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PURE
MATHEMATICS
Paper 1
July/August 2024
3hours

*KAMSSA* JOINT MOCK EXAMINATIONS

**Uganda Advanced Certificate of Education**

PURE MATHEMATICS

**Paper 1**
3hours

## Instructions to candidates:

- *Answer **All** the **eight** questions in section A and **five** questions from section **B**.*
- *Any additional question **(s)** answered will not be marked.*
- ***All** working **must be** shown clearly.*
- *Begin each answer on a fresh page.*
- *Graph paper is provided.*

*Silent non-programmeble, scientific calculators and mathematical tables with atleast of formaulae may be used.*

- *State the degree of accuracy at the end of each answer given. If a calculator or a mathematical table is used , indicate **Cal** for calculator or **Tab** for mathematical tables.*

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## SECTION A

*Answer **all** the questions in this section*

1) Given $x + 2x^3 + 4x^5 + 8x^7 + \cdots = \frac{3}{7}$

i) Find the value of $x$. (03 marks)

ii) Find the 20th term. (02 marks)

2) The directional vectors $\boldsymbol{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -2\boldsymbol{i} + \boldsymbol{j} + p\boldsymbol{k}$ and $\boldsymbol{c} = 9\boldsymbol{i} + 9\boldsymbol{j}$ are such that $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}$. Find the

i) Value of the scalar $\boldsymbol{p}$. (02 marks)

ii) Angle between $\boldsymbol{b}$ and $\boldsymbol{c}$. (03 marks)

3) If $y = 2\sin\theta$ and $x = \cos 2\theta$, show that $\frac{d^2y}{dx^2} = \frac{-1}{y^3}$ (05 marks)

4) Given $A(7,3)$ and $B(-5,0)$, the line through points $A$ and $B$ meets the line $3x + 5y = 19$ at point $C$. What are the coordinates of $C$. (05 marks)

5) The roots of the equation $x^2 + 6x + 3 = 0$ are $a$ and $\beta$. Find the equation whose roots are $\frac{a+\beta}{a}$ *and* $\frac{a+\beta}{\beta}$ (05 marks)

6) Show that the area bounded by the curve $y = 10x - x^2$ and the line $y = 4x$ is 36 square units. (05 marks)

7) Solve $\tan^{-1} x + \tan^{-1}(x-1) = \tan^{-1} 3$ (05 marks)

8) Find the value of $x$ and $y$ such that

$3\log_8 xy = 4\log_2 x$ *and* $\log_2 y = 1 + \log_2 x$ (05 marks)

## SECTION B

*Answer any **five** questions from this section. **All** questions carry equal marks*

9 a) The binomial expansion of $(1 + kx)^n$ is $1 - 6x + 30x^2 + \cdots$

find the value of $k$ and $n$ (05 marks)

b) Expand $\sqrt{1 + 8x}$ up to the term in $x^3$. Using $x = \frac{1}{100}$, show that $\sqrt{3}$=1.73205

(07 marks)

10 (a) integrate $e^x \sin 2x$ with respect to $x$. (06 marks)

(b) Using $t$ =tan$x$ or otherwise, evaluate $\int_0^{\frac{\pi}{3}} \frac{1}{1+8\cos^2 x}\,dx$. = 0.17453' (06 marks) $\frac{1}{2}$



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(11) The lines $x = \frac{y-1}{2} = \frac{z-21}{-3}$ and $\boldsymbol{r} = (12 + 3\lambda)\boldsymbol{i} + (7 - 3\lambda)\boldsymbol{j} + (1 - \lambda)\boldsymbol{k}$ lie in the same plane.

(a) Show that the lines intersect and calculate the angle between them (06 marks)

(b) Find the equation of the plane where both lines lie. (06 marks)

12)(a) Show that. $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$.

Hence solve $\frac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \leq \theta \leq \pi$. (06 marks)

b) (i) Express $7\cos x - 24\sin x$ in the form $R\cos(x + a)$. (04 marks)

(ii) Write down the maximum and minimum value of the function

$f(x) = 12 + 7\cos x - 24\sin x$. (02 marks)

13)(a) Given $z = -1 + i\sqrt{3}$, find the value of the real number $p$ such that

$Arg(z^2 + Pz) = \frac{5\pi}{6}$ (05 marks)

(b) if $\frac{|w-1|}{|w+2|} < 2$ where $w = x + iy$, represent the locus of $w$ on the argand diagram.

(14) Sketch the curve of the equation $y = \frac{x^2+3}{x-1}$ . (12 marks)

(15) An ellipse has a Cartesian equation

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

The general point $P(4\cos\theta, 2\sin\theta)$ lies on the ellipse.

(a) Show that the equation of the normal to the ellipse at P is

$2x\sin\theta - y\cos\theta = 6\sin\theta\cos\theta$ (06 marks)

b) The normal to the ellipse at $P$ meets the x axis at the point $Q$ and $O$ is the origin. Show clearly that as $\theta$ varies, the maximum area of the triangle $OPQ$ is $4\frac{1}{2}$. (06 marks)

16) (a) solve $xy\frac{dy}{dx} = x^2 + y^2$, using $y = ux$. (04 marks)

(b) A liquid is pouring into a container at a constant rate of $20cm^3s^{-1}$ and is leaking out at a rate proportional to the volume, $v\ cm^3$ of the liquid already in the container.

The volume $v\ cm^3$ of the liquid in the container at any time $t$ has a differential equation

$$\frac{dv}{dt} = 20 - kv$$

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## SECTION A

1. The resultant of the forces **(pi+j), (2qi+3pj) and (i+qj) is -6i, find the values of p and q?** **(05 marks)**

2. A random variable x has a binominal distribution with n=200 and p=0.7, find $p(138 \leq x \leq 148)$. (05 marks)

3. Use trapezium rule with six ordinates to evaluate $\int_5^6 \log x \cos x dx$ correct to 3 decimal places. (05 marks)

4. The table below represents values of x and corresponding value of f(x)

| | | | | |
|---|---|---|---|---|
| x | 0.886 | 0.752 | 0.627 | 0.500 |
| f(x) | -2.35 | 0.26 | 2.58 | 3.45 |

Using interpolation or extrapolation, determine the;
(i) Value of x such that f(x) =0, (ii) f(0) (05 marks)

5. A car accelerates from rest until it gains a speed of $20ms^{-1}$ in 10 seconds. It maintains this speed for 30 seconds, it then doubles its acceleration and moved for 20 seconds
Find
(i) Its final speed. (02 marks)
(ii) The total distance covered. (03 marks)

6. MULEME offered Physics, Economics, Mathematics and Agriculture (PEM/A) and obtained aggregates B, A, C and D respectively. He wishes to offer textile engineering at Kyambogo University PEM/A is weighted as follows for textile engineering, 3 for Physics, 2 for Economics, 1 for Math and 0.5 for Agriculture. Calculate his weighted average score using the current Uganda Education grading system. (05 marks)

7. Given that $p(x) = \frac{1}{2}$, $p(y)=\frac{1}{4}$ and $p(x/y^1)=3/$[illegible]
Find
(i) $p(x \cap y)$ (03 marks)
(ii) $p(x^1/y)$ (02 marks)

8. The rectangle ABCD has AB=20cm and AD=30cm. Particles of mass 4kg,4kg,5kg and 2kg are placed at the points A, B, C and D respectively. Find the position of the Centre of gravity of the system of particles. (taking **g = 10ms⁻²**) (05 marks)

## SECTION B (60 MARKS)

**Answer any five questions from this section**

9. A random variable t has a probability density function given by

$$f(t) = \begin{cases} 2bt, 0 \leq t \leq 1 \\ b(3-t), 1 \leq t \leq 3 \\ 0, elsewhere \end{cases}$$

a. (i) Find the value of b. (03 marks)

(ii) Sketch the graph of f(t). (04 marks)

b. (i) Obtain the median. (03 marks)

(ii) Use the distribution function above to find p(0.7< t <2.4). (02 marks)

10. A basketball is released from a player's hands with a speed of $8ms^{-1}$ at an inclination of $\alpha$ degrees above the horizontal so as to land in the Centre of the basket, which is 4m horizontally from the point of release and a vertical height of 0.5m above it.
Taking the origin, O, to be the point of release, and taking g=$10ms^{-2}$, show that $\alpha$ satisfies the quadratic equation $5\tan^2\alpha - 16\tan\alpha + 7 = 0$. (Taking **g = 10m/s$^{-2}$**) (06 marks)

b. Given that the player throws the ball at the larger angle of projection.
Find

(i) $\alpha$ correct to the nearest degree. (03 marks)

(ii) The time taken from the moment of release of the ball from the player's hands until the ball lands in the basket. (03 marks)

11(a) *Show graphically that the equation* $e^x - 2x - 1 = 0$ has a root between x=1 and *x=1.5.* (03 marks)

(b) Show that Newton Rophonson's iterative formula (N.R.M) for finding the root of the above equation is $X_{n+1} = \frac{e^{x_n}(x_n-1)+1}{(e^{x_n}-2)}$. (03 marks)

c. Use the results in part (a) and (b) above to perform a dry run to find the root of the equation, by drawing a flow chart that reads the initial approximation $X_0$ of the root computes and prints the root to two decimal places. (06 marks)

12. Lengths of rolls of Binding wire have a normal distribution with mean 75m and 15% of the rolls have length less than 73
(i) Find the standard deviation of the length. (04 marks)
(ii) Find the probability that randomly selected roll has a length greater than 74. (03 marks)

b. If a random sample of 25 rolls was picked, what is the probability that the sample mean is more than 74.5m . (05 marks)

13. A light inelastic string has natural length 2m and modulus 15gN. One end of the strings is attached to fixed point and a body of mass 3kg hangs from the other end.

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b. The body is pulled down a further 10cm and then released. Use the principle of conservation of energy to determine the speed of the body as it passes through the equilibrium position. (06 marks)

14. Two numbers A and B are approximated by a and b with errors $e_1$ and $e_2$ respectively.

a. Show that the maximum relative error in $a^2\, b$ is given by $2\left|\frac{e_1}{a}\right|+\left|\frac{e_2}{b}\right|$ (05 marks)

b. If a=2.32 and b=2.031 are each rounded off the given number of decimal places. Calculate

(i) Percentage error in $\frac{a}{b}$

(ii) Limits with in which $\frac{a}{b}$ is expected to lie. Give your answer correct to 3 decimal places. (07 marks)

15. The table below show the weights (kg) of 60 students of senior six in a certain school in Uganda.

| Weights (kg) | Frequency density |
|---|---|
| 10 - < 15 | 2 |
| 15 - < 20 | 1 |
| 20 - < 30 | 2 |
| 30 - < 35 | 3 |
| 35 - < 50 | 0.2 |
| 50 - < 60 | 0.6 |
| 60 - < 70 | 0.1 |

a. Calculate the modal weight. (04 marks)

b. Plot an O' give for the above data and use it to estimate the;

*(i) Median.*

*(ii) The middle 10% weight range.* (08 marks)

16. A particle is projected from point **O** at time t=0 and performs SHM with **O** as the Centre of Oscillation. The motion is of amplitude 20cm and time period 4seconds. Find

a. The speed of projection. (04 marks)

b. the speed of the particle when t =1.5seconds. (04 marks)

c. The value of t when the particle is first at a point 10cm from **O**. (04 marks)

**END**

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