WAVE THEORY OF LIGHT

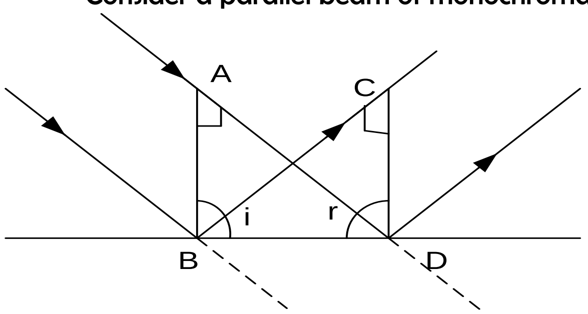
Huygens' principle

It states that every point on a wave front may be regarded as a source of secondary spherical wavelets which spread out with the wave velocity. The new wavefront is the envelope of these secondary wavelets.

Aplications of Huygens principle

(i) Reflection at palne surfaces

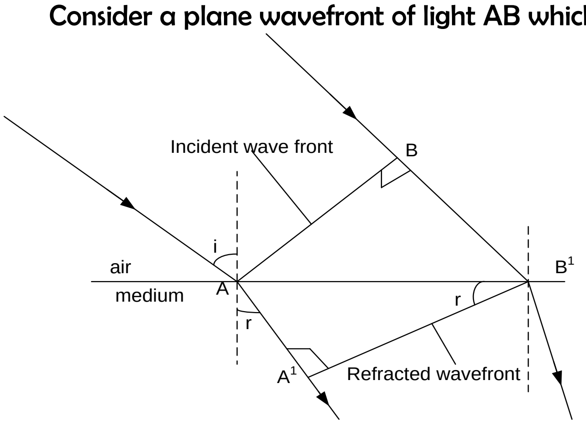
Consider a parallel beam of monochromatic light incident on a plane surface



If particles A and B on the same wave fornt in time t, B travels to C while A travels to D
$BC = AD$ and $< BAD =< BCD = 90°$
Since BD is common then $< i =< r$

(ii) Refraction at plane boundary

Consider a plane wavefront of light AB which is about to cross form one medium into another



Let C and V be the velocities of light in air and the medium respectivelt.

If the wave particle at B takes time t to move to B^1, then the distance B$B^1 = Ct$.
In the same time interval wave particle at A moves to A^1, distance A$A^1 = Vt$
From triangle ABB^1 and AA^1B^1

$$\frac{\sin i}{\sin r} = \frac{\left(\frac{BB^1}{AB^1}\right)}{\left(\frac{AA^1}{AB^1}\right)} = \frac{BB^1}{AA^1} = \frac{Ct}{Vt} = \frac{C}{V}$$

But $\frac{C}{V} = n$, refractive index of the material

$$V = \frac{C}{n}$$

Note

When light moves from one medium to another, the frequency of light remains the same
If f_a and f be the frequencies of light in the vacuum (air) and in the medium then

$$f_a = f$$

$$n = \frac{C}{V} = \frac{f_a \lambda_a}{f \lambda}$$

$$n = \frac{C}{V} = \frac{\lambda_a}{\lambda}$$

Example

If the wavelenght of light in air is 620nm, find its wavelenght in a material of refractive index 1.6

Solution

$$n = \frac{C}{V} = \frac{f_a \lambda_a}{f \lambda}$$

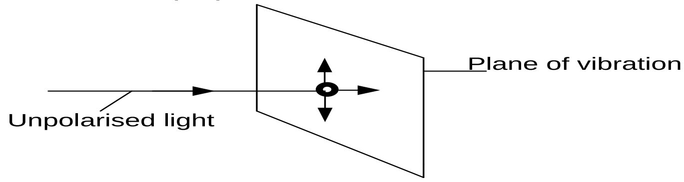
$$n = \frac{\lambda_a}{\lambda}$$

$$1.6 = \frac{620}{\lambda}$$

$$\lambda = 387.5nm$$

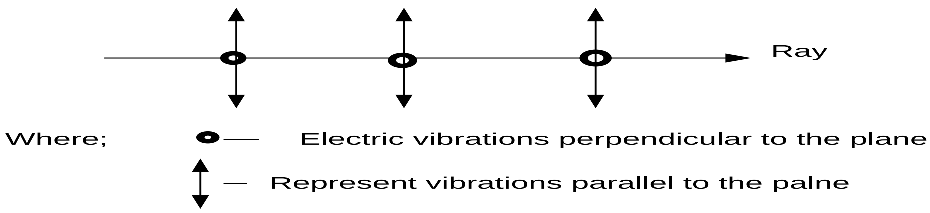
POLARISATION

Light is a transverse wave so its vibrations of electric vector occur in all directions perpendicular to the diection of propagation of light



Unpolarised light

This is light whose vibrations of the electric vectors occur in all direcions perpendicular to the direction of propagation of the wave.
Unpolarised light can be represented as below.

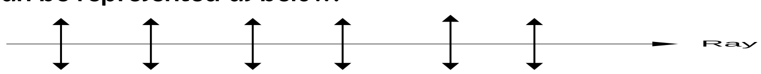


Why sound waves can not be polarised

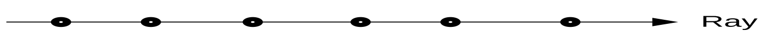
Sound waves are longitudinal waves, so its vibrations are parallel to the direction of propagation

Plane Polarised light

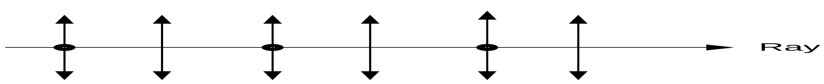
This is light whose vibrations of the electric vectors are confined to one plane perpendicular to the direction of propagation of the wave.
Polarised light can be represented as below.



Or

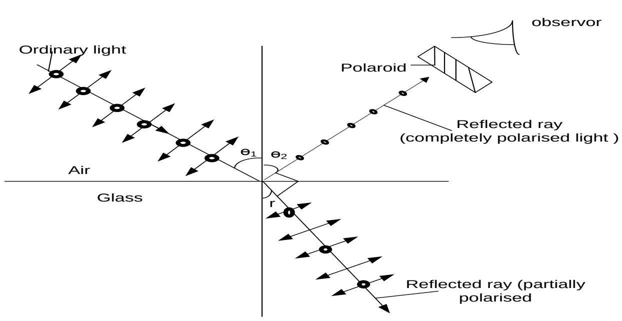


However light can under go partial polarization as shown below



PRODUCTION OF POLARISED LIGHT

(a) Reflection



- When light is incident on a boundary between air and glass, part of the light is partially reflected and the other partially transmitted into the denser medium.
- At one angle of incidence (polarizing angle), the reflected ray is completely plane-polarised while the refracted ray is partially plane polarized and the two rays are perpendicular to each other and vibrations in the reflected ray are parallel to the reflecting surface.

From sneils law: $n \sin i = constant$

$$n_a sin\theta_1 = n_g sinr \text{.........................(1)}$$
$$r + 90 + \theta_2 = 180$$
$$r = 90 - \theta_2$$
By law of reflection $\theta_1 = \theta_2 = \theta$
$$r = 90 - \theta$$
From (1) $n_a sin\theta_1 = n_g sin(90 - \theta_1)$
$$1xsin\theta = n_g cos\theta$$
$\boxed{n_g = tan\theta}$ Brewster's law

Where $\theta - polarising\ or\ Brewster\ angle$

Examples

1. The polarizing angle of ight incident in air on a glass plate is $56.5°$. What is the refractive index of glass

Solution

$n_g = tan\theta$	$n_g = tan(56.5)$	$n_g = 1.51$

2. A parallel beam of unpolarized light incident on a transparent medium of refractive index 1.62 is reflected as plane polarized light. Calculate the angle of incidence in air and the angle of refraction in the medium.

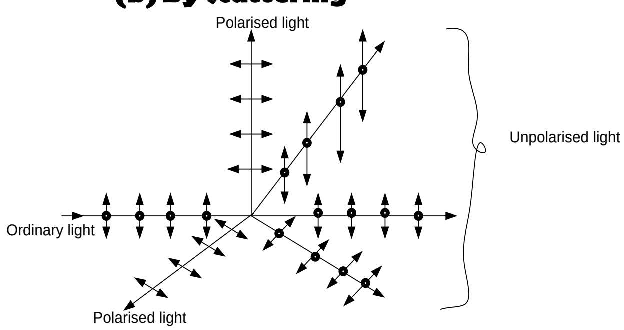
Solution

$n_g = tan\theta$ $\theta = \tan^{-1}(1.62)$ $\theta = 58.3°$	$r = 90 - \theta$ $r = 90 - 58.3$ $r = 31.7°$

Exercise

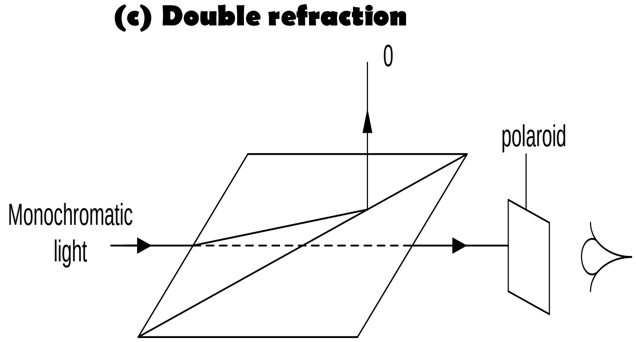
The polarising angle for light in air incident on a glass plate is 57.5°, what is the refractive index of the glass? **An(1.57)**

(b) By scattering



- When plane unpolarised light is incident on air molecules part of it is scattered.
- The light that passes through the air molecules is unpolarised and the light that is sattered in the direction perpendicular to the incident ray is polarized totally.

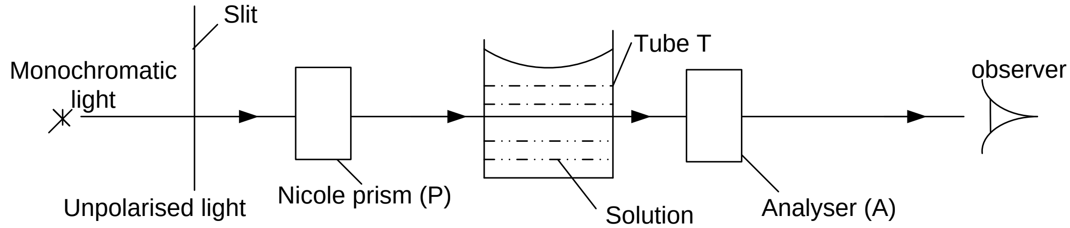
(c) Double refraction



- A narrow beam of ordinary light is made incident on a nicol prism and viewed through the analyser.
- The angle of incidence is gradually increased.
- For each angle of incidence, the emergent light is viewed through the analyser while rotating it about an axis perpendicular to the palne of the analyser.
- At a certain angle of incidence light gets cut off completely. At this point the emergent light is completely plane polarized.

Applications of polarization

(a) **Measurement of concentration of sugar in solution**



- Apparatus is arranged as above.
- With out tube T in place, the analyser A is rotated until the emergent light from T is completely cut off. The position of A is noted.
- Tube T is filled with a solution to be tested and on looking through A, light can now be seen
- Viewing through the analyzer A, it is rotated until when light is cut off and note this point.
- Measure the angle of rotation θ of the analyser.
- The concentration of the solution is proportional to the angle of rotation therefore the concentration can be determined

(b) **Reducing glare in sun glasses**

Polaroids are used in sun glasses to reduce intensity of incident sunlight and eliminate the reflected light.

When a polarized quoting is applied on sun glasses, the reflected light is partially or completely polarized and thus glare is reduced

Other applications include

- Holography
- In phot elasticity for stress analysis
- Used in L.C.D's

INTERFERNCE

Interference of waves is the superposition of waves from different two coherent soruces resulting into alternate regions of maximum and minimium intensity.
Where the path difference is an odd multiple of half a wavelength, cancellation occurs resulting itno minimum intensity. Where the path difference is an integral multiple of a full wavelength, reineforcement occurs resulting into maximum intensity.

Coherent sources

These are sources whose waves have the same frequency but nearly the same amplitude and a constant phase difference.

Conditions for observable interference to take place

- Wave trains must have nearly equal amplitudes
- There must be a constant phase relationship between the two wave trains (Wave sources must be coherent).
- The coherent sources must be close to each other.
- The screen should be as far as possible from the source

Types of interference

- Constructive interference
- Destructive interference

(a) Constructive interference

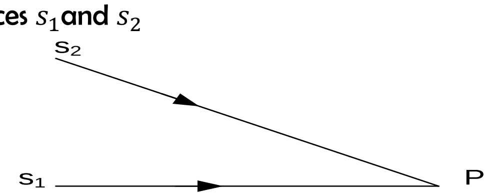
This is the re-enforcement of the intensities of two coherent sources to give maximum intensity when two wave distrubances from two sources are superimposed.
It takes place when a crest of one wave meets a crest of another wave and a troug meets a trough resulting into a large resultant amplitude.

(b) Destructive interference

This is the cancellation of two intensities of two coherent sources to give minimum intensity when two wave distrubances from two sources are superimposed.
It takes place when a crest of one wave meets a trough of another wave resulting into a small resultant amplitude.

Path difference

This is the difference in the length of the path taken by two waves from the source to a point of over lap.
Where they meet, the two waves superpose leading to reinforcement or cancellation. Where the path difference is an integral multiple of a full wavelength constructive interfence takes place. Where the path difference is an odd multiple of half a wavelength, destructive interference takes place.
Consider two coherent sources $s_1$and s_2



Wave forms from $s_1$and s_2 meet at P after taveling difffernt distances.
Waves from $s_1$travel a distance s_1P while waves from s_2 travel a distance s_2P
But $s_2P \ > s_1P$
$s_2P - \ s_1P = path\ difference$

If the path difference is zero or a whole number of wavelength. Then the brght band (constructive interference) will be formed

$\boxed{s_2P - s_1P = n\lambda}$ $n = 0,1,2,3 - - - - - - - - - - - - -$

If the path difference is an odd number of half wavelength. Then the darkband (destructive interference) will be formed

$\boxed{s_2P - s_1P = \left(n - {}^1\!/_2\right)\lambda}$ $n = 1,2,3 - - - - - - - - - - - - -$

Optical path

It is the length in a medium that contains the same number of waves as a given length in a vacuum. OR

This is the product of the geometrical path length in air and refractive index of the medium

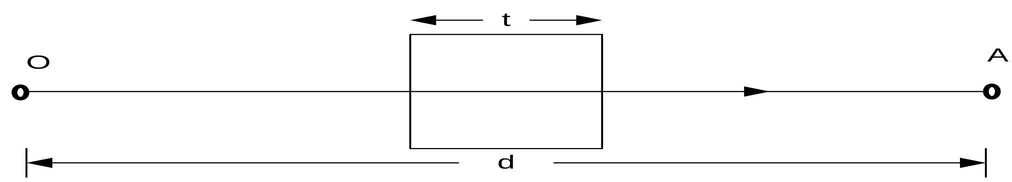
Consider a light travellign from O to A a distance d in air.

Optical path $= n_a d$

But $n_a = 1$

Optical path $= d$

If a thin transparent slab of thickness t and refractive index n is placed between O and A



Optica path between O and A is

Optical path $= n_a(d - t) + nt$

$$n_a = 1$$

Optical path $= d + (n - 1)t$

Phase diffference

This is the difference in the phase angles of two wave at a given time

Consider light travelling a distance x in the medium of refractive index n. if the wavelength of the medium is λ then, the phase difference, Ø is given by

$$\boxed{\text{Ø} = \frac{2\pi x}{\lambda}}$$

$\Rightarrow$ Phase difference $= \frac{2\pi}{\lambda}(optical\ path\ difference)$

When crests of two waves meet, then waves are said to be in phase

$\Rightarrow$ Phase difference $= 0$

Hence constructive interference occurs

When crest and trough of two waves meet, then waves are said to be out of phase

$\Rightarrow$ Phase difference $= \pi$

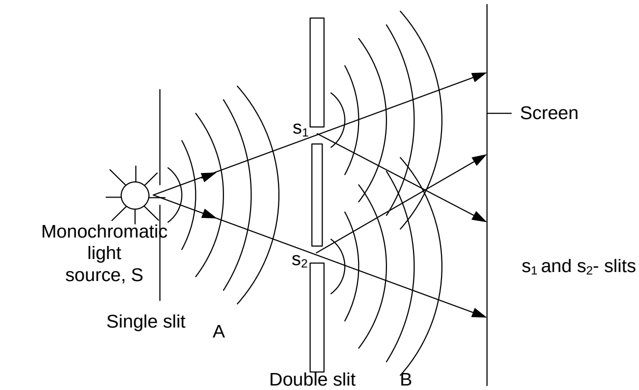
Hence destructive interference occurs

Production of coherent sources from a single source of light

(a) By division of a wave front

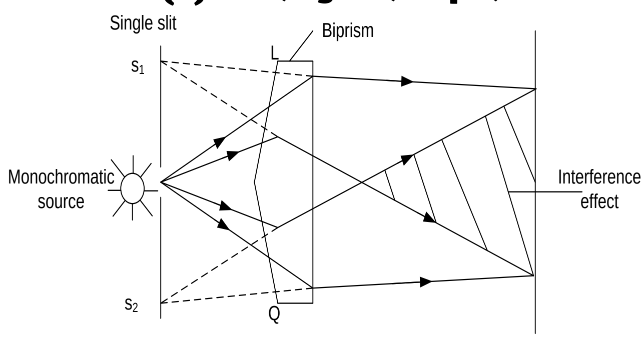
This is the process of obtaining <u>two coherent wave sources</u> from <u>a single wavefront</u>

(i) Using a double slit



- ❖ S, s_1 and s_2 are narrow slits which are parallel to each other.
- ❖ Waves from source, s diffreact into region and travel towards s_1 and s_2
- ❖ Diffraction also takes place at s_1 and s_2 and interference occurs in the region where the light from s_1 overlaps that from s_2
- ❖ Since s is narrow, the light which emerges from s_1 and s_2 comes form the same wave front as that which emerges from s. thus s_1 and s_2 are coherent

(ii) Using Fresnel prism



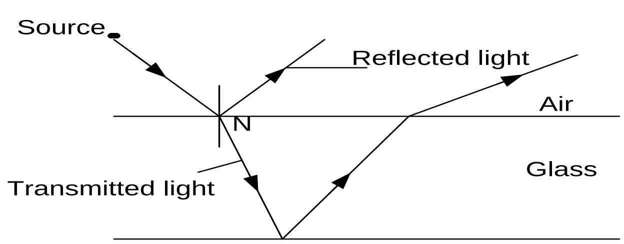
- ❖ A biprism of very large angle is placed with its refracting edge facing a narrow source of monochromatic light, s
- ❖ Light incident on face L is refracted and appear to come from a point S_1 and that incident on Q appears to come from S_2 due to refraction.
- ❖ The two sources are thus coherent since the light which emergent originates from the same wave front

Note : Biprism method is always preferred because it produces brighter fringes since the biprism converges most of the light on to the screen

(b) By division of amplitude

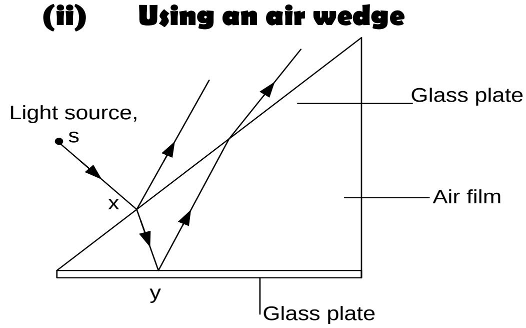
This is the process of dividing the amplitude into two parts by successive reflections

(i) When light is incident on a boundary of two media



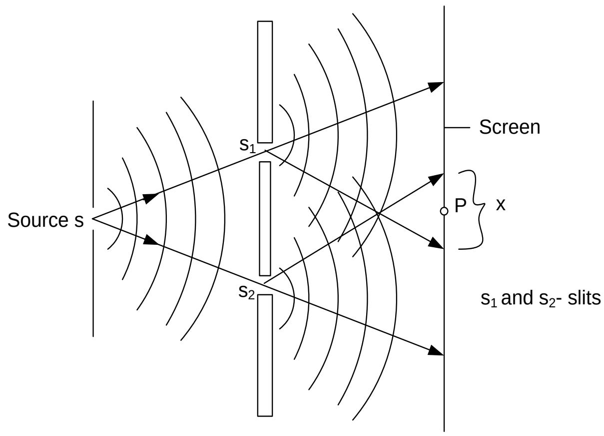
- ❖ When light is incident on a boundary between air and glass, part of the light is partially reflected and the other partially transmitted into the denser medium.
- ❖ At N, there is divison of intensity and since intensity is proportional to the square root of amplitude, then division of amplitude at N takes place.

(ii) Using an air wedge



- ❖ Monochromatic light is mad incident almost normally onto the upper glass slide.
- ❖ It is partly reflected at X and partly transmitted in the air film and reflected at Y.
- ❖ The light reflected at X and Y are coherent. When they overlap above the upper slide, they interfere
- ❖ Where the path difference is an odd mu;tiple of half a wavelenght, bright fringe is formed and where the path difference is an integral multiple of a full wavelength, a dark fringe is formed .

Young's double slit interference



When a wave front from the source, s is incident on a double slit s_1 and s_2, division of wave front take place and therefore s_1 and s_2 act as coherent sources. Waves from s_1 and s_2 superimpose in region x and interference takes place.

When a crest from s_1 meet a crest from s_2 and a trough from s_1 meets a trough from s_2 then maximum interference is achieved and a brght fringe is formed (constructive interference).

When a crest from s_1 meet a trough from s_2 and a trough from s_1 meets a cest from s_2 then minimum interference is achieved and a dark fringe is formed (destructive interference).

This results into a series of alternating dark and bright bands which areequally spaced and are parallel to the slits.

At the central point P waves from s_1 and s_2 travel equal distances and they arrive at the same time (they are in phase). This implies constructive interference hence a bright fringe is formed at P

Note;

(i) When one of the double slits is covered, no interference takes place

(ii) When the source of monochromatic light is moved close to the slits, the intensity increases and bands become brighter

(iii) When the distance between the double slits and single slit is reduced, fringe separation remains the same but bands become bright since the intensity increases

(iv) When the doube slit separation is reduced, the fringe seapraion increases and when the slit separation is increased, the fringe separation decreases unitl a stage is reached when no fringes are observed

Effect of using white light other than monochromatic light

Sets of coloured fringes are seen on the screen. The central fringe is white, with coloured fringes on either side. For each set, blue fringes is nearest to the central one while red is furthest.

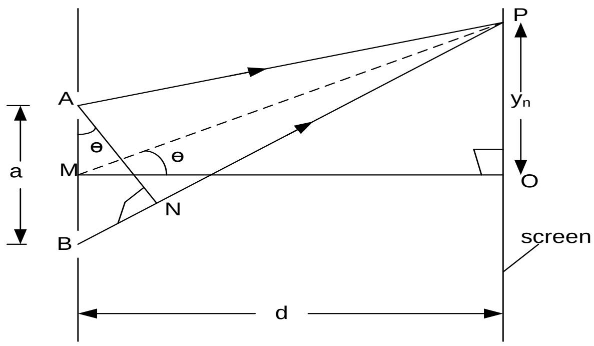
Effect of widening the single slit,s

The fringes gradually disappear. The slit s is now equivalent to a large number of slits each producing its own fringe system on the screen. The fringe systems overlap producing uniform illumination

Effect of narrowing the double slit separation

- When the slit separation is large $(a \gg \lambda)$, bright band of approximately the same width as the slit is observed.
- As the slit width is reduced so that $a \approx \lambda$, a diffraction pattern is observed. A central white band having dark bands on either sides is obtained. The dark bands have coloured fringes running from blue to red, the blue fringes being nearest to the direction position
- As the slit width is reduced further $(a < \lambda)$, the central bright band widens and extends well into the geometrical shadow of the slit.
- When the slit finally closes, no light passes through.

Derivation of fringe separation



Suppose waves from A and B superpose at P to form a bright fringe

$path\ difference, BN = BP - AP = asin\theta \dots (\mathbf{1})$

For $d \gg a$, θ is very small in radians and $sin\theta \approx tan\theta$

$BN = atan\theta = \frac{ay_n}{d}$........................(2)

For n^th bright fringe at P

$path\ difference, BN = n\boldsymbol{\lambda} \dots (\mathbf{3})$

where $\lambda - wavelenght$

$\Rightarrow \frac{ay_n}{d} = n\boldsymbol{\lambda}$

$\boxed{y_n = \frac{n\boldsymbol{\lambda}}{a}d}$............................(4)

For $(n+1)^{th}$ bright fringe

$y_{n+1} = \frac{(n+1)\boldsymbol{\lambda}}{a}d$............................(5)

Fringe separation

$$\boldsymbol{y} = y_{n+1} - y_n$$

$$\boldsymbol{y} = \frac{(n+1)\boldsymbol{\lambda}}{a}d - \frac{n\boldsymbol{\lambda}}{a}d$$

$$\boxed{\boldsymbol{y} = \frac{\boldsymbol{\lambda d}}{a}}$$

For dark fringes

Suppose waves from A and B superpose at P to form a dark fringe

$path\ difference, BN = BP - AP = asin\theta \dots (\mathbf{1})$

For $d \gg a$, θ is very small in radians and $sin\theta \approx tan\theta$

$BN = atan\theta = \frac{ay_n}{d}$........................(2)

For n^th dark fringe at P

$$path\ difference, BN = \left(n + \frac{1}{2}\right)\boldsymbol{\lambda} \dots (\mathbf{3})$$

where $\lambda - wavelenght$

$\Rightarrow \frac{ay_n}{d} = \left(n + \frac{1}{2}\right)\boldsymbol{\lambda}$

$\boxed{y_n = \left(n + \frac{1}{2}\right)\frac{\lambda\, d}{a}}$............................(4)

For $(n+1)^{th}$ dark fringe

$y_{n+1} = \left(n + 1 + \frac{1}{2}\right)\frac{\lambda\, d}{a}$............................(5)

Fringe separation

$$\boldsymbol{y} = y_{n+1} - y_n$$

$$\boldsymbol{y} = \left(n + 1 + \frac{1}{2}\right)\frac{\boldsymbol{\lambda}\, d}{a} - \left(n + \frac{1}{2}\right)\frac{\boldsymbol{\lambda}\, d}{a}$$

$$\boxed{\boldsymbol{y} = \frac{\boldsymbol{\lambda d}}{a}}$$

Examples

1. In Youngs double slit experiment, 21 bright fringes occupying a distance of 3.6mm were visible on the screen. The distance of the screen form the double slit was 29cm and the wavelength of light used in the experiment was $5.5x10^{-7}m$. Calculate the separation of the slits.

Solution

$$y = \frac{3.6x10^{-3}}{21}$$

y= $0.171x10^{-3}m$

$$\frac{\boldsymbol{y}}{\boldsymbol{d}} = \frac{\boldsymbol{\lambda}}{a}$$

$$\boldsymbol{a} = \frac{5.5x10^{-7}x29x10^{-2}}{0.171x10^{-3}}$$

$$\boldsymbol{a} = 9.327x10^{-4}m$$

2. In Youngs double slit experiment, the slits are separated by 0.28mm and the screen is 4m away. The distance betwee the 4^th bright fringe and the central fringe is 1.2cm. Find the wavelength of light used in the experiment.

Solution

Fringe separation;

$$y_n = \frac{n\boldsymbol{\lambda}}{a}d$$

$$y = y_4 - y_0$$

$$y = \frac{4\boldsymbol{\lambda}}{a}d - \frac{0\boldsymbol{\lambda}}{a}d$$

$$\Delta y = \frac{4\boldsymbol{\lambda}}{a}d$$

$$1.2x10^{-2} = \frac{4x4x\boldsymbol{\lambda}}{0.28x10^{-3}}$$

$$\lambda = 2.1x10^{-7}m$$

3. In Youngs double slit experiment, the 6^th bright fringe is formed 4mm away from the center of the fringe system when the wave length of the light used is $6.0x10^{-7}m$. Calculate the separation of the two slits if the distance from the slits to the screen is 60cm.

Solution

$$y_n = \frac{n\lambda}{a}d$$
$$y = y_6 - y_0$$
$$y = \frac{6\lambda}{a}d - \frac{0\lambda}{a}d$$

$$y = \frac{6\lambda}{a}d$$
$$4x10^{-3} = \frac{6x6.0x10^{-7}x0.6}{a}$$
$$a = 5.4x10^{-4}m$$

4. In Youngs double slit experiment, the 8th bright fringe is formed 5mm away from the center of the fringe system when the wave length of the light used is $6.2x10^{-7}m$. Calculate the separation of the two slits if the distance from the slits to the screen is 80cm.

Solution

$$y_n = \frac{n\lambda}{a}d$$
$$y = y_8 - y_0$$
$$y = \frac{\mathbf{8}\boldsymbol{\lambda}}{a}d - \frac{0\lambda}{a}d$$

$$y = \frac{\mathbf{8}\boldsymbol{\lambda}}{a}d$$
$$5x10^{-3} = \frac{8x6.2x10^{-7}x0.8}{a}$$
$$a = 7.94x10^{-4}m$$

5. In Youngs double slit experiment, the slits 0.2mm apart and are placed a distance of 1m from the screen. The slits are illuminated with light of wavelength 550nm. Calculate the distance between the 4th and 2nd bright fringes of interference patterns.

Solution

Bright fringe position;

$$y_n = \frac{n\lambda}{a}d$$
$$y = y_4 - y_2$$

$$y = \frac{4\lambda}{a}d - \frac{2\lambda}{a}d$$
$$y = \frac{2\lambda}{a}d$$

$$y = \frac{2x550x10^{-9}x1}{0.2x10^{-3}}$$
$$y = 5.5x10^{-3}m$$

6. In Youngs double slit experiment, the distance between adjacent bright fringes is $10^{-3}m$. If the distance between the slits and the screen is doubled, the slit separation halved and light of wavelength 650nm changed to light of wavelenght 400nm. Find the new separation of the fringes.

Solution

$$y = \frac{\lambda}{a}d$$

Case 1:

$$10^{-3} = \frac{650x10^{-9}d}{a} \quad ....(1)$$

Case 2:

$$y = \frac{400x10^{-9}(2d)}{\left(\frac{1}{2}a\right)} \quad ....(2)$$

$$equation\ 2 \div equation\ 1$$

$$\frac{y}{10^{-3}} = \frac{\left[\frac{400x10^{-9}(2d)}{\left(\frac{1}{2}a\right)}\right]}{\left[\frac{650x10^{-9}d}{a}\right]}$$
$$y = 492m$$

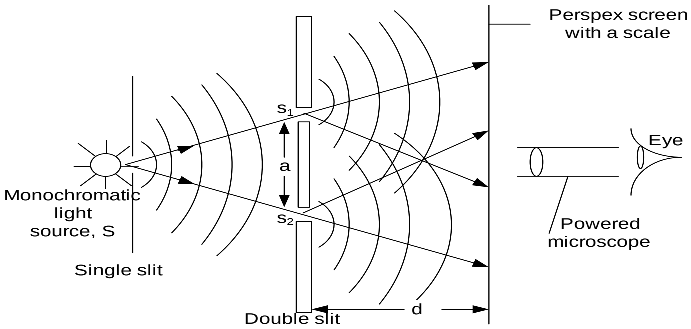
Exercise

1. In Young's double-slit experiment, the 5th bright fringe is formed 7 mm away from the centre of the fringe system when the wavelength of light used is $4.6\ x\ 10^{-7}$ m. Calculate the separation of the two slits if the distance from the slits to the screen is 90 cm. **An**$(\mathbf{2.96\ x\ 10^{-4}m})$

2. Two slits 0.5mm apart are placed at a distance of 1.1m from the screen. The slits are illuminated with light of wavelength 580nm. Calculate the distance between the sixth and second bright fringes of the interference pattern. **An**$(\mathbf{5.1\ x\ 10^{-3}m})$

3. In Young's experiment, an interference pattern in which the tenth bright fringe was 3.4 cm from the centre of the pattern was obtained. The distance between the slits and the screen was 2.0m while the screen separation was 0.34mm. Find the wavelength of the light source **An**$(\mathbf{5.78\ x\ 10^{-7}m})$

Experiment to measure wavelength of light using Youngs double slit interference



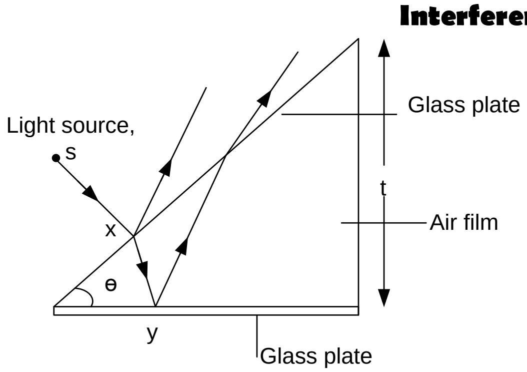
- Apparatus is arranged as above.
- A monochromatic light is used to illuminate double slits s_1 and s_2.
- The microscope is placed at such a distance d that fringes are observed in its field of view
- The number of bright fringes in a fixed length on the screen is counted and the fringe separation y is determined
- Measure the distance d using a meter rule.
- Measure the slit separation a using a travelling microscope.
- Wavelelnght of light can be calculate from

$$\boldsymbol{\lambda} = \frac{ya}{d}$$

Comparing wavelenght of red light and blue light

- Apparatus is arranged as above.
- A source of white light is used and a red filter filter is palced infornt of the slit, s
- The number of bright fringes in a fixed length on the screen is counted and the fringe separation y_r is determined
- The filter is now replaced by a blue one and the experiment repeated, and the fringe separation y_b determined.
- It is found that $y_r > y_b$ and since $\boldsymbol{\lambda} = \frac{ya}{d}$ then $\lambda \propto y$
- Wavelenght of red light is greater than that of
- blue light

Interference in thin films



- Monochromatic light is mad incident almost <u>normally</u> onto the upper glass slide.
- It is partly <u>reflected at the bottom part of X</u> and partly transmitted into the air film and <u>reflected at the top surface of</u> Y.
- The light reflected at X and Y are coherent. When they overlap above the upper slide, they <u>interfere</u>
- Where the path difference is an odd multiple of half a wavelenght, <u>bright fringe is formed</u> and where the path difference is an integral multiple of a full wavelength, <u>a dark fringe is formed.</u>

Note : when white light is used coloured fringes are observed

Derivation of fringe separation

Consider two slides inclined at an angle θ

For n dark fringes

Path difference, $2t_n = n\boldsymbol{\lambda} \ldots\ldots\ldots\ldots (\mathbf{1})$

where n=1,2,3.....

For $(n+1)^{th}$ dark fringes

$2t_{n+1} = (n+1)\boldsymbol{\lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{2})$

where n =0,1,2.....

Eqn 2- Eqn 1

$$2t_{n+1} - 2t_n = (n+1)\boldsymbol{\lambda} - n\boldsymbol{\lambda}$$

$$t_{n+1} - t_n = \frac{\boldsymbol{\lambda}}{2}$$

$$\boldsymbol{tan\ \theta} = \frac{t_{n+1} - t_n}{y_{n+1} - y_n}$$

$$y_{n+1} - y_n = y$$

$$\boldsymbol{tan\ \theta} = \frac{\boldsymbol{\lambda}}{2y}$$

Since θ is very small in radians, $tan\ \theta \approx\ \theta$

$$\boxed{y = \frac{\lambda}{2\theta}}$$

For n bright fringes

Path difference, $2t_n = \left(n - \frac{1}{2}\right)\lambda \ldots\ldots\ldots\ldots(1)$

For $(n+1)^{th}$ dark fringes

$$2t_{n+1} = \left(n + 1 - \frac{1}{2}\right)\lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Eqn 2- Eqn 1

$$2t_{n+1} - 2t_n = \left(n + 1 - \frac{1}{2}\right)\lambda - \left(n - \frac{1}{2}\right)\lambda$$

$$t_{n+1} - t_n = \frac{\lambda}{2}$$

$$\tan\theta = \frac{t_{n+1} - t_n}{y_{n+1} - y_n}$$

$$y_{n+1} - y_n = y$$

$$\tan\theta = \frac{\lambda}{2y}$$

Since θ is very small in radians, $\tan\theta \approx \theta$

$$\boxed{y = \frac{\lambda}{2\theta}}$$

Examples

1. Two glass slides in contact at one end are separated by a wire to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.6x10^{-7}m$ a total of 20 firnges occupying a distance of 15mm are obtained. Calculate the angle of the wedge.

Solution

$$y = \frac{15x10^{-3}}{20} = 0.75x10^{-3}$$

$$\tan\theta = \frac{\lambda}{2y}$$

$$\theta = \tan^{-1}\left(\frac{5.6x10^{-7}}{2x0.75x10^{-3}}\right)$$

$$\theta = 0.021°$$

2. Two glass slides in contact at one end are separated by a wire of diameter 0.04mm at the other end to form awave fringes observed when light of wavelenght $5x10^{-7}m$ is incident normally onto the slides. Find the number of dark fringes tha can be observed

Solution

For dark fringes

$2t_n = n\lambda$

$$n = \frac{2x0.04x10^{-3}}{5x10^{-7}}$$

$$n = 160\ dark\ fringes$$

3. Two glass slides in contact at one end are separated by a sheet of paper 16cm from the the line of contact, to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.8x10^{-7}m$ interference firnges of separation 2.0mm are obtained in reflection. Calculate the thickness of the paper.

Solution

$$\tan\theta = \frac{\lambda}{2y} = \frac{t}{16x10^{-2}}$$

$$\frac{\lambda}{2y} = \frac{t}{16x10^{-2}}$$

$$y = \frac{5.8x10^{-7}x16x10^{-2}}{2x2.0x10^{-3}}$$

$$y = 2.32x10^{-5}m$$

4. Two glass slides in contact at one end are separated by a metal foil 12.5cm from the the line of contact, to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.4x10^{-7}m$ interference firnges of separation 15mm are obtained. Calculate the thickness of the metal foil.

Solution

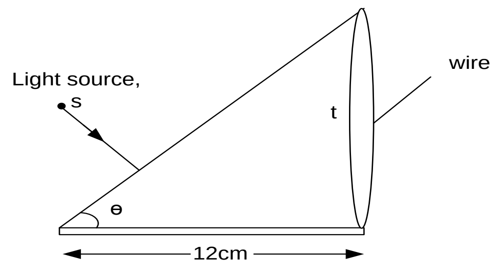
$$\tan\theta = \frac{\lambda}{2y} = \frac{t}{12.5x10^{-2}}$$

$$\frac{\lambda}{2y} = \frac{t}{12.5x10^{-2}}$$

$$y = \frac{5.4x10^{-7}x12.5x10^{-2}}{2x1.5x10^{-3}}$$

$$y = 2.25x10^{-5}m$$

5. Two glass slides 12cm long are in contact at one end and separated by a metal wire of diameter $2.5x10^{-3}cm$ at the other end. When the slides are illuminated normally as shown below with the light of wavelenhgt 500nm, a fringe system is observed



Calculate;

(i) Fringe separation
(ii) Number of dark fringes formed
(iii) Number of bright finges formed

Solution

(i) $\tan\theta = \frac{\lambda}{2y} = \frac{t}{12x10^{-2}}$

$$\frac{\lambda}{2y} = \frac{t}{12x10^{-2}}$$

$$y = \frac{500x10^{-9}x12x10^{-2}}{2x2.5x10^{-5}}$$

$$y = 1.2x10^{-3}m$$

(ii) For dark fringes

$$2t_n = n\lambda$$

$$n = \frac{2x2.5x10^{-5}}{500x10^{-9}}$$

$$n = 100\ dark\ fringes$$

(iii) For bright fringes

$$2t_n = \left(n + \frac{1}{2}\right)\lambda$$

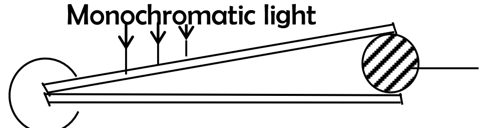
$$n = \frac{2t_n}{\lambda} - \frac{1}{2}$$

$$n = \frac{2x2.5x10^{-5}}{500x10^{-9}} - \frac{1}{2}$$

$$n = 99\ bright\ fringes$$

Exercise

1. Two glass slides in contact at one end are separated by a metal foil 12.50 cm from the line of contact, to form an air-wedge. When the air-wedge is illuminated normally by light of wavelength 5.4 x 10 $^{-7}$ m interference fringes of separation 1.5 mm are found in reflection. Find the thickness of the metal foil. **An** $(\mathbf{2.25x10^{-5}m})$
2. An air wedge is formed by placing two glass slides of length 5.0 cm in contact at one end and a wire at the other end as shown in figure 2



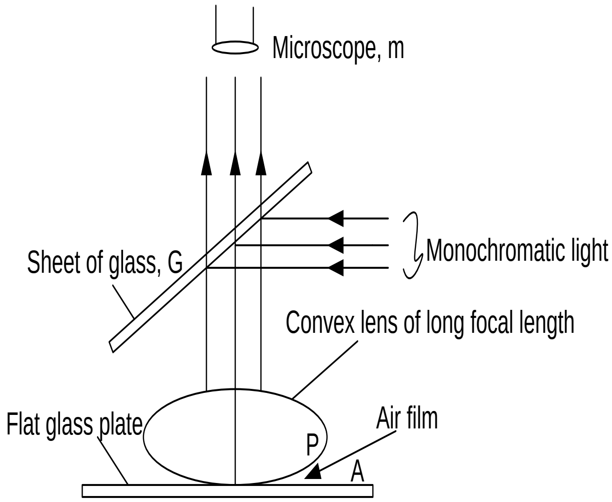
Viewing from vertically above, 10 dark fringes are observed to occupy a distance of 2.5 mm when the slides are illuminated with light of wavelength 500 nm.

(i) Explain briefly how the fringes are formed
(ii) Determine the diameter of the wire

Briefly explain why interference effect are not observed in thick films (air wedges)

- Bright fringes occur when the path difference for the wavelength is equal to $\left(n - \frac{1}{2}\right)\lambda$ where $n = 1,2,3$...
- When the film is thick, each colour attains this path difference forming bright band. The different colours thus overlap leading to uniform white illumination (blurring of the fringes).

NEWTON'S RINGS



- When a convex lens of long focal length is made to rest on an optical flat glass plates, a layer of air between the lens and the plate acts as an air wedge.
- Monochromatic light is reflected by glass plate G such that it falls normally on an air film formed between the convex lens and the flat glass plate.
- Light reflected upwards and transmitted through G is observed through a travelling microscope M. A series of dark and brght rings is observed. Light rays from P and A interfere constructively if the path difference is

$$2t_n = \left(n - \frac{1}{2}\right)\boldsymbol{\lambda}, \boldsymbol{n} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots \ldots$$

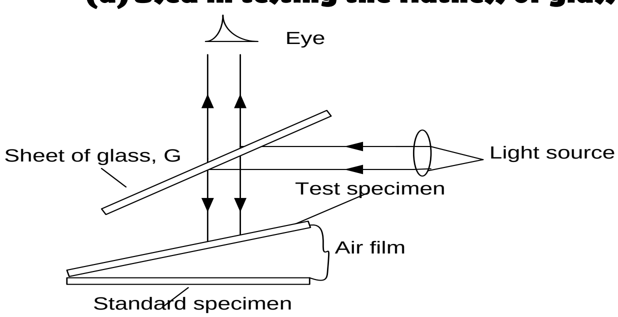
- If rays interfere destructively if the path difference is $2t_n = n\boldsymbol{\lambda}, \boldsymbol{n} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots \ldots$
- Thus interference patterns observed consist of a series of dark and bright rings with a central sport being dark

Appearance of colours on an air film

- Colours on an oil film on a water surface appear due to interference of light
- When light from the sky meets the oil film, it is partially reflected and partially refracted. The refracted light is totally internally reflected at the oil- water boundary.
- When the colours reach the eye, they interfere. The interference colours for which the waves are in phase are seen while those for which they are out of phase are not seen. The particular colour seen depends in the positon of the eye

Applications of interference

(a) Used in testing the flatness of glass surface



- The surface under test is made to for an air wedge with a plane galss surface of standard smoothness
- When a parallel beam of monochromatic light from source s is reflected from the glass G, it falls almost normally to the air wedge
- Interference fringes caused by the air wedge between the plate are observed
- Irregularities in the surface of the test specimen will show up when unparalllel, equal spaced fringes are formed.

(b) Blooming of lenses

- When light is incident on a lense, some percentage of the light is reflected from each surface. This reults into reduction in intesniyt of light due to losss of light being transmitted. This reduces clarity of the finaal image produced.
- This defect therefore can be reduced by evaporating a thin coating of magnesium fluoride unot the lense surface. This process is called blooming

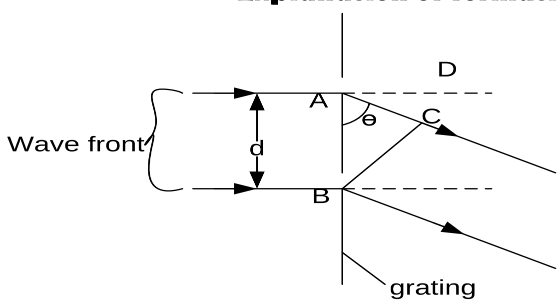
DIFFRACTION OF LIGHT

This is the spreading of waves beyond geometrical boundaries leading to interference

Diffraction grating

This is a transparent plate with many equidistant small parallel lines drawn on it using a diamond pencil

Explanation of formation of fringes by transmission grating



- Consider a transmission grating of narrow slits AB whose width is compared to the wavelength of light illuminated normally by monochromatic light
- Light is diffracted through spaces of the grating into region D where they superpose.
- Where the resultant path difference of wave through a pair f consecautive slits is an integral multiple of full wavelength, constructive interference occurs and a bright band is formed. Where the resultant path difference is an odd multiple of half a wavelenght, destructive inteference occurs and a dark band is formed
- This spreading of light along the obstacle beyond the geometrical shadow leading to interference pattern is called diffraction.

Effect of increasing the number of narrow slits in the diffraction grating on intensity

- When number of slits are increased, the intensity of the principal maxima increases and the subsidiary decreases.
- The interference at the principal maxima are always constructive hence intensity increases. Interference at the subsidiary maxima are destructive hence intensity decreases

Note;

(i) For diffraction grating
  - Lines are ruled on glass
  - The spaces transmit light

(ii) For reflection grating
  - Lines are ruled on a polished metal
  - The spaces reflect light

Condition for diffraction maxima

Consider a transmission diffraction grating of spacing d illuminated normally with light of wavelength λ.
Path difference between waves form A and B(distance BC)$= dsin\theta$
For diffraction maxima, path difference $= n\lambda$
$dsin\theta = n\boldsymbol{\lambda}$ **where** $\boldsymbol{n = 0, 1, 2}$

Examples

1. Sodium light of wavelenght 589nm falls normally on a diffraction grating which has 600 lines per mm. calculate the angle between the directions in which the first order maxima, on the same side of the straight through positions are observed.

Solution

$dsin\theta = n\boldsymbol{\lambda}$

$$\boldsymbol{\theta = sin^{-1}\left(n\lambda x\frac{1}{d}\right)}$$

But $\frac{1}{d} = \frac{600}{10^{-3}}$ $lines\ per\ meter$ and for first ordr maxima $n = 1$

$$\theta = sin^{-1}(1x589x10^{-9}x600x10^{3})$$

$$\theta = 20.70°$$

2. When monochromatic light of wavelenght 600nm is incident normally on a transmission grating, the second order diffraction image is observed at an angle of 30°. Determine the number of lines per centimeter on the grating

Solution

$dsin\theta = n\boldsymbol{\lambda}$

$\frac{\mathbf{1}}{\boldsymbol{d}} = \frac{sin\theta}{n\boldsymbol{\lambda}}$

$\frac{\mathbf{1}}{\boldsymbol{d}} = \frac{sin30}{2x\mathbf{600x10^{-9}}}$

$\frac{1}{d} = 4.17x10^5 \ lines \ per \ meter$

$\frac{1}{d} = 4.17x10^3 \ lines \ per \ cm$

3. A diffraction grating of 600 lines per mm is illuminated normally by monochromatic, the first order maxima is observed at an angle of 20°. Find the;

(i) The wavelenght of the light

(ii) number of diffracton maxima possible

Solution

(i) $dsin\theta = n\boldsymbol{\lambda}$

$\boldsymbol{\lambda} = \frac{dsin\theta}{\boldsymbol{n}}$

$\frac{1}{d} = \frac{600}{10^{-3}} \ lines \ per \ meter$ and for first order $n = 1$

$\boldsymbol{\lambda} = \frac{\left(\frac{10^{-3}}{600}\right) sin20}{\mathbf{1}}$

$\boldsymbol{\lambda} = 5.7x10^{-7}m$

(ii) $dsin\theta_{max} = n_{max}\boldsymbol{\lambda}$

But $sin\theta_{max} = 1$

$n_{max} = \frac{d}{\boldsymbol{\lambda}}$

$n_{max} = \frac{\left(\frac{10^{-3}}{600}\right)}{5.7x10^{-7}} = 2.92$

Maxima value $n = 2$

4. A diffraction grating of 500 lines per mm is illuminated normally by light of wavelenght 526nm. Find the total number of images seen

Solution

$dsin\theta_{max} = n_{max}\boldsymbol{\lambda}$

But $sin\theta_{max} = 1$

$n_{max} = \frac{d}{\boldsymbol{\lambda}}$

$\frac{1}{d} = \frac{500}{10^{-3}} \ lines \ per \ meter$

$n_{max} = \frac{\left(\frac{10^{-3}}{500}\right)}{526x10^{-9}} = 3.8$

Total number of images seen is 7

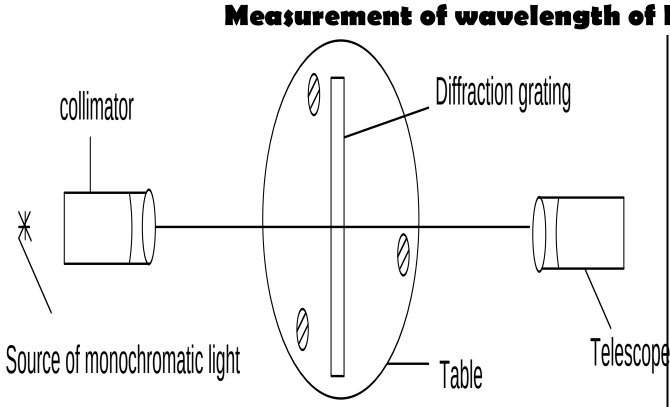
Uses of diffraction

(i) Measurement of the wavelenght of light usng a diffraction graitn

(ii) Used in spectrographic studies

(iii) Used in holograms(3-D photographs)

Measurement of wavelength of light using diffraction grating



- The telescope is adjusted to focus parallel light. The collimator is adjusted to produce parallel light and the table is leveled.
- The grating is placed on the table so that its plane is perpendicular to the incident light
- Zero order image is now received at the telescope. This position on T_1 on the scale is noted. The telescope is now turned in one direction until the first order image is obtained. The angle θ_1 of rotation form position T_1 is recorded.
- The telescope is restored to position T_2 and rotated in the opposite direction until the first

1 Reference Books

- 1. Optics by Eugene Hecth(5th Edition)
- 2. Introduction to Optics by Frank Pedrotti and Leno Pedrotti(2nd edition)
- 3. Modern Optics by Robert Guenther
- 4.Introduction to modern Optics by Grant R. Fowles(2nd edition)

2 WAVE MOTION

What is a wave? A wave is a disturbance or a form of energy, which propagates through a medium without any transfer of matter or change of form. The particles of the medium do not travel along with the wave. They instead oscillate to and fro about the equilibrium position as the wave passes by. Only the disturbance is propagated. When the disturbance arrives at a point, it sets into motion the particles at that location. The disturbance gives the particles kinetic energy and momentum.

2.1 Classes of Waves

There are generally two classes of waves. These are

- (i) Transverse waves: the particles of the medium oscillate perpendicularly to the direction of propagation of the waves. Examples include waves on stretched strings, electric waves, magnetic waves, etc.
- (ii) Longitudinal waves: the particles of the medium oscillate parallel to the direction of propagation of the wave. Examples include sound waves, waves in pipes.

2.2 Wave equation

Let us consider a wave pulse of arbitrary shape traveling on a stretched string along the x-axis as sown in Figure 1. Let the shape of the wave be described by $y^{'} = f(x^{'})$ fixed to a coordinate system $O^{'}(x^{'}, y^{'})$. Let the wave move with a uniform speed c along the x axis with respect to the fixed coordinate system $O(x, y)$. As the wave pulse moves, it is assumed that its shape is unchanged. A point N on the wave pulse can be described by either of the coordinates x or $x^{'}$, where $x^{'} = x - ct$.
The y- coordinate is identical in either coordinate system. In the stationary coordinate system's frame, the moving coordinate can be described by the function $y = y^{'} = f(x^{'}) = f(x - ct)$. Hence the function now describes the wave is of the form $y = f(x–ct)$. If the pulse moves to the left, the sign of velocity c changes and hence $y = f(x + ct)$. Note: We have assumed that at $t = 0$, $x = x^{'}$.
Generally, we may write $y = f(x \pm ct)$. Since y is a function of two variables x and t, we can use the chain rule of partial differentiation.
Let S be given by Equation(1).

$$S = x \pm ct \longrightarrow y = f(S) \tag{1}$$

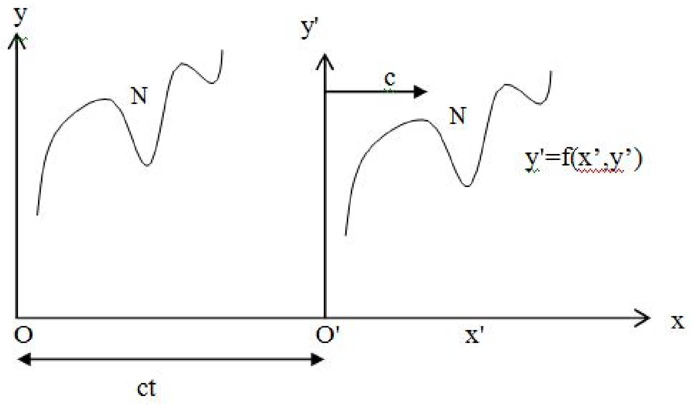


Figure 1: A wave of arbitrary shape traveling along a stretched spring with a velocity c in the + x – direction

and

$$\frac{\partial S}{\partial x} = 1 \longrightarrow \frac{\partial S}{\partial t} = \pm c \tag{2}$$

$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial S} \times \frac{\partial S}{\partial x}.$$

$$\longrightarrow \frac{\partial y}{\partial x} = \frac{\partial f}{\partial S} \tag{3}$$

Take second derivative of Equation(3)

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial}{\partial S}(\frac{\partial f}{\partial S})\frac{\partial S}{\partial x}.$$

$$\Longrightarrow \frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial S}\left(\frac{\partial f}{\partial S}\right) = \frac{\partial^2 f}{\partial S^2} \tag{4}$$

also

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial S} \times \frac{\partial S}{\partial t} = \pm c\frac{\partial f}{\partial S} \tag{5}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial S}\left(\frac{\partial y}{\partial t}\right)\frac{\partial S}{\partial t} = \frac{\partial}{\partial S}(\pm c\frac{\partial f}{\partial S}) \times \pm c = c^2\frac{\partial^2 f}{\partial S^2}.$$

$$\Longrightarrow \frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 f}{\partial S^2}.$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial x^2} \tag{6}$$

Equation(6) is the wave equation.
In general

$$\nabla^2 \psi = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2},$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial x^2},$$

$c = v_p$ is the phase velocity

2.3 Harmonic waves

Harmonic waves involve the sine or cosine functions. These waves can be written in a uniform way as

$$\psi(x,t) = A\sin(k(x \pm vt) + \varepsilon),$$

or

$$\psi(x,t) = A\cos(k(x \pm vt) + \varepsilon) \tag{7}$$

where A is known as the amplitude of the wave, k is the propagation number, and ε is the initial phase, or epoch angle. These are periodic waves, representing smooth pulses that repeat themselves endlessly. Such waves are often generated by undamped oscillators undergoing simple harmonic motion. More importantly, the sine and cosine functions together form a complete set of functions; that is, a linear combination of terms like those in (7) can be found to represent any actual periodic wave form. Such a series of terms is called a Fourier series. The argument of the sine or cosine, which is an angle that depends on space and time, is called the phase,ϕ. So, in Equation(7) we have

$$\phi = k(x \pm vt) + \varepsilon \tag{8}$$

When x and t change together in such a way that ϕ is constant, the displacement $\psi = A\sin\phi$ is also a constant. The condition of constant phase evidently describes the motion of a fixed point on the wave form. Thus, if ϕ is constant

$$d\phi = 0 = k(dx \pm vdt) \Rightarrow \frac{dx}{dt} = \pm v \tag{9}$$

This is the speed of the profile or phase velocity. Take an initial boundary condition $t = 0$, $x = 0$, $\psi = \psi_o \Rightarrow \psi_o = A\sin\varepsilon$

$$\varepsilon = \arcsin(\frac{\psi_o}{A}) \tag{10}$$

Common forms of harmonic waves

$$y = A\sin(k(x \pm vt).$$

$$v = f\lambda.$$

$$y = A\sin(\frac{2\pi}{\lambda}(x \pm \lambda ft).$$

$$y = A\sin 2\pi(\frac{x}{\lambda} \pm ft).$$

$$k = \frac{2\pi}{\lambda},$$

$$y = A\sin 2\pi(\frac{x}{\lambda} \pm \frac{t}{T}).$$

$$\omega = 2\pi f.$$

$$y = A\sin(kx \pm wt).$$

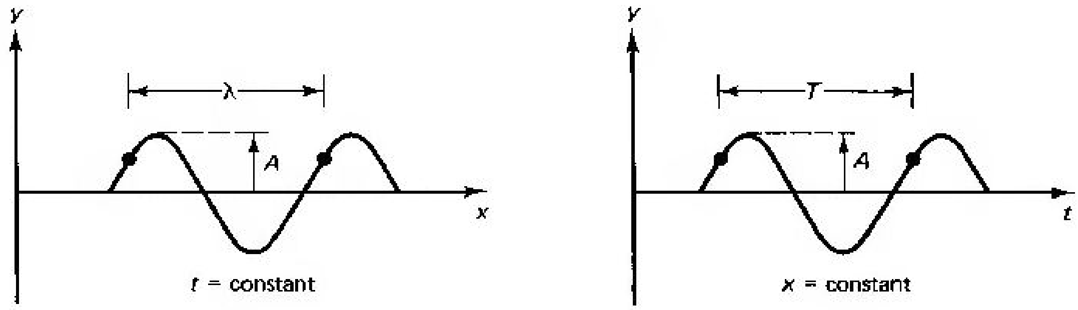


Figure 2: λ is the wave length. It is the distance between two adjacent points, which are in the same state of disturbance. A is the amplitude, the maximum displacement of a wave from its equilibrium position.

2.4 Solution to the wave equation

The mathematical expression fora sinusoidal wave travelling along a stretched string can be written in various ways such as

$$y = A\sin(k(x - vt)) \tag{11}$$

Sinusoidal waves are periodic or repetitive $\rightarrow$ increasing all x by λ in Equation (7) should reproduce the same wave.Mathematically, the wave is reproduced because the argument of the sine function is advanced by 2π.

$$A\sin(k \cdot x + k \cdot \lambda + k \cdot vt + \varepsilon) = A\sin(k \cdot x + k \cdot vt + \varepsilon + 2\pi).$$

Comparing the LHS and RHS gives: $k \cdot \lambda = 2\pi$

$$\Rightarrow k = \frac{2\pi}{\lambda} \tag{12}$$

Alternatively, if the wave is viewed from a fixed position, it is periodic in time with a repetitive temporal unit called the period,T. Increasing all t by T, the wave form is exactly reproduced, so that

$$A\sin(k \cdot x + k \cdot vt + k \cdot T + \varepsilon) = A\sin(k \cdot x + k \cdot vt + \varepsilon + 2\pi).$$

$\Rightarrow k \cdot vT = 2\pi \Rightarrow v = f\lambda$ The frequency is given by

$$f = \frac{1}{T} \tag{13}$$

The angular frequency is given by

$$\omega = 2\pi f \tag{14}$$

The wave number is given by

$$\kappa = \frac{1}{\lambda} \tag{15}$$

2.5 Examples

A sinusoidal wave moving along a string is described by the equation $y(x,t) = 0.0020\sin(10x - 120t)$ in S.I units, where y is in meters is the transverse displacement in the distance along the string in meters and t is the time in seconds. Find: the amplitude of the transverse displacement of the string, the wavelength of the travelling wave, the frequency of oscillation, speed of propagation of the wave

2.6 Representation of Waves using Complex numbers

Take a complex number $z = x + iy$, where i or $j = \sqrt{-1}$. The absolute value of

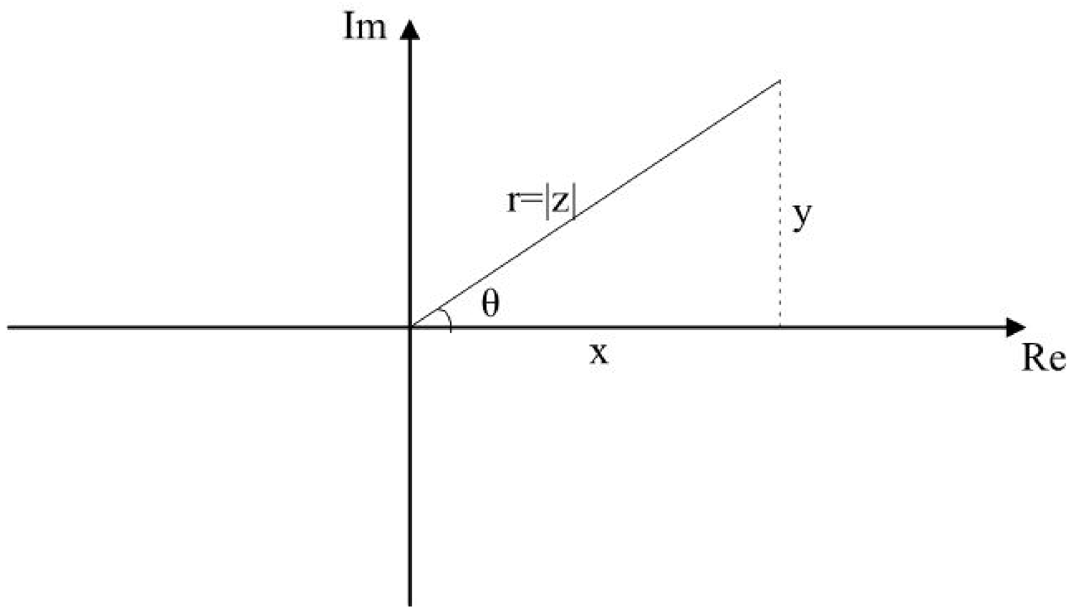


Figure 3: Argand diagram.

modulus is given by

$$r^2 = x^2 + y^2.$$

Resolving x and y on the Argand diagram

$$\Rightarrow x = r\cos\theta, y = r\sin\theta.$$

$$\Rightarrow z = r(\cos\theta + i\sin\theta).$$

By Euler's equation

$$z = re^{i\theta} = r(\cos\theta + i\sin\theta).$$

where $\theta = tan^{-1}\frac{y}{x}$
Complex conjugate

$$z^* = x - iy = r(\cos\theta - i\sin\theta) = re^{-i\theta}.$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}.$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

Euler's formula for simple harmonic waves

$$\psi = Ae^{i(\vec{k}\cdot\vec{x} - \omega t + \varepsilon)} = A\cos(\vec{k}\cdot\vec{x} - \omega t + \varepsilon) + iA\sin(\vec{k}\cdot\vec{x} - \omega t + \varepsilon).$$

2.7 Plane waves

A plane wave exists at a given time when all the surfaces on which a disturbance has a constant phase form a set of planes, each perpendicular to the direction of propagation. The equation of a plane perpendicular to a vector $\vec{k}$ and passing through a point (x_o, y_o, z_o) is given by

$$(\vec{r} - \vec{r_o}) \cdot \vec{k} = 0 \tag{16}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

$$\vec{r_o} = x_o\hat{i} + y_o\hat{j} + z_o\hat{k}.$$

and

$$\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}.$$

$$k_x(x - x_o) + k_y(y - y_o) + k_z(z - z_o) = 0.$$

$$k_x x + k_y y + k_z z = c(\text{constant}).$$

Therefore the equation of the plane perpendicular to $\vec{k}$ is

$$\vec{k}\cdot\vec{r} = \text{constant}.$$

A set of planes over which $\psi(\vec{r})$ varies in space sinusoidally given as

$$\psi(\vec{r}) = A\sin(\vec{k}\cdot\vec{r})$$

or

$$\psi(\vec{r}) = A\cos(\vec{k}\cdot\vec{r})$$

or

$$\psi(\vec{r}) = Ae^{i(\vec{k}\cdot\vec{r})}.$$

We can also have lane harmonic waves and are periodic. Introducing time dependency we have

$$\psi(\vec{r}, t) = Ae^{i(\vec{k}\cdot\vec{r} - \omega t)} \tag{17}$$

2.8 Spherical waves

Surfaces of constant phase are spheres and the direction of waves depends on $\hat{r} \to \vec{k} = k\hat{r}$.

$$r^2 = x^2 + y^2 + z^2,$$

where $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$

$$\theta = tan^{-}\frac{\sqrt{x^2 + y^2}}{z}.$$

$$\phi = tan^{-}\left(\frac{y}{x}\right).$$

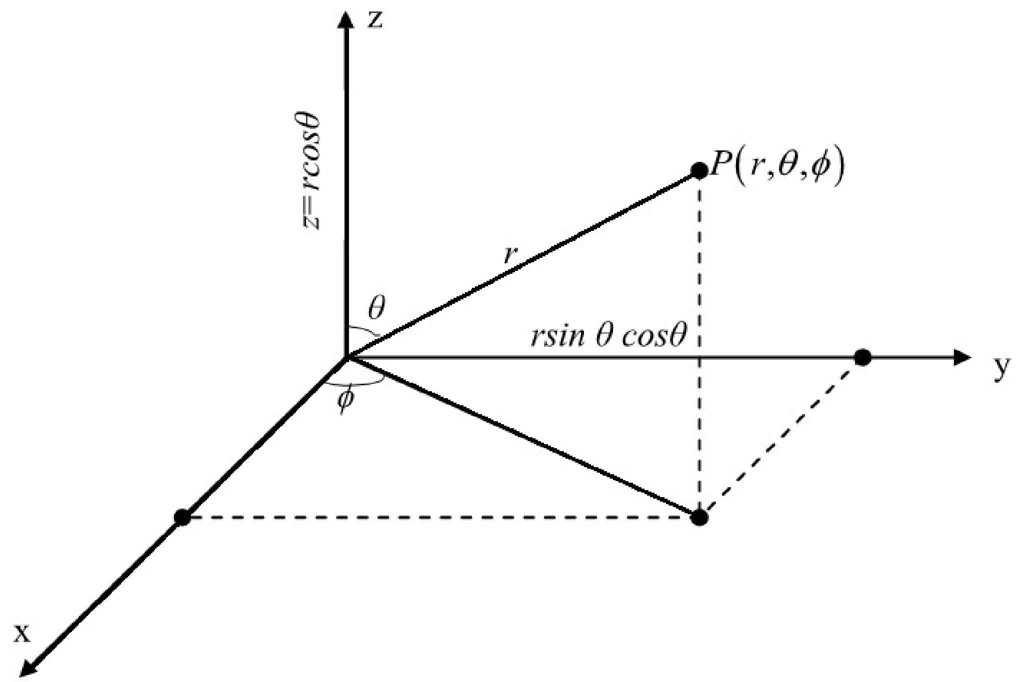


Figure 4: Geometry of spherical coordinates.

The Laplacian operator in spherical coordinates is given by

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}.$$

We consider waves that are spherically symmetric

$$\rightarrow \psi(\vec{r}) = \psi(r,\theta,\phi) = \psi(r).$$

Then, the Laplacian of $\psi(r)$ is given by

$$\nabla^2\psi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) \tag{18}$$

The Equation 18 can be expressed as

$$\nabla^2\psi(r) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi).$$

The differential equation can be written as

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi) \tag{19}$$

This is a one dimensional wave equation whose space variable is r and the wave function is $r\psi$ The solution to equation (19) is

$$r\psi(r,t) = f(r-vt).$$

or

$$\psi(r,t) = \frac{f(r-vt)}{r}.$$

This represents a spherical wave progressing radially outward from the origin at a constant speed v, with an arbitrary function form f. Another solution for the wave converging towards the origin is given by

$$\psi(r,t) = \frac{g(r+vt)}{r}.$$

The general solution to Equation(19) is of the form

$$\psi(r,t) = C_1\frac{f(r-vt)}{r} + C_2\frac{g(r+vt)}{r} \tag{20}$$

A special case of the general solution is the harmonic spherical wave

$$\psi(r,t) = \frac{C}{r}e^{i(kr-\omega t)}.$$

where C is the source strength. The amplitude is a function of r and the term $\frac{1}{r}$ serves as the attenuator. The intensity of the wave is proportional to the square of the amplitude $\rightarrow$

$$\text{Intensity} \propto \frac{\mathrm{C}^2}{\mathrm{r}^2}.$$

$$\text{Powerdensity(powerperunitarea)ofthewave} \propto \frac{1}{\mathrm{r}^2}.$$

WAVE OPTICS

1. Wave Theory of Light

(by Huygens, Fresnel, Young, etc…)

- In geometric optics we learnt light is a stream of straight-going particles (Newton proposed that first)
- Then we learnt light is a form of EM wave.
- But we had learnt all waves have common characteristic properties such as:

Reflection, refraction, interference, diffraction…

- Therefore light waves must have all these properties. Now we will learn:

* Reflection,
* Refraction,
* Dispersion,
* Interference,
* Diffraction and
* Polarization of light waves.

- Actually Huygens had already said light was a form of wave motion, long before Maxwell speculated about EM waves.

2. Properties of Light Waves

- Light waves are transverse [we already know this from EM waves]
- Amplitude of light wave can mean amplitude of electric or magnetic field component, because they are always proportional (E=cB) [But when we speak about amplitude of light waves we generally have electric field component in mind. This is because most of the optical phenomena are caused by this component]
- Color of light is determined by frequency (or wavelength) light waves.

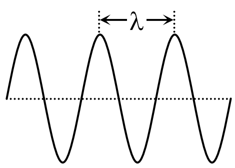
red orange yellow green blue violet	
←	→
lower frequency longer λ	higher frequency shorter λ

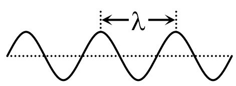
- Brightness of a light wave is determined by amplitude of light wave.

Brightness ~ (Amplitude)^2

So:

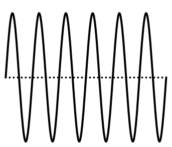
Bright red light: Dim red light:

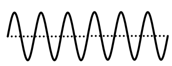




f = f

Bright blue light: Dim blue light:

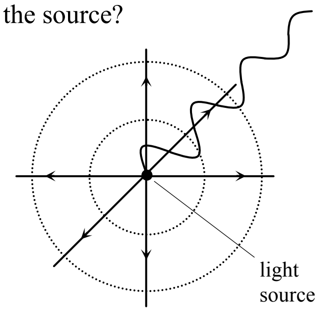




f = f

Ex: [We know that if spring wave loses energy due to friction while traveling v, f , λ do not change. Amplitude decreases.]

For EM wave there is no friction. So why is the brightness (amplitude) decreasing as we go away from the source?



Note: Frequency, wavelength and speed of light waves do not change as they propagate away from the source. Only amplitude decreases.

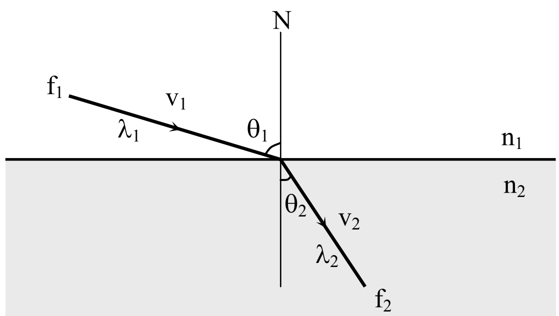
[Otherwise a blue light source would be observed as red from far away]

Ex: Can we say “intensity” in place of “brightness”?

{Remind definition and unit of intensity if needed. Also remind energy transmitted by a wave on a coil spring was proportional to amplitude squared}

Ex: Find the relation between the intensity of light and distance from the source.

3. Refraction of Light Waves



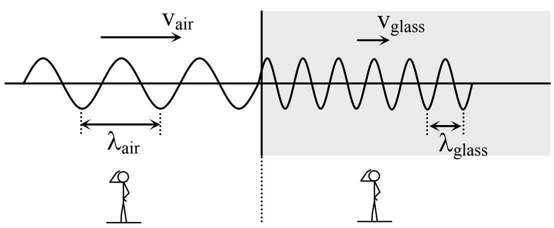
$$\boxed{\frac{Sin\theta_1}{Sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = n_{12}}$$

Rule: When a wave changes medium,

a) Frequency does not change

b) Speed changes

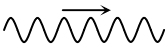
Therefore:



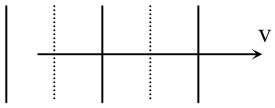
Both observers count the same number of wave crests in one second.

Special case: If light is coming from air

$$\lambda_{glass} = \frac{\lambda_{air}}{n_{glass}} \quad \text{because } n_{air}=1$$

Caution: Drawing this figure  for a light wave does NOT mean that light rays move up and down in the air. [The figure is trying to say that electric field at a point is increasing and decreasing (oscillating) as the light passes by. This oscillation itself is called light.

Remember: In water waves,



each water molecule is moving up and down as a crest or trough passes by. But we do not say the water wave is following a sinusoidal path. When we are asked to draw the path of the wave, we draw a straight arrow showing direction of motion in general, not motion of particles. And since the wave is transverse, direction of motion is perpendicular to up-down motion of particles.]

4. Dispersion of Light Waves

Dispersion means dependence of index of refraction of a medium on the frequency (or wavelength) of the incident light.

That is, for example:

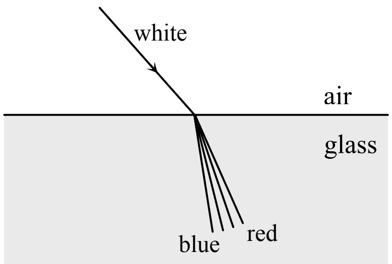
$n_{glass} \neq 1.5 = \text{constant}$

$n_{glass} = n(\lambda)$

for red light $n_{glass} = 1.513$

for blue light $n_{glass} = 1.528$

Therefore:



5. Interference of Light Waves

Coherence:

If two wave sources are "coherent" they always have the same phase difference between them. [If they are in phase at the beginning, they are always in phase. If they start 180° out of phase they will still be 180° out of phase 10 minutes later]

[If two waves are coherent at a point in space, they always have the same phase difference at this point in space]

- If two wave sources are coherent, the interference pattern is stable and observable. [Nodes and antinodes will always be at the same place, we will be able to see them]
- If two wave sources are incoherent, the interference pattern is not observable. [Think about central line in ripple tank. Suppose now the sources are in phase, and central line is an antinode, (say) 0.23 second later sources become completely out of phase, central line becomes a node, (say) 0.36 second later they become in phase again and central line is an antinode. So we will not see any interference pattern]

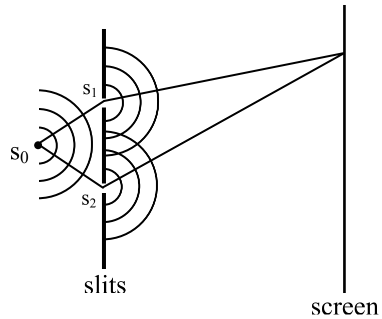
Question: [When we have two wave sources on water, we see several nodes on water surface where waves from two sources cancel]. Why don't we ever see light waves from two lamps cancel each other and some points in the room become dark (= node)?

Answer: {Explain the reason, why two light bulbs (or any other ordinary light sources) can never be coherent, then ask the students to find a way for obtaining two coherent light sources}

5.a. Young's Experiment

To obtain two coherent light sources Thomas Young used one single light source and made the light pass through two slits (slit = a long narrow opening). Now each slit is like a light source.

Note: To obtain a better visible interference pattern sources must be monochromatic (of one color = having single λ).



On the screen we see: We simply draw:

Note: These dark and bright bands are called "fringes".

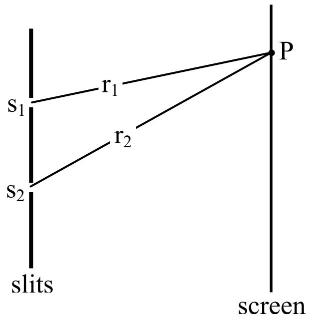
Remember: Node for water ⇒ Dark for light

Antinode for water ⇒ Bright for light

[Actually there are no definite boundaries between dark and bright fringes. Only the center of a dark fringe is totally dark and center of a bright fringe is maximum bright]

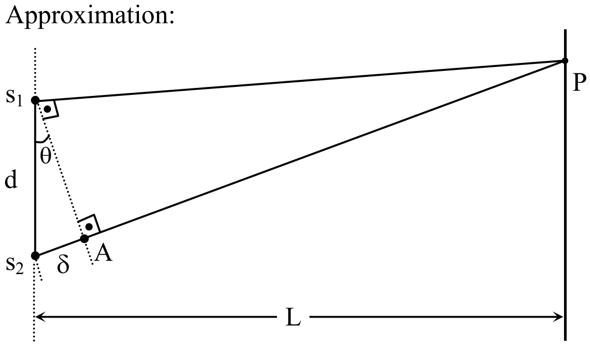
Class demo: Hold a thin glass plate over the flame of a candle. When it is black enough draw two slits with a razor blade. Illuminate the slits with a laser beam. Use a white paper 1 meter away as the screen.

Path difference (δ):



$$\boxed{\delta \equiv |r_1 - r_2|}$$

Finding δ from geometry:
Approximation:



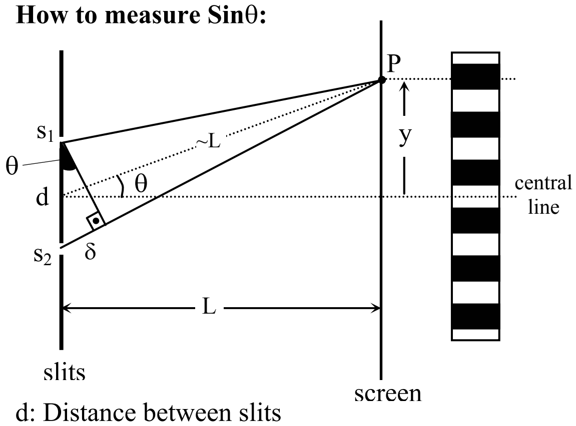
[d: Distance between sources
L: slits-screen distance]
Since L ~ 1 meter
d ~ 0.1 mm;
L>>>d so we can take $s_1P \parallel s_2P$

therefore: $|s_1P| \approx |AP|$
therefore: $|s_2A| = |r_1 - r_2| = \delta$

therefore: $$\boxed{\delta = d\ Sin\theta}$$

[This formula seems to be totally useless, because we can not even see θ let alone measuring it. But:]

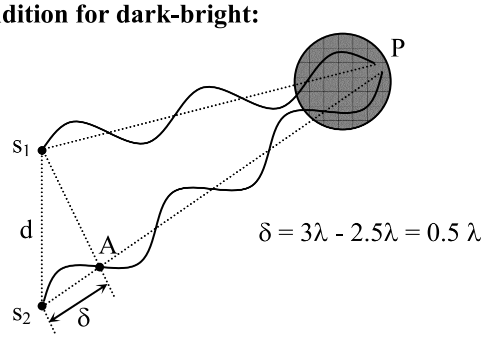
How to measure Sinθ:



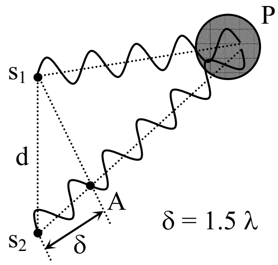
d: Distance between slits
L: Slits-screen distance
y: Distance from central line to a point on a fringe

$$\boxed{\delta = d\ Sin\theta} \Leftrightarrow \boxed{\delta = d\ \frac{y}{L}}$$

Condition for dark-bright:



δ = 3λ - 2.5λ = 0.5 λ



δ = 1.5 λ

dark	bright
δ = 0.5 λ	δ = 0
δ = 1.5 λ	δ = 1 λ
δ = 2.5 λ	δ = 2 λ

<u>Dark</u>

$$\delta = \left(m - \frac{1}{2}\right)\lambda$$

$$d\ Sin\theta = \left(m - \frac{1}{2}\right)\lambda$$

$$d\ \frac{y}{L} = \left(m - \frac{1}{2}\right)\lambda$$

$$\boxed{y_m = \left(\frac{\lambda L}{d}\right)\left(m - \frac{1}{2}\right)}$$

$$m = 1, 2, 3 ...$$

<u>Bright</u>

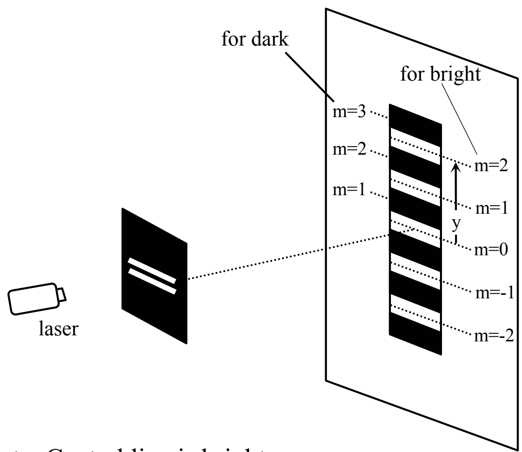
$$\delta = m\lambda$$

$$d\ Sin\theta = m\lambda$$

$$d\ \frac{y}{L} = m\lambda$$

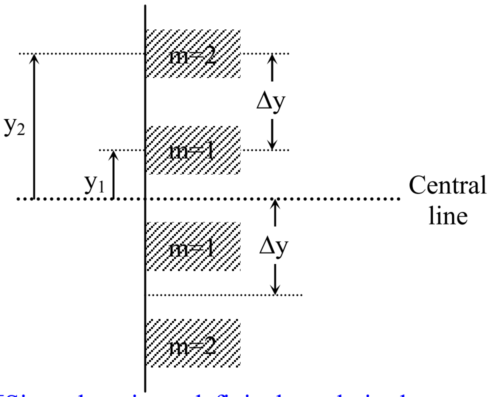
$$\boxed{y_m = \left(\frac{\lambda L}{d}\right)m}$$

$$m = \underbrace{0}_{\text{central line}}, 1, 2, 3 ...$$



Note: Central line is bright
Note: We have m=2 for dark, m=2 for bright
Note: y starts from central line
Note: m is always integer

Fringe width (Δy):



[Since there is no definite boundaries between dark and bright fringes, we take the region between two absolute darks (at the center of the dark fringe) as the width of a bright fringe.]

$\Delta y = y_2 - y_1$

$$\Delta y = \left[\frac{\lambda L}{d}\left(2-\frac{1}{2}\right)\right]-\left[\frac{\lambda L}{d}\left(1-\frac{1}{2}\right)\right]$$

$$\boxed{\Delta y = \frac{\lambda L}{d}}$$

Note: Units used for λ.
1 μm = 10^{-6} m (micrometer)
1 nm = 10^{-9} m (nanometer)
1 Å = 10^{-10} m (angstrom)

Ex:

Color	A	nm	m
red	6000	600	6×10^{-7}
blue	4000	400	4×10^{-7}

Ex: 6000 A laser light passes through two slits 0.1 mm apart and reaches the screen placed 2 m away.
a) Find fringe width
b) Find position of second dark
c) Find position of third bright
{Draw figure after solution}

Ex: Laser light (5000 A) passes through a double slit arrangement 0.05 mm apart. The screen is 1 m away from slits.
a) Find fringe separation (=fringe width)
b) Find distance between 2nd bright and 3rd dark on opposite sides.
{Draw figure during solution}

Ex: What can we do to obtain a better visible pattern in Young's experiment?
{Explain effect of changing λ, d and L on Δy. Draw two example patterns for small and large d}

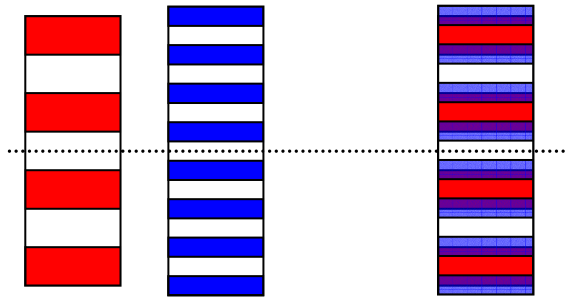
[Result: Slits closer, fringe centers distant. Slits distant, fringe centers closer]

Ex: Suppose while performing double-slit experiment, the space between the slits and the screen is filled with water. How does the interference pattern change?

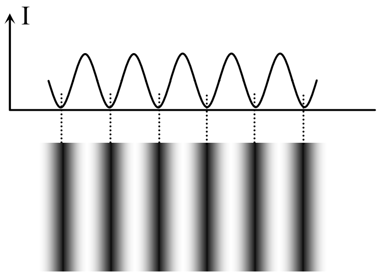
Ex: A double-slit arrangement is illuminated first with red, then with blue light.
a) Which one has wider fringes?
b) Which one produces more fringes?

Ex: What happens if we use white light in place of monochromatic light in Young's experiment?
Answer: Think about light of two color only (red-blue)

only red: only blue: together:



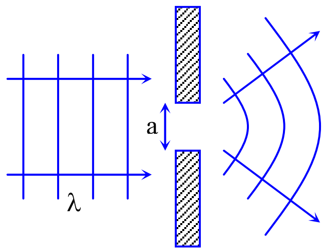
Intensity Distribution:



Rule: In double-slit interference, all fringes are equally bright and wide. {Actually we are neglecting diffraction effects for the time being. We take the slits sufficiently small themselves so as to make diffraction effects negligible. See N-slit diffraction}

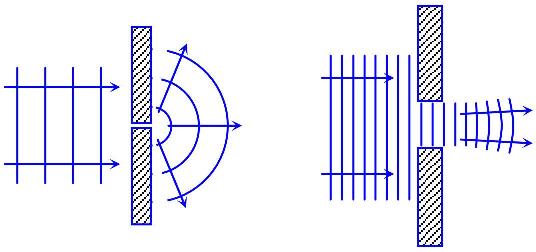
6. Diffraction
Diffraction is bending of waves around an obstacle (barrier) [or spreading of waves passing through a narrow slit]

[We had seen diffraction with water waves



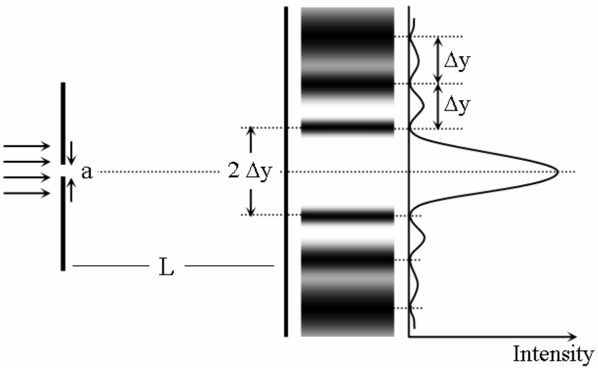
Diffraction amount depends on $\frac{\lambda}{a}$ proportion.

If $a >> \lambda$ diffraction is negligible.



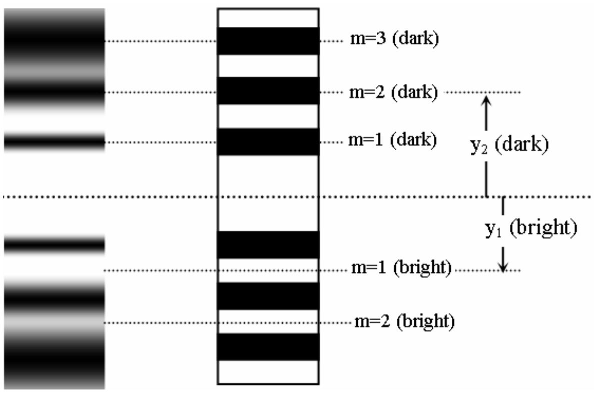
Same phenomenon is observable with light waves. Since λ of light is very small, the openning must also be very small, something like 0.1 mm]

Single slit diffraction:



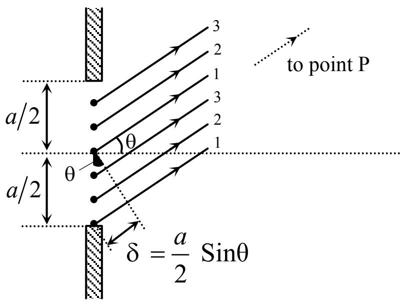
[Most of the light energy is concentrated at the central maximum. Actually it is possible to say that all the light passing through the slit is spread as wide as the central maximum simply omitting the other bright fringes]

Actual pattern: We simply draw:



[We still have dark fringes althought there is only one slit. Therefore light waves coming from different portions of the slit must be canceling]

If we divide the slit into two equal portions:



Now condition for first dark:

$$\delta = \frac{\lambda}{2} \Rightarrow \frac{a}{2} Sin\theta = \frac{\lambda}{2}$$

$$\boxed{a\ Sin\ \theta = \lambda \text{ (first dark)}}$$

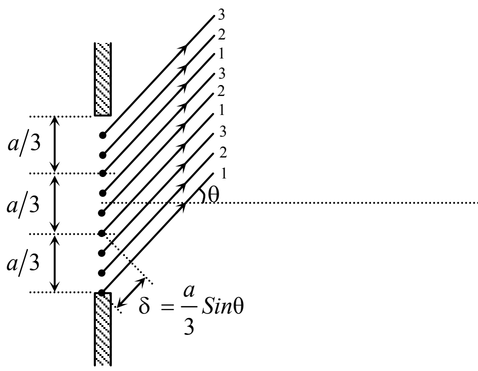
[First dark is important, because between two first darks we have the central bright, which receives nearly all the light energy passing through the slit]

if we divide the slit into 4, 6, 8, (even number) equal parts [and set δ=λ/2 we will have (a/4)Sinθ=λ/2, (a/6)Sinθ=λ/2, (a/8)Sinθ=λ/2…] we get condition for other darks {explain relation between even number and dark}:

Dark	Bright
a Sinθ = mλ	a Sinθ=(m+1/2)λ
$a\frac{y_m}{L} = m\lambda$	$a\frac{y_m}{L} = \left(m + \frac{1}{2}\right)\lambda$
$\boxed{y_m = \frac{\lambda L}{a} m}$	$\boxed{y_m = \frac{\lambda L}{a}\left(m + \frac{1}{2}\right)}$
m=1,2,3…	m=1,2,3…

[We don't have m=0 for central bright. Central bright is determined by position of first darks]

<u>Ex:</u> How have we found condition for brights?
Sol: Divide the slit into 3, 5, 7, (odd number) parts:
For first bright (m=1) we divide the slit into 3 equal portions. [Because "dividing" into 1portion gives us the central bright]



For first bright (m=1) $\delta = \frac{a}{3}\ Sin\theta = \frac{\lambda}{2}$ waves from two portions cancel but the remaining third portion illuminates the point on screen. So for first bright

$$m = 1 \Rightarrow a\ Sin\theta = \frac{3}{2}\lambda = \left(m + \frac{1}{2}\right)\lambda$$

<u>Ex:</u> Derive fringe seperation formula Δy=?

$\Delta y = \frac{\lambda L}{a}$ [same between centers of brights and darks, only central bright 2Δy]

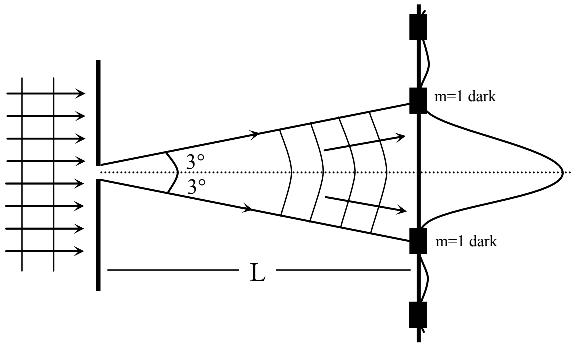
<u>Ex:</u> 5000 A monochromatic light passes through a slit having 0.05 mm width. How much does it spread?

Sol: θ_1 for first dark:
a $\sin\theta = m\lambda$

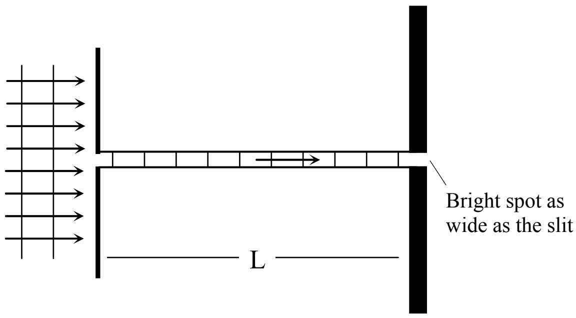
$m=1 \Rightarrow a \sin\theta = \lambda$

$$\boxed{\sin\theta_1(dark) = \frac{\lambda}{a}}$$

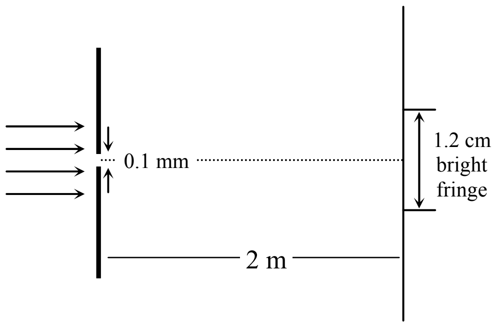
$\sin\theta_1 = 5 \times 10^{-7} / 10^{-5} = 0.05$
$\theta_1 \approx 3°$



If there wasn't diffraction:



Ex: Monochromatic light (λ=6000 A) passes through a slit 0.1 mm wide and illuminates a screen 2 m away. Find width of central bright on screen.
Answer:
$2\Delta y$= 12 mm =1.2 cm



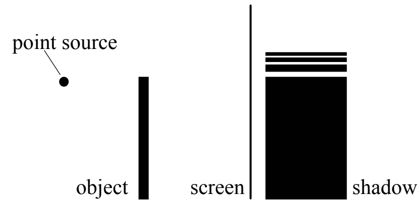
If there wasn't diffractionthere would be a bright spot 0.1 mm wide on the screen.

Ex: Laser light having 6000 A wavelength passes through a slit 0.2 mm wide. On a screen placed 1 m away find
a) Distance from central line to second bright
b) Distance between second dark and third bright on different sides.

Ex: What are the effects of diffraction?
* We can not send a light ray along a straight path for a long distance. It will spread and lose intensity.
[Actually this is the case for any type of EM wave]
[Imagine otherwise, we would be able to send mors code messages to an astronout on the moon by using a simple diode laser.]

* We don't have sharp shadows of objects even with a point light source.

Ex: Diffraction from an edge (not a slit)

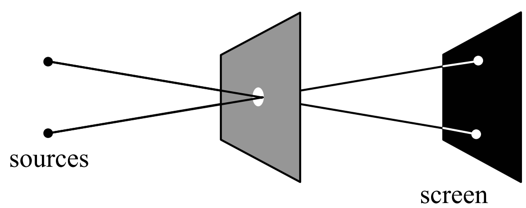


Ex: What is the minimum slit width for no diffraction minimum (dark fringe) to be observed?

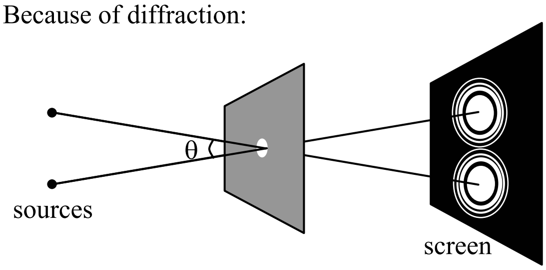
Note: Boundary between geometric optics and wave optics:
There is no definite limit. Depends on:
- Width of light source
- Distance light travels
{Explain using the example below}

Ex: What is the maximum slit width for diffraction?
[Answer: If the light source is coherent, diffraction always occurs for all openings even if the slit is large.
But according to formula: $Sin\ \theta = \frac{\lambda}{a}$ (first dark) , if $a >> \lambda$, then θ is very small. So diffraction effect becomes negligible over small distances. The light follows nearly a straight path as wide as the slit for small distances if the slit dimension is large. But over large distances a small angle causes a a large seperation. If we are trying to send a 5 mm wide laser ray from earth to moon for example, the spreading of the beam will be ~ 0.01°, which is negligible at the beginning. But when it reaches the moon, the beam will be as wide as ~ 80 km!.]

7. Resolving Power
Two light sources are seen as a single source if they are far away enaugh. [Many bright dots in the night sky are actually star pairs – not sinle stars. Enother example can be the two headlights of a car approaching from a distance]
The reason is diffraction. When light from the sources passes through the pupil of the eye, [which is a circular opening of ~2-3 mm] diffraction occurs. The retina acts as a screen.
If there was not diffraction:



Because of diffraction:

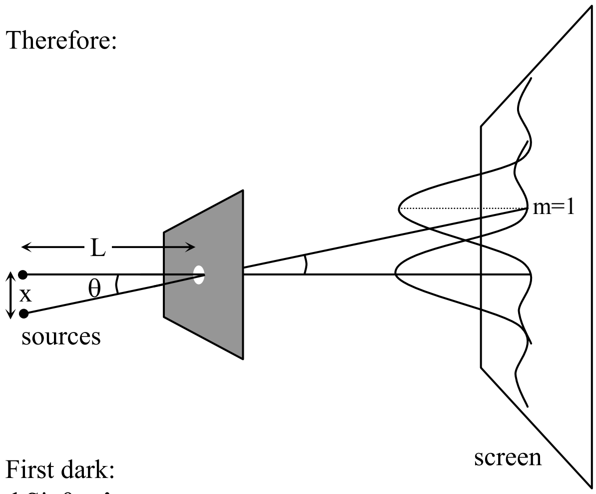


When θ gets smaller patterns overlap and seen as one:

resolved | not resolved (seen as one)

Rule: Two sources seen as one when central bright of one pattern is on the first dark of the other.

Therefore:



First dark:

d Sinθ = λ ⇒

$Sin\theta = \frac{\lambda}{a}$ $\qquad \frac{x}{L} > \frac{\lambda}{a}$ two sources seen

$\frac{x}{L} = \frac{\lambda}{a}$ (just resolved) $\qquad \frac{x}{L} < \frac{\lambda}{a}$ seen as one source

{Actually these formulas are for slits, and can be used for a cat for example. For circular apertures we have a factor of 1.22 which we neglected here}

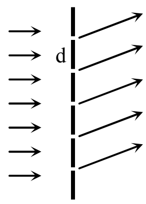
Ex: From what distance can we see two headlights of a car as two?
Distance between lights 1.5 m, take λ=5000A.
[The actual distance is much smaller due to other (such as atmosheric conditions. Diffraction is the ultimate limit in our 'seeing' power and since there are always other factors limiting our vision we are seldom limited by diffraction effects]

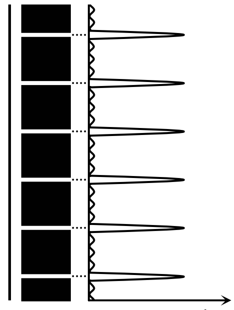
[**Ex:** Explain whys we can't ever see an atom with normal light no matter how powerful a microscope we use. That is, explain how diffraction puts a limit to seeing small objects]

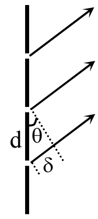
Ex: Explain why very large dishes are used for radiotelescopes.

8. Diffraction Grating

{Demo: N-slit diffraction java applet}
[The diffraction grating is a more useful device to analyse lights sources, because the interference maximums (bright fringes) are thin lines, making the measurements easier]







δ = d Sinθ
Therefore; m'th BRIGHT fringe:
$\boxed{d\ Sin\theta = m\ \lambda}$ $\qquad$ (m = 0, 1, 2, 3 …)

[Therefore we can use two slit formulas]
[We are not writing formula for dark fringe because dark friges are actullly wide dark bands between two bright lines]

Ex: A diffraction grating has 500 slits in 1 cm.
a) Find slit spacing
b) Find λ of monochromatic light if first maximum (bright fringe) occurs 3.5 cm from the central line on a screen 1 m away.

Ex: Monochromatic light of 650 nm wavelength is incident on diffraction grating having $2x10^{-6}$ m slit spacing.
a) How many bright lines will be observed?
b) What is the angular position of the first diffraction fringe ?

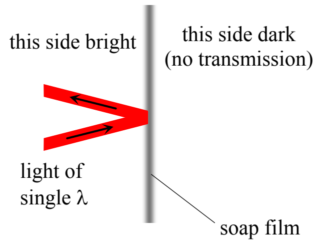
Ex: What is the path difference for the light waves forming the bright fringe at 30° from the central bright? Slit spacing of the diffraction grating is 0.05 mm.

Ex: A diffraction grating is illuminated by mixed red and blue light. Second bright of red coincides with the third bright of blue. Find λ_{blue}=?, if λ_{red}= 6000 A.

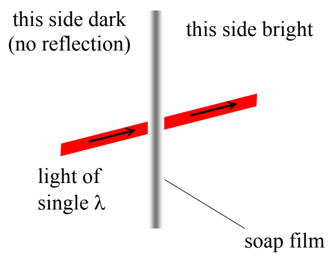
Ex: How many slits in 1 mm must a diffraction grating have, if it is to be used to analyse light having wavelength around 0.5 μm?

9. Interference in thin films

Extreme case 1:

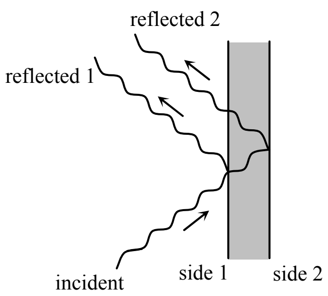


Extreme case 2:



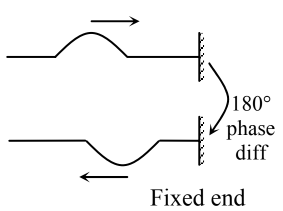
In intermediate cases light is partly transmitted, partly reflected.
{Ask students: soap bubble is normally transparent, how can it stop light. Why soap bubble? Is it because it is very thin?}

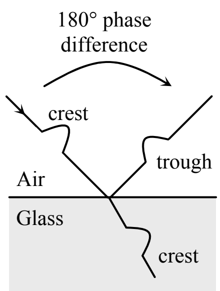
A soap bubble has two sides:

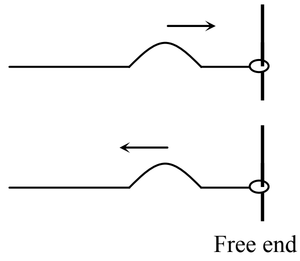


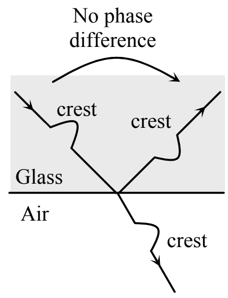
Light rays reflecting from two sides can cancel or reinforce according to phase difference between them.

Rule:
Remember waves on a spring. Light waves have the same property.



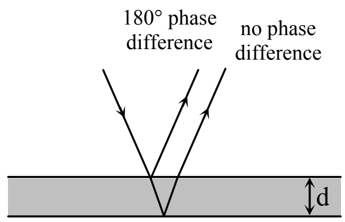






[Rule: light rays undergo 180° phase change upon reflection from an optically denser (with greater index of refraction) medium.]

Formula:
For observer looking from above:



[So between two reflected rays there is 180° phase diff]

$$180^{\circ}\text{ phase difference} \Leftrightarrow \frac{\lambda}{2}\text{ path difference}$$

For destructive interference (dark):

$$\text{Path difference} = (m-\frac{1}{2})\lambda_{film} \qquad \boxed{\lambda_{film} = \frac{\lambda_{air}}{n_{film}}}$$

{Explain 2d}

$$\underbrace{2d \pm \frac{\lambda_{film}}{2}}_{\text{equivalent path difference}} = (m-\frac{1}{2})\lambda_{film}$$

[We can add or subtract λ/2. This just means we take one wave as being λ/2 in front of or behind the other, which is not important because the situation is symmetrical. We will use the minus sign, because when we use the minus sign, we can start from m=0 ⇒ zero thickness. Otherwise we would start from m = -1,which is possible but not very nice. Remember m is just a counting number, 1st order dark, 2nd order dark etc.]

Looking from above:
[Looking from above means the light source and the observer are on the same side of the soap film]

$$\boxed{2d = m\lambda_{film} \Leftrightarrow \text{Dark}} \quad m = 0, 1, 2, 3, 4\ldots$$

Therefore:

$$\boxed{2d = \left(m-\frac{1}{2}\right)\lambda_{film} \Leftrightarrow \text{Bright}} \quad m = 1, 2, 3, 4\ldots$$

[m=0 ⇒ zero thickness. How can this happen? We will see in a minute]

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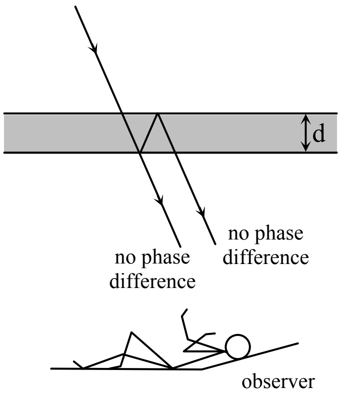
For observer looking from below:



For destructive interference (dark):

Path difference $= (m - \frac{1}{2})\lambda_{film}$

$$\boxed{2d = \left(m - \frac{1}{2}\right)\lambda_{film} \Leftrightarrow \text{Dark}} \quad m = 1, 2, 3, 4\ldots$$

Therefore:

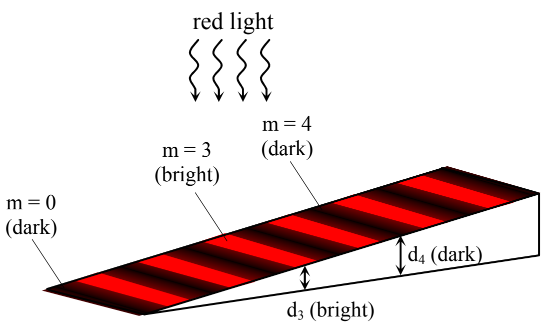
$$\boxed{2d = m\lambda_{film} \Leftrightarrow \text{Bright}} \quad m = 0, 1, 2, 3, 4\ldots$$

Formulas changed place
Therefore:
One side dark ⇔ other side bright.

Ex: 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to be able to pass to other side.
Ex: 6000 A laser light is incident on a soap film (n=1.5). What is the minimum thickness of the film for the light not to reflect back from the film surface.

Ex: 6000 A laser light is incident on a soap film (n=1.5). Find three different thicknesses the film might have, if the light is not reflecting back.

Ex: Film of changing thickness. [If you hold a soap film vertically lower side becomes thicker.]



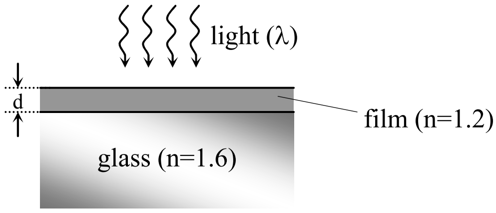
Ex: Explain why we see many different colors over a soap film.

Ex: Lenses used in a camera are generally coated with a thin film of definite thickness. Why?

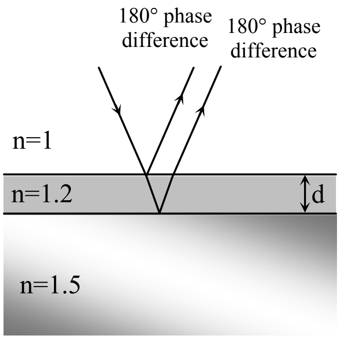
{Film thickness is adjusted to wavelength of yellow light, since it is the most intense component of sunlight}

Ex: Solar cells are also coated with thin films. Why?

Ex: **Thin coating.** Find the formula for thickness of film, if no light of wavelength λ is to reflect back.



Solution:



Dark:

$$2d = \left(m - \frac{1}{2}\right)\lambda$$

$$m = 1, 2, 3...$$

10. Air Wedge

α is the angle subtended at an unaided eye by the object.

NOTE:

Unaided eye is when the object is viewed without using an instrument.

Microscopes

These are used to view near objects

Angular magnification of microscopes $m = \frac{\alpha^1}{\alpha}$

α is the angle subtended at the eye object at the near point when microscope is not used.

α^1 is the angle subtended at the eye by image when microscope is used.

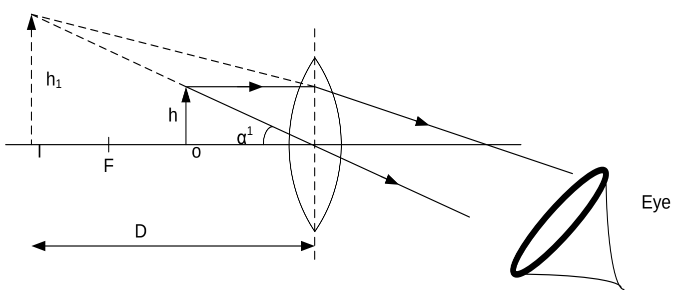
In normal adjustment or use, the microscope forms the image at the near point.

Simple microscope / magnifying glass

This consists of a single convex lens with the distance between the object and the lens less than or equal to the focal length of the lens.

Simple microscope with image at near point (normal adjustment)

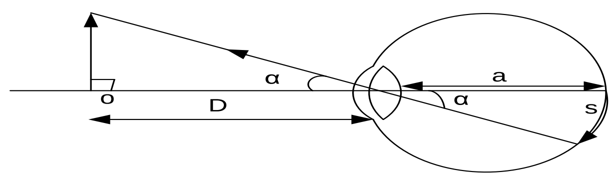
A simple microscope in normal adjustment consists of a converging lens set in such a way that it forms a virtual magnified erect image of an object placed between the principal focus and the optical centre of the lens at the least distance of distinct vision as shown.



For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{D}$

If α is the angle subtended at the eye by the object at the near point

Before using a microscope, the object is first viewed at the near point of the eye by unaided eye as shown:



$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{D}\right)}{\frac{h}{D}} = \frac{h_1}{h}$$

hence

$$\boxed{m = \frac{h_1}{h}}$$

but $m = \frac{v}{f} - 1$

There fore angular magnification, $\boxed{m = \frac{D}{f} - 1}$

where $v = D$

EXAMPLE:

Calculate the angular magnification produced by a magnifying glass of focal length **5cm** adjusted such that an image is formed at a distance of **25cm** in front of it.

Solution:

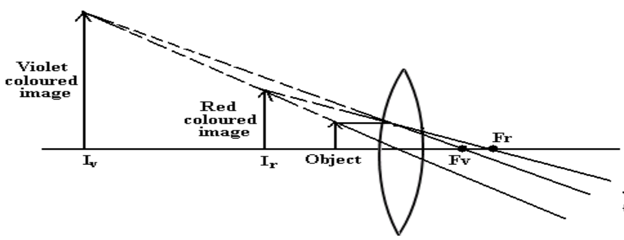
$m = \frac{D}{f} - 1$ but $D = -25cm$

$$m = \frac{-25}{5} - 1$$

$$\boldsymbol{m = -6}$$

Thus the required angular magnification is **6**

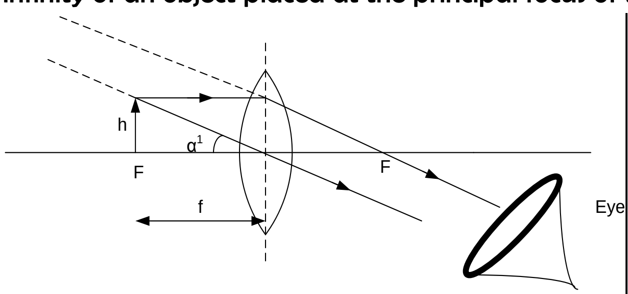
Explain why chromatic aberration is not experienced in magnifying glass



When an object **O** is viewed through a converging lens used as a magnifying glass, various coloured virtual images corresponding to say red and violet rays are formed at slightly different positions $\mathbf{I_r}$ and $\mathbf{I_v}$ respectively as shown. These images subtend the same angle at the eye and therefore appear superimposed. Thus the virtual image seen in a simple microscope is almost free from chromatic aberration.

Simple microscope with final image at infinity (not in normal adjustment)

This simple microscope consists of a converging lens which forms an erect virtual magnified image at infinity of an object placed at the principal focus of the lens as shown.



Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{f}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{f}\right)}{\frac{h}{D}} = \frac{D}{f}$$

Hence angular magnification, $\boxed{m = \frac{D}{f}}$

Note: (i) Angular magnification is higher when a simple microscope forms the image at infinity

(ii) For higher magnification, use lenses of short focal length.

Example

1. A thin converging lens of focal length 10.0cm is used as a magnifying glass. In one instance it is required that the final image to be formed at infinity and the other to be formed at 30.0cm from the lens. Find;
   (i) Angular magnification when the image is at infinity
   (ii) Position of the object when the image is at 30cm from the lens and its angular magnification

Solution

(i) $m = \frac{D}{f}$

$$m = \frac{-25}{10} = -2.5$$

(ii) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{10} = \frac{1}{u} + \frac{1}{30}$$

$$v = 15cm$$

$$m = \frac{D}{f} - 1$$

$$m = \frac{-25}{10} - 1 = -3.5$$

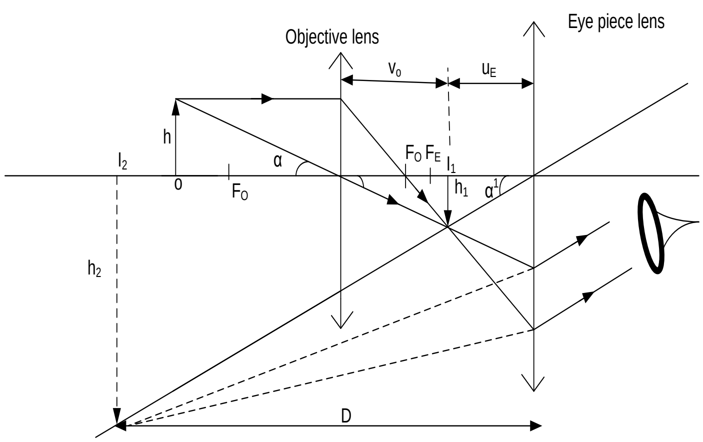
Compound microscopes

This is used to give a greater magnifying power than the simple microscope. It of two converging lenses, namely the objective (which is near the object) and the eye piece, near the eye.

Compound microscope in normal adjustment.

A compound microscope consists of two converging lenses of short focal lengths. This enables a high angular magnification to be obtained.

In normal adjustment, the objective of a compound microscope forms a real inverted image of the object at a point distance less than Fe from the eyepiece. This intermediate image formed acts as a real object for the eye piece which thus forms a virtual magnified image at a distance of distinct vision from the eye piece as shown.



The objective lens forms a real image I_1, of the object O . I_1 is formed at a point nearer the eye piece than the principal focus f_e of the eye piece.

The eye piece acts as a magnifying glass. It forms a virtual image I_2 of I_1. The observer's eyes should be taken to be close to the eye piece so that α' is the angle subtended at the eye by the final image I_2.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_2}{D}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\frac{h}{D}} = \frac{h_2}{h}$$

Multiplying h_1 and dividing by h_1

$$m = \frac{h_2}{h} x \frac{h_1}{h_1}$$

$$m = \frac{h_2}{h_1} x \frac{h_1}{h}$$

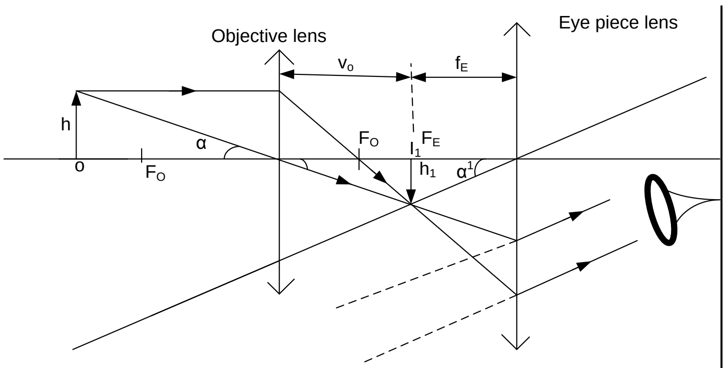
$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{v_o}{f_O} - 1\right) x \left(\frac{D}{f_E} - 1\right)}$$

Note : For higher angular magnification, both the eye piece and the objective should have short focal lengths.

Compound microscope not in normal adjustment

The objective forms a real inverted image of the object at the principle focus Fe of the eye piece which thus forms a final virtual magnified image at infinity as shown.



The separation of the object and the eye piece is such that the object forms an image of the object at the principle focus Fe of the eye piece, hence the eye piece focuses the final image at infinity.

The angle α' subtended by the final image by the eye piece is

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{f_E}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{f_E}\right)}{\frac{h}{D}} = \frac{D}{f_E} x \frac{h_1}{h}$$

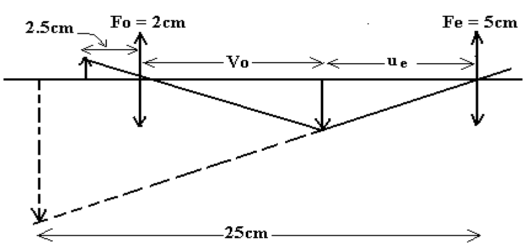
$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{v_O}{f_O} - 1\right) x \left(\frac{D}{f_E}\right)}$$

EXAMPLES:

1. The objective of a compound microscope has a focal length of **2cm** while the eyepiece has a focal length of **5cm**. An object is placed at a distance of **2.5cm** in front of the objective. The distance of the eyepiece from the objective is adjusted so that the final image is **25cm** in front of the eyepiece. Find the distance between the lenses and the magnifying power of the microscope.

Solution:



Consider the action of the eyepiece

$V_E = -25cm$ and $f_E = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{u_E} + \frac{1}{-25}$$

$$u_E = 4.167cm$$

Consider the action of the objective

$u_o = 2{\cdot}5cm$ and $f_O = 2cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{2} = \frac{1}{2.5} + \frac{1}{v_O}$$

$$v_O = 10cm$$

$\therefore$ **The required lens Separation**

$$= v_O + u_E = (10 + 4 \cdot 2)\ cm = \mathbf{14{\cdot}2cm}$$

The required magnifying power

$$m = m_o x m_E$$

$$m = \frac{10}{2.5} x \frac{25}{4.167}$$

$$m = 24$$

Alternatively

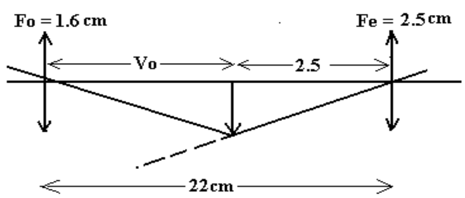
$m = \left(\frac{v_o}{f_O} - 1\right) x \left(\frac{D}{f_E} - 1\right)$ where **D = -25cm**

$$m = \left(\frac{10}{2} - 1\right) x \left(\frac{-25}{5} - 1\right)$$

$\Rightarrow$ $\therefore$ $\boldsymbol{M} = -\mathbf{24}$ Thus the required magnifying power **M = 24**

2. A compound microscope has an eyepiece of focal length **2·5cm** and an objective of focal length **1·6cm**. If the distance between the objective and the eye piece is **22cm,** calculate the magnifying power produced when the object is at infinity.

Solution



For the image to be at infinity, the object must be at the focal point of the eyepiece

Thus the image distance in the objective

$= (22 - 2{\cdot}5)\ cm\ =\ \mathbf{19{\cdot}5cm}$

$m = \left(\frac{v_o}{f_O} - 1\right) x \left(\frac{D}{f_E}\right)$ where $\boldsymbol{D}\ =\ \mathbf{-25cm}$

$$m = \left(\frac{19.5}{1.6} - 1\right) x \left(\frac{-25}{2.5}\right)$$

$\Rightarrow \quad \therefore \quad \boldsymbol{M}\ =\ -111{\cdot}875$

Thus the required magnifying power

$$\boldsymbol{M}\ =\ \mathbf{111{\cdot}875}$$

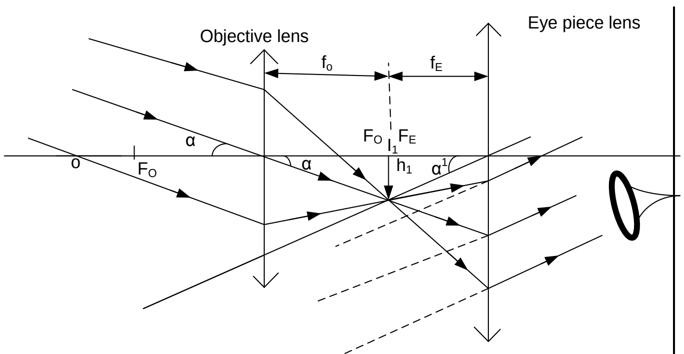
TELESCOPES

Telescopes are used to view distant objects. The angular magnification of a telescope is the ratio of the angle subtended by the final image at the aided eye to the angle subtended by the object at the un aided eye. In normal adjustment, the final image is at infinity.

REFRACTING ASTRONAMICAL TELESCOPE IN NORMAL ADJUSTMENT

A telescope is in normal adjustment when the final image of a distant object is formed at infinity.
An astronomical telescope consists of f two converging lenses; one is an objective of long focal length and the other an eyepiece of short focal length. This enables a high angular magnification to be obtained.

In normal adjustment, the objective forms a real inverted image of a distant object at its focal point $\mathbf{F_o}$ situated exactly at the principal focus $\mathbf{F_e}$ of the eyepiece. This intermediate image acts as a real object for the eyepiece to give rise to a final virtual image at infinity as shown.



In normal adjustment, the image of the distant object formed by the objective lens lies in the focal plane of both the objective and the eye piece.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{f_E}$

$$\alpha \approx tan\alpha = \frac{h_1}{f_O}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{f_E}\right)}{\left(\frac{h_1}{f_O}\right)} = \frac{f_O}{f_E}$$

$$\boxed{m = \frac{f_O}{f_E}}$$

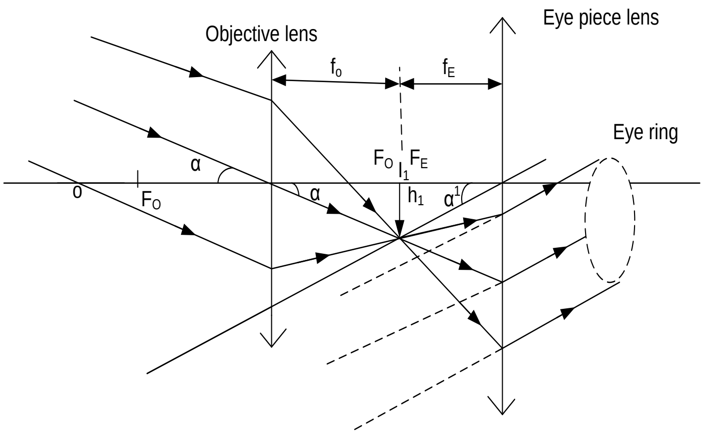
For a high magnifying power, the objective should have a long focal length and the eyepiece a short focal length.

Hence separation between the lenses $= f_o + f_E$

Eye ring/ Exit pupil

Eye ring is the best position for the eye when viewing an image through the instrument.

At the exit pupil, the eye receives a maximum amount of light entering the objective from out side so that its field of view is greatest



Note: when determining the eye ring, the separation is taken as the object distance and focal length of the eye piece is used in calculations. Hence from the above

hence ***v***, which is the eye ring can be obtained.

The eye ring, and relation to angular magnification

$u = f_o + f_E$, $f = f_E$

Then use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f_E} = \frac{1}{f_o + f_E} + \frac{1}{v}$$

$$v = \frac{f_E}{f_o}(f_E + f_o)$$

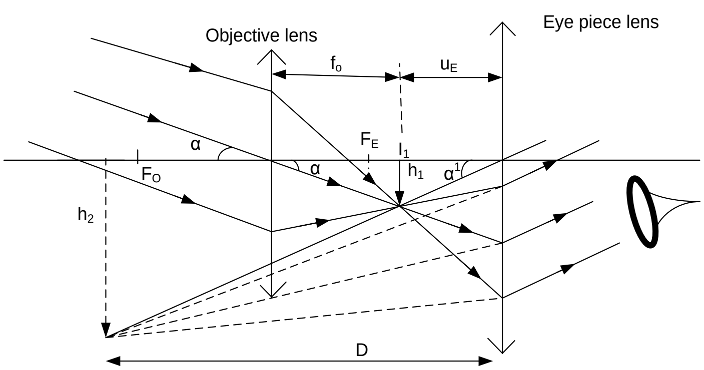
$$\frac{diameter\ of\ the\ eye\ ring}{diameter\ of\ the\ objective} = \frac{v}{u} = \frac{\frac{f_E}{f_o}(f_E + f_o)}{(f_o + f_E),} = \frac{f_E}{f_O}$$

hence angular magnification,

$$m = \frac{diameter\ of\ the\ objective}{diameter\ of\ the\ eye\ ring} = \frac{f_o}{f_E}$$

The above expression for the magnifying power is only true for a telescope in normal adjustment with lens separation $f_o + f_E$.

Astronomical telescope with image formed at near point (not in normal adjustment)



The intermediate image should be formed in front of the focal point of the eye piece.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_2}{D}$

$$\alpha \approx tan\alpha = \frac{h_1}{f_O}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\left(\frac{h_1}{f_O}\right)} = \frac{f_O}{D} x \frac{h_2}{h_1}$$

$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{D}{f_E} - 1\right) x \left(\frac{f_O}{D}\right)}$$

The lens separation $= \boldsymbol{f_o + u_E}$

DISADVANTAGES OF AN ASTRONOMICAL TELESCOPE

It forms an inverted final image.

NOTE:

The structure of an astronomical telescope can be modified to over come the above disadvantage by use of a terrestrial telescope which forms an erect image.

EXAMPLES:

1. An astronomical telescope has an objective and an eyepiece of focal length **75.0cm** and **2.5cm** respectively. Find the separation of the two lenses if the final image is formed at **25cm** from the eyepiece, calculate the:

Solution

Consider the action of the eyepiece

$v = -25cm$ and $f_E = 2.5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{2.5} = \frac{1}{u} + \frac{1}{-25}$$

$$u_E = 2.27cm$$

$\Rightarrow$ **The lens separation** $= \boldsymbol{f_o + u_E}$

$= (75 + 2 \cdot 27)\ cm = \boldsymbol{77 \cdot 27cm}$

2. An astronomical telescope has an objective and an eyepiece of focal length **100cm** and **5cm** respectively.
   (a) Find the angular magnification of the telescope if arranged in normal adjustment.
   (b) If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the:
      (i) angular magnification of the telescope in this setting.
      (ii) separation of the objective and eyepiece.

Solution:

(a) In normal adjustment, **magnifying power** $m = \frac{f_o}{f_E} = \frac{100}{5} = 20$

(b) (i) With the final image at near point**,**

$m = \left(\frac{D}{f_E} - 1\right) x \left(\frac{f_O}{D}\right)$

$$m = \left(\frac{-25}{5} - 1\right) x \left(\frac{100}{25}\right)$$

$$m = -24$$

(ii) Consider the action of the eyepiece

$v = -25cm$ and $f_E = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{u} + \frac{1}{-25}$$

$$u = 3.57cm$$

$\Rightarrow$ **The lens separation** $= \boldsymbol{f_o + u}$

$= (100 + 3 \cdot 57)\ cm = \boldsymbol{103 \cdot 57cm}$

3. The objective of an astronomical telescope in normal adjustment has a diameter of **12cm** and focal length of **80cm**.
   (a) If the eyepiece has a focal length of **5cm,** find the:
      (i) magnifying power of the telescope in this setting.
      (ii) Position of the eye-ring
      (iii) diameter of the eye-ring

(b) State the advantage of placing the eye at the eye ring.

Solution

(a) (i) In normal adjustment, **magnifying power** $m = \frac{f_o}{f_E} = \frac{80}{5} = 16$

(ii) Consider the action of the eyepiece

$u = \boldsymbol{f_o} + \boldsymbol{f_E} = (80 + 5)$ = 85cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{85} + \frac{1}{-25}$$

$$v = 5.313cm$$

∴ The eye-ring is **5•313cm** from the eyepiece

(ii) In normal adjustment,

$$\frac{diameter\ of\ the\ objective}{diameter\ of\ the\ eye\ ring} = \frac{f_o}{f_E}$$

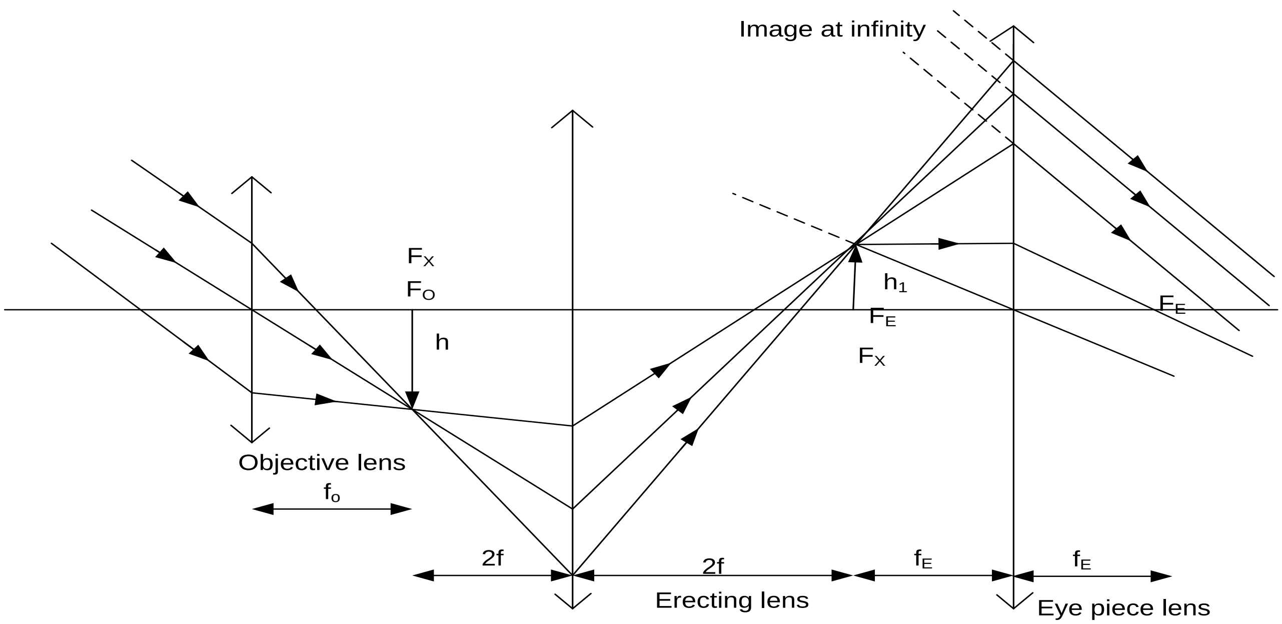
$$\frac{18}{diameter\ of\ the\ eye\ ring} = \frac{80}{5}$$

$$diameter\ of\ the\ eye\ ring = 1.125cm$$

(iii) The eye placed at the eye ring has a wide field of view since most of the light entering the objective passes through the eye ring.

Terrestrial telescope

It is a refracting telescope with an intermediate erecting lens of focal length **f**, which is place between the objective lens and the eyepiece. The erecting lens should be a distance **2f** after the principal focus of the objective lens and a distance **2f** before the principal focus of the eyepiece. The objective lens forms a real inverted image of a distant object at its focal point F_o. This act as a real object for the erecting lens which forms a real erect image of the same size as the inverted image formed by the objective.



ADVANTAGE OF A TERRESTRIAL TELESCOPE

It forms an erect final image.

DISADVANTAGES OF A TERRESTRIAL TELESCOPE

(i) It is bulky since its length is increased by **4f** compared with an astronomical telescope.

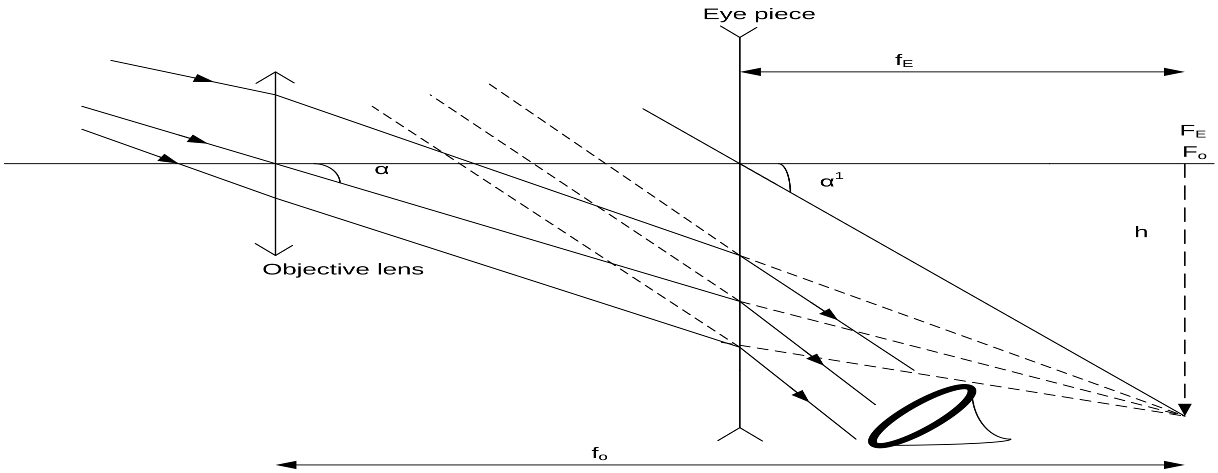
(ii) It reduces the intensity of light emerging through the eyepiece. This is due to light losses at several lens surfaces

GALILEAN TELESCOPE:

This telescope provides an erect image of a distant object with the aid of an objective which is a converging lens of long focal length and an eyepiece which is a diverging lens of short focal length.

GALILEAN TELESCOPE IN NORMAL ADJUSTMENT

A converging lens is arranged coaxially with a diverging lens such that their focal points are at the same point. The converging lens forms a real image of a distant object at its focal point F_o situated exactly at the principal focus F_e of the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final virtual image at infinity as shown.



Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{f_E}$

$$\alpha \approx tan\alpha = \frac{h}{f_O}$$

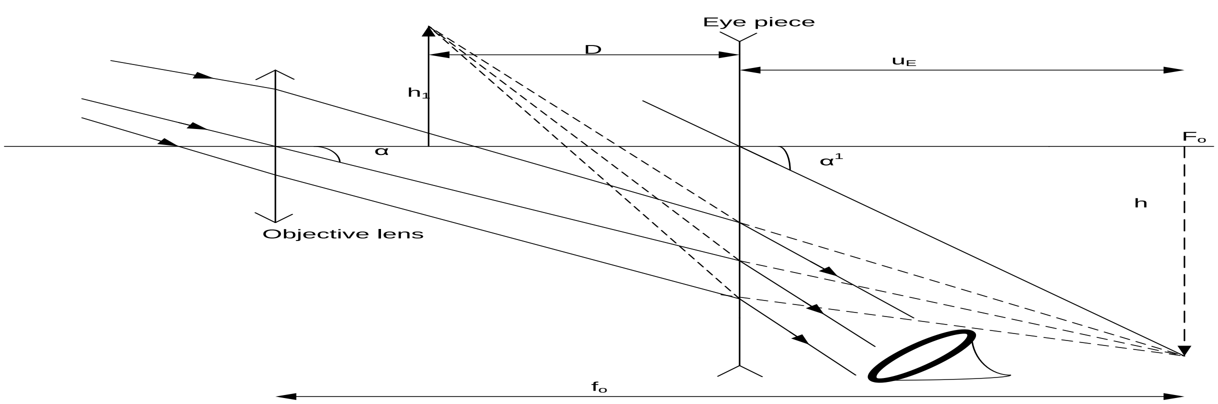
$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{f_E}\right)}{\left(\frac{h}{f_O}\right)} = \frac{f_O}{f_E}$$

$$\boxed{m = \frac{f_O}{f_E}}$$

$$Separation\ of\ the\ lens = f_O - f_E$$

GALILEAN TELESCOPE WITH FINAL IMAGE AT NEAR POINT

A converging lens arranged coaxially with a diverging lens forms a real image of a distant object at its focal point **F_o** situated a distance **u** beyond the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final erect virtual image between the converging lens and the diverging lens at an image distance **D** as shown.



Let h be the height of the image formed at F_O.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{U_E}$

$$\alpha \approx tan\alpha = \frac{h}{f_O}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{U_E}\right)}{\left(\frac{h}{f_O}\right)} = \frac{f_O}{U_E}$$

$$\boxed{m = \frac{f_O}{U_E}}$$

Consider the action of the eyepiece

Then use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$V = D$ and $f = f_E$

$\frac{1}{f_E} = \frac{1}{U_E} + \frac{1}{D}$

$$U_E = \frac{f_E D}{D - f_E}$$

$$m = \frac{f_O}{U_E}$$

$$m = \frac{f_O}{\left(\frac{f_E D}{D - f_E}\right)}$$

$$\boxed{m = \frac{f_O}{f_E}\left(1 - \frac{f_E}{D}\right)}$$

NOTE:

(i) There is need to consider the signs of **f_e** and **D** while using the above expression and are taken to be negatives.

Advantages of Galilean Telescope:

(i) It is shorter than astronomical telescope when in normal adjustment, hence it used for *opera glasses*

(ii) The final image is upright or erect.

Disadvantages of Galilean Telescope:

(i) it has a virtual eye ring not accessible to the observer.

(ii) it has a narrow field of view.

EXAMPLE

1. A Galilean telescope has a convex lens of focal length **50cm** and a diverging lens of focal length **5cm.**

(a) Find the angular magnification of the telescope if arranged in normal adjustment.

(b) If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the:

(i) angular magnification of the telescope in this setting.

(ii) separation of the objective and eyepiece

Solution

(a) (i) In normal adjustment,

magnifying power $m = \frac{f_O}{f_E} = \frac{50}{5} = 10$

(b) (i) With the final image at near point**,**

magnifying power $m = \frac{f_O}{f_E}\left(1 - \frac{f_E}{D}\right)$

$$m = \frac{50}{-5}\left(1 - \frac{-5}{-25}\right) = -8$$

Thus the required angular magnification is 8

(ii)

The lens separation $= f_O - \boldsymbol{u}$

Consider the action of the eyepiece

$v = -25cm$ and $f_E = -5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-5} = \frac{1}{u} + \frac{1}{-25}$$

$$u = -8.33cm$$

The required lens separation $= \boldsymbol{fo - u}$

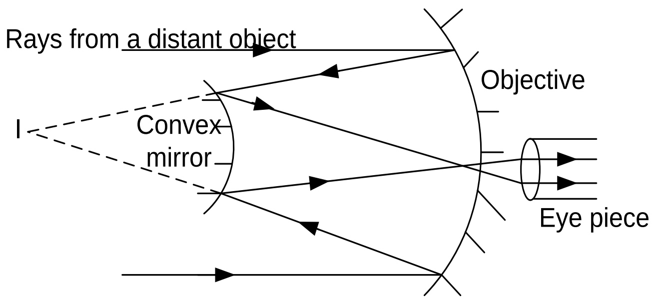
$= (50 - 8 \cdot 33)\ cm = 41 \cdot \boldsymbol{67cm}$

REFLECTING ASTRONOMICAL TELESCOPE

The objective of a reflecting telescope is a concave mirror with long focal length.
There are three types of reflector telescopes namely:

(i) Cassegrain Reflector Telescope
(ii) Newton Reflector Telescope
(iii) Coude Reflector Telescope

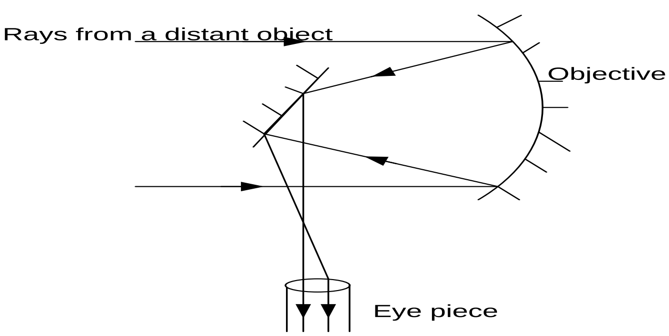
Case grain reflecting telescope



- The objective consists of a concave mirror with a long focal length
- Parallel rays of light from a distant object are first reflected at concave mrirror and then at a small convex mirror to form a real image **I** at a hole situated at the pole of the concave mirror
- The eyepiece is set such that **I** coincide with its principal focus thus forming a magnified virtual image at infinity

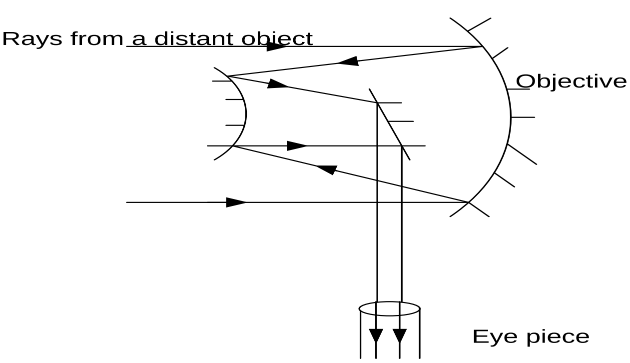
Newton's reflecting telescope

It consists of a concave mirror of long focal length as the objective instead of a convex lens, a plane mirror and a convex eye piece.



- Parallel rays of light form a distant object are first reflected at objective and then at a small slanting plane mirror to form a real image at **I**.
- The plane mirror helps to bring intermediate image to a more convenient focus
- The plane ,mirror is small so that it can not affect the effective focal length of the objective
- The eye piece is adjusted until the magnified virtual image is formed at infinity

Coude reflecting telescope



- The objective consists of a concave mirror of long focal length.
- Light from a distant object is reflected first at a concave mirror and then at a small convex mirror which then reflects it on to a slanting plane mirror to form a real image at I.
- The eyepiece is set such that **I** coincide with its principal focus thus forming a virtual magnified image at infinity as shown.