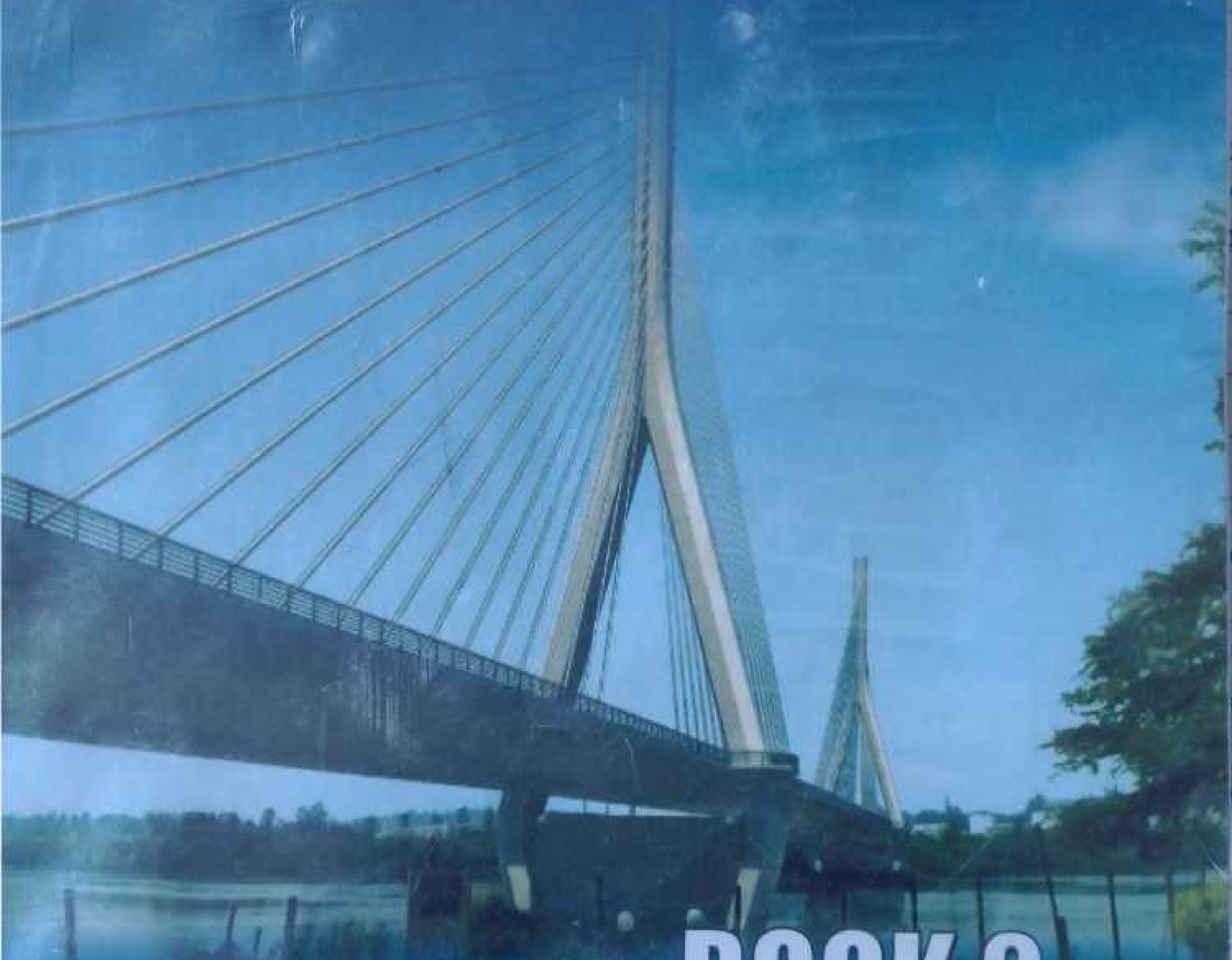
New Lower Secondary School Curriculum

MATHEMATICS

Approved by NCDC and MoES

New Generation Books



BOOK 2

Senior Two Learner's Book

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Preface

Baroque Senior Two Mathematics Learner's Book has been developed in response to the new competence-based ***Lower Secondary Curriculum*** for Uganda. The curriculum was developed by the Ministry of Education and Sports (MoES) under the National Curriculum Development Centre (NCDC) and launched in 2020.

The book is a result of extensive research from several credible Mathematics resources and input from experienced teachers and experts.

Baroque Senior Two Mathematics Learner's Book entails;

- an active competence-based and learner-centred approach
- appropriate and accurate content
- adequate and relevant activities and projects that trigger discovery, critical thinking, creativity, problem-solving and interactivity
- acceptable, appropriate, standard and grammatically correct English, which encourages vocabulary development as well as correct representation of technical terms
- accurate, relevant, clear, and adequate illustrations that enhance learning
- intuitive methods, illustrations, activities, and projects that have been explored to instill the principles of Mathematics.

In pursuit of a knowledge-based society, there is need for new generation learning books that are learner-centred, sufficiently researched and innovatively developed.

Baroque Senior Two Mathematics Learner's Book lays a firm foundation for learners who would like to pursue a career in Mathematics-related fields and seeks to equip all learners with the ability to apply Mathematics knowledge in day-to-day activities.

THEME: Patterns and Algebra

Chapter 1 Mappings and Relations

Keywords

- domain
- function
- mapping
- range
- relation

By the end of this chapter, you should be able to:

- use arrow diagrams or mappings to represent relations and functions.
- identify domain and range of a mapping.
- describe function and non-function mappings.
- distinguish between function and non-function mappings.

Introduction

In this chapter, you will learn how to use arrow diagrams or mappings to; represent relations and functions, identify domain and range of a mapping, describe function and non-function mappings, and distinguish between function and non-function mappings. This will help you to appreciate real-life examples of mappings and relations; for instance, in religion, family, friendships.

Using the knowledge of mappings and relations, you will be able to solve some societal problems. For example, domestic violence, spread of sexually transmitted diseases, and many others.

1.1 Relations

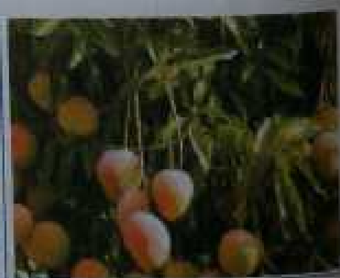
Activity 1.1(a)

1. In groups, write down some related pairs of objects you know and state the relationship between those objects in each pair.
2. In groups, discuss and state the relation between the following:
   a) Mr. and Mrs. Opio
   b) Kenya and Uganda
   c) Kampala and Nairobi
   d) 3 and 4
   e) A and E
   f) Africa and Asia
   g) Cat and Kitten

Arrow Diagrams

In Mathematics, arrow diagrams are used to show a relation between two or more objects, persons or numbers in different sets.

Activity 1.1(b)

Plant	A 	C 	E 	G 
Fruit	B 	D 	F 	H 

a) In your notebook, name the pictures shown.
b) Using letters indicated, draw an arrow diagram showing the relationship(s) between the pictures named.
c) Describe the relationships presented in the arrow diagram.

Exercise 1.1

1. Given two sets A and B below, draw an arrow diagram to show the relation "is a cube of".
   $A = \{2, 3, 4, 5\}$, and
   $B = \{8, 64, 125, 27\}$
2. Draw a diagram to illustrate the relation "is more than by three," given the two sets
   $A = \{6, 12, 13, 17\}$ and
   $B = \{3, 9, 10, 14\}$.
3. Given two sets $A = \{^-2, ^-1, 0, 1\}$ and $B = \{0, 1, 4\}$, draw an arrow diagram to show the relation "is a square of".
4. It is known that Angella, Robert and Phillip are children of Twinomugisha; and Edward, Andrew and Alice are children of Apio. Represent this information on an arrow diagram.

Mappings

These are special types of relations which associate each element of a set with an element(s) of another set. Two objects map if they have a certain relation; for example, two persons, A and B, map if they are of the same sex, or speak the same language, or are of the same age or some other relationship.

Activity 1.1(c) (Work in groups)

a) List down countries in the East African Community.
b) List down capital cities in the East African Community.
c) Draw an arrow diagram to illustrate the relationship between the East African countries and capital cities listed.
d) Describe the type of mapping illustrated in (c).

Did you know?

A monogamous marriage is a *one-to-one relation*.

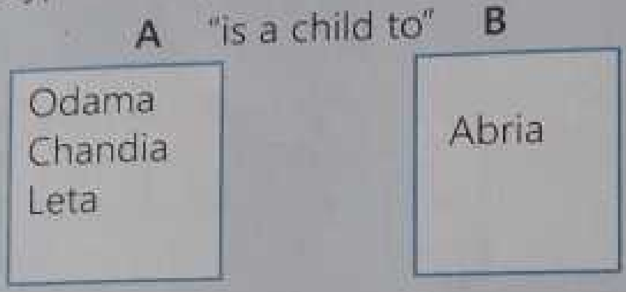
Activity 1.1(d) (Work in groups)

Mr. and Mrs. Matovu's family has 3 children (Nansamba, Nalubega, Lubega). Using the relation "is a parent to", copy and complete the arrow diagram below, and identify the type of mapping illustrated.

A	"is a parent to"	B
Mr. Matovu Mrs. Matovu		Nansamba Nalubega Lubega

Activity 1.1(e) (Work in groups)

Consider a mother (Abiria) of 3 children (Odama, Chandia, Leta). Using the relation "is a child to", copy and complete the arrow diagram below and identify the type of mapping illustrated.



Did you know?

Many people believe in one God. Therefore, the relation of believers to God is a *many-to-one*.

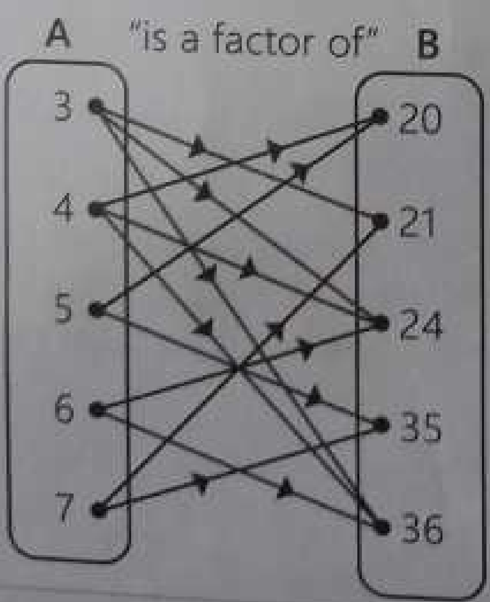
Activity 1.1(f) (Work in groups)

Use the relation "is greater than" to draw an arrow diagram to relate sets ***A*** and ***B*** and identify the type of mapping given that A = {5, 7, 9} and B = {1, 4, 5, 6}.

Example 1

Given two sets **A** = {3, 4, 5, 6, 7} and **B** = {20, 21, 24, 35, 36}, use an arrow diagram to show the relation "is a factor of."

Solution:

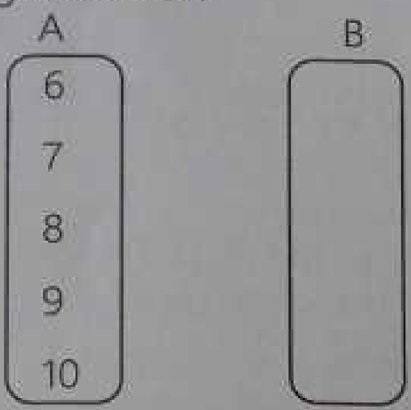


Example 1 is a many-to-many mapping.

Task: List any four more relations that give many to many mappings.

Exercise 1.2

1. If set A = {6, 7, 8, 9, 10}, use the relation "divide by two, subtract 3" to complete the arrow diagram below



2. Give four examples for each case below and in each case, draw an arrow diagram.
   a) One-to-one mapping
   b) One-to-many mapping
   c) Many-to-one mapping
   d) Many-to-many mapping

1.2 Identifying Domain and Range of a Mapping

In Mathematics, a domain is a set of objects (starting numbers) and range is the set of elements (images) to which objects are mapped.

Activity 1.2

In groups, study *Example 1* and identify the domain and range for the relation represented.

Example 2

Using the relation $3x - 2$, what is the domain if the range is {4, 10, 16, 22}.

Solution:

Let y be an element of the range. Then, y can be expressed as $3x - 2$, since each element of the range is of the form $3x - 2$ (x is an element in the domain). So, $y = 3x - 2$.

The set of the elements y is {4, 10, 16, 22}.

That implies that:

$4 = 3x - 2$, $6 = 3x$, $\therefore x = 2$

$10 = 3x - 2$, $12 = 3x$, $\therefore x = 4$

$16 = 3x - 2$, $18 = 3x$, $\therefore x = 6$

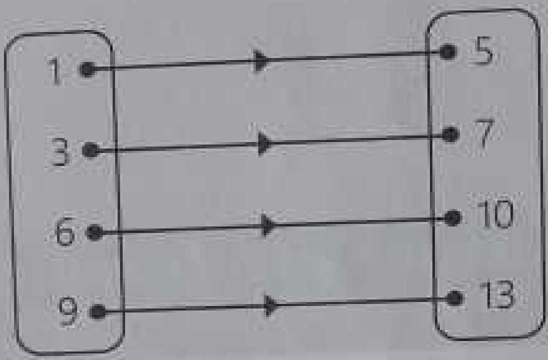
$22 = 3x - 2$, $24 = 3x$, $\therefore x = 8$

Therefore, the domain is {2, 4, 6, 8}.

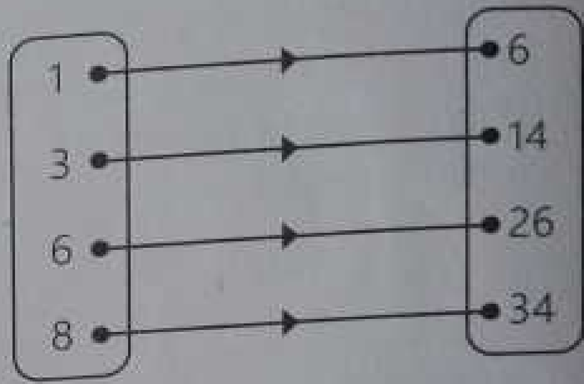
Exercise 1.3

1. Use the relation "multiply by two, add 7" to find the range of the domain {3, 4, 5, 6}.
2. The set {2, 5, 11, 13, 20} is mapped by the relation "add two, multiply by five". Draw an arrow diagram and find the range.
3. Define the relation for each of the arrow diagrams below.

a)



b)



4. The set {3, 4, 5, 6, 7} is mapped onto the set {8, 9, 10, 11, 12}.
   a) What is the relation?
   b) Draw an arrow diagram for the above relation.

1.3 Function and Non-Function Mappings

For every relation in which each object in the domain is mapped to only one object in the range is a ***function.*** In fact, in function mappings, the element in the domain corresponds to only one element in the range.
However, in *non–function mappings*, the element in the first set (domain) corresponds to more than one element in the second set (range).

Activity 1.3 (Work in groups)

a) List any 2 mappings that are not functions.
b) List any 3 examples of functions.

Exercise 1.4

1. Using the relation "add 1";
   a) find the range, given the domain A= {0, 1, 2, 3}.
   b) draw an arrow diagram illustrating this relation and classify it as either a function or non-function mapping.
2. Given the set B = $\{^-3, ^-2, ^-1, 0, 1, 2, 3\}$, using the relation "is a square of", draw an arrow diagram illustrating this relation, and classify it as either a function or non-function mapping.

ICT Activity

a) In groups, using your knowledge of word processing, type the solution to *Exercise 1.4*. Use font size 12 and font type Cambria Math. Save the word document as "Relations".

b) Create a folder on the desktop and name it your group name. Copy and paste the saved document into that folder.

Revision Questions:

1. Adam is a brother to Mariam, Sarah, and Hajarah. Using the relation "is a brother to", draw an arrow diagram relating Adam to his three sisters.
2. An electrician charges a basic fee of UGX 5,000, plus UGX 2,000 for each hour of work. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work.
   a) Identify the domain and the range in this relation.
   b) Is this relation a function?

Sample Activity of Integration

The rainy season is coming soon. Three farmers need to prepare their land for planting.

Support:

a) A table showing tools, productivity of available farm tools and cost per day

Tools	Productivity (Hectares per day)	Cost per day (UGX)
Hand hoe	0.1	5,000
Ox plough	0.5	87,500
Tractor	5	250,000

b) A table showing resources that farmers have at a time

Farmer	Resources available	
	Land in Hectares	Money in UGX
Mr. Outa	1	250,000
Mrs. Kamu	3	1,200,000
Mr. Wonialla	5	800,000

Resource:
- Knowledge of mappings and relations

Task:

As a Senior Two learner, advise these farmers on the most cost-effective method to use.

Chapter Summary

In this chapter, you have learnt that:
- A domain is a set of objects and range is a set of elements to which objects are mapped.
- In ***function mappings***, the element in the first set corresponds to only one element in the second set. In other words, each object in the domain is mapped to only one element in the range.
- In a ***non–function mapping***, the element in the domain corresponds to more than one element in the range.
- ***A one-to-one mapping*** is a mapping in which each and every element (object) in the domain is mapped onto one and only one element (image) in the range.
- ***A one-to-many mapping*** is a mapping in which at least one element (object) in the domain is mapped onto more than one element (image) in the range.
- ***A many-to-one mapping*** is a mapping in which more than one element (object) in the domain is mapped onto one and only one element (image) in the range.
- ***A many-to-many mapping*** is a mapping in which more than one element (object) in the domain is mapped onto at least one element (image) in the range, and more than one element in the range is mapped onto by at least one element in the domain.

THEME: Patterns and Algebra

Chapter 2

Vectors and Translations

Keywords

- translation
- vector
- vertical direction
- horizontal direction
- geometric
- displacement
- scalar
- kinematics

By the end of this chapter, you should be able to:

- define a translation with a vector.
- identify scalars and vectors.
- use vector notation.
- represent vectors, both single and combined, geometrically.

Introduction

In real life, vectors are widely used across the globe in different areas, such as navigation and kinematics. Vectors are even used at experimental level in engineering laboratories to model real-life problems. They are also useful in showing the paths that one can take while moving to different locations.

This chapter will help you to understand the nature of vectors, and how to manipulate and represent them in order to define translation.

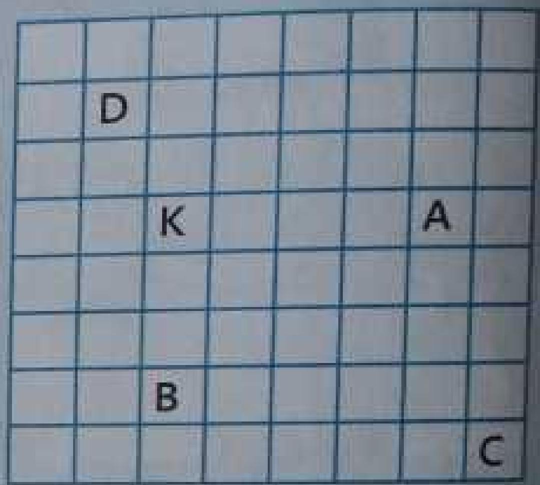
2.1 Translation

Activity 2.1(a) (Work in groups)

Suggested materials:

- bottle tops
- a pen
- a plane paper
- a ruler

Instructions:



a) Draw a grid board similar to the one shown.
b) Mark off points A, B, C, D and K.
c) Place a bottle top at point K and displace it to point A.
d) Measure and record both the horizontal and the vertical displacements to point A from K.
e) Repeat (c) and (d) when the bottle top is displaced to B, C and D respectively.
f) Tabulate your results in the table below:

Position	A	B	C	D
Horizontal displacement				
Vertical displacement				

From *Activity 2.1(a)*, to whichever position you displace the bottle top, its size and shape do not change, and whenever that holds, a body has undergone a ***translation***.

A translation tells us how far an object is displaced horizontally and vertically. The translation of an object can be represented in a column form as $\binom{x}{y}$, where x represents the horizontal displacement and y represents the vertical displacement of the object. The process of displacing the bottle top from K to A, B, C and D is called a ***translation***.

Activity 2.1(b)

From *Activity 2.1(a)*, write down the translations when the bottle top is moved to positions A, B, C and D from K.

2.2 Vectors

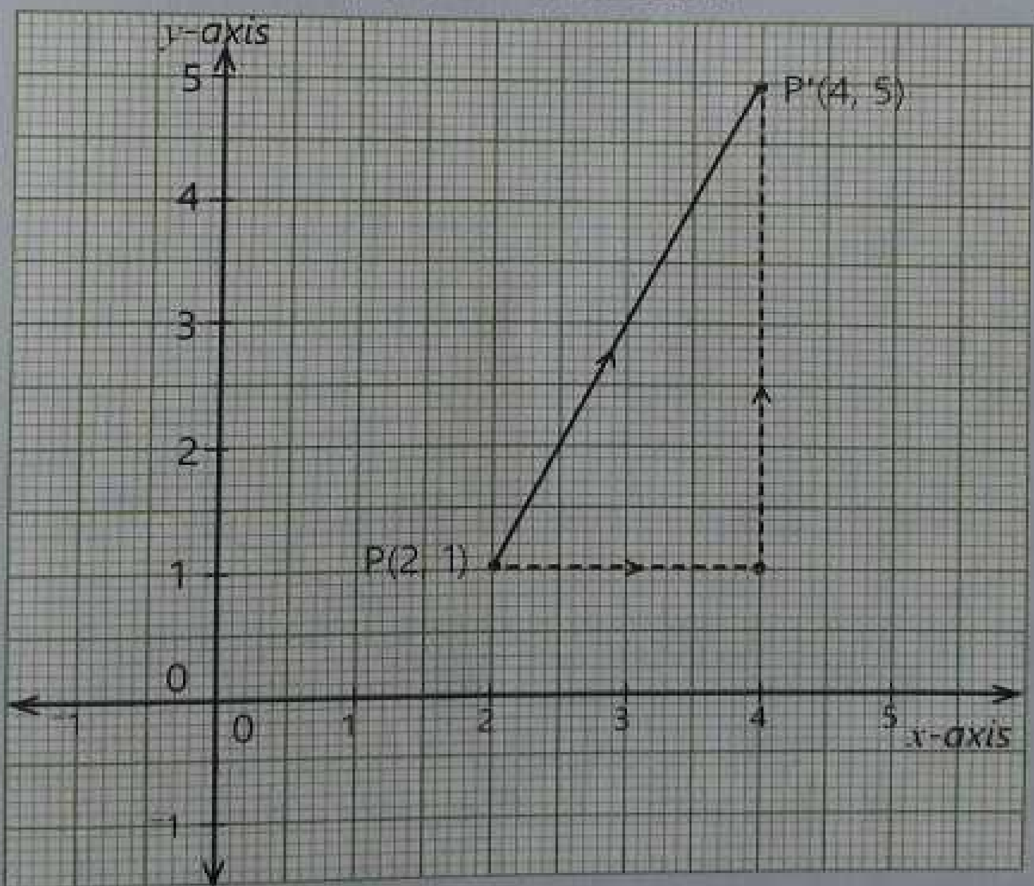
In *Activity 2.1(a)*, the bottle top covered different distances in the vertical and horizontal directions as it was being displaced to different positions. In all cases, it covered a particular distance (magnitude) in a given direction. Such physical quantities that have both *magnitude* and *direction* are called **vectors.**

When this physical quantity is represented in column form, then it is called a *column vector.*

Note: Quantities with only magnitude are called ***scalar quantities***; for example, distance.

Activity 2.2(a) (Work in groups)

Consider points P(2, 1) and P'(4, 5) as shown below.



Tasks:

a) How far is P' from P:
   i) horizontally
   ii) vertically

b) State the movement of P' from P as a column vector.

Activity 2.2(b) (Work in groups)

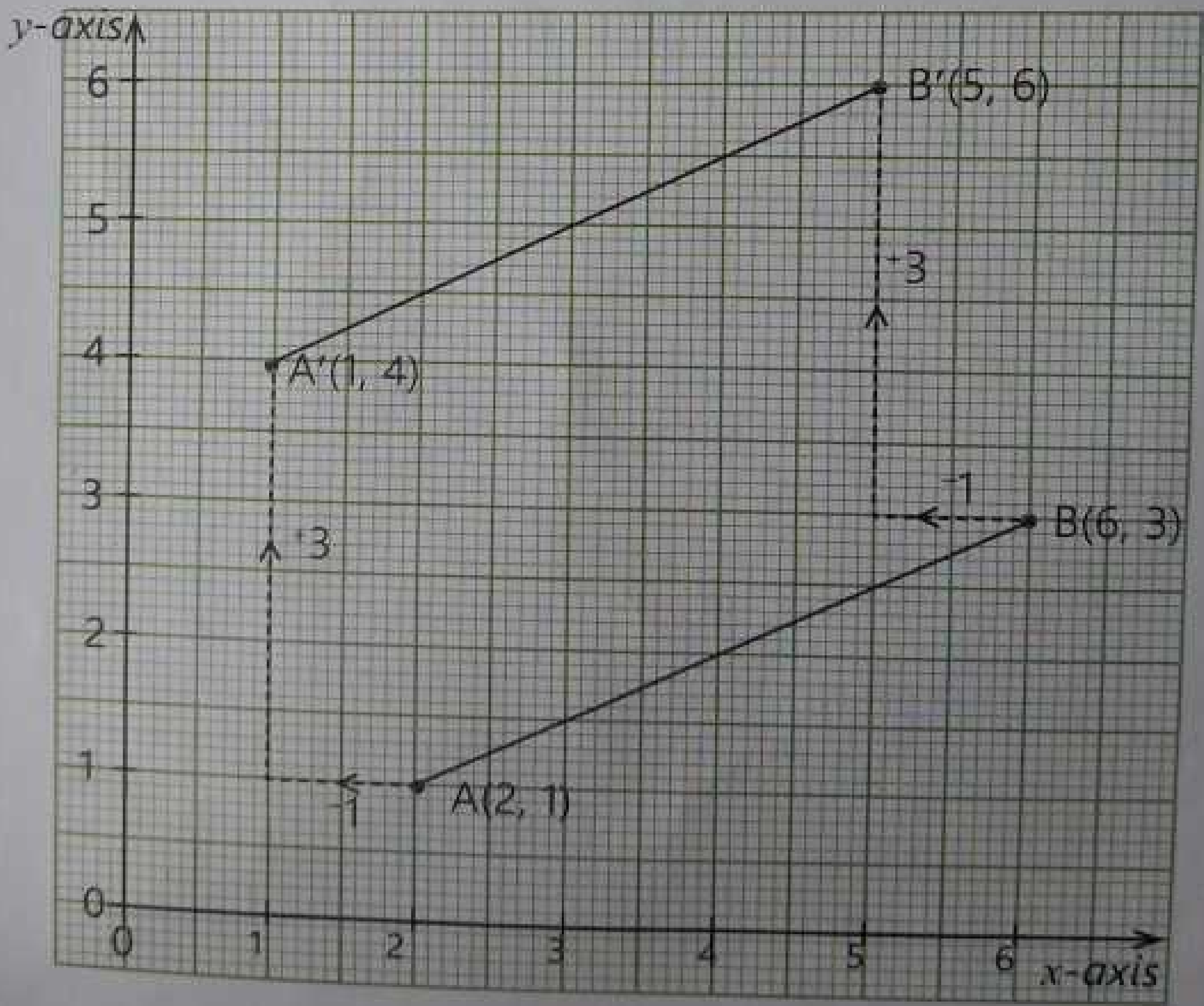
a) Plot points A(7, 5) and A′(3, 2) on a graph paper.
b) Draw the displacement vector AA′.
c) State the displacement of A to A′.
d) What is the vector that translates A′ to A?
e) Measure distance AA′.

Example 1

Line AB is translated by a vector T to give line A′B′. If A(2, 1), B(6, 3), A′(1, 4) and B′(5, 6), find the translation vector T.

Solution:

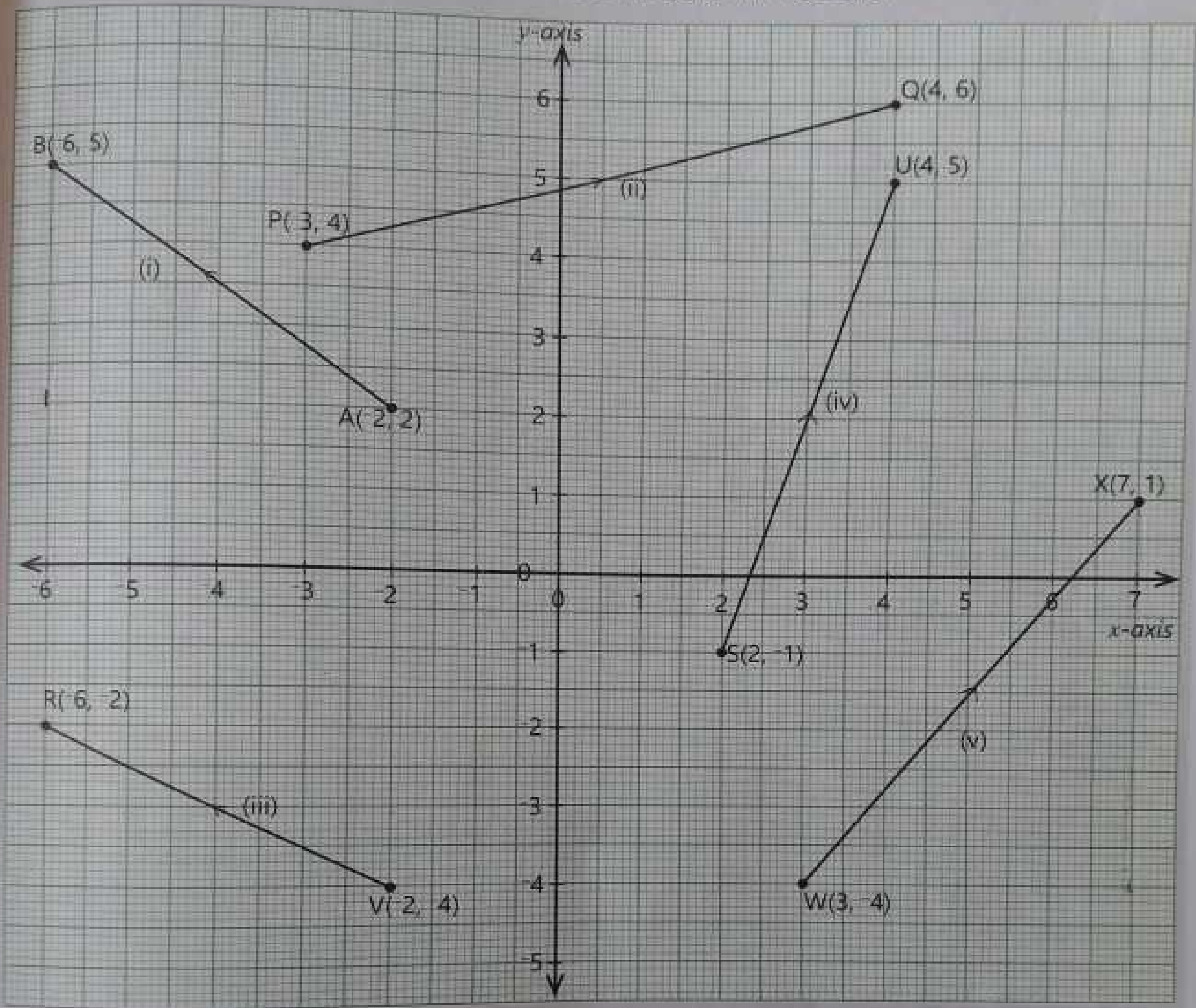
With A and its image A′, vector AA′ gives the displacement from A to A′ which represents the translation vector.



From A to A′, move ⁻1 horizontally and ⁺3 vertically. This gives T = $\begin{pmatrix} ^-1 \\ ^+3 \end{pmatrix}$. Similarly, from B to B′, make the same movements.

Exercise 2.1

Write the following displacement vectors as column vectors.



2.3 Vector Notation

Vector notation is a commonly used mathematical notation for working with mathematical vectors. Vectors are sometimes denoted using bold letters, such as **a** or **b**. But when writing where one can not easily write in bold, vectors are sometimes denoted using arrows or say, $\underset{\sim}{a}$. Other times, the vector between A and B is denoted by $\overrightarrow{AB}$, where A is the ***initial point*** (sometimes called the ***tail***) and B is the ***terminal point*** (sometimes called the ***head***) of the vector.

Geometrical Representation of Vectors

A vector is represented by a directed line segment as shown in the figure below.

The direction of the vector from A to B is shown by an arrow and its magnitude as the length of the line AB. The magnitude of vector $\vec{AB}$ is denoted by $|\vec{AB}|$.

Activity 2.3 (Work in groups)

a) Plot the points A(2, 3) and B(7, 4). Hence show vector $\vec{AB}$.

b) Represent vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} ^-3 \\ 2 \end{pmatrix}$ geometrically on a graph paper.

Hence, using your graph, find;

i) $\underset{\sim}{a} + \underset{\sim}{b}$

ii) $\underset{\sim}{a} - \underset{\sim}{b}$

Exercise 2.2

1. Given $\underset{\sim}{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} ^-4 \\ 4 \end{pmatrix}$, geometrically represent:

   i) $\underset{\sim}{a} + \underset{\sim}{b}$

   ii) $\underset{\sim}{b} - \underset{\sim}{a}$

2. Given that $\underset{\sim}{a} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 26 \\ 7 \end{pmatrix}$, geometrically represent $\underset{\sim}{a} + \underset{\sim}{b}$.

3. Given that vector $\underset{\sim}{p} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$, geometrically represent $\underset{\sim}{p} - \underset{\sim}{q}$.

ICT Activity

a) In groups, use an online dictionary to search for the meanings of the keywords in this chapter.

b) Using a word processing software, type and print your findings.

c) Present your findings to the rest of the class.

Revision Questions:

1. Point P(1, 2) is translated by a vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$. Find the new position of P.
2. Write down the coordinates of the image of each of these points under the translation given by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ using a graph paper.

   a) P(2, ⁻1) b) Q(6, 0) c) R(⁻5, 7) d) S(6, ⁻3)

3. Write down the column vectors for the translations that map each of the following points onto their images.

   a) A(2, 3) to A′(3, 7)

   b) B(⁻1, 6) to B′(3, 0)

   c) C(0, 5) to C′(5, 0)

   d) P(⁻5, 3) to P′(0, 0)

   e) Q(⁻3, 2) to Q′(2, ⁻3)

4. Find the coordinates of the object point P, if the image P′ under the translation given by vector $\begin{pmatrix} ^-4 \\ -3 \end{pmatrix}$ is:

   a) P′(5, 3)

   b) P′(⁻1, 3)

   c) P′(⁻7, ⁻3)

   d) P′(0, 6)

   e) P′(2, ⁻5)

5. On a set of coordinate axes, draw triangle PQR with vertices P(2, 3), Q(3, 4) and R(5, 3). If U and V are translations $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -7 \end{pmatrix}$, respectively, draw and label the image of triangle PQR under translations:

   a) **U** b) **V** c) **UV**

6. A is the point (⁻3, 3). **W** and **X** are translations $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$, respectively. Write down the coordinates of the following images of;

   a) **W**(A)

   b) **WX**(A)

   c) **X**(A)

   d) **XW**(A)

Hint: *W(A) means the image of A under translation* **W**, **WX**(A) *means image of A under translation* **X** *followed by* **W**.

Sample Activity of Integration

A young couple randomly placed three pictures in their living room. With time, some of the pictures became exposed to damage by the children as they grew up and as a result, it was necessary to shift them.

Support:
Pictures were originally placed as shown in the illustration below.

Resource:

- Knowledge of translations and vectors

Task:

As a Senior Two Mathematics learner, describe in terms of vectors and translations, the possible new positions of the pictures.

Chapter Summary

In this chapter, you have learnt that a translation of any object can be represented as a vector. You can now manipulate vectors and represent them geometrically.

THEME: **Data and Probability**

Chapter 3

Graphs

Keywords

- displacement
- scale
- speed
- time

By the end of this chapter, you should be able to:

- tabulate values from given relations.
- plot and draw lines through given points.
- choose and use appropriate scales to draw graphs.
- draw, read and interpret graphs; for example, distance-time and speed-time graphs, to estimate distance, speed and time.

Introduction

A set of data can be represented in many forms. One of the forms is ***graphs***. Graphs are becoming increasingly significant as they are applied in areas of Mathematics, Science and Technology.

This chapter guides you through plotting and interpretation of graphs to solve problems.

3.1 Tabulating values from given Relations

Activity 3.1(a) (Work in groups)

Draw a table of values for the given relations.

a) $n = m + 1$, starting with $m = 0$ up to $m = 8$

b) $t = d - 2$, starting with $d = 2$ up to $d = 10$

Activity 3.1(b) (Work in groups)

a) I am a farmer producing maize. In the coming harvest season, I look forward to harvesting my maize at a faster rate. The number of sacks (y) harvested in a day is related to the number of workers (x) employed by the relation $y = 2x$. Starting with $x = 1$, to $x = 10$, draw a table for the values of x and y.

b) Given the relation $y = 5x + 7$ and values of x as 0, 1, 2, 3, 4, 5 and 6, draw a table for the values of y and x.

c) Present your work to the rest of the class.

3.2 Plotting and Interpreting Graphs

Activity 3.2(a) (Work in groups)

I am a cyclist travelling such that in every minute, I cover 300 m. The following is a table showing the distance that I cover at different times.

Distance (m)	0	300		900	1200	1500		2100	2400
Time (minutes)	0	1	2	3		5	6	7	8

a) Copy and complete the table above.
b) Draw a distance-time graph for my journey.
c) Are there obstacles which may stop me from travelling 300 m per minute? If yes, mention and describe how they can stop me from moving at that speed.

Example 1

The table below shows the results obtained by a group of Senior Two learners when carrying out an experiment to determine the boiling point of water.

Temperature (^oC)	Time (minutes)
20	0
34	5
47	10
60	15
73	20
87	25
100	30
100	35
110	40

a) Using a suitable scale, plot a graph of temperature against time.
b) At what temperature was the water after heating for 23 minutes?
c) Using the graph, determine the time when the temperature was 90^oC.
d) What happened between 30 and 35 minutes?

Solution:

a) To plot a graph, you need the vertical and the horizontal scales.
To get the vertical scale, you may use;

$$\text{Vertical scale} = \frac{\textit{Biggest value} - \textit{smallest value}}{\textit{Number of boxes}}$$

- Now, counting the number of 1 cm boxes on the vertical axis of your graph paper, you realise that there are more than 20, but less than 30.

- Here, use 20 boxes for simplicity, though the choice of boxes depends on the range of values you are dealing with.

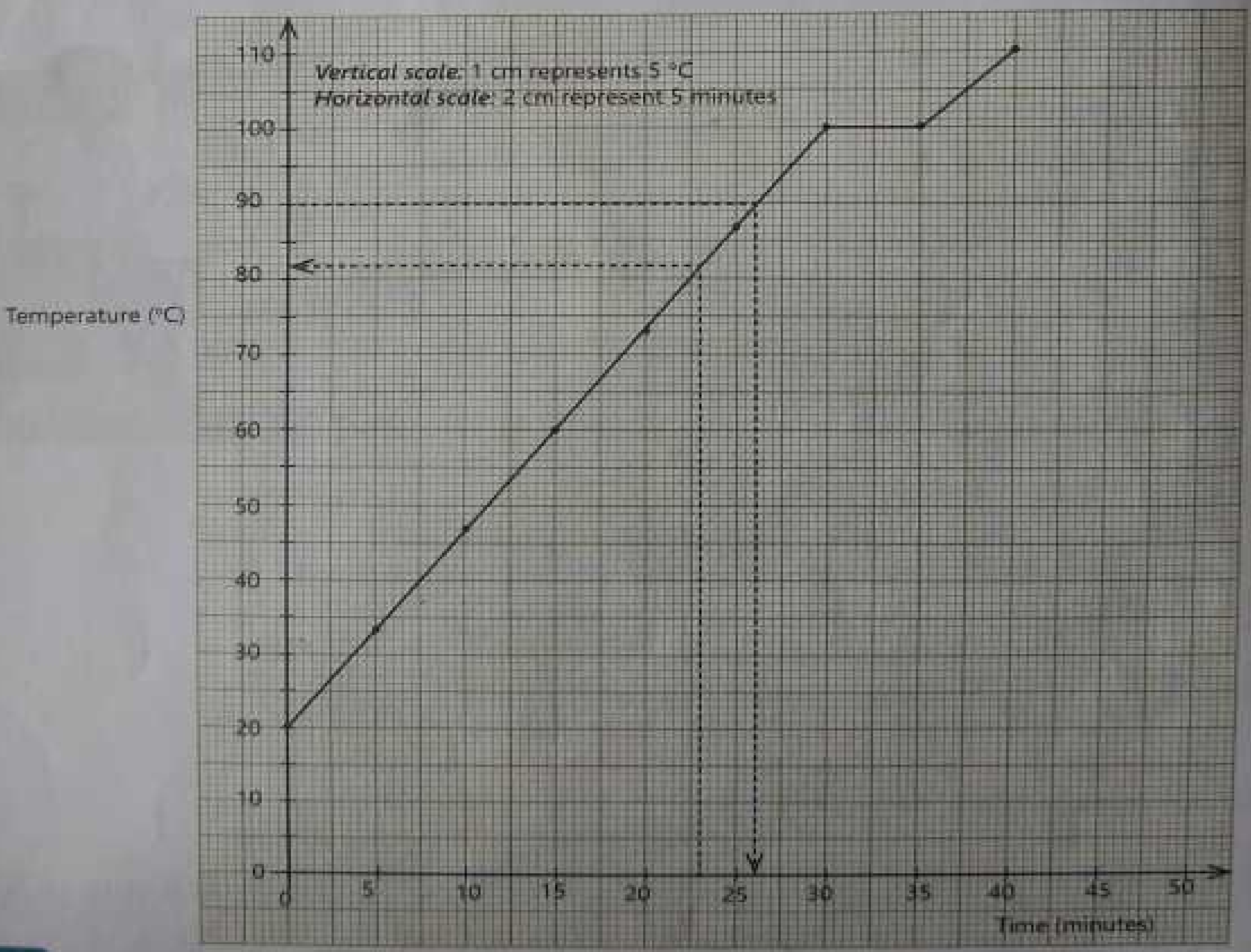
$$\text{Vertical scale} = \frac{\textit{Biggest value} - \textit{smallest value}}{\textit{Number of boxes}}$$

$$= \frac{110 - 20}{20}$$

$$= 4.5^{0}\text{C}$$

- Recall that the best scales are: a particular number of units (usually in cm) represents 1, 2, 5, 10, 20, 50, 100, 200, 500,.... (actual) quantity(ies).
- Now since 4.5 is close to 5, take 5°C for every 1 cm as our vertical scale.
- In regular intervals of 5°C, indicate temperature on the vertical axis.
- Similarly, Horizontal Scale = $\frac{40 - 0}{18} = 2.\bar{2}$ minutes.
- $2.\bar{2}$ is close to 2, but when you use a scale of 1 cm to represent 2 minutes, the values of time will not fit. To that end, increase from 2 to 5 minutes. But this would give a small graph. Therefore, the appropriate scale is: 2 cm represent 5 minutes.
- In regular intervals of 5 minutes, indicate time on the horizontal axis.

A Graph of Temperature against Time



b) Look for 23 on the horizontal axis. Draw a straight dotted line upwards until you reach the line, then horizontally towards the vertical axis, note the temperature at that point, which is 81.5°C.

c) Locate 90°C on the vertical axis, then draw a straight dotted line horizontally towards the line, then downwards to the horizontal axis and read off the time at the point of intersection or contact of the dotted line and the horizontal axis. Therefore, the time is 26 minutes.

d) Between 30 and 35 minutes, the water was at a constant temperature of 100°C, which indicates its boiling point.

Note

A good graph should cover at least ¾ of the page so as to be more clear, visible and readable.

Distance-Time Graphs

Given distance covered during certain time intervals, you can plot a distance - time graph.

Note

- Distance is plotted on the vertical axis, while time is plotted on the horizontal axis.
- The speed can be obtained by getting the slope between two points on the drawn graph.
- A rest is indicated by a horizontal line on the distance-time graph.

Activity 3.2(b)

The following table shows distances covered and time taken by a given bus moving from Kampala to Malaba.

Distance (km)	0	30	50	90	150	214	250
Time (minutes)	0	50	75	105	180	240	270
Town	Kampala	Mukono	Lugazi	Jinja	Iganga	Busitema	Malaba

Plot the points on a graph paper. Draw an appropriate line through the plotted points and from your graph, calculate;

a) the speed of the bus between;
   i) Mukono and Lugazi   ii) Mukono and Jinja   iii) Iganga and Malaba.

b) the distance covered after exactly;
   i) 1 ¼ hours   ii) 3½ hours

Example 2

At 8:00 a.m., a car is moving at 40 kmh^{-1} and a bus 30 km behind it, is moving in the same direction at 60 kmh^{-1}.

a) On the same graph paper, draw the travel graph of the car and the bus.

b) Use the graph in (a) to find;

i) the time and distance at which the bus overtakes the car.

ii) how far apart the bus and the car are, at 11:00 a.m.

Solution:

a) Moving at 40 kmh^{-1} means covering 40 km in every hour.

Time	8:00	9:00	10:00	11:00
Distance (km)	30	70	110	150

Moving at 60 kmh^{-1} means covering 60 km in every hour.

Time	8:00	9:00	10:00	11:00
Distance (km)	0	60	120	180

Vertical scale $= \dfrac{180 - 0}{20}$

$= 9$ km

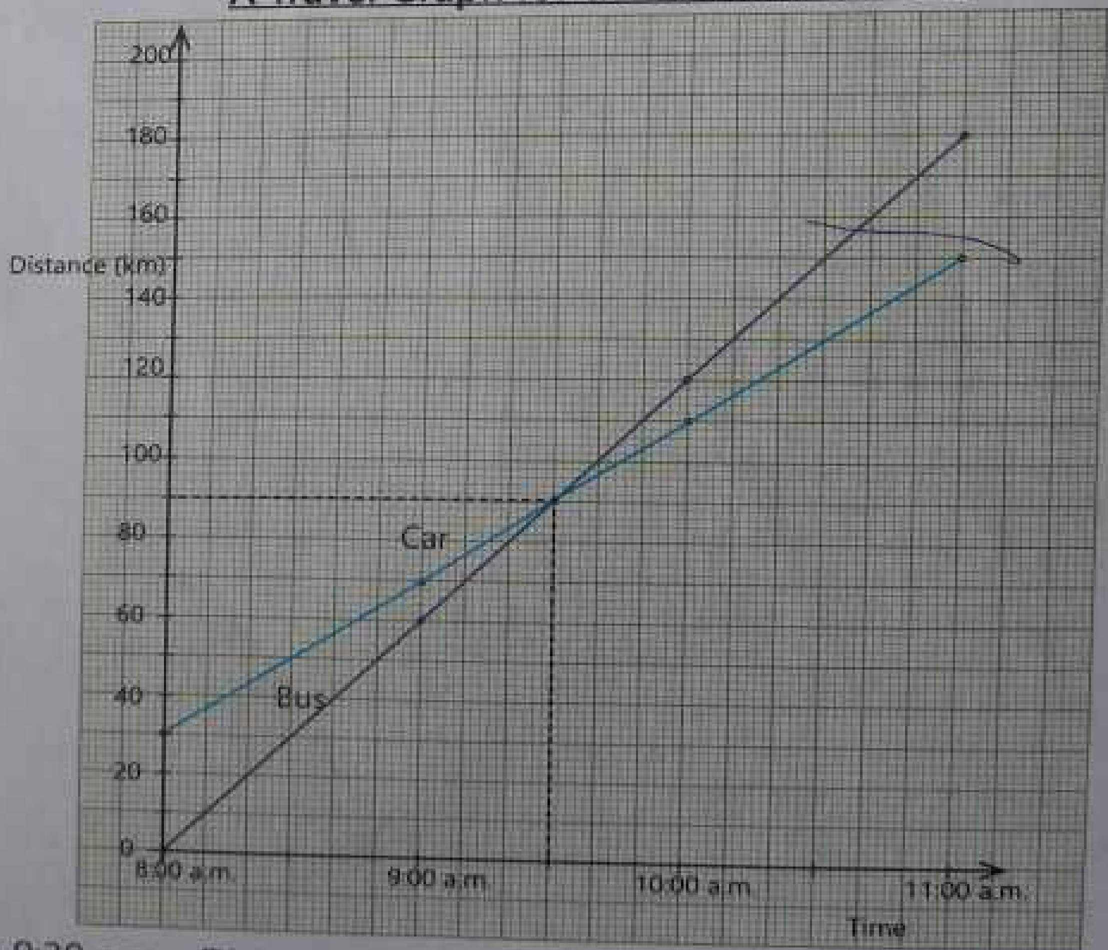
You will use 1 cm for 10 km.

Horizontal scale $= \dfrac{11{:}00 - 8{:}00}{18}$

$= 0.1\overline{6}$ hours $= 10$ minutes

You will use 1 cm for 10 minutes.

A Travel Graph for the Car and the Bus



b) i) Time: 9:30 a.m., Distance: 90 km

ii) (180 − 150) km = 30 km

Exercise 3.1

1. A cyclist travels a distance of 54 km from Kyazanga to Masaka town in 3 hours. He rests for 1 hour to take some food and water. He proceeds to Buwama which is 60 km from Masaka for 4 hours. Without resting, he proceeds to Nsangi which is 45 km from Buwama in 2 hours. He again takes a rest of 30 minutes. He then arrives at Kyengera, which is 7 km from Nsangi in 40 minutes. He then cycles to his destination, Nateete, which is 3 km from Kyengera, taking 30 minutes.
   a) Summarise the cyclist's journey in a suitable table and use it to plot a distance-time graph.
   b) Use the graph plotted to find the average speed between;
      i) Kyazanga and Masaka
      ii) Nsangi and Nateete
      iii) Masaka town and Buwama
      iv) Buwama and Kyengera
   c) Find the distance covered after;
      i) 3 hours
      ii) 6 hours
      iii) 9 hours
2. The table below shows a bus journey from Kotido to Mbale.

Bus station	Distance (km)	Depart (a.m.)	Arrive (a.m.)
Kotido		7:00	
Moroto	100	8:30	8:24
Katakwi	120	9:45	9:40
Soroti	65	10:50	10:45
Kumi	50	11:40	11:35
Bukedea	20	12:05	12:00
Mbale	40		12:45

   a) From the table above, use a suitable scale to plot a distance-time graph showing the journey of the bus.
   b) Using your graph, determine the;
      i) distance travelled by the bus after 2¾ hours
      ii) distance covered between 10:30 a.m. and 11:17 a.m.
      iii) time it takes the bus to cover 150 km from Kotido
   c) What is the average speed of the bus?
   d) How long would it take the bus to move from Kotido to Mbale if its average speed was increased by 10 kmh^{-1}?
3. Mugisha and Alupo live 80 km apart. At 7:00 a.m., Mugisha left his home cycling towards Alupo's home at 20 kmh^{-1}. At 8:00 a.m., Alupo left her home cycling towards Mugisha's home at 8 kmh^{-1}.
   a) On the same axes, draw the travel graphs for Mugisha and Alupo.
   b) Using your graph in (a), find the distance and time from Mugisha's home where the two people met.

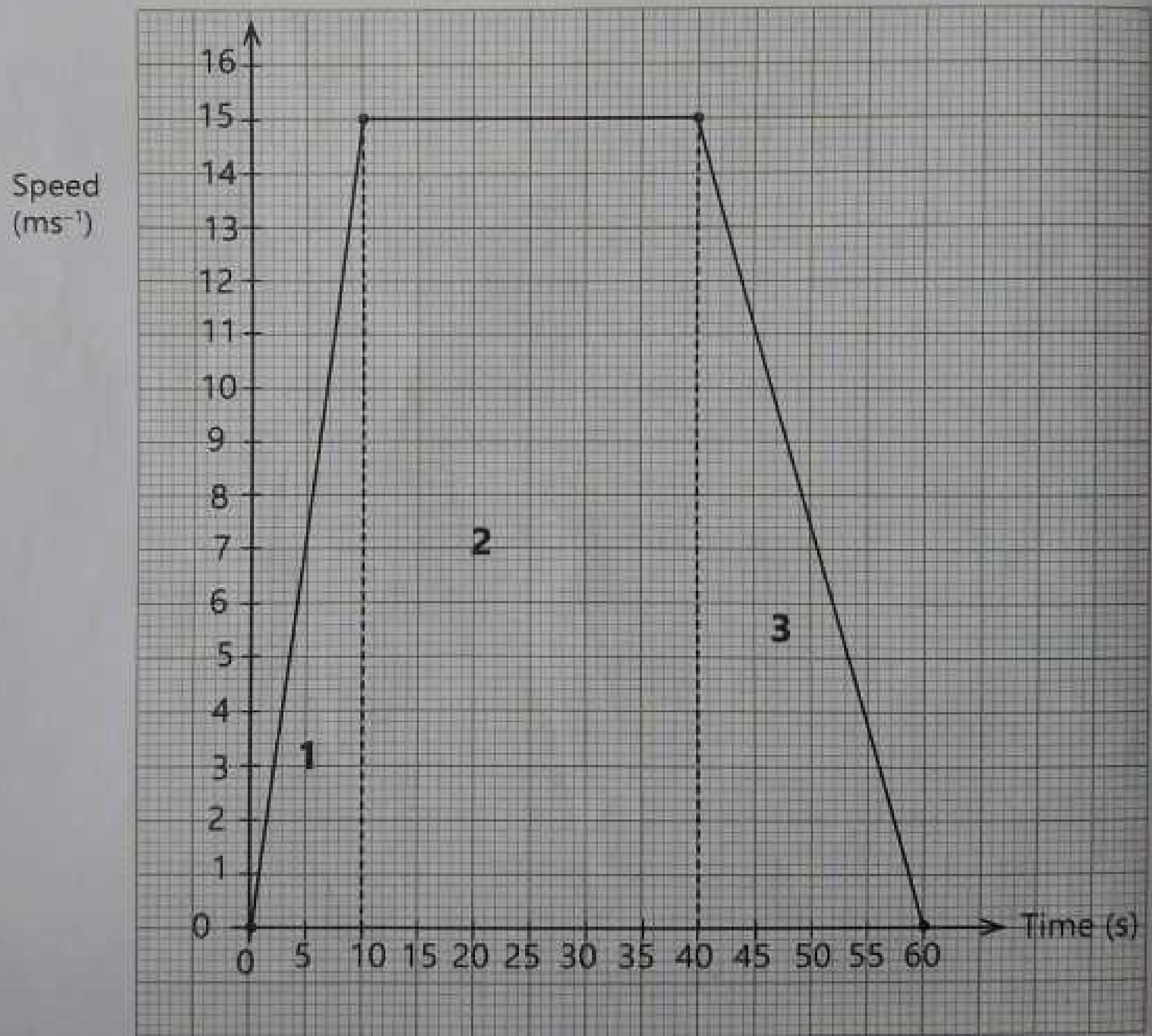
Speed-Time Graphs

Given the speed at particular time intervals, you can plot a speed–time graph. The speed is represented on the vertical axis, while the time is on the horizontal axis. From this graph;

- distance can be got by getting the area under the graph.
- average speed can be got by dividing the total distance covered by total time taken to cover the whole journey.

Example 3

Below is a speed-time graph.



Use it to find the;

a) total distance travelled

b) average speed for the whole journey

Solution:

a) Total Distance = Area of Triangle 1 + Area of Rectangle 2 + Area of Triangle 3.

$$= (\tfrac{1}{2} \times b \times h) + (l \times w) + (\tfrac{1}{2} \times b \times h)$$
$$= (\tfrac{1}{2} \times 10 \times 15) + (40 - 10) \times (15 - 0) + (\tfrac{1}{2} \times (60 - 40) \times 15)$$
$$= 75 + (30 \times 15) + 150$$
$$= 75 + 450 + 150$$
$$= 675 \text{ m}$$

b) Average speed = $\dfrac{\textit{total distance covered}}{\textit{total time taken}}$

- From a), the total distance is 675 m.
- From the graph, the total time taken is 60 seconds.
- So, average speed = $\dfrac{675}{60} = 11.25 \text{ ms}^{-1}$.

Exercise 3.2

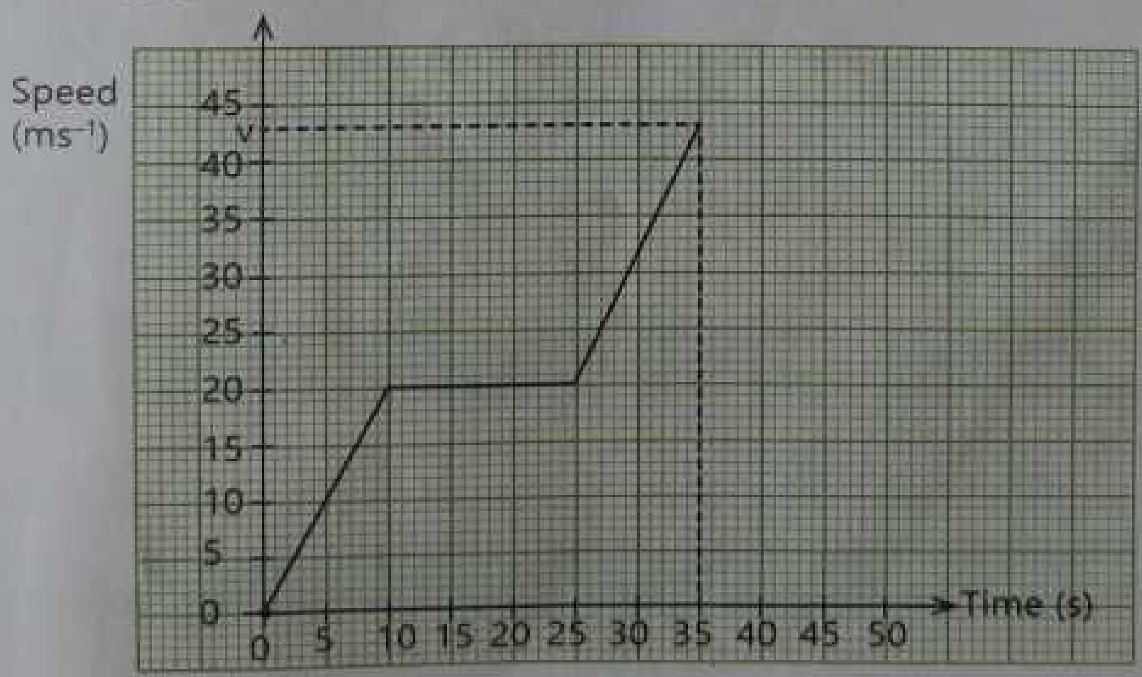
1. The speed of a train is measured at regular intervals of time from t = 0 to t = 60 s, as shown below.

Time (s)	0	10	20	30	40	50	60
Speed (ms^{-1})	0	10	16	19	22	24	25

   a) Draw a speed-time graph to illustrate the motion. Use scale: Horizontal scale = 1 cm : 5 s and Vertical scale = 1 cm : 2.5 ms^{-1}.

   b) Use the graph to estimate the distance travelled by the train from t = 30 s to t = 60 s.

2.



The graph on the previous page shows the motion of a cyclist from one town to another. Given that the total distance travelled is 710 m, find;

a) the value of v
b) the rate of change of speed when t = 30 s
c) the time taken to travel the first 420 m of the journey

ICT Activity

In groups;

a) use a spreadsheet program to re-attempt *Activity 3.2*(a) parts (a) and (b)
b) print a copy and compare it with the one you did on the graph paper.
c) of the 2 graphs, which one is more accurate, and why?

Revision Question:

The government of Uganda, through the *Operation Wealth Creation* programme, piloted with 10 farmers in Kiryandongo district with the aim of increasing the production of maize by providing them with different quantities of fertilisers. The yields obtained by the different farmers after using the fertilisers are summarised in the table below.

Farmers	Amount of Fertilisers received (Kg)	Yields obtained (Kg)
Lutaaya	20	500
Aku	30	540
Byamugisha	35	585
Nabirye	45	632
Wameri	50	670
Nambeshe	57	699
Achieng	60	725
Okello	64	765
Bwambale	71	865
Ayela	79	915

a) Plot the above information using a suitable scale on a graph paper, OR using the Microsoft Excel spreadsheet program.
b) Discuss the relationship between the amount of fertilisers used and the yields obtained using the graph.
c) As a Senior Two Mathematics learner, what advice would you give to the government regarding the above program?

Sample Activity of Integration

group of young women have been invited by the school administration to be part the school Women's Day celebrations. In their presentation, they want to talk bout teenage pregnancy with the help of a graph.

Support:

n their talk, the women want to present the following table about teenage pregnancy and motherhood showing percentages of women aged 15-19.

Age	Percentage of women Aged 15-19 who:		Percentage who have began child bearing	Number of women
	Have had a live birth	Are pregnant with first child		
15	1.6	1.6	3.1	871
16	5.5	3.8	9.4	966
17	15.0	7.0	22.1	792
18	33.0	7.2	40.2	851
19	45.8	8.0	53.9	785

Source: *Uganda DHS 2016*

Resources

- Knowledge of scales
- Knowledge and skill of plotting and interpreting graphs

Task:

As a learner of Mathematics, help them to make a clear presentation.

Chapter Summary

In this chapter, you have learnt that when given any relation, you can always tabulate the values of that relation. These tabulated values can be plotted on a graph paper using an appropriate scale. From the graph, one can interpret and analyse the relation between the sets of values plotted.

THEME: Numbers

Chapter 4

Numerical Concepts 1 (Indices)

Keywords

- approximation
- indices
- decimal place
- standard form
- estimation
- significant figures

By the end of this chapter, you should be able to:

- give approximate answers to calculations.
- write numbers to a given number of significant figures.
- differentiate between significant figures and decimal places.
- express numbers in standard form.
- identify base number and index.
- state and apply the laws of indices in calculations.
- use a calculator to find powers and roots.

Introduction

Indices and *standard form* are a convenient way of writing either very large numbers like the distance (in metres) between the sun and the earth, or very small numbers like the weight (in grams) of a human cell.

In this chapter, you will learn how to use indices and standard form to approximate answers to calculations.

4.1 Approximations

Activity 4.1 (Work in groups)

Suggested materials:

- circular objects, like bottle tops
- a thread
- a ruler

Instructions:

a) Pick any object with a circular end.
b) Measure the length around the circular end.
c) Measure the diameter of the circular end.
d) Divide the value in b) by the one in c), leave your answer as a fraction.
e) Using different objects (with a circular end), repeat procedures b) to d). What are your observations?
f) Use a calculator to find the fraction in d) as a decimal.
g) How many decimal places did you use?
h) How many digits are in your answer in g)?

Significant Figures

Significant figures are the digits of a number starting with the first non-zero digit on the left.

Examples:

a) 0.02540 has 4 significant figures.
b) 9110 has 3 significant figures.
c) 0.390 has 3 significant figures.

There are special cases when zero is considered a significant figure. These include;

a) when it is between non-zero digits; for example, 207 (3 s. f.).
b) when it is on the right side of a non-zero digit after a decimal point, for example, 2.80 (3 s. f.) and 13.120 (5 s. f.).
c) when it comes immediately after a decimal point. For example, 12.0 (3 s. f.), 44.00 (4 s. f.), 10.0 (3 s. f.) and 200.00 (5 s. f.).

> **Note** Leading zeros (in any number) and trailing zeros (in whole numbers) are non-significant; for example, 0.25 (2 s. f.), 0.0025 (2 s. f.) and 219000 (3 s. f.).

Example 1

Express the following numbers to;

a) 2 significant figures
(i) 436 (ii) 28990

b) 3 significant figures
(i) 1084 (ii) 49.96

c) 1 significant figure
(i) 0.00509 (ii) 1.908

Solution:

a) i) In 436, digit 3 is the rounding digit and 6>5, hence add 1 to 3 to become 4 and make 6 to become 0.

That is, $4\overset{+1}{\widehat{36}}$

Therefore, $436 \approx 440$ (2 s. f.).

(ii) In 28990, 8 is the rounding digit and 9>5, so add 1 to 8 and 8 becomes 9.

That is, $2\overset{+1}{\widehat{89}}90$

Therefore, $28990 \approx 29000$ (2 s. f.).

b) (i) In 1084, the rounding digit is 8, and 4<5, hence 8 remains as digit 8.

That is, $10\overset{+0}{\widehat{84}}$

Therefore, $1084 \approx 1080$ (3 s. f.).

(ii) In 49.96, the rounding digit is 9 and 6>5. So, add 1 to 9.

That is, $49.\overset{+1}{\widehat{96}}$

Therefore, $49.96 \approx 50.0$ (3 s. f.).

c) (i) In 0.00509 the digits 0.00 before digit 5 are not counted as significant. The rounding digit is 5,and 0<5.

That is, $0.00\overset{+0}{\widehat{50}}9$

Hence, $0.00509 \approx 0.005$ (1 s. f.).

(ii) In 1.908, 1 is the rounding digit and 9>5, so add 1 to the rounding digit 1.

That is, $\overset{+1}{\widehat{1.9}}08$

Hence, $1.908 \approx 2$ (1 s. f.).

Approximating Answers to Calculations

Example 2

Work out 3.8×4.1, giving both the rough estimate when rounding off and the actual value.

Solution:

Rounding off: From 3.8×4.1, we round off each value to obtain;

$4 \times 4 = 16$

Accurate value: $3.8 \times 4.1 = 15.58$

Example 3

Work out $6.8 \div 2.4$ as a rough estimate - by rounding off, and then give the actual value.

Solution

Rounding off: $7 \div 2 = 3.5$

Actual value: $6.8 \div 2.4 = 2.8\overline{3}$

Exercise 4.1

1. Work out the following by rounding off to the nearest whole number and giving the actual value.
   a) $426 \div 11.56$
   b) 1009.3×1.998
   c) $10.5 - 3.4$

2. State the number of decimal places to which the following numbers have been written.
   a) 4.672
   b) 100.7001
   c) 67.24
   d) 0.07245

3. Round off the following numbers to the given number of decimal places:
   a) 1.053 (2 decimal places)
   b) 67.4378 (3 decimal places)
   c) 100.0145 (2 decimal places)
   d) 9.42999 (4 decimal places)

4. Round off the following to the given number of significant figures:
   a) 1. 053 (3 significant figures)
   b) 100.0145 (4 significant figures)
   c) 0.00527 (1 significant figure)
   d) 48,000 (1 significant figure)

4.2 Expressing Numbers in Standard Form

Activity 4.2(a)

In groups, you are required to watch a video about the galaxy. From this video, answer the following questions.

a) How many particles are in the universe?
b) How far is it from the earth to the; moon, sun and stars?

In everyday life, large numbers are often dealt with. For instance, the population of Uganda was estimated to be 42,860,000 in 2017. The speed of light is 671,000,000 miles per hour.

These figures can be written in a short form called standard form or scientific notation. For example, $42{,}860{,}000 = 4.286 \times 10^7$ and $671{,}000{,}000 = 6.71 \times 10^8$.

To express a number in standard form, you write it in such a way that one digit is before the decimal point. This digit before the decimal point should be an integer in the interval $1 \leq x \leq 9$, and multiplied by 10^n where n is the number of steps the decimal point moves.

Activity 4.2(b) (Work in groups)

a) Match the numbers in A to their respective standard form in B.

A	B
679,000	6.79×10^1
0.679	6.79×10^{-3}
0.00679	6.79×10^5
67.9	6.79×10^{-1}

b) Write; i) 0.00002 in standard form.

ii) 8.72×10^4 as a decimal number without using powers.

Note: When the decimal point moves to the left, the power of 10 is a positive and negative when it moves to the right as illustrated in the following examples.

Example 4 (Discuss in groups)

Express the following numbers in standard form.

a) 243000 b) 3.569 c) 0.52

Solution:

a) 243000 = 2.43×10^5 (the decimal point moved five steps to the left)

b) 3.569 = 3.569×10^0 (the decimal point did not move)

c) 0.52 = 5.2×10^{-1} (the decimal point moved one step to the right)

Example 5 (Discuss in groups)

Express the following numbers in standard form.

a) 0.0000735 b) 0.0546

Solution:

a) 0.0000735 = 7.35×10^{-5} (the decimal point moved five steps to the right)

b) Here, the decimal point is moved 2 places to the right. So, $0.0546 = 5.46 \times 10^{-2}$.

Exercise 4.2

1. Write the following in standard form.
   a) 26
   b) 105
   c) 1.24003
   d) 246.8434
   e) 123400
   f) 63467.0015
   g) 3408917645
   h) 95434378910
   i) 5345.9653
   j) 1.989345
   k) 1000
   l) 503457
   m) 100000000
   n) 10934356

2. Express the following numbers in standard form.
   a) 0.034
   b) 0.000598
   c) 0.0000000018
   d) 0.00001589576

3. Write the following in standard form:
   a) $2.5 \times 10 \times 2 \times 10$
   b) $4.0 \times 10 \times 3 \times 10$
   c) $565 \times 10000 \times 10$
   d) $8798 \times 0.0001 \times 10$

4.3 Using a Calculator to find Powers and Roots

Activity 4.3

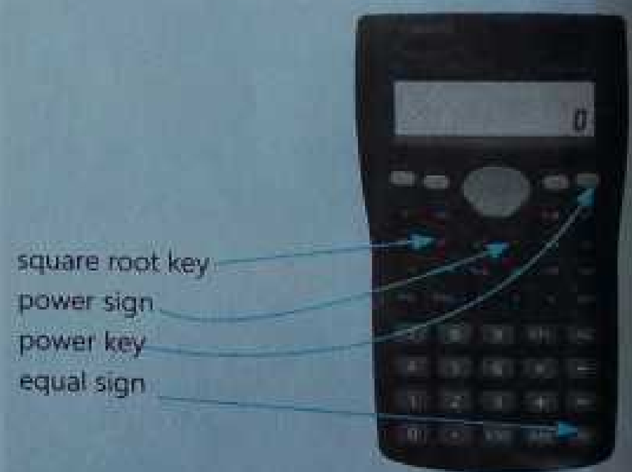
In groups, and using a calculator, evaluate;

a) 5^6

Instructions:

- Switch on the calculator.
- Press the keys in the stated order: 5, power sign, 6 and then equal sign.

b) the square root of 4



Exercise 4.3

Evaluate the following using a calculator.

a) 4^2 b) 5^3 c) 5^7 d) 7^6

e) 8^4 f) 4^{13} g) $\sqrt{7}$ h) $\sqrt{64}$

i) $\sqrt[4]{49}$ j) $\sqrt[3]{8}$ k) $\sqrt[5]{64}$ l) $\sqrt[7]{49}$

4.4 Indices

A message is spread such that if one person tells other 10 people, and then each of these 10 people tells other 10 people, and so on; the message gets spread rapidly. This message could be a video, photo, item, or product across the internet.

Level	0	1	2	3	4	...
Number of recipients	1	10	100	1,000	10,000	...
Power form	10^0	10^1	10^2	10^3	10^4	...

Question:
Write down the relationship between the number of recipients and the level.

Squares, cubes and higher powers are indicated as small digits called ***indices*** just like 10^0, 10^1, 10^2, 10^3 and 10^4 shown above.

Activity 4.4(a) (Work in groups)

If you have one rabbit and it has four offsprings, and then each of these offsprings has four offsprings of their own, and so on, you get the following exponential population growth.

Generation	0	1	2	3	4	5	6
Offsprings	1	4	16	___	___	___	___

a) Copy and complete the table above.
b) Use the table values to write the relation between number of offsprings and generation.

Activity 4.4(b) (Work in groups)

a) Prime factorise 36.
b) Express 36 as a product of its prime factors in power form.

Generally, for a^n, a is the base and n is the index (power).

a) For 2^6, 6 is the index (power) of 2 and it is read as "2 raised to the power of 6".
b) For 10^8, 8 is the index (power) of 10 read as "10 to the power 8".
c) 7^2 is read as "7 squared".
d) 5^3 is read as "5 cubed".

From the examples on the previous page, you see that an <u>index</u> is the number of times the base is multiplied by itself. For example, a^4 means "a is multiplied by itself four times", that is, $a^4 = a \times a \times a \times a$.

Exercise 4.4

Write the following numbers as a product of their prime factors.

a) 72　　b) 81　　c) 100　　d) 12

e) 16　　f) 125　　g) 40　　h) 32

Laws of Indices

Activity 4.4(c) (Work in groups)

a) Use a calculator to evaluate and compare your answers in each of the following:

i) $5^2 \times 5^6$ and $5^{(2+6)}$

ii) $3^3 \div 3^2$ and 3^{3-2}

iii) 11^1, 12^1 and 13^1

iv) $(2^3)^2$ and $2^{(3\times2)}$

v) $7^4 \div 7^4$ and $7^{(4-4)}$

vi) $2^3 \div 2^4$ and $2^{(3-4)}$

vii) $(\frac{2}{3})^{-2}$ and $(\frac{3}{2})^2$

b) Using your results in (a), complete the following:

i) $a^m \times a^n$ = ______

ii) $a^m \div a^n$ = ______

iii) a^1 = ______

iv) $(a^m)^n$ = ______

v) a^0 = ______

vi) a^{-n} = ______

vii) $(\frac{a}{b})^{-n}$ = ______

Exercise 4.5

1. Write the following numbers with a single index.

a) $2 \times 2^4 \times 2^5$

b) $a^6 \div a^3$

c) $(a^5)^{-2}$

d) $24a^7 \div 6a^3$

e) $(0.6)^6 \div (0.6)^3$

f) $3^{17} \times 3^6$

g) $(a^6)^2$

h) $12a^7 \times 3a^5$

i) $a^{23} \times a^{10}$

j) $\frac{13^{21}}{13^{14}}$

k) $a^5 \times a^7$

l) $1.5^6 \times 1.5^{19}$

m) $(m^2)^{20} \times (m^3)^{15}$

2. Solve for x in the following:

a) $2^x = 8$　　b) $3^x = 81$　　c) $a^5 \times a^x = a^{10}$　　d) $(m^2 \times n^3)^4 = (mn)^x$

3. Simplify:

a) $a^{\frac{1}{2}} \times a^{\frac{5}{2}}$　　b) $x^{3.7} \div x^{2.5}$　　c) $\frac{p^4 \times p^5}{p^{\frac{1}{2}}}$

4. Using laws of indices, simplify:

a) $6^5 \div 6^5$　　b) $a^3 \div a^3$　　c) $a^7 \div a^{15}$

Exercise 4.6

1. Simplify the following:
   a) $2^{10} \times 2^{6} \div 2^{1}$
   b) $(0.0001)^{0}$
   c) $100^{9} \times 25^{0}$
   d) $4^{5} \times 4^{9} \div 4^{14}$
   e) $8^{-3} \times 8^{-14}$
   f) $(^{-}3)^{10} \times (^{-}3)^{15}$
2. Express the following in the form a^{-n}.
   a) $\frac{1}{9}$
   b) $\frac{1}{a^{3}}$
   c) $\frac{1}{10000}$
   d) $\frac{x^{5}}{25}$

Fractional Indices

Activity 4.4(d) (Work in groups)

a) Work out the following:
   i) $3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$
   ii) $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}}$
   iii) $5^{\frac{1}{4}} \times 5^{\frac{1}{4}} \times 5^{\frac{1}{4}} \times 5^{\frac{1}{4}}$

b) Using your results in (a), relate the following:
   i) a and $a^{\frac{1}{2}}$
   ii) a and $a^{\frac{1}{3}}$
   iii) a and $a^{\frac{1}{4}}$
   iv) a and $a^{\frac{1}{n}}$

c) Work out the following:
   i) $7^{\frac{2}{3}} \times 7^{\frac{2}{3}} \times 7^{\frac{2}{3}}$
   ii) $6^{\frac{3}{4}} \times 6^{\frac{3}{4}} \times 6^{\frac{3}{4}} \times 6^{\frac{3}{4}}$

d) Using your results in (c), relate the following:
   i) a^{2} and $a^{\frac{2}{3}}$
   ii) a^{3} and $a^{\frac{3}{4}}$
   iii) a^{m} and $a^{\frac{m}{n}}$

Example 6

Evaluate: a) $8^{\frac{1}{3}}$ b) $16^{\frac{3}{4}}$ c) $27^{\frac{2}{3}}$

Solution:

a) $8^{\frac{1}{3}} = (2\times2\times2)^{\frac{1}{3}}$
$= (2^{3})^{\frac{1}{3}}$
$= 2^{\frac{3\times1}{3}}$
$= 2^{1}$
$= 2$

b) $16^{\frac{3}{4}} = (2^{4})^{\frac{3}{4}}$
$= 2^{\frac{4\times3}{4}}$
$= 2^{3}$
$= 8$

c) $27^{\frac{2}{3}} = (3^{3})^{\frac{2}{3}}$
$= 3^{\frac{3\times2}{3}}$
$= 3^{2}$
$= 9$

Example 7

Simplify: $(16x^6)^{\frac{1}{2}}$

Solution:

$$
\begin{aligned}
(16x^6)^{\frac{1}{2}} = 16^{\frac{1}{2}} \times (x^6)^{\frac{1}{2}} &= (2^4)^{\frac{1}{2}} \times (x^6)^{\frac{1}{2}} \\
&= 2^{\frac{4\times 1}{2}} \times x^{\frac{6\times 1}{2}} \\
&= 2^2 \times x^3 \\
&= 4 \times x^3 \\
&= 4x^3
\end{aligned}
$$

Exercise 4.7

Simplify the following by removing the fractional powers.

a) $\dfrac{81^{\frac{1}{4}}}{(x^3)^{\frac{4}{3}}}$ b) $\dfrac{9^{\frac{1}{2}}}{(3^3)^{\frac{1}{3}}}$ c) $\dfrac{8^{\frac{1}{3}}}{(5^3)^{\frac{1}{3}}}$ d) $64^{\frac{3}{18}} \times 27^{\frac{2}{3}}$

ICT Activity

a) In groups, using a word processing software, type and print a copy of *Exercise 4.5.*

b) Present your work to the teacher.

Revision Questions:

1. What is the value of;

   a) 0^2 b) 2^0

2. Simplify the following without using a calculator.

   a) $(2^6)^2 \div 4^5$ c) $a^0 \times b^3$ e) $\dfrac{8a^{11}}{2a^9}$ g) $\dfrac{8q^2 \times 3q^7}{6q^2}$

   b) $(32x^5)^{\frac{-2}{5}}$ d) $x\left(2x^{\frac{-1}{4}}\right)^4$ f) $\dfrac{(2x^2)^3}{4x^2}$

3. Write the following in standard form.

   a) 0.1 c) 0.3×10^4 e) 11×10^{-2}

   b) 0.5×10^{-1} d) 10.1×10^{-2} f) 5×10^{-2}

4. Evaluate:

   a) 4^0 c) $32^{\frac{1}{5}}$ e) 3^{-2} g) $\dfrac{\sqrt{200}}{\sqrt{8}}$

   b) 0^7 d) $8^{\frac{5}{3}}$ f) $16^{\frac{3}{4}}$

5. Express each of the following in the form 7^k.

a) $\sqrt[4]{7}$ b) $\dfrac{1}{\sqrt[3]{7}}$ c) $7^4 \times 49^{10}$

6. Given that $32\sqrt{2} = 2^n$, find the value of n.

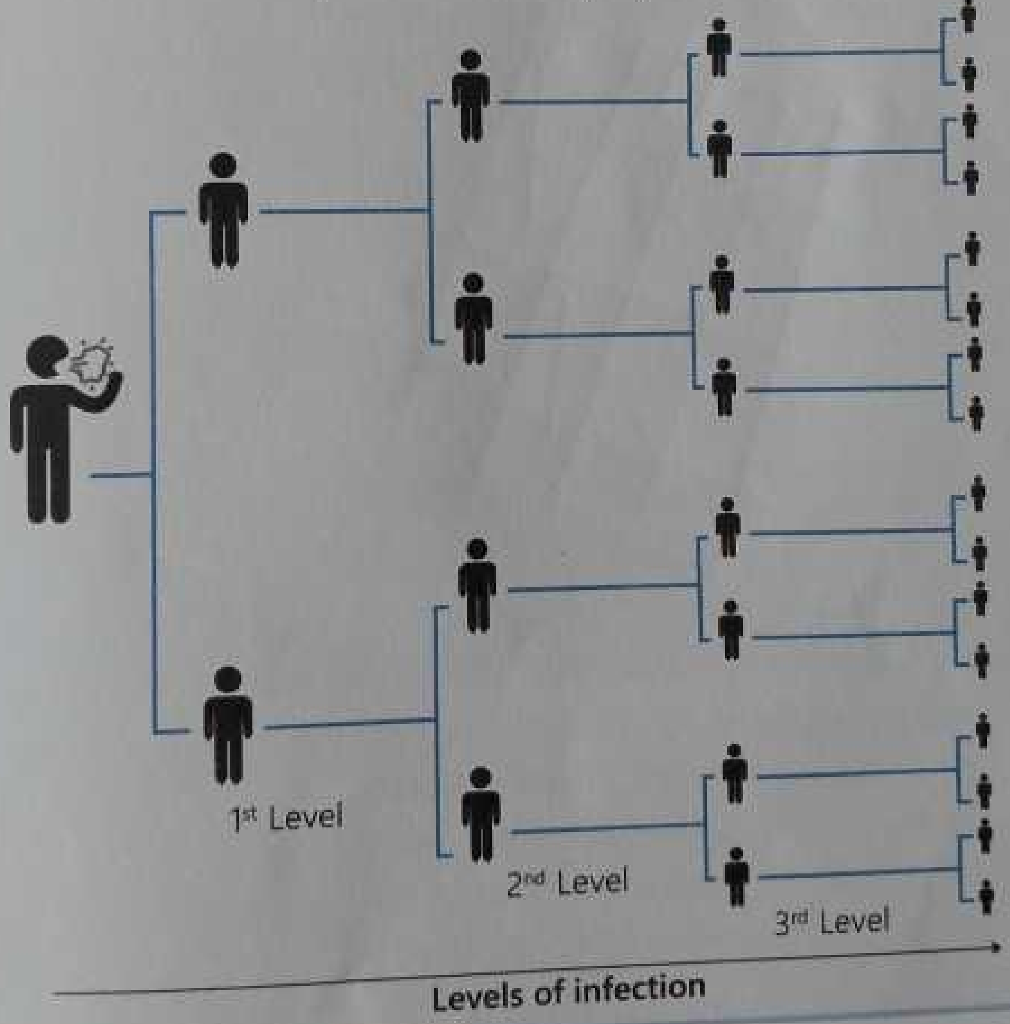
7. Express 8^{2x+3} in the form 2^y, stating y in terms of x.

Sample Activity of Integration

On 9th January, 2020, China reported and confirmed 15 cases of a novel coronavirus. A few months later, Uganda also confirmed her first case on 22nd March, 2020. However, by the end of April, 2020, over 2 million people were infected with coronavirus worldwide.

Support:

An infected person is likely to infect two people as illustrated below.



Resources:

- Knowledge of indices
- Knowledge of standard form
- Knowledge of using a calculator

Task:

If you are a member of a village health team, use the numbers infected at the 10th, 20th and at the 25th level of infection to explain to your community why the corona virus outbreak turned into a global pandemic.

Chapter Summary

In this chapter, you have learnt that:

- A number like 2.666666... can be approximated to any number of decimal places; for example, to one decimal place as 2.7.
- A number like 3.1416 has 4 decimal places but has 5 significant figures.
- Numbers can be written in standard form; for example, the total area covered by Uganda is 241,037 km^2, which can be written in standard form as 2.4×10^5.
- For a^b, a is the base and b is the power (index).
- Properties of indices can be summarised as below.

  a) $a^x \times a^y = a^{x+y}$ b) $a^0 = 1$ c) $a^x \div a^y = a^{x-y}$

  d) $\frac{1}{\sqrt[n]{m}} = m^{\frac{-1}{n}}$ e) $(a^x)^y = a^{xy}$ f) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

THEME: Patterns and Algebra

Chapter 5 Inequalities and Regions

Keywords

- inequality symbols
- linear inequalities
- regions

By the end of this chapter, you should be able to:

- identify and use inequality symbols.
- illustrate inequalities on the number lines.
- solve linear inequalities in one unknown.
- represent linear inequalities graphically.
- form simple linear inequalities for regions on a graph.

Introduction

You basically use inequalities everyday, sometimes without noticing. Think about the following situations: speed limit on a highway and the amount of time it will take you to get from home to school. All these, and many others, can be represented as mathematical inequalities.

This chapter will enable you to represent and solve problems involving inequalities.

5.1 Identifying and Using Inequality Symbols

Activity 5.1(a)

Consider two baskets, A and B, each having oranges as shown below.

A

B

Individually;

a) count the number of oranges in each basket.

b) what would you say about the number of oranges in the two baskets, **in one statement**?

Now, think about statements such as *"less than"*, *"not less than"*, *"at most"*, *"at least"*, *"greater than"*, *"greater or equal to"*, *"not more than"*, and many others. The statements of this form that you will encounter can be expressed as mathematical symbols.

Activity 5.1(b) (Work in groups)

a) Collect your pencils, pens and books to form groups A, B and C, respectively.

b) Count the number of items in each group formed in (a).

c) Using at least one of the symbols "$>$", "$<$", "$\leq$", "$\geq$" and "$=$", state the relationship between the number of items in groups:

(i) A and B

(iii) A and C

(iii) B and C

Consider the following statements.

a) There are less than 100 km from Kampala to Mukono. If x represents the distance from Kampala to Mukono, then $x < 100$.

b) There are at least 20 oranges in the supermarket. If r represents the number of oranges, then $r \geq 20$.

c) There are more than 10 boys in Senior Two. If b represents the number of boys in Senior Two, then $b > 10$.

d) There are at most 40 cars in the school parking lot. If the number of cars is represented by c, then $c \leq 40$.

Illustrating Inequalities on a Number Line

For you to illustrate inequalities on a number line, use either a filled dot (•) or an empty dot (◦) to show that the digit is either inclusive or exclusive respectively.

Activity 5.1(c)

Individually, draw a number line and on it, illustrate each of the statements below.

a) The distance (x) from Kampala to Mukono is less than 100 km.

b) There are at most 40 cars in the school parking lot.

c) There are more than 10 boys in the Senior Two class.

d) There are at least 20 oranges in the supermarket.

Example 1 (Discuss in groups)

Interpret and then represent the following statements on a number line.

a) b is at most 5.

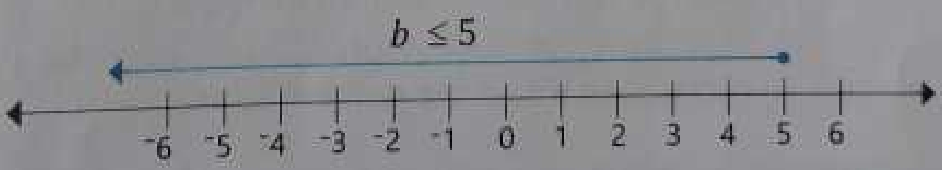
b) c is non-negative.

c) d is positive but not more than 6.5.

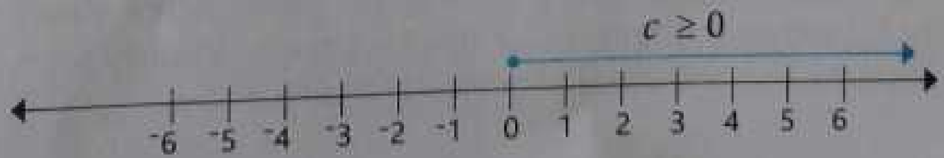
d) x is negative and greater than $^-3$.

Solution:

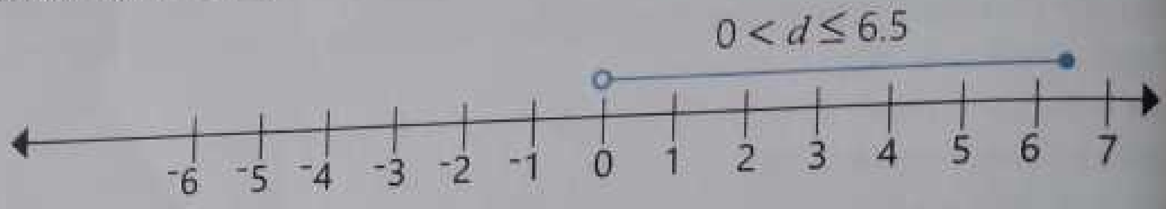
a) "b is at most 5" can be written mathematically as $b \leq 5$.



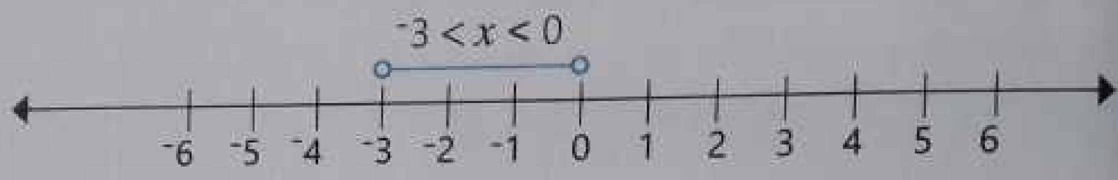
b) "c is non-negative" means that c is greater than or equal to zero, and can be written as $c \geq 0$.



c) "d is positive" can be written as $0 < d$ and "d is not more than 6.5" can be written as $d \leq 6.5$. Combining these two inequalities, you have $0 < d \leq 6.5$.



d) "x is negative" can be written as $x < 0$ and "x is greater than $^-3$" is written as $^-3 < x$. Combining the two inequalities, you get $^-3 < x < 0$.



Exercise 5.1

1. Interpret and represent the following inequalities on a number line.

   a) $x \leq 2$ b) $x > {}^-5$ c) $^-2 \leq x < 3$

2. Use < or > to copy and complete the following:

   i) 1 ______ 4.5 ii) $^-3.5$ ______ 1

   iii) 1 ______ $^-4$ iv) $^-3$ ______ 7

3. Use the inequality notation to denote the given expressions.
   a) x is a negative number.
   b) y is greater than 5 and less than or equal to 12.
   c) Betty's age is at least 27 years.
   d) The harvest is not more than 60 sacks.
   e) Uganda's rate of inflation is expected to be at least 6%, but not more than 9% in the coming year.

5.2 Solving Linear Inequalities in One Unknown

In solving linear inequalities, you may at some point need to manipulate the inequality by either multiplying or dividing the inequality with an integer. Each of these operations on the inequality affects it differently.

Activity 5.2(a): Adding inequalities with integers

In groups, given the inequality $4 < 5$;
a) add 3 on either side of the inequality.
b) state whether the inequality still holds.
c) add $^{-}3$ on either side of the inequality.
d) state whether the inequality still holds.
e) what conclusion can you make? Present to the class.

Activity 5.2(b): Subtracting inequalities with integers

In groups, given the inequality $8 > 7$;
a) subtract 5 on either side of the inequality.
b) state whether the inequality still holds.
c) subtract $^{-}5$ on either side of the inequality.
d) state whether the inequality still holds.
e) what conclusion can you make? Present to the class.

Activity 5.2(c): Multiplying inequalities with integers

In groups, given the inequality $2 < 3$;
a) multiply by 4 on either side of the inequality.
b) state whether the inequality still holds.
c) multiply by $^{-}4$ on either side of the inequality in a).
d) state whether the inequality still holds.
e) what conclusion can you make? Present to the class.

Activity 5.2(d): Dividing inequalities with integers

In groups, given the inequality $90 > 66$;
a) divide by 3 on either side of $90 > 66$.
b) state whether the inequality still holds.
c) divide by $^{-}3$ on either side of the inequality $90 > 66$.
d) state whether the inequality still holds.
e) what conclusion can you make? Present to the class.

Activity 5.2(e)

In groups, solve the following inequalities.
a) $r - 3 > 2$
b) $a + 4 < 5$
c) $2y > 12 - 4y$
d) $2x + 3 \leq 7$
e) $8 - 3x > 14$
f) $4x - 2 \geq 2 - 8x$

Example 2

Joseph owns a chapatti-making business. In his business, an employee is paid UGX 3,000 per day and the rent fee is UGX 6,000 per day. If he requires UGX 200 to make a chapatti, how many chapattis does he have to produce to make a profit if a chapatti is sold at UGX 500?

Solution:

Sales from x chapattis = $500x$

Total cost of Inputs = $3{,}000+6{,}000+200x$

You know that for one to make profits, the total sales should be greater than the total inputs.

$500x > 9{,}000 + 200x$

$300x > 9{,}000$

$x > 30$

Joseph has to produce more than 30 chapattis a day to make profits.

Example 3

Solve the following inequalities and write the integral values of x if;

a) $5 - 3x \geq 11$ b) $5x + 2 > 4x + 8$

Solution:

a) $5 - 3x \geq 11$

$5 - 5 - 3x \geq 11 - 5$

$\frac{^-3x}{^-3} \leq \frac{6}{^-3}$

$x \leq {}^-2$

$x = \{\ldots, {}^-5, {}^-4, {}^-3, {}^-2\}$

b) $5x + 2 > 4x + 8$

$5x - 4x > 8 - 2$

$x > 6$

$x = \{7, 8, 9, \ldots\}$

Exercise 5.2

Solve the following inequalities and represent the solution set on a number line.

a) $\frac{1}{2}r < 3$

b) $3y < 12$

c) $^-4p < 8$

d) $3m + 8 \geq {}^-1$

e) $6 - 2p < 14$

f) $^-3 < 3(x - 5)$

g) $^-9 < 6 - 3r$

h) $3(2y + 6) < 8y + 10$

5.3 Word Problems involving Inequalities

Activity 5.3

a) If the length of the blackboard is six times its width and its perimeter is at most 1050 cm. Find the maximum possible dimensions of the blackboard.

b) Fisher's goal is to find a job that provides an income of at least 45 million a year. A furniture mart offers him a job paying a basic salary of 17 million a year plus a commission of 6% of his sales. Determine what Fisher's total sales will need to be for him to have a yearly income greater than or equal to 45 million.

Example 4

The sum of two numbers is at least 20. If one number is 3 times the other, find the least possible values of the two numbers.

Solution:

Let one of the numbers be y.

$\Rightarrow$ The other number will then be $3y$

Now, the sum of two numbers being at least 20 means;

$$y + 3y \geq 20$$

Then $\frac{4y}{4} \geq \frac{20}{4}$

Hence, $y \geq 5$

The other number then is $3y \geq 3(5)$

$$3y \geq 15$$

Hence, the least possible values for the two numbers are 5 and 15.

Example 5

Fatuma is three times as old as Susan, but the sum of their ages is not more than 32 years. Calculate their greatest possible ages.

Solution:

Let Susan's age be x.

Then Fatuma's age will be $3x$.

Also the sum of their age is "not more than 32", implying $x + 3x \leq 32$

$$\Rightarrow 4x \leq 32$$

$$x \leq 8$$

Fatuma's greatest possible age is $3 \times 8 = 24$ years.

Hence, Fatuma and Susan's greatest possible ages are 24 years and 8 years respectively.

Exercise 5.3

1. The sum of two numbers is at least 9. If one of the numbers is twice the other, find the least possible values of the numbers.
2. Jane is four times as old as Alice. If the difference between their age is at least 27, find their least possible ages.
3. The length of a rectangle is three times its width. If its perimeter is at most 72 metres, determine the maximum possible dimensions of the rectangle.
4. Two sides of an isosceles triangle are five times the other. If the perimeter of the triangle is not more than 49.5 cm, calculate the maximum possible dimensions of the triangle.
5. Abdul Karim is given UGX 50,000 as pocket money while reporting for a new term. He wants to have at least UGX 20,000 at the end of the term, and he uses UGX 3,000 per week.
   a) Write an inequality that represents Abdul Karim's situation.
   b) How many weeks should a term at his school have?

5.4 Graphical Representation of Linear Inequalities

Activity 5.4(a): (Work in groups)

a) Draw the lines $x = 3$ and $y = 4$ on separate pairs of axes.
b) Shade the regions $x > 3$ and $y < 4$.
c) Present your results to the rest of the class.

Example 6

Show the region for which $y \geq x$ by shading the unwanted region.

Solution:

Step 1: Draw the line $y = x$.

For $y = x$, if $x = 0$, $y = 0$; you obtain the point (0, 0). And if $y = 1$, $x = 1$; you obtain (1, 1). So, you have (0, 0) and (1, 1) as the points on the line $y = x$.

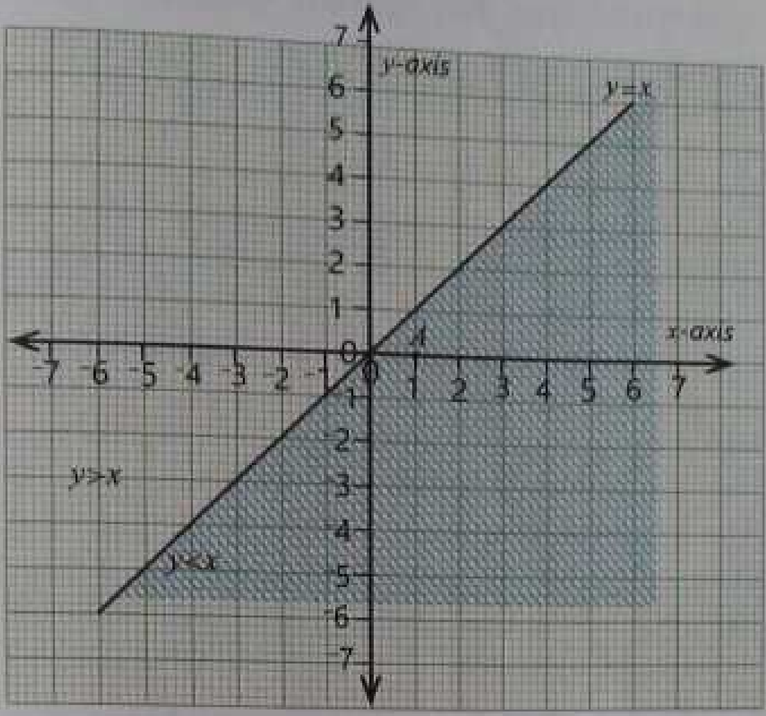
Step 2: Get any point not on the line and substitute it in the inequality $y \geq x$

Now, pick a point, say, A (1, 0) not on the line, that is, $x = 1$, $y = 0$.

Substitute the point in $y \geq x$.

Step 3: If the point chosen satisfies the inequality, you shade the opposite side of the line to the point and if it is not satisfying the inequality, you shade that side of the line.

0 is not greater than or equal to 1, hence, you shall shade this region since $y \geq x$ is false in this region.



Example 7

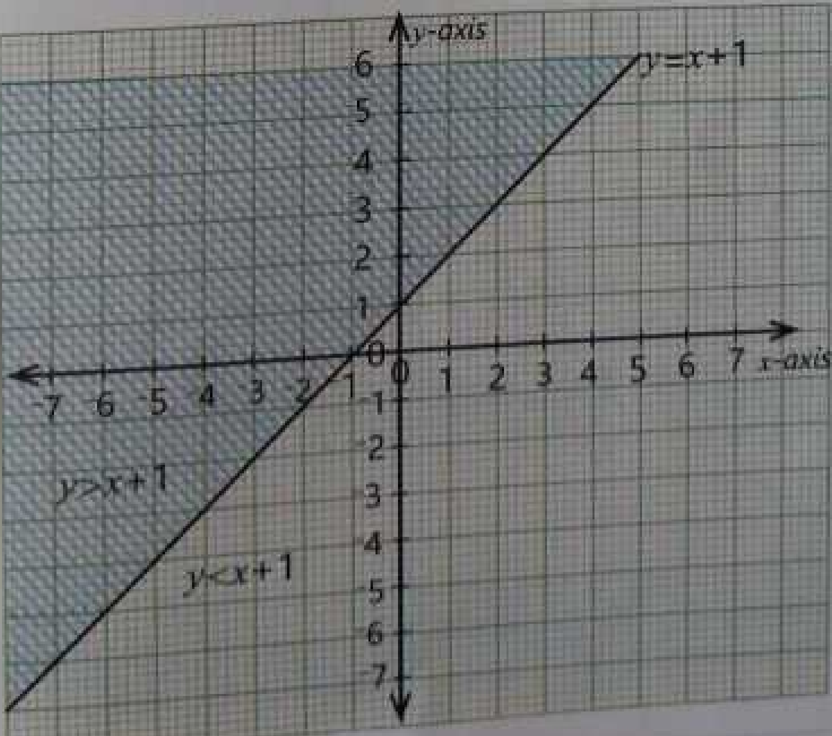
Show the region for which $y \leq x + 1$ by shading the unwanted region.

Solution

Step 1: Draw the line $y = x + 1$. For $x = 0$, $y = 1$. Hence (0, 1) is a point on the line. Also for $y = 0$, $x = {}^{-}1$. Hence (${}^{-}1$, 0) is another point on the line.

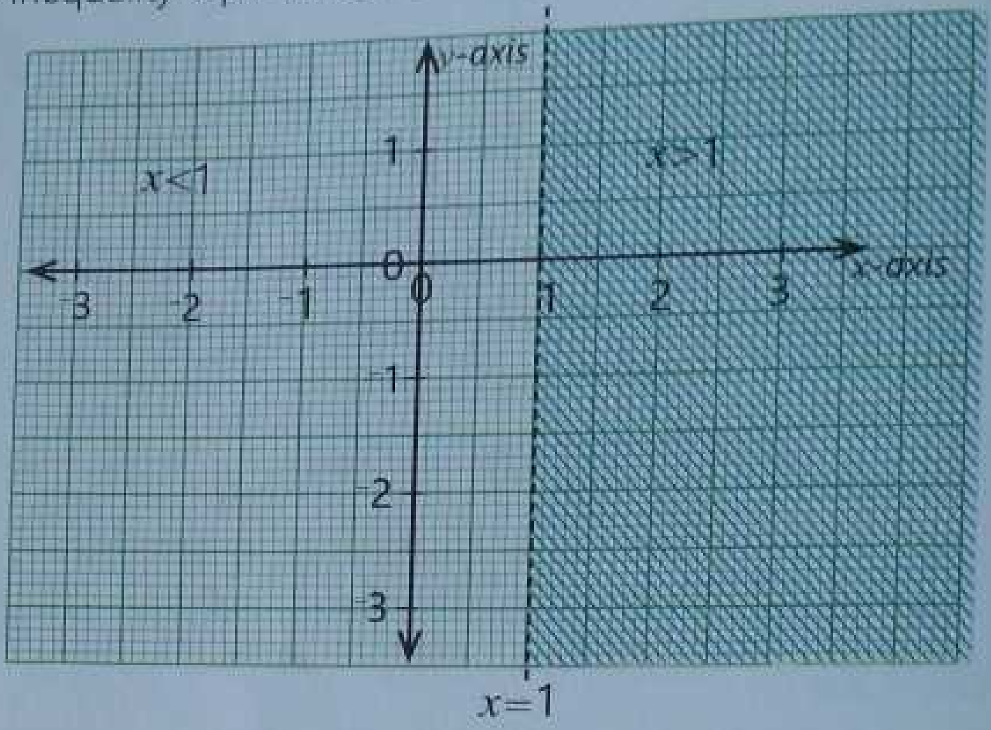
Step 2: Now let's pick a point, say, A (1, 0) not on the line and substitute it in the inequality.
$y \leq x + 1$, then you have $0 < 2$, which is true.

Step 3: Shade the upper side of the line $y = x + 1$.



Forming Linear Inequalities for given Regions on a Graph

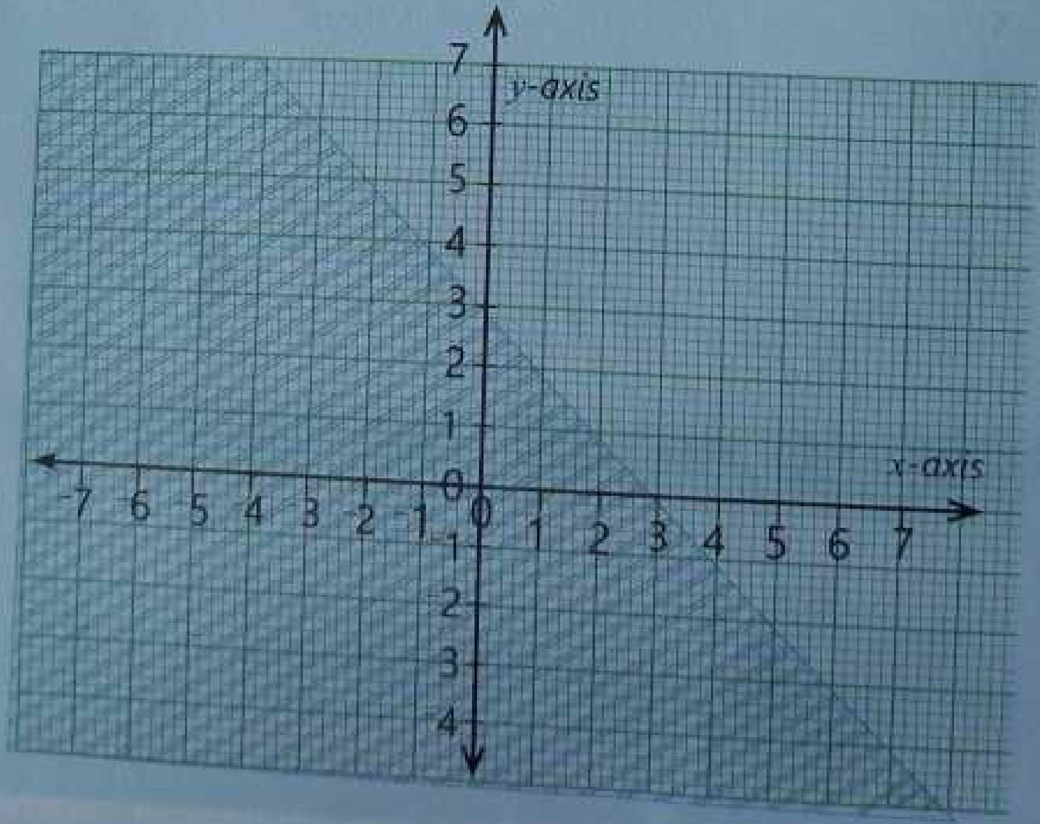
Consider the inequality represented on the graph below.



Since the region for $x > 1$ is shaded, this implies that it is the unwanted region and $x < 1$ becomes the wanted region. Hence, the inequality is $x < 1$.

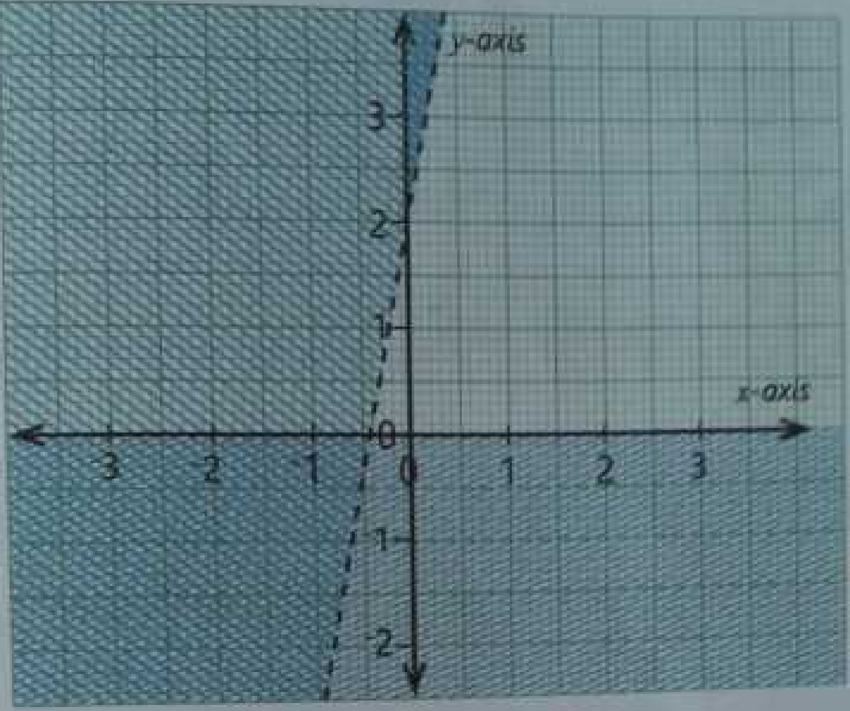
Activity 5.4(b): (Work in groups)

Find the inequality represented by the graph below.



Activity 5.4(c): (Work in groups)

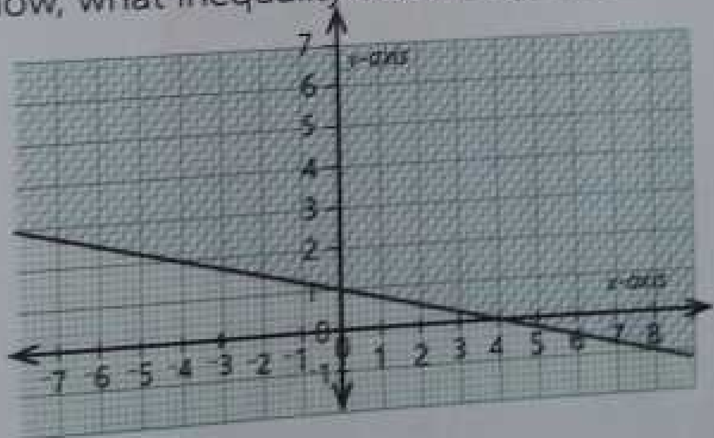
(a) Study the graph.



(b) What inequalities does it represent?

Example 8

Given the graph below, what inequality does it represent?



Solution:

From the graph, the following points do lie on the line: (4, 0), (0, 1), (8, ⁻1), (⁻4, 2), etc. Using the points, a relation between x and y can be obtained, that is,

	+4	+4	+4	
x	⁻4	0	4	8
y	2	1	0	⁻1
	−1	−1	−1	

From the pair of differences in x and y, it is evident that x is ⁻4 times y, that is, $x = {}^{-}4y$.

Let us now find $x + 4y$:

x	$^-4$	0	4	8
$4y$	8	4	0	$^-4$
$x + 4y$	4	4	4	4

Therefore, $x + 4y = 4$ is the equation of the line.

Task: Pick a point (1, 0) and substitute it in the equation, $x + 4y = 4$:

You obtain $1 + 0 \neq 4$. But $1 < 4$

Since (1, 0) is on the opposite side of the line that is shaded, then (1, 0) is in the required region.

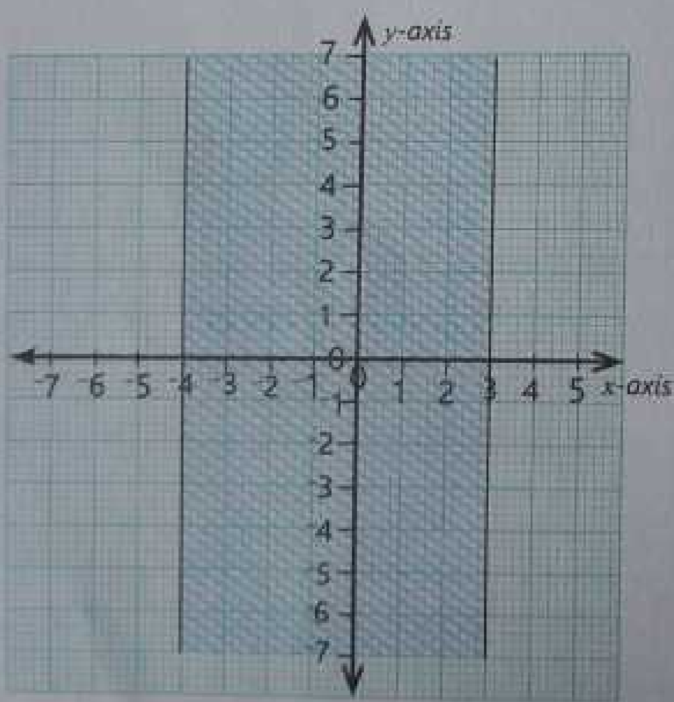
Note that the line is a solid one (not dotted). This means that the inequality is

$x + 4y \leq 4$.

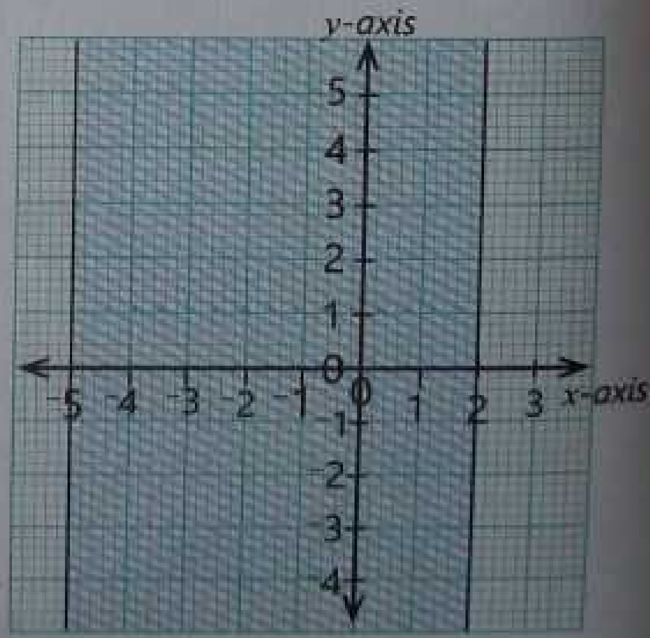
Exercise 5.4

1. Use a graph book to show the regions described by the inequalities below.
   a) $x > 3$ b) $x \leq 3$ c) $y < x + 4$ d) $y \geq 2x + 2$
2. In each case, show the region where the inequalities are true.
   a) $x < y$ and $x + y > 5$
   b) $x > y$ and $x + y \leq 16$
3. Given the following graphs, find the inequalities represented by the graphs.

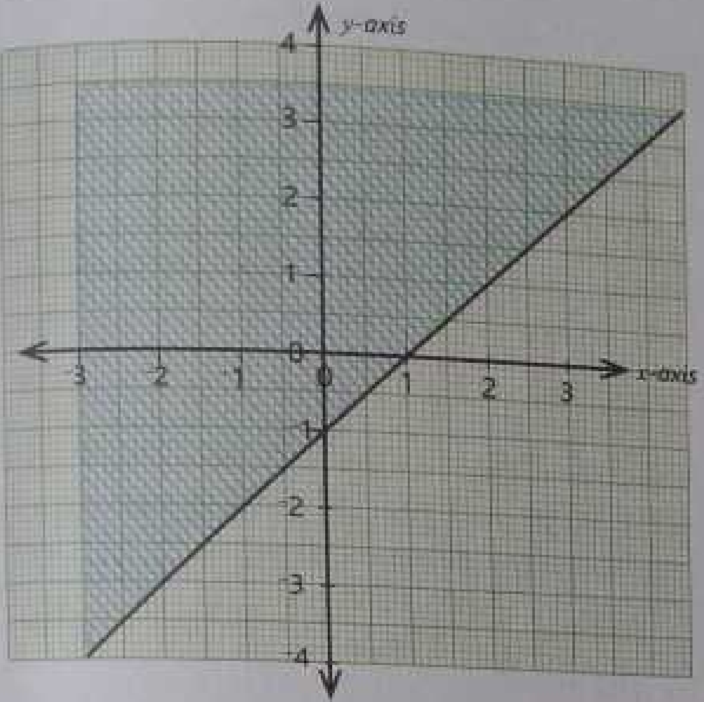
a)



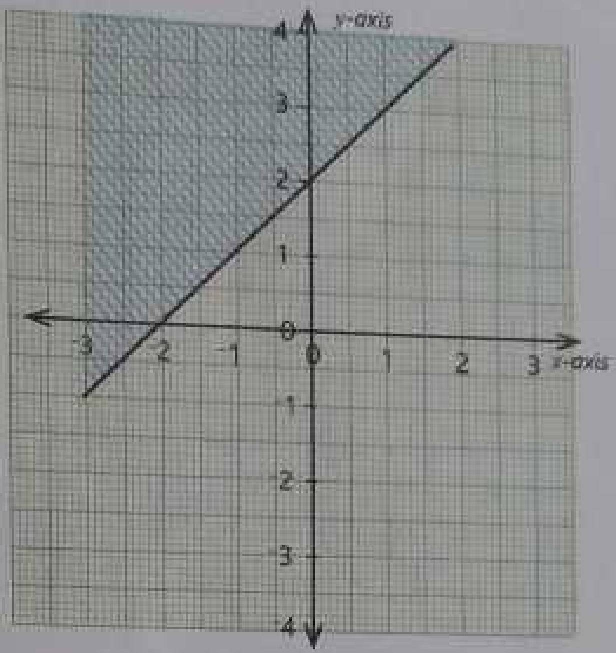
b)



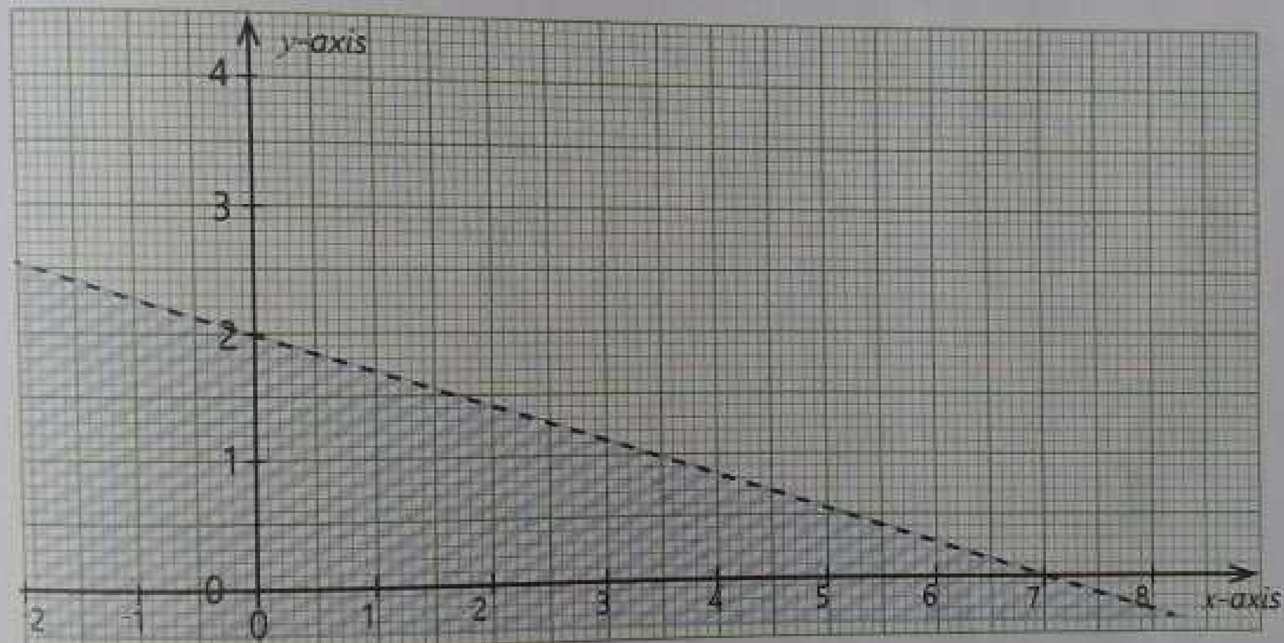
c)



d)



e)



ICT Activity

a) Use the link https://www.mathsisfun.com/data/inequality-grapher.html to graph the following:

i) $y > x + 2$ ii) $y < 2x$ iii) $y > 3x - 4$

b) Share your findings with the rest of the class.

Revision Questions:

Copy the following into your exercise book and put a ring on the correct answer.

1. What does the closed circle represent on the inequality graph?

a) equal to the number that is graphed
b) not equal to the number that is graphed

2. Which of the following represents the value of x in $6 - 4x \leq 26$?

a) $x \leq {}^{-}8$ c) $x \geq {}^{-}8$
b) $x \leq {}^{-}5$ d) $x \geq {}^{-}5$

3. Solve the inequality $^{-}8(4x + 6) < {}^{-}24$

a) $x < \frac{^{-}3}{4}$ c) $x < \frac{3}{4}$
b) $x > \frac{^{-}3}{4}$ d) $x > \frac{3}{4}$

4. Let x represent any number in the set $\{3, 6, 7, 9\}$. Which inequality is true for all values of x?

a) $x > 5$ c) $x < 5$
b) $x > 10$ d) $x < 10$

Sample Activity of Integration

Your Entrepreneurship club has been given a chance to supply the school canteen with pancakes.

Support:
- Use current prices of ingredients as on the market
- Use current price of pancakes
- Estimate the cost of labour

Resources:
- Knowledge of forming inequalities
- Knowledge of solving inequalities

Task:

Determine the price at which you would sell your product to the canteen in order to make a profit.

Chapter Summary

In this chapter, you have learnt how to:
- Represent statements using inequalities.
- Represent inequalities on the number line.
- Determine the wanted and unwanted regions given a particular inequality.

THEME: **Patterns and Algebra**

Chapter 6 Algebra 2

Keywords

- coefficient
- factorise
- perfect squares
- quadratic equations
- quadratic expressions

By the end of this chapter, you should be able to:

- recognise equivalent quadratic expressions.
- expand algebraic expressions.
- identify perfect squares.
- factorise quadratic expressions.
- solve quadratic equations where the quadratic expression can be factorised.

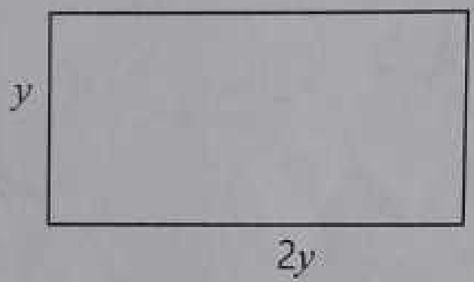
Introduction

In Senior One, you learnt about linear algebraic expressions and equations. You will build on that prior knowledge to now learn about quadratic expressions and equations. This will enable you to form and use quadratic expressions and equations in solving real-life problems.

Quadratic expressions and equations are commonly applied in describing paths of objects thrown at an angle under gravity, and shapes of cable suspension bridges. They also provide valuable tools for making decisions in businesses.

6.1 Quadratic Expressions

If a field is rectangular such that its length is twice its width as illustrated below,

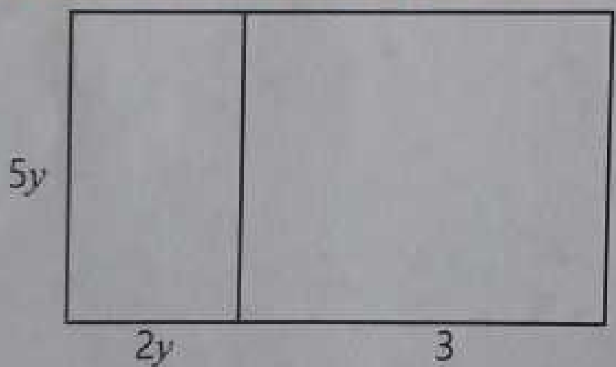


then its area is expressed as

$y \times 2y = 2y^2$ (where y is the width).

The above expression is an example of a quadratic expression because the highest index (power) of the variable is 2.

Equivalent Quadratic Expressions:



Consider a rectangular field divided into two plots each measuring $5y$ units by $2y$ units and 3 units by $5y$ units respectively, as shown above.

From this, you can generate two expressions for the total area, that is to say:

a) $(5y \times 2y) + (3 \times 5y) = 10y^2 + 15y$

b) $5y(2y + 3)$

The two expressions $5y(2y + 3)$ and $10y^2 + 15y$ are equivalent because they give the same value, no matter which number is used for the variable.

Activity 6.1 (Work in groups)

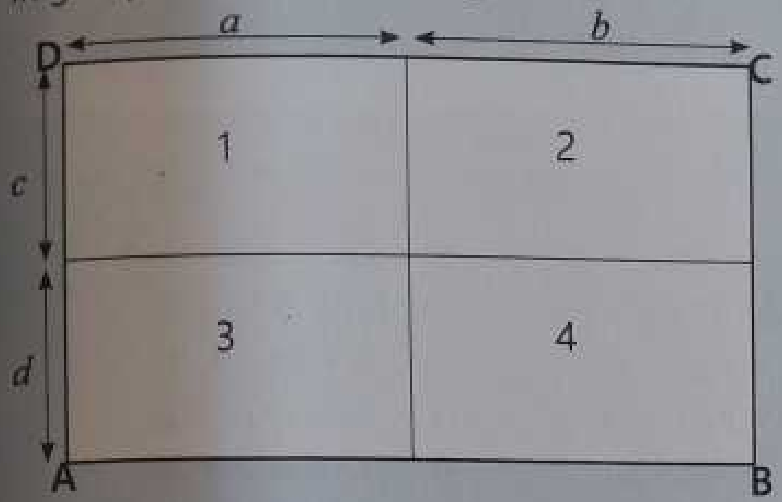
By removing brackets, identify equivalent quadratic expressions.

a) $2y(27y + 12x)$
b) $^-10a(6a - 11b)$
c) $(^-3t + s)5s$
d) $(m - 5n)3m$
e) $(22b - 12a)5a$
f) $4a(3a - b)$
g) $(4x + 9y)6y$
h) $^-3x(8x - 5y)$
i) $^-b(^-4b - 5t)$
j) $(^-9n + \frac{9}{5}m)\frac{5}{3}m$
k) $b(4b + 5t)$
l) $3x(5y - 8x)$
m) $3s(s - 5t)$
n) $4(3a^2 - ab)$

6.2 Expanding Algebraic Expressions

Activity 6.2(a)

In groups, study the rectangle below.



Procedures:

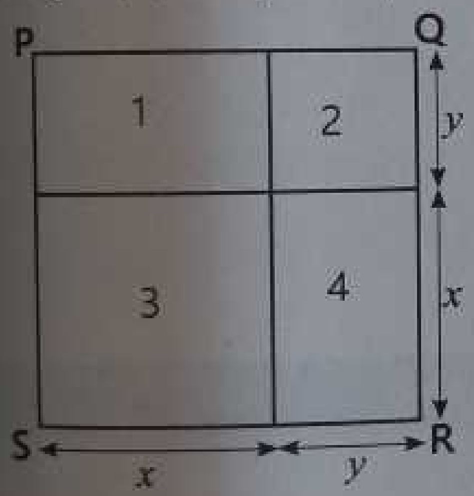
a) Find the areas of rectangles 1, 2, 3 and 4. Hence add these areas together.

b) Find the equivalent area of rectangle ABCD. Write down the expression for the area of this rectangle in terms of $(a + b)$ and $(c + d)$.

c) Relate the sum of areas in (a) to the area in (b).

Activity 6.2(b)

In groups, study the square below.



Procedures:

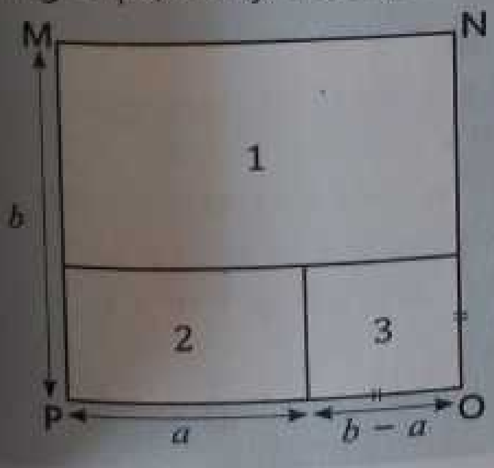
a) Find the areas of rectangles 1, 2, 3 and 4. Hence add these areas altogether.

b) Find the equivalent area of square PQRS. Write down the expression for the area of this square in terms of $(x + y)$.

c) Hence, expand $(x + y)(x + y)$.

Activity 6.2(c)

In groups, study the square below.



Procedures:

a) Find the areas of rectangles 1, 2 and that of the square MNOP.

b) Write the area of square 3 in terms of $(b - a)$.

c) Find the area of square 3 in terms of the areas in (a).

d) Expand $(b - a)(b - a)$.

Activity 6.2(d)

Find the following products and state the coefficient of each of the outcomes.

i) $(1 - 2t)(2 + 5t)$ ii) $(y - 6)(y - 10)$ ii) $(2x - 3y)(x - 5y)$

Exercise 6.1

1. Expand these expressions.

a) $(x - 1)(y - 1)$
b) $(3 + k)(2 - l)$
c) $(6 + r)(s + 2)$
d) $4b(6 - x)(x - b)$
e) $(p + 8 + q)(c + q)$
f) $(x + 2y)(x + 2y)$
g) $(4d - b)(4a - b)$
h) $(2u - v)(2u - v)$
i) $(5f - 2k)(5f - 2k)$
j) $(2 - p)(2 - p)$
k) $(8 + f)(8 - f)$
l) $(5 - a)(5 + a)$
m) $(2y + x)(2y - x)$
n) $(z - y)(y - z)$
o) $(2m - n)(m - 2n)$
p) $3(f - 5)(g + 4)$
q) $(p + q)(p + k)$
r) $(6t - u)(6t - u)$
s) $(a - 2b)(a - 2b)$
t) $(3x - 1)(3x + 1)$

2. Expand the following and state the coefficient of the unknown terms.

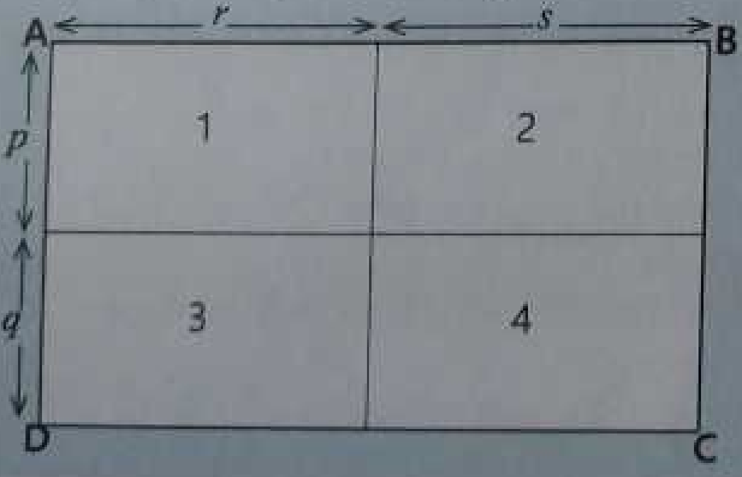
a) $(2x + 3)(4x + 1)$
b) $(a - x)(x - a)$
c) $(10 - x)(2 - x)$
d) $(4x + y)(x - y)$
e) $(x - 4)(x + 2)$
f) $(8 + f)(8 - f)$
g) $(u - 4)(2u - 7)$
h) $(2x + 3y)(2x + 3y)$
i) $(3x - 1)(3x + 1)$
j) $(2a + 7)(6a - 5)$
k) $(a + b)(b + a)$
l) $(5 - a)(5 + a)$
m) $(2y + x)(2y - x)$
n) $(x + y)(^{-}y + x)$

6.3 Factorising Quadratic Expressions

Factorisation by Grouping

Activity 6.3(a)

In groups, study the rectangle below.



Procedures:

a) Find the areas of rectangles 1, 2, 3 and 4.
b) Find the area of rectangle ABCD in terms of $(r + s)$ and $(p + q)$.
c) Hence, factorise $rp + rq + sp + sq$.

Example 1

Factorise:

a) $a^2 + ab + b^2 + ba$

b) $4x^2 - 4xy - y^2 + xy$

Solution:

a) $a^2 + ab + b^2 + ba$

You group the terms in the expression to obtain:

$(a^2 + ab) + (b^2 + ab) = a(a + b) + b(b + a)$ taking out a common factor, $(b + a)$, you obtain:

$a(a + b) + b(b + a) = (a + b)(b + a) = (a + b)^2$

b) $(4x^2 - 4xy) + (xy - y^2)$

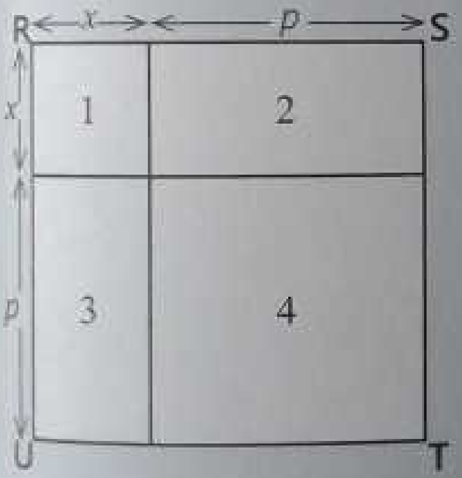
$= 4x(x - y) + y(x - y)$

$= (4x + y)(x - y)$

Identifying Perfect Squares

Activity 6.3(b)

In groups, study the square below.



Procedures:

a) Find the areas of squares 1, 4 and of rectangles 2 and 3.

b) Find the area of square RSTU in terms of $(x + p)$.

c) Factorise $x^2 + 2xp + p^2$.

Example 2

Factorise the following perfect squares.

a) $d^2 + 10d + 25$

b) $x^2 - 10x + 25$

Solution:

a) $d^2 + 10d + 25$

Common factors are 5 and 5,

since $5 + 5 = 10$ and $5 \times 5 = 25$.

You substitute in the given equation:

$d^2 + 5d + 5d + 25$

$= d(d + 5) + 5(d + 5)$

$= (d + 5)(d + 5)$

$= (d + 5)^2$

b) $x^2 - 10x + 25$

Common factors are -5 and $^{-}5$,

since $^{-}5 + {}^{-}5 = {}^{-}10$, while $^{-}5 \times {}^{-}5 = 25$.

You substitute in the given equation:

$= x^2 - 5x - 5x + 25$

$= x(x - 5) - 5(x - 5)$

$= (x - 5)(x - 5)$

$= (x - 5)^2$

Exercise 6.2

1. Identify perfect squares from the following algebraic expressions.

a) $4x^2 - 4x + 1$
b) $16 + 24y + 9y^2$
c) $2p^2 - 7p - 16$
d) $1 + 14g + 49g^2$
e) $4b^2 - 12b + 9$
f) $z^2 - 21z + 9$
g) $25x^2 + 40xy + 16y^2$
h) $64x^2 - 32xy + 4y^2$
i) $4m^2 - 20m + 100$
j) $81w^2 - 126uw + 49u^2$
k) $e^2 + 8e + 16$
l) $m^2 + 20m + 100$
m) $y^2 + 6y + 9$
n) $z^2 + 14z + 49$
o) $p^2 - 18p + 81$
p) $f^2 - 26f + 169$
q) $9 - 6s + s^2$
r) $36 - 12h + h^2$
s) $q^2 + 4q + 4$
t) $k^2 + 12k + 6$
u) $36 - 12h + h^2$

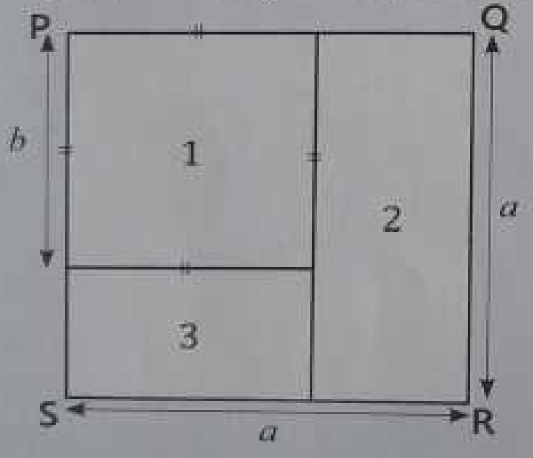
2. Factorise the following perfect squares.

a) $e^2 + 8e + 16$
b) $y^2 + 6y + 9$
c) $m^2 + 20m + 100$
d) $z^2 + 14z + 49$
e) $f^2 - 26f + 169$
f) $p^2 - 18p + 81$
g) $q^2 + 4q + 4$
h) $36 - 12h + h^2$
i) $9 - 6s + s^2$
j) $k^2 + 12k + 36$

Factors of $a^2 - b^2$ (Difference of two Squares)

Activity 6.3(c)

In groups, study the square below.



Procedure:

a) Find the areas of shapes 1, 2, 3 and PQRS.
b) Write down the sum of areas of shapes 2 and 3 in terms of $(a + b)$ and $(a - b)$.
c) Write down the difference of area of shape 1 from that of shape PQRS.
d) Hence factorise $a^2 - b^2$.

Note

$(a + b)(a - b) = a^2 - b^2$ finds its application in evaluating difference of two 'squares'. For instance, to evaluate $953^2 - 47^2$ would take a little bit longer to first evaluate 953^2 and 47^2, then substitute. But with the difference of two squares, it takes less time, that is,

$$
\begin{aligned}
953^2 - 47^2 &= (953 - 47)(953 + 47) \\
&= 906 \times 1{,}000 \\
&= 906{,}000
\end{aligned}
$$

Example 3

Factorise the following:

a) $n^2 - 49$ b) $x^2 - 9$ c) $y^2 - 1$ d) $16k^2 - 49m^2$

Solution:

a) $n^2 - 49$

From the identity

$(a^2 - b^2) = (a + b)(a - b)$,

$n^2 - 49 = n^2 - 7^2$

$= (n + 7)(n - 7)$

b) $x^2 - 9 = x^2 - 3^2$

$= (x - 3)(x + 3)$

c) $y^2 - 1 = y^2 - 1^2$

$= (y + 1)(y - 1)$

d) $16k^2 - 49m^2 = 4^2k^2 - 7^2m^2$

$= (4k)^2 - (7m)^2$

$= (4k - 7m)(4k + 7m)$

Exercise 6.3

1. Using the difference of two squares, evaluate the following:

a) $823^2 - 77^2$ d) $114^2 - 14^2$ g) $63^2 - 37^2$ j) $42^2 - 58^2$

b) $521^2 - 479^2$ e) $1003^2 - 3^2$ h) $741^2 - 259^2$

c) $653^2 - 347^2$ f) $313^2 - 687^2$ i) $520^2 - 480^2$

2. Factorise the following expressions completely.

a) $a^2 - b^2$ d) $4x^2 - 49a^2$ g) $100xy^2 - x$ j) $4x^2 - 100$

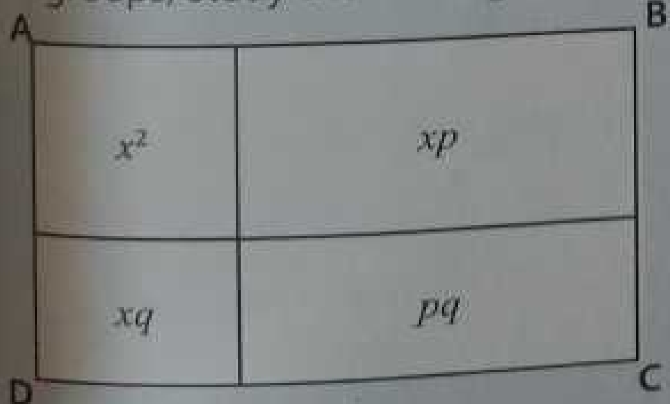
b) $m^2 - n^2$ e) $64n^2 - p^2$ h) $4y^2 - 16$ k) $ax^4 - a$

c) $9k^2 - 1$ f) $1 - 16p^2$ i) $uv^2 - uw^2$ l) $(xy^3 - xy)$

Factorisation of other Expressions

Activity 6.3(d)

In groups, study the rectangle below.



Procedures:

a) Find the area of rectangle ABCD in terms of $(x + p)$ and $(x + q)$.

b) Hence, factorise $x^2 + xp + xq + pq$.

Example 4

Factorise the following expressions:

a) $k^2 + 10k + 24$ b) $x^2 + 2x - 8$ c) $t^2 - 3t + 2$ d) $z^2 - 17z + 30$

Solution:

a) $k^2 + 10k + 24$

You look for two terms (factors) whose sum is the coefficient of k and their product equals the product of the constant term of the expression and coefficient of k^2, that is,

Coefficient of k is 10 ....(i)

Coefficient of k^2 is 1 ....(ii)

Constant term is 24 ....(iii)

The product of (ii) and (iii) is $1 \times 24 = 24$.

Terms whose sum is 10 and product is 24 are 6 and 4.

Therefore, $k^2 + 10k + 24$

$= k^2 + 6k + 4k + 24$

$= (k^2 + 6k) + (4k + 24)$ (on grouping)

$= k(k + 6) + 4(k + 6)$ (on factorising)

$= (k + 6)(k + 4)$

(on complete factorisation)

b) $x^2 + 2x - 8$

You look for two terms whose sum is 2 and product is $^-8$.

The terms are 4 and $^-2$,

since, $4 + {^-2} = 2$ and $4 \times {^-2} = {^-8}$.

Thus, $x^2 + 2x - 8 = x^2 + 4x - 2x - 8$

$= (x^2 + 4x) + (^-2x - 8)$

$= x(x + 4) - 2(x + 4)$

$= (x + 4)(x - 2)$

d) $z^2 - 17z + 30$

The two terms whose sum is $^-17$ and product 30 are $^-15$ and $^-2$,

since $^-15 + {^-2} = {^-17}$ and $^-15 \times {^-2} = 30$

So, $z^2 - 17z + 30 = z^2 - 15z - 2z + 30$

$= (z^2 - 15z) + (^-2z + 30)$

$= z(z - 15) - 2(z - 15)$

$= (z - 2)(z - 15)$

c) $t^2 - 3t + 2$

The two factors whose sum is $^-3$ and product is 2 are $^-2$ and $^-1$,

since $^-2 + {^-1} = {^-3}$ and $^-2 \times {^-1} = 2$.

Therefore, $t^2 - 3t + 2 = t^2 - 2t - t + 2$

$= (t^2 - 2t) + (^-t + 2)$

$= t(t - 2) - 1(t - 2)$

$= (t - 1)(t - 2)$

Exercise 6.4

Factorise the following expressions completely.

a) $y^2 - 2y - 3$ b) $k^2 + 4k + 3$ c) $y^2 - 7y - 8$

d) $z^2 - 4z - 5$ e) $u^2 - u - 6$ f) $e^2 + 3e + 2$

g) $q^2 - 11q + 30$ h) $h^2 - 8h + 12$ i) $d^2 - 8d - 9$

j) $w^2 - 8w + 7$

6.4 Solving Quadratic Equations using Factorisation

Activity 6.4(a) (Work in groups)

a) Find the following products.

i) 8×0 ii) 0×7 iii) 0×0

b) State the possible values of a and b given that;

i) $2 \times a = 0$

ii) $a \times b = 0$

iii) $(a + 1)(b - 2) = 0$

Activity 6.4(b) (Work in groups)

By factorisation, solve the following quadratic equations.

a) $x^2 - x = 0$ b) $x^2 + x - 20 = 0$ c) $x^2 + 6x + 8 = 0$

d) $x^2 + 4x + 3 = 0$ e) $x^2 + 6x + 9 = 0$ f) $x^2 - x - 12 = 0$

Example 5

Solve the following equations.

a) $(x - 1)(x - 3) = 0$ b) $(x + 4)(x - 3) = 0$

Solution:

a) $(x - 1)(x - 3) = 0$

Either $(x - 1) = 0$ or $(x - 3) = 0$.

(*zero product property*)

So, $x = 1$ or $x = 3$.

b) $(x + 4)(x - 3) = 0$

Either $(x + 4) = 0$ or $(x - 3) = 0$.

$\Rightarrow x = {}^{-}4$ or $x = 3$.

Note

- The final values of the variables are called the roots (solutions) of the equation.
- A quadratic equation has two roots which can either be different (distinct) or the same (repeated roots).

Exercise 6.5

Solve by factorisation the following quadratic equations.

a) $x^2 - 3x = 0$

b) $x^2 - 36 = 0$

c) $x^2 + 8x + 12 = 0$

d) $x^2 + x - 6 = 0$

e) $x^2 - 2x - 8 = 0$

f) $x^2 - 8x + 16 = 0$

g) $x^2 = 3x + 10$

h) $x^2 - 8x + 15 = 0$

i) $x^2 - 5x + 6 = 0$

j) $x^2 - 4x + 3 = 0$

ICT Activity

a) In groups, use the internet to search about the relevance of Algebra in real-life.

b) Hence, prepare a presentation using Microsoft Powerpoint and share it with the entire class.

Revision Questions:

1. Identify the perfect squares in:
   a) $x^2 + 6x + 9$ b) $x^2 + 8x + 16$ c) $x^2 + 10x + 25$
2. Use the difference of two squares to evaluate the following:
   a) $505^2 - 495^2$ b) $102^2 - 98^2$
3. Solve the following quadratic equations.
   a) $x^2 + 2x - 8 = 0$ b) $x^2 + 6x + 8 = 0$ c) $x^2 - 7x + 12 = 0$
4. The sum of John's age and his sister's is 10. The product of their ages is 21. What is;
   a) John's age b) his sister's age

Sample Activity of Integration

A farmer wants to make a rectangular paddock for his cows. He has 60 m of fencing material to cover three sides, with the other side being a brick wall.

Support: The farmer's objective is to construct a paddock with an area of 450 m^2.

Resources:

- Knowledge of formulating quadratic equations
- Knowledge of solving quadratic equations

Task:

Advise the farmer on how to choose the length and width of the paddock to achieve his objective.

Chapter Summary

In this chapter, you have learnt that; $(a + b)(a + b) = a^2 + 2ab + b^2$, $(a - b)(a + b) = a^2 - b^2$ and the application of these expressions in factorisation so as to solve quadratic equations.

You have also learnt that when the product of the coefficient of the square term and the constant of the expression is:

- Positive, then the required factors must have the same sign, either all positive or all negative, depending on the sign of the coefficient of the non-square term.
- Negative, then the required factors must have opposite signs.

THEME: Geometry and Measures

Chapter 7 Similarities and Enlargement

Keywords

- area scale factor
- linear scale factor
- similar figures
- volume scale factor

By the end of this chapter, you should be able to:

- identify similar figures.
- state and use the properties of similar figures.
- define enlargement.
- state the properties of enlargement to construct objects and images.
- understand and use the relationships between linear, and area, and volume scale factors.

Introduction

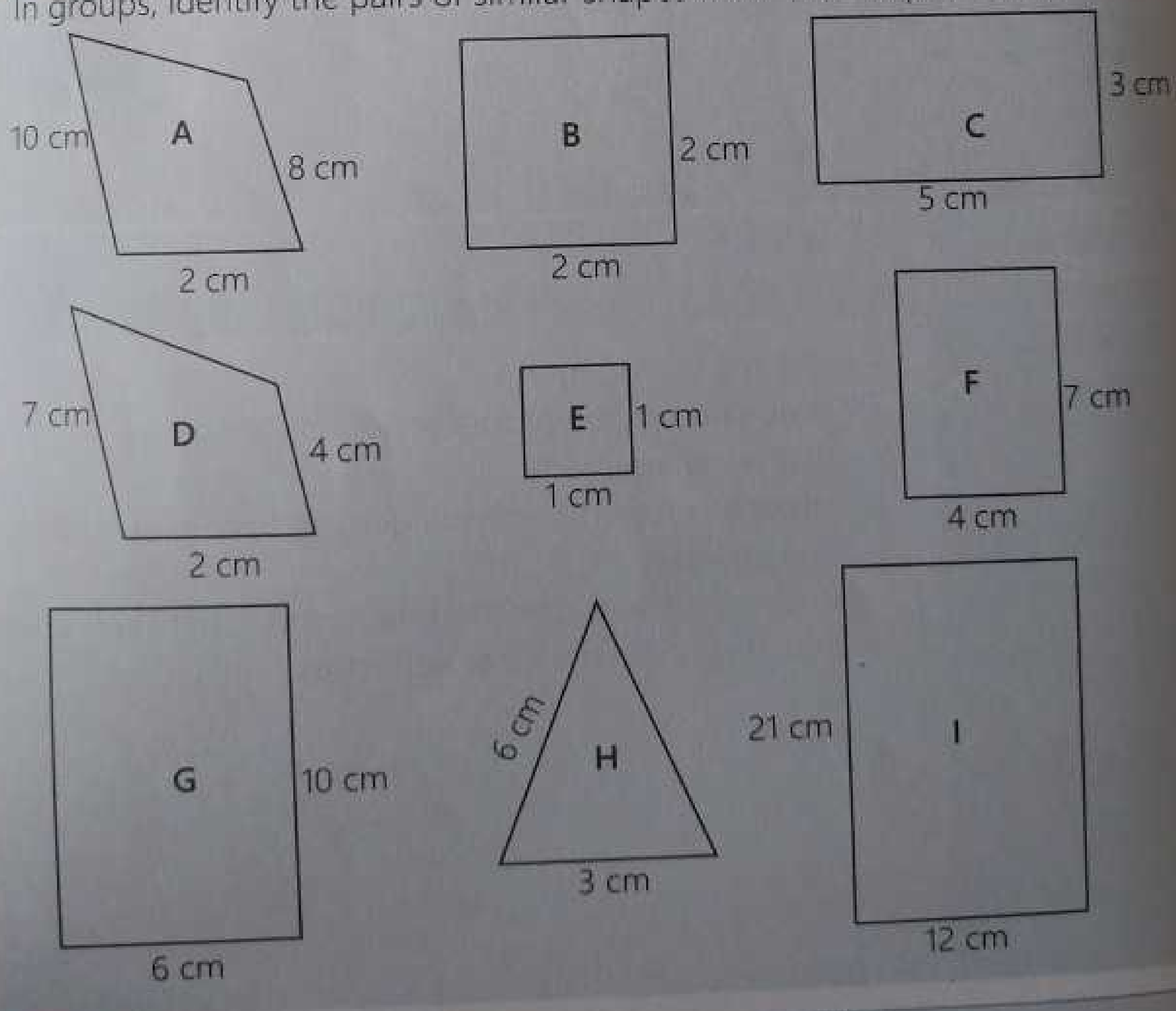
You probably know about photo albums, right?
Normally, in photo albums, there are representations of real objects but not the actual objects themselves. For instance, if one takes a photograph or captures a video of a party and sends it to another person (in a different place), what the other person views on opening the sent item is just a representation in which the images are similar to the actual objects.

In this chapter, you will be able to understand further and apply relationships among lengths, areas and volumes of similar shapes and objects.

7.1 Identifying Similar Figures

Activity 7.1(a)

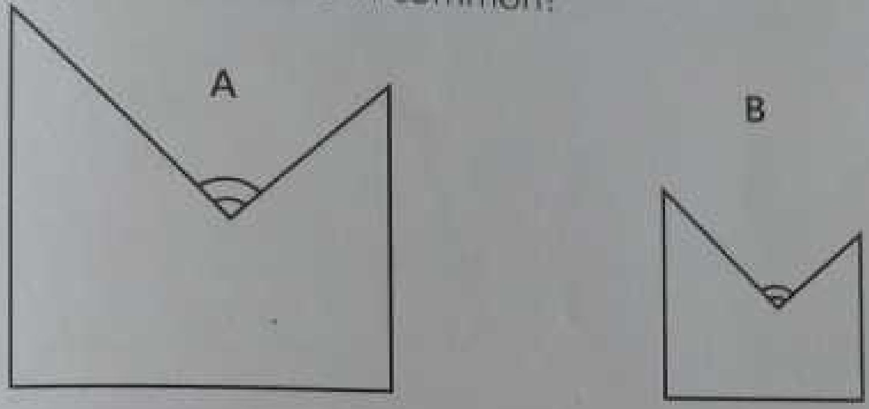
In groups, identify the pairs of similar shapes from the shapes below.



Properties of Similar Figures

Activity 7.1(b)

What do the shapes below have in common?



Activity 7.1(c) (Work in groups)

a) Draw two rectangles MNHT and PQRS such that length MN = 2 cm, NH = 4 cm, PQ = 6 cm and QR = 12 cm.

b) Find the value of $\frac{MN}{PQ}$ and $\frac{NH}{QR}$.

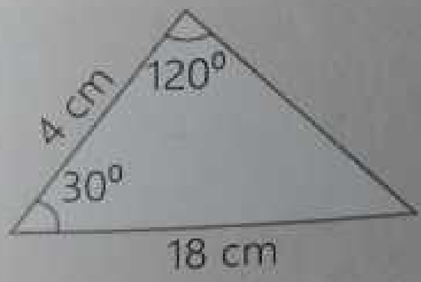
c) What can you say about the results obtained?

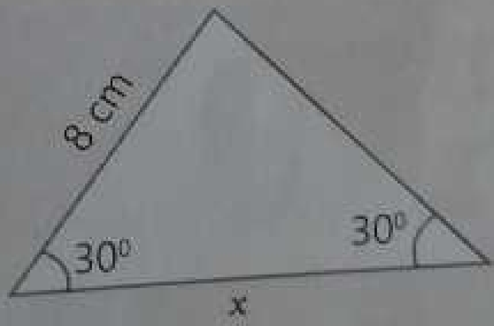
d) State the properties of similar figures.

Exercise 7.1

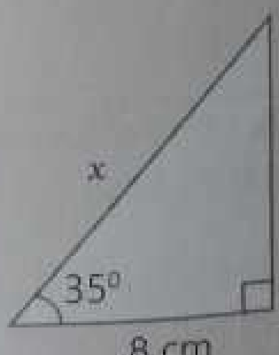
Using the knowledge of similar figures, find the value of x in the following similar triangles.

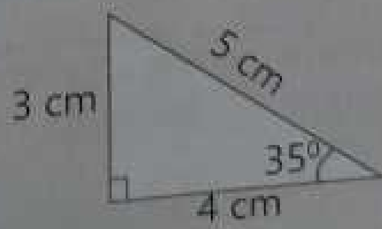
a)



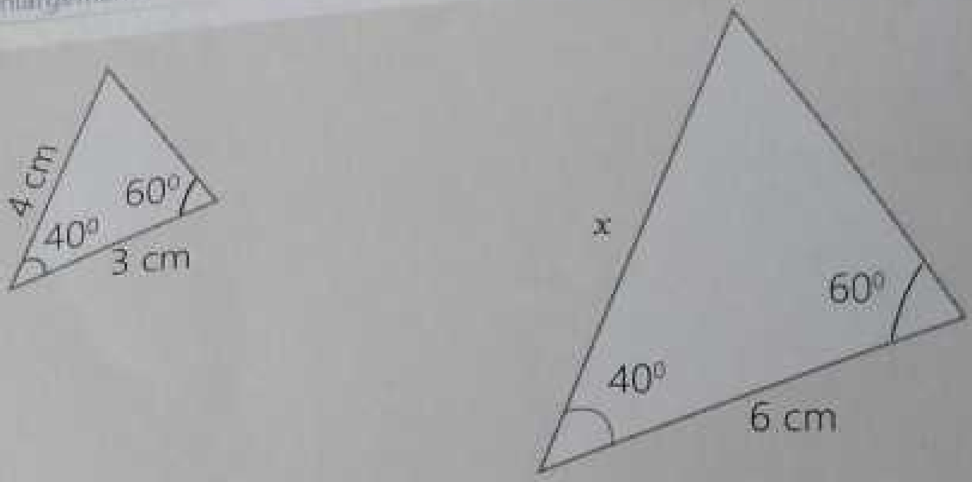


b)

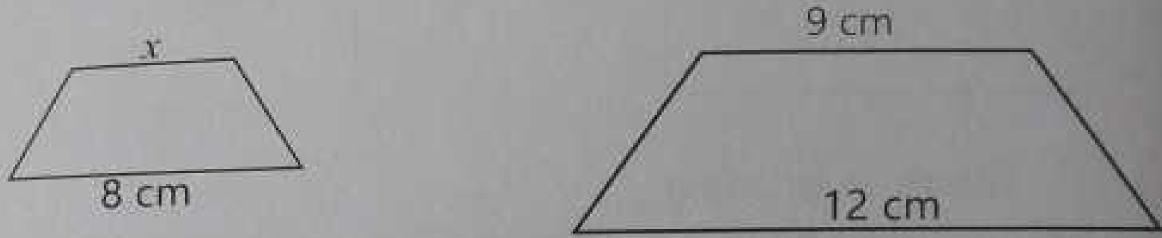




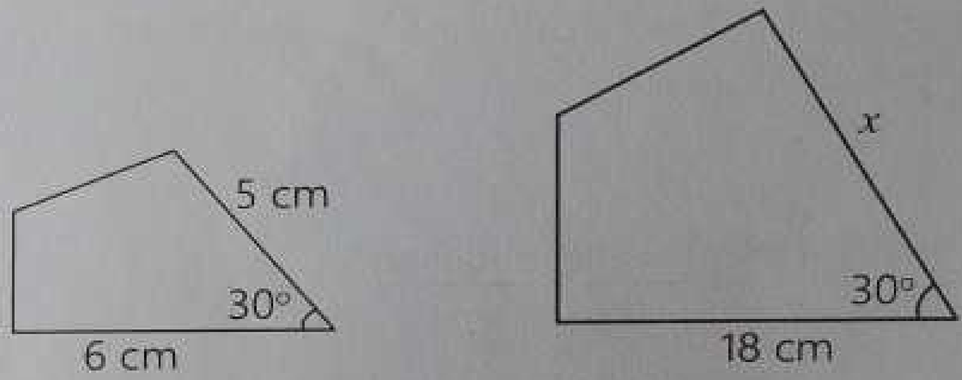
c)



d)



e)



7.2 Enlargement

Activity 7.2(a) (Work in groups)

Suggested materials:

- a bean seed
- a magnifying lens

Procedure:

View a bean seed under a magnifying lens.

Task:

State your observations.

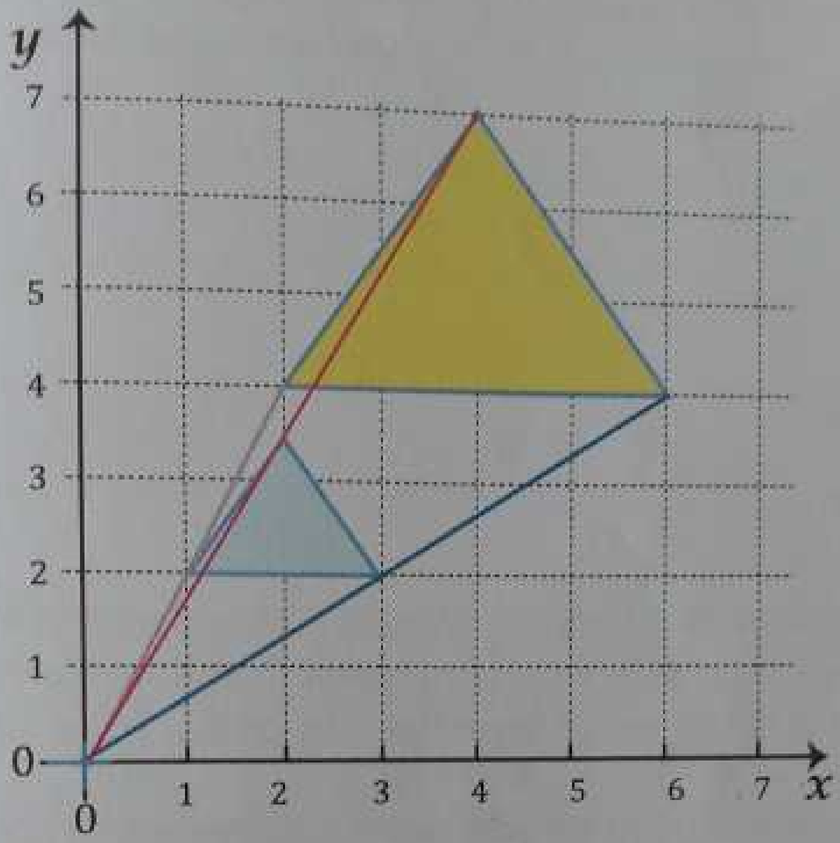
Note

Enlargement is a transformation in which the ratio of the image distance (length) to the object distance (length) is a constant about a fixed point (centre).

It is a transformation where an object maps to an image of the same shape, but different size.

From *Activity 7.2(a)*, the bean seed viewed under a magnifying lens undergoes an enlargement since the lens uses a uniform magnifying power.

The graph below shows an enlargement of a shape. The shape before enlargement (called the ***Object***) is in light-blue. The shape after enlargement (called the ***Image***) is in yellow.



In the graph above, the shape has been enlarged about the origin. The point where a given shape is enlarged is called the ***centre of enlargement***.
An enlargement is complete if its ***centre*** and ***scale factor*** are defined.

Properties of Enlargement

Activity 7.2(b) (Work in groups)

a) Plot a triangle ABC with coordinates A(2, 4), B(3, 1), and C(4, 3).
b) On the same graph, plot its image A′B′C′ with coordinates A′(4, 8), B′(6, 2), and C′(8, 6).
c) Measure the lengths AB, BC, AC, A′B′, B′C′ and A′C′.
d) Compute the ratios $\frac{A'B'}{AB}$, $\frac{B'C'}{BC}$ and $\frac{A'C'}{AC}$.
e) State the relationship among $\frac{A'B'}{AB}$, $\frac{B'C'}{BC}$ and $\frac{A'C'}{AC}$.
f) Prolong lines AB and A′B′. Comment on the nature of lines obtained.
g) Measure angles BAC and B′A′C′; thereafter compare your results.
h) Present your findings to the rest of the class.

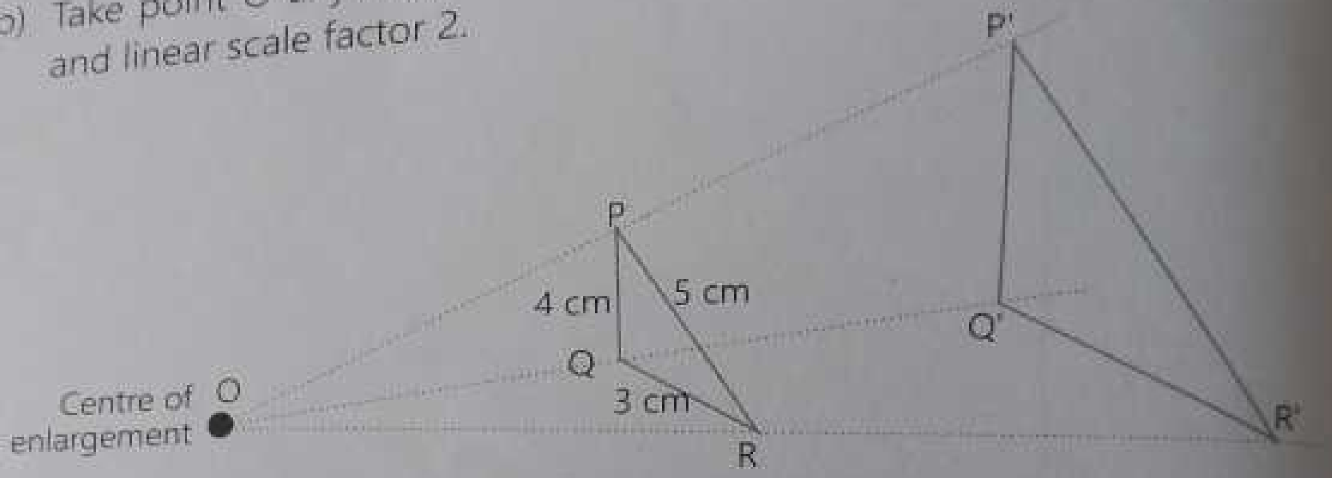
Finding the Image Under an Enlargement

1. Positive Linear Scale Factor

Activity 7.2(c) (Work in groups)

a) Draw a triangle PQR in which PQ = 4 cm, QR = 3 cm and PR = 5 cm.

b) Take point O anywhere outside the triangle as the centre of enlargement and linear scale factor 2.



c) Join the centre to each vertex of triangle PQR and extend the lines.

d) Measure each object distance from point O and multiply it by the L. S. F. This gives you the image distance from the centre. For example, OP' = L. S. F. × OP; OR' = L. S. F. × OR.

e) Mark off the image point for each point considered and join the vertices to form the image.

f) Repeat procedures (a) to (e) with the centre of enlargement anywhere inside triangle PQR.

Activity 7.2(d) (Work in groups)

a) Draw triangle ABC with vertices A(1, 3), B(5, 2) and C(3, 6) on a graph paper.

b) Transform ABC to A'B'C' under an enlargement of scale factor 3 and centre of enlargement (0, 0).

c) Hence, state the coordinates of the vertices of triangle A'B'C'.

2. Negative Linear Scale Factor

If you were standing in front of the class and your teacher requests you to move three steps, how would you perform it?

Your movement will depend on your interpretation of the direction. You think it can be forward, not so? But this may not be in the interest of the teacher. So, it is important that the teacher should have given the direction of movement; forward or backwards or otherwise (sideways) left or right.

Back to enlargement, when the L. S. F. = $^{+}2$, the centre was joined to the object point and extended the line in the direction of the object point, that is, the image and object were on the same side of the centre of enlargement.

What about now using a L. S. F. = $^{-}2$?

By definition, Linear Scale Factor: L. S. F. = $\dfrac{\textit{Image Distance}}{\textit{Object Distance}} = \dfrac{I.\,D.}{O.\,D.}$

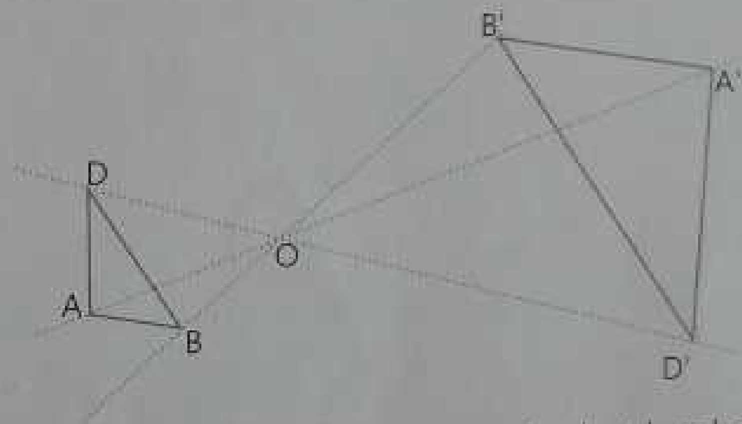
That is, I.D. = $^{-}2 \times$ O. D.

In order to find the image distance, the object distance is multiplied by $^{-}2$. This means that distance has been enlarged by two, but in the opposite direction from the centre of enlargement, that is, the image will be formed on the opposite side of the object.

Consider enlarging the triangle ABD given using L. S. F. = $^{-}2$ with the centre outside the triangle:

OA' = $^{-}2 \times$ OA

OB' = $^{-}2 \times$ OB



You observe that the image under L. S. F. = $^{-}2$ is twice the object in size but has been inverted.

Activity 7.2(e) (Work in groups)

a) Draw a triangle of your choice.
b) Transform the triangle drawn in (a) using a scale factor $^{-}1$ with the centre of enlargement anywhere inside the triangle.
c) Share your work with other groups.

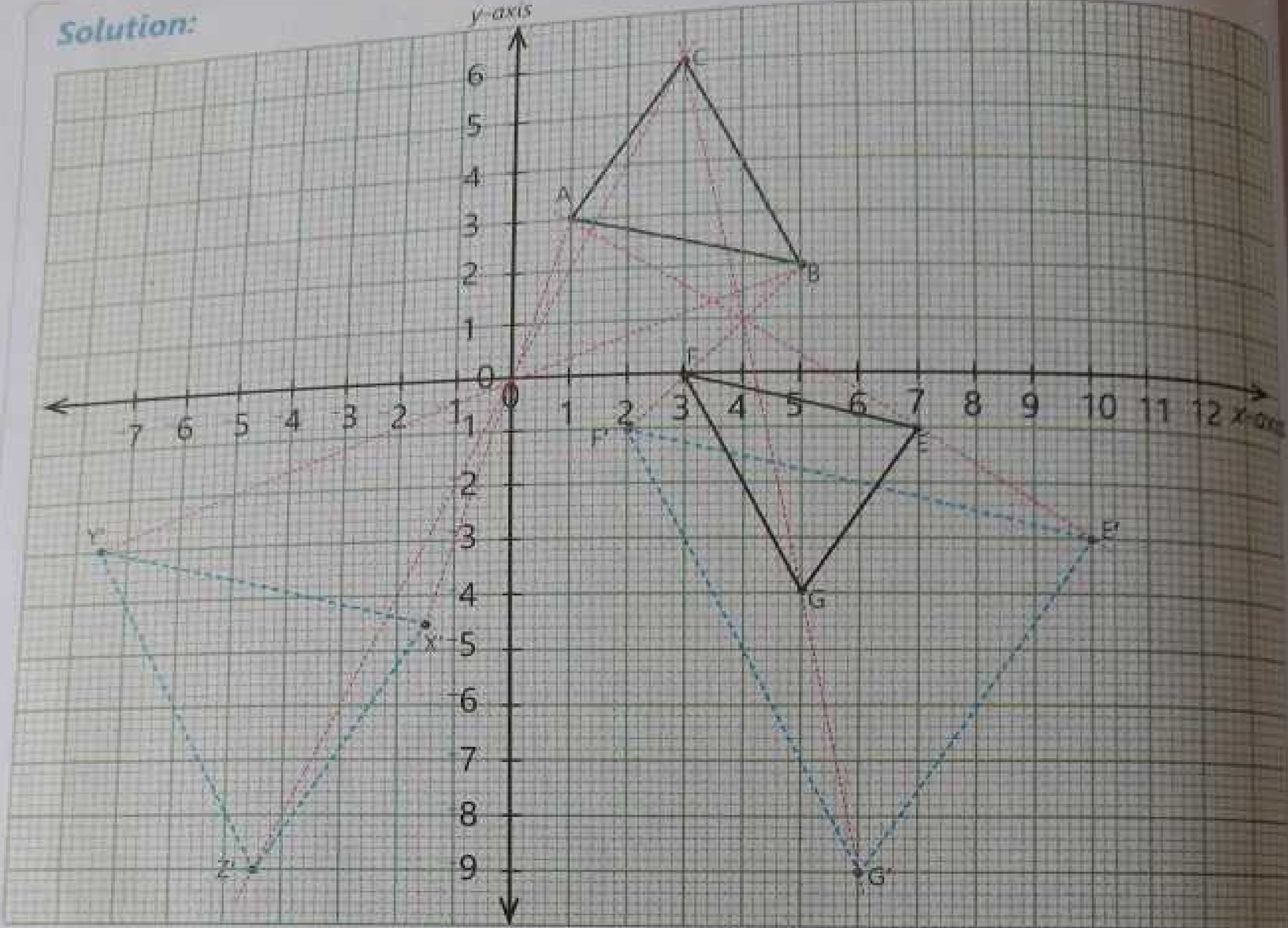
Example 2

A(1, 3), B(5, 2), C(3, 6) are vertices of a triangle ABC.

a) EFG is the image of ABC under enlargement of scale factor $^{-}1$ and centre of enlargement (4, 1).
b) E'F'G' is the image of ABC under enlargement of scale factor $^{-}2$ and centre of enlargement (4, 1).
c) X'Y'Z' is the image of ABC under enlargement $^{-}1.5$, centre (0, 0). State the coordinates of triangles:

i) EFG (ii) E'F'G' (iii) X'Y'Z'

Solution:



c) (i) E(7, $^-$1), F(3, 0), G(5, $^-$4)

Question: Discuss c) (ii) and (iii) in groups.

Determining the Centre of Enlargement and Linear Scale Factor

Given both the image and object under a certain enlargement, the centre and linear scale factor of enlargement can be obtained.

Activity 7.2(f) (Work in groups)

Given that L′(1, 9), M′($^-$7, 11) and N′($^-$3, 3) are vertices of triangle L′M′N′, the image of triangle LMN where L(1, 3), M(5, 2) and N(3, 6);

a) plot the triangles LMN and L′M′N′ on the same graph paper.
b) join the object point to the corresponding image point. Do so for all points.
c) determine the point of intersection of these three lines.
d) measure the image distance and the corresponding object distance from the point of intersection.
e) determine the centre of enlargement and state its coordinates.
f) find the linear scale factor and assign the correct sign.
g) present your findings to the rest of the class.

7.3 Relationships between Scale Factors

a) Linear Scale Factor

Let us consider a length x enlarged to a new length kx.

Then, Linear Scale Factor (L. S. F.) $= \dfrac{\text{New length}}{\text{Original length}}$

$= \dfrac{kx}{x} = k$ The linear scale factor is k.

b) Area Scale Factor

Activity 7.3(a) **(Work in groups)**

a) Let each group member draw two squares, 1 and 2, of different sizes.

b) Let each group member calculate the areas A_1 and A_2 of squares 1 and 2, respectively.

c) Record each group member's results in the table below:

Length		Area		Ratio	
L_1	L_2	A_1	A_2	$\frac{L_1}{L_2}$	$\frac{A_1}{A_2}$

d) Comment on the ratios $\frac{L_1}{L_2}$ and $\frac{A_1}{A_2}$.

e) Hence, relate area scale factor (A. S. F.) to linear scale factor (L. S. F.).

f) Share your results with other groups.

Example 3

A square of area 16 cm² is an enlargement of a square of side x. If the area scale factor is 4, find the value of x.

Solution: Area scale factor (A. S. F.) $= \dfrac{\text{New Area}}{\text{Original Area}}$

So, $4 = \dfrac{16}{x \times x}$

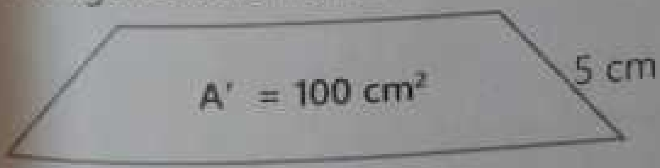
$x^2 = \dfrac{16}{4}$

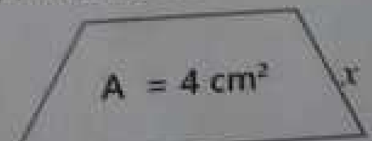
$x^2 = 4\text{ cm}^2$

$x = 2\text{ cm}$

Example 4

The figures below show two similar trapezia. Find the value of x.





Solution:

Area scale factor, A. S. F. $= \dfrac{A'}{A}$

$= \dfrac{100}{4}$

$= 25$

But, A. S. F. $= (\text{L. S. F.})^2$

$25 = (\text{L. S. F.})^2$

L. S. F. $= \sqrt{25}$

$= 5$

Therefore, $5 = \dfrac{5}{x} \Rightarrow x = 1\text{ cm}$

c) Volume Scale Factor

Activity 7.3(b) (Work in groups)

a) Let each group member draw two cubes, 1 and 2, of different sizes.
b) Let each group member calculate the volumes V_1 and V_2 of cubes 1 and 2 respectively.
c) Record each group member's results in the table below.

Length		Volume		Ratio	
L_1	L_2	V_1	V_2	$\frac{L_1}{L_2}$	$\frac{V_1}{V_2}$

d) Comment on the ratios $\frac{L_1}{L_2}$ and $\frac{V_1}{V_2}$.
e) Hence, relate volume scale factor (V. S. F.) to linear scale factor (L. S. F.).
f) Share your results with other groups.

Example 5

Find the volume of the image of a cuboid of 20 cm^3 under an enlargement with linear scale factor 2.

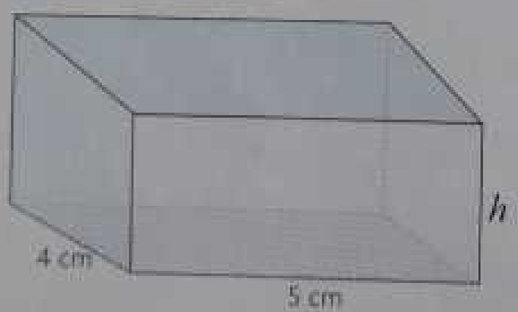
Solution:

Given, linear scale factor (L. S. F.) = 2.
But, Volume scale factor, (V. S. F.) = $(\text{L. S. F.})^3 = 2^3 = 8$.
Volume of image = $k \times 20\ cm^3$
Where k is volume scale factor.
Therefore, volume of image = $8 \times 20\ cm^3 = 160\ cm^3$

Example 6

Cuboid A has a volume of 810 cm^3 and cuboid A′ is an image of cuboid A under an enlargement of scale factor $\frac{1}{3}$. Find the height of cuboid A′ if its length and width are 5 cm and 4 cm respectively.

Solution:



Cuboid A′

$$\text{Volume scale factor} = \frac{\text{New volume}}{\text{Original volume}}$$

$$\left(\frac{1}{3}\right)^3 = \frac{VA'}{810}$$

$$VA' = \frac{810}{27}$$

$$= 30\ cm^3$$

But $VA' = \text{length} \times \text{width} \times \text{height}$

$$30 = 5 \times 4 \times h$$

$$h = \frac{30}{20} \qquad h = 1.5\ cm$$

Exercise 7.2

1. Given a triangle ABC with area 1 cm^2 and another triangle A'B'C' with area of 9 cm^2, find the length AB if length A'B' is 4 cm where A'B'C' is the image of ABC under an enlargement.
2. A circle of area 81 cm^2 and radius 9 cm is mapped onto a circle of area 16 cm^2 and diameter d. Calculate the value of d.
3. A polygon of area A = 4900 cm^2 is mapped onto a polygon of area 1600 cm^2 with one of the sides as x cm. Find x if the corresponding length is 10 cm.
4. Two cuboids A and B have corresponding lengths 15 mm and 20 mm respectively. Find the volume of B given that the Volume of A = 192 mm^3.
5. A sphere A is enlarged to sphere B. If the radius of A is 3 cm, find the radius of B, if the volume scale factor is 8.
6. A cone has a volume of 81 cm^3. If under an enlargement of linear scale factor ½, it forms a cone of volume v, find the value of v.
7. On a map, Kamuli is 5.5 cm from Namungoona. If the scale on the map is 1 : 1,500,000, calculate the actual distance between Kamuli and Namungoona in km.
8. An area of 10 cm^2 is representative of a certain city on a map. If the scale on a map is 1 : 500,000, calculate the actual area of the city in km^2.
9. A cylindrical can of radius 5 cm and height 10 cm contains water filled to capacity. If by accident, the can leaked and its volume reduced by 9 cm^3, calculate the new height of the water left in the can.
10. Kwagala wants to measure the height of the tallest tree in her school compound, but without climbing it. What should she do?

ICT Activity

a) In groups, using a camera or smart phone, take at least three photographs of objects in your school environment.
b) Compare the sizes of the photographs with the real objects.
c) Discuss your observations in relation to similarity and enlargement.

Revision Questions:

1. Find the actual length represented by;
   a) 4.5 m when the scale is 1 : 5000
   b) 2.5 m when the scale is 1 : 100
   c) 1200 km when the scale is 1 mm to 1 km
2. The length of a rectangle is as twice as the length of the other. Is one necessarily an enlargement of the other? Explain.

3. Given the points A(7, $^{-}$8), B(4, $^{-}$8), C(7, $^{-}$4), A′(8, $^{-}$9), B′(2, $^{-}$9) and C′(8, $^{-}$1),
   a) plot the points on a graph paper and find the;
      i) centre of enlargement
      ii) linear scale factor of enlargement
   b) using the linear scale factor above, find the area of A′B′C′.

Sample Activity of Integration

The Fine Art department has organised an art exhibition. Your class lacks drawings and sketches to display on that day.

Support:

- a pencil
- a paper
- a classroom

Resources:

- Knowledge of similar figures
- Knowledge of enlargement

Tasks:

You have been chosen to represent your class in this exhibition. Sitting in your position, draw the following objects in your classroom in their actual proportions.

a) chalkboard
b) window
c) door

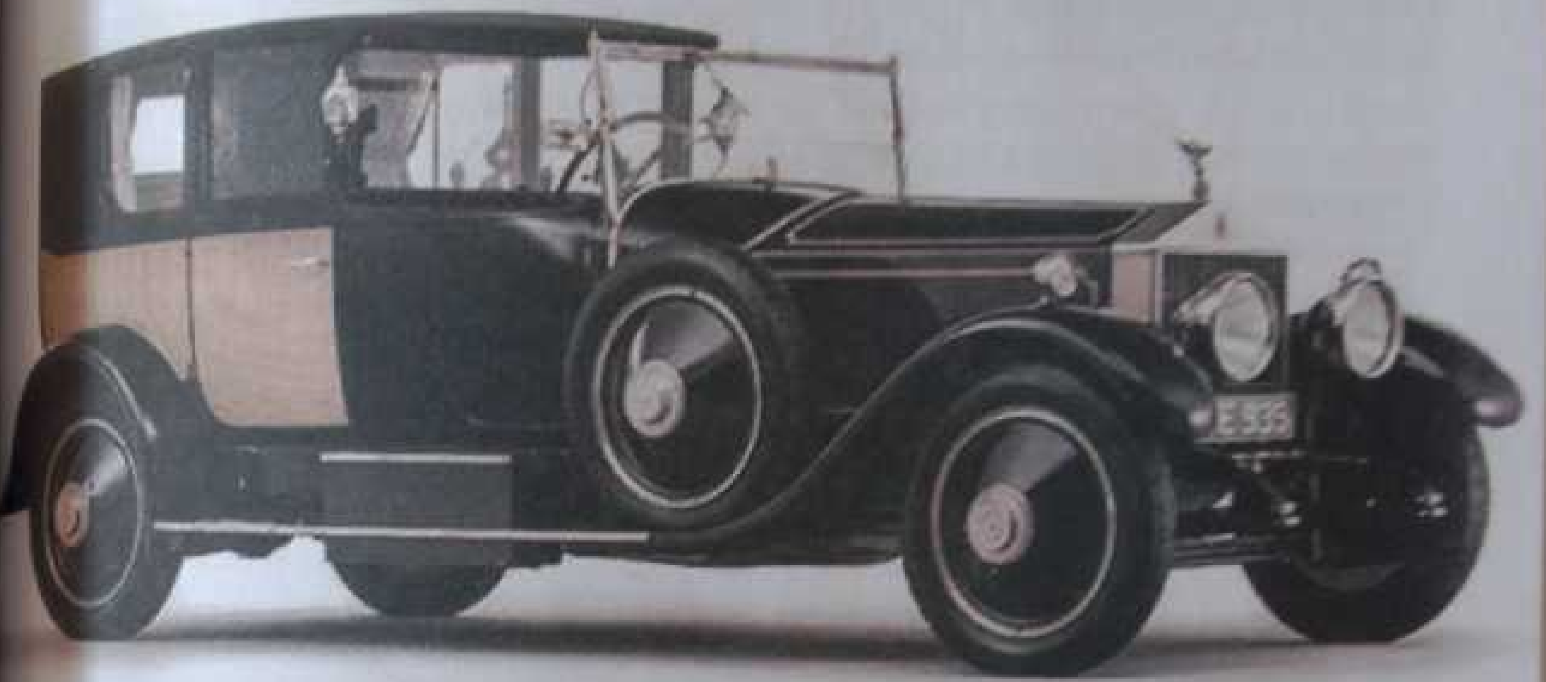
Chapter Summary

In this chapter, you have discovered that:

- Two objects are similar if they have the same shape, but not necessarily the same size.
- For similar shapes, all the angles are identical, but their side lengths can be different.
- Two similar shapes are related by a scale factor.
- When two figures are similar, their angles are equal and the ratios of the corresponding lengths are also equal.
- When two figures are similar, one can be considered an enlargement of the other.
- Linear scale factor is positive if both the image and object points are on the same side of the centre of enlargement, while it is negative if the two are on the opposite sides of the centre of enlargement.

THEME: Geometry and Measures

Chapter 8 The Circle



Keywords

- arc
- area
- chord
- circumference
- radius

By the end of this chapter, you should be able to:

- identify various parts of the circle.
- state and use the formulae for circumference and area enclosed by a circle.

Introduction

In Senior One, you learnt about a circle as a locus. Building on that knowledge, you will appreciate and justify the use of circles in daily life. You will, therefore, be able to understand, justify and apply the formulae for the area and circumference of a circle.

Circles occur in different natural forms in the world. In fact, most situations that are concerned with rotation involve circles; for example, as you turn around and face in your original direction, you become the centre of a circle.

8.1 The Circle and its Parts

Activity 8.1(a) Identifying various parts of a circle (Work in groups)

Suggested materials:

- manila papers
- a cutter
- a mathematical set

Precaution: Be careful not to cut yourself, or your friend when using the cutter.

Instructions:

a) Draw a circle on the manila paper.
b) Cut out the circle drawn.
c) Observe the cut-out and identify any two parts of the circle.
d) Cut the circle into two equal parts.
e) Observe the two and identify any two parts of the circle.
f) Cut out a piece from the half circle.
g) Identify which part of the circle is cut out.
h) How can the circle be cut so as to obtain a chord?

Activity 8.1(b) (Work in groups)

a) Draw a circle on a paper.
b) Draw different lines across the circle.
c) Identify the chords, diameters, circumference, arcs and radii.

Sector

A *sector* is a portion of the area of a circle enclosed by two radii.

8.2 Circumference and Area of a Circle

Activity 8.2(a) (Work in groups)

a) Gather five different objects with circular ends, and measure the circumference and diameter of each circular end.
b) Tabulate your results in a suitable table.
c) Plot a graph of circumference against diameter.
d) What do you notice?
e) Deduce the relationship between circumference and diameter of a circle.

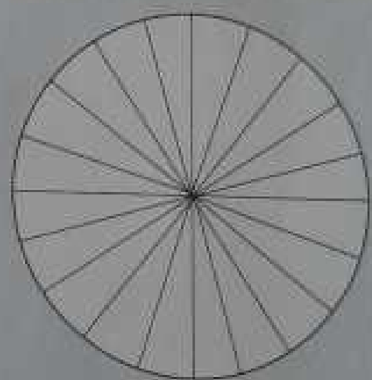
Activity 8.2(b) Deducing the formula for the area of a circle (Work in groups)

Suggested materials

- a manila paper
- a pair of scissors
- glue
- a paper
- a geometry set

Procedures:

a) Let each group draw on a manila paper a circle of radius(r) equal to 3 cm, 4 cm, 5 cm, 6 cm and 7 cm respectively.
b) Divide the circle into 20 sectors as shown.
c) Cut out the sectors, arrange and glue them on a manila paper to form a rectangular shape.
d) Measure the width w and length l of the formed rectangle.
e) Collect other values of w and l from other groups and tabulate them as shown below.

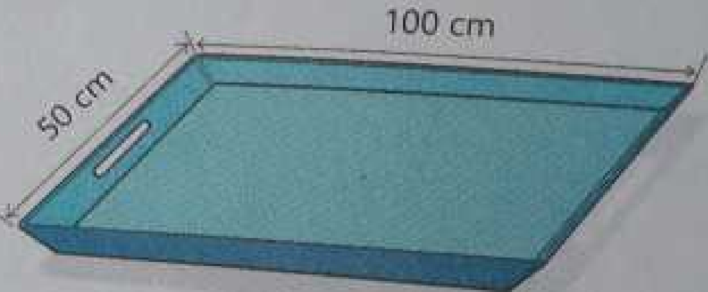


r (cm)	w (cm)	l (cm)	$\frac{l}{r}$
3			
4			
5			
6			
7			

f) Compare values of w with r.
g) Compare the values in the column $\frac{l}{r}$.
h) Comment on the values in the column $\frac{l}{r}$. What can you deduce?
i) Find the area of the rectangular shape formed in (c).
j) Hence, deduce the formula for the area of a circle of radius r.

Exercise 8.1

1. A circular disk is made to rotate from point A to point B and it is said to cover a distance of 400 cm in one rotation. Calculate the area covered by the disc. (Take $\pi = 3.1429$)
2. The area of a circle is 56 cm^2. Find its radius and diameter. (Take $\pi = 3.1429$)
3. A rectangular tray measures 100 cm by 50 cm. How many cakes can fit on the tray if each cake has a circular base of uniform radius 3 cm?



4. A bicycle wheel with radius 30 cm made 1000 revolutions. What distance did it cover?

5. An Athletics track is in a circular form with the inner circle having a radius of 60 m and the outer circle having a radius of 65 m. Find the area of the track.
6. The time shown in a circular clock is 2:00 p.m. The length of the minute hand is 20 cm. Find the distance traveled by the tip of the minute hand when the time is 4:00 p.m.

ICT Activity

In groups;

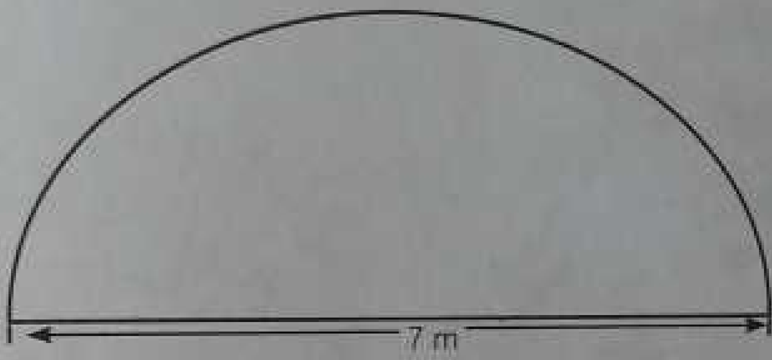
a) use a spreadsheet program to re-attempt *Activity 8.2(a)*. Compare your graph with the one you did on a graph paper.

b) read more about area and circumference of a circle using the internet on "mathisfun.com/geometry/circle.html". Type your findings using a word processor and save it on the desktop as "circles".

c) make a printout of your findings and share with the rest of the class.

Revision Questions:

1. A man wants to make circles each of radius 5 cm from an area of 3925 cm^2. How many circles can he make from this area? (Take $\pi = 3.1429$)
2. A circular piece of timber of radius 3.5 cm was cut out from a rectangular piece of timber measuring 8 cm by 9 cm. Find the area of the timber left.
3. Wanjiko walked around a circular field, which has a diameter of 100 m, once. How long did she walk?
4. A circular hat has a radius of 7 cm. What is the circumference of the hat?
5. The diagram below shows a semicircular carpet with diameter 7 m. What is the area of the carpet?



6. A circular flower garden has a radius of 21 m. The owner wants to put a plastic edge around the garden. How long should this plastic edge be?

Group Project

a) Get a thread measuring 12 cm long.

b) Place the thread on your desk to form:

i) An equilateral triangle	ii) A square	iii) A regular hexagon	iv) A circle
			

c) Calculate the area of each figure formed.

d) Compare the areas.

e) Make a conclusion.

Sample Activity of Integration

Nseko, a tailor in Bundibugyo town, is hired to design a round table-cloth from a beautiful piece of cloth.

Support:

- a beautiful piece of cloth measuring 3.14 m by 1 m
- a pair of scissors
- sewing machine / a needle and thread

Figure 1

Figure 2

Resources:

- Knowledge of units and measurements
- Knowledge about circles

Task:

Explain how Nseko should design the table-cloth such that close to 100% of the material is used.

Chapter Summary

In this chapter, you have learnt that:

- A circle has various parts, that is to say; centre, circumference, arc, radius, diameter, chord, sector and segment.
- Circumference (C) of a circle can be calculated using $C = 2\pi r$ or $C = \pi D$, and Area (A) of a circle can be calculated using $A = \pi r^2$ or $A = \frac{1}{4}\pi D^2$.

THEME: **Geometry and Measures**

Chapter 9 Rotation

Keywords

- angles of rotation
- anticlockwise rotation
- centre of rotation
- clockwise rotation
- congruence
- order of rotational symmetry
- plane figures

By the end of this chapter, you should be able to:

- identify the order of rotational symmetry of a plane figure.
- distinguish between clockwise and anticlockwise rotation.
- state properties of rotation as a transformation including congruence.
- determine the centre and angle of rotation.
- apply properties of rotation in the Cartesian plane.

Introduction

In everyday life, you usually see several objects turning around. These objects include the door that you turn in order to open or close; your head as you turn to see things that are at the sides; hands of a clock; a propeller of an aeroplane as it rotates; a steering wheel of a car; and a bicycle wheel as one rides the bicycle.

The transformation that turns a figure about a fixed point in the above examples is referred to as ***rotation***. In this chapter, you will understand rotation further and be able to apply it as a transformation.

9.1 Order of Rotational Symmetry of Plane Figures

Activity 9.1 (Work in groups)

Suggested materials:

- a tracing paper
- a pencil
- a cutter
- a pin
- a pair of dividers

The table below shows figures of different objects.

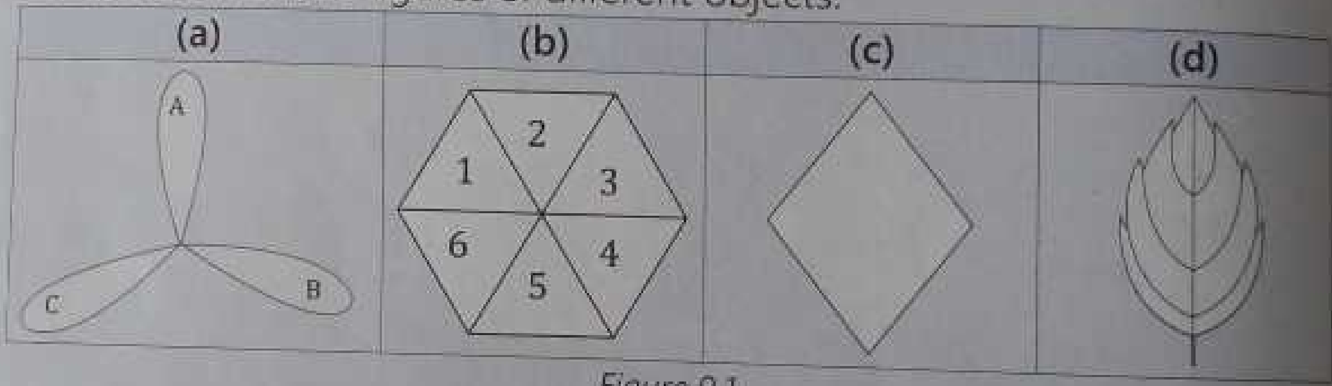


Figure 9.1

Instructions:

a) Trace out object **(a)** in *Figure 9.1* above.
b) Cut out the trace and label the lobes A, B and C.
c) Put the trace on top of the figure in your textbook so that they exactly fit onto one another.
d) Pin the centre using a pair of dividers or a pin.
e) Turn the tracing paper on top such that its point A fits on point B in the text book.
f) Through what angle does it turn?
g) Continue turning the tracing paper. How many times does it fit onto itself in one turn (360°)?
h) Repeat the procedures for objects **(b)**, **(c)** and **(d)** in *Figure 9.1* above.
i) Copy and fill the following table using your results.

Object	Angle of first turn	Order of rotational symmetry
(a)		
(b)		
(c)		
(d)		

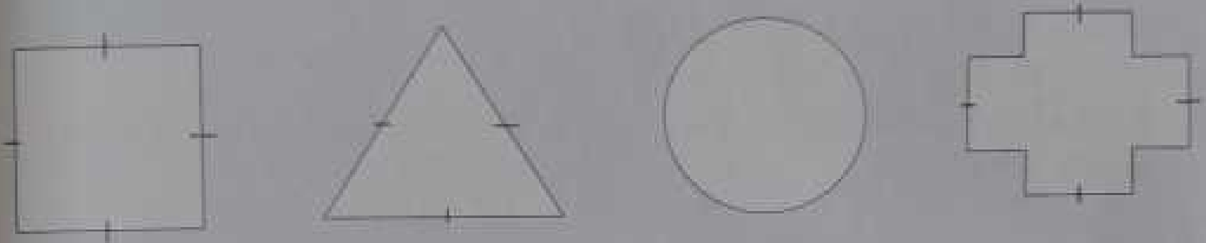
The kind of symmetry exhibited here is called a ***rotational symmetry***. Rotational symmetry is an act in which an object maps onto itself a number of times about a point or an axis. The number of times it fits onto itself in one complete turn is called the *order of rotational symmetry*.

9.2 Congruence

Activity 9.2(a) (Work in groups)

a) For each of the following figures, determine the order of rotational symmetry using their centres as the centre of rotation.

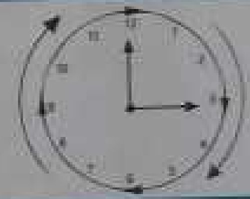
(i) (ii) (iii) (iv)



b) Draw five objects of your own (not mentioned in this chapter). State the order of rotational symmetry in each of them.

Note

A rotation that proceeds in the same direction as a clock's hands is referred to as *clockwise rotation*, and the opposite sense of rotation is *anticlockwise rotation*.



Activity 9.2(b) (Work in groups)

Suggested materials: • a tracing paper • a pin or pair of dividers

a) Draw a triangle ABC, in which AB = 3 cm, BC = 4 cm and AC = 5 cm.
b) Make a trace of triangle ABC on a transparent piece of paper.
c) Mark a point P (the centre), outside triangle ABC, in your exercise book.
d) Make the trace overlap onto ABC and also mark point P, on this paper.
e) Using a pin (or a pair of dividers), pin the two centres P, together and carry out the following:
   i) rotate the trace through 360°.
   ii) join P to A and make an angle of 60° in the anticlockwise direction at P. Turn the tracing paper such that A now takes the new position A'. Mark the new positions of B as B' and C as C'. Find angles BPB' and CPC'.
   iii) check where the trace will be if the angle turned through is 60° in the clockwise direction.
   iv) compare the sizes of the object and its outcome (image) after a rotation.

With these observations, ***rotation*** can be defined as a transformation in which every object point moves through the same angle in the same direction, about a fixed point, to form the image. The image is exactly as the object in size and shape. This condition is called ***congruence***.

Note

- The fixed point is called *the centre of rotation*.
- The angle of turn is called *the angle of rotation*.
- The direction of rotation is either *clockwise* or *anticlockwise*.
- A clockwise turn is assigned negative, for example, $^-60°$; and an anticlockwise turn is assigned positive, for example, $^+90°$.

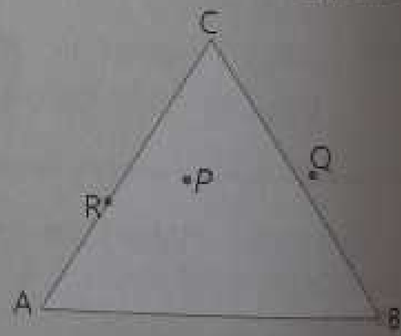
Activity 9.2(c) (Work in groups)

a) Plot points A(2, 4), B(6, 4) and C(4, $^-2$).
b) Join points A, B, C to form triangle ABC.
c) Rotate triangle ABC about the origin O through $^-60°$. Find the image triangle A′B′C′.

Exercise 9.1

Draw separate images of triangle ABC after rotations about the following points.

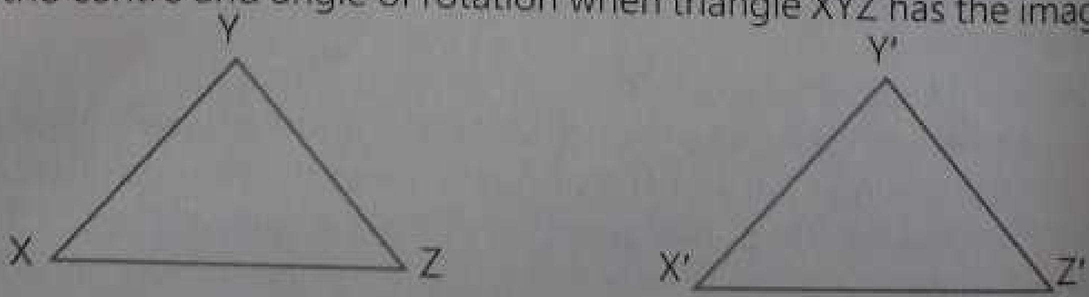
a) P through 70°
b) Q through $^-270°$
c) B through $^-90°$
d) R through 130° (for each rotation, trace triangle ABC and the centre of rotation)



Determining the Centre and Angle of Rotation

Example 1 (Discuss in groups)

Obtain the centre and angle of rotation when triangle XYZ has the image X′Y′Z′.



Procedures:

a) Trace the triangles XYZ and X'Y'Z'
To find the centre and angle of rotation, you need to first identify the object point and the corresponding image point, that is to say, X and X', Y and Y', and Z and Z'.

b) Join Z to Z' with a straight line.

c) Construct the perpendicular bisector of ZZ'.

d) Join Y to Y' with a straight line.

e) Construct the perpendicular bisector of YY'.

f) The point of intersection of the two perpendicular bisectors of lines ZZ' and YY' is the centre of rotation, O.

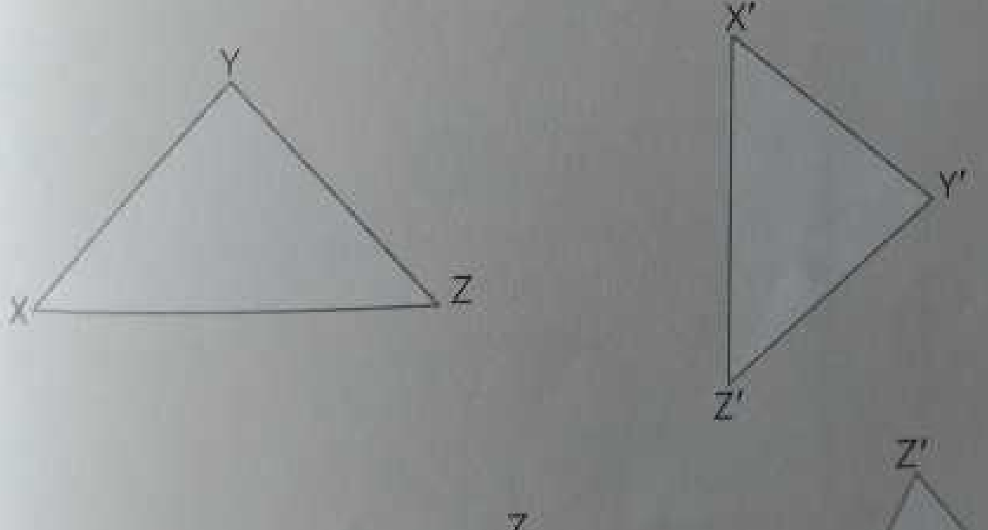
g) Join both the image point, say, Z' and object point Z to point O.

h) Measure from the object line (ZO) at O to the image line (Z'O), angle ZOZ'. Angle ZOZ' is the angle of rotation. Measure and record it.

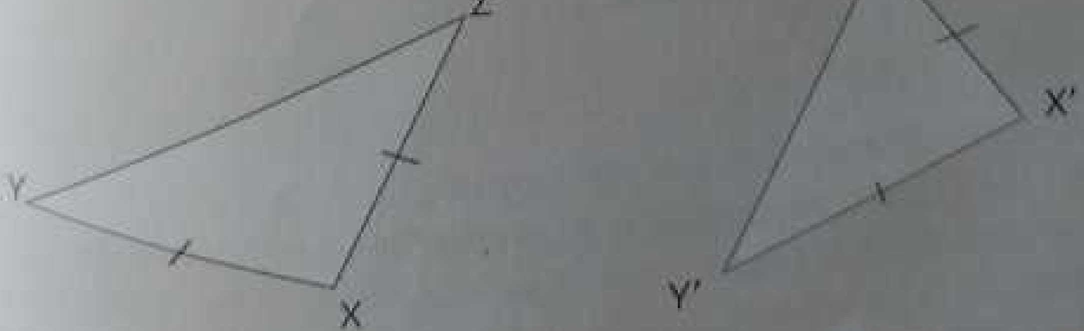
Exercise 9.2

Trace the following triangles and find the centre and angle of rotation when XYZ is mapped onto X'Y'Z' by a rotation.

a)



b)

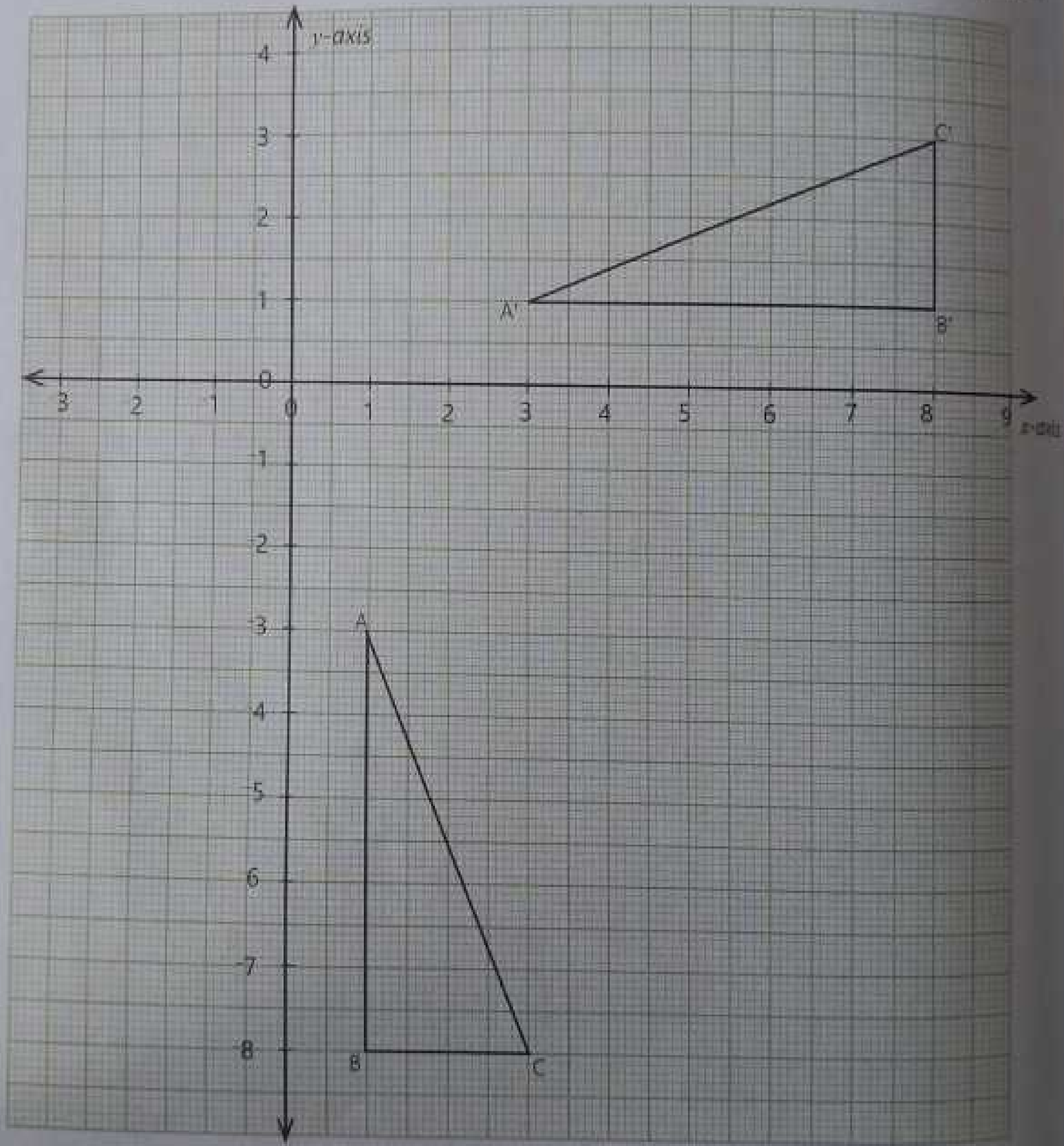


9.3 Rotation in the Cartesian Plane

The same skills of rotating a plane shape apply, but the only difference is that the vertices have a specific coordinate location.

Activity 9.3(a) (Work in groups)

In the following figure, pre-image (object) triangle ABC has been rotated to create image triangle A'B'C'. Produce a copy of the same on your graph paper.



a) Construct line bisectors of lines AA' and BB'.
b) Identify the point of Intersection, K, of the line bisectors in (a).
c) Read and record the coordinates of point K. This is the *centre of rotation*.
d) Join both the image point, say, A' and the object point A to point K.
e) Measure and record the angle AKA', starting from the object line AK.

Activity 9.3(b) (Work in groups)

a) Plot the triangles XYZ and X'Y'Z' on the same pair of axes, with points X(2, 6), Y(6, 6), Z(2, 12), X'(14, 8), Y'(10, 8) and Z'(14, 2).

b) Find the centre and angle of rotation.

c) Share with other groups.

Exercise 9.3

1. Find the centres and angles of rotation when triangle ABC is rotated to give the following images (A'B'C'), given the coordinates of A, B, and C as A(1, 7), B(4, 7) and C(1, 10), respectively.

   a) A'(6, 10), B'(6, 7), C'(9, 10)
   b) A'(4, 4), B'(1, 4), C'(4, 1)
   c) $A'(7, {}^-5)$, $B'(4, {}^-5)$, $C'(7, {}^-8)$
   d) $A'({}^-2, {}^-2)$, $B'({}^-2, {}^-5)$, $C'({}^-5, {}^-2)$
   e) $A'({}^-4, {}^-7)$, $B'({}^-4, {}^-4)$, $C'({}^-7, {}^-7)$
   f) $A'({}^-4, 3)$, $B'({}^-7, 3)$, $C'({}^-4, 0)$
   g) $A'({}^-2, 5)$, $B'({}^-5, 5)$, $C'(-2, 2)$

2. Find the image of triangle ABC (used in question 1 above) if it is rotated about;

   a) (1, 0) through $^+90°$
   b) (6, 7) through $^+57°$
   c) (6, 5) through $^-30°$
   d) (0, 0) through $^-60°$
   e) (1, 7) through $^-270°$

3. Given the coordinates of triangles PQR and P'Q'R' as (4, 0), (7, 0), (4, 4) and (0, 4), (0, 7), $({}^-4, 4)$ respectively, find the centre and angle of rotation.

ICT Activity (Work in groups)

a) Discuss the properties of rotation as a transformation. Make a presentation, using the Microsoft Power Point program to the class.

b) Use Encarta or Encyclopedia to research and find out more about rotation in Mathematics.

c) Share your findings with other groups.

Revision Questions

1) Given that the coordinates of triangles A'B'C', the image of ABC, after a rotation about the origin through 85° are (6, 0), (10, 0), and (6, 7) respectively, find the coordinates of the object ABC.

2) A rectangle of vertices (3, 3), (6, 3), (6, 5) and (3, 5) is to be rotated through 90° clockwise about the origin. Determine the vertices of the image.

Sample Activity of Integration

A community is constructing a hall. The construction management team is looking for a knowledgeable community member to provide sketches of the circular windows on the hall.

Support

- a paper
- a pencil
- a construction kit

Resource:

- Knowledge of rotational symmetry

Tasks:

You have been elected as a knowledgeable member of this community. Draw three sketches from which the team can pick to take to the fabrication workshop.

Chapter Summary

In this chapter, you have learnt that:

- In clockwise rotation, the angle of rotation is negative.
- In anticlockwise rotation, the angle of rotation is positive.
- If required to find the centre of rotation;
  - join at least two corresponding pairs of points (object and its image).
  - find their perpendicular bisectors; where the bisectors meet is the centre of rotation.
- If required to find the angle of rotation, join the object point to the centre and its image point to the centre; the angle in between is the angle of rotation.
- The object size is the same as the image size, after a rotation.

THEME: **Geometry and Measures**

Chapter 10

Length and Area Properties of 2-Dimensional Geometrical Figures

Keywords

- area
- length
- Pythagoras' theorem
- width

By the end of this chapter, you should be able to:

- describe the length of 2-dimensional geometric figures.
- develop, understand and state Pythagoras' theorem.
- apply Pythagoras' theorem to right-angled and isosceles triangles.
- understand the meaning of area in 2-dimensional geometrical figures (triangles, rectangles).

Introduction

At primary school level, you learnt how to find the areas and perimeters of different shapes. Quite often, length and area properties are used in daily life; for example, if a farmer wants to fence off a certain piece of land, he needs to know the dimensions of the plot so that he can purchase the required fencing materials.

In this chapter, you will understand further, justify and apply area and perimeter formulae for different figures.

10.1 Length of 2-Dimensional Geometrical Figures

Activity 10.1 (Work in groups)

Suggested materials: • a manilla paper • a pencil • a cutter • a protractor

Procedures:

a) Draw;

i) an equilateral triangle CDH of length 5 cm

ii) a square DEGH of length 5 cm

iii) a rectangle ABDC of length AB = 5 cm and width AC = 3 cm

iv) trapezium BFED where BF = 6 cm, BD = 3 cm, DE = 5 cm and angle DEF = angle EFB = 90°

b) Cut out the four figures.

c) Join the four different figures to form a compound figure ABFEGHC.

d) Find the perimeter of the compound shape formed in (c).

10.2 Pythagoras' Theorem

Activity 10.2 (Work in groups)

a) Locate point A at the centre of a graph paper.

b) From A, draw a vertical line that extends up 3 cm to B.

c) On the same graph paper, draw a horizontal line AC that extends 4 cm to the right from A.

d) Join points B and C, and name the shape obtained.

e) Draw a square ABPM that touches the left side of the shape obtained in (d). Draw another square ACTG that touches the bottom of the shape.

f) Shade each square drawn in a different colour.

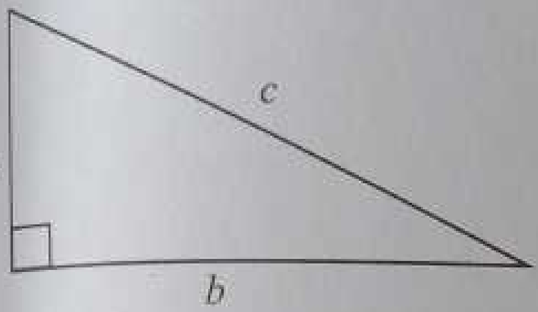
g) Measure the length of the longest side of the shape formed in (d), and use it to draw a square BCRS along that longest side.

h) Determine the area of each square.

i) What is the relationship among these 3 areas of the squares?

Now given the length of any two sides of a right-angled triangle (a triangle with one of its angles equal to 90^0), the length of the other side can be found using the Pythagoras' theorem.

Consider the right-angled triangle below with sides $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, where $\boldsymbol{c}$ is the longest side.



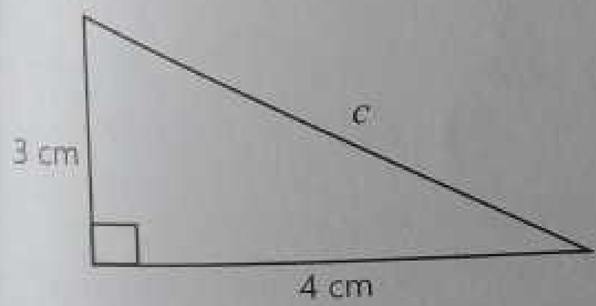
Sides a, b and c are related using the rule; the square of the hypotenuse equals to the sum of the squares of the other two sides.

$\boldsymbol{a^2 + b^2 = c^2}$, which is known as the ***Pythagoras' theorem***.

If you are given a right-angled triangle with two known sides, the length of the other side can be determined using the above theorem, as in the examples below:

Example 1

Find the unknown length of the triangle.



Solution:

Using the Pythagoras theorem,

$a^2 + b^2 = c^2$;

since $a = 3$ and $b = 4$, substituting for a and b yields:

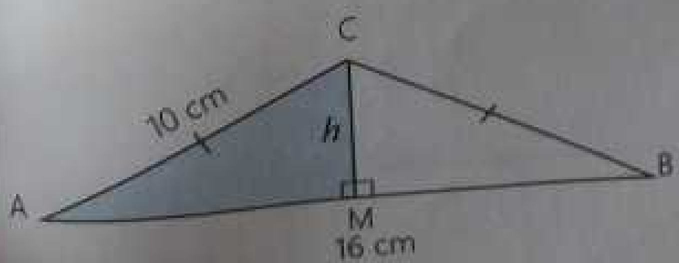
$3^2 + 4^2 = c^2$

$\Rightarrow c^2 = 25$

Now, taking the square root on both sides, you obtain $c = 5$ cm.

Example 2

Determine the length h.



Solution:

Since M divides AB into two equal lengths, then length $\overline{AM} = 8$ cm.

Considering the right-angled triangle AMC, you have;

$\overline{AM}^2 + \overline{MC}^2 = \overline{AC}^2$ (Pythagoras' theorem)

$8^2 + h^2 = 10^2$

$64 + h^2 = 100$

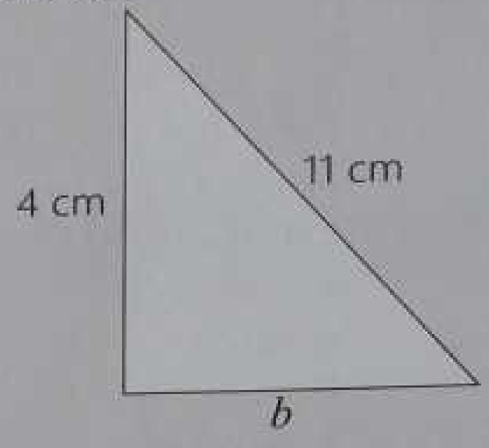
Subtracting 64 on both sides, you obtain

$h^2 = 36$.

Taking the square root on both sides, you obtain $h = 6$ cm.

Exercise 10.1

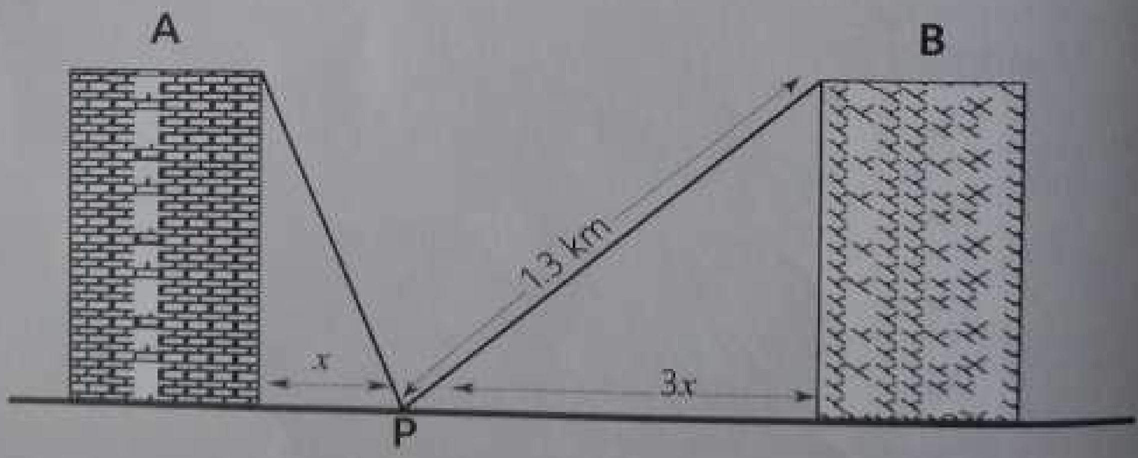
1. Given the right-angled triangle below, find the length of side b.



2. Calculate the missing side (correct to three (3) significant figures) if x, y and z are sides of a right-angled triangle; with x as the height, y as the base and z as the hypotenuse.
   a) $x = 15$ cm, $y = 9$ cm, $z = ?$
   b) $x = 0.65$ km, $y = 0.94$ km, $z = ?$

3. A tree is midway between points E and F, 14 m apart. A is a point on top of the tree 7 m above the ground.
   a) How far is E from the bottom of the tree?
   b) How far are points E and F from the top of the tree?

4. Two flathouses, A and B, of the same height are 1.6 km apart as shown in the diagram below. John stands at a point P, 3 times far from flathouse B as he is from A, so that his actual distance from point P to the top of B is 1.3 km.



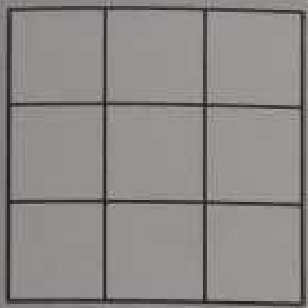
Calculate the;
   a) value of x
   b) height of the flathouses

10.3 Areas of Regular Figures

Activity 10.3(a) (Work in groups)

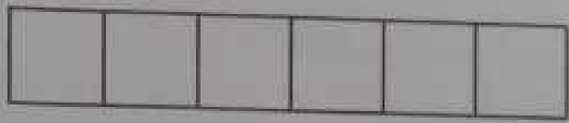
a) Count and write down the number of 1 cm square boxes in the shape below.

i)

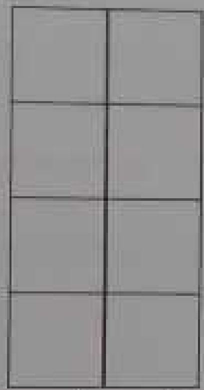


b) Repeat procedure (a) on the following shapes.

ii)



iii)



c) Count the number of square boxes in the first column for each shape. Write it down.

d) Count the number of square boxes on the first row for each shape. Write it down

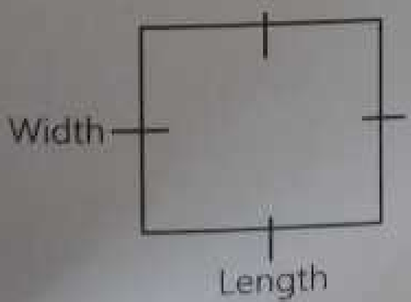
e) Multiply the answer obtained in (c) with that in (d) for each shape.

f) Compare the answers obtained in (e) with those in (a), and (b). What do you observe?

From *Activity 10.3(a)*, you discover that the number of square boxes is the same; whether you count them one by one, or use algebra to compute the total number of boxes obtained by multiplying those in the column with those in the row. The measure of the number of unit squares contained inside a figure is what is referred to as the *area of the figure*.

Area of a Square

A square is a quadrilateral whose sides are of equal lengths and corresponding opposite sides are parallel, with right angles formed on adjacent lines.



Length = width

Its area can be calculated using:

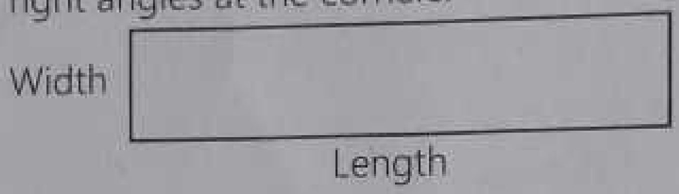
Area $= \text{length} \times \text{width}$

$= \text{length} \times \text{length}$

$= (\text{length})^2$

Rectangle

A rectangle is a quadrilateral whose opposite sides are equal and parallel, with right angles at the corners.

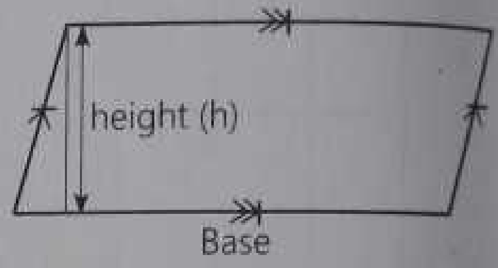


So, Area of rectangle = length × width

Parallelogram

It is a quadrilateral with the opposite sides equal and parallel.

The opposite angles are equal in measure, but there are no right angles at the corners.

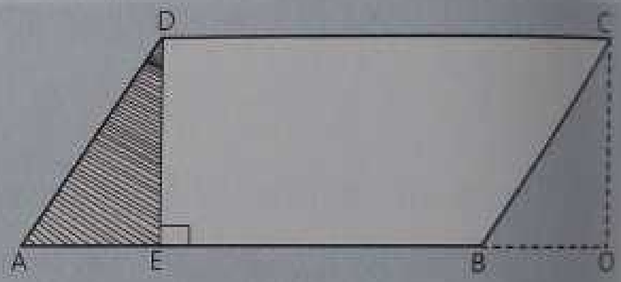


Activity 10.3(b) (Work in groups)

a) Draw a parallelogram ABCD on a manila paper.

b) Cut out the shaded area AED.

c) Fit the cut-out area AED in the area BOC.



Note

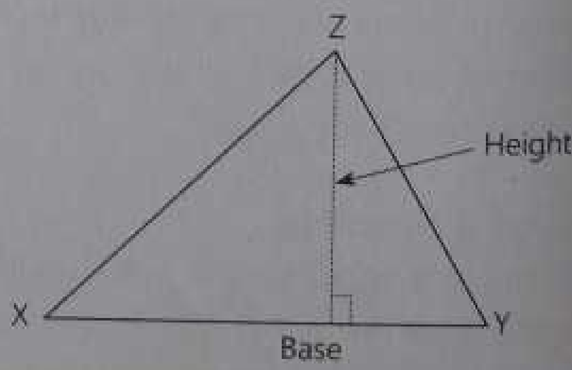
The new figure formed above is a rectangle whose area can be found using, Area = length × width.

In the new figure DEOC, the length is the base and the width is the height. Area of DEOC = Base × Height

Area of parallelogram = base × perpendicular height.

Triangles

Triangles are made up of three enclosing sides.



Activity 10.3(c) (Work in groups)

Suggested material:

- plain paper
- a ruler

Procedures:

a) Get a plain paper.

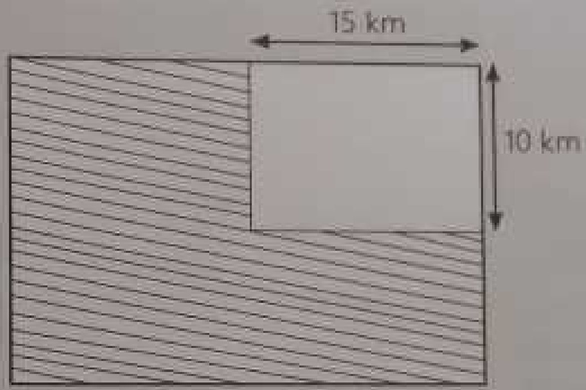
b) Measure and record its length, b and width, h.

c) Find its area.

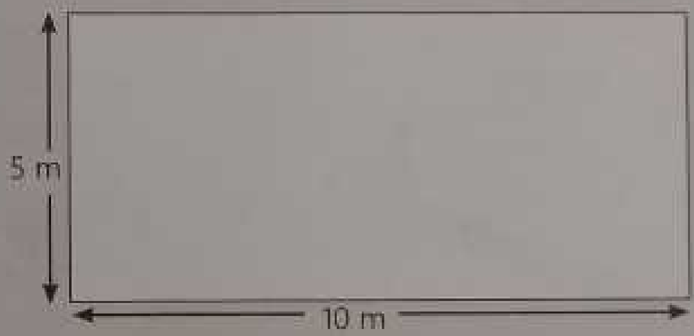
d) Fold the paper to form 2 triangles of equal dimensions.
e) Write down the area of each triangle formed in (d).
f) Hence, state the formula for the area of the triangle.

Exercise 10.2

1. Find the area of the shaded surface if the area of the whole surface is 600 km^2.

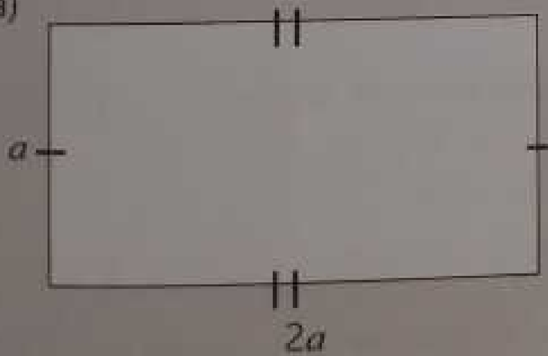


2. During a rainy season, a farmer planned to make 6 terraces in a 900 m^2 rectangular plot of land. Each terrace had dimensions of 10 m by 5 m. Calculate the remaining piece of land that was left for cultivation.

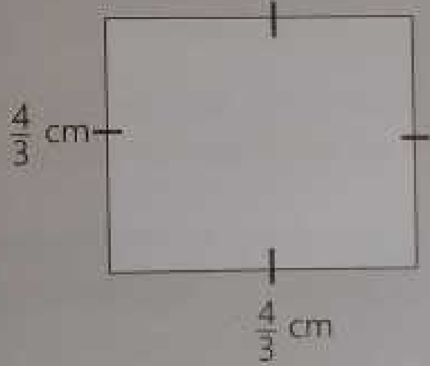


3. Calculate the areas of the following:

a)



b)



ICT Activity

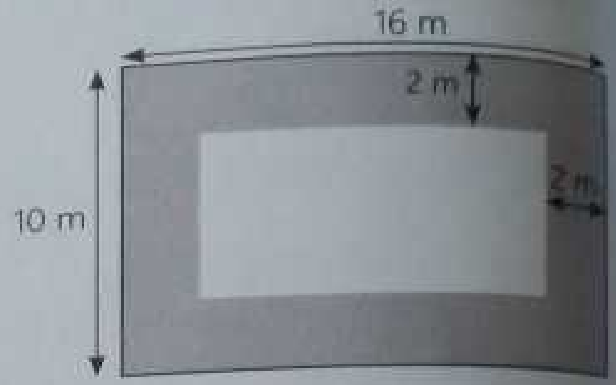
a) In groups, watch videos on YouTube; for example, on the VividMath channel, about Pythagoras' theorem in order to enhance your understanding of this chapter.
b) Share your findings with the rest of the class.

Revision Question:

Given the figure:

a) calculate the area of the;
   i) outer rectangle
   ii) inner rectangle

b) what area remains when the inner rectangle is cut out?



Sample Activity of Integration

Your parents are on a site to set up a rectangular ground floor of a poultry house measuring 12 m by 5 m.

Support:
- a tape measure / ruler
- a hammer
- a string / cord
- pegs

Resource:
- Knowledge of Pythagoras' theorem

Task:

As a Senior Two Mathematics learner, advise your parents on how to accurately set up the ground floor of the poultry house.

Chapter Summary

- In this chapter, you have learnt that; for any right-angled triangle with sides a, b, and c, where c is the longest, you can always relate these sides using Pythagoras' theorem, which is stated as $a^2 + b^2 = c^2$.
- You have also dealt with the following formulae:
  - Area (A) of a triangle is given as; $A = \frac{1}{2}bh$, where b is the base and h is the height
  - Area of a rectangle is given as; $A = l \times w$, where l is the length and w is the width
  - Area of a square is given as; $A = s^2$, where s is the length of the side of a square
  - Area of a parallelogram is given as; Area = base × perpendicular height

THEME: Geometry and Measures

Chapter 11 Nets, Areas and Volumes of Solids

Keywords

- cone
- edges
- faces
- nets
- surface area
- units of measures
- vertices
- volume

By the end of this chapter, you should be able to:

- form nets of common solids.
- identify common solids and their properties, including faces, edges and vertices.
- state units of measures.
- convert units from one form to another.
- calculate the surface areas of 3-dimensional figures.
- calculate volumes of cubes and cuboid.

Introduction

In the previous chapter, you looked at 2-dimensional figures. These 2-dimensional figures can be modelled to form solids; for example, different squares can be combined to form a cube, which is a 3-dimensional shape, and different rectangles can be combined to form a cuboid, which is also a 3-dimensional shape.

In this chapter, you will be exposed to different 3-D shapes and learn how to determine their volumes, surface areas, and form their nets. This will enable you to make and draw 2-D and 3-D shapes and explore their properties.

11.1 Forming Nets of common Solids

Net of a Cone

Activity 11.1(a) (Work in groups)

Suggested materials: • a manilla paper • a cutter • sellotape / paper glue

a) Draw a circle.
b) Draw a sector in the circle in a) and measure its arc length.
c) Draw another circle adjacent to the sector and this circle's circumference must be equal to the arc length in b).
d) Cut out the sector and the circle in c).
e) Fold the sector along the circle.
f) What 3-D shape is formed? What observations do you make?
g) Identify the number of faces, edges and vertices of the shape formed.
h) Explore other nets for this shape.
i) Present to the rest of the class.

Net of a Cylinder

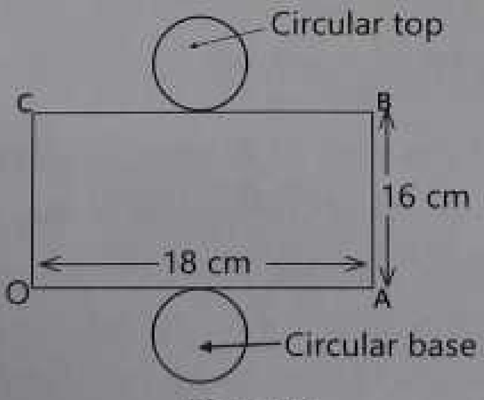


Figure 1

Activity 11.1(b) (Work in groups)

Suggested materials: • a manilla paper • a cutter • sellotape / paper glue

a) Draw a rectangle with dimensions 18 cm by 16 cm.
b) Using the length OA, compute the radius of the circular ends, that is, by equating the circumference of the circular ends to 18 cm.
c) Draw a perpendicular bisector of lines OA and CB.
d) Using the two points of contact of this bisector to lines OA and CB, draw two circles using the radius obtained in b).

e) Cut out the outline and use it to form a 3-D shape.
f) Identify the number of faces, edges and vertices of the shape formed.
g) Explore other nets for this shape.
h) Present to the rest of the class.

Net of a Cube

A cube is a shape with six faces whose dimensions are all of equal length. The net for a cube whose dimension is 7 cm is as shown in *Figure 2*.

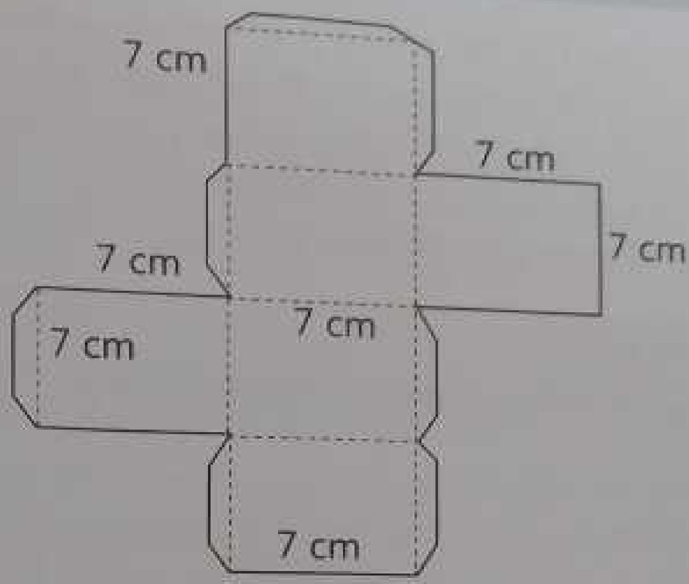


Figure 2

Activity 11.1(c) (Work in groups)

Suggested materials: • a manilla paper • a cutter • sellotape / paper glue

a) Draw four parallel lines each located 7 cm from the other.
b) Mark out 6 squares of length 7 cm as shown in the figure.
c) Cut out the outline and fold along the dotted lines.
d) Fasten the solid with sellotape. What observations do you make?
e) Identify the number of faces, edges and vertices of the shape formed.
f) Explore other nets for this shape.
g) Present to the rest of the class.

Net of a Cuboid

The net of a cuboid whose dimensions are 9 cm × 8 cm × 7 cm is as shown below.

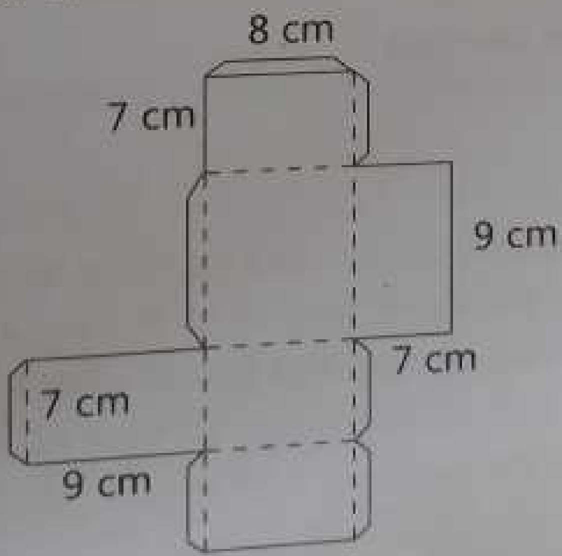


Figure 3

Activity 11.1(d) (Work in groups)

Suggested materials: • a manilla paper • a cutter • sellotape / paper glue

a) Draw four parallel lines, a distance of 9 cm, 8 cm and 7 cm apart.
b) Mark the lengths shown in the figure above.
c) Cut out the outline and fold along the dashed lines.
d) Then fasten them with sellotape. What observations do you make?
e) Identify the number of faces, edges and vertices of the shape formed.
f) Explore other nets for this shape.
g) Present to the rest of the class.

Question

In your group, discuss situations where cubes and cuboids are used in real life.

Net of a Pyramid

The figure below is a net of a pyramid.

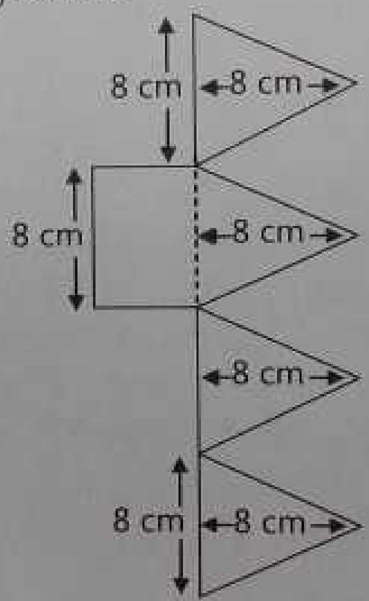


Figure 4

Activity 11.1(e) (Work in groups)

Suggested materials:

- a manilla paper
- a cutter
- sellotape / paper glue

a) Draw three parallel lines 8 cm apart and mark the lengths shown in Figure 4.
b) Cut out the outline of the figure. Fold to form a pyramid.
c) What is the slant length and height of the pyramid formed?
d) Identify the number of faces, edges and vertices for the pyramid.
e) Explore other nets for the pyramid.
f) Present to the rest of the class.

Net of a Tetrahedron

This is a net of a tetrahedron.

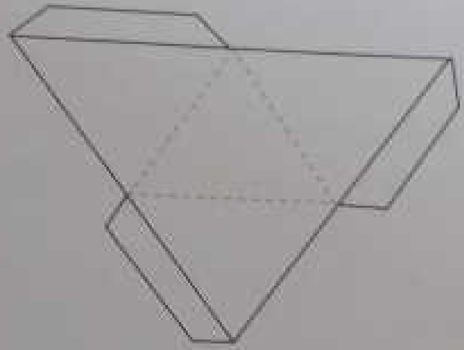


Figure 5

Activity 11.1(f) (Work in groups)

Suggested materials: • a manilla paper • a cutter • sellotape / paper glue

a) Draw an equilateral triangle of length 20 cm.
b) Put a mark 10 cm from the vertices of the triangle along each side and join the points with dotted lines as shown in the figure.
c) Cut out the outline.
d) Fold along the dotted lines and fasten with sellotape.
e) What is the slant length and the height of this tetrahedron?
f) Identify the number of faces, edges and vertices of a tetrahedron.
g) Explore other nets for the tetrahedron.
h) Present to the rest of the class.

Net of a Prism

A prism is a 3-dimensional shape with a uniform cross-section. The net of a prism is as shown in *Figure 6*.

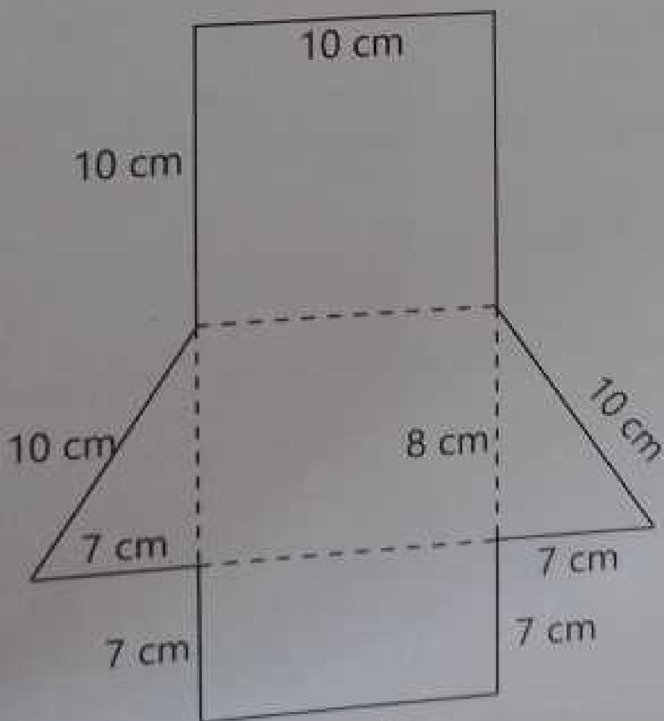


Figure 6

Activity 11.1(g) (Work in groups)

Suggested materials:
- manilla paper • cutter • sellotape / paper glue.

a) Draw four parallel lines a distance of 10 cm, 8 cm and 7 cm apart.
b) Mark or measure the distances shown in *Figure 6*.
c) Cut out the outline.
d) Fold along the dotted lines and fasten with sellotape.
e) Describe the shape formed.
f) Identify the number of faces, edges and vertices of the shape formed.
g) Explore other nets for this shape.
h) Present to the rest of the class.

Activity 11.1(h) (Work in groups)

Suggested materials:
- a manilla paper • a cutter • sellotape / paper glue.

a) Draw 4 parallel lines a distance of 8 cm, 7 cm and 10 cm apart and mark the lengths as shown in *Figure 7*.
b) The vertex of the triangle lies along the perpendicular bisector of the 8 cm line.
c) Cut out the outline. Fold along the dotted lines.
d) What is the length of the line from the vertex to the base of the solid?

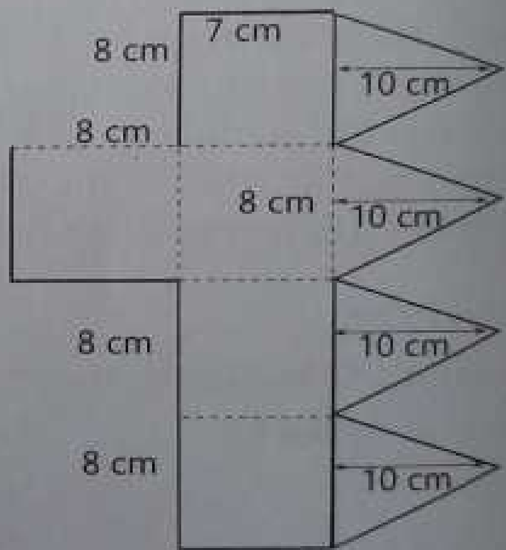
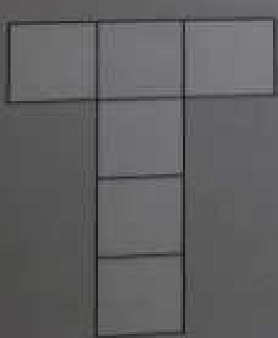
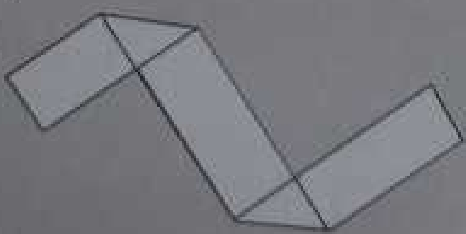


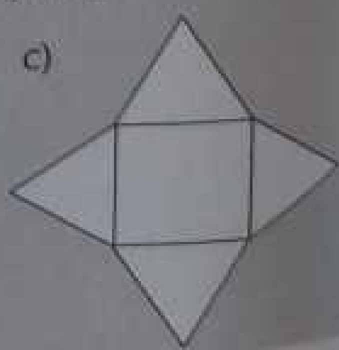
Figure 7

Exercise 11.1

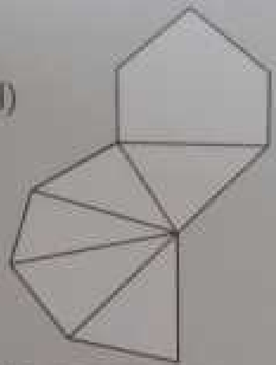
1. Identify the solid that corresponds to each of the following nets.

a)


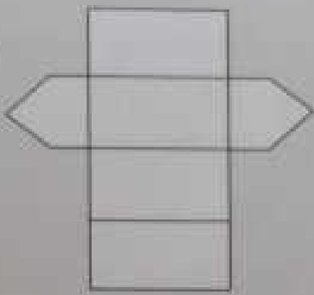
b)


c)


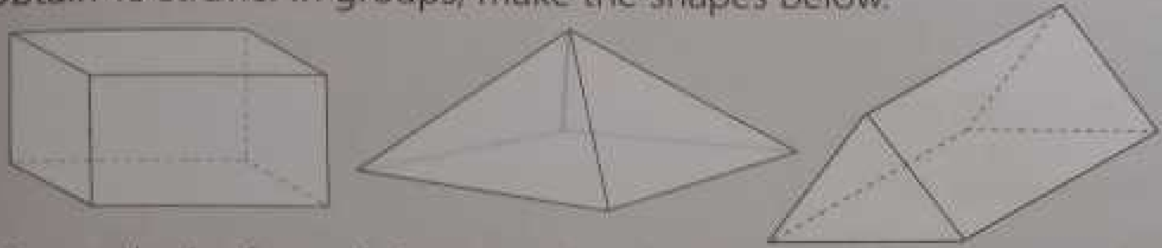
d)



e)



2. a) Obtain 16 straws. In groups, make the shapes below.



b) Dismantle the formed shapes and make nets.

c) For each shape, how many different nets have you formed?

d) Present to the rest of the class.

11.2 Units of Measures

Area and volume can be measured in various unit; for example, area can be measured in cm^2, m^2 and in many other units. Volume can be measured in m^3, cm^3, mm^3 and many other units as well. You can always convert the units of measurement from one to another.

By doing this, you get one unit of measurement written in terms of the other. Remember that in Senior One Physics, you learnt conversion of units from one to another, for example, converting metres to centimetres. Here, you will expound on the knowledge that you already have.

Activity 11.2(a) (Work in groups)

Convert the following:

a) 20 m to cm   b) 10 mm to m   c) 2 cm to mm   d) 10 cm to m

Example 1

Convert 20 m^2 to cm^2.

Solution:

Since $1 \text{ m} = 100 \text{ cm},$

$1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm}$

Implying that: $1 \text{ m}^2 = 10{,}000 \text{ cm}^2$

Therefore, $20 \text{ m}^2 = 20 \times 10{,}000 \text{ cm}^2$

$= 200{,}000 \text{ cm}^2.$

Activity 11.2(b)

In groups, convert the following:

a) $4\ m^2$ to cm^2

b) $30\ cm^2$ to m^2

c) $7\ m^2$ to cm^2

d) $45\ cm^2$ to m^2

Example 2

Convert $40\ mm^3$ to m^3.

Solution:

Since 1 m = 1,000 mm,

$$1\ mm = \frac{1}{1{,}000}\ m$$

Implying that: $1\ mm \times 1\ mm \times 1\ mm = \frac{1}{1{,}000}\ m \times \frac{1}{1{,}000}\ m \times \frac{1}{1{,}000}\ m$

$$\therefore 1\ mm^3 = \frac{1}{1{,}000{,}000{,}000}\ m^3$$

$$40\ mm^3 = \frac{40}{1{,}000{,}000{,}000}\ m^3$$

$$= 0.00000004\ m^3$$

$$= 4.0 \times 10^{-8}\ m^3$$

Activity 11.2(c)

In groups, convert the following:

a) $100\ cm^3$ to m^3

b) $1000\ cm^3$ to mm^3

c) $5\ m^3$ to cm^3

11.3 Surface Areas of 3-Dimensional Figures

Surface area is defined as the total area of the external surface of a solid.

Surface Area of a Cube

Activity 11.3(a) (Work in groups)

a) Make a net of a cube.

b) Calculate the area of each face of the net.

c) Find the total surface area of the net.

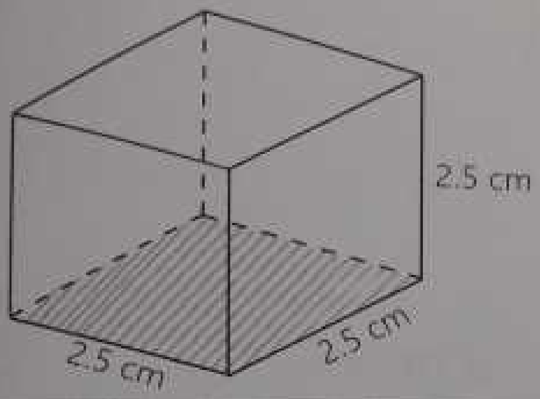
d) Deduce the formula for the surface area of a cube.

e) Share with other groups.

Example 3

A cube has sides of length 2.5 cm each. Find its surface area.

Solution:



Since each side $(s) = 2.5$ cm, from surface area (S. A.) $= 6s^2$, if you substitute for s, you obtain;

$$\text{S. A.} = 6 \times 2.5^2$$
$$= 37.5 \text{ cm}^2$$

Surface area of a Cuboid

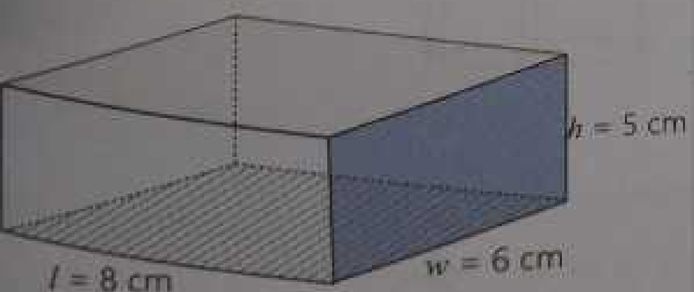
Activity 11.3(b) (Work in groups)

a) Explore your environment and collect a hollow cuboid.
b) Dismantle the cuboid collected in (a) to form a net.
c) Measure the dimensions of all the faces formed in the net.
d) Calculate the area of each face of the net.
e) Find the total surface area of the net.
f) Deduce the formula for the surface area of a cuboid.

Example 4

Find the surface area of a cuboid with dimensions 8 cm by 6 cm by 5 cm.

Solution:



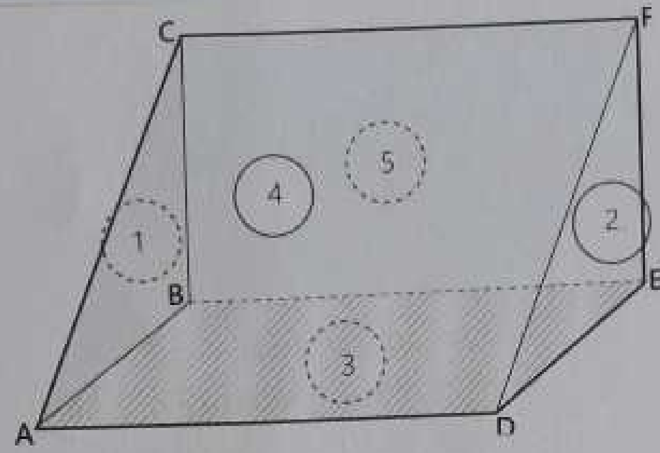
$l = 8$ cm, $w = 6$ cm and $h = 8$ cm

From surface area $= 2(wl + wh + hl)$, if you substitute for l, w and h, you will obtain;

$$\text{Surface area} = 2(8 \times 6) + 2(6 \times 5) + 2(5 \times 8)$$
$$= 236 \text{ cm}^2$$

Surface Area of a Prism

For most prisms, there is no specific formula for finding the surface area. It is simply determined by calculating the areas of the faces that make up the prism and summing up the result.



Activity 11.3(c) (Work in groups)

a) Draw a net of a prism using dimensions of your choice.

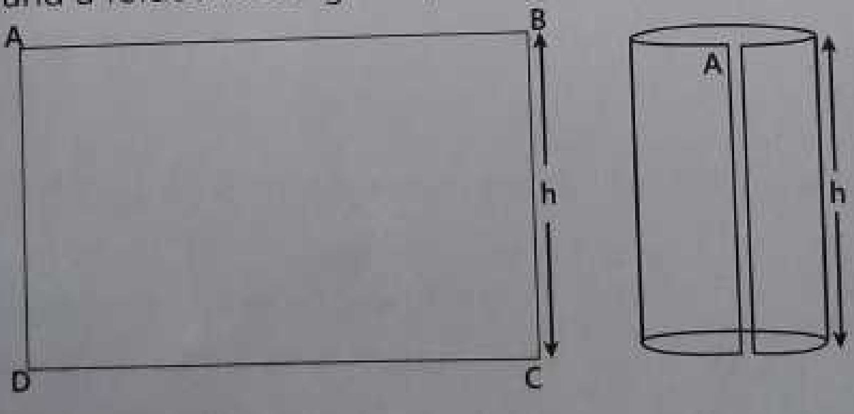
b) Calculate the area of each face of the net.

c) Find the total surface area of the net.

d) Deduce the fomula for finding the surface area of the prism whose net was drawn in (a).

Surface Areas of Cylinders

A cylinder is made up of a rectangular sheet that has been folded along one of its length to obtain a curved surface. The cylinder (closed) consists of two circular surfaces, and a folded rectangular (curved) surface.



Activity 11.3(d) (Work in groups)

a) Draw a net of a cylinder using dimensions of your choice.

b) Calculate the area of each face of the net.

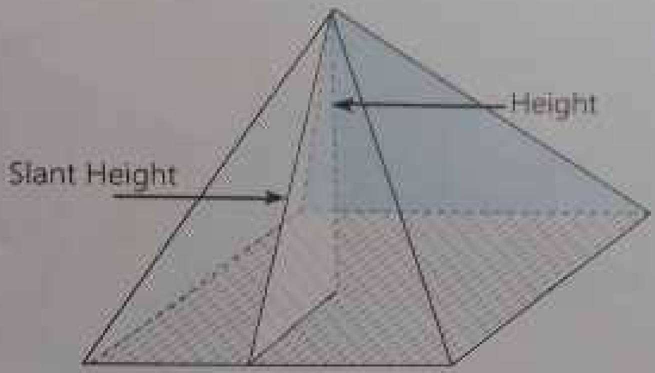
c) Find the total surface area of the net.

d) Deduce the fomula for finding the surface area of the cylinder.

Surface Area of a Pyramid

Activity 11.3(e)

In groups, consider the right pyramid below:



a) Draw the net of a right pyramid using dimensions of your choice.
b) Find the area of each face of the net.
c) Find the total surface area of the net.
d) Deduce the formula for finding the surface area of a right pyramid.

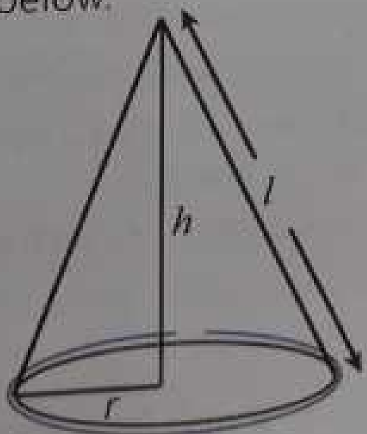
Surface Area of a Cone

A cone does not have triangular faces; you therefore, have to devise a formula for determining the surface area of its curved part. This is commonly referred to as the *curved surface area of a cone*. The total surface area will then be given by:

Total surface area of a closed cone = Curved surface area + area of base.

$$= \pi r(r + l)$$

where r is the radius of a circular base and l is the slant length of the curved circles. Considering the cone below.



The relation among the height, h, of the cone, and r and l is $l^2 = r^2 + h^2$. (Pythagoras' theorem)

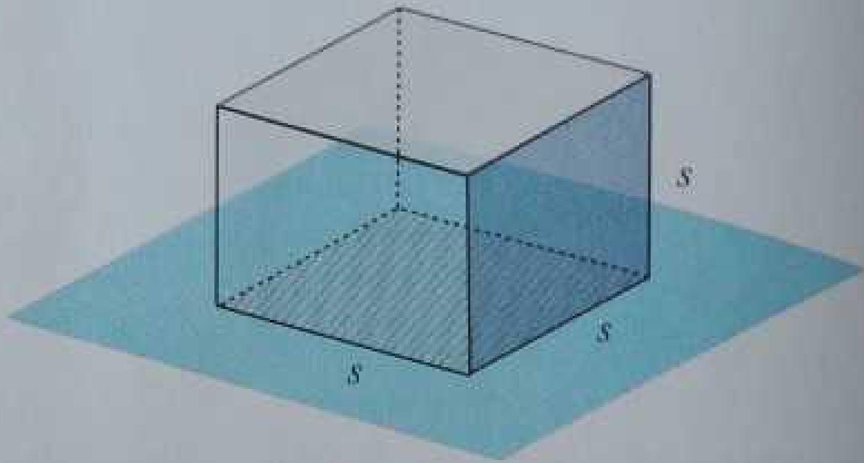
Question:

What is the surface area of an open cone?

11.4 Volumes of Cubes and Cuboids

Cubes

Suppose you have a cube of length **s**, as shown below.



The volume of this cube is given by: Volume $= s \times s \times s = s^3$ (cubic units)

Example 5

A cube has sides of length 3.5 cm each. Find its volume.

Solution:

Since volume of a cube $= s^3$ and $s = 3.5$ cm,

then, volume $= 3.5^3$

$= 42.875 \text{ cm}^3$

Activity 11.4 (Work in groups)

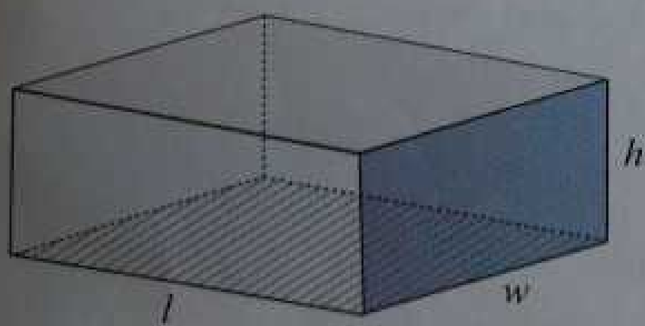
a) If the volume of a cube is 512 m^3, what are the dimensions of its sides?

b) You are provided with a die. Take the measurements of its dimensions using a ruler and hence determine its volume.

c) If the perimeter of a face of a cube is 16 mm, what is its volume?

Cuboids

Consider a cuboid of dimensions l, w and h as shown below.



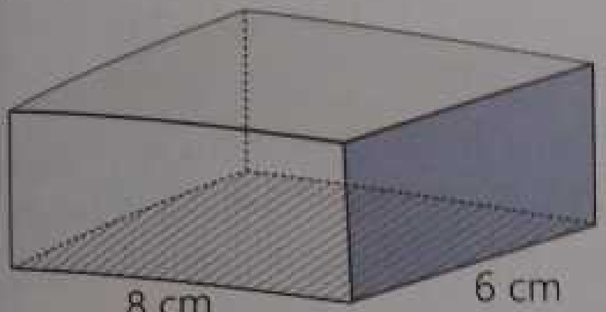
Its volume (V) is given by, $V = l \times w \times h$. The unit of measurement of volume depends on the units of the dimensions, that is, it can be m^3, cm^3 and many others.

Example 6

Find the volume of a cuboid with dimensions 8 cm by 6 cm by 5 cm.

Solution:

$l = 8$ cm , $w = 6$ cm, $h = 5$ cm



$$\text{Volume} = l \times w \times h$$
$$= 8 \times 6 \times 5$$
$$= 240 \text{ cm}^3$$

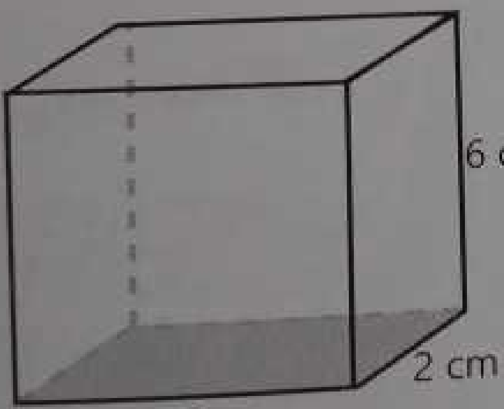
ICT Activity

a) In groups, watch a video about "**Nets of Solids**" on YouTube.

b) Share your findings with the rest of the class.

Revision Questions:

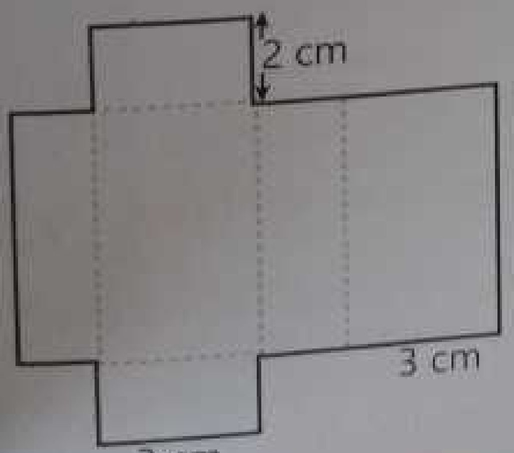
1.



a) How many nets can be formed from the figure shown?

b) How many litres of water can fill the figure to its capacity?

2. Determine the surface area of a solid whose net is shown below.



3. Find the volume of a cuboid whose dimensions are;
   a) length = 10 cm, breadth = 6 cm, height = 4 cm
   b) length = 80 m, breadth = 45 m, height = 21.5 m
4. The length, breadth and height of a chocolate box are in the ratio 7 : 5 : 3. If its volume is 7,500 cm^3, find its dimensions.
5. The length, breadth and depth of a pond are 23.5 m, 19 m and 11 m respectively. Find the capacity of the pond in litres.
6. The dimensions of a brick are 24 cm × 12 cm × 8 cm. How many such bricks will be required to build a wall of 20 m length, 48 cm breadth and 6 m height?
7. The volume of a container is 1440 m^3. The length and breadth of the container are 15 m and 8 m respectively. Find its height.
8. Find the volume of a cube whose length of side measures:
   a) 5 cm b) 3.5 m c) 21 cm.

Sample Activity of Integration

You have decided to use part of your holiday time to make a poultry feeding trough.

Support:

- a manila paper
- paper glue
- a pair of scissors

Resources:

- Knowledge of area and volume of solids
- Knowledge of mathematical operations

Task:

Using the manilla paper, design a model of an appropriate poultry feeding trough that would minimise wastage of material.

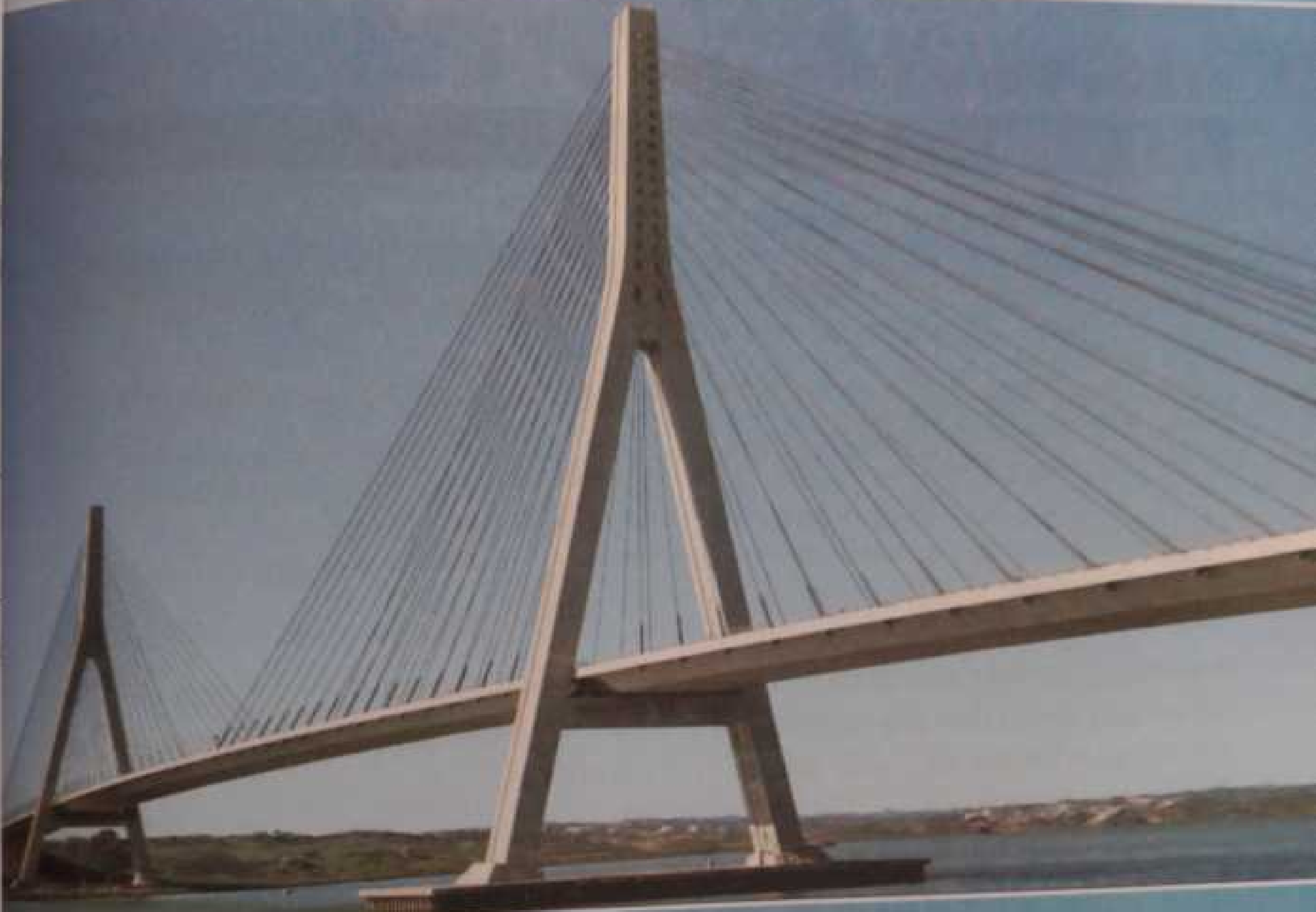
Chapter Summary

- In this chapter, you were exposed to properties of common 3-D shapes like faces, edges and vertices.
- You have also learnt about the surface area and volume of the common 3-D shapes and their units of measurements.

THEME: Numbers

Chapter 12

Numerical Concepts 2 (Surds)



Keywords

- conjugate
- irrational number
- like surds
- rational number
- root
- unlike surds

By the end of this chapter, you should be able to:

- use surds to represent roots that cannot be represented exactly as decimals.
- manipulate and simplify expressions with surds.

Introduction

A *surd* is a number left in root form to express its exact value. A surd has an infinite number of non-recurring decimals. Many times, surds are used in real life to make sure that important calculations are precise; for example, when engineers are building bridges and many other infrastructure.

In this chapter, you will learn how to manipulate surds.

12.1 Surds

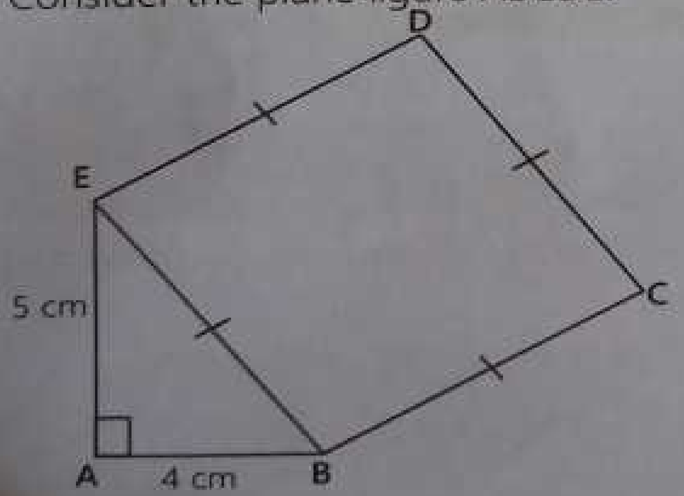
Activity 12.1 (Work in groups)

In Mukono district, a land surveyor measured Atim's plot of land and found out that it is 60 m in length and 50 m wide.

a) Using a ruler and a sharp pencil, draw an accurate diagram to represent the above dimensions.
b) Suggest the name you would give the above shape.
c) Using a ruler, measure the length of the diagonal.
d) Without using a ruler, obtain the length of the diagonal.
e) Comment on your values in c) and d).

A surd is a number whose root is not exact. The presence of the root sign does not necessarily mean that the number is a surd; for example, $\sqrt{4}$,$\sqrt{9}$, $\sqrt[3]{8}$ and $\sqrt[5]{32}$ are not surds. Whereas $\sqrt{8}$, $\sqrt{41}$ and $\sqrt[3]{49}$ are surds.

Consider the plane figure ABCDE.



ABE is a right-angled triangle and BCDE is a square. To find the area of the square, you need length BE.

$\overline{BE}^2 = 5^2 + 4^2$

$\overline{BE}^2 = 41$

$\overline{BE} = \sqrt{41} = 6.40312$ cm

Area of square $= \sqrt{41} \times \sqrt{41} = 41$ cm^2

Or area of square $= 6.40312 \times 6.40312$

$= 40.9999$ cm^2.

Using the precise value of $\overline{EB}$, that is ($\sqrt{41}$), gives a more accurate area of the square.

Surds are normally used in calculations rather than approximating the answers so as to reduce on errors in calculations. You can only approximate when you have reached the final value.

12.2 Like and Unlike Surds

A surd such as $m\sqrt{n}$ has two factors; the *rational factor* m and the *irrational factor* $\sqrt{n}$. Also, a surd $x\sqrt{n}$ has two factors; x and $\sqrt{n}$.

Since the two surds $m\sqrt{n}$ and $x\sqrt{n}$ have the same irrational factor $\sqrt{n}$, then they are *like surds*. But $m\sqrt{n}$ and $n\sqrt{m}$ are *unlike surds* since they have different irrational factors.

Activity 12.2

Identify like and unlike surds from the following set of numbers:

$\{\sqrt{5}, 5\sqrt{2}, \sqrt{3}, 2\sqrt{7}, 4\sqrt{5}, 7\sqrt{2}, 10\sqrt{3}, 3\sqrt{7}, \sqrt{10}\}$

12.3 Simplifying Surds

When you are to simplify a surd, the number under the root sign should be expressed as a product of two factors; one of the factors being the largest perfect square which factors that number.

Example 1

Simplify:

a) $\sqrt{18}$ b) $\sqrt{48}$ c) $3\sqrt{32}$

Solution:

a) $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

b) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

c) $3\sqrt{32} = 3\sqrt{4 \times 4 \times 2} = 3 \times \sqrt{4} \times \sqrt{4} \times \sqrt{2} = 3 \times 2 \times 2 \times \sqrt{2} = 12\sqrt{2}$

Example 2

Express the following as roots of a single compound number.

a) $5\sqrt{3}$ b) $4\sqrt{6}$ c) $2\sqrt{7}$

Solution:

a) $5\sqrt{3} = \sqrt{5^2} \times \sqrt{3} = \sqrt{25 \times 3} = \sqrt{75}$

b) $4\sqrt{6} = \sqrt{4^2} \times \sqrt{6} = \sqrt{16 \times 6} = \sqrt{96}$

c) $2\sqrt{7} = \sqrt{2^2} \times \sqrt{7} = \sqrt{4 \times 7} = \sqrt{28}$

Addition and Subtraction of Surds

Activity 12.3(a)

In groups, discuss the following:

a) if $x + x = 2x$, find $\sqrt{2} + \sqrt{2}$.

b) if $3y + y = 4y$, find $3\sqrt{5} + \sqrt{5}$.

c) hence, simplify $a\sqrt{p} + r\sqrt{p}$.

Activity 12.3(b)

In groups, discuss the following:

a) if $5m - 3m = 2m$, find $5\sqrt{7} - 3\sqrt{7}$.

b) if $7t - 6t = t$, find $7\sqrt{3} - 6\sqrt{3}$.

c) hence, simplify $a\sqrt{p} - r\sqrt{p}$.

Example 3 (Discuss in groups)

Simplify the following:

a) $\sqrt{27} + 2\sqrt{12} - \sqrt{108}$

b) $2\sqrt{45} - 2\sqrt{80} + 3\sqrt{5}$

Solution:

a)
$$\sqrt{27} + 2\sqrt{12} - \sqrt{108} = 3 \times \sqrt{3} + 2 \times 2 \times \sqrt{3} - (6 \times \sqrt{3})$$
$$= \sqrt{3}$$

b)
$$2\sqrt{45} - 2\sqrt{80} + 3\sqrt{5} = 6\sqrt{5} - 8\sqrt{5} + 3\sqrt{5}$$
$$= \sqrt{5}$$

Exercise 12.1

1. Simplify:

a) $\sqrt{120}$ b) $\sqrt{500}$ c) $4\sqrt{162}$

2. Simplify:

a) $\sqrt{5} + \sqrt{145}$

b) $\sqrt{45} - \sqrt{20}$

c) $\sqrt{63} + 3\sqrt{28} - \sqrt{847}$

d) $4\sqrt{20} - \sqrt{32} + 3\sqrt{180}$

e) $4\sqrt{2} + \sqrt{450}$

f) $7\sqrt{128} - 2\sqrt{200}$

g) $\sqrt{200} - \sqrt{32} + 3\sqrt{18}$

h) $\sqrt{150} + \sqrt{24}$

i) $3\sqrt{32} - 2\sqrt{8}$

Multiplication and Division of Surds

When multiplying or dividing surds, rational or irrational factors are multiplied or divided separately.

Activity 12.3(c)

In groups, discuss the following:

a) find: $ab \times cb$

b) using your result in (a), simplify:

i) $\sqrt{7} \times 2\sqrt{7}$ ii) $2\sqrt{3} \times 4\sqrt{3}$ iii) $5\sqrt{6} \times 7\sqrt{6}$

c) hence, simplify $a\sqrt{p} \times r\sqrt{p}$.

Example 4

Simplify:

a) $\sqrt{12} \times \sqrt{27}$ b) $(3 + \sqrt{2})(4 + \sqrt{2})$ c) $\sqrt{\frac{20}{80}}$

Solution:

a) $\sqrt{12} \times \sqrt{27}$

$= \sqrt{4 \times 3} \times \sqrt{9 \times 3}$

$= 2\sqrt{3} \times 3\sqrt{3}$

$= 2 \times 3 \times \sqrt{3} \times \sqrt{3}$

$= 6\sqrt{9}$

$= 6 \times 3$

$= 18$

b) $(3 + \sqrt{2})(4 + \sqrt{2})$

$= 3(4 + \sqrt{2}) + (\sqrt{2})(4 + \sqrt{2})$

$= 12 + 3\sqrt{2} + 4\sqrt{2} + 2$

$= 14 + 7\sqrt{2}$

$= 7(2 + \sqrt{2})$ (On factorisation)

c) $\sqrt{\frac{20}{80}} = \frac{\sqrt{4 \times 5}}{\sqrt{16 \times 5}}$

$= \frac{\sqrt{4} \times \sqrt{5}}{\sqrt{16} \times \sqrt{5}}$

$= \frac{2 \times \sqrt{5}}{4 \times \sqrt{5}}$

$= \frac{1}{2}$

Exercise 12.2

Simplify the following:

a) $\frac{\sqrt{252}}{\sqrt{150}}$ b) $\sqrt{242} \times \sqrt{98}$ c) $(\sqrt{2} + \sqrt{5})(3\sqrt{5} - \sqrt{2})$

12.4 Rationalisation of Surds

When $\sqrt{4}$ is multiplied by itself, you get 4, that is, $\sqrt{4} \times \sqrt{4} = 2 \times 2 = 4$. So, when a surd is multiplied by itself, the product is a *rational number*. For example, $\sqrt{3} \times \sqrt{3} = 3$. This is what is called ***rationalisation of the surd***. Now, if a fraction has a surd in its denominator, it is the denominator which is rationalised by multiplying both the numerator and the denominator by a surd that makes the denominator rational.

For example, $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{10}}{5}$.

You should note that $\frac{\sqrt{5}}{\sqrt{5}} = 1$, so you are multiplying the fraction by 1. Any number multiplied by 1 is that number. Therefore, the fraction remains the same, but written in another form.

Example 5

Rationalise: a) $\frac{3}{\sqrt{8}}$ b) $\frac{7}{5\sqrt{3}}$

Solution:

a) $\frac{3}{\sqrt{8}} = \frac{3}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{3\sqrt{8}}{8} = \frac{3\sqrt{4 \times 2}}{8} = \frac{3 \times 2 \times \sqrt{2}}{8} = \frac{3\sqrt{2}}{4}$

b) $\frac{7}{5\sqrt{3}} = \frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{15}$

In chapter six, you learnt that:

$(a + b)(a - b) = (a^2 - b^2)$ is called a ***difference of two squares***. If you are to expand $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$ using the difference of two squares, you get:

$(\sqrt{7})^2 - (\sqrt{3})^2 = 7 - 3 = 4$.

Also $(\sqrt{5} - 4)(\sqrt{5} + 4) = (\sqrt{5})^2 - 4^2 = 5 - 16 = {}^-11$.

The pair of Surds $(\sqrt{7} + \sqrt{3})$ and $(\sqrt{7} - \sqrt{3})$ are called *Conjugate Surds*.

Question:

State the conjugate surd of $(4\sqrt{5} - 5\sqrt{3})$.

Hence, show that $(4\sqrt{5} + 5\sqrt{3})(4\sqrt{5} - 5\sqrt{3}) = 5$.

Example 6

Rationalise the following:

a) $\frac{1}{5+\sqrt{3}}$ b) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Solution:

a) $\frac{1}{5+\sqrt{3}} = \frac{1}{(5+\sqrt{3})} \times \frac{(5-\sqrt{3})}{(5-\sqrt{3})} = \frac{5-\sqrt{3}}{5^2-(\sqrt{3})^2} = \frac{5-\sqrt{3}}{22} = \frac{5}{22} - \frac{\sqrt{3}}{22}$

b) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \frac{3+\sqrt{6}+\sqrt{6}+2}{3-2} = 5+2\sqrt{6}$

Exercise 12.3

1. Rationalise the following:

a) $\frac{4-\sqrt{2}}{4+\sqrt{3}}$ b) $\frac{1-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ c) $\frac{1+4\sqrt{5}}{2\sqrt{3}-\sqrt{2}}$ d) $\frac{3}{\sqrt{3}-\sqrt{7}}$

2. Express $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ in the form $p + q\sqrt{r}$, where p, q and r are constants. Hence, find the values of p, q and r.

ICT Activity

Use a word processor to type your solutions to *Exercise 12.1* and email them to your teacher.

Revision Questions:

1. Simplify the following:

a) $\sqrt{96}$ c) $\sqrt{75}$ e) $3\sqrt{40}$

b) $\sqrt{28}$ d) $\sqrt{1000}$ f) $\frac{\sqrt{20}}{\sqrt{80}}$

2. Simplify the following:

a) $\sqrt{125}+\sqrt{145}$ b) $\sqrt{200}-\sqrt{128}$ c) $(\sqrt{3}+\sqrt{5})(2\sqrt{5}-\sqrt{4})$

3. Rationalise the following:

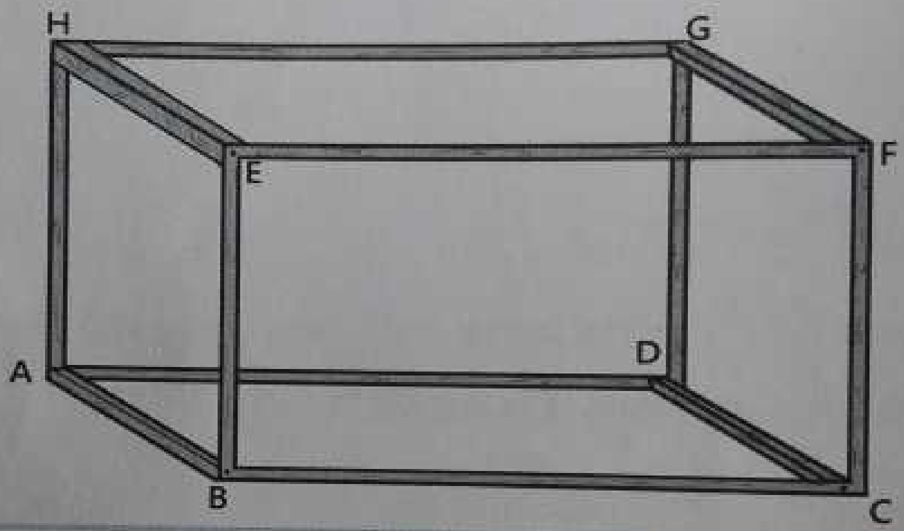
a) $\frac{3+\sqrt{5}}{1+\sqrt{6}}$ b) $\frac{3-\sqrt{2}}{1-\sqrt{3}}$ c) $\frac{2\sqrt{3}+\sqrt{6}}{3-\sqrt{3}}$ d) $\frac{4-\sqrt{7}}{4+\sqrt{7}}$

Sample Activity of Integration

In an attempt to comply with government's guidelines of washing hands regularly with soap to combat the spread of coronavirus, a non-governmental organisation (NGO) intends to provide tanks with their support frames to vulnerable communities. The tank support frames are to be fabricated in the factory and delivered for assembly on site.

Support:

Each of the support frames should be 3 m long, 2 m wide and 1 m high. Sketch of the support frame is as shown below.



Resource:

- Knowledge of measurements
- Knowledge of surds

Task:

If the support frame is to have diagonal metal bars AF, BG, CH and DE to maximise stability, applying your knowledge of surds, advise on the actual engineering lengths of the diagonal metal bars required.

Chapter Summary

In this chapter, you learnt that surds are irrational numbers that cannot be represented accurately as a fraction or recurring decimal, so they are left as roots; for example, $\sqrt{3}$.

For like surds, only numbers outside the root are the ones that are added or subtracted.

THEME: **Data and Probability**

Chapter 13 Set Theory

Keywords

- complement
- disjoint set
- empty set
- intersection
- subset
- union
- universal set
- Venn diagram

By the end of this chapter, you should be able to:

- describe a set and identify its elements.
- identify different types of sets and their symbols.
- determine the number of elements in a set.
- represent and show different operations on sets by shading the different regions on a Venn diagram.
- apply sets in practical situations using two and three sets.

Introduction

In daily life, items are often grouped, and appropriately; for instance, plates are kept separate from bowls and cups. Sets of similar utensils are kept separately. Grouping related items in such a way is an application of *Set Theory*.

This chapter will help you to discover more applications of the knowledge of sets in solving real-life challenges.

13.1 Identification of Set Elements

Activity 13.1(a) (Work in groups)

a) Explore your school environment.
b) Identify items that can be categorised as;
   i) living things
   ii) non-living things
c) Hence, identify items in the set B = {Food stuffs prepared at your school}
d) Present your findings to the rest of the class.

Activity 13.1(b) (Work in groups)

Name the following sets.

A

B

Example 1

A set of all even numbers less than 10 can be written as
{all even numbers less than 10} or {2, 4, 6, 8}.
A set of the first five prime numbers can, similarly, be written as
{the first five prime numbers} or {2, 3, 5, 7, 11}.

- The use of curly brackets is very essential to denote the start and end of a set. For example, a set of vowels is written as {a, e, i, o, u}.
- "Member of" is denoted by $\in$.
- If an object belongs to a particular set, then that object is a member or an element of that set; for example, if set A is such that A = {40, 64, 90, 45}, you can say that "40 is a member of A"; written as $40 \in A$. Also, you can say that "*50* is not an element of A"; written as $50 \notin A$.
- The number of elements in a given set is denoted by *n* ( ). Therefore, *n* (A) stands for "*number of elements in set A.*"
- So, if n (A) = 4, it means that set A contains 4 elements. This is done by counting the number of elements in the given set. For example, the number of vowels, n (V) is 5.

13.2 Description of a Set

A set can be described by:

1. mentioning a condition or rule which all the members of the set satisfy or obey. For example, A = {set of prime numbers}. This implies that set A is comprised of all prime numbers.
2. listing the elements it contains; for example, A = {2, 3, 5, 7,...}.

Example 2

Give the conditions that the following sets fully satisfy.

a) {w, i, r, e, d}

b) {3, 6, 9, 12}

Solution:

a) The letters that form the word "wired" or "weird".

b) The first four multiples of three.

Example 3

List the members of the following sets.

a)The first six letters of the English alphabet.

b)The integers between 2 and 11.

Solution:

a) {a, b, c, d, e, f}

b) {3, 4, 5, 6, 7, 8, 9, 10}

Exercise 13.1

1. a) Give examples of objects that could be grouped into a set.
   b) For each of the sets you have mentioned above, mention the condition that all the members in the set must satisfy.
2. State the sets to which the following items belong.
   a) Pajero, Ipsum, Corona, Audi, BMW
   b) The Nile, Aswa, River Senegal
   c) Tanzania, Rwanda, Uganda, Kenya
   d) Queen Elizabeth, Kidepo, Murchison falls
   e) Kigali, Juba, Gitega, Dar es salaam, Kampala

13.3 Types of Sets and their Symbols

Activity 13.3(a) **(Work in groups)**

a) Identify any five sets in your school environment.
b) Count and record the number of elements in each of the sets in (a).
c) What do you conclude about the sets in (a)?
d) Present your work to the rest of the class.

Finite Set

This is a set with an exhaustively countable number of elements; for example,

a) A = {1, 2, 3, 4, 5, 7, 8, 9}
b) E = {Uganda, Kenya, Tanzania, Rwanda, Burundi, South Sudan}

Infinite Set

A set with an uncountable number of elements is an infinite set. Its elements cannot be listed to completeness. For example, a set of whole numbers.

Equivalent Sets

Two or more sets are equivalent if they have the same number of elements.

Activity 13.3(b)

Given the sets: X = {a, e, i, o, u} and Y = {2, 4, 6, 8, 10};

a) count the number of elements in set X.
b) count the number of elements in set Y.
c) what are your observations?
d) what relationship exists between sets X and Y?

Equal Sets

Two sets, A and B, are equal if every element in set A is also an element in B and every element in set B is also an element in A. For example, sets A = {1, 2, 3} and B = {3, 2, 1} are equal sets, written as A = B.

Empty Sets

Activity 13.3(c) (Work in groups)

a) Identify elements in the following sets.
   i) The set of dogs with six legs
   ii) The set of integers between 1 and 10
   iii) The set of squares with 5 sides
   iv) A set of domestic animals
   v) A set of three-headed animals

b) What are your observations?

> Note: An empty set is one which does not have any element. It is denoted by { } or ∅, but not {∅}.

Disjoint Sets

Activity 13.3(d) (Work in groups)

Given the following pairs of sets;

i) A = {all natural numbers} and B = {all the letters in the English alphabet}
ii) G = {odd numbers less than 20} and H = {prime numbers less than 20}
iii) E = {all positive integers} and F = {all negative integers}

a) list down the members in each pair of sets.
b) does any pair of sets have elements in common?
c) what do you conclude about each pair of sets?
d) hence, write down your own definition of disjoint sets.
e) present your work to the rest of the class.

Subsets

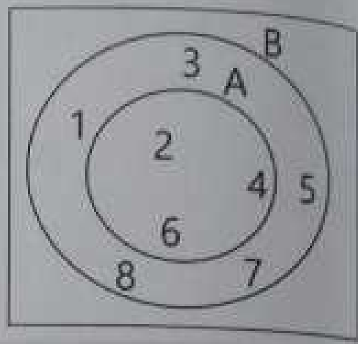
Activity 13.3(e) (Work in groups)

a) List down all possible subsets that can be formed from the set Z = {2, 5, 8}.
b) What do you conclude about the subsets listed in (a)?

> Note: An empty set is always a subset of any given set and the given set is also a subset of itself.

Set A is a subset of set B if every element of A is also an element of B. It is denoted by "$\subset$". That is, $A \subset B$.
$A \subset B$ is read as "A is a subset of B".
For example, if $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then $A \subset B$.

The following Venn diagram illustrates $A \subset B$.



Exercise 13.2

1. Copy and complete the given table.

	Set A	Number of members in set A	List of subsets of set A	Number of subsets of set A
a)	{1}			
b)	{2, 3}			
c)	{3, 4, 5}			

2. Given that n (Q) = 4, state the number of subsets of set Q.

Intersection of Sets, denoted by "$\cap$"

The intersection of sets is that set which consists of all members which are common to the given sets.

Activity 13.3(f) (Work in groups)

a) Form two sub groups.
b) Let each sub group explore the school environment and identify items they can use to form a set.
c) Let each sub group list down the items of the set formed in (b).
d) Compare the sets formed by the two sub groups.
e) What is your observation?
f) Hence, given that set A = {a, b, c, d, e} and set B = {f, g, d, a, h}, identify the member(s) of the intersection of the two sets.

Example 4

Given the sets $C = \{1, 3, 4, 6, 7\}$ and $D = \{2, 3, 5, 7, 9\}$, list all the members of $C \cap D$.

Solution:

$C \cap D = \{3, 7\}$

Union of Sets, Denoted by "∪"

The union of sets is that set comprising of all elements which belong to either sets without repetition of any element.

Activity 13.3(g) (Work in groups)

a) Let each group member list down their favourite food items.
b) Using the lists made in (a), form a set E.
c) Hence, given that A = {a, b, c, d, e} and B = {f, g, d, a, h}: Write down the elements of $A \cup B$.

Universal Set (ε)

This is the superset, because it contains all the members of the sets under consideration. For example, given A = {even numbers less than 15}, B = {odd numbers less than 15} and C = {whole numbers less than 15}, sets A and B are subsets of set C. Therefore, C is the universal set here.

The Complement of a Given Set

Activity 13.3(h) (Work in groups)

a) Pick a mathematical set.
b) Open it and identify all items.
c) Form two sets: A = {items that can be used for geometrical construction} and B = {all items identified in (b) that do not belong to set A}.
d) Comment on the relationship between sets A and B.
e) Hence, given that ε = {whole numbers less than 10} and A = {2, 3, 5, 8}, state elements of the complement of set A.

The complement of a set is a set consisting of all elements in the universal set that are not in the given set.
The complement of A is denoted as A′.

13.4 Determining the number of elements in a Set

The number of elements in a given set, say, A is denoted by n (A). This is found out by simply counting the number of elements in the given set.

Activity 13.4

Given that A = {2, 4, 6, 8}, B = {m, a, t, h, e, m, a, t, i, c, s} and C = {a, e, i, o, u}, find;

a) n (A)
b) n (B)
c) n (C)
d) $n(B \cap C)$
e) $n(B \cup C)$
f) $n(A \cap B)$

Exercise 13.3

1. Describe the sets to which the following belong.
   a) {4, 8, 12, 16, 20, 24} b) {w, a, r}
   c) {January, June, July} d) {a, b, c, d, e, f, g, h}
2. List the members belonging to the following sets.
   a) {the colours of your school uniform}
   b) {all odd numbers between 3 and 25}
   c) {all prime numbers between 10 and 36}
   d) {the colours of your school flag}
3. Using the symbol '$\in$', describe three sets to which you belong at your school.
4. For the following sets, write down the members and state the number of members in them.
   a) {all vowels}
   b) {all prime numbers less than 21}
   c) {all even numbers between 25 and 47}
5. Find the union and intersection of the following sets. Use the set notation to write the answer.
   a) A = {m, a, k, e, r } and B = {e, r, u, n}
   b) P = {2, 3, 5, 7, 11 } and Q = {1, 3, 5, 9, 13}
   c) D = {u, n, i, v, e, r, s, t, y} and E = { c, h, a, l, e, n, g}
   d) V = {vowels} and A = {first ten letters of the English alphabet}

13.5 Venn Diagrams

A ***Venn diagram*** is an illustrative diagram which helps to clearly spell out the relationship between given sets. In it, each set is represented by an enclosed shape. Supposing you take the responses of learners in a class about the games they like watching, namely, football and basketball, you would have F = {all learners who like watching football} and B = {all learners who like watching basketball}. These can be represented using Venn diagrams in three ways, as shown in the figures below.

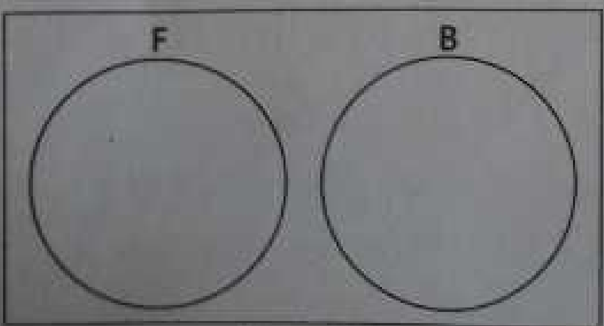


Figure 13.1

In *Figure 13.1*, the sets are apart, meaning that no learner likes watching both games. We say that sets F and B are disjoint sets, that is,

$F \cap B = \{\ \}$ or $\varnothing$. So, $n(F \cap B) = 0$, and
$n(F \cup B) = n(F) + n(B)$.

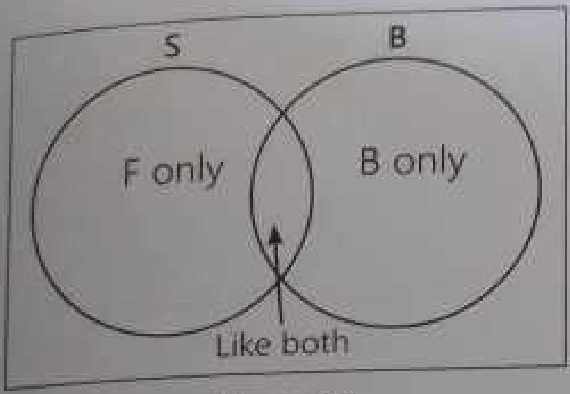


Figure 13.2

In *Figure 13.2*, some learners like to watch both football and basketball, so they belong to the intersection.

$n(F \cup B) = n(F) + n(B) - n(F \cap B)$.

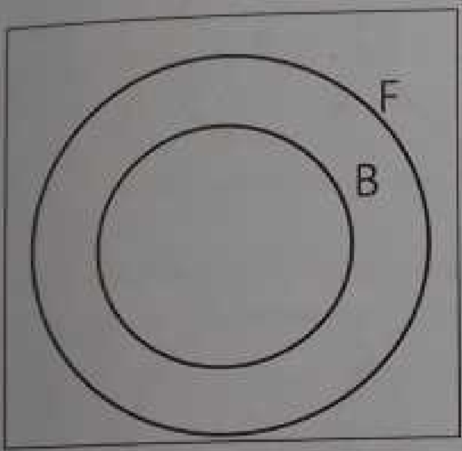


Figure 13.3

In *Figure 13.3*, you see that B is wholly within F. This means that all those who like watching basketball also like watching soccer. You say that B is a subset of F, written as $\boldsymbol{B \subset F}$.

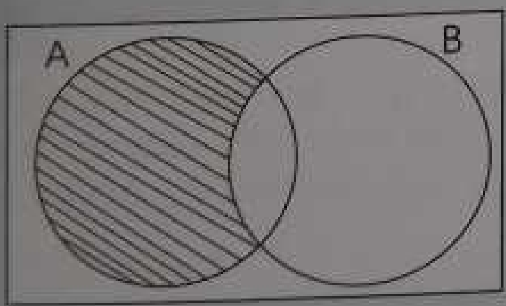
The outer enclosure forms what is called the universal set, which contains all members in the class. It is denoted by $\mathcal{E}$.

$F \cap B = B$, $n(F \cap B) = n(B)$, $F \cup B = F$, and $n(F \cup B) = n(F)$.

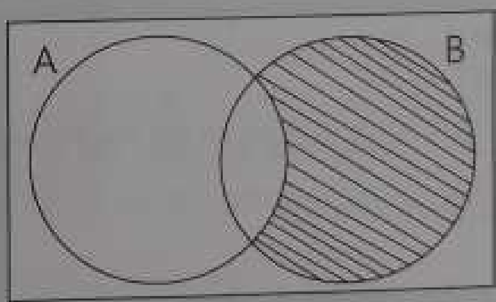
Activity 13.5(a)

In groups, using set symbols, describe the shaded regions.

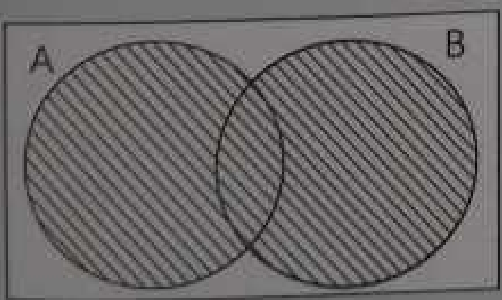
a)



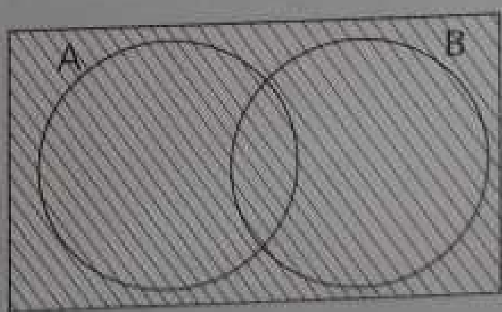
b)



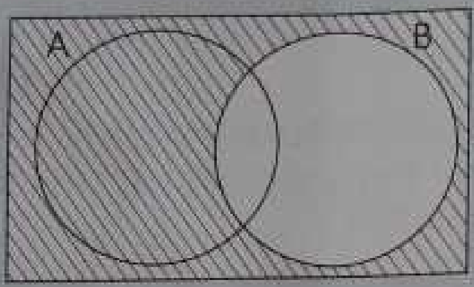
c)



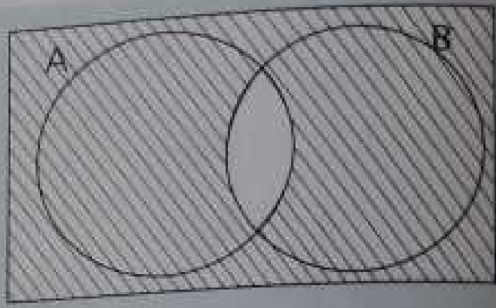
d)



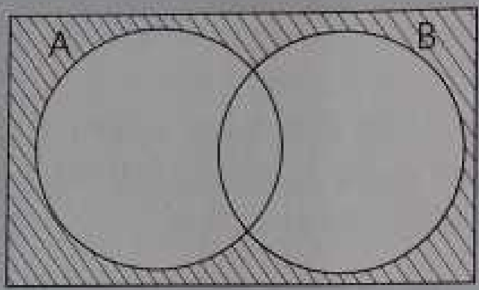
e)



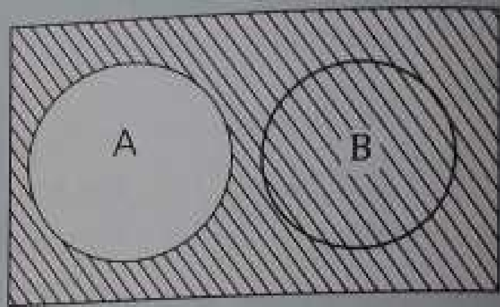
f)



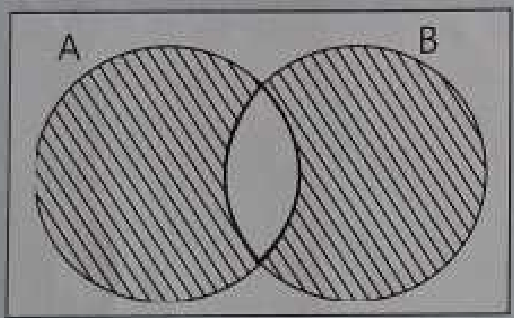
g)



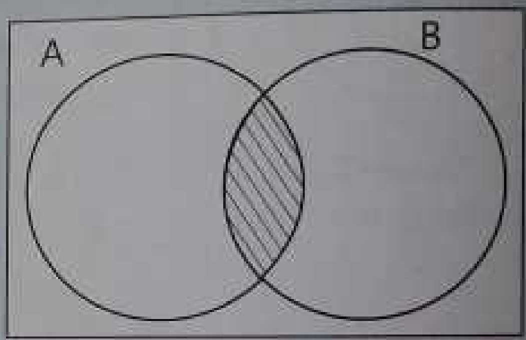
h)



i)



j)



Example 5

Draw a Venn diagram to represent set A = {1, 2, 3, 4}.

Solution:

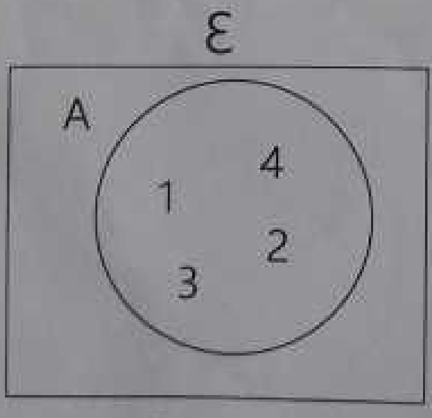


Figure 13.4

Example 6

Given that set A = {all even numbers between 5 and 20} and B = {all multiples of 3 between 2 and 20}, find $A \cap B$ and show this on a Venn diagram.

Solution:

Listing the members found in each of the sets, you obtain;

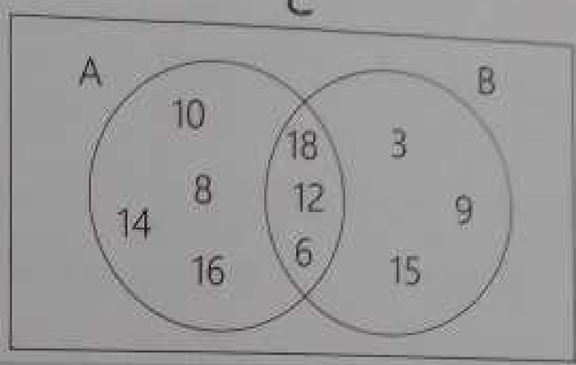
A = {6, 8, 10, 12, 14, 16, 18}

B = {3, 6, 9, 12, 15, 18}

Identifying the common members in A and B, you have $A \cap B$ = {6, 12, 18}.

Representation on the Venn diagram:

ℇ



Activity 13.5(b) (Work in groups)

a) Explore your classroom environment.
b) Identify items you can use to form two sets.
c) Count the items in each set formed in (b).
d) Represent your information in (c) on a Venn diagram.
e) Present your work to the rest of the class.

Example 7

At a class party, learners were asked what drink each preferred; Coca-Cola (C) or Fanta (F). 15 preferred both, while 35 preferred Coca-Cola. Each learner preferred at least a drink. If the class has 45 learners, find how many preferred Fanta.

Solution:

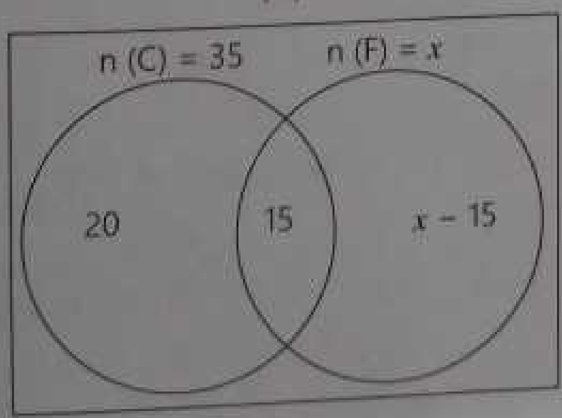
$n(C) = 35$

$n(C \cap F) = 15$

$n(F) = x$

$n(\mathcal{E}) = 45$

$n(\mathcal{E}) = 45$



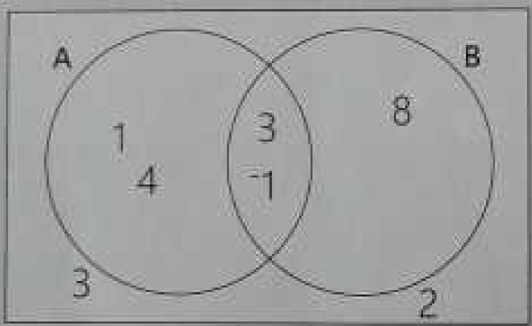
$$20 + 15 + x - 15 = 45$$
$$35 + x - 15 = 45$$
$$x + 20 = 45$$
$$x + 20 - 20 = 45 - 20$$
$$x = 45 - 20$$
$$x = 25$$

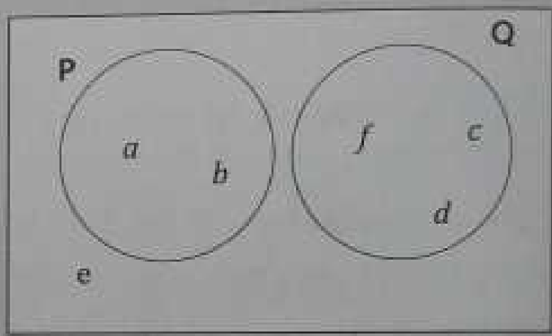
Therefore, those who preferred Fanta were 25.

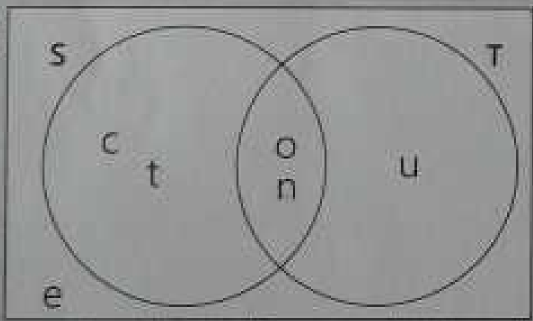
Exercise 13.4

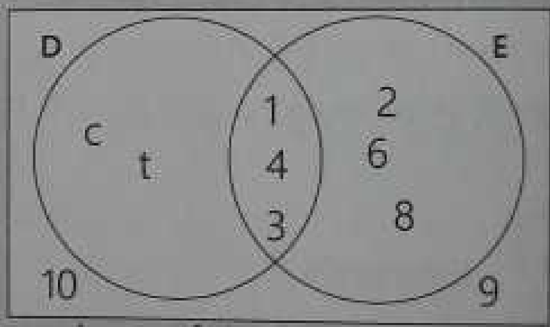
1. Draw Venn diagrams to represent the following pairs of sets and from the Venn diagrams, list the members of the union and intersection of the sets.
   a) $A = \{^-1, 2, ^-3, 4\}$ and $B = \{6, ^-3, 4, 8, 1\}$
   b) $D = \{f, g, h, i, j\}$ and $E = \{s, k, i, n, g\}$
   c) $P = \{f, o, r, c, e\}$ and $Q = \{m, o, e, n, t\}$
   d) $S = \{r, o, t, a, e\}$ and $T = \{t, u, r, n\}$

2. From the following Venn diagrams, list the members of;
   a) each set   b) the intersection of the sets   c) the union of the sets

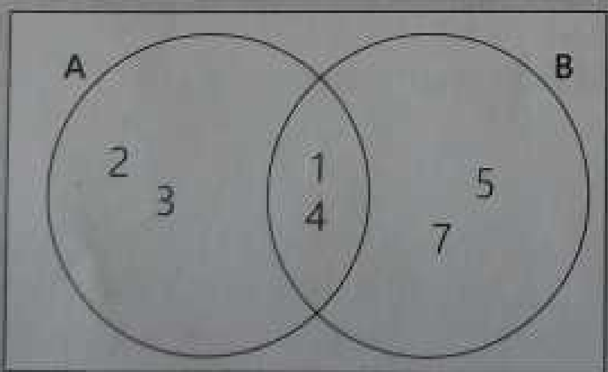
i)


ii)


iii)


iv)


3. From the Venn diagram below, list the members of the following sets.



a) A'
b) B'
c) $(A \cap B)'$
d) $(A \cup B)'$
e) $A \cup B'$
f) $A' \cap B$
g) $A' \cap B'$
h) $A' \cup B$

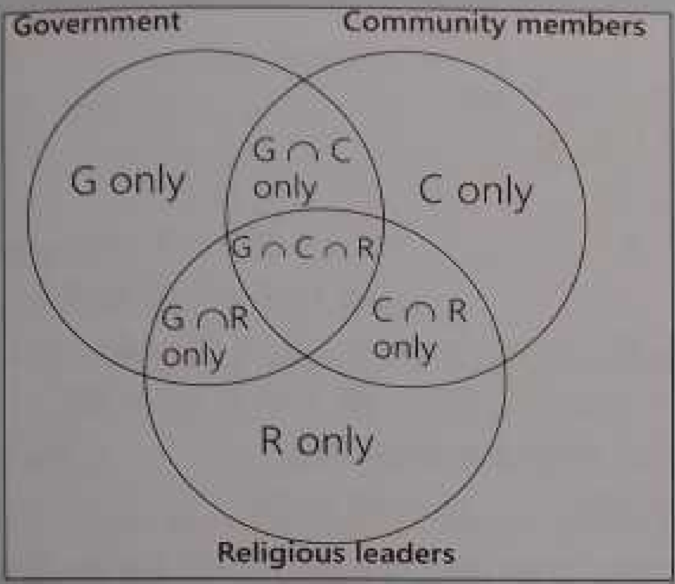
4. Given that $n(A) = 10$, $n(B) = 20$, $n(\varepsilon) = 30$ and $n(A \cap B) = 5$, find;
   a) $n(A \cap B')$ b) $n(A' \cap B)$ c) $n(A \cup B)'$

13.6 Applying Sets in Practical Situations using Three Sets

Project Work (Work in groups)

In your community, there are 3 key stakeholders involved in its development, namely; government (G), community members(C) and the religious leaders (R). These groups have roles that can be represented on a Venn diagram.

Roles of stakeholders



Tasks:

a) List the roles for each of the stake holders.
b) List any 2 roles that are common to:
   i) government and community.
   ii) government and religious leader.
   iii) community and religious leaders.
   iv) all the three stakeholders.
c) Identify any 2 roles that are particular to each stakeholder.
d) Represent the above information on a Venn diagram.

Activity 13.6(a) (Work in groups)

Your school community is comprised of learners (L), teachers (T) and non-teaching staff (N). Each group faces different challenges. However, there are challenges that are common to two or all the groups.

Tasks:
(a) List the challenges faced by each of the above groups.
(b) Represent the challenges listed in a) on a Venn diagram.

Activity 13.6(b) (Work in groups)

A number of communities are often threatened by malaria, cholera, and typhoid. A survey was conducted by health workers in these communities and they found out that 50, 50 and 40 respondents were more worried about malaria, cholera and typhoid, respectively. 10 respondents were worried about all the three diseases, 15 about malaria and cholera, 20 about cholera and typhoid, and 15 about malaria and typhoid.

a) Using a Venn diagram, determine the number of respondents who were worried about;
   i) malaria only   ii) cholera only   iii) typhoid only
b) Find the total number of respondents in the survey.
c) Present your work to the rest of the class.

ICT Activity (Work in groups)

a) Using the knowledge of inserting objects you studied in ICT, draw *Figure 13.4* under *Example 5* using a word processor program.

b) The internet is very essential in carrying out research in Set Theory. Use it to read more about this chapter. Discuss the safety practices you need to observe while using the internet.

Revision Questions:

1. Answer True or False. Given that sets A and B are:
   A = {10, 15, 14, 12, 6, 7, 9} and B = {2, 14, 6, 9}.
   a) $14 \in A$
   b) $A \cap B = \{9, 0\}$
   c) $A \cup B = B \cup A$
   d) $B \in A$
   e) $n(B) = n(A)$
   f) $A = A \cap B$
   g) $\{9\} \subset B$
   h) $A \cup B = \varnothing$
   i) $\{14\} \subset A$
   j) $n(A) = 8$
   k) $n(A \cup B) \subset A$
   l) $A \cap B' = A$
2. Given that $n(A' \cap B') = 4$, $n(B' \cap A) = 5$ and $n(A' \cap B) = 6$, $n(\varepsilon) = 24$, represent the information using a Venn diagram and find:
   a) $n(A \cap B)$ b) $n(A)$ c) $n(B)$ d) $n(A \cap B)'$
3. During a Commonwealth Heads of States' meeting, 53 Heads of State visited Uganda. The following data was collected: 40 visited the source of the Nile (N) and 30 visited Queen Elizabeth National Park (E). It was discovered that 8 visited none of the sites. Represent the information on a Venn diagram. How many Heads of State visited both sites?
4. 40 learners were asked what they know about Obama Barack and their responses were recorded. 11 responded that he was a basketball player (B), 29 said that he was a politician (P) and 5 did not know anything about him. Represent the information on a Venn diagram and determine:
   a) $n(B \cup P)$ b) $n(P' \cap B)$
5. In a class of 80 learners, 35 like Mathematics, 50 like Physics and 11 like both Mathematics and Physics. Find the number of learners who;
   a) like none of the 2 subjects
   b) do not like mathematics
   c) do not like physics
   d) like only one subject
6. Given two sets such that $n(H \cap F') = 21$, $n(H \cap F) = 15$, $n(H \cup F)' = 13$ and $n(\varepsilon) = 95$, represent the information on a Venn diagram and use it to find
   a) $n(H' \cap F)$ b) $n(F)$ c) $n(H)$
7. Given that ε = {natural numbers less than 14},
   A = {Odd numbers less than 14},
   B = {Even numbers less than 14} and C = {Prime numbers less than 14}.
   list all the elements in set A, B, and C, and represent all the sets on the same Venn diagram.

Sample Activity of Integration

A group of donors has visited your school and you have been selected to be part of the beneficiaries. The donors have the capacity to fully facilitate your welfare at school but they are very strict about honesty.

Support:

- Learners' welfare includes;
  - i) items a learner needs to attend classes
  - ii) items required in the dormitory
  - iii) items required during leisure time

Resources:

- Knowledge of Set Theory
- Knowledge of mathematical operations

Tasks:

Using your knowledge of Venn diagrams, present your list of requirements for consideration.

Chapter Summary

In this chapter, you have learnt about different types of sets, their representations and symbols. Some of these set symbols are:

- { } - Empty set
- $\in$ - is a member of
- ε - Universal set
- $\cup$ - Union
- $\cap$ - Intersection

Two sets A and B can be represented using a Venn diagram as shown below: