P425/2
APPLIED MATHEMATICS
Paper 2
Sept. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) SET 5

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take $\mathrm{g}\ to\ be\ 9.8\ ms^{-2}$.

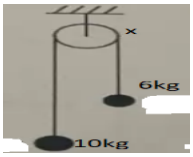
SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Two events A and B are such that $P(A) = 0.7,\ P(A'nB') = P(A'uB') = 0.2$. Find

   (i) $P(A'nB)$ (ii) $P\left(A'/_B\right)$

2. Two particles of masses 10kg and 6kg are connected by a light inelastic string passing over a smooth fixed pulley, x as show below.



   Find;

   a. Acceleration of the particles b. The tension in the string

3. Seven students were given two separate aptitude tests in their order of merit.

First test	G	F	A	D	B	C	E
Second test	D	F	E	B	G	C	A

   Calculate the rank correlation co-efficient and comment at 5% level of significance.

4. The percentage error in measuring the area of a circle is 5. Determine the corresponding percentage error in the;

   (i) Radius. (ii) circumference.

5. Derive the simple iterative formula based on Newton Raphson formula for finding the square root of 5 and show that its given by $x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right)$, n=0, 1, 2…

6. A bullet of mass 20g is fired with a velocity of $400ms^{-1}$ into a wooden block of mass 2kg lying on a smooth surface. If the bullet is embedded into the block, find the

   (i) Common speed with which they move.
   (ii) Loss in kinetic energy after collision.

7. A train travels along two stations A and B. The train starts from rest at A and accelerates at $3ms^{-2}$ up to a speed of $40ms^{-1}$. It then travels for a distance of 1.6 km at a steady speed and it then decelerates at $2ms^{-2}$ to come to rest at B. Determine the;

(i) Distance from A to B (ii) Total time taken

8. The table below shows the prices(shs) and amounts of item bought in 2004 and 2005

Item	Price (shs)		amount
	2004	2005	
A	635	887.5	6
B	720	815	4
C	730	1045	3
D	362	503	7

Taking 2004 as the base year;

(a) Calculate the average weighted price index correct to 2 decimal places

(b) Calculate the price of an item in 2005 costing 50,000 in 2004 using the weighted price index above

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. A continuous random variable T has probability density function given by

$$f(t) = \begin{cases} a + bt; & 0 \leq t \leq 1 \\ 0; & otherwise \end{cases}$$

Where a and b are constant, Given that $F\left(\frac{1}{2}\right) = \frac{3}{5}$, determine;

(i) Values of a and b.

(ii) $P\left(0 < t < \frac{1}{4}\right)$ $(iii) E(T)$

10. The germination time for a certain species of seeds is known to be normally distributed. If for a given batch of the seeds, 20% take more than 6 days to germinate and 10% take less than 4 days to germinate, determine the

(i) Mean and standard deviation of the germination time

(ii) Probability that the seeds germinate in less than 10 days

11. (a) Use the trapezium rule with 6 sub-intervals to estimate $\int_1^3 x\, lnx\, dx$, correct to 3 decimal places

(b) Find the exact value of the above expression and correct it to 3 decimal places.

(c) Hence determine the percentage error and state how the error may be reduced

12. At 12:00 noon the position vector $(\tilde{r})$ and velocity vector $(\tilde{v})$ for two objects P and Q are as follows.

Objects	Position vectors (r)	Velocity vector(v)
P	$r_P = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} km$	$v_P = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} kmhr^{-1}$
Q	$r_Q = \begin{pmatrix} 7 \\ 5 \\ -2 \end{pmatrix} km$	$v_Q = \begin{pmatrix} -3 \\ -15 \\ 18 \end{pmatrix} kmhr^{-1}$

(i) Find the position vector of P relative to Q at any time t.

(ii) Show that if the velocities remain constant, a collision between P and Q will occur and find the time of collision.

(iii) Find the position of collision. (12 marks)

13. In an agricultural experiment, the gain in mass (in kg) of 100 pigs during a certain period were recorded as follows.

Gain in mass (kg)	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34
Frequency	2	29	37	16	14	2

(a) Calculate the

(i) mean mass (ii) standard deviation (iii) median

(b) find the probability that a pig selected at random gained mass greater 14 kg

14. (a) Two numbers A and B are approximate to a and b with errors e_1 and e_2 respectively. Show that the maximum relative error made in approximation of $\frac{A^2}{B}$ by $\frac{a^2}{b}$ is given by $2\left|\frac{e_1}{a}\right|+\left|\frac{e_2}{b}\right|$

(b) Given that $a = 2.13$ and $b = 2.0125$

(i) state the maximum possible errors in a and b

(ii) find the limits within which the exact value of $\frac{a^2}{b}$ lies correct to 3 decimal places

15. Four forces of magnitude 4 N, 13N, 20N and 3N act along the sides AB, BC, DC and DA respectively of a square of side a metres. Find the;

(i) Magnitude of the resultant force and the direction of resultant force

(ii) Distance in terms of a at which the resultant cuts AB form point A.

16. (a) A particle of mass 0.4 kg has position vector after t seconds given by $r = 3t^2i + 2j$. Find

(i) its velocity after t seconds (ii) acceleration after t seconds

(iii) force required to produce the above acceleration

(b) Four forces $(a\hat{\imath} + (a-1)\hat{\jmath})$, $(3\hat{\imath} + 2a\hat{\jmath})N$, $(5\hat{\imath} - 6\hat{\jmath})N$ and $(-\hat{\imath} - 2\hat{\jmath})N$ act on a particle. The resultant of the forces make an angle of 45° with horizontal. Find the value of a and hence determine the magnitude of the resultant.