SENIOR FOUR TEST-PAPERS WITH MARKING GUIDES

SENIOR FOUR TEST-PAPERS WITH MARKING GUIDES

456/1
MATHEMATICS
PAPER 1
Feb 2023
$2\frac{1}{2}$ hours

S.4 MATH 1 MOCK SET 1 2023

Time: 2 Hours 30 Minutes

NAME: **STREAM:**

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Factorise completely: $q^2 - x^2 + 4x - 4$. [4]

Qn 2: The bearing of B from A is 230°. What is the bearing of A from B? [2]

Qn 3: Solve simultaneously the equations:

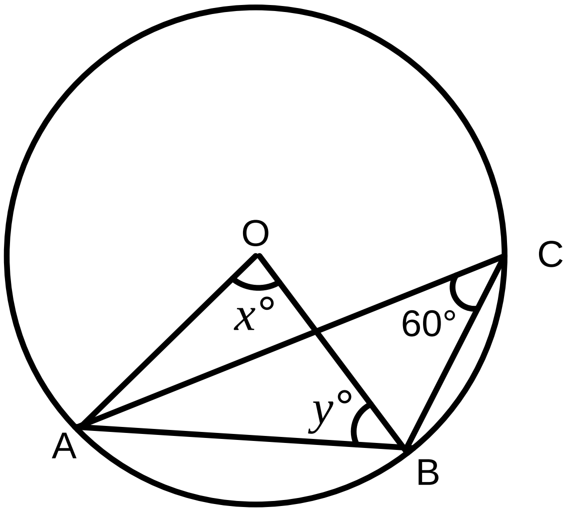
$$x = 6 - y$$
$$2x - 8 + y = 0$$

[4]

Qn 4: Given that $p * q = 3p - 2q$, find the value of y for which $(2 * 1) * y = 0$. [4]

Qn 5: Mukiibi calculated the area of a circle of radius 100 cm using $\pi = 3.142$ instead of $\frac{22}{7}$. What was the percentage error in his answer?[4]

Qn 6: In the diagram shown, O is the centre, angle $ACB = 60°$.



Find:

(i). angle x.

(ii). angle y. [4]

Qn 7: A water tank is $\frac{3}{7}$ full. After adding 52 litres, it is $\frac{4}{5}$ full.What is its total capacity? [4]

Qn 8: Find the integral solution set of: $1 \leq 3x - 3 < 7$. [4]

Qn 9: Given that P varies directly as the square of Q and that $P = 18$ when $Q = 6$,

(i). Express P in terms of Q.

(ii). Calculate the value of P when $Q = \frac{2}{3}$. [6]

Qn 10: Given that matrices $\boldsymbol{P} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$ and $\boldsymbol{R} = \begin{pmatrix} -2 & 6 \\ -1 & 9 \end{pmatrix}$, find matrix $\boldsymbol{Q}$such that $\boldsymbol{PQ} = \boldsymbol{R}$. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

40 students carried out an experiment and recorded the following measurements.

3.2	4.1	2.6	3.1	3.8	1.7	3.9	3.6
4.3	2.9	2.8	2.0	1.8	3.5	4.9	2.9
3.5	3.2	2.1	3.7	3.1	4.2	4.7	2.8
2.4	4.0	1.6	3.3	3.6	3.4	2.7	3.7
4.4	3.3	1.4	3.8	1.1	4.5	2.3	4.6

(a). Draw a frequency distribution table starting with $1.0 - 1.4$ as the first class.

(b). State the class interval.

(c). Calculate the mean using a working mean of 3.2. [12]

Question 12:

(a). Find the inverse of $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$.

(b). Hence, use the matrix method to solve simultaneously:

$$x + y = 3$$
$$2y + x = 4$$

(c). Given that matrices $\boldsymbol{K} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$ and $\boldsymbol{L} = \begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$, find:

(i). $\boldsymbol{K}^2$.

(ii). $2\boldsymbol{L} + 3\boldsymbol{K}$. [12]

Question 13:

Two fair dice are designed in such a way that the first one is green in colour with its six faces numbered 1, 1, 2, 2, 3, and 4 while the second one is yellow in colour with its faces numbered 1, 2, 3, 4, 4, and 5.

(a). Show the possibility space when both dice are rolled once.

(b). Hence, calculate the probability that the two scores will:

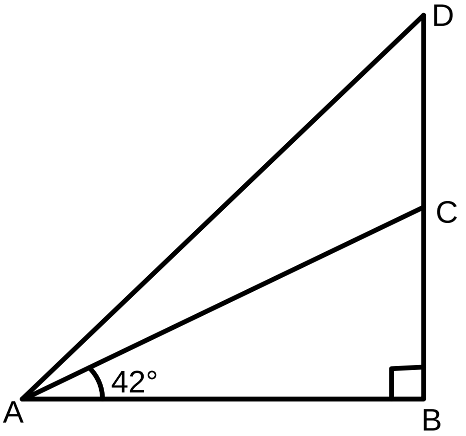
(i). be the same.

(ii). have a sum of more than 8. [12]

Question 14:

(a). Given that $3\cos\theta = 1$ and that $0° \leq \theta \leq 90°$, find the value of $\sin\theta$ and $\tan\theta$, without using tables or a calculator. (leave surds in your answers).

(b). In the given diagram below, $BC = 5.9$ cm, $\angle ABC = 90°$, $\angle BAC = 42°$ and $\angle BAD = 62°$. Calculate the lengths $\overline{AB}$ and $\overline{CD}$.

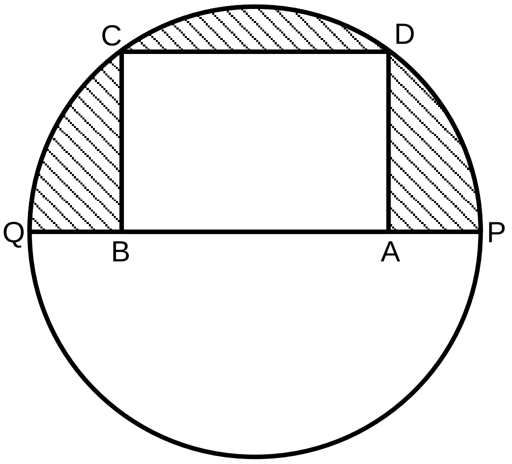


Calculate the lengths $\overline{AB}$ and $\overline{CD}$. [12]

Question 15:

(a). A room, whose width is 3 metres less than the length, has an area of $108\ \text{m}^2$. Find the dimensions and perimeter of the room.

(b).

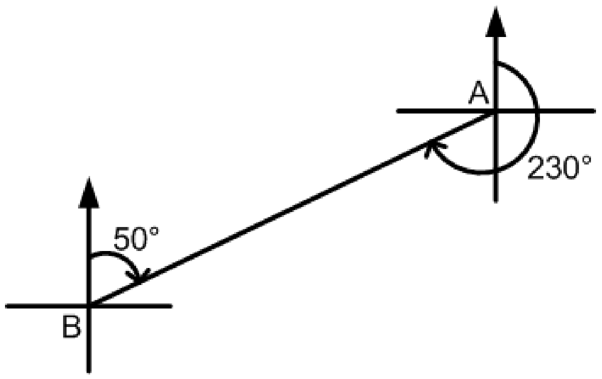


In the given diagram, $\overline{QP}$ is the diameter of the circle$PQCD$, and $ABCD$ is a rectangle with $\overline{AB} = 12$ cm and $\overline{CB} = 8$ cm. Calculate the area of the shaded region (correct to 2 d.p). [12]

*****END*****

MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	$q^2 - x^2 + 4x - 4 = q^2 - (x^2 - 4x + 4)$ $= q^2 - (x - 2)^2$ $= [q - (x - 2)][q + (x - 2)]$ $= (q - x + 2)(q + x - 2)$	**M1** **M1** **M1** **A1**
		04
2	 The bearing of A from B is 050°.	**B1** **A1**
		02
3	$2x - 8 + y = 0$ $2(6 - y) - 8 + y = 0$ $12 - 2y - 8 + y = 0$ $4 - y = 0$ $4 = y$ $y = 4$ $x = 6 - 4 = 2$	**M1** **A1** **M1 A1**
		04
4	$(2 * 1) * y = 0$ $(3 \times 2 - 2 \times 1) * y = 0$ $4 * y = 0$ $3 \times 4 - 2 \times y = 0$ $12 = 2y$ $6 = y$ $y = 6$	**M1** **B1** **M1** **A1**
		04
5	Using $\pi = 3.142$ Area, $A_1 = 3.142 \times 100^2 = 31420 \text{ cm}^2$ Using $\pi = \frac{22}{7}$	**B1** **B1**

	Area, $A_2 = \frac{22}{7} \times 100^2 = \frac{220000}{7} = 31428.57\ \text{cm}^2$ $\%\text{error} = \frac{32428.57 - 31420}{32428.57} \times 100 = 0.0273$	**M1 A1**
		04
6	(i). $x = 2 \times 60^\circ = 120^\circ$ (ii). $x + 2y = 180^\circ$ $120^\circ + 2y = 180^\circ$ $2y = 180^\circ - 120^\circ$ $2y = 60^\circ$ $y = 30^\circ$	**M1 A1** **M1** **A1**
		04
7	$\frac{3}{7}x + 52 = \frac{4}{5}x$ $L.C.D = 35$ $35 \times \frac{3}{7}x + 35 \times 52 = 35 \times \frac{4}{5}x$ $15x + 1820 = 28x$ $1820 = 28x - 15x$ $1820 = 13x$ $140 = x$ $x = 140\ \text{litres}$	**M1** **M1** **M1** **A1**
		04
8	$1 \leq 3x - 3 < 7$ $1 + 3 \leq 3x - 3 + 3 < 7 + 3$ $4 \leq 3x < 10$ $\frac{4}{3} \leq x < \frac{10}{3}$ $1.33 \leq x < 3.33$ The integral solution set is: $\{2, 3\}$.	**M1** **M1** **B1** **A1**
		04
9	(i). $P = KQ^2$ $18 = K \times 6^2$ $18 = 36K$ $\frac{1}{2} = K$ $P = \frac{1}{2}Q^2$ (ii). $P = \frac{1}{2}\left(\frac{2}{3}\right)^2 = \frac{2}{9}$	**B1** **M1** **B1** **A1** **M1 A1**

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<tr><td>10</td><td>

$$\text{Det } \boldsymbol{P} = 2 - 0 = 2$$

$$\boldsymbol{P}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\boldsymbol{PQ} = \boldsymbol{R}$$

$$\boldsymbol{Q} = \boldsymbol{P}^{-1}\boldsymbol{R} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 6 \\ -1 & 9 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -2 & 6 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$$

</td><td>M1

M1

M1 A1</td></tr>
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<tr><td>11</td><td>

(a). Let $d = x - 3.2$.

<table>
<tr><th>Class</th><th>Tally</th><th>$\boldsymbol{f}$</th><th>$\boldsymbol{x}$</th><th>$\boldsymbol{d}$</th><th>$\boldsymbol{fd}$</th></tr>
<tr><td>1.0 – 1.4</td><td>//</td><td>2</td><td>1.2</td><td>– 2.0</td><td>– 4</td></tr>
<tr><td>1.5 – 1.9</td><td>///</td><td>3</td><td>1.7</td><td>– 1.5</td><td>– 4.5</td></tr>
<tr><td>2.0 – 2.4</td><td>////</td><td>4</td><td>2.2</td><td>– 1.0</td><td>– 4.0</td></tr>
<tr><td>2.5 – 2.9</td><td>~~////~~ /</td><td>6</td><td>2.7</td><td>– 0.5</td><td>– 3</td></tr>
<tr><td>3.0 – 3.4</td><td>~~////~~ //</td><td>7</td><td>3.2</td><td>0</td><td>0</td></tr>
<tr><td>3.5 – 3.9</td><td>~~////~~ ////</td><td>9</td><td>3.7</td><td>0.5</td><td>4.5</td></tr>
<tr><td>4.0 – 4.4</td><td>~~////~~</td><td>5</td><td>4.2</td><td>1.0</td><td>5.0</td></tr>
<tr><td>4.5 – 4.9</td><td>////</td><td>4</td><td>4.7</td><td>1.5</td><td>6.0</td></tr>
<tr><td>Total</td><td></td><td>40</td><td></td><td></td><td>0</td></tr>
</table>

(b). The class interval is 0.5.
(c).

$$\text{Mean,} \qquad \bar{x} = 3.2 + \frac{\sum fd}{\sum f} = 3.2 + \frac{0}{40} = 3.2$$

</td><td>B2-for Class

B2-for Tally

B2-for $\boldsymbol{f}$

B1-for $\boldsymbol{x}$

B1-for $\boldsymbol{d}$

B1-for $\boldsymbol{fd}$

B1

M1 A1</td></tr>
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<tr><td>12</td><td>

(a).

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \qquad \Longrightarrow \text{Det } \boldsymbol{A} = 4 - 2 = 2$$

$$\therefore \boldsymbol{A}^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1/2 \\ -1 & 1/2 \end{pmatrix}$$

(b).

$$\begin{array}{c|c|} 1 & x + y = 3 \\ 2 & x + 2y = 4 \\ \hline \end{array}$$

$$x + y = 3$$

$$2x + 4y = 8$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1/2 \\ -1 & 1/2 \end{pmatrix}\begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

</td><td>B1

M1 A1

B1

M1
B1</td></tr>
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$$\Longrightarrow x = 2, \qquad y = 1$$

(c). (i).

$$K^2 = \begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}\begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}$$

$$= \begin{pmatrix}1 & -2\\ 6 & -3\end{pmatrix}$$

(ii).

$$2L + 3K = 2\begin{pmatrix}0 & 3\\ -1 & 1\end{pmatrix} + 3\begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & 6\\ -2 & 2\end{pmatrix} + \begin{pmatrix}6 & -3\\ 9 & 0\end{pmatrix} = \begin{pmatrix}6 & 3\\ 7 & 2\end{pmatrix}$$

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A1

M1

A1

M1
M1 A1

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12

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<td>13</td>
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(a).

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<tr><td>Die 1 / Die 2</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>1</td><td>1,1</td><td>1,1</td><td>1,2</td><td>1,2</td><td>1,3</td><td>1,4</td></tr>
<tr><td>2</td><td>2,1</td><td>2,1</td><td>2,2</td><td>2,2</td><td>2,3</td><td>2,4</td></tr>
<tr><td>3</td><td>3,1</td><td>3,1</td><td>3,2</td><td>3,2</td><td>3,3</td><td>3,4</td></tr>
<tr><td>4</td><td>4,1</td><td>4,1</td><td>4,2</td><td>4,2</td><td>4,3</td><td>4,4</td></tr>
<tr><td>4</td><td>4,1</td><td>4,1</td><td>4,2</td><td>4,2</td><td>4,3</td><td>4,4</td></tr>
<tr><td>5</td><td>5,1</td><td>5,1</td><td>5,2</td><td>5,2</td><td>5,3</td><td>5,4</td></tr>
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(b). (i). $\frac{7}{36}$.

(ii). $\frac{1}{36}$.

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B8

M1 A1

M1 A1

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12

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<td>14</td>
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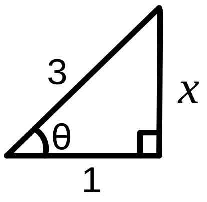
(a).

$$3\cos\theta = 1, \qquad \Longrightarrow \cos\theta = \frac{1}{3}$$



$$x = \sqrt{3^2 - 1^2} = \sqrt{8}$$

$$\sin\theta = \frac{\sqrt{8}}{3}$$

$$\tan\theta = \frac{\sqrt{8}}{1} = \sqrt{8}$$

(b).

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M1 B1

M1 A1

M1 A1

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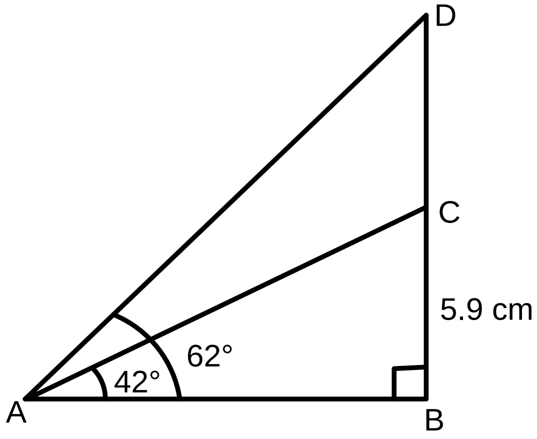
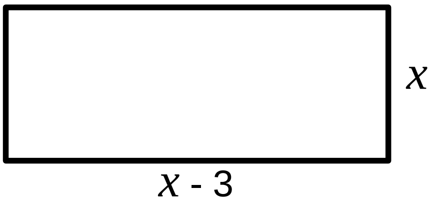
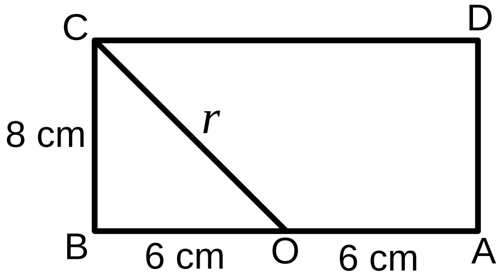
</table>

		**M1**
	$\tan 42° = \dfrac{5.9}{\overline{AB}}$	**A1**
	$\overline{AB} = \dfrac{5.9}{\tan 42°} = 6.553 \text{ cm}$	**M1**
	$\tan 62° = \dfrac{\overline{BD}}{6.553}$	**B1** **M1 A1**
	$\overline{BD} = 6.553 \tan 62° = 12.3244 \text{ cm}$	
	$\overline{CD} = 12.3244 - 5.9 = 6.444 \text{ cm}$	
		12
15	(a).	
		
	$x(x-3) = 108$	**B1**
	$x^2 + 3x - 108 = 0$	
	sum $= 3$, product $= -108$, factors $= -12, 9$	
	$x^2 - 12x + 9x - 108 = 0$	**M1**
	$x(x-12) + 9(x-12) = 0$	
	$(x+9)(x-12) = 0$	
	either $(x+9) = 0$, or, $(x-12) = 0$	
	$x = -9$, or, $x = 12$	**B1**
	Since x is length, it must be positive.	
	$\therefore x = 12 \text{ m}$	
	Length, $l = 12 \text{ m}$	**A1**
	Width, $w = 12 - 3 = 9 \text{ m}$	**A1**
	Perimeter $= 2(l + w) = 2(12 + 9) = 42 \text{ m}$	**M1 A1**
	(b).	

	 $r = \sqrt{8^2 + 6^2} = 10 \text{ cm}$ Area of semicircle, $A_1 = \frac{1}{2}\pi r^2$ $A_1 = \frac{1}{2} \times \frac{22}{7} \times 10^2 = 157.1429 \text{ cm}^2$ Area of rectangle, $A_2 = l \times w$ $A_2 = 12 \times 8 = 96 \text{ cm}^2$ Area of shaded part, $A = A_1 - A_2$ $A = 157.1429 - 96$ $A = 61.1429 \approx 61.14 \text{ cm}^2$	**B1** **B1** **B1** **M1** **A1**
		12

*****END****

456/2
MATHEMATICS
PAPER 2
Feb 2023
$2\frac{1}{2}$ hours

S.4 MATH 2 MOCK SET 1 2023

Time: 2 Hours 30 Minutes

NAME: ______________________ **STREAM:** __________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Find the L.C.M of 12, 18 and 42. [4]

Qn2: Given that $\underset{\sim}{a} = \begin{pmatrix}1\\2\end{pmatrix}and\underset{\sim}{b} = \begin{pmatrix}2\\-6\end{pmatrix}$, find:

(i). $\underset{\sim}{a} + \underset{\sim}{b}$.

(ii). $\left|\underset{\sim}{a} + \underset{\sim}{b}\right|$. [4]

Qn 3: Simplify: $2\log_{10} 2 + \log_{10} 75 - \log_{10} 3$. [4]

Qn 4: Solve the inequality: $10 - 3x < 4(x - 1)$. Hence represent on a number line. [4]

Qn 5: Find the equation of a line that passes through point $A(-2, 7)$ and the origin. [4]

Qn 6: Solve for x: $\frac{1}{2}(x - 4) - \frac{1}{3}(3 - 2x) = \frac{1}{6}(x - 1)$. [4]

Qn 7: Simplify: $\dfrac{\left(3\frac{1}{3} - 1\frac{5}{6}\right)}{\left(2\frac{3}{4} + 1\frac{1}{6} + \frac{1}{3}\right)}$. [4]

Qn 8: Solve for $x$and y:

$$\begin{aligned} -x + 2y &= 10 \\ y - 4 &= x \end{aligned}$$ [4]

Qn 9: A man bought a shirt at 20% discount. If he paid shs 20,000, find the original price of the shirt. [4]

Qn10: Make x the subject: $T = 2\pi\sqrt{\frac{m}{x-a}}$. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

Of the 35 candidates in S.4, 13 registered for Biology (B), 20 registered for History (H) and 17 registered for Fine Art (F).

9 registered for both Biology and Fine Art.

3 registered for both Biology and History.

8 registered for only History and Fine Art.

2 registered for all the three subjects.

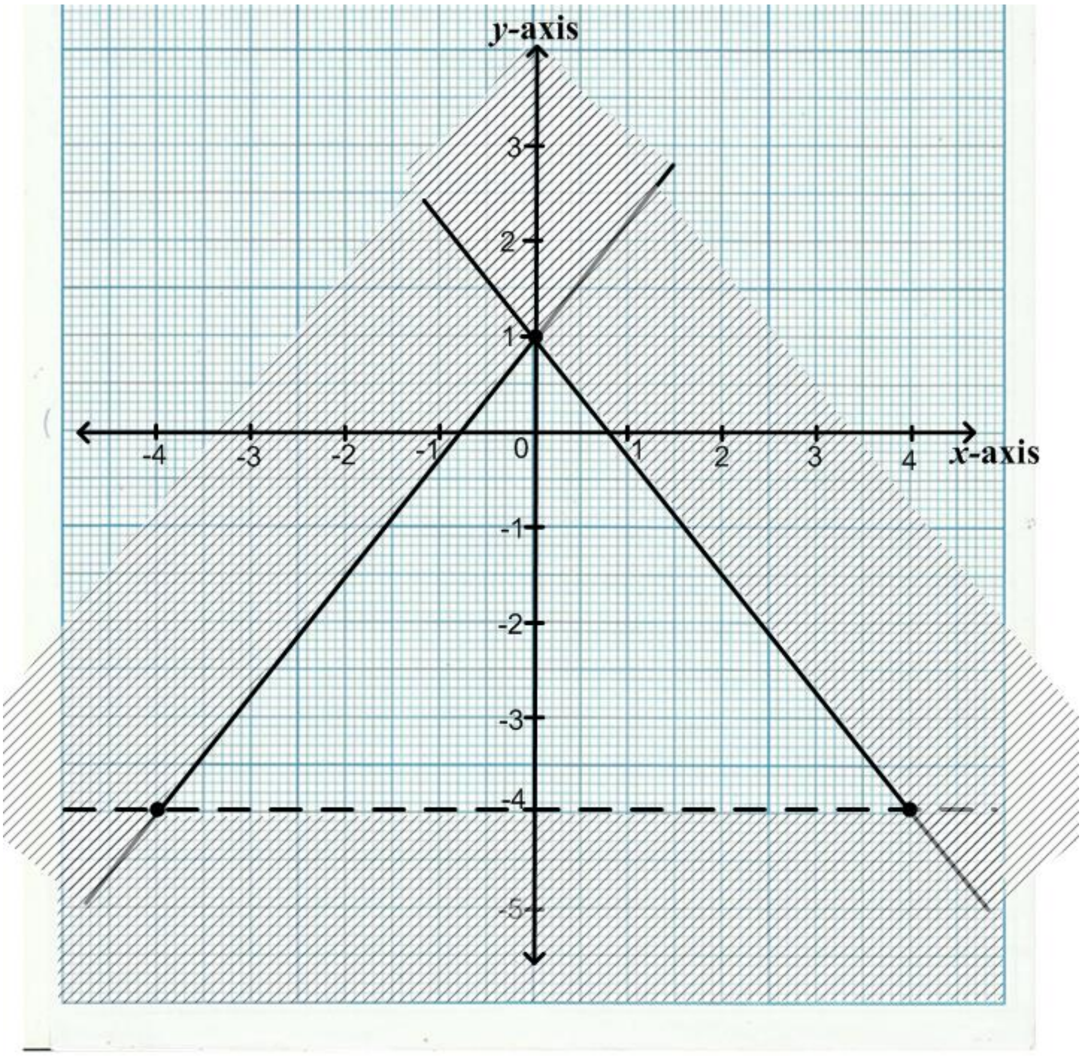
(a). Represent the information on a Venn diagram.

(b). Find:

- (i). the number of candidates who registered for History only.
- (ii). the number of candidates who registered for at least two subjects.

(c). How many candidates did not take any of the three subjects? [12]

Question 12:

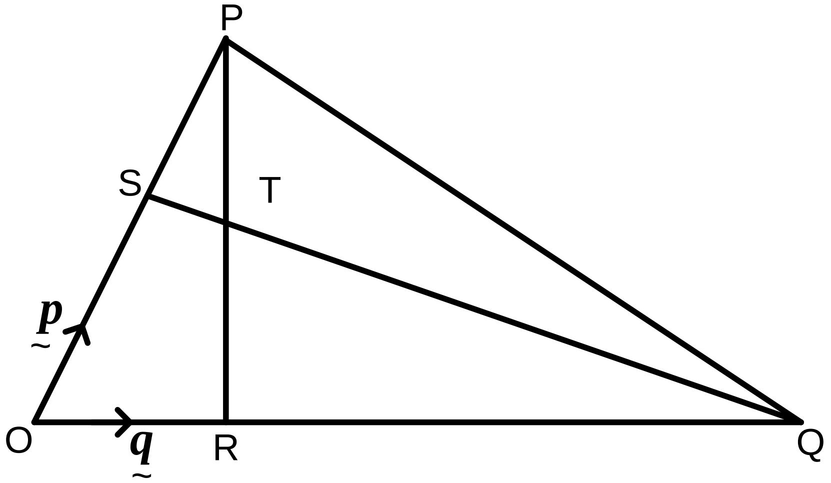


Write down the inequalities which satisfy the unshaded region in the graph above. [12]

Question 13:

A man 1.6 m tall observed the angle of elevation of a bird on top of the tree from P as 30°. He moved in a straight line a distance of 10 m towards point Q nearer to the tree and observed the angle of elevation of the bird as 55°. Determine the height of the tree. [12]

Question 14:



In the figure above, OPQ is a triangle in which $\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OP}$and $\overrightarrow{OR} = \frac{1}{3}\overrightarrow{OQ}$. $T$is a point on $\overrightarrow{QS}$ such that $4\overrightarrow{QT} = 3\overrightarrow{QS}$. If $\overrightarrow{OP} = \underset{\sim}{\mathbf{p}}$and $\overrightarrow{OQ} = \underset{\sim}{\mathbf{q}}$,

(a). Express the following in terms of $\underset{\sim}{\mathbf{p}}$and $\underset{\sim}{\mathbf{q}}$:

(i). $\overrightarrow{QS}$, (ii). $\overrightarrow{SR}$, (iii).$\overrightarrow{PT}$, (iv). $\overrightarrow{TR}$. [8]

(b). Hence, show that P, T and R lie on a straight line. [4]

Question 15:

Copy and complete the table below for $y = x^2 - 4x + 2$.

(a).

x	-2	-1	0	1	2	3	4	5	6
x^2									
$-4x$									
2	2	2	2	2	2	2	2	2	2
y									

(b). Draw the graph of $y = x^2 - 4x + 2$; use a scale of 1 cm to represent 1 unit on both axes.

(c). Use your graph to solve:

(i). $x^2 - 4x + 2 = 0$,

(ii). $x^2 - 6x + 2 = 0$. [12]

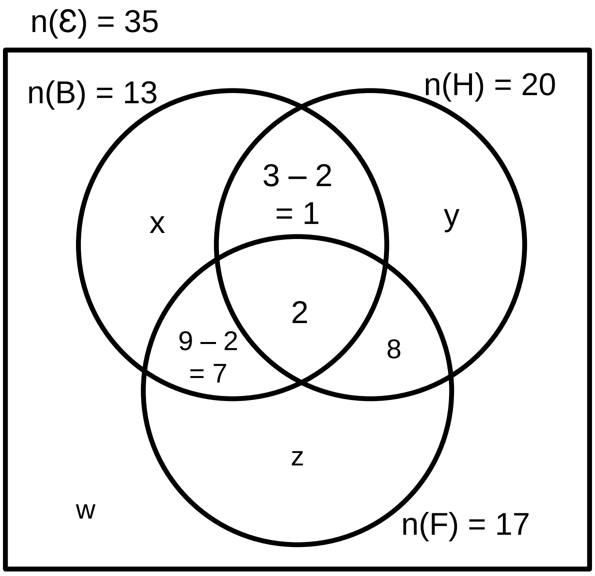
*****END*****

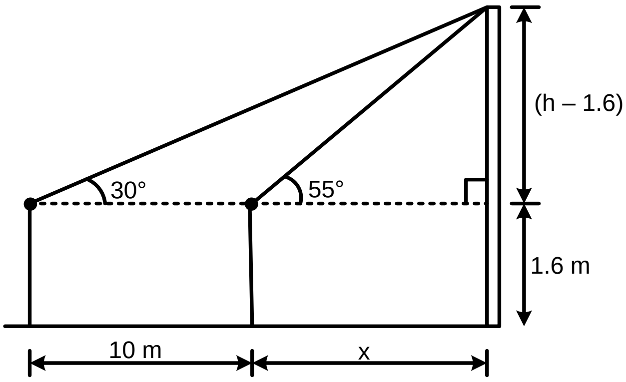
MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	$\begin{array}{c\|c\|c\|c} 2 & 12 & 18 & 42 \\ \hline 2 & 6 & 9 & 21 \\ \hline 3 & 3 & 9 & 21 \\ \hline 3 & 1 & 3 & 7 \\ \hline 7 & 1 & 1 & 7 \\ \hline & 1 & 1 & 1 \end{array}$ $\mathrm{L.C.M} = 2^2 \times 3^2 \times 7 = 252$	**B1 B1** **M1 A1**
		04
2	$\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -6 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $\left\| \underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}} \right\| = \sqrt{3^2 + (4)^2} = 5 \text{ units}$	**M1 A1** **M1 A1**
		04
3	$2\log_{10} 2 + \log_{10} 75 - \log_{10} 3$ $= \log_{10} 4 + \log_{10} 75 - \log_{10} 3$ $= \log_{10}\left(\frac{4 \times 75}{3}\right)$ $= \log_{10} 100$ $= 2\log_{10} 10 = 2$	**M1** **M1** **A1** **B1**
		04
4	$10 - 3x < 4(x - 1)$ $10 - 3x < 4x - 4$ $10 + 4 < 4x + 3x$ $14 < 7x$ $2 < x$ $x > 2$	**M1** **M1** **M1** **A1**
		04
5	Points are: $A(-2, 7)$ and $O(0, 0)$, Gradient of line, $m = \frac{7 - 0}{-2 - 0} = -\frac{7}{2}$ $\mathrm{y-intercept}, \quad c = 0$ The equation of the line is: $y = mx + c$ $y = -\frac{7}{2}x + 0$ $y = -\frac{7}{2}x$	**M1 B1** **B1** **A1**

		04
6	$\frac{1}{2}(x-4)-\frac{1}{3}(3-2x)=\frac{1}{6}(x-1)$ $L.C.D=6$ $6\times\frac{1}{2}(x-4)-6\times\frac{1}{3}(3-2x)=6\times\frac{1}{6}(x-1)$ $3(x-4)-2(3-2x)=(x-1)$ $3x-12-6+4x=x-1$ $3x+4x-x=-1+12+6$ $6x=17$ $x=\frac{17}{6}$	**M1** **M1** **M1** **A1**
		04
7	$3\frac{1}{3}-1\frac{5}{6}=\frac{10}{3}-\frac{11}{6}=\frac{20-11}{6}=\frac{9}{6}=\frac{3}{2}$ $2\frac{3}{4}+1\frac{1}{6}+\frac{1}{3}=\frac{1}{4}+\frac{7}{6}+\frac{1}{3}=\frac{33+14+4}{12}=\frac{17}{4}$ $\frac{\left(3\frac{1}{3}-1\frac{5}{6}\right)}{\left(2\frac{3}{4}+1\frac{1}{6}+\frac{1}{3}\right)}=\frac{3}{2}\div\frac{17}{4}=\frac{3}{2}\times\frac{4}{17}=\frac{12}{34}=\frac{6}{17}$	**B1** **B1** **M1 A1**
		04
8	$-x+2y=10$ $-(y-4)+2y=10$ $-y+4+2y=10$ $-y+2y=10-4$ $y=6$ $x=y-4=6-4=2$ $\therefore x=2, \quad y=6$	**M1** **A1** **M1 A1**
		04
9	Let the original price be x. $100\%-20\%=80\%$ $\frac{80}{100}\times x=20000$ $0.8x=20000$ $\frac{0.8x}{0.8}=\frac{20000}{0.8}$ $x=25000$ The original price of the shirt is shs 25,000.	**M1** **M1** **M1** **A1**
		04
10	$T=2\pi\sqrt{\frac{m}{x-a}}$ $T^2=4\pi^2\left(\frac{m}{x-a}\right)$	**M1**

	$T^2 = \frac{4\pi^2 m}{x-a}$ $T^2(x-a) = 4\pi^2 m$ $T^2 x - T^2 a = 4\pi^2 m$ $T^2 x = T^2 a + 4\pi^2 m$ $x = \frac{T^2 a + 4\pi^2 m}{T^2}$	**M1** **M1** **A1**
		04
11	(a).  (b). (i). $n(H) = y + 8 + 2 + 1 = 20$ $20 = y + 11$ $y = 20 - 11 = 9$ $\therefore n(\text{only History}) = 9 \text{ students}$ (ii). $n(\text{at least two subjects}) = 1 + 2 + 7 + 8 = 18 \text{ students}$ (c). $n(B) = x + 7 + 2 + 1 = 13, \quad \Rightarrow x = 13 - 10 = 3$ $n(F) = z + 8 + 2 + 7 = 17, \quad \Rightarrow z = 17 - 17 = 0$ $n(\varepsilon) = 13 + y + 8 + z + w$ $35 = 13 + 9 + 8 + 0 + w$ $35 = 30 + w$	**B1**-for entry 1 **B1**-for entry 2 **B1**-for entry 7 **B1**-for entry 8 **M1** **A1** **M1 A1** **B1** **B1** **M1**

	$w = 5$ $n(\text{none of the three subjects}) = 5$ students	**A1**
		12
12	For $(0, 1)$ and $(-4, -4)$ $y = mx + c$ $m = \frac{-4-1}{-4-0} = \frac{-5}{-4} = \frac{5}{4}$ $1 = \frac{5}{4} \times 0 + c, \quad \Rightarrow c = 1$ $y = \frac{5}{4}x + 1$ $4y = 5x + 4$ Suppose the inequality: $4y \leq 5x + 4$. Test it using $(0, 0)$. $0 \leq 0 + 4, \quad \Rightarrow 0 \leq 4, \quad \text{True}$ $\therefore$ The required inequality is: $4y \leq 5x + 4$. For $(0, 1)$ and $(4, -4)$ $y = mx + c$ $m = \frac{-4-1}{4-0} = \frac{-5}{4} = -\frac{5}{4}$ $1 = -\frac{5}{4} \times 0 + c, \quad \Rightarrow c = 1$ $y = -\frac{5}{4}x + 1$ $4y = -5x + 4$ $4y + 5x = 4$ Suppose the inequality: $4y + 5x \leq 4$. Test it using $(0, 0)$. $0 + 0 \leq 4, \quad \Rightarrow 0 \leq 4, \quad \text{True}$ $\therefore$ The required inequality is: $4y + 5x \leq 4$. For $(-4, -4)$ and $(4, -4)$ $\therefore$ The required inequality is: $y > -4$.	**M1 B1** **M1** **B1** **A1** **M1 B1** **M1** **B1** **A1** **B1 B1**
		12
13	 $\tan 30° = \frac{h - 1.6}{x + 10}$	**B1 B1** **B1**

	$(x+10)\tan 30^\circ = h - 1.6$	**M1**
	$0.577x + 5.77 = h - 1.6$	
	$0.577x = h - 7.37$	
	$x = \dfrac{h-7.37}{0.577} \longrightarrow (1)$	**M1**
	$\tan 55^\circ = \dfrac{h-1.6}{x}$	**B1**
	$x\tan 55^\circ = h - 1.6$	**M1**
	$1.428x = h - 1.6$	
	$x = \dfrac{h-1.6}{1.428} \longrightarrow (2)$	**M1**
	Equating the two equations;	
	$\dfrac{h-7.37}{0.577} = \dfrac{h-1.6}{1.428}$	**M1**
	$1.428(h-7.37) = 0.577(h-1.6)$	
	$1.428h - 10.524 = 0.577h - 0.9232$	**M1**
	$1.428h - 0.577h = -0.9232 + 10.524$	
	$0.851h = 9.6008$	**A1**
	$h = 11.282 \text{ m}$	
		12
14	(a). (i).	
	$\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OP} = \frac{1}{3}\underset{\sim}{\boldsymbol{p}}$	
	$\overrightarrow{QS} = \overrightarrow{OS} - \overrightarrow{OQ}$	**B1**
	$= \frac{1}{3}\overrightarrow{OP} - \overrightarrow{OQ} = \frac{1}{3}\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}$	**M1**
	$= \frac{1}{3}\left(\underset{\sim}{\boldsymbol{p}} - 3\underset{\sim}{\boldsymbol{q}}\right)$	**A1**
	(ii).	
	$\overrightarrow{OR} = \frac{1}{3}\overrightarrow{OQ} = \frac{1}{3}\underset{\sim}{\boldsymbol{q}}$	
	$\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS}$	**B1**
	$= \frac{1}{3}\underset{\sim}{\boldsymbol{q}} - \frac{1}{3}\underset{\sim}{\boldsymbol{p}}$	**M1**
	$= \frac{1}{3}\left(\underset{\sim}{\boldsymbol{q}} - \underset{\sim}{\boldsymbol{p}}\right)$	**A1**
	(iii).	
	$\overrightarrow{PT} = \overrightarrow{OT} - \overrightarrow{OP}$	
	but, $4\overrightarrow{QT} = 3\overrightarrow{QS}, \quad \Rightarrow \overrightarrow{QT} = \frac{3}{4}\overrightarrow{QS}$	
	$\overrightarrow{QT} = \frac{3}{4}\times\frac{1}{3}\left(\underset{\sim}{\boldsymbol{p}} - 3\underset{\sim}{\boldsymbol{q}}\right) = \frac{1}{4}\left(\underset{\sim}{\boldsymbol{p}} - 3\underset{\sim}{\boldsymbol{q}}\right)$	**B1**
	$\overrightarrow{OT} = \overrightarrow{OQ} + \overrightarrow{QT} = \underset{\sim}{\boldsymbol{q}} + \frac{1}{4}\left(\underset{\sim}{\boldsymbol{p}} - 3\underset{\sim}{\boldsymbol{q}}\right)$	**M1**
	$= \dfrac{4\underset{\sim}{\boldsymbol{q}} + \underset{\sim}{\boldsymbol{p}} - 3\underset{\sim}{\boldsymbol{q}}}{4} = \frac{1}{4}\left(\underset{\sim}{\boldsymbol{q}} + \underset{\sim}{\boldsymbol{p}}\right)$	

$$\overrightarrow{PT} = \overrightarrow{OT} - \overrightarrow{OP} = \frac{1}{4}\left(\underset{\sim}{\boldsymbol{q}} + \underset{\sim}{\boldsymbol{p}}\right) - \underset{\sim}{\boldsymbol{p}}$$

$$= \frac{\underset{\sim}{\boldsymbol{q}} + \underset{\sim}{\boldsymbol{p}} - 4\underset{\sim}{\boldsymbol{p}}}{4} = \frac{1}{4}\left(\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{p}}\right)$$

A1

(iv).

$$\overrightarrow{TR} = \overrightarrow{OR} - \overrightarrow{OT} = \frac{1}{3}\underset{\sim}{\boldsymbol{q}} - \frac{1}{4}\left(\underset{\sim}{\boldsymbol{q}} + \underset{\sim}{\boldsymbol{p}}\right)$$

M1

$$= \frac{4\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{p}}}{12} = \frac{1}{12}\left(\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{p}}\right)$$

A1

(b).

$$\frac{\overrightarrow{PT}}{\overrightarrow{TR}} = \frac{\frac{1}{4}\left(\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{p}}\right)}{\frac{1}{12}\left(\underset{\sim}{\boldsymbol{q}} - 3\underset{\sim}{\boldsymbol{p}}\right)}$$

$$\frac{\overrightarrow{PT}}{\overrightarrow{TR}} = 3$$

$$\overrightarrow{PT} = 3\overrightarrow{TR}$$

B1

Since $\overrightarrow{PT} = 3\overrightarrow{TR}$ and T is common to both lines $\overrightarrow{PT}$and $\overrightarrow{TR}$, then points P, T and R lie on a straight line.

12

15 (a).

x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
$-4x$	8	4	0	-4	-8	-12	-16	-20	-24
2	2	2	2	2	2	2	2	2	2
y	14	7	2	-1	-2	-1	2	7	14

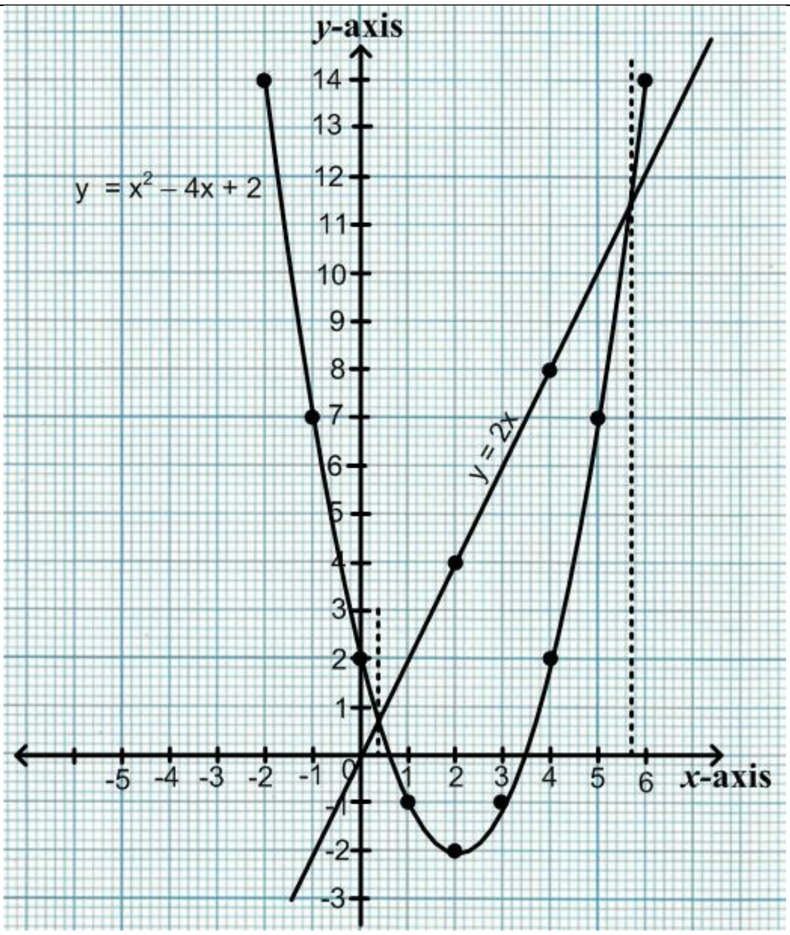
B1
B1
B1
B1

(b).

B1-for plotting

B1-for line

B1-for smooth curve

	 (c). (i). $\begin{array}{r\|c\|} & y = x^2 - 4x + 2 \\ - & 0 = x^2 - 4x + 2 \\ \hline & y = 0 \end{array}$ The roots are: $x = 0.6$and $x = 3.5$. (i). $\begin{array}{r\|c\|} & y = x^2 - 4x + 2 \\ - & 0 = x^2 - 6x + 2 \\ \hline & y = 2x \end{array}$ The roots are: $x = 0.4$and $x = 5.7$.	**A1 A1** **B1** **A1 A1**
		12

***END**

SENIOR FOUR
MATHEMATICS
March 2023
$1\frac{1}{2}$ hours

S.4 MATH BI-WEEKLY TEST 1 2023

Time: 1 Hour 30 Minutes

NAME: ______________________________ **STREAM:** __________

INSTRUCTIONS:
Attempt ALL questions in this paper.
Show your working clearly.

Qn 1: Simplify:

$$\frac{2x-3}{3}-\frac{3x+2}{5}$$

Qn 2: Suppose that after being given a discount of 12% of the marked price, David paid 5,280/= for a shirt. What was its marked price?

Qn 3: If $n(A) = 6, n(B) = 5$ and $n(A \cap B) = 2$, what is $n(A \cup B)$?

Qn 4: Find the size of each angle of a triangle if they are in the ratio 1:3:5.

Qn 5: Express 0.666…… as a rational number in its simples form.

Qn 6: Find the equation of the line whose gradient is $-\frac{1}{2}$ and passes through the point $(-4, 5)$.

Qn 7: Given that $\boldsymbol{OA} = \begin{pmatrix}-1\\2\end{pmatrix}$ and $\boldsymbol{OB} = \begin{pmatrix}-2\\3\end{pmatrix}$, find:

(i). $\boldsymbol{AB}$,
(ii). magnitude of $\boldsymbol{AB}$.

Qn 8: Given that $\log a = n$ and $\log b = m$, express the following in terms of m and n.

(i). $\log ab$
(ii). $\log\left(\frac{b}{a}\right)$
(iii). $\log a^2$

Qn 9: Use logarithm tables to evaluate:

$$\frac{0.00479}{548 \times 0.00984}$$

Qn 10: Given that $2^{2y} = \frac{1}{8}$, find the value of y.

Qn 11: The marked price of a dress is 80,000/= . However, by hire purchase, this price is increased by 5% and distributed into 10 equal monthly installments. Calculate the:

(i). Hire purchase price,
(ii). Amount of each installment,
(iii). Difference between the marked and hire purchase price.

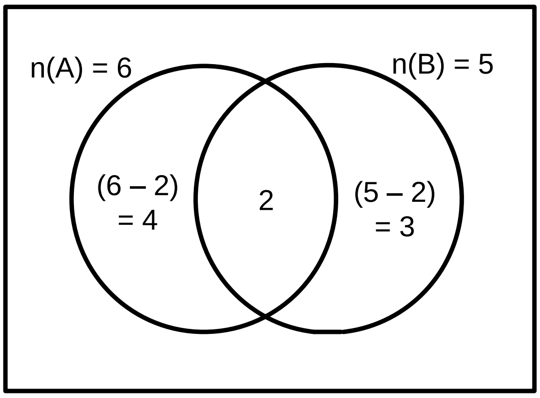
Qn 12: A man drives from town P to Q, which is 200 km away and on a bearing of 030° from P. From Q, he drives for 150 km to town R, whose bearing from P is 060°. Using a scale of 1 cm for 50 km, construct a plan for his journey. Hence find:

(i). the bearing of P from R.
(ii). distance $\overline{PR}$.

*****END*****

MARKING GUIDE

[Total Marks = 60]

SNo.	**Working**	**Marks**
1	$\frac{2x-3}{3}-\frac{3x+2}{5}=\frac{5(2x-3)-3(3x+2)}{15}$ $=\frac{10x-15-9x-6}{15}=\frac{x-21}{15}$	**M1** **M1 M1 A1**
		04
2	Let the marked price be x. $100\%-12\%=88\%$ $88\%\text{ of }x=5280$ $\frac{88}{100}\times x=5280$ $0.88x=5280$ $\frac{0.88x}{0.88}=\frac{5280}{0.88}$ $x=6000$ The marked price was shs 6,000.	**M1** **M1** **A1**
		03
3	 $n(A\cup B)=4+2+3=9$	**B1** **M1 A1**
		03
4	$\text{Total ratio}=1+3+5=9$ $\text{First angle}=\frac{1}{9}\times 180=20°$ $\text{Second angle}=\frac{3}{9}\times 180=60°$ $\text{Third angle}=\frac{5}{9}\times 180=100°$	**B1** **B1** **B1**
		03
5	Let $x=0.666\ldots\ldots$	

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$$10x = 0.666\ldots\ldots \times 10$$
$$10x = 6.666\ldots\ldots$$
$$\begin{array}{r|rcl|} & 10x & = & 6.666\ldots\ldots \\ - & x & = & 0.666\ldots\ldots \\ \hline & 9x & = & 6 \end{array}$$
$$x = \frac{6}{9} = \frac{2}{3}$$
$$\therefore 0.666\ldots = \frac{2}{3}$$

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<td>B1
M1
M1 A1</td>
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$$y = mx + c$$
$$5 = -\frac{1}{2} \times (-4) + c$$
$$5 = 2 + c$$
$$c = 2$$
∴ The required equation of the line is: $y = -\frac{1}{2}x + 2$.

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M1 A1</td>
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(i).
$$\boldsymbol{AB} = \boldsymbol{OB} - \boldsymbol{OA} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
(ii).
$$|\boldsymbol{AB}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \approx 1.414$$

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M1 A1</td>
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(i).
$$\log ab = \log a + \log b = n + m$$
(ii).
$$\log\left(\frac{b}{a}\right) = \log b - \log a = m - n$$
(iii).
$$\log a^2 = 2\log a = 2n$$

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M1 A1
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<tr><td></td><td></td><td>06</td></tr>
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<td>9</td>
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<table>
<tr><th>Number</th><th>Standard form</th><th>Logarithm</th></tr>
<tr><td>0.00479</td><td>4.79×10^{-3}</td><td>$\bar{3}.6803$</td></tr>
<tr><td>548
0.00984</td><td>5.48×10^{2}
9.84×10^{-3}</td><td>2.7387
$+\ \bar{3}.9930$
0.7317</td></tr>
<tr><td colspan="2"></td><td>$\bar{3}.6803$
$-\ 0.7317$
$\bar{4}.9486$</td></tr>
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<td>B1
B1
B1
B1
B1
M1 A1</td>
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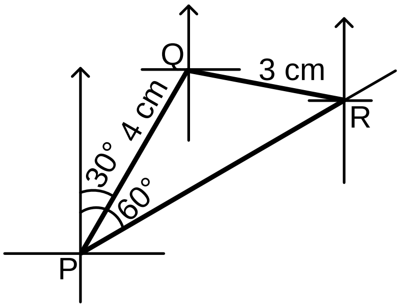
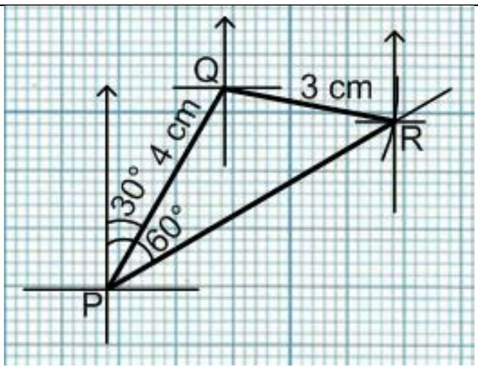
	$\text{antilog}(0.9486) \times 10^{-4} \approx 8.884 \times 10^{-4} = 0.0008884$ $\therefore \dfrac{0.00479}{548 \times 0.00984} \approx 0.0008884$	
		07
10	$2^{2y} = 8^{-1}$ $2^{2y} = (2^3)^{-1}$ $2^{2y} = 2^{-3}$ $2y = -3$ $y = -\dfrac{3}{2}$	**M1** **M1** **M1** **A1**
		04
11	(i). $100\% + 5\% = 105\%$ $\text{Hire purchase price} = \dfrac{105}{100} \times 80000 = 84000/=$ (ii). $\text{Each installment} = \dfrac{84000}{10} = 8400/=$ (iii). $\text{Difference} = 84000 - 80000 = 4000/=$	**M1 A1** **M1 A1** **M1 A1**
		06
12	$\overline{PQ} = 200\text{ km} \equiv \dfrac{200}{50} = 4\text{ cm}$ $\overline{QR} = 150\text{ km} \equiv \dfrac{150}{50} = 3\text{ cm}$ **Sketch:**  **Accurate diagram:**	**B1** **B1** **B2**

	 (i). Bearing of P from R $= 240° \pm 001°$ (ii). Length $\overline{PR} = (5.8 \pm 0.1)$cm Distance $\overline{PR} = (290 \pm 5)$km	**B1**-for north lines **B2**-for dimensions **B1**-for 30° **B1**-for 60° **B1** **B1** **B1**
		12

*****END*****

SENIOR FOUR
MATHEMATICS
March 2023
$1\frac{1}{2}$ hours

S.4 MATH BI-WEEKLY TEST 2 2023

Time: 1 Hour 30 Minutes

NAME: **STREAM:**

INSTRUCTIONS:
Attempt ALL questions in this paper.
Show your working clearly.

Qn 1: Solve for x:

$$\frac{1}{3x-4}+\frac{x}{x+1}=1$$

Qn2: A line is given by the equation $45-15x+3y=0$. Find the coordinates of its x-intercept.

Qn3: A trade made a 35% profit after selling a goat at shs 45,900. What was the cost price of the goat?

Qn4: The height of a small box is 2 cm and its volume 10 cm^3. If the height of a similar box is 6 cm, what is its volume?

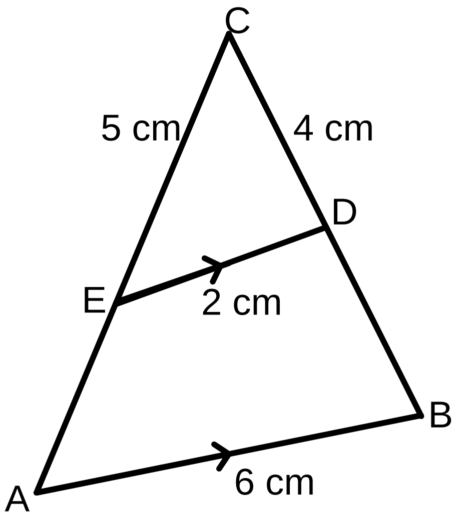
Qn5: Under an enlargement scale factor 3, the image of the point $P(0,3)$is $P'(4,5)$. Find the coordinates of the centre of enlargement.

Qn6: Show that the points $(3x,3y)$, $(2x,y)$ and $(0,7y)$ lie on a straight line.

Qn7: Solve the simultaneous equations:

$$x^2+4y^2=4 \quad \longrightarrow (i)$$
$$y-x=-1 \quad \longrightarrow (ii)$$

Qn8: In the figure ABC, $\overline{\text{AB}}=6$ cm, $\overline{\text{ED}}=2$ cm, $\overline{\text{CD}}=4$ cm and $\overline{\text{CE}}=5$ cm.



If $\overline{\text{ED}}$ is parallel to $\overline{\text{AB}}$, find length AE.

Qn 9: (a). Copy and complete the following table of values for the curve $y = (x-1)(x-3)$between$x = -1$ and $x = 5$.

x	-1	0	1	2	3	4	5
$x-1$							
$x-3$							
$y = (x-1)(x-3)$							

(b). Use your table to draw a graph of $y = (x-1)(x-3)$for $-1 \leq x \leq 5$. Use a scale of 1 cm to represent 1 unit on both axes.

(c). Use your graph to solve:

(i). $x^2 - 4x + 3 = 0$,

(ii). $x^2 - 4x + 1 = 0$.

(d). Using dotted line, indicate the line of symmetry on your graph in (b) above. Hence state the equation of the line of symmetry.

(e). From your graph, find the:

(i). minimum value of the function.

(ii). range of values for which $(x-1)(x-3) < 0$.

Qn 10: Use the inverse matrix method to find the values of $x$and y.

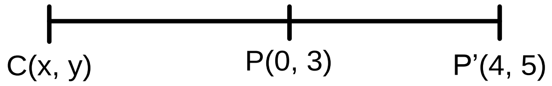
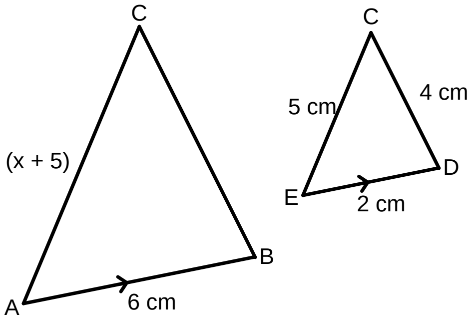
$$2x - 3y = 12 \quad \longrightarrow (i)$$
$$x + 2y + 1 = 0 \quad \longrightarrow (ii)$$

*****END*****

MARKING GUIDE

[Total Marks = 50]

SNo.	Working	Marks
1	$\frac{1}{3x-4}+\frac{x}{x+1}=1$ $\text{L.C.M for the denominator} = (3x-4)(x+1)$ $(x+1)\times 1+(3x-4)\times x = 1\times(3x-4)(x+1)$ $x+1+3x^2-4x = 3x^2+3x-4x-4$ $x+1 = 3x-4$ $5 = 2x$ $x = \frac{5}{2} = 2.5$	**M1** **M1** **M1** **A1**
		04
2	$45-15x+3y=0$ when $y=0,\quad 45-15x+3\times 0=0$ $45 = 15x$ $\frac{45}{15}=\frac{15x}{15}$ $x=3$ $\therefore$ The coordinates of its x-intercept is $(3, 0)$.	**M1** **M1** **A1** **B1**
		04
3	Let the cost price be x. $100\% + 35\% = 135\%$ 135% of $x = 45900$ $\frac{135}{100}\times x = 45900$ $1.35x = 45900$ $\frac{1.35x}{1.35}=\frac{45900}{1.35}$ $x = 34{,}000$ The cost price of the goat is shs 34,000.	**B1** **M1** **M1** **A1**
		04
4	$h_1 = 2\text{ cm},\quad v_1 = 10\text{ cm}^3,\quad h_2 = 6\text{ cm}$ $L.S.F = \frac{h_2}{h_1} = \frac{6}{2} = 3$ $V.S.F = \frac{v_2}{v_1} = \frac{v_2}{10}$ but, $\quad V.S.F = (L.S.F)^3$ $\frac{v_2}{10} = 3^3$ $v_2 = 27\times 10 = 270\text{ cm}^3$	**B1** **M1 B1** **A1**
		04

5	Let $P(x, y)$ be the centre of enlargement.  $\overrightarrow{CP'} = 3\overrightarrow{CP}$ $\begin{pmatrix}4-x\\5-y\end{pmatrix} = 3\begin{pmatrix}0-x\\3-y\end{pmatrix}$ $\begin{pmatrix}4-x\\5-y\end{pmatrix} = \begin{pmatrix}0-3x\\9-3y\end{pmatrix}$ $4 - x = -3x, \quad \Longrightarrow 2x = -4, \quad \therefore x = -2$ $5 - y = 9 - 3y, \quad \Longrightarrow 2y = 4, \quad \therefore y = 2$ $\therefore$ Centre, $C(-2, 2)$	**M1** **A1** **A1** **B1**
		04
6	For $(3x, 3y)$ and $(2x, y)$, $\text{Gradient} = \frac{y-3y}{2x-3x} = \frac{-2y}{-x} = \frac{2y}{x}$ For $(2x, y)$ and $(0, 7y)$ $\text{Gradient} = \frac{7y-y}{0-2x} = \frac{6y}{-2x} = \frac{3y}{x}$ The points don't lie on a straight line.	**B1** **B1** **B1**
		03
7	$y = x - 1$ $x^2 + 4(x-1)^2 = 4$ $x^2 + 4(x^2 - 2x + 1) = 4$ $x^2 + 4x^2 - 8x + 4 = 4$ $5x^2 - 8x = 0$ $x(5x - 8) = 0$ $x = 0, \quad \text{or}, \quad (5x - 8) = 0$ $x = 0, \quad \text{or}, \quad x = \frac{8}{5}$ $\text{when } x = 0, \quad y = 0 - 1 = -1$ $\text{when } x = \frac{8}{5}, \quad y = \frac{8}{5} - 1 = \frac{3}{5}$ Hence $(0, -1), \left(\frac{8}{5}, \frac{3}{5}\right)$.	**M1** **M1** **B1** **B1** **A1 A1**
		06
8	Let $\overline{\text{AE}} = x$  By similarity,	

$$\frac{x+5}{5}=\frac{6}{2}$$

M1 B1

$$x+5=15$$
$$x=15-5$$
$$x=10$$
$$\Rightarrow \overline{\mathrm{AE}}=10\text{ cm}$$

A1

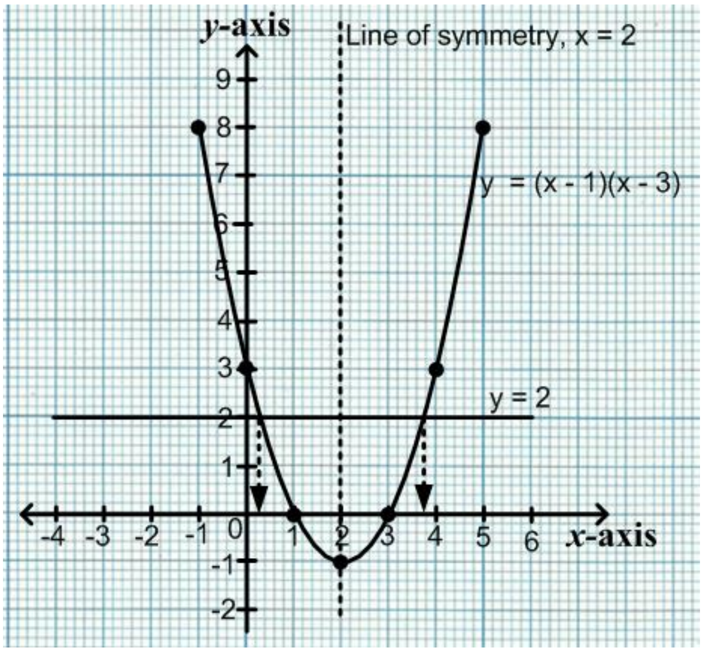
03

9 (a).

x	-1	0	1	2	3	4	5
$x-1$	-2	-1	0	1	2	3	4
$x-3$	-4	-3	-2	-1	0	1	2
y	8	3	0	-1	0	3	8

B1
B1
B1

(b).



B1-line $x=2$ and $y=2$

B1-curve

(c). (i).

$$y=(x-1)(x-3)=x^2-4x+3$$

$$\begin{array}{r|c|}
 & y=x^2-4x+3 \\
- & 0=x^2-4x+3 \\
\hline
 & y=0
\end{array}$$

B1

The roots are: $x=1$and $x=3$.

A1 A1

(ii).

$$\begin{array}{r|c|}
 & y=x^2-4x+3 \\
- & 0=x^2-4x+1 \\
\hline
 & y=2
\end{array}$$

B1

The roots are: $x=0.3$ and $x=3.7$. **A1 A1**

(d). The line of symmetry is $x=2$. **A1**

(e). (i). Minimum value is: -1. **A1**

(ii). The range of values is: $1<x<3$. **A1**

14

10	$2x - 3y = 12 \quad \longrightarrow (i)$ $x + 2y = -1 \quad \longrightarrow (ii)$ $\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$ $\text{Det} = 4 + 3 = 7$ $\frac{1}{7}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 21 \\ -14 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $x = 3, \qquad y = -2$	**M1** **M1** **A1 A1**
		04

END

SENIOR FOUR
MATHEMATICS
April 2023
$1\frac{3}{4}$ hours

S.4 MATH BI-WEEKLY TEST 3 2023

Time: 1 Hour 45 Minutes

NAME: **STREAM:**

INSTRUCTIONS:

Attempt ALL questions in this paper.
Show your working clearly.

Qn 1: Without using tables or calculator, simplify
$\sqrt{243} - \sqrt{108} + \sqrt{75}$. [4]

Qn 2: The midpoint of the segment $\overline{AB}$ is T. Given that the coordinates of B are $(6,5)$ and $T$are $(2,3)$, determine the coordinates of A. [4]

Qn 3: Given the function $f(x) = \frac{1}{x}$ and $g(x) = 2x - 1$. Determine an expression for $gf(x)$ and find the value of x for which $gf(x) = 0$. [4]

Qn 4: The base areas of two similar tins are 24 $\text{cm}^2$and 54 cm^2. If the volume of the smaller tin is 144 cm^3, determine the volume of the larger tin. [4]

Qn 5: Given that the position vectors of **A** and **B** are $\begin{pmatrix}-2\\4\end{pmatrix}$ and $\begin{pmatrix}7\\7\end{pmatrix}$ respectively and also that X is on **AB** such that $\overrightarrow{\boldsymbol{AX}}:\overrightarrow{\boldsymbol{XB}} = 1:2$. Determine the column vector:

(i). $\overrightarrow{\boldsymbol{AB}}$

(ii). $\overrightarrow{\boldsymbol{OX}}$. [4]

Qn 6: Two quantities x and y are such that y is partly constant and partly varies inversely as x and that, $y = 11$, when $x = 2$ and $y = 7$when $x = 6$. Determine the value of y when $x = 4$. [4]

Qn 7: On a map, a forest of area 7.2 km^2 is represented by 5 cm^2. Find the length of a road represented by 9 cm on the map. [4]

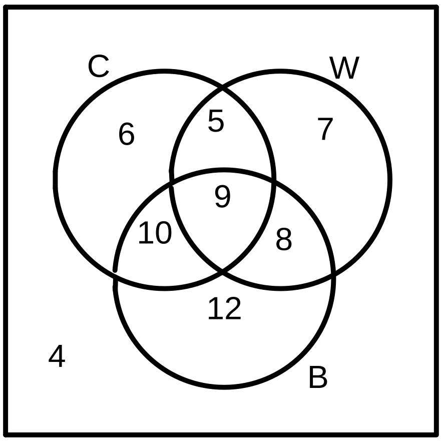
Qn 8: A sum of money is put to compound interest; the first year's interest is shs 75,000 and the second year's is shs 82,500. Find:

(a). The rate per annum, [6]

(b). The sum invested, [2]

(c). the interest for the third year. [4]

Qn 9: The diagram shows how children come to school, by walking (W), by bicycle (B) ot by car (C).

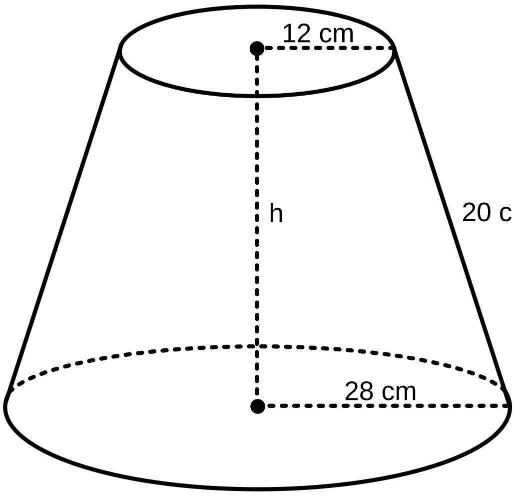


Use the information on the Venn diagram to find:

(i). $n(C \cup W \cup B)$,

(ii). $n(C \cup W')$.

Qn 10: The diagram below shows a lampshade made out of the lower part of a cone. The base radius is 28 cm, the top radius is 12 cm and the slant height is 20 cm.



Calculate the :

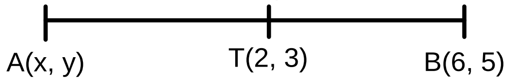
(a). height h, of the lamp shade. [3]

(b). surface area of the lampshade. (Use $\pi = 3.14$) [2]

*****END*****

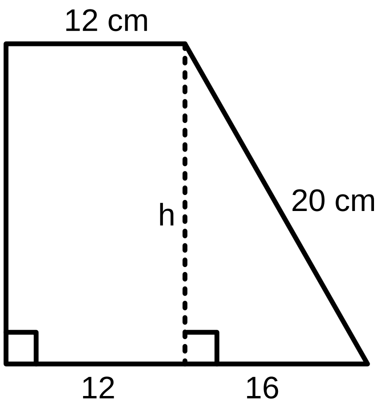
MARKING GUIDE

[Total Marks = 64]

SNo.	Working	Marks
1	$\sqrt{243}-\sqrt{108}+\sqrt{75}=\sqrt{81\times 3}-\sqrt{36\times 3}+\sqrt{25\times 3}$ $=9\sqrt{3}-6\sqrt{3}+5\sqrt{3}$ $=(9-6+5)\sqrt{3}=8\sqrt{3}$	**M1** **M1** **A1**
		03
2	Let $P(x,y)$ be the centre of enlargement.  $\frac{x+6}{2}=2, \quad \Rightarrow x+6=4, \quad \Rightarrow x=-2$ $\frac{y+5}{2}=3, \quad \Rightarrow y+5=6, \quad \Rightarrow y=-1$ $\therefore A(-2,-1)$	**M1** **M1** **A1**
		03
3	$gf(x)=g\left(\frac{1}{x}\right)$ $=2\left(\frac{1}{x}\right)-1$ $gf(x)=\frac{2}{x}-1$ for, $\quad gf(x)=0$ $\frac{2}{x}-1=0$ $\frac{2}{x}=1$ $2=x$ $x=2$	**M1** **B1** **M1** **A1**
		04
4	$A.S.F=\frac{54}{24}=2.25$ $L.S.F=\sqrt{2.25}=1.5$ $V.S.F=(1.5)^3=3.375$ $V.S.F=\frac{\text{volume of larger tin}}{\text{volume of smaller tin}}$ $3.375=\frac{v}{144}$ $v=3.375\times 144$ $v=486\text{ cm}^3$ The volume of the larger tin is 486 cm^3.	**B1** **B1** **M1** **M1** **A1**
		04
5	(i).	

	$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}} = \begin{pmatrix}7\\7\end{pmatrix} - \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}9\\3\end{pmatrix}$ (ii). $\overrightarrow{\boldsymbol{AX}}:\overrightarrow{\boldsymbol{XB}} = 1:2$ $\overrightarrow{\boldsymbol{AX}} = \frac{1}{3}\overrightarrow{\boldsymbol{AB}}$ $\overrightarrow{\boldsymbol{OX}} - \overrightarrow{\boldsymbol{OA}} = \frac{1}{3}\begin{pmatrix}9\\3\end{pmatrix}$ $\overrightarrow{\boldsymbol{OX}} - \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{\boldsymbol{OX}} = \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}-2\\4\end{pmatrix}$ $\overrightarrow{\boldsymbol{OX}} = \begin{pmatrix}1\\5\end{pmatrix}$	**M1 A1** **M1** **A1**
		04
6	$y = a + \frac{b}{x}$ when $x = 2, y = 11$ $11 = a + \frac{b}{2}$ $22 = 2a + b$ $22 - 2a = b$ $b = 22 - 2a$ when $x = 6, y = 7$ $7 = a + \frac{b}{6}$ $42 = 6a + b$ $42 = 6a + (22 - 2a)$ $42 = 4a + 22$ $42 - 22 = 4a$ $20 = 4a$ $5 = a$ $b = 22 - 2a = 22 - 2 \times 5 = 22 - 10 = 12$ $\therefore y = 5 + \frac{12}{x}$ When $x = 4, \quad y = 5 + \frac{12}{4} = 5 + 3 = 8$	**B1** **B1** **M1** **B1** **M1 B1** **B1** **A1**
		08
7	The area scale is $5\text{ cm}^2 : 7.2\text{ km}^2$ $1\text{ cm}^2 : \frac{7.2}{5}\text{ km}^2$ $1\text{ cm}^2 : 1.44\text{ km}^2$ The linear scale is $\sqrt{1}\text{ cm} : \sqrt{1.44}\text{ km}$ $1\text{ cm} : 1.2\text{ km}$ $9\text{ cm} : 1.2 \times 9\text{ km}$ $9\text{ cm} : 10.8\text{ km}$	**M1** **B1** **M1** **A1**

	The actual length of the road is 10.8 km.	
		04
8	(a). Interest for the 1st year, $I_1 = P_1 \times \frac{R}{100} \times T$ $7500000 = P_1R$ $P_1 = \frac{7500000}{R}$	**B1**
	Ammount at the end of the 1st year, $A_1 = I_1 + P_1 = 75000 + P_1$	**B1**
	Interest for the 2nd year, $I_2 = P_2 \times \frac{R}{100} \times T$ $82500 = (75000 + P_1) \times \frac{R}{100} \times 1$	**M1**
	$8250000 = (75000 + P_1)R$ $8250000 = 75000R + P_1R$ $8250000 = 75000R + \frac{7500000}{R} \times R$	**M1**
	$8250000 = 75000R + 7500000$ $8250 = 75R + 7500$ $8250 - 7500 = 75R$ $750 = 75R$	**B1**
	$10 = R$ $\therefore R = 10\%$	**A1**
	(b). $P_1 = \frac{7500000}{R} = \frac{7500000}{10} = 750,000$	**M1 A1**
	Sum inested is shs 750,000. (c). $P_2 = I_1 + P_1 = 7500 + 750000 = 825000$	**B1**
	$P_3 = I_2 + P_2 = 82500 + 825000 = 907500$	**B1**
	Interest for the 3rd year, $I_3 = P_3 \times \frac{R}{100} \times T$ $I_3 = 907500 \times \frac{10}{100} \times 1 = 90750$	**M1 A1**
		12
9	(i). $n(C \cup W \cup B) = 6 + 5 + 9 + 10 + 12 + 8 + 7 + 4 = 57$	**M1 A1**
	(ii). $n(C \cup W') = 12 + 10 + 6 + 5 + 9 + 4 = 46$	**M1 A1**
		04
10	(a).	

	 $20^2 = h^2 + 16^2$ $h^2 = 20^2 - 16^2$ $h^2 = 144$ $h = 12\text{ cm}$ (b). $S.A = \pi(R + r)l = 3.14 \times (28 + 12) \times 20 = 2512\text{ cm}^3$	**B1** **M1** **A1** **M1 A1**
		05

*****END*****

456/1
MATHEMATICS
PAPER 1
April 2023
$2\frac{1}{2}$ hours

S.4 MATH 1 MOCK SET 2 2023

Time: 2 Hours 30 Minutes

NAME: ______________________ **STREAM:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Given that $a * b = a^3 - b^2$, find the value of $4 * (3 * 5)$. [4]

Qn 2: Solve the equation $2x^2 + 3x - 27 = 0$. [4]

Qn 3: Solve for x:

$$\frac{x+1}{3} - \frac{2-x}{2} = \frac{x}{4}$$

[4]

Qn 4: A translation $\boldsymbol{T}$ maps $(4, 0)$ onto $(-2, 2)$. Determine the coordinates of the image of $(0, 1)$ under $\boldsymbol{T}$. [4]

Qn 5: Without using tables of calculators, simplify leaving your answer in surd form.

$$\frac{\sin 45^\circ + \cos 45^\circ}{\tan 60^\circ}$$

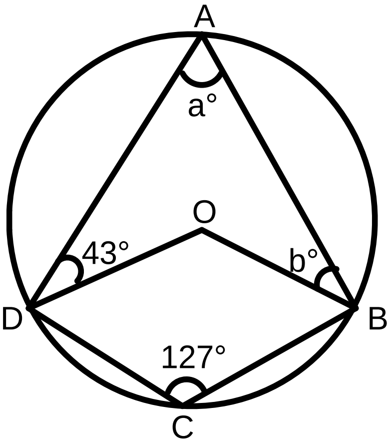
[4]

Qn 6: A bag contains blue, green and red balls. The probability of picking a blue ball is $\frac{1}{4}$ and the probability of picking a green ball is $\frac{7}{12}$.

(a). Find the probability of picking a red ball. [2]

(b). If the bag contains 84 balls, find the number of red balls in the bag. [2]

Qn 7:



In the figure shown above, O is the centre of the circle. Find:

(i). angle a.

(ii). angle b. [4]

Qn 8: Factorise completely:

$$25a^3 - ab^2 - b^3 + 25a^2b$$

[4]

Qn 9: Given that $f(x) = x + 3$ and $g(x) = 2 - x$, find:

(i). $gf(x)$,

(ii). $gf(-2)$. [4]

Qn 10: A man of height 1.6 m is 15 m from the foot of a tree. When he looks at the top of the tree, the angle of elevation is 50°. Determine the height of the tree. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

(a). If $\boldsymbol{A} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -1 & 5 \\ 6 & -6 \end{pmatrix}$, find $2\boldsymbol{A} - \boldsymbol{BA}$. [4]

(b). Determine the inverse of matrix $\boldsymbol{P} = \begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix}$. [4]

(c). Given that matrix $\boldsymbol{A} = \begin{pmatrix} 2 & 5 \end{pmatrix}$, matrix $\boldsymbol{B} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ and matrix $\boldsymbol{C} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. Find $\boldsymbol{AB} + \boldsymbol{AC}$. [4]

Question 12:

(a). Draw a table for values of x and y for the curve $y = x^2$ where $^{-}5 \leq x \leq 5$. [2]

(b). Using a scale of:

Horizontal axis 1 cm : 1 unit
Vertical axis 1 cm : 5 units,

draw a graph of $y = x^2$. [3]

(c). Use your graph to solve the equation:

$x^2 - 2x - 8 = 0$. [6]

(d). State the minimum value of the graph $y = x^2$. [1]

Question 13:

The table below shows the weights of 100 boys in Ndejje S.S.S.

Weights	**Number of boys**
10 – 19	3
20 – 29	8
30 – 39	12
40 – 49	8
50 – 59	15
60 – 69	20
70 – 79	15
80 – 89	10
90 – 99	9

(a). Using an assumed mean of 54.5, calculate the mean weight. [6]

(b). (i). Draw a histogram for the data.

(ii). Use the histogram to estimate the mode. [6]

Question 14:

A transformation represented by the matrix $\begin{pmatrix} 6 & 10 \\ 1 & 2 \end{pmatrix}$ maps the vertices A, B, C and D of a rectangle onto the points $A'(22, 4)$, $B'(62, 12)$, $C'(80, 15)$ and $D'(40, 7)$ respectively.

(a). Find the:

(i). inverse of the matrix.

(ii). coordinates of A, B, C and D using the inverse matrix. [7]

(b). (i). Plot the points A, B, C and D on a squared paper.

(ii). Find the area of the rectangle $ABCD$.

(iii). Use the area of the rectangle $ABCD$ to determine the are of $A'B'C'D'$. [5]

Question 15:

Using a ruler, a pencil and a pair of compasses only,

(a). Construct a triangle ABC, with $\overline{AB} = 8$ cm, $\overline{BC} = 12$ cm and angle $BAC = 120°$. [4]

(b). (i). Draw a perpendicular line to BC from A. The perpendicular meets BC at point D.

(ii). Measure the distance $\overline{AD}$ and find the area of triangle ABC. [4]

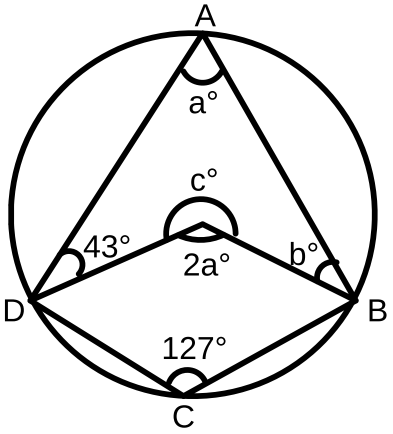
(c). Inscribe triangle ABC and state the radius. [4]

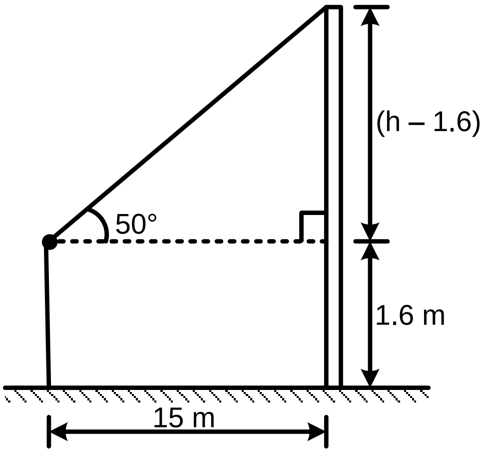
*****END*****

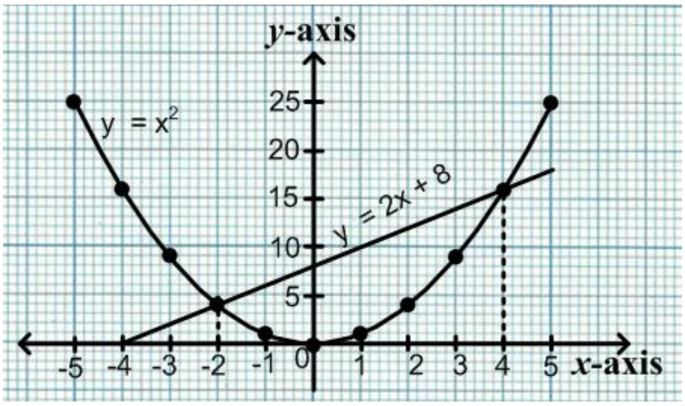
MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	(i). $3 * 5 = 3^3 - 5^2 = 27 - 25 = 2$ (ii). $4 * (3 * 5) = 4 * 2 = 4^3 - 2^2 = 64 - 4 = 60$	**M1 A1** **M1 A1**
		04
2	$2x^2 + 3x - 27 = 0$ $\text{sum} = -54, \quad \text{product} = 3, \quad \text{factors} = -9, 6$ $2x^2 - 9x + 6x + 27 = 0$ $x(2x - 9) - 3(2x - 9) = 0$ $(x - 3)(2x - 9) = 0$ $(x - 3) = 0, \quad \text{or,} \quad (2x - 9) = 0$ $x = 3, \quad \text{or,} \quad 2x = 9$ $x = 3, \quad \text{or,} \quad x = -\frac{9}{2}$	**M1** **M1** **A1 A1**
		04
3	$12 \times \frac{(x + 1)}{3} - 12 \times \frac{(2 - x)}{2} = 12 \times \frac{x}{4}$ $4(x + 1) - 6(2 - x) = 3x$ $4x + 4 - 12 + 6x = 3x$ $10x - 8 = 3x$ $7x = 8$ $x = \frac{8}{7}$	**M1** **M1** **M1** **A1**
		04
4	$\boldsymbol{T} = \begin{pmatrix}-2\\2\end{pmatrix} - \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}-6\\2\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}-6\\2\end{pmatrix} = \begin{pmatrix}-6\\3\end{pmatrix}$ $\therefore$ The image of $(0, 1)$ under $\boldsymbol{T}$is $(-6,\ 3)$.	**M1** **M1 B1** **A1**
		04
5	but, $\quad \sin 45° + \cos 45° = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ $\therefore \frac{\sin 45° + \cos 45°}{\tan 60°} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{6}}{3}$	**B1** **M1 M1 A1**
		04
6	(a). $\frac{1}{4} + \frac{7}{12} + P(R) = 1$ $\frac{5}{6} + P(R) = 1$	**M1**

	(b). $P(R) = 1 - \frac{5}{6} = \frac{1}{6}$ Number of red balls $= \frac{1}{6} \times 84 = 14$	**A1** **M1 A1**
		04
7	(i). $a + 127 = 180$ $a = 180 - 127$ $a = 53°$ (ii).  $c = 360 - 2a = 360 - 2 \times 53 = 254°$ $b + 43 + 53 + 254 = 360$ $b + 350 = 360$ $b = 10°$	**M1** **A1** **M1** **A1**
		04
8	$25a^3 - ab^2 - b^3 + 25a^2b$ $= a(25a^2 - b^2) + b(25a^2 - b^2)$ $= (a + b)[(5a)^2 - b^2]$ $= (a + b)(5a - b)(5a + b)$	**M1** **M1** **M1 A1**
		04
9	(i). $gf(x) = g(x + 3) = 2 - (x + 3) = 5 - x$ (ii). $gf(-2) = 5 - {}^{-}2 = 5 + 2 = 7$	**M1 A1** **M1 A1**
		04
10		

	 $\tan 50° = \frac{h-1.6}{15}$ $15\tan 50° = h - 1.6$ $h = 15\tan 50° + 1.6$ $h = 19.476 \text{ m}$ ∴ The height of the tree is 19.476 m.	**B1** **M1** **B1** **A1**
		04
11	(a). $2\boldsymbol{A} - \boldsymbol{BA} = 2\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 6 & -6 \end{pmatrix}$ $= \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} -2+24 & 10-24 \\ -1+18 & 5-18 \end{pmatrix}$ $= \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} 22 & -14 \\ 17 & -13 \end{pmatrix} = \begin{pmatrix} -18 & 22 \\ -15 & 18 \end{pmatrix}$ (b). $\boldsymbol{P} = \begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix}$ $\text{Det } \boldsymbol{P} = 6 - {}^{-}2 = 8$ $\boldsymbol{P}^{-1} = \frac{1}{8}\begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3/8 & -1/4 \\ 1/8 & 1/4 \end{pmatrix}$ (c). $\boldsymbol{AB} + \boldsymbol{AC} = (2 \quad 5)\begin{pmatrix} 4 \\ 6 \end{pmatrix} + (2 \quad 5)\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $= (8+30) + (4-15) = 38 - 11 = 27$	**M1** **B1** **M1 A1** **B1** **M1 B1 A1** **M1** **M1 M1 A1**
		12
12	(a). $y = x^2$ x: -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 y: 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25 $y = 2x + 8$ x: 0, -4 y: 8, 0	**B1** **B1** **M1 M1**



B1-smooth curve

B1-corret plotting

B1-line y=2x+8

$$\begin{array}{r|c|}
 & y = x^2 \\
- & 0 = x^2 - 2x - 8 \\ \hline
 & y = 2x + 8
\end{array}$$

The roots are: $x = -2$and $x = 4$.

M1
B1
A1 A1

(d).

Minimum value $= 0$

A1

12

13 (a). Let $d = x - 54.5$.

Class	f	x	d	fd	Class boundary
10 – 19	3	14.5	– 40	– 120	9.5 – 19.5
20 – 29	8	24.5	– 30	– 240	19.5 – 29.5
30 – 39	12	34.5	– 20	– 240	29.5 – 39.5
40 – 49	8	44.5	– 10	– 80	39.5 – 49.5
50 – 59	15	54.5	0	0	49.5 – 59.5
60 – 69	20	64.5	10	200	59.5 – 69.5
70 – 79	15	74.5	20	300	69.5 – 79.5
80 – 89	10	84.5	30	300	79.5 – 89.5
90 – 99	9	94.5	40	360	89.5 – 99.5
Total	100			480	

B1-for x

B1-for d

B1-for fd

B1-for class boundary

Mean, $$\bar{x} = 54.5 + \frac{\sum fd}{\sum f} = 54.5 + \frac{480}{100} = 59.3$$

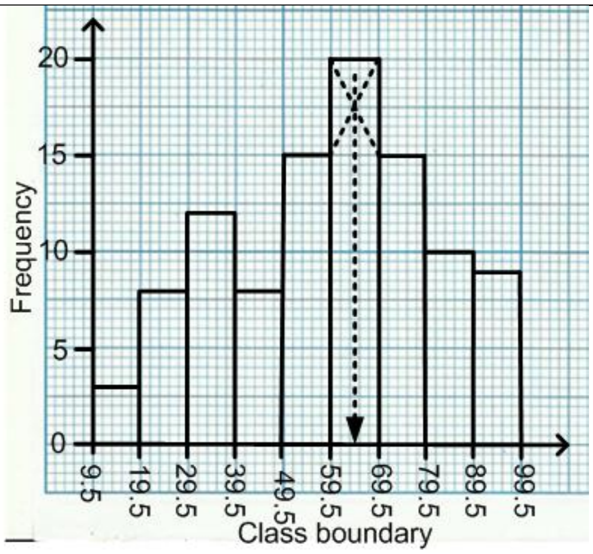
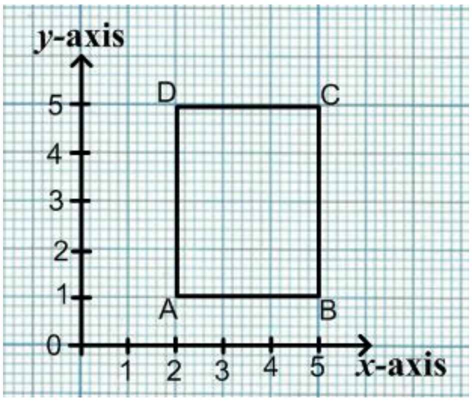
M1 A1

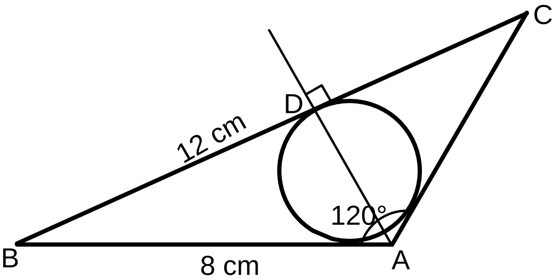
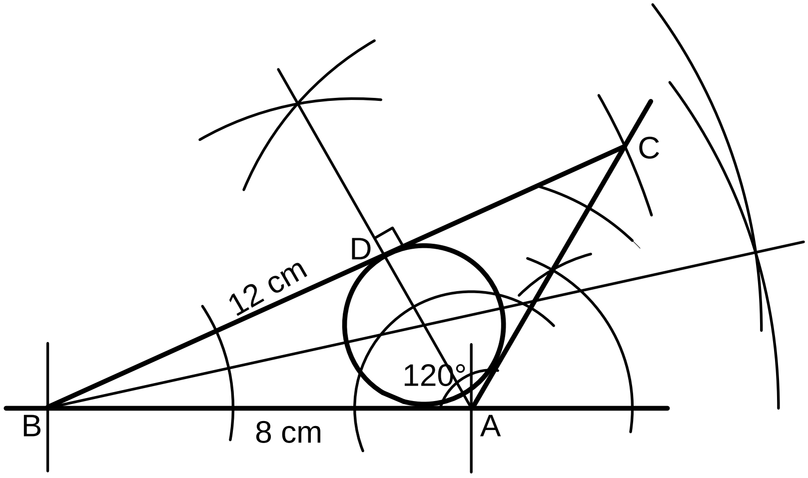
(b). (i).

B1 B1-labelling axes

B1-bars

B1-correct scaling

	 (ii). From the graph, Mode $= 59.5 + 5 = 64.5$.	**M1-**locating mode **A1**
		12
14	(a). (i). $\text{Inverse matrix} = \frac{1}{12-10}\begin{pmatrix}2 & -10\\ -1 & 6\end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix}2 & -10\\ -1 & 6\end{pmatrix} = \begin{pmatrix}1 & -5\\ -0.5 & 3\end{pmatrix}$ (ii). $\begin{pmatrix}1 & -5\\ -0.5 & 3\end{pmatrix}\begin{pmatrix}22 & 62 & 80 & 40\\ 4 & 12 & 15 & 7\end{pmatrix} = \begin{pmatrix}2 & 2 & 5 & 5\\ 1 & 5 & 5 & 1\end{pmatrix}$ $\therefore A(2,1), \quad B(2,5), \quad C(5,5), \quad D(5,1)$ (b). (i).  (ii). From the graph, the area of the square ABCD is 12 sq. units. (iii). $\text{Determinant } A'B'C'D' = \lvert\text{Det } \boldsymbol{M}\rvert \times 12$ $= \lvert(12-10)\rvert \times 12 = 2 \times 12 = 24 \text{ sq. units}$	**B1** **B1** **B1** **A1 A1A1A1** **B1** **B1 B1** **M1 A1**
		12
15	(a). **Sketch:**	

		**B1 B1**
	Accurate diagram: 	**B1**-length BC **B1**-angle 120° **B1**-dropping perpendicular **B1 B1**-any two angle bisectors **B1**-circle
	(b). (ii). $\overline{AD} = (3.35 \pm 0.1)\text{ cm}$	**A1**
	$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times 12 \times 3.35 = 20.1\text{ cm}^2$	**M1 A1**
	(c). $\text{Radius} = (1.65 \pm 0.1)\text{ cm}$	**A1**
		12

*****END****

456/2
MATHEMATICS
PAPER 2
April 20023
$2\frac{1}{2}$ hours

S.4 MATH 2 MOCK SET 2 2023

Time: 2 Hours 30 Minutes

NAME: **STREAM:**

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Evaluate without using calculators:

$$(0.4)^2 \times (0.125)^{1/3} \div \left(2\frac{1}{2}\right)^{-3}$$

[4]

Qn 2: Given that $\frac{\sqrt{8}-\sqrt{18}}{1-\sqrt{2}} = a + b\sqrt{2}$, determine the value of $a$and$b$. [4]

Qn 3: Set A and B are such that$n(A) = 12, n(B) = 13, n(A \cup B) = 20$ and $n(\varepsilon) = 24$.

(a). Draw a Venn diagram to represent the given information.

(b). Find$n(A \cup B')$. [4]

Qn 4: Find the equation of line passing through the points$(1, -3)$ and $(7, 6)$. Hence determine the coordinates of a point where the line cuts the x-axis. [4]

Qn 5: Express$0.4\overline{2}$ in the simplest form of$\frac{x}{y}$; hence evaluate$(y - x)$. [4]

Qn 6: The value of a car depreciates by 12% per year. If the value is now shs 6,195,200, what was the value of the car two years ago. [4]

Qn 7: Given that $g(x) = x + 3$and $fg(x) = 2x - 1$, determine:

(i). $f(x)$,

(ii). $f(4)$. [4]

Qn 8: The position vectors of **A** and **B** are $\begin{pmatrix}-2\\-1\end{pmatrix}$ and $\begin{pmatrix}4\\4\end{pmatrix}$ respectively. **T** divides$\overrightarrow{\boldsymbol{AB}}$ in the ratio 2: 1. Determine:

(a). Column vector$\overrightarrow{\boldsymbol{AB}}$.

(b). coordinates of $\boldsymbol{T}$. [4]

Qn 9: Using logarithmic tables, evaluate:

$$\frac{2460 \quad \times 8.72}{63.1 \quad \times 204}$$

[4]

Qn 10: John deposited shs 56,000 in a bank . The bank gives a compound interest of 15% per annum. Find the amount of money he had in the bank after 3 years. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

(a). Simplify:

$$\frac{\left(3\frac{5}{6} \div 2\frac{2}{15}\right) \times \frac{3}{23}}{5\frac{1}{3} - 2\frac{7}{12}}$$

(b). A forest reserve covering an area of 605 $\text{km}^2$is represented on a map by a green area of24.2 cm^2. Determine the scale of the map.

Question 12:

(a). Given that $f(x) = x^2 - 4x + 3$and $g(x) = \frac{1}{x}$, find:

(i). $gf(x)$.

(ii). $gf(-2)$. [5]

(b). If $h(x) = 5x + 7$, find:

(i). $h^{-1}(x)$.

(ii). $h^{-1}(8)$.

(iii). The value of x for which $h^{-1}(x) = 0$. [7]

Question 13:

A group of 24 students were asked who played Football (F),Volley ball (V) and Hockey (H). The response were; 10 play football, 14 play volley ball and 9 play hockey. The number of student who play all the three games equals thenumber of those who do not play any of these games. 5 students play both football and hockey, 6 play both football and volley balland 2 play only volley ball and hockey.

(a). Draw a Venn diagram representing the above information

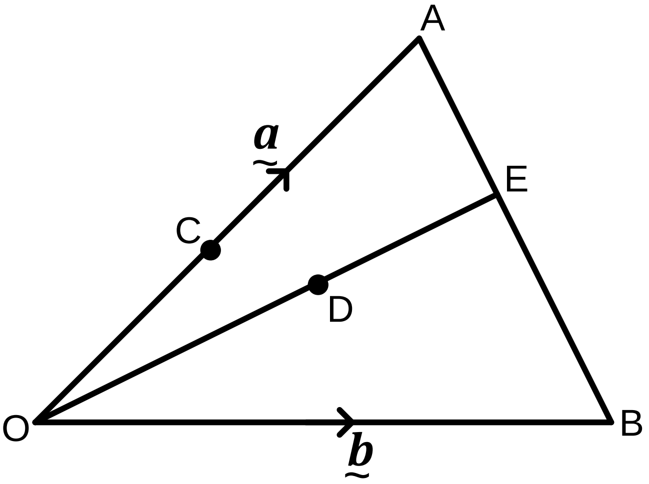
(b). Determine the number of students who play

- (i). all the three games .
- (ii). only one game

(c). Find the probability that a student selected at random from the group plays at least two games.

Question 14:

The figure below is a triangle $\boldsymbol{OAB}$ where$\overrightarrow{\boldsymbol{OA}} = \underset{\sim}{\boldsymbol{a}}$ and$\overrightarrow{\boldsymbol{OB}} = \underset{\sim}{\boldsymbol{b}}$. Points $\boldsymbol{C}$ and $\boldsymbol{E}$ are points on line. $\overrightarrow{\boldsymbol{OA}}$ and$\overrightarrow{\boldsymbol{AB}}$ such that they divide the lines$\overrightarrow{\boldsymbol{OA}}$ and$\overrightarrow{\boldsymbol{AB}}$ in the ratios $1: 2$ and$3: 1$respectively. Point $\boldsymbol{D}$ lies on $\overrightarrow{\boldsymbol{OE}}$ such that$\overrightarrow{\boldsymbol{OD}} = 2\overrightarrow{\boldsymbol{DE}}$.



(a). Find the vectors$\overrightarrow{\boldsymbol{AB}}$, $\overrightarrow{\boldsymbol{OE}}$ and $\overrightarrow{\boldsymbol{CB}}$in terms of vectors $\underset{\sim}{\boldsymbol{a}}$and $\underset{\sim}{\boldsymbol{b}}$.

(b). Show that $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{C}$ are collinear. [12]

Question 15:

Given that $\log_{10} 2 = 0.3010$and $\log_{10} 3 = 0.4771$.

(a). Express 12 as a product of primes.

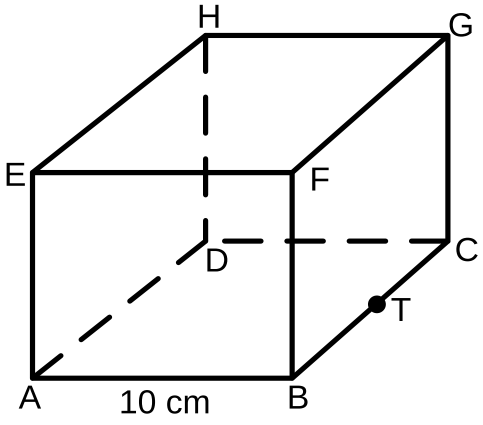
(b). Use the given information and the result in (a) above to write down:

- (i). $\log_{10} 12$,
- (ii). $\log_{10} 0.12$.

(c). Find x if$\log_{10} x = 3.6020$. [12]

Question 16:

Below is a cube of sides 10cm. $\boldsymbol{T}$ is the midpoint of $\overrightarrow{\boldsymbol{BC}}$.



Find:

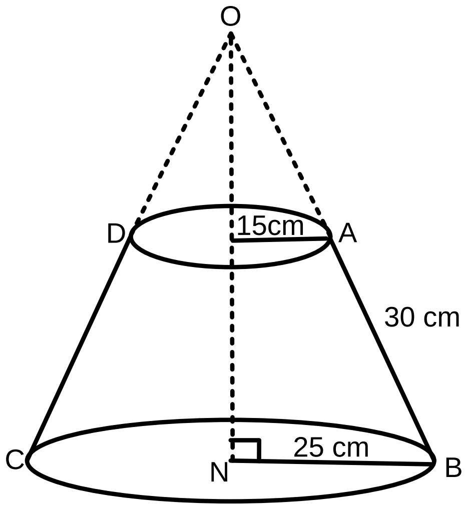
(a). (i). length $\overline{\boldsymbol{AC}}$,

(ii). Length $\overline{\boldsymbol{BH}}$.

(b). (i). angle between $\overline{\boldsymbol{BH}}$ and plane $\boldsymbol{ADHE}$,

(ii). angle between planes$\boldsymbol{ETH}$ and $\boldsymbol{ADHE}$. [12]

Question 17:



The figure above (in thick, heavy lines) shows a lampshed ABCD bounded by circles of radii 15 cm and 25 cm. The slanting side AB is 30 cm . If the lampshed was cut from an original figure OABCD, of a complete cone, calculate the

(a). (i). Slanting length of the cone OAB

(ii). The angle formed by producing CD and BA to O.

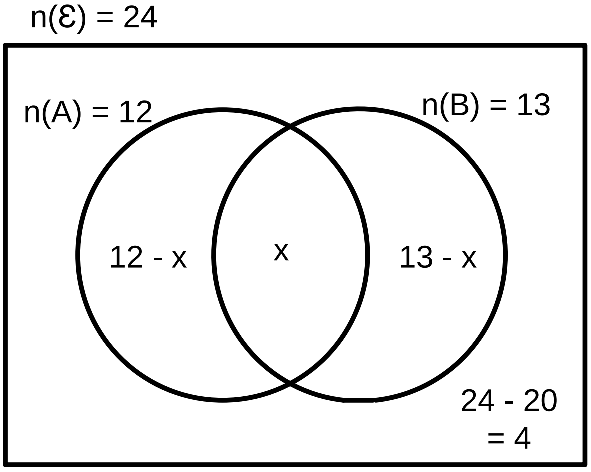
(b). (i). Vertical height of the lampshed

(ii). Volume of the lampshed. [12]

END

MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	$\left(\frac{4}{10}\right)^2 \times \left(\frac{125}{1000}\right)^{1/3} \div \left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^2 \times \left(\frac{5^3}{10^3}\right)^{1/3} \div \left(\frac{2}{5}\right)^3$ $\frac{2^2}{5^2} \times \frac{5}{10} \div \frac{2^3}{5^3} = \frac{2^2}{5^2} \times \frac{1}{2} \times \frac{5^3}{2^3}$ $= 2^{(2-1-3)} \times 5^{(3-2)} = 2^{-2} \times 5^1 = \frac{1}{2^2} \times 5 = \frac{5}{4}$	**M1** **M1** **M1 A1**
		04
2	$\frac{\sqrt{8} - \sqrt{18}}{1 - \sqrt{2}} = \frac{\sqrt{4} \times \sqrt{2} - \sqrt{9} \times \sqrt{2}}{1 - \sqrt{2}} = \frac{2\sqrt{2} - 3\sqrt{2}}{1 - \sqrt{2}} = \frac{-\sqrt{2}}{1 - \sqrt{2}}$ $= \frac{-\sqrt{2} \times (1 + \sqrt{2})}{(1 - \sqrt{2}) \times (1 + \sqrt{2})}$ $= \frac{-\sqrt{2} - 2}{1 - 2} = \frac{-\sqrt{2} - 2}{-1} = \sqrt{2} + 2$ $\Longrightarrow a = 2, \quad b = 1$	**M1** **M1** **B1** **A1**
		04
3	(a). Let $n(A \cap B) = x$  $n(\varepsilon) = 12 - x + x + 13 - x + 4$ $24 = 29 - x$ $x = 29 - 24$ $x = 5$ $\therefore n(A \cap B) = 5$ (ii). $n(A \cup B') = 12 - x + x + 4 = 16$	**B1** **B1** **A1** **A1**
		04
4	Points are: $(1, -3)$and $(7, 6)$, Gradient of line, $m = \frac{-3-6}{1-7} = \frac{-9}{-6} = \frac{3}{2}$	

	Using $y = mx + c$ and $(7, 6)$, $6 = \frac{3}{2} \times 7 + c$ $6 - \frac{21}{2} = c$ $c = -\frac{9}{2}$ $\therefore$ The required equation of the line is: $= \frac{3}{2}x - \frac{9}{2}$. At the x-axis, $y = 0$ $0 = \frac{3}{2}x - \frac{9}{2}$ $\frac{3}{2}x = \frac{9}{2}$ $3x = 9$ $x = 3$ $\therefore$ The coordinates are: $(3, 0)$	**M1** **A1** **M1** **A1**
		04
5	Let $w = 0.42222\ldots\ldots$ $10w = 0.42222\ldots\ldots \times 10$ $10w = 4.22222\ldots\ldots$ $- \begin{array}{lcr} 10w & = & 4.22222\ldots\ldots \\ w & = & 0.42222\ldots\ldots \\ \hline 9w & = & 3.8 \end{array}$ $w = \frac{3.8}{9}$ $w = \frac{3.8 \times 10}{9 \times 10}$ $w = \frac{38}{90} = \frac{19}{45}$ $\therefore (y - x) = 45 - 19 = 26$	**M1** **M1** **A1** **A1**
		04
6	Current price $= P\left(\frac{100 - R}{100}\right)^n$ $6195200 = P\left(\frac{100 - 12}{100}\right)^2$ $6195200 = 0.7744P$ $\frac{6195200}{0.7744} = P$ $P =$ shs 8,000,000	**M1** **M1** **M1** **A1**
		04
7	let, $f(x) = ax + b$ $fg(x) = f(x + 3) = a(x + 3) + b = ax + 3a + b$ $ax + 3a + b = 2x - 1$ By comparison,	**B1**

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$$a = 2$$
$$3a + b = -1$$
$$3 \times 2 + b = -1$$
$$6 + b = -1$$
$$b = -7$$
$$\therefore f(x) = 2x - 7$$

(ii).

$$f(4) = 2 \times 4 - 7 = 8 - 7 = 1$$

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B1
A1</td>
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<td>04</td>
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<td>8</td>
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(a).

$$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}} = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}6\\5\end{pmatrix}$$

(b).

$$\overrightarrow{\boldsymbol{AT}} : \overrightarrow{\boldsymbol{TB}} = 2:1, \qquad \Rightarrow \overrightarrow{\boldsymbol{AT}} = \left(\frac{2}{2+1}\right)\overrightarrow{\boldsymbol{AB}}$$
$$\overrightarrow{\boldsymbol{OT}} - \overrightarrow{\boldsymbol{OA}} = \frac{2}{3}\begin{pmatrix}6\\5\end{pmatrix}$$
$$\overrightarrow{\boldsymbol{OT}} - \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}4\\10/3\end{pmatrix}$$
$$\overrightarrow{\boldsymbol{OT}} = \begin{pmatrix}4\\10/3\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}2\\7/3\end{pmatrix}$$
$$\therefore \text{The coordinaes are } T\left(2, \frac{7}{3}\right)$$

</td>
<td>M1 A1
M1 A1
B1</td>
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<td>05</td>
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<td>9</td>
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<table>
<tr><th>Number</th><th>Standard form</th><th>Logarithm</th></tr>
<tr><td>2460
8.72</td><td>2.46×10^3
8.72×10^0</td><td>3.3909
+ 0.9405</td></tr>
<tr><td></td><td></td><td>4.3314</td></tr>
<tr><td>63.1
204</td><td>6.31×10^1
2.04×10^2</td><td>1.8000
+ 2.3096
4.1096</td></tr>
<tr><td colspan="2">$10^{0.2218} = 1.6665$</td><td>4.3314
− 4.1096
0.2218</td></tr>
</table>

$$\therefore \frac{0.0347}{0.014 \times 0.984} \approx 1.6665 \text{ (4 d.p)}$$

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A1</td>
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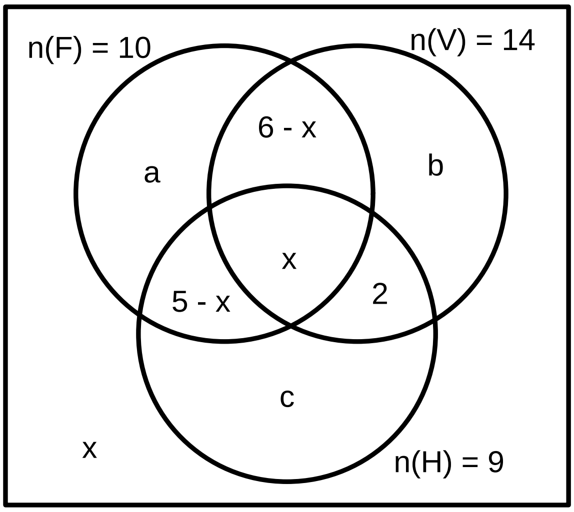
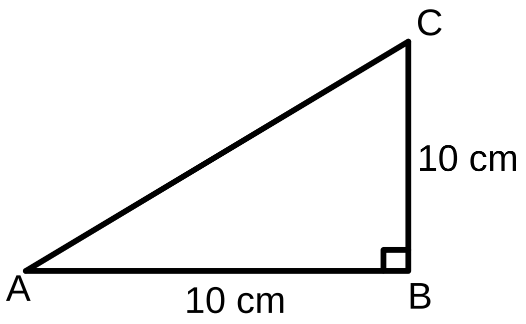
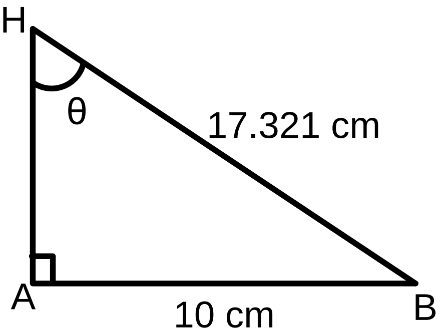
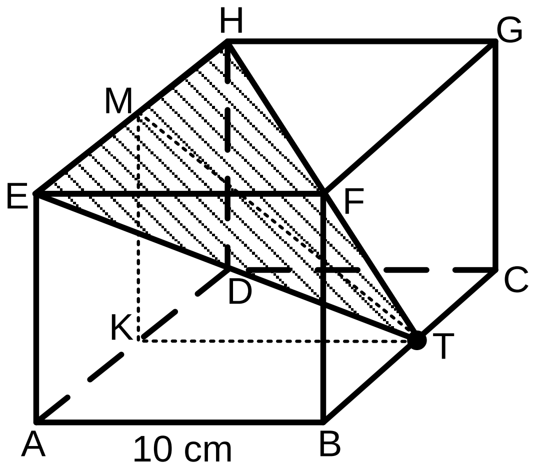
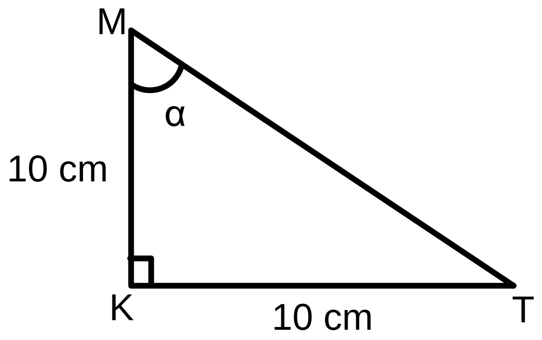
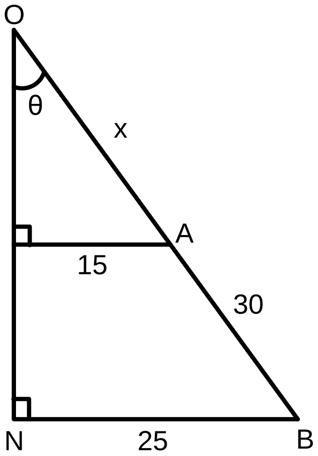
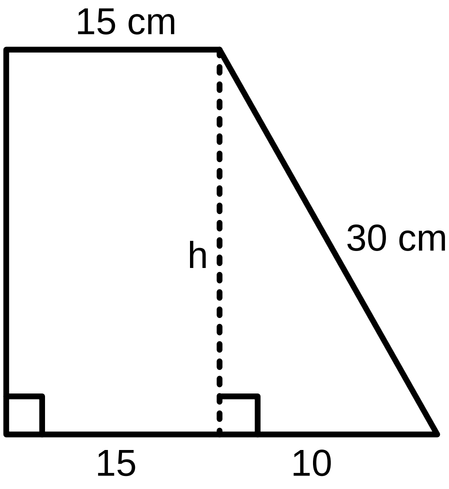
10	Ammount, $A = P\left(\frac{100+R}{100}\right)^n = 56000\left(\frac{100+15}{100}\right)^3$ $= \text{shs } 85{,}169$	**M1 M1** **A1**
		03
11	(a). $\frac{\left(3\frac{5}{6} \div 2\frac{2}{15}\right) \times \frac{3}{23}}{5\frac{1}{3} - 2\frac{7}{12}} = \frac{\left(\frac{23}{6} \div \frac{32}{15}\right) \times \frac{3}{23}}{\frac{16}{3} - \frac{31}{12}} = \frac{\left(\frac{23}{6} \times \frac{15}{32} \times \frac{3}{23}\right)}{\left(\frac{64-31}{12}\right)}$ $= \frac{15}{64} \div \frac{33}{12} = \frac{15}{64} \times \frac{12}{33} = \frac{15}{176}$ (b). The area scale is $24.2 \text{ cm}^2 : 605 \text{ km}^2$ $1 \text{ cm}^2 : \frac{605}{24.2} \text{km}^2$ $1 \text{ cm}^2 : 25 \text{ km}^2$ The linear scale is $\sqrt{1}\text{cm} : \sqrt{25}\text{km}$ $1 \text{ cm} : 5 \text{ km}$ Representative fraction is $1 \text{ cm} : 5 \times 100{,}000 \text{ cm}$ $1 \text{ cm} : 500{,}000 \text{ cm}$ The scale of the map is $1: 500000$	**B1 B1B1** **M1 M1 A1** **M1** **B1** **M1** **B1** **M1** **A1**
		12
12	(a). (i). $gf(x) = g(x^2 - 4x + 3) = \frac{1}{x^2 - 4x + 3}$ (ii). $gf(-2) = \frac{1}{(-2)^2 - 4 \times {}^{-}2 + 3} = \frac{1}{4 + 8 + 3} = \frac{1}{15}$ (b). (i). $h(x) = 5x + 7$ $Y = 5X + 7$ $Y - 7 = 5X$ $X = \frac{Y-7}{5}$ $\therefore h^{-1}(x) = \frac{x-7}{5}$ (ii). $h^{-1}(8) = \frac{8-7}{5} = \frac{1}{5}$ (iii). $h^{-1}(x) = 0$ $\frac{x-7}{5} = 0$	**B1B1** **M1 B1 A1** **B1** **M1** **A1** **M1 A1** **M1**

	$x-7=0$ $x=7$	**A1**
		12
13	(a). Let $n(F\cap V\cap H)=x$	
	n(Ɛ) = 24 	**B3**
	(b).	
	$n(F)=a+6-x+x+5-x$ $10=a+11-x$ $a=x-1$	**B1**
	$n(V)=b+6-x+x+2$ $14=b+8$ $b=6$	**B1**
	$n(H)=c+5-x+x+2$ $9=c+7$ $c=2$	**B1**
	(i).	
	$n(\varepsilon)=10+b+c+2+x$	**M1**
	$24=12+6+2+x$ $24=20+x$ $x=24-20=4$	**A1**
	$\therefore n(\text{all the three games})=4\text{ students}$	
	(ii).	
	$\therefore n(\text{only one game})=a+b+c=(4-1)+6+2$ $=11\text{ students}$	**A1**
	(c).	
	$n(\text{at least two games})=6+5-x+2=13-4=9\text{ students}$	**B1**
	$P(\text{at least two games})=\frac{9}{24}=\frac{3}{8}$	**M1 A1**
		12
14	(a). (i).	
	$\overrightarrow{\boldsymbol{AB}}=\overrightarrow{\boldsymbol{AO}}+\overrightarrow{\boldsymbol{OB}}=-\underset{\sim}{\boldsymbol{a}}+\underset{\sim}{\boldsymbol{b}}=\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{a}}$	**B1**
	(ii).	

	$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$	
	but, $\overrightarrow{AE}:\overrightarrow{EB} = 3:1, \quad \Rightarrow \overrightarrow{AE} = \left(\frac{3}{3+1}\right)\overrightarrow{AB} = \frac{3}{4}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$	**B1**
	$\therefore \overrightarrow{OE} = \underset{\sim}{a} + \frac{3}{4}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) = \frac{4\underset{\sim}{a} + 3\underset{\sim}{b} - 3\underset{\sim}{a}}{4} = \frac{\underset{\sim}{a} + 3\underset{\sim}{b}}{4} = \frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)$	**B1 B1**
	(iii).	
	$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$	
	but, $\overrightarrow{OC}:\overrightarrow{CA} = 1:2, \quad \Rightarrow \overrightarrow{CA} = \left(\frac{2}{1+2}\right)\overrightarrow{OA} = \frac{2}{3}\underset{\sim}{a}$	**B1**
	$\therefore \overrightarrow{CB} = \frac{2}{3}\underset{\sim}{a} + \left(\underset{\sim}{b} - \underset{\sim}{a}\right) = \frac{2\underset{\sim}{a} + 3\underset{\sim}{b} - 3\underset{\sim}{a}}{4} = \frac{\underset{\sim}{a} + 3\underset{\sim}{b}}{4} = \frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)$	**B1 B1**
	(b).	
	$\overrightarrow{OD} = 2\overrightarrow{DE}, \quad \Rightarrow \overrightarrow{DE} = \frac{1}{2}\overrightarrow{OD}$	
	$\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$	
	$\frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right) = \overrightarrow{OD} + \frac{1}{2}\overrightarrow{OD}$	
	$\frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right) = \frac{3}{2}\overrightarrow{OD}$	**B1**
	$\overrightarrow{OD} = \frac{2}{3} \times \frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right) = \frac{1}{6}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)$	**B1**
	$\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB} = -\frac{1}{6}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right) + \underset{\sim}{b} = \frac{-\underset{\sim}{a} - 3\underset{\sim}{b} + 6\underset{\sim}{b}}{6}$	**B1**
	$= \frac{3\underset{\sim}{b} - \underset{\sim}{a}}{6} = \frac{1}{6}\left(3\underset{\sim}{b} - \underset{\sim}{a}\right)$	
	$\frac{\overrightarrow{CB}}{\overrightarrow{DB}} = \frac{\frac{1}{3}\left(3\underset{\sim}{b} - \underset{\sim}{a}\right)}{\frac{1}{6}\left(3\underset{\sim}{b} - \underset{\sim}{a}\right)}$	**B1**
	$\frac{\overrightarrow{CB}}{\overrightarrow{DB}} = \frac{1}{3} \div \frac{1}{6}$	
	$\frac{\overrightarrow{CB}}{\overrightarrow{DB}} = 2$	
	$\overrightarrow{CB} = 2\overrightarrow{DB}$	**B1**
	Since $\overrightarrow{CB}$ can be expressed as a multiple of $\overrightarrow{DB}$, then points ***B***, ***D*** and ***C*** are collinear.	
		12
15	(a).	
	$\begin{array}{rl} 2 & \mid\; 12 \\ \hline 2 & \mid\; 6 \\ \hline 3 & \mid\; 3 \\ \hline & \mid\; 1 \end{array}$	**M1**
		B1

	$12 = 2 \times 2 \times 3 = 2^2 \times 3$ (b). (i). $\log_{10} 12 = \log_{10}(2^2 \times 3) = 2\log_{10} 2 + \log_{10} 3$ $= 2 \times 0.3010 + 0.4771 = 1.0791$ (ii). $\log_{10} 0.12 = \log_{10}\left(\frac{12}{100}\right) = \log_{10} 12 - \log_{10} 100$ $= 1.0791 - 2\log_{10} 10 = 1.0791 - 2$ $\begin{array}{r\|l\|} & 1.0791 \\ - & 2.0000 \\ \hline & \bar{1}.0791 \end{array}$ $\therefore \log_{10} 0.12 = \bar{1}.0791$ (c). $\log_{10} x = 3.6020$ $\log_{10} x = 3 + 2 \times 0.3010$ $\log_{10} x = 3\log_{10} 10 + 2\log_{10} 2$ $\log_{10} x = \log_{10} 1000 + \log_{10} 4$ $\log_{10} x = \log_{10}(4 \times 1000)$ $\log_{10} x = \log_{10} 4000$ $x = 4000$	**B1** **M1 A1** **B1** **M1** **M1** **A1** **B1** **M1** **A1**
		12
16	(a). (i).  $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$ $\overline{AC}^2 = 10^2 + 10^2$ $\overline{AC} = \sqrt{200} = 14.142$ cm (ii). $\overline{BH}^2 = \overline{BD}^2 + \overline{DH}^2$ but, $\overline{BD} = \overline{AC} = \sqrt{200}$ $\therefore \overline{BH}^2 = 200 + 10^2$ $\overline{BH} = \sqrt{300} = 17.321$ cm (b). (i).	**B1** **M1** **A1** **B1** **M1** **A1**

	 $\sin\theta = \frac{10}{17.321}$, $\Longrightarrow \theta = 35.263°$ (ii).   $\tan\alpha = \frac{10}{10}$, $\Longrightarrow \alpha = 45°$	**B1** **M1 A1** **B1** **M1 A1**
		12
17	(a). (i).	

		**B1** **M1**
	By similarity,	
	$\frac{x+30}{25}=\frac{x}{15}$	**B1**
	$15(x+30)=25x$	**A1**
	$15x+450=25x$	
	$450=25x-15x$	
	$450=10x$	
	$x=45\text{ cm}$	**B1**
	$\therefore \text{length OAB}=45+30=75\text{ cm}$	**M1 A1**
	(ii).	
	$\sin\theta=\frac{15}{45}\ ,\qquad \Longrightarrow \theta=19.471°$	
	$\text{Required angle}=2\times 19.471=38.942°$	
	(b). (i).	
		**M1** **A1**
	$30^2=h^2+10^2$	
	$h^2=30^2-10^2$	
	$h^2=800$	
	$h=28.284\text{ cm}$	
	(ii).	**M1**
	$\text{Volume}=\frac{1}{3}\pi h(r^2+rR+R^2)$	**M1 A1**

	$= \frac{1}{3} \times \frac{22}{7} \times 28.284 \times (15^2 + 15 \times 25 + 25^2)$ $= 29.631 \times 1225 = 36297.975 \text{ cm}^3$	
		12

*****END****

456/1
MATHEMATICS
PAPER 1
June 2023
$2\frac{1}{2}$ hours

S.4 MATH 1 MOCK SET 3 2023

Time: 2 Hours 30 Minutes

NAME: ____________________ **STREAM:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: An increase of 15% in salaries makes the monthly expenditure on salaries for a factory to be shs 22,425,000. Find the expenditure before the increase. [4]

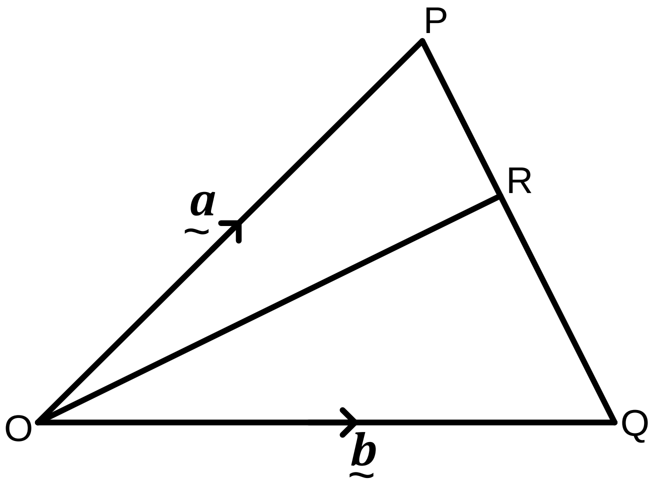
Qn 2: The distance between the two points $A(2, 2)$ and $B(6, y)$ of a line is 5 units. Calculate the possible values of y. [4]

Qn 3: Given that $\log_{10} 3 = 0.4771$, without using tables or calculator, evaluate $\log_{10} 8.1$. [4]

Qn 4: Given that M and N are two sets such that $n(\varepsilon) = 39$, $n(M \cap N) = 12$, $n(M) = 26$ and $n(M' \cap N') = 5$, find:

(i). $n(M' \cap N)$,
(ii). $n(N')$. [4]

Qn 5: In the figure below $\boldsymbol{OP} = \underset{\sim}{\boldsymbol{a}}$, $\boldsymbol{OQ} = \underset{\sim}{\boldsymbol{b}}$and $\boldsymbol{PR} = \frac{1}{3}\boldsymbol{PQ}$.



Express $\boldsymbol{OR}$ in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$. [4]

Qn 6: The force (F) which acts between two magnetic poles in inversely proportional to the square of the distance (d) between them. If $F = 18$ when $d = 4$, find F when $d = 3$. [4]

Qn 7: The total surface area of a cuboid measuring 4 cm by 0.05 cm by $\boldsymbol{x}$cm is 76 cm^2. Calculate the value of $\boldsymbol{x}$. [4]

Qn 8: Given that $f(x) = \frac{1-2x}{3x}$, find the value of

(i). $f^{-1}(x)$

(ii). $f^{-1}(0)$. [4]

Qn 9: Express $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ in the form $a + b\sqrt{c}$; hence state the values of a, b and c. [4]

Qn 10: An employee's gross salary is shs 6.72 million per month. He pays an income tax of 15% of his gross monthly income. Find his net income per month. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

In a certain school, there are students who play football (F), Tennis (T) or Volleyball (V). 24 play Football, 25 play Tennis and 29 play Volleyball. 11 play both F and T, 10 play both T and V while 13 play both F and V. the number of students who play Tennis or Volleyball but not Football is equal to twice those who play neither of the three games. If those who play neither of the three games are 12,

(a). Represent the above information on a Venn diagram. [7]

(b). Find the:

  (i). total number of students in the school.

  (ii). Number of students who play only two games. [3]

(c). Find the probability that a student chosen at random plays not more than one game. [2]

Question 12:

(a). Given that $g(x) = px^2 - qx + 1$, $g(2) = 11$ and $g(1) = 2$; find the values of p and q. [5]

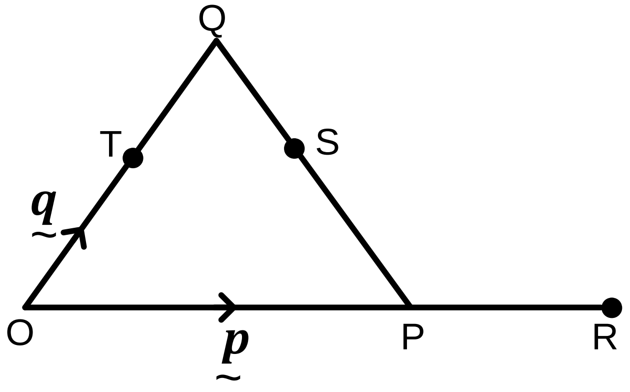
(b). Given that $f(x) = \frac{x+5}{6}$ and $fg(x) = \frac{7-x}{2}$, find

  (i). $f(-17)$,

  (ii). An expression for $g(x)$ and hence evaluate $g(4)$. [7]

Question 12:

In the diagram below, $\underset{\sim}{\boldsymbol{p}}$ and $\underset{\sim}{\boldsymbol{q}}$ are position vectors of $\boldsymbol{P}$ and $\boldsymbol{Q}$ respectively. Point $\boldsymbol{R}$ lies on $\overrightarrow{\boldsymbol{OP}}$ produced such that $\overrightarrow{\boldsymbol{OP}} = \frac{1}{2}\overrightarrow{\boldsymbol{OR}}$ and point $\boldsymbol{T}$ lies on $\overrightarrow{\boldsymbol{OQ}}$ such that $\overrightarrow{\boldsymbol{OT}} = 2\overrightarrow{\boldsymbol{TQ}}$.



If point $\boldsymbol{S}$ lies on $\overrightarrow{\boldsymbol{PQ}}$ such that $\overrightarrow{\boldsymbol{QS}} = \overrightarrow{\boldsymbol{SP}}$,

(a). Express in terms of $\underset{\sim}{\boldsymbol{p}}$ and $\underset{\sim}{\boldsymbol{q}}$ the vectors:

  (i). $\overrightarrow{\boldsymbol{QP}}$,

  (ii). $\overrightarrow{\boldsymbol{TS}}$,

  (iii). $\overrightarrow{\boldsymbol{TR}}$,

  (iv). $\overrightarrow{\boldsymbol{SR}}$.

(b). Show that $\boldsymbol{T}$, $\boldsymbol{S}$ and $\boldsymbol{R}$ are collinear. [12]

Question 14:

The time, T, taken to dig a spring well partly varies as the depth, D, of the well and partly varies as the square of the depth, D. If $T = 80, D = 20$ and when $T = 150, D = 30$.

(a). Write down an expression connecting T and D. [8]

(b). Find T when $D = 40$. [4]

Question 15:

(a). Mr. Okello bought three cars; Audi, Benz and Corsa for a total of shs 150,000,000. The amounts he paid for these cars were in the ratio 3: 5: 7. Calculate the amount he paid for each car. [6]

(b). The scale of the map is 1: 250,000. Find the actual perimeter in km of a rectangular plot which measures 15 cm by 9 cm on the map. [6]

Question 16:

A cyclist covered a journey of 48 km from station $\boldsymbol{A}$to station $\boldsymbol{B}$in $5\frac{1}{2}$ hours. The cyclist rode at 12 km h^{-1} for the first $2\frac{1}{2}$ hours are changed speed for the remaining part of the journey.

(a). (i). Determine the speed of the cyclist for the remaining part of the journey. [6]

(ii). Represent the cyclist journey on a distance-time graph. [4]

(b). Calculate the average speed of the cyclist from station $\boldsymbol{A}$to $\boldsymbol{B}$. [2]

Question 17:

The base of a right pyramid ABCDV is a square ABCD of side 24 cm. the slant edges are each 20 cm long.

(a). Draw the pyramid. [2]

(b). Calculate the:

(i). Height of the pyramid, [6]

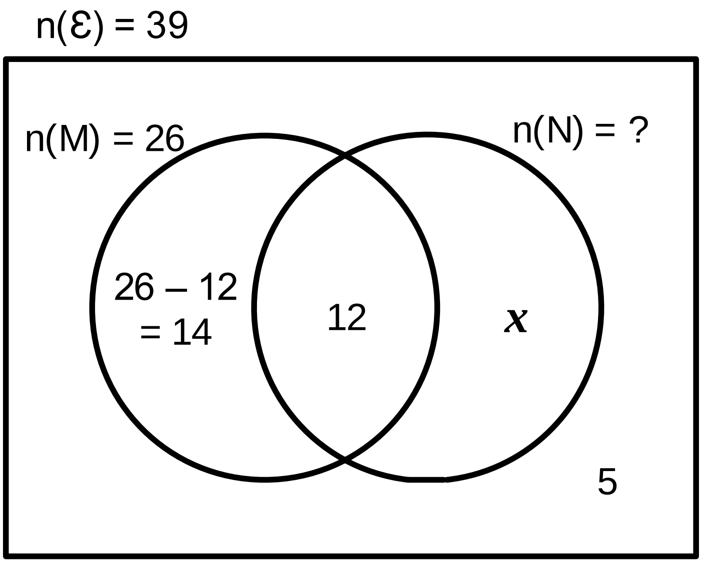
(ii). Volume of the pyramid. [4]

*****END*****

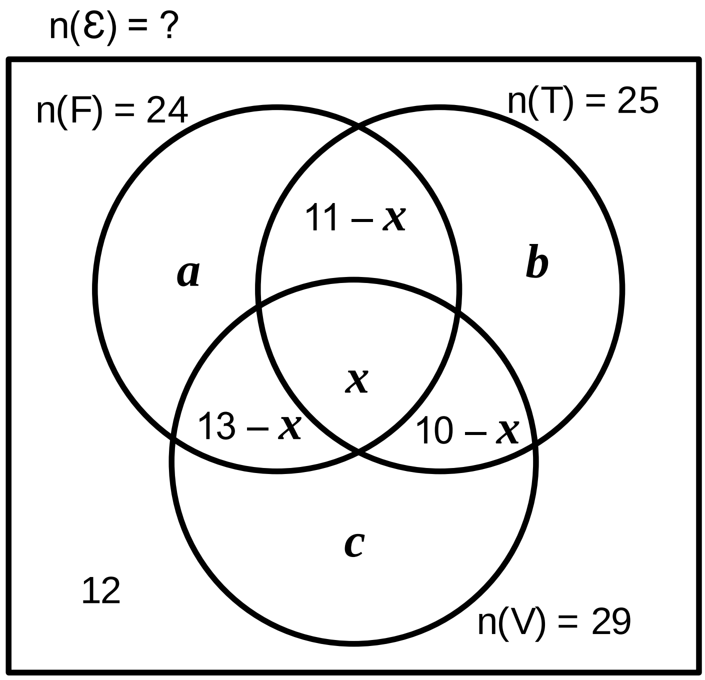
MARKING GUIDE

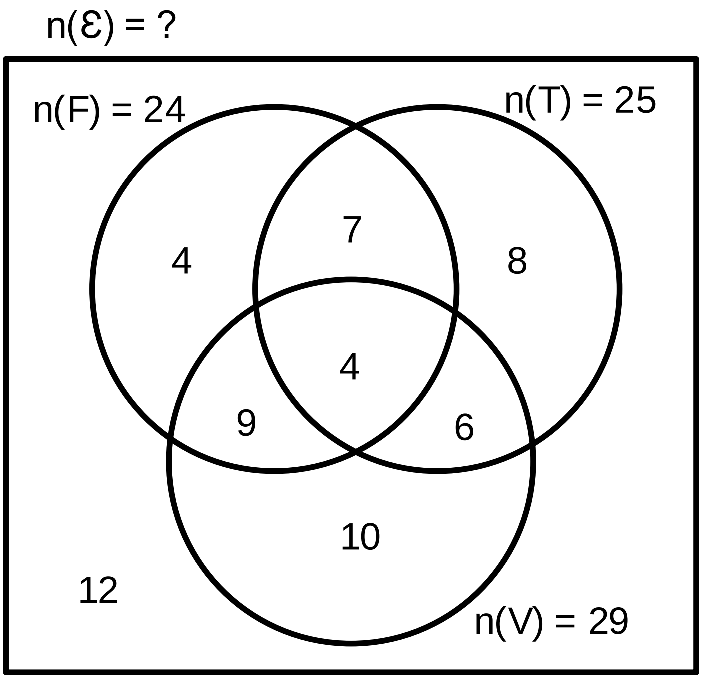
[Total Marks = 100]

SNo.	Working	Marks
1	Let the expenditure before the increasebe x. $100\% + 15\% = 115\%$ $\frac{115}{100} \times x = 22{,}425{,}000$ $1.15x = 22{,}425{,}000$ $\frac{1.15x}{1.15} = \frac{22{,}425{,}000}{1.15}$ $x = 19{,}500{,}000$ The expenditure before the increase was shs 19,500,000.	**B1** **M1** **M1** **A1**
		04
2	$\text{Length } AB = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $5 = \sqrt{(y-2)^2 + (6-2)^2}$ $5^2 = (y-2)^2 + 4^2$ $25 = y^2 - 4y + 4 + 16$ $25 = y^2 - 4y + 20$ $y^2 - 4y - 5 = 0$ $\text{sum} = -4, \quad \text{product} = -5, \quad \text{factors} = -5, 1$ $y^2 - 5y + y - 5 = 0$ $y(y-5) + (y-5) = 0$ $(y+1)(y-5) = 0$ $(y+1) = 0, \quad \text{or,} \quad (y-5) = 0$ $y = -1, \quad \text{or,} \quad y = 5$	**M1** **M1** **M1** **A1**
		04
3	$\log_{10} 8.1 = \log_{10}\left(\frac{81}{10}\right)$ $= \log_{10} 81 - \log_{10} 10$ $= 4\log_{10} 3 - 1$ $= 4 \times 0.4771 - 1$ $= 1.9084 - 1$ $= 0.9084$	**M1** **M1** **M1** **A1**
		04
4	(i). Let $n(M' \cap N) = x$	

		**B1**-entry 14, 12 **B1**-entry 39, 26, 5
	$n(\varepsilon) = 14 + 12 + x + 5 = 39$	**M1**
	$31 + x = 39$	
	$x = 39 - 31$	
	$x = 8$	**A1**
	(ii).	
	$n(N') = 14 + 5 = 19$	**A1**
		04
5	$\boldsymbol{PR} = \frac{1}{3}\boldsymbol{PQ} = \frac{1}{3}\left(\underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}\right)$	**B1**
	$\therefore \boldsymbol{OR} = \boldsymbol{OP} + \boldsymbol{PR}$	
	$= \underset{\sim}{\boldsymbol{a}} + \frac{1}{3}\left(\underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}\right)$	**M1**
	$= \frac{3\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}}{3}$	**M1**
	$= \frac{1}{3}\left(2\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}}\right)$	**A1**
		04
6	$F \propto \frac{1}{d}$	
	$F = \frac{k}{d}$	**B1**
	But $F = 18$ when $d = 4$,	
	$18 = \frac{k}{4}$	**M1**
	$k = 18 \times 4$	
	$k = 72$	**B1**
	$\therefore F = \frac{72}{d}$	
	when $d = 3$, $F = \frac{72}{3} = 24$	**A1**

		04
7	$T.S.A = 2(lw + lh + wh) = 76$ $2(4 \times 0.05 + 4x + 0.05x) = 76$ $0.4 + 8x + 0.1x = 76$ $8.1x = 75.6$ $\frac{8.1x}{8.1} = \frac{75.6}{8.1}$ $x = 9.3333$ cm	**M1** **B1** **M1** **A1**
		04
8	(i). $f(x) = \frac{1-2x}{3x}$ $Y = \frac{1-2X}{3X}$ $3XY = 1 - 2X$ $3XY + 2X = 1$ $X(3Y + 2) = 1$ $X = \frac{1}{3Y + 2}$ $\therefore f^{-1}(x) = \frac{1}{3x + 2}$ (ii). $\therefore f^{-1}(0) = \frac{1}{3 \times 0 + 2} = \frac{1}{5}$	**M1** **B1** **A1** **A1**
		04
9	$\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{2} \times (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2}) \times (\sqrt{3}-\sqrt{2})}$ $= \frac{\sqrt{6}-2}{3-2}$ $= \frac{\sqrt{6}-2}{1}$ $= -2 + \sqrt{6}$ $= a + b\sqrt{c}$ $\therefore a = -2, \quad b = 1, \quad c = 6$	**M1** **M1** **B1** **A1**
		04
10	$\text{Income tax} = \frac{15}{100} \times 6{,}720{,}000 = \text{shs } 1{,}008{,}000$ $\text{Net income} = 6{,}720{,}000 - 1{,}008{,}000 = \text{shs } 5{,}712{,}000$	**M1 B1** **M1 A1**
		04
11	(a). Let $n(F \cap T \cap V) = x$	

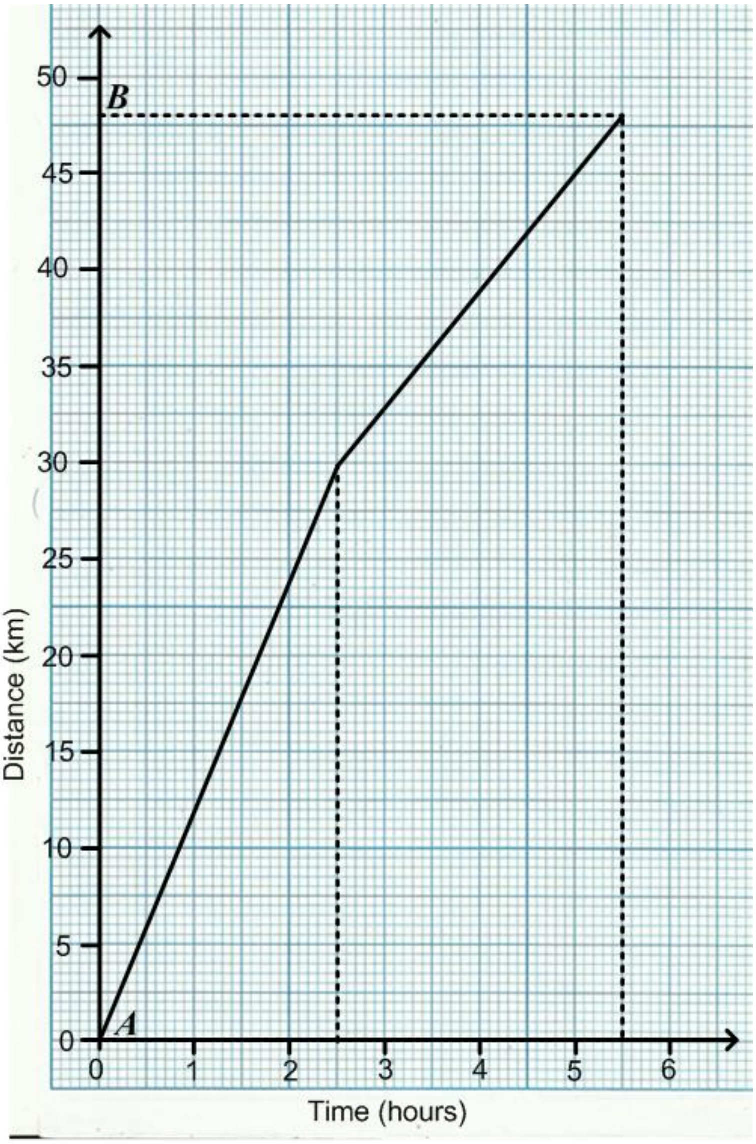
		**B1**-entry 24, 25, 29 **B1**-entry x, 12 **B1**-entry $(11-x)$, $(13-x)$, $(10-x)$
	$n(F) = a + 11 - x + 13 - x + x = 24$	
	$a + 24 - x = 24$	
	$a = x$	**B1**
	$n(T) = b + 11 - x + 10 - x + x = 25$	
	$b + 21 - x = 25$	
	$b = 4 + x$	**B1**
	$n(V) = c + 13 - x + 10 - x + x = 29$	
	$c + 23 - x = 29$	
	$c = 6 + x$	
	Since the number of students who play Tennis or Volleyball but not Football is equal to twice those who play neither of the three games, then,	**B1**
	$b + c + 10 - x = 2 \times 12$	
	$4 + x + 6 + x + 10 - x = 24$	**M1**
	$20 + x = 24$	
	$x = 24 - 20$	
	$x = 4$	**A1**

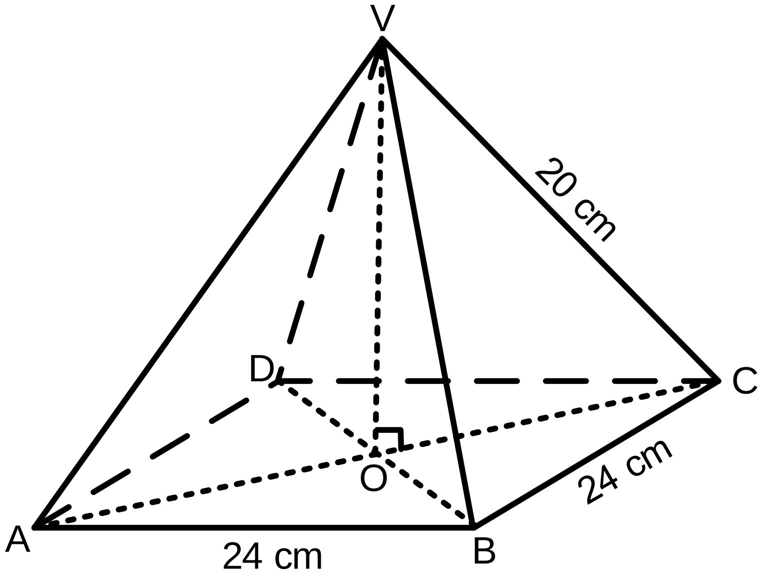
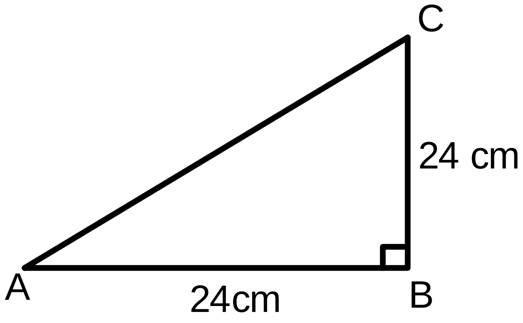
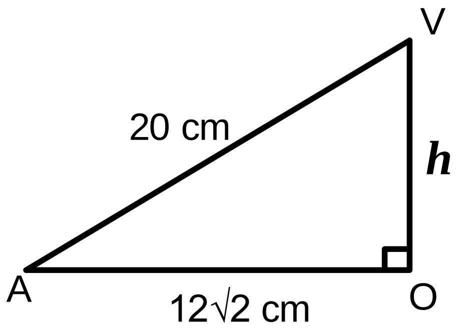
	 (b). (i). $n(\varepsilon) = 24 + 10 + 6 + 8 = 48$ students (ii). $n(\text{only two games}) = 6 + 7 + 9 = 22$ students (c). $n(\text{not more than one game}) = 12 + 10 + 8 + 4 = 34\text{students}$ $\therefore P(\text{not more than one game}) = \frac{34}{48} = \frac{17}{24}$	**A1** **A1** **B1** **A1**
		12
12	(a). $g(x) = px^2 - qx + 1$ $g(2) = p \times 2^2 + q \times 2 + 1 = 11$ $4p + 2q = 10$ $2p + q = 5 \quad \longrightarrow (1)$ $g(1) = p \times 1^2 + q \times 1 + 1 = 2$ $p + q = 1 \quad \longrightarrow (2)$ $\begin{array}{c\|c\|} 1 & 2p + q = 5 \\ 2 & p + q = 1 \\ \hline \end{array}$ $\begin{array}{c\|c\|} & 2p + q = 5 \\ - & 2p + 2q = 2 \\ \hline & -q = 3 \end{array}$ $q = -3$ and, $p = 1 - q = 1 - (-3) = 4$ (b). (i). $f(-17) = \frac{-17 + 5}{6} = \frac{-12}{6} = -2$	**B1** **B1** **M1** **A1** **A1** **M1 A1**

	(ii). $\text{let } g(x) = ax + b$ $fg(x) = f(ax + b)$ $= \frac{(ax + b) + 5}{6}$ $= \frac{ax + b + 5}{6}$ $= \frac{ax}{6} + \frac{b + 5}{6}$ also, $fg(x) = \frac{7 - x}{2} = \frac{7}{2} + \frac{-1}{2}x$ By comparing coefficients, $\frac{b + 5}{6} = \frac{7}{2}$ $2b + 10 = 42$ $2b = 32$ $b = 16$ $\frac{a}{6} = \frac{-1}{2}$ $2a = -6$ $a = -3$ $\therefore g(x) = -3x + 16$ $g(4) = -3 \times 4 + 16 = -12 + 16 = 4$	**B1** **B1** **B1** **B1** **A1**
		12
13	(a). (i). $\overrightarrow{\mathbf{QP}} = \overrightarrow{\mathbf{QO}} + \overrightarrow{\mathbf{OP}}$ $= -\underset{\sim}{\mathbf{q}} + \underset{\sim}{\mathbf{p}}$ $= \underset{\sim}{\mathbf{p}} - \underset{\sim}{\mathbf{q}}$ (ii). $\overrightarrow{\mathbf{QS}} = \frac{1}{2}\overrightarrow{\mathbf{QP}} = \frac{1}{2}\left(\underset{\sim}{\mathbf{p}} - \underset{\sim}{\mathbf{q}}\right)$ $\overrightarrow{\mathbf{OT}} = 2\overrightarrow{\mathbf{TQ}}$ $\overrightarrow{\mathbf{TQ}} = \frac{1}{3}\overrightarrow{\mathbf{OQ}}$ $= \frac{1}{3}\underset{\sim}{\mathbf{q}}$ $\therefore \overrightarrow{\mathbf{TS}} = \overrightarrow{\mathbf{TQ}} + \overrightarrow{\mathbf{QS}}$ $= \frac{1}{3}\underset{\sim}{\mathbf{q}} + \frac{1}{2}\left(\underset{\sim}{\mathbf{p}} - \underset{\sim}{\mathbf{q}}\right)$ $= \frac{2\underset{\sim}{\mathbf{q}} + 3\underset{\sim}{\mathbf{p}} - 3\underset{\sim}{\mathbf{q}}}{6}$ $= \frac{1}{6}\left(3\underset{\sim}{\mathbf{p}} - \underset{\sim}{\mathbf{q}}\right)$ (iii).	**B1** **B1** **B1** **M1** **A1**

	$\overrightarrow{\boldsymbol{OP}} = \frac{1}{2}\overrightarrow{\boldsymbol{OR}}$ $\overrightarrow{\boldsymbol{OR}} = 2\overrightarrow{\boldsymbol{OP}}$ $= 2\underset{\sim}{\boldsymbol{p}}$ $\overrightarrow{\boldsymbol{OT}} = 2\overrightarrow{\boldsymbol{TQ}}$ $= 2 \times \frac{1}{3}\underset{\sim}{\boldsymbol{q}}$ $= \frac{2}{3}\underset{\sim}{\boldsymbol{q}}$ $\therefore \overrightarrow{\boldsymbol{TR}} = \overrightarrow{\boldsymbol{TO}} + \overrightarrow{\boldsymbol{OR}}$ $= -\frac{2}{3}\underset{\sim}{\boldsymbol{q}} + 2\underset{\sim}{\boldsymbol{p}}$ $= \frac{-2\underset{\sim}{\boldsymbol{q}} + 6\underset{\sim}{\boldsymbol{p}}}{3}$ $= \frac{1}{3}\left(6\underset{\sim}{\boldsymbol{p}} - 2\underset{\sim}{\boldsymbol{q}}\right)$ $= \frac{2}{3}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)$ (iv). $\overrightarrow{\boldsymbol{SR}} = \overrightarrow{\boldsymbol{ST}} + \overrightarrow{\boldsymbol{TR}}$ $= -\frac{1}{6}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right) + \frac{2}{3}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)$ $= \left(\frac{1}{6} + \frac{2}{3}\right)\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)$ $= \frac{5}{6}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)$ (b). $\frac{\overrightarrow{\boldsymbol{TS}}}{\overrightarrow{\boldsymbol{SR}}} = \frac{\frac{1}{6}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)}{\frac{5}{6}\left(3\underset{\sim}{\boldsymbol{p}} - \underset{\sim}{\boldsymbol{q}}\right)}$ $\frac{\overrightarrow{\boldsymbol{TS}}}{\overrightarrow{\boldsymbol{SR}}} = \frac{1}{6} \div \frac{5}{6}$ $\frac{\overrightarrow{\boldsymbol{TS}}}{\overrightarrow{\boldsymbol{SR}}} = \frac{1}{6} \times \frac{6}{5}$ $\frac{\overrightarrow{\boldsymbol{TS}}}{\overrightarrow{\boldsymbol{SR}}} = \frac{1}{5}$ $\overrightarrow{\boldsymbol{SR}} = 5\overrightarrow{\boldsymbol{TS}}$ Since $\overrightarrow{\boldsymbol{SR}}$ can be expressed as a multiple of $\overrightarrow{\boldsymbol{TS}}$, then points $\boldsymbol{T}, \boldsymbol{S}$ and $\boldsymbol{R}$ are collinear.	**B1** **B1** **M1** **A1** **M1** **A1** **B1**
		12
14	(a). $T = k_1D + k_2D^2$ Where k_1 and k_2 are constants of proportionality.	**B1**

	but, $T = 80$ when $D = 20$,	
	$80 = 20k_1 + k_2 \times 20^2$	**M1**
	$80 = 20k_1 + 400k_2$	
	$4 = k_1 + 20k_2 \quad \longrightarrow (1)$	**B1**
	also, $T = 150$ when $D = 30$,	
	$150 = 30k_1 + k_2 \times 30^2$	
	$150 = 30k_1 + 900k_2$	
	$5 = k_1 + 30k_2 \quad \longrightarrow (2)$	
	Equation $(2) - (1)$ gives:	**B1**
	$\begin{array}{rl} & 5 = k_1 + 30k_2 \\ + & 4 = k_1 + 20k_2 \\ \hline & 1 = 10k_2 \end{array}$	**M1**
	$\frac{10k_2}{10} = \frac{1}{10}$	**M1**
	$k_2 = \frac{1}{10}$	**B1**
	From equation (1),	
	$k_1 = 4 - 20k_2 = 4 - 20 \times \frac{1}{10} = 4 - 2 = 2$	**M1 B1**
	$\therefore T = 2D + \frac{1}{10}D^2$	**B1**
	(b). when $D = 40$	
	$T = 2 \times 40 + \frac{1}{10} \times 40^2 = 80 + 160 = 240$	**M1 A1**
		12
15	(a).	
	$\text{Total ratio} = 3 + 5 + 7 = 15$	**B1**
	Ammount paid for Audi $= \frac{3}{15} \times 150{,}000{,}000 = \text{shs}30{,}000{,}000$	**M1 B1**
	Ammount paid for Benz $= \frac{5}{15} \times 150{,}000{,}000 = \text{shs } 50{,}000{,}000$	**M1 B1**
	Ammount paid for Benz $= \frac{7}{15} \times 150{,}000{,}000 = \text{shs } 70{,}000{,}000$	**M1 B1**
	(b).	
	Perimeter on map $= 15 + 9 = 24$ cm	**M1 B1**
	Map scale is	
	$1 \text{ cm} : 250{,}000 \text{ cm}$	
	The linear scale is	
	$1 \text{ cm} : \frac{250{,}000}{100{,}000} \text{km}$	**M1**
	$1 \text{ cm} : 2.5\text{km}$	
	$24\text{cm} : 2.5 \times 24\text{km}$	**M1**
	$24 \text{ cm} : 60\text{km}$	**A1**
	The actual perimeter is 60km.	
		12
16	(a). (i).	

	For the first part of the journey, $\text{Distance} = S \times T = 12 \times 2\frac{1}{2} = 12 \times \frac{5}{2} = 30 \text{ km}$ For the remaining part of the journey, $\text{Distance} = 48 - 30 = 18 \text{ km}$ $\text{Time} = 5\frac{1}{2} - 2\frac{1}{2} = 3\text{hours}$ $\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{18}{3} = 6 \text{ km h}^{-1}$ (ii).  (b). $\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{48}{5.5} = 8.7273\text{km h}^{-1}$	**M1 B1** **B1** **B1** **M1 B1** **B1**-vertical axis **B1**-horizontal axis **B1**-1st motion **B1**-2nd motion **M1 A1**
		12
17	(a).	

		**B1 B1**
	(b). (i).  $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$ $\overline{AC}^2 = 24^2 + 24^2$ $\overline{AC} = \sqrt{1152} = 24\sqrt{2}\text{cm}$ $\Rightarrow \overline{OA} = \frac{1}{2} \times 24\sqrt{2} = 12\sqrt{2}\text{cm}$	**M1** **B1** **A1**
	 $\overline{VA}^2 = \overline{VO}^2 + \overline{OA}^2$ $20^2 = h^2 + \left(12\sqrt{2}\right)^2$ $400 = h^2 + 288$ $h^2 = 400 - 288$	**B1** **M1**

	$h^2 = 112$ $h = \sqrt{112} = 4\sqrt{7} \approx 10.583\text{cm}$ $\therefore \text{Height} = 10.583 \text{ cm}$ (ii). $\text{Volume} = \frac{1}{3} \times (\text{base area}) \times (\text{height})$ $= \frac{1}{3} \times (24 \times 24) \times 4\sqrt{7}$ $= 768\sqrt{7}$ $\approx 2031.937\text{cm}^3$	**M1** **A1** **M1 M1** **A1**
		12

*****END****

SENIOR FOUR
MATHEMATICS
June 2023
$1\frac{1}{2}$ hours

S.4 MATH BI-WEEKLY TEST 1 2023

Topic: Business Mathematics

Time: 1 Hour 30 Minutes

NAME: **STREAM:**

INSTRUCTIONS:
Attempt ALL questions in this paper.
Show your working clearly.

Qn 1: A dealer in Owino market adds 10% to the cost price of the goods he sells. A pair of bed sheets costs him shs 56,000. Calculate:
(i). the profit.
(ii). the selling price.

Qn 2: During Christmas season, a dealer in ready-made garments, announced a discount of 20% on cash sales. Peter bought a shirt and paid shs 48,000 in cash. Find:
(i). the marked price of the shirt.
(ii). the amount discount allowed.

Qn 3: Three friends Albert, Benjamin and Chris decide to buy a car. Albert pays $\frac{1}{4}$ of the cost; Benjamin pays $\frac{1}{3}$ of the cost and Chris pays the rest. Benjamin pays shs 1,500,000 more than Albert. Calculate the cost of the car.

Qn 4: An estate agent arranged for a sale of a house and got a commission of $1\frac{1}{2}\%$ on the selling price. If the amount of commission he received was shs 3,600,000, find the selling price of the house.

Qn 5: Find the principal that will amount to shs 100,000, when invested at simple interest of 10% p.a for 8 months.

Qn 6: Jane bought a house and later sold it at shs 21,000,000 thereby making a profit of 5%. Calculate:

(i). the cost price of the house.
(ii). the amount profit.

Qn 7: Find by how much the compound interest exceeds the simple interest on shs 60,000 invested for 2 years at a 12% p.a.

Qn 8: Find the cost of covering a floor 5 m by 4.5 m with a carpet costing shs 17,000 per m^2.

Qn 9: The marked price of an article is shs 2,500,000. Opio bought the article by paying a deposit of shs 500,000 and a number of equal installments of shs 250,000 each. If the hire purchase price is 20% higher than the marked price, calculate the number of installments.

Qn 10: Amongin bought a photocopier at shs 3,500,000. If the depreciation rate of the machine is 10.5% p.a, calculate the value of the copier after 2 years.

*****END*****

MARKING GUIDE

[Total Marks = 48]

<table>
<tr><th>SNo.</th><th>Working</th><th>Marks</th></tr>
<tr><td>1</td><td>(i).
$$\text{Profit} = \frac{10}{100} \times 56,000 = \text{shs } 5,600$$
(ii).
$$\text{Selling price} = 56,000 + 5,600 = \text{shs } 61,600$$</td><td>M1 A1
M1 A1</td></tr>
<tr><td></td><td></td><td>04</td></tr>
<tr><td>2</td><td>(i).
Let the marked price be x.
$$100\% - 20\% = 80\%$$
$$\frac{80}{100} \times x = 48000$$
$$0.8x = 48000$$
$$\frac{0.8x}{0.8} = \frac{48000}{0.8}$$
$$x = 60,000$$
The marked price of the shirt is shs60,000.
(ii).
$$\text{Discount} = 60,000 - 48,000 = \text{shs } 12,000$$</td><td>B1
M1
M1
A1</td></tr>
<tr><td></td><td></td><td>04</td></tr>
<tr><td>3</td><td>Let the cost price be x.
<table><tr><th></th><th>Albert</th><th>Benjamin</th><th>Chris</th></tr><tr><td>Amount paid</td><td>$\frac{1}{4}x$</td><td>$\frac{1}{3}x$</td><td>$\left(1 - \frac{1}{4} - \frac{1}{3}\right)x$
$= \frac{5}{12}x$</td></tr></table>
$$\frac{1}{3}x - \frac{1}{4}x = 1,500,000$$
$$\left(\frac{4-3}{12}\right)x = 1,500,000$$
$$x = 12,000,000 \times 12$$
The cost of the car is shs12,000,000.</td><td>B1
M1
M1 A1</td></tr>
<tr><td></td><td></td><td>04</td></tr>
<tr><td>4</td><td>Let the selling price be x.
$$1\frac{1}{2}\% \text{ of } x = 3,600,000$$
$$\frac{1.5}{100} \times x = 3,600,000$$
$$0.015x = 3,600,000$$
$$\frac{0.015x}{0.015} = \frac{3,600,000}{0.015}$$</td><td>M1
M1
M1</td></tr>
</table>

	$x = 240{,}000{,}000$ The selling price of the house is shs240,000,000.	**A1**
		04
5	$\text{Ammount} = P + \frac{PRT}{100}$ $10{,}000 = P + P \times \frac{10}{100} \times \frac{8}{12}$ $10{,}000 = P + \frac{1}{15}P$ $10{,}000 = \left(\frac{15+1}{15}\right)P$ $10{,}000 = \frac{16}{15}P$ $10{,}000 \times \frac{15}{16} = P$ $9{,}375 = P$ The principal is shs 9,375.	**M1** **M1** **M1** **A1**
		04
6	(i). Let the cost price be x. $100\% + 5\% = 105\%$ $\frac{105}{100} \times x = 21{,}000{,}000$ $1.05x = 21{,}000{,}000$ $\frac{1.05x}{1.05} = \frac{21{,}000{,}000}{1.05}$ $x = 20{,}000{,}000$ The cost price of the house is shs 20,000,000. (ii). $\text{Profit} = 21{,}000{,}000 - 20{,}000{,}000 = \text{shs } 1{,}000{,}000$	**B1** **M1** **M1** **A1** **M1 A1**
		06
7	Simple interest, $S.I = \frac{PRT}{100} = 60{,}000 \times \frac{12}{100} \times 2 = 14{,}400$ Amount obtained using compound interest $A = P\left(\frac{100+R}{100}\right)^n = 60{,}000 \times \left(\frac{100+12}{100}\right)^2$ $= 60{,}000 \times (1.12)^2 = 75{,}264$ Compound interest, $C.I = 75{,}264 - 60{,}000 = 15{,}264$ $\therefore C.I - S.I = 15{,}264 - 14{,}400 = \text{shs } 864$ The compound interest exceeds the simple interest by shs 864.	**M1 B1** **M1** **B1** **M1 B1** **M1 A1**
		08
8	$\text{Area of floor} = 5 \times 4.5 = 22.5 \text{ m}^2$ $\text{Total cost} = 22.5 \times 17000 = \text{shs } 382{,}500$	**M1 B1** **M1 A1**
		04

9	$100\% + 20\% = 120\%$ $\text{Hire purchase price} = \frac{120}{100} \times 2{,}500{,}000 = \text{shs } 3{,}000{,}000$ Let x be the number of equal installments. $\text{Hire purchase price} = 500{,}000 + 250{,}000x$ $3{,}000{,}000 = 500{,}000 + 250{,}000x$ $3{,}000{,}000 - 500{,}000 = 250{,}000x$ $2{,}500{,}000 = 250{,}000x$ $\frac{2{,}500{,}000}{250{,}000} = \frac{250{,}000x}{250{,}000}$ $x = 10$ equal installments	**B1** **M1 B1** **M1** **M1** **A1**
		06
10	$\text{Current price} = P\left(\frac{100 - R}{100}\right)^n = 3{,}500{,}000 \times \left(\frac{100 - 10.5}{100}\right)^2$ $= 3{,}500{,}000 \times (0.895)^2 = \text{shs } 2{,}8803{,}587.5$	**M1 M1** **B1 A1**
		04

*****END*****

SENIOR FOUR
MATHEMATICS
June 2023
$1\frac{1}{2}$ hours

S.4 MATH BI-WEEKLY TEST 2 2023

Topic: Functions & Vectors
Time: 1 Hour 30 Minutes

NAME: ________________________________ **STREAM:** __________

INSTRUCTIONS:
Attempt ALL questions in this paper.
Show your working clearly.

Qn 1: Given that $g(x) = 4x^2 + 3x + c$ and $g(4) = 0$, find the value of c.

Qn 2: A function $f(x) = \frac{2}{x} + 3$, find the value of x for which $f(x) = 4$.

Qn 3: If $f(x) = x - 1$ and $g(x) = x^2 - 5x + 4$, find the value of x for which:
(i). $\frac{f(x)}{g(x)}$ is undefined.
(ii). $f(x) \bullet g(x) = 0$.
(iii). $gf(x) = 0$.

Qn 4: Given that $(x) = \frac{4x+9}{x+4}$, find $g^{-1}(x)$; hence evaluate $g^{-1}(3)$.

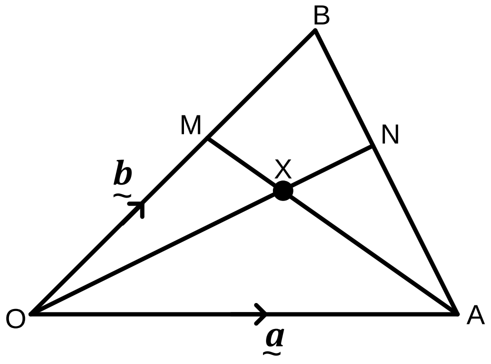
Qn 5: Given that $g(x) = px + q$ and $g(2) = 17$, then $g(-1) = 2$. Find:
(i). the values of p and q.
(ii). the values of x for which $g(x) = 0$.

Qn 6: Show that the points $A(-2, -2)$, $B(2, 1)$ and $C(10, 7)$ are collinear.

Qn 7: Given that $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $Q(7, 5)$. Find the coordinates of P.

Qn 8: Given $A(2, 3)$ and $B(5, 7)$, find:
(i). $\overrightarrow{AB}$,
(ii). $|\overrightarrow{AB}|$,

Qn 9: OAB is a triangle such that $OM:OB = 1:4$, $AN:NB = 1:2$, $\overrightarrow{OA} = \underset{\sim}{\boldsymbol{a}}$ and $\overrightarrow{OB} = \underset{\sim}{\boldsymbol{b}}$. $\overrightarrow{ON}$ and $\overrightarrow{AM}$ meet at X.

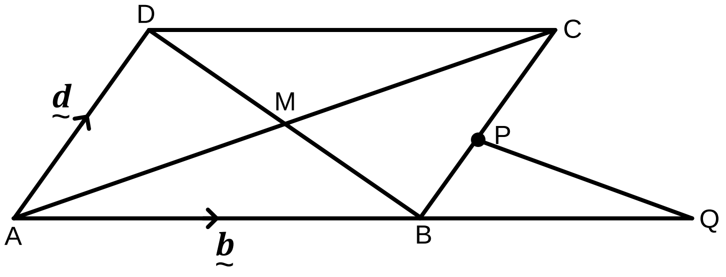


(a). Find, in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$, the vectors:

- (i). $\overrightarrow{AB}$
- (ii). $\overrightarrow{AM}$
- (iii). $\overrightarrow{ON}$

(b). Given that $\overrightarrow{OX} = h\,\overrightarrow{ON}$ and $\overrightarrow{AX} = k\,\overrightarrow{AM}$ where h and k are scalars, find the values of h and k. Hence find the ratio $\overrightarrow{AX}:\overrightarrow{AM}$.

Qn 10: The diagram below is a parallelogram. $2\overrightarrow{BC} = 3\overrightarrow{PC}$, $\overrightarrow{AQ} = 2\overrightarrow{AB}$, $\overrightarrow{AB} = \underset{\sim}{\boldsymbol{b}}$ and $\overrightarrow{AD} = \underset{\sim}{\boldsymbol{d}}$.



(a). Express, in terms of $\underset{\sim}{\boldsymbol{b}}$ and $\underset{\sim}{\boldsymbol{d}}$,

- (i). $\overrightarrow{AC}$,
- (ii). $\overrightarrow{BD}$
- (iii). $\overrightarrow{BP}$,
- (iv). $\overrightarrow{AP}$,
- (v). $\overrightarrow{PQ}$.

(b). Show that the points M, P and Q lie on a straight line.

*****END*****

MARKING GUIDE

[Total Marks = 70]

SNo.	Working	Marks
1	$g(x)=4x^2+3x+c$	
	$g(4)=4\times 4^2+3\times 4+c=0$	**M1**
	$64+12+c=0$	
	$76+c=0$	
	$c=-76$	**A1**
		02
2	$f(x)=\frac{2}{x}+3=4$	
	$\frac{2}{x}=4-3$	**M1**
	$x\times\frac{2}{x}=1\times x$	**M1**
	$2=x$	**A1**
	$\therefore x=2$	
		03
3	(i).	
	$\frac{f(x)}{g(x)}=\frac{x-1}{x^2-5x+4}$	**B1**
	$\frac{f(x)}{g(x)}$ is undefined when,	
	$x^2-5x+4=0$	**M1**
	sum $=-5$, product $=4$, factors $=-1,-4$	
	$x^2-x-4x+4=0$	
	$x(x-1)-4(x-1)=0$	
	$(x-4)(x-1)=0$	**M1**
	$(x-4)=0$, or, $(x-1)=0$	
	$x=4$, or, $x=1$	**A1**
	(ii).	
	$f(x)\bullet g(x)=0$	
	$(x-1)(x^2-5x+4)=0$	**M1**
	$(x-1)(x-4)(x-1)=0$	
	$(x-1)^2(x-4)=0$	**M1**
	$(x-1)^2=0$, or, $(x-4)=0$	
	$x=1$, or, $x=4$	**A1**
	(iii).	
	$gf(x)=g(x-1)$	
	$=(x-1)^2-5(x-1)+4$	**M1**
	$=x^2-2x+1-5x+5+4$	
	$=x^2-7x+10$	**B1**
	but, $gf(x)=0$	
	$x^2-7x+10=0$	**M1**
	sum $=-7$, product $=10$, factors $=-2,-5$	

	$x^2 - 2x - 5x + 10 = 0$ $x(x-2) - 5(x-2) = 0$ $(x-5)(x-2) = 0$ $(x-5) = 0, \quad \text{or,} \quad (x-2) = 0$ $x = 5, \quad \text{or,} \quad x = 2$	**M1** **A1**
		12
4	$g(x) = \frac{4x+9}{x+4}$ $Y = \frac{4X+9}{X+4}$ $Y(X+4) = 4X+9$ $XY + 4Y = 4X + 9$ $XY - 4X = 9 - 4Y$ $X(Y-4) = 9 - 4Y$ $X = \frac{9-4Y}{Y-4}$ $\therefore g^{-1}(x) = \frac{9-4x}{x-4}$ **For the hence part:** $g^{-1}(3) = \frac{9 - 4 \times 3}{3-4} = \frac{-3}{-1} = 3$	**M1** **M1** **B1** **B1** **M1 A1**
		06
5	(i). $g(x) = px + q$ $g(2) = p \times 2 + q = 17, \quad 2p + q = 17 \quad \longrightarrow (1)$ $g(-1) = p \times (-1) + q = 2, \quad -p + q = 2 \quad \longrightarrow (2)$ Equation $(1) - (2)$ gives, $- \begin{array}{c} 2p + q = 17 \\ -p + q = 2 \\ \hline 3p = 15 \end{array}$ $\frac{3p}{3} = \frac{15}{3}$ $p = 5$ and, $\quad q = 2 + p = 2 + 5 = 7$ (ii). $g(x) = 5x + 7 = 0$ $5x = -7$ $\frac{5x}{5} = \frac{-7}{5}$ $x = -\frac{7}{5}$	**B1** **B1** **M1** **M1** **A1** **M1 A1** **M1** **M1** **A1**
		10
6	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \binom{2}{1} - \binom{-2}{-2} = \binom{4}{3}$ $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \binom{10}{7} - \binom{2}{1} = \binom{8}{6} = 2\binom{4}{3}$	**B1**

	$\Rightarrow \overrightarrow{BC} = 2\overrightarrow{AB}$ Since $\overrightarrow{\boldsymbol{BC}}$ can be expressed as a multiple of $\overrightarrow{\boldsymbol{AB}}$, then points ***A***, ***B*** and ***C*** are collinear.	**B1** **B1** **B1**
		04
7	$\overrightarrow{\boldsymbol{PQ}} = \overrightarrow{\boldsymbol{OQ}} - \overrightarrow{\boldsymbol{OP}}$ $\begin{pmatrix} \mathbf{5} \\ \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{7} \\ \mathbf{5} \end{pmatrix} - \overrightarrow{\boldsymbol{OP}}$ $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\therefore P(2, 3)$	 **M1** **B1** **A1**
		03
8	(i). $\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (ii). $\lvert\overrightarrow{\boldsymbol{AB}}\rvert = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ units}$	 **M1 A1** **M1 A1**
		04
9	(i). $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}$ (ii). $OM:OB = 1:4$ $\overrightarrow{OM} = \frac{1}{4}\overrightarrow{OB} = \frac{1}{4}\underset{\sim}{\boldsymbol{b}}$ $\therefore \overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM} = -\underset{\sim}{\boldsymbol{a}} + \frac{1}{4}\underset{\sim}{\boldsymbol{b}}$ $= \frac{-4\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}}}{4} = \frac{1}{4}\left(\underset{\sim}{\boldsymbol{b}} - 4\underset{\sim}{\boldsymbol{a}}\right)$ (iii). $AN:NB = 1:2$ $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AB} = \frac{1}{3}\left(\underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}\right)$ $\therefore \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \underset{\sim}{\boldsymbol{a}} + \frac{1}{3}\left(\underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}\right)$ $= \frac{3\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}}{3} = \frac{1}{3}\left(2\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}}\right)$ (b). $\overrightarrow{\boldsymbol{OX}} = h\,\overrightarrow{\boldsymbol{ON}} = \frac{1}{3}h\left(2\underset{\sim}{\boldsymbol{a}} + \underset{\sim}{\boldsymbol{b}}\right) = \frac{2}{3}h\underset{\sim}{\boldsymbol{a}} + \frac{1}{3}h\underset{\sim}{\boldsymbol{b}}$ $\overrightarrow{\boldsymbol{AX}} = k\,\overrightarrow{\boldsymbol{AM}} = \frac{1}{4}k\left(\underset{\sim}{\boldsymbol{b}} - 4\underset{\sim}{\boldsymbol{a}}\right) = \frac{1}{4}k\underset{\sim}{\boldsymbol{b}} - k\underset{\sim}{\boldsymbol{a}}$ but, $\overrightarrow{\boldsymbol{OX}} = \overrightarrow{\boldsymbol{OA}} + \overrightarrow{\boldsymbol{AX}}$ $\frac{2}{3}h\underset{\sim}{\boldsymbol{a}} + \frac{1}{3}h\underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{a}} + \frac{1}{4}k\underset{\sim}{\boldsymbol{b}} - k\underset{\sim}{\boldsymbol{a}}$	 **B1** **B1** **B1** **B1** **B1** **B1** **B1**

	$\frac{2}{3}h\underset{\sim}{\boldsymbol{a}}+\frac{1}{3}h\underset{\sim}{\boldsymbol{b}}=(1-k)\underset{\sim}{\boldsymbol{a}}+\frac{1}{4}k\underset{\sim}{\boldsymbol{b}}$	**M1**
	Comparing coefficients of $\underset{\sim}{\boldsymbol{a}}$, $\frac{2}{3}h=(1-k)$ $2h=3-3k$ $2h+3k=3 \quad \longrightarrow (1)$	**B1**
	Comparing coefficients of $\underset{\sim}{\boldsymbol{b}}$, $\frac{1}{3}h=\frac{1}{4}k$ $4h=3k$ $4h-3k=0 \quad \longrightarrow (2)$	**B1**
	Equation $(1)+(2)$ gives, $\begin{array}{r\|c\|} & 2h+3k=3 \\ + & \underline{4h-3k=0} \\ & 6h=3 \end{array}$	**M1**
	$\frac{6h}{6}=\frac{3}{6}$ $h=\frac{1}{2}$	**A1**
	and, $k=\frac{4}{3}h=\frac{4}{3}\times\frac{1}{2}=\frac{2}{3}$	**A1**
	For the hence part, $\overrightarrow{\boldsymbol{AX}}=k\,\overrightarrow{\boldsymbol{AM}}$ $\overrightarrow{\boldsymbol{AX}}=\frac{2}{3}\overrightarrow{\boldsymbol{AM}}$	**M1**
	$\frac{\overrightarrow{\boldsymbol{AX}}}{\overrightarrow{\boldsymbol{AM}}}=\frac{2}{3}$ $\therefore \overrightarrow{\boldsymbol{AX}}:\overrightarrow{\boldsymbol{AM}}=2:3$	**A1**
		15
10	(a). (i). $\overrightarrow{\boldsymbol{AC}}=\overrightarrow{\boldsymbol{AB}}+\overrightarrow{\boldsymbol{BC}}=\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}$	**B1**
	(ii). $\overrightarrow{\boldsymbol{BD}}=\overrightarrow{\boldsymbol{BA}}+\overrightarrow{\boldsymbol{AD}}=-\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}=\underset{\sim}{\boldsymbol{d}}-\underset{\sim}{\boldsymbol{b}}$	**B1**
	(iii). $2\overrightarrow{\boldsymbol{BC}}=3\overrightarrow{\boldsymbol{PC}}$ $\overrightarrow{\boldsymbol{PC}}=\frac{2}{3}\overrightarrow{\boldsymbol{BC}}=\frac{2}{3}\underset{\sim}{\boldsymbol{d}}$	**B1**
	but, $\overrightarrow{\boldsymbol{BC}}=\overrightarrow{\boldsymbol{BP}}+\overrightarrow{\boldsymbol{PC}}$ $\underset{\sim}{\boldsymbol{d}}=\overrightarrow{\boldsymbol{BP}}+\frac{2}{3}\underset{\sim}{\boldsymbol{d}}$ $\overrightarrow{\boldsymbol{BP}}=\underset{\sim}{\boldsymbol{d}}-\frac{2}{3}\underset{\sim}{\boldsymbol{d}}=\frac{3\underset{\sim}{\boldsymbol{d}}-2\underset{\sim}{\boldsymbol{d}}}{3}=\frac{1}{3}\underset{\sim}{\boldsymbol{d}}$	**B1**
	(iv).	

	$\overrightarrow{\boldsymbol{AP}}=\overrightarrow{\boldsymbol{AB}}+\overrightarrow{\boldsymbol{BP}}=\underset{\sim}{\boldsymbol{b}}+\frac{1}{3}\underset{\sim}{\boldsymbol{d}}=\frac{3\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}}{3}=\frac{1}{3}\left(3\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}\right)$	**B1**
	(v). $\overrightarrow{\boldsymbol{AQ}}=2\overrightarrow{\boldsymbol{AB}}=2\underset{\sim}{\boldsymbol{b}}$ $\overrightarrow{\boldsymbol{PQ}}=\overrightarrow{\boldsymbol{PA}}+\overrightarrow{\boldsymbol{AQ}}=-\frac{1}{3}\left(3\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}\right)+2\underset{\sim}{\boldsymbol{b}}$ $=\frac{-3\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}+6\underset{\sim}{\boldsymbol{b}}}{3}=\frac{1}{3}\left(3\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}\right)$	**B1**
	(b). $\overrightarrow{\boldsymbol{AM}}=\frac{1}{2}\overrightarrow{\boldsymbol{AC}}=\frac{1}{2}\left(\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}\right)$	**B1**
	$\overrightarrow{\boldsymbol{MQ}}=\overrightarrow{\boldsymbol{MA}}+\overrightarrow{\boldsymbol{AQ}}=-\frac{1}{2}\left(\underset{\sim}{\boldsymbol{b}}+\underset{\sim}{\boldsymbol{d}}\right)+2\underset{\sim}{\boldsymbol{b}}$ $=\frac{-\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}+4\underset{\sim}{\boldsymbol{b}}}{2}=\frac{1}{2}\left(3\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}\right)$	**B1**
	$\frac{\overrightarrow{\boldsymbol{MQ}}}{\overrightarrow{\boldsymbol{PQ}}}=\frac{\frac{1}{2}\left(3\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}\right)}{\frac{1}{3}\left(3\underset{\sim}{\boldsymbol{b}}-\underset{\sim}{\boldsymbol{d}}\right)}$ $\frac{\overrightarrow{\boldsymbol{MQ}}}{\overrightarrow{\boldsymbol{PQ}}}=\frac{1}{2}\div\frac{1}{3}$ $\frac{\overrightarrow{\boldsymbol{MQ}}}{\overrightarrow{\boldsymbol{PQ}}}=\frac{3}{2}$ $2\overrightarrow{\boldsymbol{MQ}}=3\overrightarrow{\boldsymbol{PQ}}$	**M1**
	Since $2\overrightarrow{\boldsymbol{MQ}}=3\overrightarrow{\boldsymbol{PQ}}$ and $\boldsymbol{Q}$ is common to both lines $\overrightarrow{\boldsymbol{MQ}}$and $\overrightarrow{\boldsymbol{PQ}}$, then points$\boldsymbol{M}$, $\boldsymbol{P}$ and $\boldsymbol{Q}$lie on a straight line.	**B1**
		10

*****END*****

456/1
MATHEMATICS
PAPER 1
July 2023
$2\frac{1}{2}$ hours

S.4 MATH 1 MOCK SET 4 2023
Time: 2 Hours 30 Minutes

NAME: ____ **STREAM:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Simplify: $\left(1\frac{2}{3}-\frac{1}{4}+2\frac{1}{2}\right)\div\left(\frac{1}{3}+\frac{1}{4}\right)$. [4]

Qn 2: Let the operation $(\sim)$ be defined as "*Add the square of the first number to twice the second one*". Express $(\boldsymbol{p}\sim\boldsymbol{q})$ algebraically. Hence evaluate $(-3\sim4)\sim1$. [4]

Qn 3: Solve equation $x^2-5x=14$ by factorization. [4]

Qn 4: (i). Use a suitable identity to expand and simplify: $(x+3)^2$.
(ii). Use the identity $(\boldsymbol{a}-\boldsymbol{b})^2=\boldsymbol{a}^2-2\boldsymbol{ab}+\boldsymbol{b}^2$ to evaluate $(999)^2$. [4]

Qn 5: Find the actual distance of a road section represented by a length of 3.5 cm on a map of scale 1:250000. [4]

Qn 6: Find the equation of a straight line passing through point $(3,-2)$ and is parallel to the line whose equation is $2y=6x-3$. [4]

Qn 7: Express the inequality $[(3y-2)<(y+10)<(5y+2)]$ in the form $a<y<b$. Hence state the integral values of y. [4]

Qn 8: If $\begin{pmatrix}4 & 1\\ x & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}4\\ 8\end{pmatrix}$, determine the value of x and y. [4]

Qn 9: Given that $\sin(\theta+30°)=0.700$, evaluate $\cos\theta$. [4]

Qn 10: With the use of a diagram, express the following as *3-figure bearings:*
(i). North West,
(ii). $S\ 70°\,W$. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

(a). A bag contains **8** red, **4** black and **6** blue identical pens. Three pens are drawn at random from the bag in succession. Find the probability that:

(i). the three pens are all black in colour.

(ii). the first two pens are red in colour.

(b). In a school of **300** boys and **200** girls, the number of boys andgirls is increased in the ratios **4**: **3** and **3**: **2**, respectively.

(i). Find the new school enrollment.

(ii). Suppose that the students were proportionately distributed in each class according to gender, what would be the expected number of girls in a new class of **56** students? [12]

Question 12:

(a). If transformation matrix $\begin{pmatrix} 1 & n \\ k & -4 \end{pmatrix}$ maps point $P(3,-2)$ onto $P^{'}(-1,17)$; find the values of n and k.

(b). Triangle $A'B'C'$ is the image of ΔABC under transformation "$\boldsymbol{T}$", where: $A(1,1)$, $B(1,3)$, $C(4,1)$, $A^{'}(-1,1)$, $B^{'}(-1,3)$ and $C^{'}(-4,1)$.

(i). Fully describe transformation "$\boldsymbol{T}$".

(ii). Find the matrix representation for transformation "$\boldsymbol{T}$" above. [12]

Question 13:

(a). Consider a matrix $\boldsymbol{A} = \begin{pmatrix} y-3 & 1 \\ 4 & y \end{pmatrix}$. Find:

(i). an expression for $|\boldsymbol{A}|$, the determinant of the given matrix.

(ii). the value(s) of 'y' for which matrix $\boldsymbol{A}$ is *singular*.

(b). A triangle whose vertices are at $A(1,0)$, $B(1,2)$ and $C(2,3)$ has its enlargement as $A^{'}(3,-2)$, $B^{'}(3,2)$ and $C^{'}(5,4)$. Find the centre and scale factor of enlargement. [12]

Question 14:

Ntake Transporter's company plans to transport cartons of soap from Kampala to Masaka using the 'Fuso-4 wheel drive' and a 'Daina truck'. When the Fuso makes **6** journeys and the Diana **10** journeys, the number of cartons delivered must not exceed **60**. The number of cartons carried by the Fuso must not exceed those of the Daina by **2**. For each carton, a Fuso makes a profit of shs 2,500 while the Diana makes shs 1,000. Let 'x' and 'y' be the number of cartons a Fuso and Diana can load at a time, respectively.

(i). Write down four inequalities for the given constraints.

(ii). Write down the expression that maximizes the profit.
(iii). Draw a graph for the inequalities in part (i) above.
(iv). Hence, find the maximum profit that the company can make. [12]

Question 15:

Use the frequency table below to answer the accompanying questions:

Class	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79
f	2	1	7	3	4	1	2

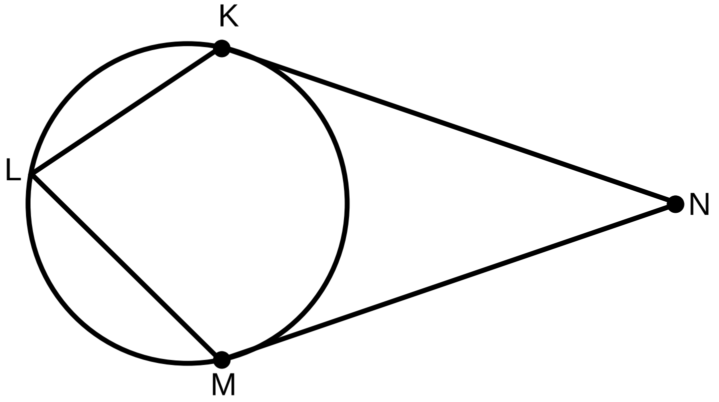
(a). Calculate the:
  (i). Mean score, using a *Working mean* of 34.5.
  (ii). Modal score.
(b). If the given data represents the marks scored by students in a Mock examination, determine the percentage number of students who passed the examination, given that 50% was the pass mark. [12]

Question 16:

(i). Construct a ΔABC such that $\overline{AB} = 6.2$ cm, $\overline{AC} = 7.1$ cm and $\angle BAC = 90°$. Hence, measure and state the length of $\overline{BC}$.
(ii). Then construct a circle whose centre is equidistant from all the vertices of ΔABC. Measure and state the size of this radius.
(iii). Also calculate the area of the circle in part (ii) above. [12]

Question 17:

(a). In the diagram below, $\overline{NK}$ and $\overline{NM}$ are tangents to the circle at points $K$and M, respectively. If $K\hat{N}M = 48°$, calculate the size of $K\hat{L}M$.



(b). How many complete revolutions must be made on a circular track of radius **35** metres in running a **4500** metre-race?
(c). A sector of a circle of radius 14 cm, has an angle of 60° at the centre. Find its perimeter. [12]

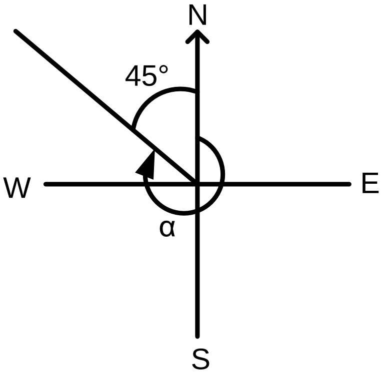
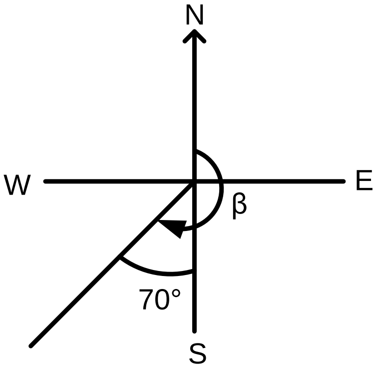
END

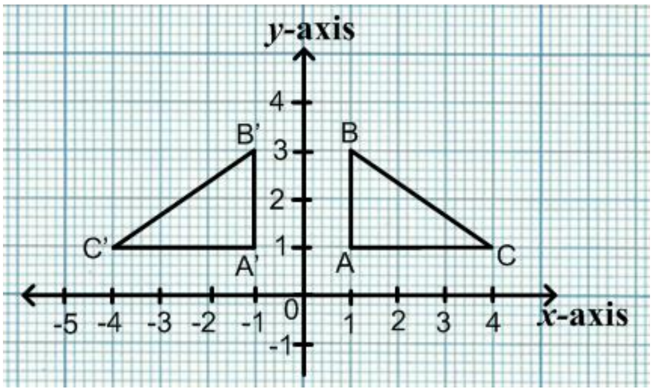
MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	$\left(1\frac{2}{3}-\frac{1}{4}+2\frac{1}{2}\right)=\frac{5}{3}-\frac{1}{4}+\frac{5}{2}=\frac{20-3+30}{12}=\frac{47}{12}$ $\left(\frac{1}{3}+\frac{1}{4}\right)=\frac{4+3}{12}=\frac{7}{12}$ $\frac{47}{12}\div\frac{7}{12}=\frac{47}{12}\times\frac{12}{7}=\frac{47}{7}=6\frac{5}{7}$	**M1** **M1** **M1 A1**
		04
2	**Question 2:** $p\sim q=p^2+2q$ $-3\sim 4=(-3)^2+2\times 4=9+8=17$ $(-3\sim 4)\sim 1=17\sim 1=17^2+2\times 1=289+2=291$	**B1** **B1** **M1 A1**
		04
3	$x^2-5x=14$ $x^2-5x-14=0$ sum $=-5$, product $=-14$, factors $=-7,2$ $x^2-7x+2x-14=0$ $x(x-7)+2(x-7)=0$ $(x-7)(x+2)=0$ $(x-7)=0$, or, $(x+2)=0$ $x=7$, or, $x=-2$	**M1** **B1** **A1 A1**
		04
4	(i). $(x+3)^2=x^2+2(3x)+3^2=x^2+6x+9$ (ii). $(999)^2=(1000-1)^2=1000^2-2\times 1000\times 1+1^2$ $=1000000-2000+1=998{,}001$	**M1 A1** **M1** **A1**
		04
5	The representative fraction is 1 cm $\leftrightarrow$ 250000 cm The linear scale is 1 cm $\leftrightarrow \frac{250{,}000}{100{,}000}$ km 1 cm $\leftrightarrow$ 2.5 km The area scale is 3.5 cm $\leftrightarrow 3.5\times 2.5$ km 3.5 cm $\leftrightarrow$ 8.75 km The actual distance is 8.75 km.	**B1**-correct interpretation **M1**-dividing by 100,000 **M1** **A1**
		04
6	$2y=6x-3,\quad\Rightarrow y=3x-\frac{3}{2}$	**M1**

	Gradient, $m = 3$ From $y = mx + c$ $-2 = (3 \times 3) + c$ $c = -11$ $\therefore y = 3x - 11$ **Alternatively:** $\frac{y - (-2)}{x - 3} = 3$ $y + 2 = 3(x - 3)$ $y + 2 = 3x - 9$ $y = 3x - 11$	**B1** **B1** **A1**
		04
7	$3y - 2 < y + 10$ $\quad$ $y + 10 < 5y + 2$ $3y - y < 10 + 2$ $\quad$ $10 - 2 < 5y - y$ $2y < 12$ $\quad$ $8 < 4y$ $y < 6$ $\quad$ $2 < y$ $\therefore 2 < y < 6$ hence, $y = 3, 4, 5$	**M1**-collecting like terms **M1**-simplifying **B1** **A1**-for both correct
		04
8	$\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x + y = 4 \\ x^2 - y = 8 \end{pmatrix}$ $4x + y = 4 \quad \longrightarrow (1)$ $x^2 - y = 8 \quad \longrightarrow (2)$ Adding: $\begin{array}{r\|c\|} & 4x + y = 4 \\ + & x^2 - y = 8 \\ \hline & x^2 + 4x = 12 \end{array}$ $x^2 + 4x - 12 = 0$ $x^2 - 2x + 6x - 12 = 0$ $x(x - 2) + 6(x - 2) = 0$ $(x - 2)(x + 6) = 0$ $(x - 2) = 0$, or, $(x + 6) = 0$ $x = 2$, or, $x = -6$ From equation (1), if $x = 2$, $y = -4$ and if $x = -6$, $y = 28$	**B1**-both eqns correct **B1**-correct Q.E **B1**-correct factors **B1**-correct pairs
		04
9	$\sin(\theta + 30°) = 0.700$ $\theta + 30° = \sin^{-1}(0.7) = 44.43°$ $\theta = 44.43° - 30° = 14.43°$	**B1**-for 44.43°

	$\cos\theta = \cos 44.43° \approx 0.9685$	**M1-** subtracting 30° **M1 A1**
		04
10	(i).  $\alpha = 360 - 45 = 315°$ $\therefore N\,W = 225°$ (ii).  $\beta = 180 + 70 = 250°$ $\therefore S\,70°\,E = 250°$	**B1-**correct sketch **B1** **B1-**correct sketch **B1**
		04
11	(a). $n(\text{Red}) = 8, \quad n(\text{Black}) = 4, \quad n(\text{Blue}) = 6$ (i). $P(\text{All Black}) = \frac{4}{18} \times \frac{3}{17} \times \frac{2}{16} = \frac{1}{204}$ (ii). $P(1^{\text{st}} \text{ two are red}) = \left(\frac{8}{18} \times \frac{7}{17} \times \frac{4}{16}\right) + \left(\frac{8}{18} \times \frac{7}{17} \times \frac{6}{16}\right)$	**M1 A1** **M1**

	$=\frac{7}{153}+\frac{7}{102}=\frac{35}{306}$	**A1**
	(b).	
	Original poulation $= 300 + 200 = 500$	**B1**
	(i).	
	New number of boys $=\frac{4}{3}\times 300 = 400$	**B1**
	New number of girls $=\frac{3}{2}\times 200 = 300$	**B1**
	New enrollment $= 400 + 300 = 700$	**M1 A1**
	(ii).	
	Proportion of girls $=\frac{300}{700}=\frac{3}{7}$	**B1**
	Expected number of girls $=\frac{3}{7}\times 56 = 24$	**M1 A1**
		12
12	(a).	
	$\begin{pmatrix}1 & n\\ k & -4\end{pmatrix}\begin{pmatrix}3\\ -2\end{pmatrix}=\begin{pmatrix}-1\\ 17\end{pmatrix}$	**B1**-correct matrix eqn
	$3-2n=-1 \quad \longrightarrow (1)$	**B1**
	$3k+8=17 \quad \longrightarrow (2)$	**B1**
	From equation (1),	
	$2n = 4, \quad \Longrightarrow n = 2$	**B1**
	From equation (2),	
	$3k = 9, \quad \Longrightarrow k = 3$	**B1**
	(b).	
		**B1**-good scale **B1**-for ABC **B1**-for A'B'C'
	(i). T represents reflection in the y-axis.	**B1**
	(ii). Using points $I(1,0)$ and $J(0,1)$ or otherwise,	**M1**
	$I(1,0) \longrightarrow I'(-1,0)$	**M1**
	$J(0,1) \longrightarrow J'(0,1)$	
	$\therefore$ Matrix $=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$	**A1**
		12
13	(a). (i).	
	$\lvert \boldsymbol{A}\rvert = y(y-3)-4 = y^2-3y-4$	**M1 A1**
	(ii).	
	for singular matrix, $\quad \lvert \boldsymbol{A}\rvert = 0$	

$$y^2 - 3y - 4 = 0$$

$$y^2 - 4y + y - 4 = 0$$

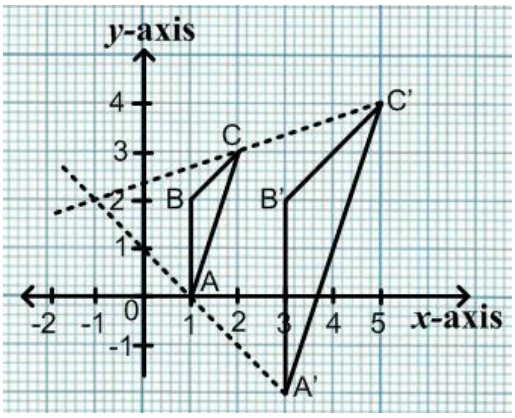
$$y(y-4) + (y-4) = 0$$ **M1**

$$(y-4)(y+1) = 0$$ **M1**

$$(y-4) = 0, \quad \text{or,} \quad (y+1) = 0$$

$$y = 4, \quad \text{or,} \quad y = -1$$ **A1**-for both correct

(b).



B1-for ABC

B1-for A'B'C'

B1-locating the centre

Centre is $(-1, 2)$ **B1**

$$\text{scale factor} = \frac{A'B'}{AB} = \frac{4}{2} = 2$$ **M1 A1**

12

14

Type	Number of cartons	Profit
Fuso	x	2500
Diana	y	1000

(i).

$$x \geq 0, \quad y \geq 0, \quad x \leq y + 2$$ **B1 B1 B1**

$$6x + 10y \leq 60, \quad \Longrightarrow 3x + 5y \leq 30$$ **B1**

(ii).

$$\text{Profit} = 2500x + 1000y$$ **B1**

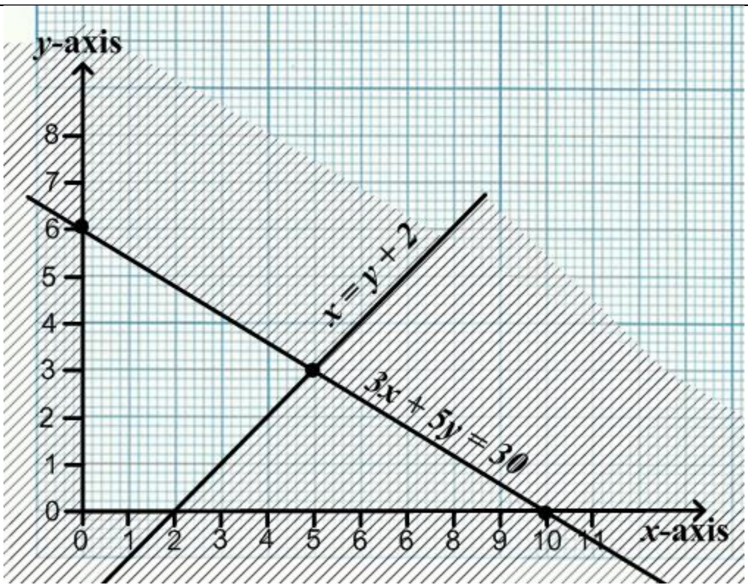
(iii).

Region	Border line	Coordinates
$x \leq y + 2$	$x = y + 2$	$(2, 0), (4, 2)$
$3x + 5y \leq 30$	$3x + 5y = 30$	$(0, 6), (10, 0)$
$x \geq 0$	$x = 0$	y-axis
$y \geq 0$	$y = 0$	x-axis

B1-any two correct points on 1st line

B1-any two correct points on 2nd line

B1-correct plotting of



$x = y + 2$

B1-correct plotting of $3x + 5y = 30$

B1-correct shading

(iv). Maximizing profit

(x, y)	$(2500x + 1000y)$
$(1, 5)$	$(2500 \times 1 + 1000 \times 5) = 7{,}500$
$(2, 2)$	$(2500 \times 2 + 1000 \times 2) = 7{,}000$
$(3, 4)$	$(2500 \times 3 + 1000 \times 4) = 11{,}500$
$(4, 3)$	$(2500 \times 4 + 1000 \times 3) = 13{,}000$
$(5, 3)$	$(2500 \times 5 + 1000 \times 3) = 15{,}500$

$\therefore$ Maximum profit = shs 15,500

M1-any two correct substitution including that of point $(5, 3)$

A1

12

15 (a).

Class	f	x	d	fd	$C.F$	Class boundaries
10 – 19	2	14.5	-20	-40	2	9.5 – 19.5
20 – 29	1	24.5	-10	-10	3	19.5 – 29.5
30 – 39	7	34.5	0	0	10	29.5 – 39.5
40 – 49	3	44.5	10	30	13	39.5 – 49.5
50 – 59	4	54.5	20	80	17	49.5 – 59.5
60 – 69	1	64.5	30	30	18	59.5 – 69.5
70 – 79	2	74.5	40	80	20	69.5 – 79.5
Total	20			170		

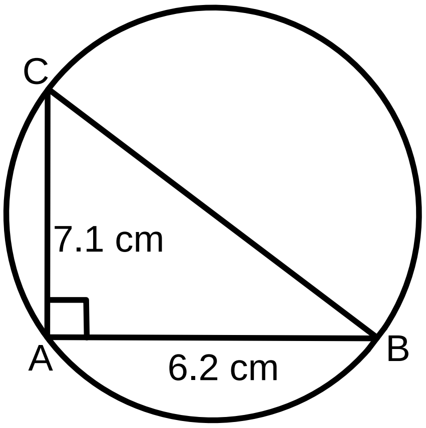
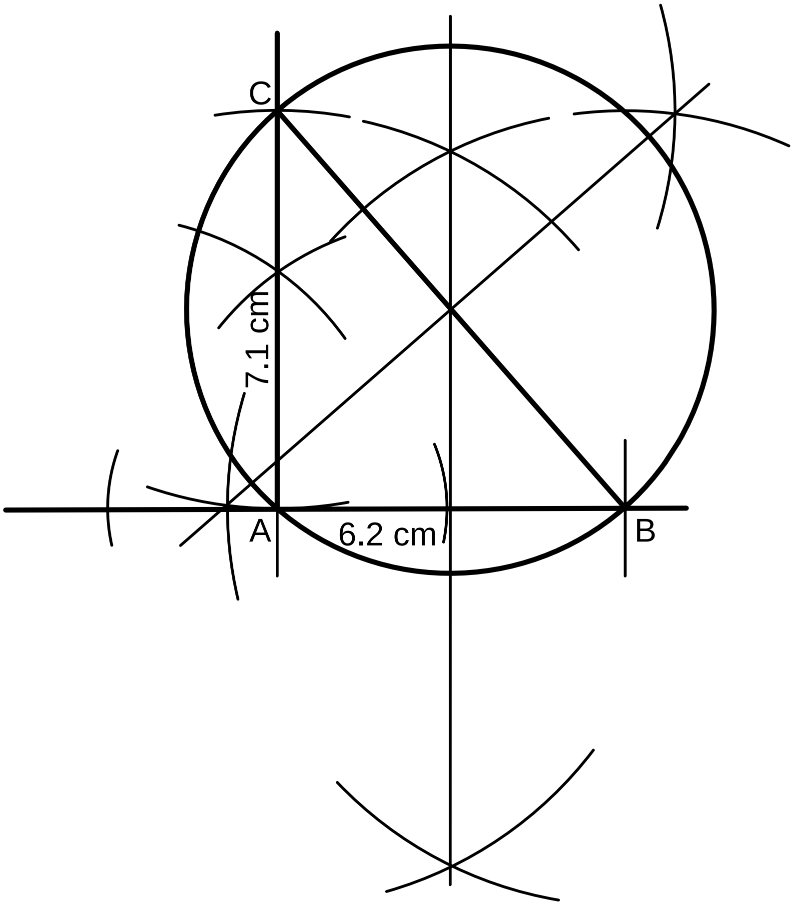
B1- for $\sum f$
B1-for $\sum fd$
B1- for "x" column
B1- for "d" column
B1- for "fd" column
B1- for "$C.F$" column

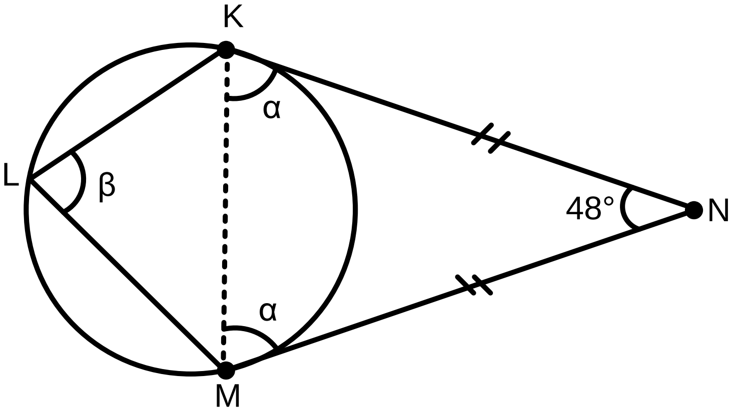
(i).

$$\text{Mean} = \text{A} + \frac{\sum fd}{\sum f} = 34.5 + \frac{170}{20} = 34.5 + 8.5 = 43$$

M1 A1

(ii).

	$$\text{Mode} = L_m + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)c = 29.5 + \left(\frac{6}{6+4}\right) \times 10$$ $$= 29.5 + 6 = 35.5$$ (b). $$\text{Percentage number that passed} = \left(\frac{4+1+2}{20}\right) \times 100\%$$ $$= \frac{7}{20} \times 100\% = 35\%$$	**M1** **A1** (accept 36) **M1** **A1**
		12
16	(i). **Sketch:**  **Accurate diagram:**  	**B1-**correct sketch (seen or implied) **B1-** angle 90° at A. **B1-** for AB=6.2 cm **B1-**for AC=7.1 cm **B2-**for perpendicular bisectors **B2-**for circumcircle

	$\text{Length } \overline{BC} = 9.4\text{ cm} \pm 0.2\text{ cm}$ (ii). $\text{Radius} = 4.6\text{ cm or } 4.7\text{ cm}$ (iii). $\text{Area} = \pi r^2 = \frac{22}{7} \times (4.6)^2 \approx 66.5\text{ cm}^2$	**B1** **B1** (accept 4.5 - 4.8 cm) **M1 A1**
		12
17	(a).  $\alpha + \alpha + 48 = 180$ $2\alpha = 132$ $\alpha = 66°$ angle KLM = angle NKM (alternate seegment theorem) ∴ angle KLM = 66° (b). $c = 2\pi r = 2 \times \frac{22}{7} \times 35 = 220\text{ m}$ $\text{Number of revolutions} = \frac{4500}{220}$ $= 20.45\ (2\text{ d.p}) \approx 20\text{ revolutions}$ (c). $\text{Perimeter} = 2r + \frac{60}{360} \times 2\pi r = 2 \times 14 + \frac{60}{360} \times 2 \times \frac{22}{7} \times 14$ $= 28 + 14\frac{2}{3} = 42\frac{2}{3} = 42.7\text{ cm}$	**M1** **A1** **B1** **M1 A1** **M1** **A1** **M1 M1** **M1 A1**
		12

*****END****

456/2
MATHEMATICS
PAPER 2
July 2023
$2\frac{1}{2}$ hours

S.4 MATH 2 MOCK SET 4 2023

Time: 2 Hours 30 Minutes

NAME: ______________________ **STREAM:** __________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Simplify: $\left(3\frac{3}{8}\right)^{\frac{-2}{3}} + \frac{1}{2}$. [4]

Qn 2: Simplify: $\frac{x^2+3x-10}{x+5}$. [4]

Qn 3: In a group of 20 students, 7 did not pass Math (M), 11 did not pass English (E), and 5 passed both subjects.
(a). Represent the information on a Venn diagram.
(b). How many passed Math but not English. [4]

Qn 4: Without using a calculator, simplify $\frac{1}{2-\sqrt{3}} - \frac{1}{2+\sqrt{3}}$. [4]

Qn 5: Given $A(x, 7)$ and $B(5, 4)$ and that $\left|\overrightarrow{AB}\right| = 5$ units, find the possible values of x. [4]

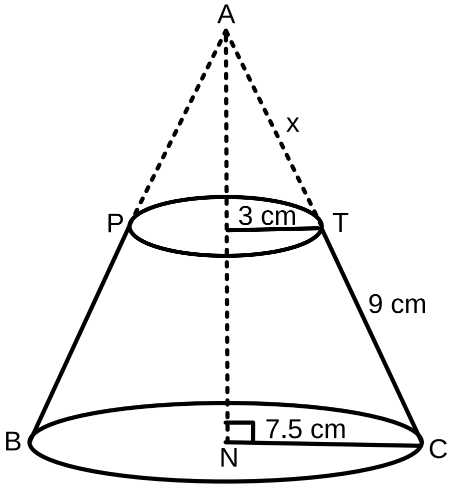
Qn 6: Determine the equation of the line parallel to $3x + 2y = 8$, which passes through $(-1, 2)$. [4]

Qn 7: Given $g^{-1}(x) = \frac{2x+1}{3}$, determine the:
(a). expression for $g(x)$.
(b). value of $g(5)$. [4]

Qn 8: On a map of area 75 km^2 is represented by 12 cm^2. Determine the scale of the map in form of $1{:}\,n$. [4]

Qn 9: A bus set off from town P at 8:30 pm for town Q at an average speed of 80 km/hr. It arrived at Q at 3:15 am. Determine the distance $\overline{PQ}$. [4]

Qn 10: The figure below shows a cone ABC with circular end of radius 7.5 cm from which cone APT is cut off at radius 3 cm.



Determine the ratio of the volume for the cut off cone to the volume of the frustum BCTP. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 11:

(a). Simplify: $\dfrac{2\frac{1}{2} + 1\frac{1}{3} \times 2\frac{1}{4}}{\frac{5}{6} + 1\frac{2}{3}}$.

(b). The cost (c) of hiring a car is partly constant and partly varies as the distance (d). When $d = 10$, $c =$ shs 45,000 yet when $d = 35$, $c =$ shs 82,500. Determine:

(i). an equation relating c and d.
(ii). the value of c when $d = 50$.
(iii). The value of d when $c =$ shs 72,000. [12]

Question 12:

(a). A machine costs shs 3,500,000. It depreciates at a rate of 5% per annum. Calculate its value after two years.

(b). The tax structure of a certain country is as follows:

Taxable income (shs)	Tax rate (%)
1 – 150,000	Free
150,001 – 400,000	5
400,001 – 700,000	8
Above 700,000	12

Ofono has an allowance of shs 50,000 which is exempted from tax, but pays tax of shs 58,100. Calculate Ofono's;

(i). gross pay

(ii). net pay. [12]

Question 13:

(a). Given $\log_{10} x = 1.3586$ and $\log_{10} y = 2.1428$. Use the information to find $\log_{10}\left(\frac{\sqrt{x}}{y}\right)$.

(b). The distance between two towns A and B is 20 km. Peter walked from town A to town B, covered two-fifth of the journey in 2 hours and the remaining journey he moved at 3 km h^{-1}. Calculate:

(i). the speed for the first part of the journey.

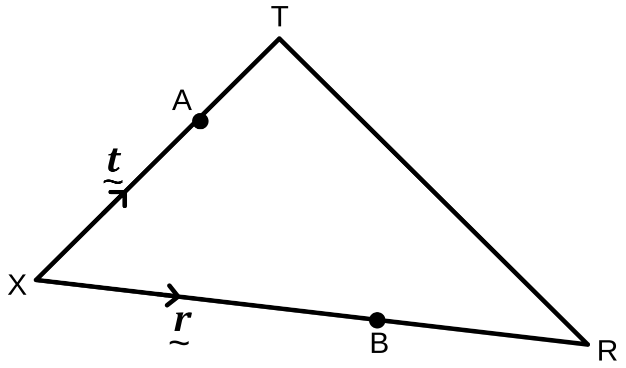
(ii). the time taken to cover the remaining journey.

(iii). Average speed for the whole journey.

(c). Draw a distance time graph showing the route of Peter. [12]

Question 14:

In the diagram below, A divides $\overrightarrow{XT}$ in a ratio $1:1$. B is on $\overrightarrow{XR}$, such that $\overrightarrow{BR} = 3\overrightarrow{XB}$. If $\overrightarrow{XA} = \underset{\sim}{\boldsymbol{t}}$ and $\overrightarrow{XB} = \underset{\sim}{\boldsymbol{r}}$.



(a). Express the following vectors in terms of $\underset{\sim}{\boldsymbol{r}}$ and $\underset{\sim}{\boldsymbol{t}}$.

(i). $\overrightarrow{XT}$,

(ii). $\overrightarrow{BT}$,

(iii). $\overrightarrow{TR}$.

(b). If $\overrightarrow{AT} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\overrightarrow{XB} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, express $\overrightarrow{TR}$ as a column vector. Hence determine $|\overrightarrow{TR}|$. [12]

Question 15:

A group of 40 students were asked whether they were members of scripture union (S), school choir (C) or interact club (I). 18 belonged to interact club; the number of those in scripture union was equal to the number of those in school choir. 10 belonged to S and C, 3 belonged to S and I only, 8 belonged to C and I only, 4 belonged to the three clubs, 7 do not belong to any of these clubs.

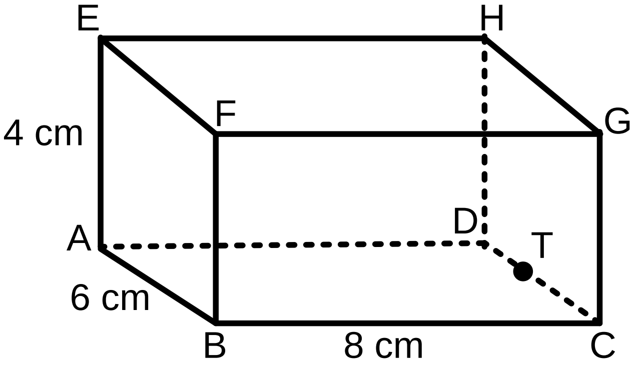
(a). Represent the information on a Venn diagram.

(b). How many students belong to S only?

(c). What is the probability of picking one who does not belong to the church choir? [12]

Question 16:

Below is a cuboid with $\overline{BC} = 8$ cm, $\overline{AB} = 6$ cm, $\overline{AE} = 4$ cm. T is the midpoint of DC.



Calculate:

(a). length

(i). $\overline{AT}$,

(ii). $\overline{TE}$.

(b). angle between line TE and plane $DCGH$.

(c). angle between planes EFT and $EFGH$. [12]

Question 17:

Line T has x and y −intercepts −2 and 4 respectively. Line R is perpendicular to $y + 5 = 3x$ and passes through $(2, 1)$. Determine:

(a). equation of line

(i). T, (ii). R.

(b). point of intersection of T and R.

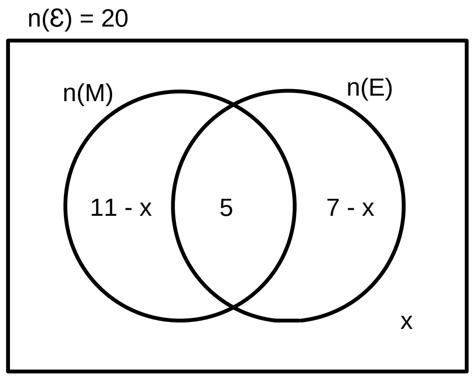
(c). x −intercept of line R.

(d). area between T, R and the x −axis. [12]

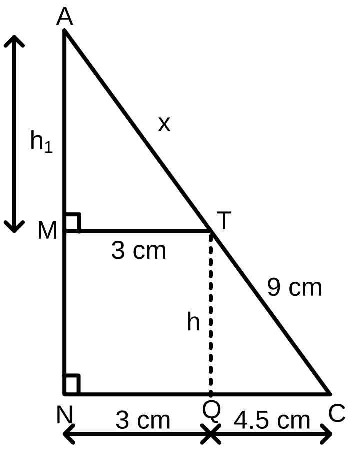
*****END*****

MARKING GUIDE

[Total Marks = 100]

SNo.	Working	Marks
1	$\left(3\frac{3}{8}\right)^{\frac{-2}{3}} = \left(\frac{27}{8}\right)^{\frac{-2}{3}} = \left(\frac{3^3}{2^3}\right)^{\frac{-2}{3}} = \left(\frac{3}{2}\right)^{3\times\frac{-2}{3}}$ $= \left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$ $\therefore \left(3\frac{3}{8}\right)^{\frac{-2}{3}} + \frac{1}{2} = \frac{4}{9} + \frac{1}{2} = \frac{8+9}{18} = \frac{17}{18}$	**B1** **B1** **M1 A1**
		04
2	$x^2 + 3x - 10 = x^2 - 2x + 5x - 10$ $= x(x-2) + 5(x-2)$ $= (x+5)(x-2)$ $\therefore \frac{x^2 + 3x - 10}{x+5} = \frac{(x+5)(x-2)}{x+5} = x - 2$	**B1** **B1** **B1 B1**
		04
3	(a). Let $n(M \cap E) = x$  $n(\varepsilon) = 11 - x + 5 + 7 - x + x$ $20 = 23 - x$ $x = 23 - 20$ $x = 3$ (b). $n(\text{passed math but not english}) = 11 - x = 11 - 3 = 8$	**B1 B1** **B1** **B1**
		04
4	$\frac{1}{2-\sqrt{3}} - \frac{1}{2+\sqrt{3}} = \frac{(2+\sqrt{3}) - (2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ $= \frac{2+\sqrt{3}-2+\sqrt{3}}{4-3}$ $= 2\sqrt{3}$	**B1** **B1** **B1** **B1**
		04

5	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} x \\ 7 \end{pmatrix} = \begin{pmatrix} 5-x \\ -3 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(5-x)^2 + (-3)^2}$ $5 = \sqrt{25 - 10x + x^2 + 9}$ $5^2 = 25 - 10x + x^2 + 9$ $x^2 - 10x + 9 = 0$ $x^2 - x - 9x + 9 = 0$ $x(x-1) - 9(x-1) = 0$ $(x-9)(x-1) = 0$ $(x-9) = 0, \quad \text{or,} \quad (x-1) = 0$ $x = 9, \quad \text{or,} \quad x = 1$	**B1** **M1** **M1** **A1**
		04
6	$3x + 2y = 8$ $2y = -3x + 8$ $y = \frac{-3}{4}x + 2$ $m = \frac{-3}{4}$ Using $y = mx + c$ for $(-1, 2)$, $2 = -1 \times \frac{-3}{4} + c, \qquad \Rightarrow c = \frac{5}{2}$ $y = \frac{-3}{4}x + \frac{5}{2}$ $4y = -3x + 10$	**B1** **M1 B1** **A1**
		04
7	(a). $g^{-1}(x) = \frac{2x+1}{3}$ $X = \frac{2Y+1}{3}$ $3X = 2Y + 1$ $3X - 1 = 2Y$ $\frac{3X-1}{2} = Y$ $\frac{3x-1}{2} = g(x)$ (b). $g(5) = \frac{3 \times 5 - 1}{2} = 7$	**B1** **B1** **M1 A1**
		04
8	map : ground $12\ \text{cm}^2 \ : \ 75\ \text{km}^2$ $\frac{12}{12}\ \text{cm}^2 \ : \ \frac{75}{12}\ \text{km}^2$ $1\ \text{cm}^2 \ : \ 6.25\ \text{km}^2$ $\sqrt{1}\ \text{cm} \ : \ \sqrt{6.25}\ \text{km}$	**B1**

	1 cm : 2.5 km 1 cm : 2.5 × 100,000 cm 1 cm : 250,000 cm 1 : 250,000 $\Rightarrow n = 250{,}000$	**B1** **M1** **A1**
		04
9	$\begin{array}{r\|l\|l} & 24:00 & \text{hours} \\ - & \underline{20:30} & \underline{\text{hours}} \\ & 03:30 & \text{hours} \end{array}$ $\begin{array}{r\|l\|l} & 03:30 & \text{hours} \\ + & \underline{03:15} & \underline{\text{hours}} \\ & 06:45 & \text{hours} \end{array}$ Total time taken, $t = 6 + \frac{45}{60} = 6.75$ hours Distance $= 80 \times 6.75 = 540$ km	**B1** **B1** **M1 A1**
		04
10	 By similarity, for ΔTMA and ΔCQT, $\frac{h_1}{h} = \frac{3}{4.5}, \quad \Rightarrow h_1 = \frac{3}{4.5}h = \frac{2}{3}h$ Volume of cut off cone : Volume of frustrum $\frac{1}{3}\pi r^2 h_1 \; : \; \frac{1}{3}\pi h(r^2 + rR + R^2)$ $r^2 \times \frac{2}{3}h \; : \; h(r^2 + rR + R^2)$ $\frac{2r^2}{3} \; : \; (r^2 + rR + R^2)$ $\frac{2 \times 3^2}{3} \; : \; (3^2 + 3 \times 7.5 + 7.5^2)$	**B1** **B1**

	$6 : \frac{207}{4}$ $24 : 207$	M1 A1
		04
11	(a). $\left(2\frac{1}{2}+1\frac{1}{3}\times 2\frac{1}{4}\right)=\frac{5}{2}+\frac{4}{3}\times\frac{9}{4}$ $=\frac{5}{2}+3=\frac{5+6}{2}=\frac{11}{2}$ $\left(\frac{5}{6}+1\frac{2}{3}\right)=\frac{5}{6}+\frac{5}{3}=\frac{5+10}{6}=\frac{15}{6}=\frac{5}{2}$ $\therefore \frac{2\frac{1}{2}+1\frac{1}{3}\times 2\frac{1}{4}}{\frac{5}{6}+1\frac{2}{3}}=\frac{11}{2}\div\frac{5}{2}=\frac{11}{2}\times\frac{2}{5}=\frac{11}{5}=2\frac{1}{5}$ (b). (i). $c=k_1+k_2d$ When $d=10, c=$ shs 45,000 $45000=k_1+10k_2 \quad\longrightarrow (1)$ When $d=35, c=$ shs 82,500 $82500=k_1+35k_2 \quad\longrightarrow (2)$ $(2)-(1)$ gives, $37500=25k_2$ $k_2=1500$ From equation (2), $k_1=45000-10\times 1500=30000$ $\therefore c=30000+1500d$ (ii). $c=30000+1500\times 50=$ shs 105,000 (iii). $72000=30000+1500d$ $72000-30000=1500d$ $42000=1500d$ $28=d$ $d=28$	**B1** **B1** **B1** **M1 A1** **B1** **B1** **B1** **B1** **M1 A1** **M1** **A1**
		12
12	(a). $A=P\left(1-\frac{R}{100}\right)^n=3500000\times\left(1-\frac{5}{100}\right)^2$ $=3500000\times 0.95^2=$ shs 3,158,750 (b).	**M1** **M1 A1**

Let taxable income be y.

Taxable income	Tax rate (%)	Tax
1st 150,000	0	0
Next 250,000	5	12,500
Next 300,000	8	24,000
Last $(y-700{,}000)$	12	$\frac{3}{25}(y-700{,}000)$

B1

B1

$$\frac{3}{25}(y-700{,}000)=58100-(12{,}500+24{,}000)$$

M1

$$\frac{3}{25}(y-700{,}000)=58100-36500$$

$$\frac{3}{25}(y-700{,}000)=21600$$

M1

$$3y-2100000=540000$$

A1

$$3y=2640000$$

$$y=880{,}000$$

(i).

M1 A1

$$\text{Gross pay}=\text{taxable income}+\text{allowances}$$

$$=880{,}000+50{,}000=930{,}000$$

M1 A1

(ii).

$$\text{Net pay}=930{,}000-58{,}100=871{,}900$$

12

13 (a).

$$\log_{10}\left(\frac{\sqrt{x}}{y}\right)=\frac{1}{2}\log_{10}x-\log_{10}y$$

B1

$$=\frac{1}{2}\times 1.3586-2.1428$$

M1

$$=0.6793-2.1428$$

$$\begin{array}{r|r|} & 0.6793 \\ - & 2.1428 \\ \hline & \bar{2}.5365 \end{array}$$

M1 A1

$$\therefore \log_{10}\left(\frac{\sqrt{x}}{y}\right)=\bar{2}.5365$$

(b). (i).

For the first part of the journey,

$$\text{Speed}=\frac{\text{Distance}}{\text{Time}}=\frac{20}{2}=10\text{ km h}^{-1}$$

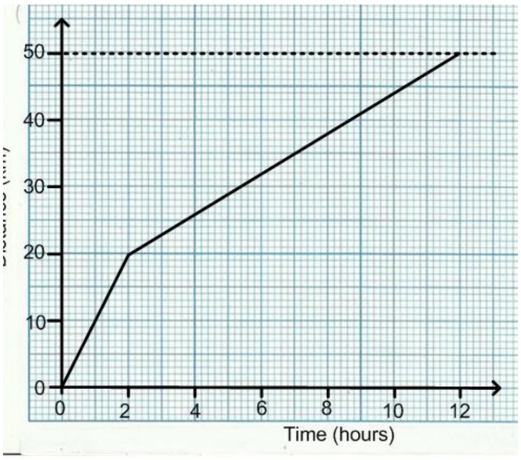
M1 A1

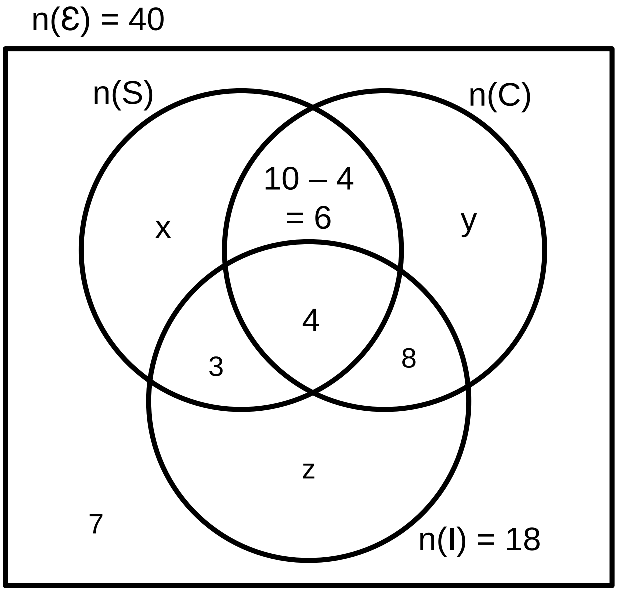
(ii).

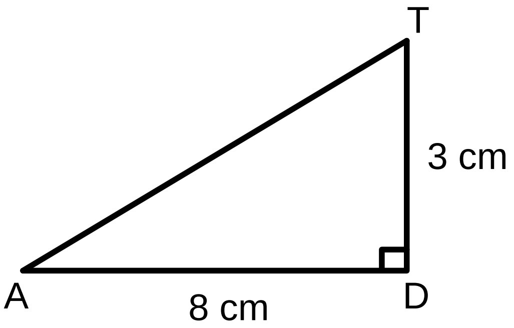
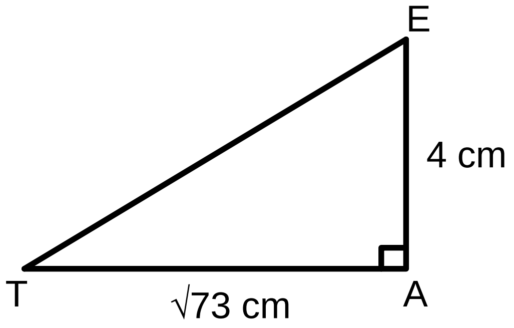
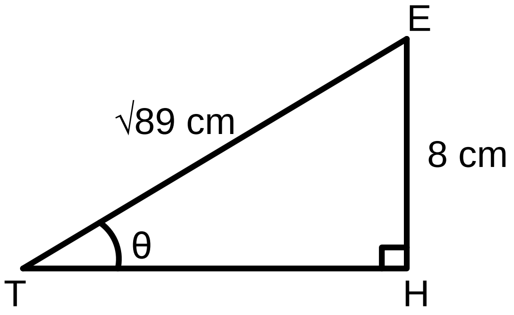
Let x be the total distance for the whole journey.

$$\frac{2}{5}\text{ of }x=\overline{AB}$$

$$\frac{2}{5}x=20,\qquad \Longrightarrow x=50\text{ km}$$

	For the second part of the journey, $$\text{Distance} = 50 - 20 = 30\text{ km}$$ $$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{30}{3} = 10\text{ hours}$$ (iii). $$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{50}{2+10} = 4.1667\text{ km h}^{-1}$$ (c). 	**M1** **A1** **M1 A1** **B1 B1**
		12
14	(a). (i). $$\overrightarrow{XA}:\overrightarrow{AT} = 1:1$$ $$\overrightarrow{XA} = \frac{1}{2}\overrightarrow{XT}$$ $$\overrightarrow{XT} = 2\overrightarrow{XA} = 2\underset{\sim}{\boldsymbol{t}}$$ (ii). $$\overrightarrow{BT} = \overrightarrow{BX} + \overrightarrow{XT}$$ $$= -\underset{\sim}{\boldsymbol{r}} + 2\underset{\sim}{\boldsymbol{t}}$$ $$= 2\underset{\sim}{\boldsymbol{t}} - \underset{\sim}{\boldsymbol{r}}$$ (iii). $$\overrightarrow{BR} = 3\overrightarrow{XB} = 3\underset{\sim}{\boldsymbol{r}}$$ $$\overrightarrow{TR} = \overrightarrow{TX} + \overrightarrow{XB} + \overrightarrow{BR}$$ $$= -2\underset{\sim}{\boldsymbol{t}} + \underset{\sim}{\boldsymbol{r}} + 3\underset{\sim}{\boldsymbol{r}}$$ $$= 4\underset{\sim}{\boldsymbol{r}} - 2\underset{\sim}{\boldsymbol{t}}$$ (b). $$\overrightarrow{XA} = \overrightarrow{AT}, \quad \Rightarrow \underset{\sim}{\boldsymbol{t}} = \begin{pmatrix}4\\4\end{pmatrix}$$ $$\overrightarrow{XB} = \begin{pmatrix}2\\-1\end{pmatrix}, \quad \Rightarrow \underset{\sim}{\boldsymbol{r}} = \begin{pmatrix}2\\-1\end{pmatrix}$$ $$\overrightarrow{TR} = 4\underset{\sim}{\boldsymbol{r}} - 2\underset{\sim}{\boldsymbol{t}} = 4\begin{pmatrix}2\\-1\end{pmatrix} - 2\begin{pmatrix}4\\4\end{pmatrix} = \begin{pmatrix}8\\-4\end{pmatrix} - \begin{pmatrix}8\\8\end{pmatrix} = \begin{pmatrix}0\\-12\end{pmatrix}$$ **Alternatively:** $$\overrightarrow{XT} = 2\overrightarrow{AT} = 2\begin{pmatrix}4\\4\end{pmatrix} = \begin{pmatrix}8\\8\end{pmatrix}$$	**B1** **B1** **B1** **B1** **B1** **B1** **B1** **B1** **M1 B1**

	$\overrightarrow{BR} = 3\overrightarrow{XB} = 3\begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}6\\-3\end{pmatrix}$ $\overrightarrow{TR} = \overrightarrow{TX} + \overrightarrow{XB} + \overrightarrow{BR} = \begin{pmatrix}-8\\-8\end{pmatrix} + \begin{pmatrix}2\\-1\end{pmatrix} + \begin{pmatrix}6\\-3\end{pmatrix} = \begin{pmatrix}0\\-12\end{pmatrix}$ $\lvert\overrightarrow{TR}\rvert = \sqrt{0^2 + (-12)^2} = 12$	**M1 A1**
		12
15	(a).  $n(S) = n(C)$ $x + 3 + 4 + 6 = y + 6 + 4 + 8$ $x + 13 = y + 18$ $y = x - 5$ $n(\varepsilon) = 18 + 7 + x + 6 + y$ $40 = 31 + x + (x - 5)$ $14 = 2x$ $x = 7$ $\Rightarrow y = 7 - 5 = 2$ $n(I) = z + 3 + 4 + 8 = 18, \quad \Rightarrow z = 18 - 15 = 3$ (b). $n(\text{S only}) = 7$ students (c). $n(\text{not in C}) = x + z + 3 + 7 = 7 + 3 + 10 = 20$ students $P(\text{not in C}) = \frac{20}{40} = 0.5$	**B3** **M1** **B1** **M1** **B1** **B1** **A1** **B1** **M1 A1**

		12
16	(a). (i).  $\overline{AT}^2 = \overline{AD}^2 + \overline{DT}^2$ $\overline{AT}^2 = 8^2 + 3^2$ $\overline{AT}^2 = 64 + 9$ $\overline{AT} = \sqrt{73} = 8.544 \text{ cm}$ (ii).  $\overline{TE}^2 = \overline{AT}^2 + \overline{AE}^2$ $\overline{TE}^2 = 73 + 4^2$ $\overline{TE}^2 = 73 + 16$ $\overline{TE} = \sqrt{89} = 9.434 \text{ cm}$ (b).  $\sin\theta = \frac{8}{\sqrt{89}}$, $\Longrightarrow \theta = 58.0^\circ$ (c).	**M1** **M1 A1** **M1** **M1 A1** **B1** **M1 A1**

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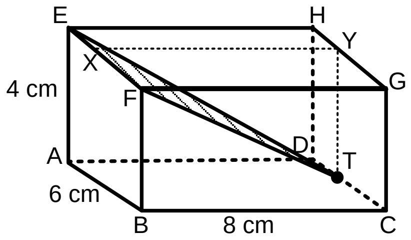

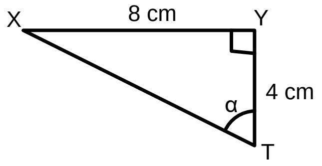

$$\tan\alpha = \frac{8}{4}\ , \qquad \Longrightarrow \alpha = 63.435°$$
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B1

M1 A1
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<td>**12**</td>
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<td>17</td>
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(a). (i).

Line T passes through the points $(-2,0)$ and $(0,4)$

$$\text{Gradient,} \qquad m = \frac{0-4}{-2-0} = 2$$

$$y \text{ intercept,} \qquad c = 4$$

Using $y = mx + c$,

$$y = 2x + 4$$

(ii).

$$\text{for } y + 5 = 3x, \qquad m_1 = 3$$

$$m_1 m_2 = -1$$

$$3m_2 = -1, \qquad \Longrightarrow m_2 = -\frac{1}{3}$$

Using $y = mx + c$ and $(2,1)$,

$$1 = -\frac{1}{3} \times 2 + c, \qquad \Longrightarrow c = \frac{5}{3}$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

(b). At the point of intersection,

$$2x + 4 = -\frac{1}{3}x + \frac{5}{3}$$

$$6x + 12 = -x + 5$$

$$6x + x = 5 - 12$$

$$7x = -7$$

$$x = -1$$

$$\text{when } x = -1, \qquad y = 2 \times (-1) + 4 = -2 + 4 = 2$$

The point of intersection is $(-1,2)$.

(c).

$$y = -\frac{1}{3}x + \frac{5}{3}$$

When $y = 0$,

$$0 = -\frac{1}{3}x + \frac{5}{3}$$

$$\frac{1}{3}x = \frac{5}{3}$$

$$x = 5$$

The x −intercept of line R is 5.

(d).
</td>
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B1

A1

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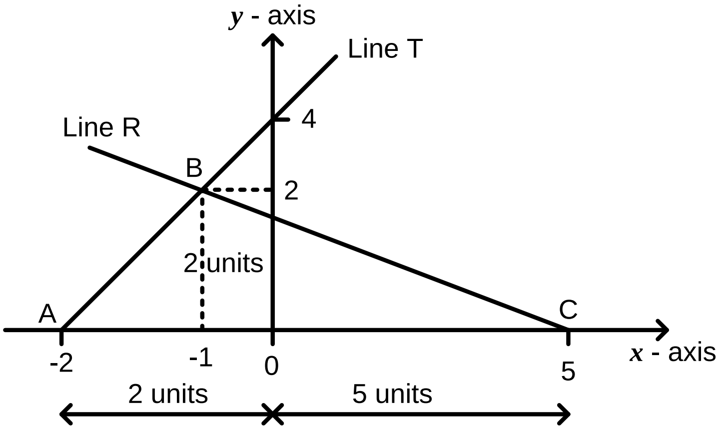
<tr><td></td><td>

$$\text{Required area} = \text{area ABC}$$
$$= \frac{1}{2}bh = \frac{1}{2} \times 7 \times 2 = 7 \text{ sq units}$$</td><td>**B1**
M1 A1</td></tr>
<tr><td></td><td></td><td>**12**</td></tr>
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*****END****