GRAPHICAL FUNCTIONS, MATRICES & TRANSFORMATIONS

1(a) Using matrix methods find the values of x and y which satisfy the equations.
$2x - y = 1$
$3x + 2y = 12$

(b) Given that M = $\begin{pmatrix} 3 & -1 \\ 4 & 6 \end{pmatrix}$, find a matrix N such that MN = $\begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix}$

Hence or otherwise find the inverse matrix for M.

2. (a) Given the equation $ax^2 + bx + c = 0$, $(a \neq 0)$ derive the formula;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For finding the roots of the above equation, use the formula to solve the equation $3x^2 + 14x + 24 = 0$.

(b) To print wedding cards at the diamond printery, one as to pay a deposit of Shs 50 and an amount which is directly proportional to the number of cards to be printed. Te table below gives the total cost c, required to print d cards.

d	1	3	6	8
c	100	200	350	450

Find (i) c in terms of d,
(ii) The total cost of printing 248 cards.

3. Use graph paper for this question.
Scale: 1cm to 1 unit on the x-axis,
1cm to 0.5 units on the y-axis.
(i) Plot the triangle PQR: P(1, 2), Q(0, 0), R(2, 0).
(ii) Write down the coordinates of PQR as a 2 by 3 matrix A.
(iii) Multiplying A on the left, by T_1 the transformation matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ to give the image of triangle PQR under T_1.
(iv) Plot P′Q′R′
(v) Find the coordinates of P″Q″R″, the image of P′Q′R′ under T_1 whose matrix is $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
(vi) Plot P″Q″R″
(vii) Write down the matrix of a single transformation which would map PQR onto P″Q″R″.

4. . (a) Given that matrices
A = $\begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$, B = $\begin{pmatrix} 9 & 9 \\ 1 & 1 \end{pmatrix}$ and C = $\begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$
Find $(ABC)^{-1}$.

(b) If $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ Determine the values of x and y

5. (a) By plotting suitable graphs on the same axes, find the solution of the equations.
$-3x + 2y = -16$
$x + y = 7$
(b) Plot the graph of $x^2 - 5x - 24$ for $-5 \leq x \leq 10$.
Use your graph to find the roots of the equation
$x^2 - 5x - 24 = 0$.

6. . Given that $\begin{pmatrix} 2 & 4 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 24 \\ 6 \end{pmatrix}$, find the value of a and b.

7. Plot the points A (-2, 1), B (-1, 2), C (2, 2) and D (0, -1) on a graph paper. The quadrilateral ABCD is enlarged to another one whose points are P (1, 3), Q (3, 5), R (9, 5) and S (5, -1) respectively.
(i) Determine the coordinates of the centre T and the scale factor of the enlargement.
(ii) Determine the area of the quadrilateral PQRS.

8. A transformation represented by the matrix $\begin{pmatrix} 4 & 6 \\ 1 & 2 \end{pmatrix}$ maps the vertices A, B, C of a triangle onto the points $A^I(6, 2)$, B^I (16, 7) and C^I (22, 9) respectively.
Find
(i) The coordinates of A, B ad C
(ii) The determinant of the matrix
(iii) The areas of ABC and its image $A^IB^I C^I$.

9. Triangle ABC has its vertices at A (2, 0), B (4, 0) and C (4, 3). The triangle is given a positive quarter turn about (0, 0) to produce $A^1B^1C^1$ the image of ABC; followed by a reflection in the line x + y = 0 to produce $A^{11}B^{11}C^{11}$, the image of $A^1B^1C^1$.
(i) Determine the co-ordinates of $A^1B^1C^1$ and $A^{11}B^{11}C^{11}$. ***(mks)***
(ii) Describe fully a single transformation which maps ABC onto $A^{11}B^{11}C^{11}$. ***(mks)***

10. **(a)** Find the inverse of A = $\begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$
(b) Tom bought 2 eggs and 3 tomatoes at a total cost of shs. 370. The cost of 4 tomatoes is shs. 90 more than that of one egg.
(i) Write down this information as a pair of simultaneous equations.
(ii) Find the cost of one egg.
(iii) Calculation the cost of one tomato.
(iv) Determine the number of eggs and tomatoes shs. 1470 fetched if twice as many tomatoes as eggs were obtained.

11. On the same axes draw the graphs of $y = x^3 - 2$ and $y = 3x + 2$ for $-3 \leq x \leq 3$.
From your graph, estimate
i)The value for $x^3 - 2 = 0$
ii)The solution of the equation
$x^3 - 2 = 3x + 2$

12. On the same axes draw the graphs of the lines y -2x = 1, and y + 3x = 6 for $-3 \leq x \leq 3$. Use your graphs to solve the equations.
y – 2x– 1 = 0,
y + 3x – 6 = 0.
Hence determine the Equation of the line passing through the point of intersection of the two equations of the two equations above whose y – intercept is 2.

13. A triangle ABC where A, B, C are points (2, 3), (6, 3) and (4, 6) respectively is given a transformation representative by the matrix
M = $\begin{pmatrix} 0 & -3 \\ -1 & 2 \end{pmatrix}$ followed by the matrix
N = $\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ to give the final image A^I B^I and C^I.
i) find the image points A^I, B^I and C^I.
ii) describe the single matrix transformation that is represented by the combined matrix transformation M followed by N.
iii) Obtain a single matrix that would map A^I,B^I and C^I back onto ABC.

14. Draw on the same co-ordinate axes, graphs of
$y = x(2x - 3)$ and $y = 2(x-1)$ for $-3 \leq x \leq 4$.

(i)Using graph determine the point of intersection $y = x(2x-3)$ and $y = 2(x-1)$
(ii)Use your graph to find the roots of
$2x^2 - 3x = 0$

15. Using matrix methods find the values of x and y which satisfy the equations.
$2x - 3y = 12$
$x + 2y + 1 = 0$

b) Given that matrix A $= \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix}$

Matrix B is such that AB $= \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

Hence otherwise find the inverse matrix of A.

16. The points A(-2,1), B(-2,4), C(1,4) and D(1,) are vertices of a square ABCD. The images of A,B,C and D under a reflection in the line x-y =0 areA^1 B^1C^1 D^1 The points A^1,B^1,C^1 and D^1 are then mapped onto the points A^{11},B^{11},C^{11} and D^{11} respectively under an enlargement with scale factor 2 and centre of enlargement the origin O(0,0).

(a) write down the matrices of the reflection and enlargement
(b) Find the coordinates of the points
(i).$A^1B^1C^1D^1$
(ii).$A^{11}B^{11}C^{11}D^{11}$
(c) Determine the matrix c of a single transformation that would map A B C D onto $A^{11}B^{11}C^{11}D^{11}$

17. The points P(0, 2) Q(1, 4)and R(2, 2) are vertices of a triangle PQR. The images of P,Q and R under a reflection in the line x - y =0 are P^1,Q^1 and R^1 respectively. The points P^1,Q^1 and R^1 are then mapped onto the points P^{11},Q^{11} and R^{11} respectively, under the enlargement with scale factor -2 and centre of enlargement O(0,0).
a) Write down the matrix for the
(i) Reflection
(ii) Enlargement
(b) Determine the coordinates of the points
(i) P^1 Q^1, R^1
(ii) P^{11}, Q^{11}, P^{11}
(c) Find the matrix of a single transformation which would map triangle PQR onto P^{11}, Q^{11}, and R^{11}

18. a) Copy and complete the following table of values for the curve $y = x^2 - 2x - 6$ and $y = 4x - 5$ for values of x between $x = -4$ and $x = 7$

x	-4	-3	-2	-1	0	1
x^2	-	9	-	-	-	1
-2x	8	-	4	2	-	-2
x^2-2x- 6	18	-	2	-	-	-
4x	-	-	-8	-4	0	-
y = 4x-5	12	-	-13	-	-5	-

x	2	3	4	5	6	7
x^2	-	-	-	-	36	49
-2x	-	-	-	-	-12	-14
x^2-2x- 6	-				18	29
4x	8				24	28
y = 4x-5	-	-	-	-	19	23

a) On the same axes plot the graph of the curve $y = x^2 - 2x - 6$ and the line $y = 4x - 5$ for $-4 \leq x \leq 7$
b) Using your graph estimate the
i) Coordinates of the points of intersection of the curve and the line.
ii) Roots of the equation $y = x^2 - 2x - 6 = 0$

19. . Draw the graph of the curve $x^2 - 2x + 1$ for $-3 \leq x \leq 3$. Use your graph to find the solutions of the following equations.
(i) $x^2 - 2x + 1$ 0
ii) $x^2 - x - 6 = 0$

20. The image of the vertices P (2, 3); Q (2, 2) and R (4, 2) of a triangle PQR under a rotational transformation $P^1(-1,2)$, $Q^1(0,2)$ and $R^1(0,4)$ respectively. The image of PQR, $P^1Q^1R^1$ then undergoes a further rotation of 52° to give the image $P^{11}Q^{11}R^{11}$.
(i) Represent triangle PQR and its images on the same coordinate axes (use a scale of 2cm to 1 unit).
(ii) Determine the centre and angle of rotation of PQR.
(iii) Find the coordinates of the final image $P^{11}Q^{11}R^{11}$. State the angle formed between PQR and $P^{11}Q^{11}R^{11}$.

21. a) Plot the graph of $y = 3x^2 + 2x - 16$ for values $x: -3 \leq x \leq 3$
(b) Use your graph to solve the equation $3x^2 + 2x - 8 = 0$
22. (a) Musa is a businessman who deals in an agricultural produce business. He visited four markets in a certain week:
In market A he bought 3 bags of beans, 5 bags of maize, 10 bags of potatoes and 3 bags of millet.
In market B, he bought 1 bag of beans, 4 bags of potatoes and 2 bags of millet,
In market C, he bought 5 bags of beans, 1 bags of maize.
In market D he bought 4 bags of beans, 3 bags of maize, 6 bags of potatoes and 1 bag of millet.
He bought each bag of beans at Shs. 45,000, a bag of maize at Shs 30,000, a bag of potatoes at Shs 15,000 and a bag of millet at Shs 50,000. He later sold all the produce he had bought at Shs. 50,000 per bag of beans; Shs 35,000 per bag of maize, Shs 18,000 per bag of potatoes and Shs 55,000 per bag of millet.

a) Form a 4 × 4 matrix to show the produce Musa bought from the four markets.

b) i) Form a cost matrix for the price of the produce,

(ii) By matrix multiplication, find the amount of money spent on the produce in each market.

(c) Find also the amount of money he got from the sale of the produce.
(d) Find Musa's profit.

23. Four schools participated in a football tournament which was played in two rounds. The results were as given below;
1st Round
- Bakulu S.S. won one, drew three and lost two matches.
- Dodo S.S won two, drew two and lost two matches
- Kawunga S.S won three, drew two and lost one match
- Oronga S.S. won none, drew two and lost four matches

2nd Round
- Bakulu S.S. won one, drew two and lost three matches.
- Dodo S.S won two, drew one and lost three matches
- Kawunga S.S won two, drew three and lost one match
- Oronga S.S. won one, drew four and lost one matches

a) Write down a 4 x 3 matrix which shows the performance of the schools in
(i) each of the two rounds ***(04 mks)***
(ii) both rounds. ***(03 mks)***
b) Three points are awarded for a win, one point for a draw and no point for a loss.
(i) write down a 3 x 1 matrix to represent the award of points ***(01 mk)***
ii) using matrix multiplication, determine which school won the tournament . ***(04 mks)***