RATIOS AND PROPORTIONS

A ratio is a type of fraction which is used to compare two or more quantities of the same kind.

If two packets of detergent contain 375g and 500g respectively then the ratio of the smaller to the larger is $\frac{375g}{500g} = \frac{125\times3}{125\times4} = \frac{3}{4}$

We write the ratio as 3: 4. Note that the quantities must be of the same unit and that this unit is omitted in the final ratio.

Examples

1. Express each of the ratios in its simplest form.

a) Sh. 6,000 : sh. 9,000

$$\frac{sh.6,000}{sh.9,000} = \frac{6}{9} = \frac{2}{3} = \frac{2}{3}$$

b) $\frac{2}{3}$ kg: 600g $= \frac{\frac{2}{3}}{0.6} = \frac{0.7\times10}{0.6\times10} = \frac{7}{6}$

c) 200m : 1km $= \frac{2\cancel{00}}{1\cancel{000}} = \frac{2}{10} = \frac{1}{5}$.

Division in a given ratio.

Examples

1. Divide Shs. 12,000 between Adhiambo , Angrero and Asanda in the ratio 3: 5 : 7.

Soln.

Total ratio = 3+ 5 + 7

= 15

Adhiambo : Angrero : Asanda.

3 : 5 : 7

For Adhiambo $= \frac{3}{15} \times 12{,}000$

$= 3 \times 800$

$= \underline{\underline{2400/=}}$

For Angwero $= \frac{5}{15} \times 12{,}000$

$= 5 \times 800$

$= \underline{\underline{4{,}000/=}}$

For Asanda $= \frac{7}{15} \times 12{,}000$

$= 7 \times 800$

$= \underline{\underline{5{,}600/=}}$

2. Divide shs. 2,400 in the ratio 6: 3: 1.

<u>Soln.</u>

Total ratio $= 6 + 3 + 1.$

$= 10$

For 6 : $\frac{6}{10} \times 2400$

$= 6 \times 240$

$= \underline{\underline{1{,}440/=}}$

For 3 : $\frac{3}{\cancel{10}} \times 2400$

$= 3 \times 240$

$= \underline{\underline{720/=}}$

For 1 : $\frac{1}{10} \times 2400$ = 1×240

= $\underline{\underline{240/=}}$

3. A cake requires sugar, margarine, and flour to be mixed in the ratio 2: 3: 5. If the cake contains 750g of flour, how much sugar does it contain?

<u>Soln.</u>

Total ratio = 2 + 3 + 5

= 10

Let the quantity for the all cake be x

For flour : $\frac{5}{10} \times x = 750$

$$\frac{5x}{10} = \frac{750}{1}$$

$5x = 7500$

$$x = \frac{7500}{5}$$

$x = 1500$

The cake contains <u>1,500g</u>

So , for sugar : $\frac{2}{10} \times 1500$

= 2×150

= $\underline{\underline{300g}}$

¬¬¬ For Daniel ; $\frac{\frac{3x}{2}}{5x} \times 6{,}000$

$$= \frac{\cancel{3x}}{\cancel{10x}} \times 6{,}000$$

$$= \frac{3}{\cancel{10}} \times 6\cancel{,000}$$

For Daniel = <u>1,800/=</u>

For Diana ; $\dfrac{\frac{5x}{2}}{5x} \times 6{,}000$

$$= \frac{5\cancel{x}}{10\cancel{x}} \times 6{,}000$$

$$= \frac{5}{1\cancel{0}} \times 6{,}00\cancel{0}$$

$= 5 \times 600$

For Diana = <u>3,000/=</u>

In the ratio form : David ; Daniel ; Diana

$x \; ; \; \frac{3}{2}x \; ; \; \frac{5}{2}x$

$; \; 1{,}200 \; ; \; \frac{3}{2} \times 1200 \; ; \; \frac{5}{2} \times 1{,}200$

; 1,200 ; 1,800 ; 3000

4. Shs. 6,000 is to be shared among David, Daniel and Diana. Daniel is to get one and a half as much as David, while Diana is to get three and a half times as much as David. Determine the ratio in which the money is to be shared.

<u>Soln.</u>

Let the share got by David be x.

now, Daniel; $1\frac{1}{2} \times x = \frac{3}{2} \times x = \frac{3}{2}x.$

Diana; $3\frac{1}{2} \times x = \frac{5}{2} \times x = \frac{5}{2}x.$

In the ratio form ; David : Daniel : Diana

$$x \quad : \quad \frac{3}{2}x \quad : \quad \frac{5}{2}x.$$

Now, total ratio $= \frac{x}{1} + \frac{3x}{2} + \frac{5x}{2}$

$$= \frac{2x + 3x + 5x}{2}$$

$$= \frac{10x}{2}$$

Total ratio $= 5x$

For David; $\frac{x}{5x} \times 6{,}000$

$$= \frac{1}{5} \times 6000 = \underline{\underline{1{,}200/=}}$$

$$\frac{1{,}2\cancel{00}}{1\cancel{00}} : \frac{1{,}8\cancel{00}}{1\cancel{00}} : \frac{3{,}0\cancel{00}}{1\cancel{00}}$$

$$\frac{12}{6} : \frac{18}{6} : \frac{30}{6}$$

$$\underline{\underline{2 : 3 : 5}}$$

∴ the ratio in which the money was shared is

$\underline{\underline{2 : 3 : 5}}$

5. Three boys; Lugemwa, Brandon and Gyavira shared shs. 10,500. Gyavira got twice as

much as Brandon and Brandon got twice as much as Lugemwa. Find how much money the three boys got.

Soln.

Let the amount of money received by Lugemwa be y.

So Brandon got $2 \times y = 2y$.

And Gyavira got $2 \times (2y) = 4y$.

The total amount of money shared =10,500.

$$\Rightarrow \quad y + 2y + 4y = 10{,}500.$$

$$7y = 10{,}500$$

$$y = \frac{10{,}500}{7}$$

$$\underline{y = 1{,}500/=}$$

$\Rightarrow$ so Lugemwa got shs. 1,500

Brandon got shs. $2 \times 1500 = \underline{\underline{3{,}000/=}}$

and Gyaviira got shs. $4 \times 1500 = \underline{\underline{6{,}000/=}}$

Increasing and Decreasing ratios.

Increasing Ratios;

To increase a given ratio say a : b means new/big value over old value;

$$\text{So new value} = \frac{a}{b} \times \text{given value.}$$

Examples

1. Increase shs. 6,000 in the ratio 6 : 5

Soln.

So new shillings = $\frac{6}{5} \times 6{,}000$

= $6 \times 1{,}200$

$\Rightarrow$ new shillings = $\underline{\underline{7{,}200/=}}$.

2. Increase $48cm^3$ in the ratio 4: 3.

Soln.

New value = $\frac{4}{3} \times 48$

$\Rightarrow$ new value = 4×16

$\underline{\underline{= 64cm^3}}$.

Decreasing Ratios.

Example.

1. Decrease shs. 6,000 in the ratio 5: 6.

Soln.

So new shillings = $\frac{5}{6} \times 6{,}000$

= 5×1000

$\Rightarrow$ new shillings = $\underline{5{,}000/=}$.

2. Decrease $48cm^3$ in the ratio 3: 4.

soln.

new value = $\frac{3}{4} \times 48$

$\underline{\underline{= 36cm^3}}$.

Percentage Increase and Decrease.

Examples.

1. Increase shs. 1,200 by 25%

%tage increase = (100+25)%

$$= \frac{125}{100} \times 1,200$$

$$= 125 \times 12$$

$$= \underline{\underline{\text{shs. } 1500.}}$$

2. Increase 600cm3 by $33\frac{1}{3}\%$

Soln.

%tage increase $= \left(33\frac{1}{3} + 100\right)\%$

$$= 133\frac{1}{3}\%$$

$$= \frac{133\frac{1}{3}}{100} \times 600$$

$$= 799.9999$$

$$\approx \underline{\underline{800cm^3}}$$

<u>Percentage decrease.</u>

<u>Examples.</u>

1. Decrease shs. 16,000 by $17\frac{1}{2}\%$.

%tage decrease $= \left(100\text{-}17\frac{1}{2}\right)\%$

$$= \left(\frac{100}{1} - \frac{35}{2}\right)\%$$

$$= \left(\frac{200 - 35}{2}\right)\%$$

$$= \frac{165}{2}\%$$

$$= \frac{\frac{165}{2}}{100} \times 16{,}000$$

$$= \frac{165}{200} \times 16{,}000$$

$$= \underline{\underline{13{,}200/=}}$$

2. Decrease 7200kg by 15%

   %tage decrease = (100-15)%

   $$= \frac{85}{100} \times 7200$$

   $$= 85 \times 72$$

   $$= \underline{\underline{6120\text{kg}}}$$

3. The price of a car in a show room is shs. 8,000,000. Determine the price of the car of the price is used by 20%.

   Soln.

   %tage decrease = (100– 20) %

   $$= \frac{80}{100} \times 8{,}000{,}000$$

   $$= 80 \times 80{,}000$$

   $$= \underline{\underline{\text{shs. } 6{,}400{,}000}}$$

4. Senior Kats bought a premio new module at Shs. 12,000,000. He wants to sell it at a 15% increment in its price. Determine how much he sells the car.

   Soln.

   Percentage increase = (100+15) %

   = 115%

$$= \frac{115}{\cancel{100}} \times 12{,}000{,}\cancel{00}0$$

$$= 115 \times 120{,}000$$

He sold the car at = <u>shs. 13,800,000</u>

REPRESENTATIVE FRACTIONS.

At the bottom of every map, there is a scale usually given as a ratio e.g. 1: 200,000. This means that every unit on the map represents 200,000 units on the ground. Sometimes the scale will be given as a fraction. This is called the representative fraction (R.F).

If 1cm on a map represents 10,000cm on the ground, then the ratio 1: 10,000 is called the representative fraction (R.F).

Representative fraction can also be written as $\frac{1}{10{,}000}$.

Representative fractions are expressed in the form 1: n, where n is a positive integer.

Examples

1. The scale on a map of Uganda 5cm represents 1km. what is the representative fraction of this map?

Soln.

$\Rightarrow$ 5cm represent 1km

Written as 5cm : 1km.

But 1km = 100,000cm

So 5cm : 100,000cm.

Dividing through by a smallest divisor on both sides;

$$\frac{\cancel{5}\text{cm}}{\cancel{5}\text{cm}} : \frac{100{,}\cancel{0}00\text{cm}}{\cancel{5}\text{cm}}$$

$\therefore$ The representative fraction $= \frac{1}{20{,}000}$.

2. On a map of Africa, the distance between two cities which are 700km apart is

represented by a straight line o length 3•5cm. Find the representative fraction of this map.

Soln.

3•5cm represent 700km (converting to the same units).

But 1km = 100,000cm

$\Rightarrow$ 3•5cm : 700× 100,000cm

3•5cm : 70,000,000cm

$$\frac{\cancel{3\cdot 5}cm}{\cancel{3\cdot 5}cm} : \frac{70,00\cancel{0},000cm}{\cancel{3}\cdot 5cm}$$

1 : 20,000,000

$\Rightarrow$ 1cm represents 20,000,000cm

$\therefore$ The representative fraction $= \dfrac{1}{20,000,000}$

3. The straight line distance on an atlas whose representative fraction (R.F) is 1 : 10,000,000 between Kampala and Nairobi is 5•1cm. what does this give for the actual distance in km between the two cities.

Soln.

Representative fraction; R.F = 1 : 10,000,000

= 1cm : 10,000,000cm.

$\Rightarrow$ if 1cm : 10,000,000cm.

5• 1cm. y

1×y : 5•1× 10,000,000

y : 51,000,000cm.

$\Rightarrow$ 5•1cm = 51,000,000cm.

But 1km = 100,000cm

x = 51,000,000cm

$$100{,}000x = 51{,}000{,}000.$$

$$x = \frac{51{,}\cancel{000{,}000}}{\cancel{100{,}000}}$$

$$x = 510\text{km}.$$

$\therefore$ the actual distance in km = <u>510km.</u>

4. The scale of map is 1: 25,000. The distance between two schools on the map is 8 cm. find the actual distance in km, between the two schools.

<u>Soln.</u>

Representative fraction (R.F) = 1: 25,000

= 1cm: 25,000cm.

$\Rightarrow$ 1cm : 25,000cm.

8cm : x

X : 8× 25,000

X : 200,000cm.

$\Rightarrow$ 8cm : 200,000cm.

But 1km = 100,000cm.

Y = 200,000cm

$$100{,}000y = 200{,}000$$

$$Y = \frac{\cancel{200{,}000}}{\cancel{100{,}000}}$$

$$Y = 2\text{km}.$$

$\therefore$ the actual distance in km = <u>2km.</u>

5. A map is drawn to a scale of 1: 10,000. What length in cm on the map represents 5km on land.

<u>Soln.</u>

1 : 10,000

$\Rightarrow$ 1cm: 10,000cm.

But 1km = 100,000cm.

5km = y

Y = 5× 100,000.

= 500,000cm.

But 1cm : 10,000cm.

x : 500,000cm

10,000x = 500,000

$$x = \frac{500000}{10000}$$

x = 50cm.

6. Given that the representative fraction of a map is $\frac{1}{250,000}$, find the length of a horizontal road on the map whose length on the ground is 25km long.

Soln.

Representative fraction (R.F) $= \frac{1}{250,000}$,

= 1 : 250,000

$\Rightarrow$ 1cm : 250,000cm.

But 1km = 100,000cm.

66.25km = y

y = 66.25km × 100,000

= 6625, 000cm.

$\Rightarrow$ 66.25km = 6,625,000cm.

But 1cm : 250,000cm

$$x \quad : \quad 6,625,000$$

$$250,000x \quad : \quad 6,625,000$$

$$x \quad : \quad \frac{6,6\cancel{25,}000}{2\cancel{50,000}}$$

$$x = 26.5 \text{ cm.}$$

$\therefore$ the length of a horizontal road = $\underline{\underline{26 \cdot 5\text{cm.}}}$

AREAS:

Examples.

1. If an area of $4cm^2$ on a map represents an area of 576km on land, find the representative fraction (R.F) of the map.

Soln.

$4cm^2$ represents $576km^2$, since they are in square

$$\Rightarrow \sqrt{4cm^2} : \sqrt{576km^2}$$

$$\sqrt{4} \times \cancel{\sqrt{cm^2}} \quad : \quad \sqrt{576} \cancel{\times} \sqrt{\cancel{cm}^2}$$

$$2\times cm \quad : \quad 24\times km$$

$$2cm \quad : \quad 24km. \text{ (Converting to the same units)}$$

$$1km = 100,000cm.$$

$$24 = y$$

$$Y = 24 \times 100,000$$

$$\Rightarrow 24km = 2400,000cm.$$

So, 2cm : 2400, 000cm.

$$\frac{\cancel{2cm}}{\cancel{2cm}} : \frac{2400,\cancel{0}00cm}{\cancel{2}cm}$$

$$1 \quad : \quad 1,200,000$$

The representative fraction (R.F) $= \dfrac{1}{1,200,000}$

2. Given that $25cm^2$ are represented by $49km^2$ on the map. Determine the representative fraction (R.F).

$25cm^2$ representative $49km^2$

$\Rightarrow \quad \sqrt{25cm^2} \quad : \quad \sqrt{49km^2}$

$\sqrt{25} \times \cancel{\sqrt{cm^2}} \quad : \quad \sqrt{49} \times \cancel{\sqrt{km^2}}$

$5 \times cm \quad : \quad 7 \times km$

$5cm \quad : \quad 7km$ (converting to the same units)

But $1km = 100,000cm$.

$7km = x$

$x = 7 \times 100,000cm$.

$\Rightarrow \quad 7km = 700,000cm$.

So, $5cm : 700,000cm$.

$$\frac{\cancel{5cm}}{\cancel{5cm}} : \frac{700,\cancel{000}cm}{\cancel{5}cm}$$

$1 : 140,000$.

$\therefore$ the representative fraction $= \dfrac{1}{140,000}$.

3. A map has a scale of 1: 250,000. The area of the swamp on the map is $12cm^2$, what is the actual area of the swamp in km^2?

<u>Soln.</u>

Representative fraction (R.F) = 1: 250,000.

$\Rightarrow$ 1cm: 250,000cm.

In terms of area,

$1cm \times 1cm: 250,000cm \times 250,000cm$.

$1cm^2 \quad : \quad (250,000\times 250,000)$ cm2

$12cm^2 \quad : \quad y$

$y \quad : \quad 12\times 250,000\times 250,000$

But $\quad 1km \quad = \quad 100,000cm$

$1km^2 \quad = \quad (100,000\times100,000)\ cm^2$

$y \quad = \quad 12\times250,000\times250,000$

$(100,000\times100,000)\ y \quad = \quad 12\times250,000\times250,000$

$$y = \frac{12\times 25\cancel{0,000}\times 25\cancel{0,000}}{10\cancel{0,000}\times 10\cancel{0,000}}$$

$$y = \frac{12\times25\times25}{10\times10}$$

$$y = \frac{75\cancel{00}}{1\cancel{00}}$$

$y \quad = \quad 75km^2$

$\therefore$ Actual area of the swamp $= 17km^2$.

4. The scale on a map is 1: 200. A building is represented on the map by an area of 3cm. Find the actual area in km^2 occupied by the building.

Soln.

R.F $\quad = 1 \quad : \quad 200$

$\Rightarrow \quad 1cm \quad : \quad 200cm.$

In terms of area

$1cm \times 1cm: \ 200cm \times 200cm$

$1cm^2 \quad : \quad (200 \times 200)\ cm^2$

$1cm^2 \quad : \quad (200\times200)cm^2$

$3cm^2 \quad : \quad x$

$x = 3\times200\times200$

$\Rightarrow \quad 3cm^2 : (3\times200\times200)cm^2$

But $\quad 1km = 100{,}000cm$

$1km^2 = (100{,}000\times100{,}000)\ cm^2$

$y = 3\times200\times200$

$(100{,}000\times100{,}000)y = 3\times200\times200$

$$y = \frac{3\times\cancel{200}\times\cancel{200}}{100{,}\cancel{0}00\times10\cancel{0}{,}000}$$

$$y = \frac{3\times2\times2}{1000\times1000}$$

$$y = \frac{12}{1{,}000{,}000}$$

$y = 0{\cdot}000012$

$\therefore$ Actual area in km^2 = $\underline{0{\cdot}000012km^2}$

Or $\underline{1{\cdot}2\times10^{-5}km^2}$.

5. The farm is on the piece of land whose area is $5{\cdot}6km^2$. What would be the area of this arm in cm^2 on a map whose scale is 1: 40,000?

<u>Soln.</u>

Representative fraction (R.F) = 1: 40,000.

$\Rightarrow \quad 1cm: \ 40{,}000cm.$

In terms of area,

$1cm\times1cm \ : \ 40{,}000\times40{,}000$

$1cm^2: (40{,}000\times40{,}000)\ cm^2.$

But $\quad 1km = 100{,}000cm$

$1km^2 = (100{,}000\times100{,}000)\ cm^2$

$5{\cdot}6km^2 = y$

$$y = 5{\cdot}6 \times 100{,}000 \times 100{,}000$$

$$\Rightarrow \quad 5{\cdot}6km^2 = (5{\cdot}6 \times 100{,}000 \times 100{,}000).$$

If $1cm^2 \quad : \quad (40{,}000 \times 40{,}000)cm^2$

$x \quad : \quad 5{\cdot}6 \times 100{,}000 \times 100{,}000$

$$(40{,}000 \times 40{,}000)\, x = 5{\cdot}6 \times 100{,}000 \times 100{,}000$$

$$x = \frac{5{\cdot}6 \times \cancel{100{,}000} \times \cancel{100{,}000}}{\cancel{40{,}000} \times \cancel{40{,}000}}$$

$$x = \frac{5{\cdot}6 \times 100}{4 \times 4}$$

$$x = \frac{560}{16}$$

$$x = 35cm^2$$

$\therefore$ The area of arm in cm^2 $= 35cm^2$

6. The scale of a map is 1: 500. The distance between A and B is $180km^2$. Find the actual area of the land in cm^2.

<u>Soln.</u>

1: 500

$\Rightarrow$ 1cm: 500cm

In terms of area

1cm ×1cm : 500cm×500cm

$1cm^2 \quad : \quad (500 \times 500)cm^2$

But 1km = 100,000cm

$1km^2 = (100{,}000 \times 100{,}000)cm^2$

$180km^2 = y$

$$y = 180 \times 100{,}000 \times 100{,}000$$

$\Rightarrow \quad 180km^2 = 180 \times 100{,}000 \times 100{,}000$

If $\quad 1cm^2 \quad : \quad (500 \times 500)cm^2$

$x \quad : \quad (180 \times 100{,}000 \times 100{,}000)\ cm^2$

$(500 \times 500)x = 180 \times 100{,}000 \times 100{,}000$

$$x = \frac{180 \times 100{,}000 \times 100{,}000}{500 \times 500}$$

$$x = \frac{180 \times 1000{,}000}{25}$$

$$x = 7{,}200{,}000cm^2$$

$\therefore$ The Actual area of land = $\underline{\underline{7{,}200{,}000cm^2}}$

PROPORTIONS.

When we look at proportions, we are looking at things which can change. We need to know how things change. This is called the rate of change.

Examples

1. A car travels 240km in 4 hours at a steady speed. Find the rate.

Soln.

The rate is $= 240 \div 4$

$= \underline{\underline{60km/hr.}}$

2. Tomatoes cost shs. 120 per pile. How much work 5 piles cost.

Soln.

5 piles will cost shs. 5×120

= sh. 600 .

3. A school generator uses 5 litres of fuel in a 12hour day. Find the rate.

Soln.

The rate is $5 \div 12$

$$= \frac{5}{12} \text{ litres per hour}$$

Using the Rate

1. 15 litres of kerosene costs Sh. 8250. How much will 8 litres cost.

Soln.

15 litres cost sh. 8250

The rate is $8250 \div 15$

$$= \frac{8250}{15}$$

= sh. 550 per litre.

So, 8 litres will cost 8×550

= sh.4400

2. A girl walks 80 metres in 20 seconds. At the same rate , how far will she walk in

(a) 10 seconds. (b) 50 seconds

Soln.

So metres in 20 seconds.

The rate = $80 \div 20$

= 4 metres per second.

(a) 10 seconds = 4×10

= 40 metres

(b) 50 seconds = 4×50

= 200 metres

FRACTIONAL METHOD

Examples.

1. A motor cycle travels 240km in 3 h0urs. How far does it go in 2 hours ?

Soln.

2hour is less than 3 hour.

Hence , required distance is less than 240km.

Let the distance in 2 hour to be x

3hrs → 240km

2hrs → xkm.

$$\frac{3x}{3} = \frac{2\times240}{3}$$

$$x = \left(\tfrac{2}{3}\times240\right)\text{km}$$

$$x = \underline{\underline{160\text{km.}}}$$

2. It takes 4 hours to drive 480km. how long would it takes to drive 1080km at the same speed.

Soln.

Let the time it takes to drive 1080 km be x.

⇒ 4hours → 480km.

x → 1080km.

An cross multiplying

$$480x = 4\times1080$$

$$x = \frac{4\times\cancel{1080}}{\cancel{480}}$$

$$x = 9\text{ hours.}$$

So the time taken to drive 1080km = $\underline{\underline{\text{9 hours.}}}$

3. Five men take 14 days to do a job. How long would seven men take to do the same job?

Soln.

5men take 14 days

1man takes $\left(\frac{5\times14}{1}\right)$ days.

$\therefore$ 7men take $\left(\frac{5\times14}{7}\right)$ days.

$= \frac{70}{7}$

$\therefore$ 7 men = $\underline{\underline{\text{10 days.}}}$

4. A family of 4 people has just enough food to last 15days. How long will the food last if 2 more people join the family?

Soln.

4 people take 15 days.

1 person will $\left(\frac{4\times15}{1}\right)$ days.

$= \left(\frac{4\times15}{1}\right)$ days.

Also (4+2)people will take $\left(\frac{4\times15}{1}\right)$ days.

6 people $= \frac{4\times15}{6}$

$= \frac{60}{6}$

It will take = $\underline{\underline{\text{10days.}}}$

5. If 16 girls can sort 800 letters in 15 minutes, how many girls would be needed to sort

1400 letters in 21minutes?

Soln.

Method1

	Girls	minutes	letters
1^{st} Information;	16	15	800
	1	1	$\frac{800}{15}$

$$\frac{\frac{800}{15}}{16} = \frac{800}{15\times16} = \frac{800}{240}$$

	Girls	minutes	letters
2^{nd} Information;	x	21	1400
	1	1	$\frac{1400}{21}$

$$\frac{\frac{1400}{21}}{x} = \frac{1400}{21x}.$$

$$\Rightarrow \quad \frac{1400}{21x} = \frac{800}{240}$$

$21x \times 800 = 240 \times 1400$

$16800x = 336000$

$$x = \frac{336000}{16800}$$

$x = 20$ girls.

$\therefore$ 20 girls will be needed.

6. If 25men together can clear $4800m^2$ of bush in three days. What area would 33 men

clear in five days?

<u>Soln</u>.

1st information;	men	days	bush.
	25	3	4800
	1	1	$\frac{4800}{3}$

$$= \frac{\frac{4800}{3}}{25}$$

$$= \frac{4800}{3\times 25} = \frac{4800}{75}$$

2nd information;	men	days	bush
	33	5	y
	1	1	$\frac{y}{5}$

$$\frac{\frac{y}{5}}{33}$$

$$= \frac{y}{5\times 33} = \frac{y}{165.}$$

$$\Rightarrow \frac{y}{165} = \frac{4800}{75}$$

$75y = 165\times 4800$

$75y = 792{,}000$

$$y = \frac{792{,}000}{75}$$

$y = 10560m^2$

$\therefore$ Area will be 10560m^2 of bush.

Direct and Indirect / Inverse proportions.

Direct proportion.

When two quantities, say x and y are n direct proportion, they can be written as

$y = mx + c$. Where m is the gradient which is also the rate of proportion.

The y intercept c is always O.

So,

when two quantities are in direct proportion they can be written;

$y = kx$ (where k is the rate of proportion)

Or k is called the proportionality constant.

Also we can write $y \propto x$,

Where $\propto$ is a sign of proportion.

We read $y \propto x$ as "y is directly proportional to x".

Examples.

1. Two quantities x and y are directly proportion when x = 3.5, y = 28. Find the value of x. when y = 20.

Soln.

We know that $y \propto x$

So, $y = kx$ for some no k.

$\Rightarrow \quad y = kx$

$28 = k \times 3{\cdot}5$

$$\frac{28}{3{\cdot}5} = \frac{3{\cdot}5k}{3{\cdot}5}$$

$K = {}^{28}\!/_{3{\cdot}5} = 8$

Now, the value of x when y = 20 is

$y = kx$

$y = 8x$

$$\frac{20}{8} = \frac{\cancel{8}x}{\cancel{8}}$$

$$x = \frac{20}{8} = 2{\cdot}5.$$

So $x = 2{\cdot}5$ when $y = 20$.

2. If p varies directly as q^2 and $p = 6$ when $q = 3$. Find p when $q = 18$.

<u>Soln.</u>

We know that $p \propto q^2$

So, $p = kq^2$

$6 = k(3)^2$

$$\frac{6}{9} = \frac{9k}{9}$$

$$K = \frac{6}{9} = \frac{2}{3}$$

Now, $p = kq^2$

$p = \frac{2}{3} \times (18)^2$

$p = \frac{2}{3} \times 324$

$p = \underline{\underline{216.}}$

3. If p varies as a cube of q and $p = 4$ when $q = 2$, find p when $q = 4$.

<u>Soln.</u>

$p \propto q^3$

$p = kq^3$

$4 = k(2)^3$

$$\frac{4}{8} = \frac{8k}{8}$$

$$k = \cancel{\frac{4}{8}} = \frac{1}{2}$$

Now, $p = kq^3$

$P = \frac{1}{2}(4)^3$

$= \frac{1}{2} \times 64$

$P = \underline{\underline{32.}}$

4. The resistance to motion in the air varies directly as the square o the speed of the object travelling through it. If a bullet is travelling at 360km|hr. when the air resistance is 2000N. Find the air resistance when an object is travelling at 600km|hr.

Soln.

$\propto s^2$

$= ks^2$

$2000 = k\,(360)^2$

$$K = \frac{2000}{(360)^2}$$

$$= \frac{2000}{129600}$$

$K = 0{\cdot}015432098$

$= ks^2$

$= 0{\cdot}015432098\,(600)^2$

$= 5555{\cdot}5555556.$

Air resistance $= \underline{\underline{5556N}}$

Inverse proportion.

When one quantity increases, the other quantity decreases.

Direct proportion involves one quantity being the product of the other and a fixed constant.

ie, $y \propto \frac{1}{x}$

$y = \frac{k}{x}$, for some constant k.

$\Rightarrow$ $y \propto \frac{1}{x}$, for " y is inversely proportional to x"

Examples.

1. is inversely proportional to b. when = 12, b = 3, find;

a. The constant of the proportionality and

b. The value of when b = 10.

<u>Soln.</u>

(a) $\propto \frac{1}{b}$

$= \frac{k}{b}$, for some k,

$\frac{12}{1} = \frac{k}{3}$

K = 3×12 = 36

So, the constant of proportionality is <u>36.</u>

(b.) $= \frac{k}{b}$

$= \frac{36}{10}$

$= \underline{3{\cdot}6.}$

2. A quantity y is inversely proportional to x^3. When x = 2, y = 5. Find y when x = 2•5 and x when y = 0•625

Soln.

$$y \propto \frac{1}{x^3}$$

$$y = \frac{k}{x^3}$$

$$5 = \frac{k}{2^3}$$

$$\frac{5}{1} = \frac{k}{8}$$

$$K = 5 \times 8$$

$$K = 40$$

$$\Rightarrow \quad y = \frac{k}{x^3}, \quad \text{but } x = 2{\cdot}5$$

$$y = \frac{40}{(2{\cdot}5)^3} = \frac{40}{15{\cdot}625}$$

$$y = \underline{\underline{2{\cdot}56.}}$$

$$\text{Also,} \quad y = \frac{k}{x^3}$$

$$0{\cdot}625 = \frac{40}{x^3}$$

$$x^3 = \frac{40}{0{\cdot}625}$$

$$x^3 = 64$$

$$\sqrt[3]{x} \qquad \sqrt[3]{64}$$

$\underline{\underline{x \quad = \quad 4.}}$

3. It takes ten men eight hours to build a wall. How long would it take three men to build the same wall at the same rate?

Soln.

Method 1;

10 men take 8 hours.

1 man takes $\frac{(10\times8)}{1}$ days.

$\therefore$ 3 men take $\frac{(10\times8)}{3}$

$= \frac{80}{3}$

$\Rightarrow$ 3men will take 26 hours 40 minutes.

Method II.

- No of men has decreased, so the no of hours will increase.
- Let the three men take t hours. So, No of men is inversely proportional to the number of hours worked.
- Let m = men and H = hours.

$$m \propto \frac{1}{H}$$

$$m = \frac{k}{H}$$

$$10 = \frac{k}{8}$$

$$K = 8\times 10 = 80.$$

$$\Rightarrow m = \frac{k}{H}$$

$$\frac{3}{1} = \frac{80}{H}$$

$$3H = 80$$

$$H = \frac{80}{3}$$

$$= 26\tfrac{2}{3}$$

$\Rightarrow$ 3 men will take <u>26 hours 40 minutes.</u>

4. The height of triangles of the same area varies inversely as the length of their bases. When the base = 6cm long, the height is 12cm. find the height of a triangle when the base is 15cm.

<u>Soln.</u>

Let h = height and L = length / base

$$h \propto \frac{1}{h}.$$

$$h = \frac{k}{h}.$$

$$\frac{12}{1} = \frac{k}{6}.$$

$$K = 6 \times 12 = 72.$$

So, $$h = \frac{k}{h}$$

$$h = \frac{72}{15}$$

$$h = 4{\cdot}8$$

$\Rightarrow$ height $\simeq$ <u>5cm.</u>

<u>JOINT AND PARTIAL VARIATION / PROPORTION</u>

JOINT VARIATION.

- When the quantity y varies jointly as p and n,

We can write; $y \propto p$ ————(i)

Also; $y \propto n$ ————(ii).

Combining equations (i) and (ii) jointly.

$\Rightarrow$ $y \propto pn.$

$y \propto pn$ ————(iii)

$y = kpn$

Where k is a constant of proportion

This is called a joint variation.

- A quantity may be proportional to more than one other quantity. eg z is directly proportional to x and is inversely proportional to yQ.

$\Rightarrow$ $z \propto x$ ------------(i)

And $z \propto \frac{1}{y^2}$ ----------(ii)

Combining equations (i) and (ii) jointly

$$z \propto \frac{x}{y^2} \text{-----------(iii).}$$

$$z = \frac{kx}{y^2} \text{, for some constant k,}$$

Examples

1. If z varies directly as the square of x and inversely as y and z = 5 when x =2 and

y =6, Find x when z = 27 and y 2• 5.

Soln.

$z \propto x^2$----------(i)

$$z = \frac{1}{y} \text{ ----------(ii)}$$

Combining the two equations (i) and (ii) jointly;

$$z \propto \frac{x^2}{y} \qquad \text{(iii).}$$

$$\Rightarrow z = \frac{kx^2}{y} \text{, for some constant } k$$

$$5 = \frac{k\,(2)^2}{6}$$

$$\frac{5}{1} = \frac{4k}{6}$$

$$4k = 5 \times 6$$

$$4k = 30$$

$$k = \frac{30}{4}$$

$$\underline{k = 7{\cdot}5}$$

$$\Rightarrow \quad z = \frac{kx^2}{y}$$

$$\frac{27}{1} = \frac{7{\cdot}5x^2}{2{\cdot}5}$$

$$7{\cdot}5x^2 = 27 \times 2{\cdot}5$$

$$7{\cdot}5x^2 = 67{\cdot}5$$

$$x^2 = \frac{67{\cdot}5}{7{\cdot}5}$$

$$x^2 = 9$$

$$\sqrt{x^2} = \pm\sqrt{9}$$

$$\underline{\underline{x = \pm 3.}}$$

2. It is given that P varies directly as a cube of Q and inversely as R and P =2 when Q =3 and R= 9, find Q when P =81 and R = 27 and P when Q =25 and R = 14.

<u>Soln.</u>

$P \propto Q^3$ ---------------(i)

$P \propto \frac{1}{R}$ ------------(ii)

Combining eq<u>n</u>s (i) and (ii) jointly;

$P \propto \frac{Q^3}{R}$ ------------- (iii)

$$P = \frac{KQ^3}{R}$$

$$\Rightarrow \quad 2 = \frac{K(3)^3}{9}$$

$$\frac{2}{K} = \frac{27K}{9}$$

$$27K = 2\times 9$$

$$K = \frac{18}{27}$$

$$K = {}^{2}\!/_{3} = 0{\cdot}667$$

$$\Rightarrow \quad P = \frac{0{\cdot}667Q^3}{R}$$

$$P = 81, \quad R = 27$$

$$\Rightarrow \quad \frac{81}{1} = \frac{0{\cdot}667Q^3}{27}$$

$$0{\cdot}667Q^3 = 27\times 81$$

$$0{\cdot}667Q^3 = 2187$$

$$Q^3 = \frac{2187}{0{\cdot}667}$$

$$Q^3 = 3278{\cdot}86057$$

$$\sqrt[3]{\cancel{Q^3}} = \sqrt[3]{3278{\cdot}86057}$$

$$\underline{\underline{Q = 14{\cdot}856.}}$$

Q = 25 and R = 14.

$$\Rightarrow \quad P = \frac{0{\cdot}667Q^3}{R}$$

$$P = \frac{0{\cdot}667(25)^3}{14}$$

$$= \frac{0{\cdot}667 \times 15{,}625}{14}$$

$$= \frac{10{,}421{\cdot}875}{14} \qquad P = \underline{\underline{744{\cdot}420.}}$$

Partial Variation.

- ✓ If y is partly constant and partly vares as x. then;

$$y = k \text{ ——— (i)}$$

and $y \propto k$ ——— (ii)

from eqn (ii) $y = Nx$ ——— (iii)

combining equation (i) and (iii)

partially;

$$y = k + Nx \text{ ——— (iv)}$$

This is a partial variation.

where k and N are constant.

Note;

(i) In partial variation, we introduce two constants which are not similar.

(ii) We add the two equations after removing the proportionality sign.

- ✓ If y varies partly as x and partly as the square of x.

Then ; $y \propto x$

And $y \propto x^2$

From eqn (i), $y = kx$ ——————(i)

From eqn (ii), $y = mx^2$ ——————(ii)

Combining equations (i) and (ii) partially

$$Y = kx + mx^2$$

Where k and m are constants.

- ✓ If P varies partly inversely to Q partly directly the square root of R.

Then ; $P \propto \frac{1}{Q}$

$P \propto \sqrt{R}$

$P = \frac{K}{Q}$ ——————(i)

$P = N\sqrt{R}$ ———(ii)

Combining equations (i) and (ii) partially

$$P = \frac{k}{Q} + N\sqrt{R}$$

Where k and N are constants.

<u>Examples.</u>

1. Given that y is partly constant and partly varies directly as the square of x and that y = 82 when x 5 and y = 19 when x 2, find y when x = 4.

<u>Soln.</u>

$y = k$ (i) and $y \propto x^2$

$y = Nx^2$ ——(ii)

Combining equations (i) and (ii), partially

$\Rightarrow \quad y = K + Nx^2$ (where k and N are constants)

$82 \quad K + N(5)^2$

$82 \quad K + 25N$ ———— (i)

$y = 19, \quad x = 2.$

$\Rightarrow \quad 19 = K + N(2)^2$

$19 = K + 4N$ ——— (ii)

Solving equation (i) and (ii) simultaneously.

$$\begin{array}{rl} & 82 = K + 25N \\ - & 19 = K + 4N \\ \hline & 63 = 21N \end{array}$$

[eliminating K from the equations.

$21N = 63$

$$N = \frac{63}{21}$$

$\underline{N = 3}$

$\Rightarrow$ substitute or N in eqn (ii)

$19 = K + 4N$

$19 = K \; 4 \times 3$

$19 = K + 12$

$K = 19 - 12$

$K = 7$

so, $\underline{y = 7 + 3x^2}$

when $x = 4$

$\Rightarrow \quad y = 7 + 3(4)^2$

$= 7 + 3 \times 16$

$y = \underline{\underline{55.}}$

2. The cost © of printing a copy of a news paper is partly constant and is also inversely

proportional to the number (n) of copies printed. When 200 copies are printed , the cost per copy is shs. 850. When 300 copies are printed the cost per copy is shs. 750.

a. Form an equation relating c and n.

b. Calculate the cost per copy when 150 copies are printed.

Soln.

$$c = k \qquad (i)$$

and $$c \propto \frac{1}{n}$$

$$c = m \times \frac{1}{n} = \frac{m}{n}$$

$$c = \frac{m}{n} \qquad (ii)$$

combining equations (i) and (ii),

partially ;

$$c = k + \frac{m}{n} \qquad (iii).$$

When n = 200, c = 850.

$$\Rightarrow \quad 850 = k + \frac{m}{200} \qquad (iv)$$

Also when n = 300, c = 750.

$$\Rightarrow \quad 750 \quad k + \frac{m}{300} \qquad (v)$$

Solving equations (iv) and (v) simultaneously.

$$850 = K + \frac{m}{200}$$ [eliminating K from the equations.

$$- \quad 750 = K + \frac{m}{300}$$

$$850 - 750 = \frac{m}{200} - \frac{m}{300}$$

$$100 = \frac{m}{200} - \frac{m}{300}$$

$$100 = \frac{300m - 200m}{60,000}$$

$$\frac{100}{1} \quad \frac{100m}{60,000}$$

$$100m = 6000,000$$

$$m = \frac{600,\cancel{0}00}{\cancel{1}00}$$

$\underline{m = 60,000}$

Substitute. For m into equation (v)

$$750 = k + \frac{m}{300}$$

$$750 = k + \frac{60,\cancel{0}00}{\cancel{3}00}$$

$$750 = k + 200$$

$$750 - 200 = k$$

$\underline{K = 550}$

a. $\Rightarrow \quad C = 550 + \frac{60,000}{n}$

b. When n = 150.

$$\Rightarrow \quad c = 550 + \frac{60,000}{150}$$

$$= 550 + 400 \qquad c = \underline{\underline{950/=}}$$

<u>Worked examples.</u>

1. Given that p varies directly as q2 and inversely as r and q = 4, r = 3, p = 24. Find the values of p when q = 6, r = 5.

<u>Soln.</u>

$p \propto q^2$ and $p \propto \frac{1}{r}$

$p = kq^2$ -------- (i) and $p = \frac{k}{r}$ ------------- (ii)

$$p = \frac{kq^2}{r}.$$

$p = 24, r = 3, q = 4$

$$p = \frac{kq^2}{r}.$$

$$24 = \frac{k(4)^2}{3}$$

$$\frac{16k}{16} = \frac{24\times3}{16}$$

$$k = 4{\cdot}5$$

$$\Rightarrow \quad p = \frac{kq^2}{r}.$$

$$= \frac{4{\cdot}5 \times (6)^2}{5}$$

$p \propto q^2$

$p \propto \frac{1}{r}$

$P \propto \frac{q^2}{r}$

$$P = \frac{kq^2}{r}$$

$$P = \frac{4{\cdot}5 \times 36}{5} \qquad = \quad 32{\cdot}4$$

2. If A varies directly as the cube of B and inversely as C and A=5 when B = 2 and C =6 , find B when A = 27 and C =7•5 .

<u>Soln.</u>

$A \propto B^3$ and $A \propto \frac{1}{C}$.

$A = kB^3$----------(i) and $A = \frac{K}{C}$ -----------(ii).

Combining ;

$$A = \frac{kB^3}{C}$$

$A \propto B^3$

$A \propto \frac{1}{C}$.

$A \propto \frac{B^3}{C}$

A =

Now,

$$5 = \frac{k(2)^3}{6}$$

$$5 = \frac{8k}{6}$$

$$\frac{8k}{8} = \frac{5 \times 6}{8}$$

$$k = 3{\cdot}75.$$

$$\Rightarrow \quad A = \frac{kB^3}{C}$$

$$27 = \frac{3{\cdot}75 \times B^3}{7{\cdot}5}$$

$$\frac{\cancel{3{\cdot}75}B^3}{\cancel{3{\cdot}75}} = \frac{7{\cdot}5 \times 27}{3{\cdot}75}$$

$$B^3 = \frac{202 \cdot 5}{3 \cdot 75} = 54.$$

$$\sqrt[3]{B^3} = \sqrt[3]{54}.$$

$$\underline{\underline{B = 3 \cdot 8.}}$$

3. I have coins of the same denomination and no other money. The number of burns b that I can buy varies directly as the number of coins c and inversely as the price p shillings per burn.

   It is given that b = 3 when c = 6 and p = 200. Find b when c = 5 and p = 100.

<u>Soln.</u>

$$b \propto c \text{ and } b \propto \frac{1}{p}$$

$$b = kc \text{------ (i) and } b = \frac{k}{p} \text{ ---------- (ii)}$$

Combining;

$$b = \frac{kc}{p}$$

$$3 = \frac{k \times 6}{200}$$

$$b \propto c$$

$$b \propto \frac{1}{p}$$

$$b \propto \frac{c}{p}$$

$$b =$$

$$\frac{6k}{6} = \frac{3 \times 200}{6}$$

$$= 100.$$

$$\Rightarrow \quad b = \frac{kc}{p}$$

$$= \frac{100 \times 5}{100}$$

$$\underline{\underline{b = 5}}$$

4. The cost per child in Kampala family is partly constant and partly inversely proportional to the number of children in the family. Given that the cost per child for a family of 10 is sh. 350 and or a family of 20 is sh. 300, find the cost per child for a family of;

   (i) 50 children (ii) n children.

<u>Soln</u>.

Let c = daily cost

n = family size.

$$c = d \text{ ---------- (i) and } c \propto \frac{1}{n}$$

$$c = d \text{ ----------(ii) and } c = \frac{k}{n} \text{ ------(iii)}$$

Combining the two equations partially;

$$C = d + \frac{k}{n}\text{, where d and k are constants.}$$

When n = 10 , c = 350

$$\Rightarrow \quad c = d + \frac{k}{n}$$

$$300 = d + \frac{k}{10} \text{ ------- (i)}$$

When n =20, c = 300

$$\Rightarrow \quad 300 = d + \frac{k}{20} \text{ ---------- (ii)}$$

Subtract (i) – (ii).

$$350 - 300 = \left(d + \frac{k}{10}\right) - \left(d + \frac{k}{20}\right)$$

$$50 = d - d + \frac{k}{10} - \frac{k}{20}$$

$$50 = \frac{2k-k}{20}$$

$$50 = \frac{k}{20}$$

$$k = 20 \times 50$$

$$= 1000$$

$\Rightarrow$ substitute k =1000 into eqn (i)

$$350 = d + \frac{1000}{10}$$

$$350 = d + 100$$

$$350 = 350 - 100$$

$$= 250.$$

From $c = d + \frac{k}{n}.$

$d = 250$, $k = 1000$

when $n = 50$,

$$c = 250 + \frac{1000}{n}$$

$$= 250 + \frac{100\cancel{0}}{5\cancel{0}}$$

$$c = 250 + 20$$

$$\underline{\underline{= 270.}}$$

5. Given that y varies as the cube of x and y = 8 when x = 4, find the value of x when y = 1.

Soln.

$Y \propto x3$

$Y = kx3$

$8 = k(4)3$

$8 = k \times 64$

$$\frac{8}{64} = \frac{64k}{64}$$

$$k = \frac{1}{8}$$

so , $y = kx3$

$$8 \times 1 = \frac{1x^3}{\cancel{8}} \times \cancel{8}$$

$8 = x3$

$X3 = 8$

$$\sqrt[3]{\cancel{x^3}} = \sqrt[3]{8}$$

$x = 2.$

6. The depth (H) is directly proportional to the volume (V) and inversely proportional to the area of the cross section (A) of the tank. If (H) = 100 when V = 60 and A = 2•88, find H when V = 35 and A =3.

Soln.

$$H \propto V \text{ and } H \propto \frac{1}{A}.$$

$$H = Kv \text{ ---------- (i) and } H = \frac{k}{A.} \text{ --------- (ii).}$$

Combining the two eqns.

$$H = \frac{kV}{A.}$$

When H =100, V = 60, A = 2•88.

$$\Rightarrow \quad 100 = \frac{k \times 60}{2{\cdot}88}$$

$$60k = 100 \times 2{\cdot}88$$

$$\frac{\cancel{60}k}{\cancel{60}} = \frac{288}{60}$$

$$K = 4{\cdot}8$$

So, $H = \frac{kV}{A.}$.

$$= \frac{4{\cdot}8 \times 35}{3}$$

$$H = \underline{\underline{56}}$$

7. Ouma takes 20 days to plough a garden; Mukasa takes 30 days to plough the same garden. How long will take the two men to plough the garden if they worked together.

<u>Soln.</u>

In 20 days Ouma ploughs the whole garden.

In 1 day, Ouma ploughs $\frac{1}{20.}$ of the garden.

In 30 days, Mukasa ploughs the whole garden

In 1 day Mukasa ploughs $\frac{1}{30.}$ of the garden.

So, in 1 day, Ouma and Mukasa together plough $\left(\frac{1}{20} + \frac{1}{30}\right)$ of the garden.

Now, $\frac{1}{20} + \frac{1}{30} = \frac{3+2}{60} = \frac{5}{60}$

$$= \frac{1}{12}$$

Thus in 1 day Ouma and mukasa together plough $\frac{1}{12}$ of the garden.

Now, $\frac{1}{12}$ of the garden is ploughed by Ouma and Mukasa in 1 day.

So,

We can write.

$\frac{1}{12}$ Corresponds to 1 day.

Let 1 (whole garden) correspond to x days

$$\frac{1}{12} = \frac{1}{x}$$

$$\frac{1}{12} = \frac{1}{x}$$

$$1 \times x = 12 \times 1$$

$$x = 12.$$

Hence if Ouma and Mukasa worked together, it would take them 12 days to plough the garden.

8. (a). A man gave half of his welfare allowance to his wife, $\frac{1}{5}$ to each of his two sons and the rest to his daughter.

Find

(i) The fraction given to the daughter

(ii) His welfare allowance if each son was given shs. 16,000.

(b) The difference btn the values of y when x = 6 and when x = 10 is 16. Given that y is inversely proportional to the square of x. Find the equation relating x and y.

Soln.

a (i). Let the fraction given to the daughter be x

$$\Rightarrow \quad x + \frac{1}{2} + \left(\frac{1}{5}\times 2\right) = 1$$

$$x + \frac{1}{2} + {}^{2}/_{5} = 1$$

$$x = 1 - \left(\frac{1}{2}+\frac{1}{5}\right)$$

$$x = 1 - \left(\frac{5+4}{10}\right)$$

$$= \frac{10-5-4}{10}$$

$$= \frac{10-9}{10}$$

$$x = \frac{1}{10}$$

Hence the fraction given to the daughter is $\frac{1}{10}$

(iii) Let his welfare allowance be y.

$$\text{So, } \frac{1}{5} \text{ of } y = 16000$$

$$5 \times \frac{1}{5} \times y = 16000 \times 5$$

$$y = 16000 \times 5$$

$y = 80,000$

so, his welfare allowance = <u>sh. 80,000.</u>

(b). let $y = y_1$ when $x = 6$

and $y = y_2$ when $x = 10$

so, y1 and y2 = 16------------ (i)

Given that $y \propto \frac{1}{x^2}$

$$y = \frac{k}{x^2}, \text{ k is a constant of proportionality.}$$

$$\therefore y = \frac{k}{x^2} \text{ --------------- (ii)}$$

Now when $x = 6$, $y = y1$.

Substitute $x= 6$ and $y = y1$ in equation (2)

$$\Rightarrow y_1 = \frac{k}{6^2}$$

$$y_1 = \frac{k}{36}$$

when $x = 10$, $y = y_2$.

Substitute $x = 10$ and $y = y_2$ in eqn. (2)

$$y_2 = \frac{k}{10^2}$$

$$y_2 = \frac{k}{100}$$

now, substitute for y_1 and y_2 in eqn (i)

$$\Rightarrow \frac{k}{36} - \frac{k}{100} = 16$$

$$\frac{25k-9k}{900} = 16$$

$$900 \times \frac{16k}{900} = 16 \times 900$$

$$16k = 16 \times 900$$

$$\frac{16k}{16} = \frac{14400}{16}$$

$$k = 900$$

thus substitute for k in eqn (2) given the required equation relating x and y as

$$y = \frac{900}{x^2}.$$

Exercise

1. A piece of string 3m long is cut into 3 lengths which are in the ration 7:5:3. Find the length of the shortest piece.

2. When a sum of money is divided in the ratio 2:3:7, the smallest share is shs.1, 500. What is the original sum of money?

3. Hellen won shs.42 million in a lottery she shared the money parents in the ratio 5:2 respectively. Find how much money she gave her parents.

4. The representative fraction of a map is $\frac{1}{200,000}$. Two towns on this map are 16cm apart. Find the distance between the two towns in km.

5. A stretch of land on a map of 1:15,000 has an area of $300cm^2$. Determine the actual area of the land in $km^{2.}$

6. A map is drawn to a scale of 1:250,000 find the actual distance in km of a piece of a road represented by 3.6cm on the map.

7. A piece of land measures 33.6m by 16.5m. find the area of this land in cm^2, on map whose scale is 1:120

8. A lake of area $120km^2$ is represented by an area of $4.8cm^2$ on a map. Find the length (in km) of a horizontal road measuring 6cm on the map.

9. A lake occupies an area of $43.75km^2$. what would be its area in cm^2 on a map whose scale is 1:250,000

10. A forest reserve covering an area of $807.5km^2$ is represented on a map by a green area of 32.3 cm^2. Determine the scale of the map

11. If it takes 24 Lorries in 21 days to shift 1400 tonnes of rubbish. How much rubbish would 18 Lorries shift in 32 days.

12. A food store has enough food to feed 200 students for 15 days. For how long will the food last if 30 more students join the group?

13. Six carpenters make 32 boxes in 4 days. How long will 5 carpenters take to make 20 boxes?

14. A quantity P is partly constant and partly varies as the square of Q. when Q = 2, P = 40, when Q = 3, P = 65.

    (a) Form an equation relating P and Q

    (b) Determine the values of P when Q = 100

15. If M is directly proportional to the square of n and n = 2 when M = 1, Find the value of M when n = -5

16. Two quantities y and x are related by the equation y = a+bx. When y = 4, x = 2 and when y = 6, x = 4, Find the values of a and b

17. Given that y is directly proportional to $x^3$and that y is 250 when x = 10, Find the equation connecting x and y, Hence find the value of y when x = 4

18. (a) It is given that y varies partly as x and partly inversely as the square of x. it is also given that y = 3 when x = 1 and that y = 5 when x = 0.5. Find y when x = 1.5s

    (b) If takes 8 days for 9 men to weed a field of maize. How long would it take 6 men to weed the same size of field if they work at the same rate?

19. The time (T – hours) taken to dig a spring well partly varies as the depth (D-metres) of the well and partly varies as the square of the depth. If T = 80, D = 20 and when T = 150, D = 30.

(a) Write down and expression connecting T and D.
(b) Find T when D = 40

20. Senior Kats has cows and goats on his farm. The cost of feeding them per day partly varies as the number of the goats and as the number of cows on the farm. He spent shs.180,000 on 60 goats and 20 cows. When the number of goats increases to 100 and that of cows to 50, he spends shs.325, 000
(a) Write down an equation that relates the cost and the number of cows and goats on the farm
(b) Find how much he spends on;
(i) Each cow and goat
(ii) 70 goats and 50 cows.