P425/2
APPLIED MATHEMATICS
Paper 2
Sept. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) SET 3

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g $to\ be$ $9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Events A and B are independent such that P(A')= 0.6 and P(AuB) = 0.8. Find the;
   (i) P(B) (ii) P(AuB')

2. A constant horizontal force of 35N causes a particle of mass 10 kg to move across a rough horizontal plane, a distance of 25m from $5ms^{-1}$ to $10ms^{-1}$. Find the;
   (i) Acceleration of the particle
   (ii) Coefficient of friction between the particle and the plane

3. A car consumed fuel amounting to Shs 14,800, Shs 15,600, Shs 16,400 and Shs 17,200 in covering distances of 10km, 20km, 30km and 40km respectively. Estimate the;
   (a) Cost of fuel consumed for a distance of 45km
   (b) Distance travelled if fuel of Shs 46,000 is used

4. A continuous random variable X is uniformly distributed over the interval $\alpha \leq x \leq \beta$. Given that E(X)=2 and P(X$\leq$ 3)= $\frac{5}{8}$. Find the;
   (a) Value of α and β. (b) P.d.f of x

5. One end of a light inextensible string of length 75cm is fixed to a point on a vertical. A particle of mass 1.2kg is attached to the other end of the string. The particle is kept in equilibrium 21cm away from the pole by a horizontal force, P Newtons. Find the;
   (i) Tension in the string (ii) Magnitude of ***P***

6. Real numbers A and B are rounded off to give numbers a and b with maximum possible errors e_A and e_B. Show that the maximum possible relative error made in the product AB is $\left|\frac{e_A}{a}\right| + \left|\frac{e_B}{b}\right|$. State any assumptions made

7. The following table gives the order in which six candidates were marked in two tests X and Y

X	E	C	B	F	D	A
Y	F	A	D	E	A	C

   Calculate the rank correlation co-efficient and comment at 1% level of significance

8. A truck of mass 4m kg moving with a velocity of 54kmh^{-1} makes a head on collision with a car of mass m kg moving with a velocity of 36kmh^{-1}. If the truck moves in the same direction with the car embedded in it after collision, find;
   (a) Common velocity after collision (b) Loss in kinetic energy

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The table below shows the marks obtained by students of principal mathematics in a certain school

Marks	Frequency density
$10 < x \leq 15$	0.4
$15 < x \leq 25$	0.8
$25 < x \leq 30$	3.4
$30 < x \leq 35$	5.2
$35 < x \leq 45$	2.4
$45 < x \leq 50$	3.2
$50 < x \leq 60$	0.6
$60 < x \leq 80$	0.05

(a) Calculate the;

(i) Mean mark (ii) Standard deviation

(b) Construct the cumulative frequency curve and use it to estimate the median

10.(a) The table below shows the heights and weights to the nearest units of ten men

Height (cm)	145	155	156	130	160	150	153	158	153	153
Weight (cm)	63	71	72	68	75	66	68	76	71	70

(i) Plot the points on a scatter diagram, draw the line of line best fit

(ii) Estimate from your graph, the weight of a man who is 162cm tall

(b) A bag contains 4 white balls, 3 black balls and 1 red ball. Two balls are picked in succession at random without replacement. Find the probability that

(i) Both are of the same colour

(ii) At least one black ball is picked

11. Forces of magnitude 2N, 4N, 6N, 1N, and 5N act along the sides AB, BC, DC, ED and EA respectively of a regular pentagon of length 1.5m. Taking AB as the reference x-axis, find the:

(a) magnitude of the resultant force and its direction

(b) line of action by taking moments about point A

12.(a) By sketching graphs of $y = lnx$ and $y = 2 - x$ in the interval $1 \leq x \leq 2$,

show that the equation $lnx + x - 2 = 0$ has a root. State it to 1 decimal place.

(b) Hence use the Newton Raphson method to find the root of the equation correct to 3 decimal places

13.A jet fighter and a cruiser have the following position and velocity vectors

	Displacement (s)	Velocity vector (v)	Time (t)
Jet	$s_1 = (-6i + 12j)$km	$v_1 = (16i - 4j)kmh^{-1}$	11:30am
Cruiser	$s_2 = (12i - 15j)$km	$v_2 = (8i + 16j)kmh^{-1}$	12:00pm

(a) Find the position vector of the jet fighter at noon

(b) If these velocities remain constant

(i) Show that collision occurs and find time of collision

(ii) Hence find the position of collision

14.A student used trapezium rule with 6 sub-intervals to estimate $\int_0^{\frac{\pi}{4}} xsinx\ dx$, correct to 3 decimal places

(a) determine the actual value of the integral

(b) find the error the student made in the estimate and state how the error may be reduced

15.A continuous random variable X has probability density function

$$f(x) = \begin{cases} 6(x - x^2); & 0 \le x \le 1 \\ 0: & else\ where \end{cases}$$

(a) (i) show that X is a random variable

(ii) find F(x), the cumulative distribution function of X

(b) Hence, find;

(i) $P(X > \frac{1}{4})$ (ii) The median

16.(a) A car of mass 2 tones has a maximum speed of 72kmh^{-1} up a hill inclined at $sin^{-1}\left(\frac{1}{7}\right)$ to the horizontal when the engine is working at 64kW. Find the resistance to motion of the car

(b) A car of mass 500kg tows a van of mass 300kg up a hill inclined at 30^0 to the horizontal. The resistances to motion of the car and the van are 200N and 180N respectively. If the power output of the car is 196kW, find the acceleration of the vehicles and the tension in the tow rope at the instant when the speed of the car is 40ms^{-1}