Introduction

to

Ordinary level

Physics

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Preface

This book is a continuation of the first book of Mechanics but not a continuation of mechanics. It covers the remaining chapters to complete all that is required in Ordinary level Physics. These two books cover all the five chapters or branches of physics;

1. Mechanics
2. Light
3. Heat
4. Electricity and magnetism
5. Modern Physics

This book and the first one are meant to introduce the the subject; Physics to the students from primary level and prepare them for advanced level physics. The topics are arranged such that the first chapters and sections are a prerequisite for the next chapters and sections.

Acknowledgement

All errors and omissions in this book are due to the author. The author wish to thank the following colleagues for the wonderful work they did, otherwise it would be impossible to complete this book: the late Dr.J.M.V.Ngaboyisonga, Thomas Williams, Goeg Horn, Joern Winkelmann, GAutier de Montmollin, John W Leis, Oren Texing, Pehong Chem and Micheal A. Harrisson, Sebastian Rahtz and Loenor Barroca, Norman Gray, Rainer Schöpf, Hubert Gäblien and Rolf Niepraschk, N.J.H.M Van Beurden, David Hyde Computing company, Umar Mwidu, Nandwa Wyclife and Kyambadde Abubaker.

Special thanks go to the One who laid the foundation of the subject in the author's intellectual mind at an early age, Mr. Tunuura Sam and at a later stage Professor E.J.K.B. Banda.

May the Almighty reward them abundantly.

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Dedicated to

Matovu Ibrahim,

Tunuura Sam and Asimwe Sufian

Science is an important part of your life and your world.

Introduction to Ordinary level Physics will help you see how important Physics is a part of your life and your world. You will seek and explore many parts of the your world. This book like the first one, will show you how scientists study the world. You will find questions like;

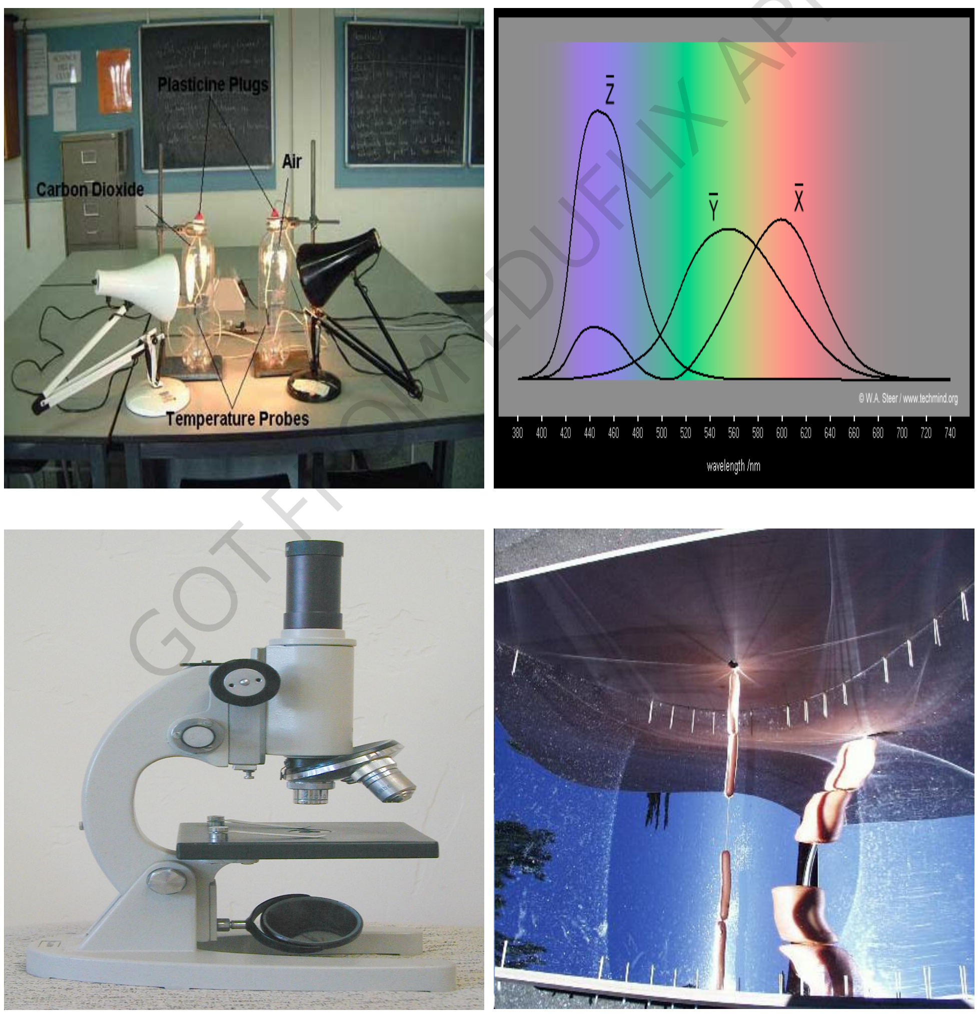
1. Why are objects coloured?
2. How can light be made to light a bulb?
3. Is it true that a liquid which boils always cools down?
4. Why does water behave differently between the temperature ranges of $0°C$ and $4°C$ or why does ice float on water?
5. Does a transformer really produce electric energy if not what does it do in the grid system?
6. Do you really know what happens when lightening strikes?
7. How destructive are atomic bombs or how useful are nuclear reactors?
8. Can electricity exist without magnetism or the reverse?
9. How is electricity generated and transferred to your home?

I will guide you through your adventure, this time we are to talk physics beyond Newtonian physics (found in the first book). We will discover many new and really exciting ideas. Hope you will **learn some more science** on your own. **Turn the page** and let us begin.

Contents

Chapter 2

Light:Optics



The Photos above show; a microscope , an optical setup, graph showing the intensities of different colours and a picture showing a sample of polarized light.

Objectives

After studying this topic, you should:

- Be able to define the following terms;
  - ⊛ Light
  - ⊛ light ray
  - ⊛ Beam of light
  - ⊛ Shadow
  - ⊛ Umbra
  - ⊛ Penumbra
  - ⊛ Pole of a mirror
  - ⊛ Principle axis of a mirror or lens
  - ⊛ centre of curvature of a mirror or lens
  - ⊛ Focal length of a mirror or lens
  - ⊛ radius of curvature of a mirror or lens
  - ⊛ principal focus of a mirror
  - ⊛ Conjugate foci
  - ⊛ Real and virtual images
  - ⊛ Refraction
  - ⊛ Absolute and relative refractive index
  - ⊛ Total internal reflection
  - ⊛ Critical angle
  - ⊛ Optical centre of a lens
  - ⊛ Dispersion
  - ⊛ Accommodation
- Be able to explain briefly what is meant by;
  - ⊠ Transparent material
  - ⊠ Opaque object
  - ⊠ Translucent object
  - ⊠ Annular eclipse
  - ⊠ Parallax
  - ⊠ Magnification or linear magnification
  - ⊠ Power of a lens
  - ⊠ Colour
  - ⊠ Light filters
- Be able to give;
  - ⊝ 2 types of reflection
  - ⊝ 2 types of shadows
  - ⊝ 3 types of beams
  - ⊝ 2 types of eclipse
  - ⊝ 4 properties of images formed by plane mirrors
  - ⊝ 2 types of reflectors, curved mirrors and lenses
  - ⊝ 4 applications of total internal reflection
  - ⊝ 2 types of colours
- Be able to describe;
  - ⋇ An experiment to verify the law of reflection
  - ⋇ An experiment determine the focal length of a concave mirror or convex lens
  - ⋇ An experiment to verify the laws of refraction
  - ⋇ How a magnifying glass, Compound microscope, projector and lens camera works.
  - ⋇ Briefly the differences between traditional, warm, cool and complementary colours

- Be able to state;
  - ⊝ the laws of reflection
  - ⊝ the formula for the number of images formed by two mirrors inclined at an angle θ to each other.
  - ⊝ the rules for drawing ray diagrams for lenses and for mirrors
  - ⊝ the principle of reversibility of light
  - ⊝ 4 uses of curved mirrors
  - ⊝ the law of refraction
  - ⊝ the relation between critical angle and absolute or relative refractive index
- Be able to;
  - ⊛ construct an accurate ray diagram to show how an image is formed in a mirror or a lens
  - ⊛ draw a diagram to show that a convex mirror has a wider field of view
  - ⊛ explain
    - ⊛ why light bends towards the normal and at times away from the normal when moving from one median to another.
    - ⊛ how the mirage is formed
    - ⊛ the magnitude of the field of view of the fish's eye
    - ⊛ how multiple images are formed in a thick plane mirror
    - ⊛ why objects are coloured
  - ⊛ obtain refractive index of a transparent material from real and apparent depth of the material
  - ⊛ answer questions about a triangular prism
  - ⊛ determine the colour of an object in coloured light

Contents

2.1 Introduction

What is light?

2.1.1 Nature of light

Light is an electromagnetic wave. Since a wave is a disturbance in a medium that transfers energy from one point to another, then we can define light as a form of energy that enables us to see or a form of energy that stimulates the sensation of vision. This is because with out light we can not see. In modern physics we shall see that light at times behaves as a particle (wave-particle duality of matter and light: This is used to explain photoelectric effect in modern physics). More so, light travels at a speed of $3.0 \times 10^8 ms^{-1}$. Combining the above we can now define light;

Definition 1.

Light is a form of energy believed to be consisting of tiny wave-like particles called photons moving at a speed of $3.0 \times 10^8 ms^{-1}$.

It is believed because no one has ever seen these photons but we feel their effects.

2.1.2 Sources of light

Where does light come from?

Light is produced when atoms of heated or very hot substances or excited atoms are returning to their ground states. Since light is a form of energy, then light is produced by a substance that posses energy more than it can hold, the excess is lost as light (energy). There are two types of light sources namely;

1. Luminous or self-luminous sources and
2. Non-luminous sources

1. **Luminous sources of light:**

   These are objects or substances that produce light of their own. Examples of such objects include the sun, the glow-worm, the stars in the sky, candles, most burning substances, flames, electric bulbs and others.

2. **Non-luminous substances:**

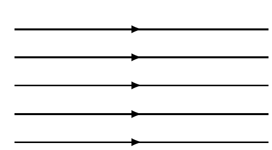
   These are substances, which do not produce light of their own, but just reflect it from others. They include the moon which reflects light from the sun, the car reflectors, mirrors (or shinning mirrors or objects), and others. When light is produced how does it move?

2.1.3 Direction of light

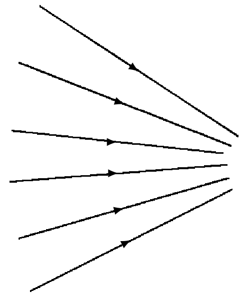
The direction or path taken by light energy is called a **light ray** or just a **ray**. A straight line with an arrow represents it. Its symbol is shown below;

We use a straight line because it is known that light travels in a straight line, we shall describe an experiment to prove this later. A collection of rays is called a **beam**, it is represented by a number of rays. There are three types of light beams;

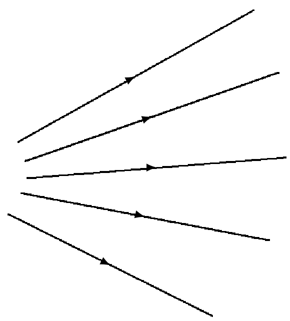
1. **Parallel beam**: a group of parallel lines with arrows pointing in the same direction.



2. **Convergent beam**; consisting of rays of light towards a fixed point.



3. **Divergent beam**; a group of light rays moving from a point.

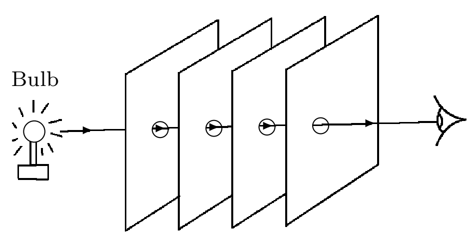


2.1.4 Rectilinear propagation of light

This is the process by which light travels in a straight line from one point to another. It is known that light travels in a straight line and that is why we can not see corners clearly.

Below we describe an experiment to show that light travels in a straight line. The following steps are taken;

- Drill holes in about four similar cardboards
- Arrange them as shown below;



- Place a candle at one end of the cardboards so that, one at the other end can see the light from the candle.
- When one of the cardboards is pulled out of the array or line then one can not see the light from the candle.

This shows that light travels in a straight line.

2.1.5 Materials and light

It is known that some material allow light to pass through them, such are called **transparent materials**. These include glass, water, paraffin and others. Materials which do not allow light to pass through them are called **opaque objects or materials**. These include stones, all metals, wood and bricks among others. The rest of the materials i.e. those that allow some light to pass through them are called **translucent materials**. These include papers, some plastics, e.t.c.

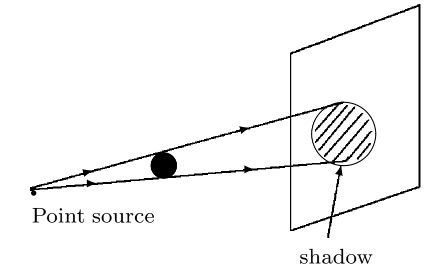
Let us look at the various properties of light which support the statement; light travels in a straight line, like shadows, eclipses and the pin hole camera.

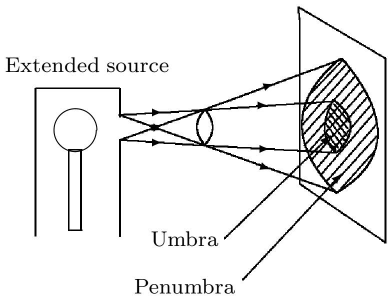
Shadows:

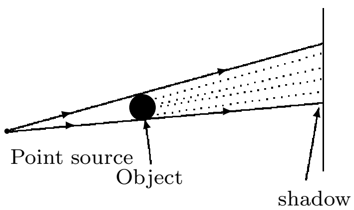
A **shadow** is something dark formed when light meets an opaque or translucent substance. Since the two kinds of sources of light (point source and extended source) do not produce the same shadows, then we say that there are two types of shadows;

1. **Penumbra:** this is the lighter part of a shadow and
2. **Umbra:** this is the dark part of a shadow.

These two shadows can be formed at the same time by an extended source, the diagrams below illustrate this;







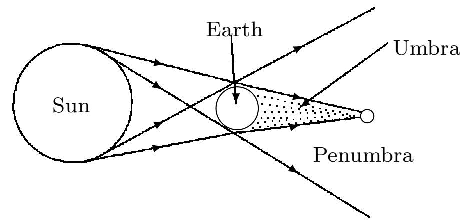
The amount of light received in the penumbra is more than that received in the umbra. It is wrong to say that the umbra receives no light because the light received by the umbra is too little to stimulate our eyes (the retina of the eye).

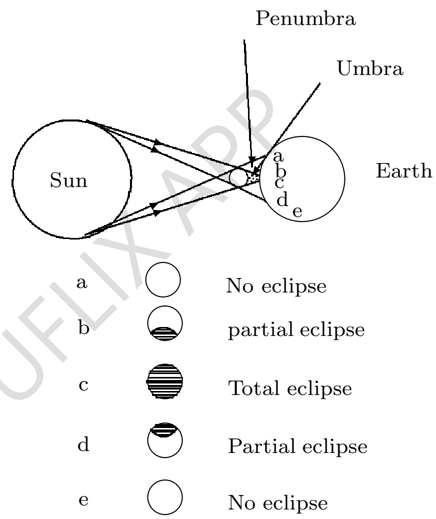
Eclipses

An eclipse is what you see when you are not observing something clearly or it is a state of unclearness. Here in light, it is highly associated with the heavenly bodies; the sun and the moon. When the moon blocks the light from the sun and we can not see the sun properly (but part of it) then we are seeing the eclipses of the sun (solar eclipses). And when the earth is between the moon and the sun, the earth blocks the light reflected by the moon, then we say we are seeing the eclipsed of the moon. Hence in light generally we say that, there are two types of eclipses;

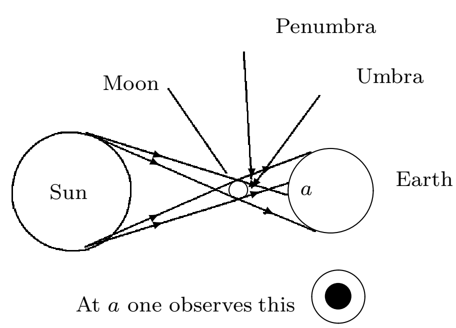
1. Solar eclipses: eclipses of the sun and
2. Lunar eclipses: eclipses of the moon.

The diagrams below illustrate these eclipses.





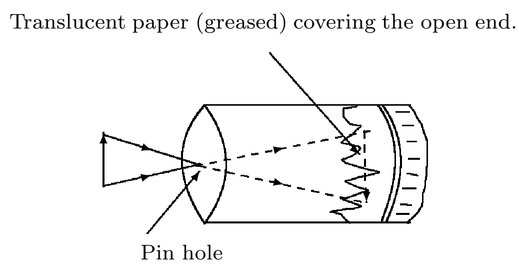
Note the following; In lunar eclipses, the earth is between the sun and the moon and in the solar eclipses the moon is between the earth and the sun. There is a special type of solar eclipses when the umbra (from the moon) fails to reach the earth's surface. Such an eclipse is called an **annular eclipse**. The diagram below shows this;



This is when the moon is further way from the earth. Note that the lunar eclipse lasts longer than the solar eclipse because of the relative distance of the sun from the two (earth and the moon).

2.1.6 The pin hole camera

The pin hole camera is an Arabic invention of the 17th century designed to view eclipses of the sun without loss of eyesight. This camera is cylindrical or square based tin with a small hole at the centre of the base and a transparent paper (greased or oiled paper) covering the other open end of the tin as shown below;



When one places the oiled paper end in a dark room and the other pointing to an object in light like day light, an image of the object is formed on the translucent paper. The hole should be capable of allowing enough light from the object to pass through it so as to form a clear image of the object. The image formed on the translucent paper has the following properties;

- It is inverted
- It is real or it can be formed on a piece of paper (or screen) unlike that formed by plane mirrors commonly used in homes.
- It is laterally inverted:- its right side is the left side of the object and the image's left is the object's right.
- It has an infinite depth of focus:- all its parts are equally sharp or there are no parts of the image which are brighter than others or clearer than others.

When the size of the pin hole is increased, the image becomes blurred or whitish due to too much light.

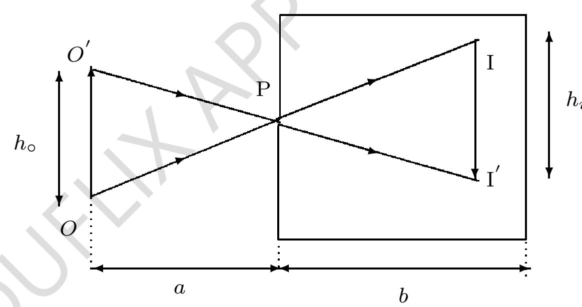
Determining the size of the object or the image formed by a pin hole camera.

Magnification:

we need to know how big the image is. Magnification is the measure of the size of the image in comparison to the size of the object. It is defined as;

$$\text{Magnification} = \frac{\text{height of image}}{\text{height of the object}} \tag{2.1}$$

Consider the pin hole camera's ray diagram below;



From the diagram we have; $M = \frac{h_i}{h_\circ}$ With the knowledge of similar triangles, we have

$$\frac{h_i}{h_\circ} = \frac{PI^1}{PO^I} \tag{2.2}$$
$$= \frac{PI}{PO} \tag{2.3}$$
$$= \frac{b}{a} \tag{2.4}$$
$$\text{Our interest is } \frac{h_i}{h_\circ} = \frac{b}{a} \tag{2.5}$$

Knowing the length of the camera, b, distance of the object from the pin hole, a, and height of the object $h_\circ$, we can obtain the height of the image, h_i as $h_i = \frac{b}{a} \times h_\circ$.

Example

1. If a pin hole camera of length 12cm forms an image 2mm in height, determine the height of the object if the object is 1.5m from the pin hole?

Solution

$$\begin{aligned}
\text{Image's height}, h_i &= 2mm = 0.2cm \\
\text{Object height}(h_\circ) &= ? \\
\text{Image distance}, b &= 12cm \\
\text{Object distance}, a &= 1.5m = 1500cm \\
\text{Using} \frac{h_i}{h_\circ} &= \frac{b}{a} \\
\frac{h_\circ}{h_i} &= \frac{a}{b} \\
h_\circ &= \frac{a}{b} \times h_i \\
&= \frac{1500}{12} \times 0.2 \\
&= 25cm
\end{aligned}$$

the height of the object is 25cm.

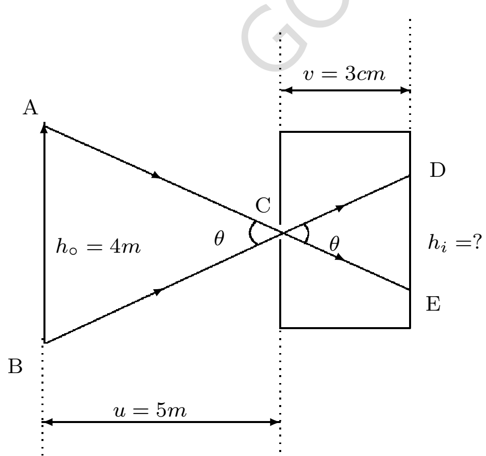
2. A camera forms an image of an object 4m high and 5m from the pin hole on a screen 3cm from the pin hole. Determine the

   (i) the height of the image

   (ii) the magnification of the image

Solution

consider the diagram below;



(i)the triangles ABC and CDE are similar triangles, then these ratios are true, taking v as the image's distance and u as the object's distance, then we can have;

$$\begin{aligned}
\frac{AB}{BC} &= \frac{DE}{CD} \\
\frac{h_\circ}{u} &= \frac{h_i}{v} \\
\Rightarrow h_i &= \frac{h_\circ}{u} \times v \\
&= \frac{4m}{5m} \times 3cm \\
&= 2.4cm
\end{aligned}$$

(ii)magnification (M) is given by;

$$\begin{aligned}
M &= \frac{\text{image's height}}{\text{object's height}} \\
&= \frac{h_i}{h_\circ} \\
&= \frac{2.4cm}{4m} \\
&= \frac{2.4}{400cm} \\
&= \frac{\cancel{2}4}{\cancel{4}000} \\
&= \frac{6}{1000} \\
&= 0.006
\end{aligned}$$

This pin hole camera can be used to take photographs, if the photographic plate is placed in the position of the screen(translucent paper). At this position a chemical reaction between light and the silver salts on the plate takes place and the image on the screen is formed permanently on the plate.

Note that from the above example about the pin hole camera, using similarity of triangles; it was showed that

$$\frac{h_i}{h_\circ} = \frac{v}{u} \quad (2.6)$$

$$\text{if } M = \frac{h_i}{h_\circ} \quad (2.7)$$

$$\text{and } \frac{h_i}{h_\circ} = \frac{v}{u}, \text{ then} \quad (2.8)$$

$$\text{also } M = \frac{v}{u} \quad (2.9)$$

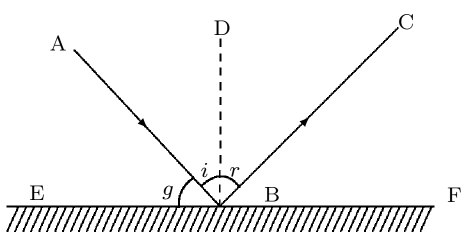
2.2 Reflection of light

2.2.1 Introduction

Reflection is one of the properties of the waves like light. A wave is said to be reflected if it meets a surface and changes direction towards the same side from which it is coming. Waves can be reflected on different surfaces (plane and curved surfaces) here we discuss plane surfaces.

2.2.2 Reflection in plane mirror

Considering a plane mirror, a ray of light approaching it is reflected as shown below;



AB - is the incident ray

BC - reflected ray

DB - normal

EF - the mirror

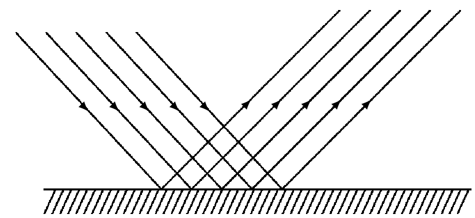
$$
\begin{aligned}
i &= \text{Angle of incidence} \\
r &= \text{angle of reflection} \\
g &= \text{glancing angle} \\
g + i &= 90^\circ
\end{aligned}
$$

2.2.3 Regular and diffuse reflection:

When a parallel beam of light is incident on a given surface, the reflected beam may be parallel or not parallel depending on the nature of the surface (smooth or rough).

Reflection on a smooth surface

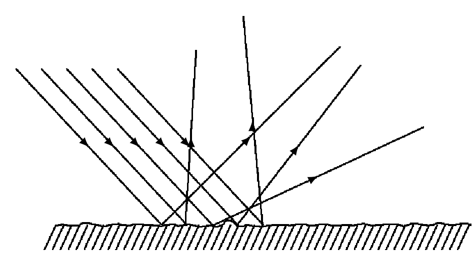
A parallel beam of light is reflected as a parallel beam and this type of reflection is called **regular reflection**. It occurs only on the smooth plane surfaces.



Regular reflection

Reflection on a rough surface

A parallel beam of light is reflected not as a parallel beam of light, this type of reflection is known as **diffuse reflection**. It occurs on rough surfaces like papers, cracked glasses and others.



Diffuse reflection

2.2.4 laws of reflection

Studying the above diagrams for various reflected rays, the following properties as laws were concluded; Laws of reflection state that;

Law 1.

1. the incident ray, reflected ray and the normal at the point of incident all lie in the same plane.
2. The angle of incident is equal to the angle of reflection

All the laws, principles, theories and others in physics have to be verified. To verify the above laws we shall concentrate on the second law as the first is not easy to prove now.

An experiment to verify the laws of reflection

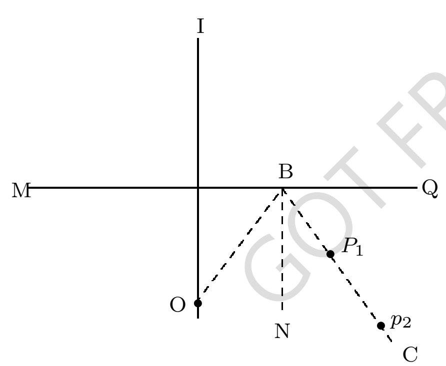
The second law of reflection

Apparatus: we need a plane mirror, plane sheet of paper, three pins, a protractor, a ruler and a soft board.

Method or procedure:

After obtaining the apparatus, the following steps should be followed to verify the law.

- Draw two perpendicular intersecting lines OI and MQ
- Place a pin on point O and the mirror with its reflecting surface along line MQ facing point O.



- On the same side as O, observe the image of point O in the mirror and pin two other pins in line or to appear in line with the image of point O.
- Remove the two last pins with the mirror and join their holes with a line (CB) to meet line MQ at a point B.
- Join point B to the point O with a line
- Draw a line BN perpendicular to MQ at point B, from point B to the side where point O is.
- Measure the angle of incident (OBN) and the angle of reflection (CBN),
- Repeat the above procedure for various positions of O and the corresponding angles of incident and reflection obtained.
- Plotting a graph of the angle of incident against the angle of reflection, a straight line with a slope of unity is obtained, this shows that always the angle of incident is equal to the angle of reflection.

2.2.5 Parallax

When seated in a moving car, the objects you see like trees and buildings appear to be moving. But in actual sense they do not move or they are not moving. And when they are too close to each other they do not appear to move relative to each other. This apparent relative movement between two objects due to the movement of the observer is called **parallax.**

In physics parallax is used to locate the position of images formed by various mirrors, lenses and other optical instruments. This is done by holding another object in the position of the image and then you move your head to and fro (side ways). if they appear to move or to change position then that is parallax hence they are not in the same position but when they do not appear to change position then there is no parallax hence they are in the same position. In this case the position of the object is the position of the image hence determining the position of the image.

2.2.6 Nature of the images formed by a plane mirror

This is the same as properties or characteristics of images formed by plane mirrors. When you observe critically the images in a plane mirror, they are;

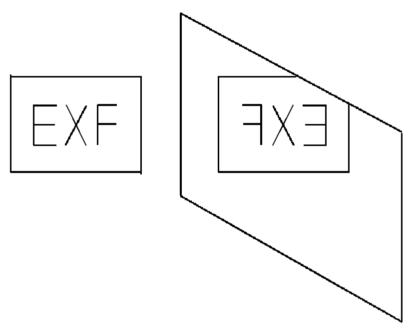
(i) Of the same size as their objects

(ii) At the same distance behind the mirror as the object is in front of it.

(iii) Virtual i.e. they can not be formed on the screen (white sheet of paper or cloth).

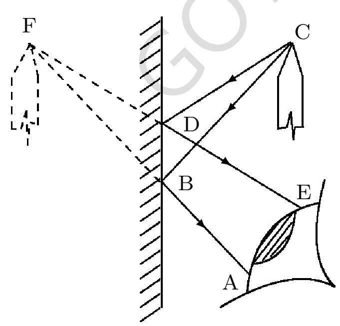
(iv) Laterally inverted (lateral inversion). This means that the right side of the image is the left side of the object and the left side of the image is the right side of the object.

The diagram below illustrates lateral inversion



How do we see images in a plane mirror

Consider some one seeing the image of the pencil in a plane mirror



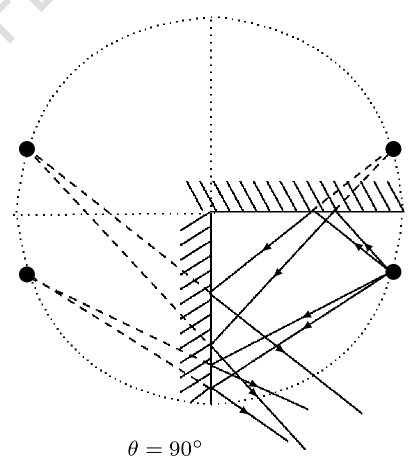
The eyes look at the rays as if they are coming from a point F behind the mirror hence the image of the pencil appears to be at that point and that is where the image is observed to be. The eye sees the image by the complete cone of rays ABCDE called the "pencil of rays".

Images formed by two inclined plane mirrors

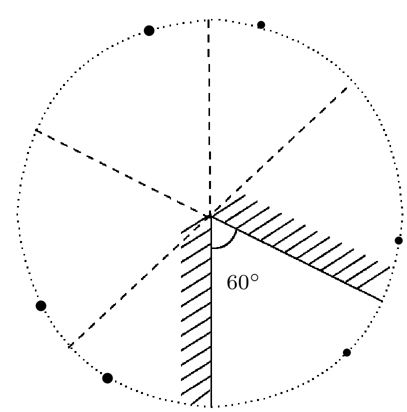
The images formed are the same as those formed in plane mirrors but how many are they? The number, N, of images formed can be given by the following expression;

$$N = \frac{360}{\theta} \tag{2.10}$$

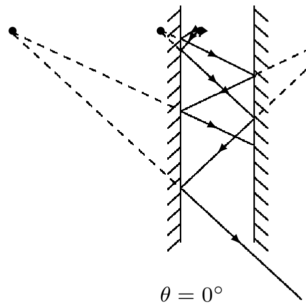
where θ is the angle between the mirrors
You can check this; When $\theta = 90°$; 3 images are formed, when $\theta = 60°$; 5 images are formed (as in a kaleidoscope used for advertising), and when $\theta = 0°$; i.e for parallel mirrors, $N = \infty$ (infinity) very many images are formed.



$\theta = 90°$

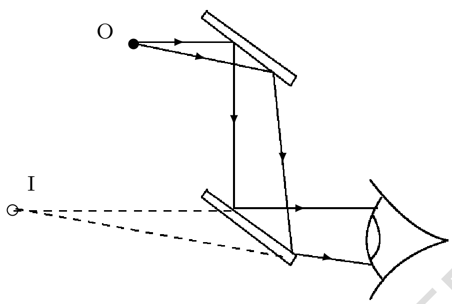


$\theta = 60°$



2.2.7 The periscope

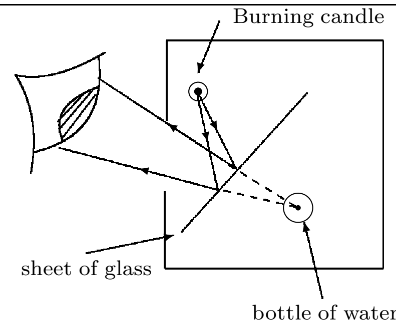
A simple periscope consists of two-fixed plane mirrors facing one another at an angle of 45° to the line joining them. It enables the user to view over a crowd of people or the top of an obstacle. A near object with this periscope appears to be far. The periscopes are commonly used in the submarines.



2.2.8 Peppers ghost

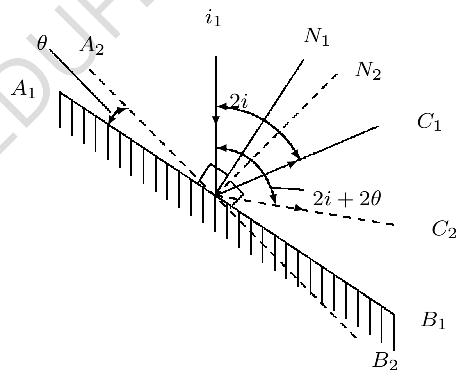
This is a method of producing an illusion of ghost of a theatrical stage named in favour of the John pepper who first experimented it. These illusions are commonly performed on theatrical stages to make people view the impossible.

Below we describe one among the many possible illusions "a candle burning in a bottle of water". Place a large sheet of polished glass diagonally across the stage, with a burning candle in front of it and a bottle of water behind it. At a certain angle the candle can be seen burning in the bottle of water. That is to say the image of the burning candle is positioned at the same point where the bottle of water is physically placed.



2.2.9 Relation between angle of rotation of the reflected ray and that of the mirror

Consider a mirror reflecting a given beam of light incident on it, when the mirror is rotated keeping the incident ray unchanged, through what angle does the reflected ray under go? Study the diagram below;



If the angle of rotation of the mirror is θ, then the angle of incidence increases by θ and the angle of reflection increased by 2θ. This can be shown when you consider the angle between the incident ray and the reflected ray before and after rotation. i.e.

Initial angle between i_1 and $r_1 = 2i$ Final angle between i_2 and $r_2 = 2i + 2\theta$. If the incident ray is fixed then the angle between the reflected rays $= (2i + 2\theta) - 2i = 2\theta$ Hence the reflected ray rotates twice the angle through which the mirror rotates.

2.3 Curved mirrors

2.3.1 Introduction

A mirror is more scientifically known as a reflector or is part of a reflector. A reflector is any thing that can change the direction of a wave (like light, radio waves and others) incident on it without passing through it. There are two types of reflectors;

1. Plane reflector and
2. Curved reflector

Plane reflectors

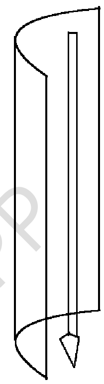
:

These are plane surfaces or straight surfaces that can change the direction of a wave without allowing the wave to pass through it. Like, the plane mirrors used in homes and salons to view the face hair, the walls of building that reflect sound and others.

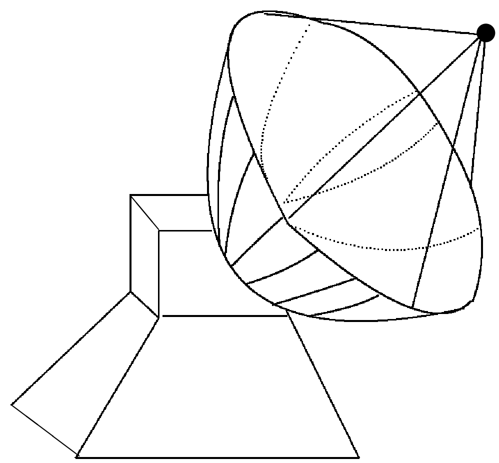
Curved reflectors

These are curved surfaces that can change the direction of the wave with out allowing the wave to pass through it. These are not common, most of them are made for a specific purpose like these below;

- The dishes (paraboloidal material) usually placed on the house so as to collect waves used to watch inaccessible TV stations like DSTV, M-net, Arab-sat, Euro-sat and others.
- Concave and convex mirrors are used in telescopes, microscopes, in schools and university laboratories to teach and in optical instruments.
- Television, radio and mobile phone transmitters and receivers (depending on the nature of the curvature of the surface, it can either produce (transmit) waves in all direction or receive waves from any direction. These curved reflectors with other electronic circuits, are the equipments that led to information technology.



Cylindrical parabola
(common mobile phone antenna)



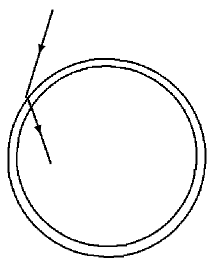
Satellite dish
(paraboloid-rotated parabola)

Modern technology has led to the invention of various types of antennas with various shapes and designs.

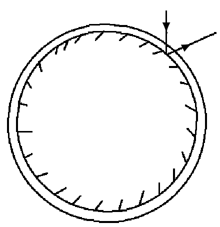
In this topic we are concerned with curved reflectors of light and they are commonly known as curved mirrors.

2.3.2 Origin of curved mirrors

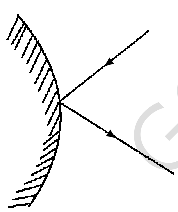
Consider a hollow spherical object made from a glass of small thickness. If the inside is silvery, then it can reflect light from anywhere outside it (is appears silvered). If this sphere breaks, its parts are curved mirrors (or spherical mirror because they have come from a sphere) such a mirror is called a convex mirror and it is shown below;



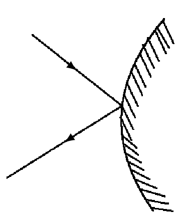
Hollow sphere
(light passes through it)



Hollow sphere whose inside is silvered
(it reflects light outside it)

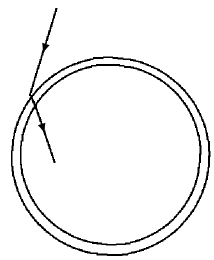


convex mirror reflecting light

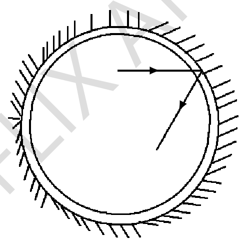


convex mirror reflecting light

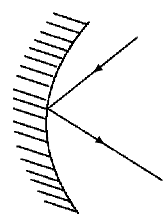
Consider a similar hollow sphere made from glass having a thin wall. If the outside surface of this sphere is silvered, then it can reflect light from inside it. If this sphere breaks, its pieces that result are examples of curved mirrors. Such a mirror that results is a concave mirror. Concave mirror diagrams are shown below;



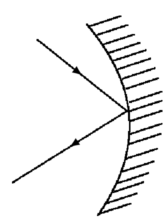
Hollow sphere
(light passes through it)



Hollow sphere whose outside is silvered
it reflects light inside it



concave mirror reflecting light

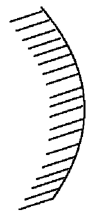


concave mirror reflecting light

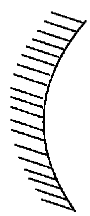
In conclusion; there are two types of curved mirrors or spherical mirrors;

1. concave mirrors and

2. convex mirrors which are silvered as shown below;



convex mirror

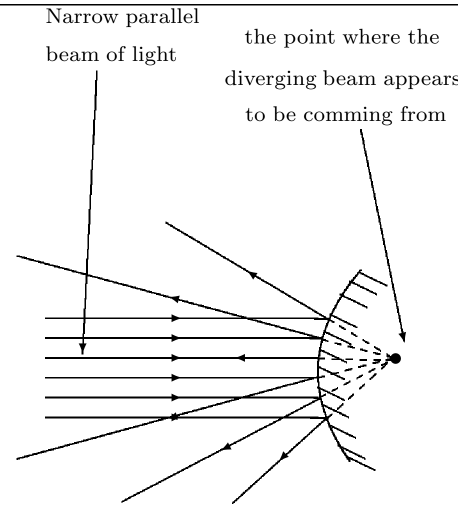


concave mirror
(Remember: cave-like mirror)

2.3.3 Effect on a parallel beam of light by spherical mirrors

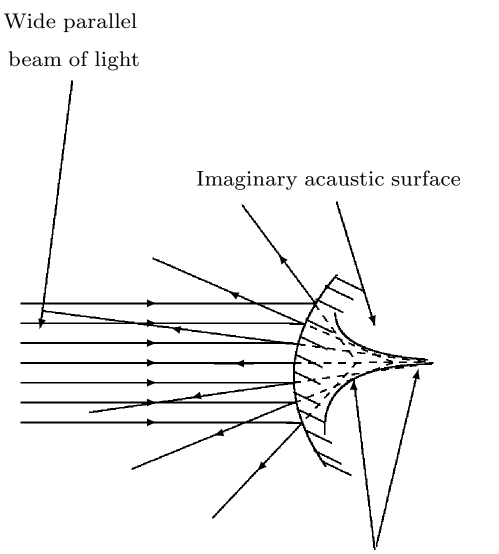
(a) Convex mirror

(a) **A narrow parallel beam of light**: when a convex mirror is placed in the path of a narrow parallel beam of light, the incident parallel beam is reflected as diverging beam of light. The reflected diverging beam appears to be coming from a point F, inside the convex mirror.



diverging beam

(b) **A wider parallel beam of light**: when a convex mirror is placed in the path of a wider parallel beam of light, the parallel beam is reflected as a diverging beam of light. The reflected diverging beam appears to be coming from a certain region (not a point) inside the mirror. This is shown below;



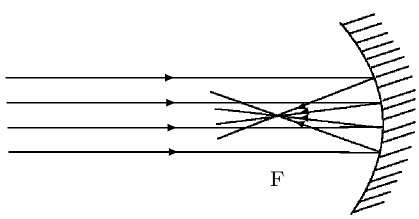
Conclusions from the above experiment with a convex mirror are;

(i) The parallel beams whether wide or narrow are reflected as a diverging beam of light hence a convex mirrors is also known as a **diverging mirror**.

(ii) It is only the narrow beam that is reflected as a diverging beam that appears to be coming from a point (not a region). This point F is known as the **principal focus of the convex mirror or diverging mirror**

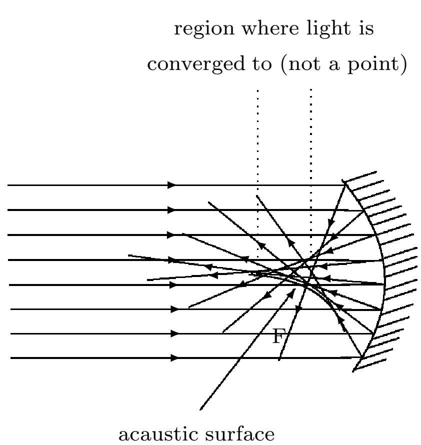
(b) A concave mirror

(i) **A narrow parallel beam of light**: when a concave mirror is placed in the path of a narrow parallel beam of light, it reflects it as a converging beam of light towards a fixed point in front of the mirror. Note that for the convex mirror this point was behind the convex mirror.

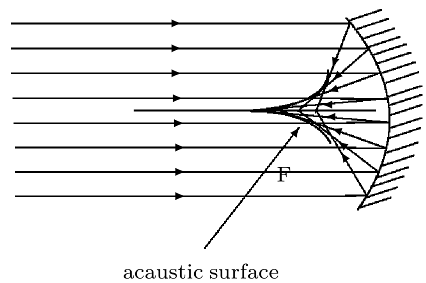


This Point F is known as the **principal focus of the concave mirror**.

(ii) **A wide parallel beam of light**: when a concave mirror is placed in the path of a wide parallel beam of light, the mirror reflects it as a converging beam of light towards a certain region (not a point) in front of it.



When the parallel beam of light is wide enough, an imaginary curved surface known as an **acoustic surface** can be traced as shown below;



From the above experiment about a concave mirror, we conclude that;

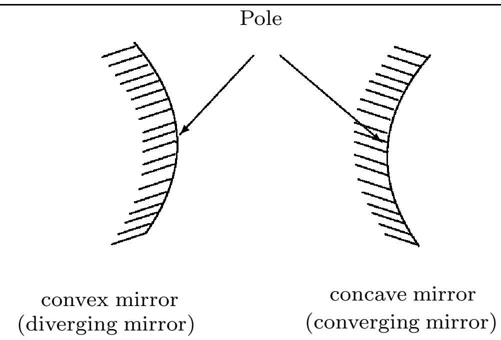
1. A parallel beam of light whether narrow or wide is reflected by a concave mirror as a converging beam of light. Hence a concave mirror is known as a **converging mirror**.
2. It is only the narrow parallel beam of light that is reflected as a converging beam towards a fixed point ,**the principal focus of a concave mirror**.

The phenomenon that occurs when a wide parallel beam is not reflected as a beam **converging to a fixed point** for concave mirror or as **diverging from a fixed point** for a convex mirror is known as **spherical aberration**. Then spherical aberration does not occur for narrow beams of light or for rays close to the principal axis.

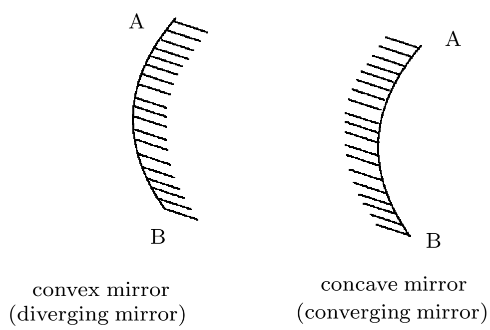
2.3.4 Terms used in curved mirrors

We can now describe curved mirrors when all its major characteristics are known. Below are the terms to use;

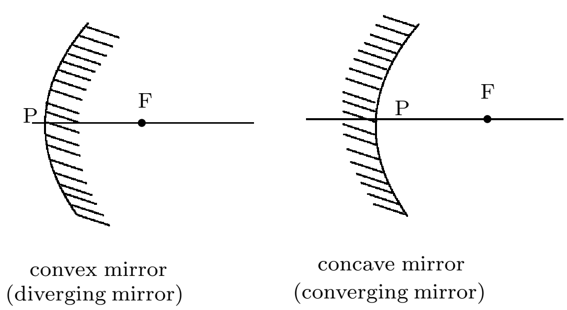
1. **A pole** (P) is the centre of a curved mirror(spherical mirror i.e. concave or convex mirror)



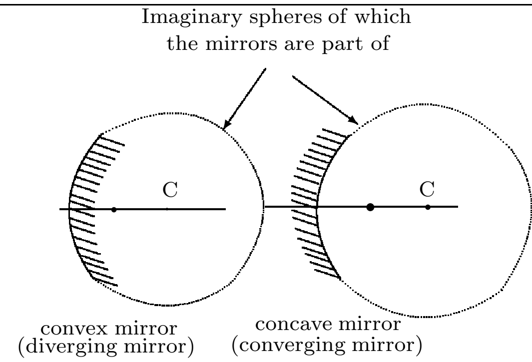
2. **Aperture of a curved mirror** is the length of the curved mirror. In the diagram below AB is the aperture.



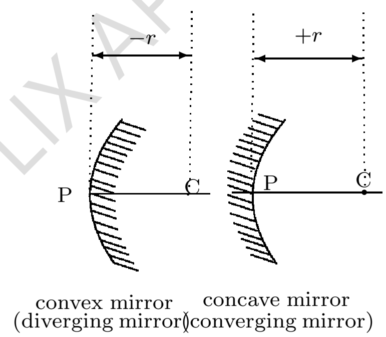
3. **Principal axis of a spherical mirror** is an imaginary line perpendicular to the mirror passing through its pole. Or it is an imaginary line passing through the pole and the principal focus of the mirror. The line PF continuing in either direction is the principal axis in the diagrams below;



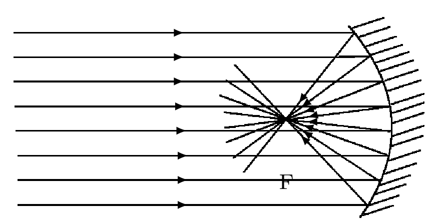
4. **Centre of curvature (C) of a curved mirror** is the centre of the sphere of which the mirror is part of. The point C in the figures below is the centre of curvature.



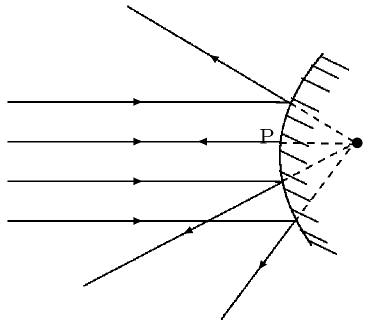
5. **Radius of curvature of a curved mirror (r)**: is the distance between the centre of curvature and the pole of the mirror. The distance r is shown in the diagrams below;



6. **Principal focus of a concave mirror** or principal focus of a converging mirror is a point on the principal axis where all rays parallel and close to the principal axis converge to after reflection on the mirror. In the diagram below , F is the principal focus;

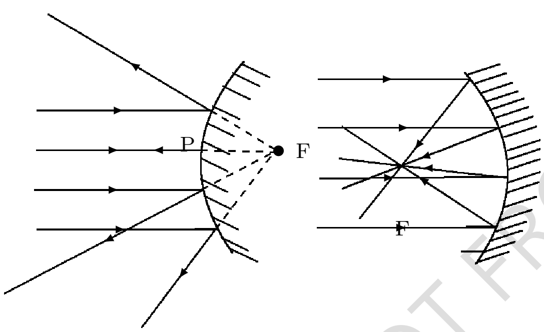


7. **Principal focus of a convex mirror** or the **principal focus of a diverging mirror** is a point on the principal axis behind the mirror where all rays parallel and close to the principal axis appear to be diverging from after reflection on the mirror.



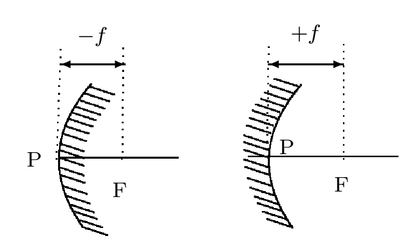
Note that we said 'parallel and close' to mean 'a narrow beam'.

8. **Principal focus of a curved mirror or spherical mirror** is a point on the principal axis where all rays parallel and close to the principal axis converge to or appear to be diverging from after reflection in the mirror. They converge to F for concave mirrors and appear to diverge from F for convex mirrors.



rays appears to diverge from F for convex mirrors   rays converge to F for concave mirrors

9. **Focal length of a curve mirror, f,** is the distance between the principal focus and the pole of the mirror.

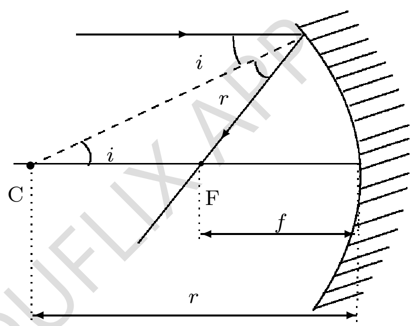


convex mirror (diverging mirror) f is negative   concave mirror (converging mirror) f is positive

Relation between focal length, f, and radius of curvature, r, of a curved mirror.

In more advanced books it can be shown that the radius of curvature is twice the focal length for a given curved mirror.

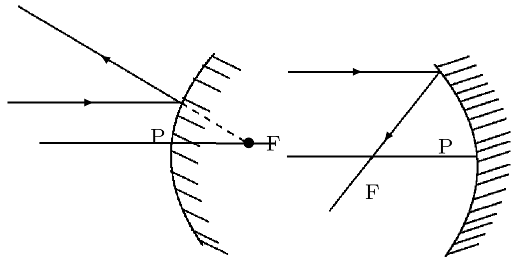
$$r = 2f \tag{2.11}$$
$$\text{or } f = \frac{r}{2} \tag{2.12}$$



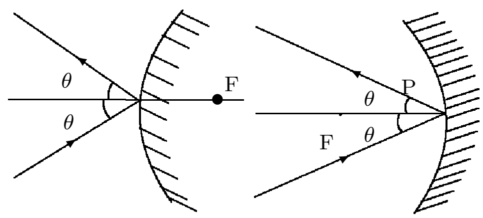
2.3.5 Rules for drawing ray diagrams with curved mirrors

A ray diagram is any diagram where arrowed lines are used to represent the path taken by light or a wave. When light is reflected by a curved mirror, it forms an image as a plane mirror does, but in a different way. The question at hand is that how can some one predict that the final image of an object placed at a given distance from the mirror is formed at a given point from the mirror, is real, magnified, inverted or otherwise? To answer this question we need to know some rules regarding curved mirrors. And these are;

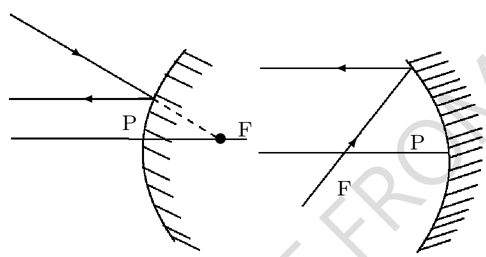
1. All rays parallel and close to the principal axis converge to or appear to diverge from the principal focus after reflection on the concave and convex mirrors respectively.



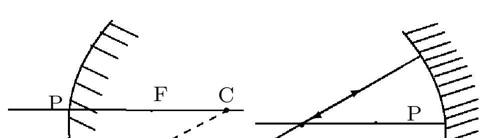
2. All rays towards the pole of a curved mirror are reflected making the same angle with the normal as their incident ray.



3. All rays passing through the principal focus for concave mirrors or towards the principal focus for convex mirrors are reflected parallel to the principal axis



4. All rays passing through or towards the centre of curvature of a concave or convex mirror respectively are reflected in their own path.



2.3.6 Images formed by curved mirrors

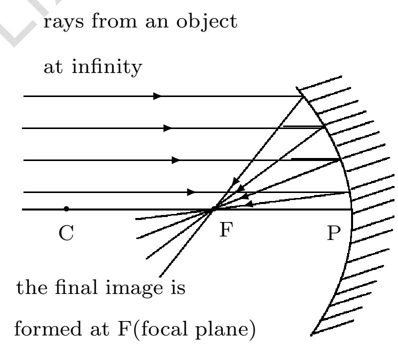
To determine the position and nature of the images formed by curved mirrors, you have to use at least two of the above 4 rules to keep the ray diagram neat. Below are some of the ray diagrams showing the position and nature of the final image of the object when placed at various positions. Let us discuss one mirror at ago, we shall use a vertical arrow to represent a vertical object.

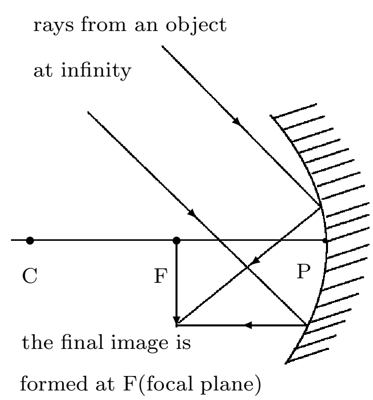
Concave mirrors

we shall use symbols C, F and P to represent centre of curvature, principal focus and pole of the mirror respectively.

(a) **Object at infinity (or very far away)**

Rays from an object at infinity are always parallel to each other. The diagram below shows how they are reflected;





The final image is;

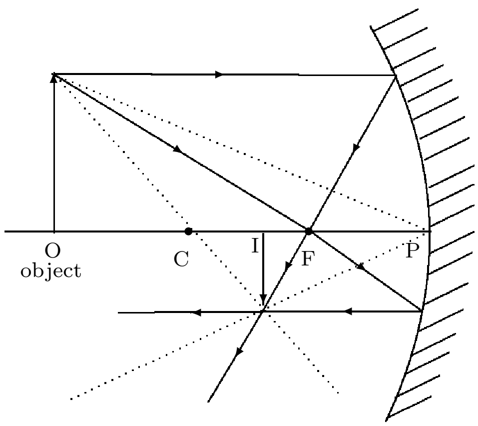
- inverted
- real, and
- formed at F (or in the mirror's focal plane).

Definition 2.

Focal plane is an imaginary plane parallel to the mirrors but perpendicular to its principal axis, passing through its principal focus.

This method is used to determine the focal length of the concave mirror and in telescopes to observe objects like planets, stars, satellites which are very far a way (at infinity).

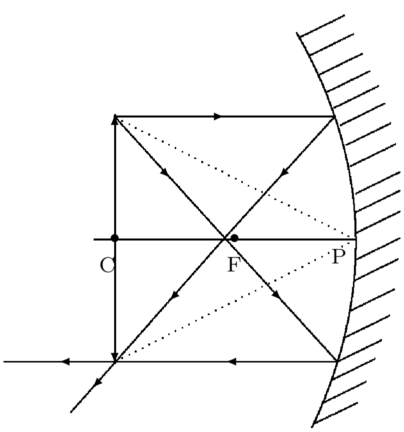
(b) **Object beyond C (but not far a way:**



The final image is;

- Formed between C and F
- real (formed by actual intersection of light rays)
- diminished (smaller than the object)
- inverted (upside down)

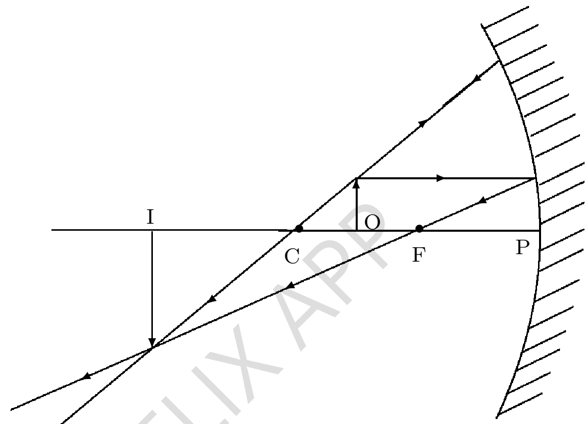
(c) **Object at C:**



The final image is;

- formed at C
- real
- same size as the object (not magnified nor diminished i.e. magnification is unity)
- inverted

(d) **Object between C and F:**

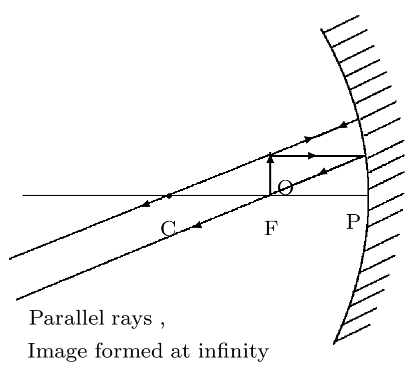


The final image is;

- formed beyond C
- real
- magnified (bigger than the object)
- inverted

This can be used in a projector but the lens[1] does it better.

(e) **Object at F:**

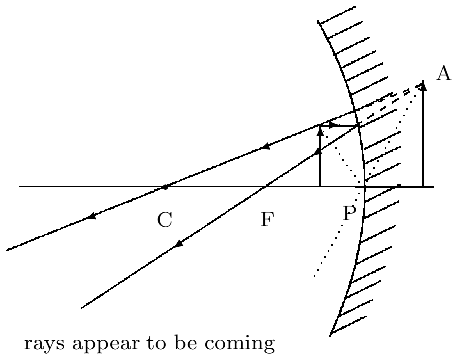


The final image is formed at infinity and the nature of the image at infinity is not yet known. This is applied in car head lamps, search light e.t.c as parabolic[2] mirrors.

[1]lenses are discussed on page 55

[2]curved mirrors are made parabolic to enable distant rays to meet on the principle focus

(f) **Object between F and P:**



rays appear to be coming
from point A

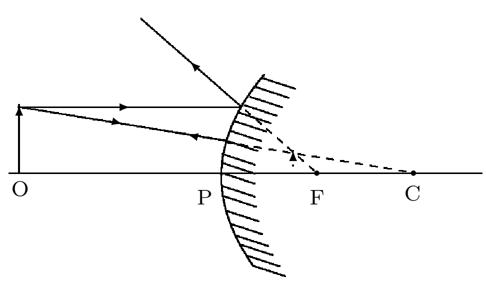
The final image is;

- formed behind the mirror
- virtual (formed not by the actual intersection of light rays but by the apparent intersection of the rays)
- magnified
- erect (upright)

This is applied in dental surgery where a very small concave mirror is placed in the patients mouth behind the tooth and its magnified image viewed. This image can now be examined (diagnosed) carefully by the dentist.

Convex mirrors

Unlike the concave mirror, which can produce real, and virtual images depending on the position of the object on the principal axis, the convex mirrors forms only virtual images. these images are always erect, diminished and formed between P and F. the diagram below shows this;



The final image is;

- virtual
- diminished
- erect or upright
- formed between P and F

The images formed by the diverging mirrors are of the same nature. (by nature we mean it is real or virtual, diminished or magnified, formed at what point and then is it erect or inverted).

Conjugate foci: (foci; is plural and focus; is singular)

Analysing the images formed by the concave mirror, note that when the object is placed in the position of the image, its image will be formed at the initial position of the object. These two points on the principal axis whereby, when an object is placed at one, its image is formed at the other are called **conjugate foci.**

Definition 3.

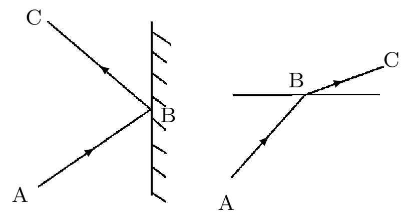
> **Conjugate foci** are any pair of points such that an object placed at one of them gives rise to its real image at the other.

Since convex mirrors form only virtual images, they can not have conjugate foci. The fact that an object and its real image can be interchanged is a consequence of the principal of reversibility of light.

Definition 4.

> **The principal of reversibility of light** states that, when the direction of light is reversed, it follows its original path.

For instance, when light from A to C passed through point B also when it is from C to A it still passes through point B.



in reflection in refraction

2.3.7 Mirror formula

You observe that when the object is placed at a specific distance from the mirror, its image is formed at a specific distance from the mirror. This shows that there exists a relationship between the distance of the image from the mirror ,v, and the distance of the object from the mirror, u, for a given spherical mirror of focal length, f,. This relation is called the mirror formula, it is given below;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{2.13}$$

Where;

u is the object distance (from the pole of the mirror)

v is the image distance (from the pole of the mirror) and

f is the focal length of the mirror.

When any two of the three are known, then the other one can be calculated using this formula.

Using the mirror formula

(sign conventions)

Note that the image of an object can be real or virtual.

Definition 5.

A real image is an image that can be viewed or put on a screen.

It is formed by the actual intersection of the two or more light rays. A screen can be a white sheet of paper or cloth.

Definition 6.

A virtual image is an image that can not be viewed or put on a screen.

It is formed by the apparent intersection of two or more light rays. It is formed not by the actual intersection of the light rays but when light rays are extended backwards. When one is to use the mirror formula, the image and object distances take different signs (positive or negative). There are two major (sign conventions) formats or methods of using the mirror formula. These are;

1. the **"real is positive"** sign convention and
2. the **"new Cartesian"** sign convention.

The real is positive sign convention says that, when using the mirror formula;

- All distances are measured from the pole of the mirror.
- All distances of real objects and real images are positive
- Distances of virtual objects and virtual images are negative.

In this new Cartesian sign convention and from the definition of the principal focus of a curved mirror, a convex mirror has a virtual principal focus and centre of curvature hence its focal length and radius of curvature are negative. A concave mirror has a real principal focus and centre of curvature (they are accessible) hence its focal length and radius of curvature are positive distances.

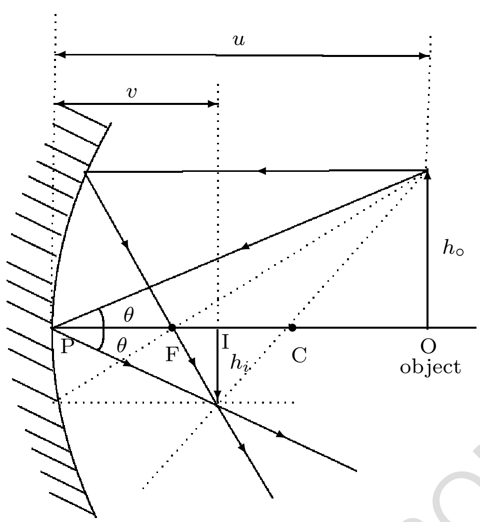
The new Cartesian sign convention says that: in using the mirror formula, the following are ensured;

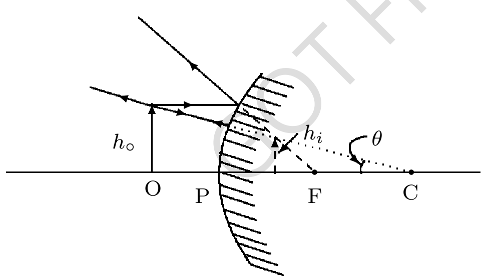
- All distances are measured from the pole of the mirror as the origin
- All distances measured against the incident light are negative
- All distances measured in the direction same as that of the incident light are positive

In this sign convention, the focal length of a concave mirror is negative and that of the convex mirror is positive. These two conventions should not be used together at the same time, they will confuse you, choose only one that is easier for and always use that. In this book we shall use the "real is -positive" sign convention. This is the convention adopted by the Uganda National Examination Board (UNEB).

Magnification

Consider the ray diagrams below for a convex and concave mirrors





from any of the diagrams

$$\tan\theta = \frac{h_i}{v} \tag{2.14}$$

$$\tan\theta = \frac{h_\circ}{u} \tag{2.15}$$

$$\Rightarrow \frac{h_i}{v} = \frac{h_\circ}{u} \tag{2.16}$$

$$\Leftrightarrow \frac{v}{u} = \frac{h_i}{h_\circ} \tag{2.17}$$

hence also for curved mirrors

$$\text{magnification} = \frac{h_i}{h_\circ} \tag{2.18}$$

Or

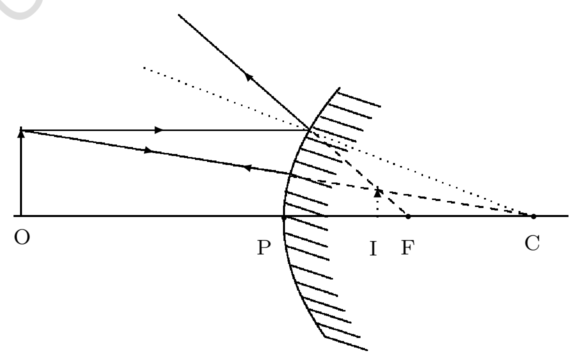
$$\text{magnification} = \frac{v}{u} \tag{2.19}$$

where v is the image distance and u is the object distance from the pole of the mirror.

Examples

1. A small object is placed on the principal axis of a convex mirror of focal length 10cm. Determine the position of the image when the object is 15cm from the mirror?

Solution sketch



Calculations

$$\begin{aligned}
\text{Using } \frac{1}{f} &= \frac{1}{v} + \frac{1}{u} \\
\text{where } f &= -10cm \text{ and} \\
u &= 15cm \\
\frac{1}{v} &= \frac{1}{f} - \frac{1}{u} \\
&= \frac{1}{-10} - \frac{1}{15} \\
&= \frac{-25}{150} \\
v &= \frac{-150}{25} \\
\Longrightarrow &= -6.0cm
\end{aligned}$$

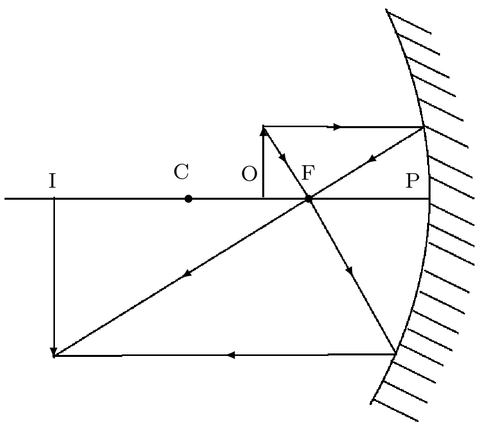
negative means that the image is virtual.

Hence the final image is formed at a point 6cm behind the pole of the convex mirror

2. A concave mirror has a radius of curvature of 20cm. Find the position , magnification and nature of the image of a small pin placed on the axis and at right angles to it and 15cm from the pole?

Solution

Sketch



Calculations

$$\begin{aligned} \text{Using } \frac{1}{f} &= \frac{1}{v}+\frac{1}{u}, \\ f &= \frac{+r}{2} \\ &= \frac{20}{2} \\ &= 10cm, \\ u &= 15cm \end{aligned}$$

$$\begin{aligned} \frac{1}{v} &= \frac{1}{f}-\frac{1}{u} \\ &= \frac{1}{10}-\frac{1}{15} \\ &= \frac{15-10}{150} \\ &= \frac{5}{150} \\ &= \frac{1}{30} \\ \Longrightarrow v &= 30cm \end{aligned}$$

$$\begin{aligned} \text{Magnification} &= \frac{v}{u} \\ &= \frac{30}{15} \\ &= 2 \end{aligned}$$

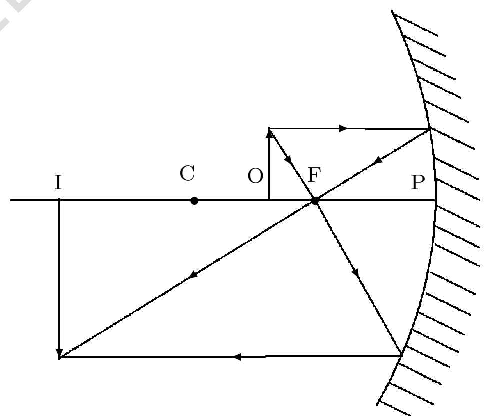
Hence the final image is formed at a point 30cm from the concave mirror, it is real, magnified, inverted and formed beyond C with a magnification of 2.

3. An image of a certain object is formed at a point 18cm from the concave mirror with a radius of curvature of 14cm. Determine the position of the object?

Solution

Sketch

Using the principle of reversibility of light, i.e we can take the image as the object and the final image as the objects position and then reverse the direction of light. This is what should be done in case one is to determine this distance not using the mirror formula



Calculations

$$\begin{aligned} \text{Using } \frac{1}{f} &= \frac{1}{v}+\frac{1}{u} \\ f &= \frac{r}{2} \\ &= \frac{14}{2} \\ &= 7cm, \\ v &= 18cm, \\ u &= ? \end{aligned}$$

$$\begin{aligned}\frac{1}{u} &= \frac{1}{f} - \frac{1}{v}\\ &= \frac{1}{7} - \frac{1}{18}\\ &= \frac{11}{126}\\ \Longrightarrow u &= \frac{126}{11}\\ &= 11\frac{5}{11}\\ &= 11.4545\end{aligned}$$

hence the position of the object is 11.455cm from the pole of the concave mirror.

2.3.8 Accurate construction of ray diagrams

We have been drawing sketches of ray diagrams, the measurements obtained from them are not exact or accurate. To obtain accurate results from a ray diagram, the following should be ensured;

1. Use a graph paper to draw a ray diagram to scale
2. Represent the mirror with a straight vertical line with a small concave mirror or convex mirror (showing the type of mirror) at the pole (where the principal axis meets this mirror line).
3. Use any two of the four rules for drawing ray diagrams with curved mirrors in a given ray diagram.
4. Arrowed ray diagrams are the only true ray diagrams. The direction of the arrow represent the direction of light. Ray diagrams without arrows are not considered to be ray diagrams.

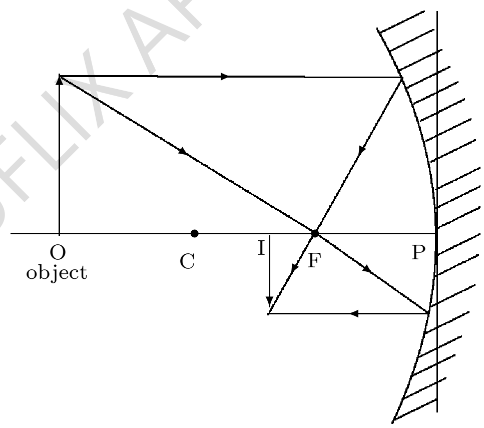
You should know how to obtain either the image distance, V, or the object distance, u, using an accurate ray diagram (or with out using the mirror formula). Ask your teacher for more about what UNEB requires.

Example

1. A cup 4cm high is placed 12 cm from a concave mirror of focal length 5cm. Using an accurate ray diagram, determine;
   (i) the position,
   (ii) the height,
   (iii) the magnification and
   (iv) nature of the image formed.

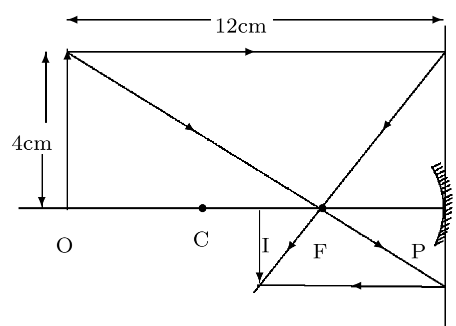
Solution

The sketch is



Here the object is represented by an upright arrow at O. Its image is formed at I.

Accurate ray diagram



Please redraw this diagram on a graph paper and obtain your own values and then compare with my values.

(i) the image is formed at a point 8.6cm the exact value is (8.57cm) from the mirror

(ii) the height of the image is 2.9cm (the exact value is 2.857cm).

(iii) the magnification

$$
\begin{aligned}
M &= \frac{H_i}{H_\circ} \\
&= \frac{2.9}{4} \\
&= 0.725
\end{aligned}
$$

[3]

$$
\begin{aligned}
\text{or } M &= \frac{V}{U} \\
&= \frac{8.6}{12} \\
&= 0.717
\end{aligned}
$$

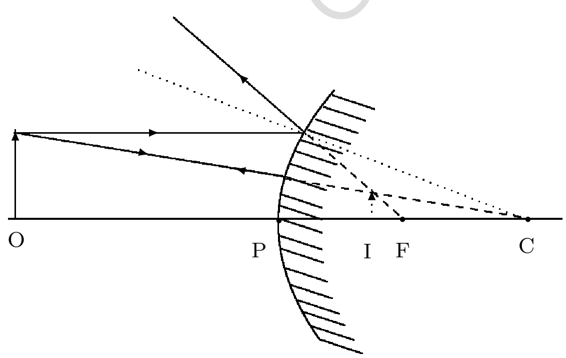
, hence the magnification is 0.72

(iv) the nature of the image is

- Real
- Diminished
- Inverted and
- Formed between the centre of curvature and the principal focus.

2. A convex mirror of focal length 12cm forms an image at a distance of 9cm. Determine the position of the object and the magnification of the image?

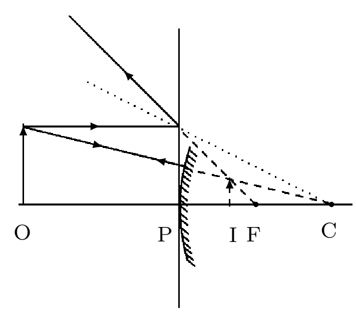
Solution



To construct it

- Draw the convex mirror and the principle axis with points F, C, P with PF = 12cm.
- Indicate on the principal axis the image distance (9cm) and draw a small image with any small chosen height
- Draw a full line parallel to the principal axis through any point (not the pole) on the convex mirror and name the point A.
- Draw dotted lines from point F to A and extend it as a full line through the mirror
- Measure 9cm from the mirror to C and draw a vertical line touching the line AF as the image
- Draw dotted line from C through the top of the image to the mirror and extend it as a full line to where it meets the line parallel to the principal axis. This point is the top of the object.
- Draw now the object and put the arrows on the diagram to complete it as a ray diagram

(i have listed these steps so that you can follow them to construct your own ray diagram and use it to answer the following)



From the accurate ray diagram, Image distance is 9cm, Object distance is ....cm, Magnification $=\frac{v}{u}=$ ...

Questions

1. An object is placed 20cm in front of a concave mirror of focal length 12cm. By means

[3] The exact value is 0.71 to 2d.p, but always no one gets it because of a number of reasons like not using sharp pencils, not drawing straight lines-light rays among others.

of an accurate graphical construction, determine the position and nature of the final image?

2. A convex mirror of focal length 18cm produces an image of an object on its axis at a point 6cm away from the mirror. determine the position of the image

   (i) by graphical construction

   (ii) by calculation

   Give reasons why your answers in (i) and (ii) may differ

3. An object is placed on the axis of a converging mirror of focal length 0.2m. The image produced is inverted with a magnification of 1.5. By calculation or by scale drawing on a graph paper, determine the position of the object?

4. An object 3cm long is placed in front of a concave mirror of focal length 15cm so that it is perpendicular to and has one end resting on the axis of the mirror. Find by means of a ray diagram, drawn to a suitable scale, the size and position of its image?

5. A concave mirror is to be used to form an image of an object pin. Where must the object be placed to obtain

   (a) an upright , enlarged image

   (b) an image same size as the object

   (c) a real magnified image

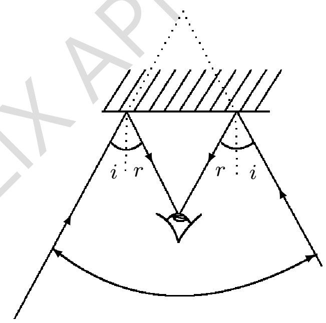
   (d) a real diminished image ?

6. A small object is placed on the principal axis of a convex mirror of focal length 10cm. Determine the position of the image when the object is 15cm from the mirror?

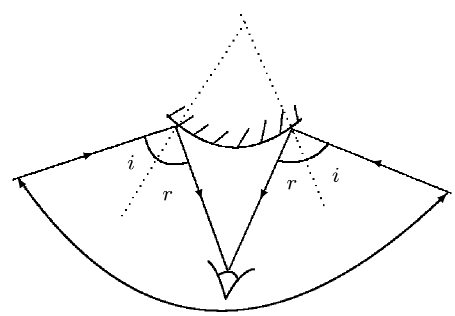
7. Draw a ray diagram to show how a spherical mirror can produce a real, magnified image of an object?

2.3.9 Comparing the field of view of a plane mirror and a convex mirror

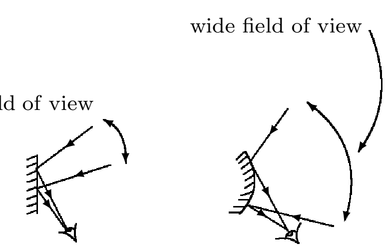
For a fair comparison to be made, the two mirrors must be of the same size and the eye must be positioned at the same distance from each of them. The angle of view is maximum when the normals are at the extreme edges of the mirror. The normals to the convex mirror are lines, which are a continuation of the radii at the edges of the mirror. Once the angles of reflection have been drawn, equal angles must be drawn on the other side of the normal to give the position of the incident rays. This is shown below for the two mirrors.



narrow field of view



a wide field of view



Note that the angle of incident for the convex mirror is much greater than the angle of incidence for the plane mirror, hence the convex mirror has a greater field of view than the plane mirror. Because of this, convex mirrors are very convenient for use as driving and side mirrors, since they always give an erect image and a wider field of view.

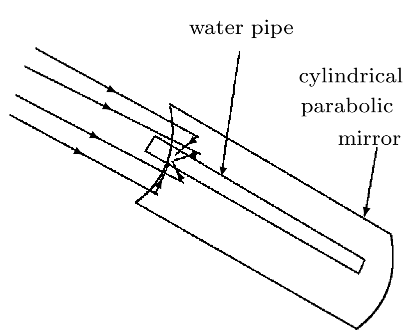
2.3.10 Uses of curved mirrors

Curved mirrors have been used in various optical instruments (and instruments used to guide waves). These are briefly described below;

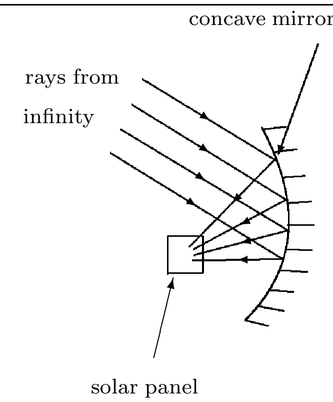
Concave mirrors

Parabolic mirrors or paraboloidal mirrors

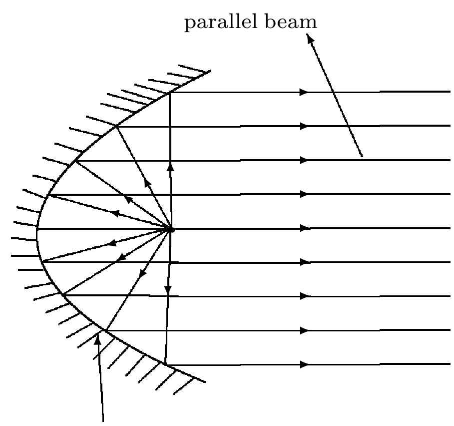
- They have been used as solar concentrators i.e. since parabolic mirrors or concave mirrors converge a parallel beam of light to a given point (principal focus). Also the sun rays are made to converge to a certain point of interest using parabolic mirrors. In general all waves are brought to one point, this point of interest may be a water pipe containing water. The water in this pipe becomes hot due to the heat rays from the sun hence solar concentrators are used to boil water. The point of interest may be a solar panel, which when heated by the sun rays, the heat energy from the sun rays is converted to electric current. This charge (electric current) is stored in special batteries. The ray diagrams below illustrate this;



all rays converge to the pipe at the principal focus of the mirror

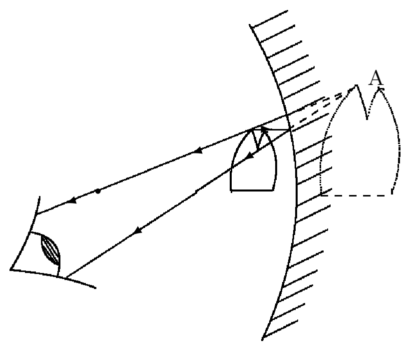


- They are used in searchlights, torches and car head lamps to produce a parallel beam of light. Recall that a parallel beam is converged to a fixed point by concave mirrors, but from the principle of reversibility of light, when the light source is placed at the principal focus, the reflected light rays are a parallel beam. In car head lamps, the filament of the bulb is positioned at the principal focus of the concave reflector to produce a parallel beam of light. Similarly to the searchlights, which are rotating torches supervising a given area (like in a camp).



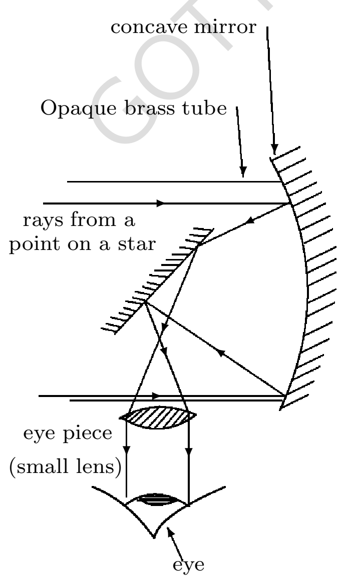
Parabolic reflector of a carhead lamp, search light, torch etc

- They are used as shaving mirrors and by dentists to examine patients teeth. Recall that when an object is placed at a point between the pole and the principal focus of the concave mirror, its image is formed behind the mirror, magnified, virtual and erect: this is made use of by people who shave hair or in saloon. Here a large concave mirror is used and the customers head is positioned close to the mirror, a very large image of the head is formed in the mirror and hence the adjustments in shaving are done properly. Dentists do analyse the defects on the patient's teeth using the same property. Dentists always have small concave mirrors which when placed behind the patients tooth: any dot, decay or hole behind the tooth can be seen.



the final image (tooth) is magnified

- They are used as reflectors in reflector telescopes: telescopes are optical instruments used to view objects that are very far away, for instance the sun, comets, stars and other heavenly bodies. Rays from a point on the distant body arrive at the mirror as a parallel beam which is reflected towards a fixed point where the final image is formed. There are various types of telescopes one is shown below;



One of the largest reflecting telescopes in use today, called the Hale telescope, is at mount Palomar observatory in California where the atmosphere is not dusty and it is cloud free. Its mirror is of diameter just over 5m, made of special glass coated with aluminium. The world's largest reflecting telescope with a mirror 6m in diameter is in Soviet Union, six of them are used to have a light collecting capacity equivalent to an 18m diameter mirror.

- In projectors to prevent loss of light and make the light brighter. More details about this can be found in the topic optical instruments.[4]

Convex mirrors

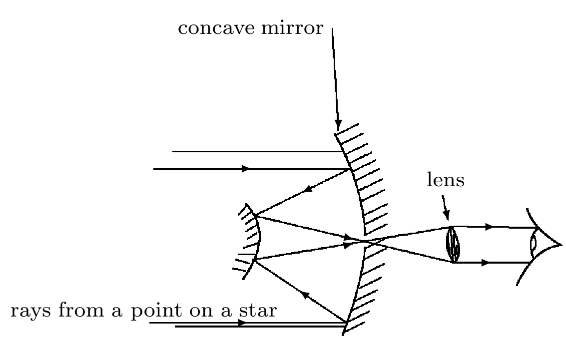
Since convex mirrors have a wider field of view, this is utilised by using it as;

(a) A driving mirror for the driver to be able to view all passengers in the small mirror.

(b) A side mirror for the driver to be able to view other automobiles, bicycles and pedestrians behind the car.

(c) Supervising mirrors or security mirrors in super markets where the customer picks all the items he needs before paying the seller (or manager). Some security persons watches the customer in this small convex mirror but these days electronic cameras are replacing the use of convex mirrors.

(d) Reflectors in some telescopes, this is shown below;



[4] see page69 for the projector

2.3.11 Experiments to determine the focal length of curved mirrors

At this level we shall discuss how to determine the focal length of converging mirror and not diverging mirrors. The methods we are to use are based on three major principles (rules or ray diagrams);

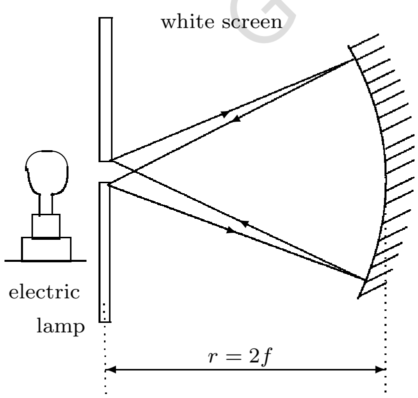
- Rays from a distant object meets at the principal focus of a concave mirror.
- Rays from an object at C meet at the same point
- The object and image distances are related according to the mirror formula.

Experiments to determine the focal length of concave mirrors (converging mirrors);

Method 1 : Object at C

(a) **Using an illuminated object at C (the centre of curvature)**

The object used in this experiment is a small hole cut in a white screen made from a metal sheet. The hole is illuminated from behind by an electric lamp and the concave mirror mounted in a lens-holder is moved to and fro in front of the screen as shown below;



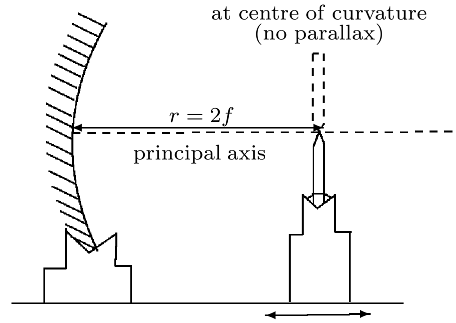
The concave mirror is moved to and fro until a sharp image of the object is formed on the screen adjacent to the object. When this has been done the object (the hole) and the image (the sharp point on the screen) are at the same distance from the mirror (equal to its radius of curvature). The distance, r, between the mirror and the screen is measured. Half this distance, r, is the focal length, f, of the mirror

$$f = \frac{r}{2} \tag{2.20}$$

Sharp focusing of the object is made easy or greatly enhanced, if thin cross wires are placed across the hole. Note that when the object is placed at the centre of curvature of a concave mirror, a real inverted image is formed at the same place where the object is.

(b) **Using an optical pin at the centre of curvature or the "no parallax method"**

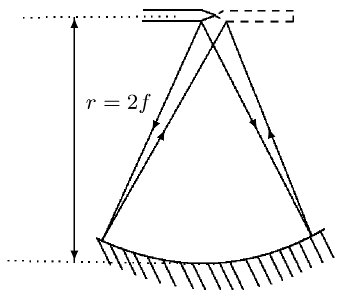
Support a concave mirror vertically on a suitable holder. Stick a pin in a cork held in a lamp-stand such that the tip of the pin is at the same level as the centre of the mirror. Move the pin to and from the mirror until a real image of the pin is seen some where in front of the mirror. Then move the pin to and fro until when there is no parallax between the pin and its image (when this adjustment has been properly made then, the pin and its image will remain in the same straight vertical line when the eye is moved side to side).



At the no parallax position measure the distance, r, between the pin and the mirror. Half this distance is the focal length, f. i.e.

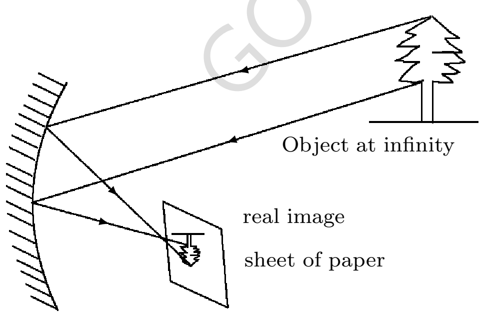
$$f = \frac{r}{2} \tag{2.21}$$

Some times the concave mirror is placed on the table facing up and the horizontal pin is moved up and down until its image coincides with the object (pin). At this point, half the distance between the mirror and the pin is equal to the focal length of the concave mirror.



Method 2 :Distant object

Recall that the image of a distant object is formed in the focal plane of the mirror. This principle is used to determine the focal length of concave mirrors. An object like tree or window at some distance a way or very far a way, is focused by the concave mirror and its image is formed in its focal plane. When a white piece of paper (or any other screen) is placed in the mirror's focal plane and moved to and from the mirror, at a given point a sharp image of the object (window or tree) is formed on it. At this point the distance between the mirror and the white sheet of paper, is equal to the focal length of the concave mirror.



Note that the screen should be positioned on one side of the principle axis so as to leave space for the light rays from infinity to be reflected by the mirror.

Method 3 : Mirror formula

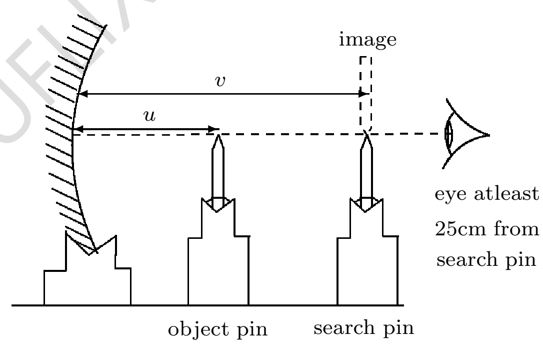
Image and object distance method:

Recall that the object and image distances are related to the focal length of the mirror according to the formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

, hence if u and v are measured (or are known) the value of f can be calculated. Here we discuss the most commonly used methods.

Two pins are required one to act as an object and the other to act as a search pin. The object pin is placed in front of the mirror (beyond F, the principal focus) and its image will be formed beyond F . The search pin is then positioned at a point so that there is no parallax between the search pin and the real image as shown below;



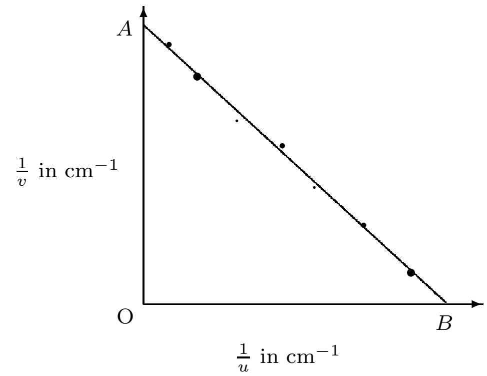
The distance of the object pin from the mirror gives the object distance, u, and that of the search pin from the mirror at the point where there is no Parallax with the image gives the image distance, v. Several pairs of object and image distances are obtained in this way and the results tabulated as shown below;

Table of results

u(cm)	v(cm)	$\frac{1}{u}cm^{-1}$	$\frac{1}{v}cm^{-1}$

Table 2.1: Object-image distance method

Plotting a graph of $\frac{1}{v}$ against $\frac{1}{u}$, we have



Measure the intercepts OA and OB, and then the focal length of the concave mirror is equal to the average of the reciprocals of these intercepts.

$$f = \frac{1}{2}\left(\frac{1}{OA} + \frac{1}{OB}\right) \tag{2.22}$$

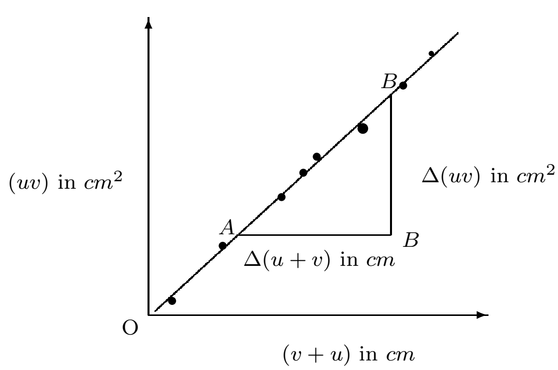
Some times the results are tabulated in the table like this below;

Table of results

u(cm)	v(cm)	$(v+u)$	(vu)

Table 2.2: Object-image distance method

Plotting a graph of uv against $v+u$, we have



The focal length is obtained as the slope

$$f = \text{slope} \tag{2.23}$$

$$= \frac{\Delta(vu)}{\Delta(v+u)} \tag{2.24}$$

Note that from

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{2.25}$$

,

$$\Rightarrow f = \frac{vu}{v+u} \tag{2.26}$$

. For more details you can contact the physics practical book for ordinary level.

$$\text{from}\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{2.27}$$

$$= \frac{v+u}{vu} \tag{2.28}$$

$$f = \frac{vu}{v+u} \tag{2.29}$$

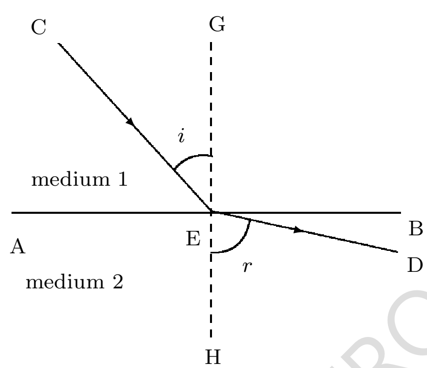
$$\Longrightarrow f = \frac{\Delta(vu)}{\Delta(v+u)} \tag{2.30}$$

$$= \text{slope of the graph} \tag{2.31}$$

2.4 Refraction

2.4.1 Introduction

Refraction is the bending of light when it crosses the boundary between two media. When a light ray meets the boundary between two media, part of it is reflected, part absorbed and the rest refracted at that point . If the bigger percentage of light energy is refracted, then we say that light has been refracted and if the biggest percentage of light energy is reflected, then we say that light has been reflected. The diagram below shown a ray of light refracted at the boundary between medium 1 and medium 2;



In the diagram; GH is the normal

AB is the boundary between the two media

CE is the incident ray

ED is the refracted ray

r is the angle of refraction and

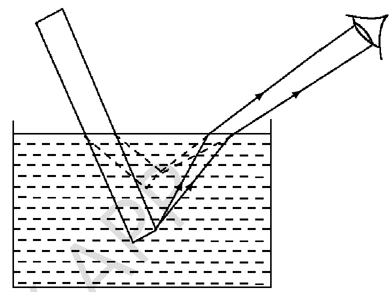
i is the angle of incident

As light crosses the boundary AB, the following happens; Its wave fronts change direction, The speed of light changes, the wavelength of light changes (remember light is a wave), the frequency of the light wave does not change, since this is determined by the source of the wave (light).

2.4.2 Effects of refraction

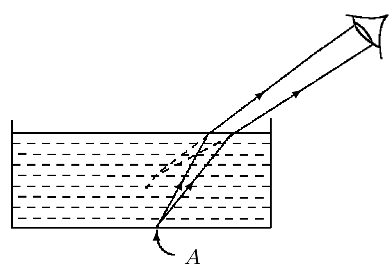
Due to refraction there are very many activities or phenomenon that occur, some of them are briefly described below;

1. When a pen or rod is dipped in a transparent liquid, it appears bent. This is due to refraction. The figure below shows this;



   The rays reach the eye after being refracted at liquid's surface and hence they appears to be coming from a point just above the actual point. Here the rod appears to be near the surface of the liquid yet it is deep in the liquid, to our eyes it appears bent.

2. The swimming pool appears shallower than it is due to refraction.



Point A appears to be raised

Scientists tried to look for a relation between the angle of incidence and the angle of refraction, finally Snell made the final conclusion on how they are related. His statement constitutes one of the laws of refraction.

2.4.3 Laws of refraction

Whenever a wave is refracted, it obeys certain rules, which are the laws of refraction these are stated as follows;

Law 2.

The laws of refraction state that;

1. The incident ray, the refracted ray and the normal at the point of incident all lie in the same plane.
2. The ratio of the sine of the angle of incident to the sine of the angle of refraction is a constant for any two media. i.e

$$\frac{\sin\ i}{\sin\ r} = \text{a constant} \tag{2.32}$$

This is some times known as Snells law and the constant is known as the refractive index of the second medium with respect to the first, always abbreviated as n. i.e.

$$\frac{\sin\ i}{\sin\ r} = n \tag{2.33}$$

2.4.4 Refractive index

Using the above diagram, if medium 1 is a vacuum or air, then the constant ,n, is known as **absolute refractive index**. When the first medium is not a vacuum or air then the constant or refractive index is known as **relative refractive index** of the second medium with respect to the first medium. Absolute refractive index is always written as n and relative refractive index of the second medium with respect to the first, is written as ${}_1n_2$.

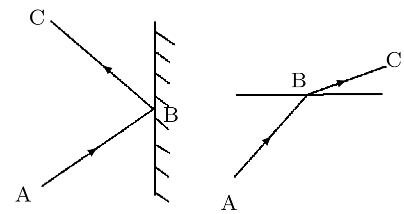
For an experiment to verify Snell's law or the laws of refraction see the physics practical book for ordinary level.

2.4.5 The principle of reversibility of light

If light travels from point A to point C along a certain path, then also when the light source is at C it travels from C to A along the same path. This is the principle of reversibility of light.

Principle 3.

The principle of reversibility of light states that, if a light ray is reversed it travels along its original path.

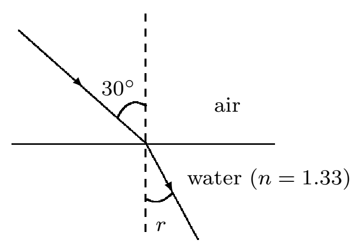


in reflection in refraction

Examples

1. A ray of light is incident at the air-water interface at an angle of incident of 30° in air. Determine the angle of refraction?

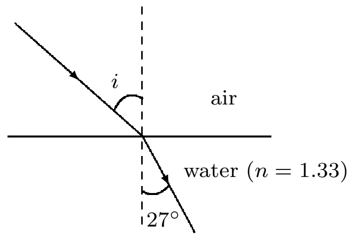
Solutions



$$\begin{aligned}
\frac{\sin i}{\sin r} &= n \\
\frac{\sin 30}{\sin r} &= 1.33 \\
\frac{0.5}{sinr} &= \frac{1.33}{1} \\
\sin r &= \frac{0.5}{1.33} \\
\sin r &= 0.3759 \\
\Rightarrow r &= 22.08^\circ
\end{aligned}$$

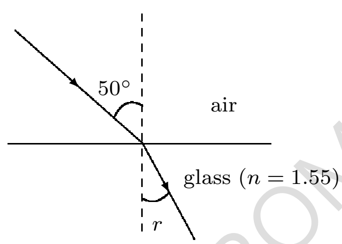
2. Calculate the angle of incident, i, and the angles of refraction, r, in the diagrams below;

   (a)



$$\frac{\sin\ i}{\sin\ r} = {}_a n_w$$
$$\frac{\sin\ i}{\sin\ 27} = 1.33$$
$$\sin i = 1.33 \times \sin 27^\circ$$
$$\sin i = 1.33 \times 0.454$$
$$= 0.6038$$
$$i = 37.14^\circ$$

(b)



$$\frac{\sin\ i}{\sin\ r} = {}_a n_g$$
$$\frac{\sin\ 50}{\sin\ r} = 1.55$$
$$\sin r = \frac{\sin\ 50}{1.55}$$
$$= \frac{0.766}{1.55}$$
$$\sin\ r = 0.4942$$
$$r = 29.62^\circ$$

3. The angles of incident and refraction for a ray moving from water to glass are 40° and 35° respectively. Calculate the refractive index of glass relative to water?

Solution

$$\frac{\sin i}{\sin r} = {}_w n_g$$
$${}_w n_g = \frac{\sin i}{\sin r}$$
$$= \frac{\sin 40}{\sin 35}$$
$$= \frac{0.6428}{0.5736}$$
$$= 1.121$$

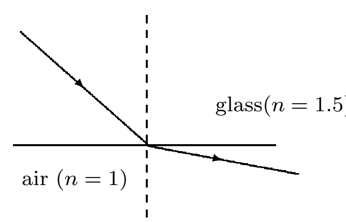
hence the refractive index of glass with respect to water is 1.121

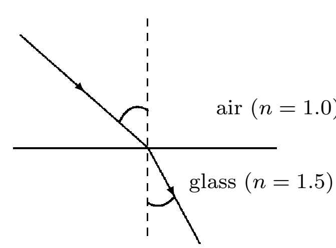
2.4.6 When does light bend towards or a way from the normal

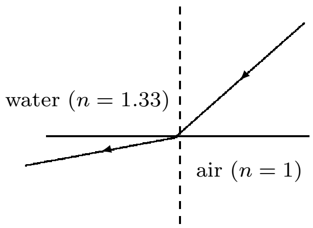
Optical density

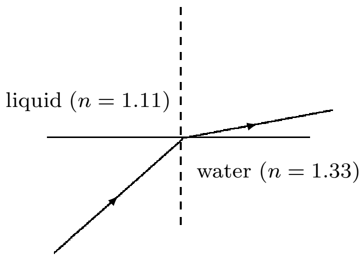
A medium that has a greater refractive index is said to be more optically dense.

1. If light moves from a more optically dense medium to a less optically dense medium, it bends away from the normal. The diagrams below show this trend;

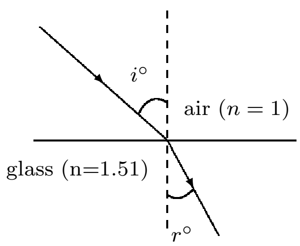


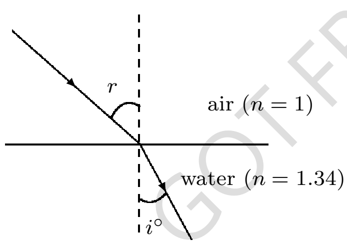


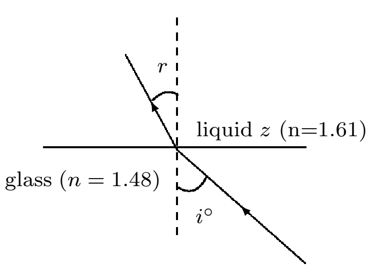




2. Light bends towards the normal when it is moving from a less optically dense medium to a more optically dense medium. The diagrams below illustrate this;



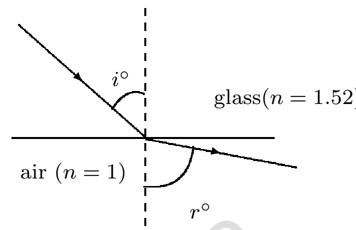




Therefore when you are sketching any ray diagram where light is refracted, you must indicate whether light is bending a way from the normal or towards the normal in accordance to the above concepts.

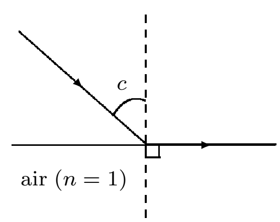
2.4.7 Critical angle and total internal reflection

It is known that when light is moving from a more optically dense medium to a less optically dense medium, it bends away from the normal as shown below;



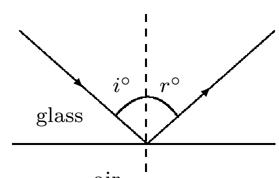
refraction

When the angle of incidence, i, is increased, the angle of refraction, r, also increases. When i is increased further and further at a certain point, the angle of refraction, r, becomes $90°$, at this point the angle of incidence is known as critical angle, c.

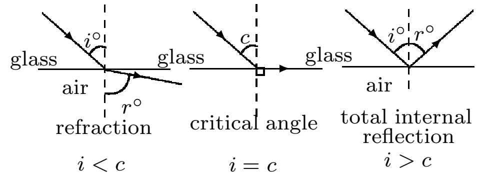


critical angle

When $i = c$, and i is increased further what happens? The ray of light is reflected (not refracted). Unlike other normal reflections where just a greater percentage of the incident light is reflected, here 100% of the incident light is reflected, such a reflection is called **total reflection** or **total internal reflection** i.e. light is totally reflected inside glass (laws of reflections are obeyed).



total internal reflection



Definition 7.

Critical angle of a medium is the angle of incident in a more optically denser medium whose an angle of refraction in the less optically dense medium is 90°.

State two conditions for the angle of incidence to be equal to the critical angle for any two media.

Definition 8.

Total internal reflection is the type of reflection which occurs when light is moving from a more optically dense medium to less optically dense medium and the angle of reflection is greater than the critical angle.

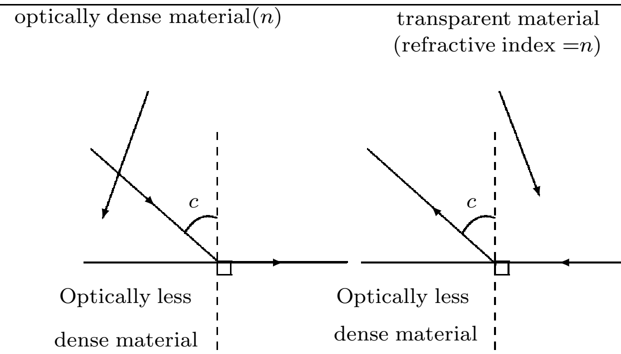
The two conditions for total internal reflection to occur are;

1. The light or wave must be moving from a more optically dense medium to a less optically dense medium

2. The angle of incidence in the more optically dense medium, must be greater than the critical angle for that media.

Relation between critical angle and absolute refractive index for a given medium

Consider the diagram above where the angle of incidence is equal to the critical angle. Using the principle of reversibility of light; when the direction of light is reversed, the angle of incidence becomes 90° and that or refraction becomes c. This change is done because absolute refractive index is defined when the first media is air i.e



$$\frac{\sin i}{\sin r} = n \quad (2.34)$$

$$\frac{\sin 90}{\sin c} = n \quad (2.35)$$

$$\frac{1}{\sin c} = n \quad (2.36)$$

$$n = \frac{1}{\sin c} \quad (2.37)$$

$$\text{Or } \sin c = \frac{1}{n} \quad (2.38)$$

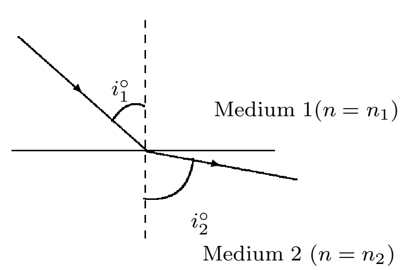
This is true for any other media

Using the above formula; the table below shows the critical angles for various media. Note that

Medium	Refractive index	Critical angle
Water	1.33	48.75°
Glass	1.5	41.81°
Water	$\frac{4}{3}$	48.59°
Diamond	2.46	23.99°

Table 2.3: Critical angle and refractive index

from $\sin c = \frac{1}{n}$, $n \sin c = 1$. This is at critical angle, for any other angle, i, in a medium of refractive index n, $n \sin i$ = a constant. Hence for any two media as shown below;



In general at the point of incidence

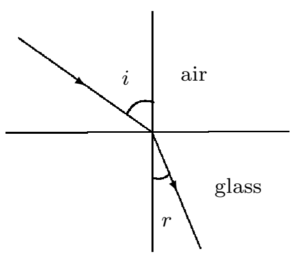
$$n_1 \sin i_1 = n_2 \sin i_2 = aconstant \tag{2.39}$$

this expression is easier to use for any two media than the one we had at the start, i.e

$$\frac{\sin i}{\sin r} = n \tag{2.40}$$

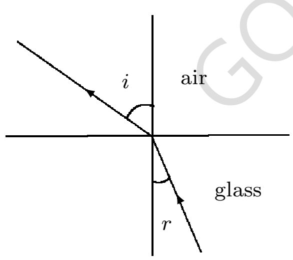
2.4.8 Prove that ${}_an_g = \frac{1}{{}_gn_a}$

For a ray of light moving from air to glass as shown in the diagram below, we have



$$\frac{\sin i}{\sin r} = {}_an_g \tag{2.41}$$

When the ray of light is reversed, then we have



$$\frac{\sin r}{\sin i} = {}_gn_a \tag{2.42}$$

$$\text{since } \frac{\sin i}{\sin r} = \frac{1}{\frac{\sin r}{\sin i}} \tag{2.43}$$

$$\text{then } {}_an_g = \frac{1}{{}_gn_a} \tag{2.44}$$

2.4.9 Showing that $n \sin i$ = constant, for any two media

Consider the diagrams below;

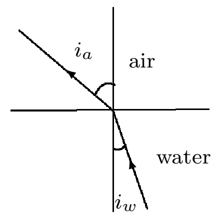


Fig 1

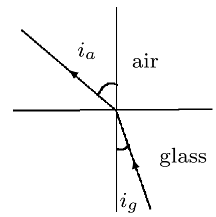


Fig 2

If the angles of incidence are the same in the two diagrams; then

For fig 1

$$\frac{\sin i_a}{\sin i_w} = {}_an_w \tag{2.45}$$

$$\Longrightarrow \sin i_a = {}_an_w \sin i_w \tag{2.46}$$

For fig 2

$$\frac{\sin i_a}{\sin i_g} = {}_an_g \tag{2.47}$$

$$\Longrightarrow \sin i_a = {}_an_g \sin i_g \tag{2.48}$$

rewriting it in terms of absolute refractive index: for water n_w, for glass n_g and for air n_a , we have

$$\sin i_a = n_w \sin i_w = n_g \sin i_g = n_a \sin i_a$$

, since $n_a = 1$; this shows that, when a ray of light is refracted from one medium to another, the boundaries being parallel;

$$n \sin i = \text{ a constant} \tag{2.49}$$

where n is the absolute refractive index and i is the angle made by the light ray with the normal in that medium.

Examples

1. The critical angle of a certain medium with air at the interface is 53.1°, calculate its refractive index?

Solution

$$\begin{aligned}
\text{Using } n &= \frac{1}{\sin c}\\
n &= \frac{1}{\sin 53.1}\\
&= \frac{1}{0.7997}\\
n &= 1.25
\end{aligned}$$

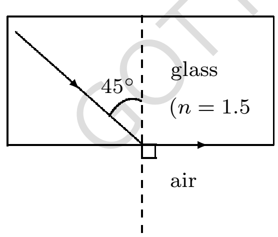
hence its refractive index is 1.25

2. The refractive index of flint glass is 1.513, calculate its critical angle?

Solution

$$\begin{aligned}
\text{Using } \sin c &= \frac{1}{n}\\
\sin c &= \frac{1}{1.513}\\
&= 0.6609\\
c &= 41.37^\circ
\end{aligned}$$

3. Describe what would happen to the incident ray in the figure below at the boundary i.e is it reflected or refracted?



Solution

Lets find the critical angle of this glass first

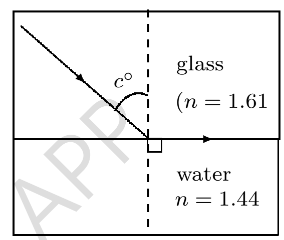
$$\begin{aligned}
\sin c &= \frac{1}{n}\\
\sin c &= \frac{1}{1.5}\\
&= 0.667\\
c &= 41.81^\circ
\end{aligned}$$

Since $i = 45$ and $c = 41.81$ then $i > c$ hence we shall have total internal reflection. The ray will be totally internally reflected.

4. Determine the critical angle for a ray of light moving from glass (n =1.61) to water (n = 1.44)?

Solution

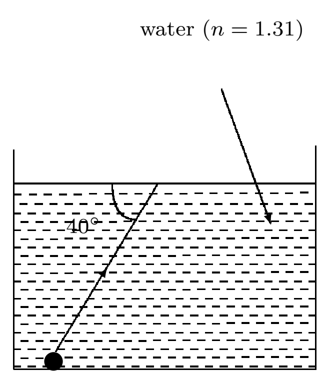
The figure below shows what happens when $i = c$



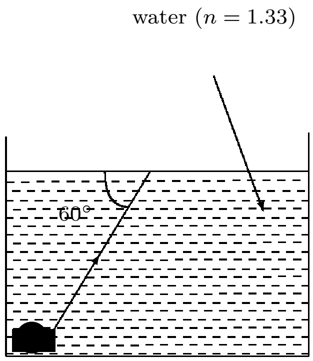
$$\begin{aligned}
\text{Here } i &= c \text{ and } r = 90^\circ\\
\text{Using } n\sin i &= \text{constant}\\
n_g \sin i_g &= n_w \sin i_w\\
1.61 \sin c &= 1.44 \sin 90^\circ\\
\sin c &= \frac{1.44}{1.61}\\
&= 0.8994\\
c &= 63.43^\circ
\end{aligned}$$

Exercise

1. The figure below shows a ray of light from a torch at the bottom of lake. Describe what would happen to it at the air-water surface

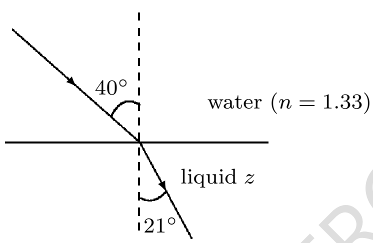


2. The figure below shows a ray of light from a submarine at a glancing angle of 60°. Determine whether the ray will be refracted or reflected and at what angle?

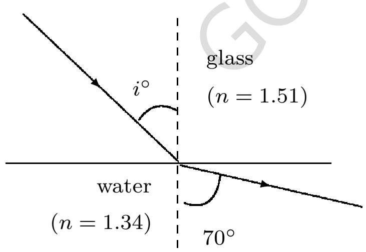


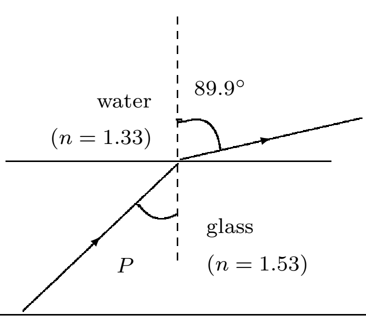
3. A ray of light is refracted glazing (angle of refraction is 90°) when moving from water (n = 1.41) to air. Determine the angle of incidence?

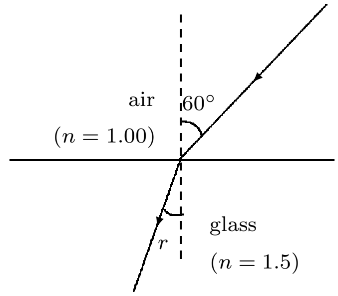
4. Using the diagram below determine the refractive index of liquid z?

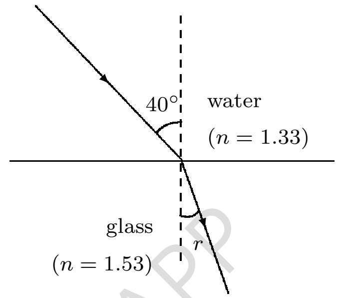


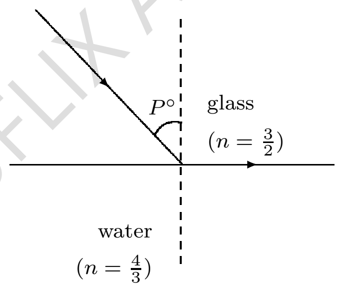
5. In the figures below determine the unknown angles











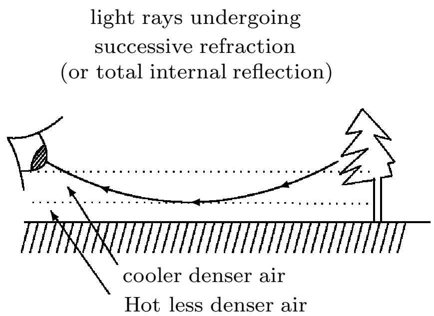
Natural phenomenon due to total internal reflection

1. **Mirage or mirage effect**

   On a sunny day or in desert areas, it is common to observe some thing like water on the ground, but when you travel to that point there is no water. This is what we call a mirage (an optical illusion).

   Explanation

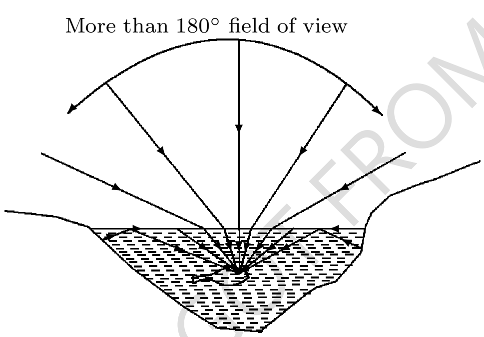
   On a hot day, air near the ground is hotter than that above it hence it is less dense than that above it i.e. the refractive index of air increases gradually from the ground up wards. Light from the sky moving towards the ground, refracts (or bends) gradually away from the normal near the hot ground (the hotter air). It begins to move from the ground towards ones eye. A person sees this light as if it is coming from the sky (like a reflection of the sky from a water surface). The diagram below illustrates this; some scientists say that this is not truly reflection but successive refraction.



Read about Newton's corpuscular theory about refraction and reflection

2. **The fish's eye field of view**

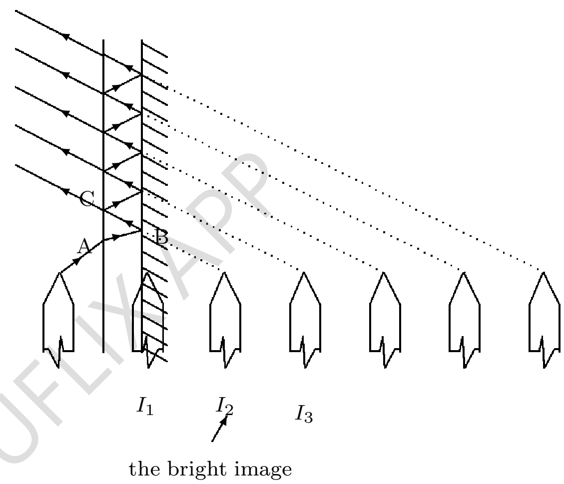
   A fish can see every thing above the water surface (a 180° field of view) with in an angle of about 98° (twice the critical angle of water i.e. 48.75 x 2 = 97.5°). The diagram below illustrates this;



   According to the diagram above, the fish can see objects with in the lake as a reflection (total internal reflection) at the water-air interface. But note that for a lake or a large water body the water surface is not smooth hence the ray can not be refracted grazing as shown on the diagram. Finally we conclude that a fish has a field of view more than 180°. . What is the field of view of a normal person? is it really 104°?

3. **Multiple images in a plane mirror**

   It is known that when you place a plane mirror in front of you, you obtain an image of your self in the plane mirror. Mirror are of different types; one type is made by silvering the back of a plane piece of glass, this is a thick plane mirror. When an object (like a pen) is placed in front of this mirror, several images of the pen are formed in the mirror but out of all, only one is brighter than the rest (the second one from the mirror). These images are formed due to the multiple reflections in the thick glass as shown below;



   Studying the diagram; At point A, a greater percentage of light is refracted, at point B it is reflected (a greater percentage) and at C a greater percentage of the same light is refracted. It is this ray that produces the brightest image (I_2). This first image is faint, the second is the brightest, the third is just brighter and the rest are very faint with the degree of brightness decreasing as you move a way from the mirror.

4. Other phenomenon include;

   - Empty test tubes appear silvery when looked at from above.
   - A coin in water in a transparent beaker, may appear and disappear when viewed from the bottom of the sides of a beaker at certain angles.
   - Pieces of glass always appears silvery or scatter light i.e cracked glass is silvery
   - Diamond is very bright because it totally reflects nearly every light ray that approaches it since it has a very low critical angle of 24°.

Applications of total internal reflection

There are very many applications of total internal reflection, a few of which are;

- Reflecting prisms
- Erecting prisms
- Transmission of radio waves
- Optical fibres used in communication

Let us describe each separately;

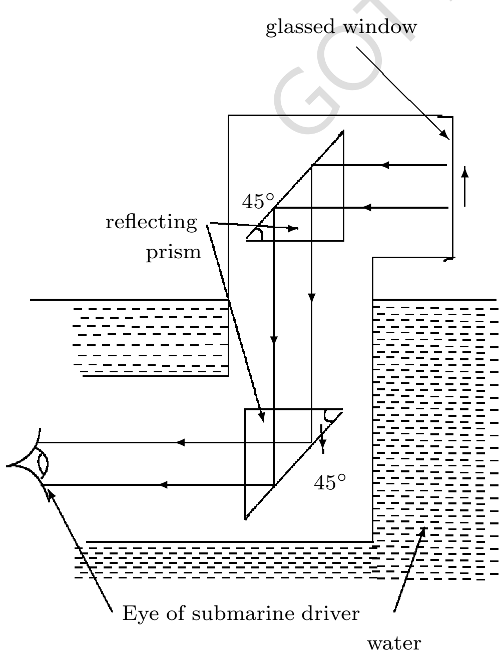
1. **Reflecting prisms (periscopes and binoculars)**

   These are triangular prisms with angles of 45°, 45° and 90°. Their use depends mainly on the angle of 45° which is more than the critical angle of the glass material used to make it, hence light is always totally internally reflected in them (since always $i > c$). They are always used in pairs in

   (a) Prism binoculars and

   (b) In submarine periscopes and in other types of periscopes.

   The diagram below illustrates this;

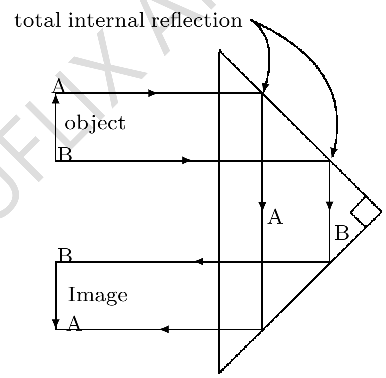


For prism binoculars: also as above the angle of incidence is 45° more than the critical angle of 41.81° hence we have total internal reflection. The advantage of these prisms over the usual plane mirrors is that they do not produce multiple reflections (images) as in plane mirrors hence the final image produced by them are more clear compared to those produced by the usual common plane mirrors.

2. **Erecting prisms:**

   These are right angle 45° prisms used to reverse light rays or invert light rays. The diagram below illustrates this property;

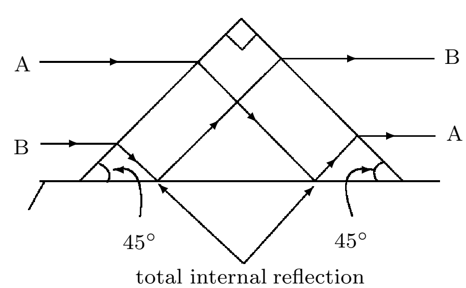
   (i) Reversing prisms



   Here the incident rays AB towards the prism are reversed by successive reflections and they leave the prism when they are inverted (upside down), study the above diagram properly.

   (ii) Erecting prisms

   These invert light rays with out deviating them. These are 45° right angles prisms used to invert light rays or to produce an upright (erect) image from an inverted one. The diagram below illustrates this;

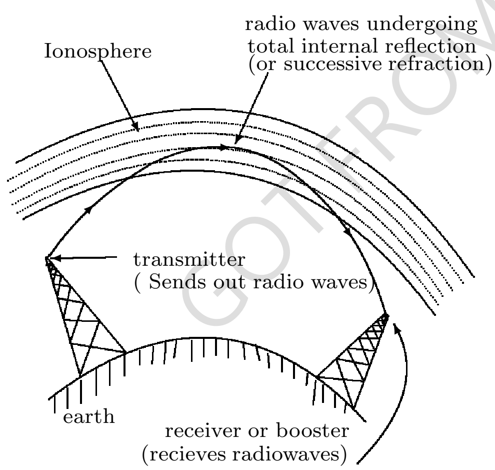


Note that if the rays AB form an inverted mage, then when they are passed through this erecting prism, they will form an erect image (upright image), that is why they are called **erecting prisms.**

3. **Transmission of radio waves (or electromagnetic waves)**

   Electromagnetic waves is one of the two kinds of waves (electromagnetic and mechanical waves). Electromagnetic waves all travel at the same speed of light ($3.0 \times 10^8 ms^{-1}$) and can not be felt easily. Examples of electromagnetic waves are light, TV waves, radio waves, microwaves, infrared radiations, x-rays, gamma rays and others. Here we are concerned with how radio waves are transmitted from one part of the earth to another.

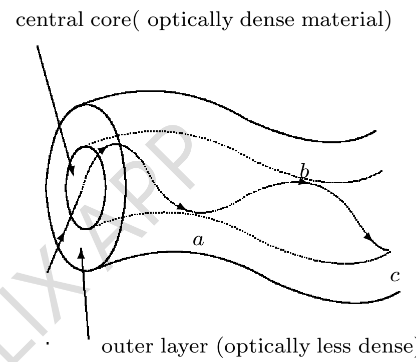
   Radio waves are produced at a certain point on earth by a transmitter in all directions. Those that move upwards into the sky are reflected (totally internally reflected) at the ionosphere to the other part of the earth as shown below;



   Ionosphere is a certain layer at a given height above the earth surface that contains ions (usually electrons) produced by the radiations from the sun. It is due to the presence of the ionosphere that enables radio waves to be reflected to the other part of the earth. This mode of communication is still used today in the short wave (SW) and medium wave (MW) transmission bands (not FM). With the modern technology, artificial satellite are used to transmit most of the electromagnetic waves.

4. **Optical fibres**

   These are fibres made from transparent materials like rubber and glass. An optical fibre is made up of two types of materials; the central core made from a more optically dense material surrounded by a an outer less optically dense material (called a cladding) as shown below;



   When light enters the core of an optical fibre at an angle as shown above, it will meet the boundary of the two materials at an angle greater than their critical angle and hence total internal reflection occurs. The same happens to the other points a, b, c, and hence the ray of light is transmitted with in the inner cladding from one end of the optical fibre to the other.

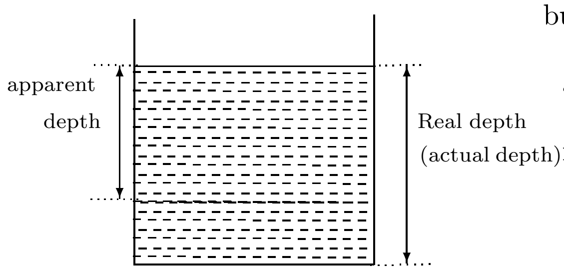
   Optical fibres are used to transmit pictures or pictures images. A special type of an optical fibre known as an endoscope is used to study the inner layers of the alimentary canal by passing it through the patient's mouth or otherwise and the degree of infection due to stomach ulcers can be determined. This is just an example.

2.4.10 Real and apparent depth

We said that refraction causes a number of effects, for instance

- a pencil dipped in water at an angle appears bend towards the water surface

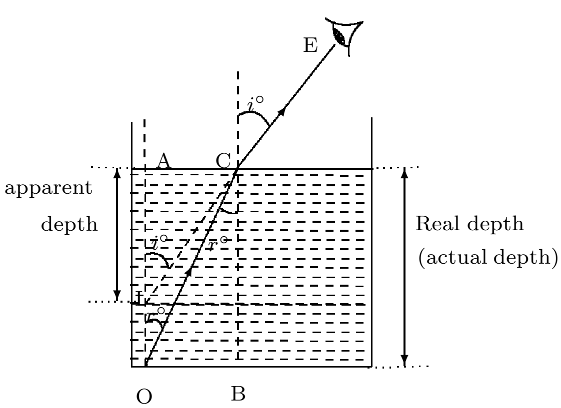
- the bottom of a pond or swimming pool appears to be raised or shallower than its actual depth (real depth). This new depth is known as apparent depth. The diagram below shows this;



The amount by which the bottom of pond is raised depends on the refractive index of the water in that pond and its depth, hence there exists a relationship between real depth, apparent depth and the refractive index of the material (like water , glass, diamond and others).

Relation between real depth, apparent depth and refractive index of a material

Consider the ray diagram below where a ray of light from an object at the bottom of tank full of water is refracted at the air-water interface;



I is the image of O

$$\begin{aligned}
\text{Using } n\sin i &= \text{a constant} && (2.50)\\
\text{we have } n_a \sin i &= n_w \sin r && (2.51)\\
\sin i &= n_w \sin r && (2.52)\\
n_w &= \frac{\sin i}{\sin r} && (2.53)
\end{aligned}$$

but also angle AIC = i

and angle AOC = r

from the triangle AIC

$$\sin i = \frac{AC}{IC} \qquad (2.54)$$

from triangle AOC

$$\sin r = \frac{AC}{OC} \qquad (2.55)$$

$$\begin{aligned}
n_w &= \frac{\sin i}{\sin r} && (2.56)\\
&= \frac{\frac{AC}{IC}}{\frac{AC}{OC}} && (2.57)\\
&= \frac{AC}{IC} \div \frac{AC}{OC} && (2.58)\\
&= \frac{AC}{IC} \times \frac{OC}{AC} && (2.59)\\
&= \frac{OC}{IC} && (2.60)\\
n_w &= \frac{OC}{IC} && (2.61)
\end{aligned}$$

when point C is very close to point A, then we have these approximations $OC \approx OA$ and $IC \approx IA$ hence

$$\begin{aligned}
n_w &= \frac{OC}{IC} && (2.62)\\
&= \frac{OA}{IA} && (2.63)\\
&= \frac{\text{real depth}}{\text{apparent depth}} && (2.64)
\end{aligned}$$

hence refractive index, n, of any transparent material like glass and water can be obtained using the expression;

$$\text{refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}} \qquad (2.65)$$

Examples

1. Water is poured in a burette to a depth of 4cm. A microscope that was previously focused at a point at the bottom of the beaker had to be raised 1.5cm upwards so as to focus the same point after pouring water into the beaker. Use this data to determine the refractive index of this water?

Solution

From the question, Real depth = $4cm$. After pouring in water in the beaker, it is known that the bottom of the beaker appears to have raised to a depth known as the apparent depth. The difference between the real depth and the apparent depth is known as the displacement.

$$\begin{aligned}
\text{displacement, } d &= 1.5cm \\
\text{Apparent depth} &= \text{Real depth} - d \\
&= 4cm - 1.5cm \\
&= 2.5cm \\
\text{if } n &= \text{refractive index} \\
n &= \frac{\text{Real depth}}{\text{Apparent depth}} \\
&= \frac{4cm}{2.5cm} \\
&= 1.6
\end{aligned}$$

hence the refractive index of water is 1.6

2. A microscope is focused upon an object. When a glass cover slip is put over the object, it is found that the eye piece of the microscope has to be moved $0.8mm$ upwards for clear focusing. Find the thickness of the cover slip (refractive index of glass =1.5)?

Solution

from the question

$$\begin{aligned}
n &= 1.5 \\
\text{displacement, } d &= 0.8mm \\
\text{Real depth}(R) &= ? \\
\text{Apparent depth}(A) &= R - d \\
R &= A + d....(i)
\end{aligned}$$

$$\begin{aligned}
n &= \frac{\text{Real depth}}{\text{Apparent depth}} \\
&= \frac{R}{A} \\
nA &= R \\
A &= \frac{R}{n}....(ii)
\end{aligned}$$

$$\begin{aligned}
\text{from } (i) R &= A + d \\
R &= \frac{R}{n} + d \\
R &= \frac{R}{1.5} + 0.8mm \\
R &= \frac{2}{3}R + 0.8mm
\end{aligned}$$

$$\begin{aligned}
R - \frac{2}{3}R &= 0.8mm \\
\frac{3}{3}R - \frac{2}{3}R &= 0.8mm \\
\frac{1}{3}R &= 0.8mm \\
3 \times \frac{1}{3}R &= 0.8 \times 3 \\
R &= 2.4mm
\end{aligned}$$

hence the thickness of the cover slip is $2.4mm$

Exercise

1. Explain why when looking vertically down into a vessel containing a transparent liquid, the bottom of the vessel is apparently raised?

2. The results in Table 2.4 were obtained in an experiment to determine the refractive index of water in a large tank ; Plot a graph of real

Real depth(cm)	Apparent depth(cm)
8.1	5.9
12.0	9.1
16.0	12.0
20.0	15.1

Table 2.4: Real and apparent depth

depth against apparent depth and use it to determine the refractive index of the water in the tank.

3. A microscope is focused on a mark at the bottom of beaker. Some liquid was added to the beaker and the microscope had to be raised 5mm to focus the mark again. It had to be raised another 10mm to focus the surface of the liquid. Calculate the refractive index of the liquid and draw a suitable diagram to illustrate this?

4. The following results were taken for rays moving from air into a glass block.

i°	r°
20	13
30	19
40	26
50	31
60	35

Table 2.5: Refractive index determination

Plot a graph of $\sin i$ against $\sin r$ and use it to determine the refractive index of glass?

5. What are the advantages of using totally reflecting prisms instead of plane mirrors in optical instruments. Illustrate your answer with two diagrams?

6. A fish which is $12cm$ long is swimming directly towards the plane glass front of an aquarium. What is the apparent length of the fish as seen by a person looking through the glass at the moment when the fish is 3feet from it? The index of refraction of water relative to air is $\frac{4}{3}$. (1 foot = 12inches, 1inch=2.54cm)

7. Calculate the apparent depth of
   (a) a pond 10feet deep
   (b) a glass block 1 feet thick

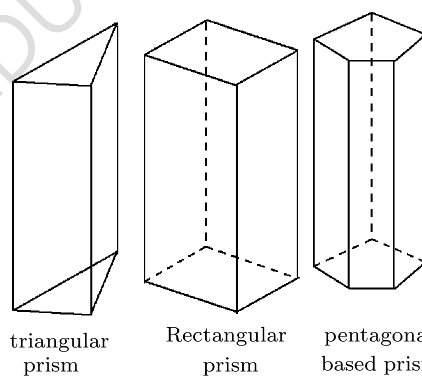
   when viewed by an eye placed vertically above(refractive index of water is 1.33 and of glass 1.5)

8. A transparent cube of $15cm$ edge contains a small air bubble. Its apparent depth when viewed through one face of the cube is $6cm$ and when viewed through the opposite face, it is $4cm$. What is the actual distance of the bubble from the first face and what is the refractive index of the substance of the cube? (ans;9cm, 1.5)

2.5 Prisms

2.5.1 Introduction

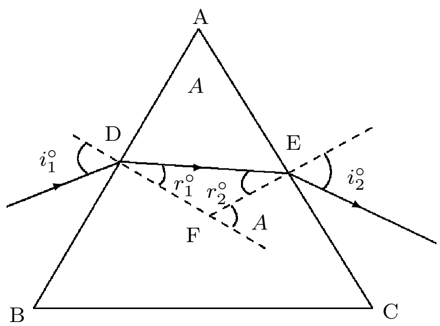
A prism is any solid or hollow three-dimensional figure with a uniform cross section. For instance a cylinder is a prism whose base is a circle, other examples of prisms are drawn below;



In physics we pay more attention to the triangular prism. Historically, sir Isaac Newton had a transparent triangular prism in his room which had a small hole or slit in the roof. When he made the light ray from the slit in the roof to pass through the triangular prism, bright colours were displayed. The colours were red, orange, yellow, green, blue, indigo and violet i.e. rainbow colours. Sometimes these colours are not formed due to some reasons. Here we shall concentrate on the case when the white beam of light is refracted through a triangular prism un dispersed (with out splitting into the rainbow colours).

2.5.2 Refraction of light in the triangular prism

Consider the ray of light passing through the triangular prism as shown below;



i_1° = angle of incidence
i_2° = angle of emergence

i_1 is the angle of incident and i_2 is the angle of emergency.

On face BA, the ray bends towards the normal after refraction, i.e. $r_1 < i_1$.

On face AC, the light ray bends away from the normal. We can obtain the angle of refraction, r_1 using

$$\frac{\sin i_1}{\sin r_1} = n. \tag{2.66}$$

But the sum of two interior angles of a triangle is equal to the opposite exterior.

$$r_1 + r_2 = A \tag{2.67}$$

where A is the refracting angle of the prism.

After obtaining the value of r_1 the value of r_2 can be obtained using the equation, $r_2 = A - r_1$. Then finally i_2 can be obtained using

$$\frac{\sin i_2}{\sin r_2} = n. \tag{2.68}$$

When the prism is not in air but in another medium (like a liquid) of refractive index n, then we use the expression $n \sin i^\circ$ = constant and the above equation becomes

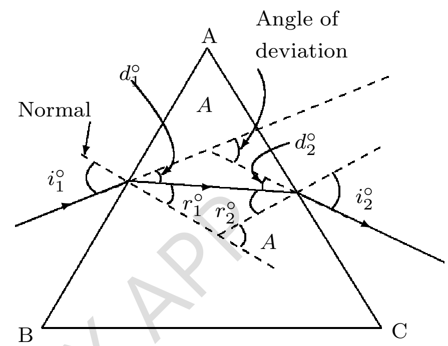
$$n \sin i_1 = n_1 \sin r_1 \tag{2.69}$$

and the second one to

$$n \sin i_2 = n_1 \sin r_2 \tag{2.70}$$

where n is the refractive index of the liquid and n_1 is the refractive index of the material of the prism.

In general, light is refracted in the triangular prisms according to the diagram below;



i_1° = angle of incidence
i_2° = angle of emergence

AB and AC are refracting surfaces

A is the refracting angle of the prism

i is the angle of incidence (i_1 and i_2)

r is the angle of refraction (r_1 and r_2)

d is the angle of deviation (d_1 and d_2)

i_2 is the angle of emergence

d is the total deviation,

$$\begin{aligned} d &= d_1 + d_2 && (2.71)\\ d_1 &= i_1 - r_1 && (2.72)\\ d_2 &= i_2 - r_2 \text{ and} && (2.73)\\ A &= r_1 + r_2 && (2.74) \end{aligned}$$

$$\begin{aligned} d &= d_1 + d_2 && (2.75)\\ &= i_1 - r_1 + i_2 - r_2 && (2.76)\\ &= i_1 + i_2 - r_1 - r_2 && (2.77)\\ &= i_1 + i_2 - (r_1 + r_2) && (2.78)\\ d &= i_1 + i_2 - A && (2.79) \end{aligned}$$

And the following statements are true about the triangular prism;

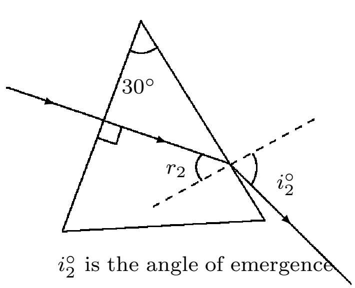
- For the prism in air, The incident ray is refracted towards the normal when entering the prism and the refracted ray is bent a way from the normal when leaving the prism.
- The emergent ray is not parallel to the original incident ray i.e. the ray is deviated.
- Light is deviated towards the base of the prism. The angle of deviation depends on the angle of incident (i), refracting angle (A) and the refractive index of the material of the prism.
- Prisms with small refracting angle (A), produce a smaller angle of deviation compared to those with a large refracting angle.
- Sometimes white light is split[5] up into colours when it passes through a triangular prism.

Examples

1. A ray of light is incident normally on a triangular prism with a refracting angle of 30°. Determine the angle of emergence? (Refractive index of the material of the prism is 1.61).

Solutions

Consider the diagram below;

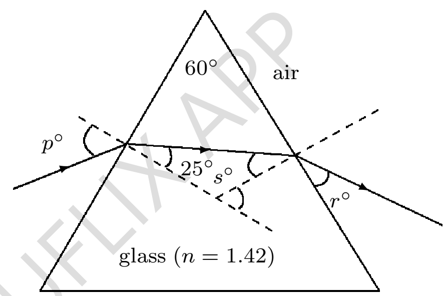


[5]This is discussed in the section 2.8.1 on page 76

Using the relation

$$\begin{aligned} n \sin i &= \text{constant} \\ n_g \sin r_2 &= n_{\text{air}} \sin i_2 \\ 1.61 \sin 30 &= 1 \sin i_2 \\ \Longrightarrow \sin i_2 &= 1.61 \times 0.5 \\ &= 0.805 \\ i_2 &= 53.61^\circ \end{aligned}$$

2. Use the diagram below to determine



(i) the angle p

(ii) the angles s and r (refractive index of the material of the prism is 1.42)

Solutions

(i) using the relation n sin i = constant

$$\begin{aligned} n_{\text{air}} \sin p &= n_g \sin 25 \\ \sin p &= 1.42 \times 0.4226 \\ &= 0.600 \\ p &= 36.88^\circ \end{aligned}$$

(ii) recall that $r_1 + r_2 = A$

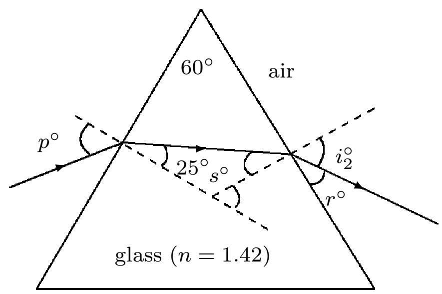
$$\begin{aligned} 25 + s &= 60 \\ s &= 60 - 25 \\ s &= 35^\circ \end{aligned}$$

at this point we need to know whether the ray will be refracted or reflected.

The Critical angle can be obtained from

$$\begin{aligned}
\sin c &= \frac{1}{n}, \\
\sin c &= \frac{1}{1.42} \\
\sin c &= 0.7042 \\
c &= 44.77^\circ,
\end{aligned}$$

since the angle of incidence, s, is less than the critical angle then we shall have refraction and not total internal reflection. Then what is the angle of refraction? Let the angle of refraction be i_2



Using the relation

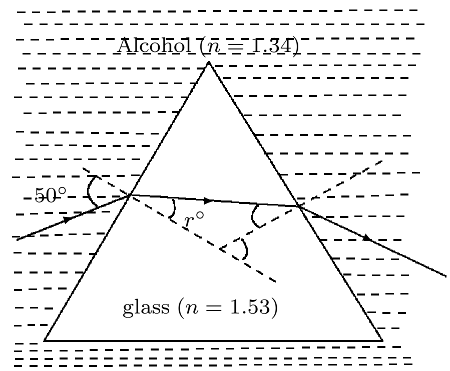
$$\begin{aligned}
n \ \sin i &= \text{constant} \\
n_g \sin s &= n_a \sin i_2 \\
1.42 \sin 35 &= 1 \times \sin i_2 \\
\sin i_2 &= 1.42 \times \sin 35 \\
&= 1.42 \times 0.5736 \\
\sin i_2 &= 0.8145
\end{aligned}$$

$$\begin{aligned}
i_2 &= 54.5^\circ \\
\text{but } i_2 + r &= 90 \\
r &= 90 - i_2 \\
&= 90 - 54.5 \\
&= 35.5^\circ
\end{aligned}$$

hence $r = 35.5^\circ$ and $s = 35^\circ$

3. The angle of incident on one of the faces of a triangular prism dipped in alcohol is 50°. Given the refractive indices of the material of the prism and alcohol are 1.53 and 1.34 respectively, determine the angle of refraction in the prism?

Solutions

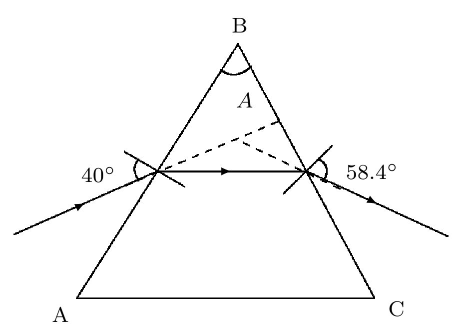


Using the relation

$$\begin{aligned}
n \sin i &= \text{constant} \\
n_{\text{alcohol}} \sin 50 &= n_g \sin r \\
1.34 \times 0.766 &= 1.53 \sin r \\
1.026 &= 1.53 \text{ sin } r \\
1.53 \sin r &= 1.026 \\
\frac{1.53}{1.53} \sin r &= \frac{1.026}{1.53} \\
\sin r &= 0.6709 \\
r &= 42.14^\circ
\end{aligned}$$

hence the angle of refraction in the prism is 42.14°.

4. A ray of light incident on a glass prism of refractive index 1.5 at an angle of incident of 40° emerges out of it, at an angle of emergence of 58.4° as shown below;



Determine the

(i) the angle of refraction and deviation on refracting face AB

(ii) the angle of refraction and deviation on the refracting face BC

(iii) the refracting angle of the prism

Solutions

Using $n_1 \sin i_1 = n_2 \sin i_2$

(i) On Face AB

$$
\begin{aligned}
n_1 \sin i_1 &= n_2 \sin i_2 \\
1 \times \sin 40 &= 1.5 \times \sin r_1 \\
\sin r_1 &= \frac{\sin 40}{1.5} \\
&= \frac{0.6428}{1.5} \\
&= 0.4285 \\
r_1 &= 25.37
\end{aligned}
$$

$$
\begin{aligned}
\text{deviation} &= d_1 \\
d_1 &= i_1 - r_1 \\
&= 40 - 25.37 \\
&= 14.63^\circ
\end{aligned}
$$

(ii) On face BC

$$
\begin{aligned}
n_1 \sin i_1 &= n_2 \sin i_2 \\
1.5 \sin r_2 &= 1 \times \sin 58.4 \\
\sin r_2 &= \frac{\sin 58.4}{1.5} \\
&= \frac{0.8517}{1.5} \\
&= 0.5678 \\
r_2 &= 34.60^\circ
\end{aligned}
$$

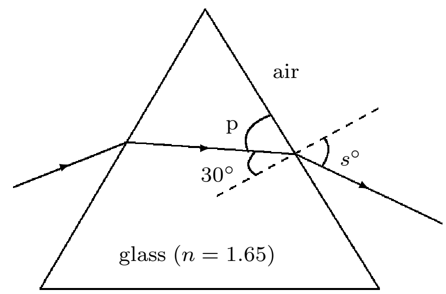
$$
\begin{aligned}
\text{deviation} &= d_2 \\
d_2 &= i_2 - r_2 \\
&= 58.4 - 34.6 \\
&= 23.8
\end{aligned}
$$

(iii) The Refracting angle (A)

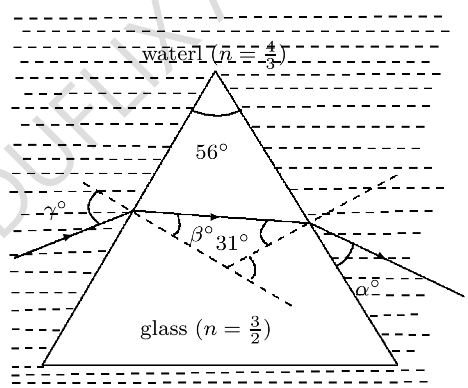
$$
\begin{aligned}
A &= r_1 + r_2 \\
&= 23.37 + 34.6 \\
&= 57.97^\circ
\end{aligned}
$$

Exercise

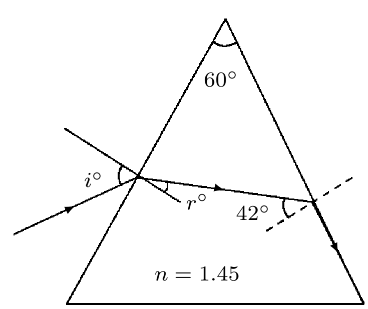
1. Determine the angles S and P in the diagrams below;



2. Use the ray diagram below to determine the angles α, β and γ



3. Light of the same wavelength is incident at an angle i on a glass prism. The light is refracted and follows the path as shown below



find

(i) the angle r

(ii) the angle i

2.6 Lenses

2.6.1 Introduction

Definition 9.

> A lens is a transparent material bounded by at least a curved surface.

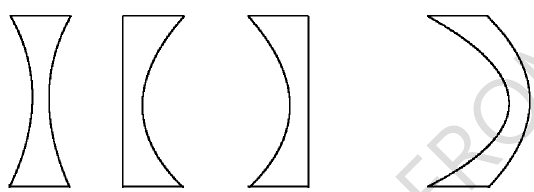
Usually a transparent material like glass, water or diamond is used.

Types of lenses

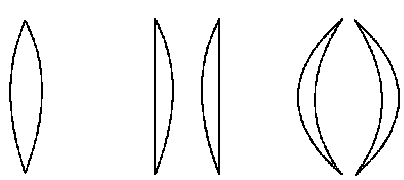
There are two types of lenses

1. Concave lens- thinner in the middle
2. Convex lens - thick in the middle.

Concave lenses are generally of three types, these are shown below;



Convex mirrors are generally of three forms, these are shown below;



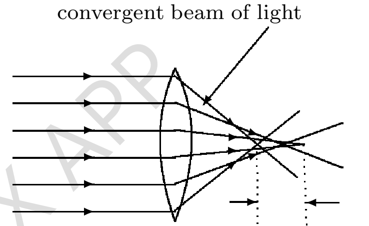
In this book, whenever we talk of concave and convex lens, we mean bi-concave and bi-convex lens respectively.

2.6.2 Effect of lenses on a parallel beam of light

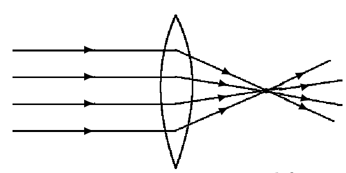
Lets handle one type of lens at a time;

Convex lenses

1. When a convex lens is placed in the path of a **wide parallel beam** of light, light rays converge to a certain small region as shown below;



2. When a convex lens is placed in the path of a **narrow parallel beam** of light, it converges to a fixed point F (principal focus) as shown below;

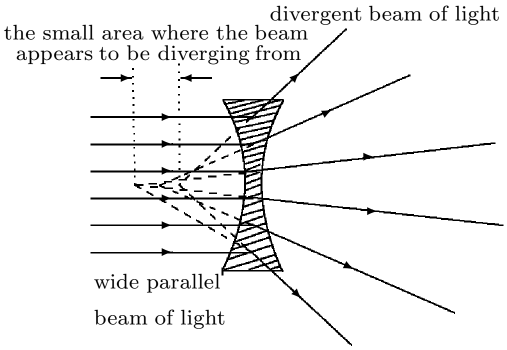


From these two illustrations it was concluded that whenever a parallel beam of light passes through a convex lens, it becomes or is refracted as a converging beam of light hence a convex lens is sometimes known as a **converging lens**

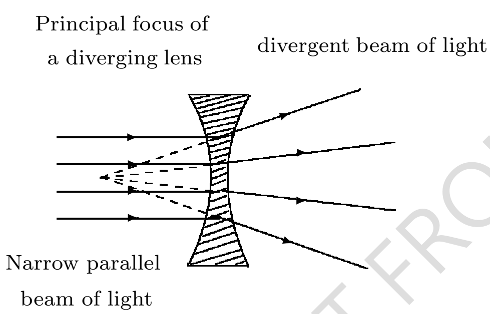
Concave lens

1. When a **wide parallel beam** of light is passed through a concave lens, it is refracted

as a diverging beam of light. It appears to be diverging from a certain small region behind the concave lens as shown in the diagram below;



2. When a concave mirror is placed in the path of a **narrow parallel beam** of light, it is refracted as a diverging beam of light, which appears to be diverging from a certain point F behind the lens. This point F is known as the **principal focus**, the diagram below illustrates this;



Because of this a concave lens is also known as a **diverging lens**.

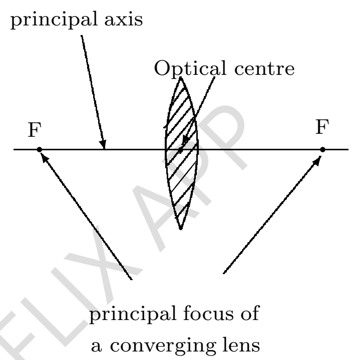
Note that a wide parallel beam of light does not converge to a fixed point or appear to be coming from a fixed point like the narrow beam of light for both lenses and curved mirrors; this effect is known as **spherical aberration**.

2.6.3 Terms used in the optics of lenses

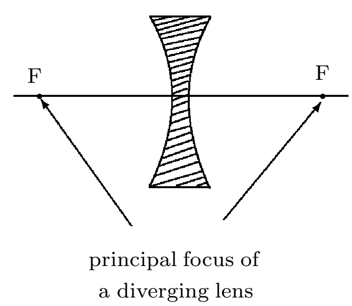
1. **Optical centre** of a lens(O) is the centre or mid point of the lens

2. **Principal axis** of a lens is an imaginary line passing through the optical centre but perpendicular to the lens or the principal axis of a lens is an imaginary line passing through the optical and principal focus of a lens.

3. **Principal focus of a convex lens** is a point on the principal axis to which all rays parallel and close to the principal axis converge to after refraction in the lens.

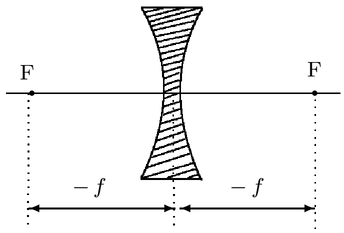


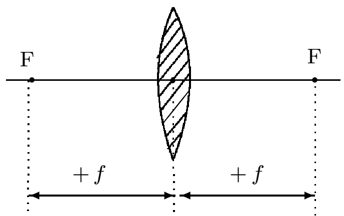
4. **Principal focus of a concave lens** is a point on the principal axis from which all rays parallel and close to the principal axis appear to diverge from after refraction in the lens.



5. **Principal focus of a lens** (F) is a point on the principal axis for which all rays parallel and close to the principal axis converge to or appear to diverge from after refraction in the lens.

6. **Focal length** (f) is the distance between the principal focus and the optical centre of a lens.



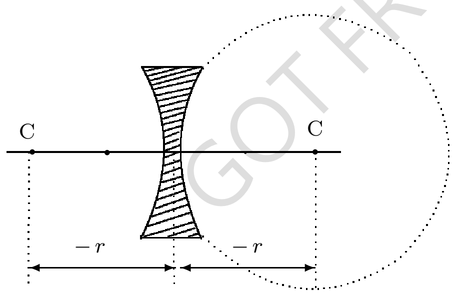


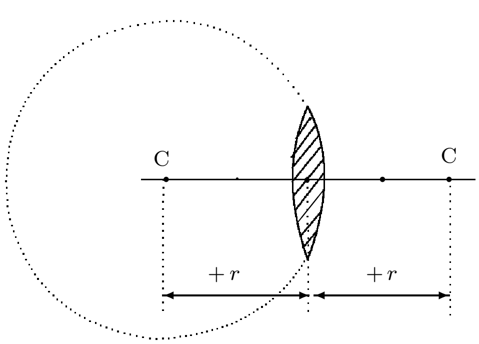
7. **Centre of curvature of a lens**. Whenever the curved surfaces of a lens are extended, they make a circle or a sphere, the lens is part of this sphere. The centre of this sphere is called the centre of curvature of the lens. Hence the

Definition 10.

center of curvature of a lens is the center of a sphere of which the lens is part.

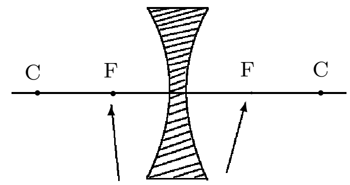
8. **Radius of curvature of a lens** (r) is the radius of a sphere of which the lens is part of.



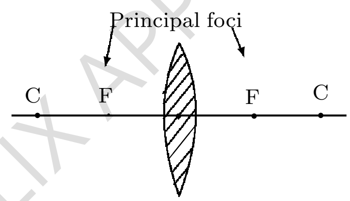


Two foci of a lens

Unlike the concave and convex mirrors which have only one principal focus, the lens (concave and convex) have two foci, they are on both sides of the lens as shown below;

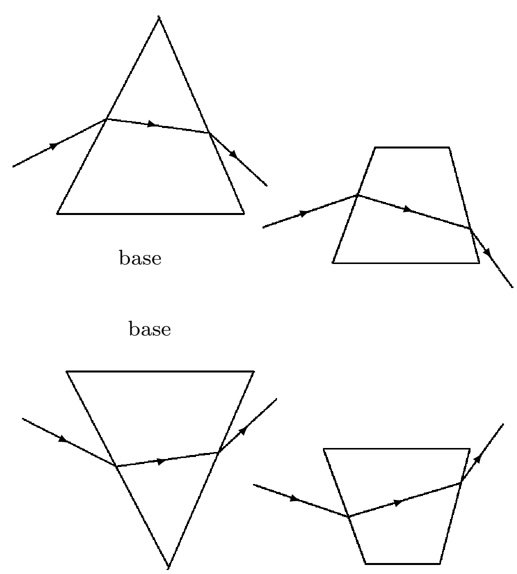


principal foci

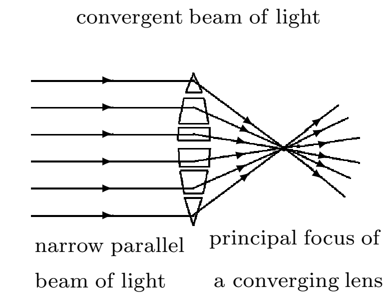


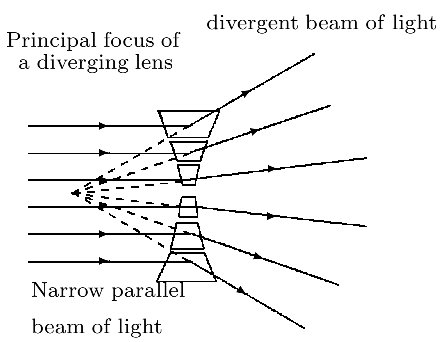
2.6.4 Prism effect of a lens:

When light is refracted in the triangular prism, it bends towards the base of the prism as shown below;



With this in mind, consider the lens as a combination of prisms. The individual prisms refract light to produce a lens effect of either diverging or converging a parallel beam of light as shown below;

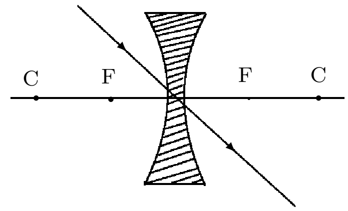


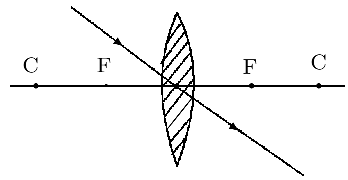


When the spaces between these prisms is minimized (made too small or removed), we obtain the true lens.

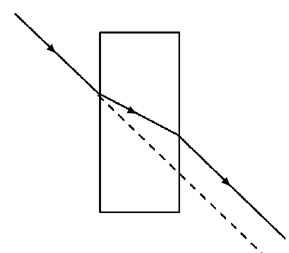
2.6.5 Rules for drawing ray diagrams with lenses

To determine the position of an image formed by a lens, we need to know certain rules or procedures to follow for lenses. These are;

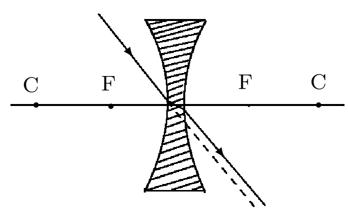




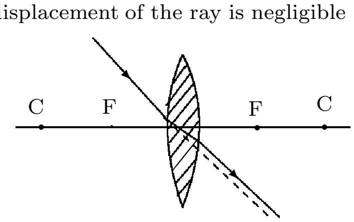
But remember that when light is incident at an angle to a parallel-sided body or glass block, the light ray is displaced slightly depending on the angle of incident and thickness of the glass block. Therefore when light passes through the optical centre of a lens, this light ray is slightly displaced but this displacement is negligible and hence neglected, that is why most lenses are made with a smaller thickness.



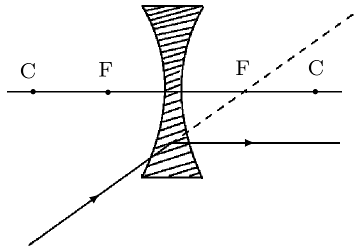
light ray is displaced when it passes through a glass block

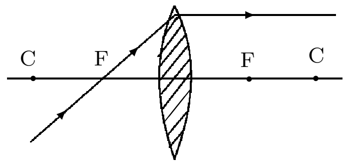


displacement of the ray is negligible

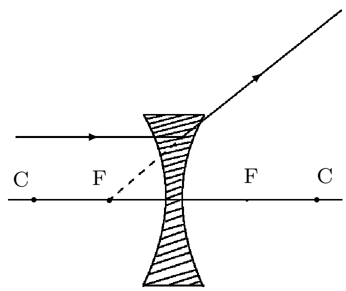


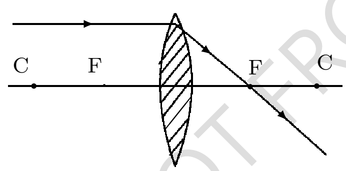
1. All rays passing through the optical centre of a lens are not refracted (they pass through it undeflected or un-refracted)
2. All rays passing through or heading towards the principal focus of a lens, are refracted parallel to the principal axis of the lens.





3. All light rays parallel and close to the principal axis are refracted towards or away from the principal focus of the lens. This is shown below (comparing this rule with the second one , one is the opposite of the other according to the principle of reversibility of light)





Note that these rules are similar to those used in curved mirrors except the rule for rays passing through the centre of curvature is not considered here. These rules can now be used to determine the position and nature of the final image of an object placed at any point on the principal axis of a lens in a given ray diagram. In a given ray diagram we shall use only two rules just to keep our ray diagram neat.

2.6.6 Images formed by lenses:

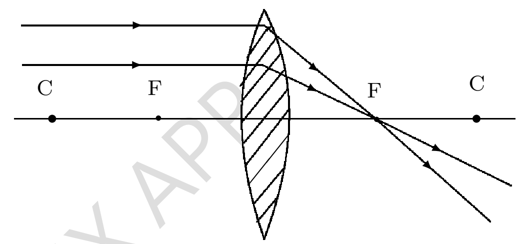
Since we have concave and convex lenses, let us discuss each separately. In a given ray diagram, we shall use an arrow to represent an object and its image, the points C, F and O represent centre of curvature, principal focus and optical center respectively.

Convex lenses

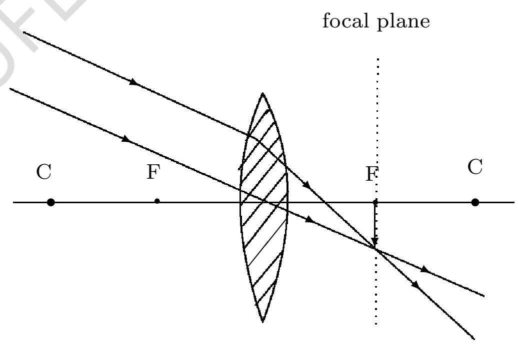
1. **Object at infinity**

   Light rays from an object at infinity (far away) arrive at the lens as parallel rays.

light rays from an object at infinity



light rays from an object at infinity

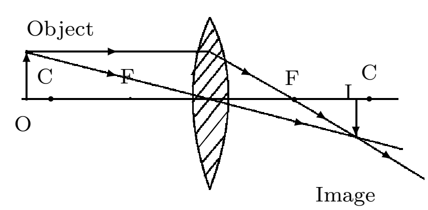


   The final image is

   - Formed at the principal focus or in the focal plane.
   - Inverted

   This result is applied in the determination of the focal length of a lens by measuring the distance of the lens from the final image as the focal length of the convex lens.

2. **Object just beyond C**

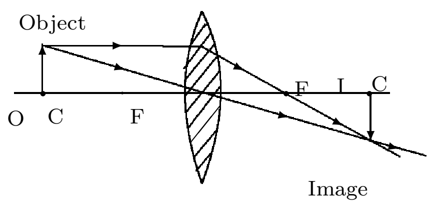


The final image is

- Real
- Diminished (smaller than the object)
- Inverted (upside down)
- Formed between F and C on the other side of the lens

This is applied in photography where the final image, a photograph on a small piece of paper, is smaller than the object (a full human being or buildings).

3. **Object at C**

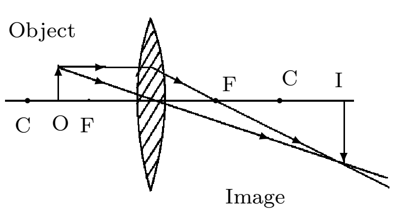


The final image is;

- real
- same size as the object (magnification is unity)
- inverted (upside down)
- formed at the other C

This is applied only when the inverted image is to be made upright.

4. **Object between F and C**

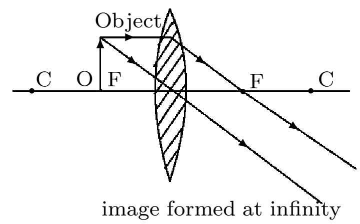


The final image is;

- Real
- Magnified (bigger than the object)
- Inverted (upside down)
- Formed beyond the other C

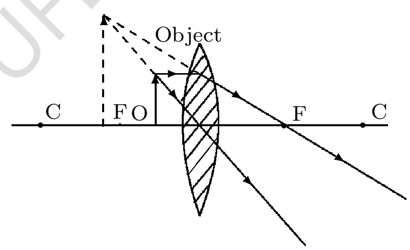
This is applied in the projector where the object is a slide whose image is formed on a large white screen (white cloth or paper). Usually projectors are used to show films, teach, demonstrate among other uses.

5. **Object at F**



The nature of the image formed at infinity is not known (since no one is sure of what is at infinity).

6. **Object between F and O**



The final image is

- Virtual (not formed by the actual intersection of the rays)
- Magnified (bigger than the object)
- Erect (upright)
- Formed on the same side of the object

This ray diagram is applied in magnifying glasses where a small convex lens is placed just near a small object and seeing the object through the lens, it appears to be larger than the object(i.e. magnified). This image is virtual, it can not be viewed on or placed on the screen (a white sheet of paper).

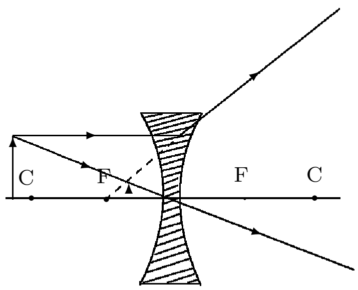
When the object is placed on the other side of the lens, we shall just be repeating the already finished ray diagrams.

Concave lenses

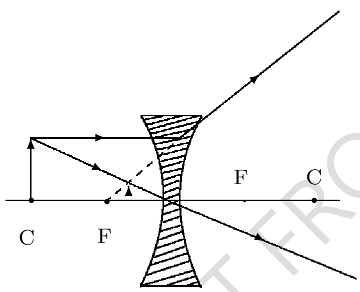
Images formed by concave lenses are all of the same nature irrespective of the position where the object is.

Consider the ray diagrams below;

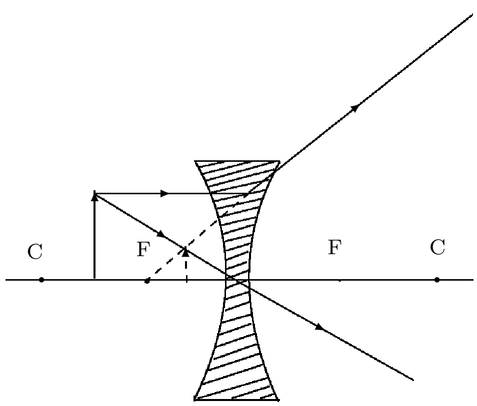
1. **Object just beyond C**



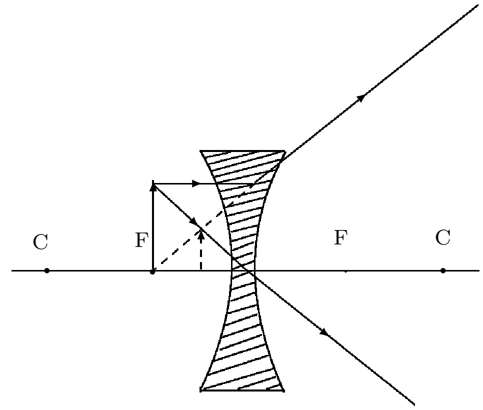
2. **Object at C**



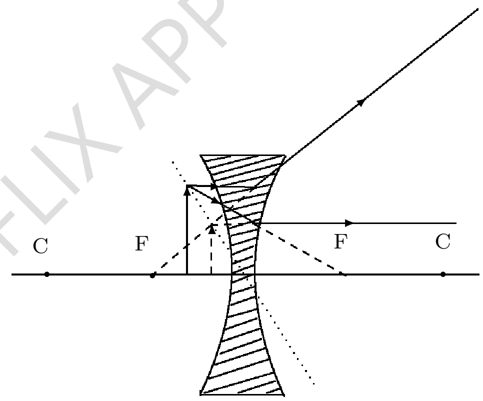
3. **Object between C and F**



4. **Object at F**



5. **Object between F and O**



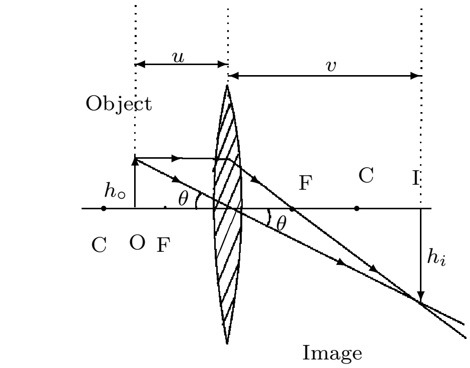
Note that all the images formed by diverging lenses are always;

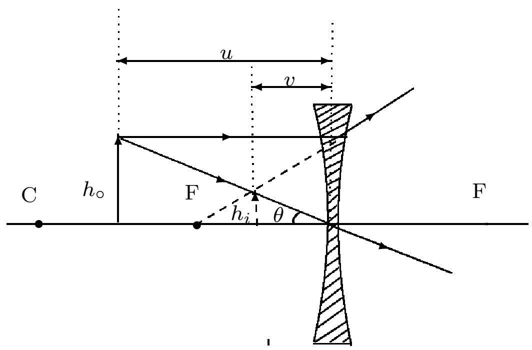
- Virtual (not formed by the actual intersection of the rays)
- Diminished (smaller than the object)
- Erect (upright)
- Formed between the object and the lens

6. Can you sketch a ray diagram you would use to determine the position of the final image formed by a concave lens due to rays from an object at infinity?

Magnification

Consider the ray diagrams below;





from these diagram, note that;

$$\tan\theta = \frac{h_i}{v} \quad (2.80)$$

$$\text{also } \tan\theta = \frac{h_\circ}{u} \quad (2.81)$$

$$\Rightarrow \frac{h_i}{v} = \frac{h_\circ}{u} \quad (2.82)$$

$$\Leftrightarrow \frac{h_i}{h_\circ} = \frac{v}{u} \quad (2.83)$$

since

$$\text{Magnification} = \frac{h_i}{h_\circ} \quad (2.84)$$

$$\text{and } \frac{h_i}{h_\circ} = \frac{v}{u} \quad (2.85)$$

then also

$$\text{magnification} = \frac{v}{u} \quad (2.86)$$

hence for lenses, mirrors, and pin-hole cameras

$$\text{magnification} = \frac{h_i}{h_\circ} = \frac{v}{u} \quad (2.87)$$

where v and u are the image and object distances.

2.6.7 Construction of accurate ray diagrams:

Note that when the object placed at one point on one side of the lens its image is formed at a fixed point on the other point of the lens. Hence there exists a special relationship between the object distance (distance of the object from the optical centre of the lens) and the image distance (distance of the image from the optical centre of the lens).

One can determine the position and nature of the final image given the object distance and a lens of known focal length using an accurate ray diagram. All the ray diagrams drawn so far are sketches, to construct an accurate ray diagram;

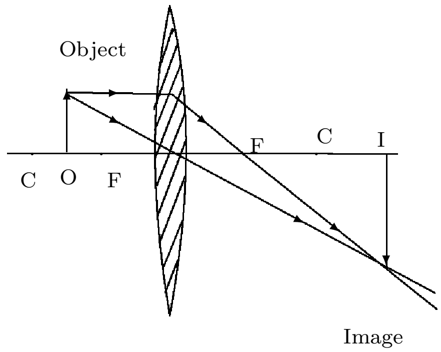
1. You use any two of the rules for constructing ray diagrams using lenses
2. You represent the lens with a vertical straight line with the symbol of the lens (which indicates its type; convex or concave) at the point where it meets the principal axis.
3. Label your ray diagram properly with the object or image distance, F, O, C, and others.
4. Use a graph paper or a certain scale so as to be able to measure the require distances from the ray diagram.

Example

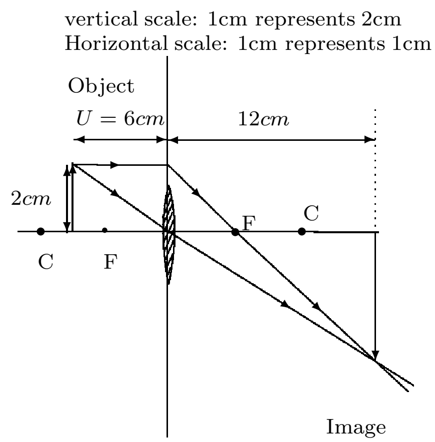
An object 4cm high is placed 6cm from a convex lens of focal length 4cm. Construct an accurate ray diagram and use it to determine the position and height of the image?

Solution

sketch



accurate ray diagram



Please draw your own ray diagram using a graph paper

From the ray diagram on the graph paper, the position of the image is 12cm and its height is 8cm.

2.6.8 Lens formula:

Note that; when an object is placed at a certain distance, u, from the lens, its image is formed at a specific distance, v, from the same lens, if this is true then there exists a relationship between u and v. This relation is called the lens formula shown below;

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{2.88}$$

where

u is object distance

v is the image distance

f is the focal length of the lens.

To use the lens formula, one should use either

1. the **"real-is-positive"** sign convention or
2. the **"new Cartesian"** sign convention

The details of these sign conventions can be found on page 26. We are to use the real is positive sign convention.

In this convention the principal focus and the centre of curvature of a diverging lens are virtual and therefore its focal length $(-f)$ and radius of curvature $(-r)$ are negative.

For the convex lens; the principal focus and the centre of curvature are real, hence its focal length $(+f)$ and radius of curvature $(+r)$ are positive.

Examples

1. A diverging lens forms an image at a distance of 6cm from the lens with a magnification of 0.6. Determine the position of the object and the focal length of the lens?

Solution

Diverging lenses form virtual images.

$$\begin{aligned} v &= -6cm \\ M &= 0.6 \\ \text{But } M &= \frac{v}{u} \\ u &= \frac{v}{M} \\ &= \frac{6}{0.6} \\ &= 10cm \end{aligned}$$

Hence the object is 10cm from the lens using the real is positive sign convention

Using

$$\begin{aligned}\frac{1}{f} &= \frac{1}{u}+\frac{1}{v},\\ \frac{1}{f} &= \frac{1}{10}+\frac{1}{-6}\\ &= \frac{-6+10}{-60}\\ &= \frac{4}{-60}\\ f &= \frac{-60}{4}\\ &= -15cm\end{aligned}$$

(negative because it is virtual focal length) hence the focal length of the diverging lens is 15cm.

2. A convex lens forms a virtual image 8cm high from an object 2cm high placed 12cm in front of the lens. Determine the focal length of the lens?

Solution

let M be the Magnification,

$$\begin{aligned}M &= \frac{h_i}{h_\circ}\\ &= \frac{8}{2}\\ &= 4\\ \text{But also } M &= \frac{v}{u}\end{aligned}$$

Hence

$$\begin{aligned}M=4 &= \frac{v}{12}\\ v &= 4\times 12\\ &= 48cm \text{ but is virtual so}\\ v &= -48cm\end{aligned}$$

Using $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$$\begin{aligned}\frac{1}{f} &= \frac{1}{u}+\frac{1}{v}\\ &= \frac{1}{12}+\frac{1}{-48}\\ &= \frac{-48+12}{12\times -48}\\ &= \frac{-36}{12\times -48}\\ &= \frac{-1}{-16}\\ \frac{1}{f} &= \frac{1}{16}\\ \Rightarrow f &= 16cm\end{aligned}$$

Hence $f = 16$cm (positive focal length) Therefore the focal length of the convex lens is +16cm.

Question

1. An object is placed 6cm from a converging lens of focal length 15cm. Determine the nature, position and magnification of the image?
2. Where must an object be placed so that the image formed by a converging lens will be
   (i) at infinity
   (ii) the same size as the object
   (iii) erect
   (iv) inverted and enlarged
   (v) near the object as possible .
3. A simple magnifying glass produces an enlarged erect image when an object is situated 15cm from the lens. Given the length of the image is twice that of object, calculate;
   (a) the distance of the image from the lens
   (b) the focal length of the lens
4. Draw a ray diagram to show how a convex lens of focal length 10cm may be used to obtain
   (i) magnified real image
   (ii) a magnified virtual image

2.6.9 Power of a lens

Power of lens is defined as the reciprocal of the focal length of the lens in metres.

$$\text{Power of a lens} = \frac{1}{\text{focal length in metres}} \quad (2.89)$$

SI units of power of a lens is a **Dioptre**. For instance the power of a lens of focal length 2cm is

$$\begin{aligned} P &= \frac{1}{f} \\ \text{but } f &= 2cm \\ f &= \frac{2}{100}m \end{aligned}$$

$$\text{but } P = \frac{1}{f \text{ in metres}}$$

$$\begin{aligned} P &= \frac{1}{\frac{2}{100}} \\ &= 1 \div \frac{2}{100} \\ &= 1 \times \frac{100}{2} \\ &= \frac{100}{2} \\ P &= 50 \text{ Dioptres or } 50D \end{aligned}$$

The power of a lens is a measure of the magnifying strength of lens. The smaller the focal length of a lens the greater is its magnifying power.

Note that the power of a convex lens is positive and that of a concave lens is negative. This means that a concave lens can not produce a magnified image, all images produced by concave lenses are diminished, the smaller the focal length of a concave mirror the more diminished are its images.

Definition 11.

Dioptre is the power of a lens with a focal length of 1m.

Example

Determine the power of the following lenses;

(a) a convex lens of focal length 4cm

(b) a concave lens of focal length 5cm

Solution

(a) Using the formula for power of a lens

$$\begin{aligned} f &= 4cm \\ f &= \frac{4}{100}m \\ &= \frac{1}{25}m \\ P &= \frac{1}{f} \end{aligned}$$

$$\begin{aligned} P &= \frac{1}{\frac{1}{25}} \\ &= 1 \div \frac{1}{25} \\ &= 1 \times \frac{25}{1} \\ &= 25 \\ P &= 25D \end{aligned}$$

(b) Using the formula for power of a lens

$$\begin{aligned} f &= -5cm \\ &= \frac{-5}{100}m \\ &= \frac{-1}{20} \end{aligned}$$

$$\begin{aligned} P &= \frac{1}{f} \\ &= \frac{1}{-\frac{1}{20}} \\ &= 1 \div \frac{-1}{20} \\ &= 1 \times \frac{20}{-1} \\ &= -20D \end{aligned}$$

2.6.10 Determination of the focal length of lenses

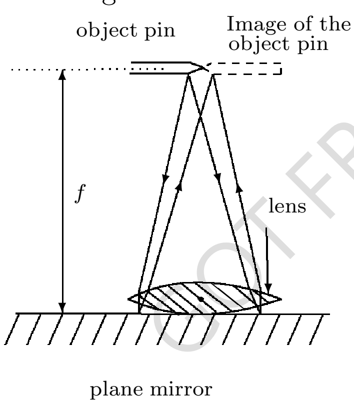
In this book we are to discuss determination of focal length of convex lenses. Determination of the focal length of diverging lenses will be found in advanced textbooks.

Below are the various methods one could use to determine the focal length of a convex mirror.

1. **Distant object method**

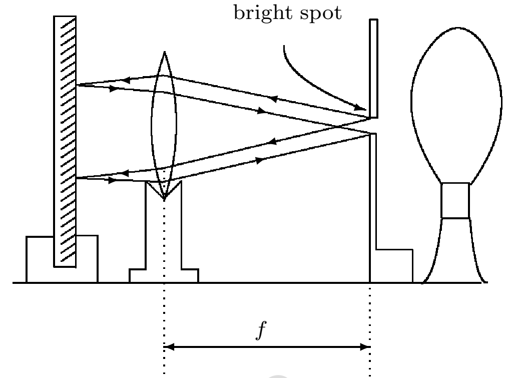
   Recall that when an object is at infinity (very far a way) its image is formed at the principal focus or in the focal plane of the lens.

   (a) Place the lens on a plane mirror facing up and move the tip of an optical pin up and down along the principal axis of the lens. At a certain height, there will be no parallax between the object pin an its image. At this point measure the distance of the object pin from the lens as the focal length of the lens.

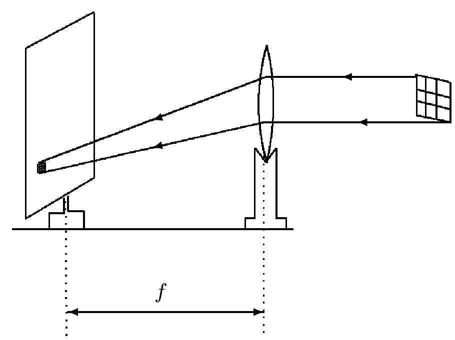


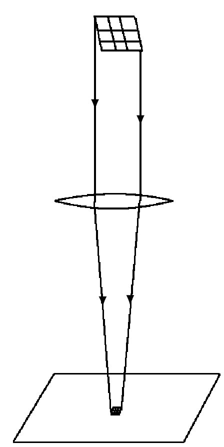
   Rays reflected from the mirror are parallel hence they must converge at the principal focus.

   (b) Place the lens on a lens holder with a plane mirror behind it and cardboard with a hole at its centre, is placed in front of the lens. When a light source (like a bulb) is placed behind the cardboard and both (the source and the cardboard) are moved to and from the lens, at a certain position, a bright spot (image) is formed on the cardboard. When this occurs, then measure the distance between the cardboard and the lens as the focal length of the lens,



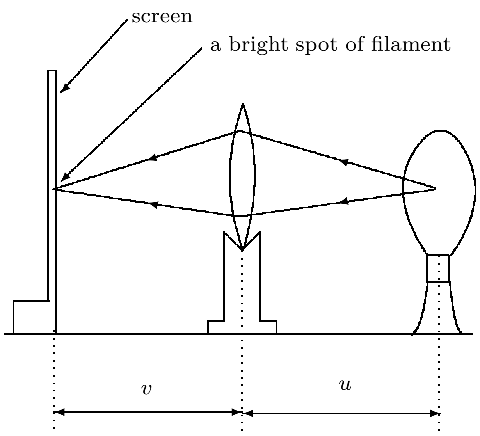
   (c) Focus an object at infinity (like a tree or window) using the convex lens and move a white screen behind the lens to and from it. At a certain position, a clear image of the object at infinity will be formed on the screen. Measure the distance between the lens and the screen as the focal length of the convex lens. The screen may be a white sheet of paper or cloth.





2. **Object and image distance**

   In this case one can use a small lighting bulb as an object and a white screen to trace the position of its image, which is always a bright spot formed on the screen. The apparatus is arranged as shown below;



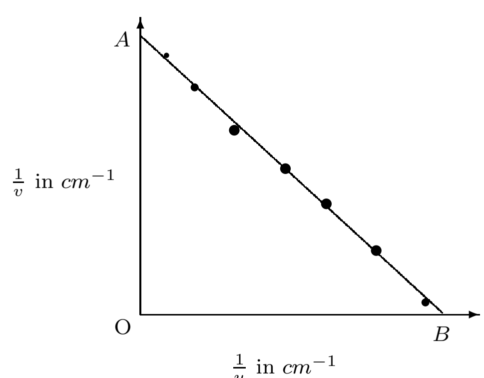
   Switch the bulb on and place it a certain distance, u, from the lens and move the screen to and from the lens until a bright spot (image of the bulb's filament) is formed on the screen. Then measure the distance of the screen from the lens ,v, as the image distance. Repeat this for more values of u (about six) and tabulate your results as shown in table 2.6;

   Table of results

u(cm)	v(cm)	$\frac{1}{v}$ in cm^{-1}	$\frac{1}{u}$ in cm^{-1}

Table 2.6: Object and image distance

   Plot a graph of $\frac{1}{v}$ against $\frac{1}{u}$ (you will get graph like this below)



Measure the intercept OA and OB on the $\frac{1}{v}$ and $\frac{1}{u}$ axes respectively. Obtain the focal length, f, as;

$$f = \frac{2}{(OA+OB)} \tag{2.90}$$

Theory [6]

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, we can plot a graph of $\frac{1}{v}$ against $\frac{1}{u}$ which represents the equation ;

$$\frac{1}{v} = \frac{-1}{u} + \frac{1}{f} \tag{2.91}$$

its slope is -1,

The vertical intercept i.e when $\frac{1}{u} = 0$, is $\frac{1}{v} = \frac{1}{f} = OA$.

The horizontal intercept i.e when $\frac{1}{v} = 0$ is $\frac{1}{u} = \frac{1}{f} = OB$.

The average value of the intercepts is given by;

$$\frac{(OA+OB)}{2} = \frac{\frac{1}{f}+\frac{1}{f}}{2} \tag{2.92}$$

$$= \frac{1}{f} \tag{2.93}$$

$$\implies f = \frac{2}{(OA+OB)} \tag{2.94}$$

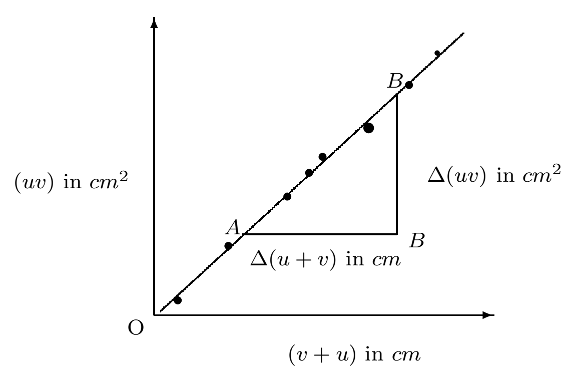
Some times the table of results is drawn as in table 2.7;

And plotting a graph of uv against $(u+v)$

u(cm)	v (cm)	$(v+u)$ in cm	vu (cm^2)

Table 2.7: Object and image distance

we have



[6] This is not always required in practicals

The focal length (f) of the lens is obtained as the slope of the graph,

$$f = \text{slope}$$

Theory [7]

From the lens formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (2.95)$$

$$\frac{1}{f} = \frac{(u+v)}{vu} \quad (2.96)$$

$$f = \frac{vu}{(v+u)} \quad (2.97)$$

$$f(u+v) = vu \quad (2.98)$$

$$vu = f(v+u) \quad (2.99)$$

hence for a graph of vu against $(u+v)$, its slope is the focal length (compare with $y = mx + c$) therefore,

$$f = \text{slope of the graph} \quad (2.100)$$

Note there are many methods of determining the focal length of the a convex lens, I have only described the commonly used and examinable ones.

Exercise

1. In an experiment to determine the focal length of a convex lens, the results in table 2.8 were obtained;

Object distance(cm)	Image distance(cm)
7.0	17.5
9.0	11.3
11.0	9.2
13.0	8.1
17.0	7.1
19.0	6.9

Table 2.8: Object and image distance

Plot a suitable graph and use it to determine the focal length of the lens

[7] This is not always required in practicals

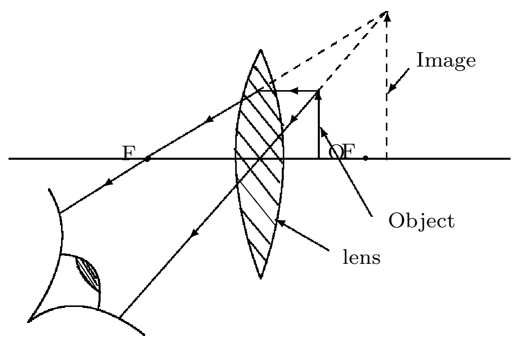
2. Describe an experiment to determine accurately the focal length of a converging lens

3. Describe how you would determine the focal length of a convex lens using a mirror and a pin.

2.7 Optical instruments

2.7.1 Magnifying glass

Recall that when an object is placed at a point between the principal focus and the optical centre of a convex lens, its image is erect, magnified, virtual and on the same side of the object. The ray diagram below illustrates this;



Note that, a diverging lens can not be used because it forms only diminished images. Hence a simple magnifying glass is a convex lens with a small focal length. To acts as a magnifying glass, it is placed close to the object such that the distance between the object and the lens is less than the focal length of the lens. A magnifying glass is the simplest optical instruments.

2.7.2 Compound microscope

An attempt to increase the magnifying power of the simple magnifying glass, instead of using only one convex lens, two were used and this resulted into the compound microscope. This microscope was invented in 1590 by Zacharias Jansen. It is used to look at very small objects at short distances or used to provide a greater angular magnification for nearby objects.

Construction

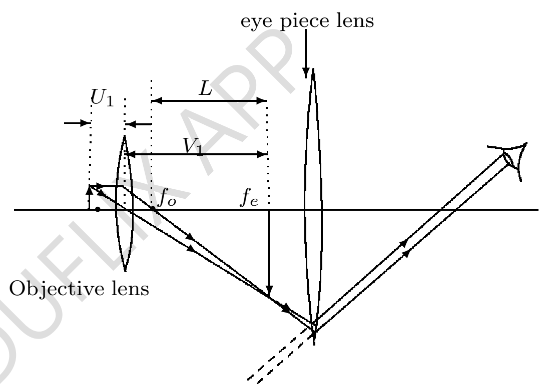
It consists of two lenses

1. One with a shorter focal length placed near the object is called the **objective lens** and
2. One placed near the eye. This one is known as the **ocular** or **the eyepiece**.

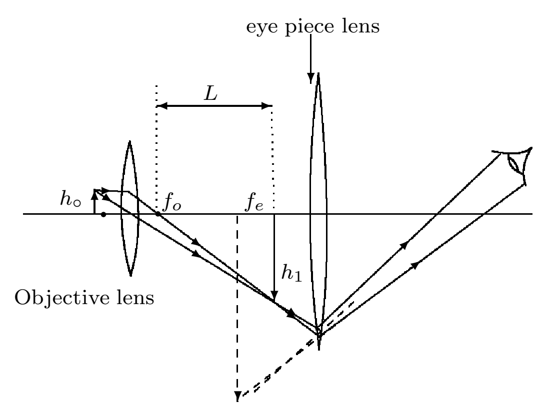
Ray diagram:

The objective and the eyepiece can be arranged in two ways with either the final image formed at infinity or at the near point. The diagrams below show this;

(image formed at infinity)



(final image formed at the near point)



Note that the distance between the closest principal foci of the two lenses is called the **tube length**, l, and is usually fixed.

How it works:

The objective lens is placed near the object so that it is just outside or beyond its principal focus (between F and C). A magnified, real and inverted image is formed on the other side of the lens beyond C. this image formed by the objective lens acts as the object for the ocular or the eyepiece. This eyepiece acts as a simple magnifying glass forming a further magnified virtual image, which is the final image seen by the eye.

Sometimes the image of the objective is formed in the focal plane or at the principal focus of the eyepiece and hence the final image is formed at infinity. This is shown in the above diagrams Or the image of the objective is formed at a point such that the final image (eyepiece image) is formed at a point about 25cm from the near point of the eye

Definition 12.

Near point is a point near the eye at a distance from the eye called the near distance or distance of distinct vision.

The point called the near point for the normal eye it is about 25cm from it. Objects closer than this point to the eye are not seen by the eye, those further than this point to the eye are seen by the eye though not clearly. But the images formed at the near point of the eye are seen by the eye clearly, that is why it is recommended that all the images in optical instruments be formed at the near point.

Uses of the compound microscope

1. it is used to observe small near objects
2. used in scientific researches to study the nature and characteristics of small animals and plants
3. used by doctors in hospitals to diagnose and treat diseases.
4. Used in schools to study the structures of small animals like amoeba, paramecia, insects and others.

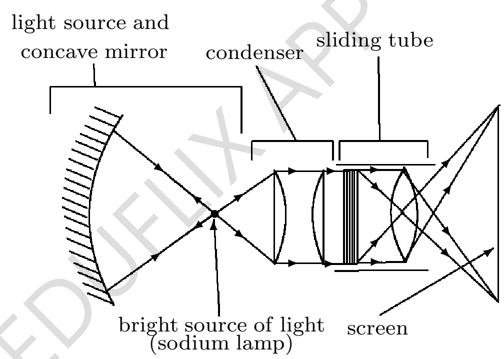
2.7.3 Projector

This is an optical device used to produce a number of images of a slide on the screen continuously so that they appear to be moving.

It is made up of four parts;

- the light source with a concave mirror,
- the condenser,
- the sliding tube and
- the screen.

These are arranged as shown in the diagram below;



Construction

The light source with a concave mirror

Light from the illuminant or light source (usually a carbon-electric arc or quartz iodine lamp) is produced in all directions. On the other side, the concave mirror is placed there to reflect the light back in its own path this is when the light source is positioned at the centre of curvature of the concave mirror. Hence the light produced by the source is towards only one direction and a diverging beam of light is directed to the condenser.

The condenser

The simplest condenser is made up of two plano-convex lenses as shown in the diagram above. When a divergent beam enters it, it emerges as a parallel beam of light directed to the sliding tube. The condenser may contain several lenses, which finally convert a diverging beam to a parallel beam.

The sliding tube

The sliding tube contains the slide that acts as the object and the projection lens mounted in it. This lens can be moved to and fro so as to focus a sharp image of the slide on the screen.

The screen

This is just a white sheet of paper or cloth on to which the final image of the slide is formed.

How it works

A very bright bulb (like the sodium lamps) is placed at the centre of curvature of the concave mirror as shown above. Rays towards the mirror are reflected back in their own path to the lamp. This mirror helps to bring back the light, which would have been wasted away. On the other side of the lamp, the diverging beam enters the condenser which changes it to a parallel beam towards the slide. The slide here acts an object for the projection lens whose image is formed on the screen as shown above. The slides are exchanged continuously so as produce more images on the screen which are perceived as moving images hence called the cinema. The continuity of the images formed by the projector is due to persistence of vision.

Example

In a projector, a square slide of length 4cm is used to produce its image 4m high when placed 5cm from the projection lens, determine;

(a)the magnification produced

(b)the distance of the screen from the lens

(c) the focal length of the projection lens

Solution

(a) from the definition of magnification

$$\begin{aligned} M &= \frac{h_i}{h_\circ} \\ &= \frac{4m}{4cm} \\ &= \frac{400cm}{4cm} \\ &= 100 \end{aligned}$$

(b) Object distance, $u = 5$cm

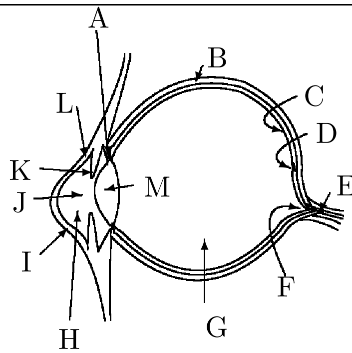
$$\begin{aligned} \text{But } M &= \frac{h_i}{h_\circ} \\ \text{and } u &= 5cm \\ \text{also } M &= \frac{v}{u} \\ \text{but } M &= 100 \\ \text{hence } M &= \frac{v}{u} \\ 100 &= \frac{v}{5cm} \\ v &= 100 \times 5cm \\ &= 500\text{cm} \\ &= 5m \end{aligned}$$

(c) Using the lens formula

$$\begin{aligned} \frac{1}{f} &= \frac{1}{u} + \frac{1}{v} \\ &= \frac{1}{5\text{cm}} + \frac{1}{500\text{cm}} \\ &= \frac{1}{5} \times \left(1 + \frac{1}{100}\right) \\ &= \frac{1}{5}\left(\frac{101}{100}\right) \\ \frac{1}{f} &= \left(\frac{101}{500}\right) \\ f &= \frac{500}{101} \\ &= 4.950cm \end{aligned}$$

2.7.4 Eye

The eye is the organ of sight found in front and on one side of your head, it has a number of parts, these are shown in the diagram below



A-Ciliary muscle

B-choroid (black)

C-retina

D-yellow spot (fovea)

E-Optic nerve

F-Blind spot

G-Vitreous humour (jelly)

H-Aqueous humour (watery liquid)

I-Cornea (transparent)

J-Pupil

K-Iris (eye colour)

L-Sclerotic (white of eye)

M- Eye lens

These include;

The eye lens: this forms the image of an object in front of the eye in front of the retina. The retina is found at the back of the eye. In good light, the yellow spot (or fovea) is most sensitive to details of the image and the image is automatically formed there. The numerous nerve endings on the retina transmit light impulses to the optic nerve, to the brain. The brain interprets them and tells whether it is a living thing, a non-living thing or otherwise.

The cornea is the front transparent part of the eye. Most refraction (bending) of light takes place here. There is a watery liquid (aqueous humour) in front of the eye lens and a jelly-like liquid (vitreous humour) behind the lens. These two liquids keep the eye in shape and enable it to withstand slight pressure changes without damage. The coloured or black part of the eye is called the **iris** and the aperture or the central hole in it, is the **pupil**. The iris automatically adjusts its size so that the round pupil is wider in dim or dull light and smaller in size in bright light. This prevents excess light from damaging the retina.

The ciliary muscles around the outside of the soft gelatinous convex lens can change the thickness and therefore the focal length of the lens.

Generally the eye lens thickens or fattens when a person views a nearby object and it becomes thinner or flattens when the eye is focused on distant objects. The change in the focal length of the eye lens so as to form the image of an object on the retina is called **accommodation.**

Most refraction of light rays from an object occurs at the cornea and more occurs in the watery liquid next to the cornea and the jelly like liquid behind the lens. The shape of the lens changes so that its relatively small refraction forms a clear image on the retina. The point where the optic nerve leaves the retina is the **blind spot**. When the image is formed at the blind spot, you do not see it. The black coat (choroid) behind the retina contains blood vessels that supply food and oxygen to the eye. It thickens and forms muscles at the front, which controls the lens. The outside coat protects all the parts inside the eye.

Accommodation :

The **near point** is the nearest point on which the eye can focus an object positioned there without strain. It is on average 25cm for the normal adult eye.

The **far point** is the furthest point to which the fully relaxed eye can focus an object at that point without strain . It is at infinity (very far a way) for people with good sight i.e. his eyes focus parallel rays to the retina when relaxed. We can also say that accommodation is the process of altering the shape of the eye lens so as to form the image of an object on the retina.

A person with good sight can focus images of objects at distances between 25cm and infinity. As a person ages his eye muscles may loose their power to alter the focal length of the eye lens so as to be able to focus either near objects only (short sight) or far objects only (long sight) clearly. Older people sometimes need spectacles for reading or for seeing distance objects or for both purposes, this change in the eyes's ability is called loss of accommodation.

Defects of vision and their correction

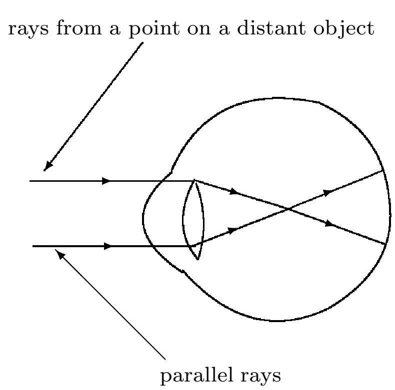
There are several eye defects but the major ones caused by the weakness of the ciliary muscles are short-sightedness and long-sightedness.

Short sightedness (myopia):

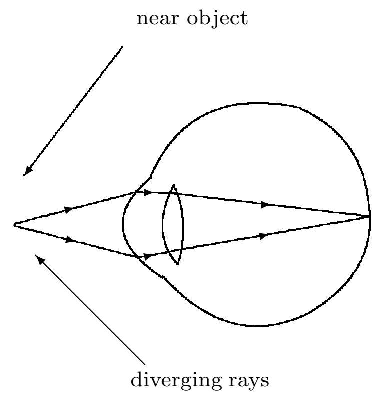
This is when some one can see clearly near objects but can not see those that are far away. that is to say:

- (a)Here a person can see near objects clearly but his far point is closer than infinity
- (b)The eye ball is too long for the lens
- (c)The final image is formed in front of the retina

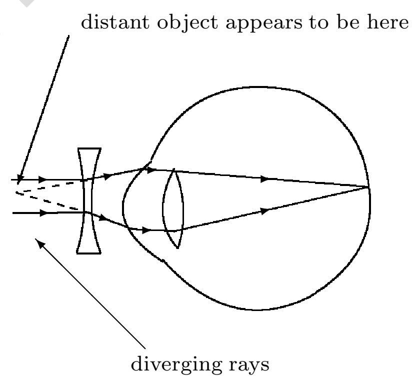
(a)Rays from a distant object



(b)Rays from a near object



To make these parallel rays meet on the retina, they must be some how diverging as in the case when the object is close to the eye hence we use a diverging lens to correct short sightedness.

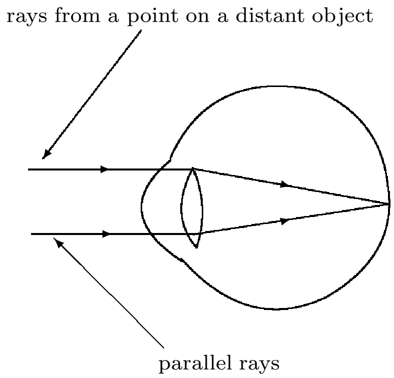


Long sightedness (hypermetropia):

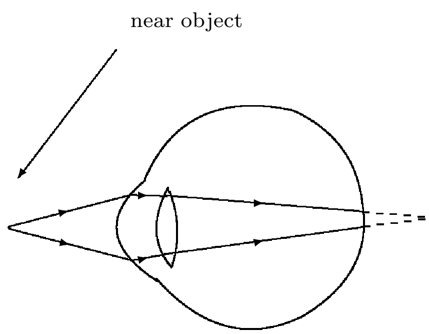
This is when some one can see distant objects clearly but not the near objects. In this case;

- The person see distant objects clearly but his near point is beyond 25cm.
- The eye ball is too short for the eye lens
- The final image is formed behind the retina

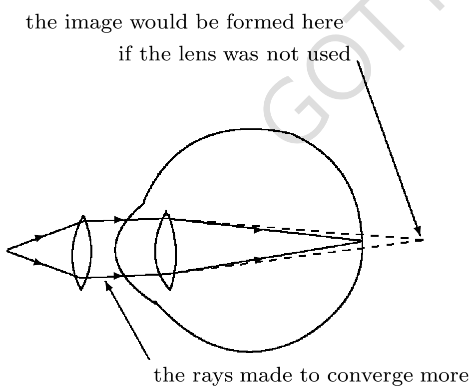
(a)Rays from a distant object



(b)Rays from a near object



To make the rays from the near object meet on the retina, they have to be converged a bit such that they meet earlier. Hence we have to use a converging lens to correct long sightedness.

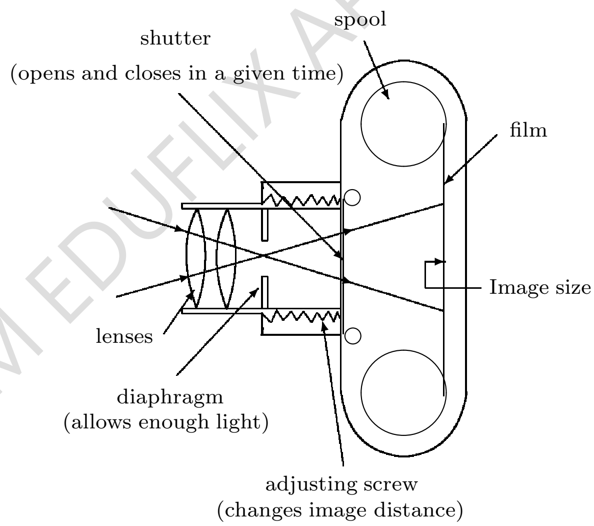


Loss of accommodation:

Many older people have both short sightedness and long sightedness because of the loss of elasticity of the eye lens and its ciliary muscles. They need spectacles for reading and seeing near objects and a different pair of spectacles for seeing distant objects or spectacles with bifocal lenses. Some times this loss can be delayed or eliminated by eating the relevant food stuff.

2.7.5 The lens camera

A lens camera is a light- tight box in which a convex lens at the front forms a real inverted image on a photo-sensitive paper, film or plate at the back. The inside is painted black to prevent stray light from blurring the image. Below is simplified diagram of a simple lens camera;



The brightness or clearness of the image is determined by the amount of light passing through the lens or entering the camera. This is controlled in two ways;

1. The **diaphragm** or a circular aperture or stop behind the lens; controls the areas of the lens through which the light can pass. A camera can have two apertures a narrow one for bright days and a larger one for dull days. The ratio of the diameter of this aperture to the focal length of the lens is called the f-number of the lens or camera.

2. The **shutter** is a small device which opens and closes in a very short period of time. i.e.

it controls the period for which light can pass. In taking a photograph, the shutter opens for a certain time and exposes the film to light entering the camera so as to form the image of the object on it and then closes. The exposure periods vary and are given in fractions of a second. For instance $\frac{1}{1000}$s, $\frac{1}{60}$s, and others. Fast moving objects need a shorted exposure time to photograph them or high-speed shutters are required.

3. **Focusing**

   Focusing or adjusting screw:

   There are two types of cameras; fixed focus cameras and focusing cameras. In fixed focus cameras; the distance between the lens and the film is fixed and is usually 2 to 5 cm which is very slightly more than the focal length of the lens. Images of objects 3m or more from the camera are sufficiently clear for ordinary photographs.

   In focusing cameras; The distance of the lens from the films can be altered by screwing the lens in or out using the adjusting screw. If the object is near, the lens is moved further away from the film to obtain a clear image.

4. **The lens**

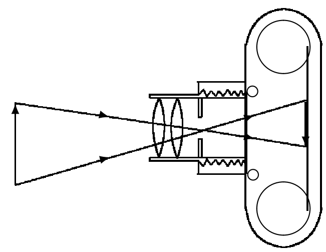
   Expensive cameras have a combination of two or more lenses which form images that are more perfect than those formed by single lenses. Single lenses may not form sharp images as required but a combination of lenses can do that.

5. **The photographic film**

   This film or plate contains chemicals (silver compounds) that change or react on exposure to light. It is "developed" to give a negative and from the negative a photograph can be printed.

How the lens camera works

Light from the object is refracted through the lens and then only a required amount of light passes through the diaphragm for a fixed period of time when the shutter is open. Light meets the film where an image is formed behind the camera. Then the film can be developed into a photograph.



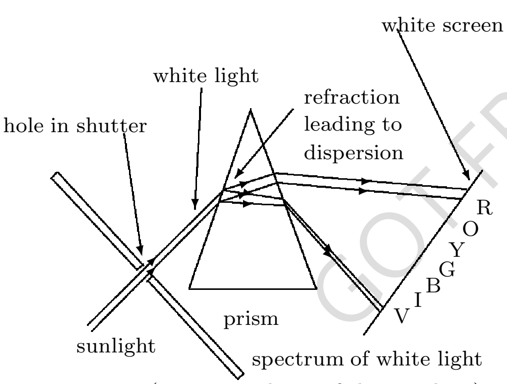
Differences between an eye and a camera

1. Focal length of the eye lens can change while that of the camera is fixed i.e. it can not change
2. Image distance for the eye is constant while that for the camera can be changed using the adjusting screw
3. Eye is made up of living cells(human fresh) while the camera is made of non-living material (glass)
4. Amount of light entering the eye is controlled by the iris while that entering the camera is controlled by the diaphragm and shutter
5. Image formed by the eye lens is formed on the retina while that formed by the camera is formed on the photographic film.

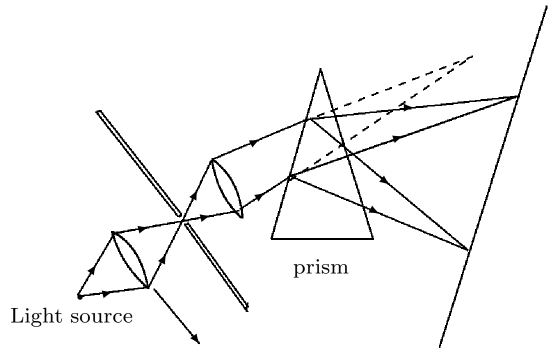
2.8 Colour and dispersion

2.8.1 Introduction:

Dispersion is the splitting of white light into the seven colours of a rainbow. These colours are red, orange, yellow, green, blue, indigo and violet. Or dispersion is the splitting of white light into its component colours. Generally light is known as white light. White light is made up of seven colours known as the spectrum of white light. They are red, orange,....as listed above. Each of these colours has a different value of refractive index of light for a given material or each of these colours at the same angle of incident is refracted at a different angle of refraction hence separating them into the spectrum. Red is the least deviated colour and violet the most deviated. This is observed when white light is refracted on two surfaces of the triangular prism. This is shown below;



Sir Isaac Newton first observed this accidentally when he was in a dark room on a sunny day. Newtons work with a small slit on this subject arose out of the need for finding away of removing the coloration from the images seen through a telescope. The spectrum produced by the above arrangement was not a pure spectrum. To produce a pure spectrum, the arrangement was modified as shown below;



Note that this is not the best pure spectrum. To produce sharp colours, which do not overlap, we should use a chromatic lens but this is above our level (chromatic aberration).

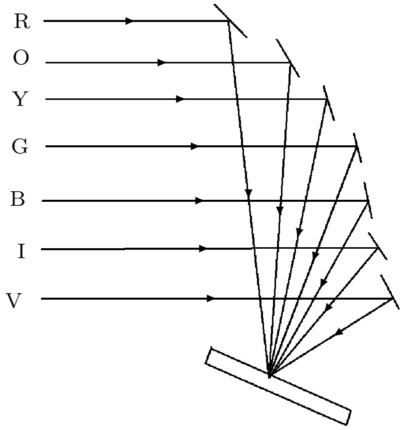
2.8.2 Combination of the colours of the spectrum

A question was raised that, if white light can split into seven colours. When the seven colours are combined or shone on one spot cant they produce white light or make a white spot? Yes, when all the seven colours are combined they form white light, this can be demonstrated by ;

1. Combining the seven colours of white light on a given spot

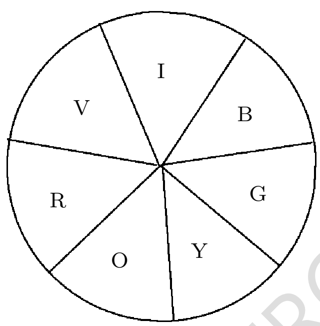
2. Rotating a disc with the seven colours (Newtons disc), this is described below;

1. Using seven different mirrors for each of the seven colours, they can be focused to one point as shown below. The spot where they are all converged to, will be a white spot.



2. **Newton's disc**

   Newton showed that the combination of the seven colours forms white light using a disc on to which those colours were painted. He painted the seven colours of the spectrum in sectors on a disc as shown below:



   When the disc is rotated at high speed it appears white.

   Some may say that the colours of the rotating disc in not pure white but Grey. It is only fair to say that the whiteness obtained is greyish owing to the difficulty of obtaining pigments that can produce pure colours. The experiment works by reason of persistence of vision. The impression of the image on the retina of the eye is retained for a small fraction of a second after the image has disappeared. Consequently the brain sums up and blends together the rapidly changing coloured images on the disc and thus produces the sensation of a stationery white image. Persistence of vision is also a responsible for the absence of flicker in the picture formed by a cine-projector.

2.8.3 Colour

A colour is a special kind of light wave with a specific frequency and wavelength. Colour is a group of light waves with a range of frequencies and wavelengths. There are two types of colours

1. Primary colours and

2. Secondary colours

Primary colours

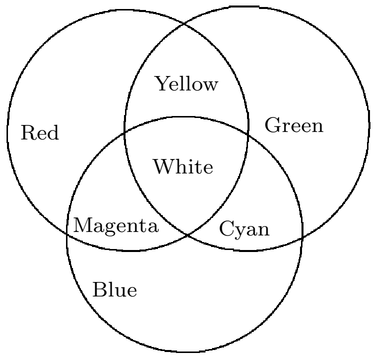
These are colours that can not be obtained by mixing or combining other colours. There are only three primary colours; **red**, **green** and **blue** and they are the primary art colours.

Secondary colours

These are colours that can be obtained by mixing other colours. Apart from red, green and blue all the other colours are secondary colours. For instance yellow, pink, purple, violet, Grey, cyan, magenta to mention but a few.

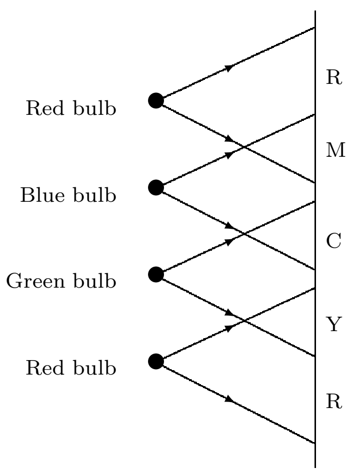
2.8.4 Mixing coloured lights

When we mix primary colours which colours do we get? The figure below illustrates the answers to the above question.



magenta is reddish purple

Cyan is peacock-blue
(Greenish blue)



Note that

Red + Green = Yellow
Red + Blue = Magenta
Blue + Green = Cyan
Red + green + blue = white

Colours that can be mixed to produce white are known as **complementary colours**. For instance red and cyan, magenta and green, blue and yellow are complementary colours. This depends on the first equation, that all primary colours when mixed produce white.

Red + green + blue = white
Red + cyan = white
Magenta + green = white
Blue + yellow = white
Where
yellow = R+G
Magenta = R+B
Cyan = G+B

All secondary colours can be produced by the addition (or mixing) of primary colours (red, green and blue) in the correct proportions and intensities.

In the coloured television monitor, there are three cameras in one, each uses one of the primary colours. And when their images are displayed on the screen they mix accordingly to produce the original picture of the object.

2.8.5 Why objects are coloured

Colours of objects in white light

When white light falls on an object, part of it is reflected, part absorbed and the rest transmitted through the object. The light that reaches our eyes is that which has been reflected and the colour of the reflected light that reaches our eyes is the colour of the object or the colours the object appears to have.

Objects are coloured because they reflect that colour they appear to be. For instance a red shirt is red because it absorbs all the other colours except red which it reflects and a blue skirt is blue because it absorbs all the other colours and reflects only the blue colour or blue light.

In general an object is coloured because it absorbs all the other colours except the colours it appears to have or because it reflects only its colour and absorbs all the other colours. When an object reflects all the colours of white light is appears white and when it absorbs all the colours in white light, i.e. reflects none, it appears black. White is a colour but black is not a colour, black is the

Reflected colour	Absorbed colours	Colour of object
None	All	Black
Some colours	Some colours	Coloured
All	None	White

Table 2.9: Objects in white light

absence of colour.

The table 2.9 summarises this; When all the colours are reflected by the objects, it appears white. When all the colours are absorbed, the object appears black. Hence a black trouser is black because it absorbs all the colours falling on it. A white browse is white because, it reflects all colours falling on it. Note that the combination of all the colours is white.

Appearance of coloured objects in coloured lights

An object, which is coloured, absorbs all the colours except that which it appears to be. If this object is placed in a coloured light which is among those it reflects, then it reflects it.

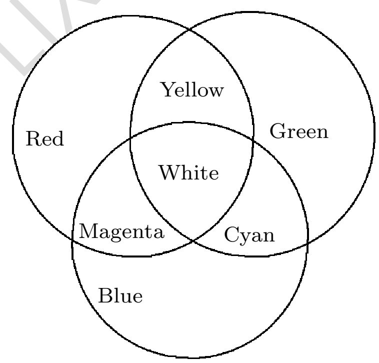
What is the colour of a yellow dress in ;

(a) Blue light? Since yellow colour is made up of red and green, it means that a yellow dress can only reflect red and green lights and absorbs all the other colours. Hence blue colour will be absorbed and no colour is reflected, therefore the dress will appear black. (Recall black is the absence of colour).

(b) Red light? A yellow dress reflects only yellow light or reflects only red and green light. Therefore in red light, red light is reflected by the yellow dress and hence it appears red. Table 2.10 showing the colours of various coloured objects in coloured light. Study and the possible explanations for each of the results in it;

colour of light in the dark room	colour of the object in white light (day light)	resultant colour of the object in dark room
Red	White	Red
Green	Yellow	Green
Red	Yellow	Red
White	Yellow	Yellow
White	Black	Black
Blue	Black	Black
Red	Black	Black
Green	Magenta	Black
Cyan	Red	Black
Blue	Cyan	Blue

Table 2.10: Net colours of objects

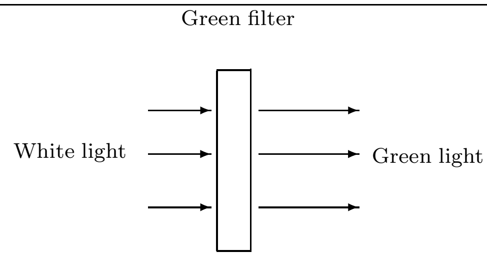


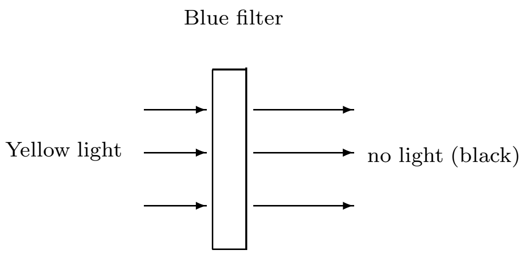
magenta is reddish purple

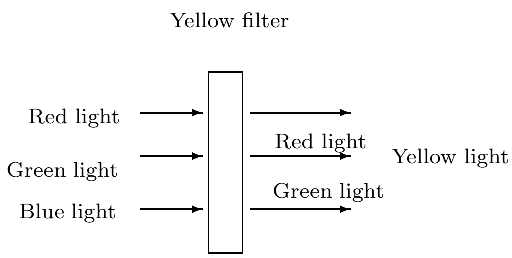
Cyan is peacock-blue
(Greenish blue)

2.8.6 Light filters

These are substances that allow only one colour of light to pass through them. Light filters are substances that transmit only one colour through them. Light filters are named according to the colour they transmit. A red filter allows only red light to pass through it. A yellow filter allows only yellow, red and green light to pass through it. A blue filter transmits only blue light. The diagrams below illustrate this;







Note that yellow colour is of two types:

(i) pure yellow and

(ii) compound yellow.

Yellow colour from yellow filters is pure yellow, but the yellow colour from petals of flowers and most paints is not pure yellow, but **compound yellow**.

2.8.7 Mixing colours by subtraction

In art lessons, a students learns that blue paint and yellow paint when mixed together form green paint yet blue and yellow are complementary colours. Note that mixing lights is not the same as mixing paints. For instance mixing many coloured paints together, each pigment subtracts something from the reflected light and the final colour is almost black. How can this difference be explained?

	Reflects	Absorbs
Yellow paint	Red yellow Green	Blue
Blue paint	Blue Peacock blue Green	Red

Table 2.11: Colour of mixed paints

First most colours from pigments are not pure colours, but instead each is a compound colour. Yellow paint on examination with a spectroscope will be seen to reflect red and green light as well as yellow light hence yellow paint absorbs blue light. Blue paint will be seen similarly to reflect blue and green light and absorb red light, because it is not primary blue but peacock blue, i.e

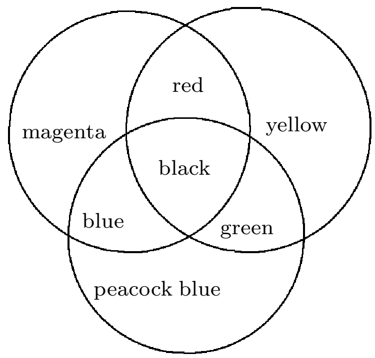
Therefore yellow paint absorbs blue light and blue paint absorbs red light. The only light that can be reflected by the mixed pigments of yellow and blue paint is green, hence the mixed pigments appear green. If a spectrum of yellow and a spectrum of blue light are used in the pigments, no light would be reflected, all would be absorbed and the appearance of the mixed pigments would be black. Remember Black is not a colour, it is the absence of colour. For instance in a dark room a peacock blue filter in one compartment is used to illuminate a screen. When yellow light is placed in front, the screen becomes green. Repeating this with ;

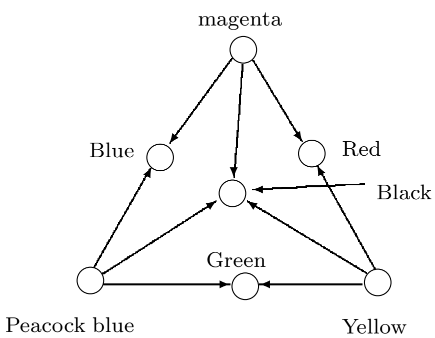
(a) yellow and magenta filter, the screen become red

(b) Peacock blue and magenta filter, the screen become blue

(c) Yellow magenta and peacock blue filters, the screen become black

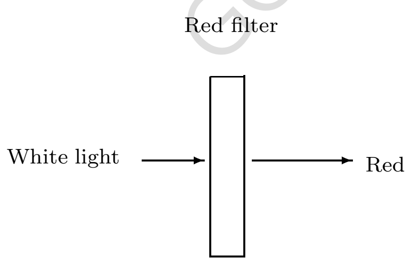
These results are summarised in the figure below

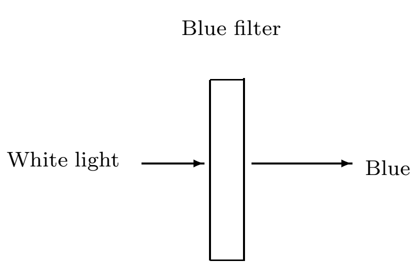


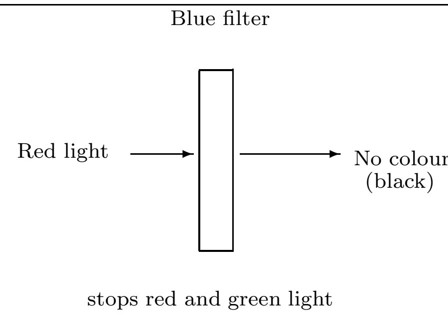


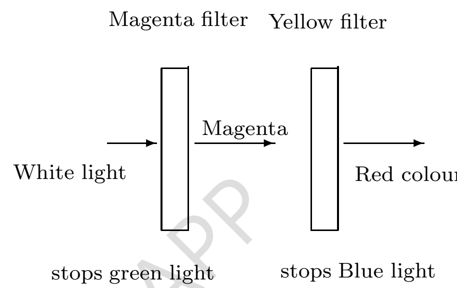
Note that;

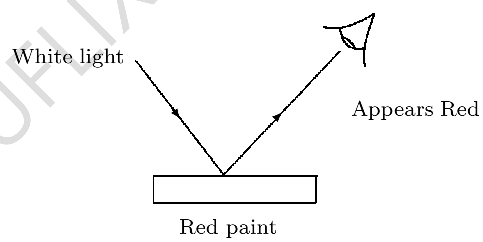
1. Each secondary colour filter allows that colour to pass through it
2. When two secondary colours overlap, a primary colour passes through
3. When all the three secondary colour overlap, no light passes through and the result is black

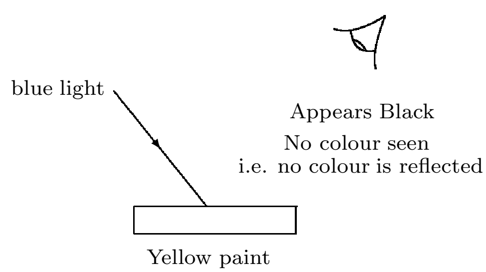












2.8.8 Traditional, warm and cool colours

Traditional colours

Traditionally black and white are not regarded as colours. They are achromatic, they are used to produce various shades and tints of colours. For instance when white is added to any colour, it brightens it up. e.g. **red + white = pink**. If black is added to any other colour, it makes it darker e.g.

Red + black = maroon
Green + black = army-green
Blue + black = navy-blue.

Intermediate colours are colours obtained by combining a primary colour and a secondary colour. They are a sub set of secondary colours for instance **red + orange = orange-red**.

Warm and cool colours

Colours can be regarded as cool or warm. Warm colours are also known as active colours or expanding colours, they are colours related to sunshine, fire or hotness. They include yellow, orange and red. These colours are said to be exciting and stimulating. **Red is the most exciting** while yellow is regarded as the warmest and brightest. **Warm colours expand and reach out**. They make an object appear larger than it actually is and they also make an object appear closer than it actually is.

Cool colours have an aspect of blue related to the cool water and the sky. They also have an aspect of green related to the green grass hence these colours give a feeling of calmness or restfulness. These colours make an object appear smaller than they are and they also seem to be far away than they are. So they are called **passive colours** or **receding colours**.

All colours have got 3 quantities in common namely; **hue**, **value** and **intensity**. **Hue** means the name of the colour for instance red, green, yellow and others. **Value** refers to the degree of lightness or darkness of the colour and **intensity** refers to the energy content of a given colour.

2.8.9 Electromagnetic spectrum

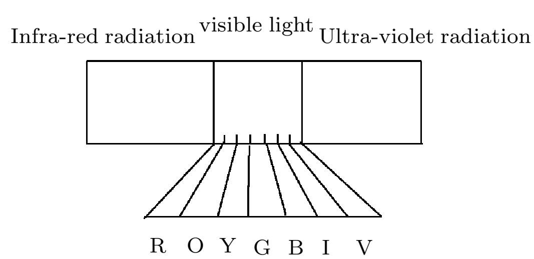
A spectrum is a group of waves indicated on a certain scale of either frequency or wavelength of the waves. Waves are of two types;

wave	f in Hz	λ in m
microwaves	$> 10^{23}$	$< 10^{-15}$
Gamma rays	$10^{18} - 10^{23}$	$10^{-10} - 10^{-15}$
x-rays	$10^{16} - 10^{21}$	$10^{-8} - 10^{-13}$
ultra-violet	$10^{15} - 10^{16}$	$10^{-7} - 10^{-9}$
violet indigo blue green yellow orange red	$10^{14} - 10^{-15}$	$10^{-6} - 10^{-7}$
Infra-red	$10^{12} - 10^{14}$	$10^{-4} - 10^{-6}$
short radio waves	$10^{7} - 10^{12}$	$10^{0} - 10^{-6}$
TV and FM radio waves	$10^{7} - 10^{8}$	$10^{1} - 10^{0}$
long radio waves	$< 10^{16}$	$> 10^{3}$

Table 2.12: Part of the electromagnetic spectrum

1. Mechanical waves (those we can feel like water waves, vibrational waves, waves on ropes and others)
2. Electromagnetic waves. These can not be felt, only specially designed electronic devices can detect them. Examples of electromagnetic waves include radio waves, gamma rays, x-rays, radar and microwaves, infra-red waves, ultra-violet radiations, visible light, TV waves and others.

Table 2.12 is part of the electromagnetic spectrum showing the band (range) in which light waves lies;



Infrared radiation

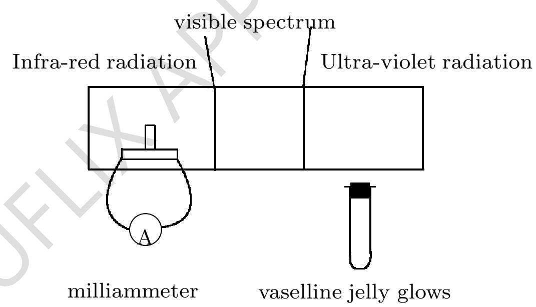
This is a type of radiation that transfers heat energy. When they meet a substance, the energy in the infrared radiations is automatically converted to heat energy. Infrared radiations is the name given to that radiation just beyond the red end of the visible spectrum. This radiation is produced by all hot bodies such as the sun, red hot or white hot metal furnaces, electric fires and others. The eye can not detect this radiation but the skin detects it as heat. These facts are known about infrared radiation;

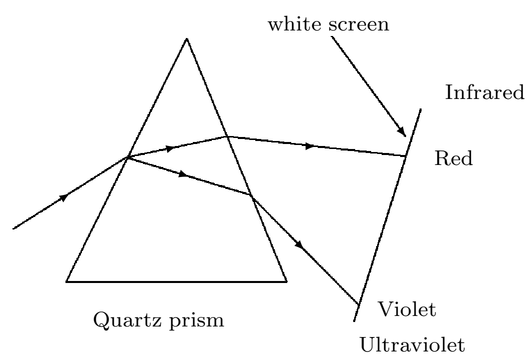
1. Infra-red lamps are used to treat sore muscles in hospitals and in homes.
2. Infra-red grills are sometimes used in restaurants to cook food.
3. The paint on cars is dried quickly by using infrared lamps
4. Clouds, mist and fog easily scatter visible radiations but not infra-red radiations and therefore photography using plates and films sensitive to infrared is possible when the ordinary photography can not be used like at night (taking photographs in darkness).
5. Some missiles have devices that guide them to their targets such as ships or aeroplanes by "homing" on the infra-red from their engines.
6. Infra-red radiation from the sun especially that of short wave length readily passes through the glass of plastic on green houses. The radiation is absorbed and warms the objects in the green house. The objects then also do radiate infra-red radiation but since their temperature is relatively low, their radiation is of such a long wavelength that it can not penetrate glass or plastic. Therefore the heat energy remains in the green house where it is wanted. Green house is a house made of glass, used by agriculturalists to grow plants under study.

Ultra-violet radiation

This is the radiation just beyond the violet end of the visible spectrum. Very hot objects like mercury vapour lamps, "steamer", and electric sparks produce ultra-violet radiation. The eyes do not respond to this radiation, but strong ultra-violet radiation (like that from the sun) can damage our eyes.

More so, it can make the skin sore or burn it. This radiation is absorbed by green plants in the process of photosynthesis by which plants convert carbon-dioxide and water to sugar (glucose). Therefore animal life entirely depends on this radiation for food supply. Ultra-violet radiation can penetrate and kill certain bacteria and do other things in bacteriology.





White light

Ordinary white light from the sun and other hot bodies is a mixture of lights of different colours (red, orange, yellow, green, blue, indigo and violet). The speed of all the colours of white light is the same as in a vacuum and is $3 \times 10^8 ms^{-1}$, this speed is less in transparent substances such as air, glass, water and others. These substances are optically denser than a vacuum, similarly water is optically denser than air. When light passes from one medium to another in which its speed is less, the incident light rays are refracted towards the normal. The frequency of a given colour can never change in moving from one medium to another. A triangular prism refracts each colour by a different amount. The spreading of colours or splitting of white light by a prism called dispersion produces a continuous spectrum of the seven colours from red to violet.

Questions

1. A red bus with blue letters on it stops in front of yellow light at night. Describe briefly the appearance of the bus?

2. Explain why clothes made of blue cloth appear black when viewed in a red room?

3. A thin slit cut in a piece of metal is illuminated by white light from a ray box. Light from the slit is then passed through a glass prism and afterwards received on a white screen. Describe the appearance of the screen and sketch a diagram showing the course of light from the slit to the screen. What would be the effect of;

   (a) replacing the white screen with a red one

   (b) placing a sheet of green glass between the prism and the red screen

4. Explain each of the following;

   (a) the appearance of a red flag with green stripes when viewed in day light through a sheet of green glass

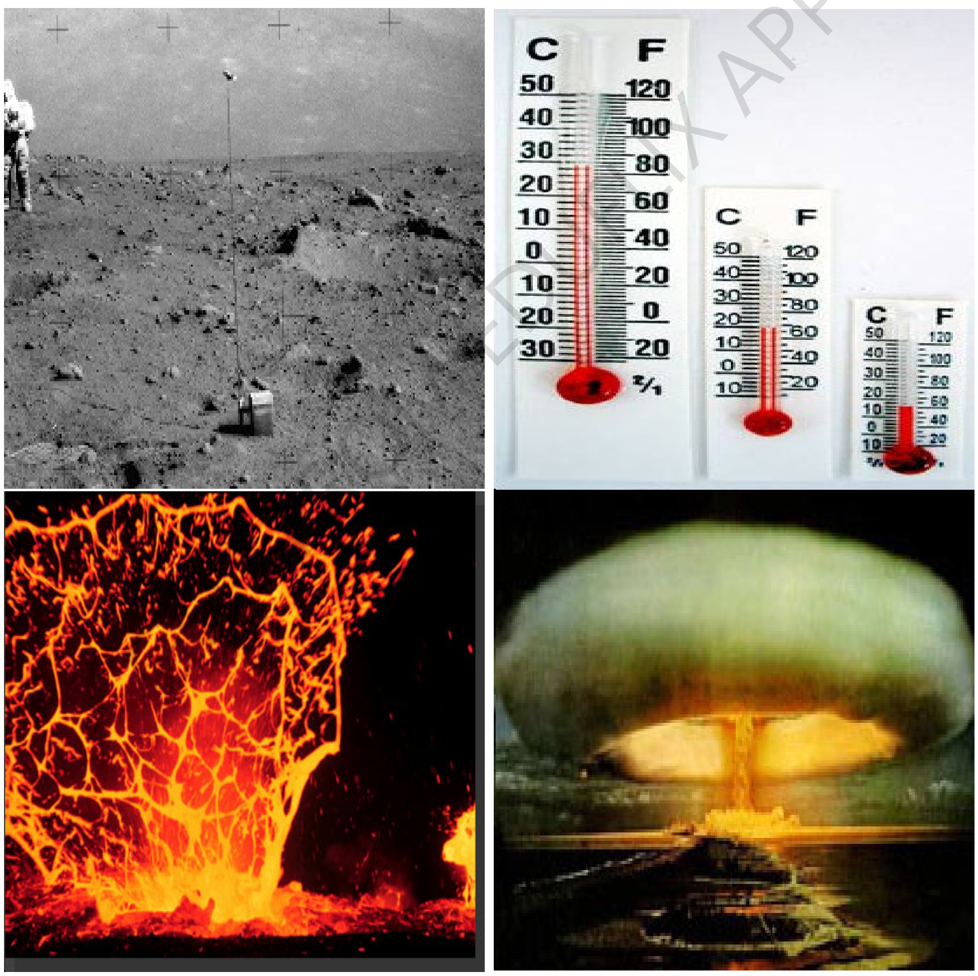
   (b) the appearance of the same flag when viewed in day light through a sheet of yellow-glass

   (c) the appearance of a bus painted red when viewed in the pure yellow-sodium light.

5.

Chapter 3

Heat



The Photos above show; the surface of the moon, a thermometer on different scaled views, larva from a volcano and a nuclear bomb test explosion.

Objectives

After studying this topic, you should:

- Be able to define the following terms;
  - ⊛ Temperature
  - ⊛ Thermometer
  - ⊛ Thermometric property
  - ⊛ Conduction
  - ⊛ Convection
  - ⊛ Radiation
  - ⊛ Heat capacity
  - ⊛ Specific heat capacity
  - ⊛ Latent heat
  - ⊛ Specific latent heat
  - ⊛ Latent heat of vapourisation
  - ⊛ Specific latent heat of fusion
  - ⊛ Specific latent heat of vapourisation
  - ⊛ Evaporation
  - ⊛ Boiling
  - ⊛ A vapour
  - ⊛ Saturated vapour
  - ⊛ Saturated vapour pressure
  - ⊛ Dew point
- Be able to give;
  - ⊛ 5 advantages of mercury over alcohol as a thermometric liquid
  - ⊛ 4 advantages of alcohol over mercury as a thermometric liquid
  - ⊛ 4 disadvantages of water as a thermometric liquid
  - ⊛ 3 factors affecting the expansivity of a substance
  - ⊛ 4 advantages and 3 disadvantages of thermal expansion
  - ⊛ 3 factors affecting heat transfer by radiation
  - ⊛ 5 properties of radiant heat
  - ⊛ 3 factors affecting evaporation and 2 affecting boiling
  - ⊛ the difference between a vapour and a gas or a volatile and non volatile liquid
  - ⊛ 3 applications of boiling at reduced pressure

  the kinetic theory explanations for each of the three gas laws
- Be able to describe;
  - ⊛ how the sensitivity of a glass thermometer can be increased.
  - ⊛ how a clinical, resistance and thermoelectric thermometers operate.
  - ⊛ how a double glazed window or door prevents heat transfer.
  - ⊛ how land and sea breezes are formed.
  - ⊛ what is meant by the term global warming.
  - ⊛ how a vacuum flask, car radiator and domestic hot water system works.
  - ⊛ how one can make ice by evaporation of ether.
  - ⊛ how construct and calibrate a mercury or alcohol thermometer.

- ⊛ the effect of temperature on saturated vapour pressure.
- ⊛ how a refrigerator or hygrometer operates
- ⊛ briefly how a petrol or diesel engine operates.;
- ⊛ an experiment to;
  - ⁎ show that a liquid, a solid or a gas expands when heated.
  - ⁎ what is meant by anomalous expansion of water.
  - ⁎ determine whether a wire gauze is a conductor or an insulator.
  - ⁎ determine which metal is the best conductor from a set of them.
  - ⁎ show that water is a bad conductor of heat.
  - ⁎ show the heat conducting powers of a two or more liquids.
  - ⁎ show that glass absorbs heat radiations.
  - ⁎ determine the specific heat capacity of a liquid or solid by the mixtures method or the electrical method.
  - ⁎ determine the saturated vapour pressure of a small quantity of a liquid.
  - ⁎ verify any of the three gas laws.

- Be able to state;
  - ⊛ the 3 units of temperature
  - ⊛ the kinetic theory of matter
  - ⊛ the 3 factors affecting heat transfer by conduction
  - ⊛ the relationship between specific heat capacity and heat capacity
  - ⊛ the 3 gas laws

- Be able to;
  - ⊛ convert one unit of temperature to another.
  - ⊛ determine the lower and upper fixed point
  - ⊛ use an uncalibrated thermometer to measure temperature
  - ⊛ distinguish between forced and free convection.
  - ⊛ explain why;
    - ⁎ some rods can become red hot and other can not.
    - ⁎ black shoes are always polished.
    - ⁎ in desert areas white clothes, white houses or white objects are common.
    - ⁎ a silvery teapot keeps tea hot for a longer period of time.
    - ⁎ ice is slippery
    - ⁎ regelation is possible in ice
    - ⁎ why evaporation is always accompanied by cooling.
  - ⊛ determine the initial or final temperature of a mixture of two or more substances by calculation given the necessary constants.
  - ⊛ sketch the heating and cooling curves for a pure and impure substance and explain the shape of the curves.
  - ⊛ distinguish between real and ideal gases

Contents

3.1 Introduction and thermometry

3.1.1 Introduction

In everyday life, we use words like warm, cold, tepid e.t.c. to describe how hot an object is. All we are describing is heat, a form of energy.

Heat

Heat is a form of energy, sometimes called internal energy or micro-mechanical energy. Heat energy in a substance cannot be measured but we can measure heat transferred from one body to another, this is measured in terms of temperature (hotness or coldness of a substance). Heat travels from regions of high temperature to regions of low temperature i.e. heat flows.

Before, heat was believed to be an invisible liquid called calorie which flows from a hot substance to a cold or cooler substance hence heat was (and sometimes today is) measured in calories. This caloric theory was disapproved by Joule (Joule's experiment has the details).

3.1.2 Temperature and thermometry

Temperature

Definition 13.

> **Temperature** is defined as a measure of hotness or coldness of a substance as a number on some chosen scale.

On some chosen scale because it may be in centigrade i.e. Celsius, Fahrenheit, Kelvin, Rankine and 0°C is not equal to 0K, e.t.c . Also we call it a number because 2°C is not equal to 2°F or 2K. Temperature can be estimated by touching but this is a rough method, to obtain an accurate value of temperature, we use a thermometer.

Thermometer

A thermometer is an instrument used to measure temperature. There are various types of thermometers depending on the **thermometric properties** used.

Thermometric property and thermometric substance

Definition 14.

> **A thermometric property** is a property of a substance, which changes with change in temperature and remains constant at constant temperature.

And the substance is called a thermometric substance (a substance which can be used in a thermometer) Examples of thermometric properties are;

1. Length of a metal (when a metal is heated its length increases).
2. Volume of a gas or liquid at constant pressure (used in liquid-in-glass thermometer)
3. Pressure of a gas at constant volume (used in the constant volume gas thermometer)
4. Density of a gas
5. Resistance of a metal (used in the platinum resistance thermometer)

Definition 15.

> A **thermometer** is an instrument, which uses a certain thermometric property to measure temperature.

There are various types of thermometers depending on the thermometric property they use, these include;

- Liquid-in-glass thermometer
- Constant volume gas thermometer

- Thermo-electric or thermal couple thermometer (thermopile)
- Platinum resistance thermometer and many others.

These are the commonly used ones but the most commonly used is the liquid-in-glass thermometer.

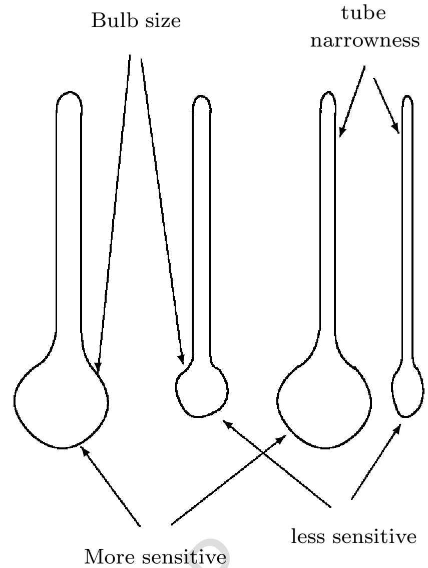
Liquid-in-glass thermometer

Here a liquid (mercury, alcohol, etc) is sealed in a glass tube. Depending on how it expands (its volume increases) when heated, the increase in length of the liquid in the tube is a measure of temperature when it is calibrated. (Major assumption is that a liquid expands uniformly when heated)

Sensitivity of a thermometer:

A sensitive thermometer is one that can record very small changes in temperature as soon as they occur. For a liquid-in-glass thermometer to be sensitive, it should have;

- A larger bulb because the greater the volume of mercury the greater the expansion (change in volume) for a given change in temperature.
- A narrow capillary tube so that a small volume of the liquid fills a greater length of the stem.



Quick response of a thermometer

A quick acting thermometer is one that records temperature changes at once as they occur. It should have;

- A bulb of thin glass (smaller thickness) so that heat can pass through it readily to and from the liquid.
- A small bulb so that there is little liquid to warm or cool.
- A liquid which is a good conductor of heat so that the whole of it quickly attains the temperature of its surroundings.

Units of temperature

Temperature has various units but the common ones are

- Degree centigrade or Celsius(C).
- Degree Fahrenheit (F) and
- Kelvin (K).

Converting one unit to another

Temperature in Kelvin = 273 + temperature in centigrade or Celsius

For instance 30° is equal to (30 + 273)K = 303K, and 27° = 273 + 27 = 300K.

If the temperature in Celsius or centigrade is C then

$$C = \frac{5}{9}(F - 32) \tag{3.1}$$

where F is the temperature in Fahrenheit i.e

$$\begin{aligned} 114^\circ F &= \frac{5}{9}(114 - 32) \\ &= \frac{5 \times 92}{9} \\ &= 51.11^\circ C \end{aligned}$$

Summary

$$K = 273 + C \tag{3.2}$$

$$F = \frac{5}{9}C + 32 \tag{3.3}$$

3.1.3 Mercury thermometer

This is a liquid-in-glass thermometer where the liquid used is mercury

How to make a mercury thermometer

1. Get a clean capillary tube
2. Heat one end in a Bunsen flame, the tube will become soft and then seal it that end by pressing it.
3. Blow at the other cold end, a small bulb will be formed at the sealed end (hot end). The size of the bulb can be determined depending on the intended temperature range and the bore of the bulb.
4. Place the open end in mercury and heat the bulb gently, leave it to cool while in mercury. On cooling the air contracts and some mercury runs up into the tube.
5. Take out the thermometer and heat the mercury in it; the mercury vapour formed expels air out.
6. Quickly dip the open end in mercury and leave thermometer to cool in it. The mercury level rises and completely fills the bulb and the tube.
7. Heat the tube now to a temperature higher than the one it is intended to measure and then seal it off(as before).
8. Now it is ready to be calibrated i.e. to mark on it the upper fixed point and the lower fixed point.

CAUTION Mercury is poisonous and secondly glass goes on contracting slowly for a considerable time. It is advisable to put the thermometer a side for a few months before calibrating it.

Calibrating a mercury thermometer

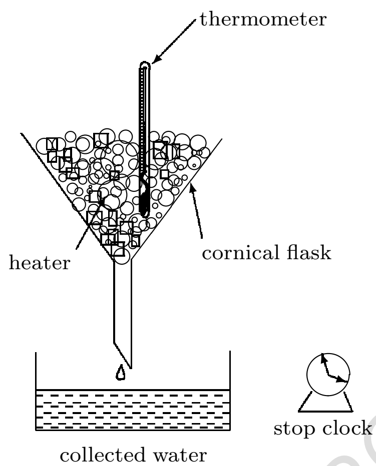
Here we mark the steam point (i.e. upper fixed point) 100°C and the ice point (lower fixed point) 0°C then divide the interval between these points called (fundamental interval) into 100 equal parts, which we now call degrees (i.e. each interval is equal to one degree).

Lower fixed point or freezing point

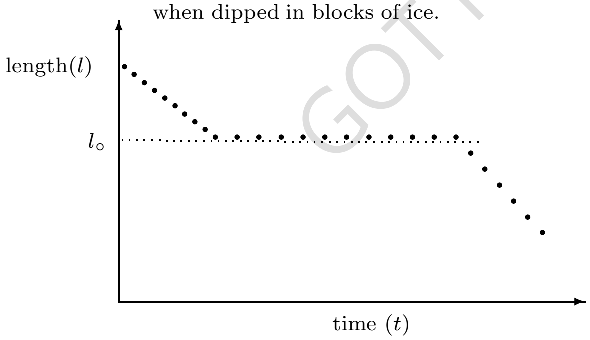
This is the temperature of pure melting ice or the temperature at which pure ice melts, it is 0°C. When water has some impurities it melts at a lower temperature because the presence of impurities in a pure substance lowers its melting point.

To label the lower fixed point on a thermometer

1. Place the thermometer in a funnel containing crushed blocks of pure ice.
2. Start the stop clock and note or record the length of the mercury thread in the thermometer.
3. This length will decrease with time. After some time this length becomes constant ($l_\circ$) and this is when ice is melting i.e. at ice point 0°C. This is marked as a point on the thermometer called ice point.



length (l) of mercury thread versus time (t) when dipped in blocks of ice.

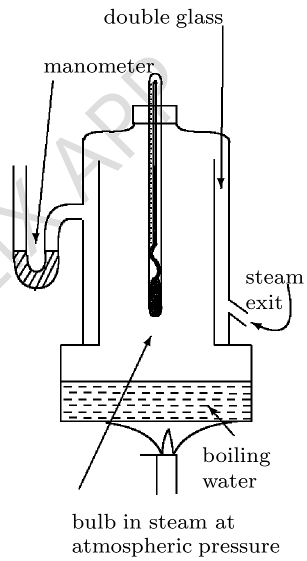


Upper fixed point (steam point or boiling point)

This is the temperature of steam from pure water under standard atmospheric pressure of 760mmHg. Pure water at 760mmHg boils at 100°C and this is the steam point. The presence of impurities in water raises its boiling point (for instance up to 104°C)

To label the upper fixed point on a thermometer

Place the thermometer in steam from pure boiling water at a pressure of 760mmHg or **just** on the surface of boiling water. This is best done using a hypsometer shown below;



When length of mercury thread remains steady for some time, the level of the surface of the mercury thread is marked on the thermometer as steam point i.e. (100°C).

Origin of the Celsius scale

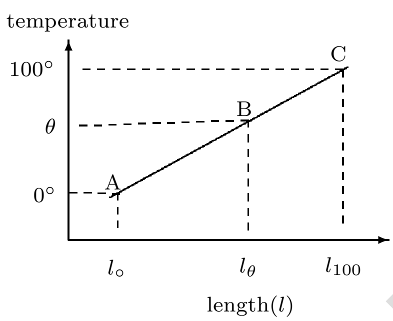
The difference between the boiling point and the melting point of a substance is called its fundamental interval. The fundamental interval for water is 100°C(100°C - 0°C = 100°C). This difference (fundamental interval) can be measured as the length on the thermometer between the ice point and the steam point is divided into 100 equal parts each equivalent to a degree. This method of subdividing was suggested by a Swedish astronomer Celsius hence called the Celsius scale. This thermometer measures temperature in degrees Celsius or degrees centigrade. (Each division is a degree).

Using an un-calibrated thermometer to measure temperature

The un-calibrated thermometer is placed in pure melting ice, in steam from pure boiling water at 760mmHg and in a substance at an unknown temperature (θ) and the length of the mercury thread in each is measured and recorded as $l_\circ$, l_{100} and l_θ respectively.

Assuming[1] that the increase in length are equal for each change in temperature, then the graph of temperature against length of the mercury thread is a straight line as shown below;

Graph of temperature θ against length l



The slope is the same hence

For the line AC

$$\begin{aligned} \text{slope} &= \frac{\Delta\theta}{\Delta l} && (3.4)\\ &= \frac{100-0}{l_{100}-l_\circ} && (3.5)\\ &= \frac{100}{l_{100}-l_\circ} && (3.6)\\ \text{and slope} &= \frac{\theta-0}{l-l_\circ} && (3.7)\\ &= \frac{\theta}{l-l_\circ} && (3.8)\end{aligned}$$

The above two slopes are equal, since it is a straight line

$$\begin{aligned} \frac{\theta}{l-l_\circ} &= \frac{100}{l_{100}-l_\circ} && (3.9)\\ \theta &= \frac{l-l_\circ}{l_{100}-l_\circ}\times 100 && (3.10)\end{aligned}$$

Hence the unknown temperature is given by

$$\begin{aligned} \theta &= \left(\frac{l-l_\circ}{l_{100}-l_\circ}\right)\times 100^\circ\mathrm{C} && (3.11)\\ \text{Or } \theta &= \frac{x}{y}\times 100^\circ C && (3.12)\end{aligned}$$

Where x is distance between the ice point and the unknown temperature point

$$x = l_\theta - l_\circ \qquad (3.13)$$

and y is the distance between the ice point and the steam point

$$y = l_{100} - l_\circ \qquad (3.14)$$

y is the fundamental interval

Examples

The fundamental interval of an un-calibrated thermometer is 12cm. In a certain liquid, the top of the mercury thread is 8cm above the ice point. Determine the temperature of the liquid?

Solution

the fundamental interval $=y$

$$\begin{aligned} y &= 12cm\\ x &= 8cm\\ \theta &= \frac{x}{y}\times 100^\circ C\\ &= \frac{8}{12}\times 100^\circ C\\ &= \frac{2}{3}\times 100^\circ C\\ &= \left(\frac{200}{3}\right)^\circ C\\ \theta &\approx 66.67^\circ C\end{aligned}$$

hence the temperature of the liquid is $66.67^\circ C$

[1] It is assumed because not all substances show a linear relationship with temperature

Exercise

1. The fundamental interval of a given thermometer is 8cm. When it is placed in alcohol at an unknown temperature, the top of the liquid's thread is 5cm above the ice point. Determine the temperature of alcohol?

2. A constant volume air thermometer records a pressure of 75cmHg when immersed in melting ice , 105cmHg when in steam from pure boiling water and 112.5cmHg, when in a certain boiling liquid. What is the centigrade temperature of the boiling liquid?

3. For a given thermometer, the length of its mercury thread was measured at various temperatures and the results were;

   In melting pure ice,
   length = 4cm
   In steam from pure boiling water,
   length = 7.5cm
   In water at room temperature,
   length = 5cm.

   Estimate the room temperature?

4. The length of the alcohol thread in a thermometer when dipped in melting ice and when in liquid x are $10cm$ and $7cm$ respectively. Determine the temperature of the liquid x?(fundamental interval is $20cm$)

3.1.4 Alcohol thermometer

This is the same as the mercury thermometer with the only difference of having alcohol as the thermometric liquid and not mercury. Since mercury is opaque and alcohol is transparent, alcohol is always coloured red to make it visible.

Advantages of mercury over alcohol as a thermometric liquid

1. Mercury is easily seen, because it is opaque and silvery and alcohol is colourless, thought it can be coloured easily. mercury does not need a dye to colour it.

2. Mercury does not wet glass yet alcohol does. No mercury remains on the sides of the tube when the mercury level falls. But for alcohol, it sticks on the sides leading to low readings.

3. Mercury is a good conductor of heat and alcohol is a poor conductor, hence mercury reaches the same temperature as that of the surrounding as soon as possible.

4. Mercury does not easily boil(boiling point is 357°C) and alcohol boils easily (boiling point is 78°C). In the alcohol thermometer, alcohol forms a vapour, which condenses on the cooler upper parts of the capillary tube at room temperature.

5. Mercury has a lower specific heat capacity [2](i.e mercury thermometer takes only a little heat from substances whose temperature it is measuring) and alcohol has a higher specific heat capacity, about 18 times the specific heat capacity of mercury (i.e. it cools substances whose temperature it is measuring.

6. Mercury has a greater fundamental interval, its fundamental interval is 357°C - -39°C = 396°C, and for alcohol it is 190°$(78 - -112 = 190)$.

Advantages of alcohol over mercury as a thermometric liquid

1. Alcohol does not solidify easily as compared to mercury, its melting point is -112°C and that of mercury is -39°C. Because of this alcohol thermometers are preferred when measuring low temperatures or for use in temperate regions (arctic and Antarctic regions)

2. Alcohol expands more than mercury. Alcohol expands about 6 times more than mercury. This means that alcohol thermometers are more sensitive.

[2]Specific heat capacity is discussed on page 123

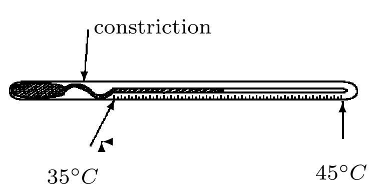
3. Mercury is more poisonous compared to alcohol.
4. Alcohol is cheap and mercury expensive.

Disadvantages of water as a thermometric liquid.

1. It is not easily seen (because it is transparent though it can be coloured).
2. It wets glass i.e. it sticks to the walls of the glass tube.
3. It is a poor conductor of heat.
4. It boils and freezes readily (i.e. at temperatures of about 50°C, it condenses to the cooler upper parts of the capillary tube)
5. It has a larger heat capacity i.e. it reduces the temperature of the substance under study.
6. Its expansion is not uniform (Read about the anomalous expansion of water).

Clinical thermometer

This is a thermometer used to measure human temperature. It has a fine stem divided into fifth or tenths of a degree and calibrated over a small range $95°F - 110°F$ or 35°C-45°C. The stem is thickened on the side remote from the graduation so that it acts as a lens, to magnify the fine mercury thread.



clinical thermometer

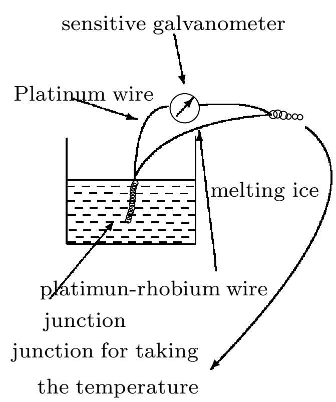
At room temperature, the mercury retreats into the bulb. Between the bulb and the graduation, is a fine kink called the **constriction**. When the object (human body) whose temperature is to be measured warms the bulb, the mercury in it expands forcing itself through the kink into the stem to a certain value. When expansion stops, the mercury does nor flow back past the kink, a space is created at the kink leaving it to stay in the stem, so that the temperature can be read at leisure (this is the purpose of the kink or constriction). When it is to be used again, it is seized back into the bulb

[3]

Thermocouple, thermopile or thermo-electric thermometer.

This uses the thermoelectric effect i.e. when two wires of different metals are twisted and the junction heated, an electric potential difference is produced. When a sensitive galvanometer is connected across the other ends of the two wires, it shows a deflection. This thermo-electric effect is observed in;

1. Iron and copper wires.
2. Bismuth and antimony wires.
3. Constantan (copper-nickel alloy) and Nichrome (nickel-chromium alloy) wires and many others.



The two wires used in this way are what we call a **thermocouple** and the combination of thermo-

[3]Other thermometers like the minimum, maximum and sixth thermometer can be found in Geography textbooks

couples is called a **thermopile**(combined to obtain a measurable value of the emf) and the thermometer which measures temperature directly on the galvanometer of the thermocouple is called a **thermo-electric thermometer**.

A thermo-electric thermometer is a thermocouple usually made from platinum and rubidium alloys chosen because their melting points are very high, one junction is kept at 0°C (or some other convenient temperature) and the other is used to record the temperature being measured. The meter (galvanometer) is calibrated so that it reads temperature directly and not merely potential differences.

Advantages of a thermo-electric thermometer

1. It can measures very high temperatures such as those in the furnaces of molten metal, bricks's tunnel and others.
2. The junction is very small and it is a quick acting thermometer i.e can measure rapid changes of temperature.
3. The recording meter can be in an office far from the furnace whose temperature is being measured.
4. It can measure temperature of a point i.e localised temperatures in a hot substance. This is because it has a very low heat capacity.

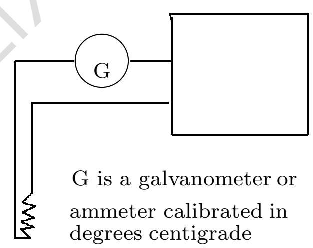
Resistance thermometers

Electricity is the flow of electrons in a substance or materials. These electrons are not free to move, their speed is reduced by the collisions they make with the atoms or molecules of the material in which they flow. This opposition to the flow of electrons in a substance is called its resistance. A material with a low resistance allows electrons to pas through it easily and that with a high resistance does not allow electrons to pass through it easily. The resistance of a material usually varies directly as the temperature of the material, i.e $R \quad \alpha \quad T$. Hence resistance of a metal or material is a thermometric property.

Using the centigrade or Celsius scale of temperature; for a given platinum wire, measure its resistance, $R_\circ$ when placed in or dipped in pure melting ice. Its resistance, R_{100}, when placed in steam from pure boiling water at a standard atmospheric pressure of 760mmHg. Its resistance, R_θ, when placed in a substance at an unknown temperature,θ, this unknown temperature, θ, it given by

$$\theta = \left(\frac{R_\theta - R_\circ}{R_{100} - R_\circ}\right) \times 100°\text{C} \tag{3.15}$$

below is the block diagram of a platinum resistance thermometer



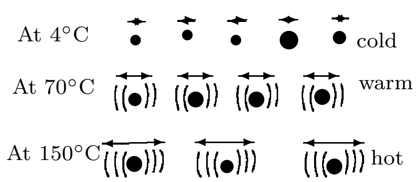
The thermodynamic scale of temperature

Apart from the Celsius scale of temperature , there exists another scale of temperature called the thermodynamic scale of temperature, which is quite independent of the properties of any substance used in a thermometer. Temperatures on this scale are measured not in degrees but in Kelvin denoted by K not °K, in honour of Lord Kevin who devised this scale. On this temperature scale a change of 1 Kelvin in temperature is equal to a change in temperature of 1 degree on the Celsius or centigrade scale. Ice point on this scale is 273K and boiling point of water is 373K on his scale. A temperature in 0°C is changed to Kelvin by adding 273 to its value.

3.2 Heat expansion

3.2.1 Introduction

If we heat an object, the atoms or molecules in it begin to vibrate vigorously, they will need more room or space to vibrate. What they do is simply to move outwards and because of this the object expands. When the object is cooled, the vibrations become less vigorous and molecules come close together, i.e. the object contracts.



This occurs in solids, liquids and gasses. Expansion depends on three properties

1. Size of the expanding substance
2. Increase in the temperature of the body
3. type or nature of the material (i.e. its expansivity)

Factors affecting expansivity of a substance

1. Size of the expanding substance. A smaller quantity expands by a smaller value and a larger quantity expands by a much larger value for a given change in temperature. The quantity may be volume, length, e.t.c.
2. Increase in the temperature. If the increase is from 10°C to 20°C i.e. increase in temperature = 20 - 10 = 10°C and also from 10°C to 80°C the increase in temperature = 80 - 10 = 70°C. In the first case expansion is smaller than in the second.
3. Type of materials (its expansivity). For instance when equal rods made from copper and lead are heated from room temperature(25°C) to a much higher temperature, their new lengths will not be equal implying that they have expanded differently. They had the same size(length) and same increase in temperature hence here the nature of the material determines the degree of expansion or how much a body will expand.

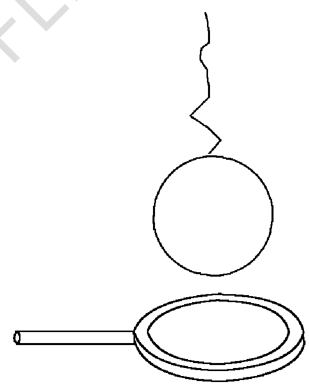
3.2.2 Expansions and contractions in solids

Experiments to show that solids expand

(a)**Ball ring experiment**

Apparatus you need to have;

- a metallic small sphere.
- metallic circular ring.
- Source of heat.



Method

- check whether the ball fits or passes through the ring at room temperature. It is expected to pass through.
- using a Bunsen - burner or other means heat up the ball (not the ring.)
- after about three or four minutes of heating, use tongs to pass the ball through the ring.

Observation

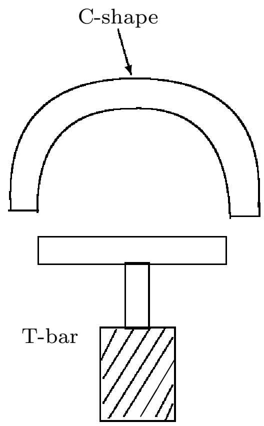
- the ball will not pass through the ring because it has expanded
- When you cool it after a few minutes it passes through.

Conclusion

when the ball was heated, it expanded thats why it could not pass through the ring.

(b)**The D- gauge experiment.**

Apparatus You need; a T-bar and a C-shaped bar (i.e. D-gauge). Sown below



this is the D-gauge

Method

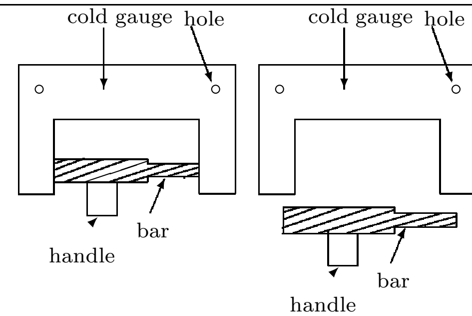
- check to ensure that the T-bar fits between the jaws and the D-gauge. It should fit between the jaws.
- Heat the T-bar, After about three minutes, fit it in between the jaws of the D-gauge (it will not fit in).
- cool it and also fit it in between the jaws again(it will fit in).

Observation

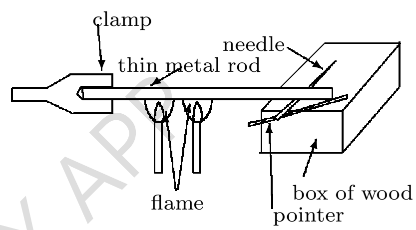
- after the heating, the T - bar could not fit in the D-gauge, this is because it had expanded and hence it was longer.
- after cooling, it fitted in between the D - gauge, this is because it contracted i.e. its length has shortened.

Other experiments that demonstrate expansion in solids include

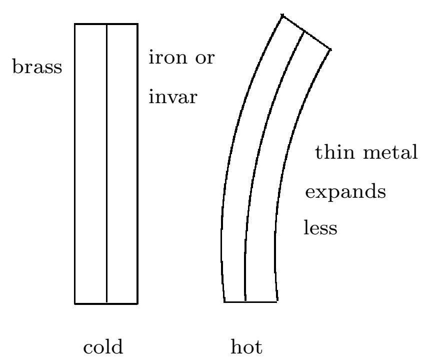
(b)Bar and gauge experiment



(c)Metal rod and needle experiment.



(d)Compound bar (bimetal bar) experiment.



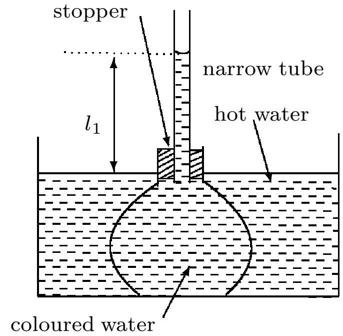
(e)Glass and silica (breaking) experiment.

(f)Bar breaking experiment.

3.2.3 Expansion and contraction in liquids

An experiment to determine whether water expands when heated

The apparatus is arranged as shown below



(a) Note the length, l_1, of the water thread in the narrow tube above.

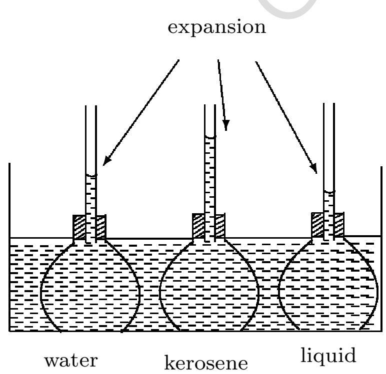
(b) Heat the water in the beaker for a few minutes (about 3 to 4 minutes)

(c) Measure the length of the water thread in the narrow tube again. l_2. It will be greater than its initial value.

(d) Since the amount of water in the beaker is constant i.e. the same mass then an increase in the length of the water thread (l) indicates an increase in the volume of the water since cross sectional area is constant, hence water expands when heated.

An experiment to show that liquids do not expand at the same rate

Place different liquids in the beakers dipped in the basin of water as shown below

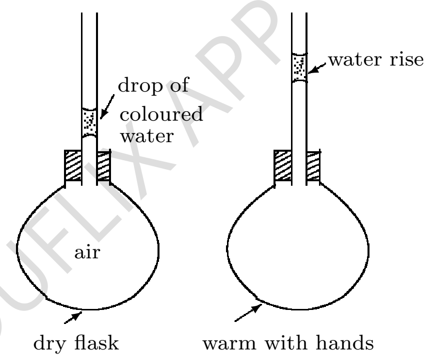


Note the length of each liquid in its capillary tube. Heat the water in the basin. After a few minutes you will find that the length of the liquid thread have increased, meaning that the liquids expanded, but the increase In the lengths will not be the same, this shows that the liquids do not expand at the same rate.

3.2.4 Expansion of gases

An experiment to show that air expands on warming

- Hold a capillary tube in the opening of a flask as shown below ensuring that it is airtight.



- Trap a thread of water or any other liquid in the capillary tube as shown above and heat the air in the beaker by placing it in hot air or above fire for a few minutes.

- You will observe that the liquid thread is pushed slightly out of the flask. This shows that the volume of air in the flask has increased hence the air in it has expanded.

Describe an experiment to show whether a given group of gasses expand at the same rate or at a different rate.

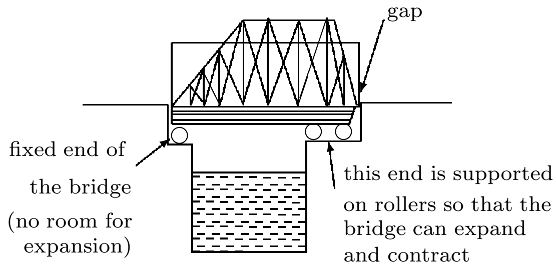
3.2.5 Advantages and disadvantages of thermal expansion

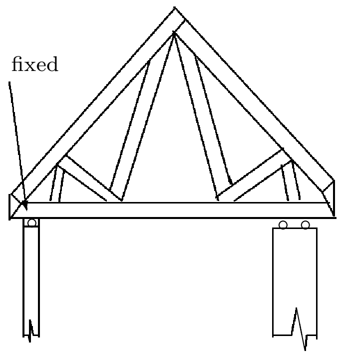
The expansion or contraction of matter can be a nuisance and at times useful in our daily life.

Below, this is described and solutions provided.

1. **Steel bridges and roofs are fixed at one end.**

   Why are gaps left in bridge ends? and what is the purpose of rollers in bridges?. Metallic bridges and roofs (of steel) are constructed with a gap (space) and roller as shown below.

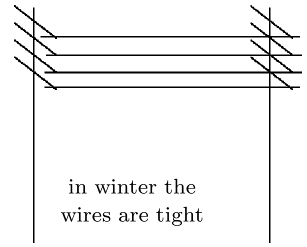


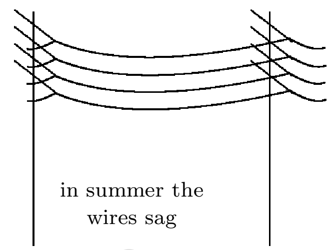


   This is because on a hot day , the metal of the roof expand. If there is no gap between it and the wall, it would push the wall outwards making it inclined which is risky as it will make the wall collapse. The roller allows easy movement of the roof or bridge as it expands and contracts. The bridge without that gap will expand on a hot day pushing the ground; which may bulge and collapse i.e. causing accidents. The purpose of the gap is to leave space for expansion and the rollers to allow easy movement of the bridge as it expands.

2. **Metal fences are made slightly longer and the telephone or electric wires are made slightly loose.**

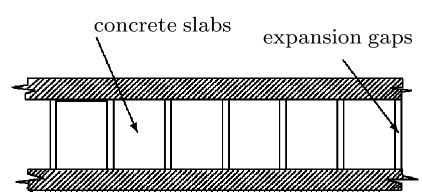
   This is because on a hot day (warm weather) they expand and sag (this has no problem) but at night when the weather is cool or on a cold day , they would contract making them very tight, this would break the fence poles or the telephone wires would break(snap). That is why they are always made slightly loose.



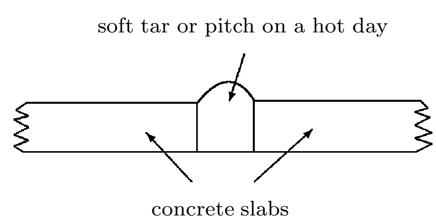


3. **Gaps are left in between the slabs used in road construction:**

   Roads are sometimes made of large slabs of concrete which are not continuous. On a hot day the slabs will expand, if small gaps are not left in between, they would crack and very soon they would need repair. The expansion gap is always filled with soft tar or pitch as shown below. In hot weather the concrete parts expand and compress some of the pitch. If the slabs were continuous, they would expand in hot weather then contract and crack in cold weather.

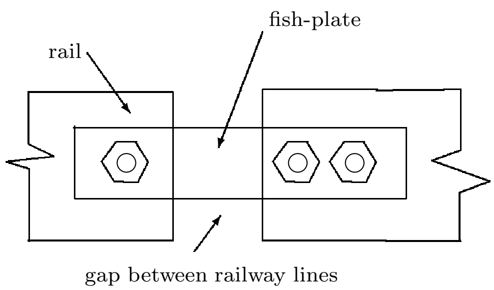


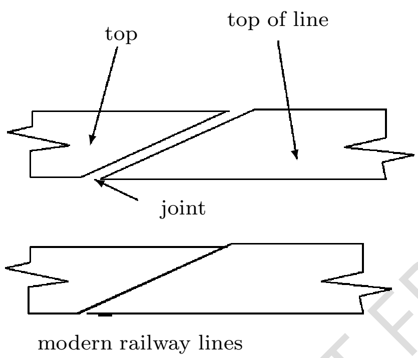
   the expansion gaps are always filled with soft tar or pitch.

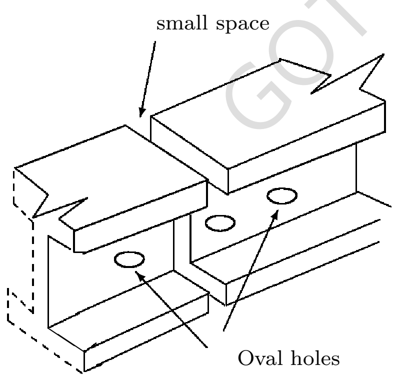


4. **Railway lines have gaps between them or are made of sliding joints for the same reason as above:**

Small spaces about a few millimeters are always left between the lengths of rails (about 30m) in railway lines so as to create room for expansion. The holes in the ends of the rails for the bolts that hold them are oval in shape so that the rails can move without breaking the bolts that hold them (this is shown below). Modern railway lines are made of long special steel, which has sliding joints, this helps the train to run on them smoothly without noise and bumps. Even when they expand, they just slide over each other.

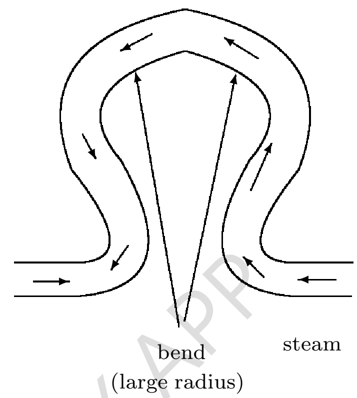






5. **Steam pipes in workshops and factories are made with bends of large radius (why large radius?):**

Steam pipes used in steam factories expand when steam passes through them, this expansion is catered for by creating expansion-bends in the steam pipes as shown below; otherwise the pipes may crack, break and start leaking. When these bends are of small radius, the stress-strain forces are greater, to reduce these forces, the bends are made of large radius hence the pipes can expand with little tension (stress and strain).



6. **Hot liquid vessels are made of thin and not thick ordinary glass:**

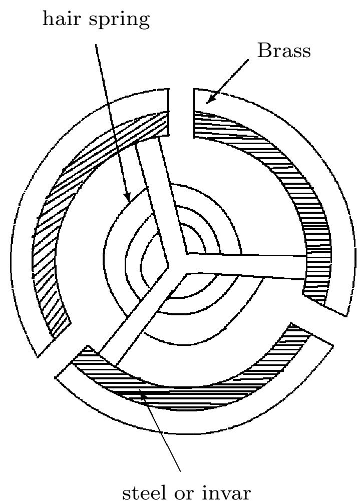
We observe that when hot liquids are poured in vessels made of thick glass, the thick glass breaks but when they are made of thin glass they do not easily break as when they are made of thick glass.

When a hot liquid is poured inside a glass vessel, the inside is heated and it expands while the out side remains cold and of the same size. This creates stress-strain forces in the glass which breaks it. For a thin glass, when it is heated , heat is quickly transferred to the outside and both sides expand equally reducing the stress-strain forces hence it can not easily break.

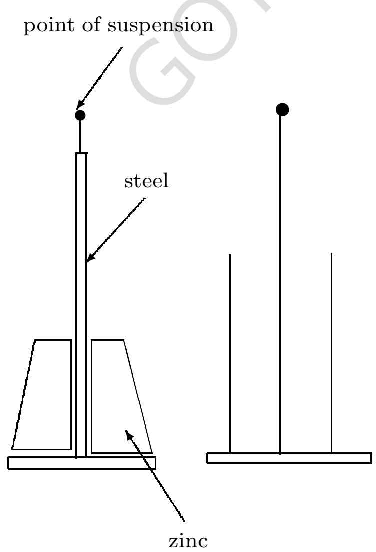
Note that Pyrex glass expands only one-third as fast as ordinary glass. Some cooking vessels and combustion tubes are made of silica, which expands only one-sixth as much as Pyrex glass.

7. **Why compensated balance-wheel of clocks and compensated pendulum are made of bimetal strip?** so as not to loose time.

Watches and many clocks are controlled by balance wheels and hair springs. When temperature rises a watch loses time because the radius of its wheel increases. A compensated balance wheel has a rim made of bimetallic strips with the less expansible metal on the inside. As the spokes expand on warming, the segments curl inwards and the time of swing remains constant. Modern balance wheels made of Invar (an alloy), need not be compensated.



A compensated pendulum of a clock is designed so that the center of gravity of the light pendulum rod combined with the heavy bob at the end remains at the same distance from the point of suspension. If the length of the rod increases due to expansion. The change of length of zinc (or brass) compensates that made by the steel rod.



8. **Why the tyre of racing cars burst or why shouldnt one pump a bicycle tyre too hard on a cool morning:**

   Racing cars move at high speeds, applying brakes at various points, this creates a lot of heat due to friction between the tyre and the road. The friction produces heat, which is transmitted to the air in the tyre that expands, making the tyre burst.

   When a bicycle tyre is pumped too hard on a cool morning, and the sun shines; at noon, it is too hot, the tyre expands the more and hence it easily bursts.

9. **How can one remove a finger stuck in the neck of a bottle?**

   Heat near the neck of the bottle either by rubbing it with your fingers or other wise. The bottle will get heated by friction and expands slowly. Depending on how thick it is, the finger can be pulled out easily when it has expanded enough. This is because when the bottle neck is heated, its radius increases.

10. **why glass cracks when it is heated ?**

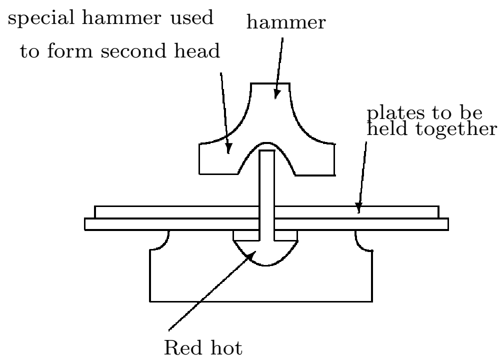
    Glass is a bad conductor of heat. When a piece of glass is heated in one place, the neighbouring parts of the glass do not at first warm up with it. The expansion of the heated part while the other has not expanded creates stress-strain forces in it, which set up cracks in the glass hence breaking it. Care must be taken to warm the whole region around the place to be made hot. Similarly glass which has been heated must be cooled slowly and uniformly.

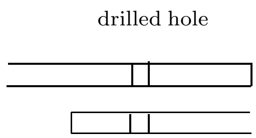
Uses of thermal expansion and contraction

There are a number of applications, some of which include;

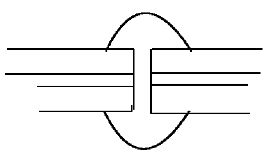
1. **Rivets:**

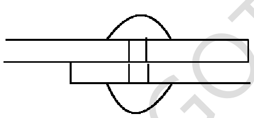
   These are used in shipbuilding, boiler making, joining metal sheets or plates, e.t.c. Usually the joining metal is made red hot, pushed through holes on two metal plates placed together and hammered until the ends are rounded. The rivet cools, contracts and grips or pulls the metal plates tightly together. The seal is so tight that it is water proof hence used in ship building and the making of boilers.





plates to be joined together


the rivet has been formed but still hot


as the rivet cools it contracts
and pulls the plates together

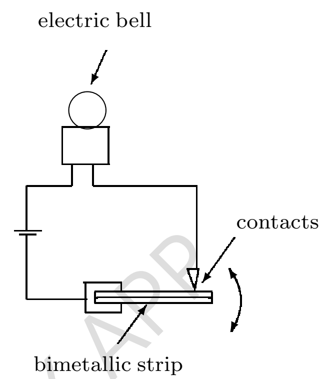
2. Metal hoofs on a beer barrel, or wire case, or glass stopper (in bottling) are heated before they are put on the wooden section: On cooling they contract to form a good tight fit. Cant you now suggest what happens in the soda bottling companies?

3. **Opening bottle tops:**

   It is easy to open or loosen a metal top of a bottle by heating it or pouring on it hot or warm water. This is because it expands and becomes loose or less tight.

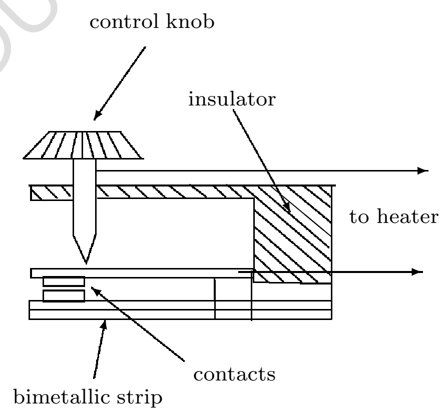
4. **In fire alarms and control of heat flow:**

   The bimetallic strip is used to control the flow of heat energy in thermostats and in fire alarms by completing an electric circuit. When a bimetallic strip is heated, it bends completing or breaking an electric circuit. This is shown below;



(It completes the circuit when heated)

A simple alarm system



A simple thermostat

5. **Soil made by weathering of rocks:**

   Hot sunshine makes the out side of a rock expand and pieces of the rock break off due to the forces of expansion. Sometimes cool rain falls on a hot rock and the outside tries to contract but the inside does not contract, as it is still hot. The forces of contraction gradually breaks the rock into small pieces which finally form particles of soil.

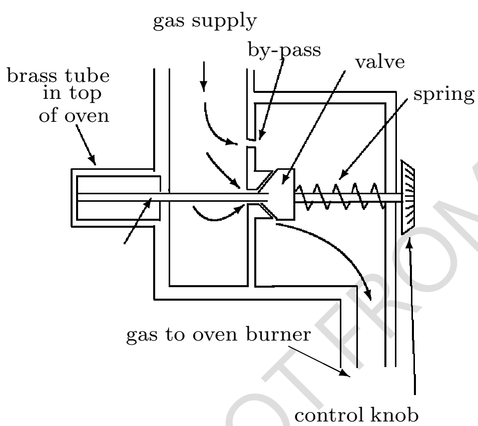
6. **Steel tyres and rubber tyres easily fit into their rims after being heated**

This is because on heating them they expand and become large and then they can fit on their rims. On contracting, they become tight on the rims as required.

7. **Gas thermostats:**

   These help to control the temperature of (and gas flow in) gas ovens. Its action depends on the expansion of metals or alloy in it.

   In the diagram below; as the temperature of the oven rises, the brass tube expands to the left pulling the Invar rod with it. Since the expansion of the Invar is negligible, the gas supply through the valve to the burner is reduced. The control knob alters the original position of the valve and selects the steady temperature reached.

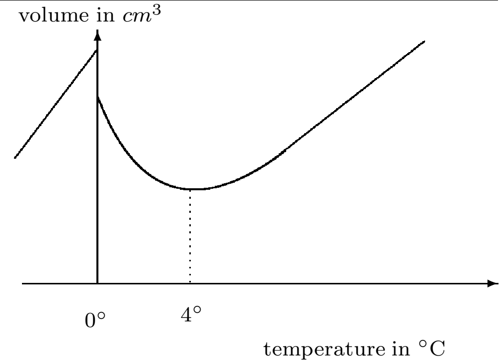


8. **Anomalous expansion of water:**

   In terms of volume

   It is known that whenever any thing is heated, it expands such that for every increase in temperature there is an increase in the volume of the substance. The reverse is also true i.e. when it cools down its volume decreases. Water behaves in a different way especially in the range between 0°C - 4°C. The graph below illustrates this anomalous expansion of water.

   Graph of volume against temperature for water

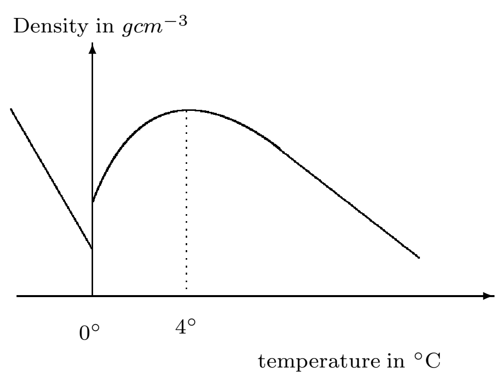


Consider ice at about -10°C, when it is heated, its volume increases uniformly to 0°C, at 0°C its volume decreases abnormally as it changes to water, from 0°C to 4°C it again decreases in volume slowly to the minimum volume at 4°C. From 4°C to the higher temperatures volume then increases. Where is the anomaly? between 0°C and 4°C, the volume of water was expected to increase when temperature is increased but instead the volume of water decreases as temperature is increased. This is the anomaly or abnormality in the expansion of water.

In terms of density, ρ

Since volume is inversely proportional to density for a given mass, the graph below illustrates how density changes.

Graph of density (ρ) against temperature for water



Note that water has its highest density at 4°C and this is what is recorded in textbook as 1000kgm^{-3} or 1gcm^{-3}

3.3 Heat transfer

3.3.1 Introduction

There are three ways by which heat can be transferred in matter (solid, liquid and gases). These are;

1. Conduction
2. Convection and
3. Radiation (and evaporation)

Note; you are supposed to know how molecules behave in solids, liquids and gases before continuing.(Brownian motion and what was deduced from its discovery).

What are the heat transfer methods?

1. **Conduction**

   Definition 16.
   Conduction is the process by which heat is transferred from regions of high temperature to regions of low temperature by movement of electrons and vibrations of molecules.

   The definition of Conduction as the process by which heat is transferred from regions of high temperature to regions of low temperature with out the movement of matter or material of the substance is not correct because it can be a definition for radiation where also there is no transfer of matter.

   From the kinetic theory of matter and Brownian motion we concluded that molecules of matter are ever in a state of motion (continuous).

   In liquids and gases, the molecules are free to move, in solids they can just vibrate in a fixed position (mean position).

   When one end of a substance is heated, it molecules vibrate more vigorously and they knock each other making the neighbouring molecules also vibrates more faster. In this way the hot molecules pass on some of their energy to the cooler molecules next to them, hence heat passes or travels from one molecule to another. Even electron may evaporate or may be transferred from one molecule and knock the other molecule increasing its vibrations. Movement of electrons transfers Heat, hence in conduction heat is transfered by vibration of molecules or atoms and movement of electrons.

   Note that molecules do not move in conduction (and in radiation).

2. **Convection**

   Definition 17.
   Convection is the process by which heat is transferred from regions of high temperature to regions of low temperature by movement of matter or material of the substance.

   Since it is in only fluids i.e. liquids and gases that can flow, convection takes place in fluids. When part of a fluid is heated its molecules move faster. The faster molecules (hot molecules) collide with the cooler molecules and pass on some of their energy to them. Heat is transfered by movement of a molecule from one point to another . Convection is also an application of Archimedess principle, whereby the hot molecules (which are less dense) in the cooler molecules (more dense), experiences a greater upthrust (upward force), that is why hot gases always rise up (smoke rises up).

3. **Radiation**

   Definition 18.
   Radiation is the movement of heat from regions of high temperature to regions of low temperature by the propagation of electromagnetic waves.

   These electromagnetic waves are produced by the vibrating charged particles of hot substances. A wave is a disturbance in a medium, when a stone is dropped into water, we see water waves moving; these are mechanical waves. The other type of waves are electromagnetic waves which we can not see or even hear, for instance those that are detected by a radio. The only electromagnetic wave that we can see is light. We always say that there are radiations that came from the sun, radiations are electromagnetic waves.

A hot body gives out invisible rays called infrared radiations. They can pass through air and in a vacuum, just as light does. When the rays fall on matter their energy changes into heat i.e. the body is heated, so we say that heat has been transferred by radiation. Heat from the sun is transferred to the earth by radiation.

Note the heat radiation or radiant heat requires no material medium to be transmitted. This is true for all electromagnetic radiations. All electromagnetic radiations do not produce enough heat, the heat radiations are the infra-red radiations. examples of electromagnetic radiations include light, x-rays, gamma rays, radio waves, micro-waves and many others. Those that produce too much heat are infrared radiation

3.3.2 Conduction

(a) Conduction in solids

Solids that can conduct heat are called conductors and those that cannot conduct heat are called insulators or bad conductors. (What of semiconductors [4] ?)

Examples of conductors

All metals and some liquids like mercury are conductors. Conductors have a number of uses for instance they are used in the manufactures or making of source pans, kettles, boilers, soldering iron, motor car engine parts. Solder iron is made of copper because it conducts heat quickly to the tip where it is required. Aluminium is commonly used in the manufacture of most cooking utensils this is because it is a better conductor as compared to steel, its weight is less and does not easily become too hot (or red hot).

[4] See page 347 for details about semiconductors

Examples of insulators

These include rubber, all glasses, cork (carbon material), wood and many more. Insulators are commonly used as handles on an iron box or any other hot object or device, as table mats, in lagging fridges, cookers and walls (i.e. to prevent heat loss).

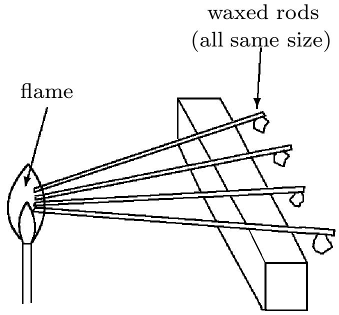
Refrigerators and iron boxes have an air space between their double walls because air is a good insulator. Ice blocks are usually covered with a cloth, paper and saw dust when being moved or stored, the air spaces in the material reduce heat loss by conduction. Cloth is a poor conductor but wool is a better insulator because it contains more tiny air spaces. Woolen clothes are more useful in cold climates and cotton clothes are better for hot countries.

Heat conductivity of a material or the amount of heat transfered from one end to another depends on

1. Cross sectional area (i.e. a thick material conducts more heat than a thin one).
2. The length of the conductor
3. Temperature difference across its ends.

An experiment to determine which metals are the best conductors of heat

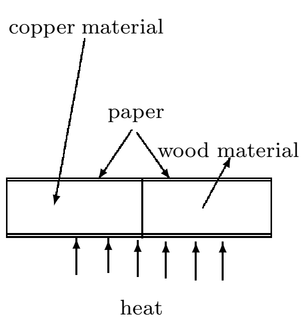
- Take rods of copper, glass, iron and those for other metals with the same length and thickness.
- Attach similar equal sized pieces of paraffin wax at the end of each rod.
- Place the other end of each in a moderate flame as shown below and note the wax that melts first is from which rod



The one whose wax melts first is the best conductor among those available followed by the next like that. The order was found to be silver, copper, brass, zinc, tin, and lead.

An experiment to determine which is a better conductor, copper and wood;

- Take a copper pipe (or brass) that fits on the end of round piece of wood both of equal external diameter.
- Wrap one turn of paper tightly around them.
- Warm in a flame the position on the paper where the two materials meet. Where does the paper burn more quickly and why?
- The paper will burn quickly on the side of copper because the flame heats the paper on the copper and wood equally but heat passes quickly through the copper (a good conductor) and the paper does not scotch. The paper on the side of wood (a bad conductor) soon scorches (or is charred) this shows that copper is a better conductor of heat as compared to wood.

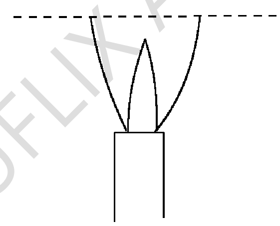


copper is a good conductor and wood is a bad conductor of heat

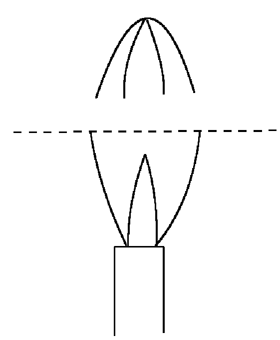
An experiment to determine whether a given wire gauze is a conductor or an insulator.

Hold a wire gauze about 5cm above the Bunsen burner. Turn on the gas and light it underneath the gauze. Does the flame pass through the wire gauze?

If the wire gauze is a conductor, the heat from the flame underneath is conducted to other parts of the gauze (i.e. good conductor) hence the gas above the gauze does not receive enough heat (high temperature) to ignite it, therefore it will not burn or the flame will not pass through. This is shown below;



If the wire gauze is an insulator, the heat from the flame underneath will not be conducted away (i.e. an insulator), hence it heats up that place or part to higher temperatures igniting the gas above the wire gauze which burns i.e. the flame passes through the wire gauze. This is shown below;



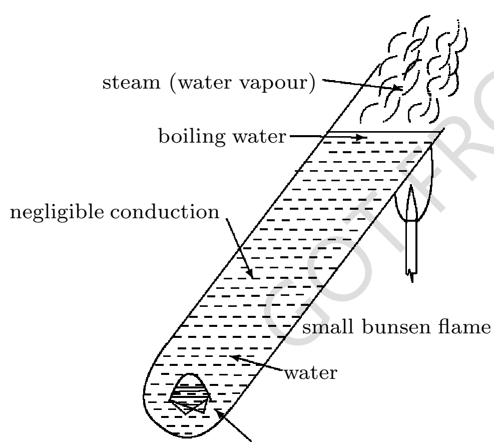
Definition 19. Ignition point of a flammable gas *is the temperature at which or above which the gas burns.*

For instance the ignition point of petrol is very low. For Davy's safety lamp[5] we need a gas whose ignition point is higher.

(b) Conduction in liquids

Experiment to show that water is a bad conductor of heat

- Fill a test tube with ice water or cold water (water that will not easily melt the ice)
- Wrap a piece of ice in a wire gauze to make it sink at the bottom of the test tube with ice cold water (recall, that pure ice floats on water so that gauze makes it sink)
- By holding the top of the test tube in a Bunsen flame the water at the top may be boiled vigorously for some time while the ice at the bottom remains in the solid state.



Ice wraped in a gauze remains unmelted

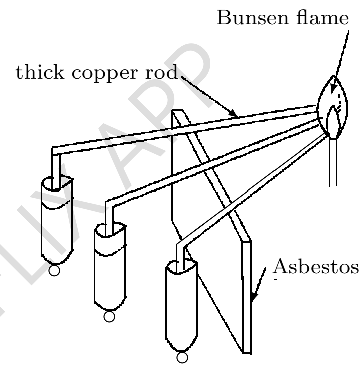
The experiment shows that heat is not easily conducted from the top of the test tube to the bottom through water; hence water is a bad conductor.

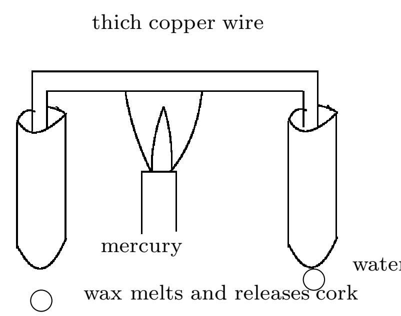
Note that heat can only travel from the top of water to the bottom by conduction and not by convection.

[5]check ordinary level physics by A.F. Abbot page 186

Experiment to compare the heat conduction powers of two or more liquids:

- Coat the bottom of the given thin test tubes with wax (like candle wax).
- Fill the test tube with equal volumes of the different liquids whose conducting powers we are to compare.
- Dip identical copper rods from the Bunsen flame into the liquids in the test tubes as shown below.





The liquid whose wax falls off fast is the best conductor of heat because heat from the Bunsen flame travelled through the rod and liquid to the wax. The liquid may be alcohol, oil, mercury, and water. e.t.c. If mercury and water are compared, the wax on mercury drops or melts first, it shows that mercury is a better conductor of heat

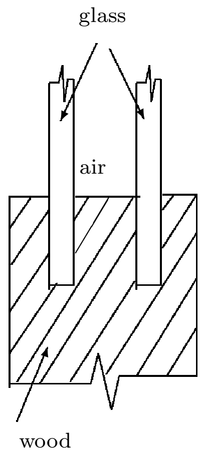
(c) Conduction in gases

Gases (at rest) are not good conductors of heat, they are very good insulators.

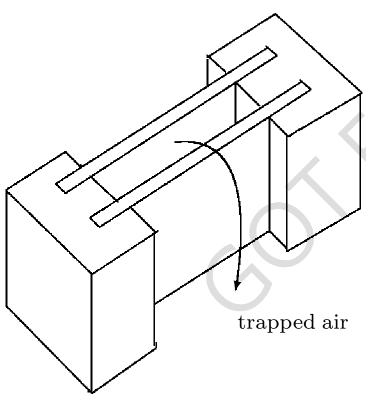
Air is used as an insulating substance to reduce heat loses in building during cold climates and prevent over-heating during hot climates by trapping air in the space between the double walls.

(i) **Double-glazing:** in temperate regions, houses are made with double glassed (double glazed) windows, walls and doors.

These trap air in between them to prevent heat loss or heat gain because air is a good insulator.



a. Double glazing



(ii) **Fibre glass in roofing:** fibreglass is another material that can trap air in it; this makes it a suitable insulator in the roof of a house. Always it is poured in the sealing; on a sunny day the heat from the roof (iron sheets) is trapped in this material keeping the room cool.

(iii) **Feathers, wool and fur keep animals warm:** nature keeps animals warm by using their materials like feathers, wool and fur. It is not the materials that keep them warm but the air trapped by them prevents heat loss hence keeping the animal warm. For instance on a cold day the feathers of birds or fur of animals tend to stand or become firm with more spaces, this is to create room for air , the insulator so as to reduce greatly the heat loss.

3.3.3 Convection

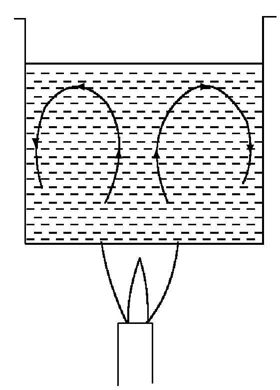
We defined convection, but does it exist i.e. Does the material of the substance actually move? Convection occurs in fluids (materials that can flow i.e liquids and gases).

Experiment to demonstrate convection currents in liquids

1. Drop one crystal of coloured substance (like aluminium paint or potassium permanganate, e.t.c.) into a test tube of cold water.

2. Heat the bottom of the test tube with a small flame.

3. Note how the colours move. The movement of the colours is the movement of the hot liquid, i.e the movement of heat.



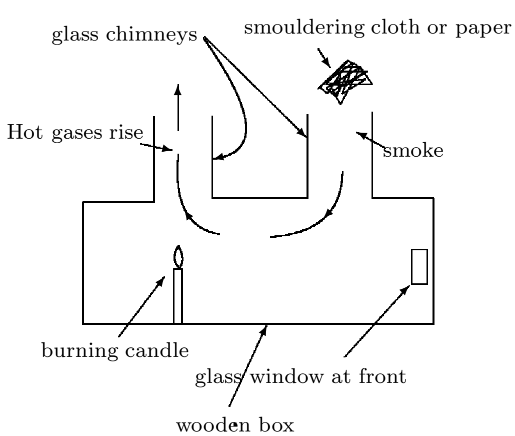
Why is it that when the liquid is heated at the bottom it moves and when heated at the top it does not move?

This is because when a liquid is heated at the bottom, it expands and becomes less dense hence it rises up (from Archimedes principle ;upthrust is greater than its weight). But when it is heated at the top, the water expands and becomes less dense and a less dense substance always floats on the top hence the liquid will not move but remains floating leaving the denser liquid below i.e the liquid does not move:

convection is an application of Archimedes principle

Experiment to demonstrate convection in gases.

1. Place two chimneys on top of a box with a glass window and two holes on top of it as shown below.
2. Place a short burning candle under one of the chimneys and close the window.
3. Hold a smouldering cloth or paper over the other chimney.
4. The smoke from the paper will be observed to move as shown below; the smoke shows the movement of hot air i.e. the movement of heat.



Note that these two experiments explain why;

1. We have a small opening below the charcoal level in a charcoal stove ("sigiri") or why a chimney is placed on top of it when igniting it.
2. Coal miners used a candle to ventilate or aerate their mines or take oxygen below the mines for breathing.
3. A fume cupboard in chemistry laboratories has chimneys (they take poisonous gasses from the chemicals in the laboratory up to the atmosphere).

Why is it that convection occurs in liquids and gases and not in solids?

Convection occurs in materials whose molecules can move or slide over each other, like a large number of balls in corner i.e. they slide over each other till each ball touches the floor. In liquids and gases; molecules can move hence convection is possible. In solids; molecules cannot move, they just vibrate in their fixed position hence convection is impossible in solids.

Forced and free natural convection

A gas or liquid may carry away heat from a hot body by convection. If the flow of the liquid or gas is simply due to its being heated by a certain hot body and hence rising, the convection is said to be free or natural. But if the gas or liquid is flowing in a stream maintained by some other means (e.g. by a fan, a motor, wind, e.t.c.) then the convection is said to be forced. Cooling ones porridge in the obvious way is an example of forced convection, it can cause a more rapid loss of heat than does natural convection.

Convection currents in our daily life.

1. **Chimneys.** (Why do we have chimneys?)

Smoke and gases from fires in houses and factories rise up in chimneys and this flow is a convection current. The drought (need for air up the chimney) through the fire is due to the pressure difference between the columns or cold air outside and the hot air in the chimney. High chimneys have a greater pressure difference hence gases pass up a tall chimney faster than a short chimney.

2. **Winds.** Why do we have wind on land? Where do they come from?

   Winds are convectional currents. When some parts of the earth are hotter than other parts, warm air rises over the hot surface and its replaced by the cooler more dense air from the cooler parts of the earth. The movement of the cooler dense air to the evacuated hot area is felt as wind.

   Trade winds are as a result of hot air near the equator rising and the cooler air from the north and the south replacing it.

3. **Land and sea breeze.**

   During the day, the sun shines and heats equally the land and the sea or water bodies. The surface of the land warms at a faster rate because;

   (a)It has a lower heat capacity (ability to store heat)

   (b)It is a good conductor of heat.

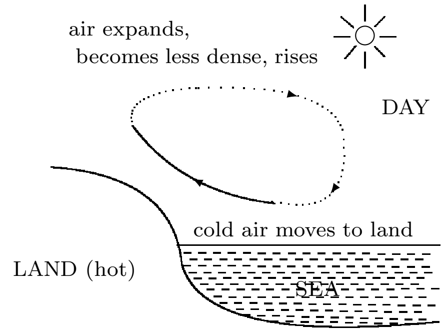
   The sea warms at a slower rate because;

   (a)It has a high heat capacity (i.e. can store more heat for a small change in temperature)

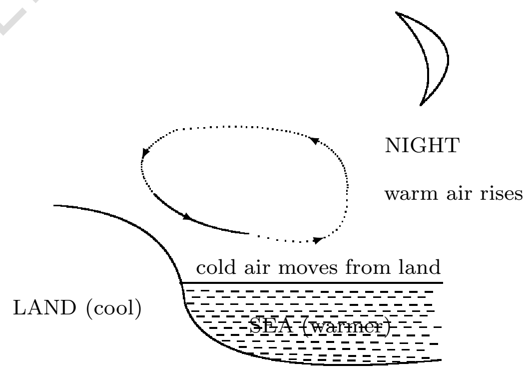
   (b)Water is transparent therefore sun rays penetrate deeply warming also the deeper water not only that on the surface.

   (c)Water waves mix the warm surfaces water with the cooler water below it.

   The air above the land is heated by the land, it expands, becomes less dense and it rises up leaving an air drought, which is occupied by the cooler dense air from over the sea. This cool air moving from the sea to land during the day is known as **sea breeze**. The diagram below illustrates this.



   During the night, the land cools very fast (because of its low heat capacity) and the sea cools at a slower rate. The air above the sea during the night is warm (warmed by the sea) and less dense hence it rises up leaving an air drought, which is replaced by cool denser air from land. This cool air moving from the land to the sea during the night is known as **land breeze.** The diagram below illustrates this.



   The monsoon winds that blow over the Indian ocean and south East Asia are land and sea breezes on a large scale.

4. **Ventilation in houses.**

   Why are ventilators of a house always near the roof?

   Houses and cinemas have opening in and near their ceiling, when one breathes out air, it is damp, warm and less dense than ordinary air, so it rises and can escape through opening near the roof. If the openings were not there, the air would condenser to water drops on the roof or become dense and flow back to the flour, such air can make some one suffocate as it has little or no oxygen. In some rooms, the warm air near the ceiling is blown out and cold air in at the window-level or door-level.

5. **Ocean currents.**

   These are convection current in the sea (the movement of water in the oceans and seas) like the banguella currents.

6. A fume cardboard in chemistry laboratories has a chimney, because hot gases from a burner rise creating a drought, which sucks chemical fumes out of the laboratory.

7. How a radiator (heater) warms air around it (or in a room) and heating water using solar concentrators.

8. Motorcar cooling system. (Car radiator or convector will be discussed after radiation because it involves heat loss by radiation)

9. Hot water supply system; will also be discussed after radiation.

3.3.4 Radiation

This is the process by which heat is transferred from regions of high temperature to regions of low temperature by electromagnetic waves emitted by vibrating particles of hot substance. These waves are also called radiations. They posses the following properties;

(a) They travel at the speed of light. (i.e. $3.0 \times 10^8 ms^{-1}$)

(b) They are invisible; apart from only light.

(c) They can travel through a vacuum (space without matter)

(d) Heat waves are absorbed by glass (i.e. infrared radiations)

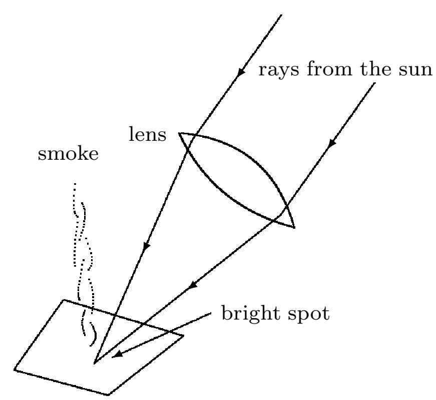
Radiant heat is a mixture of radiation but with the highest percentage of infrared radiations, other radiations include x - rays, gamma rays, micro waves, TV waves, radio waves, ultra violet radiations, e.t.c.

Experiment to show that radiations from the sun can produce a lot of heat that can burn a small piece of paper.

1. Get a glass lens (one with a large surface area is better)

2. On a sunny day, position the lens in sunshine above a piece of paper at a height such that a sharp bright spot is formed on the paper.

3. Leave it in that position for about 10 to 15 minutes, the bright spot will be observed to produce smoke (why?) the radiations from the sun have produced that heat which made the paper burn or produce smoke.



Infrared radiations that are making this paper burn can be detected by the thermopile discussed on page **??**

When one sits near a charcoal stove, he or she feels warm, why?

This is because the vibrating hot particles in the charcoal stove emit the heat radiations (electromagnetic waves). When these waves reach your body, they convert their energy back to heat energy hence your body is heated and you feel warm.

Absorbers and emitters

Recall that temperature i.e. a measure of the heat content in a substance, is also a measure of the average kinetic energy of the molecules of a substance (internal energy). When a radiation meets an object, part of it is absorbed, part reflected and the rest transmitted through it. The energy of the absorbed radiation is converted to kinetic energy of the molecules of that substance i.e. its molecules move faster hence the body's temperature increases.

Not all objects absorb these radiations at the same rate, therefore we have **good and bad absorbers**; also not all substances when heated do emit radiations at the same rate, so we have **bad and good emitters (or radiators)**

How can we differentiate between bad and good absorbers or emitters?

Let us look at these surfaces;

(i) polished and un polished surfaces

(ii) White and black surfaces

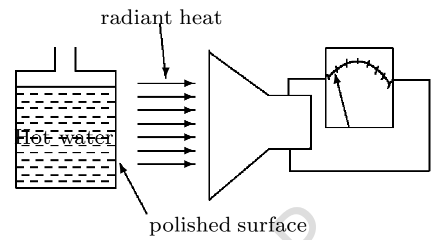
It was observed that the rate at which a body radiates energy depends on

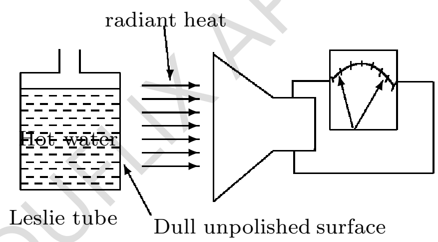
- Its temperature.
- Nature of its surface.
- Surface area of the body.

An experiment to determine which surface is the best radiator, a polished or an unpolished surface?

Two hollow copper cubes were filled with equal amounts of hot water at the same temperature. One cube has a side highly polished and the other cube has a side coated with lamp black (a dull rough surface or an un polished surface).

Using two thermopiles on each surface of the cubes. The deflection indicated by the thermopile tells which surface is emitting more heat than the other.





The thermopile on the dull surface shows a greater deflection, this shows that the dull un polished surface is a better emitter or radiator compared to a polished surface. Hence

An un polished surface is a good emitter and a polished surface is a bad emitter

Note that if the experiment is repeated with a white and black surface, a black surface gives a greater deflection i.e.

a black surface is a good emitter.

If the experiment is repeated with a polished black surface and an un polished white surface, the un polished white surface gives a greater deflection. Sometimes it may not be the case but in general it was discovered that **the texture of the surface appears to be a more important factor than its colour when reflecting**

or emitting heat. Also one could use a single Leslie cube with two surfaces polished and two un polished with two thermometers (thermopiles). They are called Leslie tube because this experiment was first performed by Sir John Leslie.

An experiment to show which surface is the best absorber (polished or un polished surface)

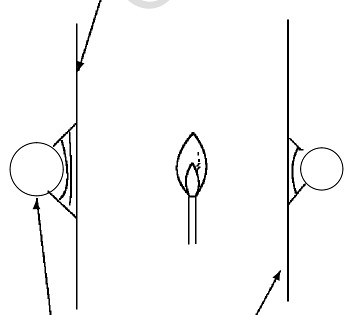
.

or

Experiment to compare the absorbing powers of dull and polished surface.

Knowing that radiations falling on a surface are partly absorbed, partly reflected and transmitted; we continue with the experiment.

1. Get two sheets of tin plate; one polished and the other un polished (painted dull black).
2. Using melted paraffin wax, fix cork on the reverse side of each plate.
3. The plates are then set up vertically, a short distance apart, with a Bunsen burner midway between them as shown below.



4. When the burner is ignited, both surfaces receive equal quantities of heat radiations.
5. In a very short time, the wax on the dull surface melts and the cork slides off while the polished plate remains cool with the paraffin wax on it un melted for some longer time.
6. This shows that the dull back surface is a much better absorber of radiations than the polished surface.

Therefore

a polished surface is a good reflector of radiations and an un polished surface is a bad reflector of radiations

We may also use thermometers with dull and bright coloured bulbs in sunshine. The dull coloured thermometer gives a greater reading implying that the dull surface is a good absorber. .

In conclusion we have

(a) A good radiator is also a good absorber of radiations

(b) A polished (shiny) surface is a good reflector and an un polished surface is a bad reflector.

(c) An un polished surface is a good emitter and a polished surface is a bad emitter.

(d) A polished surface is a bad absorber and an un polished surface is a good absorber.

(e) The texture of a surface appears to be a more important factor than its colour, that is why black shoes are polished.

Does glass absorb radiations?

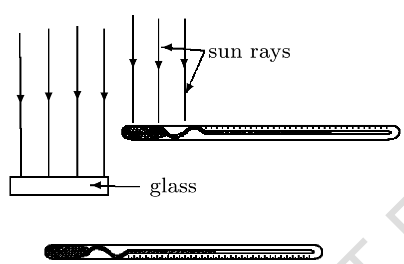
An experiment to show that glass absorbs infrared radiations (heat radiations).

1. Blacken the bulbs of the two thermometers to enable them absorb more infrared radiations.

2. Hold them in direct sunlight

3. Place a sheet of glass over one of the bulbs

4. Note whether the rise in the temperature of the two bulbs is the same. The thermometer with the glass above its bulb shows a lower rise in temperature compared to the one with out the glass sheet. This shows that glass absorbs radiations (heat radiations) otherwise the two thermometers would show that same rise in temperature.



The heat transfered from one body to another by radiation or the heat emitted by a body depends on;

1. The temperature of the hot body or difference in temperature

2. Nature or texture of its surface i.e polished or un polished

3. Colour of its surface

4. Nature of the material of the body i.e is it a conductor or an insulator

3.3.5 Green house effect

Global warming

(a) **Green house**

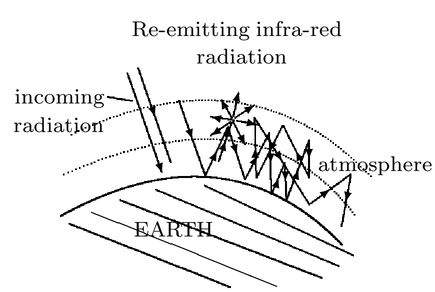
This is a house made of glass used to grow plants (hence called green house). Usually it is hot inside (why?)

Shorter wavelength infrared rays and light rays from the sun can pass through the glass or plastic panes of a green house and are absorbed by the soil and plants inside, raising their temperature. The soil and plants in turn emit longer wavelength infrared rays, which cannot penetrate glass. This keeps the inside of the green house warm or hot.

Note that very hot object like the sun emit short wavelength infrared rays which are more penetrating and less hot objects like the soil emit longer wavelength infrared rays which have a less penetrating power.

(b) **Warming of the atmosphere.**

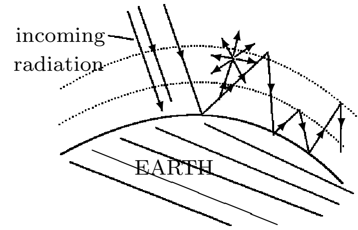
The carbon dioxide and other gases in the atmosphere behave like the green house (how?). The atmosphere lets infrared and light rays from the sun pass through it to the earth surface where they are absorbed and converted to internal energy of the plants and the land mass. The warmer land now radiates longer wavelength infrared radiations. These gases are warmed up as a result and they in turn radiate infrared rays in all directions; some of which is radiated back to the earth keeping it warm.



Without the carbon dioxide, the water vapour and other gases, this energy in the heat radiations would have been lost and the earth would be cooler. The warming of the earth due to the presence of gases and increase of specifically carbon dioxide in the atmosphere is known as **global warming** or **green house effect**.

(c)**Predicted danger**

As a result of the burning of fossils, fuels, coal, petroleum products and others, the amount of carbon dioxide in the atmosphere is increasing. These increasing amounts of carbon dioxide are thought by many scientists to be responsible for the rising temperature of the atmosphere and the sea. Such changes are likely to alter the earth's climate and raise the sea level because the high temperatures could lead to melting of all the ice or snow in the temperate regions and snow capped mountains which will flow to the sea as running water in the rivers. Dont you think that at one time land will not be enough for us as the amount of water in the oceans and seas increase or there will be a lot of carbon dioxide in the atmosphere, much more than the oxygen we breath or the temperature will be too high for us to live or keep life on earth. since we are still producing more carbon dioxide in the atmosphere by the automobiles, factories and industries , in wars and in other activities, at one time the climate on earth will not be able to favour life and that will be the end of the world. But when will this time come? this is food for thought, ask the environmentalists for more details.



Properties or radiant heat

These are some times known as **laws of radiant heat**;

1. Heat radiation travels in a straight line
2. Heat radiation warms a body only if the radiation fall normally on the body, this is because it can be reflected.
3. Bodies radiate radiant heat only if their temperature is higher than that of their surrounding.
4. Good absorbers of heat radiation are good emitters of heat radiation.
5. Heat radiation does not require a material medium for transmission hence molecular (or kinetic) theory can not explain its transmission except the wave theory.
6. It travels at the speed of light.

Questions and answers on absorbers and emitters

1. **Why is it that; some rods can become red hot and others cannot?**

   A rod that can become red hot is one that is not a good emitter (it is a good absorber or a bad emitter), therefore it can retain a lot of heat which then raises it to high temperatures making it red hot. A rod that can not become red hot is a good emitter because when it becomes hot, it emits away that heat leaving itself cold hence it cannot attain higher temperatures. Therefore a rod can become red hot only when it is a bad emitter.

2. **Why are black shoes always polished?**

   Since they are black it means that they are good absorbers of heat i.e. the shoes easily become hot or warm. They are polished to make them good reflectors of radiant heat especially from the sun hence keeping shoe cool. Since the texture of the surface over shadows its colour in reflecting radiations, a black polished shoe reflects radiant heat away keeping the shoe cool. Black shoes are polished to keep them cool.

3. **Why is it that when you go to a fireplace you feel it burning you and later comfortable?**

   This is because when your body is near the fire, the fire sends heat radiations to your body and you feel like it is burning you. After some time your body is hot and it also begins to emit heat radiations, at this time you feel comfortable.

4. **Why is it that in desert areas white clothes, houses and other objects are preferred or are common?**

   Since a white object is a good reflector of radiant heat; white clothes and houses are slightly cooler than other clothes (dark coloured clothes) on a hot day.

5. **Why is it that with a dark and a silvery teapot, a silvery one keeps tea hot for a longer time?**

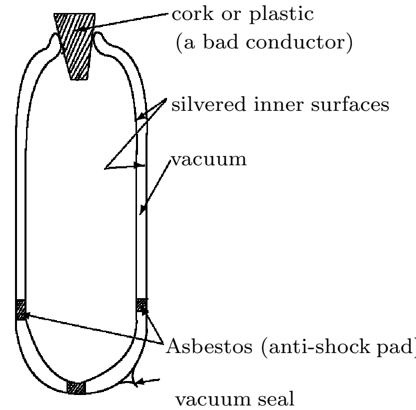
   Because a silvered teapot has a polished surface, it is a good reflector of heat radiation hence the heat from tea inside it, is reflected at the tea pot's back to the tea inside keeping it hot. A dark teapot is a good absorber and radiator, it absorbs and radiates radiant heat and hence it cools down faster.

3.3.6 Applications of conduction, convection and radiation

Conduction, convection and radiation processes are applied in a number of devices, these include;

1. **Vacuum flask** (Thermos or Dewar flask-Named after its inventor 1894)

   This is a double walled glass vessel with vacuum between the double walls. Both walls are silvered on the vacuum side. It is used to keep hot liquids hot and cold substances cold. These may be tea, coffee, soup, food, etc. Below is its diagram.



How does it minimise heat transfer.

Heat loss by Conduction is minimised by the

(a) The Vacuum space has no material through which heat can be conducted, therefore there is no heat transfer or loss by conduction.

(b) Cork is a bad conductor, it minimises heat loss or conducts little or negligible heat to the surrounding (outside the flask).

(c) Asbestos as an insulator, it just minimises heat loss or transfer from the inner wall to the outside wall of the vacuum.

Heat loss by Convection is minimised because;

(a) The liquid or gas in the flask is set up at a constant temperature i.e. it cannot move i.e. no convection currents are set up, hence no heat transfer by convection in the liquid.

(b) The cork being airtight, no air can leak out of the flask to carry heat energy with it outside the flask leading to heat loss.

(c) The vacuum contains no air, which could transfer heat from inner surface to outer surface of the double walls by moving air molecules (convectional current).

Heat loss by Radiation is minimised by

(a) Silvering the outer surface of the inner wall. This makes it a bad emitter so it emits negligible heat leading to little or no heat loss.

(b) Silvering the inner surface of the outer wall. This makes it a bad absorber i.e. good reflector in case the inner wall emits radiant heat, it is reflected back to the inner wall by the inner surface of the outer wall.

Heat loss by Evaporation is minimised by letting the stopper touch the liquid in the flask. This prevents the energetic molecules from evaporating and cooling the liquid. (Cooling by evaporation)- whenever a liquid evaporates it losses heat or it cools.

How heat is lost in a vacuum flask (thermos flask).

Heat lost from the flask, is by;

(a) Little conduction in the cork.

(b) Small radiations loss through the vacuum.

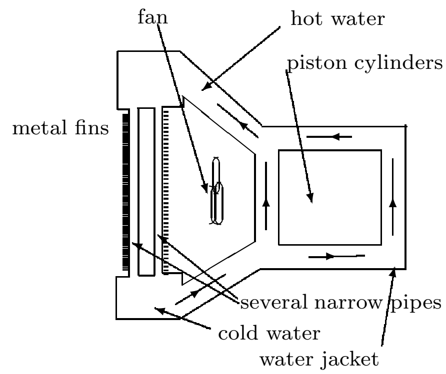
(c) Conduction through the asbestos that hold the inner vacuum wall near the outer wall.

(d) Evaporation on the surface of the liquid in the flask when it is not full.

The vacuum flask is a device that was used to reduce transfer of heat to a minimum. In a good flask, the main cause of heat loss is conduction through the cork. If the vacuum is good, convection is almost inhibited. The goodness of a vacuum determines the goodness of the flask.

2. **Motor car cooling system (car radiator)**

Car radiator cools the petrol or diesel engine by circulating water the piston cylinders where the petrol-air mixture is exploded. Usually there are about 4 piston cylinders surrounded by water. These cylinders become hot during action (i.e. when the petrol air mixture is ignited) warming the water, which expands and rises to the top, Cold water from the bottom, replaces this hot water to prevent the cylinder from becoming too hot.



The hot water from the car engine passes through the radiator, which has several narrow pipes with metal fins. These metal fins radiate away the heat to the surrounding by;

(a) Radiation at a faster rate because of having a larger surface area provided by the fins and because of being painted black, since a black body is good radiator of heat.

(b) Convection in air because, the fan in the engine blows air through the radiator fins where the heat is radiated to the surrounding air. This radiated heat was originally in the hot water heated by the car engine.

(c) Evaporation (little).

When the water reaches the bottom of the piston cylinder, it is already cooled, the cycle continues.

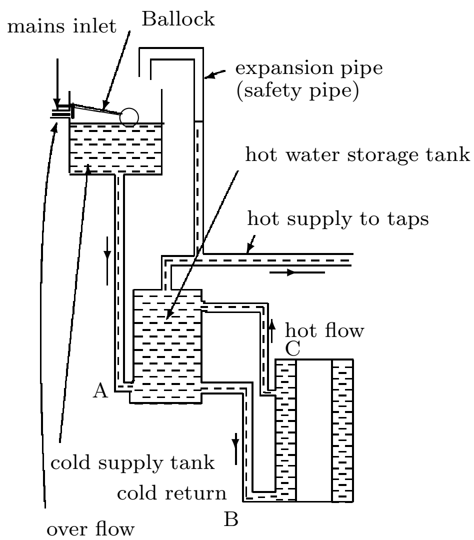
Why is a car radiator made of metal fins painted black?

(a) It is made of metal fins to increase their surface area so as to loose more heat (i.e. to make it a better radiator) and made of a metal because metals are good conductors of heat.

(b) It is painted black to make it a good radiator i.e. to let it radiate more heat a way.

3. **Domestic hot water supply system.**

This consists of a boiler, a hot water storage tank and a cold supply tank interconnected by pipes as shown below.



How it works

Hot water from the boiler rises up the flow pipe C, as cold water descends to the boiler through the return pipe B, where it is heated in turn. This cycle continues till all the water in the hot water storage tank is hot. (How?) This is by taking the cold denser water from the storage tank to the bottom of the boiler and replacing it on top with hot water from the boiler(note that cold water sinks in hot water). When the hot water taps are opened water flows from the storage tank as more replaces it equally from the cold supply tank. The system is kept constantly full of water and no air can enter. Water level in cold tank is maintained by supply from the main that enters through ball cock.

An expansion pipe bent to the cold tank can;

(a) Discharge hot water to the cold tank in case the boiler burns the water so fiercely

(b) Permits the escape of discharged air, which comes out of water when heated.

Note how the pipes are arranged, the arrangement can be changed but it must ensure a convectional flow of heat by water from the boiler to the hot water storage tank.

Questions

1. Which of the following should be dull black and which should be bright and shinning. A teapot, an electric iron, the roof of a house in a hot country, a refrigerator, a car radiator? Explain your answers.

2. An open vessel containing a hot liquid is placed on a table. State the different ways by which the liquid cools.

3. Draw a well labelled diagrams of the vacuum flask and explain why a liquid placed in such a flask remains hot for a longer period?

4. How would you demonstrate that a shiny white surface is a poorer absorber of heat than a dull black surface. Give one use of each surface depending upon its properties, in every day life.

5. The cylinder of an air-cooled petrol engine has iron fins projecting from the surface. When clean, these are painted black. Explain how these fins are useful in cooling the engine when it is working.

3.4 Heat measurement

3.4.1 Introduction

Kinetic theory of matter

It states that matter is composed of tiny particles, which are either atoms, or molecules. These particles are ever in a state of motion due to this motion[6] they posses kinetic energy. At the same time these particles are ever in a state of attraction and repulsion; due to this attraction and repulsion they posses potential energy. The mixture or sum of the potential energy and kinetic energy of the molecules or atoms of a substance is called internal energy or micro mechanical energy (not macro mechanical). Usually this internal energy is what we called heat energy.

Some times back in the beginning when heat was just being discovered, heat was thought to be a liquid, which flows from a hot substance to a cooler substance, and a number of questions could not be answered, some of which are;

- What happens when nails get heated till they are red-hot? which explanation can one give?
- Can water be boiled with out heating it (yes! how?)
- When a gas is compressed it gets heated (it temperature raises) why?.
- Does the water on a waterfall get heated?
- when a gas is compressed what happens?

All these were leading to the conclusion that, heat is a form of energy. One could say, heat is a measure of energy transfer;

[6]Recall Brownian motion

Does every substance posses heat energy (internal energy)?

Particles of every substance are ever in a state of motion (i.e. they are vibrating in solids, they can slide over each other in liquids and are free to move independently in gasses) hence every substance has a certain amount of heat energy in it, even ice at 0°C or -100°C has heat energy in it.

N.B We cannot measure heat content or heat in a substance at any cost but we can only measure heat transferred from; one form to another, for instance from light, electric, or mechanical form of energy to heat or the reverse or from an object or substance at a higher temperature to another one at a lower temperature.

Factors affecting heat transfer

The a mount of heat transferred from one body to another when they are in contact depends on;

1. **Temperature difference**

   If temperature difference is larger, a lot of heat is transferred, i.e. heat transferred is directly proportional to the temperature difference.

   $$\text{Heat} \quad \alpha \quad \text{temperature difference} \tag{3.16}$$
   $$H \quad \alpha \quad \Delta T \tag{3.17}$$

2. **Mass**

   A body with a greater mass contains a lot of heat or can absorb or lose a greater amount of heat. That is heat transferred is directly proportion to the mass of the substance.

   $$\text{Heat} \quad \alpha \quad \text{mass} \tag{3.18}$$
   $$H \quad \alpha \quad m \tag{3.19}$$

3. **Nature of the substance.**

   A Substance like the metal copper can transfer heat more easily than substance like rubber, because copper is a good conductor and rubber is a bad conductor (insulator) of heat. Hence the nature of a substance also affects the amount of heat transferred from one body to another.

$$\text{Heat} \quad \alpha \quad \text{a constant} \tag{3.20}$$
$$H \quad \alpha \quad c \tag{3.21}$$

This constant is known as the specific heat capacity of the solid.

N.B. A numerical change of temperature in Kelvin is equal to the same numerical change of temperature in degrees centigrade. For instance $2°C$ change in temperature is equal to $2k$ change in temperature i.e.$\Delta 2°C = \Delta 2k$ but $2°C$ is not equal to $2k$, i.e. $2°C \neq 2k$. Since

$$H \quad \alpha \quad m, \tag{3.22}$$
$$H \quad \alpha \quad \Delta T, \tag{3.23}$$
$$\text{then } H \quad \alpha \quad m\Delta T, \tag{3.24}$$
$$\text{hence } H \quad = \quad mc\Delta T \tag{3.25}$$

where c is a constant of proportionality which depends on the nature of the material known as the specific heat capacity of the material.

3.4.2 Heat measurement without change of state.

Heat capacity.

Since the amount of heat transfered from one point to another is proportional to temperature, $H \alpha \Delta T$, then we can have a constant of proportionality, this constant is known as heat capacity. $H = C\Delta T$,Therefore

Definition 20.

Heat capacity is a mount of heat energy transferred (gained or lost) when a given fixed mass of a substance is heated through a temperature change of 1k or 1°C.

Or

Heat capacity is the heat energy required to raise the temperature of a substance by 1 Kelvin (or 1°C).

For instance if H Joules of heat are supplied to a fixed mass of a substance and its temperature changes from T_1 to T_2 its heat capacity, C, is given by;

$$C = \begin{array}{c}\text{energy supplied per}\\ \text{unit temperature change}\end{array} \tag{3.26}$$
$$= \begin{array}{c}\text{energy per unit}\\ \text{temperature change}\end{array} \tag{3.27}$$
$$= \frac{\text{Heat supplied}}{\text{temperature change}} \tag{3.28}$$

hence,

$$C = \frac{H}{T_1 - T_2} \tag{3.29}$$
$$= \frac{H}{\Delta T} \tag{3.30}$$
$$C = \frac{H}{\Delta T} \tag{3.31}$$
$$\Rightarrow H = C\Delta T \tag{3.32}$$
$$\Longleftrightarrow H = C\left(T_2 - T_1\right) \tag{3.33}$$

S.I unit of heat capacity is Jk^{-1} or $J°C^{-1}$

Note: ΔT should never be negative. If temperature change is from 20°C to 40°C, then $\Delta T =$ 40-20 = 20°C, or If temperature change is from 30°C to 15°C, then $\Delta T = 30 - 15 = 15$°C.

Example

A body is supplied with 400joules of heat energy and its temperature changes from 30°C to 50°C. Calculate its heat capacity?

Solution

$$
\begin{aligned}
\text{Since } H &= C\Delta T, \text{ then} \\
C &= \frac{H}{\Delta T} \\
C &= \frac{H}{T_2 - T_1} \\
&= \frac{400J}{50-30} \\
&= \frac{400}{20}Jk^{-1} \\
C &= 20Jk^{-1}
\end{aligned}
$$

Hence the heat capacity of the substance is $20Jk^{-1}$ or 20J/$°C$

Specific heat capacity(S.H.C = c)

This is just a special kind of heat capacity.

Definition 21.

Specific heat capacity is the amount of heat required to raise the temperature of one-kilogram mass of a substance by 1 Kelvin (Or by 1°C).

For instance if H joules of heat energy are supplied to m kg of a substance and its temperature changes from T_1 to T_2, then its specific heat capacity is given by; From the definition of specific heat capacity,

$$c = \frac{H}{m\Delta T} \tag{3.34}$$

i.e heat transfered per unit mass per unit temperature change hence

$$H = mc(T_2 - T_1) \tag{3.35}$$
$$H = mc\Delta T \tag{3.36}$$

This constant (specific heat capacity) depends on the nature of the material. For instance that for water is different from that of paraffin, copper, zinc and others.

S.I unit of specific heat capacity is $Jkg^{-1}k^{-1}$ or $Jkg^{-1}°C^{-1}$, that is; Joules per kilogram Kelvin.

Table 3.1 gives the most common material and their corresponding specific heat capacities; S.h.C

Material	Specific heat capacity in $Jkg^{-1}k^{-1}$
Water	4,200
Ethanol	2,500
Paraffin oil	2,130
Turpentine	1,760
Hydrogen	14,300
Air	993
Helium	5,240
Oxygen	913
Granite	820
Beryllium	1,970
Copper	385-400
Lead	126-130
Aluminium	900-913
Sodium	1,240
Iron	106
Steel	420
Concrete	3,350
Polypropylene	2,100
Marble	900
Methylated spirit	2400
Mercury	140
Sea water	3900
Zinc and brass	380
glass(ordinary)	670
Ice	2100

Table 3.1: Specific heat capacity

is a measure of the insulating nature of a material i.e insulators have a very high S.H.C while conductors have a very low S.H.C. according to table 3.1

Example

A body of mass 4kg is supplied with 4000 joules of heat energy and its temperature changes from 3.4°C to 5.9°C. Calculate its specific heat capacity?

Solution

$$\begin{aligned}\text{From } H &= mc\Delta T\\ \Longleftrightarrow c &= \frac{H}{m\Delta T}\end{aligned}$$

From the above question

$$\begin{aligned}H &= 4000J,\\ m &= 4Kg,\end{aligned}$$

$$\begin{aligned}\Delta T &= T_2 - T_1\\ &= 5.9 - 3.4\\ &= 2.5k \quad or \quad 2.5^\circ C. \text{ Hence}\\ c &= \frac{H}{m\Delta T}\\ c &= \frac{4000J}{4kg \times 2.5k}\\ &= 400Jkg^{-1}k^{-1}.\end{aligned}$$

Therefore the specific heat capacity of the material of the body is $400Jkg^{-1}k^{-1}$.

Relation between heat capacity and specific heat capacity for a given body.

From the formulas of heat loss or heat gain in terms of specific heat capacity and heat capacity,

$$H = mc\Delta T \text{ but also} \quad (3.37)$$
$$H = C\Delta T, \quad (3.38)$$

where the symbols have the usual meaning. we observe that for these to be equal then $C = mc$, that is

$$\text{Heat capacity} = \text{mass} \times \text{s.h.c.} \quad (3.39)$$

This is how the two quantities are related. That is why sometimes some one may define specific heat capacity numerically as the heat capacity of a kilogram mass of a substance.

Example

20 grams of a certain ore is supplied with 500J of heat from a heater and its temperature changes from 20°C to 60°C. Calculate

(i) Its heat capacity

(ii) Its specific heat capacity

Solution

(i)Heat capacity,

$$\begin{aligned}\text{From } H &= C\Delta T\\ C &= \frac{H}{\Delta T}\\ C &= \frac{500}{(60-20)}\\ &= \frac{500}{40}Jk^{-1}\\ &= 12.5Jk^{-1}\end{aligned}$$

Hence heat capacity of the ore is $2.5Jk^{-1}$

(ii)Specific heat capacity

$$\begin{aligned}\text{From} H &= mc\Delta T\\ c &= \frac{H}{m\Delta T}\\ c &= \frac{500}{\frac{20}{1000} \times (60-20)}\\ &= 625Jkg^{-1}k^{-1}\end{aligned}$$

. Hence specific heat capacity of the ore is $625Jkg^{-1}k^{-1}$.

Note that;

$$\begin{aligned}\text{heat capacity} &= \text{mass} \times \text{s.h.c.}\\ &= \frac{20}{1000} \times 625\\ &= 12.5Jk^{-1}\end{aligned}$$

Which is the same as that obtained in the question above.

Revision questions

1. 920,000J of energy are supplied to 4kg of iron at a temperature of 21°C. If its final temperature was 50°C. Calculate the specific heat capacity of this iron?

2. Calculate the amount of heat required to raise the temperature of 10kg of brass from 10°C to 90°C? (Specific heat capacity of brass is $377Jkg^{-1}k^{-1}$).

3. A heater rated 500W is dipped in water at 20°C and switched on. After 8 seconds its temperature had risen to 56°C, determine the mass of water? (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$)

4. Calculate the amount of heat required to rise the temperature of 0.2kg of lead by 60°C. Take the Specific heat capacity of lead to be $130Jkg^{-1}{}^{\circ}C^{-1}$.

5. If 74000J of heat is given out when a brass ball of mass 1kg cools from 300°C to 100°C, calculate the specific heat capacity of brass?

6. 2200J of heat raises the temperature of paraffin in a plastic beaker from 25°C to 35°C, calculate the mass of the paraffin? (Specific heat capacity of the paraffin $=2200Jkg^{-1}k^{-1}$).

7. How many joules of heat are given out when a piece of iron of mass 60g and specific heat capacity $460Jkg^{-1}k^{-1}$ cools from 80°C to 20°C?

8. Calculate the specific heat capacity of paraffin if 22000Joules of heat are required to raise the temperature of 1.5kg of paraffin from 20°C to 30°C?

9. When 4000J of heat energy are supplied to a lump of metal. Its temperature increases from 25°C to 45°C. Determine;
   (i) its heat capacity
   (ii) its specific heat capacity (mass of the lump of metal is 800g)

10. Calculate the amount of heat energy required to raise the temperature of 5kg of water from 20°C to 100°C? (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$).

3.4.3 Finding heat loss, heat gain, initial and final temperature.

When we mix two liquids a hot one and a cold one, the hot one looses heat and the cold one gain that heat. From the law of conservation of energy assuming there is no heat loss (or heat loss is negligible)

Heat lost from the hot body is equal to the heat gained by the cold body

If there is any loss of heat energy when a hot and cold liquid are mixed then,

Heat loss from hot liquid is equal to the sum of the heat gained by cold liquid and the heat loss to calorimeter or to surrounding air.

If a hot solid is dropped into a liquid,

Heat loss from the hot solid is equal to the sum of the heat gain by the cold liquid and the heat lost to the surrounding air or calorimeter

Example 1

In an experiment 920,000J of energy are transferred to 2kg of iron. The initial temperature of the iron is 25°C, what is the final temperature of the iron? (Specific heat capacity of iron is $460Jkg^{-1}k^{-1}$).

Solution

Let the final temperature of the mixture which is greater than the initial temperature be T_f

$$\begin{aligned}
\text{Heat supplied} &= \text{heat gained by iron} \\
H &= mc\Delta T \\
920,000 &= 2 \times 460 \times (T_f - 25) \\
\frac{920,000}{(2 \times 460)} &= T_f - 25 \\
T_f - 25 &= 1000 \\
T_f &= 1000 + 25 \\
T_f &= 1025^{\circ}C
\end{aligned}$$

Hence the final temperature of the mixture is 1025°C

Example 2

Find the final temperature; when 80kg of iron pellets at 100°C are dropped into 200kg of water at 20°C? (Specific heat capacity of water and iron are $4200Jkg^{-1}k^{-1}$and $440Jkg^{-1}k^{-1}$ respectively).

Example 3

40g of lead and 50g of iron are logged together and heated in a Bunsen burner flame to a constant temperature. The two masses are dipped in $250cm^3$ of water at 23°C in a copper calorimeter. After a thorough stirring, the final temperature of the water was 51°. Determine the temperature of the Bunsen burner frame(specific heat capacities of lead, iron, copper and water are 126, 106, 400 and 4200 $Jkg^{-1}k^{-1}$, 1g of water occupies $1dm^3$, mass of the copper calorimeter is 250g and $1cm^3 = 1dm^3$)

Solution to example 2

Let the final temperature of the mixture be T_f, T_f is more than 20°C but less than 80°C.

$$\begin{aligned}
\begin{array}{c}\text{Heat loss from}\\ \text{the iron pellets}\end{array} &= \text{heat gained by water}\\
mc\Delta T &= mc\Delta T\\
m_i c_i (T_i - T_f) &= m_w c_w (T_f - T_w)\\
80 \times 440 \times (80 - T_f) &= 200 \times 4200 \times (T_f - 20)\\
35200(80 - T_f) &= 840000(T_f - 20)\\
2816000 - 35200 T_f &= 840000 T_f - 16800000\\
2186000 + 16800000 &= (840000 + 35200) T_f\\
18986000 &= 875200 T_f\\
\frac{1896000}{875200} &= T_f\\
21.69 &= T_f\\
T_f &= 21.69°C
\end{aligned}$$

Hence the final temperature of the mixture is 21.69°C.

Solution to example 3

$$\begin{aligned}
\text{Heat loss by iron and lead masses} &= \begin{array}{c}\text{Heat gained by}\\ \text{the water and calorimeter}\end{array}\\
mc\Delta T_{iron} + mc\Delta T_{lead} &= mc\Delta T_w + mc\Delta T_{cal}\\
\frac{50}{1000} \times 126 \times (T - 51) + mc\Delta T_{lead} &= mc\Delta T_{water} + \frac{250}{1000} \times 400 \times (51 - 23)\\
6.3 \times (T - 51) + \frac{40}{1000} \times 106 \times (T - 51) &= \frac{250}{1000} \times 4200 \times (51 - 23) + 2800\\
6.3(T - 51) + 4.24(T - 51) &= 29400 + 2800\\
10.54(T - 51) &= 32200\\
10.54\,T - 537.54 &= 32200\\
10.54\,T &= 32737.54\\
T &= 3106.03°C
\end{aligned}$$

hence the temperature of the Bunsen burner flame is 3106.03°C

Revision exercise

1. What is the final temperature of the mixture if 100g of water at 70°C is added to 200g of cold water at 10°C and well stirred? (Neglect heat absorbed by the container, specific heat capacity of water is $4200Jkg^{-1}k^{-1}$).

2. 100g of a metal at 100°C are dropped into 50g of a certain liquid at 20°C and the final temperature of the mixture after a thorough stirring is 40°C. Calculate the specific heat capacity of the metal? (Specific heat capacity of the liquid is $3000Jkg^{-1}k^{-1}$).

3. The temperature of 500g of a certain metal is raised to 100°C and then placed in 200g of water at 15°C. If the final temperature was 20°C, calculate the specific heat capacity of the metal? (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$).

4. 0.5kg of water at 85°C is poured into a plastic bucket containing 0.6kg of water at 20°C. What is the temperature of the mixture?

5. 3.0 kg of cold water at 0°C is added to 1.3kg of hot water and the final temperature of the warm mixture is 30°C. Calculate the original temperature of the hot water?

6. 25 grams of copper at 80°C are dropped into 2kg of water at 60°C. Find the final temperature after thorough stirring? (Specific heat capacities of water and copper are $4200Jkg^{-1}k^{-1}$ and $400Jkg^{-1}k^{-1}$ respectively).

7. The temperature of a piece of copper of mass 250g is raised to 100°C and it is then transferred to a well lagged aluminium can of mass 10.0g containing 120g of methylated spirit at 10°C. Calculate the final steady temperature after the spirit has been well stirred? (Neglect the heat capacity of the stirrer and any losses from evaporation, assume where necessary the specific heat capacities of water, copper and aluminium as 4200J/kgk, 400J/kgk and $900Jkg^{-1}k^{-1}$ respectively) answer 32.7°C

8. 40g of water at a temperature of 20°C is mixed with 20g of water at 90°C and the mixture stirred thoroughly. Determine the final temperature of the mixture?

9. A heater rated 800W is dipped in 200g of water at 30°C and switched on for 2minutes. Determine the final temperature of the water?(specific heat capacity of water is $4200Jkg^{-1}k^{-1}$)

10. In the laboratory, 400g of water at 50°C are poured in a copper calorimeter containing 100g of oil at 23°C and the mixture stirred thoroughly. Determine the final temperature of the mixture (mass of the copper calorimeter is 400g, the specific heat capacities of copper, water and oil are 400, 4200 and $2400Jkg^{-1}k^{-1}$ respectively).

11. 400g of water at 80°C are poured into 500g of water at 26°C. After stirring the mixture thoroughly, determine the final temperature of the mixture?(Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$).

12. 4 kg of paraffin are mixed with 2Kg of water at 20°C. If the final temperature of the mixture is 32°C, determine the initial temperature of the paraffin (the specific heat capacities of paraffin and water are 2130 and $4200Jkg^{-1}k^{-1}$).

13. In an experiment, 200kg of aluminium at 100°C are dropped into 256kg of kerosene at 15°C, the mixture attained a maximum temperature of 50°C. What is the specific heat capacity of kerosene? (Specific heat capacity of aluminium is $880Jkg^{-1}k^{-1}$)

14. A lump of metal was transferred to 1kg of water and the temperature of the water increased from 20°C to 60°C. Find

    (a)The heat energy lost by the metal?

    (b) Heat energy gained by the water.

    (c)Mass of the metal if it was at a temperature of 80°C and its specific heat capacity is $500Jkg^{-1}k^{-1}$.

15. A certain liquid has a specific heat capacity of $3000Jkg^{-1}k^{-1}$. What quantity of that liquid may be heated from 20°C to 50°C by 630J of heat? (Assuming there is no heat loss)

16. A kettle containing 2kg of water is placed on top of an electric heater with a correct power rating of 500W. If it was switched on for 6 minutes, calculate the change in temperature? (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$)

17. An immersion heater rated at 100W supplies heat to 2kg of paraffin oil for 440s. Calculate the rise in temperature of the paraffin in °C? Assume that the Specific Heat Capacity of paraffin oil is $2.2 \times 10^3\ Jkg^{-1}k^{-1}$ and that all the heat from the heater is used to heat the paraffin oil.

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18. In order to reduce the diameter of a wire, it is pulled through a small hole in a metal plate. If a steady force of 500N is required to do this and on emerging from the hole the wire has a mass of 0.5kgm^{-1}, what would be the rise in temperature of the wire? (The wire is made of metal whose specific heat capacity is $400Jkg^{-1}k^{-1}$ and assume that all the heat generated is retained in the wire).(Hint: Consider 1m length of the wire).

19. A kettle containing 1kg of water (specific heat capacity is $4200Jkg^{-1}k^{-1}$) is placed on top of an electric heater with a power rating of 1000W. It takes 5minutes to raise the temperature of water from 20°C to 90°C, Find

    (a)The energy released by the heater.

    (b)The energy absorbed by the water.

    (c)Account for any losses in energy or why are the two values above different?

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20. A hot water tank in a house contains 140kg of water at 15°C. Find how long it will take an immersion heater to raise temperature of the water to 50°C if the tank is well insulated and the power of the heater is 2200W? (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$ and the tank its self has a heat capacity of $6000Jk^{-1}$)

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21. A paddle wheel is used to churn 1.0kg of water in an insulated can. A total load of 200N turns the paddle wheel by falling through a distance of 1.0 m. The experiment is repeated 90 times and a total temperature rise of 4°C is produced. Calculate the specific heat capacity of water? Why is the final answer different from the commonly known value?

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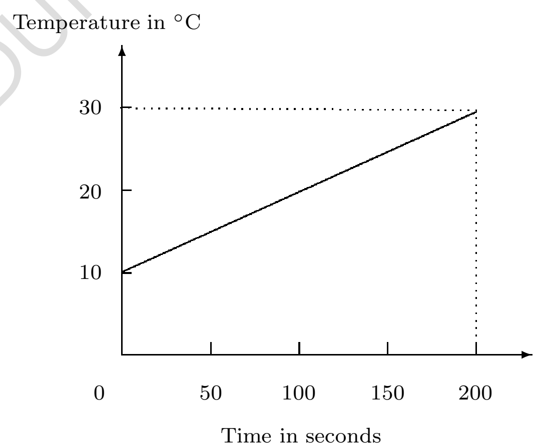
22. When 1.2 kg of paraffin is heated. The graph of temperature against time obtained is shown



if the power of the heater is 300W, determine the

(i) amount of heat supplied by the heater in 200s

(ii) the slope of the above graph

(iii) the specific heat capacity of paraffin wax

(iv) the time taken by the heater to heat paraffin wax from 30°C to its boiling point of 55°C.

23. A Bunsen burner heats an aluminium can containing 4.0kg of water for 600s. the temperature rise of the water is 10°C;

Time(s)	Temperature(°C)
0	10
30	27
60	42
90	56
120	69
150	79
180	87

Table 3.2: Values of temperature and time

(i) Calculate a value of the power rating of the burner?

(ii) Explain whether the true power rating is greater or less than this value?(specific heat capacity of water is $4200Jkg^{-1}k^{-1}$)

24. A car of mass 1000kg is brought to rest from a speed of 20ms^{-1}. Assuming that all its kinetic energy is changed into heat in the disc brakes, find the temperature rise produced, if each of the 4 brakes has a mass of 5kg.(specific heat capacity of the iron from which the brakes are made is $480Jkg^{-1\circ}C^{-1}$).

25. An immersion heater is immersed in a large beaker full of water at an initial temperature of 10°C. The heater supplied heat at a constant rate. The heater is switched on and the temperature of the water is taken every 30 seconds, the water being stirred each time before reading the thermometer. The results are in the table 3.2

(a) Plot a graph of temperature against time

(b) from the above graph estimate how long the water takes to reach its boiling point of 98°C from the time when the heater was switched on(show on your graph how you made your estimate).

(c) Explain why the graph is not a straight line?

(d) Which part of the graph would you use to estimate as accurately as possible the rate at which heat is given out by the heater? Explain your choice.

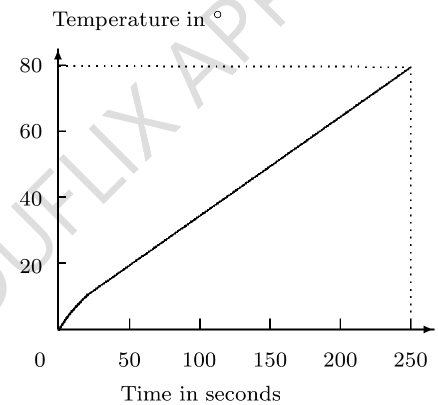
26. An immersion heater placed in 0.2 kg of water contained in a well-insulated can, raised the temperature of the water from 20°C to 35°C.

(i) How much heat was supplied to the water by the immersion heater.

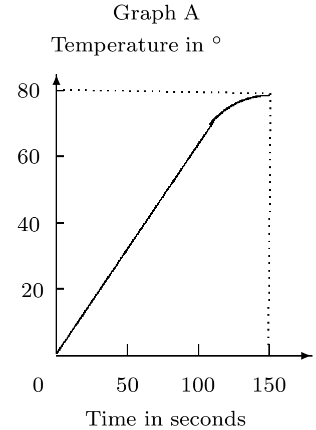
(ii) What is the power rating of the heater, if it took 12.6 seconds?

(iii) If the immersion heater was rated 50W, for how long was it switched on?(specific heat capacity of water is $4200Jkg^{-1}k^{-1}$).

27. A textbook contained a graph showing temperature plotted against time for 10g of paraffin heated by a 50W-immersion heater (graph A). A pupil tried this experiment and obtained graph B.



Graph A



Graph B

(a) Using the text book graph A, calculate the specific heat capacity of the paraffin

(b) Suggest one reason, which could account for graph B bending over (an incorrectly rated heater in not a reason).

(c) Explain how the experiment could be modified so that the graph looked more like graph A.

28. In an energy conversion experiment, an electric motor rotates a copper cylinder at a steady speed against two friction pads. The cylinder is initially at room temperature. The results obtained during a time interval test run were;

Power input by motor	=	$1.2W$
Mass of copper cylinder	=	$0.3kg$
Rise in temperature of the copper cylinder	=	$2.5°C$

(a) How much energy does the motor use during the test run of 5 minutes supply.

(b) How much heat is generated in the cylinder during the test run.

(c) Explain why the answer to (b) is less than the answer to (a).

(d) Discuss whether a ten-minute interval test run under the same conditions would produce a temperature rise of 5°C in the copper cylinder.

29. A metal cylinder of mass 0.3kg is rotated between two friction pads by pulling a cord with a steady force of 60N. While rotating, the cylinder rubs against the friction pads and as a result, the temperature of the cylinder rises. When the cord is pulled a distance of 4m, a rise in temperature of 2°C is produced in the cylinder. Assuming no heat loss;

(a) Calculate the specific heat capacity of the metal

(b) It is a good experimental procedure to have a small quantity of oil surrounding the thermometer bulb, give a reason for this answer?

(c) Suggest two ways in which a higher rise in temperature would be produced in the metal cylinder in this experiment.

(d) Discuss whether a higher rise in temperature would lead to a more accurate value for the specific heat capacity of the metal.

30. A body is heated in boiling water to a temperature of 100°C and quickly then transferred to a copper calorimeter containing 50g of water at 15°C. After a thorough stirring, the final temperature is found to be 21.6°C, calculate the specific heat capacity of the body? (Specific heat capacity of water and copper are $4200Jkg^{-1}k^{-1}$ and $400Jkg^{-1}k^{-1}$ respectively and mass of the body is 150g). Answer $126.8Jkg^{-1}k^{-1}$.

31. The engine of a motor car has a cooling system, which contains 10dm^3 of water. The water emerging from the engine jacket is at a temperature of 92°C and the water returning to the engine jacket is at 45°C. Assuming that the water in the cooling system circulates twice a minute, how much heat is taken from the engine in one minute? Calculate the loss of power in watts due to the cooling system(Specific heat capacity of water is $4.2\text{k}Jkg^{-1}k^{-1}$).

32. 1 kilogram of water in a beaker is heated from 25°C to 40°C in 20 minutes by an electric heater placed in the water. Calculate the power of the heater? The water is replaced by an equal mass of glycerine and the temperature rises from 25°C to 40°C in 12 minute. Calculate the specific heat capacity of glycerine? (Specific heat capacity of water is $4.2\text{k}Jkg^{-1}k^{-1}$).(Hint: Form two equations which are solved simultaneously)

33. The temperature of the freezing mixture was found by putting a piece of copper of mass 400g in it, allowing it to cool to the temperature of the mixture then transferring it to a lagged copper calorimeter containing water at a temperature of 25°C. The final temperature of the water in the mixture was 15°C. The mass of the copper calorimeter is 100g and contained 100g of water. Calculate the temperature of the freezing mixture correct to the nearest degree Celsius?(specific heat capacity of copper is $400Jkg^{-1}k^{-1}$ and of water is $4.2\text{k}Jkg^{-1}k^{-1}$).

34. 900g of water at 80°C are poured in 1200g of paraffin in a lagged beaker at 40°C and the mixture stirred thoroughly. Determine the final temperature of the mixture(specific heat

capacity of the water and paraffin are 4200 and $2000Jkg^{-1}k{-1}$ respectively).

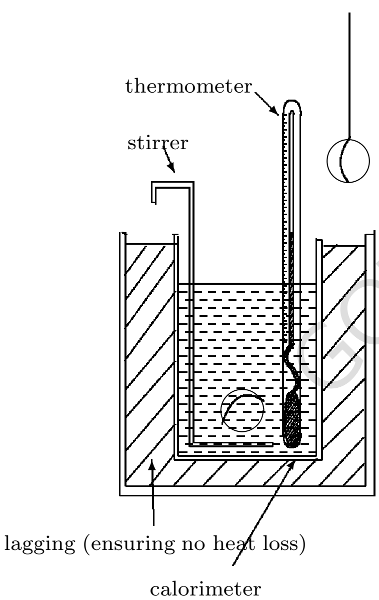
3.4.4 Determining specific heat capacities of solids and liquids

Mixtures method

An experiment to determine the specific heat capacity of a solid by the mixtures method.

Heat the solid of known mass, m_s, to a known temperature, T_1, using a Bunsen burner (or any source of heat).

Drop the solid in a lagged copper calorimeter containing water at room temperature T_2 as shown below;



Stir the mixture, it will attain a final temperature of T_f Let the masses of the solid, water, and calorimeter be m_s, m_w, and m_c with specific heat capacities c_s, c_w, and c_c respectively.

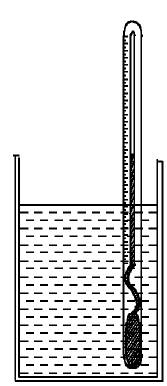
$$\begin{array}{c}\text{Heat loss}\\ \text{from the solid}\end{array} = \begin{array}{c}\text{heat gained by the}\\ \text{water and the calorimeter}\end{array} \tag{3.40}$$

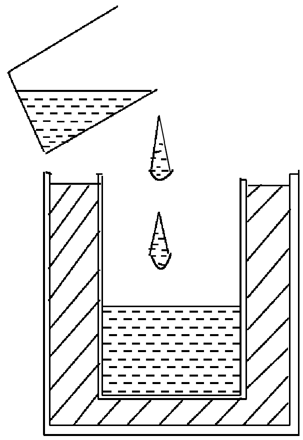
$$\begin{aligned}
H_{loss} &= m_s c_s (T_1 - T_f) \\
H_{gain} &= m_w c_w (T_2 - T_f) + m_c c_c (T_2 - T_f) \\
\Rightarrow c_s &= \frac{m_w c_w (T_2 - T_f) + m_c c_c (T_2 - T_f)}{m_s (T_1 - T_f)} \\
c_s &= \frac{(m_w c_w + m_c c_c)(T_2 - T_f)}{m_s (T_1 - T_f)}
\end{aligned}$$

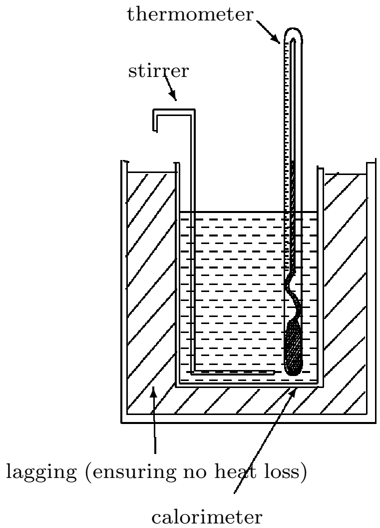
Substituting in all these values, the specific heat capacity, c_s, of the solid is calculated using the above expression.

An experiment to determine the specific heat capacity of a liquid by the mixtures method.

- Heat the liquid of known mass up to a certain temperature T_1 using any source of heat.
- Pour the hot liquid to another cooler liquid in a lagged calorimeter at a known temperature T_o (room temperature).
- With the thermometer in the mixture, record the final temperature T_2 as the maximum temperature of the mixture.







If the masses of the hot liquid, cool liquid and calorimeter are m_H, m_{cl} and m_{ca} and their specific heat capacities are c_H, c_{cl}, and c_{ca} respectively.

If H_{hl} is the heat lost by the hot liquid,

H_{cl} the heat gained by the cooler liquid and

H_{cal} the heat gained by the calorimeter, then

$$
\begin{aligned}
H_{hl} &= H_{cl} + H_{cal} \text{ but} \\
H_{hl} &= m_H c_H (T_1 - T_2) \\
H_{cl} &= m_{cl} c_{cl} (T_2 - T_o) \\
H_{cal} &= m_{ca} c_{ca} (T_2 - T_o) \\
H_{cl} + H_{cal} &= (m_{cl} c_{cl} + m_{ca} c_{ca})(T_2 - T_o) \\
c_H &= \frac{H_{hl} + H_{cl}}{m_H (T_1 - T_2)} \\
c_H &= \frac{(m_{cl} c_{cl} + m_{ca} c_{ca})(T_2 - T_o)}{m_H (T_1 - T_2)}
\end{aligned}
$$

The specific heat capacity of the liquid is obtained by substituting m_{cl}, m_{ca}, c_{cl}, T_2, T_o, T_1, and m_H in the above expression.

Short tube method

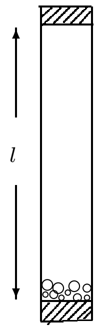
This method is not simple to perform because of the difficulty in measuring accurately the temperature of the lead beads. This method can only be tried on solids.

- Get a short tube of length l
- Place the crystals or beads of the solid (for instance lead beads) of known mass (m)
- Note their temperature (usually room temperature $T_\circ$)
- Shake them to move up and down the tube N times till their temperature increases.
- Record their new temperature T_2
- Taking c as the specific heat capacity of the material of the solids, then

The loss in potential energy is equal to the sum of heat gained by the solids and energy used to overcome friction, but the energy used to overcome friction can not be considered, we just assume that it is negligible.

$$
\begin{aligned}
\text{Loss in potential energy} &= \text{heat gained by the solids} &(3.41) \\
Ngmh &= mc\Delta T &(3.42) \\
Ngl &= c(T_2 - T_1) &(3.43) \\
c &= \frac{Ngl}{(T_2 - T_1)} &(3.44)
\end{aligned}
$$

c is the specific heat capacity of the material of the solid.



Electrical method

These methods are best understood when you have covered current electricity. If you don't mind skipping them for the time being you can go to section 3.17 on page 305

3.4.5 Latent heat

Change of state or phase

When does a substance change state i.e from liquid to gas or from liquid to solid states? The kinetic theory can explain this; according to this theory.

Note that

- **Temperature** is directly proportional to the average kinetic energy of the atoms or molecules of a substance and
- **internal energy** is the sum of the potential and kinetic energy of the particles of a substance.
- The state of a substance is a measure of the potential energy of the atoms or molecules of a substance.

When a substance is heated, the energy supplied may be stored as;

(a) **Internal kinetic energy**; here the average kinetic energy of the particles of that substance increases but the inter-atomic or intermolecular spacing remains averagely constant. In this case the temperature of the substance increases steadily as the substance is heated.

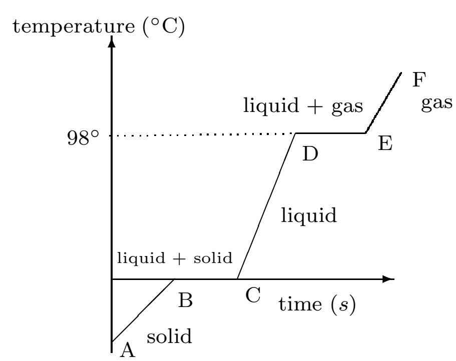
(b) **Internal potential energy;** here the heat energy supplied is used to overcome the intermolecular or inter-atomic attraction (weakened).

For instance, if the particles are just vibrating as in solids, the forces holding them are weakened up to enabling them to move by sliding over each other (like in liquids). In this case the substance changes from solid to liquid state as it is heated at constant temperature.

For a pure substance energy cannot be stored simultaneously as internal kinetic energy and as internal potential energy, it is either stored as internal potential energy or as internal kinetic energy. This can be shown by the cooling curves of a pure substance like distilled water and that for an impure substance like candle wax.

Heating and cooling curves for a pure and an impure substance

If we heat ice crystals at a temperature of -10°C continuously at a constant rate recording their temperature till they evaporate, the graph below is obtained.



This is a heating curve and not a cooling curve.

What happens in the various regions in the above graph is described below;

Observations

In the regions

- AB -The ice would just change in temperature from -10°C to 0°C. (Is remains in the solid state.
- BC - Here the ice could be seen changing from solid to liquid state, in this process the temperature remains constant. This constant temperature is the melting point of ice.
- CD -The temperature of the water increases, and the state of the substance does not change.
- DE -Here the water starts boiling i.e. changing from liquid to gaseous state (temperature remains constant). This constant temperature is the boiling point.
- EF -Here the temperature of the steam only increases (the state remains unchanged).

Explanation of the observations using the kinetic theory of matter;

- **In region AB**

  It is in the solid state, when it is heated, it stores the energy absorbed as its internal kinetic energy hence having an increase in temperature. This is because the temperature is proportional to the average kinetic energy of a substance. No energy is stored as internal potential energy.

- **In region BC**

  The absorbed energy is stored as internal potential energy of the body i.e. it is used to overcome intermolecular forces, hence the substance changes state from solid to liquid. No energy is stored as internal kinetic energy, therefore temperature remains constant.

- **In region CD**

  It is in the liquid state. Energy supplied is stored as internal kinetic energy hence temperature increases.

- **In region DE**

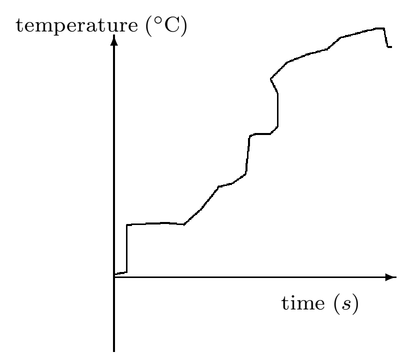
  Energy supplied to this liquid is stored as internal potential energy, this weakens the intermolecular or inter-atomic forces of the substance, and hence it changes state from liquid to Gaseous State at constant temperature; the boiling point.

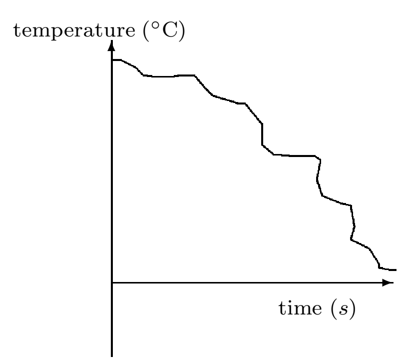
- **In region EF**

  It is in the gaseous state. The energy supplied is stored an internal kinetic energy hence only temperature of the substance changes (increases).

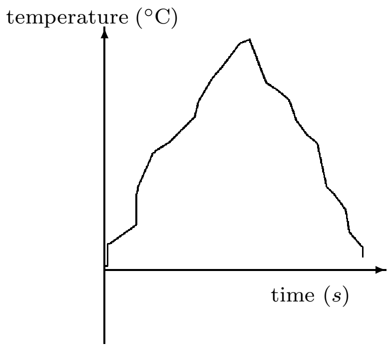
All pure substances have the same type of cooling and heating curves i.e. with platforms when changing state and inclined lines when temperature is increasing or decreasing.

Heating and cooling curves for an impure substance





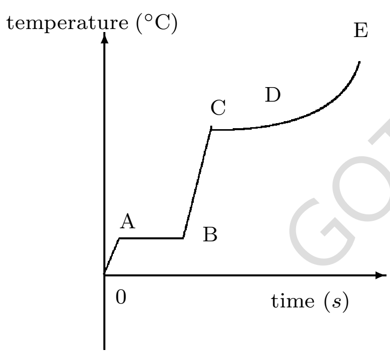
Temperature against time graph for an impure substance heated and cooled



These graphs show various constant temperatures and bad behaviours. They are graphs of mixtures of very many compounds that have different melting and boiling points.

Question

The graph below of temperature against time, is for a substance heated at a constant rate from low to high temperature. Use letters to answer the following questions about it;



(a) Which part or parts of the graph corresponds to a substance existing in two states at the same time?

(b) Over which part is the substance increasing in temperature at the fastest rate? Which point of the graph corresponds to the molecules of the substance having the greatest average kinetic energy?

(c) Between which regions is the substance in the

(i) Liquid state

(ii) Solid state

(iii) Gaseous state

Discussion

In this sub-topic we are concerned about how to measure the amount of heat stored as internal potential energy of the molecules or atoms of a substance i.e. the energy required to free the fixed particles of a solid from the forces that hold them in fixed definite positions. Recall that when a substance is changing state, its temperature remains constant; that is the thermometer can not indicate the change of state. The energy required to do this is hidden hence called latent heat (latent means hidden). This latent heat absorbed by the particles enables them to move freely in the liquid or gaseous state. Latent heat of a liquid is the energy required to increase the distance between the liquid particles (molecules or atoms). The latent heat given to a boiling liquid or absorbed by a boiling liquid (like in a freezer), enables the liquid molecules to do work against the forces of attraction that hold them together, leading to an increase in the intermolecular spacing hence changing state from liquid to gaseous state. With all the above ideas we conclude that;

Definition 22.

latent heat is the amount of energy required to change the state of a substance (solid, liquid and gas) at a constant temperature (melting or boiling point).

Definition and SI units of terms to be used.

Latent heat

Definition 23.

Latent heat is the amount of heat energy required to change the state of a pure substance at either its melting point or boiling point.

The SI units of latent heat are *Joules* (since it is energy)

Specific latent heat

Definition 24.

Specific latent heat is the amount of heat energy required to change the state of 1 kg mass of a pure substance at either its melting point or boiling point.

Or Specific latent heat is the heat required to change the state of a 1kg mass of a substance at constant temperature (this constant temperature is either the melting point or boiling point of the pure substance) .

SI unit of specific latent heat is Jkg^{-1} *(Joules per kilogram)*

There are 3 states of matter. Hence a substance can change;

(a)from liquid to solid or from solid to liquid.(freezing and melting)

(b)from liquid to gas or from gas to liquid.(evaporation and condensation) and

(c)from solid to gas or from gas to solid.(sublimation)

We shall ignore sublimation (change from solid to gas or vapour). This means that, we are to consider two forms of latent heat;

1. Latent heat of fusion (to change liquid to solid or solid to liquid).
2. Latent heat of vapourisation (to change liquid to gas or gas to liquid).

Latent heat of fusion:

Definition 25.

Latent heat of fusion is the amount of heat energy required to change the state of a pure substance from solid to liquid or from liquid to solid state at its melting point.

SI unit of latent heat of fusion is **Joules.**

Specific latent heat of fusion

Definition 26.

Specific latent heat of fusion is the amount of heat energy required to change the state of a 1kg mass of a pure substance from liquid to solid or from solid to liquid state at its melting point.

SI unit of specific latent heat of fusion is Jkg^{-1} **(Joules per kilogram)**

Really; what is latent heat of fusion?

Explain what is meant by the term latent heat of fusion using the kinetic theory of matter?

Answer. This is the energy required to free the fixed particles of a solid from the forces that hold them in fixed positions i.e to overcome the forces of attraction between molecules hence changing the state of the substance to the liquid state where particles are free to move (i.e. slide over each other) or it is the energy required to hold the molecules of a liquid so that they are not free to move i.e. to change it to the solid state.

Calculations

From the definition of specific latent heat of fusion, l_f, when H joules of heat energy are supplied to a body of mass m kg and it changes state from solid to liquid state without change in temperature (or at its melting point), then

$$H = \frac{\text{heat supplied}}{\text{mass of the body}} \quad (3.45)$$

$$l_f = \frac{H}{m} \quad (3.46)$$

$$\Rightarrow H = ml_f \quad (3.47)$$

The table 3.3 below gives the specific latent heat of fusion of certain materials

Examples

The material	Specific latent heat of fusion in Jkg^{-1}
Aluminium	390,000
Water	330,000
Iron	270,000
Copper	210,000
Naphthalene	150,000
Solder	70,000
Lead	2,600
Mercury	1,300

Table 3.3:

1. A brick thrown into a bucket full of ice at $0°C$ gives up 600Joules of heat. What is the greatest weight of ice that can be melted?($l_f = 336000$)

Solution

from the question

$$\begin{aligned} H &= 600J \\ l_f &= 336000Jk^{-1} \\ m &= ? \\ \text{using, } H &= ml_f \end{aligned}$$

$$\begin{aligned} \Rightarrow m &= \frac{H}{l_f} \\ &= \frac{600}{336000}kg \\ &= \frac{1}{560}kg \\ &= \frac{25}{14}g \\ &\approx 1.786g \end{aligned}$$

2. How many kilograms of ice can be melted by 52kJ of heat energy ($l_f = 336000Jkg^{-1}$)

Solution

from the question

$$\begin{aligned} H &= 52kJ \\ &= 52000J \\ l_f &= 336000Jk^{-1} \\ H &= ml_f \end{aligned}$$

$$\begin{aligned} \Rightarrow m &= \frac{H}{l_f} \\ &= \frac{52000}{336000}kg \\ &= 0.155kg \end{aligned}$$

3. How much heat would be necessary to convert 250g of ice at 0° to water at 0° ($l_f = 336000Jkg^{-1}$)

Solution

from the definition of specific latent heat and the question;

$$\begin{aligned} m &= 250g \\ &= \frac{250}{1000}kg \\ &= \frac{1}{4}kg \\ l_f &= 336000Jkg^{-1} \end{aligned}$$

$$\begin{aligned} H &= ml_f \\ &= \frac{1}{4} \times 336000 \\ &= 84000J \end{aligned}$$

4. Calculate the amount of heat required to change 4g of ice at $-4°C$ to water at $4°C$? (Specific heat capacity of ice is $2100Jkg^{-1}k{-}1$, specific heat capacity of water is $4200Jkg^{-1}k{-}1$ and specific latent heat of fusion of ice is $336000Jkg^{-1}$).

Solution

The water heated from $-4°C$ to $4°C$ undergoes through the following steps;

- change ice at $-4°C$ to ice at $0°C$ ($H = mc\Delta T$)

$$\begin{aligned} H &= mc_i\Delta T \\ &= \frac{4}{1000} \times 2100 \times (4-0) \\ &= \frac{4}{1000} \times 2100 \times 4 \\ &= 336J \end{aligned}$$

- change ice at $0°C$ to water at $0°C$ ($H = ml_f$)

$$\begin{aligned} H &= ml_f \\ &= \frac{4}{1000} \times 336000 \\ &= 1344J \end{aligned}$$

- change water at $0°C$ to water at $4°C$ ($H = mc\Delta T$)

$$\begin{aligned} H &= mc_w\Delta T \\ &= \frac{4}{1000} \times 4200 \times (4-0) \\ &= \frac{4}{1000} \times 4200 \times 4 \\ &= 672J \end{aligned}$$

hence the total amount of heat ,H, required is

$$\begin{aligned} H &= mc_i\Delta T + ml_f + mc_w\Delta T \\ &= 336 + 1344 + 672 \\ &= 2352 Joules \end{aligned}$$

5. Calculate the amount of heat required to change 2kg of ice at $0°C$ into water at $10°C$. (Specific heat capacity of water is $4200Jkg^{-1}k^{-1}$ and specific latent heat of fusion of ice is $3.34 \times 10^5 Jkg^{-1}$).

Solution

The steps or changes that take place are;

- changing ice at $0°C$ to water at $0°C$
- changing water at $0°C$ to water at $10°C$

$$\begin{aligned} \text{Heat} &= ml_f + mc\Delta T \\ &= 2 \cdot (3.34 \times 10^5) + 2 \cdot (4200) \cdot 10 \\ &= 668000 + 84000 \\ &= 752000J \end{aligned}$$

hence the heat required is 752,000J or 752kJ.

Latent heat of vapourisation:

Definition 27.
Latent heat of vapourisation is the amount of heat energy required to change the state of a pure substance from liquid to gaseous state or from gaseous to liquid state at its boiling point.

[7]

SI unit of latent heat of vapourisation is Joules

Specific latent heat of vapourisation

Definition 28.
Specific latent heat of vapourisation is the amount of heat energy required to change the state of a 1kg mass of a pure substance from liquid to gas or from gaseous to liquid state at its boiling point

SI unit of specific latent heat of vapourisation is Jkg^{-1} (Joules per kilogram)

Really; what is latent heat of vapourisation ?

Use the kinetic theory of matter to explain what is meant by the term latent heat of vapourisation?

Answer. This is the amount of heat required to overcome the forces of attraction between molecules of a pure liquid i.e. energy required to increase the intermolecular spacing of the liquid, hence changing state from liquid to vapour or gaseous state. We can also describe it as the energy required to hold the molecules of a gas or increase the forces of attraction between gas molecules so as to change its state from gaseous to liquid state.

Calculation

[7] some books may say; at constant temperature, yes! it can be ok to say so but to be more specific let us adopt the actual constant temperature because the actual constant temperature is either melting point or boiling point.

If H joules of heat are supplied to m kg of a pure substance and it changes state from liquid to gaseous (or vapour) state at constant temperature (its boiling point), then the specific latent heat of vapourisation, l_v, of the substance is given by;

$$L_v = \frac{\text{heat supplied}}{\text{mass of the body}} \quad (3.48)$$

$$L_v = \frac{H}{m} \quad (3.49)$$

$$\Rightarrow H = ml_v \quad (3.50)$$

Table 3.4 gives the values of the specific latent heat of vapourisation for the commonly used materials The specific latent heats of different

Material	Specific latent heat of vapourisation in Jkg-1
Turpentine	270,000
Ether	350,000
Benzene	400,000 or (395,000)
Ethanol	850,000
Trichloromethane	247,000
Water	2,260,000

Table 3.4:

liquids indicate the strength of the force of cohesion, hence turpentine and ether posses a much lower specific latent heat of vapourisation than water indicating a weaker cohesive force.

Examples

(a) Given water at its boiling point, How much heat would be required to change the state but not temperature of $25g$?($l_v = 2260000Jk^{-1}$)

Solution

$$\begin{aligned} \text{Using } H &= ml_v \\ &= 25g \times l_v \\ &= \frac{25}{1000}kg \times 2260000 \\ &= 56500J \end{aligned}$$

(b) Dry steam from a kettle boiling at $212°F$ is passed through directly into 10kg of water at 50°F and raised the temperature to 104°F. How much steam has condensed , if the latent heat of vapourisation is $2260000Jk^{-1}$?

Solution

from the question

$$\begin{aligned} m &= 10kg \\ T_1 &= 212°F \\ &= (212-32)\frac{5}{9}°C \\ &= 100°C \end{aligned}$$

$$\begin{aligned} T_2 &= 50°F \\ &= (50-32)\frac{5}{9}°C \\ &= 10°C \end{aligned}$$

$$\begin{aligned} \Delta T &= 100°C - 10°C \\ &= 90°C \\ l_v &= 2260000Jk^{-1} \end{aligned}$$

$$\begin{aligned} \text{Using } H &= ml_v \\ \Rightarrow m &= \frac{H}{l_v} \\ &= \frac{mc\Delta T}{l_v} \\ &= \frac{10 \times 4200 \times 90}{2260000} \\ &= 1.673kg \end{aligned}$$

(c) Calculate the thermal energy required to convert 5kg of ice at −15°C to steam at $100°C$? ($c_{ice} = 2100Jkg^{-1}k^{-1}$, $c_{water} = 4200Jkg^{-1}k^{-1}$, $L_f = 3.3 \times 10^5 Jkg^{-1}$ and $L_v = 2.3 \times 10^6 Jkg^{-1}$).

Solution

For ice to be converted to steam, a number of processes occur and each requires energy; these processes and their corresponding energies include;

- heat required to change ice at $-15°C$ to ice at $0°C$

$$\begin{aligned} H &= mc_i\Delta T \\ &= 5 \times 2100 \times (0 - {}^{-}15) \\ &= 5 \times 2100 \times 15 \\ &= 157,500J \end{aligned}$$

- heat required to change ice at $0°C$ to water at $0°C$

$$\begin{aligned} H &= ml_f \\ &= 5 \times 3.3 \times 10^5 \\ &= 5 \times 330000 \\ &= 1,650,000 \end{aligned}$$

- heat required to change water at $0°C$ to water at $100°C$

$$\begin{aligned} H &= mc_w\Delta T \\ &= 5 \times 4200 \times (100 - 0) \\ &= 5 \times 4200 \times 100 \\ &= 2,100,000J \end{aligned}$$

- heat required to change water at $100°C$ to steam at $100°C$

$$\begin{aligned} H &= ml_v \\ &= 5 \times 2.3 \times 10^6 \\ &= 5 \times 2,300,000 \\ &= 11,500,000J \end{aligned}$$

The total heat, H, required is the sum of the above

$$\begin{aligned} H &= mc_i\Delta T + ml_f + mc_w\Delta T + ml_v \\ &= 157,500 + 1,650,000 \\ &\quad +2,100,000 + 11,500,000 \\ &= 15407500J \\ &= 15.4075MJ \\ &\approx 15.4MJ \end{aligned}$$

Questions

1. 1 kg of water at 10°C is placed in a refrigerator. How much heat is extracted before all the water turns to ice?(L$_{ice}$ $= 2100Jkg^{-1}k^{-1}$
2. A few pieces of dry ice had been added to 100g of water at $40°C$ and the final temperature of the mixture was found to be $0°C$, what was the mass of ice added?
3. Explain why a steam burn is so much more serious than that caused by boiling water?
4. Calculate the thermal energy required to melt 2kg of ice at 0°C? ($L_f = 3.35 \times 10^5 Jkg^{-1}$).
5. Calculate the specific latent heat of fusion of naphthalene given that $2.98 \times 10^5 J$ of heat are given out when 2.0kg of naphthalene at its melting point changes into solid.
6. How long does it take a 500W heater to melt 1.0kg of ice at 0°C? ($L_f = 3.35 \times 10^5 Jkg^{-1}$).
7. Calculate the amount of heat required to change 0.5kg of liquid - ethanol to vapor at its boiling point (specific latent heat of vapourisation of ethanol is $8.5 \times 10^5 Jkg^{-1}$).
8. A 200W heater is used to melt ice at 0°C in a filter funnel. After 300s the mass of water collected is 0.188kg. if 0.018kg of the ice is melted purely due to heat taken in from the surrounding(and not from the heater) calculate the specific latent heat of fusion of ice?
9. 5kg of water at 0°C are changed by a refrigerator to ice at 0°C, determine the work done by the refrigerator? ($L_f = 3.35 \times 10^5 Jkg^{-1}$).

10. 3kg of ice at 0°C was supplied with 1,509,000J of heat. Given that the specific latent heat of ice is $335KJkg^{-1}$ and the specific heat capacity of water is $4.2KJkg^{-1}k^{-1}$, then;

    (a) How much heat energy is used in melting the ice?

    (b) How much heat is used to warm the water?

    (c) What is the expected final temperature of the water?

11. How many joules of heat are required to change $2g$ of ice at $-10°C$ to steam at $100°C$(assume the usual constants of water) Ans:6114J

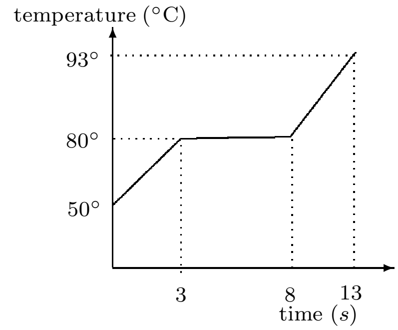
12. An Aluminium tray of mass 400g containing 300g of water is placed in a refrigerator. After 80 minutes the tray is removed and it is found that 60g of the water remain unfrozen at 0°C. If the initial temperature of the tray and its contents was 20°C, determine the average amount of heat energy removed per minute by the refrigerator (assume that $C_{\text{Aluminium}} = 1000Jkg^{-1}k^{-1}$, $C_{\text{water}} = 4.2Jg^{-1}k^{-1}$ and $L_{\text{ice}} = 340Jg^{-1}$).

13. 50g of dry ice is put in a thick walled copper calorimeter of mass 1kg if the initial temperature of the calorimeter is 25°C. How much ice will be melted?($C_{copper} = 400Jkg^{-1}k^{-1}$, and $L_{ice} = 330kJkg^{-1}$).

14. 50g of dry ice are put in a thick walled copper calorimeter of mass 1kg. If the initial temperature of the calorimeter is 25°C. How much ice will be melted? (Specific heat capacity of copper $=400Jkg^{-1}k^{-1}$, specific latent heat of fusion of ice $= 330kJkg^{-1}k^{-1}$).

15. 50g of water wax at its melting point (62°C) were poured into a copper calorimeter of mass 40g with 80g of water at 15°C. After stirring the final temperature of the mixture was 44°C. Calculate the specific latent heat of fusion of wax? (Specific heat capacities of the wax and copper are $1600Jkg^{-1}k^{-1}$ and $400Jkg^{-1}k^{-1}$).

16. Use the heating curve of naphthalene below to calculate;



    (i) the specific heat capacity of solid naphthalene.

    (ii) the specific heat capacity of liquid naphthalene.

    (iii) the latent heat of fusion of naphthalene

    (iv) the specific latent heat of fusion of naphthalene

    (v) the melting point of naphthalene.

17. The results in table 3.5 were obtained in an experiment where a substance of mass 50g was heated with an electric heater rated 1.2K Watts until it melted and then temperature recorded as shown below as it cooled;

Temperature (^oC)	Time (minutes)
81.0	0
73.0	1
68.5	2
65.2	3
63.2	4
62.8	5
62.0	6
62.0	7
62.0	8
61.7	9
60.8	10
59.5	11
58.5	12

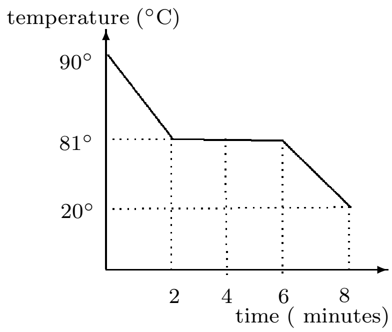
Table 3.5: latent heat measurement

    (a) Plot a cooling curve from these results.

    (b) Explain the nature or shape of the curve

(c) Use the curve to estimate
   (i) the specific latent heat,
   (ii) Specific heat capacity and
   (iii) the melting point of the substance.

18. 10g of a substance X at 90°C were placed in a cup with a thermometer in it and its temperature recorded at a 2-minute interval. The cooling curve below was obtained,



Use it to determine the following;

(i) Specific heat capacity of liquid X

(ii) Specific latent heat of fusion of liquid X

(iii) Specific heat capacity of solid X, given the rate of heat loss to the surrounding was $500Js^{-1}$.

(iv) State its melting point

(v) Explain what takes place at its melting point .

19. 40g of a cubical copper material at temperature of 140°C is dropped in water boiling 100°C. Determine the quantity of water that evaporates (specific heat capacity of copper and water is $400Jkg^{-1}k^{-1}$ and $4200Jkg^{-1}k^{-1}$ respectively and the specific latent heat of vapourisation of water is $2{,}260{,}000Jkg^{-1}$).

20. 5 kg of water were poured in a refrigerator which extracts heat from it at a constant rate P. It took 4 minutes to change this water to ice at -15°C. Determine the;

(a) the total heat absorbed by the refrigerator

(b) The power rating, P, of the refrigerator or heat absorbed by the refrigerator per second .

21. Determine the amount of heat required to change 2kg of water at 80°C to steam at 100°C(specific heat capacity of water is $4200Jkg^{-1}k^{-1}$ and the specific latent heat of vapourisation of water is $2{,}260{,}000Jkg^{-1}$).

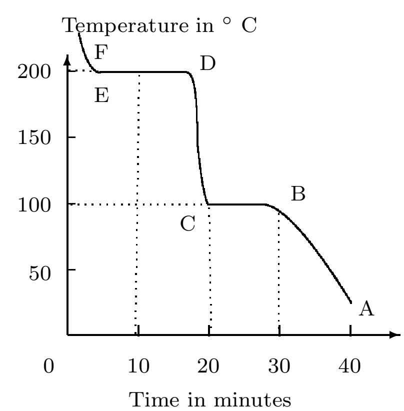
1. An immersion heater is placed in a vacuum flask containing a small quantity of a liquid and switched on. The temperature of the liquid is recorded every 50 seconds and the results in table 3.6 were obtained;

time(s)	temperature(°)
0	20
50	51
100	52
150	114
200	130
250	130

Table 3.6: Latent heat measurement

(a) Plot a suitable graph of these results
(b)Using the graph, estimate or calculate the
   (i) the boiling point of the liquid
   (ii)the time it takes for the liquid to start boiling.
(c)if the mass of the liquid in the flask is 120g, calculate;
   (i)its heat capacity
   (ii)its specific heat capacity

2. The graph below represents the cooling curve of a certain substance cooling from a higher temperature down to room temperature as shown below

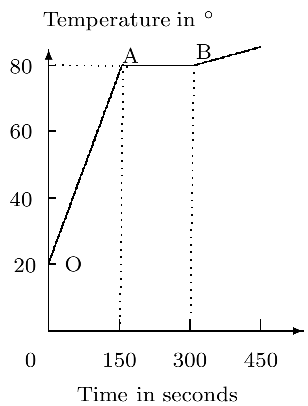


(a) what is the boiling point of the substance

(b) what is happening in the region CB

(c) why is region BC longer than region DE

3. A test tube containing 0.1kg of a powder is heated for several minutes by a Bunsen burner which is supplying heat at a constant rate of 50joules per second. The temperature of the content is noted at equal time interval and a graph of temperature against time plotted; this is shown below;



(a) what is happening to the substance during the interval OA

(b) what is the significancy of the time AB being greater than OA

(c) calculate the specific heat capacity and the specific latent heat of fusion of the powder assuming all the heat supplied by the Bunsen burner is transfered to the powder?

4. Water was heated in a vessel with an immersion heater of power 40W. The heater was used to boil the water for 100s during which time the mass of water decreased by 0.002kg. Assuming all the heat energy was given to the water, what is the specific latent heat of vapourisation of water?

5. A heater power 200W was used to keep the liquid boiling in a vessel. During a period of 50 seconds, the mass of the liquid decreased by 0.005kg determine the specific latent heat of vapourisation of the liquid in Jkg^{-1} and in Jg^{-1}?

6. Calculate the thermal energy required to boil completely 0.2kg of water at 100°C? ($L_v = 2.3 \times 10^6 Jkg^{-1}$).

7. When heat is supplied at the rate of 450W to an electric kettle containing boiling water, steam escapes at such a rate that the loss of water is $0.15gs^{-1}$, when heat is supplied now at a rate of 680W, the rate of water loss becomes $0.25gs^{-1}$. calculate;

   (a) The specific latent heat of steam

   (b) The rate of loss of heat from the kettle due to other factors other than evaporation. (Hint: form two simultaneous equations and solve them)

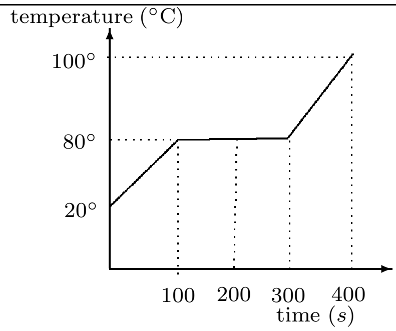
8. How long does it take a 1000W heater to convert 10kg of water at 15°C to steam at 100°C? ($c_{water} = 4200Jkg^{-1}k^{-1}$ and $L_v = 2.3 \times 10^6 Jkg^{-1}$)

9. How many joules of energy are required to heat 250g of water from 80°C to steam at 100°C? (Specific heat capacity of water $= 4200Jkg^{-1}k^{-1}$, specific latent heat of vapourisation of water $= 2260000Jkg^{-1}$).

10. Determine the amount of heat required to change 49g of ice at -10°C to steam at 98°C, the boiling point of the water?($c_{ice} = 2100Jkg^{-1}k^{-1}$, $c_{water} = 4200Jkg^{-1}k^{-1}$, $L_f = 3.3 \times 10^5 Jkg^{-1}$ and $L_v = 2.3 \times 10^6 Jkg^{-1}$).

11. 200g of a metal were heated in a flame to a temperature of 600°C and dropped into a boiling liquid. It is found that 20g of the liquid vapourised. Given the specific heat capacity of the metal is $500Jkg^{-1}k^{-1}$ and latent heat of vapourisation of the liquid is $5 \times 10^7 Jkg^{-1}$, determine the boiling point of the liquid?

12. When a 100W heater is used to heat a 1kg solid wax, the temperature of the wax is observed to change with time as shown below;



(a)what is the melting point of solid wax

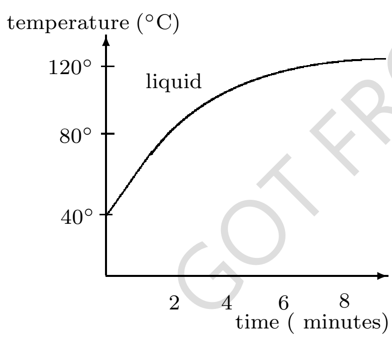
(b)calculate the specific latent heat of fusion of the solid wax

(c)explain the shape of the graph

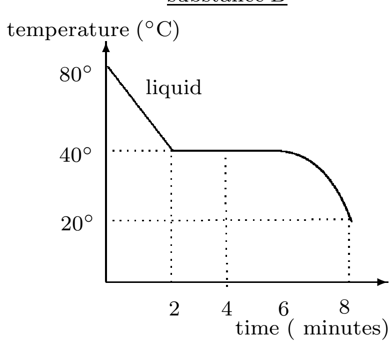
13. For how long will a 200W heater take to change 100g of water at -40°C to water at 100°C?(c_{ice} = 2100Jkg -1k -1, c_{water}= 4200Jkg -1k -1, $L_f = 3.3 \times 10^5 Jkg^{-1}$ and $L_v = 2.3 \times 10^6 Jkg^{-1}$).

14. In a certain experiment the following cooling and heating curves were obtained. Determine the boiling and melting point of these substances?

substance A



substance B



15. 1.0kg of water in a beaker, is heated from 25°C to 40°C in 20 minutes by an electric heater placed in the water. Calculate the power of the heater? The water is replaced by an equal mass of glycerine and the temperature rises from 25°C to 40°C in 12 minutes. Calculate the specific heat capacity of the glycerine? (Specific heat capacity of water is 4.2k$Jkg^{-1}k^{-1}$).

16. The-steam pipe leading from a boiler to an engine made of copper, is of mass 5kg and is at room temperature of 30°C. When steam is passed through the pipe, its temperature rises to 100°C.How much steam has condensed? (The specific heat capacity of copper is 400$Jkg^{-1}k^{-1}$ that of water is 4.2K$Jkg^{-1}k^{-1}$, and the specific latent heat of vapourisation of water is 2.1MJkg^{-1}).

17. Some powdered ice at -10°C is placed in a beaker and heated at a constant rate. After 2 minutes the temperature has risen to 0°C. The temperature remains constant for 32minutes and then begins to rise taking 40 minutes to reach 100°C. Using m, c and l to represent the mass of ice, specific heat capacity of ice and the specific latent heat of fusion of ice respectively. Write down expressions for the heat supplied per minute at each stage and hence calculate c and l. (specific heat capacity of water = 4.2k$Jkg^{-1}k^{-1}$).

18. While a beaker of water is boiling over a Bunsen burner, heat is being supplied at a steady rate and the temperature remains constant. Explain as fully as you can what happens to the heat that is supplied.

19. A closed metal vessel of mass 200g is fitted with an inlet and an outlet for steam and connected to a steam generator. Initially the vessel is at 15°C and after a short time steam at 100°C issues **freely from it**. If it is found that 3.5g of steam has been condensed in the vessel, how much heat has been lost to the surrounding? (Take specific heat capacity of the metal to be 0.38$Jg^{-1}k^{-1}$ and specific latent heat of steam to be 2270Jg^{-1}).

20. The temperature of a freezing mixture was found by putting a piece of copper of mass

400g in it, allowing it to cool to the temperature of the mixture then transferring it to a lagged copper calorimeter containing water at a temperature of 25°C. The final temperature of the water in the calorimeter was 15°C. The mass of the copper calorimeter is 100g and contained 100g of water. Calculate the temperature of the freezing mixture correct to the nearest degree Celsius?(the specific heat capacity of copper $=400Jkg^{-1}k^{-1}$ and of water is 4.2k$Jkg^{-1}k^{-1}$).

21. The cooling system of a refrigerator extracts 0.7kW of heat. How long will it take to convert 500g of water at 20°C into ice?

22. The mass of a copper calorimeter and stirrer is 60.0g, it contains 100g of water, the initial temperature being 25°C. Steam is passed until the temperature of water raises to 45°C. The calorimeter is re-weighted and the total mass is 163.5g. Calculate the specific latent heat of vapourisation of water? Why is it less than the accepted value?

23. The steam pipe leading from a boiler to an engine made of copper is of mass 5kg and is at room temperature of 30°C. When steam is passed through the pipe, its temperature rises to 100°C, how much steam has condensed?($C_{copper} = 400Jkg^{-1}k^{-1}$, $C_{water} = 4.2kJkg^{-1}k^{-1}$ and L_v for steam $= 2.1MJkg^{-1}$).

24. While a beaker of water is boiling over a Bunsen burner, heat is being supplied at steady rate and the temperature remains constant. Explain as fully as you can what happens to the heat that is supplied.

25. A closed metal vessel of mass 200g is fitted with an inlet and an outlet for steam and connected to a steam generator. Initially the vessel is at 15°C and after a short time steam at 100°C issues freely from it. If it is found that 3.5g of steam has been condensed in the vessel, how much heat has been lost to the surrounding? (take specific heat capacity of the metal to be $0.38Jg^{-1}k^{-1}$ and specific latent heat of steam to be $2270Jg^{-1}$).

26. (a) 100g of water at 80°C were put into a vacuum flask. The flask was corked and shaken thoroughly. The temperature of the water in the flask had fallen to 60°C.
    (i) what quantity of heat had the water lost to the vacuum flask?
    (ii) what is the heat capacity of the vacuum flask?

    (b) Explain why the temperature of the water in the flask remains at 60°C for some time ($c_{water} = 4.2KJkg^{-1}k^{-1}$)

27. An immersion heater is placed in a vacuum flask containing a quantity of liquid and switched on. The temperature of the liquid is recorded every 50 seconds and the following table of results is obtained;

Time(s)	Temperature in °C
0	20
50	51
100	62
150	114
200	130
250	130

Table 3.7: latent heat measurement

    (a) Plot a suitable graph of these results
    (b) Use the above graph to estimate ;
        (i) the boiling point of the liquid
        (ii) the time taken by the liquid to boil.
    (c) Explain how that above information on the graph may be used to calculate the specific heat capacity of the liquid listing any extra data needed to do this.

28. In a certain- heating system, steam at 100°C enters a radiator and water at 100°C leaves the radiator. Explain whether this process can warm a room

29. Answer the following questions;
    (a) what is meant by a change of state
    (b) how are changes of state explained by the molecular theory.

3.4.6 Determining the specific latent heat of fusion of ice

There are about three methods;

(a) Mixtures method (For advanced level)

(b) Electrical method, see page 307

(c) Graphical or cooling curve method, this is best decribed by working out question or item 12 on page 144.

3.5 Evaporation

3.5.1 Introduction

After a fall of rain, the pools of water left behind slowly dry up. The water in the pools has obviously changed to water vapour at atmospheric pressure and temperature. Hence water or indeed any other liquid can change to vapour without the liquid being boiled. The vapour exerts pressure above the liquid.

Therefore,

Definition 29.
Evaporation is the process by which a liquid changes to vapour at any temperature.

In terms of the kinetic theory of matter,

Definition 30.
Evaporation is the process by which energetic molecules leaves the surface of a liquid at any temperature.

An energetic molecule is one that has the highest energy or a lot of energy. This energy is usually in the form of kinetic energy hence an energetic molecules has the highest speed or an energetic molecule is one with the highest kinetic energy. This molecule can easily escape from the attraction of its neighbouring molecules hence it is the one that accelerates the rate of escape of molecules from the liquid, which is evaporation.

Evaporation

(a) takes place at the surface of a liquid

(b) takes place at any temperature

Factors affecting evaporation

The rate of evaporation can be increased by or is affected by;

(a) Increasing the surface area of the liquid

(b) Exposing the liquid surface to wind or increasing the speed of wind and

(c) Increasing the temperature of water.

It is observed as a common phenomenon that whenever there is evaporation, there is cooling for instance;

- When one is bathing outside on a windy dry day, he or she feels cooler than when he or she baths in a bathroom or when there is no wind.
- When water is placed on a tray and placed in an open space for sometime, the amount of water will be less than the initial quantity, this is because the difference has evaporated or changed to water vapour in the atmosphere.
- Petrol left in an open cup for about 30 minutes the cup may not contain any petrol or may contain a small quantity of cold petrol (Why?) because some of it has evaporated.
- When a cloth is dried in water or is washed and hanged on a wire and left there for some time, it dries i.e it is found not to contain any quantity of water. This is because it has evaporated to the atmosphere.

That is why the atmosphere always contains water as water vapour.

All these take place at any temperature or room temperature, i.e. evaporation takes place at any temperature and it is always accompanied by cooling (cooling by evaporation).

3.5.2 Cooling by evaporation

Evaporation is the process by which energetic molecules leaves the surface of a liquid at any temperature.

Why is it that energetic molecules are the ones that leave the liquid at its surface? and why is evaporation always accompanied by cooling?

A liquid has molecules, which are free to move (though not very free i.e. they can slide over each other). All these molecules do not have exactly the same velocity but their average velocity is the same. For instance if their velocities were 4.1 4.2, 4.15, 4.18. 4.17, e.t.c. Then the average velocity is 4.2. Since they are molecules of a substance in the same state (liquid state), their internal potential energy is constant and their internal kinetic energy is partially constant. But their kinetic energy is proportional to the velocity of the molecules of a substance and an energetic molecule is one with higher kinetic energy i.e. with a higher (highest) velocity. since these molecules attract each other, the slow molecules can not escape the attraction of their neighbouring molecules but those with high speed can escape the attraction.

Brownian motion proved to us that molecules of a liquid are ever in a state of motion. Therefore in a liquid an energetic molecule may have a large enough velocity enabling it escape from the attraction of its neighbour molecules. When this happens, the average kinetic energy of the molecules decreases. For instance if the velocities of the molecules are 4.9, 4.2, 4.3, 4.1 and 4.0. The average velocity is $\frac{21.5}{5} = 4.3$. When the molecule with velocity 4.9 leaves this liquid, the new average is $\frac{16.6}{4} = 4.15$, this is a decrease. When one energetic molecule leaves, the average velocity decreases, also average of the square of the velocities will show a decrease. Hence the average kinetic energy of the molecules decreases, leading to cooling as wind sweeps energetic molecules from the surface of the liquid.

When an energetic molecule leaves a liquid surface the average kinetic energy of the molecules of the liquid decreases.

Recall that temperature is directly proportional to the average kinetic energy of the molecules of a substance, according to the kinetic theory of matter. Therefore

when energetic molecules leave the surface of a liquid, the temperature of the liquid decreases

or the liquid cools that is evaporation causes cooling or there exists **cooling by evaporation.**

Brief explanation: why a liquid cools as it evaporates

In evaporation, energetic molecules leave the liquid surface, this lowers the average kinetic energy of the molecules. Since average kinetic energy of the molecules is proportional to the temperature of the liquid, the temperature lowers that is, it cools hence having cooling by evaporation.

3.5.3 Factors affecting evaporation

(a) **Wind**

Whenever there is wind the rate of evaporation increases because it sweeps energetic molecules from the liquid surface, therefore wind increases the rate of evaporation. On a windy day one feels more coldness

(b) **Temperature**

An increase in temperature increases the rate of evaporation. This increases the kinetic energy of molecules i.e more energetic molecules are created which escape , leaving less number of liquid molecules i.e increasing the rate of evaporation.

(c) **Nature of the liquid** or volatility of the liquid:

Volatility is the ability of a liquid to evaporate at room temperature or any lower temperature. There are two types of liquids; volatile and the non-volatile liquids.

A *volatile liquid* is one, which easily evaporates at room temperature. Most of the volatile liquids are organic compounds. Like petrol, ether, benzene, perfumes and others.

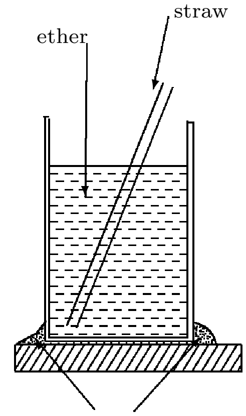
A *non-volatile liquid* is one, which does not easily evaporate at room temperature. For instance paraffin, water, diesel and others. Volatility is a measure of the strength of the molecular forces, volatile liquids have weak molecular forces.

(d) **Surface area;**

Since evaporation takes place at the surface of a liquid, then the larger the surface area the greater the number of molecules leaving the liquid surface or the greater the rate of evaporation. Hence the rate of evaporation increases when the surface area of a liquid is increased; for instance water on a tray evaporates very fast compared to when the same quantity of water is placed in a cup (why?).

3.5.4 To make ice by evaporation of ether

- Pour or sprinkle some water on a flat piece of wood
- Place a beaker half filled with ether on the wet piece of wood
- Using a straw, bubble air through the ether (ether is volatile). This increases the surface area of the liquid hence the rate of evaporation increases hence the cooling process is faster.
- After some time a small quantity of water around the beaker starts to freeze (to solidify), this is when temperature lowers to the melting point of water.



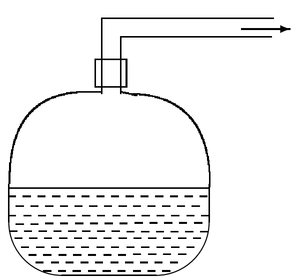
- When water an wood solidifies, we conclude that we have made ice by bubbling air in ether, when you lift the beaker it is attached to the table or piece of wood by the ice hence ice has been made by evaporation of ether.
- Note that the beaker has to be put on an insulator like wood because without it, heat will be transferred from the other parts of the conducting body to the ether preventing its temperature from lowering to low temperatures like 0°C and ice may not be formed.

Explanation using the kinetic theory of matter

When a bubble of air is passed (bubbled) through ether in a cup, the surface area of ether exposed to air increases further. When the rate of bubbling is increased (i.e. more bubbles are passed though ether) the surface area of ether exposed to air increases, increasing the rate of evaporation hence the rate of cooling is increased. Therefore the ether cools (loses heat via the ether vapour i.e. as latent heat of vapourisation). On cooling further, the ether absorbs the heat required to evaporate (latent heat of vapourisation) from the beaker and the objects surrounding it (water). Since water is on an insulator (wood), more of the heat is absorbed from this water continuously as the bubbling continues till it freezes hence forming ice.

3.5.5 Cooling by evaporation under reduced pressure

Take this to be an imaginary experiment. If i get water, put it in a container with only one outlet as shown below; When the vacuum pump is switched on for some time, what will happen?



Explanation:

On the surface of every liquid, there is a small quantity of that liquid's vapour. This vapour occupies space previously occupied by air. In this container from the top, there is air, or a mixture of air and water vapour and then the water.

When the pump is on, the air is sucked out and the vapour occupies the space previously occupied by the air. The water vapour is reduced (also sucked), this in effect causes more of the liquid to change to the vapour reducing the quantity of the liquid in it. This process continues until all the vapour is sucked creating a vacuum, but a vacuum can not exist in the presence of the liquid (recall crushing can experiment) it just changes into a vapour until all the liquid has changed to vapour.

For a liquid to change to the vapour state, energy (latent heat of vapourisation) is required. For this case; heat is absorbed from the liquid itself, hence the liquid cools. If you touch the walls of the container, it is so cold. If the temperature goes beyond the freezing point of water when a small quantity of the liquid is still there, it changes state to ice, which may remains in the container. If by the time the temperature of the liquid approaches its melting point, the whole of it is in the vapour state, the container remains empty. This experiment was performed earlier by a scientist who they claimed was boiling water in his hands.

Finally in conclusion, when air is sucked out of the container, the pressure exerted by the vapour reduces. Since this leads to more of the liquid to evaporate, then it is **cooling by evaporation under reduced pressure**. Since evaporation leads to cooling, then it is **evaporation under reduced pressure.** The refrigerator uses this principle.

Latent heat of vapourisation

It is known that a liquid becomes colder when it evaporates. Heat is required to cause evaporation, and this is called latent as before. The latent heat needed for evaporation is taken from the liquid, thus causing it to become cold.

The **specific latent heat of evaporation** of a liquid are the number of joules of heat needed to evaporate one kilogram of a liquid at a given temperature below its boiling point. This latent heat varies with temperature. For instance at a lower temperature more heat is needed to vapourise a liquid, note the specific latent heat of vapourisation of water is;

$2.5MJkg^{-1}$ at 0°C,

2.38MJkg^{-1} at 50°C and

$2.26MJkg^{-1}$ at 100°C.

The original definition of latent heat is still applicable to evaporation as the definition mentioned constant temperature. The specific latent heat measured earlier (that of vapourisation) are at the boiling point (a constant temperature). In most experiments, an approximate value of the specific latent heat of evaporation can be found and strictly speaking, this is not at a constant temperature but is within an average range of 5°C.

3.5.6 Advantages of the high latent heat of vapourisation of water

(a) The high specific latent heat of vapourisation of water accounts for the fact that rivers and lakes do not dry out rapidly in tropical climates and do not attain high temperature in very hot days. Otherwise if the lakes contained ether instead of water, during the dry seasons they would always dry up.

(b) Steam posses a large quantity of energy due to the high specific latent heat of vapourisation of water. This fact is used in various types of heat engines where this energy is changed to other forms of energy easy to use (like the electric form of energy).

(c) The evaporation of sweat is the most important factor in body temperature control of mammals. The rate of evaporation of sweat depends on the relative humidity (amount of water in air) of the atmosphere. In conditions where the relative humidity is high, the rate of evaporation is reduced, causing physical discomfort as little or negligible heat is absorbed from the body to cool it. When your body has the sweat, the sweat evaporates in turn cooling your body and you fell comfortable.

questions

1. What is meant by evaporation
2. Give two daily phenomenon that show that evaporation takes place everyday
3. Describe how the rate of evaporation can be increased
4. Describe how a wet cloth dries up when hanged on a wire
5. Explain why evaporation is always accompanied by cooling
6. Why is it that always energetic molecules leave the liquid surface first?
7. Explain why the temperature of ether lowers very first when air is bubbled in it
8. What is the difference between the specific latent heat of vapourisation and the specific latent heat of evaporation?
9. Explain the following;
   (a) On a rainy day after sometime, the water previous on the ground will not be there.
   (b) When one takes a bath from outside the bathroom he or she feels more coldness than when in the bathroom
10. Describe briefly how ice can be made by evaporation of ether
11. Explain using the kinetic theory of matter why increase in pressure exerted on a liquid lowers its rate of evaporation.(Hint: Look at increase in pressure as increase in the number of gas or vapour molecules)
12. Explain why; when water is poured on your body, you feel more coldness than when alcohol or paraffin is poured on you.

3.6 Vapours

3.6.1 Introduction

Definition 31.

A **vapour** is the gaseous state of a substance that is a solid or liquid at room temperature.

[8]. Room temperature is averagely about $20°C - 25°C$. A **gas** is not the same as a vapour, for instance we can have water vapour, sodium vapour, mercury vapour and oxygen gas or hydrogen gas and not oxygen vapour or hydrogen vapour. Liquids are of two types

(i) Volatile liquids and

(ii) Non-volatile liquids.

A **volatile liquid** is one that easily evaporates to vapour at room temperature. For instance petrol, ether and other organic compounds.

A **non-volatile liquid** is a liquid that does not easily change to a vapour at room temperature. For instance water, mercury, paraffin and others. More of the vapour can be formed when that liquid is heated because when temperature is increased, the average kinetic energy of the molecules of the liquid increases, increasing the rate of evaporation and hence more of the vapour is formed.

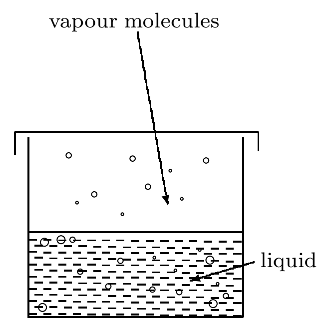
Literature review

When a liquid boils, it gives off vapour but a liquid does not have to boil to give off vapour, for instance a beaker of ether can be readily smelt, due to the presence of ether vapour even though the ether is not boiling. The vapour given off by the liquid must exert a pressure called vapour pressure. Only molecules with a large enough velocity (i.e sufficient kinetic energy), do escape to form vapour. There is a critical velocity below which molecules do not escape from the liquid surface. Increasing the temperature of the liquid raises the mean velocity and hence increases the proportion of the molecules with velocity greater than this critical velocity. This increases the number of vapour molecules or vapour pressure. If the vapour above the liquid is removed, further evaporation takes place as molecules obtain more energy from the heat supplied to the remaining liquid. This latent heat of vapourisation is absorbed from the liquid and hence it cools.

[8]room temperature is the temperature read by a thermometer when you hold it in the air in a room

Terms used in vapours

Consider a liquid in a closed container heated slowly continuously. Just above the liquid's surface there is a vapour i.e. molecules that have left the liquid.



When we continue to heat, more energetic molecules leave the liquid to the gaseous state. The vapour molecules move about in all directions and exert a pressure (vapour pressure) when they bounce off the walls of the bottle. When they strike the liquid surface, many re-enter it and eventually an equilibrium will be reached where the number of molecules leaving the liquid is equal to the number of molecules re-entering the same liquid in the same time. This equilibrium is called a **state of dynamic equilibrium** in which the rate at which molecules leave the liquid is equal to the rate at which others return to it. It is dynamic because the molecules are in a state of continuous motion. In this state the vapour is said to be saturated (saturated vapour) and before attaining this state it was an **unsaturated vapour**.

The pressure it exerts is called **saturated vapour pressure (S.V.P)** and before it was exerting **unsaturated vapour pressure.**

Definition 32.
Saturated vapour is one, which is in a state of dynamic equilibrium with its own liquid (or solid).

Definition 33.
Saturated vapour pressure (S.V.P) is the pressure exerted by a vapour in dynamic equilibrium with its own liquid or solid.

Definition 34.
Saturated vapour pressure (S.V.P) is the pressure exerted by a saturated vapour in contact with its own liquid.

Definition 35.
Dynamic state of equilibrium is a thermal equilibrium attained when the number of liquid molecules changing from liquid state to vapour state is equal to the number of vapour molecules changing to the liquid state for a given substance.

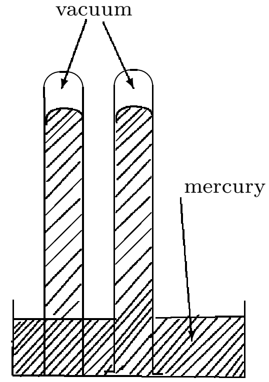
What is meant by the following terms; unsaturated vapour pressure, unsaturated vapour, and dynamic equilibrium.

Note we need to know how S.V.P can be measured, does it depend on volume of the vapour what is the effect of temperature on S.V.P? All these lead to how does a liquid boil.

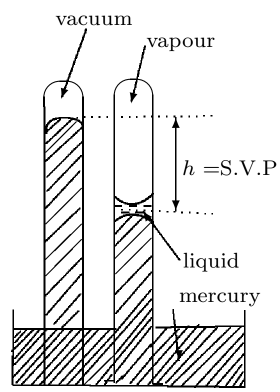
3.6.2 Measuring S.V.P of a small quantity of a liquid

An experiment to measure the Saturated vapour pressure(S.V.P) of a small quantity of a liquid

- Construct a mercury barometer with two columns of mercury as shown below;(note the barometric height i.e. atmospheric pressure)



- Using a small pipette, place a small quantity of the liquid (drop) at the bottom of one of the barometric tube. The liquid will rise up and evaporate at the top to form a vapour. (Initially this top space was a vacuum). If all the liquid evaporates then the vapour it forms is not saturated (unsaturated vapour). Add in more of the liquid till a small quantity of it remains at the top of mercury as shown below; the small quantity of the vapour is said to be saturated and the pressure it exerts is now the S.V.P..



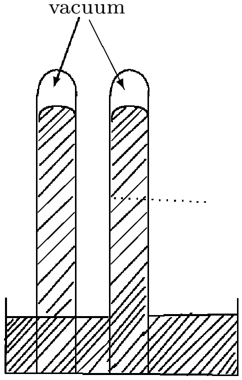
- The decrease in the barometric height (h) is the saturated vapour pressure (S.V.P) of the liquid at room temperature(or at that temperature).

- Hence the saturated vapour pressure of this liquid at this temperature (room temperature) is h mmHg.

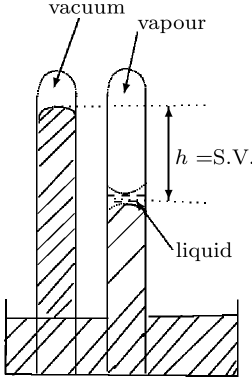
3.6.3 Does S.V.P of a vapour depend on its volume?

An experiment to show that saturated vapour pressure(S.V.P of a liquid does not depend on the volume of the saturated vapour of that liquid

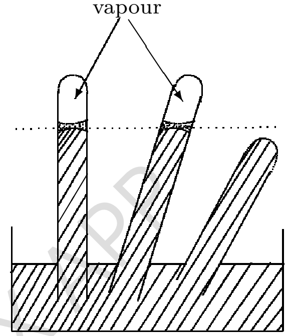
Construct a mercury barometer with two columns of mercury as shown below;



Using a small pipette, place (or drop) a small liquid at the bottom of one of the barometric tubes. The liquid rises up and evaporates to a vapour, exerting pressure. Add in more liquid till a small quantity of it remains at the top of mercury column to make it saturated (or to confirm the pressure exerted by the vapour is S.V.P. this causes the mercury height to lower, the decrease in the height is its S.V.P.



When we bend the tube, the pressure finds its level i.e. the mercury rises to nearly fill the tube (why?) because as the volume decreases the excess vapour condenses back to the liquid and the vapour pressure returns to its initial value (saturated vapour pressure) i.e. S.V.P remains constant. Hence **S.V.P. does not depend on the volume of the saturated vapour**.



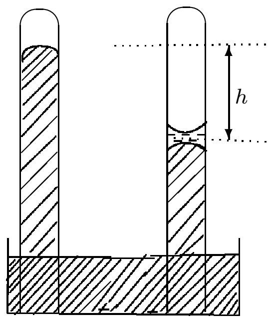
3.6.4 Effect of Temperature on S.V.P.

For a given liquid at a constant temperature, its saturated vapour pressure is always the same, irrespective of the existence of air above or in the vapour. We used the kinetic theory to explain how molecules escape from the liquid. If a liquid is heated, the energy (heat supplied) which goes into it becomes the mechanical energy of the molecules (i.e micro-mechanical energy). More and more of the molecules gain enough kinetic energy to enable them escape from the attraction of their neighbours and eventually escape from the liquid (i.e. evaporate).

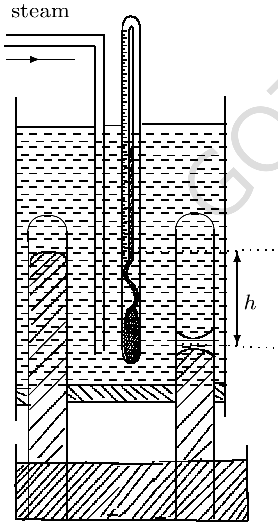
A rise in temperature is therefore accompanied by an increase in the rate of evaporation. Since vapour pressure (S.V.P) is due to the bombardment of the vapour molecules with the walls of the container, then an increase in the number of vapour molecules due to a temperature rise, increases the vapour pressure (S.V.P). Therefore saturated vapour pressure increases with temperature.

An experiment to show how S.V.P varies with temperature.

Construct a mercury barometer with two barometric tubes arranged in a water bath as shown below;



Drop a small quantity of the liquid at the bottom of one of the barometric tubes. The liquid rises to the top where it evaporates to form a saturated vapour, if a small quantity of it remains at the top of the mercury column. The decrease in the barometric height is measured as the saturated vapour pressure, record the reading of the thermometer (temperature of the water or vapour). The diagram below is used for higher temperatures;

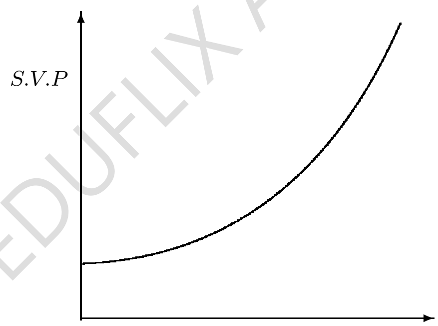


Steam is bubbled in the water ensuring that a small quantity of the liquid remains on top of the mercury thread, measure the temperature and saturated vapour pressure of the vapour. Repeat this for various temperatures and record your results as shown in table 3.8 below;

Temperature(°C)	S.V.P.(mmHg)
...	...
...	...
...	...
...	...

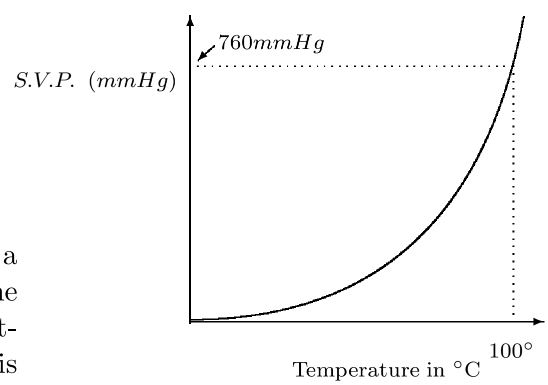
Table 3.8: S.V.P and temperature

Plotting these values, we have



The graph shows that S.V.P. increases with an increase in temperature (not linearly).

Ideally the graph is



3.7 Boiling

3.7.1 Introduction

When a liquid is heated at a constant rate, its temperature begins to rise, therefore its S.V.P will also increase. On continuous heating, ultimately the S.V.P. becomes equal to the external atmospheric pressure. At this stage, further addition of heat causes bubbles of vapour to form inside (the body of) the liquid, which rise to the surface of the liquid. This process is called boiling or ebullition. It occurs at a constant temperature known as the boiling point of the liquid.

Definition 36.
Boiling is the process by which a liquid changes into a vapour or gas when its saturated vapour pressure is equal to the external pressure (atmospheric pressure).

Definition 37.
Boiling point of a liquid is the temperature of the vapour from the liquid, when its saturated vapour pressure becomes equal to the external atmospheric pressure.

A graph of temperature against S.V.P is the same as the graph of Boiling point against S.V.P.

Bumping in boiling

Note that when heat is supplied to a liquid, evaporation takes place at a more rapid rate. When a liquid boils, "rapid evaporation takes place in all parts of the liquid": this occurs when the liquid vapour on the liquid exerts pressure equal to the external pressure acting on the liquid surface. Recall that evaporation only takes place from the surface of the liquid.

Boiling depends up on the ease with which the bubbles of vapour can form in the liquid. In a perfectly smooth or silvery container or vessel, bubbles of vapour form with difficulty, when a bubble does form, it develops rapidly and causes the liquid to boil bumping. The violence with which the bubble is expelled from the liquid causes the vessel to bump. Pieces of porous pot are usually added to liquids in glass flasks or other smooth vessels to provide a rough surface on which bubbles can form and thus prevent bumping. Containers other than glass vessels usually have a rough enough surface to permit a liquid to boil smoothly.

3.7.2 Factors affecting boiling

There are two major factors that affect boiling or boiling point

(i) Impurities in the liquid and

(ii) Pressure exerted on the surface of the liquid.

Effect of impurities on boiling or boiling point

Impurities are of two types; soluble impurities and insoluble impurities. When an impurity is in a certain substance it increases it boiling point. For instance, a water solution boils at a temperature higher than 100°C. [9]

Effect of pressure on boiling point.

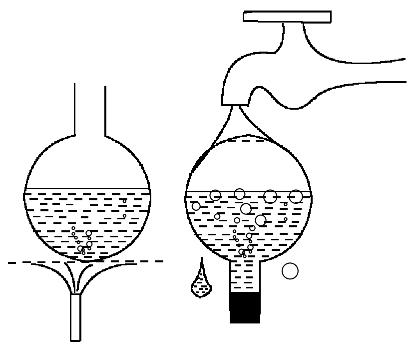
When a liquid is exposed to high pressure, for it to boil its S.V.P. has to be equal to this high pressure. Since saturated vapour pressure increases with temperature for the liquid to attain that high pressure it has to be at a high temperature (its boiling point), hence pressure increases the boiling point of a substance. That is why at the top of mountains foods takes longer to be ready because water boils at a lower temperature. (Recall pressure decreases with altitude).

[9] Details are discussed in advanced chemistry where you talk of boiling point elevation.

3.7.3 Boiling under reduced pressure.

An experiment to illustrate boiling under reduced pressure

1. half-fill a round-bottomed flask with water.
2. Boil the water until steam ensues or moves out freely and almost all the air is driven out.
3. stop heating and cork the flask.
4. Now turn the flask upside down and place it under a stream of water from a tap.
5. When the flask cools, note that the water boils.

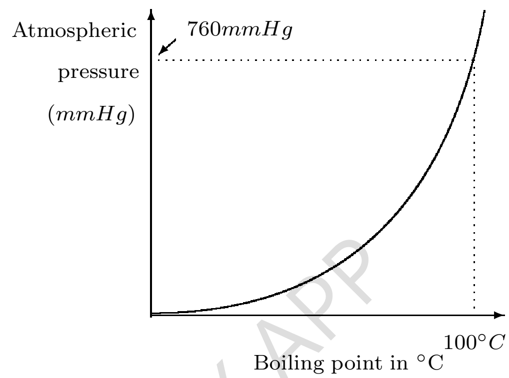


Explanation

Cooling the flask cause the water vapour to condense. As there is very little air in the flask, the condensation of steam creates partial vacuum and lowers the pressure. The water boils at reduced pressure. Hence lowering the external pressure lower the boiling point of water.

3.7.4 Variation of boiling point with the external pressure

S.V.P. is constant at a given temperature, which is the boiling point when S.V.P. is equal to atmospheric pressure. Then boiling point increases with atmospheric pressure (or external pressure). Pure water at a pressure of 760mmHg boils at 100°C, water at a pressure of 700mmHg boils at a lower temperature (lower than 100°C). Since our area or region is not at sea level (atmospheric pressure is less than 760mmHg). But water does not always boil at 100°C but at a certain lower temperature like 97°C or 98°C; Recall atmospheric pressure decreases as one rises up a hill.



On the graph in the region where pressure is 760mmHg, the S.V.P of water changes by approximately 0.037Kelvin per mmHg change in pressure.

3.7.5 Can water boils at room temperature

Water can be made to boil without heating it simply by reducing the atmospheric pressure above it to a value less than its S.V.P. at that temperature, this may be done with the aid of a vacuum pump or filter pump. In this case water-cools as it boils (this was explained in cooling by evaporation).

Therefore water can boil at room temperature at reduced pressure and in the process it cools. In general a liquid boils at a given temperature when its S.V.P at that temperature is equal to or greater than the external pressure (the pressure acting on the surface of a liquid) which is always atmospheric pressure when the liquid is in the open, but in a closed space, this pressure can be reduced using a vacuum pump

3.7.6 Application of boiling at reduced pressure

(In summary this helps to boil liquids at low temperatures or increase the boiling point of liquids). These applications include;

1. **Desalination.**

   This is the process by which salt is removed from salty water. This is used in the extraction of sodium chloride (common salt mined at lake katwe in Uganda).

2. **Distillation.**

   Boiling under reduced pressure is used in the rapid distillation of seawater in ships so as to obtain drinking water.

3. **In the refrigerator**

   There is a liquid in a copper tube which boils at a certain point under reduced pressure (reduced by the electric pump), since this boiling causes cooling (in the freezing box), hence the refrigerator cools objects in the freezing box.

4. **In the manufacture of condensed milk**;

   Boiling or cooking milk causes it to clot and it thus changes its nature. Condensed milk is made by cooking milk at a reduced pressure, this removes some of the water, forming condensed milk, but does not cause the milk to clot as the temperature is too low to cook the milk. Powdered milk is condensed milk.

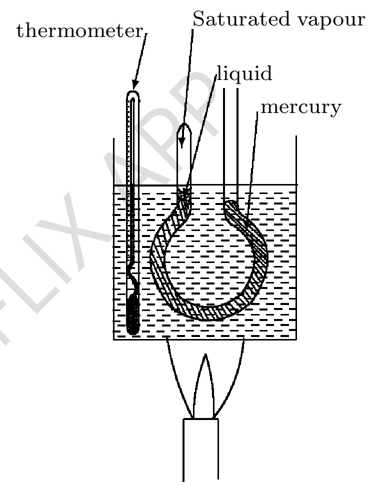
5. **Pressure cookers**.

   This one uses higher or increased pressure. It helps to cook food in a very short time. It consists of slots with an air tight fitting lids in which a valve is mounted. Water is put in the cooker, and food in baskets above the water. The water is boiled, when steam issues freely from the valve, the valve is closed. The water now boils at a much higher temperature due to the excess pressure and food is cooked in 2 to 4 minutes.

3.7.7 Determine the boiling point of a small quantity of a liquid.

An experiment to determine the boiling point of a small quantity of a liquid.

Arrange the apparatus as shown below; The levels of mercury will not be the same at the closed end, it should be higher than at the open end.

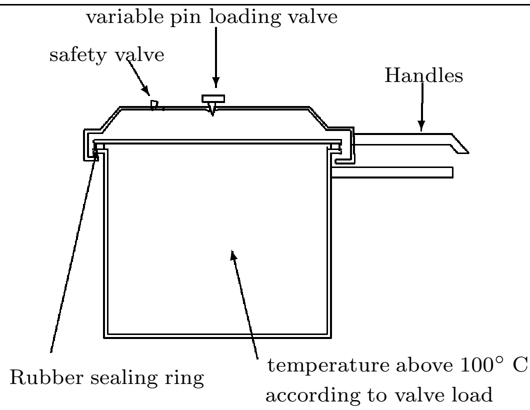


Heat the water in the beaker till the mercury levels are the same. At this point record the temperature of the vapour(equal to the temperature of the water bath). This is the temperature when the saturated vapour pressure is equal to the external pressure and it is the boiling point of the liquid.

3.7.8 The pressure cooker (digester)

The time required to cook food can be greatly reduced if the boiling point of water is raised. This is done by the pressure cooker.

Structure of the pressure cooker.



The pressure cooker has the following parts as shown above;

A variable loading pin valve; this tries to keep the pressure inside high, depending on the load. A smaller load keeps less pressure inside (and water boils at a lower temperature) and a heavy load keeps a high pressure inside hence higher boiling point.

A rubber sealing ring;it keeps an air tight contact to ensure that any air(vapour) lost passes via the pin valve. The safety valve is to ensure that pressure inside is below a certain value. This is because at a very high pressure, the pressure cooker can cause an accident incase it burst.

Action

The pin-valve with the load keeps the vapour inside at a much higher pressure compared to the external pressure. Since vapour pressure increases with boiling point, water will boil at a much higher temperature i.e. 120°C, hence less time will be required to cook food in it.

Why are pressure cookers commonly used in mountainous regions

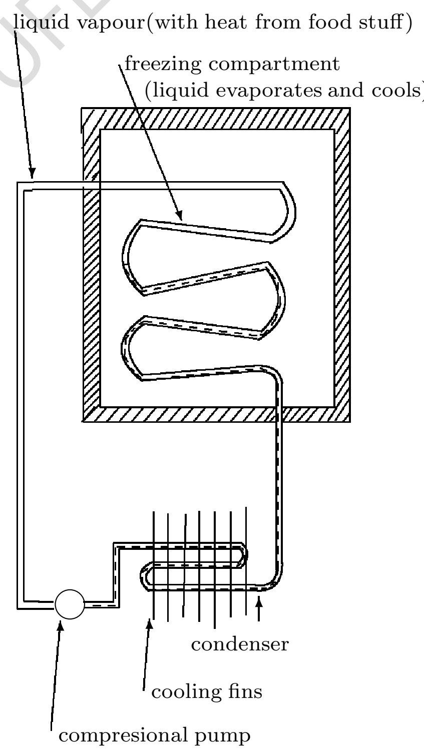
This is because, on mountains the atmospheric pressure is less than its value in the valleys, since pressure is proportional to boiling point, On the mountain water boils at a lower temperature. so food takes longer to be ready. When the pressure cooker is used, it increases the pressure in it hence increasing the boiling point of water, therefore less time is required to cook food. thats is why they are commonly used in places with less atmospheric pressure.

3.7.9 The refrigerator

A refrigerator is a device in which a volatile liquid evaporates inside a copper coil that moves around the freezing box (where foods and beverages are placed to be cooled). The refrigerator is made up of three parts;

1. the pump
2. the freezing box
3. the condenser

These are shown in the diagram below;



The pump;

This compressor pump operated by electricity is used to suck the vapour of the liquid (refrigerant) from the copper coil in one direction reducing vapour pressure so as to increase the rate of evaporation (rate of cooling) and compressing it on the other side (the condenser side). The electric current through this pump is controlled by a thermostat i.e. the rate of evaporation is controlled by a thermostat, which switches on and off the pump at intervals which can be set on a certain temperature scale. This pump, pumps the vapour to the condenser where the whole amount of heat from the freezer or foodstuffs (or heat in the vapour) is dissipated to the atmosphere.

The freezing box (ice box) or evaporator

This is a box like compartment surrounded by copper coils with the liquid surface some where in these coils. The electric pump removes this vapour on the liquid (refrigerant) as soon as it is formed. (This reduces the pressure above the liquid) under the reduced pressure, the liquid evaporates or boils rapidly. The necessary latent heat of vapourisation required to change the liquid to the vapour state is absorbed from the liquid itself which cools (i.e. the liquid evaporates at the expense of its own internal energy and hence it cools). This liquid absorbs more heat from the food staff placed in the freezing box, which also cools.

The condenser

This is the continuation of the copper coil back to the freezing box. It is in this condenser where the vapour is compressed. When the vapour is compressed it condenses giving out the absorbed heat (latent heat of vapourisation). The condenser is air-cooled

Note that some refrigerators may use a gas flame or other means instead of a pump but the principle remains the same, cooling by evaporation under reduced pressure. The refrigerator is heavily lagged with insulating material to prevent heat gain from the surrounding. The refrigerant normally used is Freon. This substance is readily liquefied by pressure, a necessary condition for the refrigerant as it passes through the condenser. The latent heat of vapourisation required to evaporate the volatile Freon is absorbed from the Freon itself and the nearby foodstuffs. This latent heat is then given out to the condenser as the Freon vapour condenses. Ammonia, carbon-dioxide and sulphur dioxide have been used as refrigerants.

In commercial refrigeration, the evaporator is placed in a tank containing a concentrated solution of brine (salt solution). This salt solution freezes at a temperature below 0°C. The cold brine solution is then pumped round the cold store or hold of a ship.

3.7.10 Volume changes on solidification

Does the volume of a substance change when it solidifies? Most substances contract when they solidify, for instance paraffin wax, candle wax and others. Other substances expand when they solidify, for instance water and the molten type metal. For water $100cm^3$ of water becomes $109cm^3$ of ice i.e the percentage increase is 9.0%. That is why; ice floats on water or ice is less dense than water. Water pipes burst during the winter due to the same reason.

Effect of pressure on the change of state

When a liquid is boiled in a beaker or test tube, the external pressure, pressure above the liquid is always the atmospheric pressure. Pressure affects the boiling point of liquids but does pressure have any effect on the melting point of solids?

3.7.11 Effect of pressure on melting point of a substance

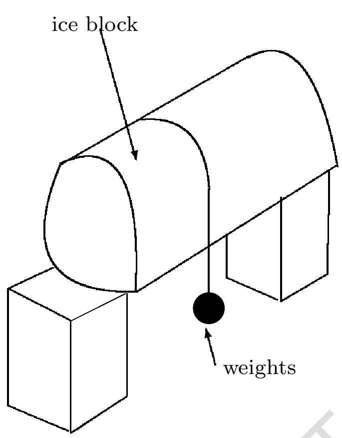
Note that the melting point of a substance can change because of the two reasons;

(i) The impurities in the substance and

(ii) the pressure acting on it

An experiment to illustrate the effect of pressure on the melting point of water (ice)

- Support a block of ice between two retort stands.
- Attach to each end of the copper wire a 4kg weight
- Place the wire over the ice block
- Observe the wire, it slowly sinks through the ice and the ice freezes over and melt below the wire.
- Also observe that the wire eventually cuts through the block of ice and the block of ice is not separated into two parts i.e ice was melting and re-freezing (solidifying).



This experiment shows that; for the wire to pass through the ice-block, the ice-block must melt. Ice at atmospheric pressure of $760mmHg$ melts at $0°C$, i.e. under pressure, water remains liquid at 0°C but the ice block is not at exactly 0°C but at about $-2°C$, and the ice or ice block melts at this temperature, a lower melting point, hence application of pressure lowers the melting point and the wire cuts through the ice block. After the wire has passed through the ice-block, pressure is no longer applied on water above it, and the temperature is less than $0°C$ hence the water freezes back (not at $0°C$)to reform ice, this re-freezing is called **regelation**. In doing so it gives out latent heat and this heat is conducted down through the wire to provide heat for further melting of the ice beneath it, hence the wire cuts through the block which is not separated into two pieces. Pressure thus lowers the melting point of ice.

The effect of pressure on the melting point of water differs from other substances. Water expands on freezing. Other substances contract on freezing and for these, pressure raises their melting point. Therefore application of pressure on some thing which expands on solidifying like water, lowers its melting point and the application of pressure on some thing which contracts on solidifying rises its melting point (like paraffin). The effect is small, i.e. a very large pressure is needed to alter the melting point of a substance by a small difference in temperature. Freezing point of water is lowered by 0.007K per atmospheric increase in pressure and that of paraffin is risen by 0.04K per atmospheric decrease in pressure. Compare this with the effect of pressure on boiling point, a small change in pressure produces a much greater effect on the boiling point of a substance.

Note that a thin string of high thermal conductivity (made from a good conductor) will cut through the ice block faster and an iron wire of smaller thermal conductivity cuts through the ice block at a low rate(slowly). A wire made from an insulator like a sisal rope can not work(why?)

How are snowballs made?

Regelation (re-freezing) is an important factor in the making of snowballs. Pressing snow by your hand causes slight melting of the ice crystals, and when the pressure is removed re-freezing occurs and binds the snow together. In very cold weather the pressure exerted is insufficient to melt the snow and so it fails to bind.

Why is ice slippery or why is ice-skating or ice skiing possible?

This is because of a thin film of water formed between the ice and the foot of the person standing on it (skater, e.t.c.). the ease with which a skater glides over the ice depends on this. What brings this water is not yet clear! It is not as a result of melting under reduced pressure, because a simple calculation shows that the pressure exerted by a skater is about 1000kPa and this would lower the melting point by less than 0.1K yet skating is possible even when the temperature is several degrees below zero. It was pointed out by F.Bowden and later J. Fremlin, that the water film in this case, is more likely to be brought about by the work done against friction. This work becomes transferred to internal energy and the ice melts as a result.

Explain the following;

1. It takes a longer period to cook food on mount Kenya than at Mombasa town.
2. It takes a short time to cook Salted food than unsalted food?
3. A dog keeps itself cool by sticking out its tongue and breath rapidly across it?
4. When air is blown through ether in a can standing on a wet wooden block, after some time it is found that the block and the can stick together?

3.7.12 Humidity

When air is dry, it tends to absorb a lot of water, lips of people dry and break, pieces of wood tend to become brittle e.t.c. This dry air increases the rate of evaporation of water where it passes. This water (evaporated water) turns into water vapour in the air. There is a fixed quantity (mass) of water vapour that a given quantity or volume of air can hold at a given temperature, it decreases as temperature is increased i.e. air at a higher temperature can hold little water vapour.

Since it is not easy to measure the quantity of water vapour present in the air and it is not so important to know this quantity, what is important is how close to saturation air is. Saturated air is one that can not absorb or retain any more water, this closeness of the air to saturation is measured as *relative humidity*. Air which contains a lot of water vapour is called **humid air**. **Humidity** is a measure of the amount of water vapour actually present in air. By definition

Definition 38.

Relative humidity is the ratio of the mass of water vapour in a given volume of air to the mass of water vapour required to saturate the same volume of air at the air temperature.

If m_{vapour} is mass of a water vapour in a given volume of air and $m_{tosaturate}$ is the mass of water vapour required to saturate the same volume of air at a given air temperature then

$$\text{Relative humidity} = \frac{m_{vapour}}{m_{tosaturate}} \tag{3.51}$$

Water has a definite vapour pressure (its S.V.P.) at a given temperature. If the vapour pressure of water vapour in the atmosphere has the value of its S.V.P. at that temperature, then the air is saturated with water, i.e. humidity (or relative humidity) is 100%. In these conditions water will no longer lose bulk by evaporation, as the rate of condensation is the same as the rate of evaporation, hence a pool of water will remain the same size since there is no net loss of water by evaporation.

The atmosphere is usually unsaturated with water vapour and the degree of unsaturation is measured as the relative humidity in terms of the ratio of vapour pressures. The fact that air is unsaturated; with a table of S.V.P. with the corresponding temperature, the degree of unsaturation can be obtained with accurate dew temperature measurements.

Dew point

Definition 39.

Dew point is the temperature at which water vapour present in air is just sufficient to saturate it.

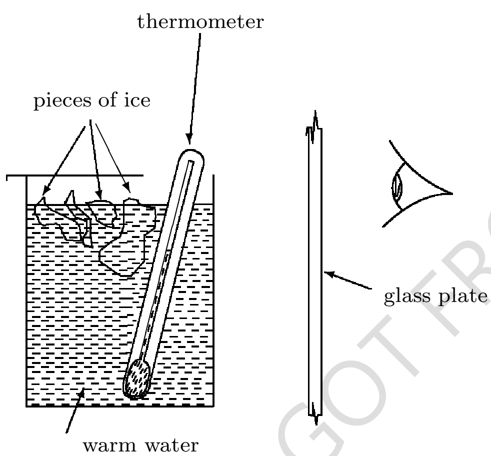
At this temperature any excess vapour present in air just condenses back to water. That is why the outside of a cup of ice is always wet.

Experiment to determine the dew point.

Get an empty milk tin with a clean shiny surface, put in water at a temperature about 3°C to 5°C above room temperature.

Observe the shiny surface through glass plate so that no condensation takes place on the surface from your breath.

Add pieces of ice to the water in the tin and stir continuously until the surface of the tin becomes misty or till dew is firmed on it, read the temperature immediately dew is formed. This temperature is the dew point.



Explanation

Note that air, when cooled reaches a temperature at which it is saturated. This temperature is called the dew point, the saturated air contains all the water vapour it can retain. When temperature of saturated air at dew point is reduced, water vapour condenses back to water from the air and forms dew (this explains how rainfall is formed and the water always found on the leaves of plants in the morning). Then humidity (relative humidity) can be measured in terms of the dew point, as the ratio of S.V.P. of air at dew point to S.V.P. of the same air at room temperature. Relative humidity , $R.H$

$$R.H = \frac{\text{S.V.P of water at dew point}}{\text{S.V.P of water at the original air temperature}} \quad (3.52)$$

For instance, suppose that experimental observations give dew point of 21°C with a room temperature of 25°C. Since the S.V.P. of water is 2.48kpa at 21°C and 3.16kpa at 25°C, then; Relative humidity, $R.H.$ is given by;

$$\begin{aligned} R.H. &= \frac{\text{S.V.P at dew point}}{\text{S.V.P. at room temperature}} \\ &= \frac{2.48kpa}{3.16kpa} \times 100\% \\ &= 78\% \end{aligned}$$

The atmospheric conditions were room temperature 25°C and vapour pressure 2.48kpa. When the air was cooled to 21°C, the vapour pressure of water remained constant and the air became saturated at 21°C, whence dew was deposited at that temperature. If the same air were warmed to 30°C, what would happen to the humidity? The dew point would be the same, but the room temperature would be 30°C(S.V.P. of water at 30°C is 31.8mmHg); then

$$\begin{aligned} \text{Relative humidity} &= \frac{18.8\text{mmHg}}{31.8\text{mmHg}} \times 100\% \\ &= 58\% \end{aligned}$$

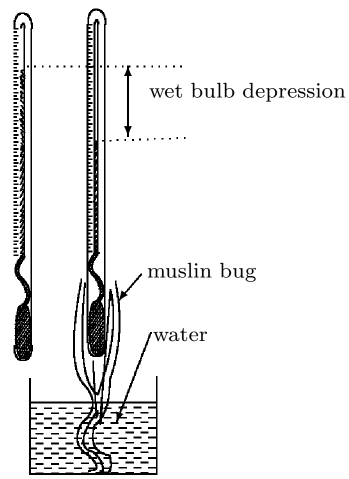
i.e. warming air reduced the humidity. A low relative humidity leads to a high rate of evaporation hence dryness.

Determination or measurement of relative humidity

There are very many methods of measuring relative humidity by instruments called hygrometers (not hydrometers). The most common one is the wet and dry bulb hygrometer.

Wet and dry bulb hygrometer

These are two thermometer set up with the bulb of one closed with a wet cloth dipped in water (not ether) and that of the other in air. The temperature of the two thermometers are measured and used to calculate relative humidity using a table of humidity factors that gives approximate values of relative humidity.



This hygrometer is usually used for meteorological purposes and measures humidity to a reasonable degree of accuracy. The smaller the difference in their readings, the greater the humidity.

Theory

The bulb surrounded by a wet cotton wool or cloth has a lower temperature due to evaporation (evaporation cools it). If the relative humidity is 100% i.e. the air is saturated with water vapour, then the two thermometers will have the same reading, there will be no cooling effect as there is no evaporation taking place from the water on the cotton wool. The difference in temperature between the two thermometers will thus depend upon the relative humidity. The wet and dry bulb hygrometer is used for quick measurement of relative humidity. Table 3.9 of humidity factors gives approximate values.

		Dry bulb reading			
		20°	25°	30°	35°
Wet bulb reading	20°	100	65%	40%	24%
	22°	-	80%	750%	30%
	24°	-	95%	60%	35%
	26°	-	-	80%	45%
	28°	-	-	90%	57%
	30°	-	-	100%	70%
	32°	-	-	-	82%
	34°	-	-	-	95%

Table 3.9: Table of humidity factors

Humidity control and air conditioning

Need for humidity control

Humidity is either a disadvantage or an advantage depending on the activities done by man. Humidity control is required in the following;

(a) In cotton industries. This is because in dry conditions cotton fibres become brittle and may break ocassionally especially in spinning industry where there is electrification by friction.

(b) In electrical appliances factories where a dry atmosphere is needed especially where they are assembled.

(c) In food factories especially driers, maize mills e.t.c. to ensure durability of food

(d) In wood industry. During seasoning where wood is to be dried by circulating dry air through the wooden pieces.

There are various ways of controlling humidity. Sometimes it is called air conditioning. It is commonly used in ships, theaters, concert halls, e.t.c.

Air conditioning

This is the process of increasing or reducing humidity of air for a certain purpose. Usually air conditioning aims at reducing the humidity (to about 60%) and lowering room temperature by about 2°C.

A simple domestic air conditioner essentially consists of a system for a refrigeration cycle where air is passed over an evaporator (to cool it) and blown into a room. It is really essential to have an air-conditioned room. Consider this case; the atmospheric condition in a room at a temperature of 30°C and relative humidity 95%, such a condition is uncomfortable owing to the high humidity i.e. sweat does not evaporate, when humidity is reduced, sweat can then evaporate to cool the body. The vapour pressure at that temperature is 30.1mmHg, this corresponds to the dew point of 29°C. Air is drawn into the air- conditioner and cooled to a temperature of 20°C, the water is condensed from the air at the evaporator and drains away. The result is cool air (at 20°C) which is saturated with water vapour. The S.V.P of water at 20°C is 17.5mmHg. The cold air passes into the room and is warmed up to 28°C, it's water content remains the same, but relative humidity now is 62% (i.e.

$$
\begin{aligned}
R.H &= \frac{\text{S.V.P. at}20°C}{\text{S.V.P. at}28°C} \\
&= \frac{17.5mmHg}{28.3mmHg)} \times 100\% \\
&= 62\%.
\end{aligned}
$$

This produces atmospheric conditions which are more tolerable.

Natural phenomenon due to evaporation

Humidity measurements are important factors in weather forecasting, as humidity is the cause of many natural phenomena.

(a)Dew

During the night, the air temperature drops as the earth cools, and if the temperature falls below the dew point, then dew is deposited on grass and other plant leaves. The lower the temperature fall, the more the quantity of dew formed. This shows that in places where a lot of dew is formed the night temperatures are low and the air contains more water vapour.

(b)Clouds

Warm humid air rises from the ground and gradually becomes cool as it ascends. If the temperature of the air falls below the dew point, condensation takes place and clouds are formed. (These clouds can stay in space with the help of upthrust - Archimedes principle). The cloud base is at a height where the temperature is that of the dew point. In some seasons air near the ground in the morning may be still at dew point, in this case the clouds can reach the ground and the whole atmosphere is full of mist (a white stuff in the atmosphere) but when the sun rises it disappears. Sometimes warm air blowing on the ground may be forced to raise over a mountain, this cools the air and forms a cloud if the air had water vapour.

(c)Mist

A mist is the cloud formed at ground level by the condensation of water into droplets in the air. Rapid cooling produces a mist. That is why when one opens the door of a refrigerator, he or she observes mist (like a white smoke) moving from it or when you hold a very cold liquid like a bottle of soda or ice you observe mist moving or rising from it.

(d)Fog

Fog is formed when a mist condenses in a dusty town (or just a town) or any other dusty locality. When water condenses on a dust or sooty particles present in air, the atmosphere becomes more opaque to light. That substance causing this opaqueness is called fog.

(e)Rain

If condensation is sufficiently great, the water droplets in a cloud become too heavy for the upward air currents and up-thrust to support them. The droplets then fall as rain.

Exercise

1. The atmospheric conditions on a certain day were temperature 20°C, relative humidity 90%. A building on the same day had the following conditions; temperature 23°C, relative humidity 65%. Explain how the conditions in the building were made different from the atmospheric conditions.

2. Benzene is a substance with a boiling point of 80°C and a melting point (or freezing point) of 5°C.

   (a) is benzene a solid , liquid or gas at room temperature(23°C)

   (b) Describe the apparatus you would use to determine the melting point of pure benzene

   (c) Beginning with liquid benzene, sketch the expected graph of temperature against time starting from 23°C and indicate on it the melting point of benzene

   (d) Give reasons for what is observed at the melting point of benzene

   (e) Benzene is an inflammable liquid. How would you determine the boiling point of pure benzene

   (f) beginning with liquid benzene at 23°C, draw a graph to show the change of temperature with time and indicate on the graph the boiling point of benzene

   (g) Naphthalene is soluble in benzene. If 5g of naphthalene were dissolved in $100cm^3$ of benzene. What effect would this have on the boiling point of pure benzene

   (h) A specimen of benzene is found to have a boiling point of 79.5°C. What explanation would you give as to why the boiling point is not 80°C?

3. A characteristic of certain cooking pots is that when they are removed from the source of heat, the contents may continue to boil for a short period of time. Suggest an explanation for this effect?

4. Explain briefly how the following are formed

   (i) Dew

   (ii) Mist

   (iii) Clouds

   (iv) Rain and

   (v) Fog

5. (a) what is meant by the terms humidity, relative humidity, humid air and saturated air

   (b) Explain briefly why one bulb of the hydrometer is kept with a wet cloth

   (c) Define humidity in terms of S.V.P

6. What is the effect of pressure on melting point and boiling point of a substance

7. Describe an experiment to show that high pressure lowers the melting point of ice

8. Explain briefly the following

   (i)How snow balls are made

   (ii)Why ice is always slippery

   (iii)A dog keeps its tongue out on a hot day

   (iv)It takes a short time to cook salted food compared to unsalted food

9. Explain briefly how a refrigerator works. What is the purpose of the following to a refrigerator

   (a)condenser

   (b)pump

   (c)Cooling fins

10. Describe an experiment to determine the S.V.P of a small quantity of a liquid

11. What is the effect of impurities in a substance to its melting point and boiling point

12. (a) Define the terms boiling and evaporation

    (b) state the differences between boiling and evaporation

    (c) Describe an experiment to determine the boiling point of a small quantity of a liquid

13. (a)Define the terms saturated vapour, saturated vapour pressure and unsaturated vapour

    (b) When is a liquid or vapour said to be in dynamic equilibrium

    (c) The S.V.P of air at 20°C and 29°C are 17.5mmHg and 30.1mmHg respectively. Saturated air at 20°C is heated to 29°C , determine the humidity of air at 29°C?

    (d) Explain why S.V.P does not depend on the volume of the saturated vapour

    (e) Explain why on a rainy or cold day ones nose it sometimes wet yet He/She is not suffering from flu?

3.8 Gas laws

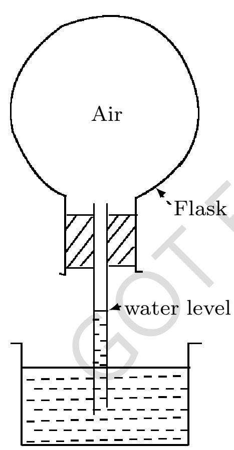
3.8.1 Introduction

The properties of gases are based on four quantities mass (or number of moles), volume, temperature and pressure. The behaviour of gases when these quantities are varied differently constitutes this topic; gas laws. There are three basic gas laws; Boyle's law, Charles law and pressure law, all of which can be combined in one equation, the general gas equation.

3.8.2 Boyle's law

Galileo's experiment

Galileo Galili attempted to make an air thermometer, but it could not work well (why?) He arranged an apparatus as shown below;

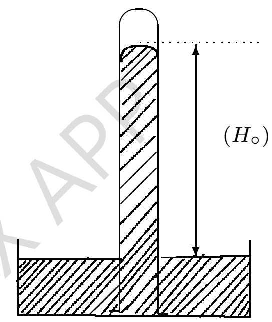


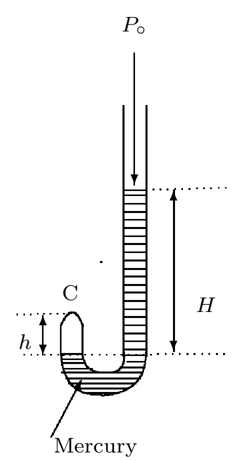
Placing warm hands over the flask, the air inside the flask is heated and expands. Bubbles are seen oozing out of the tube. When the hands are removed, the air in the flask contracts and the water rises in the tube as shown above. Now it can act as a thermometer, because subsequent changes in temperature will causes the water to either rise or fall. This thermometer could not be reliable because pressure changes could affect it i.e when the atmospheric pressure increases , the height of water in the tube will be greater.

Once Robert Boyle was interested in the volume changes of a gas as its pressure is varied at constant temperature. He used the apparatus in the experiment described below;

An experiment to verify Boyle's law

Get a J-tube and pour mercury in it so as to trap some air in it as shown below;





The air at place C has volume, V, given by

$$\begin{aligned} V &= \text{cross sectional area} \times \text{height} \\ &= Ah, \text{ since A is constant} \end{aligned}$$

we shall measure h to represent V

The net pressure or gas pressure, P, exerted by the trapped gas is the sum of atmospheric pressure, $P_\circ$, and pressure, $h\rho g$, due to the mercury column is given by

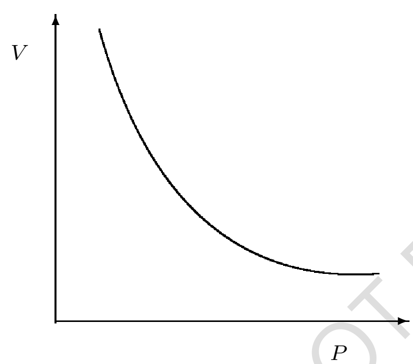
$$\begin{aligned} P &= P_{\circ} + H\rho g & (3.53)\\ &= H_{\circ}\rho g + H\rho g & (3.54)\\ &= (H_{\circ} + H)\rho g, & (3.55)\end{aligned}$$

$H_{\circ}$ is the constant barometric height
More values of h and H are obtained by adding in more mercury and recording the values as shown in table 3.10;

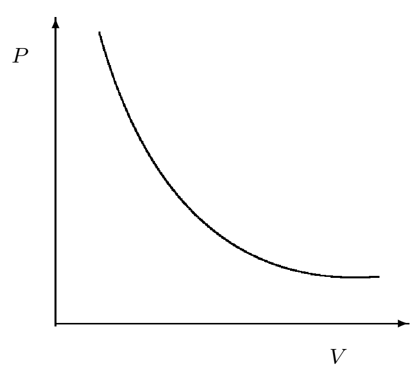
h	$H_{\circ}$	V(= Ah)	P	$\frac{1}{P}$

Table 3.10: Boyle's law

Plotting a graph of V against P we obtain a graph like this below

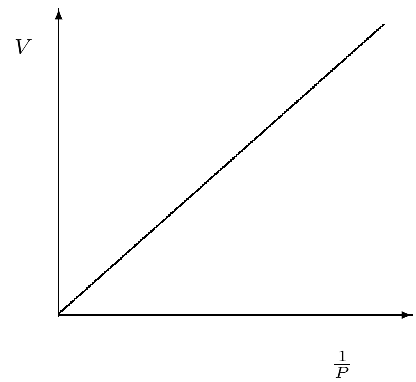


if it was a graph of P against V we would obtain



this shows that V varies inversely as P

Plotting V against $\frac{1}{P}$, we get (a straight line as shown)



this shows that Volume is directly proportional to the reciprocal of pressure

$$\begin{aligned} V &\;\alpha\; \frac{1}{P} & (3.56)\\ V &= \kappa\frac{1}{P} & (3.57)\\ PV &= \kappa & (3.58)\end{aligned}$$

where κ is a constant of proportionality

This relation is known as Boyle's law,

Law 4.

Boyle's law states that the pressure of a fixed mass of a gas is inversely proportional to its volume provided its temperature remains constant.

Mathematically

$$\begin{aligned} PV &= k, \text{ a constant i.e.}\\ P_1V_1 &= P_2V_2 = P_3V_3\end{aligned}$$

Kinetic theory explanation of Boyle's law

At constant temperature, the average kinetic energy of the molecules is constant. Geometrically, if the volume of a fixed mass of a gas is halved, the number of molecular impacts per second per unit area on the container walls is doubled (i.e. pressure is doubled). If the volume is reduced by a third, the molecular impacts increase three times (i.e. pressure is tripled). This confirms the experimental result that the pressure of a fixed mass of a gas at constant temperature is inversely proportional to its volume, which is Boyle's law.

Examples

1. The weight of a litre of air at $0°C$ under pressure of 760cm of mercury is 1.29g. What volume does this air occupy under a pressure of 19cm of mercury at the same temperature?

solution

from the question

$$\begin{aligned} V_1 &= 1\ litre \\ P_1 &= 760cmHg \\ T_1 &= 0°C \\ P_2 &= 19cmHg \\ V_2 &= ? \end{aligned}$$

Using Boyle's law since temperature is constant

$$\begin{aligned} P_1V_1 &= P_2V_2 \\ P_2V_2 &= P_1V_1 \\ V_2 &= \frac{P_1V_1}{P_2} \\ &= \frac{76 \times 1}{19} \\ &= 4\ litres \end{aligned}$$

2. A fixed mass of air has a volume of $12m^3$ at a pressure of $3.6 \times 10^5 Pa$. If the volume becomes $8cm^3$, determine its pressure in Pa at the same temperature?

Solution

from the question

$$\begin{aligned} V_1 &= 12cm^3 \\ P_1 &= 3.6 \times 10^5 Pa \\ V_2 &= 8cm^3 \\ P_2 &= ? \end{aligned}$$

Using Boyle's law, since temperature is constant

$$\begin{aligned} P_1V_1 &= P_2V_2 \\ P_2V_2 &= P_1V_1 \\ P_2 &= \frac{P_1V_1}{V_2} \\ &= \frac{3.6 \times 10^5 \times 12}{8} \\ &= 5.4 \times 10^5 Pa \end{aligned}$$

Exercise

1. The pressure of 2 litres of oxygen is reduced from 4 atmospheres to 1 atmosphere at a constant temperature. Calculate the new volume of oxygen?

2. What is the relationship between the pressure, p, and Volume, v, of a fixed mass of a gas at constant temperature

3. A gas kept at a constant temperature occupies $100m^3$ at a pressure of 15Pa. What would be its new volume when the pressure increases to 60Pa?

4. A gas occupies a volume of $10cm^3$ at 100kPa. If the pressure is increased to 200kPa, what is the new volume if the temperature remains at 27°C?

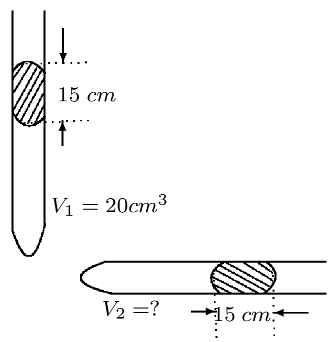
5. An air bubble at the bottom of a lake 90m deep has a volume of $1.5cm^3$. What will be its volume just below the water surface, if the atmospheric pressure is equal to 10m of water? Ans is $15cm^3$

6. A gas occupies a volume of $3.6m^3$ under a pressure due to 75cm of mercury. What will be its;
   (a) Volume, if the pressure changes to 125cm of mercury?
   (b) Pressure, if its volume changes to $1.8m^3$?

7. A gas occupies a volume, v, under a pressure, p, what will be its volume if the pressure is

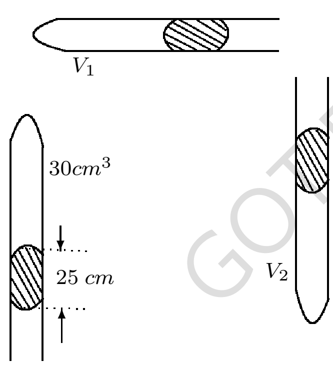
(a) doubled
(b) halved

8. A gas in the tube in the diagram below has a volume of $20cm^3$. What will be the volume of the gas if the tube is held horizontally as shown? Atmospheric pressure is equivalent to 75cmHg. Ans is 24cm^3



9. A gas at $1.0 \times 10^5 Pa$ occupies a volume of $6.0 \times 10^3 cm^3$, what volume would it occupy if the pressure changes to $3 \times 10^5 Pa$.

10. A tube has a thread of mercury 25cm trapping air of volume $30cm^3$. Calculate the volume V_1 and V_2 when the tubes lie in the positions shown in the figure below;



Assume that atmospheric pressure is equivalent to 75cmHg. (Ans is $V_1 = 20cm^3$, $v_2 = 15cm^3$).

11. A uniform narrow-bored tube, closed at one end contains some dry air, which is sealed by a thread of mercury 15.0cm long. When the tube is held vertically with the closed end at the bottom, the air column is 20cm long, but when it is held horizontally, the air column is 24cm long. Calculate the atmospheric pressure? (Ans;75cmHg)

12. A narrow uniform glass tube contains air closed by a thread of mercury 15.0cm long. When the tube is vertical with the open end upper most, the air column is 30cm long. When the tube is inverted the length of the column becomes 45cm. Explain why the length of the air column changes and calculate the pressure of the atmosphere? (Hint: $P_1 = P_\circ + h_1\rho g$, $P_2 = P_\circ - h_2\rho g$

13. A J-tube of uniform cross section is held vertically with its longer limb open to the atmosphere, by adding mercury air is trapped in the shorter limb, which is closed. The length of the air column in the closed limb is 16cm. When the difference in the vertical heights of the meniscus levels in the two limbs is 20cm. When the difference is increased to 77cm by adding more mercury, the air column is only 10cm long. Use this data to calculate the height of the mercury barometer, which may be considered to have remained constant during the experiment. Explain why the assumption of Boyle's law may not be strictly valid in this case?(Harder)

14. How would you expect the pressure exerted by a gas in a vessel to change if a quarter of the mass of the gas in the vessel escapes, given the volume and temperature of the system does not change? (Ans: the pressure would change to $\frac{3}{4}$ of its initial pressure)

15. In an experiment to verify Boyle's law, a gas syringe containing some air was connected to bourdon pressure gauge. The pressure P was measured for different volumes, V and the results tabulated in table 3.11;

V in cm^3	P in kPa
10	198
12	167
14	142
16	125
18	112

Table 3.11: Boyle's law

(a)Plot a suitable graph and use its slope to write an equation relating P and V

(b) what is the pressure when the volume is $15cm^3$?

(c) what would be the volume of the enclosed air given that the piston is set free and the atmospheric pressure on that day is 101kPa.

16. The results in table 3.12 were obtained for a sample of butane at 16.7° C

Pressure, P in atmospheres	Volume, V in cm^3
0.50	30.0
0.70	21.5
1.00	15.0
1.50	9.0
2.00	6.0
3.00	1.0

Table 3.12: Butane

17. The results shown in the table 3.13 were obtained in an experiment to verify Boyle's law

Pressure in kNm^{-2}	volume in mm^3	$\frac{1}{\text{Volume}}$ in mm^{-3}
400	2.0	0.5
320	2.5	
160	5.0	
80	10.0	

Table 3.13: Boyle's law

(a) Copy and complete the table

(b) Plot a graph of pressure on the y-axis against $\frac{1}{\text{volume}}$ on the x-axis

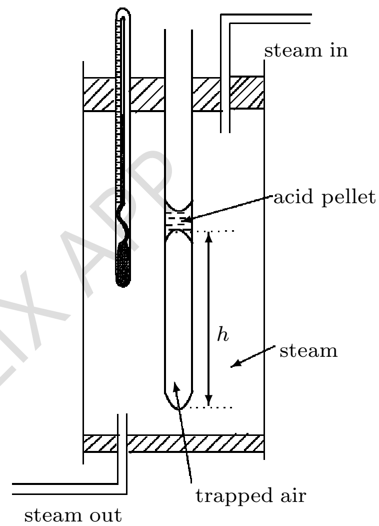
(c) State the relationship shown by this graph.

3.8.3 Charles's law

Another scientist Jacques Charles was interested in studying the relation between volume and temperature of a gas at constant pressure, i.e. if a gas at a given constant pressure is heated, does its volume increase or decrease and by how much?

An experiment to verify Charles's law

Trap air in a tube by a layer of concentrated sulphuric acid (or any other viscous liquid) and heat this trapped air in a steam chest as shown below;



Note that the trapped air is at constant pressure (the atmospheric pressure). Length of this mercury thread should be small and hence neglected.

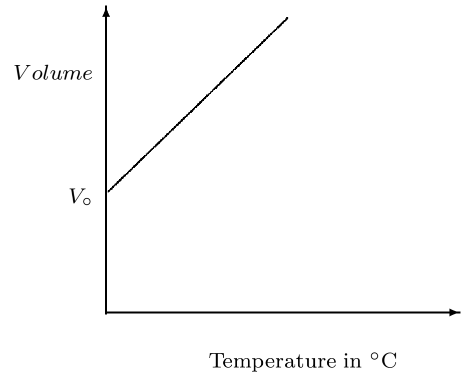
Measure and record the length, l, of the trapped air and its temperature, T, as the reading of the thermometer.

Pass steam through the apparatus and record more values of l and T simultaneously, as shown in table 3.14; (given A is the cross section area of the tube)

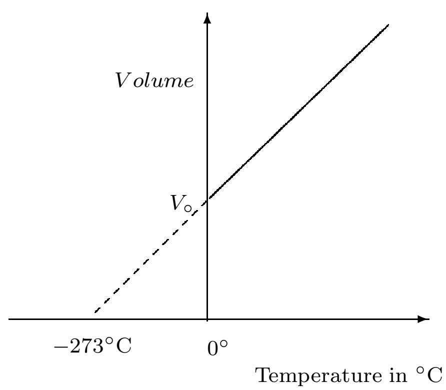
T in °C	l in cm	Volume (= Al) in cm^3

Table 3.14: Charles's law

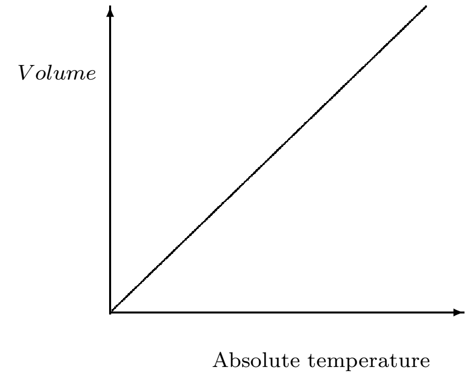
Plotting a graph of V against T, We have



Extrapolating the graph, we can estimate that the volume of the gas is zero at a temperature of -273°C. (Absolute zero of temperature)



The slope of the line in the above graph is $\frac{V_\circ}{273}$, where $V_\circ$ is the volume of the gas at 0°C. This fraction is called the cubic expansivity of a gas at constant pressure: it is the change in the volume of a gas when its temperature changes by 1°C or 1K. The volume of a gas at any temperature T is given by $V = V_\circ\left(1 + \frac{T}{273}\right)$ i.e the slope of the line is $\frac{V_\circ}{273}$, so we say that the volume of a gas expands by $\frac{1}{273}$ of its volume at 0°C per rise in temperature (this is another version of Charles's law). These proved that there exist a certain scale of temperature whose origin starts at - 273°C. This scale was called the thermodynamic scale of temperature, and the temperature measured on this scale is called the **absolute temperature** with units of Kelvin. A given temperature in °C is converted to the thermodynamic scale[10] or Kelvin by adding 273 to it, i.e. 20°C = 293K, 0°C = 273K. If we had plotted a graph of Volume against temperature in Kelvin, we would have



This graph shows that

> The volume of a fixed mass of a gas is directly proportional to its absolute temperature[11] provided pressure remains constant. This is Charles's law.

hence

Law 5.

> **Charles's law** states that The volume of a fixed mass of a gas is directly proportional to its absolute temperature provided pressure remains constant.

Kinetic theory explanation of Charles's law

If the pressure of a fixed mass of a gas is to remain constant, as the temperature is raised, the rate of change of momentum must remain constant (because it is proportional to pressure). But when the temperature of the molecules is increased their velocity also increases: to keep the rate of change of impacts on the walls (pressure) the same, it is necessary to make fewer impacts. This can be achieved only when the volume of the gas is increased so that the molecules have to travel longer distances between collisions with the walls. Thus, to maintain the pressure of the gas constant the when temperature is increased the volume of the gas has to be increased. And this condition applies if the volume varies directly as the thermodynamic temperature which is Charles's law.

[10]see page 91 for more about units of temperature

[11]This is the thermodynamic scale of temperature

Mathematical treatment of Charles's law

Charles's law shows that the volume of a gas is directly proportional to its absolute temperature

$$V \quad \alpha \quad T \tag{3.59}$$
$$V = \text{constant} \times T \tag{3.60}$$
$$V = \kappa \quad T \tag{3.61}$$
$$\frac{V}{T} = \kappa, \text{ a constant} \tag{3.62}$$
$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{V_3}{T_3} \tag{3.63}$$

Examples

1. At what temperature must $455cm^3$ of oxygen at $0°C$ be heated to in order to make its volume $605cm^3$, if the pressure is to remain constant?

Solution

from the question

$$\begin{aligned} V_1 &= 455cm^3 \\ T_1 &= 0°C \\ &= (0+273)k \\ &= 273k \\ V_2 &= 605cm^3 \\ T_2 &= ? \end{aligned}$$

Using Charles's law, since pressure is constant

$$\begin{aligned} \frac{V_1}{T_1} &= \frac{V_2}{T_2} \\ T_2 &= \frac{T_1}{V_1} \times V_2 \\ &= \frac{273}{455} \times 605 \\ &= 363Kelvin \\ &= (363-273)°C \\ &= 90°C \end{aligned}$$

2. $100cm^3$ of carbondioxide are collected at a temperature of $27°C$, find what volume this gas would occupy at the same pressure if cooled to $0°C$?

Solution

from the question

$$\begin{aligned} V_1 &= 100cm^3 \\ T_1 &= 27°C \\ &= (27+273)k \\ &= 300k \\ V_2 &= ? \\ T_2 &= 0°C \\ &= (0+273)k \\ &= 273k \end{aligned}$$

from Charles's law, since pressure is constant

$$\begin{aligned} \frac{V_1}{T_1} &= \frac{V_2}{T_2} \\ T_2 &= \frac{V_1}{T_1} \times T_2 \\ &= \frac{100}{300} \times 273 \\ &= 91cm^3 \end{aligned}$$

3. $48cm^3$ of nitrogen at $15°C$ are heated without alteration of pressure to $45°C$, what is the increase in volume?

Solution

from the question

$$\begin{aligned} V_1 &= 48cm^3 \\ T_1 &= 15°C \\ &= (15+273)k \\ &= 288k \\ T_2 &= 45°C \\ &= (45+273)k \\ &= 318k \\ V_2 &= ? \\ \Delta V &= ? \end{aligned}$$

From Charles's law, since pressure is constant

$$\begin{aligned}
\frac{V_2}{T_2} &= \frac{V_1}{T_1} \\
V_2 &= \frac{V_1}{T_1} \times T_2 \\
&= \frac{48}{288} \times 318 \\
&= 53cm^3 \\
\Delta V &= V_2 - V_1 \\
&= 53 - 48 \\
&= 5cm^3
\end{aligned}$$

hence the increase in volume is 5cm^3

4. At what temperature will a gas occupy three times the volume it occupies at $0°C$, if pressure remains constant?

Solution

from the question

$$\begin{aligned}
V_1 &= V_1 \\
T_1 &= 0°C \\
&= (0 + 273)k \\
&= 273k \\
V_2 &= 3V_1 \\
T_2 &= ?
\end{aligned}$$

Using Charles's law, since pressure remains constant

$$\begin{aligned}
\frac{V_1}{T_1} &= \frac{V_2}{T_2} \\
T_2 &= \frac{T_1}{V_1} \times V_2 \\
&= \frac{273}{V_1} \times 3V_1 \\
&= 3 \times 273k \\
&= 819 Kelvin \\
&= (819 - 273)°C \\
&= 546°C
\end{aligned}$$

Exercise

1. $1000cm^3$ of air at 10°C are heated to 80°C. What will be the new volume if the pressure remains atmospheric?

2. A gas occupies a volume of $2m^3$ at a temperature of 27°C and at atmospheric pressure. What will be its volume at the same pressure if the temperature changes to;

   (a) 47°C

   (b) -30°C (Ans:2.13m^3, 1.8m^3)

3. To what temperature must 2 litres of air at 17°C be heated to at constant pressure in order to increase its volume to 3 litres?

4. A gas at 27°C expands from a volume of $5.0m^3$ to 7.5m^3 at constant pressure. Determine its final temperature?

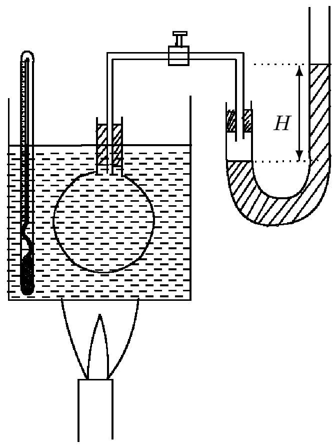
5. At what temperature will a mass of a gas occupying $200cm^3$ at 0°C have a volume of $300cm^3$ if the pressure remains constant?

6. The density of hydrogen at 0°C and standard pressure is 0.009g/litre. Find the volume occupied by 1g of hydrogen at 200°C and standard pressure? and then the density of hydrogen at 200°C

3.8.4 Pressure law

Here we look at the variation of pressure of a gas with temperature at constant volume. How can we keep a gas with a constant volume and vary its pressure or temperature? The apparatus below was designed and used as described below;

An experiment to verify pressure law



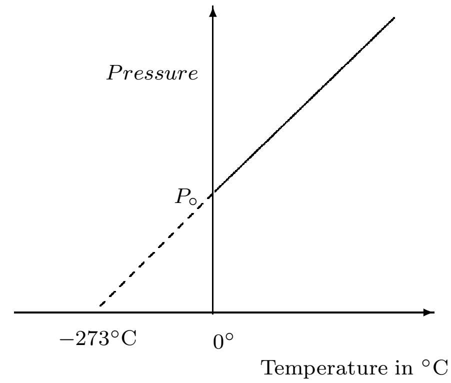
- A bulb containing the gas is placed in a beaker containing melting ice, and then connected to a manometer.
- Heat the beaker to increase the temperature of its contents.
- Record its temperature and pressure simultaneously as the ice melts up to room temperature, ensuring that volume is constant.

Tabulate the results as shown in table 3.15;

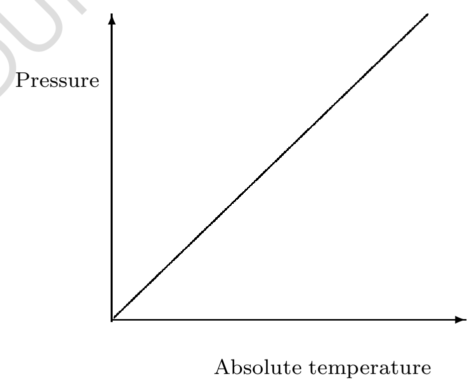
Temperature in °C	Pressure in Pa
...	...
...	...
...	...
...	...
...	...

Table 3.15: pressure law

Plotting the obtained values, the graph below was obtained;



The graph did not begin or start from the origin. Extrapolating it, it meets the Temperature - axis at -273°C i.e. when pressure is zero (ideally). The pressure of a fixed mass of a gas increases by $\frac{1}{273}$ of its pressure at 0°C per rise in temperature. This is another version of the pressure law i.e $P = P_\circ\left(1 + \frac{T}{273}\right)$ where T is in ° C. If one plots the same graph against temperature in Kelvin. The graph below is obtained;



The graph shows that the pressure of a fixed mass of a gas varies directly as its absolute temperature provided its volume remains constant which is pressure law.

Law 6.

Pressure law states that the pressure of a fixed mass of a gas is directly proportional to its absolute temperature, provided its volume remains constant.

The apparatus used here is sometimes known as a **constant volume gas thermometer.**

Calculations

Pressure law

$$\begin{aligned}\text{Pressure} &\quad \alpha \quad \text{Absolute temperature} && (3.64)\\ P &= \kappa\ T && (3.65)\\ \frac{P}{T} &= \kappa,\ \text{a constant} && (3.66)\\ \frac{P_1}{T_1} &= \frac{P_2}{T_2} = \frac{P_3}{T_3} && (3.67)\end{aligned}$$

Example

1. $40cm^3$ of air are contained in an inelastic vessel with a valve which opens at a pressure of 73mmHg. If the initial temperature and pressure of air were $20°C$ and 60mmHg, at what temperature will the valve open?

Solution

from the question

$$\begin{aligned}V_1 &= 4cm^3\\ P_1 &= 60mmHg\\ T_1 &= 20°C\\ &= (20+273)k\\ &= 293k\\ V_2 &= 4cm^3\\ P_1 &= 73mmHg\\ T_2 &= ?\end{aligned}$$

since $V_1 = V_2$ i.e. volume is constant, we can use pressure law

$$\begin{aligned}\frac{P_1}{T_1} &= \frac{P_2}{T_2}\\ T_2P_1 &= P_2T_1\\ T_2 &= \frac{P_2T_1}{P_1}\\ &= \frac{73\times 293}{60}\\ &= 356.48k\\ &= (356.48-273)°C\\ &= 83.48°C\end{aligned}$$

2. Early in the morning the pressure in a car tyre was $2.0\times 10^5 Pa$, at a temperature of 7°C. What would you expect the pressure in the tyre to be when the temperature rises to 27°C in the afternoon? Assume that the volume of the tyre does not change.

Solution

from the question

$$\begin{aligned}P_1 &= 2\times 10^5 Pa\\ T_1 &= 7°C\\ &= (7+273)k\\ &= 280k\\ T_2 &= 27°C\\ &= (27+273)k\\ &= 300k\end{aligned}$$

from pressure law, since we expect the volume of the tyre to remain constant

$$\begin{aligned}\frac{P_1}{T_1} &= \frac{P_2}{T_2}\\ T_1P_2 &= P_1T_2\\ P_2 &= \frac{P_1T_2}{T_1}\\ &= \frac{2\times 10^5\times 300}{280}\\ &= 214285.7143Pa\\ &= 2.143\times 10^5 Pa\end{aligned}$$

the pressure in the tyre would be approximately $2.143\times 10^5 Pa$

Exercise

1. Before starting a long journey, a motorist checked the tyre pressure and found it to be $3.0\times 10^5 Pa$. At the end of the journey, the pressure was found to be $3.3\times 10^5 Pa$. The temperature of the tyres and contained air at the start of the journey was 17°C. Assuming the volume of the tyre remains constant, determine the temperature of the air in the tyre at the end of the journey?

2. A steel cylinder of volume $300cm^3$ is filed with oxygen at a pressure of 15 atmospheres, the temperature being $12°C$, what does the pressure of the oxygen become when the cylinder is placed in a room at a temperature of $20°C$?

3. A thin-walled steel cylinder used for storing compressed air is fitted with a safety valve, which opens at a pressure of $1.0 \times 10^6 Pa$. It contains air at 17°C and $0.8 \times 10^6 Pa$. At what temperature does the valve open?

4. When tested in a cool garage at 12°C a motor tyre is found to have a pressure of 190Kpa. What would you expect the pressure to become after the tyre has been allowed to stand in the sun so that the temperature rises to 32°C? Assume that atmospheric pressure = 100kPa. (A:210kPa)

The kinetic theory explanation of the pressure law

Raising the temperature of a fixed mass of a gas at constant volume increases the average kinetic energy of the molecule so that they make more frequent impacts with the walls at higher velocity. Thus the rate of change of momentum on impact is increased this consequently increases the pressure exerted by the gas(as force acting on the wall increases;2^{nd} law of motion).

Can a gas cool to absolute zero of temperature

0k or zero Kelvin is a very low temperature, from the graph of volume against temperature, volume of the gas is zero at 0K. it is believed that, at this temperature the molecules of any substance do not move or vibrate. For a gas, since we are dealing with gasses, before a gas approaches 0k it would have condensed to a liquid and the liquid freezed to a solid. But the exact absolute zero of temperature has never been attained because of this, gasses are classified into two;

(a) real gases and

(b) Ideal gases

Real gases

These are gases that we can see, feel, or touch physically. They do not exactly obey all the gas laws, only with in certain limits or ranges of temperature and pressure. There gases will never have zero volume as predicted by Charles's law i.e. they do not exist at Absolute zero. Examples of real gases are the gases we know; like oxygen, nitrogen, carbon dioxide, hydrogen and the rest of the gases as we know them. Lets look at oxygen more critically; at room temperature it is a gas, when it is cooled to its boiling point, it changes to liquid-oxygen, cooling it further, it changes to a solid at a temperature still above 0k. Therefore there is no gas that can exist at 0k.

Ideal gases:

These are model gases used to illustrate most properties of real gases. They do not actually exist. They obey all gas laws. They have a volume of zero and also exert a pressure of zero at 0k. All assumptions made about gases are true for ideal gases, whenever we talk of zero Kelvin (0k) we must stress for an ideal gas.

Note that vapours exist in gaseous states,

Do vapours obey gas laws?

saturated vapours do not obey gas laws, it is only unsaturated vapours that do obey gas laws.

3.8.5 General gas equation

This is the relation between pressure, volume and thermodynamic temperature (or absolute temperature). We summarise the Gas laws as:

$$PV = \text{constant (Boyles law)} \quad (3.68)$$
$$\frac{V}{T} = \text{constant (Charles's law)} \quad (3.69)$$
$$\frac{P}{T} = \text{constant (Pressure law)} \quad (3.70)$$

We can combine them into one equation;

$$\frac{PV}{T} = \text{constant} \quad (3.71)$$
$$\Longrightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad (3.72)$$

Modifying this general gas equation, we have;

If temperature is constant, then

$$PV = \text{constant, Boyle's law}$$

If pressure is constant, then

$$\frac{V}{T} = \text{constant, Charles's law}$$

,

If volume is constant, then

$$\frac{P}{T} = \text{constant, Pressure law}$$

Hence the equation

$$\frac{PV}{T} = \text{constant}$$

includes all the three gas laws.

Standard temperature and pressure

In most experiments, a gas is collected at room temperature and pressure. It is then necessary to calculate the volume the gas would occupy at 0°C and 760mmHg pressure. These conditions; temperature of 0°C and pressure of 760mmHg or 1.01 × 103Pa are called standard temperature and pressure, usually abbreviated as S.T.P. Before, it was known as Normal temperature and pressure. Hence S.T.P means; $T = 0$°C or 273K and $P = 760mmHg$ or $1.01 \times 10^5 Pa$ or 1 atmosphere.

Example

1. At the beginning of the compression stroke of a gas engine, the gas occupies $42cm^3$ at pressure of $15Pa$ and temperature of 27°. At the end of the compression stroke, when the volume is $6cm^3$, the pressure is found to be $255Pa$, what is the temperature of the gas?

Solution

from the question

$$\begin{aligned} V_1 &= 42cm^3 \\ P_1 &= 15Pa \\ T_1 &= 27°C \\ &= (27+273)k \\ &= 300k \\ V_2 &= 6cm^3 \\ P_2 &= 255Pa \\ T_2 &= ? \end{aligned}$$

since all the three parameters (P, V and T) change, then we use general gas equation i.e.

$$\begin{aligned} \frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \\ T_2P_1V_1 &= P_2V_2T_1 \\ T_2 &= \frac{P_2V_2T_1}{P_1V_1} \\ &= \frac{255 \times 6 \times 300}{15 \times 42} \\ &= 728.57k \\ &= (728.57 - 273)°C \\ &= 455.5714°C \end{aligned}$$

2. In an experiment to determine the density of hydrogen, $200cm^3$ of hydrogen at 750mmHg and $27°C$ were found to weigh 0.0162g. Find the density of hydrogen at S.T.P?

Solution

from the question

$$\begin{aligned} V_1 &= 200cm^3 \\ P_1 &= 750mmHg \\ T_1 &= 27°C \\ &= (27+273)k \\ &= 300k \end{aligned}$$

$$\begin{aligned} m &= 0.0162g \\ P_2 &= 760mmHg \\ V_2 &= ? \\ T_2 &= 0°C \\ &= (0+273)k \\ &= 273k \\ D &= \frac{m}{v} \end{aligned}$$

Using the general gas equation

$$\begin{aligned} \frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} \\ P_2V_2T_1 &= T_2P_1V_1 \\ V_2 &= \frac{T_2P_1V_1}{P_2T_1} \\ &= \frac{273 \times 750 \times 200}{760 \times 300} \\ &= 179.605cm^3 \end{aligned}$$

The density of hydrogen, D of hydrogen at S.T.P is given by;

$$\begin{aligned} D &= \frac{m}{v} \\ &= \frac{0.0162g}{179.605cm^3} \\ &= 0.000090gcm^{-3} \\ &= 0.00009 \times 1000kgm^{-3} \\ &= 0.09kgm^{-3} \end{aligned}$$

hence the density of hydrogen at S.T.P is $0.09kgm^{-3}$

Exercise

1. $125cm^3$ of a gas were collected at 15°C and 755mmHg pressure. Calculate the volume of the gas at S.T.P? (A:118cm^3)

2. A container is filled with $70cm^3$ of air at 100kPa and 7°C. What is the final pressure when the air is compressed to $30cm^3$ at 27°C ? (A:250)

3. 2 litres of a gas at a temperature of 27°C and a pressure of 1 atmosphere is compressed to a volume of 1litre and heated to 127°C. What is the final Pressure of the gas in atmospheres?

4. A bicycle pump, with its exit hole closed, contains $80cm^3$ of air at atmospheric pressure of 760mmHg and a temperature of 280k. When the air has been compressed to $38cm^3$, the temperature rises to 301k, what is the pressure of the compressed air?

5. Calculate the ratio of the final pressure to the initial pressure when a fixed mass of a gas at 17°C is heated to 100°C and compressed to half its initial volume?

6. The temperature of $2.5m^3$ of a gas at 75cmHg is 27°C :

   (a) what would be the volume at 87°C, if pressure remains the same

   (b) what would be its temperature if the volume changes to $7.5m^3$

7. The volume of a gas at 27°C and pressure of 120cmHg is $3cm^3$;

   (a) what would be its temperature if pressure changes to 100cmHg and volume changes to $4cm^3$?

   (b) What would be its pressure if the temperature changes to 67°C and volume remains the same?

8. A cylinder of volume 0.4 litres contains a mixture of petrol vapour and air at atmospheric pressure and temperature of 27°C. The gas is compressed suddenly by a piston to a volume of 0.05litres and pressure 20 times atmospheric pressure. What is the new temperature of the gas?

9. $125cm^3$ of a gas are stored at 15°C and 755mmHg of mercury pressure. Determine the volume of the gas at S.T.P?

10. A gas occupies $4cm^3$ under a pressure of 152cmHg and temperature of 57°C. What will be its volume at S.T.P? (6.62cm^3)

11. A gas at 7°C and 100kpa occupies 20dm^3. The gas is heated to 27°C at a pressure of 120kpa. Find the new volume?

12. $100cm^3$ of dry air at a pressure of 1 atmosphere and temperature of 27°C are compressed to 5 atmospheres and then heated to 77°C. Determine its new volume?

13. A fixed mass of a gas occupies a volume of $1000cm^3$ at a pressure P and absolute temperature T. what would its volume be when the pressure is 2P and temperature is 2T

14. Use the kinetic theory of gases to explain the following;
    (a) the volume of a gas is proportional to its absolute temperature
    (b) work is done in compressing a gas

15. What happens to the energy supplied to a gas when it is heated at
    (a) constant pressure.
    (b) constant volume.

16. A fixed mass of a gas is enclosed in a vessel. Explain in terms of the molecular theory
    (a) how the pressure exerted at the wall of a vessel is produced by the gas molecules
    (b) how the pressure would be affected, if the temperature increased, keeping the volume of the vessel constant
    (c) how the pressure would be affected if the volume of the vessel is reduced at constant temperature.

17. $80cm^3$ of hydrogen gas are collected at 15°C and 75cmHg. What is its volume at S.T.P.?

3.9 Heat engines

3.9.1 Introduction

Heat engines are devices that transform or convert heat energy into mechanical energy (kinetic or potential energy). This is by using a fluid known as a **working fluid**, usually water is used although any other fluid could be used. The hot air engine for instance uses air as a working fluid. Heat engines are usually some forms of steam engines that are either a reciprocating engine (with pistons) or a turbine. Details about it are above our level.

Steam engines (reciprocating engines):

In these engines, steam is allowed to expand in a nozzle before entering a given casing. In this expansion, heat energy is transformed to kinetic energy of the molecules and they emerge from the nozzle with a high velocity (about 100ms^{-1}). The rotor blades are made to deflect the steam so that the molecules hit the blades with the maximum change of momentum on impact. Thus most of the kinetic energy of the steam is changes to the kinetic energy of the rotor which runs a turbine to generate electricity.

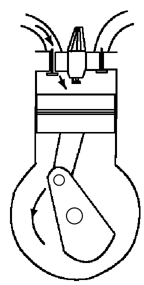
Internal combustion engine

These engines burn fuel (in the solid, liquid or gaseous state) and the combustion produces gaseous products at high temperature and high pressures. The expansion of the gases transforms heat energy into kinetic energy and work is performed in the process. The common ones include the petrol and diesel engines.

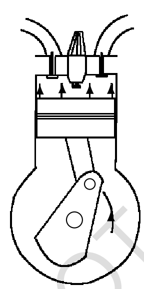
3.9.2 The petrol engine

The first four-stroke petrol engine was invented in 1876 by Otto a German scientist and the energy cycle used in this petrol engine is called the Otto cycle. The piston makes four strokes for one complete cycle, these are described below;

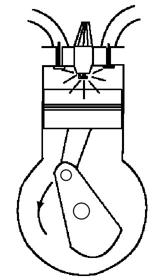
1. **The inlet stroke;** here the piston moves down, the inlet valve opens and a mixture of petrol and air mixture enters the cylinder. At the bottom (end) of the stroke the inlet valve closes.



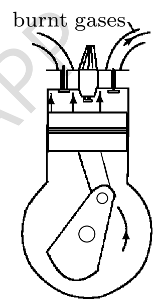
2. **The compression stroke;** the piston moves up and the gaseous mixture is compressed



3. **The explosion stroke;** a sparking plug (spark plug) which is connected to a car battery and the induction coil[12] produces a spark when an electric current passes through the plug, at the top (beginning) of the stroke. This explodes the gaseous(petrol-air) mixture and the explosion pushes down the piston, this is where heat or internal energy is converted to mechanical energy. This is the only working stroke in the cycle, because here the piston pushes the crankshaft which moves the wheel of the car.



4. **The exhaust stroke;** the piston rises and the exhaust valve opens allowing the products of combustion to escape through the exhaust tube. At the top (beginning) of the stroke the exhaust valve closes.



The cycle:

A heavy flywheel is attached to the engine shaft. The kinetic energy (rotational kinetic energy) of the fly-wheel enables it to continue to rotate and move the piston during the non-working stokes, hence it is advantageous to have four cylinders operating the crankshaft, arranged to have their explosion strokes in turn, so that every half revolution, the crankshaft has a working stroke in one of the cylinders. Such an engine is a four-piston engine. We also have the two-stroke engine like the boat and motor cycle engine. Modern engines with six and sixteen pistons are available.

Energy losses

The cylinder of the internal combustion engine becomes very hot during operation, this heat is conducted away by a water jacket (car radiator) or else metal fins project from the cylinder casing to conduct and radiate as much heat as possible.

[12]The induction coil is discussed on page 335

This is an important energy loss because if the heat is not conducted away, the piston expands and seizes in the cylinder (i.e. it will not move in the cylinder freely).

- One source of energy loss is incomplete combustion; exhaust fumes from the petrol engine contain carbon monoxide due to incomplete combustion, therefore the fuel produces less energy than it should.
- Pressure also escapes in the exhaust stroke and produces heat and noise resulting into further loss.
- Finally friction in the bearings produces some mechanical or heat losses as it opposes motion.

3.9.3 Diesel engine

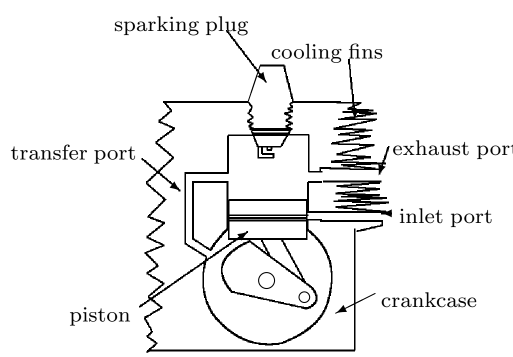
This engine has a cycle of four strokes same as those of the petrol engine. The difference is that air is admitted on the inlet stroke, then compressed and heated on the compression stroke. Fuel (oil in this case) is then injected into the cylinder using an oil injector. The oil injector is at the position of the spark plug in the petrol engine. The air is sufficiently hot to raise the oil above its ignition temperature and the mixture explodes during the explosion stroke. The exhaust stroke then follows.

As said above, instead of taking in the petrol-air mixture during the induction stroke, the diesel engine simply draws in air. On compression stroke work is done on the air and this becomes stored as internal molecular energy accompanied by a considerable rise in temperature. This rise in temperature is sufficient to ignite a spray of fuel-air mixture, which is forced into the cylinder through the injector just before the piston reaches the top of its stroke. No sparking plug is required, for this reason the diesel engine is sometimes called a **compression ignition engine.**

You may wonder why compression ignition does not normally occur in the petrol engine, the reason is that the petrol engine has a smaller compression ratio. The compression ratio is the ration of the volume of the cylinder space when the piston is at the bottom to that when it is at the top. A diesel engine has a compression ratio twice that of the petrol engine in order to achieve a gas temperature sufficiently high to ignite the fuel. The name **diesel engine** is derived from that of its inventor Rudolf Diesel, a German engineer though the current diesel engines function in somewhat different manner from their late 19^{th} century types. The diagrams for the diesel engine for the four strokes are the same as those for the petrol engine with the only difference of having an oil injector instead of the spark plug.

3.9.4 Two-stroke engine

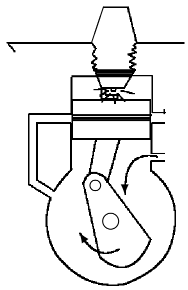
These are the engines in devices that require a smaller output power. These include small boats, Motor-bikes and motorcycles, lawnmowers, mopeds. In them, the valves are replaced by the ports (or slots) on the sides of the cylinder which are opened and closed by the piston as it moves. The diagram below shown the arrangement of its piston.



The piston only performs two steps for a complete cycle. These are describe below;

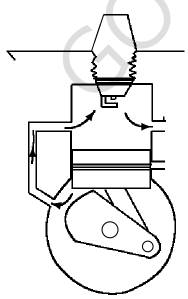
Upstroke:Compression and inlet

Here the piston rises up, closing the transfer port and the exhaust port, leaving the inlet port open. This allows the fresh mixture (usually petrol-air mixture) to enter the crank-case via the inlet port. The spark plug produces a spark. If there was already the petrol-air mixture, it is ignited and expands pushing the crankshaft to continue rotating. This stroke constitutes the inlet and compression stroke when compared with the petrol and diesel engine.



Downstroke:Exhaust and power

This starts with the sparking of the compressed mixture. This transfers energy from the internal energy of the petrol-air mixture to the rotational kinetic energy of the crankshaft. The piston moves down opening the transfer port and the exhaust port, but closing the inlet port. The burnt gases are sent out of the cylinder via the exhaust port as the new mixture replaces it via the transfer port. This stroke replaces the power stroke and exhaust stroke of the four-stroke engine.



The piston moves up again for the inlet compression stroke and down again for the power and exhaust stroke and the cycle continues. The shape of the piston helps to stop flesh fuel and burnt gases mixing. Two strokes hence called a two-stroke engine. Recall the efficiency of the petrol engine is about 30% which means that only 30% of the heat energy supplied becomes kinetic energy; much of the rest is lost with the exhaust gases.

Energy cycle

Chemical energy in petrol is transformed to heat in the explosion. Heat energy is transformed to kinetic energy in the explosion stroke and work is done during this transformation. As seen above not all the heat energy is transformed to kinetic energy. The thermal efficiency is determined from the expression;

$$\text{Efficiency} = \frac{\text{useful work done}}{\text{energy value of the fuel used}} \times 100\% \tag{3.73}$$

Differences between diesel and petrol engine

These differences are given in table 3.16

Petrol engine	**Diesel engine**
Fuel used is petrol	Fuel used is diesel oil
Uses spark plug	Does not use spark plug
Has a low compressional ratio	Has a high compression ratio
Has a less efficiency	Has a greater efficiency

Table 3.16: Petrol and diesel engine

Efficiency of heat engines

A comparison of the thermal efficiency of different types of heat engines is useful. Approximate values of thermal efficiencies of some engines are;

Steam engine with out condenser -about 8%

Steam engine with condenser -about 20%

Steam turbine with condenser -about 30%

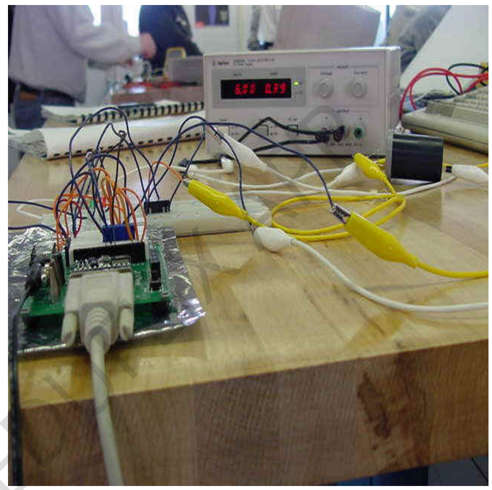
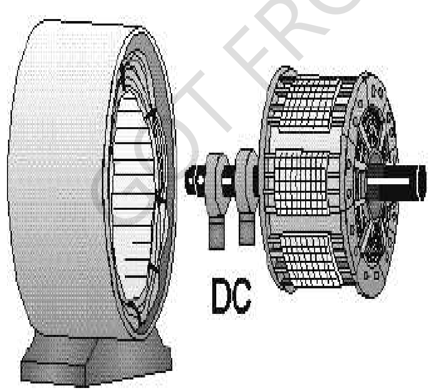
Internal combustion engine - about 35%

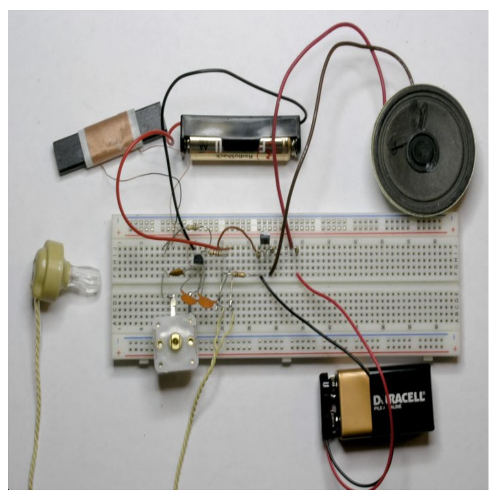
As a further comparison, human beings have a thermal efficiency of about 25%, i.e. only 25% of the energy value of food eaten is available for useful work. The remainder of the chemical energy in food (75%) is transformed into heat.

Exercise

1. Describe briefly what is meant by the term heat engine?
2. Describe briefly how a two stroke engine operates?
3. Differentiate between a working fluid and fuel
4. Give two examples of internal combustion engine
5. Name the four steps in the Otto cycle
6. Describe briefly how
   (a) A petrol engine and
   (b) a diesel engine operates
7. Discuss briefly the energy changes that take place in the internal combustion engine
8. Give 3 reasons why the efficiency of the internal combustion engine is less than 100%
9. Give 4 differences between a petrol engine and a diesel engine.
10. what are the basic differences between a four-stroke engine and a two-stroke engines
11. Most car engines are called four-stroke petrol engines;
    (a) Name the four strokes in the correct sequence
    (b) How many times does the crankshaft revolve during the four strokes.

Electricity and Magnetism



The Photos above show; some one using solar energy using a solar panel, a picture taken from an electronic laboratory, parts of a practical DC motor and a connected electronic circuit.

Objectives

After studying this topic, you should:

- Be able to define or explain what is meant by the terms;

  ⊛ electrostatics
  ⊛ surface density
  ⊛ electric field
  ⊛ electric field line
  ⊛ direction of an electric field or field lines
  ⊛ equipotential surface
  ⊛ primary cell
  ⊛ secondary cell
  ⊛ electric potential difference
  ⊛ electric current
  ⊛ An ampere
  ⊛ a volt
  ⊛ a coulomb
  ⊛ Emf of a cell
  ⊛ an Ohm
  ⊛ resistance of a conductor
  ⊛ Internal resistance of a cell
  ⊛ magnetic pole
  ⊛ magnetic field
  ⊛ direction of a magnetic field
  ⊛ magnetic axis
  ⊛ magnetic neutral point
  ⊛ magnetic induction
  ⊛ induced magnetism
  ⊛ magnetic shielding
  ⊛ an electromagnet
  ⊛ electromagnetic induction
  ⊛ mutual induction
  ⊛ eddy currents
  ⊛ alternating current
  ⊛ non-inductive resistance coils
  ⊛ rectification
  ⊛ forward biased diode
  ⊛ reverse biased diode

- Be able to give;

  ⊞ six precautions given to lead-acid cells
  ⊞ two advantages of alkaline cells over lead-acid cells
  ⊞ two types of electric circuits
  ⊞ two uses of internal resistance
  ⊞ two properties of magnetic field lines
  ⊞ three major differences between the magnetic properties of iron and steel.
  ⊞ 3 advantages of nichrome as a heating element.
  ⊞ the 4 major causes of energy loss in a transformer
  ⊞ 3 uses of transformers
  ⊞ 2 uses of the induction coil
  ⊞ 2 types of fuses
  ⊞ 2 types of semiconductors
  ⊞ 2 types of extrinsic semiconductors

⊞ 2 kinds of rectification

- Be able to describe;
  - ⊛ the difference between insulators and conductors
  - ⊛ the paper-comp experiment
  - ⊛ briefly how a potential divider or current divider operates.
  - ⊛ briefly how temperature affects the internal resistance of a cell
  - ⊛ the domain theory of magnetism
  - ⊛ briefly how the following devices operate; electromagnets, magnetic relays, lifting magnets, reed switch, electric bell, magnetic separators, telephone receivers, electric motors, loudspeakers, and moving coil galvanometer.
  - ⊛ the direction of the forces acting on a charged particle moving in a magnetic field.
  - ⊛ the importance of back emf in a motor
  - ⊛ how the sensitivity of a moving coil galvanometer can be increased
  - ⊛ how to convert a milliammeter or microammeter to an ammeter
  - ⊛ how to convert an ammeter to a voltmeter
  - ⊛ how an ammeter that can measure alternating current operates (moving iron ammeter).
  - ⊛ how the following devices operate; electric fire, electric kettle, electric iron, filament bulb, discharge lamp, fluorescent tube, and an electric fuse.
  - ⊛ how the efficiency of a transformer can be increased.
  - ⊛ how the following devices operate; electric motor, transformer, microscope, speedometer, crack detector, induction furnace, choke, electric brakes, record play pickup.
  - ⊛ the motor effect in a dynamo
  - ⊛ how one can run a low voltage dc device from a high voltage dc voltage.
  - ⊛ how to run a low voltage ac device from the ac mains supply.
  - ⊛ briefly how electric power is transferred from the power station to the consumer or industries
  - ⊛ the colour code of the plug cables used in household wiring
  - ⊛ how full wave and half wave rectification can be achieved.
  - ⊛ what is meant by wheat-stone bridge and derive its balance condition
  - ⊛ how to charge;
    - ⊡ a body positively or negatively by electrostatics induction.
    - ⊡ two bodies both negatively by electrostatic induction
    - ⊡ two bodies both positively by electrostatic induction
    - ⊡ two bodies oppositely by electrostatics induction
    - ⊡ an electroscope positively or negatively by electrostatics induction or by contact
    - ⊡ lightening occurs and how a lightening conductor works
    - ⊡ how people can protect themselves from lightening
    - ⊡ Faraday's ice pail experiment
    - ⊡ how a dry cell and a lead-acid cell operates.
  - ⊛ an experiment to

  - Ⓢ verify Ohm's law
  - Ⓢ determine the internal resistance of a cell
  - Ⓢ magnetise a steel bar by either single touch method, double touch method, heating and cooling, magnetic induction, hammering or by an electrical method.
  - Ⓢ demagnetise a bar magnet by either a mechanical, thermal or electrical method.
  - Ⓢ determine the heat capacity or specific heat capacity of a given metal in the form of a solid block.
  - Ⓢ determine the heat capacity or specific heat capacity of a given quantity of a liquid.
  - Ⓢ determine the specific heat capacity of a substance using a calibrated calorimeter.
  - Ⓢ measure the specific latent heat of fusion of ice by direct heating.
  - Ⓢ measure the specific latent heat of vapourisation by an approximate method and state the possible sources of errors in this experiment.
  - Ⓢ verify Faraday's law of electromagnetic induction
  - Ⓢ demonstrate either mutual induction, self induction or eddy currents.
  - Ⓢ to measure the electrical resistance of a conductor by;ammeter-voltmeter method, resistance box method or meter-bridge method.

- Be able to;

  - ⊛ draw a well labelled diagram of a gold leaf electroscope
  - ⊛ sketch the electric field lines of ;
    - Ⓢ isolated positive and negative charges
    - Ⓢ a charged near a neutral earthed surface
    - Ⓢ two or more like or unlike charges near a neutral (or earthed) surface.
    - Ⓢ a charge in a neutral or charged pail (Faraday's ice pail)
    - Ⓢ a charged electroscope
    - Ⓢ two charged parallel plates
    - Ⓢ two charges near each other
  - ⊛ draw a well labelled diagram of a dry cell or lead-acid cell
  - ⊛ relate the charge storage capacity, charging current and charging time for a given cell or battery.
  - ⊛ obtain an expression for the work, W, done in moving a charge ,Q through a potential difference, V.
  - ⊛ obtain the quantity of charge, Q, that has flown in circuit carrying an electric current, I in time, t.
  - ⊛ sketch;
    - ⊚ the IV curves for a metallic wire, semiconductor diode, thermionic diode, filament bulb, and for copper sulphate solution.
    - ⊚ the magnetic field lines for a single north pole, a single south pole, 2 like poles, 2 unlike poles, a bar magnet, 2 bar magnets parallel and when perpendicular to each other.
    - ⊚ the magnetic flux pattern of a short current carrying circular coil
    - ⊚ the magnetic flux pattern of a solenoid carrying an electric current.
  - ⊛ derive the expressions for the total resistance of resistors and cells in series and in parallel.
  - ⊛ explain why ammeters are always connected in series and voltmeters in parallel with the devices whose current or voltage is being determined respectively.

- ⊛ test for polarity of a bar magnet
- ⊛ plot the magnetic flux pattern using iron fillings or compass needle.
- ⊛ differentiate or distinguish between;
  - ⊚ magnetic and geographic meridian
  - ⊚ angle or inclination and angle of declination
  - ⊚ magnetic domain and magnetic dipole
  - ⊚ soft and hard magnetic material
- ⊛ determine;
  - ◊ the direction of the magnetic field produced by an electric current.
  - ◊ whether like currents or unlike currents attract or repel
  - ◊ the direction of the force acting on a current carrying conductor in a magnetic field

- Be able to state;
  - ⋊ the first law of electrostatics
  - ⋊ the 3 methods of charging a body
  - ⋊ 3 uses of an electroscope
  - ⋊ 4 applications of electrostatics
  - ⋊ the SI unit of potential, electric current
  - ⋊ Ohm's law
  - ⋊ the three formula for electric power
  - ⋊ the first law of magnetism
  - ⋊ the factor affecting the force acting on a current carrying conductor in a magnetic field
  - ⋊ Fleming's left hand rule (Motor rule)
  - ⋊ Fleming's right hand rule (Dynamo rule)
  - ⋊ Maxwell's screw rule
  - ⋊ Faraday's law and Lenzi's law of electromagnetic induction
  - ⋊ the factors affecting the magnitude of the induced current.
  - ⋊ 4 factors that affect the resistance of a conductor

- Be able to explain;
  - ★ why an electric current produces heat when flowing in an electric device, using the kinetic theory of matter.
  - ★ how electromagnetic induction is applied in galvanometer damping.
  - ★ why voltage is stepped up by transformers before transmission
  - ★ why alternating current and not direct current is preferred in electric power transmission.
  - ★ why household wiring circuits are always in parallel and not in series.
  - ★ why most rectifier circuits do poses capacitors

Contents

3.10 Static Electricity

or Electrostatics

3.10.1 Introduction

Electricity is the study of charges. The charges may be at rest (stationary or static) or moving. Electricity is made up of two-section

(a) Static electricity

(b) Current electricity

In this section we are to study the properties of electric charges at rest i.e. static electricity or electrostatics.

We know that atoms are made up of a positive nucleus surrounded by negative electrons. These signs, positive and negative refer to charges. The charges may be moving or at rest.

Definition 40.

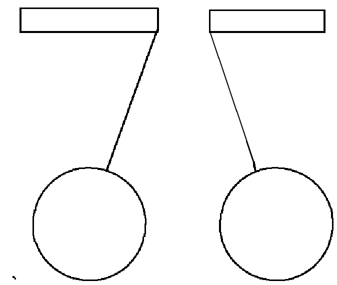
> **Static electricity** or **Electrostatics** is defined as the study of charges at rest.

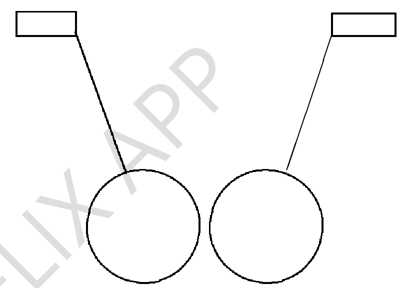
Electricity is a form of energy. Also we can say that **Static electricity** or **Electrostatics** is the form of energy a body acquires or posses due to stationary charges. For instance, if a plastic pen is rubbed in the hair, it will afterwards attract small pieces of paper. This phenomenon is called electric attraction or electrostatic attraction and the rubbed material is said to be charged electrostatically or posses static electricity, a form of energy i.e. it can do work.

Electrostatic force

Suspend two inflated rubber balloons by a length of cotton from stands. Rub them together with your hands and bring them towards each other. The balloons will be observed to be repelling or attracting each other.

These balloons have acquired electric charge by friction.





If you bring towards one of these charged balloons, first a rod of polythene, which has been rubbed with, a piece of woolen cloth and then next to a rod or a strip of Perspex charged in the same way. The polythene rod will repel the balloon and Perspex will attract the balloon. This experiment shows that there exist two types of charges (those being attracted and those being repelled)

Types of charges

We have two types charges;

1. positive charges (+) and
2. negative charges (-)

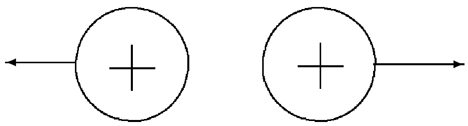
3.10.2 First law of electrostatics.

Law 7.

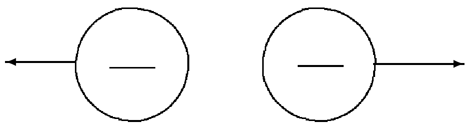
> The **first law of electrostatics** states that like charges repel and unlike charges attract.

i.e. like charges (positive and positive or negative and negative) repels each other and unlike charges(positive and negative) attract each other.

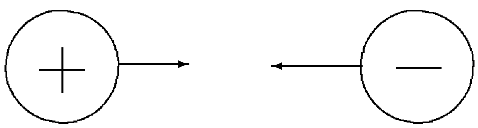
The diagram below illustrates this;



Repulsion



Repulsion



Attraction

3.10.3 Insulator and conductor

An **insulator** is a material whose electrons are firmly bound to its atoms and will not move on their own accord (i.e. are not free to move), for example rubber, nylon, wood, plastics and many more. They are usually bad conductors of heat and electricity.

In **conductors** the electrons are able to move freely from one atom to another. Most metals, e.g copper, iron, aluminum, zinc and others are conductors. They are usually good conductors of heat and electricity

Charging a body

Charging is transferring electrons or any other charged particle from one body to another. A body can be charged in two ways. From the atomic structure, an atom is made up a central positively charged nucleus surrounded by electrons revolving around it, like the moon to the earth. Since in insulators the electrons are not free to move then they are not easily charged but for conductors the electrons are free to move, i.e. they can be transferred from one atom (or conductor) to another. Therefore it is only the conductors that can easily be charged. Charging is transferring electrons or any other charged particle from one body to another, thence, the electrons can be transferred from one body to another in two ways or a body may be charged in two ways;

1. by friction and

2. by electrostatic induction

3. the other method may be by contact

3.10.4 Electrification by friction or charging by friction

When two different materials are rubbed together a small number of electrons from the atoms of one material are transferred to the atoms of the other. The material, which gains electrons, acquires a negative charge (-) and the one which looses electrons becomes positively charged (+). For instance when a polythene rod (plastic rod) is rubbed with cloth, cloth's outer electrons are transferred to the rod, making the rod negatively charged and the cloth positively charged (electron deficient).

A glass rod rubbed with a silk cloth, the rod becomes positively charged and the cloth negatively charged because the rod did lose electrons that were received by the silk cloth.

A glass rod rubbed with asbestos becomes negatively charged and asbestos positively charged. The table 3.17 gives the signs of various materials when rubbed with each other or charged by rubbing (friction);

Materials being rubbed	Sign of charge on the materials after rubbing them together	
	+ tively charged	- tively charged
Polythene and cloth(-)	Polythene	Cloth
Glass and silk (-)	Glass	Silk
Glass (-) and asbestos	Asbestos	Glass
Ebonite (-) and fur	Fur	Ebonite
Perspex and cloth (-)	Perspex	Cloth
Acetate and cloth (-)	Acetate	cloth

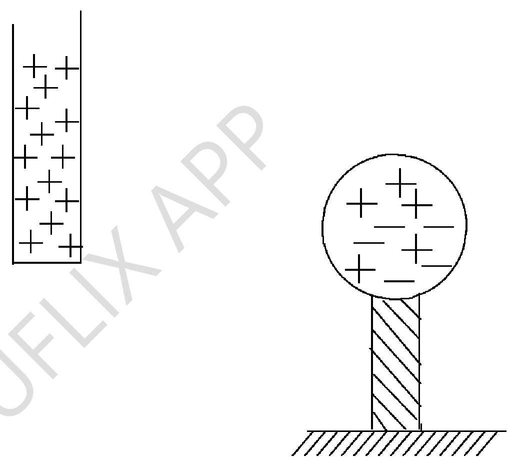
Table 3.17: Charging by friction

Comb and paper experiment

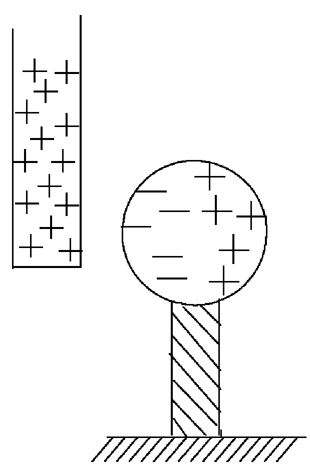
When a dry comb is rubbed in the dry hair on the head, some electrons are transferred from either the comb to the hair or from the hair to the comb depending on the type of the materials of each. Incase the comb acquired a negative charge, and it is brought near small pieces of paper, they will be observed to rise and fall continuously for a certain time (why?). This is because the comb repels some of the electrons on the papers on the table that are close to it and the papers acquire a positive charge, since positive charges are attracted by negative ones (the comb) the papers are attracted by the comb so they rise up. On touching the comb, the papers acquire a negative charge (the charge on the comb is shared with the paper, which acquires a negative charge) and they are now repelled by the rod back to the table (like charges repel). On touching the table, the papers lose the negative charge to the other papers on the table; electrons are repelled further by the rod further making the paper positively charged. The papers are now gain attracted by the rod and the process continues until the rod becomes uncharged or discharged (loses all the negative charge).

3.10.5 Charging by induction:

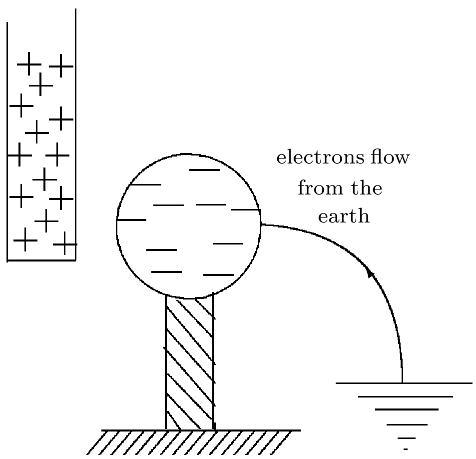
In this method we use a charged body to charge other bodies with out having a transfer of charge from the charged body to the uncharged body. In electrostatic induction, a charged body is brought near a neutral material conductor supported on an insulator. A neutral body always has charges except it has an equal number of positive and negative charges.



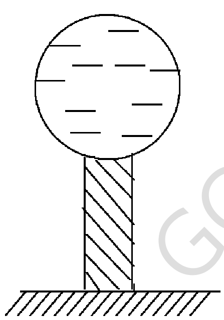
When a positively charged body is brought near a neutral conductor, it attracts the electrons towards the side where the charged body is, leaving the other part with less electrons i.e. positively charged.



If this other end is earthed (i.e. connected it with a wire to the earth or soil) by touching it with a finger, more electrons are attracted from the earth to the electron deficient region of the conductor, thus giving the conductor excess electrons.



If the connecting wire is disconnected when the positively charged body is still in the same position, the Excess electrons are trapped on the conductor hence it remains negatively charged. (i.e. the body has been charged negatively by electrostatic induction[13])



Explain how a body may be charged negatively by electrostatic induction?

Explain how a body may be charged positively by electrostatic induction? (Hint: instead of the initially positively charged body, take a negatively charged body, here electrons flow from the neutral conductor to the earth).

[13]Note that the positive charges do not move, it is only the electrons that can move and only on the surface of the conductor.

There are a number of ways by which a body may be charged; below are the various ways of charging bodies by electrostatics induction;

Ways of charging by induction

1. Charging a conductor negatively by electrostatic induction
2. Charging a body positively by electrostatic induction
3. Charging two bodies simultaneously by electrostatic induction (3 methods)

    3.1 Charging two bodies (conductors) to have opposite charges by electrostatic induction

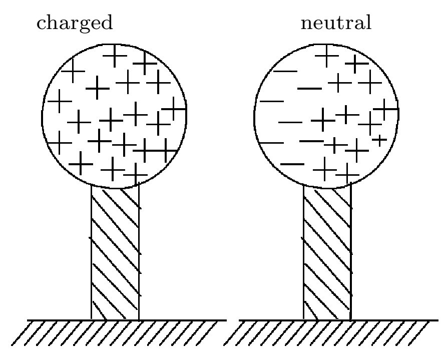
    3.2 Charging two bodies negatively by electrostatic induction

    3.3 Charging two bodies positively by electrostatic induction.

1. **Charging a conductor negatively by electrostatic induction**

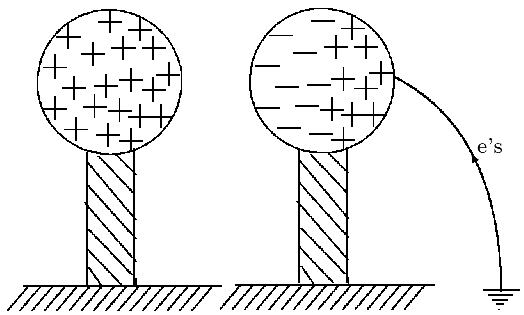
- Bring a positively charged body (or a rod) near a neutral body.

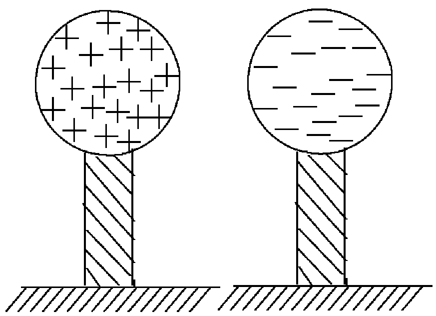
  This makes the side near the charged body of the neutral body negatively charged and the other end positively charged (i.e. it causes unequal distribution of charge on the surface of the neutral conductor).



- Connect and disconnect (not very fast) an earth wire to the other side (positively charged side) of the neutral body.

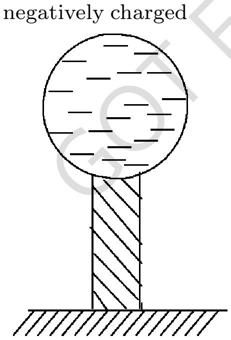
This causes the electrons to flow from the earth to the electron deficient side (positively charged side) making the initially neutral body acquires an excess of negative charges.





- Taking the positively charged body away, the initially neutral body is left with an excess of negative charges (electrons) which distribute evenly now on it, hence it has been charged negatively by electrostatic induction.

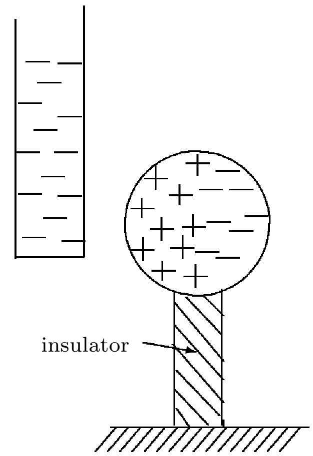
negatively charged



2. **Charging a body positively by electrostatic induction**

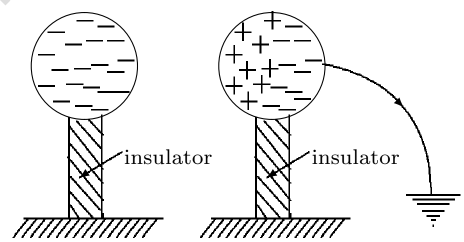
- Bring a negatively charged body (or rod) near a neutral conductor)

  This makes the side near the charged body acquire a positive charge and the other side a negative body.



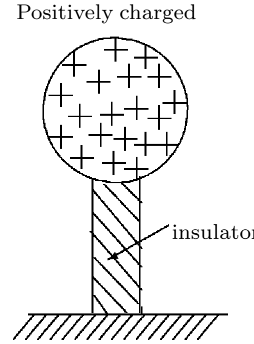
- Connect and disconnect (not very fast) an earth wire to the other side (negative side) with the negatively charged body still in the previous position.

  This causes the electrons to move from the neutral body to the earth, leaving the body with a deficiency of electrons i.e. with a positive charge.



- Taking the negatively charged body away, the initially neutral body will be left with a positive charge(a deficiency of electrons)which distribute themselves evenly now, hence the body has been charge positively by electrostatic induction

Positively charged



3. **Charging two bodies simultaneously by electrostatic induction**

Here we need a charged body and the two neutral bodies to be charged. These bodies can be charged such that they;

3.1 Have Opposite charges (one positively and the other negatively charged)

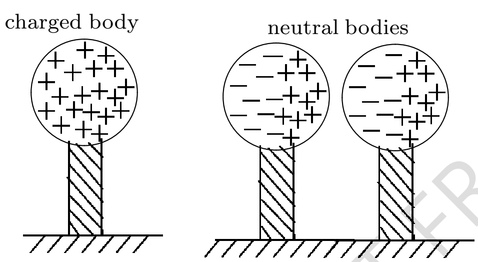
3.2 Are negatively charged or

3.3 Are positively charged

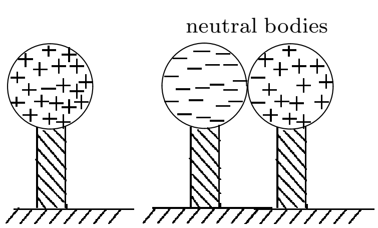
3.1 **Charging two bodies (conductors) to have opposite charges by electrostatic induction**

- Place the neutral bodies close to one another and the charged body near one of them (you may use either a positively charged or a negatively charged body, for this case we have chosen a positively charged body).

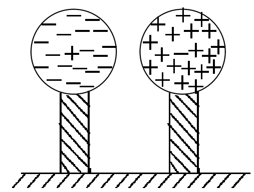
  These causes charges to redistribute on the neutral bodies as shown below



- Make the neutral bodies touch each other by contact. This makes the one near the charged body(+) negatively charged and the other positively charged



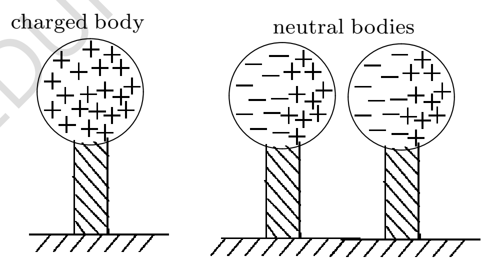
- Separate the two bodies when the charged body is still in the initial position and then take the charged body away. This traps the positive charge on one body and the negative charge on the other body. Hence the two bodies (conductors) have been charged; one positively and the other negatively by electrostatic induction.



3.2 **Charging two bodies negatively by electrostatic induction.**

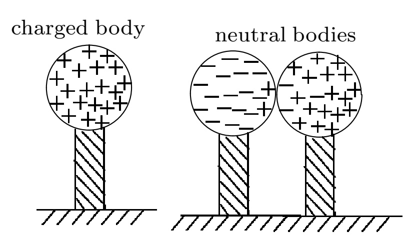
- Place the two neutral bodies close to each other and a positively charged body close to one of them as shown below.

  The existence of the positively charged body causes charge to re-arrange on the surface of the neutral bodies as shown.



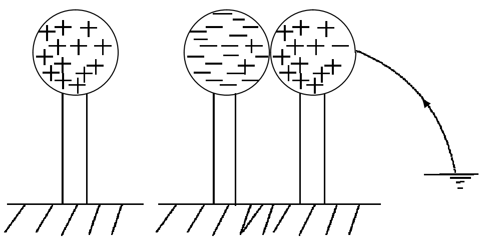
- Make the two neutral bodies to touch each other.

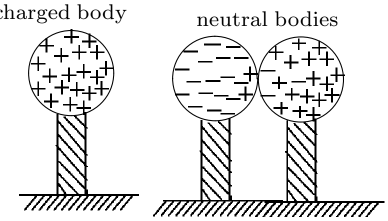
  This creates a path way for the negative charges to flow from one body to the near body hence the two bodies acquire opposite charges.



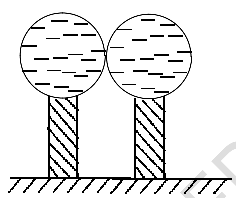
- Connect and disconnect (not very fast) an earth wire to the positive side of the neutral bodies with the positively charged body still in the previous position.

This provides a path way for the electrons to flow from the earth to this electron-deficient region. The electrons flow from the earth to the bodies hence providing the bodies with an excess of electrons(negative charges) as shown below;

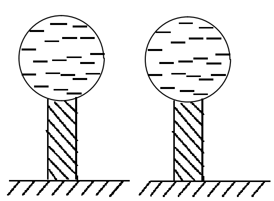




- The positively charged body is taken away, this causes the negative charges on the two bodies distributes equally on their surface.

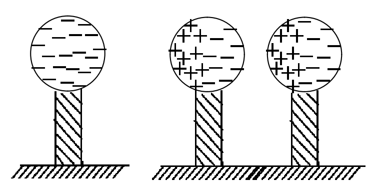


- Separating the two bodies, they separate with their negative charges and hence they have been charged negatively by electrostatic induction.



3.3 Charging two bodies positively by electrostatic induction.

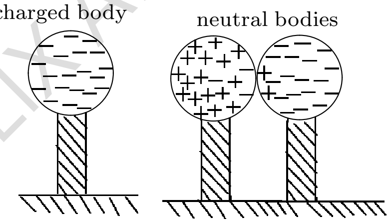
- Place the two neutral bodies close to each other and a negatively charged body close to one of them as shown below.



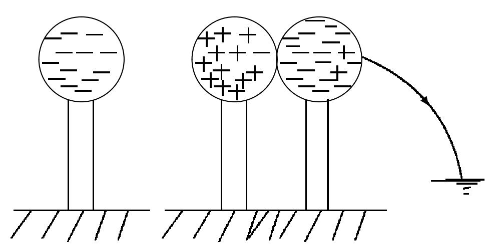
The existence of the negatively charged body causes charge to re-arrange on the surface of the neutral bodies as shown above.

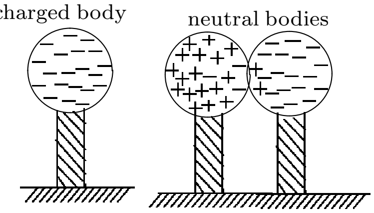
- Make the two neutral bodies to touch each other.

  This creates a pathway for the negative charges to flow from the near body to the other body hence the two bodies acquire opposite charges when still in contact.

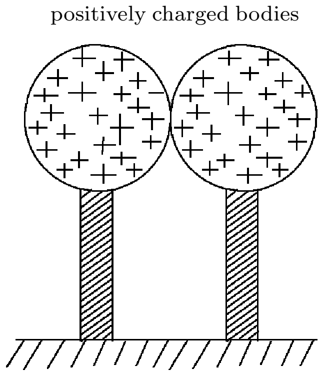


- Connect and disconnect (not very fast) an earth wire to the negative side of the neutral bodies with the negatively charged body still in the previous position. This provides a path way for the electrons to flow from the electron-deficient region to the earth. The electrons flow from the bodies to the earth hence leaving the bodies with a deficiency of electrons(positively charged) as shown below;

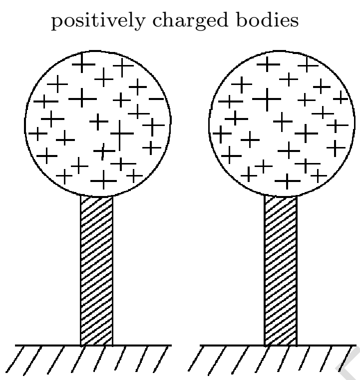




- The negatively charged body is taken away, this causes the positive charges on the two bodies distributes equally on their surface.



- Separating the two bodies, they separate with their positive charges and hence they have been charged positively by electrostatic induction.



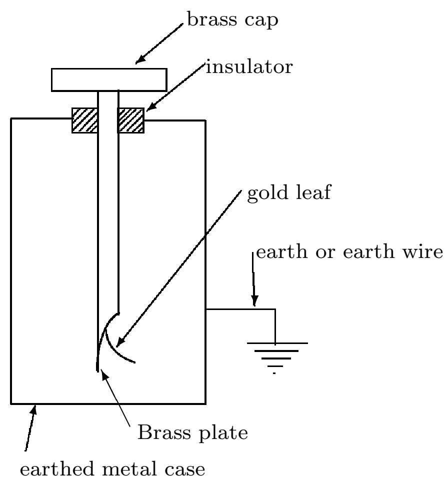
Note that when a charged body in connected to a neutral body, the charge on the charged body is shared with the neutral body, which in turn also acquires charge of the same sign, this method can be called **charging by contact**. This method can only be used to charge a body with the same charge as that of the available charged body. It has one major disadvantage that the net charge on the charging body goes on decreasing, which is not the case with the other methods of charging. The table below illustrates them briefly

		Methods of charging a neutral body electrically		
		Charging by friction	**Charging by electrostatic induction**	**Charging by contact or by sharing charge**
Charge to be on the initially neutral bodies	Positive charge	You need two neutral bodies	You need a negatively charged body	You need a positively charged body
	Negative charge	You need two neutral bodies	You need a positively charged body	You need a negatively charged body
	Positive charge	Impossible	You need a negatively charged body	You need a positively charged body
	Negative charge	Impossible	You need a positively charged body	You need a negatively charged body
	Oppositely charged bodies	You need two neutral bodies	Any charged body and two neutral bodies	impossible otherwise by friction

Table 3.18: Compare the electrical charging methods

3.10.6 The gold leaf electroscope

This is a sensitive electric device used for detecting, testing and at times used to measure small quantities of electric charges. It consists of a metal disc and a rod whose lower part is attached to small strip of gold leaf. (Some times Aluminium leafs are used). The conducting part of the instrument is mounted in a block of insulating material which forms a lid of a metal case with a window so that the gold leaf can be seen. The metal case is earthed.



Effect of a charged body and neutral body on a neutral gold leaf electroscope

It is known that a neutral body has an equal number of negative and positive charges. But the negative charges (the electrons) are the ones that are free to move from one point to another on the surface of a conductor.

A negatively charged electroscope is one that has an excess of electrons and a positively charged electroscope is one that has a deficiency of electrons.

When a negatively charged body is brought near a neutral gold leaf electroscope, is repels some electrons on its cap to the gold leaf (or the aluminum leaf) which diverges (due to the repulsion between the leaf and the metal plate since they have the same charge). Hence when a negatively charged body is brought near a neutral gold leaf electroscope, its leaf diverges.

When a positively charged body is brought near the cap of a gold leaf electroscope, it attracts the electrons on the electroscope to the metal cap leaving the leaf and the metal plate with a deficiency of electrons (i.e. positively charged). Since the leaf and the metal plate have the same charge, they repel each other and hence the leaf diverges.

When a neutral body is brought near a neutral gold leaf electroscope, the gold leaf appears to shake a bit but finally it does not diverge even when the body had touched the metal cap. This can only be observed with very sensitive electroscopes. Explain?

Therefore, in general, when a charged body is brought near a neutral gold leaf electroscope , its leaf diverges.

To charge a gold leaf electroscope

There are two ways by which an electroscope can be charged i.e. either positively or negatively, these are;

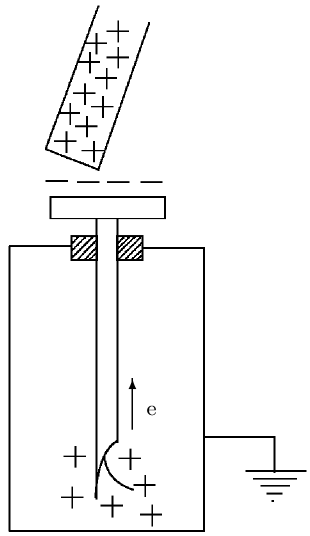
1. By electrostatic induction, this is so far the best method where the charged body and the body being charged do not come into contact and
2. By contact (where the charged body and the body to be charged are just brought into contact).

Note that charging by contact is sometimes called by **charging by conduction**. It is not a good method and often gives a charge opposite to that expected. It is better to use the method of induction.

(a) Charging a gold leaf electroscope negatively

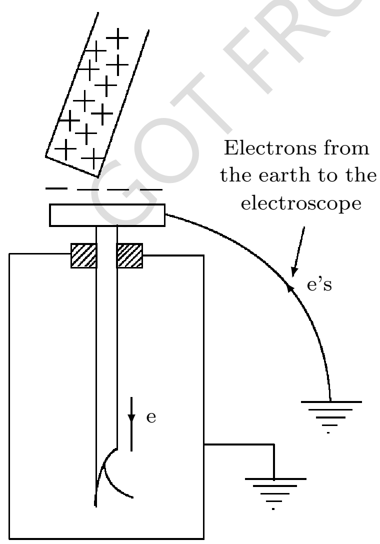
(i)**By electrostatic induction**

Here you bring a positively charged body near the metal cap of the electroscope



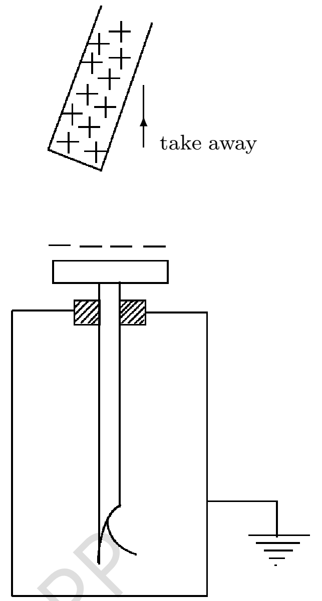
pitively charged body brought near

and connect an earth wire to the cap and then disconnect it.

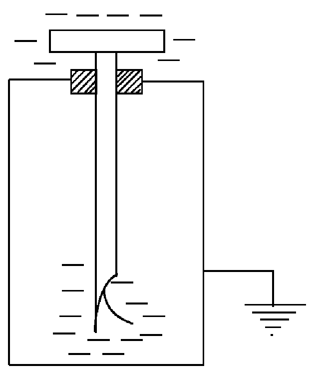


metal cap earthed and attract electrons from earth

Taking away the positively charged body,



the electroscope will be left negatively charged (i.e. with an excess of electrons).



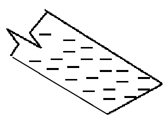
a negatively charged electroscope

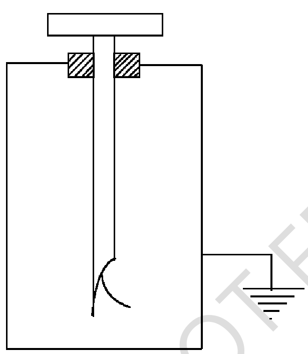
Explanation

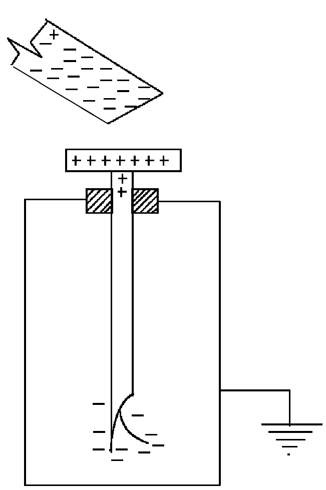
When a positively charged body is brought near the metal cap, it attracts the free electrons from the metal plate and gold leaf to the metal cap (holding them there). This leaves the metal plate and the gold leaf positively charged hence the gold leaf diverges. At this point when an earth wire is connected to the metal cap, it allows the excess electrons to flow from the earth to the metal plate and the leaf, which become neutralized. Disconnecting the earth wire, the electrons are trapped and taking away the positively charged body, the electroscope is left with excess electrons initially held by the positively charged body at the metal cap hence negatively charged.

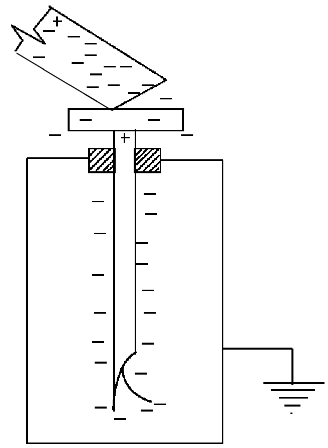
(ii)**by contact(not by friction)**

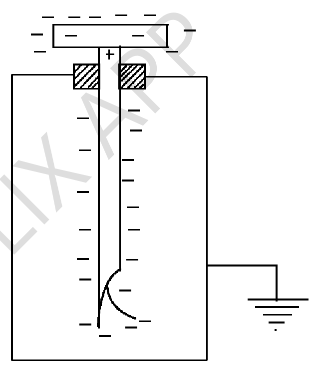
Here you get a negatively charged body and roll it on the metal cap of the neutral electroscope, then take it away. The electroscope will be found to posses a negative charge i.e. negatively charged.











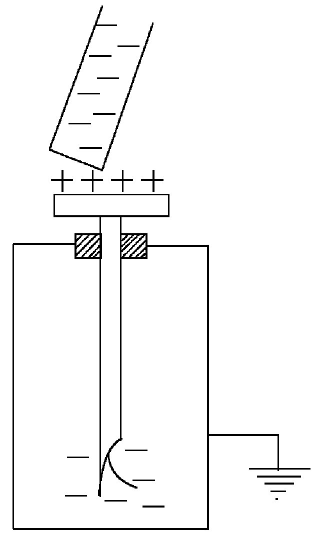
Explanation

A negatively charged body has an excess of electrons (or negative charges). When it is placed in contact with the cap of the gold leaf electroscope, the charges on the body (the excess electrons) are shared with the electroscope. Therefore the electroscope gains an excess of electrons and these make it negatively charged.

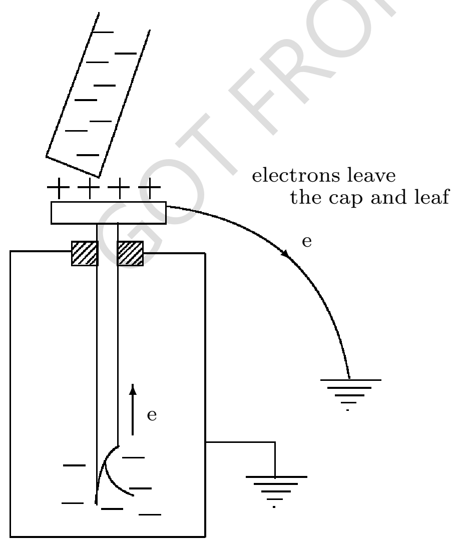
(b) Charging a gold leaf electroscope positively

(i) By electrostatic induction

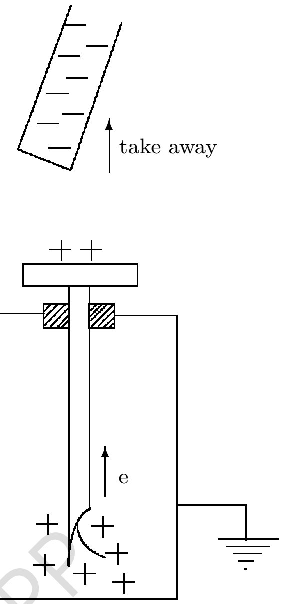
Here you bring a negatively charged body near the metal cap of the neutral electroscope



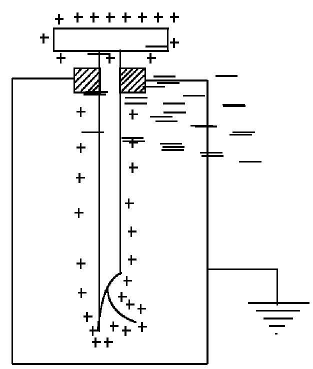
Connect an earth wire to the cap and then disconnect it.



Taking away the negatively charged body,



The electroscope will be left positively charged (i.e. with a deficiency of electrons).



Explanation

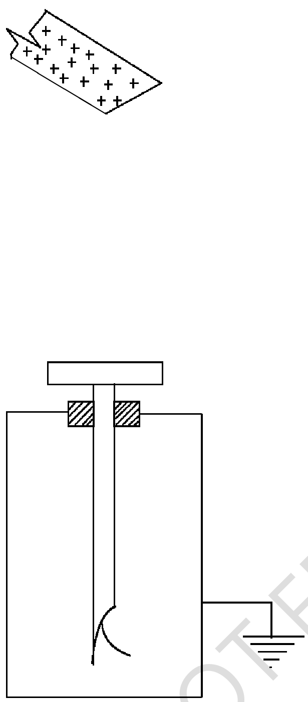
when a negatively charged body is brought near the metal cap, it repels the free electrons on the electroscope from the metal cap to the metal plate and the gold leaf. The metal cap is left with a deficiency of electrons i.e. positively charged and the metal plate with the gold leaf with an excess of electrons i.e. negatively charged hence the leaf diverges. At this point when an earth wire is connected to the metal cap, it provides an easy path or escape route for the repelled electrons on the metal plate and gold leaf to the earth [14]. The gold

[14] The earth can never get filled up of electrons i.e has a high charge storing capacity or high capacitance

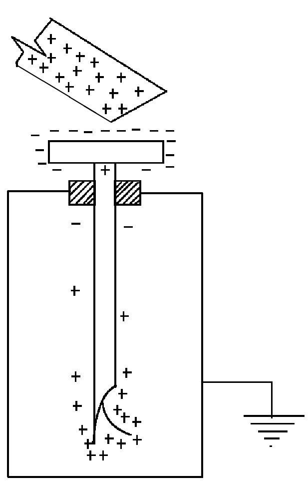
leaf collapses since it is now neutralized by the removal of the excess electrons (the metal cap is still electron deficient). Disconnecting the earth wire and taking away the negatively charged body respectively, the electroscope is generally left with a deficiency of electrons hence the electroscope is positively charged.

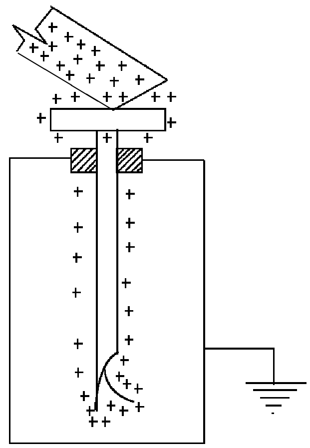
(ii) By contact

Here you get a positively charged body and an a neutral electroscope

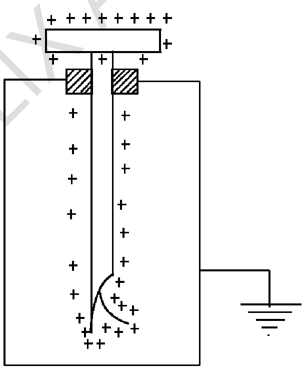


Make it touch the electroscope,





and then take it away. The electroscope will be found to posses a positive charge i.e. positively charged.



Explanation

A positively charged body has a deficiency of electrons. When it is placed in contact with the metal cap of the neutral gold leaf electroscope, the few electrons on the neutral electroscope are shared with the positively charged body. Taking away the positively charged body, it goes with some electrons initially with the neutral electroscope; hence the electroscope is now left with a deficiency of electrons i.e. positively charged.

Note that the leaf of a charged electroscope is ever diverged.

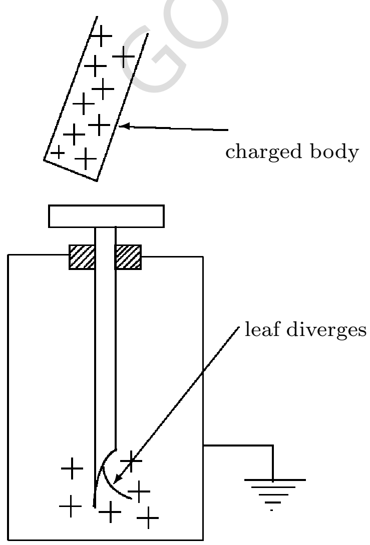
Explain how you can charge a gold leaf electroscope (a) positively and (b) negatively.

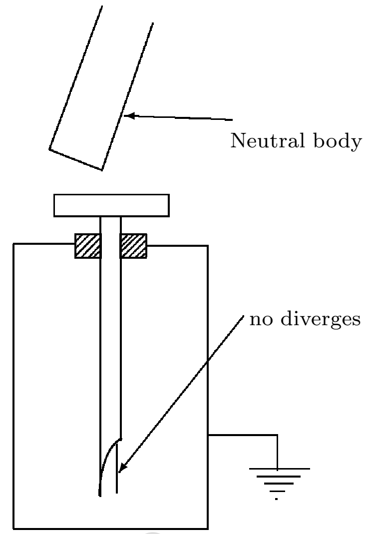
Uses of a gold leaf electroscope

1. Test or detect the presence of charge.
2. To test sign of charge on a body.
3. To test insulating properties of various materials.

1. To detect the presence of charge on a body using a gold leaf electroscope

The body under investigation is brought near the cap of the neutral electroscope. If the body is charged it will induce a similar charge on the gold leaf and the metal plate, consequently repulsion occurs, this causes the leaf to diverge. On taking away the body, the leaf collapses, hence Divergence of the leaf of a neutral electroscope shows that the body is charged.





2. To test the sign of the charge on a body (i.e. positive or negative)

To test the sign of charge on a charged body, we use a charged gold leaf electroscope, the various experiments below show how they are performed and how the sign of charge on the body is concluded;

(a) **1. A highly negatively charged body lowered to a positively charged gold leaf electroscope**

When the charged body is high above the metal cap of a positively charged electroscope and it is lowered slowly to it, the **leaf divergence decreases** as the positive charge on the leaf and the plate become partially neutralized by the free electrons which are repelled down wards. As the charged rod is gradually lowered, more electrons will be repelled to the plate and the leaf until the **leaf completely collapses**, this is when its positive charge becomes exactly neutralized by the repelled electrons.

After this state has been reached a further lowering, will cause the **leaf** to **diverge again** since the plate and the leaf now acquire an excess of electrons (negative charge-hence repelling each other).

Note that the charge must be brought from a good height slowly. Note that the leaf divergence decreases, finally collapses and diverges again. This is true for highly charged bodies, they are not always used.

(b) **A negatively charged body lowered to a neutral electroscope**

It repels the electrons on the metal cap of the electroscope to the metal plate and the gold leaf. The metal plate repels the gold leaf and hence it diverges showing the presence of charge on the body near the metal cap. **Leaf diverges.**

(c) **A negatively charged body lowered to a negatively charged electroscope**

It repels the few electrons on the metal cap making the gold leaf and the metal plate more negative. Hence the **leaf divergence increases** or increases slightly

(d) **A negatively charged body lowered to a positively charged electroscope**

A negatively charged electroscope has a deficiency of electrons (i.e. it has very few electrons) and its golf leaf is ever diverged so long as it is charged. The lowered negatively charged body, repels the few electrons to the metal plate and the gold leaf, making them less positively charged, this reduces the force of repulsion and consequently, the divergence of the leaf decreases. **Leaf divergence decreases**

(e) **A positively charged body lowered to a positively charged electroscope**

The lowered body attracts the few electrons to the metal cap making the metal plate and the gold leaf more positive i.e. increasing the force of repulsion. Hence the golf **leaf divergence increases**.

(f) **A positively charged body lowered to a negatively charged electroscope**

The lowered body attracts more electrons to the metal cap making the metal plate and the gold leaf less negative reducing the repulsive force between them. Hence the **leaf divergence decreases.**

If the magnitude of the positive charge on the lowered body is much greater than the magnitude of the negative charge on the electroscope, the leaf diverges and finally collapses and then rises. This is because its negative charge decreases to zero and then becomes positively charged.

(g) **A positively charged body lowered to a neutral electroscope**

The lowered body attracts the electrons to the metal cap leaving the metal plate and the gold leaf with less or a deficiency of electrons i.e. positively charged and they repel each other. Hence the **gold leaf diverges**.

The results from the above experiments are summarized in the table 3.19;

Charge on electroscope	Charge on the body brought near the cap	Effect on leaf divergence
+	+	Increases
-	-	Increases
+	-	Decreases
-	+	Decreases
- or +	Uncharged	Decreases
Neutral	neutral	no effect
Neutral	Positive	Diverges
Neutral	Negative	Diverges

Table 3.19: Testing sign of charge

Table 3.19 can be summarised in three sentences;

- If the divergence of the leaf decreases it means that the body near its metal-cap has either charge opposite to the electroscope's charge or it is neutral.
- If the leaf divergence increases, then the sign of charge on the body is the same as that on the electroscope.

- If the electroscope is neutral, we can not tell the sign of charge on the body [15].

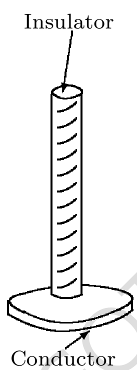
Precaution when using gold leaf electroscope

1. it must not be in damp conditions
2. it must not be touched by any thing when in use, unless otherwise.

3.10.7 Distribution of charge over the surface of a conductor

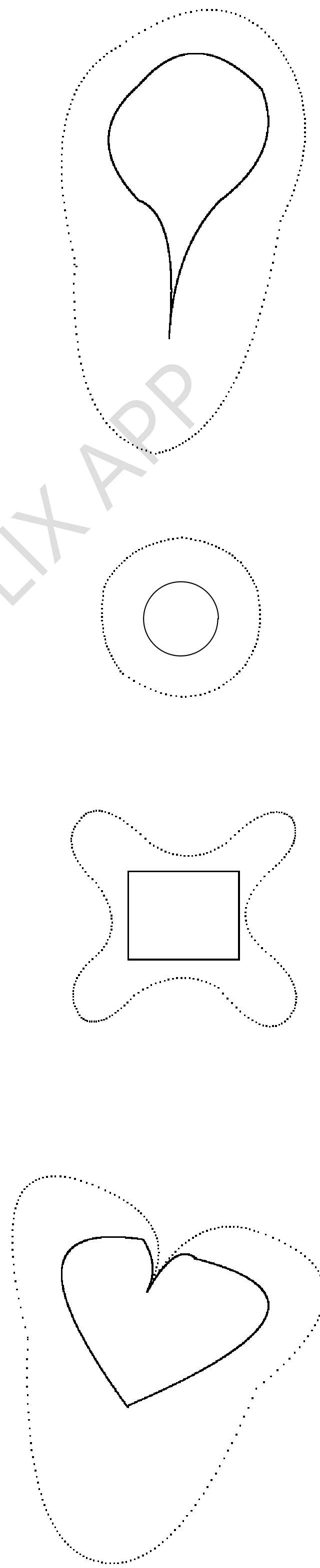
Surface charge density is the quantity of charge per unit area on the surface of a conductor. Surface density is estimated using a proof-plane.

A proof plane is a small metallic conductor glued or fixed on an insulator.



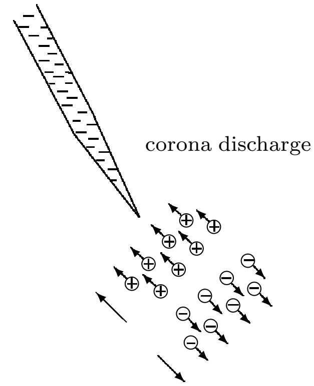
Place the proof plane into contact with various points on the conductor, and then transferring it to the metal cap of a neutral gold leaf electroscope. The gold leaf of the electroscope will diverges to show the presence of charge or will not diverge to show the absence of charge. The degree of divergence of the leaf is proportional to the quantity of charge transferred or the charged density of that area of the conductor.

The most important fact discovered was that, charge in highly concentrated where the surface is sharply curved (outwardly). The diagrams below illustrate this;



[15] You must analyse this property properly, it is not always a walkover

Action of highly charged sharp points

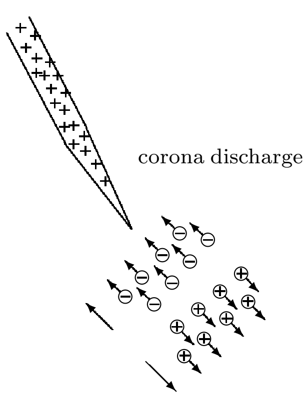


Consider a highly charged conductor with a sharp point. Ordinary air contains a certain number of negative and positive ions. A highly charged point conductor repels ions of the same sign, strongly. These fast moving ions collide with the air molecules in front of it, knocking electrons out of them, thus creating more ions in a short period of time. This stream of ions and some molecules moving away from this point carry some of the surrounding air molecules setting up a small wind known as an **electric wind**. At the same time, ions of opposite sign are attracted towards the same point (i.e. in the opposite direction) where they neutralize the charge on it.

Definition 41.

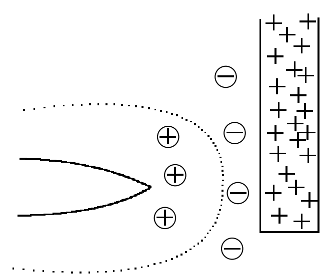
The **electric wind** is the collection of moving ions and air molecules towards and away from a highly charged point.

This process continues until the initially charged sharp point is neutralized (the whole conductor is discharged). All conductors with sharp points get neutralized very first.



This process of forming an electric wind resulting to loss (or gain) of charge is known as **corona discharge**

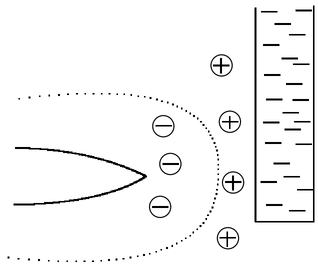
Not only does a sharp point enable a conductor to loose charge but can also act as a charge collector. In the diagram below the sharp point action takes place, such that positive ions are repelled to sharp pointed conductor while the negative ions do leave the conductor to the charged body which gets neutralized.



Corona discharge

As a result the sharp pointed conductor acquires positive charge i.e. collects charges from the positively charged one. Therefore a sharp point is also a charge collector.

Explain, what would happen if a highly-negatively charged conductor is placed near sharp point as shown below.



Corona discharge

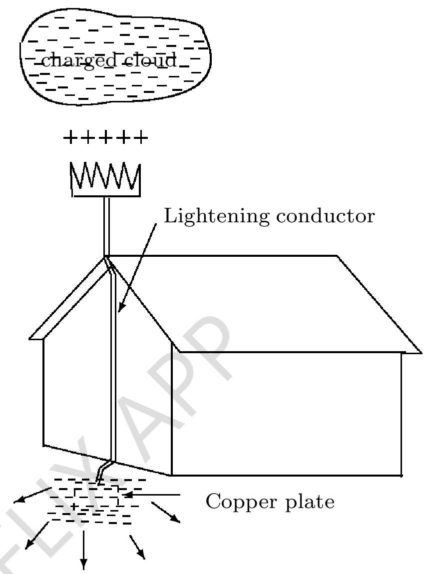
3.10.8 Lightening

Lightening is the spark between two bodies carrying unlike charges, usually from cloud to cloud or cloud to ground. Suppose a negatively charged cloud is passing over the ground, it induces a positive charge on the ground. As the charge on the negatively charged cloud builds up, it finally becomes great enough to strike through the air insulation to the ground as a spark. This giant spark is known as **lightening**. It heats the air where it passes, which expands suddenly. The expansion of air sends out a giant compression wave (or sound wave) we call **thunder**. Usually lightening (i.e. light from the lightening) is seen first and then after a few minutes the thunder (sound) is heard, this is because light travels faster than sound.

Explain how a positively charged cloud can cause lightening?

Protection from lightening (lightening conductor)

Lightening (a spark) descending from a cloud to the earth (or vise verse) tends to strike or pass through the nearest conductor. e.g. a tall building, a tree, a house or a man in an open space. The best way to safe guard a building is to provide a safe easy path-way for the charge (Usually electrons) to reach the ground, incase lightening strikes. In the present day a lightening conductor serves this purpose. A lightening conductor takes the form of a thick copper strip or wire running from the ground (earth) to the highest point on the building and ending in several sharp spikes. When a negatively charged thundercloud passes over head as shown below; it acts inductively on the conductor, charging the sharp points positively and the copper plate negatively.



The negative charge on the lightening conductor is immediately repelled to the copper plate in the ground (earth). At the same time, positive ions are attracted to the spikes by the negative cloud forming a space charge. This space charge breaks the air insulation letting the charges on the cloud jump to the spikes of the lightening conductor, the highest point of the building, which conducts the charge harmlessly to the earth. If the lightening conductor was not on the house, the charge would flow through the house causing all the air spaces in its walls to expand, some particles are over heated to red hot and the net effect is the collapse of the building.

People can protect them selves from lightening by

1. Taking shelter in a house or an enclosed space
2. Not using telephones or umbrellas during thunder storms
3. Avoiding tall solitary trees or other isolated objects in open areas

4. Avoiding electronic equipments that use the electromagnetic waves that pass through air e.g. mobile phones, radios e.t.c. during rain

3.10.9 Experiment with hollow charged conductors

An insulated hollow brass sphere with a small hole on it, is charged by induction or by transferring charge to it using a proof plane. When the proof plane is placed inside the hollow conductor and transferred to the electroscope, it showed no divergence, (absence of charge). But when placed on the outside surface of the hollow conductor and transferred to the electroscope, it showed a divergence (presence of charge).

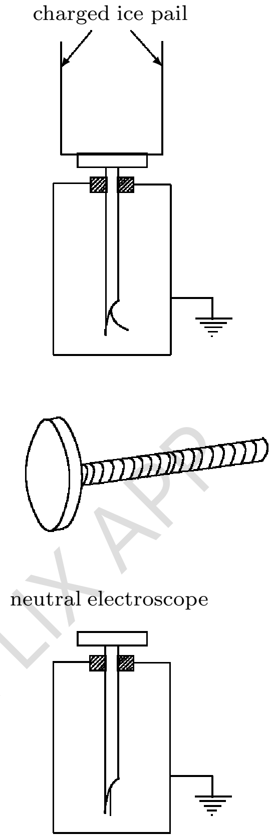
This experiment shows that charge does not reside on the inside wall of a hollow conductor but on the outside surface of the conductor. Very many experiments were conducted to verify that charged resides on the surface of the conductor not inside it. Some of these are:

1. Faraday's ice pail experiment.
2. Coulomb's hemispheres.
3. Butterfly net.

All these experiments describe the same principle. In general, charge does not reside inside a hollow conductor whether hollow or solid.

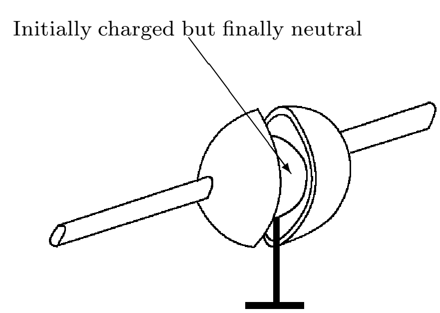
1. **Faraday's ice pail experiment**

   Faraday used pails for storing ice (ice-pails), he charged the pail and placed it on a neutral electroscope whose leaf diverged, as expected. When he placed a proof plane into the charged ice pail on the electroscope and transferred it to another neutral electroscope, its leaf did not diverge (he expected it to diverge). This shows that the proof plane did not transfer any charge and this was because the inside of the charged pail did not posses any free charge. This showed that charge does not reside on the inside surface of a charged pail.



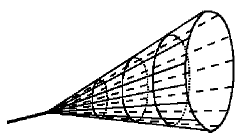
2. **Coulomb's hemispheres**

   These were two hollow hemispheres which could just fit on a certain solid conducting sphere. When the solid sphere is charged, and then two neutral hemispheres made to just fit it, the solid sphere was left with no charge. The reason being that, the solid sphere touched the inside of the hemisphere where charge can not reside and hence it became neutral.



3. **Butterfly net**

These were nets used to trap butterflies during the dry season. The net was made from an electrical conducting material. The butterfly-net was charged expecting it to electrocute the butterflies and the butterflies could not receive any effect. On touching the inside of the charged butterfly net with a proof plane, it had no charge. Still confirming that; charge does not reside inside a hollow conductor.



This principle is applied in **electrostatic shielding** i.e. preventing a given space or region from receiving electric fields or electric field lines

3.10.10 Electric fields

Electric field is a place or region around any charged body where an electric force can be exerted or experienced. An electric field is usually represented by imaginary lines known as electric field lines.

An electric field line is an imaginary line drawn in an electric field with an arrow pointing in direction of the electric field at that point.

The direction of electric field lines at any point is the direction of the force on a small positive charge placed at that point.

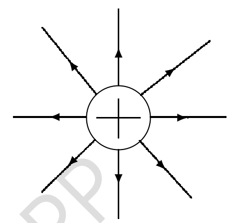
Properties of electric field lines

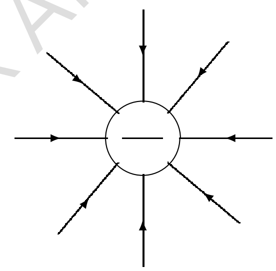
Using the above definitions and direction of an electric field line, we have

1. They begin and end on electric charges. (from positive charge to negative charge)
2. They are ever in a state of tension, longitudinally.
3. They repel one another side ways i.e. laterally.
4. Their direction is from positive to negative charges.

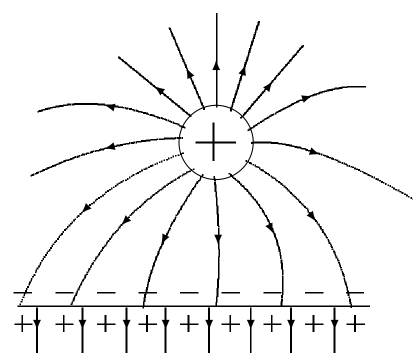
Electric field diagrams

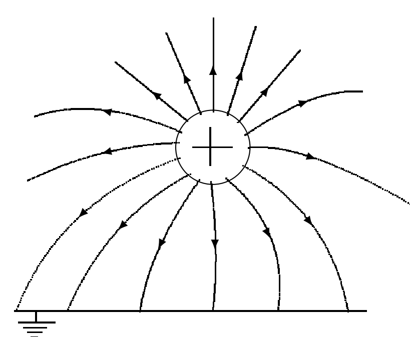
1. Isolated positive and negative charges

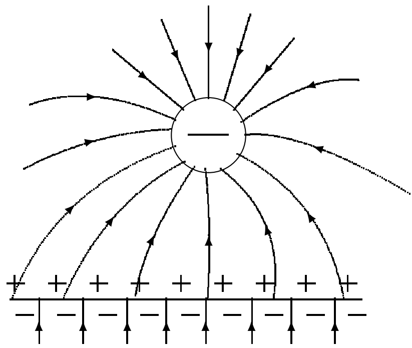


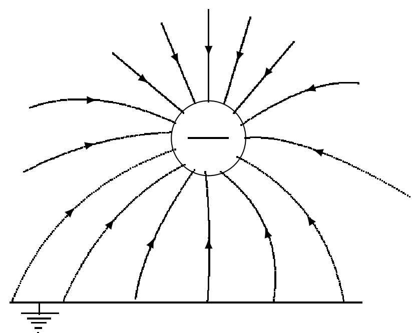


2. Isolated charges near neutral surface earthed and unearthed

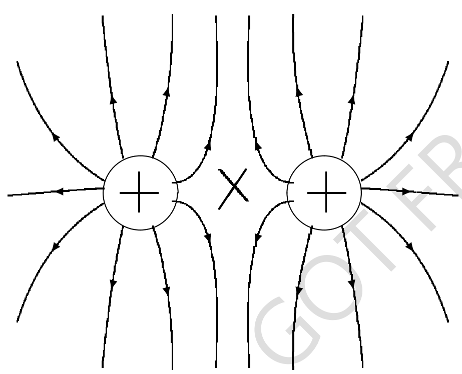




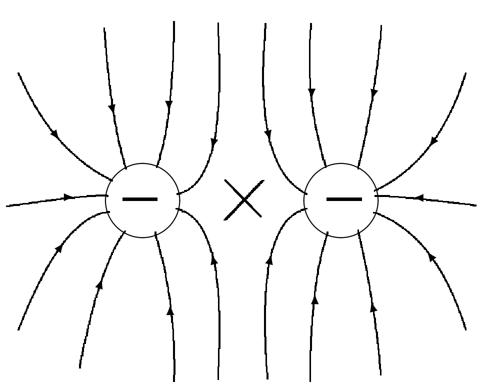




3. Like charges near each other



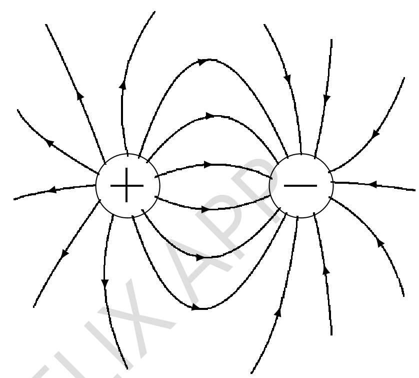
X is neutral point.



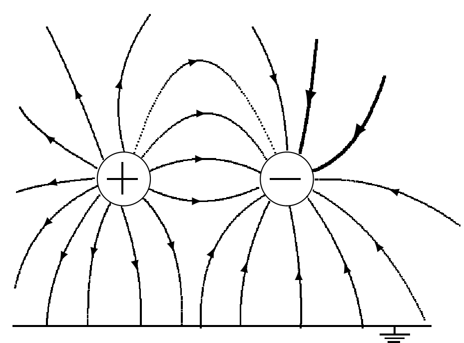
Definition 42.

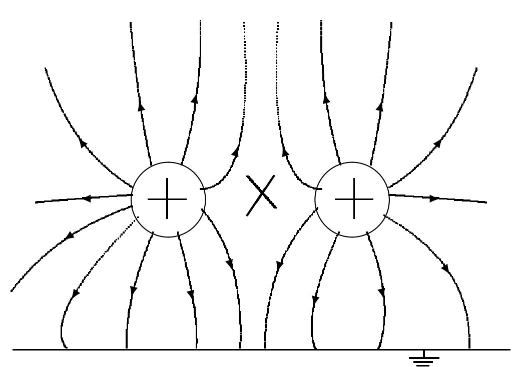
Neutral point is a point where there is no or zero electric field.
A neutral point is a point in an electric field where a charge experiences no force.

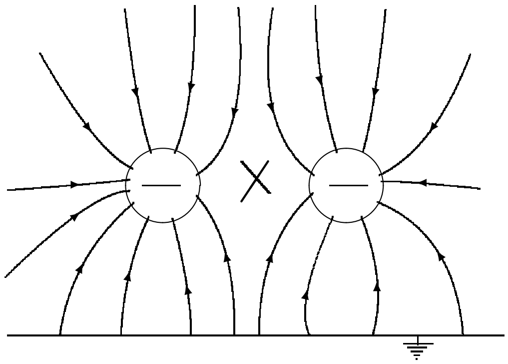
4. Unlike charges

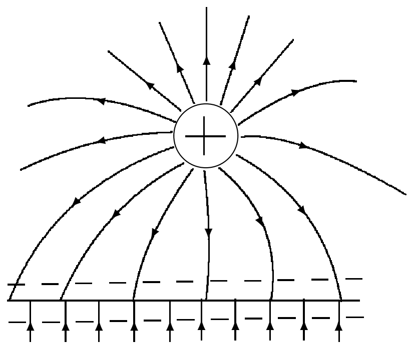


5. Like and unlike charges near an earthed and an unearthed plane surface

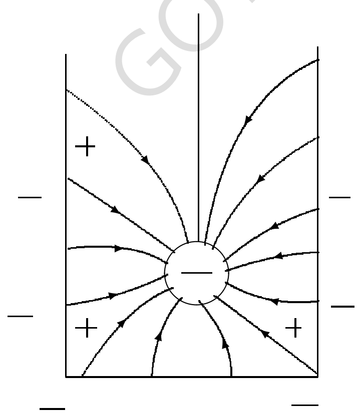




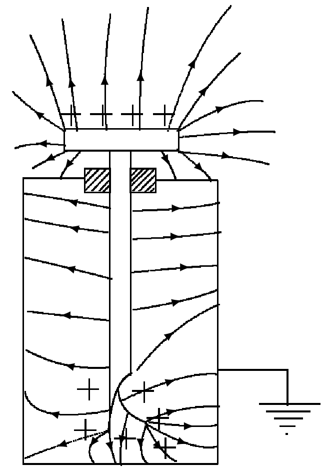




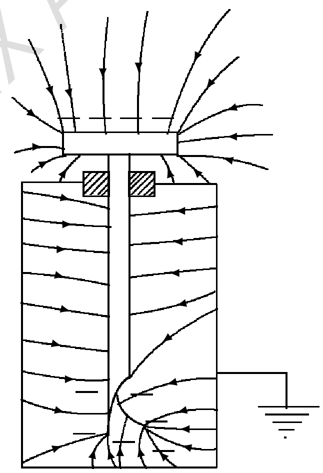
6. Electric field for a negatively charged ball in a metallic pail (ice pail experiment)



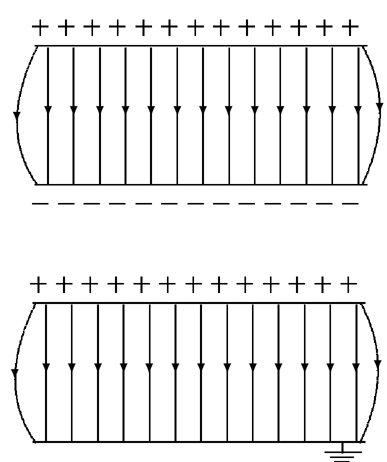
7. Electric field lines for a positively charged electroscope

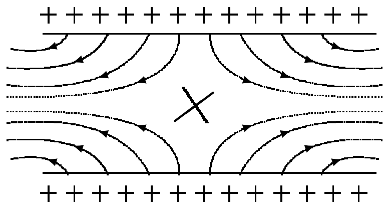


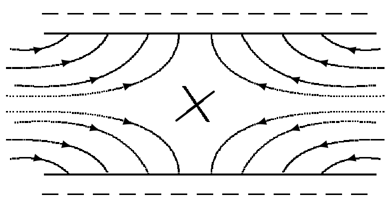
8. Electric field lines fora negatively charged electroscope.



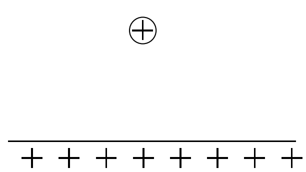
9. Electric field lines between two charged parallel plates

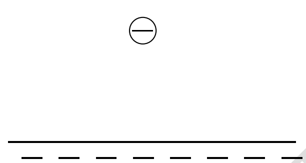






Sketch the electric field lines due to the charged bodies below;

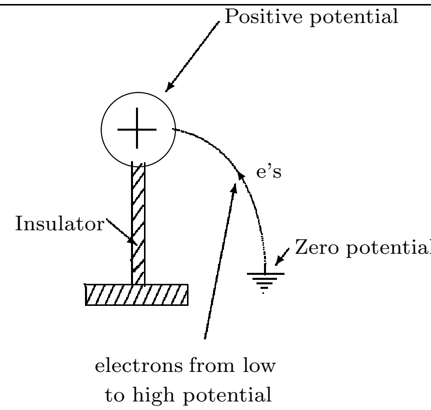


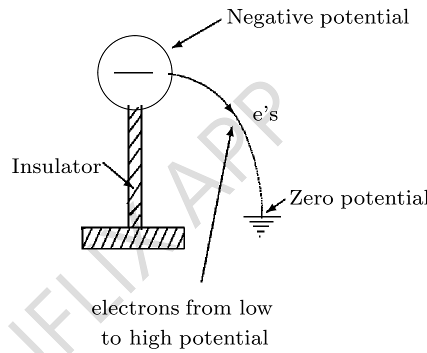


3.10.11 Potential difference

This may be defined for the time being as the electrical condition, which governs the direction of flow of electric charges, especially electrons from one point to the other. The electrical potential of the earth is taken to be zero, while a positively charged body has a potential above that of the earth (a positive potential) and a negatively charged body has a lower one (i.e. a negative potential).

Consider a positively charged and negatively charged body, when each is earthed.

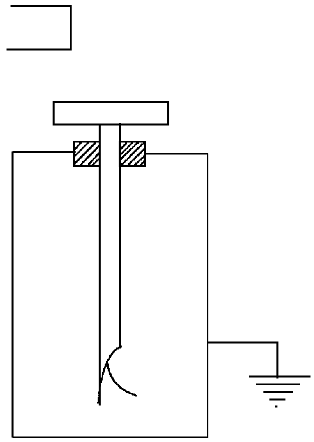




For a positively charged body, when earthed electrons flow from the earth to the body, while for the negatively charged one, electrons flow from the body to the earth. The earth is taken to be at zero potential meaning that it can absorb very many electrons and at the same time it can supply very many electrons. [16] From this we observe that, electrons tend to move up the potential gradient (i.e. from lower potential to higher potential).

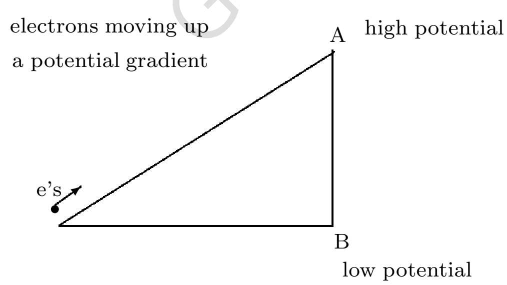
A negatively charged body and the positively charged one are not at the same potential with the earth, hence there is a potential difference between each of the conductors and the earth. Whenever there is a potential difference between two bodies, electrons do flow from a body with a lower potential to the one with a higher potential. The potential of a charged body does not depend on only its charge only but also on the presence of other charges in its neighborhood. These other charges do raise or lower the potential or potential difference by a certain amount.

[16] Practically from lightening experiments, the earth has a negative charge.



To explain more, consider when a positive charge is held near the metal cap of the gold leaf electroscope. The potential difference of the cap (with respect to the earth) raises while that of the metal case remains constant (since it is earthed). This causes the gold leaf to diverge i.e. the leaf diverges, in terms of potential difference it diverges due to the potential difference between the gold leaf and the metal case.

Note that electric field lines are drawn from region of high potential to region of low potential. Potential is like temperature, heat flows from region of high temperature to those of low temperature, also electrons flow from region of low potential to the region of high potential.(i.e. electrons move up the potential gradient)



Absolute zero of potential or a potential of zero

The earth is taken to be at zero potential (just taken) but actually it is negatively charge. So when does a body have actual zero potential or how can we obtain a potential of zero?

Absolute potential of zero is not simple to obtain. Absolute potential of zero is the potential of points at infinity. Infinity is some where very far for our senses to know what is there, whoever tries to explain what is at infinity is expressing his or her maximum ignorance. If am here in Uganda and there are no aeroplane, then America is at infinity i.e. walking from here to America is an impossibility. All in all, we assume that no one has control over what is at infinity and when electrons are at infinity, their movement is not governed by anything hence they move on their own accord, i.e there is no electrostatics potential or they are at an absolute zero of potential (Potential is zero) [17].

Equipotential surfaces

Definition 43.

An **equipotential surface** is a surface whose points on it all have the same potential.

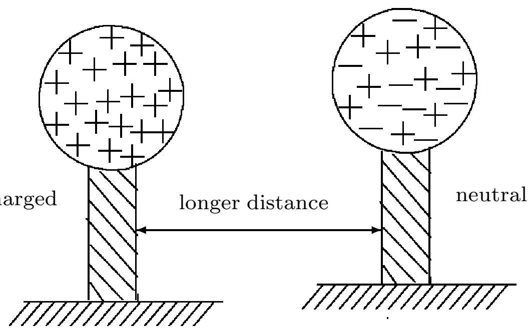
Remember electrons are free to move on the surface of a conductor as they wish with out being forced. This shows that the potential of every point on the surface of a conductor is the same; hence the surface of a conductor is an equipotential surface. If two points or places on the surface of a conductor are at different potentials, then electrons would flow from one of them to the other, i.e. we would have a continuous flow of electrons on the surface of a conductor and this would be against the law of conservation of energy.

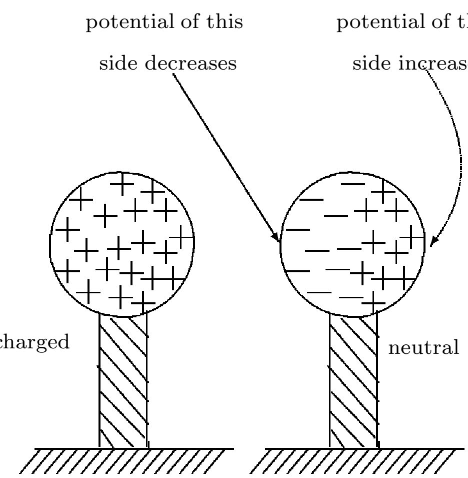
Electrostatic induction

Recall in this process, a charged conductor is brought near a neutral one. This increases or decreases the potential of one side and decreases or increases that of the other side respectively depending on the charge on the near conductor. As

[17] In reality, infinity is just forced to be real

a result, a potential difference is created on the surface of the conductor and hence electrons flow from one side to another on the same surface of the conductor





But the net potential of the neutral conductor as a whole remains constant.

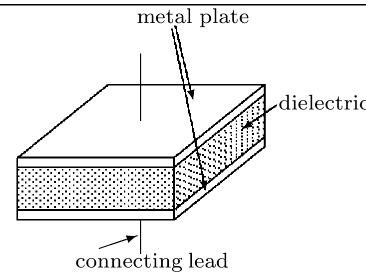
Electrostatic machines

[18]

3.10.12 Capacitance

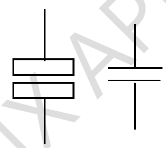
A capacitor is any arrangement of two conductors placed close to each other. The simplest capacitor consists of two small parallel metal plates separated by an insulator called a dielectric.

[18] Machines like the Electrophorus, Wirmshirst machines and Van De graafe generators will be found in advanced text books



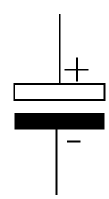
(A dielectric is any insulator). Capacitors are devices used especially in electronics to store electric charge or electric energy in the form of electric fields.

Symbol of capacitor is either



capacitor symbol

Or



capacitor symbol

The measure of the ability of a capacitor to store charge is called **Capacitance**. Capacitance of a capacitor is defined as the ratio of the charge on any of its conductors to the potential difference between them.

$$\text{capacitance} = \frac{\text{charge on one of its conductors}}{\text{p.d across its conductors}}$$

Factors affecting the capacitance of a capacitor

These are the factors affecting the charge storing capacity of a capacitor. These are;

1. Area of the conductor or Area of overlap of the plates. The larger the area the higher the capacitance

2. The distance between the conductors or the parallel plates. The closer they are the higher the capacitance

3. The insulator or dielectric. The material between the conductors (plates) may increase or decrease the capacitance.

Details about capacitors can be found in advanced books.

3.10.13 Applications of electrostatics

Electrostatics is applied in a number of fields, some are described below;

1. **In spraying cars**.

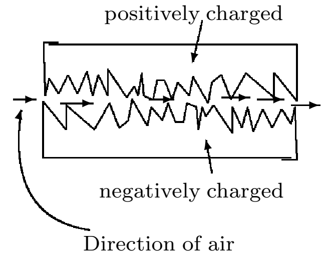
   The car's body is given a certain electric charge. Paint from the Guns becomes charged by friction or inductively and the charged paint droplets are then attracted to the car instead of missing it hence reducing loss and saving money.

2. **In preventing lightening from striking buildings**.

   Connecting lightening conductor (thick copper wire) from the top of tall buildings to the earth does this. The conductor provides an easy path for the large current, which would other wise burn the building, when lightening strikes.

3. **In collecting ash, dust or fume particles from the air in factory chimneys and power stations by electrostatic precipitation**. (How?)

   Wires inside the chimney are negatively charged and give a similar charge to the ash particles. The negatively charged particles are attracted to the positively charged metal plates inside the chimney walls. This removes the ash particles from air hence clearing air, i.e. reducing pollution.



4. **Preventing accident in aeroplanes, fuel-transporting cars and electric shocks**. (How?)

   When one sits in a car for a longer time like for a long journey on getting out, he or she at times gets an electric shock. This is because the electric charges accumulate on the surface of the car as it moves through the air and along the road. When one moves out, he conducts this charge to the earth via him and hence he gets an electric shock. This is prevented or minimized by

   - Connecting a conducting strip to the car and dragged on the road. (i.e. touching the ground).
   - By using conducting rubber tyres.

   If charges are allowed to build up on tracks carrying flammable goods like petroleum, a very small spark can cause an explosion.

5. **In electrostatic photocopying machines**,

   Here a pattern of reflected light from the origin copy containing bright and dark regions is allowed to fall onto positively charged plate. Charge leaks away from the area exposed to light. The unexposed area, that is the dark part of the original copy, still carries positive charge. This positively charged image attracts negatively charged carbon powder (black). This blackened image on the plate is then attracted onto a piece of paper with the help of a positive charge placed under the paper. The paper is then heated to fuse the powder to the paper permanently.

Questions

1. What is meant by the terms electrostatics, conductors, insulators, corona discharge and lightening as applied in electrostatics.
2. Give two types of charges and hence use them to state the law of electrostatics
3. Differentiate charging by friction and by electrostatic induction
4. Describe briefly how a neutral electroscope can be charged positively using a positively charged rod.
5. (a) With the aid of a labelled diagram describe how you would charge a neutral electroscope negatively by electrostatic induction.
   (b) State three uses of an electroscope
   (c)Describe briefly how an electroscope could be charged positively using a negatively charged rod on an insulator.
   (d) State three conditions that could lead to the decrease in the leaf divergence of a charged electroscope.
6. The action of a lightening conductor depends on the action of a "highly charged sharp points" Explain
7. Describe briefly how; a lightening conductor works or protects a high building
8. (a) Describe briefly how charge is distributed in a charged hollow conductor
   (b) Describe briefly a simple experiment to verify your answer in (a) above
9. (a) What is meant by electric field, neutral point and the direction of an electric field line as used in electrostatics
   (b) State two properties of electric field lines
10. (a) What is meant by the terms potential difference and equipotential surface as applied in electrostatics
    (b) Give one example of an equipotential surface
11. State three applications of electrostatics
12. Give one use of capacitors

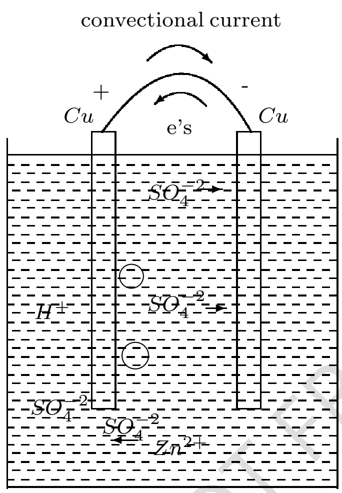
3.11 Electric Cells

3.11.1 Introduction

An **electric cell** is a device that produces moving charges (usually electrons) or which produces an electric current. An **electric current** is the flow of charges. The path taken by an electric current is called an **electric circuit**.

3.11.2 A simple cell

It is made of copper plate and zinc plate dipped in dilute sulphuric acid.



When copper and zinc are joined by a wire, the zinc slowly begins to dissolve in sulphuric acid (H_2SO_4) and bubbles of hydrogen (H_2) are formed on the copper plate, at the same time a current of electrons drift through the wire from zinc to copper, after a short time a current stops flowing.

Theory

Sulphuric acid in solution dissociates according to this equation;

$$H_2SO_{4(aq)} = 2H^+(aq) + SO_4^{-2}(aq) \qquad (3.74)$$

Zinc atoms dissolve into solution in form of zinc ions, each atom leaving two electrons behind the plate.

$$Zn(s) = Zn^{2+}(aq) + 2e^- \qquad (3.75)$$

These electrons flow through the wire from the zinc plate to the copper plate, thus forming an electric current. Zinc ions (Zn^{+2}) combine with sulphate ions, (SO_4^{-2}), forming zinc sulphate .

$$Zn^{2+}(aq) + SO_4^{2-}(aq) = ZnSO_4(aq) \qquad (3.76)$$

When zinc is being dissolved into solution, internal molecular energy is produced, thus making the solution warm. As zinc ions tend to enter the solution an equivalent number of hydrogen ions leave the solution and deposit them selves on the copper plate. When the hydrogen ions receive electrons from the copper plate, they are neutralised and bubbles of hydrogen gas are formed,

$$2H^+(aq) + e^- = H_2(g) \qquad (3.77)$$

When the copper plates lose electrons they become positively charged hence attracting more electrons from the zinc plate through the wire.

Any device such as a cell or dynamo which can deliver an electric current through a circuit(or a wire) is said to posses an Electromotive Force (Emf). Emf of a cell may be taken as being equal to the potential difference (p.d.) a cross its terminals, when it is not producing current in circuit. The direction of the electric current was not known when the first experiments about electric cells were made, and they agreed that electric current (that is electrons or charge carriers) flows from the positive terminal to the negative terminal.

After some time it was discovered and proved beyond reasonable doubt that the electrons flows from the negative terminal to the positive terminal. This created a confusion, which was solved by leaving the two ideas in existence up to today. The previous electric current is termed as the **conventional electric current**, it flow from the positive to the negative electrode but the electron producing it to flow from the negative to the positive electrodes. i.e. Conventional direction of an electric current is from the positive to the negative electrode.(But electrons flows from negative to positive).

Faults in a simple cell

when a cell is in use or when a wire is connected across the two electrodes of the simple cell, it is found that current quickly falls to a very small value, this is because of two major faults or reasons;

1. Polarisation

This is the formation of hydrogen bubbles on the positive electrode (copper plate) of a cell. This is because, when hydrogen ions are neutralized by the negative electrons, hydrogen gas is formed.

It has two effects.

(a) The gas partially insulates the plate, hence increasing the **internal resistance** of a cell (opposition to the flow of electric current through the cell).

(b) The layer sets up a back emf (a counter emf-opposing the emf of the cell). This emf (electron-driving force) opposes that due to the copper and zinc plate, i.e. hydrogen bubbles and copper plate form a simple cell, hydrogen as positive and copper a negative plate. This sets up current in the opposite direction to that of the cell, though this is very negligible at the copper plate.

Correction

Polarizations is corrected or reduced by oxidizing the hydrogen gas or bubbles to water using a depolarizing agent or oxidizing agent. This is added to the electrolyte. Examples of the common depolarizers or oxidising agents are manganese dioxide (manganese (iv) oxide);used in dry cells, potassium dichromate. e.t.c.

2. Local action

This is the set up or formation of tiny local cells at the zinc electrode. This is due to the impurities in the commercial zinc used, these include iron, carbon, sulphur, e.t.c. These impurities act as the positive plate and bubbles of hydrogen are formed and the zinc as negative plate forming the small local cells. These local cells formed opposes the main current.

Correction

Local action can be reduced (or solved) by using

1. Chemically pure zinc
2. Amalgamated zinc (i.e. zinc coated with mercury)
3. Ammonium chloride (NH_4Cl) instead of the acid.

Note that two metal rods dipped in a lemon fruit can act as a simple electric cell, which can produce up to 5V (as emf of the cell), same applies to two different metal coins dipped in a salt solution[19].

3.11.3 Types of cells

There are two types of electric cells

1. Primary cells and
2. Secondary cells.

Primary cells

These are cells, which can not be re-charged when they have been discharged or used up. In

[19] Let us borrow some chemistry: If metal A and B are used as electrodes of a simple cell, and metal B is above metal A in the metal reactivity series i.e metal B is more reactive i.e it easily losses an electron; then metal B must be the negative electrode and metal A the positive electrode

these cells, current is produced as a result of non-reversible chemical changes or reaction. Due to this they can not be re-charged once used up. An example of a primary cell is a dry cell commonly used in torches and radios.

Secondary cells

These are cells, which can be re-charged when they are used up or discharged. In these cells current is as a result of a reversible chemical reaction. After they have run down, by passing an electric current through it in the opposite direction i.e. from the cathode through the cell to the anode, using a dynamo or any other source of electric current, the cells can be used again. They are also known as **storage cells** or **accumulator**. Examples of secondary cells include

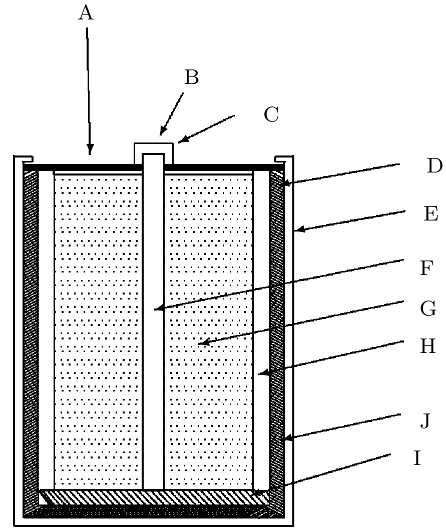
1. Lead-acid cells (most car batteries are of this type)
2. Nickel-cadmium alkaline cells(NiCad cells). Most digital cameras and watches use this type of cell.

Their major advantage is that they have a very low internal resistance, therefore they can supply a larger electric current with very little drop in terminal potential difference, i.e. with out being affected[20].

3.11.4 The dry cell

Here the positive plate (anode - positive electrode) is a carbon rod held in between (or in the center) of a cylindrical zinc plate which acts as the negative plate (cathode). The rod is surrounded by black paste of carbon and manganese(iv)oxide (MnO_2) acting as depolarizing agent. The depolarization action is slow compared to the secondary cells, if a large current is drawn from the cell continuously. The polarisation taking place reduces the Emf of the cell.

[20]this point will be understood better after the next topic of current electricity



A-Insulating top seal

B-Positive terminal (Anode)

C-Brass cap

D-Zinc case

E-insulating outer cover (always a paper with labels)

F-carbon rod

G-Mixture of carbon and Manganese(iv)oxide

H-Ammonium chloride jelly or paste

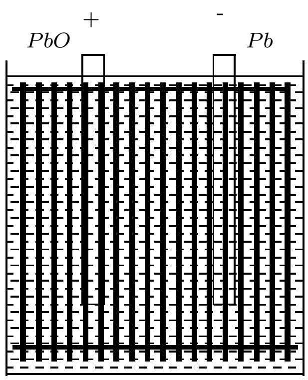
J-Negative terminal (Cathode)

I-cardboard disc

In modified dry cells, ferric chloride ($FeCl_3$) is used as a depolarizer. To prevent local action, amalgamated zinc plate (cathode) is used and ammonium chloride is used as an electrolyte instead of the acid. This electrolyte is fully prevented from drying up by sealing the top of the cell.

3.11.5 Lead acid cells

A lead acid cell is composed of lead-antimony grids filled with a paste (electrolyte) under hydraulic pressure. Reddish brown lead oxide (Pb_2O_3) is used as the positive plate and lead (ii) oxide (PbO) as negative plate, these are immersed in dilute sulphuric acid. The cell can not provide any current, it has to be charged first. On charging i.e. the positive plate becomes lead (iv) oxide (PbO_4) while the negative plate lead (Pb). For the cell to have a low internal resistance, the sets of plates should have a large surface area and placed very close to each other.



Dilute
Sulphuric acid

Theory

Charging

During charging, Lead atoms dissolve in solution in form of lead ions (Pb^{2+})

$$Pb(s) = 2e^{-} + Pb^{2+}_{(aq)} \tag{3.78}$$

And the electrons ($2e^{-}$) are left behind on the plate, thus making it negatively charged (cathode).

As Pb^{2+} comes to solution, hydrogen ions (H^{+}) are attracted to the negative plate where they are oxidized by lead (iv) oxide to a hydrogen gas (H_2), and this plate acquires a positive charge (electron deficient).

$$2H^{+}(aq) + 2e^{-} = H_2(g) \tag{3.79}$$

at the other electrode, the hydroxyl ions in the acid are oxidised to water and oxygen,

$$4OH^{-}_{(aq)} = 2H_2O_{(l)} + O_{2(g)} + 4e^{-} \tag{3.80}$$

hence during charging oxygen and hydrogen gas are evolved(the volume of hydrogen produced is twice that of oxygen)i.e. the reaction is the dissociation of water[21]

$$2H_2O_{(l)} = 2H_{2(g)} + O_{2(g)} \tag{3.81}$$

Using the cell

When a wire is now connected across the terminals of a charged lead-acid cell, the positive electrode draws electrons from negative plate through the wire i.e. an electric current flows.

In the solution (electrolyte), lead ions (Pb^{2+}) combine with sulphate ions (SO_4^{2-}) of the acid forming lead sulphate ,$PbSO_4$

$$Pb^{2+}(aq) + SO_4^{2-}(aq) = PbSO_4(s) \tag{3.82}$$

but this sulphate does not dissolve easily, after some time it coats on the plate forming a white insulation on them and the cell stops working and it need re-charging. Usually the cell is charged or distilled water added to the acid[22].

When a lead acid cell is discharging(being used up) both plates become coated with lead sulphate ($PbSO_4$) and the acid becomes more dilute due to the reduction in the a number of sulphate ions. The relative density of acid should be 1.25 when fully charged with an emf of 2.2V and 1.15 when discharged. This relative density is measured by a bulb hydrometer[23].

Recharging the lead acid cell

Connect the terminals of the cell to a direct current source with a higher voltage such that the positive terminal of the source is connected to the

[21] For more details contact chemistry:the law of volumes, Gay Lussac's law

[22] In some cells water is added to them because their discharge process is equivalent to the change of water to oxygen and hydrogen

[23] For more about the hydrometer check in Book I under pressure

positive plate of the cell and the other to the other. The re-charging current produces electrolysis of water in the cell, thus creating hydrogen and oxygen ions.

$$H_2O_{(l)} = 2H^+_{(aq)} + O^{2-}_{(aq)} \tag{3.83}$$

The hydrogen ions (H+) combine with the sulphate ions (SO_4^{2-}) to bring back or form dilute sulphuric acid.

$$2H^+_{(aq)} + SO_4{}^{2-}_{4(aq)} = H_2SO_{4(aq)} \tag{3.84}$$

the electrodes return back to their original type of oxides.

Care given to lead-acid cell

1. The electrolyte should be inspected regularly to ensure that the plates are fully immersed in it.
2. The acid should never be added to the cell, except in rare cases where spillage occurs.
3. Keep the battery fully charged all the time.
4. The specific gravity (relative density) of the electrolyte should not be below 1.15 and above 1.25.
5. When charging use the recommended charging current and old batteries should not be ruined by charging for a short time.
6. When the battery is not in use, give it the "topping up" charge monthly.
7. When the cell is fully charged, during charging; there is gassing i.e. evolution of hydrogen and oxygen gas from the negative and positive plate respectively. they are flammable gasses (can burn) so keep away from sparks.
8. Do not allow the electrolyte to touch the electrodes or else will be corroded i.e. electrodes must be sealed or amalgamated.
9. Ensure the terminals are connected correctly to the plate.
10. Clear the battery terminals to remove corrosion material using a wire brush.
11. Terminals should be coated with grease or Vaseline to prevent corrosion. (corrosion is the process by which the metals in the electrodes get oxidized by oxygen in air, the oxides are seen as a white substance on the electrode.)
12. Over discharging and shorting or short-circuiting (connecting a wire directly across the terminals) are very detrimental. This causes the swelling and buckling of plates as a result, the active material(lead sulphate on the plates) become loosened and falls to the bottom as sludge (some times called anode slime). If the cell is left in this condition, the active material changes to crystalline form, which can not be converted to lead oxide and lead respectively. The cell is said to be "sulphated" for this case, there is no redeem except to renew the plate.
13. The plate should always be covered to prevent the electrolyte from spilling out and to avoid contamination.

Summary

1. Do not spill acid on the top.
2. Do not short-circuit i.e. current is too great, much heat is generated which do damage the plates
3. Do not take large currents from it
4. Keep it charged to avoid sulphating
5. Do not drop or knock the cell
6. Keep terminals clean(remove corrosive material and cover them with grease or Vaseline)
7. Acid must always cover the plates or "top up" with distilled water and not acid because only water evaporated

3.11.6 Alkaline batteries

The electrolyte is usually caustic potash (or any alkali), the negative plate is usually iron or cadmium and positive plate is sometimes nickel hydroxide. Cadmium has an advantage over iron though addition of little iron to cadmium improves the efficiency of the cell. The active material is in powder form enclosed in perforated pockets, the most common types of alkaline cells are NiCad cells (Nickel-Cadmium cells) and Nife cells (Nickel -iron cells).

Advantages of alkaline cells over lead-acid cell

1. They can provide large current with out ill effects.
2. They can be left in the discharged condition for a longer period with no harm.
3. They have a greater emf, Its Emf is about 1.25V, which tends to fall continuously on discharge. 5 alkaline cells in series give an emf approximately equal to that of 3 lead acid cells.

Comment or compare storage capacity and internal resistance of the two secondary cells.

Storage capacity of an electric cell

The storage capacity or charge storing capacity of an accumulator is measured in Ampere-hours (Ah) 10 Ah cell can supply a current of 1Ampere for 10 hours or 2Ampere for 5hours. The above cell also can be charged fully by supplying to it a current 1Ampere for 10hours or 2 Amperes for five hours.

Table 3.20 shows the charge storing capacity and the possible charging current and time.

High charging currents are not recommended.

Note that the table is filled using the formulae

Charge storing capacity(Ah)	Charging current (A)	Charging time(hrs)
10Ah	1A	10hrs
10Ah	2.5	4
10Ah	5	2
10Ah	10	1
30Ah	30A	1hrs
30Ah	15	2
30Ah	10	3
30Ah	6	5
30Ah	3	10
30Ah	2	15
30Ah	1	30

Table 3.20: Charging cells

Storage capacity = charging current × charging time

Exercise

1. What is an electric cell
2. A given cell failed to supply a steady current of 2A for 2 minutes, instead the current decreased with time to 0A;
   (a) give two reasons why the current decreases
   (b) what could be done to ensure that the cell supplied that steady current of 2A
3. Give two types of cells with at-least two examples of each
4. Draw a well labelled diagram of a dry cell and use it to describe briefly how it works
5. How can a ruined rechargeable cell be recharged
6. state five precautions to abide by when using a lead-acid cell.
7. Give 2 advantages of alkaline cells over lead-acid cells

8. How long will it take to charge a cell with a storage capacity of 40Ah with a charging current of 5A

9. What is the capacity of a cell which can supply a current of 250mA for 4 hours

3.12 Current electricity

3.12.1 Introduction

This topic is concerned with the movement of electric charges. What determines whether a charge should move or not? (Electric potential)

3.12.2 Electric potential

In current electricity, we are concerned with moving charges (moving electrons). Recall we defined potential as a condition, which governs the flow of electrons. We shall compare potential with temperature and height above he ground.

Heat flows from a region of high temperature to a region of low temperature i.e. heat flow is due to a temperature difference.

Water flows from a region above or of greater height (higher gravitation potential) to a low region (lower potential energy) i.e water flow is due to a difference in height, i.e. Gravitational potential difference.

Similarly, but a bit difference, for electrons. Electrons move from a region with lower electrical potential energy to a region with higher electrical potential energy, i.e. due to the potential difference. The two terminals of a battery, are two points which are at different potentials. The difference in their electric potential is called potential difference (p.d). If a wire joins the two terminals, electrons flow from one terminal (-) to another (+) until the potential difference is zero.

Note that the difference in potential drives the electric charges i.e. positive or negative charge carriers (electrons in this case; of conductors).

Meaning of the word potential difference

Electric charge is measured in coulombs.

Definition 44.

The **electric potential** of a point is the work done in moving a coulomb of charge from infinity to that point against the electric field.

Definition 45.

Electric Potential difference (p.d) between two points is the work done in moving a charge of one coulomb from one point to the other.

i.e. p.d is work per coulomb. Potential difference(p.d), V, is equal to the ratio of the work done, W, to the quantity of electric charge,Q, transferred.

$$V = \frac{W}{Q} \tag{3.85}$$

Electricity is described in terms of charges Charges are measured in coulombs.

The S.I. units of electric charge is Coulombs. Potential difference and potential are measured in volts. Work is measured in joules

$$\begin{aligned} V &= \frac{W}{Q} \\ W &= VQ \qquad (3.86) \\ \text{work done} &= \text{p.d} \times \text{ quantity of charge.} \end{aligned}$$

Hence the S.I unit of p.d is Joules per coulomb i.e J/C. If 1 volt = $1JC^{-1}$, then the SI unit of p.d is Volts or JC^{-1}

Questions

1. Two large isolated parlance plates are given equal and opposite charges, the p.d between them is 12V. How much work is done is moving a charge of 2C from one plate to the other against the electric field? (48J)
2. Two points A and B are at electric potentials +16V and - 4V respectively determine;.
   (i) The point at a higher potential?
   (ii) The potential difference between the two points?

(iii) The work done in moving a charge of 5C from A to B. (Ans; A, 20V, 100J)

3. The work done in moving 12Coulombs of charge from point X to point Y is 4800J. Calculate the potential difference between the two points?
4. A given quantity of charge was transferred from an insulator (p) to a conductor (q) the P.d between them is 40V, if 360J of work were dissipated. Determine the quantity of electricity transferred?

Sources of electric potential or electric potential difference

Sources of electric potential or potential difference are ;

1. Batteries or cells
2. Dynamos[24], e.t.c.

Units of potential can be;

millivolts = $mV = 10^{-3}V$,

Megavolts =$MV = 10^{6}V$.

The instrument used to measure potential difference directly is called a **voltmeter**.

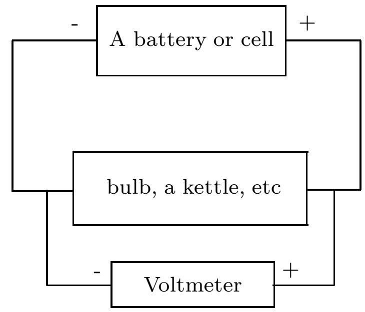
The p.d between the terminals of a battery is called its **electromotive force** (Emf), but this is not how EMF is defined.

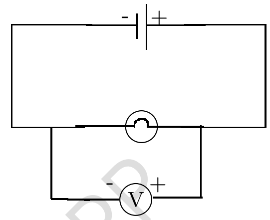
Measuring potential difference

Note that, in this topic we shall use the initials **P.d** to mean electric potential difference.

A voltmeter must be connected across a component in a circuit so that its positive terminal is connected towards the positive terminal of the battery and negative to negative as shown below.

[24]Dynamos are discussed on page 321





3.12.3 Electric current

This is the flow of charges. Usually in metals they are electrons (negatively charge particles). For the above electric connection, the electrons flow from the cell's cathode through the electric component (i.e. a conductor like a heater or bulb, e.t.c.) and back to the cell. The path along which an electric current passes is called **an electric circuit.**
Since electric current is the flow of electrons , in general it is the flow of charges. It is measured as the rate of flow of charge through a given point in a circuit. or a number of charges passing through a given point per second. Hence we can define **electric current** as the rate of flow of charges. Electric current is measured in **Amperes**. Electric current, I;

$$(I) = \text{rate of flow of charge.} \quad (3.87)$$
$$I = \frac{\text{charge passed a point}}{\text{time it takes}} \quad (3.88)$$
$$I = \frac{\Delta Q}{\Delta t} \text{ or} \quad (3.89)$$
$$I = \frac{Q}{t} \quad (3.90)$$
$$\text{Hence } Q = It, \text{ i.e.} \quad (3.91)$$
$$Q = \text{Quantity of electricity} \quad (3.92)$$

For instance, if 2.5C of charge pass through a given point in a circuit in every 0.5 seconds. Then the current in the circuit is given as;

$$
\begin{aligned}
I &= \frac{Q}{t} \\
&= \frac{2.5C}{0.5s}, \\
I &= 5Cs^{-1},
\end{aligned}
$$

i.e. 5 coulombs per second.

Therefore the S.I. unit of electric current is coulombs per second or Cs^{-1}. in favour of a certain physicist by the names Andre Marie Ampere (Who lived between 1775-1836A.D).

The **S.I unit of electric current is Ampere or A** and 1A = 1C/s = Cs^{-1}) hence I = 5 Cs^{-1} = 5Amperes = 5A

Questions

1. Find the size of the electric current flowing through a wire when 0.8C of charge passes a point in the wire in 2s. (answer = 0.4A)
2. A self-starter of a motor-car uses a current of 20A for 10s. What is the quantity of electricity that has flown through the starter? (answer = 200C)
3. A current of 5A passes in a circuit for 25s;
   (a) How much charge has passed any one point in the circuit in that time?
   (b) How long must a current of 10mA flow in order to pass the same amount of charge? (answer: 125C, 12500s)
4. In an electric circuit, 16C of charge passes through a point in 4s. What is the size of the current in the circuit? (Answer: 4A)
5. A current of 2A flows for 10s through a wire in a motor;
   (a)How many coulombs of charge passed through the wire in this time?
   (b)How many electrons flow through the wire in this time? (1 electron carries a charge of $1.6 \times 10^{-19}C$. (answer; 20C, 1.25×10^{19})

Defining units of electric current

How do we define the units of electric current (i.e. what is an ampere?) Note that from the equation $I = \frac{Q}{t}$, when a charge of 1C passes through a given point in a circuit in 1 second, then the electric current in the circuit is 1A, i.e. an ampere

Definition 46.

An ampere is the electric current in a circuit through which 1coulomb of charge passes through every point in a second.

The above definition can be restated as follows;

Definition 47.

An ampere is the electric current in a circuit through which charges flow at a rate of 1 coulomb per second.

After a series of practicals, it was concluded that:

Definition 48.

An ampere is the current of which when flowing between two thin infinite wires placed 1meter apart in vacuum, they experience a force of 2×10^{-7}Newtons per meter length of the wire.

If an electron carries a charge of 1.6×10^{-19} coulombs, how many electrons are required to have a charge of 1 coulomb? (6.25×10^{19} electrons). For one Ampere of current to flow through a given circuit, 1 coulomb or 6.25×10^{19} electrons have to pass through every point in a second, hence we can also say

Definition 49.

An ampere is the current in a circuit in which 6.25×10^{-18} electrons pass through each point in it per second.

[25]

Electric current is measured using an instrument known as an **Ammeter**.

[25]This last definition is always ignored because it is not practical to count these electrons.

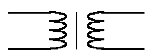
Definition 50.

A coulomb is the quantity of electricity that has passed through a given point in a circuit in one second, if the electric current flowing in the circuit is 1Ampere.

Electric symbols

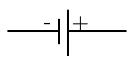
When describing or drawing a circuit we do not draw exact instrument or electric devices. We use symbols to represent various electric instruments or device, below are the common symbols.

1. Transformer



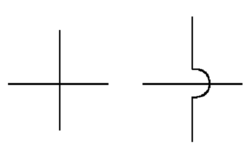
2. Connecting wire

3. Single electric cell

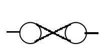


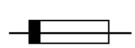
4. A battery or group of cells

5. Connecting wires with no connection between



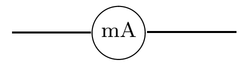
6. Fuse



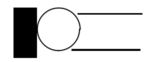


7. Inductor or solenoid or electromagnet

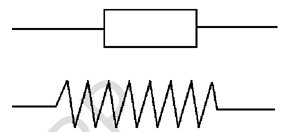
8. Milliammeter



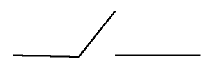
9. Microphone



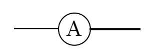
10. Fixed resistors



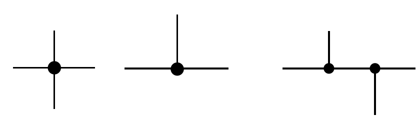
11. Switch



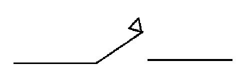
12. Ammeter



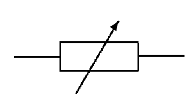
13. Wires crossing with a connection



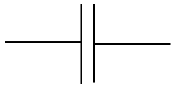
14. Tapping switch



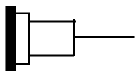
15. Resistance box



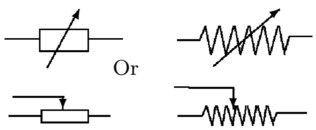
16. Capacitor



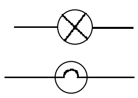
17. Receiver



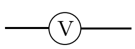
18. Variable resistor



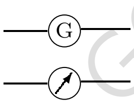
19. Electric bulb or filament bulb



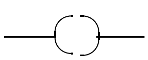
20. Voltmeter



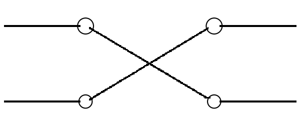
21. Galvanometer



22. Plug key

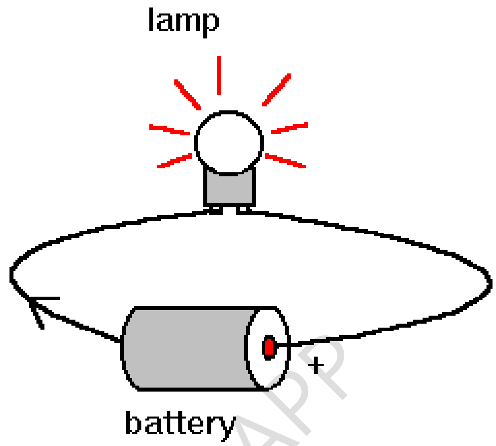


23. Reversing key

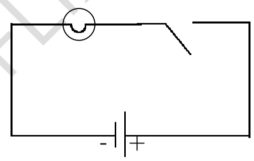


Electric circuit

Symbols simplify electric circuit diagrams



circuit diagram



Any circuit connection, with an electric current flowing, is composed of two parts;

1. **External circuit** (circuit out side the cell)
2. **Internal circuit** (circuit inside the cell)

Open circuit and short circuit

When the terminals of a cell are not connected to an external circuit(or any thing), there is a gap and the circuit is said to be an **open circuit** and no current flows. If copper wire or any other **good** conductor connects the two terminals of a cell, the cell is said to be **short-circuited** (**short circuit**), a large current flows and may damage the cell. Also connecting a wire from one terminal of an electrical device (e.g. a bulb) to the other is also short-circuiting. In general short-circuiting is providing an easy path for an electric current or to by-pass a given resistance.

The electric current (**conventional current**) flows from the positive terminal of a cell to the negative terminal, but electrons flows from the negative to positive terminal. The terminals have an electron-moving effect or **electromotive force (EMF),** i.e.

Emf is the pd across the cell when no current is drawn from it. By definition;

Definition 51.

> **E.M.F** is the total work done in joules by the cell or battery in moving one coulomb of charge through out the electric circuit.

Definition 52.

> **Potential difference** is the work done in moving one coulomb of charge from one point to another..

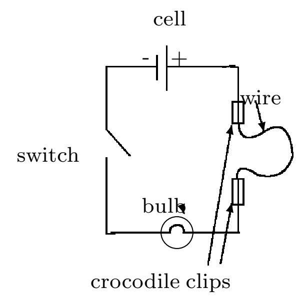
Why conventional current flows in a direction opposite to the direction of flow of the electrons

Long ago when the simple cell was discovered in 1800, its action was not yet understood and scientist had no way of finding out in which direction the charges (electrons) were flowing. Therefore, they decided to adopt the convention of regarding the current as flow of positive electricity from copper (positive terminal) to the zinc (negative terminal). After sometime they understood how the simple cell works; that is electrons flow from zinc plate (-) to the copper plate (+), they could not disseminate this true fact, after they had told the population that electrons flow from the positive terminal to the negative terminal. So they called their previous convention of the flow of electricity as the **direction of the conventional current** but in actual sense the electrons move in a direction opposite to that. There fore from now onwards we say current (convectional current) flows from (+) terminal to (-) terminal but electrons flows from (-) terminal to (+) terminal.

If the electrons were positively charged, then they would flow from the positive terminal to the negative terminal as the electric current also flows from the positive to the negative terminal.

3.12.4 Electrical resistance and Ohm's law

Using the circuit below, we can let electric current pass through various materials.



For wires of different materials, the bulb's brightness was different and at times could not even light. This shows that the opposition or resistance to the flow of electrons or electric current is different for various materials.

Since every pipe resists the flow of water through it depending on its diameter (a thin pipe allows less water and a thick pipe allows more water), similarly every material resists the flow of an electric current. Some substances like paper, air, and most gasses, wood, glass, rubber, porcelain, plastic, and others do not readily let electrons pass through them, they are insulators or non-conductors. Other substances like metals and solution of acids, bases and salts, do let electrons pass through them: they are conductors. Silver is the best conductor known and pure copper the second best. Impurities in copper greatly increase its resistance. Materials of high resistance (resistor) are used when a very low current is required and materials (resistor) of low resistances are used when a high electric current is required. Some currents may be too small to be indicated by the bulb in the above circuit. In this case a millimeter or micrometer is used to indicate such currents in such an investigative circuit.

Ohm's law

In 1826 a German scientist George Simon ohm, a teacher of physics at cologne published a book containing details of some experiments he had made to investigate the relationship between the current passing through a wire and the potential difference across the ends of the wire. He observed that for most materials of a wire the current (I) passing through a wire at constant temperature (or constant physical conditions) in direct proportional to the potential difference (V) between its ends. (This is ohm's law) i.e. when I and V are plotted a straight line graph is obtained

Definition 53.

Ohm's law states that the current passing through a conductor is directly proportional to the potential difference across it, provided temperature or all physical conditions remain constant.

Mathematically

Current (I) is proportional to P.d.(V) across it or the reverse. That is

$$I \quad \alpha \quad V \tag{3.93}$$

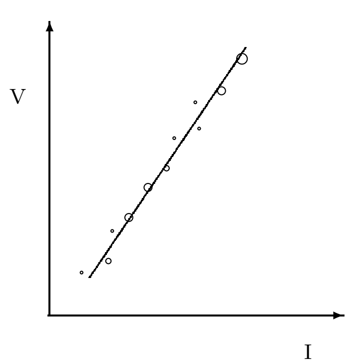
$$\text{or } V \quad \alpha \quad I \tag{3.94}$$

$$\text{If } V \quad \alpha \quad I, \text{ then} \tag{3.95}$$

$$V = RI \tag{3.96}$$

where R is a constant of proportionality which depends on the type of the material of the conductor. This constant is known as the resistance of the conductor.

If one plots a graph of P.d(V) against current(I) through a given conductor which obeys Ohm's law , a straight line is obtained.



For a given constant potential difference, a high resistance will pass a small current and a low resistance allows a large current. The value of the constant (R) which is high when current is small and low or small when current is large, can be used as a measure of electric resistance of the wire. Since From the formula $V = RI$, resistance can be given by $R = \frac{\Delta V}{\Delta I}$, then;

Definition 54.

Resistance of a conductor(R) is defined as the ratio of the potential difference across its ends in volts to the current flowing through it in amperes.

The unit of an electric resistance is called ohm (Ω) (in favour of Ohm).

Note that when a cell with an emf of 1V is connected across the terminals of a given resistor and a current of 1 amperes flows through the circuit, then the resistance of the resistor is 1Ω. From this we can define an ohm;

Definition 55.

An ohm is the resistance of a conductor which allows a current of 1 Ampere to pass through it when a potential difference of 1V is applied across its ends.

Limitations of Ohm's law

The resistance of some materials will alter (or change) when;

1. They are bent or put under tension or compression forces.
2. They are placed at an angle to a strong magnetic field.
3. Their temperature is increased or decreased.

In general, when a wire is carrying an electric current and any of the above factors is changed, the value of the electric current flowing through the wire changes at constant p.d. This shows that the electrical resistance has changed.

Conductors which obey Ohm's law (i.e. have a constant resistance) are called **Ohmic conductors** and those which do not obey Ohm's law (have variable resistance) are called **non-Ohmic conductors**.

Examples

1. A battery of emf 12V is connected across a security bulb, if the current through the bulb was 1.5 Amps, determine the resistance of the bulb?

solution

from Ohm's law the P.d across the bulb is equal to the product of the bulb's resistance and the current flowing through it. The pd across the bulb is equal to the emf of the cell=

$$\begin{aligned} E &= RI \\ R &= \frac{E}{I} \\ &= \frac{12}{1.5} \\ &= 8\Omega \end{aligned}$$

2. Determine the potential difference across 4Ω resistor, when a steady current of 1.5A is passing through it?

solution

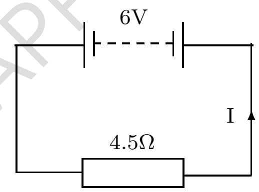
from Ohm's law

$$\begin{aligned} V &= IR \\ V &= 1.5 \times 4 \\ &= 6Volts \end{aligned}$$

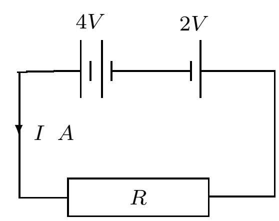
Questions

1. A cell of emf 1.5V in connected across a resistor of resistance 2Ω. Calculate the current that flows in the circuit?
2. A 20Ω resistor is connected across the terminals of 12V battery; determine the current flowing through the resistor?
3. A total current of 3A in flows in an electric circuit, given the source of current has an emf of 4.5V. Calculate the net resistance in the circuit.
4. A certain resistor 2Ω in a circuit receives a current of 1.50A. Determine the potential difference across it?
5. Use the circuits below to find the unknown values;

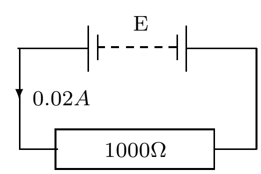
(a) Determine I



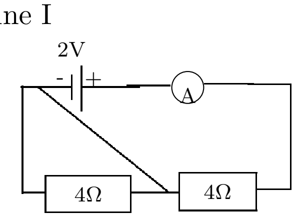
(b) Determine R



(c) Determine E



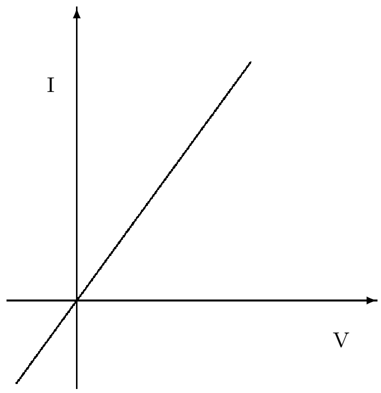
(d) Determine I



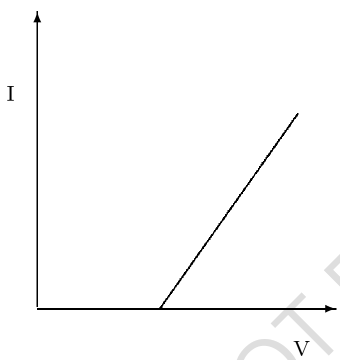
Curves or graphs of Ohmic and non-Ohmic conductors

Not all current-voltage graphs are straight lines; All Ohmic conductors have straight line I-V graphs and non-Ohmic conductors do not have straight line I-V graphs; these are shown below;

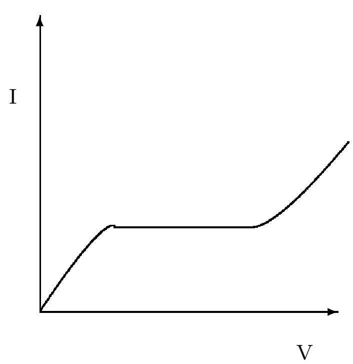
(a) For a metal



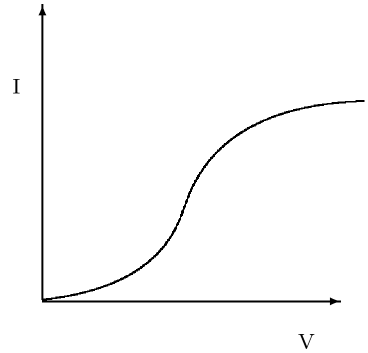
(b) For a water voltameter



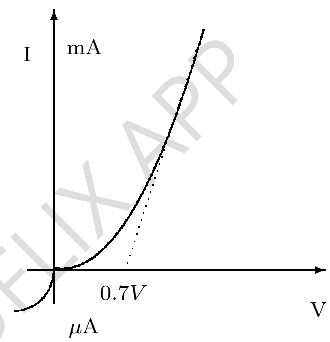
(c) For a gas discharge tube (neon lamp)



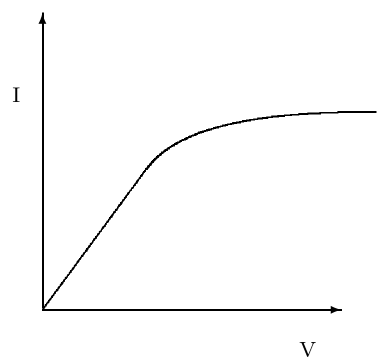
(d) Thermionic diode



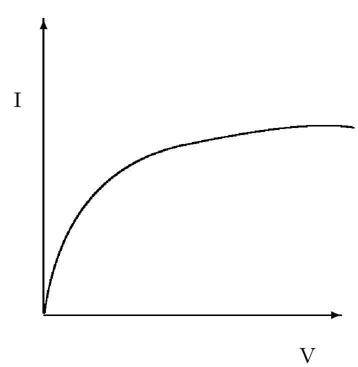
(e) Junction diode or semi-conductor diode or p-n diode

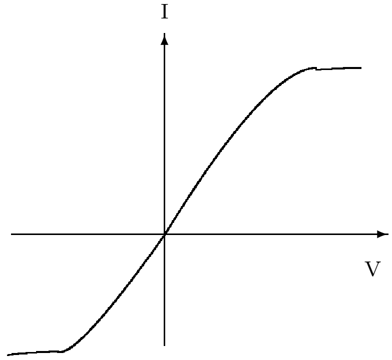


(f) Thermistor

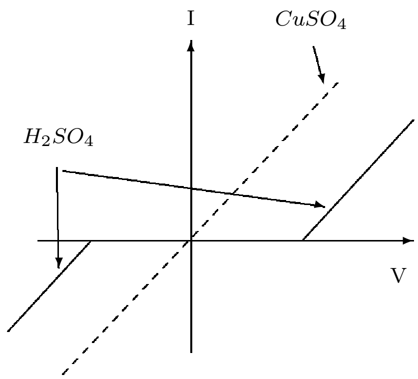


(g) A filament bulb

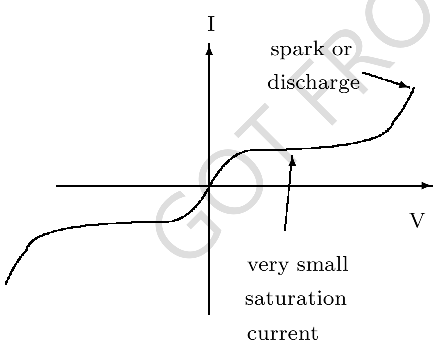




(h) Ionic solution (electrolyte) like Sulphuric acid with carbon electrodes or copper sulphate solution with copper electrodes.



(h) Gases



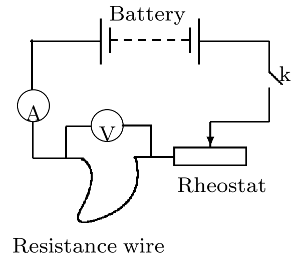
Can you explain how the resistance varies as current or voltage is increased?

An experiment to verify ohm's law

Apparatus Resistance wire, ammeter or millimeter, voltmeter, connecting wires, battery or cells, Switch, Rheostat[26].

[26]A rheostat is a variable resistor

The apparatus is arrange as shown below



The switch k is closed and the rheostat is adjusted to a value where the ammeter shows the lowest possible reading.

The readings of the ammeter (I) and voltmeter (V) are recorded.

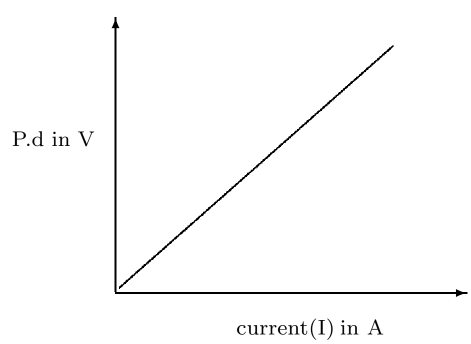
This is repeated for various values of V and I obtained by adjusting the Rheostat and the results tabulated as shown in table 3.21.

Voltage (V)	Current (I)

Table 3.21: Ohm's law

A graph of V against I is plotted. The graph will be a straight line as shown below.

Graph of V against I



This shows that the voltage across the resistance wire is proportional to the current passing through it, which is ohm's law, hence verified.

To describe an experiment to measure the resistance of a wire, you describe the above experiments but lastly get the slope $\frac{\Delta V}{\Delta I}$ as the resistance of a wire. This is the ammeter-voltmeter method of measuring electrical resistance discussed on page 311.

Circuits connections

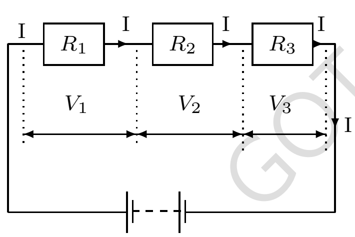
There are two types of electric circuit.

(a) Series circuit (or connection) and

(b) Parallel circuit (or connection).

These are shown below;

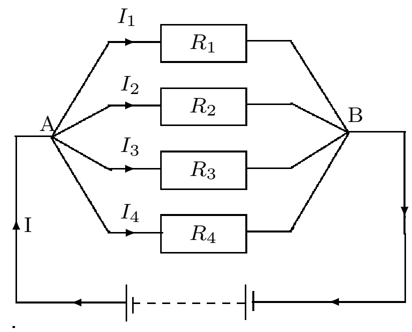
Series connection



Here the electrons which pass through R_1, pass through R_2 and R_3 i.e. electric current passing through all resistors in series is the same. The p.d a cross each resistor is different i.e. $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. All there p.ds are provided by the battery hence their sum must be equal to the emf(V) of the battery (why not pd?) i.e.

$$V = V_1 + V_2 + V_3 \tag{3.97}$$

Parallel connection



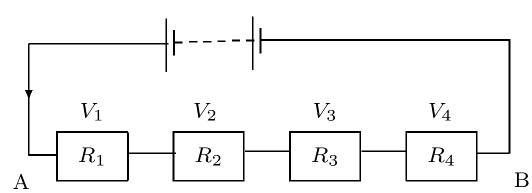
Here since each end of the resistor is connected to the terminals of the battery, the p.d across all resistors in parallel is the same (as that of the battery for this case) but the current (I) at junction (A) splits up into four currents depending on resistance of each resistor. Least current flows in the highest resistance resistor and the highest current flows in the resistor with the lowest resistance, but all the currents (I_1, I_2, I_3, and I_4) re-unite back.

$$I = I_1 + I_2 + I_3 + I_4 \tag{3.98}$$

We need to know that when several resistors are connected in series or in parallel what is the total or net resistance in the circuit.

Net resistance of resistors in series

Consider the circuit below.



Let the net p.d across all the resistors be i.e. across AB be V and current through them to be I. (it is the same). Net p.d, V, is equal to the sum of the potential drops across each resistor.

$$V_{AB} = V_1 + V_2 + V_3 + V_4 \tag{3.99}$$

$$V = V_1 + V_2 + V_3 + V_4 \tag{3.100}$$

From Ohm's law

$$\begin{aligned}
V &= IR \quad i.e. && (3.101)\\
V_1 &= I_1R_1, && (3.102)\\
V_2 &= I_2R_2, && (3.103)\\
V_3 &= I_3R_3 \text{ and} && (3.104)\\
V_4 &= I_4R_4 && (3.105)
\end{aligned}$$

where R is the value of the resistor that would replace the four resistors and the current I remains the same (constant)

$$\begin{aligned}
V &= V_1 + V_2 + V_3 + V_4\\
V &= I_1R_1 + I_2R_2 + I_3R_3 + I_4R_4\\
\text{But } I_1 &= I_2 = I_3 = I_4 = I\\
V &= IR_1 + IR_2 + IR_3 + IR_4 && (3.106)\\
V &= I(R_1 + R_2 + R_3 + R_4) && (3.107)\\
\frac{V}{I} &= R_1 + R_2 + R_3 + R_4 && (3.108)
\end{aligned}$$

Since V is the net p.d and I is the current through all the resistors or the net resistance, R, then R is given by

$$\begin{aligned}
R &= \frac{V}{I} && (3.109)\\
&= R_1 + R_2 + R_3 + R_4 && (3.110)\\
\Rightarrow R &= R_1 + R_2 + R_3 + R_4 && (3.111)
\end{aligned}$$

i.e the net resistance of resistors in series is equal to the total sum of the resistance's of each.

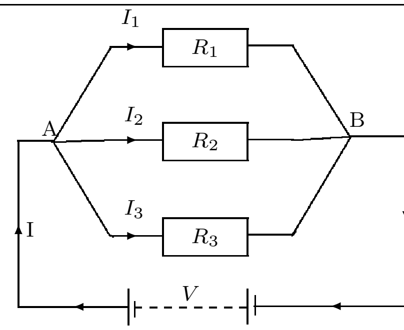
In General,

$$R = R_1 + R_2 + \ldots + R_n \qquad (3.112)$$

, for n resistors in series.

Net resistance of resistors in parallel

Consider the circuit below.



Let I be the net current and V_{AB} be V;

$$\begin{aligned}
I &= I_1 + I_2 + I_3 && (3.113)\\
\text{from Ohm's law } V &= IR && (3.114)\\
I &= \frac{V}{R}, && (3.115)\\
I_1 &= \frac{V_1}{R_1}, && (3.116)\\
I_2 &= \frac{V_2}{R_2}, && (3.117)\\
I_3 &= \frac{V_3}{R_3}. && (3.118)
\end{aligned}$$

Since V is net pd and I is the sum of the currents through all and the net resistance R is given by; $R = \frac{V}{I}$

$$\begin{aligned}
I &= I_1 + I_2 + I_3 && (3.119)\\
I &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}, && (3.120)\\
\text{but } V_1 &= V_2 = V_3 = V, && (3.121)
\end{aligned}$$

since they are in parallel

$$\begin{aligned}
I &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right), && (3.122)\\
\frac{I}{V} &= \left(\frac{1}{R1} + \frac{1}{R_2} + \frac{1}{R_3}\right), && (3.123)\\
\text{since } V &= IR && (3.124)\\
\Rightarrow \frac{I}{V} &= \frac{1}{R} && (3.125)
\end{aligned}$$

where R is the net resistance, i.e. the resistor which can replace all the resistors without changing the current and voltage.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (3.126)$$

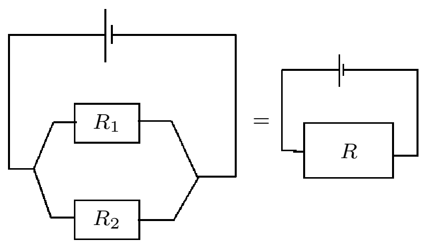
i.e. the reciprocal of the net resistance of resistors in parallel is equal to the total sum of the reciprocals of the resistance of each resistor.

In Generally, for n resistors in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}. \qquad (3.127)$$

Special case of 2 resistors in parallel.

For two resistors in parallel as shown below;where



$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} && (3.128) \\ \frac{1}{R} &= \frac{R_1 + R_2}{R_1 R_2} && (3.129) \\ \Rightarrow R &= \frac{R_1 R_2}{R_1 + R_2} && (3.130) \\ R &= \frac{\text{product of the two resistor's resistances}}{\text{sum of the two resistor's resistances}} \end{aligned}$$

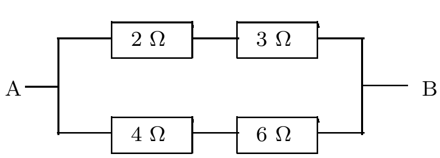
i.e. for two resistors in parallel, their net resistance is equal to the ratio of their product to their sum.

The net resistance of resistors in parallel is less than the resistance of the smallest resistor among them.

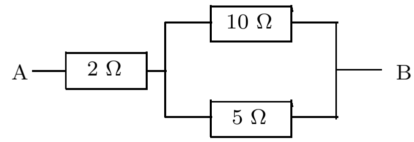
Examples

Determine the net resistance across the points AB in the circuit below;

(a)



(b)



Solution

(a) this is like two resistors R_1 and R_2 in parallel, where $R_1 = 2+3 = 5\Omega$ and $R_2 = 4+6 = 10\Omega$,

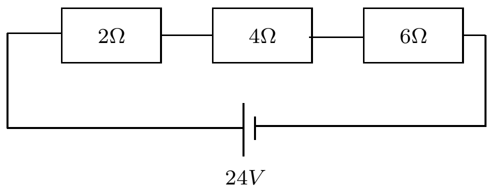
$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{R_1 + R_2}{R_1 R_2} \\ R &= \frac{R_1 R_2}{R_1 + R_2} \\ &= \frac{5 \times 10}{5 + 10} \\ &= \frac{50}{15} \\ &= 3.33\Omega \end{aligned}$$

(b) The net resistance is like that of two resistors R_1 and R_2 in series, where $R = R_1 + R_2$ $R_1 = 2\Omega$ and $R_2 =$ parallel combination of 10Ω and 5Ω

$$\begin{aligned} R_2 &= \frac{10 \times 5}{10 + 5} \\ &= \frac{50}{15} \\ &= \frac{10}{3}\Omega \\ R &= R_1 + R_2 \\ &= 2 + \frac{10}{3} \\ &= \frac{6 + 10}{3} \\ &= \frac{16}{3} \\ &= 5.33\Omega \end{aligned}$$

Exercise(series connection)

1. Resistors of 2Ω, 4Ω and 6Ω are connected in series and a voltage of 24V applied across them as shown below.

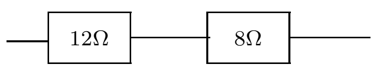


   Find

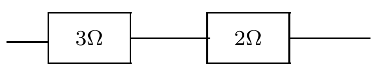
   (a) The total resistance
   (b) The current flowing
   (c) The voltage across each resistor? ans (12Ω, 2A, 4V, 8V,12V)

2. For each of the arrangements below what is the total resistance?

   (a)



   (b)



3. Two resistors of 10Ω and 5Ω are connected in series in a circuit. A voltage of 30V is applied across them, find

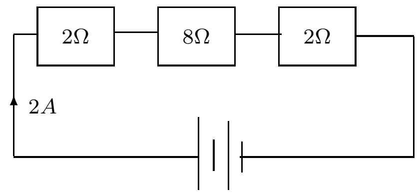
   (a) the total resistance
   (b) the current flowing
   (c) the voltage cross each resistor ? ans(15Ω, 2A, 20V, 10V)

4. A voltage of 4V is applied across two resistors (6Ω and 2Ω) connected in series. Find

   (a) the total resistance
   (b) the total current flowing through them
   (c) the p.d. across the 6Ω resistor.
   Ans (8Ω, 0.5A, 3V)

5. A 15Ω electrical resistance network is connected in series with an unknown resistor. The current flowing in the circuit is found to be 0.2A when a voltage of 40V is applied across them. Find the unknown resistance. Ans (185Ω)

6. The circuit diagram below shows resistors and an electric source which supplies current of 2A through the circuit.

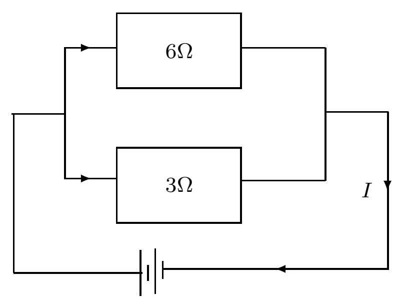


   Calculate

   (a) the voltage drop across each resistor
   (b) The *emf* of the source
   (c) The total resistance of the resistors in the circuit? Ans (4V, 2V, 6V, 32V,16Ω)

Questions (Parallel connection)

1. Two resistors of 6Ω and 3Ω are connected in parallel as shown below and an *emf* of 12V is applied across them by a battery. Find

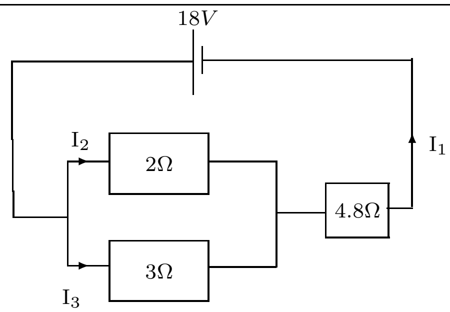


   (a) The total resistance of the circuit
   (b) The current in the main circuit (total current)
   (c) The current through each resistor (2Ω, 6A, 2A, 4A)

2. A resistor of 6Ω is connected in parallel with a resistor of 30Ω. If a current of 2A flows through the main part of the circuit, find

   (a) The total resistance
   (b) The voltage applied? Ans (5Ω, 10V)

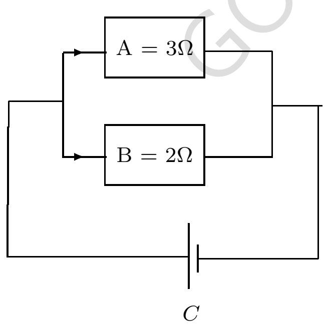
3. Use the diagram below to calculate the current in each of the resistors shown;



4. Two resistors in parallel, each 10Ω are connected to a 20V battery. Find
   (a) The total resistance of the circuit
   (b) The current through each resistor
   (c) The voltage across each resistor

5. In a car radio circuit, a loud speaker of 4Ω resistance is connected in parallel with another loud speaker of 8Ω resistance. A current of 1.0A flows through the 4Ω speaker and a current of 0.5A flows through the 8Ω speaker. Find
   (a) The voltage across each speaker
   (b) The voltage applied to the circuit
   (c) The total resistance of the circuit.

6. The figure below shows a cell C supplying current to resistors A and B. The resistance of each is shown. The current through A is 1.2A. find



   (a) The current in C
   (b) The current in B (Hint: First get the p.d. across resistor A which is equal to the p.d. across the parallel resistor and then equate it to that across resistor B, $2I_B$

7. Two resistors one of resistance 1000Ω and the other of unknown resistance are connected in parallel. This combination is then placed in a circuit and a current passing through the combination is measured for various p.ds across the combination. The results of the experiment are given in table 3.22;

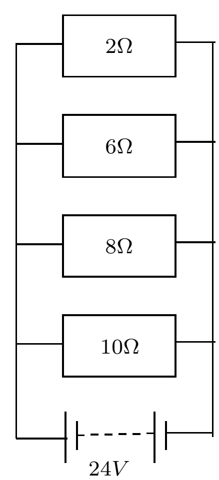
P.d(V)	Current (A)
1.5	0.075
3.0	0.15
4.5	0.225
6.0	0.30
7.5	0.375

Table 3.22: Parallel connection

Draw a labeled diagram of the circuit you would use to perform the experiment.

   (i) Plot a graph of p.d against current
   (ii) From the graph calculate the total resistance of the combination of resistors.
   (iii) Using the resistance of the combination obtained in (ii) above, calculate the unknown resistance.

8. The circuit diagram below shows 4 resistors in parallel and an emf source. Calculate



   (a) Total resistance in the circuit
   (b) The current through the 6Ω and 8Ω resistor.

Questions (series and parallel)

1. One 4Ω resistor and two 2Ω resistors are available. All three are to be connected together in two different arrangement such that the total resistance is

   (i) less than 2Ω.

   (ii) more than 4Ω but less than 8Ω.

   Draw a circuit diagram for each arrangement and calculate the total resistance in each case.

2. (a) You are provided with 3 resistors of values 2Ω, 4Ω and 8Ω respectively.

   (i) Draw a circuit diagram with a battery showing all the 3 resistors in series with each other.

   (ii) Calculate the total resistance of the circuit

   (b)

   (i) Draw a circuit diagram showing 4Ω and the 8Ω resistors in parallel with each other and with the battery.

   (ii) Calculate the current through the 8Ω and 4Ω resistors if the pd across the battery terminals in 6V and the battery is in series with a 2Ω resistor.

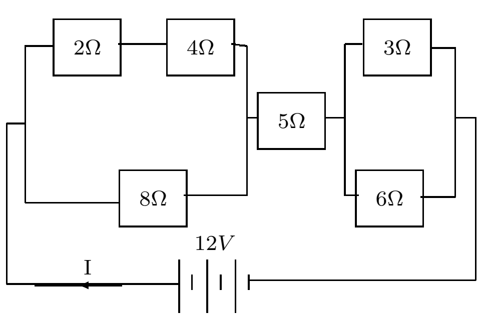
3. Draw a circuit diagram to show how you would connect four 12V electric bulbs in

   (a) In parallel

   (b) In series

   What voltage would be needed to supply the lamps in each case such that they work normally.

4. Resistors are connected in a circuit diagram as shown below. Calculate



   (a) The total resistance in the circuit

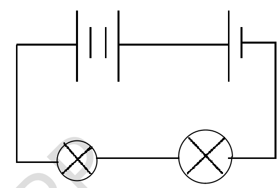
   (b) The total current, I, in the circuit

   (c) The current in the 3Ω resistor

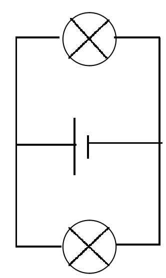
   (d) The current through the 8Ω resistor

5. Use the lamps in the circuit diagram below to answer the questions that follow;

   (a) bulb in series



   (b) bulbs in parallel

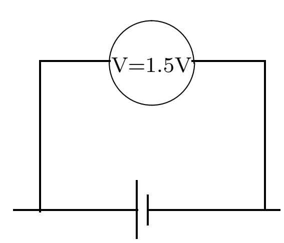


   (i) What will you observe in the lamps in (a) and in (b) in terms of brightness

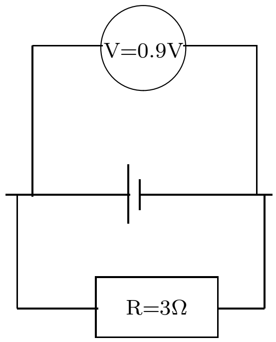
   (ii) In which arrangement are the lamps in (b)? Could this arrangement affect the brightness, comment?

3.12.5 Internal resistance

When a voltmeter (Usually of high-resistance) is connected across a battery of emf 1.5V, it reads approximately 1.5V As shown below;



When a lamp of 3Ω resistor is connected to the terminals of the cell so that the cell drives a current through it. It is noticed that the voltmeter leading dropped to 0.90V.



Why does the voltage decrease from 1.5V to 0.9V or why lose the 0.6V (1.5-0.90) = 0.6V = the "**lost volts**"), where do they go?

Explanation

It was discovered that the cell its self has a resistance it offers to the current in the circuit i.e. a resistance in the internal circuit[27], this is called the internal resistance of the cell (r).
When a 3Ω resistor was connected across the cell, the total resistance in the circuit became $R = (3 + r)\ \Omega$, if a current (I) flows in the circuit.

$$
\begin{aligned}
\text{the emf} : E &= (3 + r)I \\
E &= 3I + Ir \\
Emf &= \text{pd across the cell} + \text{lost volts} \\
Emf &= \text{p.d across R} + \text{p.d across r} \\
E &= V + Ir \\
E &= IR + Ir \\
E &= I(R + r)
\end{aligned}
$$

where R is the external resistance[28] and r is the internal resistance[29]. hence The terminal p.d of a cell is always less than the emf of the cell and the difference between them or lost volts represent the pd required to drive the current through the internal resistance of the cell. This (**lost volts**) can not be read directly from a voltmeter, but can be obtained only by subtracting the terminal Pd from the emf.

[27]See page 234 about internal circuit
[28]Total resistance in the external circuit
[29]Total resistance in the internal circuit

Therefore a voltmeter gives only a close approximation for the emf of a cell. This is because even a very high resistance voltmeter must take some current and hence a small part of the emf will be lost in driving this current through the cell.

Practical importance of the internal resistance of a cell

The internal resistance of a cell can be useful or useless depending on what the cell is being used for. The most common important uses are;

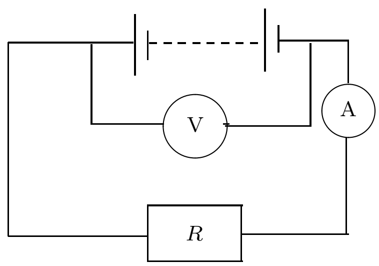
1. Avoid accident e.g. if a wire is connected across a 2V cell with an internal resistance (r) of 0.01Ω, a current of 200A(2/0.01) flow which is very dangerous, it even burns the wire connecting the terminals, this can lead to loss of life. But if r =2Ω, a current of 1A ($\frac{2V}{2\Omega)}$ would flow which can at least be sustained i.e. can not cause serious sparks or even burn the wire in less than a second.

2. A cell having a relatively very low internal resistance can run an electric starter motor this is because here a very large current is required for a car to start and this can only be obtained from a battery having low internal resistance. (a very small battery can provide a very large current only when its internal resistance is very low)

An experiment to determine the internal resistance of a cell.

Apparatus; a voltmeter, ammeter, connecting wires. Five or more known resistors (R's) and a battery of known *emf* are required.

Connect them as shown below;



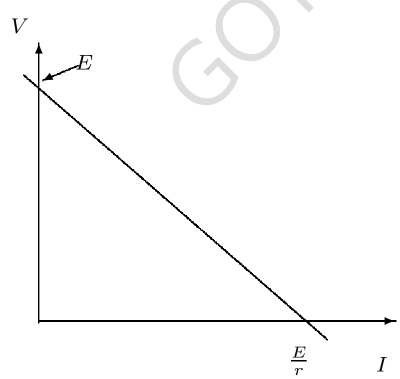
Read and record the readings of the voltmeter (V) and the ammeter (I)

Repeat the above procedure with difference resistors($R's$), measure and record the readings of the voltmeter (V) and Ammeter (I) in the table as shown below;

V(Volts)	I(Amperes)

Table 3.23: Internal resistance

Plot a graph of V against I, you will obtain a graph of the form



Calculate the slope, s, of the graph

Internal resistance(r) of the wire, will be obtained as:

$$r = -s \tag{3.131}$$

Theory

$$\begin{aligned} E &= I(R+r) && (3.132)\\ E &= IR + Ir && (3.133)\\ E &= V + Ir && (3.134)\\ V &= E - Ir && (3.135)\\ \text{When } I &= 0, V = E \\ \text{When }, V &= 0, I = E/r \end{aligned}$$

The slope for the equation

$$\begin{aligned} \text{from } V &= E - Ir, \text{i.e.} && (3.136)\\ V &= -Ir + E, \text{by comparing with} \\ y &= mx + c. && (3.137)\\ \text{slope} &= -r && (3.138)\\ \text{hence } r &= -\text{ slope} && (3.139) \end{aligned}$$

Note. The internal resistance of a cell may also be obtained by connecting a voltmeter directly across it to obtain the *emf* of the cell (E). Then the same voltmeter is connected across the same cell when it is driving a current (I) through a known resistor R and it gives a reading, V. then we can say;

$$\begin{aligned} \text{Lost volts} &= E - V, && (3.140)\\ \text{Current} &= I \\ I &= \frac{E}{R+r} && (3.141)\\ \text{also } I &= \frac{V}{R} && (3.142)\\ r &= \frac{\text{lost volts}}{I} && (3.143)\\ &= \frac{E-V}{I} && (3.144)\\ r &= \frac{R}{V}(E-V) && (3.145)\\ & && (3.146) \end{aligned}$$

This is a simple way of estimating the internal resistance of a cell.

Example

1. A battery has an internal resistance of 0.30Ω. The p.d between its terminals is 1.48V when a current of 0.20A is being drawn from it. What is its *emf?*

Solution

$$\begin{aligned} \text{from } E &= I(R+r) \\ &= IR + Ir \\ &= V + Ir \\ &= 1.48 + 0.2 \times 0.3 \\ &= 1.48 + 0.06 \\ &= 0.54V \end{aligned}$$

hence its emf is 0.54V

2. A cell drives a current of 0.6A through 2Ω coil and currents of 0.2A through a 7Ω coil, calculate the emf and internal resistance of the cell. (Hint: you must know how to solve simultaneous equations, to answer this question we shall use the elimination method)

Solution

from E = I(R + r) and 0.6A flows through 2Ω

$$\begin{aligned} E &= 0.6(2+r) \\ E &= 1.2 + 0.6r......(\text{i}) \end{aligned}$$

Since 0.2A flows through 7Ω, then

$$\begin{aligned} E &= 0.2(7+r) \\ E &= 1.4 + 0.2r......(\text{ii}) \end{aligned}$$

$$\begin{aligned} E &= 1.2 + 0.6r......(\text{i}) \quad \times 1 \\ E &= 1.4 + 0.2r......(\text{ii}) \quad \times 3 \end{aligned}$$

$$\begin{aligned} 3E &= 4.2 + 0.6r \\ E &= 1.2 + 0.6r \\ \hline 2E &= 3 \\ E &= 1.5V \end{aligned}$$

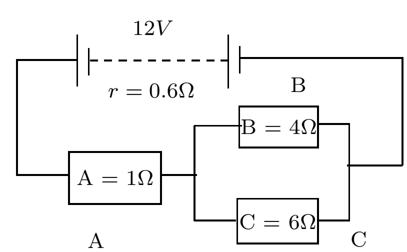
$$\begin{aligned} \text{From } E &= 1.2 + 0.6r \\ r &= \frac{E - 1.2}{0.6} \\ &= \frac{1.5 - 1.2}{0.6} \\ &= \frac{0.3}{0.6} \\ &= 0.5\Omega \end{aligned}$$

Or

$$\begin{aligned} \text{from } E &= 1.4 + 0.2r \\ r &= \frac{E - 1.4}{0.2} \\ &= \frac{1.5 - 1.4}{0.2} \\ &= \frac{0.1}{0.2} \\ &= 0.5\Omega \end{aligned}$$

Hence the emf of the cell is 1.5V and its internal resistance is 0.5Ω

3. A 12V battery of internal resistance 0.6Ω connected to three resistors A, B and C as shown below. Find the current in each resistor.



Solution

$$\begin{aligned} \text{total resistance} &= R \\ R &= R_A + R_{BC} + r \\ &= 1 + \frac{6 \times 4}{6+4} + 0.6 \\ &= 1 + \frac{24}{10} + 0.6 \\ &= 1 + 2.4 + 0.6 \\ R &= 4\Omega \end{aligned}$$

$$\text{Net current}, I = \frac{E}{R} = \frac{12}{4} = 3A$$

hence the current through the 1Ω resistor (A) is 3A. The potential difference between the 4Ω and $6\ \Omega$ resistor is the same . Let us call it V_p
From Ohm's law ($V = IR$)then we have

$$\begin{aligned} V_p &= \text{I} \times \text{total resistance} \\ &= \left(\frac{12}{4}\right) \times \left(\frac{6 \times 4}{6+4}\right) \\ &= 3 \times 2.4 \\ &= 7.2V \end{aligned}$$

$$\begin{aligned} V_p &= I_B \times 4\Omega \\ I_B &= \frac{V_p}{4} \\ &= \frac{7.2}{4} \\ &= 1.8A \end{aligned}$$

also from V= IR

$$\begin{aligned} V_p &= I_C \times 6\Omega \\ I_C &= \frac{V_p}{6} \\ &= \frac{7.2}{6} \\ &= 1.2A \end{aligned}$$

Hence the current through the 1Ω resistor(A) is 3A, 4Ω resistor(B) is 1.8A and that through the 6Ω resistor (C) is 1.2A .

Note that we expect

$$\begin{aligned} I_A &= I_B + I_C \quad or \\ I_{1\Omega} &= I_{6\Omega} + I_{4\Omega} \\ 3 &= 1.2 + 1.8 \end{aligned}$$

and it is true

4. When a cell having an emf of 1.5V is giving a current of 0.5A, a high resistance voltmeter a cross its terminals reads 1.20V

   (a) Explain why the voltmeter reading in less than 1.5V.

   (b) Find what the voltmeter will read when the cell is giving 0.60A

Solution

(a)**Note that** the pd across the battery is the pd across the external resistance and not total resistance, since the internal resistance is not zero and;

$$\begin{aligned} Emf &= Ir + IR \\ E &= Ir + V \end{aligned}$$

then V is always less than E or because r is not negligible or zero

(b) From above

$$\begin{aligned} r &= \frac{E-V}{I} \\ &= \frac{\text{lost volts}}{\text{current}} \\ \text{from above}, r &= \frac{1.5-1.2}{0.5} \\ &= \frac{0.3}{0.5} \\ &= 0.6\Omega \end{aligned}$$

$$\begin{aligned} \text{when } I &= 0.6A, \\ \text{Lost Volts} &= Ir \\ &= 0.6 \times 0.6 \\ &= 0.36\Omega \\ P.d &= V, \text{across the cell} \\ V &= E - Ir \\ &= E - \text{lost volts} \\ &= 1.5 - 0.36 \\ &= 1.14V \end{aligned}$$

Hence the voltmeter will read 1.14V

3.12.6 Cells in series and in parallel

A group of cells connected together is known as a battery. They can be connected in series or in parallel.

Series connection of cells

Here cells are connected such that the positive of one cell is connected to the negative of the next cell and so on. Generally speaking a series connection in used when the circuit resistance is high compared with that of the cells (or battery) if a reasonable current is to be maintained.

When cells are in series, the total emf of the battery is equal to the sum of the *emfs* of the separate individual cells i.e.

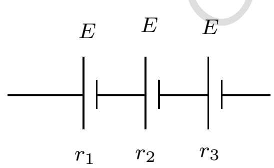
$$E_n = E_1 + E_2 + \dots \quad (3.147)$$

, and the resistance of the battery (its internal resistance) is equal to the sum the internal resistances of the individual cells, i.e.

$$r_n = r_1 + r_2 + \dots \quad (3.148)$$

In series connection, the same main current in supplied by each cell.

For the three similar cells in series below;



Cells in series

$$\text{Net } emf = 3E \quad (3.149)$$

$$\text{net internal resistance} = r_1 + r_2 + r_3 \quad (3.150)$$

Note that, In general the emf of cells in series is equal to the algebraic sum of their individual emfs. This is one way of obtaining high dc voltage.

Parallel connection of cells

Here the cells are connected such that all their positive terminals are connected together and the negatives also connected together to form the two terminals of the battery. When cells of equal emf and terminal resistance are connected in parallel, the resultant emf of the battery is equal to the *emf* of one of the cells and the terminal resistance (net internal resistance) of the battery is calculated from the formula for resistors in parallel. i.e.

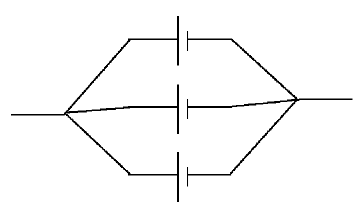
$$\text{Total } emf = E \quad (3.151)$$

$$\text{net internal resistance} = \frac{r}{n} \quad (3.152)$$

where E is the net Emf and r is the internal resistance of one of the cells.

One advantage of connecting cells in parallel is that, there is less drain on the individual cells since they share the total current. Unlike in series connection where the same main current is supplied by each cell, in parallel connection a cell supplies (or receives) only part of the main current.

Cells should never be left connected in parallel when not in use, for if the emf of one is slightly greater than that of another, current will circulate in the battery itself and the cells (or battery) become exhausted (ruin out). This can not happen when they are in series.



cells in parallel

$$\begin{aligned} \text{Total Emf} &= E \quad (3.153) \\ \text{net Internal resistance} &= \frac{r}{3} \quad (3.154) \end{aligned}$$

Examples

A cell is joined in series with a resistance of 2Ω and a current of 0.25A flows through it. When a second resistance of 2Ω is connected in parallel with the first, the current through the cell increases to 0.3A. what is ;

(a) The emf.

(b) The internal resistance of the cell.

Solution

(a)Using $E = I(R + r)$

In the first case

when $R = 2\Omega$, $I = 0.25$

$$\begin{aligned} E &= I(R+r) \\ &= 0.25(2+r) \\ E &= \frac{1}{4}(2+r) \\ 4E &= 2+r..........(i) \end{aligned}$$

In the second case

when

$$\begin{aligned} R &= \text{net parallel resistance} \\ &= \frac{\text{product of } R}{\text{sum of } R} \\ &= \frac{2\times 2}{2+2} \\ &= \frac{4}{4} \\ &= 1\Omega \end{aligned}$$

$I = 0.3A$ hence

$$\begin{aligned} E &= I(R+r) \\ &= 0.3(1+r) \\ E &= \frac{3}{10}(1+r) \\ 10E &= 3(1+r) \\ \frac{10}{3}E &= 1+r.........(ii) \end{aligned}$$

subtracting equation (ii) from (i), we have

$$\begin{aligned} 4E &= 2+r \\ \tfrac{10}{3}E &= 1+r \end{aligned}$$

$$\begin{aligned} 4E - \tfrac{10}{3}E &= 1 \\ \frac{12-10}{3}E &= 1 \\ \frac{2}{3}E &= 1 \\ 2E &= 3 \\ E &= \frac{3}{2} \\ &= 1.5V \end{aligned}$$

Hence the emf is 1.5V

(b)Using **Either** (i)

$$\begin{aligned} 4E &= 2+r \\ \Longrightarrow r &= 4E-2 \\ r &= 4\times 1.5-2 \\ &= 4\times\frac{3}{2}-2 \\ &= 6-2 \\ &= 4\Omega \end{aligned}$$

OR using (ii)

$$\begin{aligned} \frac{10}{3}E &= 1+r \\ r &= \frac{10}{3}E-1 \\ &= \frac{10}{3}\times 1.5-1 \\ &= \frac{10}{3}\times\frac{3}{2}-1 \\ &= 5-1 \\ &= 4\Omega \end{aligned}$$

hence the internal resistance is 4Ω and the Emf is $1.5V$

Questions

1. Two cells each having an emf of 1.5V and an internal resistance of 2Ω are connected

   (i) in series

   (ii) in parallel

   Find the current in each case, when the cells are connected across a 1Ω resistor. If the 1Ω resistor is substituted by a 11Ω resistor, find the new current in both cases?What do you conclude? (0.6, 0.2, 0.75, 0.125A)

2. In an experiment to determine resistance of a given fixed resistor, the results in table 3.24 were obtained;

I/Amperes	V/volts
0.75	1.5
0.50	3.0
2.25	4.5
3.00	6.0
3.75	7.5

Table 3.24: Ohm's law

   Plot a graph of V against I and use it to determine the resistance of the fixed resistor?

3. A battery made up of six cells each having an *emf* of 2V and an internal resistance of 0.1Ω is connected in series with an ammeter of negligible resistance, a 1.4Ω resistor and a metal filament lamp. The ammeter reads 3A. find

   (a) The p.d across the lamp

   (b) The resistance of the lamp

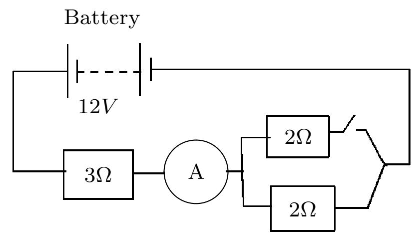
   (c) What reading you expect from a high resistance voltmeter connected a cross the battery terminals.Ans $6V$, 2Ω, $10.2V$

4. A battery consists of three accumulators in series, each having an emf of 2V. A second battery consists of four dry cells also in series each having an emf of 1.5V;

   (a) What is the emf of each battery?

   (b) Why do you get a bigger current from the battery of accumulator?

5. In the circuit below. Find the reading of the ammeter A when the switch is;



   (a) open

   (b) Closed (Assume battery resistance is negligible).

6. A cell with an emf of 1.5V and an internal resistance of 1Ω and is connected across two resistors of resistances 2Ω and 3Ω in series. Find the current flowing and p.d across each resistance?

7. Two cells connected in parallel each having an emf of 1.5V and internal resistance of 1Ω, are connected to a 4Ω resistor. What is the current flowing in this resistor?($0.33A$)

8. The values of I and V in table 3.25 were obtained in an experiment to determine the internal resistance of a cell using a standard resistor;

Voltage(V)	Current(I)
1.4	0.3
1.3	0.325
1.2	0.35
1.1	0.375
1.0	0.4
0.9	0.425
0.8	0.45

Table 3.25: Internal resistance

   (a) Plot a graph of V against I

   (b) Use your graph in (a) above to determine the emf and internal resistance of the cell

9. A cell can supply a current of 1.2A through two 2Ω resistors connected in parallel. When they are connected in series the value of the current is 0.4A. Calculate the emf and the internal resistance of the cell? (Ans 1.8V, 0.5Ω)

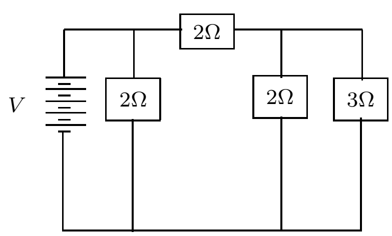
10. A cell is joined in series with a resistance of 2Ω and a current of 0.25A flows through it. When a second resistance of 2Ω is connected in parallel with the first, the current through the cell increases to 0.3A. what is ;

    (a) The emf.

    (b) The internal resistance of the cell.

11. A cell of 6.0V *emf* and negligible internal resistance is connected to a resistor and drives a current of 3A through it. Another cell of *emf* 1.5V is inserted in the circuit in series with the first one. The current remains at 3.0A. What is the internal resistance of the second cell? (Ans 0.5Ω)

12. The figure below shows 4 resistors and a source of voltage of 6V with internal resistance $R_s = 0.20\Omega$

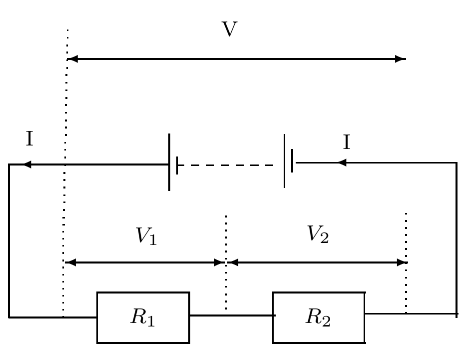


    (i) Find the effective resistance of the circuit

    (ii) Calculate the current through the internal resistance.

3.12.7 Potential divider

Consider resistors in series, the current is the same through each resistor but their resistances are different.



Therefore, there must be a different potential different across each resistor given by;

$$V_1 = IR_1, V_2 = IR_2 \tag{3.155}$$

Where $V_1 + V_2 = V$ the total potential drop in the circuit.

Re-arranging them, we have:

if $V_1 = IR_1$, then $I = \frac{V_1}{R_1}$

Substituting into V_2, we have

$$V_2 = IR_2 \tag{3.156}$$

$$= \frac{V_1}{R_1}R_2 \tag{3.157}$$

$$V_2 = \frac{R_2}{R_1}V_1 \tag{3.158}$$

Therefore $\frac{V_2}{V_1} = \frac{R_2}{R_1}$, i.e. the ratio of the p.ds across resistors in series is equal to the ratio of their resistances.

This is a use full way to produce a specific or required p.d in a circuit (how?).

$$\begin{aligned}
\text{Since} \quad V &= V_1 + V_2 \\
&= V_1 + \frac{R_2}{R_1}V_1 \\
&= V_1\left(1 + \frac{R_2}{R_1}\right) \\
V &= V_1\left(\frac{R_1 + R_2}{R_1}\right) \\
VR_1 &= V_1(R_1 + R_2) \\
V_1 &= V \times \frac{R_1}{R_1 + R_2} \\
V_1 &= V\left(\frac{R_1}{R_1 + R_2}\right) \qquad (3.159)
\end{aligned}$$

This shows that the p.d across R_1 is a fraction of the supply p.d (supplied voltage); where the fraction is the ratio of R_1 to the total resistance in the circuit.

It can also be shown that

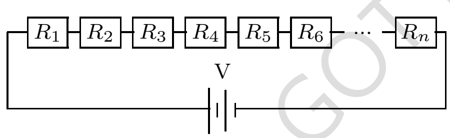
$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \tag{3.160}$$

i.e.

$$\begin{aligned}
V &= V_1 + V_2 \\
\text{from } V_2 &= \frac{R_2}{R_1} V_1 \\
\Longrightarrow V_1 &= \frac{R_1}{R_2} V_2 \\
V &= V_1 + V_2 \\
&= \frac{R_1}{R_2} V_2 + V_2
\end{aligned}$$

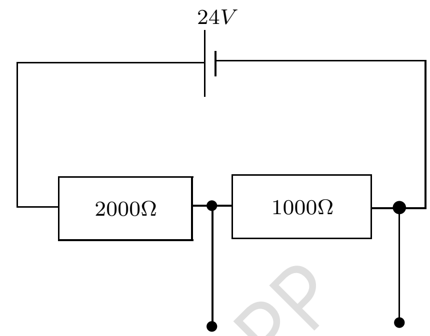
$$\begin{aligned}
&= \left(\frac{R_1}{R_2} + 1\right) V_2 \\
V &= \left(\frac{R_1 + R_2}{R_2}\right) V_2 \\
V R_2 &= (R_1 + R_2) V_2 \\
\left(\frac{R_2}{R_1 + R_2}\right) V &= V_2 \\
V_2 &= \left(\frac{R_2}{R_1 + R_2}\right) V
\end{aligned}$$

In general for;



$$\begin{aligned}
\text{If } \sum R &= R_1 + R_2 + \ldots + R_n \text{ then} \\
V_1 &= \left(\frac{R_1}{\sum R}\right) \times V && (3.161) \\
V_2 &= \left(\frac{R_2}{\sum R}\right) V && (3.162) \\
V_3 &= \left(\frac{R_3}{\sum R}\right) V && (3.163) \\
\vdots &= \vdots && (3.164) \\
V_n &= \left(\frac{R_n}{\sum R}\right) V && (3.165)
\end{aligned}$$

This is a use-full way of producing a specific p.d in a circuit. For instance if a circuit is powered by a 24V powered supply and we wish to pick up a p.d of 8V so as to use it to run an 8V loudspeaker, This can be done using two resistors[30] where one is half the other as shown in the figure below;



p.d across the 1000Ω resistor, $P_{1000\Omega}$

$$\begin{aligned}
P_{1000\Omega} &= 24 \times \left(\frac{1000}{1000 + 2000}\right) \\
&= 24.\frac{1000}{3000} \\
&= \frac{24}{3} \\
&= 8V
\end{aligned}$$

NB. The total resistance can be chosen to limit the current or potential difference to any desired value. Any device which uses resistors to provide a specific lower voltage from a larger voltage using only resistors is called a **potential divider**.

Questions

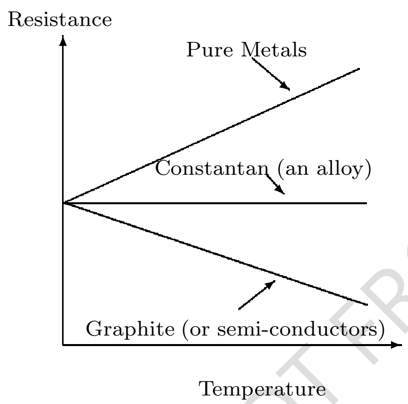
1. Resistors of 2Ω, 4Ω, and 24Ω are connected in series and a voltage of 24V applied across them. Find
   (a) the total resistance.
   (b) the current flowing in the circuit.
   (c) the voltage across each resistor. Ans (12Ω, 2A, 4V, 8V, 12V).

[30] Large resistors are used because they are more durable

2. A battery with emf 10V is connected across 3 resistors of 1Ω, 2Ω and 3Ω in series. Calculate the p.d across each resistor?(Ans; 1.67V, 3.33V, 5V)

3.12.8 Effects of temperature on resistance

Consider a group of atoms or molecules, when they are heated they begin to vibrate vigorously (violently), this means that an electron does not easily pass through them hence increased resistance. This is what we expect and it is true for pure metals, its observed that the resistance of a pure metal increases with temperature but the resistance of certain other conducting materials e.g. carbon, graphite, silicon, germanium, selenium, i.e. semiconductors decreases with temperature.



Various metals alloys notably manganin, and constantan which are used for making standard resistors, show very small resistance changes with temperature under normal conditions. Semiconductors are used in manufacture of solid-state electronic components such as transistors, diodes, and others which are the building block in any electronic circuit like an amplifier, radio, TV, phones, computer chips and others.

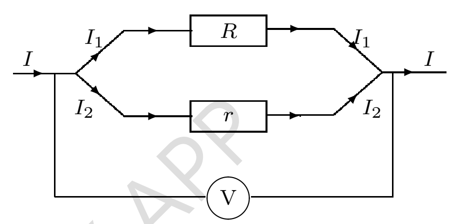
3.12.9 Why ammeters are connected in series and voltmeters in parallel.

Voltmeters (as current dividers)

Consider a resistor with a very high resistance R connected in parallel with a very low value resistor. (r) they;

(a) have the same potential difference across each

(b) share the total current in the circuit.



For each resistor $V = I_1R$ and also $V = I_2r$ and at the junction $I = I_1 + I_2$. We need to express the currents I_1 and I_2 in terms of I and compare the ratios $\frac{I_1}{I}$ and $\frac{I_2}{I}$.Which of the two is negligible?

$$\text{From } V = I_1R \tag{3.166}$$
$$\Rightarrow I_1 = \frac{V}{R} \tag{3.167}$$
$$\text{From },V = I_2r \tag{3.168}$$
$$\Rightarrow I_2 = \frac{V}{r} \tag{3.169}$$
$$\text{But } I = I_1 + I_2 \tag{3.170}$$

This shows that, they share the current. Equating V in the above equations we have

$$I_1R = I_2r \tag{3.171}$$
$$\Rightarrow \frac{I_1}{I_2} = \frac{r}{R} \tag{3.172}$$
$$\Longleftrightarrow I_1 = \left(\frac{r}{R}\right) I_2 \text{ and} \tag{3.173}$$
$$I_2 = \left(\frac{R}{r}\right) I_1 \tag{3.174}$$

Substituting equation (3.174) into equation

(3.170) to eliminate I_2, we have

$$
\begin{aligned}
I &= I_1 + I_2 && (3.175)\\
&= I_1 + \left(\frac{R}{r}\right) I_1 && (3.176)\\
&= I_1 \left(1 + \frac{R}{r}\right) && (3.177)\\
&= I_1 \left(\frac{r+R}{r}\right) && (3.178)\\
I &= I_1 \left(\frac{r+R}{r}\right) && (3.179)\\
\left(\frac{r}{R+r}\right) I &= I_1 && (3.180)\\
I_1 &= \left(\frac{r}{R+r}\right) I && (3.181)
\end{aligned}
$$

this means that we can have a current divider, since $R \gg r$, then I_1 current through the high value resistor is very small i.e. $I_1 \ll I$(because $\frac{r}{r+R} \cong \frac{r}{R} \cong 0$, when $R \gg r$). Substituting equation (3.181) into equation (3.170) to eliminate I_1; we have

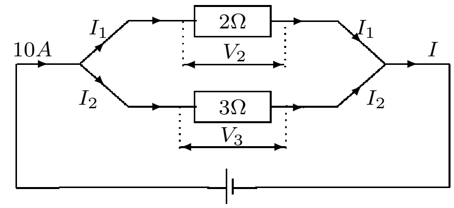
$$
\begin{aligned}
I &= I_1 + I_2 && (3.182)\\
&= \left(\frac{r}{R}\right) I_2 + I_2 && (3.183)\\
&= I_2 \left(\frac{r}{R} + 1\right) && (3.184)\\
I &= I_2 \left(\frac{r+R}{R}\right) && (3.185)\\
\left(\frac{R}{r+R}\right) I &= I_2 && (3.186)\\
I_2 &= \left(\frac{R}{r+R}\right) I && (3.187)
\end{aligned}
$$

since $R \gg r$, then I_2, the current through the low value resistor is very close to I (the total current). The voltmeter always has or must have, a very high resistance, its connected in parallel with a device, such that it does not affect the current in the circuit since from above $I_2 \cong I$, that is why voltmeters are always connected in parallel.

Example

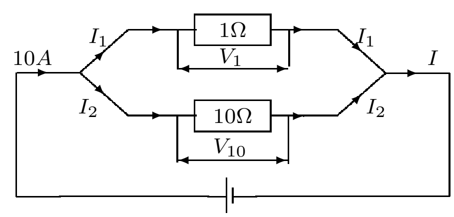
Use the diagrams below to find I_2 and I$_1$;

1. circuit 1



$$
\begin{aligned}
V_2 &= V_3\\
I_1 \times 2 &= I_2 \times 3\\
I_1 &= \frac{3}{2} I_2\\
\text{but } I_1 + I_2 &= 10\\
\frac{3}{2} I_2 + I_2 &= 10\\
\frac{5}{2} I_2 &= 10\\
\Rightarrow I_2 &= \frac{20}{5}\\
&= 4.0A\\
I_1 &= \frac{3}{2} I_2\\
&= \frac{3}{2} \times 4.0\\
&= 6.0A
\end{aligned}
$$

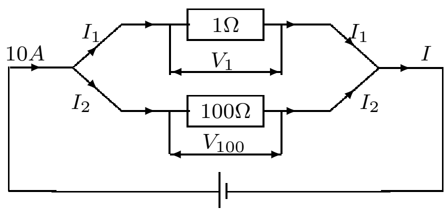
2. circuit 2



$$
\begin{aligned}
V_1 &= V_{10}\\
I_1 \times 1 &= I_2 \times 10\\
I_1 &= 10 \times I_2\\
\text{but } I_1 + I_2 &= 10\\
10 I_2 + I_2 &= 10\\
11 I_2 &= 10\\
I_2 &= \frac{10}{11}\\
&\cong 0.9091A
\end{aligned}
$$

$$\begin{aligned} I_1 &= I - I_2 \\ &\cong 10 - 0.9091 \\ &\cong 9.091A \\ \text{OR } I_1 &= 10 \times I_2 \\ &\cong 10 \times 0.9091 \\ &\cong 9.091 \end{aligned}$$

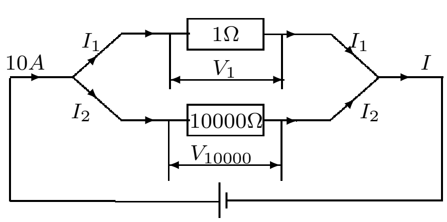
3. circuit 3



$$\begin{aligned} V_1 &= V_{100} \\ I_1 \times 1 &= I_2 \times 100 \\ I_1 &= 100I_2 \\ \text{but } I_1 + I_2 &= 10 \\ 100I_2 + I_2 &= 10 \\ 101I_2 &= 10 \\ I_2 &= \frac{10}{101} \\ &= 0.099A \end{aligned}$$

$$\begin{aligned} I_1 &= I - I_2 \\ &= 10 - 0.099 \\ &= 9.901A \end{aligned}$$

4. circuit 4



$$\begin{aligned} V_1 &= V_{10\,000} \\ I_1 \times 1 &= I_2 \times 10\,000 \\ I_1 &= 10\,000I_2 \\ \text{but } I_1 + I_2 &= 10 \\ 10\,000I_2 + I_2 &= 10 \\ 10001I_2 &= 10 \\ I_2 &= \frac{10}{10\,001} \\ &= 0.00A(0.000999) \end{aligned}$$

$$\begin{aligned} I_1 &= I - I_2 \\ &= 10 - 0.00 \\ &= 10.00A \end{aligned}$$

OR

$$\begin{aligned} I_1 &= 10\,000I_2 \\ &= 10000 \times 0.000999A \\ &= 9.999A \\ &\cong 10A \text{ as above} \end{aligned}$$

All these results are summarised in table 3.26

Summary table

I	R_1	R_2	I_1	I_2
10	2	3	4.0	6.0
10	1	10	9.091	0.909
10	1	100	9.901	0.099
10	1	10 000	9.999	0.00
10	1	10 000	10	0.00

Table 3.26: Current divider

Note that when one resistor is mush higher than the other, the current through it is very small i.e. negligible. The voltmeter should have a very high resistance value so as to take a negligible current hence not affecting the total current in the circuit. Even we say it can not measure the emf of a cell because it consumes a very small current due to a high resistance and hence the value of *emf* it gives is slightly less than the actual *emf*.

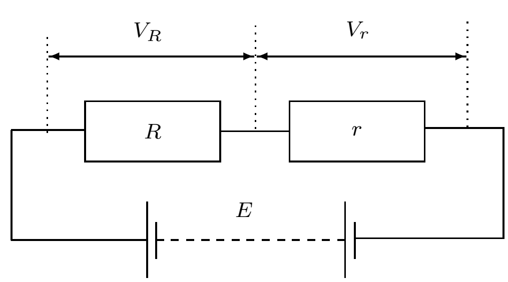
Any device which uses resistors to divide current supplied to a given device is called a current divider. Volume control knobs on radios, TV and on most electrical devices are either current divider or voltage divider depending on whether the device need voltage or current.

What is wrong with a voltmeter in series?

When voltmeter (high resistance) is connected in series with other components in a circuit, the current I, in the circuit is greatly reduced, due to high resistance. This is avoided by connecting it in parallel i.e. that is why voltmeters are connected in parallel).

Ammeters

Consider a high and low value resistors connected in series in a circuit;



The current I through each resistor is the same, from equation (3.165)

$$\begin{aligned}
\text{The potential drop across } R &= \left(\frac{R}{R+r}\right)E \\
\text{The potential drop across } r &= \left(\frac{r}{R+r}\right)E, \\
\text{if } R &\gg r \Longrightarrow R+r \approx R \\
\Rightarrow V_R &= \left(\frac{R}{R+r}\right)E \\
&\cong \frac{R}{R}E \\
&= E \\
\Rightarrow V &= \left(\frac{r}{R+r}\right)E \\
&\cong \frac{r}{R}E \\
&\cong 0 \quad or \quad \text{negligible}
\end{aligned}$$

For instance if $R = 400\Omega$, $r = 0.5\Omega$ and $E = 3.0V$ then

$$\begin{aligned}
V_R &= \frac{400}{400+0.5} \times 3 \\
&= \frac{400}{400.5} \times 3 \\
&= 2.996 \\
&\cong 3.0V \text{ to 2d.p.}
\end{aligned}$$

and

$$\begin{aligned}
V_r &= \frac{0.5}{400+0.5} \times 3 \\
&= \frac{0.5}{400.5} \times 3 \\
&= 0.003745 \\
&\cong 0.00V \text{ to 2d.p.}
\end{aligned}$$

An ammeter is supposed to have a very low resistance such that p.d across it is negligible. And hence if does not affect either current or voltage in the circuit when connected in series with other components in the circuit. That is why ammeters which always has or must have a very low resistance, are always connected in series.

What is wrong with an Ammeter in parallel?

Note that When wiring and by mistake an ammeter is connected in parallel with any resistor, the resistance of the circuit is highly reduced therefore current increases to a very high value and this will burn the ammeter instead because it provides an easy path for the electrons i.e. low electrical resistance and this can lead a serious electrocution.

3.12.10 Electric power

We have seen that the electric work done or energy dissipated is given by;

$$\begin{aligned}
\text{Work done} &= \text{Quantity of electricity} \times p.d \\
W &= QV \qquad (3.188)\\
\text{but } Q &= It \qquad (3.189)\\
\text{hence } W &= VIt \qquad (3.190)\\
\text{Power} &= \frac{\text{Work done}}{\text{Time taken}} \qquad (3.191)\\
P &= \frac{VIt}{t} \qquad (3.192)\\
\Longrightarrow \mathbf{P} &= \mathbf{VI} \quad *** \qquad (3.193)\\
\text{using } V &= IR \text{ i.e. Ohm's law} \qquad (3.194)\\
\Longrightarrow \mathbf{P} &= \mathbf{I^2R} \quad *** \qquad (3.195)\\
\text{using } I &= \frac{V}{R} \text{ we have} \qquad (3.196)\\
\Longrightarrow \mathbf{P} &= \frac{\mathbf{V^2}}{\mathbf{R}} \quad *** \qquad (3.197)
\end{aligned}$$

Hence the three formulae for electric power are

$$\begin{aligned}
\mathbf{P} &= \mathbf{VI} \qquad (3.198)\\
\mathbf{P} &= \mathbf{I^2R} \qquad (3.199)\\
\mathbf{P} &= \frac{\mathbf{V^2}}{\mathbf{R}} \qquad (3.200)\\
& \qquad (3.201)
\end{aligned}$$

This power is dissipated in the resistor or conductor as heat energy. Also the three formulae for electric work done are

$$\begin{aligned}
\mathbf{W} &= \mathbf{VIt} \qquad (3.202)\\
\mathbf{W} &= \mathbf{I^2Rt} \qquad (3.203)\\
\mathbf{W} &= \frac{\mathbf{V^2}}{\mathbf{R}}\mathbf{t} \qquad (3.204)\\
& \qquad (3.205)
\end{aligned}$$

Examples

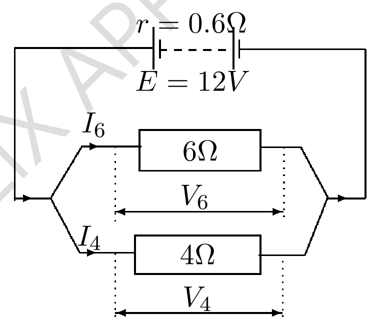
A battery of $emf = 12V$ and internal resistance, $r = 0.6\Omega$ is connected across 2 resistors 4Ω and 6Ω in parallel. Determine;

(i) the power dissipated in the 6Ω resistor and the 4Ω resistor

(ii) The total heat dissipated in the 6Ω resistor in 6 minutes

Solution

(i)The circuit below best describes this question



Total resistance , $\mathbf{R}$, in the circuit is equal to the sum of the internal resistance($\mathbf{r = 0.6\Omega}$) and the external resistance, $\mathbf{R_P}$(equal to the total parallel resistance). If also the total current is $\mathbf{I}$, then we can write

$$\begin{aligned}
\mathbf{R} &= r + \mathbf{R_P}\\
&= 0.6 + \left(\frac{R_6 \times R_4}{R_6 + R_4}\right)\\
&= 0.6 + \left(\frac{6 \times 4}{6+4}\right)\\
&= 0.6 + 2.4\\
\mathbf{R} &= 3\Omega
\end{aligned}$$

The total current, I, is

$$\begin{aligned}
\mathbf{I} &= \frac{E}{R}\\
&= \frac{12}{3}\\
\mathbf{I} &= 4A
\end{aligned}$$

$$\begin{aligned}
\textbf{But } V_6 &= V_4(\text{for Parallel}) \\
I_6 \times 6 &= I_4 \times 4 \\
6I_6 &= 4I_4 \\
I_6 &= \frac{4}{6}I_4 \\
I_6 &= \frac{2}{3}I_4
\end{aligned}$$

$$\begin{aligned}
\textbf{But also } I &= I_6 + I_4 \texttt{ At the junction} \\
4 &= I_6 + I_4 \\
&= \frac{2}{3}I_4 + I_4 \\
&= \left(\frac{2}{3} + 1\right) I_4
\end{aligned}$$

$$\begin{aligned}
4 &= \frac{5}{3}I_4 \\
\Longrightarrow I_4 &= \frac{12}{5}A \\
I_4 &= 2.4A \\
I_6 &= I - I_4 \\
&= (4 - 2.4)A \\
\Longrightarrow I_6 &= 1.6A
\end{aligned}$$

since for the 6Ω resistor, $I_6 = 1.6A$, $R_6 = 6\Omega$, then the power dissipated in the 6Ω resistor is given by;

$$\begin{aligned}
P &= I^2R \\
P_6 &= I_6^2R_6 \\
&= (1.6)^2 \times 6 \\
&= 15.36 \text{ Watts}
\end{aligned}$$

And for the 4Ω resistor, $I_4 = 2.4A$, and $R_4 = 4\Omega$, then the power, P_4 dissipated in the 4Ω resistor is given by

$$\begin{aligned}
P &= I^2R \\
P_4 &= I_4^2r_4 \\
&= 2.4^2 \times 4 \\
P_4 &= 23.04 \quad \texttt{Watts} \\
&\textbf{OR} \\
V_4 &= I_4R_4 \\
&= 2.4 \times 4 \\
&= 9.6Volts \\
P_4 &= V_4I_4 \\
&= 9.6 \times 2.4 \\
P_4 &= 23.04\ W \text{ as above}
\end{aligned}$$

(ii) Since we know the power dissipated in the 4Ω and 6Ω resistors, we shall use the formula

$$\textbf{Power} = \frac{\textbf{Work done}}{\textbf{Time}} \tag{3.206}$$

$$P = \frac{W}{t} \tag{3.207}$$

$$\Longrightarrow W = Pt \tag{3.208}$$

this is equal to the heat dissipated in each resistor. Heat dissipated in the 6Ω resistor is the work done in it, is given by;

$$\begin{aligned}
W &= Pt \\
&= P_6t \\
&= 15.36 \times (6 \text{ minutes}) \\
&= 15.36 \times (6 \times 60 \text{ seconds}) \\
&= 15.36 \times 360 \text{ Joules} \\
&= 5529.6\text{Joules}
\end{aligned}$$

Recall that heat is energy and work is energy

3.13 Magnetism I

3.13.1 Introduction

It was observed long ago that a certain magnetic ore, which was common in a town of magnetite, attracted small pieces of iron and a bar of it pointed in the same direction when freely suspended in air. This specific ore mined at magnetite was called **magnetite**, it helped the Chinese sailors in determining the direction to sail. This ore, which attracted other pieces of iron, was called a **lode-stone**, which means, "leading stone ".

Magnetic poles

If a piece of lode-stone is dipped into iron-fillings, it is observed that the fillings cling at two places near its ends and a few in the middle. This means that, the forces of attraction (and repulsion) are concentrated at these ends of the stone (a magnet). The two places or points in the lode stone where the resultant attractive forces appear to be concentrated are called **poles** of the magnet. Any substance that has property of attracting such materials like iron-fillings is called a **magnet**. These points called poles were named North and South poles because they would point in the Geographic north and south directions whenever freely suspended in free space, this will be explained later. Magnets especially the modern ones can be given any shape and these shapes gives them varies names like horseshoe magnets, bar magnet, e.t.c.

Do magnets attract any material?

3.13.2 Magnetic and non-magnetic materials

It was found that a magnet can attract some materials and not all. Materials that can be attracted by a magnet are called **magnetic materials** or **ferromagnetic materials**. These include iron, black iron oxide, cobalt, nickel and few alloys like steel, which containing these metals among others. They can be made into magnets or into a magnetic powder used in the magnetic tapes or tape recorders.

Materials that can not be attracted by a magnet are called **non-magnetic materials** (i.e. paramagnetic or diamagnetic). They can not be magnetised (i.e. made into magnets).

Non-magnetic materials are of two types; **paramagnetism** and **diamagnetism**.

Diamagnetic materials: These have an equal number of electrons spinning and orbiting in opposite direction so that in the absence of some external magnetic field, these effects cancel out. A magnetic field applied slightly disturbs it and a small magnetic effect (called diamagnetism) is produced.

Paramagnetic material: they have unbalanced number of spinning electrons so that the individual atoms or molecules act like very tiny magnets. In absence of an external magnetic field, they are randomly arranged (no magnetic field). Applying an external magnetic field, molecular magnets partially align with it, increasing its strength. This effect is called paramagnetism.

Do magnets always attract or they can also repel? If so When?

3.13.3 Action of a magnet on another

Laws of magnetism

Given that we know the north and south poles of two magnets and we place them close to each other, you will observe that certain sides of the magnets attract and others repel each other. If the North Pole of a magnet is brought near the north pole of a suspended magnet, they repel each other similarly when a south pole is brought near the south pole of another, they repel each other. But when the South Pole of one is brought near the North Pole of the other, they attract each other.

This experiment leads to the **first law of magnetism** which states that

Definition 56.

like poles repel and unlike poles attract.

That is to say north and north poles repel, south and south poles repel i.e like poles repel and north and south poles attract i.e unlike poles attract.

Can a magnet repel an unmagnetised magnetic material?

Testing for polarity of a magnet or whether a given material is a magnet

If a given iron bar is suspected to be a magnet. We suspend it freely and bring near it a magnet whose poles are known. If attraction is observed, then we can conclude the following;

1. They are unlike poles i.e the iron bar has a magnetic pole (magnetic poles exist in pairs) or,
2. The iron bar is just a magnetic material (unmagnetised).

If repulsion occurs, then there are two like-poles i.e. North and North or South and South poles.

Note that **magnetic repulsion is only due to like poles** .

This is the only test that confirms that the suspended iron bar is actually a magnet hence

Repulsion is the only sure test for a magnet

.

3.13.4 A magnetic field

It is known that when a magnetic material is brought near a magnet or in the a space near a magnet is experiences a force of attraction and when it is a magnet then we can also have a repulsion.

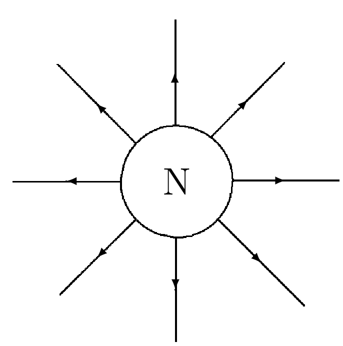
Definition 57.

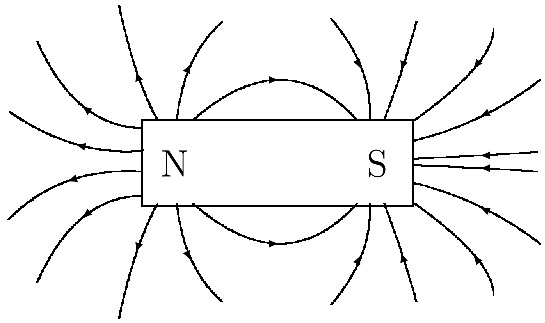
A magnetic field is the space where a magnetic material or a magnet experiences a force of attraction or repulsion.

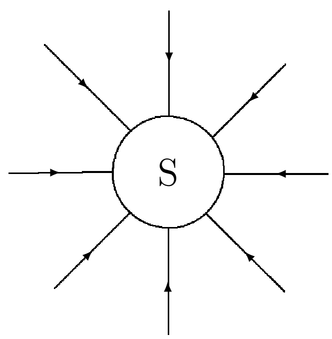
A magnetic field is a vector quantity. It is always represented by imaginary lines called **magnetic field lines** with an arrow pointing in the direction of the field line. A group of magnetic field lines is called **magnetic flux**.

Direction of the magnetic field, Magnetic field line or magnetic flux

It was decided that the direction of the magnetic field or magnetic field line or magnetic flux at a given point is the **direction of the magnetic force acting on a north pole** placed at the very point. Since a magnet's North Pole repels another North Pole then the **direction of the magnetic flux is away from the North Pole** and since a south pole attracts a north pole towards itself, then the **direction of the magnetic field is towards the south pole**. The diagram below shows the magnetic field lines near North pole, south pole and a bar magnet;







Properties of magnetic field lines

1. They are ever in a state of tension longitudinally
2. They repel each other sideways or laterally

Due to the above two properties, these line;

(i)Should never be drawn crossing each other,

(ii)Should not be drawn with kinks,

(iii)Should not touch each other but can be very close to each other.

When a magnet is freely suspended does it rest in a specific direction or it rests in any direction at random?

3.13.5 Suspending a magnet

The earth as a magnet

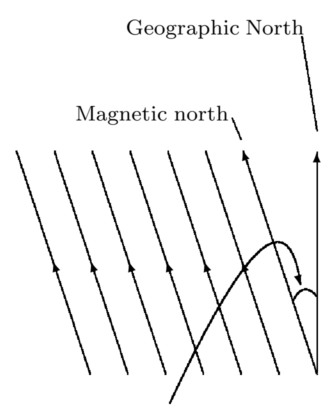
When a magnet is freely suspended in space, it oscillates to and fro for a short period of time and then comes to rest in an approximately north-south direction (why?).

Definition 58.

The imaginary line joining the two north and south poles of a magnet is called **magnetic axis.**

The suspended magnet rests with its magnetic axis in a vertical plane called a **magnetic meridian** of the earth.
Since a magnet can only be forced to face in a given direction by another magnet or by a magnetic field, then there exists a magnetic field in the earth's atmosphere. Where it comes from remains a mystery! Some scientists claim that since a magnet produces a magnetic field, then the earth contains a large magnet in its core or at its centre. It is this magnet which provides the magnetic field that aligns the freely suspended magnet in the north and south direction.

This earth's magnet has its north poles facing the geographic south and its South Pole facing the geographic North Pole, hence the earth's magnetic field lines are drawn from Geographic south to Geographic North pole as shown below;

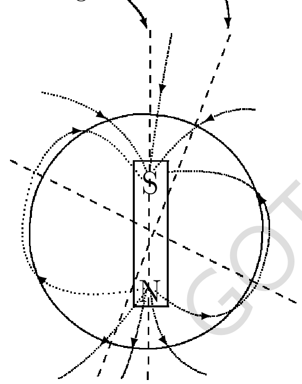


The earth's magnetic field lines run from the geographic south to the geographic north poles, implying that; they are due to a magnet whose North Pole is in the geographic south and magnetic South Pole in the geographic North Pole.

This can be an experiment to determine the poles of a magnet (how?). If i have a bar magnet whose polarity is not known, by suspending it horizontally we can identify the poles. The side that points in the geographic north is the North Pole of the magnet and the side that points in the geographic south is the South Pole of the magnet).

Describe an experiment to determine the north and south poles of a magnet?

The geographic north and the geographic south poles are not exactly in the same direction as the earth's magnetic north and south poles respectively. Analysing a magnet suspended at exactly its centre of gravity, its north poles will point in a direction known as **north seeking pole** and the south poles to a direction known as the **south seeking pole**. It always rests with its magnetic axis making a constant angle with the horizontal known as the **angle of dip or inclination**. Its magnetic axis rests in an imaginary plane called the **magnetic meridian**. The diagram below illustrates the terms defined below properly;



Definition 59.

Magnetic meridian is any imaginary vertical plane containing the magnetic axis of a freely suspended magnet at rest.

Or

Definition 60.

Magnetic meridian is any imaginary vertical plane passing through the earth's magnetic axis.

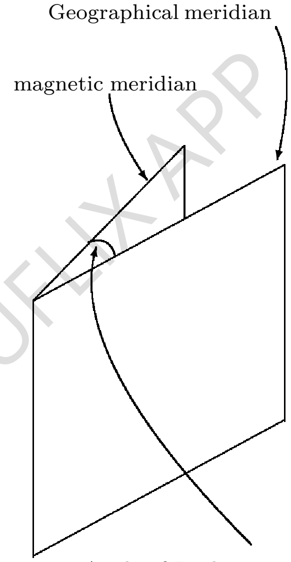
Definition 61.

Geographic meridian is any imaginary vertical plane passing through the earth's geographic north and south poles.

Or

Definition 62.

Geographic meridian is an imaginary vertical plane containing the earth's axis of rotation.



Definition 63.

Angle of declination is the angle between the geographic and the magnetic meridians.

Or

Definition 64.

Angle of declination is the angle between the magnetic north and the geographic north poles

or

Definition 65.

Angle of declination is the angle between the magnetic south and the geographic south poles.

The diagram above shows this. On a map lines joining places with the same angles of declination are known as **isogonal** lines. The angle of declination for some places is observed to change for instance at London it was $11°15^1$ in 1580 and $24°30^1$ in 1659.

Definition 66.

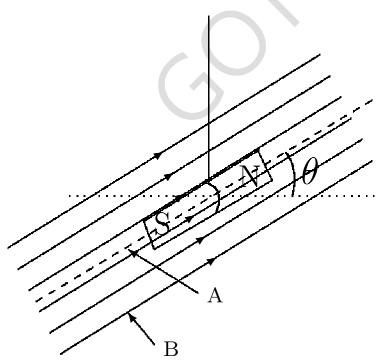
Angle of dip or **angle of inclination** (θ) is the angle between the earth's magnetic field and the horizontal.

Or

Definition 67.

Angle of dip it is angle between the magnetic axis of a **freely** suspended magnet and the horizontal.

[31] This is because a freely suspended magnet rests in the same direction as the direction of the earth's magnetic field. The angle of dip is not the same all over the earth. At the equator angle of dip ,$\theta = 0°$, at poles, $\theta = 90°$, at the rest of the intermediate places the angle of dip is between 0° and 90° below (in the North hemisphere) or above (in the Southern hemisphere) the horizontal.



B-Earth's magnetic field

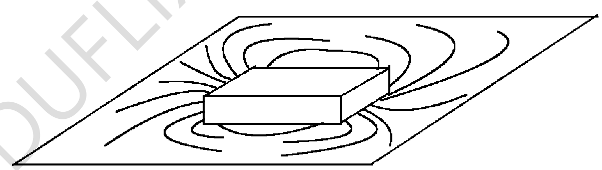
A-Magnetic axis of a freely suspended magnet

[31]Suspended at its centre of gravity

3.13.6 How to plot the magnetic flux patterns

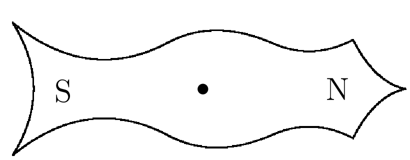
Method I - using iron fillings

- Place a cardboard on top of a table resting horizontally
- Sprinkle some iron-fillings uniformly over the cardboard and place the magnet on the cardboard [32]
- Knock the cardboard slightly and note the patterns of the iron fillings. They arrange themselves in a uniform pattern of lines, these are the field lines of the magnet.

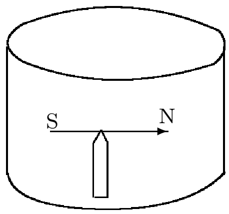


Method II - using the plotting compass or compass needle

A compass needle is a thin small magnetic needle or magnetised needle balanced at its centre of gravity. When you spin it and let it come to rest several times, it comes to rest with its ends pointing in the magnetic north and south poles, when there is no magnet in its neighbourhood. Usually its north poles is arrow-like or is marked north pole, this can be used to show the direction of the magnetic field.



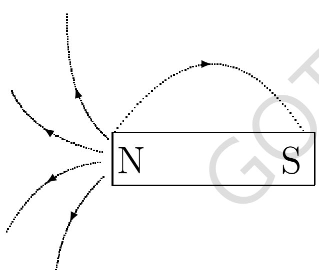
[32]Some times it is held under the cardboard



To plot the magnetic flux pattern using the compass needle

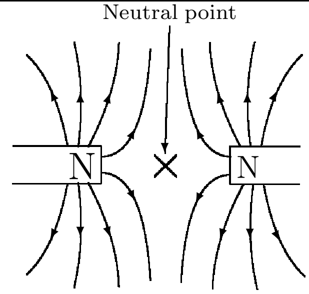
you;

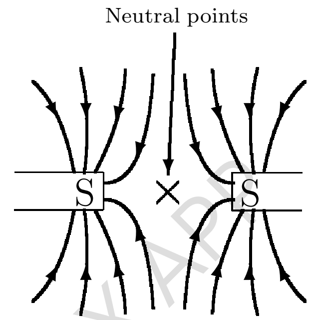
- Place a bar magnet on a piece of paper
- Starting at one pole of the magnet, mark on the paper with the compass needle; the positions of the ends N and S of the needle are marked by pencil dots A on the paper.
- The compass is moved towards the other pole making the new positions of the north poles with these dots. This is continued and a series of dots obtained are joined giving a magnetic field line that represents the direction of the magnetic field or flux.



At times there may be a point where the magnetic field is zero or has no direction i.e. when a north poles is placed there, it experiences no force, that point is called a **neutral point**[33]. It is always formed in the middle of two north poles or two south poles when two magnets are placed close to each other. The diagrams below show two neutral points;

[33]Do not confuse this neutral point in magnetism with that in electrostatics on page 216





Hence

Definition 68.

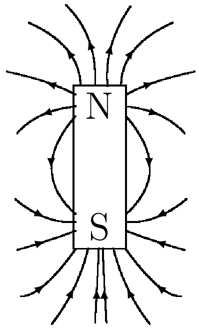
A neutral point is a point in a magnetic field where a pole or magnetic material experiences no magnetic force.

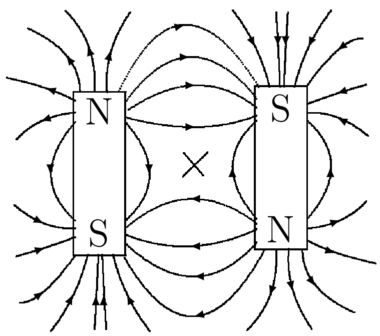
Or

Definition 69.

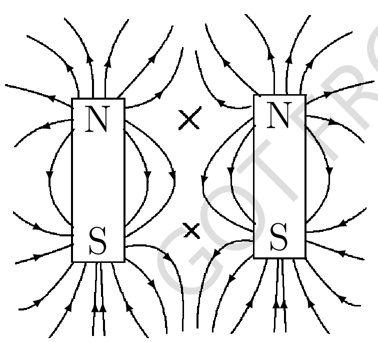
A neutral point is a point in the magnetic field where the resultant magnetic field is zero.

At the neutral point the magnet's flux density is exactly equal and opposite and the resultant flux is zero. Here we have defined the **magnetic neutral point** and not the **electrostatic neutral point**.
Below are some magnetic flux patterns of some arrangements of magnets

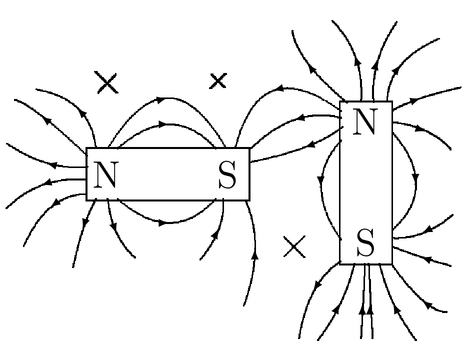


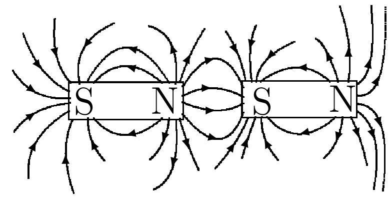


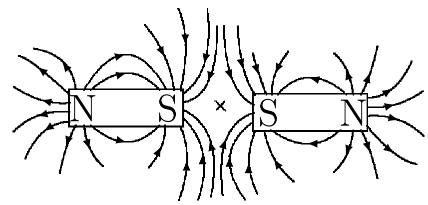
Unlike poles together

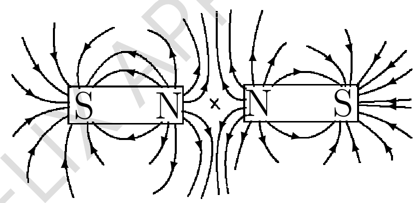


Like poles together



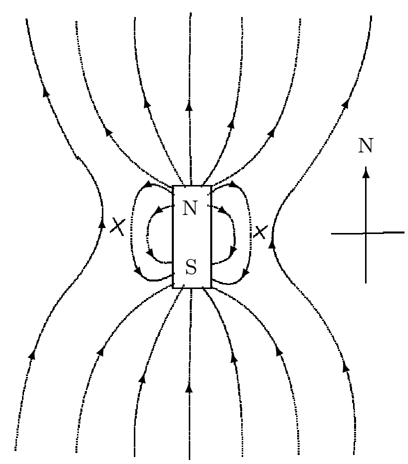




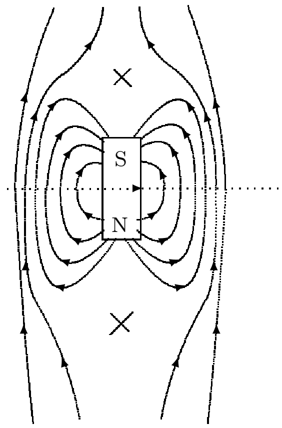


The magnetic field lines of a magnet in the earth's magnetic field with

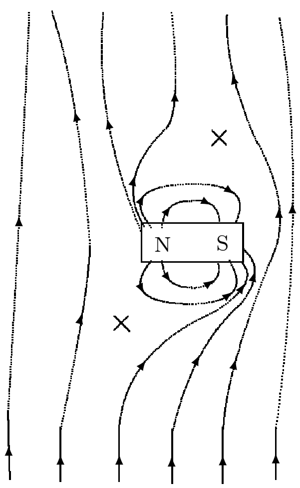
(i) north pole pointing in the geographic or magnetic north pole



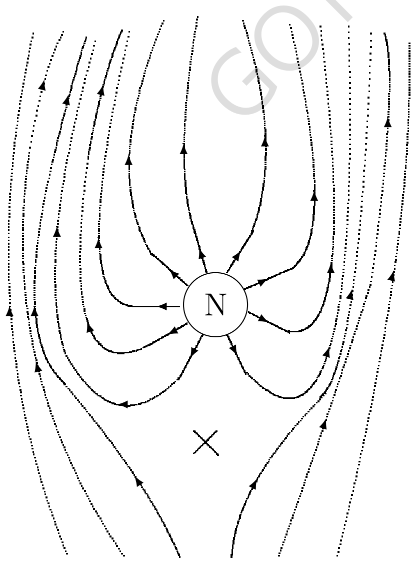
(ii) north pole pointing in the geographic or magnetic south pole



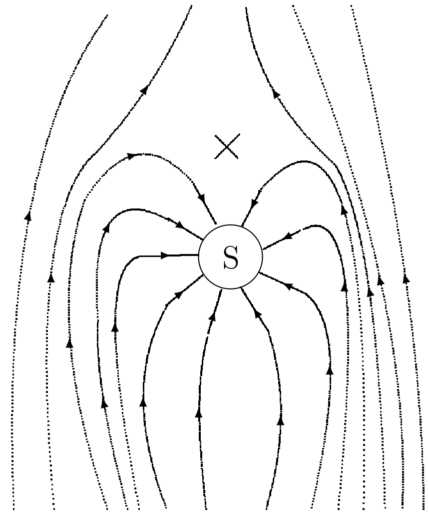
(iii)with magnetic axis of a magnet perpendicular to the earth's magnetic filed



(iv) North pole in the earth's magnetic field



(v) South pole in the earth's magnetic field



3.13.7 Domain theory of magnetism

This is a theory that explains nearly all the properties of permanent magnets. The domain theory states that magnetic materials are made up of small atomic magnets called **dipoles**. These dipoles do not act independently but in-groups called **magnetic domains**. All the dipoles in a given magnetic domain have their north poles pointing in the same direction. When all or more than half the magnetic domains in a given material are pointing in the same direction then the material is magnetised otherwise it is not magnetised.

Note that; Non magnetic materials do not posses dipoles or magnetic domains.

Definition 70.

A magnetic dipole is a magnetic atom or molecule.

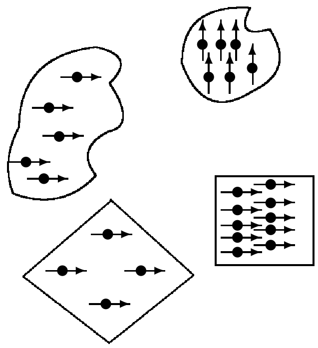
Definition 71.

A magnetic domain is a group of atomic magnets or dipoles with their north poles pointing in the same direction.

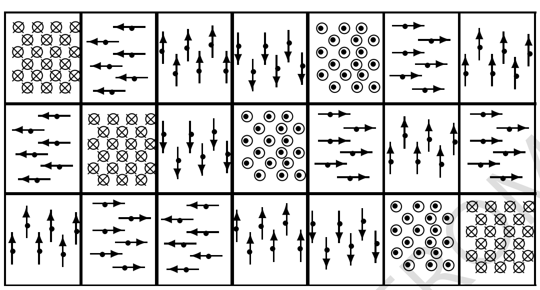
The symbol for a magnetic dipole or magnetic atom is any short arrow or

$\otimes$ when pointing into this paper and $\odot$ when pointing out of this paper.

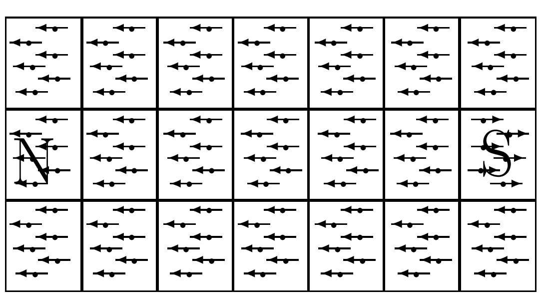
The symbol of a magnetic domain can be any shape with magnetic dipoles pointing in the same direction, like these below;



In an "**unmagnetised**" state, the resultant magnetic axes of all the domains in a magnetic material point in all or any direction at random and so the bar (or ferromagnetic material) as a whole shows no polarity. The diagram below shows this;



In the magnetised state, all the magnetic domains in the material align or have their north poles pointing in the same direction. When all the domains are pointing in the same direction then the material will posses a north pole at one end (where the domains are pointing) and a south pole at the other end. (All the magnetic domains in the material of the magnet do not always point in the same direction). The diagram below shows this;



The magnetic domains are not necessarily regular.

3.13.8 Magnetic phenomena explained by the domain theory

1. **Demagnetisation**

Definition 72.

Demagnetisation is the process of disrupting the magnetic domains of magnetised material so that they change their direction to no specific direction.

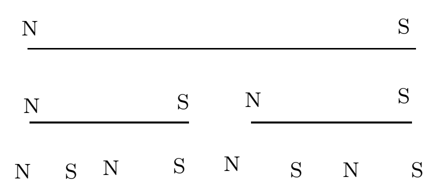
Any thing that does this weakens or destroys the magnetism of the magnet as a whole. This can be caused by

(i) A mechanical vibration like hammering a magnet. This weakens or destroys its magnetism.

(ii) An increase or rise in temperature; this is because it increases the amplitude of vibration of the atoms which disorganise the domains hence demagnetising the magnet as a whole.

2. **Breaking a magnet**:

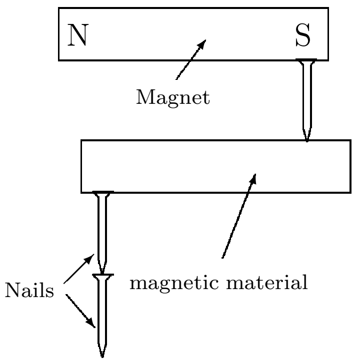
When a magnet is broken down into two pieces, the two pieces are also magnets. Even if I continue to break the pieces into other smaller pieces, they are also magnets i.e. each pieces has two magnetic poles. The diagram below shows this;



The smallest magnet that can be obtained is called a **magnetic domain**. This shows that a magnet is made up of smaller magnets (domains) according to the domain theory of magnetism.

3. **Magnetic induction:**

It is known that when a magnetic is made to touch an unmagnetised magnetic material like iron or steel, the magnetic material also starts to attracts other pieces of iron. When the magnet is pulled off the magnetic material, the magnetic material losses its magnetism. This **temporary magnetism** is called **induced magnetism** and the process, **magnetic induction**. The diagrams below show this;

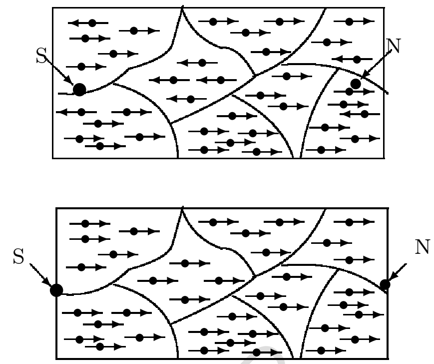


Explanation

When a magnetic material is exposed to a magnetic field from a magnet in its neighbourhood or in contact with it, its domains (or dipoles) turn so that their N-poles point in the direction of the external magnetic field. Just like how a freely suspended magnet changes to the direction of the magnetic field in its neighbourhood.

Due to the crystalline forces pulling the domains to their initial directions, when the magnetising field or the contact with the magnet is removed, they turn to their initial directions or positions and hence losing the magnetism

4. **Magnetic poles are not exactly at the poles of a magnet where they should be:**
 If all the magnetic domains of a magnet are pointing in the same direction then the poles should be exactly at the extreme end of the magnet. But if some are not in the same direction as that of the majority, then the poles can not be at the extreme end.



Explanation

The Major cause of this is the repulsion. When a magnet repels another magnet, some of its magnetic domains change direction (though not always). And this leads to the shift of the net pole position some where in the magnet and not at the extreme surface or end.

5. **Magnetic saturation**

It is known that when a magnetic material is being magnetised, the strength of its magnetism depends on the magnetising field. But the strength of the magnetism attained by the magnetic material has a maximum limit it can not exceed; this property of having a maximum limit is called magnetic saturation.

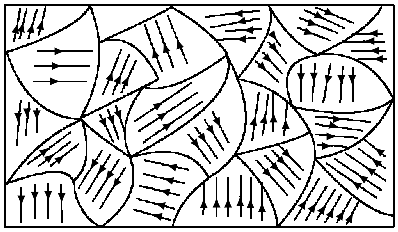
Definition 73.

Magnetic saturation is when the strength of a magnet can not exceed a certain limit.

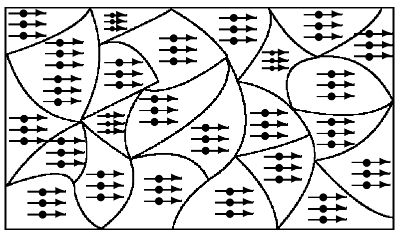
Why?

Explanation

When all the magnetic domains of a magnet are not yet aligned, then the magnet can be magnetised further by aligning the rest and the magnetic strength increases. But when all the magnetic domains have been brought into alignment with the magnetising field or in the same direction, then the strength of the magnet can not be increased any more and we say that the magnet is saturated, it can not be magnetised any more.



Unsaturated state

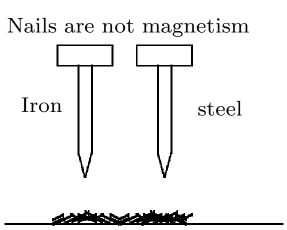


Saturated state (fully magnetised state)

3.13.9 Magnetic properties of iron and steel

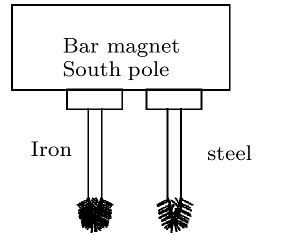
The most common magnetic materials are iron and steel. Steel is an alloy of iron, carbon and other metals like chromium. Are they the same or do they have the same magnetic properties? Let us compare them.

1. Suspend one iron nail and one steel nail (of equal sizes) over iron fillings



The nails do not attract any iron filling since they are not magnetised.

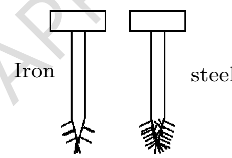
2. Hold one pole of a bar magnet near the heads of the nails.



(Iron attracts more nails)

Which nail attracts more iron-fillings, or is strongly magnetised?(iron)
Which nail is more easily magnetised (by induction)?(iron)

3. Take the bar magnet away.



(Steel retains more magnetism)

Which nail still attracts more of the iron fillings with out the magnetising field or the magnet?(steel)

From this experiment we conclude that;

1. **Iron can easily be magnetised and demagnetised than steel** because iron attracted iron fillings and had less iron fillings when the magnet was removed. That is why iron is used in making temporary magnets or electromagnets or as cores in solenoids and transformer.

2. Under the same conditions, **iron forms a stronger magnet than steel** because it attracted more iron fillings hence when a stronger magnetic force is required, iron is used for instance in lifting magnets.

3. **Steel keeps its magnetism much longer than iron** (it is more retentive) because it remained with more iron fillings than iron when the magnet was taken away. That is why steel is used in making permanent magnets.

3.13.10 Soft and hard magnetic materials

Magnetic materials are of two types soft and hard magnetic materials. These are;

Definition 74.

A **soft magnetic material** is a magnetic material that can easily be magnetised and demagnetised.

For instance iron and soft iron.

Definition 75.

A **hard magnetic material** is a magnetic material that is not easily magnetised and demagnetised.

For instance steel.

3.13.11 Magnetising a magnetic material

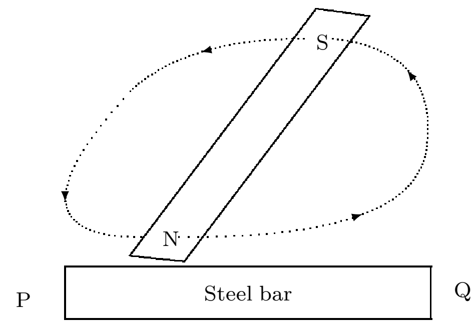
There are six, major methods of magnetising a magnetic material, these are;

1. Single touch method
2. Double touch or divided touch method
3. Hammering
4. Heating and cooling
5. Magnetic induction
6. Electrical method

1. Single touch method;

In this method a magnet is used to magnetise a steel bar by stroking on it. The following procedures are followed;

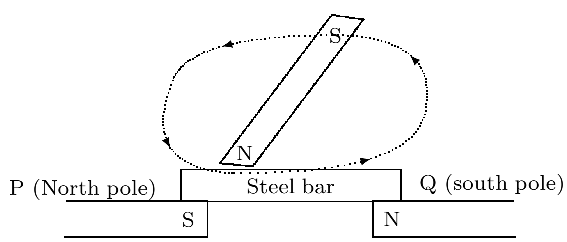
- Place the steel bar on a table
- Holding a magnet in your hand stroke it (press and slide it) on the steel bar several times (about ten) with one pole of the magnet in one direction as shown below;



End P of the steel bar becomes a North Pole and Q a South Pole.

- The steel bar now attracts other pieces of iron i.e. it is now a magnet.

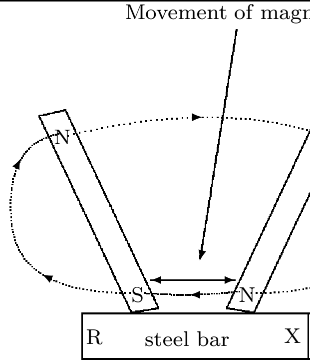
NB. A better method would be laying the steel bar on two magnets and stroke the other on it as shown below; P and Q attain the same polarity as before.



2. Double touch or divided touch method;

In this method, we use two magnets instead of using one. Its described below;

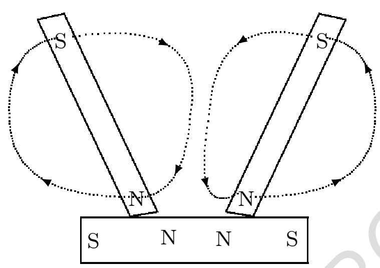
- Place the steel bar to be magnetised on the table or horizontal surface
- Stroke the two magnets several times with their polarity in the directions shown below;



R becomes a North Pole and X a south pole

- Testing the steel bar, it attracts pieces of iron i.e. it is now a magnet.

If the magnets are stroked on the steel bar with like poles facing each other, the steel bar will posses like poles at its ends and like pole at its centre. Such poles are called **consequent poles.**

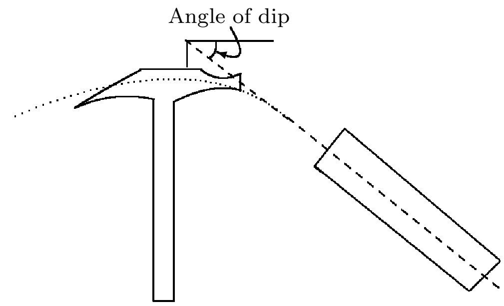


The two methods (single touch and double touch) depend on the a magnetic field directed to the magnetic material whose domains get aligned in the direction of the magnetising field.

3. Hammering

In this case the steel bar is held pointing in the direction of the earth's magnetic field i.e. in the position and angle a freely suspended magnet would rest. The following steps are followed.

- A steel bar is held in a position in which a bar magnet rests when freely suspended.
- In this direction the steel bar is hammered on one side for a short time.
- On testing the steel bar with iron fillings it attracts them hence it has been magnetised.

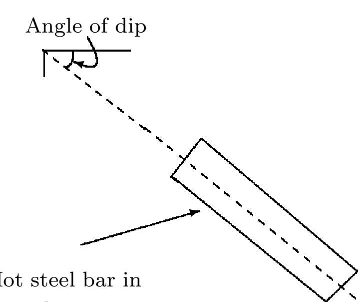


This method depends on the assumption that the magnetic domains are made to change their directions to that of the earth's magnetic field on hammering.

4. Heating and cooling method

Knowing the angle of dip of the place where we are, the following procedures are followed;

- The steel bar to be magnetised is heated till it is red hot.
- The red-hot steel bar is cooled down while facing in the direction of the earth's magnetic field.
- In testing the steel bar it attracts pieces of iron hence it is magnetised. This is because the magnetic domains aligns themselves in the same direction as that of the earth's magnetic field.



5. Magnetic induction

Definition 76.

Magnetic induction is the process by which a piece of magnetic material becomes a temporary magnet when it is near or touching a permanent magnet.

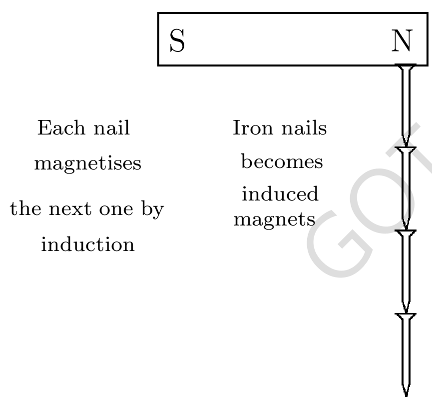
The magnetism attained in this way is called **induced magnetism**.

Definition 77.

Induced magnetism is a type of temporary magnetism attained by a magnetic material when it is near or touching a permanent magnet.

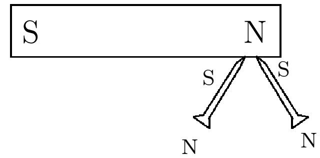
An experiment to demonstrate it

Let a magnet attract a small iron nail. Try to pick a second nail with the free end of the first one, then try to pick a third nail with the free end of the second one and so on as shown below;



When the magnet inducing the magnetism is removed, the nails are no longer magnetised hence they fall apart. This shows that the the nails are magnetised (magnets) so long as the magnet is touching the magnetic material (the nails).

When two nails are placed near each other on one pole, they diverge as shown below, why?



This is because the lower ends of the nails are of the same polarity so they repel each other hence forming the divergent rest position.

Electrical method:

To describe this method clearly, you need to know what is direct current and how is it different from alternating current. If you don't mind, let us skip all electrical methods because you need to know the magnetic effect of an electric current but if mind then move to page 299and read only the electrical method of magnetisation.

3.13.12 Demagnetisation of a magnet (in the form of a steel bar)

There are basically three methods of demagnetising a magnet, the best among the three is the electrical method. They are described below;

1. Electrical method

2. Mechanical treatment

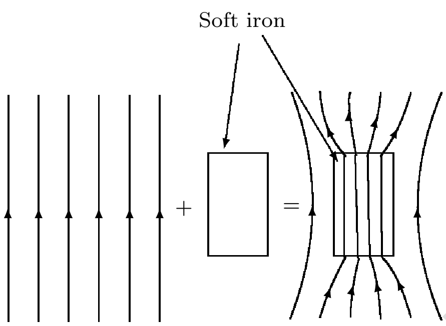
- Hold the magnet pointing in the east-west direction.
- Hit it with a hammer on one end several times (about 20 or 50)
- On testing the magnet, its strength will be less than its previous one: i.e. it can attract few pieces of iron or non-at all.

3. Heating

- Heat the magnet till it is red hot
- Let it cool down while pointing in the east-west direction
- After cooling down, it is found to have lost all its magnetism or it can not attract other pieces of iron.

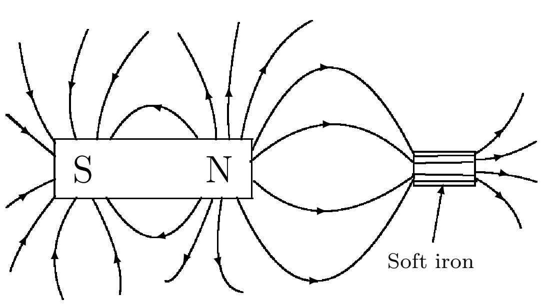
3.13.13 Magnetic shielding

When a magnetic material called **soft iron** is placed in a magnetic field, nearly the whole field passes through soft iron. The diagram below shows how the earth's magnetic field changes when soft iron is placed in it.



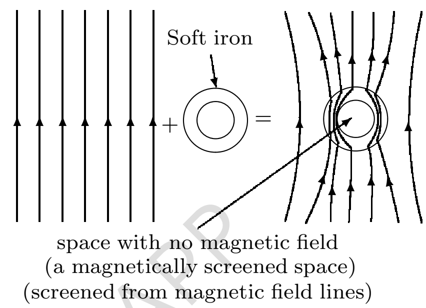
The magnetic field appears to be drawn into the iron and concentrated through it hence soft iron is said to be more permeable to magnetic flux than air or to posses a higher magnetic permeability.

When it is placed near a magnet, the magnetic field line may alter as shown below;



This property of soft iron is used to prevent a magnetic field from reaching certain regions (of interest) i.e. shielding those regions from the magnetic field or screening the magnetic field from that region hence the name **magnetic screening** or **magnetic shielding**.

The diagram below shows how a ring made from soft iron performs the **magnetic screening effect**.



All the flux passes through the ring and no flux crosses the space inside it. The inside of the ring is said to be shielded or screened from the magnetic flux.

Definition 78.

> **Magnetic shielding** is the use of soft iron material to prevent or stop the magnetic field (or flux) from reaching a certain region of interest.

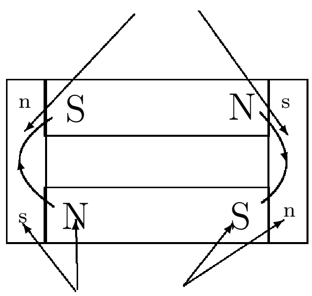
This is applied in watches where the pointers are protected from the influence of the earth's magnetic field.

3.13.14 Magnetic keepers

Bar magnets always become weaker with age due to self-demagnetisation. This is caused by the poles at the ends of the magnet, which tend to reverse the direction of the atomic dipoles inside it.

To prevent this, the bar magnets are stored in pairs, with their opposite poles adjacent and with small pieces of soft iron called **keepers** across their ends, or stored in pairs to ensure that they are in a state of a closed magnetic field lines.
These keepers become strong induced magnets

and the opposite induced poles at their ends neutralise the poles of the bar magnet i.e the magnetic dipoles in the domains of both magnets and keepers, form closed loops with no free end, this consequently cancels the self demagnetisation effect.



3.13.15 Sintering

This is the name given to the conversion of a magnetic powder into solid blocks by application of heat and high pressure. The resulting solid blocks can be magnetised to the strongest permanent magnets known. Various metallic powders; either very soft or very hard magnetic materials are used, the common ones are alloys like magnadur and mumetal: mumetal has a special property that it becomes strongly magnetised when in a magnetic field than either iron or steel because of this mumetal is said to posses a greater **susceptibility**.

Questions

1. (a) What is meant by induced magnetism? Describe a simple experiment to demonstrate it.

   (b) Explain why two steel needles hanging from the N-pole of a magnet are not parallel

2. (a) Define soft and hard magnetic materials

   (b) Give one example of each of the soft and hard magnetic materials

3. (a) Describe an experiment to demonstrate the differences in the magnetic properties of iron and steel and then list them.

   (b)Which of the metals (iron and steel) would you select for use;

   (i)in the core of an electromagnet

   (ii)as a compass needle

   (iii)in magnetic screening

   (iv)as magnetic keepers

   Give reasons for your answer

   (c) why do some watches have a casing made of iron

4. (a) What do the terms magnetic field and neutral point mean.

   (b) Draw diagrams of the magnetic fields due to

   (i)A bar magnet placed horizontally with its axis in the magnetic meridian with its north pole pointing in the geographical south pole

   (ii)A bar of unmagnetised soft iron placed similarly in the earth's magnetic field as that magnet in (i) above.

5. Explain as fully as you can why

   (a)A pivoted compass needle always sets in one particular direction

   (b)Some iron fillings on a piece of paper form a definite pattern after the paper has been tapped or shaken slightly.

   (c)Vertical retort stand rods in laboratories, are found to be magnetised and the polarity of such rods in Russia is different from those found in Uganda.

6. State 3 differences between the magnetic properties of iron and steel

7. You are given three metal rods. One is a magnet, one is an unmagnetised zinc rod and the other is a brass rod[34]. How would you use iron fillings to identify the rods?

[34]brass is an alloy of zinc and iron

8. Use the domain theory of magnetism to explain why?

   (a)an iron nail can be magnetised by induction

   (b)iron readily losses its magnetism and steel does not.

   (c)a bar magnet losses its magnetism if dropped several times

   (d)when a magnet is broken into two pieces, they also behave as magnets

   (e)A magnet can not be magnetised beyond a certain limit

   (f)A magnetic material attached to a magnet attracts pieces of iron.

9. State the difference between the following ;

   (a) Soft and hard magnetic material

   (b) Magnetic and non magnetic material

   (c) A magnetic material and a magnet

   (d) Magnetic flux and magnetic field

   (e) Angle of declination and angle of dip

   (f) Single touch and double touch methods of magnetisation

   (g) Magnetic domain and magnetic dipole

   (h) Magnetic saturation and magnetic shielding

   (i) Magnetic meridian and geographic meridian

   (j) Ferromagnetic material and paramagnetic or diamagnetic material

   (k) the electrical methods of magnetisation and demagnetisation

   (l) Geographic north and magnetic north poles

10. What is meant by the terms

    (a)magnetic sintering

    (b)magnetic axis

    (c)magnetic keepers

    (d)A neutral point as applied to magnetism

11. A magnet attracts

    (a) plastic

    (b) any metal

    (c) iron and steel

    (d) aluminium (Ring the correct option)

12. To test a piece of metal to determine whether it is a magnet, one would check if it

    (a)attracted steel fillings

    (b)repelled a metal bar

    (c)repelled a known magnet

    (d)attracted a magnet (Ring the correct option)

13. Why does heating reduce the strength of a magnet ?

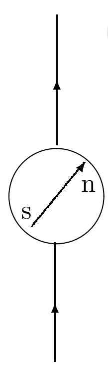
3.14 Magnetic effect of an electric current

3.14.1 Introduction

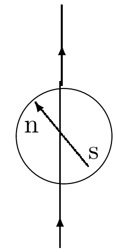
In 1819 Hans Christian Oersted, a professor of physics at Copenhagen discovered accidentally that a wire carrying an electric current was able to deflect a pivoted magnetic needle. It happened during the course of a lecture (lesson) about the simple cell. Having shown that a wire connected to a cell had no effect when placed at right angle to the needle. Immediately to his surprise and that of his students, the needle deflected out of the meridian. On further investigation, Oersted found that the direction of the deflection depended on the direction of the current and also on whether the wire was above or below the compass needle.

Oersted's experiment or discovery.

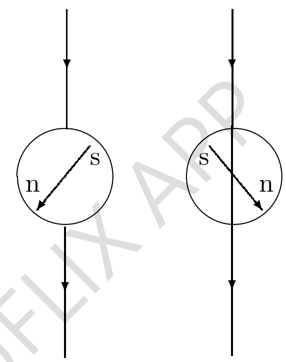
When a compass needle is placed on top of a current carrying conductor like a wire, the compass needle deflects as shown below.



When the wire carrying the current is under the magnetic compass, it deflected in the opposite direction.



When the current was reversed, the magnetic compass deflected in the opposite direction (change of flow of charge) as shown below



All in all this shows that every electric current is accompanied by a magnetic field.

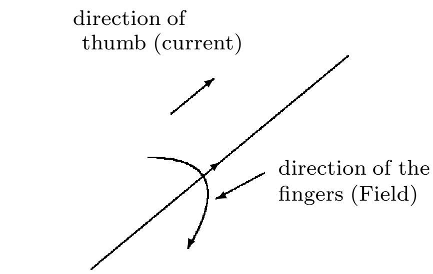
The question now is in which direction is the magnetic field?

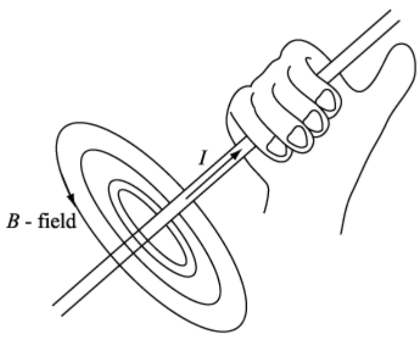
3.14.2 Direction of the magnetic field due to an electric current.

Here various models were proposed to determine this direction and two of them are;

(a) **Maxwell's screw rule. (Proposed by James clerk Maxwell):**
It states that, if a right handed screw is turned so that its tip travels along the direction of the current, then the direction of rotation of the cork screw gives the direction of the magnetic field.

(b) **Right hand grip rule:** It states that for a right hand holding a current carrying conductor or wire, if the thumb points in the direction of the electric current then the other fingers point in the direction of the magnetic field.

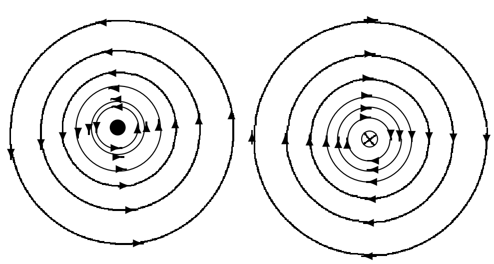




N.B. You must choose one, which is easier for you our interest is; can you find direction of the magnetic field given the direction of the electric current?.

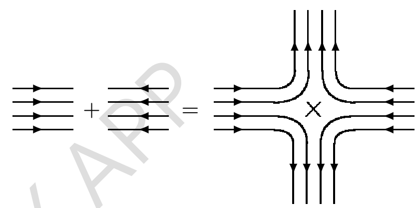
Magnetic field pattern of an electric current.

Let $\odot$ mean current flowing out of the paper (i.e. you are facing the arrow moving out of the paper). And $\otimes$mean current flowing into the paper (i.e. you are seeing the tail of the arrow into the paper). Using right-hand grip rule, the magnetic fields due to the electric current are shown below;

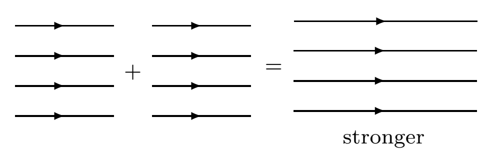


Here we would like to determine what happens when two wires carrying electric currents are close to each other! Do they repel or attract each other? If so when and why?
To answer this we should know these properties of magnetic field lines;

1. *They repel each other side ways or laterally* i.e. they should never cross each other; When two magnetic field lines are to meet each other head on, they bend to a direction perpendicular to their initial direction and a neutral point is formed.

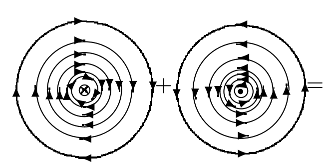


2. *They are ever in a state of tension or attract each other longitudinally* i.e. they should never be drawn with kinks; If they are to meet each other moving in the same direction, they just continue as attraction is experienced.

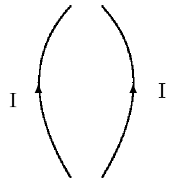


Attraction and repulsion of like and unlike currents

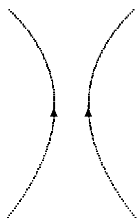
(a) **Unlike currents;** they flow in opposite direction



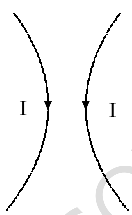
Since magnetic field lines repel each other side ways, these currents experience the forces (F) away from each other i.e. repulsion. Hence *unlike currents repel each other*



(b) Like currents (flow in the same directions)



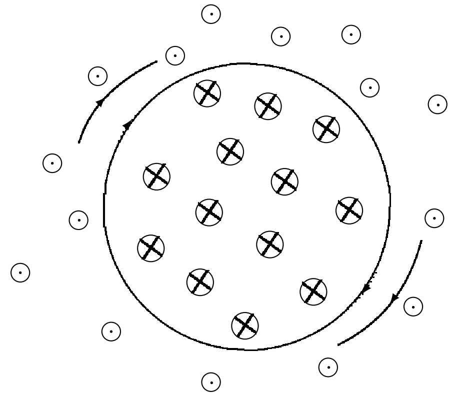
Since magnetic field lines attract each other longitudinally. These currents attract each other; Similarly.



these two diagrams show that like currents attract each other.

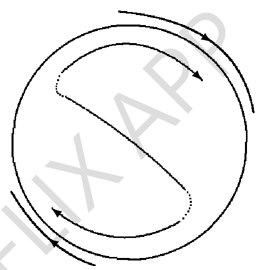
Magnetic field pattern (or magnetic flux) due to a short circular coil.

Study the currents in the coils below; their magnetic field lines are as shown; (a)



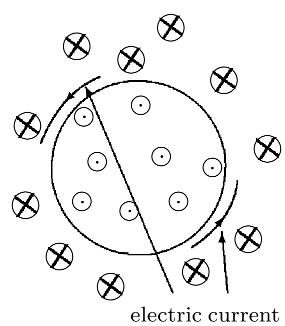
Current is clockwiSe

In the coil it is a South pole

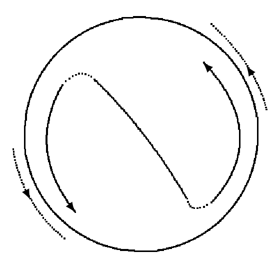


Clockwi**S**e

(b)

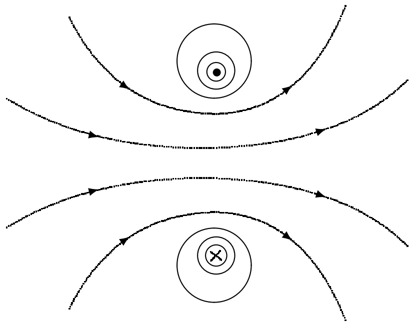


Current is aNticlockwise
In the coil it is a North pole



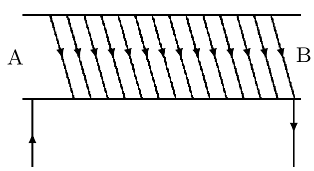
A**N**ticlockwise

If the circular coil is observed on the sides, the pattern is as shown below;



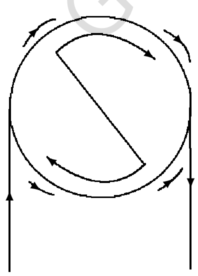
Magnetic flux pattern due to a rectangular or circular solenoid

A solenoid is a cylindrical or rectangular coil. When an electric current passes through it, it behaves like a magnet (known as an Electro-magnet). This is shown below



If one looks through the solenoid at side A, he observes current flowing in the clockwi*S*e direction.

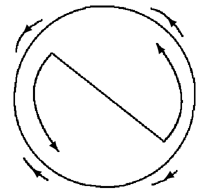
side A



hence south pole

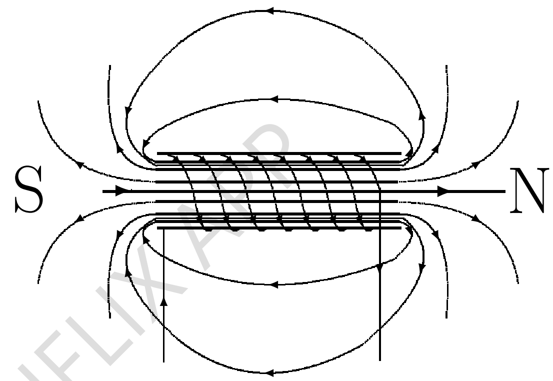
If one looks through the solenoid at side B, he observes current flowing in the A*N*ti-clockwise direction or sense.

side B



hence North pole

Hence the magnetic flux pattern of a solenoid is



The magnetic flux pattern of a solenoid is similar to that of a bar magnet.

Question: Sketch the magnetic flux pattern of a solenoid carrying an electric current

NB. An *Electro-magnet* is a *solenoid* carrying an electric current. Some time its inner part or space if filled with a ferromagnetic material like iron, which easily gains and losses magnetism (i.e. a soft magnetic material). This ferromagnetic material (soft iron) only increases the strength of the electromagnet. When current is switched off, it losses its magnetism; that is why it is sometimes called a temporary magnet, hence *a temporary magnet is an electromagnet*. When a solenoid is used in the generation of electricity (i.e. in electromagnetic induction) it is called an *inductor*.

3.14.3 Applications of the magnetic effect of an electric current

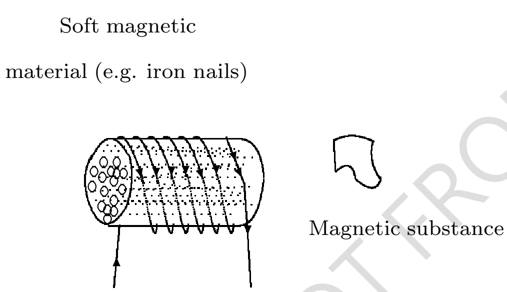
This effect is applied in several electric devices; these devices include;

1. Electromagnets
2. Lifting magnets
3. Electric bell
4. Magnetic relay
5. Reed switch
6. Magnetic separator
7. Telephone receiver

below we describe how each works;

1. **Electromagnets**

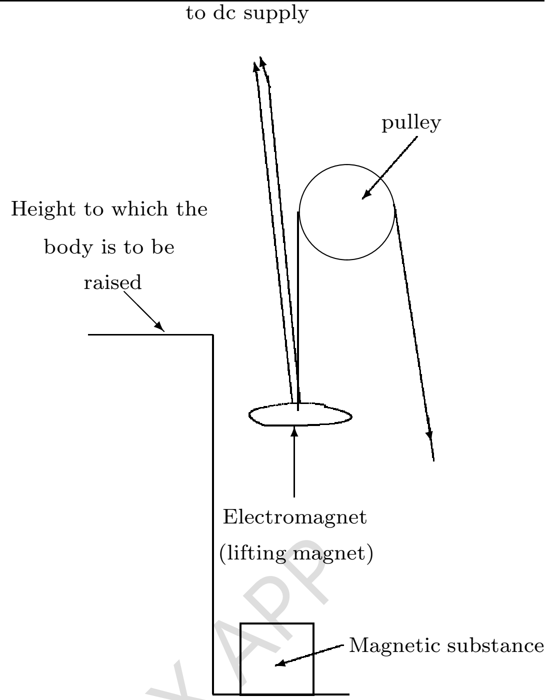
   These are temporary magnets or it is a solenoid carrying an electric current. Some times they contain soft magnetic materials (i.e. materials that are easily magnetised and demagnetised) e.g. soft iron (or just iron). Soft iron may be in the form of nails; the diagram below shows this;



   When current flows in the coils or solenoid, the magnetic substance is attracted, when current is switched off no force acts on the magnetic substance.

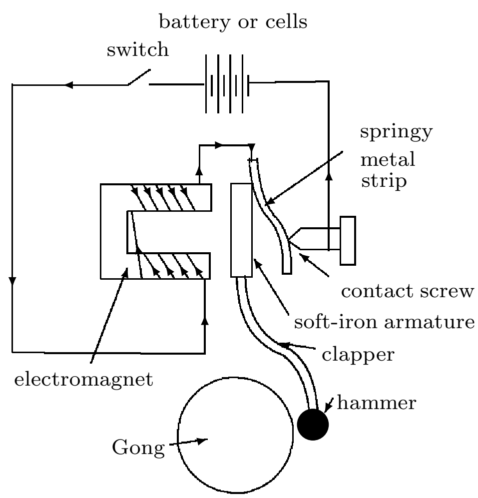
2. **Lifting magnets**

   These are electromagnets with their magnetic fields pointing in the vertical direction i.e. upwards or downwards. They are used for lifting magnetic objects like metallic scraps, and others car body parts. Using a pulley or otherwise, current is switched on, and this creates the temporary magnetism and hence it attracts the other metals where it was placed. On lifting up the coil, it is lifted with the metals to the required height.



3. **Electric bell**

   The electric bell also uses the same principle of an electromagnet, i.e. it only rings when the electromagnet is energized by the current. Below is its diagram;



   NB. Usually the switch of the bell is the push-type switch.

   Action (how it works)

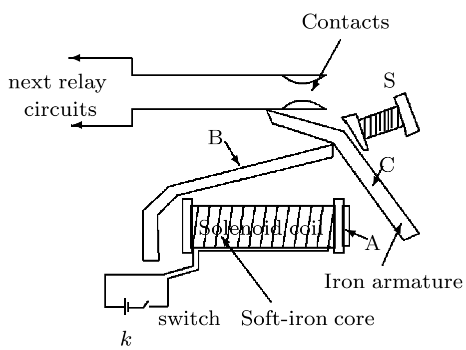
   When the switch is closed or the pull-push switch is pressed, the electric circuit becomes complete and current flows as indicated in the diagram. The electromagnet becomes magnetized and attracts the soft iron armature.

This breaks the circuit and current stops flowing and the electromagnet becomes demagnetized as the hammer hits the gong and sound is produced. The springy metal strip pulls the armature back touching the contact screw completing the circuit and the electromagnet attracts the armature again, this repeats itself and each time the hammer hits the gong thus producing a ringing sound whose frequency depends on the tension in the springy metal strip.
Usually the contact screw it tipped with tungsten or any other metal with a high melting point, so that the spark which passes across the gap when the contact is made and broken does not affect it. This electric bell is used in ding-gong door bells, chimes, school-bells, e.t.c.

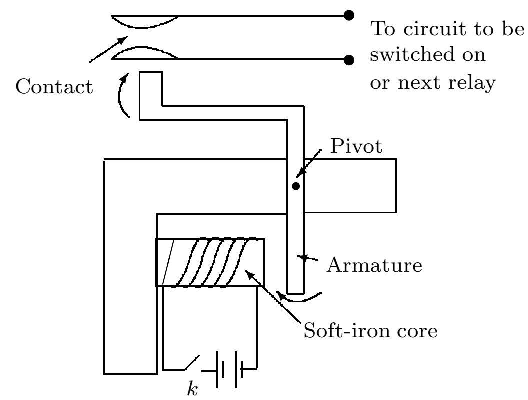
Question Can you sketch the magnetic fields in the armature, when the electromagnet is attracting it?

4. **Magnetic relay**

A Relay is a device, which uses current in one circuit to switch on current in another, especially if the current is large in the second circuit. It is an example of an "**electromagnetic**" switch. Relays are used in telephone handles, call boxes, skin lamps, car-starters, grinding mill motors e.t.c. Below is the principle circuit diagram;



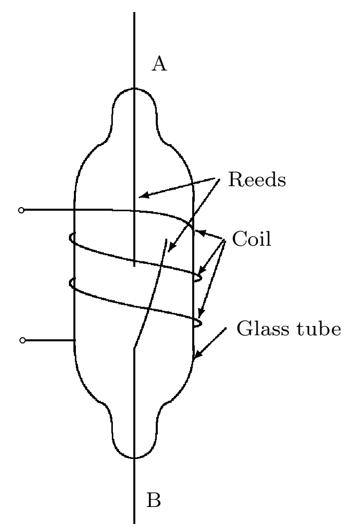
Or



When switch k is closed the soft iron core becomes magnetized and attracts the armature, which with the help of the lever system, completes the other circuit via the contact, by pushing the contacts to touch each other. A complex system of a telephone circuit through a country requires millions of relays for passing messages. The smallest known relay is the reed switch.

5. **Reed switch**

This is a switch, which consists of two small magnetic materials called reeds acting as contact makers, operated by some other nearby circuit. It is shown below;



Action (how it works)
When electric current flows in the coil, the magnetic field produced magnetizes the strips (called "reeds") since they are made from magnetic materials. Their ends become opposite magnetic poles and one reed is attracted

to the other, hence completing the circuit connected to A and B, hence reed switches are used to control current in another circuit using a smaller current in another nearby circuit (like the above magnetic relay). Reed switches can also be operated by permanent magnets. They are used;

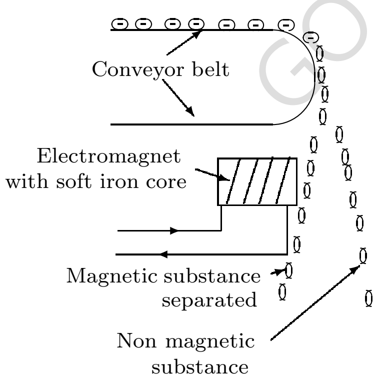
(a) in Burglar alarms

(b) as Door switches or alarms

(c) in any device where a magnetic relay can be used

6. **Magnetic separator**

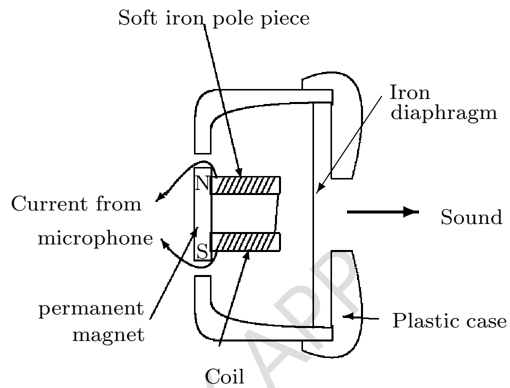
A magnet is commonly used whenever we are to separate magnetic materials from non-magnetic materials. The magnet used may be a permanent magnet like an iron-steel bar or a temporary magnet like an electromagnet with an iron core. The electromagnets are usually the best to use because they produce strong magnetic fields.This type of separation (magnetic separation) is commonly used in the extraction or purification of iron by separating it from other substances, which are not magnetic.When the impure or crushed iron ore is being transferred from one place to another via the conveyor (or belt) during mining, on leaving the conveyor, the crushed ore is dropped as shown below;



A magnetic field from the electromagnet is applied on the falling crushed ore, here the electromagnet attracts the magnetic substance in the ore i.e. iron hence separating it from the rest of the crushed ore which does not contain iron or any magnetic material.

7. **Telephone receiver**

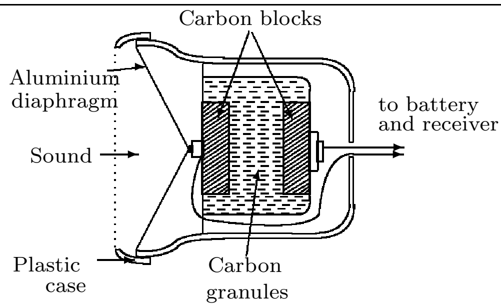
This is the piece of the telephone handle that is placed near the ear. It converts electric energy to sound energy like a loud speaker or it converts speech current along the telephone line to sound waves in the air. Below is its circuit diagram;



When no current flows, the iron diaphragm is under tensional forces (i.e. it is not loose). When the varying current from the microphone (via the amplifier) passes through the coils that acts as an electromagnet wound around the permanent magnet, the tensional forces on the iron diaphragm also change. This is because the resultant magnetic force acting on the diaphragm changes and hence it is pulled towards the poles and then released i.e. it is displaced by a distance proportional to the magnitude of the current and at a frequency equal to that of the changing current. As a result the diaphragm moves in and out of the permanent magnet's port setting the surrounding air into vibration producing sound waves that have the same frequency as that of the sound that entered the microphone at the other end.

Carbon microphone (it does not use the magnetic effect of an electric current)

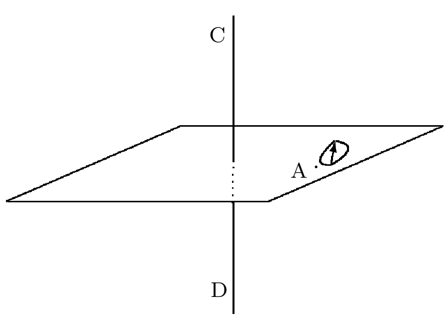
This instrument depends on the principle that, when pressure on carbon is increased, its electrical resistance decreases and hence the current passing through it also increases.When some one speaks facing this carbon microphone shown below;



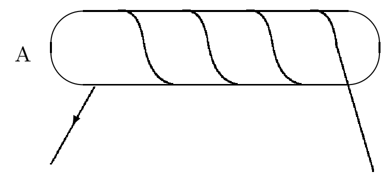
Sound waves produced cause the aluminium diaphragm to move backward and forward. This varies the pressure exerted on the carbon granules between the front carbon block, which is attached to the diaphragm, and the back one, which is fixed. When the pressure increases, the granules are squeezed closer and the electrical resistance decreases. A decrease in pressure has the opposite effect (increase in electrical resistance). If there is a current passing through the microphone (from a battery), it will vary in a similar way as the sound wave variations i.e. the frequency of the new variations in the current is equal to the frequency of the sound waves sent to the aluminium diaphragm. The sound produced by this current via the loud speaker has the same frequency as the original sound, hence they are the same.

Questions

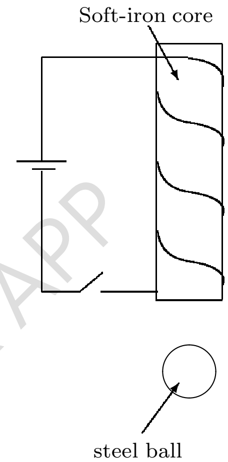
1. In the diagram below, the compass needle at point A deflects as shown. In which direction is the current flowing in the wire.



2. Determine the pole on side A in the figure below when current flows in the direction shown.



3. A small electromagnet used for lifting and then releasing a small steel ball, is made in the laboratory as shown below;



(a) Explain why soft iron is a better material than steel to use as the core

(b) In order to lift a slightly large ball it is necessary to make a stronger electromagnet. State two ways in which an electromagnet could be made more powerful?

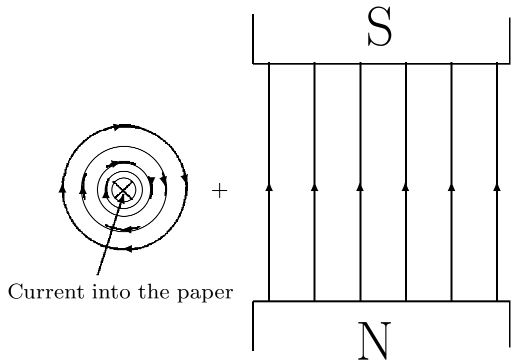
3.14.4 Force on a straight current carrying conductor in a magnetic field

When a current carrying conductor is placed in a magnetic field, it experiences a force (why?)A study about this force shows that its magnitude depends on;

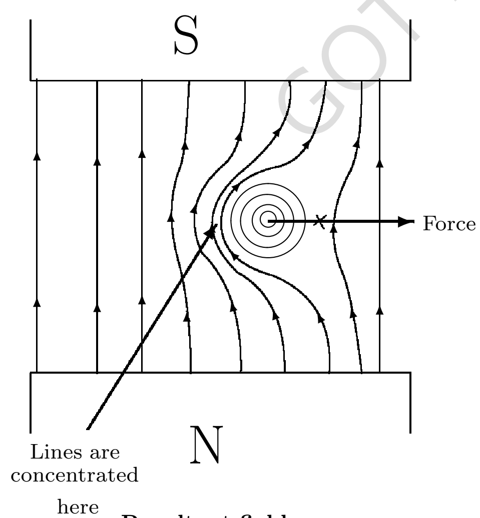
1. the magnitude of the electric current in the conductor
2. the angle between the conductor or current and the magnetic field.
3. the strength of the magnetic field.

4. Length of the conductor exposed to the magnetic field
5. The type of media where they are situated i.e are they in water, air, paraffin, soil or any thing else.

We need to explain this. A useful way is to consider the interaction between the, magnetic field due to the current and the magnetic field in which the conductor is placed. Putting into consideration the two properties of the magnetic field lines. Consider the current below with its magnetic field placed in the uniform magnetic field.



⊗ means the tail of an arrow, i.e electric current flows into this page. When the wire is placed in the uniform magnetic field above, the resultant field is



Resultant field

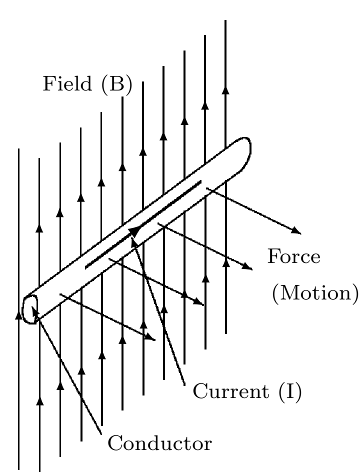
NB. You can take these magnetic field lines to be straight elastic threads that repel each other. The properties of the magnetic field lines used here are;

1. They attract each other longitudinally and
2. Repel each other sideways or laterally.

NB. Various experiments have been performed (like the Faraday's rotating experiment, kicking wire experiment, barrow wheel experiment, e.t.c[35]. They have all showed that the maximum value of the force is obtained when the three quantities (magnetic field, current and the force produced) are perpendicular to each other. And the force is zero, when the current and the magnetic field are parallel even though they are of large magnitudes.

3.14.5 Direction of the force on a current carrying conductor

This direction is given by;**Fleming's left hand rule**, which states that: if the left hand is held with the first three fingers mutually at right angles to each other; with the fore **F**inger in the direction of the **F**ield (B) and the se**C**ond finger in the direction of the **C**urrent (conventional current), then the thu**M**b points in the direction of the force acting on the conductor or its motion. Some times it is known as the **motor rule.**



[35] check barrow wheel experiment

3.14.6 Applications

This property that a current carrying conductor experiences a force when placed in a magnetic field, has several applications, the few of our interest among the many are;

1. Simple electric motor
2. Loud speaker (same as telephone receiver)
3. Force on charged particles in a magnetic field.
4. Moving coil galvanometer

1. Simple electric motor

An electric motor is a device, which converts electric energy into mechanical energy. There are two types of electric motors

(a) Dc motors (Direct current motors) and

(b) Ac motors (Alternating current motors).

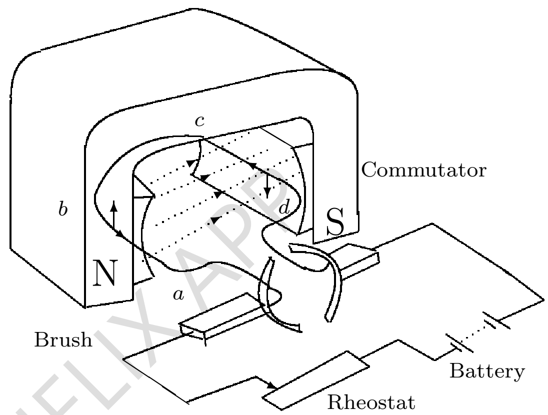
Dc motor

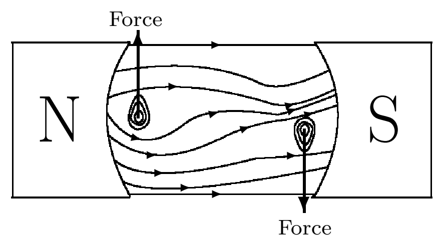
Construction

It consists of a coil or coils of wire mounted on a spindle so that it can rotate between the curved poles of a U-shaped permanent magnet. The two ends of the coil are soldered respectively to the two halves of the copper split ring, called the **commutator**. The brushes are made such that they press lightly against the commutator, usually by means of springs. And when the brushes are connected in a circuit with a battery and a rheostat the coils rotate.

Action : How it works

Suppose the coil is in the horizontal position carrying current in the direction shown below;





On side ab; applying Fleming's left hand rule, the force acting on length **ab** is upwards.
On side bc; since **bc** is parallel to the magnetic field, no force acts on it.
On side cd; applying Fleming's left hand rule the force on the side **cd** acts down wards.
On side da; since **da** is parallel to the magnetic field, no force acts on it.
The two forces on **ab** (upward) and on **cd** (downward) form a couple, which causes the coil to rotate in a clockwise direction until it reaches the vertical position. In this position, the brushes are in the gaps between the commutators, therefore no current flows through the coil hence no force or couple acts on it. But due to its momentum, it does not stop moving i.e. it continues to move. Past this vertical position there is a change in contact to the commutator i.e. current now flows in

the opposite direction in the coil as the sides have also exchanged positions. The previous cycle repeats itself and the motor continues to run or the coil remains rotating.

This motor is run by only direct current (Dc) i.e. current from batteries, dry cells and not from the common Alternating current (Ac) generators and dynamos. The Ac motors will be discussed after analysing what is meant by alternating current and in advanced textbooks.

Practical Dc electric motor

This simple Dc motor described is not a powerful or efficient motor. Its efficiency or power can be improved by;

1. Increasing the number of turns in the rotating coil
2. Winding the coil on a soft iron armature (why?)
3. Using strong magnets or powerful magnets.

Commercial Dc motors; have cylindrical armatures built up from soft-iron discs. Usually the magnetic fields in which the armature runs are produced by an electromagnet. The coils that produce the magnetic field are called **field coils**. The coils that produce a turning couple are wound around the armature and are called **armature coils**. The field coils may be connected in series or in parallel with the armature coils.

Series wound motors are motors in which the armature and field coils are connected in series. They produce a high torque or turning force but revolve at a low speed. Therefore they are used in lifting or moving heavy loads like electric trains, as starter motors in cars, e.t.c.

Shunt wound motors are motors in which the armature and field coils are wound in parallel. They produce a weaker or less strong torque but its advantage is; it runs at a constant higher speed under varying loads. Therefore it is used in driving powerful tool like drills, radio cassettes, grinding mills and others that require a constant speed.

These two types of motors, neither of them can produce a high torque at high speed or revolve at a low speed with a low torque, a combination of the two is required to achieve this. Compound motors are designed for this purpose. ***Compound wound motors*** are motors in which part of the field coil(s) is in series and the other part in parallel with the armature coil. This is a combination of the other two types of motors, this is an attempt to attain the advantage of both shunt wound motors and series wound motors.

Back emf in an electric motor;

Dynamo effect in an electric motor

When a motor is running it also acts like a dynamo and so sets up an *emf* or voltage called **Back emf** E_b. This **back emf** produced, opposes the applied *emf* E, Hence for a motor, the resultant current, I is given by;

$$I = \frac{\text{net emf}}{\text{total resistance}} \quad (3.209)$$

$$I = \frac{E - E_b}{R} \textbf{ not } \frac{E}{R} \quad (3.210)$$

The work done by the *back-emf* is converted into useful work done by the motor.

$$\begin{pmatrix}\text{work done}\\ \text{by the motor}\end{pmatrix} = \text{power} \times \text{time} \quad (3.211)$$

$$W = P \times t \quad (3.212)$$

$$= \frac{E_b^2 t}{R} \quad (3.213)$$

$$\begin{pmatrix}\text{Supplied}\\ \text{emf}\end{pmatrix} = \begin{pmatrix}\text{back emf}\\ \text{p.d. across R}\end{pmatrix} \quad (3.214)$$

$$E = E_b + IR, \quad (3.215)$$

$$IE = IE_b + I^2R \quad (3.216)$$

$$\begin{matrix}\text{Power supplies}\\ \text{to motor}\end{matrix} = \begin{matrix}\text{Power done}\\ \text{by motor}\end{matrix} + \begin{matrix}\text{Power loss as}\\ \text{heat in wires}\end{matrix} \quad (3.217)$$

where R is the total resistance in the circuit hence Electric power supplied is equal to the sum of the power delivered by motor and the heat power dissipated in the circuit.

CAUTION, If the motor armature is brought to rest while current is still flowing the back *emf* becomes zero and all-electrical energy is converted into internal energy which rises the temperature of the windings. This must be avoided because the motor will burns out (the winding burn as the rising temperature approaches the melting point of the material from which they were made). Therefore, whenever the speed of the motor decreases, its windings are more likely to be overheated or to melt.

Example

A motor is connected to a battery of an *emf* 12V in series with a 1Ω resistor and a current of 1.0A flows in the circuit. If the resistance of the motor is 6Ω calculate;

(a)The back *emf* in the circuit

(b)The electric power supplied to the motor

(c)The work done by the motor in 2 seconds

(d)The efficiency of the motor

Solution

(a) If back *emf* in the circuit is E_b. Then net *emf* in the circuit is $= 12 - E_b$, net resistance = R = 6 + 1 = 7Ω

$$\begin{aligned} \text{Net current} &= \frac{\text{net emf}}{\text{net resistance}} \\ I &= \frac{12 - E_b}{7} \\ 1 &= \frac{12 - E_b}{7} \\ 7 &= 12 - E_b \\ E_b &= 5V \end{aligned}$$

Hence back emf is 5V

(b) Electric power supplied to the motor

$$\begin{aligned} P &= IE \\ &= 1 \times 12 \\ &= 12W \end{aligned}$$

Therefore the power supplied to the motor is 12W.

(c) Work done by the motor in 2 seconds

$$\begin{aligned} W &= IE_b \times t \\ &= 1 \times 5 \times 2 \\ &= 10J \end{aligned}$$

Or

$$\begin{aligned} W &= \left(IE - I^2R\right)t \\ &= \left[(1 \times 12) - (1^2 \times 7)\right] \times 2 \\ &= 10J \end{aligned}$$

thence the work done by the motor is 10J.

(d) Efficiency of the motor (E)

$$\begin{aligned} E &= \frac{\text{work out put}}{\text{work input}} \times 100\% \\ &= \frac{\text{Power done by motor}}{\text{power in put}} \times 100\% \\ &= \frac{5W}{12W} \times 100\% \\ &= 41.67\% \end{aligned}$$

Or

$$\begin{aligned} E &= \frac{\text{Back emf}}{\text{Supplied emf}} \times 100\% \\ &= \frac{5}{12} \times 100\% \\ &= 41.67\% \end{aligned}$$

Exercise

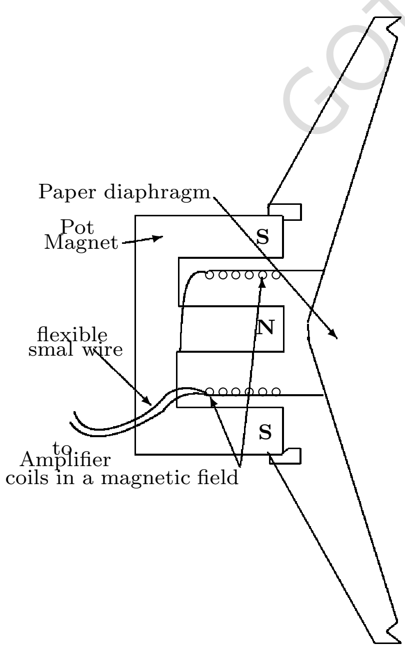
A motor rated at 750W drives a pump which pumps 300 litres of water per minute from a well to a channel 4m above the well. Calculate; the efficiency of the motor and account for the lost energy.

Ac motors will be discussed after differentiating Ac and Dc but their details are not in this book, they are considered in advanced textbooks.

2. Moving coil loud-speaker

Construction
A moving coil loud speaker is a device for converting electrical energy to sound energy. It is commonly used in radio receivers, record players, e.t.c. It works by the principle of a force exerted on a current carrying coil situated in a magnetic field.

Varying electric current which corresponds (in frequency) with the sound to be reproduced, is passed through a short cylindrical coil called **voice coil**. This coil is attached on a paper cone that is free to move in the radial magnetic field set up by a permanent pot-magnet.



This magnet produces a radial magnetic field that cuts through the turns of the coil at right angles, and consequently as the current passing through the coil varies, it will move to and fro in accordance to Fleming's left hand rule. The coil is attached to a cone made of a special material paper which moves with the coil and sets the surrounding air in vibration producing the suitable sound, hence converting a suitably varying current to a suitable sound with a frequency numerically equal to that of the original sound.

Example

Aloud speaker of resistance 4 Ω is connected across a battery of *emf* 3.0V and internal resistance 1Ω, a steady current of 0.15A flows in the circuit. Calculate;

(i) The total resistance in the circuit

(ii) The back emf induced in the loud speaker

(iii) The electric power changed into sound waves by the loud speaker.

(iv) Discuss whether it is scientifically wise to have such a circuit

Solutions

(i) total resistance

$$\begin{aligned} \text{total resistance }, R_t &= R + r \\ &= 4 + 1 \\ R_t &= 5\Omega \end{aligned}$$

(ii) Back emf

$$\begin{aligned} \text{from } Emf &= \text{back emf} + IR \\ E &= E_b + IR \\ \Rightarrow E_b &= E - IR \\ &= 3.0 - (0.15 \times 5) \\ &= 3.0 - 0.75 \\ E_b &= 2.25V \end{aligned}$$

(iii) Power, $P = IE_b$

$$
\begin{aligned}
P &= IE_b \\
P &= 0.15 \times 2.25 \\
&= 0.3375W
\end{aligned}
$$

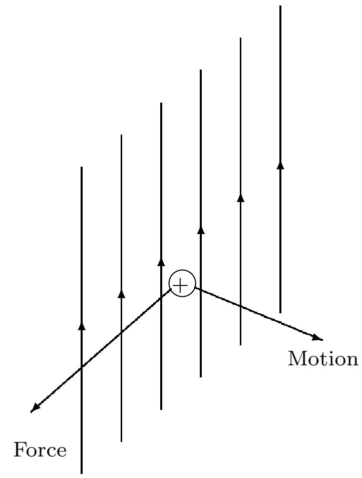
(iv) Efficiency,

$$
\begin{aligned}
E &= \frac{P_o}{P_I} \times 100\%, \\
\text{but } P_i &= EI = 3.0 \times 0.15 \\
\text{hence } E &= 0.45W \\
&= \frac{0.3375}{0.45} \times 100\% \\
E &= 75\%
\end{aligned}
$$

This is a higher efficiency i.e. only 25% of the supplied energy is wasted, less than a half, hence the circuit is acceptable scientifically if there is no other option that can produce a higher efficiency.

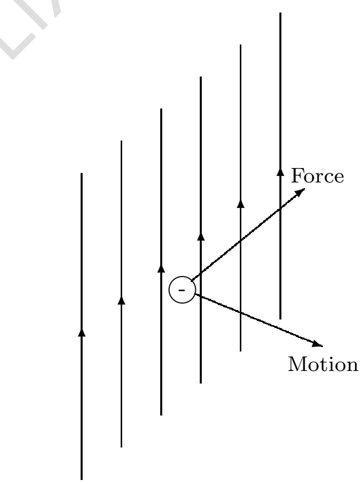
4. Force on a charged particle

Electric current is as a result of moving electrons. Which are negatively charged, and move in a direction opposite to the direction of the electric current i.e the conventional direction of electric current. When a charged particle like an electron moves in a magnetic field, it experiences a magnetic force, we can determine the direction of this force acting on it which is always perpendicular to the direction of its motion and the magnetic field by applying Fleming's left hand rule. A positive charge moving in a given direction is treated like electric current moving in that same direction.



Magnetic field

A negatively charged particle moving in a given direction is treated as electric current flowing in the opposite direction.



Magnetic field

knowledge of what happens when a charged particle moves in a magnetic field will be required in modern physics when determining the sign of charge on a given particle especially in radioactivity.

Moving coil galvanometer

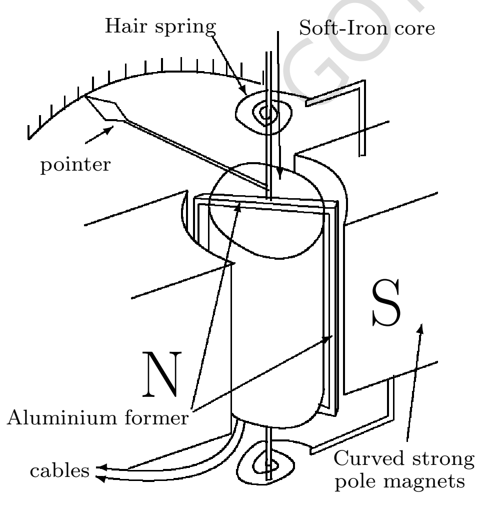
A moving coil galvanometer consists of a coil (rectangular coil) of many turns of enamelled copper wire wound on an aluminum former, which moves around a soft-iron core. This coil carries a pointer pivoted on Jewelled bearings such that it is free to rotate. This coil is placed between the curved pole pieces of a strong magnet.

When a current flows through the coil in the magnetic field, it sets up forces on the sides of the coil. Accordance to Fleming's left hand rule, equal and opposite forces act on the two vertical sides of the coil. These forces together form a deflecting couple which causes the coil to rotate until the deflecting couple or electromagnetic couple is just balanced by the control couple set up by the hairsprings. Since the control couple (or mechanical couple) is proportion to the angle through which the coil has turned i.e obey Hooke's law, then the angle through which the coil has deflected in proportional to the current. Thus the divisions on the scale are equally spaced , it is a linear scale.

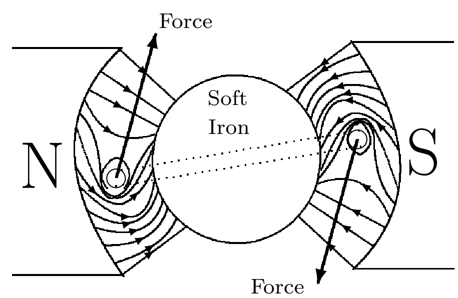
$$\theta \quad \alpha \quad I \tag{3.218}$$
$$I \quad \alpha \quad \theta \tag{3.219}$$
$$I \quad = \quad \kappa\theta \tag{3.220}$$

When the current is switched off the electromagnetic couple becomes zero and the mechanical couple of the hairsprings returns the coil to its equilibrium position with the pointer at the zero mark.

The current can then pass in either direction in the instrument and the coil deflects in the corresponding direction



The function of the soft-iron cylinder is to concentrate the magnetic flux radially in the annular space. Thus for all positions of the coil the magnetic flux density is uniform (constant) and is in the plane of the coil and hence the force on the sides of the coil will be proportional to the current.



The coil itself is wound on a light aluminum former. The function of this former is to damp the movements of the coil and so make it *dead beat*, i.e. the pointer swings out a deflection and comes to rest immediately instead of oscillating to and fro this is the case of eddy current damping.[36]

This galvanometer (moving coil) can not be used to measure alternating current i.e. Ac. This is because the frequency of the Ac is greater than frequency at which the coil can oscillate (natural frequency) hence the galvanometer just records the average value of the alternating current which is zero.
It is essential that the positive terminal on a direct-reading instrument to be connected to the positive terminal of the battery, whether the instrument in center-reading or direct-reading, other wise the instrument may be damaged as the coil tries to rotate in the opposite direction.

A mirror galvanometer is used to measure very small currents, i.e. it is a moving coil galvanometer with a different type of suspension i.e. instead of being pivoted. The coil is suspended by phosphorus-bronze springy strip to which a mirror is attached. A beam of light falls onto the mirror and is reflected to give a spot of light on the screen.
The only major advantage of this instrument is that, when the coil or mirror rotates, the reflected

[36]Eddy currents are discussed on page 318 and Galvanometer damping on page 334

light turns through twice the angle the mirror rotates. Thus depending upon the distance of the lamp and the scale from the mirror, a small deflection of the mirror can give a quite large deflection of the spot of light, this increases the sensitivity of the instrument i.e it can indicate or measure very small currents. The sensitivity of the moving coil galvanometer is the deflection per unit current i.e. cm per ampere or degrees per ampere. It can be increased by;

1. Using a coil of large area,

$$\text{sensitivity } \alpha \text{ A}$$

2. Using a coil with more number of turns,

$$\text{sensitivity } \alpha \text{ N}$$

3. Using a special alloy permanent magnet which can provide a strong magnetic filed,

$$\text{sensitivity } \alpha \text{ magnetic strength}$$

4. Using weak hair springs to give a small control couple,

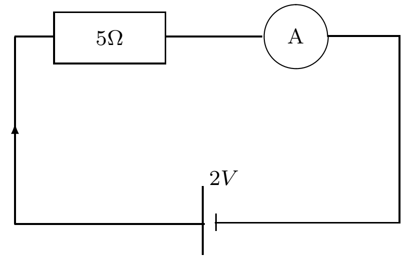
$$\text{sensitivity } \alpha \frac{1}{\tau}$$

Where τ is the torsional constant.

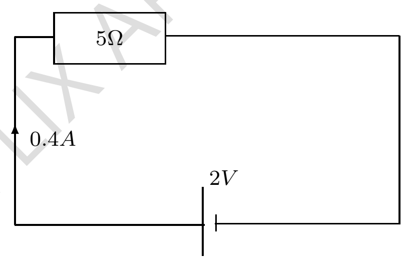
Construction of Ammeters and Voltmeters

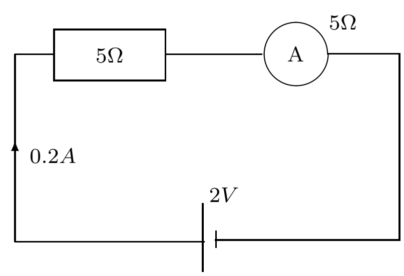
Galvanometer is a name given to an electrical device, which deflects whenever there is a flow of charge or current in it or in the circuit where it is connected. It is used to test the presence of electric current or flow of charge in electric circuits. The moving coil galvanometer described above, can be used as an ammeter or a voltmeter when designed and calibrated for the intended purpose.

Ammeters measure current and must be connected in series, so that the current to be measured passes through it. It always has a small electrical resistance so that it does not alter the current it is measuring, the diagram below shows how it is connected;



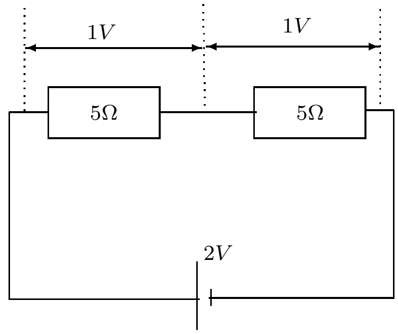
The ammeter should have a very low resistance compared to that in the circuit so as not to affect the current in the circuit. Diagrams below show the effect of the ammeter's resistance to the total current[37].

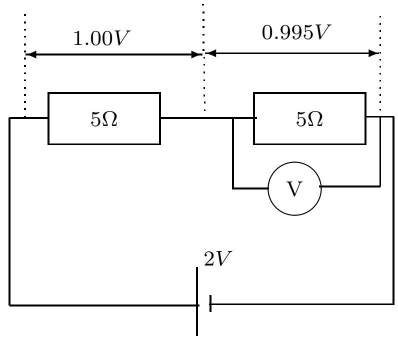




A voltmeter is used to measure the potential difference between two points. It should not affect the p.d. across the two points. It is always connected in parallel with the device or in parallel with whatever is between the points. The above conditions for the voltmeter are possible only when no current or negligible current passes through the voltmeter and this is only possible when it has a very high resistance.

[37]See page 255-259 for details





Note using voltmeter of resistance 1000 ohms the potential difference between the resistor changes slightly (a negligible change).

For practical purposes, if a very large resistance like a voltmeter is connected in parallel with a small resistor, the total resistance of the combination is that of the small resistor.

The rules for using ammeters and voltmeters are

1. An ammeter should have a very small resistance and should be connected in series
2. A voltmeter should have a very large resistance and should be connected in parallel.

Conversion of ammeters

Any moving coil instrument like a moving coil galvanometer can be considered as an ammeter whose resistance is equal to the small resistance of the coil. Small current measuring instruments include;

1. Microammeters; which measure current in Microamperes ($\mu A : 1\mu A = 10^{-6}A$) and
2. Milliammeters which measure current in milliamperes ($mA : 1mA = 10^{-3}A$)

In this section we are concerned with converting Microammeters to Ammeters or Millimeters to Ammeters i.e increasing their scale of reading. There is little difference between a moving coil ammeter and a moving coil voltmeter. The movement is the same for both, they differ only in the way they are connected to a special resistor in them.

To increase the scale reading of any ammeter, one connects a low-valued resistor in parallel with the ammeter. Let us discuss what happens; You are given a millimeter with a resistance of 40Ω which gives a full scale deflection or reading when a current of 5mA is passing through it. How can we use it to measure current in amperes? If you allow 1A to pass through it, it will burn, this problem is solved by connecting a suitable small scale resistor a cross its terminals, i.e. in parallel with it. This resistor or such a resistor is called a **shunt** because it diverts (shunts) most of the current a way from the delicate coil of the ammeter.

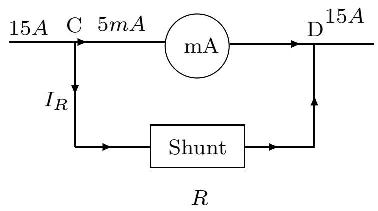
A shunt is a resistor of a very small resistance usually connected to an ammeter to increase its scale reading. If a shunt of 0.004Ω is used, the resistance of the converted ammeter (combined resistance of the shunt and millimeter in parallel) will be 0.0039996. This is a very low resistance which is required for an ammeter not to affect the total current in the circuit. Most of the current passes through the shunt. By using different resistors as shunts, the ammeter or milliammeter may be used to measure current in different ranges, such an ammeter is called a **multimeter**.

Example 1

Find the value of the shunt required to convert the millimeter with a resistance of 20Ω, which reads 0-5mA into an ammeter reading 0-15A?

Solution

First draw a circuit diagram like this below;



C and D are the terminal of the milliammeter and only 5mA which is 0.005A, can pass through it without damaging it. The rest to the current i.e.

$$(15A - 0.005A) = 14.995A$$

must pass through the shunt of resistance R.

$$I_R = 14.995A$$

Since they are in parallel, the potential differences across each are the same, which ever path the current takes. using Ohm's law.

$$\begin{aligned}
\text{p.d. a cross the ammeter} &= \text{p.d. a cross the shunt} \\
0.005 \times 20 &= I_R R \\
0.1 &= 14.995R \\
\Rightarrow R &= 0.00667\Omega
\end{aligned}$$

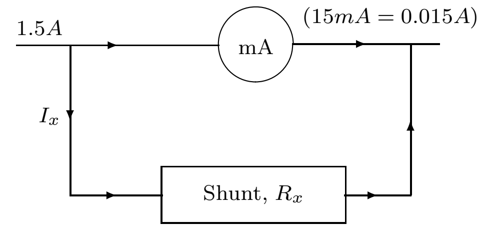
the value of the required shunt is 0.00667Ω (to be connected in parallel with the ammeter).

Example 2

Describe how a millimeter of resistance 5Ω and full scale deflection 15mA can be used for purposes of measuring current to 1.5A?

Solution

Consider the circuit below



The current through the shunt I_x is given by;

$$\begin{aligned}
I_x &= 1.5A - 0.015A \\
&= 1.485A. \\
\text{but p.d across ammeter} &= \text{p.d. across the shunt} \\
0.015 \times 5 &= 1.485 \times R_x \\
R_x &= 0.0505\Omega
\end{aligned}$$

A 0.0505Ω resistor is connected in parallel with the millimeter and this changes its full-scale deflection from 15mA to 1.5A. (i.e. each reading on the previous scale is multiplied by 1000 and called an ampere instead of a milliampere)

In general, an ammeter of resistance R_A and full scale deflection I_A can be changed into an ammeter with a higher full-scale deflection I_s by connecting a low resistance resistor (shunt) of resistance R_s given by

$$R_s = \frac{I_A R_A}{I_s - I_A} \qquad (3.221)$$

Questions

1. What value of a shunt is needed to convert a millimeter whose resistance is 40Ω and reads 0 - 5mA into an ammeter reading 0 - 5.0A.
2. Find the value of shunt necessary to convert the millimeter resistance 8Ω and f.s.d 20mA into an ammeter reading 0 - 30A.
3. A moving coil meter resistance 5Ω gives a full scale deflection when 1mA passes through it. Find the value of the resistor required to convert it to an ammeter with a f.s.d. of 0.1A?
4. A current of 10A utmost is measured by an ammeter which initially would read up to only

2A, the resistance of this ammeter is 20Ω. Find the minimum value of the resistor they could have used?

5. A milliammeter of resistance 80Ω and full-scale deflection 0.5A is connected in parallel with a resistor of resistance 2Ω, calculate the maximum value of current it can now measure? (Ans; 2.5A)

6. An ammeter of resistance 40Ω which reads 0 - 1A is to be converted to one which reads up to 5A, find the value of the resistor required to do this?

Converting an ammeter to a voltmeter

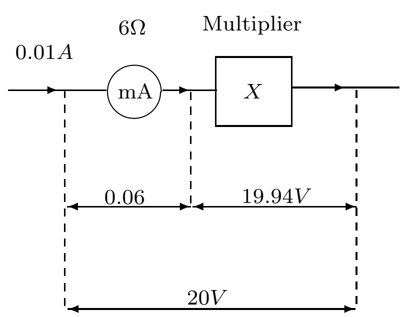
The ammeter or millimeter can also be converted into a voltmeter by connecting a high resistance resistor called a **multiplier** in the series with it. Here you must ensure that at maximum deflection the current through the ammeter does not exceed its full-scale deflection and the maximum p.d it can measure must be the total p.d. across the combination at maximum deflection. The example below illustrates;

Example

Find the value of X in Ω of the resistor which must be connected in series to convert a millimeter of resistance 6Ω reading 0-10mA into a voltmeter reading $0 - 20V$.

Solution

Consider the circuit below



The p.d. across the millimeter at f.s.d is

$$\begin{aligned} V &= IR \\ &= 10mA \times 6\Omega \\ &= 0.01 \times 6\Omega \\ &= 0.06V \end{aligned}$$

The rest of the p.d. (V_m) at f.s.d. must be the P.d across the multiplier (required resistor)

$$\begin{aligned} V_m &= 20 - 0.06 \\ &= 19.94V \end{aligned}$$

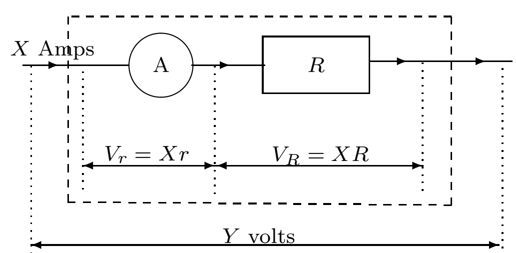
since they are in series current is the same. The value of the multiplier is obtained using Ohm's law; from $V = IR$,

$$\begin{aligned} R_m &= \frac{V}{R_m} \\ &= \frac{19.94}{0.01} \\ &= 1994\Omega \\ R_m &= 1994\Omega \end{aligned}$$

For this case note that the readings of the milliammeter are multiplied by 2 and called volts. That is why the resistor required to do this is called a multiplier.

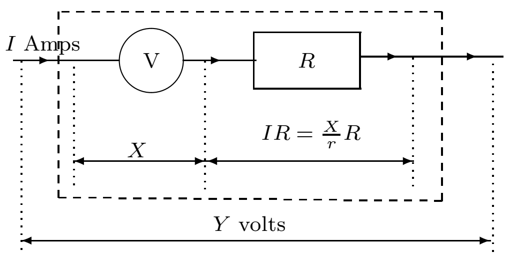
General use of multipliers

If an ammeter has a resistance of r Ω and a full scale deflection of X amperes, then the value R of the required multiplier to convert it to a voltmeter with a f.s.d. of Y volts is given by;



$$\begin{aligned} Y &= Xr + XR \\ \Rightarrow R &= \frac{Y - Xr}{X} \end{aligned}$$

For this voltmeter, we are to increase its f.s.d. from X Volts to Y volts , then the required multiplier R is given by;



$$\begin{aligned} Y &= X + IR \\ &= X + \frac{X}{r}R \\ R &= (Y - X)\frac{r}{X} \end{aligned}$$

Exercise

1. Find the value of the resistor required to change an ammeter of resistance 5Ω and f.s.d 5A to a voltmeter, which can measure up to 75V.
2. An ammeter of resistance 2Ω and f.s.d 0.5A is used to measure voltage up to 10V. find the value of resistor used to do this?
3. A milliammeter of resistance 15Ω and f.s.d 10A is connected in series with a resistor of resistance 75Ω, find the maximum value of voltage it can measure if used as a voltmeter?
4. An ammeter of resistance 8Ω is connected in series with a 64Ω resistor and used as a voltmeter to measure voltage up to 900V. Find the f.s.d of the ammeter before this connection?
5. An ammeter of f.s.d 0.8A is connected in series with a 600Ω resistor and used to measure maximum voltage of 900V. Calculate the resistance of the ammeter.

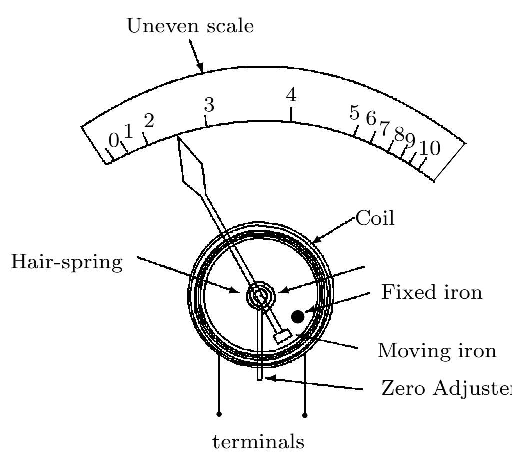
Moving-iron ammeter

Recall that the moving coil ammeter can not be used to measure alternating current it only measures direct current. But a moving iron ammeter can be used to measure alternating current as well as direct current.
The moving iron ammeters are of two types;

1. Repulsion - type ammeter
2. Attraction - type ammeter

Repulsion type moving iron ammeter

It is made up of two iron bars one fixed and the other free to move. The moving bar is attached to a pointer pivoted at the center as shown below,



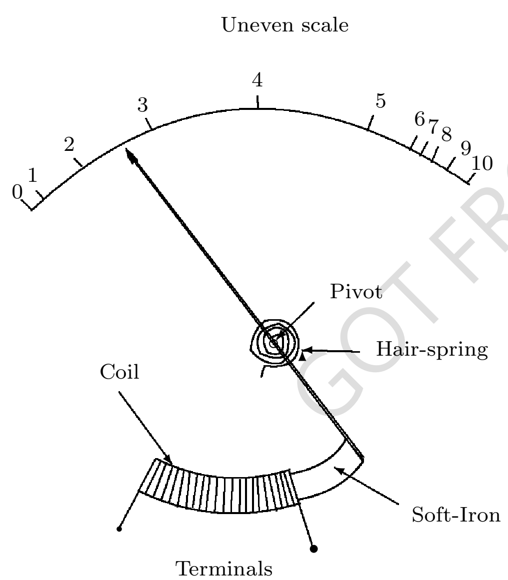
Both iron bars are placed in side a solenoid. When current flows through the solenoid both iron bars get magnetized in the same direction with the North Pole at the same end. Here they repel each other and the moving iron bar deflects the pointer through a certain angle.
The strength of each magnet depends upon the strength of the current and thus the force of repulsion between them is proportional to the square

of current. This makes its scale uneven as shown above. The divisions start very close together then widen in the middle. They are very close together again at the other end of the scale.
Since similar poles are always at the same ends of the bars, then the direction of the current does not matter, it even makes no difference if the current changes direction continuously. This instrument is not suitable for measuring very small currents, but it is very useful for large currents.

Attraction type moving-iron ammeter

It is made up of a movable soft iron metal attached to a pointer but near another fixed one all in the solenoid. When current flows through the solenoid it becomes magnetized and attracts the soft iron, which deflects the pointer to a deflection or angle proportional to the square of the current passing through the solenoid.

$$\text{Deflection } \alpha \ I^2$$



Both types of these moving iron instrument can be used as ammeters and voltmeters with suitable shunts and multipliers. They work in whichever direction the current flows and so can measure both alternating current and direct current. The only disadvantages of moving iron-ammeters are;

1. They have an uneven scale, which is closer at the start, and end.
2. And are not suitable for measuring very small currents i.e. can not make a good microammeter).

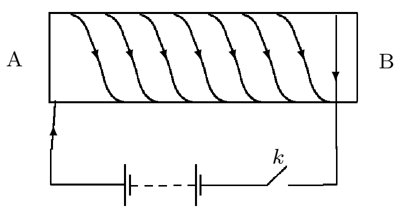
3.15 Magnetism II

Electrical methods

1. Electrical method of magnetising a magnetic material

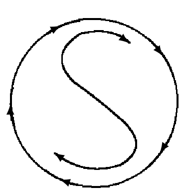
Here in this method of magnetisation we need strictly **direct current**. Ensure the following procedures to magnetise a steel bar;

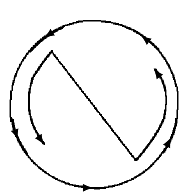
- Wound a wire around a steel bar to be magnetised as shown below;



- Connect the two ends of the wire to a **direct current** source like a cell or battery.
- Close the switch κ for a short period of time
- Testing the steel bar, it attracts pieces of iron hence it has been magnetised with side A a south pole and side B a North Pole.

The side of the steel bar where when one is facing it, the current flows in an a**N**ti-clockwise direction, becomes a **N**orth pole and the other **S**outh pole (clockwi**S**e).





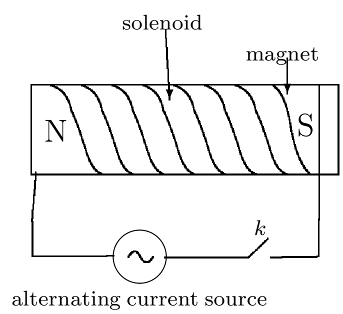
Explanation

When a direct electric current passes through a current carrying coil or solenoid, it produces a magnetic field. This magnetic field due to the solenoid (or coil) forces the domains of the steel bar to face in the same direction as that of the field hence magnetising the steel bar. The magnetising field is that due to the current carrying coil or solenoid.

2. Electrical method of demagnetising a magnetic material

In this method the following steps are taken

- Place the bar magnet in a solenoid.
- Connect the ends of the solenoid to an **alternating current** source as shown below and close the switch κ for a very short period of time.



- Testing the magnet, it can not attract other pieces of iron hence it has been demagnetised.

The switch should be connected for a very short period of time because this current produces a lot of heat in the solenoid and if it lasts longer in it, its temperature may approach its melting point and the whole circuit blows: this should be avoided.

In this electrical method the magnet totally losses all its magnetism, that is why it is the best method.

Exercise

1. Describe briefly how electricity may be used to;

    (a)make a temporary magnet

    (b)make a permanent magnet

    (c)demagnetize a magnet

3.16 Heating effect of an electric current.

3.16.1 Introduction

Using the kinetic theory: how does electric current produce heat?

Electric current is the flow of electrons or charge. When they pass through a material it means that the electrons collide with the atoms or molecules of that substance. These molecules or atoms are ever vibrating and their vibrations are increased by the electrons collision, since temperature is the average kinetic energy of the molecules of a substance, the temperature of the substance increases. Therefore the flow of current through a substance increases its temperature. A measure of heat energy in a substance.

Generally

Electric current produces heat in a clear, fast and convenient way. All heaters e.g electric fire i.e hot plate or electric coil, kettles, iron boxes, ovens, immersion heaters and others, contain a nichrome heating wire or element. Nichrome is an alloy of nickel, chromium and iron. An alloy is a mixture of two or more metals.

Advantages of nichrome as a heating element

(a) It has a low electric resistance.

(b) It has a high melting point.

(c) It does not react with air.

(d) It does not wear-out easily.

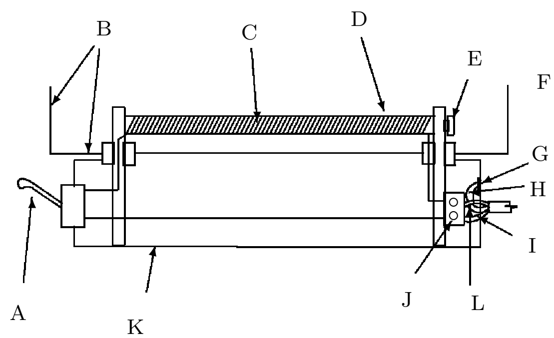
The heating element in an ordinary radiant electric fire is a length of resistance wire whose temperature rises to 900°C when current is passed through it. The wire is supported on a fire-clay rod or bar coiled inside a fused silica tube. The alloy (nichrome) from which the wire is made, is a mixture of nickel and chromium which resists oxidation in air when red-hot.

3.16.2 Applications of the heating effect of an electric current.

This heating effect of an electric current has been applied in various electric devices, these are; electric kettle, filament lamp, fluorescent tubes, fuses, laundry iron, toaster, electric blanket, etc.

1. **Electric Fire (Cooker or hot plate).**

   A nichrome wire is wound round a groove similar to a screw thread on a rod of fire-clay or ceramic (the rod must be a good insulator). Some times the wire is cooled inside the silica tube, which is a good insulator. The ends of the wire are connected to metal screws, which are fixed to metal supports. The supports are insulated from the chassis of the electric fire other wise any one touching the chassis wound receive an electric shock. The hot element transmits or losses heat mainly by radiation and a little by convection through the air. The curved polished metal behind the element is a good reflector of radiant heat.



A-switch

B-curved connector

C-Nichrome element

D-Fire clay or ceramic tube

E-screw

F-support

G-chassis connection

H-green/yellow wire (earth wire)

I-brown (live wire)

J-terminal block

K-chassis

L-blue (neutral wire)

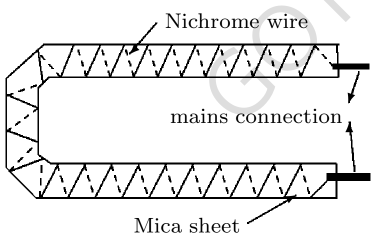
2. **Electric kettle**

This is used to heat or raise temperature of water or other liquids. The heating element or wire is well insulated and enclosed in a metal cube or sheath made from mica or asbestos, which protects it from the water.

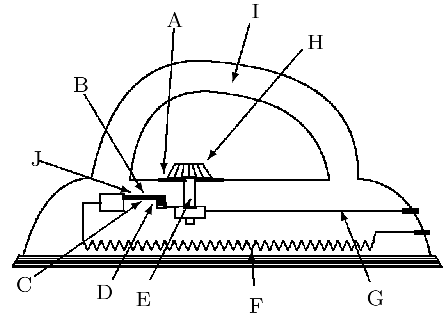
It is important not to let the element come into direct contact with the water in the kettle, other wise the water will become live and hence very dangerous (can shock whoever touches it). The element and its cover are near the bottom of the kettle because hot water rises and the cold water is always at the bottom because it is denser. It should not touch the metal-case of the kettle.

3. **Electric iron;**

The element is a flat nichrome strip wound on mica. The strip has a larger surface than ordinary wire and it heats the bottom of the iron uniformly. The mica with the nichrome around it, is between two thin sheets of mica, which insulate the element from the metal of the electric iron.



Asbestos above the element reduces the heat losses ensuring that most of the heat warms the bottom of the iron. The handle is made of wood or a poor conductor of heat, the handle therefore remains cool. A bimetal thermostat is installed to control the temperature of the electric iron so that it can be used on different materials, since various materials like silk, nylon, cotton, and others require different temperatures when ironing them..



A - Temperature scale

B - Bimetal strip

C - Brass

D - Contact points

E - Screw

F - heating element

G - Flexible wire (conductor)

H - Temperature setting knob

I - Handle

J - Iron

Filament lamp;

The first electric lamp made in 1879 had a thin wire (filament) of the carbon in a glass bulb with no air inside it. Some of the hot carbon at about 1300^{o}C, formed vapour, which condensed on the cold glass of the lamp making it black.

At the beginning of the 20th century, it was found that a tungsten filament wound give a much brighter light. Tungsten is a metal with high melting point of 3400^{o}C and therefore can burn at a temperature of 2000^{o}C , before the glass blackens as compared with 1300^{o}C for carbon.

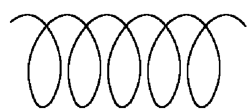
Earlier filament lamps were vacuum filled, but most lamps today are gas filled i.e. a mixture of nitrogen and 95% argon at low pressure. This gas prevents;

(a) Vapourisation of metal (tungsten) from the filament.

(b) Enables the lamp to run at a much higher temperature.

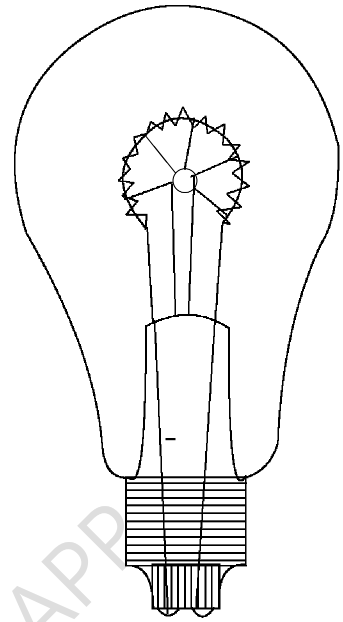
(c)Prevent blackening of the glass as the tungsten vapour now does not condense on the cooler sides of the bulb as was the case of vacuum filled lamps.

Very efficient lamps contain krypton at 3mmHg pressure. Only 10% of electric energy is changed to light. The filament can be either a straight wire, a coil which is hotter or a coiled coil which is hotter still.



coil

The disadvantage of gas filling is that a convection of gas molecules is set up in the bulb, this cools the filament. But by constructing the filament in the form of a coiled coil it occupies a smaller space and therefore the rate of loss of heat by convection is greatly reduced.
Improvements go on all the time. For instance tungsten tends to evaporate and condense on the inside of the bulbs there by darkening it. In projector lamps introducing a little iodine, which forms tungsten-iodide from the tungsten vapour eliminates this darkening, and this (vapour of tungsten Iodide - WI) remains as vapour when the lamp is in use.

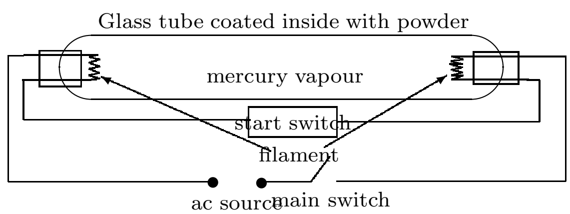


4. **Discharge Lamps**

If some gas at low pressure in a tube is fitted with metal electrodes at each end, the gas glows with a characteristics colour when a high voltage is applied to the electrodes. The electric field set up inside the tube causes ions present in the gas to move with high speed (negative ones to the positive electrode and positive ones to the negative electrode).
As a result of collision between ions and the gas molecules, light is emitted of a colour which depends on the nature of the gas. e.g. neon lamp gives the familiar bright orange-red light seen in advertising signs. Successive production of these coloured gas tubes turned the manufacturers intention to developing discharge tubes for general illumination. Their research is seen in orange sodium vapour lamps and blue-green mercury vapour lamps used for street lighting.
Although these emit coloured light, they give five times more illuminous energy per watt than the best filament lamps and they are cheaper to run.

5. **Fluorescent tube**

A fluorescent tube contains mercury vapour.



When the current is switched on, it passes through the starter switch and makes the two filaments hot; the starter switch then breaks the circuit (this creates a ***back-emf*** [38]). The electric current now pass through the mercury vapour in the tube and produce ultraviolet radiation.[39] The ultraviolet radiation strikes the fluorescent powder on the inside walls of the tube; which changes the radiation to visible light radiation. Depending on the type of powder coated inside the tube, they may give out either white or tinted light. Some of these mixtures of powder contain beryllium compounds, which are highly poisonous, if they enter a cut on the skin. Great care should be taken when handling broken tubes.

Owing to the fact that fluorescent tubes require a special starting equipment, fluorescent tubes were initially more expensive than filament lamps. On the other hand, they are three times more efficient (i.e. 50% efficient) than filament lamps which waste most of the energy as heat.

6. **Fuses**

A fuse is a device that contains a thin wire that burns and breaks the electric circuit when current exceeds a certain value. The wire is always a thin copper wire covered with tin or lead-tin alloy. Tin and lead have high resistance and low melting points (melting point for lead is 327°C and that for tin is 232°C).

A fuse of 5A melts or breaks the circuit (to open circuit) when the current passing through it, is equal to or greater than 5A. In a house or building a current of 5A melts the fuse in lighting circuits (a circuit that contains all bulbs or lamps) and 13A in power circuits; which contain the cooker and sockets. [40]

Exercise

(a) Use the kinetic theory of matter to explain how heat is generated in a current carrying conductor

(b) Describe briefly how the following instruments operate

(a) Electric fuse
(b) Electric cooker
(c) Fluorescent tube
(d) Neon lamp
(e) Electric iron
(f) Electric kettle
(g) Filament lamp

[38] According to Faraday's law of electromagnetic induction on page 316

[39] For details read about waves in Book I

[40] Read page 342 for more about fuses

3.17 Electrical calorimetry

Electrical methods

Introduction

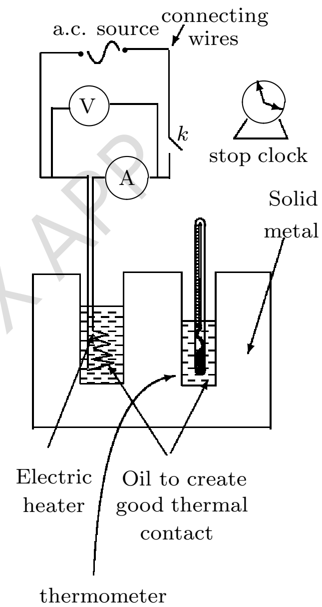
Recall that in the subtopic of heat measurements in the branch of heat, we did not consider the electrical methods. In the previous chapter we explained how the work done by an electrical current when it passes through a simple resistor may be measured in joules and is always transferred to internal energy or heat. So we are in position now to deal with some elementary methods based on electrical heating for measurements of specific heat capacities and specific latent heats.
In this chapter, we assume that you are well equipped with the definition and calculations discussed in the subtopic; heat measurement. The electrical methods to be discussed are;

1. to measure the heat capacity of a metal in the form of a solid-block.
2. to determine the specific heat capacity of a metal in the form of a solid-block.
3. to determine the heat capacity of a given quantity of a liquid
4. to measurement the specific heat capacity of a liquid.
5. to measure the specific heat capacity of a substance using a calibrated calorimeter.
6. to measure the specific latent heat of ice by direct heating.
7. to measure the specific latent heat of vapourisation of steam by an approximate method

Note that in such experiments, it is better to use metals or substances that can easily absorb and conduct away heat energy so as to save time. If insulators are used the experiment lasts for a very long period as you wait for the heat to be transferred from one end to another, this is particular to solids.

An experiment to determine the heat capacity of a solid metallic block

- Drill two holes in the solid and pour in them a small quantity of oil. The oil is to ensure good thermal contact. It is better to use oil with a low specific heat capacity
- Dip in one of the two holes a heater and in the other a thermometer as shown below;



- Close the switch, k, in the circuit the ammeter and the voltmeter must deflect
- After a certain time, when the system has attained a **steady state**[41], read and record;

  the reading of the ammeter $= I$ and

  the reading of the voltmeter $= V$
- Start the stop clock then Read and record the reading of the thermometer as T_0
- After a certain time,t,from when the stop clock was switched on, stop it then read and record the new reading of the thermometer,T_1

neglecting heat losses, the heat energy absorbed by the solid, H_{solid} is equal to the electrical energy

[41] This is a state when all parts of the system are at constant temperature

supplied by the heater, E_{heater}.

$$H_{solid} = H_{heater} \quad (3.222)$$
$$C\Delta T = Pt \quad (3.223)$$
$$\Rightarrow C = \frac{Pt}{\Delta T} \quad (3.224)$$
$$= \frac{Pt}{T_1 - T_0} \quad (3.225)$$
$$\text{or} \quad C = \frac{VIt}{T_1 - T_0} \quad (3.226)$$

where P is the power rating of the heater.
With I in amperes , V in volts, t in seconds, T_1 and T_0 in $°C$ or Kelvin, heat capacity ,C, is obtained in Jk^{-1}.

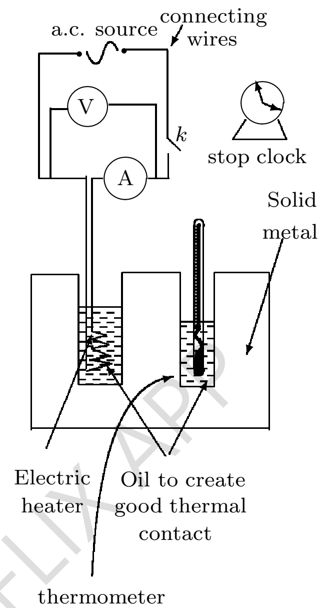
Sources of errors

In this experiment the value of heat capacity obtained may not be the exact value due to the following reasons;

1. Some Heat is absorbed by the oil which provides a good thermal contact and this is not catered for in the calculations
2. The apparatus is hot, it losses heat to the surrounding air
3. Performing this experiment in a windy atmosphere leads to very wrong values of heat capacity, because a lot of heat is lost due to convection and conduction.
4. The apparatus used like the thermometers , heater and others also absorb heat supplied by the heater and this is not catered for in our calculation.
5. The ammeter or voltmeter may be wrongly read or calibrated

An experiment to determine the specific heat capacity of a solid block

- Drill two holes in the solid block provided
- Dip in one, the heater and in the other a thermometer
- Add in the two holes a small quantity of oil to provide good thermal contact
- Connect the heater to an electric source as shown below;



- Close the switch, k
- Measure and record the steady readings of the voltmeter, V, and ammeter, I, when the system has attained a **steady state**.
- Read and record the reading of the thermometer, T_0, and start the stop clock
- After a certain time, t, open the switch and stop the stop clock.
- Take the new reading of the thermometer, T_1 Assuming negligible heat loss, the heat absorbed by the solid, H_{solid} is equal to the electric energy supplied by the heater, H_{heater}

$$H_{solid} = H_{heater} \quad (3.227)$$
$$mc\Delta T = Pt \quad (3.228)$$
$$c = \frac{Pt}{m\Delta T} \quad (3.229)$$
$$= \frac{Pt}{m(T_1 - T_0)} \quad (3.230)$$
$$c = \frac{VIt}{m(T_1 - T_0)} \quad (3.231)$$

Where P is the power rating of the heater in watts (P=IV) and m is the mass of the solid in kg. The specific heat capacity of the solid, c, is obtained using the expression, in J/kgk or $Jkg^{-1}k^{-1}$.

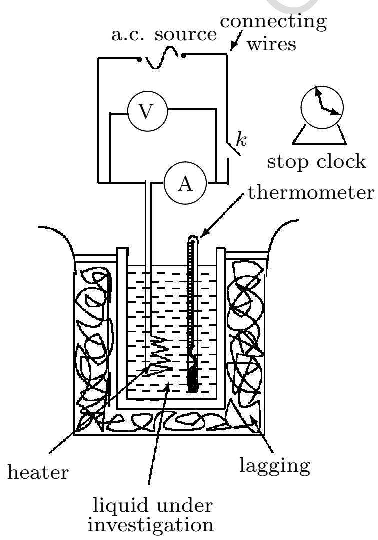
Sources of errors

The value of c will be less or more than the exactly true value due to the following reasons;

- The whole mass of the solid is not heated equally
- There is heat loss to the surrounding air
- Heat is absorbed by the apparatus or equipments like the thermometer, heater, and the oil that was used for good thermal contact
- The presence of wind increases the rate of heat loss to the surrounding air by convection.

An experiment to determine the heat capacity of a given quantity of a liquid

- Pour that liquid in a lagged copper calorimeter or any other properly lagged container
- Dip in that liquid an electric heater of known power rating, P, and a thermometer as shown below;



- Close the switch, k, and start the stop clock
- Wait for some time till when the system has attained a **steady state**.
- Measure the **steady** readings of the ammeter, I, Voltmeter, V and thermometer T_0.
- After a certain time, t, open the switch, k, stop the clock clock and record the reading of the thermometer, T_1

Assuming negligible heat loss, Heat absorbed by the liquid, H_{liquid} is equal to the electric energy supplied by the heater, E_{heater}

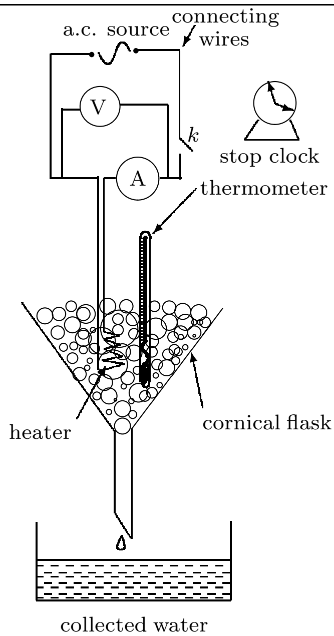
$$\begin{aligned}
\begin{array}{c}\text{Heat absorbed}\\ \text{by the liquid}\end{array} &= \begin{array}{c}\text{Heat supplied}\\ \text{by the heater}\end{array} && (3.232)\\
H_{liquid} &= E_{heater} && (3.233)\\
C\Delta T &= Pt && (3.234)\\
C(T_1 - T_0) &= Pt && (3.235)\\
\Rightarrow C &= \frac{Pt}{T_1 - T_0} && (3.236)\\
Or,\ \ C &= \frac{VIt}{T_1 - T_0} && (3.237)
\end{aligned}$$

the heat capacity C is obtained using the above expression. With V, I, t T_1, and T_0 in SI units C will be obtained in J/kgk or $Jkg^{-1}k^{-1}$

Sources of error in this experiment are the same as those in the previous experiment.

An experiment to determine the specific latent heat of fusion of ice

- get a dry ice block and crush it into small pieces
- Place the small pieces of ice in a conical flask as shown below;



- dip in, a thermometer and a heater of known power rating, P
- Switch on the heater by closing the switch k and watch the reading of the thermometer, it will decrease slowly, till it becomes steady i.e. when it stops decreasing, the temperature is constant.
- When the thermometer reading is steady(i.e. constant) read and record the ammeter reading, I and voltmeter reading, V, start the stop clock and start collecting the dropping water.
- After some time, t, during which temperature was constant, stop the stop clock and measure the mass, m_1, of the water collected in this time.
- Repeat the above step using another similar set of ice blocks without the heater and collect the mass m_2 of the water in the same time, t.

Assuming constant heat loss ; heat gained or absorbed by the ice pieces at constant temperature, H_{ice} is equal to the heat supplied or lost by the heater, E_{heater}.

$$H_{ice} = E_{heater} \tag{3.238}$$
$$(m_1 - m_2)l_f = Pt \tag{3.239}$$
$$ml_f = Pt \tag{3.240}$$
$$\text{where } m = m_1 - m_2 \tag{3.241}$$
$$l_f = \frac{Pt}{m} \tag{3.242}$$
$$= \frac{VIt}{m} \tag{3.243}$$

where P, is the power rating of the heater or $P = IV$. substituting V, I, t, and m in the above expression in SI units, the specific latent heat of fusion (l_f) is obtained in J/kg or Jkg^{-1}

Sources of errors

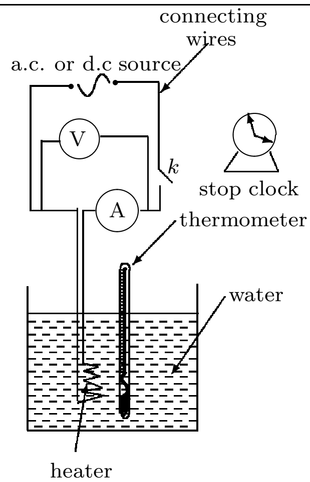
The value of the specific latent heat of fusion obtained above may be either greater or less than the actual value because of the following reasons;

1. It is not easy to collect the exact amount of water that has melted due to the exact heat supplied in the measured time
2. The funnel, heater, and the thermometer are also heated and the heat they absorb is not considered.
3. The thermometer readings may decrease at a very slow rate and one can not tell whether the reading it constant or still rising to start or stop the stop clock.

An experiment to determine the specific latent heat of vapourisation of steam

This is an approximate method

- Pour water in the lagged container and dip in it a heater connected to the electric source as shown below;



- close the switch, k,
- When the water starts boiling Read and record the ammeter reading, I, and Voltmeter reading,V
- Measure the net mass of the lagged container with water , m_0 and start the stop clock for a few minutes(about 10 minutes) and then open the switch.
- Record the time, t, for which the stop clock was on.
- Measure the mass m_1 of the system again record the difference ,$m = (m_1 - m_0)$

Assuming no heat loss, heat used to vapourise the water, E_{vapour} is equal to the electric energy supplied, $E_{supplied}$

$$E_{vapour} = E_{supplied} \quad (3.244)$$

$$ml_v = Pt \quad (3.245)$$

$$ml_v = VIt \quad (3.246)$$

$$l_v = \frac{VIt}{m} \quad (3.247)$$

$$\text{or} \quad l_v = \frac{VIt}{m_1 - m_0} \quad (3.248)$$

where m is the mass in kg of water that evaporated.
I and V are the ammeter and Voltmeter readings in Volts and Amperes respectively. t is time in seconds taken for m kg of water to evaporate.

Note that this method does not always give a reasonable value of the specific latent heat of vapourisation because of reasons similar to those given in the previous experiment.

Exercise

1. The heating coil in the apparatus for determining specific latent heat of vapourisation was supplied with 2.1 Amperes at 10.5V. The current was passed for 6 minutes 32 seconds. The original mass of the beaker and water was 71.8g and the final mass 68.2g.

   (a)Calculate the specific latent heat of vapourisation of water

   (b)The experiment was repeated by passing a current of 1.8A for 39.2 seconds and the voltmeter reading was 9.45V. 2.7g of the steam condensed. Determine the new value of the specific latent heat of vapourisation?

   (c)If in(a) and (b) above; there was a constant heat loss, determine the rate of heat loss and the true specific latent heat of vapourisation? (Hint: this involves solving two simultaneous equations)

3.18 Measurement of electrical resistance

3.18.1 Introduction

Electrical resistance is the opposition to the flow of electric current. From Ohm's law, it was found that for some materials this resistance is constant (Ohmic conductors) and for others it is not constant (Non- Ohmic conductors). Ohm's law only assisted us in knowing the relationship between the resistance of a conductor(R), current through the conductor(I) and the potential difference across the conductor(V) i.e.

$$V = IR \quad (3.249)$$

$$\Longleftrightarrow R = \frac{V}{I} \quad (3.250)$$

$$\text{Or } R = \frac{\Delta V}{\Delta I} \quad (3.251)$$

But now which factors affect the resistance of a conductor?

3.18.2 Factors which affect the resistance of a conductor

There are four major factors that affect the resistance of a conductor;

(a) the cross sectional area of the conductor

(b) the length of the conductor

(c) the temperature of the conductor

(d) the type of material from which the conductor was made.

but how does each of these factors affect the resistance of a conductor? A number of experiments were performed so as to know how each of the above factors affect the resistance of the conductor, below i give the result from those experiments;

1. The resistance of a conductor is directly proportional to its length;

$$\text{resistance} \quad \alpha \quad \text{length of the conductor}$$
$$R \quad \alpha \quad l \quad (3.252)$$

2. Resistance of the conductor is inversely proportional to the cross sectional area of the conductor;

$$\text{resistance} \quad \alpha \quad \frac{1}{\text{cross sectional area}}$$
$$R \quad \alpha \quad \frac{1}{A} \quad (3.253)$$

3. Resistance of the conductor depends on the nature of the material from which it was made. When all the above properties are considered, we come up with one formulae relating all of them, this is;

$$\text{resistance} \quad \alpha \quad \frac{\text{conductor's length}}{\text{cross sectional area}}$$
$$= \rho \times \frac{\text{conductor's length}}{\text{cross sectional area}}$$
$$R = \frac{\rho l}{A} \quad (3.254)$$

the constant of proportionality ρ is the constant which depends on the nature of the material that constitute the conductor, and it is called the **resistivity** of the material of the conductor.

4. Temperature is not easy to discuss here as it depends on the environment where the conductor is.

There are three major methods of measuring electrical resistance, these are;

1. Ammeter-voltmeter method
2. Substitution method (i.e using the resistance box) and
3. Wheatstone or metre-bridge method

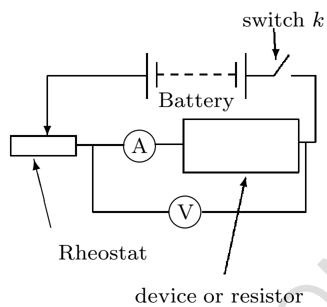
let us discuss each separately;

1. **Ammeter-voltmeter method**

We know that an ammeter can measure electric current passing though a given device by connecting it in series with that device, in which case the same current passes through the device and the ammeter. The reading of the ammeter is the current passing through the device or conductor. More so, we know that the potential difference across a given conductor is measured using a voltmeter by connecting it across the conductor i.e. in parallel, here the reading of the voltmeter is the potential difference across it or across the conductor

In this method, the following steps are followed;

- Connects the circuit as shown below;



- close the switch k
- Read and record the readings of the ammeter (I) and voltmeter (V)
- Substitute the above values in the expression below to obtain the resistance, R, of the conductor or device;

$$\text{R} = \frac{\text{voltage across the device}}{\text{current through the device}}$$

$$R = \frac{V}{I} \tag{3.255}$$

$$\tag{3.256}$$

Some times various values of I and V are obtained for the same resistor by altering V using more batteries or voltage divider, a graph of V against I plotted and the slope obtained as the value of the resistance of the resistor used.

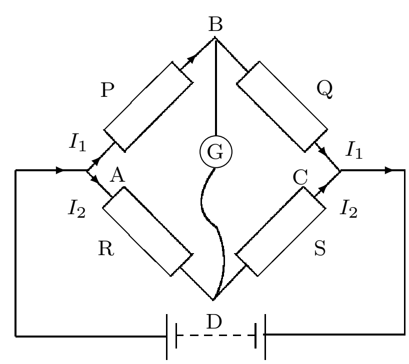
2. **Substitution method: resistance box**

This is best explained when you are looking at the resistance box with the help of you teacher. But in general, the resistance box consists of very many resistors of various values and each is at the slots you see on it. The total resistance of the resistance box is equal to the sum of the resistances of each i.e. of the open slots.

3. **Wheatstone bridge or metre Bridge**

Wheatstone bridge is a special type of circuit which was first suggested by Charles Wheatstone, a Professor of physics in King's College London in 1843.

It consists of three known resistors P, Q, S and a fourth unknown resistor of resistance R. A source of emf like a battery is connected between points A and C and a galvanometer between points B and D as shown below;



Two of the known resistors say P and Q are fixed and the third resistor S varied until the galvanometer shows zero deflection (null deflection or zero current)i.e no current flows in BD.

when this happens the bridge is said to be balanced

This condition occurs when the points B and D are at the same potential i.e. current through resistor P and through resistor Q is the same, lets call itI_1.

Similarly the current passing through resistor R and resistor S is the same, let us call it I_2. From these two conditions about current, we have;

$$\begin{aligned}
\text{p.d across } AB &= \text{p.d. across AD} \\
I_1P &= I_2R && (3.257) \\
\Longrightarrow \frac{I_1}{I_2} &= \frac{R}{P} && (3.258) \\
\text{p.d. across } BC &= \text{p.d. across } DC \\
I_1Q &= I_2S && (3.259) \\
\Longrightarrow \frac{I_1}{I_2} &= \frac{S}{Q} && (3.260) \\
\text{equating the two, } \frac{I_1}{I_2} &, \text{ we have} \\
\frac{R}{P} &= \frac{S}{Q} \text{ or} \\
\frac{R}{S} &= \frac{P}{Q} && (3.261) \\
\Longleftrightarrow R &= \frac{R}{Q} \times S && (3.262) \\
& && (3.263)
\end{aligned}$$

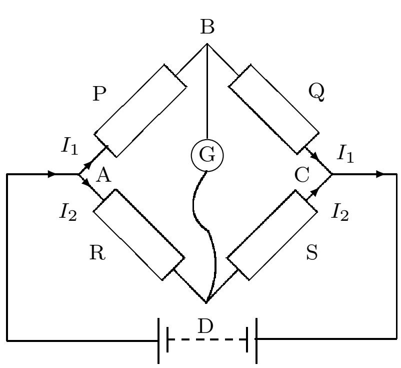
Here the unknown resistance R can be obtained in terms of the known resistances S, Q and P at balance point.

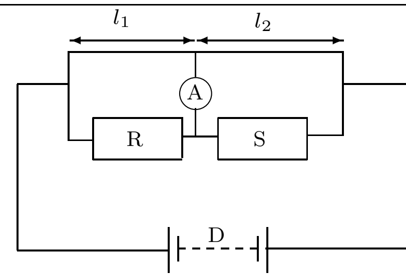
This Wheatstone bridge circuit method is the most accurate method of measuring resistances.

Because of the inconveniences in changing the values of S, this Wheatstone bridge circuit is modified into another one called the meter bridge.

Metre-bridge

The fixed resistors P and Q in the wheat stone bridge were replaced by a uniform resistance wire which could be divided into two parts by a movable contact. Usually a wire 1 metre long is used hence the name metre-bridge.





Since, we only need the ratio of the resistance of two resistors a wire of a given resistance per cm, hence the two resistors P and P are redefined.

Here in the above diagram;

P is equivalent to resistance of the length l_1 of the metre wire.

Q is equivalent to resistance of the length ,l_2, of wire

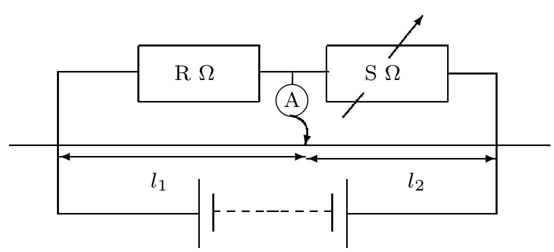
hence the ratio of the two resistors becomes;

$$\frac{P}{Q} = \frac{l_1}{l_2} \qquad (3.264)$$

Compare the diagram below with that above to realise the modification;

Construction

A Meter bridge consists of a straight uniform resistance wire AB 1m long stretched over a wooden box with a scale graduated in millimetres.



The ends of the wire are joined to the ends of the two resistors and across a source of emf as show above. Between the two resistors a galvanometer is connected whose other end is to a jockey which slides freely on the 1m wire till a balance point (null deflection) is obtained.

since at balance point

$$\frac{R}{S} = \frac{P}{Q} \quad , \text{ and} \qquad (3.265)$$

$$\frac{P}{Q} = \frac{l_1}{l_2} \qquad (3.266)$$

$$\text{then } \frac{R}{S} = \frac{l_1}{l_2} \qquad (3.267)$$

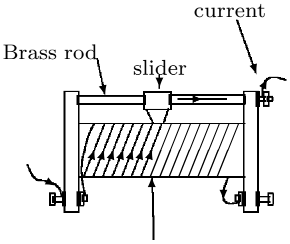
$$\Longrightarrow R = \frac{l_1}{l_2}S \qquad (3.268)$$

S could be any standard resistors of known resistance and it is simple to measure the length l_1 and l_2 compared to P and Q.

A rheostat or variable resistor

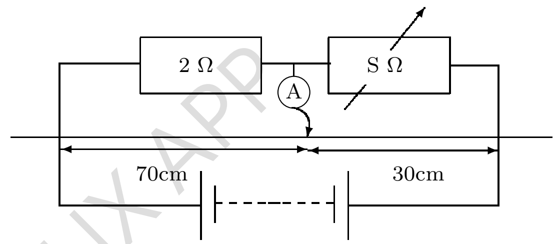
This is a device used to alter gradually the resistance in a circuit. It consists of a wire or a coil of constantan wire (a resistance wire) wound on a slate or a porcelain cylinder and has a sliding contact on a brass bar. Moving the slider alters the length of the wire in a circuit. Since the resistance of a wire is directly proportional to its length (i.e. $R \ \alpha \ l$). Then changing the length of the wire in the circuit changes its resistance and therefore also current changes. Here length of the wire in the circuit changes by moving the slider on the coiled wire on the cylindrical porcelain. The volume controls on television and radio sets are rheostats.

In general the rheostat depends on the fact that the resistance of a wire is directly proportional to the length of the wire in the circuit.



Exercise

1. Describe how the resistance of the conductor can be measured using
   (a)the ammeter-voltmeter method
   (b)the resistance box
   (c)the meter bridge
2. What is meant by a Wheatstone bridge. Derive its balance condition
3. Briefly describe how a rheostat works
4. In the circuit diagram below, the ammeter reads 0A at that balance point. Use it to determine the electrical resistance of resistor S



3.19 Electromagnetism

3.19.1 Introduction

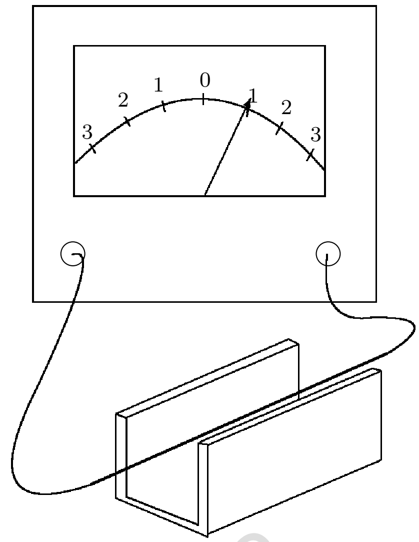
In 1831 Michael Faraday discovered that an electric current is produced when a magnet is moved into or out of a coil of wire.

The current is called **induced current**, the voltage causing it is called an **induced emf** and the process of producing this current or emf is called **electromagnetic induction**. Faraday was trying to reverse the effect that; if a force acts on a current carrying conductor in a magnetic field, then can't the force and the magnetic field produce a current???

Experiment to demonstrate electromagnetic induction

Experiment 1

- Connect a straight copper wire across the ends of a sensitive center-zero galvanometer.
- Hold it between the pole of a powerful magnet.
- Now remove it quickly between the poles.
- Note whether the pointer of the galvanometer deflects (it really moves).
- Move the wire in the opposite direction, does the galvanometer pointer deflect in the opposite direction? (Yes).
- If the wire is moved up and down the pointer oscillates about the point of zero deflection.

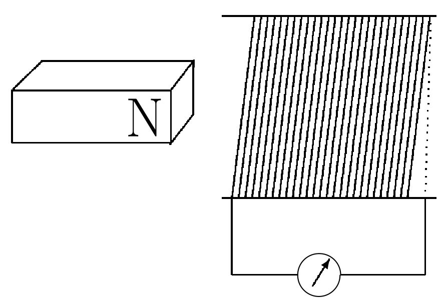


This shows that, there is a current produced when the wire is moved

Experiment 2

Magnet and coil experiment (dynamo principle)

- Connect a galvanometer across the two ends of a solenoid with more turns as shown below;



Center zero galvanometer

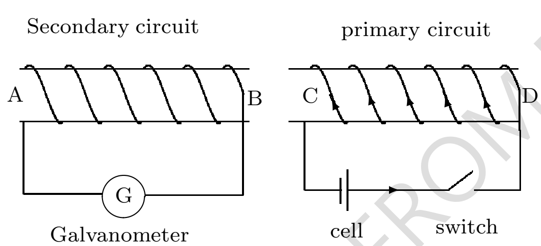
- Push the north-pole of a strong bar magnet into the coil and observe the galvanometer pointer while the magnet is moving. it deflects when the magnet is moving and shows no deflection when the magnet stops moving.
- Move the magnet slowly and then quickly and note how the deflection changes. When moving slowly the deflection is low, when moving faster the deflection is large.

- Now move the magnet out of the coil and observe the pointer. It deflects in the opposite direction.
- Push the south pole of the magnet into the coil and observe the pointer. (this time it deflects in the opposite direction).
- Repeat using a more powerful magnet, note the difference, the deflection is greater
- Move the magnet in and out of the coil. The deflection pointer oscillates to both sides of the zero deflection point.

Experiment 3

Principle of the transformer- Mutual induction

- Using two coils which are electrically connected and separated as shown below



- Close the switch to complete the primary circuit, observe the pointer of the galvanometer, it deflects (why?).

  Here the current flows in the primary as soon as the switch is closed. This makes C a North pole, and the magnetic field links to the secondary circuit. When the magnetic field changes from zero to a certain value, an emf is induced in the secondary circuit hence the galvanometer shows a deflection.

- Open the switch and note the deflection of the galvanometer, It points in the opposite direction.

Conclusion of the observations in experiment 1, 2 and 3

- The pointer of the galvanometer moves momentary in one direction when the primary circuit is completed and in the opposite direction when the primary circuit is broken.
- According to experiment 3, the pointer remains at zero when either no current or a steady current is flowing in the primary circuit.
- The deflection is very much bigger when iron is inside in the solenoid.
- There is a deflection only when the magnet is moving, this is according to experiment 2 or when the wire is moving according to experiment 1. In general, there is a deflection whenever there is relative movement between the coil or any conductor and the magnetic field.

Mutual induction is the induction of an emf in one circuit by changing the current in another near by circuit or coil.

Experiment 4

(**Self induction**)

Circuit containing a large electromagnet or solenoid made of many turns of wire wrapped around an iron core is broken by a knife switch. As the switch is broken a spark jumps the gap a cross the switch. i.e. the changing magnetic field (from its value when the solenoid is carrying current to zero when the circuit is broken) caused by breaking the circuit induces a large emf a cross the switch resulting into the electric break down of the air insulation as indicated by the spark.

3.19.2 Factors affecting the induced emf.

1. the rate at which the magnet is moved or how fast i.e. velocity of the magnet or the rate of change of the magnetic flux.

2. the strength of the magnet i.e. magnetic flux density.
3. the area of the coil or conductor exposed to the magnetic field or the length of the conductor exposed to the changing magnetic field.
4. the number of turns of the coil.
5. the angle at which the magnet is introduced or the angle between the magnetic field and the conductor.

Recall the presence of a soft iron core in the solenoid or coil increases the value of the induced emf. All the above are summarized in the two laws of electromagnetic induction.

3.19.3 Laws of electromagnetic induction.

Definition 79.

Faraday's law of Electromagnetic induction states that, whenever there is a change in the magnetic flux linked with a circuit, an emf is induced in it; the strength of which is proportional to the rate of change of the magnetic flux linked with the circuit.

Induced emf is proportional to the rate of change of the magnetic flux.

The first statement is the law, the second sentence only talks about one factor that affects the induced emf.

Definition 80.

Lenzi's law of electromagnetic induction states that, the induced current or emf acts in such away as to oppose the change causing it.

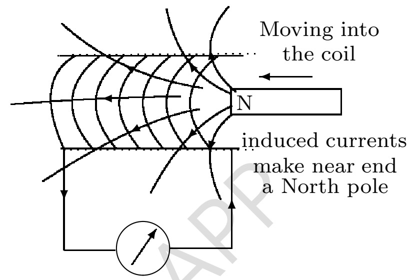
Or

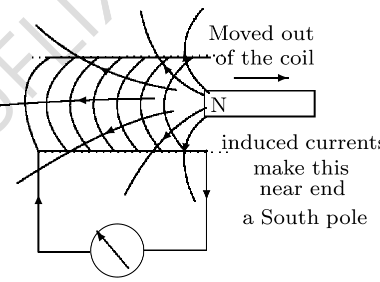
the direction of the induced current or emf flows in a direction as to oppose the change causing or producing it.

This is observed when a magnet is drawn into a solenoid connected to a center zero galvanometer when you apply the right hand grip rule.

Experiment to demonstrate Lenzi's law

Connect the circuit as shown below;





When a permanent bar magnet is moved towards the solenoid, its experiences an opposing force implying that side of the solenoid attains the same magnetic polarity like that of the incoming pole. That is the induced current indicated by the deflected galvanometer flows in a direction to create a polarity that opposes the motion of the magnet.

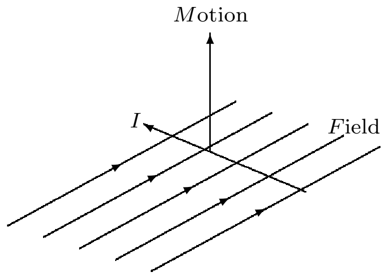
Direction of the induced emf

This problem was solved by Fleming when he designed the formula for determining this direction , that is by using Fleming' right hand rule.

Fleming's right hand rule: Dynamo rule

Hold the thumb, first finger and second finger of the right hand at right angles to each other, if the thu**M**b points in the direction of **M**otion, the first **F**inger in the direction of the magnetic

Flux or **F**ield then, the se**C**ond finger points in the direction of the induced **C**urrent.



This should not be confused with Flemings left hand rule: the motor rule.

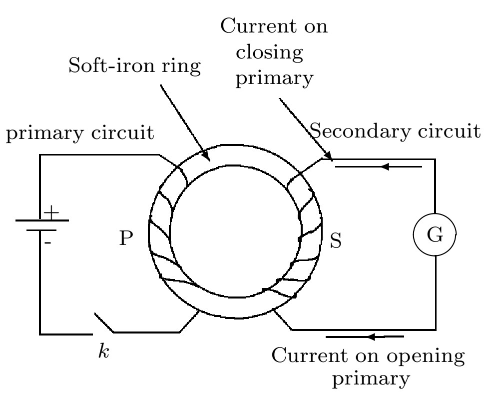
3.19.4 Mutual induction and self-induction.

Definition 81.

Mutual induction is the process by which an emf is induced in one circuit due to a change in the electric current in another nearby circuit.

Experiment to demonstrate mutual induction

- Get two coils
- Connect one to a complete circuit with only a center zero galvanometer
- Connect the other coil to a circuit with a cell and a switch
- Place the coils close to each other or the two coils may be wound on the same soft magnetic ring as shown below;



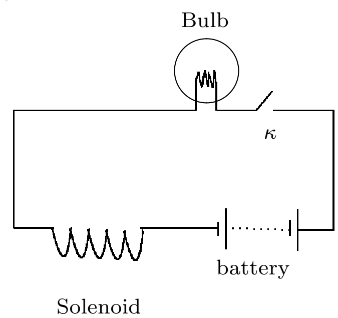
- Close the switch or open the switch and note or look at the galvanometer's pointer.
- The galvanometer deflects. This deflection indicates that there is an induced emf or current flowing due to the change in the current in a nearby coil, which is mutual induction hence verified.

Definition 82.

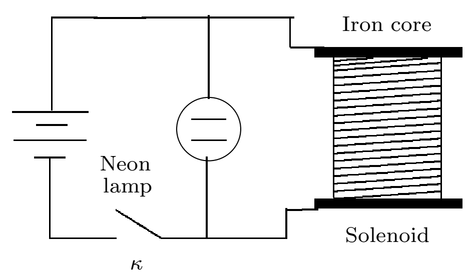
Self-induction is the process by which an emf is induced in a coil or circuit due to a change of the current in the same coil or circuit.

An experiment to demonstrate self-induction

- Connect the circuit as shown below; It is better to use iron cored solenoid if available.



Or



- Close the switch, the neon bulb is observed to light.
- Open the switch, the neon lamp is observed to light much brighter and then goes off.

Why does it first shine brighter before going off? This is due to self-induction, an emf is induced in this circuit due to a changing magnetic field in the solenoid. Since this induced emf is more than the supplied voltage, the bulb has to produce a brighter light. This bright light when the switch is opened inform us of the existence of a larger emf due to self-induction.

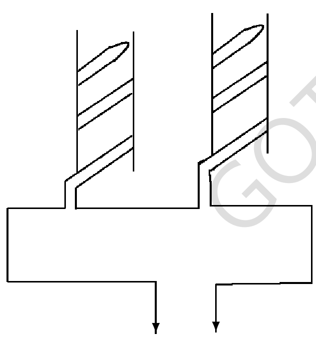
Some times the existence of sparks at the switch when it is closed confirms the existence of self-induction in the circuit.

3.19.5 Non-inductive resistance coils.

These coils are used when self-induction is not required or when self-induction is a disadvantage and needs to be eliminated.

How can it be eliminated?

The circuit is made to contain a part where two wires carrying the same current in opposite directions are placed too close to each other either in a linear form or in a coiled form.



By this winding of the coils, they have zero resultant magnetic fields around them and so consequently there is no self-induction or no counter emf set up when the current through them changes. Such coils are used in the construction of resistance boxes because in them current should only change due to change in resistance not due to induced emf.

3.19.6 Eddy currents

If the magnetic flux through a piece of metal changes, an emf is induced in it. This emf produces circular electric currents in the metal, these currents are called **eddy currents.** These currents are produced by either moving the metal in a constant magnetic field or exposing the metal to varying magnetic fields.

Definition 83.

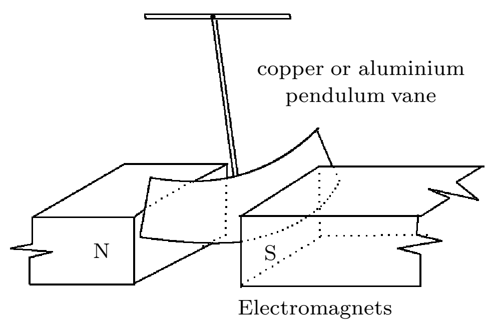
Eddy currents are currents induced in a metal or any conductor when it is moved in a magnetic field or is exposed to a varying magnetic field.

Why do they flow in a circular path?

Experiment to demonstrate eddy currents.

Eddy current damping or Waltenhofen's pendulum

- The apparatus include; A pendulum having a plane strip of copper or aluminum at its lower end, arranged as shown below;



Eddy currents cause damping of the pendulum swings

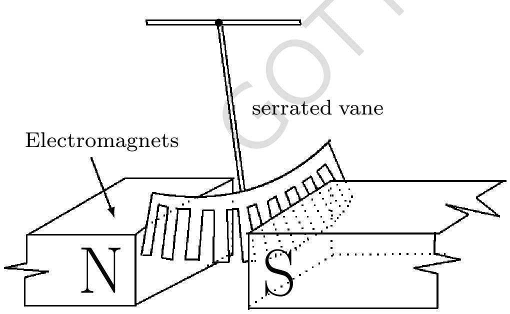
- Switch off the electromagnets and push the pendulum, it will swing freely in a vertical plane between the electromagnets.
- When still swinging, switch on the electromagnet, the vane of the pendulum come to rest very fast or the oscillations of the pendulum are heavily damped and die away rapidly (why)? This is because when the pendulum

swings to and fro, it cuts the magnetic flux from the electromagnet, and an emf is induced in it.

- This induced emf sets up eddy currents inside the metal vane, which in accordance to Lenzi's law sets up magnetic fields which oppose the motion of the pendulum hence a braking or damping effect.
- This confirms that the induced current (eddy currents) produces a magnetic field which oppose the motion of the metal, hence eddy currents exist.

Note. If this metal vane is made to have slots i.e. is replaced by a serrated vane, and the experiment repeated, it is found that switching on the electromagnet, the damping of the oscillations is weak. (Why?) A smaller emf is induced in each part of the serrated vane. This is because of

- the vane now has a much smaller area which cuts the magnetic flux.
- the resistance of each small segment is lighter than when combined and so the eddy currents induced are small and have only a very small effect on the vane's motion.



Slots reduce Eddy currents
and prevent damping

Also remember that the coil in the moving coil galvanometer is wound on a metal former. When this former moves in a magnetic field quite a large current is induced in it and this opposes the motion of the coil. Hence the coil and the pointer come to rest immediately and do not oscillate. This is called eddy current damping. Eddy currents are directly applied in the following instruments;

1. Crack detector
2. Speedometer
3. Galvanometer damping
4. Electric furnaces e.g. solder iron
5. Induction coil

3.19.7 Applications of electromagnetic induction:

Faraday's law, Lenzi's law and Eddy currents
Electromagnetic induction has been applied in very many electric devices and fields, these include;

1. In generation of electricity (generators)
2. Making electricity enough by use of transformers
3. Microphones
4. Speedometers
5. Crack detectors
6. Galvanometer damping
7. Induction furnace
8. Induction coil
9. Choke
10. Record play pick-up
11. Electric brakes

1. **Generating electricity**

The ability to produce an electromotive force by changing the magnetic field linked with a coil or solenoid is used to generate electricity. This is done in a device known as a generator. A **generator** is a device which produces an emf when its coil or conductor is rotated in a magnetic field or exposed to a changing magnetic field. It changes mechanical energy to electric energy. There are two types of generators;

1. Alternating-current generators.
2. Direct-current generator.

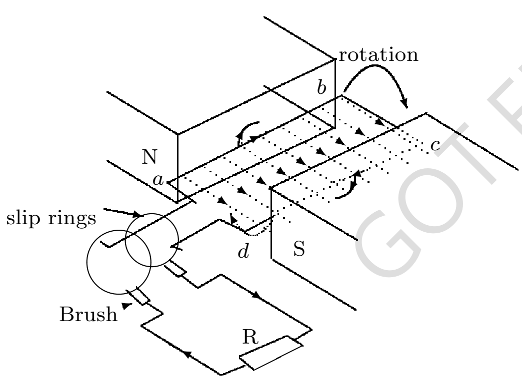
Let us look at each separately.

Alternating current (Ac) generator

The alternating current generator or alternator produces current or emf, which changes every after a certain interval of time or frequency. Here we consider the simple Ac generator.

Construction:

A simple Ac generator consists of rectangular coil of wire which is rotated in the magnetic field provided by a permanent magnet(s) as shown below.



When this coil rotates, it cuts the magnetic field lines and from Faraday's law of electromagnetic induction an emf is induced across ites ends. If the brushes are connected to an external circuit, an induced current flows. (How?).

Analysing the induced current in each side;

Side *bc* and **side** *ad* are always parallel to the magnetic field. Because of this, they do not cut the magnetic field hence no emf is induced in them.

Side *ab* is moving up wards. Applying Fleming's right hand rule, the induced current flows from *a* to *b*.

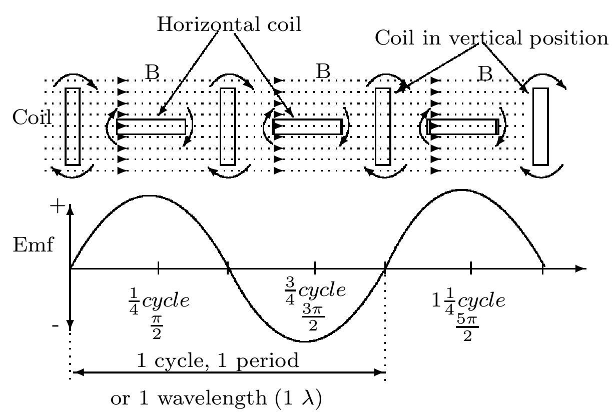
Side *cd* is moving down wards. Applying Fleming's right rule the induced current flows from *c* to *d*.

These currents are shown in the above diagram. When side *ab* replaces the position of side *dc*, current in the coil changes direction as the coil passes the vertical position. As the rotation continues, the current keeps on changing direction by increasing in one direction and then reduces to zero, here it changes direction and increases and decreases in magnitude and the cycle continues. It keeps on alternating hence the name alternating current.

Definition 84.

Alternating current is a type of current, which changes its magnitude and direction of flow in a conductor continuously at a regular internal of time or frequency.

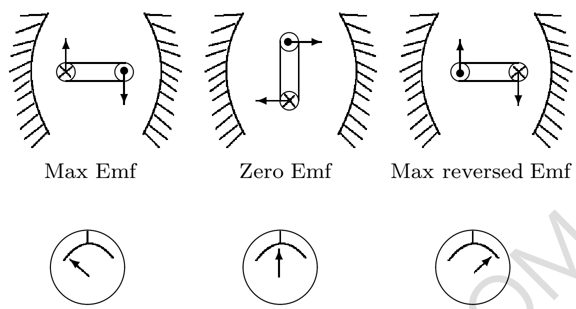
Below are diagrams to show how the induced emf varies with various positions of the rectangular coil.



Note that, the emf is a maximum in either direction when the coil is horizontal (why?). Because since magnitude of induced emf is proportional to the rate of change of the linked magnetic flux, when the coil is horizontal it cuts more magnetic

field lines (magnetic flux) than when it is vertical hence maximum induced emf occurs when the coil is horizontal.
The induced emf is zero when the coil is vertical because in this position is does not cut any magnetic field lines (magnetic flux) hence induced emf $\left(= \frac{\text{magnetic flux change}}{\text{time}}\right)$ is zero. though there is maximum flux linkage.

Note that the graph of induced emf against time is a sinusoidal graph (a sine or cosine curve). Its induced emf has a certain period(T), or frequency related to T by ($f = \frac{1}{T}$). Alternating current is usually produced at a frequency of 50Hz or 60Hz. We can observe the alternating nature of this induced current by rotating the coil connected to a center zero galvanometer and the coil observed at the following positions.



Pointer of the Galvanometer as the coil rotates
Note the reversed Emf

The deflections of the galvanometer shows that really the current reverses directions of flow. Note that it does not only vary in direction but also in magnitude. The magnitude of the induced emf depends on;

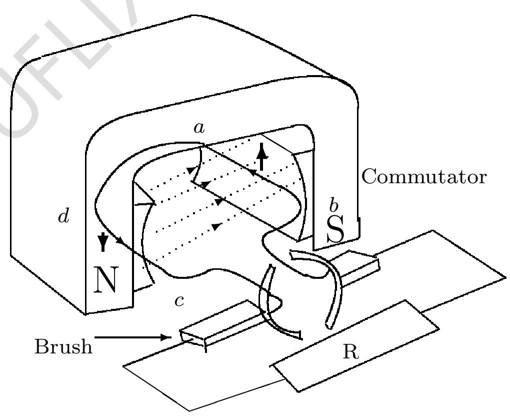
1. The rate of change of magnetic flux or rate of magnetic flux cutting which depends on the speed of rotation of the coil which should be high enough for a high rate.
2. The number of turns of wire in the coil
3. The strength of the magnetic field intensity i.e. strength of the permanent magnets (field magnets)
4. Whether the coil is wound on a soft iron armature or not. This is because the soft iron armature increases the magnetic flux through the coil.

Direct current generator (or dynamo)

If the slip rings and brushes of the simple ac generator are replaced by a ring with two diametrically opposed brushes, the Ac generator becomes a simple Dc generator or dynamo. The major difference is that a split ring (commutator) is used in a Dc generator instead of slip rings.

Construction

It is made up of a coil, usually rectangular, which is rotated in a magnetic field provided by permanent magnets as shown below.



How is the induced emf produced?

In the direction of rotation of the coil shown,

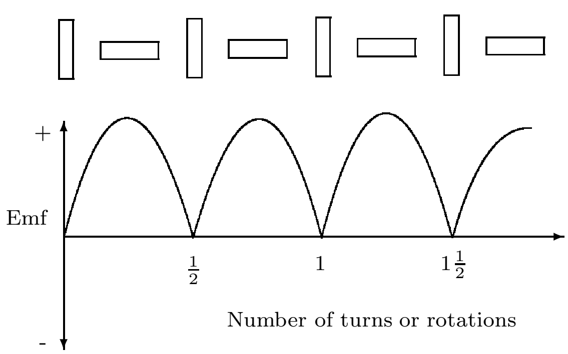
Side *da* and *cb* are always parallel to the magnetic field and so it does not cut the magnetic field hence no emf is induced in them.

Side *dc* is moving downwards. Applying Fleming's right hand rule, the induced current flows in the direction from **c** to **d**.

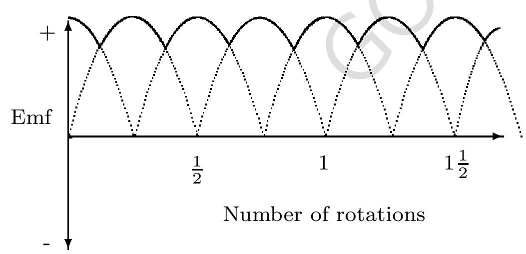
Side *ba* is moving upwards. Applying Fleming's right hand rule, the induced current flows in the direction; **b** to **a**.

Generally current flows in that direction as the coil rotates, till it is vertical. In this position no emf is induced and the brushes are just in the spaces between the commutator (and no current flows). Due to the momentum of the coil it continues moving and the brushes exchange positions of the coil.

Now **ab** replaces the former position of **dc** and **dc** replaces that of **ab**, as a result current in the coil changes direction but its direction to the brushes does no change.The plot of induced emf against time or angle of rotation is shown below



The current is a pulsating direct current. When the commutator is made of 4 segments a smoother out put like this below can be obtained



Motor effect in a dynamo

The motor acts on the principle that; when current flows through a conductor placed in a magnetic field, it experiences a force. Now in the dynamo the coils are also in a magnetic field and carrying the induced current. This creates a force on the conductor or coil as in the motor whose direction is given by Fleming's left hand. This force is felt as an opposition to the motion of the dynamo coil. Some times it is interpreted as Lenzi's law (another version of the law of conservation of energy).
In summary, motor effect in a dynamo is the effect felt as the opposition to the rotation of the armature or coil in the dynamo.

OR, when current is produced by the dynamo, current forces act on the sides of the coil, which oppose the motion slowing down the rotating coil.

Recall dynamo effect in the motor on page 288, where the coil in the motor rotates cutting the magnetic field and an emf called back-emf is induced in it which opposes the emf supplied by the cell. The work done by this back-emf is the mechanical work done by the motor.

Comparing a motor and a dynamo

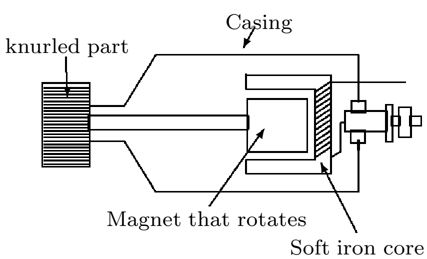
There is no structural difference between a d.c. Motor and a d.c. generator i.e. the same machine can be used for either purpose. But;

1. In the motor, electrical energy is used to rotate the armature and the rotating armature is used to produce mechanical work while in a generator, mechanical energy is used to rotate the armature and the rotating armature produces electrical energy.
2. There is a motor effect in a dynamo similar to the dynamo effect in a motor.
3. It is not essential to have a rotating coil in a generator, this is because in a generator only flux-cutting is ensured. In some generators the coils are fixed and a magnet rotates between them. This is because the brushes and the coils are not needed since the coil does not move.

   In a bicycle dynamo, a circular magnet is made to spin or rotate inside an iron core on which a coil is wound.

A bicycle dynamo

This dynamo is used to light lamps on the bicycle. It consists of a casing in which a coil on a soft iron core is fixed and at its centre a circular permanent magnet rotates with the tyre. When the bicycle is at rest, the magnet is also at rest and its field lines pass through the coils. When the wheel turns the magnet rotates between the coils, its magnetic field lines (flux) are cut by the coils. This changing magnetic field through the coil induces an emf according the Faraday's law and hence current flows in the coils. This current can make the bulbs on the bicycle light up at night increasing the visibility.



Practical dynamos

Note that the described dynamo has one disadvantage, the emf is reduced to zero every half rotation. This problem is overcome by having a number of coils wound in slots in an iron cylinder called the armature. Each coil has its own pair of segments in a multi-segment commutator. The iron armature is built up of discs insulated from one another either by vanish or an oxide coating so as to reduce or prevent large eddy currents from being set up in the armature.

Usually electromagnets are used and not permanent magnets because permanent magnets are heavy and bulky. The coils of this electromagnet are called the **field coils.** The dynamo itself supplies the necessary current to them. These coils may either be in series or in parallel with the armature coils. When the field coils and the armature coils are in parallel, the dynamo is said to be **shunt wound** and when connected in series it is said to be **series wound**. When a combination of the two windings is used, the dynamo is said to be a **compound wound** where there are two sets of field coils, one in parallel and the other in series with the armature coils.

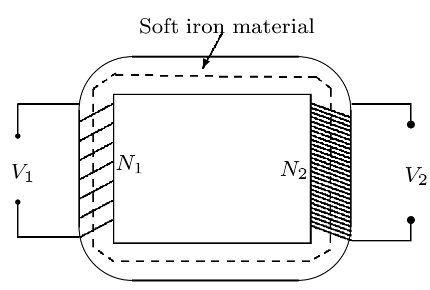
2. **The transformer**

Introduction

This is a device, which uses mutual induction to produce an emf in a coil when current changes in another nearby coil. The emf induced in the coil will be larger or smaller than that in the second coil, depending on whether the number of turns in the first coil (the primary) are more or less than those in the second coil (the secondary coil) and secondly on the rate of change of current in the primary coil.

How a transformer works

A transformer is made up of two coils, the primary coil connected to the primary circuit and the secondary coil connected to the secondary circuit; the primary coil is linked to the secondary circuit by a soft magnetic material. The soft magnetic material ensures that the magnetic field produced by the primary circuit is transferred directly to the secondary circuit.



When current in the primary circuit or voltage (V_1) across it changes, also the magnetic field in the soft magnetic material also changes. This changing magnetic field is linked to the secondary circuit and an emf is induced in the secondary coil, this is due to the relative movement between the field and the coil (or magnetic flux cutting).

The induced emf (V_2) in the secondary coil depends on the number of turns(N_2) in the secondary coil. These are related to the applied emf(V_1) and the number of turns N_1 in the primary coil as shown below;

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \text{ or} \tag{3.269}$$

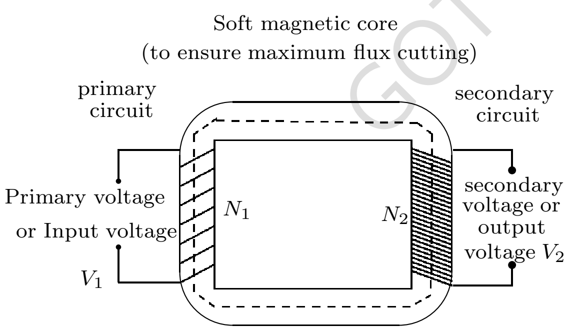
$$\frac{V_2}{N_2} = \frac{V_1}{N_1} \tag{3.270}$$

Explain briefly how a transformer works?

Note that depending on whether $V_1 < V_2$ or $V_1 > V_2$ a transformer can either be a step up transformer ($V_1 < V_2$) or a step down transformer ($V_1 > V_2$).

Step up transformer

When the number of turns in the primary circuit (N_1) is less than those in the secondary circuit (N_2), the transformer is called a step up transformer, for instance;



Step-up transformer ($N_2 > N_1$

For a step up transformer, $N_2 > N_1$, Since the induced emf depends on the rate of flux cutting which is proportional to the number of turns in the linked coil(N) i.e. induced emf α number of turns in the coils

$$E \quad \alpha \quad N \tag{3.271}$$

$$\frac{E}{N} = \text{constant} \tag{3.272}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \text{ i.e.} \tag{3.273}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \text{ or} \tag{3.274}$$

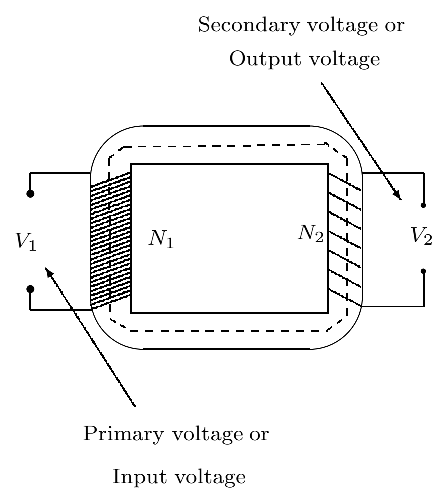
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \tag{3.275}$$

hence when $N_2 > N_1$ then also $V_2 > V_1$. for a step up transformer the output voltage is greater than the supplied voltage.

$$\frac{\text{secondary emf}}{\text{Primary emf}} = \frac{\text{number of turns in secondary}}{\text{number of turns in primary}}$$

Step down transformer

Here, the number of turns in the secondary coil is less than those in the primary coil. When an A.c. voltage is applied to the primary circuit a smaller emf is produced across the secondary circuit, $V_2 < V_1 \Rightarrow N_1 > N_2$.



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \tag{3.276}$$

CAUTION Since a transformer uses mutual induction i.e. an emf is induced in a coil when current (or voltage) in the nearby coil changes, then a transformer can only use changing current or voltage i.e. it can use only alternating currents and not direct currents (or steady currents).

Energy conservation in a transformer

Stepping up or stepping down alternating electric current

Although the transformer steps up or increases the supplied voltage from a lower voltage to higher voltage or Steps down or decreases the supplied voltage from higher voltage to lower value, it does not create or destroy energy, it only obeys the law of conservation of energy. Since energy is fed in through the primary circuit and gets out via the secondary circuit. Then,

$$\begin{matrix}\text{energy input via}\\ \text{primary circuit}\end{matrix} = \begin{matrix}\text{energy output via}\\ \text{secondary circuit}\end{matrix} \quad (3.277)$$

In one second; energy fed into the primary circuit per second is equal to the energy given out from the secondary circuit per second.

$$\begin{matrix}\text{Power in put}\\ \text{at primary}\end{matrix} = \begin{matrix}\text{power out put}\\ \text{at secondary}\end{matrix} \quad (3.278)$$

Since electric power is given by;

$$\begin{aligned} P &= IV && (3.279)\\ P_{in} &= P_{out} && (3.280)\\ V_1I_1 &= V_2I_2 && (3.281)\\ \Leftrightarrow \frac{V_1}{V_2} &= \frac{I_2}{I_1} && (3.282)\\ \text{since } \frac{V_1}{V_2} &= \frac{N_1}{N_2} && (3.283)\\ \text{then } \frac{I_2}{I_1} &= \frac{N_1}{N_2} && (3.284) \end{aligned}$$

These equations shows that; a transformer also can be used to step-up or step-down electric current. Hence when a step-up transformer steps up voltage, it automatically steps down current.

A step-down transformer step down voltage and automatically steps up current, i.e. if voltage is stepped up ten times (from 10V to 100V or 1V to 10V) the current is stepped down ten times (for instance from 10A to 1A or from 1A to 0.1A).

Efficiency of a transformer

In practice, the energy put into the transformer via the primary circuit is not equal to the energy out put from the transformer via the secondary circuit, this is because some energy is wasted, lost or used up in the transformer by,

- Heating up the material in the core of the transformer by eddy currents.
- To produce the sound heard from a transformer, etc hence the energy out put from the transformer is always less than energy in put.

$$\begin{aligned} \text{Energy in put} &> \text{Energy out put} && (3.285)\\ \text{Power in put} &> \text{Power out put} && (3.286) \end{aligned}$$

This inefficiency of device is calculated as efficiency of the device or machine. By definition ;

$$\text{Efficiency} = \frac{\text{work done by the machine}}{\text{work done on the machine}}$$

In a given time interval we use power , hence

$$\begin{aligned} \text{Efficiency} &= \frac{\text{power output}}{\text{power input}} && (3.287)\\ E &= \frac{P_{out}}{P_{in}} && (3.288) \end{aligned}$$

Hence the efficiency, E, of a transformer

$$\begin{aligned} E &= \frac{P_{out}}{P_{in}} = \frac{V_2I_2}{V_1I_1} && (3.289)\\ \Leftrightarrow EV_1I_1 &= V_2I_2 && (3.290)\\ \text{hence } V_2I_2 &= EV_1I_1 && (3.291)\\ E &< 1 && (3.292)\\ Or E &< 100\% && (3.293) \end{aligned}$$

Generally $E \leq 1$, The current modern transformers have a very high efficiency of about 96% to 99.99%. The efficiency of a transformer is always less than unity due to energy losses , what are they?

Energy losses in a transformer.

The efficiency of a transformer is always less than unity because of the following;

1. **Flux leakage**. The magnetic flux generated in the primary circuit does not as a whole pass through the secondary coil, hence some energy is conducted some where else. Recall, that energy is stored in the magnetic field. This can be minimized by;
   (a) Winding the primary on the top of the secondary or vise verse.
   (b) Having a closed soft iron core i.e. the center of the two coils (primary and secondary) should be filled with a soft iron material.

2. **Heating effect of eddy currents**

   Eddy current losses

   The iron core in the coils is a conductor and the magnetic field varies continuously. The changing magnetic field in the conductor induces an emf hence an induced current (called eddy current) flows, Due to low resistance the currents are large, this leads to an enormous heating effect. This heat energy comes from the energy or power put in, this reduces the efficiency. Increasing the resistance can solve this problem. This is done by;
   (a) Using laminated iron core; for instance sheets of iron insulated from each other.
   (b) Using a bundle of fine thin wires, insulated from each other

   This reduces the current to a smaller value which just oscillates in a small lamina or wire hence producing less heat.

3. **Energy loss in heating the copper coils of the windings;**

   Since any conductor carrying an electric current gets heated, the primary and secondary windings get heated by an amount given by I^2R. This can be reduced by using thick low resistance copper coils.

4. **Hysteresis loss**

   when an iron cored coil is used in the transformer, the iron core gets magnetised in one direction and demagnetised in half a cycle and then magnetised and demagnetised in the opposite direction in the next half of the cycle.

   Energy is required to bring about these magnetic reversals, the magnetic reversals are called hysteresis. The energy required to do this is obtained from that put into the transformer hence reducing its efficiency. The hysteresis energy is greater for hard magnetic materials and smaller for soft magnetic materials (easy to magnetise and demagnetise). Hysteresis loss is reduced or minimised by using a core made from a soft magnetic material so as to use less energy in bringing about the magnetic reversals.

Uses of a transformer

The uses of a transformer depend on whether it is stepping up or stepping down voltage or current; These uses include;

1. they are used for stepping up voltage where a high voltage is required like in TV sets, x-ray tubes and other devices.
2. they are used for stepping up current (in step down transformers) where a large current is required like in electric furnaces, welding, e.t.c.
3. they are used in running low voltage devices from a high voltage supply like the mains. The low voltage devices bulbs, small motors e.t.c.
4. they are used in the transmission of mains electricity from the generator to the consumer at low energy losses. With out the transformers the electricity produced in a given country would not be enough.

Examples

1. A 60W12V bulb is run normally on a 240V mains supply. Using a step-down transformer with 400 turns in the secondary, determine the
   (a) the number of turns of coils in the primary circuit, it the efficiency of the transformer is 90%
   (b) the electric current supplied by the mains
   (c) Power supplied by the mains
   (d) the total electrical resistance in the primary and secondary circuit?

Solution

(a) using

$$\begin{aligned} \frac{N_P}{N_s} &= \frac{V_p}{V_s} \\ N_p &= \frac{V_p}{V_s} \times N_s \\ &= \frac{240}{12} \times 400 \\ &= 8000 \text{ turns} \end{aligned}$$

There are 8000 turns in the primary windings.

(b) Using the formula for efficiency

$$\begin{aligned} \text{efficiency} &= \frac{P_{out}}{P_{in}} \\ E &= \frac{P_{out}}{P_{in}} \\ &= \frac{60W}{V_p I_p} \\ I_p &= \frac{60}{EV_p} \\ &= \frac{60}{\frac{90}{100} \times 240} \\ &= \frac{10}{36} \\ &\simeq 0.2778A \\ &\simeq 277.778mA \\ &\simeq 277.8mA \end{aligned}$$

Hence the electric current supplied by the mains is 277.8mA

(c) from the formula for efficiency

$$\begin{aligned} E &= \frac{P_{out}}{P_{in}} \\ &= \frac{P_s}{P_p} \\ \Rightarrow P_p &= \frac{P_s}{E} = \frac{60}{\frac{90}{100}} \\ &= \frac{60}{0.9} \\ &= 66.67\text{Watts} \end{aligned}$$

Therefore the power supplied by the mains is 66.67 Watts.

(d) Applying Ohm's law in the secondary and primary circuit.
In the secondary circuit;

$$\begin{aligned} \text{from } V &= IR \\ \text{and } P &= \frac{V^2}{R} \\ \Rightarrow R &= \frac{V^2}{P} \\ &= \frac{12^2}{60} \\ &= 2.4\Omega \end{aligned}$$

hence the secondary circuit resistance is 2.4Ω.
In the primary circuit,

$$\begin{aligned} \text{using } V &= IR \\ R_p &= \frac{V}{I} \\ &= \frac{240}{0.2778} \\ &= \frac{240}{\frac{10}{36}} \\ &= 864\Omega \end{aligned}$$

Hence the primary circuit resistance is 864 Ω

2. A certain transformer connected to a 240V mains is used to step up electric current from 5mA to 40mA so as to have normal lighting of a 24V lamp, Determine;

(a) the power loss in the circuit

(b) the heat generated by the whole system in 2 minutes

(c) the efficiency of the transformer

Solution

(a)Using the expression, $P = VI$ for the power supplied by the source P_p and the power dissipated P_s is given by;

$$\begin{aligned} P_p &= I_pV_p \\ &= 5mA \times 240 \\ &= (5 \times 10^{-3}) \times 240 \\ &= 1.2W \\ P_s &= I_sV_s \\ &= (40 \times 10^{-3}) \times 24 \\ &= 0.96W \\ P_{loss} &= P_p - Ps \\ &= 1.2 - 0.96 \\ &= 0.24W \end{aligned}$$

(b) Power loss P_{loss} is always dissipated as heat in the resistors in the circuit;

$$\begin{aligned} P_{loss} &= \text{Power loss} \times \text{time} \\ E &= P \times t \\ &= 0.24 \times (2 \times 60) \\ &= 28.8\ Joules \end{aligned}$$

(c) From the expression for efficiency;

$$\begin{aligned} \text{Efficiency} &= \frac{P_{out}}{P_{in}} \\ &= \frac{I_sV_s}{I_pV_p} \\ &= \frac{0.96}{1.2} \\ &= 0.8 \\ \text{Or}\ , E &= \frac{P_\circ}{P_i} \times 100 \\ &= 0.8 \times 100 \\ &= 80\% \end{aligned}$$

Exercise

(a) A step up transformer is designed to operate from 20V supply and deliver energy at 250V. If the transformer is 90% efficient, determine the current in its primary windings when the output terminals are connected to a 250V100W lamp?

(b) A transformer is used on a 240V a.c. supply to deliver 9.0A at 80V to a heating coil. If 10% of the energy taken from the supply is dissipated in the transformer itself, what is the current in the primary winding?

(c) A step down transformer is used to light a 12V24W lamp from the 240V mains. The current through the primary is 250mA. What is the efficiency of the transformer?

(d) Find the power wasted as internal energy in the cable of resistance 0.5Ω when 10kW are transmitted through the cable at;
(a)200V
(b)200,000V
(c)When are the energy losses minimised ? at low voltage or at high voltage.

(e) A step down transformer has coils of 1000 and 4000 turns. The input voltage is 240V. What is the output voltage? Why do transformers often have extra connections (or tappings) on the primary and secondary windings?

(f) The voltage and current in the primary of a transformer are 240V and 1.5A and in the secondary are 24V and 14A. calculate the power;
(a)supplied to the primary
(b)supplied by the secondary
(c)wasted in the transformer

(g) Draw and label the circuit you would use to recharge a 12V battery from the 240V mains. What should be the ratio of primary to secondary turns in the transformer used?

(h) Calculate in kilowatts the heat generated in cables of resistance 5Ω when conveying 1000kW of electrical power
(a) at 100kV and
(b) 10kV. What does this calculation illustrate about the voltage used in the grid system of electrical supply from the power station?

(i) Water behind a dam flows through a pipe to a hydro-electric power station. The water drives a water wheel, which generates power to cables that take electricity to appliances in homes and factories. Discuss the energy changes that occur?

(j) A potential difference of 120kV is applied to cables of resistance 3Ω, and a current of 10A flows. Calculate

(a)the power input to the cables

(b)the power wasted in the cables

(c)Why is it not practical to transmit this power at 240V instead of 120kV?

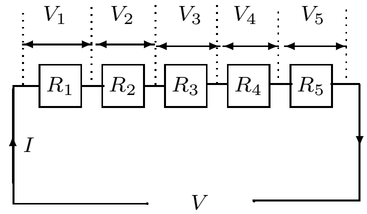
(k) A transformer, which is 80% efficient, gives an output of 10V and 4A. What is the input power?

How to run a low voltage device from the main or a high voltage source

Voltage sources are of two types;those that produce alternating current or alternating voltage and those that produce direct current or direct voltage. Let us discuss how each can be used to run a low voltage device from the mains (a high voltage supply);

(a) Using a high voltage direct current source to run a low voltage device:

Here we use the principle of a potential divider. Recall that if we have a number of electrical devices or resistors connected in series as shown below



Voltage source

then

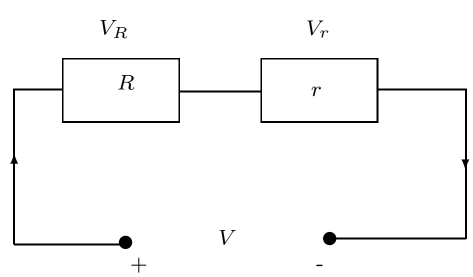
$$\begin{aligned} V &= V_1 + V_2 + V_3 + V_4 + V_5 \\ V &= IR_1 + IR_2 + IR_3 + IR_4 + IR_5 \\ V &= I(R_1 + R_2 + R_3 + R_4 + R_5) \end{aligned}$$

Then the ratio of the voltage across one of the devices to the supplied voltage (voltage of the source) is given by;

$$\begin{aligned} \frac{V_1}{V} &= \frac{IR_1}{I(R_1 + R_2 + R_3 + R_4 + R_5)} \\ \frac{V_1}{V} &= \frac{R_1}{R_1 + R_2 + R_3 + R_4 + R_5} \end{aligned}$$

i.e. is equivalent to the ratio of the resistance of that resistor to the total resistance in the circuit. Therefore to obtain a fraction of the supplied voltage one has to connect another resistor in series with the one available such that the required voltage is across the available resistor and the rest of the voltage is across the connected resistor.

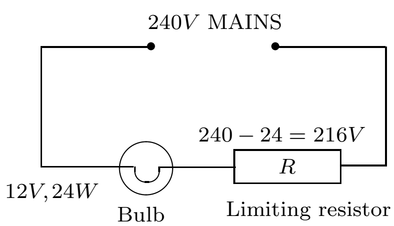
Hence to run a low voltage device on a high d.c. voltage main, one has to connect the low voltage device of resistance, r, in series with the a high resistance, R, across the high voltage source with voltage, $V_\circ$; such that the ratio of the required potential difference (V_r) to the source voltage is equal to the ratio $\left(\frac{r}{r+R}\right)$ of the resistances.



$$\text{Since } \frac{V_r}{V_o} = \frac{Ir}{V_r + V_R} \quad (3.294)$$
$$= \frac{Ir}{IR + Ir} \quad (3.295)$$
$$= \frac{Ir}{I(R + r)} \quad (3.296)$$
$$= \frac{r}{r + R} \quad (3.297)$$
$$\text{Then } V_r = \left(\frac{r}{r + R}\right) \times V_\circ \quad (3.298)$$

Let us consider how we could run a 12V 24W lamps from a 240V d.c mains supply.

In this case it is necessary to connect a limiting resistor in series with the lamp. The limiting resistor acts as a potential divider to divide the 240V between itself and the lamp in the ratio of their resistances as shown below



The value of the resistor is calculated as follows;

the potential difference across the lamps must be 24V, hence the P.d across the limiting resistor(R) will be;

$$240V - 24V = 216V$$

Since the lamp and the resistor are in series, the same current passes through both. The current taken by the lamp may be calculated from the formula

$$\text{Power} = \text{Voltage} \times \text{Current}$$
$$\text{Current} = \frac{\text{power}}{\text{voltage}}$$
$$I = \frac{P}{V}$$
$$= \frac{12W}{24V}$$
$$= 0.5A$$

the value of the resistor, R, required is given by;

$$R = \frac{\text{p.d. across the resistor}}{\text{current through it}}$$
$$R = \frac{V}{I}$$
$$= \frac{216}{0.5}$$
$$= 432\Omega$$

Note that the mains current is the same as that through the lamp, $0.5A$ and so the total power output from the mains is given by

$$\text{Power} = \text{voltage} \times \text{current}$$
$$P = VI$$
$$= 240 \times 0.5$$
$$= 120W,$$

Out of the 120W, the lamp uses only 12W, the remaining 108W is wasted in raising the temperature of the limiting resistor. This is the disadvantage of direct current.

Example

(a) A 48V60W lamp is to be operated on a 240V dc mains. Calculate;

(i)the current through the lamp

(ii)the value of the resistor required

(iii)the power wasted

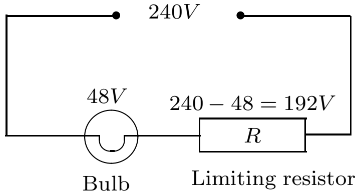
Solution

(i)from $P = VI$

$$I = \frac{P}{V}$$
$$= \frac{60W}{48V}$$
$$= 1.25A$$

Thence current through the lamp is 1.25A

(ii)from the circuit diagram below,



If the p.d across the resistor is V_r, then

$$\begin{aligned} V_R &= 240 - 48 \\ &= 192V \end{aligned}$$

the same current of 1.25A passes thorough the resistor, R, since they are in series

$$\begin{aligned} R &= \frac{V}{I} \\ &= \frac{192V}{1.25A} \\ &= 153.6\Omega \end{aligned}$$

Whence the required resistance is 153.6 Ω

(iii) power wasted P_w is power dissipated in the limiting resistor (153.6Ω)

$$\begin{aligned} ,P_w &= VI \\ &= 192 \times 1.25 \\ &= 240W \\ \text{or } P_w &= \text{total power} - 60W \\ &= 240 \times 1.25 - 60 \\ &= 300 - 60 \\ &= 240W \end{aligned}$$

hence Power wasted is 240W

(b) A 230.4Ω resistor is used in operating a 60W 96V motor from a 240V d.c. mains. Calculate;

(a) the current through the 230.4Ω resistor

(b) the resistance of the motor

(c) the voltage across the 230.4 Ω resistor

(d) the power dissipated in the 230.4Ω resistor

(e) the power wasted

Solution

(a) the current through the 230.4Ω resistor is the same as that through the motor (Why?) when operating normally.

$$\begin{aligned} \text{From } P &= IV \\ I &= \frac{P}{V} \\ \text{For the motor, } I &= \frac{60W}{96V} \\ &= 0.625W/V \\ I &= 0.625A \end{aligned}$$

Hence current through the 230.4Ω resistor is 0.625A

(b) Using $P = \frac{V^2}{R}$

$$\begin{aligned} R &= \frac{V^2}{P} \\ \text{For the motor, } R &= \frac{(96)^2}{60} \\ &= 153.6\Omega \\ \text{Or } R &= \frac{P}{I^2} \\ &= \frac{60}{0.625^2} \\ R &= 153.6\Omega \end{aligned}$$

Hence the motor's resistance is 153.6 Ω

(c) Using Ohm's law;

$$\begin{aligned} V &= IR \\ &= 0.625 \times 230.4 \\ V &= 144V \\ \text{Or } V &= 240 - 96 \\ &= 144V \end{aligned}$$

(d)Power, $P = VI$

$$\begin{aligned} P &= VI \\ &= 144 \times 0.625 \\ &= 90W \\ \text{Or } P &= \text{total power} - 60 \\ &= 240 \times 0.625 - 60 \\ &= 150 - 60 \\ &= 90W \end{aligned}$$

(e)it is the same as the power dissipated in 230.4Ω resistor $= 90W$ Or

$$\begin{aligned} P &= VI \\ &= 0.625 \times 144 \\ &= 90W \end{aligned}$$

Exercise

(a) Tom has a small bulb rated 0.5W4V. He uses a 12V battery to light it. Calculate the value of the resistor he should use. Sketch the circuit he connected.

(b) A certain unknown electrical device has a label $0.8W, 8\Omega$. If a 12V battery is the only source of emf available , calculate
  (i)the current that passes through the device when operating normally
  (ii)the p.d across the device when operating normally
  (iii)the value of the resistor required to use the device on the 12V battery
  (iv)the power wasted in this connection

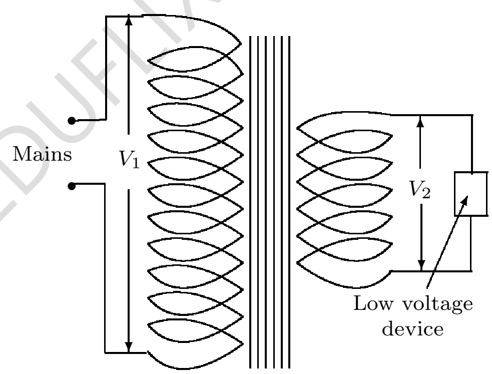
(c) A 50W electric knife is operated on a 240V d.c supply using a limiting resistor of 400Ω. An ammeter in series with the resistor reads 0.5A. calculate
  (i)the p.d across the 400Ω resistor
  (ii)the p.d across the electric knife
  (iii)the power wasted in using this electric knife

(d) An electric room-heater 60Ω is operated on a 110V d.c mains by connecting a limiting resistor in series with it. The current through the resistor was measured as 0.75A. calculate;
  (i)the power rating of the heater
  (ii)the value of the limiting resistor used
  (iii)the energy wasted if it operates for 15hours

How to run a low voltage device from a high voltage a.c supply

Here we use the step down transformer, this steps down voltages such that the output voltage (V_2) is less than the input voltage (V_1) i.e. $V_2 < V_1$ the ration $\frac{V_1}{V_2} =$ a constant, which constant is determined by the manufacturer.

Below is the circuit for running a low voltage device from the mains



Suppose we wish to run a $12W24V$ lamp from the $240V$ Ac mains. The required voltage may be obtained by using a 10:1 step down transformer. The current taken by the lamp is got from $P = VI$ as;

$$\begin{aligned} I &= \frac{P}{V} \\ &= \frac{12W}{24V} \\ &= 0.5A \end{aligned}$$

assuming no energy loss in the transformer, then

power input = power output

$$\begin{aligned} P_{in} &= P_{out} \\ 240 \times \text{primary current} &= 12W \\ \text{primarycurrent} &= \frac{12}{240} \\ &= 0.05A \end{aligned}$$

the current taken from the mains is 0.05A

Example

A transformer operating from a 240V Ac supply gives an output of 8V for a ringing doorbell. What is the primary current if a current of 2.4A flows through the bell and the efficiency of the transformer is 98%?

Solution

$$\begin{aligned} \text{Efficiency} &= \frac{\text{power output}}{\text{power input}} \\ E &= \frac{P_{out}}{P_{in}} \\ EP_{in} &= P_{out} \\ E \times P_{in} &= P_{out} \end{aligned}$$

$$\begin{aligned} \text{But} E &= \frac{98}{100} \\ P_{in} &= I_p V_p \\ &= I_p \times 240 \\ P_{out} &= I_s V_s \\ &= 2.4 \times 8 \end{aligned}$$

$$\begin{aligned} EP_{in} &= P_{out} \\ \frac{98}{100} \times I_p V_p &= I_s V_s \\ \frac{98}{100} \times I_p \times 240 &= 2.4 \times 8 \\ I_p &= \frac{2.4 \times 8 \times 100}{98 \times 240} \\ I_p &= 0.082A \end{aligned}$$

Hence the primary current is $0.082A$ or $82mA$.

Exercise

(a) A 60% efficient transformer operates a 12W 48V-discharge tube on a 240 Ac main. Calculate the primary current?

(b) A transformer operates a 12V60W bulb on a 240 Ac main with a primary current of 0.2A. Calculate the efficiency of the transformer?

3. **Microphones**

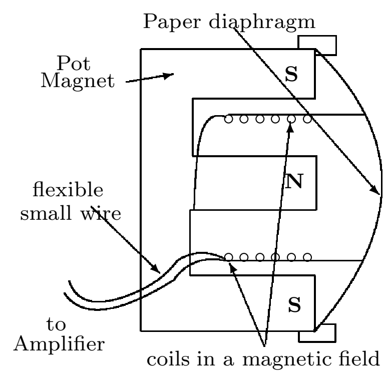
A microphone is an electrical devices which converts sound energy to electrical energy. In their construction, they are of two types;

(a) moving coil type microphone

(b) carbon granule microphone

Moving coil type microphone

Here circular coils attached to a paper diaphragm are placed in a radial magnetic field provided by a cylindrical pot magnet as shown below;



When one speaks near the paper diaphragm, the sound waves produced cause the diaphragm to vibrate and the coil moves back and forth in the magnetic field. This moving or vibrating coil in the magnetic field produces an alternating emf in it (according to Faraday's law). The frequency of the induced alternating emf or current is the same as that of the sound wave producing the vibrations.

This small induced emf or current is fed into an amplifier by a light flexible cable. If one wishes to hear the converted sound, you connect the loudspeaker to the output of the amplifier.

Comparison between the loudspeaker and the microphone

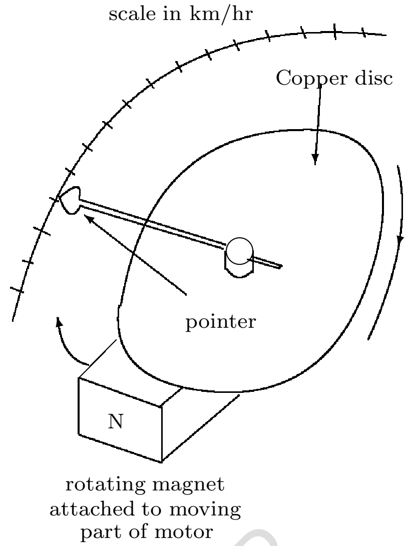
(a) Loudspeaker converts electrical energy to sound energy while a microphone converts sound energy to electrical energy.

(b) The loudspeaker uses the principle of force on a current carrying conductor while the microphone (moving coil type) uses electromagnetic induction to convert sound to electrical energy.

Carbon microphone see appendix page 399

4. **Speedometer**

If a copper disc is rotated so that it cuts the magnetic flux of a magnet pivoted above it, eddy currents are set up in the disc, which in accordance to Lenzi's law oppose the relative motion between the magnet and the disc (i.e. they tend to move together). Consequently the magnet tends to rotate and follow the motion of the disc.
In one type of speedometers, a magnet is rotated near a pivoted aluminium disc by a flexible cable. This rotating magnet induces or sets up eddy currents in the disc. In accordance to Lenzi's law, the eddy currents sets up a couple on the disc, which varies with the speed of the car. The disc therefore rotates until an opposing couple set up by the hairsprings just counter balances its electromagnetic couple. A pointer attached to the disc spindle indicates the speed of the car on a suitably calibrated scale.



5. **Crack detector**

Invisible cracks may become fractures and endanger life, especially when found in or on certain automobiles like; the aero plane body or engine, the railway line and others. These machines can not be easily re-assembled.

To detect a crack on a railway line: a crack detector probe which has a coil carrying alternating current is passed over the rail. This sets up eddy currents in the rails over which it is passed. The presence of a crack will reduce the eddy currents by electromagnetic induction since electrical resistance has increased. This leads to a rise in the voltage across the coil, which is indicated or read on an ammeter, thereby giving a warning of the presence of a crack. [42]

6. **Galvanometer damping**

A galvanometer is an electric device, which consists of a rectangular coil with N turns in a radial magnetic field. And when current passes through it, it deflects through a given angle. When it is counter balanced by hair springs, the deflection (θ) of the coil (or pointer) is directly proportional to the current passing through coil.

$$\theta \ \alpha \ I \qquad (3.299)$$

[42]**Note that** when a large current is taken from an emf source like a battery the voltage across it drops and when a small current is taken, the p.d. across it increases. See the graph on page 247

How is electromagnetic induction applied in it?

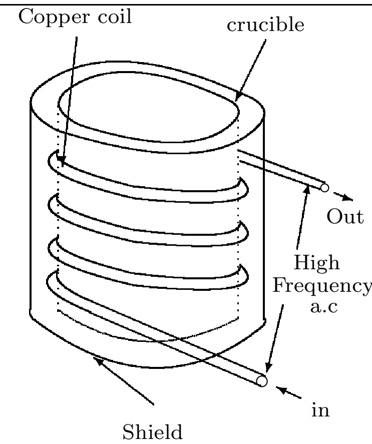
When an electric current passes through it, the pointer is supposed to oscillate before it settles. When it rotates, it (the coil or conductor) moves in a radial magnetic field, here an emf is induced in it according to Faraday's law. But this induced emf or current is in such a way or direction as to oppose the change causing it i.e. to oppose the motion of the pointer, hence it does not oscillate as it would have oscillated if it was not in a magnetic field. This effect of reducing the oscillations of the pointer and it just reads the exact value is called damping. Damping here is caused by electromagnetic induction.

7. **Induction furnace**

The large eddy current in a solid metal can produce so much heat that can melt the metal. Usually the substance to be heated is exposed to a rapidly changing or high frequency magnetic field, which produces large eddy current in the substance.

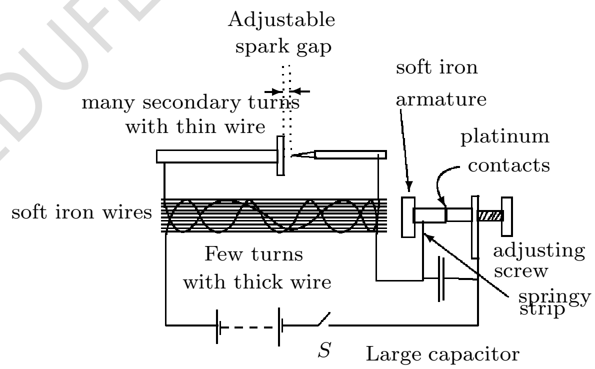
Basically an induction furnace is a crucible containing the metal to be melted, which is surrounded by a coil of copper tubing. When a very high frequency alternating electric current is passed through the coil, the rapidly changing magnetic flux through the metal induces eddy current in it. The heat produced causes the metal to melt. The cold water is passed through the coil to keep it cool.

Note that this same furnace can be used as a metal extractor i.e. to separate the pure metal from its ore.



8. **Induction coil**

A German instrument maker Heinrich Ruhmokorff showed how to produce a 5cm spark using a battery of only a few volts using a coil known as **induction coil** or **Ruhmkorff spark coil** shown below;



It consists of a thick wire wound on a core consisting of a bundle of soft iron wires on top of which is wound a secondary coil with many turns of thin wire. The ends of the secondary coil are connected to an adjustable spark gap. The primary is connected in series with a make and break device like that used in the electric bell. When the switch, s, is closed, current flows through the primary coil and magnetises the iron core which attracts the soft iron armature and causes the platinum contacts to separate and break the circuit. When this occurs the core becomes demagnetised and the armature is pulled back again by the springy strip on which it is mounted completing the circuit.
The whole process is repeated. As a result the

armature vibrates to and fro causing a rapid make and breaks of the primary current.

Each time the primary current is broken the magnetic flux through the core collapses suddenly and the emf is induced in the secondary. Because the secondary has a large number of turns, the induced emf is very high indeed and is sufficient to cause a spark to jump across the gap.
A small arc or spark tends to form between the make and break contacts every time they open, this is due to the self induction of the primary coil, this makes the contacts wear away. This is prevented by connecting a large capacitor across the contacts. The use of soft iron wires instead of solid material core reduces eddy currents and consequently energy losses due to hysteresis.

Uses of the induction coil

(a) To produce electric sparks of desired length using direct current sources.

(b) To ignite the petrol-air mixture in the petrol internal combustion engine during the explosion stroke in each of the four or two sparking plugs. See page 182

(c) To study the electric discharge through gasses i.e. how high voltages can make some gasses at low pressure to emit light, for instance the fluorescent tubes, discharge tubes and others.

(d) To operate **some** x-ray tubes

9. **Choke**

Recall that when running a low voltage device from the mains, current is controlled by means of a rheostat, but this method is very wasteful of energy. Rheostats can also control the current in an a.c circuit but for maximum efficiency it is better to use a choke.

What is a choke? It is a coil or an inductor or a solenoid.

A choke consists of a low resistance copper-coil wound on a laminated iron-core. When alternating current passes through it, it sets up a counter emf due to self-induction. This emf acts so as to oppose the flow of current through it. Therefore this counter emf impedes (opposes) the current flowing through it without leading to wastage of energy. This property of the choke to impede current in this way is called inductive reactance to distinguish it from the ordinary electrical resistance.

Note that sometimes if not always, the back emf or induced emf in the choke is greater than that supplied to it. This property is used in lighting fluorescent tube where high voltages are required using a special type of choke; the starter.

10. **Record play-pickup**

A tape is a plastic coated with a magnetic oxide like chromium oxide on the side nearest the heads. This tape in the radio passes over the heads that play. Usually there are 3 heads; for erase, record and play. Sometimes one head is used to record and to play. These heads are electromagnets i.e. each consists of a coil of wire a round an iron core. When the tiny magnets in a pre-recorded tape pass over the play head, they induce an alternating current in it which can be played out by the loudspeaker.

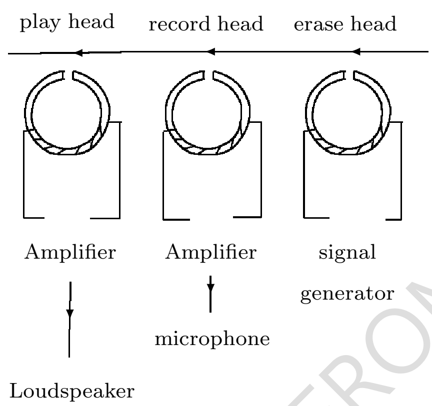
Recording(magnetising)

A tape recorder consist of a motor mechanism that pull the magnetic tape at a constant speed past three (sometimes two) heads. During recording, an alternating current corresponding to the required sound is obtained from a microphone or directly from a record play or radio. The a.c passes through the record coil and magnetises the tape. Each complete oscillation of the Ac produces two tiny magnets on the tape and their north poles face opposite directions. The number of magnets per cm of the tape is proportional to the pitch of the sound because the frequency of the a.c depends on the pitch. The strength of the magnets depends on the loudness of the sound because the amplitude of the Ac produced depends on the loudness.

Playing(electromagnetic induction)

; During playing or playback of the tape, the magnetic tape passes over a play head and the magnets induce an a.c in the coil around the head by electromagnetic induction. The induced a.c is amplified and then fed to a loudspeaker which produces the original sound of the microphone, record or radio.

Note that electromagnetic induction is applied in the production of the a.c in the coil when the tiny magnets (on the tape) pass over the coil i.e. their magnetic flux cuts the conductor (the coils).



11. **Electric brakes**

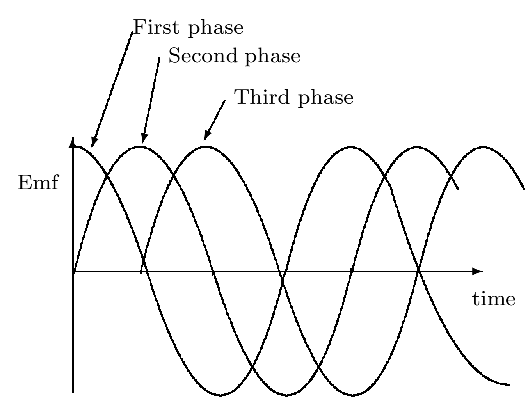
Electric Brakes or eddy current brakes are used in stopping electric trains. The principle of these brakes is that, when a magnetic field is suddenly applied to a rotating metallic disc like the wheel of a train, eddy currents are set up in the disc, these currents in turn exert a torque on the disc so as to stop its motion. This is in accordance to Lenzi's law, hence the disk stops rotating or its speed is reduced.

3.19.8 Production and distribution of electricity

The Grid system:

Electricity is transmitted from one part of a country (Jinja) to another by the grid system. Grid system is a network of cables supported on pylon, which connect very many power stations in a country. Electricity is generated at power stations using oil, coal, nuclear materials, running water, etc. Countries like Kenya, New Zealand, and others make use of running-water-power (hydropower). Hydropower accounts for 80% of the total electricity supplies of such countries. The electricity is moved all over the country by overhead and underground cables. All power stations are linked so that if one breaks down or is stopped for maintenance work the other stations replace its supply. This method of interconnecting power stations is called **the grid system**.

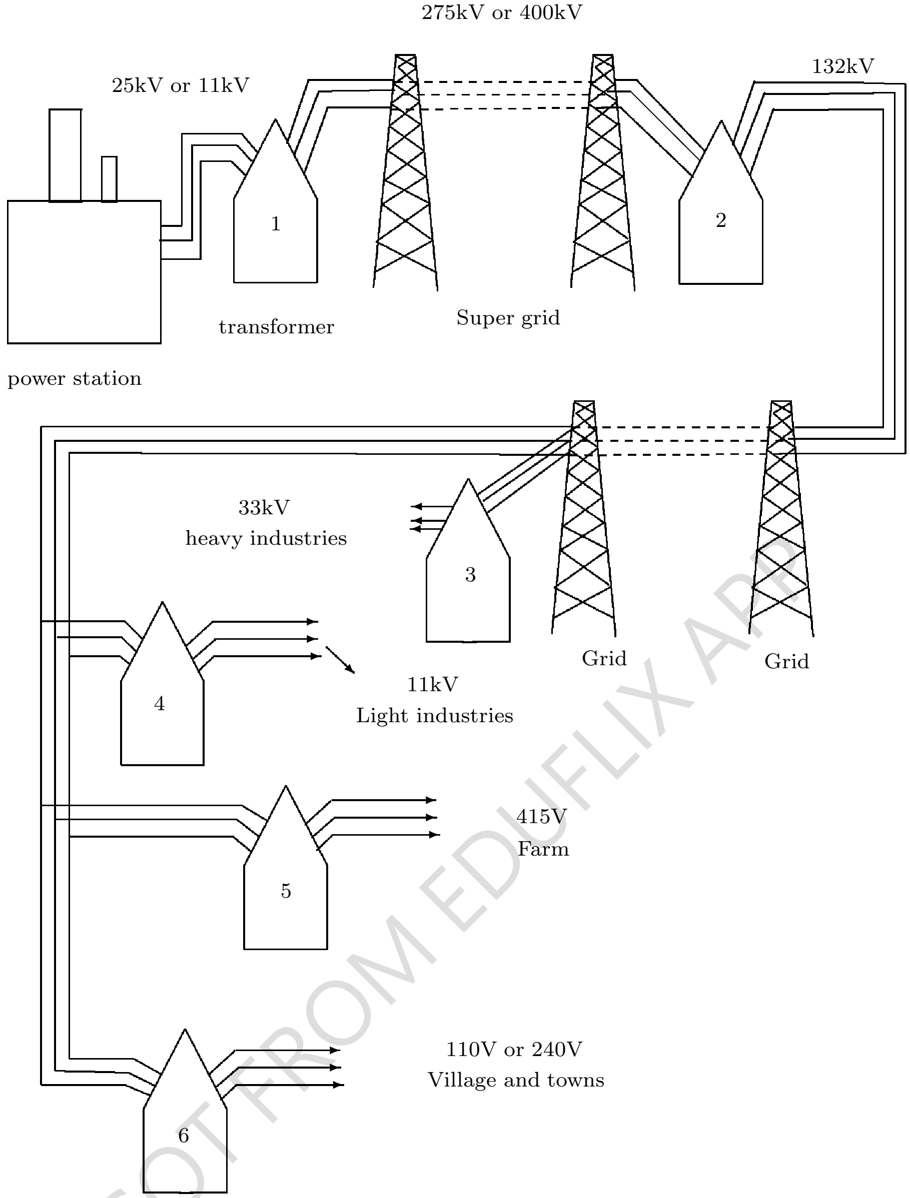
Electricity is produced at the power stations by several means but usually at a voltage of 25000V(25kV) or 11kV. This voltage is fed into the input (or primary) of the step up transformer which steps up the voltage at once to 275kV or 400kV. The generator at the power station produces electricity in three phases, this means that the three separate coils on the armature produce emfs of the same frequency and peak value, but the peaks do not coincide i.e. all of them do not attain maximum value at the same time, this is the meaning of not being in phase.



At this high voltage electricity is sent over long distances on the super-grid or cables on pylons or underground cables. Underground cables are of copper, insulated with rubber and plastics. Overhead cables are of aluminium because it is less dense compared to copper hence the cables are light. They're bare and not insulated because they are out of reach and surrounded only by air (a good insulator). They are supported on high pylons and are fitted to large porcelain insulator to keep the wire from the metal of the pylons. The pylons carry four wires, the one connecting the tops of the pylons in the earth (or neutral) and each of the other three carry a separate phase. When electricity has been transferred over a large resistance (of the wires) and it is near the consumer, it has to be stepped down for consumer use. It is stepped down to various voltages depending on the type of consumer, for instance; It is stepped down to 132kV for transportation to the nearby transformer station (district transformer),

> For heavy industries it is stepped down to 33kV,
> For light industries it is stepped down to 11kV,
> For farms it is stepped down to 415V and
> For villages and towns its stepped down to 240V.

Industries and certain farms use all the three phase wires like the maize grinding mills, they first switch on all the three bulbs (red, green and blue) to test whether all the three phases are on, the machine can not work when one of them is not on. In towns and villages we only need one phase. But usually we see two wires to a homestead, one it the neutral (which is earthed at the nearby transformer station) hence live and neutral.
Note: we shall not talk about the use of power in industries (this is still advanced) but we shall talk about the use of power at our homes (like in towns and villages).



Transformer 1 is a step-up transformer and the rest (i.e. transformer 2,3,4,5 and 6) are step-down transformers.

The Super grids are used when power is to be transported over very long distances.

The values we have used especially for the voltages stepped up and stepped down may vary slightly from country to country depending on a number of factors, one of which can be how the power is generated i.e. nuclear, hydropower, e.t.c.

3.19.9 Why is voltage stepped up before transmission?

To repeat the grid system in summary, the voltage generated at power stations is about 11,000V(11kV or 25kV). It is stepped up by a transformer to about 132kV or 273kV or 400kV, this high voltage is stepped down by district transformers to about 240V or 110V before going to houses and other buildings. Transmission at high voltages reduces energy loss in the transmission wires. The numerical example below shows this;
Consider the transmission of 1 Mega watt, that is 1,000,000W (= 1MW = 1000kW) of electric power over the grid system, this can be done at either 10kV,50kV, 100kV, 200kV or 400kV e.t.c since power, $P = VI$ we can get the current in the cables at each voltage.
Suppose that the resistance of the cables is 10Ω. The heat loss per second in the cables is equal to I^2R i.e. $P = I^2R$.

Power (in kW)	Voltage (in kV)	Current (in A)	Heat loss in the cables, $R = 10\Omega$ i.e. $P = I^2R$ in watts
1000	10	100	100,000 = 100kW
1000	50	20	4000
1000	100	10	1000
1000	200	5	250
1000	400	2.5	62.5

Table 3.27:

Note that, as transmission voltage is increased from 10kV to 400kV the heat loss per second or power loss reduces i.e. it decreases from 100kW to 62.5W, since our aim or objective is to transmit more electric power, should it be at high voltage or low voltage?
At low voltage (10kV);
Power to be transmitted is 1,000,000W,
Power loss in the cables is 100,000W hence
The power transmitted is 900,000W.
At high voltage (400kV)
Power to be transferred is 1,000,000W
Power loss in the cables is 62.5W
Power actually transferred is 999,937.5W

Since more electric power is transferred at high voltage or heat loss in the transmission cables is low at high voltage, electric power is always transmitted at high voltage because of that.

Why is alternating voltage preferred for electricity transmission and not direct current.

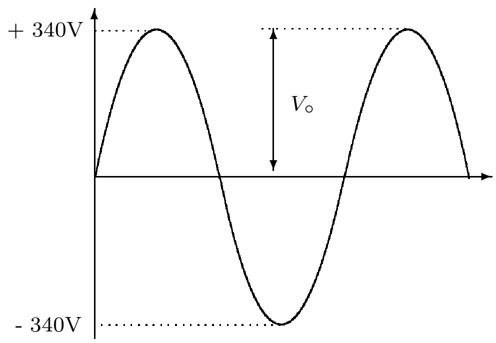
This is because for minimum heat loss, the transmission voltage has to be high and a transformer can only step up voltage when it is an alternating voltage (a.c) and not direct or steady voltage hence a.c. is preferred.

3.19.10 Domestic electric installation

Electricity supply to houses and buildings

Electricity from the grid is fed into the house by a heavy cable, which contains two wires, which are well insulated from each other. One of these wires is the live wire (one of the 3 phases from the power station) and the other is the neutral wire (a wire connected to a metal plate in the ground, near the local step-down transformer or district step down transformer(s)).

The potential of the neutral wire is earth potential (i.e. zero potential) since it is earthed at the power station and the potential of the live wire varies between about + 340V and -340V peak value at 50Hz or 60Hz. This means that the root mean square value (r.m.s.) of the potential of the live wire is 240V, because by definition;



$$V_{r.m.s.} = \frac{V_\circ}{\sqrt{2}} \tag{3.300}$$
$$= \frac{340}{\sqrt{2}}$$
$$= 240.416$$
$$\cong 240V$$

at 50Hz. [43]

When an electrical appliance like an electric fire or lamp is connected to the live and neutral wires, Charge (or current) flows through it. When live is positive, charge flows from live to neutral and when live is negative, charge flows from neutral to live.

Electricity is fed into the house via the two wires entering the company meter box (main fuse) which should never be opened except by the company employees, the wire passes through the meter, to the main switch then to the distribution box. Switches are of two types ;

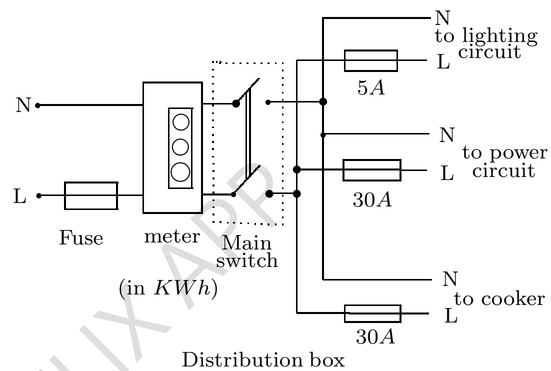
1. Single pole switch which cuts just one wire and
2. Double pole switch which cuts two wires

The main switch is a double pole switch. From the distribution box, there are normally three circuits namely;

[43] The peak value and the root mean square value are discussed again on page 374

1. Lighting circuit.
2. Power circuit (The circuit of sockets)
3. Circuit to the cooker, which is separate from the power circuit.

The diagram below shows how the two wires (live and neutral) form the three circuits.



Wiring in the home

At home we use electric devices of various wattages for instance electric fire (or cooker) 3kW, a lamp 60W and many others. All these need different currents to operate normally. For instance at 240V, a 3kW electric fire takes a current of 12.5A whereas a 60W lamps takes 0.25A. Thus these can not be connected in series (to have the same current). This is the major reason why series circuits are not used and parallel circuits used in wiring a house.

Why house wiring circuits are parallel circuits and not series circuits

1. Electric devices used at homes especially those having different wattages requires different current which is not possible for series circuits but possible with parallel circuits
2. In a series circuit when many appliances are to be used the required voltage across all of them would be too large, this is not simples to obtain. But for parallel circuits all of them are set to the same potential difference (this would be 240V for our case in Uganda)

3. In parallel, if one lamp fails to operate or blows up the other lamps are not affected but in series all the lamps are affected at once (since no current flows)

Because of these few reasons among others all the electrical devices used at home are connected in parallel.

Note that although it is possible to connect a 60W lamp and 3kW electric fire in parallel (because the p.d across each is the same), separate circuits are used for lighting and heating. Less current is used for lighting compared to that used in heating, hence lighting circuits have fine gauge cables (thin ones) and so low value fuses are used and the heating circuits have thick wire cables and high value fuses are required.

Fuses

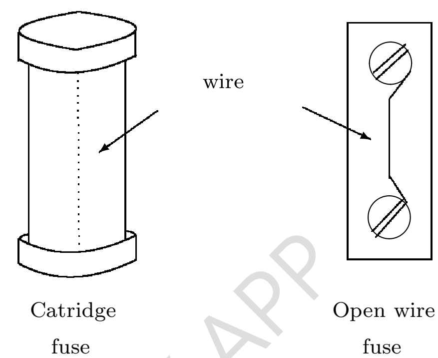
A fuse is a thin wire which burns (or melts) and breaks the circuit when the electric current passing through it exceeds a certain value. The most common reasons for having a fuse in a given circuit are to prevent;

1. **"Short circuit"** i.e. this is when a low resistance or zero resistance wire is connected across two points at a certain potential difference. A large current passes through the wire and can cause an accident. This may be due to worn insulation on connecting wires or otherwise
2. **"Overloading"** This is when high voltages are applied to a device or circuit which is supposed to operate at a lower voltage, . For instance connecting a touch bulb which should use 3.0V to the a.c mains that supplied 240V.

With out the fuse the wiring would become hot and cause a fire. Basically the fuse it meant to break the circuit incase there is a flow of large electric current.
There are two types of fuses;

1. Cartridge fuses and
2. Open wire fuses

These are shown below;



See page 233 for the symbol of a fuse. Always switch off the electric current or power before replacing a fuse and the fault must be rectified before fitting a new fuse. Depending on the size of the fuse wire and other factors, the fuse manufacturer performs a number of experiments on each gauge of the fuse wire and determines the maximum current or current above which the fuse blows and he labels them accordingly. So we have fuses of 30A, 5A, 15A, e.t.c.

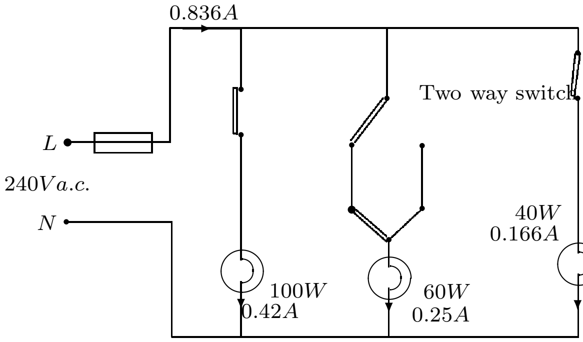
How many 60W lamps can be switched on at the same time on a 240V supply, if there is a 5A fuse in the lighting circuit?

[44]

Study circuit - lighting circuit

The circuit below shows three different wattage lamps connected in parallel, the current in each lamp must be different and they would not function properly if connected in series.

[44] **An alternative for a fuse is a circuit breaker** but how a circuit breaker works is above our level at the moment (the circuit breaker is an advancement of the electric bell).



Two-way switch In the above circuit a two-way switch operates the 60W lamp. This is usually in wiring devices like a lamp on a stairway, this means that, it can be switched on or off from both the top and bottom of the stair.
There are two important points worth noting about the circuit;

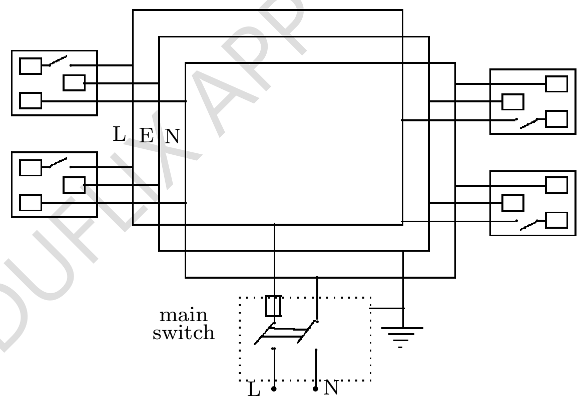
1. The switches are all on the live side of the lamps, hence when the lights are switched off the light socket is not live. If the switches were on the neutral side, the socket would be live when the switch is open (off) and any one touching the bulb casing would get an electric shock.

2. The fuse is on the live side of the circuit, thus when the fuse blows the metal case holding the bulb is safe. If the fuse is on the neutral side, and it blows the appliance would remain live, and any one touching it receives an electric shock.

3. The total current in the circuit is the sum of the individual currents ($0.836 = 0.42 + 0.25 + 0.166$) and so a 1A fuse is adequate. Although it is unlikely that all the lamps in a house are switched on at once, the fuse must allow for this possibility.

Power circuit

This power circuit has a third terminal, the earth wire. Usually at the meter box just in the ground, a metal case (sometimes a bag of certain salts) is buried and a wire connected from it to the three circuits. This wire which is earthed just at the meter box is called the **earth wire**. It is connected to all metal casings or bare metals on electrical appliances like lamp holders and others to avoid electric shock.
Power circuits are usually on a ring-mains. The advantage of the ring-main is that it saves one from a much more expensive and clumsy way of wiring the house where one connects separate circuits from the distribution box to each room. The circuit below shows a simple ring-main in the power circuit;

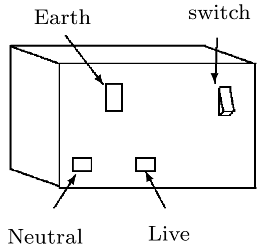


Three-pin socket

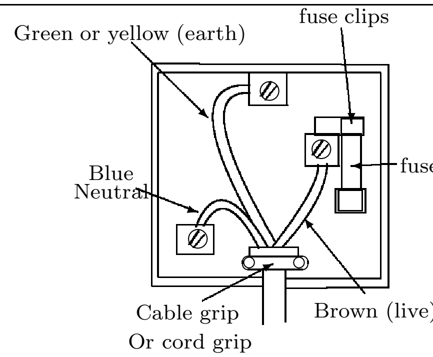
These three pin sockets are connected to the ring mains at different points. The live terminal of each socket is connected to the live wire, the neutral to the neutral terminal and the earth terminal to the earth wire. There are two important safety precautions in the socket;

1. The switch in the socket breaks the live wire (not neutral)

2. The sockets have shutters which only open when a plug is inserted (how). The earth pin of the plug, is usually longer than the other two pins.

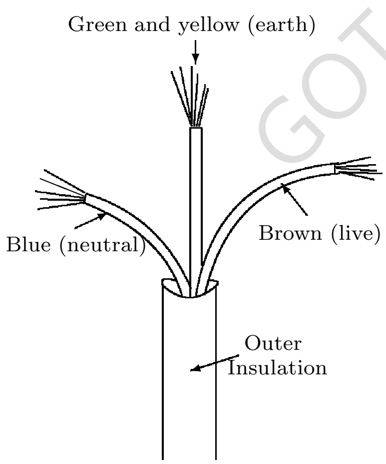


Three Pin Socket



The wire from the ring main to the socket is a three-core cable, this same cable is always used to connect electric appliances such as electric fires to the ring main in the socket. The three wires in this cable are identified by the colour of the insulation on them;

- **Live wire** is covered with **brown insulation**
- **Neutral wire** is covered with a blue insulation and
- **Earth wire** is covered with **green or green-yellow insulation**.



When an electrical appliance is to be used its three-pin cable is connected to the plug as shown below;

This figure shows the three-pin plug with the wires attached correctly. The blue wire is connected to the neutral (N) terminal, the green and yellow to the earth (E) terminal and the brown wire to the live (L) terminal. The N and E terminals are connected directly to the plug pins but the L terminal is connected to one end of a fuse, the other end of which is connected to the live pin. There are three other safety precautions to note in the plug;

1. Each plug contains a separate fuse, so that if a fault occurs in the appliance. This fuse will blow instead of the main fuse. This also has the advantage of pin pointing the fault in the appliance. If the main fuse blows each appliance has to be tested separately to find out the cause.
2. The fuse breaks the live wire so that the appliance is not live when the cause is not yet rectified.
3. The cable grip prevents the wire from being pulled out of the terminal as this can cause short circuiting.
4. More so the appropriate fuse must be chosen like 13A for electric fires.

Special circuits

The electric cooker is not run from the ring main, but has its own separate circuit. This is because it takes a large current.

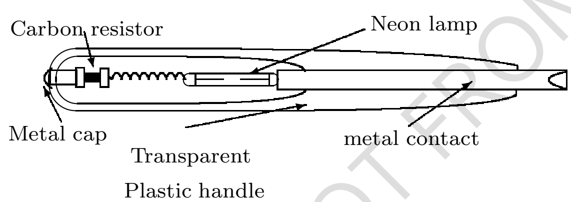
Other devices that take large current include instant showers, domestic hot water heaters, and specials like the lighting systems used on the fences and others. These may also be connected separately to the distribution box.

CAUTION: Over tightening serves no useful purpose apart from breaking the insulation which is very dangerous.

Live mains lead indicator

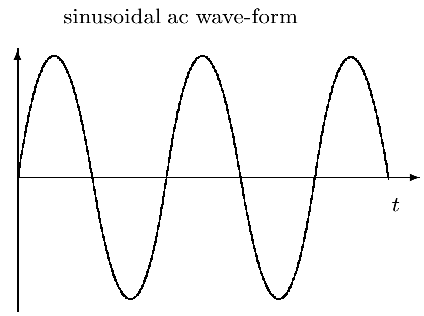
These are used for indicating the live wire, if it is not insulated. They are usually made in the form of a small screwdriver.

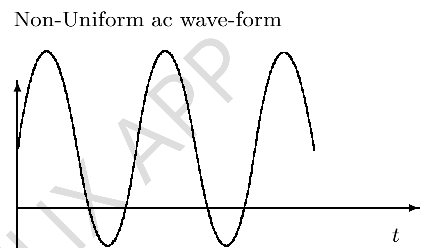
In general it takes the form of a small probe with a hollow-insulating handle containing a small neon discharge lamp. One electrode of the neon tube is in contact with the probe (header metal) and the other is connected through a high carbon resistance to a metal cap on the handle.
When the probe is inserted into the live socket, leakage of current to the earth takes place through your body and the neon lamp glows. Owing to high resistance the current is negligible and hence there is no danger of an electric shock but only indicating the livened socket.

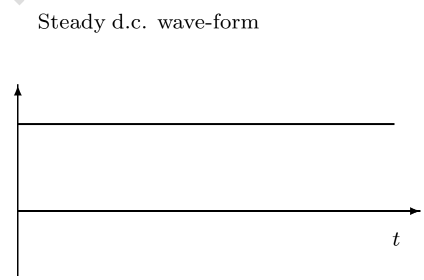


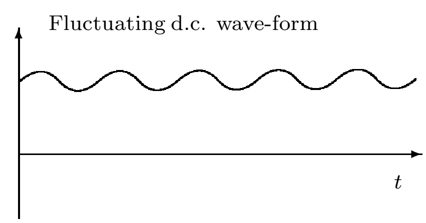
3.19.11 What is an alternating current?

How different is it from direct current?









Direct current in a wire is the flow of electrons from one end at low potential to the other end at high potential. The flow is like that of water in water pipes, it flows in only one direction, it does not reverse. An alternating current or voltage changes in magnitude and direction regularly every after a given period of time. In every cycle its value is zero twice and also peaks twice, one negative and the other positive. The diagrams below illustrate this;

Alternating current in a wire is a surge of electrons, first in one direction and then in the opposite direction. A surge occurs in the whole circuit at exactly the same instant. The frequency of our a.c supply is 50Hz. This means that the period $T = \frac{1}{f} = \frac{1}{50} = 0.02s$. it also means that there is zero current every after 0.01s and also a peak value of current every after 0.01s i.e. the electrons flow in one direction, then stops, then changes direction of flow and flows in the opposite direction

and the cycle continues.
A.c is used in power stations because it can be stepped up or down as required using a transformer. D.c voltages can not be stepped up or down easily and cheaply. However practically all modern electrical appliances (radios, TV sets, computers and others) requires d.c and therefore the a.c of the supply must be converted to d.c before it is of use inside the appliance. The process of converting a.c to d.c is called **rectification**. In rectification we use semiconductor devices which are the basic components of the modern and demanding 'branch' of physics called **electronics**.

3.20 Electronics and Rectification

3.20.1 Introduction

This is a "branch" of physics, which encompasses different activities in banks, factories, homes and hospitals. The development of semi-conductor devices such as diodes, transistors and integrated circuits (IC) or chips has led to the availability of pocket calculators, clocks, programmable musical instruments, heart pacemakers (measures heart beat), microcomputers, robots, TV games, machines for teaching spelling and arithmetic and even recognizing signatures. Let us handle this 'branch' briefly as we head to our purpose of understanding rectification.

3.20.2 Semiconductors

As the word suggest, semiconductors (half conductor or half insulator). They are neither very good conductors nor very good insulators, its known that elements of group I, II, and III are conductors and those of group V, VI, VII are insulators. Semiconductors are elements from group IV in the periodic table. The most commonly used semiconductor elements are carbon, silicon [45] and germanium. They are used to make electronic devices because their conductivity can be increased by adding tiny small but controlled amounts of other substances usually elements of group V or group III. This process of adding these substances (called impurities) is called "**doping**". The pure elements of group IV are called **intrinsic semiconductors** and after doping they are called **extrinsic semiconductors**, therefore there are two types of semiconductors;

1. **Intrinsic semiconductors**; these are the pure semiconductors or pure group IV elements (i.e. without any impurity).
2. **Extrinsic semiconductors** or **doped semiconductors**; these are the semiconductors containing measured or controlled amounts of impurity elements.
   These are of two types of extrinsic semiconductors;

   (i) n - type semiconductors and

   (ii)p - type semiconductors

[45] Silicon is the main constituent of sand

n-type semiconductors:

These are obtained by doping the intrinsic semiconductor with a small quantity of elements from group V. they have an excess electron compared to group IV elements in their outer most shell, hence conduction is by negative electrons. n in n-type stands for negative but the n-type materials are not negatively charged.

p-type semiconductors:

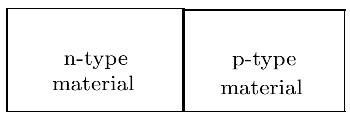
These are obtained by doping the intrinsic semiconductor with small quantities of group III elements. This creates a situation **like** a deficiency of an electron compared to group IV elements i.e. they do not have enough electrons for conduction. Conduction is by imaginary positive charges called "**holes**". p in p-type stands for positive, but the p-type materials are not positively charged.

If semiconductors are heated their conductivity or conductance (reciprocal of resistivity or resistance respectively) increases rapidly or resistance decreases rapidly and the current though it may no longer be controllable, this increased current causes a lot of heating in the semiconductor. Therefore, there must be a way of conducting this heat (I^2R) away, this is done by connecting a small or larger metal case called a "**heat sink**" which conducts the heat easily to the surrounding air hence keeping its temperature or working temperature constant.

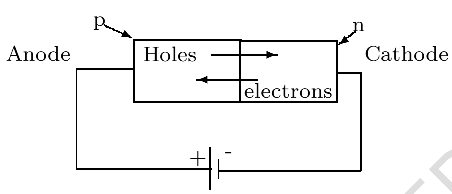
Germanium devices heat up faster than the silicon devices.

3.20.3 Junction diode

When the p-type and the n-type materials are brought together (in a special contact), a p-n junction diode is formed. Note that the p-type material is not charged similarly n-type material is not charged, they are neutral, though they have different concentrations of conduction electrons.

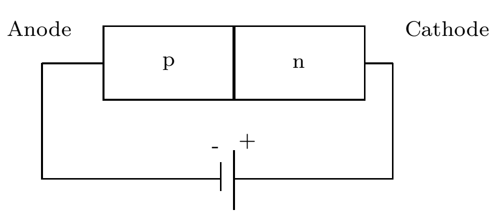


If the positive terminal of a battery is joined to the p-type and the negative to the n-type, the diode conducts current due to the "**holes**" from the p to n-type material and the electron from n to p-type material. In such a connection (where p-type is to positive terminal and n-type to negative terminal) the diode is said to be **forward biased** and the current flowing is **larger**, increases with voltage applied, but not linearly.



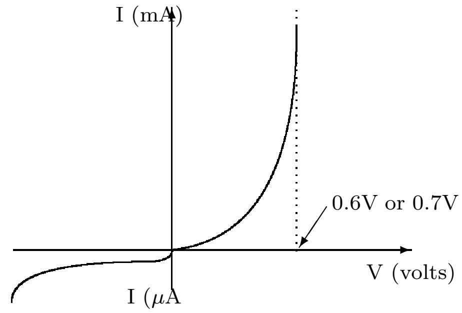
Forward biased

If the battery is connected to the diode in the other way i.e. positive electrode to n-type and negative electrode to p-type, the diode is said to be **reverse biased** and little or no current flows.



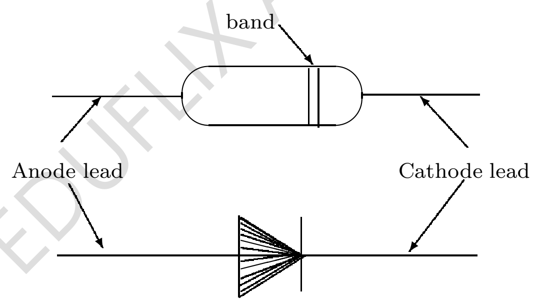
Reverse biased

The graph below shows the current - voltage curve for a semiconductor diode. Current and voltage in forward biased direction is taken to be positive and that for reverse biased negative.



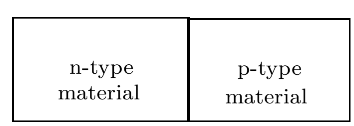
Such a device, which allows current to flow through it in only one direction, is called a diode or valve. This type is called a semiconductor diode not thermionic diode [46].

How does a diode look like and its symbol



symbol of diode

The arrow is in the direction of the conventional current i.e from the anode to the cathode (when it is forward biased). Anode and cathode are the p-type and n-type sides for material of the diode respectively.



3.20.4 Rectification and smoothing

Practically, all modern appliances (radios, TV sets, computers, calculators, e.t.c.) require direct current (d.c) and therefore the alternating current (a.c) supplied must be converted to d.c before it is

[46] Thermionic diodes are discussed on page 359

of use inside the appliance. This process of changing a.c to d.c is called **rectification**.

Definition 85.

Rectification is the process of converting alternating current or voltage to direct current or voltage.

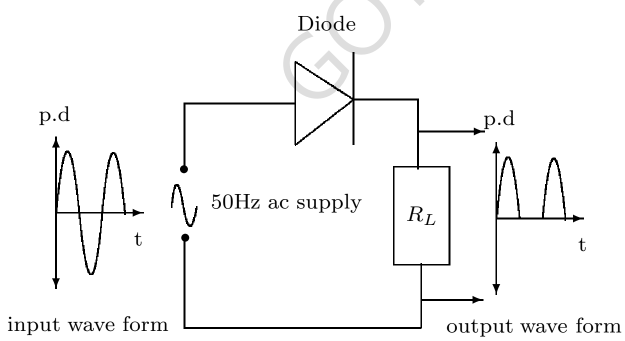
A semiconductor diode or thermionic diode can rectify a.c i.e. convert a.c to d.c. There are two kinds of rectification;

1. Half-wave rectification and
2. Full-wave rectification

Half-wave rectification

Since the diode conducts only in one direction(when forward biased), then when connected to an a.c, it removes the negative half cycles of the a.c to give a pulsating direct current or p.d across the load, R,.

A half-wave rectifier consists of a transformer, diode, and a head resistor. The primary coil of the transformer is connected to the a.c mains and its secondary coil connected to a load resistor, R_L, through a diode as shown below;

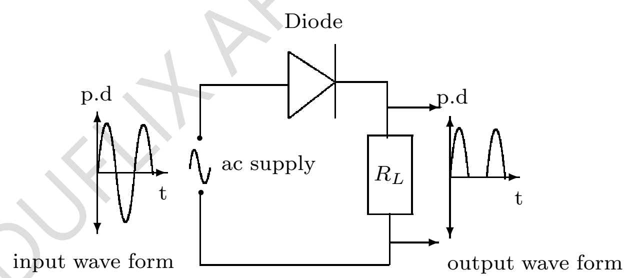


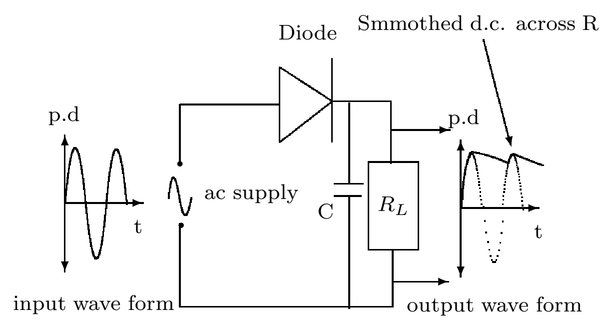
During the positive half cycle of the input sine wave, the diode is forward biased and hence it conducts through R_L. The current flowing in the circuit develops a voltage, V_L, across the load which has the same shape as the positive half-cycles of the input.

During the negative half-cycle of the input voltage, the diode is reverse biased, no current flows and no voltage is developed across the load resistor. This type of rectifier circuit, which allows current to flow during the positive half cycles of the input voltage and block the current during the negative half-cycle, is called a **half-wave rectifier**. The a.c input is converted into a pulsating d.c voltage as shown above.

smoothing

the "**humps**" in the d.c are smoothed by connecting a large capacitor, C, across R_L as shown below;





On the positive half cycle of the a.c supply, the diode conducts and current flows through R_L and also charge is stored in the capacitor. A capacitor stores charge, a capacitor are just two plates or conductors very close to each other but not in contact, when charged the electric field between them stores energy.

On the negative half cycle of the Ac supply, the diode is reverse biased and therefore does not conduct. The capacitor, C, is partly discharged through R_L i.e. it supplies the charges it had stored as current to R_L, keeping the voltage across R_L positive. Thus the charge storing action of the capacitor maintains current in R_L and a steadier voltage across it, hence a smoothing action. Although, there is still a weaker vibration heard.

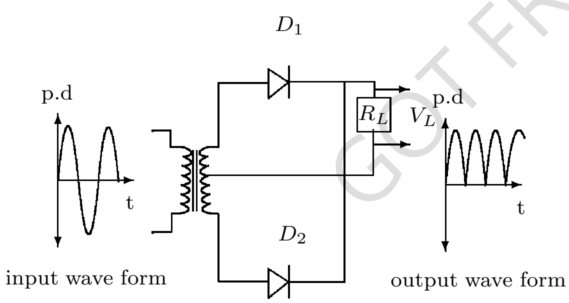
3.20.5 Full-wave rectification

In this case the negative cycles canceled by half-wave rectifier are converted into positive cycles, instead of losing them as in the half-wave rectifier. There are two possible ways of doing this, these are;

1. centre-tap full-wave rectification and
2. bridge full wave rectification

Centre-tap full-wave rectifier

This full-wave rectifier consist of two diodes D_1 and D_2 connected to the center tap of the secondary of the transformer, in such a way that the diodes conduct during alternating half-cycles of the input supply voltage as shown below;

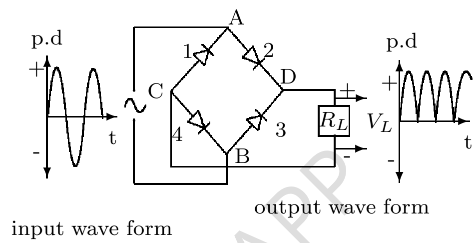


During the positive half-cycle of the input a.c voltage, V_I, the diode D_1 is forward biased and the diode D_2 is reverse biased. Hence the current flows through the load resistor, R_L, such that its upper end becomes positive with respect to the lower point.

During the negative half-cycle, only D_2 conducts. But the current flows in the same direction through the load as before. Therefore the current flows through the load resistor in the same direction for both half-cycles of the input. This circuit is called **Center-tap full-wave rectifier** [47].

Bridge rectifier circuit

This consists of basically four diodes connected as shown below;

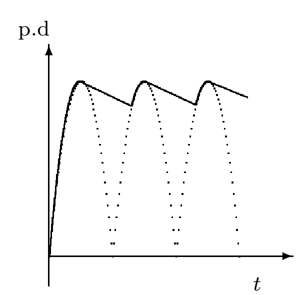


During the positive half-cycle when point A is positive, diode 2 conducts and diode 1 is reverse biased, this makes point D positive.

During the negative half-cycle when point B is positive, diode 3 conducts and diode 4 is reverse biased, still keeping point D positive. As a result of this point D is positive in the two half-cycles, hence the a.c input has been converted to the d.c output across the load resistor.

Smoothing:

If a capacitor is connected between points D and C or across the load resistor, the output is smoothed as shown below;



[47]It involves tapping from the center of the transformer

Exercise

1. Give five fields or places where electronics is applied

2. Discuss briefly what is meant by the term semiconductors

3. There are two types of semi-conductors, name them giving an example of each

4. Which elements are added to the pure semiconductors to make them

   (i) n-type semiconductors

   (ii)p-type semiconductors

   Give reasons for your answers in (i) and (ii) above

5. Give two types of extrinsic semiconductors

6. Semiconductor devices always have a metallic substance in contact with it. what is its name and purpose?

7. When comparing the germanium and silicon devices, which one heats up faster?

8. Describe briefly how a diode can be made from p-type and n-type materials?

9. What is meant by the terms reverse biased and forward biased as applied to semiconductor diodes?

10. Sketch the typical I-V graph of a semiconductor diode

11. Answer the following questions;

    (i) what is meant by the term rectification

    (ii) state the two kinds of rectification

    (iii) state the two possible ways of attaining full-wave rectification

12. With the aid of a well labelled diagram, describe briefly how

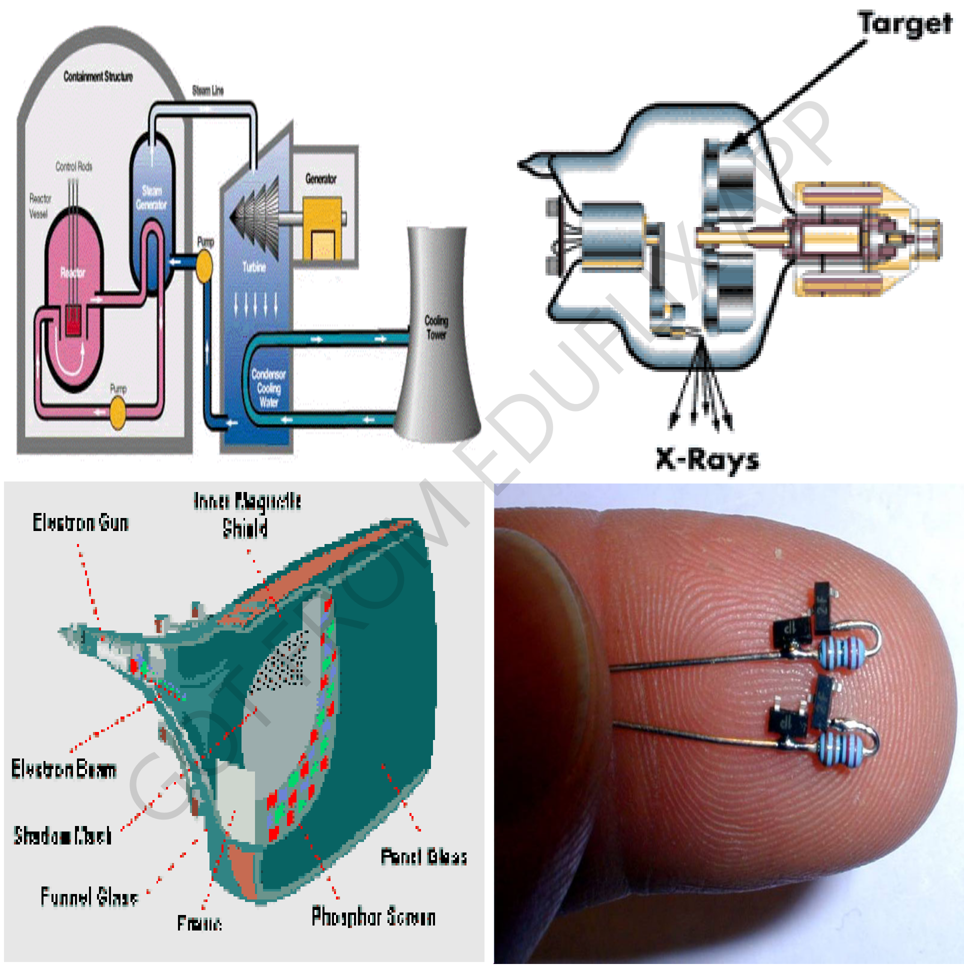
    (i) a half-wave rectifier operates

    (ii) a full-wave rectifier operates

13. Give two reasons as to why rectification is necessary?

14. Smoothing the output of any rectifier is very important, why?

Modern physics



The Photos above show; a block diagram of a nuclear reactor, a small simple x-ray tube, cathode ray tube like a TV screen and two small photo-transistors.

Objectives

After studying this topic, you should:

- Be able to define;
  - ⊛ atomic number
  - ⊛ proton number
  - ⊛ mass number
  - ⊛ nucleon number
  - ⊛ thermionic emission
  - ⊛ photo-electric effect
  - ⊛ nuclear fission
  - ⊛ nuclear fusion
  - ⊛ isotope
  - ⊛ radioactivity
  - ⊛ transmutation
  - ⊛ half-life
- Be able to give;
  - ⊛ 3 examples of isotopes.
  - ⊛ 2 types of x-rays.
  - ⊛ 2 characteristics of photoelectric effect.
  - ⊛ 2 advantages of a cathode ray tube over a moving coil galvanometer.
- Be able to describe;
  - ⋇ briefly how an x-ray tube operates
  - ⋇ an experiment to demonstrate photoelectric effect
  - ⋇ briefly how a cathode ray tube operates
  - ⋇ briefly the nature of radioactive radiations
  - ⋇ how a geiger muller tube works
  - ⋇ how a scintillation counter operates
  - ⋇ the three major properties used to detect radioactive radiations.
- Be able to state;
  - ⊡ 5 properties of cathode rays
  - ⊡ 6 properties of x-rays
  - ⊡ the health hazards caused by x-rays
  - ⊡ the safety precautions that should be taken to a void the health hazards caused by x-rays.
  - ⊡ the agricultural, medical and industrial uses of x-rays.
  - ⊡ the 3 types of photocells
  - ⊡ 4 uses of photocells
  - ⊡ 3 properties of each of the 3 radioactive radiations
  - ⊡ 2 uses of nuclear fission
  - ⊡ 4 medical and 4 industrial applications of radioactivity
  - ⊡ 5 health hazard causes by radioactive radiations

- ⊡ 4 safety precautions that should be ensured to reduce the health hazards caused by radioactive radiations.

- Be able to;
  - ⊛ explain what is meant by;
    - ⊖ thermionic valves
    - ⊖ grid and anode current
    - ⊖ edison current
    - ⊖ quality and quantity of x-rays
  - ⊛ identify alpha,α, beta, β and gamma,γ rays and particles from their cloud chamber pictures.
  - ⊛ relate peak value and root mean square values of voltage or current.
  - ⊛ identify the radioactive radiations from sketches showing their deviations in electric and magnetic fields.
  - ⊛ identify the type radioactive radiations emitted or absorbed from a decay equation.
  - ⊛ determine the half-life of a radioactive sample from some simple given date or from its decay curve.

Contents

3.21 Atomic structure

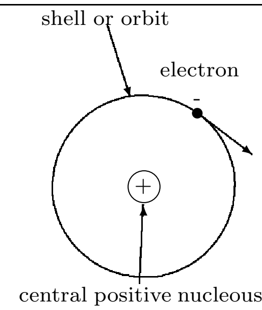


Figure 3.1: Model of an atom

Introduction

This section of atomic physics deals with the physics of very small particles like atoms and molecules. It is sometimes known as modern physics or nuclear physics. When you get any object like a stone, chalk, a piece of wood, a piece of metal or any other object and you break it into two pieces, then one of the two into two and you continue, you end up with an indivisible particle known as an atom. The word atom came from a Greek language meaning indivisible. Experiments were done to prove the existence of atom and today every one believes that atoms exist just because we feel their effects though we have never seen them with our naked eyes. An atom can be positively or negatively charged but what is it composed of?

Atomic structure

Several experiments were performed that led to the discovery of an **electron** as a component of an atom. Various models of the atom (the smallest indivisible particle) were suggested e.g. the Plum-pudding model which regards the atom a positively charged sphere in which the negative electrons were distributed in sufficient numbers to make the atom electrically neutral. (There are no doubts about this model).
All the experiments showed that an atom is the smallest indivisible particle of a substance; it is made up of a small central positive nucleus, small like a pea in the center of a football pitch. This nucleus is surrounded by moving electrons, concentrated in certain circular regions called shells or orbits as shown in figure 3.1 The nucleus consists of two types of particles called protons and neutrons , all have approximately the same masses.

Protons are positively charged and neutrons are neutral (have no charge). Protons and neutrons are collectively called **nucleons** (particles found in the nucleus). The number of protons and nucleons in the nucleus of an atom is called **nucleon number**.

A proton is the nucleus of a hydrogen atom, or a hydrogen atom which has lost an electron (a hydrogen ion). Its charge is equal in size but opposite in sign to that of an electron. Its mass is about 2000 times greater than that of an electron i.e $m_e = \frac{1}{1480}m_p$. The symbol of a proton is P or H. The number of protons in the nucleus of an atom is called **atomic number** or **proton number** . It is equal to the number of electrons in a neutral atom.

A neutron is an uncharged particle found in the nucleus of an atom. It has almost the same mass as a proton. Its symbol is $^1_0$n , we take mass of a proton and that of a neutron to be equal.

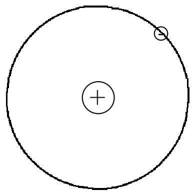
Protons account for the positive charge of the nucleus. In a neutral atom, the number of protons equals the number of electrons surrounding the nucleus. The electrons determine the chemical properties of an atom.

The **mass number** or **nucleon number (A)** of an atom is the number of nucleons (protons and neutrons) in the nucleus of an atom. The atomic nuclei of atoms are represented by symbols .e.g.

Hydrogen is written as ^1_1H
Helium is written as ^4_2H
Lithium is written as ^7_3H

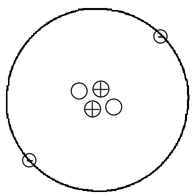
In general atom X is written as ^A_zX where A is the mass number and Z the atomic number .

Note that Mass number (A) of an atomic is not the same as its relative atomic mass (R.A.M.) rela-



Hydrogen

Figure 3.2: Hydrogen atom



Helium

Figure 3.3: Helium atom

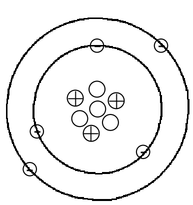
tive atomic mass is the number of times that atom is greater than mass of a hydrogen atom. Since 1960 the carbon scale has been adopted where R.A.M is defined as the number of times an atom is heavier than $\frac{1}{12}^{th}$ of the mass of a carbon-12 isotope.

Isotopes

The homogeneity of atoms is known as an **element**. Examples of elements include copper, zinc, iron and others, So far there are 107 elements known in the world and more are still being discovered.

The numbers of protons in an atom define the type of element the atom is, some times an element may have two or more atoms i.e. having the same atomic numbers but different atomic masses or different number of neutrons, these are called isotopes.

Definition 86.



Lithium

Figure 3.4: Lithium atom

Isotopes of an element are atoms, which have the same number of protons but different number of neutrons.

i.e. their atomic numbers are the same but not their mass numbers. Isotopes have identical chemical properties since they have the same number of electrons and occupy the same place in the periodic table (in Greek iso - means same and tops means place). Isotopes occupy the same position or place in the periodic table.

Few elements consist of identical atoms, most are mixtures of isotopes e.g.

Chlorine has 2 isotopes. Chlorine -17 and chlorine -15, these are represented as $^{37}_{15}Cl$ and $^{35}_{15}Cl$ respectively

These are present in ordinary chlorine in the approximate ration of three atoms to one atom, giving chlorine an average atomic mass of 35.5.

Hydrogen has 3 isotopes: Normal hydrogen $^{1}_{1}H$, Deuterium $^{2}_{1}H$, and Tritium $^{3}_{1}H$

Deuterium has 1proton and 1neutron. Tritium has 1 proton and 2 neutrons. Water nucleus from deuterium is called heavy water (D_2O), it has a density of $1.108gcm^{-3}$, it freezes at 3.8°C and boil at 101.4°C

Questions

1. What is an atom
2. Describe briefly how protons , neutrons and electrons are arranged in an atom.
3. What is meant by nucleon number, proton number and isotopes.
4. Given the atom $^{29}_{11}X$, determine its;

   (i) Atomic number

   (ii) Mass number

   (iii) Number of protons in it

3.22 Thermionic emission

Introduction

These are several ways of removing electrons from a substance; especially a metal surface, one of them is by heating the substance. When a substance is heated, the atoms in it begin to vibrate at a very high speed or frequency with greater amplitudes. This makes the electrons more loose on an atom hence they loose electrons and create an electron cloud just on the surface of the metal called a **space charge**. This process is called thermionic emission.

Definition 87.

> **Thermionic emission** is the emission of electrons from a metal surface when it is heated.

A metal surface may be heated directly or indirectly.[48] The electrons in some of the hot atoms of the metal have enough kinetic energy to escape from the surface just as some molecules escape from the surface of a hot liquid by evaporation. The emitted electrons in the space charge leave the metal with a positive charge equal in magnitude to that of the negative space charge. At any particular temperature, there is equilibrium between the electrons leaving the metal surface by thermionic emission and those returning. Metal surfaces coated with certain oxides like barium oxide and strontium oxide, show thermionic emission more readily than pure metals.

Thermionic valves or diodes

A thermionic valve or diode is a device, which uses thermionic emission to conduct electric current in only one direction. It consists of two metals or wire mesh that has been sealed in an evacuated glass envelope.

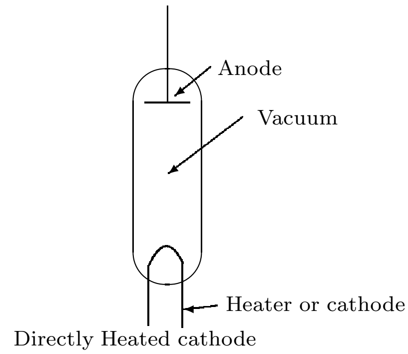


Figure 3.5: Directly heated cathode

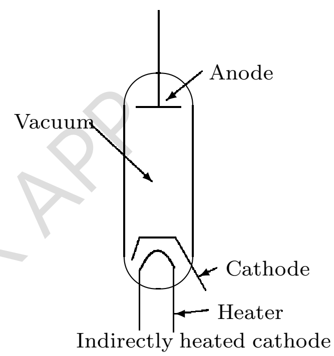


Figure 3.6: Indirectly heated cathode

The cathode may be in a separate heater, some times a tungsten filament is used. Tungsten is used because thermionic emission occurs from it well below its melting point ($3380°C$), whereas other metals emit well only near their melting point.
The cathode may be heated directly or indirectly. In the indirectly heated one, it is coated with an oxide, this is because a filament coated with an oxide gives plenty of thermionic electrons even at dull red heat of about $750°C$.

The anode and cathode are in a highly evacuated glass bulb or tube so that air molecules do not interfere with the movement of electrons.

There are two kinds of thermionic diodes, namely;

1. Those with directly heated cathode and
2. Those with indirectly heated cathode

These are shown in figure 3.32 and figure 3.33 In

[48] Direct heating is heating the metal surface directly and indirect heating is heating some other substance attached to the metal surface

the indirectly heated cathode, an electrical insulator separates the cathode and the heater, usually aluminium oxide, which has a high melting point. Alternating current can be used to heat the indirectly heated cathode but can not be used with a directly heated cathode because it would vary the cathode potential continually, this results into continuous attraction and repulsion of the emitted electrons. A typical heater supply voltage is 6.3 Volts A.c.

When the anode is positive with respect to the cathode, it attracts the negative electrons from the space charge and a current of a few Milliamperes flows. The electrons move from the cathode to the anode but the current they produce is called anode current. When the anode is negative it repels the electrons and no current flows since the diode allows current to flow through it only in one direction.

The valve (diodes) current or anode current can be controlled in two ways;

1. by varying the heater voltage V_h(i.e. amount of emitted electrons) or
2. by varying the anode voltage (i.e. amount of accelerated electrons)

Increasing the heater voltage raises the temperature of the heater , more electrons are emitted and thus increasing the size of the space charge. At a given heater voltage, there is an equilibrium of charges escaping back to the cathode by its slight positive potential. A very sensitive Microammeter between the anode and cathode can indicate (or detect) an extremely small current known as the **Edison current** produced by the electrons with sufficient kinetic energy that crossed the gap and reached the anode.

When the heater voltage is fixed producing a steady space charge, the flow of electrons to the anode or anode current depends on the potential difference, V_a, between the anode and the cathode.

When the anode is at a negative potential with respect to the cathode ($V_a = 0A$), no current

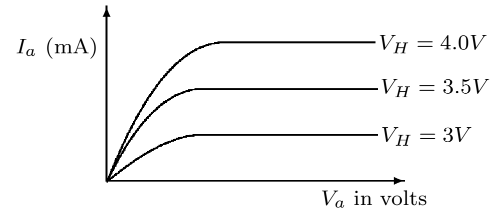


Figure 3.7: Variation of anode current with heater voltage

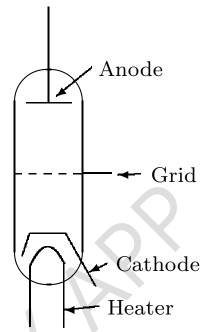


Figure 3.8: Thermionic Triode

flows through the valve. However when the anode is positive, electrons are accelerated through the gap between the anode and cathode and current flows (large current). Increasing the value of V_a increases the rate of flow of electrons through the valve. The anode current, I_a, increases as the anode potential is increased from zero to a certain value and there after remains almost at a constant value known as the **saturation current**, I_s. The graph in figure 3.34 shows this; Another terminal is always added to the thermionic diode to create more control over its anode current. This third terminal between the anode and cathode is called the **grid**. Figure 3.35 shows it; This type of thermionic diode with three terminals is called a **triode**.

How does the grid function?

When the grid is at zero potential, the anode current is just less than what is would be without the grid.

When the **grid** is **at a positive potential**, it attracts electrons from the space charge on the cathode to the anode and hence the anode current, I_a, increases to a value determined by the

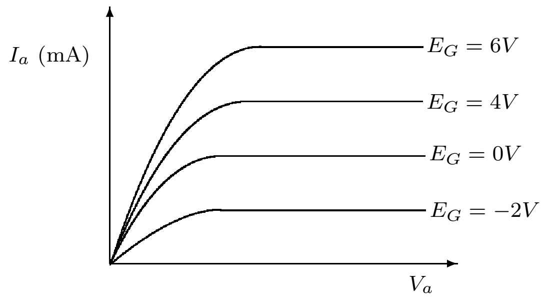


Figure 3.9: Anode current and Grid Voltage

magnitude of the grid's potential.

When the **grid** is **at a negative potential**, it repels the electrons hence reducing the anode current, I_a. hence in general, by just altering the grid's potential, the anode current is either increased or decreased i.e. set to the required value. The graph in figure 3.36 shows the effects of the grid voltage to the anode current

3.23 Cathode rays

Introduction:

In the beginning scientists never know any thing about electrons. Certain experiments were performed; one of them is described below. A metal plate and a larger metal are placed in an evaluated glass tube with the larger metal in contact with a heating element connected to a certain A.c. voltage source, as shown in figure 3.37; The voltage source heats the cathode and electrons will be emitted by thermionic Emission. When the anode and cathode are connected to a very high voltage with the cathode to the negative terminal and anode to the positive terminal, after a certain time, certain striations, whitish in colour were observed running from the cathode to the anode. These striations, which were not known, were called **cathode rays** because they come from the cathode. After subjecting the Cathode rays to electric fields, magnetic fields e.t.c, it was found that they are negatively charged particles moving at high speed, electric current is now known to be moving electrons.

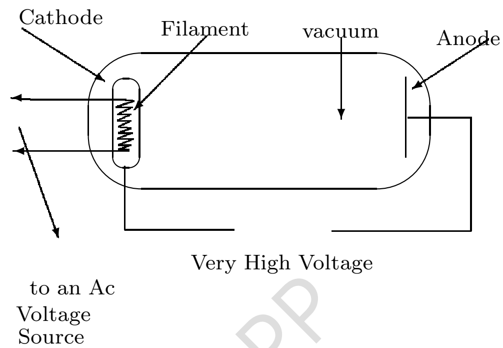


Figure 3.10: Cathode ray tube

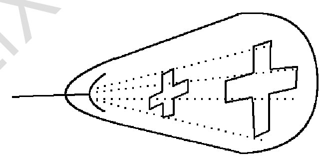


Figure 3.11: Maltese cross

Cathode rays are high-speed electrons.

3.23.1 Properties of cathode rays

Early experiments with cathode rays tubes showed that cathode rays have these properties;

1. They travel in straight lines; an object placed in the path of the cathode rays forms or casts a sharp shadow on a florescent screen. Figure 3.11 shows a Maltese cross as a shadow.

2. They cause fluorescence when they strike certain substances, **Fluorescence** is when a substance gives out light when exposed to certain rays or radiations or electromagnetic waves. The light given out can be of any colour depending on the nature or type of material in the substance it hits. E.g. a screen

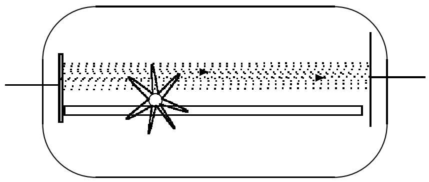


Figure 3.12: Cathode rays and kinetic energy

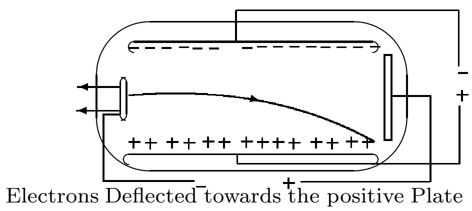


Figure 3.13: cathode rays and electric fields

coated with zinc sulphide gives out green light when stricken by cathode rays.

3. They can penetrate hard matter e.g. a thin sheet of metal like a thin aluminium foil. Hydrogen molecules and helium atoms ,the smallest molecules or atoms can not penetrate such a film.

4. They posses kinetic energy and therefore have mass. This is because the rays rotate a small light paddle wheel pivoted between the anode and cathode (and reversing the voltage reverses the rotation in some cathode ray tube). Figure 3.12 shows a simple arrangement.

5. They are deflected by an electric field. Cathode rays are deflected away from the negative to wards the positive plate. This again shows that the rays have a negative charge. Figure 3.13 shows this deflection.

6. They are deflected by a magnetic field. The deflection is at right angles to the direction of the rays and the direction of magnetic field. Applying Flemings left hand rule it can be shown that the rays have a negative charge. See figure 3.14

7. They affect a photographic film or photographic plate, i.e. when a photographic film (e.g. negative of a photograph) or a photographic plate (like a photograph) is placed in

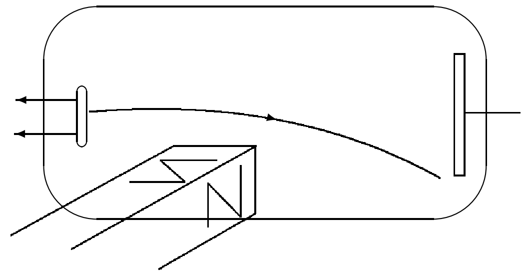


Figure 3.14: Cathode rays and magnetic field

the pathway of these rays, they become forged (or become whitish or black).

8. They produce X-rays when they strike matter. When Roentgen was near a cathode ray tube with some minerals, these minerals could glow giving out certain colours.
He suggested that this must be due to certain rays from the cathode ray tube and these rays were called X-rays (unknown rays)

3.24 X-rays

3.24.1 Introduction and production

In 1895, a German physicist Wilhelm Röentgen discovered that when a cathode ray tube was operating near some barium platinocyanide crystals, they glowed brilliantly even when the tube was covered. More so, wrapped photo graphic plates become fogged like those exposed to light. Later research showed that the electron stream in a cathode ray tube produced invisible and very penetrating rays when the electrons struck the walls of the glass tube. The radiation was called X-rays because it was not known whether it was a stream of particles or a wave or radiation similar to light. It is now known that X-rays are a form of electromagnetic radiation of short wave length and therefore of very high frequency and energy ,they are produced in modern X-rays tube by firing high energy or high speed electrons to a hard metal target inside an evacuated tube.

This process of producing x-rays is the opposite of the Photo-electric effect in which electrons are

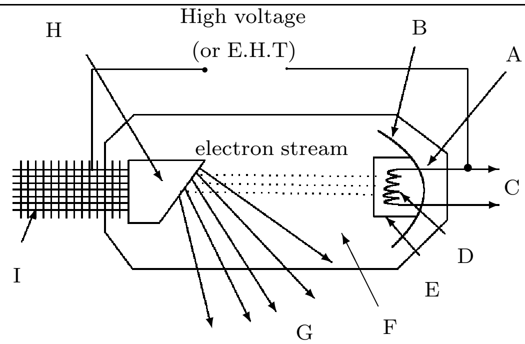


Figure 3.15: X-ray tube

released from metals by various kinds of radiation.

X-ray tube

An x-ray tube is a cathode ray tube in which the electrons produced by thermionic emission are accelerated to high speed by a high voltage and made to hit a hard metal target where x-rays are produced. Figure 3.15 shows an x-ray tube

A-filament leads

B-Focusing cap (concave metal or reflector)

C-Low voltage supply

D-tungsten filament wire

E-Hot cathode (lower block)

F-vacuum

G-X - rays

H-tungsten target (or hard metal target)

I-cooling copper fins (some times a cooling liquid is used)

Action ;how it works

The filament connected to a low voltage Ac source, is used in heating up the hot cathode that emits electrons by thermionic emission.

The concave metal behind the cathode concentrates, or collects the stream of electrons to the anode.i.e. directs all the electrons to the metal target. The emitted electrons in the electron cloud are accelerated to a very high speed by the very high voltage or the Extra High Tension (E.H.T) of the anode, these strike a hard metal target producing x- rays. 99% - 99.9% of the kinetic energy of the electrons is changed into heat energy at the target and the rest to x-rays. since most of the energy changes to heat the, target becomes very hot and it may even melt, hence it should be made from a metal with a very high melting point (e.g. tungsten , platinum or molybdenum).

A tungsten target is used because it has a very high melting point that can withstand the heat generated on impact. The generated heat is transferred away to the surrounding air by the cooling copper fins. Some times in some x-ray tubes, moving water or oil is used to transfer the heat a way. This would be more effective because it cools down fast due to the larger specific heat capacity of water compared to that of copper but the problem is how to have a continuous supply of water or oil.

3.24.2 Intensity and quality of x-rays

Definition 88.

The intensity of x-rays or **the quantity of the x-rays**; is the energy transferred by the x-rays per second per unit area of exposure.

This depends on the temperature of the tungsten filament, determined by the number of electrons emitted by the filament by thermion emission. In turn the temperature depends on the current passing through the filament, the variable resistor in the low voltage A.c. source circuit in series with the filament therefore controls the intensity of the rays.

Definition 89.

The quality of x-rays is the penetrating power of x-rays.

It depends on the velocity of the electrons when they are just hitting the target. In turn the velocity depends on the accelerating voltage , the

EHT, across the tube. The variable resistor in the primary circuit of the step up transformer whose secondary is supplying the EHT, therefore controls the quality of x-rays.

Types of x-rays

There are two types of x-rays

1. **Soft x-rays**; These are the less penetrating x-rays produced when a lower accelerating voltage is used. They are usually for medical purposes.
2. **Hard x-rays**; These are the more penetrating x-rays produced when a high accelerating voltage is used. These are usually for industrial purposes.

Differences between hard and soft x-rays

	Hard x-rays	**Soft x-rays**
Production	high voltage is required	Low voltage is required
wave-length	Short	long
Energy	are high energy x-rays	are low energy x-rays
Ionising power	High	Low
Penetrating power	high	low

Table 3.28: Hard and Soft x-rays

3.24.3 Properties of x-rays

The important properties of x-rays are:

1. They travel in straight lines. This is because they cast sharp shadows of metals and other objects in their path on a florescent screen .
2. They readily penetrate light matter such as flesh, wood, cloth, cardboard and even some thin metals. [49]
3. They affect a photographic plate or film in the same way as ordinary light does; hence x-ray photo-graphy is possible.
4. They cause fluorescence in certain substances. For instance a screen coated with barium platinum cyanide can be used to detect and view x-rays by observing the light colours it gives out.
5. They cause ionisation especially in gases, i.e. they ionise a gas and increase its conductivity. Therefore when x-rays fall on a charged electroscope it becomes discharged due to loss of charge from it[50]. They also ionise and destroy body tissues and the effect is cumulative i.e. neighbouring tissues also become affected with time.
6. They are not deflected by either electric or magnetic fields hence they have no charge..
7. They cause photo-electric emission, the ejection of electrons from metal surfaces when they are exposed to x-rays. Not all metals can show photoelectric emission easily like most group I metals and few others. Group III and group IV elements **may** not show any emission due to their high work function.
8. They are diffracted by crystal atoms.
9. They are electromagnetic waves with a very short wavelength in the range of 10^{-8}m to 10^{-13}m.

Note that; x-rays are detected by using properties 3, 4 and 5 i.e photo-graphic effect, florescence and ionisation of gases.

[49] Lead glass (sodium lead silicate) is a good absorber of x-rays. But ordinary glass (sodium calcium silicate) is not.

[50] By photo-electric emission in addition to ionising the surrounding air

3.24.4 Health hazards caused by x-rays

These are the damages or effects caused by x-rays on a human body. They include;

1. Sterility i.e. inability to produce.
2. Genetic mutations;change in the genes of the offsprings.
3. Eye cataracts; the eyes are affected.
4. Deep seated burns; the skin acquires patches as if it was burnt.
5. Weakens up white blood cells or reduces body immunity to diseases.

3.24.5 Precautions

To prevent or reduce on the health hazards caused by x-rays the following precautions should always be taken.

1. Always wear a lead coat whenever going to an x-ray laboratory or room. This is because they are made from lead a high-density metal which is has a high absorptive power for x-rays.
2. You should not over expose your body to x-rays (even if you are a patient).
3. Always put on a dose meter, a dose meter measures a mount of x-rays taken up by a person. Beyond a certain dose you are supposed to leave the laboratory (for some time or be given a leave, if you are to live longer).
4. Always stand behind thick walls of concrete.
5. Always cover the x-ray tube with lead shields to prevent stray x-rays.

3.24.6 Uses of x-rays

Uses of x-rays can be classified into three;

1. Agricultural uses.
2. Medical uses.
3. Industrial uses.

1. **Agricultural uses:**

   They are being used in agricultural research centres (e.g. Kawanda research centre in Uganda) to bring a bout change in spices of plants and animals, e.g. oranges, passion fruits, e.t.c. This is part of genetic Engineering.
   item **Medical use:**

   - They are used in detecting broken parts, metallic particles or fractures in the bone by blackening out a photo graphic plate.
   - Detecting metallic equipment that may be harm full to your body for security reasons.
   - In detecting Tuberculosis in the lungs using x-ray photography.
   - In the treatment of cancerous and malignant growth in the human body.

2. **Industrial use:**

   - Detecting of cracks and flaws in metal castings and welded joints which are invisible to the eye.
   - In art paint analysis to find out whether the paintings are genuine or imitated.
   - In detecting defects in motor tyres.
   - In investigating the structure of crystals

Questions on x-rays

1. On an x-ray tube what is the purpose of

   (a)The low voltage supply

   (b)The extra high tension

   (c)Focusing cap

2. Explain why

(a)Tungsten metal is used at the target,

(b)Tungsten target is put in a copper pot,

(c)The x-ray tube is evacuated

(d)Oil can be a substitute for cooling fins.

3. What is meant by thermionic emission?
4. What percentage of the kinetic energy is converted to x-rays and where does the rest go?
5. How is heat dissipated to the cooling fins transferred to the surrounding air?
6. Lead absorbs x-rays, how does thickness of a lead sheet affect its absorption capacity of x-rays
7. If x-rays are waves, wont the reflected x-rays affect some one in an x-ray laboratory discuss?
8. (a) Describe with the aid of a diagram the action of the modern hot cathode x-ray tube.

   (b) Draw the diagram of a circuit;

   - (i) Suitable for heating the filament and varying its temperature
   - (ii) Suitable for producing the high voltage (E.H.T) on the anode and varying it.
9. (a)What is meant by the intensity and quality of x-rays?

   (a)How are they varied in an x-ray tube,

   (a)What are the differences between soft and hard x-rays?
10. In an x-ray tube heat is produced at the target, how is this heat removed from the target;
11. How does the penetrating power of x-rays depend on the voltage applied across the x-ray tube?
12. How is the penetrating power of x-rays affected by an increase in their frequency?
13. State three important facts about x-rays (not their uses) other than their penetrating power.
14. Explain how the properties of x-rays make them suitable for both medical and industrial photo-graphy
15. Out line the ways in which x-rays are a danger to human health.
16. State how operators of x-ray machines are protected from the bad reflects of x-rays.
17. What is the effect of;

    (a) Strong magnetic field and

    (b) A strong electric field on x-rays.
18. Mention three ways in which x-rays can be detected.
19. How is the wavelength of x-rays compared with that of visible light rays?

3.25 Photo-electric effect

Introduction

It was found that when certain radiations (e.g. x-rays, ultra violet light, e.t.c.) fall on metal surfaces, electrons are emitted. This process is called **photoelectric effect** or emission and the electrons emitted photo-electrons.

Definition 90.

Photoelectric emission is the emission of electrons from a metal surface when hit by a radiation with a frequency more than the cut-off frequency of the metal surface.

How was it verified?

3.25.1 An experiment to demonstrate photoelectric effect

1. Clean a Sheet of zinc with emery paper, which removes a film of zinc oxide.
2. Place the zinc on the cap of an electroscope.

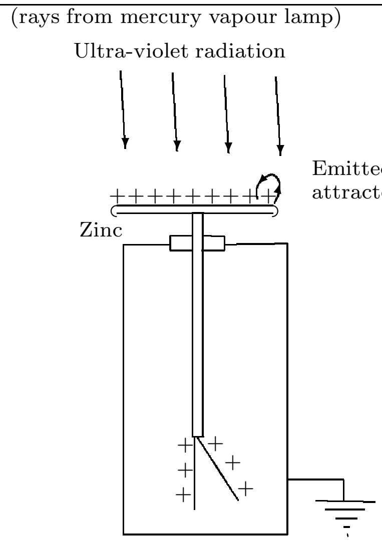


Figure 3.16: Positively charged electroscope

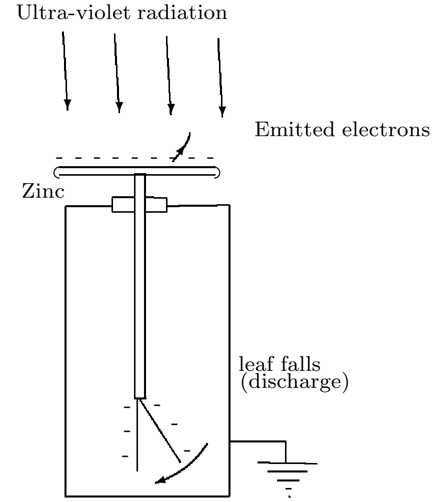


Figure 3.17: Positively charged electroscope

3. Charge the electroscope positively and then shine ultraviolet radiation from a mercury vapour lamp on the zinc. There was no effect; the electroscope never lost charge i.e. the leaf divergence remained constant.

4. The electroscope is now charged negatively and then the ultraviolet radiation is shine on to the zinc. It was observed that the leaf divergence decreased which means that the electroscope has lost negative charge, a discharge. (How?) (It has lost electrons).

5. A sheet of glass between the lamp and negative zinc plate stops the discharge.

Explanation of the observations

1. These is emission of photo electrons from the metal surfaces when x-rays, ultraviolet radiations, light or infrared radiations fall on them.

2. When the zinc sheet is positively charged the negatively charged photoelectrons emitted from it by the ultraviolet radiations are attracted back to the positive zinc and fails to escape (hence no discharge) see figure 3.16.

3. When the zinc sheet is negatively charged, the negative charge on it repels the negative photoelectrons and therefore both the zinc and electroscope lose charge. The electroscope is therefore discharged. This was indicated by a decrease in the leaf divergence in figure 3.17.

4. A sheet of glass between, absorbs the ultraviolet radiation and stops it from reaching the zinc: this shows that it is the ultraviolet radiation and not the violet light that causes this effect. (Why?) This is because each metal surface has a minimum frequency below which no photo electrons can be emitted. For instance if the cut-off frequency (minimum frequency) is 35000Hz then any radiation with a frequency more than this causes photo-electric emission and any radiation with a frequency less than this value can not cause photo-electric emission. Figure 3.18 shows this The leaf does not fall when no radiation reaches the zinc plate. Glass absorbs ultraviolet radiation

N.B: photoelectrons are the same as all other electrons.

3.25.2 Properties of photoelectric effect

1. All metals show the photoelectric effect.

2. The number of photoelectrons emitted per

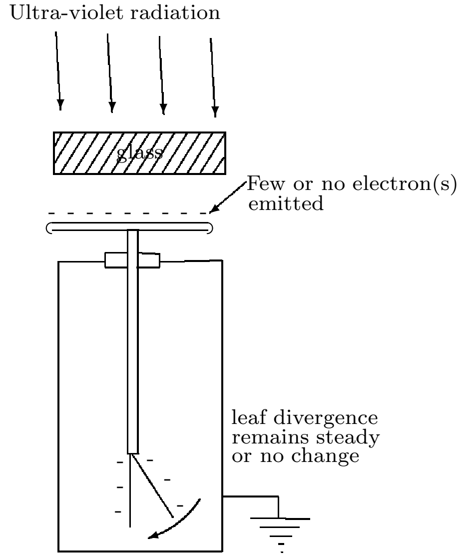


Figure 3.18: Effect of glass

second is proportional to the brightness or intensity of the incident radiation.

3. The velocities and energies i.e. kinetic energy of the photoelectrons, ranges from zero to a maximum value which is independent of the intensity of the radiation but which increases with its frequency.

   The intensity of the radiation determines the number of photoelectrons emitted. And the frequency of the radiation determines their energy.

   For a given metal, there is a minimum frequency of radiation called the threshold frequency, below which no photoelectrons are emitted no matter how high the intensity of the incident radiation is. For instance for the metal rubidium, a bright red light emits more electrons than a dull red light or a dull blue light, but the blue light produces electrons with a greater maximum velocity. This is because frequency of blue light is higher than that of red light.

Note that for most metals the threshold frequency is in the ultraviolet region and for some metals it is even in the x-ray region (10^{17} Hz).

Photoelectric effect is strongest in all Alkalis metals, for instance;

- Zinc emits electrons when exposed to x-rays or ultraviolet radiations.
- Sodium emits electrons when exposed to x-rays (10^{17}Hz), ultraviolet radiation (10^{15}Hz) and with all colours of light accept orange and red (colours with the lowest frequency)
- Rubidium responds even to red light
- Caesium responds to all the radiations mention and also to infrared radiation (3×10^{11}Hz)

3.25.3 Applications of the photoelectric effect

A photoelectric effect to be used, we need a photocell or a photoelectric cell. It changes radiations energy into an electric current. Photo-cells are used in intruder or burglar alarms, automatic counting and control systems, to produce sound from a cine film and in light meters and television cameras. There are three types of photocell.

1. **Photo emissive cell; the common photocell.**
2. **Photoconductive cell.**
3. **Photo voltaic cell.**

1. **Photo emissive cell:**

   It consists of a cathode as a curved metal plate coated with an emissive surface such as caesium and an anode or mesh of metal for the anode as shown in figure 3.19 and figure 3.20; The anode and cathode) are in an evacuated glass bulb, but some bulbs contain an inert gas. When radiations fail on the cathode, photoelectrons are emitted and move towards the positive anode and hence a current flows. The current in the circuit is a few microamperes and increases with the intensity of the radiation, this current flows through the circuit connected to the cell. Change in the incident radiation changes the

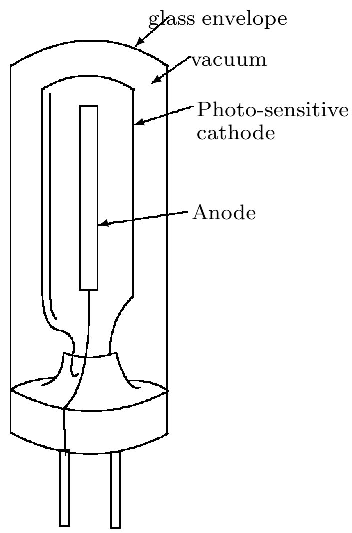


Figure 3.19: Photo-emissive cell

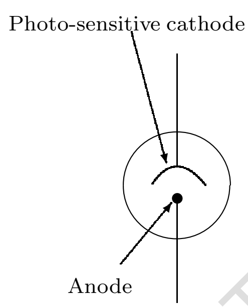


Figure 3.20: Symbol of photocell

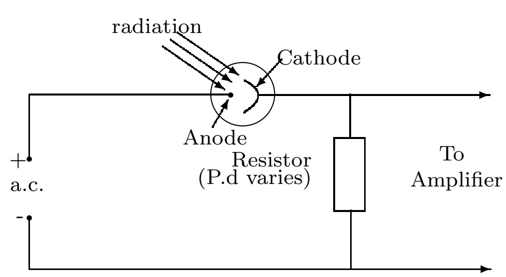


Figure 3.21: Electric circuit with Photocell

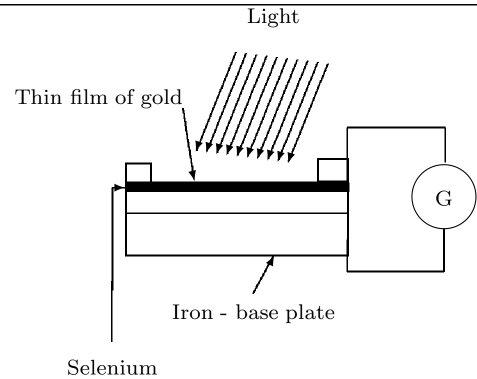


Figure 3.22: Photovoltaic cell

current, which flows through a resistor and operate an amplifier. This type of photo-cell is commonly used in Bulgar alarms (used in security systems).

2. **Photoconductive cell:**

   The cathode is a photo conductor (e.g. cadmium sulphide) whose resistance decreases enormously when incident light or infrared radiation falls on it. For instance the resistance may fall from over one million ohms to less than one thousand ohms. The current in the cell increases with the intensity of radiation. Infrared sources can be detected even several miles a way using this type of photo-cell. Figure 3.21 shows this.

3. **Photovoltaic cell:**

   An Emf is generated when light fails on a thin layer of selenium on iron (or a translucent film of gold deposited on the selenium). Copper(i)oxide and copper can replace the selenium and iron. Variation in the intensity of the incident light varies the emf and the variations are shown by a galvanometer as shown in figure 3.22;

3.25.4 Uses of photo cells

1. A photovoltaic cell is used as a light meter in photography and helps the photographer to determine the exposure (amount of light) required for best pictures or photographs.

2. In a burglar alarm; radiation from an exciter lamp falls continuously on an infrared photocell and a steady current is produced. Any one moving between the lamp and the cell reduces the radiation and starts the alarm.

3. In counting systems, objects such as bottles or tins on a conveyor belt in a factory cut off the radiation momentarily and operate a counting mechanism, which counts the number of objects. For instance the number of Pepsi cola bottles that have passed a given position.

4. A photoconductive cell is used to switch on and off artificial lighting on the streets according to the prevailing light conditions in the atmosphere. Formally, clock mechanisms were used which depended on time rather than the more important natural lighting available. Photo-cells have one problem of switching on street lighting during thunderstorms or when thick clouds cause darkness but clock mechanisms do not.

5. A sound track on the cine film: a cine film is a dark substance which varies the intensity of light passing through it from an exciter lamp, the variations are caused by changes in either areas or thickness of the film, when light falls on a photocell, current is produced whose value changes with the intensity of the light. This current resembles exactly that produced by a microphone or by the recording apparatus when making the cine film. The current produces a varying p.d across a resistor in a circuit, the p.d is amplified and the amplified p.d produces sound in a loudspeaker.

6. A television camera allows light from a scene being photographed to fall on a thin cathode, which emits photoelectrons. The photoelectrons produce a varying electric current, which varies a according to the intensity or colour of the light received, in effect, the camera converts light image into an electrical image, which is used to produce a signal for transmission to another part of the world.

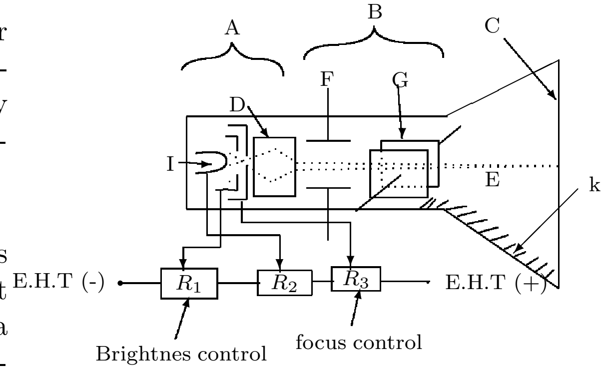


Figure 3.23: C.R.O.; for practical purposes

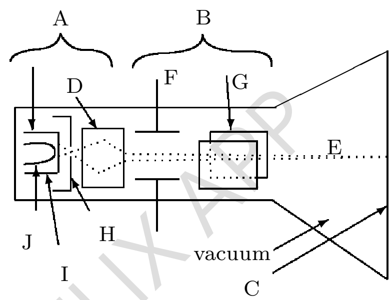


Figure 3.24: C.R.O.: for examination purposes

3.26 Cathode ray oscilloscope

Introduction:

It is an evacuated tube having two electrodes at its ends with a high potential difference between them, which drives electrons (i.e. cathode rays) from the negative electrode (cathode) to the positive electrode (anode).

In this cathode ray oscilloscope (C.R.O.), the electrons are deflected by electric fields. It consists of three parts;

1. The electron gun.
2. A deflection system and,
3. A fluorescent screen

A-Electron gun

B-Deflection system

C-Fluorescent screen

D-Accelerating and focusing electrodes

E-Electron beam

F-Y-plates (Deflect the electron beam vertically)

G-X-plates (Deflect the electron beam horizontally)

H-control grid

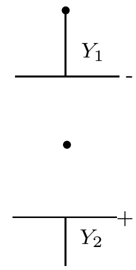
I-Hot cathode

J-Heater

K-Graphite coating

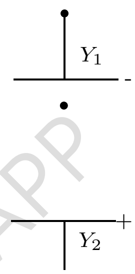
The electron gun

The electron gun produces a narrow beam of electrons (cathode rays); which then produces a bright spot of light on the fluorescent screen. The electron gun, consists of a hot indirectly heated cathode, which produces electron by thermionic emission. The electrons are accelerated to the anode by the E.H.T. (Extra high tension) This high voltage makes the anode positive and the cathode negative. When the electrons pass through the anode, they move with an averagely constant velocity (if there is no or there is negligible collisions). The more positive the anode voltage the sharper or the more clear the spot on the screen becomes, hence the resistor (voltage divide) which controls its voltage is called the **focus control**. (It can make the spot sharp or blurred). The number of electrons reaching or hitting the screen (those passing through the anode) is controlled by the cylinder called **grid control**. It is always at a negative potential with respect to the cathode hence electrons are just repelled back to the cathode by this cylinder. Since the brightness of the spot on the screen depends on the number of electrons hitting it, then the resistor controlling the p.d. of the grid is called **brightness control**. When the grid is sufficiently negative no electrons pass through and there is no spot on the screen. Anode may consist of more than one cylinder and they act as a lens for the electrons i.e. electron lens.



Dot undeflected (centered)

Figure 3.25: (a)



Dot deflected up

Figure 3.26: (b)

Deflection system.

The electron beam from the electron gun passes between two horizontal metal plates (y-plate), which deflect it vertically and between a similar pair of vertical plates (x-plates), which deflect it horizontally.

For the horizontal plates (Y-Plates)

If no potential is applied on either of the plates the spot is at the center as shown in figure 3.25. If plate Y_1, is at a higher potential than Y_2 the spot is deflected upwards from its central position on the screen as shown in figure 3.26. If plate Y_2, is at a higher potential the spot is deflected down wards as shown in figure 3.27. If an alternating voltage is applied on the Y-plates, the spot is deflected up and down continuously and if the frequency is high enough, we see a vertical line due to persistence of vision. This is shown in figure 3.28.

For the vertical plates (X-Plates)

When the X-plates are at different voltages the

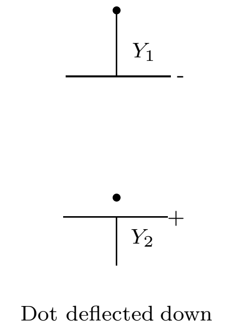


Figure 3.27: (c)

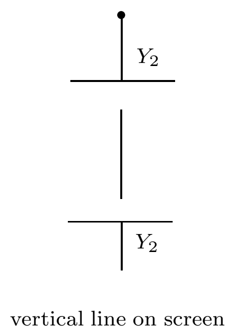


Figure 3.28: (d)

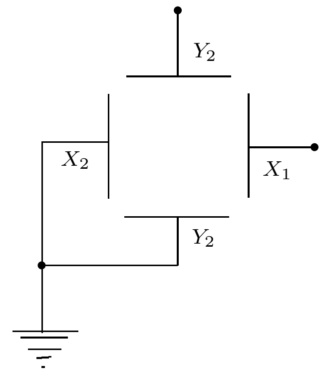


Figure 3.29: C.R.O.: X and Y plates

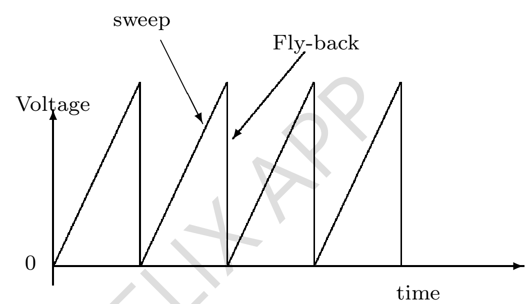


Figure 3.30: Time-base output waveform

electron beam or the bright spot on the screen is deflected horizontally. Usually voltages are applied simultaneously and one A.c voltage to the Y-plates and another on the X-plates, a waveform appears on the screen. Plates Y_2 and X_2 are connected inside the C.R.O. and brought to a single terminal connected to the earth marked E (Earth). The deflecting voltages or voltages to be displayed are then applied to E and Y_2 (usually marked Y or input) for a one screen C.R.O. and to E and X_2 (marked X). This is when the C.R.O. is to compare any property of two voltages.

Fluorescent screen.

The screen of a C.R.O. is usually coated with zinc sulphide, a fluorescent material which gives a blue trace and no after glow. Other substances (phosphors)[51] with a marked **after glow** are used, if the C.R.O. tube has to show a voltage effect that occurs momentarily and does not recur. The electron stream is a current of few microamperes. The graphite coating inside the tube provides a return path for the electrons to or from the earth and thus prevent the negative charge from accumulating on the screen.

Time-base of C.R.O.

A cathode ray oscilloscope contains a time-base circuit which applies a certain voltage to the x-plates so that the bright spot moves horizontally across the screen from far left to the right with a steady speed, returns to the left (fly back) in negligible time (fly back time) and repeats the process continuously at steady frequency as shown in figure 3.30 and figure 3.31 This frequency can be varied from 0.1Hz to 100,000Hz. During fly back the grid of the electron gun is made very negative with respect to the cathode and it cuts off the spot when the time base control is switched on there can be no other input to the X-plates.

Any alternating voltage or input signal to be examined, is connected to the Y-plates, it deflects the spot vertically. The spot moves up and down

[51]phosphors and fluorescent materials are nearly the same with the only difference of: phosphors emits light even after removing or cutting off the radiation causing the glow i.e. after glow yet the fluorescent materials instantly stop emitting light, on cutting off the radiation

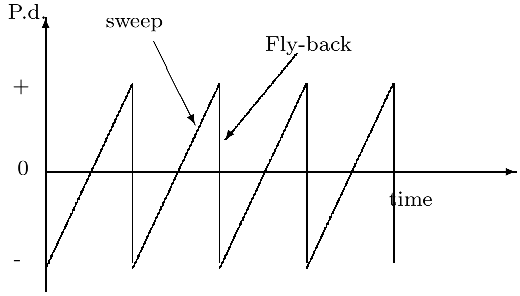


Figure 3.31: Time-base output waveform

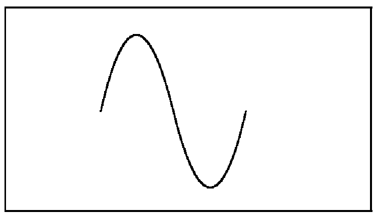


Figure 3.32: One wave

instantly. A stationary wave form i.e. a line or curve appears on the screen, if the light spot moves over the same path each time it crosses the screen from left to right. That is; when the time-base voltage has the same frequency as the input voltage, one complete wave is formed on the screen. See figure 3.32. If the time-base frequency is half that of the input voltage, two waves are formed on the screen, see figure 3.33 This means that the frequency of the input signal must be either equal to a whole number multiplied of the time-base frequency, if not, each waveform is not in the same place as the previous one and the picture on the screen is not still(i.e. mixed curves).

Other controls on the C.R.O

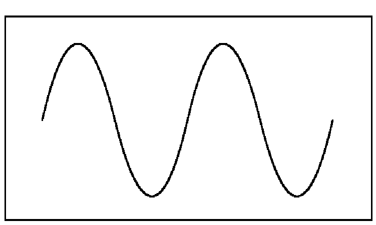


Figure 3.33: Two waves

X-shift and Y-shift controls always found on the C.R.O. are used to move the spot or trace on screen horizontally and vertically respectively. They apply a small voltage to one of the plates according to the shift required.

X-gain and Y-gain controls; control amplifiers in the C.R.O., which amplify voltages. They amplify the input signal like an input alternating voltage, which is too small to produce a suitable trace on the screen. In this way input between a few millivolts and many volts can display waveforms on the C.R.O..

Uses of a Cathode Ray Oscilloscope.

It is used to;

1. Display waveforms of various electrical signals.
2. Measure time-intervals between two events or pulses like the period of a wave, collision time, e.t.c.
3. Measure frequency of alternating electrical signals by comparison.
4. Measure voltages (a.c and d.c. voltages)

Using a C.R.O. as a voltage meter (Ac and Dc voltmeter)

Switch off the time base, connect the x-plates together so that they have no effect, and apply a known Dc voltage to the y-input terminal, measure in centimeters the vertical deflection of the spot from its central position. Repeat with different voltages draw a graph of voltage against deflection. It is a straight line through the origin; if the slope is κ v/cm,it means that k volts deflect the bright spot by 1cm.

Apply an unknown d.c voltage to the y-input and measure the deflection produced, read off from your graph the value of the unknown voltage or multiply the deflection by κ.

Now apply an unknown alternating voltage, which produces a vertical line trace on the screen.

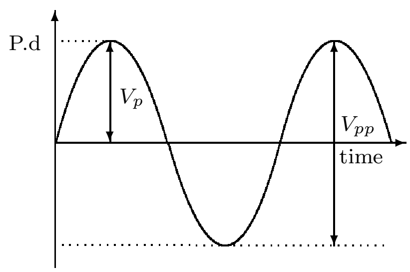


Figure 3.34: Showing peak and peak to peak values

The voltage has a peak positive value and a peak negative value every Cycle, therefore the length of the line corresponds to the peak-to-peak value of the alternating voltage i.e. twice the peak voltage. Measure the length of the line, divide the length by two and use your graph to read off the unknown peak voltage. The voltage reading given on the electric devices is called the **root-mean square value**. (r.m.s value) e.g. the common 240V is a r.m.s value. The r.m.s. value is related to the peak value as shown below;

For current

$$I_{r.m.s} = \frac{I_{\texttt{peak}}}{\sqrt{2}}$$
$$I_{r.m.s} = \frac{I_{\circ}}{\sqrt{2}}$$

For Voltage

$$V_{r.m.s} = \frac{V_{\texttt{peak}}}{\sqrt{2}}$$
$$V_{r.m.s} = \frac{V_{\circ}}{\sqrt{2}}$$

Note also that

$$\begin{aligned} V_{pp} &= V_{\texttt{peak to peak}} \\ &= 2 \times V_{\texttt{peak}} \\ &= 2V_p \end{aligned}$$

Note that from the C.R.O, we can only get or read the peak value of the alternating voltage measured as $V_{\circ}$. The root-mean square voltage is only obtained using the above formulas.

Advantages of a C.R.O. as a voltmeter over moving coil voltmeter

(a) The electron beam in the C.R.O. acts as a pointer with negligible inertia and responds instantly. i.e. it has a dead-beat response unlike the pointer on the moving coil galvanometer..

(b) The C.R.O. can measure Dc and Ac voltages[52], even if the frequency of the a.c. is several megahertz (MHz) unlike the moving coil voltmeter which can measure only Dc voltages because at low frequency the pointer oscillates and at high frequency it points to the zero mark.

(c) It passes or consumes no current from the circuit being measured. This is because a C.R.O. has an infinite resistance hence draws zero current from the circuit.

Television receiver

A television tube is a special kind of Cathode Ray Tube, it has an electron gun and a fluorescent screen but the deflection system has a different function. The system causes the electron beam to scan the whole screen horizontally from top to bottom. A time base voltage on the x-plates moves the beam across the screen. This time base makes 625 horizontal lines in $\frac{1}{25}$ of a second. A second vertical or frame time base voltages on the y-plates make each successive line lower, until the screen is covered by the 625 lines and returns the spot instantaneously to the top. The screen is covered 25 times a second. A more clear screen makes more lines.

The television aerial receives a small alternating signal voltage, which is amplified, and then applied to the grid. The signal therefore alters the intensity and brightness of the electron beam and causes a black and white picture on the screen.

[52] Like the moving iron instrument on page 299

There are 25 still pictures on the screen every second but persistence of vision causes our brains to see a moving or continuous pictures.

Differentiate between fluorescent and reflection?

In fluorescence the radiation has a different wavelength an compared to the incident radiation while in reflection the radiation from the object or mirror is the same as the incident radiation in frequency and wavelength.

3.27 Radioactivity

3.27.1 Discovery or origin of radioactivity.

Radioactivity was discovered by accident with Uranium ore. It is known that when x-rays fail on uranium salts they fluorescence (emitting visible light). In 1896 Becquerel wondered whether this process could happen in reverse i.e. sunlight (visible light) could cause uranium salts emit x-rays. To test this he wrapped a photographic plate in an opaque paper so that it was protected from the sunlight, placed uranium salt (as a hard metal target) on the paper and then in the sunlight. The plate become forged. He assumed that since the plate become forged, x-rays were produced by the Uranium salt in the sun, fortunately he also found that the photographic plate become forged. If the photographic plate with the uranium salt upon it were placed in a closed drawer where no light could enter then it is the uranium salt was emitting the radiation which passed through the protective covering and forged the plate. Substances which emit such radiations were called radioactive substances. Several experiment were performed on there radiations emitted by uranium salts which we cannot go through to even repeat the mistakes and corrections they made. We shall now start with the conclusions made.

In 1898 Pierre and Marie curie discovered two new elements, polonium and radium, in the ore pitchblende from which uranium was extracted. Radium glowed in the dark and kept it self warmer than its surroundings. Some radium atoms split up to form the noble gas radon, which in turn splits up into other elements and finally into lead. These discoveries proved that atoms are not indivisible and indestructible. This process i.e. spontaneous splitting up of the atoms of certain elements is called **radioactivity**. Natural radioactivity is shown only by few elements with large mass numbers and the most common ones are uranium-235, uranium-238, and thorium-232 all of which finally decay to lead. Radioactive decay is spontaneous and its rate cannot be changed. The rate of decay is not altered by any change in the physical conditions such as temperature, pressure, e.t.c. The energy released during a radioactive change is millions of times more than that released during a similar chemical change or from several kilograms of coal.

Emission from radioactive substances

The curies (wife and husband) found that radioactive substances, just like x-rays, ionises air and other gases. In early research both photographic and ionisation methods were used to examine the types of emission from the radioactive substances. Three kinds of radiations were detected and Rutherford called them, alpha , beta and gamma rays. (These α, β and γ are the first three letters of the Greek alphabet). These radiations come from the nucleus of radioactive atoms (and therefore are called nuclear radiations), The masses, charges and nature of the three nuclear radiations are so different that a single experiment can not be done on all three. They are emitted by unstable nuclei or nucleus.

Definition 91.

> **Radioactivity** is the spontaneous disintegration of an unstable nucleus by emitting alpha, beta or gamma rays.

Types of radiations

1. **Alpha particle** α

   (a) Properties of alpha particles

   - They are the least penetrating of the three radiations, i.e. they can not easily pass through a material like paper.
   - They have a range of few centimeters in air (i.e. they can be stopped by a few cm in air).
   - A thin aluminium foil, paper even human skin, stops them.
   - They produce much ionisation of the gas through which they pass, i.e. the have a high ionizing power.

- They are deflected by electric and magnetic fields. This deflection shows that the particles are relatively heavy (heaviest among the three types.).
- The direction of the deflection by electric and magnetic fields show that they are positively charged.

(b) Nature of alpha particle

- An alpha particle is a helium nucleus He^{2+} i.e. consists of two protons and two neutrons (i.e. a helium atom which has lost two electrons).
- Its symbol is α or ^4_2He

2. **Beta particle,** β

(a) Properties of beta particles

- They are more penetrating compared to alpha particles but less compared to gamma rays. And can pass through several meters of air and a few millimetres of aluminium foil.
- They ionise gases but not strongly ionising as alpha particles do.
- Magnetic and electric fields deflects them more easily because they are light i.e. not heavy. (Most deflected)
- The direction of deflection by electric and magnetic fields shows that they are negatively charged.

(b) Nature of beta particles

- They are electrons moving with a very high velocity and some are emitted with velocities close to that of light $\sim 10^7$.
- Beta particles are high-speed electrons.
- Its Symbol is β or $^{\;\,0}_{-1}e$

3. **Gamma rays,** γ

(a)Properties of gamma rays

- These are most penetrating and can pass even through several millimeters of lead.
- They practically have no ionisation effect.
- They are not deflected by electric and magnetic fields. hence they have no charge.
- they travel at the speed of light

(b) Nature of gamma rays

- They are a form of electromagnetic radiation, with wavelength corresponding to that of very hard x-rays i.e. have high frequency but very short wave length.
- Its symbol is γ or ^0_0n(no mass, no charge)

Summary of the properties of nuclear or radioactive radiations

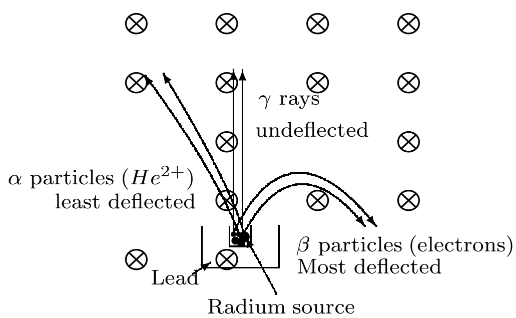
	Nuclear radiations		
	Alpha-particle, α	Beta-particle ,β	Gamma rays γ
Penetrating power	Least	Medium i.e. between α and β powers	Highest
Ionising power	Highest (e.g. discharges electroscope rapidly)	Less (discharges electroscope slowly)	Least (may not discharge the electroscope at all)
Charge	Positive(+2e)	Negative(-e)	Uncharged
Electric and magnetic fields	Least deflected (massive)	Deflected most (light)	Undeflected
Nature	Helium nucleus i.e. ${}^{4}_{+2}He$	High speed electrons	Electromagnetic wave
Range or stopped by	By about 5cm of air	Few mm of aluminium sheet	Several mm of lead
Cloud chamber tracks	Short straight thick tracks	Irregular long thin tracks	Very faint short tracks

Table 3.29: General Properties of radioactive particles or emissions

3.27.2 Effect of Magnetic and Electric field on nuclear radiations:

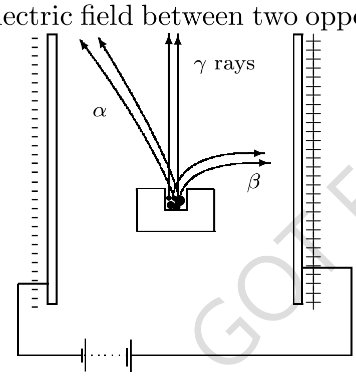
Effect of magnetic field on nuclear radiations:

For a magnetic field perpendicular to and into this sheet of paper, the radiations are deflected as shown below;



Effect of electric field on nuclear radiations

The diagram below shows how they are deflected in an electric field between two oppositely charged



plates.

3.27.3 Nuclear changes during radioactivity

Artificial nuclear transmutations

Alpha decay.

When the nucleus loses an alpha particle, it loses two protons and two neutrons i.e. mass number (nucleon number) becomes four less and its proton number two less than those of the original atom,

General Alpha decay equation

$$ {}^{A}_{Z} \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y \tag{3.301}$$

for instance

(a) The equation of the decay of uranium-238 to thorium is

$$ {}^{238}_{92}U \longrightarrow {}^{4}_{2}He + {}^{234}_{90}Th \tag{3.302}$$

The product thorium is called a daughter nucleus (daughter atom).

(b) Radium-226 decays to Radon ,

$$ {}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He \tag{3.303}$$

Beta decay

When a nucleus loses a beta-particle its mass number does not change but its atomic number increases by 1.

General beta decay equation

$$ {}^{A}_{Z}X \longrightarrow {}^{0}_{-1}e + {}^{A}_{Z+1}Y \tag{3.304}$$

Beta decay occurs when a neutron changes into an electron and a proton,

$$ {}^{1}_{0}n \longrightarrow {}^{0}_{-1}e + {}^{1}_{1}H \tag{3.305}$$

(a) Radium-228 changes to actinium by emitting a beta-particle,

$$ {}^{228}_{88}Ra \longrightarrow {}^{0}_{-1}e + {}^{228}_{89}Ac \tag{3.306}$$

(b)Radioactive lead changes to bismuth by under going a beta decay.

$$ {}^{210}_{82}Pb \longrightarrow {}^{210}_{83}Bi + {}^{0}_{-1}e \tag{3.307}$$

Gamma decay

The changes that cause Gamma decay are above our level.

3.27.4 Artificial nuclear transmutation

Now it is clear that radioactivity transforms the radioactive elements into different elements. This change of an element into another by emitting either alpha or beta particles is called **transmutation of elements** (a kind of mutation, physical mutation and not a biological mutation). For instance Nitrogen changes to oxygen when bombed with alpha particles.

$$ {}^{14}_{7}N + {}^{4}_{2}He \longrightarrow {}^{17}_{8}O + {}^{1}_{1}p \tag{3.308} $$

Several other elements were produced by bombarding stable atoms with atomic particles (like α and β particles) and the resulting elements are called **artificial elements**. Thats why even the process is called **artificial transmutation**. For instance

$$ {}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{239}_{92}U \tag{3.309} $$
$$ {}^{239}_{92}U \longrightarrow {}^{239}_{93}Np + {}^{0}_{-1}e \tag{3.310} $$
$$ {}^{239}_{93}Np \longrightarrow {}^{239}_{94}Pn + {}^{0}_{-1}e \tag{3.311} $$
$$ \tag{3.312} $$

These elements or isotopes produced in this manner are also known as **radioisotopes**

Note that hitting the uranium-238 salt with a neutron leads to or ignites several other radioactive processes. Radiactive processes that occur naturally leading to the formation of elements are called natural transmutaions of elements. Other artificial transmutations equations are;

$$ {}^{4}_{2}He + {}^{27}_{13}Al \longrightarrow {}^{30}_{15}P + {}^{1}_{0}n \tag{3.313} $$
$$ {}^{24}_{12}Mg + {}^{1}_{0}n \longrightarrow {}^{24}_{11}Na + {}^{1}_{1}H \tag{3.314} $$
$$ {}^{24}_{11}Na \longrightarrow {}^{0}_{-1}e + {}^{24}_{12}Mg \tag{3.315} $$

3.27.5 Why radioactivity occurs

Nuclei undergo radioactivity when they are unstable, imagine having a lot of proton e.g.82 in one small space the nucleus, they have to repel each other strongly and so it has to split into other particles. But this reason of splitting due to repulsion is one reason out of many i.e. like a drop of water into a basin of water. But the other reasons are based on nuclear stability and nuclear force which is above our academic level, but in summary the nuclei splits due to nuclear instability and so it splits up to form a stable daughter nuclei. These processes or series through which an unstable nuclei under goes till a stable nuclei is reached, are of 3 major types

1. Uranium series.
2. Thorium series and
3. Actinium series,

These series are named after the element from which they originate in nature. For instance for uranium-238 we have

$$ {}^{238}_{92}U \xrightarrow{\texttt{alpha decay}} {}^{226}_{88}Ra \xrightarrow{\texttt{other emissions}} {}^{286}_{82}Pb $$

This Pb is Non-radioactive lead

The mathematics of these changes is more complex but it led to the convenient way of expressing the degree of stability of radioactive element that is in terms of half-life

3.27.6 Nuclear stability: Binding energy and fission

The nucleus is made up of neutrons and protons. What causes them to stick together especially the protons? Why don't the protons repel each other? Surprisingly the mass of the nucleus is less than the sum of the masses of the individual protons and nucleons that make up the nucleus. The lost mass (mass defect) has been changed into the energy necessary to bind the nucleus together called **binding energy**. Suppose the mass defect is Δm then the binding energy of the nucleus is given by;

$$ E = \Delta mc^2 \tag{3.316} $$

Where c is the speed of light

When E is divided by the numbers of nucleons in the nucleus to obtain the **binding energy per nucleon**. The higher this value the more stable is the nucleus (i.e. it can not easily disintegrate into the α, β and γ radiations). The most stable nucleus is that of Iron-56.

Stable nuclides

Since the positive protons in the nucleus repel each other, neutrons are required to bind the nucleus together. For higher elements, the number of protons and neutrons increases but the ratio of neutrons to protons is greater than one. For heavier elements the number of neutrons in the nucleus is approximately $1\frac{1}{2}$ times the number of protons. stable nuclides are those with the same number of protons and neutrons

Unstable nuclides

Whenever a beta particle is emitted, a neutron is lost and a proton is gained therefore the neutron to proton ratio is reduced by beta radiation. When an alpha particle is emitted the nucleus looses two protons and two neutrons, since there were originally more neutrons than protons, it follows that the neutron proton ratio is increased. And thus if the neutron to proton ratio is more than the stability ratio (of 1, i.e. one to one) a beta particle is emitted (to lower the ratio) bringing it closer to the stability ratio. When the neutron -proton ratio is lower than the stability ratio of one to one, an alpha particle is emitted to raise the ratio to the stability ratio, e.g. Nitrogen $^{14}_{7}N$ is stable and the neutron-proton ratio is one, when it is bombarded by neutrons (like cosmic rays) it changes to carbon,

$$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}H \qquad (3.317)$$

Thus $^{14}_{6}C$ is formed and the neutron -proton ratio is $\frac{8}{6}$, which is greater than one. Carbon -14 then emits a beta particle to form the stable $^{14}_{7}N$ by the equation;

$$^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e \qquad (3.318)$$

this is what takes place in the atmosphere and it is used in carbon-dating.

Nuclear fission

Natural uranium is a mixture of 0.7% uranium 235 and 99.3% uranium 238, and both nuclides can be split by high speed neutrons. In 1939 it was found that the uranium-235 split into two roughly equal nuclides or atoms (barium and krypton). This process is called **nuclear fission**, which means the splitting of one atom into two or more nuclides or simpler atoms. Hence

Definition 92.

Nuclear fission is the process by which an unstable nuclide is split into two or more lighter nuclides.

The total mass of the fission products of uranium is less than the original mass (the difference in mass has been changed into energy) and a large amount of heat energy is produced. The equation of the process is ;

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n \qquad (3.319)$$

Or

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

Note that the fission of one uranium nucleus by one neutron caused the emission of other neutrons called **fission neutrons**. Under suitable conditions, the fission neutrons can cause further fission of uranium-235 i.e. the three neutrons released may collide with other uranium nuclides splitting them in turn producing nine other neutrons, the nine neutrons produced may then split nine other nuclides and so on. The figure 3.35 shows this; In this way a chain reaction may occur, it can either be a steady reaction or an explosive reaction. The quantity of energy released may be very large, for instance a few kilograms of uranium can produce as much energy as that produced by thousands of tonnes of coal, this form of energy is called **nuclear energy**.

This energy may be released in an uncontrolled manner e.g. in a nuclear bomb (atomic bomb)

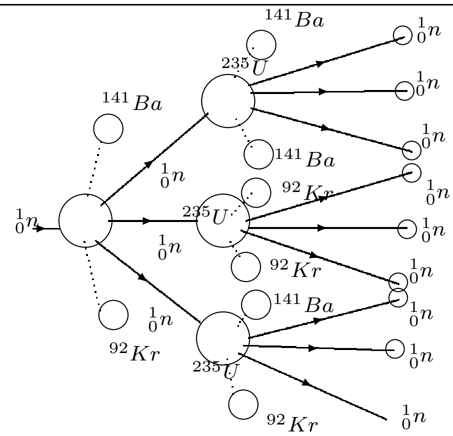


Figure 3.35: Nuclear fission;chain reaction

or in a controlled manner like in a nuclear power station (nuclear reactor) where boron rods are inserted into the uranium substance to absorb two out of the three neutrons emitted. This allows the reaction to continue at steady rate. The energy released is used to drive steam turbines and produce electricity. Note that Uranium -238 does not easily undergo nuclear fission like Uranium-235.

Uses of nuclear fission

Nuclear fission is used mainly in;

1. The manufacture of atomic bombs or nuclear bombs and other nuclear weapons.
2. Production of electric energy.

1. **Nuclear energy in bombs:**

   Consider a sphere of uranium, neutrons can escape through the surface of the uranium - 235 and do not cause fission. Its mass is directly proportional to the cube of its radius $radius^3$ but its surface area is proportional the square of its radius, $radius^2$ If the mass of the sphere is doubled, the mass increases by 2^3 times i.e. 8 times but the surface area only increases by 2^2 times i.e. 4 times. Therefore during fission of the larger sphere, the number of fission neutrons increases 8 times but the surface through which escape is possible increases only 4 times.

   For a certain mass of the sphere, the number

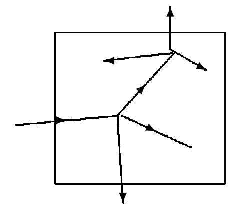


Figure 3.36: Steady fission reaction

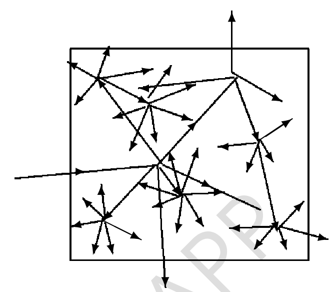


Figure 3.37: Explosive fission reaction

of the emitted neutrons retained in the sphere will be greater than the number of emitted neutrons that escape. Here a rapid chain reaction starts or results and the sphere bursts producing a lot of heat, the size of the sphere (or any other shape) for this to occur is called its **critical size**, its the minimum size of a radioactive substance for an atomic explosion to occur, and its mass is called the **critical mass**.
In an atomic bomb two or more pieces of uranium - 235 (or plutonium -239) are suddenly brought together to form a mass greater than the critical size and an explosion occurs. see figure 3.36 and figure 3.37.

   2 out of 3 fission neutrons escape.

2. **Nuclear reactor, Atomic reactor, Nuclear power station or Thermal reactor**

   A nuclear reactor is a device that converts the energy released in nuclear fission to electric energy. How special is it in releasing energy? The burning of 1g of carbon (coal) produces about 34KJ of energy but in nuclear fission the conversion of 1g of matter into energy would produce 90×10^9KJ, this is a large amount of energy. The destruction of matter

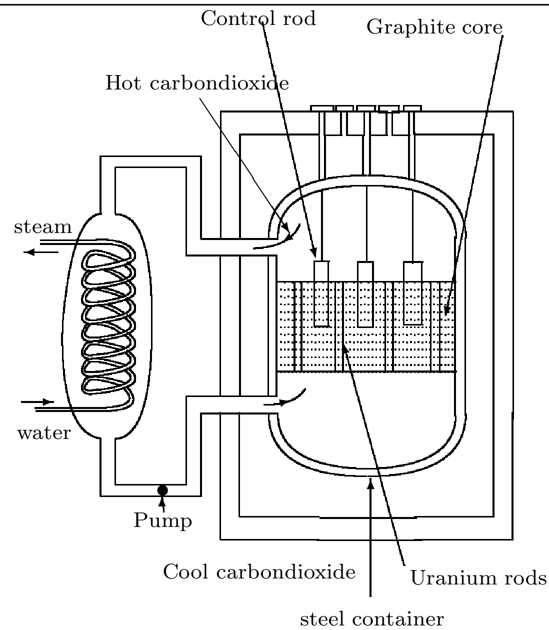


Figure 3.38: Nuclear reactor

during nuclear fission of uranium or plutonium is used to provide heat and then electricity in a nuclear power station. Nuclear fuel (uranium or plutonium) replaces coal, oil and running water in electricity generating station.

Construction

A nuclear reactor is made up of uranium rods encased in a graphite core. The graphite core contains holes or slots for the movable boron rods, all enclosed in a strong thick concrete container as shown in figure 3.38
All the time uranium is emitting neutrons (fast ones), Uranium rods or fuel rods are placed in a graphite core which reduces the neutrons speed. In it, slow neutrons react with uranium oxide fuel. The fuel rods are in vertical channels inside the graphite core. The graphite is called a **moderator** because it slows down the fission neutrons to enable them produce more fission (fast neutrons do not cause further fission).
Control rods made from cadmium or boron can be moved into or out of the graphite core so as to stop or accelerate the chain reaction respectively. They are good absorbers of neutrons (slow and fast neutrons) and therefore control the rate of fission. In any emergency they automatically move down into the graphite to absorb all neutrons and put the chain reaction to a halt. The whole reactor is in a stronger steel or concrete pressure vessel, which absorbs any escaping neutron and other radioactive radiations. In the reactor there is carbon dioxide which carries away the heat generated and is used to change water to steam that operates a generator of electricity.

Note 1: A moderator is any type of material that can slow down the fast neutrons in fission of uranium in a nuclear reactor.

Note 2: There is what we call a breeder reactor; this is a type of reactor which produces fuel inside it i.e. it may use uranium and among its products we have elements like plutonium which can also begin another chain reaction. In a fast breeder reactor, the reactor forms or breads plutonium from uranium-238. Fast neutrons escaping from the uranium-235 cause fission of the uranium-238, which forms plutonium as shown below;

$$^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{239}_{94}Pu + 2^{0}_{-1}e + \text{energy}$$

The plutonium can itself be fissioned by neutrons

3.27.7 Nuclear fusion

When lighter nuclides fuse together usually at high temperatures to form a heavy nuclide, the process is called nuclear fusion.

Definition 93.
Nuclear fusion is the process by which two or more lighter nuclides or atoms combine to form heavy nuclides or atoms.

In this process mass is lost and energy is produced, it takes place at high temperatures. For instance two atoms of hydrogen may fuse to form helium and a neutron.

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + ^{1}_{0}n \qquad (3.320)$$

The sum of the exact masses of the helium atom and the neutron is less than the sum of the exact masses the two hydrogen atoms. This lost mass is released as energy. It is thought that the sun's energy is produced by nuclear fusion. (How?). At temperatures of millions of degrees the basic reaction taking place is as follows; four hydrogen atoms combine to form one helium atom or two deuterium atoms form one helium atom.

$$4^1_1H \longrightarrow {}^2_1He + 2^0_{-1}e \qquad (3.321)$$

Or

$${}^2_1H + {}^2_1H \longrightarrow {}^4_2He \qquad (3.322)$$

The combination of two nuclei is very difficult because their positive charges repel. The hydrogen atoms must move with greater speed in order to overcome the strong repulsive forces between the positive charges. The great speed can be achieved at very high temperatures. The very high temperatures required for fusion reactions would melt any known material. The atoms could be held in plasma (hot ionised gas) in an intense magnetic field so that they do not touch the walls of the container. **It has not yet been possible to produce a controlled reaction**, but in future this problem may well be overcome. An uncontrolled fusion reaction has been produced in the hydrogen bomb.

In a hydrogen bomb, an ordinary atomic bomb is used to start this fusion and then the energy released by the fusion keeps the reaction going. It is a thermal nuclear fusion and of course the reaction is uncontrolled and explosive. If the reaction could be controlled then the deuterium obtained from the seas would meet the worlds energy needs forever.

Note that the energy released in nuclear fusion is more than that released in nuclear fission.

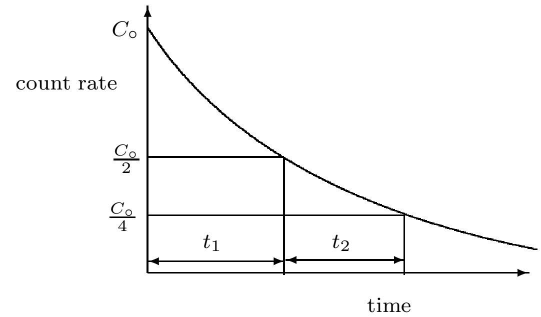


Figure 3.39: count rate variations

3.27.8 Half-life

The most convenient way of expressing the degree of stability of a radioactive element is to state its half-life. **Half-life** is the time required for the radioactivity to fall to half its observed value at a given moment. (This is not a definition). Radioactivity of a radioactive substance can be measured in terms of;

1. The mass of the radioactive substance remaining after a certain time or
2. The number of counts[53] or scintillation it gives at a given time.

There is an instrument called a **counter**, which counts the number of particles (or atoms) it is receiving from the radioactive substance and the number it gives is called the **count rate**. It was found that the number of counts or mass of the radioactive substance at a given time obeys an exponential decay law. i.e. it has a curve of the form shown in figure **??** and figure **??**. The times t_1, t_2 and others were found to be the same, hence called **half-life**.

Half-life is defined as;

(i) The time it takes half the mass of a radioactive substance to decay. (Time for M_o to become $\frac{M_o}{2}$)

[53] counts may be number of α or β particles emitted per second

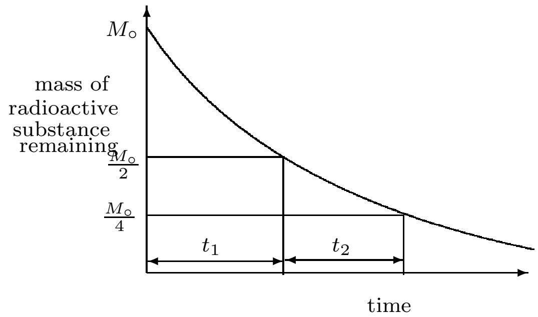


Figure 3.40: radiactive mass changes

(ii) The time it takes for the count rate or activity of a radioactive substance to decrease to half its initial value. (i.e. time for C or A to become $\frac{C_o}{2}$ or $\frac{A_o}{2}$ respectively).

is the time taken for any quantity of a radioactive substance to decay to one half of the quantity originally present.

Half-life of a substance ranges from fractions of a second to millions of years.

Consider table 3.30;

Element	**Half-life**
Thorium	10^{10} years
Radium	1620 years
Bismuth (210)	5 days
Polonium (218)	3 minutes
Polonium (214)	10^{-6} seconds

Table 3.30: Range of half life(s)

How to determine half-life.

There are two methods; here of determining half-life;

1. Graphical method:

   Here you may be given values of mass or count-rate of a radioactive substance in a table, you plot them on a graph, to obtain a decay curve and then draw a horizontal line at $\frac{C_o}{2}$ or $\frac{M_o}{2}$ and determine the time $t_{\frac{1}{2}}$ Where the line cuts the curve. That time is the half-life. Look at the above decay curve

2. Calculation method.

   Here you may be given initial mass or count-rate and mass or count-rate after a given time (t) then you are required to determine half-life, in that case we use any of these equations;

$$M = M_\circ(\frac{1}{2})^n \tag{3.323}$$

or

$$C = C_\circ(\frac{1}{2})^{\frac{t}{t_{\frac{1}{2}}}} \tag{3.324}$$

where M is the mass or activity of the radioactive sample at any time , t, and $M_\circ$ its initial mass or activity. n number of half-life(s) and $t_{\frac{1}{2}}$ half-life of the sample.

$$n = \frac{t}{t_{\frac{1}{2}}}$$

Examples

1. When a source is placed in front of a Geiger-muller tube (G.M tube) and a scaler. The initial count rate after the background count has been deducted is 4000, after 20 minutes the count rate after deducting the background count is 125. What is the half-life of the source?

Solution

since

$$\begin{aligned} C_\circ &= 4000 \\ t &= 20\,min \\ C &= 125 \end{aligned}$$

(i) Using the logical method

$4000 \xrightarrow{t_{\frac{1}{2}}} 2000 \xrightarrow{t_{\frac{1}{2}}} 1000$
$1000 \xrightarrow{t_{\frac{1}{2}}} 500 \xrightarrow{t_{\frac{1}{2}}} 250 \xrightarrow{t_{\frac{1}{2}}} 125$

in total the count of 4000 to reduce to 125, it requires 5 half-life(s) , i.e. it takes time t given by

$$\begin{aligned} t &= 5 \times t_{\frac{1}{2}} \\ \texttt{but} \quad t &= 20 \ min \\ 20 &= 5 \times t_{\frac{1}{2}} \\ \Longrightarrow t_{\frac{1}{2}} &= 4 \ mins \end{aligned}$$

hence the half-life of the source is 4 minutes

(ii) Using the formular $C = C_\circ \left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$, here

$$\begin{aligned} C_\circ &= 4000 \\ C &= 125 \\ t &= 20 \ \texttt{minutes} \\ C &= C_\circ \left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}} \\ 125 &= 4000 \left(\frac{1}{2}\right)^{\frac{20}{t_{\frac{1}{2}}}} \\ \frac{125}{4000} &= \left(\frac{1}{2}\right)^{\frac{20}{t_{\frac{1}{2}}}} \\ \frac{1}{32} &= \left(\frac{1}{2}\right)^{\frac{20}{t_{\frac{1}{2}}}} \\ \frac{1}{2^5} &= \left(\frac{1}{2}\right)^{\frac{20}{t_{\frac{1}{2}}}} \\ \left(\frac{1}{2}\right)^5 &= \left(\frac{1}{2}\right)^{\frac{20}{t_{\frac{1}{2}}}} \\ \Longrightarrow 5 &= \frac{20}{t_{\frac{1}{2}}} \\ t_{\frac{1}{2}} &= \frac{20}{5} \\ &= 4 \ \texttt{minutes} \end{aligned}$$

hence the halflife is 4 minutes

Note: Do you realise that, the logical method is easier, short with less mathematical expressions?

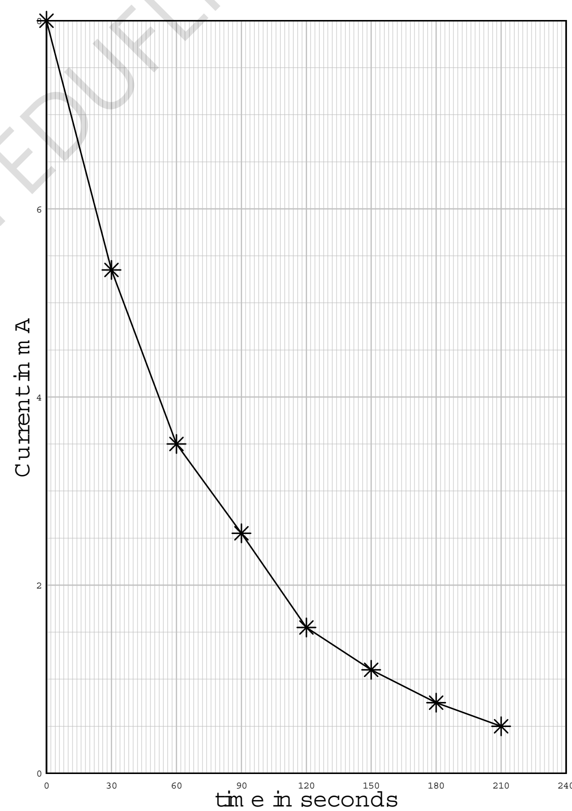
2. In an experiment to determine the half-life of thorium using a G.M. tube the following values of current in the table below were obtained;

Current/mA	Time/seconds
8.00	0
5.35	30
3.50	60
2.55	90
1.55	120
1.10	150
0.75	180
0.50	210

Plot a graph of current, I, against time, t, and use it to determine the half-life of thorium?

Solution

On plotting the values, we have the graph below



from the graph, half life $t_{\frac{1}{2}}$ is approximately 52 minutes [54]

[54] from the graph we used time taken for the current to drop from $8mA$ to $4mA$, this time is equal to; $30 + \frac{11}{15}$ of $60 - 30$ which is $30 + 22 = 52$ minutes

Note that for any graph the start point is not all that important when finding the half-life , all that is required is to note the time on the graph when any quantity (count rate, mass e.t.c.) is at a certain value I_o and then note the time when it is at half this initial value i.e. $\frac{I_o}{2}$, the difference between these two times is the half life ($t_{\frac{1}{2}}$) .

3.27.9 Uses of radioactivity

These uses can be classified as either medical or industrial;

1. Industrial applications.
2. Medical applications

Industrial applications.

1. **In detecting leakages in pipes**

   Leaks in pipes may be traced by introducing a small quantity of a radioisotope into the fluid in the pipe. Then a radiation detector can be used to determine whether the radioisotope is escaping or not, this is by observing the amount of radiations detected, when it increases then there is leakage and if not , then there is no leakage.

2. **It can be used to examine cracks in welds**

   Here cobalt-60 which emits high-energy gamma radiation is used as a source. The source should be shielded properly. But it has the great advantage over x-rays that it is much more potable and needs no power supply.

3. **Carbon-dating** is used to determine the age of fossils (remains of plants and animals) there are a small number of radioisotopes of low proton numbers, which occur naturally. They are produced by bombardment of radiations from outer space (cosmic rays). The most well known of these is radioactive carbon-14, which is produced when neutrons bombard nitrogen.

   $$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}H \qquad (3.325)$$

   The unstable Carbon - 14 decay with the emission of a beta - particle and returns to nitrogen,

   $$^{14}_{6}\mathrm{C} \longrightarrow ^{0}_{-1}\mathrm{e} + ^{14}_{7}\mathrm{N} \qquad (3.326)$$

   using the half-life of carbon-14 and its proportion in the dead plant or animal , the age of fossils is determined.

4. Radioisotopes are **widely used in industry**. E.g. to check that the thickness of a material being produced is constant (For instance Paper, iron sheets, plywood and others). As the material passes between the radioactive source and a counter, any variation in thickness causes a change in the count-rate (if the thickness decreases, the count-rate increases) and the irregularities can be pin pointed.

5. Radioisotopes can be used to estimate the amount of wear in bearings. (Radioactive iron is used) if the radioisotope is impregnated into the bearings or pistons, the weared bearing fillings are radioactive and are carried away by the oil. If the sample of oil containing the fillings is tested for radioactivity, the amount of wear can be estimated from the results, this is by comparing their count rate with the count rate we expect (like in carbon dating).

6. In dissipation of electrostatic charge or electrostatic precipitation so as to reduce the quantity of dust particles in air. Dust particles are always charged and are passed in small openings between two charged plates which attract all the charged dust particles. This reduces pollution. These plates are charged by placing a radioactive substance on one of them and the other is earthed.

7. Production of electricity using nuclear reactors using uranium-235.

Medical use or applications

1. They are used in the treatment of cancerous growth, this is by exposing the cancerous part of the body to the γ- rays from the radioactive material. The commonly used samples for this purpose are Radium or cobolt-60 got from radioactive cobalt-59 when bombarded with neutrons.

$$^{59}_{27}Co + ^{1}_{0}n \rightarrow ^{60}_{27}Co \qquad (3.327)$$

   It is used in cancer therapy instead of the more elaborate high-energy x-ray radiation. The source has to be shielded properly, but has the great advantage over x-rays that it is much more portable and needs no power supply. (Half-life is about 5.23 years).

2. Radioactive phosphorous $^{32}_{15}P$ has been tried in plants for treatment of leukaemia. Radioactive phosphorous-32 taken in by plants and animals provides information about
   (i) Growth and decay of teeth in animals.
   (ii) Its inclusion in fertilizers indicates how they are absorbed by the plant and hence choose the appropriate fertilizer.

3. Radioactive-iodine is used in treating thyroid diseases like goiter.

4. **They are used as tracers** i.e. can be used to study the movement of various compounds in a plant or animal or human body. Usually radio phosphors (P-32) and radioiodine (I-131) are among the many artificial radio nuclides used as tracers. Compounds containing small proportions of the radioactive isotopes along with the stable isotopes are used so that the path of the element in the body of plants, animals or human being can be followed. The position of the radioactive isotope in the body can be detected from out side the body by the radiation, which it emits. This allows some medical diagnosis about internal organs to be made with out surgery.

3.27.10 Health hazards due to radioactive radiations

Dangers of radioactive radiations

The first and only time atomic bombs were used was in World War II in 1945[55] when Hiroshima and Nagasaki towns of Japan were devastated. Widespread destruction and damage was produced immediately by the blast. Some of the damage caused by the emitted radiations became evident with in days, but some did not come evident until years after the event. Some of the effects on humans by the radioactive exposure of large doses, or prolonged small doses of radiations were very hazardous to the human body for instance:

1. **Burns. (Deep-seated burns)**. They cause burning on the skin similar to that caused by fire i.e. redness, blistering and soles and strong radiation may later cause radiation sickness and death.

2. They cause **leukaemia** (cancer of the blood).

3. **Eye cataracts.**

4. **Genetic mutations**, (hereditary defects in children) a mutation is a sudden change in a gene, it may be a gene for the eyes, taste, arm, legs, reproductive organs, e.t.c. And when it changes the organ controlled by that gene becomes disabled, Permanently to your descendants (off springs), grandchildren. E.g. lameness, albinism, gigantism, blindness .e.t.c

5. **Sterility** ,inability to produce or have offsprings.

6. Some children born with serious abnormalities. Like deformed nose and breathing.

7. They damage blood cells due to the ionisation they cause, and this lowers the body's immunity to normal disease. The body's immunity is reduced.

[55] When united states of America invaded Japan because it declared to be supporter of German which was the strongest nation by that time, attacking all nations against it. Other countries like America resorted to attacking the weak supporting nations like Japan. As soon as these bombs were dropped on those cities of Japan, Japan declared "war free" and the second world war ended with that fear

Note that the last two can be summarized in the fourth point.

Safety precautions

The growth and use of radioactive products has increased considerable since about 1930. Radioactive sources have become part of normal school equipment and although the sources are very weak, it is essential to take stringent safety precautions. These are:

1. The sources should only be handled by the forceps or long tweezers, provided and never be touched by bare hand.
2. The sources should be transported in containers of thick lead.
3. The sources should be stored in lead containers locked in a fireproof cabinet and clearly marked with an international warning sign.
4. The sources should never be pointed towards a person.
5. Food should not be taken where the sources are being used, because it may become contaminated.
6. Never smoke near a radioactive source.
7. The user should wear rubber gloves before touching them, and hands should be washed after the sources have been put away safely.

In places where the quantities of radioactive materials used are greater, special clothing is worn and photographic emissions or some means of monitoring the radiation are used. e.g. Beta particles cause intense ionisation but protective clothing easily absorbs them. Thus they are not likely to do much damage unless they enter the body or contaminated food e.t.c they will then do a lot of damage because they have been absorbed internally. β particles have a greater range, but they are too easily absorbed and a Perspex screen may be used as protection. Gamma rays are most dangerous because of their high penetrating power, and great care should be taken when they are used. Radioisotopes increase the level of radiation, but the effects can be minimized by using those with a fairly short half-life.

3.27.11 Radioactive waste

Waste products from nuclear power stations, laboratories, and other sources are becoming serious problems they should be put where the radiation can do no harm. Unfortunately there is no way of stopping a radioactive nucleus from emitting radiation. Radioactive material escapes into the atmosphere during nuclear bomb tests, which should therefore be done under strict control. The material enters the soils and oceans and can appear in the foods we eat (e.g. in Kampala yams grown in the swamps were found to be radioactive). All the three types of radiations are dangerous to the human body. People working with radioactive powders, liquids, and gases must wear special protective clothing and always wash themselves carefully every time they leave the radiation area. Regular tests are done to ensure that the workers have not absorbed a larger dangerous amount of the radiations otherwise they should be given a leave (a small holiday).

3.27.12 Detection and counting of radioactivity

Nuclear radiation is detected in three ways.

1. Scintillations (tiny flashes of light) of phosphors in a spinthariscope.
2. The ionisation of gases as in
   (a) Cloud chambers and
   (b) Geiger- Muller tubes
3. The blackening of photographic emissions.

The ionising property of alpha particles, beta particles, and gamma rays are used in the detection and counting of the radiations.

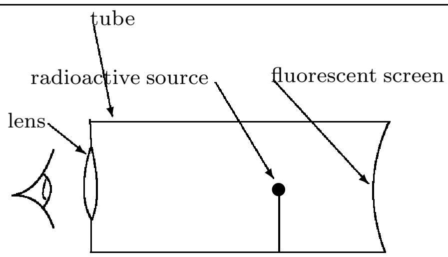


Figure 3.41: Sparinthariscope

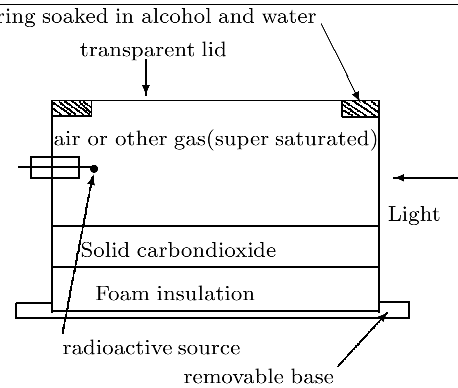


Figure 3.42: Cloud chamber

1. **The sparinthariscope or Scintillations method**

   This is a device that enables scintillations produced when radioactive emissions strike a suitable phosphor so as to be observed.

   Definition 94.

A phosphor is a chemical substance which gives out light when a radiation strikes it even after for a few seconds

   , e.g. **zinc sulphide** is used for alpha-particles. **Anthracene** (sometimes known as anthracite) for beta particles and **sodium iodide** for gamma rays. The arrangement of a sparinthariscope is as shown in figure 3.41: The source is in a tube and the observer looks at the fluorescent screen through a lens. The observer can count the flashes of light caused by each kind of particle or radiation. The sparinthariscope is very useful for gamma counting. Every alpha or beta particle that hits the screen, a flash of light is formed and counted.

2. **Ionisation instruments or detectors**

   (a) Cloud chamber

   The cloud chamber makes the path or track of an ionizing radiation visible. Figure 3.42 shows a diagram of a cloud chamber.

   Definition 95.

A **cloud chamber** is a container containing a super saturated vapour in a dust tree atmosphere

   Constructions.

   It has a felt ring at the top soaked in a mixture of alcohol and water and solid carbon dioxide at $-78°C$ is at the bottom between them is air or other gas and at some part there is layer of air super saturated with water and alcohol vapour.

Action.

The radioactive emission from the source, ionize the air or gas and the super saturated vapour condenses and forms drops of liquid on the ions (emitted particles α, β, or γ) This condensed liquid forms some form of reflection making it appear whitish. Hence a whitish trail (track) is formed and it can be seen and photographed.

Results.

α particles

1. α-particles forms short, straight thick tracks
2. β-particles form long-irregular thin tracks
3. γ-rays form only very faint tracks.

A **track** is the path taken by a particle

α-particle tracks.

(a) Short thick straight tracks in figure 3.43 They are all approximately of the same length, indicating that they are emitted with same energy.

(b) In figure 3.44 Here they are of different lengths indicating that the two sets of

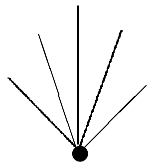


Figure 3.43: Same energy α tracks

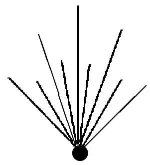


Figure 3.44: Different energy α tracks

particles are emitted with different energies.

Beta particles.

(a) Figure 3.45Track of light β-particles, they are thin but more less straight. They are thinner than those of alpha particles because they make ionising collisions much less frequently.

(b) Figure 3.46 shows the normal tracks of β-particles are thin and tortuous because the particles are easily deflected and cause little ionisation. This is shown in

(c) Irregular, Thin tracks of figure 3.47 are obtained. Some are long.

Gamma particles. Very faint

The path of gamma rays may be inferred. Gamma rays do not produce tracks, but they may collide with an electron in the shell of an atom, giving sufficient energy to this electron to enable it cause ionisation and produce

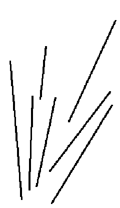


Figure 3.45: Low and high energy α tracks

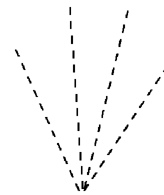


Figure 3.46: β tracks

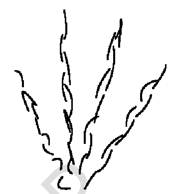


Figure 3.47: less energy β tracks

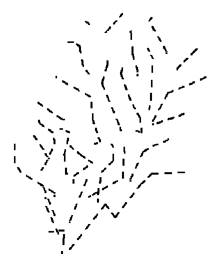


Figure 3.48: Different energy β tracks

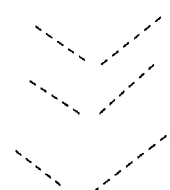


Figure 3.49: γ tracks

Figure 3.50: Different energy γ tracks

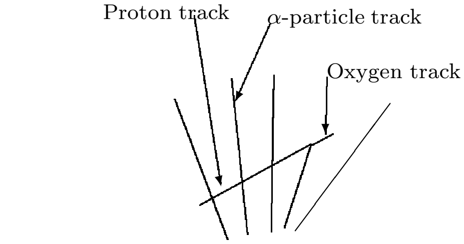


Figure 3.51: Collision tracks

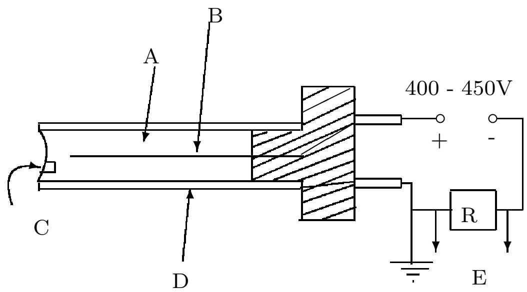


Figure 3.52: Geiger-Muller tube

same short track. Thus a number of short thin tracks are seen starting from the same straight line; this line is the path of the γ-ray. Figure3.48,3.49 and 3.50 shows this.

Cloud chambers are of two types depending on the way the super saturated vapour is produced i.e. Wilson cloud chamber and diffusion cloud chamber. The purpose or use of a cloud chamber or cloud chamber tracks is to give a quick indication of the type of radiation, which is being emitted. The observer would be very lucky to see any unusual track in a cloud chamber. The probability of seeing α-particles collide with another nucleus is very small. It may be used to show collisions between two nuclei e.g. between helium nuclear, alpha particle and nitrogen nuclei, to produce oxygen and hydrogen as shown in figure 3.51.

$$^{14}_{7}N + ^{4}_{2}He \longrightarrow ^{17}_{8}O + ^{1}_{1}H \qquad (3.328)$$

The short thick track is that of oxygen and, the long thin track is that of the emitted proton.

(b) Geiger Muller tube:

This apparatus is shown in figure3.52.

A - Argon gas with a trace of bromine

B - Tungsten wire (+ve)

C - Thin mica window

D - Aluminium cylinder (-ve)

E - To amplifier, then to scaler and loudspeaker

This is sensitive ionizing chamber. The cathode is a cylindrical aluminium tube and a tungsten wire in the middle of the cylinder is the anode. The potential difference between them is about 400V (or 450V) high. The tube contains argon or neon at pressure of about 150mmHg (or $\frac{1}{7}^{th}$ of the atmospheric pressure). The gases are present to imitate the discharge. Also a small quantity of halogen is present to quench or stop the discharge. Rapid quenching is essential if each discharge is to be recorded separately.

When a radioactive particle enters through the mica window, ionisation is produced in the gas and there is an electrical discharge between the wire and tube, i.e. if radioactive emission ionises one atom of the gas to form an ion pair, the electrons produced are accelerated towards the anode and can produce millions of electrons in a few tenths of a microsecond. There is a small pulse of anode current; this electrical impulse produces a voltage across the resistor R that is amplified and recorded on a scalar (or rate-meter). The scalar usually contains the voltage supply for the tube, amplifier and counter. The digital counter records the number of lionizations and radiations entering the tube. Gamma rays can penetrate the wall of the aluminium tube, but alpha particles and slow moving beta particles enter through a window of thin mica. The tube detects almost every beta particle entering but is much less efficient for gamma rays (scintillation counter replace Geiger Muller tubes for gamma counting).

N.B. A scaler records the total number of pulses received in a given time, a rate meter records the pulses per second or per minute and aloud speaker indicates each pulse by a clicking noise.

Background radiation

When the Geiger Muller tube is set up in position for an experiment with radioactive sources, the counter starts to count before the radioactive sources is brought near, (why) this is known as the background count and is due to background radiation received by all human beings, they are radiations from;

1. Outer space i.e. heavenly bodies, they are called Cosmic rays.
2. Radioactive minerals in the earth,
3. Radon in the atmosphere and potassium-40 in the body
4. Radiations from the sun
5. Naturally occurring radioisotopes
6. Artificial radioisotopes
7. Products made from nuclear explosion e.g. strontium-90, which has a half-life of 28 years.
8. Rocks in the earth which contain traces of radioactive substances.

Practical sample

If you place no radioactive substance at the end of the tube, the count rate increases and decrease considerably as shown in table 3.31;

Time interval (mins)	Number of counts
1	22
2	12
3	16
4	32
5	10
6	26
7	15
8	13
9	23
10	29

Table 3.31: Background counts

Note that the count rate;

(a).Varies considerably showing the random nature of the radiations.

(b).Is small.

(c).Has an average of about 20 counts per minute.

There is no need to count every minute separately when determining the background count. The average count per minute can be found from the total count over the ten minutes. The average background count must be taken in all the experiments involving measurements in radioactivity. In accurate work, it is usual to find the average background count before starting and after finishing the experiment. The average of these two is taken and subtracted from each reading during the experiment to obtain the true count due to the radioactive substance under study.

3.The photographic emulsions

The effect of nuclear radiations on silver bromide and silver iodide is the same as that of light (visible light), i.e. it changes them to silver, which blackens on developing. The tracks of ionising radiation can be photographed on plates, the more radiations it receives the greater degree of blackening on these plates. People exposed to radiations wear badges of film emulsions and the degree of blackening of the film when developed indicate if they are in danger of high dose of the radiations.

Note that these devices can be used to ;

1. Determine the penetrating power of different radiations
2. The range of different radiations
3. Demonstrate deflection of beta-particles
4. Measure the intensity and absorption of gamma rays
5. Identify the radiation from a radioactive source, e.g. radium -226

Practical sources of alpha, beta and gamma rays

The following sources may be available for various kinds of radiations

(a). Plutonium -239 or Americium-241 for alpha particles only (b). Strontium -90 for beta-particles only

(c). Cobolt-60 for gamma rays

(d). Radium-226 for alpha-particles, beta-particles and gamma rays together

3.27.13 Questions: Half-life

1. The half-life of radioactive substance is 12days. What fraction of the original activity will remain after 36 days ? [$\frac{7}{8}$]

2. If radon decays to 25% of its original activity in 8 days, what is the half-life of Radon?

3. Radon $^{226}_{88}Ra$ losses 5 α-particles and 4 β-particles and it is converted to a new stable element, an isotope of lead, Pb. Find the mass number and atomic number of the isotope?

4. Thorium $^{232}_{90}Th$ is converted into Radium $^{224}_{88}Ra$ by radioactive transformation. How many α and β emissions have taken place?

5. The results in table 3.32 were obtained from the readings using a ratemeter for the emissions from a radioactive isotope of iodine;

Time in days	Average count-rate
0	1200
1	1120
2	1040
5	830
10	540
15	330
20	220
25	140
30	90

Table 3.32: Half life measurement

Plot a graph of count-rate against time and from the graph estimate the half-life of the isotope of iodine.

6. A sample of radioactive substance has a mass of 2g and a half-life of 7 days. What fraction of the original sample would be present after 21 days? could the half-life be confirmed by reweighing the sample after 14 days? Explain

7. The half-life of Radon gas is 4 days. if some radon gas is accidentally released into a closed laboratory, How long would it take for the concentration of the gas to fall below 1% of its original value?

8. The half-life of a radioactive substance is 10 hours. the Original activity of the same is measured and found to be 1200 counts per minute.

   (i) what would be the count-rate after 10 hours and 20 hours

   (ii) After what time will the activity be 12.5% of the original activity

9. Use table 3.33 to calculate the half-life of a radioactive element and complete the missing part of the table;

Time(s)	Count-rate per min
0	1600
102	400
155	200 100
308	....

Table 3.33: Half life measurement

10. In an experiment to measure the half-life of a radioactive element the following results in table 3.34 were obtained;

Count rate or counts per minute	1000	250	125
Time in seconds	0	110	160

Table 3.34: Half life measurement

   (a)State clearly what is meant by the term half-life of a radioactive element

   (b)From the results in the table calculate;

      (i) Two different values for the half-life of the element, and

      (ii) The average half-life of the element

11. (a)What changes in the mass of the nucleus of an atom which emits

      (i) An alpha-particle

(ii) A beta particle and

(iii) Gamma rays

(b) Explain how it is possible for atoms of the same element to have different mass numbers.

(c) Describe an experiment to determine the half-life of a radioactive material, indicate how the result is obtained from the observations

(d) If the half-life of thorium is 52 second, how long will it take for the activity of a thorium sample to be reduced to $\frac{1}{32}$ of its initial value?

12. In an experiment to determine the half-life of radon-220, $^{222}_{86}Ra$, the results in table 3.35 were obtained after allowing for the background count;

Time(s)	Count rate (s^{-1})
0	30
10	26
20	23
30	21
40	18
50	16
60	14
70	12
80	12

Table 3.35: Half life measurement

(a) By plotting the count rate against the time, determine the half-life of $^{220}_{86}Ra$. Show clearly on your graph how you obtain your answer.

(b)

(i) What is the origin of the background count?

(ii) How is the background count determined?

(c) Radon -220 emits an alpha -particle

What are alpha -particles?

When radon emits an alpha particle, it becomes an isotope of the element polonium (Po). Write an equation to represent this change?

(d) When carrying out experiments with radioactive sources, students were instructed that

The sources should never be held close to their eyes.

No eating or drinking is allowed in the laboratory. Why is it important to follow these instructions?

13. A certain nuclide F decays by emission of β-particle to form a daughter product G. The half-life of F is 20 seconds,

(a) Name two types of particles that form the nucleus of the nuclide F.

(b) How does the nucleus of G differ from the nucleus of F?

(c) How long will it take for three quarters of a sample of F to decay?

(d) What effect if any would it have on the rate of decay of F if its temperature is raised to $100°C$. Explain.

(e) The daughter product G subsequently undergoes alpha emission to form nuclei H. How is the atomic number of nuclide H compared to nuclide G?

14. Radon is a radioactive element, which may be represented by $^{222}_{86}Ra$

(a)State the meaning of the numbers 222 and 86.

(b)Radon disintegrates with the emission of an alpha particle to form a different element.

(i) How may this element be represented?

(ii) What are the constituents of the nucleus of this element? The element, which has been formed from radon, is an isotope of polonium,

(c)In what ways will the nuclei of other isotopes of polonium differ from the nuclei of the element formed from radon?

(d)Radon disintegrates with a half-life of 3.8 days. How long will it take for the number of emissions per second of a sample of radon to be reduced to 25% of the original value? Show how you obtain your answer?

15. (a) Give two properties, in each case of alpha , beta and gamma radiations

    (b) Explain what is meant by, half-life of a radioactive substance?

    (c) The number of particles counted per second of a certain samples of a material, which radiates alpha particles, is recorded regularly for a period of time. The measurements are given in table 3.36; use these values to determine the half-life of the material;

Time in Hrs	Number per second
0	500
1	305
2	186
3	118
4	62

Table 3.36: Half life measurements

    (d) Explain the meaning of the terms; isotopes, proton number and nucleon number.

    (e) Thorium -227 (Atomic number - 90) decays by alpha emission to an isotope of radium. Give the atomic number and atomic mass number of the radium isotope formed?

16. (a) A nitrogen nuclei is written as $^{14}_{7}N$, what information about the structure of the nitrogen atom can be deduced from this symbol for the nuclei?

    (b) A nuclide whose symbol is $^{16}_{7}N$, is an isotope of nitrogen, in what way is an atom of this type of nitrogen different from the atom in (a).

    (c) The nuclide $^{16}_{7}N$, decays to become an oxygen nuclide by emitting an electron, write down an equation for this process?

    (d) The half-life of the nuclide $^{16}_{7}N$, is 7.3 seconds,
    What does this mean?

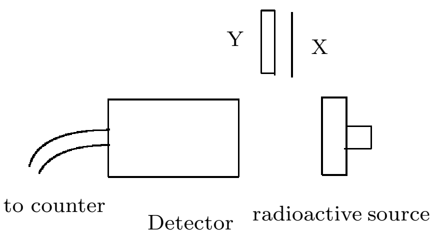


Figure 3.53: Radioactive detector

A sample of this type of nitrogen is observed for 29.2 seconds, calculate the fraction of the original radioactive isotope remaining at this time?

17. (a) Explain what is meant by the spontaneous nature of radioactivity.

    (b) Explain what is meant by half-life and how the concept depends on the random nature of radioactive decay.

    (c) A sample of a certain nuclei, which has a half-life of 1500 years, has an activity of 32000 counts per hour at the present time;
    Plot a graph of the activity of this sample over the period in which it will reduce to $\frac{1}{16}$ of its present value.
    If the sample of the nuclei could be left for 2000 years, what would be its activity then?

18. The mass of a radioactive sample was measured every after 5 days and the following results were obtained 64, 32, 16 and 8g. Determine the half-life of the radioactive sample?

3.27.14 Questions:Detection of radiations

1. A radioactive source is placed in front of a detector as shown in figure 3.53 and a high-count rate is observed. When a sheet of paper X is lowered between the source and detector, there is a reduction in the count rate when an aluminium sheet Y is lowered next to X, there is practically no further reduction in the count rate. The radiation is most likely to be

   A.Alpha radiation only.

B. Beta radiation only.

C. A mixture of alpha and beta radiation only.

D. A mixture of beta and gamma radiation.

E. A mixture of alpha and gamma radiation.

2. Various sheets are placed between a radioactive source and a detector, which is capable of indicating the presence of alpha, beta and gamma emissions. The observations in table 3.37 were recorded;

Material of sheet	Effect on the count rate
Paper	Slight drop
Paper + aluminium	No further drop
Paper + aluminium + lead	A further slight drop

Table 3.37: Emissions detection

The emissions from the source are;

1.Gamma only.

2.Beta and Gamma.

3.Alpha and Beta.

4.Alpha and Gamma.

5.Alpha, Beta and Gamma.

3. Certain atoms emit gamma radiation, because

A. They have a high nucleon number

B.Their nuclei emit electrons

C.Their nuclei contain protons and neutrons

D.Their nuclei are unstable

E.Their nuclei are at a high temperature

4. (a) Describe the mode of operation of one type of nuclear radiation detector

(b) a nuclear radiation detector is connected to a scaler which counts the number of particles detected. The scaler indicates 12 counts during period of one minute. During two subsequent periods of one minute the scaler indicates 11 and 15 counts. When a source of radioactive material is placed near the detector; the counts over three periods of one minute are 1480,1508,1496. A piece of thick paper is placed between the source detector and the counts reduce to 1216, 126 and 1230 over one minute periods. Finally when a piece of lead 5mm thick is placed between the source and the detector, the counts are 12,11,and 14 over one minute period;

(i) Why is a count obtained without the source?

(ii) Why do the counts obtained in any group differ?

(iii) Which of the three types of radiations α, β and γ are emitted by the source?

5. State the changes that take place in the nucleus of a radioactive atom, when an alpha particle is emitted by that atom. Illustrate your answer by an equation for the emission of an alpha particle from the nuclide ${}_{z}^{A}X$.

6. Explain the following;

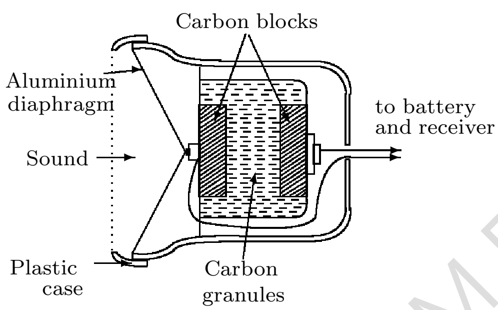
(a) Alpha particles produce much denser tracks in a cloud chamber than do beta particles

(b) Beta particles are much more readily deflected by an electric field than are alpha particles.

Appendix, index and tables

1. **Carbon microphone**

   (it does not use the magnetic effect of an electric current) This instrument depends on the principle that, when pressure on carbon is increased, its electrical resistance decreases and hence the current passing through it also increases.When some one speaks facing this carbon microphone shown below;

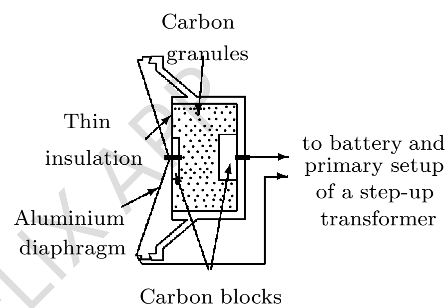
   Sound waves produced cause the aluminium diaphragm to move backward and forward. This varies the pressure exerted on the carbon granules between the front carbon block, which is attached to the diaphragm, and the back one, which is fixed. When the pressure increases, the granules are squeezed closer and the electrical resistance decreases.

   If there is a current passing through the microphone (from a battery), it will vary in a similar way as the sound wave variations i.e. the frequency of the new variations in the current is equal to the frequency of the sound waves sent to the aluminium diaphragm.

   The sound produced by this current via the loud speaker has the same frequency as the original sound, hence they are the same.

2. **Carbon microphone**

   This type consists essentially of two polished carbon discs placed a short distance apart, the space between them being filled with small carbon granules. One of the discs is fixed and the other connected to a metal diaphragm.

   When a person speaks in front of the microphone, the diaphragm is set in motion by the sound waves and so the carbon granules are subjected to variable compressions. Consequently the electrical resistance between the carbon blocks varies in a corresponding manner (i.e. with the same frequency).

BASIC TRIGONOMETRIC TABLES

TABLE OF SINES

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2233
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
30	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934

TABLE OF SINES (cont.)

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE OF COSINES

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
22	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206

TABLE OF COSINES (cont.)

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
45	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017

TABLE OF TANGENTS

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623

TABLE OF TANGENTS (cont.)

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	8.1443	8.2636	8.3863	8.5126	8.6428	8.7769	8.9152	9.0579	9.2052	9.3572
84	9.5144	9.6768	9.8448	10.018	10.198	10.385	10.578	10.779	10.988	11.204
85	11.430	11.664	11.908	12.163	12.428	12.706	12.996	13.299	13.617	13.950
86	14.300	14.668	15.055	15.463	15.894	16.349	16.831	17.343	17.886	18.464
87	19.081	19.740	20.446	21.205	22.021	22.903	23.859	24.897	26.030	27.271
88	28.636	30.144	31.820	33.693	35.800	38.188	40.917	44.066	47.739	52.080
89	57.290	63.656	71.615	81.847	95.489	114.58	143.23	190.98	286.48	572.96

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All power lies in the will