

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

(**SMATA**)



O'LEVEL MATHEMATICS SEMINAR

**THE 7TH ANNUAL GRAND MATHS ZONAL SEMINAR
ORGANIZED AT**

ST. JOSEPH OF NAZARETH HIGH SCHOOL

Saturday 8th July, 2023

PAPER ONE

- Statistics
- Matrices
- Transformations
- Linear Programming
- Quadratics
- Inequalities
- Construction
- Bearings
- Trigonometry
- Probability
- Operations
- Fractions
- Factorisation
- Equations & Formulae
- Circle Properties

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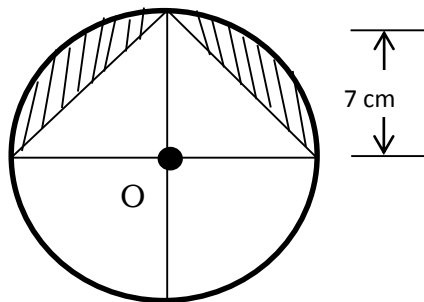
PAPER TWO

- Set theory
- Vectors
- Business Math (Taxation)
- Kinematics
- Mensuration
- 3 Dimensions
- Mapping
- Functions
- Coordinate Geometry
- Ratios & Proportions
- Variations
- Decimals
- Surds
- Indices
- Logarithms



GENERAL QUESTIONS

1. Without using tables or calculators determine the value of $\sin A - \cos A$; if $\tan A = \frac{8}{15}$ in the range $90^\circ < A < 270^\circ$
2. A straight line passes through points A(5,4) and B(3,8). Find the equation of a line which is a perpendicular bisector of line AB.
3. Simplify $(2x - 1)(2x + 1) - (7x - 4)$ and hence factorize the expression.
4. The figure below shows a circle with centre, O.



Find the area of the shaded region.

5. Given that $a * b = 2a + b - 3$
 - i). Find the value of $-1 * 1$
 - ii). Find the value of n if $3 * n = n * 2$
6. Mukiibi is four times as old as his brother Mutyaba. Five years ago the sum of their ages was 25. What will be the product of their ages ten years from now?
7. If $y = a \left(\sqrt[4]{\frac{x^2 - n}{m}} \right)$ Make x the subject of the formula.
8. A flag pole casts a shadow 28 feet long. A person standing just at the foot of the pole casts a shadow eight feet long. If the person is six feet tall, how tall is the flag pole.

9. Mr. Kibuuka spends $\frac{1}{4}$ of his salary on school fees. He spends $\frac{2}{3}$ of the remainder on food and a fifth of what is left on transport. He saves the balance. If he saved Sh. 3400 in a certain month, what was his salary?
10. If P is (-7, -5) and Q is (-9, -1) and M is a Mid-point of PQ; Find the
- Position vector of M
 - $|OM|$

MATRICES

- 11 a) Find a matrix A such that its inverse $A^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$
- b) Show the $A \bullet A^{-1} = I$ where I is an identity matrix of order 2x2.
- c). Find the value of x for which the matrix $\begin{pmatrix} x & 1 \\ 3 & x-2 \end{pmatrix}$ is singular.
12. In one of the sports analysis on the international scene, it was found out that Kiprotich had 5 gold medals, 6 silver and 4 bronze. His counterpart Kiplagat had 3 gold medals, 6 silver and 8 bronze.
- The weights of the medals was as below by then:
- A gold medal weighed \$800, Silver weighed \$600 while as a bronze weighted \$400.
- From the above information, write down
 - a 2x3 matrix for the medals
 - A column matrix for the weights.
 - Use the matrices in (a)i) and (ii) above, to calculate the total amount for each person.

INDICES, SURDS AND LOGARITHMS

13. Solve for the value of n in the equation;

$$4^{n+1} x \left(\frac{1}{256} \right)^{2-n} = 64$$

14. Express $\frac{3+2\sqrt{5}}{1+\sqrt{5}}$ in the form $p+q\sqrt{r}$ and write down the values of p , q and r .

15. Without using tables or calculators find the values of m if;

(a) $\log_2(m-2) + \log_2 2m - \log 6m = 1$

- (b) Use mathematical tables to evaluate;

$$\sqrt[3]{\frac{7.25 \times 86.7}{0.00813}}$$

QUADRATICS

- 16 a). Draw a graph of $y = 2x^2 - x - 3$ for $-3 \leq x \leq 3$.

Use scales of 2cm to represent 1 unit on the x-axis and 1cm to represent 1 unit on the y-axis.

- b) On the same axes, draw the line $y = x + 1$

- c) i). Use the two graphs to solve the equation $x^2 - x - 2 = 0$
ii). Find the minimum value of the function $y = 2x^2 - x - 3$

SET THEORY AND LOGIC

17. In a school of 100 teachers, 16 like Football (F) and Netball (N); 6 like Volleyball (V) and Netball (N); 8 like Football (F) and Volleyball (V). 26 like more than one game. 30 like Football. 40 like Netball. 14 like only volley ball.

- a). Use a Venn diagram to represent the above information.

- b). Find the number of teachers' who like

- i). all the three games
ii). none of the games
iii). volley ball.

- c). What is the probability that a teacher chosen at random likes;
- atleast one game
 - atmost one game

18. Given that sets; A, B and C contains items such that;

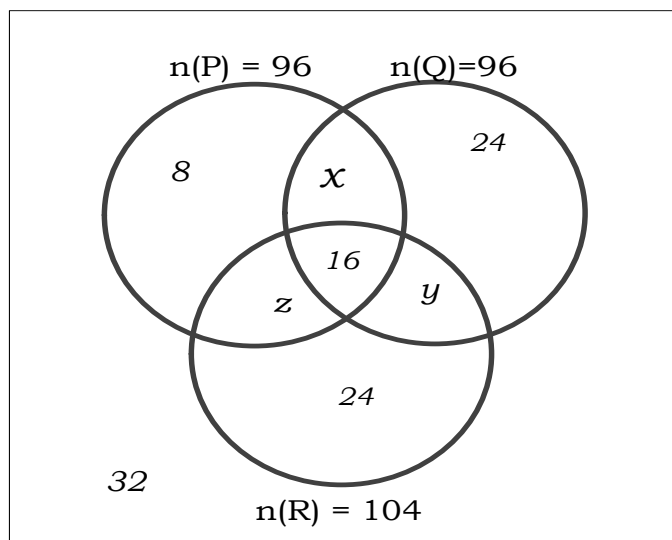
$$\begin{array}{ll} n(A) = 22 & n(A^1 n B^1 n C^1) = 14 \\ n(B) = 38 & n(A n C) = 12 \\ n(C) = 32 & n(B n C) = 18 \\ n(A n B) = 14 & n(A n B n C) = x \end{array}$$

If $n(A^1 n B^1 n C)$ is twice $n(A n B^1 n C^1)$

Represent the above information on a Venn diagram.

- Find the;
 - value of x
 - Universal set
- Find the probability that if an item is selected at random, it belongs to;
 - only two of the three sets
 - A or B

19. Consider the Venn diagram below with three sets; P, Q and R.



- Using the Venn diagram above find the values of ; x , y and z.
- Find the;
 - $n(P n Q)$
 - $n(P \text{ or } R)$
 - $n(P n R n Q^1)$

20. Given that; $U = \{x: x \in N, 1 \leq x < 14\}$
 $A = \{x : x \text{ is an odd number}\}$
 $B = \{x : x \text{ is Prime}\}$
 $C = \{x : x \text{ is a multiple of 3}\}$

Find the;

- i). $A \cup B$
- ii). $A \cap C$
- iii). $n(A \cap B)$

STATISTICS

21. The following marks (in percentages) were obtained by candidates in a mathematics exam.

43	70	50	35	64	62	50	53
46	62	65	83	59	54	58	64
55	54	32	59	48	54	35	48
40	58	64	40	71	74	55	70
72	48	75	45	55	40	57	55

- a). Starting with 30 - 34 as the first class and using equal class intervals of 5 marks, form a frequency distribution table for the data.
- b). State the i). number of students in the class.
ii). median class
iii). modal class.
- c). calculate the mean mark.
- d). Draw a cumulative frequency curve and use it to estimate the median mark.
- e). Find the probability that a student picked at random scored 60% and above.

PROBABILITY

22. A bag contains “n” white balls and $(n - 8)$ Black balls. The probability of picking at random a black ball is $\frac{1}{4}$,
- Find the number of balls in the bag.
 - Find the probability of picking at random a white ball.
23. A box contains 8 Red marbles and a number of Blue marbles. The probability of drawing a blue marble is $\frac{1}{5}$.
- How many Blue marbles are in the box.
 - What is the probability of picking a Red one.
24. A bag contains 3 black beads, 3 white beads and 4 Red ones. If a bead is picked at random and a second one is picked without replacement;
- Draw a probability tree diagram for the above.
 - Find the probability of picking a bead whose colour is
 - the same as the first one.
 - different from the first bead picked.
 - Calculate the probability of picking a bead which is not Red.

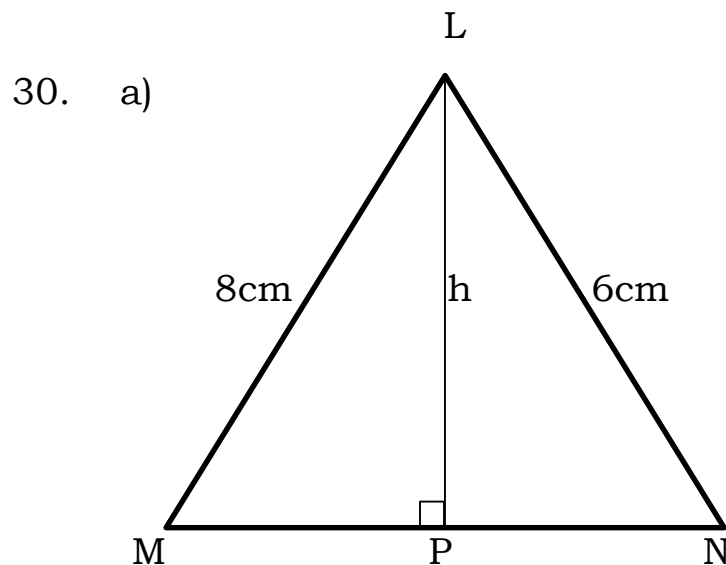
MAPPING, RELATIONS AND FUNCTIONS

25. a.) Given that $f^{-1}(x) = \sqrt{ax+b}$ and $g(x) = x - 2$ and that $f^{-1}\left(\frac{2}{3}\right) = 2$, when $f^{-1}\left(-\frac{1}{3}\right) = 1$. Find the value of a and b, hence $f(-3)$
- b). Determine values x , if $fg(x) = -\frac{1}{3}$

26. The functions h and g are defined by $h(x) = \frac{x}{x-5}$ and $g(x) = x + 4$. Find the values of x for which;
- $h(x)$ is undefined or meaningless
 - $hg(x) + gh(x) = 0$
27. If $f(x) = 3 - x$ and $fg(x) = -2x$. Find $g(x)$, hence $g(0)$
28. Given the mapping $x \rightarrow x^2 - 4$ and its range as $\{0, -3\}$.
- find the domain for the mapping.
 - Draw an arrow diagram hence state the type of mapping.

TRIGONOMETRY

29. Calculate the size of the smallest angle and the area of an obtuse angled triangle whose sides are 8cm, 6cm and 10cm.



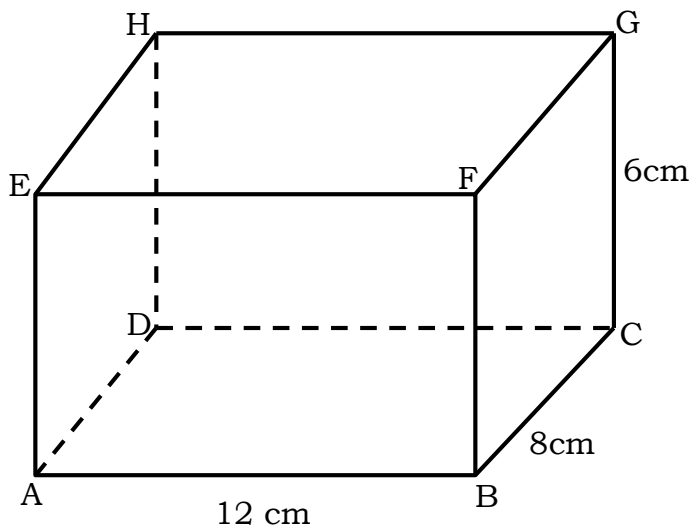
In the figure LP is the altitude of triangle LMN, angle MLN = 90° . Find;

- The value of h
 - The area of triangle LMN.
- b). Building A is 40 metres high. The angle of depression of the top of building B from the top A is 26° . Given A and B are 10m apart, find the height of building B.

(Give your answers to 2 decimal places)

THREE DIMENSIONS (3D SHAPES)

31. The diagram below shows a cuboid ABCDEFGH in which $AB = 12\text{cm}$, $BC = 8\text{cm}$ and $CG = 6\text{cm}$.



a) Calculate the length to 4 significant figure

- i) AC ii). AG

b) Determine the angle between;

- i) line AG and the base $ABCD$
ii) plane $DCFE$ and plane $EFGH$

INEQUALITIES, REGIONS & LINEAR PROGRAMMING

32. a) Solve the inequality below and represent the solution on a number line $x^2 - 3x - 4 \leq 0$

b). Omega was given shs. 160 to buy files and exercise books. He had to buy at least four files and more than 5 exercise books, and also the ratio of files to exercise books was more than two to three. How many files and exercise books did he buy if a file costs shs 20 and an exercise book cost shs. 8.

33 a).By shading the unwanted regions, show on a graph the region satisfying the inequalities below:

i). $x \geq 0$

ii). $y \geq 0$

iii). $x + y \leq 6$

iv). $x + 2y \leq 8$

b).Use your graph to find the values of x and y which gives the maximum value for both $x + y$ and $x + 2y$.

34. Mr. Kiyangi has organized an agriculture study tour for 234 students of JONAS. Two types of vehicles are available for hire, they are ISUZU buses and FUSO Mini-Buses. The capacity of each ISUZU bus is 65 passengers while that of the FUSO Mini-bus is 26 passengers. The number of FUSO Mini-buses will be more than the number of ISUZU buses. The number of FUSO Mini-buses will be less than six.

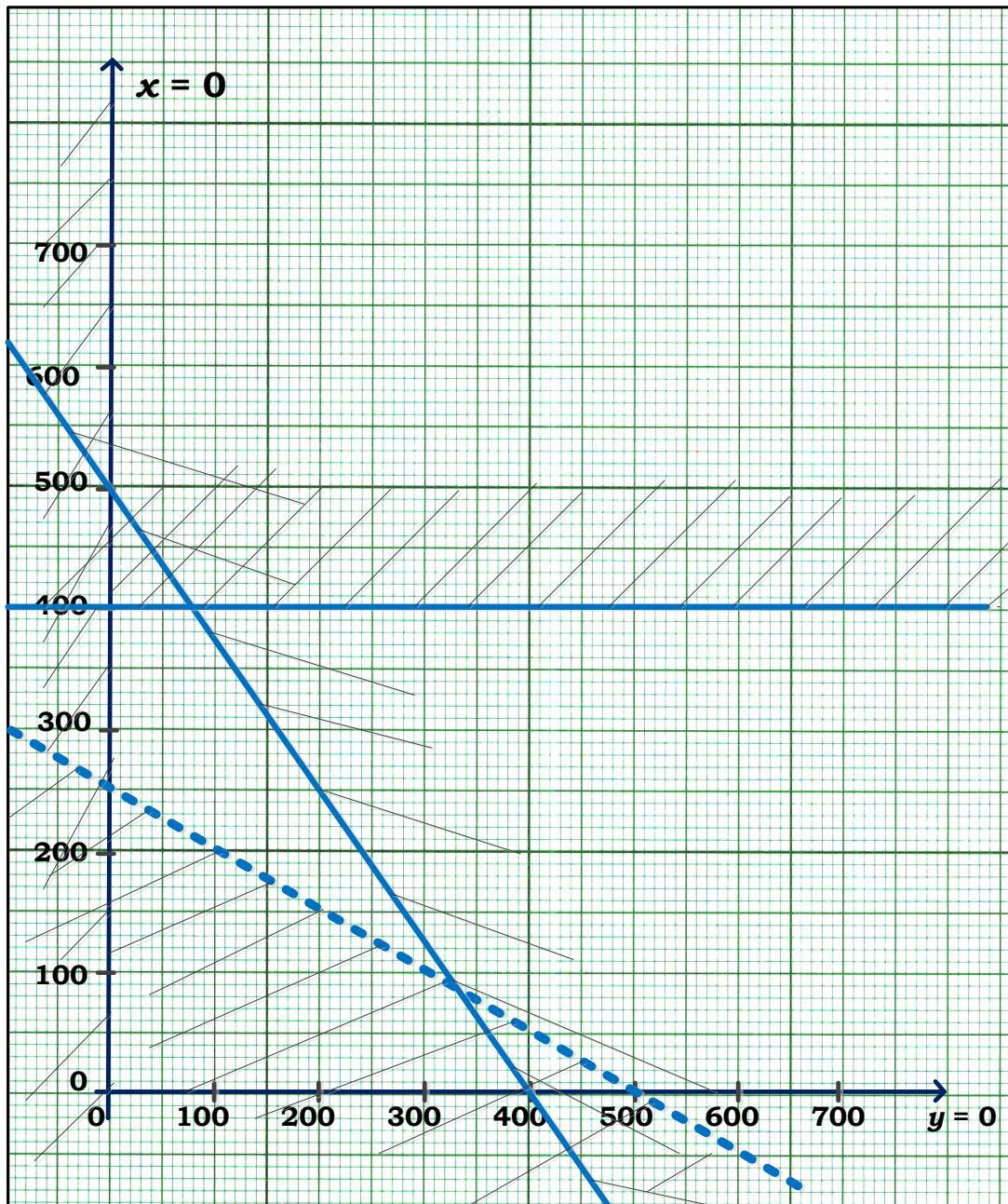
The cost of hiring each ISUZU bus is shs. 100,000 while that of the FUSO mini-bus is shs. 70,000. There is only shs 700,000 available for transporting the students.

a) If X represents the number of ISUZU- bus to be ordered and y the number of FUSO min-buses to be ordered, write down ix inequalities for the given information.

b) Using a scale of 2cm to represent 1unit on both axes, represent the inequalities on a graph and shade the unwanted regions.

c) Use your graph to find the number of ISUZU bases and FUSO min-buses to be ordered so that all the students are transported at a minimum cost. Hence find the amount of money saved.

35. Determine the inequalities represented by the unwanted region in the diagram below:



BUSINESS ARITHMETIC

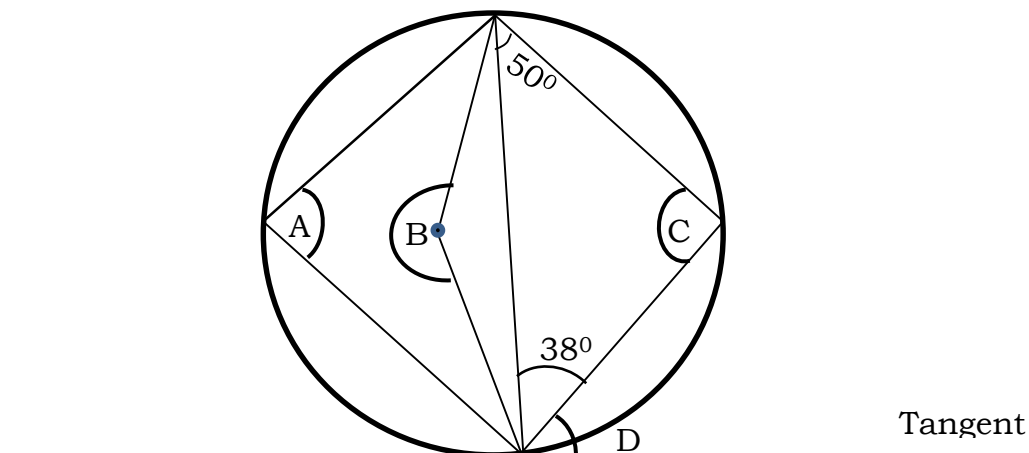
36. By selling an article at shs 3000 a seller makes a profit of 25%. How many articles must he sell in order to make a profit of shs 10,800.
37. In her Vacation Benitah hopes to work with her Dad in a restaurant. If she is promised a fixed pay of Shs. 7,000/= per day which is equivalent to the cost of a plate of food in their restaurant and an additional commission of 5% on each plate of food she sells to a customer. If she serves 10 customers each day. Determine Benitah's income in a week.
38. a).The cash price of a gas cooker is Shs. 200,000. A customer opts to buy it on hire purchase terms by paying a deposit of shs. 50,000. A simple interest of 10% p.a is charged on the balance and the customer is require to repay by 24 equal monthly installments. Calculate the amount of each installment hence find the hire purchase price.
- b). The table below shows part of the tax table structure for monthly income of employees for the year 2021.

Taxable income (shs)	Tax rate (%)
01 – 90,000	10
90,001 – 200,000	15
200,001 and above	25

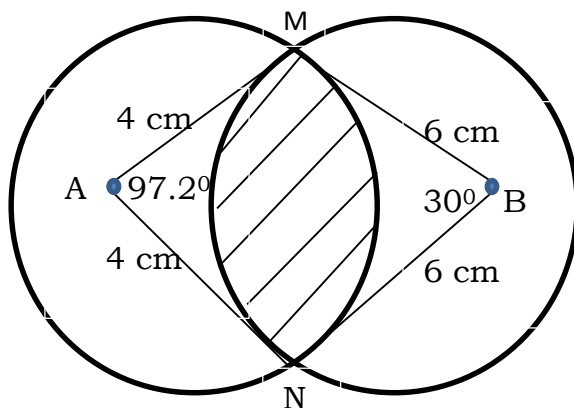
- i). If Mr. Masembe in that year paid a tax of shs 29,500 per month find his monthly income.
- ii). Given that 1 \$ = 4000 UG SHS. What is Mr. Masembe's income in dollars.

CIRCLE PROPERTIES

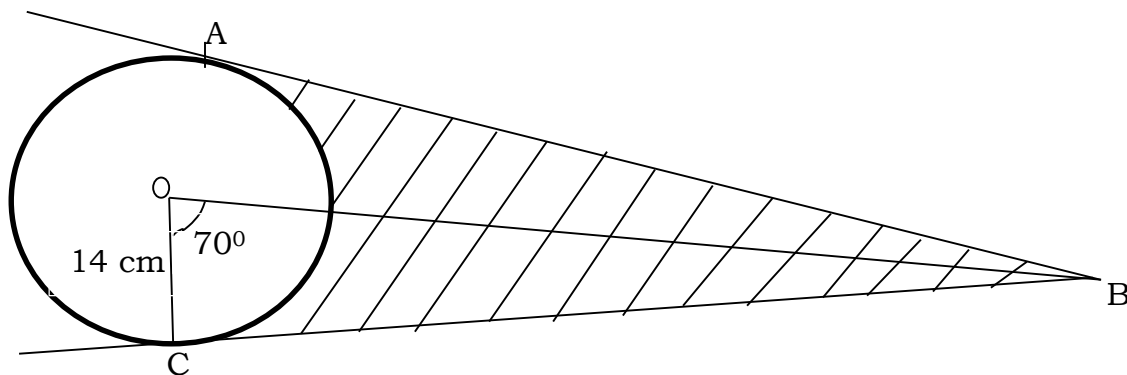
39. Find $\angle A$, $\angle B$, $\angle C$ and $\angle D$ in the diagram below where TP is the tangent to the circle at point P.



40. Find the area between the intersecting circles (Shaded region)
Take $\log_{10} \pi = 0.4972$.

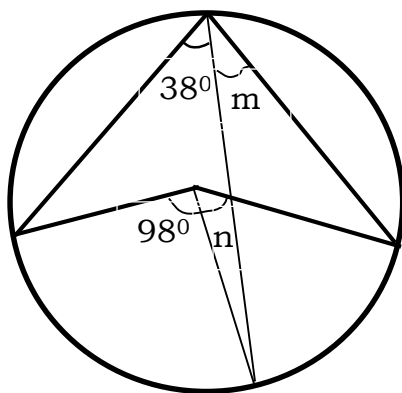


41. In the figure below BC and BA are tangent to the circle centre O. If $OC = 14$ cm and angle $COB = 70^\circ$.

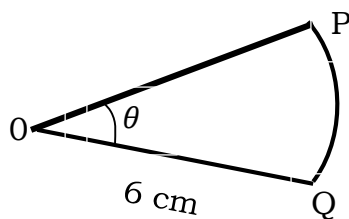


Calculate the area of the shaded region (take $\pi = \frac{22}{7}$)

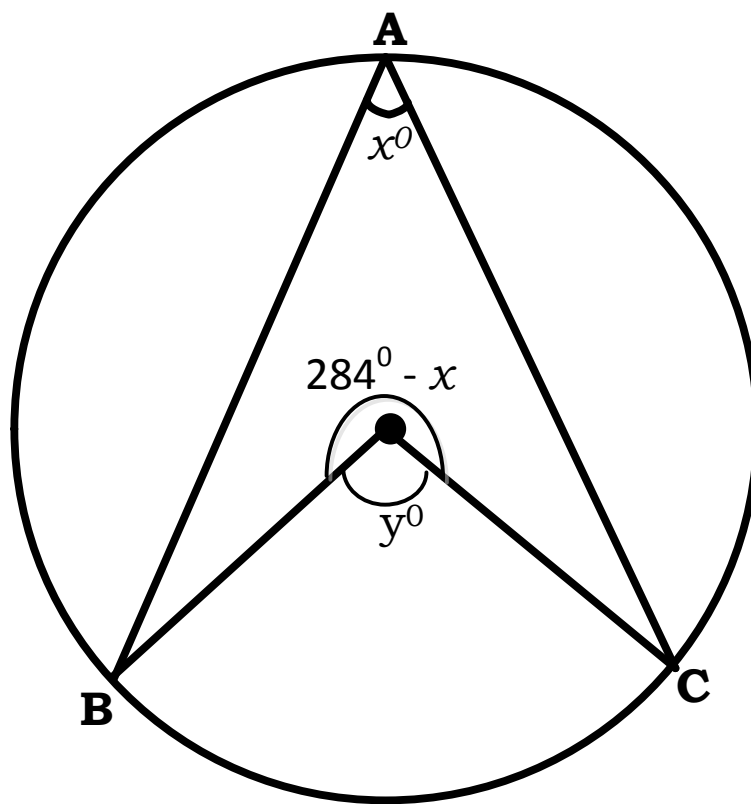
42.a). Find the angle m , then use your answer to work out angle n .



b). The area of the sector is 24.2 cm^2 . Find the value of θ .
(Take $\pi = \frac{22}{7}$).

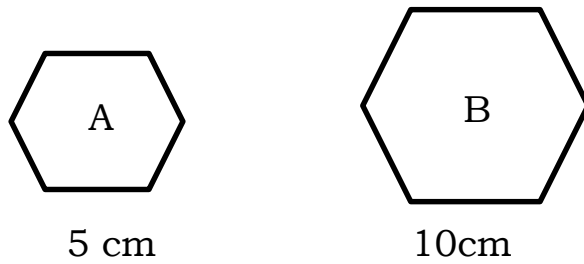


c). Find the size of angle X and Y in the diagram below.

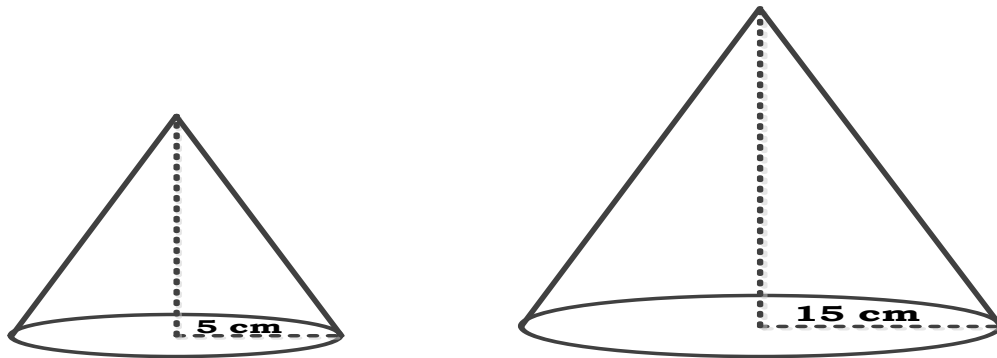


SIMILARITIES

43. These two figures are similar. The area of shape A is 60cm^2 . Find the area of shape B.



44. The volume of small cone A is 400cm^3 .



Find the volume of the big cone.

45. Find the value of x .
-
- Diagram showing two similar triangles. The top triangle has a base of 3 cm and a side of 4 cm. The bottom triangle has a base of 12 cm and a side of x . The triangles are similar.

46. Two similar cones are such that the volume of cone A is 32cm^3 . The volume of cone B is 108cm^3 , Given that the height of cone A is 10 cm. Find the height of the cone B.

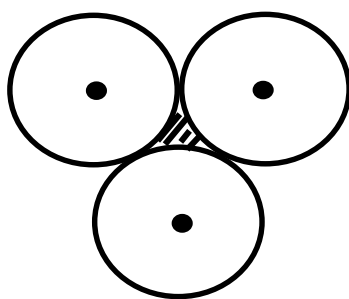
MENSURATION

47. A water tank has a square base of length 48cm and height 35 cm. It is filled with water to a height of 25cm.

When a solid sphere is placed in the tank, the water level rises by π cm. Assuming the sphere fully submerged, determine the radius of the sphere.

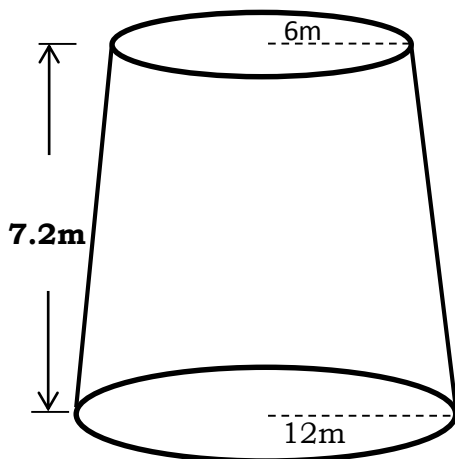
48. A cow is tied on the corner of a rectangular field of size 30m x 20m by a 14m long rope. Determine the area of the region that the cow can graze on this field.

49. Three circles of equal radius 8 cm touch each other as shown.



Find the area of the shaded

50. A school water tank is in the shape of a frustrum of a cone. The height of the tank is 7.2m and the top and bottom radii are 6m and 12m respectively.



- a). Calculate the area of the curved surface of the tank, correct to 2 decimal places.
- b). Find the capacity of the tank, in litres correct to the nearest litre.

- c). On a certain rain day, the tank was filled with water. If the school has 500 students and each student uses an average of 40 litres of water per day, determine the number of days that the students would use the water.

BEARING

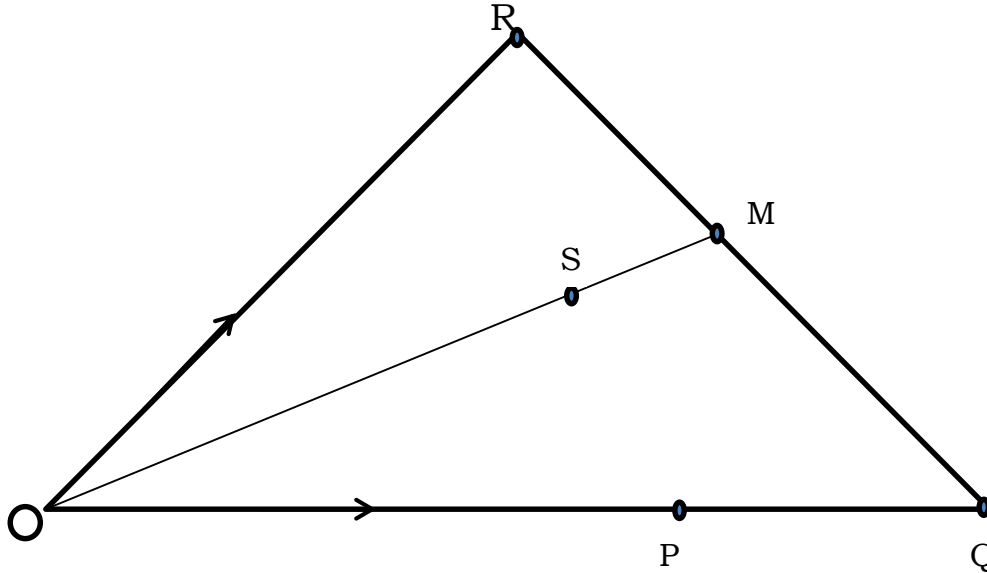
51. Town B is 240 Km on a bearing of 240° from town A Town C is 180 km on a bearing of 330° from A.
- a).i). Show the positions of A, B and C on a sketch.
- ii). Find the bearing of C from B by calculation from the sketch.
- b). If town D is half way between B and C, using a scale of 1cm to represent 20 km;
- i) Draw an accurate diagram showing positions A,B,C and D.
- ii) Find the distance and bearing of A from D
- c). Calculate the time taken by a Saloon Car travelling at a speed of 45km per hour to go through all the four towns and back to the starting town.

VECTORS

52. a). Given the points $L (0, -2)$ $M (2, 0)$ $N (4, 2)$; show that L, M and N are Collinear.
- b). If A is $(-2, 4)$ and $B (x, y)$
 $\mathbf{AB} = 3\mathbf{BM}$; where $M(2, 0)$, Find the values of x and y
53. If $\mathbf{r} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$, $\mathbf{S} = \begin{bmatrix} 26 \\ 32 \end{bmatrix}$ and $\mathbf{t} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$
 Find the scalar k if $3\mathbf{r} + k\mathbf{t} = \mathbf{s}$

54. In the figure below; $OQ = \underline{q}$; $OR = \underline{r}$ $OP : PQ = 3 : 2$ and $OS : SM = 3 : 1$

M is the midpoint of RQ.



a). Express the following Vectors in terms of \underline{q} and \underline{r}

i). **OM**

ii). **PR**

b). Show that ; P,S, and R are collinear.

55. In triangle OAB ; $OA = \underline{a}$ and $OB = \underline{b}$ point X lies on OB

such that $OX = \frac{1}{4} OB$. Point Y lies on AB such that;

$YB = \frac{3}{5} AB$. Lines OY and XA intersect at point Z.

Given that, $\vec{OZ} = h(\vec{OY})$ and $\vec{XZ} = k(\vec{XA})$

a). Draw triangle OAB and use it to find vectors;

i). **AB**

ii). **XA**

iii). **OY**

b). Express the position Vector of Z in terms of

i). h , a and b .

ii). k , a and b , hence, find the values of constants k and h .

TRANSFORMATION MATRICES

57. A triangle PQR with vertices $P(-2, -4)$, $Q(-4, -4)$ and $R(-5, -2)$ is mapped onto triangle $P^I Q^I R^I$ by a transformation described by a matrix $E = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$. $P^I Q^I R^I$ is again mapped onto triangle $P^{II} Q^{II} R^{II}$ by matrix $M = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. The coordinates of P^I , Q^I , and R^I are $(4, 8)$, $(8, 8)$ and $(10, 4)$ respectively.

Determine the:

- Matrix E and describe it fully.
- Single Matrix of transformation that Maps triangle PQR onto triangle $P^{II} Q^{II} R^{II}$.
- Coordinates of triangle $P^{II} Q^{II} R^{II}$
- Area scale factor for the transformation that Maps triangle PQR onto triangle $P^{II} Q^{II} R^{II}$.
- Area of triangle PQR if that triangle $P^{II} Q^{II} R^{II}$ is 16 square units

KINEMATICS

58. Kabiite covered a distance of 15km in 3 hours from her home to Madrine's home. She walked part of the Journey at 3 kmh^{-1} and ran the rest of the Journey at 9 kmh^{-1} . Find the distance she covered by running.
59. Kasozi arrives early to school by 10 minutes when he rides from home at a steady speed of 9 kmh^{-1} , when he rides at a steady speed of 7.5 kmh^{-1} , he arrives late by 6 minutes. Calculate;
- How far the school is from his home.
 - the speed enables him to be just punctual.
60. Towns P and Q are 180km apart. A car left town P for Q travelling at a steady speed of 40 kmh^{-1} . At 09:30am after travelling for two hours it stopped for $1\frac{1}{2} \text{ hours}$ and there after proceeded with its Journey at a steady speed of 50 kmh^{-1} .

- a). On the same axes show the Journeys of the two vehicles using a scale of 2cm for 20 Km and 2cm for 1 hour.
 - b). Use your graphs to find
 - i). when and where from Q the two vehicles met.
 - ii). distance between the two vehicles at 9:30 am.
 - iii). times of arrival of the two vehicles at their destinations.
 - iv). time the bus would have arrived at P if it had not reduced speed.
61. Towns Masaka and Kampala are 150 km apart. At 11:00 hours a car left Masaka for Kampala travelling at a steady speed of 50kmh^{-1} . After half an hour a bus left Masaka for Kampala at a steady speed of 120 kmh^{-1} but after travelling 30 Km it stopped for 15 minutes at Lukaya and then resumed its journey at the original speed. The bus arrived in Kampala and rested for 30 minutes before returning to Masaka by the same road where it arrived at 14:42 hours.
- a). On the same graph show the journeys of the cars using an appropriate scale.
 - b). Use your graph to determine;
 - i). time and distance from Masaka where the bus overtook the car on its way to Kampala.
 - ii). time and distance from Kampala where the bus met the car on its way back to Masaka.
 - iii). Average speed of the bus on its return journey.
 - iv). Average speed of the bus on its way to Kampala.
 - v). Average speed of the bus for the entire journey.

CONSTRUCTION

62. a) Construct a parallelogram PQRS in which line $PQ = 6\text{cm}$,
Line $QR = 4\text{cm}$ and $\angle PQR = 75^\circ$. Measure $\angle QRP$.
b). Inscribe a circle in triangle PQR. Measure the radius of the circle.
c). If the circle is cut out, what is the area of the remaining part of the parallelogram.

COORDINATE GEOMETRY

63. The lines $mx + 2n = 3$ and $mx - ny = 5$ intersect at point $(1, 2)$. Find the values of m and n .
64. The line $y = \frac{6-3x}{4}$ cuts the x -axis at $(P, 0)$ and y -axis at $(0, Q)$
i). Find the values of P and Q
ii). Find the area enclosed by the line and the two axes.
65. (a) Find the equation of a line passing through the point $(-2, 3)$ and is parallel to line $2y + 3x - 7 = 0$
(b). The points $(2n, 5/2)$, $(0, -7/2)$ and $(1, -2)$ lie on the same line. Find the value of n .
66. a). Draw the graphs of the lines $3y - 6x + 12 = 0$ and $3x + 2y = 6$ on the same axes.
b). Use your graph to state the coordinates of the point of intersection of the two lines.

RATIOS, PROPORTIONS AND VARIATIONS

67. A cylinder whose volume is constant has its radius r inversely proportional to the square root of its height h . If $r = 32\text{ cm}$ when $h = 9\text{ cm}$, find r when $h = 16\text{ cm}$.

68. The scale of a map is 1:250000. The area of the land on the map is 16cm^2 . Find the actual area of the land in Km^2 .
69. If it takes 200 workers 9 months to construct a 15 km road. How long would it take 300 workers to construct a 20 km road.
70. Mr. Opiyo has sheep and chicken on his farm. The cost of feeding them per day party varies as the number of the chicken and as the number of sheep on the farm. He spent Shs. 180,000/= on 60 chicken and 20 sheep. When the number of chicken increases to 100 and that of the sheep to 50, he spends Shs. 325,000.
- a). Find how much he spends on;
- i). each chicken and sheep
 - ii). 50 sheep and 60 chicken
- b). If he gets a loan of shs 1,800,000 and spends shs 900,000 on chicken and 900,000 on sheep. Determine the number of chicks and goats he has on farm.

THE END

THANK YOU FOR YOUR PARTICIPATION

“SMATA”

Together for Mathematics

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