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Binomial distribution

A binomial distribution is a special type of a discrete random variable in which an experiment gives rise to only two outcomes either success or failure.

Conditions for binomial distribution

(i) The experiment has a finite (repeated) number of trials, n
(ii) The trials are independent
(iii) The outcome of each trial is either a success or a failure
(iv) The probability, p of successful outcome is constant for all trials

If a discrete random variable X is the number of **successful** outcomes in n trials and satisfies the above conditions, then X follows a binomial distribution written as X~B(n, p) or X~Bin (n, p)

Formula for Binomial distribution

If X~B(n, p), the probability of obtaining, r success in n trials P(X = r) where

$P(X = r) = {}^nC_r p^r q^{n-r}$ for r = 0, 1, 2, 3, …,n where q = 1-p

$$= \frac{n!}{(n-r)!r!} p^r q^{n-r}$$

Example 1

The random variable X is distributed B(7, 0.2). find

(i) P(X=3) (ii) P(1<X ≤ 4) (iii) P(X>1)

Solution

(i) n = 7, p= 0.2, q= 1- 0.2 = 0.8

$P(X=3) = {}^7C_3 \times 0.2^3 \times 0.8^4$

$= \frac{7!}{(7-3)!3!} x 0.2^3\ x\ 0.8^4$

$= \frac{7\ x\ 6\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{(4\ x\ 3\ x\ 2\ x\ 1)(3\ x\ 2\ x\ 1)} x 0.2^3\ x\ 0.8^4 = 0.115$

(ii) P(1<X ≤ 4) = P(x=2) + P(X= 3) + P(X=4)

$= {}^7C_2 \times 0.2^2 \times 0.8^5 + {}^7C_3 \times 0.2^3 \times 0.8^4 + {}^7C_4 \times 0.2^4 \times 0.8^3$

= 0.275 + 0.115 + 0.029 = 0.419

(iii) P(X>1) = 1- (P(X≤ 1) = $[1 - P(X = 0) + P(X = 1)]$

$= 1 - [{}^7C_0 \times 0.2^0 \times 0.8^7 + {}^7C_1 \times 0.2^1 \times 0.8^6]$

=[1 – (0.210 + 0.367)

= 0.423

Example 2

At freedom city super market, 60% of the customers shop on Saturday. Find the probability that in a randomly selected sample of 10 customers

(i) Exactly 2 shop on Saturday

n= 10, p = 0.6 q = 1- 0.6 = 0.4

$P(X = 2) = {}^{10}C_2 \times 0.6^2 \times 0.4^8 = 0.011$

(ii) More than 7 shop on Saturday

$P(X > 7) = P(X = 8) + P(X = 9) + P(X = 10)$

$= {}^{10}C_8 \times 0.6^8 \times 0.4^2 + {}^{10}C_9 \times 0.6^9 \times 0.4^1 + {}^{10}C_{10} \times 0.6^{10} \times 0.4^2$

$= 0.121 + 0.040 + 0.006 = 0.167$

Example 3

The probability that a marble drawn from a box is red is 0.4. if a sample of 6marbles is taken, find the probability that it will contain;

(i) No red marble

n = 6, p= 0.4, q= 1 – 0.4 = 0.6

$P(X = 0) = {}^{6}C_0 \times 0.4^0 \times 0.6^6 = 0.047$

(ii) $P(X = 5 \text{ or } 6) = P(X= 5) + P(X = 6) = {}^{6}C_5 \times 0.4^5 \times 0.6^1 + {}^{6}C_6 \times 0.4^6 \times 0.6^0$

$= 0.037 + 0.004 = 0.041$

(iii) Less than half red marbles

$P(X<3) = P(X = 0) + P(X = 1) + P(X = 2)$

$={}^{6}C_0 \times 0.4^0 \times 0.6^6 + {}^{6}C_1 \times 0.4^1 \times 0.6^5 + {}^{6}C_2 \times 0.4^2 \times 0.6^4$

$= 0.047 + 0.187 + 0.331 = 0.545$

Example 4

A biased con is such that the chance of a head appearing upper most when tossed is twice of the tail appearing uppermost. If the coin is tossed 10 times. Find the probability that

(i) Exactly 6 heads will appear

$P(H) + P(T) = 1$

$2x + x = 1$

$x = \frac{1}{3}$

$n = 10,\ p = \frac{2}{3};\ q= \frac{1}{3}$

$P(X = 6) = {}^{10}C_6 \times \left(\frac{2}{3}\right)^6 x \left(\frac{2}{3}\right)^4 = 0.228$

(ii) $P(5<X<8) = P(X = 6) + P(X = 7) = {}^{10}C_6 \times \left(\frac{2}{3}\right)^6 x \left(\frac{2}{3}\right)^4 + {}^{10}C_7 \times \left(\frac{2}{3}\right)^7 x \left(\frac{2}{3}\right)^3$

$= 0.228 + 0.260 = 0.488$

Example 5

A box contains a large number of pens. The probability that a pen is faulty is 0.1. How many pens would you need to select to be more than 95% certain of picking one faulty pen?

Solution

n? p= 0.1, q = 0.9

$P(X \geq 1) = 1- P(X = 0)>0.95$

$= 1 - {}^nC_0 \times 0.1^0 \times 0.9^n > 0.95$

$= 0.05 > 0.9^n$

$n > \frac{\log_{10} 0.05}{\log_{10} 0.9}$

n >29

The least value n = 29

Using Cumulative binomial probability table

The table give values of $P(X \geq x)$ for values of n and p.

(i) $P(X \leq X) = 1- (P(X \geq (X+1)$
(ii) $P(X = x) + P(X \geq x) – (PX \geq x+1)$

Example 6

The random variable is distributed B(5, 0.3). Find

(i) $P(X \geq 3)$ (ii) $P(X > 1)$ (iii) $P(X \leq 4)$ (iv) $P (X < 3)$ (v) $P(X = 2)$

Solution

n = 5, p = 0.3

(i) $P(X \geq 3) = 0.1631$
(ii) $P(X > 1) = P(X \geq 2) = 0.4718$
(iii) $P(X \leq 4) = 1 – (P(X \geq 5) = 1 – 0.0024 = 0.9976$
(iv) $P(X < 3) = P(X \geq 2) = 1 – P(X \geq 3) = 1 – 0.1631) = 0.8369$
(v) $P(X = 2) = P(X \geq 2) – P(X \geq 3)\ 0.4718 – 0.1631 = 0.3087$

Example 7

An unbiased coin is tossed 15 times. Find th probability that

(i) Exactly eight heads will appear upper most
(ii) Between 6 and 10 heads will appear
(iii) Between 6 and 10 heads inclusive will appear

Solution

n = 15, p = 0.5

(i) $P(X = 8) = P(X \geq 8) – P(X \geq 9) = 0.5000 – 0.3036 = 0.1964$
(ii) $P(6< X < 10) = P(X \leq X \leq 9) = P(X \geq 7) – P(X \geq 10) = 0.6964 – 0.1509 = 0.5455$
(iii) $P(X \leq X \leq 10) = P(X \geq 6) – P(X \geq 11) = 0.8491 – 0.0592 = 0.7899$

Example 8

A student attempts 20 objective questions by guest work. Each questions has got four possible alternatives out of which one is correct. Find the probability that he gets

(i) Exactly 9 correct answers

(ii) At least 12 correct answers
(iii) At most 6 correct answers
(iv) Between 6 and 14 correct answers inclusive
(v) Exactly 7 correct answers.

Solution

For correct answers, $n = 20$, $p = 0.25$, $q = 0.75$

(i) $P(X = 9) = P(X \geq 9) - P(X \geq 10) = 0.0409 - 0.0139 = 0.027$
(ii) $P(X \geq 12) = 0.0009$
(iii) $P(X \leq 6) = 1 - P(X \geq 7) = 1 - 0.2142 = 0.7858$
(iv) $P(X \leq X \leq 14) = P(X \geq 6) - P(X \geq 15) = 0.3828 - 0.0000 = 0.3828$

For incorrect answers, $n = 20$, $P = 0.75$, $q = 0.25$

Revision exercise 1

1. A biased coin is such that a head is three times as likely to occur as a tail. The coin is tossed 5 times. Find the probability that at most two tails occur = 0.8965
2. Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, calculate the probability he passes
(i) Only two examinations = 0.2963
(ii) More than half of the examinations = 0.5926
3. A fair die is rolled 6 times, calculate the probability that
   (i) A 2 or 4 appears on the first throw $= \frac{1}{3}$
   (ii) Four 5's will appear in the six throws= 0.0080
4. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all= 0.0003
   (ii) Exactly 5 games= 0.3277
5. The probability that Alex wins a chess game is $\frac{2}{3}$. He plays 8 games, what is the probability that he wins
   (i) At least 7 games= 0.1951
   (ii) Exactly 5 games= 0.2731
6. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all= 0.0003
   (ii) He record at least 2 success= 0.9933
   (iii) If he is successful in 5 practice runs, he makes two additional runs. The probability of success in either of the additional runs is 0.7. Determine the probability that Bolt will make 7 successful runs = 0.1606
7. In a test there are 10 objective questions each with a choice of five possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets at least two answers correct. = 0.6242
8. 30% of the students in the school are day scholars. From a sample of 10 students chosen at random, find the probability that

(i) Only 3 are day scholars= 0.267
(ii) Less than half are day scholars=0.850

9. The probability that a shopper buys a cake is 0.25. Find the probability that in a random sample of 9 shoppers
   (i) Exactly 3 buys a cake=0.2334
   (ii) More than 7 buy a cake= 0.0001
10. A bag contains counter books of which 40% are blue and the rest are black. A counter book is taken from the bag, its colour is noted then replaced. This is performed 8 times in all. Calculate the probability that
    (i) Exactly 3 will be blue= 0.279
    (ii) At least one will be blue= 0.983
    (iii) More than half will be black= 0.594

11. At a certain school, the records taken from admission's office the ratio of male to female s. 5 applicants is 4:6. Basing on this experience, what is the probability that will be more female applicants in a random collection of a dozen applicants? = 0.665
12. The random variable X is B(6, 0.42). find
    (i) $P(X = 6) = 0.00549$ (ii) $P(X = 4) = 0.157$ (iii) $P(X \leq 2) = 0.503$
13. An unbiased die is thrown seven times. find the probability of throwing at least 5 sixes.= 0.002
14. In a family a couple is likely to produce a girl or a boy. Find the probability that in a sample of 5 children there will be more boys than girls = 0.5
15. The probability that it will rain on any given day during examination period is 0.3. calculate the probability that in a given week during examination period, it will rain on;
    (i) Exactly 2 days = 0.318
    (ii) At most two days = 0.647
    (iii) Exactly three days that are consecutive = 0.0324
16. A fair coin is tossed 6 times. find the probability of throwing at least four heads = 0.344
17. The random variable x is B(4, p) and $P(X = 4) = 0.0256$. find $P(X = 2) = 0.3456$
18. In agriculture lab, Silvia plants bean seeds and the probability that they germinate successfully is $\frac{1}{3}$.
    (a) She takes 9 seeds. Find the probability that
        (i) More than five seeds germinate = 0.0424
        (ii) At least three seeds germinate = = 0.623
    (b) Find the number of seeds that she needs to take in order to 99% certain at least one germinate = 12
19. In a shooting competition, the probability of hitting the target with a single shot is 0.6, if 7 shots are taken; find the probability that the target is hit more than twice. = 0.9037
20. In mass production of shirts, it is found that 5% are defective. Shirts are selected at random of put into packets of 10.
    (a) A packet is selected at random. Find the probability that it contains
        (i) Three defective shirt = 0.0105
        (ii) Less than three defective shirts = 0.9885
    (b) Two packets are selected at random. Find the probability that there are no defective shirts in either packet = $({}^{10}C_0(0.05)^0(0.95)^{10} \times ({}^{10}C_0(0.05)^0(0.95)^{10} = 0.358$

21. A biased coin is such that it is twice as likely to show a head as a tail. If the coin is tossed 5 times. find the probability that
    (i) Exactly three heads are obtained = 0.329
    (ii) More than three heads are obtained = = 0.3333
22. The probability that a target is hit 0.3. Find the probability the least number of times shots should be fired if the probability that the target is hit is at least once is greater than 0.95. =9
23. 1% of the light bulbs in a box are faulty. Find the largest sample size which can be taken if it required that the probability that there is no faulty bulb in the sample is greater than 0.5
    (0.99)n > 0.5
    n = 68
24. In a test there are 10 multiple choice questions. Each question has got four possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets
    (i) More than 7 correct answers = 0.0004
    (ii) More than half correct answers = 0.0197
25. X$\sim B(n, o.3)$. find the value of n such that P(X ≥ 1) = 0.8 :
26. The random variable X is B(n, 0.6). find the value of n such that P(X<1) = 0.0256

Solutions to revision exercise 1

1. A biased coin is such that a head is three times as likely to occur as a tail. The coin is tossed 5 times. Find the probability that at most two tails occur
   Solution
   n = 5
   p + q = 1
   x + 3x = 1, x = 0.25
   p = 0.25, q = 0.73
   P(X ≤ 2) = P(X = 0) + P(X =1) + P(X = 2)
   = $^5C_0$ x 0.25^0 x 0.75^5 +$^5C_1$ x 0.25^1 x 0.75^4+ $^5C_2$ x 0.25^2 x 0.75^3
   =0.2373 + 0.3955 + 0.2637 = 0.8965

2. Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, calculate the probability he passes

(i) Only two examinations
   n = 4, p = $\frac{2}{3}$, q =$\frac{1}{3}$
   P(X = 2) = $^4C_2$ x $\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$ = 0.2963

(ii) More than half of the examinations
   P(X < 2) = P(X= 3) + P(X =4) = $^4C_3$ x $\left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^1$ + $^4C_4$ x $\left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^0$ = 0.5926

3. A fair die is rolled 6 times, calculate the probability that
   (i) A 2 or 4 appears on the first throw
   P(X = 2 or 4) = P(X =2) + (X = 4) = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
   (ii) Four 5's will appear in the six throws
   n = 6, P= $\frac{1}{6}$, q = $\frac{5}{6}$
   P(X = 4) = $^6C_4$ x $\left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^2$ = 0.0080

4. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all
   n = 5, p = 0.8, q = 0.2
   P(X = 0) = ${}^5C_0$ x $(0.8)^0(0.2)^5$ = 0.0003
   (ii) Exactly 5 games
   P(X = 5) = ${}^5C_5$ x $(0.8)^5(0.2)^0$ = 0.3277
5. The probability that Alex wins a chess game is $\frac{2}{3}$. He plays 8 games, what is the probability that he wins
   (i) At least 7 games
   n = 8, p = $\frac{2}{3}$, q = $\frac{1}{3}$
   P(X ≤ 7) = P(X = 7) + P(X= 8) = ${}^8C_7$ x $\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^1$ + ${}^8C_8$ x $\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^0$
   =0.1561+ 0.390 = 0.1951
   (ii) Exactly 5 games
   P(X =5) = ${}^8C_5$ x $\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3$ = 0.2731
6. Usain Bolt makes 5 practice ran in the 100m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful. Find the probability that;
   (i) He records at least no success at all
   n = 5, p = 0.8, q = 0.2
   P(X = 0) = ${}^5C_0$ x $(0.8)^0(0.2)^5$ = 0.0003
   (ii) He record at least 2 success
   P(X ≥ 2) = 1 –P(X <2) = 1- [P(X=0) + P(X = 1)
   = 1 – [${}^5C_0$ x $(0.8)^0(0.2)^5$+ ${}^5C_1$ x $(0.8)^1(0.2)^4$) = 1- (0.0003 + 0.0064) = 0.9933
   (iii) If he is successful in 5 practice runs, he makes two additional runs. The probability of success in either of the additional runs is 0.7. Determine the probability that Bolt will make 7 successful runs
   Probability = ${}^5C_5$x 0.8^5 x 0.2^0 and ${}^2C^2$ x $0.7^2(0.3)^0$
   = 0.32768 x 0.49= 0.1606
7. In a test there are 10 objective questions each with a choice of five possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets at least two answers correct.
   Solution
   n = 10, p = 0.2, q = 0.8
   P(X ≥ 2) = 1- P(X<2)
   = 1 – [P(X = 0) + P(X = 1)
   = 1 – [${}^{10}C_0$ x 0.2^0 x 0.8^{10} x ${}^{10}C_1$ x 0.2^1 x 0.8^9)
   = 1- (0.1074 + 0.2684) = (1 – 0. 3758) = 0.6242
8. 30% of the students in the school are day scholars. From a sample of 10 students chosen at random, find the probability that
   (i) Only 3 are day scholars
   n = 10, p = 0.3, q = 0.7
   P(X = 3) = ${}^{10}C_3(0.3)^3(0.7)^7$ = 0.267
   (ii) Less than half are day scholars

$P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= {}^{10}C_0(0.3)^0(0.7)^{10} + {}^{10}C_1(0.3)^1(0.7)^9 + {}^{10}C_2(0.3)^2(0.7)^8 + {}^{10}C_3(0.3)^3(0.7)^7$

$+ {}^{10}C_4(0.3)^4(0.7)^6$

$= 0.850$

9. The probability that a shopper buys a cake is 0.25. Find the probability that in a random sample of 9 shoppers
   (i) Exactly 3 buys a cake

   $n = 9, p = 0.25, q = 0.75$

   $P(X = 3) = {}^9C_3(0.25)^3(0.75)^6 = 0.2334$

   (ii) More than 7 buy a cake

   $P(X > 7) = P(X = 8) + P(X = 9)$

   $= {}^9C_8(0.25)^8(0.75)^1 + {}^9C_9(0.25)^9(0.75)^0 = 0.0001$

10. A bag contains counter books of which 40% are blue and the rest are black. A counter book is taken from the bag, its colour is noted then replaced. This is performed 8 times in all. Calculate the probability that
    (i) Exactly 3 will be blue

    $n = 8, p = 0.4, q = 0.6$

    $P(X = 3) = {}^8C_3(0.4)^3(0.6)^5 = 0.279$

    (ii) At least one will be blue

    $P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - {}^8C_0(0.4)^0 (0.6)^8 = 0.983$

    (iii) More than half will be black

    $P(X \leq 3) = P(X = 0) + P(X= 1) + P(X = 2) + P(X = 3)$

    $= {}^8C_0(0.4)^0 (0.6)^8 + {}^8C_1(0.4)^1 (0.6)^7 + {}^8C_2(0.4)^2 (0.6)^6 + {}^8C_3(0.4)^3 (0.6)^5$

    $= 0.594$

11. At a certain school, the records taken from admission's office the ratio of male to female s. 5 applicants is 4:6. Basing on this experience, what is the probability that will be more female applicants in a random collection of a dozen applicants?

    Solution

    $n = 12, p = 0.4, q = 0.6$

    $P(X \leq 5) = P(X = 0) + P(X =1) + P(X =2) + P(X= 3) + P(X = 4) + P(X = 5)$

    $= {}^{12}C_0(0.4)^0 (0.6)^{12} + {}^{12}C_1(0.4)^1 (0.6)^{11} + {}^{12}C_2(0.4)^2 (0.6)^{10} + {}^{12}C_3(0.4)^3 (0.6)^9 +$

    ${}^{12}C_4(0.4)^4 (0.6)^8 + {}^{12}C_5(0.4)^5 (0.6)^7$

    $= 0.6652$

12. The random variable X is B(6, 0.42). find
    (i) $P(X = 6)$

    $n = 6, p = 0.42, q= 1- 0.42= 0.58$

    $P(X = 6) = {}^6C_6(0.42)^6(0.58)^0 = 0.00549$

    (ii) $P(X = 4)$

    ${}^6C_4(0.42)^4(0.58)^2= 0.157$

    (iii) $P(X \leq 2)$

    $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X= 2)$

    $= {}^6C_0(0.42)^0(0.58)^6 + {}^6C_1(0.42)^1(0.58)^6 + {}^6C_2(0.42)^2(0.58)^4 = 0.503$

13. An unbiased die is thrown seven times. find the probability of throwing at least 5 sixesn = 7, $p = \frac{1}{6}, q = \frac{5}{6}$

    $P(X \leq 5) = P(X = 5) + P(X = 6) + P(X = 7)$

14. $= {}^7C_5(\frac{1}{6})^5(\frac{5}{6})^2 + {}^7C_6(\frac{1}{6})^6(0.58)^1 + {}^7C_7(\frac{1}{6})^7(\frac{5}{6})^0. = 0.002$

15. In a family a couple is likely to produce a girl or a boy. Find the probability that in a sample of 5 children there will be more boys than girls

    $n = 5, p = 0.5, q = 0.5$

    $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X= 5)$

    $= {}^5C_3(0.5)^3(0.5)^2 + {}^5C_4(0.5)^4(0.5)^1 + {}^5C_5(0.5)^5(0.5)^0 = 0.5$

16. The probability that it will rain on any given day during examination period is 0.3. calculate the probability that in a given week during examination period, it will rain on;

    (i) Exactly 2 days

    $n = 7, p = 0.3, q = 0.7$

    $P(X = 2) = {}^7C_2(0.3)^2(0.7)^5 = 0.318$

    (ii) At most two days = 0.671

    $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

    $= {}^7C_0(0.3)^0(0.7)^7 + {}^7C_1(0.3)^1(0.7)^6 + {}^7C_2(0.3)^2(0.7)^5 = 0.647$

17. A fair coin is tossed 6 times. find the probability of throwing at least four heads

    $n = 6, p = 0.5, q = 0.5$

    $P(X \leq 4) = P(X = 4) + P(X =5) + P(X =6)$

    $= {}^6C_4(0.5)^4(0.5)^2 + {}^6C_5(0.5)^5(0.5)^1 + {}^6C_6(0.5)^6(0.5)^0 = 0.344$

18. The random variable x is B(4, p) and P(X = 4) = 0.0256. find P(X = 2)

    $n = 4, p = p, q = (1-p)$

    $P(X = 4) = {}^4C_4(p)^4(q)^0 = 0.0256$

    $(p)^4 = 0.0256; p = 0.4$

    $P(X = 2) = {}^4C_2(0.4)^2(0.6)^2 = 0.3456$

19. In agriculture lab, Silvia plants bean seeds and the probability that they germinate successfully is $\frac{1}{3}$.

    (a) She takes 9 seeds. Find the probability that

    (i) More than five seeds germinate

    $P(X<5) = P(X =6) + P(X = 7) + P(X = 8) + P(X = 9)$

    $= {}^9C_6(\frac{1}{3})^6(\frac{2}{3})^3 + {}^9C_7(\frac{1}{3})^7(\frac{2}{3})^2 + {}^9C_8(\frac{1}{3})^8(\frac{2}{3})^1 + {}^9C_9(\frac{1}{3})^9(\frac{2}{3})^0 = 0.0424$

    (ii) At least three seeds germinate

    $P(X \geq) = 1 - [P(X=0) + P(X = 1) + P(X = 2)]$

    $= 1 - [{}^9C_0(\frac{1}{3})^0(\frac{2}{3})^9 + {}^9C_1(\frac{1}{3})^1(\frac{2}{3})^8 + {}^9C_2(\frac{1}{3})^2(\frac{2}{3})^7] = 0.623$

    (b) Find the number of seeds that she needs to take in order to 99% certain at least one germinate

    $P(X \leq 1) = 1 - P(X = 0) = 1 - {}^nC_0(\frac{1}{3})^0(\frac{2}{3})^n = 0.99$

    $(\frac{2}{3})^n = 0.01$

    $n = 11.36$

    therefore the number = 12

20. In a shooting competition, the probability of hitting the target with a single shot is 0.6, if 7 shots are taken; find the probability that the target is hit more than twice.

    $n = 7, p = 0.6, q = 0.4$

    $P(X > 2) = 1 - P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X =2)]$

    $= 1 - [{}^7C_0(0.6)^0(0.4)^7 + {}^7C_1(0.6)^1(0.4)^6 + {}^7C_2(0.6)^2(0.4)^5]$

    $= 0.9037$

21. In mass production of shirts, it is found that 5% are defective. Shirts are selected at random of put into packets of 10.
    (a) A packet is selected at random. Find the probability that it contains
        (i) Three defective shirt
        $P(X= 3) = {}^{10}C_3(0.05)^3(0.95)^7= 0.0105$
        (ii) Less than three defective shirts
        $P(X < 3) = P(X = 0) + P(X = 1) + P(X=2)$
        $= {}^{10}C_0(0.05)^0(0.95)^{10} \text{ x } {}^{10}C_1(0.05)^1(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8= 0.9885$
    (b) Two packets are selected at random. Find the probability that there are no defective shirts in either packet = $({}^{10}C_0(0.05)^0(0.95)^{10} \text{ x } ({}^{10}C_0(0.05)^0(0.95)^{10} = 0.358$
22. A biased coin is such that it is twice as likely to show a head as a tail. If the coin is tossed 5 times. find the probability that
    (i) Exactly three heads are obtained
    $n = 5, p = \frac{2}{3}, q = \frac{1}{3}$
    $P(X = 3) = {}^5C_3(\frac{2}{3})^3(\frac{1}{3})^2 = 0.3292$
    (ii) More than three heads are obtained = 0.3333
    $P(X > 3) = P(X = 4) + P(X = 5)$
    $={}^5C_4(\frac{2}{3})^4(\frac{1}{3})^1 + {}^5C_5(\frac{2}{3})^0(\frac{1}{3})^5 =$
23. The probability that a target is hit 0.3. Find the probability the least number of times shots should be fired if the probability that the target is hit is at least once is greater than 0.95. =9
    n = ? p = 0.3, q = 0.7
    $P(X \geq 1) = 1 – P(X = 0) = 1 - {}^nC_0(0.3)^0(0.7)^n > 0.95$
    $(0.7)^n > 0.05$
    $n = \frac{\log 0.05}{\log 0.7} = 8.399$
    therefore n =9
24. 1% of the light bulbs in a box are faulty. Find the largest sample size which can be taken if it required that the probability that there is no faulty bulb in the sample is greater than 0.5
    (0.99)n > 0.5
    n = 68
25. In a test there are 10 multiple choice questions. Each question has got four possible alternatives out of which only one is correct. If a student guesses each of the answers, find the probability that he gets
    (i) More than 7 correct answers = 0.0004
    (ii) More than half correct answers = 0.0197
26. $X \sim B(n, o.3)$. find the value of n such that $P(X \geq 1) = 0.8 : 0.7^n = 0.2$. = 5
27. The random variable X is B(n, 0.6). find the value of n such that P(X<1) = 0.0256
    $0.4^n = 0.0256$, n = 4

Expectation, variance and standard deviation

If $X \sim B(n, p)$

E(X) = np

Var(X) = npq where q = 1- p

s.d = $\sqrt{npq}$

Example 9

The random variable x is B(4, 0.8). the mean, variance and standard deviation

Mean = np = 4 x 0.8 = 3.2

Variance = npq = 4 x0.8 x 0.2 = 0.64

s.d = $\sqrt{npq}$ = $\sqrt{0.64}$ = 0.8

Example 10

The probability that, it will be a sunny day is 0.4. Find the expected number if sunny days in a week and also find the standard deviation

Solution

E(X) = np = 7 x 0.4 = 2.8

s.d = $\sqrt{npq} = \sqrt{(7 \ x \ 0.4 \ x \ 0.6)}$ = 1.296

Example 11

X is B(n, p) with mean 5 and standard deviation2. Find the value of n and p.

E(X) = np

np = 5 ………………….. (i)

s.d = $\sqrt{npq}$

$\sqrt{npq}$ =2

npq = 4 …………………(ii)

Eqn. (ii) ÷ eqn. (i)

$\frac{npq}{np} = \frac{4}{5} = 0.8$

q = 0.8

p = 0.2

n = $\frac{5}{0.2}$ = 25

Mode of the binomial distribution

The mode is the value of X that is most likely to occur. The value of X with the highest probability and its close to the mean gives the mode.

Example 12

The probability that a student is awarded a distinction in mathematics examination is 0.15. In a randomly selected group of 15 students, what is the most likely number of students awarded a distinction

Solution

E(X) = np = 15 x 0.15 = 2.25

$P(X = 2) = {}^{15}C_2(0.15)^2(0.85)^{13} = 0.286$

$P(X = 3) = {}^{15}C_3(0.15)^3(0.85)^{12} = 0.216$

The most likely number of students awarded a distinction = 2

Example 13

In a school 80% of the students find difficulties in Physics. If a sample of 12 students is chosen

(i) What is the most likely number of students who find it difficult in physics.
E(X) = np = 0.8 x 12 = 9.6
$P(X = 9) = {}^{12}C_9(0.8)^9(0.2)^3 = 0.236$
$P(X = 10) = {}^{12}C_{10}(0.8)^{10}(0.2)^2 =0.294$
The most likely number is 10

(ii) Find the probability that fewer than half find difficulty in Physics.
$P(X<6) = 0.0004$

Revision exercise 2

1. 10% of drugs at a certain Pharmacy are expired. A sample of 25 drugs is taken. Find the expected number and standard deviation of expired drugs (25,1.5)
2. The probability that a student scores above 60% in mathematics test is0.5. In a random sample of 15 students, what is the most likely number of students who score above 60%. (7 and 8)
3. The probability that an apple picked at random from a sack is bad is 0.15.
   (a) Find the standard deviation of the number of bad apples in a sample of 15 apples.=1.38
   (b) What is the most likely number of bad apples in a sample of 30 apples = 4
4. The random variable X is B(n, 0.3) and E(X) = 2.4. Find n and standard deviation (8, 1.30)
5. In a group of people, the expected number who wear glasses is 2 and the variance is 1.6, find the probability that;
   (i) A person chosen at random from the group wear glasses = 0.2
   (ii) 6 people in the group wear glasses. =0.00551
6. New vision publishes a governance article in its newspaper each day of the week Sunday. A man is able to read 8 out of ten articles
   (i) Find the expected value and the standard deviation of the number of read articles in a given week= 4.8
   (ii) What is the probability that he will read at least 5 articles in a given week? = 0.98
7. A die is biased and probability, p of throwing a six is known to be less than $\frac{1}{6}$. An experiment consists of recording the number of sixes n 25 throws of the die. The standard deviation of the number of sixes I 1.5. calculate the
   (i) value of p. = 0.1
   (ii) the probability that exactly three sixes are recorded during a particular experiment = 0.23
8. The random variable X is B(10, p) where p < 0.5. The variance of X is 1.875. find
   (i) Value of p. = 0.25
   (ii) E(X) = 2.5
   (iii) P(X = 2) = 0.282

9. In a bag there are 6 red pens, 8 blue pens and 6 black pens. an experiment consists of taking a pen at random from the bag, noting its colour and then replacing it in the bag. This procedure is repeated 10 times in all. find
   (i) Expected number of red pens drawn=3
   (ii) Most likely number of black pens drawn = 3
   (iii) Probability that not more than four blue pens are drawn = 0.633
10. The random variable X is distributed binomially with mean 2 and variance 1.6, Find
   (i) the probability that x is less than 6 = 0.994
   (ii) the most likely value of X = 2
11. Each day a bakery delivers the same number of loaves to a certain shop which sells on average98%of them. Assuming that the number of loaves sold per day has a binomial distribution with standard deviation 7. Find the expected number of loaves the shop would expect to sell per day = 2500
12. On average 20% of the bolts produced by a machine are faulty. Samples of 10 bolts are to be selected at random each day. Each bolt will be selected and replaced in the set of bolts which have been produced on that day.
   (i) Find the probability that in any one sample, two bolts or less will be faulty = 0.68
   (ii) Calculate the expected value and variance of the number of bolts in a sample which will not be faulty. = 8, 1.6
13. An experiment consists of taking 12 shots at a target and counting the number of hits. The expected number of hit was found to be 3. Calculate
   (i) The probability of hitting the target with a single shot. = 0.25
   (ii) Standard deviation of the number of hits in an experiment = 1.5
14. In a certain family, the probability that they will have a baby boy is 0.6. If there are 5 children in a family determine
   (a) Expected number of girls = 2
   (b) The probability that there are at least three girls = 0.317
   (c) The probability they are all boys. = 0.0778
15. The probability of winning a game is 0.8. Ten games are played. What is the;
   (a) Mean number of success and variance. = 8, 1.6
   (b) The probability of at least 8 success in the ten = 0.6778
16. In a test there are 10 multiple choices questions. Each question has four possible alternatives out of which one is correct. If a student guesses each of the answers, find the
   (i) The probability that at least fur answers are correct = 0.2241
   (ii) Most likely number of correct answers = 2
17. A biased coin is such that a head is twice as likely to occur as a tail. the coin is tossed 15 times. Find the
   (i) The expected number of heads = 10
   (ii) Probability that at most two tails occur = 0.0793

Solutions to revision exercise 2

14. In a certain family, the probability that they will have a baby boy is 0.6. If there are 5 children in a family determine
   (a) Expected number of girls
      n = 5, p = 0.4, q = 0.6
      E(X) = np = 5 x 0.4 = 2

(b) The probability that there are at least three girls

$P(X \leq 3) = P(X= 3) + P(X = 4) + P(X = 5)$

$= {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6)^1 + {}^5C_5(0.4)^5(0.6)^0 = 0.317$

(c) The probability they are all boys.

$P(X = 0) = {}^5C_0(0.4)^0(0.6)^5 = 0.0778$

15. The probability of winning a game is 0.8. Ten games are played. What is the;

(a) Mean number of success and variance.

Mean = np = 10 x 0.8 = 8

Variance = npq = 10 x 0.8 x 0.2 =1.6

(b) The probability of at least 8 success in the ten

$P(X \leq 8) = P(X = 8) + P(X = 9) + P(X = 10)$

$= {}^{10}C_8(0.8)^8(0.2)^2 + {}^{10}C_9(0.8)^9(0.2)^1 + {}^{10}C_{10}(0.8)^{10}(0.2)^0 = 0.6778$

16. In a test there are 10 multiple choices questions. Each question has four possible alternatives out of which one is correct. If a student guesses each of the answers, find the

(i) The probability that at least four answers are correct

n = 10, p = 0.25, q = 0.75

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - [P(X= 0) + P(X= 1) + P(X = 2) + P(X = 3)$

$= 1 - [{}^{10}C_0(0.25)^0(0.75)^{10} + {}^{10}C_1(0.25)^1(0.75)^9 + {}^{10}C_2(0.25)^2(0.75)^8 + {}^{10}C_3(0.25)^3(0.75)^7]$

$= 0.2241$

(ii) Most likely number of correct answers

E(X) = np = 0.25 x 10 = 2.5

$P(X = 2) = {}^{10}C_2(0.25)^2(0.75)^8 = 0.2816$

$P(X = 3) = {}^{10}C_3(0.25)^3(0.75)^7 = 0.2503$

∴ the most likely number of correct answers = 2

17. A biased coin is such that a head is twice as likely to occur as a tail. The coin is tossed 15 times. Find the

(i) The expected number of heads

$n = 15$, $p = \frac{2}{3}$, $q = \frac{1}{3}$

$E(X) = np = \frac{2}{3}\, x\, 15 = 10$

(ii) Probability that at most two tails occur

$n = 15$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= {}^{15}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{15} + {}^{15}C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{14} + {}^{15}C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{13} = 0.0793$

Thank you

Dr. Bbosa Science