

Answers section I & II

## I. L.C.M

1.	a) G.C.D of $81xy^4$ and $144x^3y^2$ is $9xy^2$ $144x^3y^2 - 81xy^4$ $9xy^2(16x^2 - 9y^2)$ $9xy^2(4x - 3y)(4x + 3y)$	M1  M1  A1	
		3	
2.	Let the other number be x $LCM = \frac{\text{Product of the two numbers}}{\text{GCD of the numbers}}$ $140 = \frac{20 \times x}{7}$ $x = \frac{140 \times 7}{20}$ $= 49$	M1  A1	
		02	
3.	$LCM = 7920 = 24x32x5x11$ $GCD = 12 = 2^2 \times 3$ $48 = 24 \times 2$ $264 = 23 \times 3 \times 11$ $X = 22 \times 32 \times 5$ $X = 180$	M1  M1  A1	Factor notation GCD and LCM Factor notation 2numbers  C.A.O
		03	
4.	$9 = 3 \times 3$ $15 = 3 \times 5$ $20 = 2 \times 2 \times 5$ $24 = 2 \times 2 \times 2 \times 3$ $LCM = 3^2 \times 2^3 \times 5$ $= 9 \times 8 \times 5$ $= 9 \times 8 \times 5$ $= 360 \text{ sweets}$	M1  M1  A1	Correct factors  Accept other alternative correct method for getting the L.C.M
		03	
5.	L.C.M = $\begin{array}{c cccc} 2 & 27 & 30 & 45 \\ 3 & 27 & 15 & 45 \\ 3 & 9 & 5 & 15 \\ 3 & 3 & 5 & 5 \\ 5 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{array}$ $2 \times 3^3 \times 5 = 270$	M1	✓LCM

	$= 270 + 3 = 273.$	A1	
		02	

6.

2	20,	24,	26,	28
2	10	12	13	14
2	5	6	13	7
3	2	3	13	7
5	5	1	13	7
7	1	1	13	7
13	1	1	13	1
1	1	1	1	

$$\text{Size of the land} = (2^3 \times 3 \times 5 \times 7 \times 13) + 7 \text{ aces}$$

$$= 10920 + 7 = 10,927 \text{ aces}$$

7.

2	30	45	54
3	15	45	27
3	5	15	9
3	5	5	3
5	5	5	1
	1	1	1

$$\text{Least volume of } x = 2 \times 3 \times 5 + 21$$

$$= 270 + 21 = 291$$

8. L.C.M. of 30, 36 and 45

2	30	36	45	
2	15	18	45	
3	15	9	45	
3	5	3	15	
5	5	1	5	
	1		1	

$$\text{L.C.M.} = 2^2 \times 3^2 \times 5 = 180$$

$$\text{The number } m = 180 + 7 = 187$$

9.

$$x^2 + x = x(x + 1)$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$x^2 - x = x(x - 1)$$

$$x(x+1)(x-1)$$

$$x^3 - x$$

## 2. Integers

1.	Let the number be x and y $x + y = xy - 1$ $y - x = xy - 5$ $x + y + 1 = xy$ $y - x + 5 + = xy$ $x + y + 1 = y - x + 5$ $2x = 4 \quad x = 2$ $Y - 2 + 5 = 2y = y = 3$	B1  B1  B1	
		03	

2.  $X > -1$   
 $X \geq 3$

3.  $2x 2^3 x 8 x 8^2 = 128$   
 $2x \div 2^3 x 2^3 x x 8^2 = 128$   
Let  $2^x$  be  $y$   
 $y/8 x y^3 x 64 = 128$   
 $8y/8 = 128/8$   
 $y^4 = 16$   $M1$   
 $y^4 = 24$   $M1$   
 $\therefore y = 2$   $A1$   
 $-5 x 6 \div 2 + (-5)$

4.  $-12 - 3 = 4$   
 $4 x 4 + 15$   
Numerator  $16 + 15 = 31$   
Denominator  $-5 x 3 + -5 = 31$   
 $-15 + -5$   
 $-15 + -5$   
 $= -20$   
 $\frac{31}{-20}$   
 $= -1 \frac{11}{20}$

5.  $= \frac{(-8) - (-4)}{-9 + 15} + \frac{(-16) + (-6)}{46 - 13}$   
 $= \frac{-12}{6} + \frac{-22}{33}$   
 $= -2 - \frac{2}{3}$   
 $= -2 \frac{2}{3}$

6.  $P^{-1} = \begin{pmatrix} 4 & -3 \\ 1 & -2 \end{pmatrix}$   
 $\frac{-1}{5} \begin{pmatrix} 4 & -3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{-6}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

$$P^{-1} R = \begin{pmatrix} \frac{-4}{5} & \frac{-3}{5} \\ \frac{-1}{5} & \frac{2}{5} \end{pmatrix} \quad \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{5} & \frac{-6}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

7.  $\frac{-8 \div 2 + 12 x 9 - 4 x 6}{56 \div 7 x 2}$   
 $= \frac{-4 + 108 - 24}{16}$

$$\frac{80}{16} = 5$$

### 3. Fractions

<b>1.</b>	$\frac{(5)^{\frac{3 \times 2}{3}} \div 3^4}{3^{\frac{-3 \times 5}{5}}} = \frac{5^2 \div 3^4}{3^{-3}}$ $= 5^2 \div 3^7$ $= \frac{25}{2187}$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	Simplifying numerator simplify
<b>2.</b>	<p>Num <math>\left(\frac{1}{5} \times 20\right)^{\frac{1}{2}}</math></p> $= 4^{\frac{1}{2}}$ $= 2$ <p>Denom. <math>8 \times 1 \times 25</math></p> $= 200$ $= \frac{2}{200}$ $= \frac{1}{100}$	M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub>	Or equivalent 0.01
		03	
<b>3.</b>	$\frac{\left(\frac{10}{7} - \frac{5}{8}\right) \times 2/3}{3/4 + \frac{12}{7} \times 7/4 \times 7/3}$ $= \frac{\frac{45}{56} \times 2/3}{3/4 + 1}$ $= \frac{15/28}{7/4}$ $= \frac{15/28 \times 4/7}{7/4} = \frac{15}{49}$	M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub>	✓ Application of bodmas  Simplification of both numerators and denominator
		03	
<b>4</b>	<p>Let the digits be x and y      The number becomes xy  <math>= 10x + y</math>      and <math>x + y = 10</math>      Reserved <math>yx = 10y + x</math>  <math>(10y + x) - (10x + y) = 54</math>  <math>10y + x - 10x - y = 54</math>  <math>9y - 9x = 54</math>  <math>y - x = 6</math>  <math>y - x = 6</math>  <math>y + x = 10</math>  <math>2y = 16</math></p>	M <sub>1</sub>  M <sub>1</sub>  1M	Splitting of ones & tens and the reverse  Solving of the simultaneous

	y = 8 x = 8 - 6 = 2 . . . The number is 28	A1	eqn. Answer
		3	
5	$\frac{3}{8} \left( \frac{38}{5} - \frac{55}{36} \times \frac{12}{5} \right)$ $= \frac{3}{8} \times \frac{59}{15}$ $= \frac{59}{40} = 1 \frac{19}{40}$	M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub>	
6	2.181818 .... × 100 = 218.1818 .... 2.181818 .... × 1 ..... 2.1818 ....  Difference = 216  Difference of multipliers = 100 – 1 = 99  Fraction $= \frac{216}{99} = \frac{72}{33} = \frac{24}{11}$	B1  M1A1 3	
7	$\left( \frac{10^5}{3^5} \right)^{\frac{2}{5}} \times \left( \frac{3^2}{10^4} \right)^{\frac{1}{2}}$ $\frac{100}{9} \times \frac{3}{100}$ $\frac{1}{3}$	M <sub>1</sub>  M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub> — 3	For +ve index
8.	$\frac{6}{7} \text{ of } \frac{14}{3} \div 80 \times -\frac{20}{3}$ $-2 \times 5 + (14 \div 7) \times 3$ $4 \div 80 \times -\frac{20}{3}$ $-\frac{2}{2} \times 5 + 2 \times 3$ $\frac{1}{20} \times -\frac{20}{3}$ $-\frac{10}{6} + 6$	M1  M1  M1  M1	

	$\begin{array}{r} -1/3 \\ \hline -4 \\ \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} \end{array}$	A1	
		04	
9.	<p>Let the number of chicken be x          Turkeys will be <math>x + 6</math>  <math>\frac{1}{4}x + \frac{1}{3}(x + 6) = 30</math>  <math>\frac{1}{4}x + \frac{1}{3}x + 2 = 30</math>  <math>\frac{7}{12}x = 28</math>  <math>= 48</math>          Number of chickens = 48          Number of turkeys = <math>48 + 6 = 54</math>          Total number of birds = <math>54 + 48</math>  <math>= 102</math></p>	B1	For 48
		B1	For 102

11. 
$$\begin{array}{r} -4 \text{ of } (-4-3)+-3-2 \\ \hline -12+3+5 \end{array}$$

$$\begin{array}{r} -4 \text{ of } (-7-3-2) \\ \hline -4 \end{array}$$

$$\begin{array}{r} = \frac{48}{-4} \\ M1 \end{array}$$

$$= -12$$

A1

3

12.	<p>(a) (i) ratio</p> $r = \frac{3}{100} \div \frac{3}{10}$ $r = \frac{3}{100} \times \frac{10}{3} = \frac{1}{10}$ $ii) S_n = \frac{\frac{3}{10} \left(1 - \frac{1}{10}\right)^{n-1}}{1 - \frac{1}{10}}$ $S_n = \frac{\frac{3}{10} \left(\frac{9}{10}\right)^{n-1}}{\frac{9}{10}}$ $S_n = \frac{3}{10} \times \frac{10}{9} \left(\frac{9}{10}\right)^{n-1}$ $S_n = \frac{1}{3} \left(\frac{9}{10}\right)^{n-1}$ $iii) T_8 = \frac{3}{10} \left(\frac{1}{10}\right)^{8-1}$ $= \frac{3}{10} \left(\frac{1}{10}\right)^7$ $= \frac{3}{10} \times \frac{1}{10^7} = \frac{3}{100,000,000}$	M1	
		A1	
		M1	
		M1	
		A1	
		M1	
		M1	
		A1	
		M1	

	(b) 1 <sup>st</sup> bounce 30m 2 <sup>nd</sup> $\frac{3}{4} \times 30 = 22.5\text{m}$ 3 <sup>rd</sup> $\frac{3}{4} \times 22.5 = 16.85\text{m}$ 4 <sup>th</sup> $\frac{3}{4} \times 16.85 = 12.64\text{m}$ 5 <sup>th</sup> $\frac{3}{4} \times 12.64 = 9.48\text{m}$ 6 <sup>th</sup> $\frac{3}{4} \times 9.48 = 7.11\text{m}$ 7 <sup>th</sup> $\frac{3}{4} \times 7.11 = 5.3325\text{m}$ 8 <sup>th</sup> $\frac{3}{4} \times 5.3325 = 3.9993\text{m}$ 9 <sup>th</sup> $\frac{3}{4} \times 3.9993 = 2.9995\text{m}$ 10 <sup>th</sup> $\frac{3}{4} \times 2.9995 = 2.2496\text{m} \approx 2.25$ Or using formula $T_{10} = 30(\frac{3}{4})^{10-1} = 30(\frac{3}{4})^9$ = $30 \times 0.07508$ $\approx 2.2524\text{m} \approx 2.25\text{m}$	M1 M1 A1 2 d.p M1 A1	Every four Every four
13.	$\frac{5}{\frac{12}{43}} = \frac{25}{43}$ $\frac{25}{43}$ $20$	<b>M1</b> <b>M1</b> <b>A1</b>	For num For den
14.	$\text{Numerator} \rightarrow 7 + -6 = 1$ $\text{Denominator} \rightarrow -9 + 2 + 4 = -3$ $= -\frac{1}{3}$	<b>M1</b> <b>M1</b> <b>A1</b>	03
15	$\begin{array}{r} -8-39+5 \\ -1-3x2 \\ \hline = \frac{-42}{-7} \\ = 6 \end{array}$	M1  <b>A1</b>  $\frac{2}{2}$	Numerators & Denominators
16	$\begin{array}{r} \sqrt[3]{13824-4} \\ 3+2-35 \\ \hline = \frac{\sqrt[3]{13824}-4}{-30} \\ 13824 = \sqrt[3]{2^9 \times 3^3} = 2^3 \times 3^1 = 24 \\ \hline \frac{24-4}{-30} = \frac{-2}{3} \end{array}$	M1 M1 A1	Simplified denominator $\sqrt[3]{ }$ Method shown Show how to get factors of 13824
		3 mark s	

17	$  \begin{array}{r}  100r = 193.3\dot{3} \dots \\  10r = 19.\dot{3}\dot{3} \dots \\  \hline  90r = 174 \\  r = \frac{174}{90} \\  100r = 25.\dot{2}\dot{5} \dots \\  r = 0.\dot{2}\dot{5} \dots \\  \hline  99r = 25 \\  r = \frac{25}{99} \\  \frac{174}{90} + \frac{25}{99} \\  = \frac{2164}{990} \\  = 2 \frac{92}{445}  \end{array}  $	B1	
		B1	
		B1	3

18.  $\frac{1}{2}x\frac{7}{2} = \frac{3}{2}x\frac{1}{2} - \frac{3}{4}x\frac{5}{2}xX$

$$\begin{array}{r}
 2 2 6 \\
 7 + 3x \frac{11}{2} = \frac{15}{4} \\
 4 2 2 \\
 7 + \frac{11}{4} = \frac{18}{4} \\
 \therefore \frac{18}{4} \div \frac{15}{4} \\
 \frac{18}{4} \times \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5} \quad \text{A1}
 \end{array}$$

19. 
$$\begin{aligned}
 & \frac{2}{5} \div \frac{1}{2}0f^4/9 - 1^l/10 \\
 &= \frac{2}{5} \div \frac{1}{2}X^4/9 - 1^l/10 \\
 &= \frac{2}{5}x\frac{9}{2} - 1^l/10 \\
 &= \frac{9}{5} - 1^l/10 = 1^8 - 1^l/10 = 7/10
 \end{aligned}$$

$$\frac{1}{8} - \frac{1}{6}X^3/8 = \frac{1}{8} - \frac{1}{16} \\
 = 2^{-1}/16 = 1/16$$

$$\begin{aligned}
 & \frac{2}{5} \div \frac{1}{2}0f^4/9 - 1^l/10 = \frac{7}{10} \\
 & \frac{1}{8} - \frac{1}{6}of^3/8 = 1/16 \\
 & = 7/10 X^{16/1} \\
 & = 56/5 = 11^l/5
 \end{aligned}$$

20.  $BODMAS$

$$\begin{array}{r}
 \frac{3}{7}X\frac{7}{3} = 1 \\
 \frac{9}{7}X1 = 9/7 \\
 \frac{3}{4} + \frac{9}{7} = 21 + 36 = 57 \quad \text{M1} \\
 28 28 \\
 \frac{9}{7} - \frac{3}{8} = 72 - 21 = 51 x \frac{2}{3} = 17/28 \quad \text{M1} \\
 \frac{57}{28} x \frac{28}{17} = 3 \frac{6}{17} \quad \text{A1}
 \end{array}$$

$$\begin{array}{r}
 21. \quad \frac{2}{5} x \frac{9}{2} - \frac{11}{10} \\
 \underline{\frac{1}{8}} - \underline{\frac{1}{16}} \\
 = \frac{7}{10} x \frac{16}{1}
 \end{array}$$

$$= \frac{56}{5} = 11 \frac{1}{5}$$

$$22. \quad \frac{3}{8} \left( \frac{3^8}{5} - \frac{5^5}{3^6} x^{12/5} \right)$$

$$\frac{3}{8} x^{59/15} = \frac{59}{40} = 1^{19/40}$$

$$\begin{array}{l}
 23. \quad \text{Numerator} \\
 \frac{9/5 X^{25/18}}{7/3 - (\frac{1}{4} x 12)} \div \frac{5/2 X 24}{5/3}
 \end{array}$$

$$\frac{9}{5} x^{25/18} = \frac{5}{2} \div \frac{5}{3} x 24$$

$$\frac{5}{2} x^{3/5} x 24 = 36$$

$$\frac{7}{3} - \frac{1}{4} x 12 \div \frac{5}{3}$$

$$\frac{7}{3} - 3 x \frac{3}{5}$$

$$\therefore \frac{36}{\frac{8}{15}} = 67.50 \\
 \frac{8}{15} = 67 \frac{1}{2} \\
 \frac{7}{3} - 3 x \frac{3}{5}$$

24. Let  $X$  be money raised

$$\text{Teachers house} = \frac{1}{7}x$$

$$\text{Classrooms} = \frac{2}{3}x \frac{6}{7} = \frac{4}{7}x$$

$$\text{Remainder} = \frac{1}{3}x \frac{6}{7} = \frac{2}{7}x$$

$$\frac{2}{7}x = 300000$$

$$x = \text{Shs. } 1050000$$

#### 4. Decimals

1	$  \begin{array}{r}  0.0168 \times 2.46 \times 7 \\  \hline  5.74 \times 0.112  \end{array}  $ $  \begin{array}{r}  0.0003 \times 0.03 \\  \hline  0.01 \times 0.002  \end{array}  $ $  \begin{array}{r}  0.3 \times 3 \\  \hline  2  \end{array}  = 0.45  $	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	÷ 0.41 or 4.1 or 41 ✓ attempt to simpify
2	$x + y = 10$ $(10y + x) - (10x + y) = 54$	3	
		M <sub>1</sub>	M <sub>1</sub>

	$\begin{array}{r} -9x + 9y = 54 \\ -x + y = 6 \\ x + y = 10 \\ \hline 2y = 16 \\ y = 8 \\ -x + 8 = 6 \\ -x = -2 \\ x = 2 \end{array}$ <p style="text-align: right;">: No. is 28</p>	A1	
3.	$\begin{array}{r} \sqrt[3]{0.064} \\ \sqrt[3]{0.512} \\ \sqrt[3]{\frac{64}{512}} \\ \sqrt[3]{\frac{1}{8}} \\ \frac{1}{2} \end{array}$	M1 M1 A1	

4. a) 471331.512

b) 7.273352

c) 40.16649692

5. Let  $r = 5.722222\dots$   
 $10r = 57.22222\dots$   
 $100r = 572.22222\dots$   
 $1000r = 5722.2222\dots$   
 $\underline{10r = 57.222\dots}$   
 $90r = 515$

6. 
$$\begin{array}{r} 38x 23x 27x 100x 100000 \\ 114x 575 \\ \hline = 36 \end{array}$$
 For elimination of decimals      For correct answer only

7. 
$$\begin{array}{r} 84x 132x 35 \\ 41 \quad 4 \quad 16 \\ \hline -287 \quad 560 \\ \hline 99 \\ 41 \quad 1 \end{array}$$

8. 
$$\begin{array}{r} 12x 0.25 - 12.4 \div 0.4x 3 \\ \frac{1}{8} \text{ of } 2.56 + 8.68 \\ \underline{3 - 31x 3} \\ 0.32 + 8.68 \\ \underline{-90} \\ 9 \\ = -10 \end{array}$$

### 5. Squares and square roots

1.	$\begin{aligned} & \sqrt[3]{\frac{0.125 \times \sqrt{64}}{0.064 \times \sqrt{629}}} \\ &= \sqrt[3]{\frac{0.125 \times 8}{0.064 \times 27}} \\ &= \sqrt[3]{\frac{0.5^3 \times 2^3}{0.4^3 \times 3^3}} \\ &= \frac{0.5 \times 2}{0.4 \times 3} \\ &= \frac{1.0}{1.2} \\ &= \frac{1 \times 10}{1.2 \times 10} \\ &= \frac{10}{12} \\ &= \frac{5}{6} = 0.83(2dps) \end{aligned}$	M1 M1 M1 A1	
		<b>04</b>	
2.	$\begin{aligned} & 19.901 \times 10^2 \\ & 19.901 + 1 = 1991.1 \\ & \underline{1991.1} \\ & 0.07245 \\ & 1991.1 \times 0.1380 \times 10^2 \\ &= 165.77 \end{aligned}$	<b>M1</b>  <b>M1</b>  <b>A1</b>	For ✓ square  For ✓ rec
		<b>03</b>	
3.	$\begin{array}{r} \sqrt{(1.800324)^2} \\ 0.8462 \\ 1.800324 \\ \hline 0.8462 \\ 2.127539589 \\ \hline \approx 2.128 \end{array}$	<b>M1</b>  <b>M1</b>  <b>A1</b>	$\sqrt{3.241166505}$ 0.8462
		<b>03</b>	
4.	$\begin{aligned} & 2 \times 10 \times 0.01697 \times -1.06 \times 0.1182 \times 10^{-2} \\ &= 3.393 \\ &= \sqrt{3.393} \\ &= 1.842 \end{aligned}$	B1  B1  B1	Both reciprocals

5. (a) (i) 24.78  
(ii) 0.0316

$$(b) 24.78 - 0.0316 = 24.75 \quad M1 \quad A1$$

6.  $3x \frac{1}{1.36x10^{-2}} - 2x \frac{1}{13.84}$

$$\begin{aligned} & 3x 8.575 - 2x 0.07224 \\ &= 25.725 - 0.14448 \\ &= \underline{\underline{25.58052}} \\ &= \underline{\underline{25.58}} \end{aligned}$$

7.  $\frac{153x1.8}{0.68x0.32}$

$$\begin{aligned} & \sqrt{\frac{158x1.8 \times 10000}{0.68x0.32 \times 10000}} \\ &= \sqrt{\frac{158x18000}{68x32}} = \sqrt{\frac{9x9000}{4x16}} \\ &= \sqrt{\frac{9x9x10^3}{4x16}} = \frac{9x10^{3/2}}{8} \\ &= 1.125x10^{3/2} \end{aligned}$$

## 6. Algebraic expressions

1.	$\begin{aligned} & \frac{3Z-12}{3-(1+z)} = \frac{3(Z^2-4)}{3-1-Z} \\ &= \frac{3(Z-2)(Z+2)}{2-Z} \\ &= \frac{3(Z-2)(Z+2)}{-1(Z-2)} \\ &= -3(Z+2) \end{aligned}$	M1	
		A1	<b>03</b>

2. Let the daughter's age 5yrs ago be  $x$

Mother  $4x$

come;

Daughter =  $x + 9$

Mother =  $4x + 9$

$4x + 9 = \frac{5}{2}(x + 9)$

$4x + 9 = 2.5x + 22.5$

$1.5x = 13.5$

$x = 9$

Mother = 41yrs

$14 + 41 = 55$

3.  $B.P = 160 \times 50 = 24000$

$$\begin{aligned}
 S.P &= \frac{((160x8) - (20 + 12)) \times 180}{8} \\
 &= 28080 \\
 \text{Profit} &= 28080 - 24000 = \text{Shs.} 4080
 \end{aligned}$$

4. a)  $6a + 7a - 2b - 4b + 2$   
 $= 13a - 6b + 2$

$$\begin{aligned}
 b) \frac{2x - 2 - 3x + 2}{2x} &= \frac{2(2x - 2) - (3x + 2)}{4x} \\
 &= \frac{4x - 3x - 4 - 2}{4x} \\
 &= \frac{x - 6}{4x}
 \end{aligned}$$

5.  $6u^2y^2 + 13uy - 5 = (2uy + 5)(3xy - 1)$   
 $3u^2y^2 - 13uy + X = (uy - 4)(3xy - 1)$   
 $\frac{(2uy + 5)}{(uy - 4)} \frac{(3xy - 1)}{(3xy - 1)}$   
 $= \frac{2xy + 5}{uy - 4}$

6. a) From  $x + y$  and  $x^2 = y^2 = 34$   
 $X = 8 - y$   
Substituting for  $x$  in  $x^2 - y^2 = 34$   
 $(8 - y)(8 - y) + y^2 = 34$   
 $64 - 8y - 8y + y^2 + y^2 = 34$   
 $64 - 16y + 2y^2 = 34$   
 $2y^2 - 16y + 64 - 34 = 0$   
 $2y^2 - 16y + 30 = 0$   
 $y^2 = 8y + 15 = 0$   
 $y(y - 3) - 5(y - 3) = 0 \quad (y-5)(y-3)$   
 $y$  is either 5 or 3  
but  $x - y = 8$   
 $x$  is either 5 or 3  
 $\therefore x^2 + 2xy + y^2 = 32 + 2x3x5 + 25$   
 $= 9 + 30 + 25 = 64$

b)  $2xy = 2x3x5 = 30$   
c)  $x^2 - 2xy + y^2 = 9 - 2x3x5 + 25 = 4$

d)  $x = y = 8$  and  $x^2 + y^2 = 34$   
 $x = 8 - y$   
 $(8 - y)^2 + y^2 = 34$   
 $y^2 - 8y + 15 = 0$   
 $y^2 - 3y - 5y + 15 = 0$   
 $y(y - 3) - 5(y - 3)$   
 $(y-3) = 0 \quad y = 3$   
 $(y-5) = 0 \quad y = 5$

$$\begin{aligned}
 x + 3 &= 8, x = 5 \text{ or } x + 5 = 8 \\
 x &= 3 \\
 \therefore x &\text{ is either 3 or 5} \\
 y &\text{ is either 3 or 5}
 \end{aligned}$$

7.

$$\begin{aligned}
 &\frac{6x^2 + 35x - 6}{2x^2 - 72} \\
 &= \frac{6x(x + 6) - 1(x + 6)}{2(x^2 - 36)} \\
 &= \frac{(6x - 1)(x + 6)}{2(x - 6)(x + 6)} \\
 &= \frac{6x - 1}{2(x - 6)}
 \end{aligned}$$

8.

$$\begin{aligned}
 &\frac{2/5(3x - 2) - 3/4(2x - 2)}{12} \\
 &= \frac{8(3x - 2) - 9(2x - 2)}{12} \\
 &= \frac{24x - 16 - 18x + 18}{12} \\
 &= \frac{6x + 2}{12} \\
 &= \frac{2(3x + 1)}{12} \\
 &= \frac{3x + 1}{6}
 \end{aligned}$$

$$\begin{array}{r}
 124x - 2x \\
 \hline
 385
 \end{array}
 \quad x$$

9. Numerator:

$$4y^2 - x^2 = (2y + x)(2y - x)$$

Denominator :

$$\begin{aligned}
 &2x^2 + 4yx + 3yx - 6y^2 \\
 &= (2x^2 - 4yx) + (3yx - 6y^2) \\
 &= 2x(x - 2y) + 3y(x - 2y) \\
 &= (2x + 3y)(x - 2y)
 \end{aligned}$$

Combining :  $(2y + x)(2y - x)$   
 $(2x + 3y)(x - 2y)$

$$\frac{-2x + 3y \text{ or } -2x - 3y}{2y + x}$$

10.

$$\begin{aligned}
 &\frac{3(x + y) - (x - y)}{x^2 - y^2} \\
 &= \frac{3x + 3y - x + y}{x^2 - y^2} \\
 &= \frac{2(x + 2y)}{x^2 - y^2}
 \end{aligned}$$

11.

$$\begin{aligned}
 x^2 + 2x - 5 &= 3x + 1 \\
 x^2 - x - 6 - 6 &= 0 \\
 (x+2)(x-3) &= 0 \\
 x = -2 \text{ or } x &= 3 \\
 \text{When } x = -2, \quad y &= 3x - 2 + 1 = -5 \text{ Point } (-2, -5) \\
 \text{When } x = 3, \quad y &= 3x + 3 + 1 = 10 \text{ Point } (3, 10)
 \end{aligned}$$

12. (a)  $y(y + 2)$

$$\frac{y(y^2 - y - 60)}{y(y^2 - y - 6)} = \frac{y + 2}{(y+2)(y-3)}$$

$$(b) \frac{y+2}{(y+2)(y-3)} = \frac{1}{4}$$

$$4y + 8 = y^2 - y - 6$$

$$y^2 - 5y - 14 = 0$$

$$(y-7)(y+2) = 0$$

$$y=7$$

$$y=-2$$

13.  $\frac{104.6}{2.4} = 44 \times 2$

$$\frac{63.9}{2.4} = 26 \times 2$$

$$= 88 + 54 = 142$$

14.  $3(25x^2 - 9y^2)$   
 $3(5x + 3y)(5x - 3y)$

15. i)  $d = 8.4$        $r = \frac{1}{2}$   
 $6^{th} \text{ jump} = 8(\frac{1}{2})^{6-1}$   
 $\frac{8.4}{32}$

$$= 0.2625 = 0.26 \text{ cm}$$

$$\text{ii) } 56 = \frac{9.4(1 - (\frac{1}{2}))6}{1 - \frac{1}{2}}$$

$$= \frac{8.4 \times 63 \times 2}{64}$$

$$= 16.54 \text{ cm}$$

16. Factorizing the numerator

$$= p(p^2 - q^2) + q(p^2 - q)$$

$$= (p+q)(p^2 - q^2)$$

$$= (p+q)(p+q)n(p-q)$$

Factorising the denominator

$$(p+q)(p+q)$$

$$\frac{\text{Numerator}}{\text{Denominator}} = p - q$$

17.  $\frac{(3x + 2y)(3x - 2y)}{(3x + 2y)(3x - 2y)}$   
 $\frac{3x + 2y}{4x + 3y}$

18.  $(x-3)(AX^2 + BX + C) = x^3 - 7x - 6$   
 $AX^3 + BX^2 + CX - 3AX^2 - 3BX - 3C = x^3 - 7x - 6$   
 $A = 1$   
 $B - 3A = 0$   
 $B - 3 \times 1 = 0$

$$\begin{aligned}B &= 3 \\-3c &= -6 \\c &= 2\end{aligned}$$

19. a)  $8(2^y)^2 = 6x 2^y - 1$   
 $\text{let } t = 2^y$   
 $8t^2 = 6t - 1$   
 $8t^2 - 6t + 1 = 0$   
 $(4t - 1)(2t - 1) = 0$   
 $t = \frac{1}{4} \text{ or } \frac{1}{2}$   
 $\therefore t = 2^y = \frac{1}{4} = 2^{-2}$   
 $\therefore y = -2$   
 Or  $t = 2^y = \frac{1}{2} = 2^{-1}$   
 $\therefore y = -1$   
 $\therefore y = -2 \text{ or } -1$

b) Numerator  $= 2x^2 - 98$   
 $= 2(x^2 - 49)$   
 $= 2(x+7)(x-7)$   
 Denominator  $= 3x^2 - 16x - 35$   
 $= 3x^2 - 21x + 5x - 35$   
 $= 3x(x-7) + 5(x-7)$   
 $= (x-7)(3x+5)$   
 $\therefore \frac{2x^2 - 98}{3x^2 - 16x - 35} \div \frac{x+7}{3x+5} = \frac{2(x+7)(x-7)}{(3x+5)(x-7)} \cdot \frac{x(3x+5)}{(x+7)}$

$$= 2$$

20. 
$$\frac{(2x-y)(2x+y)}{(x-3y)(2x-y)} \sqrt{\frac{2x+y}{x-3y}}$$

21.  $P^2 - 2pq + q^2 = (p-q)^2$   
 $P^3 - pq^2 + p^2q - q^3$   
 $= p(p^2 - q^2) + q(p^2 - q^2)$   
 $= (p+q)(p^2 - q^2)$   
 $\frac{(p-q)^2}{(p+q)(P^2 - q^2)} = \frac{(p-q)^2}{(p+q)^2(p-q)}$   
 $= \frac{p-q}{(p+q)^2}$

22. Let the numbers be  $a$  and  $b$   
 $a + b = 15 - x3$   
 $5a - 3b = 19 \times 1$

$$\begin{array}{r} 3a + 3b = 45 \\ 5a - 3b = 19 \\ \hline 8a = 64 \\ a = 8 \\ b = 7 \end{array}$$

$$4 \quad 3 \quad 2$$

$$23. \quad \frac{3(2x-5) - 4(1-x) - 6(x-4)}{12}$$

$$\frac{6x - 15 - 4 + 4x - 6x + 24}{12}$$

$$\frac{4x - 5}{12}$$

$$\begin{aligned}
 24. \quad & \frac{3a^2 + 4ab + b^2}{4a^2 + 3ab - b^2} = \frac{3a^2 + 3ab + ab + b^2}{4a^2 + 4ab - ab - b^2} \\
 &= \frac{3a(a+b) + b(a+b)}{4a(a+b) - b(a+b)} \\
 &= \frac{(3a+b)(a+b)}{(a+b)(4a-b)} \\
 &= \frac{3a+b}{4a-b}
 \end{aligned}$$

## *7. Rates Ratio and percentages*

1.	<p>Let ten shillings coins be t  <math>\therefore</math> five shilling coins <math>2t</math>  <math>\therefore</math> one shilling coins <math>21 - 3t</math>  <math>(10xt) + (5x2t) + 1(21-3t) = 72</math>  <math>20t + 21 - 3t = 72</math>  <math>17t = 51</math>  <math>t = 3</math></p>	B1  M1  A1	
		03	
2.	<p>(a)</p> $\frac{1}{4} : \frac{1}{2} : \frac{1}{5} = 5 : 10 : 4$ $\frac{1}{4} = \frac{5}{19} \times 1000 = 263$ $\frac{1}{2} = \frac{10}{19} \times 1000 = 526$ $\frac{1}{5} = \frac{4}{119} \times 1000 = 710$ <p>(b) Let volume of 45% concentration be x  Therefore 25% will be <math>(100 - x)</math></p> $\frac{0.45x + 0.25(100-x)}{100} = 30\%$ $0.45x - 0.25x + 25 = 30$ $0.20x = 5.0$ $x = 25$ $x = 25\text{cm}^3$ <p>vol of 45% = <math>25\text{cm}^3</math>  vol of 25% = <math>75\text{cm}^3</math>  ratio 1 : 3</p> <p>(c) (i) Cost of 1 kg mixture</p> $\frac{2}{5}x 140 + \frac{3}{5}x 160$ $152$ $\text{Profit} = 240 - 152$ $= \text{sh } 88$ $\text{Gain } \frac{88}{152} \times 100 = 57.9\%$ <p>(ii) <math>140 \quad 160</math></p> 	B1  B1  B1  M1  A1  B1  B1  M1	
			Follow through for alternative



	Senjeni's share = $\frac{4}{11} \times \text{sh } 130900 = \text{sh } 47600$ (c) Mkimwa = sh 450000 + sh 59500 = sh 509500 Kuku = sh 180000 + sh 23800 = sh 203800 Senjeni = sh 360000 + sh 47600 = sh 407600	M1 M1 A1	
		<b>10</b>	
8.	$M \quad Hrs \quad Days$ 15        8        24 16        ?        20  Number of hours reduces in ratio 15:16 from increase in the number of men.  No. of hrs increase in ratio 24:20 from reduction in the days $\frac{15}{16} \times \frac{24}{20} \times 8$ $3 \times 3$ = 9 hrs	<b>M1</b> <b>M1</b> <b>A1</b>	Both ratio
		<b>03</b>	
9.	Mwashuma takes X days Mwandime takes $X - 2$ days  $\frac{1}{x} + \frac{1}{x-2} = \frac{5}{12}$  $5x^2 - 34x + 24 = 0$  $x = \frac{34 \pm 26}{10}$  = $\frac{60}{10}$ or $\frac{8}{10}$ Ignore = 6  Mwandime $6 - 2$ = 4 days	M1 M1 M1 A1	

10	$= \frac{4 \times 20}{8} =$ 10 hours	M1 A1	
----	---	----------	--

11. Men cottages days

5	2	21
x	6	21
$\frac{6}{2} \times \frac{21}{21} \times 5$		= 15

$$\text{more men} = 15 - 5 = 10$$

12. a) i) In 1 hr; Tap A fills  $\frac{1}{3}$

13. Max Perimeter  $= 2(18.5 + 12.5)$

$$= 62 \text{ cm}$$

Working Perimeter  $= 2(18 + 12)$

$$= 60 \text{ cm}$$

$$\% \text{ error} = \frac{2}{60} \times 100 = 3.33\%$$

$$B - \frac{1}{4}$$

Capacity filled in 1 hr  $= \frac{1}{3} + \frac{1}{4}$

$$= \frac{7}{12}$$

$$\frac{7}{12} = 1 \text{ hr}$$

$$1 = 1 \times 1 \times \frac{12}{7}$$

$$= 1 \frac{5}{7} \text{ hrs.}$$

ii)  $\frac{1}{3} + \frac{1}{4} - \frac{1}{6} = \frac{5}{12} \Rightarrow \text{in one hr}$

$$\frac{5}{12} = 1 \text{ hr}$$

$$1 = 1 \times 1 \times \frac{12}{5}$$

$$= 2 \frac{2}{5} \text{ hrs}$$

14. (a)  $\frac{144000 - 144000}{n-5} R$

$$= \frac{720,000}{n(n-5)}$$

(b)  $720,000 = 2400$

$$n(n-5)$$

$$300 = n(n-5)$$

$$n^2 - 5n - 300 = 0$$

$$(n-20)(n+15) = 0$$

Either  $n = 20$ ,  $n = -15$  m = 20

(c) contributed =  $\frac{144000}{20} = 7200$

(d) % increase =  $\frac{2400 \times 100}{7200} = 33.33\%$

15. (a) In 1 hour  $\frac{1}{40} + \frac{1}{15} + \frac{1}{20}$  of the tank will be filled

$$= \frac{17}{120}$$

$$\text{In 5 hours} = \frac{17}{120} \times 5$$

$$= \frac{17}{24}$$

(b) In two hours taps x and y

$$\left( \frac{1}{40} + \frac{1}{15} \right) \times 2 \text{ of the tank to be filled}$$

$$= \frac{11}{60}$$

$$\text{In 7 hours} = \left( \frac{11}{60} + \frac{17}{24} \right)$$

$$= \frac{107}{120}$$

$$(c) \text{ Remaining fraction} = 1 - \frac{107}{120}$$

$$= \frac{13}{40}$$

In  $\frac{1}{40}$  hour proportion, time taken

$$= \frac{13}{120} \times 40h$$

$$= 4\frac{1}{3}$$

$$\text{Time taken} = 7 + 4\frac{1}{3} = 11 \text{ hrs } 20 \text{ min.}$$

Tank will be full at 8.00 + 11hrs 20 min

1920 hrs or 7.30 p.m

16. Let Philip take  $x$  days to finish the job alone.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$6(x+5) 6x = x(x+5) \checkmark$$

$$6x + 30 + 6x = x^2 + 5$$

$$x^2 - 7x - 30 = 0$$

$$(x-10)(x+3) = 0 \checkmark$$

$$x = 10 \text{ and } x = -3$$

17.  $\begin{array}{ccc} 16 & 9 & 14 \\ X & 7 & 12 \end{array}$

$$X = 16 \times \frac{9}{7} \times \frac{14}{12}$$

= 24 men

$$\text{Extra men} = 24 - 6$$

$$= 8 \text{ men}$$

18. a) Let the original no. of people be  $x$

Originally each would contribute

$$\frac{180000}{X}$$

New contribution per person

$$\frac{180000}{X-3}$$

$$\frac{180000}{X-3} - \frac{180000}{x} = 3000$$

$$180000x - 180000x + 540000 = 30000 - 9000$$

$$30x^2 - 90x - 5400 = 0$$

$$3x^2 - 9x - 540 = 0$$

$$X^2 - 3x - 180 = 0$$

$$(x-15)(x+12) = 0$$

$$X = 15 \text{ or } -12$$

Original number of people 15

b)  $\frac{180000}{15} = \frac{180000}{15}$

c) Original contribution per person  
Shs. 12000

New contribution per person  
 $= \frac{180000}{12} = 15000$

% increase  
 $\frac{15000 - 12000}{12000} \times 100\% = 25\%$   
 $\frac{3000}{12000} \times 100\% = 25\%$

19. a) cost of running the business

$$\frac{20}{100} \times 43200 = \text{Shs. } 8640$$

b) 15% of profit

$$\frac{15}{100} \times 43200 = \text{Shs. } 6480$$

Rest of the profit

$$= 43200 - (8640 + 6480) = 28080$$

Ratio of contribution

$$40000 : 64000 \\ 5 : 8$$

Mue received

$$\frac{1}{2} \times 6480 = \text{Shs. } 3240$$

$$\frac{8}{13} \times 28080 = \text{Shs. } 17280 \\ = \text{Shs. } 20320$$

c) Konie received

$$\text{Shs. } 3240 + 10800 = 14040$$

$$\frac{14040}{1800} = 7.8 \\ = 7 \text{ cows}$$

20.  $(7x - 3y) : 2x + 3y$

$$x = 2 \quad y = 1$$

$$14 - 9 : 4 + 9$$

$$5 : 13$$

21. a)  $\begin{array}{r} B \text{ ____ bulls} \\ G \text{ ____ Goats} \end{array}$

$$5B + 30G = \text{Kshs. } 117000 \dots\dots\dots \text{Equation (i)}$$

$$4B + 25G = \text{Kshs. } (117000 - 22250)$$

$$4B + 225G = \text{Kshs. } 94750 \dots\dots\dots \text{Equation (ii)}$$

From equation (i)  $5B + 30G = \text{Kshs. } 117000$  (dividing through by 5)

$$= (B + 6G = 23400) \times 4$$

$$= 4B + 24G = 93600 \dots\dots\dots \text{(iii)}$$

$$\begin{aligned} \text{Equation (ii)} - q(\text{iii}) &= 4B + 24G = 94750 - \\ 4B + 24G &= 93600 \\ G &= 1150 \end{aligned}$$

$$\begin{aligned} \therefore 1 \text{ goat costs Kshs. } 1150 \\ \text{Substituting in (i)} \\ 5B + 30(1150) &= 117000 \\ 5B + 34500 &= 117000 \\ 5B &= 825000 \\ B &= \text{Kshs. } 16500 \end{aligned}$$

b) Abduls selling price  
 $\text{Bull } \frac{140}{100} \times 16500 = 23100 \times 5 = \text{Kshs. } 115,500$

$$\text{Goat } \frac{130}{100} \times 1150 = 1495 \times 30 = \text{Kshs. } 44850$$

$$\begin{aligned} \text{Total } 44850 + 115500 &= \text{Kshs. } 160350 \\ &= \text{Kshs. } 160350 \\ \text{Ali's selling price} \\ \text{Bulls } \frac{150}{100} \times 16500 &= 24750 \times 4 = \text{Shs. } 99000 \end{aligned}$$

$$\text{Goats } \frac{140}{100} \times 1150 = 1610 \times 25 = \text{Shs. } 40250$$

$$\text{Total } 99000 + 40250 = \text{Kshs. } 139,250$$

$$\begin{aligned} \text{Profit made} \\ \text{Abdul } \underline{\quad} \text{ Kshs. } (160350 - 117000) &= \text{Kshs. } 43350 \end{aligned}$$

$$\text{Ali } \underline{\quad} \text{ Kshs. } (139250 - 94750) = \text{Kshs. } 44500$$

Ali made more profit by Kshs. 1150/=

22. Original costs

$$\begin{aligned} T &= \frac{8}{24}x = \frac{x}{3} \\ L &= \frac{4}{24}x = \frac{x}{6} \\ R &= \frac{12}{24}x = \frac{x}{2} \end{aligned}$$

$$\begin{aligned} \text{New } T &= \frac{x}{3} \times 1.12 = 0.3733x \\ L &= \frac{x}{6} \times 1.18 = 0.1967x \\ R &= \frac{x}{2} \times 1.4 = 0.7x \end{aligned}$$

$$\begin{aligned} \text{Therefore \% change} \\ (\underline{0.3733x + 0.967x + 0.7x}) - x \times 100 \end{aligned}$$

$$\begin{aligned} X \\ = 0.27 \times 100 \\ = 27\% \end{aligned}$$

23. Let Mary's yrs be  $x$   
 Mothers age =  $2 \frac{1}{2}x$   
 4yrs ago Mary was  $x - 4$   
 4yrs ago mother was  $2 \frac{1}{2}x - 4$

$$\begin{aligned} \frac{2\frac{1}{2}x - 4}{x - 4} &= \frac{3}{1} \\ \frac{5}{2}x - 3x &= -12 \\ -\frac{1}{2}x &= -12 \\ x &= 24 \text{ yrs} \\ \text{mother's age is } &= (\frac{5}{2}x 24) \\ &= 60 \text{ yrs} \end{aligned}$$

24.  $\frac{16x 9x 14}{7x 12} = 24$   
 Extra men =  $24 - 16$   
 $B1 = 8 \text{ more men}$

- 25.. Ratio  $K : B = 3 : 4$   
 a) Kongo got  $\frac{3}{7} x \frac{35}{100} x 181300 = 27195/=$   
 Beatrice got  $\frac{4}{7} x \frac{35}{100} x 181300 = 36260/=$   
 b) Kongo got  $\frac{3}{7} x \frac{60}{100} x 181300 + 9000 = 136,620/=$   
 Beatrice got  $\frac{4}{7} x \frac{60}{100} x 181300 + 120000 = 182,160/=$

26. Let no. be  $mn$   
 $M + n = 9 \dots (i)$   
 $10m + n, \text{ reversed } 10n + m$   
 $10n + m - 10m + n = 27$   
 $1n - 9m$

27.  $Vl = \pi r^2 h$   
 $R = 130r = 1.3r$   
 $H = \frac{80h}{100} = 0.8h$   
 $V_2 = \pi R^2 h = (1.3r)^2 x 0.8h = 1.352V_l$   
 $\% \text{ change} = \frac{V_2 - V_l}{V_l} x 100 = \frac{(1.352 - 1)V_l}{V_l} x 100$

$$0.352 \times 100 = 35.2\%$$

28. In 1hr both fills =  $1 + 1 - 10 = 23$   
 Tina to fill =  $120 = 5 \frac{5}{23}$   
 5hrs 13min

29.

16	9	14
$X$	7	12
$X = 16 \times \frac{9}{7} \times \frac{14}{12}$		
$= 24 \text{ men}$		

Extra men =  $24 - 6$   
 = 8men

30. a) Expenses =  $\frac{30}{100} \times 600,000$   
 = sh. 180,000  
 Business =  $\frac{15}{100} \times 420,000$   
 = sh. 63,000  
 Rest of profit = 357,000  
 Ratio 160 : 200 : 240  
 4 : 5 : 6

(i) Langat received = sh  $\frac{4}{15} \times 357,000$

$$= \text{sh } 95,200$$

(ii) Korir received = sh  $\frac{5}{15} \times 357,000$   
 = sh 119,000

(iii) Koech received = sh  $\frac{6}{15} \times 357,000$   
 = 142,800

(b) % =  $\frac{119,000}{600,000} \times 100$   
 = 19.83

31. a)  $125 : 100 = 5 : 4$

b)  $\frac{5}{4} \times 400 = 500$

32. Alcohol A =  $\frac{25}{120}$   
 =  $30 \text{ cm}^3$   
 Alcohol in B =  $\frac{20}{100} \times 180$   
 =  $36 \text{ cm}^3$   
 Results =  $\frac{36 + 30}{120 + 180}$   
 =  $\frac{66}{300} \times 100$   
 = 22%

$$\text{Remaining} = 300-x$$

$$\text{Volume of alcohol} = (300-x) \times \frac{22}{100} = 66 - 0.22x$$

$$\text{Total volume of alcohol} = 66 - 0.22x + x$$

$$= 66 + 0.78x$$

$$\% \text{ alcohol} = \frac{66 + 0.78x}{300} \times 100 = 35$$

$$= 66 + 0.78x = 105$$

$$0.78x = 39$$

$$x = 50$$

33. Max Perimeter  $= 2(18.5 + 12.5)$   
 $= 62 \text{ cm}$

Working Perimeter  $= 2(18 + 12)$   
 $= 60 \text{ cm}$

$$\% \text{ error} = \frac{2}{62} \times 100 = 3.33\%$$

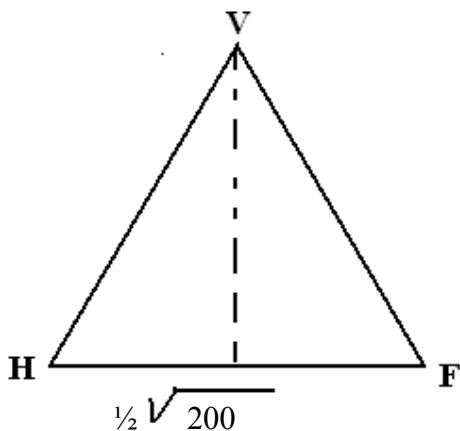
34.  $a:b = 1:2$   
 $b:c = 3:4$   
 $a:b = 3:6$   
 $b:c = 6:8$   
 $\therefore a:b:c = 3:6:8$

### 8. Length

1.	Mass = Density x volume But Density is constant. $x$ $y$  Vol $(270000 \times 2.8) : x \cdot 2.1$  $= \frac{270000 \times 2.8}{2.1} = 360m$	M1  M1  M1  A1	
		4	

2. a.)	$DF_1 = \sqrt{6^2 + (\sqrt{200})^2}$ $= 15.362291$	M1  A1	
-----------	--	--------------	--

b.)



$$\sqrt{144 - \frac{1}{2}\sqrt{200}}^2$$

$$= 9.6953597$$

$$+ \frac{6}{15.6953597}$$

$$\tan \Theta = \frac{15.6953597}{7.0710678}$$

$$= 2.219659$$

C.

$$\Theta = 65.747499^\circ$$

M1

A1

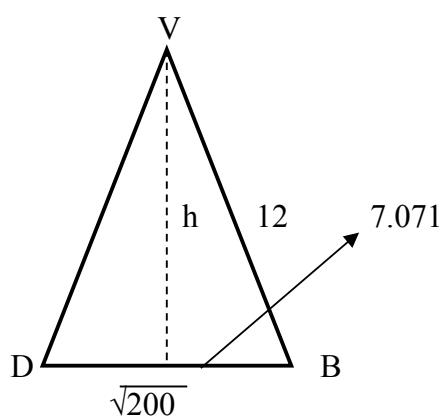
B1

$$\frac{1}{2} HF$$

$$\frac{1}{2} \times \sqrt{200}$$

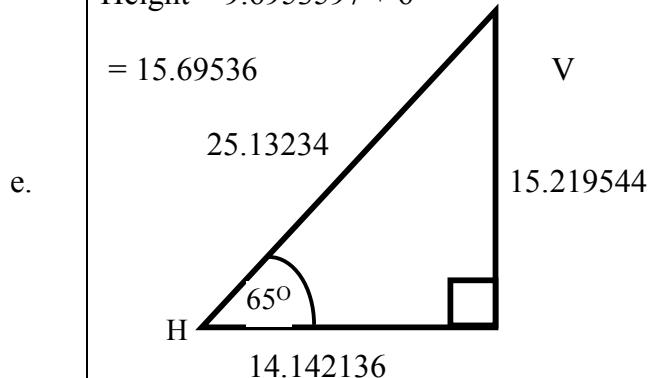
$$= 7.0710678$$

d.

M1  
B1

$$\sqrt{144 - 50} = 9.6953597$$

$$\begin{aligned}\text{Height} &= 9.6953597 + 6 \\ &= 15.69536\end{aligned}$$



$$\begin{aligned}&\sqrt{14.142136^2 + 15.219544^2} \\ &= 25.13234\end{aligned}$$

B1

&lt; BDF

3

f.

$$\begin{aligned}Row \triangle BDF \text{ mn} &= \frac{960}{10-x} \\ \frac{960}{10-x} &+ 20 \\ 15.362291 &+ 20 \\ (110-x)960 &= 960x + 20x(110-x) \\ x^2 - 206x + 5380 &= 0 \\ \frac{\Theta}{6} &= \frac{90}{15.362291} \\ 176x + 5380 &= 0 \\ x(x-30) - 176(x-30) &= 0 \\ \Theta &= \frac{90}{15.362291} \\ x &= 176 \\ &= 35.15^\circ r \\ x &= 30 \\ \text{Dimensions} & 30 \text{ by } 80 \text{ cm}\end{aligned}$$

M1

No of tiles =

$$\begin{aligned}\frac{960 \times 960}{30 \times 30} \\ &= 384\end{aligned}$$

A1  
10

Cost =

$$\frac{364}{12} \times 1500 + 3000$$

$$= sh.483000$$

$$4. \frac{(3x+2y)(5x-3y)}{(5x-3y)(x-y)} = \frac{3x+2y}{x-y}$$

$$5. 3N + \frac{1}{2}(R-M)$$

$$\begin{aligned} &= 3 \begin{pmatrix} 2/3 & 1 \\ 2 & 4 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 3 \\ 6 & 12 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -4 & 2 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 3 \\ 6 & 12 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 0.5 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 6.5 & 10 \end{pmatrix} \end{aligned}$$

## 9. Area

1	$\frac{1}{2} \times 14 \times 22 \sin 75^\circ - \frac{75}{360} \times \frac{22}{7} \times 14 \times 14$ $7 \times 22 \sin 75 - \frac{55 \times 7}{3}$ $= 20.42$	M1 M1 M1 A1	
2.	LSF 1 cm rep 50000cm 1cm rep 500m ASF 1cm <sup>2</sup> rep 250000m <sup>2</sup> $\text{Area} = \left( \frac{6.16 \times 250000}{10000} \right)$ $= 154\text{ha}$	B1 M1 A1	ASF given
		03	
3.	$\text{Area} = 4 \times 4 \sin 420 - \frac{42}{360} \times \frac{22}{7} \times 4 \times 4$ $= 10.71 - 5.867$ $= 4.796$	M1 M1 A1	✓area of rhombus & sector ✓difference in area
		03	

$$5. (a) \tan 60^\circ = \frac{AC}{5\text{cm}} \quad M1$$

$$AC = 8.6605\text{CM} \quad A1$$

$$(b) A = \frac{1}{2} \times 5 \times 8.6605 \quad M1$$

$$A = 21.65125 \quad A1$$

$$(b) \frac{60}{360} \times \pi r^2 \quad M1$$

$$\frac{60}{360} \times 3.142 \times 25 \quad M1$$

$$= 13.091\text{cm}^2$$

A1

(d) Area of shaded part

$$\Delta \text{COA} = \Delta \text{OBA}, \text{sector OCD} = \text{OCB}$$

$$21.65 \times 2 = 43.3025\text{cm}^2$$

M1

$$13.091 \times 2 = 26.182\text{cm}^2$$

M1

 $\therefore$  Area of shaded part

$$43.3025 - 26.182$$

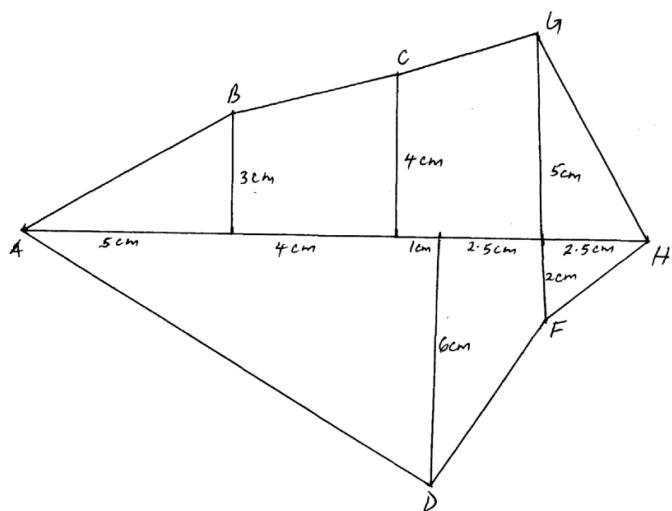
M1

$$= 17.11225\text{cm}^2$$

A1

10

6.



$$\frac{1}{2} \times 200 \times 120 = 12,000$$

**B1** For AH

$$160 \times \frac{1}{2} \times 50 = 4000$$

**B1** for✓ offset

$$\frac{1}{2} \times 50 \times 40 = 1000$$

**B1** ✓ div for AH

$$\frac{1}{2} \times 50 \times 100 = 2500$$

**B1** Offsets  $\perp$  to AH

$$\frac{1}{2} \times 180 \times 70 = 6300$$

**B1** Complete diagram

$$\frac{1}{2} \times 140 \times 80 = 5600$$

**M1**

$$\frac{1}{2} \times 100 \times 60 = 3000$$

**M1**

$$\underline{34400\text{m}^2}$$

$$= 3.44\text{ha}$$

**M1****A1****B1****10**

7.	$S = \frac{5.7 + 4.2 + 6.3}{2} = 8.1$ $= \sqrt{8.1(8.1 - 5.7)(8.1 - 4.2)(8.1 - 6.3)}$ $= \sqrt{8.1 \times 2.4 \times 3.9 \times 1.8} = 11.68$ $\text{Shaded area} = 18.05 - 11.68$ $= 6.368 \text{ cm}^2$	B1 M1 A1 B1	
		04	

8	<p>Area of the rectangle = <math>60k^2</math></p> <p>Area of unshaded part parts</p> $= \frac{1}{2}6k \times 5k + \frac{1}{2} \times 15k^2 + \frac{1}{2} \times 30k^2$ $= 15k^2 + 7.5k^2 + 15k^2$ $= 37.5k^2$ <p>Area of shaded part = <math>60k^2 - 37.5k^2</math></p> $= 22.5k^2$ <p>b)</p> $\frac{1}{2} \times 15k^2 = 30$ $k^2 = \frac{30 \times 2}{15}$ $k = 2$ <p>Dimensions = 20m by 12 cm</p> $\tan \theta = \frac{12}{10}$ $\theta = 50.19^\circ$ $\tan \beta = \frac{6}{10}$ $\beta = 30.96^\circ$	M1 M1 M1 A1 M1 A1 A1 M1 A1 M1 A1
---	--	--

		10	
9	<p>a) =</p> $\frac{160}{360} \times \frac{22}{7} \times 10.5^2$ $= 154 \text{ cm}^2$	M1 A1	
	$\sin 80 = \frac{h}{10.5}$ $h = 10.5 \sin 80 = 10.34 \text{ cm}$ $\cos 80 = \frac{x}{10.5}$ $x = 10.5 \cos 80 = 1.823$ $\text{length } CD = 21 - (1.823 \times 2)$ $= 17.354$	B1 B1 M1 A1	
	<p>Area of AX YBCD</p> $= \frac{1}{2} \times 10.34(17.354 + 21) \times 2$ $= 396.58 \text{ cm}^2$ <p>b) Area of the shaded part</p> $= 396.58 - 154$ $= 88.58 \text{ cm}^2$	M1 A1 M1 A1	
		10	

10.  $Mx m \text{ value} = \frac{2.655 + 6.415}{6.405 - 2.655}$

$= \frac{9.07}{3.75}$

$= 2.4187$

11. (a) Number of tiles  $= \frac{10.5 \times 6}{0.3 \times 0.3}$   
to cover the room  $= 700 \text{ tiles}$

(b) (i) 15 x 700 tiles  
 $\underline{15 \times 700} \text{ cartons}$

$$\begin{aligned} & 20 \\ & Cost = \frac{15 \times 700}{20} \times 800 \\ & Cost = Kshs. 420,000 \end{aligned}$$

$$\begin{aligned} (ii) \text{ Other expenses} &= 2000 + 600 = 2600/ \\ \text{Total expenses} &= Kshs. 420,000 + 2600 \\ &= Kshs. 422600 \end{aligned}$$

$$\begin{aligned} \text{Selling price} &= \frac{112.5}{100} \times 422600 \\ &= Kshs. 475,425 \end{aligned}$$

$$\begin{aligned} \text{Selling price per tile} &= \frac{475,425}{525 \times 20} \\ &= 45.27 \\ &= Kshs. 45.00 \end{aligned}$$

12.  $\frac{AC}{\sin 60^\circ} = 10 = AC = 8.66$   
 $\angle A 70^\circ, \frac{BC}{\sin 70^\circ} = 10 = BC = 8.91$   
 $\text{Area} = \frac{1}{2} \times 8.66 \times 8.91 \sin 50^\circ$   
 $= 27.28$

13.  $S = \frac{1}{2} (170 + 190 + 210)$   
 $S = 285$   
 $\sqrt{\text{Area}} = \sqrt{285 (285 - 170) (285 - 190) (285 - 210)}$   
 $= \sqrt{2865 \times 115 \times 95 \times 75}$   
 $= \frac{15281m^2}{10,000}$   
 $= 1.528ha$

14. LCM of 30, 50 and 35 mins  
 $30 = 2 \times 3 \times 5$   
 $35 = 5 \times 7$   
 $50 = 2 \times 5^2$  } L.C.M =  $2 \times 3 \times 5 = 1050$   
 $\text{Into hrs } \frac{1050}{60} \text{ hrs} = 17.5 \text{ hrs}$   
 $\text{Next wail together at } \begin{array}{r} 7:18 \\ + 17:30 \\ \hline 24:48 \end{array}$   
 $= \text{at } 1.48 \text{ a.m on Tuesday}$

15. Maize -  $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$   
 $\text{Remainder} - \frac{2}{3} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$   
 $\text{Beans} - \frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$   
 $\text{carrots} - \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

Let total area of farm be  $x$  acres

$$\frac{1}{10}x = 0.9$$

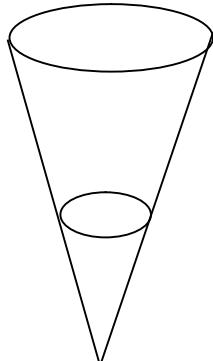
$$10$$

$$x = 0.9 \times 10 = 9 \text{ acres}$$

### 10. Volume and capacity

1.	$\frac{1}{2} \times 14 \times 22 \sin 75^\circ - \frac{75}{360} \times \frac{22}{7} \times 14 \times 14$ $7 \times 22 \sin 75^\circ - \frac{55 \times 7}{3}$ $= 20.42$	M <sub>1</sub> M <sub>1</sub> M <sub>1</sub>  A <sub>1</sub>	
		4	
2.	LSF 1 cm rep 50000cm 1cm rep 500m ASF 1cm <sup>2</sup> rep 250000m <sup>2</sup> $\left( \frac{6.16 \times 250000}{10000} \right)$ Area = $= 154 \text{ha}$	B1  M <sub>1</sub>  A <sub>1</sub>	ASF given
		03	
3.	Area = $4 \times 4 \sin 42^\circ - \frac{42}{360} \times \frac{22}{7} \times 4 \times 4$ $= 10.71 - 5.867$ $= 4.796$	M <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>  03	✓ area of rhombus & sector ✓ difference in area
4.	a) (i) $\frac{30 + h}{h} = \frac{40}{30}$ $h = 90$ $\frac{1}{3} \Pi \times 1600 \times 120 - \frac{1}{3} \Pi \times 900 \times 90$ $(64000 \Pi - 27000) \div 1000$ $37 \Pi \text{ litres}$  (ii) Volume of water = $\frac{2}{5} \times \Pi \times 1.44 \times 1.35$ $= 777.6 \Pi \text{ litres}$  b) $\frac{777.6 \Pi}{37 \Pi}$ $= 22$	M <sub>1</sub>  A <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub>  Divide by 1000  Mult by 1000	

5. 16 12



$$\frac{12}{16} = \frac{L}{30 + L}$$

$$L = 90$$

B1 for 90

$$h = \sqrt{90^2 - 12^2} \\ = 89.2$$

$$H = \sqrt{120^2 - 16^2} \\ = 118.9$$

B1 for both 89.2  
1189

$$\text{Vol. big core} = \frac{1}{3} \times 3.142 \times 16^2 \times 118.9 \\ 31879.151$$

M1

$$\text{Small cone} = \frac{1}{3} \times 3.142 \times 12^2 \times 89.2 \\ = 13452.789$$

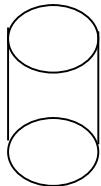
M1

$$\therefore \text{Volume of water} \\ 31879.151 - 13452.789 \\ = 18426.3645$$

M1

A1

(b) 4.5 12



$$3.142 \times 12^2 \times h = \\ 18426.364$$

M1

$$h = 40.73$$

A1

$$S.A = 2 \times 3.142 \times 12 (45 - 40.73) \\ = 321.99 \text{ cm}^2$$

M1

A1

10

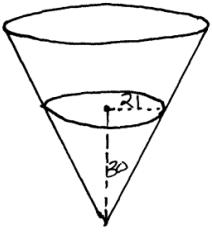
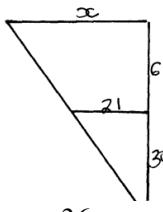
6.	(a) $(300 \times 5) + (140 \times 8)$ $= 1500 + 1120$ $= 2620$ fans (b) Cost of fuel Boeng 747 $= 120 \times 10.5 \times 60 \times 5 \times 2 \times 0.3$ $= 226800$ dollars Boeng 740 $= 200 \times 10.5 \times 60 \times 8 \times 2 \times 0.3$ $= 604,800$ dollars (c) Total collection Boeng 747 $= 300 \times 5 \times 800$ $= 1,200,000$ dollars Boeng 740	M1  A1  M1 A1  M1 A1  B1	
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	= 140x8x800 = 896,000 dollars (d) Net profit Boeng 747 = 1200000 – 226800 = 973,200 dollars Boeng 740 = 896,000 – 604,800 = 291,200 dollars	B1 B1 B1	
		10	

7. a.)	<p style="text-align: center;">50</p> $\frac{50}{15} = \frac{70+x}{x}$ $50x = 15(70 + x)$ $50x = 1050 + 15x$ $35x = 1050$ $= 30 \text{ cm}$ $\text{Total height} = \sqrt{100^2 - 50^2}$ $= \sqrt{7500}$ $= 86.60 \text{ am}$ $\frac{50}{15} = \frac{86.60}{Y} \quad Y = \frac{86.6 \times 15}{50}$ $= 25.98$ $\text{Height} = 86.60 - 25.98$ $= 60.62$	M1 A1 B1 M1 A1 A1
b.)	Volume $= (\frac{1}{3} \pi r^2 h) \times 50^2 \times 86.60 - (\frac{1}{3} \pi r^2 h) \times 15^2 \times 25.98$	M1

	25.98)  = $\frac{1}{3} \times \frac{22}{7} (216500 - 5845.5)$ = 220685.67am <sup>3</sup> = 221litres	M1  M1 <u>A1</u> 10
--	--	---------------------------------

8.	a)		
----	----	--	--

	 $\text{volume} = \frac{\pi}{3} r^2 h$ $= \frac{3.142}{3} \times 21 \times 21 \times 30$ $= 13856.22 \text{ cm}^3$ <p>b)</p>  $\frac{x}{21} = \frac{36}{30}$ $x = 25.2 \text{ cm}$ <p>ii) New volume = <math>\frac{1}{3} \times 3.142 \times 25.2 \times 25.2 \times 36</math>  <math>= 23943.55 \text{ cm}^3</math></p> <p>Volume change = <math>23943.55 - 13856.22</math>  <math>= 10087.33 \text{ cm}^3</math></p> <p>iii) <math>2/3\pi r^3 = 10087.33</math>  <math>r^3 = 10087.3 \times 3/2 \times 1/\pi</math>  <math>r^3 = 4815.72</math>  <math>r = \sqrt[3]{4815.72}</math>  <math>r = 16.89 \text{ cm}</math>  diameter = <math>16.89 \times 2</math>  <math>= 33.78 \text{ cm}</math></p>	M1  A1  M1  A1  B1  M1  A1  B1	Attempt
		10	

9.  $L.s.f. = \frac{18}{24} = \frac{3}{4}$

$$A.s.f = \frac{9}{16}$$

$$v.s.f = \frac{27}{64}$$

$$\frac{h}{3.2} = \frac{3}{4} \Rightarrow 4h = 3h + (3 \times 3.2)$$

$$h = 9.6$$

(i) surface area of small cone:

$$L = \sqrt{9^2 + 9.6^2} = 13.16 \text{ m}$$

$$S.A = (3.142 \times 9 \times 13.6) = 384.581$$

Curved area of frustum

$$= \frac{7 \times 3.142 \times 9 \times 13.16}{9}$$

$$= 289.4$$

$$\text{Top area} = (3.142 \times 9^2) = 254.5\text{cm}^2$$

$$\therefore \text{Total area} = 543.9\text{m}^2$$

$$(ii) \text{Volume of smaller cone} = \frac{3.142 \times 9^2 \times 9.6}{3}$$

$$= 814.41$$

$$\text{Volume of frustum} = \frac{(37 \times 814.41)}{27}$$

$$= 1116.043\text{m}^3$$

$$= 1116043L$$

$$\text{Litres used per day} = (15 \times 15 \times 40) + (116 \times 65) = 16540L$$

$$\text{No. of days} = \frac{1116043}{16540} = 67.5\text{days}$$

10.  $L.S.F = \frac{3}{2} = \frac{28+h}{h}$

$$56 + 2h = 3h$$

$$h = 56\text{cm}$$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} r^2 H - \frac{1}{3} r^2 h \\ &= \frac{1}{3} \pi 22/7 \times 15 \times 15 \times 56 - \frac{1}{3} \pi 22/7 \times 10 \times 10 \times 28 \\ &= 13200 - 29331/3 \\ &= 10.2667\text{litres} \end{aligned}$$

$$(b) \text{Slant height} = 152 + 562 = 3361$$

$$= 57.97\text{cm}$$

$$\text{Curved surface} = RL - rl$$

11.  $2.6 \times 4.8 \times 3.2 = 39.936\text{m}^3$

$$1\text{m}^3 = 1000\text{litres}$$

$$\begin{aligned} 39.936\text{m}^3 &= 39.936 \times 1000 \\ &= 39936 \text{ litres} \end{aligned}$$

12. The top surface of the frustum is 2/3 way up the vertical height of the original one.

$$\Rightarrow VX:XY = 1:3; h = 1:3$$

Using similar triangle we have

$$\frac{R}{r} = \frac{VX}{VY} = \frac{1}{3}$$

$$R:r = 1:3$$

$$\frac{r}{R} = \frac{1}{3} \Rightarrow R = 3r$$

$$R = 3 \times 7 = 21\text{cm}$$

(c) height of removed cone is 1/3 height of original cone

$$h = \frac{1}{3} \times 45 = 15\text{cm}$$

$$\begin{aligned} \text{volume of removed cone} &= \frac{1}{3} r^2 h \\ &= 1 \times \frac{22}{7} \times 7 \times 7 \times 15 \\ &= 770\text{cm}^3 \end{aligned}$$

$$\text{Now L. S. F} = \frac{1}{3}$$

$$V. S. F = (\frac{1}{3})^3 = \frac{1}{27}$$

Hence ratio of volumes = 1:27

$$\begin{aligned} \text{Volume of original cone} &= 27x \text{ Vol. of small cone} \\ &= 770x 27 = 20790\text{cm}^3 \end{aligned}$$

Capacity of frustum

$$\begin{aligned} &= \text{vol. of original cone} - \text{vol. of removed cone} \\ &= 20790 - 770 = 20020\text{cm}^3 \end{aligned}$$

$$\frac{20020}{1000} = 20l$$

$$(d) \text{ capacity of tank} = \frac{150 \times 120 \times 80}{1000} = 1440l$$

$$\text{No. of buckets} = \frac{1440}{20} = 72 \text{ buckets}$$

13. Mass of water =  $1 \times 3000 \text{ cm}^3 = 3000 \text{ g}$

Mass of alcohol =  $0.8 \times 1200 = 9600\text{g}$

Mass of mixture =  $12,600\text{g}$

Volume of mixture =  $15,000 \text{ cm}^3$

$$\begin{aligned} \text{Density of mixture} &= \frac{12600}{15000} \\ &= \underline{0.84\text{g/cm}^3} \end{aligned}$$

14. (a) Vol. of tank =  $22 \times 144 \times 1.7 = 5.236$

Vol. of milk =  $\frac{3}{5} \times 5.236 = 3.146\text{m}^3$

Vol. in liters =  $3.1416 \times 1000 = 3141.6 \text{ litres}$

$$\begin{aligned} (b) (i) \text{ Vol. of packet} & ( \frac{1}{3} \times 10 \sin 60 ) \times 13.6 \\ &= 26.97 \times 13.6 \\ &= 3.66.75\text{cm}^3 \\ &= 367\text{cm}^3 \end{aligned}$$

$$(ii) \text{ No. packets} = \frac{(3141.6 \times 1000)}{367}$$

$$\begin{aligned} (iii) \text{ Amount} &= 8560.2 \times 20 \\ &= 171204.3597 \\ &= \text{Shs. } 171,204.40 \end{aligned}$$

15. Volume of culvert

$$= \frac{22}{7} (76^2 - 64^2) \times 300 \times 10^{-6}$$

$$= \frac{22}{7} \times \frac{1680 \times 300}{10000000000}$$

$$= 1.584\text{m}^3$$

## 11. Mass, weight and density

2.	$4 \times 3.142 \times r^2 = 18$ $\rightarrow R \approx 1.197$ $\text{Vol.} = \frac{4}{3} \times 3.142 \times 1.197$	M1	
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	= 7.185cm <sup>3</sup> Density = $\frac{100}{7.185}$ = 13.92g/cm <sup>3</sup>	M1 A1	
		03	
3.	(a) (i) $800 \times 0.2$ = 160m <sup>3</sup> (ii) $160 \times 2000$ = 320,000kg (iii) Cement = $\frac{2}{8} \times 320,000$ = 80,000 (b) Bags = $\frac{80,000}{50}$ = 1600 (c) Ballast = $\frac{3}{8} \times 320,000$ = 120,000 tonnes = 120 lorries	M1 A1 M1 A1 M1 A1 B1  B1 M1 A1 B1	
		10	
4.	$4\pi r^2 = 18$ $4 \times \frac{22}{7} \times r^2 = 18$ $r = 1.197\text{cm}$ $vol = \frac{4}{3} \times \frac{22}{7} \times 1.197^3$ = 7.187cm <sup>3</sup> $D = \frac{m}{v} = 100\text{g} / 7.187\text{cm}^3$ = 13.91g/cm <sup>3</sup>	B1  B1  B1	
		03	

5. Density =  $\frac{300 \times 1,000,000}{200 \times 1000}$   
= 15,000 kg/m<sup>3</sup>

6.  $D = \frac{M}{V}$   
Mass =  $D \times V$   
=  $\frac{1\text{g}}{\text{cm}^3} \times 2500\text{cm}^3$   
= 2500g.....(i)  
Mass =  $0.8 \times 8000$   
= 6400g .....(ii)  
total mass =  $(2500 + 6400)\text{g}$   
= 8900g  
Density of mixture =  $\frac{8900\text{g}/\text{cm}^3}{10500}$

## 12. Time

<b>1</b> $T = \frac{100}{40} + \frac{60}{30}$ $= 2.5\text{hrs} + 2\text{hrs}$ $= 4.5\text{hrs}$ $\text{Average speed} = \frac{160}{4.5} \text{ km/h}$ $= 35.56\text{ km/h}$	M <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>	✓ exp for total time
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2. Time between Monday 0545hr and Friday 1945

$$= 4 \times 24 + 14 = 110 \text{ hrs}$$

$$\text{Time lost} = 0.5 \times 110 = 55 \text{ min.}$$

Time in 12 hr system

$$(1945 - 55 - 1200)$$

6.50 p.m.

3. Time between Monday 0445h and Friday 1845h

$$= 4 \times 24 + 14 = 110h$$

$$\text{Time lost} = 0.5 \times 110$$

$$= 55\text{min}$$

Time shown in 12 hour system

$$1845 - 55 = 1750 h$$

= 5.50 p.m

4. (a)  $1600h - 830h = 7\text{hrs } 30\text{min or } 7 \frac{1}{2} \text{ hours}$

(b) Average speed =  $\frac{300}{7\frac{1}{2}} = 40\text{km/h}$

### 13. Linear

<b>1</b> (0,3), (3,0) $\frac{3-0}{0-3} = -1$ $\frac{y-0}{x-3} = -1 \quad y = -x + 3$ $y < -x + 3$ $x \geq 1$ $y \geq 0$	B <sub>1</sub> B <sub>1</sub> B <sub>1</sub>	
<b>2.</b> (a) $x \geq -4$ (b) $y = -x$ $y + x \leq 0$ $\frac{0 - -9}{8 - -9} = \frac{9}{17} = \frac{3}{4}$ (c) Grad = $\frac{3}{4}$ $y = mx + c$ $0 = \frac{3}{4}(8) + c$ $c = -6$ $y = \frac{3}{4}x - 6$	B1  B1	M1 M1

	$y - \frac{3}{4}x > -6$		
		04	
3.	$2x + 3 \geq 5x - 3$ $-3x \geq -6$ $x \leq 2$ $5x - 3 > -8$ $5x > -5$ $x > -1$ $-1 < x \leq 2$ Integral values 0, 1, 2	B1 B1 B1	
		03	
4.	a) $4x - 9 < 6 + x$ $x < 5$ $8 - 3x < x + 4$ $1 < x$ b) $1 < x < 5$	M1 M1 A1	
5.	$-2x + 1 < x - 5$ $-3x < -6$ $x > 2$ $x - 5 < 5 - x$ $2x < 10$ $x < 5$ $\therefore 2 < x < 5$		M <sub>1</sub>  $\frac{A_1}{2}$
7	$-3x + 2 < x + 6$ $x > 1$ $x + 6 \leq 17 - 2x$ $x \leq 3\frac{2}{3}$ 2, 3	B <sub>1</sub> B <sub>1</sub> B <sub>1</sub> <hr/> $\frac{3}{3}$	
8.	$\frac{3a + 2}{4} \leq \frac{2a + 3}{5} \leq \frac{4a + 15}{6}$ $\frac{3a + 2}{4} \leq \frac{2a + 3}{5}$ $5(3a + 2) \leq 4(2a + 3)$ $15a + 10 \leq 8a + 12$ $7a \leq 2$ $a \leq 0.2857$ $\frac{2a + 3}{5} \leq \frac{4a + 15}{6}$ $6(2a + 3) \leq 5(4a + 15)$ $-8a/-8 \leq 57/-8$ $a \leq -7.125; -7.125 \leq a \leq 0.28$ Integral values -7, -6, -5, -4, -3, -2, -1	1  M <sub>1</sub>  B <sub>1</sub> B <sub>1</sub>	
		03	

9.  $\frac{1}{2}x - 2 \leq 3 - 2 ; 3x - 2 < + \frac{1}{2}x$

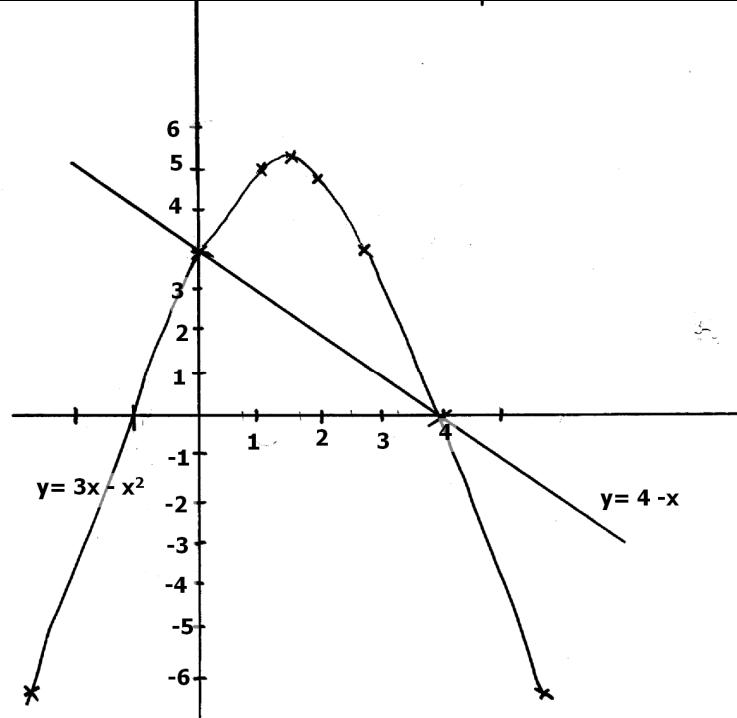
$$O \leq {}^7/{}_2X$$

$$O \leq x - B1$$

$$x = 0, 1$$

$$\frac{5}{2}x < 4$$
$$x < \frac{8}{5}$$

B1  
A1  
3

**B1****B1****B1****B1**

b) turning point 1.5, 6.25

c) i) Line  $y = 0$   $x =$   
 $-1$  or  $x = 4$   
 $x = -1$  or  $x = 4$

ii)  $4 + 3x - x^2 = y$

$$\begin{array}{r} 4x - x^2 = 0 \\ 4 - x = y \\ \hline x & | 0 & 4 \\ y & | 4 & 0 \end{array}$$

$$x = 0 \text{ or } x = 4$$

**10**

14  $\frac{2x}{3} + \frac{3x}{4} \leq 9.5 + 5.5$

$$\frac{8x + 9x}{12} \leq 15$$

$$17x \geq 180$$

$$x \leq \frac{180}{17}$$

$$-\frac{x}{3} - \frac{3x}{4} < 18 - 9.5$$

**M1**

$\frac{-4x - 9x}{12} < 8.5$ $-\frac{13x}{12} < \frac{17}{2}$ $-13x < 102$ $x > -\frac{102}{13}$	M1  A1	
		3marks

15. The diagram below shows the graphs of

$$Y = \frac{3}{10}x - \frac{3}{2}, \quad 5x + 6y = 30 \text{ and } x = 2$$

By shading the unwanted region, determine and label the region  $R$  that satisfies the three inequalities;

$$Y \geq \frac{3x}{10} - \frac{3}{2}, \quad 5x + 6y \geq 30 \text{ and } x \geq 2 \quad (2 \text{ mks})$$

$$L_I y = \frac{3x - 3}{10} \quad \text{at} \quad \begin{matrix} (0, 0) \\ 0 \geq 2 \end{matrix} *$$

Picking  $P(0,0)$

$$0 \geq -\frac{3}{2}$$

$$L_2 \quad 5x + 6y = 30$$

$$At (0, 0) \quad 5x + 6y \geq 30$$



$$35s + 15t = 14750$$

$$9s + 15t = 8250$$

$$26s = 6500$$

$$s = 250$$

*t = 275*

$$3(100) + 3(250)$$

$$2t + 2s = 2(400) + 2(250) \\ = \text{shs. } 1,300$$

17. Let the cost of a biro be  $b$

*Pencil be p*

$$2b + 5p = 120 \times 3$$

$$3b + 2p = 114 \times 2$$

$$6b + 15p = 360$$

$$\frac{6b + 4p}{11p} = 228$$

$$11p = 132$$

$$P = 121$$

$$2b + 60 = 120$$

$$2b = 60$$

$$b = 30$$

$\therefore$  The cost of 1 biro is 30/=

The cost of 1 pencil is 12/=

18. Let son's present age be  $n$  yrs

Father's age is  $2n$  yrs

Ten years ago: son's age  $\Rightarrow n - 10$

Father's age  $\Rightarrow 2n - 10$

Son's present age = 30yrs

Father's present age =  $2 \times 30 = 60$ yrs

19.  $2x + 21 > 15 - 2x$

$$15 - 2x \geq x + 6$$

$$4x > 0.6$$

$$-3x \geq -9$$

$$x > -1 \frac{1}{2}$$

$$x \leq 3$$

$$\Rightarrow -1 \frac{1}{2} < x \leq 3$$

Values are -1, 0, 1, 2, 3.

20.  $y = -2x + 4$

gradient of h line is  $\frac{1}{2}$

$$\text{Equation } \frac{y+4}{x+1} = \frac{1}{2}$$

$$2y + 8 = x + 1$$

$$2y - x + 7 = 0$$

21.  $2s + 3t = 1750$

$$3s + 2t = 1500$$

$$4s + 6t = 3500$$

$$9s + 6t = 4500$$

$$2t = 1500 - 600$$

$$t = 450$$

$$5s = 1000$$

$$s = 200$$

Shirt = sh 200

Trouser = sh 450

22. Let  $r = 3.818181\dots$

$$100r = 381.818181$$

$$99r = \underline{378} = \underline{42}$$

$$99 \quad 11$$

$$= 3^9/11$$

23. (a) Let cost of pencils be  $x$  and biro pens to be  $y$

$$4x + 6y = 66$$

$$2x + 5y = 51$$

$$4x + 6y = 66$$

$$4x + 10y = 102$$

$$4y = 96$$

$$y = 24$$



	$x^2 - 4x + x - 4 = 0$ $x(x - 4) + 1(x - 4) = 0$ $(x + 1)(x - 4) = 0$ $x = 4$ $x = -1$ <p style="text-align: center;">When <math>x = 4 y = 5</math> <math>x = -1 y = -5</math></p>	B1	✓ both answers
		4	
2.	Length of the frame $(x + x + 5) = (2x + 5)$ Width of the frame $(\frac{1}{2}x + \frac{1}{2}x + 5) = (x + 5)$ Area $= (2x + 5)(x + 5)$ (a) $75 = 2x^2 + 10x + 5x + 25$ $2x^2 + 15x - 50 = 0$ (b) $2x^2 + 15x - 50 = 0$ $2x^2 + 5x + 20x - 50 = 0$ $x(2x - 5) + 10(2x - 5) = 0$ $(x + 10)(2x - 5) = 0$ $x = -10 \text{ or } 2 \frac{1}{2}$ length $(2 \times 5/2 + 5) = 10\text{cm}$ width $(5/2 + 5) = 7.5\text{cm}$ (c) Area not covered = area of frame – area of photo $= 75 - 25$ $= 50\text{cm}^2$ % area $= 50/75 \times 100$ $= 66.67\% / 66 \frac{2}{3}\%$	B1 M1 A1 M1 A1 B1 B1 M1 M1 A1 M1 M1 A1 10	Dimension with unknowns Factorization Difference in area
3.	$24x\left(\frac{1}{4x}\right) = 24x\left(\frac{5}{6x}\right) - 7(24x)$ $6 = 20 - 168x$ $\frac{-14}{-168} = x$ $\frac{1}{12} = x$	M1 M1 A1	Multiplication by LC.
4	$3(25x^2 - 9y^2)$ $3(5x - 3y)(5x + 3y)$	M1 A1 2	For 3 out
5.	$\frac{x-3}{5} = 4 - \frac{x-2}{2}$ $10\left(\frac{x-3}{5}\right) = 10 \times 4 - 10\left(\frac{x-2}{2}\right)$ $2(x-3) = 10(4) - 5(x-2)$ $2x - 6 = 40 - 5x + 10$ $7x = 56$ $x = 8$	M1 M1 A1	
		03	

8.	$\frac{(p+2m)(p-2m)}{2m-p}$	B1	$\checkmark$ factors for numerator $\checkmark$ factors for denominator
	$= -\frac{p+2m}{m-3p}$	B1	
		3	
9.	Let " of goats be a " chicken be b $a + b = 45 \quad \} \quad B1$ $4a + 2b = 100 \quad \} \quad \text{for both equations}$		
	$\begin{array}{r} a + b = 45 \\ -2a + b = 50 \\ \hline -a = -5 \\ a = 5 \\ b = 40 \end{array}$	M1	method for solving any of the unknown
	Goats were 5 Chicken were 40 } A1 3		
10.	Ken – suit; Let the number of suits bought be x and the cost per suit be y		
	Then $xy = 57600$ $y = \frac{57600}{x}$	M1	Eq
	Umoja No. of suits bought is $(x + 4)$ Cost per suit is $(y - 480)$		
	$\begin{aligned} &= P(x + 4)(y - 480) = 57600 \\ &y = (x + 4) \left( \frac{57600}{x} - 480 \right) \\ &- 480x^2 - 1920 + 230400 = 0 \end{aligned}$	M1 M1	Eq Sub of y
	$x^2 + 4x - 480 = 0$	M1	formation of quadratic eq
	$(x - 20)(x + 24) =$ No of suits = 20	M1 A1	fact
	(b) Cost per suit = $\frac{57600}{20}$	M1	exp of cost
	Sh. 2880	A1	
	Profit per suit = Sh. 720		

$$\therefore \% \text{ profit} = \frac{720}{2880} \times 100$$

M1

exp

$$= 25\%$$

A1

10

11.	$13824 = 2^9 \times 3^3$ $0.000125 = \frac{1}{8000} = \frac{1}{2^6 \times 5^3}$ $\therefore = \left( \frac{2^9 \times 3^2}{2^6 \times 5^3} \right)^{\frac{1}{3}}$ $= \left( \frac{2^6 \times 5^3}{2^9 \times 3^3} \right)^{\frac{1}{3}}$ $= \left( \frac{5}{2 \times 3} \right)$	M1 M1 Reciprocal A1 Prime products	
		03	
12.	$64^x + 4^{3x} = 128$ $(4^3)^x + 4^{3x} = 128$ $4^{3x} + 4^{3x} = 128$ $2(4^{3x}) = 128$ $4^{3x} = 64 = 4^3$ $\therefore 3x = 3$ $X = 1$	M1 M1 A1	
		03	
13.	$4x = 3y \Rightarrow y = \frac{4}{3}x$ $\frac{\frac{1}{3}x^2 - 4x\left(\frac{4}{3}x\right) + \left(\frac{4}{3}x\right)^2}{4x^2 + \left(\frac{4}{3}x\right)^2}$ $= \frac{\frac{1}{3}x^2 - \frac{16}{3}x^2 + \frac{16}{9}x^2}{4x^2 + \frac{16}{9}x^2}$ $= \frac{-\frac{29}{9}}{\frac{52}{9}} = -\frac{29}{52}$	<b>M1</b> <b>M1</b> <b>A1</b>	$x = \frac{3}{4}y$ For ✓ subst For ✓ num For ✓ den
		04	
14.	$\frac{3^5 \times 3^{2y}}{3^6 \times 3^{y-2y+1}} = 3^4$ $\frac{3^{5+2y}}{3^{7-y}} = 3^4$ $y = 2$	<b>M1</b> <b>M1</b> <b>A1</b>	For ✓ factorization For ✓ simplification

		<b>03</b>	
15.	$5(9a^2 - 4b^2)$ $5(3a - 2b)(3a + 2b)$ <p>When <math>a = 5; b = 3</math></p> $= 5(3 \times 5 - 2 \times 3)(3 \times 5 + 2 \times 3)$ $= 5(15 - 6)(15 + 6)$ $= 5 \times 9 \times 21$ $= 945$	<b>B1</b> <b>M1</b> <b>A1</b>	For ✓ simplified factorization  ✓ substitutes and simplification
		<b>03</b>	
16.	$\frac{3^3 \times 3^n - 3 \times 3^n}{4 \times 3^2 \times 3^n}$ $\frac{3^n(27 - 3)}{3^n \times 36}$ $\frac{24}{36}$ $\frac{2}{3}$	<b>M1</b> <b>M1</b> <b>A1</b>	Condone 0.667
17.	$27^{-m} \times \frac{1}{81} = 243$ $(3^3)^{-m} \times \frac{1}{3^4} = 3^5$ $3^{-3m-4} = 3^5$ $-3m - 4 = 5$ $-3m = 9$ $m = -3$	M1 M1 A1 	Exp. in powers of 3  Equating powers of 3
18.	$4p + 6q = 184 \dots \times 3$ $3p + 8q = 222 \dots \times 4$ $12p + 18q = 552$ $12p + 32q = 888$ $\frac{14q}{14} = \frac{336}{14}$ $q = 24$ $4p + 144 = 184$	M1 M1	Formation of Equation  Elimination of p or q or equivalent

	4p = 40  P = 10	<u>A1</u> 3	In both
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19.	Old area = $80 \times 60 = 4800\text{m}^2$ New area = $(80 - 2x)(60 - 2x) = 2$ $\Rightarrow 4x^2 - 280x + 4800 = 2/3 \times 4800$ $4x^2 - 280 + 1600 = 0$ $X = 6.28\text{m}$	M1 M1 M1 A1	✓ old area ✓ exp ✓ solving CAO
20.	04	B1	2 equations

$$\begin{array}{rcl} 4t + 3n & = & 4250 \\ 6t + 2n & = & 4000 \\ \hline 3t + n & = & 2000 \\ 9t + 3n & = & 6000 \\ 4t + 3n & = & 4250 \\ \hline 5t & = 350 \\ n & = 2000 - 1050 \\ & = 950 \\ 3 \times 350 + 3 \times 950 & = 3900 \end{array}$$

		M1	solving
		A1	For t and n
		B1	

21. Through A/C in Kenya  
 $1000000 \times 76.84 = \text{Shs. } 7684000$   
 through A/C in UK  
 $\frac{1000000 \times 115.70}{1.53} = \text{Shs. } 7,562,091.15$   
 Through UK less by  
 $768400 - 7562091.85 = 121,908.85$

22. 6000 turn \_\_\_\_\_  $6000 \times 84.15$   
 $= \text{Kshs. } 504900$   
 $\text{Balance} = 504900 - 300000$   
 $= 204900$   
 $\therefore \text{sterling pound} = \frac{204900}{121.47}$   
 $= 1686.8$

23. In Rand =  $\frac{2800265}{10.0166} = 279562.4264$

$$\text{Expenses} = (115,700 + 97000 + 53689)$$

$$= 266389 \text{ Rand}$$

$$\text{Remainder} = 279562.4264$$

$$\underline{266,389.000}$$

$$13,174.4264$$

$$\text{Amount in Kshs.} = 13174.4264 \times 9.9399$$

$$= 130,942.50$$

24. Kshs.  $(3000 \times 1.89) = 5670$

$$\text{Remain} = 5670 - 4695 = 75$$

$$\text{Francs} = \frac{(975)}{1.95} = 500$$

25. Amount in dollars  $= 75 \times 40 = 3,000$

$$\text{Amount in Ksh} = 3000 \times 81.40 = 244,200/$$

$$\text{Less commission} \quad \underline{4,000}$$

Total received sh 240,200

26. Hong Kong  $8105,000 \times 9.74 = \text{ksh.} 1022700$

$$\text{Amount spent in Kenya} = 403879$$

$$\text{Balance} = 1,022,600 - 403,879 = 618,821$$

$$\text{Amount in South Africa} = \frac{618821}{12.11} = 51100 \text{ rands}$$

27.  $500000 \text{ J yen into Kshs.} = \frac{(500000 \times 66.5)}{100}$

$$= \text{Kshs.} 330,250$$

$$\text{Amount spend in Kenya} = \text{Kshs.} 16200$$

$$\text{Remained with Kshs.} (330250 - 16200) \\ = \text{Kshs.} 314,040$$

Kshs. 314040 into Euros:

$$= \frac{(314040)}{78.15}$$

$$= 4,018.554063 \text{ Euros}$$

He left Kenya with = 4,019 Euros (nearest Euro)

28. 1 \$ ————— Kshs. 77.43

$$5600\$ = (5600 \times 77.43)$$

$$= 433608$$

Spent 201,367

$$\text{Remainder} = (433608 - 201367)$$

$$= 232241$$

ISR ————— shs. 9.51

$$\left. \begin{array}{r} \text{Shs.} 232241 \\ 1 \times 232241 \\ \hline 9.51 \end{array} \right\}$$

$$= \text{shs.} 24420.715$$

29. 1 UK £ = 125.30

$$9000 \text{ UK £} = 125.30 \times 9000$$

$$= 1,127,700$$

$$\text{Commission} = 5/100 \times 1,127,700 = 56,385$$

He got 1,071,315

$$\text{Expenditure} = \frac{3}{4} \text{ of } 1,071,315 = 803,486.25$$

$$\text{Amt. left} = 267,828.75$$

$$\text{In US \$} = \underline{267,828.75}$$

$$63.20$$

$$= 4237.7966$$

$$\simeq 4237 \text{ US \$}$$

30. 1 sterling pound = Kshs. 120

$$? = \text{Kshs. } 100000$$

$$100000/120 = 833.3 \text{ sterling pounds}$$

$$1 \text{ sterling pound} = 1.79 \text{ U.S dollars}$$

$$833.3 = ?$$

$$= 833.3 \times 1.79 = 1491.7 \text{ dollars}$$

$$1 \text{ U.S dollar} = \text{Kshs. } 78$$

$$1491.7 \text{ dollars} = \text{Kshs. } ?$$

$$1491.7 \times 78 = 116350 \text{ Kenya shillings}$$

31. Amount received in Kenya shillings

$$= \underline{\sum 50,000 \times \text{Shs. } 120.7131}$$

$$\sum = \text{Kshs. } 6035655$$

$$\text{Amount received in sterling pound}$$

$$= \underline{1 \sum x \text{ Kshs. } 6035655}$$

$$120.9294 = \sum 49910.568$$

32. Sh(20000 x 147.86) = sh. 2957200

$$\text{To US Dollars} = \underline{44700}$$

$$74.5$$

$$= 6000$$

He received 6000 US Dollars

33. a)  $6a + 7a - 2b - 4b + 2$

$$= 13a - 6b + 2$$

b)  $\underline{2x - 2} - \underline{3x + 2} = \underline{2(2x - 2)} - \underline{(3x + 2)}$

$$2x \quad 4x \quad 4x$$

$$= \underline{\frac{4x - 3x - 4 - 2}{4x}}$$

$$= \underline{\frac{x - 6}{4x}}$$

### 15. Commercial arithmetic

1	(a) $48,000 - 20,000 = 28,000$ $\frac{8}{100}x = 28,000$ $x = 28,000 \times \frac{100}{8}$ $= \text{Ksh. } 350,000$ $\text{Sales } 350,000 + 100,000 = 450,000$ (b) (i) Sales in February	M <sub>1</sub> M <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub> M <sub>1</sub>	✓ exp
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	$\frac{118}{100} \times 450,000$ $= Ksh.531,000$ $531,000 - 100,000$ $431,000 \times \frac{8}{100}$ $= 34,480$ (ii) $\frac{75}{100} \times 531,000$ $= Ksh.398,250$ Commission $298,250 \times \frac{8}{100}$ $= Ksh.23,860$ $= Ksh.23,860 + 20,000$ $= Ksh.43,860$	M <sub>1</sub> A <sub>1</sub>  M <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>	
2	Total expense $In Ush. 1050000 + 60 \times Ush 1016 + \frac{55}{100} \times 1050000$ (a) $= 1050000 + 60960 + 577500$ $= Ush 1688460$ $\frac{1688460}{24.83}$ In Ksh. $Ksh. 68000.81$ $\frac{60960 + 577500}{1688460}$ (b) $= 37.81\%$ $Ksh. \frac{68000.81}{0.0714}$ (c) $= Tsh 952,392.30$ $\frac{68000.81 - \frac{1050000}{24.83}}{\frac{105000}{24.83}} \times 100$ (d) $= 60.81\%$	M <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>  M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	
3	$A = 3P$ $\therefore 3P = P \left(1 + \frac{20}{100}\right)^n$ $3 = (1.2)n$ $\log 3 = n \log 1.2$	M <sub>1</sub>  M <sub>1</sub> A <sub>1</sub>	✓ exp ✓ attempt to simplify

	$\begin{aligned} n &= \frac{\log 3}{\log 1.2} \\ &= 6.0254 \\ \therefore & 6 \text{ years} \end{aligned}$		
		3	
4.	$\begin{aligned} 100 \text{ Yens} &= 63.16 \text{ (Bank sold Yen)} \\ 36632.8 \text{ Yen} &= \left( \frac{36632.8 \times 63.16}{100} \right) \\ &= 23137.27648 \\ &= \text{ksh } 23137. \end{aligned}$	M1 A1	
		02	
5.	$\begin{aligned} \text{No of good eggs} &= 24 \times 30 - 54 \\ &= 666 \\ \text{Total cost} &= 24 \times 225 \\ &= \text{shs } 5400 \\ \text{Cost with profit of } 22\% \text{ expected} &= \\ &\quad \left( \frac{122}{100} \times 5400 \right) \\ &= 6588 \\ \text{New price per egg} &= \frac{6588}{666} \\ &= 9.892 \\ &= \text{sh } 10.00 \end{aligned}$	M1 M1 M1 M1 A1	
		04	
6.	$\begin{aligned} A &= P \left( 1 + \frac{r}{100} \right)^n \\ n &= 6 \ r = 4\% \ p = 10,000 \\ A &= 10,000 \left( 1 + \frac{4}{100} \right)^6 \\ &= 10,000(1.04)^6 \\ &= 12,653.19 \\ \text{interest} &= 12,653 - 10,000 \\ &= \text{sh. } 2,653 \end{aligned}$	M1 A1 M1 A1	
		04	
7	$\begin{aligned} \frac{1}{2} \text{ of } 2400E &= 1200E \\ \text{In ksh.} &= 1200E \times 95.65 \\ &= \text{Ksh. } 114,780 \\ \text{Number of dollar} &= \text{Kshs. } 114,780 \\ &\quad 76.50 \\ &= \text{sh } 1500.39 \end{aligned}$	M1 M1 A1	
		3	
8	<p>SECTION B (50 MARKS)</p> <p>Selling price = 88/100 of marked price</p> <p>(a)</p>		

	<p>(i) <math>4800 = 88/100 \text{ of m.p}</math>  <math>4800/88 \times 100 = \text{m.p}</math>  <math>= \text{sh.}5454.54</math></p> <p>(ii) <math>145/100 \text{ of buying price} = 4800</math>  <math>\text{buying price} = \frac{48000 \times 100}{145}</math>  <math>= 3310.34</math></p> <p>(b) <math>\frac{5454.54 - 3310.34}{3310.34} \times 100</math>  <math>= 0.6477 \times 100</math>  <math>= 64.77\%</math></p> <p>C) <math>\frac{87.5}{100} \text{ of } 3310.34</math>  <math>= 2,896.55</math></p>	
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10	$\frac{100}{72} \times 1440 = 2000$ $\frac{100}{2000} \times 100$ $5\%$	M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub> — 3	
11	$200s + 600t = 22\ 000$ $250s + 510t = 22\ 700$  $1000s + 3000t = 110\ 000$ $1000s + 2040t = 90800$ $960t = 19\ 200$ $t = 20$ $s = 40$ $(0.5 \times 200 \times 20) + (0.3 \times 510 \times 40)$ $2500 + 6120 = 8620$ $(0.5 \times 200 \times 20) + (0.3 \times 600 \times 40)$ $2000 \times 7200$ $\frac{9200}{48000} \times 100$ $19\%$	B1  M1  A1 B1 M1 M1 A1 M1 M1 M1 M1 M1 M1	For both  Elim  For shorts For trousers
12.	(a) Cost of tonne of rice is US \$ 500 Shipping cost of rice = $20/100 \times \text{US\$} 500 =$ US\$ 100 Transport to Nairobi = $5/100 \times \text{US\$} 500 =$ US\$ 25 Custom duty = $10/100 \times \text{US\$} 500 = \text{US\$} 500 =$ US\$ 50 Total cost in dollars = $500 + 100 + 50 + 25 =$	B1 B1 B1 A1 B1	

	US\$ 675 Converting this to ksh = $76.60 \times 675 = \text{ksh } 51705$ (b) To make profit of 20% he must sell rice at 120% of the cost price per tonne Selling price of a tonne of rice = $120/100 \times \text{sh } 51705$ = ksh 62046 1 tonne has 1000kg, selling price of 1kg of rice = 62.046 = sh 62.00 (c) Total collection = sh 62046 in US dollars this becomes $\frac{62046}{78.20} = \text{US\$ } 793.4271$ Actual profit = US\\$ 793.4271 – 675 = 118.4271 Actual percentage = $118.4271/625 \times 100$ = 17.545%	B1 B1 B1 B1 A1	
		10	

15. Total exp =  $600000 + 100000$  M1

$$= \text{Sh. } 700000$$

$$\text{Profit} = 108000 - 700000$$

$$\% \text{ profit} = \frac{380000}{60000} \times 100$$

$$63\frac{1}{3}\%$$

A1

3

16. (a) Swiss Franc =  $\frac{52}{1.28}$  M1  
= 40.625 A1

$$(b) \text{Kenya shillings} = 40.625 \times 45.21$$

$$= 1837$$

B1

3

17.	Let the rates be x% and y% respectively $\frac{x}{100} \times 50000 = 2250$ $\frac{y}{100} \times 45000 = 2250$ $\therefore x = 4.5\%$ $y = 5\%$	M1 M1 A1	
		03	
18.	Sitienei – x Lagat – 1.1x Rotich – 1.155x Diff = $1.155x - 1.1x = 0.55x = 110,000$ $x = 2,000,000$ Rotic paid $1.155 \times 200,000 = 231000$	M1 M1 A1	
		03	
19.	$5000 \times 84.15$ = sh. 420750	<b>M1</b>	

	420750 - 289850 = sh. 130900 Amount = $\frac{130900}{65.45} \times 100$ = 200,000 Japanese Yen	M1  A1  03	
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20 a.)	$A = 1200000 (1 - \frac{8}{100})^2$ $1200000 (0.92)^2$ = 1,051,680  $1015680 (1 - \frac{12}{100})^4$ $1015680 (0.88)^4$  = 609098.58	M1  M1  M1  A1	
b.)	$125/100 \times 609098.58$  = 761373.23  $761373.23 = 1200000(1 - r/100)^{72}$ $(1 - r/100)^{72} = 0.6344776$  $72/72 \log(1 - r/100) = -0.1975836/72$ $\log(1 - r/100) = -0.0027442173$  $1 - r/100 = 0.9937011$  $r/100 = 0.0062988$ $r = 0.62988 \% \text{ p.m.}$	M1  A1  M1  M1  M1  A1  10	Follow thro' if A1 is missing  A1✓ if A above is lost.

21.	$50x + 25y = 200$ $2x + y = 8 \dots\dots (i)$	M1	✓ eqn (i)
-----	--	----	-----------

	$50x + \frac{28}{100} + \frac{24}{100} \times 25y = 53.50$ $14x + 6y = 53.50$ $\therefore 2x + y = 8$ $14x + 6y = 53.50$ $\therefore 2x + y = 8$ $14x + 6y = 53.50$ $\therefore y = 2.50, x = 2.75$ $\text{milk} = \text{Ksh. } 2.75, \text{ salt} = \text{Ksh. } 2.50$	M1 M1 A1	✓ eqn (ii) ✓ solving ✓ both
		04	
22.	$\text{Yen} = 1000 \times 105 \times \frac{105}{100} + 1260 = 111510 \text{ Yen}$ $\text{Ksh} = 111510 \times \frac{63}{105} = \text{Ksh. } 66906$	M1 A1 B1	
		03	
23	$C.P = \frac{240.50}{125} \times 100$ $= \text{Sh} 192.40$ $\text{R.P} = 240.50 - 22.90$ $= \text{Sh} 217.60$ $\% \text{ profit} = \frac{217.60 - 192.40}{192.40} \times 100$ $= 13.1\%$	M1 M1 A1	
		3	
24	$7 \left[ x^2 - \frac{1}{4}(y+1)^2 \right]$ $= 7 \left( x - \frac{1}{2}(y+1) \right) \left( x + \frac{1}{2}(y+1) \right)$ When $x = 5$ and $y = 8$ $7 \left( 5 - \frac{9}{2} \right) \left( 5 + \frac{9}{2} \right)$ $= 7 \times \frac{1}{2} \times \frac{19}{2}$ $= 33.25$	B1 B1 B1	
		3	
25	$1500 \times 78.43 = \text{sh. } 117645$ $\frac{117645}{79.25} = \$1484.5$ $\text{loss: } 1500 - 1484.5 = \$15.5$	M1 M1 A1	
		3	

26.  $2x - 3y + 6 = 0$

$$-3y = -2x - 6$$

$$y = \frac{2x + 6}{3}$$

$$\text{When } y = 0 \quad x = -3$$

$$x = 0 \quad y = 2$$

$\therefore$  Co-ordinate of  $y$ -intercept is  $(0, 2)$

" "  $x$ -intercept is  $(-3, 0)$

$$\therefore \angle CAO = \tan^{-1} \frac{2}{3}$$

$$= 33.69^\circ$$

$$\therefore \angle \theta = 180 - 33.69^\circ$$

$$= 146.31^\circ$$

27. Point  $y$   $(\frac{4+(-2)}{2}, \frac{7+(-1)}{2}) = (1, 3)$

$$\text{grad } AB = \frac{7+1}{4+2} = \frac{8}{6}$$

$$\text{grad } xy = -\frac{3}{4}$$

$$\text{grad } xy = -\frac{3}{4}$$

$$\frac{y-3}{x-2} = -\frac{3}{4}$$

$$x-2$$

$$y = -\frac{3}{4}x + \frac{15}{4}$$

28.  $Y = 3x - 1$

$$M = 3$$

$$M_1 m_2 = -1$$

$$M_2 = -\frac{1}{3}$$

$$\frac{y-3}{x-2} = -\frac{1}{3}$$

$$3y - 9 = -x + 2$$

$$\frac{3y}{3} = \frac{-x}{3} + \frac{11}{3}$$

$$Y = \frac{x}{3} + \frac{11}{3}$$

29. Pt T is  $\frac{1+5}{2}, \frac{4+10}{2} = (-2, 7)$

$$\text{grad. of grid } xy = \frac{10-4}{-5-1} = \frac{14}{-6} = -\frac{7}{3}$$

$$\therefore \text{grad of } L_2 = \frac{3}{7}$$

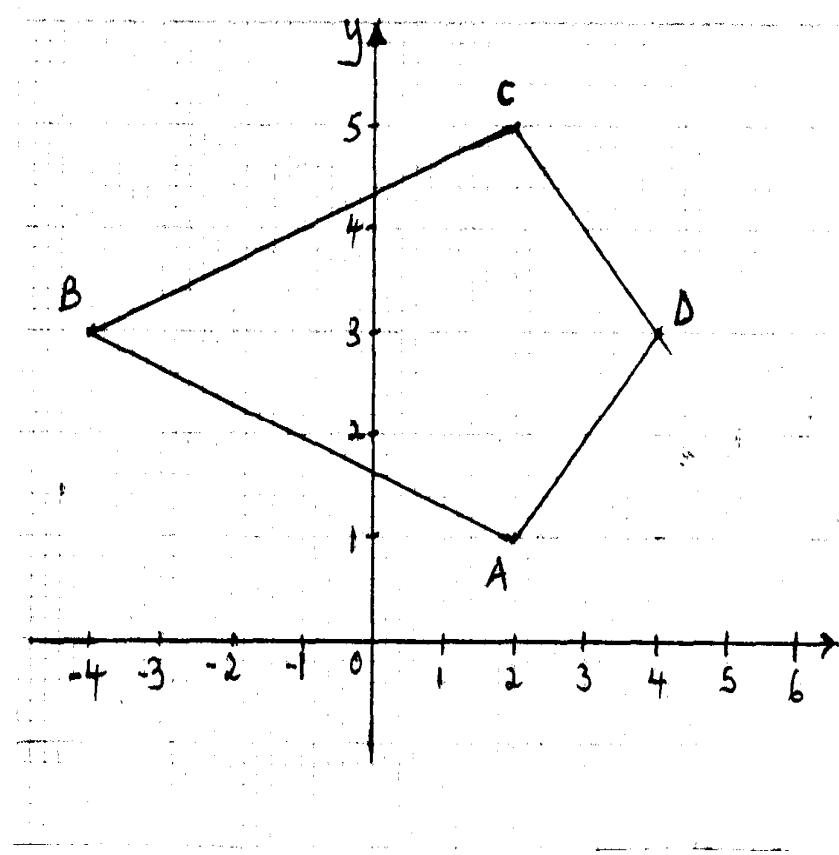
Take a general pt P(x,y) on  $L_2$

$$\Rightarrow \frac{y-7}{x-2} = \frac{3}{7}$$

$$\begin{aligned} \Rightarrow 7y - 49 &= 3x + 6 \\ 7y &= 3x + 55 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \text{Equation of } L_2$$

$$\text{Or } y = \frac{3x + 55}{7}$$

30. a, b



(c) Name : a kite

31. (a) Grad of line  $QP = \frac{4-2}{1-3} = \frac{2}{-2} = -1$

Grad of line  $QR = 1$

Take a pt  $Q(1, 4)$  and  $T(x, y)$  on line  $QR$

$y - 4 = 1$

$x - 1$

$y - 4 = x - 1$

$y = x + 3$  .....equ. of  $QR$

(b)  $y = x + 3$  ... (i) Equ of  $QR$

$y = 3x - 7$  ... (ii) Equ. of  $Pr$

Solving simultaneously ;:

$x + 3 = 3x - 7$

$2x = 10$

$x = 5$

Substituting ;  $y = 8$

$\therefore R$  is the pt  $(5, 8)$

(c)  $\overset{\sim}{PS} = \overset{\sim}{QR} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$\overset{\sim}{OS} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

*S is the point (7,6)*

32. a) Gradient  $OA = \text{Gradient of } CB$

$$\frac{-1 - 0}{2 - 0} = -\frac{1}{2}$$

Gradient of  $CB$ 

$$\frac{y - 3}{0 - 4} = -\frac{1}{2}$$

$$2y - 6 = 4$$

$$2y = 10$$

$$y = 5$$

b) i)  $AN = ON - OA = \frac{1}{2} OM - OA$

$$OM = OA + \frac{1}{2} AB = (2) + \frac{1}{2} (2)$$

$$= 3$$

$$AN = \frac{1}{2} \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -1 \end{pmatrix}_3$$

ii)  $NC = OC - ON$

$$= \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$$

iii)  $AC = OC - OA = \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

c)  $AN = \frac{1}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$NC = \frac{3}{2} \begin{pmatrix} -1/3 \end{pmatrix}$$

$4AN = AC$  And A is a common point hence A, N, C lie on a straight line.

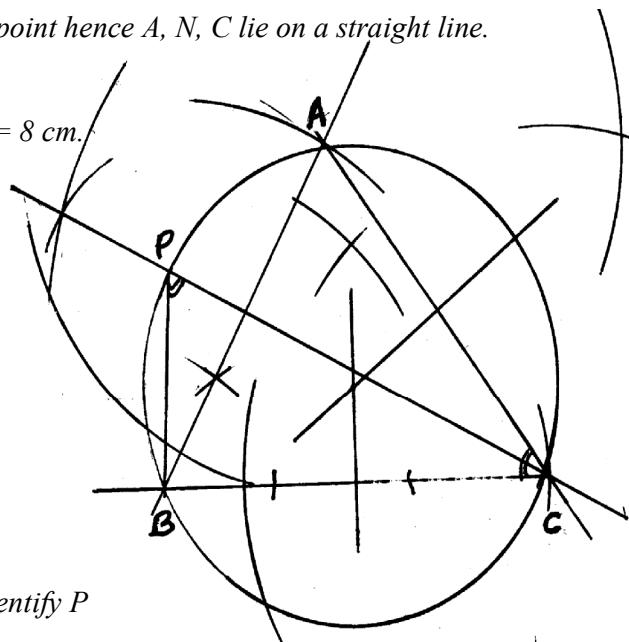
33. a)  $\Delta ABC$  line  $AB = 7 \text{ cm}$  and  $BC = 8 \text{ cm}$ .

Construction of  $\triangle ABC$ 

(b)  $AC = 7.6 \pm 0.1$  and

$\angle ACB = 53 \pm 1^\circ$

(c) 2 sides bisector

Circle drawn radius  $4.4 \pm 0.1$ 

(d) Bisect  $\angle ACB$

Bisection line to cut the circle to identify P

$\angle PBC$  measure =

(a)  $AB = 7 \text{ cm}$ ,  $BC = 8 \text{ cm}$ 

$$\angle ABC = 60^\circ$$

(b)  $AC = 7.6 \pm 0.1 \text{ cm}$ 

$$\angle ABC = 53^\circ \pm 0.1$$

(c) Perpendicular bisectors of any two sides.

Circle drawn

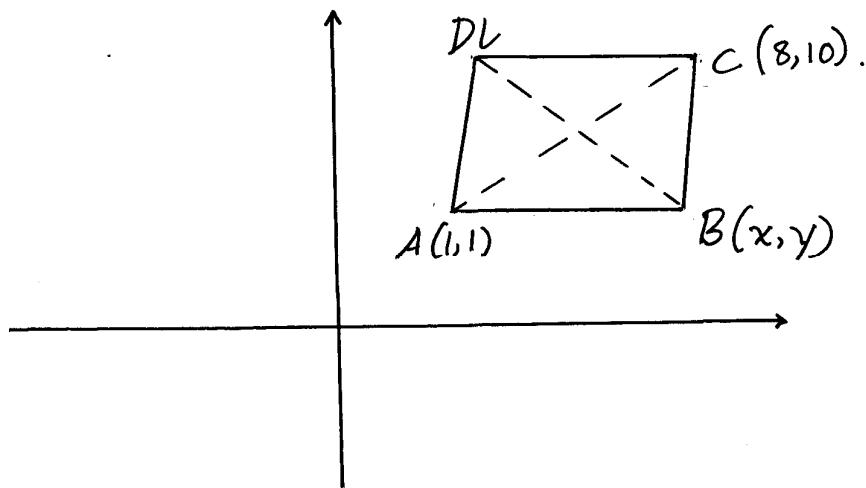
Radius =  $4.4 \pm 0.1 \text{ cm}$ (d)  $\angle ACB$  bisected

Bisection line drawn to cut circle at P

$$\angle BPC = \angle BAC = 67^\circ$$

$$\angle PBC = 88 \pm 0.1^\circ$$

34.



$$M\left(\frac{1+8}{2}, \frac{1+10}{2}\right) = M(4.5, 5.5)$$

$$b) AB: 4x - 5y = -1 \quad x \neq 2$$

$$BC: 5x - 2y = 20 \quad x \neq 5$$

$$8x - 10y = -2$$

$$\underline{25x - 10y = 100}$$

$$-17x = -102$$

$$x = \frac{-102}{-17} = 6.0$$

$$24 - 5y = -1$$

$$5y = -25$$

$$Y = 5$$

$$\therefore B(6, 5)$$

$$\frac{x + 6.0}{2} = 4.5 \quad x = 3$$

$$\begin{aligned} \frac{y+5}{2} &= 5.5 \\ y &= 6 \\ \therefore D(3,6) \end{aligned}$$

c)  $AB = \sqrt{(16-1)^2 + (5-1)^2}$   
 $\sqrt{25+16}$   
 $\sqrt{41} = 6.40 \text{ (units)}$

35. *Mid ordinate*

$$\begin{aligned} \text{Area} &= 1.2(6.2 + 4.3 + 2.6) \\ &= 15.72 \end{aligned}$$

## 16. Coordinates and graphics

1.	<p>(i)</p> $k\left(\frac{3-7}{2}, \frac{4+2}{2}\right) = (-2, 3)$ $p\left(\frac{3+1}{2}, \frac{4-2}{2}\right) = (2, 1)$ <p>(ii)</p> $G_1 = \frac{3-2}{-2-2} = \frac{-1}{2}$ $G_2 = 2$ $\text{Mid } p+kp = \left(\frac{-2+2}{2}, \frac{3+1}{2}\right) = (0, 2)$ $\therefore \text{equation } y = 2x + c$ $\text{when } x = 0, y = 2, \text{then } c = 2$ $\text{hence, } y = 2x + 2$	<p>B<sub>1</sub> for both p and k ✓</p> <p>B<sub>1</sub> for both G<sub>1</sub> and G<sub>2</sub> ✓ r identified</p> <p style="text-align: right;"><math>\frac{B_1}{3}</math></p>
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2. Let the exterior ∠ be x

$$6.5x + x = 180$$

$$7.5x = 180^\circ$$

$$x = 24$$

$$\begin{aligned} \text{No. of sides} &= \frac{360}{24} \\ &= 15 \text{ sides.} \end{aligned}$$

3.  $\frac{(2n-4)90}{(2(n/2)-4)90} = \frac{3}{4}$   
 $\frac{2n-4}{2n} = \frac{3}{4}$   
 $8n - 16 = 6n$   
 $2n = 16$

$$n = 8$$

$$(2(8) - 4) 90$$

$$= 12 \times 90 = 1080$$

4.  $\frac{15}{2} b = 60$

$$\frac{15}{2} b = 60$$

$$15b = 60 \times 4$$

$$b = 16\text{cm (diagonal)}$$

$$\begin{array}{l} \diagdown \\ \Rightarrow \end{array} = \sqrt{8^2 + 7.5^2}$$

$$\therefore \text{per} = 4 \sqrt{8^2 + 7.5^2}$$

$$= 43.86\text{cm}$$

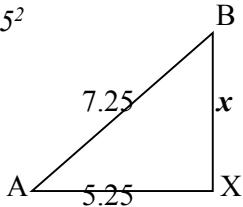
5.  $x^2 = 7.25^2 - 5.25^2$

$$x = \sqrt{7.25^2 - 5.25^2}$$

$$= 52.5625$$

$$\frac{27.5625}{\sqrt{25}}$$

$$= 5\text{cm}$$



$$BC = 15.25 + 5 = 22.25\text{cm}$$

$$\text{Arc } CD = \frac{90}{360} \times 3.142 \times 2 \times 22.25$$

$$= 34.65475$$

$$\begin{aligned} \text{Perimeter} &= AB + BC + CD + DE + EA \\ &= 15.25 + 7.25 + 22.25 + 34.95 + 5.25 \\ &= 84.95\text{cm} \end{aligned}$$

6.  $AB^2 = 10^2 - 8^2 = 100 - 64$

$$AB^2 = 36$$

$$AB = 6\text{cm}$$

$$\cos(90^\circ - x^\circ) = \frac{4}{5}$$

*Attempt to get x by using t+e = 180°*

$$e = \frac{(2n-4)90}{n}$$

*number of sides*

7.  $x - 20 + 3x = 180^\circ C$

$$4x = 200$$

$$x = 50^\circ$$

8.  $2x + 40 + x - 25$

$$3x + 15 + 9 = 180$$

$$3x + 15 = 29$$

$$9 = \frac{1}{2}(3x + 15)$$

$$3x + \frac{3x}{2} = 180 - 15 - \frac{15}{2}$$

$$x = 35^\circ$$

$$x = 35 = 10^\circ$$

$$\frac{1}{2}(10 + 110) = 60^\circ$$

9.  $\frac{1260}{90} = 14rt \angle s$

$$90$$

*Sum of interior ∠s*

$$(2n - 4) rt \angle s$$

$$2n - 4 = 14$$

$$n = 9$$

*9 sided polygon*

10.  $N = 50 + 40 = 90^\circ$

*Alternative angles*

11.  $5^{3(y+1)} + 5^{3y} = 630$

Let  $x = 5^{3y}$

$$5^3 x + 5^{3y} = 630$$

$$125x + x = 630$$

$$x = 5$$

$$5^{3y} = 5^1$$

$$3y = 1$$

$$y = \frac{1}{3}$$

12.  $\frac{360}{n} + 108 = 180 - \frac{360}{n}$

$$360 + 108n = 180n - 360$$

$$-72n = -720$$

$$n = 10$$

13. Let exterior angle be  $x$

$$\frac{4x}{4} = \frac{180^\circ}{4}$$

$$x = 45^\circ$$

$$n = 360$$

*Exterior angle*

$$n = \frac{360}{45}$$

$$= 8 \text{ sides}$$

14. a) Let  $\angle BDC = \phi$

$$A^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos \phi$$

$$\cos \phi = \frac{89 - 16}{80} = \frac{73}{80} = 0.9125$$

$$\phi = \frac{24^\circ 9}{1} = 24^\circ 8$$

b) Area of ABD

$$= \frac{1}{2} \times 8 \times 10 \sin 24^\circ 9^\frac{1}{1}$$

$$= 40 \times 0.4091$$

$$= 16.36 \text{ cm}^2 \quad 16.37 \quad 16.38$$

15. (a)  $\angle CDF = 100 - 60 = 40^\circ$  (exterior angle of a  $\Delta$ )

(b)  $\angle BDE = 20^\circ$  (DE is bisector of BDG)

$\therefore \angle ABD = 20^\circ$  (alternate angles)

16.  $4x + x - 30 = 180$

$$5x = 210^\circ$$

$$x = 42$$

$$(x - 30)n = 360^\circ$$

$$12n = 360^\circ$$

$$n = \frac{360^\circ}{12}$$

$$n = 30$$

17.  $180(n-20) = 1440$

$$n-2 = \frac{1440}{180} = 8$$

$$180$$

$$n = 10$$

Decagon

18.  $\angle PQR = \angle SRT = x$  (Alt  $\angle$ s of  $\triangle SPQ \parallel RS$ )

$$\therefore 5x + 3x + x = 180^\circ$$

$$9x = 180^\circ$$

$$X = 20^\circ$$

$$\therefore 5x + 20 + y = 180$$

$$y = 180 - 120 = 60$$

19. Let the interior  $\angle$  be  $x$  and exterior be  $y$

$$\therefore x + y = 180$$

+

$$\frac{x-y=132}{2x=312}$$

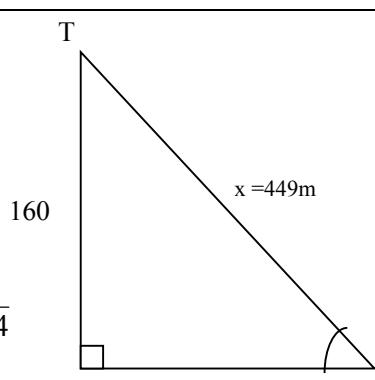
$$x = 156$$

$$y = 180 - 156 = 24^\circ$$

$$\text{No. of sides } (n) = \frac{360^\circ}{24} = 15$$

$$= 15 \text{ sides}$$

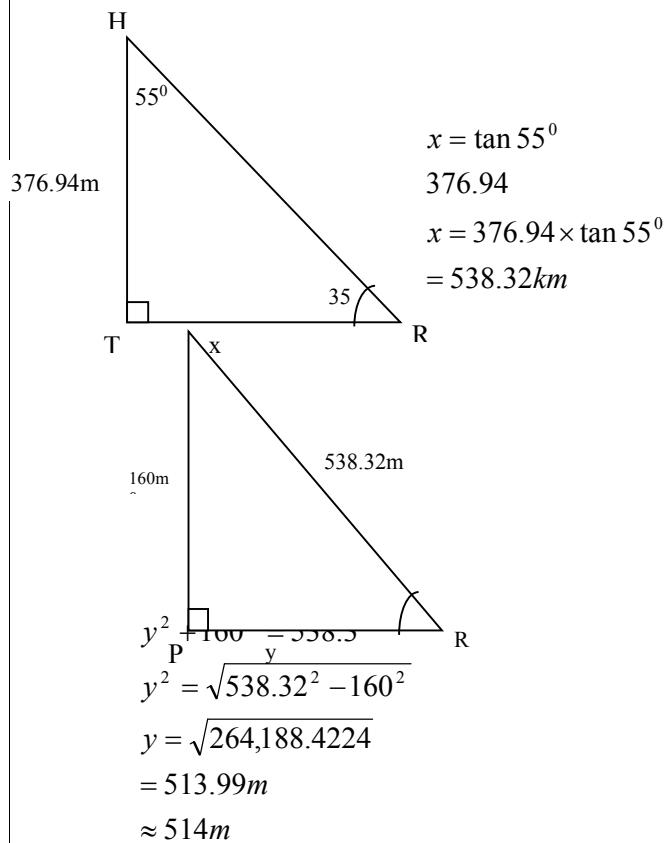
### 17. Angles and Plane figures

<b>1</b> $2a+b=180$ $13a-b=360$ $15a=540$ $a=\frac{540}{15}=36$ $72+b=180$ $b=180-72-108^\circ$	M <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	✓ formation of the equations ✓ attempt to solve
<b>2</b> $\angle XAD = 30^\circ$ $180 - (50+30)$ $= 180 - 80 = 100^\circ$	B <sub>1</sub> B <sub>1</sub>	
<b>3</b> $\frac{h}{100} = \tan 67^\circ$ $h = 160 \times \tan 67^\circ$ $= 376.94\text{m}$  $x^2 = \sqrt{160^2 + 420^2} = \sqrt{25600.1764}$ $= 449\text{m}$	 M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	

$$\frac{376.94m}{449m}$$

$$\tan \theta = 0.8395$$

$$\theta = 40^\circ$$

M<sub>1</sub>M<sub>1</sub>  
A<sub>1</sub>M<sub>1</sub>  
M<sub>1</sub>M<sub>1</sub>  
A<sub>1</sub>

4.	$aA^2 = b^2 + c^2 - 2bc \cos A$ $4^2 = 3^2 + 6^2 - 2 \times 3 \times 6 \cos \theta$ $-29 = -36 \cos \theta$ $-29 = \cos \theta$ $-36$ $36.34^\circ = \theta$	M <sub>1</sub>  M <sub>1</sub>  A <sub>1</sub>	Substitution  Attempt to simplify
5.	$\frac{1}{3} \left( \frac{180(n-2)}{n} \right) = \frac{360}{n}$ $180n - 360 = 1080$ $180n = 1440$ $n = 8$ The polygon is an octagon		M <sub>1</sub>  M <sub>1</sub>  $\frac{A_1}{3}$

6.	$y = 180^\circ - 130^\circ = 50^\circ$ $x = 180^\circ - (50^\circ + 83^\circ) = 47^\circ$ $z = 180^\circ - 47^\circ - 133^\circ$	B1 B1 B1 3
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9. Let the ex  $\angle$  be  $x^\circ$

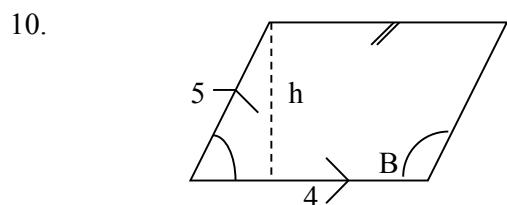
In  $\angle < 8x^\circ$

$x + 8x = 180$ .....  
 $x = 20$

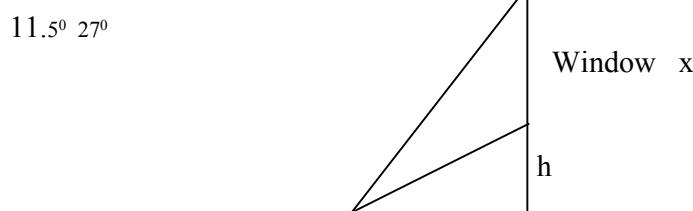
M1  $\frac{n-2}{n} 180 = 8 \left( \frac{360}{n} \right)$  M1M1  
 $n = 18$  sides A1

No of sides =  $\frac{360}{20}$  M1  
= 18 sides A1

3



Area =  $5 \times \sin \alpha = 12$  M1  
 $\alpha = 36.87^\circ$  A1  
 $B = 143.13^\circ$  A1  
3



$\tan 27^\circ = \frac{h}{20}$  M1  
 $h = 10.19m$   
 $\tan 32^\circ = \frac{x}{20}$  M1  
 $x = 12.50m$   
Window height = 2.31m A1

3

12.	$\frac{360^\circ}{n} = 18^\circ$ $n = \frac{360^\circ}{18^\circ} = 20$ sides Area = $(\frac{1}{2} \times 16 \times 16/2 \tan 81^\circ) \times 20$ = $(8 \times 8 \times 6.3138) \times 20$ = $8081.66\text{cm}^2$	B1 M1 A1 03
13.	(a)	B2

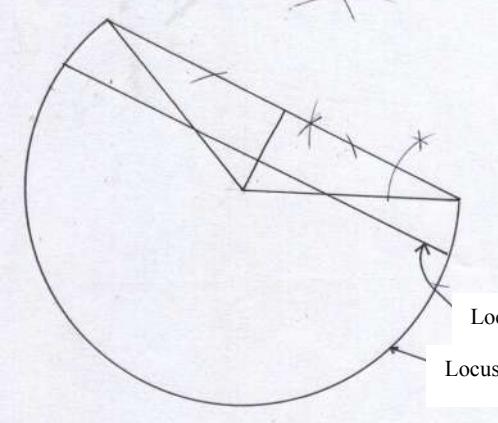
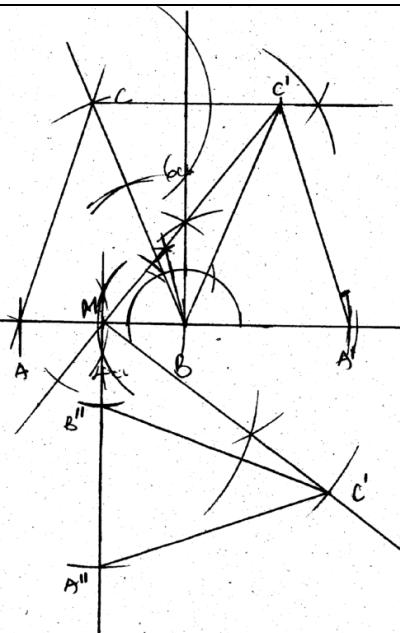
		M1		
	Sketch (b) i)	M1		
	$h = \tan 25^\circ \Rightarrow h = x \tan 25^\circ$	M1		
	x			
	$h = \tan 18^\circ \Rightarrow h = \tan 18^\circ (x + 70)$	M1		
	$x + 70$			
	Equating the two equations	M1		
	$x \tan 25^\circ = x \tan 18^\circ + 70 \tan 18^\circ$			
	$x(\tan 25 \tan 18^\circ) = 70 \tan 18^\circ$	A1		
	$x = \frac{70 \tan 18^\circ}{\tan 25^\circ - \tan 18^\circ}$	B1		
	$x = \frac{22.744}{0.1414} = 160.8$			
	$h = 160.8 \tan 25^\circ = 75m$	M1		
	(c) Distance of A to the front of post			
	$= x + 70$	A1		
	$= 160.8 + 70$			
	$= 230.8m$			
		10		
14.	{2(8) - 2} x 90 $14 \times 90$ $1260^\circ$	M1 $\frac{A_1}{2}$		
15.				
	$\tan 57^\circ = \frac{h}{x} \Rightarrow h = x \tan 57^\circ$			
	$\tan 48^\circ = \frac{h}{50-x} \Rightarrow h = (50-x) \tan 48^\circ$	M1		
	$x \tan 57^\circ = (50-x) \tan 48^\circ$			
	$1.53986x = 55.53 - 1.1106x$	A1		
	$x = 20.95$			
	distance = $50 - 20.95 = 29.045m$ or $20.95m$	M1		
	$h = x \tan 57^\circ = 20.95 \tan 57^\circ$ $= 32.26m$	A1		
		04		

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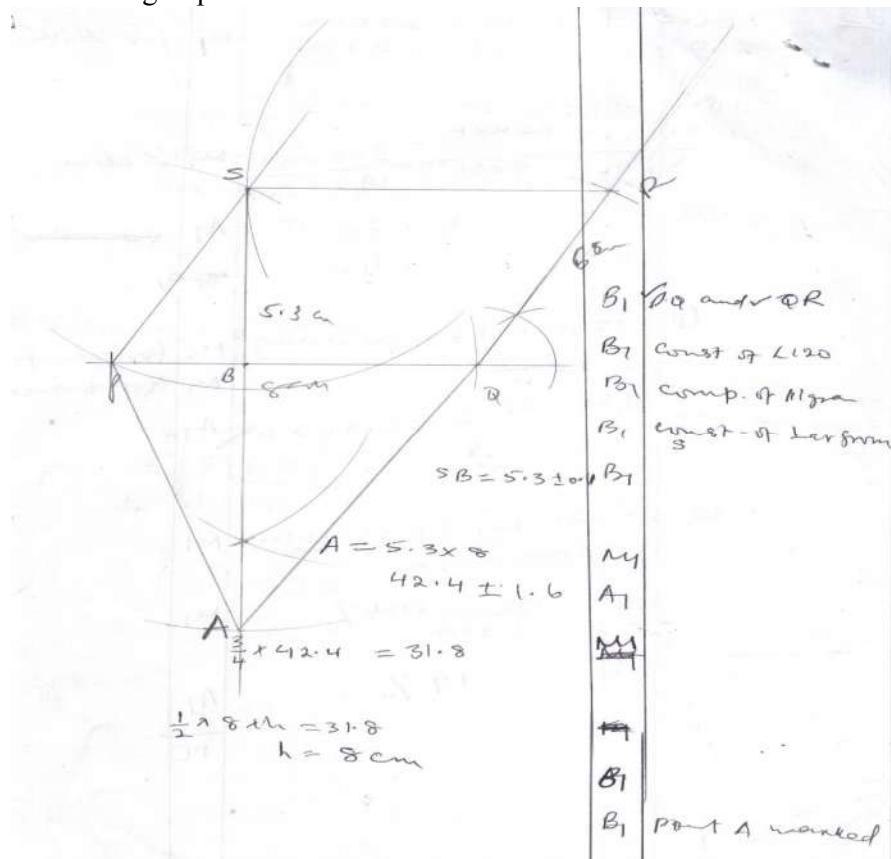
### 18. Geometrical constructions

1	<p> <math>CD = 3.7\text{cm} \pm 0.1</math>          Area of <math>\Delta ABC = \frac{1}{2} \times 9 \times 3.7\text{cm}</math>  <math>= 16.65\text{cm}^2</math> AE =       </p>	B <sub>1</sub> B <sub>1</sub> B <sub>1</sub> B <sub>1</sub> B <sub>1</sub>  B <sub>1</sub> M <sub>1</sub> A <sub>1</sub> B <sub>1</sub> B <sub>1</sub>	✓conct $30^\circ$ ✓conct $15^\circ$ ✓AB 9cm ✓AC 6cm ✓ $\Delta ABC$  ✓CD  Loci of E For AE
---	---	--	--

2		B <sub>1</sub> B <sub>1</sub> B <sub>1</sub>	✓construction of $90^\circ$ at x ✓bisection of line XC and location of centre O ✓circle drawn
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		3	
3	 <p style="text-align: center;"><math>BC = 9.4\text{cm} (\pm 0.1)</math>  <math>AD = 2.7\text{cm} (\pm 0.1)</math></p>	3	B1 ✓length AB = 5.4cm B1 ✓construction of 300 at B B1 ✓location of C and ABC B1 ✓length of BC stated B1 ✓identification of A as centre B1 ✓Locus of P drawn. (Bo if circle completed) B1 ✓dropping of perpendicular ✓length AD stated ✓his height ✓locus of Q drawn
		10	
4.		10	B1 <67% constructed B1 ABC complete B1 AC = 5.7 ± 0.1 B1 C1Drawn B1 A1Drawn B1 A1BC1 completed B1 Locating M (midpoint M of AB) B1 B11 and A11 rotated B1 C1 rotated A1B1C11 completed
		10	

6.



7.	<p><math>6 \text{ cm}</math></p> <p><math>8 \text{ cm}</math></p> <p><math>45^\circ</math></p> <p><math>AM = 4.2 \text{ cm}, AC = 5.6 \text{ cm } (\pm 0.1 \text{ cm})</math></p>	<p><math>B_1</math> Construction of <math>45^\circ</math></p> <p><math>B_1</math> <math>\Delta ABC</math></p> <p><math>B_1</math> <math>\perp</math> dropped from A to BC</p>
9.	(a) $\tan 60^\circ = \frac{AC}{AM}$	M1

9. (a)  $\tan 60^\circ = \frac{AC}{AM}$  M1

$AC = 8.6605 \text{ CM}$  A1

(b)  $A = \frac{1}{2} \times 5 \times 8.6605$  M1  
 $A = 21.65125$  A1

(b)  $\frac{60}{360} \times \pi r^2$

$$\frac{60}{360} \times 3.142 \times 25$$

M1

$$= 13.091 \text{ cm}^2$$

A1

(d) Area of shaded part

$$\Delta COA = \Delta OBA, \text{ sector OCD} = OCB$$

$$21.65 \times 2 = 43.3025 \text{ cm}^2$$

M1

$$13.091 \times 2 = 26.182 \text{ cm}^2$$

M1

 $\therefore$  Area of shaded part

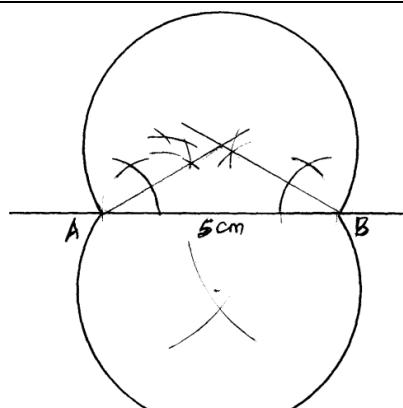
$$43.3025 - 26.182$$

M1

$$= 17.11225 \text{ cm}^2$$

A1

10.



Constant angle locus

10

B1  
B1

B1

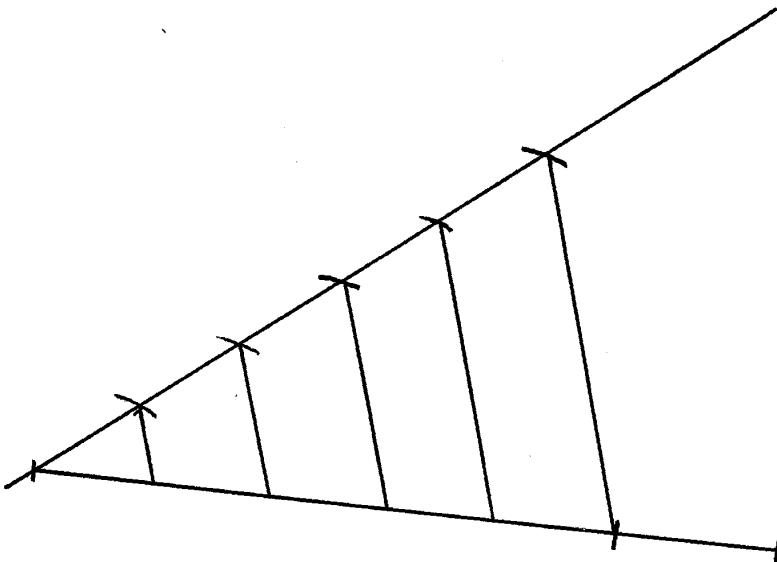
B1

Const of  $30^\circ$  at A  
Const. of  $30^\circ$  at B

For one arc constructed

For lower arc  
constructed.

11.



B1

A line drawn slant to  
touch the given line at  
one end.

B1

Subdivided to 5 equal  
Sections

B1

Parallel lines drawn  
from slant line to  
touch the given line  
.All complete

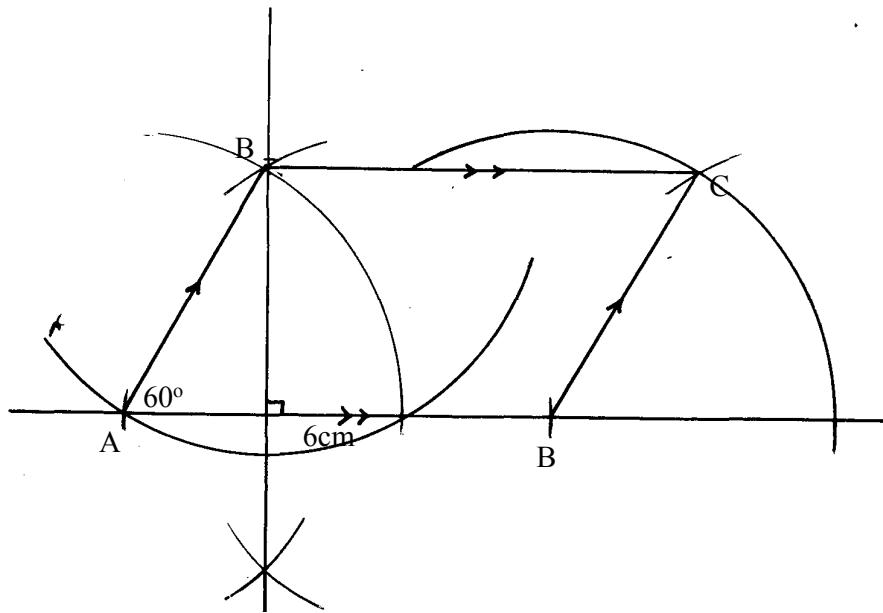
03

12.	<p>a) length of ON = 3.9cm b) Area = <math>6 \times 3.9 = 23.4\text{cm}^2</math></p>	B1 B1 B1 B1 B1 B1 B1 B1 A1	Both $90^\circ$ & $60^\circ$ at A 75° at A 90°&60° at B 75° drawn at point B Both AB=6cm and BC = 4cm Parallelogram completed $\perp$ drawn
		10	

$$13. A = 120000 (1 + \frac{8}{100} x \frac{1}{4})^3$$

$$120000 (1.02)^3 = 127344.95$$

14.

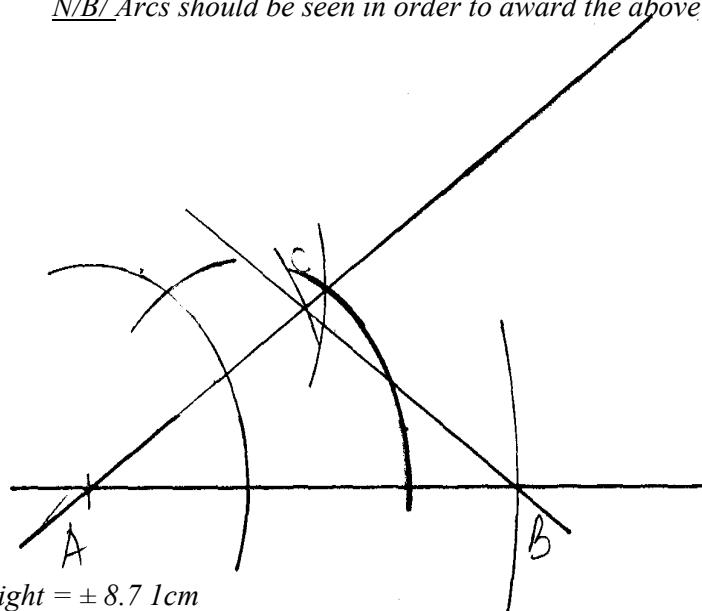


15.  $BC = 3.5 \text{ cm} \pm 0.1$        $B_1$

$B_1$  construction of  $\angle CAB$ .

$B_1$  completion of triangle.

N/B/ Arcs should be seen in order to award the above marks.

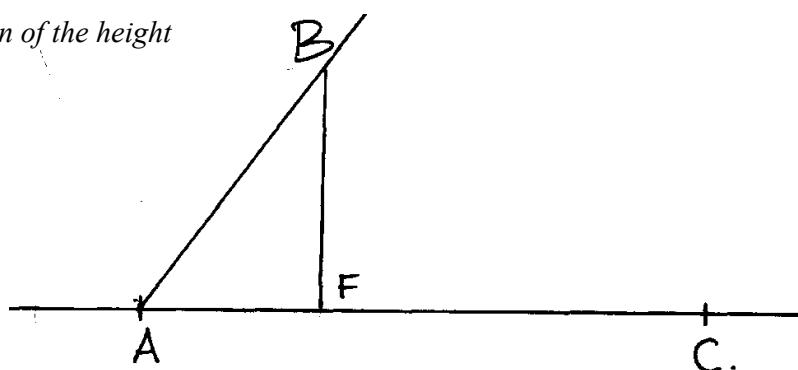


16. Height =  $\pm 8.7 \text{ 1cm}$   
 $(\frac{1}{2} \times 7 \times 8.7) 30.45 \text{ cm}^2$

$2 \pm 1 \text{ cm}$

17. Give 1m of correct and complete triangle  
 Correct angle  
 Correct construction of the height

18.



19. Marked price =  $\frac{100}{90} \times 450 = \text{shs.} 500$

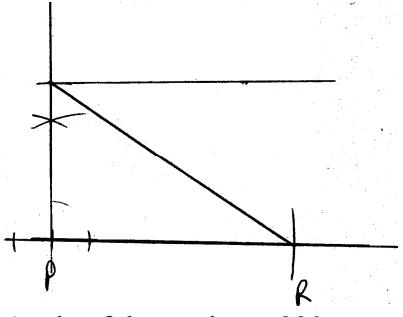
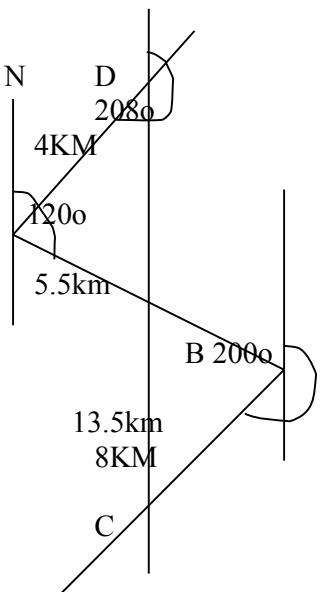
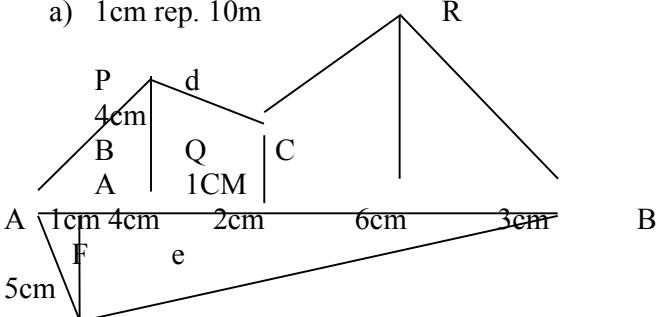
Cost =  $\frac{100}{25} \times 450 = \text{shs.} 360$

Profit =  $500 - 360$   
 $= \text{shs.} 140$

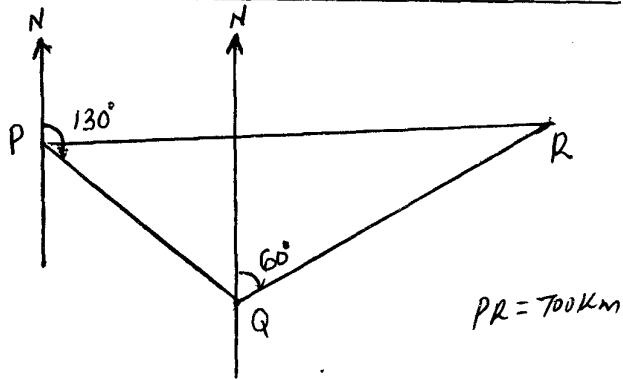
### 19. Scale drawing

1	(i)	$B_1$	✓ sketch not on scale
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	<p>(ii) <math>MC = \sqrt{1200^2 + 900^2}</math>  <math>= \sqrt{1440000 + 810000}</math>  <math>= \sqrt{2250000}</math>  <math>= 1500m</math></p>	M1 A1 3	
2.	<p><math>\frac{\sin 30^\circ}{15} = \frac{\sin Z}{12}</math>  <math>\sin Z = 0.4</math>  <math>\angle Z = 23.580</math>  <math>\angle x = 1800 - 300 - 23.58</math>  <math>= 126.420</math>  Compass bearing  N53.580W</p>	M1 A1 M1 A1	
3.	<p>Distance of R from S  <math>3.8\text{cm} \pm 0.1</math>  <math>3.8 \times 4 = 15.2\text{m}</math>  Bearing of R from S  <math>0680 \pm 10</math></p>	B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1	600 bearing from P and 7cm drawn South of Q and 5cm drawn from P 1400 bearing from P and 4cm drawn Completed diagram.

	 <p>Angle of depression = 330</p>	B1 B1 B1	Award of 3 digits only  Posts P drawn  Position P and R shown and triangle completed  Angle of depression given
		10	
4	<p>1cm represent 1km</p>  <p>Bearing 030o 4km from starting point</p>	1M 1M 1M A1	Bearing of starting point A  Use of scale correctly and plotting of points  Use of bearing correctly
5	<p>a) 1cm rep. 10m</p>  <p>b) Area</p>	B1 B1	Correct scales  Correct drawing

	$A = \frac{1}{2} \times 5 \times 2 = 5\text{cm}^2$ $B = \frac{1}{2} \times 2(2+1) = 3\text{cm}^2$ $C = \frac{1}{2} \times 6 (1+4) = 15\text{cm}^2$ $D = \frac{1}{2} \times 3 \times 4 = 6\text{cm}$ $E = \frac{1}{2} \times 5 \times 15 = 75/2 = 37.5$  $f = \frac{1}{2} \times 1 \times 5 = 2.5$ Total = $69\text{cm}^2$  Area = $69 \times \frac{1000000}{10000} = 690\text{m}^2$	M1	
7.	<p>(a) <math>324^\circ</math>  (b) <math>(7.2 \times 5)\text{km} = 36\text{km}</math></p>	A1	
8.	<p>(a)</p>	04	
		B1	Z accurately located wrt Y
		B1	X accurately located wrt Y
		B1	Bearing of X from Z
		B1	Distance of X from Z
8.		S1	Scale
		B1	Base line
		B2	Offsets (all - offsets) A want B1 for at least 2 ✓
		B1	

	(b) Area1 = $\frac{1}{2} \times 2 \times 3 = 3\text{cm}^2$ Area2 = $\frac{1}{2} \times 4(5+3) = 16\text{cm}^2$ Area3 = $\frac{1}{2} \times 5 \times 4 = 10\text{cm}^2$ Area4 = $\frac{1}{2} \times 2 \times 4 = 4\text{cm}^2$ Area5 = $4 \times 4 = 16\text{cm}^2$ Area6 = $\frac{1}{2} \times 4 \times 4 = 8\text{cm}^2$ Total area = $(3+16+10+4+16+8)\text{cm}^2$ = $57\text{cm}^2$ Actual area = $(57 \times 100)\text{m}^2$ = $5700\text{m}^2$ (c) $10,000\text{m}^2 = 1\text{ha}$ $5700\text{m}^2 = ?$ $1 \times 5700$ $10,000$ = $0.57\text{ha}$	B1 M1 A1 M1 A1	(3 areas) (3 areas) Addition of all six areas
		10	
9.	 $PR = 700\text{km}$	B1 B1 B1	For North line at 600 may be simplified ✓ location of R ✓ for $700\text{km} \pm 10\text{km}$
10.	(a) $\sin \theta = 8/12$ $\text{DOC} = 41.81 \times 2$ = 83.620 (b) Area of $\Delta \text{APCO} = (16 \times 20) - (\frac{1}{2} \times 122 \times \sin 83.62)$ = $320 - 71.15$ = 248.45 (c) $83.62 \times 22 \times 122$ $360 \quad 7$ = 105.09 $\text{cm}^2$ (d) $248.45 - 105.09$ = 353.54	M1 M1 A1 M1 M1 A1 M1 A1 M1 A1 M1 A1	
		03	
		10	

11.	<p><math>1 \text{ cm} = 2 \text{ km}</math></p> <p><math>20 \text{ km} = 10 \text{ cm}</math></p> <p><math>12 \text{ km} = 6 \text{ cm}</math></p> <p>i) <math>10.4 \text{ cm} = \times 2 = 20.8 \text{ km}</math></p> <p>ii) <math>042^\circ</math></p> <p>iii) <math>154^\circ</math></p> <p>c) Area of <math>PQR = \frac{1}{2} ab \sin C</math></p> $= \frac{1}{2} \times 20 \times 12 \sin 78^\circ$ $= 120 \times 78^\circ$ $= 117.38 \text{ km}^2$	B1 B1 B1	Locating Q Locating P Locating R
		10	

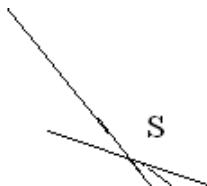
12.	<p>1:50,000 L.S.F. 1:25,000,000 A.S.F. <math>17 \text{ cm}^2 : 425000000 \text{ cm}^2</math> <math>42500 \text{ m}^2</math></p> $\frac{42500}{10,000}$ $= 4.25 \text{ ha}$	M1	
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13.

Positions

Q	B1
R	B1
S	B1
<	

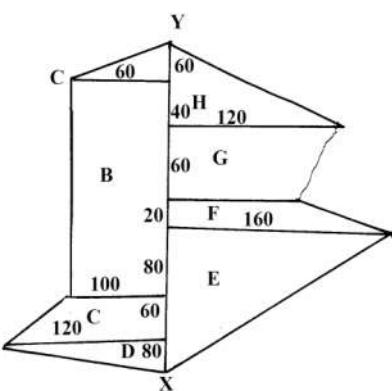
Const 300 B1



- b.) i.  $7.8 \times 50 = 390$  km. B1  
 ii.  $7.10 \times 50 = 355$  km B1  
 iii.  $320^{\circ}$  B2

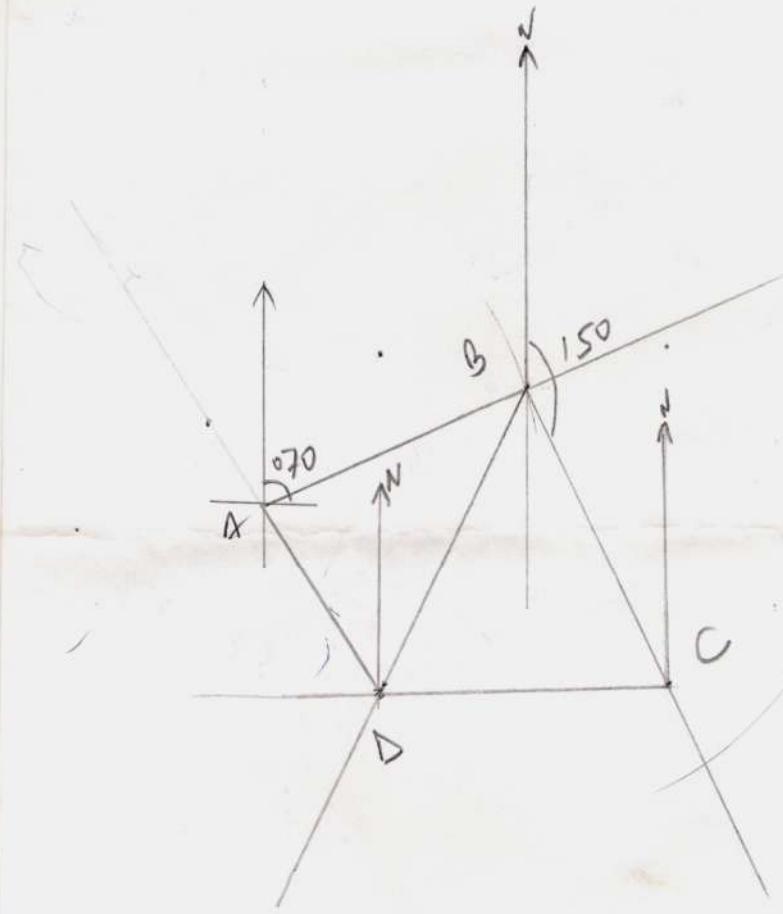
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10

14.	$\text{Area} = 35100000\text{m}^2$ $= 351000000000\text{cm}^2$ $\text{Area} = 2.6 \times 1.5 = 3.9\text{cm}^2$ $\text{Scale} = 3.9 : 351000000000$ $= 90000000000$ $\therefore n = 9 \times 10^{10}$	M1  A1 B1	✓ finding area  ✓ area (actual)
		03	
15.	 Areas.	B3	3 for at least 6. 2 for at least 4, 1 for at least 2

	$A = \frac{1}{2} \times 60 \times 60 = 1800m^2$ $B = \frac{(60+100)}{2} 200 = 16000m^2$ $C = \frac{(100+120)}{2} 60 = 6600m^2$ $D = \frac{1}{2} \times 120 \times 80 = 4800m^2$ $E = \frac{1}{2} \times 160 \times 220 = 17600m^2$ $F = \frac{(160+100)}{2} \times 20 = 2600m^2$ $G = \frac{(100+120)}{2} 60 = 6600m^2$ $H = \frac{1}{2} \times 120 \times 100 = 6000m^2$ <p>Total area = <math>62000m^2 = \frac{62000}{10000} = 6.2\text{ha}</math></p> <p><math>1\text{ha} = 80,000</math>  <math>6.2\text{ha} = 80000 \times \frac{6.2}{1}</math>  <math>= \text{ksh } 496,000.00</math></p>	M1  M1  B1  M  A1	
		10	

16



$$\text{i) } AD = 3.5 \pm 0.1 \times 50 = 175 \pm 5 \text{ m.}$$

$$\text{ii) } BD = 5.2 \pm 0.1 \times 50 = 260 \pm 5 \text{ m}$$

iii) Bearing of A from D  $323^\circ \pm 1^\circ$

B1 Locating A

B1 Locating B

B1 Locating C

B1 Locating D

B1 North at D

M1

A1

M1

A1

B1

10

$$17. \quad a) \quad \frac{YZ}{\sin 28^\circ} = \frac{13.5}{\sin 100^\circ}$$

$$\text{Duration of travel} = 8:55\text{a.m} - 7:35\text{a.m} \\ = 4/3$$

$$\text{Speed} = \frac{6.436}{4/3}$$

$$= 4.827 \text{ km/hr}$$

$$(b) \quad \frac{13.5}{\sin 10^\circ} = 6.436 + ZQ$$

$$\sin 10^\circ \sin 118^\circ$$

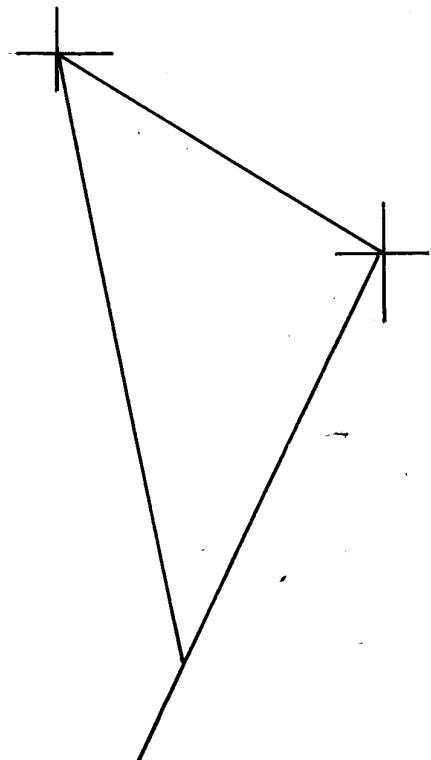
$$6.436 + ZQ = 13.5 \times \sin 118^\circ = 68.659$$

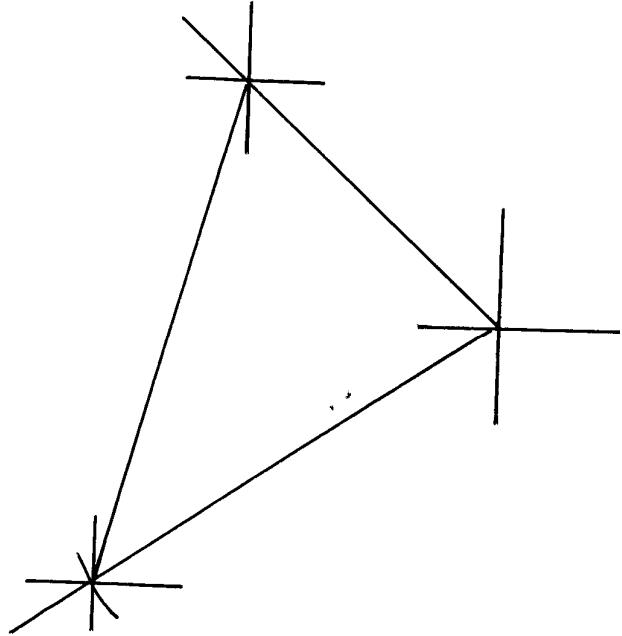
$$ZQ = 68.659 - 6.436$$

$$= 62.223$$

18.

1cm rep 100km



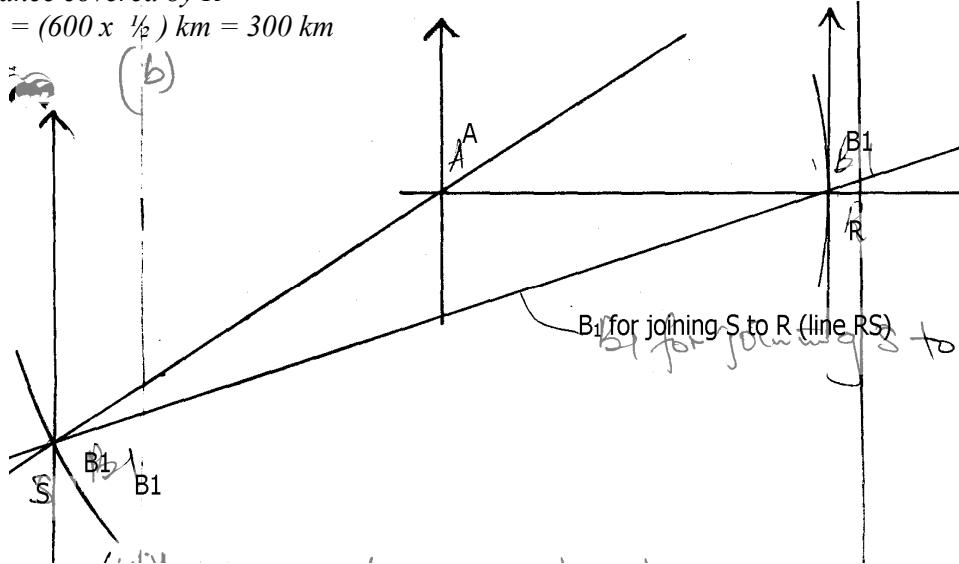


- b) i)  $049 \pm 1$   
 ii)  $190 \pm 1$   
 c)  $6.7 \pm 0.1$   
 $670 \pm 10$

19. a) (i) Distance covered by s  
 $= (750 \times \frac{1}{2}) \text{ km} = 375 \text{ km}$

Distance covered by R

$$= (600 \times \frac{1}{2}) \text{ km} = 300 \text{ km}$$



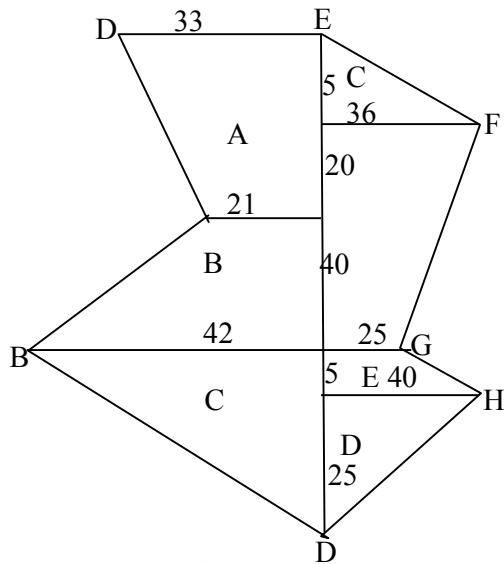
(b) (i) Distance between the two aeroplanes  
 $= 12.5 \times 50 = 625 \pm 5 \text{ km}$

(ii) Speed =  $\left( \frac{625 \times 60}{\text{Mocks Topical Analysis}} \right) \text{ km/hr}$

$$\begin{aligned} 45 \\ = 833 \frac{1}{3} \text{ km/h} \end{aligned}$$

- (c) (i) Bearing of S from R =  $225^\circ$   
(ii) The bearing of R from S =  $72^\circ$

20.



$$\text{Area } A: \frac{1}{2} \times 25 (33 + 21) = 675$$

$$\text{Area } B: \frac{1}{2} \times 40 (21 \times 42) = 1260$$

$$\text{Area } C: \frac{1}{2} \times 30 \times 42 = 630$$

$$\text{Area } D: \frac{1}{2} \times 25 \times 40 = 500$$

$$\text{Area } E: \frac{1}{2} \times 5 (40 + 25) = 162.5$$

$$\text{Area } F: \frac{1}{2} \times 60 (25 + 36) = 1830$$

$$\text{Area } G: \frac{1}{2} \times 5 \times 36 = 90$$

$$= 5,147.5 \text{ m}^2$$

21. A to C =  $96 \pm 1 \text{ km}$ Bearing =  $300^\circ$ 

$$(i) \quad 62 \pm 1 \text{ km}$$

$$(ii) \quad 97 \pm 1 \text{ km}$$

$$a. \quad 304^\circ$$

$$030^\circ$$

22. Graph

b) i) 80 km

ii) 11.06 a.m

c) Average speed of the 2<sup>nd</sup> train

$$\text{Time taken} = 80 \div 1\frac{11}{12} = \frac{80 \times 12}{23}$$

$$= 41.74 \text{ km/h}$$

$$\begin{aligned} 23. \quad L.S.F &= \frac{4}{2000000} = \frac{1}{500000} \\ A.S.F &= \underline{1}^2 = \underline{1} \end{aligned}$$

$$5 \times 10^5 \quad 2.5 \times 10^{11}$$

$$\text{Area of rectangle} = (2.4 \times 1.5) \text{ cm}^2 \\ = 3.6 \text{ cm}^2$$

$$\text{Actual area} = \frac{3.6 \times 2.5 \times 10^{11}}{100 \times 10000} \text{ ha} \\ = 9 \times 10^5 \\ = 900,000 \text{ ha}$$

24. a)  $\triangle ABD$   $\sqrt{ly}$  constructed

$\triangle ABP$

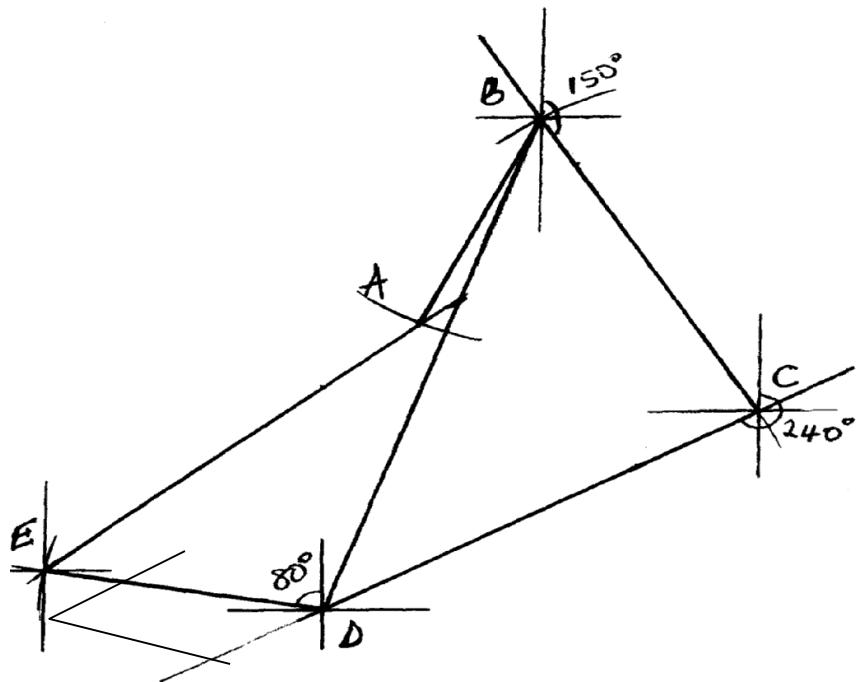
b) i)  $AD = 4.5 + 0.1 \text{ cm}$   
 $\text{Distance } A + D$   
 $= 4.5 \times 10 = 45 \text{ km}$

ii) Bearing of (i) from B  
 $= 241 + 1$

iii) Bearing P from D  
 $= 123 = 2$

iv)  $Dp = 12.9 + 0.2 \text{ am}$   
 $\text{Distance } D + P = 12.9 \times 10$   
 $= 129 \text{ km}$

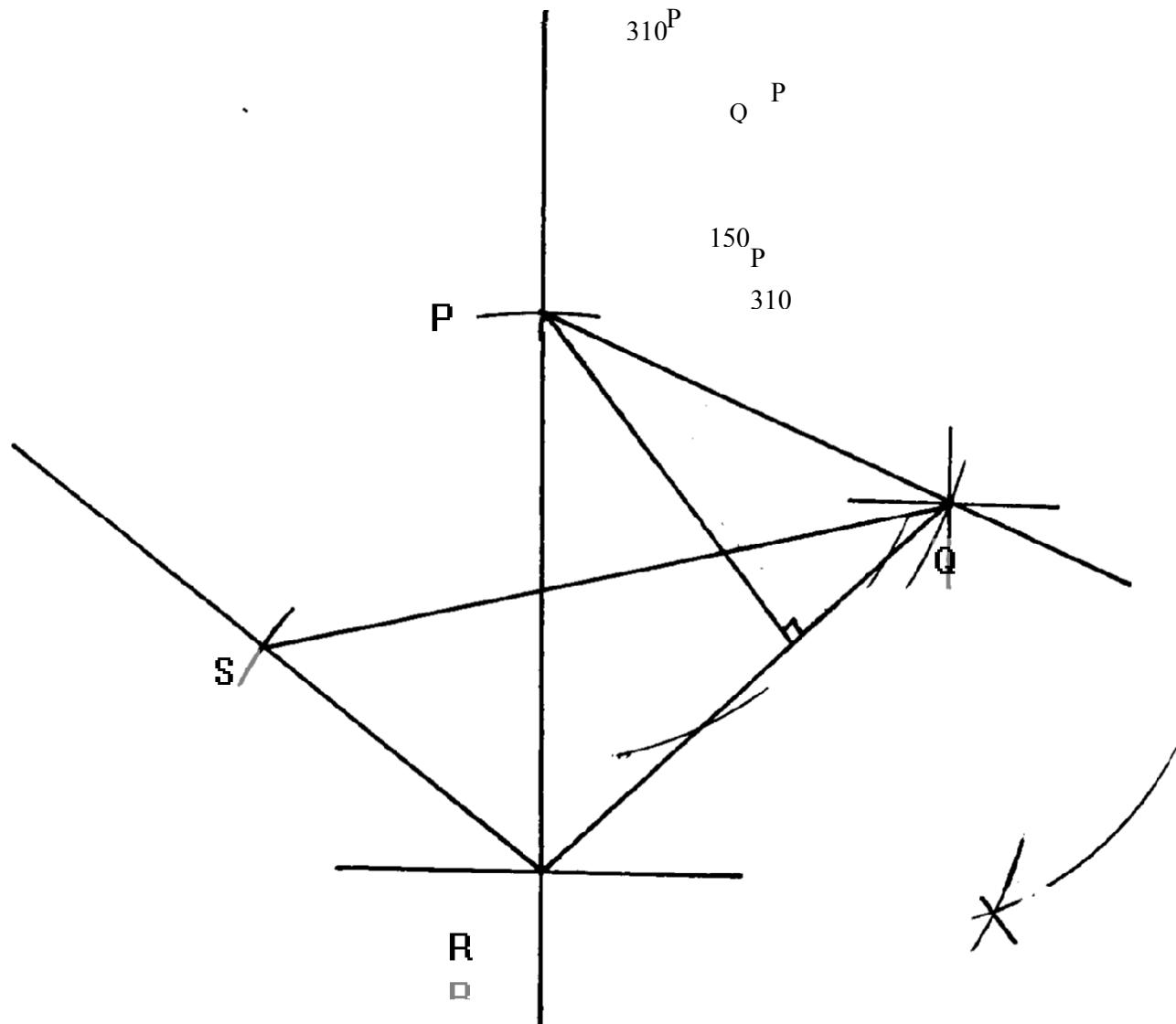
25. a)



b) i)  $6.8 + 0.1 \text{ cm}$   
 $\text{Distance } Ae = 340 + 5 \text{ km}$

ii)  $180 + 18 = 198 + 2$

26. a)

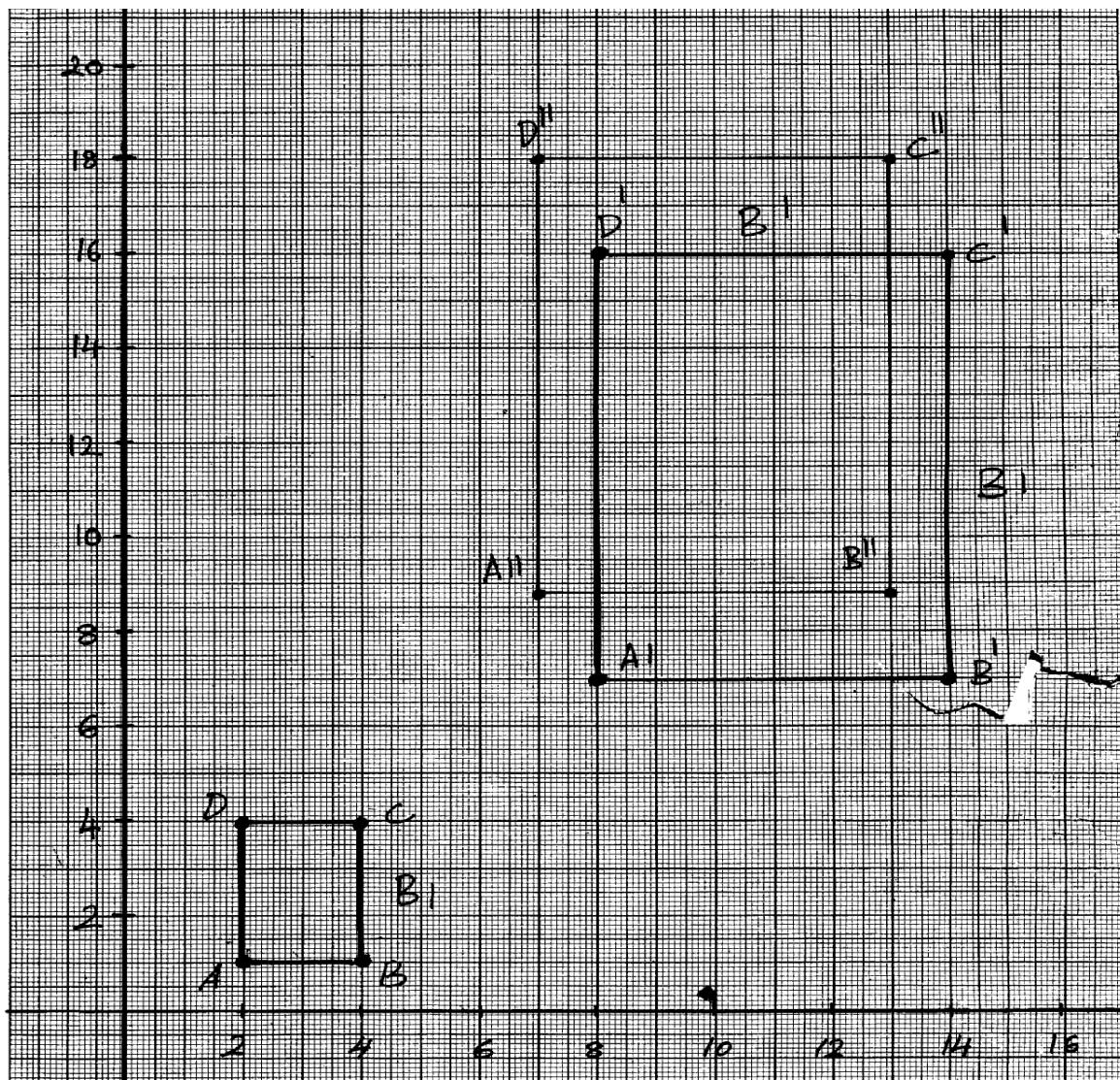


- b) (i)  $SP = 7.8 \times 50 = 390 \text{ km} \pm 5 \text{ km}$   
(ii)  $S \& Q = 255^\circ \pm 1^\circ$   
(iii)  $4 \times 50 = 200 \text{ km} + 5 \text{ km}$

27. (a) Scale = 50km  
Drawing accurately  $\angle NCE = 25^\circ$   
 $\angle NCT = 145^\circ$   
 $\angle NTY = 90^\circ$   
Lines drawn //

- (b) By measurement:  
(i) Distance  $SY = 6.9 \times 50 = 345 \pm 5 \text{ km}$   
Bearing Y For S =  $360^\circ - 114^\circ = 246 \pm 1^\circ$   
(ii) distance  $ST = 7.9 \times 50 = 39.5 \pm 5 \text{ km}$

$$(iii) \quad \text{distance } YT = 9.8 \times 50 = 490 \text{ km}$$



$$XY = 250m$$

$$\text{Area of } A = \frac{1}{2} \times 50 \times 60 = 1500m^2$$

$$B = \frac{1}{2} \times 70 \times 60 = 2100m^2$$

$$C = \frac{1}{2} (60 + 80) \times 120 = 11050m^2$$

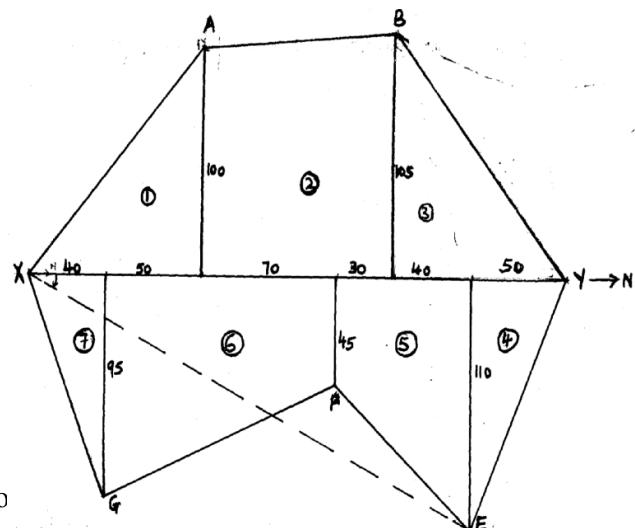
$$D = \frac{1}{2} \times 80 \times 80 = 3200m^2$$

$$F = \frac{1}{2} \times 10 \times 70 = 350m^2$$

$$\text{Total area} = 26600m^2$$

$$\text{Ha} = \frac{26600}{10,000} = 2.66\text{ha}$$

29.



(b) Total area = area (1) + (2) + (3) + (4) + (5) + (6) + (7)

$$\text{Area (1)} = \frac{1}{2} \times 90 \times 100 = 4500 \text{ m}^2$$

$$(2) = \frac{(100+105)10}{2} = 10250 \text{ m}^2$$

$$(3) = \frac{1}{2} \times 90 \times 105 = 4725 \text{ m}^2$$

$$(4) = \frac{1}{2} \times 50 \times 110 = 2750 \text{ m}^2$$

$$(5) = \frac{1}{2} \times (110+45)70 = 5425 \text{ m}^2$$

$$(6) = \frac{(45+95)120}{2} = 8400 \text{ m}^2$$

$$(7) = \frac{1}{2} \times 40 \times 95 = 1900 \text{ m}^2$$

$$\text{Total area} = 37,950 \text{ m}^2$$

$$\text{In hectares} = \frac{37950}{10,000} \text{ ha} = 3.795 \text{ ha}$$

(c) (i) bearing of E from x is  $0.25 \pm 1^\circ$

(ii) Distance Ex =  $(12.8 \times 0.1 \times 20 \text{ m}) = 256 \pm 2 \text{ m}$

30. Area  $A = \frac{1}{2} \times 170 \times 80 = 6800$

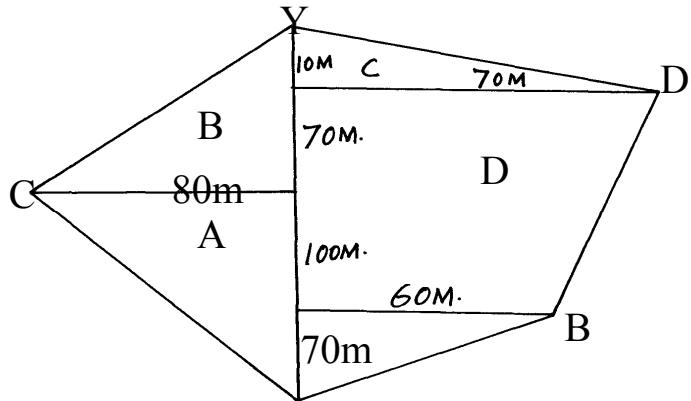
$$B = \frac{1}{2} \times 80 \times 80 = 3200$$

$$C = \frac{1}{2} \times 10 \times 70 = 350$$

$$D = \frac{1}{2} \times 170 \times 130 = 11050$$

$$E = \frac{1}{2} \times 70 \times 60 = 2100$$

$$\text{Total} = 23,500 \text{ m}^2$$



31. (a) L.s.f = 1

$$40,000$$

$$\frac{1}{40,000} = \frac{3.25}{x}$$

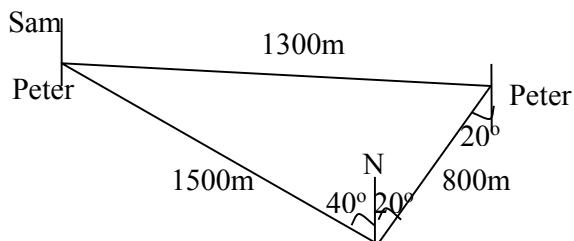
$$x = 130,000 \text{ cm}^2$$

(b) A.s.f

$$\left( \frac{1}{40,000} \right)^2 = \left( \frac{x}{36,000,000} \right)^2$$

$$x = 0.0225 \text{ cm}^2$$

32.

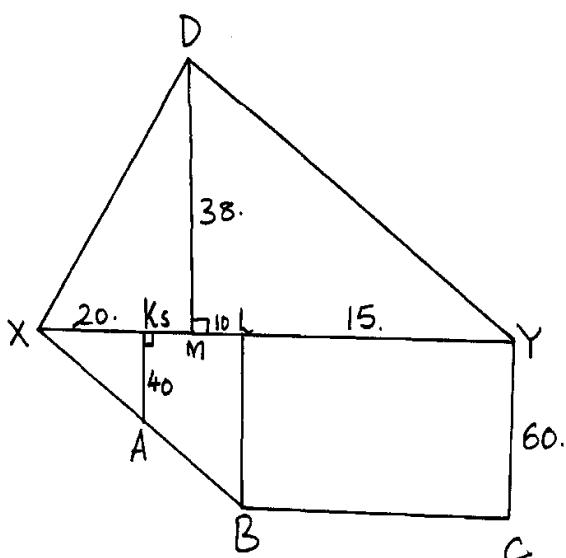


(a) bearing =  $180 + 20 = 200^\circ$

$$\begin{aligned}
 (b) \quad a^2 &= 1500 + \\
 a^2 &= b^2 + c^2 - 2bc \cos A \\
 a^2 &= 1500^2 + 800^2 - 2 \times 1500 \times 800 \cos 60 \\
 &= 2250000 + 640000 - 1200000 \\
 &= 1690000 \\
 \therefore a &= 1300m
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad \frac{1300}{\sin 60} &= \frac{1500}{\sin c} \\
 1300 \sin c &= 1500 \sin 60 \\
 \sin c &= \frac{1500 \sin 60}{1300} \\
 &= 0.9993 \\
 \therefore c &= 87.79^\circ \\
 c &= 87.80
 \end{aligned}$$

33.



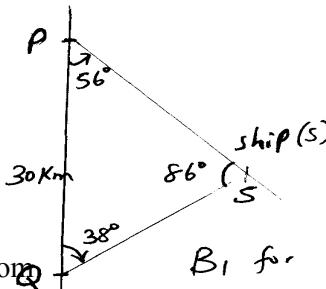
$$\begin{aligned}
 \text{Area of } \triangle XYD &= \frac{1}{2} \times 50 \times 38 = 950m^2 \\
 \text{Area of } XBCY &= \frac{1}{2} (50 + 15) 60 \\
 &= \frac{1}{2} \times 65 \times 60 \\
 &= 1950m^2 \\
 \text{Total Area} &= (950 + 1950)m^2 \\
 &= 2900m^2
 \end{aligned}$$

34.  $B1$  for  $86^\circ$

$$\begin{aligned}
 \frac{30}{\sin 86^\circ} &= \frac{QS}{\sin 56^\circ} \\
 QS &= 30 \sin 56^\circ \\
 &\quad \sin 86^\circ \\
 &= 24.93 \text{ km}
 \end{aligned}$$

35.  $1 \text{ cm for } 100000 \text{ cm}$

$$\begin{aligned}
 1 \text{ cm}^2 &= (100000 \text{ cm})^2 \\
 \text{Area} &= 5.4 \times 4.5 \times 100000 \text{ cm}^2 \\
 &= \frac{5.4 \times 4.5 \times 100000 \times 100000 \text{ Km}^2}{100000 \times 100000} \\
 &= 24.3 \text{ km}^2
 \end{aligned}$$



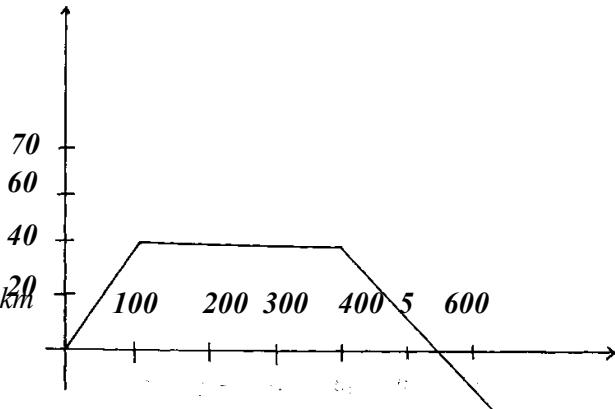
$$36. \quad \frac{\theta}{360} \times \frac{22}{7} \times 6370 \times 2 = 900 \\ = \frac{900 \times 360 \times 7}{22 \times 6370 \times 2} \\ = 8.1^\circ$$

$$\text{Latitude of } B = 8.1^\circ - 5^\circ N \\ = 3.5^\circ S$$

$$37. \quad i) acc = \frac{40 - 20}{100 - 50} \\ = \frac{20}{50} = 0.4 \text{ m/s}$$

$$ii) \frac{20 - 40}{460 - 400} = \frac{-20}{60} = 0.3333 \text{ m/s}^2$$

$$iii) \text{Area} = \frac{1}{2} (520 + 300) \times 40 \times \frac{1}{1000} = 16.4 \text{ km}$$



$$38. \quad a) \tan 11.3 = \frac{200}{x} \\ x = \frac{200}{\tan 11.3} = 100.1 \text{ m}$$

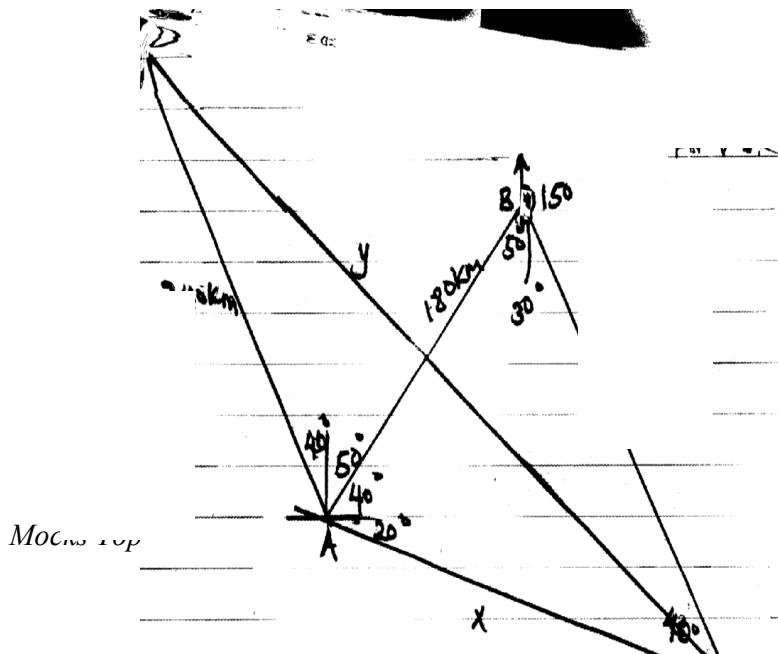
$$b) \frac{(36 \times 1000)}{60 \times 60} \text{ m/s}$$

$$D = (10 \times 5) 50 \text{ m} \quad \tan \theta = 7.590 \\ < \text{of depression} = 7.590$$

$$c) i) \sqrt{50.9^2 - 49.9^2} = 10.04 \text{ cm}$$

$$ii) \tan \theta = \frac{10.04}{200} \\ = 2.874^\circ \\ = 3^\circ$$

39. a) Make a sketch to show positive of A, B, C and D



For ✓ sketch  
 For ✓ exp. of x  
 For ✓ ans.  
 For ✓ Sub.  
 ✓ cos 150  
 For taking sq. root.  
 For exp. of BC

Use sine rule in  $\Delta ABC$

$$\frac{X}{\sin 80^\circ} = \frac{180}{\sin 40^\circ} \Rightarrow x = \frac{180 \sin 80^\circ}{\sin 40^\circ}$$

$$= 275.8$$

Hence  $AC = 276 \text{ km}$

(b) Use the cosine rule in  $\Delta AD$  when  $\angle DAC = 150^\circ$

$$y^2 = 240^2 + 276^2 - 2 \times 240 \times 276 \cos 150^\circ$$

$$= 576000 + 76180 - 132480 (-\cos 30^\circ)$$

$$= 133776 + 114731 = 248507$$

$$y = \sqrt{248507}$$

$$= 498.5$$

Hence  $CD = 499 \text{ km}$

(c) Using sine rule in  $\Delta ABC$  we have

$$\frac{BC}{\sin 60^\circ} = \frac{180}{\sin 40^\circ}$$

$$BC = \frac{180 \sin 60^\circ}{\sin 40^\circ}$$

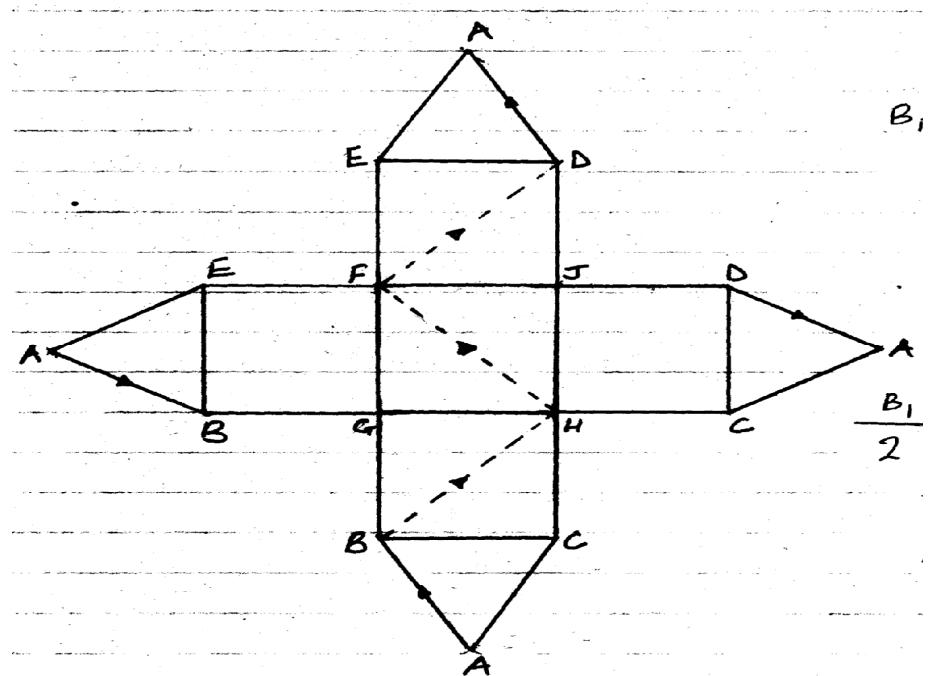
$$= 242.5$$

$$= 243 \text{ km}$$

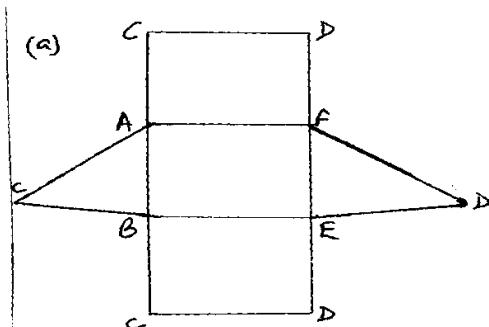
## 20. Common solids

1.	a)	 b) The figure is tetrahedron	B1	Sketch completed and the lines dotted.
			B1	02

2. Sketch of the net of the solid (not free hand) base must be square, other lengths must be within. Labeling of all verticals with the path correctly shown. AB and DA may be shown one.

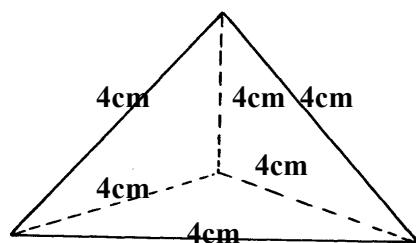


3. (a)

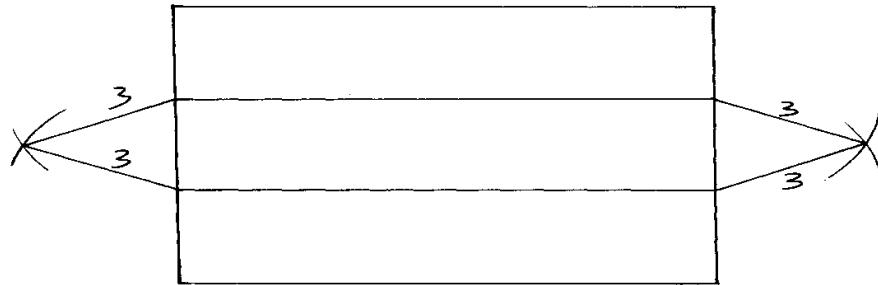


$$\begin{aligned}
 (b) \text{ Total surface area} \\
 &= 2\sqrt{9 \times 3 \times 4 \times 2} + 10(6 + 5 + 7) \\
 &= 29.39 + 180 = 209.4 \text{ cm}^2
 \end{aligned}$$

4.



5.

**21. Indices**

1.  $3^4 \times 3^{4x} + 3^{4x} = 246$

$$3^{4x}(81 + 1) = 246$$

$$\frac{82}{82} \times 3^{4x} = \frac{246}{82}$$

$$3^{4x} = 3^1 \quad \checkmark$$

$$4x = 1$$

$$x = \frac{1}{4}$$

2.  $5^{2y} x 5^1 = 4^{(5y+1)} - 15$

$$5^y x 5^y x 5^1 = 4 x 5y x 51 - 15$$

$$\text{Let } 5y = t$$

$$5t^2 = 20t - 15$$

$$t^2 = 20t - 15$$

$$t^2 - 4t + 3 = 0$$

$$(t-1)(t-3) = 0$$

$$t = 1 \text{ or } 3$$

$$5y = 1 = 5^0$$

$$\text{Or } 5y = 3 \Rightarrow y = \frac{\log 3}{\log 5} = 0.6826$$

3.  $CBD = 90 - 42 = 48^\circ$

*Angle of triangle add to  $180^\circ$* 

$$DOB = 180^\circ - 42 = 138^\circ$$

*Opposite angles of cyclic quadrilateral add to  $180^\circ$* 

$$DAB = \frac{138^\circ}{2} = 69^\circ$$

*Angle at circumference is half the angle subtended at centre by same chord**CDA*

$$ABD = 90 - 48 = 42^\circ$$

$$ADB = 180 - (69+42)$$

$$180 - 111 = 69^\circ$$

$$CDA = 90 + 69^\circ = 159^\circ$$

*Show  $\triangle ADB$  is a scalene triangle*

$$\angle DAB = 69^\circ$$

$$\angle DAB = 69^\circ$$

$$\angle ADB = 69^\circ$$

$$\angle ABD = 42^\circ$$

So two angles are equal hence it is a sccesters

$$4. \quad 25^{\frac{3}{4}} = (25^{\frac{1}{2}})^{3/2} = 5 \\ 0.9^2 = (\frac{9}{10})^2 = \frac{81}{100} \\ 2^2 = 2^2 \\ \frac{(\sqrt{5})^3 x 9^2 x 2^2}{(\sqrt{5})^5 x 10^2 x 3^3} \\ \frac{3 x 4}{(\sqrt{5})^2 x 10^2} \\ \frac{3}{5 x 25} = \frac{3}{125}$$

$$5. \quad 2^x = 0.0625 = \frac{625}{1000} \\ 2x = \frac{1}{16} = 2^{-4} \\ \therefore x = -4$$

$$6. \quad 16x^2 = 8^{4x-3} \\ 2^{4x2} = 2^{3(4x-3)} \\ = 4^{x2} = 12x - 9 \\ = 4^{x2} - 12x + 9 = 0 \\ (2x-3)^2 = 0 \\ 2x-3 = 0 \\ x = 1.5$$

No	Log
5.627	0.7503
$(0.234)^3$	$T. 3692$
	<u>x 3</u>
2.8579	
8.237	0.4779
$2.399 \times 10^{-3}$	$\frac{0.9158}{2}$
	3.3800
	$= 0.002399$

$$7. \quad 9^{x+1} + 3^{2x+1} = 36 \\ 3^{2x+2} + 3^{2x+1} = 36 \\ 3^{2x}(9+3) = 36 \\ 3^{2x} = 3^1 \\ 2x = 1 \\ x = \frac{1}{2}$$

$$8. \quad (a) 4p^2 - 3p - 10 = 0 \\ (b) 4p^2 - 8p + 5p = 0 \\ (4p+5)(p-2) = 0$$

$$\begin{aligned} p_1 &= -5/4, p = 2 \\ \text{When } y &= -5/4, \\ 4^y &= \frac{-5}{4} \\ y &= \frac{\log_4(-5)}{2} \end{aligned}$$

$$P = 2$$

$$4^y = 2$$

$$2^{-2y} = 2^1$$

$$y = -1/2$$

9.

$$\frac{1}{16^x} = \frac{1}{32}$$

$$\left(\frac{1}{2^{4x}}\right)^{x - 1/4} = \frac{1}{2^5}$$

$$2^{-4x^2 + x} + x = 2^{-5}$$

$$-4x^2 + x + 5 = 0$$

$$4x^2 - x - 5 = 0$$

$$4x^2 - 5x + 4x - 5 = 0$$

$$x(4x - 5) + 1(4x - 5) = 0$$

$$x = -1 \text{ or } x = \frac{5}{4}$$

$$10. \quad 15(ax)^4 (-^2/x^2) = 4860$$

$$60a^4 = 4860$$

$$a^4 = 81$$

$$a = 3$$

## 22. Reciprocals

1	$\begin{aligned} &\sqrt{\frac{1}{2.456} \times 0.1 + 4.346^2} \\ &\sqrt{0.04072 + 18.888} \\ &\sqrt{18.929} \\ &4.3509 \end{aligned}$	B <sub>1</sub>	✓ reciprocal
		B <sub>1</sub>	✓ square
		B <sub>1</sub>	✓ square root

$$2. \quad \begin{array}{r} 10 \quad 1 \\ 0.834 \quad \underline{-} \quad 129.64 \end{array}$$

$$\begin{aligned} &(10 \times 1.199) - (3 \times 0.007713) \\ &11.99 - 0.923139 \\ &11.966861 \\ &12.0 \end{aligned}$$

$$\begin{aligned} 3. \quad 807 &\rightarrow 0.001239 \\ 0.0591 &\rightarrow 16.92 \\ 5(0.001239) + 4(16.92) \\ &= 67.69 \end{aligned}$$

4.  $\frac{1}{3} \{ 2 \times 1.5065 + 5 \times 1.2004 \}$

$$\frac{1}{3} (3.013 + 6.002) (0.3333)$$

$$= 9.015 \times 0.3333 \\ = 3.005 \text{ (3 dp)}$$

5.  $\frac{12 \times 0.25 - 12.4 \div 0.4 \times 3}{\frac{1}{8} \text{ of } 2.56 + 8.68}$

$$\frac{3 - 31 \times 3}{0.32 + 8.68}$$

$$\frac{-90}{9}$$

$$= -10$$

6.  $\frac{\frac{4}{(8.68)^3}}{\frac{4}{653.97}} + \frac{\frac{5}{34.46}}{(0.1451)^{1/3}}$   
 $4(0.1529) + 0.5255 \\ 0.6116 + 0.5255 = 1.1371$

7.  $\frac{l}{a} = 0.007874 + 0.0869 \\ a \\ = 0.9483 \\ a = 10.55$

8.  $3.5932 = 12.91 \\ \Rightarrow \left[ \frac{1}{1.291 \times 10} \right] + 2 \left[ \frac{1}{5.26 \times 10^{-1}} \right] \\ = (0.7746 \times 10^{-1}) + 290.1901 \times 10 \\ = 0.07746 \\ + \frac{3.802}{3.87946}$

$$\sqrt{3.87946} = \sqrt{3.879}$$

$$= 1.9695 \\ = 1.970 \text{ (4s.f)}$$

9.  $No \quad s.f \quad rec$   
 $0.6638 \quad 6.638 \times 10^{-1} \quad 0.1500 \times 10 = 1.5000$   
 $0.833 \quad 8.33 \times 10^{-1} \quad 0.1200 \times 10 = 1.200$   
 $= \frac{1}{3} (2 (105) + (1.2)) \\ = \frac{1}{3} (3 + 6) \\ = \frac{1}{3} \times 9 = 3$

10.  $3 \times 1.485 + 13 \times 6.410 \\ = 4.455 + 83.33 \\ = 87.785$

$$\begin{aligned}
 ALT \\
 \frac{30}{6.735} + \frac{130}{1.56} &= 30 \times 0.1485 + 130 \times 0.641 \\
 &= 4.455 + 83.33 \\
 &= 87.785
 \end{aligned}$$

### 23. Common logarithms.

1.	$\log 31.59 = 1.4996$ $\log a = \underline{2.6182}$ $\log b^{1/3} = \underline{\overline{28.814}}$ $\log b = \underline{\overline{4.6442}}$ $b = 0.0004407$ $b = 0.0004$				M1	Subt b logs
					M1	Multip by 3
					A1	
					B1	
					04	
2.	$\log_{10} 25 - \log_{10} 4 + \log_{10} 1600$ $\log_{10} \left( \frac{25}{4} \times 1600 \right)$ $4$				M1	
					M1	
					A1	
					03	
3	No. $(0.00246)^2$ 142	Std. form $(2.46 \times 10^{-2})^2$ $1.42 \times 10^2$	Log 2.3909 $\times \underline{2}$ 4.7818 $+2.1523$ $\underline{2.9341}$		1M	Correct logs addition
	0.002 1.14	$2.0 \times 10^{-3}$ $1.14 \times 10^0$	3.3010 $+0.0569$ 3.3579		1M	Correct logs addition
			2.9341 $-3.3579$ $\underline{1.5762}$ $\underline{1.5762}$ 3		1M	Correct logs subtractions
	3.3527	$3.3527 \times 10^0$	0.5254		A1	Correct answer
					4	
4	$\frac{\log \left( \frac{1}{4} \times 64 \right)}{\log \left( \frac{1}{32} \div \frac{1}{8} \right)}$ $\frac{\log 2^4}{\log 2^{-2}}$ $\frac{4 \log 2}{-2 \log 2}$ -2				M1	
					M1	
					M1	
					A1	
					4	

5.  $\left(\frac{1}{2^3}\right)^x \cdot \left(2^6\right)^2 = \left[2^4\right]^2$  M1 for writing in index form  
 $2^{-3x} \cdot 2^{12} = 2^8$   
 $12 - 3x = 8$  M1

$$\begin{aligned} x &= \frac{4}{3} \\ &= 1 \frac{1}{3} \end{aligned}$$

3

6.	No. 0.6845 <sup>2</sup> 0.08416 0.005937 0.3459	std form 6.845x10 <sup>-1</sup> 8.416x10 <sup>-2</sup> 5.937x10 <sup>-3</sup> 3.459x10 <sup>-1</sup>	log $\bar{1.8354} \times 2$ $\bar{1.6708} \quad \bar{1.6708}$ $\underline{\bar{2.9252}}$ $\bar{3}$ $\bar{1.6417} \quad \underline{\bar{1.6417}} +$ $\bar{1.3125}$ $- \quad \underline{\bar{3.7736}}$ $\bar{1.5389}$	M1 M1 M1 M1 A1	
				04	

7	Number 8.694 0.1267 0.006974  6.039 × 10 <sup>2</sup> 602.9	log 0.9392 $\bar{1.1028} \times \frac{1}{3} = \bar{1.7009}$ $\bar{3.8434}$ $\underline{\bar{3.5443} \times \frac{3}{4}}$ $\bar{1.3861} \times 3 = \bar{2.1583}$ 0.9392 $\bar{2.1583}$ $\underline{\bar{2.7809}}$		M1 M1 M1 A1	All logs + - x of logs ÷ x of logs
				4 mark s	

8.

No.	Log
2849	3.4547
-	+
0.00574	3.7589
1.2136	
-	←
36.891	1.5669
0.023	2.3617
—————	3.2052 →

*All logs read correctly*

*Correct Addn /subst. of logs.*

$$2. \ 0084 \times \frac{1}{4}$$

$$3.178 \times 10^{-1} \leftarrow 1.5021$$

$$\rightarrow 0.3178$$

9.  $\log y = \log B + n \log x$   
 $n \log x = \log y - \log B$   
 $n = \frac{\log y - \log B}{\log x}$

10.  $= 6 \log_2 4 + 10 \log_3 3$   
 $= 12 \log_2 2 + 10 \log_3 3$   
 $= 12 + 10$

11.  $\log \frac{2x - 11}{2} = \frac{\log 3}{x}$

$$(2x - 11) = {}^3/x$$

$$2x^2 - 11x - 6 = 0$$

$$(2x + 1)(x - 6) = 0$$

$$x = -\frac{1}{2} \text{ or } 6$$

$$x = 6$$

12.

No.	Log
0.5241	7.7194
$(0.5241)^2$	$7.7194 \times 2$
83.59	$\underline{7.4388} +$ 1.9222
0.3563	1.3610
$3\sqrt[3]{0.3563}$	7.5518 $(3+2.5518) \div 3$
	7.8506
	0.3610 -
	1.8506
$3.239 \times 10^1$ $= 32.4$	1.5104

13.

No.	Log
38.32	1.5834
12.964	<u>1.1127</u> 2.6961
86.37	1.9364
6.285	<u>0.7783</u> 2.7347
-	1.9587

$$\frac{-3 + 2.9587}{3} = 0.9695$$

$$14. \quad H^3 = \frac{3d(L-d)}{10L}$$

$$\sim \frac{3dL - 10H^3L}{L} = \frac{3d^2}{3d - 10H^3}$$

No.	Log
6.195	0.7920
11.82	<u>1.0726</u>
	1.8646
83.52	<u>1.9218</u>
	<u><u>1.9428 x 1/4</u></u>
	<u>4. + 3.9428</u>
	<u>4</u>
0.9676	7.9857

$$16. \quad \log y^2 (x-1) = \log 9 \quad y^2 (x-1) = 9 \quad \dots(1)$$

$$\log(xy) \log 6 \quad xy = 6 \quad \dots 2$$

$$\text{from (2)} \quad x = 6/y$$

$$\text{substitute in (1)} \quad y(6-1) = 9$$

$$y$$

$$6y - y^2 = 9$$

$$y^2 - 6y + 9 = 0$$

$$(y-3)^2 = 0$$

$$y = 3$$

$$\therefore x = 2$$

5

$$17. \quad \frac{4}{5} \log_{10} 25 + \log_{10} 25x2 - \log 10$$

$$4 \log 2 = \log_{10} 25x2 - 3 \log 2$$

$$2 \log 10 + 2 \log 5$$

$$\log 10 \times 100$$

18.

NO	LOG
0.9895	-
$(0.9895)^2$	<u>1.9954</u> <u>1.9954 x 2</u> <u>1.9908</u>
0.004974	<u>3.6968</u> <u>3.6876 ÷ 4</u>
6.598	<u>1.4219 x 3</u> <u>2.2657</u> <u>0.8195 -</u> <u>2.2657</u> <u>2.5538</u>
3.579 X 10 <sup>2</sup> OR 357.9	←

Use sine rule

19.  $\log 3x + 8 - \log 8 = \log (x-4)$

$$\frac{\log (3x+8)}{8} = \log (x-4)$$

$$3x + 8 = x - 4$$

$$3x + 8 = 8x - 32$$

$$5x = 40$$

20.

No.	Log
36.72 →	1.5649
0.46² →	2(T.6628)
	T.3256
185.4	0.8905
	2.2682
	2.9223 x $\frac{1}{3}$ = $\frac{1}{3}$ + $\frac{1.6223}{3}$
$3.474 \times 10^{-1}$	Or 0.3474
	1.5408

No	Log
Sin 44.5	1.8457
Tan 14.9	1.4250
Cos 82	2.5686 - 1.1486 + <u>1.2772</u> 2
$10 \times 4.351$	0.6386

22. From square roots  $12.25 = 3.5$

$$\underline{3.264 \times 1.215 \times 3.5 \times 107}$$

$$\underline{1.088 \times 0.4725 \times 107}$$

$$\underline{3264 \times 1215 \times 35}$$

$$\underline{1088 \times 4725}$$

$$\sqrt{27} = 3$$

23.  $\log_8(x+5) - \log_8(x-3) = \log_8 4$

$$\frac{\log_8(x+5)}{x-3} = \log_8 4$$

$$\frac{x+5}{x-3} = 4$$

$$4x - 12 = x + 5$$

$$3x = 17$$

$$x = 17 = 5^2/3$$

$$\text{Or } \log_8 \frac{x+5}{x-3} = \frac{2}{3}$$

$$8^{2/3} = \frac{x+5}{x-3}$$

$$2^3(2/3) = \frac{x+5}{x-3}$$

$$2^2 = \frac{x+5}{x-3} \Rightarrow 4 = \frac{x+5}{x-3}$$

$$4x - 12 = x + 5 \Rightarrow 3x = 17$$

$$x = \frac{17}{3} = 5^2/3$$

24.

$No$ $6.57^2$  $4.317 \times 10^1$ $43.17 + 6.57$  $49.74$ $(7.92)^2$  $\frac{30.08}{2.636 \times 10^{-2}}$	$Log$ $0.8176$ $\frac{2x}{1.6352}$  $0.8987$ $X2$  $1.7974$ $1.4783 + \frac{3.2757}{2.4210}$ $= 0.02636$ $= 0.0264 (4 \text{ d.p})$	$1.6967$
$No$ $6.57^2$  $4.317 \times 10^1$ $43.17 + 6.57$  $49.74$ $(7.92)^2$  $\frac{30.08}{2.636 \times 10^{-2}}$	$Log$ $0.8176$ $\frac{2x}{1.6352}$  $0.8987$ $X2$  $1.7974$ $1.4783 + \frac{3.2757}{2.4210}$ $= 0.02636$ $= 0.0264 (4 \text{ d.p})$	$1.6967$

25.  $\log 120 = \log 4 + \log 3 + \log 10$   
 $= \log 2^2 + \log 3 + \log 10$   
 $= 2\log 2 + \log 3 + \log 10$   
 $= 2(0.30103) + 0.47712 + 1$   
 $= 2.07918$

26.  $\log_2(3x - 4) = \frac{1}{3} \log_2 8x^6 - \log_2 4$   
 $\log_2(3x - 4) = \log_2(2^3 x^6) - \log_2 4$   
 $\log_2(3x - 4) = \log_2 2x^2 - \log_2 4$   
 $\log_2(3x - 4) = \log_2 \left(\frac{2x^2}{4}\right)$   
 $= 3x - 4 = \frac{2x^2}{4}$   
 $2x^2 - 12x + 16 = 0$   
 $x^2 - 6x + 8 = 0$   
 $x - 2x - 4x + 8 = 0$   
 $(x - 2)(x - 4) = 0$   
 $x = 2 \text{ or } x = 4$

27.

No	Log
5.627	0.7503
$(0.234)^3$	$T. 3692$
	<u><math>x 3</math></u>
	2.8579
8.237	0.4779 <u><math>0.9158</math></u> 2
$2.399 \times 10^{-3}$	3.3800
	$= 0.002399$

28.  $\text{Det} \begin{vmatrix} 2 & -3 \\ -1 & 5 \end{vmatrix} = 5$   
 $\text{Area of } A^T B^T C^T = 5 \times 15$   
 $= 75 \text{ cm}^2$

29.  $\log_{10}(6x-2) - \log_{10} = \log_{10}(x-3)$   
 $\log_{10} \frac{6x-2}{10} = \log_{10}(x-3)$   
 $\frac{6x-2}{10} = x-3$   
 $6x-2 = 10x-30$   
 $x = 7$

30. No.                   Log  
 $0.07526^2 \quad 2.8766 \times 2 = 3.7532$   
 $6.652 \quad 0.8230 = 0.8230$   
 $4.9302$

$$\frac{4.9302}{3} = 6 + \frac{2.9302}{3}$$

$$= 2.9767$$

$$\text{Antilog} = 9.4776 \times 10^{-2}$$

$$\begin{aligned} \text{eeducationgroup.com} \\ = 0.094776 (\text{accept } 0.09478) \end{aligned}$$

No.	Log
4.283	0.6317
0.009478 <sup>2</sup>	<u>3.9767</u> X 2 + <u>5.9534</u> <u>4.5851</u> - <u>1.9964</u> <u>4.5887</u> ÷ 5
Log 9.814	
2.0785 X 10 <sup>-1</sup>	1.3177 = 0.20785

## 24. Equations of straight lines

1	$RV = 3.2 + \frac{1}{4}V$ $R = \frac{3.2}{V} + \frac{1}{4}$ $\text{Gradient} = 3.2$ $y - \text{intcept} = \frac{1}{4}$	B1 B1 B1	
		3	
2.	$y = -\frac{2}{3}x + \frac{5}{3}$ $\text{Grad of } \perp \text{line } \frac{3}{2}$ $\frac{3}{2} = \frac{1+k}{-2-2}$ $2k = -14$ $k = -7$	B1 M1 A1	Equating to grad
		03	
3.	$-y\sqrt{3} = -x - 3$ $y = \frac{1}{\sqrt{3}}x + 3$ $\text{grad} = \frac{1}{\sqrt{3}}$ $= 0.5774$ $\tan^{-1} 0.5774 = 30^\circ$	M1 M1 A1	
		03	

1. a) Length of diagonal =  $\sqrt{10^2 + 8^2}$   
 $= \sqrt{164}$

$$\text{Vertical height} = \sqrt{16^2 - (\frac{\sqrt{164}}{2})^2} = 14.66\text{cm}$$

b) Height of the slant surfaces

$$\sqrt{16^2 - 4^2} = \sqrt{240}$$

$$\sqrt{16^2 - 5^2} = \sqrt{231}$$

*Area of slant surfaces*

$$(\frac{1}{2} \times 8 \times \sqrt{240} \times 2) = 124.0 \text{ cm}^2$$

$$(\frac{1}{2} \times 10 \times \sqrt{231} \times 2) = 152.0 \text{ cm}^2$$

$$\text{Area of the rectangular base} = 8 \times 10 = 80 \text{ cm}^2$$

$$\text{Total surface area} = \underline{\underline{356 \text{ cm}^2}}$$

c) *Volume*

$$= (\frac{1}{3} \times 80 \times 14.66) = 391.0 \text{ cm}^3$$

2. Gradient of line  $AB = \frac{3 - 3k}{K + 1}$

*Equation of other line can be written as*

$$Y = \frac{-3x}{2} + \frac{9}{2}$$

$$\therefore \text{its gradient} = -\frac{3}{2}$$

$$\text{Hence } \frac{3 - 3k}{K + 1} = -\frac{3}{2}$$

$$6 - 6K = -3k - 3$$

$$-3K = -9$$

$$K = 3$$

$$A(-1, 9), \quad B(3, 3)$$

3.  $M_1 = 2x - 3x^2$

$$M_2 = 1 - 2ax$$

$$M_1 = M_2 \text{ at } x = \frac{1}{3}$$

$$2x - 3x^2 = 11 - 2ax$$

$$\frac{2}{3} - 3 \left(\frac{1}{3}\right)^2 = 1 - 2a \left(\frac{1}{3}\right)$$

$$\frac{2}{3} - \frac{1}{3} = 1 - \frac{2}{3}a$$

$$-\frac{1}{3} = -\frac{2}{3}a$$

$$\frac{1}{4} = a$$

4.  $M_1 = \frac{5 - 1}{4 - -2} = \frac{4}{6} = \frac{2}{3}$

$$M_2 = -\frac{3}{2}$$

$$\text{i.e. } -\frac{3}{2} = \frac{y - 5}{x - 4}$$

$$2(y - 5) = -3(x - 4)$$

$$2y - 10 = -3x + 12$$

$$3x + 2y = 22$$

5. Points (3, 0) and (-5, 2)

$$M = -\frac{1}{4}$$

$$y - 0 = -\frac{1}{4}$$

$$x - 3$$

$$y = -\frac{1}{4}x + \frac{3}{4}$$

7.  $\text{Grad} = \frac{2}{3}$

$$\frac{y - 4}{x + 2} = \frac{2}{3}$$

$$y = \frac{2x}{3} + \frac{16}{3}$$

8.  $3y - 5x = 4$  or equivalence

$$5y = 3x - 10$$

$$y = \frac{3}{5}x - 2$$

$\frac{5}{3}$

$$\therefore \text{Gradient} = \frac{3}{5}$$

$\frac{3}{3}$

$$5 = \frac{y-3}{x+1}$$

$$3y - 9 = 5x - 5$$

9.  $L.S.F = \frac{4}{2000000} = \frac{1}{500000}$

$$A.S.F = \frac{1}{5 \times 10^5}^2 = \frac{1}{2.5 \times 10^{11}}$$

$$\begin{aligned} \text{Area of rectangle} &= (2.4 \times 1.5) \text{ cm}^2 \\ &= 3.6 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Actual area} &= \frac{3.6 \times 2.5 \times 10^{11}}{100 \times 10000} \text{ ha} \\ &= 9 \times 10^5 \\ &= 900,000 \text{ ha} \end{aligned}$$

10.  $2y - 5x = 11$

$$Y = \frac{5}{2}x + \frac{11}{2}$$

$$g = \frac{5}{2}$$

$$\frac{5}{2}m = -1$$

$$M = -\frac{2}{5}$$

$$\frac{Y-4}{X+4} = -\frac{2}{5}$$

$$5y + 2x = 14$$

$$P(x, o)$$

$$5Xo + 2x = 14$$

$$X = 7$$

$$Q(o, y)$$

$$5y + 2Xo = 14$$

$$Y = 2.8$$

$$P(7, 0)$$

$$Q(0, 2.8)$$

11. i)  $K \left( \frac{3-7}{2}, \frac{4+2}{2} \right) = (-2, 3)$

$$P \left( \frac{3+1}{2}, \frac{4-2}{2} \right) = (2, 1)$$

ii)  $K_I = \frac{3-1}{-2-2} = -\frac{1}{2}$   
 $= 2$

12. Gradient of L1 =  $\frac{1}{5}$

*Gradient of L2 = -5*

$$Y = mx + c$$

$$2 = -5(1) + c$$

$$2 = -5 + c$$

$$C = 7$$

*Equation of L2*

$$Y = -5x + 7$$

$$13. \quad 3y - 5x = 4 \text{ or equivalence}$$

$$5y = 3x - 10$$

$$y = \frac{3}{5}x - 2$$

$$\therefore \text{Gradient} = \frac{-5}{3}$$

$$5 = \frac{y-3}{x+1}$$

$$3y - 9 = 5x - 5$$

$$14. \quad \text{Gradient} = g = \frac{m-1}{4-2} = \frac{m-1}{2}$$

$$3y = 5 - 2x$$

$$y = \frac{5}{3} - \frac{2x}{3} \quad g_l = -\frac{2}{3}$$

$$g \times g_l = \frac{m-1}{2} \times \frac{-2}{3} = -1$$

$$-2(m-1) = -6$$

$$-2m + 2 = -6$$

$$-2m = -8$$

$$M = 4$$

$$15. \quad L_1 y = -\frac{2}{3}x - \frac{4}{3}$$

$$M_1 = -\frac{2}{3}$$

$$M_2 = \frac{3}{2}$$

$$L_2 \quad y = \frac{3}{2}x + c \quad x = 1, y = 1$$

$$1 = \frac{3}{2} + c$$

$$c = -\frac{1}{2}$$

$$L_2 \quad y = \frac{3}{2}x - \frac{1}{2}$$

$$16. \quad BP = shs. \frac{144}{6} \times 100$$

$$SP = shs. \frac{140}{144} \times \frac{144}{6} \times 100$$

$$\begin{array}{r}
 100 \quad 6 \\
 \text{Let pineapples sold at shs. } 72 \text{ for every shs. } 3 \text{ be } x \\
 \therefore \text{At shs. } 60 \text{ for every } 2 \text{ will be } 144 - x \\
 \frac{x}{3} \times 72 + \frac{144-x}{3} = 3360 \\
 24x + 30(144 - x) = 3360 \\
 -6x = -960 \\
 x = 60
 \end{array}$$

17.  $\frac{x+2}{3} - \frac{x-1}{2} = \underline{5}$

$$\begin{aligned}
 2(x+2) - 3(x-1) &= 30 \\
 2x + 4 - 3x + 3 &= 30 \\
 -x + 7 &= 30 \\
 -x &= 23 \\
 x &= -23
 \end{aligned}$$

### 25. Reflection and congruence

1	Mid point (1, -4) $\frac{-3-(-5)}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$ Grd of mirror line = -2 $\frac{y+4}{x-1} = -2$ $y+4 = -2x+2$ $y+2x+4-2=0$ $y+2x+2=0$	B <sub>1</sub> B <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	
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1. (a) Dist. traveled in 3hrs s. drawing

$$\text{Plane A} - 400 \times 3 = 1200 \text{ km} - \text{cm}$$

$$\text{Plane B} - 500 \times 3 = 7.5 \text{ cm}$$

$$\text{Plane C} - 300 \times 3 = 900 \text{ km} - 4.5 \text{ cm}$$

$$(b) \text{Dist. BA} = 12.8 \text{ } 0.1 \times 200 = 2560 \text{ km } 20 \text{ km}$$

$$T = \frac{D}{S} = \frac{2560}{500} \text{ hrs}$$

$$= 5.12 \text{ hrs of } 5 \text{ hrs, } 7.2 \text{ mns}$$

$\approx 5 \text{ hrs, } 7 \text{ min (nearest min)}$

$$(c) \text{ Bearing of B from C} = 360^\circ - 20^\circ = 340^\circ$$

$$\begin{aligned}
 \text{Dist. BC} &= (10.9 \pm 0.1 \times 200) \text{ km} \\
 &= 2180 \text{ km} \pm 20 \text{ km}
 \end{aligned}$$

### 26. Rotation

1.  $V.S.F = 3^3 : 5^3 = 27 : 125$

$$\text{Volume of larger tank} = \underline{8.1 \times 125}$$

$$\frac{27}{= 37.5m^3}$$

### 27. Similarities and enlargement

1	<p>(a)</p> $512000 : 1000000$ $512 : 1000$ $64 : 125$ $4^3 : 5^3$ $\text{L.S.F } 4:5$ $5 \equiv 300\text{cm}$ $\therefore 4 \equiv \frac{4 \times 300}{5} = 240\text{cm}$ <p>(b)</p> $25 \equiv 1200\text{m}^2$ $16 = \frac{16 \times 1200}{25} \text{m}^2$ $= 768\text{m}^2$ <p>(c)</p> $64 \equiv 800\text{kg}$ $\therefore 125 \equiv \frac{125 \times 800}{64}$ $\equiv 1562.5\text{kg}$	$M_1$ $M_1$ $M_1$ $A_1$ $B_1$ $M_1$ $A_1$ $B_1$ $M_1$ $A_1$	$\checkmark A.S.F$ $\checkmark ex$ $\checkmark V.S.F$
2.	$  \text{Centre} \\ (x,y)   A(1,4)$ $3 \begin{pmatrix} 1-x \\ -4-y \end{pmatrix} = \begin{pmatrix} 2-x \\ 5-y \end{pmatrix}$ $3 - 3x = 2 - x$ $x = \frac{1}{2}$ $-12 - 3y = 5 - y$ $y = -8 \frac{1}{2}$ $\text{centre } (\frac{1}{2}, -8 \frac{1}{2})$	$A1(2,5)$ $M1$ $M1$ $A1$	
			03
3.	$10 = x + 6 \checkmark$ $5 \quad 6$ $60 = 5x + 30$ $30 = 5x$ $6 = x \checkmark$ $10 = 5 + y$ $5 \quad y$ $10y = 25 + 5y$ $5y = 25$ $Y = 5 \checkmark$	$M1$ $A1$ $B1$	Application of L.S.F

1.  $E.S.F = \frac{4-x}{0-x} = 3$

$$\begin{aligned}
 4 - x &= -3x \\
 2x &= -4 \\
 x &= -2 \\
 \underline{6 - y} &= 3 \longrightarrow 6 - y = 6 - 3y \\
 2 - y &= 0 \\
 y &= 0 \\
 \text{Centre of enlargement} \\
 &= (-2, 0)
 \end{aligned}$$

2. a) L.S.F = 1:500

$$\begin{aligned}
 \text{Height in cm} &= (500 \times 5) = 2500 \text{cm} \\
 \therefore \text{Height in m} &= \frac{2500}{100} = 25 \text{m}
 \end{aligned}$$

$$\begin{aligned}
 b) A.S.F &= 1:250000 \\
 &= 1:25 \text{ (in } m^2\text{)} \\
 \therefore \text{if } 25 &= 36 \\
 &= (3^6/25)m^2 &= 1.44m^2
 \end{aligned}$$

$$\begin{aligned}
 c) V.S.F &= 1:500 \\
 &= 1:125m^3 \\
 \text{Corresponding volume} \\
 &= (125/120)m^3 \\
 &= 1.042 m^3 &= 10420 \text{cm}^3
 \end{aligned}$$

3. Let  $DE = x \text{ cm}$

$$\begin{aligned}
 \therefore AD &= 3 + x \\
 \frac{3+x}{x} &= \frac{9}{4} \\
 12 + 4x &= 9x \\
 x &= 2.4 \text{ cm} \\
 DE &= 2.4
 \end{aligned}$$

$$4. L.S.F = \frac{12}{8} = \frac{3}{2}$$

$$A.S.F = \frac{9}{4} = \frac{336}{x}$$

$$x = 149 \frac{1}{3} \text{cm}^2$$

$$\begin{aligned}
 \text{Area of QRTS} &= 336 - 149 \frac{1}{3} \\
 &= 186 \frac{2}{3} \text{cm}^2
 \end{aligned}$$

$$5. (a) \frac{4}{3} = \frac{64}{x}$$

$$x = 48 \text{cm}$$

$$(b) \frac{3}{4} = \frac{810}{y}$$

$$\frac{27}{64} = \frac{810}{y}$$

$$27y = 810 \times 64$$

$$y = 1920 \text{grams}$$

6.  $\triangle ABC$  is similar to  $\triangle ADE$

$$\frac{DE}{4} = \frac{7}{4}$$

$$\frac{DE}{4} = \frac{(7 \times 8) \text{ cm}}{4}$$

$$= 14 \text{ cm} = 7/23$$

7. Area scale factor = 12 : 108

$$= 1 : 9$$

$$\text{Linear scale factor} = \sqrt{1 : 9}$$

$$= 1 : 3$$

$$\text{Volume scale factor} = 1^3 : 3^3$$

$$= 1 : 27$$

$$\text{Volume of the smaller cone} = \frac{810 \text{ cm}^3 \times 1}{27}$$

$$= 30 \text{ cm}^2$$

8.  $\frac{1}{2} h(a + b) = \text{Area of trap.}$

$$\frac{1}{2} x^3 (DC + 4) = 15.6$$

$$DC + 4 = \frac{15.6 \times 2}{3}$$

$$DC = 6.4$$

$$\frac{DC}{BE} = \frac{DA}{EA}$$

$$\therefore \frac{3+x}{x} = \frac{6.4}{4}$$

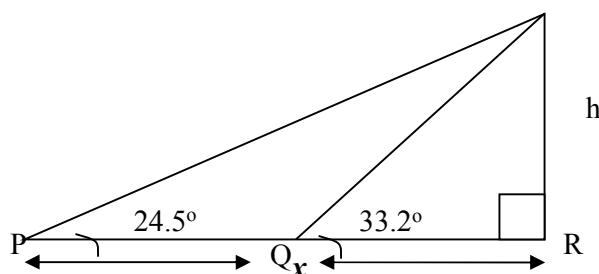
$$12 + 4x = 6.4x \checkmark$$

$$2.4x = 12 \checkmark$$

$$x = 5 \text{ cm}$$

## 28. The Pythagoras theorem

1.



$$\text{From } \triangle PTR, \tan 24.5^\circ = \frac{h}{x} \quad x = \frac{h}{\tan 24.5^\circ}$$

$$\text{From } \triangle QTR, \tan 33.2^\circ = \frac{h}{x-5} \quad x = \frac{h}{\tan 33.2^\circ} + 5$$

$$\therefore \frac{h}{\tan 24.5^\circ} = \frac{h}{\tan 33.2^\circ} + 5 \quad \circ \frac{h}{\tan 24.5^\circ} - \frac{h}{\tan 33.2^\circ} = 5$$

$$h \left[ \frac{1}{0.4557} - \frac{1}{0.6544} \right] = 5$$

$$h = (2.194 - 1.528) = 5$$

$$h = \frac{5}{0.666} = 7.508$$

$$\therefore \text{height} = 7.5 \text{ m}$$

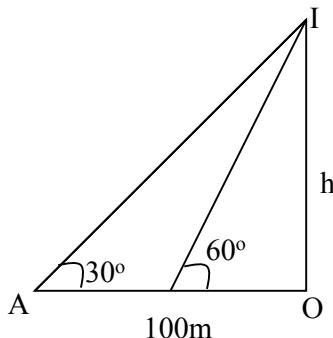
$$2. \quad L.S.F = \frac{2}{3}$$

$$V.S.F. = \left[ \frac{2}{3} \right] = \frac{8}{27}$$

$$\text{Ratio} = 8 : 27$$

## 29. The trigometric ratio I

1.



$$\tan 30^\circ = \frac{x}{100+y}$$

$$x = (100+y) \tan 30^\circ$$

$$(100+y) \tan 30^\circ = y \tan 60^\circ$$

$$\tan 60^\circ = \frac{x}{y} = x = y \tan 60^\circ$$

$$(100+y) 0.5774 = 1.1732y$$

$$57.74 = 1.155y$$

$$y = \frac{57.74}{1.155}$$

$$y = 49.99 \equiv 50m$$

$$\therefore x = 50 \tan 60^\circ$$

$$x = 86.6m$$

$$2. \quad \sin \theta = 0.70$$

$$\theta = 44.43^\circ, 135.57^\circ$$

$$3. \quad (a) (i) \text{Area of triangle } A^1B^1C^1 = \frac{1}{2} \times 4 \times 4 \\ = 8 \text{ sq. units}$$

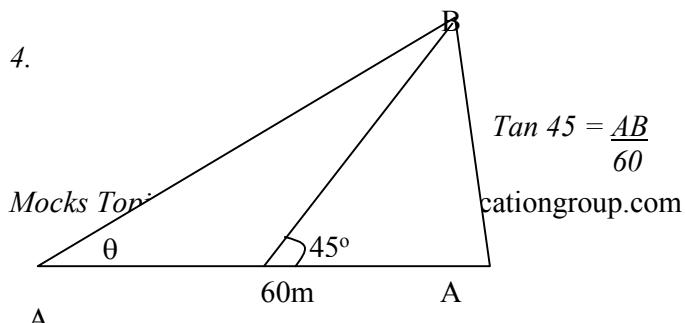
$$(b) (ii) \text{Reflection in the line } y = x$$

$$(c) \text{combine transformation} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$\text{Def} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} 0 - 2 x 2 = -4$$

$$\text{Inverse transformation} = -\frac{1}{4} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix}$$

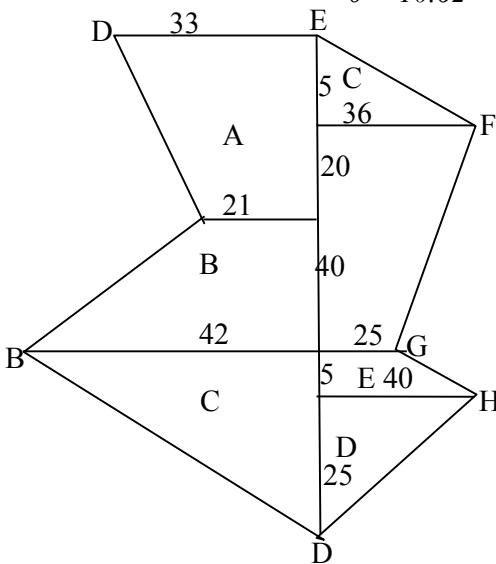
4.



$$\tan 45^\circ = \frac{AB}{60}$$

$$\begin{aligned} AB &= 45 \\ \tan \theta &= \frac{45}{240} \\ &= 0.1875 \\ \theta &= 10.62^\circ \end{aligned}$$

5.



$$\text{Area } A: \frac{1}{2} \times 25 (33 + 21) = 675$$

$$\text{Area } B: \frac{1}{2} \times 40 (21 \times 42) = 1260$$

$$\text{Area } C: \frac{1}{2} \times 30 \times 42 = 630$$

$$\text{Area } D: \frac{1}{2} \times 25 \times 40 = 500$$

$$\text{Area } E: \frac{1}{2} \times 5 (40 + 25) = 162.5$$

$$\text{Area } F: \frac{1}{2} \times 60 (25 + 36) = 1830$$

$$\text{Area } G: \frac{1}{2} \times 5 \times 36 = 90$$

$$= 5,147.5 \text{ m}^2$$

6.  $\therefore$  Philip takes 10 days.

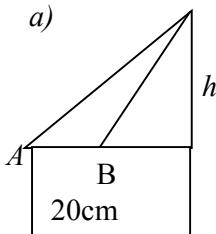
$$2\cos 2x = 0.600$$

$$\cos 2x = 0.3000$$

$$2x = 72.5^\circ, 287.5^\circ$$

$$x = 36.25^\circ, 143.75^\circ$$

7.



$$\tan 32^\circ = \frac{h}{20 + x}$$

$$h = (20 + x) \tan 32^\circ = 12.498 + 0.6249x$$

$$\tan 40^\circ = \frac{h}{x}$$

$$h = x \tan 40^\circ = 0.8391x$$

$$0.8391x = 12.498 + 0.6249x$$

$$0.8391x - 0.6249x = 12.498$$

$$0.2142x = 12.498$$

$$x = \underline{12.498} = 58.35m$$

0.2142

$$\therefore \text{The distance of } A \text{ from the house} \\ = (20 + 58.35)m = 78.35$$

$$b) h = x \tan 40^\circ = 58.35 \times 0.8391 = 48.96m$$

$$\therefore \text{The total height of the house} \\ = 1.82m + 48.96m = 50.78m$$

$$11. \quad \tan 32^\circ c = \frac{h}{20+x}$$

$$h = (20+x) \tan 32^\circ$$

$$\tan 40^\circ = \frac{h}{x}$$

$$h = \tan 40^\circ$$

$$\therefore x \tan 40^\circ = (20+x) \tan 32^\circ$$

$$0.8391x = (20+x) 0.6249$$

$$0.8391x = 12.498 + 0.6249x$$

$$0.8391x - 0.6249x = 12.498$$

$$x = 58.35m$$

$$20 + 58.35 = 78.35m$$

(b) The height of the house

$$\tan 40^\circ = \frac{h}{58.35} = h = 58.35 \tan 40^\circ$$

$$h = 58.35 \times 0.8391$$

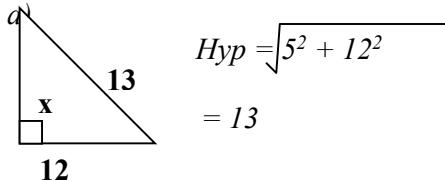
$$h = 48.96 + 1.82$$

$$h = 50.78$$

$$12. \quad \frac{24}{\sin 48} = 2R \Rightarrow R = 16.15 \text{ cm}$$

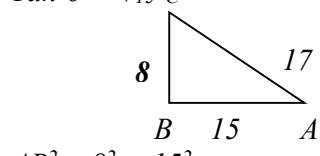
$$\text{Area} = 3.14 \times 16.15^2 \\ = \underline{\underline{819.26 \text{ cm}^2}}$$

13.



$$\cos x = 12/13$$

$$(b) \sin^2 90^\circ - x = (12/13)^2 = 144/169$$

14.  $\tan \theta = 8/15$ 

$$AB^2 = 8^2 + 15^2$$

$$AB = \sqrt{289} = 17$$

$$\sin \theta = 8/17, \cos \theta = 15/17$$

$$\frac{\sin \theta - \cos \theta}{\cos \theta + \sin \theta} = \frac{8/17 - 15/17}{15/17 + 8/17} = -7/17 \times 17/23$$

$$= -7/23$$

### 30. Area of a triangle

1. a)  $BC^2 = 50^2 + 80^2 - 2 \times 50 \times 80 \cos 30^\circ$   
 $= 2500 + 6400 - 6928.20 = 1971.8$   
 $\therefore BC = \sqrt{1971.8}$   
 $= 44.40m$   
 $= 44m$

b) Area of the plot  
 $= \frac{1}{2} \times 50 \times 80 \times \sin 30^\circ = 1000m^2$   
 $= \frac{(1000)}{10000} ha$   
 $= 0.01ha$

c) i) Length of wire required  
 $= (50 + 80 + 44) \times 4 = 696m$   
ii) Complete rolls to be bought = 2  
iii) Cost ( $2 \times 4000$ ) = Shs.8000

### 31. Area of polygons

1.	<p>(a) AC</p> <p>(b) <math>AC = \sqrt{6^2 + 6^2}</math>  <math>= \sqrt{72}</math>  <math>= 8.485</math></p> <p><math>\bar{AO} = \frac{1}{2} \times 8.485 = 4.243</math></p> <p><math>VA = \sqrt{4.243^2 + 10^2}</math>  <math>= \sqrt{118.003}</math></p>	<b>B1</b>  <b>M1</b>  <b>M1</b>	
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	<p><math>= 10.863.</math></p> <p>(c)</p> <p><math>\cos \theta = \frac{4.243}{10.863}</math></p> <p><math>\cos \theta = 0.39059</math></p> <p><math>\theta = 67.01^\circ</math></p>	A1	
	<p><math>\tan \theta = \frac{10}{3}</math></p> <p><math>\theta = 73.30^\circ</math></p>	M1 A1	
	<p>(d)</p> <p><math>\tan \theta = \frac{10}{3}</math></p> <p><math>\theta = 73.30^\circ</math></p>	M1 A1	
	<p>(e) <math>\text{Vol} = \frac{1}{3} \times 6 \times 6 \times 10</math>  <math>= 120\text{cm}^3</math></p>	M1 A1	10

$$1. \quad \frac{180(n-2)}{180(n-1-2)} = \frac{4}{3}$$

$$540n - 1080 = 720n - 2160$$

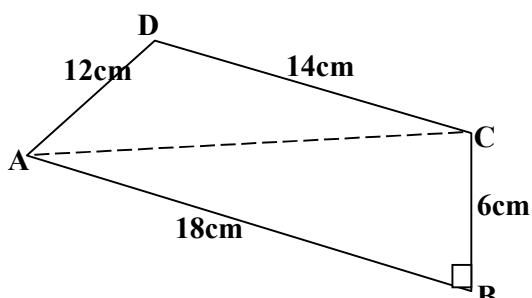
$$720n - 540n - 2160 - 1080$$

$$180n = 1080$$

$$n = 6$$

$$\text{Area of hexagon} = 6 (\frac{1}{2} \times 10 \times \sin 60^\circ)  
= 6 \times 43.30 = 259.81 \text{ cm}^2$$

2.



$$\text{Area } \angle rt \Delta = \frac{1}{2} \times 8 \times 6$$

$$S = \frac{12 + 14 + 10}{2}$$

$$A = \sqrt{\frac{18(18-12)(18-14)(18-10)}{16}}$$

$$\begin{aligned}
 &= 18 \times 6 \times 4 \times 8 \\
 &\equiv \boxed{3456} \\
 &= 58.79
 \end{aligned}$$

$$\text{Total area} = 24 + 58.79 = 82.79$$

### 32. Area of part of a circle

1. (a)  $A = \frac{120}{360} \times \pi \times 10^2 - \frac{1}{2} \times 100 \times 10 \sin 12$

$$\begin{aligned}
 &= 104.72 - 43.30 = 61.42 \text{ m}^2
 \end{aligned}$$

(b) (ii)  $\frac{120}{360} \times 2 \times 10 \times 20$

$$\begin{aligned}
 &= 418.9 \text{ m}^2
 \end{aligned}$$

(b) Total area =  $61.42 + 61.42 + 418.9$   
 $= 541.74 \text{ m}^2$   
 $\text{Cost} = 541.74 \times 310 = 167,939$

2. a)  $\cos 54^\circ = \frac{x}{10}$   
 $X = 5.878$   
 $\therefore \text{size} = 2 \times 5.878 = 11.756$   
 $\text{Area of } \Delta = \frac{1}{2} \times 10^2 \sin 72^\circ = 47.55$   
 $\text{Total area of } \Delta s = 47.55 \times 5 = 237.8 \text{ cm}^2$

b) Area of circle =  $\frac{22}{7} \times 10 \times 10 = 314.8$

$$\begin{aligned}
 \text{Shaded region} &= \frac{3}{5} (3.143 - 237.8) \\
 &= 45.9 \text{ cm}^2
 \end{aligned}$$

3. (a)  $7.8^2 = 6.6^2 + 5.9^2 - 2 \times 6.6 \times 5.9 \cos R$   
 $\cos R = \frac{6.6^2 + 5.9^2 - 7.8^2}{2 \times 6.6 \times 5.9}$   
 $= \frac{78.37 - 60.84}{77.88}$   
 $= 0.2251$

$$\begin{aligned}
 \angle R &= 77^\circ \\
 \frac{7.8}{\sin 77} &= 2r
 \end{aligned}$$

$$\begin{aligned}
 r &= \frac{7.8}{2 \sin 77} \\
 &= 4 \text{ cm}
 \end{aligned}$$

(b)  $\frac{5.9}{\sin p} = \frac{7.8}{\sin 77}$   
 $\sin P = \frac{5.9 \sin 77}{7.8}$   
 $= 0.7370$

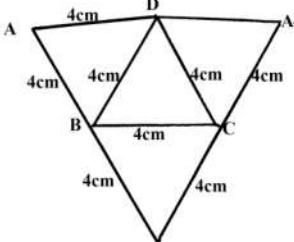
$$\begin{aligned}\angle P &= 47.5^\circ \\ \angle Q &= 180 - (77 + 47.5) = 55.5^\circ\end{aligned}$$

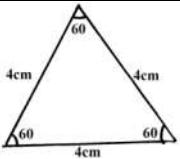
(c) Area of shaded region  
 $= 3.142 \times 4^2 - \frac{1}{2} \times 6.6 \times 5.9 \sin 77$   
 $= 50.27 - 18.97 = 31.30$

4.  $(\frac{60}{360} \times 22/7 \times 24 \times 24) - (\frac{60}{360} \times 22/7 \times 12 \times 12)$

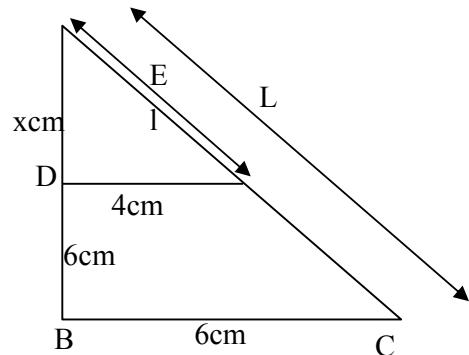
$$301.71 - 75.43 = 226.26$$

### 33. Surface area of solids

1.	(a) $(10+2x)(8+2x)=168$ $80 + 20x + 16x + 4x^2 = 168$ $4x^2 + 36x - 88 = 0$ $x^2 + 9x - 22 = 0$ $p = -22$ $s = 9$ $-2, 11$ $x^2 - 2x + 11x - 22 = 0$ $x(x-2) + 11(x-2) = 0$ $(x+11)(x-2) = 0$ $\therefore x = 2$ $2m$	M <sub>1</sub>	✓ equation
		M <sub>1</sub>	✓ quad equation
		A <sub>1</sub>	
		M <sub>1</sub>	✓ partial fact
		A <sub>1</sub>	
		M <sub>1</sub>	✓ exp. for area path
	(b) (i) Area of the path $168 - 80 = 88m^2$ Area of the path excluding corners $88 - 4 \times 4m^2$ $= 88 - 16$ $= 72m^2$ No of slabs = $\frac{72 \times 100 \times 100}{50 \times 50}$ $= 288$	M <sub>1</sub>	✓ exp. for area of the slabs excluding corners
		M <sub>1</sub>	✓ exp for No. of slabs
		A <sub>1</sub>	✓ exp total cost
	(ii) $4 \times 600 + 288 \times 50$ $= 2400 + 14400$ $= Ksh.16800$		
2.	 $S.A = \frac{1}{2} \times 4 \times 4 \sin 60 \times 4$ $= 27.713 \text{ cm}^2$	B <sub>1</sub>	
		M <sub>1</sub>	
		A <sub>1</sub>	

			
		03	

1. (a)



$$\frac{x}{x+6} = \frac{4}{6}$$

$$6x = 4x + 24$$

$$x = 12 \text{ cm}$$

$$L = \sqrt{12^2 + 4^2}$$

$$= \sqrt{160}$$

$$= 12.65 \text{ (2 d.p)}$$

$$L = \sqrt{18^2 + 6^2}$$

$$\sqrt{360}$$

$$= 18.97$$

$$SA = \pi(RL - rL)$$

$$= 3.142(6 \times 18.97 - 4 \times 12.65)$$

$$= 3.142 \times 63.22 = 198.64 \text{ cm}^2$$

(b) Cost of material for one lamp shape

$$= \frac{198.64}{10000} \times 800$$

$$10000$$

$$= Sh15.90$$

$$\text{Cost of 10 lamp shape} = 2 \times 10 \times 15.90 = sh 318$$

2. Area of the remaining cross-section

$$= 4.22 \times \pi$$

$$= (17.64\pi)\text{cm}^2$$

Area of the curved surface

$$= (8.4\pi \times 150)$$

$$= \frac{1260\pi \text{ cm}^2}{2}$$

Area of the flat surface

$$= (150 \times 8.4) \text{cm}^2$$

$$= 1260 \text{cm}^2$$

$$\text{Total area} = (1260 + 630\pi + 17.64\pi)$$

$$= (1260 + 647.64\pi) \text{cm}^2$$

$$= 3295 \text{cm}^2 / 3295.44 \text{cm}^2$$

3.  $\text{Surface area} = 2(0.6 \times 2.8)m^2 + 2(0.6 \times 3.2)m^2$   
 $= (3.36 + 3.84)m^2$   
 $= 7.2m^2$

4. a)  $\text{Area of hemispherical part}$   
 $= \frac{1}{2} \times 4 \pi R^2$   
 $= 2 \times \frac{22}{7} \times 35 \times 35$   
 $= 7700 \text{cm}^2$

b) Slant height for original cone

$$\frac{L}{L-60} = \frac{35}{14}$$
 $L = 100 \text{cm}$

c) Surface area of frustum  
 $= \pi RL - \pi r_1^2$   
 $= \frac{22}{7} \times 35 \times 100 - \frac{22}{7} \times 14 \times 40$   
 $= 11000 - 1760 = 9240 \text{ cm}^2$

d) Area of base  
 $\frac{22}{7} \times 14^2 = 616 \text{ cm}^2$

e) Total surface  
 $= 7700 + 9240 + 616 = 17556 \text{cm}^2$

5. a)  $TA = 2 \times 6.8 \times 3.5 + 2 \times 4.2 \times 3.5 m^2$   
 $= 47.6 + 29.4 m^2 = 77 m^2$

b)  $77 - (\frac{75}{100} \times 2.5 \times 2 + \frac{400}{100} \times 1.25) m^2$   
 $77 - (3.75 + 5) m^2$   
 $77 - 68.25 m^2 = 8.75 m^2$

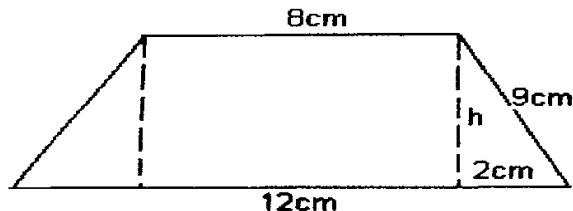
c) i) Cost of paint A  
 $= 68.25 \times 0.8 \times 80 = \text{Kshs. } 43681$

ii) Cost of paint B  
 $\frac{68.25 \times 35}{0.5}$   
 $= \text{Kshs. } 4777.5$

d) No of tins  
 $= \frac{54.6 \times 1000}{400}$   
 $= \frac{136.5}{1.25} = 137 \text{ tins}$   
 $No. of tins$   
 $= \frac{136.5}{1.25}$   
 $= 109.2 = 110 \text{ tins}$

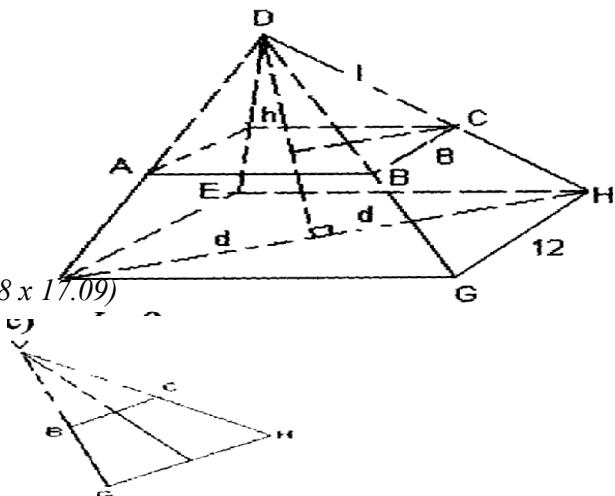
6.  $\text{Top surface area} = 8 \times 8 = 64 \text{ cm}^2$   
 $\text{Bottom surface area} = 12 \times 12 = 144 \text{ cm}^2$   
 $\text{Height of slanting faces}$   
 $H = 9^2 - 2^2 = 8.775 \text{ cm}$   
 $\text{Area of slanting face} = \frac{1}{2} (12 + 8) \times 8.775 \times 4$   
 $= 351 \text{ cm}^2$   
 $T.S.A = 64 + 144 + 351 = 559 \text{ cm}^2$

**For both**  
**Attempt to solve area for**  
**slant face**



$$\begin{aligned} \frac{1}{8} &= \frac{1+9}{12} \\ I &= 18 \\ D &= \frac{1}{2}\sqrt{12^2 + 12^2} = 8.485 \\ H &= \sqrt{27^2 - 8.485^2} = 25.63 \\ \frac{h}{25.63} &= \frac{8}{12} \\ h &= 17.09 \text{ cm} \\ V &= \left(\frac{1}{3} \times 12 \times 12 \times 25.63 - \left(\frac{1}{3} \times 8 \times 8 \times 17.09\right)\right) \\ &= 865.7 \text{ cm}^2 \end{aligned}$$

$$(c) \tan \theta^{25.63/6} = 4.272 \\ \theta = 76.82^\circ$$



### 34. Volume of solids

<b>1.</b> Volume of cube = $4.4 \times 4.4 \times 4.4$ Volume of sphere $\frac{22}{7} \times r^3 = 4.4 \times 4.4 \times 4.4$ $r^3 = 4.4 \times 4.4 \times 4.4 \times \frac{7}{22} \times \frac{3}{4}$ $r^3 = 20.328$ $r = 2.73 \text{ cm} \quad (3 \text{ s.f})$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	
<b>2.</b> Vol. of sphere =	M <sub>1</sub>	Follow through $\frac{22}{7}$ as $\pi$ is

$\begin{aligned} & \frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} (2.3^3 + 3.86^3) \\ &= \frac{88}{21} \times 69.679456 \\ &= 291.990 \end{aligned}$ <p>Remaining material</p> $\left( \frac{19}{20} \times 291.990 \right)$ $= 277.297$ <p>No of slabs =</p> $\frac{277.297}{3.142 \times 0.8^2 \times 7}$ $= 19.699$ $= 19 \text{ slabs}$	M1  M1  A1	used
	04	

1. a) Length of diagonal =  $\sqrt{10^2 + 8^2}$   
 $= \sqrt{164}$

$$\begin{aligned} \text{Vertical height} &= \sqrt{16^2 - (\sqrt{164})^2} \\ &= \sqrt{16^2 - 164} \\ &= \sqrt{14.66} \end{aligned}$$

b) Height of the slant surfaces

$$\sqrt{16^2 - 4^2} = \sqrt{240}$$

$$\sqrt{16^2 - 5^2} = \sqrt{231}$$

Area of slant surfaces

$$(\frac{1}{2} \times 8 \times \sqrt{240} \times 2) = 124.0 \text{ cm}^2$$

$$(\frac{1}{2} \times 10 \times \sqrt{231} \times 2) = 152.0 \text{ cm}^2$$

$$\text{Area of the rectangular base} = 8 \times 10 = 80 \text{ cm}^2$$

$$\text{Total surface area} = \underline{356 \text{ cm}^2}$$

c) Volume

$$= (\frac{1}{3} \times 80 \times 14.66) = 391.0 \text{ cm}^3$$

2. Volume of the cylinder

$$= (\frac{22}{7} \times 6 \times 6 \times 12) \text{ cm}^3 = 1357.71 \text{ cm}^3$$

Volume of a sphere

$$= (\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3) \text{ cm}^3 = 113.14 \text{ cm}^3$$

∴ No. of spheres formed

$$= \frac{1357.71}{113.14 \text{ cm}^3}$$

$$= 12 \text{ spheres}$$

3. Let the smaller length be  $x$  cm

∴ Dimensions are  $x, 2x, 3x$

$$x \cdot 2x \cdot 3x = 1024$$

$$\begin{aligned}6x^3 &= 1024 \\x^3 &= \frac{1024}{6} \\x &= \sqrt[3]{\frac{1024}{6}}\end{aligned}$$

Dimensions are 5.547, 11.09, 16.64

4.  $(60/360 x 22/7 x 24 x 24) - (60/360 x 22/7 x 12 x 12)$

$$301.71 - 75.43 = 226.26$$

5. (a)(i)  $2\pi rh + 2r\pi^2 + \pi r^2$   
 $= 2 \times 22/7 \times 1.4 \times 1.4 + 2 \times 22/7 \times 1.42 + (22/7 \times 1.42)m^2$   
 $= (12.32 + 12.32 + 6.16)m^2 = 30.8m^2$

OR  $r(2h + 2r + r)$   
 $= 22 \times 1.4 (2 \times 1.4 + 3 \times 1.4) = 30.8m^2$

(ii) shs.  $(75 \times 30.8) = \text{Shs. } 2,310$

(iii) Total vol.  
 $= 22/7 \times 1.42 \times 1.4 + (\frac{1}{2} \times \frac{4}{3} \times 22/7 \times 1.42)m^3$   
 $= 8.624 \times 4.106 = 12.7306m^3$   
 $\text{capacity} = (12.7306 \times 1000) \text{liters} = 12730.6 \text{litres}$

(b) First 2 days =  $185 \times 2 = 370 \text{litres}$   
 $\text{Remaining amount} = (12730.6 - 370) \text{liters}$   
 $= 12360.6 \text{litres}$

Days to use =  $\frac{12,360.6}{200}$   
 $= 61.803 \text{days}$

In all it takes =  $(61.803 + 2) \text{days} = 63.803 \text{days}$

6. a)  $\frac{h+3}{h} = \frac{9}{6} \checkmark$   
 $6h + 18 = 9h$   
 $h = 6 \text{ cm} \checkmark$   
 $\text{height} = \underline{6 + 3 = 9 \text{ cm}}$

b) Base =  $9 \times 9 = 81 \text{ cm}^2$

Top =  $6 \times 6 = 36 \text{ cm}^2$

Sides =  $3.67 \times 15 \times \frac{1}{2} \times 4$   
 $= 110.15 \text{ cm}^2$

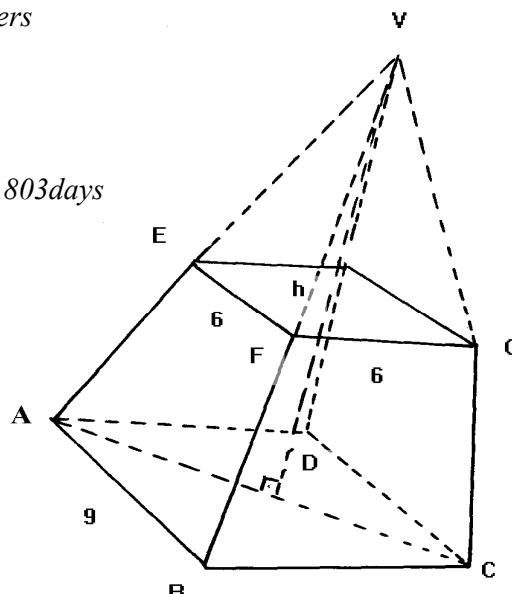
Total = 227.15 cm<sup>2</sup>

c) Vol. of bigger =  $\frac{1}{3} \times 81 \times 9$   
 $= 243$

Vol. of smaller =  $\frac{1}{3} \times 36 \times 6$   
 $= 72$

Vol. of frustum = 171 cm<sup>2</sup>

d)  $\sin \theta = \frac{9}{11.02}$   
 $\theta = 54.8^\circ$



7. *Volume of a hemisphere*

$$\begin{aligned} \frac{2\pi r^3}{3} &= \frac{2}{3} \times \frac{22}{7} \times 12 \times 12 \times 12 \\ &= \frac{176}{7} \times 144 \\ &= 3620.571429 = 3620.57 \end{aligned}$$

*Volume of a cone*

$$\begin{aligned} \frac{2/3\pi r^2 h}{3} &= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times h = 36.20.57 \\ \frac{6 \times 44h}{7} &= 3620.57 \\ 264h &= 3620.57 \times 7 \\ h &= \frac{3620.57 \times 7}{264} \\ &= 95.9981 = 95.998 \end{aligned}$$

$$8. V = \left[ \frac{22}{7} \times 2 \times 2 \times 1.5 \right] + \left[ \frac{22}{7} \times 3 \times 3 \times 1.5 \right] + \left[ \frac{22}{7} \times 4.4 \times 1.5 \right]$$

$$= \frac{132}{7} + \frac{297}{7} + \frac{528}{7}$$

$$V \text{ of hole} = \frac{22}{7} \times 1 \times 1 \times 4.5$$

$$= \frac{99}{7}$$

$$V = \frac{957}{7} - \frac{99}{7} = \frac{858}{7} = 122.57 \text{ cm}^3$$

$$\begin{aligned} \text{Mass} &= 2.8 \times 122.57 \\ &= 343.196g \\ &\simeq 343.2g \end{aligned}$$

$$9. \text{ Volume of hemisphere} = \frac{1/2}{3} \times \frac{22}{7} \times \frac{7}{1} \times 7 \times 7$$

$$\text{Vol. of cylinder} = \pi r^2 h = \frac{22}{7} \times \frac{1}{1} \times 7 \times 5 = 770 \text{ cm}^3$$

$$\begin{aligned} \text{Vol of frustum} &= \frac{1/3}{7} \times \frac{22}{7} \times 7 \times 7 \times h_1 - \\ &\quad \frac{1/3}{7} \times \frac{22}{7} \times 3.5 \times 3.5 \times h_2 \end{aligned}$$

$$\text{Height of cone} \Rightarrow h_1 = \frac{7}{h_2} \times \frac{7}{3.5}$$

$$\frac{h_2 + 6}{h_2} = \frac{7}{3.5} \Rightarrow 7h_2 = 3.5h_2 + 21$$

$$3.5h_2 = 21$$

$$h_2 = 6 \text{ cm}$$

$$h_1 = 12 \text{ cm}$$

$$\therefore \text{Vol. of frustum} = \frac{1}{3} \pi \frac{22}{7} \times 7 \times 12 -$$

$$\begin{array}{r} 11 \\ \cancel{1} \cancel{4} \cancel{x} \cancel{22} \cancel{x} 3.5 \times 3.5 \times 6 \\ \cancel{1} \cancel{2} \cancel{1} \end{array}$$

$$= 616 - 77 = 539 \text{ cm}^3$$

$$\begin{aligned} \text{Total volume} &= 718.67 \text{ cm}^3 + 770 \text{ cm}^3 + 539 \text{ cm}^3 \\ &= 2027.67 \text{ cm}^3 \end{aligned}$$

$$a) \text{ S.A of top} = \pi r^2 \frac{22}{7} \times 3.5 \times 3.5 = 38.5 \text{ cm}^2$$

$$\text{S.A of curved part of frustum} = \frac{22}{7} \times 7 \times 13.89 -$$

$$\begin{array}{r} 22 \times 3.5 \times 6.945 \\ \cancel{7} \\ 305.580 \\ - 76.395 \\ \hline 229.185 \text{ cm}^2 \end{array}$$

$$\begin{aligned} \text{S.A of curved part of cylinder} &= 2\pi r \times h = 2\cancel{\pi} \times \cancel{22} \times 7 \times 5 \\ &= 2220 \text{ cm}^2 \end{aligned}$$

$$\text{S.A of hemisphere} = \frac{1}{2} \times 4 \pi r^2 = \frac{22}{7} \times 7 \times 7 = 308 \text{ cm}^2$$

$$\text{Total S.A} = \underline{795.685 \text{ cm}^2}$$

$$\begin{aligned} 10. \quad L/S.F &= \frac{2.2}{3.3} = \frac{2}{3} \\ \frac{4.8}{4.8+h} &= \frac{2}{3} \\ h &= 24 \end{aligned}$$

$$\begin{aligned} \text{volume of smaller cone} \\ \frac{1}{3} \times \frac{22}{7} \times 2.2 \times 2.4 \\ = 12.169 \end{aligned}$$

$$\begin{aligned} \text{Volume of large cone} \\ \frac{1}{3} \times \frac{22}{7} \times 3.3 \times 3.3 (4.8 + 2.2) \\ \therefore V \text{ of frustum} \\ 82.14 - 12.17 = 69.97 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} 11. \quad (a) \text{ Volume} &= \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 \times \frac{3}{2} r = 31.5 \pi \\ 4r^3 + 3r^3 &= 31.5 \times 6 \\ r &= \sqrt[3]{\frac{31.5 \times 6}{7}} \\ &= 3 \text{ cm} \end{aligned}$$

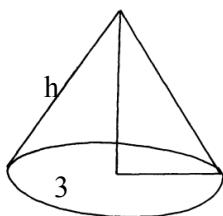
$$\begin{aligned} (b) \text{ slant height of con} &= \sqrt{4.5^2 + 3^2} \\ &= 5.408 \text{ cm} \\ \text{Surface are} &= 2\pi r \times 3^2 + \pi r \times 3 \times 5.408 = 107.5 \text{ cm}^2 \end{aligned}$$

$$(c) \text{Height} = \frac{31.5}{4^2 \pi} \\ = 1.969 \text{cm}$$

$$(d) \text{Density} = \frac{144}{231.5 \pi} \\ = 1.46 \text{g/cm}^3$$

12. Volume of cube side  $x$  cm =  $(x\text{cm})^3$   
 $\therefore x^3 \text{cm}^3 = \frac{1280}{20} \text{cm}^3$   
 $x = \sqrt[3]{\frac{1280}{20}}$   
 $= \sqrt[3]{64}$   
 $= 4 \text{ cm}$

13.  $\frac{9}{3} = l^4 + h/h$



$$9h = 42 + 3h$$

$$6h = 42$$

$$h = 7$$

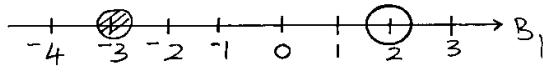
$$\text{volume of the frustum} = \left(\frac{1}{3} \pi r^2 h\right) \left(\frac{r_1^2 + r_1 r_2 + r_2^2}{3}\right) \\ = \left(\frac{1}{3} \pi 3^2 \times 7 \times (3^2 + 3 \times 7 + 7^2)\right) \text{cm}^3 \\ = 1782 - 66 = 1716 \text{cm}^3$$

### 35. Quadratic equations

<b>1</b>	$25x^2 - 20x + k = (5x - c)^2$ $= 25x^2 - 10cx + c^2$ $- 20x = -10cx$ $c = 2$ $k = c^2 = 2^2$ $\therefore k = 4$	<b>M<sub>1</sub></b> <b>A<sub>1</sub></b>	Comparing terms or equivalent $c = \frac{b^2}{4a}$ $k = \frac{(-20)^2}{4 \times 25}$
		2	

1.  $(3x + 5)^2 + (\sqrt{611})^2 = (7x - 2)^2$   
 $(9x^2 + 30x + 25) + 611 = 49x^2 + 28x + 4$   
 $- 40x^2 + 2x + 632 = 0$   
 $20x^2 - x = 316 = 0$   
 $x = \frac{1 \pm \sqrt{2581}}{40}$   
 $= \frac{160}{40} \text{ OR } x = 4$   
 $\text{Area} = (\frac{1}{2} \times \sqrt{611} \times 17)$   
 $= 210.1 \text{cm}^2$

$$\begin{array}{l}
 2. \quad 7x - 4 \leq 9x + 2 \\
 \underline{-6} \leq \underline{2x} \\
 2 \quad 2 \\
 -3 \leq x \\
 \therefore -3 \leq x < 2
 \end{array}
 \quad \left| \quad \begin{array}{l}
 9x + 2 < 3x + 14 \\
 6x < 12 \\
 x < 2
 \end{array} \right.$$



Integral values are -3, -2, -1, 0 and 1

### 36. Linear inequalities

$$\begin{array}{l}
 1. \quad \frac{12x - 0.25 - 12.4}{0.4} \div 3 \\
 \frac{1}{8} \text{ of } 2.56 + 8.68 \\
 \frac{3 - 31x}{0.32} \times 3 \\
 \frac{-90}{9} \\
 = -10
 \end{array}$$

$$\begin{array}{l}
 2. \quad x - 9 \leq -4 < 3x - 4 \\
 x - 9 \leq -4 \\
 x \leq 5
 \end{array}$$

$$\begin{array}{l}
 3x - 4 > -4 \\
 3x > 0 \\
 x = 0 \\
 0 > x \leq 5 \checkmark
 \end{array}$$

$$\{1, 2, 3, 4, 5\} \checkmark$$

3

$$\begin{array}{l}
 3. \quad x > 3 - 2x \\
 x \leq \frac{2x + 5}{3} \\
 3 - 2x < x - 5 \\
 -2x < x - 3 \\
 -3x < -3 \\
 x < 1 \\
 2x + 5 \geq 3x \\
 -x \geq 5 \\
 x \leq -5 \\
 -5 \leq x < 1
 \end{array}$$

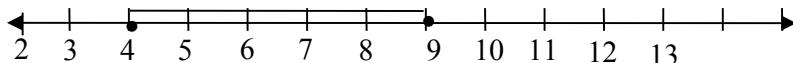
$$\begin{array}{l}
 4. \quad 3 - X \leq 1 - \frac{1}{2}X \\
 3 - 1 \leq X - \frac{1}{2}X \\
 2 \leq \frac{1}{2}X \\
 X \geq 4
 \end{array}$$

$$-x + 5 \leq 14 - 2x$$

$$2x - x \leq 14 - 5$$

$$x \leq 9$$

$$4 \leq x \leq 9$$



5.  $4x - 3 \leq 6x - 1$

$$-2x \leq 2$$

$$x \geq -1$$

$$6x - 1 < 3x + 8$$

$$3x < 9$$

$$x < 3$$



$$-1 \leq x < 3$$

6.  $2(2-x) < 4x - 9$

$$4 - 2n < 4x - 9$$

$$4 + 9 < 4x + 2n = 13 - 6x$$

$$= 13/6 < n \quad = 2/6 < n$$

$$\text{and } 4x - 9 < x + 11$$

$$4n - n < 11 + 9$$

$$3n < 20$$

$$x < 20/3 = < 2/3$$

Integral values 3, 4, 5, 6

7.  $L_3 : y \geq 1$

$$L_1 : y + x \geq -1$$

$$L_2 : y - x$$

8. a)  $x^2 + 2xy + y^2 = x^2 + xy + xy + y^2$   
 $= x(x + y) + y(x + y)$   
 $= (x + y)(x + y)$   
 $\therefore (x + y)^2 = 8 \times 8 = 64$

b)  $x^2 + 2xy + y^2 = 64$   
 $(x^2 + y^2) + 2xy = 64$   
 $34 + 2xy = 64$   
 $2xy = 30$

9. Equation of L1

$$(3.5, 4) (0, 2)$$

$$\frac{y-2}{x-4} = \frac{2}{-3.5}$$

$$x-0 = 3.5-0$$

$$3.5y - 7 = 2x$$

$$\therefore y = \frac{4}{7}x + 2$$

Inequality of

Equation of L2

$$(0, 3) (4, 2)$$

$$\frac{y-2}{x-4} = \frac{3-2}{0-4}$$

$$-4(y-2) = x-4$$

$$-4y + 8 = x - 4$$

$$-4y = x - 12$$

$$\text{inequality } y \geq -\frac{1}{4}x + 3$$

**Equation of L3**

$$\frac{y-2}{x-4} = \frac{2}{-0.5}$$

$$-0.5(y-2) = 2(x-4)$$

$$-5y + 10 = 2x - 8$$

$$-5y = 2x - 18$$

$$y = -\frac{2}{5}x + 3.6$$

in equality  $y \leq -\frac{2}{5}x + 3.6$

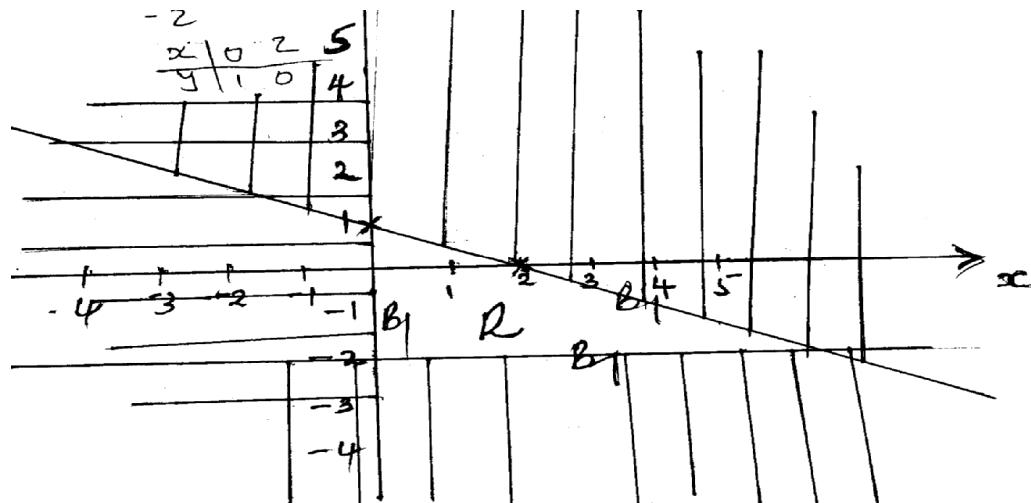
$$y \leq \frac{4}{7}x + 2$$

Or  $7y - 4x - 14 = 0$

10. Lines to be drawn  $x = 0, y = 2$

$$2y + x = 2$$

x	0	2
y	1	0



11.  $3(1+x) < 5x - 11$

$$3+3x < 5x - 11$$

$$-2x < -14$$

$$x > 7$$

$$5x - 11 < 45$$

$$5x < 56$$

$$x < 11.2$$

Integral values are 8, 9, 10, 11

12.  $y \leq x$

$$x \leq 8$$

$$y \geq 0$$

### 37. Angle properties of circles

1. $\angle QRO = 30^\circ$ Base angles of isosc. Triangle $\angle ROT = 100 - 120 = 60^\circ$ $\angle s$ on straight line $\angle ROT = 60^\circ$ $\angle ORP = 60^\circ$ Base angles of Isoc triangle $\angle QRS = 90^\circ$ diameter subtended right angle at the circumference (a) $\angle SRQ = 90^\circ - 30^\circ - 30^\circ$ $= 30^\circ$ $\angle QRO + \angle ORP + \angle SRP = 90^\circ$ Diam. Subt $90^\circ$ at circumference (b) $\angle ORP = 60^\circ$ Base angle of isosceles triangle (c) OP to MPT $\angle OPT = 90^\circ$ Radius meets tangent at $90^\circ$ $\angle RTP = 90^\circ - \angle OPR$	<b>B1</b> <b>B1</b> <b>B1 B1</b> <b>B1</b>
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	$= 90^\circ - 30^\circ$ $= 60^\circ$ (d) $\angle STP = 180^\circ - \angle OPT 90^\circ - \angle POT 60^\circ$ Angle sum of triangle $= 30^\circ$ (e) $\angle QPM = \angle QRP = 60^\circ$ Angles in alternate segment	<b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b>	
		<b>10</b>	
2.	$\angle QRO = 30^\circ$ Base angles of isosc. Triangle $\angle ROT = 100-120 = 60^\circ$ $\angle s$ on straight line $\angle ROT = 60^\circ$ $\angle ORP = 60^\circ$ Base angles of Isoc triangle $\angle QRS = 90^\circ$ diameter subtended right angle at the circumference (a) $\angle SRQ = 90^\circ - 30^\circ - 30^\circ$ $= 30^\circ$ $\angle QRO + ORP + SRP = 90^\circ$ Diam. Subt $90^\circ$ at circumference (b) $\angle ORP = 60^\circ$ Base angle of isosceles triangle (c) OP to MPT $\angle OPT = 90^\circ$ Radius meets tangent at $90^\circ$ $\angle RTP = 90^\circ - \angle OPR$ $= 90^\circ - 30^\circ$ $= 60^\circ$ (d) $\angle STP = 180^\circ - \angle OPT 90^\circ - \angle POT 60^\circ$ Angle sum of triangle $= 30^\circ$ (e) $\angle QPM = \angle QRP = 60^\circ$ Angles in alternate segment	<b>B1</b> <b>B1</b> <b>B1 B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b> <b>B1</b>	<b>10</b>

1. Area of  $\Delta AXY = \frac{1}{2} \times 4^2 \times \sin 97.2^\circ$   
 $= 7.94 \text{ cm}^2$

Area of sector  $AXY = \frac{97.2}{360} \times \pi \times 4^2$

$= 13.57 \text{ cm}^2$

Area of shaded part  $= 13.57 - 7.94 = 5.63 \text{ cm}^2$

Area of  $\Delta BXY = \frac{1}{2} \times 6^2 \sin 30^\circ$   
 $= 9 \text{ cm}^2$

Area of sector  $BXY = \frac{30}{360} \times \pi \times 6^2$   
 $= 9.42 \text{ cm}^2$

Area of shaded part  
 $= (9.42 - 9) \text{ cm}^2 = 0.42 \text{ cm}^2$

Area of shaded region  $= (5.63 + 42) \text{ cm}^2 = 6.05 \text{ cm}^2$

2. (i)  $\angle AOB = 2 \angle ACB$   
 $= 100^\circ$

$$\begin{aligned}\angle OAB &= \frac{180 - 100}{2} \text{ Base angles of Isosceles } \Delta \\ &= 40^\circ\end{aligned}$$

$$\begin{aligned}(ii) \angle ADC &= 180^\circ - 70^\circ \\ &= 110^\circ\end{aligned}$$

$$\begin{aligned}3. \quad \frac{2}{5} \div \frac{1}{2} 0f^4/9 - 1^l/10 \\ &= \frac{2}{5} \div \frac{1}{2} X^4/9 - 1^l/10 \\ &= \frac{2}{5} x^{9/2} - 1^l/10 \\ &= \frac{9}{5} - 1^l/10 = 18 - 1^l/10 = 7/10\end{aligned}$$

$$\begin{aligned}\frac{1}{8} - \frac{1}{6} X^3/8 &= \frac{1}{8} - \frac{1}{16} \\ &= 2 - \frac{1}{16} = \frac{1}{16}\end{aligned}$$

$$\begin{aligned}\frac{\frac{2}{5} \div \frac{1}{2} 0f^4/9 - 1^l/10}{\frac{1}{8} - \frac{1}{6} of^3/8} &= \frac{7/10}{1/16}\end{aligned}$$

$$= \frac{7}{10} X^{16/1}$$

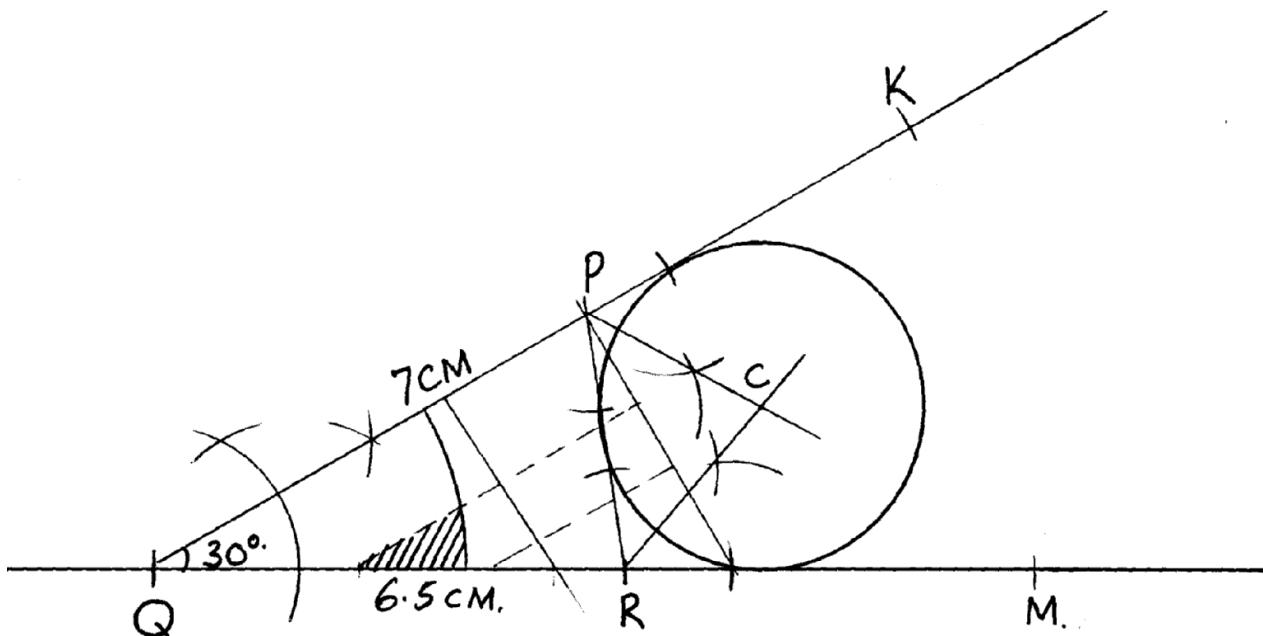
$$= 56/5 = 11^l/5$$

$$4. \quad a) \angle DAC = \angle DCA = \frac{1}{2} (180 - 100) \text{ (base angles)} = 40^\circ$$

$$(b) \angle BAC = \angle DCA \text{ alt, } \angle s AB \parallel AD \\ = 40^\circ$$

$$(b) \angle DAB = \angle DAC + \angle BAC = 40 + 40 = 80^\circ$$

$$\begin{aligned}\angle BCD &= 180^\circ - 80^\circ \\ &= 100^\circ\end{aligned}$$



$$5. \quad c) (ii) \text{ Radius} = 2.3 \pm 0.1 \text{ cm} \\ \text{Name of } \triangle PQR : \text{ Escribed circle}$$

6.
  - (i)  $\angle ACB = 10^\circ$  ( $\angle$ s subtended by chord AB)
  - (ii)  $\angle AOD = 160^\circ$  ( $\angle$  at centre line at circumference)
  - (iii)  $\angle CAB = 40^\circ$  ( $\angle$ s subtended by chord AB)

(iv)  $\angle ABC = 130^\circ$  ( Opposite  $\angle$ s of cyclic quadrilateral)(v)  $\angle AXB = 60^\circ$  (sum angle of triangle)

7. i)  $\frac{80}{360} \times \frac{22}{7} \times 9 \times 9$   
 $= 63.6429 \text{ cm}^2$

ii)  $\frac{1}{2} ab \sin C$   
 $= \frac{1}{2} \times 9 \times 9 \sin 80^\circ$   
 $= 39.8847 \text{ cm}^2$

iii)  $\frac{180}{360} \times \frac{22}{7} \times 9 \times 9$   
 $= 127.2857 \text{ cm}^2$

Segment:  $63.6429 - 39.8847$   
 $= 23.7582 \times 2 = 47.5164 \text{ cm}^2$   
 $\therefore 127.2857 - 47.5164$   
 $= 79.7693 \text{ cm}^2 = 79.77 \text{ cm}^2$

8. (a)  $\angle RST = 180^\circ - 46^\circ$  Opposite angel in cyclic quadrilateral  
 $= 134^\circ$
- (b)  $\angle SUT = 180^\circ - 46^\circ - 27^\circ$  (Sum of angles in a traingle QRU)  
 $= 180^\circ - 173^\circ = 7^\circ$
- (c)  $\angle ROT = 2 \times 46^\circ$  (angle substended by chord RT at the centre  
 $= 92^\circ$
- (d)  $\angle PST = 180^\circ - 37^\circ - 48^\circ - 53^\circ$   
Sum of angles in a triangle PST
- (e) Reflex  $\angle SOP = (2 \times 37^\circ) + 2 \times 42^\circ = 158^\circ$   
Angle subtended chord at centres is twice angle at circle

9.  $\angle POQ = 80^\circ$   
Radius =  $\frac{1}{2}$   
 $\sin 40^\circ = 2.645 \text{ cm}$   
Area of the triangle =  $\frac{1}{2} \times 2.645^2 \sin 80^\circ = 3.445 \text{ cm}^2$   
Area of the sector =  $(\frac{80}{360} \times \pi \times 2.645^2) = 4.884 \text{ cm}^2$   
Area of the shaded segment =  $(4.884 - 3.445) = 1.439 \text{ cm}^2$

10. a)  $\angle BDC = 90^\circ - 33^\circ = 57^\circ$ , 3<sup>rd</sup> angle of  
 $= 57^\circ \Delta BCD$ ,  $\angle BCD = 90^\circ$   
 $\angle ADC = \angle ADB + \angle BDC$   
 $= 48^\circ + 57^\circ = 105^\circ$

- b) Consider  $\Delta BCE$   
 $\angle AEB$  is an exterior opposite angle  
 $\therefore \angle AEB = 33^\circ + 48^\circ = 81^\circ$

## 38. Vectors

<b>1</b>	$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	M <sub>1</sub> A <sub>1</sub>	✓ exp
<b>2</b>	<p>(a) (i) <math>4p - 3q = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \times 1</math></p> $P + 2q = \begin{pmatrix} -14 \\ 15 \end{pmatrix} \times 4$ $4p - 3q = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$ $4p + 8q = \begin{pmatrix} -56 \\ 60 \end{pmatrix}$ $-11q = \begin{pmatrix} 66 \\ -55 \end{pmatrix}$ $q = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$ $p + 2\begin{pmatrix} -6 \\ 5 \end{pmatrix} = \begin{pmatrix} -14 \\ 15 \end{pmatrix}$ $p + \begin{pmatrix} -12 \\ 10 \end{pmatrix} = \begin{pmatrix} -14 \\ 15 \end{pmatrix}$ $p = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $q = \begin{pmatrix} -6 \\ 5 \end{pmatrix} \text{ and } p = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ <p>(ii) <math> p + 2q </math></p> $= \begin{pmatrix} -2 & -12 \\ 5 & 10 \end{pmatrix}$ $\begin{pmatrix} -14 \\ 15 \end{pmatrix} = \sqrt{(-14)^2 + (15)^2} = \sqrt{196 + 225} = \sqrt{421} = 20.52$ <p>(b) <math>\vec{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}</math></p> $\vec{BC} = \begin{pmatrix} 11 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $AB = kBC$ $AB = 1BC$ <p>B (3, 5) is common</p> <p>AB is a scalar multiple of BC. Hence A (1, -1), B (3, 5) and C (5, 11) are collinear</p>	M <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub> B <sub>1</sub> B <sub>1</sub> B <sub>1</sub> A <sub>1</sub>	Scalar 1  Correct pt B

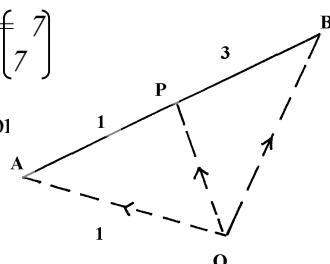
<b>3</b> i) $\begin{aligned} P &= 2 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \end{aligned}$ ii) $\begin{aligned}  P  &= \sqrt{9+1+4} \\ &= \sqrt{14} = 3.742 \end{aligned}$	M1 A1 B1	
		3
<b>4.</b> $\begin{aligned} PQ &= \begin{pmatrix} -2 \\ -1 \end{pmatrix} - \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \\ QR &= \begin{pmatrix} 6 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ 2 \sim &= \sim \text{ multiples of each other} \\ Q \text{ is common point hence } PQ \text{ and } R \text{ are collinear} \end{aligned}$	B1 B1 B1	
		03

1.

$$\begin{aligned} \sin 60^\circ &= \sqrt{3}/2 & 1 \\ \sin 45^\circ &= 1/\sqrt{2} = \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{2}} & - 1 \\ &= \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}}} = \frac{1}{\frac{\sqrt{2}}{2}} & - 1 \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6}}{4} - \frac{2\sqrt{2}}{4} \end{aligned}$$

2.

$$\begin{aligned} OP &= OA + \frac{1}{4} AB \\ &\approx OA + \frac{1}{4} (QB - QA) \\ &= QA + \frac{1}{4} QB - \frac{1}{4} QA \\ &= \frac{3}{4} QA + \frac{1}{4} QB \\ &= \frac{3}{4} \underbrace{QA}_{\text{ }} + \frac{1}{4} \underbrace{QB}_{\text{ }} \\ &= \frac{3}{4} \begin{pmatrix} 12 \\ 8 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 16 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix} \end{aligned}$$



$$\begin{array}{l}
 3. \quad m \begin{matrix} 4 \\ 3 \end{matrix} + n \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \\
 4m - 3n = 5 \dots\dots\dots (i) \times 2 \\
 3m + 2n = 8 \dots\dots\dots (ii) \times 2 \\
 8m - 6n = 10 \\
 \underline{9m + 6n = 24} \\
 17m = 34 \\
 m = 2 \\
 4 \times 2 - 3n = 5 \\
 -3n = -3 \\
 n = 1 \\
 \therefore m = 2, n = 1
 \end{array}$$

$$\begin{array}{l}
 4. \quad (a) (i) BM = \frac{\underline{2}a - b}{5} = \frac{1}{5}(2a - 5b) \\
 \quad \quad \quad (ii) AN = \frac{\underline{2}b - a}{3} = \frac{1}{3}(2b - 3a) \\
 \quad \quad \quad (b) \quad \quad \quad BX = \frac{\underline{t}}{5}(2a - 5b) \\
 \quad \quad \quad AX = \frac{\underline{h}}{3}(2b - 3a) \\
 \quad \quad \quad OX_I = OB + BX = b + t \frac{(\underline{2}a - 5b)}{5} \\
 \quad \quad \quad = (-t)b + \frac{\underline{2}}{5}a + a \\
 \quad \quad \quad OX = OA + AX = a + h(2b - 3a) \\
 \quad \quad \quad = (1-h)a + \frac{\underline{2}hb}{3} \\
 \quad \quad \quad (c) OX_I = OX_2 \\
 \quad \quad \quad \frac{\underline{2}}{5}a + a + \frac{(1-t)b}{3} = (1-h)a + 2hb \\
 \quad \quad \quad \frac{\underline{2}t}{5} = 1-h \dots\dots (i) \\
 \quad \quad \quad (1-t) = \frac{3}{4}h \dots\dots (ii) \quad t = \frac{5-5h}{2} \\
 \quad \quad \quad 1 - \frac{(5-5h)}{2} = \frac{\underline{2}h}{3} = 11h = 9 \\
 \quad \quad \quad h = \frac{9}{11} \\
 \quad \quad \quad t = \frac{5-5}{2} \left[ \frac{9}{11} \right] = \frac{5}{11} \\
 \quad \quad \quad (i) BX : XM = 1:10
 \end{array}$$

$$(ii) AX: XN = 3:8$$

5. a) i)  $MA = \frac{1}{2}a$

ii)  $AB = a$

iii)  $AC = a + c$

iv)  $AX = \frac{2}{7}AC = \frac{2}{7}(-a + c)$

b)  $MA = \frac{1}{2}a$

$$AX = \frac{2}{7}c - \frac{2}{7}a$$

$$\begin{aligned} MX &= \frac{1}{2}a + \frac{2}{7} - \frac{2}{7}a \\ &= \frac{3}{14}a + \frac{2}{7}c \end{aligned}$$

$$\begin{aligned} \text{Co-ordinates of } P &= \left( \frac{1+3}{2}, \frac{6+0}{2}, \frac{8+4}{2} \right) \\ &= (2, 3, 6) \end{aligned}$$

$$\begin{aligned} |OP| &= \sqrt{2^2 + 3^2 + 6^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} = 7 \text{ units} \end{aligned}$$

c) Co-ordinates of O (0,0,0)

Co-ordinates of A (1, 6, 8)

$$\begin{aligned} \text{Mid points of } AO &= \left( \frac{1+0}{2}, \frac{6+0}{2}, \frac{8+0}{2} \right) \\ &= (0.5, 3, 4) \end{aligned}$$

6. a)  $AB = DC \Rightarrow 1-x = 2 \Rightarrow x = -1$   
 $6-y = 4 \Rightarrow y = 2$   
 $\therefore D = (-1, 2)$

b) (i)  $\overrightarrow{RQ} = \underset{\sim}{Q} \left[ R \underset{\sim}{=} q - \frac{3}{2} \underset{\sim}{q} - \frac{1}{2} \underset{\sim}{p} \right]$   
 $\quad \quad \quad \left[ -\frac{1}{2} \underset{\sim}{q} \right] \underset{\sim}{-} p \left[ = \frac{1}{2} \underset{\sim}{p} - q \right] \quad \checkmark$

$$\begin{aligned} (ii) \overrightarrow{PR} &= \underset{\sim}{q} - \underset{\sim}{p} - P \checkmark \\ &= \frac{\underset{\sim}{q} - \underset{\sim}{p}}{\underset{\sim}{k}} \\ \Rightarrow k &= -3 \quad \text{Also } \underset{\sim}{-} \frac{3}{2} \underset{\sim}{p} = \frac{1}{2} \underset{\sim}{kp} \\ \Rightarrow k &= -3 \quad \Rightarrow k = -3 \end{aligned}$$

Hence P, Q, R, Q Collinear.

(iii)  $\overrightarrow{PQ} = q - p, \underset{\sim}{QR} \underset{\sim}{=} \frac{1}{2}(q - P)$

$PQ : QR = 2 : 1$

$$\begin{aligned} 7. \quad (a) \quad PQ &= PO + OQ = -p + q \\ Or &= OP + PR = P + 2/3 PQ \\ &= P + 2/3 (-p + q) \\ &= 1/3p + 2/3q \end{aligned}$$

$$\begin{aligned} QT &= QO + OT = -q + 1/2 OR \text{ since } OT = TR \\ &= -q + 1/2 (1/3p + 2/3q) \\ &= 1/6p - 2/3q \text{ OR } 1/6 (p - 4q) \end{aligned}$$

$$\begin{aligned} (b) \quad TS &= TO + OS = -1/2 OR + 1/4 OP \\ &= -1/2 (1/3p + 2/3q) + 1/4 p = 1/6p - 1/3q + 1/4 p \\ &= 1/12p - 1/3q \text{ or } 1/12(p - 4q) \end{aligned}$$

$QT: TS = 1/6(p - 4q): 1/12(p - 4q) = 1/6: 1/12 = 2: 1$   
 $\therefore QT = 2TS$   $OT//TS$  but T is a common point hence Q, T, S are collinear

(c) Vector OT can be expressed in 2 ways

$$\begin{aligned} \text{1st } OT &= 1/2 OR \text{ given} \\ &= 1/2 (1/3 P + 2/3q) = 1/6q + 1/3q \dots\dots\dots(i) \end{aligned}$$

2nd using OPT

$$\begin{aligned} OT &= OP + PT = P + 5/6PM \\ \text{But } PM &= PO + OM = -P + KOQ = -P + Kq \\ OT &= P + 5/6 (-P + Kq) \\ &= P - 5/6Kq \\ &= 1/6p + n^5/5kq \dots\dots\dots(ii) \end{aligned}$$

Aqn (i) and (ii) represent the same vector OT

$$1/6p + 1/3q = 1/6p + 5/6kq \dots\dots\dots(iii)$$

Comparing coefficients of q in eqn (iii) have  $5/6k = 1/3$   
 $15k = 6$

$$\begin{aligned} 8. \quad 3a &= 3(-3) \\ &\quad \quad \quad \quad 2 \quad \quad \quad \quad 6 \\ \frac{1}{2}b &= \frac{1}{2}(4) \\ &\quad \quad \quad \quad -6 \quad \quad \quad \quad -3 \\ \frac{1}{10}c &= \frac{1}{10}(5) \\ &\quad \quad \quad \quad -10 \quad \quad \quad \quad -1 \\ P &= (-9) \quad - (2) \quad \quad + (0.5) \\ &\quad \quad \quad \quad 6 \quad \quad \quad \quad -3 \quad \quad \quad \quad -1 \\ &= (-10.5) \\ &\quad \quad \quad \quad 8 \\ |P| &= \sqrt{(-10.5)^2 + 8^2} \\ &= \sqrt{110.25} = 64 \\ &= \sqrt{174.25} \\ &= 13.20037878 \\ &= 13.20 \text{ (2 d.p)} \end{aligned}$$

9. (i)  $BM = BO + OM$

$$= \frac{2}{5}a - b$$

(ii)  $AN = AO + ON$

$$= \frac{2}{3}b - a$$

(b)  $OX = OB + BX$

$$= b + k(2a - b)$$

$$\sim = \frac{2}{5}ka + b(1-k)$$

$$OX = OA + AX$$

$$= a + h(\frac{2}{3}b - a)$$

$$= a(1-h) + 2hb$$

$$= a(10h) 2hb$$

(c)  $\frac{2}{5}a = a(1-h)$  also  $b(1-k) = 2hb$

$$2k = 1-h \quad 1-k = 2h$$

$$k = \frac{5}{2} - \frac{5}{2}h$$

$$\therefore 1 - \frac{5}{2} + \frac{5}{2}h = \frac{2}{3}h$$

$$\frac{5}{2}h - \frac{2}{3}h = \frac{5}{2} - 1$$

$$1 \frac{5}{6}h = \frac{3}{2}$$

$$h = \frac{3}{2} \times \frac{6}{11} = 9$$

$$k = \frac{5}{2} - \frac{5}{2} \times \frac{9}{11}$$

$$= \frac{5}{2} - \frac{45}{22}$$

$$= \frac{5}{11}$$

10. (i)  $AN = AO + ON$

$$= -a + \frac{4}{5}b$$

(ii)  $\underset{\sim}{BM} = BO + OM$

$$\underset{\sim}{=} -b + \frac{2}{5}a$$

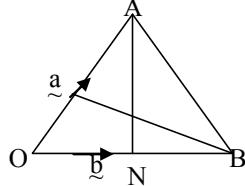
(iii)  $AB = AQ + QB$

$$\underset{\sim}{=} -a + b$$

$$\underset{\sim}{AX} = \underset{\sim}{AN}$$

$$\underset{\sim}{BX} = \underset{\sim}{tBM}$$

$$\underset{\sim}{OX} = \underset{\sim}{OB} + \underset{\sim}{BX}$$



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$$= \underline{\underline{b}} + tBM$$

$$= \underline{\underline{b}} + t(\underline{\underline{-b}} + \frac{2}{5}a)$$

$$= \underline{\underline{b}} - \underline{\underline{tb}} + \frac{2}{5}\underline{\underline{ta}}$$
$$= \underline{\underline{b}}(1-t) + \frac{2}{5}\underline{\underline{ta}}$$

$$OX \underset{\sim}{=} OA \pm AX$$

$$= \underline{\underline{a}} + sAN$$

$$= \underline{\underline{a}} + s(\underline{\underline{-a}} + \frac{4}{5}b)$$

$$= \underline{\underline{a}} - \underline{\underline{Sa}} + \frac{4}{5}s\underline{\underline{b}}$$
$$\underline{\underline{a}}(1-s) + \frac{4}{5}s\underline{\underline{b}}$$

$$b(1-t) + \frac{2}{5}ta = a(1-s) \quad \frac{4}{5}sb$$
$$b(1-t) = \frac{4}{5}sb$$

$$1-t = \frac{4}{5}s \quad \text{(i)}$$

$$a(1-s) = \frac{2}{5}ta$$

$$1-s = \frac{2}{5}ta$$

$$s = 1 - \frac{2}{5}t \quad \text{(ii)}$$

$$1-t = \frac{4}{5}(1 - \frac{2}{5}t)$$

$$1-t = \frac{4}{5} - \frac{8}{25}t$$

$$-\frac{17}{25}t = -\frac{1}{5}$$

$$t = \frac{5}{17}$$

$$s = \frac{15}{17}$$

$$\sim \quad \sim \quad \sim \quad \sim$$

$$11. \quad \frac{115800}{76.84} \times \frac{97.5}{100}$$

$$= 1469.35 \checkmark$$

$$= 1469.35 - 270$$

$$= 1199.35 \checkmark$$

$$= 1199 \text{ dollars}$$

12.

$$RM = \begin{bmatrix} -2 \\ 6 \\ 7 \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ -1 \end{bmatrix}$$

$$|RM| = \sqrt{(-3)^2 + 8^2 + (-1)^2}$$

$$74 = 8.602 \text{ units}$$

13.

$$(a) (i) \underline{\underline{OB}} \underset{\sim}{=} a \pm b$$
$$(ii) \underline{\underline{BC}} = \underline{\underline{BA}} + \underline{\underline{AO}} + \underline{\underline{QC}}$$
$$= \underline{\underline{-b}} + \underline{\underline{-a}} + \underline{\underline{2b}}$$
$$= \underline{\underline{b}} - \underline{\underline{a}}$$

$$(b) CX \underset{\sim}{=} CO \pm OA \pm AB \pm BX$$

$$\begin{aligned}
 &= -2b + a + b + hBC \\
 &= a - b + h(b - a) \\
 &= a - b + hb - ha \\
 &= (1 - h)a + (h - 1)b
 \end{aligned}$$

$$(c) CX = CO + OA + AX$$

$$= 2b + a + KAT$$

$$\text{but } AT = AO + OT$$

$$= -a + 3b$$

$$CX = 2b + a + K(3b - a)$$

$$= a - Ka + 3Kb + 2b$$

$$= (1 - K)a + 3(K + 2)b$$

$$(d) I - h = 1 - k \dots\dots\dots (i)$$

$$h - 1 = 3k + 2 \dots\dots\dots (ii)$$

from (i)  $h = k$

$$\text{sub in (ii)} \quad h - 1 = 3h + 2$$

$$h = -\frac{1}{2}$$

$$K = \frac{3}{2}$$

$$\begin{aligned}
 14. \quad a + b &= (2 - 3)i + (1 + 4)j + (-2 - 1)k \\
 &= -i + 5j - 3k
 \end{aligned}$$

$$\begin{aligned}
 |a + b| &= \sqrt{(-1)^2 + (5)^2 + (-3)^2} \\
 &= \sqrt{35} \\
 &= 5.916
 \end{aligned}$$

$$15. \quad i) BD = BA + AD$$

$$= -b + \frac{3}{5}c$$

$$AE = AB + BE$$

$$= b + \frac{1}{2}BC = b + \frac{1}{2}(c - b)$$

$$= \frac{1}{2}b + \frac{1}{2}c$$

$$ii) BF = t(\frac{3}{5}c - b)$$

$$AF = n(\frac{1}{2}b + \frac{1}{2}c) = \frac{n}{2}(b + c)$$

$$AF = AB + BF$$

$$= b + t(\frac{3}{5}c - b) = b + \frac{3}{5}tc + tb$$

$$= (1 - t)b + \frac{3}{5}tc$$

$$(1 - t)b + \frac{3}{5}tc = \frac{n}{2}b + \frac{n}{2}c$$

$$1 - t = \frac{n}{2}; 2 - 2t = n \dots\dots\dots (i)$$

$$\frac{3}{5}t = \frac{n}{2}; 6t - 5n = 0 \dots\dots\dots (ii)$$

Sub from équation (ii)

$$6t - 5(2 - 2t) = 0$$

$$6t - 10 + 10t = 0$$

$$16t = 10$$

$$t = \frac{10}{16} = \frac{5}{8}$$

$$n = \frac{3}{4}$$

$$iii) BF = \frac{5}{8} BD$$

*F divides BD in the ratio 5 : 3*

$$AF = \frac{3}{4} AE$$

*F divides AE in the ratio 3 : 1*

16.  $BA = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$

$$\frac{1}{2} BC = \frac{1}{4} \begin{bmatrix} -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \frac{1}{2} \\ -2 \end{bmatrix}$$

$$OP = \begin{bmatrix} -8 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} \frac{1}{2} = \begin{bmatrix} -9 \frac{1}{2} \\ -4 \end{bmatrix}$$

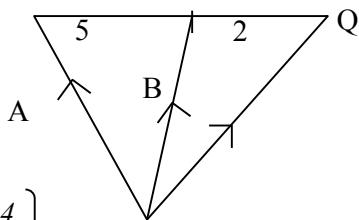
*Co-ordinates of P (-9 1/2, -4)*

—

17.  $OB = \frac{5}{7} OQ + \frac{2}{5} OA$

$$OQ = \frac{7}{5} OB - \frac{2}{5} OA$$

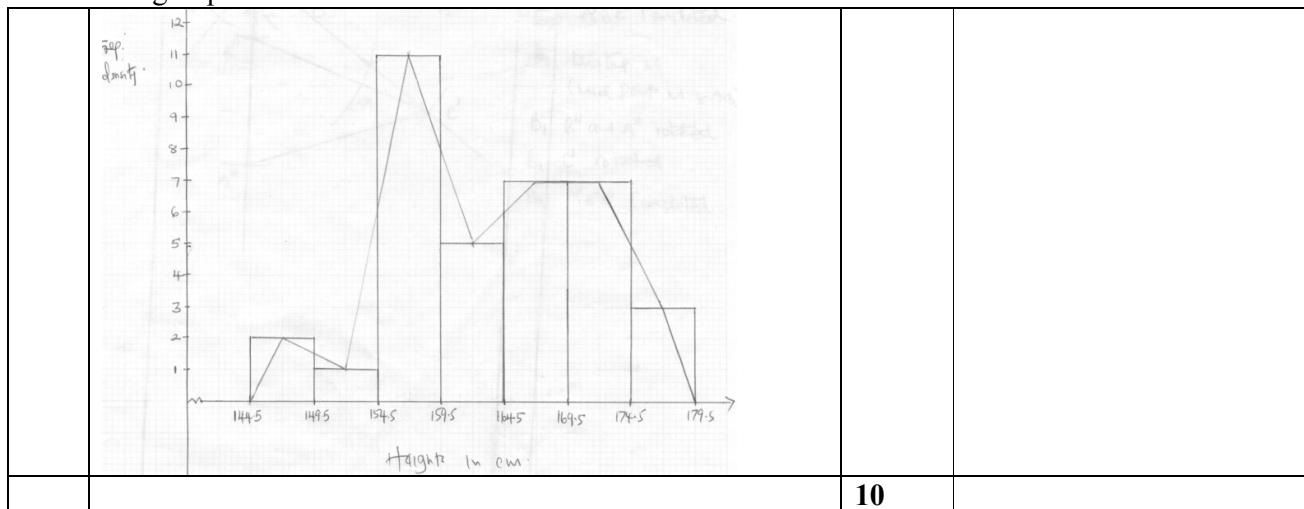
$$OQ = \frac{7}{5} \begin{bmatrix} 2 \\ -1 \end{bmatrix} - \frac{2}{5} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 14/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 20/5 \\ -15/5 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$



$$Q = (4, -3)$$

### 39. Representation of data

1.	(a) <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Class</th><th>Tally mark</th><th>Freq</th><th>Freq D</th></tr> </thead> <tbody> <tr> <td>145-149</td><td>//</td><td>2</td><td>2</td></tr> <tr> <td>150-154</td><td>/</td><td>1</td><td>1</td></tr> <tr> <td>155-159</td><td>     </td><td>11</td><td>11</td></tr> <tr> <td>169-164</td><td>   </td><td>5</td><td>5</td></tr> <tr> <td>165-169</td><td>    //</td><td>7</td><td>7</td></tr> <tr> <td>170-174</td><td>    //</td><td>7</td><td>7</td></tr> <tr> <td>175-179</td><td>///</td><td>3</td><td>3</td></tr> </tbody> </table>	Class	Tally mark	Freq	Freq D	145-149	//	2	2	150-154	/	1	1	155-159		11	11	169-164		5	5	165-169	//	7	7	170-174	//	7	7	175-179	///	3	3	<b>B1</b> Classes <b>B1</b> Tally mark column <b>B1</b> Freq. column <b>B1</b> Freq density column (can be implied) <b>B1</b> Freq. density (y axis) <b>B1</b> Height (x axis) <b>B1</b> Correct spacing as per scale <b>B1</b> Histogram drawn (bars) <b>B1</b> Joining mid point of the bars <b>B1</b> Joining mid point of first class to 144.5 <b>B1</b> Joining mid point of last class to 179.5
Class	Tally mark	Freq	Freq D																															
145-149	//	2	2																															
150-154	/	1	1																															
155-159		11	11																															
169-164		5	5																															
165-169	//	7	7																															
170-174	//	7	7																															
175-179	///	3	3																															



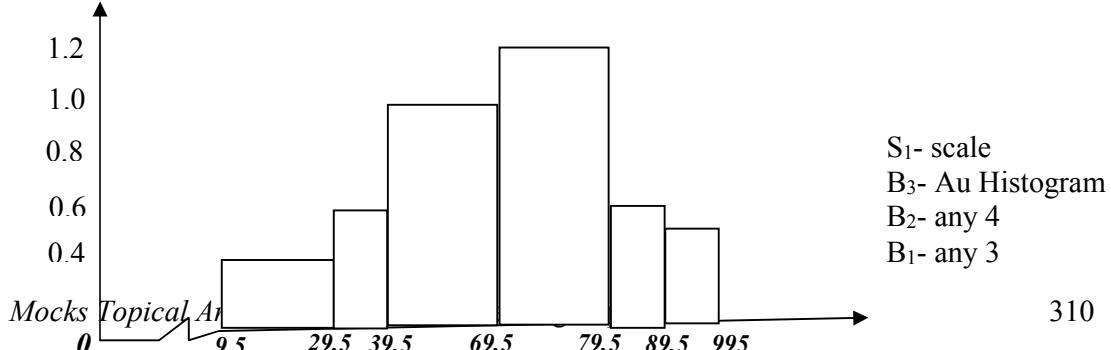
1.

Length	Frequency
$11.5 \leq x \leq 13.5$	6
$13.5 \leq x \leq 15.5$	9
$15.5 \leq x \leq 17.5$	6
$17.5 \leq x \leq 23.5$	3

2. Food:  $\frac{40}{100} \times 360 = 144^\circ$   
 Transport:  $\frac{10}{100} \times 360 = 36^\circ$   
 Education:  $\frac{20}{100} \times 360 = 72^\circ$   
 Clothing:  $\frac{20}{100} \times 360 = 72^\circ$   
 Rent:  $\frac{10}{100} \times 360 = 36^\circ$

Class	Tally	Frequency	Upper Limit
10 - 29		8	29.5 B <sub>2</sub> for
30 - 39		6	39.5 all tally
40 - 69		28	69.5 B <sub>2</sub> all
70 - 74	/   /   /   /   /	6	74.5 - frequency
75 - 89		8	89.5 - B <sub>1</sub>
90 - 99		4	99.5 B <sub>1</sub>

$\swarrow$   
*Modal class 40 - 69 B<sub>1</sub>*



0.2

|

4. See the graph paper.  
 For correct class boundaries  
 For correct class intervals.  
 All frequency densities

Correct scale  
 All the bars drawn.

Top mid pts. Of bars indicated.  
 For the mid pts. Joint to make a polygon.  
 For correctly identifying the modal mark point.  
 For reading correctly the modal mark  $\equiv 53.5 \pm 0.1$

5. (a)

Marks	Frequency
5-9	20
10-19	50
20-30	40
40-49	30

(b) Modal class is 10-19

(c)(i)

Class	x	f	fx	Cf
5-9	7	20	140	20
10-19	14.5	50	725	70
20-30	29.5	40	1180	110
40-49	44.5	30	1335	140
		$\Sigma F = 140$	$\Sigma Fx = 3380$	

$$x = \frac{\Sigma fx}{\Sigma f} = \frac{3380}{140} = 24.14$$

(ii) Median mark is at  $70 + 71 = 70.5^{\text{th}}$  position

$$\begin{aligned} \text{Median} &= 119.5 + \frac{(0.5)}{40} \times 20 \\ &= 19.5 + 0.25 \\ &= 19.75 \end{aligned}$$

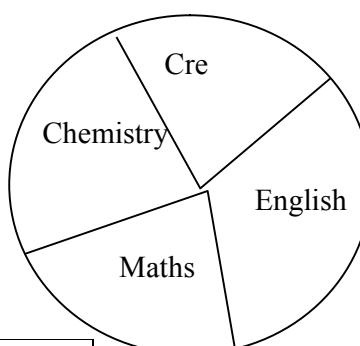
6. Total No. of sessions  
 $= 8 + 7 + 4 + 3 = 22$

Angle for:

$$\begin{aligned} \text{English} &= \frac{8}{22} \times 360 = 130.9^{\circ} \\ \text{Maths} &= \frac{7}{22} \times 360 = 114.5^{\circ} \\ \text{Chemistry} &= \frac{4}{22} \times 360 = 65.5^{\circ} \\ \text{CRE} &= \frac{3}{22} \times 360 = 49.01^{\circ} \end{aligned}$$

7.  $180 - 189$   
 Class limits

class	limits	f	cf



149.5	159.5	2	2
159.5	169.5	9	11
169.5	179.5	12	23
179.5	189.5	16	39
189.5	199.5	7	46
199.5	209.5	4	50

$$\text{Median} = \frac{50}{2} = 25$$

$$\frac{179.5 + \underline{25 - 23} \times 10}{16}$$

$$= 179.5 + \frac{\underline{20}}{16} = 180.75$$

$$\frac{179.5 + \underline{26 - 23} \times 10}{16}$$

$$\frac{179.5 + \underline{30}}{16} = 181.38$$

16

$$\frac{180.75 + 181.38}{2}$$

$$= 181.06$$

8. a) i) 145 – 153

ii) Median class

$$(\frac{40+1}{2})^{\text{th}} \text{ value} \therefore \text{median class} = 145 - 153$$

This is the 20.5<sup>th</sup> value

The value also in the 145 – 153 class

b)

Class	x	f	fx
118- 126	122	3	366
127- 135	131	4	524
136 – 144	140 B1	10 B2	1400
145 – 153	149	12	1788
154 – 162	158	5	790
163 – 171	167	4	668
172 - 180	176	2	352
		$Ef = 40$	$Efx = 5888$

B2 for all values of fx correct and B1 for 4 values of fx and above orrect

$$\text{Mean} = Efx = 5888 = 147.2 \text{ mm}$$

$$\frac{Ef}{40}$$

$$\text{Median } 20^{\text{th}} = 144.5 + (\frac{11}{12} \times 9) = 152.75$$

$$21^{\text{st}} = 144.5 + (\frac{12}{12} \times 9) = 153.5$$

$$\text{Median} = \frac{152.75 + 153.5}{2} = 153.125$$

(Alternatively one could work out the 20.5 value directly using median formula)

## 40. Measures of central tendency

$$1. \quad 4 + 6 + 10 + 14 + x + 24 + 14 + 6 = 100$$

$$78 + x = 100$$

$$(i) \quad x = 22$$

$$(ii) \quad \text{Modal class} = 55 - 59$$

Marks	x	f	fx	cf
-------	---	---	----	----

30-34	32	4	128	4
35-39	37	6	222	10
40-44	42	10	420	20
45-49	47	14	659	34
50-54	52	22	1144	56
55-59	57	24	1368	80
60-64	62	14	868	94
65-69	67	6	462	100
$B_I$		$\sum f = 100$	$\sum fx = 5210$	$B_I$
$B_I$				

$$\sum fx = 5210$$

$$(i) \quad \text{Mean} = \frac{5210}{100}$$

$$= 52.10$$

$$(ii) \text{Median} = 49.5 + \left[ \frac{50-34}{22} \right] \times 5$$

$$= 53.14$$

2.  $\log_{10} 5^2 - \log_{10} 2^3 + \log 2^5$

$$\log_{10} \left( \frac{\cancel{25} \times \cancel{32}}{\cancel{8}} \right)$$

$$1 \qquad \qquad \qquad 2$$

$$\log_{10} 100 = \log_{10} 10^2$$

$$= 2 \log_{10} 10$$

$$\text{But } \log_{10} 10 = 1$$

$$\therefore = 2$$

3. Modal class 150-154

✓ Application of logarithmic laws.  
✓ Application  
C.A.O

Height	Frequency	c.f
140-144	3	3
145-149	15	18
150-154	19	37
155-159	11	48
160-164	2	50

$$\begin{aligned} & \text{Height} \quad \text{Frequency} \quad c.f \\ & = 149.5 + \left( \frac{25-18}{19} \right) \times 5 \\ & \qquad \qquad \qquad 19 \\ & = 149.5 + \frac{7}{19} \times 5 \\ & \qquad \qquad \qquad 19 \\ & = 149.5 + 1.842 \\ & = 15.34 \end{aligned}$$

4.

H	20-24	25-29	30-34	35-39	40-44	45-49
F	3	19	25	20	18	15
CF	3	22	47	67	85	100

$$\begin{aligned} & Md = 34.5 + \left( \frac{50-47}{20} \right) \times 4 \\ & \qquad \qquad \qquad 20 \\ & = 34.5 + 12/20 = 35.1 \end{aligned}$$

5. a)  $2x^2 + 6x - 2x = 0$   
 $32 - 24 - 2x = 0$   
 $-2x = -8$   
 $x = 4$

b)  $2x^2 + 6x - 8 = 0$   
 $x^2 + 3x - 4 = 0$   
 $x^2 + 4x - x - 4 = 0$   
 $x(x - 4) - (x + 4) = 0$   
 $(x - 1)(x + 4) = 0$   
 $\therefore \text{the other root is } 1$

6.  $\sum xf = 61x10 + 65.5x20 + 71x40 + 77x15$   
 $= 610 + 1310 + 2840 + 1155$   
 $= 5915$   
 $\frac{\sum xf}{\sum f} = \frac{5915}{85}$   
 $X \text{ Mean} = 69.59$

7.

Marks	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of candidates	2	3	10	12	8	3	2
C.F	2	5	15	27	35	38	40

- a) Number who sat = 40  
b) The modal class = 60 – 69  
c)

Marks	x	f	$X - 64.5 = d$	fd
30-39	34.5	2	-30	-60
40-49	44.5	3	-20	-60
50-59	54.5	10	-10	-100
60-69	64.5	12	0	0
70-79	74.5	8	10	80
80-89	84.5	3	20	60
90-99	94.5	2	30	60
		$\sum f = 40$		$\sum fd = -20$

$$\text{Mean} = 64.5 + \frac{-20}{40} \\ = 64.0$$

d) The median mark  
 $= \frac{1}{2} (20^{\text{th}} \text{ and } 21^{\text{st}}) \text{ marks}$   
 $= \frac{1}{2} (59.5 + \frac{5}{12}x10 + 59.5 + \frac{6}{12}x10)$   
 $= \frac{1}{2} (59.5 + 4.16666 + 59.5 + 5)$   
 $= \frac{1}{2} (128.1666667) = 64.083$

8. 1, 1, 2, 2, 3, 4, 4, 6

a) Mode = 4  
b) Median = 3  
c) Mean =  $\frac{1x2 + 2x2 + 3x1 + 4x3 + 6x1}{9} = 3$

9. a) i) Modal class = 60 – 69

ii) class where medium lies  
median class 50- 59

Class	Centre X	Fd	$D = x - A$
0 - 9	4.5	-50	-50
10 - 19	14.5	-80	-40
20 - 29	24.5	-120	-30
30 - 39	34.5	-140	-20
40 - 49	44.5	-100	-10
50 - 59	54.5	0	0
60 - 69	64.5	200	10
70 - 79	74.5	120	20
80 - 89	84.5	90	30
90 - 99	94.5	40	40
		$\sum fd$	-40

$$\text{Mean} = 54.5 - \frac{40}{70} \\ = 53.93$$

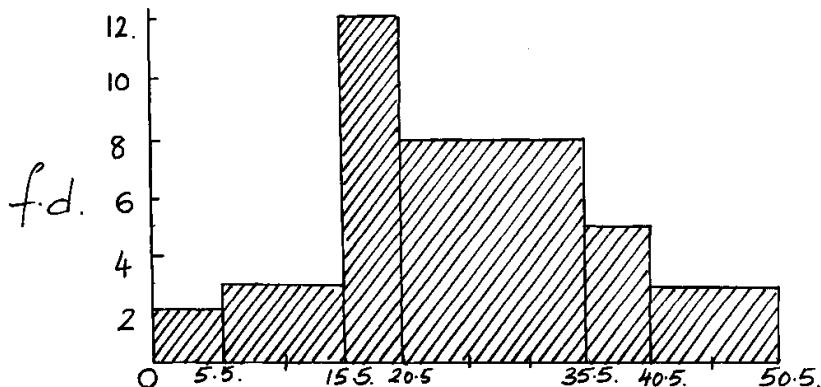
10. Cumulative frequency

3, 11, 30, 44, 50

$$\text{Median} = L1t \left( \frac{n/2 - cfa}{Fn} \right) \\ = 8 + \frac{(25 - 11)}{19} \times 4$$

$$= 10.947$$

11.



#### 41. Linear motion

1. Distance covered by Kinyua in  $1\frac{2}{3}$  hrs  
 $= 5 \times 90 = 150\text{km}$

Distance traveled by Nyaboke during the rest  $= (\frac{1}{3} \times 120) = 40\text{km}$   
 $x = \frac{390 - x}{90} \Rightarrow 120x = 90(390 - x)$

$$= 167.1\text{km}$$

$$\text{Time} = \frac{167.1}{90} = 1.86$$

$$8.33 + 1.86 = 10.19; \text{ they met at } = 10.11\text{a.m}$$

$$580 - (150 + 167.1) = 262.9\text{km from M}$$

Before the rally driver started, Nyaboke had traveled for  $1 \frac{1}{2}$  hrs

$$\left(\frac{3}{2}x 120\right) = 180 \text{ km}$$

$$\frac{x}{120} = \frac{x + 180}{80}$$

$$180x - 120x = 21600$$

$$x = 360 \text{ km}$$

$$\text{Distance from } K = 580 - (180 + 360)$$

$$x = 40 \text{ km}$$

$$\text{Time} = \frac{540}{180} = 3 \text{ hrs}$$

$$(9.30 + 3 \text{ hrs}) = 12.30 \text{ p.m}$$

2. Distance covered by the car after 15 min =  $(\frac{1}{4} \times 80) \text{ km} = 20 \text{ km}$

$$\text{Distance covered together} = 130 \text{ km}$$

$$\text{Relative speed} = (80 + 40) = 120 \text{ km/h}$$

Time taken to meet

$$= \frac{(130)}{120} \text{ hrs}$$

$$120$$

$$= 1 \text{ hr } 5 \text{ min}$$

$$\text{Time they met} = 10:15 \text{ a.m.} +$$

$$\begin{array}{r} 1:05 \\ \hline 11:20 \text{ a.m.} \end{array}$$

3. a)  $\frac{1}{2} \times 50h + \frac{1}{2} \times 100h + 150h = 2700$

$$225h = 2700$$

$$H = \frac{2700}{225} = 12 \text{ m/s}$$

$$\text{Maximum speed} = \frac{12 \times 60 \times 60}{1000}$$

$$= 43.2 \text{ km/h}$$

$$\text{b) Acceleration} = \frac{12}{50} \text{ m/s}$$

$$= \frac{6}{25} \text{ m/s}$$

$$\text{c) } \frac{1}{2} \times 50 \times 6$$

$$150 \text{ m}$$

d) Time for half of journey

$$\frac{1}{2} \times 12 (50 + t + t) = \frac{1}{2} \times 2700$$

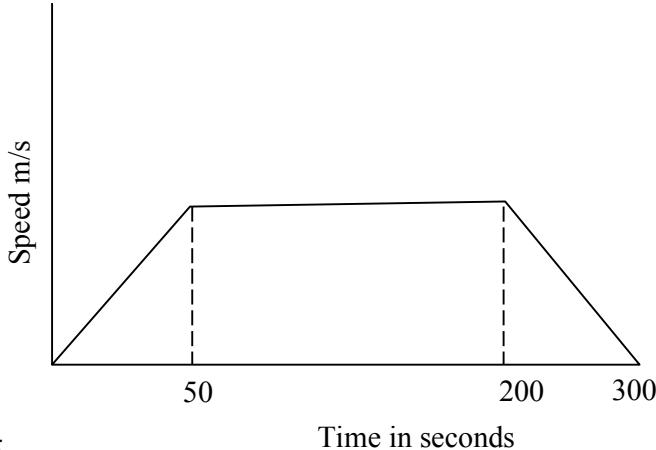
$$6(50 + 2t) = \frac{1}{2} \times 2700$$

$$50 + 2t = 225$$

$$T = \frac{225 - 50}{2} = 87.5$$

Total time

$$= 50 + 87.5 = 137.5 \text{ sec}$$



4. Time taken at 10km

$$= \frac{45}{10} = 4.5 \text{ hrs}$$

Time taken at 15km/hr

$$\frac{45}{15} = 3 \text{ hrs}$$

$$\text{Total time taken} = (4.5 + 3) = 7.5$$

$$(4.5 + 3) = 7.5 \text{ hrs}$$

$$\begin{aligned} \text{Average speed} \\ &= \frac{90}{7.5} \\ &= 12 \text{ km/hr} \end{aligned}$$

5.  $D = \frac{5}{4} \times 80 + \frac{50}{1000}$   
 $= 100.05 \text{ km}$   
 $\text{Speed} = 120 - 80 = 40 \text{ km/h}$   
 $T = \frac{D}{S} = \frac{100.05}{40}$   
 $= 2.50125 \text{ hours}$

(b)  $D = S \times T = 120 + \frac{100.05}{4000} + \frac{199}{800}$   
 $= \frac{120 \times 11000}{40000}$   
 $= 330 \text{ km}$

(c) Total time  $= \frac{330}{80}$   
 $= 4 \frac{1}{8} \text{ hrs}$   
 $\text{Time lapse} = \frac{4}{8} - \frac{5}{4} + \frac{100.05}{40000} + \frac{199}{800}$   
 $= \frac{4}{8} - 4 = \frac{1}{8} \text{ hrs}$

6. a) Distance traveled by bus before the matatu started off the journey is  
 $\text{Distance} = \text{speed} \times \text{time}$   
 $= 60 \times 2 \frac{1}{2}$   
 $= 150 \text{ km}$

Relative speed  $= 100 - 60 = 40 \text{ km/hr}$   
The matatu would cover the bus head start of 150 km in  $150/40 \text{ hrs} = 3.75 \text{ hrs} = 3 \text{ hrs } 45 \text{ min}$   
 $\therefore$  The matatu will overtake the bus after 3 hrs 45 minutes  
This will be  $1:15 + 3:45 = 5.00 \text{ pm}$

b) Time taken by the matatu to complete the remaining 350 km  $= 350/100 = 3 \frac{1}{2} \text{ hrs}$   
 $= 3 \text{ hours } 30 \text{ minutes}$

Time taken by the bus to complete the remaining 350  
 $= 350/60 = 5 \frac{5}{6} \text{ hrs} = 5 \text{ hours } 50 \text{ minutes}$   
Matatu waits for 5 hr 50 min - 3 hr 30 min = 2 hrs 20 min

7. Total distance  $= 100 + 140 + 150 = 490$   
Total speed  $= 88 + 164 = 252 \text{ km/hr}$   
 $252 \text{ km/hr into m/h} = \frac{252 \times 1000}{3600} = 70 \text{ m/h}$

Time taken  $= \frac{490}{70} = 7 \text{ sec}$

8. Distance  $= (5 + 15)m = 20m = 0.02 \text{ km}$

$$\begin{aligned}
 S &\Rightarrow \text{Bus} = 40 \text{ km/h} \\
 \text{Trailer} &= x \text{ km/h} \\
 \text{Relative speed} &= (40 - x) \text{ km/h} \\
 T &= 4.8 \text{ sec.} \quad = \frac{4.8 \text{ h}}{3600}
 \end{aligned}$$

$$\begin{aligned}
 S &= \frac{D}{T} \\
 (40 - x) &= \frac{0.02}{\frac{48}{3600}} \\
 &\simeq \frac{0.02 \times 3600}{48} \\
 &= 15 \text{ km/h} \\
 40 - x &= 15 \\
 x &= 25 \text{ km/h}
 \end{aligned}$$

9.  $L.C.M = 2^4 \times 3^2 \times 5^3 = 1800$

$$G.C.D. = 2 \times 3 \times 5^2 = 150$$

10.  $\text{Total distance} = 60 \text{ cm}$

$$\text{Total time taken} = 3 \frac{1}{5} \text{ hrs}$$

Let speed in still water be  $x \text{ km/h}$

$$\text{Speed upstream} = (x - 5) \text{ km/h}$$

$$\text{Speed downstream} = (x + 5) \text{ km/h}$$

$$\frac{30}{x - 5} + \frac{30}{x + 5} = \frac{16}{5}$$

$$30x - 150 + 30x + 150 = \frac{16}{5} (x^2 - 25)$$

$$300x = 16x^2 - 400$$

$$x = -\frac{5}{4} \text{ or } 20$$

$\therefore$  Speed in still water is 20 km/hr

11. When David left, Ojwang had covered  $15 \times \frac{3}{2} = 22.5 \text{ km}$ .

a) (i) Remaining dist. =  $40 - 22.5 = 17.5 \text{ km}$

$$\text{Relative speed} = 15 + 25 = 40 \text{ km/h}$$

$$\text{Time taken before meeting} = \frac{17.5}{40} = 0.4375 \text{ hrs}$$

$$\text{Ojwang covered } 15 \times 0.437 = 5.5625 \text{ km}$$

$$\begin{aligned}
 \text{Distance from Ojwang's house} &= 22.5 + 5.5625 \\
 &= 29.0625 \text{ km}
 \end{aligned}$$

(ii)  $0.4375 = 26 \text{ min } 15 \text{ sec}$

$$\begin{aligned}
 \therefore \text{They met at } 10.30 + 26.15 \\
 &= 10.56.15 \text{ am.}
 \end{aligned}$$

(iii)  $40 - 29.0625 = 10.9375 \text{ km}$

b) Time take =  $\frac{10.9375}{12} = 0.9115 \text{ hrs}$

$$= 54 \text{ min, } 41 \text{ sec.}$$

$$\begin{aligned} \text{They arrived at } 10.56. & 15 + 54.41 + 10 \text{ min} \\ & = \underline{12.00. 56 pm.} \checkmark \end{aligned}$$

12. (a) In 10 minutes Kamau has travelled

$$\frac{10}{60} \times 24 = 6 \text{ km}$$

$$\text{Distance left} = 42 - 6 = 36 \text{ km}$$

$$\begin{aligned} \text{Relating speed} & = 24 + 50.4 \text{ km/hr} \\ & = 74.4 \text{ km/hr} \end{aligned}$$

$$\begin{aligned} \text{Time taken to meet} & = \frac{42}{74.4} = 0.565 \text{ hrs} \\ & = 34 \text{ minutes} \end{aligned}$$

$$\text{Time for meeting is } 6.10$$

$$\frac{34}{6.44 \text{ a.m}}$$

$$\frac{34}{60} \times 50.4 = 28.56 \text{ km from R or } 13.44 \text{ km from S}$$

- (b) Kamau arrival time

$$\frac{42 \text{ km}}{24 \text{ km/hr}} = 1.75 \text{ hrs}$$

$$1 \text{ hr. } 45 \text{ minutes}$$

$$6.00 \text{ a.m}$$

$$\frac{1.45}{7.45 \text{ a.m}}$$

$$(c) \text{ Mrs Ronoh speed} = \frac{D}{T}$$

$$= 50.4 \text{ km/hr}$$

$$\text{Twice} = 50.4 \times 2 = 100.8$$

$$7.00 \text{ a.m, Mr. Kamau covered} = 1 \times 24 = 24 \text{ km}$$

$$\text{Retain speed} = 100.8 - 24 = 76.8 \text{ km/hr}$$

$$\text{So } 24 = 8.75$$

$$76.8$$

$$\begin{aligned} \text{He was overtaken at } & 7.00 \\ & + 18.75 \\ & \hline 7.18 \text{ am} \end{aligned}$$

$$\begin{aligned} \text{At distance of D} & = S \times t \\ & = \frac{100.8 \times 189.75}{60} \end{aligned}$$

$$31.5 \text{ km from S or } 10.5 \text{ km from R}$$

13. i) A gains on B at the rate of  $(72 - 56)$  Km/hr or  $16 \text{ km/h}$

$$\therefore \text{in 1 hr A gains on B } 16 \text{ km}$$

$$\text{In } 545 \text{ A gains on B}$$

$$\frac{16 \times 1000 \times 54}{60 \times 60} = 240$$

The sum of the lengths of the two trains is  $240 \text{ m}$  but the length of the first train is  $100 \text{ m}$   
The length of the second train is  $140 \text{ m}$

$$\text{ii) Relative speed} = (72 + 56) \text{ km/h} = 128 \text{ km/hr}$$

$$\text{Distance between A and B decrease at the rate of } 128 \text{ km/hr}$$

The distance decreases by 240m

$$\frac{60 \times 60 \times 240}{128 \times 1000} \text{ s} = \frac{27}{4} \text{ seconds}$$

$$= 6 \frac{3}{4} \text{ s}$$

14. (a)  $\text{Time} = \frac{D}{S}$

$$= \frac{5}{x} \text{ hrs}$$

(ii)  $\text{Time} = \frac{7}{x+24} \text{ hrs}$

(b)  $\frac{\underline{x}}{x+24} - \frac{\underline{36}}{60} = \frac{7}{x+24}$

$$\frac{7}{x+24} = \frac{25-3x}{5x}$$

$$35x = 25x - 3x^2 + 600 - 72x$$

$$3x^2 + 82x - 600 = 0$$

$$(3x+100)(x-6) = 0$$

$$x = \frac{-100}{3} \text{ or } 6$$

His speed = 6km/hr

(c)  $\text{Time} = S \times T$

$$= \frac{5}{6} \times 60$$

$$= 50 \text{ mins}$$

15. a) Relative speed = 80 - 60  
= 20 km/h

$$\text{Time} = \frac{40}{20} \text{ hrs}$$

$$= 2 \text{ hrs}$$

(b) 1.50 p.m. = 13.50 hrs.

$$\text{Time} = 13.50 + 2 = 15.50 \text{ hrs}$$

16. (a) Nairobi 400km Kisumu

Speed = 120km/h

Distance = 400km

Time taken =  $\frac{400}{120} = 10 = 3 \text{ hrs } 20 \text{ min}$

$$8.30 + 3 \text{ hrs } 20 \text{ min} = 11:50 \text{ a.m}$$

(b) at 8.30a.m distance covered by bus =  $\frac{1}{2} \times 80 = 40 \text{ km}$

Dist. Left = 360km speed = 200km/h

Time taken =  $\frac{360}{200} = 1 \text{ hr } 48 \text{ mins}$

They met at 8:30 + 1hr 48mins  
= 10:18a.m

(c) 8 - 10.18a.m is 2hrs 18mins distance =  $2 \times 80 + \frac{18}{60} \times 80$   
= 160 + 24km = 184 from Nairobi

(d) car arrived in Nairobi after 3hrs 20mins

Bus traveled a time of 3hrs 20mins + 30mins

3hrs 50mins

$$\text{Dist.} = 3 \times 80 + 50 \times 80 = 240 + 66 \frac{2}{3}$$

60

$$\text{Distance from Kisumu} = 93 \frac{1}{3} \text{ km}$$

17. Total distance = 25m

$$\begin{aligned} \text{Relative speed} &= 54 \text{ km/hr} \\ \text{To m/s} &= \left[ \frac{54 \times 1000}{60 \times 60} \right] = 15 \text{ ms} \end{aligned}$$

$$\begin{aligned} \text{Time they met} &= \left[ \frac{25}{15} \right] \\ &= 1\frac{2}{3} \text{ sec} \end{aligned}$$

## 42. Quadratic expressions and equation 2

<b>1</b> <table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>x</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr> <tr> <td><math>2x^3</math></td><td>-128</td><td>-54</td><td>-16</td><td>-2</td><td>0</td><td>2</td><td>16</td></tr> <tr> <td><math>5x^2</math></td><td>80</td><td>45</td><td>20</td><td>5</td><td>0</td><td>5</td><td>20</td></tr> <tr> <td><math>-x</math></td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>-1</td><td>-2</td></tr> <tr> <td>-6</td><td>-6</td><td>-6</td><td>-6</td><td>-6</td><td>-6</td><td>-6</td><td>-6</td></tr> <tr> <td>y</td><td>-50</td><td>-12</td><td>0</td><td>-2</td><td>-6</td><td>0</td><td>28</td></tr> </tbody> </table> $2x^3 + 5x^2 + x - 6 = y$ $2x^3 + 5x^2 x - 4 = 0$ $-2x - 2 = y$ $y = -2x - 2$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td><td>0</td><td>2</td></tr> <tr> <td>Y</td><td>-2</td><td>-6</td></tr> </table>	x	-4	-3	-2	-1	0	1	2	$2x^3$	-128	-54	-16	-2	0	2	16	$5x^2$	80	45	20	5	0	5	20	$-x$	4	3	2	1	0	-1	-2	-6	-6	-6	-6	-6	-6	-6	-6	y	-50	-12	0	-2	-6	0	28	X	0	2	Y	-2	-6	B2
x	-4	-3	-2	-1	0	1	2																																																
$2x^3$	-128	-54	-16	-2	0	2	16																																																
$5x^2$	80	45	20	5	0	5	20																																																
$-x$	4	3	2	1	0	-1	-2																																																
-6	-6	-6	-6	-6	-6	-6	-6																																																
y	-50	-12	0	-2	-6	0	28																																																
X	0	2																																																					
Y	-2	-6																																																					

<b>2</b> a) $\frac{dy}{dx} = 4x - 6$ b) $4x - 6 = 0 \therefore x = 1.5$ $y = 2(1.5)^2 - 6(1.5) + 9$ $= 4.5$ $\therefore \text{Turning point } (1.5, 4.5)$	10		
		B1 M1 M1 A1 B1  B1  B1	Equating to zero  B1  B1

	Minimum point d) (i) Gradient = 2 $\frac{y-5}{x-2} = 2$ $\therefore y = 2x + 1$	B1	
	(ii) $M_1 \times M_2 = -1$ $\therefore M_2 = -\frac{1}{2}$ $\frac{y-5}{x-2} = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}x + 6$	B1	✓ gradient of normal
		10	
3.	$A = \frac{1}{2} \times \{(6+14) + 2(6+4+16)\}$ = $\frac{1}{2}(20+32)$ = 26 units	M1 M1 A1	Use of absolute values of y
		03	

1.  
(a)

x	-2	-1	0	1	2	3	4	5	6
y	-17	-9	-3	1	3	3	1	-3	-9

(b)  $y = 5x - x^2 - 3$

$\underline{0 = 5x - x^2 - 3}$

$\underline{y = 0}$

$x = 0.75 \text{ or } 4.3 \pm 0.1$

(c)  $y = 5x - \underline{x^2} - 3$

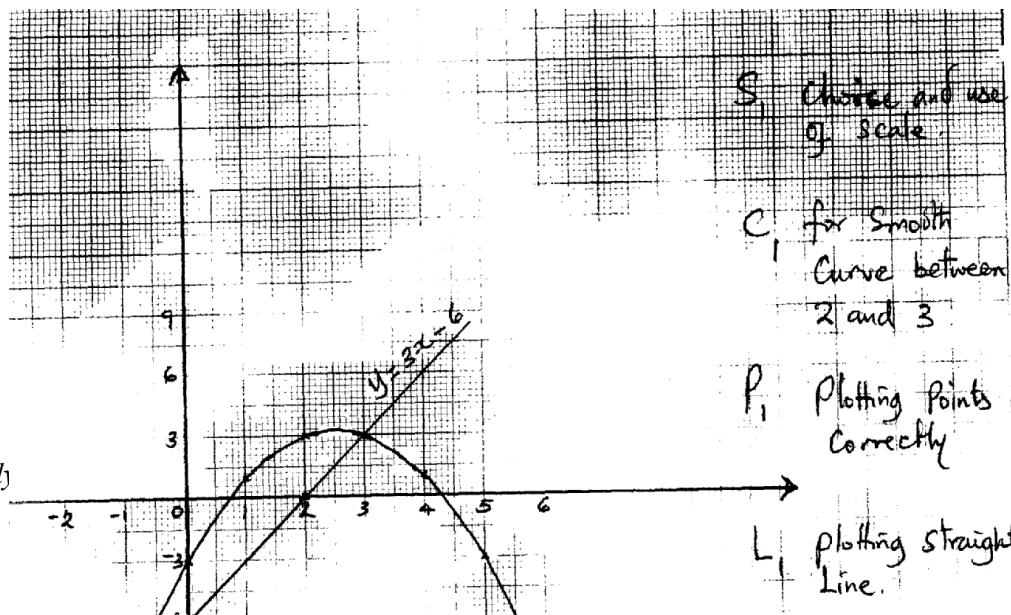
$\underline{0 = 2x - x^2 + 3}$

$y = 3x - 6$

x	0	-1	2
y	-6	-9	0

For all ✓ values of y B1  
for at least 5 ✓ values.

$x = -1 \text{ or } 3 \pm 0.1$



$$\begin{aligned}
 2. \quad & x - 2.5 - \sqrt{3} \quad x - 2.5 + \sqrt{3} = 0 \\
 & x^2 - 2.5x + x\sqrt{3} - 2.5x + 6.25 - 2.5\sqrt{3} \\
 & x\sqrt{3} + 2.5 - \sqrt{3} = 0 \\
 & x^2 - 5x + 6.25 - 3 = 0 \\
 & x^2 - 5x + 3.25 = 0 \\
 & 4x^2 - 20x + 13 = 0
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & 17.35 \times 13.85 = 240.3 \\
 & 17.35 \times 13.75 = 237.2 \\
 & \therefore 17.3 \times 13.8 = 238.7 \\
 & \text{Max err} \quad 240.3 - 238.7 = 1.5 \\
 & \text{Min err} \quad 238.7 - 237.2 = 1.6 \\
 & \text{Max err} \quad = \frac{1.6 + 1.5}{2} = \frac{3.1}{2} = 1.55
 \end{aligned}$$

$$\text{Product} \quad 238.7 \pm 1.55$$

$$\text{Last product} \quad 240$$

$$\text{Max err} = \quad 1.55$$

$$\text{Relative err} = \quad \frac{1.55}{28.1\%}$$

$$\text{error} = \frac{1.55}{28.1\%} \times 100 = 0.6\% \quad 28.1$$

$$\text{Relative err} = \quad \frac{1.55}{238.7}$$

4.

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y		04	-2		-8	-8		-2	4	12	

$$(c) (i) x^2 + 3x - 6 = 0$$

$$x = -4.5 \text{ or } 1.5 \pm 0.2$$

$$(ii) y = x^2 + 3x - 6$$

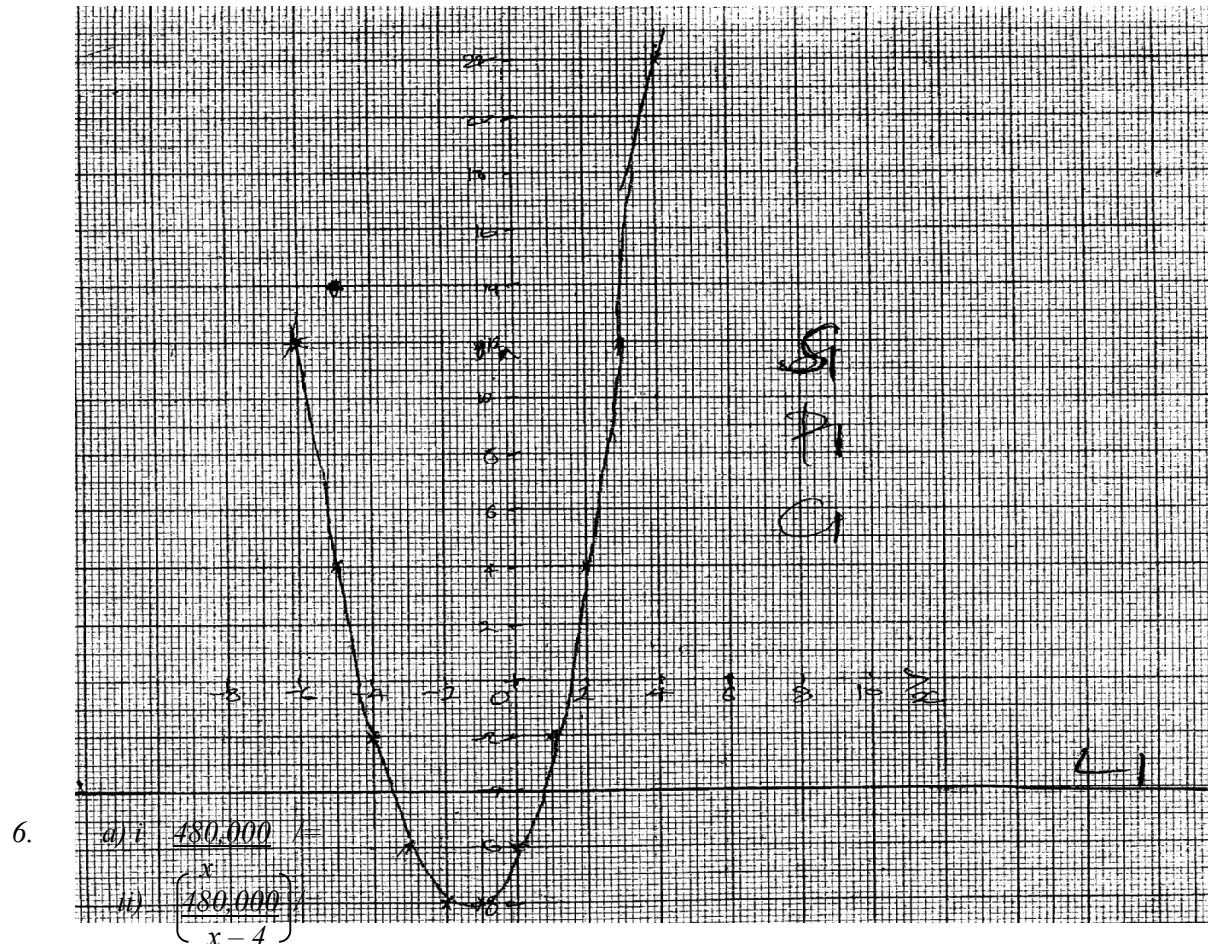
$$\begin{aligned}x^2 + 3x - 2 \\y = -4 \\x = 5 \text{ or } 4 \pm 0.2\end{aligned}$$

5.

$x$	-4	-3	-2	-1	0	1	2	3
$y$	21	10	3	0	1	6	15	28

$$\begin{aligned}(c) \quad & 2x^2 + 3x + 1 = 0 \\& \underline{2x + 4x - 3 = 0} \\& -x + 2 = y \\& x = 0.6 \text{ or } x = -2.6 \pm 0.1\end{aligned}$$

$$(d) x = 0.30 \quad -x = -1.8 \pm 0.1$$



6.

$$a) i) \frac{480,000}{x}$$

$$ii) \frac{480,000}{x-4}$$

$$b) \frac{480,000}{x-4} = \frac{480,000}{x} + 20,000$$

Multiply all hr' by L.C.M.

$$480,000x = 480,000(x - 4) + 20,000(x^2 - 4x)$$

Dividing by 10,000

$$48x = 48x - 192 + 2x^2 - 4x$$

$$48x - 48x + 4x - 2x^2 + 192 = 0$$

$$4x - 2x^2 + 192 = 0$$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \\&= \frac{-4 \pm \sqrt{1552}}{-4} \\&= \frac{-4 \pm 39.3954}{-4} \\x &= \frac{-4 + 39.3954}{-4} \quad \text{or } x = \frac{-4 - 39.3954}{-4}\end{aligned}$$

But  $x$  cannot be -ve hence

$$x = \frac{-43.3954}{-4} = 10.8489$$

$$= 11$$

c) Original : new cont.

$$\frac{480,000}{11} : \frac{480,000}{7}$$

d) Size of land bought = 6 hectares

$$\frac{6}{7} = 0.857143 \simeq 0.8571 \text{ hectares}$$

7.

$x$	-3	-2	-1	0	1	2
$y$	13	4	-1	-2	1	8

$$\begin{aligned}(iii) \quad y &= 2x^2 + x - 2 \\0 &= 2x^2 + 2x - 3 \\y &= -x + 1\end{aligned}$$

$x$	-3	-2	-1	0	1	2
$y$	5	3	2	1	0	-1

$$\begin{aligned}y &= 2x^2 + x - 2 \\0 &= 2x^2 + x - 5 \\y &= -3\end{aligned}$$

19.

8. (a) Dist. traveled in 3hrs s. drawing

$$\text{Plane A} - 400 \times 3 = 1200 \text{ km - cm}$$

$$\text{Plane B} - 500 \times 3 = 7.5 \text{ cm}$$

$$\text{Plane C} - 300 \times 3 = 900 \text{ km - 4.5 cm}$$

$$(b) \text{Dist. BA} = 12.8 \times 0.1 \times 200 = 2560 \text{ km - 20 km}$$

$$T = \frac{D}{S} = \frac{2560}{500} \text{ hrs}$$

$$= 5.12 \text{ hrs of 5 hrs, 7.2 mns}$$

$$\approx 5 \text{ hrs, 7 min (nearest min)}$$

(c) Bearing of B from C =  $360^\circ - 20^\circ = 340^\circ$ 

$$\begin{aligned} \text{Dist. BC} &= (10.9 \pm 0.1 \times 200) \text{ km} \\ &= 2180 \text{ km} \pm 20 \text{ km} \end{aligned}$$

9. a)

$x$	-2	-1.5	-1	-0.5	0	0.5	1
$x^2$	4	2.25	1	0.25	0	0.25	1
$4x$	-8	-6	4	-2	0	2	4

$$\begin{array}{cccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ y & 0 & 0.25 & 9 & 2.25 & 4 & 6.25 & 9 \end{array}$$

$$\begin{aligned} A &= \frac{1}{2} h \left\{ (y_1 + y_7) + 2(y_2 + \dots + y_6) \right\} \\ &= \frac{1}{2} \times \frac{1}{2} \left\{ (0 + 9) + 2(0.25 + 9 + 2.25 + 4 + 0.25) \right\} \checkmark \\ &= \frac{1}{4} \left\{ 9 + 4.25 \right\} \checkmark \\ &= \underline{\underline{13.25 \text{ sq. units}}} \checkmark \end{aligned}$$

$$b) \int_{-2}^0 (x^2 + 4x + 4) dx + \int_0^1 (x^2 + 4x + u) dx$$

$$\begin{aligned} &\left[ \frac{x^3}{3} + 2x^2 + 4x \right]_0^1 + \left[ \frac{x^3}{3} + 2x^2 + ux \right]_0^1 \checkmark \\ &= (-\frac{8}{3} + 8 - \frac{8}{3}) + (\frac{1}{3} + 2 + 4) \checkmark \\ &= 9 \checkmark \\ &\text{Error} = 13.25 - 9 = 4.125 \end{aligned}$$

$$\begin{aligned} \% &= \frac{4.125}{9} \checkmark \times 100 \\ &= \underline{\underline{45.84\%}} \end{aligned}$$

10. a)

$x$	-2	-1.5	-1	-0.5	0	0.5	1
$x^2$	4	2.25	1	0.25	0	0.25	1
$4x$	-8	-6	4	-2	0	2	4

$$\begin{array}{cccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ y & 0 & 0.25 & 9 & 2.25 & 4 & 6.25 & 9 \end{array}$$

$$\begin{aligned} A &= \frac{1}{2} h \left\{ (y_1 + y_7) + 2(y_2 + \dots + y_6) \right\} \\ &= \frac{1}{2} \times \frac{1}{2} \left\{ (0 + 9) + 2(0.25 + 9 + 2.25 + 4 + 0.25) \right\} \checkmark \\ &= \frac{1}{4} \left\{ 9 + 4.25 \right\} \checkmark \\ &= \underline{\underline{13.25 \text{ sq. units}}} \checkmark \end{aligned}$$

$$b) \int_{-2}^0 (x^2 + 4x + 4) dx + \int_0^1 (x^2 + 4x + u) dx$$

$$\begin{aligned} & \left[ \frac{x^3}{3} + 2x^2 + 4x \right] + \frac{x^3}{3} + 2x^2 + ux \quad \checkmark \\ & = (-\frac{8}{3} + 8 - 8) + (\frac{1}{3} + 2 + 4) \quad \checkmark \quad 0 \\ & = 9 \quad \checkmark \\ & \text{Error} = 13.25 - 9 = 4.125 \end{aligned}$$

$$\% = \frac{4.125}{9} \times 100 \\ = 45.84\%$$

11.  $y = 2x^2 - 4x - 5 \qquad \qquad \qquad y = 2x + 3$

$X$	-3	-2	0	1	2	3	4	5			$x$	-4	-2	0	2
$2x^2$	18	2	0	2	8	18	32	50			$y$	-5	-1	3	7
$4x$	-12	-8	-4	0	4	8	12	16	20						
5	5	5	5	5	5	5	5	5	5						
$y$	25	11	1	-5	-7	1	11	25	11	$B_2$					

(a)  $x = 1$

(b)  $-0.9 < x < 2.8$

$x = -1$  and  $x = 4$

12.

$X$	-1.5	-1	0	1.5	2	2.5	3.5
$Y$	-4	0	5	5	3	0	-9

$(0.75, 6.125)$

$Y = -2$

Range of values  $-1.3 < x < 2.75$

Integral values; -1, 0, 1, 2

13. a)

$x$	-4	-3	-2	-1	0	1	2
$2x^2$	32	18	8	2	0	2	8
$4x - 3$	-19	-15	-11	-7	-3	1	5
$y$	13	3	-3	-5	-3	3	13

(b) Roots for  $x = -2.6 \pm 0.1$

$x = 0.6 \pm 0.1$

$y = 2x^2 + 4x - 3$

$$\begin{array}{l} 0 = 2x^2 + 4x - 3 \\ y = 3x + 2 \end{array}$$

Roots read from the 2 pts of intersection of the line and curve.

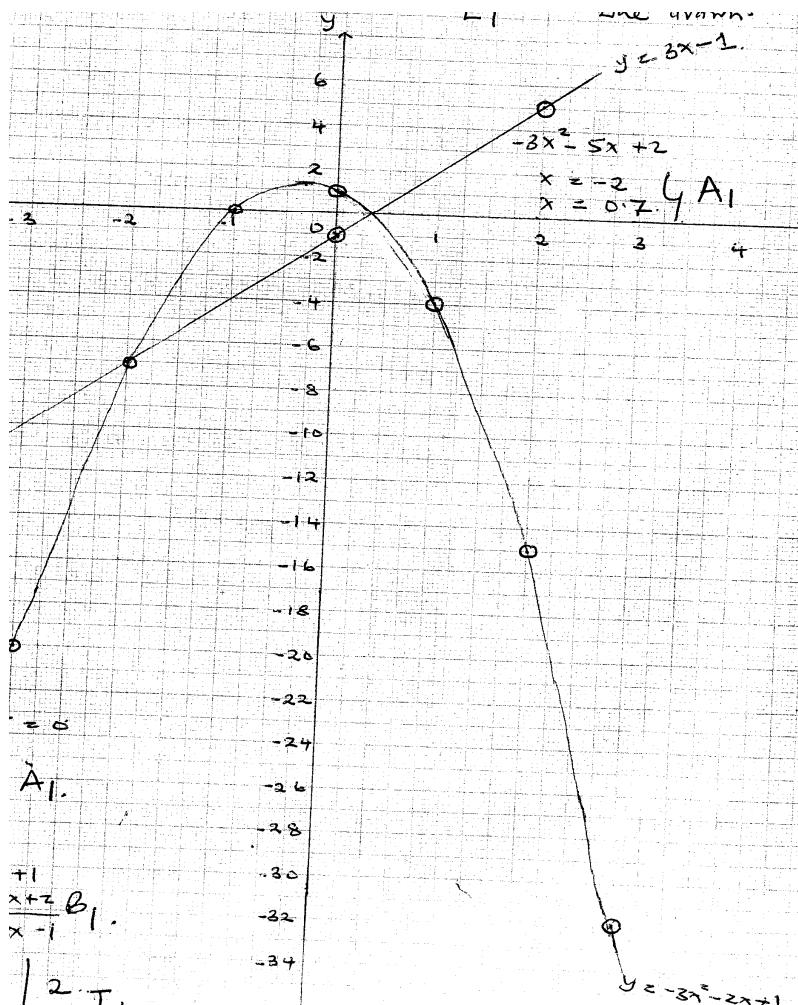
$X = -1.9 \pm 0.1$

$X = 1.4 \pm 0.1$

14.

$x$	-3	-2	-1	0	1	2	3
$-3x^2$	-27	-12*	-3	0	-3*	-12	-27*
$-2x$	6	4	2*	0	-2*	-4*	-6

$I$							
$y$	-20	-7	0	1	-4	-15	-32



$$\begin{aligned} I - 2x - 3x^2 &= 0 \\ x = -1 \\ \text{or } x = 0.7 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} A_1$$

$$\begin{aligned} y &= -3x^2 - 2x + 1 \\ 0 &= -3x^2 - 5x + 2 \\ y &= o + 3x - 1 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} B_1$$

$$\left. \begin{array}{c} \frac{x}{y} \mid \begin{array}{c|c|c} 0 & 2 \\ -1 & 5 \end{array} \\ \end{array} \right\} T_1$$

$$\begin{aligned} 15. \quad x^2 + ax - b &= 0 \\ (x-1)(x+5) &= x^2 + ax - b \\ x^2 + 4x - 5 &= x^2 + ax - b \\ a = 4, b = 5 \end{aligned}$$

16. Let  $a = 1.5 + \sqrt{2}$

$$b = 1.5 - \sqrt{2}$$

$$\therefore (x-a)(x-b) = 0$$

$$x^2 - xb - ax + ab = 0$$

$$x^2 - x(1.5 - \sqrt{2}) - x(1.5 + \sqrt{2}) + ab = 0$$

$$x^2 - 1.5x + x\sqrt{2} - x(1.5x + \sqrt{2}) = 0$$

$$x^2 - 3x + ab$$

$$x^2 - 3x + (1.5 + \sqrt{2})(1.5 - \sqrt{2}) = 0$$

$$x^2 - 3x + 2.25 - 2 = 0$$

$$x^2 - 3x + \frac{1}{4} = 0$$

$$4x^2 - 12x + 1 = 0$$

17. a) i)  $a^2 + b^2 = 89$        $a + b = 13$   
 $a^2 + 2ab + b^2 = (a + b)^2 = 13^2 = 169$

ii)  $2ab = 169 - 89$   
 $= 80$

iii)  $a^2 - 2ab + b^2 = a^2 + b^2 - 2ab$   
 $= 89 - 80 = 9$

iv)  $(a - b)^2 = 9$   
 $a - b = \pm 3$

b)  $a + b = 13$   
 $\frac{a - b = 3}{2a = 16}$

### 43. Approximation and errors

1	Error in length = $0.015 \times 15 = 0.225$ Error in breadth = $0.015 \times 12 = 0.18$ Error in perimeter = $2(0.225 + 0.18) = 0.81$	$M_1$ $M_1$ $A_1$	Finding error in both length and breadth
		3	

1. Maximum perimeter =  $2(12.05 + 8.05) = 40.2\text{cm}$

$$\text{Actual perimeter} = 2(12.0 + 8.0) = 40.0\text{cm}$$

$$\text{Error} = 40.2\text{cm} - 40.0\text{cm} = 0.2\text{cm}$$

$$\% \text{error} = \frac{(0.2 \times 100)}{40} = 0.5\%$$

2.  $A = \frac{1}{2} \times 12 \times 8 = 48$

i) Absolute error

$$= [\frac{1}{2} \times 12.5 \times 8.5 - \frac{1}{2} \times 11.5 \times 7.5]^2 = 5$$

$$\text{ii) } \% \text{error} = \frac{5}{24} \times 100\% = 10.4\%$$

3.  $A = L \times W$

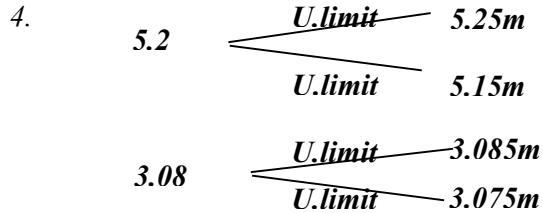
$$A = x(14-x) = 14x - x^2$$

$$\frac{dA}{dx} = 14 - 2x = 0$$

$$14 = 2x, x = 7$$

$$\text{Maximum area} = 7(14 - 7)$$

$$= 7 \times 7 = 49 \text{ cm}^2$$



$$\text{Shortest possible length of } 2^{\text{nd}} \text{ piece}$$

$$= 5.15 - 3.085 = 2.065 \text{ m}$$

5. Absolute error  $10 \pm 0.05$  and  $15 \pm 0.05$

$$\text{Max area} = 10.5 \times 15.05$$

$$\text{Min area} = 9.95 \times 14.95 = 148.7525$$

$$a.e = \frac{150.2525 - 15 + 150 - 148.7525}{2}$$

$$= 1.25$$

$$\% \text{ error} = \frac{1.25}{150} \times 100$$

$$= 0.8333\%$$

6.  $17.35 \times 13.85 = 240.3$   
 $17.35 \times 13.75 = 237.2$   
 $\therefore 17.3 \times 13.8 = 238.7$   
 $\text{Max err} = 240.3 - 238.7 = 1.5$   
 $\text{Min err} = 238.7 - 237.2 = 1.6$   
 $\text{Max err} = \frac{1.6 + 1.5}{2} = \frac{3.1}{2} = 1.55$

$$\text{Product} = 238.7 \pm 1.55$$

$$\text{Last product} = 240$$

$$\text{Max err} = 1.55$$

$$\text{Relative err} = \frac{1.55}{28.1\%}$$

$$\text{error} = \frac{1.55}{28.1\%} \times 100 = 0.6\% = 28.1$$

$$\text{Relative err} = \frac{1.55}{238.7}$$

7. 14 Kg to the nearest  $10/1000$  Kg

$$A.E = 0.01$$

$$\% E = \frac{0.01}{14} \times 100$$

$$= 0.07$$

- 8.

X	$0^\circ$	$3^\circ$	60	90	$120^\circ$	$150^\circ$	$180^\circ$	21	24	$270^\circ$	$300^\circ$	$330^\circ$	$360^\circ$
---	-----------	-----------	----	----	-------------	-------------	-------------	----	----	-------------	-------------	-------------	-------------

			°	°				0°	0°				
$\cos x$	$1$	$0.87$	$0.$ $5$	$0$	$-0.5$	$0.87$	$-1.0$	$-$ $0.$ $87$	$0.5$	$0$	$0.5$	$0.87$	$1$
$2 \cos(x + 30)$	$1.73$	$1$	$0$	$-$ $1.$ $0$	$-1.73$	$-2.0$	$-1.73$	$-$ $1.$ $0$	$0$	$1$	$1.73$	$2.00$	$1.73$

b) i) Amplitude of  $y = \cos x$  is 1 unit  
And  $Y = 2\cos(x + 30)$  2 units

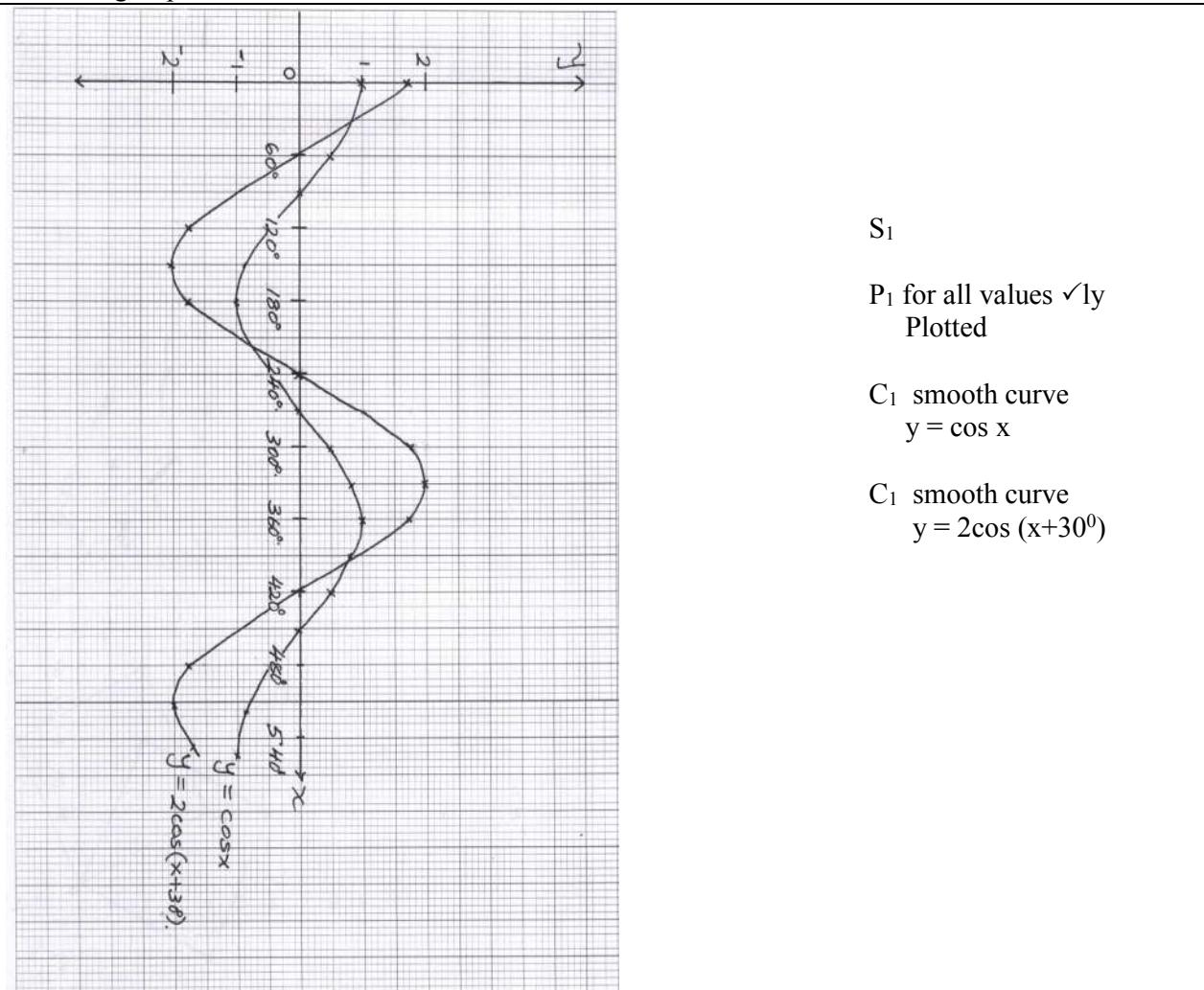
ii) period of  $y = 2 \cos(x + 30^\circ)$   
 $330^\circ$

c)  $\cos x = 2 \cos(x + 30^\circ)$   
 $x = 40^\circ \pm 1$   
 $x = 219^\circ \pm 1$

9. 
$$\begin{aligned} \frac{y+x}{y-x} &= \frac{12+6}{8-6} && \text{Correct substitution} \\ &= \frac{18}{2} && \text{Simplification} \\ &= 9 && \text{CAO} \end{aligned}$$

#### 44. Trigonometry 2

<b>1</b> $\sin \frac{5}{2}x = -\frac{1}{2}$ $\frac{5}{2}x = 210^\circ, 330^\circ, 390^\circ$ $x = 84^\circ, 132^\circ, 156^\circ$	<b>B<sub>1</sub></b> <b>B<sub>1</sub></b> Allow for any 2 ✓ angles																														
<b>2</b> a) <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X<sup>0</sup></td> <td>60<sup>0</sup></td> <td>120<sup>0</sup></td> <td>180<sup>0</sup></td> <td>240<sup>0</sup></td> <td>300<sup>0</sup></td> <td>360<sup>0</sup></td> <td>420<sup>0</sup></td> <td>480<sup>0</sup></td> <td>540<sup>0</sup></td> </tr> <tr> <td>cosX</td> <td>0.50</td> <td>- 0.50</td> <td></td> <td>-0.5</td> <td></td> <td>1.00</td> <td>0.50</td> <td>-0.5</td> <td>-1.0</td> </tr> <tr> <td>2cos(x+30)</td> <td>0.00</td> <td></td> <td>- 1.73</td> <td></td> <td>1.73</td> <td>1.73</td> <td>0.00</td> <td>- 1.73</td> <td>- 1.73</td> </tr> </table> b) i) Period = 3600 ii) Phase angle = 300	X <sup>0</sup>	60 <sup>0</sup>	120 <sup>0</sup>	180 <sup>0</sup>	240 <sup>0</sup>	300 <sup>0</sup>	360 <sup>0</sup>	420 <sup>0</sup>	480 <sup>0</sup>	540 <sup>0</sup>	cosX	0.50	- 0.50		-0.5		1.00	0.50	-0.5	-1.0	2cos(x+30)	0.00		- 1.73		1.73	1.73	0.00	- 1.73	- 1.73	<b>B<sub>2</sub></b> <b>B<sub>1</sub></b> allow B <sub>1</sub> for 7✓ values ✓ values to 2 d.p. apply ow-1 if given to other d.p <b>B<sub>1</sub></b> <b>B<sub>1</sub></b>
X <sup>0</sup>	60 <sup>0</sup>	120 <sup>0</sup>	180 <sup>0</sup>	240 <sup>0</sup>	300 <sup>0</sup>	360 <sup>0</sup>	420 <sup>0</sup>	480 <sup>0</sup>	540 <sup>0</sup>																						
cosX	0.50	- 0.50		-0.5		1.00	0.50	-0.5	-1.0																						
2cos(x+30)	0.00		- 1.73		1.73	1.73	0.00	- 1.73	- 1.73																						

S<sub>1</sub>

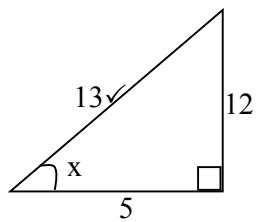
P<sub>1</sub> for all values ✓ly  
Plotted

C<sub>1</sub> smooth curve  
 $y = \cos x$

C<sub>1</sub> smooth curve  
 $y = 2\cos(x+30^\circ)$

c)	$37.5^\circ \leq x \leq 217.5^\circ$ $397.5^\circ \leq x \leq 540^\circ$	B <sub>1</sub> B <sub>1</sub>	Allow $\pm 0.5$
		10	

$$\begin{aligned}
 1. \quad & 5 \sin x + \cos x \\
 &= 5\sqrt{\frac{12}{13}} - \frac{5}{13} \\
 &= \frac{60}{13} - \frac{5}{13} = \frac{55}{13} \\
 &= \frac{12}{13}
 \end{aligned}$$



$$\begin{aligned}
 2. \quad & \frac{2\cos 3\theta}{2} = 1 \\
 & \cos^3 \theta = 0.5 \\
 & 3\theta = \cos^{-1} 0.5 \\
 & \frac{3\theta}{3} = \frac{60^\circ}{3}, \frac{300^\circ}{3}, \frac{420^\circ}{3}, \frac{66^\circ}{3}, \frac{78^\circ}{3}, \frac{102^\circ}{3} \\
 & \therefore \theta = 20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ, 340^\circ
 \end{aligned}$$

✓Identification of exact number of quadrants to satisfy the equation.  
✓Values of at least 4 soln. of  $\theta$

$$3.. \quad \frac{\frac{1}{2} X^{\frac{3}{2}}}{\frac{3}{2} X^{\frac{1}{2}}}$$

$$\frac{\frac{3}{4}}{\frac{3}{4} - \frac{1}{2}} X^{\frac{\frac{3}{2} + \frac{1}{2}}{\frac{3}{2} + \frac{1}{2}}} = \frac{\frac{3}{8} + \frac{3}{4}\sqrt{2}}{\frac{1}{4}}$$

$$\begin{aligned} \frac{\frac{3}{8} + \frac{3}{4}\sqrt{2}}{\frac{3}{4} - \frac{1}{2}} &= \frac{\frac{3}{8} + \frac{3}{4}\sqrt{2}}{\frac{1}{4}} \\ &= \frac{3}{2} + \frac{3}{2}\sqrt{2} \end{aligned}$$

4.     a)  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $b^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cos 100^\circ$   
 $= 74 - 70(-0.173648)$   
 $= 74 + 12.15537$   
 $b^2 = 86.15537$   
 $b = 9.28199$

$$AC = 9.3 \text{ km}$$

$$b) \frac{9.3}{\sin 100^\circ} = \frac{5}{\sin \theta}$$

$$\sin \theta = \frac{5 \sin 100^\circ}{9.3} = 0.529466$$

$$\theta = 31.9694$$

$$\theta \approx 32^\circ$$

$$32^\circ - 20^\circ = 12^\circ$$

$$\text{Bearing} = 360^\circ - 12^\circ = 348^\circ$$

$$c) 020^\circ$$

5.

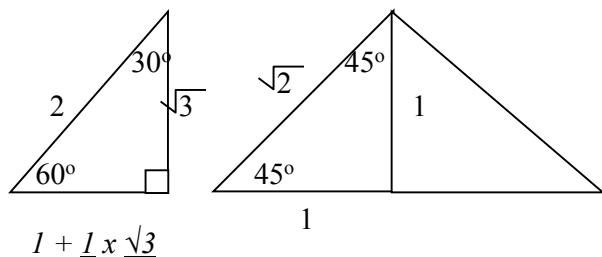
$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \sin 45^\circ &= \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \end{aligned}$$

$$= \frac{\sqrt{6}}{5} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - 2\sqrt{2}}{4}$$

6.



$$\sqrt{2} \quad 2$$

$$\frac{1 + \sqrt{3}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}}$$

$$\frac{1 + \frac{2\sqrt{6}}{4}}{1}$$

$$\frac{4 + 2\sqrt{6}}{4}$$

7.  $\frac{\sqrt{5}(2\sqrt{2} + \sqrt{5}) + \sqrt{2}(2\sqrt{2} - \sqrt{5})}{(2\sqrt{2})2 - (\sqrt{5})^2}$

$$\begin{aligned} & \frac{2\sqrt{10} + 5 + 4 - \sqrt{10}}{8 - 5} \\ & \frac{9 + \sqrt{10}}{3} \\ & 3 + \frac{1}{3}\sqrt{10} \end{aligned}$$

8. a)  $b^2 = a^2 + c^2 - 2ac \cos B$   
 $b^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cos 100^\circ$   
 $= 74 - 70(-0.173648)$   
 $= 74 + 12.15537$   
 $b^2 = 86.15537$   
 $b = 9.28199$   
 $AC = 9.3 \text{ km}$

b)  $\frac{9.3}{\sin 100^\circ} = \frac{5}{\sin \theta}$

$$\begin{aligned} \sin \theta &= \frac{5 \sin 100^\circ}{9.3} = 0.529466 \\ \theta &= 31.9694^\circ \\ \theta &\approx 32^\circ \\ 32^\circ - 20^\circ &= 12^\circ \\ \text{Bearing} &= 360^\circ - 12^\circ \\ &= 348^\circ \end{aligned}$$

c)  $020^\circ$

#### 45. Surds

<p>1.</p> $BC = \sqrt{r^2 + 1^2}$ $= \sqrt{6}$ $\sin(90^\circ - \theta) = \frac{\sqrt{5}}{\sqrt{6}}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p><b>02</b></p>
---	--	------------------

$$1.. \quad \frac{3}{\sqrt{7}-2} + \frac{1}{\sqrt{7}} = \frac{3}{\sqrt{7}-4} + \frac{\sqrt{7}}{\sqrt{7}}$$

$$\begin{aligned}
 & \frac{3}{\sqrt{7}-2} + \frac{1}{\sqrt{7}} = \frac{3\sqrt{7}+7-2}{\sqrt{7}-2} \\
 & = \frac{3\sqrt{7}+(7-2)}{\sqrt{7}-2} \\
 & = \frac{3\sqrt{7}+5}{\sqrt{7}-2} \\
 & = \frac{3\sqrt{7}+5}{7-2\sqrt{7}} \\
 & = 49-28 \\
 & = \frac{(3\sqrt{7}+5)(7+2\sqrt{7})}{21} \\
 & = \frac{(4\sqrt{7}-2)(7+2\sqrt{7})}{21}
 \end{aligned}$$

2.

$$\frac{2+\sqrt{5}}{2-\sqrt{5}} - \frac{3+\sqrt{5}}{2+\sqrt{5}} = a + b\sqrt{5}$$

$$\begin{aligned}
 & \frac{4+4\sqrt{5}+\sqrt{5}-(6-3\sqrt{5}+2\sqrt{5}-5)}{4-5} \\
 & = \frac{8+5\sqrt{5}}{-1}
 \end{aligned}$$

$$a = -8 \quad b = -5$$

$$\begin{aligned}
 3. \quad & \frac{\sqrt{14}(\sqrt{7}+\sqrt{2}) - \sqrt{14}(\sqrt{7}-\sqrt{12})}{7-12} \\
 & = \frac{\sqrt{14}\cdot\sqrt{7} + \sqrt{14}\cdot\sqrt{2} - \sqrt{14}\cdot\sqrt{7} + \sqrt{14}\cdot\sqrt{12}}{-5}
 \end{aligned}$$

4.

$$\sqrt{2-1})^2 = 2\sqrt{2} - 1 \sqrt{3} - 2$$

$$(\sqrt{2}-1)^3 = 2-1(\sqrt{2}-2)$$

$$= 5\sqrt{2} - 7$$

$$\begin{aligned} & \frac{2-\sqrt{2}}{5\sqrt{2-7}} \times \frac{5\sqrt{2+7}}{5} = 2\sqrt{2+7} - 2\sqrt{2+2} \\ & = 17\sqrt{2-6} = -6 + 1\sqrt{2} \end{aligned}$$

5.  $(2-3)(3+2)$

$3(2)2-2)2$

$\frac{3x2-3+2-2}{9x2-4x3}$

$\frac{6-3+2-6}{18-12}=6$

6. i)  $Or = 16^2 - 5^2$

$$= \sqrt{256-25}$$

$$= 15.198 \text{ cm}$$

$$\text{ii) } \tan \theta = \frac{5.066}{4} = 1.2665$$

$$\therefore \theta = 51.71^\circ$$

7.  $\log_{10} 5 - \log_{10} 10^2 + \log_{10} (2y+10) = \log_{10} (y-4)$

$$\log_{10} \left\{ \frac{5(2y+10)}{10^2} \right\} = \log_{10} (y-4)$$

$$10y + 50 = 100y - 400$$

$$90y = 450$$

$$y = 5$$

$$8. \frac{\sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2} [\sqrt{3} - \sqrt{2}]}$$

$$= \frac{3 - \sqrt{6} - \sqrt{6} + 2}{3 - \sqrt{6} + \sqrt{6} - 2}$$

$$= \frac{5 - 2\sqrt{6}}{3 - 2}$$

$$= 5 - 2\sqrt{6}$$

## 46. Further logarithms

1.

No	Log
$1934^2$	$3.2865 \times 2$
$\sqrt{0.00324}$	$= 6.5729$
	$-3.5105 : 2$
	$= 2.7553$
	$= 5.328$
2.8727	$0.4583$
	$= 4.8699$
$\text{Anti log } 4.8699 = 7.4114 \times 10$	
$= 74114$	

2.

a) monthly taxable pay;

$$\begin{aligned}15\% \text{ of monthly salary} &= \frac{15}{100} \times 20000 \\&= \text{kshs. } 3000\end{aligned}$$

$$\begin{aligned}\text{Monthly pay} &= \text{Kshs.}(20000 + 3000 - 700) \\&= \text{Kshs. } 22300\end{aligned}$$

$$\begin{aligned}\text{In Kenya pounds} &= \frac{22300}{20} \\&= \text{KE } 1115\end{aligned}$$

b) Total tax payable (Gross tax)

$$\begin{array}{r} 1 - 342 \\ 343 - 684 \\ 685 - 1026 \\ 1027 - 1368 \end{array} \begin{array}{l} 342 \times 2 = \text{Kshs. } 684 \\ 342 \times 3 = \text{Kshs. } 1026 \\ 342 \times 4 = \text{Kshs. } 1368 \\ 89 \times 5 = \text{Kshs. } 445 \end{array}$$

$$\text{Total tax} = \text{Kshs. } 3523$$

c) Net tax

$$\begin{aligned}&= \text{Gross tax} - \text{relief} \\&= \text{Kshs.}(3523 - 600) = \text{Kshs. } 2923\end{aligned}$$

d) Net pay;

$$\begin{aligned}&= \text{Kshs. } 20000 - (2923 + 2100 + 200 + 2/100 \times 20000) \\&= \text{Kshs. } (20000 - 5623) = \text{Kshs. } 14377\end{aligned}$$

3.

6 month depreciation rate = 8%

Number of periods = 8

$$400,000 (1 - 0.08)^8 = 205288$$

4.

Mid ordinate

$$\begin{aligned}\text{Area} &= 1.2(6.2 + 4.3 + 2.6) \\&= 15.72\end{aligned}$$

5.

$$\text{N. } \log \frac{2^5 \times 2^7}{3^6} = \log \frac{2^{12}}{3^6}$$

$$= \log \left( \frac{2^2}{3} \right)^6 = \left( \frac{4}{3} \right)^6$$

$$\frac{\left( D; \log \frac{2^5 \times 2^7}{3^6} \right)}{N; \log \frac{4}{3}} = \log \frac{2^6}{3^3} = \log \frac{2^2}{3} = \log \frac{4}{3}$$

$$\frac{D}{3} = \frac{4}{3}$$

$$\begin{aligned} & \log \frac{4}{3} \\ &= \frac{6 \log \left[ \frac{4}{3} \right]}{3 \log \left[ \frac{4}{3} \right]} \\ & 6/3 = 2 \end{aligned}$$

6.  $\log(x+5) = \log(4)$   
 $(x+2)$   
 $x + 5 = 4$   
 $x + 2$   
 $(x+5)(x+2) = 4$   
 $x^2 + 2x + 5x + 10 = 4$   
 $x^2 + 7x + 6 = 0$   
 $x^2 + 6x + x + 6 = 0$   
 $x(x+6) + 1(x+6) = 0$   
 $(x+1)(x+6) = 0$   
 $x = -1 \quad x = -6$

7.  $a = 100$   
 $r = \frac{200}{100} = 2$   
 $\frac{a(r^n - 1)}{r - 1} > Sn$   
 $\frac{100(2^n - 1)}{2 - 1} > 3,100$   
 $2^n - 1 > 31$   
 $2^n > 32$   
 $2^n > 2^5$   
 $n > 5$   
 $n = 6$

8. a)

2	3	5	7
2	32	52	72
3	23	53	73
5	25	35	75
7	27	37	57

b)  $P(E) = \frac{4}{16}$   
 $= \frac{1}{4}$

9.

$$\begin{aligned} x^2 + y^2 - 6x &= 3 - 4y \\ x^2 - 6x + (\frac{6}{2})^2 + y^2 + 4y + (\frac{4}{2})^2 &= 3 + (\frac{6}{2})^2 + (\frac{4}{2})^2 \end{aligned}$$

$$\begin{aligned} (x - 3)^2 + (y + 2)^2 &= 3 + 9 = 4 \\ (x - 3)^2 + (y + 2)^2 &= 16 \\ C(3, -2) \end{aligned}$$

10.

$$\begin{aligned} \frac{\text{Gradient } \Delta y}{\Delta x} &= \frac{7 - -2}{6 - 3} = 3 \\ A &= P(1 + \frac{r}{100})^n \end{aligned}$$

$$\begin{aligned}
 &= 10000 (1 + \frac{4}{100})^6 \\
 &= 10000(1.04)^6 \\
 &= 12653.19 \quad (12,653)
 \end{aligned}$$

11.

	No.	Std. Form	Log
	13.6	$1.36 \times 10^1$	1.1335
		+	
	$\cos 40^\circ$	—	1.8842
		$1.0177$	
	63.5	$6.35 \times 10^1$	1.8028
		$1.2149 \div 3$	
		$= \frac{3}{33} + \underline{2.2149}$	
	0.5474	$5.474 \times 10^{-1}$	1.7383
		0.5474	← 1.7383

12.  $\log_{10} 5^2 - \log_{10} 2^3 + \log 2^5$

$$\begin{aligned}
 &\log_{10} \left( \frac{\cancel{25} \times \cancel{32}}{\cancel{8}} \right) \\
 &1 \qquad \qquad \qquad \qquad \qquad \qquad \checkmark \text{Application of logarithmic laws.} \\
 &\log_{10} 100 = \log_{10}^{10} \\
 &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \checkmark \text{Application} \\
 &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{C.A.O} \\
 &\log_{10} 100 = \log_{10}^{10} \\
 &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad = 2 \log_{10}^{10} \\
 &\text{But } \log_{10}^{10} = 1 \\
 &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \therefore = 2
 \end{aligned}$$

13.  $\log \frac{3x+8}{2^3} = \log (x-4)$

$$\begin{aligned}
 \frac{3x+8}{8} &= x-4 \\
 3x+8 &= 8(x-4) \\
 3x+8 &= 8x-32 \\
 -5x &= -40 \\
 x &= 8
 \end{aligned}$$

✓ Application of logarithmic laws.  
 ✓ Application  
 C.A.O

*Division of logs.**Dropping logs and simplification.**C.A.O***47. Commercial Arithmetic 2**

<b>1</b>	a) $1.15 \times 54,450 + 6000$ $68,617.5 \times 12$ $sh.823,410$ b)	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>  M <sub>1</sub>  M <sub>1</sub>	✓ 1 <sup>st</sup> and 2 <sup>nd</sup> slabs  ✓ 3 <sup>rd</sup> and 4 <sup>th</sup> slabs
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	$1^{st} 116160 \times \frac{10}{100} = sh.11,616$ $next 109440 \times \frac{15}{100} = sh.16,416$ $next 109440 \times \frac{20}{100} = sh.21,888$ $next 109440 \times \frac{25}{100} = sh.27,360$ $Re \text{ maining } 378,930 \times \frac{30}{100} = sh.113,95$ Total annually = 190,959 Monthly = $\frac{190,959}{2} = sh.15,913.25$ Less relief 1100.00 Net tax payable = sh.14,813.25	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>  B <sub>1</sub> B <sub>1</sub>	✓last slab Subtraction of relief  Total deductions
c)	$\frac{20}{100} \times 54,450 = sh.10,890$ Total deductions = $10,890 + 14,813.25 = 25,703.25$ Monthly income = $60,450 - 25,703.25 = sh.34,746.75$		

1. After 1<sup>st</sup> year =  $\frac{95}{100} \times 4200000$   
= Shs.357,000  
After 2<sup>nd</sup> year =  $(\frac{87}{100} \times 357000)$   
= sh310590  
After 3<sup>rd</sup> year =  $(\frac{88}{100} \times 310590)$   
= shs.273319.20  
After 4<sup>th</sup> year =  $(\frac{91}{100} \times 273319.20)$   
=shs.248720.50  
After 5<sup>th</sup> year =  $(\frac{248720.50}{100} \times 93)$   
= shs.231310

The next 6years  
 $A = 231310 (1- 0.05)^6 = 170034.10$

Then  $140000 = 170034.10 (1-0.04)^n$

$(0.96)^n = \frac{140000}{170034.10} = 0.8234$

$n = \frac{\log 0.8234}{\log 0.96}$

$= \frac{0.0844}{0.01773} = 4.76 \text{ yrs}$

Total no. of years = 5 + 6 + 4.76yrs = 15.76years

2. Gross tax =  $4830 + 1120 + 600 = sh\ 6550$  per month

$$\text{Annual gross tax} = 6550 \times 12 = 78,600$$

$$\frac{10}{100} \times 120,000 = sh.\ 12,000$$

$$\frac{15}{100} \times 120,000 = sh.\ 18,000$$

$$\frac{25}{100} \times 120,000 = sh.\ 30,000$$

$$\frac{Re. \ tax}{100} = 78600 - (12000 + 18000 + 30000)$$

$$= 78600 - 60,000 = 18,600$$

$$\frac{35}{100} \times x = 18,600$$

$$0.35x = 18,600$$

$$x = sh\ 53142.86$$

$$\text{Taxable income p.a} = 36,000 + 53142.86$$

$$= sh.\ 412142.86$$

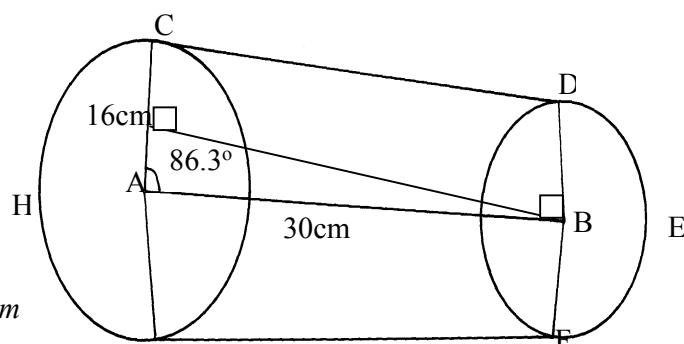
$$\text{Monthly salary} = \frac{412142.86}{12} + 12,000$$

$$= 34428.57 + 1200 = Sh\ 35628.57$$

3. a)  $\sin 86.3^\circ = \frac{XB}{AB}$

$$\sin 86.3^\circ = \frac{XB}{30}$$

$$XB = 30 \sin 86.3^\circ$$



b)  $\angle ABX = 90^\circ - 86.3^\circ$

$$= 3.7^\circ$$

$$\therefore \angle ABD = 3.7^\circ + 90^\circ$$

$$= 93.7^\circ$$

c)  $\angle DBF$  obtuse =  $360^\circ - 187.4^\circ$

$$= 172.6^\circ$$

$$\text{Arc } DEF = \frac{\theta}{360} \pi D \text{ or } \frac{\theta}{360} \times 2\pi r$$

$$\text{But } \cos 86.3^\circ = \frac{AX}{AB}$$

$$\cos 86.3^\circ = \frac{AX}{30}$$

$$AX = 1.935969248 \text{ cm}$$

$$DB = 16 - 1.935969248 = 14.06403075 \text{ cm}$$

$$\therefore \text{Arc } DEF = \frac{172.6^\circ}{360^\circ} \times \frac{22}{7} \times 14.06403075$$

$$= \frac{106807.8751}{2520}$$

$$= 42.38407742 \text{ cm}$$

$$\begin{aligned}
 & \text{Arc } CGH \\
 & <\text{reflex } CAG = 360^\circ - (2 \times 86.3^\circ) \\
 & = 187.4^\circ
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Arc } CGH &= \frac{187.4^\circ}{360^\circ} \times 2 \times \pi \times 16 \\
 &= \frac{131,929.6}{2520} \\
 &= 52.35301587 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 & \text{Total length of belt to go round the belt} \\
 & = CD + DEF + GF + CHG \\
 & = 29.93746855 + 42.38407742 + 29.93746855 + 52.35301587 \\
 & = 154.6120304 \text{ cm}
 \end{aligned}$$

4.  $\angle ABD = 31^\circ$   
 $\angle CBD = 37^\circ$
5.  $A = 15,000(1 + \frac{8}{100})^7$   
 $= \text{Ksh.} 25,707$
6.  $\text{Principle} = 30,000 - 6,000$   
 $= 24,000/$   
 $\text{Amount} = 18 \times 2000$   
 $= 36,000/$   
 $A = P \left[ \frac{1}{100} + r \right]$

$$6,000 = 24000 \left[ \frac{1}{100} + r \right]^8$$

$$\frac{36000}{24000} = \left[ 1 + \frac{r}{100} \right]$$

$$\frac{3}{2} = \left[ \frac{1}{100} + r \right]$$

$$1 + \frac{r}{100} = \sqrt[8]{\frac{3}{2}}$$

$$1 + \frac{r}{100} = 1.023$$

$$\frac{r}{100} = 0.023$$

$$\Rightarrow 2.3\%$$

7. Commission earned Kshs.  $(8368 - 6700) = \text{Kshs.} 1668/$   
let sales in 3rd bracket be  $y$   
 $\frac{10}{100} \times 5000 + (\frac{15}{100} \times 3000) + (\frac{20}{100} \times y) = 1668$   
 $500 + 450 + 0.2y = 1668$   
 $0.2y = 1668 - 950 = 718$   
 $y = \frac{718}{0.2} = 35\%$   
Total sales =  $(8000 + 3590)$   
 $= \text{shs.} 11590$
8. Find the principal which in 12 years at 5% p.a compound interest amounts to sh.450,00  
.  $A = P \left( 1 + \frac{R}{100} \right)^n$   
 $I = A - P$

$$\therefore A = (100 + \underline{R})^n$$

$$\frac{100}{100}$$

$$I = P (100 + \underline{R})^n - P$$

$$\frac{100}{100}$$

$$= P (100 + R/100)n - 1$$

$$\underline{450000} = P = \underline{450000} = 565397$$

9. a) Taxable income =  $(25000 + 12000 + 3000) = 40000$

b) Income tax

$$10164 x \frac{2}{20} = \text{Shs. } 1016.40$$

$$10164 x \frac{3}{20} = \text{Shs. } 1524.60$$

$$10164 x \frac{4}{20} = \text{Shs. } 2032.80$$

*Remaining :*

$$9508 x \frac{5}{20} = \text{Shs. } 2377$$

$$\text{Total tax payable p.m.} = 6950.8 - 1162 = \text{Shs. } 5788.80$$

$$c) \text{Annual tax payable} = 5788.80 x 12 = \text{Shs. } 69465.60$$

10. (a) taxable income = Kshs.  $25000 + \text{Kshs. } 10480$   
 $= \text{Kshs. } 35480$

b) *tax charged:*

$$1^{\text{st}} 4350 = 4350 x \frac{2}{20} = 683.25$$

$$2^{\text{nd}} 4555 = 4555 x \frac{3}{20} - 683.25$$

$$3^{\text{rd}} 4555 = 4555 x \frac{4}{20} - 911$$

$$4^{\text{th}} 4555 = 4555 x \frac{5}{20} - 1138.75$$

$$\text{Rem. } 17465 = 17645 x \frac{6}{20} - 5239$$

$$\text{Total tax} - 8407.5$$

$$\begin{array}{r} 800.00 \\ \hline 7607.50 \end{array}$$

$$(c) 40/100 x 35480 - 14.192 = 49672$$

$$\text{New income} = 35480 + 14192 = 49672$$

$$\text{Remainder} = 49672 - 18015 = 31657$$

$$\text{Tax charged} = 31657 x \frac{6}{20} = 12665.1$$

$$\text{Total tax} = 12665.1$$

$$\% \text{ increase in income ax} = 4257.6 x 100$$

$$7607.5 = 55.97\%$$

11.  $A = P(HR/100)^n$

$$500000 = P(1 + \underline{20})^5$$

$$\frac{100}{100}$$

$$500,000 = (1^{20}/100)^5$$

$$\frac{500,000}{(1.2)^5} = P$$

$$P = \text{Shs. } 200,938.786 \approx \text{shs. } 200,939$$

12. Principal =  $26,000 - 6,000 = 20,000$

$$\text{Total H.P instalments} = 1045.3 x 24 = 25087.20$$

$$25087.20 = 20,000 \left[ \frac{1}{100} + \frac{r}{100} \right]^2$$

$$1.254 = \left[ 1 + \frac{r}{100} \right]^2$$

100

$$1.120 = \frac{1}{100} + r$$

$$\frac{r}{100} = 0.12 \text{ or } 12\%$$

13. No. of periods = 12  
 $r = 4\% \text{ per period}$   
 $A = 1.0412 \times 15000 \quad \left. \right\}$   
 $= 24015.5$

14. a) i) taxable income =  $19200 + 12000 + 1300 + 2300 = 34800$   
 b) Net tax

$$8400 \times \frac{2}{20} = 840$$

$$9600 \times \frac{3}{20} = 1440$$

$$12000 \times \frac{4}{20} = 2400$$

$$4800 \times \frac{5}{20} = \underline{1200}$$

$$5800$$

$$\text{Net tax} = 5800 - 1240 \\ = 4560$$

$$c) \text{ Net salary} = 34800 - (4560 + 5530) \\ = 24710$$

15. (a)  $9000 + 350 + 800 + 1200 = 11350$

(b)  $9000 + 3000 = 12000$

(c) Total taxes =  $12000 \times 12$   
 $= \text{shs. } 144000 \text{ p.a}$

Taxes

$$450 \times 2 = \text{shs. } 9000$$

$$3000 \times 3 = \text{shs. } 9000$$

$$3000 \times 4 = \text{shs. } 12000$$

$$3000 \times 5 = \text{shs. } 15000$$

$$3000 \times 6 = \underline{\text{shs. } 18000}$$

$$\text{Shs. } 63,000$$

$$144000 - 63000 = \text{shs. } 81000$$

$$7y = 81000 \\ y = 11571$$

$$\text{Taxable income} = 4500 + 3000 \times 4 + 11571 = \text{K } 28071 \text{ p.a}$$

$$\text{Gross salary} = \text{shs. } 561420 \text{ p.a}$$

(d) Total allowances =  $12000 \times 12$   
 $= 144,000$

$$\text{Basic salary} = 561420$$

$$\underline{14400}$$

$$\text{Shs. } 417,420$$

$$\text{Monthly basic pay} = \text{shs. } 34785$$

16. (a) Net tax 5512  
 Add relief 1162  
 Tax payable 6674

$$\text{Tax on } 9680 \text{ earned} \\ 9680 \times \frac{10}{100} = 968$$

$$\begin{aligned} \text{Tax on } 9120 \text{ earned} \\ 9120 \times \frac{15}{100} &= \text{Shs. } 1368 \end{aligned}$$

$$\text{Tax on next } 9120 \times \frac{20}{100} = \text{Shs. } 1824$$

$$\text{Tax on next } 9120 \times \frac{25}{100} = 2280$$

$$\text{Total } 968 + 1368 + 1824 + 2280 = 6440$$

$$6674 - 6440 = 234$$

Let  $x$  be charged at 30%

$$\begin{aligned} \frac{30}{100} X x &= 234 \\ X &= \frac{234}{30} \times 100 = \text{Shs. } 780 \end{aligned}$$

Total chargeable Income

$$780 + (9120 \times 3) + 9680 = 37820$$

Salary  $37820 - 15220 = \text{Shs. } 2260$  per month.

$$b) \text{ Net salary } (37820 - 1270 - 6674) = \text{Shs. } 29876$$

17. a) 1<sup>st</sup> year after dep. Of 20%

$$\begin{aligned} 800\,000 \times \frac{80}{100} \\ = \text{Khs. } 640,000 \dots \end{aligned}$$

2<sup>nd</sup> year after dep. of 5%

$$\begin{aligned} &= 640000 \times \frac{95}{100} \\ &= 608,000 \dots \end{aligned}$$

The next 3 yrs

$$\begin{aligned} A &= P \left( 1 - \frac{R}{100} \right)^n = 608,000 \left( 1 - \frac{10}{100} \right)^3 \\ &= 698\,000 (0.9)^3 \\ &= \text{Sh. } 443,232 \dots \\ &800,000 - 443,232 = \text{Sh. } 356,768 \dots \end{aligned}$$

$$\begin{aligned} (b) \text{ S.I.} &= 3000 \times \frac{15}{100} \times 2 \\ &= \text{Sh. } 900 \dots \end{aligned}$$

$$A = 3000 \left( 1 + \frac{15}{100} \right)^2$$

$$\begin{aligned} &= 3000 [1.15]^2 \\ &= \text{sh. } 3967.50 \dots \end{aligned}$$

$$C.I. = \text{sh. } 967.50$$

$$967.50 - 900 = \text{sh. } 67.50 \dots$$

18. (i) Taxable Income

$$\left[ \frac{115}{100} \times 24\,800 \right] + 12000 - 1220$$

$$= 28520 + 12000 - 1220$$

$$= Ksh.39,300$$

$$= \underline{K\£ 1965 p.m.}$$

$$(ii) \text{ Tax due } 325 \times 2 = sh 650$$

$$650 \times 3 = sh 1950$$

$$325 \times 5 = sh 1725$$

$$325 \times 6 = sh 1950$$

$$340 \times 7.50 = sh 2250$$

$$\begin{aligned} \text{Total tax} &= sh. 8825 P.m \\ &\text{without relief} \end{aligned}$$

(b) (i) Total deduction

$$= sh (7280 + 2400 + 1200 + \frac{2}{100} \text{ of } 24800) \dots \dots \dots$$

$$= (7280 + 2400 + 1200 + 496) + 1220$$

$$= sh (11376 + 1220) = \underline{sh. 12,596 P.m} \dots \dots \dots$$

$$(ii) \text{ Net income} = sh (24800 + 1200 - 12596) = sh. 24,204 P.m \dots \dots \dots$$

$$19. \quad a) \text{ Total instalments} = (24 \times 1250) = Shs.30000$$

$$H.P = 7200 + 30000 = 37200$$

$$b) 124\% = 37200$$

$$100\% =$$

$$C.P = \frac{100}{124} \times 37200$$

$$= 30000$$

$$c) A = 30000 (1 + \frac{18}{100})^2$$

$$= 30000 (1.18)^2 = 41772$$

$$\text{Total interest} = 41772 - 30000 = 11772$$

$$20. \quad (a) \quad (i) (10,500 + 6,500) \times \frac{12}{20} = K\£ 10,20 p.a$$

$$(ii) 1^{st} 1980 \times 2 = Kshs. 3960$$

$$2^{nd} 1980 \times 3 = Kshs 5940$$

$$3^{rd} 2480 \times 5 = Kshs. 12400$$

$$4^{th} 1480 \times 7 = Kshs. 10360$$

$$5^{th} 1980 \times 9 = Kshs. 17820$$

$$\text{Last } 300 \times 10 = Kshs \frac{3000}{53480}$$

$$Kshs. 53480$$

$$\text{PAYE} = \frac{53480 - 300 \times 12}{12}$$

$$= Shs. 4156.70$$

(b) Net monthly pay

$$17000 - 320 + \frac{2}{100} \times 17000$$

$$= 17000 - 660$$

$$= Kshs 16340.00$$

### 48. Circles –chords and tangents

1. $6^2 = x(5 + x)$ $x^2 + 5x - 36 = 0$ $(x - 4)(x + 9) = 0$ $x = 4 \text{ or } -4$ $BC = 4\text{cm}$	<b>M1</b> <b>M1</b> <b>A1</b> <b>03</b>	Correct factorisation

1. a) i)  $\angle DCF = \frac{180 - 92}{2} = 44^\circ = \angle CAD$

ii)  $\angle BAO = 50^\circ$

Acute angle  $AOB = 80^\circ$

$\therefore$  obtuse angle  $= 360 - 80 = 280^\circ$

b) Area of the sector  $= (\frac{80}{360} \times \pi \times 7 \times 7) = 34.22\text{cm}^2$

Area of the  $\Delta = \frac{1}{2} \times 7 \times 7 \times \sin 80^\circ = 24.13\text{cm}^2$

Area of the shaded segment  $= 34.22 -$

$$\frac{24.13}{10.09\text{cm}^2}$$

2.  $\angle COB = 2 \times 50 = 100^\circ$

$\angle OCA = \frac{\angle OAC}{2} = \frac{180 - 100}{2} = 40^\circ$

$\therefore \angle BAC = 180 - (50 + 70)$

$= 60^\circ$

3.  $PB \cdot PA \cdot (PT)^2$

$$\frac{PB}{PT} = \frac{PT}{PA}$$

$$\frac{4}{12} = \frac{12}{4 + 2r}$$

$$\frac{4(4+2r)}{4} = \frac{12^2}{4}$$

$4 + 2r = 36$

$2r = 32$

$r = 16 \text{ cm}$

4. (a)  $\angle BOE = 2 \angle BCE = 2 \times 20^\circ = 40^\circ$

(b)  $\angle BOE = 40^\circ$

$\angle BEC = \frac{1}{2} (360^\circ - 60^\circ) = 150^\circ$

Angles subtended at the centre is twice at the Circumference.

c)  $\angle CEF = 90^\circ - 80^\circ = 10^\circ$

d)  $\angle BCO = \angle CBO = 60^\circ$

Base angles isosceles triangle.

$$\begin{aligned} \angle OXC &= 180^\circ - (60^\circ + 20^\circ) \\ &= 100^\circ \end{aligned}$$

e)  $\angle BCE = 20^\circ$

$\angle CXE = 180^\circ - 100^\circ = 80^\circ$

$$\angle CEX = 80^\circ$$

$$\begin{aligned}\angle OEF &= 180^\circ - (80^\circ + 50^\circ + 10^\circ) \\ &= 40^\circ\end{aligned}$$

5. (a)  $PQ = \sqrt{8^2 - 2^2}$   
 $= 60$   
 $= 7.746\text{cm}$

(b)  $\angle PAS = 2\cos^{-1}$   
 $= 151^\circ$   
 $\therefore \text{Reflex } \angle PAS = 209^\circ \text{ OR } 360^\circ - 151^\circ = 209^\circ$

(c) Length PYS =  $\frac{209}{360} \times 2 \times 6 = 21.89\text{cm}$

Length QXR =  $\frac{151}{360} \times 2 \times 4 = 10.54\text{cm}$

(d) Length of belt =  $7.74 \times 2 + 21.89 + 10.54$   
 $= 47.92\text{cm}$

6. a) i) In 1 hr; Tap A fills  $\frac{1}{3}$   
 $B - \frac{1}{4}$

Capacity filled in 1 hr =  $\frac{1}{3} + \frac{1}{4}$   
 $= \frac{7}{12}$   
 $\frac{7}{12} = 1 \text{ hr}$   
 $1 = 1 \times 1 \times \frac{12}{7}$   
 $= 1 \frac{5}{7} \text{ hrs.}$

ii)  $\frac{1}{3} + \frac{1}{4} - \frac{1}{6} = \frac{5}{12} \Rightarrow \text{in one hr}$   
 $\frac{5}{12} = 1 \text{ hr}$   
 $1 = 1 \times 1 \times \frac{12}{5}$   
 $= 2 \frac{2}{5} \text{ hrs}$

7.  $\angle ABD = 31^\circ$   
 $\angle CBD = 37^\circ$

8.  $x(x+9) = 4x9$   
 $x^2 + 9x - 36 = 0$   
 $(x^2 - 3x) + (12x - 36) = 0$   
 $x(x-3) + 12(x-3) = 0$   
 $(x+12)(x-3) = 0$   
 $x - 3 = 0$   
 $x = 3 \text{ only}$

9.  $PO \cdot OQ = BO \cdot OA$   
 $8 \times 6 = 4.5 \times y$   
 $y = \frac{8 \times 6}{4.5}$   
 $= 10.67$

10.  $\angle DGB = \angle ABG = 40^\circ \text{ (alt.seg } \angle s)$   
a)  $\angle DGE = \angle DBE = 25^\circ \text{ ( $\angle s$  in same segment)}$

b)  $\angle EFG$ 

$$\angle GEB = 40^\circ, \angle BDG \text{ and } \angle BED = 45^\circ = \angle BGD$$

$$\therefore \text{In } \triangle GED, \angle GDE = 180 - (25 + 40 + 45) = 70^\circ$$

$$\therefore \angle GFE = 180 - 70 = 110^\circ \text{ (Sup angles)}$$

$$d) \text{Angle CBD in } \triangle BGE, \text{Angle GBE} = 180 - (110) = 70^\circ$$

$$\therefore \text{Angle CBD} = 180 - (40 + 70 + 25) = 45^\circ$$

Or Angle CBD = Angle BGD =  $45^\circ$  (Angles in Alt segment)

$$e) \text{Angle BCD in } \triangle BCD, \text{Angle BDC} = 70^\circ \text{ Angles in a straight line}$$

$$\therefore \text{Angle BCD} = 180 - (70 + 45) \text{ Angles of a triangle} = 65^\circ$$

$$11. (a) \sin \theta = \frac{4.5}{8} = 0.5025$$

$$\theta = \sin^{-1} 0.5625$$

$$= 34.23^\circ$$

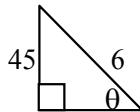
$$\angle Apb = 68.46^\circ$$

$$\sin \alpha = \frac{4-5}{6} = 0.75$$

$$\alpha = \sin^{-1} 0.75$$

$$= 48.59^\circ$$

$$\angle Aqb = 97.18^\circ$$



$$(b) \text{Area Of Segment PAB} = \frac{68.46 \times 22}{360} \times 8 \times 8 - \frac{1}{2} \times 8 \times 8 \sin 68.46$$

$$= 38.25 - 29.77$$

$$= 8.48 \text{ cm}^2$$

$$\text{Area Of Segment AQB} = \frac{97.18 \times 22}{360} \times 36 - \frac{1}{2} \times 36 \sin 97.18$$

$$= 30.65 - 17.86 = 12.68 \text{ cm}^2$$

$$\text{Area of quadrilateral APBQ} = \frac{1}{2} \times 64 \sin 68.46 + \frac{1}{2} \times 36 \sin 97.18$$

$$= 29.77 + 17.86 = 47.63$$

$$\text{Shaded area} = 47.63 - (8.48 + 12.68) = 26.47 \text{ cm}^2$$

$$12. \quad CBD = 90 - 42 = 48^\circ$$

Angle of triangle add to  $180^\circ$

$$DOB = 180^\circ - 42 = 138^\circ$$

Opposite angles of cyclic quadrilateral add to  $180^\circ$

$$DAB = \frac{138^\circ}{2} = 69^\circ$$

Angle at circumference is half the angle subtended at centre by same chord

CDA

$$ABD = 90 - 48 = 42^\circ$$

$$ADB = 180 - (69 + 42)$$

$$180 - 111 = 69^\circ$$

$$CDA = 90 + 69^\circ = 159^\circ$$

Show  $\triangle ADB$  is a scalene triangle

$$\angle DAB = 69^\circ$$

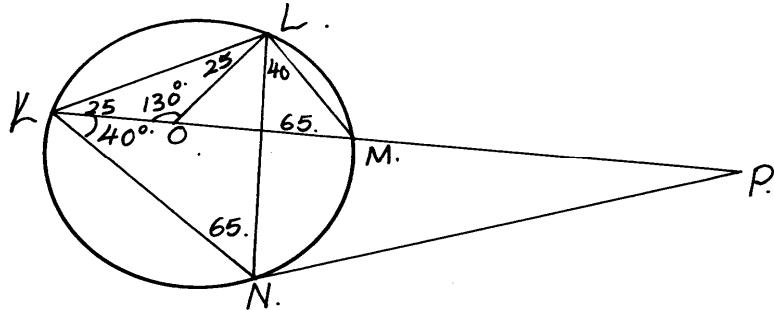
$$\angle DAB = 69^\circ$$

$$\angle ADB = 69^\circ$$

$$\angle ABD = 42^\circ$$

So two angles are equal hence it is a cyclic quadrilateral

13.



- a)  $MLN = 40^\circ$  angles subtended by same chord in the same segment are equal.
- b)  $OLN = 90 - 65 = 25^\circ$   
Angle sum of  $\Delta$  is  $180^\circ$  or angle subtended by  $>$  diameter is  $90^\circ$ .
- c)  $LNP = 65^\circ$  exterior  $\Delta$  is equal to opposite interior angle or angle btwn a chord and a tangent is equal to angle subtended by the same chord in the alternate segment.
- d)  $MPN = 180 - 170 = 10^\circ$  angle sum of a  $\Delta$  is  $180^\circ$
- e)  $LMO = 65^\circ$  angles subtended by same chord.

14. (a)

$$\begin{aligned} \sin \theta &= \frac{4}{4.6} = 0.869565 \\ &= \sin^{-1} 0.869565 = 60.408^\circ \\ ABR &= 2 \times 60.408^\circ = 120.8163^\circ C \\ &\approx 120.82^\circ \text{ (2d.p)} \end{aligned}$$

(b) Area of sector  $ABCR$

$$= \frac{120.8163^\circ}{360^\circ} \times \pi \times 4.6^2 \text{ cm}^2$$

$$= 22.30994 \text{ cm}^2$$

Area of sector  $OAPC$

$$= \frac{60^\circ}{360^\circ} \times \pi \times 8^2 \text{ cm}^2$$

$$= 33.51032 \text{ cm}^2$$

$$= 33.51 \text{ cm}^2 \text{ (2d.p)}$$

$$\text{Area of } \triangle ABC = (\frac{1}{2} \times 4.6^2 \sin 120.8163) \text{ cm}^2 = 9.08625 \text{ cm}^2$$

$$\text{Area of } \triangle AOC = (\frac{1}{2} \times 8^2 \sin 60) \text{ cm}^2 = 27.7128 \text{ cm}^2$$

$$\text{Sum of area of } \Delta s = 36.799 \text{ cm}^2 \quad 36.80 \text{ cm}^2$$

$$\therefore \text{Area of shaded part} = \text{area of sectors} - \text{area of } \Delta s$$

$$= (22.31 + 33.51 - 36.80) \text{ cm}^2 = 19.02 \text{ cm}^2 \text{ (2dp)}$$

15.

(a)  $\angle TDC = ABT$  (exterior opp. angle of a cyclic quadrilateral)  
 $= 100^\circ$

(b)  $BAT = ATB$  (base angles of isosceles  $ATB$ )  
 $= 180 - 100 = 40^\circ$

(c)  $\angle TCD = \angle XTD$  (angles in alternate segments)  
 $= 60^\circ$

Or  $\angle BTC + 40^\circ = 100^\circ$  (exterior angle of a  $\Delta$ )

$$\angle BTC = 100^\circ - 40^\circ = 60^\circ$$

$$(d) DTC = 180^\circ - (58^\circ + 100^\circ) \text{ (angles in } \triangle TDC) = 12^\circ$$

16. a)  $GBD = 90^\circ$

$$\begin{aligned} ABG &= 180 - (90 + 36) \\ &= 180 - 126 = 54^\circ \\ GEB &= ABG = 54^\circ \end{aligned}$$

b)  $BED = CBD = 36^\circ$

c)  $DGE = FEG = 20^\circ$

$$\begin{aligned} OEB &= 90 - (36 + 20) \\ &= 90 - 56 = 34^\circ \end{aligned}$$

$OBE = OEB = 34^\circ$

d)  $BGE = 36 + 20 = 56^\circ$

e)  $GFE = 180 - EDG$

$$= 180 - 70 = 110^\circ$$

17.  $XZ^2 = 13.4^2 + 5^2 - 2 \times 13.4 \times 5 \cos 57.7^\circ$

$$= 170.56 + 25 - 134 \times 0.5344$$

$$= 204.56 - 71.6096$$

$$XZ^2 = 132.9504$$

$$XZ = 11.5304 \text{ cm}$$

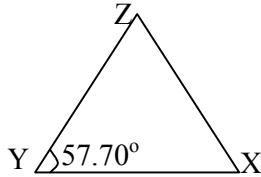
(ii)  $2R = 11.5304$

$$\sin 57.7^\circ$$

$$2R = \frac{11.5304}{0.8453}$$

$$2R = 13.60866$$

$$R = 6.08043 \text{ cm}$$



18.  $52 = 62 + 62 - 2 \times 6 \times 6 \cos A$

$$72 \cos A = 72 - 25 = 46$$

$$\cos A = 46/72 = 0.6389$$

$$A = \cos^{-1} 0.6389 = 50.29^\circ$$

Area of the minor sector  $APQ$

$$= \frac{50.29}{360} \times 3.142 \times 6^2$$

$$= 15.801 \text{ cm}^2$$

Area of the triangle  $APQ$

$$= \frac{1}{2} \times 6 \times 6 \sin 50.29 = 13.847 \text{ cm}^2$$

Area of the minor segment

$$= (15.801 - 13.847) \text{ cm}^2 = 1.954 \text{ cm}^2$$

Area of triangle  $PBQ$

$$\sqrt{6.5(6.5-4)(6.5-4)(6.5-5)}$$

$$\sqrt{6.5 \times 2.5 \times 2.5 \times 1.5} = 7.806 \text{ cm}^2$$

$$\text{Area of shaded region} = (7.806 - 1.954) \text{ cm}^2 = 5.852 \text{ cm}^2$$

19. a)  $\angle PQR = 180^\circ - 75^\circ$

$\angle = 105^\circ$ .  $NPQR$  is cyclic quadrilateral.

$$(b) \angle NRP = 90^\circ - 75^\circ \\ = 15^\circ, \text{ Third angle of } \triangle NRP.$$

$$\angle PRS = 180^\circ - 65^\circ, \text{ Angles on a} \\ = 115^\circ, \text{ straight line.}$$

$$\therefore \angle QSR = 180^\circ - (115^\circ - 35^\circ) \\ = 30^\circ, 3^{\text{rd}} \text{ angle of triangle } PRS.$$

$$(c) \text{Reflex } \angle POR = 2 \angle PQR \\ = 2 \times 105^\circ = 210^\circ$$

$$(d) \angle MQR = \angle MNR = 40^\circ \\ \text{Subtended by same chord } MR$$

20.

- (a)  $\angle TDC = 100^\circ$  (Cyclic quadrilateral)
- (b)  $\angle TCB = 40^\circ$  (Cyclic quadrilateral)
- (c)  $\angle TCD = 58^\circ$  (Cyclic quadrilateral)
- (d)  $\angle BTC = 60^\circ$  (Sum angle of a  $\Delta$  add upto  $180^\circ$ )
- (e)  $\angle DTC = 22^\circ$  (angle sum of a straight line add upto  $180^\circ$ )

21.  $4x10 = 5(5+x)$

$40 = 25 + 5x$

$3 = x$

22.  $T_{11} = a + 10d$

$T_2 = a + d$

$a + 10d = 4a + 4d \dots \dots \dots (i)$

$3a - 6d = 0$

$S7 = \frac{7}{2}\{2a + 6d\} = 175 \dots (ii)$

$2a + 6d = 50$

$\underline{3a - 6d = 0}$

$5a = 50$

$a = 10$

$d = 5$

23.  $CBE = 40^\circ$  (alt.segment theorem)

$\angle BCE = 120^\circ$  (Suppl. To  $BCD = 60^\circ$  alt. seg.)

$\therefore (40 + 120 + E) = 180^\circ$  (Angle sum of  $\Delta$ )

$\angle BEC = 20^\circ$

24.  $\text{Taxable income p.a} = 36,000 + 53142.86 \\ = sh. 412142.86$

$\text{Monthly salary} = \frac{413142.86}{12} + 12,000 \\ = 34428.57 + 1200 = Sh 35628.57$

25. a) (i)  $\angle PTQ = 180^\circ - 56^\circ = 124^\circ$

$124 + 38 = 162^\circ$

$180^\circ - 162^\circ = 18^\circ$

$90^\circ + 18^\circ = 108^\circ$

$180^\circ - 108^\circ = 72^\circ$

$180^\circ - (72^\circ + 56^\circ) = 52^\circ$

$\angle PRS = 52^\circ$

**Value of the constant.**

(ii)  $\angle RSQ = \angle RPQ = 18^\circ$

b)  $A \propto B. \underline{I}$

**Substitution Formulation**

$$\begin{aligned}
 C^3 & \\
 A &= \frac{K \cdot B}{C^3} \\
 12 &= \frac{3K}{2^3} \\
 K &= \cancel{42} \cancel{x} \cancel{8} = 32 \\
 \therefore A &= \frac{32B}{C^3} \\
 \frac{10 \times (1.5)^3}{32} &= B \\
 \therefore B &= 1.055
 \end{aligned}$$

c)  $y = K + Mx^2$  where  $K$  and  $M$  are constants

$$\begin{array}{l|l}
 7 = K + 100M & 100 \times 0.005 + K = 7 \\
 \underline{5.5 = K + 400M} & -0.5 + K = 7 \\
 \hline
 1.5 = 300M & K = 7.5
 \end{array}$$

$$\begin{aligned}
 M &= 0.005 \\
 y &= 7.5 - 0.005 \times 18^2 \\
 y &= 7.5 - 1.62 \\
 y &= 5.88 \\
 26. \quad a) PN^2 &= 5^2 - 4^2 \\
 PN &= 3\text{cm} \\
 QN^2 &= 6^2 - 4^2 \\
 QN &= 4.47\text{cm} \\
 \therefore PQ &= 3 + 4.47 = 7.47
 \end{aligned}$$

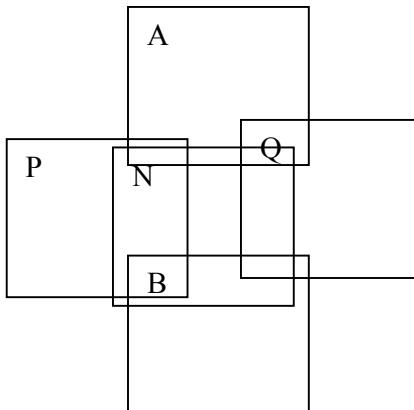
$$b)i) < APB$$

$$\sin \frac{1}{2} \theta = 0.8$$

$$\frac{1}{2} \sin \theta = 53.13$$

$$< APB$$

$$\begin{aligned}
 ii) \quad \sin \frac{1}{2} \alpha &= 4/6 = 0.6667 \\
 \frac{1}{2} \alpha &= 41.81 \\
 \alpha &= 83.62 \\
 \therefore < AQB &= 83.62^\circ
 \end{aligned}$$



$$\begin{aligned}
 c) \text{Area of the shaded region} - \text{Area of the segments} \\
 &= \frac{106.3 \times 22 \times 5^2}{360} - \frac{1}{2} \times 5 \times 5 \sin 106.3
 \end{aligned}$$

$$\begin{aligned}
 &= 23.19 - 11.998 = 11.192 \\
 &\frac{83.6 \times 22 \times 6 \times 6}{360} - \frac{1}{2} \times 6 \times 6 \sin 83.6 = 8.38 \\
 \text{Total } 11.192 + 8.38 &= 19.52
 \end{aligned}$$

27. Using cosine rule

$$\begin{aligned}
 7.8^2 &= 6.6^2 + 5.9^2 - 2 \times 6.6 \times 5.9 \cos R \\
 \cos C &= \frac{6.6^2 + 5.9^2 - 7.8^2}{2 \times 6.6 \times 5.9} \\
 &= \frac{43.59 + 34.81 - 60.84}{77.88} = \frac{78.37 - 60.84}{77.88} \\
 &= \frac{17.53}{77.88} = 0.2251 \\
 \angle C &= 77^\circ \\
 \frac{7.8}{\sin 77} &= 2r \Rightarrow r = \frac{7.8}{2 \times \sin 77} \\
 &= 4 \text{ cm} \\
 \text{Area of circle} &= 3.142 \times 4^2 = 50.27 \\
 \text{Area of } \triangle PQR &= \frac{1}{2} (6.6) (5.9) \sin 77 \\
 &= 18.97 \\
 \therefore \text{Area of shaded region} &= 50.27 - 18.97 = 31.30 \text{ cm}^2
 \end{aligned}$$

28. a)  $\angle PAQ = 2 \angle PAB = 42^\circ \times 2 = 84^\circ$   
 $\angle PBQ = 2 \angle ABQ = 30^\circ \times 2 = 60^\circ$

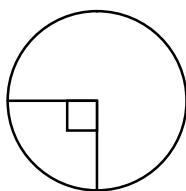
(b) (i) Area of sector $APQ = \frac{84}{360} \times \frac{22}{7} \times 6 \times 6 = 26.4 \text{ cm}^2$	$\checkmark \text{ angle}$
Area of sector $PBQ = \frac{60}{360} \times \frac{22}{7} \times 8 \times 8 = 33.5 \text{ cm}^2$	$\checkmark \text{ angle}$
(ii) Area of $\triangle APQ = \frac{1}{2} \times 6 \times 6 \times \sin 84^\circ = 18 \times 0.9945 = 17.9 \text{ cm}^2$	$\checkmark$
Area of $\triangle PBQ = \frac{1}{2} \times 8 \times 8 \times \sin 60^\circ = 64 \times 0.8660 = 54.7 \text{ cm}^2$	$\checkmark$
	$\checkmark \text{ diff. areas}$
	$\checkmark \text{ diff. areas}$
	$\text{Exp. for total}$
	$\checkmark \text{ answer.}$

(iii) For each circle, shaded area = sector area - triangle Area.

$$\begin{aligned}
 &= \text{area of sector } APQ - \text{area of triangle } APQ \\
 &= 26.4 - 17.9 = 8.5 \text{ cm}^2 \\
 &\text{2nd circle, shaded area} \\
 &= \text{area of sector } PBQ - \text{area of } \triangle PBQ \\
 &= 33.5 - 27.7 = 5.8 \text{ cm}^2
 \end{aligned}$$

$$\text{Total shaded area} = 8.5 + 5.8 = 14.3 \text{ cm}^2$$

29.  $\frac{90}{360} \times 3.142 \times 2 \times 6.5$   
 $\frac{10.2115}{10.21} \text{ cm}$



## 49. Matrices

<b>1.</b> $C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ <p>Let <math>\begin{pmatrix} 1 &amp; 3 \\ 4 &amp; 1 \end{pmatrix} \begin{pmatrix} a &amp; b \\ c &amp; d \end{pmatrix} = \begin{pmatrix} 2 &amp; 4 \\ 3 &amp; 6 \end{pmatrix}</math></p> $\begin{aligned} (11a + 3c = 2) \times 1 && (11b + 3d = 4) \times 1 \\ (4a + c = 3) \times 3 && (4b + d = 6) \times 3 \\ 11a + 3c = 2 && 11b + 3d = 4 \\ 12a + 3c = 9 && 12b + 3d = 18 \\ a = 7 && b = 14 \\ c = -25 && d = -50 \\ \therefore C = \begin{pmatrix} 7 & 14 \\ -25 & -50 \end{pmatrix} \end{aligned}$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub> 3	Alternative $C = B^{-1}A$ $\begin{pmatrix} \checkmark \text{equations } B^{-1} \\ -1 & 3 \\ 4 & -11 \end{pmatrix}$ allow any two ✓ solving of equations $\begin{pmatrix} -1 & 3 \\ 4 & -11 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ or equivalent
<b>2.</b> $\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 0 \end{pmatrix}$ $Det 4 - 9 = 13$ $\frac{1}{13} \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 13 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $x = 2, y = 3$	M1 M1 A1 03	

1.

$$\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

*Premultiplication by the inverse.*

$$\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{-3}{11} \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{-3}{11} \end{pmatrix} \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

*Simplification.**C.A.O*

$$a = 2 \checkmark \text{ and } b = 3 \checkmark$$

$$\begin{aligned}
 2. \quad & (x-3) - (2x) = 0 \\
 & x-3-2x = 0 \\
 & -2x + x - 3 = 0 \\
 & -x - 3 = 0 \\
 & x = 3
 \end{aligned}$$

$$3. \quad \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} \quad \begin{bmatrix} 7 & 3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -13 & -7 \\ -4 & -2 \end{bmatrix}$$

$$\begin{aligned}
 4. \quad & \text{Determinant} = +65 - 49 = 16 \\
 & C \begin{bmatrix} 1 & -5 \\ 7 & -13 \end{bmatrix} \\
 & \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 2 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 3a + 2c &= 9 \\
 2a + 2c &= 2 \\
 a &= 7 \\
 c &= -6 \\
 3b + 2d &= -3 \\
 2b + 2d &= 1 \\
 b &= -4 \\
 d &= 4.5 \\
 A &= \begin{bmatrix} 7 & -4 \\ -6 & 4.5 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & 20x(-3 - 8) \\
 & 100 \text{ area of } 1^{\text{st}} \text{ image.} \\
 & 100x(4 - 3) \\
 & 700 \text{ area of } 2^{\text{nd}} \text{ image} \\
 6. \quad & \text{Det. } 9 + 2 = 11
 \end{aligned}$$

$$\begin{aligned}
 A^I &= \frac{1}{11} \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} \\
 \begin{bmatrix} 3 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 10 \\ 4 \end{bmatrix} \\
 \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{11} \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \end{bmatrix} \\
 \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{11} \begin{bmatrix} 22 \\ 22 \end{bmatrix} \\
 \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 2 \\ 2 \end{bmatrix}
 \end{aligned}$$

$$P(2, 2)$$

7.  $PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & q \end{pmatrix} \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$x=1 \quad y=-2$$

8.

$$\begin{aligned} \frac{1}{2}x - \frac{1}{4}y &= 2 \\ \frac{2}{5} + \frac{1}{6} &= 6 \\ 2x - y &= 8 \\ 12x + 5y &= 180 \\ \frac{10x - 5y = 40}{22x} &= 220 \\ x &= 10 \\ \frac{1}{4}y &= \frac{1}{2}(10) - 2 \\ \frac{1}{4}y &= 5 - 2 = 3 \\ Y &= 12 \end{aligned}$$

9.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -6 \\ 1 & 4 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X^I & Y^I & Z^I \\ -1 & -2 & -6 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 9 \\ -1 & -2 & -6 \end{pmatrix}$$

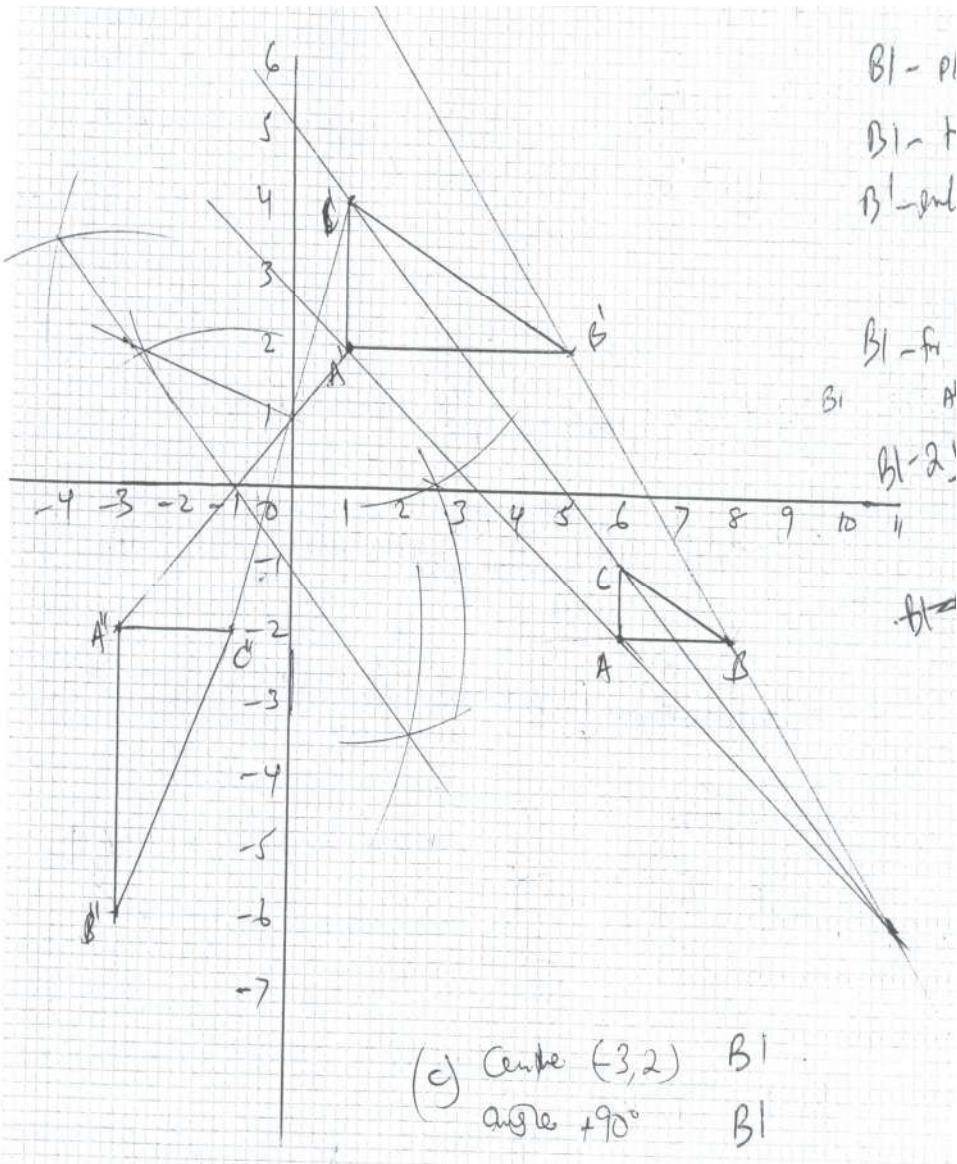
Final image  $X^{II}(1, -1)$ ,  $Y^{II}(4, -2)$ ,  $Z^{II}(9, -6)$

10.

$$a|: \begin{pmatrix} P & Q & R \\ 2 & 5 & -1 \\ 0 & -1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} A & B & C \\ 6 & 8 & 6 \\ 2 & 2 & -1 \end{pmatrix}$$

(c) Centre (-3, 2)  
Angle + 90°

a)  $\begin{pmatrix} 2 & 5 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 6 & 8 & 6 \\ 1 & -1 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 6 & 8 & 6 \\ 2 & 2 & -1 \end{pmatrix}$



11.  $\text{Det} \begin{vmatrix} 2 & -3 \\ -3 & 5 \end{vmatrix} = 5$   
 $\text{Area of } A'B'C' = 5 \times 15$   
 $= 75 \text{ cm}^2$

12.  $A.S.F = \frac{110}{10} = 11$   
 $5X(X) - -6 = 11$   
 $5X^2 + 6 = 11$   
 $5x^2 = 5$   
 $X^2 = 1$   
 $X = \pm 1$

13.  $\text{Area of the image} = \text{Area of the object} \times \text{Det.}$   
 $\text{Det. } (\Delta) = 15 - 18 = -3$   
 $54 \text{ cm}^2 = A \times -3$   
 $\underline{54} \text{ cm}^2 = A$

Area of  $\Delta ABC = 18 \text{ cm}^2$ 14. Det.  $9 + 2 = 11$ 

$$A^I = \frac{1}{11} \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 22 \\ 22 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

 $P(2, 2)$ 

## 50. Formulae and variation

<b>1.</b> $X = \frac{U^2 V}{U^2 2W}$ $U^2 X + 2WX = U^2 V$ $U^2 X - U^2 V = -2WX$ $U^2(X - V) = -2WX$ $U^2 = \frac{-2WX}{X - V}$ $U = \pm \sqrt{\frac{-2WX}{X - V}}$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	Collecting terms of $U^2$ C.A.O $U = \pm \sqrt{\frac{2WX}{V - X}}$
3		
<b>2.</b> $P = Kt + Ct^2$ $45 = 20K + 400C \dots\dots(i)$ $60 = 24K + 576C \dots\dots(ii)$ $(80C + 4K = 9)2$ $(48C + 2K = 5)4$ $160C + 8K = 18$ $192C + 8K = 20$ $-32C = -2$ $C = \frac{1}{16} \therefore K = 1$ $P = 32 + \frac{32 \times 32}{16}$ $= 96$	M1 M1 A1 B1	Allow for one ✓ equation ✓ Attempt to solve For both values
4		

<p><b>3.</b></p> $V = \frac{K}{W^2} + C$ $W = 2, V = 14$ $14 = \frac{K}{2^2} + C \Rightarrow 56 = K + 4c \dots\dots\dots(i)$ $9 = \frac{K}{3^2} + C \Rightarrow \frac{81}{9} = K + 9c \dots\dots\dots(ii) \quad (i)-(ii)$ $5 = c$ $56 = k + 20$ $36 = k$ $V = \frac{36}{W^2} + 5$ $V = \frac{36}{6^2} + 5 = 5$	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p>	
		04
<p><b>4.</b></p> $T^2 = \frac{1}{4} \left( \frac{2}{x+y} \right)$ $4T^2(x+y) = 2$ $x+y = \left( \frac{2}{4T^2} \right)$ $y = \frac{1}{2T^2} - x$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Removal of root sign</p> <p>Simplification</p> <p>Y expressed in simplified form</p>
		03

1.  $P = kr^2 ; R = MT^2$   
 $18 = 9k \quad 3 = 25m$

$$K = 2 \quad M = \frac{3}{25}$$

$$P = 2R^2 \quad R = \frac{3}{25} T^2$$

$$\left( P = 2 \right)^2 \frac{3}{25} T^2 = \frac{18}{625} T^4$$

$$P = \frac{18 \times 10000}{625} = 288$$

2.  $v^2 = \frac{r}{r+c}$   
 $v^2 \cancel{(r+c)} = \cancel{v} (r+c)$   
 $v^2 r + vc = r$   
 $r - v^2 r = vc$   
 $r(1-v^2) = vc$   
 $r = \frac{vc}{1-v^2}$

*Removing the sg. Root.*

*Factorization.*

*C.A.O*

$$I - v^2$$

3.  $X \propto \frac{Y^3}{\sqrt{Z}}$

$$6 = \frac{K(3)^3}{\sqrt{25}}$$

$$6 = \frac{27K}{5}$$

$$K = \frac{10}{9}$$

$$\therefore X = \frac{10}{9} \cdot \frac{Y^3}{\sqrt{Z}}$$

$$X = \frac{10}{9} \cdot (7)^3$$

$$= \left( \frac{10 \times 343}{27} \right)$$

$$= 127.04$$

(a)  $Y^3 = \frac{9}{10}x Z$

$$Y = \sqrt[3]{\frac{9x}{10} \cdot 4x \cdot 8}$$

$$Y = \sqrt[3]{\frac{144}{5}} = 3.07$$

(b)  $X_1 = \frac{KY^3}{\sqrt{Z}}$

$X_2 = \frac{K(1.2y)^3}{\sqrt{0.64Z}}$

$$\frac{1.728KY^3 - KY^3}{\sqrt{0.8Z}} \quad M_1$$

$$\left( \frac{\frac{2.16KY^3 - KY^3}{\sqrt{Z}}}{\frac{KY^3}{\sqrt{Z}}} \right) \times 100 \% \quad M_1$$

$$= 116 \% \quad A_1$$

4.  $K(b-a) = ab$

$$Kb - ka = ab$$

$$Kb - ab = ka$$

$$B(k-a) = ka$$

$$B = ka$$

$$K-a$$

5.  $x - 2.5 - \sqrt{3} \quad x - 2.5 + \sqrt{3} = 0$

$$\frac{x^2 - 2.5x + x\sqrt{3} - 2.5x + 6.25 - 2.5\sqrt{3}}{x\sqrt{3} + 2.5\sqrt{3}} = 0$$

$$x^2 - 5x + 6.25 - 3 = 0$$

$$x^2 - 5x + 3.25 = 0$$

$$4x^2 - 20x + 13 = 0$$

6.  $Z = \frac{Kx^2}{\sqrt{y}}$   
 $Z = \frac{(1.2x)^2 K}{\sqrt{0.64y}}$   
 $= \frac{1.44Kx^2}{0.85y}$   
 $= 1.8 \frac{Kx^2}{\sqrt{y}}$   
 $\% \text{ increase} = 80\%$

7.  $ar^3 = 48$   
 $ar^6 = 384$   
 $\therefore \frac{ar^6}{ar^3} = \frac{384}{48}$   
 $r^3 = 8$   
 $r = 2$   
 $ar^3 = 48$   
 $8a = 48$   
 $a = 6$   
 $S_n = \frac{a(r^n - 1)}{r - 1}$   
 $\frac{6(2^6 - 1)}{2 - 1}$   
 $= 6(64 - 1)$   
 $= 6 \times 63$   
 $= 378$

8.  $P = \frac{KQ^2}{R}$   
 $2 = \frac{16K}{6}$   
 $K = \frac{3}{4}$   
 $P = \frac{3}{4} \frac{Q^2}{R} = \frac{3}{4} \times \frac{64}{4} = 12$

9.  $B \& M^2 = \frac{1}{N}$   
 $B = Km^2 + \frac{Q}{N}$   
 $(96 = 4K + 2Q)^3$   
 $(46 = 3K + 0.5Q)^4$   
 $104 = 4Q$   
 $Q = 26$   
 $K = 11$   
 $\text{Expression } B = 11m^2 + \frac{26}{N}$

10.  $3x = y - 1 \quad \dots \dots \dots i$   
 $\frac{2x + 2}{y - 5} = \frac{1}{2}$   
 $4x + 4 = y - 5$   
 $4x + 9 = y \quad \dots \dots \dots ii$

$$\begin{aligned} 3x &= y - 1 \\ \underline{4x = y - 9} \\ -x &= 9 \quad x = -9 \end{aligned}$$

$$\begin{aligned} -27 &= y - 1 \\ y &= -26 \end{aligned}$$

$$11. \quad P = \sqrt[3]{\frac{x-1}{x+2}} \Rightarrow P^3 = \frac{x-1}{x+2}$$

$$\begin{aligned} P^3x - 2P^3 &= x - 1 \\ P^3x - x &= -1 - 2P^3 \\ x(P^3 - 1) &= -1 - 2P^3 \end{aligned}$$

$$\begin{aligned} x &= \left( \frac{-1 - 2P^3}{P^3 - 1} \right) - 1 \\ x &= \frac{1 + 2P^3}{1 - P^3} \end{aligned}$$

$$\begin{aligned} 12. \quad a^4 &= \frac{1+d^2}{b^2} + \frac{b}{3} \\ 3d^2 &= 3a^4b^2 - b^2 - 3 \\ d &= \sqrt{\frac{3a^4b^2 - b^2 - 3}{3}} \end{aligned}$$

$$\begin{aligned} 13. \quad (a) Z &= \frac{KX^2}{y^2} \\ Z &= \frac{100k}{16} = 15 \\ K &= \frac{12}{5} \\ Z &= \frac{12x^2}{5y^2} \end{aligned}$$

$$(b) Z = 21.90$$

$$\begin{aligned} 14. \quad R &= kn + t\sqrt{n} \\ 9k + 3t &= 42 \\ 25k + 5t &= 100 \\ 45k + 15t &= 210 \\ \underline{75k + 15t = 300} \\ -30k &= -90 \\ k &= 3 \\ t &= 5 \end{aligned}$$

$$\begin{aligned} 15. \quad R &= 3(16) + S(4) = 68 \\ a^2 &= \frac{b^2 d^2}{b^2 + d} \\ a^2 b^2 + a^2 d &= b^2 d^2 \end{aligned}$$

$$\begin{aligned} b^2 d^2 - a^2 b^2 &= a^2 d^2 \\ b^2 (d^2 - a^2) &= a^2 d^2 \end{aligned}$$

$$\begin{aligned} b^2 &= \frac{a^2 d^2}{d^2 - a^2} \\ b &= \sqrt{\frac{a^2 d^2}{d^2 - a^2}} \end{aligned}$$

16.  $P = KQ + m\sqrt{Q}$   
 $22 = K(4) + m(2) \dots \dots \dots (1)$   
 $42 = K(g) + n(3) \dots \dots \dots (2)$   
 $22 = 4K + 2m$   
 $42 = 9K + 3m$   
 $3(22) = 3(4K) + 3(2m)$   
 $2(42) = 2(9K) + 2(3)$   
 $66 = 12k + 6m$   
 $84 = 18K + 6m$   
 $18 = 6k = k=3$   
 $22 = 4(3) + 2m$   
 $22-12 = 2m$   
 $20 = 2m$   
 $M = 10$   
 $= 3(25) + 10(5)$   
 $= 75 + 50$   
 $= 125$

17.  $b = \sqrt{k-ac}$   
 $b^2 = k-ac$   
 $b^2 - k = -ac$   
 $\frac{b^2 - k}{-9} = c$   
 $C = \frac{b^2 - k}{-9} \quad \text{or } c = \frac{k-b^2}{9}$   
 $C = \frac{1-2^2}{4}$   
 $= -\frac{3}{4} = -0.75$

18.  $V = 30, r = 2$   
 $K = Ur^2$   
 $= 30 \times 22 = 120$   
 $\text{When } r = 4$   
 $V = \frac{120}{4^2} = 7.5 \text{ m/s}$

19.  $P = \sqrt[3]{\frac{XY}{z+X}}$   
 $P^3 = \frac{XY}{z+X}$   
 $Xy = P^3Z + P^3X$   
 $Xy - P^3X = P^3 z$   
 $X(y-P^3) = P^3 z$   
 $\therefore X = \frac{P^3 z}{Y - P^3}$

20.  $X \propto y + \frac{1}{z}, x = Ky + M$

$$X = 6, y = 3, z = 2 - 6 = 3k + M$$

$$X = 8, y = 5, z = 1 - 8 = 5k + M$$

$$X4 \cdot 24 = 12k + M$$

$$-16 = -7k, k = 1$$

When  $y = 10$ ,

$$z = \frac{16}{7}(10) - \frac{24}{7(64)} = \frac{160}{7} - \frac{24}{448} = \frac{10216}{448} = 22.8$$

21.  $T_{11} = a + 10d$

$$T_2 = a + d$$

$$a + 10d = 4a + 4d \dots \dots \dots (i)$$

$$3a - 6d = 0$$

$$S7 = \frac{7}{2}\{2a + 6d\} = 175 \dots (ii)$$

$$2a + 6d = 50$$

$$\underline{3a - 6d = 0}$$

$$5a = 50$$

$$a = 10$$

$$d = 5$$

22. (i)  $R = m + nI$

$$55 = M + 20n \dots \dots (i)$$

$$\underline{58 = m + 28n} \dots \dots (ii)$$

$$-3 = -8n$$

$$n = \frac{3}{8} = 0.375$$

$$55 = m + \frac{60}{8}$$

$$m = 55 - 7.5 \Rightarrow m = 47.5$$

$$R = 47.5 + 60 X \frac{3}{8}$$

$$R = 70 \text{ ohms}$$

23.

$$\left(1 - \frac{1}{2x}\right)^5 = [1 - 2x]^5$$

$$= 1^5 (-2x)^0 + 5 \cdot 1^4 (-2x)^1 + 10 \cdot 1^3 (-2x)^2 + 10 \cdot 1^2 (-2x)^3$$

$$= 1 - 10x + 40x^2 - 80x^3$$

$$(1 - 2x)^5 = (0.98)^5 = (1 - 0.02)^5$$

$$\therefore 2x = 0.02$$

$$x = 0.01$$

$$\text{Thus } (0.98)^5 = 1 - 10(0.01) + 40(0.01)^2 - 80(0.01)^3$$

$$= 1 - 0.1 + 0.004 - 0.00008 = 0.9039$$

## 51. Sequence and series

<b>1</b>	$S_n = \frac{n}{2} \{a + l\}$ $\frac{6}{2} (15 + l) = 360^0$	$M_1$ $M_1$ $A_1$	Alternative $\frac{6}{2} \{2 \times 15 + (6-1)d\} = 360^0$ $\therefore d = 18$ $15 + 5 \times 18$
----------	---	-------------------------	--

	$15 + l = 120^{\circ}$ $l = 105^{\circ}$		
		3	
2.	a = 2 nth = 32 Sn = 357 Sn = n/2 (2 + 32) 714 = 34n 21 = n	M1 M1 A1 03	Substitution Simplification

1. 
$$\begin{aligned} P \left( 1 + \frac{R}{100} \right)^3 \\ = 40,000 \left[ 1 + \frac{2}{100} \right]^3 \\ = 40,000 \times (1.02) = 42,448.32 \text{ km}^2 \\ \text{Encrouched area} \\ = 42,448.32 - 40,000 = 2448.32 \text{ km}^2 \end{aligned}$$

2. (a) 
$$\begin{aligned} \frac{9^x}{3^{2x+1}} &= \frac{81}{9^x} \\ 9^{2x} &= 3^4 (3^{2x+1}) \\ 3^{4x} &= 3^{4+2x+1} \\ 3^{4x} &= 3^{2x+5} \\ 4x &= 2x + 5 \\ 2x &= 5 \\ x &= 2.5 \end{aligned}$$

(b) Common ratio =  $\frac{81}{92.5}$   
 $= \frac{1}{3}$

(c) 
$$\begin{aligned} a &= 3^{(2x-2.5+1)} \\ &= 3^6 \\ &= 729 \\ S_{10} &= 729 \left[ 1 - \frac{1}{3} \right]^{10} \\ &\quad \frac{1-2}{3} \\ &= 1093.5 \times 0.99998 = 1093.5 \end{aligned}$$

**For both the 5<sup>th</sup> and 7<sup>th</sup> term**

(d) 5<sup>th</sup> term =  $729 \times (\frac{1}{3})^4$   
 $= 9$   
 7<sup>th</sup> term =  $729 \times (\frac{1}{3})^6$   
 $= 1$   
 $a = 9 \quad d = 1 - 9 = -8$   
 $S_{20} = \left[ \frac{20}{2} \times 9 + (20-1) \right] (-8)$   
 $= 10 (18 - 152) = -1340$

3. 
$$\begin{array}{r} -12 \pm -10 + -8 + \dots \dots \dots \\ a = -12 \quad d = z \end{array}$$

$$S_n = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$$

$$338 = \frac{n}{2} \left\{ 2(-12) + (n-1)d \right\} 2$$

$$676 = \left\{ n - 24 + 2n \right\} - 2$$

$$\frac{2n^2 - 26n}{2} - \frac{676}{2} = 0$$

$$n^2 - 13n - 338 = 0$$

$$(n-26)(n+13) = 0$$

$n = 26$  or  $n = -13$  reject

$\therefore n = 26$  terms

3. 
$$\begin{array}{r} -12 \pm -10 + -8 + \dots \dots \dots \\ a = -12 \quad d = z \end{array}$$

$$S_n = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$$

$$338 = \frac{n}{2} \left\{ 2(-12) + (n-1)d \right\} 2$$

$$676 = \left\{ n - 24 + 2n \right\} - 2$$

$$\frac{2n^2 - 26n}{2} - \frac{676}{2} = 0$$

$$n^2 - 13n - 338 = 0$$

$$(n-26)(n+13) = 0$$

$n = 26$  or  $n = -13$  reject

$\therefore n = 26$  terms

4. 
$$\begin{array}{l} 32 = 2 + (n-l)d \dots \dots \dots (i) \\ 357 = \frac{n}{2} [2 \cdot 2 + (n-l)d] \dots \dots \dots (ii) \end{array}$$

$$N(4 + (n-l)d) = 714$$

$$2 + (n-l)d = 32$$

$$N(4 + nd - d) = 714$$

$$\frac{-d + nd}{4n + n^2d - d} = 30$$

$$4n + n^2d - d = 714$$

$$nd - d = 30$$

$$d(n-l) = 30$$

5. a)  $OC = OB + BC = a + b$

$$b) OM = OA + AM = a + \frac{1}{2} b$$

$$\text{Given } OX = rOM$$

$$= r(a + \frac{1}{2} b)$$

From  $\Delta O BX$

$$Ox = OB + BX$$

$$= OB + BC + CX$$

$$= b + a + sa$$

$$= (1+s) a + b$$

$$\therefore r(a + \frac{1}{2} b) = (1 + s) a + b$$

Comparing coefficients of  $a$  and  $b$

$$r = 1 + S$$

$$\text{and } \frac{1}{2} r = 1 \Rightarrow r = 2$$

$$\text{Substitute for } r = 2 \Rightarrow 2 = 1 + s \Rightarrow s = 1$$

$$c) \text{ Now } BX = BC + CX$$

$$= a + a = 2a$$

$$\therefore BC : BX = 1 : 2$$

$$6. (a) -91 = 29 + (n-1)x - 6$$

$$-120 = -6n + 6$$

$$6n = 126$$

$$n = 21$$

$$(b) S_{21} = \frac{21}{2} [(2 \times 2a) + (20 \times -6)]$$

$$= \frac{21x - 62}{2}$$

$$= -651$$

$$7. d = p - 5 \dots \dots \dots (i)$$

$$d = q - p \dots \dots \dots (ii)$$

$$0 = 2p - q - 5$$

$$0 = 7 - 2q + p$$

$$-p + 2q = 7$$

$$2p - q = 5$$

$$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

$$-2p + 4q = 14$$

$$2p - q = 5$$

$$3q = 19$$

$$q = 19/2$$

$$p = 2q - 7 \quad 38/3 - 7$$

$$p = 17/8$$

$$S = \frac{n}{2} [2a + (n-1)d]$$

$$= 12/2 (10 + 11x^2/3)$$

$$= 6 (10 + 22/3) = 104$$

$$S_n = a(r^n - 1) = S(1.5 - 6)$$

$$r-1 \quad 1.5 - 1$$

$$= 5 \times (1.5 - 1) = 103.90$$

$$0.65 = 10.4$$

$$8. \quad a + a + d = 10 \dots \dots \dots \dots \dots (i)$$

$$\begin{aligned}
 \frac{10}{2} \left\{ 2a + 9d \right\} &= 210 \dots \dots \dots \dots \dots \dots \quad (ii) \\
 2a + d &= 10 \\
 2a + 9d &= 42 \\
 8d &= 32 \\
 d &= 4 \\
 Tl &= 3 + 6(4) \\
 &= 3 + 24 \\
 &= 27
 \end{aligned}$$

9.  $S_6 = \frac{15(1-0.56)}{1-0.5}$   
 $= 29.5314 \text{ metres}$

10.  $S_n = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$   
 $S_{51} = \frac{51}{2} (2x - 22) + (51 - 1)3$   
 $= 2703$

11.  $100 + 200 + 400 + 800 + 1600 + 3200 + 6400 + 12800 + 25600 + 51200$   
 $\underline{200} = \frac{400}{200} = \frac{800}{400}$   
 $100 \quad 200 \quad 400$   
 $= 51200 \quad 99600 \quad 108200 \quad 110,600$   
 $\underline{25600} \quad \underline{6400} \quad \underline{1600} \quad \underline{700}$   
 $76800 \quad 105,000 \quad 109,800 \quad 111,300$   
 $\underline{12800} \quad \underline{3,200} \quad \underline{800}$   
 $99,600 \quad 108,200 \quad 110,600$   
 $= 111300$

12.. a) Let  $n$  be the initial members  
 Each to contribute  $\frac{720000}{n}$

New membership  $n + 20$   
 Contributions:  $\frac{720000}{n+20}$   
 $\underline{720000} - \underline{720000} = 3000$   
 $n \quad n + 20$   
 $720000(n + 20) - 720000n = 3000n(n + 20)$   
 $4800 = n(n + 20)$   
 $n^2 + 20n - 4800 = 0$   
 $n^2 + 80n - 60n - 4800 = 0$   
 $n(n + 80) - 60(n + 80) = 0$   
 $(n-60)(n + 80) = 0$   
 $n = 60$   
 Original members = 60

b) Contributions required before recruitment

$$= \frac{720000}{60} = 120000$$

*After requirement = 720000*

13.  $n^{th}$  term is  $ar^{n-1}$   
 $a = 8, r = \frac{1}{2}$

$$n^{th} \text{ term} = ^1/_{512}$$

$$8(\frac{1}{2})^{n-1} = ^1/_{512}$$

$$8(\frac{1}{2})^{n-1} = 2^{-9}$$

$$(\frac{1}{2})^{n-1} = 2^{-9} \div 2^3$$

$$(\frac{1}{2})^{n-1} = 2^{-12} = (\frac{1}{2})^{12}$$

$$n-1 = 12$$

$$n = 13$$

14.  $3^{rd} a + 2d$   
 $9^{th} a + 8d$   
 $25^{th} a + 24d$   
 $(i) \quad \frac{a+2d}{a+8d} = \frac{a+8d}{a+24d}$   
 $a + 8d = 24d$   
 $(a+2d)(a+2d) = (a+8d)(a+8d)$   
 $a^2 + 26da + 48d^2 = a^2 + 16da + 64d^2$   
 $\frac{10da}{10d} = \frac{16d^2}{10d}$   
 $a = 1.6d \dots \dots \dots (i)$   
 $(a+6d) + 2(a+5d) = 78$   
 $3a + 16d = 78 \dots \dots \dots (ii)$   
 $\text{But } a = 1.6d$   
 $\therefore (3 \times 1.6d) + 16d = 78$   
 $4.8d + 16d = 78$   
 $4.8d + 16d = 78$   
 $\frac{20.8}{20.8} = \frac{78}{20.8}$   
 $\text{Common distance } d = 3.75$   
 $a = 1.6 \times 3.75$   
 $\text{first term } a = 6$

$$(ii) \quad S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_a = \frac{9}{2} ((2 \times 6) + (9-1)3.75)$$

$$= \frac{9}{2} (12 + 30)$$

$$\frac{9}{2} \times 42 = 189$$

15.  $T_4 = a + 3d$   
 $T_7 = a + 6d$   
 $(a + 6d) - (a + 3d) = 12$   
 $3d = 12$   
 $d = 4$   
 $\text{But } a = 9$   
 $S_5 = \underline{\frac{5}{2}} 2(9) + 4 (4)$   
 $= \underline{\frac{5}{2}} 18 + 16$

$$\begin{aligned} & 2 \\ & = \underline{5} \times 34 \\ & 2 \\ & = 85 \end{aligned}$$

## 52. Vectors 2

<p><b>1.</b></p> <p>(a) <math>BD = BA + AD</math></p> $= -\overset{\sim}{b} + \frac{3}{5}\overset{\sim}{c}$ $AE = AB + BE$ $= \overset{\sim}{b} + \frac{1}{2}BC$ $= \overset{\sim}{b} + \frac{1}{2}(\overset{\sim}{c} - \overset{\sim}{b})$ $= \frac{1}{2}\overset{\sim}{b} + \frac{1}{2}\overset{\sim}{c}$ <p>(b)</p> $BF = +\left(\frac{3}{5}\overset{\sim}{c} - \overset{\sim}{b}\right)$ $AF = n\left(\frac{1}{2}\overset{\sim}{b} + \frac{1}{2}\overset{\sim}{c}\right)$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>M1</b> AF and BF in terms of n and t</p> <p><b>M1</b> Equating the expressions</p> <p><b>M1</b> Extraction of the coefficient</p> <p><b>M1</b> Substitution/its equivalent</p> <p><b>A1</b> Any of the unknown</p>
---	--	--

$  \begin{aligned}  &= \frac{n}{2} (b + c) \\  AF &= AB + BF \\  &= b + \frac{3}{5}t c - t b \\  &= (1-t)b + \frac{3}{5}c \\  (1-t)b + \frac{3}{5}t c &= \frac{n}{2}b + \frac{n}{2}c \\  1-t = \frac{n}{2} &\dots\dots\dots(i) \\  2-2t = n & \\  \frac{3}{5}t = \frac{n}{2} &\dots\dots\dots(ii) \\  6t - 5n &= 0 \\  \text{subt } (i) \text{ in } (ii) & \\  6t - 5(2-2t) &= 0 \\  6t &= 10 \\  t &= \frac{10}{16} = \frac{5}{8} \\  n &= 2 - 2\left(\frac{5}{8}\right) \\  n &= \frac{3}{4} \\  (\text{c}) \text{ BD:BF} & \\  8 : 5 &  \end{aligned}  $	<b>B1</b> <b>B1</b>	The other unknown
		<b>10</b>

1. a) (i)  $AN = OA + ON$   
 $= \underline{a} + \frac{2}{7} \cancel{b}$   
 $= \frac{2}{7} \cancel{b} - \underline{a}$

(ii)  $\overrightarrow{AT} = \frac{7}{13} \overrightarrow{AN}$

$\frac{7}{13} \left[ -\underline{a} + \frac{2}{7} \cancel{b} \right]$

$\underline{\frac{2}{13} \cancel{b} - \frac{7}{13} \underline{a}}$

(iii)  $AM = \underline{\frac{1}{4}} AB$   
 $= \underline{\frac{1}{4}} (\overrightarrow{AO} + \overrightarrow{OB})$

$$\begin{aligned}
 &= \frac{1}{4} (b - a) \\
 \frac{4}{(b)} \quad OT &= OA + AT \\
 &\approx a \left[ \frac{2}{3} b - \frac{7}{13} a \right] \\
 &= \frac{2}{13} [3a + b] \\
 \overrightarrow{OM} &= \overrightarrow{OA} + \overrightarrow{AM} \\
 &= a + \left[ -\frac{1}{4} a + \frac{1}{4} b \right] \\
 &= \frac{3}{4} a + \frac{1}{4} b \\
 &= \frac{1}{4} [3a + b]
 \end{aligned}$$

$$\frac{OT}{OM} = \frac{2}{13} \quad (\cancel{3a + b})$$

✓ ✓ **Construction of  $\angle 60^\circ$  and  $\angle 90^\circ$**

**Bisect  $\angle$  btw  $90^\circ$  and  $60^\circ$  to obtain  $\angle 75^\circ$**

$$\overrightarrow{OT} = \frac{8}{13} \overrightarrow{OM}$$

✓ **Construction of the given sides**

$$\text{Or } \overrightarrow{OM} = \frac{13}{8} \overrightarrow{OT}$$

**Construction of  $\triangle XYZ$**

$$\text{Since } \overrightarrow{OT} = \frac{8}{13} \overrightarrow{OM}$$

$$\begin{aligned}
 \text{Then } OT : TM &= \frac{8}{13} : \frac{5}{13} \\
 &= 8 : 5
 \end{aligned}$$

2.

$$\begin{array}{c}
 \bullet 3 \qquad \qquad \bullet 5 \\
 \bullet (X, Y) \qquad \bullet T(-3, 4) \\
 TB = \frac{5}{8} AB \\
 \left( \begin{matrix} 6 \\ -5 \end{matrix} \right) - \left( \begin{matrix} 3 \\ 4 \end{matrix} \right) = \frac{5}{8} AB \\
 \left( \begin{matrix} 9 \\ -9 \end{matrix} \right) = \frac{5}{8} \left[ \left( \begin{matrix} 6 \\ -5 \end{matrix} \right) - \left( \begin{matrix} x \\ y \end{matrix} \right) \right]
 \end{array}$$

$$\begin{aligned}
 \left( \begin{matrix} 9 \\ -9 \end{matrix} \right) &= \frac{5}{8} (6-x) \\
 &\quad \frac{5}{8} (-5-y) \\
 \frac{30}{8} - \frac{5}{8} X &= 9
 \end{aligned}$$

$$\frac{-25}{8} - \frac{5}{8} y = -9$$

$$\begin{array}{l} 30 - 5x = 72 \\ -25 - 5y = -72 \\ X = -8.4 \quad y = 9.4 \end{array} \quad \left| \begin{array}{l} -5x = 42 \\ -5y = -47 \end{array} \right.$$

3.  $OX = \frac{2}{3}(3i + 2j - 4k) + \frac{1}{3}(6i + 11j + 2k)$   
 $\sim = 2i + 4j - \frac{8k}{3} + 2i + \frac{11j}{3} + \frac{2}{3}$   
 $\sim = \frac{4i + 5j - 2k}{10 \times 1} = \sqrt{16 + 25 + 4}$   
 $\sim = 6.71 \text{ units}$

4. a)  $2^5 - 5(2^4)(\frac{1}{5}) + 10(2^3)(\frac{1}{5}x)^2 - 10(2^2)(\frac{1}{5}x)^3 + 5(2)(\frac{1}{5}x)^4 - (\frac{1}{5}x)^5$   
 $32 - 16x + \frac{16}{25}x^2 - \frac{8}{25}x^3 + \frac{2}{125}x^4 - \frac{1}{3125}x^5$   
 $\frac{1}{5}x = -0.04$   
 $x = 0.2$

b)  $32 - 16(0.2) + \frac{16}{5}(0.2)^2 - \frac{8}{25}(0.2)^3 + \dots$   
 $= 32 - 3.2 + 0.128 - 0.00256$   
 $= 28.92544$   
 $= 29.925$

5.  $AS = AO + OS$   
 $= -a + 2(3c)$   
 $= 2c - a \dots \dots \dots$   
 $BC = BA + AC$   
 $= a - b + AC$   
 $\text{But } AC = AO + OC = -a + 3c$   
 $= 3c - a \dots \dots \dots$   
 $AB + \frac{2}{3}OC = \frac{2}{3}3c = 2c$   
 $BA = 2c \dots \dots$   
 $BC = -12c + 3c - a = c - a.$

b) (i)  $AT = \eta AS = \eta(2c - a)$   
 $= 2\eta c - \eta a$   
 $AT = AB + BT = 2c + K(c - a)$   
 $= 2c + Kc - Ka$   
 $= (2 + k)c - Ka$

(ii)  $2 + K = 2\eta \quad (i) \quad K = \eta \quad (ii)$   
 $2 + \eta = 2\eta$   
 $2 = 2\eta - \eta$   
 $2 = \eta, K = 2$

(c)  $BT : BC$   
 $BT = 2 BC$

6. (a) (i)  $PQ = PO + OQ$   
 $= \underline{P} + \underline{q} \text{ or } q - \underline{P}$   
*For ✓ PQ or P and q*  
*For ✓ exp. Of OR*  
*For ✓ OR in p & q*  
*For ✓ SQ in P & Q*

$$\begin{aligned}
 (ii) \quad \underline{QR} &= \underline{QP} + \underline{PR} \\
 &= \underline{\underline{P}} + \frac{2}{3} \underline{PQ} \\
 &= \underline{\underline{P}} + \frac{2}{3} (\underline{q} - \underline{p}) \\
 &= \underline{\underline{P}} + \frac{2}{3} \underline{q} - \frac{2}{3} \underline{p} \\
 &= \frac{1}{3} \underline{p} + \frac{2}{3} \underline{q}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \underline{SQ} &= \underline{SO} + \underline{OQ} \\
 &= -\frac{3}{4} \underline{OP} + \underline{OQ} \\
 &= -\frac{3}{4} \underline{p} + \underline{q} \text{ or } \underline{q} - \frac{3}{4} \underline{p}
 \end{aligned}$$

(b) Express  $OT$  in two different ways:

$$\begin{aligned}
 \text{Given } \underline{OT} &= n \underline{QR} \\
 &= n \left( \frac{1}{3} \underline{p} + \frac{2}{3} \underline{q} \right) \\
 &= \frac{n}{3} \underline{p} + \frac{2n}{3} \underline{q}
 \end{aligned}$$

From  $\triangle OST$ ,

$$\begin{aligned}
 OT &= \underline{OS} + \underline{ST} \\
 &= \frac{3}{4} \underline{OP} + M \underline{SQ} \\
 &= \frac{3}{4} \underline{p} + M \left( \frac{-3}{4} \underline{p} + \underline{q} \right) \\
 &= \left( \frac{3}{4} - \frac{3m}{4} \right) \underline{p} + mq \\
 \therefore \frac{n}{3} \underline{p} + \frac{2n}{3} \underline{q} &= \left( \frac{3}{4} - \frac{3m}{4} \right) \underline{p} + mq
 \end{aligned}$$

Compare the coefficients of  $\underline{p}$  and  $\underline{q}$

$$\frac{n}{3} = \frac{3}{4} - \frac{3}{4} m$$

$$4n = 9 - 9m$$

$$4n + 9m = 9 \dots \text{eq (1)}$$

$$\frac{2n}{3} = m$$

$$m = \frac{2n}{3} \dots \text{eq. (2)}$$

Substitutes form in equation (1)

$$4n + 9 \left( \frac{2n}{3} \right) = 9$$

$$4n + 6n = 9$$

$$10n = 9$$

$$n = \frac{9}{10}$$

Substitute for  $n$  in equation (2)

$$m = \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$$

## 53. Binominial expansion

<b>1</b>	<p>a)</p> $\left(2 - \frac{1}{x}\right)^8 = 2^8 - 8 \cdot 2^7 \left(\frac{1}{x}\right) + 28 \cdot 2^6 \left(\frac{1}{x}\right)^2 - 56 \cdot 2^5 \left(\frac{1}{x}\right)^3 + \\ 70 \cdot 2^4 \left(\frac{1}{x}\right)^4 + \dots$ $256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1792}{x^3} + \frac{1120}{x^4}$ <p>b)</p> $(1.75)^8 = 256 - \frac{1024}{4} + \frac{1792}{16} - \frac{1792}{64} + \frac{1120}{256} \\ = 256 - 256 + 112 - 28 + 4.375 \\ = 88.375$	M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	$2 - \frac{1}{x} = 1.75$ $0.25 = \frac{1}{x} \therefore x = 4$
<b>2.</b>	$(2+x)^5 \\ 25 + 5(2)4x + 10(2)3x + 10(2)2x3 + \dots \\ = 32 + 80x + 80x^2 + 40x^3 + \dots \\ (1.970)5 = (2 - 0.03)5 \\ x = -0.03 \\ (1.97)5 = 32 + 80(-0.03) + 80(-0.03)2 + 40(-0.03)3 \\ = 32 - 2.4 + 0.072 - 0.00108 \\ = 29.67092 \\ = 29.67$	M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	4

1. a)  $I^5 + 5(-3x)^1 + 10(-3x)^2 + 10(-3x)^3 + 5(-3x)^4 + (-3x)^5$

$$I - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5$$

$$I - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5$$

b)  $3x = I - 0.997$

$$x = 0.001$$

$$= I - 15(0.001) + 90(0.001)^2 - 270(0.001)^3 + 405(0.001)^4$$

$$= I - 0.015 + 0.00009 - 0.00000027 + \dots$$

$$= I + 0.00009 - 0.015 - 0.00000027$$

$$= 1.00009 - 0.01500027 = 0.98508973$$

$$= -0.9851 \text{ (4 d.p)}$$

2. (i)  $5 + \frac{x}{2}^6 = 15625 + \frac{3125}{3}X + \frac{9375}{4}X^2 + \frac{625}{2}X^3 + \dots$

(ii)  $X = \frac{1}{2}$

$$\left(\frac{1}{2}\right)^6 = 15625 + \frac{3125}{3} + \frac{9375}{4} + \frac{625}{2} \\ = 15625 + 1041.667 + 2343.75 + 312.5$$

3.

$$(\sqrt[3]{3} + 2x)^6 = (\sqrt[3]{3})^6 + 6(\sqrt[3]{3})^5(2x) + 15(\sqrt[3]{3})^4(2x)^2 + 20(\sqrt[3]{3})^3(2x)^3 \\ = 27 + 108x\sqrt[3]{3} + 270x^2 + 480x^3\sqrt[3]{3}$$

$$\begin{aligned}
 & 3+2x=3 \quad 3 \\
 & \sqrt{2x+2\sqrt{3}} \\
 & x=\frac{3}{\sqrt{27+108\sqrt{3}} \sqrt{3} + 270\sqrt{3^2} + 48\sqrt{3}(-3)^3} \\
 & = 27+324+810+4320=5481
 \end{aligned}$$

4.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{Sum odd}) = \frac{18}{36} = \frac{1}{2}$$

$$\begin{aligned}
 5. \quad \angle PQR &= 180 - (35o + 75) \\
 &= 70^o
 \end{aligned}$$

$$PR^2 = 12^2 + 8.4^2 - 2(12)(8.4) \cos 70^o$$

$$PR = 145.61 = 12.07$$

$$6. \quad (a) \text{ Terms: } 2^5, 2^4(\beta/x), 23(\beta/x)^2, 2^2(\beta/x)^3, 2^3(\beta/x)^4$$

$$\text{Co eff } 1, 5, 10, 10, 5$$

$$(2 + \beta/x)^5 = 25 + 5(2)^4(\beta/x) + (2)^3(\beta/x)^2 + 10(22)(\beta/x)^3 + 5(2)(\beta/x)^4$$

$$= 32 + 2140x^{-1} + 720x^{-2} + 1080x^{-3} + 820x^{-4}$$

$$(b) 9.5 = 2 + \beta/x$$

$$\beta/x = 7.5$$

$$x = \beta/7.5 = 0.4$$

$$(9.5)^5 = 32 + \frac{240}{0.4} + \frac{720}{(0.4)^2} + \frac{1080}{(0.4)^3} + \frac{810}{(0.4)^4}$$

$$= 53647.625(3d.p)$$

$$\begin{aligned}
 7. \quad X^5 - 5x^4(0.2) + 10x^3(0.20 - 10x^2(0.2)^3 + 5x(0.2)^4 - (0.2)^5) \\
 X^5 - 5x^4(\frac{2}{10}) + 10x^3(\frac{2}{10})^2 - 10x^2(\frac{2}{10})^3 + 5x(\frac{2}{10})^4 - (\frac{2}{10})^5 + x^5 - (\frac{2}{10})x^3 - (\frac{2}{100})x^2 + 5x16 - 2^5/10^5 \\
 X^5 - x^4x^3 - 8/100x^2 + 80x - 2^5/10^5 \\
 90,392,079
 \end{aligned}$$

$$8. \quad \log(x+24) = \log(x(9-2x))$$

$$X+24 = 81-18x$$

$$X=3$$

$$9. \quad \frac{1+\frac{x}{12}}{12} = 1 + \frac{x}{2} + \frac{5x^2}{48} + \frac{5x^3}{432}$$

$$\left[ 1 + \frac{x}{12} \right]^6 = 1 \frac{1}{4}$$

$$\frac{x}{12} = \frac{1}{4}$$

$$x = 3$$

$$\left( \frac{5}{4} \right) = 1 + \frac{3}{2} + \frac{9}{48} + \frac{27}{432}$$

$$= 2.7500$$

10.  $(a) (1 + \frac{1}{2})^8 = 1 + 8(\frac{1}{2}) + 28(\frac{1}{2}x)^2 + 56(\frac{1}{2}x)^3 + 70(\frac{1}{2}x)^4 + 567(\frac{1}{2}x)^5 + 2(\frac{1}{2}x)^6 + 8(\frac{1}{2}x)^7 + (\frac{1}{2}x)^8$

$$= 1 + 4x + 7x^2 + 7x^3 + 4.375x^4 + 1.75x^5 + 0.4375x^6 + 0.0625x^7 + \frac{1}{256}x^8$$

$(b) (1.05)^8 = 1 + 4(0.1) + 7(0.1)2 + 7(0.1)3$

$$= 1 + 0.4 + 0.07 + 0.0074\dots$$

$$= 1.48$$

11.  $81 + 27x + 9x^2 + 3x^3 + x^4$

$$81 + 108x + 54x^3 + x^4$$

$$81 + 108(0.02) + 54(0.02)^3$$

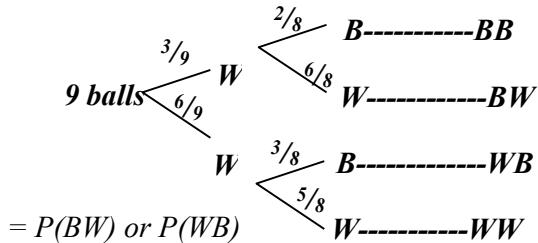
$$= 83.182$$

#### 54. Probability

<b>1</b>	<p>a) <math>P(R) = \frac{1}{8}</math> and <math>P(H) = \frac{3}{7}</math></p> $\therefore \frac{1}{8} \times \frac{3}{7} = \frac{3}{56}$ <p>b) <math>P(RH')</math> or <math>P(R' H)</math> or <math>P(RH)</math></p> $\left( \frac{1}{8} \times \frac{4}{7} \right) + \left( \frac{7}{8} \times \frac{3}{7} \right) + \frac{3}{56}$ $\frac{4}{56} + \frac{21}{56} + \frac{3}{56}$ $= \frac{28}{56} \text{ or } \frac{1}{2} \text{ or } 0.5$	B <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	
4			
<b>2.</b>	<p>(b) (i) <math>(\frac{1}{2} \times \frac{7}{10} \times \frac{2}{9}) + (\frac{1}{2} \times \frac{3}{10} \times \frac{7}{9}) + (\frac{1}{2} \times \frac{4}{9} \times \frac{5}{9}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{4}{9}) = 22/45</math></p> <p>(ii) <math>1 - \frac{22}{45} = \frac{23}{45}</math></p>	B <sub>2</sub>	

	$= 23$ $45$  (iii) $(\frac{1}{2} \times \frac{7}{10} \times \frac{1}{3}) + (\frac{1}{2} \times \frac{3}{10} \times \frac{7}{10}) + (\frac{1}{2} \times \frac{1}{2} \times \frac{5}{9})$ $+ (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ $\quad \quad \quad \frac{7}{60} + \frac{7}{60} + \frac{5}{36} + \frac{5}{36}$ $\quad \quad \quad = 23/45$  (iv) $1 - \frac{23}{45} = \frac{22}{45}$		
		U1	
		A1 M1 A1	
		M1	
		A1	
		B1 A1	
		10	

1. (a) (i) Total balls =  $3 + 6 = 9$



$$\begin{aligned}
 &= \left[ \frac{1}{3} \times \frac{2}{8} \right] + \left[ \frac{6}{9} \times \frac{3}{8} \right] \\
 &= \frac{18}{72} + \frac{18}{72} = \frac{36}{72} \\
 &= \frac{1}{2}
 \end{aligned}$$

(ii)  $= P(BW)$  or  $P(WB)$

$$\begin{aligned}
 &= \left[ \frac{3}{9} \times \frac{6}{9} \right] + \left[ \frac{6}{9} \times \frac{3}{9} \right] \\
 &= \frac{18}{81} + \frac{18}{81} \\
 &= \frac{36}{81} = \frac{4}{9}
 \end{aligned}$$

$$(b) (i) P(WW) = \frac{6}{9} \times \frac{5}{8} \\ = \frac{30}{72} = \frac{5}{12}$$

$$(ii) P(WW) = \frac{6}{9} \times \frac{6}{9} \\ = \frac{4}{9}$$

2.  $P(W) = \frac{7}{12} \quad P(B) = \frac{5}{12}$

(2 white and one brown)  
 $= (WWB \text{ or } WBW \text{ or } BWW)$   
 $= (\frac{7}{12}x^6/\frac{11}{10}) + (\frac{7}{12}x^5/\frac{11}{10}x^6/\frac{10}) + (\frac{7}{12}x^7/\frac{11}{10}x^6/\frac{10})$   
 $= \frac{22}{44}$

(ii)  $P(BBW \text{ or } WBW \text{ or } WBB)$   
 $= (\frac{5}{12}x^4/\frac{11}{10}x^7/\frac{10}) + (\frac{5}{12}x^7/\frac{11}{10}x^4/\frac{10}) + (\frac{7}{12}x^5/\frac{11}{10}x^4/\frac{10})$   
 $= \frac{7}{22}$

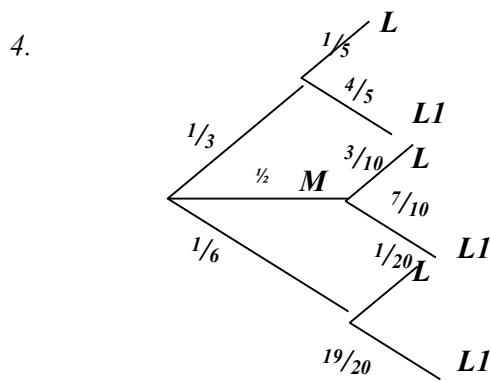
(iii)  $P(\text{at least one white cup})$   
 $= (1-P(BBB)) = 1 - (\frac{5}{12}x^4/\frac{11}{10}x^3/\frac{10})$   
 $= \frac{21}{22}$

(iv)  $P(\text{same colour}) = P(BBB \text{ or } WWW)$   
 $= (\frac{7}{12}x^6/\frac{11}{10}x^5/\frac{10}) + (\frac{7}{12}x^4/\frac{11}{10}x^3/\frac{10})$   
 $= \frac{9}{44}$

3. a)

2	3	5	7
2	32	52	72
3	23	53	73
5	25	35	75
7	27	37	57

b)  $P(E) = \frac{4}{16}$   
 $= \frac{1}{4}$

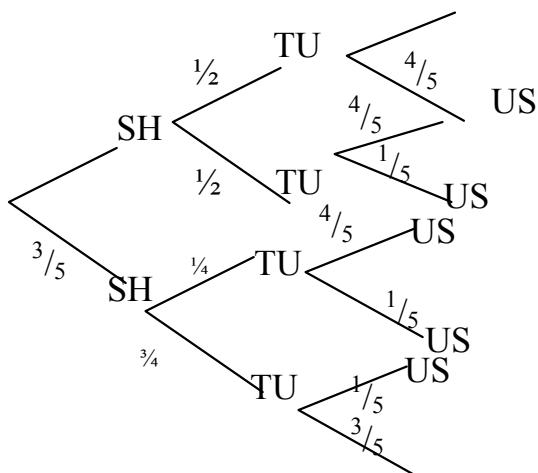


(a)  $P(\text{late}) = (\frac{1}{3}x^{1/5}) + (\frac{1}{2}x^{3/10}) + (\frac{1}{6}x^{1/20})$   
 $= \frac{1}{15} + \frac{3}{20} + \frac{1}{120}$   
 $= \frac{9}{40}$

(b)  $P = \frac{1}{3}x^{1/5} + \frac{1}{6}x^{1/20}$   
 $= \frac{1}{15} + \frac{1}{120}$

$$(c) P = (\text{not late}) = (1 - \frac{9}{40})$$

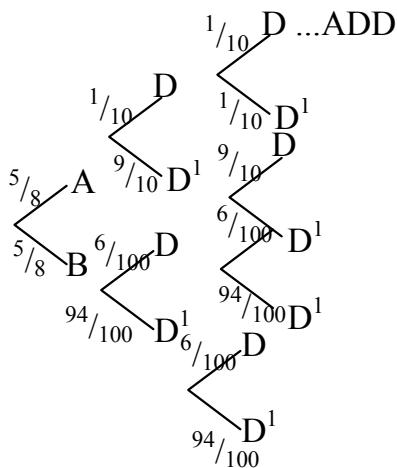
5. a)



$$b) i) P(\text{all faults}) = P(\text{SH and TU and US}) \\ = \frac{2}{5} \times \frac{1}{2} \times \frac{4}{5} = \frac{4}{25}$$

$$ii) P(\text{exactly two}) = \frac{2}{5} \times \frac{1}{2} \times \frac{1}{5} + \frac{2}{5} \times \frac{1}{2} \times \frac{1}{5} + \frac{3}{5} \times \frac{3}{4} \times \frac{1}{5}$$

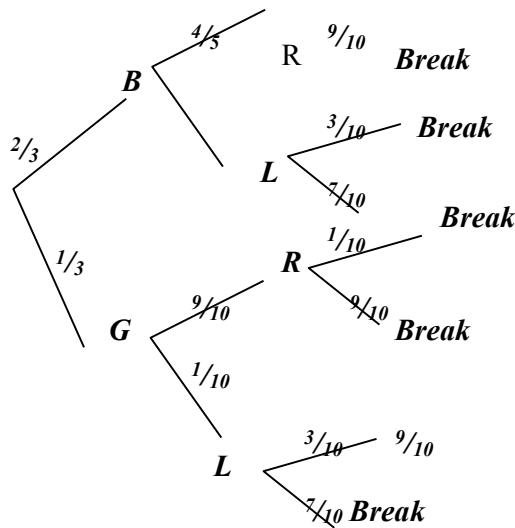
6.



Both defective

$$\begin{aligned}
 &= \frac{3}{8} \times \frac{1}{10} \times \frac{1}{10} + \frac{5}{8} \times \frac{6}{100} \times \frac{6}{100} \\
 &= \frac{3}{800} + \frac{180}{80000} \\
 &= \frac{24}{4000} \\
 &= \frac{3}{500}
 \end{aligned}$$

7. a)



$$b) i) P(BL \text{ or } GL) = \frac{2}{3}X^{\frac{1}{10}} + \frac{1}{3}X^{\frac{1}{10}} \\ = \frac{2}{15} + \frac{1}{30} = \frac{5}{30}$$

$$ii) P(BL \text{ break or GR break}) \\ = \frac{2}{3}X^{\frac{1}{5}}X^{\frac{3}{10}} + \frac{1}{3}X^{\frac{1}{10}}X^{\frac{3}{10}} \\ = \frac{2}{50} + \frac{1}{100} = \frac{4+1}{100} = \frac{5}{100}$$

$$iii) P(BR \text{ break or GR break}) \\ = \frac{2}{3}X^{\frac{4}{5}}X^{\frac{1}{10}} + \frac{1}{3}X^{\frac{9}{10}}X^{\frac{1}{10}} \\ = \frac{8}{150} + \frac{9}{300} = \frac{16+9}{300} = \frac{25}{300}$$

$$iv) 1 - (\frac{5}{100} + \frac{25}{300}) = 1 - \frac{15+25}{300} \\ = \frac{260}{300}$$

8.

1	2	3	4	5	6
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
					12

$$P(\text{a two days outing}) = \frac{10}{36} = \frac{5}{18}$$

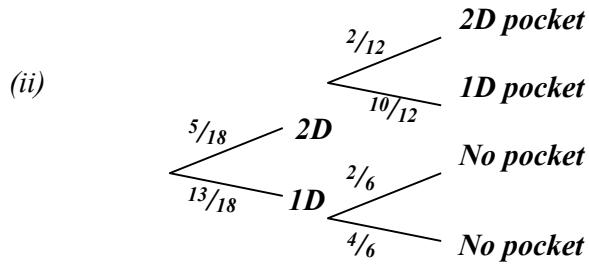
$$(b) \quad \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ H & H1 & H2 & H3 & H4 & H5 & H6 \\ T & T1 & T2 & T3 & T4 & T5 & T6 \end{matrix}$$

$$P(2\text{days and one day pocket money})$$

$$= \frac{5}{18} \times \frac{10}{12}$$

$$= \frac{25}{108}$$

$$(c) \quad (i) \frac{5}{18} x \frac{2}{12} \\ = \frac{5}{108}$$



$$P(\text{get pocket money}) \\ = \frac{5}{18} x \frac{2}{12} + \frac{5}{18} x \frac{10}{12} + \frac{13}{18} x \frac{2}{6}$$

$$9. (a) (i) P(WW) = \frac{4}{10} x \frac{3}{9} \\ = \frac{2}{15}$$

$$(ii) P(WW) \text{ or } (RR) = \frac{4}{10} x \frac{3}{9} + \frac{6}{10} x \frac{5}{9} \\ = \frac{2}{15} + \frac{1}{3} = \frac{7}{15}$$

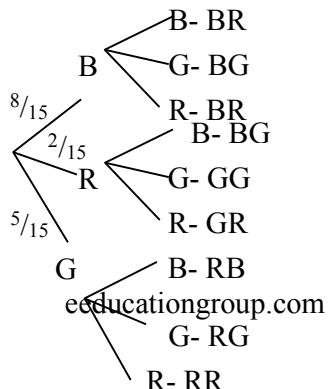
$$(iii) P(\text{at least Red}) = 1 - P(WW) \\ = 1 - \frac{2}{15} \\ = \frac{13}{15}$$

$$(iv) P(WR) \text{ or } P(RW) = \frac{3}{5} x \frac{4}{9} + \frac{2}{5} x \frac{2}{3} \\ = \frac{8}{15}$$

$$10. \quad a) \quad i) \frac{8}{15}$$

$$ii) \frac{2}{15} + \frac{5}{15} = \frac{7}{15}$$

b) i)



$$Gh = {}^2/_{15}x {}^1/_{14} = {}^2/_{21}0 = {}^1/_{105}$$

ii) RG or RB

$$\frac{{}^3/_{21} + {}^7/_{45}}{945} = \frac{45 + 147}{945}$$

$$= \frac{192}{945}$$

(c)(i)

	H	T
1	1H	1T
2	2H	2T
3	3H	3T
4	4H	4T
5	5H	5T
6	6H	6T

11. (a)

$$(b) \quad (i) \text{ same colour} = {}^5/{}_9 x {}^{4/2} x {}^{3/7} + {}^{4/9} x {}^{3/8} x {}^{2/7} \\ = {}^{5/42} + {}^{1/7} \\ = {}^{11/42}$$

$$(ii) \text{ more red balls} = {}^{5/89} x {}^{1/2} x {}^{3/7} + {}^{5/9} x {}^{1/2} x {}^{4/7} + {}^{5/9} x {}^{1/2} x {}^{4/7} \\ = {}^{5/42} + {}^{10/63} = {}^{10/63} \\ = {}^{5/42} + {}^{20/63} = \frac{15 + 40}{126} = \underline{\underline{55}}$$

(iii) at least black ball was picked

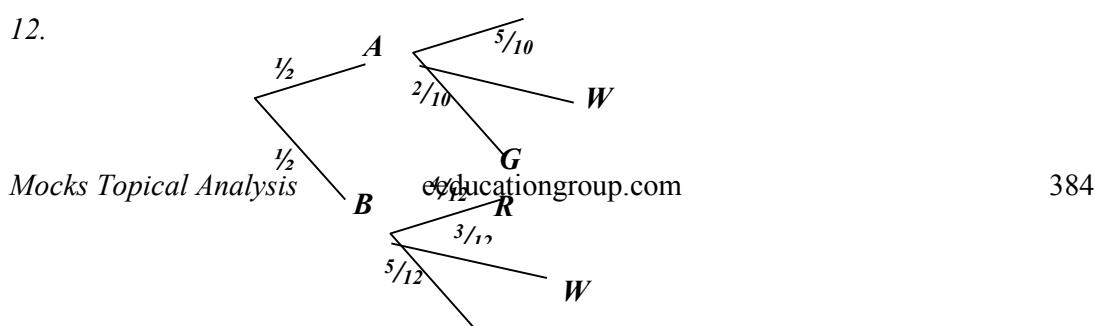
$$= 1 - {}^{5/9} x {}^{1/2} x {}^{3/7} \\ = 1 - {}^{5/21} \\ = {}^{16/21}$$

(iv) Atmost 1 red ball picked

$$= {}^{5/9} x {}^{4/2} x {}^{3/7} + {}^{4/9} x {}^{5/8} x {}^{3/7} + {}^{4/9} x {}^{3/8} x {}^{2/7} \\ = {}^{5/42} + {}^{5/92} + {}^{1/21} \\ = \frac{5 + 5 + 2}{42} \\ = {}^{12/42} \\ = {}^{2/7}$$

${}^{3/10}$

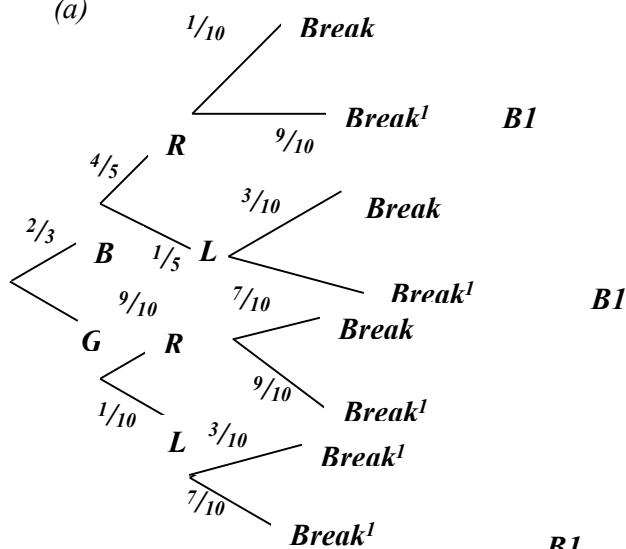
12.



$$P(\text{Red}) = \left(\frac{1}{2}x^{\frac{3}{10}}\right) + \left(\frac{1}{2}x^{\frac{4}{12}}\right) = 19/60$$

13.

(a)

bi)  $P(\text{left handed})$ 

$$\begin{aligned} &= \frac{2}{3}x^{\frac{1}{5}} + \frac{1}{3}x^{\frac{1}{10}} \\ &= \frac{2}{15} + \frac{1}{3} \\ &= \frac{5}{30} = \frac{1}{6} \end{aligned}$$

ii)  $P(\text{Right handed and will break})$ 

$$\begin{aligned} &= \frac{2}{3}x^{\frac{4}{5}}x^{\frac{1}{10}} + \frac{1}{3}x^{\frac{1}{9}}x^{\frac{1}{10}} \\ &= \frac{8}{150} + \frac{9}{300} \\ &= \frac{25}{300} = \frac{1}{12} \end{aligned}$$

c)  $P = \frac{2}{3}x^{\frac{4}{5}}x^{\frac{1}{10}} + \frac{2}{3}x^{\frac{1}{5}}x^{\frac{3}{10}} + \frac{1}{3}x^{\frac{9}{10}}x^{\frac{1}{10}} + \frac{1}{3}x^{\frac{1}{10}}x^{\frac{3}{10}}$ 

14.

$$\begin{aligned} (i) P(RRR) &= \frac{5}{15} \times \frac{5}{15} \times \frac{5}{15} \\ &= \frac{125}{3375} \\ &= \frac{1}{27} \end{aligned}$$

$$\begin{aligned} (ii) \frac{125}{3375} + \frac{64}{3375} + \frac{216}{3375} &= \frac{405}{3375} \\ &= \frac{3}{25} \end{aligned}$$

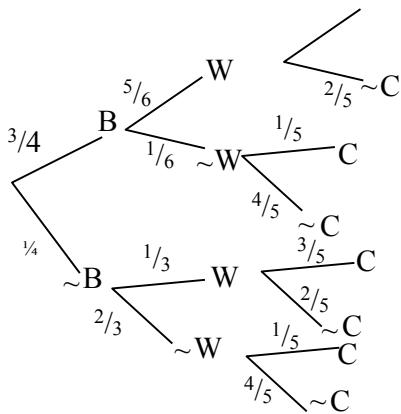
(iii)  $P(RBG) + P(GRB) + P(BGR)$

$$\begin{aligned}
 & \frac{5}{15} X \frac{4}{15} X \frac{6}{15} + \frac{6}{15} X \frac{5}{15} X \frac{4}{15} + \frac{4}{15} X \frac{6}{15} X \frac{5}{15} \\
 & = \frac{120}{3375} + \frac{120}{3375} + \frac{120}{3375} \\
 & = \underline{24}
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad & P(BBB) + P(GGG) + P(BBG) + P(GGB) \\
 & = \frac{4}{15} X \frac{4}{15} X \frac{4}{15} + \frac{6}{15} X \frac{6}{15} X \frac{6}{15} + \frac{4}{15} X \frac{4}{15} X \frac{6}{15} + \frac{6}{15} X \frac{6}{15} X \frac{4}{15} \\
 & = \frac{64}{3375} + \frac{216}{3375} + \frac{96}{3375} + \frac{144}{3375} \\
 & = \frac{520}{3375} + \frac{104}{3375}
 \end{aligned}$$

**B- To bed on time**  
**B- To bed late**  
**W- Waking upon time**  
**W- waking up late**  
**C- Getting to class on time**  
**C- Getting to class late**

15.



**B- To bed on time**  
**~B- To bed late**  
**W- Waking upon time**  
**~W- waking up late**  
**C- Getting to class on time**  
**~C- Getting to class late**

$$\begin{aligned}
 (a) \quad (i) \quad & P(Bnw) = \frac{3}{4} \times \frac{5}{6} \\
 & = \frac{5}{8}
 \end{aligned}$$

ii)  $P(\text{Waking up late})$

$$\begin{aligned}
 & \left( \frac{1}{4} \times \frac{1}{3} \right) + \left( \frac{1}{2} \times \frac{2}{3} \right) \\
 & = \frac{1}{8} + \frac{1}{6} = \frac{3+4}{24} \\
 & = \frac{7}{24}
 \end{aligned}$$

✓ tree diagram.  
 ✓ Addition of probability  
 ✓ Addition of prob.  
 ✓ Addition of prob.

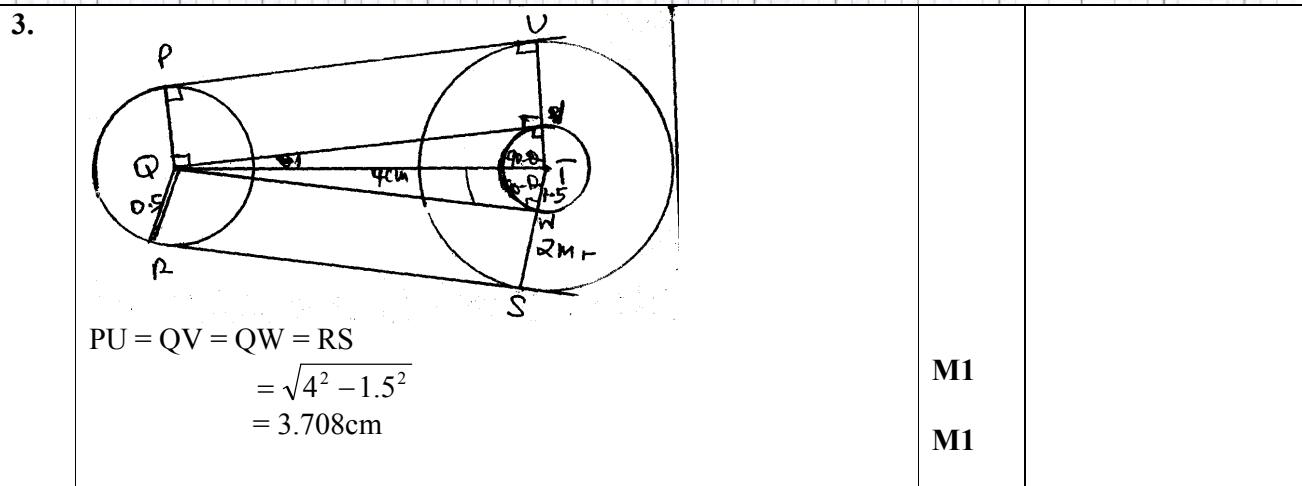
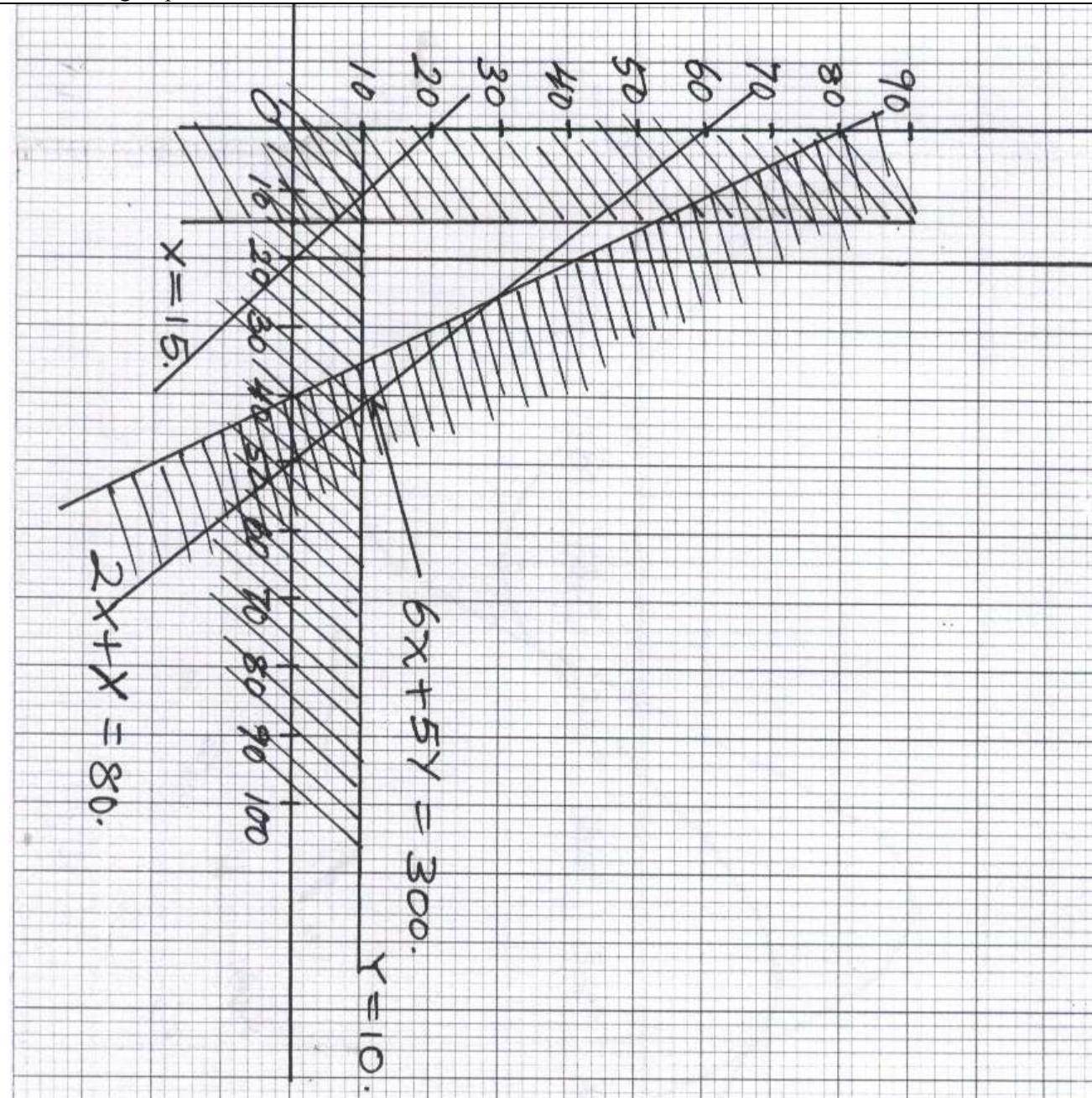
b) (i)  $P(BW\sim C)$  or  $P(B\sim W\sim C)$

$$\begin{aligned}
 & \frac{1}{10} \left( \frac{3}{4} \times \frac{1}{3} \times \frac{4}{5} \right) + \left( \frac{3}{4} \times \frac{5}{6} \times \frac{2}{5} \right) \\
 & = \frac{1}{10} + \frac{1}{4} = \frac{4+10}{40} \\
 & = \frac{7}{20}
 \end{aligned}$$

$$\begin{aligned}
 ii) \quad & P(\sim B \sim C) = \frac{1}{4} \times \frac{1}{3} \times \frac{3}{5} + \frac{1}{4} \times \frac{2}{3} \times \frac{1}{5} \\
 & = \frac{1}{20} + \frac{1}{30} = \frac{3+2}{60} = \frac{5}{60} \\
 & = \underline{\frac{1}{12}}
 \end{aligned}$$

## 55. Compound proportions, mixtures and rates of work

<b>1</b>	$\text{Cost of mixture per kg} = \frac{(5 \times 30) + (3 \times 60)}{8}$ $= \text{sh } 41.25$ $\text{Selling price} \Rightarrow \frac{130}{100} \times 41.25$ $= \text{Ksh. } 53.625$	$M_1$ $M_1$ $A_1$	Allow 53.60
		3	
<b>2</b>	a) x-Hexagonal and y-Rectangular tables $x \geq 15$ $y \geq 10$ $6x + 3y \leq 240 (2x + y \leq 80)$ $120x + 100y \leq 6000 \quad (6x + 5y \leq 300)$ b) $x \geq 15$ region ✓ shaded $y \geq 10$ region ✓ shaded $2x + y \leq 80$ region ✓ shaded $6x + 5y \leq 300$ region ✓ shaded c) (i) Search line ✓ drawn (26, 28) (ii) $(26 \times 80 + 28 \times 60)$ Ksh. 3,760	B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1	
		10	



$\sin \theta = \frac{1.5}{4} \Rightarrow \sin^{-1} 0.375 = 22.02^\circ$ $\angle PQR (\text{obtuse}) = 360^\circ - 2(90 + 22.02)$ $= 135.96^\circ$ $\text{Arc } PR = \frac{135.96}{360} \times 2 \times 3.14 \times 0.5 = 1.1866$ $\text{Arc US subtends angle } 360 - 2(90 - 22.02)$ $= 224.04$ $\text{Arc US} = \frac{224.04}{360} \times 2 \times 3.142 \times 2 = 7.8215$ $\text{Total length} = 3.708 \times 2 + 1.1866 + 7.8215$ $= 16.4241$ (b) $(\frac{4}{3} \times 16.4241)$ $21.8988$ $22.90$	<b>M1</b> <b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>B1</b>	<b>10</b>
---	--	-----------

1. a) Deposit: Total ratio  $2 + 3 + 5 = 10$

$$\text{Georgina: } \frac{2}{10} \times 30000 = 6000$$

$$\text{Gilbert: } \frac{3}{10} \times 30000 = 9000$$

$$\text{Akumu: } \frac{5}{10} \times 30000 = 15000$$

b) Balance to be paid

$$= 510000 - 30000 = 480000$$

$$\text{Each pays } \frac{480000}{3} = 160000$$

c) Profit =  $\frac{20}{100} \times 510000 = 102000$

$$\text{Georgina received: } \frac{1}{6} \times 102000 = 17000$$

$$\text{Gilbert received: } \frac{2}{6} \times 102000 = 34000$$

$$\text{Akumu received: } \frac{3}{6} \times 102000 = 51000$$

2. Men Days

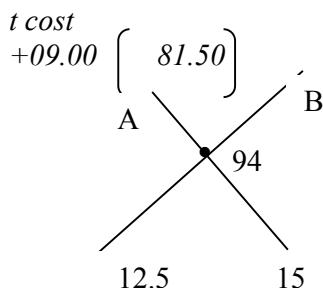
$$12 \quad 20$$

$$16 \quad ?$$

$$= \frac{(12 \times 20)}{16} \text{ days}$$

$$= 15 \text{ days}$$

3

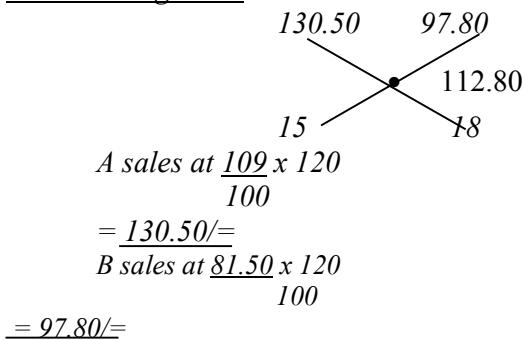


Cost of mixture

$$\text{Sh } 112.8 \times \frac{100}{120} = 94 \text{ per kg}$$

$$\begin{aligned}
 & \text{Ratio } A : B \\
 & (81.50 - 94) : (109 - 94) \\
 & 12.5 : 15 \\
 & 2.5 : 3 \\
 & 5 : 6
 \end{aligned}$$

Alt. At selling Price



$$\begin{aligned}
 & A \& B \text{ mixed sells at} \\
 & \frac{94 \times 120}{100} =
 \end{aligned}$$

sh 112.80 per kg

Ratio  $A : B$

$$(112.80 - 97.8) :$$

$$(130 - 112.8)$$

$$15 : 18$$

$$5 : 6$$

4 Let Onacha take  $x$  days.

Mogutu takes  $x + 5$  days.

$$\frac{1}{x} + \frac{1}{x+5} = 1$$

$$x^2(x+5) + 6x = x(x-5)$$

$$x^2 - x - 30 = 0$$

$$(x-10)(x+3)$$

$$x = 10, 3$$

Onacha takes 10 days.

5  $\frac{dy}{dx} = 6x^2 + x - 4$

When  $x = 1$ ,

$$\frac{dy}{dx} = 6+1-4 = 3$$

Grad of normal  $= -1/3$

$$y + \frac{1}{2} = -\frac{1}{3}(X-1)$$

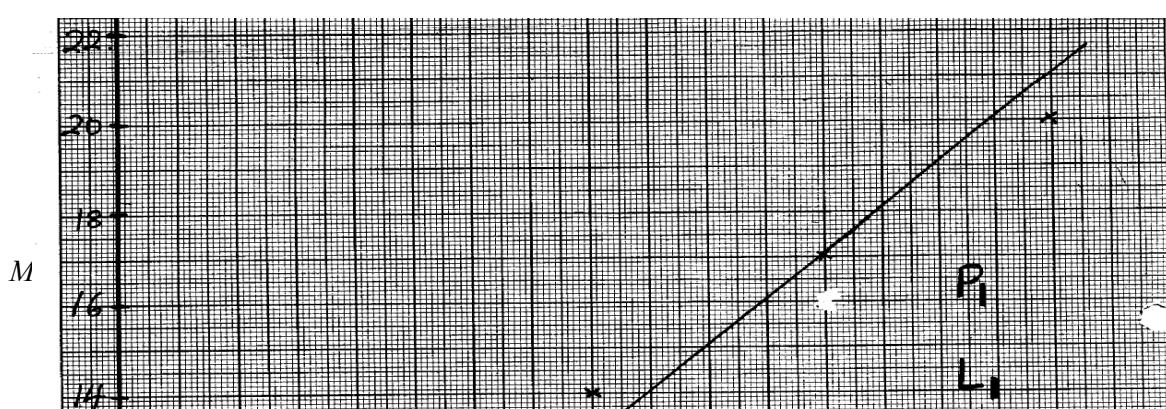
$$y = -\frac{1}{3}x - \frac{1}{6}$$

6 Gradient  $= \frac{11-8}{3-1.5}$

$$= 2$$

$$K = 2, M = 5$$

$$B = 2A + 5$$



$$\begin{aligned}
 7 \quad & (70 - 25 \times 60 = 2700 \\
 & 2700 \cos 47 \\
 & = 2700 \times 0.68 = 1841.4 \text{ nm}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad & \frac{6x 72 + 66x 4}{10} = 69.6 \\
 & 100\% = 69.6 \\
 & \therefore 105 = 73.10
 \end{aligned}$$

$$\begin{aligned}
 9 \quad (a) (i) \quad & A \quad B \quad \text{Mixture} \\
 & 150 \quad 160 \quad 156 \\
 & 1 \quad n \quad 1+n \\
 & 150 \quad 160n \quad (n+1)156 \\
 & 150 + 160n = 156(n+1) \\
 & N = \frac{6}{4} = \frac{3}{2} \\
 & = \frac{112}{100} \times 156 \\
 & = \text{shs. } 174.72
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \text{At } 11.45 \text{ a.m} \\
 & \text{Depth filled by } P \text{ in } 2 \text{ hrs} = 2.1 \text{ m} \\
 & 3 \text{ hrs} = \frac{3 \text{ hr}}{2 \text{ hr}} \times 2.1 \text{ m} \\
 & = 3.15 \text{ m} \\
 & \text{Depth filled by } q \text{ in } 7 \text{ hrs} = 2.1 \text{ m} \\
 & 3 \text{ hrs} = \frac{3 \text{ hrs}}{7 \text{ hrs}} \times 2.1 \text{ m}
 \end{aligned}$$

$$= 0.9m$$

*Depth emptied by R in 6hrs = 2.1m*

$$\frac{2\text{hrs}}{6\text{hrs}} = 2\text{hr} \times 2.1$$

$$\therefore \text{Depth at } 11.45\text{a.m} = (3.15 + 0.9) - 0.7 = 3.35m$$

- 10 Let the amount to be mixed be  $x$  kg of the lower, priced grade and  $y$  kg for higher price grade

$X$  kg of the lower priced grade cost Sh. 420x

$Y$  kg of the higher priced grade cost Sh.470y

Total cost of  $(x+y)$  kg of mixture

$$= \text{Shs.} \frac{420x + 470y}{x+y}$$

$$\text{equating } \frac{420x + 470y}{x+y} = 455$$

$$420x + 470y = 455x + 455y$$

$$470y - 455y = 455x - 420y$$

$$15y = 35x$$

$$X: y = 3:7$$

11. Cross sectional area =  $r^2$

$$= \left( \frac{22}{7} \times 35 \times 35 \right) \text{cm}^2$$

$$\text{Flow per second} = \left( \frac{22}{7} \times 35 \times 35 \times 45 \right) \text{cm}^2$$

$$\text{After } 2\frac{1}{4} \text{ hrs} = \left( \frac{22}{7} \times 35 \times 35 \times 45 \times 3 \times 60 \times 69 \right) \text{liters}$$

$$= 233887.5 \text{litres}$$

- 12 a) In 2000, Costs Shs

$$\text{Material} = \frac{8}{25} \times 1250 = 400$$

$$\text{Labour} = \frac{14}{25} \times 1250 = 700$$

$$\text{Transport} = \frac{3}{25} \times 1250 = 150$$

In 2003

$$\text{Material} = 400 \times 2 = 800$$

$$\text{Labour} = \frac{130}{100} \times 700 = 910$$

$$\text{Transport} = \frac{120}{100} \times 150 = 180$$

b) In 2004 Costs

$$\text{Material} = 800$$

$$\text{Transport} = 180$$

$$\therefore \text{labour} = 1981 - (800 + 180) = \text{Shs.} 1001$$

$$\therefore \text{Increase in labour} = 1001 - 910 = 91$$

$$\% \text{increase} = \frac{91}{910} \times 100$$

$$= 10\%$$

13. Cost price =  $100 \times 114 = \text{shs.} 95$

$$120$$

Let A: B =  $n : 1$

$$\frac{95}{1} = \frac{80n + 100}{n + 1}$$

$$95n + 95 = 80n + 100$$

$$15n = 5$$

$$n = \frac{1}{3}$$

$$n : 1 = 1 : 3$$

$$A:B = 1:3$$

14. Let the ratio be  $x:y$

$$76x + 84y = 81(x + y)$$

$$84y - 81y = 81x - 76x$$

$$3y = 5x$$

$$3 = x$$

$$5y$$

$$x:y = 3:5$$

15. a) Cost of 8kg =  $5x25 + 2x30 + 1x45 = 230$

$$\text{Cost of } 1 \text{ kg} = \frac{230}{8} = 28.75$$

$$\text{Profit/kg} = 28.75 \times \frac{20}{100}$$

$$= 5.75$$

b) i) Selling price

$$= 28.75 \times \frac{112}{100} = 32.20$$

$$32.20 \times \frac{120}{100} = 38.64$$

$$38.64$$

ii) New cost/kg

$$= 1.12 \times 28.75 = 32.20$$

$$\% \text{ Profit} = \frac{40.25 - 32.20}{32.20} \times 100$$

$$= 25\%$$

16.  $= \frac{3(5.60) + 11y}{14} = 6.70$

$$14$$

$$= 16.8 + 11y = 93.8$$

$$11y = 77$$

$$y = 7$$

1Kg costs Shs. 7.00

## 56. Graphical methods

1.  $x^2 + 4x + y^2 = 5$

$$x^2 + 4x + (\frac{1}{2} \times 4)^2 + y^2 = 5 + (\frac{1}{2} \times 4)^2$$

$$(x + 2)^2 + (y + 0)^2 = 5 + 4$$

$$(x + 2)^2 + (y + 0)^2 = 9$$

Centre (-2,0)

Radius  $\sqrt{9}$

$r = 3$  units

2.  $x^2 + 6x + (3)^2 + y^2 - 10y + (-5) = 2 + 9 + 25$

$$(x + 3)^2 + (y - 5)^2 = 36$$

$$(x - 3)^2 + (y - 5)^2 = 6^2$$

$$\therefore \text{centre } (-3, 5)$$

$$\text{Radius } 6 \text{ units}$$

3.  $CBE = 40^\circ$  (alt.segment theorem)

$$\angle BCE = 120^\circ$$
 (Suppl. To  $BCD = 60^\circ$  alt. seg.)

$$\therefore (40 + 120 + E) = 180^\circ$$
 (Angle sum of  $\Delta$ )

$$\angle BEC = 20^\circ$$

4.  $X^2 + Y^2 - 10Y + 25 = 25 - 16$

$$(X - 0)^2 + (Y - 5)^2 = 9$$

$$(X - 0)^2 + (Y - 5)^2 = 3^2$$

$$\text{Centre } (0, 5)$$

$$\text{Radius} = 3$$

5.

$x$	-5	-4	-3	-2	-1	0	1
$x^3$	-125	-64	-27	-8	-1	0	1
$6x^2$	150	96	54	24	6	0	6
$8x$	-40	-32	-24	-16	-8	0	8
$y$	-15	0	3	0	-3	0	15

$$x^3 + 6x^2 + 8x > 1$$

Between

(i)  $x = -3.85 \pm 0.1$  and  $x = -2.15 \pm 0.1$

(ii)  $x > 0.5 \pm 0.1$

6.  $y = x^3 - 3x + 2$

$$x = 0, y = 2$$

$$(0, 2) \Rightarrow y - \text{intercept.}$$

$$\frac{dy}{dx} = 3x^2 - 3 = 0$$

$$x^2 = 1$$

$$x = \sqrt{1}$$

$$x = 1, y = 0$$

Point  $(1, 0)$  min point

$$x = -1, y = 4$$

Point  $(-1, 4)$  max point.

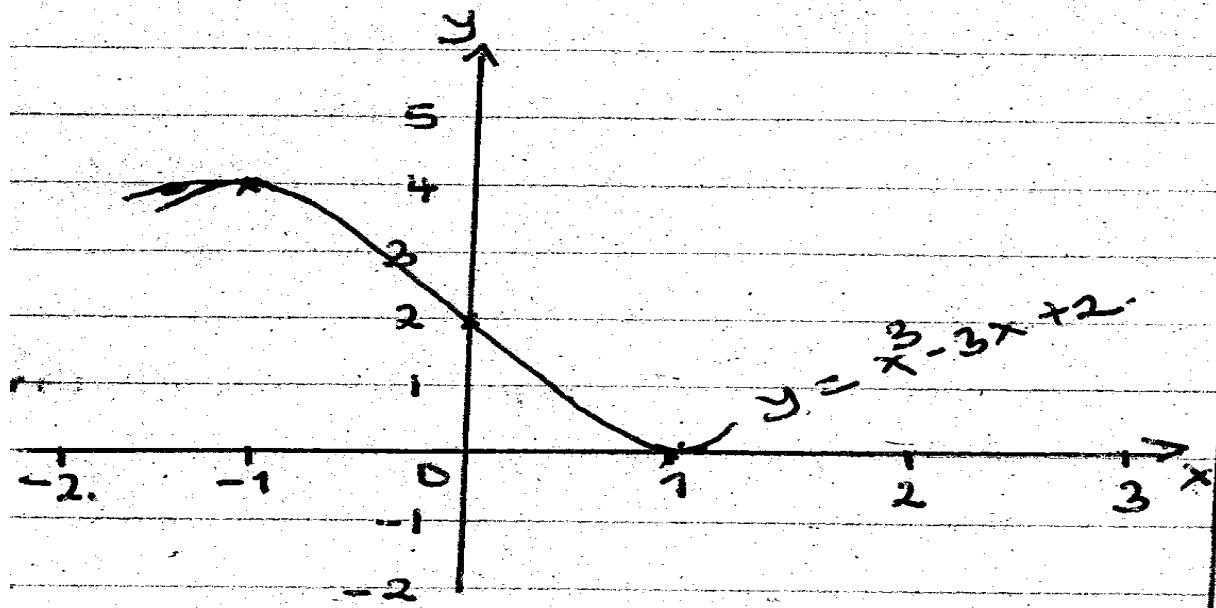
**Completing of sq. for expression in x and y.**

**✓ Expression.**

**✓ Centre**

**✓ Radius**

Point  $(-1, +)$  max point



$$\begin{aligned}
 7. \quad & 4x_2 - 12x + 4y^2 + 12y = 7 \\
 & x^2 - 3x + y^2 + 3y = \frac{7}{4} \\
 & x^2 - 3x + (\frac{3}{2})^2 + y^2 + 3y + (\frac{3}{2})^2 = \frac{7}{4} + \frac{9}{4} + \frac{9}{4} = \frac{25}{4} \\
 & (x - \frac{3}{2})^2 + (y + \frac{3}{2})^2 = \frac{25}{4} \\
 & \therefore \text{Centre } (1.5, -1.5) \quad \text{Radius } 2.5 \text{ units}
 \end{aligned}$$

$$8. \quad \log R = n \log p + \log K$$

$\log P$	0.48	0.54	0.60	0.65	0.70
$\log R$	1.56	1.69	1.81	1.91	2.00

$$\begin{aligned}
 \text{Gradient} &= \frac{2 - 0.6}{0.7} \\
 &= \frac{1.4}{0.7} = 2
 \end{aligned}$$

$$\begin{aligned}
 \log R \text{ intercepts} &= 0.6 = \log k \\
 K &= 4
 \end{aligned}$$

The law connecting  $R$  and  $P$  is  $R = 4P^2$

$$900 = 4P^2$$

$$P^2 = \frac{900}{4}$$

$$225 = P^2$$

$$\begin{aligned}
 9. \quad & (x+2)^2 (y-3)^2 = 3^2 \\
 & X^2 + 4x + 4 + y^2 - 6y + 9 = 3^2 \\
 & X^2 + y^2 + 4x - 6y + 4 = 0
 \end{aligned}$$

10.

$V$	0	2	4	6	8	10
$\frac{1}{T}$	2.04	3.33	4.17	5	6.25	7.30

$$\underline{T = a}$$

$$\begin{aligned} b + V \\ \frac{I}{T} = \frac{b+V}{a} \\ \frac{I}{T} = \frac{IV}{a} + \frac{b}{a} \\ y = mx + C \end{aligned}$$

$$\frac{b)(i)}{a} - 1 = \frac{Grad}{\Delta x} \Rightarrow \Delta y = \frac{7.3 - 5}{10 - 6} = \frac{2.3}{4} = 0.575$$

$$a = 1.739$$

$$\begin{aligned} \frac{b}{a} &= y - Intercept \Rightarrow 2.04 \\ \frac{b}{1.739} &= 2.04 \quad b = 2.04 \times 1.739 \\ &\quad = 3.547556 \\ &\quad b \simeq 3.548 \end{aligned}$$

$$(ii) T = 0.38$$

$$\frac{I}{T} = 2.63 \text{ shown on graph}$$

$$V = I$$

$$\begin{aligned} (iii) \quad \frac{I}{T} &= 4.45 \\ T &= (4.45) \\ &= 0.2247 \\ &\simeq 0.22 \end{aligned}$$

$$11. \quad y = 2x^3 + x^2 + 3x - 1$$

$$\frac{dy}{dx} = 6x^2 + 2x + 3$$

gradient at (1, -5)

$$= 6 + 2 + 3 = 11$$

$$\underline{y - (-5)} = 11$$

$$x - 1$$

$$y + 5 = 11x - 11$$

$$y = 11x - 16$$

$$12. \quad 3^5 = 3^{-4} x 3^{-x}$$

$$3^5 = 3^{-4-x}$$

$$-4 - x = 5$$

$$-x = 9$$

$$x = -9$$

$$13. \quad x^2 + 2x + 1 + y^2 - 4y + 4 = 4 + 1 + 1$$

$$(x+1)^2 + (y-2)^2 = 9$$

Centre (-1, 2)

Radius 3 units

$$14. \quad c)$$

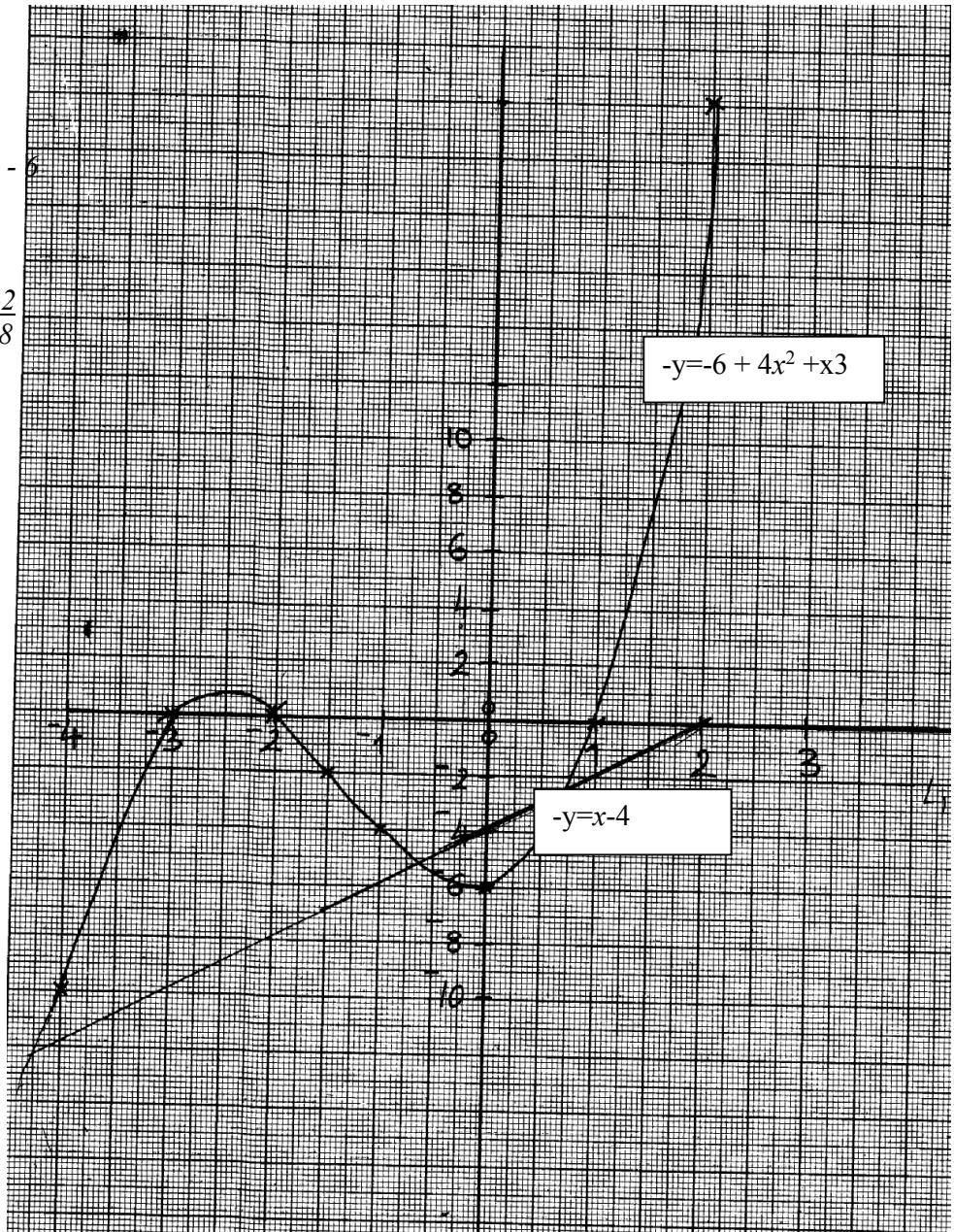
X	-4	-3	-2	-1	0	1	2
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-6	-6	-6	-6	-6	-6	-6	-6
X	-4	-3	-2	-1	0	1	2
$4x^2$	64	36	16	4	0	4	16
$X^3$	-64	-27	-8	-1	0	1	8
$Y = -6 + x + 4x^2 + x^3$	-10	0	0	-4	-6	0	20

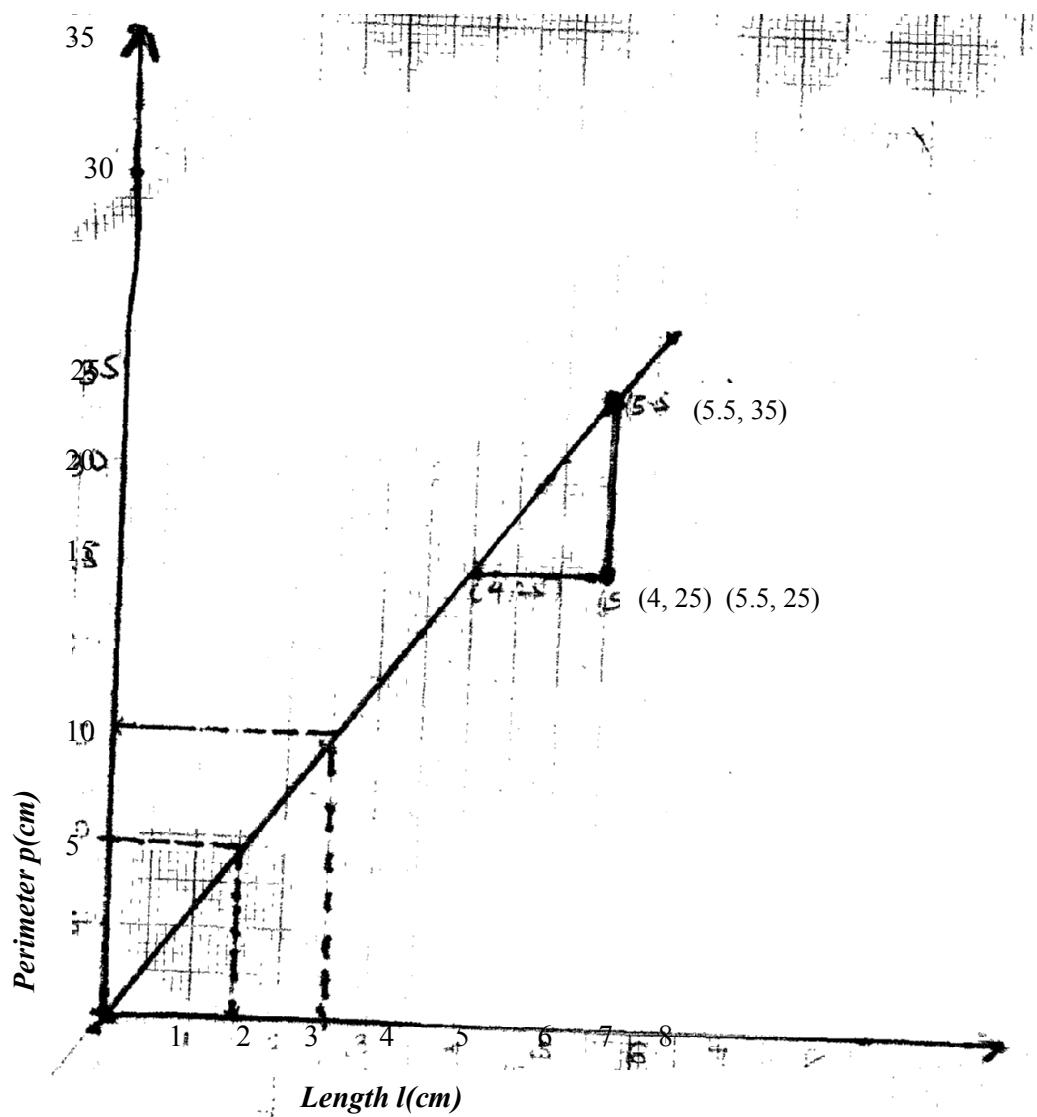
$$\begin{aligned}y &= x^3 + 4x^2 + x - 6 \\0 &= x^3 + 4x^2 + x - 4 \\y &= -2\end{aligned}$$

$$\begin{aligned}(iii) \quad y &= x^3 + 4x^2 + x - 6 \\0 &= x^3 + 4x^2 + 0 - 2 \\y &= -x - 4\end{aligned}$$

$$\begin{array}{r|rrr}x & 1 & 0 & -2 \\ \hline y & -3 & -4 & -8\end{array}$$



- c      (i) solution 0.8  
       -1.5  
       And -3.2  
       (c) 1, -2, -3



$$(i) P = 15.75 \text{ cm}$$

$$(ii) l = 1.5 \text{ cm}$$

$$(iii) m = \frac{35 - 25}{5.5 - 4.0} = \frac{10}{1.5} = 6.67$$

(c) choose  $P(5, 31.4)$

$$\frac{p - 31.4}{l - 5} = \frac{10}{1.5}$$

$$\frac{p - 31.4}{l - 5} = \frac{100}{15}$$

$$15p - 471 = 100l - 500$$

$$15p = 100l - 29$$

$$15 \ 15$$

$$2k = \frac{100}{15}$$

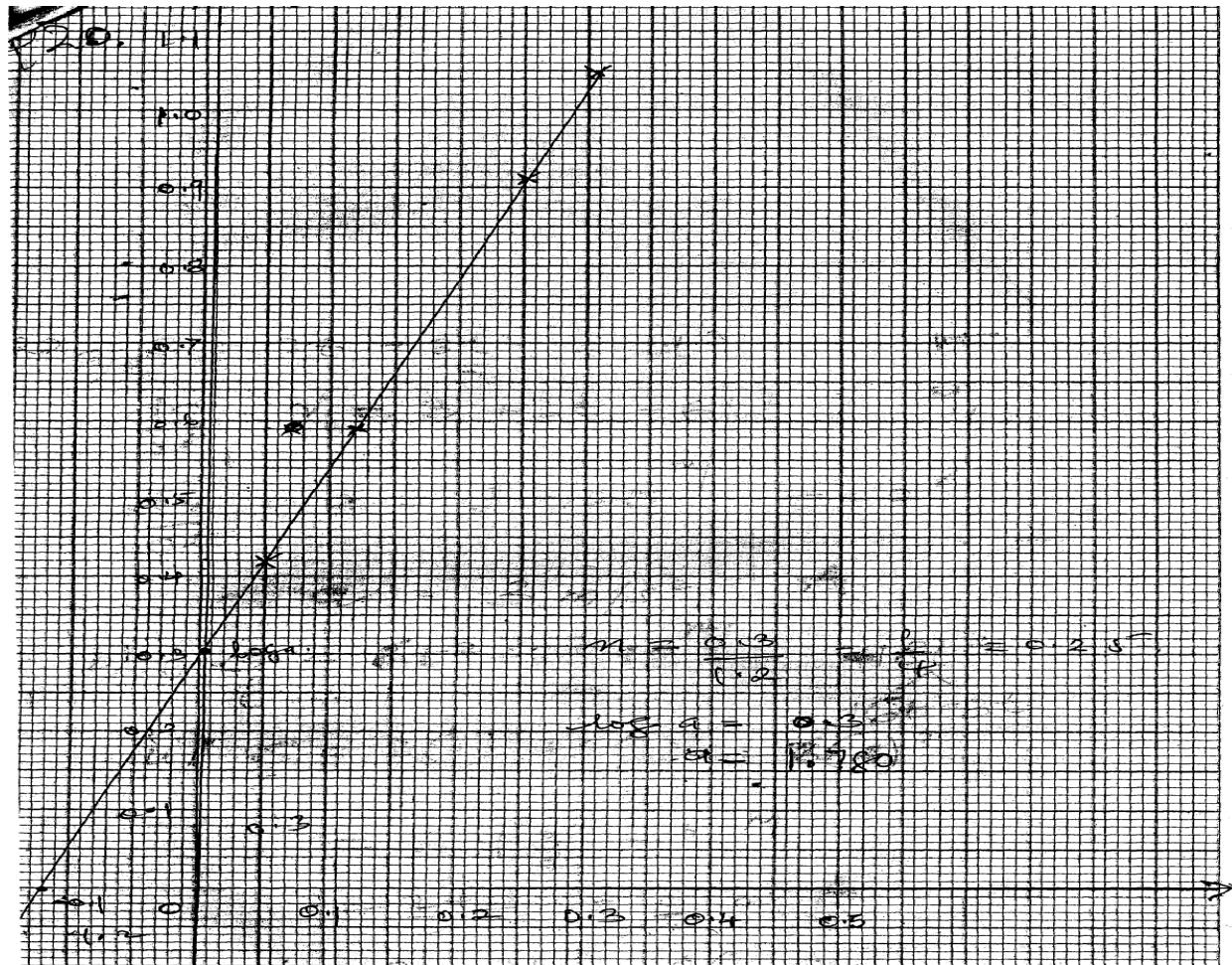
$$k = \frac{100}{2 \times 15} = 3.33$$

$$c = 1.93$$

$$P + 0.6 = ar^h$$

$$\begin{aligned} \log(P + 0.6) &= \log a + n \log R \\ &= n \log R + \log 9 \end{aligned}$$

$P + 0.6$	1.33	2.65	3.85	8.04	11.22
$\log(P + 0.6)$	-0.13	0.42	0.59	0.91	1.05
$\log R$	-0.05	0.05	0.12	0.25	0.30



$$\begin{aligned} 17. \quad x^2 + y^2 - 6x &= 3 - 4y \\ x^2 - 6x + (-6/2)^2 + y^2 + 4y + (4/2)^2 &= 3 + (-6/2)^2 + (4/2)^2 \end{aligned}$$

$$(x - 3)^2 (y + 2)^2 = 3 + 9 = 4$$

$$(x - 3)^2 (y + 2)^2 = 16$$

$$C(3, -2)$$

$$\text{Gradient } \frac{\Delta y}{\Delta x} = \frac{7 - -2}{6 - 3} = 3$$



$x$	-3	-2	-1	0	1	2	3	4
$-x^3$	27	8	1	0	-1	-8	-27	-64
$2x^2$	18	8	2	0	2	8	18	32
$-4x$	12	8	4	0	-4	-8	-12	-16
2	2	2	2	2	2	2	2	2
$y$	59	26	9	2	-1	-6	-19	-46

b) Check on the graph paper.

c)  $x = 0.5 \pm 0.1$

d)  $-x^3 + 2x^2 - 5x + 3 = 0$

Line to allow:  $y = x - 1$

$x$	0	1
$y$	-1	0

$x = 0.65$

19.  $\frac{Dy}{dx} = 12x^2 - 12$

$12x^2 - 12 = 0$

$12(x^2 - 1) = 0$

$x = 1$

$x = -1$

At  $x = 1$

	0	1	2	-2	-1	0
GRD = 12	0	36	36	0	-12	

- 0 + + 0 -

At  $x = -1$

Minimum

(1, 7)

maximum

(-1, 9)

20.

(a) table

(b) plotting

scale

smooth curve

(c) (i)  $-0.5 < x < 1$  and  $x > 1$

(iii)  $x = 2.5 \pm 0.1$

21.

$2x^2 + 2y^2 - 6x + 10y + 9 = 0$

$x^2 + y^2 - 3x + 5y + 9/2 = 0$

$x^2 + y^2 - 3x + 5y = -9/2$

$x^2 - 3x + 9 + y^2 + 5y + 25 = 8.5 - 4.5$

$\frac{4}{2}$   $\frac{4}{2}$

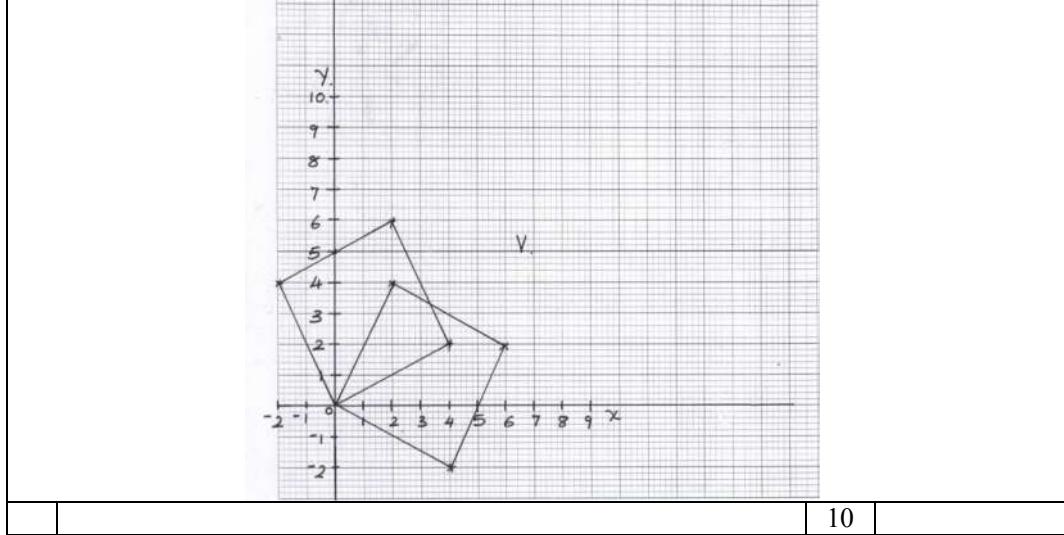
$(x - 3)^2 + (y + 5)^2 = 4$

Radius = 2 units

Centre = (1.5, -2.5)

## 57. Matrices and Transformations

<b>1</b>	a) (i) (ii) $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 0 & 4 & 2 & -2 \\ 0 & 2 & 6 & 4 \end{pmatrix}$ $T \Rightarrow (0,0), (4,-2), (2,6), (-2,4)$ (iii) $\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 6 & 2 \\ 0 & -2 & 2 & 6 \end{pmatrix}$ $U \Rightarrow (0,0), (4,-2), (6,2), (2,4)$ b) (i) $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 2 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$ $V \Leftarrow (0,0), (10,0), (10,10), (0,10)$ (ii) Enlargement center (0,0) s.f = 5	B <sub>1</sub> M <sub>1</sub>  A <sub>1</sub> B <sub>1</sub> B <sub>1</sub> B <sub>1</sub>  B <sub>1</sub>  B <sub>1</sub> B <sub>1</sub>  ✓ coordinates given Square T drawn ✓ coordinates (implied) Square U drawn  ✓ coordinates (implied) Square V drawn
----------	---	---



10

2.	<p><u>Q18</u></p> <p>a) reflection in line <math>x=0</math>    B1      B1    A <math>PQR</math>      B1    A <math>P''Q''R''</math>      B1 line <math>y=-x</math>      B1    A <math>P'Q'R'</math>      B2    A <math>P'''Q'''R'''</math></p> <p>e) Opposite Congruence  <math>PQR</math> and <math>P''Q''R''</math>  <math>PQR</math> and <math>P'Q'R'</math>  <math>P''Q''R''</math> and <math>P'''Q'''R'''</math>  <math>P'Q'R'</math> and <math>P'''Q'''R'''</math></p> <p>Total 10</p>	
		10

1. a)  $B(4, -5)$ ,  $C(3, 6 \frac{1}{2})$

$\Delta ABC$  drawn

$\Delta ABC$  drawn

a) ii) Shear maps

1

$$\text{Matrix} = \begin{pmatrix} I & (1, 1\frac{1}{2}) \\ 1 & 0 \\ 1 & \frac{1}{2} \end{pmatrix}$$

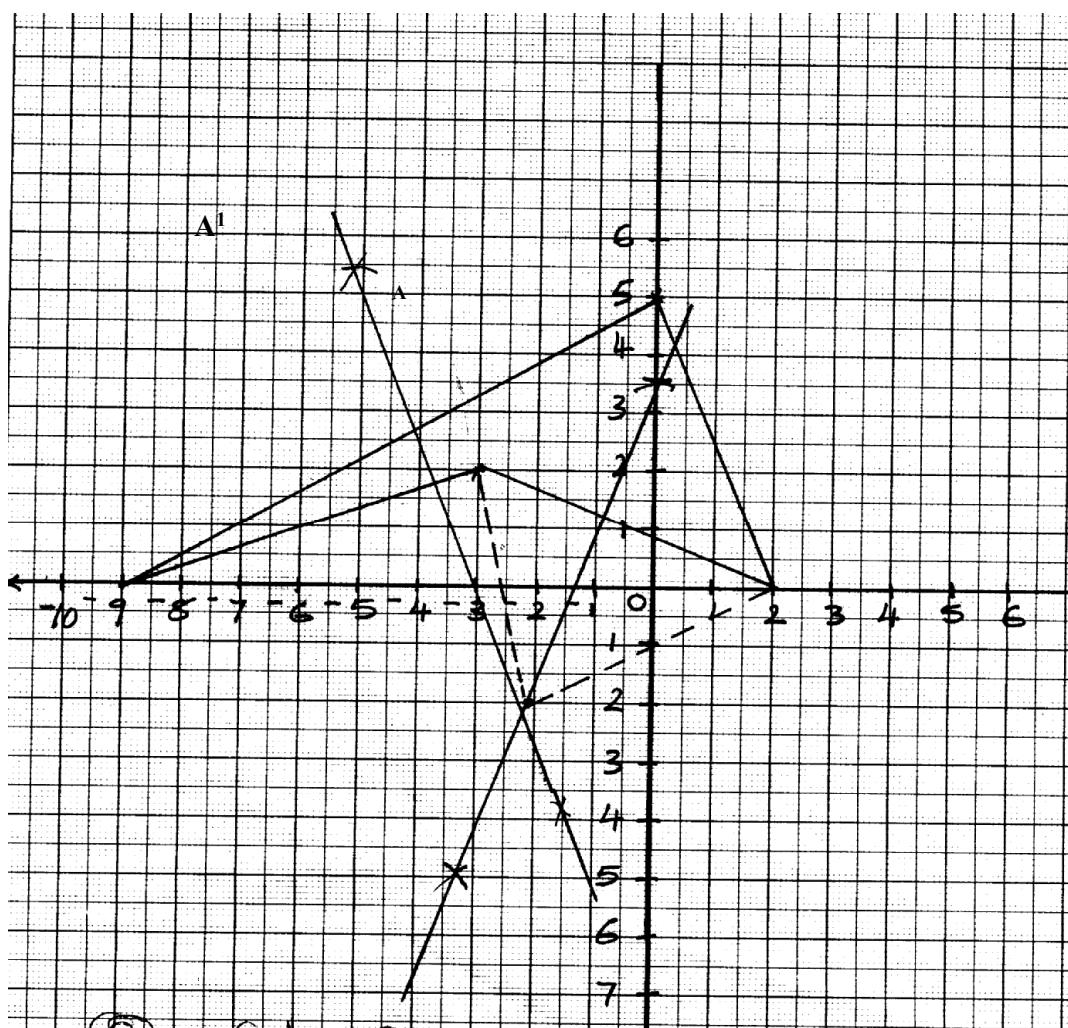
$$b) i) \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & B \\ \frac{3}{2} & -1 & C \end{pmatrix} \begin{pmatrix} A & B & C \\ -6 & -4 & 3 \\ -4 & -5 & 6 \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} A^{II} & B^{II} & C^{II} \\ 6 & 4 & -3 \\ -5 & -1 & -2 \end{pmatrix}$$

$\Delta A^{II} \ B^{II} \ C^{II} \ D^{II}$  drawn

ii) Half turn about  $(0,0)$ 

2.

 $B^1$ 

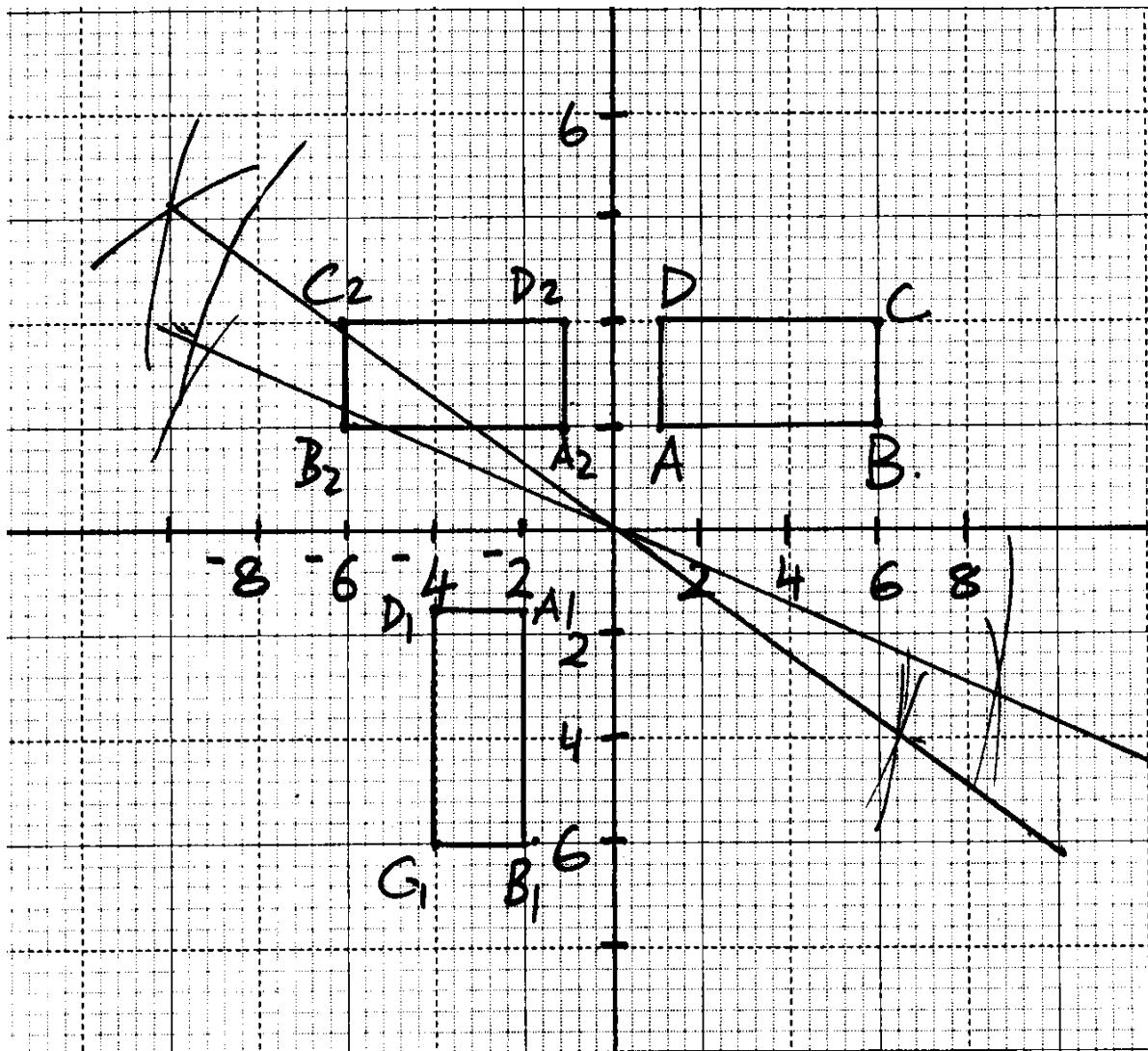
- (a) Centre  $(-2, -2)$   $90^\circ$   
 (b)  $A11(-2, -4)$ ,  $B11(0, 9)$   
 (c) Half-turn about the centre  $(0, 2)$

3.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} A & B & C & D \\ 1 & 6 & 6 & 1 \\ 2 & 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} A^1 & B^1 & C^1 & D^1 \\ -2 & -2 & -4 & -4 \\ -1 & -6 & -6 & -6 \end{pmatrix} \begin{matrix} A_1(-2, -1) \\ B_1(-2, -6) \\ C_1(4, -6) \\ D_1(-4, -1) \end{matrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} A_1 & B_1 & C_1 & D_1 \\ -2 & -2 & -4 & -4 \\ -1 & -6 & -6 & -6 \end{pmatrix} \begin{pmatrix} A_2 & B_2 & C_2 & D_2 \\ 2 & 2 & 4 & 4 \end{pmatrix} \begin{matrix} A_2(-1, 2) \\ B_2(-2, -6) \\ C_2(-6, 4) \\ D_2(-6, 4) \end{matrix}$$

(b)

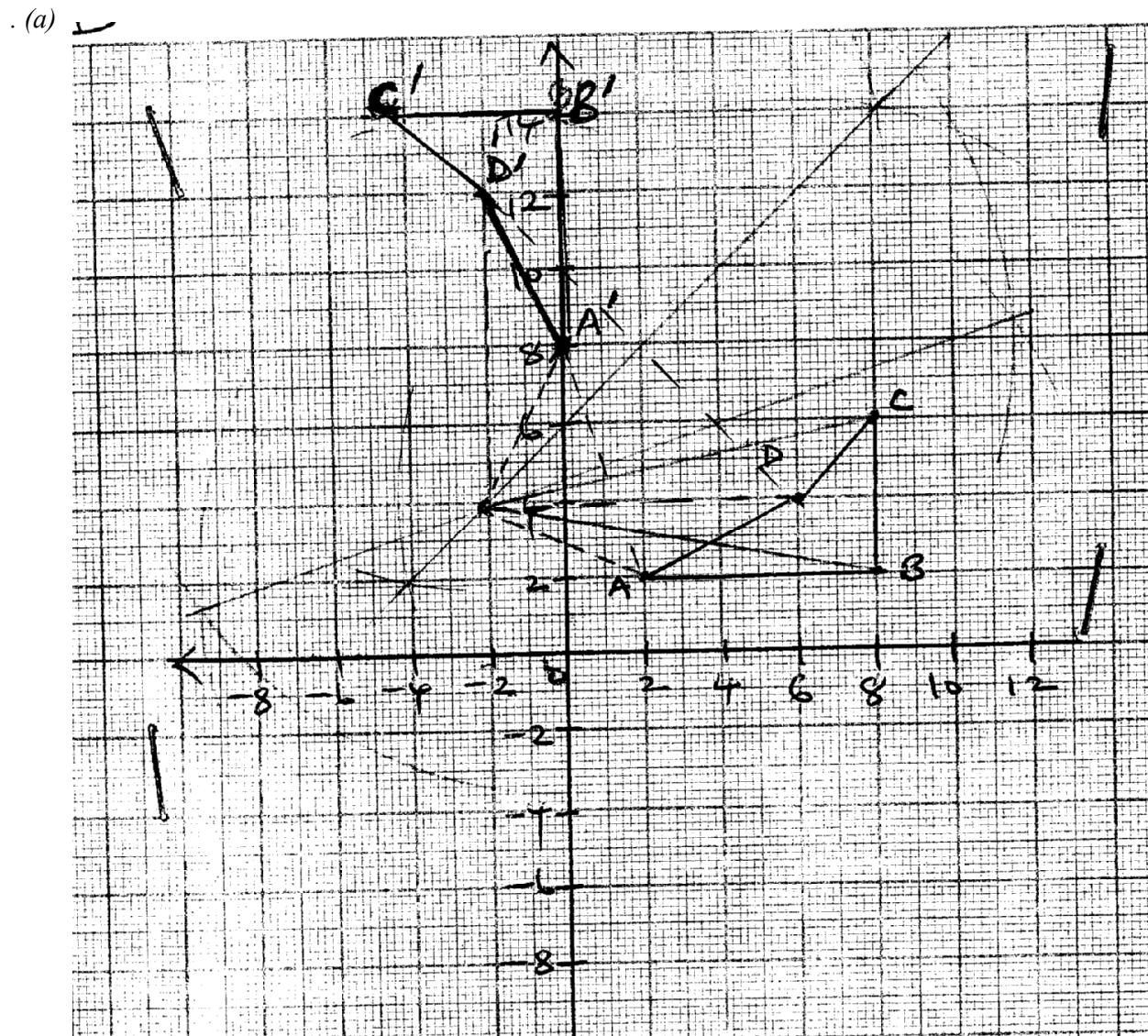


(c) (i)  $U - \text{positive three-quarter turn about the origin}$

(ii)  $UT - \text{Reflection in the line } x = 0$

(d)  $I \cap I_1 = I_2.5 \times -2 - 1 \times 0 = 5$

$$\therefore \text{Area} = 5 \times (5 \times 2) = 20 \text{ sq. units}$$



b) Centre (-2, 4)

Angle + 90°

5.  $P(5, -3)$

$$\begin{bmatrix} 5 \\ -3 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$R^I = \begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{array}{r} -3 \\ -2 \end{array}$$

$$\begin{aligned} &= -5 \\ &-5 \\ P^I R^I &= \begin{pmatrix} -5 & -2 \\ -5 & -5 \end{pmatrix} \\ &= \begin{pmatrix} -7 \\ 0 \end{pmatrix} \end{aligned}$$

Mag. = 7 units

6.  $A^I = (0+1, -1-2) = (1, -3)$   
 $B^I = (4+1, 3-2) = (4, 1)$   
 $C^I = (2+1, 2-2) = (3, 0)$

Matrix  $\begin{pmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 0 \\ 0 & 3 \end{pmatrix} A^{II} \begin{pmatrix} 3 \\ -9 \end{pmatrix} B^{II} \begin{pmatrix} 15 \\ 3 \end{pmatrix} C^{II} \begin{pmatrix} 9 \\ 0 \end{pmatrix}$

Determinant (0-9) = -9

Area =  $9 \times 24 = 216 \text{ cm}^2$ 

$$\begin{pmatrix} a & b \\ c & d \\ 5(31-9b) = 1 & 5(3c-9d) = -3 \\ -15a+3b = 5 & 15c+3d = 1 \end{pmatrix} \begin{pmatrix} 3 & 15 \\ -9 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -3 & 1 \end{pmatrix}$$

$$-48b = 0 \quad -48d = -16$$

$$b = 0 \quad d = 1/3$$

$$a = 1/3 \quad c = 0$$

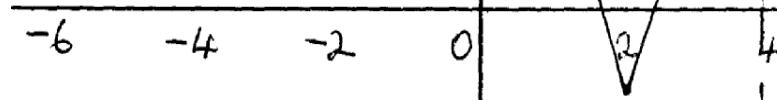
matrix  $\begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix}$

Scale used  $S_1$  $\Delta ABC$  drawn  $B_1$  $\Delta A_1 B_1 C_1$  drawn  $B_1$  $A, (6, -1), B(7, 2) C, (4, 4) B_1$ Line  $x = 4 L_1$  $\Delta A_2 B_2 C_2$  drawn  $B_1$ Two seen  $B_1$ 

Centre of rotation

Angle of centre of rotation  $B_1$  $A_3 B_3 C_3$  drawn  $B_1$ Scale used  $S_1$  $\Delta ABC$  drawn  $B_1$  $\Delta A_1 B_1 C_1$  drawn  $B_1$  $A, (6, -1), B(7, 2) C, (4, 4) B_1$ Line  $x = 4 L_1$  $\Delta A_2 B_2 C_2$  drawn  $B_1$ Two seen  $B_1$ 

Centre of rotation

Angle of centre of rotation  $B_1$  $A_3 B_3 C_3$  drawn  $B_1$ 

Centre  
L



8. (a)  $P(6, -2)$   
 $X^l = 6 - 3(-2) = 12$   
 $Y^l = 2(6) = 12$   
 $(X^l, Y^l) = (12, 12)$

- (b) (i)  $A^l(3, 4)$   
(ii)  $B^l(3, 2)$   
 $C^l(1, 4)$   
 $D^l(4, 3)$

(c) (i)  $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A^l & B^l & C^l & D^l \\ 3 & 3 & 1 & 4 \\ 4 & 2 & 4 & 5 \end{pmatrix}$

$$= \begin{pmatrix} A^{l1} & B^{l1} & C^{l1} & D^{l1} \\ -5 & -1 & -7 & -6 \\ 4 & 2 & 4 & 5 \end{pmatrix}$$

$A^{l1}(-5, 4), B^{l1}(-1, 2), C^{l1}(-7, 4)$  and  $D^{l1}(-6, 5)$

(ii) A stretch with y-axis invariant and a sketch factor (3)

$$\begin{aligned} 2h &= 6 \\ h &= 3 \\ -5a + 4b &= 4 \\ -a + 2b &= 2 \\ \hline -5a + 4b &= 4 \\ -a + 4b &= 4 \\ \hline -4a &= 0 \\ a &= 0 \\ b &= 1 \end{aligned} \quad \begin{aligned} -5c + 4d &= -3 \\ -c + 2d &= 3 \\ -5c + 4d &= -3 \\ -c + 4d &= -6 \\ \hline -4c &= 3 \\ c &= -\frac{3}{4} \\ d &= \frac{15}{8} \end{aligned}$$

9. (a)  $X_1(5, -1) Y_1(7, -1) Z_1(-2, 2)$   
 $xyz$  &  $x_1y_1z_1$  well drawn

(b) 1-3  $xyz$   $x_1y_1z_1$   
 $X_2(2, 10) Y_2(2, 14)$   
(c)  $X_2y_2Z_2$  well drawn

$$\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 5 & 7 & -2 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 7 & -2 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ 2 & -6 \end{pmatrix}$$

(d)) Area of  $\Delta X_2y_2Z_2$   
 $= 4 \times 15 = 60 \text{ cm}^2$

$$\begin{array}{l}
 10. \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{matrix} 2 & 4 & 4 & 2 \\ 1 & 1 & 4 & 4 \end{matrix} = \begin{matrix} 7 & 14 & 14 & 8 \\ 8 & 7 & 16 & 16 \end{matrix} \\
 2a + b = 8 \\
 \underline{4a + b = 14} \\
 -2a = -6 \\
 6 + b = 8 \\
 b = 2 \\
 \therefore 6 + b = 8 \\
 b = 2
 \end{array}$$

$$\begin{array}{l}
 2c + d = 7 \\
 \underline{4c + d = 7} \\
 -2c = 0 \\
 c = 0 \\
 d = 7 \\
 \therefore \begin{pmatrix} 3 & 2 \\ 0 & 7 \end{pmatrix}
 \end{array}$$

- it is an enlargement with scale factor 3 with centre (-1, -2)

$$\begin{array}{l}
 (c) \begin{pmatrix} 8 \\ 7 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix} \\
 a + 8 = 7 \quad 7 + b = 9 \\
 a = -1 \quad b = 2 \\
 \therefore T = \begin{pmatrix} -1 \\ 2 \end{pmatrix}
 \end{array}$$

11. a)  $ABCD$  drawn  $B_1$   
Name – Parallelogram  $B_1$

- b)  $A^I B^I C^I D^I$  drawn  $B_1$   
Attempt to joining any two points and bisecting.  $B_1$   
Description – Rotation +  $90^\circ$ .  $B_1$  or quarter turn about (0,0)

- c)  $A^{II} B^{II} C^{II} D^{II}$  drawn.  $B_1$   
Description – Enlargement centre (0, 0) Scale factor -Z.  $B_1$

- d)  $A^{III} B^{III} C^{III} D^{III}$  – drawn.  $B_1$   
Attempt to reflect.  $B_1$   
Coordinates

$$\begin{array}{ll}
 A^{III} = 9-2, 4 & C^{III} = (-8, 4) \quad B_1 \text{ All correct} \\
 B^{III} = (-6, 0) & D^{III} (-4, 8) \\
 12. \quad \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 4 & 0 & -2 \\ 1 & -2 & 4 \end{pmatrix} & \\
 \begin{pmatrix} -3 & -2 & 6 \\ 5 & 6 & -16 \end{pmatrix} & \\
 A^I (-3, 5) \quad B^I (-2, 6) \quad C^I (6, -16) &
 \end{array}$$

$$\begin{array}{l}
 \begin{pmatrix} 2 & -1 & -3 \\ 1 & 2 & 5 \end{pmatrix} = \begin{pmatrix} -2 & 6 \\ 6 & -6 \end{pmatrix} \\
 \begin{array}{c} A^{\parallel} \quad B^{\parallel} \quad C^{\parallel} \\ \left( \begin{array}{c} \text{Mocks} \\ \text{Topical Analysis} \end{array} \right) \end{array} \quad \text{eeducationgroup.com}
 \end{array}$$

$$\begin{array}{ccc} -11 & -10 & 18 \\ 7 & 10 & -6 \\ A^I(-11, 7) \quad B^I(-10, 10) \quad C''(18, -6) \end{array}$$

$$MN = \begin{pmatrix} 2 & -1 \\ 1 & 2 \\ 4 & 5 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$$

$$P^{-1} = \frac{1}{-12} \begin{pmatrix} 5 & -7 \\ 4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} -5/12 & 7/12 \\ 1/3 & -2/3 \end{pmatrix}$$

13.  $\text{Det} = 2 - 6$

$$= -4$$

$$A.S.F = 4$$

$$\underline{25.6} = 4$$

$$x \\ x = 6.4 \text{cm}^2$$

$$\text{Area of } \Delta ABC = 6.4 \text{cm}^2$$

14.  $T + (2) = (4)$

$$\begin{matrix} & 2 \\ -4 & 0 \end{matrix}$$

$$T = (4 - 2) = (2)$$

$$\begin{matrix} & 2 \\ 0 + 4 & 4 \end{matrix}$$

$$\therefore (2) + (-1) = (1)$$

$$\begin{matrix} 4 & 2 & 6 \\ & & \end{matrix}$$

$$Q(1, 6)$$

16.  $5x^2 + 6 = 110/10$

$$5x^2 + 6 = 11$$

$$x^2 = 1$$

$$x = \pm 1$$

## 58. Statistics II

<b>1</b> <table border="1" style="margin-left: 20px;"> <tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>f</td><td>20</td><td>8</td><td>6</td><td>4</td><td>1</td><td>1</td></tr> <tr><td>fx</td><td>0</td><td>8</td><td>12</td><td>12</td><td>4</td><td>5</td></tr> <tr><td>fx<sup>2</sup></td><td>0</td><td>8</td><td>24</td><td>36</td><td>16</td><td>25</td></tr> </table> $\sum f = 40 \quad \sum fx = 41 \quad \sum fx^2 = 109$ $s.d = \sqrt{\frac{109}{40} - \left(\frac{41}{40}\right)^2}$	X	0	1	2	3	4	5	f	20	8	6	4	1	1	fx	0	8	12	12	4	5	fx <sup>2</sup>	0	8	24	36	16	25	M <sub>1</sub> fx M <sub>1</sub> fx <sup>2</sup> M <sub>1</sub>
X	0	1	2	3	4	5																							
f	20	8	6	4	1	1																							
fx	0	8	12	12	4	5																							
fx <sup>2</sup>	0	8	24	36	16	25																							

	$= \sqrt{2.725 - 1.050625}$ $= \sqrt{1.674375}$ $= 1.294$	A <sub>1</sub>	Allow 1.293976429
		4	

1.

Mass kg	Mid term x	F	d = x A	fd	d <sup>2</sup>	fd <sup>2</sup>
50 - 54	52	19	-15	-285	225	4275
55 - 59	57	23	-10	-230	100	2300
60 - 64	62	40	-5	-200	25	1000
65 - 69	67	28	0	0	0	0
70 - 74	72	17	5	85	25	425
75 - 79	77	9	10	90	100	900
80 - 84	82	4	15	60	225	900
		$\sum f = 140$		$\sum fd = -480$		$\sum fd^2 = 9800$

Marks awarded for √ table as follows:-

$$\sum f = 140 \quad BI$$

$$\text{Column for } d \quad B1$$

$$\text{Column for } fd \quad B1$$

$$\sum fd = -480 \quad B1$$

$$\sqrt{\text{Column for } d^2} = 9800 \quad B1$$

$$\sum fd = 9800 B1$$

$$x = A + \frac{\sum fd}{\sum f}$$

$$= 67.0 + \frac{-480}{140}$$

$$= 67.0 - 3.43 = 63.57 \quad M1$$

$$= 63.6 \text{ kg} \quad A1$$

$$\text{Standard deviation} = \frac{\sum fd^2 - \sum fd}{\sum f}$$

$$= \sqrt{\frac{9800}{140} - (3.43)^2}$$

$$= \sqrt{58.24} = 7.631$$

$$= 7.6$$

$$2. = ^8/_{150} + ^6/_{150} + ^9/_{300} + ^3/_{300}$$

$$= ^{40}/_{300} = ^2/_{15}$$

a) Construction of AB B1

*Construction of BC B1**Construction of AC B1**b) Construction of bisect of AC B1**Construction of bisect BC B1**Radius 3.6 cm B1**c) Construction of bisect  $\angle CAB$  B1 OC B1**Construction of AD B1 AD = 12.8cm B1*

3.

a)

Class	f	x	$d = A - x$	fd	$d^2$	$fd^2$
41 – 50	20	45.5	15	300	225	4500
51 – 55	60	53	7.5	450	56.25	3375
56 – 65	60	60.5	0	0	0	0
66 – 70	50	68	-7.5	-375	56.25	2812.50
71 – 85	15	73	-12.5	187.5	156.25	2343.75
				$\sum fd 562.5$		$\sum fd^2 13031.25$

$$b) S = \sqrt{\frac{\sum f d^2}{\sum f} - \left( \frac{\sum fd}{\sum f} \right)^2}$$

$$\begin{aligned} S &= \sqrt{\frac{13031.25}{205} - \left( \frac{562.5}{205} \right)^2} \\ &= \sqrt{63.567 - 7.529} \\ &= \sqrt{56.038} \\ &= 7.486 \end{aligned}$$

4.  $15 (ax)^4 (\cdot^2/x^2) = 4860$

$$60a^4 = 4860$$

$$a^4 = 81$$

$$a = 3$$

5.

Marks(x)	Freq.(f)	$fx$	$d=x-x$	$d^2$	$Fd^2$
5.5	1	5.5	-40.45	1636	1636
15.5	6	99	-30.45	927.2	5563
25.5	10	255	-20.45	418.2	4182
35.5	20	710	-10.45	109.2	2184
45.5	15	682.5	-0.45	0.2025	3038
55.5	5	277.5	9.55	91.20	456
65.6	14	917	19.55	382.2	535
75.5	5	377.5	29.55	873.2	4366
85.5	3	256.5	39.55	1564	4692
95.5	1	95.5	49.55	2455	2455
	$\sum f = 80$	$\sum fx = 3676$			$\sum fx^2 33,923$

$$Mean = \frac{\sum fx}{\sum f} = \frac{3676}{80}$$

$$\begin{aligned}
 &= 45.95 \\
 (b) Q1 &= 30.5 + \frac{3}{14} \times 10 \\
 &= 62.64 \\
 S.I.R &= \frac{1}{2} (62.64 - 32) \\
 &= 15.32
 \end{aligned}$$

(c) Standard deviation

$$\begin{aligned}
 &= \sqrt{\frac{\sum fd^2}{\sum f}} = \sqrt{\frac{33923}{80}} \\
 &= 20.59
 \end{aligned}$$

6. a)  $x = 90 - (2 + 13 + 51 + 27 + 14 + 1)$   
 $= 90 - 84 = 6$

b) 15 – 19

c) i)

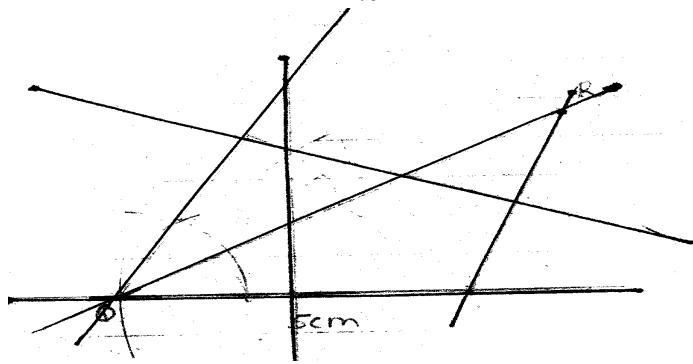
Class	$x$	$f$	$D = x - A$	$fd$	$D^2$	$Fd^2$
5-9	7	2	-15	-30	225	450
10-14	12	13	-10	-130	100	1300
15-19	17	31	-5	-155	25	775
20-24	22	23	0	0	0	0
25-29	27	14	5	70	350	4900
30-34	32	6	10	60	600	3600
35-39	37	1	15	15	225	225

$$Ef = 90 \quad Efd = 170 \quad Ef d^2 = 11250$$

$$\begin{aligned}
 \text{Mean} &= \frac{E + d}{Ef} + A \\
 &= \frac{-170}{90} + 22 \\
 &= 22 - 1.888 = 20.11
 \end{aligned}$$

$$\begin{aligned}
 ii) S.d &= \sqrt{\frac{Ef d^2}{Ef} - \left(\frac{Efd}{Ef}\right)^2} \\
 &= \sqrt{125 - (-1.888)^2} \\
 &= \sqrt{125 - 3.566} = \sqrt{121.4} \\
 &= 11.02
 \end{aligned}$$

7.



$$\begin{aligned}
 RQ &= 7.5 \pm 0.1 \\
 &< PRQ 40^\circ \pm 1
 \end{aligned}$$

$$d) \quad r = 4.1^\circ 0 \\ A = \pi r^2 \\ 22/7 \times 4.1 \times 4.1 = 52.83$$

8.

Class limits	$f$	$cf$
-0.5 - 19.5	7	7
19.5 - 39.5	21	28
39.5 - 59.5	38	66
59.5 - 79.5	27	93
79.5 - 99.5	7	100

i) from the curve median = 52. M1 A1

(ii) Inter quartile range = 66-38 = 28.

(iii) 7th 7/10 = 62.46marks

(iv) 60th percentile - 56.34

9.  $25^2 + 24^2 + 22^2 + 23^2 + x^2 + 262 + 21^2 + 23^2 + 22^2 + 27^2 = 5154$   
 $5.625 + 576 + 2(484) + 2(529) + 676 + 441 + 729 + x^2 = 5154$   
 $X^2 = 81$   
 $X = 9$

(ii)  $X = \frac{222}{10} = 22.2$

$$\begin{aligned} \Sigma(X-x)^2 &= 2.8^2 + 1.8^2 + 0.22 + 0.8^2 \\ &= 13.2^2 + 3.8^2 + 1.22 + 0.8^2 + 0.2^2 + 4.8^2 \\ (x-x)^2 &= 7.84 + 3.24 2(0.04) + 2(0.64) \\ &+ 174.24 + 14.44 + 1.44 + 23.04 \\ &= \frac{225.6}{10} \\ s.d &22.56 \\ &= 4.75 \end{aligned}$$

(b) (i) New mean =  $22.2 + 3$   
 $= 25.2$

(ii)  $s.d = 4.75$

10. a) i)  $x = A + \frac{\sum fd}{\sum f}$   
 $= 45.6 + \frac{(-74)}{40}$   
 $= 43.75$

Class	Mis-pt x	$d = (x - A)$	Frequency $f$	$fd$	$Fd^2$
1 - 10	5.5	-40.1	1	-40.1	1608.01
11 - 20	15.5	-30.1	3	-90.3	8154.05
21 - 30	25.5	-20.1	4	-80.4	6464.16

31 - 40	35.5	-10.1	7	-70.7	4998.49
41 - 50	45.5	-0.1	12	-1.2	1.44
51 - 60	55.5	9.9	9	89.1	7938.81
61 - 70	65.5	19.9	2	39.8	1584.04
71 - 80	75.5	29.9	1	29.9	894.01
81 - 90	85.5	39.9	0	0	0
91 - 100	95.5	49.9	1	49.9	2410.01

*i) Standard Deviation*

$$\begin{aligned}
 D &= \sqrt{\frac{\sum fd^2}{\sum f} - \left( \frac{\sum fd}{\sum f} \right)^2} \\
 &= \sqrt{10 \left[ \frac{34135.11}{40} - \left( \frac{-74}{40} \right)^2 \right]} \\
 &= \sqrt{10 \times 29.1531} = 29.1531
 \end{aligned}$$

*b) 30<sup>th</sup> student = 10<sup>th</sup> from bottom*

$$\begin{aligned}
 &30.5 + \left[ \frac{10-8}{7} \right] 10 \\
 &= 30.5 + 2.9 = 33.4 \text{ marks.}
 \end{aligned}$$

11. a) Mean  $45.5 + \frac{530}{60} = 54.33$

(b) Median  $= 50.5 + \left[ \frac{30.5 - 23}{14} \right] 10 = 55.86$

(c) Standard deviation  $= \sqrt{\left[ \frac{5300}{60} \right]^2 \frac{530}{60}} = 17.52$

(d) Modal class 51 - 60

12.

x	f	d	d2	fd	fd2
24.5	4	-30	900	-120	3600
34.5	26	-20	400	-520	10400
44.5	72	-10	100	-720	7200
54.5	53	0	0	0	0
64.5	25	10	100	250	2500
74.5	9	20	400	180	3600
84.5	11	30	900	330	9900
	200			-600	37200

$$\begin{aligned}
 (a) (i) \text{Mean} &= A + \frac{\sum fd}{\sum f} \\
 &= 54.5 - \frac{600}{200}
 \end{aligned}$$

$$= 51.5$$

(ii) Standard deviation

$$= \sqrt{\frac{\sum fd^2 - \sum fd^2}{\sum f}}$$

$$= \sqrt{\frac{37200 - (-3)^2}{200}}$$

$$= \sqrt{186 - 9}$$

$$\cong 13.30$$

$$(b) Q_1 = 39.5 + \frac{50 - 30}{72} \times 10$$

$$= 42.28$$

$$Q_3 = 49.5 + \frac{150 - 102}{53} \times 10$$

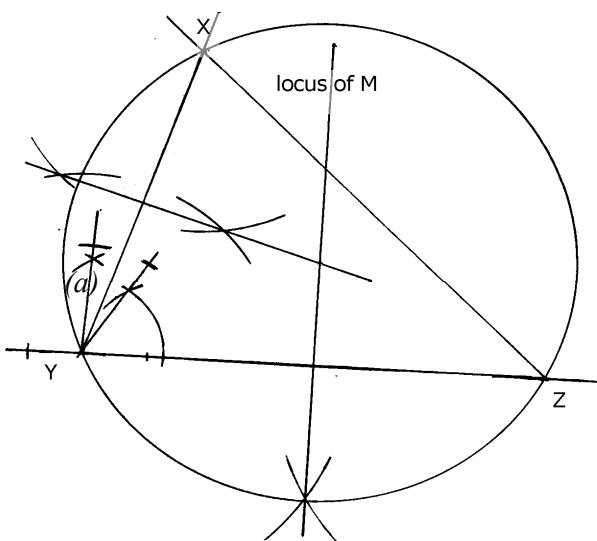
$$= 58.56$$

$$Q_3 - Q_1 = 58.56 - 42.28$$

$$= 16.28$$

### 59. Loci

1.



✓✓ Construction of  $\angle 60^\circ$  and  $\angle 90^\circ$

Bisect  $\angle$  btw  $90^\circ$  and  $60^\circ$  to obtain  $\angle 75^\circ$

✓ Construction of the given sides

Construction of  $\triangle XYZ$

$$(b) \angle XYZ = 42^\circ \pm 1^\circ$$

$$XZ = 8.8 \pm 0.1 \text{ cm}$$

c) Bisecting any two sides

Drawing the circle

(d) Perpendicular bisector of YZ

Identification of locus of M

2.  $AC = 8 \text{ cm} \pm 0.1$

$$\angle ACB = 46^\circ \pm 1^\circ$$

3. a)  $AC = 12.9 \pm 0.1\text{cm}$

b) i) Line and well shaded B2

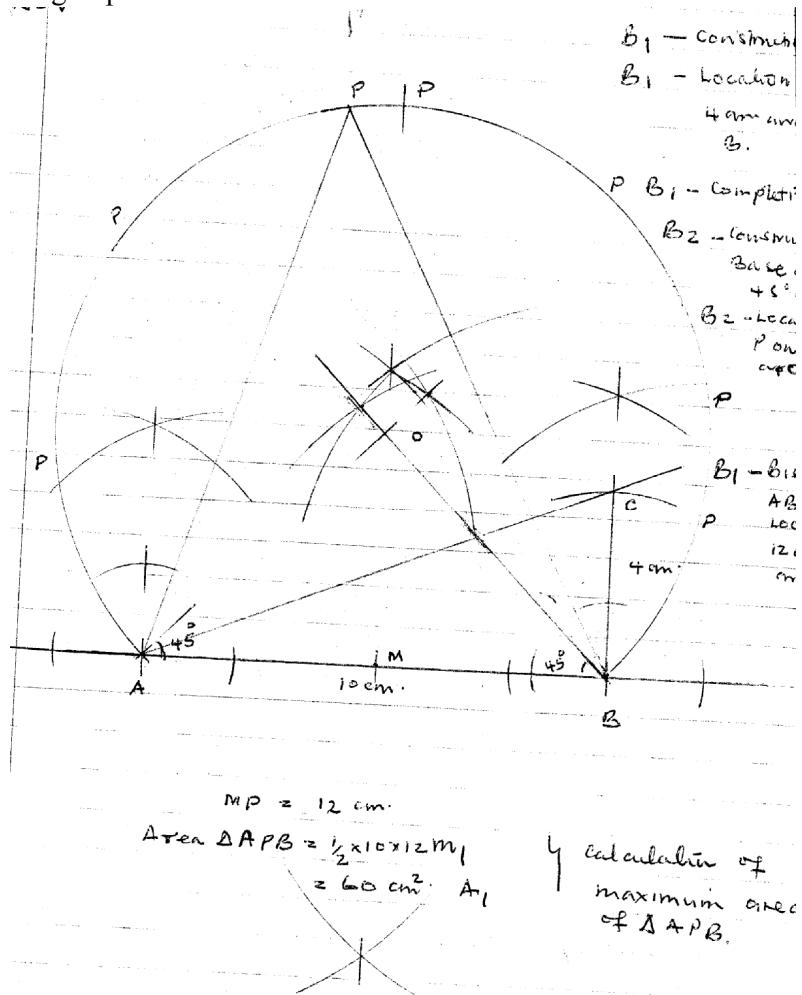
c)  $h = 7 \pm 0.1$

d)  $\Delta ABC$  \_\_\_\_\_ Area =  $\frac{1}{2} \times 8 \times 7\text{cm}$   
=  $28\text{cm}^2$

i.e.  $\frac{3}{4} \times 28 = \text{Area for } ARB$   
=  $21\text{cm}^2$

i.e.  $\frac{1}{2} \times 8 \times h = 21$   
 $h = 5.25$

4.



- Constructing of  $90^\circ$
- Location of C 4 cm away from B.

Completing  $\triangle ABC$

Construction of Base angles  $45^\circ$ .

Location of P on major arc APB

Bisecting AB to locate P 12 cm away

Calculation of maximum

area of  $\triangle APB$

B, B1

B1

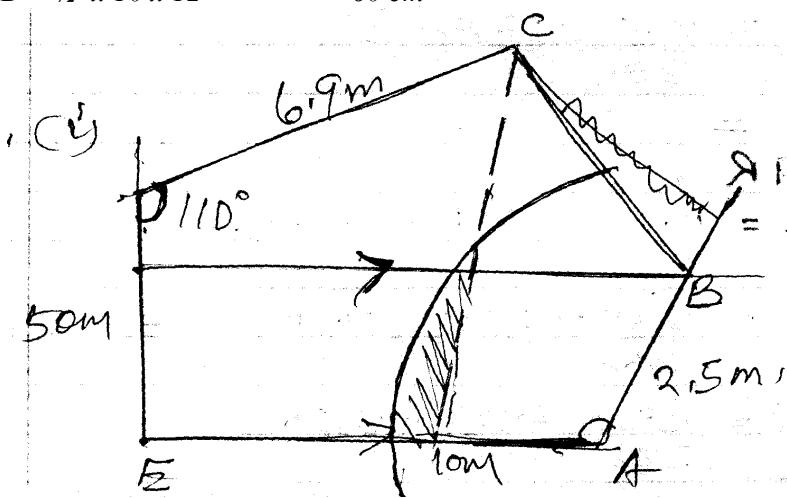
$$MP = 12 \text{ cm}$$

$$\text{Area } \triangle APB = \frac{1}{2} \times 10 \times 12 \text{ m}_1$$

calculation of  
maximum area  
of  $\triangle APB$ .

$$= 60 \text{ cm}^2$$

5. i)



ii) Yes

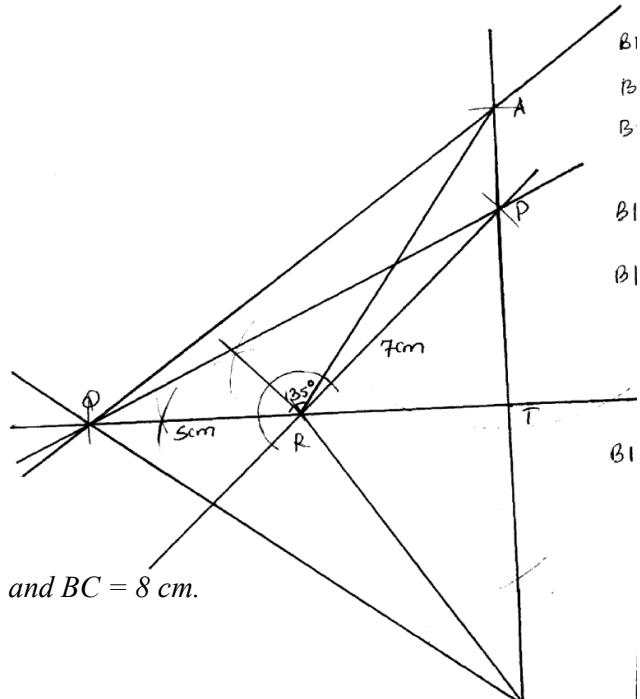
6. (a)

b)  $\angle PQR = 26^\circ + 1^\circ$

d)  $4.9 + 0.1 \text{ cm}$

e)  $AT = u = 8.7 \text{ cm}$

f)  $\angle AQR = 37 + 1$



7. a)  $\Delta ABC$  line  $AB = 7 \text{ cm}$  and  $BC = 8 \text{ cm}$ .

*Construction of  $\angle 60^\circ$*

(b)  $AC = 7.6 \pm 0.1$  and

$\angle ACB = 53 \pm 1^\circ$

(c) 2 sides bisector l

Circle drawn radius  $4.4 \pm 0.1$

(d) Bisect  $\angle ACB$

Bisection line to cut the circle to identify P

$\angle PBC$  measure =

(a)  $AB = 7 \text{ cm}, BC = 8 \text{ cm}$

$\angle ABC = 60^\circ$

(b)  $AC = 7.6 \pm 0.1 \text{ cm}$

$\angle ABC = 53^\circ \pm 0.1$

(c) Perpendicular bisectors of any two sides.

Circle drawn

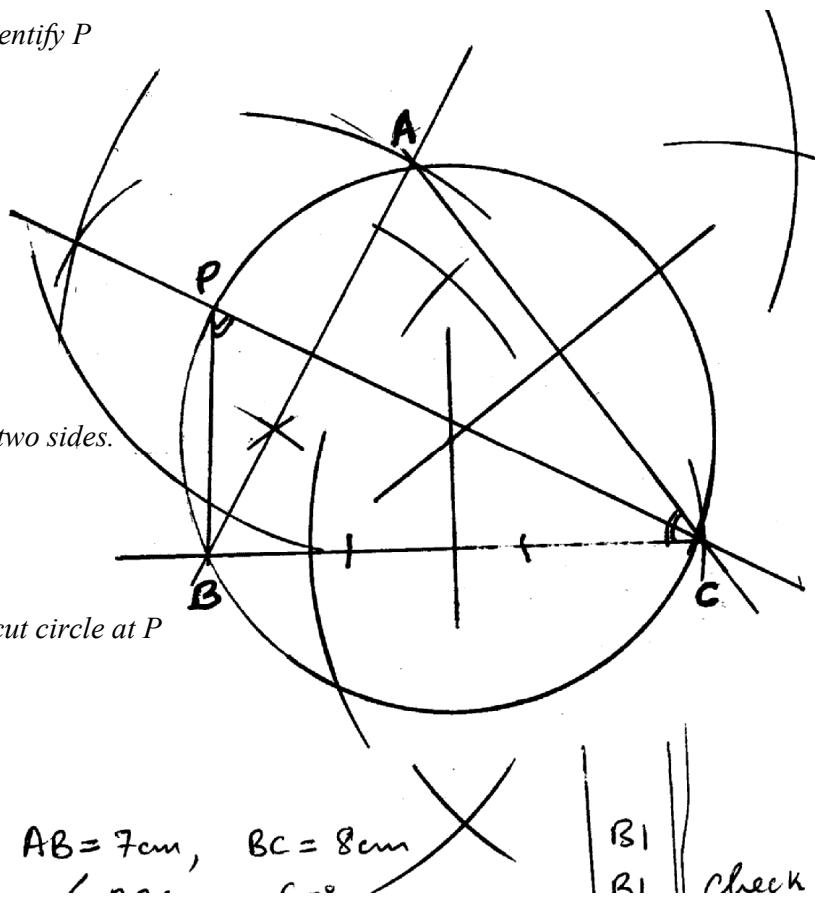
Radius =  $4.4 \pm 0.1 \text{ cm}$

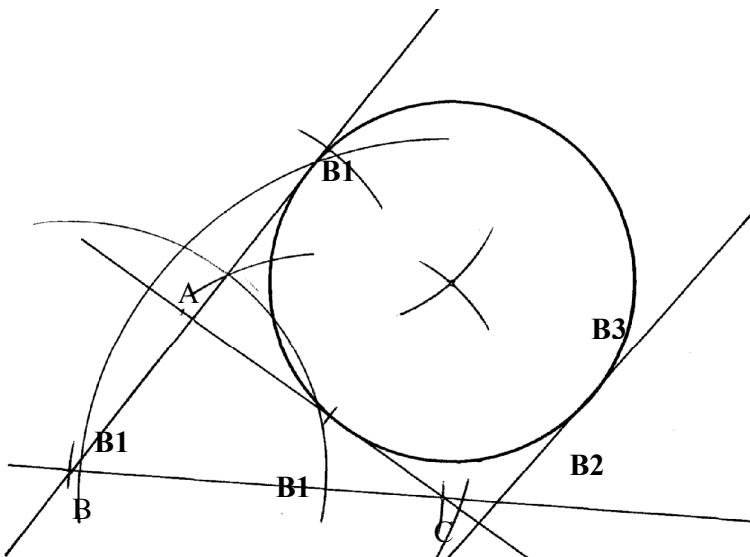
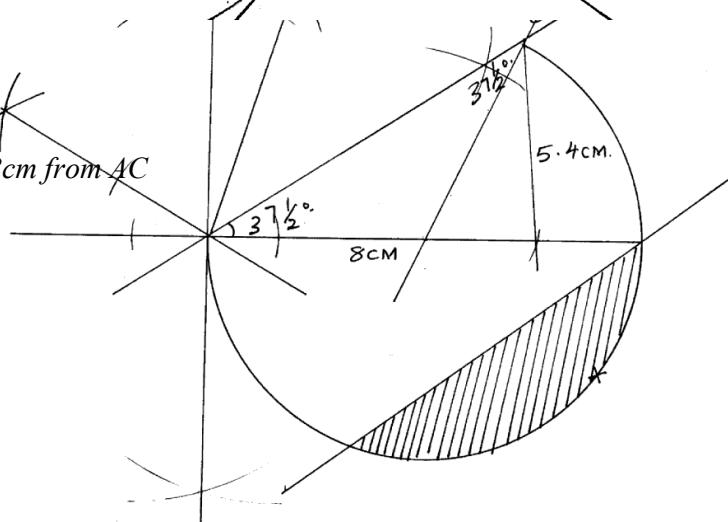
(d)  $\angle ACB$  bisected

Bisection line drawn to cut circle at P

$\angle BPC = \angle BAC = 67^\circ$

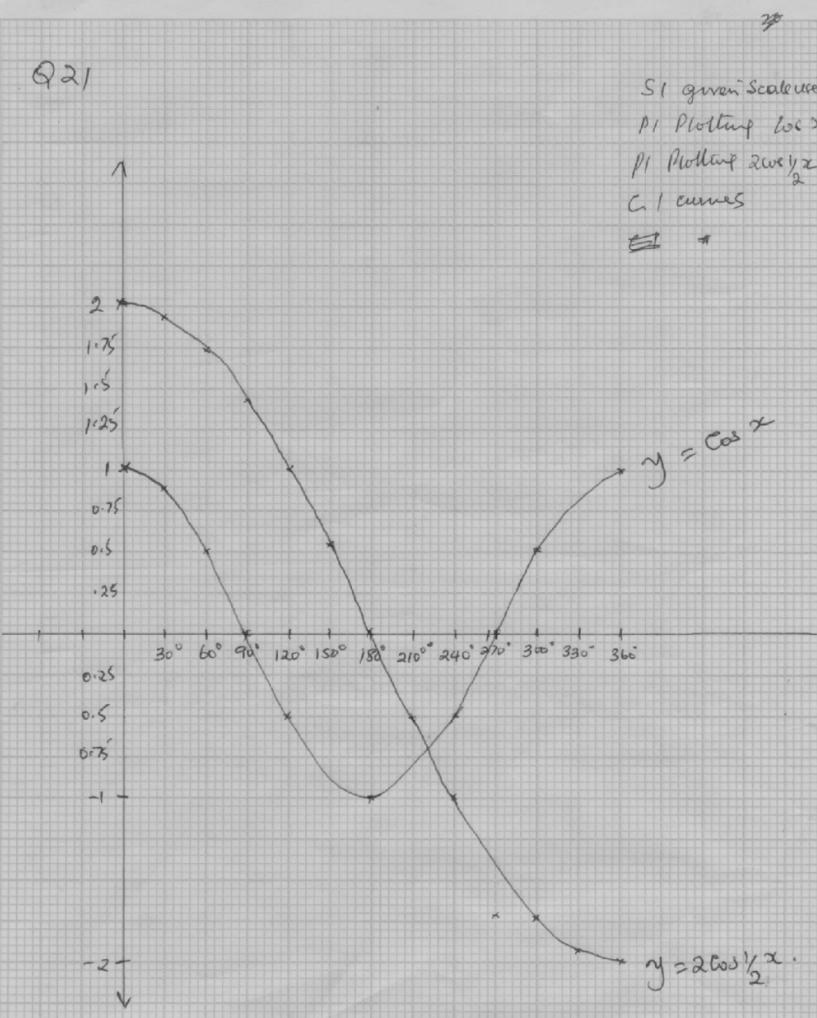
$\angle PBC = 88 \pm 0.1^\circ$



8.  $B1 - Line AC$  $B1$  Line  $AB$  $B1 AD$  $B3 - Drawing correct circle$  $B2$ - Tangent correctly drawn9. a)  $B1$  for constructing  $15^\circ$  $B1$  for constructing  $75^\circ$  $B1$  for completing tria $B1$  for  $AC = 8.8 \pm 0.1$ (b) (i)  $B1$  for locating locus centre $B1$  for locus of  $X$ (ii)  $B1$  for constructing arcs  $6.8\text{cm}$  from  $AC$  $B1$  for locus  $Y$ (c)  $B2$  for shading the locus of  $P$ 

## 60. Trigometric ratios 3

1.	$X^\circ$	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$	$210^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$330^\circ$
Cos x	1.00	0.87	0.50	0	-0.5	-0.87	-1	-0.87	-0.5	0.5	0.7	1	
$2\cos \frac{1}{2}x$	2.00	1.93	1.73	1.41	1	0.52	0.00	-0.52	-1	-1.73	-1.93	-2.00	



- (a) amplitude = 2 B1  
 period =  $720^\circ$  B1  
 (b)  $2\cos \frac{1}{2}x = \cos x$   
 $X = 222^\circ \pm 6^\circ$

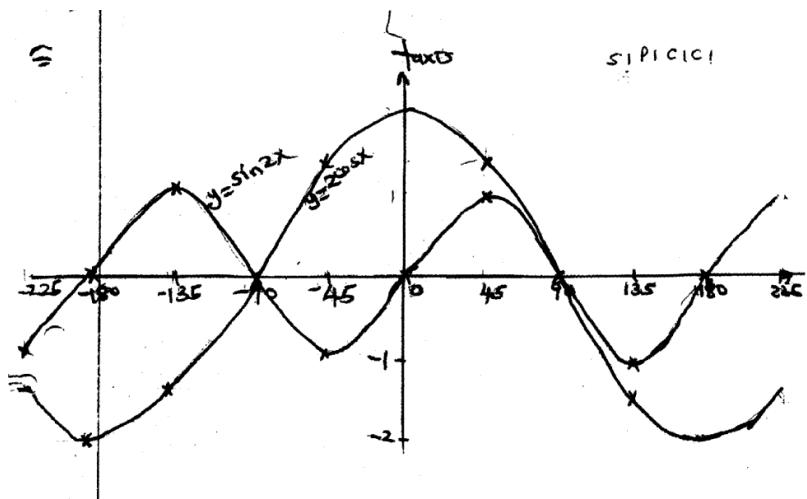
<b>B1</b>	All values of $\cos x$
<b>B1</b>	All values of $\cos \frac{1}{2}x$
<b>S1</b>	Given scale used
<b>P1</b>	Plotting $\cos x$
<b>P1</b>	Plotting $2\cos \frac{1}{2}x$
<b>C1</b>	Curve smooth continuous

<b>B1</b>
<b>B1</b>

1. a)

$X^\circ$	-225	-180	-135	-90	-45	0	45	90	135	180	225
$y = \sin 2x$		0		0	1.0	1.0	0		0		
$y = 2\cos x$		-2.0		0	1.4	1.4	0		-2.0		

b)



- (c)  $-90^\circ$  or  $90^\circ$   
 (d) (i) Highest point 1 unit  
 Lowest point - 1.4

2.

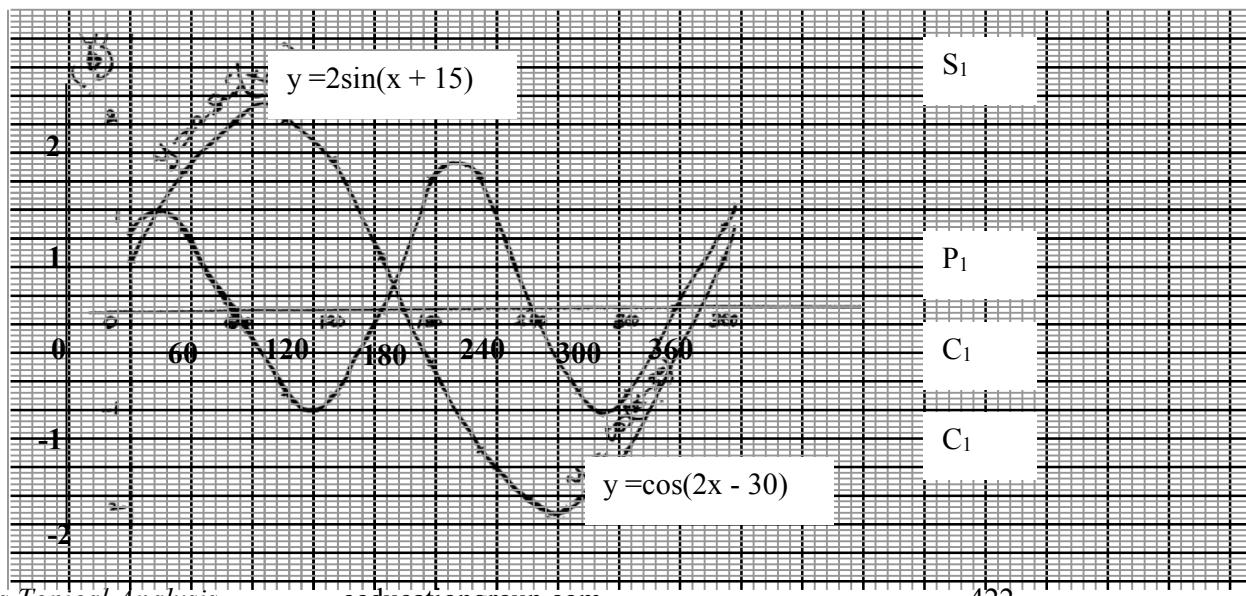
$x$	0	30	60	90	120	150	180	210
$2\sin(x+15^\circ)$	0.52	1.41	1.93	1.93	1.41	0.52	-0.52	-1.41
$\cos(2x-30^\circ)$	0.87	0.87	0	-0.87	0.87	0	0.87	0.87

$x$	240	270	300	330	360
$2\sin(x+15^\circ)$	-1.93	-1.93	-1.41	-0.52	0.52
$\cos(2x-30^\circ)$	0	-0.87	-0.87	0	0.87

B<sub>1</sub>  
B<sub>1</sub>

(i) Amplitudes:,  $y = 2 \sin (x + 15)$   
 $= 2$  units  
 $y = \cos (2x - 30)$  B<sub>1</sub>  
 $= 1$  unit B<sub>1</sub>

12°, 159°



3. Determine the

i) Altitude of the frustum

Solution

$$A^1C^1 = \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$AC = \sqrt{10^2 + 10^2}$$

$$= \sqrt{200}$$

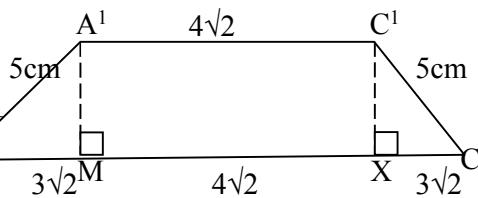
$$= 10\sqrt{2}$$

$$AM + XM = 10\sqrt{2} - 4\sqrt{2}$$

$$= 6\sqrt{2}$$

$$AM = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

$$\text{Height} = AM = \sqrt{5^2 - (3\sqrt{2})^2} = \sqrt{25 - 18}$$



$$= \sqrt{7} = 2.646$$

$\therefore$  the altitude of the frustum = 2.646 cm

ii) Angle between AC and the base

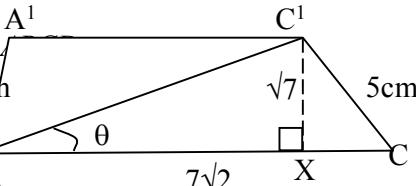
$$AX = 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$$

$$\tan \theta = \frac{CX}{AX} = \frac{\sqrt{7}}{7\sqrt{2}}$$

$$= 0.2673$$

$$\theta = \tan^{-1} 0.2673$$

$$= 14.96^\circ$$



iii) Volume of pyramid =  $\frac{1}{3}bh$

$$AC = 10\sqrt{2}$$

$$A1C1 = 4\sqrt{2}$$

$$\text{L.S.F} = 10:4$$

$$\therefore \frac{h + 2.646}{h} = \frac{10}{4}$$

$$4(h + 2.646) = 10h$$

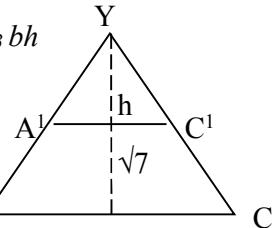
$$4h + 10.584 = 10h$$

$$6h = 10.584$$

$$h = 1.764$$

$$H = h + 2.646$$

$$= 1.764 + 2.646 = 4.410$$



$$Vf = \left(\frac{1}{3} \times 10 \times 10 \times 4.41\right) - \left(\frac{1}{3} \times 4 \times 4 \times 1.76\right)$$

$$= \frac{441.0}{3} - \frac{28.224}{3}$$

$$= \frac{413.776}{3}$$

$$= 137.592 \text{ cm}^3$$

4. ✓(a) table completed

(b)

(c) (i) 3 P1 - plotting

S1 - scale

C1 - smooth curve

(ii)  $180^\circ$

(iii) Line  $y = 1$  drawn

$$x = 4.5^\circ \text{ or } 72.8^\circ - 107.2^\circ = 175.4^\circ$$

5.  $\left(\frac{A}{B}\right)^2 = p + 33q$

$$q - 3P$$

$$A^2q - 3A^2P = BP + 3Bq$$

$$Aq^2 - 3Bq = BP + 3A^2P$$

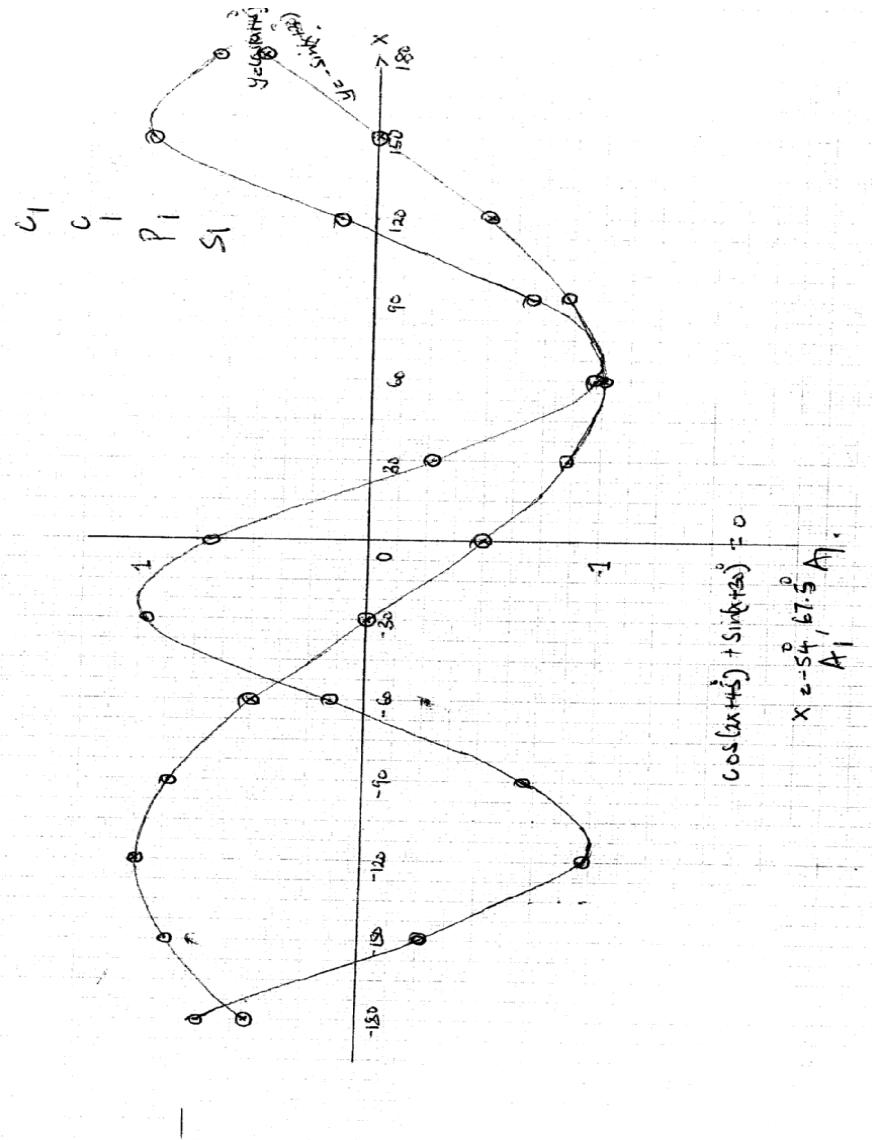
$$2(A^2 - 3B) = BP + 3A^2P$$

$$Q = \frac{BP + 3A^2P}{A^2 - 3B}$$

6.

$$\begin{aligned} x &= -150, -120, -60, 0, 30, 60, 90, 120, 150, 180 \\ y &= -\sin(45^\circ), 0.50, 0.87, 1.00, 0.87, 0.50, 0.00, -0.50, -0.87, -1.00, -0.87, -0.50, -0.26, 0.26, 0.71, 0.97, 1.26, 1.71, 1.72 \end{aligned}$$

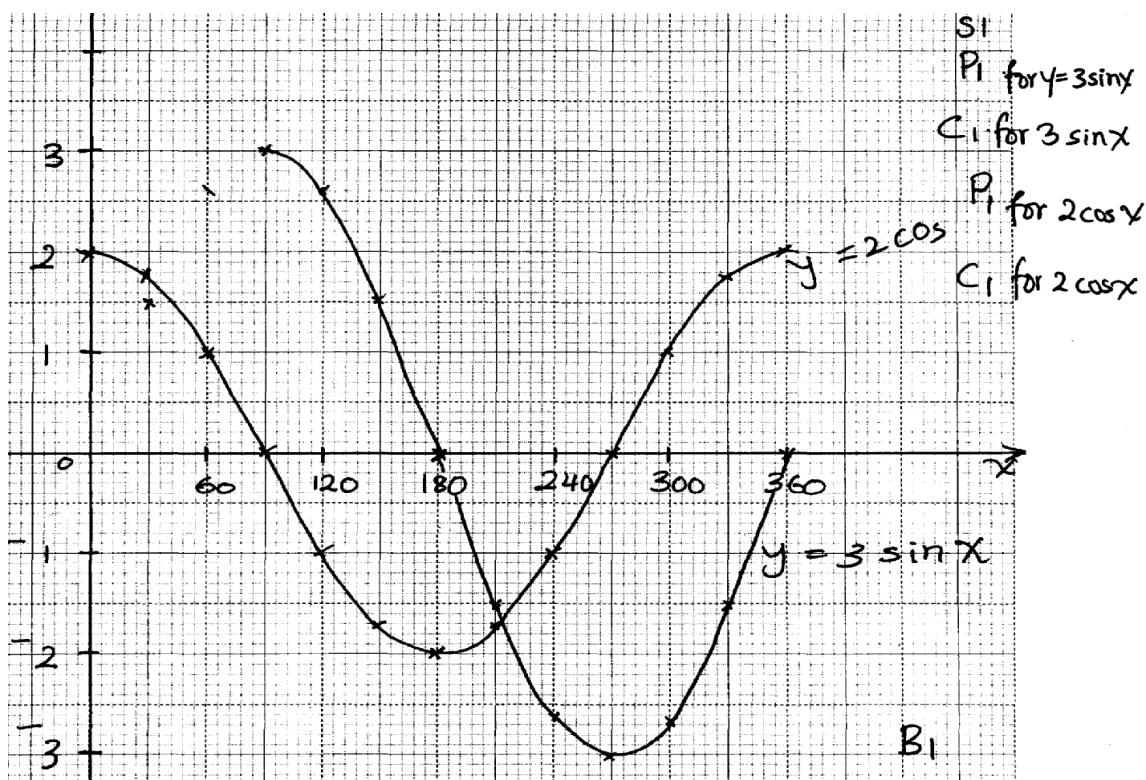
7. 7.  $\frac{\sqrt{3}}{2} x \frac{1}{2}$   
 $\frac{1}{2}$   
 $\frac{1}{\sqrt{3}} x \frac{1}{\sqrt{2}}$   
 $\frac{\sqrt{3}}{4} x \frac{\sqrt{6}}{4}$   
 $\frac{1}{4} x \frac{1}{4}$   
 $\frac{\sqrt{18}}{18}$



$$\frac{3}{4} \sqrt{2}$$

8. a)

$x$	0	30	60	90	120	150	180	210	240	270	300	330	360
$3\sin x$		1.5			2.6	1.5					-		0
$2\cos x$	2			0	-1.0			-1.7		0			



(c) (i) Amplitude = 3

(ii)  $x = 36^\circ$  $x = 216^\circ$ (iii)  $33^\circ \leq x \leq 213^\circ$ 

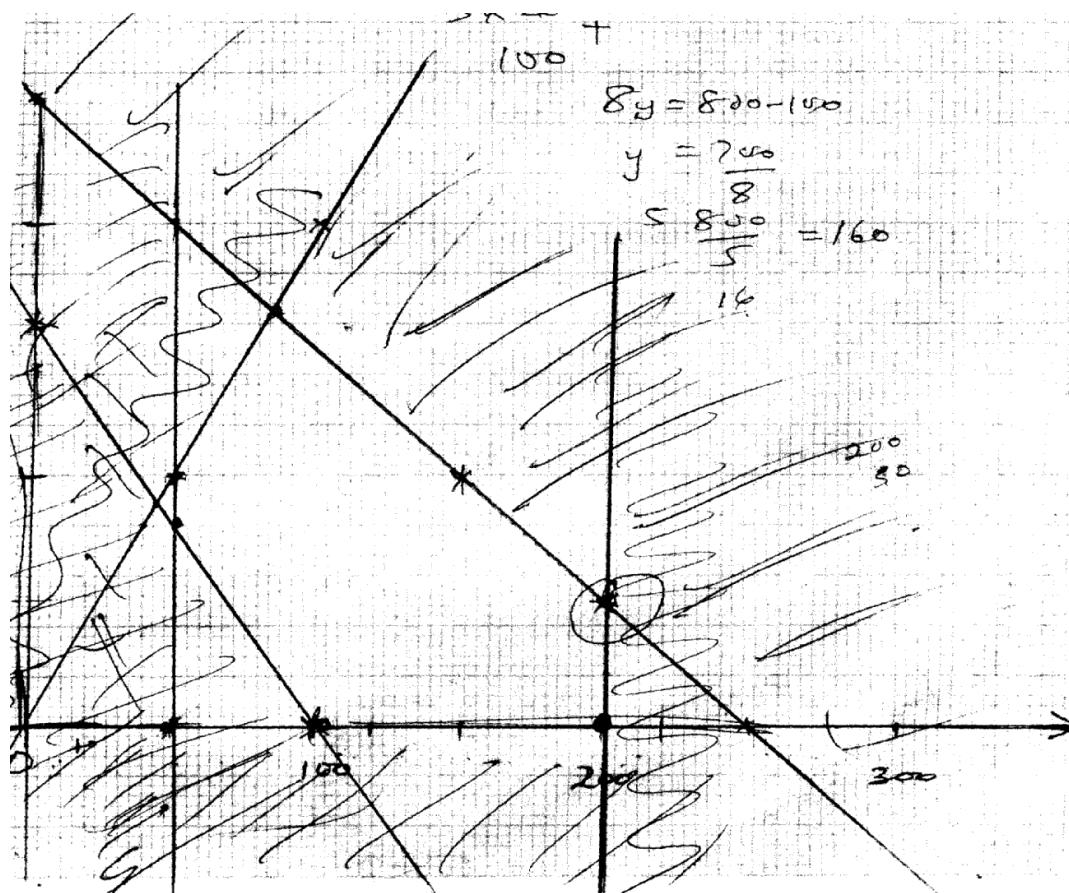
9.

$x$	0	90	180	270	360	450	540	630	720	810
$\sin \frac{1}{2}x$	0	0.71	1	0.71	0	-0.71	-1	-0.71	0	0.71
$3\sin(\frac{1}{2}x + 60)$	2.6	2.9	1.5	-0.78	-2.6	2.9	-1.5	0.78	2.6	2.9

10.

$x$	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$
$2\sin x$	0	1	1.73	2	1.73	1.00	0

$1 - \cos X$	1	0.13	0.50	1	0.06	1.87	2
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11.  $\sin(x + 30) = 0.5$

$$x + 30 = 30^\circ$$

$$x = 0$$

$$0, 180, 360$$

12. (c)  $10\sin x = -1/50 + 5$

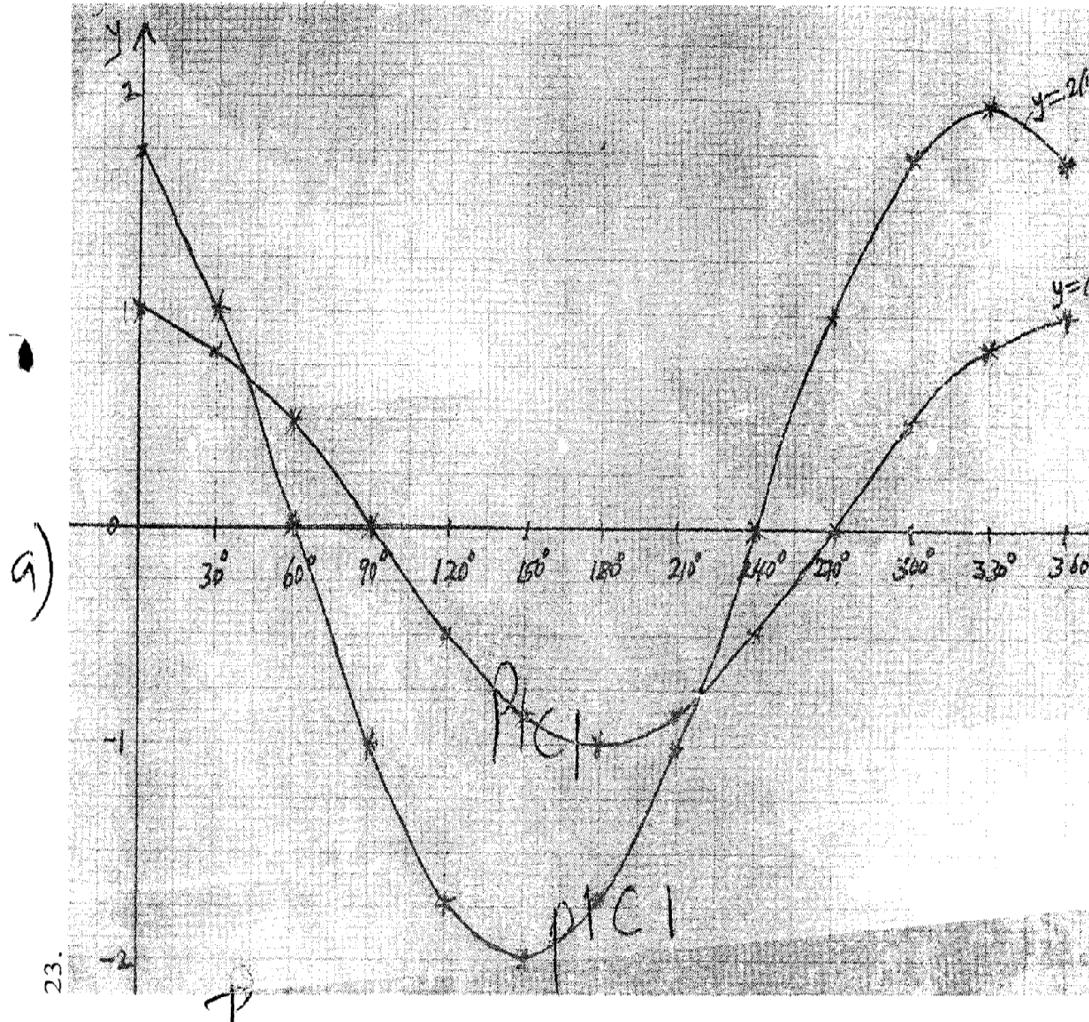
$$Y = -1/50 + 5$$

X	0	50
y	5	4

$$X_1 = 28^\circ \pm I$$

$$X_2 = 70^\circ \pm I$$

12.



- b) i) amplitude = 1  
 ii) Period =  $360^\circ$   
 iii)  $45^\circ, 219^\circ$

13.  $2\theta + 10 = 210^\circ, 330^\circ, 570^\circ, 690^\circ$

$$2\theta = 200^\circ, 320^\circ, 560^\circ, 680^\circ$$

$$= 100^\circ, 160^\circ, 280^\circ, 340^\circ$$

$$= \frac{5\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}, \frac{17\pi}{9}$$

14.  $4\sin 2x + 4\cos x - 5 = 0$

$$4(1-\cos 2x) + 4 \cos x - 5 = 0$$

$$4\cos 2x - 4 \cos x + 1 = 0$$

$$4\cos 2x - 2\cos x - 2\cos x + 1 = 0$$

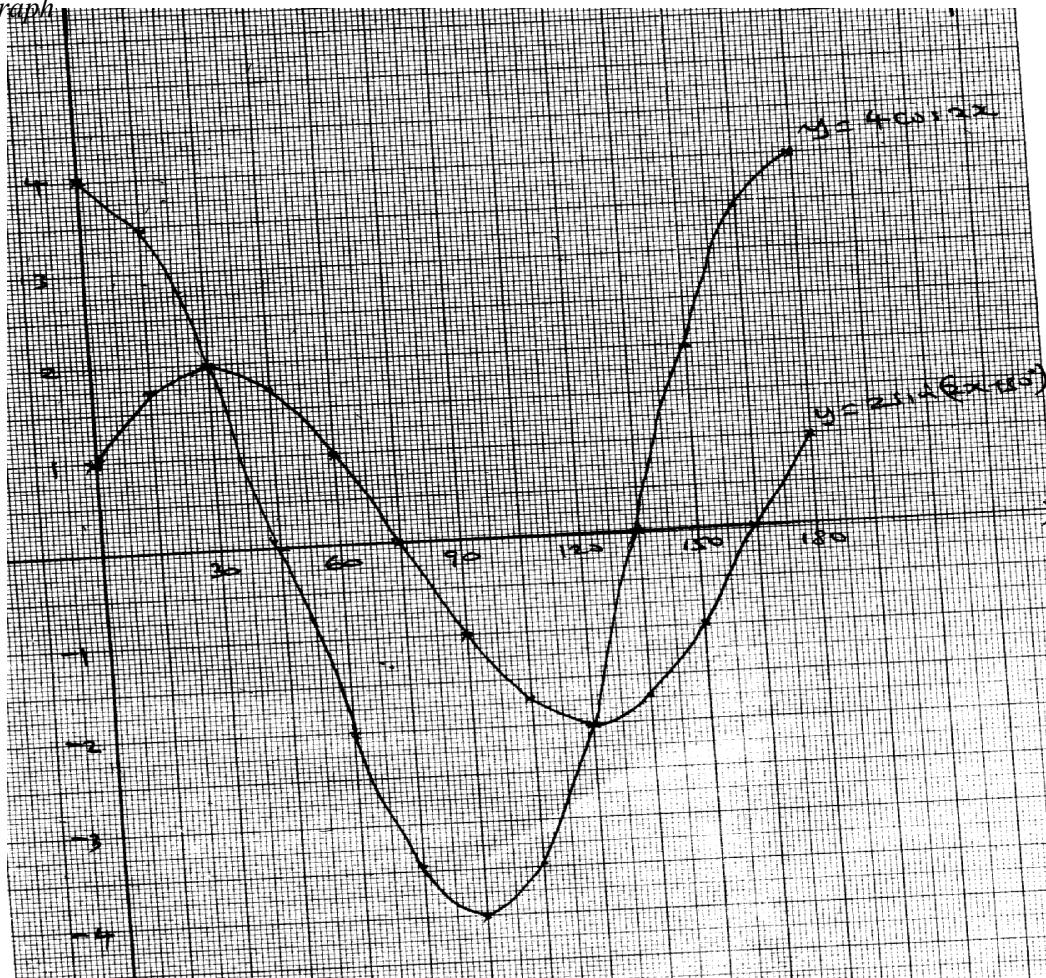
$$(2\cos x - 1)2 = 0$$

$$X = 60^\circ, 300^\circ$$

15.

$x$	$15^\circ$	$60^\circ$	$150^\circ$	$165^\circ$
$4 \cos 2x$	3.46			3.46
$2\sin(2x + 30^\circ)$		1.00	-1.00	

(b) graph

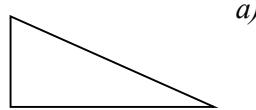


(c)(i) Amplitude = 4

(ii) period =  $180^\circ$ (d)  $x = 30^\circ, 120^\circ$

## 60. Three dimensional geometry

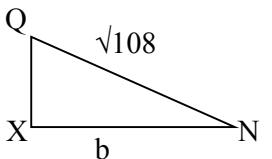
1.



a)

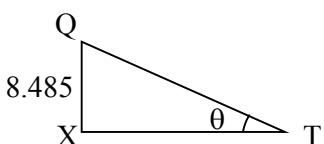
$$\begin{aligned}QN &= \sqrt{12^2 - 6^2} \\&= 10.39\end{aligned}$$

b)



$$\begin{aligned}QX &= (\sqrt{108})^2 - 6^2 \\&= \sqrt{72} \\&= 8.485\end{aligned}$$

c)



$$\tan \theta = \frac{8.485}{6}$$

$$\theta = 54.73^\circ$$

$$d) \tan \theta = \frac{6}{10}$$

$$\theta = 30.96$$

$$\begin{aligned}\frac{6}{10} \quad \text{obtuse} &= 180^\circ - 30.96 \\&= 149.04^\circ\end{aligned}$$

2. a)  $\sin 36^\circ = \frac{a}{5}$ 

$$\begin{aligned}Where \ a \ is \ the \ side \\a &= \frac{5}{\sin 36^\circ} = 8.507\end{aligned}$$

$$\begin{aligned}h^2 &= 18.2 - 8.507 \\&= 258.87\end{aligned}$$

$$H = 16.09 \text{ cm}$$

b)  $\frac{1}{2} ab \sin \theta$ 

$$\begin{aligned}\frac{1}{2} \times 8.507^2 \sin 72^\circ \times 5 \\= 172.06 \text{ cm}^2\end{aligned}$$

c)  $\frac{\tan 36^\circ}{x} = 5$ 

$$\begin{aligned}x &= 6.882 \\Tan \theta &= 16.09\end{aligned}$$

$$6.882$$

$$\theta = 66.84^2$$

$$d) \frac{1}{3} \times 172.06 \times 16.09 = 922.8 \text{ cm}^3$$

$$e) S = 23.2$$

$$\sqrt{23.2(23.2 - 18.2)(23.2 - 10)} \\ = 87.50 \text{ cm}^3$$

3. (i)  $\frac{1}{3} \times 4.2 \times 7.5h = 52.5$

$$3$$

$$h = \frac{52.5 \times 3}{4.2 \times 7.5} = 5.0 \text{ cm}$$

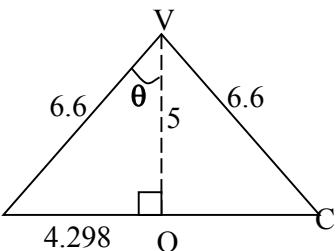
$$(ii) AC = \sqrt{4.2^2 + 7.5^2} \\ = \sqrt{17.64 + 56.25} \\ = \sqrt{73.89} \\ = 8.596$$

$$AO = 8.596 \div 2 = 4.298$$

$$AV = \sqrt{AO^2 + OV^2} \\ = \sqrt{4.298^2 + 5^2} \\ = 18.47 + 25 \\ = \sqrt{43.47} \\ = 6.6 \text{ cm}$$

$$(iii) \tan \theta = \frac{4.298}{5} \\ = 0.8596 \\ \theta = 40.68^\circ$$

$$\angle AVC = 40.68 \times 2 \\ = 81.36$$



*Alternative*

$$\cos \theta = \frac{5}{6.6} = 0.7576$$

$$\theta = 40.749^\circ$$

$$\angle AVO = 40.749^\circ$$

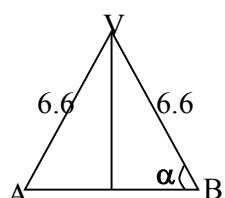
$$\angle AVC = 81.498^\circ$$

(iv)  $\cos \alpha = \frac{2.1}{6.6}$

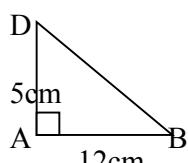
$$= 0.3182$$

$$\alpha = 71.45^\circ \text{ Acute angle}$$

$$\text{obtuse angle} = 180^\circ - 71.45^\circ \\ = 108.55^\circ$$

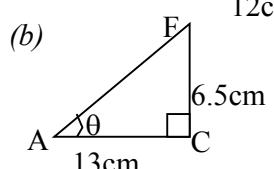


4. (a)



$$BD^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$BD = \sqrt{169} = 13 \text{ m}$$



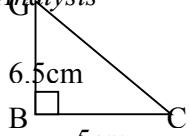
$$AF^2 = 13^2 + 6.5^2 = 169 + 42.25$$

$$= 211.25 \quad AF = \sqrt{211.25} = 14.53 \text{ cm}$$

$$\tan \theta = \frac{6.5}{13} = 0.5 \quad M1$$

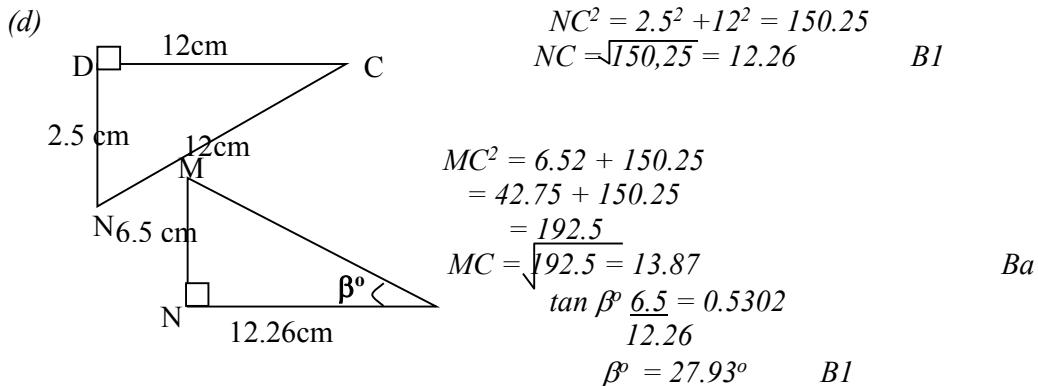
$$\theta = 26.57^\circ \quad A1$$

B1



$$(c) \quad \tan \alpha^\circ = \frac{6.5}{5} = 1.3 \quad M1$$

$$\alpha^\circ = 52.43 \quad A1$$



5.

$$i) Or = 16^2 - 5^2$$

$$= \sqrt{256 - 25}$$

$$= 15.198 \text{ cm}$$

$$ii) \quad \tan \theta = \frac{5.066}{4} = 1.2665$$

$$\therefore \theta = 51.71^\circ$$

6.

a) Height

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{10^2 + 10^2}$$

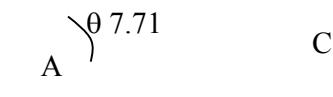
$$= \sqrt{200}$$

$$= 14.142$$

$$\therefore OA = \frac{1}{2} AC = \frac{14.14^2}{2} = 7.71$$

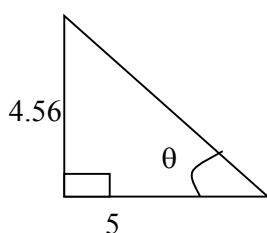
$$OE = \sqrt{AE^2 - AO^2}$$

$$\therefore \sqrt{64 - 59.44} = 4.56$$



$$b)i) \quad \tan \theta = \frac{4.56}{5.00} = 0.912$$

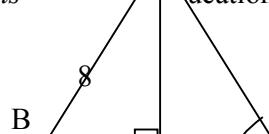
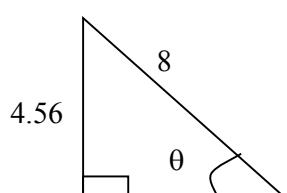
$$\theta = 65.78^\circ$$



$$ii) \quad \tan \theta = \frac{4.56}{7.71} = 0.5914$$

$$\theta = 30.6^\circ$$

c)



$$\begin{aligned} \angle AEC &= 30.6 \times 2 \\ &= 61.2^\circ \end{aligned}$$

7. Let length of cut off pyramid be meters

$$\text{Then } \frac{7+h}{H} = \frac{5.5}{2.1}$$

$$14.7 + 2.1h = 5.5$$

$$3.4h = 14.7$$

$$h = 4.3$$

Slant height of big pyramid

$$= \sqrt{11.3^2 + 2.75^2} = 11.6$$

Slant height of the pyramid cut off

$$= \sqrt{4.3^2 + 1.05^2} = 4.4\text{m}$$

$$\begin{aligned} \text{Area of } EFCD &= \frac{1}{2} \times 11.6 \times 5.5 - \frac{1}{2} \times 4.4 \times 2.1 \\ &= 27.28 \text{ m} \end{aligned}$$

$$\text{Total surface area} = 4 \times 27.28 + 2.1 \times 2.1 = 113.5$$

b)  $\frac{1}{2}$  litre paint  $10\text{m}^2$

$$4 \text{ litres paints } 80\text{m}^2$$

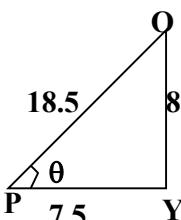
$\therefore 113.5\text{m}^2$  requires 2 tins

$$2 \times 650 = \text{Kshs. } 1300/$$

8. (a)  $PR = \sqrt{12^2 + 9^2} = 144 + 81 = 225 = 15\text{cm}$

$$h = \sqrt{380.25 - 56.25} = \sqrt{324} = 18$$

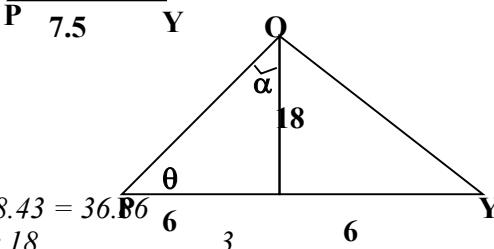
$$(b) \tan \theta = \frac{18}{7.5} = 2.4$$



$$\theta = \tan^{-1} 2.4 = 67.38^\circ$$

$$(c) \tan \alpha = \frac{6}{18} = \frac{1}{3}$$

$$\alpha = \tan^{-1} 0.3333 = 18.43^\circ$$



$$(d) \text{Volume} = \frac{1}{3} \times 12 \times 9 \times 18 = 648\text{cm}^3$$

9. a)  $AC^2 = 12^2 + 12^2 = 288$

$$\therefore AC = \sqrt{288} = 16.97$$

$$VO^2 = h^2 = 24^2 - \frac{(16.97)^2}{2} = 504$$

$$h = \sqrt{504} = 22.45\text{cm}$$

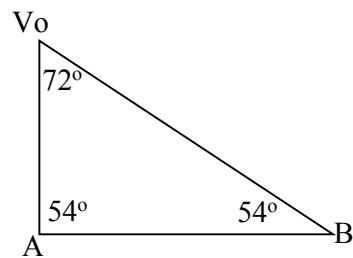
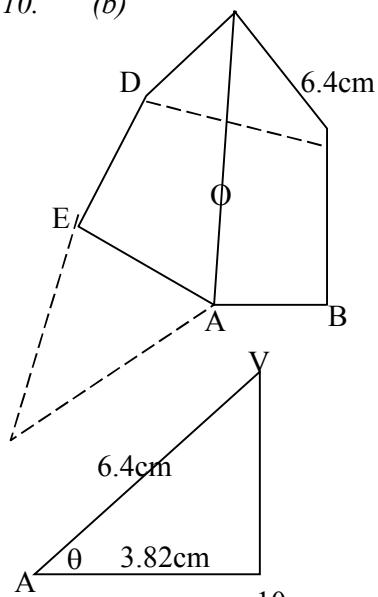
$$b) \text{Base area} = 12 \times 12 = 144\text{cm}^2$$

$$\therefore \text{Volume} = \frac{1}{3} \times 144 \times 22.45 = 1077.6\text{cm}^3$$

$$c) \text{ Slanting surface} = \sqrt{30(30-24)(30-24)(30-12)} \\ = 139.44 \text{ cm}^2$$

$$\text{Total curved S.A} = 139.44 \text{ cm}^2 \times 4 + 144 \text{ cm}^2 \\ = 701.6 \text{ cm}^2$$

10. (b)

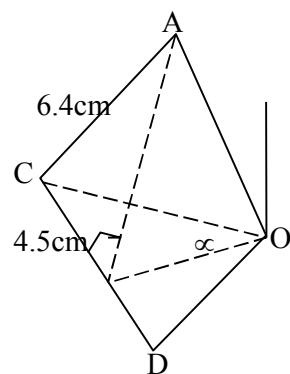


$$AO = \frac{4.5 \times \sin 54^\circ}{\sin 72^\circ} = 3.82 \text{ cm} \\ = \cos^{-1} \frac{(3.82)}{6.4} = 53.35^\circ$$

$$(c) Vo = \sqrt{6.4^2 - 3.82^2} \\ = 5.13$$

$$VX = \sqrt{6.4^2 - 2.55^2} \\ = 5.99 \text{ cm}$$

$$\alpha = \sin^{-1} \left( \frac{Vo}{Vx} \right) \sin^{-1} \left( \frac{5.13}{5.99} \right) \\ \alpha = 58.91^\circ$$



11. a) Longitude difference =  $139^\circ + 41^\circ$   
 $= 180^\circ$

b) Distance along latitude =  $\theta/360 \times 2\pi r \cos \theta$   
 $= 180/360 \times 2 \times 22/7 \times 6370 \cos 60^\circ$   
 $= 22 \times 910 \times 0.5$   
 $= 10,010 \text{ Km}$

Or via north pole (great circle)

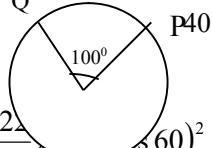
Latitude difference =  $60^\circ$

Distance =  $60/360 \times 2 \times 22/7 \times 6370$   
 $= 6673.33 \text{ Km}$

c) Distance =  $\text{long diff}/360 \times 2\pi R \cos 60^\circ$   
 $420 = \theta/360 \times 2 \times 22/7 \times 6370 \cos 60^\circ$   
 $\theta = \frac{420 \times 360 \times 7}{2 \times 22 \times 6370 \cos 60^\circ}$   
 $= 7.552^\circ$

Longitude of C =  $41^\circ - 7.55^\circ = 33.45^\circ \text{N}$

## 61. Longitudes and latitudes

<b>1</b> $\angle \text{difference}$  $P40 + 60 = 100^\circ$ $\text{Area} = \frac{100}{360} \times \frac{2}{7} \times (60)^2$ $= \frac{55}{63} \times 0.01$ $= 0.008730$ $(87.30 \text{ cm}^2)$	B <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	Angle difference
<b>2</b> a) i) $480 - 1015' = 46045'$ B(46045'N, 370E) ii) Diff in longitude $\Rightarrow 37 + 23 = 60^\circ$ $D = \frac{60}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 46.75^\circ = 4,572.45 \text{ km}$	M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> M <sub>1</sub> A <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	Subtraction ✓ position Addition Allow 4,572km
b) i) $\frac{60 \times 4}{60} = 4 \text{ hrs}$ difference $\therefore$ Time at C = $7.00 - 4 \text{ hrs} = 9.00 \text{ p.m}$ ii) Time taken $= \frac{4572.45}{840} = 5.44 \text{ hrs}$ Arrival at C = $9.00 + 5 \text{ hrs } 26 \text{ min } s = 2.26 \text{ a.m}$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	Or (1426hrs)
<b>3.</b> Angle difference btw longitudes ( $41+3$ ) = $440$ Dist = $60 \times \text{angle difference} \times \cos \text{latitude}$	10	

	$1370 = 60 \times 44 \cos P$ $\frac{1370}{60 \times 44}$ $\cos P = 0.51894 = 58.740$ $58.740$	M1  M1  A1	Subst  Cos P the subject
		03	
4.	a = 400 E b = 600 N c = 200 W (b) R (600N, 400E) P (300N, 200W) Q (300N, 400E) S (600N, 200W) PQR PQ = $600 \times 60 \cos 300$ = $3600x$ = $3117.69 \checkmark$ QR = $30 \times 60 = 1800\text{nm} \checkmark$ Total distance = $1800 + 3117.69$ = $4917.69\text{nm}$ PSR PS = $30 \times 60 = 1800\text{nm}$ SR = $60 \times 60 \cos 60 = 1800 =$ Total distance $1800 + 1800 = 3600 \checkmark$ (c) PQR speed $400\text{nm/hr}$ Time = $4917.69 = 12.294$ hrs $\frac{400}{300}$ Along PSR Time = $3600 = 12$ hrs $\frac{300}{2}$ 2nd pilot by 0.294hrs or 18 min	B1  B1  B1  M1  A1  M1  A1  B1  B1  B1  B1	$\checkmark$ values of PQ and QR  $\checkmark$ value of PS and SR
		10	

1.  $(70 - 25 \times 60 = 2700)$   
 $2700 \cos 47^\circ = 2700 \times 0.68 = 1841.4\text{nm}$

2. (a)  $\frac{22}{7} \times 6370 \times 2 \times \frac{\alpha}{360} = 1600$   
 $\alpha = 14.4^\circ$   
Position ( $4.4^\circ\text{N}, 60^\circ\text{E}$ )  
(b)  $72 \times 60 \cos 4.4^\circ = 4307\text{nm}$   
(c)  $T = \frac{D}{S} = \frac{4307 \times 1.853}{800} = 9.976 \text{ hrs}$   
(d) Difference in longitude =  $72^\circ$   
 $15^\circ - 1\text{hr}$   
 $\therefore 72^\circ = \underline{72}$   
 $15 = 4.8\text{hrs} = 4\text{hrs } 48\text{mins behind}$   
 $1300\text{hrs}$   
 $\underline{- 448}$   
 $8.12\text{a.m}$

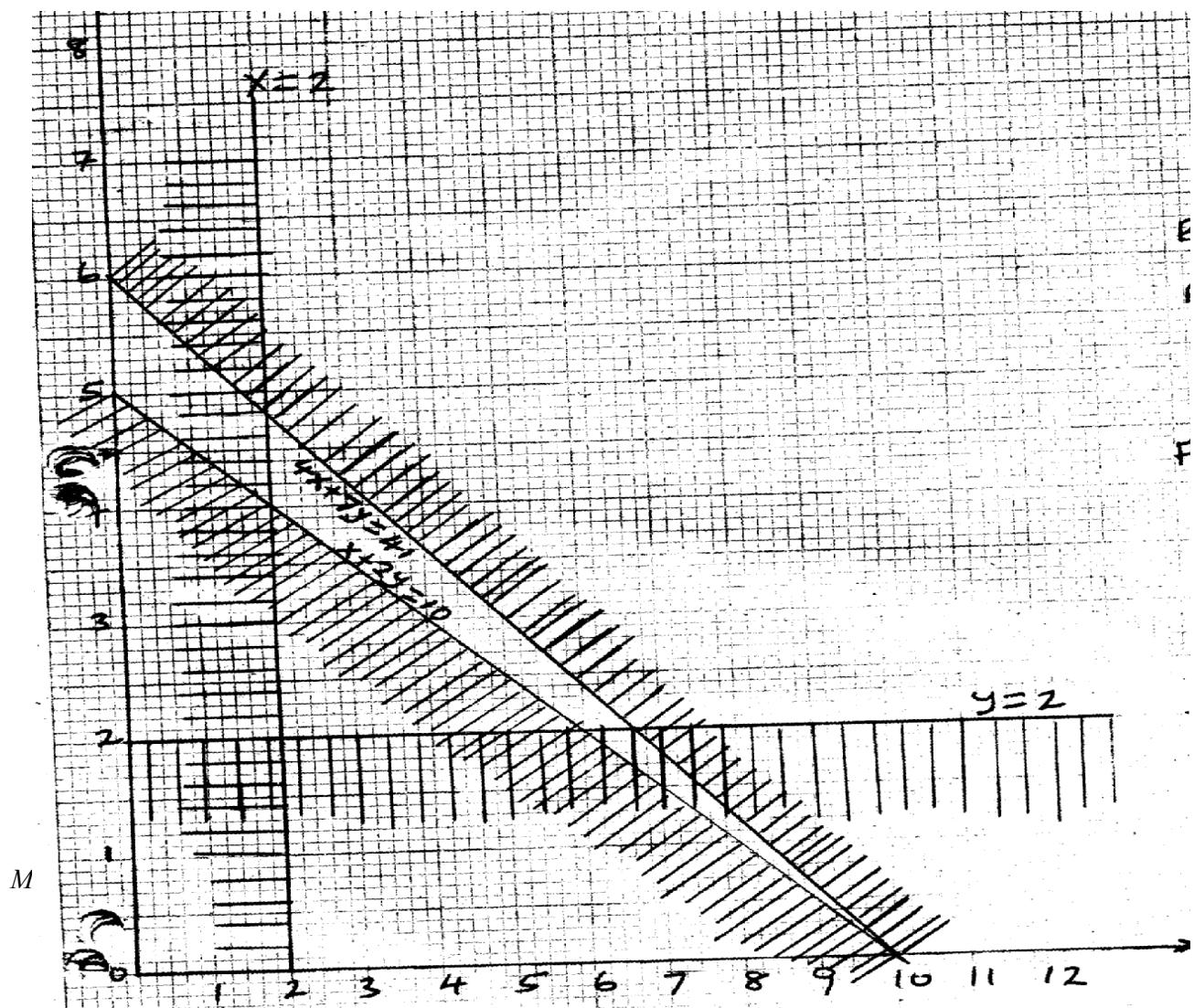
3. a)  $800x + 1600y \geq 8000$   
 $x + 2y \geq 10$

$$4x + 7y \leq 41$$

$$x \geq 2$$

$$y \geq 2$$

b)



c) For type A = 3 and B = 4  
 No. of operators =  $(3 \times 4) + (4 \times 7)$   
 4. a)  $\frac{180}{300} \times 2 \times 22/7 \times 6370 \cos 48^\circ = 13,396 \text{ Km}$

b)  $Km = \frac{(180 - 96)}{360} \times 2 \times 22/7 \times 6370 = 9342.7 \text{ km}$   
 $\text{Time} = \frac{9342}{280} = 33.36 \text{ km/hr}$

c)  $\theta = 180^\circ$   
 $\text{time} = \frac{(4 \times 180)}{60} = 12 \text{ hrs}$   
 $(14:15 - 12:00) = 2:15 \text{ a.m}$

d)  $\frac{600}{60} \text{ Nm}$   
 $60^\circ$   
 $Q = (12N, 30W)$

5. Long Difference =  $24 - 12 = 12^\circ$

$12 \times 60 \cos 34^\circ = 596.9 \text{ nm}$   
 $S = \frac{\text{"5.96"} \text{ nm}}{1.5} = 397.9 \text{ knots}$

6. (i)  $AB = \frac{80}{360} \times 2 \times 3.142 \times 25$   
 $\quad \quad \quad = \frac{4}{9} \times 25 \times 3.142$   
 $\quad \quad \quad = \frac{314.2}{9} \text{ cm}$   
 $\quad \quad \quad = 34.9111 \text{ cm.}$

(ii)  $\frac{\theta}{360} \times 2 \times 3.142 \times 25 \cos 50^\circ = \frac{314.2}{9}$   
 $\theta = \frac{314.2}{9} \times 360$   
 $\quad \quad \quad = \frac{50 \times 3.142 \times \cos 50}{9}$   
 $\quad \quad \quad = 93.35^\circ$

Longitude of BC( $93.35^\circ - 90^\circ$ )E  
 $= 03.35^\circ \text{ E.}$

(iii)  $\frac{\theta}{360} \times 3.142 \times 50 = \frac{314.2}{9}$   
 $\theta = \frac{314.2}{9} \times 360$

$$\frac{9}{3.142 \times 50} \\ = 80^\circ$$

Latitude of B ( $80^\circ - 50$ ) S

$$= 30^\circ S$$

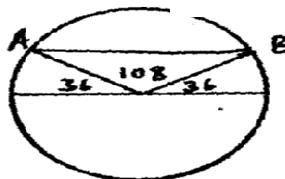
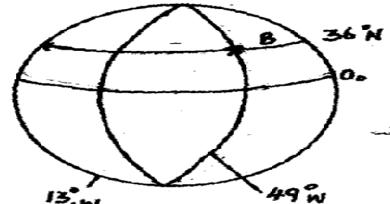
Position of B  $\Rightarrow (30^\circ S, 03.35^\circ E)$

7.

$$2133.6 = \frac{x}{360} \times 2 \times \frac{22}{7} \times 6380 \cos 70^\circ \\ \propto = \frac{21.33 \times 6 \times 360 \times 7}{44 \times 6380 \times \cos 70^\circ} \\ \propto + 15^\circ = 56^\circ \\ = 56 - 15 = 41^\circ N \\ \therefore \text{Location of B is } B(70^\circ S, 41^\circ N)$$

8.

$$(a) \text{ Longitudinal diff} = 180^\circ \\ (b) (i) \frac{180}{360} \times 2 \times \frac{22}{7} \times 6370 \times \cos 360^\circ \\ = 16196.52m \\ (ii) \frac{180}{360} \times 2 \times \frac{22}{7} \times 6370 \\ = 12012km \\ (c) \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 36^\circ = 840 \\ = 9.3353^\circ \\ = \text{position C} = 131.9.3^\circ W \\ C(36^\circ N, 121.7^\circ W)$$



$$9. a) PQ = \frac{120}{360} \pi \times 6370 \times 2 \\ = \frac{240}{360} \pi \times \frac{22}{7} \times 6370 = 13,346.6$$

$$b) 2PR \cos 60^\circ$$

$$PR = \frac{100}{360} \times 2 \pi \times 6370 \cos 60^\circ \\ = \frac{200}{360} \times \frac{22}{7} \times 6370 \cos 60^\circ = 5561.1km$$

$$c) PN = \frac{30}{360} \times 2 \times \frac{22}{7} \times 6370 \\ = 3336.67 km$$

10.

$$(a) (i) 60 (z - 50) = 1200$$

$$Z = 20$$

$$Z = 70^\circ S$$

$$(ii) xy = \frac{48}{360} \times 2 \times 6370 \cos 50^\circ \\ = 3431.629km$$

$$(b) (i) XZ = \frac{3431.627}{1.853} + 1200 \\ = 3051.9km$$

$$\text{Time} = \frac{3051.9}{400} = 7.6 \text{hrs}$$

$$(b) (ii) tie = 7.36 + 4.28 = 12.04$$

$$11. a) A - B = 45 + 35 = 800 \text{ Lat. Diff}$$

$$\begin{aligned}
 &= 80 \times 60 = 4800 \text{ nm} \\
 B - C &= 15 + 45 = 60 \text{ nm long. Diff} \\
 &= (60 \times 60) \times \cos 45 \\
 &= 3600 \times 0.7071 = 2545.56 \text{ nm} \\
 \text{Total distance} &= (4800 + 2545.56) \text{ nm} \\
 &= 7345.56 \text{ nm} \\
 &\approx 7346 \text{ nm (4.s.f)}
 \end{aligned}$$

$$\begin{aligned}
 b) \frac{80}{360} \times 2 \times \frac{22}{7} \times \frac{6370}{910} \\
 &= \frac{88 \times 910}{9} \\
 &= 8897.78 \text{ km} \\
 &\approx 8898 \text{ km (to nearest km)}
 \end{aligned}$$

$$\begin{aligned}
 c) B - C &= \frac{60}{360} \times 2 \times \frac{22}{7} \times \frac{6370}{910} \times \cos 45^\circ \\
 &= \frac{22 \times 910 \times 0.7071}{6^3} \\
 &= 471.87 \text{ km} \\
 A - C \text{ in Km} &= (8898 + 471.87) \\
 &= 13616.7 \text{ KM} \\
 \text{Time taken} &= \frac{13616.7}{840} = 16.21 \text{ hours} \\
 &= 16 \text{ hrs } 13 \text{ min} \\
 \text{Arrival time} &= 08.15 \\
 &\underline{16.13} \\
 &24.28 \\
 &= 12.28 \text{ am following morning}
 \end{aligned}$$

## 62. Linear programming

1.	<p>(a) let the No. of garments of type A be <math>x</math> and those of type B be <math>y</math></p> <p>(i) <math>3x + 2\frac{1}{2}y \leq 600</math> (material)</p> <p>(ii) <math>x \leq 1000</math>  <math>y \geq 80</math>  <math>x \geq 0</math></p> <p>(b) Lines drawn  <math>3x + 2\frac{1}{2}y = 600</math>  <math>x = 100</math>  <math>x = 80</math>  <math>x = 0</math></p> <p>(c) The object function is  <math>P = 80x + 60y</math> where <math>P</math> = total profit  Either draw a search line by choosing an appropriate value of <math>P</math>  e.g <math>12000 = 80x + 60y</math>  or inspect for maximum profit using points further from origin  maximum profit  100 garment of type A  120 garments of type B</p>	<p><b>B1</b></p> <p><b>B2</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>all✓ any two✓</p> <p>✓ lines and shading✓</p> <p>✓ lines and shading</p> <p>for✓ region indicated</p> <p>✓ objective function</p> <p>Use of search line or inspection</p> <p>✓</p>
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1.

$$\begin{aligned}30x + 20y &\leq 4800 \dots\dots(i) \\30x + 40y &\geq 3600 \dots\dots(ii) \\10x &< 30y \dots\dots(iii) \\x > 0, y > 0\end{aligned}$$

objective function  $10x + 12y = K$

$3x + 2y = 480$	$3x + 4y = 360$	$x = 3y$																								
<table border="1"> <tr> <td>X</td> <td>40</td> <td>60</td> <td>80</td> </tr> <tr> <td>y</td> <td>180</td> <td>150</td> <td>120</td> </tr> </table>	X	40	60	80	y	180	150	120	<table border="1"> <tr> <td>X</td> <td>20</td> <td>40</td> <td>60</td> </tr> <tr> <td>Y</td> <td>75</td> <td>60</td> <td>45</td> </tr> </table>	X	20	40	60	Y	75	60	45	<table border="1"> <tr> <td>X</td> <td>30</td> <td>45</td> <td>60</td> </tr> <tr> <td>Y</td> <td>10</td> <td>15</td> <td>20</td> </tr> </table>	X	30	45	60	Y	10	15	20
X	40	60	80																							
y	180	150	120																							
X	20	40	60																							
Y	75	60	45																							
X	30	45	60																							
Y	10	15	20																							

$$\begin{aligned}(ii) \text{ consider } (60, 40) \\10(60) + 12(40) &= 600 + 480 \\&= 1080 \\10x + 12y &= 1080 \\5x + 6y &= 540 - \text{search line}\end{aligned}$$

X	20	40	60
y	73	57	40

Maximum profit at ( $\alpha, 240$ )  
No queen cake, 240 marble cakes

(iii)  $240 \times 12 = \text{sh. } 2880$

(iv)  $10x + 12y \geq 600 \Rightarrow 10x + 12y = 600$   
 $5x + 6y = 300$

X	$\alpha$	12	60
y	50	40	0

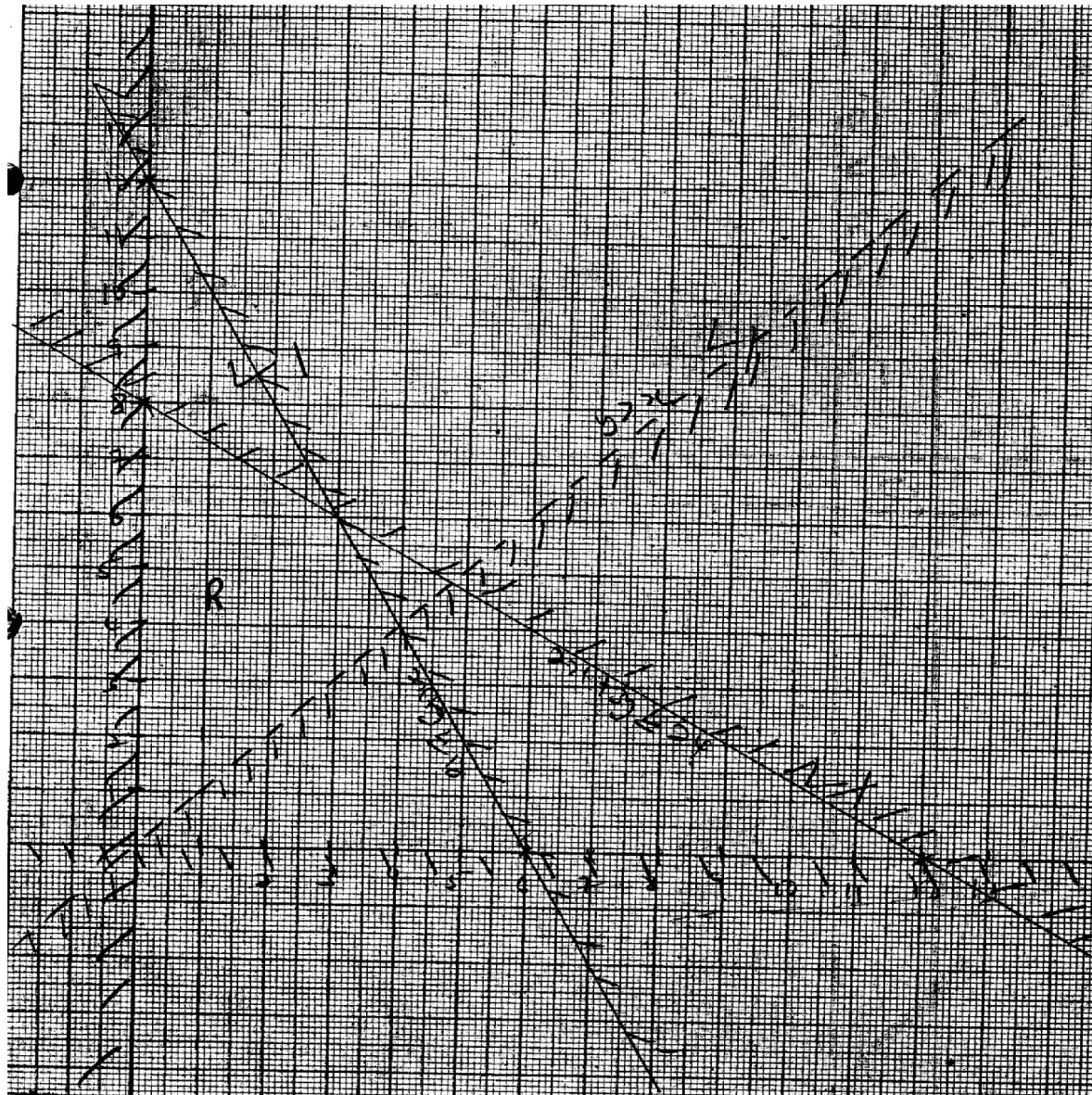
2.  $\begin{array}{ll}Machine\ A & Machine\ B \\Shirts & Jerseys \\No.\ x & y \\Hrs.\ @2hrs & @3hrs \\(i) 2x + 3y & \leq 24 \\(ii) 2x + y & \leq 12 \\(iii) y & > x \\(iv) x & > 0 \\y & > 0 \\Max\ pt(3,6) & \\Max\ profit & = 22x + 200x \\& = 600 + 1200 \\& = \text{Shs. } 1800\end{array}$

$$\begin{array}{l}(i) 2x + 3y = 24 \\(ii) 2x + y = 12 \\(iii) y = x \\(iv) y = 0 \\x = 0\end{array}$$

$$\begin{array}{r|rr}x & 0 & 12 \\y & 8 & 0\end{array}$$

$$\begin{array}{r|rr}x & 0 & 6 \\y & 12 & 0\end{array}$$





3. (a)  $3x + 7y \leq 210$   
 $x + y \geq 20$   
 $x < 2y$   
 $x > 15$
- (b) refer
- (c)  $120x + 140y = 120 \times 130 + 140 \times 10$   
 $\text{Profit} = \text{shs. } 5960$   
 $x = 31$
- $y = 16$
4. Passengers  
 $64x + 48y \geq 384$  i.e.  $8x + 6y \geq 48$   
 $x > 0$   
 $y > 0$   
 $x + y \geq 7$

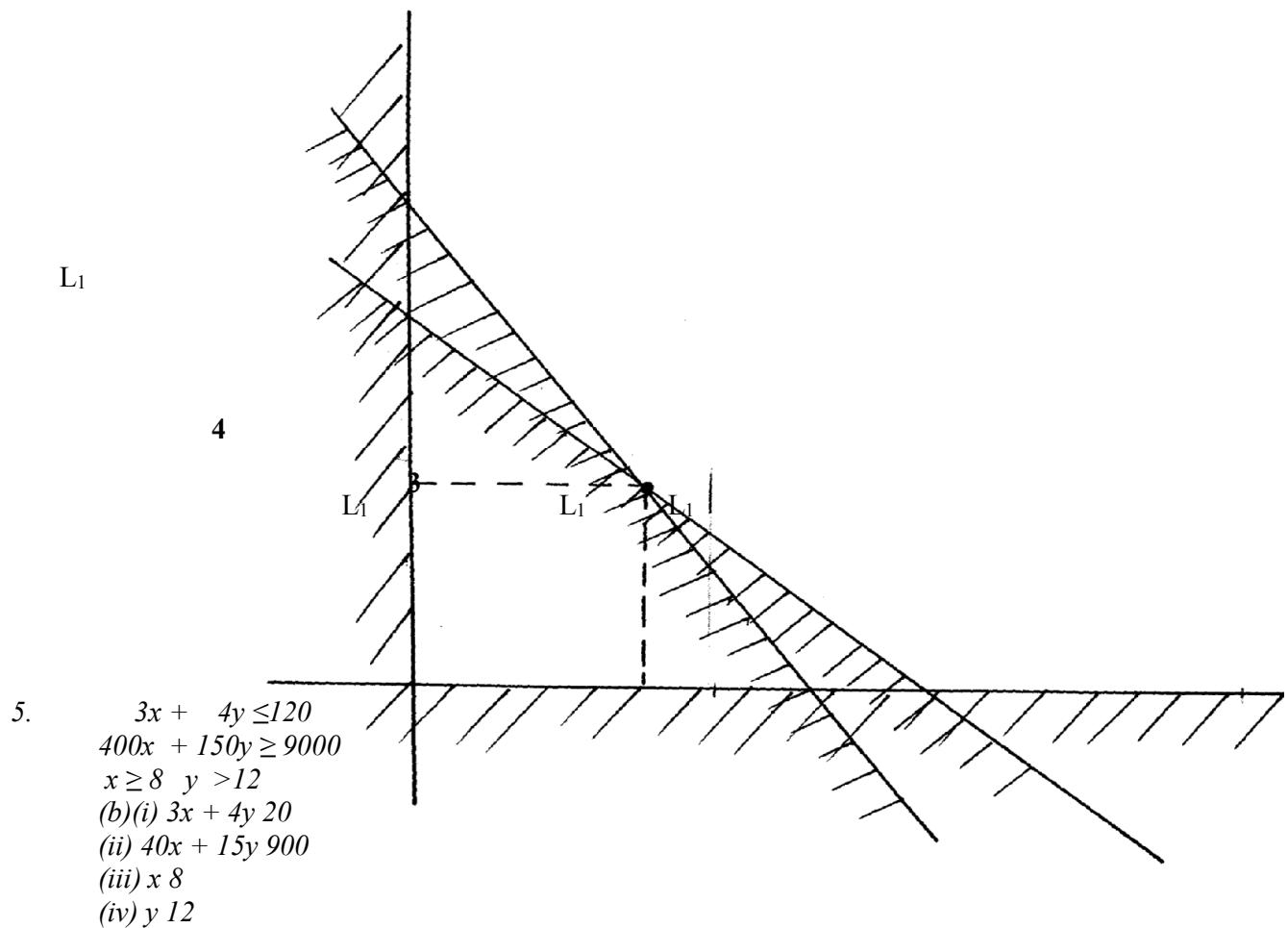
*Cost equation*

$$\text{Total cost} = 2500x + 20000y$$

(3,4)

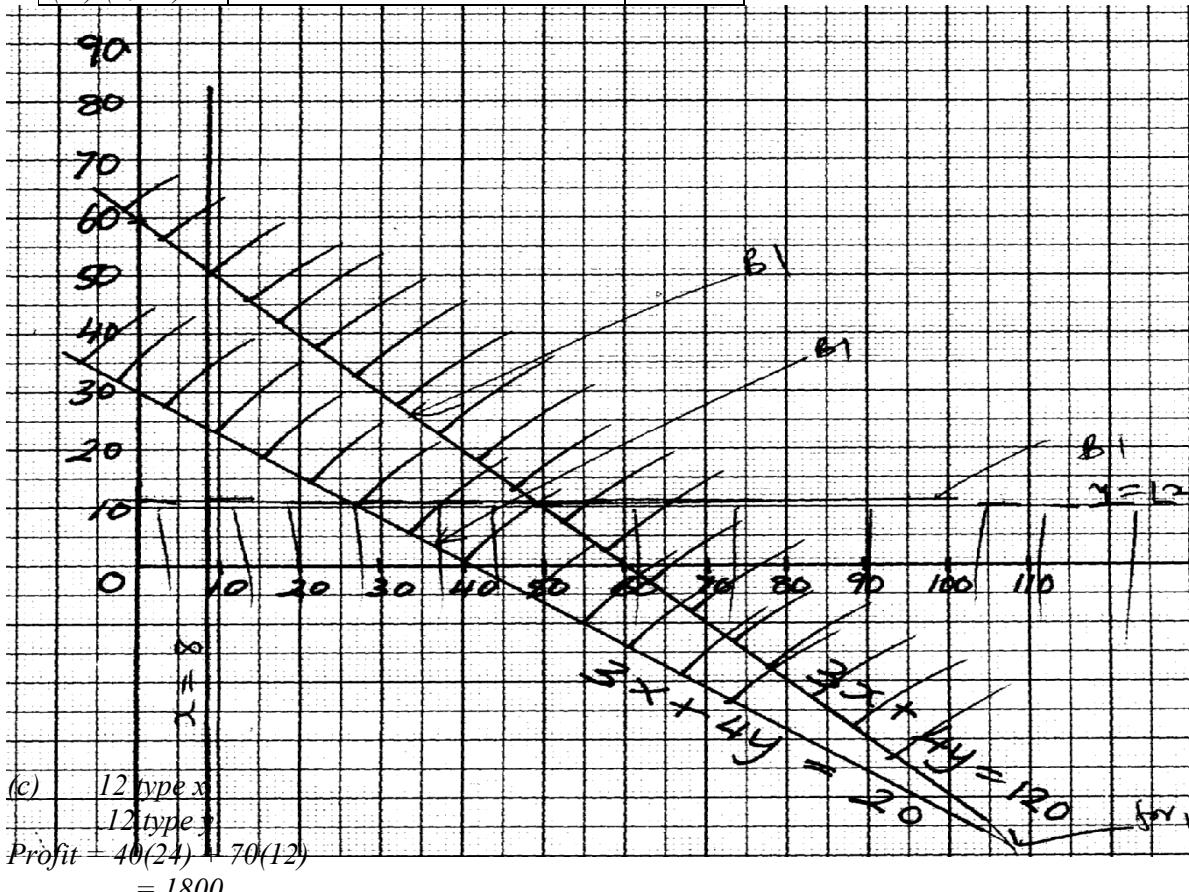
3 type x

4 type y



(table showing calculation of profit )

Points	Objective function $40x + 70y$	Profit
(i) (8,24)	$320 + 1680$	2000
(ii) (24, 12)	$960 + 840$	1800
(iii) (8, 12)	$320 + 840$	1160

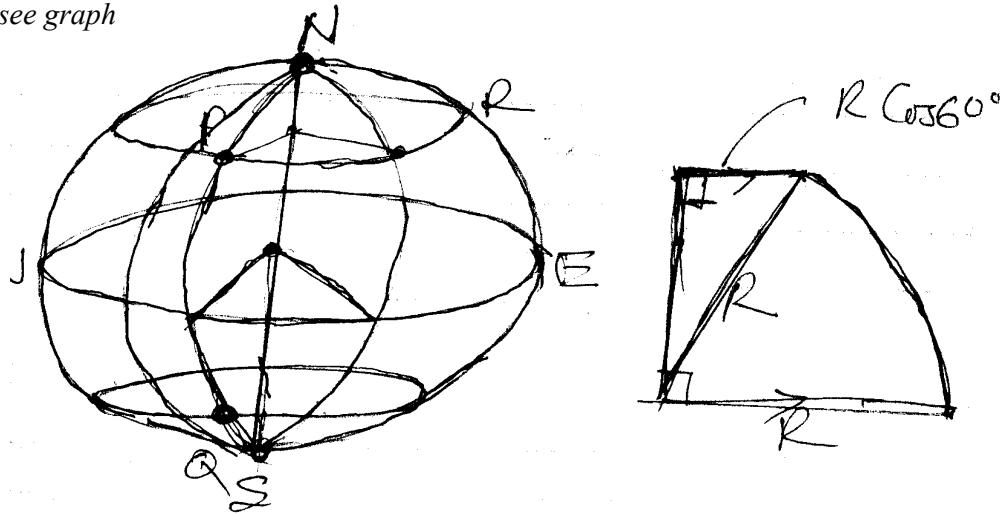


6.  $100x = 160y = 16000$        $5x 200 + 8x 50$   
 $= 100x 200 + 160x 50$        $1000 + 4000$   
 $20000 + 8000$        $10x 200 + 16x 50 =$   
 $28000/ =$        $10x + 16y = 1600$   
 $5x + 8y = 800$   
 $5x 20 + 100$   
 $8y = 800 - 100$   
 $y = \frac{700}{8}$   
 $800/5 = 160$

a)  $y < 2x$ ,  $50 \leq x \leq 200$ ,  $x > 100$   
 $y > 0$ ,  $x+y \leq 250$ ,  $100x + 160y \geq 16000$

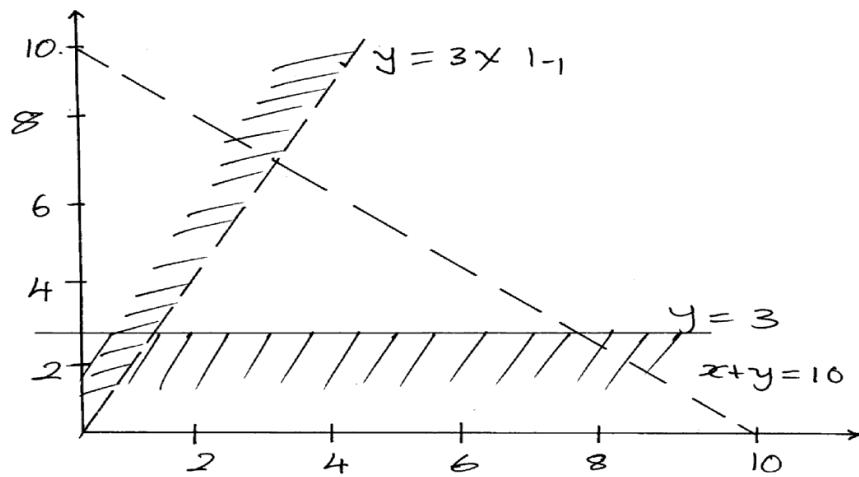
b) See graph

c) see graph



profit?

7.  $x + y < 10$   
 $y < 3x$   
 $y > 3$

(c) Objective function  $3x + 2y = I$  or use of search line

5 packets of cups and 4 packets of sticks

x	y	Profit
2	4	14
2	5	16
3	4	17
3	5	19
3	6	21
4	4	20
4	5	22
5	4	23

8. Pang - P, Jembe J

(a)  $50P + 30J = 4260$

$50P + 15J = 1290$

$50P + 30J \cancel{=} 4260$

$\underline{10P + 30J \cancel{=} 1290}$

eeducationgroup.com

$$40P = 1680$$

$$P = \frac{168}{4} = 42$$

$$50(42) + 30J = 4260$$

$$2100 + 30J = 4260$$

$$30J = 2160$$

$$J = \frac{(2160)}{30}$$

$$J = 72$$

Wholesaler

$$\frac{110}{100} \times 42 = \text{shs. } 46.50 = \text{pangas}$$

$$100$$

$$\frac{85}{100} \times 72 = \text{shs } 60 = \text{jembes}$$

$$100$$

For B

$$50 \times 46.50 + 30 \times 61.2$$

$$2310 + 1836 = 4146$$

$$\begin{aligned} \text{Saving} &= 4260 \\ &\quad \frac{4116}{144} \end{aligned}$$

$$(b) \text{ Discount } 5000 - 3500 = 1500$$

$$\begin{aligned} \% \text{ discount} &= \frac{1500}{5000} \times 100 \\ &= 30\% \end{aligned}$$

$$9. \quad a) X \geq 0, y \geq 0$$

$$10x + 20y \geq 120$$

$$4x + y \geq 20$$

b) On the graph.

$$c) i) (4,4)$$

$$\begin{aligned} 4 \times 100 + 4 \times 300 \\ 400 + 1200 = 1600 \end{aligned}$$

$$10. \quad \text{Distance Covered} = (3t^2 - 3t - 6)dt$$

$$= t^3 - \frac{3}{2}t^2 - 6t \Big|_1^4$$

$$16 - \left[ \frac{4^3 - \frac{3}{2}(4)^2 - 6(4)}{2} \right] - \left[ \frac{1^3 - 3(1)^2 - 6(1)}{2} \right]$$

### 63. Differentiation

1	$\int_1^2 (9t^2 - 6t + 2)dt$ $[3t^3 - 3t^2 + 2t + c]_1^2$	M <sub>1</sub> M <sub>1</sub> A <sub>1</sub>	
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	$(3 \times 2^3 - 3 \times 2^2 + 2 \times 2) - (3 - 3 + 2)$ $(24 - 12 + 4) - (2)$ $16 - 2 = 14m$		
2.	<p>(a) <math>V = ds/dt = 8 - 2t</math>          (i) At <math>t = 1</math> <math>V = 8 - 2 = 6m/s</math>          (ii) At <math>t = 3</math> <math>v = 8 - 6 = 2m/s</math>          (b) At maximum <math>ds/dt = 0</math>  <math>8 - 2t = 0</math>  <math>t = 4</math> secs          therefore maximum displacement  <math>s = 8t - t^2</math>  <math>S = 8 \times 4 - 4^2</math>  <math>= 16m</math>          (c) Acceleration <math>= dv/dt = 2m/s^2</math>          (d) At starting point, displacement is zero  <math>= 8t - t^2 = 0</math>  <math>t(8 - t) = 0</math>  <math>t = 0</math> or <math>t = 8</math>          body back after 8sec</p>	B1 B1 B1 M1 A1 M1 A1 B1 M1 A1	
		10	

$$1. \quad S = t^3 - 3t^2 + 2t$$

$$(a) V = \frac{ds}{dt} = 3t^2 - 6t + 2$$

When  $t = 2$   
 $V = 3(4) - 6(2) + 2$   
 $= 2m/s$

(b) At minimum velocity :

$$\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = 6t - 6$$

$$6t - 6 = 0$$

$$t = 1$$

Min-velocity  $= 3(1)^2 - 6(1) + 2$   
 $= -1m/s$

$$(c) 3t^2 - 6t + 2 = 0$$

$$t = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(2)}}{6}$$

$$= \frac{6 \pm 5.2}{6}$$

$$t = 1.58 \text{ or } 0.4\text{sec}$$

$$(d) acc = \frac{dv}{dt} = 6t - 6$$

$$a = 6(3) - 6 = 12m/s^2$$

$$2. \quad a)$$

X	2	5	8	10
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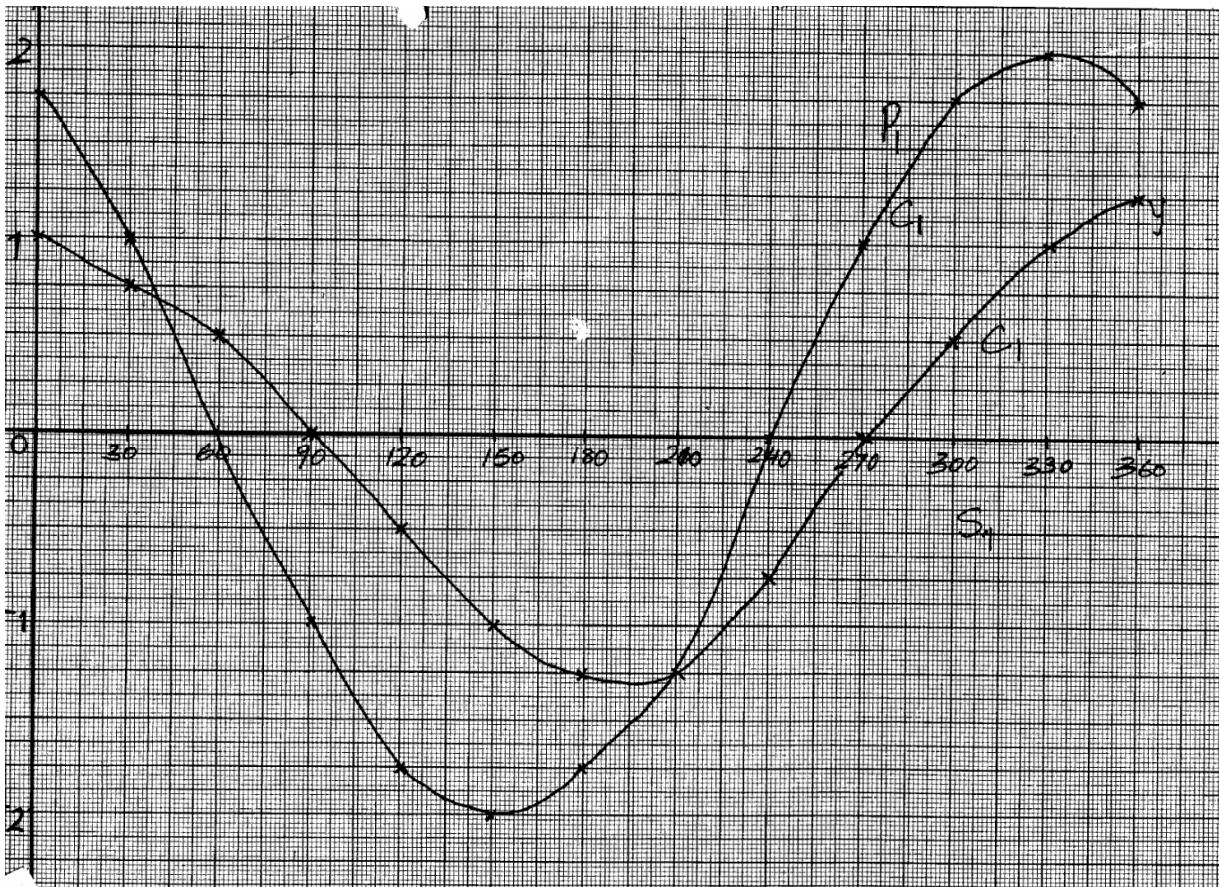
$y$	5	26	65	101
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b)  $A = h(2 + 10 + 26 + 50 + 82)$   
 $= 2 \times 170$

$= 34$  square units

c)  $A = (x^2 + 1) dx$   
 $= (1000/3 + 10) - 0$   
 $= 333.33 + 10$   
 $= 343.33$   
 $= 343.33$  square units

d) Percentage error  $= \frac{3.33}{343.33} \times 100\%$   
 $= 0.97\%$



3.  $y = \frac{2x^2 + x + c}{2}$

$a + x = -4, y = 6$

$6 = (-4)^2 - 4 + c$

$c = -6$

$y = x^2 + x - 6$

4. a)  $-2t^2 + t + 28 = 0$

$P = -56$

$S = 8, -7$

$-2t^2 + 8t - 7t + 28 = 0$

$-2t(t - 4) - 7(t - 4) = 0$

$$t = 3.5$$

$$t = 4$$

$$\begin{aligned} b) AC &= -4t + 1 \\ -4t + 1 &= 0 \\ T &= \frac{1}{4} \\ V &= -2\left(\frac{1}{4}\right)2 + \frac{1}{4} + 28 \\ V &= 28.125 \end{aligned}$$

$$\begin{aligned} c) \quad Acc &= -4t + 1 \\ At rest t &= 3.5, t = 4 \\ Acc &= -4 \times 4 + 1 \\ &= -15 \text{ m/s}^2 \\ At t &= 3.5 \\ A &= -13 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} d)(i) \quad D &= \frac{2t^3}{3} + \frac{t^2}{2} + 28t + 5 \\ Distance &= -2 \times 3^3/3 + 3^2/2 + 28 \times 3 + 5 = 75.5 \text{ m} \end{aligned}$$

$$\begin{aligned} ii) \quad D &= \frac{2t^3}{3} + \frac{t^2}{2} + 28t + 5 \\ D &= -2 \times 3^3/3 + 3^2/2 + 28 \times 3 + 5 \\ &= -18 + 4.5 + 84 + 5 \\ &= 70.5 + 5 = 75.5 \end{aligned}$$

$$\begin{aligned} 5. \quad a) i) \quad V &= 15 + 4t - 3t^2 \\ \frac{dv}{dt} &= Acc = 4 - 6t \\ ii) \quad V &= 15 + 4t - 3t^2 \\ V &= \frac{dv}{dt} = 15 + 4t - 3t^2 \\ \therefore S &= \int (15 + 4t - 3t^2) dt \\ S &= 15t + \frac{4t^2}{2} - \frac{3t^3}{3} + C \\ S &= 15t + 2t^2 - t^3 + C \\ b) i) Acc &= 0 \text{ hence } \frac{dv}{dt} = 0 \\ 4 - 6t &= 0 \\ -6 &= -4 \\ t &= \frac{2}{3} \text{ sec.} \end{aligned}$$

$$\begin{aligned} ii) \quad S &= \left[ 15t + 2t^2 - t^3 + C \right]_0^{2/3} \\ &= 15\left[\frac{2}{3}\right] + 2\left[\frac{2}{3}\right]^2 - \left[\frac{2}{3}\right]^3 \\ &= \frac{10}{1} + \frac{8}{9} = \frac{8}{27} \end{aligned}$$

$$= \frac{286}{27} = 10.5925 \simeq 10.59$$

c) Acc.  $4 - 6t$   
 $-4 = -6t$   
 $t = \frac{2}{3}$  Acc. = 0  
 $\therefore$  Time is 0 and  $\frac{2}{3}$   
 Both. 0 and  $\frac{2}{3}$  sec.

6. (a)  $x^2 = -x^2 + 8$   
 $2x^2 = 8$   
 $x = 2 \quad a = -2, \quad b = 2$

$$(b) \text{ Area of } \int_{-2}^2 x^2 = \left[ \frac{x^3}{3} \right]_{-2}^2$$

$$= \frac{8 - (-8)}{3}$$

$$= \frac{16}{3}$$

Area =  $(x^2 + 8)dx$

$$= \left[ \frac{-x^3}{3} + 8x \right]_{-2}^2$$

$$= \left[ \frac{-80 + 16}{3} \right] \left[ -\frac{8 - 16}{3} \right]$$

$$\frac{80}{3} = 26 \frac{2}{3}$$

$$(c) \text{ Area} = \frac{80}{3} + \frac{16}{3} = \frac{96}{3} = 32$$

7.  $a = \frac{d^2s}{dt^2} = \frac{d^2}{dt^2}(t^3 - 5t^2 + 2t + 5)$   
 $= \frac{d}{dt} = 3t^2 - 5t + 2$   
 $= 6t - 5$   
 If  $a = 0$   
 $6t - 5 = 0$   
 $t = \frac{5}{6}$   
 $v = \frac{ds}{dt} = 3t^2 - 5t^2 = 3 \times \frac{25}{36} - 5 \times \frac{5}{6} + 2$   
 $= -\frac{1}{12} m/s$

8. (a)  $V = 6t + 4 = 3t^2 + 4t + c$   
 $5 = 3(0)^2 + 4(0) + c$   
 $5 = c$   
 $V = 3t^2 + 4t + 5$   
 (b)  $V = 3(4)^2 + 4(4) + 5$   
 $= 69 \text{ m/s}$

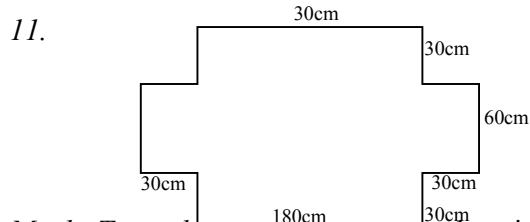
$$(c) \quad (i) \int 3t^2 + 4t + 5 \\ = t^3 + 2t^2 + 5t + c \\ \text{When } t = 0 \quad S = 0 \\ S = t^3 + 2t^2 + 5t$$

$$(ii) \quad S = t^3 + 2t^2 + 5t \quad | \\ = [(4)^3 + 2(4)^2 + 5(4)] - [(1)^3 + 2(1)^2 + 5(1)] \\ = 108 \text{ m} \\ 9. \quad a) \quad S = 3t + \frac{3t^2 - 2t^3}{2} \\ \frac{ds}{dt} = v = 3 + 3t - 6t^2 \\ \frac{dv}{dt} = a = 3 - 12t \quad t = 0 \\ a = 3 \text{ m/s}^2 \\ b)i) \quad O = -6t^2 = 3t + 3 \\ t = 1 \\ \begin{array}{r} -8t^2 \\ +6t - 3t \\ \hline \end{array} \\ ii) \quad S = 3(1) + \frac{3(1)^2 - 6(1)^3}{2} \\ = 3 + \frac{3}{2} - 2 \\ = \frac{2}{2} + \frac{3}{2} = \frac{5}{2}$$

$$c) \quad V = 3 + 3(1) - 6(1) \\ = 3 + 3 - 6 \\ = 0 \text{ m/s} \\ 10. \quad dy/dx = 12x^2 - 4x - 3 \text{ at } (2, 23) \\ = 12(4) - 4(2) - 3 \\ = 48 - 8 - 3 \\ = 40 - 3$$

$$= 37 \\ M = y - y \text{ or } y = mx + c \\ = \frac{23 - y}{2 - x} \\ 23 - y = 37(2 - x) \\ 23 - y = 74 - x \\ 23 = 37(2) + c \\ C = 23 - 74 = -51$$

Hence equation is  $y = 37x - 5$



$$\begin{aligned}
 (i) (180 \times 30 \times 2) &= 10800 \\
 (60 \times 30 \times 2) &= 3600 \\
 (180 \times 60 \times 1) &= 10800 \\
 \text{Total area} &= 25200 \text{cm}^2
 \end{aligned}$$

$$\begin{aligned}
 (ii) \text{Volume of the cuboid} &= (180 \times 60 \times 30) \text{cm}^3 = 324,000 \text{cm}^3 \\
 \text{Mass} &= (2.5 \times 180 \times 60 \times 30) \\
 &= \frac{810000}{1000} \text{g} \\
 &= 810 \text{kg} \\
 \text{Volume of water} &= (324,000 \text{cm}^3) \\
 \text{Mass of water} &= \frac{(324,000 \times 1)}{1000} \\
 &= 324 \text{kg} \\
 \text{Mass of cuboid} &= 324 + 810 \\
 \text{Full of water} &= 1,134 \text{kg}
 \end{aligned}$$

12. Let length of square cut off be  $x$   
 Length of box =  $8 - 2x$   
 Width of box =  $5 - 2x$   
 Height of box =  $x$

$$\begin{aligned}
 V &= (8 - 2x)(5 - 2x)x \\
 &= 4x^3 - 26x^2 + 40x \\
 \frac{dv}{dx} &= 12x^2 - 52x + 40 \\
 12x^2 - 52x + 40 &= 0 \\
 3x^2 - 13x + 10 &= 0 \\
 3x^2 - 10x - 3x + 10 &= 0 \\
 X(3x - 10) - 1(3x - 10) &= 0 \\
 (x - 1)(3x - 10) &= 0 \\
 x = 1 & \qquad x = 10/3
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2V}{dx^2} &= 24x - 52 \\
 x = 1 & \\
 \frac{d^2V}{dx^2} &= 24x - 52 = -28 \quad \text{maximum} \\
 x = 1 \text{ cm} & \text{ gives maximum vol} \\
 (8-2)(5-2) \times 1 &= 6 \times 3 \\
 &= 18 \text{cm}^3
 \end{aligned}$$

13. a)  $\frac{dy}{dx} = 3x^2 - 2$   
 Gradient of the tangent is 1 so, gradient of the normal is -1  
 $\frac{y-2}{x-1} = -1$   
 $\frac{y+2}{x-1} = -1$   
 $y = -x - 1$

$$(b) dy = 3x^2 - 3 = 0$$

$$3x^2 - 1 = 0$$

$$(x-1) = 0$$

$$x = 1, y = 0 \text{ & } x = -1, y = 4$$

Coordinates of turning points

$$(1, 0) \text{ and } (-1, 4)$$

For  $(1, 0)$   $x < 1$ ,  $\frac{dy}{dx}$  is -ve

$x > 1$ ,  $\frac{dy}{dx}$  is +ve

$(1, 0)$  is a minimum point for  $(-1, 4)$   $x < -1$ ,  $\frac{dy}{dx}$  is +ve

$(1, 0)$  is a minimum point for  $(-1, 4)$   $x < -1$ ,  $\frac{dy}{dx}$  is +ve

$x > -1$ ,  $\frac{dy}{dx}$  is -ve

$\Rightarrow (-1, 4)$  is a maximum point

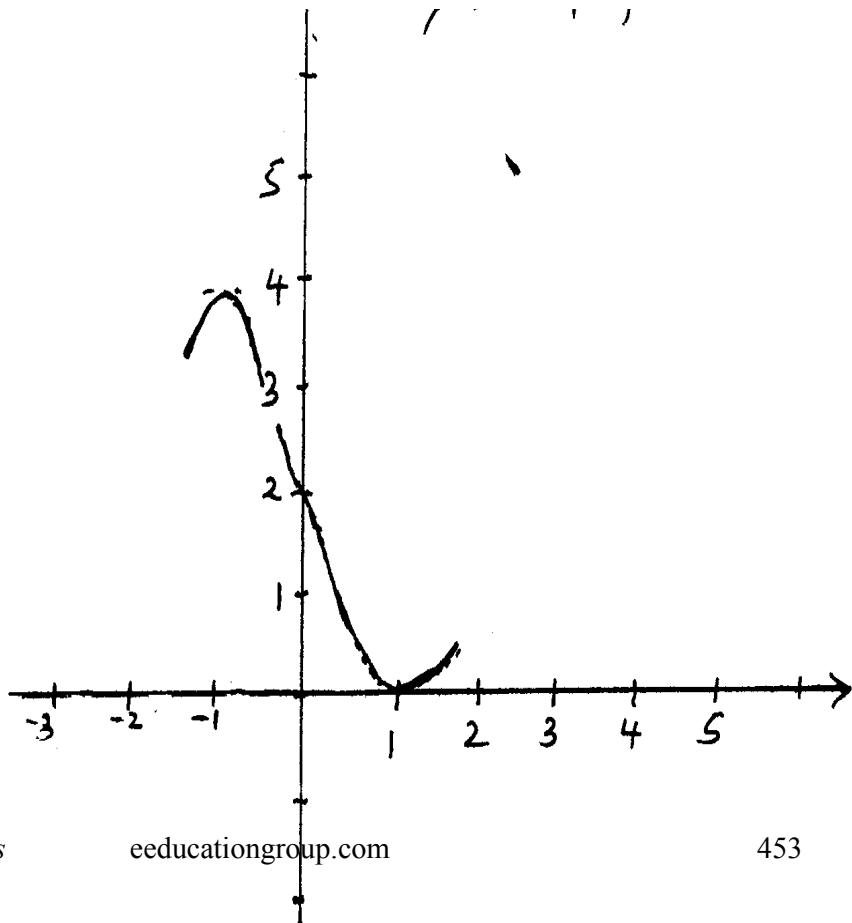
To sketch the curve we

(i) Its turning points and their nature

(ii) The points the graph cuts the x and y axis i.e the x and y-intercepts

(b)  $\Rightarrow$  Indicating that the curve turns at  $(-1, 4)$   $(1, 0)$  and cuts the y-axis at  $(0, 2)$  B<sub>1</sub>

$\Rightarrow C_1$  for correct sketch



14. a)  $-2t^2 + t + 28 = 0$   
 $t^2 - t - 28 = 0$   
 $2t^2 - 8t + (7t - 28) = 0$   
 $+ (t-4) + 7(t-4) = 0$   
 $t + 7)(t-4) = 0$   
 $t = -3.5 \text{ or } 4$   
*p.B at rest at t= 4seconds*

(b)  $a = 1-4t$   
 $1 - 4t = 0$   
 $0.25s = t$   
 $V = 28 + 25 - 2(0.25)^2$   
 $= 28.25 - 0.125$   
 $V = 28.125m/s$

(c) (i)  $S = 28t + \frac{t^2}{2} - \frac{2t^3}{3} + C$   
when  $t = 0, s = 0$   
 $\therefore S = 28t + t^2 - 2t^3$

PB at rest after 4s  
 $\therefore S = 28 \times 4 \times 42 - \frac{2}{3} \times 4^3$   
 $= 112 + 8 - 42.667$   
 $= 120 - 42.6667 = 77.33m$

15.  $S = t^3 - 3t^2 + 2t$   
(a)  $V = \frac{ds}{dt} = 3t^2 - 6t + 2$   
When  $t = 2$   
 $V = 3(4) - 6(2) + 2$   
 $= 2m/s$

(b) At minimum velocity :  
 $\frac{dv}{dt} = 0$   
 $\frac{dv}{dt} = 6t - 6$   
 $6t - 6 = 0$   
 $t = 1$   
Min-velocity =  $3(1)^2 - 6(1) + 2$   
 $= -1m/s$

(c)  $3t^2 - 6t + 2 = 0$   
 $t = \frac{6 \pm \sqrt{(-6) - 4(3)(2)}}{6}$   
 $= \frac{6 \pm 5.2}{6}$

$$t = 1.58 \text{ or } 0.4 \text{ sec}$$

$$(d) acc = \frac{dv}{dt} = 6t - 6$$

$$a = 6(3) - 6 = 12 \text{ m/s}^2$$

## 61. Approximation of area

1.  $h = \frac{3 - 1}{5} = \frac{4}{5} = 0.8$

$x$	-1	-0.2	0.6	1.4	2.2	3
$y$	5	7.56	8.84	8.84	7.56	5

$$A = 0.8 (5 + 5) + 2 (7.56 + 8.84 + 8.84 + 7.56)$$

$$\begin{aligned} &= 0.4 (10 + 2(32.8)) \\ &= 0.4 \times 75.6 \\ &= 30.24 \text{ sq. units} \end{aligned}$$

2.  $y_0 = 0$

$$y_1 = 2.5$$

$$y_2 = 6$$

$$y_3 = 10.5$$

$$y_4 = 16$$

$$y_5 = 22.5$$

$$y_6 = 30$$

$$\begin{aligned} A &= \frac{1}{2} x 1(0+30) + 2(2.5 + 6 + 10.5 + 16 + 22.5) \\ &= \frac{1}{2} x 145 = 72.5 \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{1}{2} x^2 - 2 &= \frac{x^3 - x}{6} \\ &= \frac{8^3 - 8}{6} - \frac{2^3 - 2}{6} \\ &= 77.33 - 0.67 \\ &= 78 \text{ square units} \end{aligned}$$

$$\begin{aligned} (c) \% \text{ error} &= \frac{72.5 - 78}{78} \times 100 \\ &= -7.05\% \end{aligned}$$

3.  $y_0 = 0$

$$y_1 = 2.5$$

$$y_2 = 6$$

$$y_3 = 10.5$$

$$y_4 = 16$$

$$y_5 = 22.5$$

$$y_6 = 30$$

$$\begin{aligned} A &= \frac{1}{2} x 1(0+30) + 2(2.5 + 6 + 10.5 + 16 + 22.5) \\ &= \frac{1}{2} x 145 \\ &= 72.5 \end{aligned}$$

$$(b) \quad \frac{1}{2} x^2 - 2 = \frac{x^3 - x}{6}$$

$$\begin{aligned}
 &= \frac{8^3 - 8}{6} - \frac{2^3 - 2}{6} \\
 &= 77.33 - 0.67 \\
 &= 78 \text{ square units}
 \end{aligned}$$

$$\begin{aligned}
 (c) \% \text{ error} &= \frac{72.5 - 78}{78} \times 100 \\
 &= -7.05\%
 \end{aligned}$$

4      a)

$$\begin{aligned}
 -2x^2 + 3x + 4 &= 2x + 3 \\
 -2x^2 + x + 1 &= 0 \\
 -2x^2 + 2x - x + 1 &= 0 \\
 (x-1)(-2x-1) &= 0 \\
 x = 1 \quad \text{or} \quad x = -\frac{1}{2} & \\
 \text{when } x = 1 \quad y = 2x + 3 &= 5 \\
 Q(1, 5) & \\
 (b) -2x^2 + 3x + 4 dx &- (2x+3)dx
 \end{aligned}$$

5. a)

X	-5.5	-5	-4.25	-3.75
y	16.25	12	6.56	3.56

$$\begin{aligned}
 b) A &= 0.5 (18.56 + 14.06 + 10.06 + 6.56 + 3.56 + 1.06) \\
 &= 0.5 \times 53.86 = 26.93
 \end{aligned}$$

$$\begin{aligned}
 c) i) \int x^1 + 2x - 3 \\
 [x^3 + x^2 - 3x]^{-3} \\
 \frac{3}{3} \\
 = \left[ \frac{(-3)^3}{3} + (-3)^2 - 3(-3) \right] \\
 = 9 + 18 = 27 \text{ square units} \\
 ii) \frac{27 - 26.93}{27} \times 100 \\
 = 0.25925 \% = 0.2593 \%
 \end{aligned}$$

6

x	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
y	18	28.	4	56.	7	94.	1	142	1	20	23
	25	1	25	4	25	1	.25	7	0	5	3

$$\begin{aligned}
 \therefore \text{Area} &= \frac{1}{2} n \left( y_0 + y_n \right) + 2(y_1 + \dots) \left. \right\} y_n - 1 \\
 &= \frac{1}{2} (1) \left\{ (18 + 233) + 2(41 + 74 + 55 + 170) \right\} \\
 &= \frac{1}{2} \left\{ 251 + 2(340) \right\} \\
 &= \frac{1}{2} (251 + 680) \\
 &= \frac{1}{2} (831)
 \end{aligned}$$

### 65. Integration

			<b>Diagram</b>
1	a)	$x^2 + 5 = 8 - 2x$ $x^2 + 2x - 3 = 0$ $(x-1)(x+3) = 0$ $X = 1 \text{ or } x = -3$ $C(-3, 14)$ $D(1, 6)$	$M_1$ $A_1$ $B_1$ $B_1$
	b)	$\int_{-3}^1 (x^2 + 5) dx$ $\left[ \frac{1}{3}x^3 + 5x \right]_{-3}^1$ $\left( \frac{1}{3} + 5 \right) - (-9 - 15)$	$M_1$ $M_1$ $A_1$
		$5 \frac{1}{3} + 24$ $= 29 \frac{1}{3} \text{ sq units}$	$M_1$ $A_1$
	c)	$\frac{1}{2}(14 + 6) \times 4 = \text{Area}$ $\therefore A = 2 \times 20$ $= 40 \text{ sq units}$	$B_1$
	d)	$40 - 29 \frac{1}{3}$ $= 10 \frac{2}{3} \text{ sq units}$	
			10

1.  $S_{10} = 100$   
 $\int_2^5 (x-1)(x-2) dx$   
 $x-2$   
 $= \int_2^5 x^2 - 3x + 2 dx$   
 $= \left[ \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_2^5$

2.  $\int(x^2 + 1) dx = 2a$

$$\left[ \frac{x^3}{3} + x \right]_0^a = 2a$$

$$a^3 + a - 0 = 2a$$

$$\begin{aligned}3 & \quad 1 \\a^3 + 3a &= 6a \\a^3 &= 3a \\(a^3 - 3a) &= 0 \\a(a^2 - 3) &= 0 \\a &= 0 \\&\text{or } 3 = \pm 1.732\end{aligned}$$