BURJEB PURE MATHS PROPOSED GUIDE 2024

S/N	SOLUTION	Marks
1.	$$y = x^2 - 5$$ $$\frac{dy}{dx} = 2x = 3$$ $$x = 1.5,$$ $$y = 1.5^2 - 5 = -2.75$$ The coordinates are (1.5, -2.75) $$\frac{y + 2.75}{x - 1.5} = 3$$ $$y = 3x - 7.25\ compare\ with\ y = 3x + k$$ The value of k is -7.25 Alternatively $$substituting\ (1.5, -2.75) into\ y = 3x + k$$ $$-2.75 = 3(1.5) + k, k = -7.25$$	05
2.	$$\lvert z + 2 + 3i \rvert = 3$$ $$\sqrt{(x + 2)^2 + (y + 3)^2} = 3$$ $$(x + 2)^2 + (y + 3)^2 = 3^2$$ Centre (-2, -3) and radius is 3units Mean point is (1, 1) $$\max value = 3 + \sqrt{(-2 - 1)^2 + (-3 - 1)^2}$$ $$\max value = 3 + 5 = 8units$$	05
3.	$$let\ the\ roots\ be\ \propto and\ \beta$$	

	$\propto +\beta = -\frac{b}{a}$ $\propto \beta = \frac{c}{a}$ $\propto -\beta = 4$ $\sqrt{(\propto +\beta)^2 - 4 \propto \beta} = 4$ $\left(-\frac{b}{a}\right)^2 - \frac{4c}{a} = 16$ $b^2 = 16a^2 + 4ac$ $\frac{b^2}{4a} = (4a + c)$	05
4.	$\boldsymbol{MN} = \begin{pmatrix} 1+3\lambda \\ 2-\lambda \\ 3+2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} = \begin{pmatrix} -3+3\lambda \\ 5-\lambda \\ -7+2\lambda \end{pmatrix}$ $\begin{pmatrix} -3+3\lambda \\ 5-\lambda \\ -7+2\lambda \end{pmatrix} . \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 0$ $-9 + 9\lambda - 5 + \lambda - 14 + 4\lambda = 0$ $14\lambda = 28$ $\lambda = 2$ $\lvert\boldsymbol{MN}\rvert = \sqrt{(-3+6)^2 + (5-2)^2 + (-7+4)^2}$ $= 5.1962 units$	05
5.	$RHS = \frac{sin5A - sinA}{sinA}$	

	$$=\frac{2cos3Asin2A}{sinA}$$ $$=\frac{4cos3AsinAcosA}{sinA}$$ $$=4cos3AcosA$$ $$=LHS$$	05
6.	$$\int_1^{10} xlogx^2\,dx=\frac{2}{In10}\int_1^{10} xInx\,dx$$ $$let\ u=Inx$$ $$\frac{du}{dx}=\frac{1}{x}$$ $$\int dv=\int x\,dx$$ $$v=\frac{x^2}{2}$$ $$\int_1^{10} xlogx^2\,dx=\left[\frac{2x^2Inx}{2In10}\right]_1^{10}-\frac{2}{2In10}\int_1^{10} x\,dx$$ $$=\left\{\frac{10^2In10}{In10}-0-\frac{10^2}{2In10}+\frac{1}{2In10}\right\}$$ $$=2\left(50-\frac{99}{4In10}\right)\ As\ required$$ Alternatively $$\int_1^{10} xlogx^2\,dx=\frac{1}{In10}\int_1^{10} 2xInx\,dx$$	05

sign	differentiation	integration
+	Inx	$2x$
-	$\frac{1}{x}$	x^2

$$\frac{1}{In10}\int_1^{10} 2xInx\,dx = \frac{1}{In10}\left\{\left[\frac{x^2Inx}{1}\right]_1^{10} - \int_1^{10} x\,dx\right\}$$

$$= \left\{\frac{10^2In10}{In10} - 0 - \frac{10^2}{2In10} + \frac{1}{2In10}\right\}$$

$$= 2\left(50 - \frac{99}{4In10}\right)$$

7.

$$\frac{dy}{dx} - \frac{xy}{1+x} = \frac{xe^x}{1+x}$$

$$R = e^{\int\frac{-x}{1+x}dx} = e^{\int\left(-1+\frac{1}{1+x}\right)dx}$$

$$= \frac{1+x}{e^x}$$

$$\left(\frac{1+x}{e^x}\right)\frac{dy}{dx} - \left(\frac{xy}{1+x}\right).\left(\frac{1+x}{e^x}\right)$$

$$= \left(\frac{xe^x}{1+x}\right).\left(\frac{1+x}{e^x}\right)$$

$$\frac{d}{dx}\left(y\left(\frac{1+x}{e^x}\right)\right) = x$$

$$\int d\left(y\left(\frac{1+x}{e^x}\right)\right) = \int x\,dx$$

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	$$y\left(\frac{1+x}{e^x}\right)=\frac{x^2}{2}+c$$ $$1\left(\frac{1+0}{e^0}\right)=0+c, c=1$$ $$y\left(\frac{1+x}{e^x}\right)=\frac{x^2}{2}+1$$	05
8.	$$cos\theta=\frac{x}{9},\quad sin\theta=\frac{y}{16}$$ $$using\ sin^2\theta+cos^2\theta=1$$ $$\left(\frac{x}{9}\right)^2+\left(\frac{y}{16}\right)^2=1\ is\ an\ ellipse$$ $$a=16\ and\ b=9$$ $$using\ b^2=a^2(1-e^2)$$ $$\frac{81}{256}=1-e^2, e=\frac{\sqrt{175}}{16}$$ $$ae=16.\frac{\sqrt{175}}{16}=\sqrt{175}$$ $$\frac{a}{e}=\frac{16X16}{\sqrt{175}}=\frac{256}{\sqrt{175}}$$ $$s\left(0,\sqrt{175}\right),\qquad s^1\left(0,-\sqrt{175}\right)$$ $y=\frac{256}{\sqrt{175}}\ and\ y=\frac{256}{\sqrt{175}}$ are the equations of the directrices.	05

9 (a)	**SECTION B** $p^{3-x-x-5}q^{5x-3x} = 1$ $p^{-2-2x}q^{2x} = 1$ $q^{2x} = p^{2+2x}$ $2x\log_r q = (2+2x)\log_r p$ $x\log_r q - \log_r p - x\log_r p = 0$ $x\log_r\left(\frac{q}{p}\right) - \log_r p = 0\ As\ required$	
		05
b)	$2a - 14 = b \ldots\ldots\ldots\ldots..1$ $\sqrt{a} + \sqrt{b} = 5 \ldots\ldots\ldots\ldots\ldots 2$ $\left(\sqrt{a} + \sqrt{2a-14}\,\right)^2 = 5^2$ $a + 2\sqrt{2a^2 - 14a} + 2a - 14 = 25$ $3a - 14 + 2\sqrt{2a^2 - 14a} = 25$ $\left(\,2\sqrt{2a^2 - 14}\,\right)^2 = (39 - 3a)^2$ $4(2a^2 - 14a) = 1521 - 234a + 9a^2$ $a^2 - 178a + 1521 = 0$ $a = \frac{-(-178) \pm \sqrt{(-178)^2 - 4(1)(1521)}}{2(1)}$ $a = 9\ or\ a = 169$ $verifying$ $when\ a = 9$ $LHS = \sqrt{9} + \sqrt{4} = 5 = RHS$	

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$$LHS = \sqrt{169} + \sqrt{338 - 14} = 13 + 18 \neq 5$$

$$\therefore a = 9 \ and$$

$$b = 2(9) - 14 = 4$$

$$\therefore a = 9 \ and \ b = 4$$

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<tr><td>10
a)</td><td>

$$let \ u = y^6 + 2y^3, du = 6(y^5 + y^2)dy$$

Change of limits

<table>
<tr><td>y</td><td>u</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>3</td></tr>
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$$\int_0^1 \frac{y^2 + y^5}{(y^6 + 2y^3)^7} dy = \frac{1}{6} \int_0^3 \frac{1}{u^7} du$$

$$= -\frac{1}{36} \left| \frac{1}{u^6} \right|_0^3$$

$$= -\frac{1}{36} \left\{ \frac{1}{3^6} - \frac{1}{0} \right\} = -\frac{1}{26244}$$

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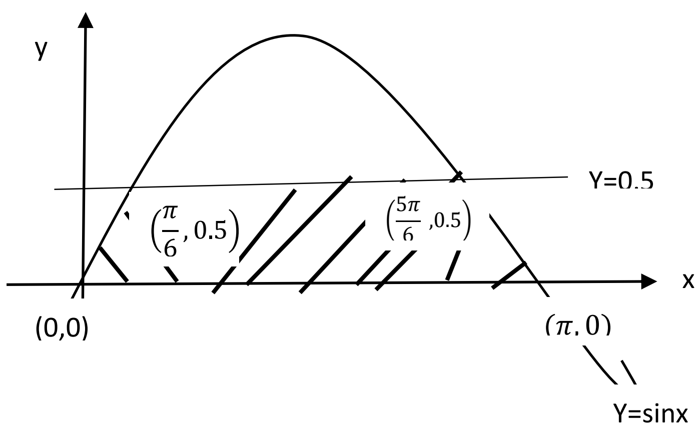
b)

X	0	$\frac{\pi}{2}$	π
Y	0	1	0



Point of intersection $sinx = 0.5$

$$x = \sin^{-1} 0.5$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \left(\frac{\pi}{6}, 0.5\right) and \left(\frac{5\pi}{6}, 0.5\right)$$

$$Area = \int_0^{\frac{\pi}{6}} sinx\, dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 0.5dx + \int_{\frac{5\pi}{6}}^{\pi} sinx\, dx$$

$$= [-cosx]_0^{\frac{\pi}{6}} + [0.5x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} + [-cosx]_{\frac{5\pi}{6}}^{\pi}$$

$$= \left\{-cos\frac{\pi}{6} + cos0 + \frac{5\pi}{12} - \frac{\pi}{12} - cos\pi + cos\frac{5\pi}{6}\right\}$$

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	$= 1.3152\ square\ units$	
11a	$$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & -\boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & 5 \\ 1 & -1 & -2 \end{vmatrix}$$ $$\boldsymbol{n} = 3\boldsymbol{i} + 9\boldsymbol{j} - 3\boldsymbol{k}$$ $$\begin{pmatrix} x \\ y \\ z \end{pmatrix}.\begin{pmatrix} 3 \\ 9 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \\ -3 \end{pmatrix}.\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ $$3x + 9y - 3z = 0$$ $$x + 3y - z = 0$$	04
b	$$cos\theta = \frac{\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}.\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}}{\sqrt{(3^2 + (-4)^2 + (2)^2).(1^2 + 3^2 + (-1)^2)}}$$ $$cos\theta = \frac{-11}{\sqrt{319}},$$ $$\theta = \cos^{-1}\left(\frac{11}{\sqrt{319}}\right) = 51.98^0$$	04
c)	$$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & -\boldsymbol{j} & \boldsymbol{k} \\ 3 & -4 & 2 \\ 1 & 3 & -1 \end{vmatrix}$$ $$\boldsymbol{n} = -2\boldsymbol{i} + 5\boldsymbol{j} + 13\boldsymbol{k}$$ $$let\ y = 0$$	

	$$\begin{array}{r\|l\|} + & 3x + 2z = 5 \ldots\ldots\ldots..1 \\ 2 & x - z = 0 \ldots\ldots\ldots\ldots\ldots..2 \end{array}$$ $$5x = 5, x = 1$$ $$from\ eqn(2)$$ $$z = 1$$ $$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 5 \\ 13 \end{pmatrix}\ is\ the\ line\ of\ intersection$$ $$Alternatively$$ $$\begin{array}{c\|c\|} \frac{1}{-3} & \begin{matrix} 3x - 4y + 2z = 5 \\ x + 3y - z = 0 \end{matrix} \end{array}$$ $$-13y + 5z = 5$$ $$z = \frac{5 + 13y}{5}$$ $$let\ y = t$$ $$z = \frac{5 + 13t}{5}$$ $$x + 3t - \left(\frac{5 + 13t}{5}\right) = 0$$ $$5x + 15t - 5 - 13t = 0$$ $$x = \frac{5 - 2t}{5}$$ $$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} \frac{-2}{5} \\ 1 \\ \frac{13}{5} \end{pmatrix} is\ the\ line\ of\ intersection$$	04
12a	$$tan4x + tan2x = 0$$ $$\frac{2tan2x}{1 - tan^2 2x} + tan2x = 0$$	

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	$2tan2x + tan2x - tan^3 2x = 0$ $3tan2x - tan^3 2x = 0$ $tan2x(3 - tan^2 2x) = 0$ $Either\ tan2x = 0$ $2x = \tan^{-1} 0$ $2x = 0^0, 180^0$ $x = 0^0, 90^0$ $Or\ tan2x = \pm\sqrt{3}$ $When\ tan2x = \sqrt{3}$ $2x = \tan^{-1}\sqrt{3}$ $2x = 60^0, 240^0$ $x = 30^0, 120^0$ $ignoring\ the\ negative\ sign\ A = \tan^{-1}\sqrt{3} = 60^0$ $2x = 120^0, 300^0$ $x = 60^0, 150^0$ $x = \{0^0, 90^0, 30^0, 120^0, 60^0, 150^0\}$	07
b	$RHS = \frac{sin\theta}{\sqrt{2}\,sin\theta cos\frac{\pi}{4} + \sqrt{2}cos\theta sin\frac{\pi}{4}}$ $= \frac{sin\theta}{\frac{2}{2}sin\theta + \frac{2}{2}cos\theta}$ $= \frac{sin\theta}{sin\theta + cos\theta}$	

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$$=\frac{\frac{sin\theta}{cos\theta}}{\frac{sin\theta}{cos\theta}+\frac{cos\theta}{cos\theta}}=\frac{tan\theta}{tan\theta+1}$$

$$=\frac{\frac{p}{q}}{\frac{p+q}{q}}=\frac{p}{q}.\frac{q}{(p+q)}$$

$$=\frac{p}{p+q}=LHS$$

Accept the use of a right angled triangle to obtain the expression for $cos\theta$ *and* $sin\theta$ instead of dividing the numerator and denominator to create $tan\theta$</td><td>05</td></tr>
<tr><td>13a</td><td>

$$y=\frac{cos\lambda x}{1+sin\lambda x}$$

$$\frac{dy}{dx}=\frac{(1+sin\lambda x)(-\lambda sin\lambda x)-cos\lambda x(\lambda cos\lambda x)}{(1+sin\lambda x)^2}$$

$$\frac{dy}{dx}=\frac{-\lambda sin\lambda x-\lambda sin^2\lambda x-\lambda cos^2\lambda x}{(1+sin\lambda x)^2}$$

$$\frac{dy}{dx}=\frac{-\lambda-\lambda sin\lambda x}{(1+sin\lambda x)^2}=\frac{-\lambda(1+sin\lambda x)}{(1+sin\lambda x)^2}$$

$$=\frac{-\lambda}{1+sin\lambda x}\ as\ required$$</td><td></td></tr>
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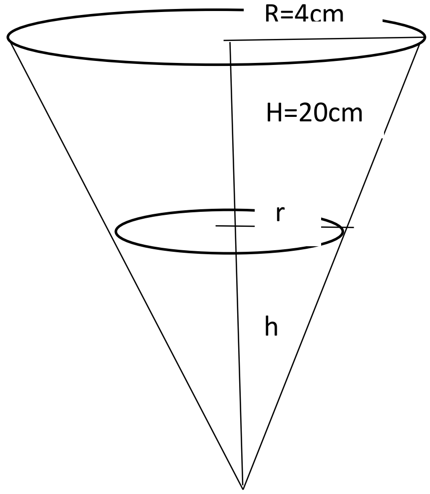
b	 Added volume, $\frac{dv}{dt} = 1.5cm^3/s$ Removed volume, $\frac{dv}{dt} = 2cm^3/s$ Resultant change in volume, $\frac{dv}{dt} = 0.5cm^3/s$ By similarity, $\frac{H}{R} = \frac{h}{r}$ $$\frac{20}{4} = \frac{h}{r}$$ $$r = \frac{h}{5}$$ $$v = \frac{1}{3}.base\ area.height$$ $$v = \frac{\pi}{3}\left(\frac{h}{5}\right)^2$$ $$v = \frac{\pi}{75}h^3, \frac{dv}{dh} = \frac{\pi}{25}h^2$$	

	$$\frac{dv}{dt}=\frac{dv}{dh}.\frac{dh}{dt}$$ $$0.5=\frac{\pi}{25}h^2.\frac{dh}{dt}$$ $$\frac{dh}{dt}=\frac{12.5}{\pi(12)^2}=\frac{25}{288\pi}=0.0276$$ $\therefore$ *the depth is changing at a constant rate of* $\frac{25}{288\pi}cms^{-1}$ *when the depth is* $12cm$	07
14a	$$y^2-2y=8x+17$$ $$y^2-2y+(-1)^2=8x+17+(-1)^2$$ $$(y-1)^2=8x+18$$ $$(y-1)^2=8(x+2.25)$$ Comparing with $Y^2=4aX$ $$4a=8,a=2$$ $$vertex(-2.25,1)focus(-0.25,1)$$ The axis of the parabola is $y=1$	05
b	$$2y\frac{dy}{dx}=4a$$ $$\frac{dy}{dx}=\frac{2a}{y}$$ $$At\ p(ap^2,2ap)$$ $$\frac{dy}{dx}=\frac{2a}{2ap}=\frac{1}{p}$$ The equation becomes $\frac{y-2ap}{x-ap^2}=\frac{1}{p}$ $$yp-ap^2=x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..eqn1$$	

	$also\ at\ Q(aq^2, 2aq)$ $\frac{dy}{dx} = \frac{2a}{2aq} = \frac{1}{q}$ The equation becomes $\frac{y-2aq}{x-aq^2} = \frac{1}{q}$ $yq - aq^2 = x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn2$ $eqn1 = eqn2$ $yp - ap^2 = yq - aq^2$ $y(p-q) = a(p^2 - q^2)$ $y = a(p+q)$ From eqn1 $x = ap(p+q) - ap^2 = apq$ $\therefore The\ coordinates\ of\ R\ (apq, a(p+q))$	

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$$x^2y + 2xy - (3y + 8) = 0$$

For real values $B^2 - 4AC \geq 0$

$$(2y)^2 + 4(y)(3y + 8) \geq 0$$

$$16y^2 + 32y \geq 0$$

$$16y(y + 2) \geq 0$$

For critical values, $16y = 0, y = 0$

$$y + 2 = 0, y = -2$$

Investigation table

<table>
<tr><td></td><td>$y < -2$</td><td>$-2 < y < 0$</td><td>$y > 0$</td></tr>
<tr><td>16y</td><td>-</td><td>-</td><td>+</td></tr>
<tr><td>y+2</td><td>-</td><td>+</td><td>+</td></tr>
<tr><td>Net sign</td><td>+</td><td>-</td><td>+</td></tr>
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The curve has real values in the ranges $y < 2$ and $y > 0$

Hence

$$when\ y = -2$$

$$-2x^2 + 2(-2)x - 3(-2) - 8 = 0$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1\ (-1, -2)\text{max}$$

$x^2(0) + 2(0)x - 3(0) - 8 = 0$ N/A

The turning point is $(-1, -2)max$

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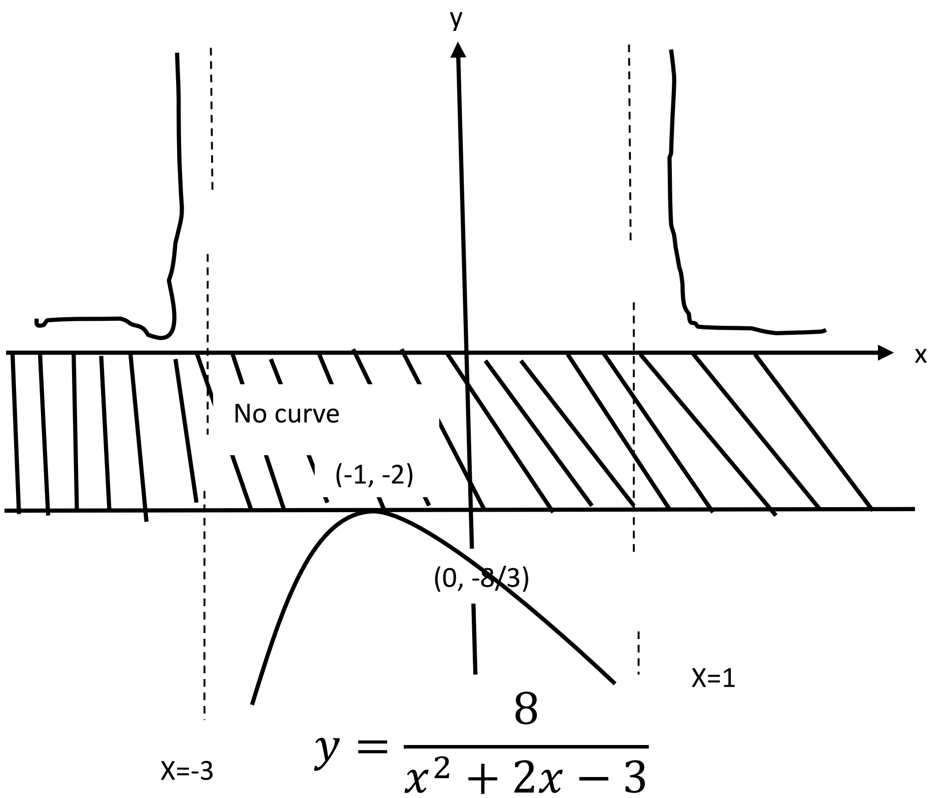
ii	$$y(x^2+2x-3)=8$$ $$y=\frac{8}{x^2+2x-3}$$ Vertical asymptotes $$as\ y\to\pm\infty$$ $$x^2+2x-3\to 0$$ $$(x+3)(x-1)=0$$ $x=-3\ and\ x$ $=1\ are\ the\ equations\ of\ vertical\ asymptotes$ Horizontal asymptotes $$y=\frac{\frac{8}{x^2}}{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}$$ $$as\ x\to\pm\infty,$$ $$y\to 0$$ $y=0\ is\ the\ equation\ of\ horizontal\ asymptote$ Hence Intercepts When $x=0, y=\frac{-8}{3}$ $\left(0,\frac{-8}{3}\right)$ When $y=0, x=$N/A The curve has no x-intercept Critical values $$x=-3\ and\ x=1$$	

Investigation table

	$x < -3$	$-3 < x < 1$	$x > 1$
8	+	+	+
$x + 3$	-	+	+
$x - 1$	-	_	+
Net sign	+	_	+

The sketch of the curve $x^2y + 2xy = 3y + 8$



$$y = \frac{8}{x^2 + 2x - 3}$$

07

16a

$$-\frac{d\theta}{dt} \propto (\theta - 25)$$

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	$$\frac{d\theta}{dt} = -k(\theta - 25)$$ $$\int \frac{d\theta}{\theta - 25} = \int -kdt$$ $$In(\theta - 25) = -kt + A$$ When $t = 0, \theta = 95^0 c$ $$In(95 - 25) = -k(0) + A,$$ $$A = In70$$ $$In(\theta - 25) = -kt + In70$$ $$In\left(\frac{\theta - 25}{70}\right) = -kt$$ $$t = 25mins, \theta = 60^0 c$$ $$In\left(\frac{60 - 25}{70}\right) = -25k,$$ $$k = \frac{1}{25} In2$$ $$In\left(\frac{\theta - 25}{70}\right) = -t\frac{1}{25} In2$$ The differential equation is $\frac{d\theta}{dt} = -(\theta - 25)\frac{1}{25} In2$	
b	When $\theta = 32^0 c$ $$In\left(\frac{32 - 25}{70}\right) = -t\frac{1}{25} In2$$ $$25In0.1 = -tIn2$$ $$t = \frac{25In0.1}{In2} = 83.0482minutes$$ $further\ time = 83.0482 - 25 = 58.0482minutes$ **END**	05

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