies the inequality.

$$2 - \frac{3y}{2} > y + 3$$

Answer

$$2 \times 2 - \frac{3y}{\cancel{2}} \times \cancel{2} > y \times 2 + 3 \times 2$$

$$= \quad 4 - 3y > 2y + 6$$

$$= \quad 4 - 4 - 3y > 2y + 6 - 4$$

$$= \quad -3y > 2y + 2$$

$$= \quad -3y - 2y > 2y - 2y + 2$$

$$= \quad -5y > 2$$

$$= \quad \frac{-5y}{-5} < \frac{2}{-5}$$

$$= \quad y < \frac{-2}{5}$$

$$= \quad y < -0.4$$

Since y < -0.4, the greatest integral of y is -1.

Exercise 5

1. Solve the following inequalities:

i) $\frac{y}{5} - 1 > \frac{y}{10}$

ii) $3 < 9 - 3p < 15$

iii) $5 - (1 + x) > 3 - (2 + 3x)$

iv) $\frac{y + 2}{3} > 2 + \frac{y}{3}$

2. Solve the following inequalities and in each case represent the solution set on a number line.

a) $x + 6 < -2$ b) $r - 5 \geq -3$

c) $9 - 2x < 16$ d) $1 - \frac{1}{7}r > 2$

e) $-1 < -2x - 3 \leq 9$

3. Find the range of integral value of x for the inequality $-2 \leq 3 - x < 5$.

4. Find the least value of x for which $2x - 7 > x - 2$.

5. Find the greatest integral value of x which satisfies the inequality $5 - 2x > x - 6$.

c) Solving problems using inequalities

Example 6

1. The sum of two numbers is at most 12. One number is five times the other. Find the greatest possible values for the two numbers.

Answer

Let one number be x

= 5 times x means 5x

= $5x + x \leq 12$

= $6x \leq 12$

= $x \leq \frac{12}{6}$

$\Leftrightarrow$ $x \leq 2$

= $5x = 2 \times 5$

= 10

$\therefore$ The greatest possible values are 2 and 10.

2. Find the least possible values of two numbers such that one number is three times the other and their sum is at least 24.

Answer

Let the smaller number be x then the second one is 3x

= $3x + x \geq 24$

= $4x \geq 24$

= $x \geq \frac{24}{4}$

= $x \geq 6$

i.e. $3x = 3 \times 6$

= 18

$\therefore$ The least possible values are 6 and 18.

3. Two numbers are such that the larger is seven times the smaller and the difference between the two numbers is not more than 48. Find the largest possible values of the two numbers.

Answer

Let the smaller number be x, then the larger number is 7x.

$= \quad 7x - x \leq 48$

$= \quad 6x \leq 48$

$= \quad \frac{6x}{6} \leq \frac{48}{6}$

$= \quad x \leq 8 \qquad$ so $7x = 7 \times 8 = 56$

$\therefore$ The largest possible values are 8 and 56.

Exercise 6

1. Find the least possible values for the two numbers such that the sum of the two numbers is at least 16.
2. The length of a rectangle is four times its width. If the perimeter is at most 80 metres, calculate the maximum possible dimensions of the rectangle.
3. Rose is five years younger than Alice. If the sum of their age is at least 36 years, find their least possible ages.
4. John is three times as old as James. If the sum of their age is not more than 32 years, calculate their greatest possible ages.

Sub-Topic 2: Regions

Inequalities can be shown on a cartesian graph.

Method:

i) Change the inequality sign to equal sign.

ii) Plot and draw the line, name it.

iii) Shade the unwanted region after testing.

iv) If the inequality sign is

a) either $<$ or $>$ a broken (or dotted) line is drawn.

b) $\leq$ or $\geq$ a continuous (or solid) line is drawn.

v) Broken or dotted line means that points on the boundary line are not included in the region.

vi) Continuous or solid line means that points on the boundary line are included in the region.

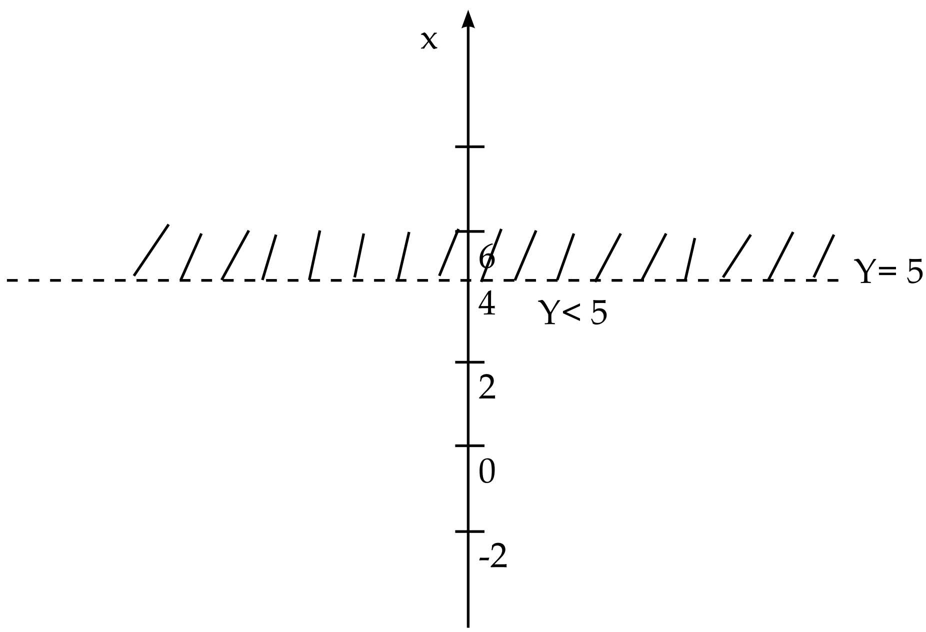
Example 7

1. Show the region $y < 5$ on a graph shading unwanted region.

Answer

$y = 5$, the line is broken to indicate that points on the line are not included as all the values of y must be less than 5.

Figure 1



Testing

i) Choose one point from the region and use it for testing.

(0, 7) $\Rightarrow$ 7 > 5 not true for y < 5.

(1, 4) $\Rightarrow$ 4 < 5 true for y < 5.

ii) Shade the unwanted region.

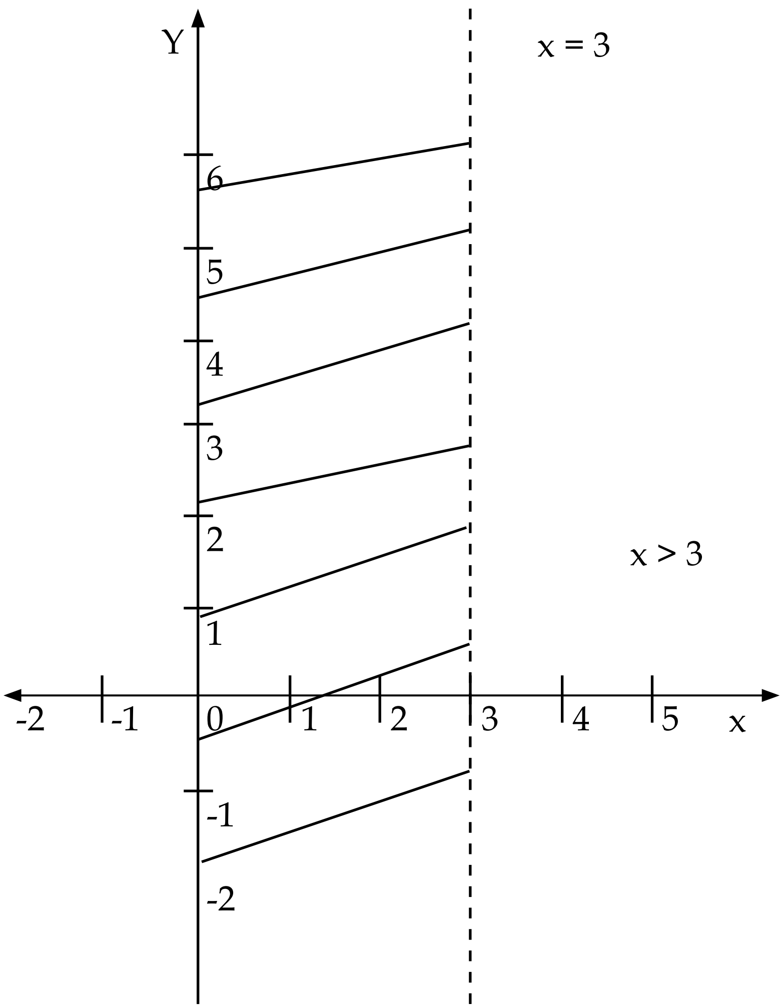
2. Show the region x > 3 on the graph.

Answer

---

x = 3

Figure 2



Testing

(2, 1) when x = 2 and y = 1, the inequality x > 3 is not true.

(5, 3) when x = 5 and y = 3, x > 3 is true.

3. Show the region for which $y \geq x + 4$ by shading out the unwanted region.

Answer

$y = x + 4$

Find two points which lie on the line when x = 0

$y = 0 + 4 = 4$

(0, 4) lies on the line $y = x + 4$

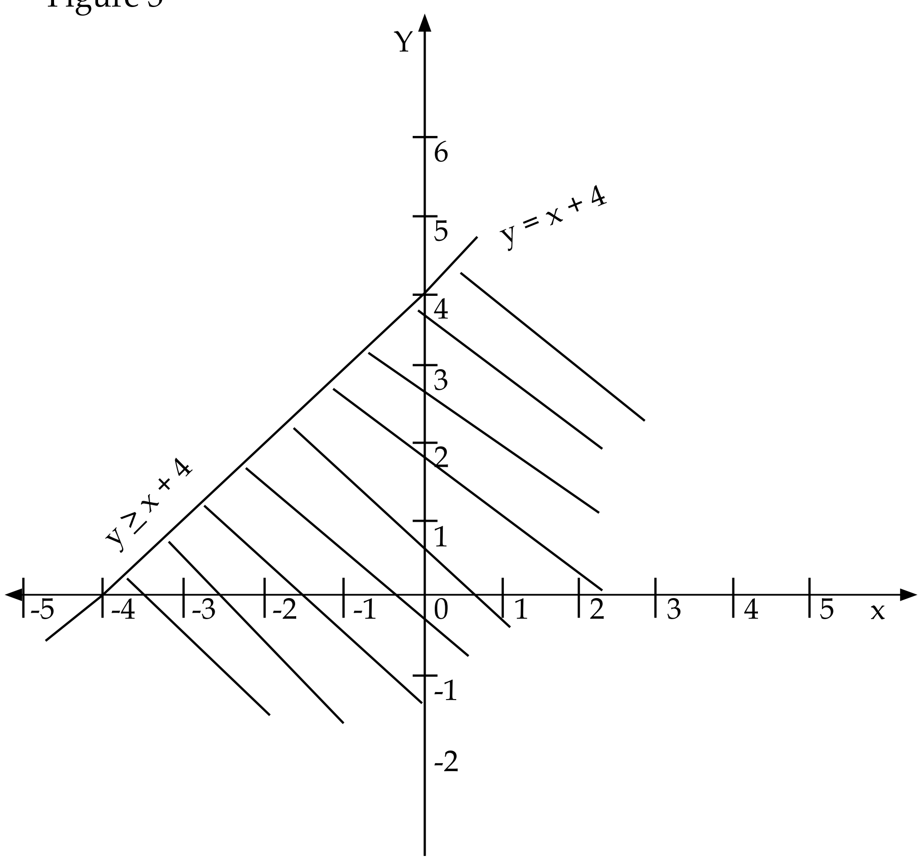
When y = 0 $\quad x + 4 = 0$

$x = -4$

(-4, 0) lies on the line $y = x + 4$.

Draw the line and it must be a solid line and use the points above.

Figure 3



Testing

(0, 0) $x = 0$ when $y = 0$

$y \geq x + 4$

$o \geq 0 + 4$

$o \geq 4$ not true.

4. Show the region $x + y < 5$ by shading the unwanted region.

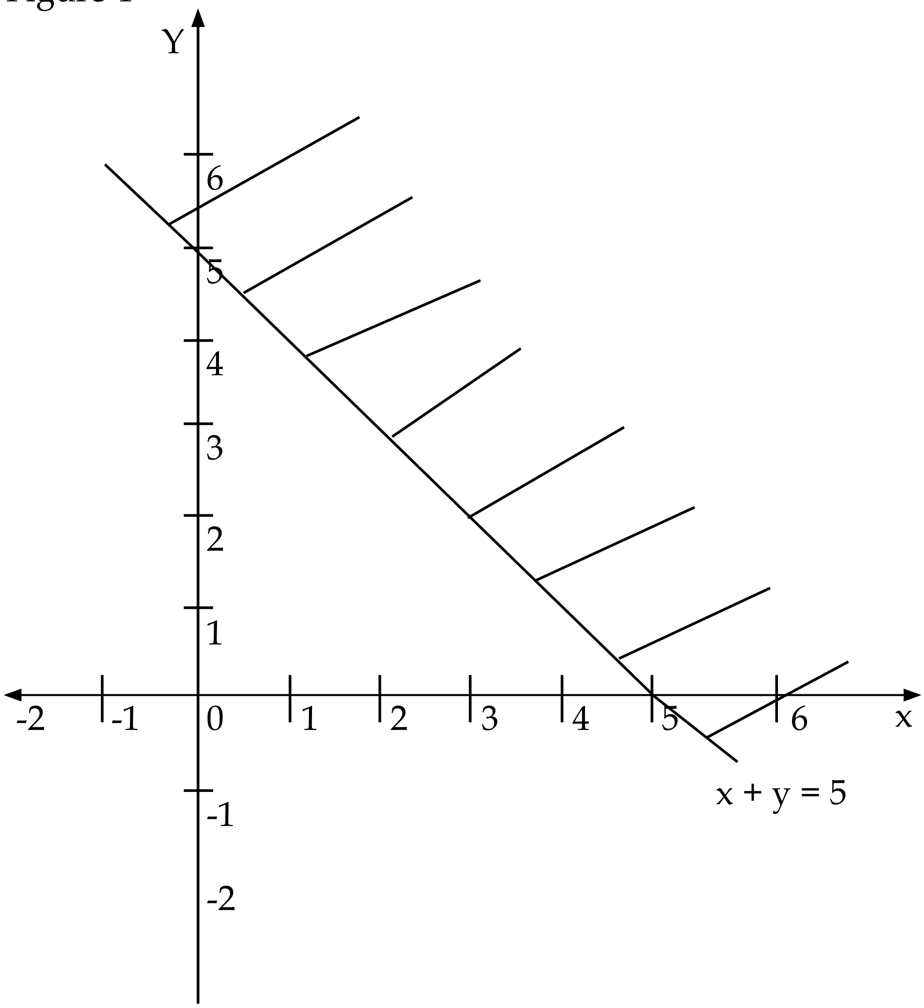
Answer

$x + y$	=	5	when y	=	0
when x	=	0	$0 + x$	=	5
$0 + y$	=	5	x	=	5
y	=	5			

(0, 5) lies on the line $x + y = 5$; (5, 0) lies on the line $x + 5 = 5$

The line $x + y = 5$ is a broken line.

Figure 4



Testing

(0,0)

$x + y < 5$

$0 + 0 < 5$

$0 < 5$ the inequality is true.

5. Draw the line $y = x$ and $x + y = 6$ and show the region in which the inequalities below are true. $x + y > 6$ and $x < y$.

Answer

x + y = 6

When x = 0, 0 + y = 6

y = 6

(6, 0) lies on the line x + y = 6

When y = 0, x + 0 = 6

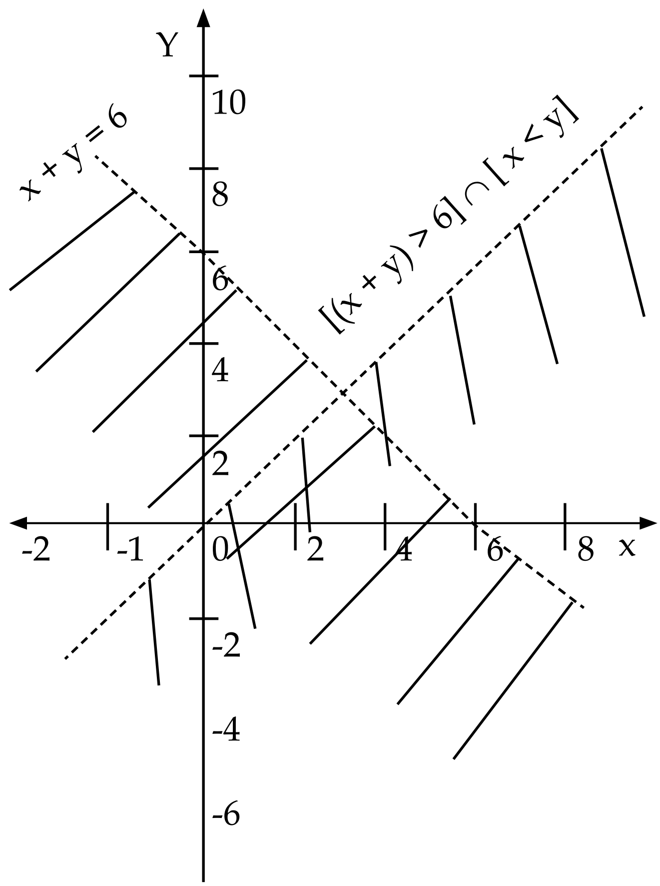
= 6

(6, 0) lies on the line x + y = 6.

The line x + y = 6 and y = x are broken lines.

(x + y > 6) n (x < y).

Figure 5



Testing

(0, 6)	$x < y$		
$x + y > 6$	(3, 4)	$3 < 4$	true
$v0 + 0 > 6$	(3, 1)	$3 < 1$	not true
		$0 > 6$	not true.

6. Show the region on the graph defined by $y \leq x + 1$, $x \leq 4$ and $y \geq 1$.

Answer

$y = x + 1$

$x = 0$ when $y = 0 + 1$

$= 1$

(0, 1) lies on the line $y = x + 1$

$y = 0$ when $x + 1 = 0$

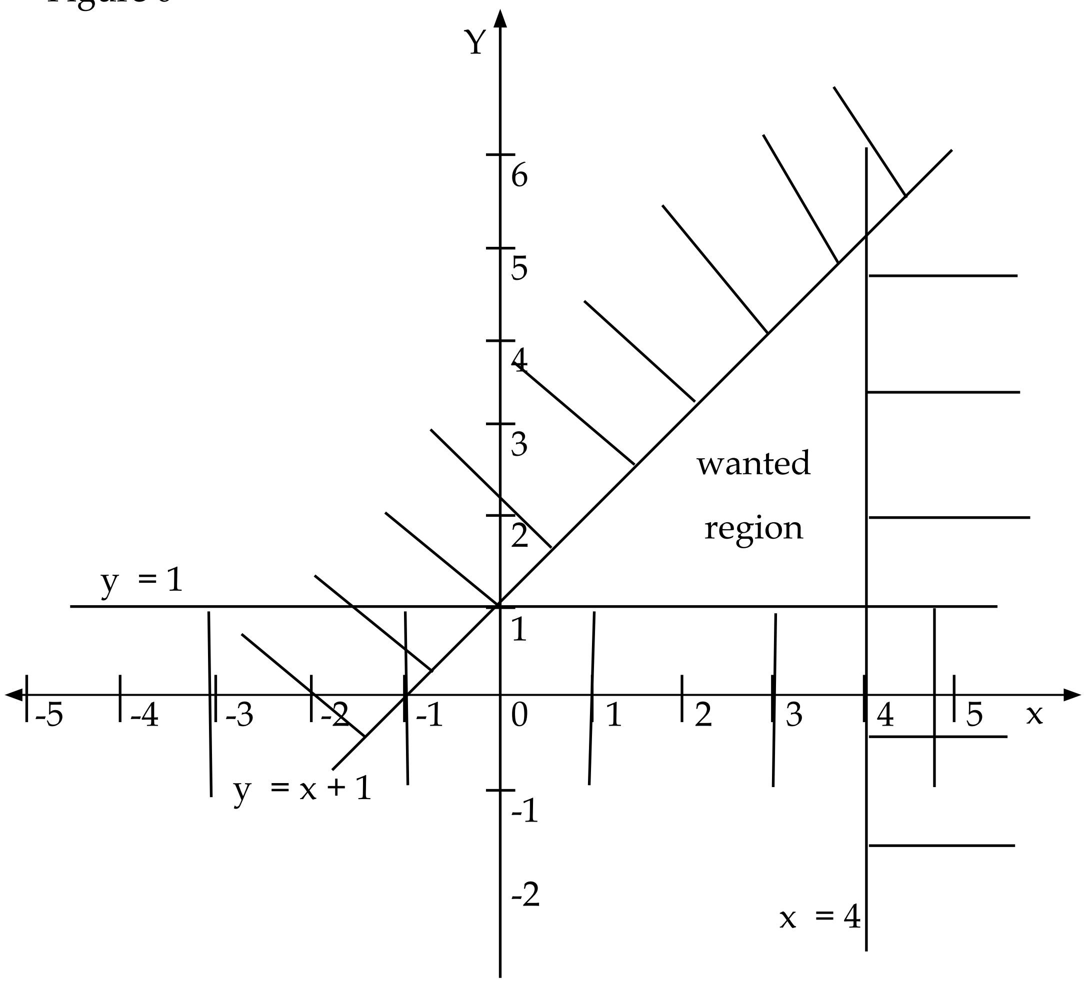
$x = -1$

(-1, 0) lies on the line $y = x + 1$

$x = 4$ and $y = 1$

All the lines are solid lines.

Figure 6



Testing

i) $y \leq x + 1$

(0, 0) $0 \leq 0 + 1$

$0 < 1$ true

ii) $x \leq 4$

(3, 7) $3 \leq 4$ true (6, 2) $6 \leq 4$ not true

Region A is the wanted region, i.e. $(y \leq x + 1)$ n $(x \leq 4)$ n $(y \geq 1)$

Exercise 7

1. Draw the given lines on a graph and show the region described by the inequality by shading the unwanted region.

    a) line $x = 4$, region $x > 4$

    b) line $x = 5$, region $x \leq 5$

    c) line $y = x + 5$, region $y < x + 5$

    d) line $y = x - 4$, region $y \geq x - 4$

    e) line $y = 2x + 6$, region $y \geq 2x + 6$.

2. Draw the line $y = x$ and $x + y = 6$ and show the region where $x + y \leq 6$ and $x < y$.

3. Shade on a graph paper the region in which both of these inequalities are satisfied $3y < 6 - 2x$ and $y < 2 + 2x$.

Activity: Calculating profits from the sell of fruits

Introduction: It is the application of linear inequalities.

Objectives: i) To formulate inequalities.

ii) To tell the difference between wanted and unwanted regions.

Materials/Information: Fruits for sale, e.g. oranges and mangoes or any fruit

i) Cost for each type

ii) Number of fruits to be bought (at least one of each type)

iii) Specific number of fruits the bag can hold.

iv) Specify the profit made on each type of fruit.

Procedure

i) Write down three inequalities from the given information.

ii) Represent the inequalities on the graph.

iii) Shade the unwanted region.

Note: the inequality signs used.

Observation: The unshaded region is the wanted region.

Required points are read from the wanted region.

The required region lies in the real quadrant.

Conclusion: Linear programming helps in profit maximisation or cost minimisation.

Application: It is applied in the transport industry, supermarkets, schools, etc.

Follow-up activity: Look for information where linear programming is being applied.

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