S6 post mock examinations 2005
Maths

SECTION A: (40 marks)

1. Solve: $6\cos 2x - 5\sin x + 1 = 0$; $0^o \leq x \leq 360^0$
2. A line segment AP = a moves in the x-y plane, remaining parallel to the x-axis so that its left end point A slides along the circle $x^2 + y^2 = a^2$. Show that the locus of p is a cirlcle and write down its centre and radius.
3. Solve the differential equation $x\ln x \frac{dy}{dx} + y = 2\ln x$, given that y(e) = 2
4. Determine the equation of a plane passing through a point (2, 2, – 2) which is parallel to the plane x – 2y – 3Z = 0 .
5. Show that $\int_o^{\frac{1}{\sqrt{2}}} \frac{x\sin^{-1} x^2}{\sqrt{1-x^4}}\,dx = \frac{\pi^2}{144}$
6. Given that $y = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}$

   Find $\frac{dy}{dx}$ at x = ½
7. Assuming that x is so small that terms in x^3 and higher powers can be neglected, show that Maclaurin's series for

   $\ln\left\{\frac{x+1}{\sqrt{1-2x}}\right\} \approx 2x + \frac{1}{2}x^2$

   State the range of values of x for which the expansion is valid.
8. Solve: $(2 + i)Z^2 - Z + (2 - i) = 0$
9. (a) Differentiate

   (i) $\frac{e^x}{x\sin x}$

   (ii) $\frac{\sin^{-1}(x)}{\sqrt{(1-x^2)}}$

   (iii) $\tan^{-1}(\sec x + \tan x)$

   (b) The gradient of the tangent to a certain curve is directly proportional to the product of the ordinate and abscissa at the point of contact. Given that the curve passes through the points (2,3), (4,12), find its equation.

10. Evaluate

(i) $\displaystyle\int_{5/4}^{2} \left(\frac{2-x}{x-1} \right)^{\frac{1}{2}} dx$

(ii) $\displaystyle\int \frac{\sin x\,dx}{9 + 16\cos^2 x}$

11. Sketch the curve $y = \dfrac{x^3}{X^2 - 1}$

12. (a) Show that $y = m(x - 1) + {}^{2}/_{m}$ is a tangent to the parabola $y^2 = 8(x - 1)$ for all values of m.

Find the;

(i) angle between the tangents to this parabola from the point (4,5),

(ii) equations of these possible tangents,

(iii) co-ordinates of the points of contact of each of the tangents with the parabola.

(b) If P is the point $(at^2, 2at)$, on the parabola $y^2 = 4ax$ and S is the focus, find:

(i) the locus of the midpoint of SP, and interpret your answer geometrically;

(ii) the normal at P to the parabola $y^2 = 4ax$ meets the curve again at Q $(aT^2, 2aT)$. Write down two expressions containing t and T. Hence express T in terms of t.

13. (a) The position vectors of the points A and B with respect to the origin O are $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ respectively.

Determine the equation of line AB

(b) Find the equation of a plane OPQ, where O is the origin and P and Q are the points (0,3,0) and (1,0,2) respectively.

(c) (i) Find the co-ordinates of point R at which line AB meets the plane in (b) above.

(ii) Show that S (1, – 1, 2) lies on OR