Sub-math Marking Guide

Section A

Answer all the questions in this section

1. From $n^{th} = a + (n-1)d$

For the third term,

$a + (3-1)d = 12$ — developing and equating the two equations — m_1

$a + 2d = 12 \ldots\ldots \ldots\ldots\ldots\ldots \ldots\ldots\ldots 1$

For the seventh term,

$a + (7-1)d = 32$

$a + 6d = 32 \ldots\ldots\ldots \ldots\ldots\ldots \ldots\ldots\ldots 2$

$eqn1 - eqn2$

$$\begin{array}{l} \ (a + 2d = 12) \\ -(a + 6d = 32) \\ \hline \ 0 - 4d = -20 \end{array}$$

obtaining the difference — B_1

$d = 5$

From $eqn1$

$a + 2(5) = 12$ — obtaining the first term — B_1

$a = 2$

From $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$ — using the formulae correctly — m_1

$S_{10} = 5(49)$

$S_{10} = 245$ — writing the correct sum — A_1

2. Matrix

a. $AB = \begin{pmatrix} -1 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} -6-6 & -3-3 \\ 6+2 & 3+1 \end{pmatrix}$ — substituting correctly — $B_1 m_1$

$AB = \begin{pmatrix} -12 & -6 \\ 8 & 4 \end{pmatrix}$ — writing the correct matrix — A_1

b. $2A - B = 2\begin{pmatrix} -1 & -3 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$

$2A - B = \begin{pmatrix} -2 & -6 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$ — substituting correctly — B_1

$2A - B = \begin{pmatrix} -8 & -9 \\ 0 & 1 \end{pmatrix}$ — writing the correct matrix — A_1

3. Vectors

a. $2a + b = 2\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ — substituting and writing — B_1

$2a + b = \begin{pmatrix} 10 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ — vectors in column form — m_1

$2a + b = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$2a + b = 7\boldsymbol{i} + 3\boldsymbol{j}$ — writing the correct vector — A_1

b. $a \cdot b = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ — correctly using dot product — m_1

$a \cdot b = -15 - 14$

$a \cdot b = -29$ — writing the correct solution — A_1

4. $\begin{pmatrix} 3 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Adjoint $= \begin{pmatrix} -2 & -1 \\ -3 & 3 \end{pmatrix}$ determinant $= -6 - 3$ — obtaining the adjoint and — B_1B_1

$= -9$ — determinant

$-{}^1\!/_9 \begin{pmatrix} 3 & 2 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -{}^1\!/_9 \begin{pmatrix} -2 & -1 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$-{}^1\!/_9 \begin{pmatrix} -9 & 0 \\ 0 & -9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -{}^1\!/_9 \begin{pmatrix} -9 \\ -18 \end{pmatrix}$ — correctly substituting — B_1

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\therefore x = 1 \,\&\, y = 2$ — writing correct solution — A_1A_1

5.

8 boys	6 girls
3 boys	4 girls

Analyzing and drawing a table — B_1B_1

Number of committees $= C_3^8 \times C_4^6$ — using combinations correctly — B_1m_1

$= 1120$ committees. — Writing correct solution — A_1

6. Probability

a. $P\left({}^C\!/_B\right) = \frac{P(B \cap C)}{P(C)}$

$P\left({}^C\!/_B\right) = \frac{9}{20} \,\&\, P(C) = \frac{4}{5}$

$\frac{9}{20} = P(B \cap C) \times \frac{5}{4}$ — substituting probabilities — B_1m_1

$36 = 100P(B \cap C)$ — correctly in the formulae

$P(B \cap C) = \frac{9}{25}$ — writing a correct solution — A_1

b. $P\left({}^A\!/_C\right) = \frac{P(A \cap C)}{P(C)}$ — substituting probabilities — B_1

$P\left({}^A\!/_C\right) = \frac{7}{25} \times \frac{5}{4}$ — correctly in the formulae

$P\left({}^A\!/_C\right) = \frac{7}{20}$ — writing a correct solution — A_1

7. $y = 2x^3 - 4x^2 + 5x - 6$

$\frac{dy}{dx} = 6x^2 - 8x + 5$ — obtaining the derivative — B_1m_1

$\left.\frac{dy}{dx}\right|_{(-2,\ 4)} = 6(-2)^2 - 8(-2) + 5$ — substituting the point in — B_1B_1

$\left.\frac{dy}{dx}\right|_{(-2,\ 4)} = 45$ — the derivative

$\therefore$ the gradient of the curve at P(-2, 4) is 45 — writing the correct gradient — A_1

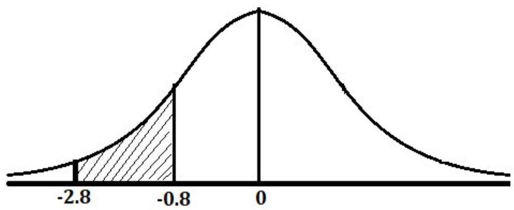
8. X ~ N (72, 25)

$Z = \frac{X-\mu}{\sigma}$ from $P(30 < x \leq 60)$

$= \frac{30-72}{\sqrt{225}}, \frac{60-72}{\sqrt{225}}$ standardizing correctly B_1B_1

$= -2.8, = -0.8$

$P(-2.8 < z \leq -0.8)$



drawing a correct graph B_1

$P(-2.8 < z \leq -0.8) = P(0 > z > 2.8) - P(0 > z > 0.8)$

$= 0.4974 - 0.2881$ correctly reading from the table B_1

$= 0.2093$ (tab) writing a correct value A_1

Section B

Answer four questions with at least one question from each part

Part I

9. Indices and logs

a. Indices

i. $9^x \cdot 3^{(x+1)} = 81$

$3^{2x} \cdot 3^{(x+1)} = 3^4$ obtaining the same base B_1m_1

$3^{2x+x+1} = 3^4$

Equating powers

$3x + 1 = 4$ equating powers B_1

$3x = 4 - 1$

$3x = 3$

$x = 1$ obtaining the correct solution A_1

ii. $2 + 3\log x = \log 0.1$

$2\log_{10} 10 + 3\log_{10} x = \log_{10} 0.1$

$\log_{10} 10^2 + \log_{10} x^3 = \log_{10} 0.1$ using power law m_1

$\log_{10}(100x^3) = \log_{10} 0.1$ using other laws of logarithms B_1

$100x^3 = 0.1$

$x^3 = \frac{1}{1000}$

$(x^3)^{\frac{1}{3}} = \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$ introducing cube root both sides B_1

$x = \frac{1}{10}$ obtaining the correct solution A_1

b. $x^2 + 3x + 2 = 0$

$x^2 + x + 2x + 2 = 0$

$(x + 2)(x + 1) = 0$

$x = -2$ or $x = -1$ — obtaining the roots of the divisor $B_1 B_1$

When $x = -2$

$(-2)^4 + a(-2)^3 + b(-2)^2 + 5(-2) + 3 = 2(-2) + 1$

$16 - 8a + 4b - 10 + 3 = -3$

$-8a + 4b + 9 = -3$ — correctly substituting in the polynomial $B_1 B_1$

$-8a + 4b = -12$

$2a - b = 3$

When $x = -1$

$(-1)^4 + a(-1)^3 + b(-1)^2 + 5(-1) + 3 = 2(-1) + 1$

$1 - a + b - 5 + 3 = -1$

$-a + b - 1 = -1$ — correctly substituting in the polynomial $B_1 B_1$

$-a + b = 0$

$a - b = 0$

$a = b$

$Eqn2\ into\ Eqn1$

$2b - b = 3$

$b = 3$

From $Eqn2$

$a = 3$ — writing correct solutions of the polynomial A_1

$\therefore a = 3\ \&\ b = 3$

10. The curve $y = (x - 2)(3 - x)$

a. Sketching

(i) $y = 5x - 6 - x^2$

x-intercept $(y = 0)$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$ — obtaining the values of the x-intercepts $B_1 B_1$

$x = 3\ \&\ x = 2$

y-intercept $(x = 0)$

$y = 5(0) - 6 - (0)^2$ — obtaining the value of the y-intercept B_1

$y = -6$

For turning points, $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 5 - 2x$ — equating the derivative and obtaining the value of the derivative $m_1 B_1$

$2x - 5 = 0$

$2x = 5$

$x = 2.5$

When $x = 2.5$

$y = 5(2.5) - 6 - (2.5)^2$

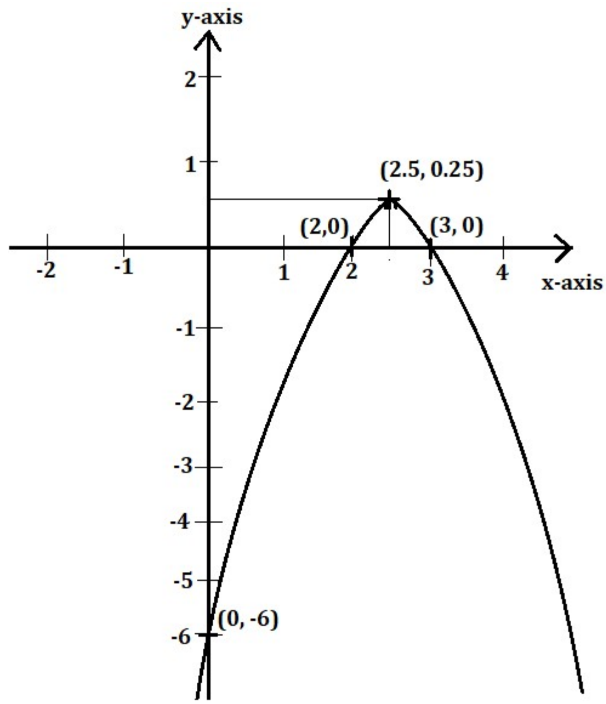
$y = 0.25$ — obtaining the y value B_1

$$\frac{d^2y}{dx^2} = -2$$

Since $\frac{d^2y}{dx^2} < 0$ then point $(2.5, 0.25)$ is a maximum point.

Ascertaining the nature of the Point A_1

(ii)



For every point indicated on the sketch curve B_1

b. $\int_{-\infty}^{\infty} y\, dx$

Obtaining limits

From $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$ obtaining the limits of the curve $B_1 B_1$

Either $x = 3$ or $x = 2$

$= \int_2^3 (5x - 6 - x^2)dx$

$= \left[\frac{5x^2}{2} - 6x - \frac{x^3}{3}\right]_2^3$ integrating and substituting limits B_1

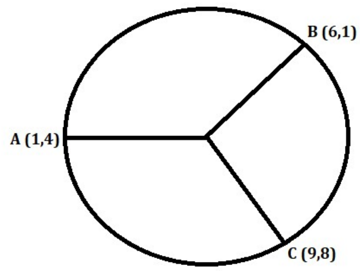
$= \left[\left(\frac{5(3)^2}{2} - 6(3) - \frac{(3)^3}{3}\right) - \left(\frac{5(2)^2}{2} - 6(2) - \frac{(2)^3}{3}\right)\right]$

$= [-4.5 + 4.67]$

0.17 Square Units writing the correct area A_1

11. Using A (1, -4), B (6, 1) and C (9, -8)

(i)



Using the equation of a circle;
$x^2 + y^2 + 2gx + 2fy + c = 0$
At A (1, -4)
$(1)^2 + (-4)^2 + 2g(1) + 2f(-4) + c = 0$
$1 + 16 + 2g - 8f + c = 0$ obtaining a correct equation
$2g - 8f + c = -17 \ldots\ldots(1)$ at point A B_1B_1

At B (6, 1)
$(6)^2 + (1)^2 + 2g(6) + 2f(1) + c = 0$
$36 + 1 + 12g + 2f + c = 0$ obtaining a correct equation
$12g + 2f + c = -37 \ldots\ldots(2)$ at point B B_1B_1

At C (9, 8)
$(9)^2 + (-8)^2 + 2g(9) + 2f(-8) + c = 0$
$81 + 64 + 18g - 16f + c = 0$ obtaining a correct equation
$18g - 16f + c = -145 \ldots\ldots(3)$ at point C B_1B_1

Eqns 1, 2, and 3 are solved simultaneously
$\therefore g = -6, f = 4, \& c = 27$ for every correct value A_1
$x^2 + y^2 - 2x + 8y + 27 = 0$ is the equation of the circle.

(ii) But $C\,(-g, -f)$ correctly substituting in the
$C\big(-(-6), -(4)\big)$ formula for the centre B_1B_1
$C(6, -4)$ writing the correct centre A_1

From $r = \sqrt{g^2 + f^2 - c}$ correctly using the formula
$r = \sqrt{(-6)^2 + (4)^2 - 27}$ for radius of a circle and
$r = \sqrt{25}$ substituting B_1m_1

$r = 5\ units$ obtaining the correct radius A_1

12. (a) (i) $Z_1Z_2 = (2 + 3\boldsymbol{i})(3 + 4\boldsymbol{i})$ correctly using properties of
$= 2(3 + 4\boldsymbol{i}) + 3\boldsymbol{i}(3 + 4\boldsymbol{i})$ complex numbers B_1B_1

$= 6 + 8\boldsymbol{i} + 9\boldsymbol{i} + 12\boldsymbol{i}$

$Z_1Z_2 = -6 + 7\boldsymbol{i}$ obtaining a correct Z_1Z_2 A_1

(ii) $\frac{Z_1}{Z_2} = \frac{2+3\boldsymbol{i}}{3+4\boldsymbol{i}}$

$= \frac{(2+3\boldsymbol{i})(3-4\boldsymbol{i})}{(3+4\boldsymbol{i})(3-4\boldsymbol{i})}$ correctly using properties of B_1

$= \frac{6-8\boldsymbol{i}+9\boldsymbol{i}-12\boldsymbol{i}^2}{3^2+4^2}$ complex numbers (conjugate) B_2

$= \frac{6+12 \quad \boldsymbol{i}}{25}$

$\frac{Z_1}{Z_2} = \frac{18}{25} + \frac{\boldsymbol{i}}{25}$ obtaining a correct Z_1/Z_2 A_1

$\left|\frac{\boldsymbol{Z_1}}{\boldsymbol{Z_2}}\right| = \sqrt{\left(\frac{\mathbf{18}}{\mathbf{25}}\right)^{\mathbf{2}} + \left(\frac{\mathbf{1}}{\mathbf{25}}\right)^{\mathbf{2}}}$ correctly using the formula

$= \sqrt{0.5184 + 0.0016}$ of magnitude B_1B_1

$= \sqrt{0.52}$ obtaining a correct value of

$= 0.7211$ (4 d.p) modulus of Z_1/Z_2 A_1

(b) $\int_0^1 x \sin x \, dx$

Let $u = x, \frac{du}{dx} = 1$ obtaining derivative B_1

Let $\frac{dv}{dx} = \sin x, v = \int sinx \, dx$

$v = -\cos x + C$ obtaining integral solution B_1

From $\int v \frac{du}{dx} = uv - \int u \frac{dv}{dx} dx$ correctly using integration B_1

$= -x \cos x + \int \cos x \, dx$ by parts

$\int_0^1 x \sin x \, dx = [-x \cos x + \sin x]_0^1$ correctly substituting B_1B_1

$= [(-(1)\cos(1) + \sin(1)) - (-(0)\cos(0) + \sin(0))]$

$= -0.982$ obtaining a correct integral

$\int_0^1 x \sin x \, dx = -0.982$ (3 d.p) solution A_1

Part II

13. Continuous random variable

a. $\int_{-\infty}^{\infty} f(x) dx = 1$

$\int_{-2}^{2} k(4 - x^2) dx = 1$

$k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$ integrating the f(x) B_1

$k\left[\left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right] = 1$ substituting in limits B_1B_1

$^{32}/_{3}\,k = 1$ obtaining the correct value

$k = {}^{3}/_{32}$ of k B_1

$f(x) = \begin{cases} \frac{3}{32}(4-x^2)\,; & -2 < x < 2 \\ 0 \quad ; & elsewhere \end{cases}$ writing f(x) correctly A_1

b. $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)dx$

$= \int_{-2}^{2} x\left[\frac{3}{32}(4-x^2)\right]dx$

$= \frac{3}{32}\int_{-2}^{2} x(4-x^2)dx$ integrating xf(x) B_1B_1

$= \frac{3}{32}\int_{-2}^{2}(4x-x^3)dx$

$= \frac{3}{32}\left[\frac{4x^2}{2}-\frac{x^4}{4}\right]_{-2}^{2}$

$= \frac{3}{32}\left[\left(\frac{4(2)^2}{2}-\frac{(2)^4}{4}\right)-\left(\frac{4(-2)^2}{2}-\frac{(-2)^4}{4}\right)\right]$ substituting limits correctly B_1B_1

$= \frac{3}{32}[4-4]$

$= 0$ writing correct solution of E(x) A_1

c. $Var\ (x) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx$

$= \int_{-2}^{2}\frac{3}{32}x^2(4-x^2)dx$ integrating x^2f(x) B_1B_1

$= \frac{3}{32}\int_{-2}^{2}(4x^2-x^4)dx$

$= \frac{3}{32}\left[\frac{4x^3}{3}-\frac{x^5}{5}\right]_{-2}^{2}$

$= \frac{3}{32}\left[\left(\frac{4(2)^3}{3}-\frac{(2)^5}{5}\right)-\left(\frac{4(-2)^3}{3}-\frac{(-2)^5}{5}\right)\right]$ substituting limits correctly B_1B_1

$= \frac{3}{32}[4.27+4.27]$

$= \frac{3}{32}\times 8.54$

$= 0.801$ (3 d.p) writing correct solution of Var(x) A_1

14. Winning

a. Hitting a target

(i) Success $p = \frac{1}{5}$, failure $q = \frac{4}{5}$, $n = 5$, identifying success & failure B_1

$P(X=3) = {}^{5}_{3}C \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^2$ using combinations B_1

$= 0.0512$ obtaining correct solution A_1

(ii) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= {}^{5}_{2}C \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^3 + 0.0512 + {}^{5}_{4}C \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^1 + {}^{5}_{5}C \cdot \left(\frac{1}{5}\right)^5 \cdot \left(\frac{4}{5}\right)^0$

Using combinations correctly B_1B_1

$= 0.2627$ obtaining correct solution B_2

b. Football tournament

(i) Matrix

	Win	Draw	Loss
Arsenal	2	1	1
Chelsea	2	0	2
Liverpool	1	2	1

$\mathbf{3 \times 3}$

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$ 3 by 3 matrix B_2

$\mathbf{1 \times 3}$

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$ column matrix B_2

(ii) $$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$ B_2

$$= \begin{pmatrix} 6+1+0 \\ 6+0+2 \\ 3+2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 8 \\ 5 \end{pmatrix}$$ B_1

∴ Chelsea was the winner with 8 points A_1

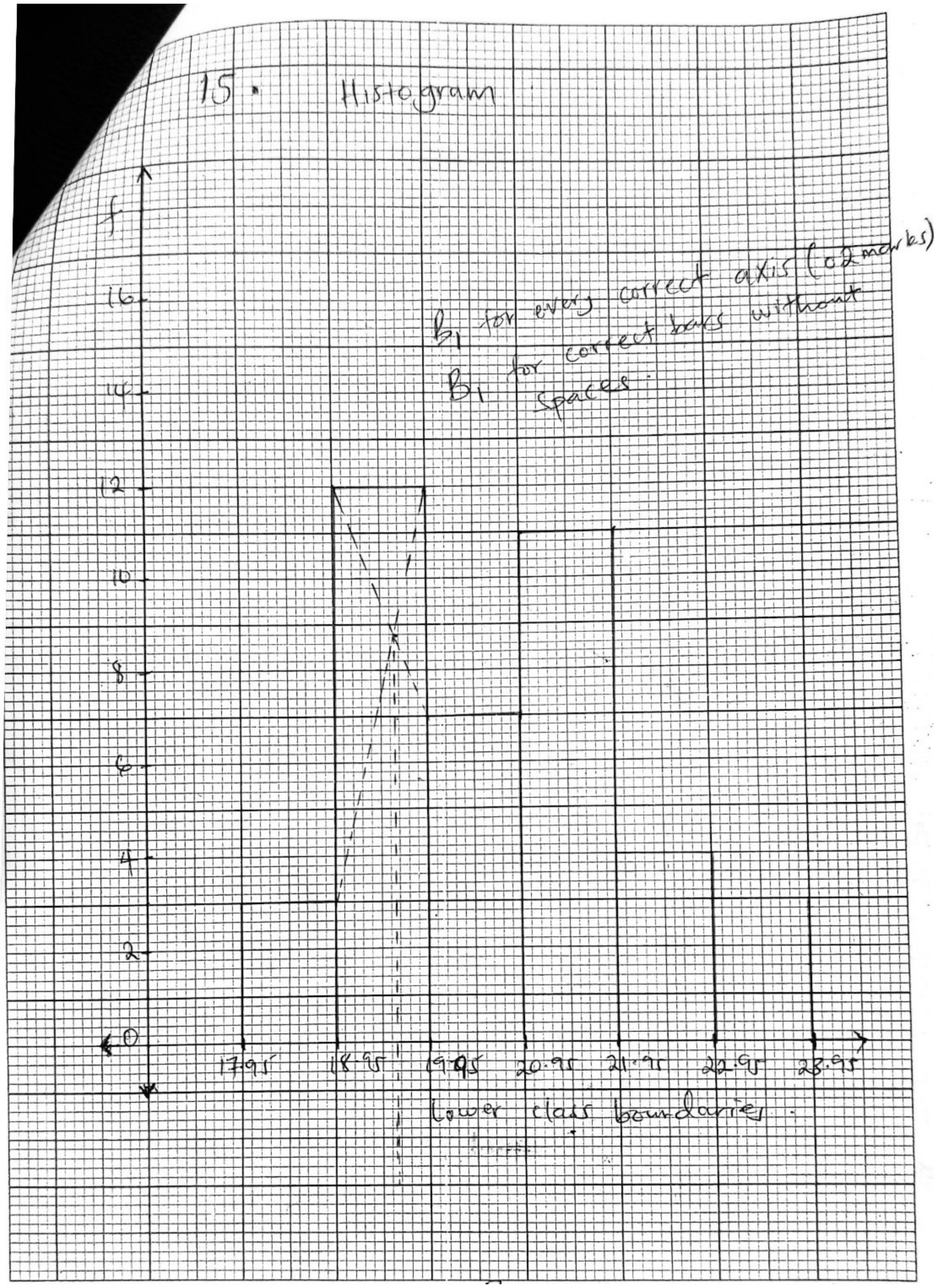
15. Agricultural survey

Length (mm)	F	x	fx	fx^2	Class boundaries
18.0 – 18.9	3	18.45	55.35	1021.2075	17.95 – 18.95
19.0 – 19.9	12	19.45	23.34	4539.63	18.95 – 19.95
20.0 – 20.9	7	20.45	143.15	2927.4175	19.95 – 20.95
21.0 – 21.9	11	21.45	235.95	5061.1275	20.95 – 21.95
22.0 – 22.9	4	22.45	89.8	2016.01	21.95 – 22.95
23.0 – 23.9	3	23.45	70.35	1647.7075	22.95 – 23.95
	$\Sigma f = 40$		$\Sigma fx = 617.94$	$\Sigma fx^2 = 17213.1$	

For every summation B_1

Correct column of class boundaries B_1

a. Histogram



For every correct axis B_1

For correct bars with no spaces B_1

From the graph,

$$\text{Modal number of leaves} = 18.95 + 7 \times 0.1$$

$$= 19.65$$ correct modal number of leaves B_1B_1

b. Mean and SD

(i) Mean $= \frac{\sum fx}{\sum f}$

$= \frac{617.94}{40}$ correctly using the formula of mean $B_1 m_1$

$= 15.4485$ obtaining a correct mean A_1

(ii) $\text{SD} = \sqrt{Variance}$

$Variance = \frac{\sum fx^2}{\sum f} - (Mean)^2$ correctly using the formula of

$= \frac{17213.1}{40} - (15.4485)^2$ variance $B_1 B_1$

$= 430.3275 - 238.6562$

$= 191.6713$

$\text{SD} = \sqrt{191.6713}$

$= 13.8445$ obtaining a correct Standard Deviation A_1

16. Sales of Computer Accessories Company

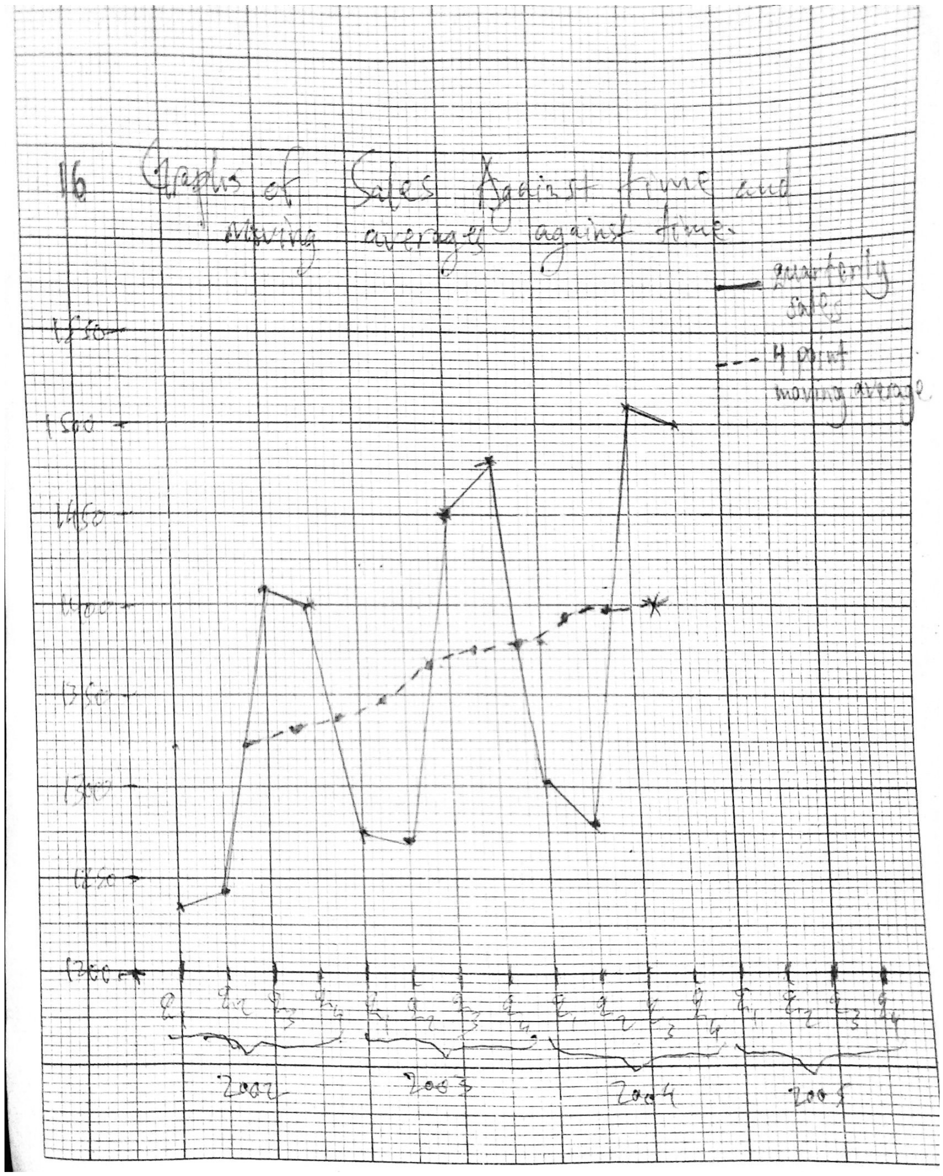
a. 4-point moving averages

quarter	**sales**	**4 point Moving totals**	**4 point Moving averages**
2002 q1	1235		
2002 q2	1242		
		5287	1321.75
2002 q3	1410		
		5327	1331.15
2002 q4	1400		
		5355	1338.75
2003 q1	1275		
		5395	1348.75
2003 q2	1270		
		5475	1368.75
2003 q3	1450		
		5502	1375.5
2003 q4	1480		
		5512	1378
2004 q1	1302		
		5572	1393
2004 q2	1280		
		5592	1398
2004 q3	1510		
2004 q4	1500		

Correct column for moving totals B_2

Correct column for moving averages B_2

b. Graphs



Correct axes B_2

Plotting sales correctly B_1

Plotting moving averages B_1

Using a correct key B_1

Comment: the number of sales increases every year correct comment B_1

c. Estimates

From the graph, the next moving average is 1400 thousand shillings.

Reading correctly from the graph B_1B_1

$\therefore 1400 = \frac{1280+1510+1500+x}{4}$ using the right formula B_1

$x + 1280 + 1510 + 1500 = 1400 \times 4$

$x = 5600 - 1280 - 1510 - 1500$ solving for x B_1

$x = 1310$ obtaining the correct value A_1

The 1st quarter of 2005 made sales worth 1310 thousand shillings

END