P425/1
PURE MATHEMATICS
3 HOURS
ADVANCED LEVEL CERTIFICATE OF EDUCTION
FEB. 2007

Answer all questions in section A and any five from section B

1. Given that $y = \sqrt{(3x^2 + 2)}$, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3$

2. Find the values of c for which the line $2x - 3y = c$ is a tangent to the curve $x^2 + 2y^2 = 2$ and find the equation of the line joining the points of contact.

3. Evaluate $$\int_o^{\pi/2} \frac{\text{Sec}^2x\, dx}{\tan^2x + 2\tan x + 2}$$

4. Solve $\left|\frac{x}{x+4}\right| < 2$

5. A polynomial P(x) is divisible by (x – 2). Show that the remainder when P(x) is divided by $(x - 2)^2$ is also divisible by (x – 2)

6. Show that the area enclosed by the two curves $y = x - x^2 + 7$ and $y = (x + 3)^2$ is $^9/_8$ square units.

7. By reducing the appropriate matrix to echelon form, solve the simultaneous equations.
$x + 3x - z = 4$
$2x + 4y + z = 8$
$3x + 6y + 2z = 10$

8. Prove that
$$\frac{\text{Sin}\ \alpha + \cos\alpha}{\text{Sin}\ (\theta - \alpha)\ \text{Sin}\alpha - \beta\ )} + \frac{\text{Sin}\beta + \text{Cos}\beta}{\text{Sin}(\theta - \beta\ )\sin(\beta - \alpha)} = \frac{\text{Sin}\theta + \cos\theta}{\text{Sin}(\theta - \alpha)\sin(\theta - \beta)}$$

SECTION B:

9. (a) Prove that $\log_b{}^n = \log_b^n \cdot \log_b{}^a$

(b) If $p = \log_a bc$, $q = \log_b ca$, $r = \log_c ab$

Show that $pqr = p + q + r + 2$

10. (a) Find the equation of a plane containing the points (3,2,1) and (– 5, 1, 2) but not intersecting the y – axis.
(b) Find the distance between the straight lines $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z+4}{-1}$ and
$x = 2t$, $y = 3t - 1$, $z = 6t - 2$.
(c) Determine the angle between the line $x = 2t$, $y = -2t + 1$, $z = 2t + 3$ and the plane in (a) above.

11. (a) Prove that in any triangle ABC, $a \cos 2B + 2b \cos A \cos B = c\cos B - b \cos c$.

(b) Solve for all pairs of x and y which lie between 0^o and 180^o, that satisfy the simultaneous equations.
$\tan x + \tan y + 3 = 0$
$\tan 2x + \tan 2y = 0$

12. (a) Show that the expression $2 \tan x - \sin x \cos x - 4\ln \cos x + x$ increases as x increases for all real values of x.

(b) Given the curve $y = \dfrac{\sin x \cos x}{1 + 2\sin x + 2\cos x}$

(i) Prove that the tangents to this curve at the points $(^{\pi}/_{2},0)$ and at the origin, meet in a point whose abscissa is $^{\pi}/_{4}$.
(ii) Show that the abscissae in the interval $0 \leq x \leq 4\pi$ at which the curve has turning points are in an arithmetic progression whose sum is 7π.

13. (a) Show that $$\int_{o}^{1} x \left(\frac{1-x^2}{1+x^2}\right)^{½} dx = {}^{\pi}/_{4} - 1$$

(b) Evaluate $$\int_{\pi/6}^{\pi/2} \frac{4\cos dx}{3 + \cos^2 x}$$

14. (a) A vessel in the shape of an inverted right circular cone contains a liquid. The rate of evaporation of the liquid is proportional to the surface area of the exposed surface to the atmosphere. The radius of the base of the cone is 9cm and the height of the cone is 15 cm. If it takes one minute for the radius to decrease from 9cm to 4.5cm, how long will it take for the liquid to evaporate completely?

(b) The diagonals of a rhombus are of lengths 2xcm and (10 – x) cm. As x varies, find
(i) the maximum area of the rhombus,
(ii) the minimum length of its perimeters.

15. (a) Given that $P(x) = 8x^3 - 12x^2 - 18x + k$, find the value(s) of k such that the equation $p(x) = 0$ has a repeated root.

(b) Express $2x^3 + 5x^2 - 4x + 3$ in the form $(x^2 + x - 2)\,Q(x) + Ax + B$; where Q (x) is a polynomial in x and A and B are constants. Determine the values of A and B and the expression for Q(x).

16. (a) The first term of a G.P is A and the sum of the first three terms is $^7/_4$ A. Show that there are two possible progressions. If A = 4, find the next two terms of each progression.

(b) Prove by induction that $x^n - y^n$ is divisible by $x - y$ for all positive integral values of n.