Physics

for Rwanda Secondary Schools

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Learner's Book 4

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Contents

Introduction

Changes in schools

This text book is part of the reform of the school curriculum in Rwanda: that is changes in what is taught in schools and how it is taught. It is hoped this will make what you learn in school useful to you when you leave school, whatever you do then.

In the past, the main thing in schooling has been to learn knowledge – that is facts and ideas about each subject. Now the main idea is that you should be able to use the knowledge you learn by developing **skills** or **competencies**. These skills or competencies include the ability to think for yourself, to be able to communicate with others and explain what you have learnt, and to be creative, that is developing your own ideas, not just following those of the teacher and the text book. You should also be able to find out information and ideas for yourself, rather than just relying on what the teacher or text book tells you.

Activity-based learning

This means that this book has a variety of **activities** for you to do, as well as information for you to read. These activities present you with material or things to do which will help you to learn things and find out things for yourself. You already have a lot of knowledge and ideas based on the experiences you have had and your life within your own community. Some of the activities, therefore, ask you to think about the knowledge and ideas you already have.

In using this book, therefore, it is essential that you **do all the activities.** You will not learn properly unless you do these activities. They are the most important part of the book.

In some ways this makes learning more of a challenge. It is more difficult to think for yourself than to copy what the teacher tells you. But if you take up this challenge you will become a better person and become more successful in your life.

Group work

You can also learn a lot from other people in your class. If you have a problem it can often be solved by discussing it with others. Many of the activities in the book, therefore, involve discussion or other activities in groups or pairs. Your teacher will help to organise these groups and may arrange the classroom so you are always sitting in groups facing each other. You cannot discuss properly unless you are facing each other.

Research

One of the objectives of the new curriculum is to help you find things out for yourself. Some activities, therefore, ask you to do research using books in the library, the internet if your school has this, or other sources such as newspapers and magazines. This means you will develop the skills of learning for yourself when you leave school. Your teacher will help you if your school does not have a good library or internet.

Icons

To guide you, each activity in the book is marked by a symbol or **icon** to show you what kind of activity it is. The icons are as follows:

Thinking Activity

Thinking Activity icon

This indicates thinking for yourself or in groups. You are expected to use your own knowledge or experience, or think about what you read in the book, and answer questions for yourself .

Practical Activity

Practical Activity icon

The hand indicates a practical activity, such as a role play on resolving a conflict, taking part in a debate or following instructions on a map. These activities will help you to learn practical skills which you can use when you leave school.

Writing/Research Activity

Writing Activity icon

Some activities require you to write in your exercise book or elsewhere.

Group Activity

Group Work Activity icon

Group work means that you are expected to discuss something in groups and report back on what your group discussed. In this way you learn from each other and how to work together as a group to address or solve a problem.

Fieldwork Activity

Fieldwork Activity icon

Field work is an enjoyable and practical part of Social Studies. For these activities, you will need to go out of the classroom to study parts of your environment, such as the way that rivers flow, or the distance between landmarks in your school grounds or any other tours in relation to the activity.

Discussion/Vocabulary Reading

Discussion Activity icon

Some activities require you to discuss an issue with a partner or as part of a group. It is similar to group work, but usually does not require any writing, although some short notes can be written for remembrance..

Computer/Internet Activity

Computer/Internet Activity icon

Some activities require you to use a computer in your computer laboratory or elsewhere.

Pair Activity

Pairing Activity icon

This means you are required to do the activities in pairs and exchange ideas

Listening Activity

Listening Activity icon

The listening activity requires learners to carefully listen to the teacher or fellow learner reading a passage, poem or extraction on the subject and then answer the questions

Observation Activity

Observation Activity icon

Learners are expected to observe and write down the results from activities including experiments or social settings overtime.

Good luck in using the book.

Content Map

Unit	No of periods	Topics	How will I do it?	Materials/ equipment I will use	
1	24	Thin lenses	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	• Convex and concave lenses. • Eye glasses. • Water. • Glass prism. • Optical pins. • Torch bulbs. • White sheets of papers. • ruler.	
2	18	Simple and optical instruments.	• Observation. • Group discussions and presentations. • Visiting places with television dishes. • Practical activities like taking pictures. • Making the instruments using local materials. • Visiting the neighbouring video halls to look at the working of a projector.	• Convex lens, concave lens. • Hand lens. • Lens camera. • Microscope. • Eye glasses. • A small box. • Charts. • Dirty water, prepared slides.	

	Activities I will do	Skills I will practice	What I will learn
	• Working in groups, observe the different types of lenses and describe their characteristics. • Carry out experiments to determine the characteristics of images formed by lenses for an object placed at different positions in front of the lens. • Construct accurate ray diagrams for images formed by lenses. • Carry out experiments to determine the focal length of a convex lens. • Use the lenses to observe some parts of organisms. • Carry out experiments to verify the laws of refraction of light. • Use a ruler and later a prism to observe the dispersion of light and chromatic aberration.	• Observing and examining different organisms using lenses. • Measuring different distances. • Carrying out experiments. • Drawing graphs on a scale. • Using internet to search. • Making comparison using physical features. • Deriving different formulae.	• How to view different objects using lenses for example small organisms. • How to locate the position of an image formed in a lens. • How to determine the focal length of the lens. • How to determine magnification of the lens. • How to correct defects of lenses .
	• Observe the fur on the skin using a hand lens. • Do the same by moving the convex lens up and down above the part of the body to be observed. • Make a pin hole camera and convert it into a lens camera by putting a drop of water on the pin hole. • Use a lens camera to take some pictures. • Discuss in groups the working of a slide projector. • Discuss in groups how a convex lens can be used as a lens camera, a simple microscope and a projector. • Design a compound microscope using two convex lenses. • Discuss in groups the characteristics of a compound microscope and make a presentation. • Use a compound microscope to view some small organisms in dirty water. • Visit the neighbouring place with a television dish and compare it with a reflecting telescope. • Discuss in groups the functioning of prism binoculars and the situations when they are applicable.	• Using a simple microscope. • lens camera, slide projector, telescopes and prism binoculars. • Presenting findings from group discussions. • Accurate measurements. • Using internet to search information. • Making comparison using physical features and from the uses of instruments. • Deriving different related formulae.	• How to view very small organisms using a hand lens(magnifying glass). • How to examine micro organisms using a compound microscope. • How to determine the magnifying power of an instrument. • How to correct the eye defects. • How to take photographs using a lens camera. • How a slide projector forms an image. • How telescopes and prism binoculars focus distant objects.

3	19	Moment and equilibrium of forces	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Metre ruler, ropes, lamina, knife edges, sea saw.	
4	19	Work, energy and power	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Balls, pendulum bob, spiral spring, chairs, tables.	

	• Make a local sea saw using a log and swing with a friend. • Balance a meter rule on a knife edge. • Discuss with friends how ladders can assist people to move up a house. • Carry out an experiment to find the centre of gravity of a body using a lamina. • Discuss in groups the difference between vector and scalar quantities. • Carry out an experiment to demonstrate equilibrium of a system.	• Making a sea saw. • Carrying out experiments to locate centre of gravity. • Team work and presentation of data from group discussions. • Solving problems involving moments and equilibrium of bodies.	• How to locate the center of gravity of an object. • How to find the resultant of two or more forces. • How to solve problems involving vectors and scalars. • How to solve problems involving equilibrium of forces.
	• Discuss in groups where the food that we eat goes. • Try to carry a bench alone and then with a friend and note the difference. • Discuss in groups the difference between work, energy and power. • Swing a pendulum bob and discuss in groups the different forms of energy it possesses at the different positions. • Make two balls to collide and observe. Then discuss in groups the different types of collisions. • Discuss in groups the impact of collisions on bodies.	• Presentation of findings. • Observation . • Comparison • Deriving expressions for kinetic and potential energy. • Calculation of potential energy and kinetic energy. • Deriving an expression for conservation of momentum.	• How to define work energy and power. • How to derive equations for potential energy and kinetic energy. • How to derive work energy theorem. • How to find the work done in deforming objects. • The difference between the two types of collisions and the impact of collision on objects. • How to solve problems related to energy conservation and conservation of linear momentum.

5	20	Kirchhoff's laws and electric circuits	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Ammeter, voltmeter, cells, holders, bulbs, cells, standard resistors, rheostat, connecting wires, galvanometer.	
6	20	Sources of energy in the world.	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Solar panels, cells, wind hawks/ mills, Fans	

	• Look at the ammeter and voltmeters provided, identify their main features and discuss their uses. • Make a simple circuit comprising of two cells, a bulb, a switch, an ammeter and a voltmeter. • Use a voltmeter to measure the terminal potential difference and compare it with the electromotive force of the cell. • Construct a circuit consisting of the battery and resistors and use it to verify Kirchhoff's laws. • Observe the electrical appliances at home and school, discuss their uses with your friends and how they are used.	• Reading an ammeter and a voltmeter. • Making a simple circuit. • Present data from a given experiment. • Manipulating apparatus for carrying out an experiment and evaluate experimental procedures.	• How to use an ammeter and a voltmeter to measure terminal potential difference and current through a circuit. • How to construct a simple circuit. • How to describe the difference between electromotive force and potential difference. • How to determine the effective resistance of resistors in series and parallel arrangement; and their advantages and disadvantages. • How to solve circuit problems using kirchhoff's laws.
	• Visiting the nearby home with solar power to observe and discuss how solar energy is generated. • Observe a fun and relate it with a wind mill. • Visit hydro electric power generation dam and observe how they operate. • Visit a nearby place where biogas is generated and discuss in groups with the help of the person generating it, how it is generated, and to what extent it can be used.	• Generating biogas from cow dung. • Extraction and generation of energy. • Generating wind energy. • Searching from internet.	• How to generate hydro electric power. • How to generate biogas. • How to generate wind energy. • How to differentiate between renewable sources and non renewable sources of energy. • Advantages and disadvantages of different energy sources.

7	20	Energy degradation (dilapidation) and power generation	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Horse shoe magnet, compass needle, Charts.	
8	20	Projectile motion and circular motion	• Group work and presentations. • Experimentation. • Practical activities. • Class outings.	Stones, balls, chalk, bicycles, conical pendulum, shot put stone, simple pendulum bob, Spinning drier.	

	• Carry out an experiment to demonstrate the motion of a compass needle when brought near a horse shoe magnet. • Discuss in groups and present on mechanisms of electrical energy production. • Discuss as a group an investigation about conversion of thermal energy into work.	• Presentation of findings from the group discussions. • Discussing in groups. • Searching for work from internet.	• How to define energy degradation/ dilapidation. • Explain mechanisms of electrical energy production. • How to convert thermal energy into work by single cyclic processes. • How to draw energy diagrams illustrating energy degradation.
	• Throw a stone vertically upwards and then at an angle, discuss the characteristics of the motions of the two. • Use the equation of linear motion to determine the horizontal and vertical velocities and displacements of projectile motion. • Tie a bob on a thread and swing it on a horizontal circle and then in a vertical circle. Release the bob and observe. • Discuss in groups and make a presentation on the relationship between linear and angular quantities. • Discuss in groups why a bicycle rider bends inwards when he/she is negotiating a corner. • Work in groups to solve problems in circular motion.	• Drawing velocity and displacement time graphs for projectile motion. • Differentiating a graph from a path taken by an object. • Deriving equations. • Applying the subject matter in daily life.	• How to use equations of linear motion to determine equations for projectile motion. • How to determine equations for time of flight, range and maximum height. • How to determine the equations for circular motion parameters. • Differentiating the motion in a horizontal circle and a vertical circle.

9	20	Universal gravitation field potential	• Observations. • Discussions. • Practical activities/ investigations. • Field work.	Globe, conical pendulum, Charts, books	
10	24	Effects of electric and potential fields.	• Discussions. • Practical activities/ investigations. • Class outings.	Ebonite rod, glass rod, a Bic pen, small pieces of papers, lightening conductor	

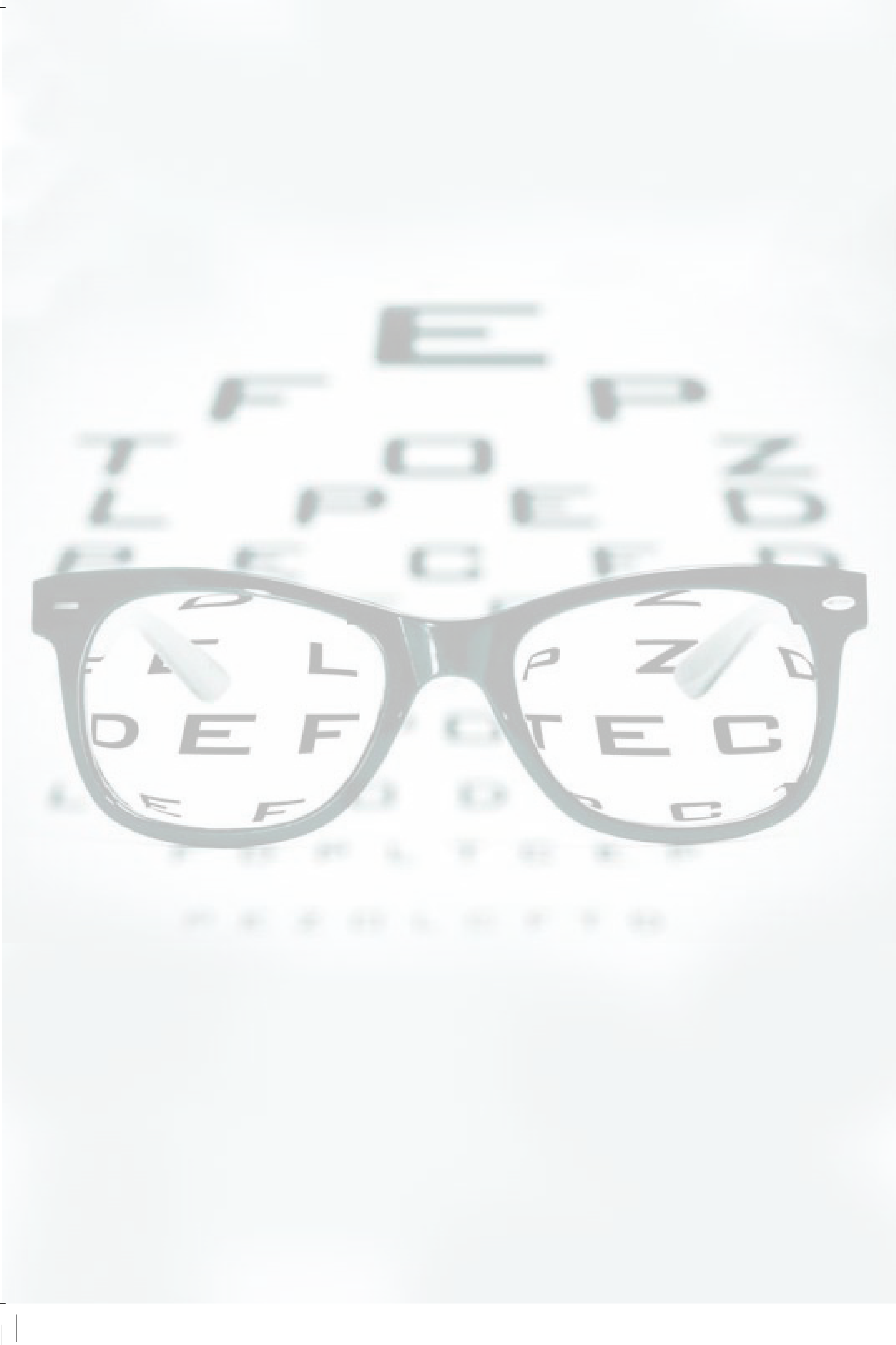
	• Discuss in groups what causes days and nights. • Discuss in groups what causes seasons in a year. • Discuss the law of universal gravitation. • Solve problems involving the law of universal gravitation. • In groups, discuss how world wide communication is achieved with the help of satellites.	• Analysis of information from internet. • Using the globe to describe the motion of bodies. • Using a conical pendulum to differentiate gravitational constant from force due to gravity.	• How to state newton's law of universal gravitation. • How to state kepler's law of planetary motion. • How to determine the relationship between universal gravitational constant and force of gravity. • How world wide communication is achieved by use of artificial satellites. • How to derive an expression for gravitational potential and solve problems on gravitational potential.
	• Rub a Bic pen with hair and use it to attract small pieces of papers; discuss why attraction occurs. • Perform an experiment to illustrate electric fields due to a point charge. • Perform an experiment to illustrate an electric field due to two parallel plates. • Visit a nearby tall building with a lightening arrestor and observe its features, and then discuss in groups how it works.	• Using simple objects to illustrate the existence of charge on a body. • Observing a lightening conductor and describe its functioning thereafter. • Deriving the relation between potential and field intensity.	• How to identify charge on an object. • How to state coulombs law. • How to show field lines due to different charged objects. • How the lightening arrestor is used to safe guard tall buildings from being struck by lightening. • How to derive an expression for electric potential and field intensity. • Establish the relationship between potential and field intensity.

11	24	Application of thermodynamic laws	• Discussions. • Practical activities/ investigations. • Field work.	Polythene bag, balloon, bicycle pump, sauce pan, syringe, refrigerator, plastic cup, thermometer.	
12	20	General structure of the solar system	• Discussions. • Practical activities/ investigations. • Observation.	Globe, telescopes and binoculars.	

	• Inflate a balloon and leave it to move up during a sunny day. • Discuss in groups why a balloon or a bicycle tube bursts when left in sunshine for long. • Discuss in groups why a loose sauce pan cover goes off during the cooking and relate it with the first law of thermodynamics. • Work in groups to investigate energy changes and work done for a thermodynamic process and present the findings. • Use a syringe and discuss changes the volume changes at constant pressure. • Discuss and derive the expressions for the work done for the different thermodynamic changes. • Visit the nearby garage to study the working of the diesel engine and the petrol engine. • Discuss in groups the operation of both petrol and diesel engines and present your findings. • Visit a nearby place and observe the working of a refrigerator. • Discus in groups the negative effects of heat engines on the environment and how the dangers can be minimised.	• Preparing for designing an experiment. • Differentiating different thermodynamic processes. • Interpreting observed circumstances. • Applying thermodynamic systems in daily life.	• How to state the first law of thermodynamics. • How to apply the law of thermodynamics to describe the gas changes; isochoric, isothermal, isobaric and idiabatic changes. • How to define the second law of thermodynamics and apply it, explain the principle of carnot engine, diesel engine and a refrigerator. • Determine the impact of heat engine on climate.
	• Work in groups and carry out an experiment to determine acceleration due to gravity on the earth's surface. • Use a telescope to observe the stars and planets. • Work in groups and discuss the number of planets in the universe and the characteristics of each. • Work in groups to discuss kepler's law of planetary motion.	• Using telescopes and binoculars to observe planets and stars. • Working in groups(team work). • Searching internet. • Illustrating the phenomenon of eclipse.	• How to describe the solar system. • How to apply kepler's laws to describe the motion of planets. • How to distinguish a planet from a star. • How to define and apply celestrial coordinates. • Explain the existence of constellations.

LIGHT

Thin Lenses



Unit 1

Thin lenses

Key unit Competence

By the end of this unit, the learner should be able to explain the properties of lenses and image formation by lenses.

My goals

By the end of this unit, I will be able to:

* explain physical features of thin lenses
* state the types of lenses and explain their properties
* differentiate between lenses and curved mirrors
* explain the phenomenon of refraction of light by lenses
* construct the ray diagrams for formation of images by lenses
* explain the defects of lenses and how they can be corrected
* describe the daily applications of lenses

Introduction

Observe and think

Look at yourself in a flat mirror and choose one of the following that identifies your observation;

a) my image is clearly seen without changes.

b) my image shows some changes.

What do you think

a) What do you think about formation of your image by the mirror?

b) What are the characteristics of this image formed?

Key concept

Image formation through a mirror.

Discovery activity

a) Look through a plain glass window and observe what happens. Discuss with your neighbor on what is observed.

b) Look through an open window and discuss with your neighbor about the observations.

c) Compare the observations in part (a) and (b) above.

d) Look through the lenses and describe the nature of image formed.

What I discover

Just curved mirrors change images, certain transparent medium called lens alter what you see through them.

A lens is a transparent medium (usually glass) bound by one or two curved surfaces. Different lenses give various natures of images depending on their characteristics.

Types of lenses and their characteristics

A lens is a piece of glass with one or two curved surfaces. The lens which is thicker at the centre than at the edges is called a convex lens while the one which is thinner at its centre is known as a concave lens. The curved surface of the lens is called a meniscus. The lens in the human eye is thicker in the centre, and therefore it is a convex lens

Activity 1

Required Materials
- Notebook
- 2 convex lenses
- 2 concave lenses
- Flashlight or a torch bulb
- White paper

Procedure
1. Look closely at the lenses and answer these questions in your notebook:
   a. How are the lenses shaped?
   b. How are the lenses alike?
   c. How are the lenses different?
2. Look through the lenses at the pages of a book, your hands, a hair, and other things. Draw what you see in your notebook and label each picture with the type of lens with which you observed the object. Be sure to answer the following questions:
   a. How does a concave lens make things look like?
   b. How does a convex lens make things look like?
3. Lenses bend light in different directions. Shine a flashlight through the lenses onto a piece of white paper and then answer the following questions in your notebook:
   a. In what direction do convex lenses bend light?
   b. In what direction do concave lenses bend light?
4. Shine the flashlight through different combinations of lenses: two convex lenses, two concave lenses, one concave and one convex lens. Draw pictures of what you see and answer these questions:
   a. What happens when you use multiple lenses at the same time?
   b. Can you use two different lenses to make things far away appear closer?
5. If you can, darken the room and place a convex lens between a sunlit window and a white piece of paper. Place the lens close to the paper and then slowly move the lens towards the window. Draw a picture of what you see in your science notebook.

Do you see that rays change the direction after the lens? How do the emergent rays from each of the lenses behave?

The light rays from the ray box change the direction after passing through the lens. They are therefore refracted by the lens. Hence, lenses form images of objects by refracting light.

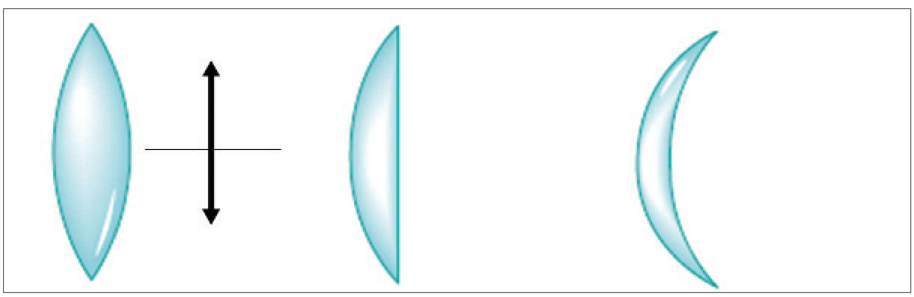
You can see that the rays from the convex lens are getting closer and closer to a point. The rays are thus converging, and hence a convex lens is called a converging lens. You can also see that the refracted rays from the concave lens are spreading out. This kind of lens is called diverging lens.

Summary:

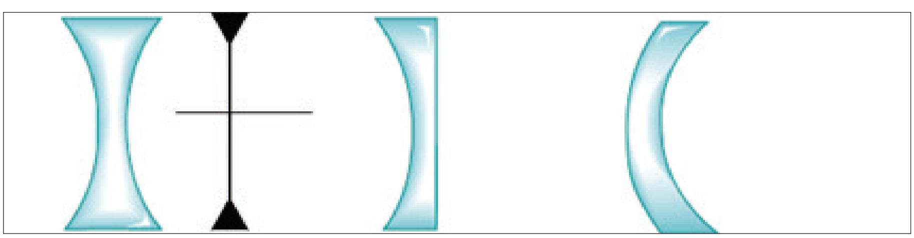
1. A lens is a transparent medium (usually glass) bounded by one or two curved surfaces. There are two types of lenses; a convex lens also called a converging lens and a concave lens also known as a diverging lens.

2. A convex lens is the one which is thicker at the centre than at the edges. A concave lens is the one which is thinner at the centre than at the edges.

The figure below shows three types of convex lenses and three types of concave lenses.



Biconvex lens *Plano convex* *Converging meniscus*



Biconcave lens *Plano concave* *Diverging meniscus*

Figure 1.1: Cross section of three types of converging and diverging lenses and their representation

Terms used in lenses

Activity 2

(i) Place a convex lens on a white sheet of paper with its sharp edge perpendicular to the paper.

(ii) Draw two parallel lines each touching the apex of the lens.

(iii) Measure the length between the two lines.

Write down in your notebook the comments about the length measured?

The length between the lines is the width of the lens. This width of the lens is called the aperture of the lens.

Activity 3

Place two similar Plano convex lenses together so that the two plane surfaces are in contact.
Write down in your notebook your observation.

When do the two Plano convex lenses form a bi-convex lens? The two plane surfaces of the Plano convex lenses form a vertical line which divides the lens into two halves. This line is called the axis of the lens.

Activity 4

(i) You have learned about symmetry in secondary mathematics. How many lines of symmetry does a convex lens have?

(ii) Place a convex lens on a white sheet of paper and perpendicular to it, draw its outline.

(iii) Draw its lines of symmetry.

(iv) Where do these lines meet?

(v) Repeat the above steps ii) and iii) but with a concave lens.

Discuss in your group and write down in your notebook the observation.

Lenses have two lines of symmetry, a vertical line and a horizontal line. The vertical line is called the axis of the lens (already seen in activity 2). The horizontal line is known as the principal axis of the lens.

Notice that these lines meet at a point. This point is the centre of the lens, called the optical centre of the lens denoted by **O.**

Activity 5

(i) Place a convex lens on a white sheet of paper and draw its outline.

(ii) Produce the outlines so as to make spheres from which the lens was cut.

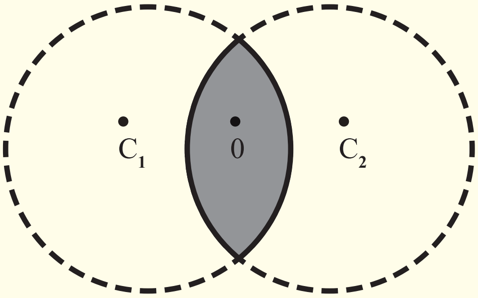


Figure 1.2: Drawing a biconvex lens

(iii) Note the centres C_1 and C_2 of the spheres formed.

(iv) Join the two centres of the spheres.

(v) Measure the distance between each centre and the optical centre.

What do you notice about the measured distance?

Can you see that the distances, OC_1 and OC_2 are equal?

Repeat the same but with a concave lens.

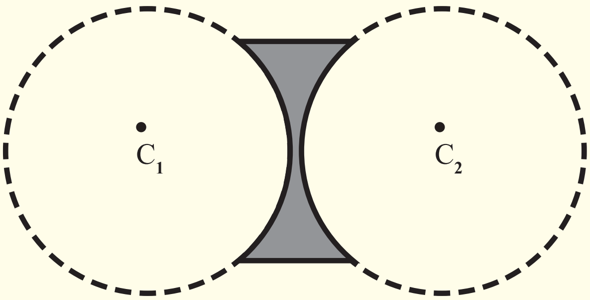


Figure 1.3: Drawing a biconcave lens

Do you see that the concave lens is not part of the spheres?

Discuss with your neighbour and write in your notebook the observation.

The centre of each sphere is called the centre of curvature of the surface of a lens and the distance from the centre of curvature to the optical centre is the radius of curvature of the surface. Since the convex lens forms part of the spheres, its centre of curvature is real and hence its radius of curvature.

Activity 6

(i) Use a torch to produce several rays of light to shine on the convex lens.

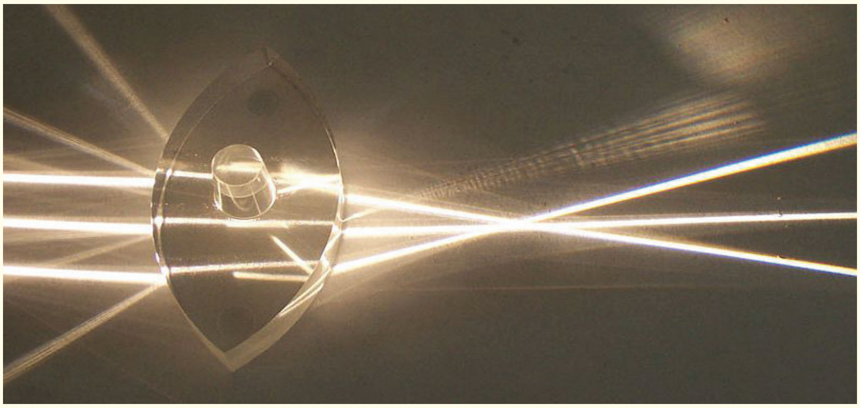


Figure 1.4: Light through a converging lens

(ii) Look at rays emerging from the lens.

Can you see that the rays converge to a point? Which name do you give to this point? Write down the observation in your notebook.

Repeat the experiment but with the concave lens. Write down the observation in your notebook.

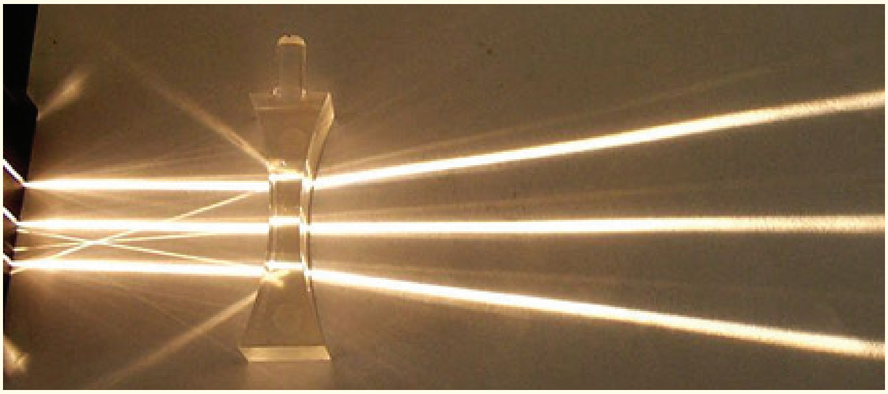


Figure 1.5: Light through a diverging lens:

This point to which all parallel rays converge after refraction by a convex lens is called the principal focus of the convex lens.

The rays emerge from the lens when they are spreading out. They are diverged and appear to come from a point. This point from which the rays appear to diverge after refraction by the concave lens is the principal focus of the lens.

Since rays converge to this point for the case of a convex lens, the principal focus of a convex lens is real. The principal focus of a concave lens is virtual as the rays appear to come from it.

Repeat the above experiments by changing the lenses so that their right sides become the left.

Do you see that the same thing happens for each?

Light can travel into the lens from the left or from the right. It therefore has two principal foci on both sides of the lens.

The principal focus of a lens is also called the focal point of the lens, and it is denoted by F.

Activity 7

(i) Hold a convex in a lens holder so that the rays of light from a distant tree are focused on a white piece of paper by moving the paper to and fro from the lens.

(ii) Measure the distance from the lens to the sheet of paper.

Repeat the above experiment with a fatter lens.

Does the fatter lens give a shorter focal length or a longer one?

Discuss on the observation and write short notes in your notebook.

Since the image forms where the refracted rays meet and because the rays from the distant tree are parallel, the piece of paper must then be at the principal focus of the lens. This distance from the lens to the image is the focal length of the lens. The focal length of the lens is thus the distance from the centre of the lens to the principal focus. It is always denoted by ***f***.

The fatter lens has a shorter focal length, implying that the thicker the lens, the shorter the focal length and vice versa.

We have already seen that the lens has two principal foci. It means that these principal foci are at equal distances on the opposite sides of the lens.

Repeat the experiment with the concave lens.

What do you notice?

The image cannot be seen. This is because the concave lens has a virtual principal focus.

Activity 8

Read and interpret the sentences below and fill in the table redrawn in your notebook.

a) The distance between the edges of the lens.

b) The line through the optical centre at right angles to the lens or the line passing through the optical centre that joins the centers of curvature of the two surfaces of the lens.

c) A point on the principal axis to which all rays parallel and close to the axis converge in case of a convex lens or from which they appear to diverge in case of a concave lens after passing through the lens.

d) The distance from the optical centre to the principal focus of the lens.

e) A plane containing a focal point in which all parallel rays close to the axis converge or appear to diverge after refraction by the lens.

f) The center of the lens or point in which vertical line through the lens meets the principal axis.

Table: Definition of terms used in lenses

Vocabulary terms in lenses	Corresponding definition
Focal point or principal focus	
Focal plane	
Aperture	
Principal axis	
Optical centre	
Focal length.	

Go Further

Visit the library and draw the diagrams in your notebook using convex and concave lenses. Indicate all terms defined in the table above on the diagrams.

Refraction of light through lenses

Lenses can be thought of as a series of tiny refracting prisms, each of which refracts light to produce an image. These prisms are near each other (truncated) and when they act together, they produce a bright image focused at a point.

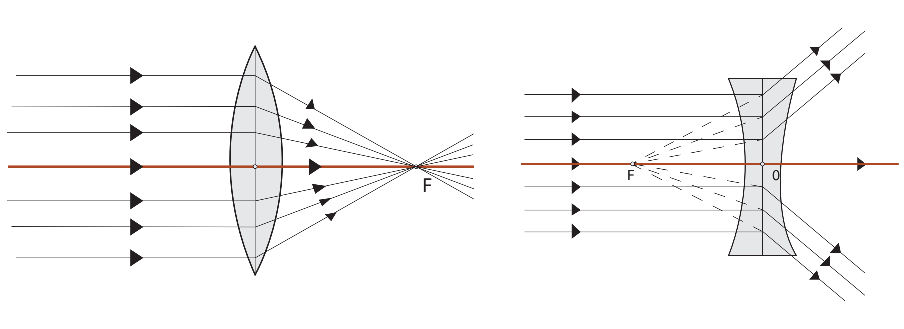


Figure 1.6: Action of converging and diverging lenses

Each section of a lens acts as a tiny glass prism. The refracting angles of these prisms decrease from the edges to its centre. As a result, light is deviated more at the edges than at the centre of the lens.

The refracting angles of the truncated prisms in a converging lens point to the edges and so bring the parallel rays to a focus.

The truncated prisms of the diverging lens point the opposite way to those of the converging lens, and so a divergent beam is obtained when parallel rays are refracted by this lens because the deviation of the light is in the opposite direction.

The middle part of the lens acts like a rectangular piece of glass and a ray incident to it strikes it normally, and thus passes undeviated.

Properties of images formed by lenses

Activity 9

(i) Hold a hand lens about 2m from the window. Look through the lens. (CAUTION: Do not look at the sun).

What do you see?

(ii) Move the lens farther away from your eye.

What changes do you notice?

(iii) Now, hold the lens between the window and a white sheet of paper, but closer to paper.

(iv) Slowly move the lens away from the paper towards the window. Keep watching the paper.

What do you see? What happens as you move the paper?

Do you see that an inverted image of trees outside is formed on the paper? How do you think the image is formed?

Rays come from all points on the objects. Where these rays meet or appear to meet after refraction by the lens is the position of the image.

Activity 10

You are provided with a lamp, a convex lens of known focal length, a lens holder and a white sheet of paper.

(i) Arrange the apparatus as shown below to investigate different images formed when the object (lamp) is placed at different positions from the lens.

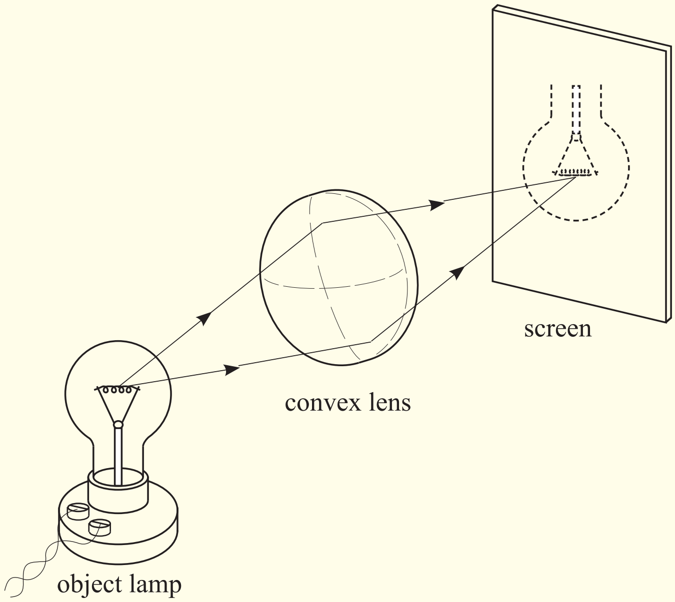


Figure 1.7: Image of a lamp formed by a convex lens

(ii) For each position of the object, move the screen (white sheet of paper) until you get a sharp image.

(iii) Fill in the table to show your results

Position of Object	**Position of Image**	**Image Real or Virtual**	**Image Inverted or Erect**	**Image smaller or larger than object**
At infinity				
Outside 2F				
Between 2F and F				
At F				
Between F and the lens				

Can you see that some images are larger than the object, some smaller and others same size as the object?

Do you notice that the images are inverted? What do you notice when an object is between F and the lens? Can you see that it is not seen on the screen?

Repeat the above experiment but with a concave lens of known focal length.

What do you notice in your observation?
Now, remove the screen and observe with the eye.

What do you notice? Do you see that the image is small and upright (erect)?

Notice that an image cannot be seen on the screen irrespective of the position of the object.

The nature of the image formed by a convex lens depends on the position of the object along the principal axis of the lens.

The principal focus of a lens plays an important part in the formation of an image by a lens since parallel rays from the object converge to it, and thus, we consider points F and 2F when describing the nature of the images formed by the lens.

These images can be larger or smaller than the object or same size as the object. When an image is larger than the object, we say that it is magnified and when it is smaller, we say that it is diminished.

Images which can be formed on the screen are Real images. Because light rays pass through these images, real images can be formed on the screen. All real images formed by the convex lens are inverted.

When an object is between F and the lens, there is no image formed on the screen. The image formed is not real and is only seen by removing the screen and placing an eye in its position. We say that it is a virtual image. For a virtual image, rays appear to come from its position.

Unlike for a convex lens where the nature of the image depends on the position of the object, a concave lens gives only an upright, small, virtual image, and is situated between the principal focus and the lens for all positions of the object.

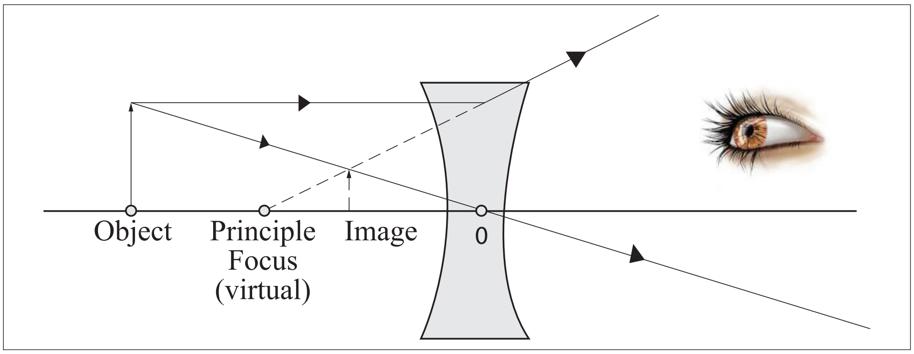


Figure 1.8: Viewing virtual image of diverging lens

Critical thinking:

1. Design an experiment to study images formed by convex lenses of various focal lengths. How does the focal length affect the position and size of the image produced?
2. Suppose you wanted to closely examine the leaf of a plant, which type of a lens would you use? Explain your decision.

Ray diagrams and properties of images formed by lenses

Activity 11

Shine on a convex lens in a dark room using a torch bulb. How many rays do you see emerging from the lens?
Notice that the emergent rays are infinite

We have already seen that an image is formed where rays from the object meet. Rays come from all points on the objects. However, for simplicity, only a few rays from one point are considered when drawing ray diagrams. Where these rays meet or appear to meet after refraction by the lens is the position of the image.

To locate the position of the image, two of the following three rays are considered.

1. A ray parallel to the principal axis which after refraction passes through the principle focus or appears to come from it.

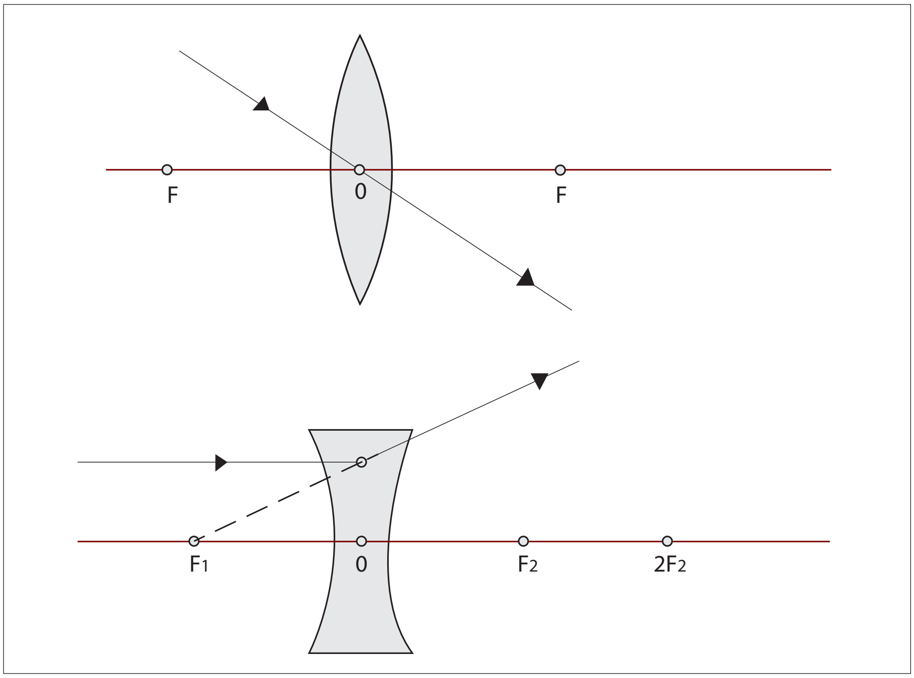


Figure 1.9: Refraction of a ray parallel to the principal axis

2. A ray through the optical centre which passes through the lens undeviated (un deflected).

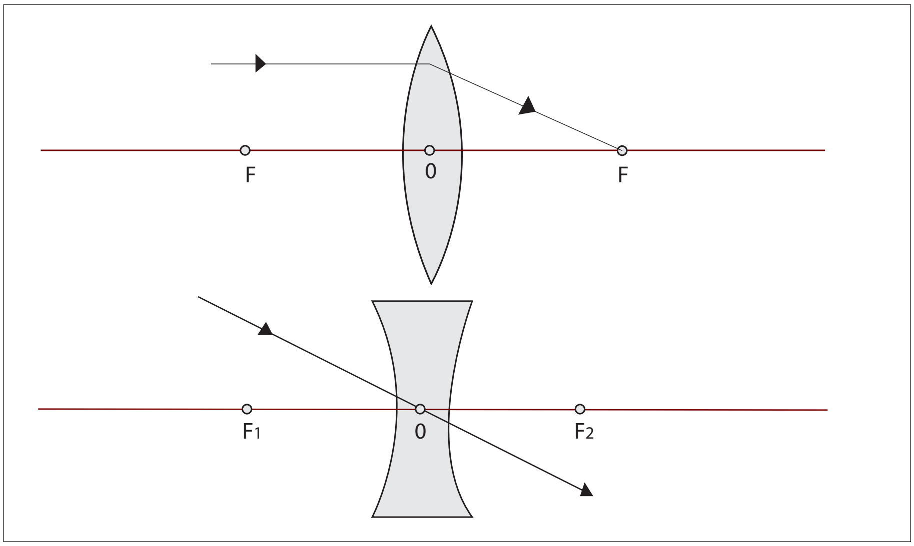


Figure 1.10: Ray passing through the optical centre

3. A ray through the principal focus which is refracted parallel to the principal axis.

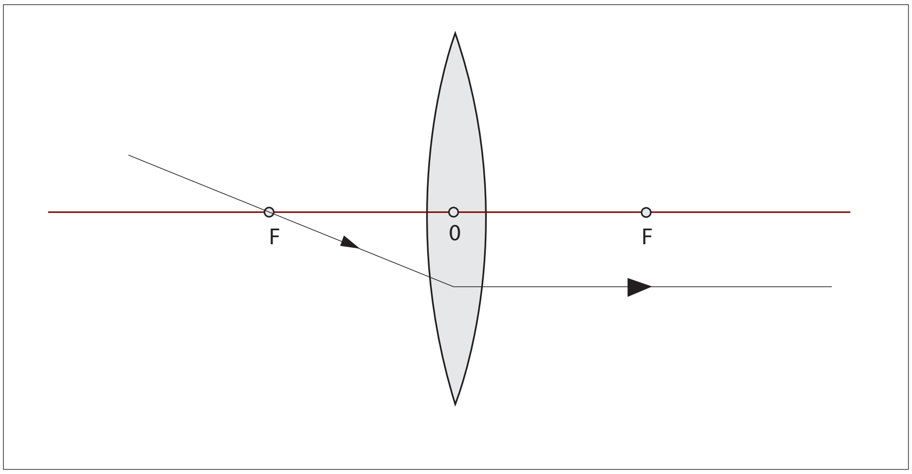
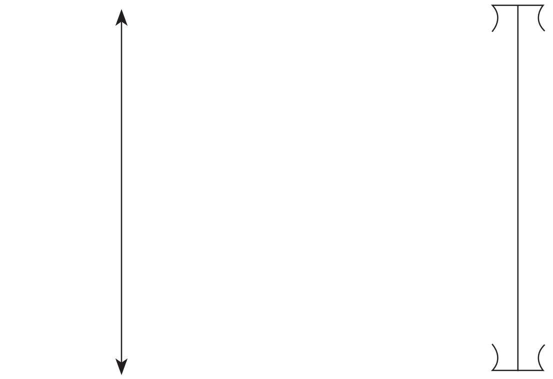


Figure 1.11: Refraction of a ray passing through the focus

The central part of a lens acts as a small parallel –sided block which slightly displaces but does not deviate a ray passing through it and for a thin lens, the displacement can be ignored.

In ray diagrams, a thin lens is represented by a straight line at which all the refraction is considered to occur. In reality, bending takes place at each surface of the lens.



(a) Converging lens (b) Diverging lens

Figure 1.12: Representation of convex and concave lenses in ray diagrams

Activity 12

(i) On a graph paper, draw a long horizontal line to represent the principal axis of the lens and a shorter line, at right angles to represent a thin lens.

(ii) Using 2cm to represent 10 units on both axes, mark the position of F on each side of the lens at 20cm from the lens. Also mark the points 2F at twice the focal length of the lens.

(iii) Mark the position of an object (pin), 20cm tall on the principal axis at a distance of 45cm from the lens.

(iv) Draw a line from the top of an object parallel to the axis that will pass through F after passing through the lens.

(v) similarly, draw a line that passes through the centre of the lens.

(vi) Mark the position on the principal axis where the two lines meet.

(vii) Measure the distance of this position from the lens.

The position where the two lines meet is the position of the image.

Is the image inverted or upright?

The tip of the image is at the point where the two lines meet. Since the object is standing on the principal axis, the bottom of the image is also at the axis, hence the image is inverted.

Measure the height of the image (using the scales).

Is the image magnified or diminished?

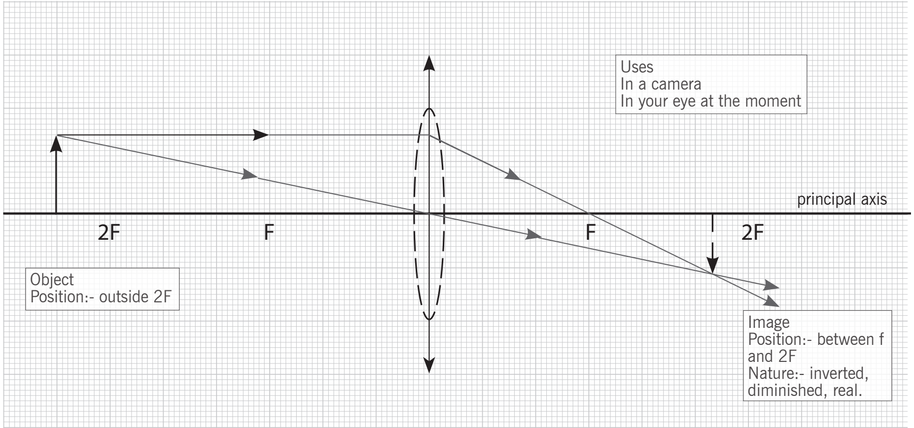


Figure 1.13: The position of the image formed by a lens

Ray diagrams for a convex lens

Object between the lens and F

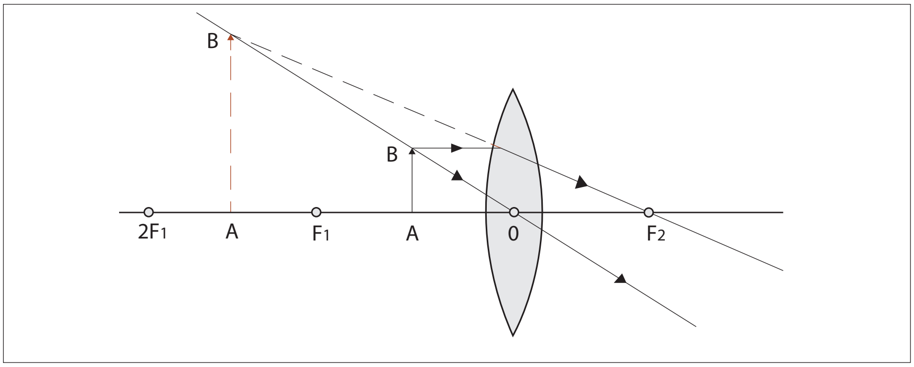


Figure 1.14: Images of an object between O and F

Nature of image

The image is virtual, erect, larger than the object and behind the object.

Exercise

How is this lens useful when the object is in this position?

Object at F

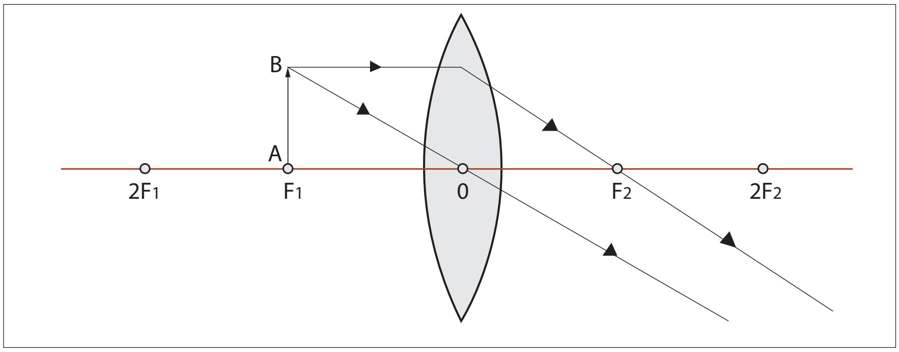


Figure 1.15: Image of an object at infinity

Nature of image

The image is formed at infinity.

Exercise

Can you think of how useful is the lens when an object is at its focal point? What is it?

Object between F and 2F

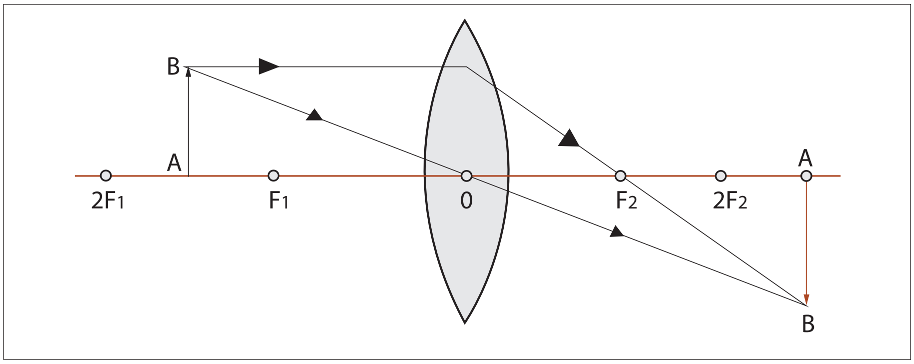


Figure 1.16: Image of an object between F and 2F

Nature of image

The image is real, inverted, larger than object (magnified) and beyond 2F.

Exercise

How is the lens useful when the object is in the above position?

Object at 2F

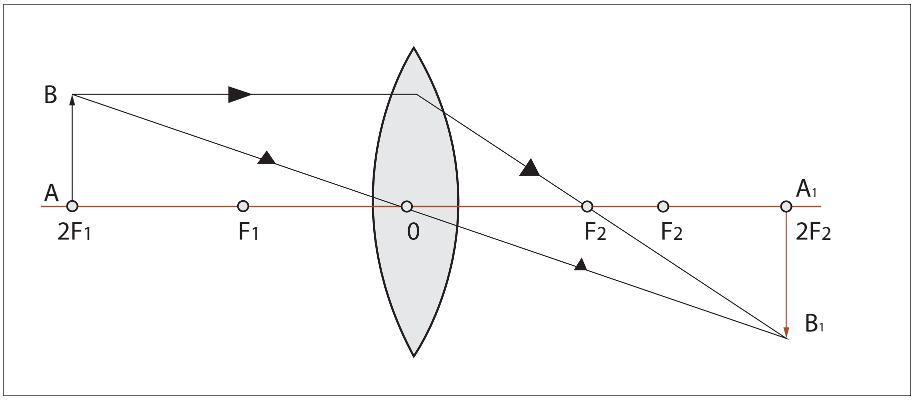


Figure 1.17: Image of an object at 2F

Nature of image

The image is real, inverted and same size as object.

Object beyond 2F

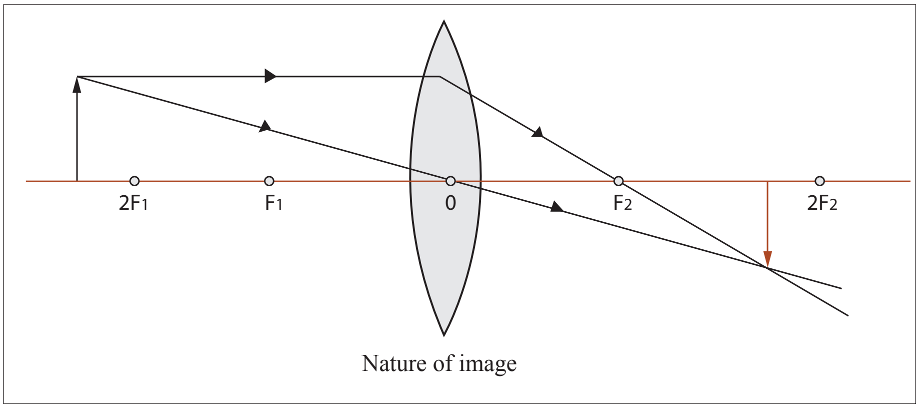


Figure 1.18: Image of an object beyond 2F

The image is real, inverted, smaller than object (dimensional) and is formed between F and 2F.

Discover

What can be a daily application of the lens when an object is in this position?

Object at infinity

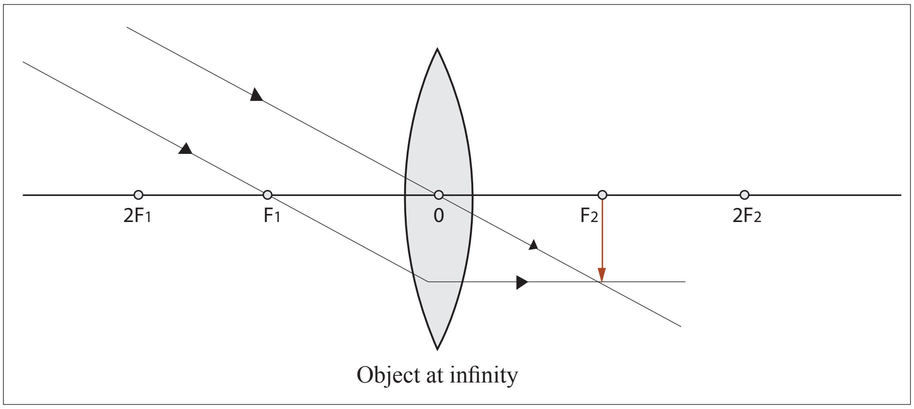


Figure 1.19: Image of an object at infinity

Nature of image

The image is real, inverted, smaller than object and is formed at F.

When an object is between the lens and the principal focus, the rays from the object never converge, instead they appear to come from a position behind the lens. In this case, the lens is used as a simple magnifying glass because it forms an upright and magnified image (Figure 1.14).

When an object is at the principal focus of the lens, refracted rays emerge from the lens parallel to each other, and the lens is used as a search light torch, and theatre spotlights (Figure 1.15).

Figure 1.16 shows that when an object is between F and 2F, the lens forms a magnified real image. In this case, a lens is used as a film projector.

When an object is beyond 2F (Figure 1.18), a lens forms real and small image. The lens is used as a camera because this small, real image can be formed on a piece of film.

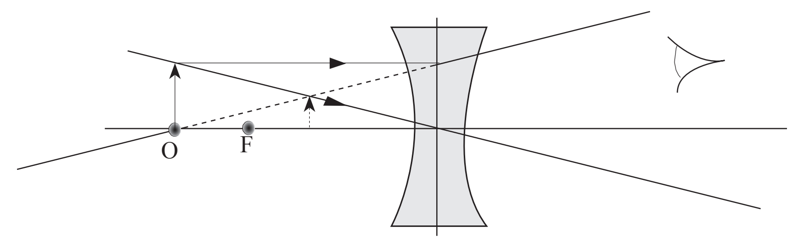


Figure 1.20: Formation of an image by a diverging lens

Accurate construction of ray diagrams

Problems for locating the position of the image can be solved by constructing a ray diagram as an accurate scale drawing on a graph paper.

Activity 13

An object 2cm high stands on the principal axis at a distance of 9cm from a convex lens. If the focal length of the lens is 6cm, what is the nature, size and position of the image.

Scale: Let 1cm on the paper represent 2cm of actual distance.

Example

1. An object is placed 40cm away from a diverging lens of focal length 20cm. If it is 2cm high, determine graphically the position, size and nature of the image.

2. Let 1cm on the paper represent 10cm on the horizontal axis and 1cm on the vertical axis of the actual distance.

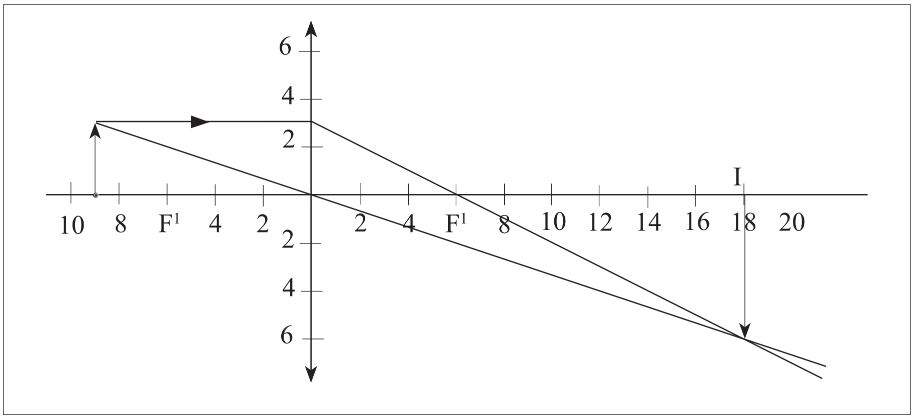


Figure 1.21: Position of an image formed by a lens

It is 4cm high and is formed at 18 cm from the lens.

The image is virtual, erect, 0.7cm tall and is formed at 13cm from the lens on the same side as the object.

The thin lens formula

Activity 14

Using the same question in the above activity (13), find the position of the image v for an object at a distance u infront of a convex lens of focal length f, using the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

What value of the image distance have you got?

Compare the value obtained with the one obtained from a ray diagram.

What do you notice in accordance to your observations?

Activity 15

(i) Draw a ray diagram to determine the nature and position of the image of an object placed 10cm from a diverging lens of focal length 15cm.

(ii) Using the above information, find the nature and position of the image using a lens formula. (assign f a negative sign during your substitution).

What is the location of the image?

The lens formula gives the relationship between the object distance, u, image distance, v, and the focal length, f of the lens.

This relation is given by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

Where u is the distance of the object from the lens, v the image distance and f the focal length of the lens.

The sign convention

From activity 13, we notice that all the distances are measured from the optical centre and in activity 14, we substituted for u, v and f using positive numerical values. It therefore follows that distances of real images and real principal focus are positive.

In activity 14, then you will notice that the image distance from the lens is negative but equal to the distance determined graphically. This distance is obtained by using a negative numerical value of the focal length. Since a concave lens has a virtual principal focus, and forms virtual images, distances of virtual images and virtual principal foci are negative.

Sign convention states that real is positive while virtual is negative. This should be put under consideration when one is using the lens formula to solve problems.

Derivation of the lens formula

Convex lens

Consider a point object O on the principal axis, at a distance, u greater than the focal length from the lens.

Suppose that a ray from O is incident on the lens at a small height h above the axis and is refracted to form an image I at a distance v from the lens.

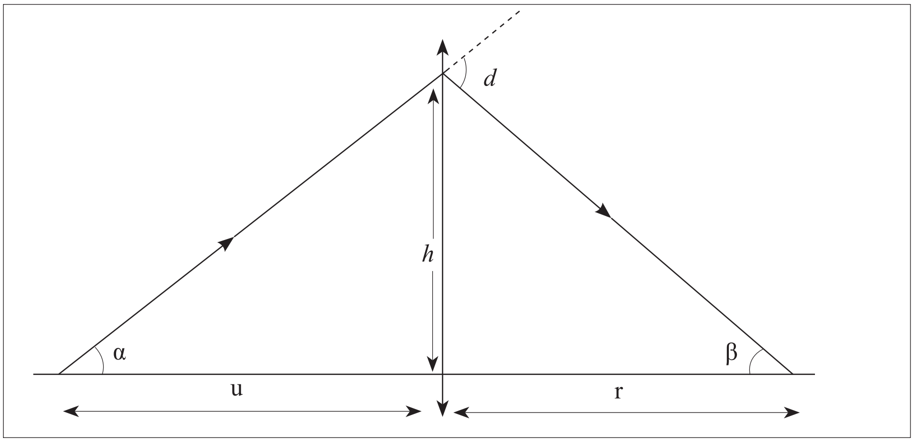


Figure 1.22: Diagram of deriving lens formula

Let α and β be angles the incident ray and the refracted ray make with the axis

If the incident ray suffers a small deviation, d, then, from fig. 1.22,

$d = \alpha + \beta$ (Two interior angles of a triangle are equal to one opposite exterior angle).

Since the ray strikes the lens at a height, h, it follows that:

$\tan \alpha = \frac{h}{u}$ and $\tan \beta = \frac{h}{v}$

For thin lenses, α and β are very small, and thus $\tan \alpha \approx \alpha$ and $\tan \beta \approx \beta$ in radians.

Therefore, $\alpha = \frac{h}{u}$ and $\beta = \frac{h}{v}$

So, $d = \frac{h}{u} + \frac{h}{v}$ (1)

Now consider a ray from a finite object parallel to the principal axis and incident on the lens at the small height, h. After refraction, this ray passes through the focal point, F, a distance, f, from the lens.

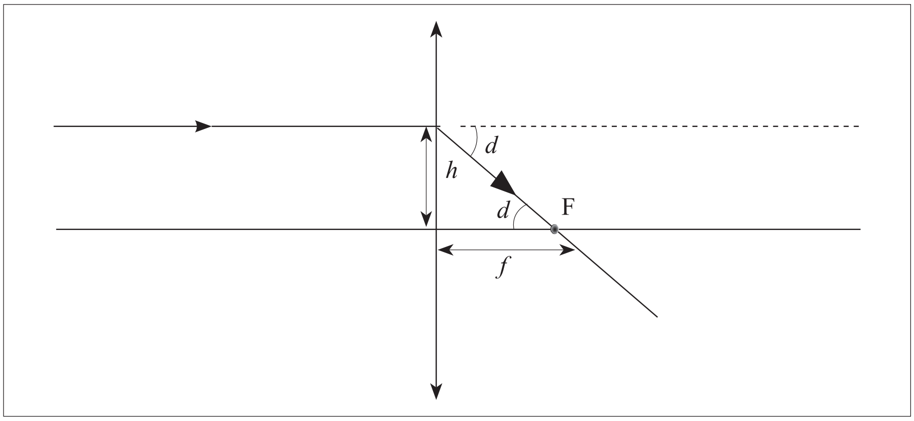


Figure 1.23: Position of an image of an object at infinity

This incident ray suffers the same deviation, d, as above since the lens is considered as a small angle prism and all rays entering a small angle of prism at small angles of incidence suffer the same deviation.

From the figure above, the deviation, d, is equal to the angle the refracted ray makes with the axis (alternate angles)

Thus $\tan d = \frac{h}{f}$

For d small, $\tan d \approx \alpha$

Therefore, $d = \frac{h}{f}$ (2)

From (1) and (2)

$\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$

Dividing by h *on* both sides, we have $\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$

Hence for any lens of focal length f, $\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$

Concave lens

Consider a point object O on the principal axis of the diverging lens at a distance, u, so that its image is formed at a distance, v.

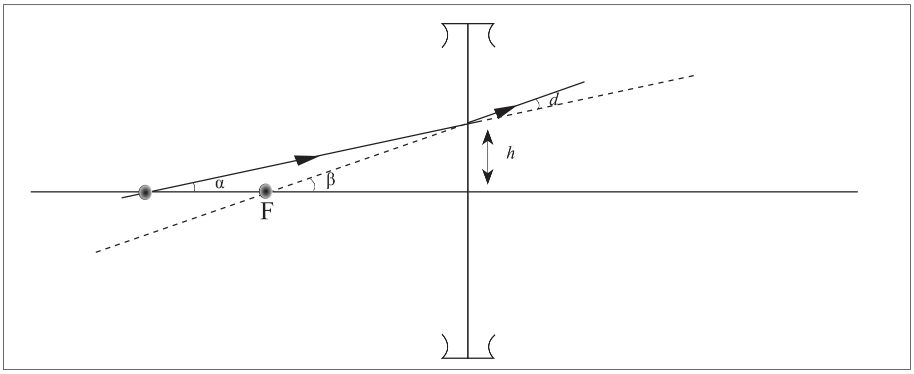


Figure 1.24: Deriving a lens formula using a diverging lens

Let angles α and β be the angles made by the incident and refracted rays with the axis respectively.

From the diagram, $\alpha + d = \beta$

Thus $d = \beta - \alpha$

But $\tan \alpha = \dfrac{h}{v}$ and $\tan \beta = \dfrac{h}{v}$ for small angles, $\tan \alpha \approx \alpha$ and $\tan \beta \approx \beta$.

Then $d = \dfrac{h}{v} - \dfrac{h}{u}$ (1)

Now consider a ray from a finite sized object parallel to the axis. This ray appears to come from the focal point F after refraction.

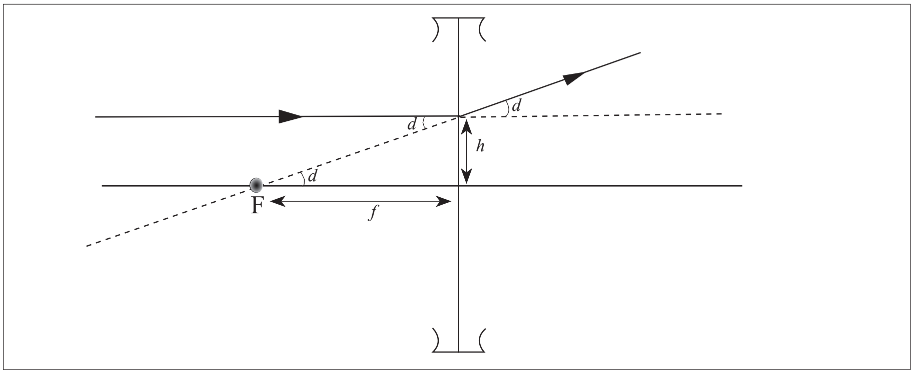


Figure 1.25: Image formed by a diverging lens of an object at infinity

From the diagram, $\tan d = \dfrac{h}{f}$

But for d very small $\tan d \approx d$

Thus $d = \dfrac{h}{v}$ (2)

If we introduce the "real is positive" sign convention, the focal length of the diverging lens is negative and the distance v in equation (1) is also negative since it's a virtual image.

Therefore, it follows from the above equations that $\frac{-h}{f} = \frac{-h}{v} - \frac{h}{u}$

Dividing by, $-h$, we have $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$

Thus this lens equation $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$ applies to diverging lenses. To all cases of real and virtual objects and images, we use the sign convention rule.

Magnification

Activity 16

(i) Using the same drawing in activity 15, measure the heights of the object and the image respectively.

(ii) Find the ratio of the image height to the object height.

(iii) How many times is the image larger than the object?

(iv) Now, find the ratio of the distance of image from the lens to the distance of object from the lens.

(v) Compare the two ratios.

What do you notice in accordance to your observation?

You can notice that the ratio of image height to object height is equal to that of image distance to object distance from the lens. This ratio is called Linear magnification of the image. It tells us the number of times the image is larger than the object. It is sometimes called Lateral or transverse magnification.

Thus, the lateral, transverse or linear magnification of an image produced by the lens is the ratio of image size to the object size or image distance to object distance.

Mathematically,m = $\frac{\textit{Image height}}{\textit{object height}}$ or $M = \frac{\textit{Image distance}}{\textit{object distance}}$

Hencem = $\frac{v}{u}$

consider a ray from a finite object at a distance u through the optical centre of a converging lens passing through a point I, the position of an image at a distance v.

The two triangles OAC and IBC are similar, and

from the geometry of similar figures, $\frac{IB}{OA} = \frac{IC}{OC}$

But IB is the height of image and OA the height of object, and

OC and IC are object and image distances respectively

Therefore $\frac{\text{Image height}}{\text{object height}} = \frac{\text{Image distance}}{\text{object distance}}$

Magnification = $\frac{IB}{OA}$

Hence magnification,m = $\frac{v}{u}$

Thus magnification can also be defined by magnificationm = $\frac{v}{u}$

Applications of the lens formula

The following examples show how to apply the lens equation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ correctly.

Example

An object is placed 20cm from a converging lens of focal length 15 cm. Find the nature, position and magnification of the image.

The object is real and therefore $u = + 20\text{cm}$

Since the lens is converging, $f = + 15\text{cm}$

Substituting in $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

We have, $\frac{1}{20} + \frac{1}{v} = \frac{1}{15}$

Therefore, $\frac{1}{v} = \frac{1}{15} - \frac{1}{20}$

$$\frac{1}{v} = \left(\frac{4-3}{60}\right)\text{cm}^{-1}$$

Hence, $v = 60\text{cm}$

Since v is positive, the image is real and it is 60cm from the lens.

Magnification,m = $\frac{v}{u}$

Thusm = 3

Therefore, the image is three times taller than the object.

Exercise

1. An object is placed 12cm from a converging lens of focal length 18 cm. Find the nature and the position of the image.
2. Find the nature and position of the image of an object placed 15cm from a diverging lens of focal length 15cm.

Critical thinking exercise

From the magnification formula and the lens formula, show that the image distance v can be related to the focal length of the lens bym = $\frac{v}{f} - 1$.

Least possible distance between object and real image with converging lens

Activity 17

You are provided with a convex lens of known focal length, a pin and a white screen.

(i) Place an object (pin) in front of a concave lens at a distance greater than the focal length.

(ii) Place the screen on the other side of the lens and move it to and fro until a clear image is seen.

(iii) Measure the distance between the object and the screen.

(iv) Repeat the above procedures for other values of object distance.

(v) Compare the corresponding distances between the object and the image with the focal length of the lens.

(vi) Are the distances between corresponding objects and images greater than four times the focal length of the lens?

(vii) Discuss in your groups and write short notes in your notebook.

Experiments show that it is not always possible to obtain a real image on a screen although the object and the screen may both be at a greater distance from a converging lens than its focal length.

Theory shows that the minimum distance between the object and the screen for an image to be formed is four times the focal length, f. Therefore, the distance between an object and a screen must be equal to or greater than four times the focal length.

Consider a point object O on the principal axis of a converging lens forming an image I.

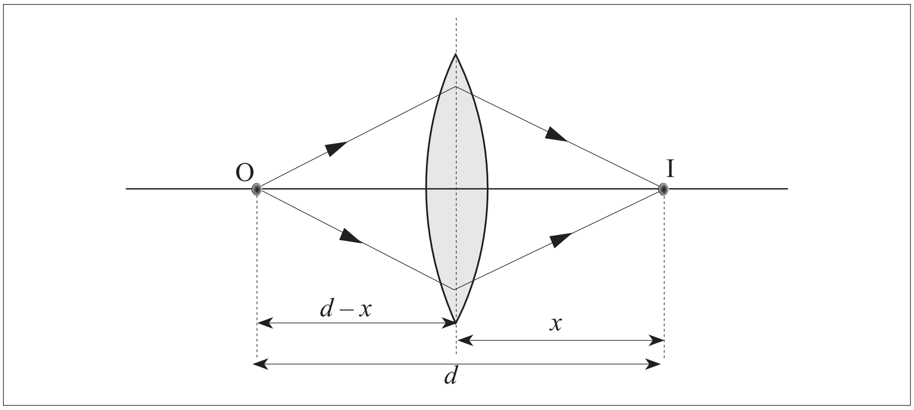


Figure 1.26: Minimum distance between object and image

Suppose that the image distance is x, and the distance between object and image is d, then the object distance $u = d - x$.

Substituting in the lens equation, $\frac{1}{f} = \frac{1}{x} + \frac{1}{d-x}$

We therefore have $\frac{1}{f} = \frac{d-x+x}{x(d-x)} = \frac{d}{dx-x^2}$

It follows that, $fd = dx - x^2$.

Hence, $x^2 - dx + df = 0$

This is a quadratic equation, and for a real image, the roots of this equation must be real.

Applying the condition for real roots $b^2 - 4ac \geq 0$ for the general quadratic equation $ax^2 - bx + c = 0$,

Then, $d^2 - 4df \geq 0$

It follows that $d^2 \geq 4df$

Thus $d \geq 4d$

Thus the distance d between the object and the screen must be greater than or equal to 4f otherwise no image can be formed on the screen.

Power of the lens

Activity 18

(i) Focus a distant object through the window with a thin lens.

(ii) Note the distance of the screen from the lens.

(iii) Repeat the above procedures with a thicker lens.

(iv) Compare the distances of the images formed for each case.

Do you notice that the image formed by the thicker lens is nearer to the lens than that formed by the thinner one?

Discuss and write short notes in your note book.

Since the image formed by the thicker lens is nearer, the thicker lens is more powerful than the thinner lens of the same material.

We have already seen that an image of a distant object forms at the focus of the lens and the thicker the lens the shorter the focal length. So the power of the lens depends on its focal length, that is, as the focal length becomes shorter, the power increases.

The power of the lens is defined as the reciprocal of its focal length in metres.

Power of a lens $= p = \frac{1}{f}$

The standard unit of power of a lens is a Dioptre.

Quick activity

1. Calculate the power of the lens of focal length of 15 cm.
2. A converging lens has a power of 0.02D, what is its focal length?

Determination of the focal length of the lens

Converging lens

Rough method

Activity 19

(i) Place a converging lens on a table while facing a window.

(ii) Place a white screen behind the lens.

Move the screen to and fro (forwards and backwards) until a sharp image of a distant object is seen on the screen.

Discuss and write down the observation in your notebook.

Measure the distance from the lens to the screen.

The distance from the lens to the screen is the focal length of the lens since rays from a distant object strike the lens when they are parallel.

Graphical determination of focal length of a convex lens

Activity 20

You are provided with a lamp, a screen with cross wires, a convex lens, a lens holder and a white sheet of paper

(i) Set up the lens in front of an illuminated object at a given distance u=15cm and adjust the screen until a sharp image is seen.

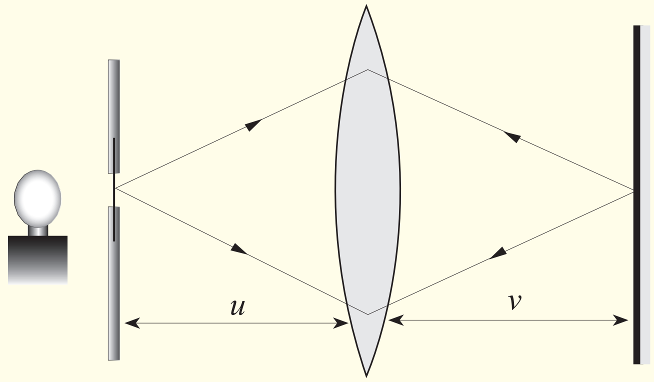


Figure 1.27: Focal length of a lens by u and v method

(ii) Measure the distance, v from the lens to the screen.

(iii) Repeat the above for values of u = 20cm, 25cm, 30cm, 35cm, 40cm and 45cm.

(iv) Record your results in a suitable table including values of uv and $u + v$.

u / cm	v / cm	$u + v$ / cm	uv / cm^2

(v) Plot a graph of uv against $u + v$.

(vi) Find the slope (gradient), of the graph.

From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

It follows that $\frac{u+v}{uv} = \frac{1}{f}$,

Thus, $f = \frac{uv}{u+v}$,

Therefore, finding the slope of the graph gives the mean value of the focal length.

Similarly, from $\frac{u+v}{uv} = \frac{1}{f}$

It implies that $uv = f(u + v)$

The above expression is an equation of a line and hence a graph of uv against $u + v$ is a straight line passing through the origin and its slope is the focal length f of the lens.

Instead of using an illuminated object, a pin may be set up in front of the lens so that it forms a real image on the opposite side whereby the position of this image can be located by the help of a search pin using the method of no-parallax.

Diverging lens

Determination of focal length of a diverging lens by Concave mirror method

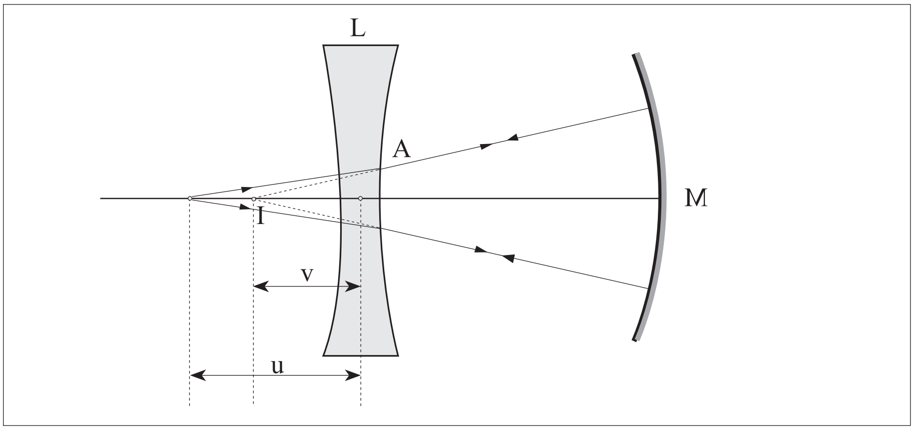


Figure 1.28: Focal length of a diverging lens

Activity 21

You are provided with a concave lens, concave mirror of known radius of curvature, a screen with cross wires and a lamp.

(i) Place an object in front of a concave lens (diverging lens) at a measurable distance from the lens.

(ii) Place a concave mirror behind the lens so that a diverging beam is incident on it.

(iii) With the object and the lens in position, move the mirror to and fro until an image coincides with the object.

(iv) Measure the object distance.

(v) Measure the distance between the lens and the mirror, L_m.

(vi) Calculate the image distance v from the lens $v = r - L_m$, where r is the radius of curvature of the mirror.

(vii) Find the focal length of the lens using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Discuss on the observation and write short notes in your notebook.

We have already seen that a concave lens forms virtual images of real images which cannot be seen on the screen. So, to determine the focal length of a diverging lens, we need to form a virtual object for the diverging lens so that a real image is produced. This is achieved in the experiment by putting a concave mirror behind the lens so as to reflect back the diverging rays from the lens.

As you saw in your lower secondary classes, when an object is placed at the principal focus of a concave mirror, the image is formed at the same position with it. Now, since the object and its image are coinciding, it means that they are at the centre of curvature of the mirror; v is negative as I is a virtual image for the lens, and as the object and image are coincident, the rays must be incident normally on the mirror M. Thus, reflected rays from the mirror pass through its centre of curvature which is the position of the virtual image.

Combination of lenses

Activity 22

Have you ever critically looked at the microscope?

Look at the microscope provided and count the number of lenses it has.

How many lenses have you seen?

In our next unit, we shall talk about instruments which use lenses to focus objects. Among others, a microscope uses a combination of two lenses to focus objects.

Effective focal length of a combination of lenses

Activity 23

a) (i) Focus a distant tree through a window using a convex lens onto a white sheet of paper.
   (ii) Measure the distance from the lens to the paper.

b) (i) With the convex lens still in position, place another convex lens similar to the above besides and in contact with it.
   (ii) Move the paper to and fro until a clear image of the tree is focused on it.
   (iii) Measure the distance from the lenses to the white sheet of paper.

Since the rays from a distant tree are parallel, they meet at the focal plane of the lens (in a) and of the combination of the lenses (in b). Therefore, the distance from the lens to the white sheet of paper is the focal length of the lens (in a) and the distance from the combination of the two lenses to the screen is the focal length of the combination.

From the above activity, let the focal length of the individual lenses be f_1 and f_2 respectively.

Substitute the values the focal lengths obtained from the a experiment in the equation $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ where F is the focal length of the combination.

What do you notice?

Can you see that the left hand side of the equation is equal to the right hand side?

The focal length of a combination of lenses in contact is obtained from the relation, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Note that sign convention still holds for the focal lengths of the lenses during substitution of the numerical values in the formula.

Exercise:

Find the focal length of a combination of a converging lens and a diverging lens of focal lengths 5cm and 10cm respectively.

Derivation of the expression of effective focal length of the lens combination. The focal length, f of a combination of two thin lenses of focal lengths f_1 and f_2 respectively can be found by considering a point object O placed on the principal axis of the lenses in contact.

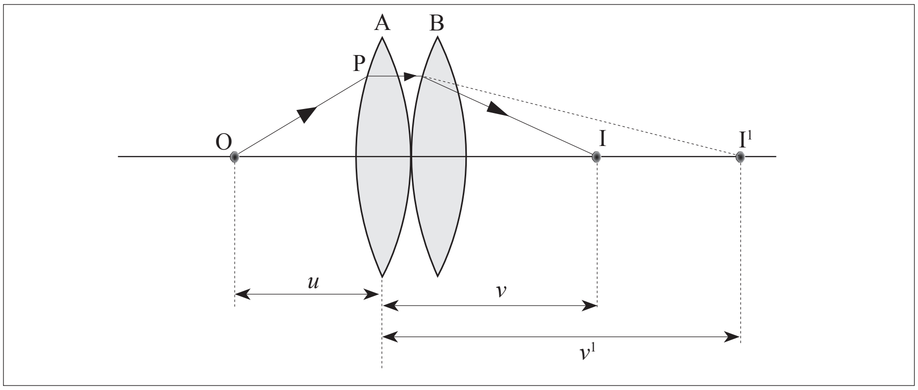


Figure 1.29: Focal length of a converging lens

In the absence of lens B, ray OP would pass through point I^1 which would be a real image of lens A.

If u is the object distance and v^1 is the image distance, then from the lens formula;

It follows that $\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v^1}$ (i)

With lens B in position, I^1 acts as a virtual object for this lens forming an image at I.

This means that for lens B, the object distance is $-v^1$ and the image distance is v. Thus using the lens formula, it follows that $\frac{1}{f_2} = \frac{1}{-v^1} + \frac{1}{v}$ (ii)

Adding (i) and (ii) to eliminate v^1, we have, $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v^1} - \frac{1}{v^1} + \frac{1}{v}$

Hence $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$

Since I is the image of O by refraction through both lenses, then using the lens formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ where f is the focal length of the combination.

Thus $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$

Therefore, the expression for the focal length of the combined lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

This formula applies to any two thin lenses in contact, such as two converging lenses, or a converging and diverging lens. When the formula is to be used, the sign convention must be applied.

Example

A thin converging lens of focal length 8cm is placed in contact with a diverging lens of focal length 12cm. Calculate the focal length of the combination.

$f_1 = +\,8$cm (Converging lens)
$f_1 = -\,12$ cm (Diverging lens)

From $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

It implies that $\left(\frac{1}{F} = \frac{1}{8} - \frac{1}{12}\right)\text{cm}^{-1}$

It follows that $\frac{1}{F} = \left(\frac{3-2}{24}\right)cm^{-1}$

Thus, F = +24cm

The positive sign shows that the combination of the two lenses acts like a converging lens.

Exercise

1. An object O is placed 12cm from a thin converging lens P of focal length 10cm and an image is formed on a screen S on the other side of the lens. A thin diverging lens, Q is now placed between the converting lens and S, 50cm from the converging lens. Find the position and nature of the final image if the focal length of the diverging lens is 15cm.
2. An object is placed 6.0cm from a thin converging lens A of focal length 5.0cm. Another thin converging lens B of focal length 15cm is placed co-axially with A and 20cm from it on the side way from the object. Find the position, nature and magnification of the final image.

Defects of lenses and their corrections

Activity 24

(i) Place a white sheet of paper on a horizontal ground.

(ii) Hold a glass ruler above the paper so as to focus rays from the sun on to the paper.

(iii) Observe carefully the image formed on the sheet of paper.

(iv) Repeat the above with the convex lens.

What have you observed?
Share the ideas about the observations.

Notice that the image has coloured patches.

This defect where by an image formed has coloured patches is called chromatic aberration.

There are two kinds of defects; spherical aberration and chromatic aberration.

Spherical aberration

This arises in lenses of larger aperture when a wide beam of light incident on the lens, not all rays are brought to one focus. As a result, the image of the object becomes distorted. The defect is due to the fact that the focal length of the lens for rays far from the principal axis are less than for rays closer to a property of a spherical surface and as a result, they converge to a point closer to the lens.

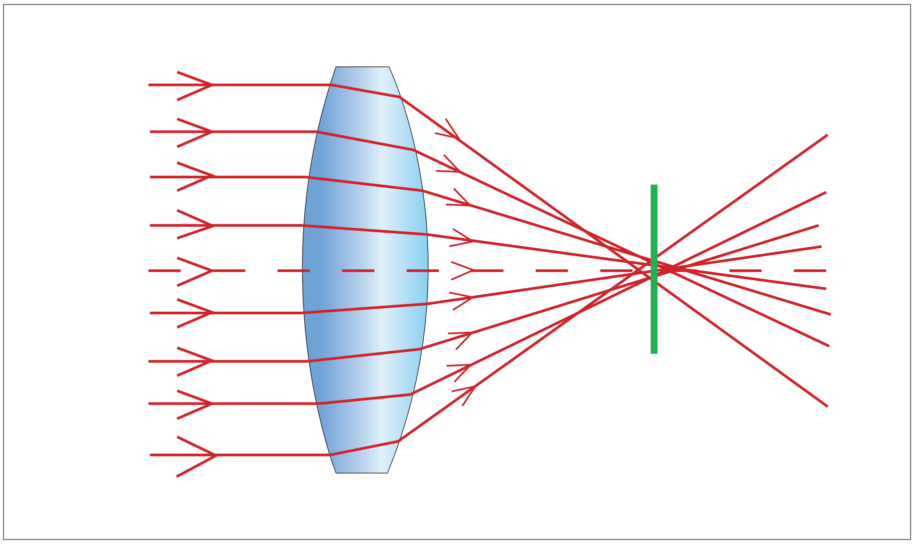


Figure 1.30: Spherical aberration

This defect can be minimised (reduced) by surrounding the lens with an aperture disc having a hole in the middle so that rays fall on the lens at a point closer to its principal axis. However, this reduces the brightness of the image since it reduces the amount of light energy passing through the lens.

Chromatic aberration

This occurs when white light from an object falls on a lens and splits it into its component colours. These colours separate and converge to different foci, and this results into an image with coloured edges.

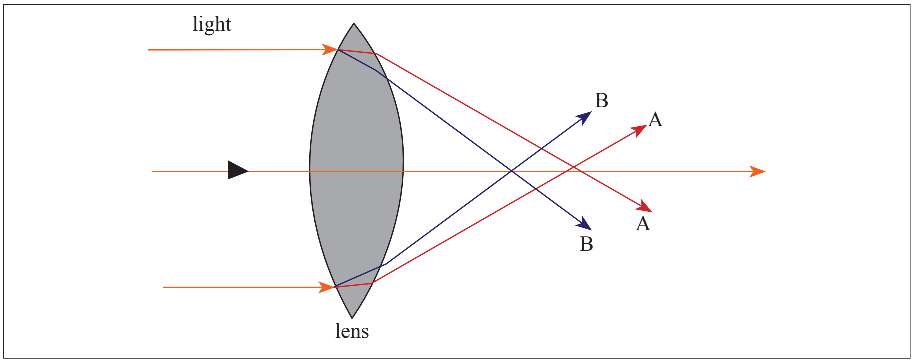


Figure 1.31: Chromatic aberration

The separation takes place because the material of a glass of a lens has different refractive indices for each colour. The colours travel at different speeds in glass: red colour with the greatest and the violet with the least. As a result, violet is deviated most and red is the least deviated

Thus, a converging lens produces a series of coloured images of an extended white object as shown in the figure above (exaggerated for clarity).

Chromatic aberration can be minimised by using an achromatic lens called an achromatic doublet. This consists of a converging lens of crown glass combined with a diverging lens of flint glass cemented together with Canada balsam.

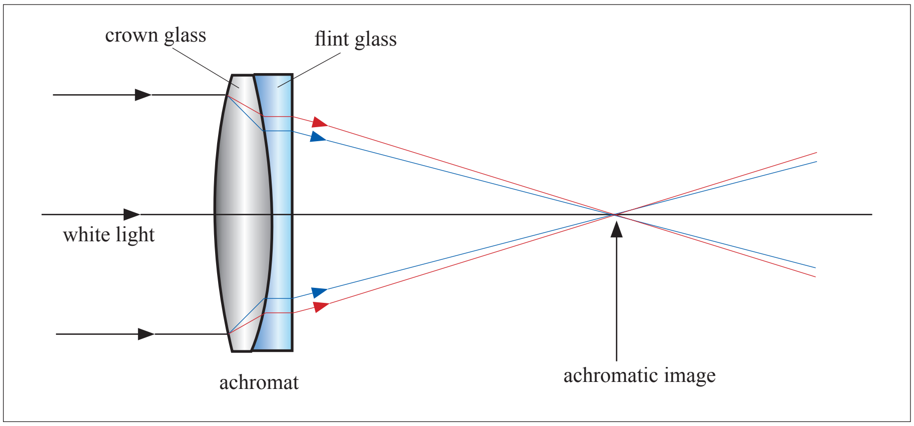


Figure 1.32: An Achromatic doublet

The flint glass of the diverging lens produces the same dispersion as the crown glass of the converging lens but in the opposite direction and the overall combination is converging. As a result, the achromatic combination converges the white light to one focus.

Activity 25

Disscuss with your neighbour the applications of lens combinations in daily lives and write short notes in your notebook.

Refraction through prisms

Activity 26

Problem

Have you ever heard of a prism?
How does it look like?

Procedures

a) Consider the shapes of the glasses provided below. Observe them clearly and identify the shape of a prism. Explain your reasoning in your notebook.

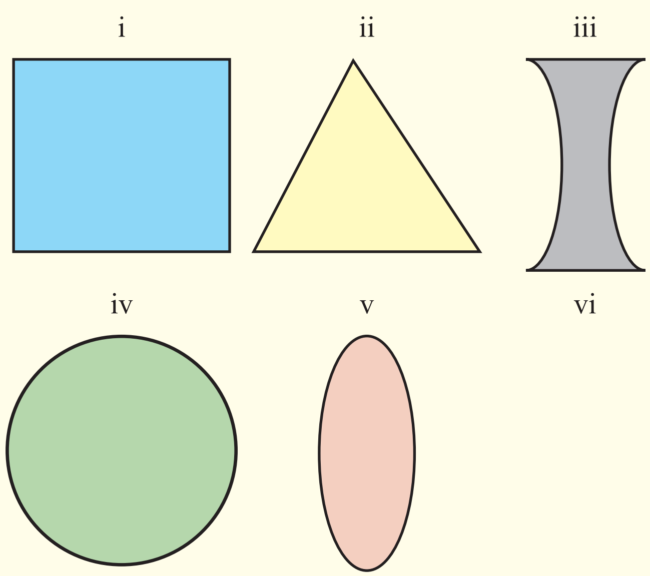


Figure 1.33: An Achromatic doublet

b) With the help of a teacher, have different shapes of glasses. Touch, observe and identify the real shape of the prism.

c) Examine the features of the one selected as a prism. Discuss them with your neighbour and write them in your notebook.

In optics, a prism is transparent material like glass or plastic that refracts light. Atleast two of the flat surfaces must have an angle less than 90° between them. The exact angle between the surfaces depends on the application.

Terms associated with refraction through prism

Activity 27

(i) Place a glass prism on a white sheet of paper fixed on a soft board and mark its outline ABC.

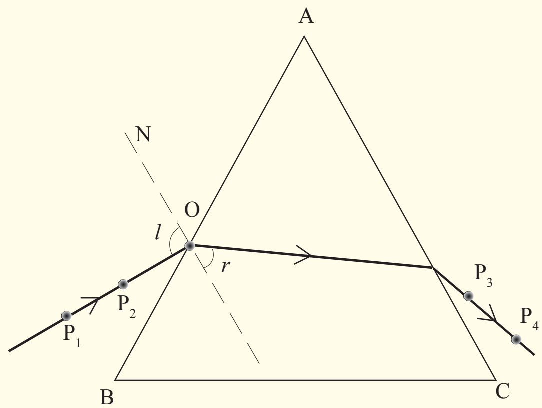


Figure 1.34: Investigating the path of a ray through a prism

(ii) Remove the glass prism and measure an angle between two slanting faces of the prism.

(iii) Draw a normal line ON, and draw a line making a given angle with the normal.

(iv) Place the glass prism back in its outline and stick two pins, P_1 and P_2 in the paper along the line. While looking through the prism through face AC, stick pins P_3 and P_4 in the paper exactly in line with image, I_1 and I_2 of the pins, P_1 and P_2.

(v) Remove the prism and join points P_3 and P_4 .

(vi) Join point O to the point where the line through P_3 and P_4 meets face AC.

(vii) Discuss the observations through presentations.

What name can you give to the angle between the line passing through pins P_1 and P_2 and the normal ON?

What do you think is the name of an angle between the normal ON and the line from the point where the line through P_3 and P_4 meets face AC?

What is the name of the line that passes through P_3 and P_4?

What do you think is the name of an angle between the line passing through P_3 and P_4 and the normal to AC?

Angle A: This is called refracting angle or angle of the prism. It is the angle between the inclined surfaces of the prism.

Angle *i*; This is the angle of incidence on the first face of the prism.

Angle r_1; This is the angle of refraction on the first face of the prism.

Angle r_2; This is the angle of refraction on the second face of the prism.

Angle l_2 ; This is the angle of emergence from second face of the prism. Sometimes this is denoted by letter e.

General formulae for the prism

Activity 28

In groups of four, use geometry of the above drawn figure and derive the relation $r_1 + r_2 = A$.

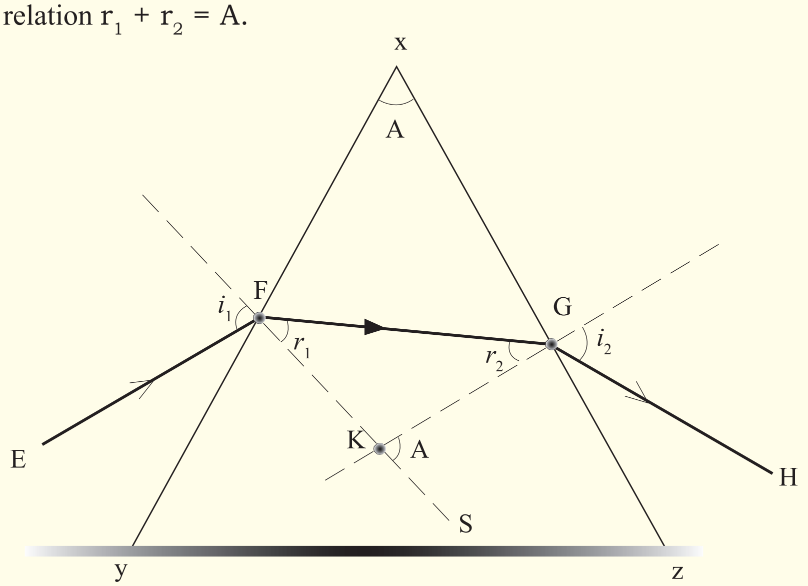


Figure 1.35: Refraction through prism

In the figure, E F is a ray incident on the refracting surface YX of the prism X Y Z from air and then to air from surface XZ of the prism. KF and KG are normals at the points of incidence and emergence of the ray respectively.

Now, from the geometry of quadrilateral XFKG,

$< X\,FK + \; < X\,GK = 180^{O}$ and

$A + <F\,K\,G = 180^{O}$ ……………………(1)

But since FKS is a straight line,

$\Delta FKG + \Delta GKS = 180^{O}$…………….(2)

Comparing equation (1) and (2), it means that, $\Delta GKS = A$.

Using ΔKFG, ΔGKS is an opposite exterior angle of r_1 and r_2

Thus, $r_1 + r_2 = \Delta GKS$.

Hence, $r_1 + r_2 = A$

Note that given i_1, r_1 and i_2, r_2 as angles of incidence and refraction at F and G as shown and n is the prism refractive index, then Snells law holds.

That is; $\text{Sin } i_1 = \text{n sin } r_1$, and

$\text{Sin } i_2 = n \sin r_2$

The position and shape of the third side of the prism does not affect the refraction under consideration and so is shown as an irregular in Fig.

Example

A ray of light falls from air to a prism of refracting angle 60° at an angle of 30°. Calculate the angle of emergence on the second face of the prism (Take refractive index of the material of glass, $n_g = 1.5$).

Solution

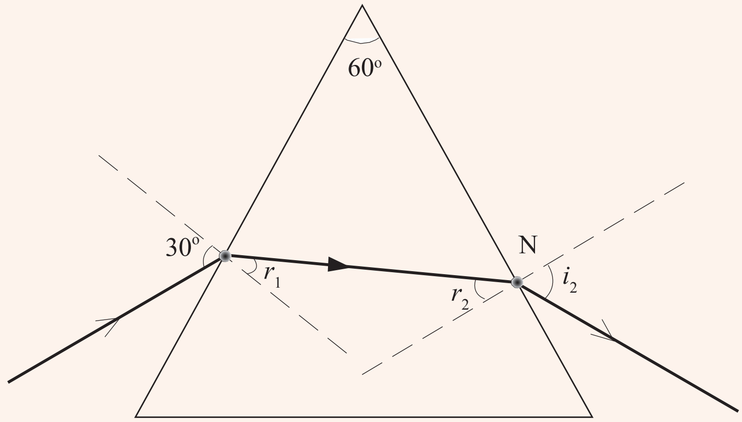


Figure 1.36: Ray through a prism

Using Snell's law, nsini = constant

Thus $n_a \sin l_1 = n_g \sin r_1$

$1 \sin 30^o = 1.5 \sin r_1$

Therefore $\sin r_1 = \frac{0.5}{1.5}$

$r_1 = \sin^{-1} \frac{0.5}{1.5}$

Hence, $r_1 = 19.5^o$

But $r_1 + r_2 = A$

It follows that $r_1 = 60^o - 19.5^o$

$= 40.5^o$

Now, on the second face, $n_g \sin r_2 = n_a \sin l_a$

Thus, $1.5 \sin 40.5^o = \sin i_2$

So, $i_2 = \sin^{-1} (0.974.7)$

Hence the angle of emergence $= 77^o$

Example

A prism of refracting angle of 67º and index of refraction of 1.6 is immersed in a liquid of refractive index 1.2. If a ray travelling through a liquid makes an angle of incidence of 53º. Calculate the angle of emergence of the ray from the second face of prism.

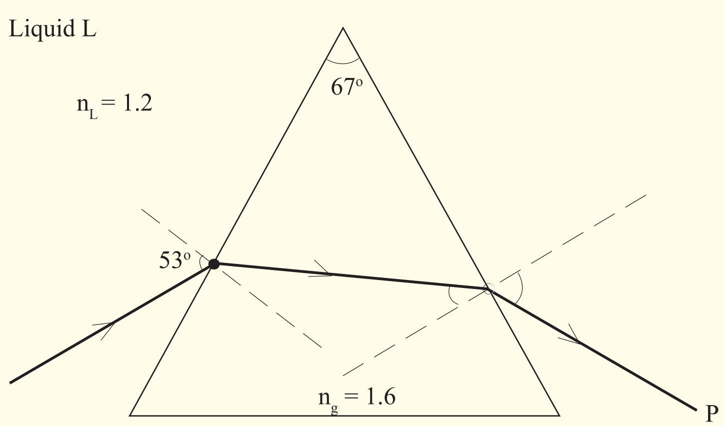


Figure 1.37: Example of a ray through a prism

Suppose that n_l is the refractive index of the liquid

From Snell's law $n_l \sin i_1 = n_g \sin r_1$

Thus, $1.2 \sin 53^o = 1.6 \sin r_1$

$r_1 = \sin^{-1} (0.5990)$

So, $r_1 = 36.8^o$

But $r_1 + r_2 = A$

It follows that $r_2 = 67^o - 36.8^o$

Hence, $r_2 = 30.2^o$

Now

$n_g \sin r_2 = n_L \sin i_2$ (on the second face)

$1.6 \sin 30.2 = 1.2 \sin \iota_2$

$\iota_2 = \sin^{-1} (0.6707)$

Thus, $\iota_2 = 42^o$

The emergent ray makes an angle of 42^o with the normal at the second face of the prism.

Exercise

A ray of light incident from air to a prism of refracting angle 60^o grazes the boundary on the second face of the prism. Find the angle of incidence of the ray on the first face. (Take $n_g = 1.52$).

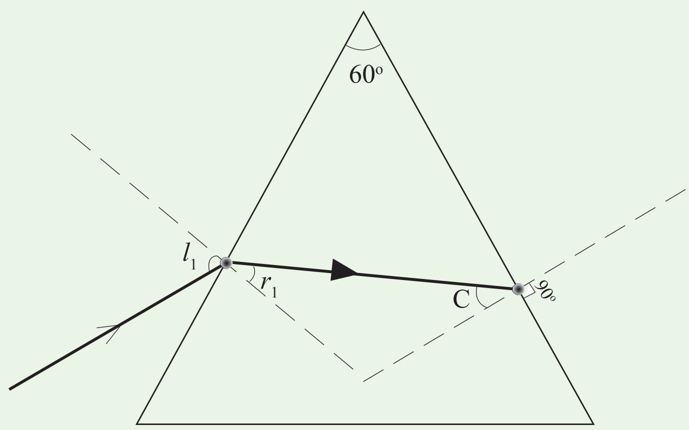


Figure 1.38: Diagram related to exercise

Since the ray grazes the boundary, r_2 = critical angle C and $\iota_2 = 90^o$.

From Snell's law, $n_g \sin C = n_a \sin 90^o$ (on the second face)

But $n_a = 1$ and $\sin 90 = 1$

Thus $\sin C = \frac{1}{1.52}$

Hence $C = \sin^{-1}\frac{1}{1.52}$ $C = 41.1^o$	On the first face, $n_a \sin i_1 = n_g \sin r_1$
Now	Thus, $\sin i_1 = 1.52 \sin 18.9^o$
$r_1 + C = A$	$i_1 = \sin^{-1}(0.4924)$
Thus	Hence, $i_1 = 29.5^o$
$r_1 = 60^o - 41.1^o$	The angle of incidence on the first face $= 29.5^o$.
$= 18.9^o$	

Deviation of light by a prism

Activity 29

Have you ever heard of the word deviation?
List down in your notebook atleast two ways in which light can be deviated.

Light can be deviated by reflection and refraction. Since a prism refracts light, it therefore changes its direction.

Activity 30

You are provided with a glass prism of refracting angle 60^o, four optical pins, a white sheet of paper, a soft board and fixing pins.

(i) Place a prism on a white sheet of paper and mark its outline ABC.

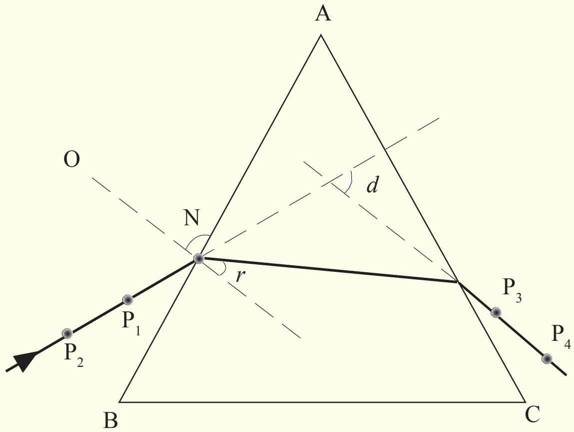


Figure 1.39: Diagram related to activity

(ii) Remove the prism and draw a normal line ON to face AB and draw a line making an angle of 10º to ON to represent the incident ray.

(iii) Place back the prism in its outline and fix pins P_1 and P_2 along the line.

(iv) While looking through the other face AC of the prism, fix pins P_3 and P_4 so that they appear in line with images of P_1 and P_2.

(v) Remove the prism and draw a line through P_3 and P_4 on to face AC of the prism.

(vi) Measure the angle of deviation d.

A prism deviates light on both faces. These deviations do not cancel out as in a parallel sided block where the emergent ray, although displaced, is parallel to the incident ray surface. The total deviation of a ray due to refraction at both faces of the prism is the sum of the deviation of the ray due to refraction at the first surface and its deviation at the second face.

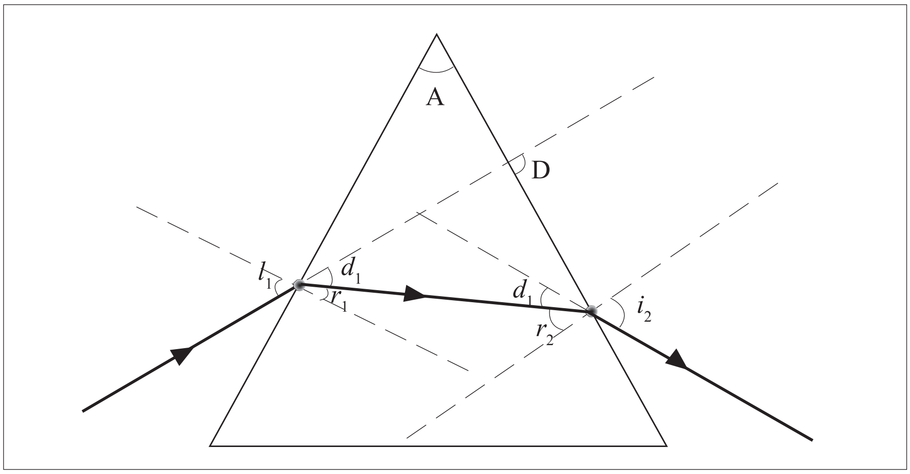


Figure 1.40: Diagram related to activity

Let d_1 and d_2 be angles of deviation at the first and second faces of the prism respectively.

Total deviation $D = d_1 + d_2$

Angle of deviation at the first face, $d_1 = i_1 - r_1$ and the angle of deviation at the second face, $d_2 = i_2 - r_2$

Thus $D = i_1 - r_1 + i_2 - r_2$

$= (i_1 + i_2) - (r_1 + r_2)$

But $r_1 + r_2 = A$

Therefore $D = (i_1 + i_2) - A$

Angle of minimum deviation and determination of refractive index *n* of a material of the prism

Activity 31

(i) Place a prism on a white sheet of paper and mark its outline ABC.

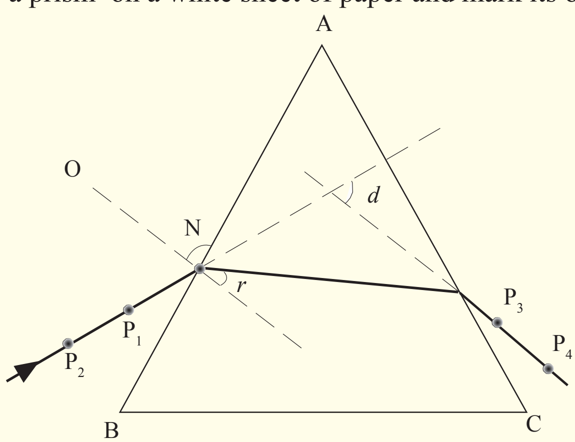


Figure 1.41: A prism on a sheet of paper

(ii) Remove the prism and draw a normal line ON, and then several lines at different angles to ON to represent the incident rays.

(iii) Place the prism back in its outline and fix pins P_1 and P_2 along one line.

(iv) While looking through the other face AC of the prism, fix pins P_3 and P_4 in such a way that they appear in line with images of P_1 and P_2.

(v) Remove the prism, and draw a line through P_3 and P_4 .

(vi) Measure angle of deviation d of the ray.

(vii) Repeat the above procedures for other values of *i*.

(viii) Record your values in a suitable table.

I/°	d/°

(ix) Plot a graph of deviation d against angle of incidence.

The graph is a U-curve and its minimum value corresponds with the angle of minimum deviation. So D_{min} can be read from the deviation axis.

Experiment shows that as the angle of incidence i is increased from zero, the deviation begins to decrease continuously to some minimum value D_{min} and then increases to a maximum as i is increased further to 90°.

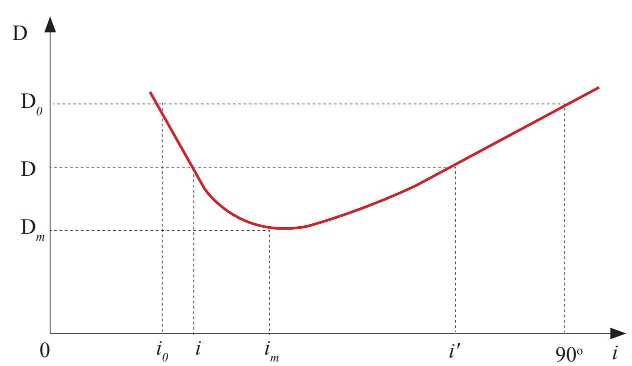


Figure 1.42: Curve of minimum deviation

From the variation fig 1.41 above, there is one angle of incidence which gives a minimum deviation. The experiment shows that this minimum deviation occurs when the angle of emergence is exactly equal to the angle of incidence and the two anternal angles of refraction are equal. At this value, a ray passes symmetrically through the prism and the ray inside the prism is perpendicular to the directing plane, see figure 1.42.

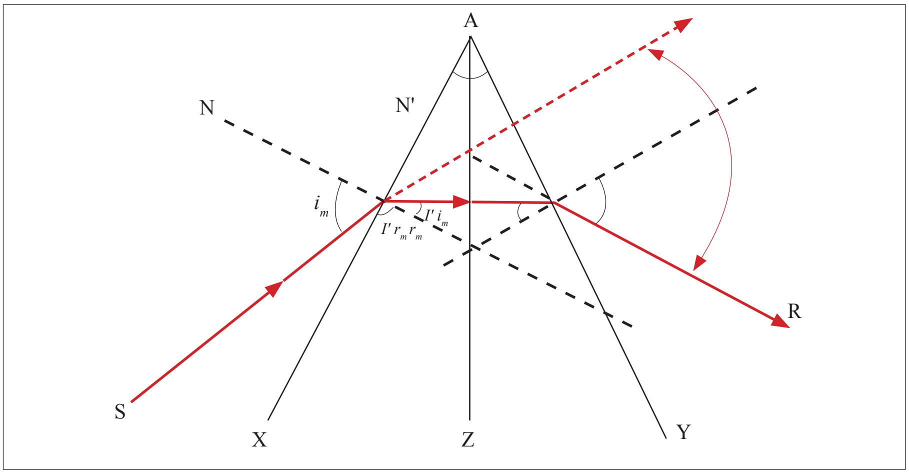


Figure 1.43: Minimum deviation

Since the angle of emergence i_2=angle of incidence i_1, it follows that

$i_1 = i_2 = i$ and $r_1 = r_2 = r$

From deviation $D = i_1 + i_2 - A$

$D_{\min} = i + i - A$

$D_{\min} = 2i - A$

Angle of minimum deviation and the refractive index *n* of the material

Experimentally, it is shown that when the angle of incidence increases, the deviation decreases, passes at a maximum then increases.

When the deviation is minimal, the angles of incidence and emergence are equal.

Considering the equation $A = r_1 + r_2$ we have: $r_1 = r_2 = r_m \frac{A}{2}$, and also equations $\sin i_1 = n\sin r_1$ and $\sin i_2 = n\sin r_2$ become identical and give: $i_1 = i_2 = i_m$

This allows us to calculate for which incidence we have the minimum deviation. Finally the last equation gives the value of that deviation: $D_m = 2i_m - A$

From these relations, we deduce: $i_m = \frac{D_m + A}{2} \Rightarrow sin\ \frac{D_m + A}{2} = nsin\ \frac{A}{2}$

Example

A glass prism of refracting angle 60° has a refractive index of 1.5. Calculate the angle of minimum deviation for a parallel beam of light passing through it.

Solution

$$\Rightarrow n = \frac{\sin\left(\dfrac{D_m + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

$$\text{Thus, } 1.5 = \frac{\text{Sin}\left(\dfrac{D_{min}+60}{2}\right)}{\sin\left(\dfrac{60}{2}\right)}$$

It follows that $D_{min} = 2\sin^{-1}(0.75) - 60$

$= 97.2^0 - 60^o$

Hence, $D_{min} = 37.2^0$.

Exercise

A glass prism of refracting angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. What is the angle of minimum deviation for a parallel beam of light passing through the prism?

Deviation of light by a small angle prism

Consider a ray incident almost normally in air in a prism of small refracting angle A (less than about 6^0 or 0.1 radian) so that the angle of incidence i is small.

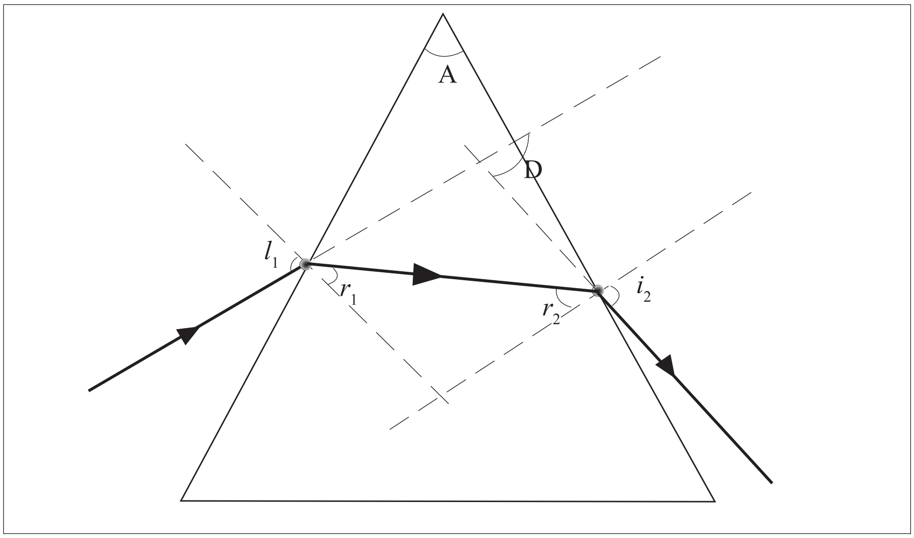


Figure 1.44: Small angle prism (Angles exaggerated for clarity)

Since l is small, r is also small.

Now if n is the refractive index of the material of the prism, $n_a \sin i_1 =$ n$\sin r_1$

For $n_a = 1$ (refractive index of air);

$$n = \frac{\text{Sin } i}{\text{Sin } r} \text{ .... (1)}$$

For small angles, the sine and tangent of an angle is nearly equal to the angle in radians.

Thus $\sin i \approx i$ and $\sin r \approx r$

Therefore $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$

Hence equation (1) becomes $i_1 = \text{n } r_1$................(2)

Also $A = r_1 + r_2$, and so if A and r_1 are small, r_2 and i_2 will also be small.

From nsinr$_{2\,=}$ n_a sin l_2 and for sin $i_2 \approx i_2$

It follows that $n\, r_2 = n_a\, i_2$

But $n_a = 1$

Thus $i_2 = n\, r_2$..........................(3)

But the deviation D of a ray passing through any prism is given by

$D = (i_1 - r_1) + (i_2 - r_2)$

Substituting for i_1 and i_2

$\text{D} = nr_1 - r_1 + nr_2 - r_2$

$\quad = (\text{nr}_1 + \text{nr}_2) - (\text{r}_1 + \text{r}_2)$

$= n\,(r_1 + r_2) - (r_1 + r_2).$

But $A = r_1 + r_2$

So $D = An - A$

Factorising out A,

Hence $D = A\,(n - 1)$

Thus the deviation D for a small angle prism is $D = A\,(n\text{-}1)$

The expression $D = A\,(n - 1)$ shows that for a given angle A, all rays entering a small angle prism at small angles of incidence suffer the same deviation.

Example

Light is incident at a small angle on a thin prism of refracting angle 5^o and refractive index 1.52^o. Calculate the deviation of the light by the prism.

For small angle prism, $D = (n - 1)$ A

$n = 1.52$ and $A = 5$

Thus $D = (1.52 - 1)5.$

$= 0.52 \times 5 = 2.6^o$

Example

A mono chromatic light is incident on one refracting surface of a prism of refracting angle 60^o, made of glass of refractive index 1.50. Calculate the least angle of incidence for the ray to emerge through the second refracting surface.

Solution

The least angle is the angle of incidence for which there is grazing emergence at the second face of the prism.

On the second face, $\sin 90 = n_g \sin C$

Thus $1 = 1.5 \sin C$

Hence $C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^o$

Since $R + C = A$

It follows that $r = 60^o - 41.8^o = 18.2^o$

Now on the first face, $1\text{x} \sin i = 1.5 \sin 18.5$

$i = \sin^{-1}(0.4685)$

Thus $i = 27.9^o$

So the least angle of incidence for a ray to emerge on the second face of a 60° prism of refractive index 1.5 is 27.9°.

Determination of refractive index of a material of a prism

Activity 32

(i) Place a glass prism on a white sheet of paper fixed on a soft board and mark its outline ABC.

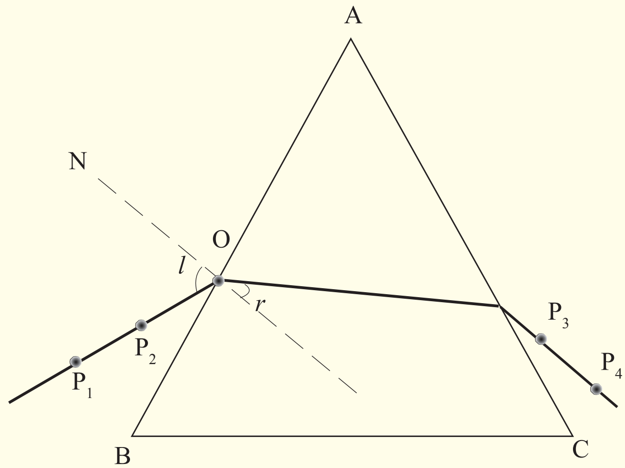


Figure 1.45: Diagram related to this activity

(ii) Remove the glass prism and draw a normal line ON, and several lines at different angles to ON to represent incident rays.

(iii) Place the glass prism back in its outline and stick two pin P_1 and P_2 in the paper along one of the lines drawn to represent an incident ray. While looking through the prism through face AC, stick pins P_3 and P_4 in the paper exactly in line with image, I_1 and I_2 of the pins, P_1 and P_2.

(iv) Remove the prism and join points P_3 and P_4 . This line represents the emergent ray.

(v) Join point O to the point where the line through P_3 and P_4 meets face AC. This ray represents the refracted ray.

(vi) Measure angle of refraction, *r*.

(vii) Repeat the above procedures for other values of *i*.

(viii) Record your results in a table including values of sin *i* and sin *r*.

i^o	r^o	sin$i$	sin$r$

(ix) Plot a graph of sini against sinr and find the slope of the graph.

(x) Discuss through group presentation about the graph obtained.

The graph is a straight line graph and the gradient represents the mean value which is the refractive index of the material.

Dispersion of light by a prism

Activity 33

(i) Place a plane mirror in a basin and then pour water into the basin.

(ii) Leave the water to settle and slowly place the basin on sunshine so that the plane mirror reflects the light rays from the sun on a white wall or iron sheet.

(iii) Observe what is formed on the wall(iron sheet).

Discuss with your group and write in your notebook about the observation.

Sunlight split into many colours when it fell into water. This process is called dispersion. Dispersion is the splitting of light into its component colours.

Activity 34

(i) Place a prism in the centre of a piece of paper so that its refracting surface is directly facing the windows in order to receive light from the sun.

(ii) Place a white screen on the far side of the prism so that the refracted rays hit it.

(iii) Observe what is formed on the screen.

(iv) In brief, write in your notebook the observation.

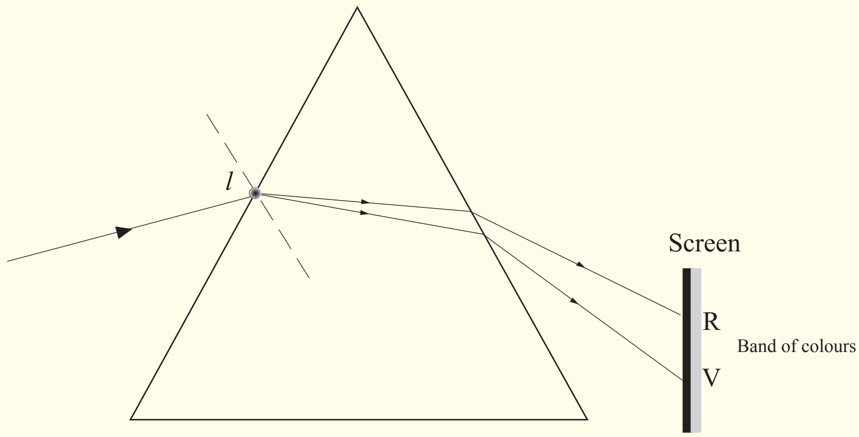


Figure 1.46: Dispersion of white light

A band of seven colours is formed on the screen. The colours are in order of Red, orange, yellow, green, blue, indigo and violet (ROYGBIV) which are colours of rainbow. This band of colours is called a spectrum.

Thus, when a narrow beam of white light falls on a glass prism, it splits into a range of colours and these colours separate to form a spectrum, a process called dispersion. This occurs because white is not a single colour but mixture of all colours of the rainbow. The prism refracts each colour by a different amount because the colours travel at different speeds in the glass and thus the glass has different refractive indices for each colour.

The speed of a red colour is greatest and that of a violet colour is the least, and so the refractive index of a material of the prism for red colour is the least and that of the violet colour is the greatest. Now it follows that since the angle of incidence in air is the same for all the colours, red in deviated least by the prism and the violet rays are the most deviated as shown in the figure above (exaggerated for clarity because the colours overlap).

Applications of total internal reflection of light by a prism

Activity 35

(i) You are provided with a glass prism with angles measuring 45o-45^{o}-90^{o}.

(ii) Place the prism on a sheet of paper and use a ray-box to shine in a ray of light as shown in the figure below.

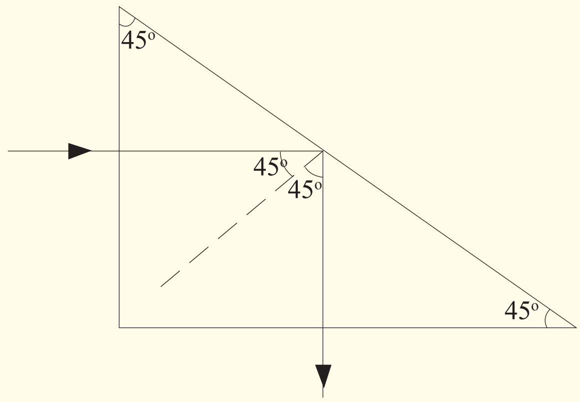


Figure 1.47: Refraction of light through a prism

(iii) What do you notice about the phenomemenon above?

Notice that light goes straight through the first surface and when it meets the second surface, it is internally reflected. So, the long side of the prism acts as a mirror and turns light through an angle of 90°. Two prisms of the same type as above can be arranged in away and used in a periscope; an instrument used to see the top of an obstruction.

Use of prisms in periscopes

Activity 36

(i) Arrange two prisms provided and shine on one of the prisms using a ray box as shown below.

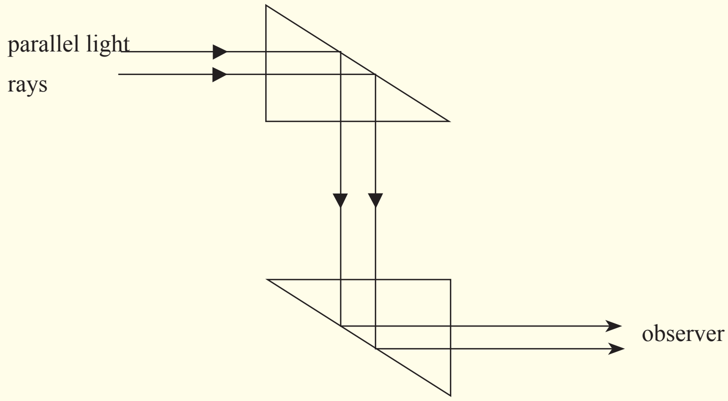


Figure 1.48: Refraction of light through a prism

(ii) Discuss in your respective group about the phenomenon.

Light is tuned through 90^o at each prism and it emerges parallel to the incident light.

In prism periscopes, light from an object is turned through 90^o at each prism ands reaches the observer at a different altitude to that of an object. So the image of the object is formed at another altitude but is same size as object.

Activity 37

(i) Place the same prism above on a piece of paper and use your ray box to shine on it as shown below.

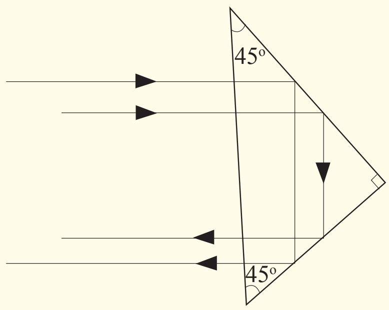


Figure 1.49: Diagram related to activity

(ii) What do you notice?

You can see that rays of light are turned through 180^o.

An arrangement of two prisms each turning light through an angle of 180^o is used in prism binoculars; instruments used to view hidden objects. This will be discussed in the next unit.

Critical Thinking Exercise

a) Give reasons why prism rather than plane mirrors are used in periscopes and prism binoculars.

b) Explain why diamonds are cut with their sides flat and others slanting.

In periscopes and prism binoculars, plane mirrors can be used but prisms are preferred because of the following reasons.

In the first place, a prism allows light to undergo total internal reflection and thus the images are formed by total internal reflection where as a mirror allows light to both reflect and refract at its surface. So for a prism, all the light

(100%) from the object is reflected but for a mirror some light is absorbed (about 95% is reflected) and thus a prism produces a brighter image than a mirror.

The silvering on the mirrors wears off with time but with prism no silvering is needed.

Some mirrors, for example, thick plate mirrors produce multiple images of one object because of reflections and refractions at the surfaces and inside the glass but a prism produces anyone image.

Diamonds are cut that way so as to make use of total internal reflection. The multiple reflections inside diamond make it bright.

Exercises

1. A ray of light incident at an angle i on a prism of angle, A, passes through it symmetrically. Write an expression for the deviation, d, of the ray in terms of i and A. Hence find the value of d, if the angle of the prism is 60^o and the refractive index of the glass is 1.48.

2. A beam of monochromatic light in incident normally on the refracting surface of a 60^o glass prism of refractive index 1.62. Calculate the deviation caused by the prism.

3. a) Define the critical angle of a medium.

   b) One side of a triangular glass prism put in a pool of water of refractive index 4/3 and the other side was left open to air. A ray of light from water was incident on the prism at an angle i = 21.7^o. The light just grazes as it emerges out of the prism. Given that the refractive index of glass 1.52, determine the refracting angle A of the prism.

4. A monochromatic light is incident at an angle of 45^o on a glass prism of refracting angle 70^o in air. The emergent ray grazes the boundary of the other refracting surface of the prism. Find the refractive index of the material of glass.

5. A prism of diamond has a refracting angle of 60^o. A ray of yellow light is incident at an angle of 60^o on one face. Find the angle of emergence if the refractive index of diamond for yellow light is 2.42.

6. A ray of light just undergoes total internal reflection at the second face of a prism of refracting angle 60^o and refractive index 1.5. What is its angle of incidence on the first face?

7. A sharp image is located 78.0mm behind a 65.0mm-focal-length converging lens. Find the object distance (a) using a ray diagram, (b) by calculation.

8. What is (a) the position, and (b) the size of the image of a 7.6cm high flower placed 1.00m from a 50.0mm focal length camera lens?

9. An object is placed 10cm from a lens of 15m of focal length. Determine the image position.

10. Two converging lenses A and B, with focal lengths fA=20cm and fB = -25cm, are placed 80cm apart, as shown in the figure (1). An object is placed 60cm in front of the first lens as shown in figure (2). Determine (a) the position, and (b) the magnification, of the final image formed by the combination of the two lenses.

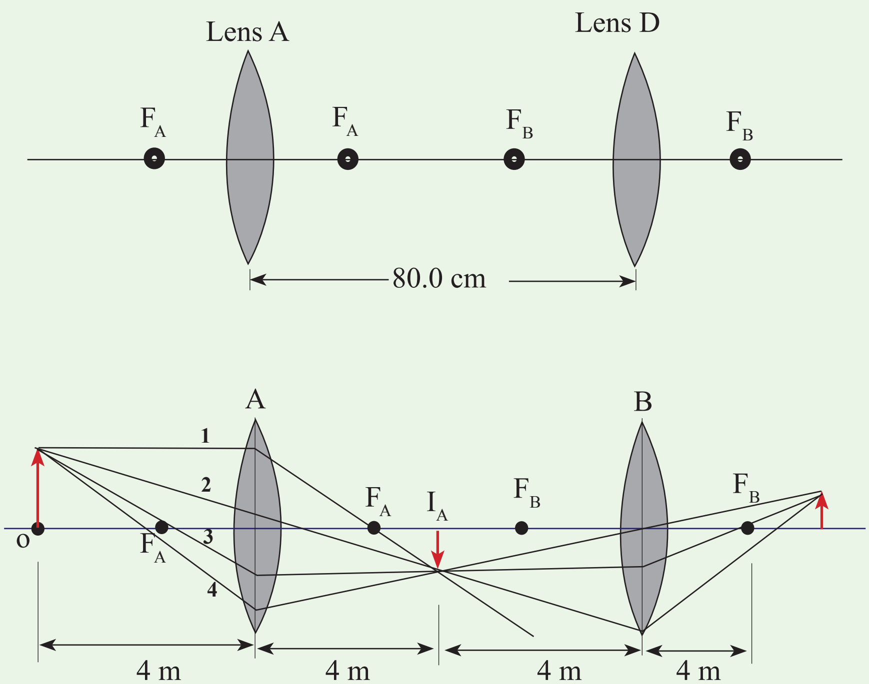


Figure 1.50

11. Where must a small insect be placed if a 25cm focal length diverging lens is to form a virtual image 20cm in front of the lens?

12. Where must a luminous object be placed so that a converging lens of focal length 20cm produces an image of size four times bigger than the object (Consider the case of a real image and the case of a virtual)

13. From a real object AB we want to obtain an inverted image four times bigger than the object. We place a screen 5m away the object. Specify the kind, the position and the focus of the lens to use. Give the graphical and the algebraic.

14. In cinematography the film is located at 30m from the screen and the image has a magnification of 100. Determine the focal length of the lens used in projection.

15. An object AB of 1cm is placed at 8cm from a converging lens of focal length 12cm. Find its image (Position, nature and the size).

16. An object of 2cm is placed at 50cm from a diverging lens of focal length 10cm. Determine its image.

17. An object located 32.0 cm infront of a lens forms an image on a screen 8.00 cm behind the lens. (a) Find the focal length of the lens. (b) Determine the magnification. (c) Is the lens converging or diverging?

18. A movie star catches the reporter shooting pictures of her at home. She claims the reporter was trespassing. To prove her point, she gives as evidence the film she seized. Her 1.72m height is 8.25mm high on the film and the focal length of the camera lens was 210mm. How far away from the subject was the reporter standing?

19. A lighted candle is placed 33cm in front of a converging lens of focal length f1=15cm, which in turn is 55cm in front of another converging lens of focal length f2=12cm. (a) Draw a ray diagram and estimate the location and the relative size of the final image. (b) Calculate the position and relative size of the final image.

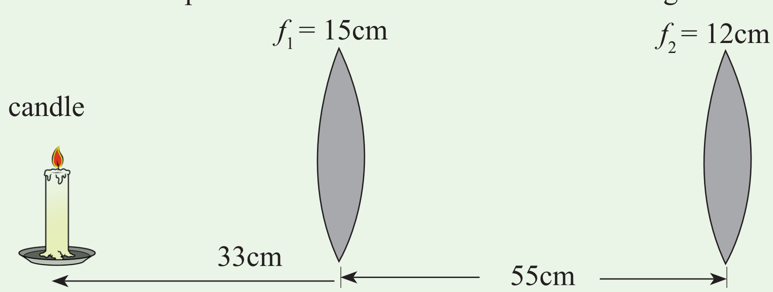


Figure 1.51

20. When an object is placed 60cm from a certain converging lens, it forms a real image. When the objet is moved to 40cm from the lens, the image moves 10cm farther from the lens. Find the focal length of this lens.

21. A converging glass lens (n=1.52) has a focal length of 40.0cm in air. Find its focal length when it is immersed in water, which has an index of refraction of 1.33.

22. Verify that the focal length f of a symmetrical biconvex lens which the two faces have a radius of curvature R and refractive index 1.5 is $f_{\text{meter}}=R_{\text{meter}}$.

23. We put in contact a converging lens of focal length 20cm and a diverging lens of focal length 50cm. What are the nature, the power and the focal length of the constituted lens?

24. To a converging lens of focal length 20cm we put in contact a second lens so that the constituted system has the power of 4 diopters. Determine the nature of the second lens and calculate the focal length.

25. In a physics lab students want to determine the focal length x of a thin diverging lens. They stick to it a converging lens of 5 diopters and they use the system to have a real and inverted image A'B' of size equal to the one of the object AB. The distance from the object AB to the screen where they watch the image is 4m. Calculate x.

26. A thin glass lens n = 1.5 has a focal length +10cm in air. Compute its focal length in water n = 1.33.

27. A prism which has a refracting angle equals 60^o and refractive index 1.5 receives a ray at an angle of incidence 45^o; calculate the angle of emergence and the deviation of the ray.

28. Calculate for the same prism (Question 1) the value of minimum deviation as well as the value of $i = i'$.

29. Let consider a prism made in glass of refracting angle $A=59^o$ and the refractive index 1.52.
    a) Calculate the deviation that makes an emerging ray with the extension of the incident ray for an incidence equal to 35^o.
    b) Calculate the angle of minimum deviation and specify the value of the angle of the corresponding angle of incidence and refraction inside the prism.

30. Given that a prism of refracting angle $A = 60^o$ and refractive index $n=\sqrt{3}$

31. Let consider a ray of light falling on a prism through an angle $i=90^o$. If it goes out the prism through an angle i', calculate i'.

32. Through what angle i must fall on the prism a ray to go out through an emergence $i =90^o$.

33. Find the refractive index of a prism $A = 60^o$ producing a minimum deviation equal to 40^o.

34. A triangular glass prism with apex angle 60.0° has an index of refraction of 1.50. (a) Show that if its angle of incidence on the first surface is $\theta_1 =48.6°$, light will pass symmetrically through the prism, (b) Find the angle of deviation D_{min} for $\theta_1 = 48.6°$. (c) What If? Find the angle of deviation if the angle of incidence on the first surface is 45.6°. (d) Find the angle of deviation if $\theta_1 = 51.6°$.

35. A triangular glass prism with apex angle $\Phi = 60.0°$ has an index of refraction $n= 1.50$. What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

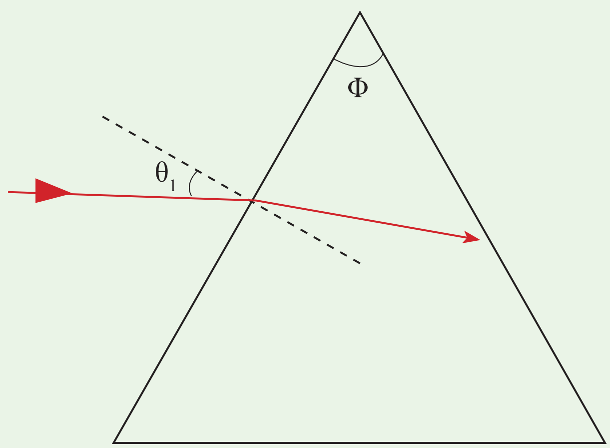


Figure 1.52: Exercise 35

36. A triangular glass prism with apex angle Φ has index of refraction n. What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

37. Place a triangular glass prism on a white sheet of paper and draw its outline.

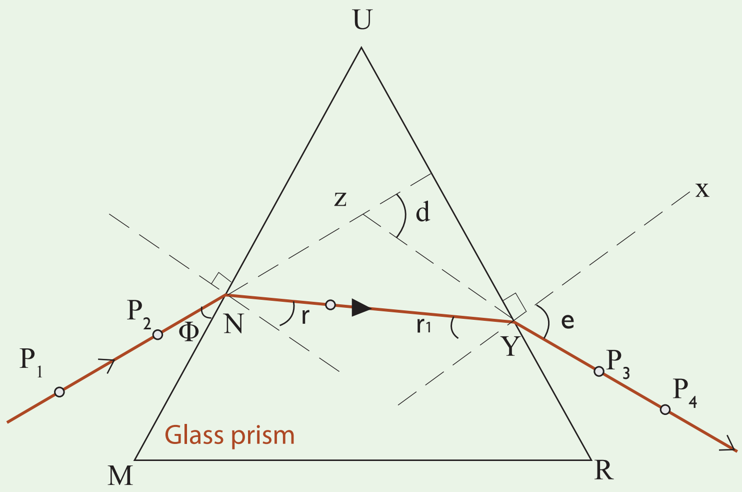


Figure 1.53: Exercise 37

(i) Remove the prism and label the outline as MUR
(ii) Draw a perpendicular line to the face MU of the prism atN.
(iii) Draw a line TN so that it makes a relatively small angle with normal at N.
(iv) Replace the prism in its outline.
(v) Place pins P_1 and P_2 along TN and perpendicular to the paper. While looking through the other face UR of the prism, fix pins P_3 and P_4 so that they are in line with images of P_1 and P_2.
(vi) Remove the prism and draw a line through P_3 and P_4.
(vii) Repeat the above procedures with the same prism but turned upside down so that its refracting angle is facing upwards. What do you notice?

Can you see that the rays are coming to a point?

38. Copy the diagram below and fill in the names and use of other optical instruments you know.

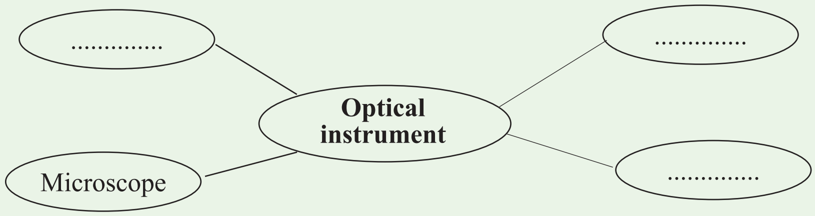


Figure 1.54: Question 38

39. Use the concept obtained from unit 1 and write the definition and main function in the table below in your notebook.

Table: Definition and functions of optical terms

Term	Definition and function
Cornea	
Pupil and Iris	
Lens	
Retina	
Rods and cones	

40. Match the following statements with corresponding name of vision problems in the table below in your notebook.

Table: Vision problems

Vision problems	**Name corresponding to vision**
The rays focus before the retina	
Distant objects appear blurry	Near sightedness
Occur either because the cornea is too curved or the eye ball is too long.	
It can be corrected using diverging (concave) lens.	Far sightedness
Near images appear blurry	
Occur either because a cornea is not curved enough or an eyeball is too short.	
Distant objects appear clearly	

LIGHT

Optical Instruments

Unit 2

Simple and compound optical instruments

Key unit Competence

By the end of the unit, the learner should be able to analyse the functioning of simple and compound instruments and determine their magnifying power.

My goals

By the end of this unit, I will be able to:

* explain an optical instrument.
* explain the physical features of a human eye.
* describe the image formation by the eye.
* identify the physical features of a simple and compound microscope.
* explain the applications of simple and compound microscopes.
* differentiate between simple and compound microscopes.
* explain the operation of a lens camera and its application.
* explain the operation of a slide projector and its applications.
* describe the physical features of a telescope.
* list different types of telescopes.
* demonstrate the operation of telescopes.
* differentiate between telescopes and microscopes.
* identify the physical features of prism binoculars.

Introduction

Once the rules for predicting how rays travel through lenses have been discussed, guide your learners to discover that; a fantastic range of practical devices began to appear which aided the development of the modern world. The simple magnifying glass became the basis for telescopes, microscopes and spectacles. These devices were modified to improve the projection of images and with the discovery and development of light-sensitive chemicals, gave birth to modern photography and cinematography.

Definition of an optical instrument

Activity 1

(i) What objects (things) do you see in the classroom?

(ii) Move outside class and observe the kind of objects there, and write down atleast five of them.

(iii) Look at the distant objects. Are you able to examine the objects in a more detailed manner? Do you think you can be able to see these objects at night?

We use our eyes to see and view different objects. The eye cannot be used to view clearly these objects at night, and some distant objects or hidden objects. Objects which cannot be viewed by the eye can be focused using other instruments. All the instruments used to aid vision are called Optical instruments.

Man has always shown interest in observing things in a more detailed manner. In your early secondary, you looked at the uses of mirrors. We have also learnt in unit 1 of this book that lenses are used to focus objects. When the lenses or mirrors or both are arranged in a way, the arrangement can be used to observe objects in a more detailed manner. The arrangement makes what we call a compound optical instrument. The compound instruments include a compound microscope, telescopes, prism binoculars etc.

Angular magnification or magnifying power of an optical instrument

The human eye

The eye is a biological instrument used to see objects at different distances. It uses a convex lens system to form a small, inverted, real image of an object infront of it.

Structure of the eye

Activity 2

(i) In groups of two, look at one another's eye.

(ii) Observe critically its external shape.

(iii) Observe it carefully and note its behaviour as one tries to see some objects in class.

Notice that the eye ball is round and fleshy.

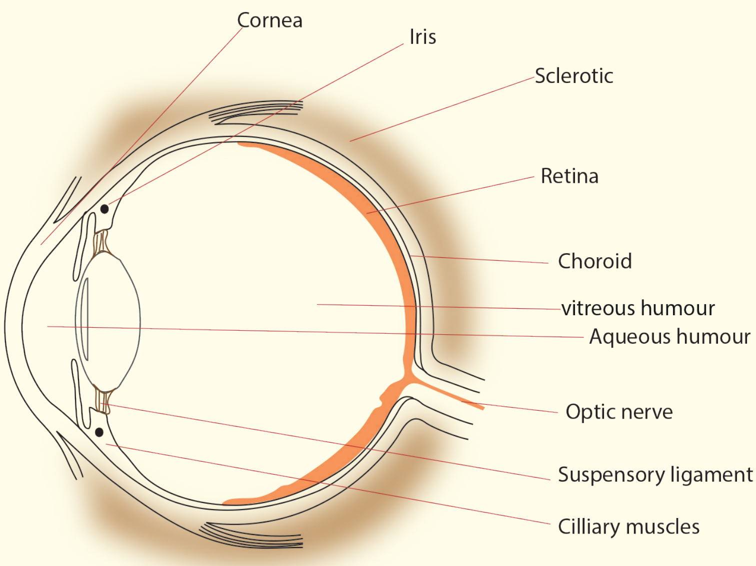


Figure 2.1: Anatomy of an eye

Functions of the parts of the eye

The cornea: It is made out of a fairly dense, jelly like material which provides protection for the eye, and seals in the aqueous humour. It also provides most of the power of the eye (59 Dioptres), having about 46 Dioptres. So it provides most of the bending of light rays.

The aqueous humour: This is a waterly liquid that helps to keep the cornea in a rounded shape, similar to that of a lens.

The iris: This controls the amount of light entering the eye. The amount of light that enters the eye is one of the factors determining how focused an image is on the retina. The brighter the light the eye is exposed to, the smaller the iris' opening will be. The brighter the light the eye is exposed to, the smaller the iris' opening will be. The iris is the coloured part of the eye as seen from the outside. The iris opening or a gap through which light passes is called a pupil.

The lens: This is used to focus an image on the retina. It controls the bending of light rays by change of its shape, a process called accommodation, which is done by the ciliary muscles.

The ciliary muscles: These control the thickness of the lens during focusing. By contracting or squeezing the lens, they make it thicker and vice versa. Because the power of the lens is directly related to its thickness, the ciliary muscles change the power of the lens by their movement.

The retina: This is the light sensitive part of the eye and it is where images are formed. It contains millions of tiny cells which are sensitive to light. The cells send signals along the optic nerve to the brain. So the retina is the screen of the eye and the image is formed by successive refraction at the surfaces between air, the cornea, the aqueous humour, the lens and vitreous humour. The retina is black, which prevents any light rays that hit it from reflections and thereby changing the image.

The vitreous humour: This is a jerry like substance that helps the eye to keep its round shape. It is very close in optical density to the lens material.

The yellow spot: This is a small area on the retina where the sharpest image, that is, the finest detail can be seen.

The optic nerve: This is the nerve that transmits images received by the retina to the brain for interpretation. The part of the eye where the optic nerve joins the retina is called the **blind spot** because no images can be observed at at this point.

Visual Angle

Activity 3

(i) Go outside class and view the trees around.

(ii) Are the trees of the same height?

Notice that some trees at a distance, look shorter than the nearby trees when it is not the case? Why do you think it is so?
Discuss and write down in your notebook about your observation.

The height of an object depends on the angle of elevation of its top from the eye. The larger the angle, the taller the objects. This angle is called the visual angle.

The visual angle is the angle subtended at the eye by an object.

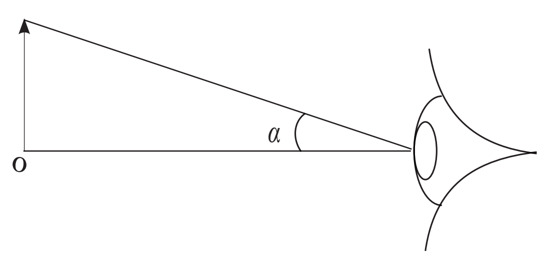


Figure 2.2: Visual Angle

Let us observe the flame of a candle: its two extremities A and B are seen by an eye at a certain angle. Expressed in radians, this angle has a measure: $\alpha = \frac{AB}{D}$

This angle decreases when the distance D increases and increases when the distance D decreases. It also increases when the length AB increases and decreases when AB decreases. We call it visual angle of the object.

Lead the learners to define the visual angle of an object as the angle between two rays of light from extremities of the object and penetrating into the eye of an observer.

Activity 4

(i) In groups of four, explain why trees in a forest appear to be of the same size.

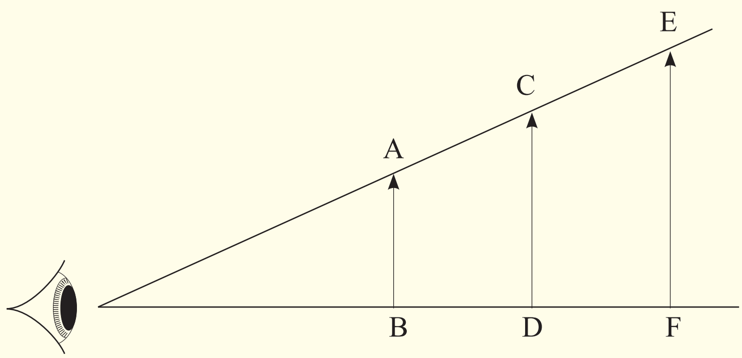


Figure 2.3: Visual Angle of trees

Objects that subtend the same angle at the eye appear to be of the same size as viewed by the eye.

The apparent size of an object depends on the size of its image on the retina. For example, the two objects above; AB and CD appear to have same size because they subtend the same angle θ at the eye. This explains why trees in a forest appear to have the same height.

It is defined as the ratio of the apparent size of the final image i.e angle subtended by the image at the position of eye to the apparent size of the object i.e angle subtended by the object at the eye.

We have seen that we can use other instruments apart from the eye to aid vision. So, angular magnification or magnifying power of an optical instrument can also be defined as the ratio of the angle subtended at the eye by the image when the optical instrument is used to the angle subtended by the object at the unaided eye (when the instrument is not used).

If β is the angle subtended at the eye by the image and α is the angle subtended by the object at some distance by unaided eye, then the angular magnification $M = \frac{\beta}{\alpha}$

Accommodation of the eye

Accommodation of the eye is the ability of the eye to see near and distant objects. The eye is capable of focusing objects at different distances by automatic adjustment of the thickness of the eye lens which is done by the ciliary muscles. To focus a distant object, the eye lens is made thinner, so less powerful, and the rays from the object are brought to focus on the retina by the eye lens. In this case, the ciliary muscles are relaxed and pull the lens. For nearer objects, the eye lens must be made thicker and hence more powerful so that the rays from the near object can be brought to a focus on the retina. In this case, the ciliary muscles tighten and squeeze the lens.

Near point and far point of the eye

Activity 5

(i) Hold a book at an arm's length and move it closer to find the nearest distance that you can focus the words clearly without straining your eyes.

(ii) Approximate the distance between your eyes and the book.

(iii) What does this distance represent?

The near point of the eye is the nearest point that can be focused by the un aided eye. It is a closest distance that the 'normal' human eye can observe clearly; without any strain to the eye. It is called the least distance of distinct vision. The near point of a normal eye is 25 cm.

Activity 6

(i) Look at the trees around your school.

(ii) Now, try to look at objects far from the school.

(iii) Are you able to focus the distant objects?

(iv) Measure this distance from the object to your eye.

(v) Write down your observation in the notebook.

Notice that you can not be able to measure this distance.

The distance from a distant object to the eye is the far point of the eye. The far point of the eye is infinity.

The far point is the farthest point that can be focused by the eye.

The distance of 25 cm from the eye is called distance of most distinct vision or least distance for distinct vision. The range of accommodation of the normal eye is thus from 25 cm to infinity. This range is based upon the average human eye which has an age of 40 years. Young persons have a much wider range but the average 70 year – old has a reduced range.

People with normal vision can focus both near and distant objects.

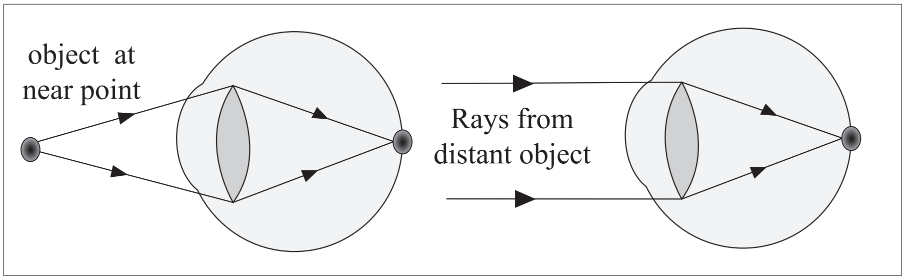


Figure 2.4: Near and far points of a normal eye

Defects of vision and their correction

Activity 7

(i) Have you seen before some people putting on eye glasses?

(ii) What do you think these glasses(spectacles) are used for.

People put on eye glasses for different reasons. Some people wear them in order to read a text, some put them on to see near objects if their eyes cannot be able to do so while others put them on so as to focus distant objects; others wear them for fan like sun goggles

Short-sightedness (myopia)

Activity 8

(i) Hold a book at an arm's length and move the lens so that the prints are read without the eye getting strained.

(ii) Now, try to read the words on a chalkboard a distance from the book.

(iii) Are you able to focus both near and distant objects?

People with normal vision can focus clearly near and distant objects. Those who only focus near objects are said to be short-sighted, meaning that they see nearer.

Short-sightedness is the defect whereby a person can see near objects clearly but cannot focus distant objects. His far point is nearer than infinity. This is because the eyeball is too long or the lens is too strong so that rays of light from a distance object are focused in front of the retina.

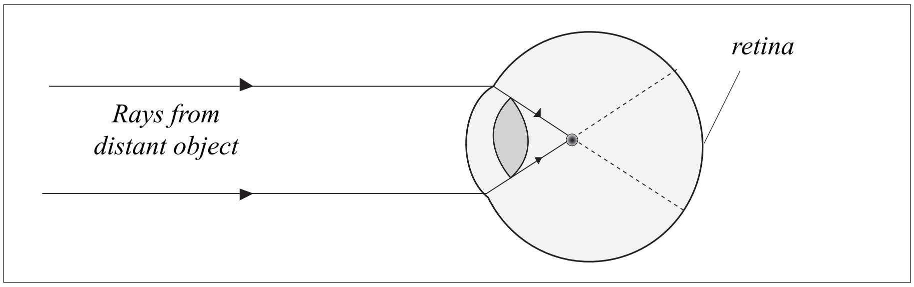


Figure 2.5: Short-sightedness

The rays are focused in front of the retina because the focal length of the eye lens is too short for the length of the eye ball. This defect can be corrected by wearing a concave (diverging) spectacle lens. The rays of light from a distant object are diverged so that they appear to come from a point near, and so they are focused by the eye.

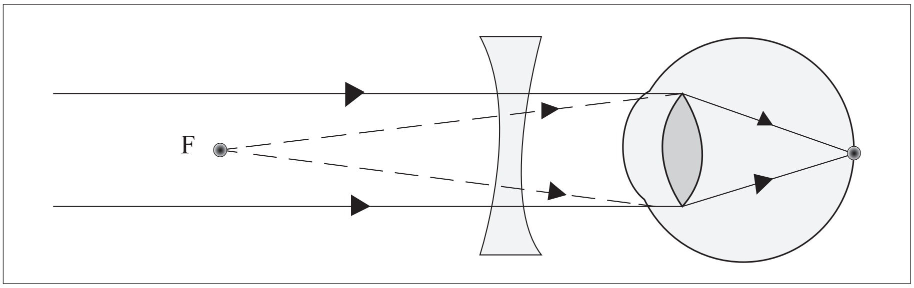


Figure 2.6: Correction of short sight

Rays from object at infinity appear to come from a near point F and converge to the retina.

Long-sightedness (hypermetropia)

This is where a person is able to see distant objects clearly but cannot focus near objects.

This is because either his eye ball is too short or the eye lens is too weak (thin) so that rays of light from a close object are focused behind the retina.

This eye's near point is further than 25 cm.

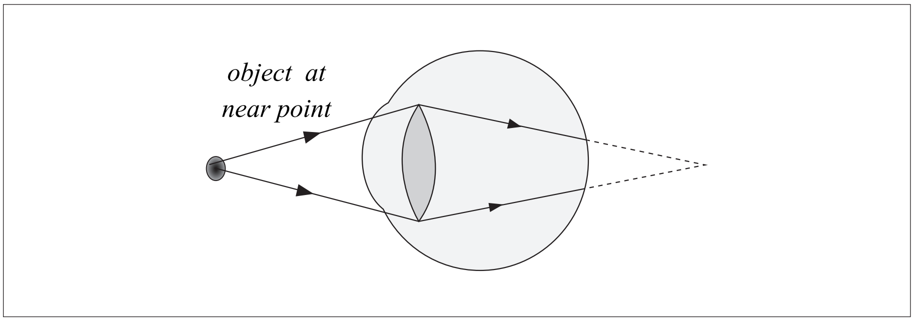


Figure 2.7: Long-sight

The image of the near object is focused behind the retina because the focal length of the eye lens is too long for the length of the eye ball.

This defect can be corrected by wearing a convex lens spectacle. The rays of light from a near object are converged so that the rays appear to come from a point far, and so are focused by the eye.

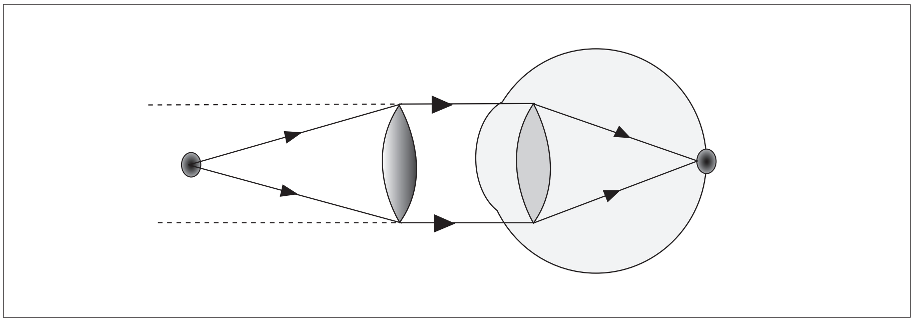


Figure 2.8: Correction of long sight

Rays from a near object O appear to come from a distant object.

Presbyopia

Activity 9

(i) How many of you still have their grandparents?

(ii) Have you ever tried to observe how grand parents observe objects?

(iii) Discuss with your neighbour and write in your notebook results of your discussion.

When people grow older, their eye lens become stiff and it becomes hard for the ciliary muscles to adjust it. Such people have a defect called Presbyopia. Presbyopia is the stiffening of the eye lens such that it is less capable of being adjusted by the ciliary muscles. This means that the eye lens becomes less flexible and loses its power (ability) to accommodate for objects at different distances.

This defect is corrected by wearing bifocals spectacles whose lenses have a top part for looking at distant objects and a bottom part for close ones. These bifocal spectacles have a diverging top part to correct for distant vision and converging lower part for reading.

Astigmatism

This is the defect that occurs if the curvature of the cornea varies in different directions so that rays in different planes from an object are focused in different positions by the eye and the image is distorted. A person suffering from astigmatism sees one set of lines more sharply than others. This defect is corrected by wearing corrected lenses. These help to bend the incoming rays to correct for irregular refraction.

Example

The far point of the defective eye is 1m. What lens is needed to correct this lens. With this lens, at what distance from the eye is its near point, if the near point is 25cm without the lens?

Solutions for problem

This far point is less than infinity, so the person is short sighted and he needs a diverging lens of $f = 1$m

This lens refracts the rays and appear to come from new near point

u = New near point

$v = -25\ cm$ because the image is virtual in diverging lens f =-1m= -100cm

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

It follows that $\frac{-1}{100} = \frac{1}{u} + \frac{-1}{25}$

Thus $\frac{1}{u} = \left(\frac{1}{25} - \frac{1}{100}\right)$ cm^{-1}

Hence u = 33.3cm

This is the new near point distance.

Formation of an image by the eye

Light enters the eye through the transparent cornea, passes through the lens and is focused on the retina. The retina is sensitive to light and sends messages to the brain for interpretation. Although the image is inverted, the brain interpretes it correctly.

A lens camera

Activity 10

(i) Make a paper box and carefully use a pin to make a tiny hole in the centre of the bottom of the paper box.

(ii) Place a piece of wax paper on the open end of the box. Hold the paper in place with the rubber band.

(iii) Turn off the room lights. Point the end of the box with a hole in a bright window.

(iv) Look at the image formed on the wax paper.

Which kind of image have you seen? Is it upside down or right side up. Is it smaller or larger than the actual object? What type of image is it?

The image is upside down. The pin hole helps you to see the image of the object. This device is called a pin hole camera.

Activity 11

(i) When you were going to register for Rwanda National Examinations, you took some photographs.

(ii) What device did the person that took your photograph use?

Figure 2.9: Taking a photo

In our daily lives, we take photographs. We use a lens camera to take these photographs.

Activity 12

(i) Enlarge the hole in the pinhole camera above at the front of the box and hold convex lens over the hole.

(ii) Adjust the position of the lens for either near or far objects to make a sharp image on the screen.

(iii) Is the image erect or inverted? If the objects are coloured, is the image coloured?

Notice that the image formed is inverted and coloured if the object is coloured. By placing a lens above the hole, you are making a lens camera from a pin hole camera.

Formation of images by a lens camera

Activity 13

(i) Draw a ray diagram for the formation of an image of an object placed at a point beyond 2F of a thin converging lens.

(ii) State the nature and size of the image.

Is the image bigger or smaller?

We have already seen that when an object is beyond 2F of a thin converging lens, the image formed is smaller than the object.

A camera consists of a light- tight box with a convex (converging) lens at one end and the film at the other end. It uses the convex lens to form a small, inverted, real image on the film at the back.

Figure 2.10: The lens camera

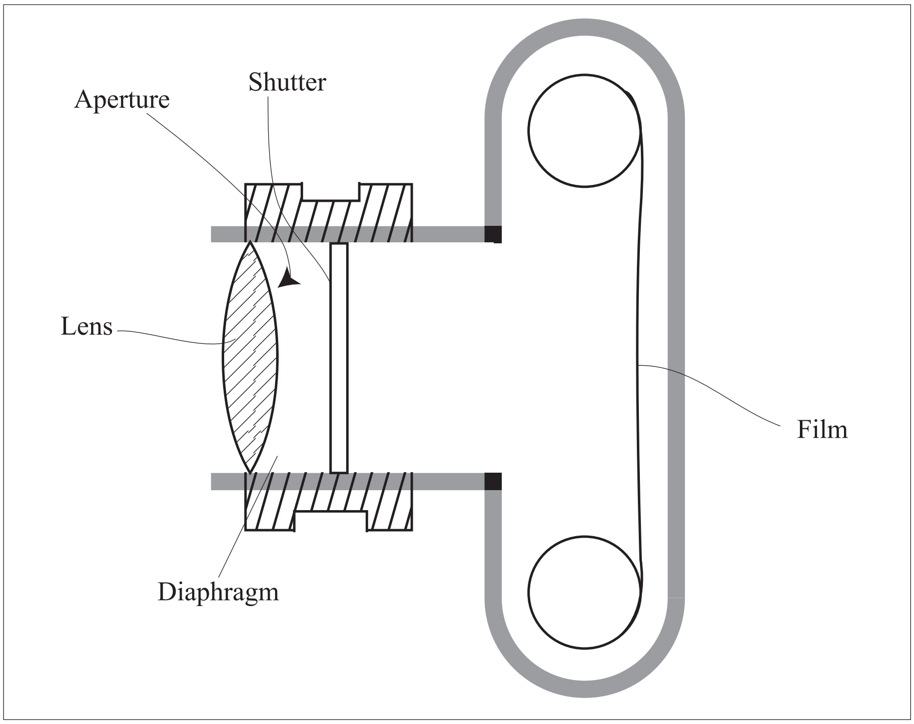


Figure 2.11: The lens camera

The lens focuses light from the object onto a light sensitive film. It is moved to and fro so that a sharp image is formed on the film. In many cameras, this happens automatically. In cheaper cameras, the lens is fixed and the photographer moves forwards and backwards to focus the object.

The diaphragm is a set of sliding plates between the lens and the film. It controls the aperture (diameter) of a hole through which light passes.

In bright light, a small aperture is used to cut down the amount of light reaching the film and in dim light, a large hole is needed. Very large apertures give blurred images because of aberrations so the aperture has to be reduced to obtain clear images.

In many cameras, the amount of light passing through the lens can be altered by an aperture control or stop of variable width. This size of the hole is marked in f – numbers i.e 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22, 32. The smaller the f-number, the larger the aperture. An f-number of 4 means the diameter d of the aperture is ¼ the focal length, f of the lens. To widen the aperture, the f number should therefore be decreased.

The aperture also controls the depth of field of the lens camera. The depth of field is a range of distances in which the camera can focus objects simultaneously. This depth of field is increased by reducing the aperture.

This large depth of field ensures a large depth of focus. The depth of focus is the tiny distance the film plane can be moved to or from the lens without defocusing the image. A large depth of focus means that both near and far objects appear to be in focus at the same time which is obtained by a small hole in the diaphragm.

The shutter controls the exposure time of the film. It opens and closes quickly to let a small amount of light into the camera.

The exposure time affects the sharpness of the image. When the exposure time is short, the image is clear (sharp) but when it is long the image becomes blurred.

The film. This is where the image is formed. It is kept in darkness until the shutter is opened. It is coated with light sensitive chemicals which are changed by the different shades and colours in the image. When the film is processed, these changes are fixed and the developed film is used to print the photograph.

Note that a diminished image is always formed on the film and that the image of distant object is formed on a film at distance f from the lens. For near objects, the lens is moved further away from the film (closer to the object) to obtain a clear image. In this case, the film is at a distance greater than f of the lens.

Activity 14

Discussion

In groups of four, discuss the differences and similarities between the lens camera and the human eye.

How would you use a lens camera to make a million francs in one year?

The slide projector

Activity 15

(i) Have you ever seen an instrument called a slide projector?

(ii) What is that instrument used for?

A slide projector is an opto-mechanical device for showing photograhic slides.

Activity 16

(i) Have you ever watched a cinema where the pictures are seen on the white wall?

(ii) What device were they using to throw the pictures on the screen (wall or white cloth)?

(iii) Where do you think the pictures came from?

Are the images small or large?

The pictures are thrown on the screen using a slide projector.

A projector is a device used to throw on a screen a magnified image of a film or a transparent slide. It produces a magnified real image of an object.

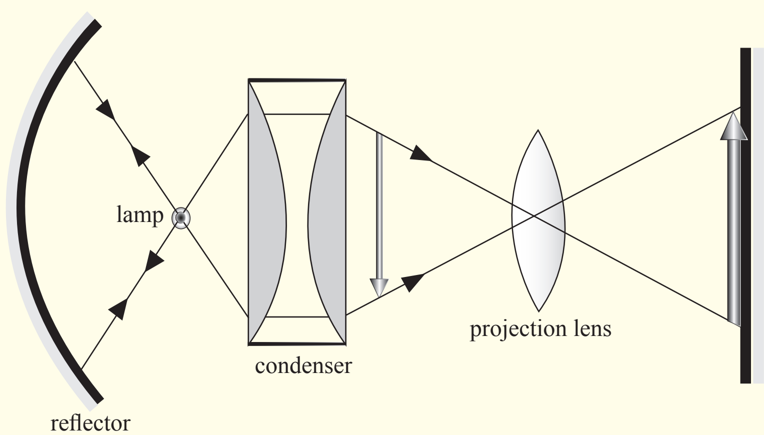


Figure 2.12: Projector

It consists of an illumination system and a projection lens. The illumination system consists of a lamp, concave reflector and the condenser. The illuminant is either a carbon electric arc or a quartz lamp to give a small but very high intensity source of light in order to make the image brighter.

The lamp is situated at the centre of curvature of the mirror so that the rays are reflected back along their original path. The concave mirror reflects back light which would otherwise be wasted at the back of the projector housing. The condenser consisting of two Plano concave lenses collects light which would otherwise spread out and be wasted, and concentrates it on to the film (slide) so that it is very bright and evenly illuminated.

The light is then scattered by the film and focused by a convex projection lens on to the film. The projection lens is mounted in the sliding tube so that it is moved to and fro to focus a sharp image on the screen.

Example

1. A slide projector has a converging lens of focal length 20.0cm and is used to magnify the area of a slide, $5cm^2$ to an area of $0.8m^2$ on a screen.
2. Calculate the distance of the slide from the projector lens.

Solution

$(\text{Linear scale factor})^2 = \text{Area scale factor}$

A linear scale factor is the linear magnification and area scale factor is the area magnification of an image.

Area scale factor is given by $\frac{\text{Area of image}}{\text{Area of object}} = \frac{0.8}{0.0005} = 1600$

Linear scale factor $= \sqrt{\text{Area scale factor}}$

Linear scale factor $= \frac{v}{u}$

So, $\frac{v}{u} = \sqrt{1600} = 40$

Thus $v = 40u$

From the lens formula, $\frac{1}{f} + \frac{1}{u} + \frac{1}{v}$

$\frac{1}{u} + \frac{1}{40u} = \frac{1}{20}$

Therefore, $\frac{40+1}{40u} = \frac{1}{20}$

$\frac{41}{40u} = \frac{1}{20}$

It follows that $40u = 820\text{cm}$

Hence, $u = 20.5\text{cm}$

Thus, the distance of the slide from the projector is 20.5cm.

Exercise

1. A colour slide has a picture area 2.4 cm x 3.6 cm. Find the focal length of the projection lens which will be needed to throw an image 1.2m x 1.8m on a screen 5m from the lens.
2. A projector projects an image of area 1 m^2 onto a screen placed 5m from the lens. If the area of the slide is 4 m^2, calculate;
   (i) The focal length of the projection lens.
   (ii) The distance of the slide from the lens

Activity 17

Make a projector on the bench using a ray box lamp, a single convex lens (focal length about 5 cm) for the condenser; a slide; a convex lens (focal length 5cm or 10cm) as the projection lens and a sheet of paper for the screen.
Is the image inverted?
By how much is it magnified?

Note that if the film is placed just after the lamp, the object would be poorly illuminated. So to give a bright picture, a condenser is included. The film O is placed between F and 2F of the projection lens so that the image I is real, inverted and magnified.

The film is put in the projector while it is upside down so that the picture on the screen is upright.

Microscope

Simple Microscope (Magnifying Glass)

Activity 18

(i) Hold a hand lens at above the word Rwanda at a distance of about 4cm from the word.
(ii) Move the lens farther away slowly from the word while observing the word through the lens.
(iii) What changes do you notice after observing?

(iv) Share ideas with your neighbour and write your observation in your notebook.

Figure 2.13: Children observing using a magnifying glass

The word Rwanda becomes larger and larger and finally disappears. This word gets larger because of the lens. We say that it is being magnified by the lens.

Activity 19

(i) Place your hand on a table and hold a hand lens above it and do the same as in activity 18.

(ii) What do you notice?

Figure 2.14: Magnifying glass

Notice that the hair (fur) and other small holes on the skin are seen clearly. These parts of the skin are made bigger by the glass lens and this enables one to see them clearly. This lens which magnifies images is called a magnifying glass or a simple microscope.

A magnifying glass consists of a thin converging lens and It is used to view very small organisms or parts of organisms which cannot be easily seen by the naked eye.

Formation of images by a magnifying glass

Activity 20

Using the knowledge from thin lenses, draw a ray diagram to show the formation of an image by a magnifying glass.
State the characteristics of the image formed.

We have already seen in unit 1 that when an object is between the lens and its principal focus, the image formed is magnified and upright. So, a magnifying glass forms a virtual, upright, magnified image of an object placed between the lens and its principal focus.

Activity 21

Making a simple microscope

(i) Use a pin or a nail to make a hole about 2 mm in diameter in a piece of a kitchen foil or glass.

(ii) Carefully let a drop of water fall on the hole so that it stays there and acts as a tiny lens with short focal length.

(iii) Use it to observe prints on a piece of paper.

Simple microscope (magnifying glass) in normal adjustment.

The magnification of a magnifying glass depends upon where it is placed between the user's eye and the object being viewed and the total distance between them.

Activity 22

(i) Carefully place a magnifying glass above some prints on a piece of paper and adjust it until they are seen clearly.

(ii) Make sure that you don't feel any strain in the eye while you are observing.

(iii) What do you think is the position of the image from the eye?

The image is at the least distance of vision since the eyes are not strained and the magnifying glass is said to be in normal adjustment.

A microscope is in normal adjustment if the final image is formed at the near point, and it is not in normal adjustment if the final image is at infinity.

Magnifying power of a simple microscope

We have already seen that the size of the image depends on the angle subtended by the object on the eye called the visual angle. Thus, the magnifying power depends on the visual angle.

It is defined as the ratio of the angle subtended by the image to the lens to the angle subtended by the object at the near point to the eye.

a) Magnifying power of a simple microscope in normal adjustment

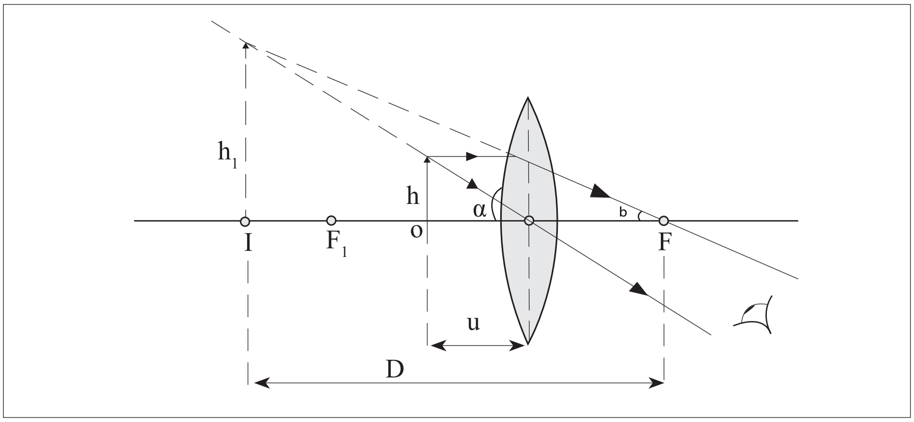


Figure 2.15: Image formed by a magnifying glass

Consider an object of height h placed at a given distance from the lens.

Let β be the angle subtended by the image I to the lens.

From the figure, $\tan\beta = \dfrac{h_1}{D}$

Assuming that rays are paraxial and that the eye is very close to the lens.

It implies that β is very small and $\tan\beta \approx \beta$.

Thus $\beta = \dfrac{h_1}{D}$ (1)

Now suppose that the object is viewed at the near point by the un aided eye and that it subtends an angle of α at the eye.

Now $\tan\ \alpha = \dfrac{h}{D}$

For α small, $\tan\alpha \approx \alpha$.

Thus $\alpha = \dfrac{h}{D}$ (2)

It follows that the magnifying power (angular magnification) M is given by

$$M = \frac{\frac{h_1}{D}}{\frac{h}{D}} = \frac{h_1}{h}$$

$$M = \frac{h_1}{h}$$

But $\dfrac{h_1}{h}$ = linear magnification produced by a lens or magnifying glass,

$$M = \frac{v}{f} - 1$$

Hence the magnifying power, $M = \dfrac{v}{f} - 1$

Since the image is at the near point (least distance of district vision), the image distance v is equal to $-D$, (negative for a virtual image).

$$M = \frac{-D}{f} - 1$$

This gives the maximum magnifying power of a simple microscope.

Note that in calculations, the value of the magnifying power is negative. The negative sign can always be neglected since magnification cannot be negative.

The object distance can take any value in the range from the focal point to the point where it lies at the near point and if the object is at the focal point, then the object distance is equal to the focal length and the image is at infinity, and the microscope is not in normal adjustment.

b) Magnifying power of a simple microscope when it is not in normal adjustment

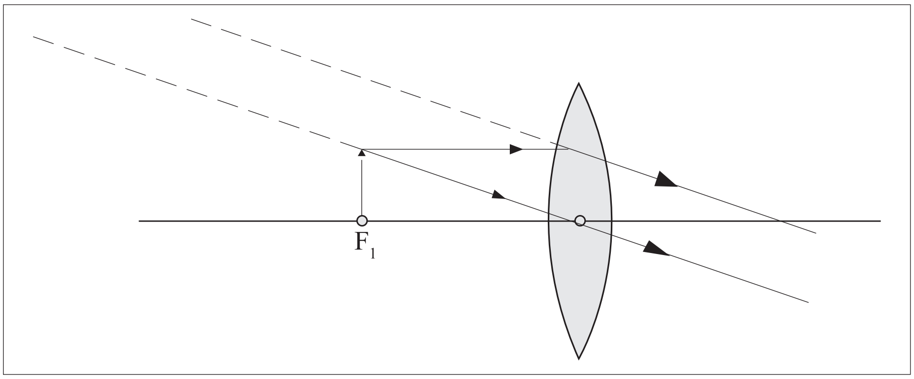


Figure 2.16: Angular magnification or magnifying power of an optical instrument

Angular magnification, $M = \frac{\beta}{\alpha}$

From the figure, $tan\ \beta = \frac{h}{f}$

For α small, $tan\ \beta \approx \beta$.

Thus, $\beta = \frac{h}{f}$

Diagram (not using a microscope)

From the figure, $tan\ \ \alpha = \frac{h}{D}$

For α small, $tan\ \alpha \approx \alpha$.

Thus, $\alpha = \frac{h}{D}$

It follows that, angular magnification, $tan\ \alpha - \dfrac{\frac{h}{f}}{\frac{h}{D}}$

Hence, $M = \frac{D}{f}$

This is the minimum magnifying power of the simple microscope.

Note that, in this case, D is positive since it is of a real image from the eye, and from the formula, angular magnification is high for a lens of short local length.

Example

A magnifying glass has a focal length of 5cm. Find the angular magnification and the position of an object if the image is formed at the position of least distinct vision of 25cm.

Solution

Since the image is formed at the position of least distinct vision, the magnifying glass is in normal adjustment.

f = 5cm, D = 25cm

$$M = \frac{D}{f} + 1$$

$$M = \frac{25}{5} + 1 = \frac{25 + 5}{5} = \frac{30}{5} = 6$$

Thus, the maximum angular magnification is 6

But since angular magnification for a magnifying glass = linear magnification

As the image is formed at the least distance of distinct vision from the lens then: $v = {}^{-}D$

It follows that $6 = \frac{v}{u} = \frac{25}{u}$

Thus, $6u = 25$

Hence $u = 4.2$cm

Exercise:

1. Find the angular magnification produced by a simple microscope of focal length 5cm when used not in normal adjustment.
2. Explain why angular magnification of a simple microscope is high for a lens of short local length.
3. Why the image formed by magnifying glass is free from chromatic abberation.

Activity 23

In groups of five, discuss why the image formed in a magnifying glass is almost free of chromatic abbreviation.

When an object is viewed through the magnifying glass, various coloured images corresponding to I_R, I_V for red and violet rays are formed but each image subtends the same angle at the eye close to the lens and therefore these colours overlap. The overlap of these colours makes a virtual image seen in a magnifying glass free of a chromatic abberation.

Group Activity 24

In groups of five, go out side class and pick different kinds of leaves. Examine, with the use of a magnifying glass, the structures of the leaves. Discuss in detail the structural characteristics of each leaf.

Group Activity 25

You are provided with dirty water in a glass container.
Use the magnifying glass provided and view some living organisms in it.
Record what you see.

Activity 26

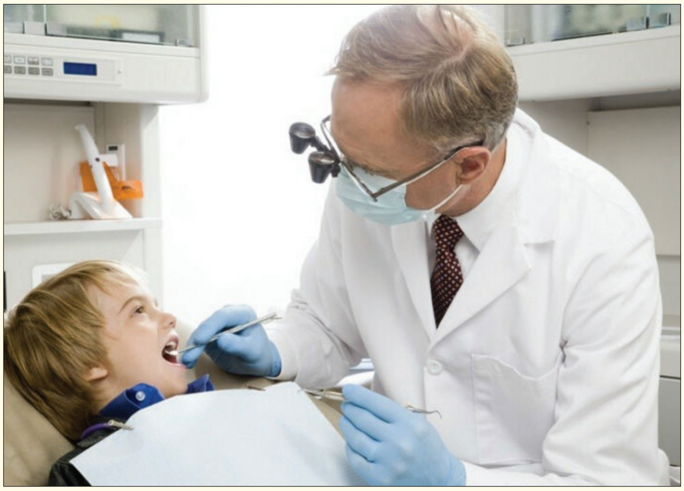


Figure 2.17: Observing a tooth with a magnifying glass

(i) Observe critically and describe the activity being done in the photograph.

(ii) State other uses of a magnifying glass.

Uses of magnifying glass: Magnifying glasses have many different uses. Some people use it for fun activities such as starting fires, or use the lens to help them read. You can start a fire with a magnifying glass when the sun rays are concentrated on the lens. Some retail stores sell reading glasses with the double convex lens. In everyday life, magnifying glasses can be used to do a variety of things. The most common use for magnifying glasses would be how scientists use them, they use magnifying glasses to study tiny germs

The compound microscope

Activity 27

Have you ever heard or seen an instrument called a compound microscope?
What is it used for?

Figure 2.18: Different Rwandans using compound microscopes

The compound microscope is used to detect small objects; is probably the most well-known and well-used research tool in biology.

Activity 28

Observe the above pictures carefully and in groups of three, discuss places where a compound microscope is used in daily life.

In daily life, microscopes are used in hospitals, in biology laboratories, etc.

Activity 29

(i) You are provided with two lenses of focal lengths 5cm and 10cm together with a half meter ruler and some plasticine.

(ii) Arrange the lenses as shown in the figure below.

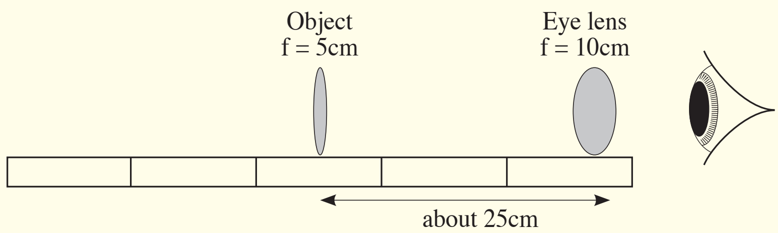


Figure 2.19: An arrangment of lenses

(iii) Move the object to and fro until it appears in focus.

What do you notice about the image? Is it distorted? Is it coloured differently in any way?`

By arranging the lenses as above, you have actually made a compound microscope.

We have already seen how a single lens (magnifying glass) can be used to magnify objects.

However, to give a higher magnifying power, two lenses are needed. This arrangement of lenses makes a compound microscope. It produces a magnified inverted image of an object.

A compound microscope is used to view very small organisms that cannot be seen using our naked eyes for example micro organisms.

Figure 2.20: A compound microscope

A compound microscope consists of two convex lenses of short focal lengths referred to as the objective and the eye piece. The objective is nearest to the object and the eye piece is nearest to the eye of the observer.

The object to be viewed is placed just outside the focal point (at a distance just greater than the focal length) of the objective lens. This objective lens forms a real, magnified, inverted image at a point inside the principal focus of the eye piece. This image acts as an object for the eye piece and it produces a magnified virtual image. So the viewer, looking through the eye piece sees a magnified virtual image of a picture formed by the objective i.e of the real image.

Image formation in a compound microscope

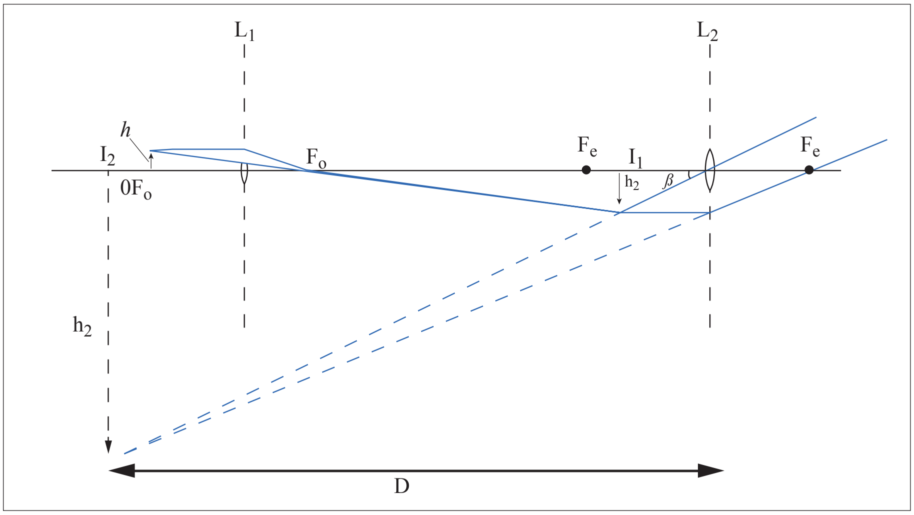


Figure 2.21: Images formed by a compound microscope

An objective lens L_1 forms a real magnified image I, of an object O just placed outside its principal focus F_0. I, is formed just inside the principal focus F_e of the eye piece L_2, which acts as a magnifying glass and produces a magnified, virtual image I_2 of I_1.

Compound microscope in normal adjustment (normal use)

Activity 30

You are provided with a bird's feather; observe it critically using a compound microscope and draw it in a fine detail.

Make sure you observe the features when your eyes are relaxed.

When the eyes are relaxed, the image is at the near point and the compound microscope is said to be in normal adjustment.

The compound microscope is in normal adjustment when the final image is formed at the near point (least distance of distinct vision), D of the eye.

Angular magnification (magnifying power) of a compound microscope

The magnifying power of a compound microscope is the ratio of the angle subtended by the final image to the eye when the microscope is used to the angle subtended by the object the unaided eye.

Angular magnification of a compound microscope in normal use

We have already seen that when a microscope is in normal use, the image I_2 is formed at the least distance of distinct vision, D from the eye. Thus $v = D$.

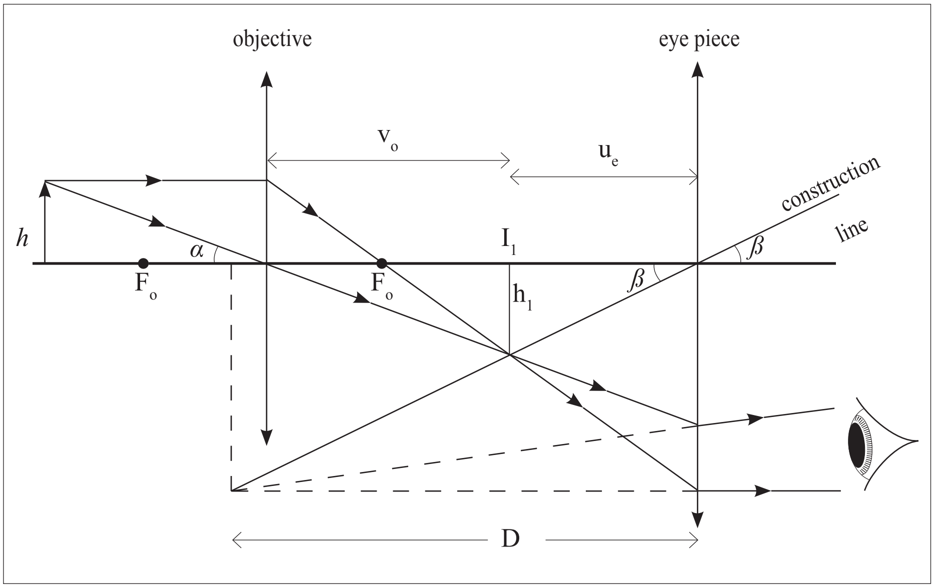


Figure 2.22: Images formed by a microscope in normal use

Consider an object of height h at a given distance slightly greater than the focal length of the objective lens.

Suppose that the final image has a height h_2 and is formed at a distance v from the eye piece and that it subtends an angle β to the eye. $M = \frac{\beta}{\alpha}$

From the figure, $\tan\beta = \frac{h_2}{D}$

Supposing that the eye is very close to the eye piece, β is very small and $\tan\beta \approx \beta$

Hence $\beta = \frac{h_2}{D}$

Now suppose that the object subtends an angle of α when placed at the near point, D, when viewed by a naked eye.

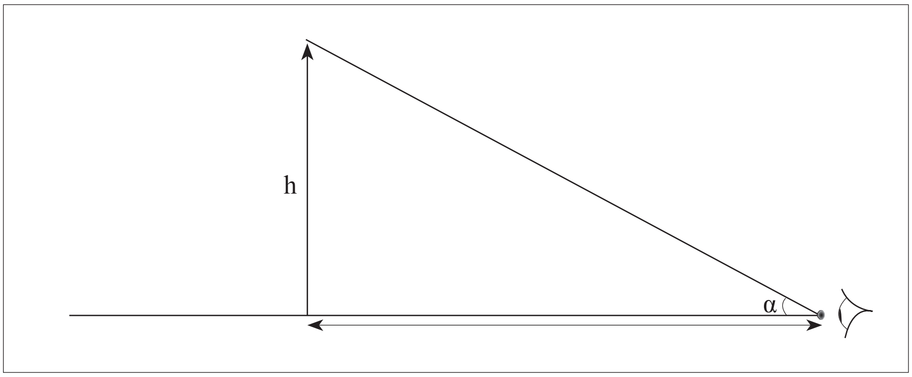


Figure 2.23: Object viewed by a naked eye

From the figure, $tan\ \alpha = \frac{h}{D}$

For α small, $tan\ \alpha \approx \alpha$

Thus $\alpha = \frac{h}{D}$

Hence, the angular magnification (magnifying power) is given by

$$M = \frac{\frac{h_2}{D}}{\frac{h}{D}} = \frac{h_2}{h}$$

Introducing the height of image due to the objective, h_1. $M = \frac{h_2}{h_1} \times \frac{h_1}{h}$

But $\frac{h_1}{h_2}$ = linear magnification, m_e of image due to eyepiece and $\frac{h_1}{h}$ = linear magnification, m_0 of image due to objective lens

It follows that M = linear magnification due to eyepiece lens x linear magnification due to objective

Thus $M = m_e \times m_0$

We have already seen that linear magnification is also given by$m = \frac{v}{f} - 1$, where v is the image distance from the lens and f is the focal length.

It follows that linear magnification due to the objective lens, $m_0 = \frac{v_0}{f} - 1$, and that due to the eye piece, $m_e = \frac{v_e}{f} - 1$

Therefore, $M = \left(\frac{v_e}{f_e} - 1\right)\left(\frac{v_0}{f_0} - 1\right)$

But $v_e = -D$ (since the Image formed by the eye piece is virtual).

For the eye piece, $v = -D$ (since it's a virtual image)

Hence $M = \left(1 + \frac{D}{f_e}\right)\left(1 + \frac{v_0}{f_0}\right)$

From the above expression, it can therefore be seen that if f_0 and f_e are small, M becomes large. So the angular magnification M can be made high if the focal lengths of the objective and eye piece are both small.

Angular magnification of a compound microscope when not in normal use:

We have already seen that when a microscope is not in normal adjustment, the final image is formed at infinity i.e $v = \infty$.

Suppose that an object of height h is at a given position from the objective lens, forming an image of height h_1.

Angular magnification, $M = \frac{\beta}{\alpha}$

From the figure, $\tan\beta = \frac{h_1}{f_e}$

Suppose that the object is viewed using the naked eye, $\tan\alpha = \frac{h}{D}$

For $v = -D$ very small, $\tan\beta \approx \beta$ and $\tan\alpha \approx \alpha$

Thus $\beta = \frac{h_1}{f_e}$ and $\alpha = \frac{h}{D}$

$$\text{Therefore, } M = \frac{\frac{h_1}{f_e}}{\frac{h}{D}} = \frac{h_1}{f_e} \times \frac{D}{h}$$

$$= \frac{h_1}{h} \times \frac{D}{f_e}$$

But $\frac{h_1}{h}$ linear magnification of the objective lens given by $\frac{v_0}{f_0} - 1$

Hence angular magnification $M = \frac{D}{f_e}\left(\frac{v_0}{f_0} - 1\right)$

Example

A compound microscope has an eye piece of focal length 2.50cm and an objective of focal length 1.60cm. If the distance between the objective and eye piece is 22.1cm, calculate the magnifying power produced when the final image is at infinity.

Solution

If the final image is at infinity, the objective forms an image at the focal point of the eye piece.

Let f_e be the focal length of the eye piece and f_0 of the objective

The position of an image of l_o from L_o = separation – focal length of eye piece v_e

$= (22.1 - 2.50)$ cm

$v_e = 19.5$cm

Magnifying power $M = \dfrac{D}{f_e}\left(\dfrac{v_e}{f_o} - 1\right)$

But for a normal eye, $D = 25$cm

Thus $M = \dfrac{25}{2.5}\left(\dfrac{19.5}{1.6} - 1\right) = 111.8$

Activity 31

Viewing specimens

The purpose of this exercise is to view micro organisms found in pond water while learning to operate a microscope.

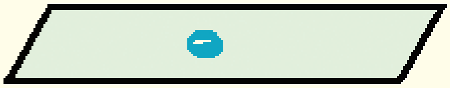
Equipment

* Microscope
* Jar of pond water
* Slide
* Coverslip
* Dropper

Procedure

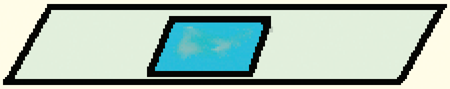
1. Collect a jar of pond water containing micro organisms. To ensure that you capture the largest number of micro organisms, do not simply scoop a jar of water from the centre of a pond. Instead, fill the jar partway with pond water and then squeeze water into the container from water plants or pond scum.

2. Prepare a specimen of pond water.



   a) Using the dropper, place a few drops of pond water onto the centre of a clean, dry slide.

   b) Hold the side edges of the coverslip and place the bottom edge on the slide near the drop of pond water.



   c) Slowly lower the coverslip into place. The water should spread out beneath the coverslip without leaving any air bubbles. If air bubbles are present, you can press gently on the coverslip to move the air bubbles to the sides.

3. Set up the microscope.

   a) Remove the dust cover from the microscope.
   b) Plug in the microscope.
   c) Turn on the microscope's light source.

4. View the specimen with the low-power objective.

   a) Move the slide around on the stage using your fingers or the control knobs until you find a micro organism.

5. View the micro organism with the high-power objective.

6. Sketch a picture of the micro organism.

7. Repeat steps 4, 5, and 6 until you have sketched atleast five different micro organisms.

8. Turn off the microscope.
   a) Carefully, lower the objective to its lowest position by turning the coarse' adjustment knob.
   b) Turn off the light source.
   c) Remove your slide. Clean the slide and cover slip with water.
   d) Unplug the microscope and store it under a dust cloth.

Telescopes

Activity 32

You have heard in your early secondary that there are some heavenly and distant earthly bodies that cannot be seen by our naked eyes. How did the people know that there exist such bodies?

Which instrument do you think is used to see these bodies and to observe what takes place on these bodies?

Why do you think it is difficult to see distant objects using our eyes?

Telescopes are instruments used to view distant objects such as stars and other heavenly bodies. Distant objects are difficult to see because light from them has spread out by the time it reaches the eyes, and since our eyes are too small to gather much light.

There are two kinds of telescopes; refracting telescopes and reflecting telescopes.

Refracting telescopes

Activity 33

(i) Hold a convex lens of focal length 5cm close to your eye.
(ii) Hold another lens of focal length 20cm at an arm's length.
(iii) Use the lens combination to view distant objects.
(iv) Adjust the distance of the farther lens until the image is clear (take care not to drop the lenses).

What type of image do you see?

The above lens combination is a refracting telescope. It is called a refracting telescope because it forms an image of the object by refracting light. Therefore, Refracting telescopes use lenses and they form images by refraction of light. Below are different kinds of refracting telescopes.

Astronomical telescope

The telescope made in the above activity is called an astronomical telescope. It consists of two convex lenses, the objective lens of long focal length and an eye piece lens of short focal length.

An astronomical telescope in normal adjustment

Activity 34

Using a telescope made in activity (30) above, view a distant object by moving the lenses so that the eyes are relaxed.

What do you think is the position of the image?

When the eyes are relaxed, the image is at infinity and the telescope is in normal adjustment. Therefore, an astronomical telescope is in normal adjustment when the final image is formed at infinity.

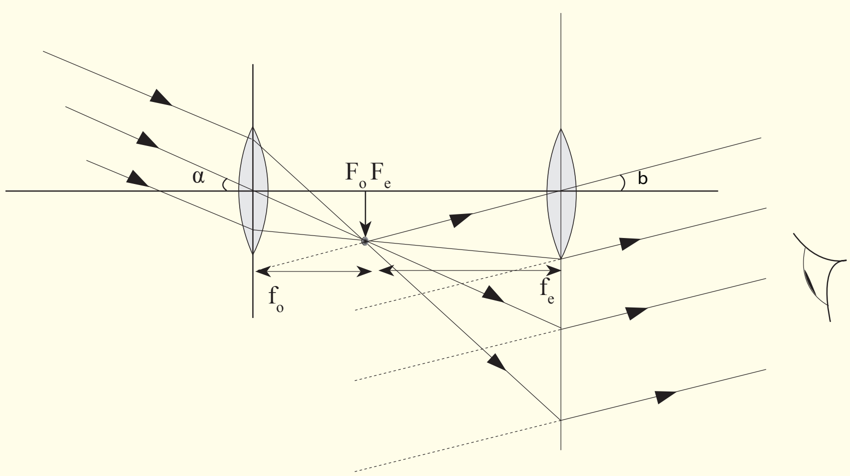


Figure 2.24: An astronomical telescope in normal adjustment

The rays of light coming from a distant object form a parallel beam of light. This parallel beam is focused by the objective lens and it forms a real, diminished image at its principal focus F_o. The eye piece is adjusted so that this image lies in its focal plane. This image acts as the object for the eye piece and the eye piece produces the image at infinity.

Note that in normal adjustment, the eye is relaxed or un accommodated when viewing the image. In this case, the eye has minimum strain.

Magnifying power or angular magnification of an astronomical telescope

The magnifying power of a telescope is the ratio of the angle subtended by the image to the eye when the telescope is used to the angle subtended at the unaided eye by the object.

Since the telescope length is very small compared with the distance of the object from either lens, the angle subtended at the unaided eye by the object is the same as that subtended at the objective by the object.

Angular magnification of an astronomical telescope in normal adjustment

In normal adjustment, the magnifying power (angular magnification) of an astronomical telescope is given by:

$$M = \frac{\text{angle subtended at the eye by the final image at infinity}}{\text{angle subtended at the objective by the object}}$$

Let β = angle subtended by the final image at the eye and α = angle subtended by the object at the objective

Hence, $M = \dfrac{\beta}{\alpha}$

Supposing that the eye is close to the eye piece and h_1 is the image of image I1 formed by the objective, then $tan\ \alpha \approx \alpha$ and $tan\ \beta \approx \beta$

It follows that $M = \dfrac{\dfrac{h_1}{f_e}}{\dfrac{h_1}{f_0}} = \dfrac{h_1}{f_e} \times \dfrac{f_0}{h_1}$

Therefore, $M = \dfrac{f_0}{f_e}$

Note

(i) From the above expression, M is high when eye piece focal length fe is short and the objective focal length f0 is long. This explains the fact why the objective lens of long focal length and the eye piece lens of short focal length are used during the construction of the astronomical telescope.

(ii) For a telescope in normal adjustment, the separation of the objective and the eye piece is $f_0 + f_e$

Activity 35

In groups of four , discuss and give a summary of differences between a compound microscope and an astronomical telescope.

The table below shows the differences between a compound microscope and an astronomical telescope.

Compound microscope	**Astronomical telescope**
In normal adjustment, the final image is at near point.	The final image is at infinity
The objective lens has a short focal length.	The objective has a long focal length.
The object is near (it is used to view near and small objects)	The object is at infinity (used to see distant objects).
The distance between the objective lens and eye piece is greater than f_0+f_e.	Distance between the objective and the eye piece is f_0+f_e.

Example

An astronomical telescope has an objective lens of focal length 120 cm and an eye piece of focal length 5 cm. If the telescope is in normal adjustment, what is;

(i) The angular magnification (magnifying power)

(ii) The separation of the two lenses?

Solution

(i) $M = \dfrac{f_0}{f_e} = \dfrac{120}{5} = 24$

(ii) Separation $= f_0 + f_e = 120 + 5 = 125$ cm

Exercise

An astronomical telescope is used to view a scale that is 300 cm from the objective lens. The objective lens has a focal length of 20cm and the eye piece has a focal length of 2 cm. Calculate the angular magnification when the telescope is adjusted for minimum eye strain.

An astronomical telecope with the final image at the near point

In this case, the image is seen in detail but the telecope is not in normal adjustment (use) because the eyes are strained.

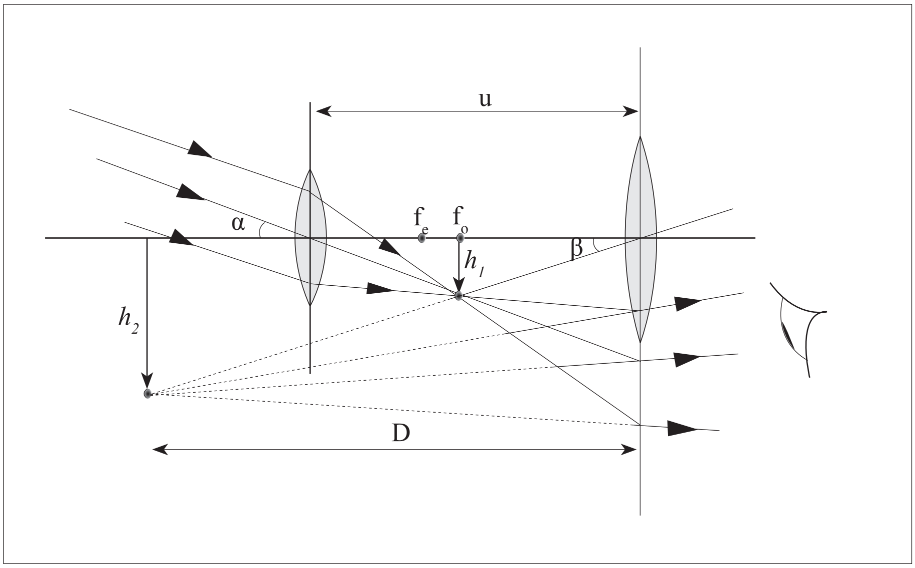


Figure 2.25: Image formed by an astronomical telescope

The objective forms an image of a distant object at its focus F_o. The eye piece is moved so that this image is at a position inside its focus. This image acts as the object for the eye piece which acts as a magnifying glass and thus forms a magnified, virtual image.

Suppose that β is the angle subtended by the final image at the eye, and h_1 is the height of the image formed by the objective lens and that the angle subtended at the unaided eye is that subtended at the objective by the object, α

For the eye close to the eye piece

$$\tan\alpha \approx \alpha = \frac{h_1}{f_0} \quad \tan\beta \approx \beta = \frac{h_2}{D}$$

Hence $M = \dfrac{\dfrac{h_2}{D}}{\dfrac{h_1}{f_0}}$

Thus $M = \dfrac{h_2}{D} \times \dfrac{f_0}{h_1} = \dfrac{f_0}{D}\left(\dfrac{h_2}{h_1}\right)$

But $\dfrac{h_2}{h_1}$ = linear magnification due to the eye piece, $m_e = \dfrac{v_e}{f_e} - 1$

It follows that

$$M = \frac{f_0}{D}\left(\frac{v_e}{f_e} - 1\right) = \frac{f_0}{D}\left(\frac{D}{f_e} - 1\right) = \frac{f_0}{D} \times \frac{D}{f_e}\left(1 - \frac{f_e}{D}\right) = \frac{f_0}{f_e}\left(1 - \frac{f_e}{D}\right)$$

Hence, it means that for an astronomical telescope with final image at near point, the magnifying power (Angular magnification is given by

$$M = \frac{f_0}{f_e}\left(1 - \frac{f_e}{D}\right)$$

As the final image is virtual, in calculation, D is negative, and note that separation of the lenses $= f_0 + u_e$.

The eye ring

The eye ring is the best position to place the eye in order to be able to view as much of the final image as possible.

The best position for an observer to place the eye when using a microscope is where it gathers most light from that passing through the objective. In this case, the image is brightest and the field of view is greatest.

In case of the telescope, all the light from a distant object must pass through the eye ring after leaving the telescope. So by placing the eye at the eye ring, the viewer is able to see the final image as much as possible.

Terrestrial telescope

Activity 36

(i) Did you notice that the final image in an astronomical telescope in activity (30) is inverted?

(ii) Place a convex lens in between the two lenses used to construct an astronomical telescope in activity (26) above.

(iii) Adjust the objective lens until the image is seen when the eyes are relaxed.

What is the nature of the image? Is it upright or inverted?

An astronomical telescope produces an inverted image, so it is not suitable for viewing objects on the earth. It is suitable for viewing stars and other heavenly bodies. A terrestrial telescope provides an erect image and this makes it suitable to view objectives on the earth.

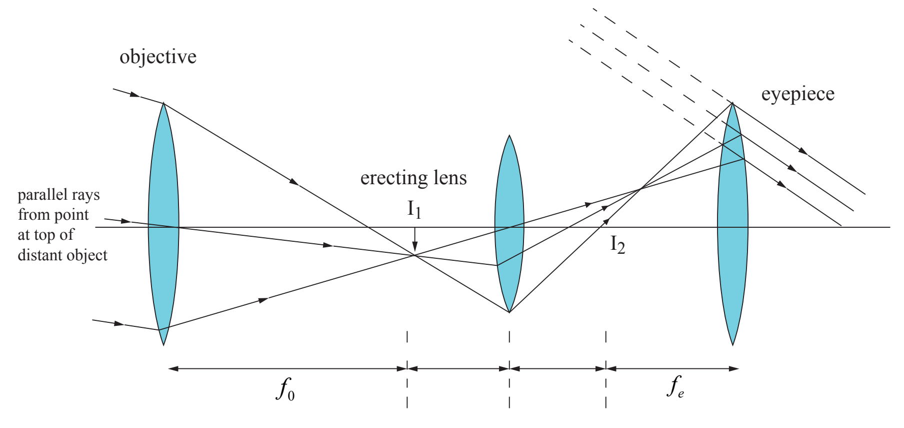


Figure 2.26: The terrestrial telescope

It consists of an erecting lens L of focal length f between the objective and the eye piece. The objective lens form an inverted image I_1. The lens L is placed at a distance of $2f$ from the image I_1. The image I_1 acts as the object and an erect image I_2 of the same size as I_1 is formed at $2f$ beyond the erecting lens. This image I_2 acts as an object for the eye piece and in the usual way the eye piece forms the final image at infinity.

Note that the angular magnification of the terrestrial telescope is similar to that of the astronomical telescope because the erecting lens has no effect on the angular magnification produced but only inverts the image l_1 so that the final image is upright.

Activity 37

In groups of four, discuss the advantages and disadvantages of a terrestrial telescope over an astronomical telescope.

The advantage a terrestrial telescope has over an astronomical telescope is that it produces an upright image.

However, the telescope is so long. It is much longer than other kinds of refracting telescopes. Its length is given by $= f_0 + f_e + 4f$.

The erecting lens also reduces the intensity of light emerging through the eye piece which makes the final image faint.

Galilean Telescope

Activity 38

Have you ever heard of a scientist called Galileo Galilee?

What is he known for?

Galileo was a great scientist well known for his discoveries in astronomy.

Activity 39

(i) Hold a concave lens of focal length 5cm close to your eye.

(ii) Hold another convex lens of focal length 20cm at an arm's length.

(iii) Use the lens combination to view distant objects.

(iv) What is the nature of the image?

The above lens combination is a Galilean telescope. A Galilean telescope consists of an objective lens which is a convex lens of long focal length and an eye piece which is a concave lens of short focal length. It forms erect images both in normal and not in normal adjustment.

Galilean telescope in normal adjustment

The objective lens would produce an image I_1 in the absence of the eye piece. With the eye piece in position at the distance f_e from I_1, I_1 acts as a virtual object to the eye piece and a virtual image of it is formed at infinity since 1_1 is at the focal point of the eye piece.

Angular magnification for a Galilean telescope in normal adjustment

Let h_1 be the height of image, l_1, β be angle subtended at the eye and α be the angle subtended at the unaided eye by the object which is very nearly equal to the angle subtended by the object at the objective lens.

$$\text{Angular magnification} = \frac{\alpha}{\beta}$$

For the eye very close to the telescope,

$$tan\ \alpha = \frac{h_1}{f_0} \text{ and } tan\ \beta \approx \beta = \frac{h_1}{f_e}$$

Therefore, angular magnification M is given by: $M = \dfrac{\frac{h_1}{f_e}}{\frac{h_1}{f_0}}$

Hence $M = \dfrac{f_0}{f_e}$, this is similar to that of the astronomical telescope.

Galilean telescope with final image at near point

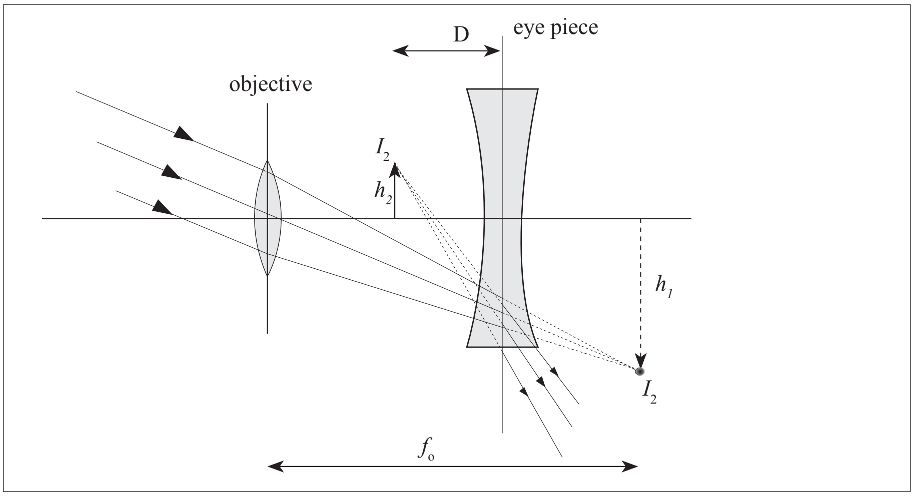


Figure 2.27: The Galilean telescope

The final image in a Galilean telescope can also be viewed at the near point of the eye when the telescope is not in normal adjustment.

The final image in a Galilean telescope can also be viewed at the near point of the eye when the telescope is not in normal adjustment.

The objective lens forms the image I_1 at a distance greater than the focal length of the eye piece. This image acts as a virtual object for the eye piece and an erect image of it is formed at a distance D. Thus $v = -D$ (since the image is virtual).

Angular magnification is thus given by $M = \dfrac{\beta}{\alpha}$

From the fig; $\tan\beta = \dfrac{h_2}{D}$ and $\tan\alpha = \dfrac{h_1}{f_0}$ where h_1 is the height of image I_1 and h_2 is the height of image I_2

From $M = \dfrac{\beta}{\alpha}$, it follows that $M = \dfrac{\frac{h_2}{D}}{\frac{h_1}{f_0}} \times \dfrac{h_2}{D} \times \dfrac{f_0}{h_1} = \dfrac{f_0}{D} \times \dfrac{h_2}{h_1}$

But $\dfrac{h_2}{h_1} = m_e =$ linear magnification due to the eyepiece

It follows that $M = \dfrac{f_0}{D} \times m_e$

But $m_e = \dfrac{v_e}{f_e} - 1$

Thus, $M = \dfrac{f_0}{D}\left(\dfrac{v_e}{f_e} - 1\right)$

Therefore, $M = \dfrac{f_0}{D} \times \dfrac{D}{f_e}\left(1 - \dfrac{f_e}{D}\right)$

But $v_e = -D$

Hence $M = \dfrac{f_0}{f_e}\left(1 - \dfrac{f_e}{D}\right)$

Activity 40

In groups of five, discuss the advantages and disadvantages of a Galilean telescope over an astronomical telescope and write them in your notebook.

Unlike in an astronomical telescope where the final image is inverted, the final image formed in a Galilean telescope is erect The telescope is also shorter than astronomical telescope and hence portable. The distance between the lenses is given by $f_0 - f_e$.

On the other hand, a Galilean telescope has a small field of view and its eye ring is virtual (since the eye piece is concave) that is, it is between the lenses and so inaccessible to the eye.

Reflecting telescopes

Activity 41

In groups of four, go outside and observe a TV satelite dish in the neighbourhood.

Discuss with your neighbour about the observation and present the report to the class.

Reflecting telescopes consist of a large concave mirror of long focal length as their objective. There are three kinds of reflector telescopes, all named after their inventors.

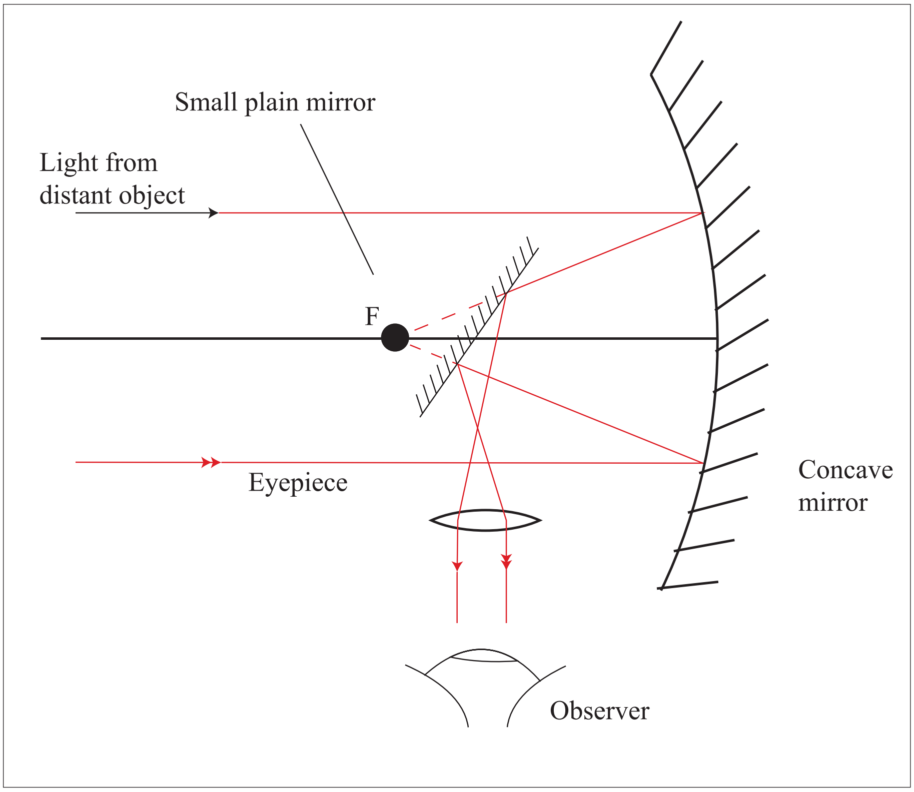


Figure 2.28: The Newtonian reflecting telescope

The Newtonian telescope is commonly used by amateur astronomers. A small plane mirror is used to direct the light from the concave mirror, which acts as an objective into an eye piece.

Rays from a distant object are reflected by the objective (concave mirror) to the plane mirror. This reflects the rays to form a real image I_1 which can be magnified by an eye piece or photographed by putting a film at I_1.

Note that the plane mirror deflects the rays of light side ways without changing the effective focal length f_0 of the objective.

In normal adjustment, the angular magnification of the Newtonian reflection telescope is given by $M = \frac{f_0}{f_e}$

Cassegrain reflecting telescope

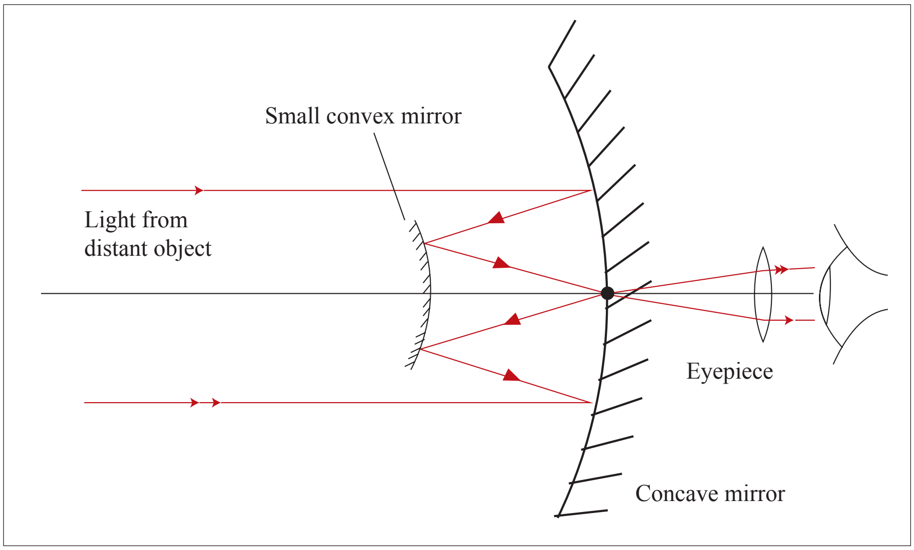


Figure 2.29: The cassegrain reflecting telescope

This is the type used in most observatories

It consists of a concave mirror which acts as an objective, a small convex mirror and the eye piece lens.

Light from a distant object is reflected by the concave mirror to the convex mirror which reflects it back to the centre of the concave mirror where there is a small hole to allow the light through. So the convex mirror forms the final image (real) at the pole of the objective.

Coude Reflector Telescope

This is a combination of Newtonian and cassegrain reflector telescopes.

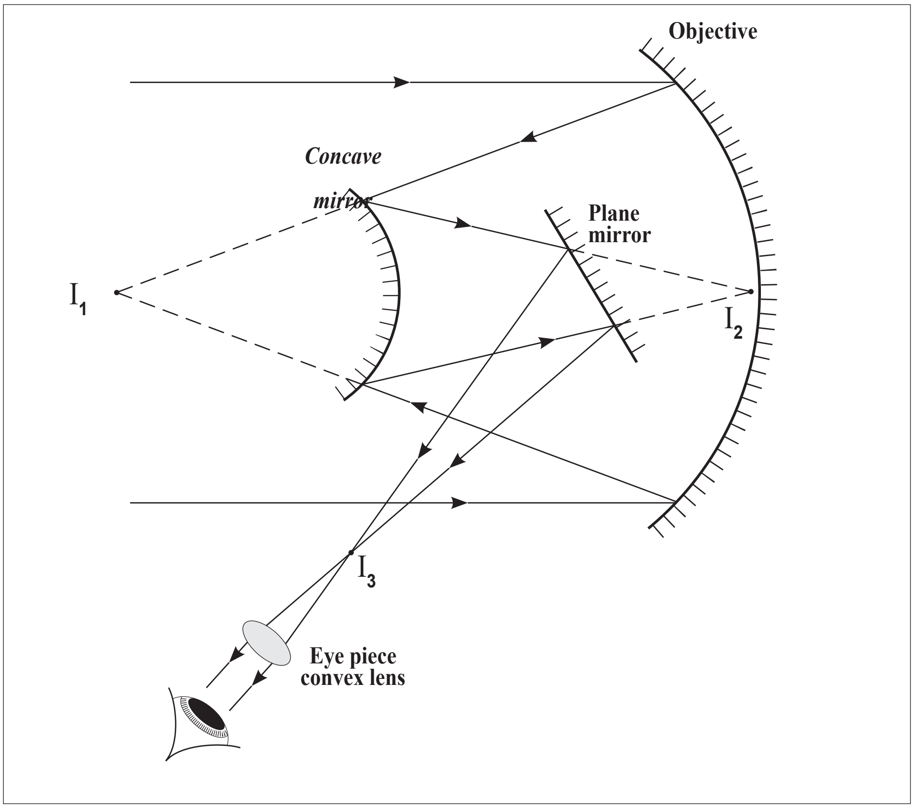


Figure 2.30: The reflecting telescope

The plane and convex mirrors used in reflecting telescopes are used to bring the light to a more convenient focus where the image can be photographed and magnified several times by the eye piece for observation.

Activity 42

In groups of five, discuss the advantages of reflecting telescopes over refracting telescopes and write them in your notebook.

The reflecting telescopes are free from chromatic aberration since no refraction occurs.

The image formed is brighter than in refracting telescopes where there is some loss of light during refraction at the lens surfaces.

Spherical aberration can be eliminated by using a parabolic mirror instead of a spherical mirror as an objective.

They have a power because of higher ability to distinguish two closely related objects because of the large diameter of the parabolic mirror. We say that they have a high resolving power.

They are easier to construct since only one surface requires to be grounded.

Critical Thinking Exercise

What is meant by the resolving power of an optical instrument? Explain its usefulness.

Explain why astronomers use reflecting telescopes rather than refracting telescopes?

Prism binoculars

Activity 43

Have you ever asked yourself how tourists and scientists are able to see distant animals and birds in a forest or any hidden places?

Discuss with your neighbour and write in your notebook the observation.

Tourists and scientists use prism binoculars to view wild animals and birds in hidden places such as caves and forests.

These consist of a pair of refracting astronomical telescopes with two totally reflecting prisms between each objective and eyepiece. The prisms use total internal reflection to invert rays of light so that the final image is seen the correct way. These prisms reflect up and down the light and by doing so, they shorten the length of the instrument.

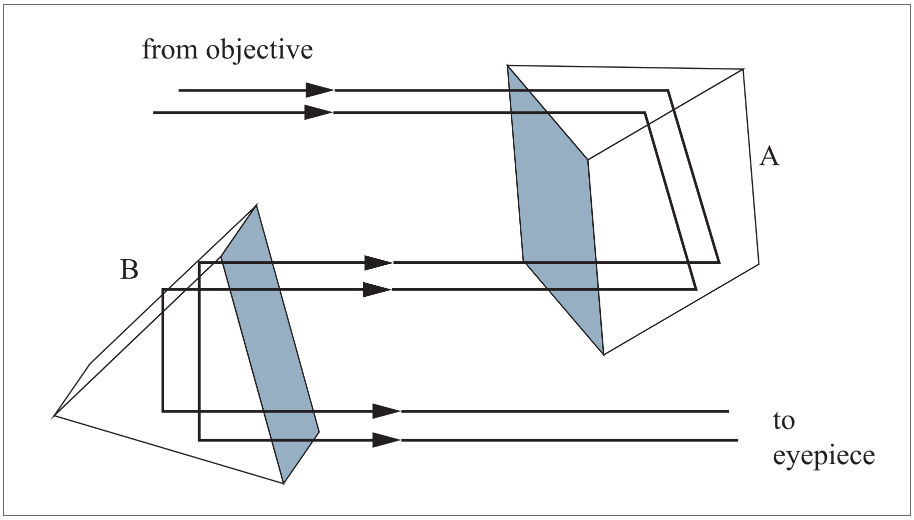


Figure 2.31: Arrangement in prism binoculars

Prism A causes lateral inversion and prism B inverts vertically so that the final image is the same way round and same way up as the object. Each prism reflects light through 180°. This makes the effective length of each telescope three times shorter than the distance between the objective and the eye piece. So good magnifying power is obtained with compactness.

Review Questions

1. (a) With the aid of a ray diagram, describe how a convex lens is used as a magnifying glass.
   (b) Explain why an image formed in a magnifying glass is almost free from chromatic aberration.
2. (a) When is a compound microscope said to be in normal use?
   (b) Derive an expression for the magnifying power of a compound microscope in normal use.
   (c) Explain why the lenses that make up a compound microscope are of short focal lengths.
3. (a) When is a telescope said to be in normal adjustment.
   (b) What is meant by the eye ring as applied to optical instruments.
   (c) What are the differences between microscope and telescopes?

4. (a) Explain why prisms are preferred to mirrors in prism binoculars.

   (b) State the advantages of reflecting telescopes over refracting telescopes.

   (c) The objective of an astronomical telescope in a normal adjustment has a diameter of 15cm and a focal length of 400cm. The eye piece has a focal length of 2.5cm. Find the magnifying power of the telescope.

5. (a) A distant objective subtending an angle of 3x 10^{-5} and in viewed with a reflecting telescope whose objective is a concave mirror of focal length 10m. The reflected light falls on a concave mirror placed 9.5cm from the pole of the objective which reflects the length back and a real image is formed a the pole of the objective where there is a hole. The image is viewed with a convex lens of focal length 5cm used as a magnifying glass which produces the final image at infinity.

6. How far must a 50mm focal-length camera lens be moved from its infinity setting to sharply focus an object 3m away?

7. Sue is far-sighted with a near point of 100cm. Reading glasses must have what lens power so that she can read a newspaper at a distance of 25cm? Assume the lens is very close to the eye.

8. A near-sighted eye has near and far point of 12cm and 17cm, respectively. (a) What lens power is needed for this person to see distant objects clearly, and (b) What then will be the near point? Assume that the lens is 2cm from the eye (typical for eye glasses).

9. What power contact lens is needed for an eye to see distant objects if its point is 25cm?

10. An 8cm focal-length converging lens is used as a "jeweler's loupe", which is a magnifying glass. Estimate (a) the magnification when the eye is relaxed, and (b) the magnification if the eye is focused at its near point N=25cm.

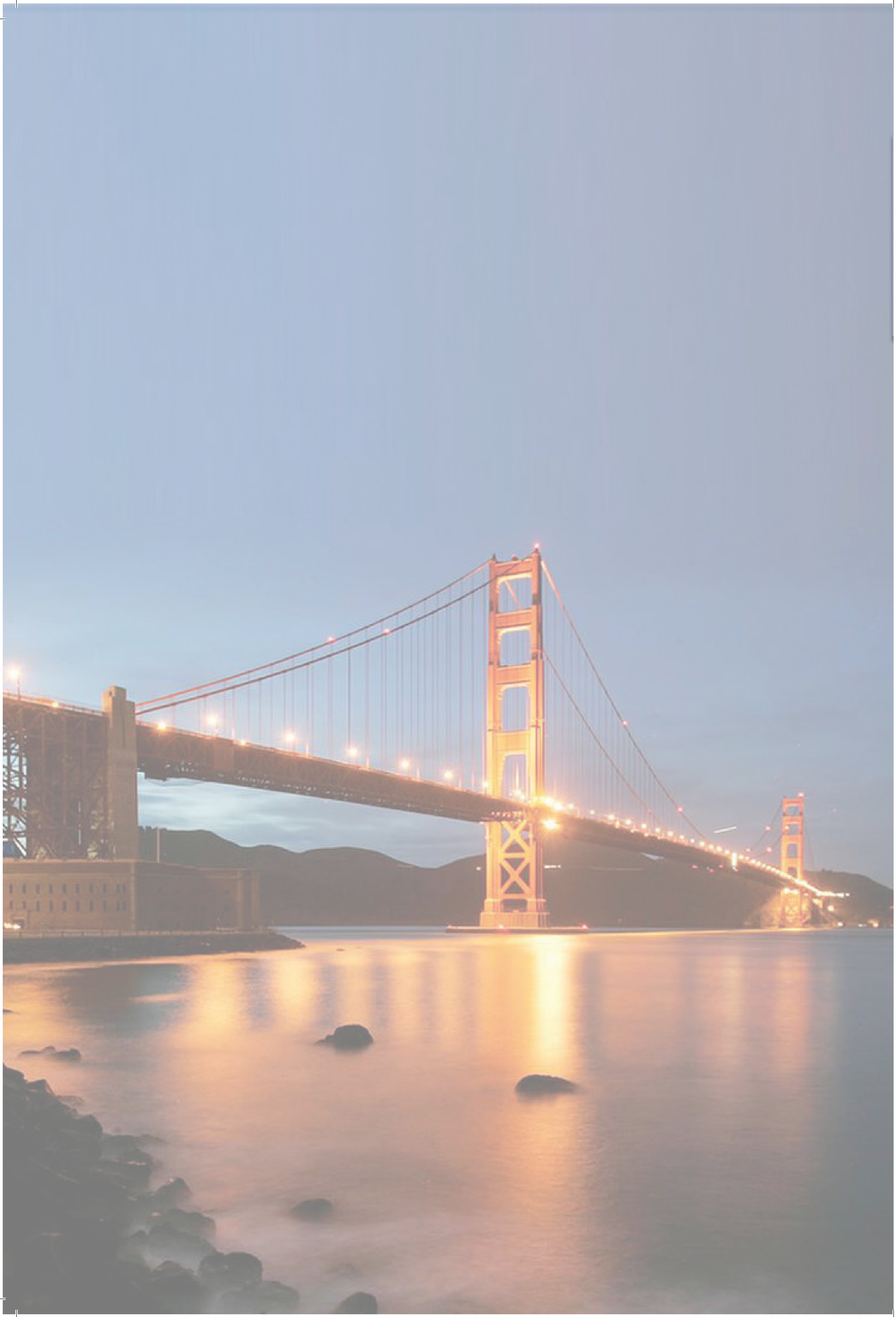
11. A compound microscope consists of a 10X eyepiece and 50X objective 17cm apart. Determine (a) the overall magnification, (b) the focal length of each lens, and (c) the position of the object when the final image is in focus with eye relaxed. Assume a normal eye, so N = 25cm.

12. A near-sighted person cannot see objects clearly beyond 25.0cm (her far point). If she has no astigmatism and contact lenses are prescribed for her, what power and type of lens are required to correct her vision?

13. Microscope uses an eyepiece with a focal length of 1.4cm. Using a normal eye with a final image at infinity the tube length is 17.5cm and the focal length of the objective lens is 0.65cm. What is the magnification of the microscope?

MECHANICS

Moments and Equilibrium of Bodies



Unit 3

Moments and Equilibrium of Bodies

Key Unit Competence

By the end of the unit, the learner should be able to explain the principle of moments and apply it the equilibrium of a body. .

My goals

By the end of this unit, I should be able to:

* Explain the principle of moments and apply it to equilibrium of a body.
* Come out with the effects of forces when applied onto a body.
* Know the effects of forces.

Introduction

In here, we shall majorly concetrate on the turning effect of force. As you know, it is very hard to close a door when you apply force near its turning point. That's why door handdles are always put at the end of the door so that the distance from the turning point to where force is applied increases. This increases the turning effect of the force applied. Which is the effect of forces on bodies one of our interrest in this unit.

Scalar and vector quantities

Activity 1

In a group of five members, try to stand in a line behind one another. Push your friend.

(i) What happens to your friend?

(ii) How do you feel?

(iii) What if in the process one stops pushing, What would happen?

In daily life, we normally pull the objects from one place to another.

What you should know

When pulling a goat that is to be tethered, obviously it will take the direction of the pull.

We can call this a force. **This is a quantity that changes body's state of rest or uniform motion.**

You noticed that after pushing your friend he/she changed position and direction. Hence, a force has both magnitude and direction.

This quantity can be termed as **a vector quantity.** This is a quantity with both magnitude and direction.

Activity 2

(i) Using the above example, discuss in groups or as a class other vector Quantities.

(ii) Analyse the effects of these physical quantities.

(iii) In daily life, how are these quantities utilised?

Ask your friend what time is it?

You will realise that he/she will tell the exact time not even indicating direction. Such a quantity is termed to be **a scalar quantity.**

A scalar quantity is a physical quantity that is defined by only magnitude (size).

Other examples of scalar quantities are **volume, mass, speed, and time intervals**. The rules of ordinary arithmetic are used to manipulate scalar quantities.

Activity Quick check!

Of the following physical quantities, group them in different sets of scalar and vector quantities: mass,energy,power,weight, acceleration,velocity, momentum, time, impulse, magnetic flux density, pressure, displacement.

Force as vector

Activity 3

1. As an individual or a group push the desk.
2. What happens to it?
3. What causes the change in position?
    a) Let as a class move to:
        (i) Football pitch.
        (ii) Net ball pitch.
        (iii) Basket ball play ground.

Try to kick a ball. What happens to it? What causes it to change its position?

Note what you observe.

Also, as you sit reading this book, you eventually feel tired. This is because of gravitational force acting on your body and yet you remain stationary.

From the above examples, we can define the "quantity force". We have to know the direction and the magnitude. For that matter, we conclude that the force is vector quantity.

We can think of force as that which causes an object to accelerate.

What happens when several forces act simultaneously on an object? In this case, the object accelerates only if the net force acting on it is not equal to zero.

The net force acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the total force, the resultant force, or the unbalanced force.)

GROUP WORK

1. .... is an example of a scalar quantity
   a) Velocity.
   b) Force.
   c) Volume.
   d) Acceleration.
2. .... is an example of a vector quantity
   a) Mass.
   b) Force.
   c) Volume.
   d) Density.
3. A scalar quantity:
   a) always has mass.
   b) is a quantity that is completely specified by its magnitude.
   c) shows direction.
   d) does not have units.
4. A vector quantity
   a) can be a dimensionless quantity.
   b) specifies only magnitude.
   c) specifies only direction.
   d) specifies both a magnitude and a direction.
5. A boy pushes against the wall with 50 kilogrammes of force. The wall does not move. The resultant force is:
   a) -50 kilogrammes.
   b) 100 kilogrammes.
   c) 0 kilogrammes.
   d) -75 kilogrammes.
6. A man walks 3 miles north then turns right and walks 4 miles east. The resultant displacement is:
   a) 1 kilometre SW
   b) 7 kilometres NE
   c) 5 kilometres NE
   d) 5 kilometres E
7. A plane flying 500km/hr due north has a tail wind of 45 mi/hr the resultant velocity is:
   a) 545 kilometres/hour due south.
   b) 455 kilometres/hour north.
   c) 545 kilometres/hour due north.
   d) 455 kilometres/hour due south.
8. The difference between speed and velocity is:
   a) Speed has no units.

b) Speed shows only magnitude, while velocity represents both magnitude (strength) and direction.
c) They use different units to represent their magnitude.
d) Velocity has a higher magnitude.

9. The resultant magnitude of two vectors
   a) Is always positive.
   b) Can never be zero.
   c) Can never be negative.
   d) Is usually zero.
10. Which of the following is not true.
    a) Velocity can be negative.
    b) Velocity is a vector.
    c) Speed is a scalar.
    d) Speed can be negative.

Table summarising Scalar and vector Quantities

Scalars	Vectors
Speed	Velocity
Temperature	Acceleration
Distance	Displacement
Area	Force/Weight
Entropy	Momentum
Volume	Drag

Moment of a force or torque

Requirements

* Asee-saw
* A knife edge
* A ruler
* Masses

Activity 4

Aim: ***To find the effect of moment of a force or couple***

(i) Pull/push the door by applying the force at its handle.

(ii) Pair yourselves and go out on a see saw balance.

(iii) Sit on one side and your friend to the next.

(iv) What happens?

A door knob is located as far as possible from the hinge line for a good reason.

If you want to open a heavy door, you must certainly apply a force, that done, however, is not enough where you apply that force and in what direction you push are also very important.

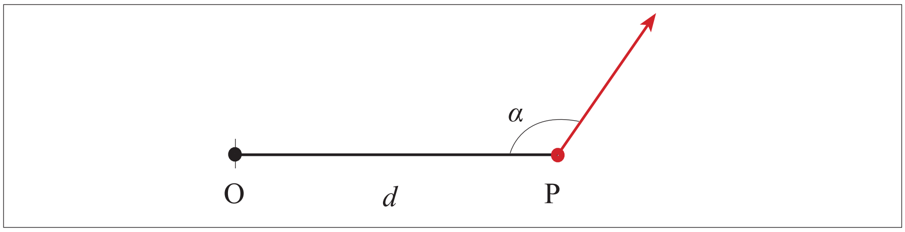


Figure 3.1: A force applied at a distance, d from a centre of rotation

This figure shows a force $\vec{F}$ acting on a body that is free to rotate about an axis. The force is applied at the point P whose position is defined by the vector $\vec{d}$. The direction of $\vec{F}$ and $\vec{d}$ make an angle α with each other.

We define the *torque* τ acting on the body from: $\tau = F \times d \times \sin\alpha$

The perpendicular distance of the line of action of the force from the axis of rotation is called the *moment arm* of the force.

The S.I unit of torque is *Newton-metre* [N m] or *metre-newton* [mN]

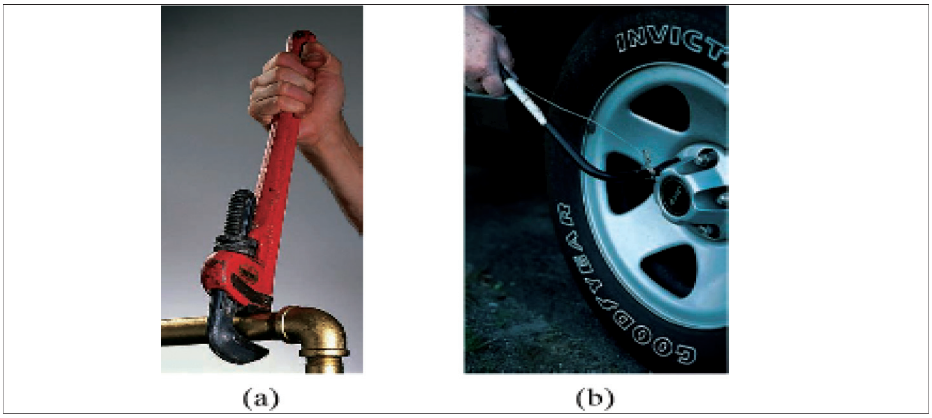


Figure 3.2: (a) A plumber can exert greater torque using a wrench with a long level arm (b) A tire iron too can have a long lever arm

Study the diagrams above

What happens when force is applied ;

(i) At the end of the tool used?

(ii) In the middle of the tool used?

Couple of forces

* In daily life, they say that a man and a woman constitute a couple. Why?
* How many of you know how to ride a bicycle? If you know, what do you do when negotiating a corner?
* If you have never ridden a bicycle ask your friend?

A *couple C* consists of two equal and opposite parallel forces whose lines of action do not coincide. It always tends to change rotation. It is clearly indicated in the figure below.

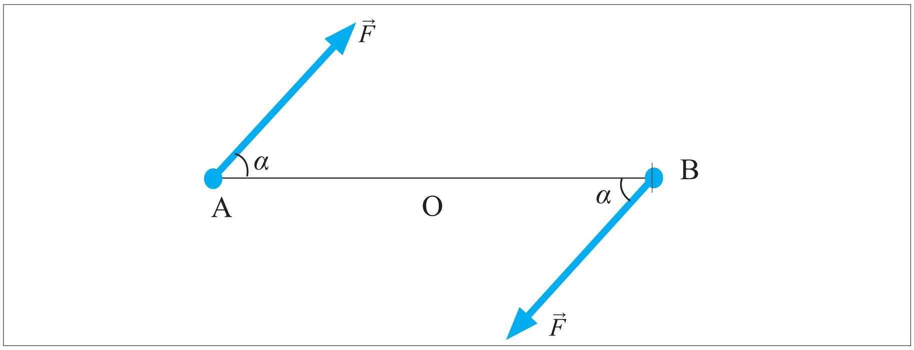


Figure 3.3: The resultant of couple of forces is zero but has an effect to rotate an object on which it's applied

$$C = \tau_1 + \tau_2 = F \times \overrightarrow{AO} \times \sin\alpha + F \times \overrightarrow{AB} \times \sin\alpha = F\,(\overrightarrow{AO} + \overrightarrow{AB})\sin\alpha$$

$$C = F \times \overrightarrow{AB} \times \sin\alpha$$

Coplanar forces

Activity 5

(i) As a class, visit a specialist in roofing.

(ii) Ask him/her what they consider when roofing.

(iii) Ask him/her the materials suitable to use while roofing and why do they opt for materials he/she told you ?.

(iv) Discuss amongst yourselves what would happen when a kinyarwanda hut (local hut) is roofed using Iron bars. What causes the effects ?

Parallelogram of forces

A force is a vector quantity. So it can be represented in size and direction by a straight line drawn to scale. The sum or resultant $\vec{R}$ of two forces $\vec{F}_1$ and $\vec{F}_2$ can be added by one of two vector methods.

(a)

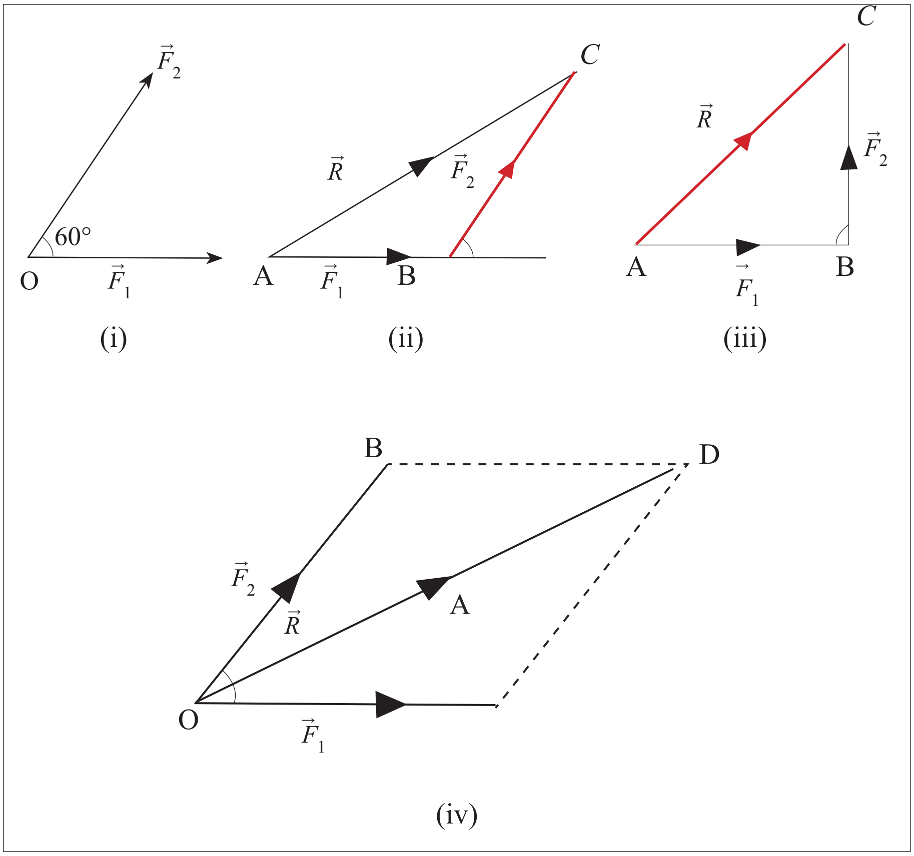


Figure 3.4: Addition of vectors

Figure (i) shows two forces $\vec{F}_1$ and $\vec{F}_2$ acting at 60° to each other.

Work

Draw a line AB to represent $\vec{F}_1$ and from B draw a line BC to represent $\vec{F}_2$. Note that AB is parallel to $\vec{F}_1$ and BC is parallel to $\vec{F}_2$. Join AC. Then AC is the resultant $\vec{R}_1$ of $\vec{F}_1$ and $\vec{F}_2$ in magnitude and direction. Note that the arrows on AB and BC follow each other.
How can you find the size of $\vec{R}$ and its direction θ to $\vec{F}_1$.

Either by accurate drawing or by calculation (using trigonometry in triangle ABC), If $\vec{F}_1$ and $\vec{F}_2$ are at 90° to each other; in this case the vector triangle ABC is a right-angled triangle, figure (iii). Applying Pythagoras' theorem, then: $R^2 = F_1^{\ 2} + F_2^{\ 2}$. So $R = \sqrt{F_1^{\ 2} + F_2^{\ 2}}$

Also the angle θ $\vec{R}$ makes with $\vec{F}_1$ is given by tan $\frac{BC}{AB} = \frac{F_2}{F_1}$, so knowing $\vec{F}_1$ and $\vec{F}_2$ θ can be found.

(b) Parallelogram of forces. The resultant $\vec{R}$ of $\vec{F}_1$ and $\vec{F}_2$ can also be found drawing a parallelogram of the forces. In figure (iv), draw OA to represent $\vec{F}_1$ and OB to represent $\vec{F}_2$ at an angle, say 60°, between $\vec{F}_1$ and $\vec{F}_2$ as in figure (i). Then complete the parallelogram OBDA. The resultant $\vec{R}$ is represented by the diagonal OD through O.

This gives the same result for $\vec{R}$ as in the previous method, since AD represents $\vec{F}_2$. If $\vec{F}_1$ and $\vec{F}_2$ are at 90° to each other, the parallelogram becomes a rectangle.

In general, if $\vec{F}_1$ and $\vec{F}_2$ make an angle θ, the resultant $\vec{R}$ is given by:

$$R = \sqrt{F_1^{\ 2} + F_2^{\ 2} + 2F_1F_2\cos\theta}$$

Resolved components

Note that in real life, we do a lot of things; name them.

Do we (you) apply physics?

If yes, how?

If no, why?

Activity 6

Aim; ***To investigate what happens when forces are added together***

* Try to lift your seat alone.
* How do you feel?
* Tell your friend to help you and you lift it together
* Are you feeling the same way as before when you were alone?
* What if your friend pulls in opposite direction to that of your force? What would happen?

Therefore, when solving daily problems, it is often helpful to replace one force by a combination of two forces with given directions. Of course, these two forces must be equivalent to the given one.

This means that their vector sum must agree with the given force. If this condition is fulfilled, we say that *the force* has been *resolved into components*.

A simple geometrical construction provides the magnitudes of the components: We can draw two lines from the end of the given force vector parallel to the given directions. In this way, we get the so-called *parallelogram of forces*. The magnitudes of the components now can be read off from the sides of this parallelogram.

Example

When pulling/saving a cow that has fallen into a pit, it is advised to pull it applying forces in one/same direction.
Why?

Equilibrium of coplanar forces

Class work

As a class, let us move outside. With the help of a teacher outside the class.

Are you seeing any object outside?

Are they stationary or in motion?

If at rest, what causes them to be at rest?

We say that an object is at rest or in motion under the influence of forces; if a body is at rest,then, there is no net force acting.

Therefore, if the resultant forces acting on a body is zero,the body is said to be in equilibrium.

The following pointers will help you to solve problems that involve a body acted on by three co-planar forces:

a) The line of action of three forces must ***all pass through the same point***.

b) The principle of moments: The sum of all clock-wise moments about ***any point*** must have the same magnitude as the sum of all anti-clock wise moments about the same point.

Centre of gravity

Activity 7

Requirements

* A stone
* A block of wood
* Retort stand
* Thread
* Pendulum bob or a plumbing bob
* A card board
* A marker

Selectively, choosing some of the apparatus, design a simple experiment to determine centre of gravity of an irregular lamina.

State reasons why you chose the apparatus.

Discuss the reasons with your friends.

Note down and present it to the whole class.

Hint Use the idea of O'level.

From the experiment, what can you say about Centre of gravity?

Is it the same as Centre of mass? Defend your answer.

Quiz. Can you locate where your Centre **of gravity is? What about your** Centre **of mass?**

Why do big ladies and gentlemen feel complications when standing up?

Fun!!!!!!!!
There are groups of people with big heads. It was found out that when they are moving and they fall down, they can't stand up by themselves. So they have to carry whistles alongside them so that when they fall down, they can blow the whistle for rescue!

Ask yourself why they are unable to help themselves.

From what we have seen above, the *centre of gravity* is the average location of the weight of an object.

The centre of mass or mass centre is the *mean* location of all the *mass* in a system. It can also be defined as a fixed point through which the entire weight of the object acts in whatever position the object may be placed.

In the case of a rigid body, the position of the centre of mass is fixed in relation to the body. In the case of a loose distribution of masses in free space, such as a shot from a shotgun or the planets of the solar system, the position of the centre of mass is a point in space among them that may not correspond to the position of any individual mass.

The term *centre of mass* is often used interchangeably with centre of gravity, but they are physically different concepts. They happen to coincide in a uniform gravitational field, but where gravity is not uniform, centre of gravity refers to the mean location of the gravitational force acting on a body

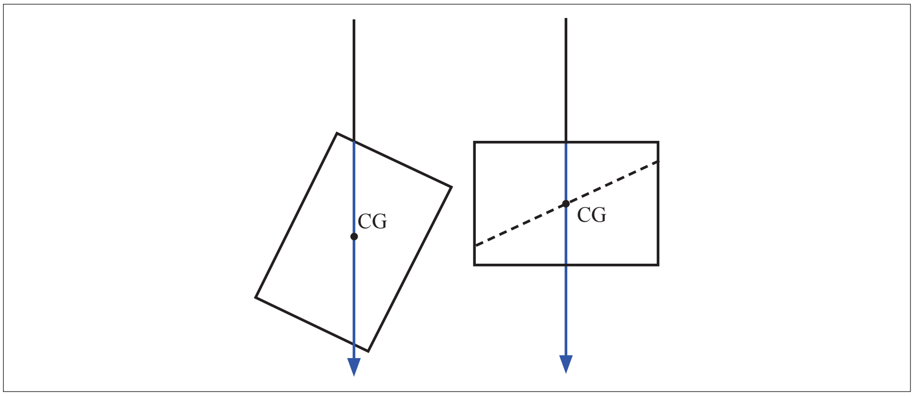


Figure 3.5: Determination of the centre of gravity

By observing the diagrams above.

* How can you build on and determine the centre of gravity of the body shown in the figure above?

* What if the object was irregular lamina .Explain an experiment you would perform to determine its Centre of gravity.
* In laboratory, perform the experiment.

Types of equilibrium

Requirements

* A log of wood
* A bottle
* A table
* A knife edge made of wood or A triangular glass prism
* A rectangular wooden block

Activity 8

Aim;*To find out the effect of application of force on to the equilibrium on the stability of a body.*

* Displace the desk. What happens when you withdraw the force you applied.
* Place a bottle on a table so that it rests on its horizontal surface. Displace or roll it. What happens?
* Place a knife edge on a table resting on its tip. Give it a small displacement. What happens to it?
* From the observations made,how do you conclude?

From what you have done above, before displacing the body, the body was at rest, and we describe this state to be in equilibrium.

Equilibrium has many different meanings, depending on what subject (chemistry or physics) or what topic (energy or forces).

Dealing with energy, there are three types of equilibrium.

From the Activity, a body is in either Stable, Unstable or in neutral equilibrium depending how it behaves when subjected to a small displacement.

Stable is when any sort of movement will raise the object's centre of gravity. When objects in stable equilibrium are moved, they have a tendency to fall back to their original position. For instance, a skateboarder at the bottom, in the middle, of a ramp. Either way the skateboarder moves, his/her potential energy will increase because he/she will be raising in height. The boarder will also roll back to the bottom of the ramp if he/she doesn't exert any sort of energy to maintain the new position.

When a body returns to its original position on being slightly disturbed, it is said to be in stable equilibrium.

Unstable is when any sort of movement will lower the object's centre of gravity. When such objects are moved, they cannot return to their original position without some exertion of energy. For instance, when a coin is placed on its side, it exhibits unstable equilibrium. Any sort of push will cause the coin to fall flat, lowering its centre of mass. The coin will not return to its side unless someone picks it up and resets it.

If the position of a body is disturbed and the body does not return to its original position, it is in unstable equilibrium.

Neutral is when any sort of movement does not affect the object's centre of gravity. For instance, a ball on a table exhibits neutral equilibrium. If the ball rolls, the centre of mass stays at the same height and thus it maintains the same equilibrium.

A body is said to be in neutral equilibrium if it moves to a new position when it is disturbed. Let us consider a cone to understand these states.

Conditions of equilibrium of objects on a horizontal plane

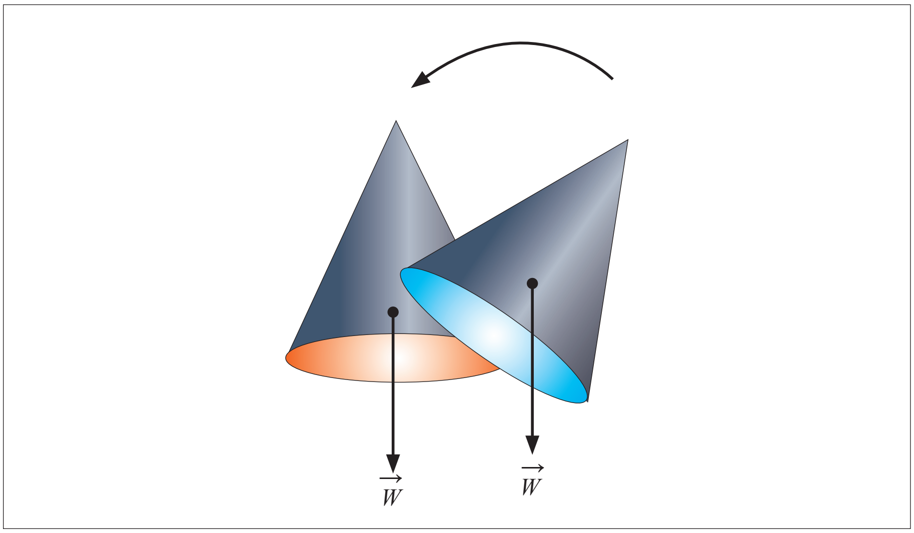


Figure 3.6: Cone in stable equilibrium

Activity 9

* Using a cone or a knife edge, or a triangular glass prism.
* Place it on a table resting it on its broad surface.
* Displace it in the direction of arrow shown.
* What happens to the cone?
* State your observations.
* Present them to the whole class.

From whatever we have done, we can conclude and say that a body is stable when:

1. The object's base is broad.
2. The centre of gravity is as low as possible.
3. The vertical line drawn from the centre of gravity should fall within the base. Lowering the centre of gravity of an object is important for stability.

Some examples

1. Articles like vases, table lamps, wine glasses, pedestal fans, glass jugs, bunsen burners and lanterns have a broad or heavy base to avoid toppling.
2. Racing cars have a broad base.
3. Motorists do not keep too much luggage in overhead carriers.
4. Cargo ships are loaded at the base.
5. Double-decker buses do not allow standing passengers in the upper deck.
6. Overloaded Lorries are stopped at check posts while going up and down mountain roads.

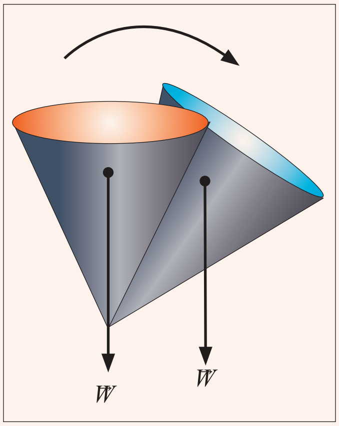


Figure 3.7: Cone in unstable equilibrium

Group work!

* Place a knife edge on a table so that it rests on its tip
* Displace it as shown in the figure.
* Note what happens to it. Can you say that it was stable,unstable or neutral? Why?

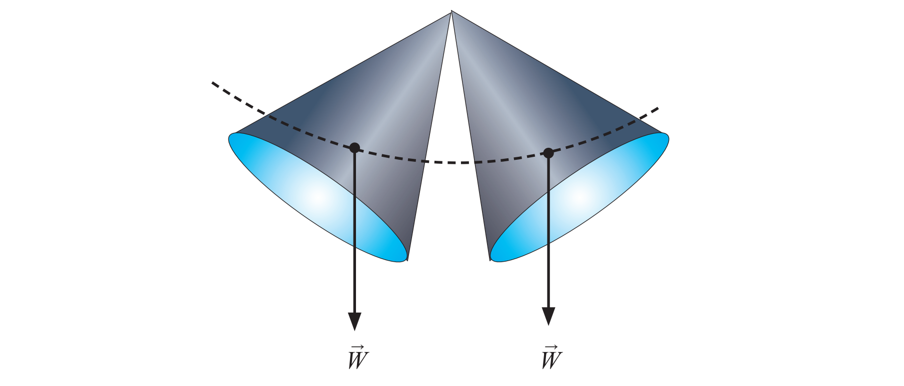


Figure 3.8: Cone in neutral equilibrium

CLASS WORK

* Roll a paper so that you come up with a cone. This should be done individually.
* Place them on a table as shown above.
* Displace it as shown in the figure 3.8.
* What did you observe?

You now know that,

* The centre of gravity of an object is a fixed point through which the entire weight of the object acts in whatever position the object may be placed.
* There are three states of equilibrium namely.
  a) stable
  b) unstable
  c) neutral.
* There are three conditions for an object to be stable:
  a) The object should have a broad base.
  b) The Centre of gravity should be as low as possible.

c) The vertical line drawn from the centre of gravity should fall within the base.

Other examples

1. In here, we shall look at what happens when a body is rolled on an incline.

Field work

* Outside the class, try to get a place that is sloping.
* Try to get a spherical body; say a used bicycle rim.
* Roll it down the incline.
* In groups of 5, discuss what happens.
* Present your finds to the class.

Considering the diagram below, we can see that the body will ascend downwards when put on the incline.

Have you ever done this alone?

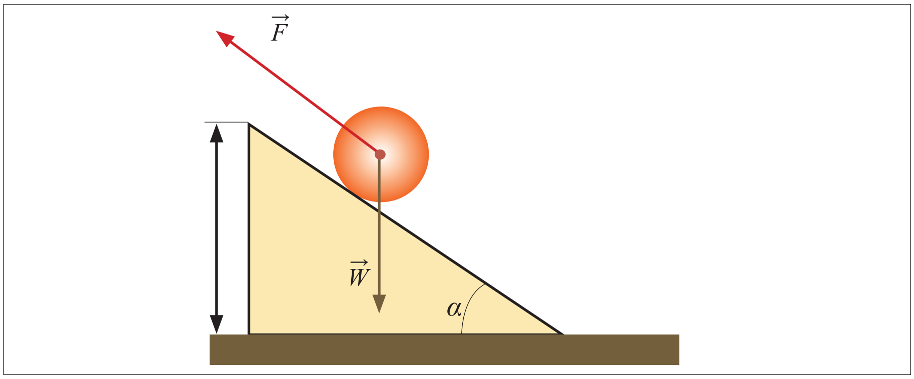


Figure 3.9: Equilibrium of an object on an inclined plane

On a plane without friction, if l is its length, h its height, the object is in equilibrium if there is a force $\vec{F}$ acting on the object upward on the plane having a magnitude: $F = W\dfrac{h}{v}$

Note that there is a frictional force.

Equilibrium of a suspended object

Let us consider a ruler such that its centre of gravity is at a point G and can be suspended at a given point O

The centre of gravity is below the point of suspension o

The immobility being realized, if we move the ruler from its equilibrium position, it has the tendency to come back. We have the case of a *stable equilibrium*.

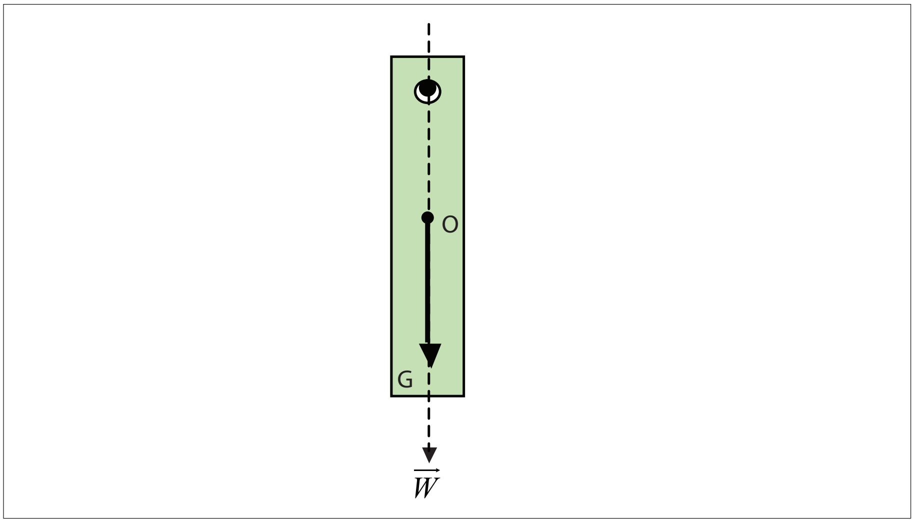


Figure 3.10: Equilibrium of a suspended object; the centre *of gravity is below the point of suspension*

The centre of gravity is above the point of suspension o

This equilibrium is difficult to realize. When you move lightly the ruler, it moves away from the equilibrium position. We have an *unstable equilibrium*

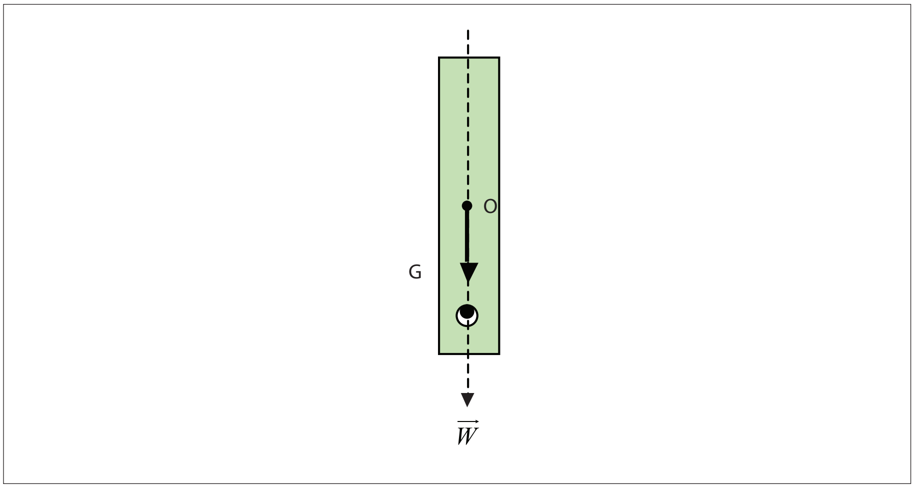


Figure 3.11: Equilibrium of a suspended object; the centre *of gravity is above the point of suspension*

The centre of gravity is at the point of suspension o

In this case, the action and the reaction being at the same point, when you move lightly the ruler from its equilibrium position, it takes a new equilibrium position. We have a *neutral equilibrium.*

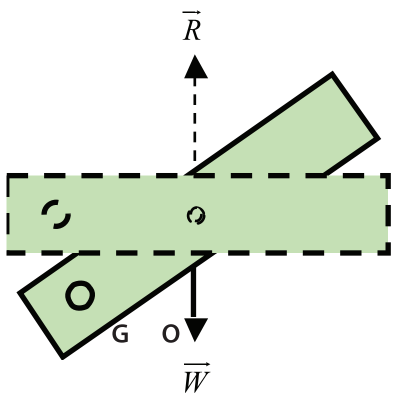


Figure 3.12: Equilibrium of a suspended object; the centre *of gravity is at the point of suspension*

Equilibrium of a solid submitted to several forces

For a body to be at rest, the sum of the forces acting on it must add up to zero. Since force is a vector, the components of the net force must each be zero. Hence, if the forces on the object act in a plane, a condition for equilibrium is that: $\Sigma F_x = 0$, $\Sigma F_y = 0$.

We must remember that, if a particular force component points along the negative axis, it must have a negative sign. Equations above are called the *first condition* for equilibrium.

Although these equations must be true if an object is to be in equilibrium, it is not a sufficient condition. If the body is to remain at rest, the net torque applied to it must be zero. Thus we have *the second condition* **for equilibrium: that the sum of the torques acting on a body must be zero:** $\Sigma\tau_y = 0$.

This will assure that the angular acceleration about any axis will be zero; if the body is not rotating initially it will not start rotating. The first and second conditions are the only requirements for a body to be in equilibrium.

This has several applications among them; we have the balancing of a seesaw, beam balance, etc.

Examples

QUIZ

1. Why do you think that when a person with huge bums after sitting down feels it hard to stand up? Do you think that it is easy for the lady below to stand up? Why?

Figure 3.13: A fat woman with a raised centre of mass

2. A bottle resting on its top falls to the next position when slightly displaced. Explain this observation.

Stevinus proof

Stevinus (some times called Stevin) proof of the law of equilibrium on an inclined plane known as the "Epitaph of Stevinus".

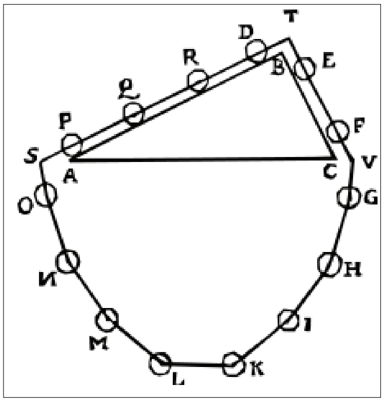


Figure 3.14: Stevenus proof diagram

He derived the condition for the balance of forces on inclined planes using a diagram with a "wreath" containing evenly spaced round masses resting on the planes of a triangular prism (see the illustration on the figure 3.14). He concluded that the weights required were proportional to the lengths of the sides on which they rested assuming the third side was horizontal and that the effect of a weight was reduced in a similar manner.

It's implicit that the reduction factor is the height of the triangle divided by the side (the sine of the angle of the side with respect to the horizontal). The proof diagram of this concept is known as the "Epitaph of Stevinus".

Archimedes and the principles of the lever

Activity 10

1. In pairs, discuss levers using knowledge of O'level Machines.
2. What is your friend telling you. Is it what you knew before?

Building up from the earliest remaining writings regarding levers date from the 3rd century BC and were provided by Archimedes. **"***Give me a place to stand, and I shall move the Earth with it***"** is a remark of Archimedes who formally stated the correct mathematical principle of lever.

Force and levers, law of the lever

A lever is a beam connected to ground by a hinge or pivot called a fulcrum. In other words we say that the **lever is a movable bar that pivots on a fulcrum attached to a fixed point.** The lever operates by applying forces at different distances from the fulcrum, or a pivot.

The ideal lever does not dissipate or store energy, which means there is no friction in the hinge or bending in the beam. In this case, the power into the lever equals the power out, and the ratio of output to input force is given by the ratio of the distances from the fulcrum to the points of application of these forces. This is known as the *law of the lever.*

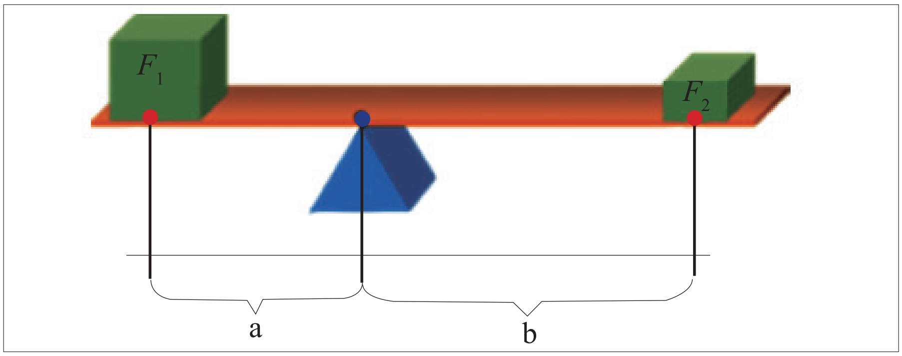


Figure 3.15: A lever in balance

Note: You can draw a diagram masses replaced by learners.

The mechanical advantage of a lever can be determined by considering the balance of moments or torque, τ, about the fulcrum, and we write:

$$MA = \frac{F_2}{F_1} = \frac{a}{b}$$

where F1 is the input force to the lever and F2 is the output force. The distances a and b are the perpendicular distances between the forces and the fulcrum.

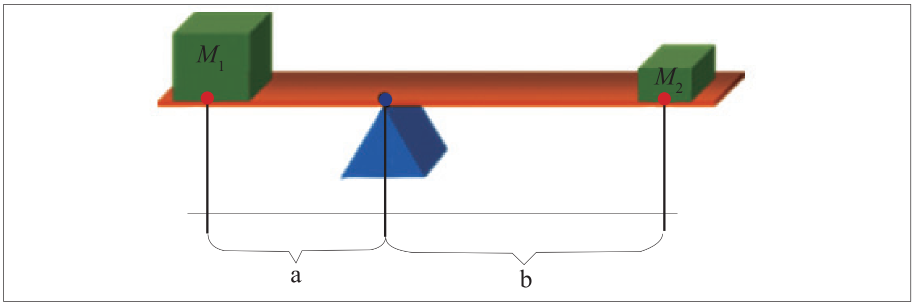


Figure 3.16: A lever in balance considering masses

For the case of the figure 3.16, knowing that the forces considered are weights of objects and W = Mg, we can write the mechanical advantage of the lever as a ratio, $MA = \frac{M_2}{M_1} = \frac{a}{b}$

Activity 11

In groups determine the mass of the object using the principle of moments.

Apparatus required

* 1 uniform meter rule
* A knife edge
* Unknown mass

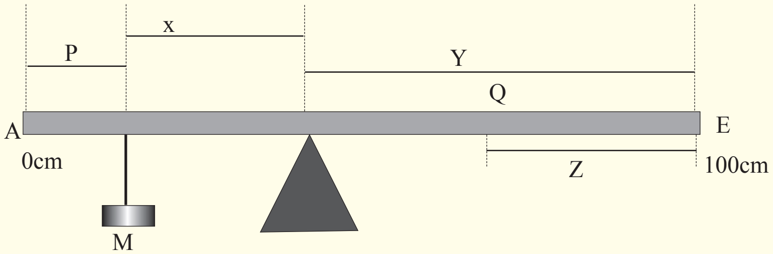


Figure 3.17: To determin the mass (m) of the object (M)

Instructions

In this experiment, the learner determines the mass *M* of the provided body (meter ruler).

a) Weigh the meter ruler provided to obtain its mass, *M*. Balance the meter ruler on a knife edge. Read Q the balance point. Find Z.

b) Balance the meter ruler with its graduated face upwards on a knife edge. With the mass *M* provided at P=10 cm from A as shown,

c) Measure and record distance X and Y.

d) Repeat procedure (b) and (c) for values of P=15,20,25,30 and 35cm.

e) Tabulate your results including values of (Y-Z) and (X-P).

f) Plot a graph of (Y-Z) against (X-P).

g) Find the slope "*S*"of your graph.

h) Calculate the mass *M* of the body from Sm=*M*.

EXAMPLE

1. A seesaw consisting of a uniform board of mass M =10kg and length l=2m supports a father and daughter with masses mf and md, 50 and 20kg respectively as shown in the Figure. The support (called the fulcrum) is under the **centre** of gravity of the board. The father is a distance d from the **centre**, and the daughter is at distance *l*/2 from the **centre**. (a) Determine the magnitude of the upward force (reaction) n exerted by the support on the board. (b) Determine where the father should sit to balance the system.

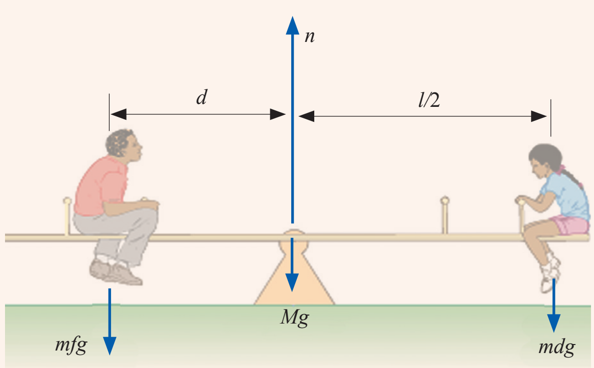


Figure 3.18: A seasaw

SOLUTION

$n = m_f g + m_d g + mg$, and

$m_f = 50\text{kgs},\ m_d = 20\text{kg},\ g = \text{m/s}^{-2}, m = 10\text{kg},$

$n = 50 \times 10 + 20 \times 10 + 10 \times 10 = (500 + 200 + 100)\text{N}$

$n = 800\text{N}$

Sum of clockwise moments = Sum of anticlockwise moments

$m_f \times d = m_d \times \frac{l}{2}$

$50 \times d = 20 \times \frac{2}{2}$

$50d = 20$

$d = \frac{20}{50}\ \text{m}$

$d = 0.4\text{m}$

Student's trials

1. A person holds a 50.0N sphere in his hand. The forearm is horizontal, as shown in Figure. The biceps muscle is attached 3.0cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.

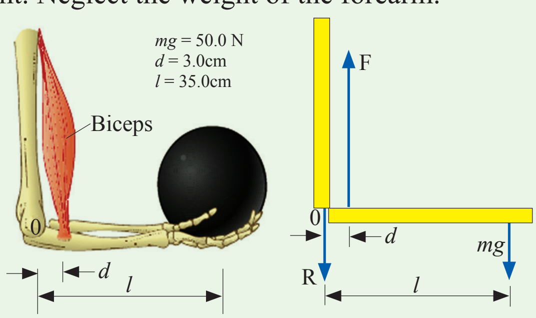


Figure 3.19: Showing forces on the arm lifting a heavy mass

2. A uniform horizontal beam with a length of 8.00m and a weight of 200N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam. If a 600N person stands 2.00m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.

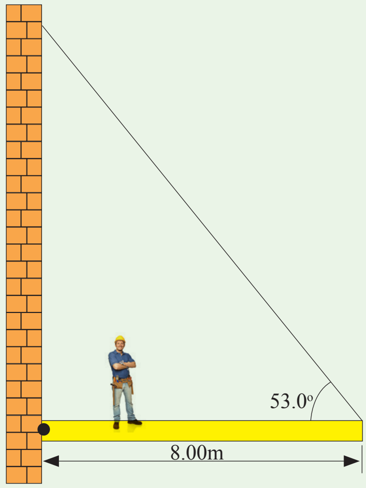


Figure 3.20: Showing the forces developed in a bar when subjected to tension

3. Calculate the magnitudes F_A and F_B of the tensions in the two cords that are connected to the vertical cord supporting the 200kg chandelier in the figure.

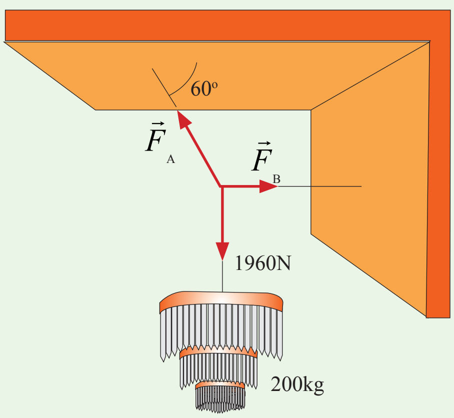


Figure 3.21: Indicating forces developed when a mass is held by two strings

4. A uniform 1500kg beam, 20m long, supports a 15,000kg printing press 5 from the right support column, see the figure. Calculate the force on each of the vertical support columns.

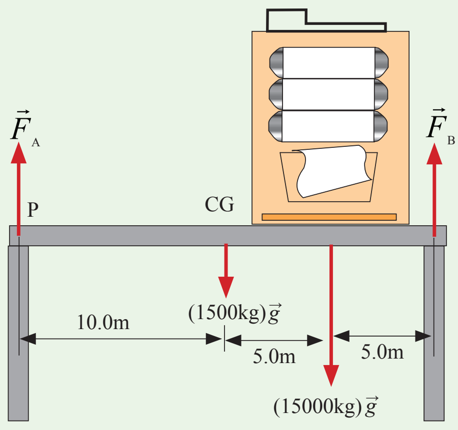


Figure 3.22: Indicating forces developed in a beam

5. The bar in the figure is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force F_p required at the long end of the bar can be quite a bit smaller than the rock's weight mg, since it is torques that balance in the rotation about the fulcrum. If, however, the leverage isn't sufficient, and the large rock isn't budged, what are the two ways to increase the leverage?

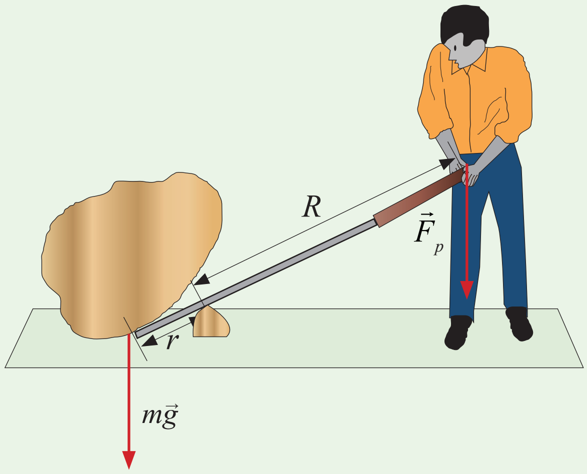


Figure 3.23: Indicating forces developed on a lever

6. The bridge on a river of a country of Central Africa is supposed to have a length of 100m and mass 10^5kg. It leans on two pillars to its extremities.

What are forces exerted on the pillars when three cars (one Mercedes of 1500kg, a Renault of 1200kg and a Fiat 1000kg) are respectively to 30, 80 and 60m from the extremity leaning on the left bank.

7. A horizontal rod AB of negligible weight, 51cm long is submitted in A and in B to two forces $\vec{F}_1$ and $\vec{F}_2$ of magnitudes respectively 14N and 7N. The force $\vec{F}_1$ makes an angle of 45° with the vertical and the force $\vec{F}_2$ is perpendicular to the rod. Their direction is oriented downward. Determine the characteristics of the force which will make the rod in equilibrium.

8. A horizontal rod AB is suspended at its ends by two strings. (See the figure below). The rod is 0.6m long and its weight of 3N acts at G where AG is 0.4m and BG is 0.2m. Find the tensions X and Y

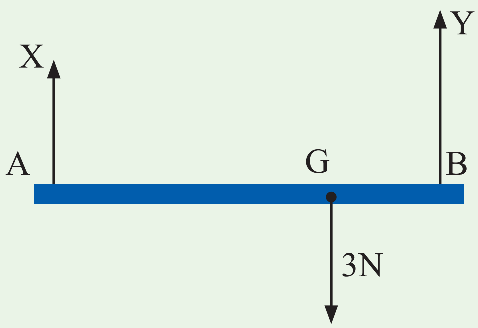


Figure 3.24: Indicating forces on abeam supported by two strings

9. A sphere of 50N stands against two inclined planes making respectively angles of 30° and 45°. Calculate the forces of reaction of the two planes on the sphere.

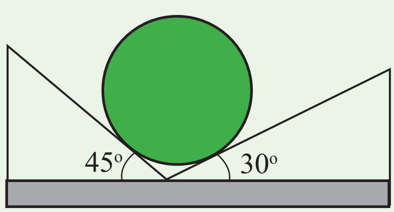


Figure 3.25: Showing asphere resting on two inclines

10. A block of mass 330kg is suspended by three unstretchable ropes as shown on the figure below. If the system is in equilibrium,

a) determine T_1,

b) If O_1 = 15°, θ_2 = 30°, find the tensions in the ropes.

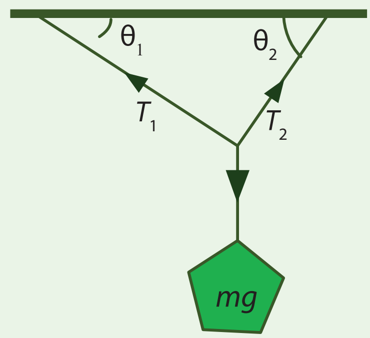


Figure 3.26: Showing a body of mass supported by 2 strings

11. A ladder AB weighing 160N rests against a smooth vertical wall and makes an angle of 60° with the ground as shown In the figure below. The ladder has small wheels at the point A such that the friction with the vertical wall is negligible. Find the forces acting on the ladder at point A and point B.

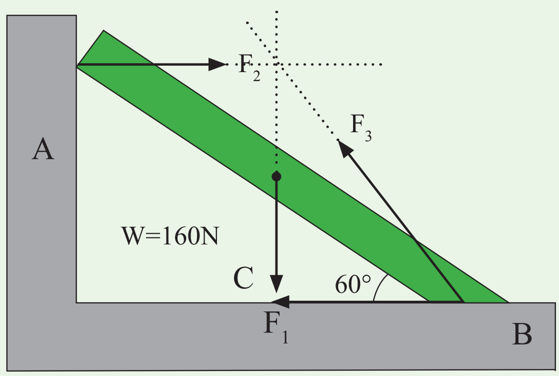


Figure 3.27: Showing a uniform bar resting on two surfaces at an angle

The coefficient of static friction between the ladder and the ground is 0.53. How far up the ladder can the firefighter go before the ladder starts to slip?

12. A homogeneous beam of length 2.20m and of massm =25.0kg is fixed on a wall by a hinge and is held in horizontal position by a metallic string making angle of $\theta = 30.0°$ as shown in the figure below. It holds a mass $M = 280$kg suspended at its extremity. Determine the components of the force $\vec{F}$ exerted by the wall on the beam at the hinge and components of the tension $\vec{T}$ in the metallic string.

13.

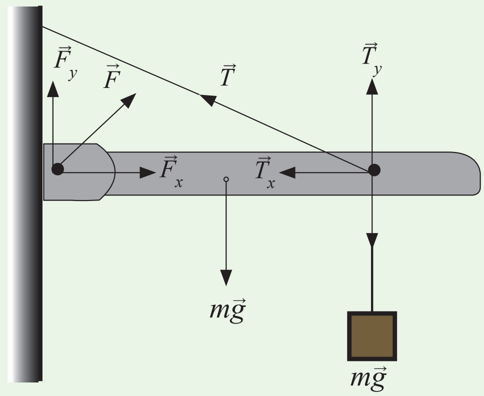


Figure 3.28: Showing forces acting on a bar fixed at one point with a mass connected at one point

Extension exercise

1. a) State the conditions under which a rigid body is in equilibrium under the action of coplanar forces.

   b) Forces of 2.83N, 4.00N and 6.00N act on an object O as shown the figure below.

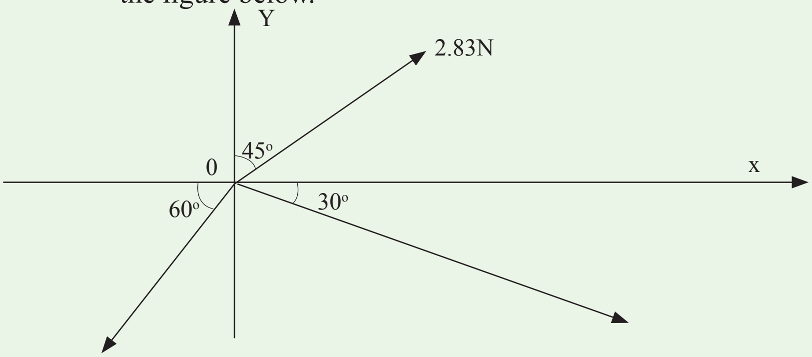


Figure 3.29: Showing forces acting on a body in different directions

2. Find the resultant force on the object.
   When three concurrent forces act on a body which is in equilibrium, the resultant of the two forces should be equal and opposite to the third force. Prove this statement.

3. A uniform ladder of massm and length L leans against a smooth vertical wall making an angle φ with a horizontal floor. The coefficient of static friction between the ladder and the floor is μ

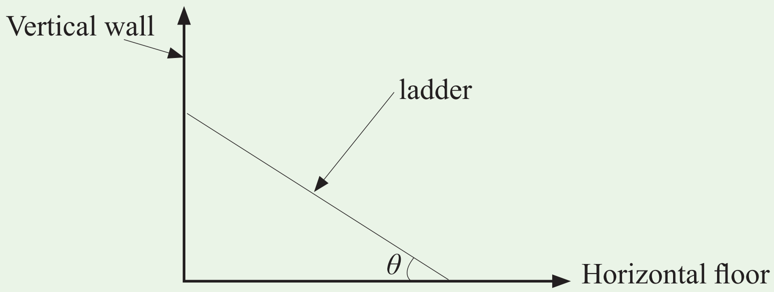


Figure 3.30: Showing a ladder (shape) resting on two surfaces

Find (in terms of μ) the minimum angle θ_m at which the ladder does not slip

MECHANICS

Work, Energy and Power


CASE
721E

Unit 4

Work, Energy and Power

Key Unit Competence

By the end of the unit, the learner should be able to evaluate the relation between work, energy and power and the resulting phenomena.

My goals

By the end of this unit, I will be able to:

* define work done, energy and power.
* state the formulae of work, energy and power.
* explain how power depends on energy.
* explain how gravitational potential energy.
* identify the difference between potential energy and kinetic energy
* describe strain and work done in deforming materials

Introduction

In real life, we always use the term **work**. Which means "task to be accomplished. But before the task to be done, one must have **energy**. Then if a given work is done in a given time, we say that one has power i.e work done in a given time.

Work

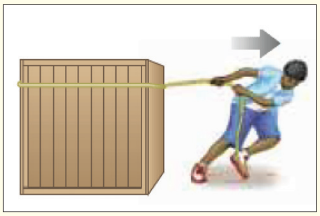
Review of the idea of work

Activity 1

Study and interprete the diagram below

CASE I

Work is done when a force moves its point of application along the direction of its action.



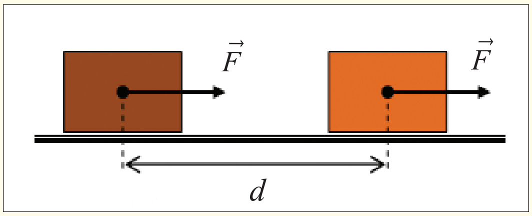


Figure 4.1: The force and the displacement have the same direction

* How is the force applied onto the body?
* Why does it change its position?
* What if the body is 10 times the mass of the boy. Would the body change its position? Why?
* State the direction of application of force.

From the fig. 4.1 and your deductions, how can you define Work?

CASE II

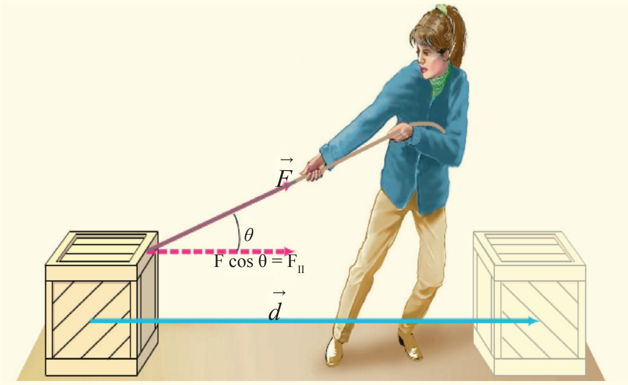


Figure 4.2: The force and the displacement make a certain angle

Activity 2

Aim; ***To relate distance,force and work***
Let us as a class visit any where people are constructing a house,a bride, road.
Ask them why they are paid?
Ask them how they measure what they do.

From the diagram (CASE I)

The work is done when a force moves its point of application along the direction of its action.

Let $W = Force \times distance$

$W = F \times d$

1Nm=1Joule

In the second case, the work done is defined as the product of the component of the force in the direction of the motion and displacement in that direction. That is: $W = F \times \text{COS}\,\theta$

There is another unit of work called kilogram-metre, which is the work done by a force of 1kg when its application point moves through a distance of 1 m

1 kgf = 9.81N

In cgs system, the work is expressed in (erg) 1erg = 10-7J, 1(erg)=1(dry n cm)

Then the ***Joule*** **is the work done by a force of 1N when its application point moves through a distance of 1 meter in the direction of force**.

Work is the scalar although force and displacement are both vectors.

Expressions of some kinds of work

Work of the gravitational force

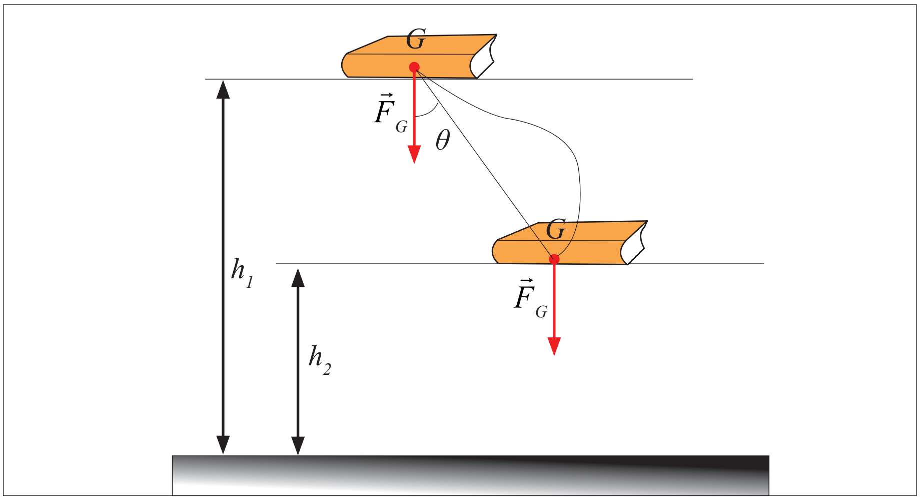


Figure 4.3: Object falling under gravity

Activity 3

a) Hold a book in your hands at a height say h.

b) Leave it to fall vertically onto the ground.

NOTE;

If it moves from h_1 to h_2 under the gravitational force W following the path GG', the work done is: $W = F_G \times \overline{GG'} \times \cos\theta$

$\overline{GG'}\ \cos\theta = h_1 - h_2$, we have: $W = F_G(h_1 - h_2)$

What if it is projected a bit, would the work done change?

Why?

From the deductions, it can be noted that:

The work done by the gravitational force does not depend on the path followed but on the change of the height.

Work done by the force of pressure

Activity 4

Requirements

* A syringe with a piston.

Aim: ***To determine work done by a piston.***

* Pull the piston through a small distance Δx as shown in the figure below.

Assuming you applied a force F, What is the work done?

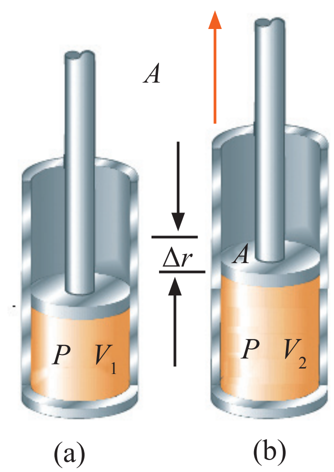


Figure 4.4: Heating the gas, the piston moves up

Note:

From Pressure being the force per unit area, we know that gas exerts pressure on the inside surface of the container.

Let us consider gas confined in a cylinder by a piston.

If p is the pressure in the cylinder A, the area of the piston, heating the gas, the piston moves up, the volume increases but the pressure remains constant.

If Δx is the displacement of the piston, the force which moves it is given by:

$F = PxA$ The work during the displacement is:
$W = F \text{ x } \Delta x$

$W = P \text{ x } A \text{ x } \Delta x$

$A \text{ x } \Delta x = \Delta V$, the change of the volume

Then we have: $W = P \text{ x } \Delta V = P(V_2 - V_1)$

Energy

Ask yourself why some times you feel like not working or bored.

What do you normally say when you are asked why you are not performing any duty?

Use what comes into your mind to define energy.

Normally we say that Energy **is ability of a body to do work.**

It's measured in Joules like work. When an interchange of energy occurs between two bodies, we can consider the work done as a measure of the quantity of energy transferred between them.

Potential energy

Quick Check 1

Use this principle to determine the blanks in the following diagram. Knowing that the potential energy at the top of the tall platform is 50 J, what is the potential energy at the other positions shown on the stair steps and the incline?

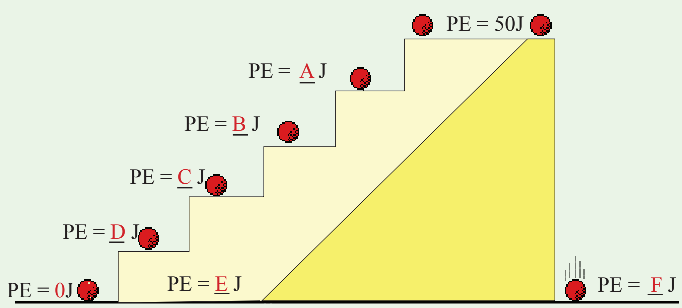


Figure 4.5: Showing energy change at different levels from the ground

Activity 5

How do we know that things have energy just because of their height? Well, let's think about the following process:

1. You lift a ball off the ground until it is above your head.
2. You drop it.
3. It is moving fast right before it hits the ground.
4. Draw a conclusion.

Energy is said to be conserved, which means that it cannot be created or destroyed, only transferred from one form into another. So whatever energy we put in, has to go somewhere.

Then we can define the *potential energy* **as the energy a system of bodies has because of the relative position of its part** .

Gravitational potential energy

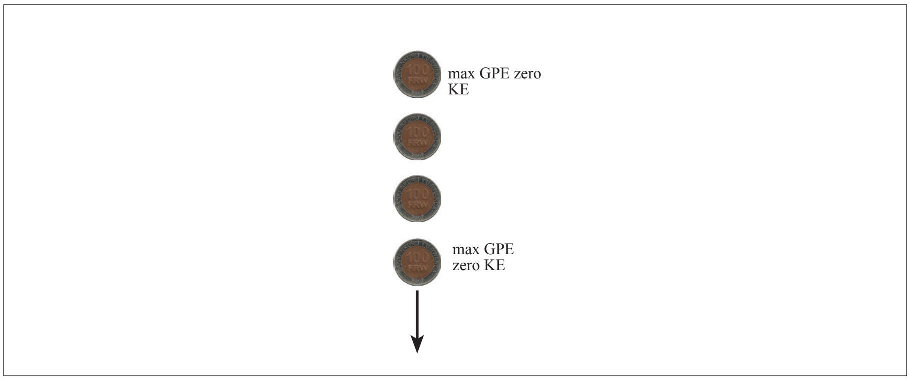


Figure 4.6: Showing change in energy as body falls

From the diagram, what can you deduce?

Still in a class, perform the same experiment using any mass.

Note:

* The potential energy when a body of massm is at height *h* above ground level equals to the work which must be done against the downward pull of gravity to raise the body to this height.

* A force equal and opposite to $W=mg$ has to be exerted on the body over displacement *h* assuming g is constant near the earth's surface.

Therefore, work done by external force against gravity:
= *force X displacement*
= *mgh*
And so $p.e = mgh$

Figure 4.7: Showing change in energy of a car at an inclined road/slope

Activity 6

Aim ; ***To determine the effect of Gravitational potential energy***

* The diagram.
* What do you think will happen when the car is made to slope down when its engine is switched off?

Check Your Understanding

Check your understanding of the concept of potential energy by answering the following questions.

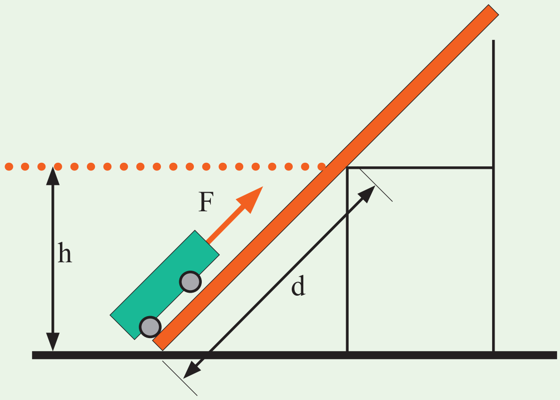


Figure 4.8: Showing a body moving up the incline

1. A cart is loaded with bricks and pulled at constant speed along an inclined plane to the height of a seat-top. If the mass of the loaded cart is 3.0kg and the height of the seat top is 0.45 metres, then what is the potential energy of the loaded cart at the height of the seat-top?

Elastic potential energy

Individual Assignment

* Try to get a rubber band or any elastic material.
* Try to stretch as shown in the diagram.
* Explain what you have felt and seen.

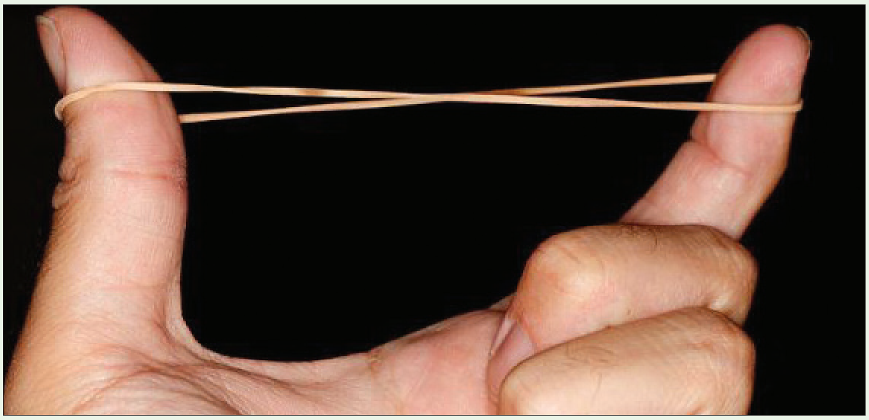


Figure 4.9: Showing energy change in an elastic rubber band being pulled by fingers

Activity 7

Aim; ***To find out whether there is energy stored in elastic Materials***

In laboratory,

* Try to perform experiment arranging your apparatus as shown in the figure below.
* What do you observe after putting a mass on the spring.
* What would happen if the mass of the body is given a small displacement downwards?

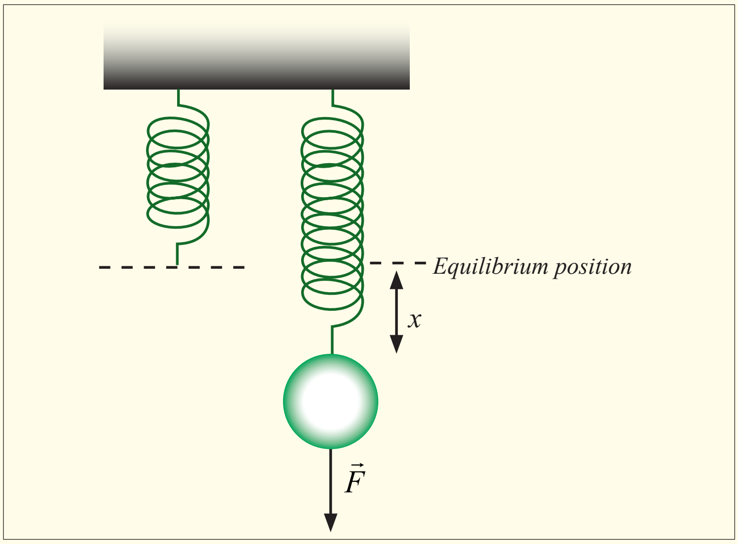


Figure 4.10: A force exerted on a mass attached on a spring extends the spring

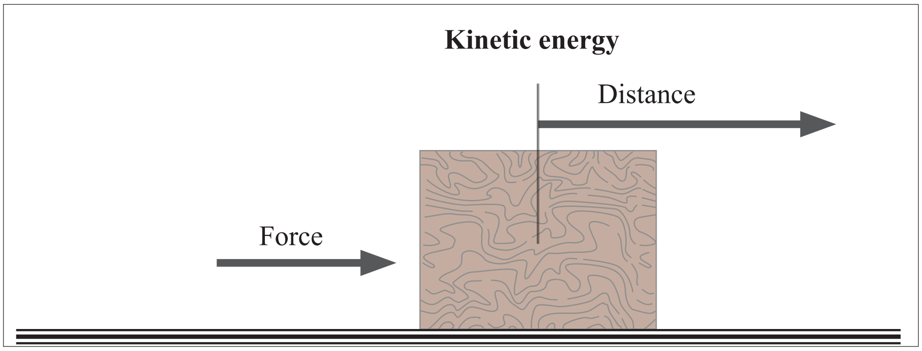
Note: On a spring on which a force $\vec{F}$ is exerted producing an extension x, according to the Hooke's law:

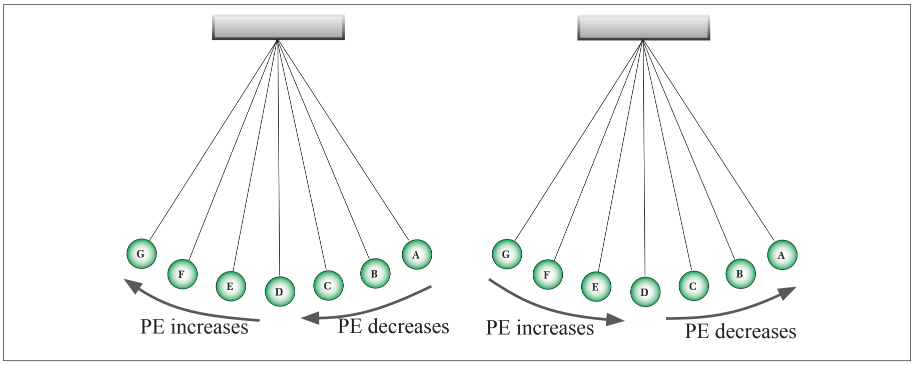
$F = kx$ where $k > 0$ is the constant depending on the string.

The potential energy stored is $p.e = \frac{1}{2}kx^2$

Kinetic energy

In all the diagrams indicated below, show the changes of energy of a moving object. study them clearly.





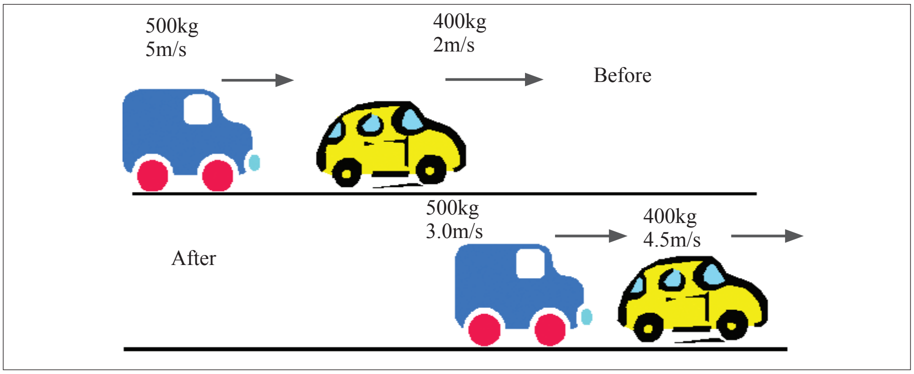


Figure 4.11: Showing energy changes as bodies are set in motion

What energy do the objects you are seeing possess?

State the reason for your answer.

In general, we can define ***Kinetic energy*** **as the energy a body has because of its motion.**

Kinetic energy of an object in translational motion

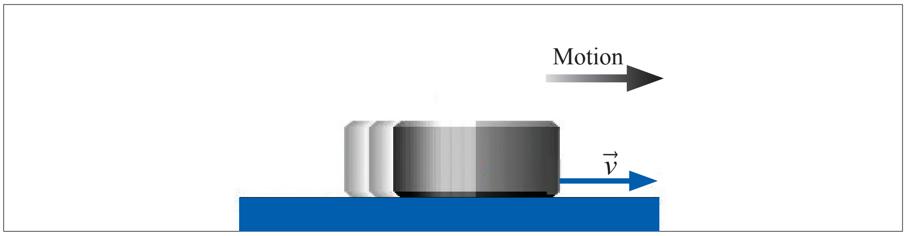


Figure 4.12: Translational motion of an object on a horizontal plane

If the motion is uniformly varied from the rest:

$$x = \frac{1}{2} g t^2 \qquad (1)$$

$v = gt$ (2)

$F = mg$ (3)

and $W = Fx \Rightarrow W = F\frac{1}{2}gt^2 = \frac{1}{2}mggt^2$

$W = \frac{1}{2}g^2t^2 = \frac{1}{2}m(gt)^2 \Rightarrow W = \frac{1}{2}mv^2$

Then if the centre of mass has a velocity *v,* the kinetic energy is given by:

$k.e = \frac{1}{2}mv^2$

Exercise

Study the graph below and answer the questions.

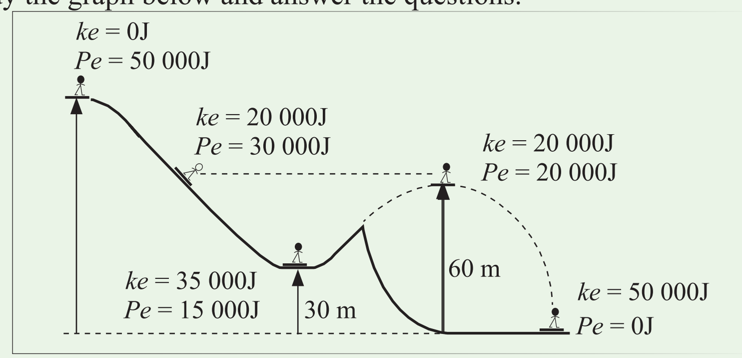


Figure 4.13: The loss of potential energy is the gain of kinetic energy

* Explain using equations where necessary how the values at different points were obtained.
* From the graph, what is the maximum energy of the system?

Kinetic energy theorem or work-energy theorem

We have seen that a moving object can do work. The opposite is true as well: work must be done on an object to give it *k.e*. To find the precise relationship, we reverse the above argument.

Suppose an object of massm is moving with an initial speed v_0 and to accelerate it (uniformly) to a speed v a net force $\vec{F}$ is exerted on it parallel to its motion over a distance x.

Then the net work done on it is $W = F \times x$

Using the second Newton's law $F = m\gamma$ and we know that $v^2 = v^2_0 + 2\gamma x$, we find: $W = F \times x$

$$W = m\gamma x = m\left(\frac{v^2 - v_0^2}{2x}\right)x = m\left(\frac{v^2 - v_0^2}{2}\right) = \frac{1}{2}m(v^2 - v_0^2) = \Delta k.e$$

We can write: $\Sigma W = \Delta k.e$

Where ΣW is the net work done.

Theorem: "The net work done on an object is equal to its change in kinetic energy"

This is known as the work-energy theorem.

Total mechanical energy

The figure below is about a falling mango from a branch study it carefully.

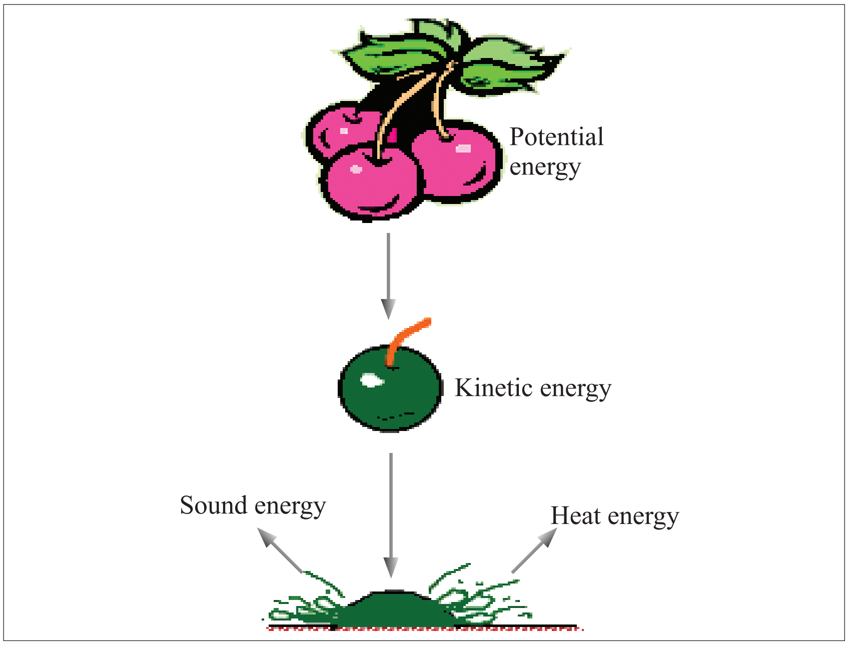


Figure 4.14: A falling fruit

* Are there energy changes?

* If yes, what are those energy changes?

We can therefore say that the total Mechanical energy is the sum of $P.e$ **and** $K.e$. $M.e = k.e + p.e$

Conservation of mechanical energy

If a body of massm is thrown vertically upwards with an initial velocity v_0 at A, it has to do work against the constant force of gravity.

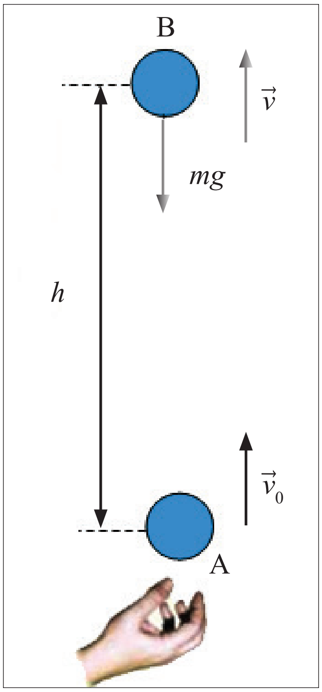


Figure 4.15: The loss of potential energy is the gain of kinetic energy

When it has risen to B, the velocity becomes v.

By definition of kinetic energy: loss of kinetic energy between A and B = work done against gravity

By definition of potential energy: gain of potential energy between A and B = work done against gravity. Then the loss of kinetic energy = gain of potential energy.

$\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = mgh$, and we can write

$\Delta k.e = \Delta p.e$

This is called the ***principle of conservation of mechanical energy*** and maybe stated as follows: ***"The total amount of mechanical energy of an isolated body is a constant"***.

Power

Activity 8

Interpret the diagrams below

Figure 4.16: A man lifting heavy bags

Figure 4.17: Ladies digging

- Have you ever done any of the two? If not, have you ever seen people doing such activities?
- If yes,how long did you take to accomplish the work?

From all the observations, power is the work done per unit time.

The concept power involved here **is the rate of doing work**. If an amount of work W is carried out a time t, the power for that time is defined to be: $p = \frac{W}{t}$

We can also express the power delivered to a body in terms of the force that acts on the body and its velocity. Thus, for a particle moving in one dimension, the relation above becomes:

$$p = \frac{W}{t} = \frac{Fx}{t}$$

In the case of uniform motion, $v = \frac{x}{t}$, then the power delivered is $p = Fv$

Units

The S.I unit of power is the *joule per second* (J/S). This unit is used so often that it has been given a special name, the *watt* abbreviated (W) and 1(W) =(J/S).

Definition: A watt is the power when one joule of work is done for a second.

Linear momentum and impulse

Momentum

Momentum (symbol: p) of an object is the product of the **mass** and **velocity** of a moving body.

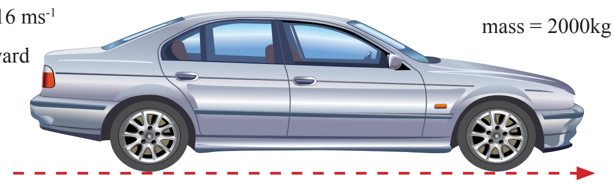


Figure 4.18: A car of massm moves with velocity v

Momentum p = m.v

= (2000) (16)

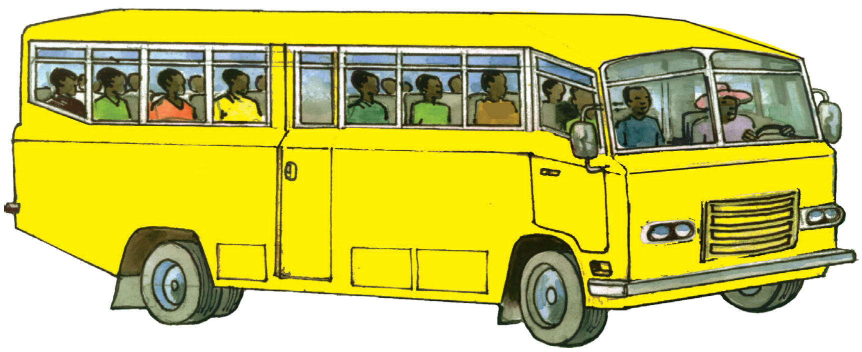
= 32000$kgms^{-1}$ forwards

Units:kg. $m.s^{-1}$

Note that since velocity is a vector quantity, momentum is also a vector quantity.

Momentum = mass × velocity, Momentum depends on the mass of the object and its velocity. Momentum does not equal mass. (see Elephant)

	
If an object is **at rest**, it has **no momentum** no matter how large its mass. Momentum is not the same as inertia. Momentum = (**Mass**)(0) = 0	A bullet can have a **large momentum** even if it has a small mass, because it is moving at **high velocity.** (Mass) (**velocity**) = **Momentum**



A bus can have a **large momentum** even if it is moving very slowly because it has a **large mass**

(**Mass**)(velocity) = Momentum

Figure 4.19: The linear momentum depends on the mass and velocity of the object

Linear momentum, p

$\vec{p} = m\vec{v}$

Which car has more momentum? A or B

A

B

The faster car, A

Figure 4.20: Comparison of linear momenta

Conservation of momentum

Activity 9: Field work

As a class, visit a place with a pool table.

Let each and every body try to hit the ball using the playing stick.

What happens when one ball hits another?

State and observe what you notice.

Draw a conclusion.

Figure 4.21: A man hitting a billiard ball

Activity 10: Fieldwork

* As a class, visit a place where there is billiard.
* Try to arrange the balls with the help of your teacher or any of the learners who have ever played it.
* Let one of you hit the white ball to strike/hit the rest.
* State what you observe after the white ball has hit the balls.
* Draw your conclusion from your observations.
* Repeat the same procedures using balls in the play grounds.

Suppose that a moving Ball A on the pool table of mass m_1 and velocity $\vec{v}_1$ collides with another ball B, of mass m_2 and velocity $\vec{v}_2$, moving in the same direction:

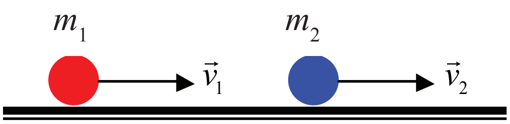


Figure 4.22: Two objects in motion having different speeds

From Newton's third law, the force F exerted by A on B is equal and opposite to that exerted by B on A. Also the time t during which the force acted on B is equal to the time during which the force of reaction acted on A.

So, if $\vec{p}_1$ is the momentum of A and $\vec{p}_2$ the momentum of B, we can write before collision: $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$

The momentum of the system constituted by the two masses is

$$\vec{p} = \vec{p}_1 + \vec{p}_2, \qquad \vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Suppose that after collision, the velocity of A becomes $\vec{v}_1{}'$ and the one of B becomes $\vec{v}_2{}'$.

So $\vec{p}_1{}' = m_1\vec{v}_1{}'$ and $\vec{p}_2{}' = m_2\vec{v}_2{}'$

$\vec{p}\,' = \vec{p}_1{}' + \vec{p}_2{}'$, and $\vec{p}\,' = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$

$\vec{p} = \vec{p}\,' \Rightarrow m_1\vec{v}_1{}' + m_2\vec{v}_2{}' = m_1\vec{v}\,'_1 + m_2\vec{v}\,'_2$

$m_2\vec{v}_2{}' + m_2\vec{v}_2 = -m_1\vec{v}\,'_1 + m_1\vec{v}\,'_1$

$m_2(\vec{v}\,'_2 - \vec{v}_2) = -m_1(\vec{v}\,'_1 - \vec{v}_1) \Rightarrow m_2(\Delta\vec{v})_2 = -m_1(\Delta\vec{v})_1$

The principle of conservation of the linear momentum law states that: *"If no external forces act on a system of colliding objects, the total momentum of the objects in a given direction before collision equals to the total momentum in same direction after collision"*.

Exercise

Truck Collision

Study the pictures below carefully

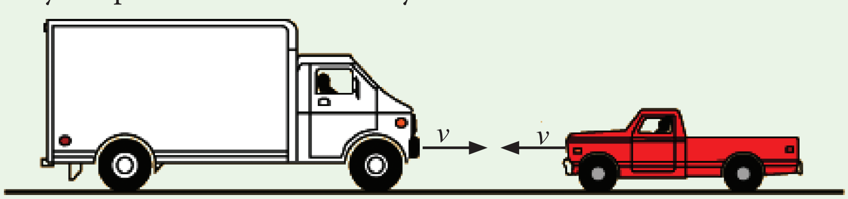


Figure 4.23: Two trucks moving towards one another

In a head-on collision:
Which truck will experience the greatest force?
Which truck will experience the greatest change in momentum?
Which truck will experience the greatest change in velocity?
Which truck will experience the greatest acceleration?
Which truck would you rather be in during the collision?

Impulse

Activity 11

* Move out the class to the play ground.
* In pairs (one pair at a time), kick a ball so that it is moving with a low speed. Let your friend stop it. Ask him/her what he/she felt. Let your friend do the same.
* For the second time, make sure that you kick the ball by applying a strong force so that it moves faster. Let your friend try to stop it. Ask him/her what this time he/she has felt?
* Go back in class and summarise what you observed and felt while in the play ground.

Definition

If one exerts a force $\vec{F}$ on moving a object in time t, the velocity of the object changes. We say that its momentum changes too.

The product of the force $\vec{F}$ and the time t in which it acts is called *impulse* represented by $\vec{I}$. $\vec{I} = \vec{F} \times \Delta t$

In S.I units, the unit of impulse is Newton-second [Ns].

Figure 4.24: Air-bags in automobiles have saved countless lives in accidents. The air-bag increases the time interval during which the passenger is brought to rest

Exercise

A ball of mass of 0.4kg is thrown against a brick wall. It hits the wall moving horizontally to the left and rebounds to the right.

a) Find the Impulse of the net force on the ball during its collision with the wall.

b) If the ball is in contact with the wall for 0.01s, find the average force that the wall exerts on the ball during the collision.

Relationship between impulse and momentum

Suppose a force F acts on a body of massm and gives it an acceleration a, the relationship between impulse and momentum can be seen by using Newton's second law.

From Newton's second law,

$$\vec{F} = m\gamma,\ \vec{\gamma} = \frac{\Delta \vec{v}}{\Delta t} \text{ then } \vec{F} = \frac{m\Delta \vec{v}}{\Delta t} \Rightarrow \vec{F}\Delta t = m\Delta \vec{v}$$

$$\vec{I} = \vec{F}\Delta t \text{ and } \Delta \vec{p} = m\Delta \vec{v}$$

$$\text{So } \vec{I} = \Delta \vec{p}$$

The impulse is equal to the total change of momentum.

Applications

Slow down of a moving object by a constant force

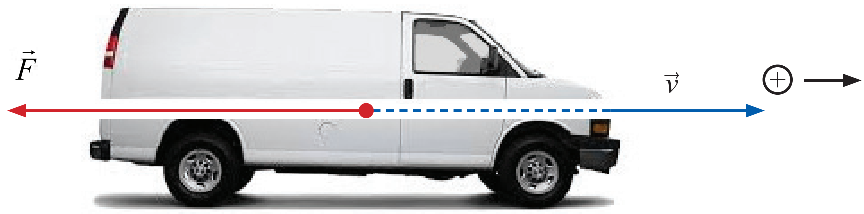


Figure 4.25: The velocity and the force have opposite directions

Let us consider a rectilinear uniform motion of velocity *v*, of a moving object of massm on which a retarding constant force acts parallel to the path. Let *t* be the time. The impulse on the object is given by: Impulse = *F.t*

The final linear momentum is zero when the initial linear momentum is *mv*, its projection on the axis is given by:

Linear momentum = ^{+}mv

Since the impulse is equal to change of momentum, it follows that Ft = 0 – mv, then t =m $\frac{v}{F}$.

Recoil back of a rifle

Suppose a bullet of mass *m*, is, fired from a riffle of mass *M* with velocity $\vec{V}$. Initially the total linear momentum is zero. From the principle of the conservation of linear momentum, when the bullet is fired, the total momentum of bullet and rifle is still zero, since external force has acted on them. So if $\vec{v}$ is the velocity of the rifle, we write: $M\vec{v} + m\vec{V} = \vec{0}$

Considering magnitude, we write $0 = - Mv + mV$ because $\vec{v}$ and, are in the opposite direction.

Then: $\frac{v}{V} = \frac{m}{M}$

Thus, if a rifle of mass *M* throws a bullet of massm with a speed *V*, it moves back with a velocity: $v = \frac{m}{M}V$

WORK

Carefully interpret the diagram below.

Why do you think after firing the cap of soldier moved away from his head?

Again, do you think that the soldier remained in same position as he was in before? Explain your answer.

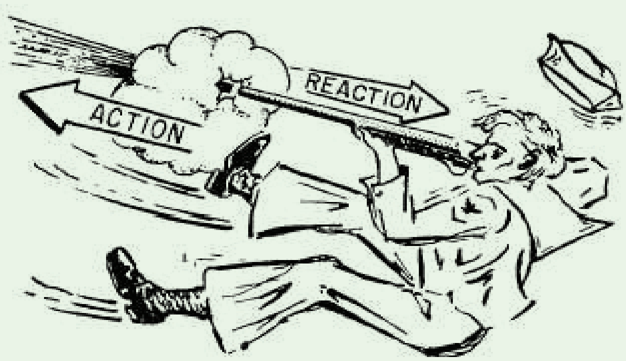


Figure 4.26: Firing the rifle and the bullets move in opposite directions

Exercise

1. Two bodies of mass 3kg and 5kg travelling in opposite directions on a horizontal surface collide. The velocities of the bodies before collision are 6m/s and 5m/s respectively. Given that after collision the two separate and move in the same direction in which the 5kg body was moving before collision, and the velocity of the 5kg mass is 1m/s, find the speed of the 3kg body after impact. Find also the loss in the energy.

2. A body of mass 2kg initially moving with a velocity of 1m/s is acted upon by a horizontal force of 6N for 3seconds. Find

   (i) Impulse given to the body
   (ii) Final speed of the body

Collisions

Activity 12

To know what happens to bodies after impact/colliding.

To know the effect of collision on the velocities and masses of bodies after colliding.

Observe the diagram below carefully assuming the black car to have a larger mass than a white one and answer the questions that follow.

Figure 4.27: Collision of two vehicles

Questions about the picture above:

a) Do you think after collision , the two cars continue moving? Explain why?

b) From what you observed, what is the effect of collision?

c) After separating the cars, do you think the masses of the cars changed? Explain why.

Note: We can define collision as an interaction between bodies in which the time intervals during which the bodies interaction is small relative to the time for which we can observe them.

In collision the total momentum of colliding objects is always conserved. Usually, however, their total kinetic energy is not conserved; some of it is changed to heat or sound energy, which is recoverable. **Such collisions are said to be inelastic**.

For example, when a lump of putty falls to the ground, the total momentum of putty and earth is conserved, that is, the putty loses momentum and the earth gains an equal amount of momentum. But all the kinetic energy of putty is changed to heat and sound on collision.

An *Inelastic collision* **is the collision where the total kinetic energy is not conserved** (total momentum always conserved in any type of collision). If the total kinetic energy is conserved, the collision is said to be *elastic*. **For example, the collision between two smooth smoker balls is approximately elastic.**

Elastic collision

In here, we shall consider objects colliding in a straight line and thereafter they move with different speeds in the same direction.

Elastic collision in one dimension

Let m_1 and m_2 be masses of two objects moving with speeds $\vec{v}_1$ and $\vec{v}_2$.

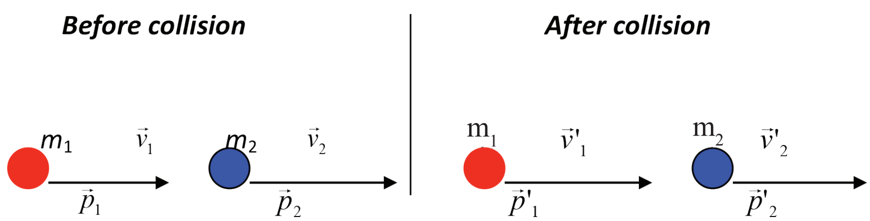


Figure 4.28: Diagram of two objects before and after collision

For a body of massm moving with a velocity *v,* the kinetic energy is given by the relation $k.e = \frac{1}{2}mv^2$ and for a system of particles, the total kinetic energy is: $k.e = \sum_i k.e_i$. Then we have:

Before collision:

$$\begin{cases} p = \Sigma p_i = m_1 v_1 + m_2 v_2 & (1) \\ k.e = \Sigma k.e_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 & (2)" \end{cases}$$

After collision

$$\begin{cases} p' = \Sigma p'_i = m_1 v'_1 + m_2 v'_2 & (3) \\ k.e' = \Sigma k.e'_i = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 & (4)" \end{cases}$$

The collision being elastic: $p = p'$ and $K.e = K.e'$

$$\begin{cases} m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2 \\ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 \end{cases}$$

$$\Rightarrow \begin{cases} m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2 \\ m_1v_1^2 + m_2v_2^2 = m_1v'^2_1 + m_2v'^2_2 \end{cases}$$

$$\begin{cases} m_1(v_1 - v'_1) = m_2(v'_2 - v_2) \\ m_1(v_1^2 - v'^2_1) = m_2(v'^2_2 - v_2^2) \end{cases}$$

$$\Rightarrow \begin{cases} m_1(v_1 - v'_1) = m_2(v'_2 - v_2) & (1)' \\ m_1(v_1 - v'_1)(v_1 + v'_1) = m_2(v'_2 - v_2)(v'_2 + v_2) & (2) \end{cases}$$

Dividing (2) and (1), we get:

$$\frac{m_1(v_1 - v'_1)(v_1 + v'_1)}{m_1(v_1 + v'_1)} = \frac{m_2(v'_2 - v_2)(v'_2 + v_2),,}{m_2(v'_2 + v_2)}$$

$$v'_2 = v_1 + v'_1 - v_2$$

Then we have: $\begin{cases} v'_2 = v_1 + v'_1 - v_2 \\ m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2 \end{cases}$

$$\Rightarrow m_1v_1 + m_2v_2 = m_1v'_1 + m_2(v'_1 + v_1 - v_2)$$

$$\Rightarrow m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_1 + m_2v_1 - m_2v_2$$

$$\Rightarrow m_1v_1 + 2m_2v_2 = v'_1(m_1 + m_2) + m_2v_1$$

$$\Rightarrow v'_1 = (m_1 + m_2) = m_1v_1 + 2m_2v_2 - m_2v_1$$

$$\Rightarrow v'_1 = \frac{m_1v_1 + 2m_2v_2 - m_2v_1}{(m_1 + m_2)}$$

$$\Rightarrow v'_1 = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}$$

In the same way, we can find v_2 with $v_2 = v_1 + v_1 - v_2$ and it's shown that:

$$v'_2 = \frac{m_2v_2 + 2m_1v_1 - m_1v_2}{(m_1 + m_2)}$$

$$\Rightarrow v'_2 = \frac{(m_2 + m_1)\, v_2 + 2m_1v_1}{m_1 + m_2}$$

Notice: If $m_1 = m_2$, $v_2 = 0$ then $v'_1 = 0$ and $v'_2 = v_1$.

Elastic collision in two or three dimensions

To understand this, use billiards as shown in the previous lesson

Figure 4.29: A man hitting a billiard ball

Some times after hitting the balls, they do not move in a straight line most especially (those who know how to play it) when you want to score in the centre hole. You must make sure that you hit the ball in target so that it moves at a certain angle.

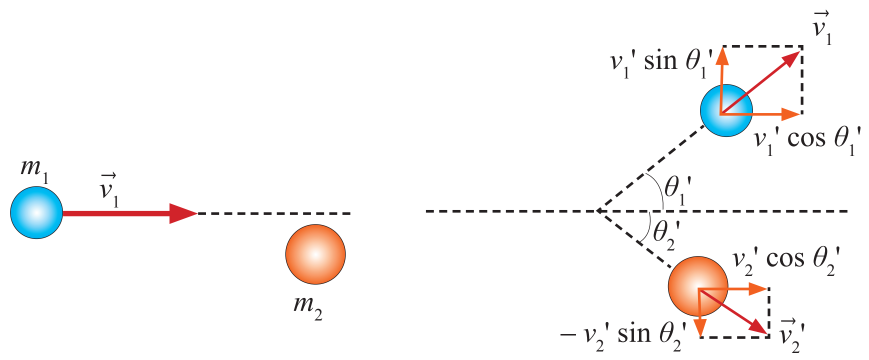
Activity 13

- Try to hit a billiard ball as shown in figure above.
- Observe what happens when one ball hits another.
- Note down your observations.
- Present your findings/observation to the whole class.

On Striking the balls ,energy and momentum is conserved.

Conservation of momentum and energy can also be applied to collisions in two or three dimensions and in this case the vector nature of momentum is important.

One common type of non-head-on collision is one for which one particle (called the "projectile") strikes a second particle initially at rest (the "tangent" particle). This is the common situation in games such as billiards.



(a) Before the collision (b) After the collision

Figure 4.30: Diagram of collision in two dimensions

The figure 4.30 shows particle 1 (the projectile m_1) heading along the x-axis towards particle 2 (the tangent m_2) which is initially at rest. If these are, say, billiard balls, m_1 strikes m_2 and they go off at angles θ_1' and θ_2', which are measured relative to m_1's initial direction (the x-axis)

Let us now apply the conservation of momentum and kinetic energy for an elastic collision like that on figure above. From conservation of kinetic energy, since $v_2 = 0$, we have:

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 \quad \frac{1}{2}m_2v_2'^2 \qquad (1)$$

We choose the xy plane to be the plane in which lie the initial and final momenta. Since momentum is a vector, and is conserved, its components in x and y directions remain constant. In the x direction

$$m_1v_1 = m_1v_1'\cos\theta_1' + m_2v_2'\cos\theta_2' \qquad (2)$$

Since there is no motion in the y direction initially, the y component of the total momentum is zero:

$$0 = m_1v_1'\sin\theta_1' + m_2v_2'\sin\theta_2' \qquad (3)$$

We have three independent equations. This means we can solve for utmost three unknowns. If we are given m_1, m_2, v (and v_2, if not zero), we cannot uniquely predict the final variables v'_1, v'_2, θ_1' and θ_2', since there are four of them, θ_2', for example, can be anything. However, if we measure one of these variables, say θ_1', then the other three variables (v'_1, v'_2, and θ_2') are uniquely determined and we can calculate them using the above three equations..

Inelastic collision

Collisions in which kinetic energies are not conserved are *called inelastic collision*. Some of the initial kinetic energy in such collision is transformed into other types of energy such as thermal or potential energy.

So the total final *K.e* is less than the total initial *K.e*, the inverse can also happen when potential energy (such as chemical or nuclear) is released and then the total final *K.e* can be greater than the initial *K.e*.

If two objects stick together as a result of a collision, the collision is said to be *completely inelastic*. Two colliding balls of putty that stick together or two railroad cars that couple together when they collide are examples.

On the diagram below, we have a collision in two dimensions of two vehicles on a road.

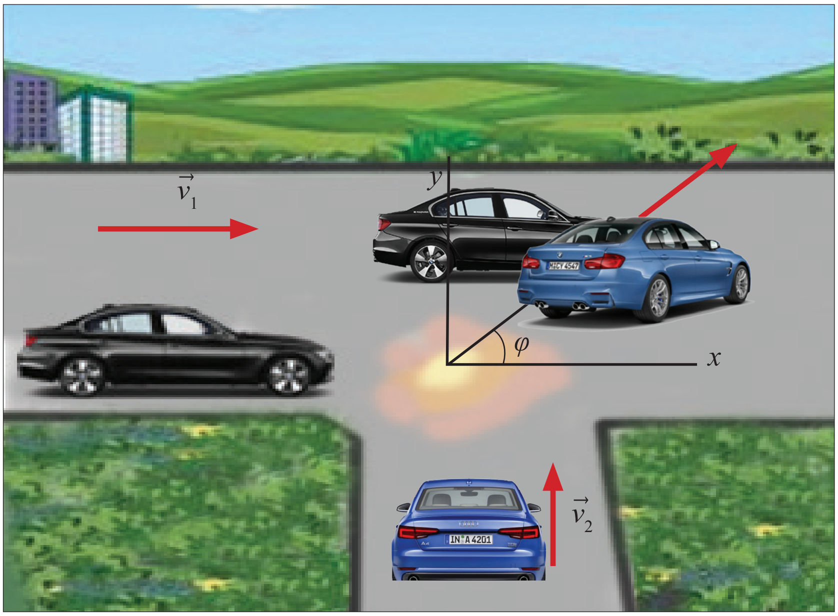


Figure 4.31: Collision of two vehicles is in two dimensions

If the velocities of the two objects make a certain angle before collision and after collision they stick together, analytically we have this situation:

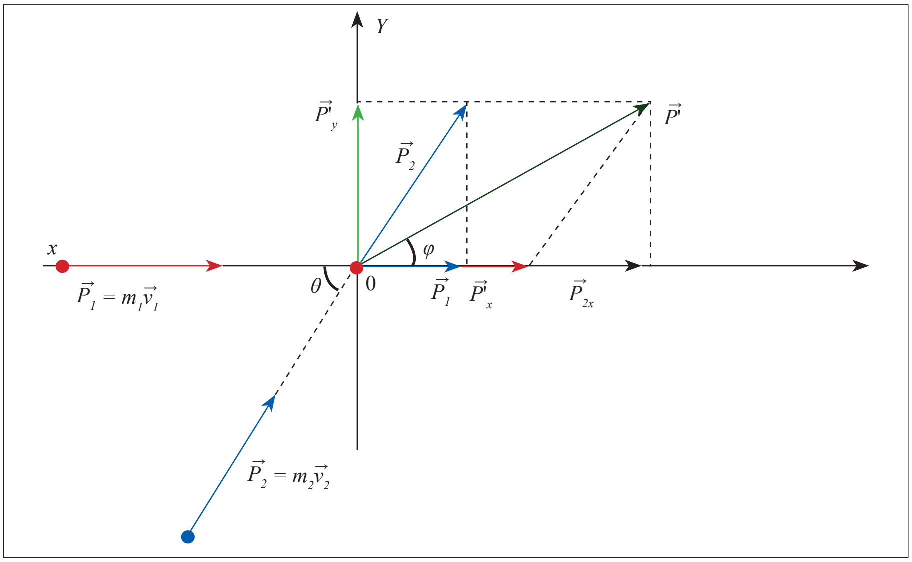


Figure 4.32: The velocity and the force have opposite directions

There is no conservation of kinetic energy; there is only conservation of momentum. $\vec{P} = \vec{P}_1 + \vec{P}_2 = \vec{P}\,' = (m_1 + m_2)\,\vec{v}$

If there is no change of direction, we have:

$$p' = \sqrt{p_x^{\,2} + p_y^{\,2}}$$

With: $\begin{cases} p_x = p_1 + p_2 \text{Cos}\ \theta \\ p_y = 0 + p_2 \sin\theta \end{cases} \Rightarrow \begin{cases} p_x = p_1 + p_2 \text{Cos}\ \theta \\ p_y = p_2 \sin\theta \end{cases}$

We have: $v' = \dfrac{p'}{m_1 + m_2}$

This velocity v^1 is the magnitude of the common velocity after collision. It's directed through an angle φ given by: $tan\varphi = \dfrac{P_y}{P_x}$

This angle is the angle formed by the direction of the common velocity and the initial direction of $\vec{P}_1$.

Division of mass

Let us consider a system constituted by two masses m_1 and m_2 which can slide without friction on a horizontal surface.

Initially, the two objects are linked and form a system. The centre of mass of the system has a uniform motion of velocity $\vec{v}_1$. The momentum is constant and equal to: $p = (m_1 + m_2)\, v$

If in the motion, there is division, the mass m_1 has the velocity v_1 and m_2 has the velocity v_2. The total momentum of the system is $\boldsymbol{p^1 = m_1v_1 + m_2v_2}$

The conservation of momentum allows us to write:

$p = p^1 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

Examples

WORK, ENERGY AND POWER

1. Determine the work done by a horse exerting a force of 60kg on a vehicle when the vehicle travels a distance of 2 km.

Solution

From work done

W = From × displacement $= F \times d$

$F = 60$ kgf $= 60 \times 9.8$ N $=$ 588N, $d = 2$ km $= 2000$m

Therefore $W = 588 \times 2000 = 1176000$ J

2. A 145-g baseball is thrown with a speed of 25m/s
   a) What is its kinetic energy?
   b) How much work was done to reach this speed starting from rest?

Solution

$K.e = \frac{1}{2}mv^2$

$K.e = \frac{1}{2} \text{ x } 0.145 \text{ x } 25^2$

$K.e = 45.3125$ J

c) Using third equation of motion

$K.e = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.145 \times 25^2 = 45.3125$ J

Using the kinetic energy theorem we write: $\Sigma W = \Delta\, k.e$

We have $W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2 = 45.3$ J

Student's trials

1. A worker lifts up a stone of 3.5kg to a height of 1.80m each 30s. Find the work done in one hour.

2. Calculate the kinetic energy and the velocity required for a 70kg pole vaulter to pass over a 5.0m high bar. Assume the vaulter's centre of mass is initially 0.90m off the ground and reaches its maximum height at the level of the bar itself.

3. Calculate the power required of a 1400kg car under the following circumstances
   a) The car climbs a 10° hill at a steady 80km/h and
   b) The car accelerates from 90 to 110km/h in 6.0s to pass another car on a level road. Assume the force of friction on the car is 700N in both parts of the problem.

4. A bullet is thrown obliquely in gravitational field, where $g = 9.8$ m/s^2 with a speed of 20m/s. Calculate its speed when it reaches the height of 10m.

5. A woman of mass 75kg walks up a mountain of height 20m.
   a) What is the work done?
   b) The walking up being done in 1.5 min, find the power,
   c) What time will be taken by this woman to walk up the 20m in order to develop a power of 73W?

6. A stone of 2000kg falls from the top of a tower of height $H = 200$m. What is the total mechanical energy? What is the *P.e* at height $h = \frac{H}{2}$ and its *K.e?*

7. Using the *K.e.* theorem, find the acceleration of the following system:

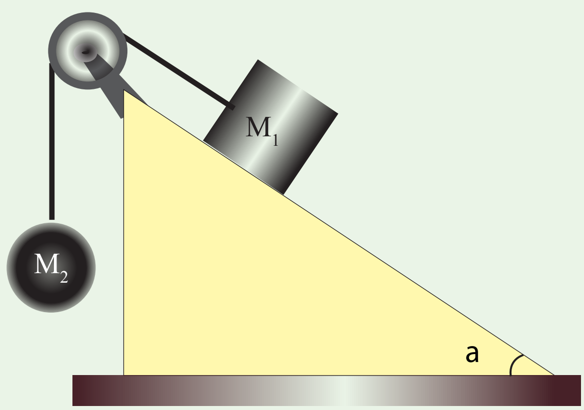


Figure 4.33: Two masses M_1 and M_2 connected together over a pulley at an incline

8. A small object A is suspended on a string of negligible mass of length OA=l making angle α with the vertical OB. One drops A without initial speed. Express, in function of l, g and α its speed when it passes in B.

9. A car travelling with a speed of 180km/hstrikes a wall. Find the height from which it will fall to produce the same energy. (g = 10m/s^2).

10. An object of 2kg falls freely during 5s. What is the kinetic energy? What will be the kinetic energy if the object is thrown downward with the speed of 4m/s? g = 10m/s^2.

11. A small object A_0 of mass 50g is suspended by a string of 80cm of length of negligible mass. It's moved away from the equilibrium position to the point A. The angle A_0OA being 60°, what is the change of the potential energy.

Linear momentum and impulse

1. What is the momentum of an 18.0g sparrow flying with a speed of 15.0m/s?

2. A moving object has an acceleration of 2.4m/s^2. It reaches in 12s a momentum of 800kgm/s. Compute the mass of that object and the force acting on it.

3. An object of mass 200g slides without friction on a horizontal surface and strikes a vertical obstacle and moves back following the same direction with a speed of 11m/s Find the impulse.

4. A system is constituted by two masses m_1=2kg and m_2=0.5kg connected by a string. The system moves on a horizontal table without friction from the rest. One makes it in motion applying an impulse of 10Ns but the string is cut. The result is, m_2 moves away with a certain speed and m_1 with a speed of 2m/s What is the impulse received by m_1? by m_2? What is the speed of m_2 at the end of the impulse?

5. An object of massm = 100g falls freely during 3s:

   a) Find the received impulse,
   b) Deduce the change of the speed.
   c) Generalize to find the law of the free fall $h = \frac{1}{2}gt^2$

6. One drops a ball of massm from a height h_0 above the ground. The ball bounces till the point situated at the height h_1. Find the impulse received by the ball from the ground. Given that h_0= 2.55m, h_1= 2m, m = 0.2kg, g = 10m/s^2.

7. A tennis ball of mass 200g is thrown horizontally with a speed of 15m/s toward the north. Assuming that the ball and the racket are in contact during 0.01s, find the force that the player has to exert to return it back with a speed of 25m/s, (a) toward the south, (b) toward the south-east.

8. A 10,000kg railroad car travelling at a speed of 24.0m/s strikes an identical car at rest. If the cars lock together as result of collision, what is their common speed afterward?

9. Calculate the recoil back velocity of 4.0kg rifle which shoots a 0.050kg bullet at a speed of 280m/s.

Extension

1. Suppose you throw a bowl of 0.4kg on a wall in bricks. It strikes the wall rolling horizontally leftward at 30m/s and rebounds horizontally rightward at 20m/s.
   a) Find the impulse of the force exerted on the bowl by the wall.
   b) If the bowl remains in contact with the wall during 0.01s, find the average force exerted on the bowl at the time of impact.

2. An automobile of massm = 749.5kg accelerates from the rest. During the first ten seconds, the net force acting on it is given by the relation $F = F_0 - kt$, where F_0 = 888.6N, $k = 44.48$N/s and t is the time elapsed in second after the departure. Find the velocity at the end of the 10s and the travelled distance during that time.

3. A ball of mass 100g is dropped from a height h = 2m above the floor. It rebounds vertically to a height h' = 1.5m after colliding with the floor.
   a) Find the momentum of the ball immediately before it collides with the floor and immediately after it rebounds.
   b) Determine the average force exerted by the floor on the ball. Assume the time interval of the collision is 10^{-2} s.

Collisions

1. Two objects of masses m_1 and m_2 slide on a horizontal table without friction. The first has a speed $\vec{v}_1$ and the second has a speed $\vec{v}_2$. They strike together. Assuming that the collision is elastic, find speeds v_1' and v_2' after collision in the following cases:

   a) $\vec{v}_1'$ and $\vec{v}_2'$ have the same direction,

   b) $\vec{v}_1'$ and $\vec{v}_2'$ have opposite direction.

2. A proton travelling with a speed 8.2×10^5m/s collides elastically with a stationary proton in a hydrogen target. One of the protons is observed to be scattered at a 60° angle. At what angle will the second proton be observed, and what will be the velocities of the two protons after the collision?

3. A 15,000kg railroad car travels alone on a level frictionless track with a constant speed of 23.0m/s A 5000kg additional load is dropped onto the car. What then will be its speed?

4. A 90kg fullback is travelling 5.0m/s and is stopped by a tackler in 1s. Calculate (a) the original momentum of the fullback, (b) the impulse imparted to the tackler and (c) the average force exerted on the tackler.

5. A billiard ball of mass m_A = 0.400kg moving with a speed v_A = 200m/s strikes a second ball, initially at rest, of mass m_B = 0.400kg. As a result of the collision, the first is deflected off at an angle of 30° with a speed of v_A = 1.20m/s . (a)Taking the x to be the original direction of motion of ball A, write down the equations expressing the conservation of momentum for the components in the x and y directions separately, (b) solve these equations for the speed, v_B', and angle α, of ball B. Assume the collision is elastic.

6. Two billiard balls of equal mass move at right angles and meet at the origin of an xy coordinates system. One is moving upward along the y axis at 3.00m/s, the other is moving to the right angle along the x axis with a speed of 4.80m/s After the collision (assumed elastic), the second ball is moving along the positive y axis. What is the final direction of the first ball, and what are their two speeds?

An explosion breaks a block of stone in three pieces A, B and C of respective masses m_1, m_2 and m_3. Immediately after explosion the speeds v_1=15m/s, v_2=7m/s and v_3=50m/s Vectors $\vec{v}_1$ and $\vec{v}_2$ form a right angle. Assuming that m^1=1.5kg and m^2=3kg, determine the direction of $\vec{v}_3$ and m_3.

Group work

1. Two masses m_1 = 5Kg and m_2 = 10Kg have velocities u_1 = 2m/s according to x positive axis and u_2 = 4m/s according to y positive axis. They collide and they get stuck. What is the final velocity after collision?

2. A lorry of transport of goods is empty and has a mass of 10000kg. When the lorry moves at 2m/s on a horizontal plane, it collides another lorry loaded, of total mass 20000kg; this last being initially at rest but with released breaks. If the two Lorries stuck together, after collision, what is their speed after collision?

3. a) With which velocity the loaded lorry must travel so that after collision the two remain at rest?

4. a) Distinguish between elastic collision and inelastic collision.

   b) Suppose two balls A and B of masses m_1 and m_2 are moving initially (in the same direction) along the same straight line with velocities u_1 and u_2 respectively. The two balls collide. Let the collision be perfect elastic. After collision, suppose v_1 is the velocity of A and v_2 is the velocity of B along the same straight line. Prove that:

   $$v_1 = \frac{(m_1 - m_2)\,u_1}{m_1 + m_2} + \frac{2m_2 v_2}{m_1 + m_2} \text{ and } v_2 = \frac{(m_1 - m_2)\,u_2}{m_1 + m_2} + \frac{2m_1 v_1}{m_1 + m_2}$$

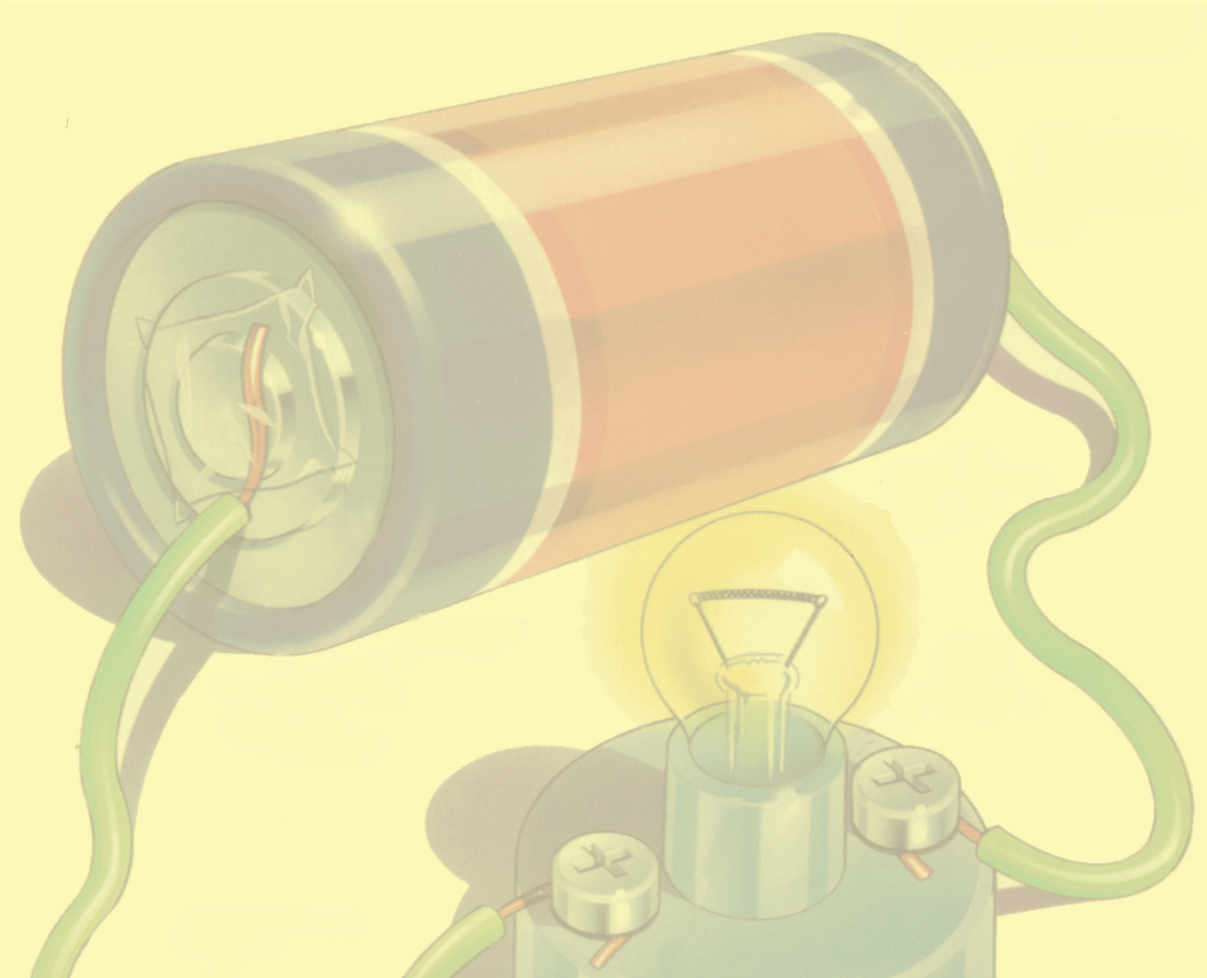
   c) A ball of 0.1kg makes an elastic head-on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed, what is the mass of the other ball?

5. a) A 40g golf ball initially at rest is given a a speed of 30m/s when a club (a specially shaped stick for striking a ball) strikes. If the club and the ball are in contact for 1.5ms. what average force acts on the ball?

   b) Is the effect of the ball's weight during the time of contact significant? Why or why not?

ELECTRICITY

Electric Current



Unit 5

Kirchhoff's Laws and Electric Circuits

Key Unit Competence

By the end of the unit, the learner should be able to analyse complex electric circuits using Kirchhoff's laws.

My goals

By the end of this unit, I will be able to:

* analyse complex electric circuits using Kirchhoff's laws.
* identify sources of electric current.
* describe components of simple electric circuits.
* state Kirchhoff's laws and apply them to solve problems in electric circuits.
* acquire practical skills to manipulate apparatus and evaluate experimental producers.
* explain the differences between the potential difference and electromotive forces.

Introduction

This unit is one of the most interesting units in Physics. Even if you ask someone who did not have enough studies in Physics he or she will tell you that People studying physics will be engineers specifically electricians. This Unit addresses the principles those electricians use in their career.

Review of elements of simple electric circuits and their respective role

An electric current consists of moving electric charges. Electric current must flow in electric devices connected by conductors (wires). The motion of electrons in a conductor is compared to water flow in a pipe. To move electrons, there must be a source of electric current, a cell, a battery, a generator which acts as a pump of water.

Making a simple circuit

Activity 1

Making a simple electric circuit with a bulb, a battery and wires

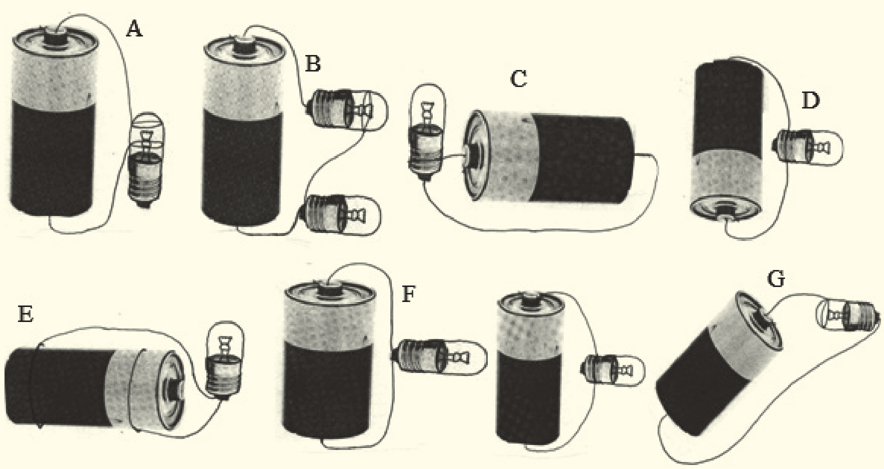
Materials:

* 2 pieces of copper wire
* 1 bulb
* 1 battery

Procedure

1. Examine diagrams A-J below. Predict whether the circuit will be complete, and record your prediction on the chart below.
2. Your teacher, with a helper, will demonstrate the arrangements to test your predictions. Record their results on the chart below.

PREDICTION CHART



Activity 1

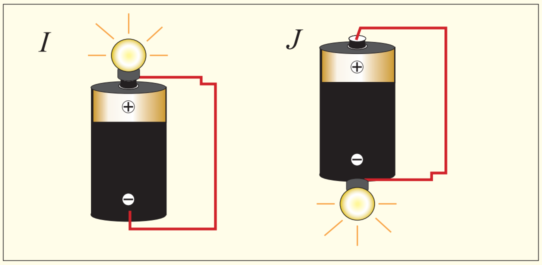


Figure 5.1: Different connections

What makes the bulb light?

You may already understand an electrical circuit, or this may seem like magic to you. Give what your teacher demonstrated some thought. Why do you think the bulb in the diagram lights?

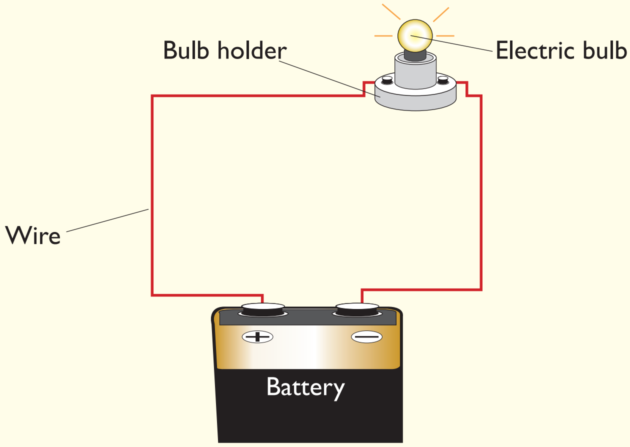


Figure 5.2: Simple circuit

It would be useful here to summarize some basic electricity points which you may know already.

a) A current flows along a metal or wire when a battery is connected to it.
b) The current is due to free electrons moving along the metal.
c) The battery has a potential difference p.d or voltage between its poles due to chemical changes inside the battery. The p.d. pushes the electrons along the metal.

One pole of the battery is called the positive (+) pole, the other is called the negative (-) pole. The "conventional" current, shown by an arrow, flows in a circuit connected from the + to the – pole. The electrons carrying the current along the circuit wires actually move in the opposite direction to the conventional current but this need not to be taken in account in calculations or circuit formulae.

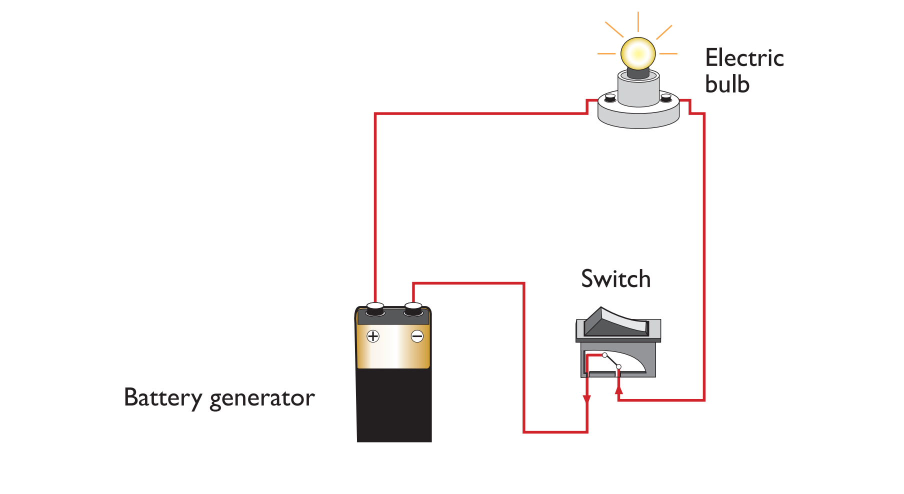


Figure 5.3: Diagram of a simple electric circuit

Any path along which electrons can flow is a ***circuit***. For a continuous flow of electrons, there must be a complete circuit with no gaps. A gap is usually provided by an electric switch that can be opened or closed to either cut off or allow energy flow.

Most circuits contain more than one device that receives electric energy from the circuit. These devices are commonly connected in a circuit in one of two ways, series or parallel. When connected in series, the devices and wires connecting them form a single pathway for electron flow between the terminals of the battery, generator or wall socket. When connected in parallel, the devices and wires connecting them form branches, each of which is a separate path for the flow of electrons.

Making a series and parallel circuit

Activity 2

Making a series circuit

Materials:

* 1 Battery .
* 3 Bulbs.
* 3 bulb holders .
* Assembled battery holder.
* 4 Pieces of copper wire (as needed).

Procedure

1. Construct a complete circuit with a battery and a bulb.
2. Using another wire, add a second bulb as shown on the picture below.

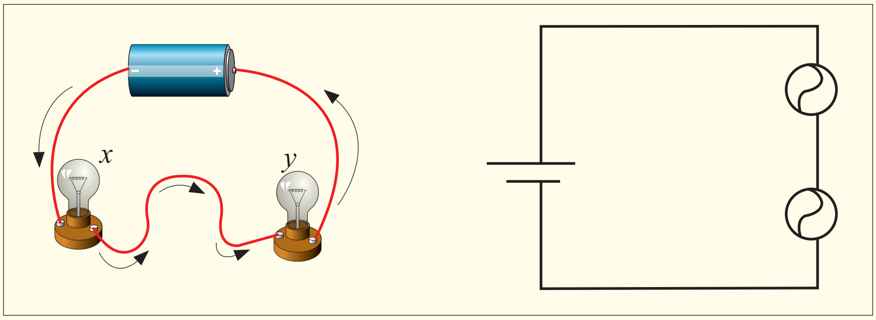


Figure 5.4: A series circuit

3. What did you notice happened to the first bulb when the second bulb was added?

____
____
____

4. Look carefully at how the series circuit is set up. Write a prediction of what you think will happen if you unscrew one of the bulbs.

Why did you make this prediction?

5. Unscrew bulb "X". Describe what happens to bulb "Y".

6. Tighten bulb "X", and unscrew bulb "Y". Describe what happens to bulb "X".

7. Add a third bulb to your series circuit. What happens to the brightness of the bulbs each time another bulb is added to the series?

8. Add a third bulb to your series circuit. What happens to the brightness of the bulbs each time another bulb is added to the series?

9. Draw a schematic diagram of the circuit you constructed with three bulbs.

Activity 3

Making a parallel circuit

Materials:

* 1 battery
* 3 bulbs
* Assembled battery holder 3 bulb holders
* 6 pieces of copper wire

Procedure

1. Construct a complete circuit with one battery and one bulb.
2. Using another two wires, add a second bulb as shown in the figure below.

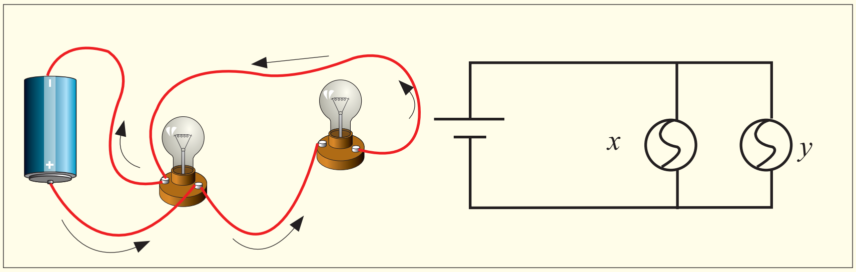


Figure 5.5: A parallel circuit

3. What do you notice happened to the first bulb when the second bulb was added?

____

____

____

4. Look carefully at how a parallel circuit is set up. Write a prediction of what you think will happen if you unscrew one of the bulbs in the parallel circuit.

____

____

____

Why did you make this prediction?

____

____

____

5. Unscrew bulb "X". Describe what happens to bulb "Y".

____
____
____

6. Tighten bulb "X" and unscrew bulb "Y". Describe what happens to bulb "X".

____
____
____

After carrying out experiments for series and parallel circuits,

* What advantages and disadvantages can you note for the two cases?
* What are the characteristics of a series connection and a parallel connection?

Conclusion

Series and parallel connections each have their own distinctive characteristics.

In a series circuit, the current is the same at all points; it is not used up. In a parallel circuit the total current equals the sum of the currents in the separate branches.

Figure 5.6: From electric lines to houses, all household lights and appliances are connected in parallel because a parallel circuit allows all devices to operate on the same voltage

Generators and receptors

Generators: Sources of electric current

Activity 4

Searching on internet about sources of electric energy

Search on internet about different sources of electric current and answer the following questions.

a) What is a source of electric current?

b) What is another name of electric source of energy?

c) List some of electric sources you have found.

d) On the picture below are some sources of electric energy. Name them and tell what the common role they have is.

e)

Figure 5.7: Some sources of electric energy

Tell what energy is changed in electric energy for each device.

Figure 5.8: At one electric energy distribution station of EUCL Rwanda: Here the produced electric energy is done by an alternator from a hydro-power station

Figure 5.9: Electric current produced by alternators are so dangerous to human beings

Figure 5.10: This woman was electrocuted by an electric current

The pictures above show the danger of electric current. So be careful when you are in their presence.

Electromotive force

Activity 5

Electromotive force of a generator

Materials

- Battery of 6V,
- Rheostat
- Voltmeter
- Ammeter
- Connecting wires

Procedure

1. Make the connection as shown in the following figure.

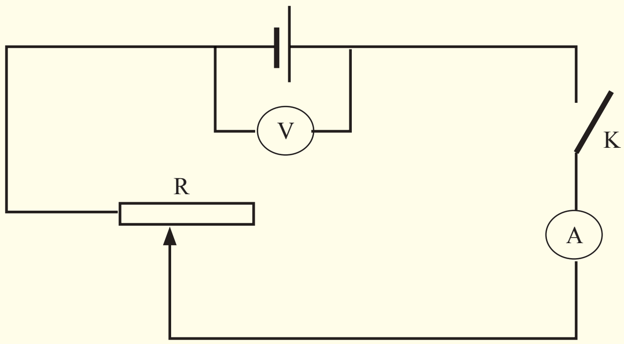


Figure 5.11: Diagram related to activity 5

2. Write down the voltage and the current indicated by the voltmeter and the ammeter when the switch is open.
3. Close the switch and vary the current in the circuit by varying the values of the rheostat and every time write down values of voltage and current indicated respectively by voltmeter and Ammeter.
4. Fill in the following table the obtained data:

Voltage V [V]						
Current I [A]						

5. When you vary the value of the resistance of the rheostat, does the intensity of current remain constant? Why?
6. Does the voltage remain constant?
7. What is the maximum voltage that you have got? How is this voltage called?
8. In general, if a charge Q (in coulombs) passes through a source of emf E (in volts) which relation will give the electrical energy W supplied by the source (in joules)?
9. Which relation will give the total power of the source?

Interpretation

A voltmeter connected to terminals of a battery measures the voltage between terminals of battery. When the switch was closed, we have noticed that there was a current across the circuit and the value of the voltage has been changed.

By varying the value of the resistance of the rheostat, current in the circuit is changed; voltage indicated by the voltmeter changes also, it decreases when current increases. Its maximum value is reached when the switch is open. Such voltage is called electromotive force E (emf) of the battery.

The *electromotive force emf E* of a source (a battery, generator, etc) is the energy transferred to electrical energy when unit charge passes through it. In other words, we can say that the emf of a source of electrical energy is its terminal p.d. on open circuit.

The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.

The unit of emf like the unit of p.d; is the volt [V] and equals the emf of a source which transfers 1 Joule of energy when 1 coulomb passes through it.

In general, if a charge Q (in coulombs) passes through a source of emf E (in volts), the electrical energy supplied by the source W (in joules) is:

$$W = QE \rightarrow W = EIt \text{ so } E = \frac{W}{Q}$$

Then the electric energy provided to the circuit by the source is given by the relation above. The electric power P, in this case is given by:

$P = EI$ so $E = \dfrac{P}{I}$ where $\boldsymbol{I}$ is the electric current in the circuit.

Examples

1. What is the power supplied by a cell of emf 4.5V, knowing that a current of 0.5A flows in the circuit?

2. Find the emf of a generator of power 12W sending a current of 1A in an external circuit.

Internal resistance

Activity 6

Existence of internal resistance in a generator

* Consider a certain number of cells which you put in an electric apparatus, like a radio…
* With your cheek, feel their temperatures before use.
* Put the cells in your apparatus and let them work for a certain time.
* Remove the cells and again with your cheek, feel the new temperature of the cells then answer the following questions:
    - Are the two temperatures of the cells equal? (Before and after use)
    - If not, what do you think is the cause of different temperatures?
    - Is one part of the current produced by the generator consumed by it? Why?
* The same observations can be made by feeling the temperature of the battery of a telephone before a call and after a call of about 10 minutes. Have you felt the increasing of temperature of a phone after using it? If yes, you think it's due to what?

Figure 5.12: The phone burnt due to the high temperature developed in the battery during the charging and use of the phone at the same time

The term internal resistance refers to the resistance within an emf. The terminal p.d. of a cell on closed circuit is also the p.d. applied to the external circuit.

In an external circuit electrical energy is changed onto other forms of energy and we regard the terminal p.d. of a cell on closed circuit as being the number of joules of electrical energy changed by each coulomb in the external circuit.

Not all the electrical energy supplied by a cell to each coulomb is changed in the external circuit. The "lost" energy per coulomb is due to the cell itself having resistance. Each coulomb has to "waste" some energy to get through the cell itself and so less is available for the external circuit. The resistance of a cell is called its internal resistance [r] and depends among other things on its size.

The electric power dissipated as heat in a cell is given by: $P_i = I^2 r$

Examples

3. The power dissipated as heat in a cell is of 7W, find its internal resistance if a current of 2A flows through it.

4. Find the power dissipated as heat in a generator of internal resistance 0.6Ω crossed by a current of 3A.

Remark: Any electrical generator, then, has two important properties, an emf E and an internal resistance r. E and r may be represented separately in a diagram, though in practice they are together between the terminals. To represent a cell, we can write (E, r). So we can think of the battery as an "electric pump", with its emf E pushing the current round the circuit through both the external (outside) resistor R and internal resistance r

In an electric circuit, a generator is then represented by the following symbol:

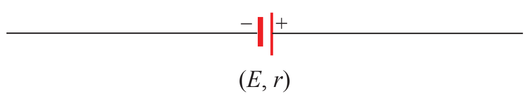


Figure 5.13: Representation of a generator in an electric circuit

Activity 7

Experiment to find the emf (E) and the internal resistance (r) of a cell

Materials

- 1.5V (approx) cell,
- Resistance box,
- Push switch,
- Digital Ammeter (0-1A).

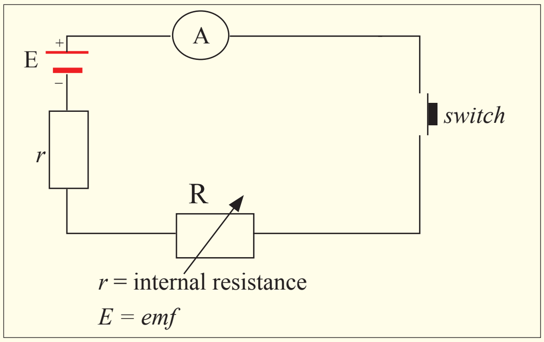


Figure 5.14: Diagram related to activity 7

Procedure

1. Check the scale on the digital Ammeter by comparing to other Ammeters.
2. Set *R* at 10 Ohms. Reduce in steps of 1 Ohm, recording resistance and current. Read the Ammeter as accurately as possible. Release switch (**not** a tap switch) after each reading, otherwise the cell will run down during the experiment.
3. Repeat the readings, increasing *R* back up to 10 Ohms. Obtain average values of ***I***, the current for each value of the resistance.
4. Calculate $\frac{1}{1}$.
5. Plot a graph of *R* against $\frac{1}{1}$. Draw the best possible straight line through the points (they might be quite scattered) in Excel, put the equation of the line on the graph.

The gradient of this graph is the emf of the cell. The negative intercept on the y-axis is the internal resistance.

Relationship between the P.d and the emf at terminals of a cell of a closed circuit

Activity 8

Relation between the Emf and the P.d

Materials

* Dry cell,
* Analogy multimetres (2),
* Rheostat and switch.

Procedure

1. Set up the circuit as shown.

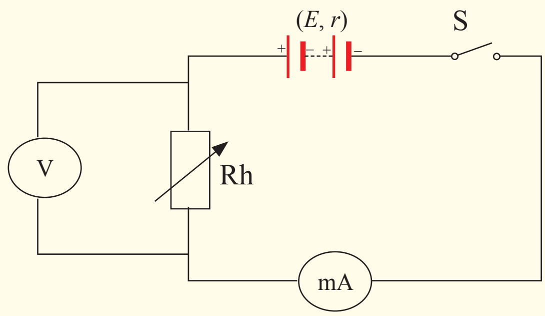


Figure 5.15: Diagram related to activity 8

2. Set the resistance of the rheostat to a large value to protect the circuit before switch on the circuit.
3. Set the milliammeter to the range 0-1 A or suitable range.
4. Set the voltmeter to the range 0-5V or suitable range.
5. Switch on the circuit. Record down the readings of the Ammeter and voltmeter.
6. Slide the rider of the rheostat to another position. Record the readings of the ammeter and voltmeter again. Tabulate the results.

Results

P.d. across the rheostat V [V]							
Current flowing in the circuit I [A]							

Questions

Verify that $V = E - I(r + R_A)$ where V is the potential difference across the rheostat, E is the emf of the cell, r is the internal resistance and R_A is the internal resistance of the Ammeter.

Interpretation

The electrical energy provided by the generator is consumed by the generator itself and by the external circuit.

Let be:

P_i: The power supplied by a cell in the internal resistance;

P_e: The power supplied by a cell in the external circuit

If P is the total power supplied by a cell, we can write: $P = P_i + P_e$

I being the intensity of the current provided by the cell, we write:

$EI = rI^2 + RI^2$
$EI = I(rI + RI)$
$E = rI + RI$
$E = rI + V$
Then: $V = E - rI$

The previous relation $\boldsymbol{V = E - rI}$ shows that $\boldsymbol{V < E}$

If $\boldsymbol{I = 0}$**,** we have $\boldsymbol{V = E}$, means the p.d. is equal to emf if the generator does not work.

Examples

1. What is the voltage at terminals of a battery of emf 3V and internal resistance 0.3 Ω when sending a current of 1.5 A in a circuit?
2. Knowing that the voltage at terminals of a cell is 1.5V and the current crossing the circuit is 1.2 A. Find its emf if its internal resistance is of 0.4 Ω.

Efficiency of a cell

Activity 9

Analysis of the relation $P = P_i + P_e$

Study the relation $P = P_i + P_e$ and answer the following questions.

a) In the relation, there are three quantities representing power. The power supplied to the external circuit, the one dissipated in the cell by Joule's effect and the one which is the total power supplied by the cell. Match each power by its symbol in the relation.

b) Is the total power supplied by the cell consumed by the external circuit? If yes, why? And if not, why?

c) In general, how do you call the ratio of the amount of power produced by a machine and the power put into it?

d) What is the unit of the physical quantity described in the question above?

e) The ratio of the power supplied by the cell to the external circuit and the total power supplied by the cell has a special name. Have you ever heard it? If yes what is it? What is its unit?

f) Is there another way to write that ratio using the power supplied by the generator in the internal circuit? If yes find it.

g) Is there a way to write the same quantity using the emf and voltage at terminal of a cell?

Since the relation above involves the total power in the circuit, which is the sum of the power supplied in the external circuit and the power dissipated in the cell itself, this one is useful. It's used to find the efficiency of the cell and how to calculate the voltage at terminals of a cell or battery.

Ohm's law for a circuit having a cell and a resistor

Activity 10

From the observation of the following diagram and analysing different elements deduce a relation

Questions:

a) State the Ohm's law.

b) Observe the following diagram and list constituting elements.

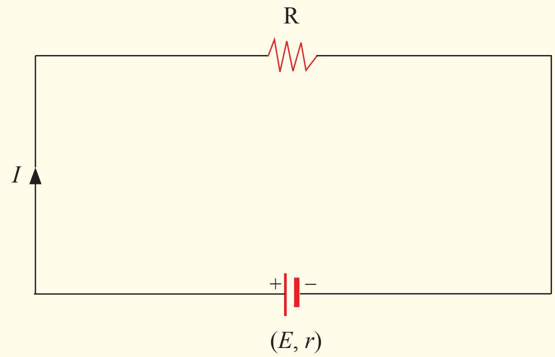


Figure 5.16: Circuit having a cell and external resistance

c) Do these elements make an electric circuit? If yes, why?
d) Are these elements connected in series or in parallel? Why?
e) How can you find the total resistance of a series connection?
f) Having the total power supplied by a cell, the power supplied by a cell in an external circuit and the power distributed by the internal resistance. Write the relation between them.
g) Write down relations for each type of power and substitute them in the relation above. (Powers dissipated in internal and external resistances must be written in terms of resistance)
h) From the relation found, deduce the emf E. The relation found expresses the Ohm's law for a circuit having a cell and a resistor.
i) Express the intensity of the current for this specific case.

Examples of application

1. The total power of a battery is of 9V and its internal resistance 3Ω. Knowing that the current crossed is of 0.4A. Find the efficiency of the battery.
2. A generator of internal resistance 2Ω sends a current of 4A in a resistor of resistance 10Ω. Calculate its power.
3. An external resistance of 4Ω is connected to an electric cell of emf 1.5V and internal resistance 2Ω. Calculate the intensity of the current flowing the external resistance.
4. An electric cell of emf 1.5V and internal resistance 2Ω is connected in series with a resistance of 28Ω. Calculate the power dissipated as heat in the cell.

Combination of cells

Activity 11

Cells wired in parallel and in series

Materials:

* 3 batteries
* 2 bulbs
* 3 assembled battery holders
* 2 bulb holders
* 6 pieces of copper wire

Procedure

1. Construct a complete circuit with one battery and one bulb.
2. Observe the brightness of the bulb.
3. Construct the circuit below. Are these batteries in series or parallel?

   ____
   ____
   ____

   How can you tell?

   ____
   ____
   ____

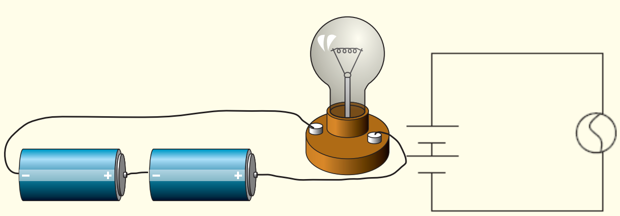


Figure 5.17: Cells in series

4. Observe the brightness of this bulb. Is the bulb brighter than it was with one battery?

   ____
   ____
   ____

5. If you added a third battery to this circuit in series, what do you think would happen to the brightness of the bulb?

____

____

____

Why do you think this?

____

____

____

6. Add a third battery to this circuit. Describe what happens to the bulb as this battery is added to this circuit in series and why you think the bulb is acting in this way.

____

____

____

7. Construct another complete circuit with one battery and one bulb. Record again what the brightness of the bulb is using your brightness metre.

8. Look at the pictures below, are the batteries in the picture in series or parallel?

____

____

____

How can you tell?

____

____

____

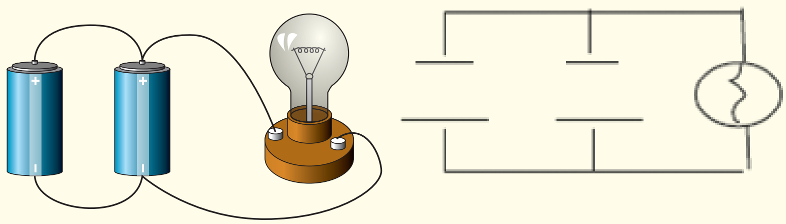


Figure 5.18: Cells in parallel

Construct the circuit in 8. Is the bulb brighter with two batteries than it was with one battery?

---

9. Add one more battery to this circuit in parallel. Describe what happens to the bulb as one more battery is added to this circuit in parallel and why you think the bulb is acting this way.

---

10. Connect then two batteries in opposition to mean the positive (negative) terminals of batteries are connected together and the two free negative (positive) terminals are connected to the bulb. What happens to the bulb?

---

11. Connect two batteries in series in opposition with one battery. When the two free ends of the combination are connected to terminals of the bulb, what happens to the brightness of the bulb?

Interpretation

Combination in series

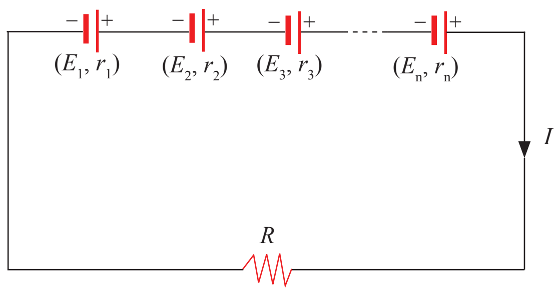


Figure 5.19: Cells combined in series

Let us consider cells combined in series of, respectively, emf and internal resistance $E_1, E_2, E_3, \ldots\ldots$, *and* $r_1, r_2, r_3, \ldots\ldots r_n$,

The p.d at the terminals of the combination is

$$V_t = V_1 + V_2 + V_3 + \ldots\ldots + V_n = E_t - Ir_t$$
$$= E_1 - Ir_1 + E_2 - Ir_2 + E_3 - Ir_3 + \ldots\ldots + E_n - Ir_n$$
$$= E_1 + E_2 + E_3 + \ldots\ldots + E_3 - I(r_1 + r_2 + r_3 + \ldots\ldots r_n)$$

Then $E_t = E_1 + E_2 + E_3 + \ldots\ldots + E_n$ and $r_t = r_1 + r_2 + r_3 + \ldots\ldots r_n$

We write $E_t = \sum_{i=1}^{n} E_i$ and $r_t = \sum_{i=1}^{n} r_i$

When two or more cells are arranged in series, the total emf is the algebraic sum of their emfs and the total internal resistance is the algebraic sum of their internal resistances

For n identical cells of emf E and internal resistance r each, we have:

$E_t = nE$ and $r_t = nr$

The intensity of the current produced is: $I = \dfrac{E_t}{R + r_t} = \dfrac{nE}{R + nr}$

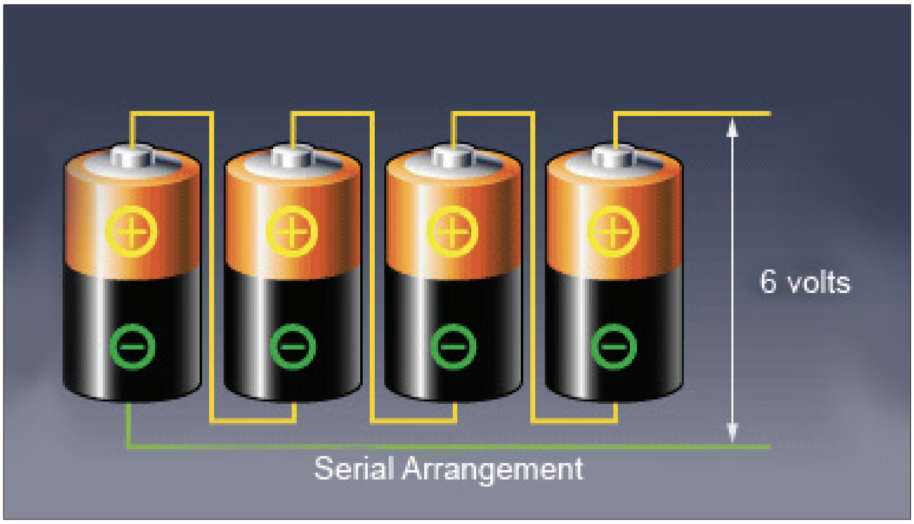


Figure 5.20: Each cell is of emf 1.5V: The emf of the combination is 6V, to mean 1.5V+1.5V+1.5V+1.5V

Note: A series arrangement is used to increase the voltage, also the total internal resistance of the circuit, so the energy loss due to internal resistance is greater than for a single cell.

Figure 5.21: In a torch, cells are in series: The three cells in the figure above act like one cell of emf which equals to the sum of the three cells

Examples

1. Four 1.5V cells are connected in series to a 12Ω lightbulb. If the resulting current is 0.45A, what is the internal resistance of each cell, assuming they are identical and neglecting wires?

2. A certain number of cells of emf 1.5V and internal resistance 2Ω are connected in series. When connected this combination to an external resistance of 10Ω, a current of 500m. A flows in this resistance. Find the number of cells used.

Combination in opposition

Let us see also what the result could be if the cells were associated in opposition

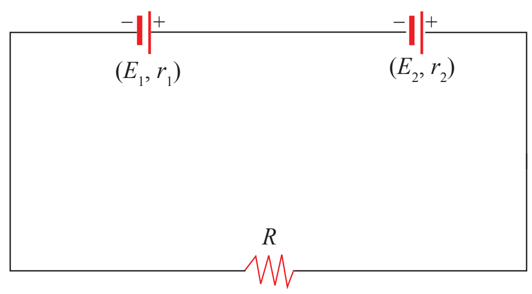


Figure5.22 : Cells combined in opposition

Let us combine two cells in opposition. Two terminals of same sign are connected together. The direction of the current in the circuit will be determined by the direction of the current produced by the cell having the higher emf. For internal resistances they are in series. So we write:

$$E_t = |E_2 - E_1|$$

$$= E_2 - E_1 \text{ if } E_2 > E_1$$

$$= E_1 - E_2 \text{ if } E_1 > E_2$$

$$= O \text{ if } E_1 = E_2 \text{ and } r_1 = r_2 + r_2$$

The intensity of the current which will flow in the circuit is: $I = \dfrac{E_2 - E_1}{r_1 + v_2 + R}$

Note: You might think that connecting batteries in opposition would be wasteful. For more purposes, that will be true. But such an opposition arrangement is precisely how a battery charger works.

Example

1. A cell of emf 2V and internal resistance 0.2Ω is associated in opposition with another cell of emf 1.5V and internal resistance 1.2Ω. Calculate the intensity of the current knowing that the external resistance is 1.1Ω.

Combination in parallel

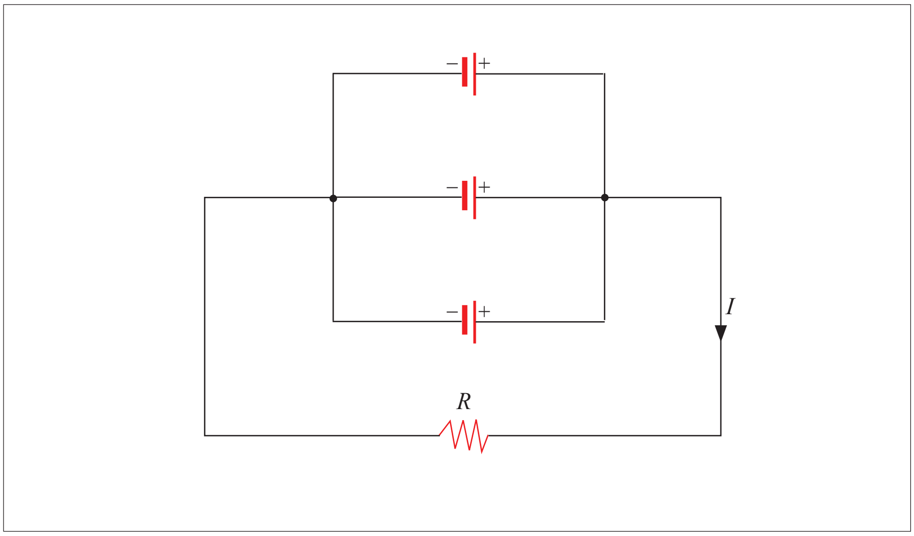


Figure 5.23: Identical cells combined in parallel

Consider a parallel arrangement of n identical cells having (E, r) as characteristics each. The total emf of the arrangement is the emf of one cell and the total internal resistance is found considering the parallel arrangement of resistors. Then, we have: $E_t = E$ and $r_t = \frac{r}{n}$

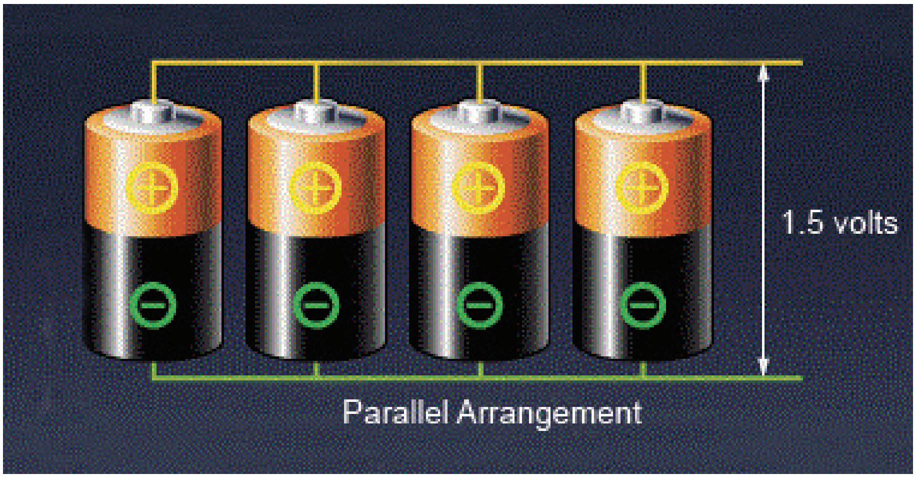


Figure 5.24: Each cell having an emf of 1.5 V, the total emf is of 1.5V

The intensity of the current produced is:

$$I = \frac{E_t}{R + r_t} = \frac{E}{R + \frac{r}{n}} = \frac{E}{\frac{nR + r}{n}}, \text{ finally } I = \frac{nE}{nR + r}$$

Note: The parallel arrangement is useful normally only if the emfs are the same. A parallel arrangement is not used to increase the voltage, but rather to provide large currents. Each of the cells in parallel has to produce only a fraction of the total current, so the energy loss due to internal resistance is less than for a single cell; the batteries will be exhausted less quickly.

Examples

1. We have 8 cells of emf 1.5V and internal resistance 2Ω. Calculate the intensity of the current flowing in an external resistance of 1Ω connected to the terminals of the 8 cells combined in parallel.

2. Six cells of unknown emf and internal resistance of 2Ω are associated in parallel. When an external resistance of 1Ω is connected to this combination a current of 1.5A is produced. Calculate the emf.

Mixing series and parallel combinations

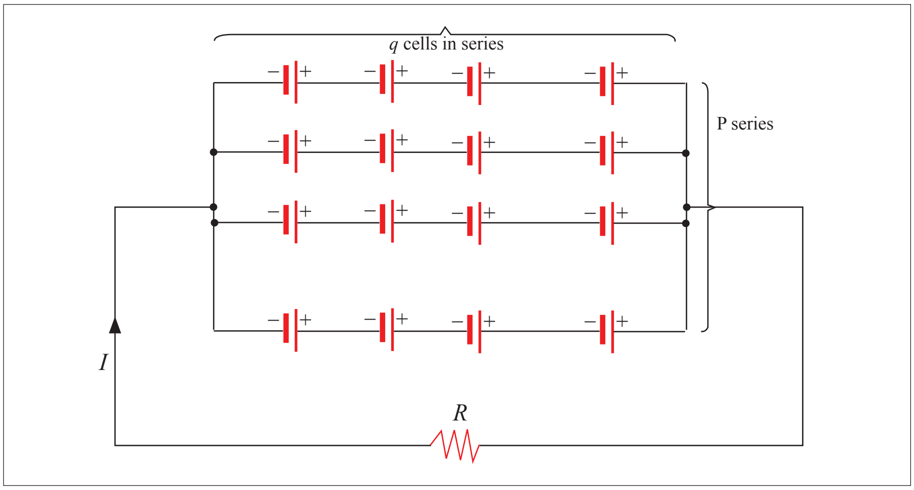


Figure 5.25: Mixing of a series and parallel combination

Consider a combination having p series having q cells each.

The total emf at terminals of the combination is the emf of one series, that means

$E_t = qE$ and $r_t = \dfrac{qr}{p}$

The intensity of the current flowing in the circuit is given by:

$$I = \frac{E_t}{R + r_t} = \frac{qE}{R + \frac{qr}{p}} = \frac{qE}{\frac{pR + qr}{p}} = \frac{pqE}{pR + qr}$$

The total number n of cells in the circuit is $n = pq$, then: $I = \dfrac{nE}{pR + qr}$

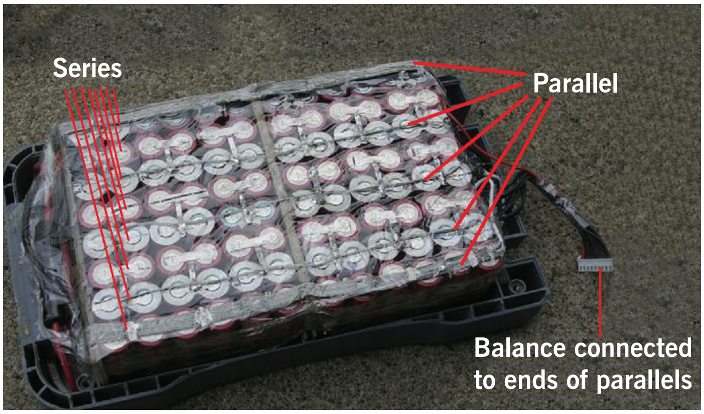


Figure 5.26: Identical series combined in parallel

Example

1. Four cells of emf 4.5V each and internal resistance 2Ω are combined in series. The combination is connected to an external resistance of 24Ω

   a) What is the intensity of the current?
   b) Same question if the cells are combined in parallel.
   c) Same question if the combination has two parallel series of two cells each.

Receptors

Activity 12

Distinguishing a receptor from a passive resistor

a) Observe the following devices and name them

Figure 5.27: Some appliances

b) What is the use of each one?

c) The flowing of the current in them produces the same effect? Explain.

d) Among them, which ones transform the whole electric energy consumed in heat and which ones transform a part of electric energy consumed in another kind of energy which is not heat?

e) As we had in the case of generators, what are characteristics of these apparatuses?

Conclusion: Among the apparatuses above, there are some which transform the total electric energy consumed into heat and some transform just a part into heat, other part transformed into another type of energy which is not heat. Those which transform the whole quantity of electric energy consumed into heat are *passive resistors* or *passive receptors* and those transforming a part of the consumed electric energy in another form of energy which is not heat are called *receptors* or *active receptors*.

The main characteristics are back electromotive force and internal resistance.

Back electromotive force

Back electromotive force (emf) is normally used to refer to the voltage that is developed in electric motors. This is due to the relative motion between the magnetic field from the motor's field windings and the armature of the motor!

Internal resistance

The *internal resistance* of a receptor *r'* is its ability to oppose electric current.

When a receptor is traversed by an electric current, part of the energy consumed is transformed into heat. The power dissipated in the receptor by joule effect is: $P_J = I^2 r$

The p.d at terminals of a receptor

Activity 13

Find the P.d at terminals of a motor

Materials

* Electric motor
* Ammeter
* Voltmeter
* Power supply

Procedure

1. Make the connection as shown in the figure below.

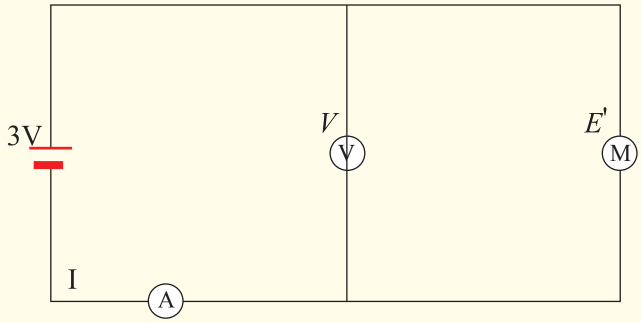


Figure 5.28: Circuit containing a receptor

Measure the voltage (*V*) between terminals of the motor (M) and the current *I* in the circuit.

Questions

a) What is the net electrical power received by the motor?
b) What becomes this power and how is it transformed?
c) What is the relation between the voltage and the back electromotive force?
d) From the relation found, how do you calculate the intensity of the current flowing?

Exercises

1. A circuit has in series a generator of emf 6V and internal resistance 0.1Ω, a receptor of back emf 1.5V and internal resistance 0.4Ω and a passive resistor of 8.5Ω. Calculate:
   a) The intensity of the current flowing in the circuit.
   b) The power supply by the generator.
   c) The quantity of heat produced in the resistor in one minute.
2. A battery has an emf of 12.0V and an internal resistance of 0.05Ω. Its terminals are connected to a load resistance of 3.00Ω. (a) Find the current in the circuit and the terminal voltage of the battery. (b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

3. Calculate the terminal voltage for a battery with an internal resistance of 0.9Ω and an emf of 8.5V when the battery is connected in series with (a) an 81Ω resistor, and (b) 810Ω.

4. A 9V battery whose internal resistance *r* is 0.5Ω is connected in the circuit shown in the figure.

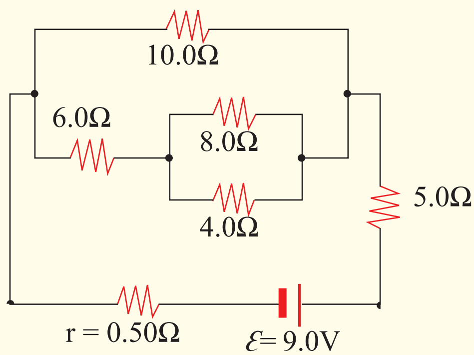


Figure 5.29: Related circuit to question 4

   a) How much current is drawn from the source?
   b) What is the terminal voltage of the battery?
   c) What is the current in the 6Ω resistor?
5. What is the internal resistance of a 12V car battery whose terminal voltage drops to 8.4V when the starter draws 75A? What is the resistance of the starter?

6. A 1.5V dry cell can be tested by connecting it to a low-resistance Ammeter. It should be able to supply atleast 22A. What is the internal resistance of the cell in this case, assuming it is much greater than that of the Ammeter?

7. A cell whose terminals are connected to a wire in nickel silver of resistivity 30 x 10 $^{-6}$ Ωcm and cross sectional area 0.25mm^2 and length 5m sends a current of 160mA. When the length is reduced to a half, the intensity of the current is of 300mA. Calculate:

   a) The internal resistance.
   b) The emf of the cell.

8. A cell (E = 1.5V, r = 1.3Ω) sends a current in an external resistance of 3Ω. Calculate:

   a) The intensity of the current in the circuit.
   b) The p.d at terminals of the cell.
   c) The power of generator.
   d) The efficiency of the cell.

9. A battery is composed by 120 cells in series. Each element has an emf of 2V and an internal resistance of 0.001Ω. The combination is connected to an external resistance of 4.8Ω. Calculate:

   a) The intensity of the current in the circuit.
   b) The voltage at terminals of the battery.
   c) The energy dissipated by joule effect when the current flows in the circuit in one hour.

Kirchhoff's rules

Activity 14

Find the equivalent Resistance

In this experiment, you will investigate three ideas using combinations of resistors in series and in parallel. Remember that the **total** or **equivalent resistance** in a series circuit is given by: $R_{eq} = R_1 + R_2 + \ldots$

For a parallel circuit, the equivalent resistance is given by:

$1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$

In this experiment you will be using a digital multimeter (DMM) which can function as either a voltmeter or an Ammeter.

The voltmeter must always be wired in parallel with the resistor whose voltage you are measuring. The ammeter, used to measure current, must always be wired in series. Disconnect the meter from the circuit before you change the function setting. Failure to follow these procedures can result in serious damage to the meter. Be sure that you use the correct units with your data.

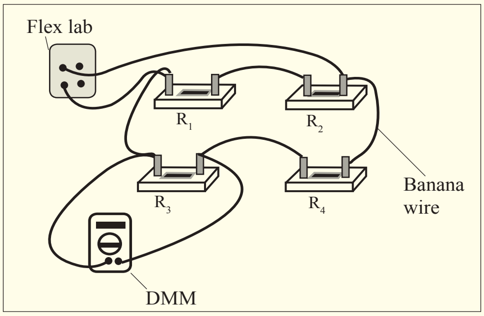
Materials

- 1 multimeter.
- 1 330 Ω or 240 Ω resistor.
- 1 1000 Ω (1KΩ) resistor.
- 1 2000 Ω (2KΩ) resistor.
- 1 3000 Ω (3KΩ) resistor or 1 3300 Ω (3.3KΩ) resistor.
- 1 0- 10K resistor substitution box.
- 2 spade lugs.
- 2 2' red banana wires.
- 2 2' black banana wires.
- 4 4" black banana wires.

Procedure

1. Set the DMM function switch to "Ohms" (Ω). Measure and record the resistance of the resistors R_1, R_2, R_3, R_4.

2. Figure 5.29 is a sketch of the components. In this sketch, the DMM is wired in parallel with R_3 in order to measure the voltage V_3. Wire the circuit shown in Figure 1, but do not connect it to the power supply until it has been approved by your lab instructor. Once it has been approved, apply power. Set the DMM to DCV. Connect the black banana wire to COM and the red wire to the V- Ω input. Measure the power supply voltage (Vps) and the voltages across R_1 (V_1), R_2 (V_2), R_3 (V_3), and R_4 (V_4) as indicated in Figure 5.30.

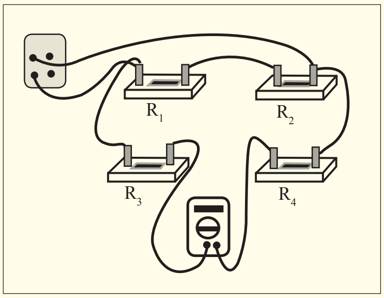


To measure voltage, set the meter to DCV and wire it in parallel with the resistors. This meter is set up to measure V_3, the voltage across R_3

Figure 5.30: Voltage measurements

3. Unplug the circuit and disconnect the meter. Change the function switch on the DMM to direct current amperes. Move the red wire to the "mA/µA terminal.

   Study Figure 5.31 and notice the way the ammeter is wired in series with the resistors. Again, have the lab instructor approve the circuit before you plug it in. Make the current measurements I_1, I_2, and I_3 indicated in Figure 4 of 5.31



To measure current, set the meter to DCmA and wire it in series with the resistors. This meter is set up to measure I_3, the current through R_3 and R_4
Note that the positive lead to the meter must be removed.

Figure 5.31: Current measurements

4. R_1 and R_2 are in series. R_3 and R_4 are also in series. The two series circuits are in parallel. Calculate the equivalent resistance, R_{eq}, for the entire circuit. Show your working.

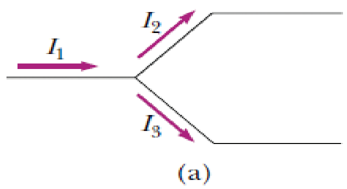
5. Set the decade resistance box to the value you calculated for R_{eq} for the circuit. Be sure the DMM is set on DCA and wire the DMM and the resistance box in series with the power supply. Measure I_{eq}.

6. Turn everything off, disconnect the components and put the equipment away neatly.

Simple circuits can be analysed using the expression $V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analysing more complex circuits is greatly simplified if we use two principles called *Kirchhoff's rules*:

Junction rule: ***The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction***: $\sum I_{in} = \sum I_{out}$

Loop rule: ***The sum of the potential differences across all elements around any closed circuit loop must be zero***: $\sum_{\substack{closed\\loop}} V = IR$



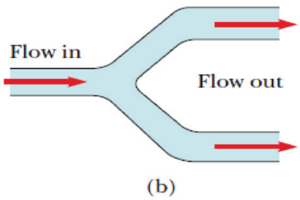


Figure 5.32: (a) Kirchhoff's junction rule: Conservation of charge requires that all charges entering a junction must leave that junction; therefore, $I_1 = I_2 + I_3$

(b) A mechanical analog of the junction rule: the amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left

Kirchhoff's first rule is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point. If we apply this rule to the junction shown in Figure 5.31. we obtain $I_1 = I_2 + I_3$

Kirchhoff's second rule follows from the law of conservation of energy. Let us imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop $-IR$ across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

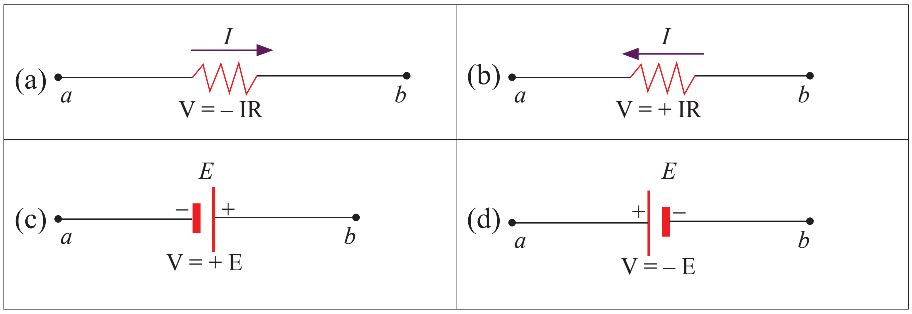


Figure 5.33: Rules for determining the potential differences across a resistor and a battery (the battery is assumed to have no internal resistance). Each circuit element is traversed from left to right

Examples

1. A single-loop circuit contains two resistors and two batteries, as shown in figure 5.34 (neglect the internal resistances of the batteries). (a) Find the current in the circuit. (b) What power is delivered to each resistor? What power is delivered by the 12V battery?

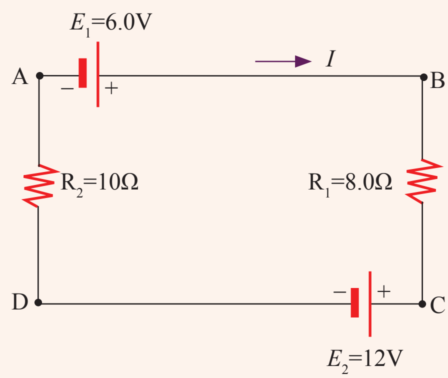


Figure 5.34: Related figure to question 1

2. Find the currents I1, I2, and I3 in the circuit shown in Figure 5.35

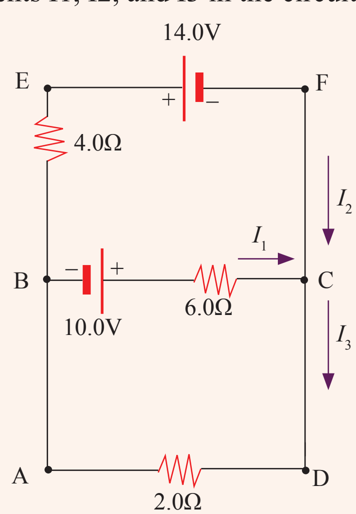


Figure 5.35: Related figure to question 2

3. (a) Under steady-state conditions, find the unknown currents I1, I2, and I3 in the multi loop circuit shown in the figure 5.36.

   b) What is the charge on the capacitor?

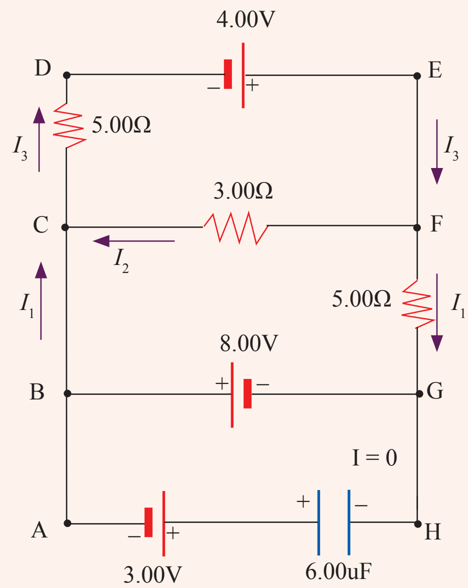


Figure 5.36: Related figure to question 3

Exercises

1. In the following circuit, using the Kirchhoff's rules find the currents I_1, I_2 and I_3.

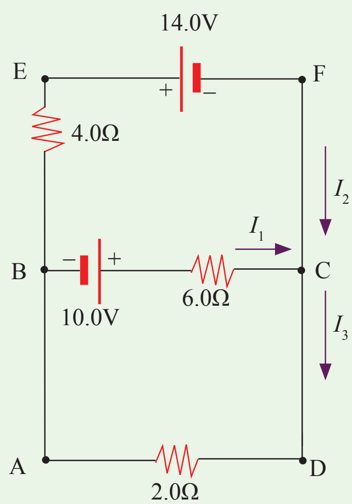


Figure 5.37: Related figure to question 1

2. (a) Determine the currents I_1, I_2 and I_3 in the figure below. Assume the internal resistance of each battery is $r = 1\Omega$.
   (b) What is the terminal voltage of the 6V battery?

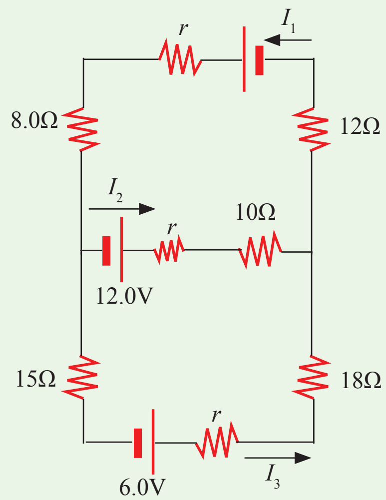


Figure 5.38: Related figure to question 2

3. In the circuit of the figure, determine the current in each resistor and the voltage across the 200**Ω** resistor.

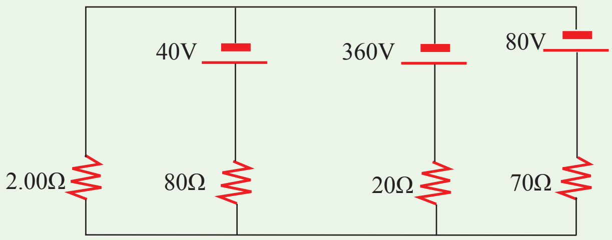


Figure 5.39: Related figure to question 3

4. Using Kirchhoff's rules, (a) find the current in each resistor in the figure bellow. (b) Find the potential difference between points C and F. Which point is at the higher potential?

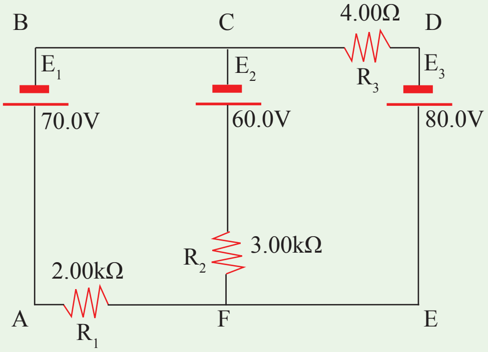


Figure 5.40: Related figure to question 4

5. Taking $R = 1.00k\Omega$ and $E = 250V$ in the figure below, determine the direction and magnitude of the current in the horizontal wire between A and E.

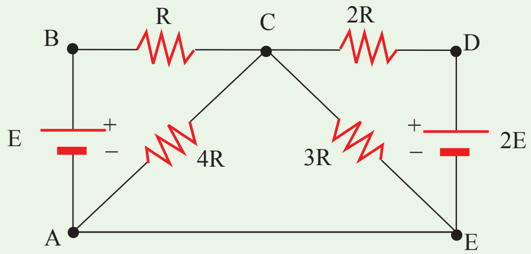


Figure 5.41: Related figure to question 5

6. A dead battery is charged by connecting it to the live battery of another car with jumper cables as shown in the figure. Determine the current in the starter and in the dead battery.

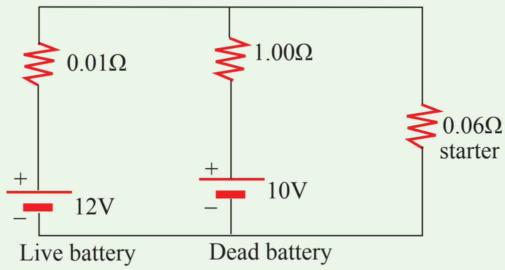


Figure 5.42: Related figure to question 6

Career guidance

Did you know that Physics is one of the subjects that will help you have a career in engineering?

In engineering, we have electrical engineering, which basically uses concepts discussed in this Unit.

Start to plan your career. Be an engineer or more specifically an electrical engineer.

ENERGY, POWER AND CLIMATE CHANGE

Sources of World Energy

Unit 6

Sources of Energy in the World

Key unit Competence

By the end of the unit, the learner should be able to evaluate energy sources in the world

My goals

By the end of this unit, I will be able:

* identify sources of energy in Rwanda.
* outline the basic features of renewable and non renewable energy sources.
* evaluate energy uses and availability in Rwanda.
* identify various advantages and disadvantages of various energy sources.
* be aware of the moral and ethical uses associated with use of energy.

Introduction

Origins of the power used for transportation, for heat and light in dwelling and working areas, and for the manufacture of goods of all kinds, among other applications. The development of science and civilization is closely linked to the availability of energy in useful forms. Modern society consumes vast amounts of energy in all forms: light, heat, electrical, mechanical, chemical, and nuclear. The rate at which energy is produced or consumed is called power, although this term is sometimes used in common speech synonymously with energy.

Activity 1

Answer these questions.

a) What do you think when you hear the word "energy"?

b) Give the definition of the word "energy" and the term "energy source."

c) Among scientists and energy professionals, a standard list of current energy sources would include: biomass (plant matter), nuclear, coal, oil, geothermal, solar, hydro (rivers), wave or tidal, natural gas, wind. Add other sources of energy which you may know.

d) Read carefully these key terms in the table below then give answers to related questions.

Key Terms

Biomass energy	Energy released from plants (wood, corn, etc) through combustion or other chemical process
Fossil fuel	A non-renewable energy resource that began to form millions of years ago from the remains of once living plants and animals. Its current forms include petroleum, coal and natural gas.
Geothermal energy	Heat energy from the earth.
Hydropower	Transformation of the energy stored in a depth of water into electricity.
Non renewable energy	Resources, such as fossil fuels that cannot be replaced by natural processes at the same rate it is consumed.
Photovoltaic	A chemical process that releases electrons from a semi-conductor material in the presence of sunlight to generate electricity.

Renewable energy	Resources, such as wind and water that can be recycled or replaced at a rate faster than they are consumed.
Solar Energy	Energy from the sun; often captured directly as heat or as electricity through a photovoltaic process.
Uranium	An element that releases heat as it undergoes radioactive decay.
Wind energy	Energy transferred with the motion of air in the lower atmosphere that arises from differential heating of the earth. The energy in the wind can be extracted as mechanical energy to do work such as grind grains (a wind mill) or generate electricity (wind turbine).
Wave energy	Wave power captures the energy of ocean surface waves, and tidal power. Converting the energy of tides, are two forms of hydropower with future potential; however, they are not yet widely employed commercially.

Use definitions in the table and decide which of energy sources in (c), include what you added, are renewable and which are non-renewable.

e) From the list given in (c) what is the major category of renewable energy?

f) Between renewable and non-renewable energy which one produces a little or no pollution or hazardous waste and pose few risks to public safety? How the other produces it?

g) Discuss in groups this consequence above.

h) List as many as you can uses of renewable energy sources.

Worldwide, wood is the largest source of biomass for non-food energy, but other sources are also used, including municipal wastes and crop wastes. Crops such as sugar cane are used to make alcohol for transportation fuel. In many developing countries, wood is the most important energy source.

Global resources of geothermal energy (the heat contained below Earth's surface) are so immense that they are usually considered to be renewable. But this classification is not strictly correct, since the heat stored in any given volume of rock or underground water is depletable. In addition, the most easily accessed geothermal resources, natural hot springs and geysers, will not last for more than a few decades if exploited for energy on a large scale.

Estimates vary widely as to how long fossil fuels, oil, coal, and natural gas will last. These estimates depend on assumptions about how much fossil fuel remains in the ground, how fast it will be used, and how much money and effort will be spent to recover it. However, most estimates agree that, if present rates of consumption continue, **proven oil and natural gas reserves will run out in this century, while coal reserves will last more than 200 years. Once they are used, these energy sources cannot be replaced.**

Activity 2

Renew-A-Bead

In this activity, you will be given a bag of "energy beads." Each bag contains energy provided by both renewable (white beads) and non-renewable (black beads) sources. You will "use" the energy provided by both types of sources by randomly picking beads from a bag – some of the "energy" you use will be renewable, some will be non-renewable. You will see what happens to the renewable/non-renewable energy sources that remain after many years of energy use.

Materials

* One plastic bag containing 100 beads. The black beads represent non-renewable energy resources and the white beads represent renewable energy resources. (The ratio of white beads to black beads will vary depending on group)
    - Group 1= 90 black beads and 10 white beads.
    - Group 2= 80 black beads and 20 white beads.
    - Group 3= 70 black beads and 30 white beads.
* Small Cloth
* Extra plastic bag
* Calculator
* Pencil

Procedure

1. Split into groups of 2-3 learners.
2. Collect all equipment and materials necessary to carry out the activity.

Part 1: Simulate the annual consumption of energy - constant rate of energy use

3. Have one person from each group pick out 10 "energy beads" from the bag, without looking. These 10 beads represent the energy that is used in one year.

4. Count the black and white beads and record the number on the attached data collection sheet for Year 1.

5. The black beads represent energy from non-renewable energy sources, so when a black bead is picked it cannot be returned to the bag (place it in the extra plastic bag). The white beads are renewable energy beads, so they should be put back into the bag each turn after counting them.

6. Let another person from the group pick 10 beads to represent energy use in Year 2. Fill in the number of black and white beads on the chart, and return the white beads as in step 5.

7. Repeat the process, returning all white beads to the bag after each person's turn, until 20 years have passed or until all the black energy beads are gone.

Part 2: Simulate the annual consumption of energy - increasing rate of energy use

8. Consider the increasing use of power and energy over time. Repeat steps 3 through 7, but increase the amount of energy use by picking out 5 additional "energy beads" each year (pick 10 beads in year 1, 15 beads in year 2, 20 beads in year 3, etc.). Record information on the attached data collection sheet.

9. Complete the discussion questions.

Discussion Questions

1. How many years did it take for the non-renewable energy sources to run out when you used 10 energy beads per year? How many years did it take for the non-renewable energy sources to run out when you increased the rate of consumption each year (part 2)? What conclusion can you draw from this about our energy use habits?

2. What are some examples of renewable and non-renewable energy sources?
3. What does this activity demonstrate about our consumption of resources - what will happen if we keep using non-renewable resources?
4. Describe what happens to the proportion of renewable vs non-renewable energy sources that remain available, as energy is used over time.
5. Compare the results from teams with different energy mixes. If each bag represents a country, what can you say about countries that currently use a greater fraction of renewable energy? Will they be able to continue to provide for their country's energy needs?

Data collection for Renew-A-bead

Part 1: 10 energy beads used each year

Year	Total beads removed	Number black beads	Number white beads	Percentage of beads that are renewable $\frac{\text{White beads}}{\text{Total beads}} \times 100\%$	Number of beads remaining
1	10				
2	10				
3	10				
4	10				
5	10				
6	10				
7	10				
8	10				
9	10				

10	10				
11	10				
12	10				
13	10				
14	10				
15	10				
16	10				
17	10				
18	10				
19	10				
20	10				

Data collection for Renew-A-bead

Part 2: Increasing use of energy each year

Year	Total beads removed	Number black beads	Number white beads	Percentage of beads that are renewable	Number of beads remaining
1	10				
2	15				
3	20				
4	25				
5	30				
6	35				
7	40				
8	45				
9	50				
10	55				
11	60				
12	65				
13	70				
14	75				
15	80				
16	85				
17	90				
18	95				
19	100				
20					

Activity 3

Learn more about sources of energy

Read these following notes, they talk more about energy sources and some sources of energy related to Rwanda. Understand them because the understanding will help you to do other activities. About different topics, you could also do research on internet. You'll do also research about advantages and disadvantages of those sources of energy.

Procedure

The learning will be always done in groups. Each member of the group has to make sure that he/she has understood the content read. The explanation must be in terms of discussion in group. For the research on internet or in the library it can be done separately then discussed in groups.

Fossil fuel

Fossil fuels are fuels formed by natural processes such as anaerobic decomposition of buried dead organisms. The age of the organisms and their resulting fossil fuels is typically millions of years, and sometimes exceeds 650 million years. Fossil fuels contain high percentages of carbon and include coal, petroleum and natural gas.

Figure 6.1: Coal, one of fossil fuel

- How it works? - Making power from fossil fuels

Burn Fuel → Heat water to make steam → Steam turns turbines → Turbines turn generators → Electric power sent around the country

Rwanda's main fossil fuel resource is methane gas. It is estimated that there are 50 billion cubic metres of exploitable methane, which is the equivalent of 40 million tons of petrol (TOE) laying at the bottom of the Lake Kivu under 250 metres of water. Of the 55 billion cubic metres (cum) Standard Temperature and Pressure, STP) of methane gas reserves, 39 billion cum (STP) are potentially extractable. This represents a market value of USD 16 billion, equivalent to 31 million Ton Oil Equivalent (TOE). The technical and economic feasibility of methane gas exploitation has been clearly demonstrated since 1963 by the small methane extraction pilot unit at Cape Rubona with a capacity of 5000 cum of methane per day at 80 % purity. The resource is estimated to be sufficient to generate 700 mW of electricity for 55 years with Rwanda's share being 350 mW.

Figure 6.2: A methane gas extraction plant on Lake Kivu

Nuclear fuel

Nuclear fuel is a material that can be 'burned' by nuclear fission or fusion to derive nuclear energy.

Figure 6.3: Uranium, one of nuclear sources

Most nuclear fuels contain heavy fissile elements that are capable of nuclear fission. When these fuels are struck by neutrons, they are in turn capable of emitting neutrons when they break apart. This makes possible a self-sustaining chain reaction that releases energy with a controlled rate in a nuclear reactor or with a very rapid uncontrolled rate in a nuclear weapon.

"…Rwanda should choose a path to renewable energy—although nuclear is the best other alternative; Rwanda does not have the technology to generate nuclear energy.

Even if Rwanda was ready to develop it despite the international laws and regulations, nuclear energy poses a great danger especially, Rwanda being located in a volcanic region. Nuclear energy for Rwanda in my opinion is a no go option". *New times May 21, 2015*

Renewable sources

Renewable energy is generally defined as energy that comes from resources that are not significantly depleted by their use, such as sunlight, wind, rain, tides, waves and geothermal heat. Renewable energy is gradually replacing conventional fuels in four distinct areas: electricity generation, hot water/ space heating, motor fuels, and rural (off-grid) energy services.

Figure 6.4: Sources of renewable energy

Generally, Rwanda is well endowed with renewable energy resources, but most potential still remains untapped. Micro hydro-power in particular constitutes a significant potential for rural power supply with many areas having ample rainfall and most streams and rivers unexploited. Solar irradiation is high - between 4-6 kWh/m^2/day - but diffusion is hampered by high initial cost and limitations on high load usage. Biogas is promising for thermal energy needs for farms and small institutions, especially considering the large number of households that own cows and other livestock.

Geothermal

Geothermal energy is from thermal energy generated and stored in the Earth. Thermal energy is the energy that determines the temperature of matter.

Figure 6.5: Source of geothermal energy

According to a study by Geothermal Energy Association, geothermal potential in Rwanda ranges from 170 - 340 MW.

In Rwanda geothermal is a main energy policy priority and forms a significant part of the 7-year electricity development strategy including a very ambitious action plan targeting 150 MW of generation capacity by 2017 (which represents up to 50% of total generation). A Geothermal Act and a geothermal exploration and development paper have been drafted although a proposal for a feed-in-tariff for geothermal still needs to be developed.

Figure 6.6: The minister of foreign affairs Mrs Louise Mushikiwabo touches water to feel the hotness from geothermal energy

Three sites (Rubavu, Karongi and Rusizi) were identified already in the 1980's with resource temperatures in excess of 150°C which could be suitable for geothermal power generation. In early 2012, test drilling commenced to explore possibilities to harness energy in Rubavu, Karisimbi, Kinigi located in western region as well as Bugarama in southern region. The Government has self-financed and contracted the first exploratory drilling in 2013.

Biomass and biofuels

Biomass is biological material derived from living, or recently living organisms. It most often refers to plants or plant-derived materials which are specifically called *lignocellulosic biomass*. As an energy source, biomass can either be used directly via combustion to produce heat, or indirectly after converting it to various forms of biofuel. Conversion of biomass to biofuel can be achieved by different methods which are broadly classified into: *thermal*, *chemical*, and *biochemical* methods.

Figure 6.7: Biological material

In Rwanda, It has been observed that if an average household used 1.8 tonnes of firewood in a year to satisfy its cooking needs with a traditional stove, the same household would use 3.5 tonnes of wood if it were to switch to charcoal with an improved stove. The use of charcoal in urban areas, in combination with high urban growth rates, therefore is a worrisome phenomenon that accelerates pressure on wood resources. Peat is also a resource the government intends to promote use of. It is estimated that there exists in Rwanda estimated reserves of 155 million tons of dry peat spread over an area of about 50,000 hectares. About 77% of peat reserves are near Akanyaru and Nyabarongo rivers and the Rwabusoro plains Potential for Peat-to-Power Generation. Peat in the Rwabusoro marshland and around the Akanyaru river can fuel 450 mW of electricity generation for 25 years. Currently, a cement plant and some prisons utilize peat for cooking.

How it works - Burning landfill or digester gas to make power

Burn Fuel → Heat water to make steam → Steam turns turbines → Turbines turn generators → Electric power sent around the country

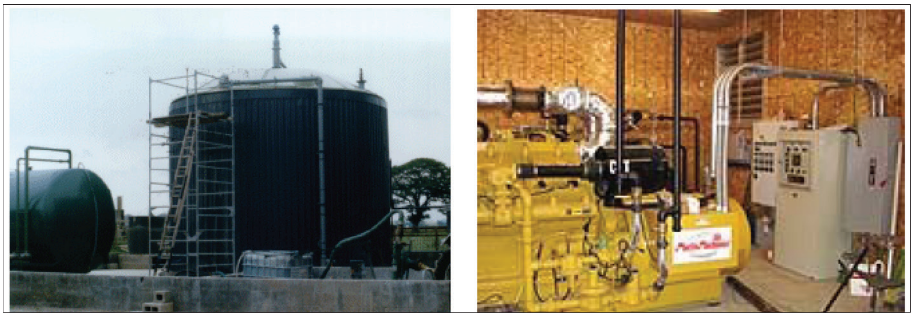


Figure 6.8: Burning landfill and digester gas to make power

Biogas has been introduced in the country many years ago and Rwanda has gained international recognition for its program in prisons and large institutions. The Government in 2008 announced a policy to introduce biogas digesters in all boarding schools (estimated at around 600 schools), large health centres and institutions with canteens to reduce the consumption of firewood. This process started in 2010 but until today the focus has been mainly on installations for schools. In total, about 50 large biogas digesters have been constructed in institutions in Rwanda and the biogas systems that have been installed in the prisons over the last decade have reduced firewood consumption by up to 40% and improved hygienic conditions.

Figure 6.9: Construction works of the digester chamber for a fixed dome bio-gas plant. Biogas will play a key role in reducing pressure on the country's forests

Activities in the domestic biogas sector started much later. It is estimated that over 120,000 households have dairy cows that are kept under zero grazing conditions to reduce soil erosion and also due to lack of grazing areas. These numbers are increasing due to the governments programs to increase the number of families with dairy cows.

Figure 6.10: Use of biogas for cooking in Gicumbi district

Figure 6.11: Use of biogas for lighting in Gicumbi district

Solar energy (photovoltaic cells and solar heating panels)

Photovoltaic Cells

Solar energy, radiant light and heat from the sun, is harnessed using a range of ever-evolving technologies such as solar heating, photovoltaic, concentrated solar power, solar architecture and artificial photosynthesis.

Figure 6.12: Focusing solar energy

The Rwandan government is set to commission the first utility-scale solar photovoltaic (PV) plant at eastern Rwanda's Rwamagana district in August 2014

The project, with a production capacity of 8.5 mW, has commenced testing, stated local reports. Dutch company Gigawatt Global is the developer of the project, while Norwegian firm Scatec Solar has agreed to operate and maintain the plant.

Figure 6.13: Israel's Energiya Global is behind a new solar project in Rwanda

Solar Heating Panels

A *solar thermal collector* collects heat by absorbing sunlight. A collector is a device for capturing solar radiation. Solar radiation is energy in the form of electromagnetic radiation from the infrared (long) to the ultraviolet (short) wavelengths.

Figure 6.14: In Rusizi, solar heating panels on a roof of a house

The term "solar collector" commonly refers to solar hot water panels, but may refer to installations such as solar parabolic troughs and solar towers; or basic installations such as solar air heaters.

Hydroelectric power, wind power and wave power

Hydroelectricity

Figure 6.15: Hydroelectric power station

Energy in water can be harnessed and used. Since water is about 800 times denser than air, even a slow flowing stream of water, or moderate sea swell, can yield considerable amounts of energy.

Hydroelectricity is the term referring to electricity generated by hydropower; the production of electrical power through the use of the kinetic energy of falling or flowing water. It is the most widely used form of renewable energy, accounting for 16% of global electricity consumption.

Figure 6.16: Hydro electric Power Plant Project on River Nyabarongo

The country currently has about 57 MW installed hydropower generating capacity. Hydroelectric power is mainly from the northern and southern parts of the country (Musanze , Rubavu and Rusizi) namely from the following power plants: Ntaruka, Mukungwa , Rubavu, Gihira as well as Rusizi 1 and Rusizi 2. A number of new sources are supposed to come on line within the coming years adding a capacity of 232 MW by 2013. This includes the hydropower site Nyaborongo with 27.5 MW in Muhanga and Ngororero Districts planned to come on line by February 2013 but currently experiencing delays, and numerous mini/micro hydro plants adding up to 35 MW. The new hydropower plant, Rukarara located in Nyamagabe district, Southern Province, with 9.5 MW and costs of US$ 23.5 million was commissioned in January 2011. Construction for this plant had already started in 2007.

Wind Power

Airflows can be used to run wind turbines. Modern utility-scale wind turbines range from around 600 kW to 5 MW of rated power, although turbines with rated output of 1.5–3 MW have become the most common for commercial use; the power available from the wind is a function of the cube of the wind speed, so as wind speed increases, power output increases up to the maximum output for the particular turbine.

Figure 6.17: Airflow can run a turbine

Wind Potential in Rwanda has not been fully exploited for power generation although potential wind power that Rwanda has in some areas may provide with possible solutions such as water pumping, windmill and electricity generation. A study of wind speed distribution has been made. (In this study, the results have been found for the average wind speeds and directions for 3 stations (Kigali, Rubavu and Huye) from 1985 to 1993.

These results can be summarised as follows:

- Direction of wind varies from 11 to 16°.
- Wind speed varies from 2 to 5.5m/s

The analysis of the wind energy possible solution for energy supply in rural areas of Rwanda, was undertaken to estimate the wind power potential. In total data from 4 stations (Kamembe, Huye, Nyagatare and Rubavu) have been analysed by the National Meteorological Division in 1989. Once again, the data from 3 synoptic sites (Kigali, Huye and Rubavu) are analysed by the Weibull function. The considered data has been used to evaluate the annual frequency of wind speed and the direction of wind, yearly variation of the monthly average, annual and daily variation, and vertical profile of wind energy potential. Nevertheless more detailed data is still required. In 2010 a wind system was put in place to serve the Rwanda office of information RBA on Mount Jali overlooking Kigali. This is the same site for the 250KW solar system feeding to the grid. There is need for more thorough assessment of the wind potential in the country.

Wave Power

Wave power captures the energy of ocean surface waves, and tidal power, converting the energy of tides, are the two forms of hydropower with future potential; however, they are not yet widely employed commercially.

Activity 4

Energy Source research

Purpose

Although most of the energy consumed in Rwanda comes from fossil fuel sources, there are many other potential sources of energy available. In all cases, there are pros and cons (advantages and disadvantages) to our use of these sources. Some of the energy sources are limited by their availability or environmental impact; others need technological improvements before they can become widely used. For scientists and engineers, research is the best way to learn about unknown topics.

In this section, we will examine information about energy sources and how those sources are used to produce electrical energy. We can use this information to help us understand the various pros and cons that affect our use of different energy sources. In this activity, each group of students will begin to become an expert on one aspect of each source of energy and report their findings back to the class.

Procedure

1. Break into a group of 2-3 learners.
2. Choose or accept an assignment to research one particular question about each source of energy.
3. Using the provided information packet, find the answer to your question for all seven energy sources.
4. Once you have answered your question for all seven sources, answer the two conclusion questions.

As a class, we will fill in the energy sources chart based on your findings.

Research Questions

1. What is this energy source? Where can we find it in Rwanda?
2. How do we harness the energy? (How does it work?)
3. Are there different types or uses of this source? If yes, what are the differences?
4. How is this energy source currently used? For example: At farms, in industry etc. Could this source be used in a family home?

Note: Prepare a report summarizing your research and present the report to the class.

Primary energy sources take many forms, including nuclear energy, fossil energy-like oil, coal and natural gas - and renewable sources like wind, solar, geothermal and hydropower. These primary sources are converted to electricity, a secondary energy source, which flows through power lines and other transmission infrastructure to your home and business.

Activity 5

Discussion Questions

1. If you had to choose an energy system to tell your community about based only on the aspect you researched, which system would you choose? Why?
2. Why do we as a nation depend so much on fossil fuels? **AND** what do you think we could do to reduce this dependence on fossil fuels?

Note: Prepare a report summarizing your research and present the report to the class.

Energy source	**"Pros"**	**"Cons"**
Biomass		
Fossil fuels		
Geothermal		
Hydropower		
Nuclear		
Solar		
Wind		

While listening to the other groups in your class present their information, list some "pros" and "cons" (advantages and disadvantages) of using their energy source to solve your problem. While listening to the students in your group present their information, list some "pros" and "cons" of using that energy source to solve the energy problem.

Advantages and disadvantages of renewable and non-renewable energies

Activity 6

Do research in the library or internet and complete the task below

1. Complete the chart below about the basic types of renewable energy resources.

Type	Definition	Examples	Advantages	Disadvantages
Solar				
Hydropower				
Wind Energy				
Geothermal				
Biomass				

2. List those energy sources that are fossil fuels.

____
____
____

3. What main advantage do fossil fuels have over the renewable energy resources?

____
____
____

4. What are two main disadvantages of fossil fuels compared to renewable energy?

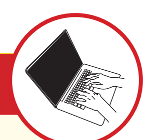
____
____
____

The sun, prime source of world energy

Solar energy comes from thermonuclear fusion; 30% of solar energy arriving on higher layers of atmosphere are reflected in space. 47% of that energy are absorbed by the ground and oceans during daytime and become the Earth's internal energy. The remaining 23% of solar energy are used in evaporation of water of oceans. When it rains, a part of energy is transformed into potential gravitational energy, stocked in mountains, lakes, which are the sources of hydroelectric power. About 0.2% is used by convection currents in atmosphere and creates wind energy. Finally 0.02% is absorbed by plants during photosynthesis and is stocked by them in form of chemical energy. Plants are sources of biomass. Photovoltaic cells transform solar energy in electrical energy.

Extraction and creation of renewable and non-renewable energy sources

Activity 7



Creation of renewable and non renewable energy

From what you have already learned, you'll do also research and tell how these energies are created: Solar energy, hydropower, wind energy, geothermal energy, and biomass. Try to find or to formulate how they are extracted.

You'll fill the table below

Energy	Creation	Extraction
Solar		
Hydropower		
Wind		
Geothermal		
Biomass		
Nuclear		
Fossil fuel		

Creation

Non-renewable resources

A non-renewable resource (also called a finite resource) is a resource that does not renew itself at a sufficient rate for sustainable economic extraction in meaningful human time-frames. An example is carbon-based, organically-derived fuel. The original organic material, with the aid of heat and pressure, becomes a fuel such as oil or gas.

Earth minerals and metal ores, fossil fuels (such as coal, petroleum, and natural gas), nuclear fuels, and groundwater in certain aquifers are all non-renewable resources.

Natural resources such as coal, petroleum (crude oil) and natural gas take thousands of years to form naturally and cannot be replaced as fast as they are being consumed. Eventually it is considered that fossil-based resources will become too costly to harvest and humanity will need to shift its reliance to other sources of energy. These resources are yet to be named.

Renewable resources

Natural resources, known as renewable resources, are replaced by natural processes and forces persistent in the natural environment. There are intermittent and reoccurring renewable and recyclable materials, which are utilized during a cycle across a certain amount of time, and can be harnessed for any number of cycles.

The production of goods and services by manufacturing products in economic systems creates many types of waste during production and after the consumer has made use of it. The material is then incinerated, buried in a landfill or recycled for reuse. Recycling turns materials of value that would otherwise become waste into valuable resources again.

The natural environment, with soil, water, forests, plants and animals are all renewable resources, as long as they are adequately monitored, protected and conserved. Sustainable agriculture is the cultivation of plant and animal materials in a manner that preserves plant and animal ecosystems over the long term. The overfishing of the oceans is one example of where an industry practice or method can threaten an ecosystem, endanger species and possibly even determine whether or not a fishery is sustainable for use by humans. An unregulated industry practice or method can lead to complete resource depletion.

Extraction

Resource extraction involves any activity that withdraws resources from nature. This can range in scale from the traditional use of preindustrial societies, to global industry. Extractive industries are, along with agriculture, the basis of the primary sector of the economy. Extraction produces raw material which is then processed to add value. Examples of extractive industries are hunting, trapping, mining, oil and gas drilling, and forestry. Natural resources can add substantial amounts to a country's wealth, however a sudden inflow of money caused by a resource boom can create social problems including inflation harming other industries ("Dutch disease") and corruption, leading to inequality and underdevelopment, this is known as the "resource curse".

ENERGY, POWER AND CLIMATE CHANGE

Energy Degradation (Dilapidation) and Power Generation

Unit 7

Energy degradation (dilapidation) and power generation

Key unit Competence

By the end of the unit, the learner be able to analyse energy degradation/ dilapidation and power generation

My goals

By the end of this unit, I will be able to:

* convert thermal energy into work by single cyclic process.
* draw energy diagrams illustrating energy degradation.
* identify mechanisms of electrical power generation.
* explain energy degradation.
* analyse energy degradation/dilapidation and power generation.

Introduction

In Rwanda cutting down of trees, burning of bushes, brick firing is dorminantly carried out especially in villages / rural areas. However, this is being regulated by the government. Remember that these are bad acts and they lead to loss of natural resource. There are so many ways how energy can be made less available to work.

Other activities that lead to loss of energy include:

- Clearing land for agriculture and construction (industries and homes).
- Using harmful insecticides and catalysts.
- Fumes from vehicles and industries. etc.

When thermal *energy* is converted to mechanical or electrical *energy*, part of the thermal *energy* has to be expelled into the environment. This *energy* is considered *degraded.*

Definition of energy degradation/ dilapidation

The *degradation of energy* is the process by which energy becomes less available for doing work. Compare conservation of energy and dissipation of energy. Degradation of energy is the process of energy transforming into disordered, spread out energy.

Thermal energy is described as the most degraded form of energy, as it is the final form energy that is 'spread out' or lost to the surroundings in any conversion, and ultimately becomes unavailable to perform useful work.

An *energy transformation* is the change of energy from one form to another. Energy transformations occur everywhere every second of the day.

Nowadays, it's seen that a high energy consumption results into development of industries.

Production of electrical energy by rotation of coils in a magnetic field

Activity 1

Generate electricity

Materials

Each learner or group of learners will need the following materials to perform this experiment:

* compass.
* powerful magnet bar.

* a small-gauge insulated copper magnet wire.

Procedure

First, use the wire to make a coil of 40 turns and about 5 or 6cm in diameter. Next, wrap about 25 turns of the wire around a compass. Connect the two coils together at both ends to make a complete circuit. Rapidly pass the magnet back and forth through the centre of the first coil. Watch the compass needle.

a) What happens when you move the magnet in one direction? In the other direction?

b) Why does this happen?

c) For more fun, you could connect the two ends of the first coil to a microampere meter that measures electrical current and repeat the experiment. What happens to the meter's needle? Why?

Activity 2

Explore the World Outside

a) List places where an electrical generator might be needed during a power failure or places where they have seen portable generators in use.

b) Why electricity is a useful form of energy?

c) Describe how electric energy is produced by rotating coils in magnetic field.

d) Discuss your observation in your groups.

e) Write down important ideas.

Disadvantage of cutting power

In the case of the power cut, there are so many disadvantages. Lights in medical operating theatre go off. The first "unexpected" problem might be encountered when you want to access the internet. How about the doorbell and traffic lights? In some areas, if the electricity fails, the domestic water supply also fails within a few minutes.

Other things

Computers and a few other devices do not shut down cleanly during a loss of power. In addition to losing data that was in use at the time of the failure, they can also have problems in restarting. Having an Uninterruptible Power Supply is a good idea. Other devices which do not resume where they left off include air conditioning, video recorders, TV (goes into standby mode), photocopier, etc.

The power shutdown gives occasion to thieves and criminals to operate. You'll remember that during the Genocide aganist Tutsi in Rwanda, killers were cutting off power to exterminate people they considered enemies.

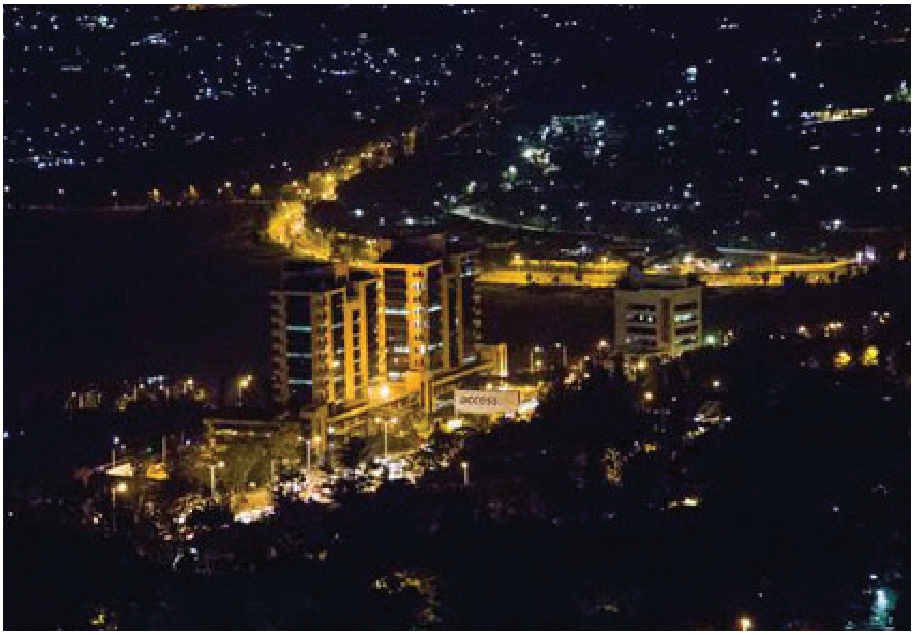


Figure 7.1: During Rwanda Genocide, killers cut off power to exterminate other people

Conversion of thermal energy into work by single cyclic processes

Activity 3

Search on internet and read in books to get information about conversion of thermal energy into work by a single cyclic processes

- Carry out research and write a report of your study.
- Present your report to the whole class.
- Hand in your report to the teacher for marking.

Now I know that

Thermodynamics is the study of the connection between thermal energy and work and the conversion of one into the other.

This study is important because many machines change heat into work (such as an automobile engine) or turn work into heat as in a fire drill (or cooling, as in a refrigerator). There are two laws of thermodynamics that explain the connection between work and heat. But first, it must be shown how mechanical energy can be equivalent to heat energy.

The first law of **thermodynamics** is a version of the law of **conservation of energy,** adapted for **thermodynamic systems**. The law of conservation of energy states that the total **energy** of an **isolated system** is constant; energy can be transformed from one form to another, but cannot be created or destroyed. The first law is often formulated by stating that the change in the **internal energy** of a **closed system** is equal to the amount of **heat** supplied to the system, minus the amount of **work** done by the system on its surroundings. Equivalently, **perpetual motion machines** of the first kind are impossible.

A process that occurs at constant temperature is called an isothermal process. The internal energy of an ideal gas is a function of temperature only. Hence, in an isothermal process, the internal energy, $\Delta E_{int} = 0$.

For an isothermal process, we conclude from the first law that the energy transfer ΔQ must be equal to the negative of the work done on the gas, that is, $\Delta Q = -\Delta W$

Any energy that enters the system as heat is transferred out of the system by work; as a result, no change in the internal energy of the system occurs in an isothermal process.

In these kinds of problems, you are asked about work done by a *heat engine*. In a heat engine, thermal *energy* is put in and *work* is out put. In other words, heat engines can be understood by tracking energy. This is a Conservation of Energy problem.

This will be explained at length in the Laws of thermodynamics.

Energy flow diagram illustrating energy degradation (sankey diagram)

Activity 4: Role play

Energy System Diagrams

Purpose

In order to use an energy system, you need to know how your system works. In this activity, you will use system diagrams to discover how your assigned energy source may be used to produce electrical energy. You should then be able to identify and name the components of the energy system. Using this knowledge, you will draw a flowchart to illustrate the path of energy conversions through the system.

Procedure

1. Break into your energy system groups.
2. Each learner will be given a card with either the name of a system component, or a description.
3. Someone else in the room has the description for the word you are given and vice versa. Now you must find that person.
4. Once you have found your partner, go to your system diagram poster and place your word and description in the spot pointing to that component.

Draw a flowchart, using the following template as a guide:

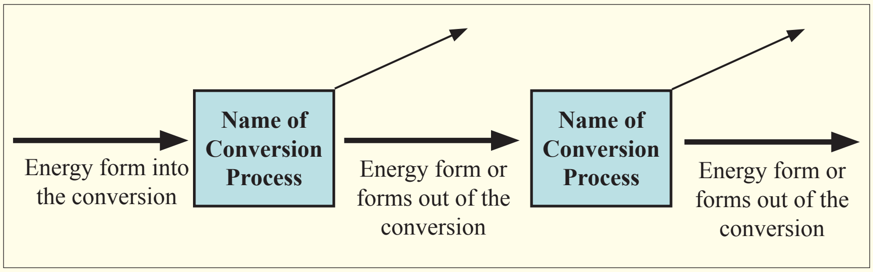


Figure 7.2: An energy system diagram

Discussion Questions

1. Is all the energy available from your source used? If not, what components contribute most to this loss?
2. After looking at all the system diagrams, which components are common to most of the systems? Why?
3. Present your suggestions to the whole class.
4. Note down important ideas in your report.

Caution!

In making groups, if it is a mixed school. It is good to work together. This will help you learn from one another.

Activity 5

Sankey Diagram

Search in the internet or from books in the school library about Sankey diagram. Write down all links used and answer the following questions

a) What is the Sankey diagram?
b) Why is Sankey diagram used in this unit?
c) How is the flow of energy illustrated on Sankey diagrams?
d) Why are Sankey diagrams better in illustrating energy flow?
e) When working with Sankey diagrams most physics Sankey diagrams are used to examine energy efficiency. What do we consider when using the idea of work?
f) Draw a Sankey diagram?

MECHANICS

Dynamics

Unit 8

Projectile and uniform circular motion

Key Unit Competence

By the end of this unit, the learner should be able to analyse and solve problems related to projectile and circular motion.

My goal

By the end of this unit, I will be able to:

* define and explain terms used in projectile motion.
* discuss the different applications of projectile motion.
* apply concepts of projectile and circular motion in real life.
* differentiate between projectile motion and circular motion.

Introduction

We have different kinds of sports, for examples; football, netball, tennis amongst others.

Using the example above

A lot of reasoning is needed while playing football to score one of which is to kick a ball at a certain angle (i.e. to move above the ground). We say that the ball is projected.

This also applies to basket ball; the ball to enter the round ring for a score it has to be thrown at a certain angle. Hence, projected.

The same principle is used by the military in shooting and launching their missiles.

Projectile Motion

Activity 1: Field study

Aim; ***To study motion of bodies in free space***

a) Out of class, (in pitch,or in school compound), throw a ball,a stone or any body upward.
b) State what happens.
c) Hold a ball in your hands and release it to fall.
d) Is the motion of the ball same as in the first case?
e) Note down your observation.
f) Relate your observations for bodies moving linearly.

Caution

While throwing a stone or any body, take care so that it does not harm you.

We can define a projectile as any body thrown into space/air. The path taken is called a trajectory.

The motion of a projectile unless taken otherwise is a free motion under gravity. We assume that air resistance is negligible in this kind of motion.

We have three cases: oblique projection, vertical projection and horizontal projection.

Projection at an angle above the horizontal

- Study the picture below carefully.
- Go outside class and try to kick the ball so that it does not roll on the ground.
- State when will the ball cover a long horizontal range. (State down the conditions for that to occur).

Figure 8.1: A football player kicking a ball at a certain angle

From the figure above, if the ball is kicked so that it does not roll on the ground, it will move at certain angle *relative to the ground.*

Activity 2

a) In the ground, kick the football individually.

b) By observing, the flight of the ball state whether it will cover a longer horizontal distance when it is projected at a large angle or a small angle.

c) Explain your observation and note down any key points in your book.

Consider a projectile having a certain mass, projected at a speed $\vec{v}_0$ at an angle α to the horizontal.

Upward projection

From the figure above, a football player can kick the ball and it takes the motion of a projectile.

Figure 8.2: An object thrown at an angle above the horizontal

This is the motion in the *x-y* plane; we consider axis OX and OY. $\vec{v}_0$ has two components even the acceleration. We have:

For the acceleration; $\begin{cases} \vec{\alpha}_x = 0 \\ \vec{g}_y = -\vec{g} \end{cases}$ Thus:

For the velocity $\vec{v}_0$: $\begin{cases} v_x = u_x - \alpha t \\ \quad = v_0 cos\alpha \\ v_y = u_y + \alpha t \\ \quad = v_0 sin\alpha - gt \end{cases}$

According to OX axis, we have the rectilinear uniform motion whose velocity is constant and has value $v_{0x} = v_0 \, cos\alpha$ = constant

According to OY axis, we have the rectilinear uniformly decelerated motion with acceleration $\gamma_0 \, sin\alpha - gt.$

Equations of the Motion

For horizontal motion,

$\alpha_x = 0$, $u_x = v_o \cos\alpha$, constant and $x = u_x + \alpha_x t$, we have

$x = x = v_o t \cos\alpha$ (1)

For vertical motion,

$$v_y = v_y + \alpha_y t$$

$$= v_o \sin\alpha - gt \quad (2)$$

$$height\ \ y = u_y t + \frac{\alpha_y t^2}{2}$$

$$= t v_o sin\alpha - \frac{gt^2}{2} \quad (3)$$

Equations (1) and (3) represent the parametric equations of the motion.

Using the equations developed above, obtain the parametric equation.

We have: $y = xtan\alpha - \frac{1}{2} g \left(\frac{x^2}{v_0^2 \cos^2\alpha}\right)$ (4)

Calculation Of The Maximum Height

Let y_{max} be the maximum height reached by the projectile.
$y = y_{max}$ if and only if $v_y = 0$ and $v_0 \sin\alpha - gt$

$0 = v_o \sin\alpha - gt \qquad t = \frac{v_0 sin\alpha}{g}$ (5)

The relation gives the time taken to reach the maximum height y_{max} (h_{max})

Let us introduce (5) in (3), we find:

$$h_{max} = 0 - \frac{g}{2}\left(\frac{v_0 sin\alpha}{g}\right)^2$$

$$= \frac{v_0^2 sin\alpha^2}{2g}$$

$$h_{max} = \frac{v_0^2 sin\alpha^2}{2g}$$

Therefore: h is negative because its direction is opposite to the direction of g.

The Horizontal Range of the Projectile

The horizontal distance travelled by a projectile from the initial position *(x = y = 0)* to the position where it passes $y = 0$ during its fall is called the *horizontal range R.*

It's the horizontal distance travelled during the time of flight t_f.

To calculate this range denoted by *R,* it's important to know that $R = x_{max}$

When $y = 0$ and $y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha}$,

$$x\tan\alpha - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha} = 0$$

We have: $x\left(tg\alpha - \frac{1}{2}g\frac{x}{v_0^2\cos^2\alpha}\right) = 0$

$x_1 = 0$, x_1 represents the initial position

$$\tan\alpha - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha} = 0 \rightarrow tg\alpha = \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha}$$

$$x_{max} = \frac{2v_0^2\sin\alpha.\cos\alpha}{g}$$

$$\Rightarrow R = x_{max} = \frac{v_0^2\sin 2\alpha}{g}$$

We can show this relation using another way:

$R = (v_o\cos\alpha)\,t_f$ where $t_f \frac{2v_0\sin\alpha}{g}$ is the total time of the flight because

$t_f = t_{um} + t_m$

$$R = v_0\cos\alpha \times \frac{2v_0\sin\alpha}{g} \times \frac{2v_0^2\sin\alpha\cos\alpha}{g}$$

$$R = \frac{v_0^2\sin 2\alpha}{g}$$

Velocity at a given point a of the trajectory

At each time: $v^1 = v_x^{\ 1} + v_y^{\ 1}$, $\sqrt{v_x^{\ 2} + v_y^{\ 2}}$, where $v_x = v_0\cos\alpha$ and $v_y = v_0\sin\alpha - gt$

$v = \sqrt{v_0^2\cos^2\alpha + (v_0\sin\alpha - gt)^2}$, $v = \sqrt{v_0^2\cos^2\alpha + v_0^2\sin^2\alpha - 2v_0\sin\alpha.gt + g^2t^2}$

$$\Rightarrow v = \sqrt{v_0^2(\cos^2\alpha + \sin^2\alpha) - 2v_0 \sin\alpha\, gt + g^2t^2}$$

$$\Rightarrow v = \sqrt{v_0^2 - 2gtv_0 \sin\alpha + g^2t^2}$$

Example

1. A particle is projected from a point on a horizontal plane and has an initial velocity of 45 ms^{-1} at an angle of elevation of $\tan^{-1}\left(\frac{3}{4}\right)$. Find the time of flight and the range of the particle on the horizontal plane.

Solution

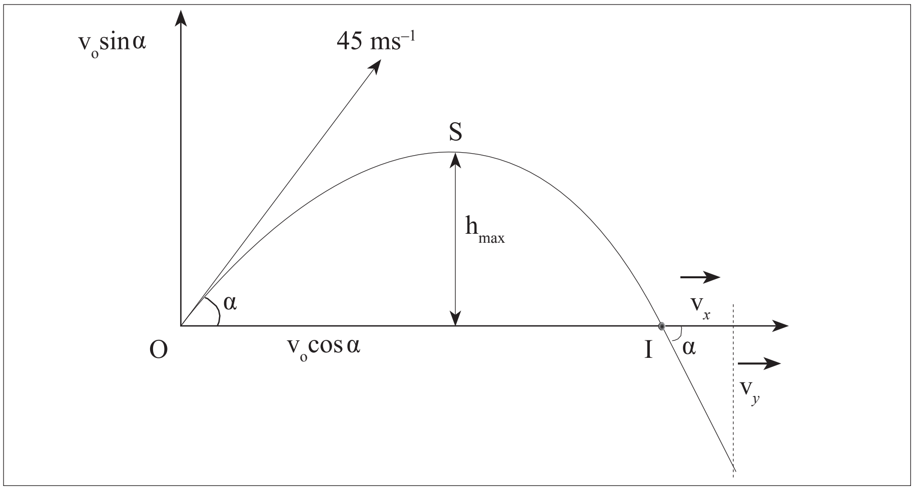


Figure 8.3: Showing movement of a particle at an angle

Vertical Motion

The total time of the flight can be found from the equation:

On Y-axis, $y = v_0\, sin\, \alpha t - \frac{1}{2}gt^2$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \alpha = 36.9, \sin\alpha = 0.60042$

$45 \times 0.60042t - \frac{1}{2} \times 4.9^2 = 0$

$t(27.009 - 4.9t) = 0$

Either $t = 0$ or $29.009 - 4.9t = 0 \Rightarrow t = \frac{29.009}{4.9} = 5.9s$

Then the total time of flight is 5.9s

The range $R = v_0 \cos\alpha t$, $\cos\alpha = 0.7997$

$R = 45 \times 0.7997 \times 5.9 = 212m$

Downward projection

Note:

The motion of the projectile has also two components:

a) A rectilinear uniform motion in the horizontal direction with velocity $v_{ox} = v_o \cos \alpha$ and parametric equation $x = (v_o \cos \alpha)t$.

A vertical accelerated motion downward in the vertical direction with initial velocity: $v_{oy} = v_o \sin a$, acceleration $\gamma = g$ and parametric equation: $y = (v_o \sin \alpha)t + \frac{1}{2} gt^2$.

Thus the parametric equations are: $\begin{cases} x = (v_0 \cos \alpha)\, t & (1) \\ y = (v_0 \cos \alpha)\, t + \dfrac{1}{2} gt^2 & (2) \end{cases}$

where t is the parameter.

From (1): $t = \dfrac{x}{v_0 \cos \alpha}$, t in $y = v_o \sin \alpha . \dfrac{x}{v_0 \cos \alpha} + \dfrac{1}{2} g \left(\dfrac{x}{v_0 \cos \alpha} \right)^2$

$y = x\tan\alpha + \dfrac{g}{2v_0 \cos^2 \alpha} x^2$ is the equation of the trajectory. The trajectory is a path taken by the projectile.

Velocity at a given point of the trajectory, $\vec{v} = \vec{v}_x + \vec{v}_y$, $v = \sqrt{v_x^{\,2} + v_y^{\,2}}$, with $v_x = v_o \cos \alpha$ and $v_y = v_o \sin \alpha + gt$

$v = \sqrt{v_0 \cos \alpha + (v_0 \sin \alpha + gt)^2}$

Horizontal projection

Activity 3

Place a stone on top of a table.
Displace it so that its motion takes the shape below.
Try to observe the motion carefully.
Note down what you observe and share it with your class members.

Take care

In throwing the stone / displacing it, you should take care so that it does not hit you because it may harm you.

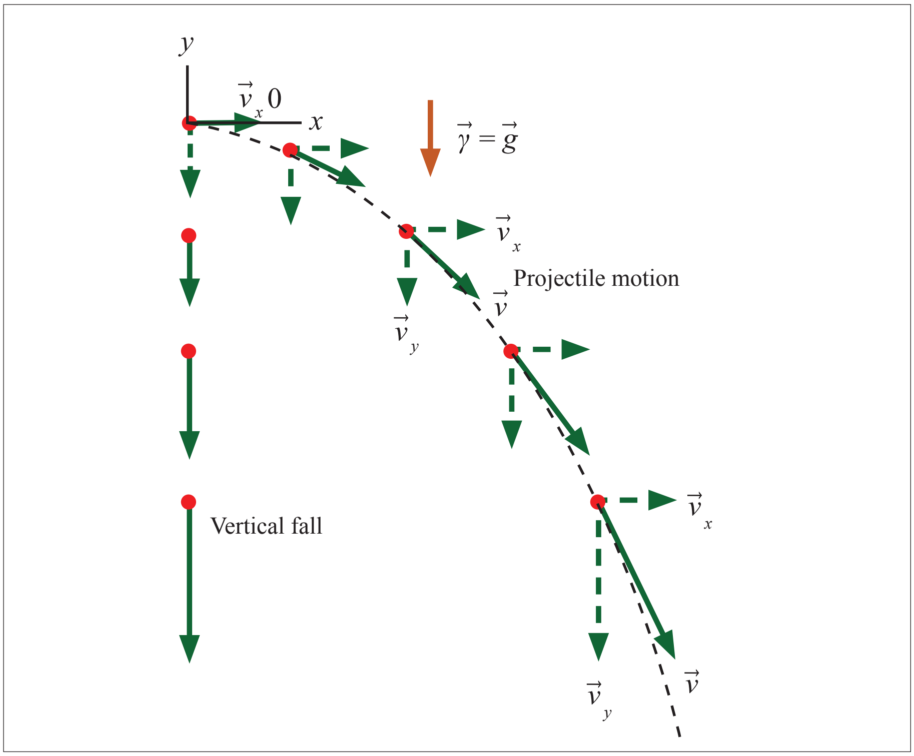


Figure 8.4: An object thrown horizontally

For horizontal motion; $v_x = v_o \cos\theta$ but $\theta = o$

According to Ox, we have RUM with $v_x = v_0$ constant

According to OY, we have a free fall with $v_{0y} = 0$ and $v_y = gt$

Parametric equations are:

$x = v_0 t$ (1)

$y = \frac{1}{2} gt^2$ (2)

From (1) $t = \frac{x}{v_0}$, in (2) $y = \frac{1}{2} g \frac{x^2}{v_0^2}$, represents the equation of a parabola

The speed at a given point of the trajectory is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + g^2t^2}$

Vertical projection

For this case, we consider the oblique projection where $\alpha = \frac{\pi}{2}$. We find the equations for vertical projection.

Activity 4

Using the information given above;

a) Derive the equations for the motion.

b) Study the picture below and do the same.

c) State the condition when the body attains maximum height.

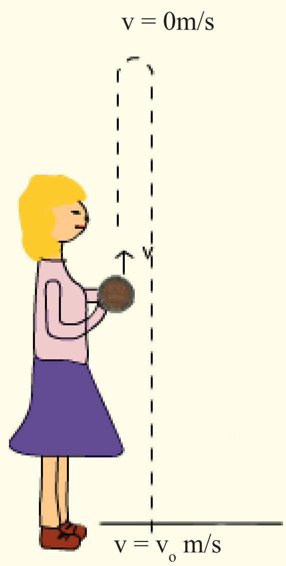


Figure 8.5: Showing the path taken by a body when thrown vertically

Circular motion

Activity 5

Study carefully the motion of the ball shown below.
State what would happen if at any point the thread holding the ball breaks?
Note and record your observation.

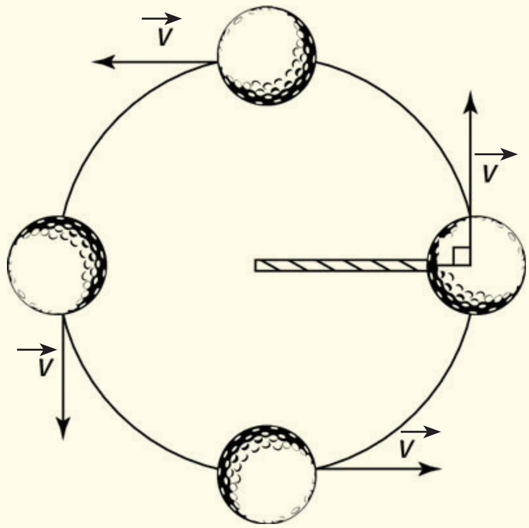


Figure 8.6: Showing the path taken by a ball moving in a circular path

A motion is said to be circular if the trajectory is a circle of constant radius R.

The motion is *uniform* if the body describes in equal angular displacements in equal times..

Even if the motion is uniform, it has an acceleration because the velocity changes after every moment since its direction keeps changing.

Angular displacement θ

its equation is: $\theta = \theta_o + \omega t$

Where θ: angular displacement of M at time t expressed radian [rad] as S.I of units.

θ_o = angular displacement of M at t =0 expressed radian [rad] as S.I of units.

ω = angular velocity of M expressed in [rad/s] in S.I of units.

The relationship between S and θ is:

$$\boxed{S = R.\theta}$$

[m] [m] [rad]

Centripetal acceleration

As we said, in a circular uniform motion, there is acceleration. This acceleration is called centripetal acceleration.(γ_x, γ_y) or $\vec{\gamma} = \gamma_x \vec{i} + \gamma_y$

Where $\gamma_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$ and $\gamma_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right)$.

$v_x = -R\omega \sin \omega t$, $v_y = -R\omega \cos \omega t$, we get:

$\gamma_x = -R\omega^2 \cos \omega t$, and $\gamma_y = -R\omega^2 \sin \omega t$

$$\vec{\gamma} = \begin{cases} \gamma_x = -\omega^2 \cos \omega t \\ \gamma_y = -\omega^2 \sin \omega t \end{cases}$$

$$\vec{\gamma} = \gamma_x \vec{i} + \gamma_y \vec{j} = -R\omega^2\, R \cos \theta \vec{i} + R \sin \theta \vec{j}$$

$$\vec{\gamma} = \gamma_x \vec{i} + \gamma_y \vec{j} = -\omega^2 (R \cos \theta \vec{i} + R \sin \theta \vec{j})$$

$\vec{\gamma} = -\omega^2 \overline{OM}$, $\vec{\gamma}$ and $\overline{OM}$ have opposite directions.

> **Conclusion**: In the uniform circular motion, $\vec{\gamma}$ is carried by the radius and directed toward the centre, reason why $\vec{\gamma}$ is called the ***centripetal acceleration***.

The magnitude of $\vec{\gamma}$ is found, proceeding like this:

$$\vec{\gamma} = \gamma_x \vec{i} + \gamma_y \vec{j} \Rightarrow \left\|\vec{\gamma}\right\| = \gamma \sqrt{\gamma^2_x + \gamma^2_y}$$

$$\gamma = \sqrt{\omega^4 R^2 \cos^2 \omega t + \omega^4 R^2 \sin^2 \omega t} \quad \sqrt{R^2\omega^4}$$

$$\gamma = R\omega^2 \qquad v = R\omega, \qquad \omega = \frac{v}{R}, \qquad \text{Then } \gamma = \frac{v^2}{R}$$

Periodic time, frequency

Activity 6

- Go to the play ground.
- Make sure you round the playground two times.
- Note and observe the time taken to make one complete revolution.
- What do you call the time taken to move around the play ground.

I should know that:

- ***Period* [*T*]**: The period is the time taken for a full revolution of the motion $\theta = \omega t$ if $\theta_0 = 0$

  For one turn, $\theta = 2\pi$ rad and $t = T$
  We have: $2\pi = \omega T$

Then

[rad]

$$T = \frac{2\pi}{\omega} \text{ and } \Omega = \frac{2\pi}{T}$$

[s] [rad/s]

From the activity 6, you made two rounds in a given time. The number of rounds made in a Unit time is called frequency.

Therefore,

- ***Frequency* [*f*]**: is the number of rotation per unit time

It's given by $f = \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f$

Notice: In S.I units, the frequency is in [rotations/sec], unit called *Hertz* [Hz].

Angular displacement	θ	$\theta = f(t)$	[rad]
Curvilinear displacement	S	$S = f(t)$	[m]
Angular velocity	ω	$\omega = \frac{d\theta}{dt}$	[rad/s]
Linear velocity	v	$v = \frac{dS}{dt}$	[m/s]
Angular acceleration	A	$A = \frac{d^2\theta}{dt^2}$	[rad/s^2]
Acceleration	γ	centripetal acceleration $\gamma_c = R\left(\frac{d\theta}{dt}\right)$ tangential acceleration $\gamma_T = R\left(\frac{d^2\theta}{dt^2}\right)$	[m/s^2] [m/s^2]

Distance time-graph of a uniform circular motion

When an object executes a circular motion of constant radius R, its projection on an axis executes a motion of amplitude a that repeats itself back and forth, over the same path.

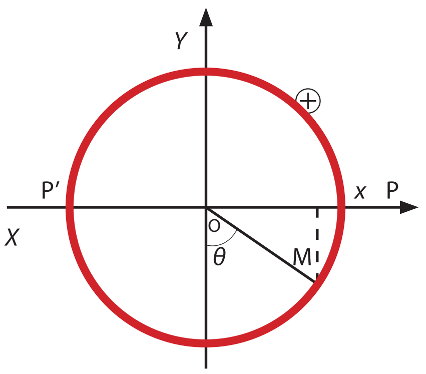


Figure 8.7: A circular motion can be projected on an axis

When M executes a uniform circular motion, its projection on X-axis executes a back and forth motion between positions P and P' about O.

Considering the displacement and the time, we find the following graph

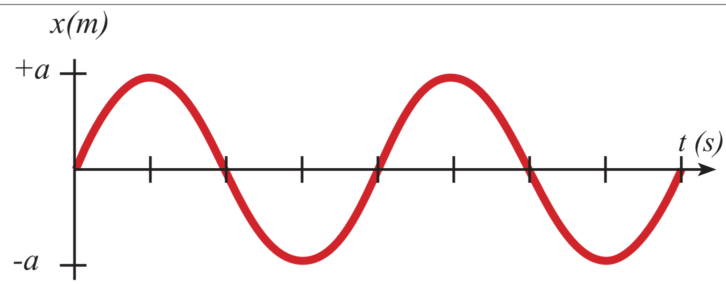


Figure 8.8: Distance-time graph of a uniform circular motion

Centripetal force

If you try to move / run in a circular path, you will finally notice that you keep moving in a circle even when you try to stop. There is a force that keeps you more in a circular path called **centripetal force**.

Since a body moving in a circle (or a circular arc) is accelerating, it follows from Newton's first law of motion that there must be force acting on it to cause the acceleration.

This force, like the acceleration, will also be directed toward the centre and is called the *centripetal force*. The value F of the centripetal force is given by Newton's second law, that is:

$F = m\gamma = \frac{mv^2}{R}$ Wherem is the mass of the body and v is its speed in circular path of radius R. If the angular velocity of the body is ω, we can also say, since$v = R\omega$,

$F = mR\omega^2$

When a ball is attached to a string and is swung round in horizontal circle, the centripetal force which keeps it in a circular orbit arises from the tension in the string.

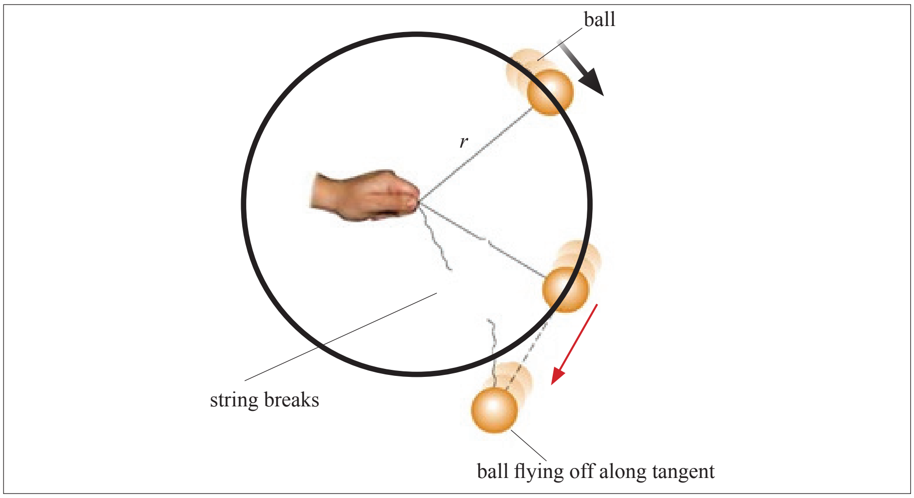


Figure 8.9: Increasing the velocity, the tension in the string increases and the string can break

Other examples of circular motion will be discussed. In all cases, it is important to appreciate that the forces acting on the body must provide a resultant force of magnitude $\frac{mv^2}{R}$ toward the centre.

Application of circular motion

Vertical and horizontal circle

Vertical circle

Taking the approach that the ball moves in a vertical circle and is not undergoing uniform circular motion, the radius is assumed constant, but the speed v changes because of gravity. Nonetheless, the formula of centripetal acceleration is valid at each point along the circle, and we use it at point 1 and 2. The free-body diagram is shown in the figure 8.10 for the positions 1 and 2.

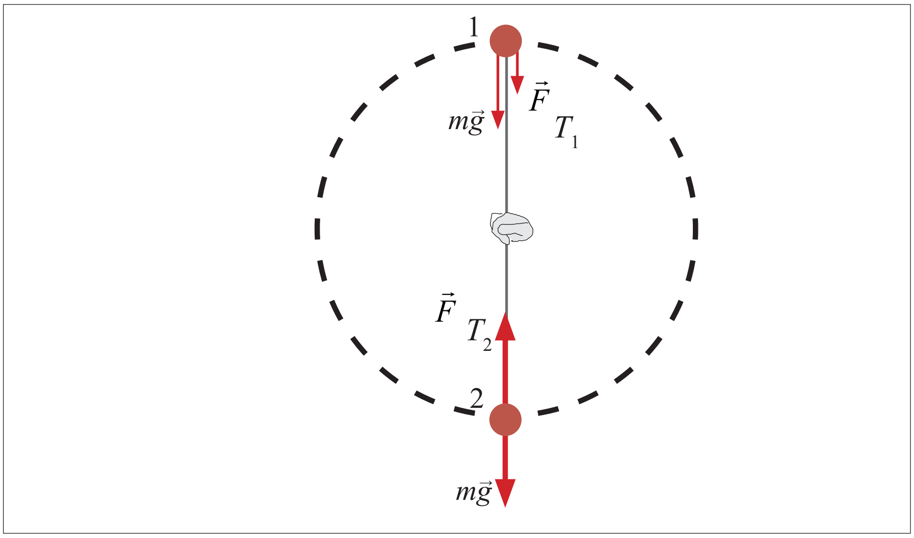


Figure 8.10. Free-body diagrams for position 1 and 2

a) At the top (point 1), two forces act on the ball: the force of gravity and the tension force the cord exerts at point 1. Both act downward and their vector sum acts to give the ball its centripetal acceleration. We apply Newton's second law, for t a vertical direction, choosing downward as positive since the acceleration is downward (toward the centre):

$$\sum\vec{F}\ \text{m}\alpha \ \Rightarrow F_{T_1} + mg = m\frac{v_1^2}{r} \quad \text{(at top)}$$

From this equation, we can see that the tension force F_{T_1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected.

But we are asked for the minimum speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But the tension disappears (because v_1 is too small), the cord can go limp and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T_1} = 0$, for which we have: $mg = m\frac{v_1^2}{r}$ (minimum speed at top)

We solve for v_1: $v_1 = \sqrt{gr}$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

b) When the ball is at the bottom of the circle (point 2 in the figure 8.10), the cord exerts its tension force F_{T_2} upward, whereas the force of gravity, still acts downward. So we apply Newton's second law, this time choosing upward as positive since the acceleration is upward (toward the centre):

$\sum F = m\gamma \Rightarrow F_{T_2} - mg = m\dfrac{v_2^{\,2}}{r}$ (at bottom)

We solve for F_{T_2}: $\sum F = m\gamma \Rightarrow F_{T_2} = mg + m\dfrac{v_2^{\,2}}{r}$

The second case is the case of a force on revolving ball (horizontal) which is: Estimate the force a person must exert on a string attached to a ball to make the ball revolve in a horizontal circle of radius *r*.

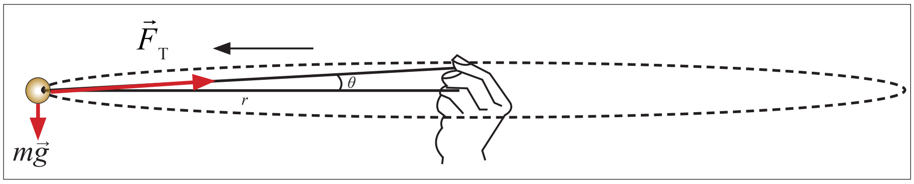


Figure 8.11: An object in a horizontal circular motion

The forces acting on the ball are the force of gravity, downward, and the tension that the string exerts toward the hand at the centre. The free-body diagram for the ball is as shown in the figure 8.11. The ball's weight complicates matter and makes it a little difficult to revolve a ball with the cord perfectly horizontal. We assume the weight is small and put θ=0 in the figure 8.11. Thus the tension will act nearly horizontally and, in any case, provides the force necessary to give the ball its centripetal acceleration.

We apply Newton's second law to the radial direction.

$\Sigma F = m\gamma$ where $\gamma = \dfrac{v^2}{r} \Rightarrow F_T = m = \dfrac{v^2}{r}$

Satellite cycling the earth

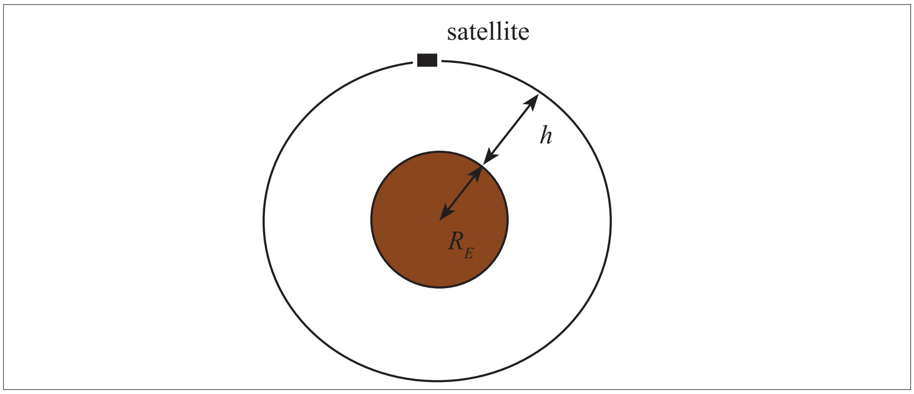


Figure 8.12: A satellite at altitude h from the earth

The centripetal force which keeps an artificial satellite in orbit round the earth is the gravitational attraction of the earth for it. For a satellite of massm travelling with speed v in circular orbit of radius (R_E+h) measured from the centre of the earth. R_E is the radius of the earth, h is the height where the satellite is.

We can write: $F_{cp} = W \Rightarrow \dfrac{mv^2}{R_E+h} = mg \Rightarrow \dfrac{v^2}{R_E+h} = g \Rightarrow v^2 = g\,(R_E+h)$

Then: $v^2 = \sqrt{g\,(R_E+h)}$

The time for the satellite to make one complete orbit (the period) is given by:

$$T = \frac{2\pi}{\omega},\ v = (R_E+h)\,\Omega \Rightarrow \omega = \frac{v}{R_E+h},$$

$$T = \frac{2\pi\,(R_E+h)}{v} = \frac{2\pi\,(R_E+h)}{\sqrt{g\,(R_E+h)}} = \frac{2\pi\,(R_E+h)\sqrt{g\,(R_E+h)}}{g\,(R_E+h)}$$

$$T = \frac{2\pi\sqrt{g\,(R_E+h)}}{g}$$

We can use a similar formula to find the velocity (speed) of a satellite cycling the earth.

$$v = \sqrt{G\frac{M_E}{(R_E+h)}}$$

In fact: $F_{cp} = \dfrac{mv^2}{R_E+h}$ *and* $W = G\dfrac{M_E m}{(R_E+h)^2}$

$$F_{cp} = W \Rightarrow \frac{mv^2}{R_E+h} = G\frac{M_E m}{(R_E+h)^2} \Rightarrow v^2 = G\frac{M_E}{R_E+h}$$

$$v = \sqrt{G\frac{M_E}{(R_E+h)}}$$

Conical pendulum

Activity 7

DO THIS!

* Tie a thread of about 50cm on retort stand.
* On a thread, tie a pendulum bob.

* Displace the bob through a certain angle.
* Displace the bob through a certain angle. What do you observe.
* Release the bob to move through a certain angle so that it moves in a horizontal circle.
* Try to investigate forces acting in the bob.
* Relate your findings to fig. 8.13.

A small object of massm is suspended from a string of length *L*. The object revolves with constant speed *v* in a horizontal circle of radius *r,* as shown in Figure 8.13. Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.

Let us find an expression for *v*.

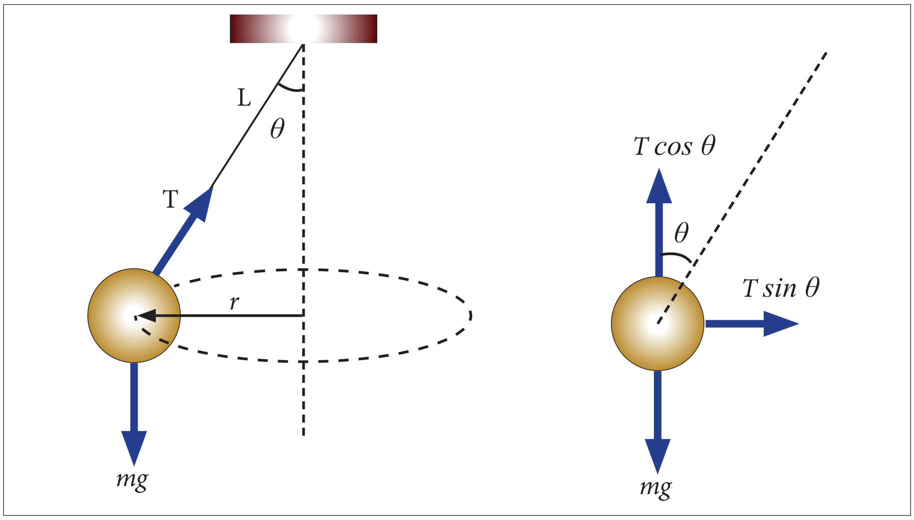


Figure 8.13: The conical pendulum and its free-body diagram

To analyse the problem, begin by letting θ represent the angle between the string and the vertical. In the free-body diagram shown, the force $\vec{T}$ exerted by the string is resolved into a vertical component $T\cos\theta$ and a horizontal component T sinθ acting toward the centre of revolution. Because the object does not accelerate in the vertical direction, $\sum F_y = m\gamma_y = 0$ and the upward vertical component of $\vec{T}$ must balance the downward gravitational force. Therefore, $T\cos\theta = mg$ (1)

Because the force providing the centripetal acceleration in this example is the component T sinθ, we can use the formula of centripetal acceleration to obtain

$$\sum F = T\sin\theta = m\alpha = \frac{mv^2}{r} \qquad (2)$$

Dividing (2) by (1) and using $\frac{sin\theta}{cos\theta} = tan\theta$, we eliminate T and find that

$$tan\ \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\ tan\ \theta}$$

From the geometry in Figure 8.13, we see that $r = L\sin\ \theta$; therefore, $v = \sqrt{Lg\sin\theta\ \ tan\ \theta}$

Note that the speed is independent of the mass of the object.

A steel ball of mass 0.5kg is suspended from a light inelastic string of length 1 m. The ball swings in a horizontal circle of radius 0.5 m. Find

(i) The centripetal force and tension in the string.

(ii) The angular Speed of the ball.

Road banking

Circular motion on JOB

Activity 8

A car negotiating a corner

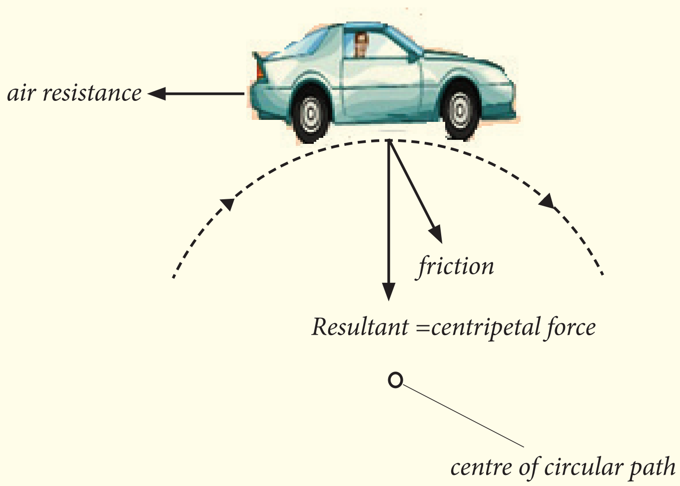


Figure 8.14: Traveling on a circular bend

The successful negotiation of a bend on a flat road therefore depends on the tyres and the road surface being in a condition that enables them to provide a sufficiently high frictional force, otherwise skidding occurs. Safe cornering that does not rely on friction is achieved by "banking" the road.

The problem is to find the angle α at which the bend should be banked so that the centripetal force acting on the car arises entirely from a component of the normal force $\vec{N}$ of the road.

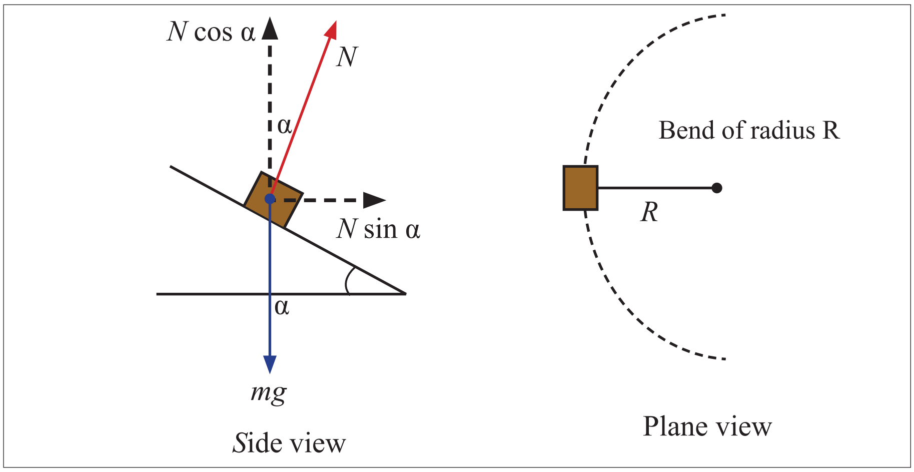


Figure 8.15: Rounding a bend

Treating a car as a particle and resolving $\vec{N}$ vertically and horizontally we have; since $N \sin\alpha$ is the centripetal force $N \sin\alpha = \frac{mv^2}{R}$ wherem and v are the mass and the speed respectively of the car and R is the radius of the bend. Also, the car is assumed to remain in the same horizontal plane and so has no vertical acceleration, therefore $N \cos\alpha = mg$.

Hence by division: $\tan\alpha = \frac{v^2}{gR} \Rightarrow v^2 = Rg \tan\alpha$

$$v = \sqrt{Rg \tan\alpha}$$

The equation shows that for a given radius of bend, the angle of banking is only correct for one speed.

Career centre

Learn more about careers in physics where projectile and circular motion are applied.

Exercises

Projectile motion

1. A projectile is thrown horizontally with a speed of 300m/s from the top of a building 78.4m high.
   a) Compute the range of the projectile.
   b) What is the time taken to reach the ground?

2. A machine gun throws a projectile with a speed of 740m/s Find the range, the maximum height reached by the projectile and the time taken to reach the ground, when it is projected through an angle of 45°.

3. The range of the motion of a projectile is $20\sqrt{3}$ m when it is projected from the ground with a speed of 20m/s What is the maximum height reached? $g = 10\text{m/s}^2$.

4. A projectile thrown through an angle of 30° reaches 50m. Find the initial speed.

5. A football player punts the ball so that it will have a "hang time" (time of flight) of 4.5s and land 45.7m away. If the ball leaves the player's foot 1.52m above the ground, what is its initial velocity (magnitude and direction)?

6. A projectile is fired with an initial speed of 400m/s at an angle of 60° above the horizontal from the top of a cliff 49m high. Determine the:
   a) time to reach the maximum height.
   b) maximum height above the base of the cliff reached by the projectile.
   c) total time it is in the air.
   d) horizontal range of the projectile.

7. A projectile is fired with a speed of 600m/s at an angle of 60°. Find:
   a) the horizontal range.
   b) the maximum height.
   c) the speed and the height after 30s.
   d) the time and the speed when the projectile reaches 10km.
   e) the time to reach the maximum height.

8. From the top A of a cliff 100m high, a projectile is fired at angle 45°. The initial speed is 400m/s and $g = 10\text{m/s}^2$. Determine:
   a) The maximum height.
   b) The horizontal distance below A and the point where it strikes the ground.
   c) The time taken to travel the total distance.
   d) The velocity when it strikes the ground.

9. A rescue plane is flying at a constant elevation of 1200m with a speed of 430km/h toward a point directly over a person struggling in the water. At what angle of sight θ should the pilot release a rescue capsule if it is to strike (very close to) the person in the water?

10. A police officer is chasing a burglar across a roof top; both are running at 4.5m/s Before the burglar reaches the edge of the roof, he has to decide whether or not to try jumping of the next building, which is 6.2m away but 4.8m lower. Can he make it? Assume that he jumps horizontally.

Extension questions

1. A bomber (plane) is at an altitude of 20km and has 400km/s of speed. When the plane is above a certain point, it releases a bomb. How long will it take the bomb and at what distance from the same point to reach the ground?

2. A gun fires a projectile of mass 2kg at initial velocity $V_0 = 200\text{m/s}$ making an angle $\theta_o = 53°$ with the ground.
   a) Find;
      (i) The kinetic energy of the projectile at the point where it leaves the gun.
      (ii) The potential energy at the highest point of the flight; deduce the velocity of the projectile at that point.
      (iii) The range of the projectile; what is the time taken for the flight?
   b) Find the position of the projectile, the magnitude and direction of the velocity when $t = 25$.

3. A ball is thrown horizontally with a speed V_0 of 243.8cm/s. Find the position and velocity after 25.

4. A ball is thrown vertically upward in air and returns in the hand from which it was 3s later. A second ball is thrown at an angle of 30º with the horizontal. At which velocity must the second ball be thrown so that it reaches the same maximum height as the first one thrown vertically?

5. a) A projectile is launched with speed $\vec{v}_0$ at angle α_o above the horizontal. The launch point is at a height h above the ground. Show that if air resistance is ignored, the horizontal distance that the projectile travels before striking the ground is $= \frac{v_0 \cos\alpha_0}{g}\left(v_0 \sin\alpha_0 + \sqrt{v_0^2 \sin^2\alpha_0 + 2gh}\right.$

   b) Determine x if h is taken to be zero.

Circular motion

1. A body describes a circumference of radius 2m and the motion is uniform. It does 2 rotations in 6s. If $\pi^2 = 9.86$, find the centripetal acceleration.

2. A moving body is in uniform circular motion. The radius of the circle is 25m. Assuming that the acceleration equals 9m/s^2, find the angular velocity.

3. How many rotations a wheel of 3.20m diameter does in one minute. Assuming that the linear speed is equal to 16m/s?

4. A ball at the end of a string is swinging in a horizontal circle of radius 1.15m. The ball makes exactly 2.00 revolutions in a second. What is its centripetal acceleration?

5. The wheel of an engine of 4m diameter does 90 rotations per minute. Calculate:

   a) the linear speed.
   b) the angular speed.
   c) the centripetal acceleration.

6. What is the angular velocity of the earth around its axis? What is the linear velocity of a point situated at the equator? The radius of the earth is supposed to be 6400km.

7. The motion of the earth is considered as a rotation about its axis. Find, for a point located in Paris (latitude 45°),
   a) The angular velocity.
   b) Linear velocity.
   c) Centripetal acceleration.
8. Two bodies A and B describe in the same direction, the same circle of radius 10cm with constant angular velocities $W_A = 10$ rad/s and $W_B = 11$ rad/s. The motion starts when they are at the origin.
   a) After how many seconds do they coincide again for the first time?
   b) What is the distance travelled by A?
9. The artificial satellite syncom appears motionless in the sky and its trajectory is circular at the height of 35,700km. What is its speed? The radius of the earth is 6400km.
10. What time is it, after 12:00 the watch hands make an angle of 180° for the first time?
11. How many minutes after 4:00, watch hands coincide for the first time?
12. What time is it, after 3:00, the watch hands make a right angle for the first time?
13. An engine having a speed of 4000 rotations per minute decelerates during 8s till the stop. How many rotations does it make in that time?
14. Let a uniformly decelerated circular motion of deceleration 0.5 rads^2. At $t = 1$ sec, the angular velocity has the magnitude of 1rad/s and the body is at the point $\frac{p}{4}$. Find the equation of the motion.
15. Let $\theta = 5 + 4t - t^2$ be the equation of a circular motion of radius 0.05m.
   a) determine the angular velocity at $t = 0$ and $t = 2$.
   b) determine its angular acceleration.
   c) determine its acceleration and the linear velocity at $t = 0$ and $t = 2$s.
16. A punctual moving object describes a circular trajectory of radius $r = 18$m. The curvilinear displacement is given by $S=3t^2$, where S is in meter and t in seconds. Calculate the acceleration at $t = 2$s.
17. During 5s a wheel doubles its angular velocity and executes 120 rotations. What are the magnitudes of the angular velocities at the beginning and the end of the process?

Extension questions

1. An electric motor is switched off and its angular velocity decreases uniformly from 900 rotations to 400 rotations in 5s.
   a) Find the angular acceleration in rotations/sec^2 and the number of rotations done by the motor in the time interval of 5s.
   b) How long does it take the motor to stop if the angular acceleration remains constant and equal to the one in (a)?

Dynaics of circular motion, centripetal force

1. Compute the centripetal force applied on a wheel of mass 1000kg, assuming that its diametre is 3m and it turns with a speed of 300 rotations per minute.
2. A train of 105kg travels with a speed of 70km/hand reaches a bend of radius 500m. Find the value of the centripetal force.
3. A frigate bird is soaring in a circular path. Its bank angle is estimated to be 25° and it takes 13s for the bird to complete one circle.
4. How fast is the bird flying?
5. What is the radius of the circle?
6. A 1000kg car rounds a curve on a flat road of radius 50m at a speed of 50 km/h. Will the car make the turn if: (a) The pavement is dry and the coefficient of static friction is 0.60, (b) The pavement is icy and the coefficient is 0.20?
   Calculate the speed required for a satellite moving in a circular orbit 200km above the earth's surface.
7. What is the maximum speed with which a 1300kg car can round a turn of radius 95m on a flat road if the coefficient of friction between tires and road is 0.55? Is this result independent of the mass of the car?
8. How large must the coefficient of friction be between the tires and the road if a car is to round a level curve of radius 62m at a speed of 55km/h?

9. If a curve of radius of 60m is properly banked for a car traveling 60km/h, what must be the coefficient of static friction for a car not to skid when traveling at 90km/h?

10. A 1200kg car rounds a curve of radius 65m banked at an angle of 14°. If the car is traveling at 80km/h, will a friction force be required? If so, how much and in which direction?

11. A vehicle of mass 1000kg is moving on a bridge which has the shape shown on the figure below. The radius is of 50m and the speed is 15m/s Find the magnitude of the force exerted by the vehicle on the bridge if the car is on the top of the bridge.

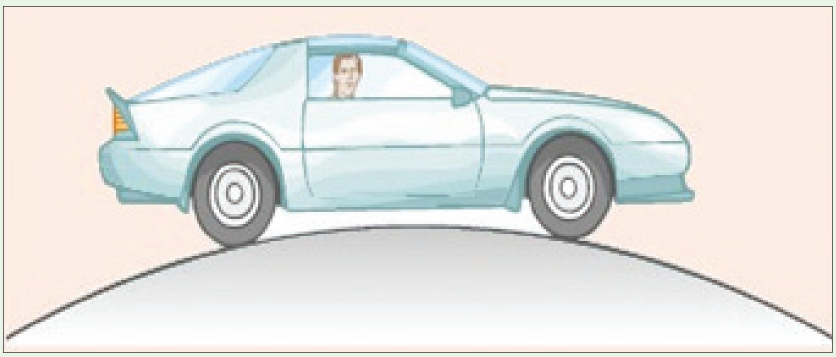


Figure 8.16. Showing a car moving on a banked path

12. A mass m_1 on a frictionless table is attached to a hanging mass m_2 by a cord through a hole in a table (see the figure). Find the speed with which m_1 must move for m_2 to stay at rest.

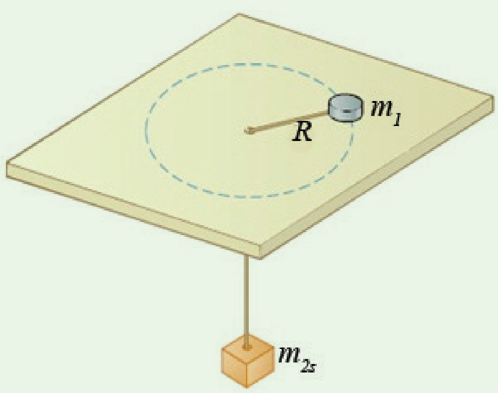


Figure 8.17. Showing two bodies (masses) connected

Extension questions

1. A mass rotates on a vertical circle at the end of a light string of 0.3m of length. Calculate:
   a) The difference in kinetic energy between the upper and the lower point of the circle.
   b) The difference in tension of the string between the upper and the lower point of the circle.
2. Read the passage below and answer the questions that follow:
   "Satellites which orbit 35,788km above the Earth's equator are said to be in geostationary orbit. Most telecommunication satellites are in this type of orbit. Other satellites are in solar orbit only and few hundreds of miles above the earth. These are said to be in low earth orbit. These are mainly military satellites which because of their low orbit can "see" things in great detail",

   What is the difference between an artificial satellite and a natural satellite of the Earth?
   c) Briefly explain three benefits of artificial satellites.
   d) Describe what each of the following words used in the passage mean: (i) Geostationary, (ii) Low earth orbit.
   e) Establish an expression for the velocity of geostationary satellites in terms of altitude h, earth radius R and acceleration due to gravity at the earth's surface g and then find its numerical value.
3. a) An object is moving in a circle with constant speed. What is the direction of the net force acting on this object?
   b) What is the net force required to make an object of 40kg accelerate at a rate of 2m/s?

MECHANICS

Universal Gravitational Field

Unit 9

Universal gravitational field

Key Unit Competence

By the end of the unit, learner should be able to explain gravitational field potential and its application in planet motion.

My goals

By the end of this unit, I will be able to:

* explain universal gravitation field.
* describe the factors affecting force of gravity.
* state and explain Kepler's laws of planetary motion.
* investigate planetory motion using computer simulation.

Link to other subjects

Geography and Astronomy (Landslides, motion of planets and satellites) Chemistry (Electrons orbiting the nucleus).

Introduction

The Universe is composed of different planets one of which is the earth.

All objects on the earth remain on it. They cannot move away unless acted on by external forces. This shows that there is a region around it that provides a force that attracts these earthly objects.

Since the earth is part of the universe it follows that a round the universe there is attracting field.

This is called universal gravitational field.

Universal gravitational field potential

To have potential is to have energy, therefore gravitational field potential is the ability of gravity to attract other objects.

Gravitational field

Questions to think about!

1. What force that unites us as Banyarwanda?
2. How do you feel if you come close to a fellow munyarwanda when you find him/her outside our country?
   Relate the situation to the force around the earth.
3. What makes you feel attracted to your fellow munyarwanda?

A field is a region of space where forces are exerted on objects with certain properties.

The diagram represents the Earth's gravitational field. The lines show the direction of the force that acts on a mass that is within the field.

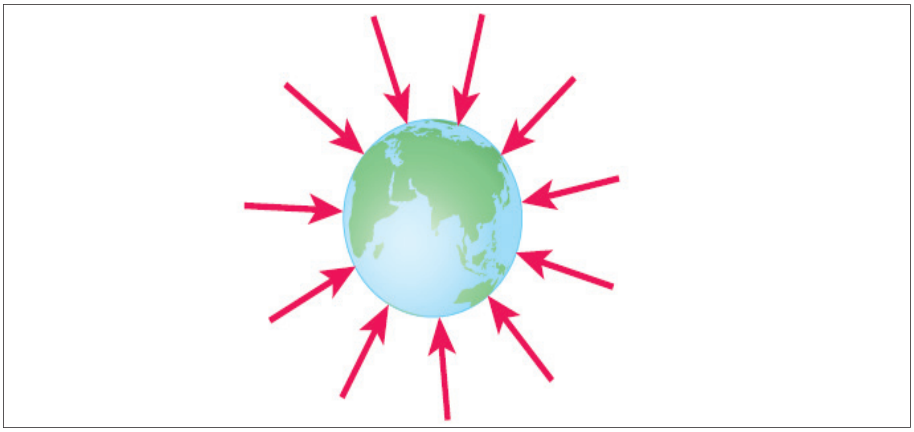


Figure 9.1: Earth's gravitational field

This diagram shows that:

- Gravitational forces are always attractive – the Earth cannot repel any objects.

- The Earth's gravitational pull acts towards the centre of the Earth.
- The Earth's gravitational field is radial; the field lines become less concentrated with increasing distance from the Earth.

The force exerted on an object in a gravitational field depends on its position.

The less concentrated the field lines, the smaller the force. If the *gravitational field strength* at any point is known, then the size of the force can be calculated.

The gravitational field strength g at any point in a gravitational field is the force per unit mass at that point: $g = \frac{F}{m}$

Close to the Earth's surface, g has the value of 9.81Nkg^{-1}, though the value of 10Nkg^{-1} is often used in calculations.

Gravitational field strength is a vector quantity: its direction is towards the object that causes the field.

In studying gravitation, Newton concluded that the gravitational attractive force that exists between any two masses:

- Is proportional to each of the masses.
- Is inversely proportional to the square of their distances apart.

The law states that '**The force of attraction between two masses m_1 and m_2 a distance r apart is directly proportional to the product of masses and inversely to the square of distance r of separation**.'

This force acts along the line joining the two particles. In magnitude, the force is given by: $F = G\frac{m_1 m_2}{r^2}$

where m_1 and m_2 are the masses of the two particles, r is the distance between the centres of mass and $G = 6.67 \times 10^{-11}$Nm^2 kg^{-2} is the universal constant of gravitation.

Dimensions of gravity

$[G] = \frac{[F][r^2]}{[M_1][M_2]}$ $\qquad [G] = \frac{MLT^{-2}L^2}{M.M}$

$[G] = M^{-1}L^3T^{-2}$

The S.I unit of G is Nm^2 kg^{-2}

A point mass is one that has a radial field, like that of the Earth.

A graph that shows inverse square law

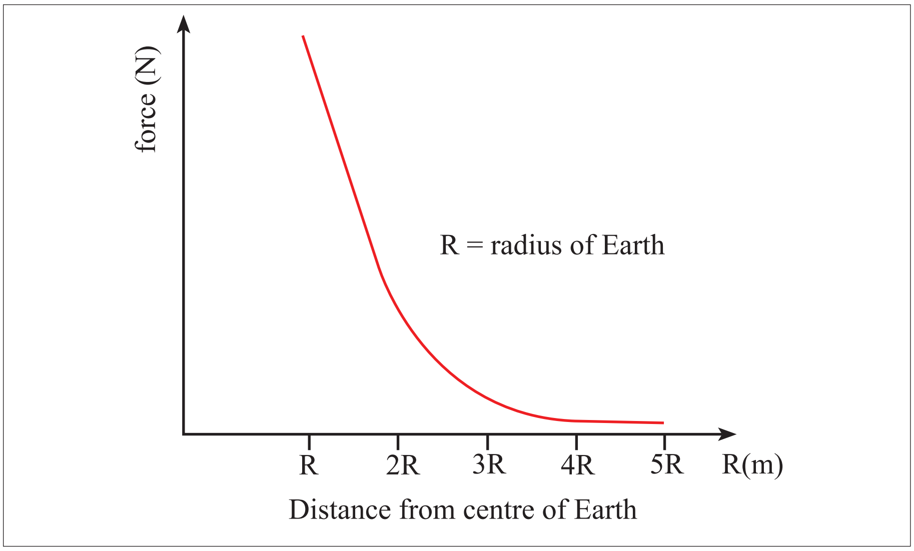


Figure 9.2: Variation of the force of gravity with the distance

Gravitational potential energy

Potential and potential energy

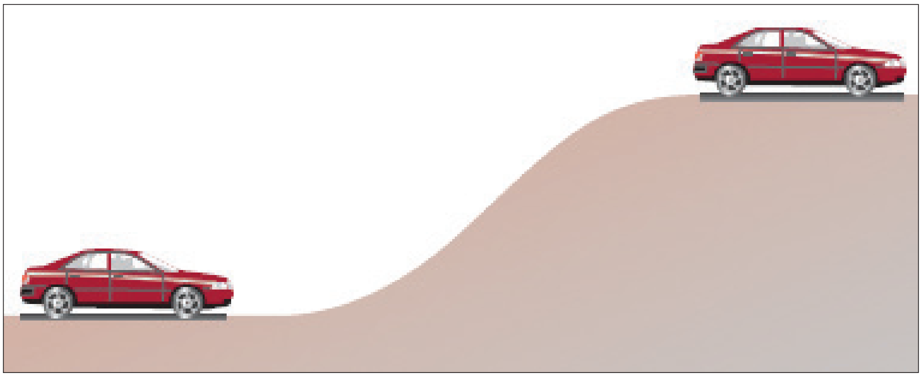


Figure 9.3: The car at the top of the hill has more potential energy than the one at the bottom

Question about fig. 9.3

- The car at the top of the hill has more potential energy than the one at the bottom, but relative to ground level they both have zero. why?
- Note and record in your notebook your analysis.

Using this reference point:

- All objects at infinity have the same amount of potential energy; zero.
- Any object closer than infinity has a negative amount of potential energy, since it would need to acquire energy in order to reach infinity and have zero energy.

The gravitational potential at a point in a gravitational field is the potential energy per unit mass placed at that point, measured relative to infinity.

Calculating potential and potential energy

When an object is within the gravitational field of a planet, it has a negative amount of potential energy measured relative to infinity. The amount of potential energy depends on:

- The mass of the object.
- The mass of the planet.
- The distance between the centres of mass of the object and the planet.

The Centre of mass of a planet is normally taken to be at its centre.

The gravitational potential energy measured relative to infinity of a mass, m, placed within the gravitational field of a spherical mass M can be calculated using: $p.e = \frac{GMm}{r}$.

Gravitational potential, V, is given by the relationship: $V = -\frac{GM}{r}$.

Gravitational potential is measured in **$\mathbf{Jkg^{-1}}$.**

Relation between the universal gravitational constant and force of gravity (*g* and *G*)

A small object of mass m, placed within the gravitational field of the Earth, mass M, experiences a force, F, given by: $F = G\frac{Mm}{r^2}$

Where r is the separation of the centres of mass of the object and the Earth.

It follows from the definition **of gravitational field strength as the force per unit mass that the field strength at that point, g, is related to the mass of the Earth by the expression**: $g = \frac{F}{m} = \frac{GM}{r^2}$

The same symbol, $\boldsymbol{g}$, is used to represent:

- Gravitational field strength.
- Free-fall acceleration.

Kepler's Laws

Activity 1: Field work

As a class, let us visit one of the roundabouts (where three roads meet). Try to see/check how cars, motorcycles, bicycles move around it.

Qn i) Does the features on a roundabout move?

Assuming a roundabout to be a sun and vehicles to be planets, what can you say?

1. Discuss your findings in groups of 5 members.
2. Present your findings to the whole class.
3. Note down the observation.
4. Present your work to the teacher for marking.

Activity 2

Check on the watch (that one with a clock hand).

Look at where the second hand is fixed.

While the hand is rotating about a fixed point,describe the shape the second hand describes!

Figure 9.4: Rotation about a fixed point

We can relate the movement of the minute hand as the movement of planets about the sun.

Kepler's first law: The path of each planet about the sun is an ellipse with the sun at one focus(or planets describe ellipse about the sun as one focus).

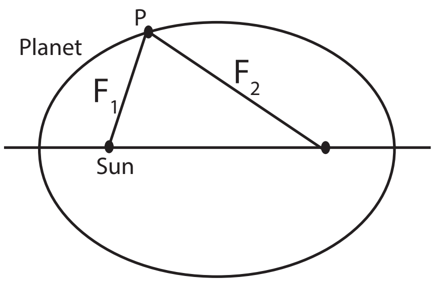


Figure 9.5: The trajectory of P is an ellipse

Kepler's second law: The line joining the sun to the moving planet sweeps out equal areas in equal times.

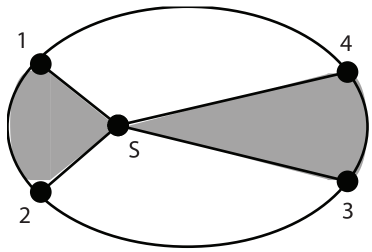


Figure 9.6: The area S12 is equal to the area S34

If planet P takes the same time to travel from 1 to 2 as from 3 to 4 then the shaded areas are equal.

Kepler's third law: The squares of the times of revolution T of the planets about the sun are proportional to the cubes of their mean distances r from it: $= \frac{T^2}{r^3} = \text{constant}$

The value of this constant is $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$, M is the mass of the sun in this case.

Proof of Kepler's third law

Activity 3

- Using Newton's law of gravitation (Formula) and the formula that keeps the planet in circular paths (Formula for centripetal force), Derive expression for Kepler's third law of planetary motion
- Put your derivation in your notebook after discussing it with your friends.

Planetary data applied to Kepler's third law

Planet	Mean radius of the planet[m]	Mass[kg]	Period of rotation[s]	Mean radius of orbit r[metres]	Period of revolution T[seconds]	$\frac{r^3}{T^2}$
Sun	$6.96\text{x}10^8$	$1.98\text{x}10^{30}$	2.3x106	-	-	-
Mercury	$2.34\text{x}10^6$	$3.28\text{x}10^{23}$	$5.03\text{x}10^6$	5.79×10^{10}	7.60×10^6	3.36×10^{18}

Venus	$2.22x10^6$	$4.83x10^{24}$	?	1.08×10^{11}	1.94×107	3.35×10^{18}
Earth	$6.37x\,10^6$	$5.98x10^{24}$	$8.62x10^4$	1.49×10^{11}	3.16×10^7	3.31×10^{18}
Mars	$3.32x10^6$	6.40×10^{23}	$8.86x10^4$	2.28×10^{11}	5.94×10^7	3.36×10^{18}
Jupiter	$6.98x10^7$	$1.90x10^{27}$	$3.54x10^4$	7.78×10^{11}	3.74×10^8	3.36×10^{18}
Saturn	$5.82x10^7$	$5.6x10^{26}$	$3.61x10^4$	1.43×10^{12}	9.30×10^8	3.37×10^{18}
Uranus	$2.37x10^7$	$8.67x10^{25}$	$3.85x10^4$	2.87×10^{12}	2.66×10^9	3.34×10^{18}
Neptune	$2.24x10^7$	$1.05x10^{26}$	$5.69x10^4$	4.50×10^{12}	5.20×10^9	3.37×10^{18}
Pluto	$3.00x10^6$	$5.37x10^{24}$	?	5.90×10^{12}	7.82×10^9	3.36×10^{18}
Moon	$1.74x10^6$	$7.34x10^{22}$	$2.36x10^8$	$3.84\ x10^8$	$2.36\ x10^6$	To find

EXAMPLES

1. Calculate the force of gravity between two bowling balls each having a mass of 8.0kg, when they are 0.50m apart.

SOLUTION

$$\text{Force} = \frac{Gm^1m^2}{r^2}$$

Wherem is mass

G gravitational constant = $6.67 \text{ x } 10^{-11}\ \text{Nm}^{-2}$

R is distance between masses

$$F = \frac{6.67 \times 10^{-11} \times 8 \times 8}{0.5^2}\ \text{N}$$

$F = 1.70752 \times 10^{-8}\text{N}$

$F = 1.7 \times 10^{-8}\,\text{N}$

2. At the surface of a certain planet, the gravitational acceleration g has a magnitude of 2.0m/s^2. A 4.0kg brass ball is transported to this planet. Give:
   a) The mass of the brass ball on the earth and on the planet; and
   b) The weight of the brass ball on the earth and on the planet.

SOLUTION

a) The mass of an object cannot change

Therefore mass remains as 4.0kg

b) Weight = *mg* (on the earth)

m = 4.0kg

g = $10.0ms^{-2}$

Then weight = 4.0 x 10.0

Weight = 40.0N

On the planet

Weight = mg wherem = 4.0kg, and $g = 2.0ms^{-2}$

Weight = 4.0 x 2.0 = 8N

Exercises

1. Calculate the effective value of *g*, the acceleration of gravity, (a) 3200m, (b) 3200km, above the earth's surface.

2. Determine the net force on the moon ($m_m = 7.36\times10^{22}$ kg) due to the gravitational attraction and both the earth (m_e=5.98×10^{24} kg) and the sun (m_s=1.99×10^{30} kg) assuming they are at right angles to each other.

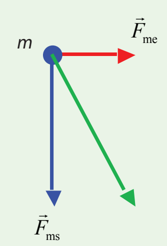


Figure 9.7: Relation to question 2

3. What is the effective value of *g* at a height of 1000km above the earth's surface? That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

4. The universal attraction is given by: $F = G\dfrac{m_1 m_2}{d^2}$ where G = 67×10^{-12} USI

   a) Find the acceleration g_o at the earth's surface in function of *R*, *M* and *G*, where *R* is the radius of the earth and *M* its mass. If *R* = 6400km, g_o = 9.81m/s, calculate *M*.

b) Find in function of g_o, R and h, the acceleration due to the gravity g at a certain height h.

c) If the satellite is at the height h,

(i) Find the speed in function of g_o, R and h.

(ii) Find its value if h = 36000km.

5. Venus is at average distance of 1.08×10^8 km from the sun. Estimate the length of the Venusian year using the fact that the earth is 1.49×10^8 km.

6. The planet Mars of massm describes around the sun of mass M, an ellipse of mean radius of orbit $a = 230\times10^6$ km in 1.8 years. The satellite Deimos of mass m' describes around the planet mars an ellipse of mean radius $a' = 28\times10^6$ km in 30h. Find the mass of the planet mars, given that $M = 2\times10^{30}$ km and 1year is 365days.

Extension Questions

1. We actually know fifteen satellites revolving around the planet Uranus. Let us denote the period of revolution of satellite by T and the mean distance to the centre of the planet by r. The five bigger than others have the following characteristics:

Satellite	Oberon	Titania	Umbriel	Ariel	Miranda
T (J)	13.46	8.706	4.144	2.520	1.414
r (10^3 km)	582.6	435.8	266.0	191.2	129.8

a) (i) For each satellite, calculate T_2 and r_3, (ii) Assume T_2 = y and r_3 = x. Trace the graph of y = f(x). What conclusion related to the nature of the graph can you get?

b) (i) Calculate the slope of the plotted segment. (ii) Deduce the mass of Uranus.

MOTION IN FIELDS

Electric Field and Electric Potential

Unit 10

Effects of electric and potential fields

Key Unit Competence

By the end of the unit, the learner should be able to analysis electric and potential fields.

My goals

By the end of this unit, I will be able to:

* define electric field and electric potential.
* explain the relationship between electric potential and electric field intensity.
* describe functioning of lightening arrestors.
* identify the dangers of lightening and how to avoid them.

Introduction

Have you ever heard sound due to lightening? If yes, what do you think was the cause?

If not, ask your friend in your class, at home, or neighbour about lightening.

Scientifically, lightening and thunder are effects of electric charges created in space (will be discussed later).

Attraction and repulsion of charges

Activity 1

In this section, you will observe the characteristics of the two types of charges, and verify experimentally that opposite charges attract and like charges repel.

Equipment

* Two lucite rods
* One rough plastic rod
* Stand with stirrup holder
* Silk cloth
* Cat's fur

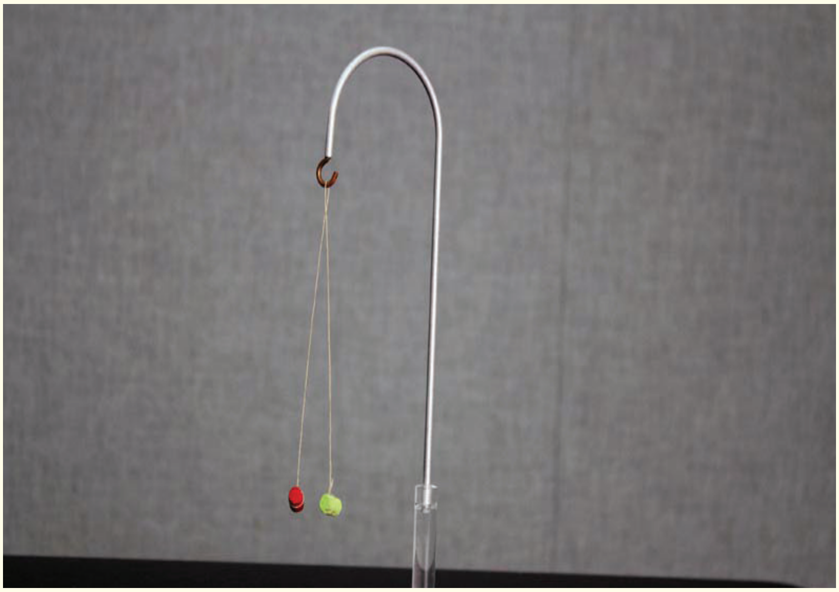


Figure 10.1: Attraction and repulsion

Procedure

1. Charge one lucite rod by rubbing it vigorously with silk. Place the rod into the stirrup holder as shown in Figure.
2. Rub the second lucite rod with silk, and bring it close to the first rod

3. . What happens? Record the observations in your notes.
4. Rub the rough plastic rod with cat's fur, and bring this rod near the lucite rod in the stirrup. Record your observations.
5. What do you conclude?
6. Note down observation in your notebook.

For reference purposes, according to the convention originally chosen by Benjamin Franklin, the lucite rods rubbed with silk become positively charged, and the rough plastic rods rubbed with cat's fur become negatively charged. Hard rubber rods, which are also commonly used, become negatively charged.

Coulomb's law

Activity 2

Materials

* Coulomb's Law apparatus
* Electrophorus (The electrophorus is a simple electrostatic induction device. It's an inexhaustible source of charge").
* Silk cloth.
* A computer for the graph and quick calculation.

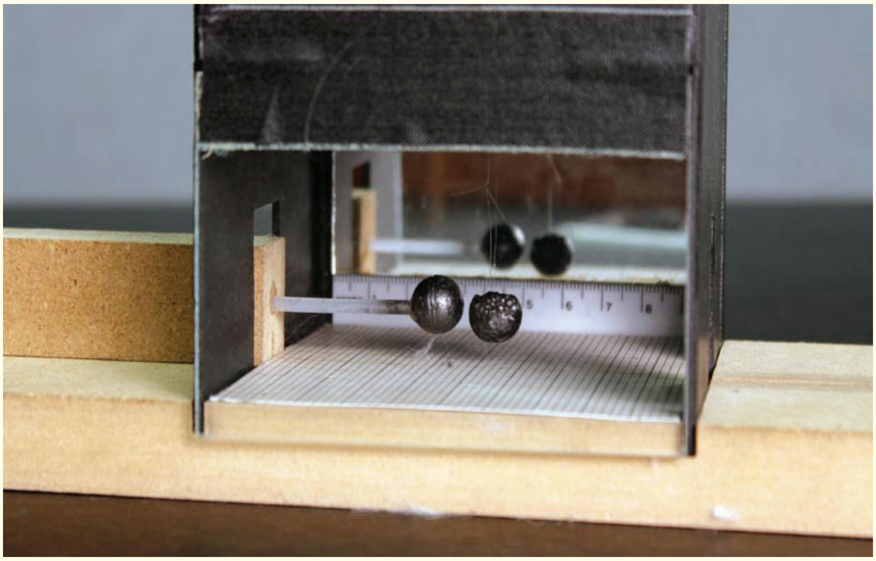


Figure 10.2: Coulomb's apparatus

Procedure

1. Take a moment to check to position of the hanging ball in your Coulomb apparatus. Look in through the side plastic window. The hanging ball should be at the same height as the sliding ball (i.e. the top of the mirrored scale should pass behind the centre of the hanging pith ball, as in Figure 10.3 below). Lift off the top cover and look down on the ball. The hanging ball should be centred on a line with the sliding balls. If necessary, adjust carefully the fine threads that hold the hanging ball to position it properly.
2. Charge the metal plate of the electrophorus in the usual way by rubbing the plastic base with silk, placing the metal plate on the base, and touching it with your finger.
3. Lift off the metal plate by its insulating handle, and touch it carefully to the ball on the left sliding block.
4. Slide the block into the Coulomb apparatus without touching the sides of the box with the ball. Slide the block in until it is close to the hanging ball. The hanging ball will be attracted by polarization, as in Section III of this lab. After it touches the sliding ball, the hanging ball will pick up half the charge and be repelled away. Repeat the procedure if necessary, pushing the sliding ball up until it touches the hanging ball.
5. Recharge the sliding ball so it produces the maximum force, and experiment with pushing it towards the hanging ball. The hanging ball should be repelled strongly.
6. You are going to measure the displacement of the hanging ball. You do not need to measure the position of its centre, but will record the position of its inside edge. Remove the sliding ball and record the equilibrium position of its inside edge that faces the sliding ball, which you will subtract from all the other measurements to determine the displacement d.
7. Put the sliding ball in, and make trial measurements of the inside edge of the sliding ball and the inside edge of the hanging ball. The difference between these two measurements, plus the diameter of one of the balls, is the distance r between their centres. Practice taking measurements and compare your readings with those of your lab partner until you are sure you can do them accurately. Try to estimate measurements to 0.2 mm.

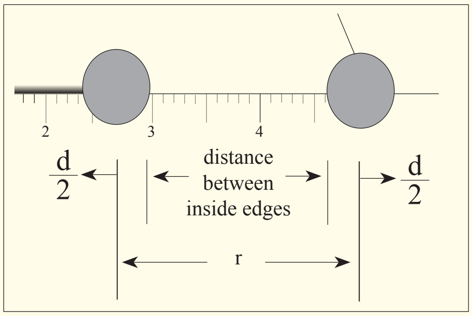


Figure 10.3: The positions of the inside edges are marked. The difference between these positions plus the diameter of one ball is the distance between the centres of the balls

8. Take measurements, and record the diameter of the balls (by sighting on the scale).

9. Remove the sliding ball, and recheck the equilibrium position of the inside edge of the hanging ball.

10. You can record and graph data in Excel or by hand (although if you work by hand, you will lose the opportunity for 2 mills of additional credit below). Recharge the balls as in steps 1 – 4, and record a series of measurements of the inside edges of the balls. Move the sliding ball in steps of 0.5 cm for each new measurement.

11. Compute columns of displacements d (position of the hanging ball minus the equilibrium position) and the separations r (difference between the two recorded measurements plus the diameter of one ball).

12. Plot (by hand or with Excel) d versus $\frac{1}{r^2}$. Is Coulomb's Law verified?

13. For an additional credit of 2 mills, use Excel to fit a power-law curve to the data. What is the exponent of the r-dependence of the force? (Theoretically, it should be −2.000, but what does your curve fit produce)?

14. For your records, you may print out your Excel file with a table and graph of your numerical observations and any other electronic files you have generated.

Interpretation

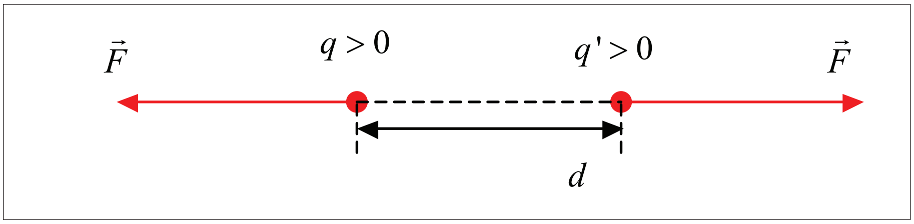


Figure 10.4: Coulomb's law, the diagram shows the force between two forces

Knowledge of the forces that exist between charged particles is necessary for an understanding of the structure of the atom and of matter. The magnitude of the forces between point charges was first investigated quantitatively in 1785 by Coulomb, a French scientist. The law he discovered is stated as follows:

"The force between two point charges is directly proportional to the product of charges divided by the square of their distance apart".

Mathematically we have: $F = k\dfrac{qq'}{d^2}$ Where, k is a positive constant.

Note that a positive F tends to increase d.

In the S.I, the unit of charge is the ***coulombs*** [C].

In S.I units, the coulomb constant k has the following value for charges in vacuum: $k \approx 9\times10^{9N}.m^2/C^2$

Often k is replaced by $\dfrac{1}{4\pi\varepsilon_0}$, where $\varepsilon_0 = 8.85\times10^{-12}\ C^2/N^2.m^2$ is called the **permittivity of a vacuum.**

In term of it, Coulomb's law becomes, for vacuum: $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qq'}{d^2}$

When the surrounding medium is not a vacuum, forces caused by induced charges in material reduce the force between point charges. If the material has a dielectric constant k, then ε_0 in Coulomb's law must be replaced by $K\varepsilon = \varepsilon$, where ε is called the permittivity of the material. Then: $F = \dfrac{1}{4\pi\varepsilon}\dfrac{qq'}{d^2}$ with $K\varepsilon_0 = \varepsilon$

For vacuum, K = 1; for air K = 1.0006, and is thus often taken to be 1.

Sometimes it's better to write: $F = k\dfrac{qq'}{d^2}$ where $k = \dfrac{1}{4\pi\varepsilon}$ and $\varepsilon = \varepsilon_r\varepsilon_0$

ε_0 : Permittivity of free space = 8.85 × 10^{-12} USI

ε_r : Relative permittivity of a given medium

The relative permittivity, ε_r, of a medium is the ratio of its permittivity ε to that of a vacuum ε_0. So: $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Although ε and ε_0 have dimensions, ε_r is a number and has no dimensions.

We can write: $F = \frac{1}{4\pi\varepsilon}\frac{qq'}{d^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{qq'}{d^2}$

In the vacuum: $\varepsilon_r = 1$, then $F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{d^2}$

$F = \frac{1}{4\pi\varepsilon_0} = 9.10^9 \Rightarrow F = 9.10^9\frac{qq'}{d^2}$

In a medium of relative permittivity ε_r, this formula is written: $F = \frac{9.10^9}{\varepsilon_r}\frac{qq'}{d^2}$

Some values of ε_r

- Air: $\varepsilon_r = 1.0006$
- Vacuum: $\varepsilon_r = 1.00$
- Water (pure): $\varepsilon_r = 80$
- Alcohol: $\varepsilon_r = 26$
- Glass: ε_r = from 3 to 9
- Polythene: $\varepsilon_r = 2.3$
- Perspex: $\varepsilon_r = 2.6$
- Paper (waxed): $\varepsilon_r = 2.7$
- Mica: $\varepsilon_r = 7$
- Barium titanate: $\varepsilon_r = 1200$

Quick check

1. If two equal charges, each of 1C, are separated in air by a distance of 1 km, what would be the force between them?
2. Determine the force between two free electrons spaced 1AO apart.
3. Two equally charged pith balls are 3cm apart in air and repel each other with a force of 4×10^{-5}N. Find the charge of each ball.
4. How many electrons are contained in – 1C of charge? What is the total mass of these electrons?

Exercises

1. Two point charges q_1 and q_2 are 3m apart, and their combined charge is 20μC.
   a) If one repels the other with a force of 0.075N, what are the two charges?
   b) If the one attracts the other with a force of 0.525N, what are the magnitudes of charges?
2. Two point charges of q and q^1 coulombs separated by a distant of 5m repel with a force of 0.072N. After having been put in contact, one replaces them at the same distance. Then they repel with a force of 0.081N. Calculate the charges before contact.
3. Two balls have identical masses of 0.1g each. When suspended to 10cm - long strings, they make an angle of 15° with the vertical. If the charges on each are the same, how large is each charge?
4. A test charge q = +2μC is placed halfway between a charge q = +6μC and a charge q = +4μC which are 10cm apart. Find the force on the charge test and its direction.
5. Three point charges are placed at the following points on the x axis: +2μC at x = 0, −3μC at x = 40cm, −5μC at x = 120cm. Find the force on the −3 μC charge.
6. Charges +2, +3 and −8μC are placed at the vertices of an equilateral triangle of side 10cm. Calculate the magnitude of the force acting on the −8μC charge due to the other charges.

Electric field

Notions and definitions

Questions to think about

a) You have learned about Coulomb's law and you have seen that when an electric charge is brought near to another, there is an attractive or a repulsive force. Does that force acts when charges are in contact or it acts even at a certain distance?
b) If so, what can be the reason?
c) Does that force increase or decrease when the distance between charges increases?

After responding to those questions, you'll see that around an electric charge is a region so that when another charge is placed in it, it undergoes an electric force. That region is called electric field created by the first charge.

An electric field can be defined as a region where an electric force is obtained. It's a region where an electric charge experiences a force.

If a very small charge, positive point charge q is placed at any point in an electric field and it experiences a force F, then the field strength E (also called the E–field) at that point is defined by the equation: $E=\frac{F}{q}$.

In words, the magnitude of E is the force per unit charge and its direction is that of $\vec{F}$ (that is to say of the force which acts on a positive charge). Electric field is therefore a vector and we can write: $\vec{E}=\frac{\vec{F}}{q}$

If F is in newtons [N] and q is in coulombs [C], then the unit of E is [$\mathbf{NC^{-1}}$]. We shall see later that a more practical unit of E is volt–meter $^{-1}$ [$\mathbf{Vm^{-1}}$].

E due to a point charge

The magnitude of $\vec{E}$ due to an isolated positive point charge $+q$ at the point P distance d away, in a medium of permittivity, ε, can be calculated by imagining a very small charge $+q$' to be placed at P.

Figure 10.5: The direction of the electric force is the same as the one of the electric field

By the Coulomb's law, the force F on q' is:

$$F=\frac{1}{4\pi\varepsilon}\frac{qq'}{d^2}$$

But E is the force per unit charge, that is; $E=\frac{F}{q'}$. So, $E=\frac{1}{4\pi\varepsilon}\frac{q}{d^2}$

$\vec{E}$ is directed away from $+q$, as shown. If a point charge $-q$ replaced $+q$, $\vec{E}$ would be directed toward $-q$ since unlike charges attract.

The following diagrams show it:

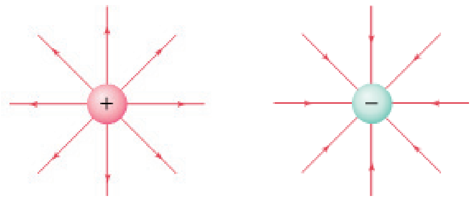


Figure 10.6: Directions of electric field of positive and negative point charge

The above expression shows that E decreases with distance from the point charge according to an inverse square law. The field due to an isolated point charge is therefore non – uniform but it has same value at equal distances from the charge and so has spherical symmetry.

Examples

1. A charge, +6μC, experiences a force of 2mN in the +x direction at a certain point in space.
   a) What was the electric field there before the charge was placed there?
   b) Describe the force a −2μC charge would experience if it was used in place of +6μC
2. Find the electric field at a distance of 0.1m from a charge of 2ηC.
3. Calculate the electric field created by a point charge of 1μC at a point situated at 2.5m in air and in water. The relative permittivity of water is 80.

Field lines (lines of force)

Activity 4: Lab zone

Existence of field lines

This shows the shape of electric fields, in much the same way that magnetic fields are demonstrated with iron filings.

Materials

* Power supply, EHT, 0-5kV.
* Electric fields apparatus.
* Semolina.
* Castor oil.

Procedure

a) Fill the electrode unit with a layer of castor oil to a depth of about 0.5cm. Sprinkle a thin layer of semolina over the surface. (A thin piece of glass tubing drawn out to give a fine pointed stirrer is helpful so that the semolina is evenly distributed.) It is better to start with too little semolina than to start with too much. You can always increase the quantity later.

b) Place the electrodes in the castor oil. Connect the positive and negative terminals of the EHT power supply to the electrodes. Adjust the supply to give 3,000 to 4,000 volts. When the voltage is switched on, the field lines will be clearly visible.

c) Try electrodes of different shapes. For example, one can be a 'point' electrode whilst the other is a plate, or two point electrodes can be used. A wire circular electrode with a point electrode at the centre will show a radial field. The field with two plates quite close together should also be shown.

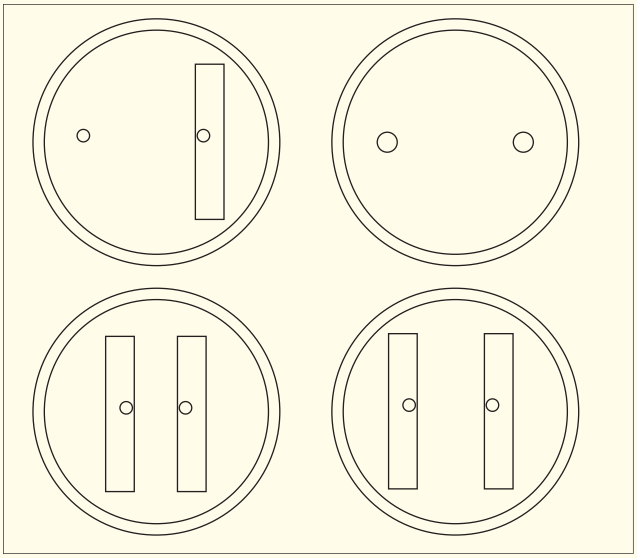


Figure 10.7: Electrodes of different shapes

A *line of force* or *field lines* is defined as a line such that the tangent to it at a point is in the direction of force on a small positive charge placed the point.

Arrows on the lines of force show the direction of the force on a positive charge; the force on a negative charge.

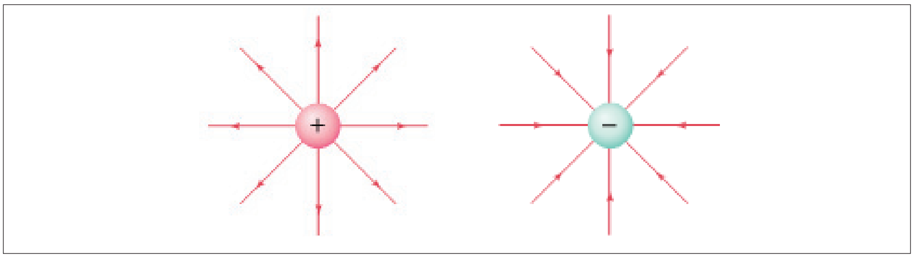


Figure 10.8: Field lines of isolated charges

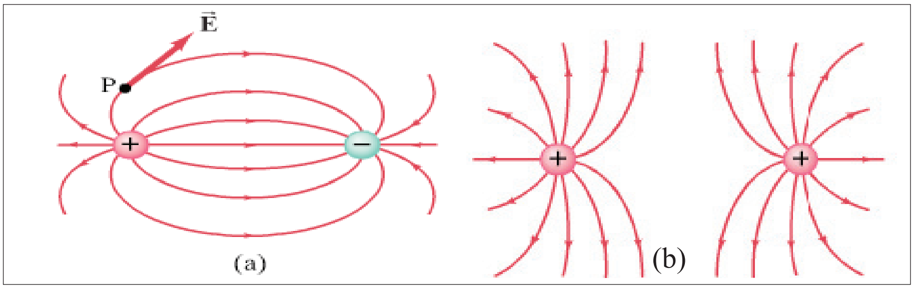


Figure 10.9: Field lines of unlike charges and like charges

Uniform electric field

A uniform electric field is one in which $\vec{E}$ has the same magnitude and direction at all points, there is a plane symmetry and the field lines are parallel and evenly spaced.

This is the case for example of electric field between two parallel – plate carrying charges which have opposite signs.

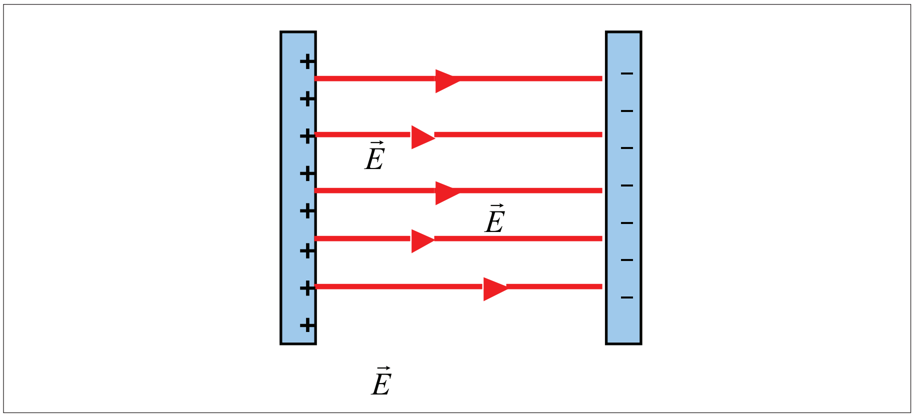


Figure 10.10: Field lines in a uniform field

Electric field due to a distribution of electric charges

Activity 5

Electric field due to a distribution of charges

Materials

* A sheet of paper
* A pen
* A ruler

Procedure

1. Represent a distribution of charges where you have charges of different signs.
2. Represent a point A where you want to find the total electric field.
3. At the point, A represents directions of electric fields vectors produced by each charge.
4. Do the sum of electric fields. Remember that an electric field is a vector. When they make a certain angle between them, use the method of parallelogram. When they have the same direction or opposite directions, use the appropriate method.
5. Establish a mathematical relation of the total electric field due to the distribution of charges.

Field strength and charge density

So far as external effects are concerned, an isolated spherical conductor having a charge q uniformly distributed over its surface behaves like a point charge q at its centre. If r is the radius of the sphere, the field strength E at its surface is therefore given by:

$$E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$$

The charge per unit area of the surface of the conductor is called the charge density σ (sigma) and since a sphere has the surface area $4\pi r^2$, we have; $\sigma = \frac{q}{4\pi r^2}$.

Therefore, q = $4\pi r^2\sigma$ and so $E = \frac{\sigma}{\varepsilon}$.

This expression has been derived by considering a sphere but it gives E at surface of any charges conductor. It's called *Gauss' theorem*.

Quick check

1. Two point charges of 1μC and 9μC respectively are situated at two points A and B 8cm apart. Find the point of the straight line AB where the electrostatic field is zero.

2. Two charges of +1μC and -1μC are placed at the corners of the base of an equilateral triangle. The length of a side of a triangle is 0.7m. Find the electric field intensity at the apex of the triangle.

Exercises

1. A uniform electrostatic field exists between two parallel plates having equal charges of opposite signs. An electron initially at the rest escapes from the surface negatively charged and strikes the surface of the other plate, situated at 2cm in $1.5\text{x}10^{-8}$s.
   a) Calculate the electric field,
   b) Calculate the speed of the electron at the time of the impact with the second plate.
2. An electron is situated in a uniform electric field of intensity or field-strength 1,200,000Vm^{-1}. Find the force on it, its acceleration, and the time it takes to travel 20mm from rest (electron mass,m = 9.1×10^{-31} kg).

3. A point charge −30μC is placed at the origin of coordinates. Find the electric field at the point $x = 5m$.

4. A 5.0μC point charge is placed at the point x = 20cm, y = 30cm, Find the magnitude of E due to it
   a) At the origin.
   b) At x =1.0m, y = 1.0m.
5. The ball of an electrostatic pendulum of mass 2.5g has a charge of 0.5μC.
   a) What must be the intensity of a horizontal electrostatic field so that the wire makes an angle of 30° with the vertical?
   b) What angle makes the wire with the vertical if the electrostatic field has an intensity of 10^4 NC^{-1}?

Potential difference

Work of electric force

Activity 6

Find the expression of the work done by an electric force

a) When can we say that we have a uniform electric field?

b) Draw a diagram showing two plates of opposite signs (the left plate is positive and the right one is negative) between which the electric field is uniform.

c) Show the direction of field lines in the electric field.

d) Between the two plates, put a positive charge at a point A which has to travel toward a point B in the field.

e) Represent the direction of the vector force on the line joining A and B.

f) Write down the expression of the force undergone by the charge.

g) What is the expression of the work done if the charge has to move from A to B (in the final formula)?

Particles that are free to move, if positively charged, normally tend towards regions of lower voltage (net negative charge), while if negatively charged they tend to shift towards regions of higher voltage (net positive charge).

However, any movement of a positive charge into a region of higher voltage requires external work to be done against the field of the electric force, work equal to that electric field would do in moving that positive charge the same distance in the opposite direction. Similarly, it requires positive external work to transfer a negatively charged particle from a region of higher voltage to a region of lower voltage.

The electric force is a conservative force: work done by a static electric field is independent of the path taken by the charge. There is no change in the voltage (electric potential) around any closed path; when returning to the starting point in a closed path, the net of the external work done is zero.

Potential in a field

Activity 7

Understanding the potential in a field

1. What kind of energy has a body when it's held above the earth? If the body has to move under the force of gravity, does it move from a point of great height to one of less or it's the inverse?

2. Do you agree or not that points in the earth's gravitational field have potential values depending on their heights?
3. According to you, can this theory be similar to the one established for electric field? Explain.
4. For charges, instead of saying gravitational potential for gravitational field, can we say electric potential for the case of electric field? Explain.
5. Can points around the charge be said to have electric potential?
6. How can we define the electric potential at a point?

Potential generally refers to a currently unrealized ability. The term is used in a wide variety of fields, from physics to the social sciences to indicate things that are in a state where they are able to change in ways ranging from the simple release of energy by objects to the realization of abilities in people.

Although the concept of electric potential is useful in understanding electrical phenomena, only differences in potential energy are measurable. If an electric field is defined as the force per unit charge, then by analogy an electric potential can be thought of as the potential energy per unit charge. Therefore, the work done in moving a unit charge from one point to another (e.g., within an electric circuit) is equal to the difference in potential energies at each point.

Potential difference, work, energy of charges

Activity 8

Potential energy, work, energy of charges

a) Consider two points A and B in an electrostatic field of strength E, and suppose that the force on a positive charge q has a component $\vec{F}$ in the direction AB. Then if we move a positively charged body from B to A, we do work against this component of the field $\vec{E}$. The potential at A and at B are not equal. How can we define the potential difference between A and B?

b) From the definition in (a), if V_A is the electric potential at the point A and V_B the electric potential at the point B. Knowing that if move a positive charge from A to B, the force $\vec{F}$ produces a work W_{AB}. With which formula can we calculate the potential difference between A and B?

c) The unit of the potential difference has a special name called volt [V]. Can you find its unit in S.I units knowing that that 1V is equal to 1 unit of what you have to find?

d) Considering potential difference theory, the energy is expressed in another unit called electron-volt [V]. How can you define an electron-volt? What is the relation between an [eV] and a joule [J]?

e) From your knowledge, what is the instrument used to measure the potential difference in the circuit and how is it connected?

Electric potential is a location-dependent quantity that expresses the amount of potential energy per unit of charge at a specified location. When a Coulomb of charge (or any given amount of charge) possesses a relatively large quantity of potential energy at a given location, then that location is said to be a location of high electric potential. And similarly, if a Coulomb of charge (or any given amount of charge) possesses a relatively small quantity of potential energy at a given location, then that location is said to be a location of low electric potential. As we begin to apply our concepts of potential energy and electric potential to circuits, we will begin to refer to the difference in electric potential between two points.

Relation between *E* and *V*

Activity 9

Relation between *E* and *V*

1. What is the relation to find the work done by an electric force to move a charge from A to B, knowing that the distance between A and B is *d*?
2. What is the relation of the work using the potential difference?
3. Equalize the two relations and deduce the value of *E*. The relation found is the one between *E* and *V*.
4. From the expression found, deduce the new unit of the electric field *E*.
5. Write down the relation between E and V found, express in equation of *V*, write the electric field produced by a charge at a point deduce the electric potential created by a charge at a point situated at a distance *d* from it.

Quick check

1. Determine the work of an electric force which moves a point charge of q = 4μC from A to B if the p.d VA −VB =10V. Find the electric potential in vacuum at 0.2m from a charge of 2μC.
2. Between two parallel plates 1cm apart is a p.d of 200V. What energy is given to a charge of 1μC to move it from one plate to another? Calculate the value of the electric field between them.
3. Between two parallel plates 10cm apart is a electric field of 300V/m. Calculate: (a) The voltage between them. (b)The work of the electric force applied to an electron to move from one plate to another.

Exercises

1. Two spheres A and B charged negatively of radii 3cm and 9cm have an electric potential 300 000 *V*. Determine the distance between them so that the spheres repel with a force *F* = 0.3N.
2. Three equal charges of+6ηC are located at the corners of an equilateral triangle whose sides are 12cm length. Find the potential at the centre of the base of the triangle.
3. Suppose metal parallel plates are spaced 0.50cm apart and are connected to a battery. Find the electric field between them and the surface charge density on the plates.
4. The charge on an electron is 1.6×10^{-19} C in magnitude. An oil drop has a weight of $3.2\times10^{13\text{N}}$. With an electric field of 5×10^{5} V/m between the plates of Millikan's oil drop apparatus this drop is observed to be essentially balanced. What is the charge of the drop in electronic charge units?
5. In the Millikan experiment, an oil drop carries four electronic charges and has a mass of 1.8×10^{-12} g. It is held almost at rest between two horizontal charged plates 1.8cm apart. What voltage must there be between the two charges plates?

6. Between two vertical parallel plates A and B exists a p.d V. The distance between the plates is 10cm. A small electrified ball of mass 0.3 g carrying a positive charge of 0.3μC is suspended to an insulating wire, of negligible mass that, the balance being realised forms an angle $\alpha=15°$ with the vertical. If $g=9.8 m/s^2$. (a) Calculate V. (b) Trace some field lines between them, indicate their direction. (c)What work would be necessary to give to move the ball P and to bring it in the position P on the vertical of O; (length of the wire OP = l = 20cm).

Motion of electric charges in an electric field

Activity 10

Figure 10.11: The inside of a TV set

a) Observe the picture and say it represents the inside of which apparatus.
b) You see a tube called cathode ray tube (CRT) Search on internet and give its main parts.
c) Doing the research, give a small idea about its principle of functioning.

In the process of functioning you'll find that charges (electrons) are produced, are sent in motion in an electric field and reach a fluorescent screen. Here we are interested in the motion of the electric field.

On the figure below, charges, here we consider electrons, with a horizontal vector velocity of magnitude v_0 entering between two horizontal plates P_1 and P_2 separated by a distance d. A p.d $V = V_{P_1} - V_{P_2}$ is applied between the plates.

We assume the electric field between the plates is uniform and acts on electrons on a horizontal distance l measured from 0. The point A is the point I where electrons get out the electric field; l is the distance through which the uniform field acts and x the horizontal trajectory travelled by electrons. In the electric field, an electric force acts vertically on the charges. So there is deflection of electrons in the electric field.

d) Why the upper plate must be charged positively and the lower plate charged negatively for this case?

e) If $l = x$, the motion being in the plane, find the equation of the horizontal motion.

f) Find the equation of the vertical motion.

g) Write down the second Newton's law of the motion of those electrons, write the electric force from which electrons are subjected and deduce the acceleration of the motion.

h) Show that the trajectory of the motion between plates is a parabola and give its equation.

i) Calculate the velocity of electrons at the point A where they leave the electric field.

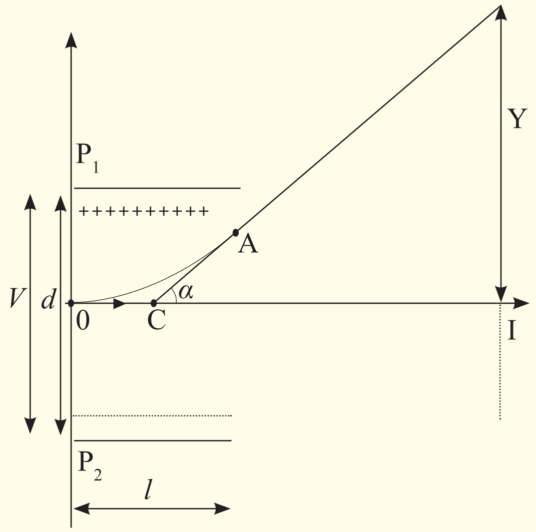


Figure 10.12: Motion of a charge in an electric field

There are so many applications of cathode ray tube which is a practical example of the motion of electrons in an electric field in daily life. For example TV sets, oscilloscope, etc. use cathode ray tubes.

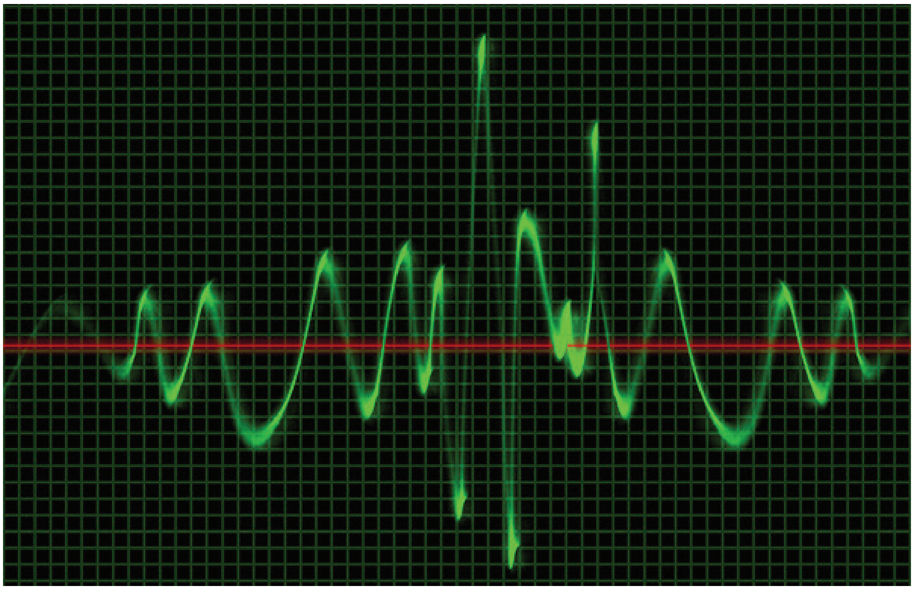


Figure 10.13: The image shows a signal received by a fluorescent screen of a CRT

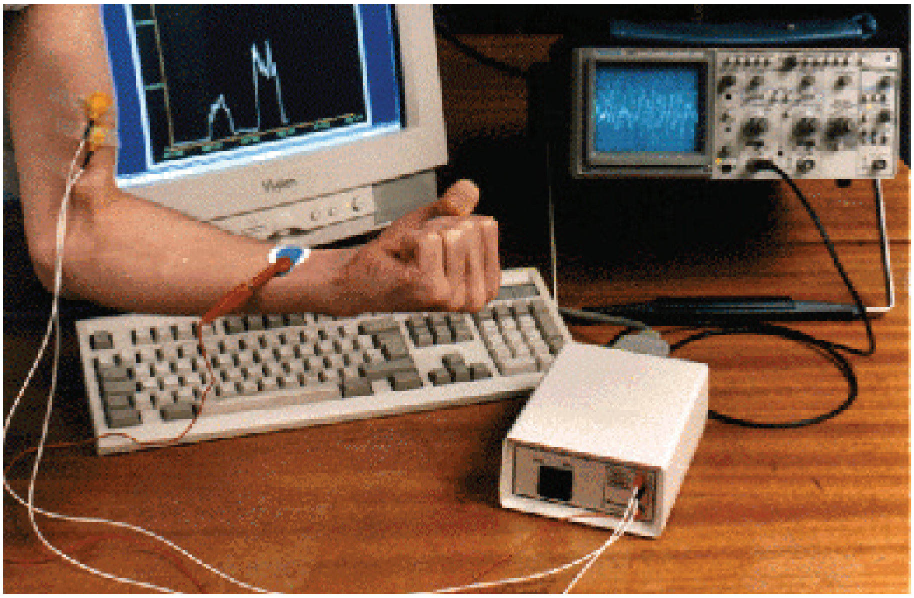


Figure 10.14: Oscilloscopes are used in hospitals for different purposes. Here it's measuring the pressure of people

Lightening and lightening arrestor

Activity 11

Lightening and lightening arrestor

a) Surely, you have heard a thunder before the rainfall. What do you observe in the sky during it?
b) According to you, this is due to what?
c) Is the fact observed dangerous?
d) If yes, do you know some consequences which you have observed or heard?
e) If yes, is there a way to be protected from it?
f) Do some research on internet to know more about it and submit the result of your research to the teacher.

Some explanation

What you observe is called *Lightening* which is a sudden electrostatic discharge (the sudden flow of **electricity** between two electrically charged objects caused by contact, an electrical short, or dielectric breakdown) during an electrical storm between electrically charged regions of a cloud (called intra-cloud lightening or IC), between that cloud and another cloud (CC lightening), or between a cloud and the ground (CG lightening). The charged regions in the atmosphere temporarily equalise themselves through this discharge referred to as a *strike* if it hits an object on the ground. Although lightening is always accompanied by the sound of thunder, distant lightening may be seen but be too far away for the thunder to be heard. Lightening strikes can be damaging to buildings and equipment, as well as dangerous to people.

Figure 10.15: A lightening flash during a thunderstorm

Figure 10.16: A thunder struck tree

Buildings often use a *lightening protection* or *lightening rod* system consisting of a lightening rod (also called a lightening conductor) and metal cables to divert and conduct the electrical charges safely into the ground. Another form of lightening protection system creates a short circuit to prevent damage to equipment. The electrically conducting metal skin of commercial aircraft is isolated from the interior of to protect passengers and equipment.

Often, the lightening protection is mounted on top of an elevated structure, such as a building, a ship, or even a tree, electrically bonded using a wire or electrical conductor to interface with ground or "earth" through an electrode, engineered to protect the structure in the event of lightening strike. If lightening hits the structure, it will preferentially strike the rod and be conducted to the ground through the wire, instead of passing through the structure, where it could start a fire or cause electrocution. Lightening rods are also called *finials*, air terminals or strike termination devices.

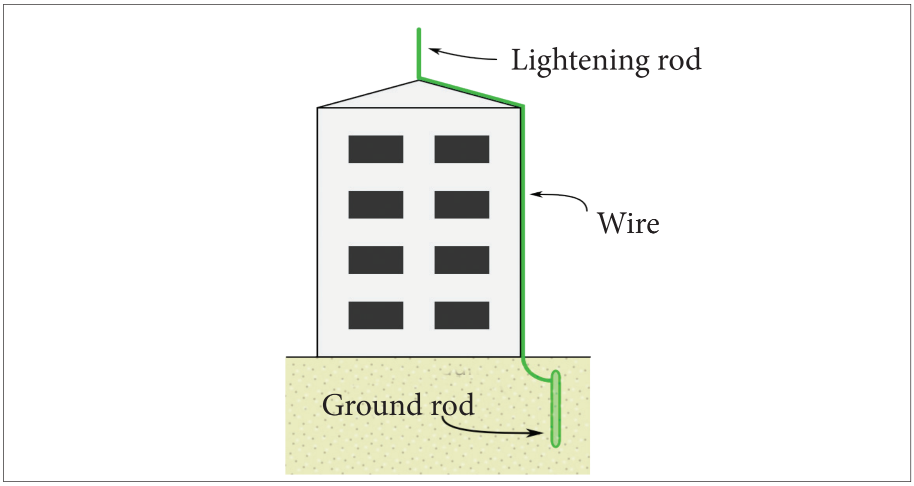


Figure 10.17: Diagram of a simple lightening protection system

In a lightening protection system, a lightening rod is a single component of the system. The lightening rod requires a connection to earth to perform its protective function. Lightening rods come in many different forms, including hollow, solid, pointed, rounded, flat strips or even bristle brush-like. The main attribute common to all lightening rods is that they are all made of conductive materials, such as copper and aluminum. Copper and its alloys are the most common materials used in lightening protection.

Exercise

1. Briefly describe how a lightening conductor can safeguard a tall building from being struck by lightening.
2. Find the force between two point charges $+4\mu C$ and $-3\mu C$ placed at a distance of 12dm apart in free space.
3. A charge of $4\mu C$ is placed in a vacuum. Determine the electric field intensity at a point P at a distance of 20cm from the charge.
4. The vertical deflecting plates in a Television set are 5.0cm and 1.0cm apart. If a potential difference of 100V is applied between the plates and the electron beam enters horizontally mid way between the plates with a speed of $2.0 \times 10^{-7} ms^{-1}$. Find the kinetic energy gained from the electric field by an electron in the beam.

5. The studied device is in an empty enclosure. The study is done with regard a mark supposed galilean; being horizontal and vertical. Electrons penetrate in O, with a horizontal velocity , inside a parallel plates capacitor. Between the parallel plates P_1 and P_2 of this capacitor separated by a distance d is applied a constant voltage $V = V_{P1}-V_{P2} = 140V$. We'll assume that the resulting electric field acts on electrons on a horizontal distance of 1m measured from O. Knowing that: Charge of an electron: $e = 1.6 \times 10^{-19}$ C;
   Mass of an electron:m = 9.1×10^{31}kg ; Acceleration due to gravity: $g = 9.8$ m.s^{-2}; $l = 15$ cm; Velocity of electrons arriving in O: $v_o = 30$ 000km s^{-1}; distance between the plates P_1 and P_2: d =3cm.

   a) Compare the values of the weight of the electron and the electrostatic force undergone inside the capacitor. Conclude.
   b) (i) Give the equations of coordinates x and y of the motion of the electron in the mark , when it passes between the plates P_1 and P_2.
      (ii) Establish the equation of the trajectory of the electron.
   c) With which vertical distance electrons are deviated at the exit of the capacitor? (ii) What is the condition that electrons move out the electric field between the plates P_1 and P_2, the initial velocity keeping the above fixed value?
   d) These electrons make a spot on a luminescent screen placed perpendicularly to and at a distance $D = 20$cm from the center C of the capacitor. What is the distance of that spot to the center I of the screen?

HEAT AND THERMO-DYNAMICS

Thermal effects

Unit 11

Applications of laws of thermodynamics

Key unit Competence

By the of this unit, the learner should be able to evaluate applications of first and second laws of thermodynamics in real life.

Unit goals

By the end of this unit, I will be able to:

* differentiate between Internal energy and total energy of a system.
* explain the work done by the expanding gas.
* state the first law of thermodynamics.
* state the second law of thermodynamics.
* explain thermodynamic processes in heat engines.

Introduction

Before, you learnt that:

- Heat is a form of energy.
- Heat can be changed / transformed from one form to another.

So, if in a system heat changes from one form to another, its called thermal dynamic system.

The systems to discuss in this unit include refrigerators, heat pumps, car engines. Remember that heat is the measure of total internal energy of a body. This means that particles of a body vibrate because of energy they have.

Thermal energy and internal energy

Activity 1

Have you ever boiled water on a sauce pan with a cover?

Describe what happens to the cover when water boils?

When water boils, the vapour pushes the cover off the sauce pan. You have already seen in your early secondary that heat is a form of energy. Therefore, when this saucepan is heated, the heat gained is used to boil off the water and extra work is done to push the sauce pan cover. This total heat energy supplied is called thermal energy.

Science in action! Discover

- In groups of five, explain why an inflated bicycle tube bursts when it is left on sunshine for a very long time.
- Similarly explain why a balloon full of air bursts as it rises in the atmosphere.
- Note down your observation in your exercise books.

You already know the characteristics of the three states of matter that is; solids, liquids and gases. In this unit, we shall be interested in studying the behaviour of molecules in matter.

When the bicycle tube is left exposed to sunshine, it gets heated and the molecules in the gas gain energy and hence its kinetic energy increases. As a result, they collide frequently with the walls of the tube and therefore exert high pressure on the walls and the tube bursts.

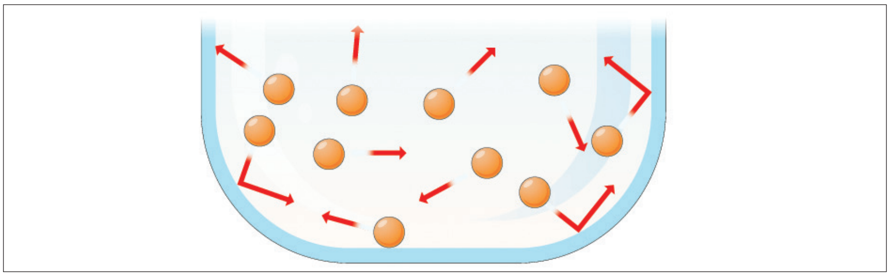


Fig 10.1: Pressure of the gas

The same thing happens with the balloon in air.

The energy possessed by the molecules of the gas is called internal energy of the gas. This energy depends on the temperature of the gas. When a gas is heated its temperature increases and hence the average speed of molecules also increases increasing the internal energy of the gas. Further increase of heat supplied means that extra energy is absorbed by the molecules of the gas, hence expanding and pushing the tyre. As a result the tyre bursts.

Activity 2

List down three utensils used for cooking food in your homes.
Describe how these utensils are used to cook the food. Are they always left open while cooking?

Fig 10.2: Preparing food at home

In all the above, there exists energy exchanges and such things are called systems. Systems can either be closed or open. When water is being boiled in an open sauce pan, vapour is allowed to escape. It is an example of an open system. When someone cooks meat using a closed container, no gas is allowed to escape. Its an example of a closed system.

Whenever heat flows to or from a system, or work is done on or by a system, there is a change in the energy of this system. The study of the processes that cause these energy changes is termed **thermodynamics.**

Thermodynamic systems

Activity 3: Discover

In groups of five, discuss how heat is transferred in the three states of matter.
Do you think heat can be transferred from one state of matter to another? For example from gas to a solid or from a solid to a liquid or from a liquid to a gas and vice versa?

Heat is the energy that flows by conduction, convection or radiation from one body to another because of a temperature difference between them. These bodies where exchange of heat to other forms of energy occurs are called thermodynamic systems.

A thermodynamic system consists of a fixed mass of matter, often a gas, separated from its surroundings, perhaps by a cylinder and a piston. For example heat engines such as a petrol engine, a steam turbine and jet engine all contain thermodynamic systems designed to convert heat into mechanical work. Head pumps and refrigerators are thermodynamic devices for transferring heat from a cold body to a hotter one.

In such devices, energy is transferred from one system to another by a force moving its point of application in its own direction.

The energy of a system, whether transferred to it as heat or work is termed as the internal energy of the system.

When there is no heat transfer between two systems, that is, the two are at the same temperature, they are said to be in thermal equilibrium.

Activity 4

Have you ever observed smoke moving in the atmosphere.
Move outside class and go towards the kitchen and observe how smoke is moving. Describe briefly how it moves.
Why does it move like that?

You have already seen in your early secondary that molecules in a gas are more further apart and are always in constant random motion while moving at high speed colliding with one another and the walls of the container, and when the gas is heated their speed increases.

Smoke particles are always in random motion and when they are moving in air, they collide with air molecules and a zigzag pattern is seen.

Similarly, when smoke is put in a container and then closed, the particles are seen to be in a random motion. Smoke is an example of a real gas.

In thermodynamics, we are mainly interested in ideal gas. At higher temperatures, a real gas behaves like an ideal gas.

Activity 5

Have you ever heard of an ideal gas?
What are the differences between a real gas and an ideal gas?

When a gas is heated, molecules move further apart and the forces of attractions between them become negligible and the gas becomes ideal.

When the molecules become further apart, the gas expands and the volume of the individual molecule becomes so small compared to the entire volume of the gas. It therefore becomes negligible compared to the volume of the gas and the gas becomes ideal.

When the molecules are colliding with one another, collisions are assumed to be perfectly elastic. In this case, the gas becomes ideal because for a real gas we expect to have time between approach and separation during collision.

Exercise

What is a perfectly elastic collision?

Work done by an expanding gas

Activity 6: Discover

Explain why a pump gets hot when one pumps air into a tyre.

When you compress air in a bicycle pump, your muscles transfer energy to the handle, which in turn transfers energy to the molecules of air in the pump. This additional energy makes the molecules move faster. As they are compressed into a smaller space, they also collide more often with the wall of the pump, so they transfer more energy to the metal wall and it becomes hot.

We have already seen how heat can be transferred, so you probably have a good idea what Q means. Work is simply a force Multiplied by distance in the direction of force.

A gas can be heated by compressing it, for example with a bicycle pump. Hence the temperature of the gas can be raised either by doing work in compressing it or by heating it. Likewise the temperature can be lowered by either making the gas do work in expanding or by extracting heat from it.

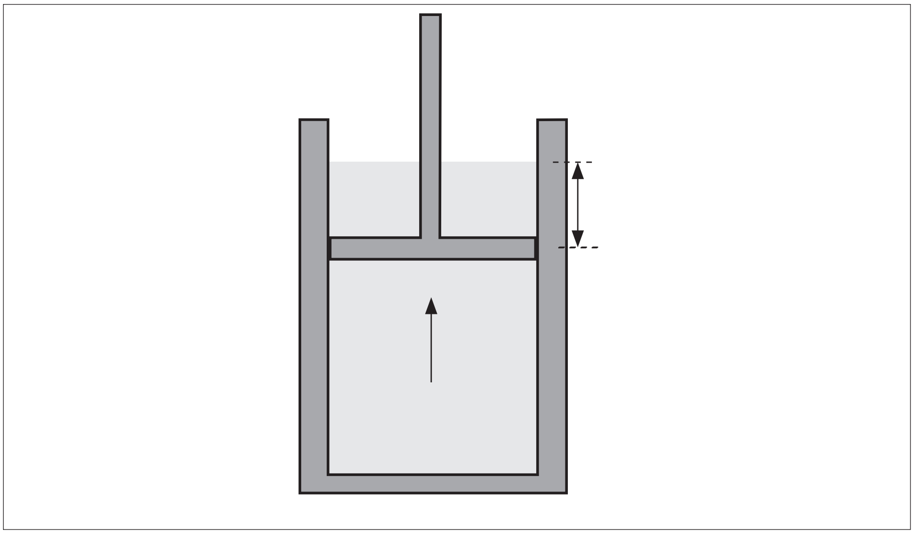


Fig 10.3: When you compress air

Consider a mass of gas enclosed in a cylinder by a frictionless piston of cross-section area A which is in equilibrium under the action of an external force, F, acting downwards (i.e pressing the gas). Let this force be infinitely reduced so that the piston is pushed up by the gas a distance Δx, which is so small that the pressure of the gas is virtually unchanged by the expansion.

If the gas is heated, it will expand and push the piston thereby doing work on the piston. If the piston is pushed down, on the other hand, it does work on the gas. This is an example on how work is done by a thermodynamic system.

The piston must be held in position by force PA (by definition of pressure).

The external work done by the gas, ΔW against F will be;
$\Delta W = F\Delta x = PA\Delta x = P\Delta V$,

$\Delta V = F\Delta x$ is the increase in volume of the gas.

Suppose that the pressure is kept constant during the expansion, and the gas expands from V_1 to V_2, then the total work done by the gas is given by calculus as; $W = \int dW = \int PdW$

It follows that; $W = P(V_2 - V_1)$

Specific heat capacities of gases

(i) Weigh a given quantity of a gas.

(ii) Confine the gas in a closed container.

(iii) Place a thermometer in the container.

(iv) Using a given source of heat, supply heat to a gas by keeping its volume constant.

(v) Record the change in temperature using the thermometer.

(vi) You can do this by closing the container.

(vii) Calculate the quantity of heat supplied.

(viii) Repeat the above procedures by keeping the pressure constant.

(ix) Calculate the heat supplied for the same temperature change as in above.

(x) Compare the two quantities of heat supplied.

Do you notice that the heat needed at constant pressure is higher than that at constant volume?

Why do you think it is so?

Gases are considered to have a number of specific heat capacities. A change in temperature of a gas is likely to cause large changes in pressure and volume of the gas but for solids or liquids, the change in pressure is neglected.

In your early secondary, you have already seen that heat energy is calculated by measuring the mass of liquids and solids. However, in gases, we replace the mass with the number of moles of a gas.

When the specific heat capacity of a gas is measured in terms of its moles, it is known as principal specific heat capacity. There are two important heat capacities: the molar heat capacity and constant volume (C_v) and molar heat capacity at constant pressure (C_P).

The principal molar heat capacity at constant volume (C_v) is defined as the heat required to increase the temperature of one mole of a gas at constant volume by one Kelvin.

The principal molar heat capacity at constant pressure (C_P) is the amount of heat required to increase the temperature of one mole of a gas at constant pressure by one Kelvin.

The molar heat capacities have units $Jmol^{-1}K^{-1}$.

Since at constant volume, the work done by a gas is zero from ($W=P\Delta V$, ΔV, $=0$), then it is evident that the principal K molar heat capacity at constant pressure, C_P is greater than that at constant volume, C_v. Why?

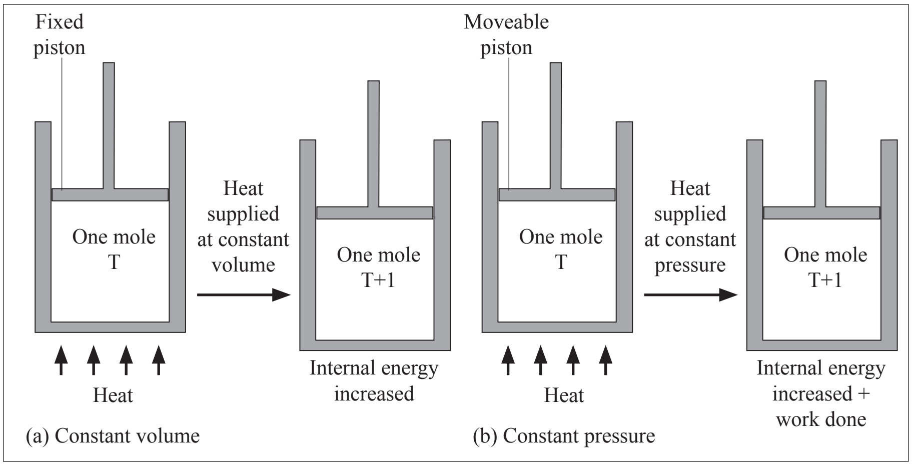


Fig 10.4: Piston in a cylinder

The reason as to why this is so can be done by considering cylinders in (a) and (b) each initially containing one mole of gas at temperature T and pressure, P. The piston in (a) is fixed and that in (b) is frictionless and can move freely but has a constant force applied to it. If heat is supplied to each until the temperature has risen by one Kelvin, the increase of internal energy must be the same in each case (Since the temperature rise is the same).

All the heat supplied in case (a) is used to increase the internal energy of the gas. In (b), however, the gas expands and work is done by it on the piston; the heat supplied in this case equals the increase of internal energy plus the work done in the expansion of the gas.

The first law of the thermodynamics

We have already seen that when a quantity of heat ΔQ is supplied to a gas, two things happen: (i) the heat supplied may increase the internal energy, U of the gas and (ii) the gas may expand and do some work, W in moving the piston.

The magnitude of internal energy depends on; the temperature of the gas i.e. the internal energy is high at a high temperature and low at low temperature.

The amount of heat supplied is equal to the change in internal energy of the gas plus the work done by the gas.
i.e. $\Delta Q = \Delta U + \Delta W$ (i)

The above is the statement of the first law of thermodynamics.

since $\Delta W = P\Delta V,$

It follows that $\Delta Q = \Delta U + P\Delta W$

When n moles of a gas are considered, the amount of heat supplied at constant pressure is $nC_p\Delta T$, whereas the amount of heat supplied at constant volume would be $nC_v\Delta T$.

Relationship between C_p and C_v.

Consider one mole of a gas at a pressure P, temperature T and volume V, heated to cause the same temperature change, ΔT first at constant volume and secondly, at constant pressure.

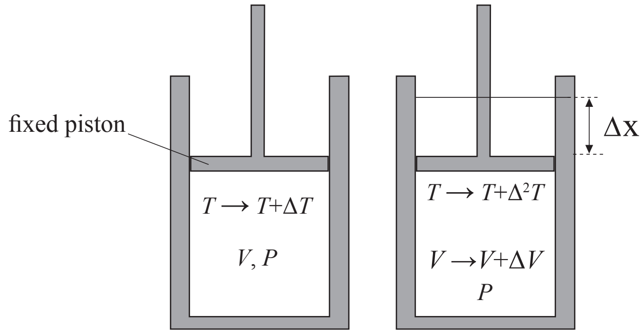


Fig 10.5: At constant volume

From the first law of thermodynamic, At constant volume,

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = 1\times CV + \Delta T$

It therefore follows that $\Delta U = CV\Delta T + \Delta W$……….......................…….(i)

At constant pressure, $\Delta Q = \Delta U + \Delta W$

In this case , $\Delta U = C_V\Delta T$; $\Delta W = P\Delta V$ and $\Delta Q = C_p\Delta T$

From equation (i)

$\Delta Q = \Delta U + \Delta W$, ……………………………………...................................…(ii)

It follows that $C_p\Delta T = = C_V\Delta T + = P\Delta V$……….......................……..(iii)

From the ideal gas equation, $PV = RT$.
If the volume of the gas changes by ΔV and the temperature by ΔT;

$P(V+\Delta V) = R(T+\Delta T) \Rightarrow PV + P\Delta V = RT + T\Delta T = P\Delta V + R\Delta T$ ……… (iv)

Substituting (iv) in (iii)

$C_p = C_v\Delta T + R\Delta T$

$C_p\Delta T = (C_v + R)\Delta T)$

$C_P = C_v + R$

Therefore, $(C_P - C_v) = R$

where R is the universal molar gas constant whose value is 8.31 $Jmol^{-1}K^{-1}$

Applications of first law of thermodynamics in particular gas changes

Isovolumetric process (Isochoric process)

Activity 7

(i) Have you ever heard of an isovolumetric or isochoric process?

Study the Figure and answer questions that follow;

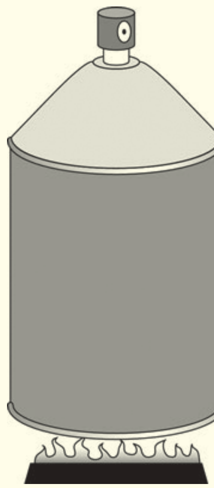


Fig 10.6: Heating a can

(ii) What substance is likely to be getting cooked using the can on the left? Give a reason for your answer.

(iii) Why is the can covered and not open?

(iv) If one tried to open it while its on fire, what do you think would happen?

The above can be used to boil liquids for example milk effectively. As the liquid inside the can heats up, its pressure increases, but its volume stays the same (unless, of course, the can explodes). It therefore means that all the processes inside the can take place at constant volume. They are called isovolumetric or isochoric processes.

When the pressure in a system changes but the volume is constant, you have what is called an *isochoric* process. An example of this would be a simple closed container, which can't change its volume as seen above.

Activity 8

How much work does the fire do on the can?

In this case, the volume is constant, from the law of thermodynamics, no work is being done since $\Delta V = 0$

This process takes place at constant volume and since
$\Delta V = 0, \Delta W = \int P\Delta W = 0$

$\Delta Q = \Delta U + C_V \Delta T$

In this process, the energy supplied is used to increase the internal energy since the internal energy is independent of the volume.

Isobaric process

Activity 9

Study the Figure here showing a woman preparing sauce.

(i) What name of the utensil is she using?

(ii) Why is the utensil open?

(iii) Do you think it is good to use an open utensil to boil liquids?

Fig 10.7: A woman preparing food

Boiling liquids in open containers is very safe for example if the container is closed, pressure may build up in the container and force it to burst. Boiling in open containers imply that the pressure of the substance is kept constant. This process is called an isobaric process. An isobaric process is the one that occurs at constant pressure.

Heating of water in an open vessel and the expansion of a gas in a cylinder with a freely moving piston are typical examples of isobaric processes. In both cases, the pressure is equal to atmospheric pressure. For example when water is being heated, its volume increases and the pressure inside the container is constant since the number of collisions between water molecules and the walls of the container is constant.

The same process occurs when a gas enclosed in a cylinder with a frictionless piston is heated such that at any time, the gas pressure equals the external pressure.

Work done by the gas in the isobaric process

When the gas expands from volume V_1 to V_2,

$\Delta W = P\Delta V = P(V_2 - V_1)$

From a PV graph, the work equals the area under the graph.

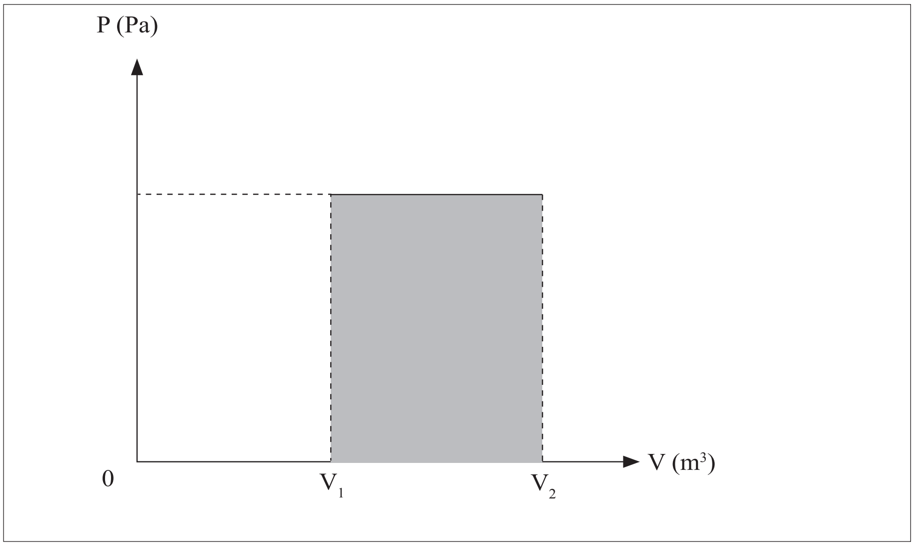


Fig 10.8: Pv graph

In this process, the energy supplied is used to increase the internal energy since the internal energy is independent of the volume.

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta Q = C_V \Delta T + P(V_2 - V_1)$$

Isothermal change (constant temperature)

Activity 10

(i) Get a polythene bag and fill it with air.

(ii) Insert a thermometre in the bag and place it in the ice-water mixture.

(iii) Note what happens.

Do you notice that the gas condenses and the volume decreases?
What happens to the temperature recorded by the thermometer?

You can notice that the temperature remains constant. This change is called Condensation and is an example of isothermal process.

Do you think this process is reversible?

An isothermal change can be reversible. An isothermal change is the change that occurs at constant temperature. It is either a compression or expansion of a gas at a constant temperature.

If the volume increases, the pressure must decrease and if the volume decreases, the pressure must increase

Since $PV = RT$;

It follows that PV = constant, since T is constant.

This equation clearly shows that for an isothermal change, Boyle's law is obeyed.

Considering states $A\ (P_1 V_1,\ T_1)$ and B $(P_2,\ V_2,\ T_2)$, represented in an isothermal expansion B- A is an isothermal compression.

Since $\Delta Q = \Delta U + P\Delta V \Rightarrow \Delta Q = C_V \Delta T + P\Delta V$

For an isothermal change, $\Delta Q = P\Delta V$, since $\Delta T = 0$, implying that all the heat supplied is used to do work in expanding or compressing the gas.

Conditions necessary for an isothermal process to occur

Activity 11: Discover

(i) On a cold day, how do you keep yourself warm?

(ii) In groups of five, describe how you can keep the temperature of the system constant.

For an isothermal process to take place, the gas must be contained in a thin –walled heat conducting vessel/container in good thermal contact with a constant temperature.

The process must be carried out slowly to allow time for heat exchange to take place.

Work done in Isothermal Change

Activity 12: Science at work

(i) Have you ever tried to boil water in a closed sauce pan?

(ii) What happens to the cover when the vapour starts to come off the water?

(iii) Notice that this vapour pushes the cover off the pan.

We say that the vapour does work on the cover.

From the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$.

When the volume of gas changes by ΔV at constant temperature then the pressure has also to change so that the ideal gas equation is satisfied.

The work done, W is then given by $W = \int PdV$ but $PV = RT$ (For 1 mole of gas)

It follows that $P = \dfrac{RT}{V}$

Thus, $W = RT\int_{v_1}^{v_2} \dfrac{RT}{V} dV = RT\,\text{In}\,[V]_{v_1}^{v_2}$

$$W = RT\,(\text{In}\,V_2 - \text{In}\,V_1) = RT\,\text{In} \int \frac{V_2}{V_1}$$

From the above equation, the following can be drawn;

(i) When the gas expands (i.e V2 > V1), then W is positive.

(ii) When the gas is compressed (i.e $V_1 > V_2$), thus W is negative, meaning that work is done on the gas in compressing it.

Adiabatic change

Activity 13

(i) Pump a bicycle tyre using a pump until it is full.

(ii) Open the tube slowly while placing your other hand in its path.

(iii) Do you notice that the the air coming out of the tyre is hotter than the surrounding air?

As one pumps, the air molecules are compressed into a smaller space. They also collide more often with the wall of the pump, so they transfer more energy to one another and become hot. No heat has been supplied to the system. It is called an adiabatic compression.

Activity: 14

(i) Now pump the tyre and leave it standing for sometime.

(ii) Make sure you don't expose it to sun shine.

(iii) Open the valve after two hours while your hand is placed in the path of air from it.

Do you notice that the air is colder than its surrounding?

Heat has been lost but not to the surroundings. When the air is left standing, expansion occurs. This is associated with a decrease in temperature. It is called an adiabatic expansion.

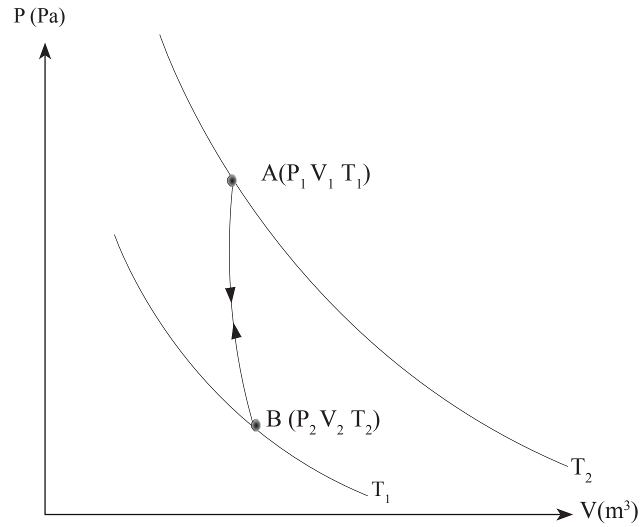
An adiabatic change is process in which no heat enters or leaves the gas system. It is either an expansion or a compression.

Since $\Delta Q = \Delta U + P\Delta V$ and $\Delta Q = 0$

$\Delta Q = C_V\Delta T + P\Delta V$ Or $\Delta U = \; P\Delta V$

If the gas expands, it does work, its internal energy is reduced and hence the temperature is lowered.

If the gas is compressed, work is done on the gas, its internal energy will increase and therefore its temperature rises.



A – B; adiabatic expansion.
B – A; adiabatic compression.

Figure 10.9: PV – diagram for an adiabatic expansion or compression

Conditions that are necessary for an adiabatic change to occur

Activity 15

How do you always protect yourself from a bad weather?

On a cold day, we always wear woolen jackets to protect ourselves from coldness. Therefore no heat is either lost to the surrounding and or gained. In this case, an adiabatic process is achieved.

For an adiabatic process to be achieved, the gas must be contained in a thick –walled and perfectly insulated isolated container.

The process must be carried out rapidly to avoid any possible heat exchanges between the gas system and the surroundings.

Equations for an adiabatic change

From the first law of thermodynamics, $\Delta Q = \Delta U + P\Delta V$

For an adiabatic process, $\Delta Q = 0$, and for 1 mole, $\Delta U = C_V\Delta T_1$, $C_V\Delta T = 0$ *(i)*

For infinitesimal small changes, but from the ideal gas equation, for one mole,

$PV = RT$, so , $P = \dfrac{RT}{V}$ (ii)

Substitute (ii) in (i) $C_V dT + \dfrac{RT}{V}\ dV = 0$

Dividing throughout by T; $C_V \dfrac{\Delta T}{T} + R\dfrac{\Delta V}{V} = 0$ (iii)

But $C_P - C_V = R$;

$$C_V \frac{\Delta T}{T} + (C_P - C_V)\frac{\Delta V}{V} = 0$$

Dividing throughout by C_V;

$$\frac{\Delta T}{T} + \left(\frac{C_p}{C_V} - 1\right)\frac{\Delta V}{V} = 0$$

Let $\gamma = \dfrac{C_p}{C_V}$ (the ratio of the principal heat capacities)

$$\frac{\Delta T}{T} = (1-\gamma)\frac{\Delta V}{V}$$

It follows that, $\dfrac{\Delta T}{T} = (1-\gamma)\dfrac{dV}{V}$ (iv)

Integrating both sides for (iv)

$$\int \frac{dT}{T} = \int (1-\gamma)\frac{\Delta V}{V}$$

$\text{In } T = (1-\gamma)\text{ In } V + \text{In } A$

$\text{In } T = \text{In } V^{(1-\gamma)} + \text{In } A$

It follows that $\text{In } T\,(A\ V^{1-\gamma})$

$T = (A\ V^{1-\gamma})$

Hence $TV^{\gamma-1}) =$ a constant (v)

From $PV = RT$, $T = \dfrac{PV}{R}$ (vi)

Substitute (vi) in (v)

$$\frac{PV}{R} \times V^{\gamma-1} = \text{a constant}$$

Therefore, $PV^{\gamma} =$ a constant

Activity 16

Derive the expression for temperature and pressure for adiabatic change i.e

$T^{\gamma} P^{1-\gamma}$ = a constant

Example I

A gas has a volume of $0.02\ m^3$ at a pressure of 2×10^5 Pa and a temperature of 27°C. It is heated at constant pressure until its volume increases to $0.03\ m^3$. **Calculate the**:

(i) External work done.

(ii) New temperature of the gas.

(iii) Increase in internal energy of the gas if its mass is 16g, its molar heat capacity at constant volume is $0.8 Jmol^{-1}K^{-1}$ and the molar mass is 32g.

Solution

$V_1 = 0.02cm^3$

$V_2 = 0.03cm^3$

$P_1 = 2\times10^5$ Pa

$P_2 = 2\times10^5$ Pa

$T_1 = 300K$

$T_1 = ?$

$C_V = 0.8\ Jmol^{-1}K^{-1}$

$m = 16g$

$M = 32g$

(i) External work done, $\Delta W = P\Delta V = 2\times10^5 (0.03 - 0.02) = 2\times10^3 J$

(ii) From $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

It follows that $\dfrac{0.02}{300} = \dfrac{0.03}{T_2}$

Hence $T_2 = 450K$

(iii) Increase in internal energy, $\Delta U = nC_V\Delta T$

But $n = \dfrac{m}{M} = \dfrac{16}{32}$

Thus $\Delta U = 0.5 \times 0.8\ (450 - 300) = 0.4 \times 150 = 60J$

Example 2

An ideal gas at 17° C has a pressure of 760mm Hg is compressed (i) isothermally (ii) a diabatically, until its volume is halved.

Calculate in each case the final temperature and pressure of the gas. Assume that $C_p = 2100\text{Jmol}^{-1}\text{K}^{-1}$ and $C_V = 1500\text{Jmol}^{-1}\text{K}^{-1}$.

Solution

$P_1 = 760$ mmHg

$P_1 = \frac{1}{2}V$

$V_1 = V$

$T_1 = 290\text{K}$

For isothermal change,

$$P_1V_1 = P_2V_2 \Rightarrow P_1V_1 = P_2 = \frac{V_1}{2} \Rightarrow P_2 = 760 \times 2 = 1520 \text{ mmHg}$$

$T_1 = 290$ K

For adiabatic change

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\text{But } \gamma = \frac{C_P}{C_V} = \frac{2100}{1500} = 1.4$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 760 \times 2^{1.14} = 2005.56\text{mmHg}$$

Using

$TV^{\gamma-1} = \text{constant}$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 290(2)^{0.4} = 328.7 \text{ K} = 109.7^0 \text{C}$$

Exercise

1. Distinguish between isothermal and adiabatic changes, clearly stating the conditions under which they occur in practice.
2. Define the two principal molar heat capacities of a gas and derive an expression relating the two. Explain the difference between these two principal molar heat capacities.

3. A quantity of oxygen is compressed isothermally until its pressure is doubled, it is then allowed to expand adiabatically until its original volume is restored. Find the final pressure in terms of its original pressure. Draw a PV diagram for the above processes.

4. $0.45m^3$ of a gas at a temperature of 15^oC expands adiabatically and its temperature falls to 4^oC.

   a) What is the new volume if $\gamma = 1.40$
   b) The gas is then compressed isothermally until the pressure returns to its original value. Calculate the final volume of the gas.

5. A vessel containing $2m^3$ of air initially at a temperature 25^oC and pressure 760mmHg, is heated at constant pressure until its volume is doubled. Find (a) the final temperature (b) the external work done by the air in expanding, (c) the quantity of heat supplied.

6. (Assume that the density of air at s.t.p is $1.293kgm^{-3}$ and that the principal molar heat capacity of air at constant volume is $20.4Jmol^{-1}K^{-1}$.

   An ideal gas at a temperature 45^o C and pressure $1.0 \times 10^5Nm^{-2}$ occupies a volume of $2.0 \times 10^{-3} m^3$. It expands adiabatically to twice its volume. Find the final temperature and pressure. Represent this process on PV- diagram.
   (Take $\gamma = 1.40$)

Second Law of thermodynamics

The second law of thermodynamics can be stated in many equivalent ways, each expressing a different facet of its meaning. There are so many different forms of it because it is of such significance. William Thomson (Lord Kelvin) in 1851 stated it in this form;

"no heat engine can perform a cyclic operation whose only result is to convert internal energy into mechanical energy"

The second law was stated by Rodolph Clausus in 1850 in this form;

"no refrigerator (or heat pump) can transfer internal energy from a cold reservoir to a hot reservoir without some external agent doing work."

Applications of the second law of thermodynamics

Heat engines

Activity 17

* Have you ever heard of an engine?
* Where exactly do we find engines?
* What do you think an engine is?
* How do you think the engine operates?

Any device which will convert heat cyclically into mechanical work is called a heat engine.The material which, on being supplied with heat, performs mechanical work is called the working substance. It is a machine, which changes heat energy, obtained by burning a fuel, to kinetic energy. In an internal combustion engine, e.g petrol. Diesel, jet engine, the fuel is burnt in the cylinder chamber where the energy change occurs. This is not so in other engines e.g steam turbines. All practical engines use one of the two working substances either water (in the reciprocating steam engine and the turbine) or air (in the internal combustion engine). The working of an engine is a thermodynamic process in which at one point no heat enters or leaves a system during expansion or compression of the fluid composing the system.

The Carnot Cycle

The cycle of operations through which the working substance has been taken is called carnot's cycle.

There are two main ways in which the carnot cycle differs from that of any practical engine.

First, the heat absorbed is all taken in at one constant temperature and all the heat rejected to the sink is given out at another constant temperature. In this manner, it is very much simpler than any practical engine.

Secondly, as no work is done at any stage in overcoming frictions, and no heat is lost to the surrounding, the cycle is completely reversible. This means that if we had carried out the whole sequency of changes in the reverse order, every operation would have been exactly reversed. This is called an ideal heat engine because in all practical engines work, is done in overcoming friction and heat is lost to the surroundings.

Otto Cycle and Diesel Cycle

Otto Cycle

An Otto cycle is an idealized thermodynamic cycle which describes the functioning of a typical spark ignition reciprocating piston engine, the thermodynamic cycle most commonly found in automobile engine.

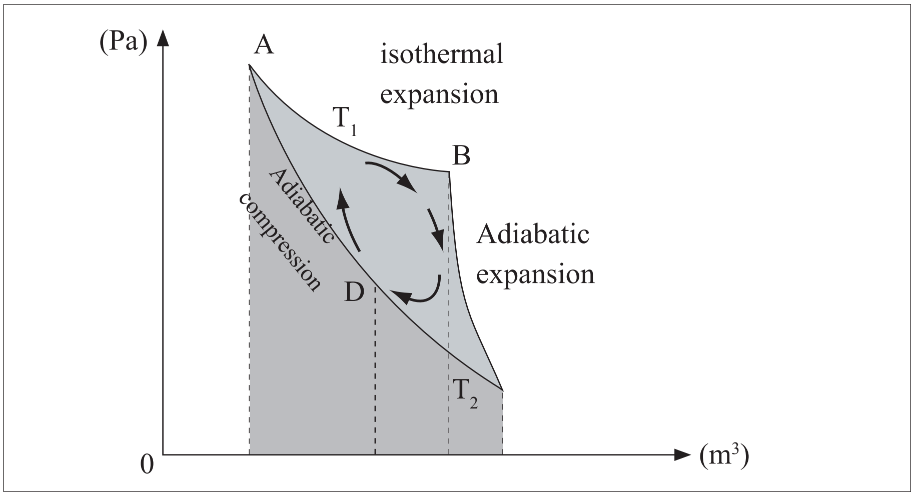


Fig 10.10: pv graph of otto cycle

The Pressure Volume diagram above represents the Otto cycle which has the following strokes; the intake (A) stroke is performed by an isobaric expansion, followed by an adiabatic compression (B) stroke (along 1-2). Through the combustion of fuel, heat is added in an isovolumetric process (2-3), followed by an adiabatic expansion process (3-4), characterising the power (C) stroke. The cycle is closed by the exhaust (D) stroke, characterized by isovolumetric cooling and isobaric compression processes.

The processes are described by:

Process 1-2 is an isentropic compression of the air as the piston moves from bottom dead centre (BDC) to top dead centre (TDC).

Process 2-3 is a constant –volume heat transfer to the air from an external source while the piston is at top dead centre. This process is intended to represent the ignition of the fuel –air mixture and the subsequent rapid burning.

Process 3-4 is an isentropic expansion (power stroke).

Process 4-1 completes the cycle by a constant-volume process in which heat is rejected from the air while the piston is a bottom dead centre.

The Otto cycle consists of adiabatic compression, heat addition at constant volume, adiabatic expansion, and rejection of heat at constant volume. In the case of a four-stroke Otto cycle, technically there are two additional processes; one for the exhaust of waste heat and combustion products (by isobaric compression), and one for the intake of cool oxygen –rich) air (by isobaric expansion); however, these are often omitted in a simplified analysis. Even though these two processes are critical to the functioning of a real engine, wherein the details of heat transfer and combustion chemistry are relevant, for the simplified analysis of the thermodynamic cycle, it is simpler and more convenient to assume that all of the waste-heat is removed during a single volume change.

Diesel Cycle

The **diesel cycle** is the thermodynamic cycle, which approximates the pressure and volume of the combustion chamber of the Diesel engine, invented by Rudolph Diesel in 1897. It is assumed to have constant pressure during the first part of the "combustion" phase $V_2$to V_2 in the diagram, below). This is an idealised mathematical model; real physical diesels do have an increase in pressure during this period, but it is less pronounced than in the Otto cycle. The idealized Otto cycle of a gasoline engine approximates constant volume during that phase, generating more of a spike in a P-V diagram.

The Idealised Diesel Cycle

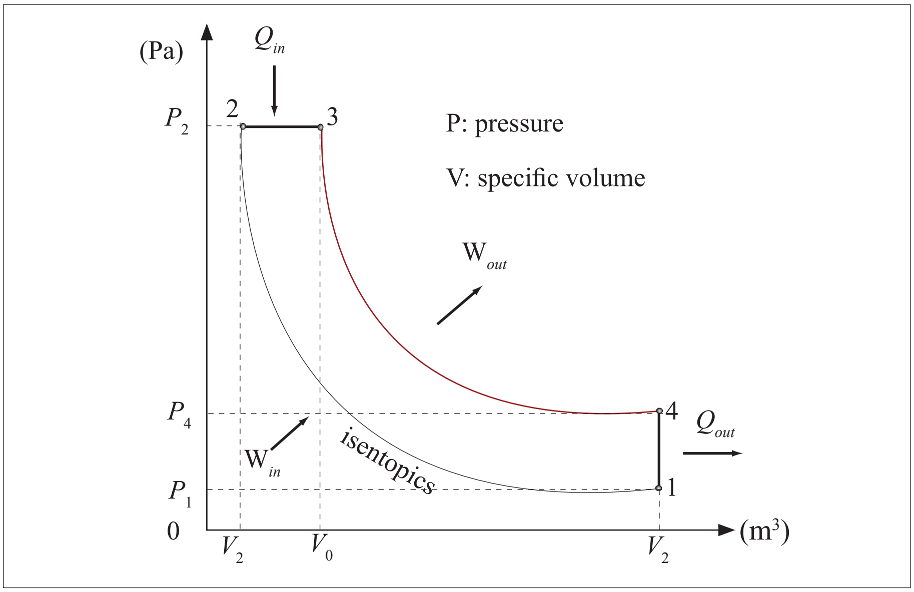


Fig 10.11: pv graph of ideal diesel cycle

From *P-V* diagram for the Ideal Diesel cycle, the cycle follows the numbers 1-4 in clockwise direction.

The image on the top shows a *P-V* diagram for the ideal Diesel cycle; where *P* is pressure and *V* is specific volume. The ideal Diesel cycle follows the following four distinct processes (the colour references refers to the colour of the line on the diagram):
Process 1-2 is isentropic (adiabatic) compression of the fluid (blue colour).
Process 2-3 is reversible (isobaric constant pressure heating (red).
Process 3-4 is isentropic (adiabatic) expansion (yellow).
Process 4-1 is reversible constant volume cooling (green).

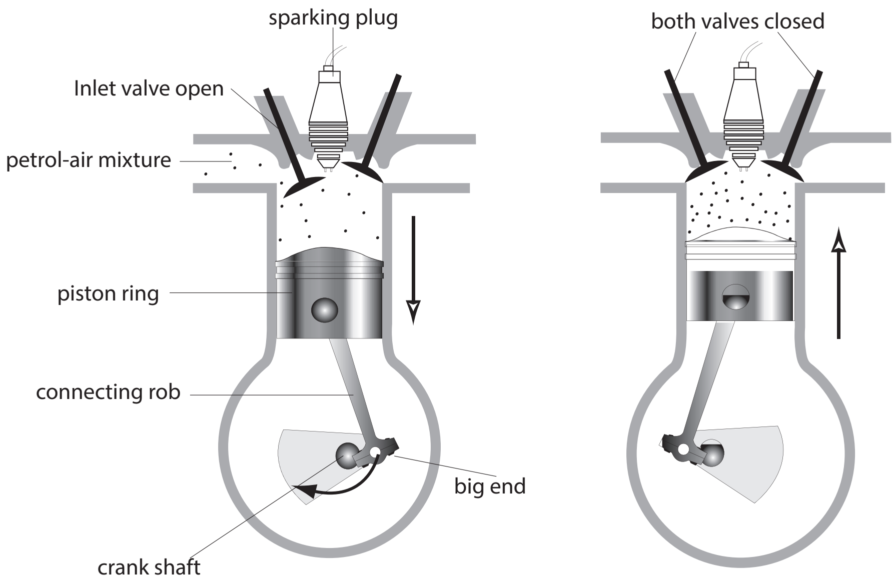
The Diesel is a heat engine; it converts heat into work. The isentropic processes are impermeable to heat; heat flows into the loop through the left expanding isobaric process and some of it flows back out through the right depressurising process, and the heat that remains does the work.

Work in (W_{in}) is done by the piston compressing the working fluid.

Heat in (Q_{in}) is done by the combustion of the fuel.

Work out (Wout) is done by the working fluid expanding on to the piston (this produces usable torque).

Heat out (Vout) is done by venting the air.



1: Intake *2: Compression*

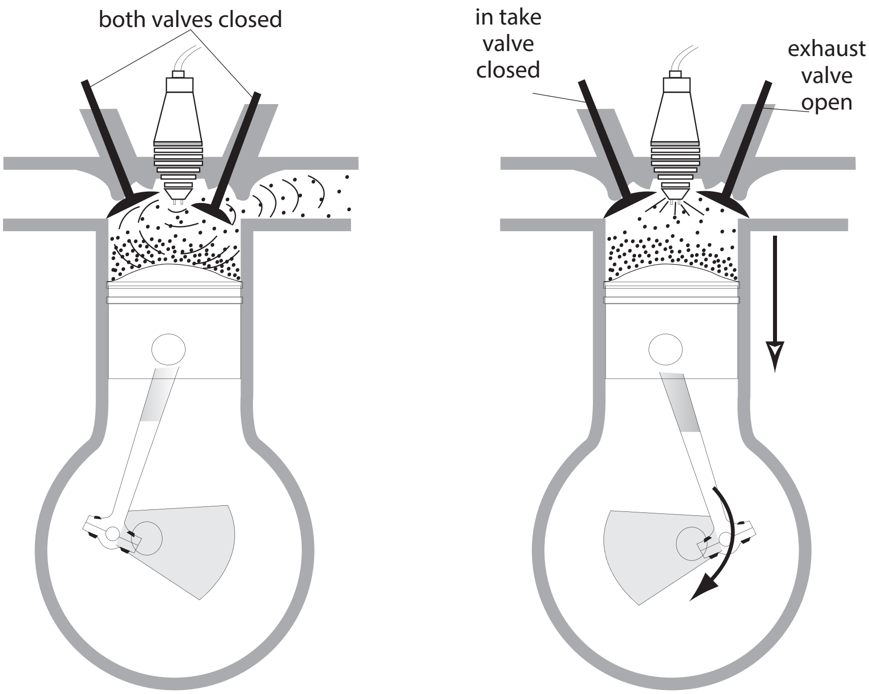


Fig 10.12: For stroke-cycle interval combustion

A heat engine is a machine, which changes heat energy, obtained by burning a fuel, to kinetic energy. In an internal combustion engine, e.g petrol, diesel, jet engine, the fuel is burnt in the cylinder of chamber where the energy change occurs. This is not so in other engines e.g steam turbine.

Petrol engine

Activity 18

(i) How many types of fuels do vehicles use to operate?

(ii) Have you ever heard of vehicles which use petrol in order to operate?

(iii) List down four vehicles which use petrol.

(iv) What type of engine do they have?

Many vehicles use petrol in order to move. Such vehicles are small cars and motorcycles. The engine they have is called a petrol engine since it uses petrol to operate. It operates by moving the piston. The upward and downward movement of the piston is called a stroke.

a) **Four – stroke engine:** On the intake stroke, the piston moves down (due to the starter motor in a car or the kick start in a motor cycle turning the crankshaft) so reducing the pressure inside the cylinder. The inlet value opens and the petrol – air mixture from the carburetor is forced into the cylinder by atmospheric pressure.

On the compression stroke, both valves are closed and the piston moves up, compressing the mixture.

On the power stroke, a spark jumps across the points of the sparking plug and explodes the mixture, forcing the piston down.

On the exhaust stroke, the outlet valve opens and the piston rises, pushing the exhaust gases out of the cylinder.

The crankshaft turns a flywheel (a heavy wheel) whose momentum keeps the piston moving between power strokes.

Most cars have atleast four cylinders on the same crankshaft. Each cylinder fires in turn in the order 1-3-4-2, giving a power stroke every half revolution of the crankshaft. Smoother running results.

b) **Two-stroke engine:** This is used in mopeds, lawnmovers and small boats. Valves are replaced by ports on the side of the cylinder which are opened and closed by the piston as it moves.

Diesel engine

Activity 19

(i) Have you ever heard of vehicles which use diesel in order to move?

(ii) What kind of vehicles are they?

(iii) What is the name of the engine in such vehicles?

The engine which uses diesel is called a diesel engine. A diesel engine can operate by making two or more strokes.

The operation of two and four stroke diesel engines is similar to that of the petrol varieties. However, fuel oils is used instead of petrol, there is no sparking plug and the carburetor is replaced by a fuel injector.

Air is drawn into the cylinder on the down stroke of the piston and on the upstroke it is compressed to about one-sixteenth of its original volume (which is twice the compression in a petrol engine). This very high compression increases the temperature of the air considerably and when, at the end of the compression stroke, fuel is pumped into the cylinder by the fuel injector, it ignites automatically. The resulting explosion drives the piston down on its power stroke. (You may have noticed that the air in a bicycle pump gets hot when it is squeezed.The same applies here.)

Activity: 20

State the advantages of a diesel engine over a petrol engine.

Diesel engines, sometimes called compression ignition (C.I) engines, though heavier than petrol engines, are reliable and economical. Their efficiency of about 40% is higher than that of any other heat engine. A disadvantage of the diesel engine is that its higher compression ratio means that it needs to be more robust, and is therefore more massive.

The Refrigerator

Activity: 21

* How many of you have seen a refrigerator?
* With the help of a teacher visit any place where there is a refrigeration and observe it carefully.
* How useful is it to our daily lives?
* Who can describe how it works?
* Write your suggestions in the notebook.

Fig 10.13: Fruits and beverages in a refrigerator

You should know that

A refrigerator is used to cool substances. It cools things by evaporation of a volatile liquid called Freon. The coiled pipe around the freezer at the top contains Freon which evaporates and takes latent heat from the surroundings so causing cooling. The electrically driven pump removes the vapour and forces it into the heat exchanger (pipes with cooling fins outside the rear of the refrigerator). Here the vapour is compressed and liquefies giving out latent heat of vapourization to the surrounding air. The liquid returns to the coils around the freezer and the cycle is repeated. An adjustable thermostat switches the pump on and off, controlling the rate of evaporation and so the temperature of the refrigerator.

Review exercise

1. (a) (i) What is meant by a reversible isothermal change?

   (ii) State the conditions for achieving a reversible isothermal change.

   (b) (i) What is meant by adiabatic change?

   (ii) An ideal gas at 27°C and a pressure of 1.01×10^5 Pa is compressed reversibly and isothermally until its volume is heated. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if a = 1.4.

2. (a) Explain why the specific heat of a gas at constant pressure is higher than that at constant volume.

   (b) The density of an ideal gas is 1.6kgm^{-3} at 27°C and 1.00×10Nm^2 pressure and specific heat capacity at constant volume is 0.312KJkg^{-1}. Find the ratio of the specific heat capacity at constant pressure to that at constant volume. Point out any significance attached to the result.

3. (a) Explain why the cooling compartment of a refrigerator is always on top.

   (b) The refrigerator cools substances by evaporation of a volatile liquid.

   Explain how evaporation causes cooling.

   (c) State the reason why water is used in the cooling system of a car engine.

4. (a) With the aid of a labelled diagram, describe how a refrigerator works.
   (b) The cooling system of a refrigerator extracts 0.7 Kw of heat. How long will it convert 500g of water at 20°C to ice?
   (c) Explain how evaporation takes place in the refrigerator.
   (d) Explain why water in a porous pot keeps at a lower temperature than that of the surrounding.

Heat engine and climatic change

Activity 22

(i) In groups of five, discuss the causes of air pollution and water pollution.

(ii) Explain how water and air pollution affect the environment and the climate.

(iii) Note down your findings as a group.

(iv) Present your findings to the whole class.

Most of air pollution is caused by the burning of fuels such as oil, natural gas etc. The air pollution has an adverse effect on the climate. Climate change is the greatest environmental threat of our time endangering our health.

When a heat engine is running, several different types of gases and particles are emitted that can have detrimental effects on the environment. Of particular concern to the environment are carbon dioxide, a greenhouse gas; hydrocarbons -- any of more than a dozen volatile organic compounds, nitrogen oxides; sulfur oxides; and particulate matter, tiny particles of solids, such as metal and soot.

Engines emit greenhouse gases, such as carbondioxide, which contribute to global warming. Fuels used in heat engines contain carbon. The carbon burns in air to form carbon dioxide. The Carbondioxide and other global warming pollutants collect in the atmosphere and act like a thickening blanket, and destroy the ozone layer. Therefore the sun's heat from the sun is received direct on the earth surface and causes the planet to warm up. As a result of global warming, the vegetation is destroyed, ice melts and water tables are reduced.

Heat engines especially diesel engines produce Soot which contributes to global warming and its influence on climate.The findings show that soot, also called black carbon, has a warming effect. It contains black carbon particles which affect atmospheric temperatures in a variety of ways. The dark particles absorb incoming and scattered heat from the sun; they can promote the formation of clouds that can have either cooling or warming impact; and black carbon can fall on the surface of snow and ice, promoting warming and increasing melting. Therefore soot emissions have significant impact on climate change .

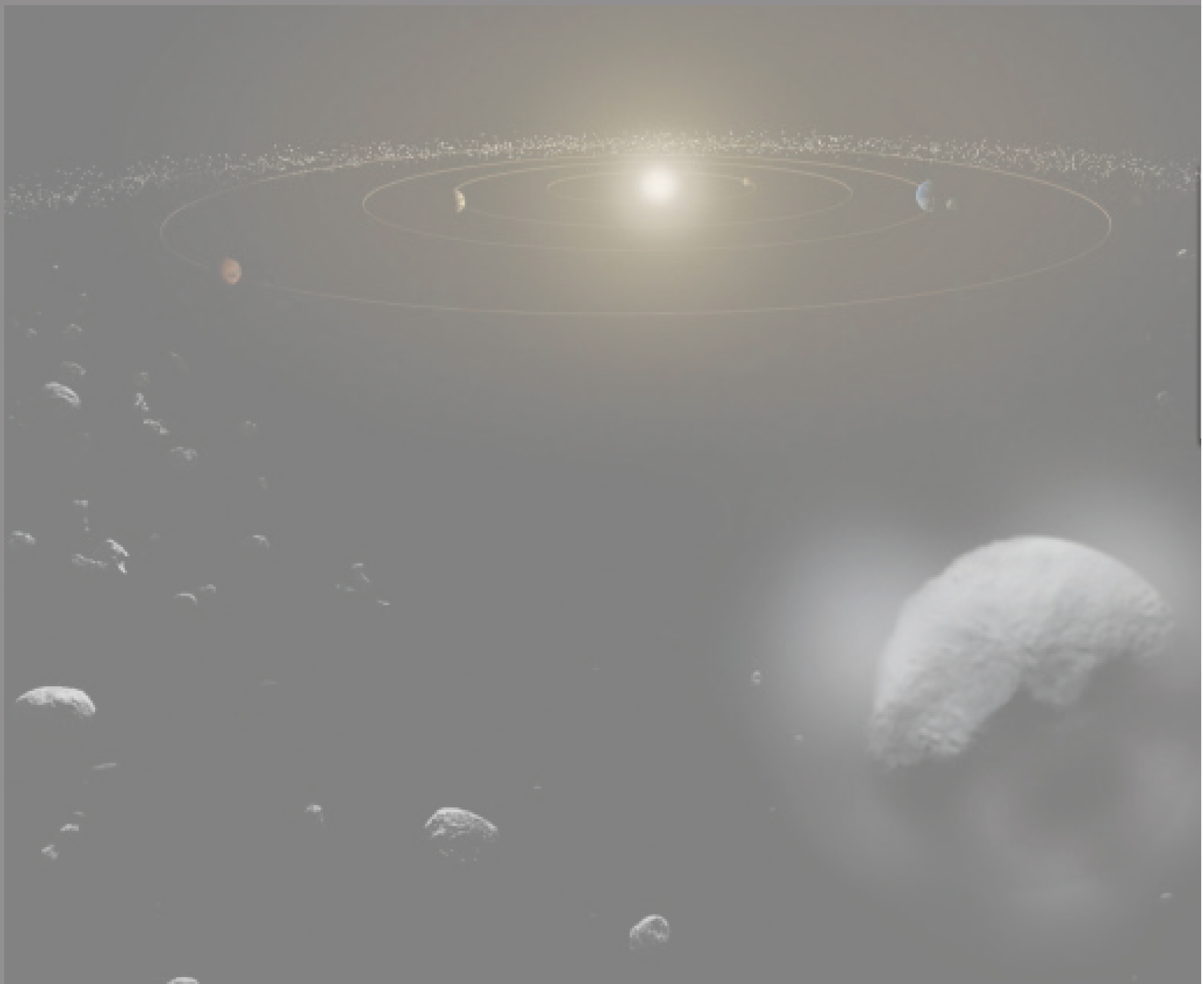
Similarly, some engines leak, for example, old car engines and oil spills all over. When it rains, this oil is transported by rain water to lakes and rivers. The oils then create a layer on top of the water and prevent free evaporation of the water.

Critical thinking exercise

In which ways can the dangers resulting from heat engines be minimised?

ASTROPHYSICS

Earth and Space



Unit 12

General Structure of the Solar System

Key unit Competence

By the end of the unit, the learner should be able to illustrate and describe the general structure of the solar system.

My goals

By the end of this unit, I will be able to:

* illustrate and describe the general structure of the solar system.
* identify and explain scales for estimate astronomical distances.
* explain the phenomenon of eclipse and explain phases of the moon.
* differentiate inner, outer planets, comets, meteorites and asteroids.
* discuss Kepler's laws and explain stars patterns.
* identify celestial coordinates.

Astronomical scales

Astronomy is the study of the universe, and when studying the universe, we often deal with unbelievable sizes and unfathomable distances. To help us get a better understanding of these sizes and distances, we can put them to scale. **Scale** is the ratio between the actual object and a model of that object. Some common examples of scaled objects are maps, toy model kits, and statues.

Maps and toy model kits are usually much smaller than the object it represents, whereas statues are normally larger than its analog.

Our solar system is ***immense*** in size. We think of the planets as revolving around the sun but rarely consider how far each planet is from the sun or from each other. Furthermore, we fail to appreciate the even greater distances to the other stars. Astronomers refer to the distance from the sun to the Earth as one "astronomical unit" or AU = approximately 150 million kilometres. This unit provides an easy way to calculate the distances of the other planets from the sun and build a scale model with the correct relative distances.

Activity 1: Role play

Solar System Bead Distance Activity

We will construct a distance model of the solar system to scale, using coloured beads as planets. The chart below shows the planets and asteroid belt in order along with their distance from the sun in astronomical units.

First, complete the chart by multiplying each AU distance by our scale factor of 10 centimetres per astronomical unit. Next, use the new distance to construct a scale model of our solar system. Start your model by cutting a 4.5 meter piece of string (5.0 metres if you are doing the Pluto extension).

Use the distances in centimetres that you have calculated in the chart below to measure the distance from the sun on the string to the appropriate planet and tie the coloured bead in place. When you are finished, wrap your string solar system around the cardboard holder. Note that the bead colours are rough approximations of the colors of the planets and the sun,

Keep two important solar system facts in mind. The first is that the planets never ever align in a straight line. Occasionally skywatchers are treated to the sight of two bright planets apparently close together as viewed from our planet.

The second fact is that your string solar system is a **radius** of the orbits of the planets. To see how large the solar system is, hold the sun in one location and swing the planets in a circle around it. If you move counter-clockwise you will be moving the planets in the direction they move as viewed from above their plane. The whole circumference of the solar system probably will not fit into your classroom.

Planet	AU	Scale value (cm)	Color
Sun	0.0	___0___	Yellow
Mercury	0.4	________	Solid red
Venus	0.7	________	Cream
Earth	1.0	___10___	Clear blue
Mars	1.5	_________	Clear red
Asteroid belt	2.8	_________	Black
Jupiter	5.2	_________	Orange
Saturn	9.6	_________	Clear gold
Uranus	19.2	_________	Dark blue
Neptune	30.0	_________	Light blue
Pluto (Closest)	29.7	_________	Brown
Pluto (average)	39.5	_________	Brown
Pluto (most distant)	49.3	_________	Brown

Materials:

* Planet beads (large craft pony beads in 11 colours) roughly. approximating the appearance of the planets and the sun.
* Five metres of string for each learner.
* Small piece of cardboard to wrap solar system string around (10 cm x 10 cm).
* Metre sticks or rulers with centimetre markings for each learner or group to share.
* Learner calculations table, one for each learner.

Background

To speed up the activity, the string may be pre-cut and a set of solar system beads may be put into a plastic zip-lock bag for each learner. Also, a measured marking grid can be put on a table top so you can mark their measured distances and then tie off the beads. If the pre-marking method is used, extra distance must be added to each planet distance to accommodate the string within each knot (approximately 4 centimetres for a double knot around the bead). Tape newspaper to the surface where you will be marking your strings so you do not mark up the counter or floor.

Procedure

1. Convert the various astronomical unit distances to centimetres and complete the chart on the student calculations table.
2. Measure and cut a piece of string 4.5 metres long.
3. Using the calculated centimetre distance, tie the bead onto the string using a double knot.
4. When finished with the activity, wrap the solar system string (with beads) around the cardboard holder.

Learner Calculations Table:

Planet	AU	Scale value (cm)	Color
Sun	0.0	___0___	Yellow
Mercury	0.4	________	Solid red
Venus	0.7	________	Cream
Earth	1.0	________	Clear blue
Mars	1.5	_________	Clear red
Asteroid belt	2.8	_________	Black
Jupiter	5.2	_________	Orange
Saturn	9.6	_________	Clear gold
Uranus	19.2	_________	Dark blue
Neptune	30.0	_________	Light blue
Pluto (Closest)	29.7	_________	Brown
Pluto (average)	39.5	_________	Brown
Pluto (most distant)	49.3	_________	Brown

Viewed from Earth it is difficult to gauge the scale of the universe but astrophysicists have developed techniques to help to do this. Stars and galaxies are so far away than a new unit of distance measurement, the *light-year (ly),* is often used. For light travelling at 3×10^8m/s, the distance traveled in one year is:

$$1\ ly = (3 \times 10^8\ \text{m/s}) \times (365 \times 24 \times 60 \times 60\ \text{s}) = 9.46 \times 10^{15}\ \text{m}.$$

For specifying distances to the Sun and Moon, we usually use metres or kilometres, but we could specify them in terms of light. The Earth-Moon distance is 384 000 km, which is 1.28 light-seconds. The Earth-Sun distance is 1.5×10^{11} m, or 150,000,000 km; this is equal to 8.3 light-minutes. Far out in our solar system, the ninth planet, Pluto, is about 6 x 10^9 km from the Sun, or 6 x 10^{-4} ly. The nearest star tous, other than the sun, is Proxima Centauri, about 4.3 ly away. (Note that the nearest star is about 10,000 times farther from us than the outer reaches of our solar system.)

The *Milky Way* or our *Galaxy* is about 100 000 ly across; our sun is located on one of the spiral arms of the galaxy at a distance of 28,000 ly from the galactic centre.

Career centre

Learn more about career in physics and engineering about the general structure of solar system.

Sun-Moon-Earth System (Eclipses and Phases of the Moon)

Eclipses (lunar and solar eclipses)

Activity 2

Eclipses in classroom

Building the Sun-Earth-Moon system described below will allow your class to discover how and why eclipses happen. They will be able to understand exactly what they are seeing if ever they see a real eclipse.

Materials

For each model, you will need:

* Adhesive tape
* Glue
* Two cardboard tubes (e.g. empty toilet rolls)
* Torch
* Scissors (suitable for cutting cardboard)
* Aluminum foil
* Sturdy but bendable wire (35-50 cm long)
* Styrofoam ball the size of a large orange
* Ping pong ball (or a Styrofoam ball of a similar size)
* Large strip of cardboard (about 60cm in length and not less than 20cm in width)
* Stack of books or magazines

Procedures

1. Divide the class into groups of three or four. Give each group their own materials to make the model.
2. Take one cardboard tube and make a series of small (2 cm) even, vertical cuts around the circumference of each end.
3. At each end, bend the cut pieces out, and then stand the tube upright. At the top, the cut edges should fan out like a flower.
4. Using adhesive tape, fasten one end of the cardboard tube to the strip of cardboard; this is the base of the model. The tube should be at least 30cm from one end of the cardboard strip.
5. Using tape or glue, attach the larger ball to the open flower of the tube. This ball is Earth.
6. Cover the smaller ball with aluminum foil, shiny side out. This is the Moon.
7. Insert one end of the wire into the top of Earth, so that the wire is vertical.
8. Measure a finger's length along the wire. Bend the wire at a right angle to give a horizontal arm.
9. Insert the other end of the wire into the Moon.
10. About halfway between Earth and the far end of the cardboard strip, measure a finger's length along the wire and bend it downwards at a right angle, toward the cardboard base. The Moon's equator should be at the same height as Earth's equator.
11. Balance the torch on a stack of books or magazines at the other end of the cardboard strip from Earth. Make sure the height is correct: the middle of the torch beam should hit Earth's equator. If the beam is too diffuse, attach the second cardboard tube to the end of the torch to direct the light horizontally. Ensure the beam hits the nearest half of Earth and the Moon directly. If the beam is not bright enough, move the stack of books closer.

Eclipse, in astronomy is the obscuring of one celestial body by another, particularly that of the sun or a planetary satellite. Two kinds of eclipses involve the earth: those of the moon, or lunar eclipses; and those of the sun, or solar eclipses. A lunar eclipse occurs when the earth is between the sun and the moon and its shadow darkens the moon. A solar eclipse occurs when the moon is between the sun and the earth and its shadow moves across the face of the earth.

Activity 3

Create lunar and solar eclipses

Materials

The required materials are ones in activity 2

Procedures

1. Set the apparatus in activity 2
2. Create a solar eclipse. Stand facing the torch and swing the wire around until the Moon casts a shadow on Earth; if necessary, dim the lights. The Moon is now between Earth and the Sun and is blocking the sunshine for some people on Earth. Point out that only people directly in the shadow see a complete eclipse of the Sun. You can show how the shadow moves by slowly rotating the wire.
3. Now create a lunar eclipse. Stand facing the torch and swing the wire so that the Moon is behind Earth. No light should be hitting the Moon: Earth is between the Sun and the Moon, casting a shadow over the entire Moon. Explain that unlike during the solar eclipse, the entire 'night side' of Earth can see the lunar eclipse.

Lunar Eclipses

The earth, lit by the sun, casts a long, conical shadow in space. At any point within that cone the light of the sun is wholly obscured.

A total lunar eclipse occurs when the moon passes completely into the umbra. If it moves directly through the centre, it is obscured for about 2 hours. If it does not pass through the centre, the period of totality is less and may last for only an instant if the moon travels through the very edge of the umbra.

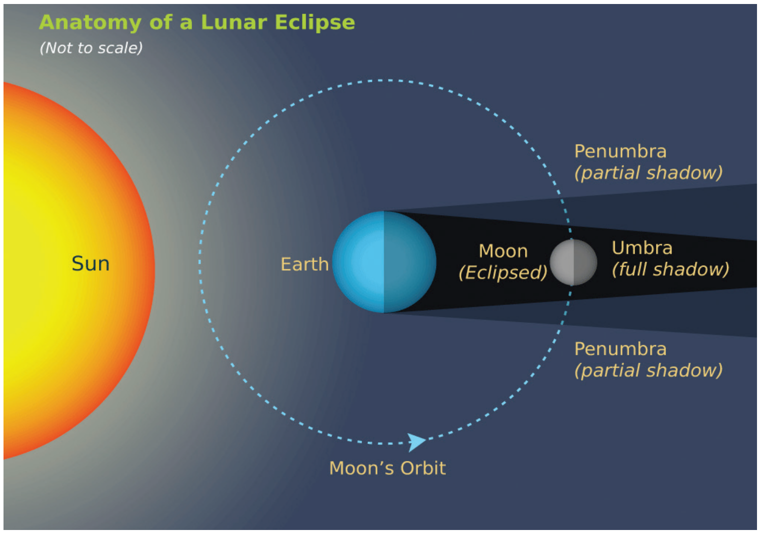


Figure 12.1: Anatomy of lunar eclipse

A partial lunar eclipse occurs when only a part of the moon enters the umbra and is obscured. The extent of a partial eclipse can range from near totality, when most of the moon is obscured, to a slight or minor eclipse, when only a small portion of the earth's shadow is seen on the passing moon. Historically, the view of the earth's circular shadow advancing across the face of the moon was the first indication of the shape of the earth.

Solar eclipses

In areas outside the band swept by the moon's umbra but within the penumbra, the sun is only partly obscured, and a *partial eclipse* occurs.

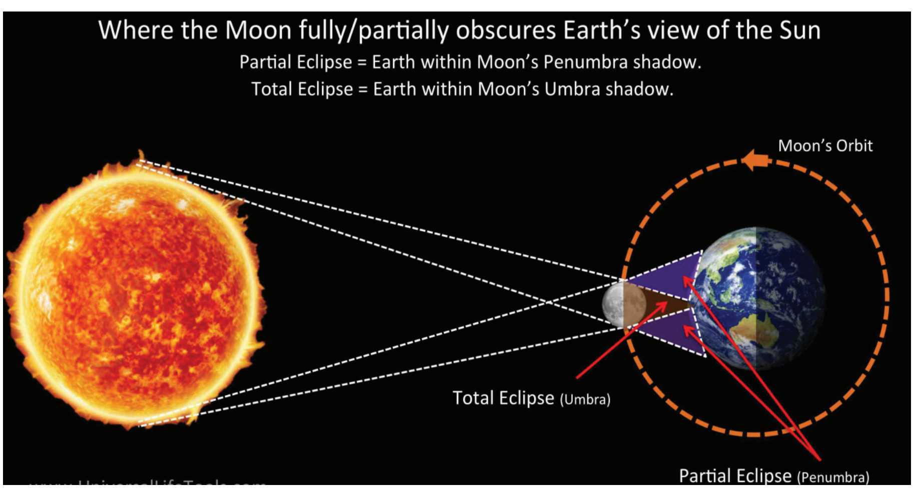


Figure 12.2: Moon phase cards

Phases of the moon

Key Facts about Space and Space Exploration/The Moon:

- There are different phases of the Moon that make it appear a little different every day, but it looks the same again about every four weeks.
- The Moon can sometimes be seen at night and sometimes during the day.

Activity 4

Moon Discussion

Materials:

* "Moon Phases Cards"
* Scissors
* Pencil or crayons (for "Moon Phases Chart")
* "Moon Phases Chart"

To Prepare before

* Print out one "Moon Phases Chart" per learner.

Print out one copy of the "Moon Phases Cards" handout for every 3-5 learners.

Discussion (Key questions)

a) Describe when the best time is to see the moon. Can the moon also be seen during the day?
b) Does the moon look the same every time when you look at it? Explain how it changes.
c) According to you how many days it takes for the moon to travel around the earth and what do we observe as different phases?
d) Hold up the "Moon Phases Cards" and point out the different phases that the moon goes through (Figure 12.3)

Fun facts to share:

- We can only see half of the moon from earth, since the other side is always turned away from us.
- As the moon travels around the earth, we see different fractions of the moon, as it is lit by the sun.
- "Waxing" means growing and is used to describe the moon as it grows from new moon to full moon.

- "Waning" means shrinking and is used to describe the moon as it gets smaller from full moon to new moon.
- The "first quarter" is when the moon has completed ¼ of its orbit around the earth. This is when the moon looks like a "half moon."
- The "last quarter" is when the moon has completed ¾ of its orbit around the earth and also looks like a "half moon" to us.

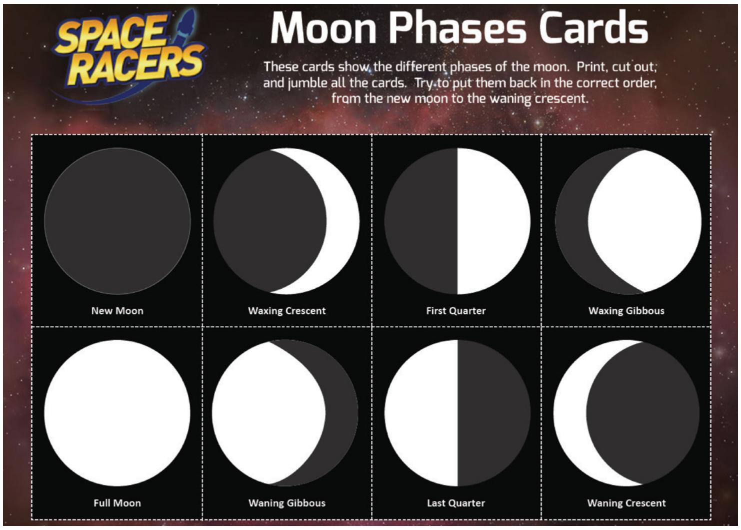


Figure 12.3: Moon phase cards

One revolution of the Moon around Earth takes a little over 27 days 7 hours. The Moon rotates on its axis in this same period of time, so the same face of the Moon is always presented to Earth. Over a period, a little longer than 29 days 12 hours, the Moon goes through a series of phases, in which the amount of the lighted half of the Moon we see from Earth changes. These phases are caused by the changing angle of sunlight hitting the Moon. (The period of phases is longer than the period of revolution of the Moon, because the motion of Earth around the Sun changes the angle at which the Sun's light hits the Moon from night to night).

The solar system

Solar System is constituted by the Sun and everything that orbits the Sun, including the planets and their satellites; the dwarf planets, asteroids, and comets; and interplanetary dust and gas...

Inner planets and outer planets

Activity 5: Role play

Inner and outer planets (Teachers and learners)

Background Information

This lesson focuses on comparing and contrasting the four inner planets with the four outer planets and one dwarf planet. You will first explore the differences using Venn diagrams to establish the groupings in the Solar System. Then you will express these differences in size, temperature and composition artistically through dance and movement. Then, you will create a dance based on information gained from this lesson.

Materials

* Music
* Colourful scarves
* Books
* Posters and charts on the solar system.
* CD/cassette player

Procedure – Venn Diagrams

1. Discussion: How many planets are in our solar system? Name them. Explain that Pluto had been considered a planet, but in August 2006 it was demoted to a dwarf planet.
2. Use the bulletin board, classroom books, and research from previous lessons to complete the Venn diagrams on pages 11-12 of the Astronomer Journal. You should place the name of each planet in its appropriate location on the Venn diagram.
3. The Terrestrial Planets (or INNER PLANETS) have compact and rocky surfaces and the Gaseous Planets (or OUTER PLANETS) have a gaseous composition.
4. Look at and discuss the differences between the four inner planets, the four outer planets and dwarf planet.

a) Size: Inner - small Outer - big (excluding Pluto)
b) Temperature: Inner - hot Outer – cold
c) Composition: Inner - rocky Outer – gaseous (excluding Pluto)

Procedure – Interpretive Dance

1. Remain sitting after discussion and demonstrate a small movement with your hand, then a big movement with your hand. Repeat with head, then shoulders, foot, elbow, etc. staying in your own personal/self space.
2. Next, move with small and big movements while travelling around the room in general/shared space. Emphasize NO TOUCHING OR BUMPING!!! Encourage movement on different levels (high, middle, low).
3. Teacher calls out, "Inner Planets" and the dancers respond by dancing with SMALL movements. Then, Teacher calls out, "Outer Planets" and the dancers respond with BIG movements. Teacher continues to alternate between "Inner and Outer Planets".
4. BIG/SMALL DANCE - Divide the class into small groups so you can watch each other. One group at a time dances while the other groups watch as audience members. Dancers begin in a frozen shape and begin moving when the music starts. Dancers should move either small or big in correspondence to what the teacher calls out (alternating between "Inner" and "Outer" Planets). When music stops, learners freeze in a SMALL or BIG shape.
5. Across the Floor Exercise - Identify one wall as the SUN and the opposite wall as furthest outer planet.
6. Review the order of the planets. Divide class into groups (size dependent on size of dance space). With music, one group at a time begins on one side of the room and moves to the other side of the room, changing the size of their movement (SMALL or BIG) representing the size of each planet they pass through along the way. Repeat travelling the other way. Try again, this time incorporating temperature changes that correspond with the planets.

7. ROCKY vs GASEOUS - Two groups dance at a time. Group One - OUTER

8. PLANETS - dances with light, flowing movement, demonstrating the composition of the outer planets, using scarves as a prop. Group Two - INNER PLANETS - dances with strong, hard, rocky, abrupt movement, demonstrating the composition of the inner planets. Drums or rhythm sticks may be used by the dancers as a prop.

9. Culminating Activity - Divide the class into small groups. Based on the previous activities, each group must work together to create its own dance about the Solar System's Inner and Outer Planets. Each group must display a clear beginning, middle and end to their dance and must contain at least one or two elements from the lesson. After working for 10-15 minutes in small groups, have each group perform their dance for the other groups. Remind the groups which are watching to be a good, respectful audience by sitting quietly without talking, laughing or playing around and to be encouraging to your classmates.

Expected Results & Explanations

Upon completion of this activity, you should understand that the 8 planets can be categorised into 2 groups quite easily. You should notice that Pluto does not fall into either of these categories. Instead, Pluto may be the first of many dwarf planets.

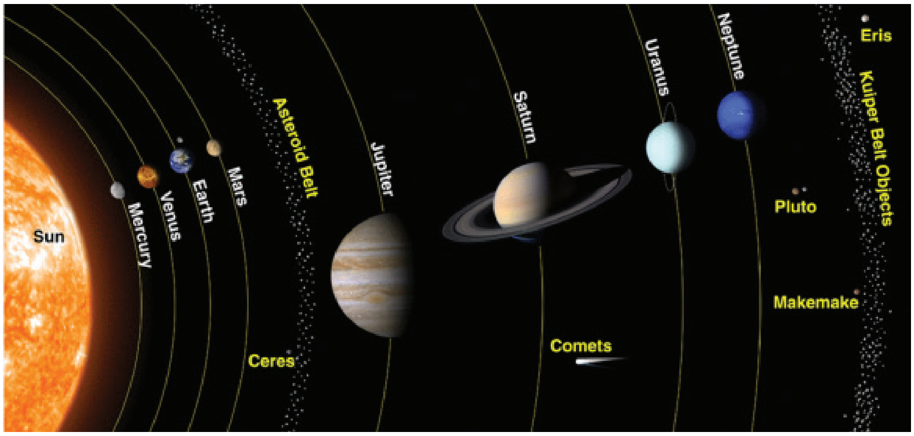


Figure 12.4: Artist's impression of the solar system showing the inner planets (Mercury to Mars), the outer planets (Jupiter to Neptune) and beyond

In our Solar System, astronomers often divide the planets into two groups — the inner planets and the outer planets. The inner planets are closer to the Sun and are smaller and rockier. The outer planets are further away, larger and made up mostly of gas.

The inner planets (in order of distance from the sun, closest to furthest) are Mercury, Venus, Earth and Mars. After an asteroid belt come the outer planets, Jupiter, Saturn, Uranus and Neptune. The interesting thing is, in some other planetary systems discovered, the gas giants are actually quite close to the sun.

This makes predicting how our Solar System formed an interesting exercise for astronomers. Conventional wisdom is that the young Sun blew the gases into the outer fringes of the Solar System and that is why there are such large gas giants there. However, some extrasolar systems have "hot Jupiters" that orbit close to their Sun.

The Inner Planets

The four inner planets are called terrestrial planets because their surfaces are solid (and, as the name implies, somewhat similar to Earth — although the term can be misleading because each of the four has vastly different environments). They're made up mostly of heavy metals such as iron and nickel, and have either no moons or few moons. Below are brief descriptions of each of these planets based on this information from National Aeronautic and Space Authority of the USA (NASA).

Mercury

Mercury is the smallest planet in our Solar System and also the closest. It rotates slowly (59 Earth days) relative to the time it takes to rotate around the sun (88 days). The planet has no moons, but has a tenuous atmosphere (exosphere) containing oxygen, sodium, hydrogen, helium and potassium. The NASA MESSENGER (Mercury Surface, Space Environment, Geochemistry, and Ranging) spacecraft is currently orbiting the planet.

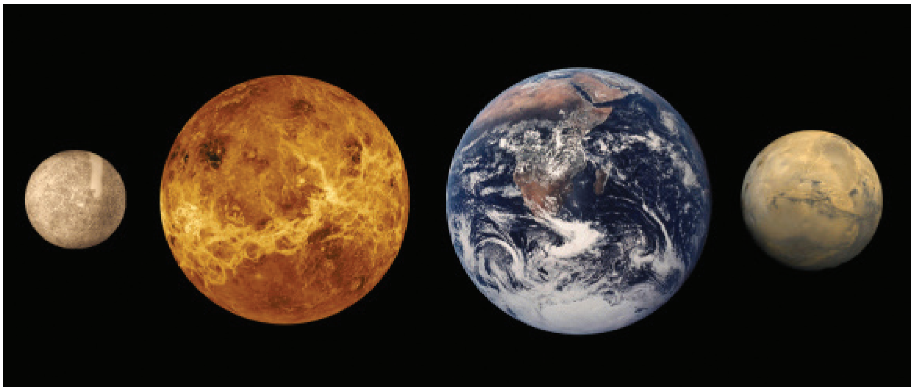


Figure 12.5: The terrestrial planets of our Solar System at approximately relative sizes. From left: Mercury, Venus, Earth and Mars

Venus

Venus was once considered a twin planet to Earth, until astronomers discovered its surface is at a lead-melting temperature of 900 degrees Fahrenheit (480 degrees Celsius). The planet is also a slow rotator, with a 243-day long Venusian day and an orbit around the sun at 225 days. Its atmosphere is thick and contains carbon dioxide and nitrogen. The planet has no rings or moons and is currently being visited by the European Space Agency's Venus Express spacecraft.

Earth

Earth is the only planet with life as we know it, but astronomers have found some nearly Earth-sized planets outside of our solar system in what could be habitable regions of their respective stars. It contains an atmosphere of nitrogen and oxygen, and has one moon and no rings. Many spacecraft circle our planet to provide telecommunications, weather information and other services.

Mars

Mars is a planet under intense study because it shows signs of liquid water flowing on its surface in the ancient past. Today, however, its atmosphere is a wispy mix of carbon dioxide, nitrogen and argon. It has two tiny moons (Phobos and Deimos) and no rings. A Mars day is slightly longer than 24 Earth hours and it takes the planet about 687 Earth days to circle the Sun. There's a small fleet of orbiters and rovers at Mars right now, including the large NASA Curiosity rover that landed in 2012.

The Outer Planets

Sometimes called *Jovian planets* or gas giants are huge planets swaddled in gas. They all have rings and all of plenty of moons each. Despite their size, only two of them are visible without telescopes: Jupiter and Saturn. Uranus and Neptune were the first planets discovered since antiquity, and showed astronomers the solar system was bigger than previously thought. Below are brief descriptions of each of these planets based on this information from NASA.

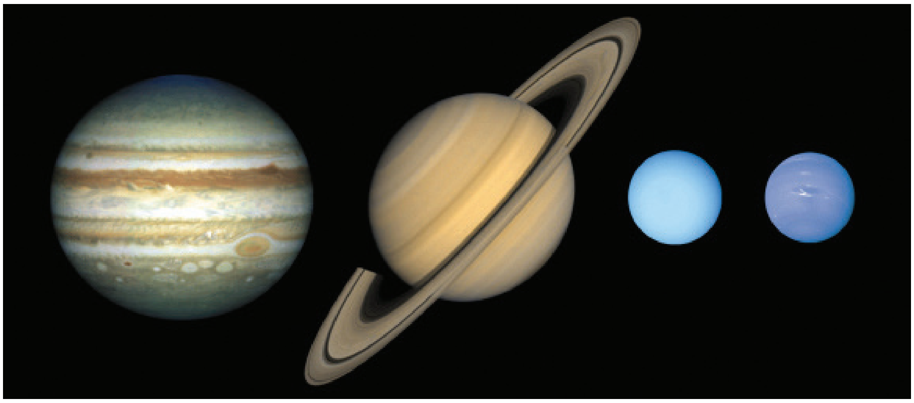


Figure 12.6: The outer planets of our Solar System at approximately relative sizes. From left: Jupiter, Saturn, Uranus and Neptune

Uranus was first discovered by William Herschel in 1781. The planet's day takes about 17 Earth hours and one orbit around the Sun takes 84 Earth years. Its mass contains water, methane, ammonia, hydrogen and helium surrounding a rocky core. It has dozens of moons and a faint ring system. There are no spacecraft slated to visit Uranus right now; the last visitor was Voyager 2 in 1986.

Jupiter

Jupiter is the largest planet in our Solar System and spins very rapidly (10 Earth hours) relative to its orbit of the sun (12 Earth years). Its thick atmosphere is mostly made up of hydrogen and helium, perhaps surrounding a terrestrial core that is about Earth's size. The planet has dozens of moons, some faint rings and a Great Red Spot, a raging storm happening for the past 400 years at least (since we were able to view it through telescopes). NASA's Juno spacecraft is en route and will visit there in 2016.

Saturn

Saturn is best known for its prominent ring system, seven known rings with well-defined divisions and gaps between them. How the rings got there is one subject under investigation. It also has dozens of moons. Its atmosphere is mostly hydrogen and helium, and it also rotates quickly (10.7 Earth hours) relative to its time to circle the Sun (29 Earth years). Saturn is currently being visited by the Cassini spacecraft, which will fly closer to the planet's rings in the coming years.

Uranus

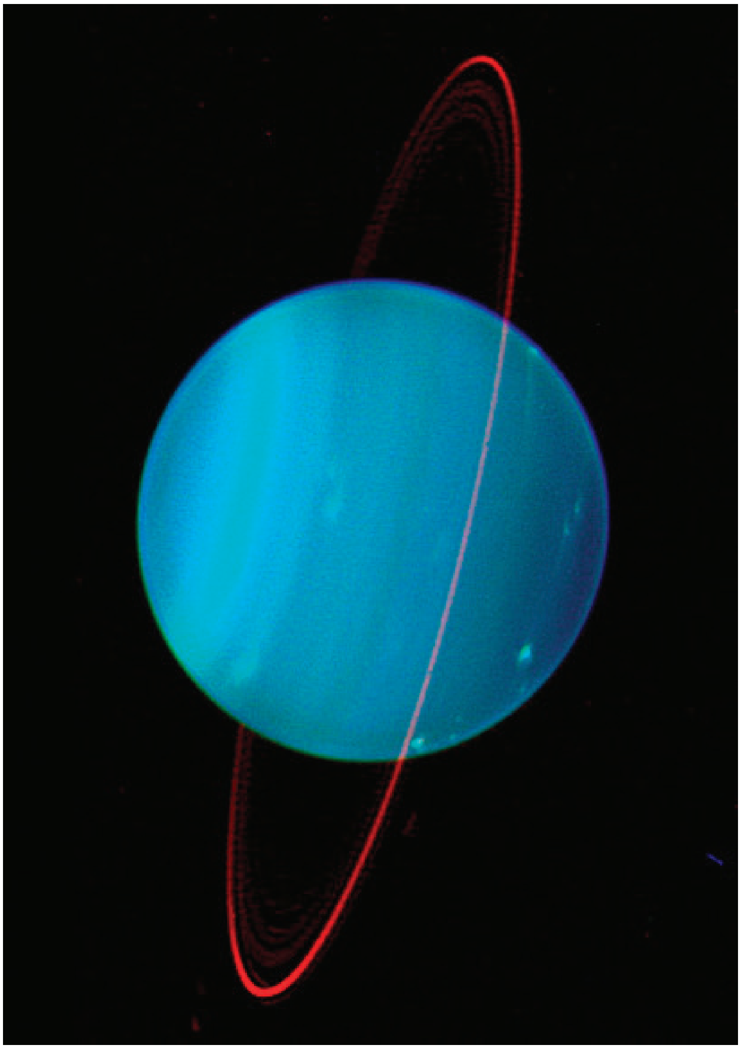


Figure 12.7: Near-infrared views of Uranus reveal its otherwise faint ring system, highlighting the extent to which it is tilted

Neptune

Neptune is a distant planet that contains water, ammonia, methane, hydrogen and helium and a possible Earth-sized core. It has more than a dozen moons and six rings. The only spacecraft to ever visit it was NASA's Voyager 2 in 1989.

Comets

Figure 12.8: Comet Hale-Bopp

Activity 6

Learn about comets

Read notes below, they talk about comets. Understand them and answer the following questions:

* What is a comet?
* What happens when a comet is heated by the sun?
* How ancient people were considering comets?

Comet, small icy body in space that sheds gas and dust. Like rocky asteroids, icy comets are ancient objects left over from the formation of the solar system about 4.6 billion years ago. Some comets can be seen from Earth with the unaided eye.

Comets typically have highly elliptical (oval-shaped), off-centre orbits that swing near the Sun. When a comet is heated by the Sun, some of the ice on the comet's surface turns into gas directly without melting. The gas and dust freed from the ice can create a cloud (coma) around the body (nucleus) of the comet. More gas and dust erupt from cracks in the comet's dark crust. High-energy charged particles emitted by the Sun, called the *solar wind*, can carry the gas and dust away from the comet as a long tail that streams into space. Gas in the tail becomes ionized and glows as bluish plasma, while dust in the tail is lit by sunlight and looks yellowish. This distinctive visible tail is the origin of the word *comet*, which comes from Greek words meaning "long-haired star."

Humans have observed comets since prehistoric times. Comets were long regarded as supernatural warnings of calamity or signs of important events. Astronomers and planetary scientists now study comets for clues to the chemical makeup and early history of the solar system, since comets have been in the deep-freeze of outer space for billions of years. Materials in comets may have played a major role in the formation of Earth and the origin of life. Catastrophic impacts by comets may also have affected the history of life on Earth, and they still pose a threat to humans.

Meteorites

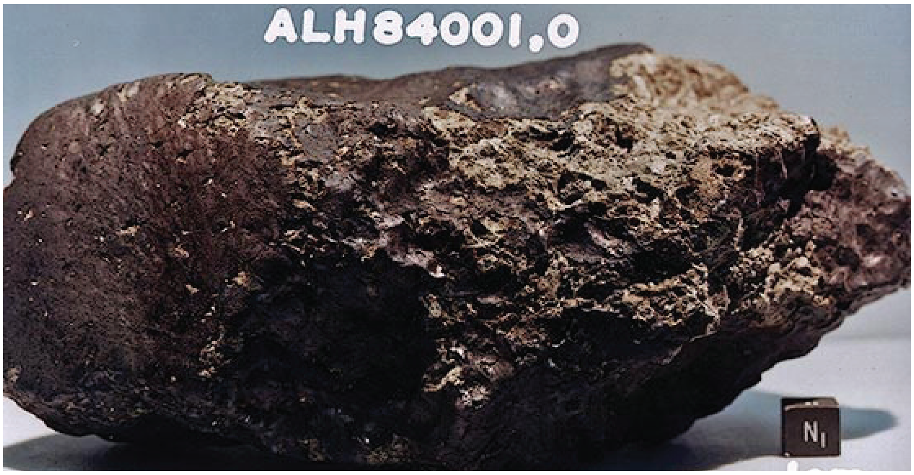


Figure 12.9: Meteorite from Mars

Activity 7

Learn about meteorites

Read notes below, they talk about meteorites. Understand them and answer the following questions:

* What is a meteorite?
* In how many types meteorites found on the earth are classified and this classification depends on what?
* Recent studies suggest that meteorites are from where and how was it done?
* Give a summary of what you have read on this topic.

A meteorite is a **rock from outer space**; it's a piece of rock that has reached Earth from outer space. It can also be defined as a **fiery mass of rock from space,** a mass of rock from space that burns up after entering the Earth's atmosphere.

Meteorite, meteor that reaches the surface of Earth or of another planet before it is entirely consumed. Meteorites found on Earth are classified into types, depending on their composition: *irons,* those composed chiefly of iron, a small percentage of nickel, and traces of other metals such as cobalt; *stones,* stony meteors consisting of silicates; and *stony irons,* containing varying proportions of both iron and stone.

Although most meteorites are now believed to be fragments of asteroids or comets, recent geochemical studies have shown that a few Antarctic stones came from the Moon and Mars, from which they presumably were ejected by the explosive impact of asteroids. Asteroids themselves are fragments of planetesimals, formed some 4.6 billion years ago, while Earth was forming. Irons are thought to represent the cores of planetesimals, and stones (other than the aforementioned Antarctic ones) the crust. Meteorites generally have a pitted surface and fused, charred crust. A meteorite that landed in Texas in 1998 was found to have water trapped in its rock crystals. The discovery helped scientists theorize about whether water exists in other parts of the solar system.

Large meteorites strike Earth with tremendous impact, creating huge craters. The largest known meteorite, estimated to weigh about 60 metric tons, is situated at Hoba West near Grootfontein, Namibia. The next largest, weighing more than 31 metric tons, is the Ahnighito (the Tent); it was discovered, along with two smaller meteorites, in 1894 near Perlernerit (Cape York), Greenland, by American explorer Robert Edwin Peary. Composed chiefly of iron, the three masses had long been used by the Inuit as a source of metal for the manufacture of knives and other weapons. Peary brought the Ahnighito to the United States, and it is now on display at the American Museum of Natural History in New York City. The three largest known impact structures are located in Vredefort, South Africa; Sudbury, Canada (north of Lake Huron); and off the coast of the Yucatán Peninsula of Mexico. The original craters from these impacts have eroded away, but the remaining structures indicate that they were all about 300 km (about 190 mi) in diammeter.

On the figure 12.9, collisions between the planet Mars and asteroids have blasted chunks of the planet into space. Occasionally, a piece of Mars will strike the Earth, as this meteorite did about 13,000 years ago. Astronomers believe that this meteorite, called ALH84001, was blasted off of Mars about 16 million years ago.

Asteroids

Figure 12.10: Three asteroids

Activity 8

Learn about asteroids

Read notes below, they talk about asteroids. Understand them and answer the following questions:

* What is an asteroid?
* What is the range of the size of asteroids?
* Give a summary about what you read.

Asteroid, small rocky or metallic body that orbits the Sun. Hundreds of thousands of asteroids exist in the solar system. Asteroids range in size from a few metres to over 500km wide. They are generally irregular in shape and often have surfaces covered with craters. Like icy comets, asteroids are primitive objects left over from the time when the planets formed, making them of special interest to astronomers and planetary scientists.

On the figure 12.10, Asteroid Mathilde, *left,* is the third and the largest asteroid ever to be viewed at close range. The Near Earth Asteroid Rendezvous (NEAR) spacecraft flew by Mathilde in late June 1997. Asteroids Gaspra and Ida, centre *and right,* photographed by the Galileo orbiter in 1991 and 1993, respectively, are smaller and more oblong-shaped than Mathilde. The three asteroids are partially obscured by shadows.

Most asteroids are found between the orbits of the planets Mars and Jupiter in a wide region called the *asteroid belt*. Scientists think Jupiter's gravity prevented rocky objects in this part of the solar system from forming into a large planet. The giant planet Jupiter's gravity also helped throw objects out of the asteroid belt. The hundreds of thousands of asteroids now in the asteroid belt represent only a small fraction of the original population.

Figure 12.11: Asteroid Collision with Earth

Thousands of asteroids have orbits that lie outside the asteroid belt. Some of these asteroids have paths that cross the orbit of Earth. Many scientists think that an asteroid that hit Earth 65 million years ago caused the extinction of the dinosaurs. Because asteroids can pose a danger to people and other life on Earth, astronomers track asteroids that come near our planet. Space scientists are also studying ways to deflect or destroy an asteroid that might strike Earth in the future.

Many scientists believe that a large asteroid or comet struck Earth about 65 million years ago, changing the Earth's climate enough to kill off the dinosaurs.

Kepler's laws

Activity 9

Investigating Kepler's law of planetary motion

Materials

* Sheet of paper
* Cardboard
* Pencil
* Tacks
* Calculator

Procedure

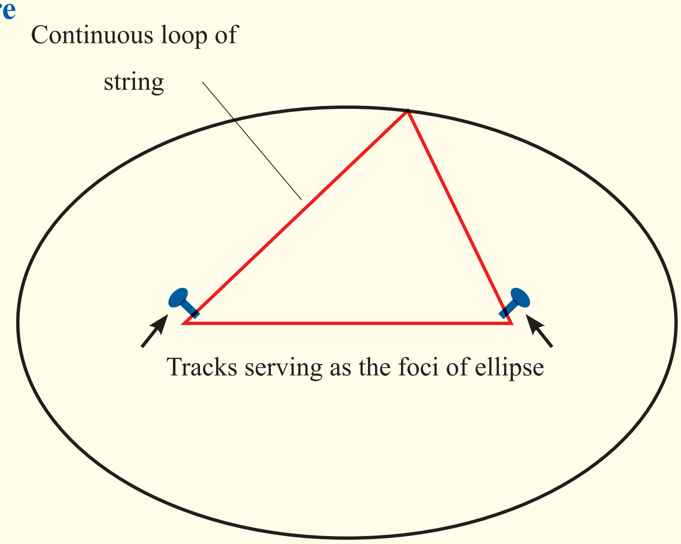


Figure 12.12: Construction of an eclipse

a) ***Construct an ellipse***. An ellipse can easily be constructed using a pencil, two tacks, a string, a sheet of paper and a piece of cardboard. Tack the sheet of paper to the cardboard using the two tacks. Then tie the string into a loop and wrap the loop around the two tacks. Take your pencil and pull the string until the pencil and two tacks make a triangle (see diagram at the right). Then begin to trace out a path with the pencil, keeping the string wrapped tightly around the tacks. The resulting shape will be an ellipse. The two other points (represented here by the tack locations) are known as the **foci** of the ellipse. The motion of the pencil is the motion of the planet about an eventual position of the sun at one tack.

b) In the diagram below are the sun and the Earth turning about it. As can be observed, the areas formed when the earth is closest to the sun can be approximated as a wide but short triangle; whereas the areas formed when the earth is farthest from the sun can be approximated as a narrow but long triangle. Can we confirm that these areas can be of same size? Why?

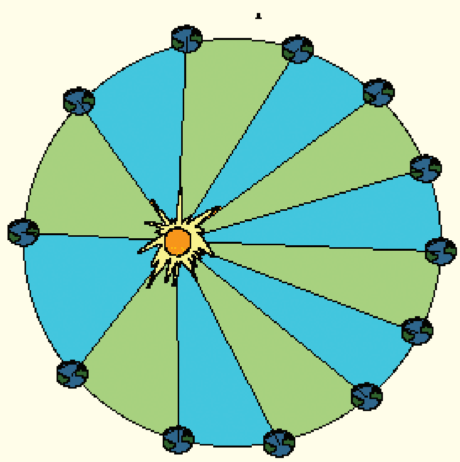


Figure 12.13: An imaginary line drawn from the sun to any planet sweeps out equal areas in equal amounts of time

c) The data given below are for the planetary motion. They represent, the planet, the period of rotation of the planet about the Sun, the average distance from the sun to the planet and a column with no value of the ratio of the square of the period and the cube of the average distance. Here the time is in second [s] and the distance in meter [m].

- Calculate and fill the value of the ratio of the squares of the periods to the cubes of their average distances from the sun.
- Compare these ratios for the two planets (Earth and Mars).

Planet	Period(s)	Average Distance (m)	T^2/R^3 (s^2/m^3)
Earth	3.156×10^7	1.4957×10^{11}	___
Mars	5.93×10^7	2.278×10^{11}	___

d) Consider again the table below; here the period is in year [yr] the distance in astronomic unit [AU]

Planet	Period [yr]	Average Distance [AU]	T^2/R^3 [yr^2/AU^3]
Mercury	0.241	0.39	___
Venus	.615	0.72	___
Earth	1.00	1.00	___
Mars	1.88	1.52	___
Jupiter	11.8	5.20	___
Saturn	29.5	9.54	___
Uranus	84.0	19.18	___
Neptune	165	30.06	___
Pluto	248	39.44	___

e) Calculate and fill the value of the ratio of the squares of the periods to the cubes of their average distances from the sun.
- Compare these ratios for the planets
- What is the final conclusion that we can find for the last column?

In astronomy, *Kepler's laws of planetary motion* are three scientific laws describing the motion of planets around the Sun.

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

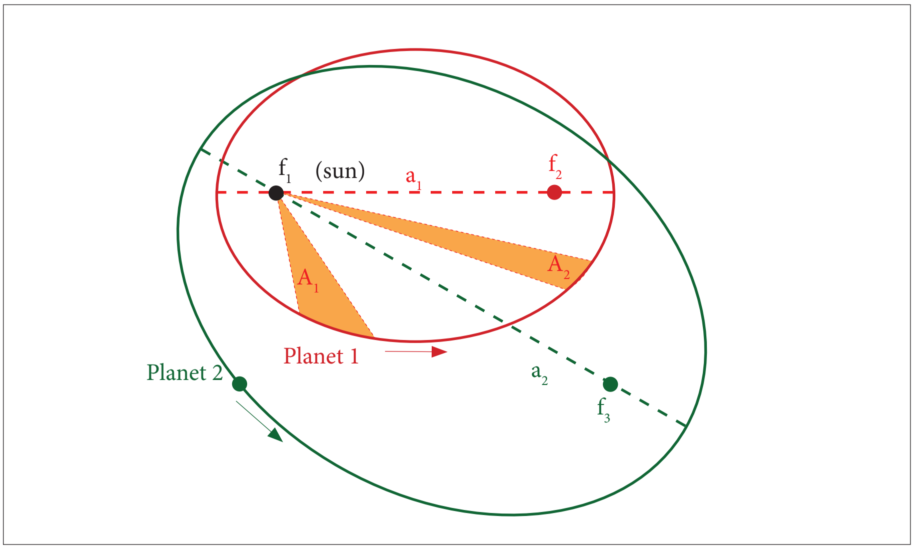


Figure 12.14: Illustration of Kepler's three laws with two planetary orbits

Exercises

4. Venus is at average distance of 1.08×10^8 km from a sun. Estimate the length of the Venusian year using the fact that the earth is 1.49×10^8 km.

5. The planet Mars of massm describes around the sun of mass M, an ellipse of mean radius of orbit $a = 230 \times 10^6$ km in 1.8 years. The satellite Deimos of mass m^1 describes around the planet mars an ellipse of mean radius $a^1 = 28 \times 10^6$ km in 30h. Find the mass of the planet mars, given that $M = 2 \times 10^{30k}$g and 1year is 365days.

6. We actually know fifteen satellites revolving around the planet Uranus. Let us denote the period of revolution of satellite by T and the mean distance to the centre of the planet by r. The five bigger than others have the following characteristics:

Satellite	Oberon	Titania	Umbriel	Ariel	Miranda
T (J)	13.46	8.706	4.144	2.520	1.414
r (10^3 km)	582.6	435.8	266.0	191.2	129.8

a) (i) For each satellite, calculate T^2 and r^3, (ii) Assume $T^2 = y$ and $r^3 = x$. Trace the graph of $y = f(x)$. What conclusion related to the nature of the graph can you get?

b) (i) Calculate the slope of the plotted segment. (ii) Deduce the mass of Uranus.

Stars patterns: Constellations

Activity 10

Learn about stars pattern

Read notes below and do research on internet about constellations and answer the following questions:

* What is a constellation?
* Up to now how many constellations are known?
* Give a list of at least 30 constellations known.

Away from city lights on a clear, moonless night, the naked eye can see 2000-3000 stars. As you look at these stars, your mind may group them into different shapes or patterns. People of nearly every culture throughout history have looked at the stars and given names to shapes they saw, they even invented stories to go with them. The pattern that the Greeks named Orion, the hunter, was also seen by the ancient Chinese who saw it as a supreme warrior named Shen. The Chemehuevi Native Americans of the California desert saw the same group of stars as a line of three sure-footed mountain sheep.

The patterns of stars seen in the sky are usually called *constellations*, although more accurately, a group of stars that forms a pattern in the sky is called an *asterism*. Astronomers use the term constellation to refer to an area of the sky.

The International Astronomical Union (IAU) divides the sky into 88 official constellations with exact boundaries, so that every place in the sky belongs within a constellation. Most of the constellations in the northern hemisphere are based on the constellations invented by the ancient Greeks, while most in the southern hemisphere are based on names given to them by seventeenth century European explorers.

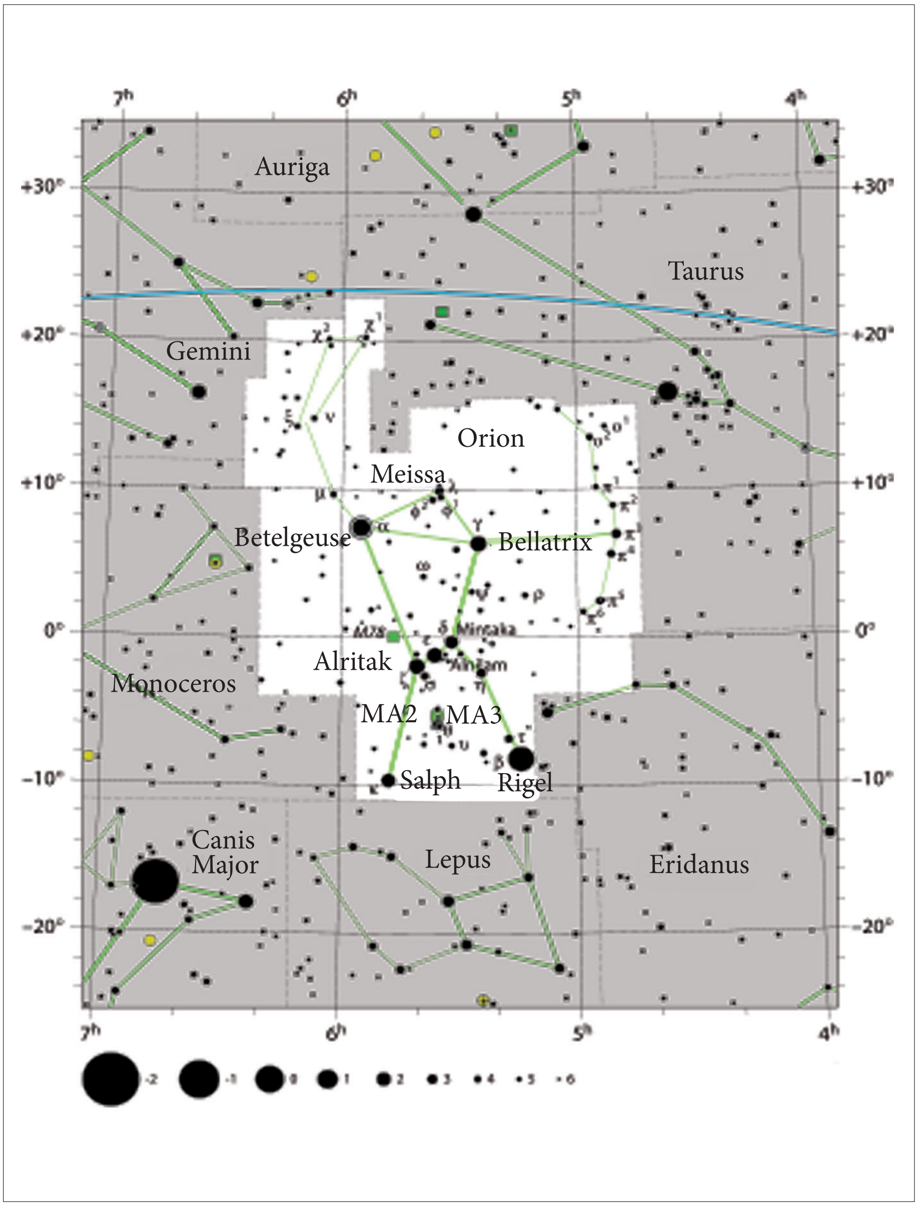


*Figure 12.15: The constellation **Orion** is one of the most recognisable in the night sky*

*Figure 12.16: The constellation **Orion** is one of the most recognisable in the night sky*

Thus, any given point in a celestial coordinate system can unambiguously be assigned to a constellation. It is usual in astronomy to give the constellation in which a given object is found along with its coordinates in order to convey a rough idea in which part of the sky it is located. For example, saying the Crab Nebula is in Taurus immediately conveys it is close to the ecliptic and best observable in winter.

Celestial coordinates

A basic requirement for studying the heavens is determining where in the sky things are. To specify sky positions, astronomers have developed several *coordinate systems*. Each uses a coordinate grid projected on the Celestial Sphere, in analogy to the Geographic coordinate system used on the surface of the Earth. The coordinate systems differ only in their choice of the *fundamental plane*, which divides the sky into two equal hemispheres along a great circle. (The fundamental plane of the geographic system is the Earth's equator). Each coordinate system is named for its choice of fundamental plane.

Equatorial coordinate system

Activity 11

Research on Equatorial coordinate system

The **equatorial system** is a coordinate system that is used to locate a body in the sky using declination and right ascension. Search on internet and answer the followings:

a) What is the difference between equatorial and geographic coordinates systems?
b) What is declination, right ascension?
c) The declination is in which unit? The inclination is in which unit?
d) Which correspondence is between the unit of declination and the one of right ascension?
e) Explain what you found in your research.

The *Equatorial coordinate system* is probably the most widely used celestial coordinate system. It is also the most closely related to the Geographic coordinate system, because they use the same fundamental plane, and the same poles. The projection of the Earth's equator onto the celestial sphere is called the *Celestial Equator.* Similarly, projecting the geographic Poles onto the celestial sphere defines the North and South *Celestial Poles.*

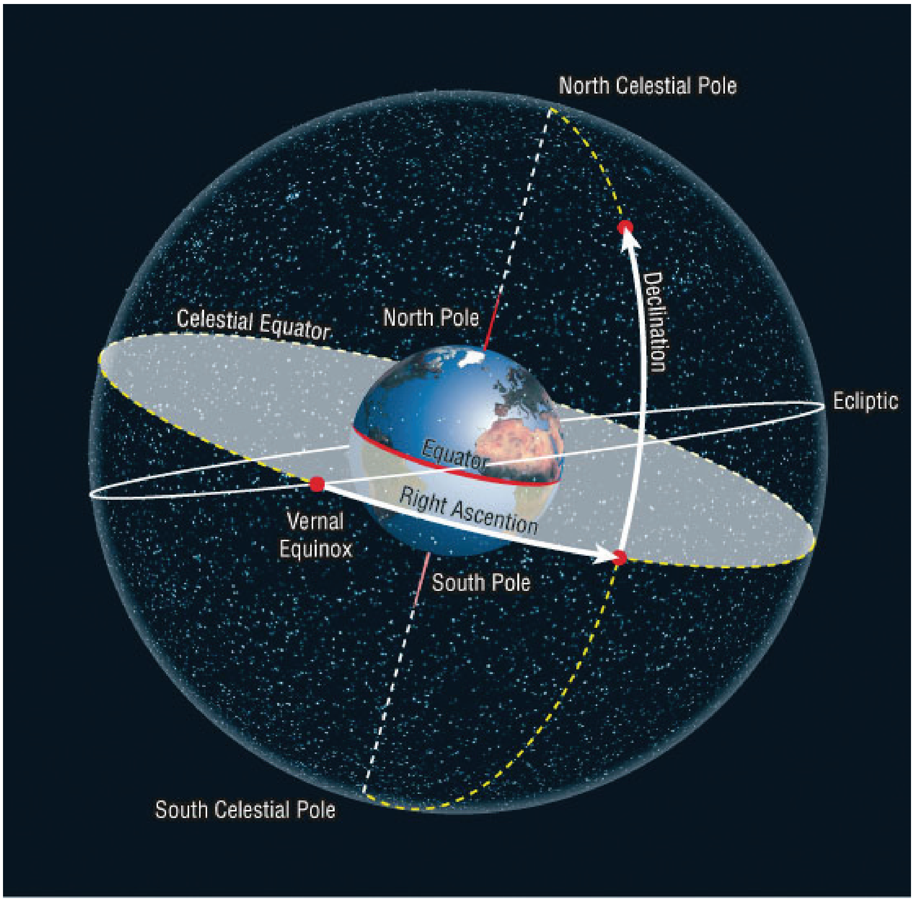


Figure 12.17: Equatorial coordinate system

Horizontal coordinates system

Activity 12

Reading and understanding about Horizontal coordinate system

Read notes below and give a summary of what you have read.

The Horizontal coordinate system uses the observer's local *horizon* as the Fundamental Plane. This conveniently divides the sky into the upper hemisphere that you can see, and the lower hemisphere that you can't (because the Earth is in the way). The pole of the upper hemisphere is called the *Zenith*. The zenith is a point in the sky that is directly above the observer. The pole of the lower hemisphere is called the *nadir*. The angle of an object above or below the horizon is called the *Altitude* (Alt for short). The angle of an object around the horizon (measured from the North point, toward the East) is called the *Azimuth*. The Horizontal Coordinate System is sometimes also called the **Alt/Az** Coordinate System.

The Horizontal Coordinate System is fixed to the Earth, not the Stars. Therefore, the Altitude and Azimuth of an object changes with time, as the object appears to drift across the sky. In addition, because the Horizontal system is defined by your local horizon, the same object viewed from different locations on Earth at the same time will have different values of Altitude and Azimuth.

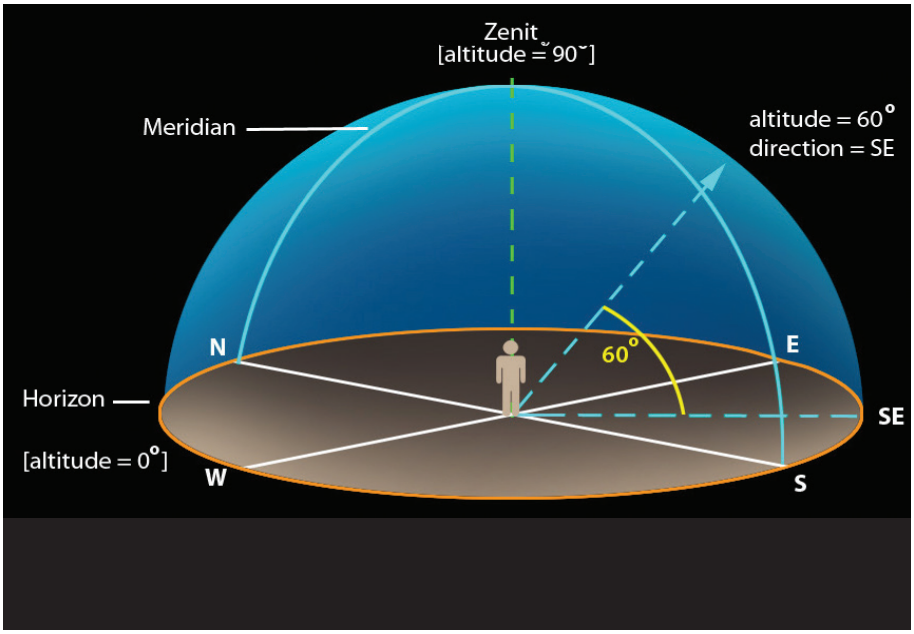


Figure 12.18: Horizontal coordinate system

Horizontal coordinates are very useful for determining the Rise and Set times of an object in the sky. When an object has Altitude = 0 degrees, it is either Rising (if its Azimuth is < 180 degrees) or Setting (if its Azimuth is > 180 degrees).

Normally, there are several celestial coordinates; we have also, the ecliptic coordinate system, the galactic coordinate system.

Activity 13

Choose the most suitable answer from the options

1. The angular distance of an object around the horizon, starting from the north, and measured eastwards around the horizon to a point on the horizon directly below the object's location on the celestial sphere is known as the:

   a) Horizon
   b) Latitude
   c) Longitude
   d) Altitude
   e) Azimuth

2. The angular distance above the celestial horizon is called the:

   a) Horizon
   b) Latitude
   c) Longitude
   d) Altitude
   e) Azimuth

3. This is a point in the sky that's located directly above the observer:

   a) Horizon
   b) Latitude
   c) Longitude
   d) Azimuth
   e) Zenith