MAGNETISM NOTES:

Magnetism – is a group of phenomena or concepts associated with the field of force that exists around a magnetic body or a current carrying conductor.

A MAGNETIC FIELD

A magnetic field is a region of space in which:

(i) A magnetic dipole (a magnet) experiences a force or

(ii) A current carrying conductor experiences a force, or moving charge experiences a force or

(iii) An e.m.f is, induced across a moving conductor.

Definition:

A magnetic field is thus defined in short as a field of force that exists around a magnet or a current carrying conductor.

The magnetic properties of a magnetic body appear to originate at certain regions in the magnet called POLES. In a bar magnet these are near the ends of the magnet.

Experiments show that:

(i) magnetic poles are of two kinds, north poles and south poles.

(ii) like poles repel each other while the unlike poles attract each other.

(iii) poles always seem to occur in equal and opposite pairs and

(iv) when no other magnet is near a freely suspended bar magnet, it rests in such a position that its magnetic axis is approximately parallel to the Earth's North – South axis.

MAGNETIC FIELD LINES.

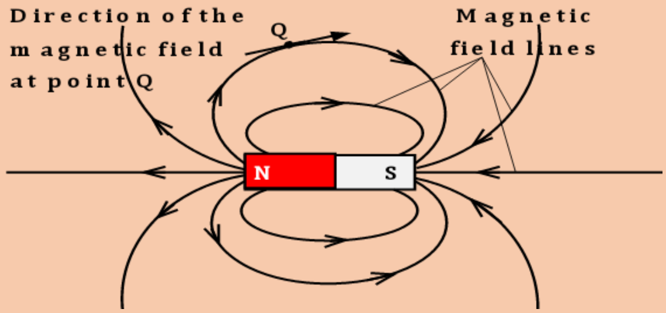
The direction of a magnetic field at a point – is taken to be the direction of force on a north pole of magnetic dipole there under the influence of the field at that point.

The path which such a pole would follow is called a magnetic field line or (line of force)

A magnetic field can therefore be represented by magnetic field lines so that;

(i) the line (or the tangent to it if it's a curved path) gives the direction of the magnetic field at that point.

(ii) the number of lines per unit cross sectional area is an indication of the strength of the field. i.e the strength of the magnetic field is proportional to the density of the field lines.



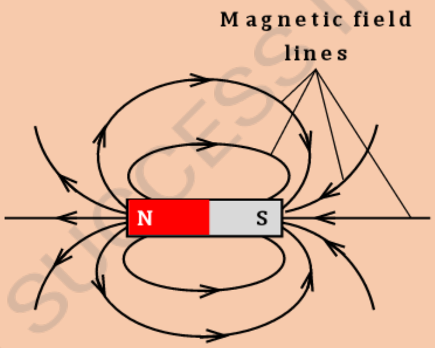
The direction of the magnetic field is always directed a way from the North pole of a magnet towards south pole.

For the case of a compass needle placed in the magnetic field of a bar magnet, the needle will be tangential to the magnetic field line at that point. The north pole of the needle points from the N-pole of the bar magnet to the South pole.

REPRESENTATIONS OF MAGNETIC FIELD PATTERNS

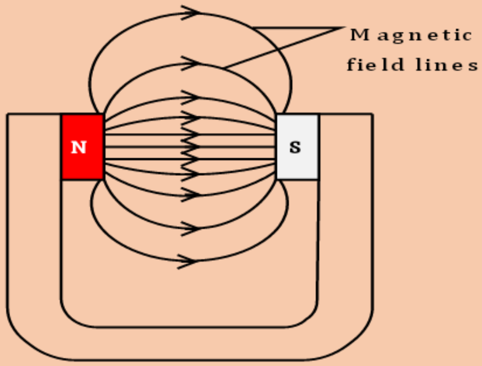
1. Around a magnet.

(i) Bar magnet.

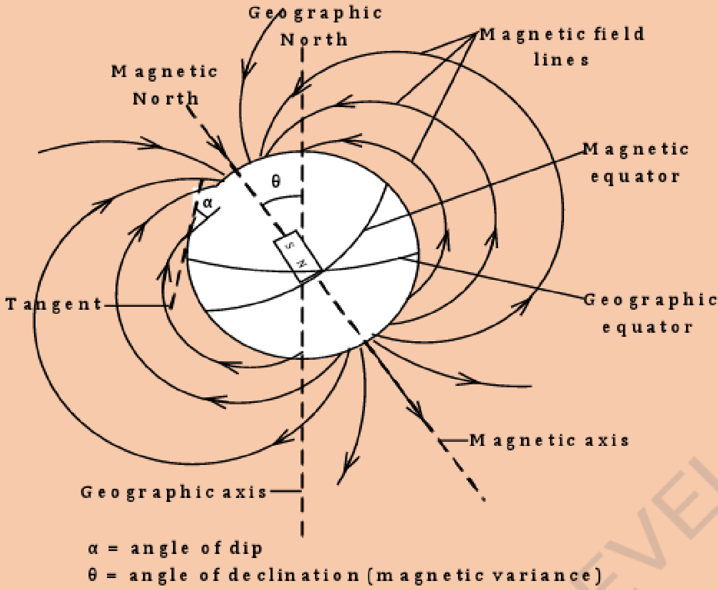


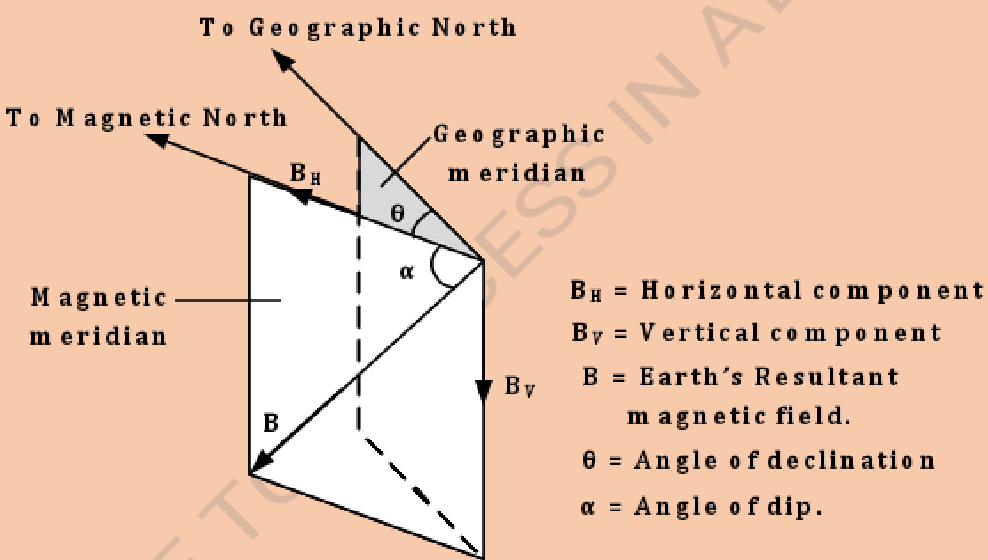
The magnetic field around a bar magnet is non – uniform i.e varies in strength and direction from point to point.

(ii) U-shaped (Horse shoe) magnet.



2. The Earth's magnetic field:





Definitions:

(i) **Magnetic Meridian:** - is a vertical plane passing through the Earth's magnetic North and south poles.

(ii) **Geographic Meridian:** - is a vertical plane passing through the Earth's geographic North and south directions.

(iii) **Angle of declination or Magnetic variance:** - is the angle between the earth's magnetic meridian and geographic meridian.

(iv) **Angle of dip**: - is the angle between the Earth's resultant magnetic field and the horizontal component of the earth's magnetic field.

Or the angle between the horizontal and the axis through the poles of a freely suspended bar magnet when it sets.

(v) **Magnetic Axis** – is a vertical line through the Centre of the earth and passing through the earth's magnetic poles.

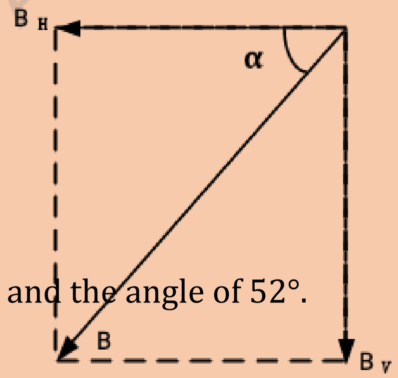
(vi) **Geographic Axis** – is a vertical line through the Centre of the earth and passing through the earth's Geographic north and south directions.

(vii) **Magnetic Equator** – is the largest horizontal circle where a freely suspended bar magnet experience zero magnetic dip.

(viii) **Geographic Equator** – is the largest horizontal circle in a plane through the Centre of the earth perpendicular to the geographic meridian.

The relationship between, B_H, B_V, B and the angle of dip, α.

At a particular location over the Earth's surface, the Earth's resultant magnetic field **B** is related the horizontal component, B_H the vertical component and the angle of dip, α by the following equation:



$B^2 = B_H^{\ 2} + B_V^{\ 2} \Longrightarrow B = \sqrt{B_H^{\ 2} + B_V^{\ 2}}$ …………(i)

Angle of dip, $\alpha = \tan^{-1}\left(\dfrac{B_V}{B_H}\right)$……………………(ii)

Example:

At a particular location over the earth's surface the Horizontal magnetic flux density has a value of 3.0×10^{-4} T and the angle of 52°. Determine the;

(i) Earth's resultant magnetic field.

(ii) Vertical component of the earth's magnetic field.

Solution:

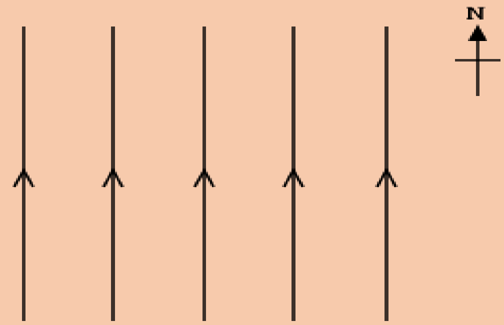
$$\cos\alpha = \frac{B_H}{B} \Longrightarrow B = \frac{B_H}{\cos\alpha} = \frac{3.0\times10^{-4}}{\cos 52°} = 4.87\times10^{-4}\ \text{T}$$

$$\tan\alpha = \frac{B_V}{B_H} \Longrightarrow B_V = B_H\tan\alpha = 3.0\times10^{-4}\tan 52° = 3.84\times10^{-4}\ \text{T}$$

The Earth's local magnetic field:

This is a representation of the earth's magnetic field pattern at a particular location over the earth's surface.

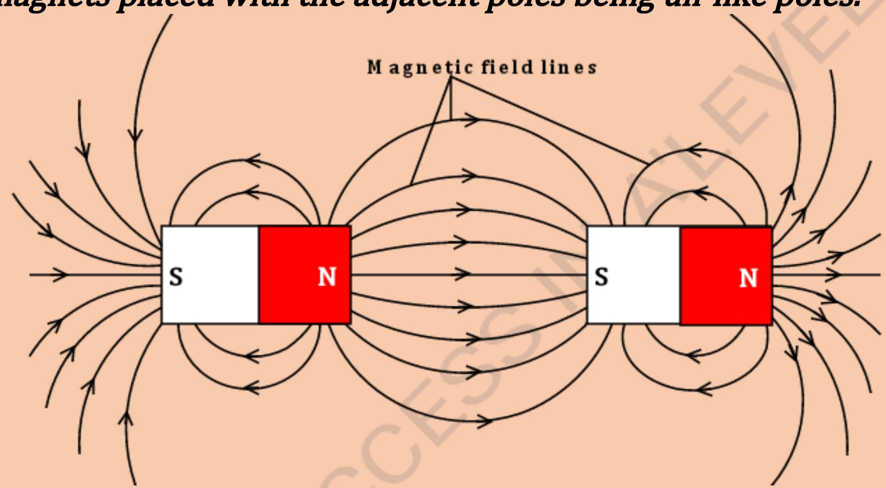
It's denoted or represented by a uniform parallel beam of the magnetic field directed towards the North.



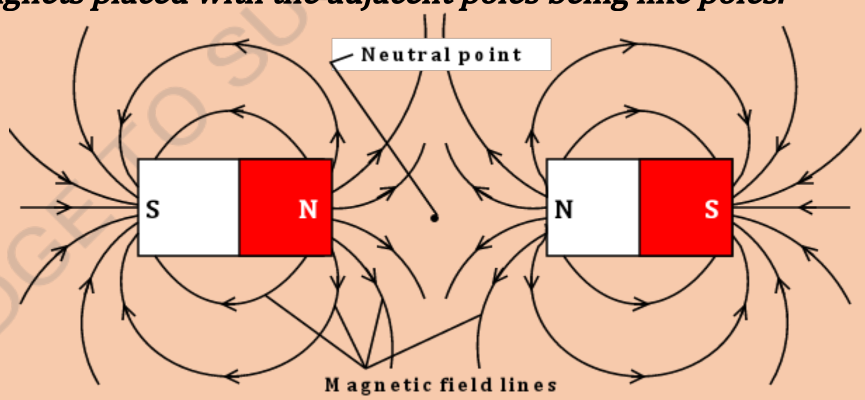
:he magnetic field lines

The Super position the magnetic fields:

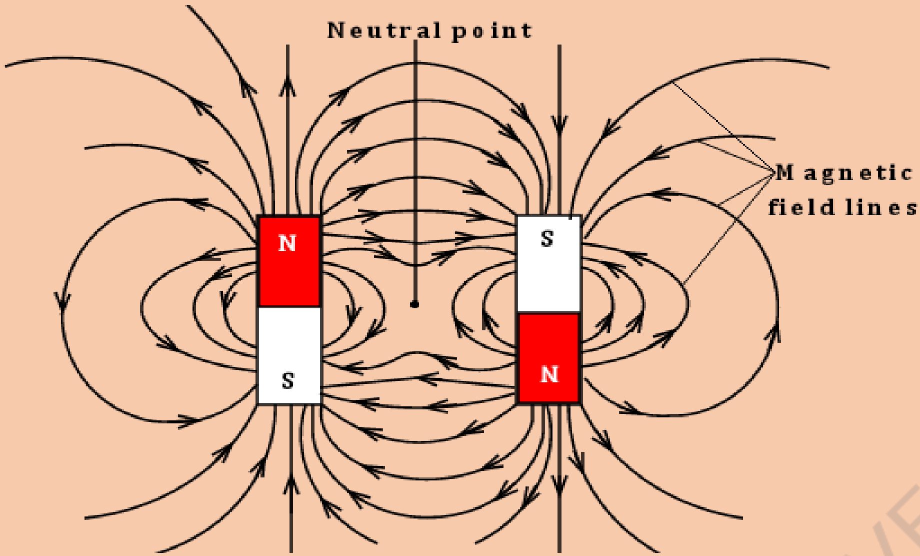
(a) *Two Bar magnets placed with the adjacent poles being un-like poles.*



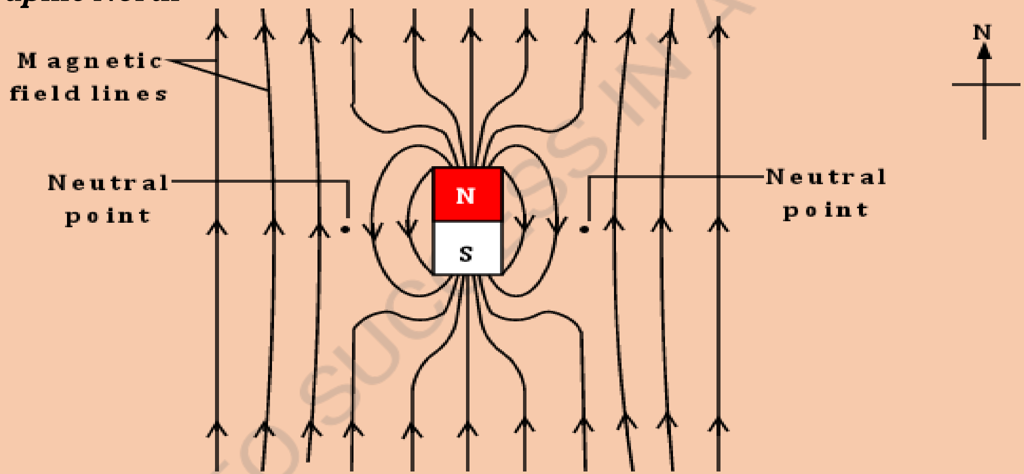
(b) *Two Bar magnets placed with the adjacent poles being like poles.*



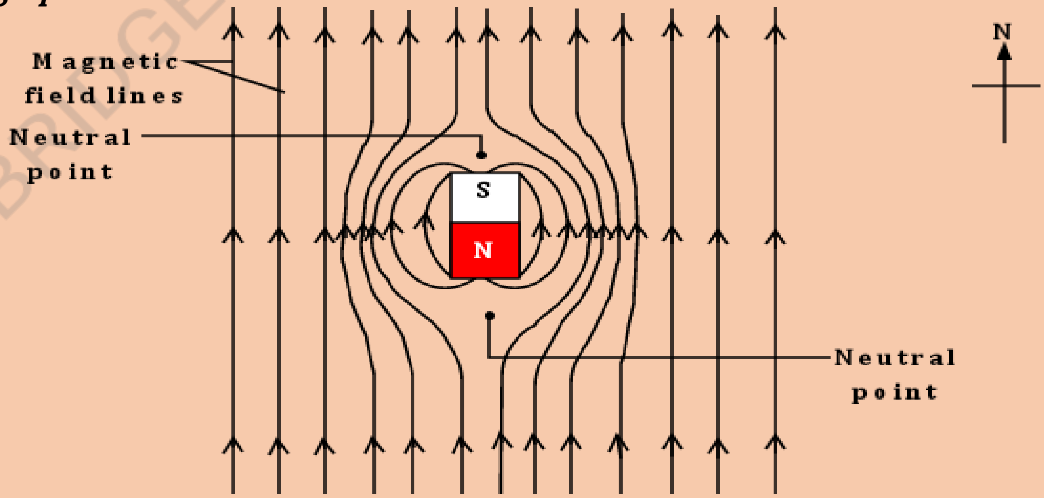
(c) Two Bar magnets placed parallel and near each other with adjacent poles being unlike poles.



(d) *A bar magnet placed in the Earth's local magnetic field with the North pole facing the Geographic North.*



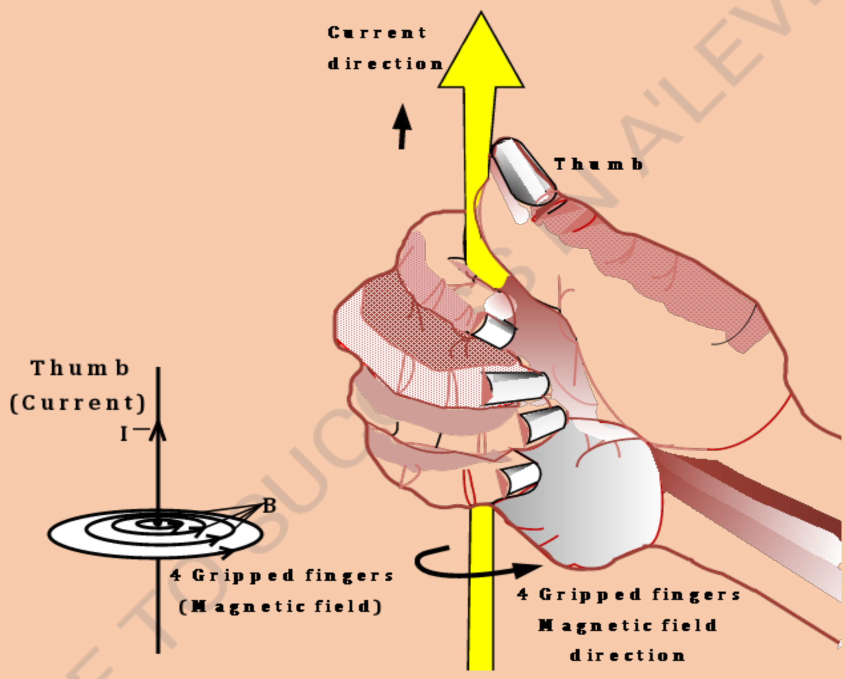
(e) *A bar magnet placed in the Earth's local magnetic field with the South pole facing the Geographic North.*



A neutral point – *is a region of space in a magnetic field where two magnetic fields have equal magnitude but opposite in their directions i.e. the resultant magnetic field is zero. At such a point, the resultant force on a magnetic dipole or freely suspended bar magnet is zero.*

THE RIGHT HAND GRIP RULE:

It is the rule used to predict the direction of a magnetic field when a current is passed through a conductor or a wire in such a way that, if the right hand is gripped, the ***Thumb*** *represents the direction of the* ***current*** *provided. While the* ***four gripped fingers*** *will provide the direction of the* ***magnetic field*** *around the current carrying conductor wire.*



It is also used to predict the direction of flow of a current if the direction of the magnetic field is provided.

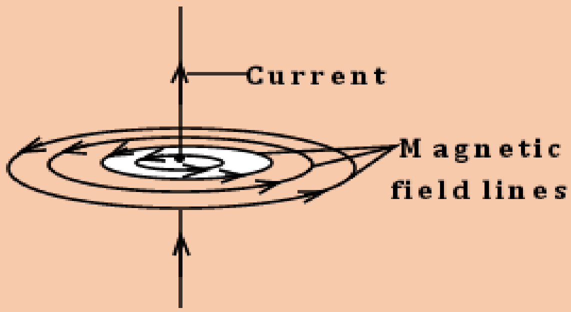
Any straight conductor carrying a current, experiences a magnetic field around it. The direction of a magnetic field around the conductor is given by the **right hand grip rule** which states that ***imagine a conductor to be griped in the right hand with the thumb pointing in the direction of the current, the four gripped fingers indicate the direction of the magnetic field around the conductor***.

NB: The representations are interchanged or reversed when dealing with a coil or solenoid, carrying a current.

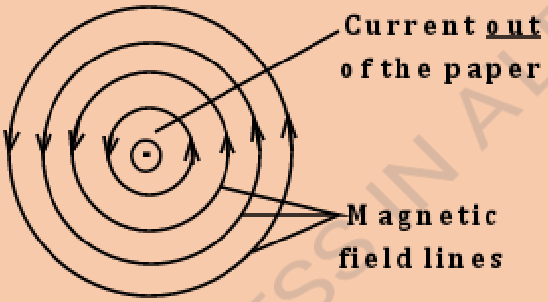
Diagram:

CONSIDER THE CODUCTORS SHOWN BELOW.

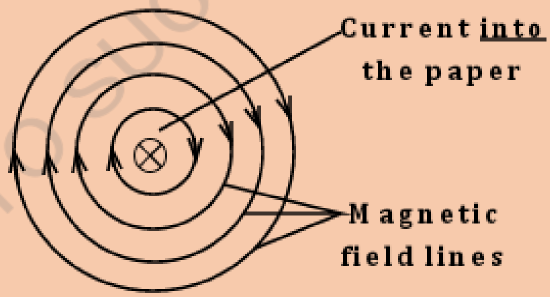
1. A Straight wire carrying a current



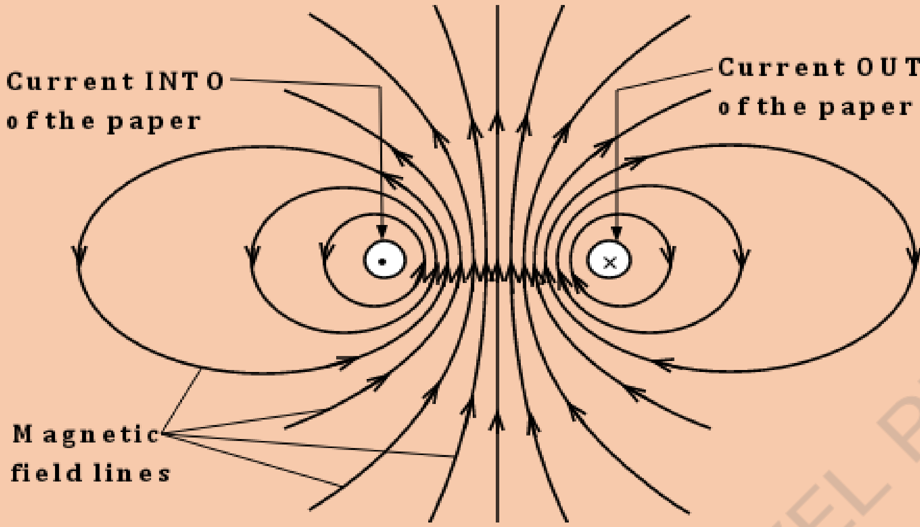
(a) A Wire carrying current up wards (out of the plane of the paper)



(b) Wire carrying current downwards (into the plane of the paper)

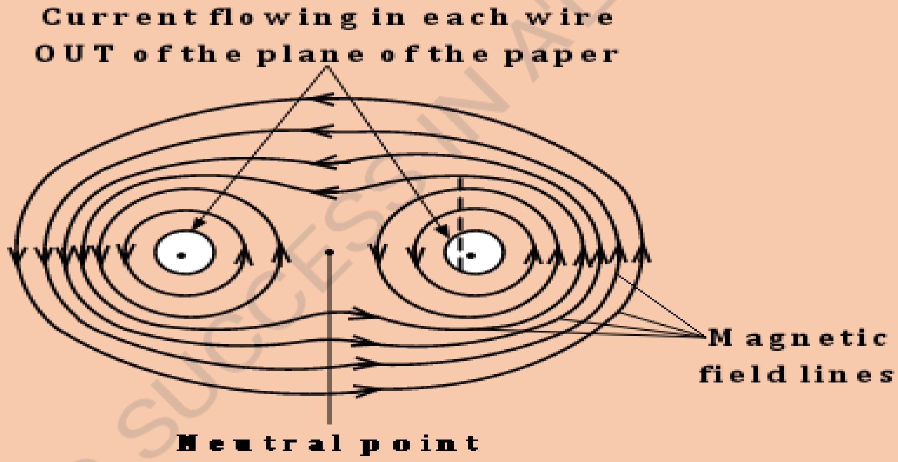


2. Magnetic fields due to two parallel wires carrying currents:
(a) In opposite directions.

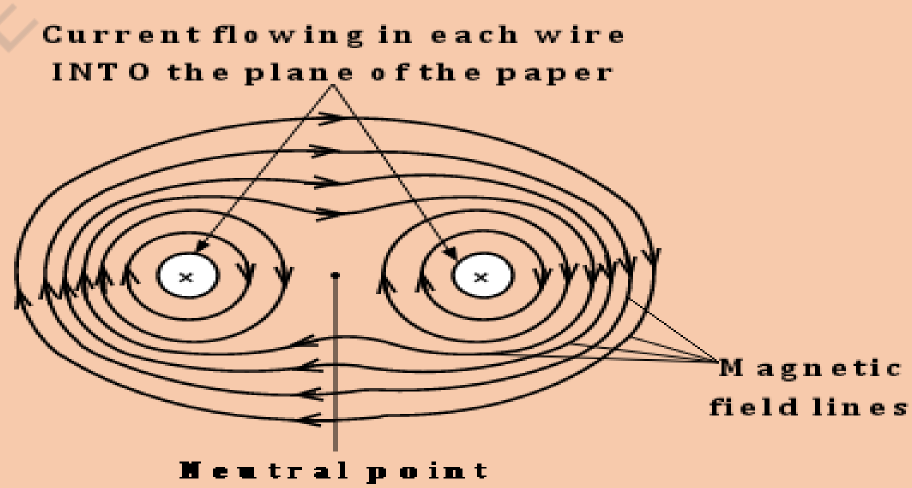


(b) In the same direction.

(i) INTO the plane of the paper.

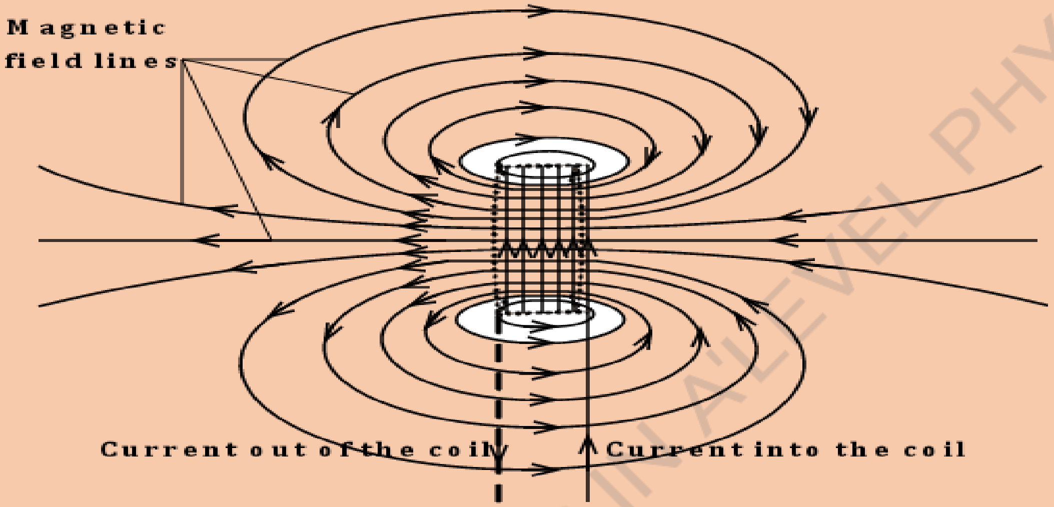


(i) OUT of the plane of the paper.

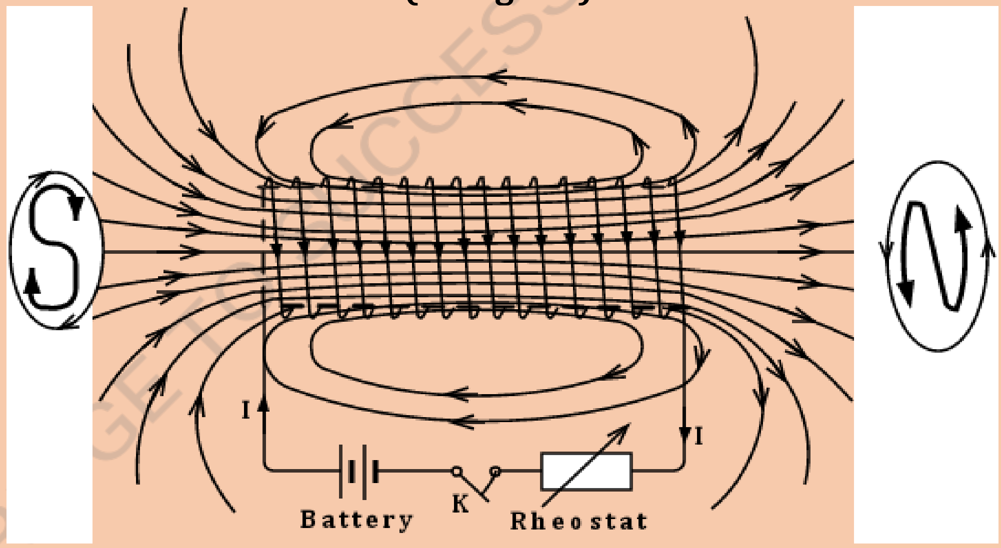


NB: *In each of the cases in (i) and (ii) above, there are more magnetic field lines at the extreme outer positions of each wire. This implies the existence of a stronger magnetic field at the outer positions and a weaker resultant field in the region between the wires. A resultant magnetic force always acts towards the centre of the two wires.*

3. Magnetic field pattern due to a plane circular coil of N – turns.



4. Magnetic field round a Solenoid (a long coil)



NB: When direct current is passed through a solenoid or coil in the direction indicated, the **four gripped fingers of** the **right hand** are used to indicate the **direction of** the several **turns of the current** carrying wire constituting the coil. While on the other hand, the **thumb** indicates the **direction of the magnetic field B**, at the **centre of such a coil**, considered to be fairy uniform.
The "S and N symbols" – at the extreme ends, help to identify the polarity of the magnetic poles at the ends of the solenoid.

Assume you are looking at the cross – section of either side of the solenoid, consider the direction of flow of the current, as clockwise or anti-clockwise. The "letter" **S** or **N** whose end arrows agree with the direction of current i.e. **same direction as that of the current through the coil of solenoid**, is the **correct pole** at that end of the solenoid.
Example, from the diagram above, on the **left hand side** of the solenoid, **current direction is clockwise**, so **"letter S"** agrees with the current direction, hence the **south pole, S**.
Likewise on the **right hand side of the solenoid**, current flows in the **anti-clockwise direction**, and **"letter, N"** agrees with **the current direction**, hence the **North pole, N**.

Magnetic Flux Density, (Magnetic induction) B:

Magnetic flux density or magnetic induction – Is the force exerted on a conductor of length 1m carrying a current of 1A in a direction normal to the magnetic field. Its SI unit is a tesla (**T**).

A tesla – is the magnetic flux density across which conductor of length **1m** and carrying a current of **1A** in a direction normal to the field experiences a magnetic force of **1N**. i.e 1 T = 1 N (A m)$^{-1}$

Derivation for the force exerted on a current carrying wire (conductor)

Suppose a straight wire of length, **L**, having N free electrons each of charge **e**, has a current, **I**, flowing through it and is placed perpendicularly across a uniform magnetic field of flux density **B**, the electrons attain an average drift velocity, v. The magnetic force acting on each electron is given by;

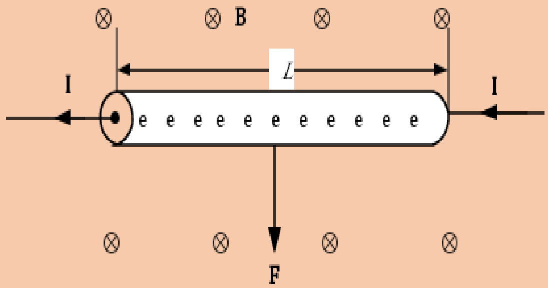
Force on each electron, $F_1 = Bev$

Total force on N-free electrons, $F = NF_1 = NBev$

But $Ne = Q$ (total charge on the conductor)

$\therefore F = BQv$, But $Q = It$

$\Rightarrow F = BIt\,v$



But $v = \dfrac{L}{t}$

$\therefore \quad F = BIt \times \dfrac{L}{t}$

Hence, $F = BIL$

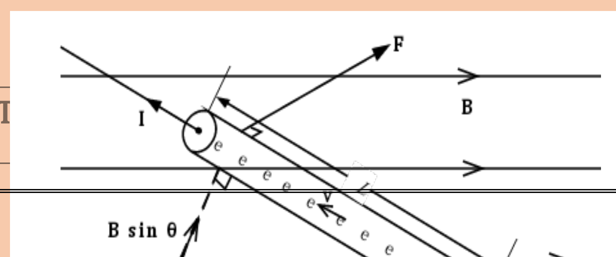
NB: Suppose the conductor is at an angle θ to the magnetic field, B

Force on each electron, $F_1 = Bev \sin\theta$

Total force on N-free electrons, $F = NF_1 = NBev \sin\theta$ but $Q = Ne$

$\therefore F = BQv \sin\theta$, But $Q = It$

$\Rightarrow F = BItv \sin\theta$



Derivation for the force exerted on an electron moving at a speed v, in a magnetic field, B, in a current carrying wire (conductor)

Total force on a conductor of length, L, carrying a current I, across a magnetic field of flux density B, is $\mathbf{F} = BIL$ but current flowing $\mathbf{I} = \frac{Q}{t}$

$\therefore \mathbf{F} = BIL = B\left(\frac{Q}{t}\right)\mathbf{L}$ where $Q = N\,e$, N = Total no. of free electrons

$\therefore \mathbf{F} = B\left(\frac{N\,e}{t}\right)\mathbf{L} = \frac{BNeL}{t}$ but $\frac{L}{t} = \mathbf{v}$ (average drift velocity of electrons)

$\therefore \mathbf{F} = BNev$ But the force on each elctron, $F_1 = \frac{F}{N} = \mathbf{Bev}$

Hence, the force on each elctron, $F_1 = \mathbf{Bev}$

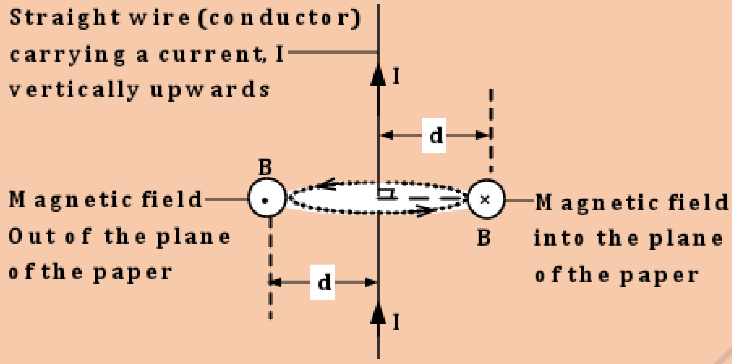
Magnetic field, B, due current carrying wires (conductors)

NB: *The derivations for the expressions for the magnetic fields due to current carrying conductors involves the use of Biot – Savat law that is out of the context of our A' level physics syllabus. However, learners are expected to memorize the expressions, and make use of them in various applications of the syllabus.*

1. **A straight wire carrying a current, I, in free space (Vacuum or Air)**
   At a perpendicular distance, d , from the straight wire carrying a current I, the magnetic flux density, B, is given by,

B   **μ₀ I**

   ....*(i)*



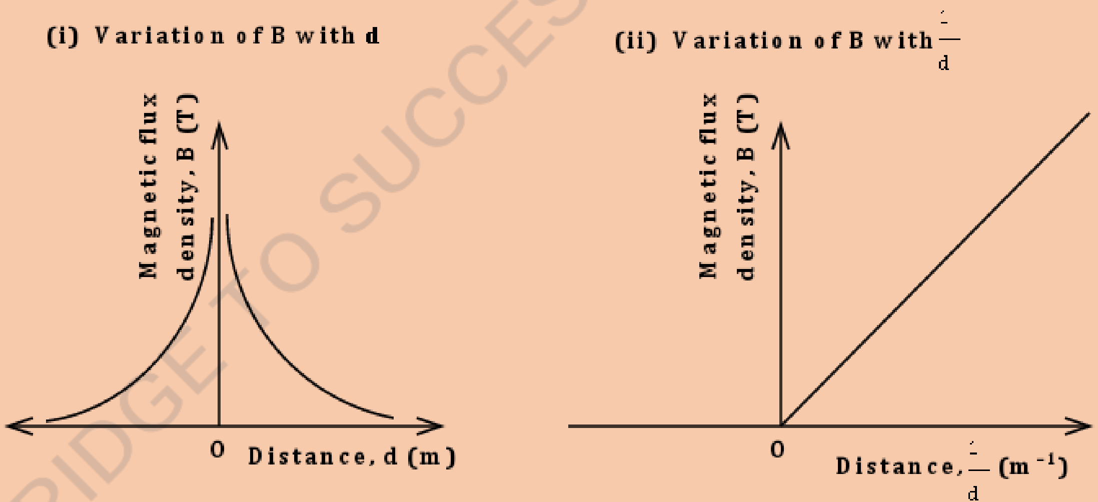
NB: If the wire is surrounded by any other medium of permeability μ, where

$\mu = \mu_o\mu_r$ where, μ_r = Relative permeability of the media.

The magnetic flux density, equals, $B = \dfrac{\mu I}{2\pi d} = \dfrac{\mu_o\mu_r I}{2\pi d}$

Magnetic flux density, B, due to a current carrying wire varies inversely with distance, d, as shown on the graph below.

A graph of magnetic flux density with distance, and magnetic flux density against reciprocal of distance from the wire.

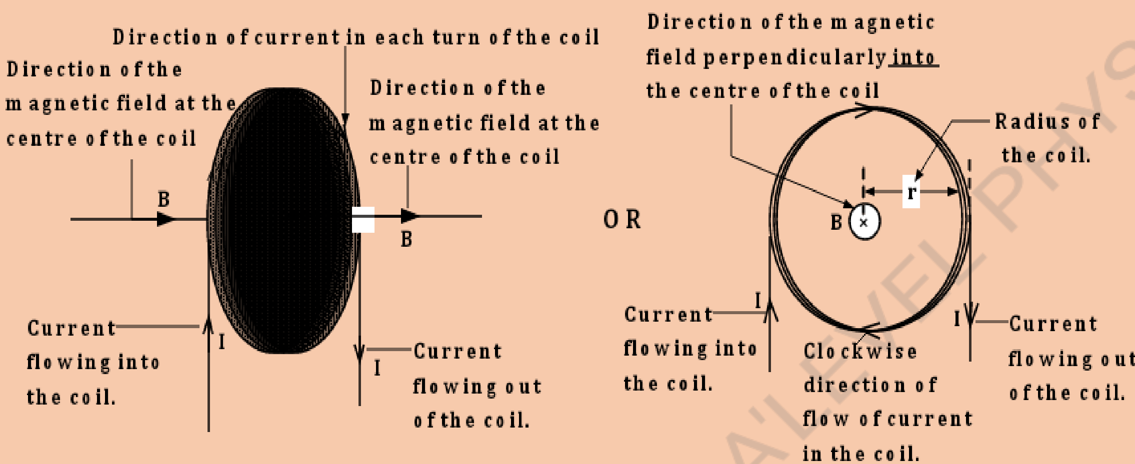


NB: Graph (i) above shows that as distance tends to zero, i.e. very close to the wire, the magnetic flux density becomes infinitely large and as the distance tends to infinity, the magnetic flux density tends to zero. While for graph (ii) **B** increases with increase in the value, $\dfrac{1}{d}$.

2. **At the centre of a plane circular coil, of radius r, and of N – turns each carrying a current, I, in free space (Vacuum or Air)**

At the centre of a plane circular coil of radius r, and of N – turns of wire each carrying a current I, the magnetic flux density, B, is given by,

} : ....(ii)

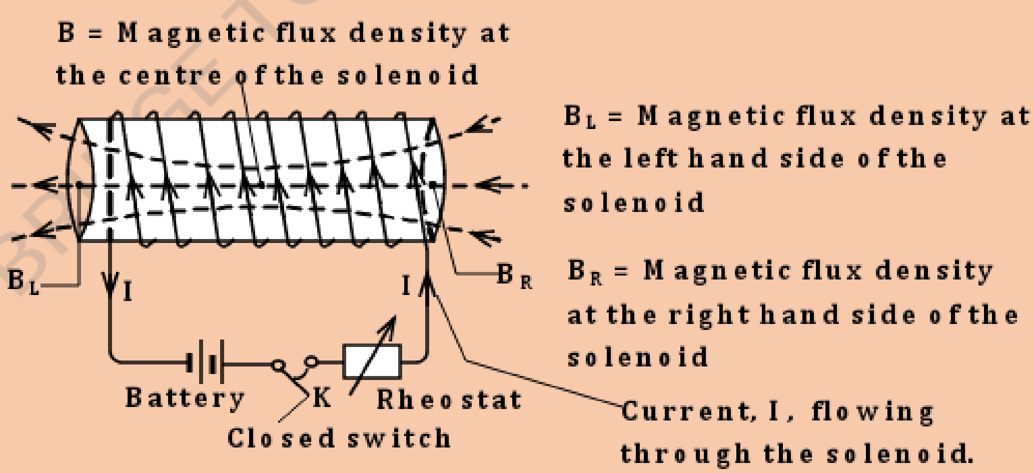


NB: We use the right hand grip rule to identify direction of the magnetic field when that of current is known and vice versa.

3. At the centre of a long coil, (Solenoid) of Length, L and N – turns or of (n – turns per metre), each carrying a current, I, in free space (Vacuum)

At the centre and along the axis of a long solenoid of n – turns per metre of wire each carrying a current I, the magnetic flux density, B, is given by,

$$\boxed{B = \mu_o n I = \frac{\mu_o N I}{L}}$$ ....(iii)



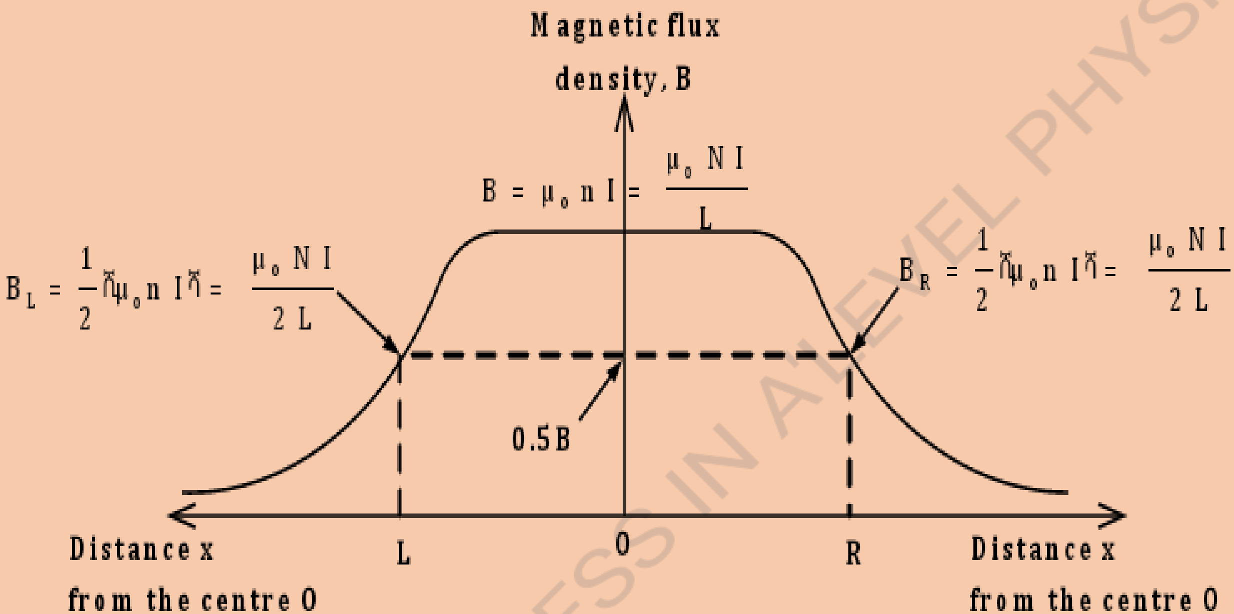
At the ends, L and R, along the axis of a long solenoid of n – turns per metre of wire each carrying a current I, the magnetic flux density, B_L and B_R, is the same and given

$$B_L = \frac{1}{2}(\mu_o n I) = \frac{\mu_o N I}{2L}$$

by,

$$\boxed{B_L = B_R = \frac{1}{2}(\mu_o n I) = \frac{\mu_o N I}{2L}} \quad \text{.................(iv)}$$

A graph of magnetic flux density, B , with distance, x, from the centre of a long solenoid, of n – turns per metre carrying a current I

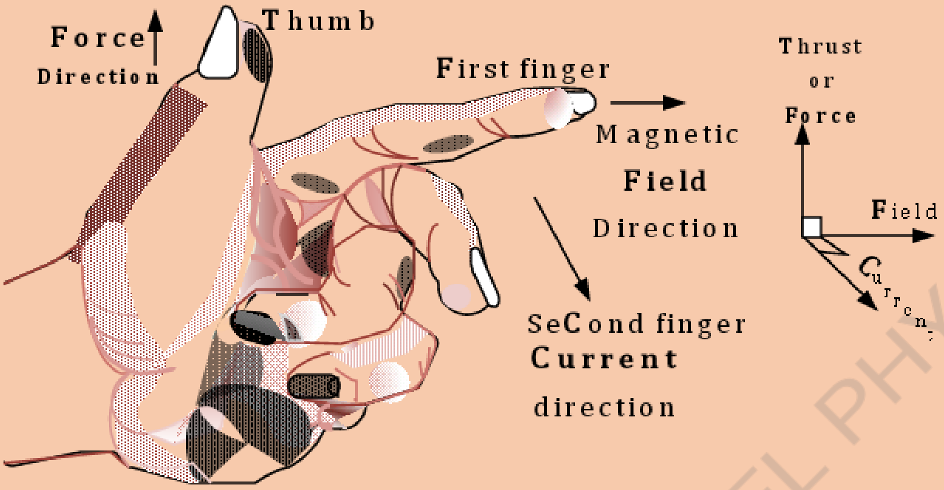


NB: *The magnetic flux density at the centre of a solenoid is approximately uniform, and is maximum at a value* $B = \mu_o n I$ *and this value progressively reduces as distance, x, from the centre O increases.*
At the extreme positions L and R of the solenoid, the value of the magnetic flux density reduces to half the value at centre O, on each side, hence,

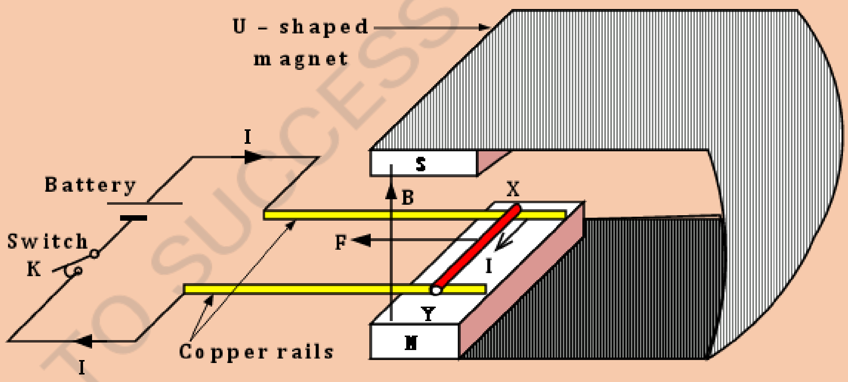
$$B_L = B_R = \frac{1}{2}(\mu_o n I)$$

Fleming's Left Hand Rule: (Often called the Motor rule)

States that – Whenever a current carrying conductor is placed perpendicularly across a magnetic field, it will experience a magnetic force F = BIL, *that acts in the direction of the* ***Thumb*** *of the left hand, while the first and second. Fingers, placed perpendicular to each other and whose plane is normal to the thumb, represent Magnetic Field,* ***B*** *and current* ***I*** *respectively, as shown in the figure below.*



FORCE ON A STRAIGHT CONDUCTOR CURRYING CURRENT IN A MAGNETIC FIELD
When a conductor carrying current is placed in an external magnetic field (produced by a permanent magnet for example), it experiences a force that will move it, if its free to do so. This can be demonstrated using the apparatus shown below. A metal rod is placed across metal rails which lie between the poles of a horse shoe (U – shaped) magnet, as shown on the diagram below.



When the switch K is closed, a current **I** is passed through the rod, XY in the direction, X towards Y. By Flemings left hand rule, a magnetic force F = BIL acts on the rod XY, acting from right to left, hence causing it to roll from the right hand side along the raids towards the left hand side direction as shown. The direction of force and hence the movement of the metal rod was predicted by Flemings left hand rule.

FACTORS AFFECTING THE SIZE OF THE FORCE:

Simple experiments show that, the magnitude of the force (F) on a wire carrying a current in a magnetic field depends on **four** basic factors as follows and the magnitude of this force can be investigated by measuring the angle θ, of swing of the wire as shown on the diagram that follows:

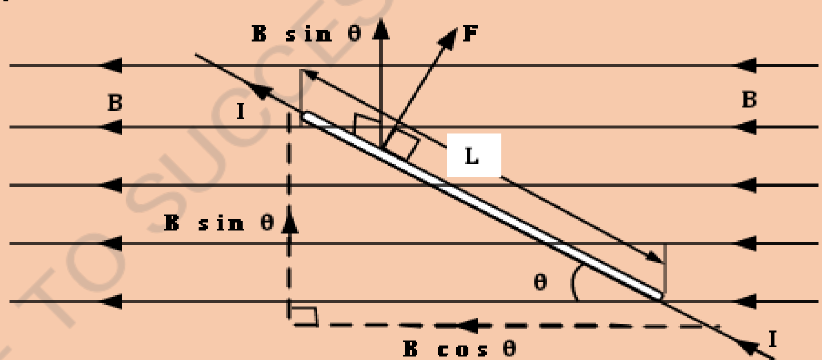
1. **The magnetic field strength often called the magnetic flux density, B.**

   The greater the magnetic field strength, the greater the force experienced by a current carrying wire and F is proportional to B. i.e. (**$F \propto$** B)

2. **The size of the current I, flowing through the wire (or conductor).**
   The greater the current flowing through the conductor the greater the force exerted on the current carrying wire i.e. (**$F \propto$** I)

3. **The length, L, of the conductor, within and across the magnetic field.**
   The greater the length, L is the greater the force i.e. ($F \propto L$).

4. **The angle θ of inclination of the conductor to the direction of the magnetic field**.
   Only the ***component of the magnetic field perpendicular to the current*** or ***vice versa,*** produces the magnetic force on the conductor, but NOT the component parallel to the current or magnetic field. It can be shown that i.e. (**$F \propto$** $B \sin\theta$)
   Or (**$F \propto$** $I \sin\theta$) as shown on the diagram below. A conductor parallel to direction of the magnetic field experiences **no magnetic force.**
   The converse is thus true, i.e. **maximum force** is experienced by the conductor when,
   $\theta = \pi/2$ or **90°** (i.e. The conductor is at right angles to the magnetic field) and becomes **zero** when, **θ = 0°.**



Combining all the four factors given above, together.
$F \propto BIL\sin\theta$
or $F = kBIL\sin\theta$ where, k = constant, and experiments show that, k = 1
$\therefore$ **$F = BIL\sin\theta$**
Explanations:
The magnetic force experienced by an electron within the conductor is given by
$F = Bev$ where v is the average drift velocity of the electrons.
Each electron interacts with one magnetic field line.

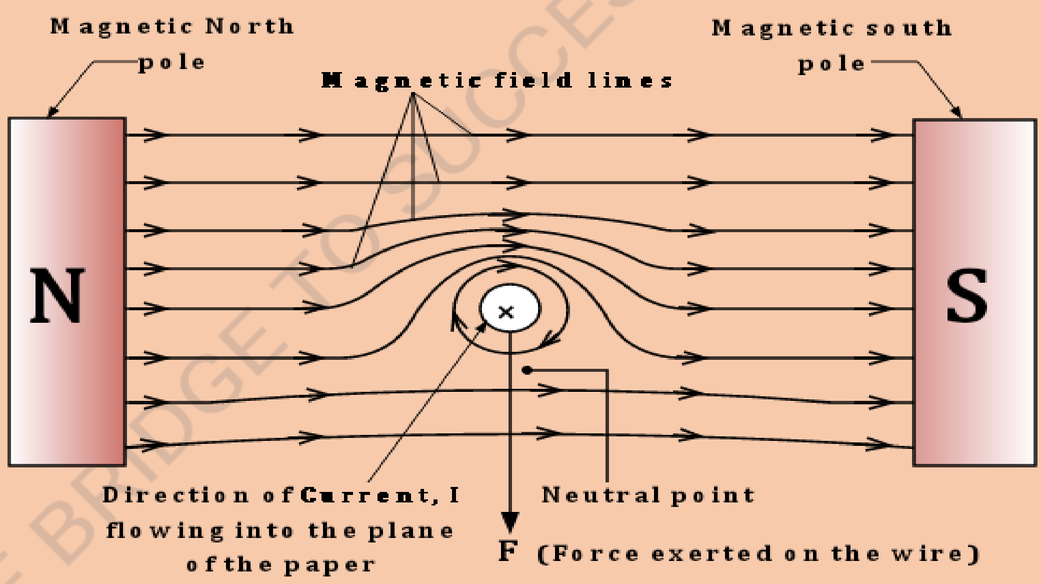
- *When the magnetic field strength is increased, the number of electrons interacting with the field increases so the total force on the conductor as a whole therefore increases.*

- *From I = nevA, I ∝ n , thus when the current flowing in the conductor increases, the number of electrons increases, and since each experiences a force, F = Bev, so the total force on the conductor as a whole therefore increases.*

- *Increasing the length, L of the conductor within the magnetic field, implies increasing the number of conducting electrons in the region of the magnetic field and since each experiences a force, F = Bev, so the total force on the conductor as a whole therefore increases.*

- *When the conductor is parallel to the magnetic field Fleming's left hand rule doesn't hold so electrons do not experience a magnetic force. When the angle of tilt θ is increased, the component of length L across the field increases, since L∝F, the force on the conductor increases reaching a maximum value when θ = 90° since the* $\sin 90° = 1$, from the expression, $F = BIL\sin\theta \Longrightarrow F \propto \sin\theta$ when all the other factors are kept constant

Explanation why a conductor carrying a current across a magnetic field experiences a magnetic force.



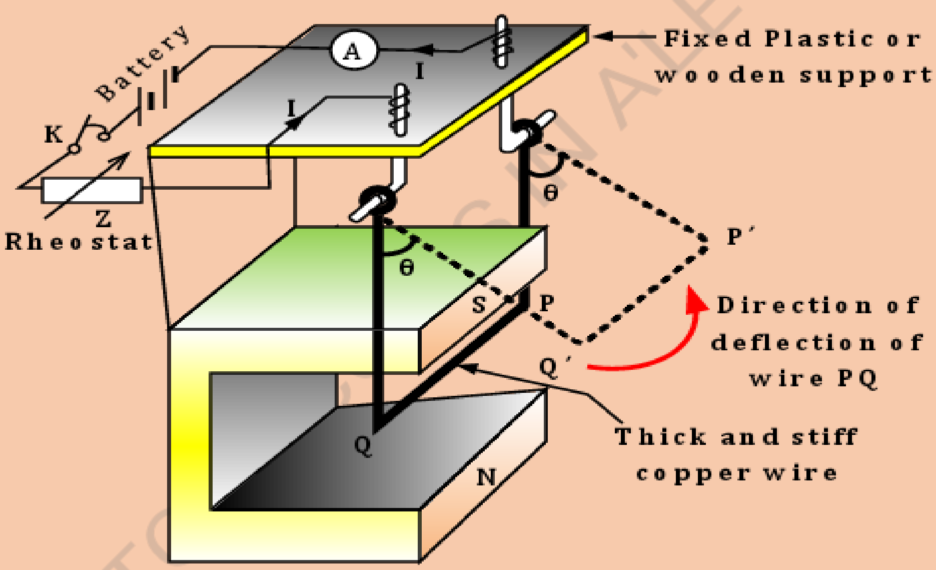
- *The wire carrying a current, produces a magnetic field around itself in a direction given by the right hand grip rule.*
- *The magnetic field of the wire interacts with the external magnetic field, B, provided by the pole pieces of the magnet.*

- *The magnetic fields interact and reinforce each other above the wire creating a*

Stronger resultant magnetic field.

- *On the other hand, a weaker resultant magnetic field is created below the wire and a neutral point is also created below the wire where two magnetic fields are equal in magnitude but opposite in directions.*
- *The wire then experiences a resultant magnetic force F = BIL acting vertically downwards Form the region of stronger magnetic field towards the region of weaker magnetic field, as shown on the diagram above.*
- *When the wire is light and free to move, this may cause motion of the wire along the direction of the force.*

Experiment to investigate some of the factors affecting the size of force exerted on a wire PQ carrying a current across a magnetic field.



- Using the set up shown above, using length PQ = L, switch K is closed, and wire PQ kicks off to the right, and the maximum angle θ_0 of deflection is noted. The switch is then opened and the wire PQ left to settle down without movement.
- Current I is increased using a rheostat Z, then switch K is closed, the new deflection θ_1 is noted. It is observed that, $\theta_1 > \theta_0$. ***This ⇒ magnetic force on wire PQ increases with increase in current flowing through the wire.***
- Bar magnets of the same size are then added on each pole of the **"C"** shaped magnet, and the experiment is repeated and the new deflection, θ_2. It is observed that, $\theta_2 > \theta_0$. ***This ⇒ magnetic force on wire PQ increases with increase in the magnetic field strength, B, across the wire.***
- The **length, L** of wire PQ, is **increased** and the experiment is repeated and the new

deflection, θ_3 is noted. It is observed that, $\theta_3 > \theta_o$. ***This ⇒ magnetic force on wire PQ increases with increase in length L of the wire, across the field.***

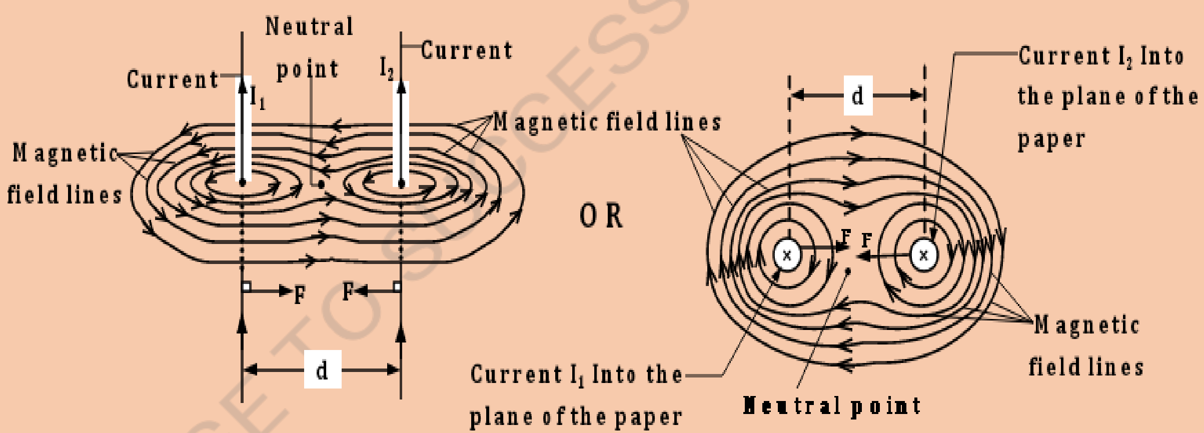
- The orientation, **β** of the wire PQ to the magnetic field, is reduced where **β < 90°** and the experiment is repeated. The new deflection θ_4 is noted. It is observed that, $\theta_4 < \theta_o$. ***This ⇒ magnetic force on wire PQ decreases with decrease in angle β of inclination of the wire to the magnetic field.***

Magnetic Forces experienced by current carrying wires:

When a current currying wire is placed across a magnetic field provided by another current carrying wire, the two magnetic fields interact creating regions of reinforced magnetic fields and reduced or cancelled out magnetic fields. This causes the wires to experience magnetic forces as demonstrated by the following set ups.

1. Two straight parallel wires W_1 and W_2 carrying currents I_1 and I_2 in the same direction.

(i) Two wires carrying currents upwards (ii) Two wires carrying currents perpendicularly into the plane of the paper.



Magnetic flux density at a perpendicular distance, d, at the position of wire 2 due to wire W_1 is given by $B_1 = \frac{\mu_0 I_1}{2\pi d}$..............................**(i)**

Similarly**,** magnetic flux density at a perpendicular distance, d, at the position of wire 1 due to wire W_2 is given by $B_2 = \frac{\mu_0 I_2}{2\pi d}$............**(ii)**

Thus, by Fleming's left hand rule, a magnetic force,

$F_1 = B_2 I_1 L$ and substituting for B_2 from equation (ii) we obtain;

$F_1 = B_2 I_1 L = \frac{\mu_0 I_2}{2\pi d} I_1 L$ **where L is the length of each wire.**

$\therefore F_1 = \left(\frac{\mu_0 I_2 I_1 L}{2\pi d}\right)$ **acting towards W_2 ....(iii)**

Likewise, by Fleming's left hand rule, a magnetic force, on wire W_2,

$F_2 = B_1 I_2 L$ **and substituting for B_1 from equation (i) we obtain;**

$F_2 = B_1 I_2 L = \frac{\mu_0 I_1}{2\pi d} I_2 L$ **where L is the length of each wire.**

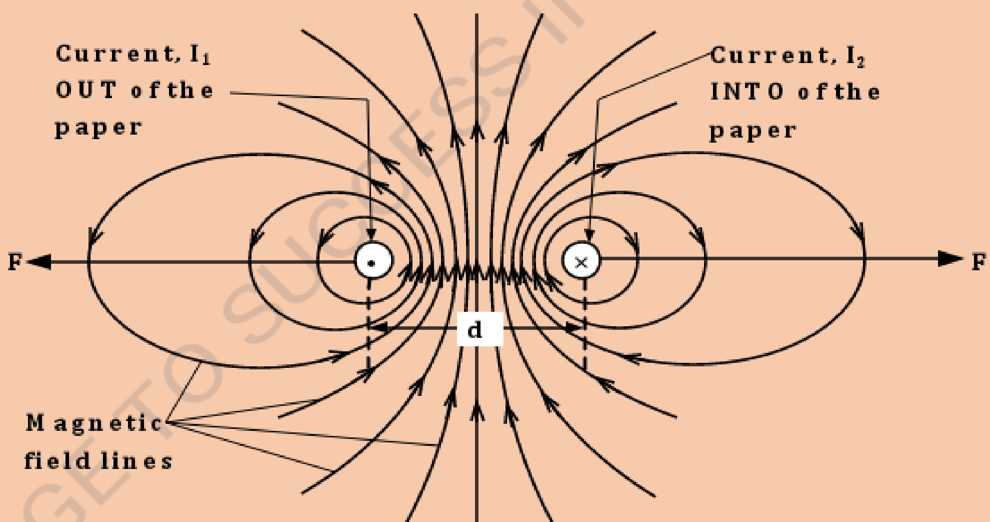
$\therefore F_2 = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$ **acting towards W_1 ....(iv)**

NB: From equations (iii) and (iv), the two wires have the same magnitude of force, **i.e.**

$$|F_1| = |F_2| = F = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$$

and i ... res together, towards each other.

2. Two straight parallel wires W_1 and W_2 carrying currents I_1 and I_2 in opposite directions.



Magnetic flux density at a perpendicular distance, **d**, at the position of wire W_2 due to wire $\mathbf{W_1}$ is given by $B_1 = \frac{\mu_0 I_1}{2\pi d}$ **...(i)**

Similarly, magnetic flux density at a perpendicular distance, d, at the position of wire 1 due to wire W_2 is given by $B_2 = \frac{\mu_0 I_2}{2\pi d}$ **...............................(ii)**

Thus, by Fleming's left hand rule, a magnetic force,

$F_1 = B_2 I_1 L$ **acting to the left of W_1 and sub.for B_2 from eqn. (ii), we obtain;**

$F_1 = B_2I_1L = \frac{\mu_0 I_2}{2\pi d} I_1L$ **where L is the length of each wire.**

$\therefore F_1 = \left(\frac{\mu_0 I_2 I_1 L}{2\pi d}\right)$ **acting away from W_2 i.e. to the left of W_1 ....(iii)**

Likewise, by Fleming's left hand rule, a magnetic force, on wire W_2,

$F_2 = B_1I_2L$ **and substituting for B_1 from equation (i), we obtain;**

$F_2 = B_1I_2L = \frac{\mu_0 I_1}{2\pi d} I_2L$ **where L is the length of each wire.**

$\therefore F_2 = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$ **acting to the Right of W_2 i.e away from W_1...(iv)**

***NB: From equations (iii) and (iv), the two wires have the same magnitude of force*, i.e.**

$$\boxed{|F_1| = |F_2| = F = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)}$$

and is a repulsive force pushing wires apart or away from each other.

3. **The definition of an ampere.**

From the expression, $F = \left(\frac{\mu_0 I_1 I_2 L}{2\pi d}\right)$, when the two parallel wires above, each of length, ***one metre***, placed ***one metre apart*** in ***free space***, and if each is to carry a current of ***one ampere***, then substituting, $\mu_0 = 4\pi\times10^{-7}\ Hm^{-1}$, $I_1 = 1\ A$, $I_2 = 1\ A$, $d = 1\ m$ **and** $L = 1\ m$ then,

$F = \left(\frac{4\pi\times10^{-7}\times 1.0\times 1.0\times1.0}{2\pi\times1.0}\right) = 2.0\times10^{-7}N$, then an ampere is defined as follows:

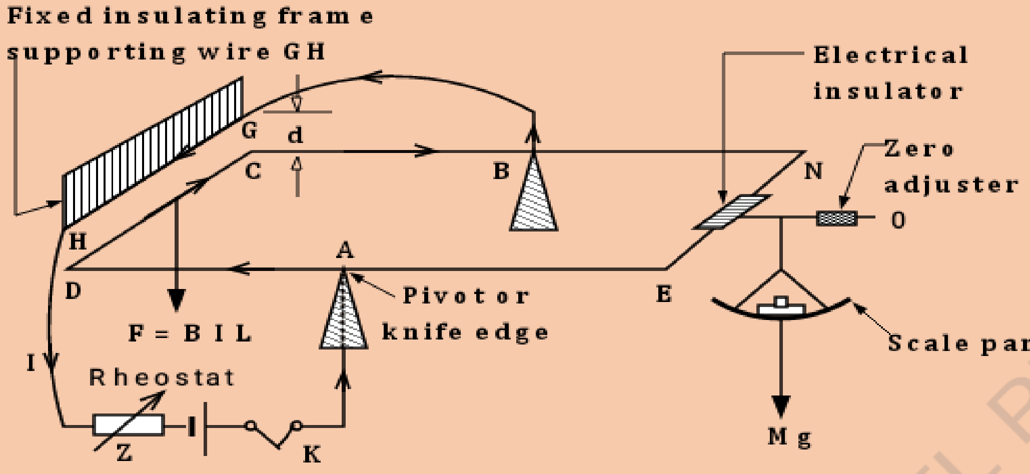
Definition:

An *ampere* – *is the steady or constant current which when flowing in each of the two straight and parallel wires of negligible cross sectional area, placed one metre apart in a vacuum, exert a force of* $2.0\times10^{-7}N$ *per metre of each other's length.*

4. Absolute measurement of current using a current balance:

The method is called **absolute**, because its accuracy is derived from the measurement based on some of the **basic quantities** like length, **L** , mass **M** and probably time **T** that do not require calibration of the instrument used in the measurement, using a slide wire potentiometer.

Diagram: of a simple current Balance.



- The apparatus is set up as shown on the diagram above.
- DCNE is a conducting copper wire frame such that **AD = AE**.
- When switch K is open, i.e. with no current flowing in the circuit, the zero screw (adjuster) is adjusted until the rigid, rectangular copper wire frame CDEN balances horizontally.
- The switch, K, is then closed and the current flows through the two parallel wires CD and GH in opposite directions, causing wire CD to be repelled vertically downwards.
- Small masses are carefully added into the scale pan until the horizontal wire frame CDEN balances horizontally again.
- The total weight Mg, in the scale pan is noted, and recorded down.
- The length L of wire CD is measured using a metre rule and recorded down.
- The distance, d, of separation of the wires CD and GH is also measured using a travelling microscope and recorded down.

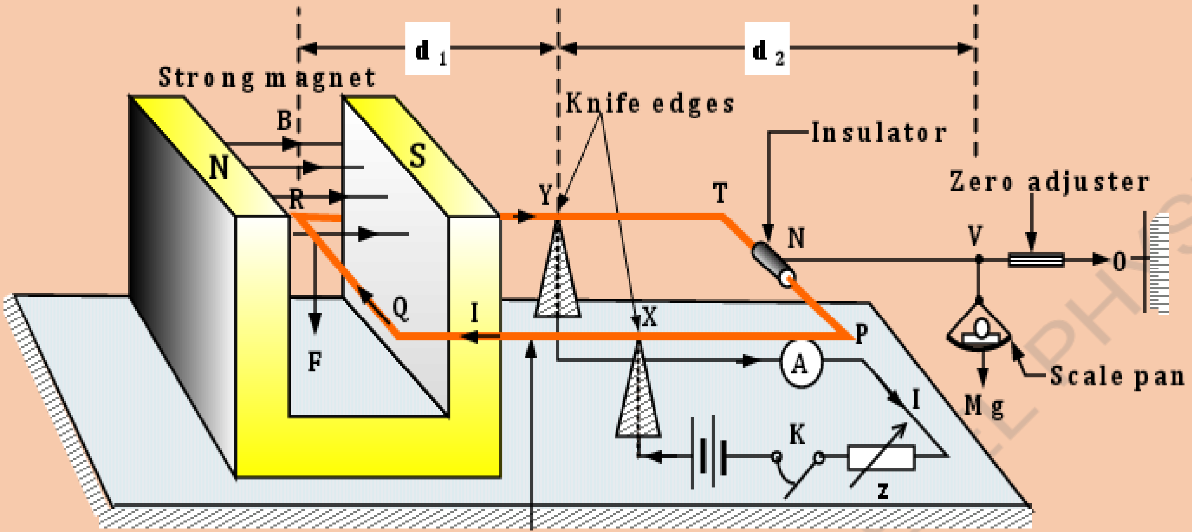
  Now since practically the distance, **AE = AD**, and **CD = *L***
- The value of the current I is calculated from the expression,

$$I = \sqrt{\frac{2\pi\, mgd}{\mu_0 L}} \qquad \text{where, } \mu_0 = 4\pi \times 10^{-7}\,\mathrm{Hm^{-1}}$$

NB: Other versions of current balances do exist but use the same concept for their operations.

This knowledge can be used to determine the size of the magnetic flux density B, provided by a given source of magnetic field. For example the magnetic flux density, B, between the pole pieces of a U – shaped magnet can be determined, the magnetic flux density at the centre of a plane circular coil and at the centre of a solenoid can similarly be determined.

5. Magnetic flux density between the pole pieces of a U – shaped magnet.



- The experiment is set up as shown on the diagram above.
- With switch K open, the rigid rectangular copper wire frame PQRT is made horizontal using the zero adjuster.
- Using a suitable setting of the rheostat Z, switch K is then closed, such that a current **I** flows through the wire QR normal to the magnetic field, B, in such a direction to cause wire QR move vertically downwards. The setting of Z may be altered until wire QR registers a reasonable downward force.
- Small weights are added into the scale pan until the wire frame PQRT balances horizontally.
- The ammeter reading **I** is then noted.
- The total weight **Mg** in the scale pan is also noted.
- The length **L** = QR of the part of the wire between the pole pieces of the magnet, is measured using a metre rule and recorded down.
- Taking the distances RY = YT = NV as known values provided on the instrument or that can be measured, the magnetic flux density, **B** is calculated from,

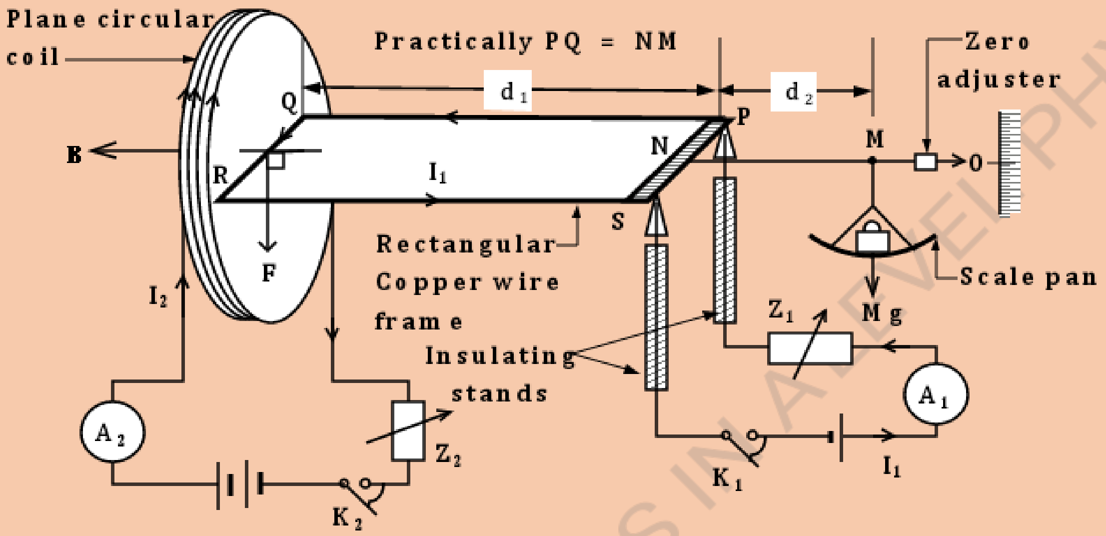
BIL× RY = Mg × (YT + NV) where RY = d_1 and (YT + NV) = d_2

$$\Rightarrow B = \left(\frac{Mg \times d_2}{d_1 I L}\right) \text{ assuming } d_1 = d_2 \text{, then, } B = \left(\frac{Mg}{I L}\right)$$

NB: In order to investigate effect of length, L on the force F exerted on the wire carrying a current in a magnetic field, a number of equally strong U – shaped magnets are placed in series so that the varying length, of the wire is between the adjacent pole pieces of the magnets.

6. Measurement of magnetic flux density at the centre of a plane circular coil carrying a current.

Diagram:



- The experiment is set up as shown on the diagram above.
- With switches K_1 and K_2 both open, the rigid rectangular copper wire frame PQRS is made horizontal using the zero adjuster.
- Using suitable settings of the rheostats Z_1 and Z_2, switches K_1 and K_2 are then closed, such that a current $\mathbf{I_1}$ flows through the wire QR normal to the magnetic field, **B**, in such a direction as to cause wire QR to move vertically downwards.
- The setting of Z_1 may be altered until wire QR registers a reasonable downward force.
- Small weights are then added into the scale pan until, the wire frame PQRS again balances horizontally.
- The ammeter reading $\mathbf{I_1}$ is then noted.
- The total weight **Mg** in the scale pan is also noted.
- The length **L** = QR of the part of the wire between the pole pieces of the magnet, is measured using a metre rule and recorded down.
- Taking the distances **PQ = NM** as known values provided on the instrument or that are measured using a metre rule, the magnetic flux density, **B** is calculated from,

$$BI_1L \times PQ = Mg \times NM \text{ where } PQ = d_1 \text{ and } NM = d_2$$

$$\Rightarrow B = \left(\frac{Mg \times d_2}{d_1 I_1 L}\right)$$

NB: The same principles are used for measuring the magnetic flux density at the centre of a long solenoid.

7. Factors affecting the size of the magnetic flux density at the centre of a plane circular coil.

The factors include the following:

(i) The number of turns N of the coil carrying a current I.

Magnetic flux density increases with increase in the number of turns of the coil. i.e. $B \propto N$

(ii) The current, I, flowing through the coil.

Magnetic flux density increases with increase in the current I flowing through the coil. i.e. $B \propto I$

(iii) The radius r, of the plane circular coil.

Magnetic flux density increases with a decrease in the radius of the coil. i.e.

$B \propto \frac{1}{r}$

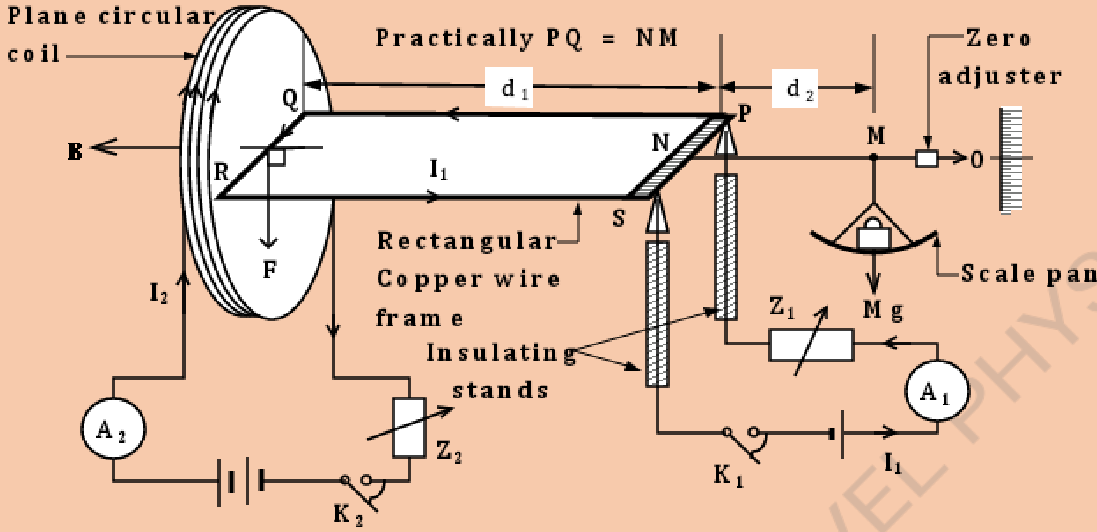
(iv) The magnetic permeability μ of the medium in the region of the coil.

Magnetic flux density increases with increase in the strength of the magnetic permeability threading the plane of the coil. i.e. $B \propto \mu$

8. Variation of the factors affecting Magnetic flux density at the centre of a plane circular coil.

(a) Variation of Magnetic flux density at the centre of a plane circular coil, with the number of turns, N of the coil.

- A plane circular coil of known number of turns N is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2.
- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame PQRS is made horizontal.



- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1. Current through the coil is also adjusted using rheostat Z_2 ¬ until a reasonable downward deflection on wire QR is registered.

- The ammeter reading I_2 is noted.

- Small weights are added into the scale pan until the wire frame **PQRS** balances horizontally.

- The length **L** of wire QR is measured using a metre rule and the total weight **Mg** in the scale pan is noted. i.e $B I_1 L d_2 = Mg\, d_1$, where, $B = \dfrac{\mu_0 N I_2}{2r}$

$$B I_1 L d_2 = Mg\, d_1 \Rightarrow B = \frac{\mu_0 N I_2}{2r} = \frac{Mg\, d_1}{2 r I_1 L d_2} \Rightarrow N \propto Mg$$

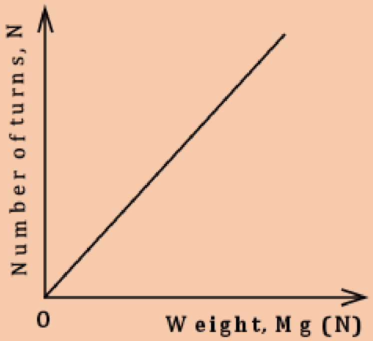
- Keeping values of currents I_1 and I_2 constant, the experiment is **repeated** using different increasing number, **N** of turns of the coil each time and in each case, the total weight Mg in the scale pan is noted.

- The results are tabulated in a suitable table including values of N and Mg.

- A graph of N against Mg is then plotted and gives a straight line through the origin. i.e. $\Rightarrow N \propto Mg$ and since, $B \propto Mg \Rightarrow B \propto N$

(i) Variation of number of turns N, with weight, Mg

(ii) Variation of Magnetic flux density, B with number of turns N



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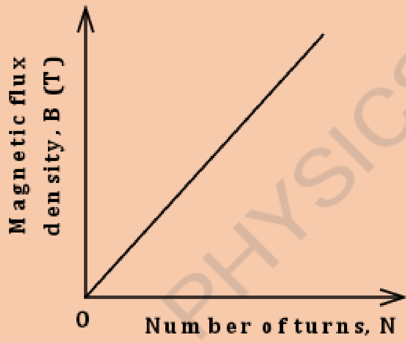
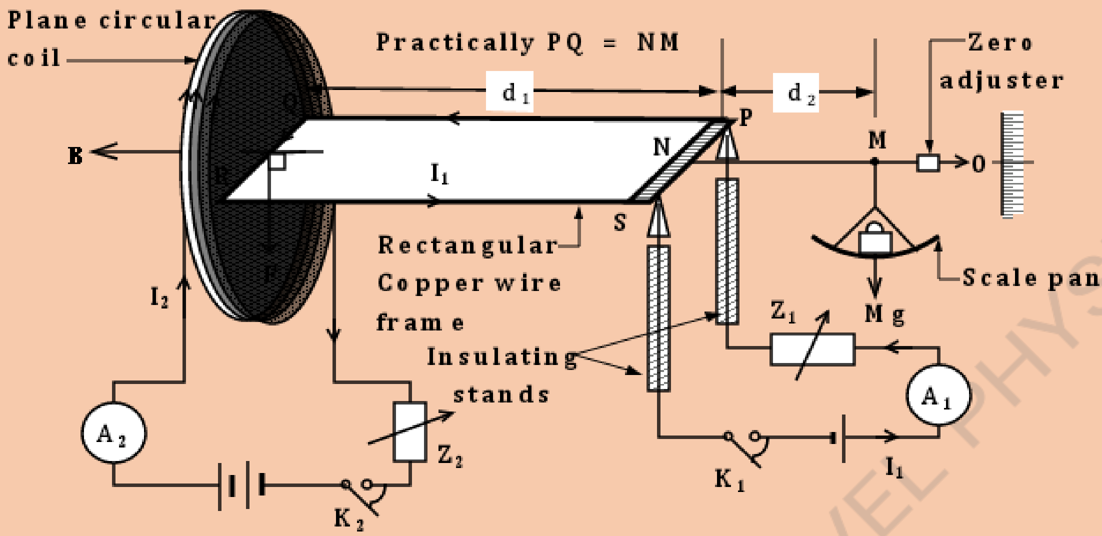


$\Rightarrow N \propto mg$ but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow mg \propto B$

$\therefore B \propto N$ i.e. magnetic flux density varies directly with the number of turns of the coil.

(b) Variation of Magnetic flux density at the centre of a plane circular coil, with the Current, I flowing through the coil.

- A plane circular coil of known number of turns N is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2.
- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame PQRS is made horizontal.
- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1. Current through the coil is also adjusted using rheostat Z_2 until a reasonable downward deflection on wire QR is registered.
- The ammeter reading I_2 is noted.
- Small weights are added into the scale pan until the wire frame **PQRS** again balances horizontally.
- The length **L** of wire QR is measured, using a metre rule and the total weight **Mg** in the scale pan is noted. i.e $B I_1 L d_2 = Mg\, d_1$, where, $B = \dfrac{\mu_0 N I_2}{2r}$
- $B I_1 L d_2 = Mg\, d_1 \Rightarrow B = \dfrac{\mu_0 N I_2}{2r} = \dfrac{Mg\, d_1}{2 r I_1 L d_2} \Rightarrow I_2 \propto Mg$



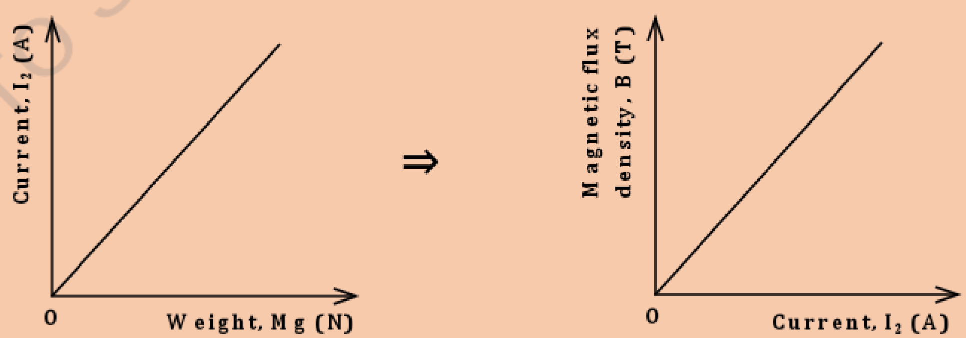
- Keeping values of currents I_1, r, d_1, d_2, L and N constant, the experiment is **repeated,** using different increasing values of current, I_2 flowing through the coil each time and in each case, adjusted using rheostat, Z_2 , the total weight Mg in the scale pan is noted.

- The results are tabulated in a suitable table including values of I_2 and Mg.

- A graph of I_2 against Mg is then plotted and gives a straight line through the origin. i.e. $\Rightarrow I_2 \propto$ Mg and since, $B \propto Mg \Rightarrow B \propto I_2$

(i) Variation of current, I_2 flowing in the coil with weight, Mg.

(ii) Variation of Magnetic flux density, B with I_2 flowing in the coil.



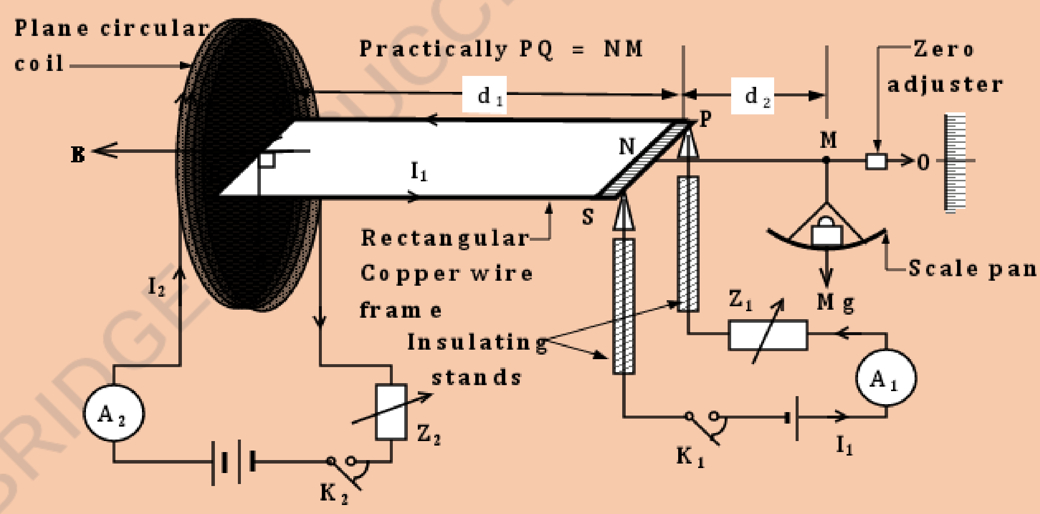
$\Rightarrow I_2 \propto mg$ but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow mg \propto B$

$\therefore B \propto I_2$ i.e. magnetic flux density varies directly with the I_2 flowing through the coil.

(c) Variation of Magnetic flux density, B, at the centre of a plane circular coil, with the radius, r , of the coil.

- A plane circular coil of known number of turns N, is connected to a d.c source via a rheostat Z_2 and an ammeter, A_2.
- When switches K_1 and K_2 are both open, the plane of the rigid rectangular copper wire frame PQRS is made horizontal.
- Switches K_1 and K_2 are then closed and current I_1 through wire QR is noted on ammeter A_1. Current through the coil is also adjusted using rheostat Z_2 until a reasonable downward deflection on wire QR is registered.
- The ammeter reading I_2 is noted.
- Small weights are added into the scale pan until the wire frame **PQRS** again balances horizontally.
- The length **L** of wire QR is measured, using a metre rule and the total weight **Mg** in the scale pan is noted. i.e $B I_1 L d_2 = Mg\, d_1$, where, $B = \dfrac{\mu_0 N I_2}{2r}$

$$B I_1 L d_2 = Mg\, d_1 \Rightarrow B = \frac{\mu_0 N I_2}{2r} = \frac{Mg\, d_1}{2 r I_1 L d_2} \Rightarrow \frac{1}{r} \propto \mathbf{Mg}$$



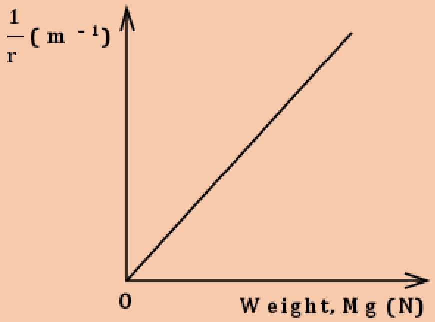
- Keeping values of currents I_1, I_2, d_1, d_2, L and N all constant, the experiment is **repeated,** using different samples of the coils of **the same wire** but of **increasing radii**, **r**, of the coil each time and in each case, the coil is connected to the circuit, and the total weight Mg in the scale pan is noted.

- The results are tabulated in a suitable table including values of, **r,** $\frac{1}{r}$ and Mg.
- A graph of $\frac{1}{r}$ against Mg is then plotted and gives a straight line through the origin.

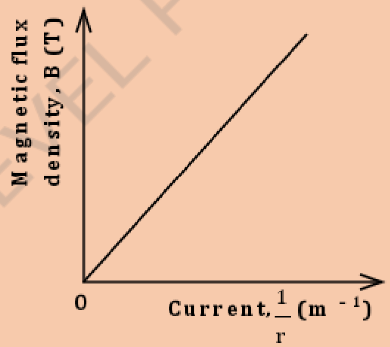
i.e. $\Rightarrow \frac{1}{r} \propto$ Mg and since, $B \propto Mg \Rightarrow B \propto \frac{1}{r}$

(i) Variation of $\frac{1}{r}$ with weight, Mg.

(ii) Variation of Magnetic flux density, B with $\frac{1}{r}$



$\Rightarrow$



$\Rightarrow \frac{1}{r} \propto$ mg but, assuming, $d_2 = d_1$, $Mg = F = BI_1L \Rightarrow mg \propto \frac{1}{r}$

$\therefore B \propto \frac{1}{r}$ i.e. ***magnetic flux density varies inversely with the radius, r*** of the coil.

Examples:

1. (a) Write down the expressions for the magnetic flu density due to each of the following current – carrying conductors placed in air.

(i) At a perpendicular distance, d, due to a straight wire carrying a current I, in air.

(ii) A plane circular coil of N – turns and of radius R, each carrying a current I.

(iii) A long solenoid, of n – turns per metre carrying a current I.

(b) Two straight and parallel wires W_1 and W_2 each of length 0.25 m are carrying currents of 2A and 5A respectively and are separated by a distance of 10.0 cm in air. Determine the;

(i) Position from W_1 for which the resultant magnetic flux density is zero.

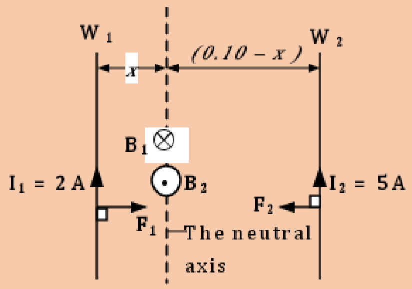
(ii) Magnetic flux density mid-way between the wires.

(iii) Magnetic force exerted by W_1 on W_2

Solutions:

(a) (i) $B = \dfrac{\mu_0 I}{2\pi d}$ (ii) $B = \dfrac{\mu_0 NI}{2R}$ (iii) $B = \mu_0 nI$

(b) *(i)*



Let the neutral axis be a distance, x from wire W1,

$\therefore B_1 = \dfrac{\mu_0 I_1}{2\pi x}$ into the paper while

$B_2 = \dfrac{\mu_0 I_2}{2\pi(0.10-x)}$ out of the paper

$$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{x} - \frac{I_2}{(0.10-x)}\right) = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{2}{x} - \frac{5}{(0.10-x)}\right] \text{ but } B = 0$$

$$\frac{4\pi \times 10^{-7}}{2\pi}\left(\frac{2}{x} - \frac{5}{(0.10-x)}\right) = 0 \Rightarrow \frac{2 \times 10^{-7}}{x}\left[\frac{2(0.10-x)-5x}{(0.10-x)}\right] = 0$$

$$\Rightarrow x = \frac{4.0 \times 10^{-8}}{14.0 \times 10^{-7}} = 2.86 \times 10^{-2} m$$

Hence, the neutral point is 2.86 cm from wire W¬1

(ii) Mid – way between the wires, $\mathbf{x = (d-x) \Rightarrow x = \frac{d}{2} = \frac{0.10}{2} = 0.05m}$

$$\therefore B_1 = \frac{\mu_0 I_1}{2\pi x} = \frac{4\pi \times 10^{-7} \times 2.0}{2\pi \times (0.05)} = 2.0 \times 10^{-6} T, \text{ into the paper}$$

$$\text{and } \therefore B_2 = \frac{\mu_0 I_2}{2\pi x} = \frac{4\pi \times 10^{-7} \times 5.0}{2\pi \times (0.05)} = 2.0 \times 10^{-5} T, \text{ out of the paper}$$

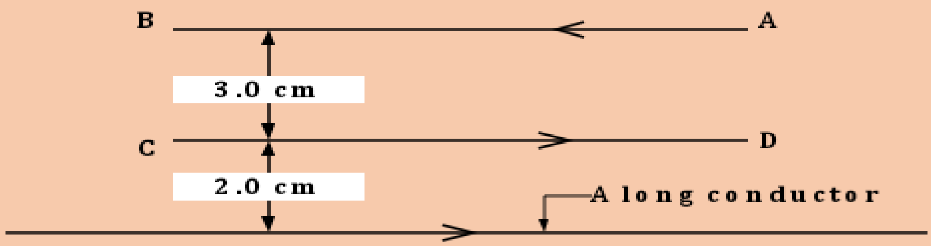
Hence, the magnetic flux density mid-way the wires W1 and W2 is $\therefore \mathbf{B = (B_2 - B_1) = (2.0 \times 10^{-5} - 2.0 \times 10^{-6}) = 1.80 \times 10^{-5} T}$ *acting* ***perpendicularly out*** *of the plane* ***of the paper****.*

(iii) Let $F_1 = B_2 I_1 L = \dfrac{\mu_0 I_1 I_2 L}{2\pi d} = \dfrac{4\pi \times 10^{-7} \times 2.0 \times 5.0 \times 0.25}{2\pi \times (0.10)}$

$\therefore \mathbf{F_1 = 5.0 \times 10^{-6} N}$ **acting from W_1 towards W_2.**

2. The figure below shows two wires **AB** and **CD** each of length 5.0 cm and each carrying of 10.0 A in the directions shown. A long conductor carrying a current of

15.0A is placed parallel to the wire **CD**, 2.0 cm below it.



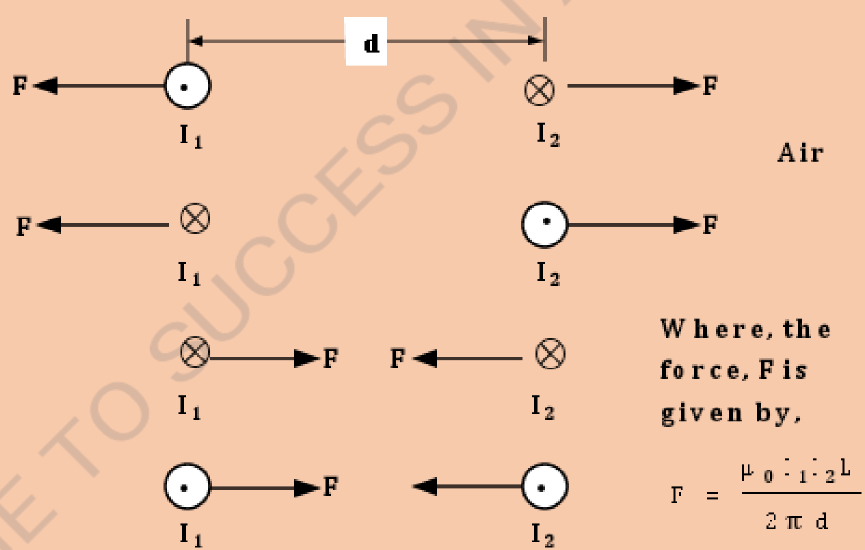
(i) Calculate the net force on the long wire.

(ii) Sketch the magnetic field pattern between the long wire and wire **CD** after removing wire **AB**. Use the field pattern to define a neutral point.

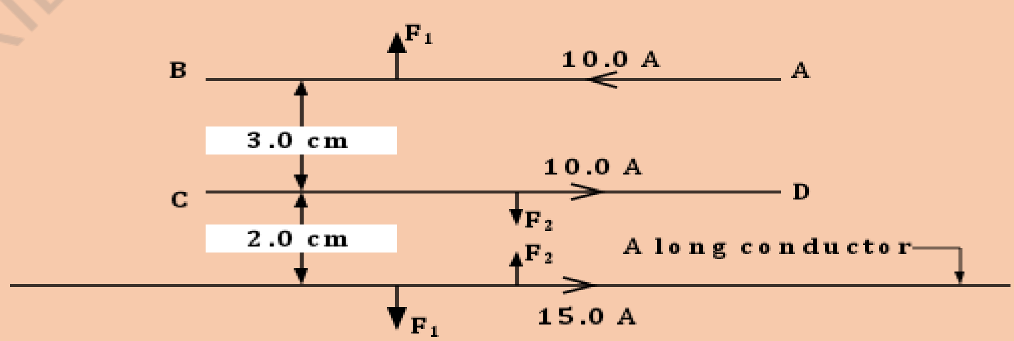
Solution:

NB: Two parallel wires carrying currents in opposite directions exert repulsive forces on each other, but of equal magnitudes.

Two parallel wires carrying currents in same directions exert attractive forces on each other, but of equal magnitudes as summarized by the diagram below.



(i)



Let F_1 be the force on the long wire due to current flowing in the wire **AB**

Let **F2** be the force on the long wire due to current flowing in the wire **CD**

$$F_1 = \frac{\mu_0 I_1 I_3 L}{2\pi d} = \frac{4\pi \times 10^{-7} \times 10.0 \times 15.0 \times 0.05}{2\pi \times (0.05)}$$

$\therefore F_1 = 3.0 \times 10^{-5}$ **N vertically downwards.**

$$F_2 = \frac{\mu_0 I_2 I_3 L}{2\pi d} = \frac{4\pi \times 10^{-7} \times 10.0 \times 15.0 \times 0.05}{2\pi \times (0.02)}$$

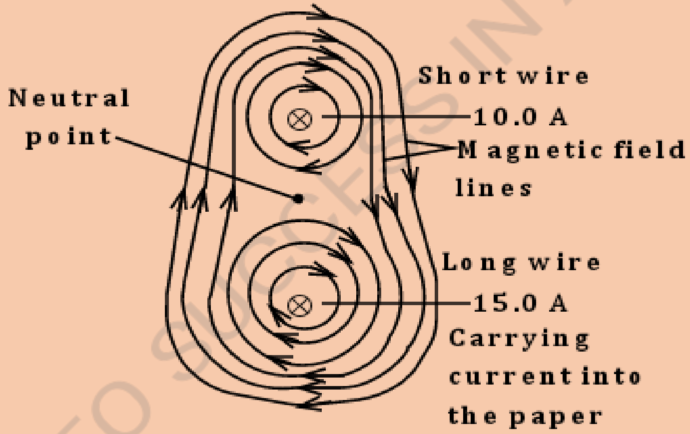
$\therefore F_1 = 7.5 \times 10^{-5}$ **N vertically upwards**

Thus the resultant force, $\therefore F = F_1 - F_2 = (7.5 - 3.0) \times 10^{-5}$

$\therefore F = 4.5 \times 10^{-5}$ **N vertically upwards**

(ii) Since wire CD and the long wire carry currents in the same direction, the magnetic field pattern has the shape below.

NB: Since the long wire carries a much bigger current than wire CD, the long wire produces more magnetic field lines than wire CD, and the neutral point lies closer to wire CD than the long wire.



The neutral point – is the region between the two wires where two magnetic fields are equal in magnitudes but opposite in directions and the resultant magnetic field is zero.

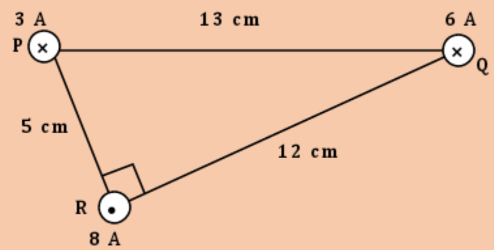
3. (a) Define the following terms:

(i) Magnetic variance.

(ii) Magnetic meridian

(b) (i) Define an ampere.

(ii) Three parallel wires P, Q and R each of length 0.8 m carrying currents of 3.0 A, 6.0A and 8.0 A respectively, are arranged at the corners of a triangle PQR of sides 5.0 cm, 12.0 cm and 13.0 cm as shown below.



Determine the resultant force experienced by wire Q.

(c) Describe an experiment to measure current accurately.

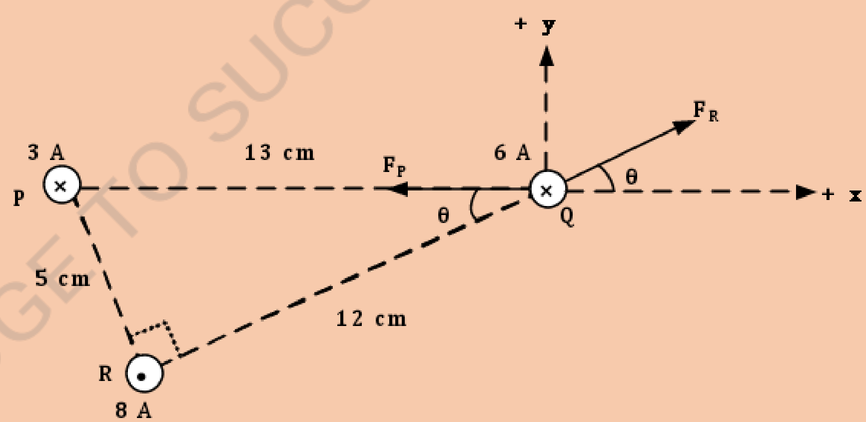
Solutions:

(a) (i) ***Magnetic variance –*** is the angle between the Earth's Magnetic and Geographic meridians.

(ii) ***Magnetic meridian*** – is a vertical plane in which a freely suspended Bar magnet sets and contains the earth's magnetic poles.

(b) (i) The **ampere** – is the steady or constant current which when flowing through each of the two straight, parallel and infinitely long wires of negligible cross-sectional area separated by a distance of 1m apart in a vacuum exert a force of $2\times10^{-7}Nm^{-1}$ on each wire.

(ii) Let F_P and F_R be the magnetic forces on the charge at Q due to charges at points P and R respectively.
NB: The triangle of charges is a right angled Δ.



$$\sin\theta = \frac{5}{13} \quad \text{or} \quad \cos\theta = \frac{12}{13} \quad \text{or} \quad \tan\theta = \frac{5}{12} \Rightarrow \theta = 22.6°$$

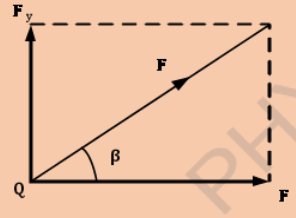
$$F_P = BI_qL_q = \left(\frac{\mu_0 I_P}{2\pi a}\right)I_qL_q = \frac{4\pi\times10^{-7}\times3\times6\times0.8}{2\pi\times0.13} = 2.22\times10^{-5}N$$

$$F_R = BI_qL_q = \left(\frac{\mu_0 I_R}{2\pi a}\right)I_qL_q = \frac{4\pi\times10^{-7}\times6\times8\times0.8}{2\pi\times0.12} = 6.40\times10^{-5}N$$

$$\uparrow F_y = F_R \sin\theta = 6.40\times10^{-5}\times\frac{5}{13} = 2.46\times10^{-5}\text{N}$$

$$\rightarrow F_x = F_R\cos\theta - F_p = \left(6.40\times10^{-5}\times\frac{12}{13}\right) - 2.22\times10^{-5}$$

$$\therefore F_x = 3.69\times10^{-5}\text{N}$$



Resultant force, $F = \sqrt{F_x^2+F_y^2}$

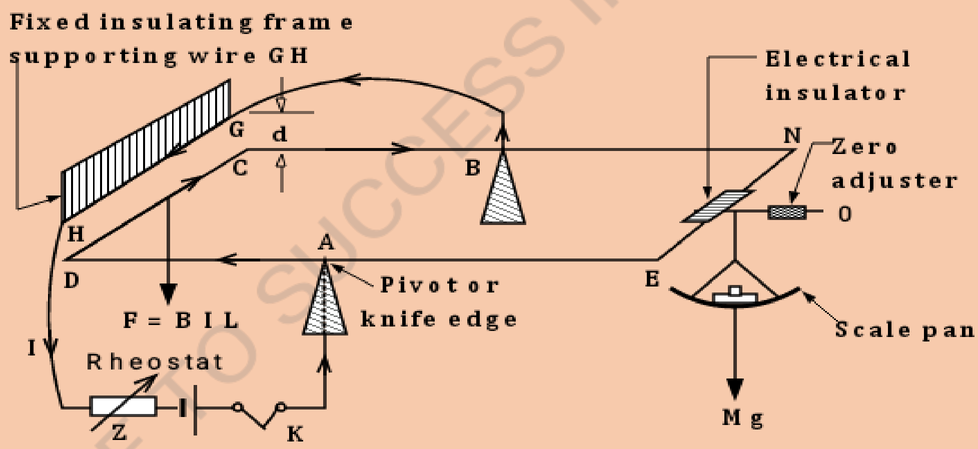
$$\Rightarrow F = \sqrt{\left(3.69\times10^{-5}\right)^2+\left(2.46\times10^{-5}\right)^2}$$

$$\therefore\quad F = 4.43\times10^{-5}\textbf{N}$$

In the direction $\beta = \left[\tan\left(\frac{2.46\times10^{-5}}{3.69\times10^{-5}}\right)^{-1}\right] = 33.7°$

***Hence, the resultant force is* 4.43×10^{-5}N *at* 33.7°**
to the + X - direction.

(d) **Diagram of <u>A current Balance</u>.**

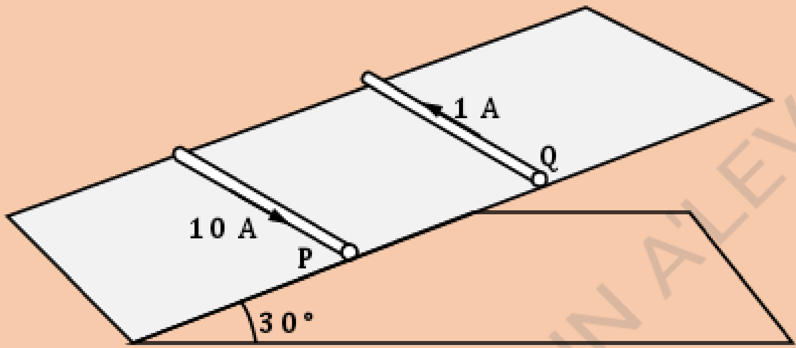


- *The apparatus is set up as shown up as shown on the diagram above.*
- *DCNE is a conducting frame such that **AD = AE**.*
- *With no current flowing, i.e. when switch K is open, the zero screw (adjuster) is adjusted until the frame CDEN balances horizontally.*
- *The switch is closed and the arm CD is repelled vertically downwards.*
- *Masses are added into the scale pan until the horizontal balance position of the frame CDEF is restored.*

- *The total value of the mass M in the scale pan is measured, together with the separation, d, between arms CD and GH and the length, L of wire CD. Now since **AE = AD**, and CD = L*
- *The value of the current I is calculated from,* $I = \sqrt{\dfrac{2\pi Mgd}{\mu_0 L}}$

  Where, $\mu_0 = 4\pi \times 10^{-7}\ H\ m^{-1}$

4. (a) Two parallel wires P and Q each of length 0.20 m carry currents of 10A and 1A respectively in opposite directions as shown on the diagram.



The distance between the wires is 0.04 m. if both wires remain stationary and the angle the plane makes with the horizontal is 30°, Calculate the weight of wire Q.

(b) A wire of thickness 0.34 mm and of length 7.85 m is wound into a circular coil of radius 0.05 m. If a current of 2A passes through the coil, find the;

(i) number of turns of the oil.

(ii) value of the magnetic flux density at the centre of the coil.

(c) A coil of 50 turns and radius 4 cm is placed with its plane in the earth's magnetic meridian. A compass needle is placed at the centre of the coil. When a current of 0.1 A passes through the coil, the compass needle deflects through 40°. When the current is reversed, the needle deflects through 43° in the opposite direction.

(i) Calculate the horizontal component of the earth's magnetic flux density.

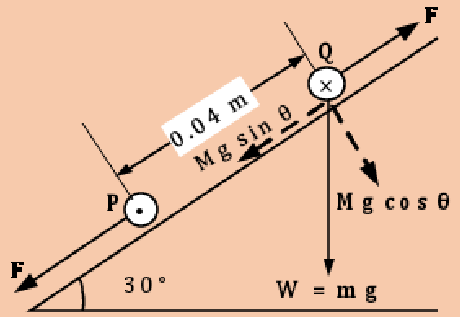
(ii) Calculate the earth's resultant magnetic flux density at the location where the angle of dip is 15°.

Solutions:

(a) *Assuming wire P is fixed, The, repulsive magnetic force F = BIL just prevents the weight component Mg sin θ from moving it downwards. i.e.*

$$W \sin\theta = \frac{\mu_0 I_P I_Q L}{2\pi d}$$

$$\therefore W = \mu_0 I_P I_Q L$$



(b) (i) One loop or turn of the coil = circumference of a circle = $2\pi R$

i.e. $(2\pi R)N = L$ (Length of the wire) $\therefore N = \dfrac{L}{(2\pi R)} = \dfrac{7.85}{2\pi \times 0.04}$

$\Longrightarrow N = 31.2$ turns

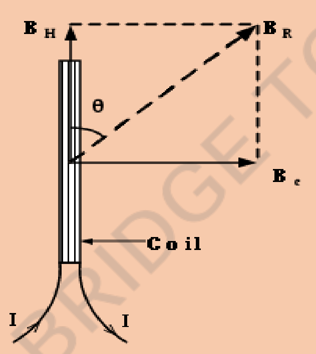
$\therefore N \cong 31$ turns.

(ii) Assuming the turns of the coil touch each other, it implies 31 turns × the thickness, d, of each wire = total length, l ,of the coil.

$\therefore N d = l$ and using $B = \mu_0 n I = \dfrac{\mu_0 N I}{l} = \dfrac{\mu_0 \cancel{N} I}{\cancel{N} d} = \dfrac{\mu_0 I}{d}$

$\therefore B = \dfrac{\mu_0 I}{d} = \dfrac{4\pi \times 10^{-7} \times 2.0}{0.34 \times 10^{-3}} = 7.39 \times 10^{-3}$ T

(c) Horizontal component of Earth's magnetic field is along the magnetic meridian.



$\theta_1 = 40°$, $\theta_2 = 43°$ $\therefore \theta = \left(\dfrac{\theta_1 + \theta_2}{2}\right) = \left(\dfrac{40° + 43°}{2}\right) = 41.5°$

Thus, $\tan 41.5° = \dfrac{B_c}{B_H} = \dfrac{\mu_0 NI}{2 r B_H} \Longrightarrow B_H = \dfrac{\mu_0 NI}{2 r \tan 41.5°}$

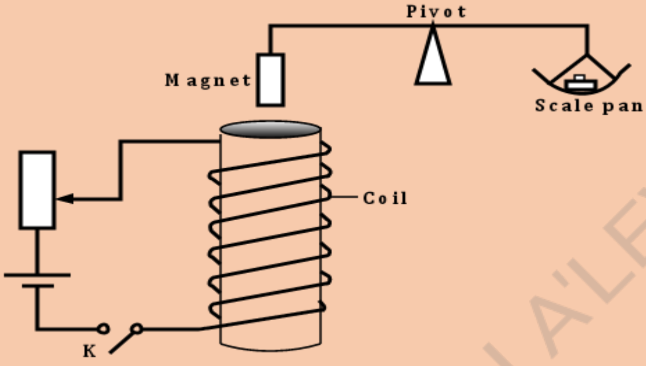
(b) At the location where angle of dip, $\alpha = 15°$ $\Longrightarrow \cos 15° = \dfrac{B_H}{B}$

$\therefore B = \dfrac{B_H}{\cos 15°} = \dfrac{8.88 \times 10^{-5}}{\cos 15°} = 9.19 \times 10^{-5}$ T

5. (a) (i) Define the ***Magnetic flux density.***

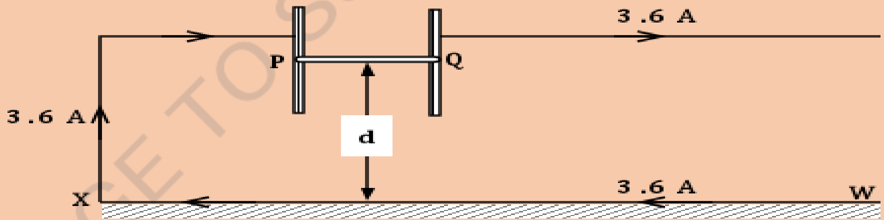
(ii) Write does the expression for the force on a conductor of length **L** carrying a current **I** when it is inclined at an angle **θ** to a uniform magnetic field of flux density **B.** Use the expression, above to derive an expression for the force experienced by one electron of charge, e , and moving with an average drift velocity, v.

(b) The figure below represents a simple current balance. When switch, K is open the force required to balance the magnet is 0.20 N. When switch, K is closed and a current of 0.50A flows, a force of 0.22 N is required for balance.



(i) Determine the polarity at the end of the magnet closest to the coil.
(ii) Calculate the weight required for the balance hen a current 2A flows through the coil.

(c) In the diagram below, a copper rod PQ of length 12.0 cm is guided between a pair of parallel vertical metal rails with perfect electrical contact. PQ is directly above wire WX fixed on a flat horizontal table and parallel to PQ.



Given that wire PQ has a mass per cm of 3.0 mg cm^{-1} and carries a current of 3.6 A.
Determine the distance, d, between the wires PQ and WX for which PQ remains stationary at equilibrium.

Solution:

(a) (i) **Magnetic flux density** – is the force exerted of a 1 m long conductor carrying a current of 1 A in a direction normal to the field.
(ii) **F = BIL sin θ**

Current flowing $I = \frac{Ne}{t}$

$\therefore F = BNe\frac{l}{t}\sin\theta$

$\therefore F = B\,Nev\sin\theta$ but $\frac{l}{t} = v$

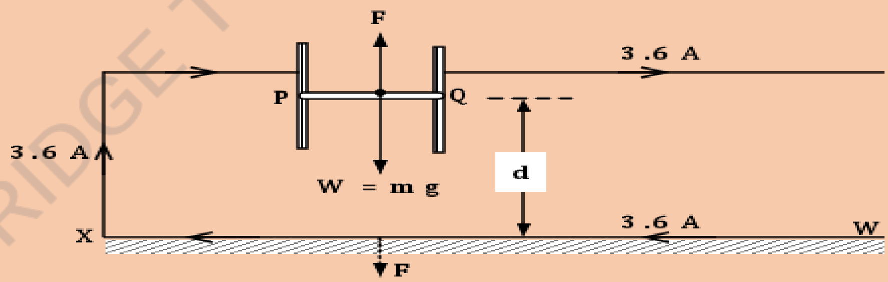
Force on one electron $F_1 = \frac{F}{N}$

$F_1 = Bev\sin\theta$

(b) (i) When switch K is closed, a current flows through the solenoid, in a clockwise direction while observing at the top. This makes the top end of the coil a **South pole**, but since there is an increase in the size of the force required to establish horizontal equilibrium, it implies the lower end of the magnet a **north pole** being attracted by the south pole of the top side of the solenoid.

(ii) When K is open, weight of the magnet = weight in the pan = 0.20 N
When K is closed, a current flows through the coil and a magnetic force created is proportional to square of current flowing i.e. $F \propto I^2$
$\Rightarrow F = kI^2 \quad \therefore \quad 0.20 + kI_1^2 = 0.22$.............(i) when, $I_1 = 0.50$ A
$\Rightarrow k = 0.08\ NA^{-2}$ thus when the current becomes, $I_2 = 2.0$ A
$\Rightarrow 0.20 + kI_2^2 = F'$..................................(ii)
$\therefore F' = 0.20 + 0.08\times 2.0^2 = 0.20 + 0.32 = 0.52$ N

(c) The current flowing in the two wires WX and PQ in opposite directions causes each wire to experience a repulsive magnetic force. This force acts against the weight of wire PQ. When the two forces equalize, the wire PQ stops rising or falling, thus horizontal equilibrium of PQ is established.



At equilibrium, $F = mg$, where, $F = \frac{\mu_0 I^2 L}{2\pi d}$ and $\frac{m}{L} = 3.0\times10^{-6}\text{kgcm}^{-1}$

$\therefore$ mass of PQ, $m = 3.0\times10^{-6}\text{kgcm}^{-1}\times 12\text{ cm} = 3.6\times10^{-5}\text{kg}$

Thus at equilibrium, $\frac{\mu_0 I^2 L}{2\pi d} = mg \Rightarrow d = \frac{\mu_0 I^2 L}{2\pi mg}$

$$d = \frac{\mu_0 I^2 L}{2\pi mg} = \frac{4\pi \times 10^{-7} \times (3.6)^2 \times 0.12}{2\pi \times 3.6 \times 10^{-5} \times 9.81} = 8.81 \times 10^{-4}\,\text{m}$$

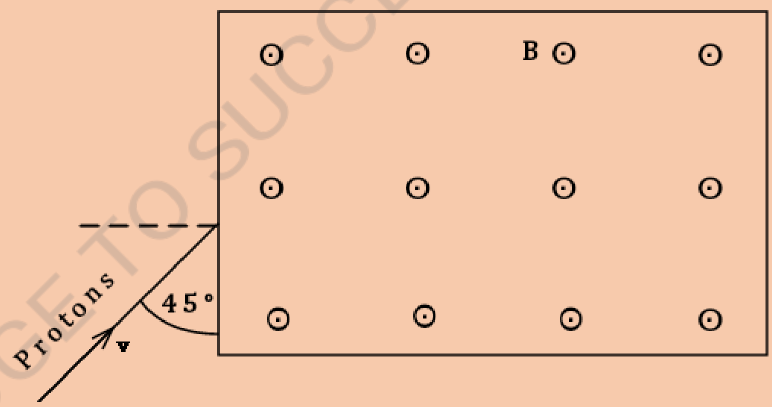
$\therefore\ d = 8.81 \times 10^{-4}\,\mathbf{m}$

6. (a) The figure below shows two parallel wires P and Q of infinite length Carrying currents of 30 A and 2 A respectively and are separated by a distance of 10.0 cm apart.

(i) Determine the resultant magnetic field midway between the wires.

(ii) At what distance from wire q is the resultant magnetic flux density zero?

(b) The figure below shows a beam of electrons directed into a region of uniform magnetic field of flux density 0.80 T, perpendicularly out of the plane. The electrons enter the magnetic field with a speed of $2.0 \times 10^6\,\text{ms}^{-1}$ at 45°.Where necessary, use $\left(\frac{e}{m} = 1.76 \times 10^{11}\ \text{C kg}^{-1}\right)$

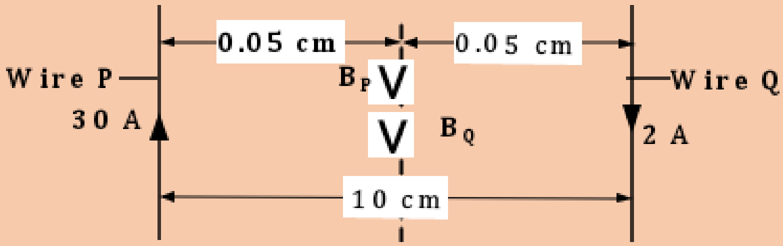


(i) Explain the motion of the electrons while inside and out of the magnetic field.

(ii) Calculate the radius of the circular path described.

Solutions:

(a) (i) Let $\mathbf{B_P}$ and $\mathbf{B_Q}$ be magnetic flux densities due to wires **P** and **Q** respectively.

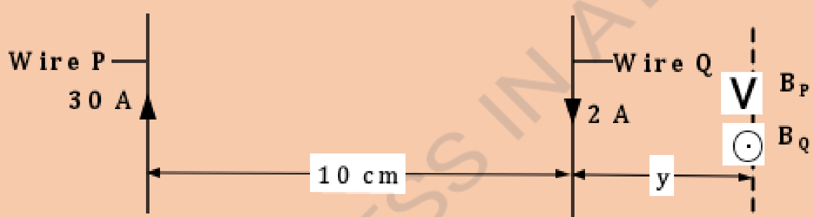


$$B_P = \frac{\mu_0 I_P}{2\pi x} \quad \text{and } B_Q = \frac{\mu_0 I_Q}{2\pi x}$$

Mid-way between the wires, d = 0.05 m and $\mathbf{B = B_P + B_Q}$

$$B = \frac{\mu_0}{2\pi x}\left(I_P+I_Q\right) = \frac{4\pi \times 10^{-7}}{2\pi \times 0.05}(30+2) = 8.96\times 10^{3}\ \text{T}$$

(ii) Let the neutral point be a distance, y, from wire Q to the right.



$$B_P = \frac{\mu_0 I_P}{2\pi(0.10 + y)} \quad \text{and } B_q = \frac{\mu_0 I_q}{2\pi y} \quad \text{since } B = 0$$

$$\Longrightarrow B_P - B_q = 0 \text{ or } B_P = B_q$$

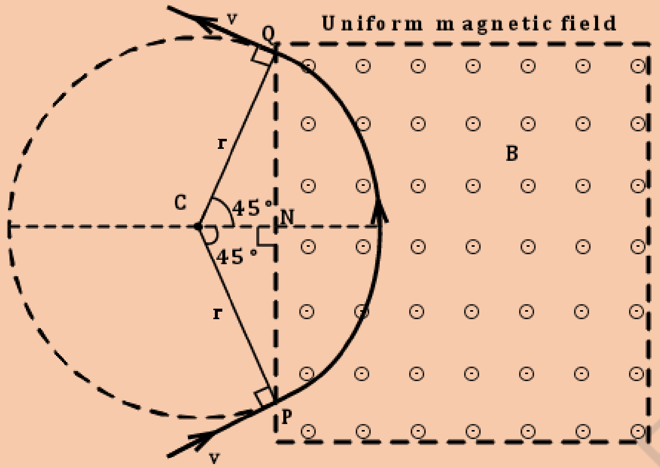
$$\therefore \frac{\mu_0 I_P}{2\pi(0.10 + y)} = \frac{\mu_0 I_q}{2\pi y} \Longrightarrow \frac{10}{(0.10 + y)} = \frac{2}{y}$$

$$5y = \left(0.10+y\right) \Longrightarrow \mathbf{y =}\ 2.5 \times 10^{-2}\,\mathbf{m}$$

(c) (i) When electrons enter the region of uniform magnetic field, by Fleming's left hand rule the electrons (charged particles) experience a magnetic force, F = Bev , and as a result of changing velocity, the electrons experience an acceleration that causes them to move in a circular path, as provided by the centripetal force, $F = \frac{m v^2}{r}$. Where, m is the mass of an electron and v is the velocity of the electron within the magnetic field, of flux density, B.

When the electrons get out of the region of uniform magnetic field, the magnetic force ceases to exist and so they continue to move in a straight line along the tangent to the final path while in the magnetic field as shown on the diagram below.

(ii) **Method I**



In the region of magnetic field, $Bev = \frac{mv^2}{r} \Longrightarrow r = \frac{mv}{Be}$..............(i)

$PN = r\sin 45°$ and also $NQ = r\sin 45° \Longrightarrow PQ = 2r\sin 45°$....(ii)

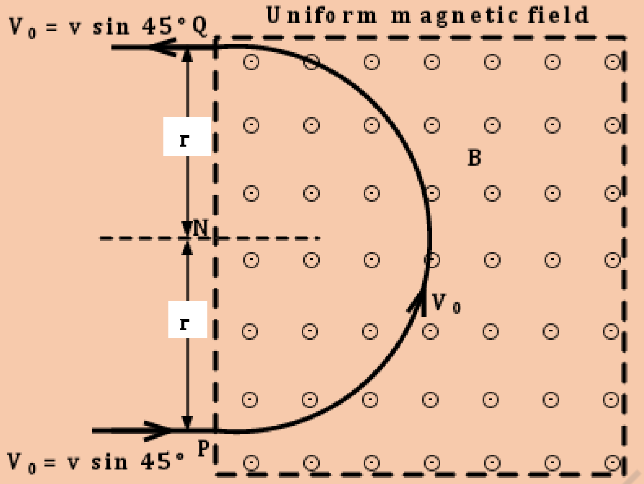
∴ substituting (i) into (ii)

$$\Longrightarrow PQ = 2r\sin 45° = 2\left(\frac{mv}{Be}\right)\sin 45° = 2\left(\frac{v}{B \times \frac{e}{m}}\right)\sin 45°$$

$$\therefore PQ = 2\times\left(\frac{2.0\times10^6}{0.80 \times 1.76 \times 10^{11}}\right)\times\sin 45°$$

Hence, **PQ = 2.01×**10^{-5} m

Alternative Method II:



Suppose, V_0 is the velocity of the electron normal to the magnetic field at P, Since the electron emerges out of the uniform magnetic field, and is not moving towards P again, it's assume to emerge normal to the magnetic field again at Q, a distance 2r away from P.

$$\Rightarrow BeV_0 = \frac{mV_0^2}{r} \text{ but, } V_0 = v\sin 45° \Rightarrow Be\cancel{(v\sin 45°)} = \frac{m(v\sin 45°)^2}{r}$$

$$\therefore r = \frac{m\, v\sin 45°}{B\,e} = \frac{v\sin 45°}{B\left(\frac{e}{m}\right)} = \left(\frac{2.0 \times 10^6}{0.80 \times 1.76 \times 10^{11}}\right) \times \sin 45° = 1.00 \times 10^{-5} \text{ m}$$

$\therefore$ **PQ** = 2 r = 2 × 1.00× 10^{-5} = **2.00** × 10^{-5}**m**

Exercise:

1. A long straight wire carries a current of 50.0 A. An electron, of charge 1.6 × 10^{-19} C travelling at 1.0× 10^{7} ms^{-1} is 5.0 cm from the wire at that instant. Determine the force (both in magnitude and direction) acting on the electron,

   (i) If the electron's velocity is directed towards the wire. Ans: [3.20×10^{-16}] parallel to the wire in the same direction with the current.

   (ii) If the electron's velocity is in the same direction as that of the current. Ans: [3.20×10^{-16}] Perpendicular to the wire away from the wire.

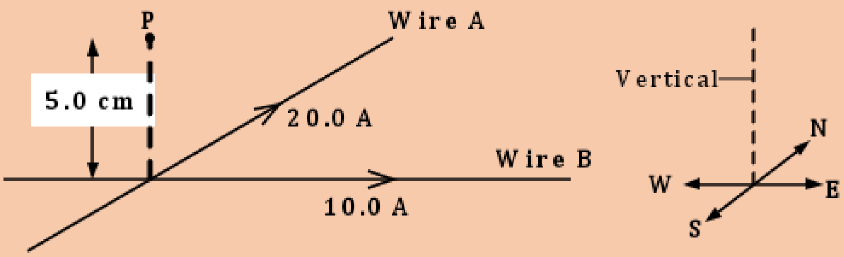
2. Two long parallel wires X and Y each carries a current of 10A and are 5 cm apart. Calculate the;

   (i) Magnetic flux density at the position of wire Y due to the current in X.

   Ans:[4.00 × 10^{-5} T]

(ii) Force per metre on wire Y. Ans:[4.00×10^{-4} Nm^{-1}]

3. Two long insulated wires lie in the same horizontal plane. A current of 20.0 A flows towards the north inside wire A while a current of 10.0 A flows towards the east in wire B as shown in the figure below.



What is the magnitude and direction of the magnetic field at a point that is 5.00 cm above the point P, where the wires cross? Ans:[8.94×10^{-5} T, in direction, **S**26.6°**E**]

4. A circular coil of **four** turns and diameter 11 cm has its plane vertical and parallel to the magnetic meridian of the Earth. Determine the resultant magnetic flux density at the centre of the coil when a current of 0.35 A flows through it. (*Take the horizontal component of the Earth's magnetic flux density to be 1.6 × 10^{-5}* T)

Ans: [1.12×10^{3} N]

5. A metal wire 10 m long lies in the East – west direction on a wooden table. What p.d. has to applied across the ends of the wire, and in what direction, in order to just make the wire rise from the surface? Assume electrical connections to the wire make no appreciable restraint. (Density of the metal =1.0×10^{4} kg m^{-3}, resistivity of the metal = 2.0×10^{-8} Ω m, Horizontal component of the Earth's magnetic field = 1.8×10^{-5} T, Earth's gravitational field strength, g = 9.81 N kg^{-1})

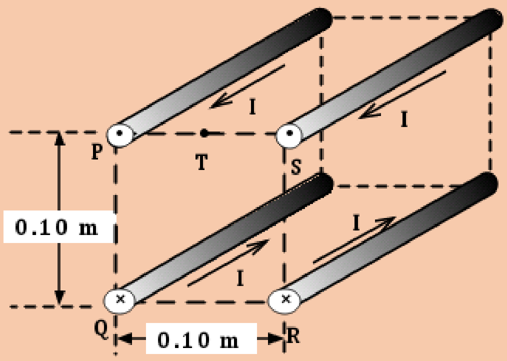
Ans: [1.09×10^{3} V]

6. A long solenoid with 3000 turns per metre carries a current of 4.0A. A horizontal wire X 4.0 cm long is in the middle of the solenoid perpendicular to its axis and also carrying a current of 4.0A.

(i) Determine the force experienced by wire X. Ans:[2.41×10^{-3} N]

(ii) Suggest a design for a current balance based on this principle.

7. Four long and parallel wires are arranged at the corners of a square of side 0.10 m. All the four wires carry the same magnitude of current I = 10.0 A in the directions indicated.



(i) Find the magnetic flux density at the centre of the square.
Ans: [80 μT to the right.]

(ii) Find the magnetic flux density at the mid point, T of side PS of the square.

Ans: [0.11 mT to the right.]

8. A rectangular coil of 20 turns and dimensions 4 cm by 2 cm is suspended with its plane and longer side vertical in a horizontal field of 2.0×10^{-2} T. If a current of 2A flows in the coil, calculate the torque or moment of the couple initially on the coil when its plane is,

(i) Parallel to the field. Ans:[6.40×10^{-4} Nm]

(ii) Inclined at 60° to the field. Ans:[3.20×10^{-4} Nm]

9. A moving coil instrument has a rectangular coil of 10 turns and dimensions of 5 cm by 2 cm situated in a radial magnetic field of 0.4 T. The coil is suspended by a torsion wire that has a restoring couple of 2.0×10^{-6} Nm per degree of twist. Calculate the;

(i) Deflection of the coil when a current of 40 μA is pass into it. Ans:[0.08°]

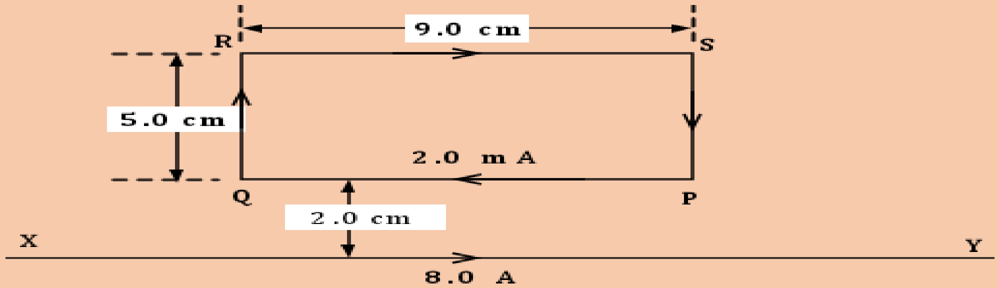
(ii) Sensitivity of the instrument. Ans:[2.0° mA

10. A flat circular coil of wire of 20 turns and of radius 10.0 cm is placed with its plane vertical and at 45° to the magnetic meridian. Assuming that horizontal component of the earth's magnetic flu density = 2.0×10^{-8} T, $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1} ,

Calculate the current in the coil, if a compass needle, free to move in a horizontal plane, points in the East – West direction, when placed at the centre of the coil.

Ans:[0.23 A]

11. A rectangular loop of wire PQRS, carrying a current I_1= 2.0 mA, is next to a very long wire XY carrying a current I_2 = 8.0 A as shown on the diagram below.



(a) What is the magnitude and direction of the magnetic force on each of the four sides of the rectangular loop of wire, due to the long wire's magnetic field?

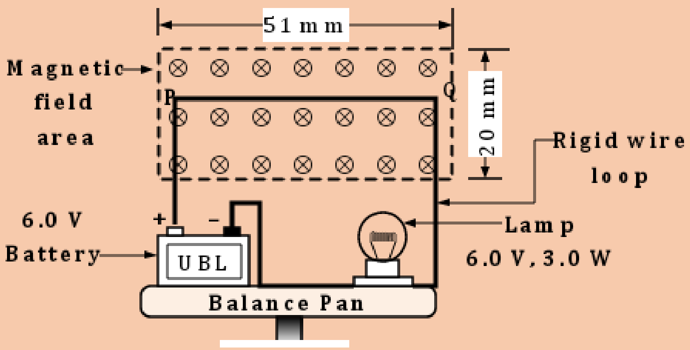
(b) Calculate the net magnetic force on the rectangular loop due to the long wire's magnetic field. [**Hint:** *The long wire does **not** produce a uniform magnetic field.*]

(a) Answers:

SIDE	CURRENT DIRECTION	MAGNETIC FIELD DIRECTION	DIRECTION OF THE FORCE
RS	Right	Out of the page	Attracted to the long wire.
PQ	Left	Out of the page	Repelled by the long wire.
QR	Up	Out of the page	To the Right.
SP	Down	Out of the page	To the left.

(b) Ans:[1.00×10^{-8} N] away from the long wire.

12. The diagram below shows a rigid conducting wire PQ connected to a 6.0 V battery through a 6.0 V, 3.0 W lamp. The circuit is standing on the top pan of a balance. A uniform horizontal magnetic field of strength 50 mT acts at right angles to the plane of the wire loop into the plane of the paper. The balance reads 153.86g.



Calculate the;

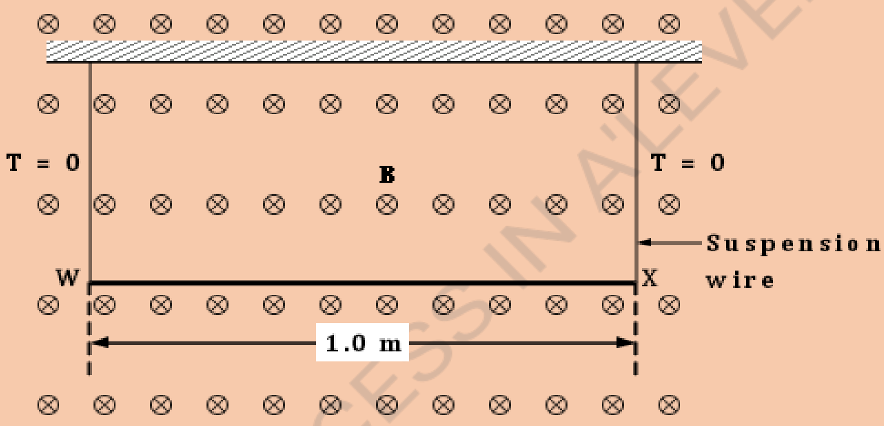
(a) Force exerted on the conducting wire PQ by the magnetic field.

Ans: [1.28×10^{-3} N]

(b) New balance reading if the direction of the magnetic field is reversed.

Ans: [1.51 N]

13. A straight stiff wire of length 1.0 m and mass 25.0g is suspended in a magnetic field of flux density, B = 0.75 T. The wire is connected to a d.c. source.



Determine the magnitude and direction of the current that must flow in the suspension wire so that the tension in the supporting wires is zero.

Ans: [0.327 A]

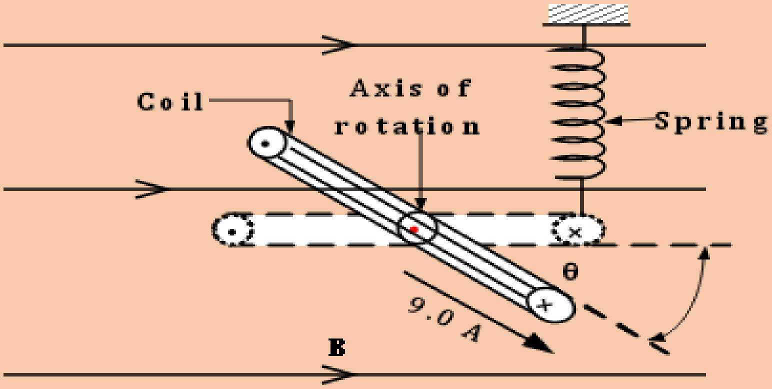
14. A light power line 125 m long is held horizontally between two pylons and carry a

current of 2500 A towards the south. The Earth's magnetic field at that location is 0.52 mT towards the north at an angle of dip of 62° . Determine the magnetic force experienced by the wire. **Ans: [0.140 N due East]**

15. A square loop of wire of side 0.60 m carries a current of 9.0 A as shown in the

figure below. When there is no applied magnetic field, the plane of the loop is horizontal and non – conducting nonmagnetic spring of force constant,

k = 550 Nm^{-1} is un-stretched. A Horizontal magnetic field of magnitude 1.3 T is now applied.



At what angle θ is wire loop's new equilibrium position? Assuming the spring remains vertical because θ is small. **Ans: [4.9°]**

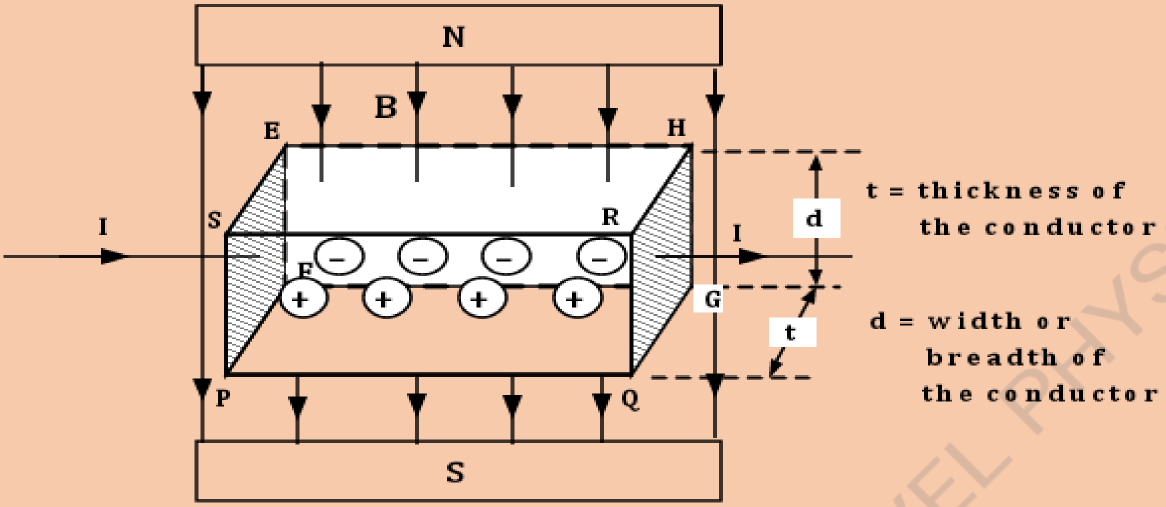
16. An electron revolves in a circular orbit of radius 2.0×10^{-10} m at a frequency of 6.8×10^{15} Hz. Calculate the magnetic flux density at the centre of the coil.

Ans: [2.43×10^{5} T]

17. A current of 3.25 A flow through a long solenoid f 400 turns and length 40.0 cm. Determine the magnitude of the force exerted on a particle of charge 15.0μC moving at 1.0×10^{3} ms^{-1} through the centre of the solenoid at an angle of 11.5° relative to the axis of the solenoid. **Ans: [1.22×10^{-5} N]**

THE HALL EFFECT (Named after **Edwin Herbert hall 1855 – 1938)**

Whenever a current carrying conductor is placed or positioned a cross a magnetic field, the majority charge carriers get urged by the field towards one side of the conductor in a direction predicted by Fleming's left hand rule. As a result a large p.d. is set across the sides of the conductor. This p.d. is called the Hall p.d.



- When current I and a magnetic field of flux density B are applied to the conducting slab at right angles to each other as shown in the diagram, the

  magnetic field urges the majority charge carriers (electrons) towards face EFGH, according to Fleming's left hand rule, leaving an equal magnitude of positive charge on the opposite face PQRS.

- A magnetic force F = Bev, acts on conduction electrons as they drift along the

  conductor and eventually drift off in the direction from face PQRS to face FGHE.

- As the process continues, face PQRS acquires excess positive charge while face EFGH acquires excess negative charge.

- When maximum separation of charge has occurred so that no more net flow of charge occurs across oppositely charged faces, a large voltage or a p.d develops across the faces PQRS and EFGH.

- This voltage is called a **hall voltage, V_H** and the effect is called the hall effect.

Derivation of the expression for the Hall Voltage, V_H

From the above setup, when there is no net flow of charge across opposite charged faces PQRS and EFGH, the magnetic force on the electrons, equals the electric force on the same electron, due to a strong electric field intensity across the charged faces.

i.e. $Bev = Ee$, but $E = \dfrac{V_H}{d}$, and $v = \dfrac{I}{n\,A\,e}$

$$Bv = E \Longrightarrow \frac{B\,I}{n\,A\,e} = \frac{V_H}{d} \text{ but } A = (d\times t) \Longrightarrow V_H = \frac{B\,I\,\cancel{d}}{n\,(d\times t)\,e}$$

$\therefore V_H = \dfrac{B\,I}{n\,e\,t}$ is the expression for the Hall voltage.

Example:

(a) Describe an experiment to measure magnetic flux density between the pole pieces of a U – shaped magnet.

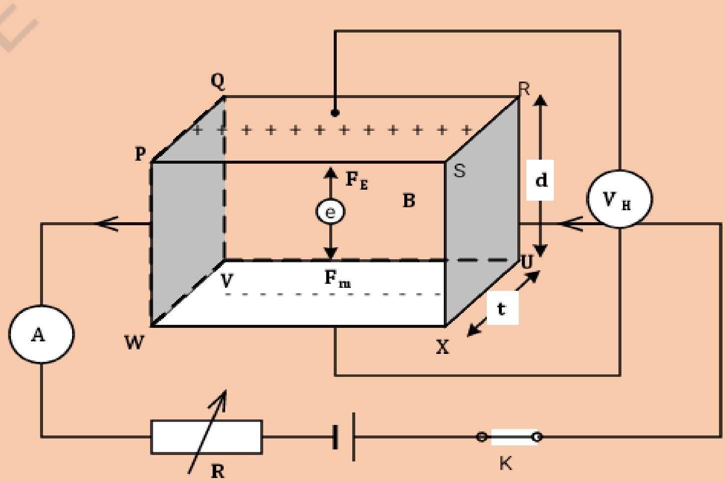
(b) An electric current of 1.0 A passed through the smallest face of a rectangular metal slab of thickness 1mm, placed with its largest face normal to a uniform magnetic field of 0.80T, causes a p.d of 0.48 mV to be set up across the slab when electron, of charge 1.6×10^{-19} C, is un-deflected. Determine the number of electrons per cubic metre of the conductor.

Solution:

(a) **Measurement of Magnetic Flux Density, B, using a <u>Hall- Probe</u>**

The Hall Probe – Is one of the applications of the Hall effect.

This device has a small wafer of Germanium semiconductor mounted along a narrow handle, so that it can be conveniently used to probe the magnetic field being examined.



- A small current I, measured by a suitable ammeter, **A or mill-ammeter, mA** is passed between two opposite faces of the semi-conductor by closing switch **K** and adjusting the rheostat **R** to a suitable value of the current **I.**
- The semiconductor being placed with the largest face **PSXW** perpendicular to a uniform magnetic field of flux density **B** has its electrons experiencing a magnetic force $\mathbf{F_m = Bev}$ acting down towards the lower face **XUVW** and positive charges left on the upper face **PQRS**.
- The opposite charges continue moving to the opposite faces until no more charges can move across the faces.
- A maximum steady p.d. $\mathbf{V_H}$ called the **Hall voltage** is then set up across the opposite charged faces PQRS and UVWX. This p.d. called the **Hall voltage** is then measured using a high impedance voltmeter, **V**.
- At the same time when the p.d. V_H registered by the voltmeter, V is steady and maximum, the ammeter reading, I is noted.
- Using a known value, **"net"** provided by the manufacturer of the semi-conductor wafer, the magnetic flux density, B, is calculated from the expression, $V_H = \frac{BI}{net}$

From which, **B =** $\frac{V_H(net)}{I}$ the magnetic flux density **B** is determined when n, e, t, I, and V_H are all known values.

(b) Using the equation, $V_H = \frac{BI}{net} \Longrightarrow n = \frac{BI}{V_H et}$

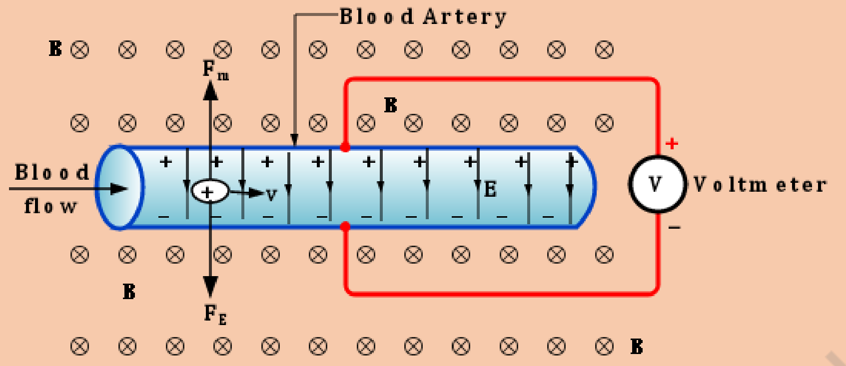
$$\Longrightarrow n = \frac{0.8 \times 1.0}{\left(1.6 \times 10^{-19} \times 1.0 \times 10^{-3} \times 0.48 \times 10^{-3}\right)}$$

$\therefore$ n = 1.042×10^{25} per m^3 Is the number of charge carriers per unit volume.

ELECTROMAGNETIC BLOOD FLOW METER

This is yet another interesting application of the Hall effect similar to experiment of J J Thompson's set up used in the determination of the charge – to – mass ratio of an electron, where an electron is accelerated into the velocity selector region. The electric and magnetic fields in the velocity selector are adjusted until the electron passes through the region un-deflected.

Diagram:



- **Electromagnetic flowmeter** measures ***the speed of blood flow*** through a major artery during **cardio-vascular surgery**.
- **Blood** contains ions; the motion of the ions can be affected by a magnetic field.
- In an electromagnetic flow meter, a magnetic field is applied perpendicular the flow direction of blood in the artery.
- The magnetic force exerted by the field on the positive ions is towards one side of the artery according to Fleming's left hand rule, while the negative ions move in the opposite direction of the artery.
- This separation of the charge with the positive charge on one side and negative charge on the opposite side creates a large p.d. across the artery, resulting in the generation of a strong electric field, E.
- Electric force is then exerted on the moving ions in the opposite direction to that of the magnetic force.
- When the magnetic force on the ions = electric force on the same ions, i.e. $F_m = F_E$

$$\Longrightarrow B\cancel{q}v = E\cancel{q} \Longrightarrow v = \frac{E}{B} \text{..........(i)}$$

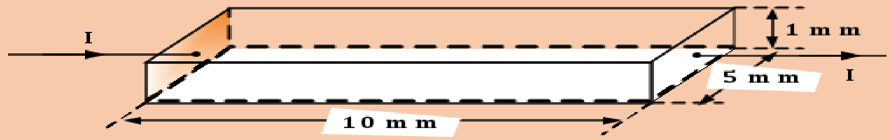
where v = average speed of the ion, and that of the blood flow.

- A voltmeter attached to the opposite sides of the artery is used to measure the potential difference, V, and knowing the magnetic field strength, B, and the electric field intensity in the region, the speed of the blood flow, v , is calculated from,

$\therefore \mathbf{v} = \dfrac{V_H}{Bd}$ ....(ii) the ions in the blood are primarily sodium, Na^+ ions.

Exercise:

1. The diagram below shows a rectangular piece of semiconductor with leads attached to metal end faces.



(i) If the resistance of the specimen is approximately 100 Ω , show that the resistivity of the material of the material is about 0.050 Ω m.

(ii) Suppose a uniform magnetic field of flux density 0.5 T is applied vertically and perpendicularly to the largest area, determine the hall voltage $\mathbf{V_H}$ if a current of 40 mA, flows across the opposite smaller faces.

2. The current in a strip of copper is given by I = nevA where, A is the cross sectional area, of the strip and n is the number of free electrons per unit volume. If d is the thickness of the strip and b is the breadth;

(i) Express ev in terms of I, a, b and d.

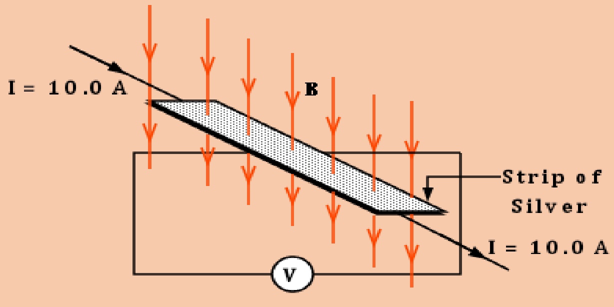
(ii) Show that the hall voltage, $V_H = \dfrac{BI}{ned}$, when a field B is applied.

(iii) Calculate the hall voltage, given that B = 1.0 T, I = 6.0 A, n = 7.5×10^{28},

d = 1 mm and e = 1.6×10^{-19}C. **Ans:[5.00×10^{-7} V]**

3. The concentration of free electrons in silver is 5.85×10^{28} per m^3. A strip of silver of thickness 0.050 mm and width 20.0 mm is placed in a magnetic field of 0.80 T.

A current of 10.0 A is sent down along the strip as shown on the diagram below.



Determine the,

(i) Drift velocity of the electrons.

(ii) Hall voltage measured by the moving coil meter.

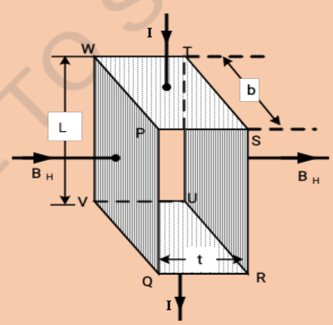
(iii) Side of the voltmeter that is at a higher potential.

4. An electromagnetic flowmeter is used is to be used to measure the blood speed. A magnetic field of 0.115 T is applied across an artery of inner diameter 3.80 mm. The Hall voltage is measured to be 88.0 μV. What is the average speed of the blood flowing in the artery? **Ans:[20.14 cm s^{-1}]**

5.(a) A circular coil of 10 turns and radius 5.0 cm caries a current of 1.0 A. Find the magnetic flux density at the centre of the coil. **Ans:[1.26 × 10^{-4} T]**

(b) A copper wire of cross sectional area 1.5 mm^2 carries a current of 5.0 A. The wire is placed perpendicular to a magnetic field of flux density 0.2 T. If the density of the electrons in the wire is 10^{29} m^{-3}. Calculate the force on each electron. Ans:[6.67 × 10^{-24} N]

6. The diagram in the figure below shows a cuboid of a conductor of length L, breadth, b and thickness, t ,placed with its largest face PQVW perpendicular to the horizontal component of the Earth's magnetic field of flux density B_H. A current, I is passed through it as shown.



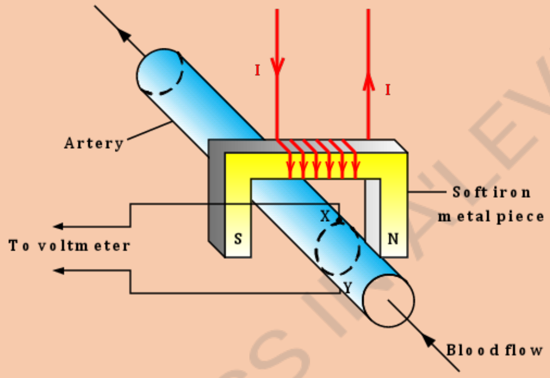
(i) Account for the occurrence of a large potential difference across faces PQRS and UVWT and derive an expression for this voltage in terms of B_H, b and the average velocity of the charge carriers, v.

Ans: $\left[V_H = \dfrac{B_H I}{n e t}\right]$

(ii) If the Earth's magnetic field at the location of the conductor is 2.0×10^{-4} T, the angle of dip is 60°, the breadth of the conductor is 5 cm and the mean speed of the electrons is 4.0×10^{-2} ms^{-1}.Calculate the potential difference across faces PQRS and UVWT.

Ans: [$V_H = 3.46 \times 10^{-7}$ V]

7. The figure below shows a model used to demonstrate the working principle of an electromagnetic flow meter used to measure the rate of flow of blood through an artery.



When a magnetic field of 2.0 T is produced by the electromagnet, a potential difference (p.d.) of 600 μV is developed between the two electrodes, X and Y. The cross-sectional area of the artery is 1.5×10^{-6} m^2 and the separation of the electrodes is 1.4×10^{-3} m

(a) Write down an expression for the force on an ion in the blood which is moving at right angles to the field. Define your symbols used. Which electrode is positive?

(b) An ion has a charge of 1.6×10^{-19} C . Determine the force on the ion due to the electric field between X and Y. **Ans: [6.86×10^{-20} N]**

(c) Given that the p.d. of 600 μV is developed when the electric and magnetic forces on an ion are equal and opposite, calculate the;

(i) Speed of the blood through the artery. **Ans: [0.214 $m\ s^{-1}$]**

(ii) Volume of blood flowing through each section of the artery per second. **Ans: [3.21×10^{-7} m^3]**

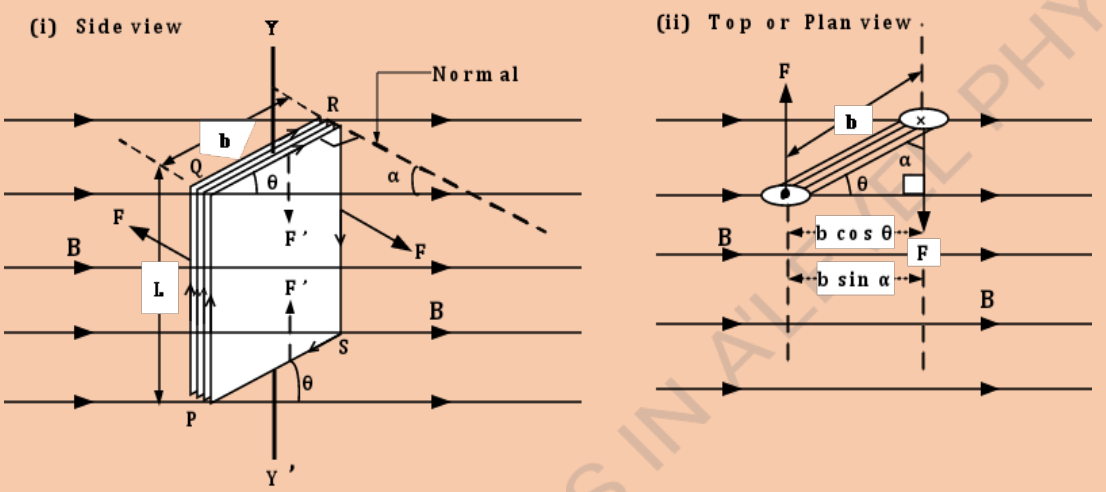
Magnetic Torque on a coil carrying a current in a magnetic field:

Definition: Magnetic Torque is the product of the magnitude of one of the forces constituting a couple and the distance between the lines of actions of the forces.
SI unit: is newton metre (Nm)

Derivation for the Torque experienced by a rectangular coil.

(a) The Plane of the coil making an angle θ with the magnetic field, B.

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the plane of the coil inclined ***at an angle θ*** to the magnetic field as shown below.



When current, **I**, flows through the coil, in the direction shown on the diagram, each side of the conductor experiences magnetic force, F, given by.

Force on side PQ, F = NBIL(into the plane of the paper)..............(i)
Force on side RS, F' = NBI b sin θ (vertically downwards)...........(ii)
Force on side RS, F = NBIL(Out of the plane of the paper)..........(iii)
Force on side SP, F' = NBI b sin θ (vertically upwards)..............(iv)

The two ***forces on sides QR and SP*** are *equal in magnitude but are in opposite* directions and so they ***cancel out*** due to the rigidity of the coil.

Side **PQ** experiences force **NBIL** perpendicularly ***into*** the page, while **QR** experiences force **NBIL** perpendicularly ***out*** of the page.

The two forces ***constitute a couple whose turning moment or torque***

$$T = F \times b\cos\theta \quad \text{or} \quad T = F \times b\sin\alpha$$

$T = NBIL \times b\cos\theta = NBIL\,b\cos\theta$ or $T = NBIL \times b\sin\alpha$ but $(L \times b) = A$

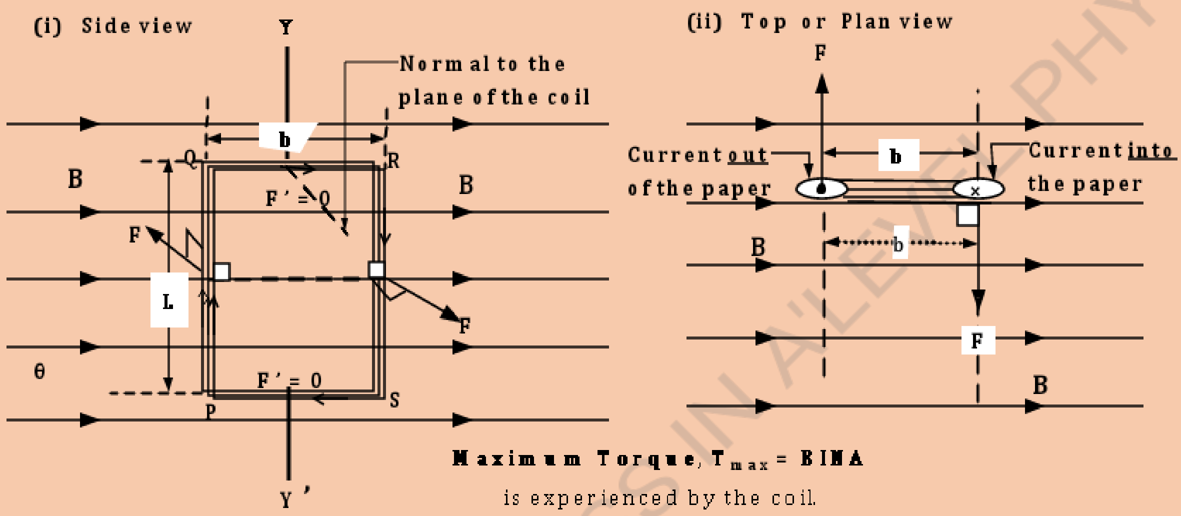
$\therefore$ $\mathbf{T = NBIA}\cos\boldsymbol{\theta}$ or $T = NABI\sin\boldsymbol{\alpha}$ is the Torque on the coil.

Torque – is the product of the magnitude of one of the forces constituting a couple and the distance between the lines of the actions of the forces.

SI unit – is newton metre (Nm)

(b) The Plane of the coil being parallel to the magnetic field, B, direction.

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the ***plane of the coil parallel*** to the ***magnetic field*** as shown below.



When current, **I**, flows through the coil, in the direction shown on the diagram, each side of the conductor experiences magnetic force, F, given by.

Force on side PQ, F = NBIL(into the plane of the paper)…………..(i)
Force on side QR, F' = NBI b sin 0° = 0 ,sincesin 0° = 0…….. ……(ii)
Force on side RS, F = NBIL(Out of the plane of the paper)……….(iii)
Force on side SP, F' = NBI b sin 0° = 0 ,since, sin 0° = 0…………..(iv)

The two ***forces on sides QR and SP*** are *each* ***zero***, *since* ***current flows*** *in each in the* ***same direction as*** *that of the* ***magnetic field*** *thus Fleming's left hand rule doesn't apply* on the sides QR and SP of the rigid rectangular coil.
Side **PQ** experiences force **NBIL** perpendicularly ***into*** the page, while **QR** experiences force **NBIL** perpendicularly ***out*** of the page.
The two forces ***constitute a couple whose moment or torque***

$T = F \times b$ or $T = \mathbf{NBIL} \times b = \mathbf{NBI}(L\times b)$, but $\mathbf{(L\times b)} = A$

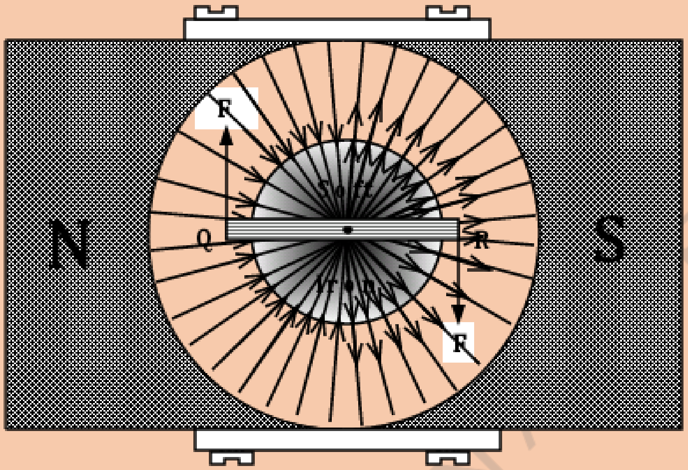
$\therefore$ **T = NBIA** or T = NABI is the Torque on the coil.

NB: This gives the **maximum torque** the same coil can experience under the conditions given or indicated.

(c) The Plane rectangular coil being placed in a radial magnetic field, B.

As opposed to a uniform magnetic field, B , in a radial magnetic field, the plane of the coil is parallel the magnetic field at all positions of the coil in the magnetic field, hence maximum torque is experienced by the coil each time a current flows in the coil.

Top or Plan view of the Coil



When current, **I**, flows through the coil, in the direction shown on the diagram, only the vertical sides of the conductor experiences magnetic force, **F**, given by.

Force on side PQ, $F = NBIL$ (perpendicularly INTO the plane of the coil)

Force on side QR, $F' = NBI\,b\sin 0° = 0$,since $\sin 0° = 0$

Force on side RS, $F = NBIL$ (perpendicularly OUT of the plane of the coil)

Force on side SP, $F' = NBI\,b\sin 0° = 0$,since, $\sin 0° = 0$

The two ***forces on sides QR and SP*** are *each **zero**, since* ***current flows*** *in each side in the* ***same direction as*** *that of the* ***magnetic field,*** *thus* ***Fleming's left hand rule doesn't apply*** on the sides QR and SP of the rigid rectangular coil.

Side **PQ** experiences force **NBIL** perpendicularly ***into*** the page, while **QR** experiences force **NBIL** perpendicularly ***out*** of the page.

The two forces ***constitute a couple whose turning moment or torque***

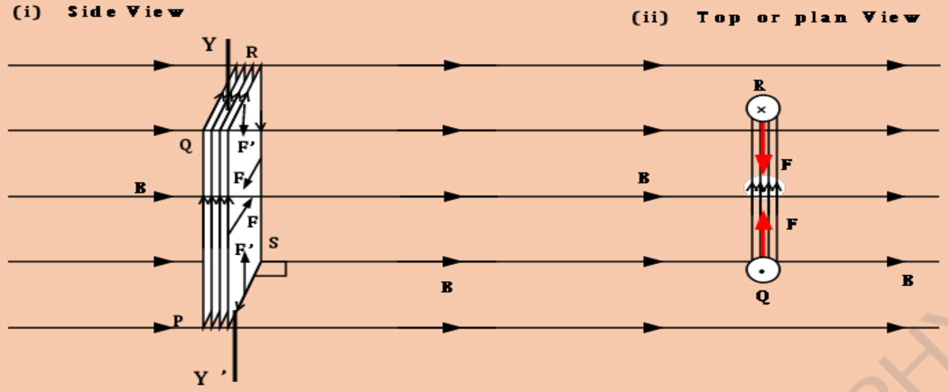
$$\mathbf{T} = F \times b \text{ or } \mathbf{T} = \mathbf{NBIL} \times b = \mathbf{NBI}(L \times b), \text{ but } (\mathbf{L \times b}) = A$$

$$\therefore \mathbf{T = NBIA} \quad \text{or} \quad T = NABI \text{ is the Torque on the coil.}$$

NB: The ***torque here is maximum*** for ***all positions*** of the coil in the magnetic field.

(d) The Plane of the coil being perpendicular to the magnetic field, B.

Consider a rectangular coil of wire of N – turns each carrying a current **I** in an external uniform magnetic filed of flux density **B**, with the ***plane of the coil perpendicular*** *to the* ***magnetic field*** as shown below.



When current, **I**, flows through the coil, in the direction shown on the diagram, **ALL** the sides of the coil experience magnetic forces, **F**, given by.

Force on side PQ, F = NBIL (towards side RS in the plane of the coil)

Force on side QR, F' = NBIb (towards side SP in the plane of the coil)

Force on side RS, F = NBIL (towards side PQ in the plane of the coil)

Force on side SP, F' = NBIb (towards side QR in the plane of the coil)

The two ***forces on sides PQ and RS*** each of magnitude F = NBIL ***cancel out*** each other since they act in ***opposite directions*** *of the rigid coil.*

Similarly, the two ***forces on sides QR and SP*** each of magnitude F' = NBIb also ***cancel out*** each other since they act in ***opposite directions*** *of the rigid coil.*

Since none of the pairs of forces acting on the coil, ***constitutes a couple, there is NO*** *turning moment of a couple or* ***torque*** *generated and so* ***the torque is zero.***

T = 0

NB: The ***torque here is minimum*** i.e. ***ZERO torque when the*** plane of the coil is normal to the magnetic field direction.

ELECTROMAGNETIC MOMENT (m) OF A CURRENT CARRYING COIL:

- It has been found convenient to define the quantity known as ***electromagnetic moment, m*** (sometimes called ***magnetic moment***) of a current – carrying coil as that property which determines;

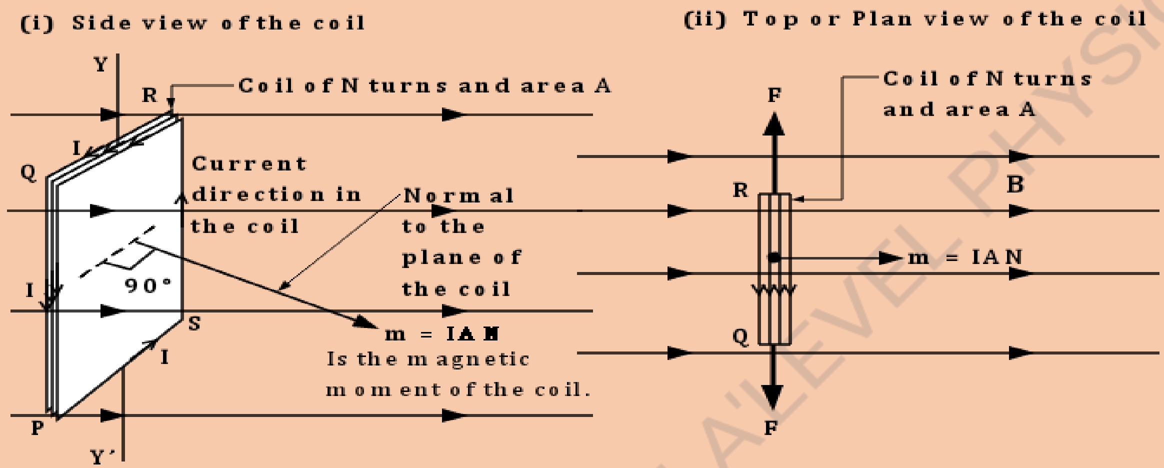
- *The magnitude of the magnetic torque that acts on the coil when it is at a given angle to the given magnetic field.*

- *The angle at which the coil ultimately comes to rest in the magnetic field.*

Definition:

Electromagnetic (magnetic) moment – ***is numerically the magnetic torque acting on a coil whose plane is parallel to a uniform magnetic field of flux density one tesla.***

Electromagnetic moment – *is a vector quantity whose* ***magnitude*** *is,* ***m = IAN*** *and whose* ***direction is along the normal to plane of the coil*** *such that it agrees with the direction of advance of Maxwell's Right Handed Advancing Screw Rule. Or it is provided by the direction of the magnetic field at the centre of the coil in the Right Hand Grip Rule in relation to the direction of flow of current in the coil.*



From the equation for magnetic torque, **T = BANI**cos θ or T = BANIsin α
Where, **θ** = Angle between the plane of the coil and the magnetic field.
and, **α** = Angle that the normal to the plane of the coil makes with the field(B),
Thus, **T = B** mcos θ or T = B msin α

The SI unit of the magnetic moment (m) is ampere metre squared (**Am^2**)

NB: - The magnetic torque on the current – carrying coil acts so as to align the electromagnetic moment, **m**, with the direction of the magnetic field, **B**.

Restoring torque on a current – carrying coil.

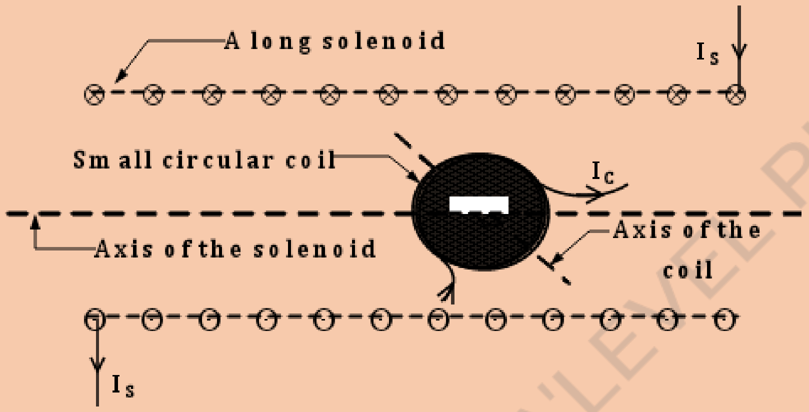
When a coil with its plane placed at an angle θ to the magnetic field direction, it experiences a ***deflection torque*** **T** = BANI cos **θ**…………………(i)
that twists the suspension wire through an **angle β** as it turns.
The suspension wire in turn, then provides ***a restoring torque,***

T' = k **β**, where k is a proportionality constant…………….(ii)
k = torsion suspension constant of the torsion wire (Nm rad^{-1})
When the coil stops turning or rotating in a magnetic field, the deflection torque equals the restoring torque. i.e. from (i) and (ii) above,
BANI cos **θ** = k **β** where θ is expressed in degrees while β is in Nm rad $^{-1}$
β = is the angle turned by the coil due to the deflection torque.
Examples:

1. A small circular coil of 10 turns and mean radius 2.5 cm is mounted at the centre of

a long solenoid of 750 turns per metre with its axis at right angles to the axis of the solenoid. If a current in the solenoid is 2.0 A. Calculate the initial torque on the circular coil when a current of 1.0 A flows through it.

Solution:



The plane of the coil is parallel to the magnetic field at the centre of the solenoid, given by,

$B = \mu_0 nI = 4\pi \times 10^{-7} \times 750 \times 2.0 = 1.88 \times 10^{-3}$ T

The initial magnetic torque experienced by the circular coil is given by,

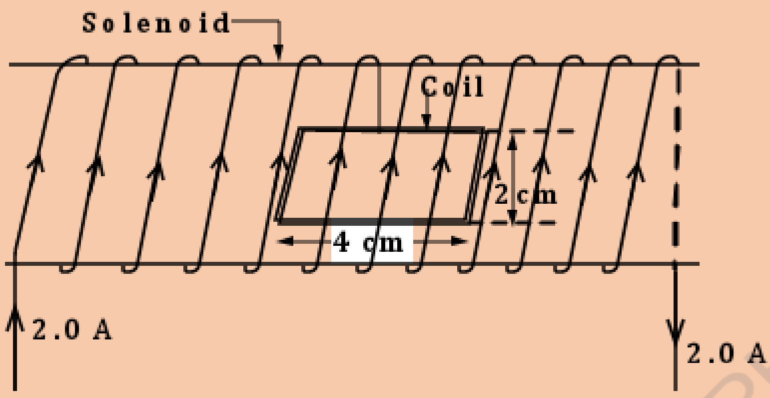
$T = BANI\cos 0° = BANI\sin 90° = BANI$

$T = 1.88 \times 10^{-3} \times \pi(0.025)^2 \times 10 \times 1.0$

$\therefore$ **T = 1.88 ×** 10^{-5} Nm

2. A small rectangular coil of 10 turns and dimensions 4 cm × 2 cm is suspended

inside a long solenoid of 1000 turns per metre so that its plane lies along the axis of the solenoid as shown in the figure below. The coil is connected in series with the solenoid. When a current of 2.0 A is passed through the solenoid, the coil deflects through 30°. Calculate the torsion constant of the suspension.



Solution:

The plane of the rectangular coil is parallel to the magnetic field at the centre of the solenoid, i.e, $B = \mu_0 nI = 4\pi\times10^{-7}\times1000\times2.0 = 2.51\times10^{-3}$ T

The initial magnetic torque experienced by the circular coil is given by,

$T = BANI\cos 0° = BANI\sin 90° = BANI$ since $\theta = 0°$ and $\alpha = 90°$

$T = 2.51 \times 10^{-3} \times(0.02\times0.04)\times10\times2.0$

$\therefore$ **T = 4.02 ×** 10^{-5} Nm

When the coil turns through angle, $\beta = 30° = \left(\frac{\pi}{180°}\times30°\right)$ radians,

The restoring torque $T' =$ k $\boldsymbol{\beta}$

But the deflection torque = restoring torque

$\therefore \mathbf{T} = T' \Rightarrow \mathbf{4.02 \times 10^{-5} = k}\times\frac{\pi}{6}$

$\therefore \mathbf{k} = \left(\frac{6\times4.02\times10^{-5}}{\pi}\right) = \mathbf{7.68\times}\,10^{-5}$ Nm rad $^{-1}$

3. A flat circular coil X of 30 turns and mean diameter 30 cm is fixed in a

vertical plane and carries a current of 3.0 A. Another coil Y of 2 cm × 2 cm and having 200 turns is suspended in a vertical plane, at the centre of the circular coil. Initially the planes of the two coils coincide. Determine the torque on coil Y when a current of 2.0 A is passed through it.

Solution:

Coil X provided a uniform magnetic field to coil Y, where $B_x = \frac{\mu_0 N_x I_x}{2R}$ B_x

$= \frac{4\pi \times 10^{-7} \times 30 \times 3.0}{2 \times 0.15} = 3.77 \times 10^{-4}$ T

But the planes of the two coils, coincide ⟹ the magnetic field B_x is normal to the plane of coil Y, ⟹ θ = 90° and β = 0°

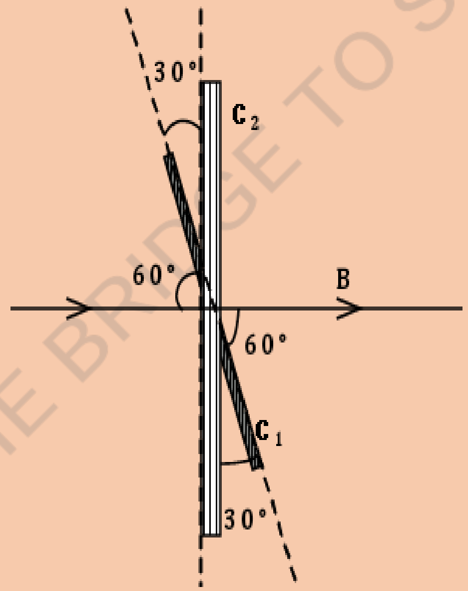
$T_Y = B_x A_Y N_Y I_Y \cos 90° = B_x A_Y N_Y I_Y \sin 0° = 0$ B'se $\cos 90° = \sin 0° = 0$

$T_Y = 3.77 \times 10^{-4} \times (0.02 \times 0.02) \times 200 \times 2.0 \cos 90°$, but $\cos 90° = 0$

$\therefore T_Y = 0$ Nm i.e. the coil Y does not experience a turning effect.

4. A rectangular coil of sides 2 cm by 4 cm having 10 turns and carrying a current of 2.0 A is freely pivoted at the centre of a large plane circular coil of 200 turns and of radius 8 cm, carrying a current of 5.0 A.
If the planes of the coils are initially at an angle of 30° to each other. Determine the torsion constant of the suspension wire supporting the rectangular coil, when it has turned through 45° .

Solution:



C_1 area $A_1 = 4 \times 2 = 8$ cm^2 $= 8.0 \times 10^{-4}$ m^2, $N_1 = 10$ turns

C_2 area $A_2 = \pi r^2 = 3.14 \times (0.08)^2 = 2.01 \times 10^{-2}$ m^2, $N_2 = 200$ turns

$I_2 = 5.0$ A , $\theta_1 = 30°$, $\theta_2 = 60°$, $\beta = 30° = \frac{\pi}{6}$ radian

$B_2 = \frac{\mu_0 N I}{2r} = \frac{4\pi \times 10^{-7} \times 200 \times 5}{2 \times 0.08} = 7.85 \times 10^{-3}$ T

$T = BANI \cos\theta = k\beta$

Exercise:

1. A flat coil of 50 turns and of mean diameter 40 cm is in a fixed vertical plane and has a current of 5 A flowing through it. A small coil , 1.0 cm square and having 120 turns, is suspended at the centre of a circular coil in a vertical plane at an angle of 30° to that of the larger coil. Calculate the magnetic torque experienced by the small coil when it carries a current of 2 mA. **Ans: [9.42 $\times 10^{-9}$ Nm]**

2. A fixed vertical circular coil has a diameter of 15.0 cm and 120 turns. At the centre of the coil is a small coil of radius 2.0 cm and 100 turns. Pivoted through the centre so that it can rotate about a horizontal axis which lies along the diameter of the larger coil. A rider of mas 0.05 g must be moved 13.0 cm from the axis of the small coil along an arm fixed to the small coil to keep the plane of the latter horizontal when the same current is passed through both coils. Determine the value of this current. **Ans: [0.734 A]**

3. A coil of radius 7.5 cm and 500 turns is suspended vertically with the plane of the coil in the east – west direction. If the horizontal component at the centre of the coil is 1.8×10^{-5} T, what current must be passed through the coil to just neutralize this field? Explain why there are two possible answers. **Ans: [4.30 $\times 10^{-3}$A]**

4. At a distance of 5.0 cm from a vertical wire carrying a current in air, the resultant magnetic field is zero as a result of the Earth's horizontal component of magnetic field of 1.8×10^{-5} T. Calculate the current flowing through the wire. **Ans: [4.5 A]**

5. A circular coil of 100 turns and mean radius 10.0 cm is set up with its plane vertical and at right angles to the magnetic meridian. A short magnetic needle suspended at its centre makes 8 oscillations per minute when slightly deflected. How many oscillations per minute will the needle make when a current of 0.5 A flows in the coil. If the horizontal component of the Earth's magnetic field is 2.0×10^{-5} T. Explain clearly why there are two possible answers.

Ans: [7.88 and 8.27 vibrations per minute].

……. TO BE CONTINUED ……..

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