456/2

MATHEMATICS

PAPER 2

July/ August 2023

2 ½ hours

# ASSHU ANKOLE JOINT MOCK EXAMINATIONS 2023

**Uganda Certificate of Education**

**MATHEMATICS**

**PAPER 2**

**2 hours 30 minutes**

**INSTRUCTIONS TO CANDIDATES**

*Answer **ALL** Questions in Section **A** and any **Five** Questions from Section **B**.*

*Any Additional Question(s) answered will **not** be marked*

*All necessary calculations must be done in the answer sheets provided.*

*Graph paper is provided.*

*Silent, Non-Programmable Scientific Calculators and Mathematical Tables with a list of formulae may be used.*

## SECTION A

1. Given that $\mathbf{p} = \begin{pmatrix} -15 \\ 40 \end{pmatrix}$ and $q = k\begin{pmatrix} n \\ -8 \end{pmatrix}$. If $\mathbf{p} = \mathbf{q}$, find the values of k and n. (4 marks)

2. Without using tables or a calculator, evaluate

$$3\log 5 - 3\log 2 + \log 32 - \log 5$$ (4 marks)

3. If $y$ is inversely proportional to $x^2 - 1$ and that when $y = 2, x = 3$, find $y$ when $x = 5$. (4 marks)

4. Simplify $4\left(\frac{4}{100}\right) - 8 \times 4^{-1} \times 16^{\frac{3}{4}}$ (4 marks)

5. Given that $n(\varepsilon) = 23$, $n(P' \cap Q) = 3, \cap (P' \cap Q') = 5$ and $n(P \cap Q') = 4$.
   (a) Represent this information on a Venn diagram
   (b) Find $n(P \cap Q)$ (4 marks)

6. The line $3x + 2y = 8$ cuts the y-axis at P(0, k). Find the
   (i) gradient of the line, (2 marks)
   (ii) value of k (2 marks)

7. A cone has a circular base of radius 5 cm and a vertical height of 12 cm. Calculate the area of the curved surface (4 marks)

8. A car travels 40 km in 30 minutes , stops for 15 minutes, and then travels a further 100 km in 1 hour 15 minutes. Find the average speed for the whole journey. (4 marks)

9. Find the simple interest on sh. 20.000 for 1 ¾ years at 1 ½ % per month. Find also, the amount after 1 ¾ years. (4 marks)

10. Sh. 4,895,000 is divided into three parts in the ratio $1 : \frac{1}{2} : \frac{1}{3}$. Find the value of the smallest part. (4 marks)

## SECTION B

11.(a) The variable y is partly constant and partly varies as x.

When x = 2, y = 16 and when x = 7, y =31. Find y in terms of x and hence value of y when x = 4 and x when y = 18. (8 marks)

(b) t varies directly as the square of d and inversely as h. If t = 2 when d =12 and h = 6, find t when d =18 and h =1.5 (4 marks)

12(a) If $f(x) = ax^2 + bx, f(1) = 9$ and $f(2) = 30$. Find

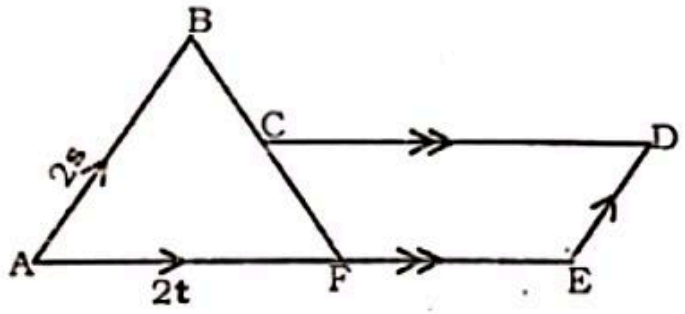
(i) the constants $a$ and $b$

(ii) f(5) (7 marks)

(b) Given that $f(x) = x^2 + 1$ and $g(x) = x + 1$, find $gf(-2)$ (2 marks)

(c) If $f(x) = \frac{24}{x} + 4$, find $f^{-1}(x)$. (3 marks)

13. In the figure, $\overline{AB}$, is parallel to $\overline{ED}, \overline{CD}$ is parallel to $\overline{AE}$. And C is the mid – point of $\overline{BF}$ and $\overline{FE} = ½\ \overline{AF}$.




Given AB =2s and AF = 2t, express in terms of s and t

(i) BF (ii) CF (iii) AC (iv) FE

Because CD is parallel to AE and $\overline{AB}$ is parallel to $\overline{ED}$ we can write CD = pt and ED = qs.

By considering AD = AC + CD and AD = AE + ED, express AD in terms of s and t and work out the values of p and q (scalars)

(12 marks)

14. In a certain school there are 87 students in Form 3. Of these, 43 play Hockey (H), 42 play Football (F) and 47 play Volleyball (V); 15 play Hockey and Volleyball, 17 play Volleyball and Football, 21 play Hockey and Football. Each student plays at least one of the three games and $x$ students play all three.

(a) show this information above on a venn diagram. (6 marks)

(b) Write down an equation in $x$ and hence find $x$. (3 marks)

(c) If a student is chosen at random from Form 3, what is the probability that he plays exactly two of these games? (3 marks)

15. A house was valued at sh. 75,000,000 after an appreciation of 15% in 1 year.

(a) Calculate the value of the house before appreciation.

(b) Mr. John paid a deposit of 15% of the appreciated value of the house, followed by 36 equal monthly instalments of shs. 2,100,000 each. How much did he pay for the house in total?

(c) If Mr. John had paid for the house in cash, how much would he have saved.

(d) Mr. Tom bought a similar house at sh. 75,000,000 by taking a loan, payable in full at the end of two years at 14% per annum Compound Interest. How much more did Tom pay for the loan than Mr. John paid for the house? (12 marks)

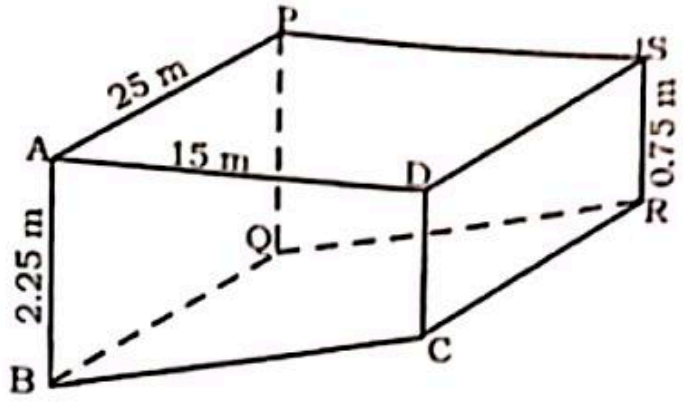
16. The Table below gives the speed, v, m/s of a boy after t seconds from the start in a race.

| Time (s) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| Speed (m/s) | 0 | 3.2 | 4.8 | 5.8 | 6.4 | 6.8 | 7 | 8 | 10 |

Plot the **speed-time graph** and

(a) Find the acceleration when t = 4.
(b) Estimate the distance travelled in 16 seconds.
(c) Calculate his average speed over the whole journey.
(d) If he actually ran a 100m race, find the percentage error in your estimated distance. (12 marks)

17.




The diagram above represents a swimming pool, with trapezium ABCD as its cross-section. **AB** = 2.25m, **AD** = 15m, **DC** = **SR** = 0.75m and **AP** = 25m.

(i) Calculate in $m^3$, the volume of the swimming pool.

(ii) The swimming pool is initially empty and is filled through a cylindrical pipe of radius 10 cm. Water flows through the pipe at 0.8 m/s. Find the time in hours and minutes, it takes to fill the swimming pool.

**END**

PROPOSED MARKING GUIDE

ASSHU ANKOLE JOINT MOCK 2023

456/2 MATHEMATICS PAPER TWO

BY GRANT. K. MP 0702741835

## SECTION A

1. $P = \begin{pmatrix} -15 \\ 40 \end{pmatrix}$ $q = k\begin{pmatrix} n \\ -8 \end{pmatrix}$

$P = q$.

$\begin{pmatrix} -15 \\ 40 \end{pmatrix} = k\begin{pmatrix} n \\ -8 \end{pmatrix}$

$\frac{-15}{40} = \begin{pmatrix} kn \\ -8k \end{pmatrix}$

$-15 = kn$ —①

$40 = -8k$ —②

from ② $k = \frac{40}{-8}$

$k = -5$

from ①

$-15 = -5n$

$n = 3$.

2. $3\log 5 - 3\log 2 + \log 32 - \log 5$

$= 2\log 5 + \log 32 - \log 2^3$

$= \log 5^2 + \log\left(\frac{32}{8}\right)$

$= \log 25 + \log 4$

$= \log(25 \times 4)$

$= \log 100$

$= 2$

3 $y \propto \frac{1}{x^2-1}$

$y = \frac{k}{x^2-1}$

$y = 2, \quad x = 3$

$2 = \frac{k}{3^2-1}$

$2 = \frac{k}{8}$

$k = 16$.

$y = ? \quad x = 5$

$y = \frac{16}{5^2-1}$

$y = \frac{16}{24}$

$y = \frac{2}{3}$

4. $4\left(\frac{4}{100}\right) - 8 \times 4^{-1} \times 16^{3/4}$

$= \frac{16}{100} - 8 \times \frac{1}{4} \times (16^{\frac{1}{4}})^3$

$= (2^4 \times 10^{-2}) - (2^3 \times 2^{-2} \times 2^3)$

$= \frac{2^4}{100} - 2^4$

$= 2^4\left(\frac{1}{100} - 1\right)$

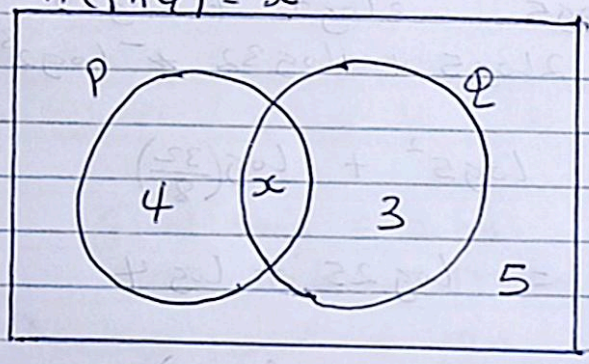
$= 2^4\left(\frac{-99}{100}\right)$

$= \frac{-396}{25}$ or $-15.84$.

5. $n(\mathcal{E}) = 23$. $n(P' \cap Q) = 3$, $n(P' \cap Q') = 5$

a) $n(P \cap Q) = 4$

Let $n(P \cap Q) = x$




b) $4 + x + 3 + 5 = 23$

$x + 12 = 23$

$x = 11$.

$n(P \cap Q) = 11$

6. $3x + 2y = 8$

$2y = -3x + 8$

$y = \frac{-3x}{2} + 4$

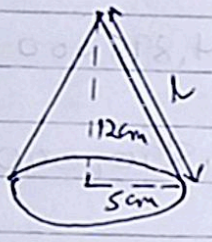
gradient $= -3/2$.

At $(0, k)$

$k = -\frac{3}{2}(0) + 4$

$k = 4$

7.




$C.S.A = \pi r l$

$L = \sqrt{12^2 + 5^2}$

$L = 13cm$.

$C.S.A = \pi \times 13 \times 5$

$= 204.2 cm^2$

8.

| Distance(km) | 0 | 40 | 40 | 140 |
|---|---|---|---|---|
| Time(hrs) | 0 | 1/2 | 3/4 | 2 |

Total time = 2 hrs.

Total distance = 140 km.

Average speed $= \frac{140}{2}$

$= 70 km/hr$.

9. $S.I = \frac{P \times R \times T}{100}$

$= \frac{20,000 \times 3/2 \times 21}{100}$

$=$ shs. 6300

Amount after $1\frac{3}{4}$ yrs $=$ 20,000 + 6300.

$=$ sh. 26,300

10. $1 : \frac{1}{2} : \frac{1}{3}$

Total ratio $= 1 + \frac{1}{2} + \frac{1}{3}$

$= \frac{11}{6}$.

Smallest portion $= \frac{1}{3}$.

$= \frac{\frac{1}{3}}{11/6} \times 4,895,000$

$= \frac{2}{11} \times 4,895000$

$= Sh.\ 890,000$.

SECTION B.

11a) $y = a + bx$.

when $x = 2$, $y = 16$.

$16 = a + 2b$ —— ①.

when $x = 7$, $y = 31$.

$31 = a + 7b$ —— ②

② − ①

$31 = a + 7b$

$16 = a + 2b$

$15 = 5b$

$b = 3$

$\Rightarrow y = 10 + 3x$.

from ①

$16 = a + 2(3)$

$16 = a + 6$

$a = 10$.

when $x = 4$, $y = ??$

$y = 10 + 3(4)$

$y = 22$

when $y = 18$, $x = ??$

$18 = 10 + 3x$

$8 = 3x$

$x = 8/3$.

11b) $t \propto \frac{d^2}{h}$

$t = \frac{kd^2}{h}$

when $t=2$, $d=12$, $h=6$.

$2 = \frac{k(12)^2}{6}$

$12 = k(12)^2$

$k = \frac{1}{12}$.

$t = \frac{d^2}{12h}$.

$d=18$, $h=1.5$. $t=?$

$t = \frac{18^2}{12(1.5)}$

$t = 18$.

12a) $f(x) = ax^2 + bx$

$f(1) = a + b = 9$.

(i) $a + b = 9$ —— ①.

$f(2) = 4a + 2b = 30$

$4a + 2b = 30$ —— ②.

$2 \mid a + b = 9$

$1 \mid 4a + 2b = 30$

$- \mid 2a + 2b = 18$

$4a + 2b = 30$

$-2a = -12$

$a = 6$

from ① $a + b = 9$

$6 + b = 9$

$b = 9 - 6$

$b = 3$.

12a ii)

$$f(x) = 6x^2 + 3x$$
$$f(5) = 6(5^2) + 3(5)$$
$$= 165.$$

b) $f(x) = x^2+1$, $g(x) = x+1$

$gf(-2)$

$$gf(x) = (x^2+1)+1$$
$$= x^2+2.$$
$$gf(-2) = (-2)^2+2$$
$$= 4+2$$
$$= 6.$$

c) $f(x) = \frac{24}{x} + 4$

Let $y = \frac{24}{x} + 4$

$$y = \frac{24+4x}{x}$$
$$yx = 24+4x$$
$$yx - 4x = 24$$
$$(y-4)x = 24$$
$$x = \frac{24}{y-4}$$
$$f^{-1}(x) = \frac{24}{x-4}.$$
$$f^{-1}(x) = \frac{24}{x-4}.$$

13.

i) $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$

$= \overrightarrow{AF} - \overrightarrow{AB}$

$= 2\underset{\sim}{t} - 2\underset{\sim}{s}$.

$= 2(\underset{\sim}{t} - \underset{\sim}{s})$

ii) $\overrightarrow{CF} = \frac{1}{2}\overrightarrow{BF}$

$= \frac{1}{2} \cdot 2(\underset{\sim}{t} - \underset{\sim}{s})$

$= \underset{\sim}{t} - \underset{\sim}{s}$.

iii) $\overrightarrow{AC} = \overrightarrow{AF} + \overrightarrow{FC}$

or $= \overrightarrow{AF} - \overrightarrow{CF}$.

$= 2\underset{\sim}{t} - (\underset{\sim}{t} - \underset{\sim}{s})$

$= 2\underset{\sim}{t} - \underset{\sim}{t} + \underset{\sim}{s}$

$= \underset{\sim}{t} + \underset{\sim}{s}$.

OR

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2\underset{\sim}{s} + \underset{\sim}{t} - \underset{\sim}{s}$

Since $\overrightarrow{BC} = \overrightarrow{CF}$.

$= \underset{\sim}{s} + \underset{\sim}{t}$.

iv) $\overrightarrow{FE}$

$\overrightarrow{FE} = \frac{1}{2}\overrightarrow{AF}$

$= \frac{1}{2}(2\underset{\sim}{t})$

$= \underset{\sim}{t}$.

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$\overrightarrow{CD} = p\underset{\sim}{t}$ and $\overrightarrow{ED} = q\underset{\sim}{s}$.

$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$

$= \underset{\sim}{s} + \underset{\sim}{t} + p\underset{\sim}{t}$

$\overrightarrow{AD} = \overrightarrow{AE} + \overrightarrow{ED}$

$= 3\underset{\sim}{t} + q\underset{\sim}{s}$.

$3\underset{\sim}{t} + q\underset{\sim}{s} = \underset{\sim}{s} + \underset{\sim}{t} + p\underset{\sim}{t}$.

$3\underset{\sim}{t} + q\underset{\sim}{s} = \underset{\sim}{s} + (1+p)\underset{\sim}{t}$.

$q = 1$

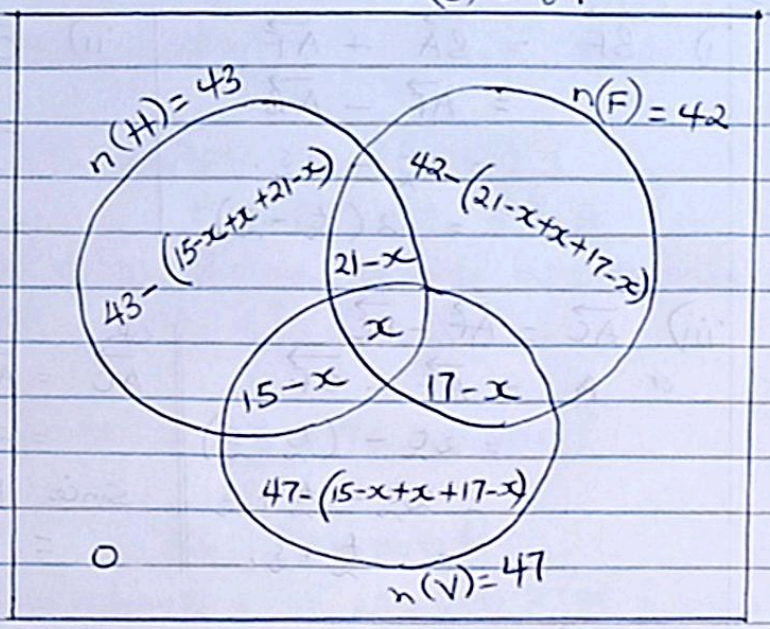
and $3 = 1 + p$

$p = 2$

$q = 1$ and $p = 2$.

14.

a)




b) $43 + 42 - (21 - x + 17) + 17 - x + 47 - (15 + 17 - x) = 87$

$43 + 42 - 38 + x + 17 - x + 47 - 32 + x = 87$

$79 + x = 87$

$x = 8$.

c) Exactly two games.

$= 15 - x + 21 - x + 17 - x$

$= 15 - 8 + 21 - 8 + 17 - 8$

$= 29$.

Probability $= \frac{29}{87}$

$= \frac{1}{3}$.

15. $A = P(1 + R)^n$.

a) $75,000,000 = P(1 + 15/100)^1$

$75,000,000 = P(1.15)$

$P = 65,217,391.3$

Value of the house before appreciation was sh. 65,217,391.3.

b) Deposit $= \frac{15}{100} \times 75,000,000$

$= 11,250,000$ ₱

Hire purchase $= 11,250,000 + (36 \times 2,100,000)$

$=$ sh. 86,850,000.

John paid sh. 86,850,000 in total.

c) Cash = ₱ 75,000,000

$86,850,000 - 75,000,000$

$=$ sh. 11,850,000.

d) $n = 2$ yrs. $R = 14\%$

$A = 75,000,000\left(1 + \frac{14}{100}\right)^2$

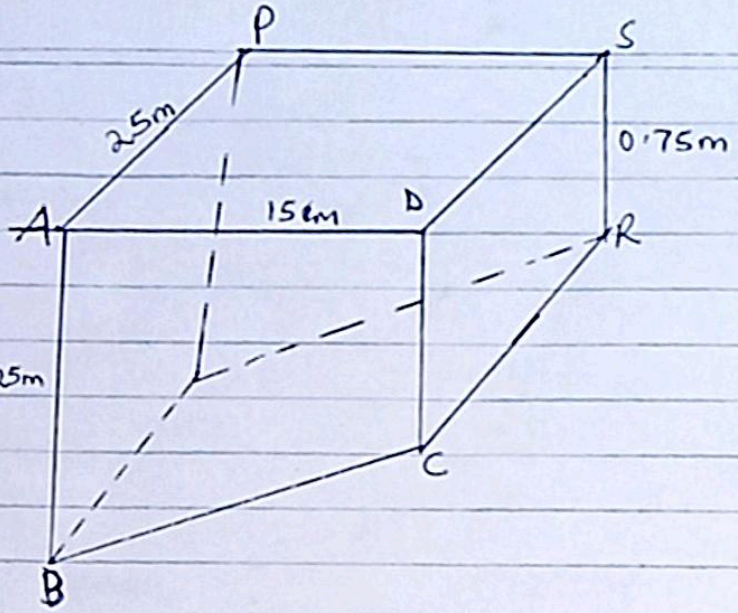
$= 97,470,000.$

$97,470,000 - 86,850,000$

$= 10,620,000$ ₱

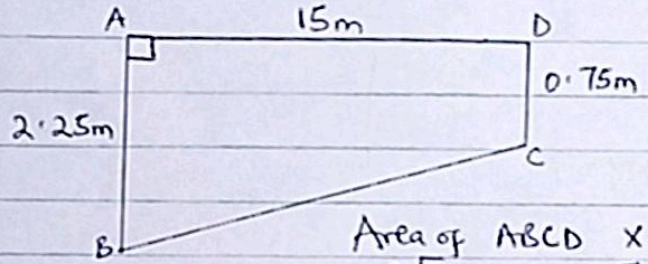
Tom paid sh. 10,620,000 more than John paid for the house.

17




ABCD is a trapezium.

i)

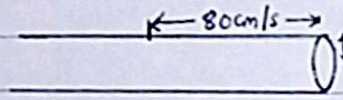



Volume of the swimming pool $= \frac{1}{2}(15)\left[0.75 + 2.25\right] \times 25$ (Area of ABCD × AP)

$= 7.5(3) * 25$

$= 562.5\ m^3$.

ii)




$h = 80cm = 0.8m$.

$r = 10cm = 0.1m$.

Volume of water in the cylindrical pipe in one second

$= \pi r^2 h$.

$= \pi \times 0.1^2 \times 0.8$

$= 0.025\ m^3/s$.

A full swimming pool $= 562.5\ m^3$ of water.

$0.025\ m^3 \longrightarrow 1s$

$562.5\ m^3 \longrightarrow \left(\frac{562.5}{0.025}\right)s$

$= 22,500s$

$\frac{22,500}{3600} = 6.25$

$= 6$ hours and 15 minutes.