P425/1, Pure mathematics
Marking guide.

1. $\sin x + \sin 3x + \sin 5x + \sin 7x = 0$

$\sin 7x + \sin x + \sin 5x + \sin 3x = 0$

$2\sin 4x \cos 3x + 2\sin 4x \cos x = 0$ — M1 → factor formulae

$2\sin 4x(\cos 3x + \cos x) = 0$

$2\sin 4x(2\cos 2x \cos x) = 0$ — M1 → factor formula

$4\sin 4x \cos 2x \cos x = 0$

$\sin 4x = 0$

$4x = \sin^{-1}(0) = 0^\circ, 180^\circ, 360^\circ$ — M1

$x = 0^\circ, 45^\circ, 90^\circ$

$\cos 2x = 0$

$2x = \cos^{-1}(0) = 90^\circ, 270^\circ$ — M1

$x = 45^\circ, 135^\circ$

$\cos x = 0$

$x = \cos^{-1}(0) = 90^\circ$

$\therefore x = 0^\circ, 45^\circ, 90^\circ.$ — A1

05

2. Let $f(x) = x^3 + ax^2 + bx - 7$.

$x^3 + ax^2 + bx - 7 = (x^2 - x - 2)Q(x) + 8x - 1$

$x^3 + ax^2 + bx - 7 = (x-2)(x+1)Q(x) + 8x - 1$

$x = 2$; $8 + 4a + b(2) - 7 = 8(2) - 1$

$4a + 2b = 14$

$2a + b = 7 \quad \text{--- i}$ — B1

$x = -1$; $(-1)^3 + a(-1)^2 + b(-1) - 7 = 8(-1) - 1$

$a - b = -1 \quad \text{--- ii}$ — B1

eqn i + eqn ii

$$\begin{aligned} &2a + b = 7 \\ +\; &a - b = -1 \\ \hline &3a = 6 \end{aligned}$$ — M1

$a = 2$ — A1

$2 - b = -1$

$b = 3$ — A1

05

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$$x^2-x-2 \overline{\big) x^3+ax^2+bx-7} \quad \text{quotient: } 1x+(a+1)$$

$$-\{x^3-x^2-2x$$

$$\{(a+1)x^2+(b+2)x-7$$

$$-\}(a+1)x^2-(a+1)x-2(a+1)$$

$$(b+2+a+1)x-7+2(a+1)$$

$$(b+a+3)x+2a-5$$ B1

But $R = 8x-1$

$\Leftrightarrow$ $b+a+3=8$

$a+b=5$ - - - - i B1

$\Leftrightarrow$ $2a-5=-1$ M1

$2a=4$

$a=2$ B1

Put $a=2$ in eqn i

$2+b=5$

$b=3$ B1

05

3 $\displaystyle\int_0^3 x\sqrt{9-x^2}\,dx$. $\quad x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

x	θ
0	0
3	$\pi/2$

B1 for Change of limits

$$\int_0^3 x\sqrt{9-x^2}\,dx = \int_0^{\pi/2} 3\sin\theta\cdot\sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\,d\theta$$ M1

$$= \int_0^{\pi/2} 3\sin\theta\cdot\sqrt{9(1-\sin^2\theta)}\cdot 3\cos\theta\,d\theta$$ M1

$$\int_0^3 x\sqrt{9-x^2}\,dx = \int_0^{\pi/2} 3\sin\theta\cdot 3\cos\theta\cdot 3\cos\theta\,d\theta$$

$$= 27\int_0^{\pi/2} \sin\theta\cos^2\theta\,d\theta$$

$$= 27\left[\frac{-1}{3}\cos^3\theta\right]_0^{\pi/2}$$ A1

$$= -9\cos^3\theta \Big]_0^{\pi/2}$$ M1

$$= -9\left((\cos \pi/2)^3 - (\cos 0)^3\right)$$

$$= -9(0-1)$$

$$= 9$$ A1

$$\therefore \int_0^3 x\sqrt{9-x^2}\,dx = 9$$

05

4

$3x+2y-3z=-18$ ---- i

$x-2y+z=12$ ---- ii

eqn i + eqn ii

$4x-2z=-6$

$2x-z=-3$ ---- i

$x = \frac{z-3}{2}$ M1

eqn i + 3 eqn ii

$3x+2y-3z=-18$

$+ \; 3x-6y+3z=36$

$6x-4y=18$

$3x-2y=9$ ---- ii M1

$x = \frac{2y+9}{3}$

$\therefore x = \frac{2y+9}{3} = \frac{z-3}{2}$ M1 A1

$x = \frac{y+9/2}{3/2} = \frac{z-3}{2}$

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ -9/2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3/2 \\ 2 \end{pmatrix}$$ B1

$$\underset{\sim}{r} = \begin{pmatrix} 0 \\ -9/2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$ (nc)

Alt: from $2x - z = -3$

$z = 2x+3$ M1

eqn i − 3 eqn ii

$$\begin{cases} 3x+2y-3z=-18 \\ 3x-6y+3z=36 \end{cases} -$$

$$8y-6z=-54$$

$$4y-3z=-27$$

$$z=\frac{4y+27}{3}$$ M1

$$\therefore \quad 2x+3 = \frac{4y+27}{3} = z$$ M1 A1

$$\frac{x+3/2}{1/2} = \frac{y+27/4}{3/4} = z$$

$$r = \begin{pmatrix} -3/2 \\ -\frac{27}{4} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1/2 \\ 3/4 \\ 1 \end{pmatrix}$$ B1

$$\underset{\sim}{r} = \begin{pmatrix} -3/2 \\ -27/4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

05 marks

Alt 2. $\underset{\sim}{b} = n_1 \times n_2$

$$\underset{\sim}{b} = \begin{vmatrix} i & -j & k \\ 3 & 2 & -3 \\ 1 & -2 & 1 \end{vmatrix} = i\begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix}$$ M1

$$\underset{\sim}{b} = -4\underset{\sim}{i} - 6\underset{\sim}{j} - 8\underset{\sim}{k}.$$ A1

Where it passes.
$$\begin{cases} 3x+2y-3z=-18 \\ x-2y+z=12 \end{cases} +$$

$$4x-2z=-6$$

$$2x-z=-3$$

When $z=1$, $2x-1=-3$

$$2x=-2$$

$$x=-1$$

Subst $x=-1, z=1$ in eqn i

$$3x+2y-3z=18$$
$$-3+2y-3=-18$$
$$2y=-12$$
$$y=-6$$

M passes thru: $(-1,-6,1)$ B1

$$\underset{\sim}{r}=\begin{pmatrix}-1\\-6\\1\end{pmatrix}+\lambda\begin{pmatrix}-4\\-6\\-8\end{pmatrix}$$ M1A1

$$\underset{\sim}{r}=\begin{pmatrix}-1\\-6\\1\end{pmatrix}+\mu\begin{pmatrix}2\\3\\4\end{pmatrix}$$

(05)

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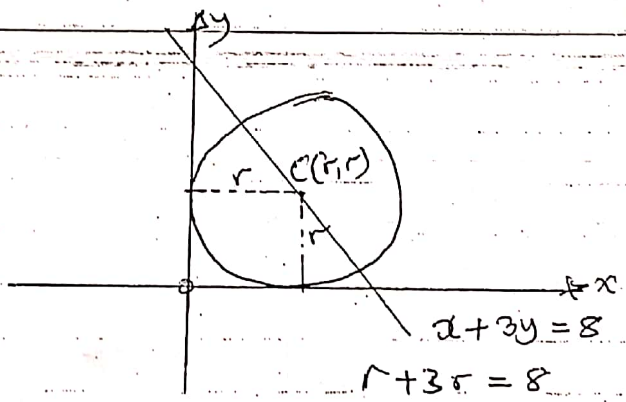


B1 — for $C(r,r)$ (Accept $C(a,a)$ ie equal values.)

$$r+3r=8$$ M1
$$4r=8$$
$$r=2 \quad \therefore C(2,2)$$ A1

$\therefore$ Equation of the circle

$$(x-2)^2+(y-2)^2=2^2$$ M1
$$(x-2)^2+(y-2)^2=4$$ A1

or $x^2+y^2-4x-4y+4=0$

(05)

6 $\quad 2(5^x)+3^{y+2}=53 \qquad$ let $5^x=m$, $3^y=n$

$$2(5^x)+3^y.3^2=53$$
$$2m+9n=53 \quad \text{---- i}$$

$$5^{x+2}-3^{y+5}=544$$
$$5^2(5^x)-3^y.3^5=544$$
$$25m-243n=544 \quad \text{--- ii}$$

B1 for both equation.

From $2m+9n=53$

$$n=\frac{53-2m}{}$$

Subst n in eqn ii

$25m - 243\left(\frac{53-2m}{9}\right) = 544$ M1

$25m - 27(53-2m) = 544$

$25m - 1431 + 54m = 544$

$79m = 1975$

$m = 25$ A1 for either m or n

$n = \frac{53-2(25)}{9} = \frac{3}{9} = 1/3$ ~~A1~~

$\therefore 5^x = 25$ $\quad 3^y = 1/3$

$5^x = 5^2$ $\quad 3^y = 3^{-1}$ — for x = 2, for y = −1

$x = 2$ $\quad y = -1$ B1 B1

05

7. Let $y = \left(\frac{(2x+3)^3}{1+x^2}\right)^{1/2} = \frac{(2x+3)^{3/2}}{(1+x^2)^{1/2}}$

$u = (2x+3)^{3/2}$ $\quad \frac{du}{dx} = \frac{3}{2}(2x+3)^{1/2}\cdot 2 = 3(2x+3)^{1/2}$ — for $\frac{du}{dx}$, $\frac{dv}{dx}$ B1

$v = (1+x^2)^{1/2}$ $\quad \frac{dv}{dx} = \frac{1}{2}(1+x^2)^{-1/2}\cdot 2x = x(1+x^2)^{-1/2}$

$\frac{dy}{dx} = \frac{(1+x^2)^{1/2}\cdot 3(2x+3)^{1/2} - (2x+3)^{3/2}\cdot x(1+x^2)^{-1/2}}{1+x^2}$ M1

$= (2x+3)^{1/2}(1+x^2)^{-1/2}\left[\frac{3(1+x^2) - x(2x+3)}{1+x^2}\right]$ M1

$= \frac{(2x+3)^{1/2}\left(3+3x^2-2x^2-3x\right)}{(1+x^2)^{3/2}}$ M1

$\frac{dy}{dx} = \frac{(2x+3)^{1/2}\left(3-3x+x^2\right)}{(1+x^2)^{3/2}}$ A1

05

1

Qu. $y = \dfrac{(2x+3)^{3/2}}{(1+x^2)^{1/2}}$

$\ln y = \ln (2x+3)^{3/2} - \ln(1+x^2)^{1/2}$ — B1 for splitting logs

$\ln y = \frac{3}{2}\ln(2x+3) - \frac{1}{2}\ln(1+x^2)$ — B1

$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2}\left(\frac{2}{2x+3}\right) - \frac{1}{2}\left(\frac{2x}{1+x^2}\right)$ — M1

$= \dfrac{3(1+x^2) - x(2x+3)}{(2x+3)(1+x^2)}$ — M1 for LCM

$\frac{dy}{dx} = \dfrac{(3+3x^2-2x^2-3x)}{(2x+3)(1+x^2)} \cdot y$

$\frac{dy}{dx} = \dfrac{3-3x+x^2}{(2x+3)(1+x^2)} \cdot \dfrac{(2x+3)^{3/2}}{(1+x^2)^{1/2}}$ — M1 for making $\frac{dy}{dx}$ the subject

$= \dfrac{(2x+3)^{1/2}(3-3x+x^2)}{(1+x^2)^{3/2}}$ — A1

(05)

8 $\frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2}$

$\frac{d}{dx}\left(\frac{y}{1-y}\right) + \frac{d}{dx}\left(\frac{x}{1-x}\right) + \frac{d}{dx}(5x) - \frac{d}{dx}(3y) = 0$ — Differentiating completely

$\frac{1}{(1-y)^2}\frac{dy}{dx} + \frac{1}{(1-x)^2} + 5 - 3\frac{dy}{dx} = 0$ — B1

at $(2,2)$ $\frac{1}{(1-2)^2}\frac{dy}{dx} + \frac{1}{(1-2)^2} + 5 - 3\frac{dy}{dx} = 0$ — M1

$\frac{dy}{dx} + 1 + 5 - 3\frac{dy}{dx} = 0$

$2\,dy = 6$

$\frac{dy}{dx} = 3$ A1

Gradient of tangent $= 3$

Equation of the tangent

$\frac{y-2}{x-2} = 3$ M1

$y - 2 = 3x - 6$

$y = 3x - 4$ A1 (05)

9a $z^3 = \frac{-(5+i)}{2+3i} \cdot \frac{2-3i}{2-3i}$

$= -\frac{(10 - 15i + 2i + 3)}{4+9}$

$= -\frac{(13 - 13i)}{13}$

$= -(1 - i)$

$= -1 + i$ B1

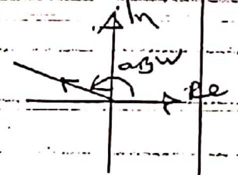
$z^3 = -1 + i$

$z = (-1+i)^{1/3}$

Let $w = -1+i$

$|w| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ B1

$\arg w = 180° - \tan^{-1}(1/1) = 135°$ B1 for $135°$ or $\frac{3\pi}{4}$



$\therefore z = (\sqrt{2})^{1/3} \left[\cos \left(\frac{135 + 360° k}{3} \right) + i \sin \left(\frac{135 + 360 k}{3} \right) \right]$

where $k = 0, 1, 2$

$k = 0$

$z = (\sqrt{2})^{1/3} \left(\cos \frac{135°}{3} + i \sin \frac{135°}{3} \right)$

$= 0.7937 + 0.7937i$ B1 for atleast 4 dps

$k = 1$

$z = (\sqrt{2})^{1/3} (\cos 165° + i \sin 165°)$

$z = -1.0842 + 0.2905i$ B1

	$k=2$ $z=(\sqrt{2})^{1/3}(\cos 285^\circ + i\sin 285^\circ)$ $z = 0.2905 - 1.0842i$	B_1
b	$z\bar{z} + 2(z - i\bar{z}) = 3+2i$ Let $z = x+iy$ $(x+iy)(x-iy) + 2(x+iy - i(x-iy)) = 3+2i$ $x^2+y^2+2(x+iy-ix-y) = 3+2i$	M_1
Re Im	$x^2+y^2+2x-2y = 3$ - - - i $2(y-x) = 2$ $y-x=1$ - - - ii $y = x+1$	B_1 — for both eqs
	Subst $y=x+1$ in eqn i $x^2+(x+1)^2+2x-2(x+1) = 3$ $x^2+x^2+2x+1+2x-2x-2=3$ $2x^2+2x-4=0$	M_1
	$x^2+x-2=0$ $(x+2)(x-1)=0$	M_1 OR ALT for solving
	$x=-2$, $x=1$	A_1 for both
	When $x=-2$, $y=-2+1=-1$ $x=1$, $y=1+1=2$ $\therefore z = -2-i$ or $1+2i$	B_1 for both
		12 marks
10	$x+2y+3z=13$ - - - i $ax+y+z=4$ - - - ii If $r\cdot n_1 = d_1$ and $r\cdot n_2 = d_2$ are perpendicular, then $\underset{\sim}{n_1}\cdot\underset{\sim}{n_2}=0$ $n_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}$ $n_2 = \begin{pmatrix}a\\1\\1\end{pmatrix}$	B_1 — for both normals
	$\therefore \begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}a\\1\\1\end{pmatrix} = 0$	M_1
	$a+2+3=0$ $a=-5$	A_1

1 1 1

$r_1 = \underset{\sim}{a} + \lambda \underset{\sim}{b}$

where $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ — B1

$\underset{\sim}{r}_1 = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ — M1 A1

iii At A; $r_1 = r_2$

$r_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 9+2\lambda \\ 13+3\lambda \end{pmatrix}$ $\quad r_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3-\mu \\ 7+2\mu \\ t-3\mu \end{pmatrix}$

$\Leftrightarrow$: $2+\lambda = -3-\mu$

$\lambda + \mu = -5$ --- i

$\Leftrightarrow$ $9+2\lambda = 7+2\mu$ — B1 for both eqns

$2\lambda - 2\mu = -2$

$\lambda - \mu = -1$ --- ii

eqn i + eqn ii

$2\lambda = -6$ — M1

$\lambda = -3$ — A1 for either λ or μ

Subst. $\lambda = -1$ in eqn (i)

$-3 + \mu = -5$

$\mu = 2$

$x = 2 + -3 = -1$

$y = 9 + 2(-3) = 3$

$z = 13 + 3(-3) = 4$

$\therefore A(-1, 3, 4)$ — B1

for t: $13 + 3\lambda = t - 3\mu$

$13 + 3(-3) = t - 3(-2)$ — M1

$4 = t + 6$

$t = -2$ — A1

12 marks

11 $T = 2\pi\sqrt{\frac{l}{10}} = \frac{2\pi\, l^{1/2}}{\sqrt{10}}$

for $\frac{dT}{dl}$

$\frac{dT}{dl} = \frac{1}{2}\cdot\frac{2\pi l^{-1/2}}{\sqrt{10}} = \frac{\pi l^{-1/2}}{\sqrt{10}}$ B1

$\delta l = 10.1 - 10.0 = 0.1\,m.$ B1 for δl

$\delta T \simeq \frac{dT}{dl}\cdot\delta l$

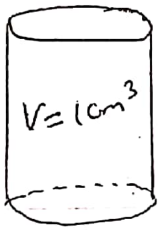
$\delta T = \frac{\pi l^{-1/2}}{\sqrt{10}}\,\delta l$

$= \frac{\pi (10)^{-1/2}\cdot(0.1)}{\sqrt{10}}$ M1

$= \frac{\pi}{100} = \frac{3.14}{100}$ M1

$\delta T \simeq 0.0314\,s$ A1

b



$V = \pi r^2 h$

$1 = \pi r^2 h$

$h = \frac{1}{\pi r^2}$ - - - i

Surface area (when open one end), $A = \pi r^2 + 2\pi r h$

$A = \pi r^2 + 2\pi r\left(\frac{1}{\pi r^2}\right)$ M1

$A = \pi r^2 + \frac{2}{r}$ # A1

for mini area ; $\frac{dA}{dr} = 2\pi r - \frac{2}{r^2}$. B1

$\frac{dA}{dr} = 0$

$2\pi r - \frac{2}{r^2} = 0$ M1

$2\pi r^3 - 2 = 0$

$2\pi r^3 = 2$

$r^3 = \frac{2}{2\pi}$

$$r^3 = \frac{1}{\pi}$$

$$r = \frac{1}{\sqrt[3]{\pi}}$$ B1

$$h = \frac{1}{\pi r^2} = \frac{1}{\pi\left(\frac{1}{\pi^{1/3}}\right)^2} = \frac{1}{\pi \cdot \frac{1}{\pi^{2/3}}} = \frac{1}{\pi^{1/3}}$$

$$= \frac{1}{\sqrt[3]{\pi}}$$ B1

$$\therefore h = r = \frac{1}{\sqrt[3]{\pi}}$$ B1

12 marks

12a $\tan^2\left(\frac{\theta}{2} + \pi/4\right) = \frac{1+\sin\theta}{1-\sin\theta}$

$$LHS = \left(\tan\left(\frac{\theta}{2} + \pi/4\right)\right)^2$$

$$= \left(\frac{\tan\theta/2 + \tan\pi/4}{1 - \tan\theta/2 \tan\pi/4}\right)^2$$ M1

$$= \left(\frac{\tan\theta/2 + 1}{1 - \tan\theta/2}\right)^2$$

$$= \left(\frac{\frac{\sin\theta/2}{\cos\theta/2} + 1}{1 - \frac{\sin\theta/2}{\cos\theta/2}}\right)^2$$ M1

$$= \left(\frac{\sin\theta/2 + \cos\theta/2}{\cos\theta/2 - \sin\theta/2}\right)^2$$

$$= \frac{\sin^2\theta/2 + 2\sin\theta/2\cos\theta/2 + \cos^2\theta/2}{\cos^2\theta/2 - 2\sin\theta/2\cos\theta/2 + \sin^2\theta/2}$$ M1

$$= \frac{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 - 2\sin\theta/2\cos\theta/2}$$ M1

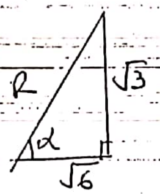
$$= \frac{1+\sin\theta}{1-\sin\theta} = RHS$$ B1

b $\sqrt{6}\sin 2x - \sqrt{3}\cos 2x = R\sin(2x-\alpha)$

$R\sin 2x\cos\alpha - R\cos 2x\sin\alpha$

$R\sin\alpha = \sqrt{3}$

$R\cos\alpha = \sqrt{6}$



$R^2 = (\sqrt{3})^2 + (\sqrt{6})^2$

$R^2 = 3+6$

$R^2 = 9$

$R = 3$ B1

$\tan\alpha = \frac{\sqrt{3}}{\sqrt{6}}$

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{6}}\right) = 35.3^\circ$ B1

$\therefore \sqrt{6}\sin 2x - \sqrt{3}\cos 2x = 3\sin(2x - 35.3^\circ)$ B1

$$\therefore \frac{1}{\sqrt{6}\sin 2x - \sqrt{3}\cos 2x + 5} = \frac{1}{3\sin(2x-35.3)+5}$$

$$\left(\frac{1}{3\sin(2x-35.3)+5}\right)_{max} = \frac{1}{3(-1)+5} = \frac{1}{2}$$ B1

$\Leftrightarrow \sin(2x-35.3) = -1$

$2x - 35.3 = \sin^{-1}(-1) = 270^\circ$

$2x = 305.3^\circ$

$x = 152.65^\circ$ B1

$$\left(\frac{1}{3\sin(2x-35.3^\circ)+5}\right)_{min} = \frac{1}{3(1)+5} = \frac{1}{8}$$ B1

$\Rightarrow \sin(2x - 35.3^\circ) = 1$

$2x - 35.3^\circ = 90^\circ$

$2x = 125.3^\circ$

$x = 62.65^\circ$ B1

12 marks

13 $\int_1^e \frac{\ln x}{x^2}\,dx$

Let $u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$. B1 for $\frac{du}{dx}$ and v ✓

$\frac{dv}{dx} = \frac{1}{x^2} = x^{-2}$ $\quad V = -\frac{1}{x}$.

$\int \frac{\ln x}{x^2}\,dx = (\ln x)\left(\frac{-1}{x}\right) - \int \frac{-1}{x}\cdot\frac{1}{x}\,dx$ M1

$= -\frac{1}{x}\ln x + \int x^{-2}\,dx$

$= -\frac{1}{x}\ln x - \frac{1}{x} + c$ M1 A1

$\therefore \int_1^e \frac{\ln x}{x^2}\,dx = \left[-\frac{1}{x}\ln x - \frac{1}{x}\right]_1^e$

$= \left(-\frac{1}{e}\ln e - \frac{1}{e}\right) - \left(-\frac{1}{1}\ln 1 - \frac{1}{1}\right)$ M1

$= \frac{-1}{e} - \frac{1}{e} + 0 + 1$

$= 1 - 2/e$ # A1

b $\int_0^4 x\sin(5x^2)\cos(7x^2)\,dx$

Let $\theta = x^2$ B1 for dx and Limits.

$d\theta = 2x\,dx$

$dx = \frac{d\theta}{2x}$

x	θ
0	0
4	16

$$\int_0^4 x\sin(5x^2)\cos(7x^2)\,dx = \int_0^{16} x\sin 5\theta\cos 7\theta \frac{d\theta}{2x}$$ M1 for substn

$$= \frac{1}{2}\int_0^{16} \sin 5\theta \cos 7\theta\, d\theta$$ ~~M1~~

$$= \frac{1}{2}\int_0^{16} \frac{1}{2}(\sin 12\theta - \sin 2\theta)\,d\theta$$ M1

$$= \frac{1}{4}\left[-\frac{1}{12}\cos 12\theta + \frac{1}{2}\cos 2\theta\right]_0^{16}$$ A1

$$= \frac{1}{4}\left[-\frac{1}{12}\cos 192 + \frac{1}{2}\cos 32\right] - \frac{1}{4}\left(-\frac{1}{12}\cos 0 + \frac{1}{2}\cos 0\right]$$ M1 (Reject 192° and 32°)

$$= 0.01959$$ A1

$$\therefore \int_0^4 x\sin(5x^2)\cos(7x^2)\,dx = 0.01959.$$

12 marks

14a $(1+2x-3x^2)^6 = (1+(2x-3x^2))^6$

$$= 1+6(2x-3x^2)+15(2x-3x^2)^2+20(2x-3x^2)^3$$ M1M1

$$= 1+12x-18x^2+15(4x^2-12x^3)+20(8x^3)$$ M1

$$= 1+12x-18x^2+60x^2-180x^3+160x^3$$ M1

$$= 1+12x+42x^2-20x^3$$ A1

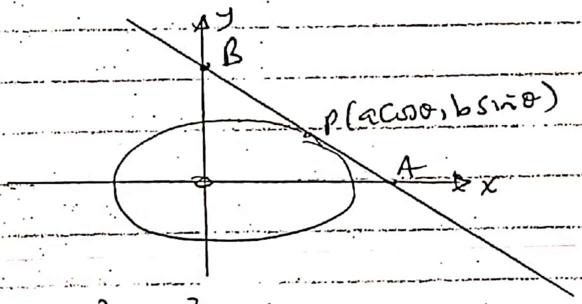
b Let a = first of G.P & A.P

Let $d = r = x$.

A.P	G.P
$u_1 = a$	$u_1 = a$
$u_3 = a+2x$	$u_2 = ax$
$u_4 = a+3x$	

	B1
$\Rightarrow$ $a+3x=10$ --- i	B1
$a+2x=ax$ --- ii	
from eqn i; $a=10-3x$	
Subst $a=10-3x$ in eqn ii	M1
$10-3x+2x=(10-3x)x$	
$10-x=10x-3x^2$	
$3x^2-11x+10=0$	
$3x^2-5x-6x+10=0$	M1
$x(3x-5)-2(3x-5)=0$	
$(x-2)(3x-5)=0$	
$x=2$; $x=5/3$	A1 for both x-values
when $x=2$, $a=10-3(2)=4$	B1
when $x=5/3$, $a=10-3\left(\frac{5}{3}\right)=5$	B1
	12 marks

15



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{2y}{b^2}\frac{dy}{dx}=\frac{-2x}{a^2}$$

$$\frac{dy}{dx}=\frac{-b^2x}{a^2y}$$

at $P(a\cos\theta, b\sin\theta)$; $\frac{dy}{dx}=\frac{-b^2(a\cos\theta)}{a^2(b\sin\theta)}$

$=\frac{-b\cos\theta}{a\sin\theta}$ B1

Equation of the tangent

$\frac{y-b\sin\theta}{x-a\cos\theta}=\frac{-b\cos\theta}{a\sin\theta}$ M1

$a y \sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$

$a y \sin\theta + bx\cos\theta = ab\sin^2\theta + ab\cos^2\theta$

$a y \sin\theta + bx\cos\theta = ab(\sin^2\theta + \cos^2\theta)$ M1

$a y \sin\theta + bx\cos\theta = ab$ A1

$\frac{ay\sin\theta}{ab} + \frac{bx\cos\theta}{ab} = \frac{ab}{ab}$

$\frac{y}{b}\sin\theta + \frac{x}{a}\cos\theta = 1$

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ # B1

At A; $y = 0$

$\frac{x}{a}\cos\theta = 1$

$x = \frac{a}{\cos\theta}$

$A\left(\frac{a}{\cos\theta}, 0\right)$ B1

At B: $x = 0$

$\frac{y}{b}\sin\theta = 1$

$y = \frac{b}{\sin\theta}$

$B\left(0, \frac{b}{\sin\theta}\right)$ B1

Area, $A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{a}{\cos\theta}\right)\left(\frac{b}{\sin\theta}\right)$ M1

$= \frac{ab}{2\sin\theta\cos\theta}$

$= \frac{ab}{\sin 2\theta}$ # B1

for M; $M\left(\frac{0 + a/\cos\theta}{2}, \frac{\frac{b}{\sin\theta} + 0}{2}\right)$

1 1 1

$$\therefore M\left(\frac{a}{2\cos\theta}, \frac{b}{2\sin\theta}\right)$$ B1

$x = \frac{a}{2\cos\theta}$ $\quad y = \frac{b}{2\sin\theta}$

$\cos\theta = \frac{a}{2x}$ $\quad \sin\theta = \frac{b}{2y}$

but $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{a}{2x}\right)^2 + \left(\frac{b}{2y}\right)^2 = 1$$ M1

$$\frac{a^2}{4x^2} + \frac{b^2}{4y^2} = 1$$ is the locus of m B1

12 marks.

16 $y = Ae^{x^2}$, $A = y/e^{x^2}$ — for $\frac{dy}{dx}$

$\frac{dy}{dx} = 2xAe^{x^2}$ B1

$\frac{dy}{dx} = 2x \cdot \frac{y}{e^{x^2}} \cdot e^{x^2}$ M1

$\frac{dy}{dx} = 2xy$ A1

b $\frac{dT}{dt} \propto (T-70)$ B1

$\frac{dT}{dt} = -k(T-70)$ B1 for D.E — (Accept only T) Any other award zero only.

$\frac{dT}{T-70} = -k\,dt$ ~~M1~~

$\int \frac{dT}{T-70} = -k\int dt$ M1 for separation of variables & integral sign

$\ln(T-70) = -kt + c$ ~~M1~~ A1

at $t=0$ $T=160$	
$\ln(T-70) = -k(t)+C$	
$\ln(160-70) = -k(0)+C$	M_1 for substn
$\ln 90 = C$	~~B_1~~ A_1
$\ln(T-70) = -kt+\ln 90$	
$kt = \ln 90 - \ln(T-70)$	
~~$kt = \ln\left(\frac{90}{T-70}\right)$~~	
at $t=20$, $T=140$	
$20k = \ln\left(\frac{90}{140-70}\right)$	
$20k = \ln\left(\frac{90}{70}\right)$	
$20k = \ln\left(\frac{9}{7}\right)$	
$k = \frac{1}{20}\ln\left(\frac{9}{7}\right)$	A_1
$\therefore \ln(T-70) = -\frac{t}{20}\ln\left(\frac{9}{7}\right)+\ln 90$	
for $T=120$, $t=?$	
$\ln(120-70) = -\frac{t}{20}\ln\left(\frac{9}{7}\right)+\ln 90$	M_1
$\ln 50 = -\frac{t}{20}\ln\left(\frac{9}{7}\right)+\ln 90$	
$\frac{t}{20}\ln\left(\frac{9}{7}\right) = \ln 90 - \ln 50$	
$t = \dfrac{20(\ln 90 - \ln 50)}{\ln\left(\frac{9}{7}\right)}$	
$t = 46.7770$ minutes.	A_1
Required time $= 46.7770 - 20$	
$= 26.7770$ minutes.	B_1
END	12 marks