NUMERICAL METHODS (REVISION)

ERRORS

1. A value of P=673.16 was obtained in a certain experiment. Given that the relative error in the measurement of this value is 0.01%, find the limits within which the value of P is expected to lie.
2. The relative error obtained in determining the value of T=873.16 is 0.02%, find
   (i) The error in the measurement of this value
   (ii) The value within which T lies
3. Given that $T = 29.7°C$ and $M = 48kg$
   (i) Write down the possible errors in each value
   (ii) Write down the ranges within which each value lie
4. Given two parcels, with A having a mass of 2kg and B having a mass of 6kg, each mass being rounded off to the given decimal places. Find the interval within which the total mass lies
5. A student measured the length and the breadth of a rectangular sheet of iron as 3.6m and 2.3m respectively.
   (i) Write down the maximum possible error in each measurement
   (ii) Find the limits within which the area of the sheet lies
6. Given that $M = \frac{4.05+2.023}{5.67-4.9312}$
   (i) Write down the maximum possible error in each of the values above
   (ii) Find the range of values within M lies
   (iii) Estimate the maximum percentage error in M
7. Find the interval within which 4^x is expected to lie, if the measured of $x = 2.4$
8. Given $A = 3, B = 12, C = 16$ all to the nearest integer, find the maximum values of:
   (i) $\frac{A+C}{B}$ (ii) $\frac{AB}{C}$ (iii) $\frac{A}{B} - \frac{B}{C}$
9. Three adjacent blocks of sugar cane with areas 400 acres, 600acres, 900 acres to the nearest fifty acres. What
   (i) The possible error in each measurement
   (ii) The greatest total area of the blocks
   (iii) The least possible area of the three blocks
10. Find the range of values within which $\frac{0.38}{4.28} + \frac{030}{2.14}$ lies, if the numbers are rounded off to the given number of decimal place.
11. Given that $X = 2.7654, Y = 3.80$, state the maximum possible error in X and Y, hence find the interval within which the following are expected to lie
    (i) X+Y
    (ii) X-Y
    (iii) $\frac{X}{Y}$
    (iv) $\frac{X+Y}{XY}$
    (v) $X\left(Y - \frac{X}{Y}\right)$
12. The numbers X and Y are measured with possible errors of ΔX and ΔY respectively.
    (a) Show that the maximum relative error in XY is $\frac{|\Delta X|Y + |\Delta Y|X}{XY}$

(b) If $X = 6.42$ and $Y = 29.3$ are each rounded off to the given number of decimal places, calculate the

(i) Write down the maximum possible error in X and Y

(ii) Find the interval in which the product XY lies.

13. Find the range of values within which $\frac{1.362(7.54-13.2)}{4.7}$ lies, if the numbers are rounded off to the given number of decimal place.

14. Find the maximum absolute error in the expression $\frac{\sqrt{A}}{B^2C^3}$ given that A=2.8, B=6.4 and C=3.4 all rounded off

15. Given that $Z = \quad |\boldsymbol{x}||\boldsymbol{y}|\boldsymbol{sin\theta}$

(a) Deduce that the maximum possible relative error in Z is given by

$$\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + cot\theta|\Delta\theta|$$

where $\Delta x, \Delta y$ *and* $\Delta\theta$ are small numbers compared to x, y and θ respectively

(b) find the maximum percentage relative error in Z, given that $x = 5.5cm$, $y = 6.8cm$ and $\theta = 45°$

16. Given that the area of a triangle whose adjacent sides are of size a and b, and the angle between the sides is θ $A = \quad |\boldsymbol{a}||\boldsymbol{b}|\boldsymbol{sin\theta}$ given that $a = 2.5cm$, $b = 3.4cm$ and $\theta = 30°$

(a) Write down the possible errors in each measurement

(b) Error made in the area

(c) Value within which the area is expected to lie,

17. Find the range of values within which the exact value of $2.6954\left(4.6006 - \frac{16.175}{0.82}\right)$ lies, if the numbers are rounded off to the given number of decimal place.

18. Given that $A = 2.5, B = 1.71$, C=16.01, state the maximum possible error in A, B and C, hence find the limits within which the following are expected to lie

(i) $\frac{A+C}{B}$ (ii) $\frac{AB}{C}$

Give your answer to 2 decimal places

19 The numbers x and y are approximated by A and B with possible errors of e_1 and e_2 respectively.

(a) Show that the absolute relative error in the quotient $\frac{x}{y}$ is given by

$$\left|\frac{e_1}{A}\right| + \left|\frac{e_2}{B}\right|$$

(b) If $A = 4.67$ and $B = 1.813$ are each rounded off to the given number of decimal places, calculate the

(i) Write down the maximum possible error in A and B

(ii) Find the absolute relative error in the quotient $\frac{4.67}{1.813}$

(iii) Find the limit within which the quotient $\frac{4.67}{1.813}$ lies

20. Given that $A = 3.3366, B = 0.559$, state the maximum possible error in A and B, hence find

(i) Absolute error in the quotient $\frac{A}{B}$

(ii) the limits within which $\frac{A}{B}$ is expected to lie

Give your answer to 3 decimal places

21. The numbers M and N are approximated with possible errors of e_1 and e_2 respectively.
    (a) Show that the maximum absolute error in the quotient MN is given by
$$\frac{|e_1|\mathrm{N} + |e_2|\mathrm{M}}{MN}$$
    (b) Given that M=6.43, N=37.2, write down the maximum possible error in M and N. Hence find the interval in which
        (i) Product MN lies
        (ii) The quotient $\frac{M}{N}$ lies

LINEAR INTERPOLATION

1. The table below shows the values of a function f(x)

x	1	2	3
f(x)	2	8	11

Find the value of
   (i) $f(1.15)$
   (ii) X corresponding to $f(x) = 9$
   (iii) X when $f(x) = 6.4$
   (iv) f(x) when x=4

2. The table below shows the distance in cm travelled by a spider on the ceiling in four seconds of its motion

t(s)	0	1	2	3	4
d(cm)	0	5	38	68	104

Use linear interpolation to estimate
   (i) Distance travelled when $t = 2.3$
   (ii) The time when the distance travelled is $100cm$

3. The distance between Kajjansi and Kampala town is 20km.Seguuku, Zaana and Kibuye are 8km, 12km and 16km respectively from Kajjansi and the taxi charges are also respectively 500/=, 800/=, 1000/= and 1500/=. Nakimboowa is going to Visit her cousin brother Opio living 11km from Kajjansi
   (i) Find how much she will be charged in this taxi
   (ii) Suppose she had only 850/= and the taxi left her at a distance worth the money, find how far from Kampala town the taxi leaves her

4. The table below shows the values of two variables x and y

x	29	-0.1	-2.9
y	12	20	30

Use linear interpolation to find the value of
   (i) x when $y = 16$
   (ii) y when $x = -1$

5. The table below shows the distance, r(m) of a particle along a straight line after time intervals of 3 seconds

t(s)	0	3	6	9	12	15	18
r(m)	8.6	7.5	4.6	3.1	2.8	2.3	1.8

Use linear interpolation or extrapolation to estimate

(i) Distance travelled when $t = 11.4s$

(ii) The time when r=6.5m

(iii) r when t=21s

(iv) t when r=1.0m

6. The table below shows the values of P and Q

P	0	8	12	20
Q	9.2	6.0	4.4	1.5

Use linear interpolation to find,

(i) Q when P=15

(ii) P when Q=5.0

7. The table below shows the values of;

x	6	10	15	20
t	13	25	39	56

Use linear interpolation to find,

(i) t when x=12

(ii) x when y=48

8. The table below shows the values of;

Ɵ	0^1	6^1	12^1	18^1	24^1	30^1
$\sin 10^0$	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822

Use linear interpolation to find,

(i) $\sin 10°16^1$

(ii) $\sin^{-1} 0.1747$

9. The table below shows the values of a function f(x)

x	10	20	30	40
f(x)	0.1708	0.1679	0.1650	0.1622

Use linear interpolation the value of;

(i) $f(36)$

(ii) X when $f(x) = 0.1685$

10. If $f(0.120) = 1.7652$, $f(0.125) = 1.7666$, find $f(0.123)$

11. A body freely moving from rest is acted on by some variable force F(N) as shown in the table below

Distance (m)	0	4	10	15	20	25	31
Force, F(N)	5	8.0	11	12	13.6	10.5	5

Find;

(i) The force when the body has travelled a distance of 25m

(ii) The distance when a force F=12.8N is acting on the body

(iii) The force when the boy has travelled a distance of 34.7m

12. The table shows values of x and f(x)

x	0.8	0.9	1.0	1.1	1.2	1.3
f(x)	0.28	0.260	0.241	0.218	0.192	0.172

Find;

(i) The value of $f(1.07)$

(ii) The value of x when f(x)=0.231

(iii) The value of $f(1.4)$

13. Given the table below

T	1	2	3	4	5	6
D	5	17	34	57	85	105

Find;

(i) The value of D when $T = 4.6$

(ii) The value of T when $D = 70$

(iii) The value of D when $T = 7$

14. Given the table below

T	0	120	240	360	480	600
ϴ	100	80	75	65	56	48

Find;

(i) The value of θ when $T = 3704.6$

(ii) The value of T when $\theta = 70.2$

(iii) The value of θ when $T = 720$

(iv) The value of T when $\theta = 38$

15. The table below shows the temperature (℃) at different times for a certain oven

Time (min)	0	10	20	30	40	50	60
Temp (^{0}C)	40	80	100	130	180	200	300

Find;

(i) Time when temperature will be at 53℃

(ii) Temperature after 54 minutes

(iii) Expected temperature after 70 minutes

16. The charges of sending parcels by a certain distributing company depend on the weights of the parcels. For the parcels of weight 500g, 1kg, 1.5kg, 2kg, 3kg and 5kg, the charges are 1000/=, 2000/=, 3500/=, 4000/= respectively. Estimate

(i) What the distributor would charge for a parcel of weight 27kg

(ii) If the sender pays 6200/= what is the weight of his parcel

17. Show that the linear extrapolation formula for approximating a value f(c), can be given by $f(c) = f(b) + \left(\frac{c-b}{b-a}\right)\left(f(b) - f(a)\right)$

18. The diameter, d(mm) of an egg produced by a hen of a certain farm depends on the mass (gm) of the layer' mash ratio it is fed as shown below

Food ratio,m(gm)	200	290	330	410	440	500
Diameter, d(mm)	30.2	34.2	36.2	40.1	41.0	46.2

Assuming the egg to be spherical;

(i) The optimum amount of the food the hen should be given if it is to produce an egg of average diameter of 38.2gm

(ii) The radius of an egg if the food ratio supplied is 540gm

19. The table below shows the values of;

x	5	10	15
t	13	24.1	38.7

Use linear interpolation or extrapolation to find,

(i) t when x=8

(ii) x when t=44

20. The table below shows the variation of temperature (°C) with time in a certain laboratory experiment

Time (s)	0	120	240	360	480	600
Temp (^{0}C)	100	80	75	65	56	48

Use linear interpolation or extrapolation to find;

(i) Time when temperature will be at 76°C

(ii) Temperature after 400 s

(iii) Expected temperature after 620s

(iv) Time when temperature will be at 40°C

Trapezium rule

1. (a) Use trapezium rule with six strips to estimate $\int_0^{\pi} xsinx\,dx$ correct to 2dp.
   (b) Determine the percentage relative error in your estimation.
2. Use trapezium rule to estimate the approximate value of $\int_0^1 \frac{1}{1+x^2}dx$ using 6 ordinates correct to 3 decimal places
3. (a) Use trapezium rule with six strips to estimate $\int_2^4 \frac{10}{2x+1}dx$ correct to 4dp.
   (b) Determine the percentage error in your estimation and suggest how this may be reduced.
4. (i) Use the trapezium rule to estimate the area of y=5^{2x} between the x-axis, $x = 0$, and $x = 1$, Using five sub intervals. Give your answer correct to 3dp.
   (ii) Find the exact value of $\int_0^1 5^{2x}\,dx$
   (iii) Find the percentage error in the calculations in (a) i) and (a) ii) above
5. **(i)** use the trapezium rule to estimate the area of $y = 3^x$ between the x-axis, $x = 1$ and $x = 2$ using five strips . give your answer correct to 4s.f
   (ii) find the exact values of $\int_1^2 3^x\,dx$
   (iii) Find the percentage error in calculation (i) and (ii) above
6. Use the trapezium rule with 7ordinates to find the value of $\int_0^{\pi} \sqrt{(1+sinx)}dx$ correct to 2dp
7. Use trapezium rule with 6 ordinates to evaluate $\int_0^1 e^{-x^2}dx$ correct to 2 decimal places
8. Use trapezium rule with 6 ordinates to estimate $\int_1^2 \frac{In\,x}{x}dx$ correct to 3 decimal places
9. Use trapezium rule with 5 strips to approximate $\int_0^1 \frac{1}{1+x}dx$ correct to 1 decimal places
10. (a) use trapezium rule to estimate the integral value of $\int_0^{\pi/3} tanx\,dx$ using five sub-intervals correct to 3dp.
    (b) (i) find the exact value of $\int_0^{\pi/3} tanx\,dx$
    (ii) find the percentage error in your estimation
11. Use trapezium rule with 4 sub-intervals to evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}dx$ correct to 3 dP
12. Use trapezium rule with 6 ordinates to evaluate $\int_0^2 \frac{1}{1+x^2}dx$ correct to 3 decimal places
13. Use trapezium rule with 5 sub-intervals to evaluate $\int_2^4 \frac{5}{1+x}dx$ correct to 3 decimal places
14. A student Used trapezium rule with 5 sub-intervals to evaluate $\int_2^3 \frac{x}{x^2-3}dx$ correct to 3 decimal places
    Determine
    (a) The value the student obtained
    (b) The actual value of the integral
    (c) The error the student made in the estimation and suggest how the student can reduce the error
15. Use trapezium rule to estimate $\int_1^{1.4}(x+tanx)\,dx$ using five ordinate
16. Using five subintervals, find the value of $\int_0^1(x^2e^x)\,dx$ correct to 2dp and hence find the percentage error in your answer

17. Use the trapezium rule to estimate the area of $y = e^{-2x}$ between the x-axis, $x = 1$ and $x = 2$. Using six ordinates , Give your answer correct to 2 significant figures
18. Use trapezium rule to evaluate
    (a) $\int_2^{8/3} x^2(x-1)\,dx$ use five sub intervals
    (b) $\int_2^3 \frac{dx}{x(x^2+x)^{1/2}}$ use five ordinates
    Give your answers to 2 decimal places
19. (a) Use trapezium rule with seven ordinates to estimate $\int_0^{\pi} x\sin x\,dx$ correct to 2dp.
    (b) Determine the percentage error in your estimation and suggest how this may be reduced.
20. Use trapezium rule with eight ordinates to estimate $\int_1^8 \log_{10} e^{3x}\,dx$ correct to 3dp.

GRAPH WORK

1. Show that the positive real root of the equation $2x^2 + 3x - 3 = 0$, lies between 0 and 1.
2. On the same axes, draw graphs of $y = 3 - 3x$ and $y = 2x^2$ to show that the root of the equation $2x^2 + 3x - 3 = 0$ lies between -3 and -2
3. Show that the positive real root of the equation $x^3 - 3x - 1 = 0$, lies between 1 and 2
4. On the same axes, draw graphs of $y = 3x - 1$ and $y = x^3$ to show that the root of the equation $x^3 - 3x - 1 = 0$ lies between 0 and 1
5. Using suitable graphs and plotting them on the same axes, find the real root of the equation $e^{2x}\sin x - 1 = 0$, in the interval x=0.1 and x=0.8
6. Show graphically that equation $e^{-x} = x$ has only one real root between 0.5 and 1.0.
7. Show graphically that equation $e^{x} = x$ has only one real root between 0.5 and 1.0.
8. On the same axes, draw graphs of $y = 9x - 4$ and $y = x^3$ to show that the root of the equation $x^3 - 9x + 4 = 0$ lies between 2.5 and 3
9. Show that the positive real root of the equation $4 + 5x^2 - x^3 = 0$, lies between 5 and 6
10. On the same axes, draw graphs of $y = x + 1$ and $y = \tan x$ to show that the root of the equation $\tan x - x - 1 = 0$ lies between 1 and 1.5
11. Using suitable graphs and plotting them on the same axes, find the real root of the equation $5e^{x} = 4x + 6$, in the interval $x = -2$ and $x = -1$
12. On the same axes, draw graphs of $y = 2x + 1$ and $y = \log_e(x+2)$ to show that the root of the equation $\log_e(x+2) - 2x - 1 = 0$ lies between -1 and 0
13. Using suitable graphs and plotting them on the same axes, find the real root of the equation $9\log_{10} x = 2(x-1)$, in the interval x=3 and x=4
14. On the same axes, draw graphs of $y = 2x$ and $y = \tan x$ to show that the root of the equation $2x = \tan x$ lies between 1.1 and 1.2. Hence use linear interpolation to find the value of the root correct to 2 decimal places
15. Show graphically that equation $\log_e x + x = \frac{1}{2}$ has only one real root between 0.5 and 1.0. Hence use linear interpolation to find the value of the root correct to 2 decimal places

NEWTON RAPHSON METHOD

1. (a) Show that the equation $2x^2-6x-3=0$ has a real root between $x=3$ and $x=4$
   (b) Using linear interpolation, find the first approximation for this root to 2dp
   (c) Show that the Newton Raphson formula for approximating the root of the equation $2x^2-6x-3=0$ is given by

$$x_{n+1}=\frac{2x_n^2+3}{4x_n-6}$$

   (d) Use the formula above, with an initial approximation of $x_0=3$, to find the root of the given equation correct to two decimal places.

2. (a) Show that the equation $f(x)=3xe^x-1=0$ has a real root between 0.2 and 0.3
   (b) Using linear interpolation, find the first approximation for this root to 3dp
   (c) Show that the Newton Raphson formula for approximating the root of the equation $3xe^x-1=0$ is given by

$$x_{n+1}=\frac{3x_n^2e^{x_n}+1}{3e^{x_n}(x_n+1)}$$

   (d) Use the formula above, to find the root of the given equation correct to 3 decimal places.

3. (a) Show that the equation $sinx-\left(\frac{x}{2}\right)=0$ has a real root between 1 and 2
   (b) Show that the Newton Raphson formula for approximating the root of the equation $sinx-\left(\frac{x}{2}\right)=0$is given by

$$x_{n+1}=\frac{2(x_n\cos x_n-\sin x_n)}{2\cos x_n-1}$$

   (c) Use the formula above, to find the root of the given equation correct to 2 decimal places.

4. (a) Show that the equation $2x^2+3x-4=0$ has a real root between $x=0.2$ and $x=1.0$
   (b) Show that the Newton Raphson formula for approximating the root of the equation $2x^2+3x-4=0$is given by

$$x_{n+1}=\frac{2x_n^2+4}{4x_n+3}$$

   (c) Use the formula above, to find the root of the given equation correct to two decimal places.

5. (a) Show that the equation $x^3-5x-40=0$ has a real root between $x=3$ and $x=4$
   (b) Show that the Newton Raphson formula for approximating the root of the equation $x^3-5x=40$ is given by

$$x_{n+1}=\frac{2x_n^3+40}{3x_n^2-5}$$

   (c) Use the formula above, to find the root of the given equation correct to two decimal places.

6. (a) Show graphically that the root of the equation $e^x\text{sinx}=1$ has a root between 0.2 and 1.2
   (b) Use the Newtown- Raphson method to find the root of the equation in (a) correct to 2 decimal places

7. (a) On the same axes, draw graphs of $y=x-0.5$ and $y=\text{Inx}$ to show that the

root of the equation $x + \text{In}x = 0.5$ lies between 0 and 1

(b) Use the Newton Raphson method to calculate the root of the equation $x + \text{In}x = 0.5$, taking the approximate root in (a) as an initial approximation. Correct your answer to 3 decimal places

8. (a) On the same axes, draw graphs of $y = x$ and $y = e^x$ to show that the root of the equation $e^x - x = 0$ lies between 0.1 and 0.9

(b) Use the Newton Raphson method to calculate the root of the equation $e^x - x = 0$, taking the approximate root in (a) as an initial approximation. Correct your answer to 3 decimal places

9. (a) On the same axes, draw graphs of $y = x^2$ and $y = 2x + 1$ to show that the root of the equation $x^2 - 2x - 1 = 0$ lies between 2 and 3

(b) Use the Newton Raphson method to calculate the root of the equation $x^2 - 2x - 1 = 0$, taking the approximate root in (a) as an initial approximation. Correct your answer to 2 decimal places

10. (a) Show graphically that the root of the equation $sinx - cosx = 0$ has a root between 0 and 1

(b) Use the Newtown- Raphson method to find the root of the equation in (a) correct to 2 decimal places

11. Show the equation $e^x + x - 4 = 0$ has a real root between 1 and 1.2.

(b) Use the NRM to find the root of the equation correct to 3 significant figures

12. (a) Show that the equation $f(x) = x^3 + 3x - 9$ has a root between $x = 1$ and $x = 2$.

(b) Using the newton Raphson formula, to estimate the root of the equation rounded off to 2 s.f.

13. (a) Show that the equation $e^x simx = 1$ has a positive root between $x = 0$ and $x = 1$.

(b) Using the newton Raphson formula once, estimate the root of the equation rounded off to 2 d.p.

14. (a) Show that the equation $e^x + x^3 - 4x = 0$ has a real root between 1.0 and 1.5

(b) Show that the Newton Raphson formula for approximating the root of the equation $e^x + x^3 - 4x = 0$is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 2x^3{}_n}{e^{x_n} + 3x^2{}_n - 4}$$

(c) Use the formula above, to find the root of the given equation correct to 2 dp.

15. (a) Show that the equation $x + 3tanx = 0$ has a real root between 2 and 3

(b) Show that the Newton Raphson formula for approximating the root of the equation $x + 3tanx = 0$ given by

$$x_{n+1} = \frac{3}{2}\left(\frac{2x_n - \sin 2x_n}{3 + \cos^2 x_n}\right)$$

(c) Use the formula above, to find the root of the given equation correct to 2 decimal places.

16. (a) Show that if x is the square root of 7, then there is only one value of x between 2 and 3

(b) Show that the Newton Raphson formula for approximating the square root of 7 is given by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{7}{x_n}\right)$$

(c) Use the formula above, to find the root, correct to 2 decimal places

17. (a) On the same axes, draw graphs of $y = x^2$ and $y = sin2x$ to show that the root of the equation $x^2 = sin2x$ for $0 \leq x \leq \pi/2$
    (b) Use the Newton Raphson method to calculate the root of the equation $x^2 = sin2x$, taking the approximate root in (a) as an initial approximation. Correct your answer to 2 decimal places

18. (a) On the same axes, draw graphs of $y = e^{-x}$ and $y = sinx$ to show that the root of the equation $e^{-x}\, sinx = 0$ lies between 0 and 1.0
    (b) Use the Newton Raphson method to calculate the root of the equation $e^{-x}\, sinx = 0$, taking the approximate root in (a) as an initial approximation. Correct your answer to 2 decimal places
19. (a) Show that the equation $e^x + x = 10$ has a positive real root between $x = 2$ and $x = 2.5$
    (b) Show that the Newton Raphson formula for approximating the root of the equation $e^x + x = 10$ is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 10}{e^{x_n} + 1}$$

    (c) Use the formula above, to find the root of the given equation correct to 2 decimal places.

ITERATIVE FORMULAS

1. Show that the iterative formula for solving the equation $x^2 - 5x + 2 = 0$ can be written in two ways as

$$\boldsymbol{x_{n+1}} = \mathbf{5} - \frac{\mathbf{2}}{\boldsymbol{x_n}} \text{ or } \boldsymbol{x_{n+1}} = \frac{\boldsymbol{x_n}^{\mathbf{2}}+\mathbf{2}}{\mathbf{5}}$$

   Starting with $\boldsymbol{x_0} = \mathbf{4},$ deduce the more suitable formula for solving for the equation and hence find he root correct to 2dp.
2. Show that the iterative formula for solving the equation $x^3 - x - 1 = 0$ is

   $\boldsymbol{x_{n+1}} = \sqrt{\left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{x_n}}\right)}$ hence starting with $x_0 = 1$find the root of the equation correct to 3s.f
3. Given two iterative formulae 1 and 2 (shown below) for calculating the positive root of the quadratic equation $f(x) = 0$

   Formula 1: $\boldsymbol{x_{n+1}} = {}^{\mathbf{1}}\!/_{\mathbf{2}}\left(\boldsymbol{x_n}^{\mathbf{2}} - 1\right)$ Formula 2: $\boldsymbol{x_{n+1}} = {}^{\mathbf{1}}\!/_{\mathbf{2}}\left(\frac{\boldsymbol{x_n}^{\mathbf{2}}+\mathbf{1}}{\boldsymbol{x_n}-\mathbf{1}}\right)$

   Taking $x_0 = 2.5$, use each formula twice to two decimal places to decide which is the more suitable formula. Give a reason for your answer
4. If b is the first approximation to the root of the equation $x^2 = a$, show that the second approximation to the root is given by $\frac{b+\frac{a}{b}}{2}$. Hence taking $b = 4$, estimate $\sqrt{17}$ correct to 3dp
5. An iterative formula for solving the equation $f(x) = 0$, **is given by**

$$\boldsymbol{x_{n+1}} = {}^{\mathbf{1}}\!/_{\mathbf{3}}\left(\frac{\mathbf{2}\boldsymbol{x_n}^{\mathbf{3}} + 12}{\boldsymbol{x_n}^{\mathbf{2}}}\right)$$

(i) Taking $x_0 = 2$, deduce the root of the equation to three decimal places

(ii) Find the equation which is solved by this iterative formula

6. Show that the iterative formula for finding the fifth root of a number N is given $\frac{1}{5}\left(4\boldsymbol{x_n} + \frac{\boldsymbol{N}}{\boldsymbol{x_n}^4}\right)$ Hence use $x_0 = 2$ to find $\sqrt[5]{50}$ correct to 2 decimal places

7. **(a) (i)** Show that the equation $e^x - 2x - 1 = 0$ has a root between $x = 1$ and $x = 1.5$

(ii) Use linear interpolation to obtain an approximation for the root

(b) (i) Solve the equation in (a)(i), using each formula below twice. Take the approximation in (a) (ii) as the initial value.

Formula 1: $\boldsymbol{x_{n+1}} = {}^{\boldsymbol{1}}/_{\boldsymbol{2}}\,(e^{x_n} + 1)$ **Formula 2:** $\boldsymbol{x_{n+1}} = \frac{e^{x_n}(x_n - 1) + 1}{e^{x_n} - 2}$

(ii) Deduce with a reason which of the two formula is appropriate for solving the given equation in (a)(i). Hence write down a better approximate root, correct to 2dp

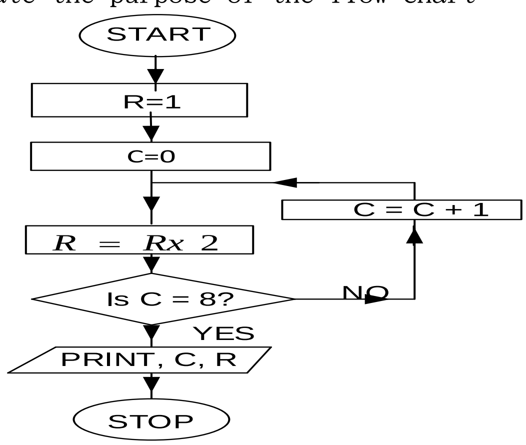
8. Given the following iterative formulas

(i) $\boldsymbol{x_{n+1}} = \boldsymbol{5} - \frac{\boldsymbol{3}}{\boldsymbol{x_n}}$ (ii) $\boldsymbol{x_{n+1}} = \frac{\boldsymbol{1}}{\boldsymbol{5}}\left(\boldsymbol{x_n}^{\boldsymbol{2}} + \boldsymbol{3}\right)$

Taking $\boldsymbol{x_0} = \boldsymbol{5}$ deduce a more suitable iterative formula for solving the equation. Deduce also the possible original equation

FLOW CHARTS

1. Perform a dry run and state the purpose of the flow chart



2. (a) Show that the iterative formula based on Newton Raphson' s method for approximating the fourth

root of a number N is given by $x_{n+1} = {}^{3}/_{4}\left(x_n + \frac{N}{3x_n^3}\right)$

(b) Draw a flow chart that;

(i) Reads N and the initial approximation x_0 of the root

(ii) Computes and prints the root after four iterations

(c) Taking $N = 39.0$, $x_0 = 2.0$, perform a dry run for the flow chart, give your root correct to three decimal places

3. (a) Show that the iterative formula based on Newton Raphson' s method for finding the natural logarithm of a number N is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + N}{e^{x_n}}, \qquad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Draw a flow chart that;

(i) Reads N and the initial approximation x_0 of the root

(ii) Computes and prints the natural logarithm after four iterations and gives the natural logarithm to three decimal places

(c) Taking, $N = 10, x_0 = 2$, perform a dry run for the flow chart, give your root correct to three decimal places

4. A shop offers a 25% discount on all items in their store and a second discount of 5% for paying cash.
   (a) Construct a flow chart for the above information
   (b) Perform a dry run for;
   (i) A shoe 75,000/= cash
   (ii) A shirt 45,000/= credit
5. Given that a man deposited 100,000/= to a bank which gives a compound interest rate of 5% . Draw a flow chart to compute the amount of money accumulated after 5 years, and perform a dry run for the flow chart
6. (a) Show that the iterative formula base on Newton Raphson' s method for approximating the cube
   root of a number N is given by
   $$x_{n+1} = {}^{1}\!/_{3}\left(2x_n + \frac{N}{x_n^2}\right) \quad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$
   (b) Draw a flow chart that;
   (i) Reads N and the initial approximation x_0 of the root
   (ii) Computes and prints the root to three decimal places
   (c) Taking $N = 54$, $x_0 = 3.7$, perform a dry run for the flow chart, give your root correct to three decimal places
7. (a) Show that the iterative formula base on Newton Raphson' s method for approximating the fourth
   root of a number N is given by
   $$x_{n+1} = {}^{3}\!/_{4}\left(x_n + \frac{N}{3x_n^3}\right) \quad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$
   (b) Draw a flow chart that;
   (i) Reads N and the initial approximation x_0 of the root
   (ii) Computes and prints the root to two decimal places
   (c) Taking $N = 35$, $x_0 = 2.3$, perform a dry run for the flow chart, give your root correct to two decimal places
8. (a) Show that the iterative formula base on Newton Raphson' s method for finding the root of the
   $x = N^{1/5}$ is given by
   $$x_{n+1} = \left(\frac{4x_n^5 + N}{5x_n^4}\right), \qquad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$
   (b) Draw a flow chart that;
   (i) Reads N and the initial approximation x_0 of the root
   (ii) Computes the root to three decimal places
   (iii) Prints the root and the number of iterations, n

(c) Taking $N = 50$, $x_0 = 2.2$, perform a dry run for the flow chart, give your root correct to three decimal places

9. (a) Show that the iterative formula based on Newton Raphson's method for finding the root of the

$2\,In x - x + 1 = 0$ is given by

$$x_{n+1} = x_n\left(\frac{2\,In\,x_n - 1}{x_n - 2}\right), \qquad n = 0, 1, 2 \dots\dots\dots\dots\dots\dots$$

(b) Draw a flow chart that;

(i) Reads the initial approximation x_0 of the root

(ii) Computes and prints the root to two decimal places

(c) Taking, $x_0 = 3.4$, perform a dry run for the flow chart

10. (a) Show that the iterative formula based on Newton Raphson's method for finding the root of the

$In x + x - 2 = 0$ is given by

$$x_{n+1} = x_n\left(\frac{3 - In\,x_n}{1 + x_n}\right), \qquad n = 0, 1, 2 \dots\dots\dots\dots\dots\dots$$

(b) Draw a flow chart that;

(i) Reads the initial approximation r of the root

(ii) Computes and prints the root of the equation, when the error is less than $1.0x10^{-4}$

(c) Taking, $r = 1.6$, perform a dry run for the flow chart.

11. (a) Show that the iterative formula based on Newton Raphson's method for finding the root of the

$x = In(x + 2)$ is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 2}{e^{x_n} - 1}, \qquad n = 0, 1, 2 \dots\dots\dots\dots\dots\dots$$

(b) Draw a flow chart that;

(i) Reads the initial approximation x_0 of the root

(ii) Computes and prints the root to three decimal places

(c) Taking, $x_0 = 1.2$, perform a dry run for the flow chart, give your root correct to three decimal places

12. (a) Show that the iterative formula based on Newton Raphson's method for finding the natural logarithm of a number N is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + N}{e^{x_n}}, \qquad n = 0, 1, 2 \dots\dots\dots\dots\dots\dots$$

(b) Draw a flow chart that;

(iii) Reads N and the initial approximation x_0 of the root

(iv) Computes and prints the natural logarithm to two decimal places

(c) Taking, $N = 45, x_0 = 3.5$, perform a dry run for the flow chart, give your root correct to two decimal places

13. (a) Show that the iterative formula based on Newton Raphson's method for finding the root of the

$2x^3 + 5x - 8 = 0$ is given by

$$x_{n+1} = \left(\frac{4x_n^3 + 8}{6x_n^2 + 5}\right), \qquad n = 0, 1, 2 \dots\dots\dots\dots\dots\dots$$

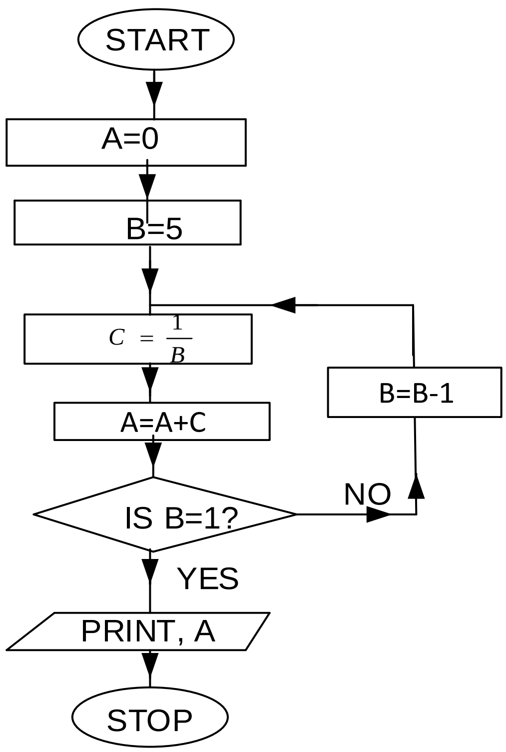
(b) Draw a flow chart that;

(i) Reads the initial approximation α of the root

(ii) Computes and prints the root of the equation, when the error is less than 0.001

(c) Taking, $\alpha = 1.1$, perform a dry run for the flow chart, give your root correct to three decimal places

14. A shop offers a 25% discount on all items in their store and a second discount of 5% for paying cash.
    (a) Construct a flow chart for the above information
    (b) Perform a dry run for;
        (i) A radio 125,000/= cash
        (ii) A T.V 340,000/= credit
15. Given that a man deposited 120,000/= to a bank which gives a compound interest rate of 15% . Draw a flow chart to compute the amount of money accumulated after 4 years, and perform a dry run for the flow chart
16. Given the flow chart below

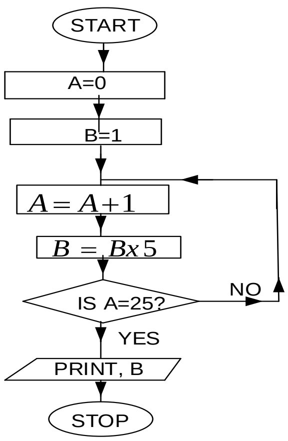


(i) Perform a dry run of the flow chart

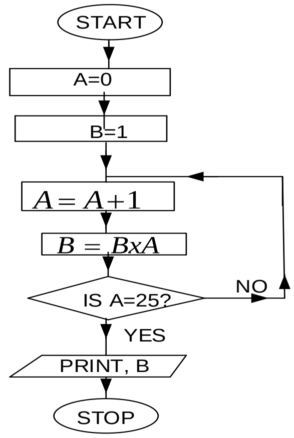
(ii) State the purpose of the flow chart

17. Study the flow charts below and perform a dry run of each flow chart and state the purpose of each flow chart

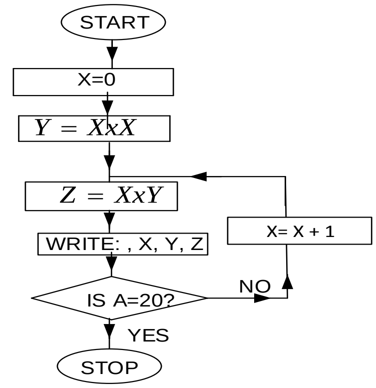
(i)



(ii)

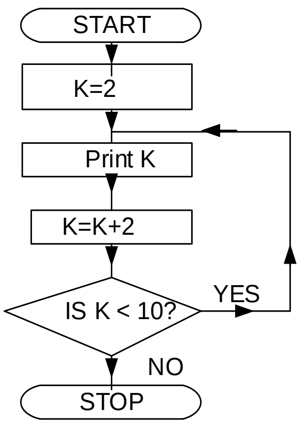


(iii)

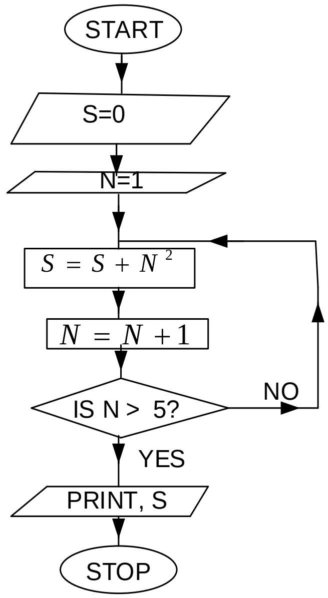


18. Study the flow charts below and perform a dry run of each flow chart

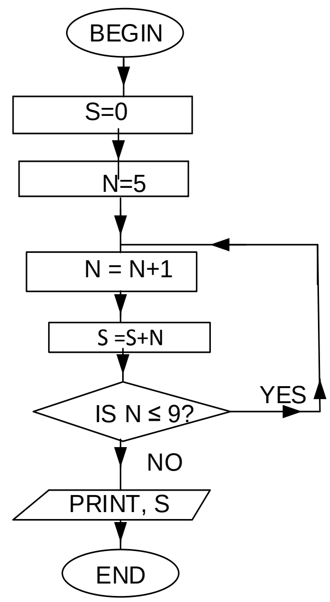
(i)



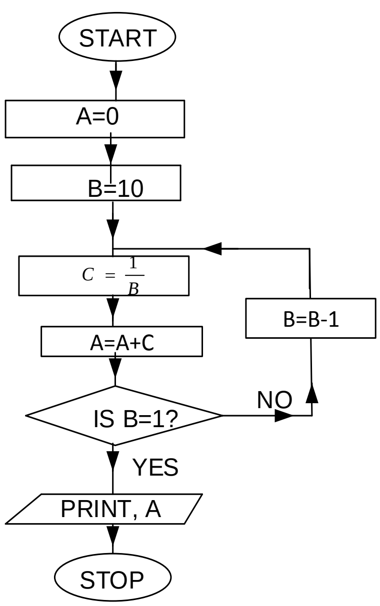
(ii)



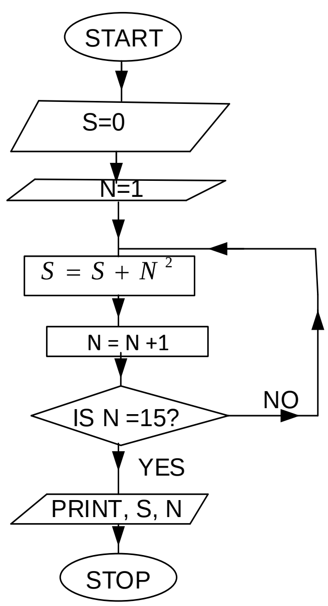
(iii)



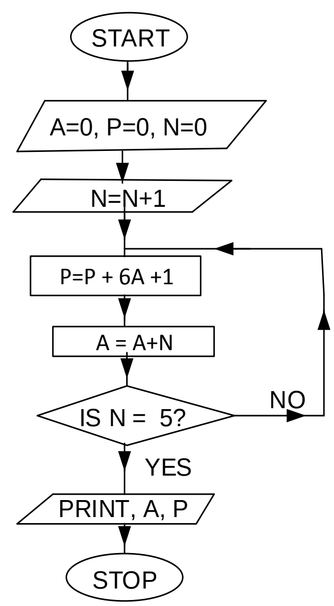
(iv)



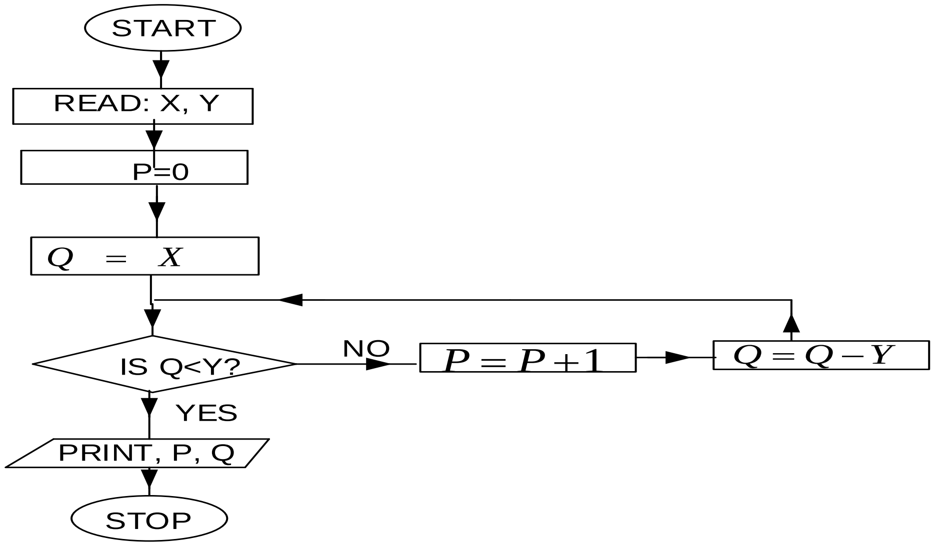
(v)



(vi)



19. Given the flow chart below



Perform a dry run by completing the tables below

(i) X=17, Y=13

P	Q
0	17
--------------	--------------
--------------	--------------
--------------	--------------
--------------	--------------

(ii) X=50, Y=7

P	Q
--------------	--------------
--------------	--------------
--------------	--------------
--------------	--------------
--------------	--------------

(iii) X=9, Y=2

P	Q
--------------	--------------
--------------	--------------
--------------	--------------
--------------	--------------
--------------	--------------

TEST 1

1. (a) Use the trapezium rule with five subintervals to estimate the area of enclosed by the curve $y^2 = 4x$, the x-axis, $x = 1$, and $x = 4$, Give your answer correct to 2dp.

   (**b**) If x_0 is the first approximation to the root of the equation $x^2 - \mathrm{b} = 0$, where b is a constant, show by Newton Raphson formula that a second approximation is given by $\frac{1}{2}\left(x_0 + \frac{b}{x_0}\right)$. (12marks)

2. Find the range of values within which the exact value of $2.6954\left(4.6006 - \frac{16.175}{0.82}\right)$ lies, if the numbers are rounded off to the given number of decimal place.
(5marks)

3. (a) Show that the equation $e^{x} - 2x = 1$ has a real root between $x = 1$ and $x = 1.5$
   (b) Using linear interpolation, find a better approximation for this root to three decimal places
   (c) Use Newton Raphson formula to find the root of the equation in (a) above, to two decimal places.
(12marks)

4. The diameter, d(mm) of an egg produced by a hen of a certain farm depends on the mass (gm) of the layer' mash ratio it is fed as shown below

Food ratio,m(gm)	200	290	330	410	440	500
Diameter, d(mm)	30.2	34.2	36.2	40.1	41.0	46.2

Assuming the egg to be spherical;

(i) The optimum amount of the food the hen should be given if it is to produce an egg of average diameter of 38.2gm
(ii) The radius of an egg if the food ratio supplied is 540gm

(5marks)

5. (a) On the same axes, draw graphs of $y = 2\mathrm{x} - 5$ and $y = \mathrm{x}\mathrm{e}^{x}$ to show the positive root of the equation $xe^{x} - 2x + 5 = 0$
   (b) Use the Newton Raphson method to calculate the root of the equation $x\mathrm{e}^{x} - 2x + 5 = 0$, Correct your answer to 2 decimal places
(12marks)

6. (a) Show that the equation $2x^3 - \frac{18}{x} + 2 = 0$ has two real roots in the interval $|x| < 3$
   (b) Use linear interpolation to find the least root to one decimal places
(7marks)

7. (a) Show that the equation $f(x) = xe^{x} + 5\mathrm{x} - 10 = 0$ has a real root between 1 and 2

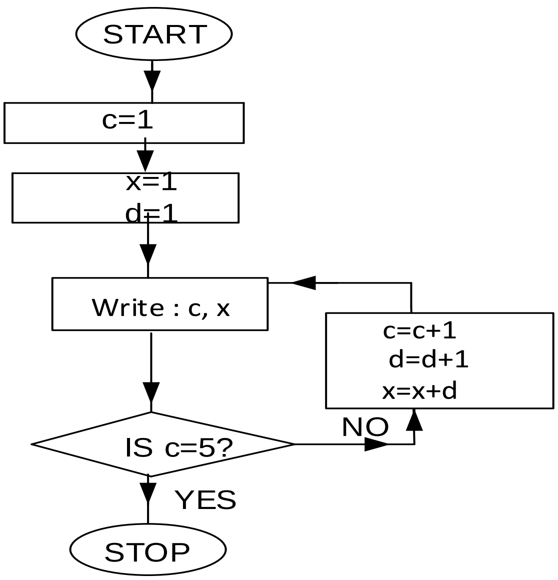
(b) Show that the Newton Raphson formula for approximating the root of the equation $xe^x + 5x - 10 = 0$ is given by

$$x_{n+1} = \frac{x_n^2 e^{x_n} + 10}{e^{x_n}(x_n + 1) + 5}$$

(c) Use the formula above, to find the root of the given equation correct to 3 decimal places.

(12marks)

8. Given the flow chart below



(i) Perform a dry run of the flow chart

(ii) State the relation between c and x

(5marks)

9. Show that the iterative formula based on Newton Raphson's method for finding the natural logarithm of a number N is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + N}{e^{x_n}}, \qquad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Draw a flow chart that;

(i) Reads N and the initial approximation x_0 of the root

(ii) Computes and prints the natural logarithm after four iterations and gives the natural logarithm to three decimal places

(c) Taking, $N = 10, x_0 = 2$, perform a dry run for the flow chart, give your root correct to three decimal places

(12marks)

GUIDE

1. (a) $h = \frac{4-1}{5} = 0.6$

x	$2x^{1/2}$	
1	2	
1.6		2.5298
2.2		2.9665
2.8		3.3466
3.4		3.6878
4.0	4	
Sum	6	12.5307

$$\int_1^4 2\,x^{1/2}\,dx \approx \frac{1}{2}x0.6[6+2(12.5307)]$$
$$\approx 18.6368 \approx 18.64$$

(b) $f(x) = x^2 - \mathrm{b} = 0 \quad f^1(x) = 2x$

$$x_{n+1} = x_n - \left(\frac{{x_n}^2 - b}{2x_n}\right)$$
$$x_{n+1} = \frac{x_n(2x_n) - ({x_n}^2 - \mathrm{b})}{2x_n}$$
$$x_{n+1} = \frac{{x_n}^2 + \mathrm{b}}{2x_n}$$
$$x_1 = \frac{{x_0}^2 + \mathrm{b}}{2x_0}$$
$$x_1 = \frac{1}{2}\left(x_0 + \frac{b}{x_0}\right)$$

2.

$$|e_{2.6954}| = 0.00005$$
$$|e_{4.6006}| = 0.00005$$
$$|e_{16.175}| = 0.0005$$
$$|e_{0.82}| = 0.005$$
$$Z = 2.6954\left(4.6006 - \frac{16.175}{0.82}\right)$$

$$Z_{max} = 2.69545\left(4.60065 - \frac{16.1745}{0.825}\right) = -40.4433$$
$$Z_{min} = 2.69535\left(4.60055 - \frac{16.1755}{0.815}\right) = -41.0966$$
$$\text{Range} = (-41.0966, -40.4433)$$

3.

(a) $f(x) = e^{x} - 2x - 1 = 0$

$f(1) = e^0 - 2x0 - 1 = -0.282$

$f(1.5) = e^{1.5} - 2x1.5 - 1 = 0.4817$

since $f(1)\,f(1.5) < 0$ then $1 <$root< 1.5

(b)

x	1	x_0	1.5
f(x)	-0.282	0	0.4817

$$\frac{x_0 - 1}{0 - -0.282} = \frac{1.5 - 1}{0.4817 - -0.282}$$
$$x_0 = 1.185$$

(c) $f(x) = e^{x} - 2x - 1$

$f^1(x) = e^{x} - 2$

$$x_{n+1} = x_n - \left(\frac{e^{x_n} - 2x_n - 1}{e^{x_n} - 2}\right)$$
$$x_{n+1} = \frac{x_n(e^{x_n} - 2) - (e^{x_n} - 2x_n - 1)}{e^{x_n} - 2}$$
$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 1}{e^{x_n} - 2}$$
$$x_1 = \frac{e^{1.185}(1.185 - 1) + 1}{e^{1.185} - 2} = 1.2632$$
$$x_2 = \frac{e^{1.2632}(1.2632 - 1) + 1}{e^{1.2632} - 2} = 1.2565$$
$$x_3 = \frac{e^{1.2565}(1.2565 - 1) + 1}{e^{1.2565} - 2} = 1.2564$$

Root= 1.26

4.

m	330	m_0	410
d	36.2	38.2	40.1

$$\frac{m_0 - 330}{38.2 - 36.2} = \frac{410 - 330}{40.1 - 36.2}$$
$$m_0 = 371.03\mathrm{g}$$

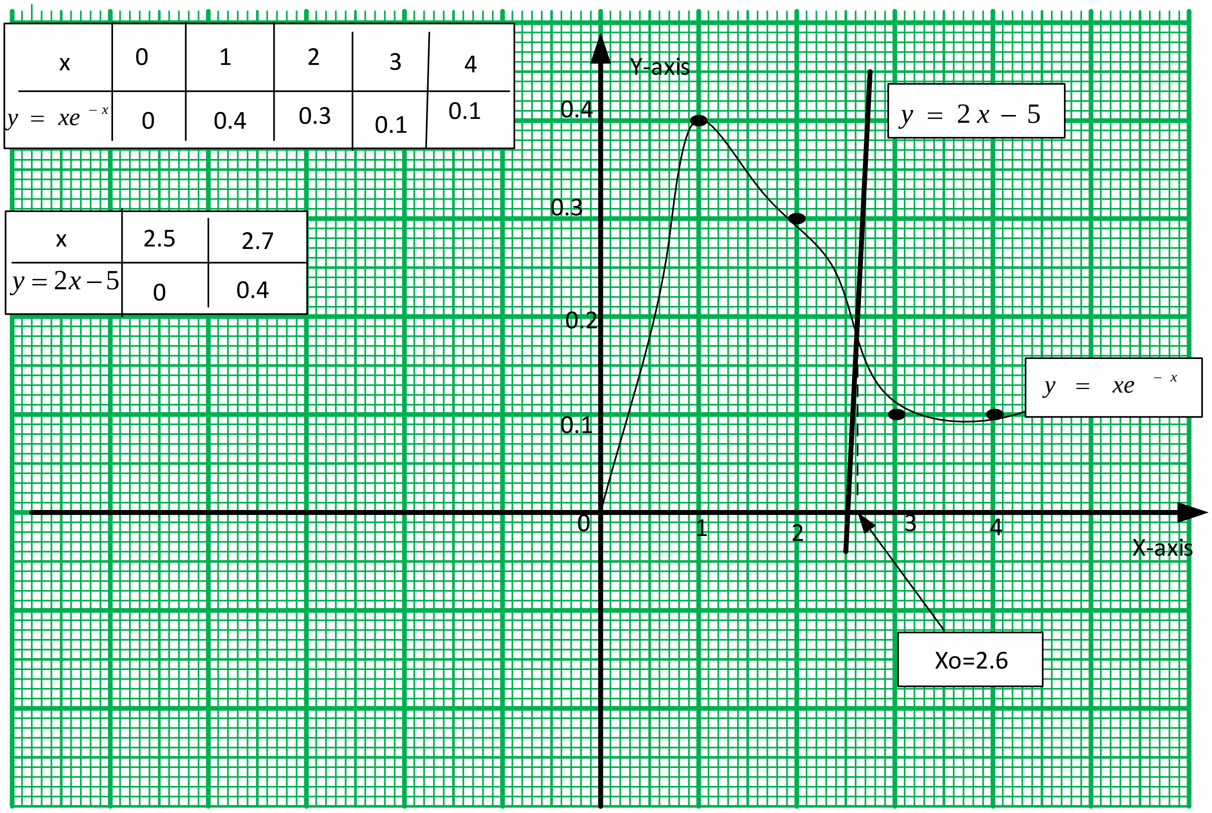
m	410	500	540
d	40.1	46.2	d_0

$$\frac{d_0 - 46.2}{540 - 500} = \frac{46.2 - 40.1}{500 - 410}$$
$$d_0 = 24.46\mathrm{mm}$$

5.

x	0	1	2	3	4
$y = xe^{-x}$	0	0.4	0.3	0.1	0.1

x	2.5	2.7
$y = 2x - 5$	0	0.4



$(b)\quad f(x) = xe^{-x} - 2x + 5$

$f^1(x) = -xe^{-x} + e^{-x} - 2$

$$x_{n+1} = x_n - \left(\frac{x_n e^{-x_n} - 2x_n + 5}{-x_n e^{-x_n} + e^{-x_n} - 2}\right)$$

$$x_{n+1} = \frac{x^2{}_n e^{-x_n} + 5}{e^{-x_n}(x_n - 1) + 2}$$

$$x_1 = \frac{(2.6)^2 e^{-2.6} + 5}{e^{-2.6}(2.6 - 1) + 2} = 2.597$$

$$x_2 = \frac{(2.597)^2 e^{-2.597} + 5}{e^{-2.597}(2.597 - 1) + 2} = 2.597$$

Root= 2.60

6.

x	-3	-2	-1	0	1	2	3
y	-46	-5	18	∞	-14	9	50

since $f(-2)\,f(-1) < 0$ then $-2 <$root< -1

and $f(1)\,f(2) < 0$ then $1 <$root< 2

x	-2	x_0	-1
y	-5	0	18

$$\frac{x_0 - -2}{0 - -5} = \frac{-1 - -2}{18 - -5}$$

$$x_0 = -1.8$$

7.

$f(x) = xe^{x} + 5x - 10$

$f(1) = 1e^1 + 5x1 - 10 = -2.282$

$$f(2) = 2e^2 + 5\text{x}2 - 10 = 14.778$$

$f(1)\,f(2) < 0$ then $1 <$root< 2

$$f^1(x) = \text{x}e^{\text{x}} + e^{\text{x}} + 5$$

$$x_{n+1} = x_n - \left(\frac{x_n e^{x_n} + 5x_n - 10}{x_n e^{x_n} + e^{x_n} + 5}\right)$$

$$= \frac{x_n(x_n e^{x_n} + e^{x_n} + 5) - (x_n e^{x_n} + 5x_n - 10)}{x_n e^{x_n} + e^{x_n} + 5}$$

$$= \frac{x_n^2 e^{x_n} + 10}{x_n e^{x_n} + e^{x_n} + 5}$$

$$x_{n+1} = \frac{x_n^2 e^{x_n} + 10}{e^{x_n}(x_n + 1) + 5}$$

$$x_0 = \frac{1+2}{2} = 1.5$$

$$x_1 = \frac{(1.5)^2 e^{1.5} + 10}{e^{1.5}(1.5+1) + 5} = 1.2394$$

$$x_2 = \frac{(1.2394)^2 e^{1.2394} + 10}{e^{1.2394}(1.2394+1) + 5} = 1.2019$$

$$x_3 = \frac{(1.2019)^2 e^{1.2019} + 10}{e^{1.2019}(1.2019+1) + 5} = 1.2013$$

Root= 1.201

8.

c	d	x
1	1	1
2	3	4
3	5	9
4	7	16
5	9	25

$$x = c^2$$

9.

$$x = \text{In N} \quad \therefore \text{e}^x = N$$

$$\text{e}^x - N = 0$$

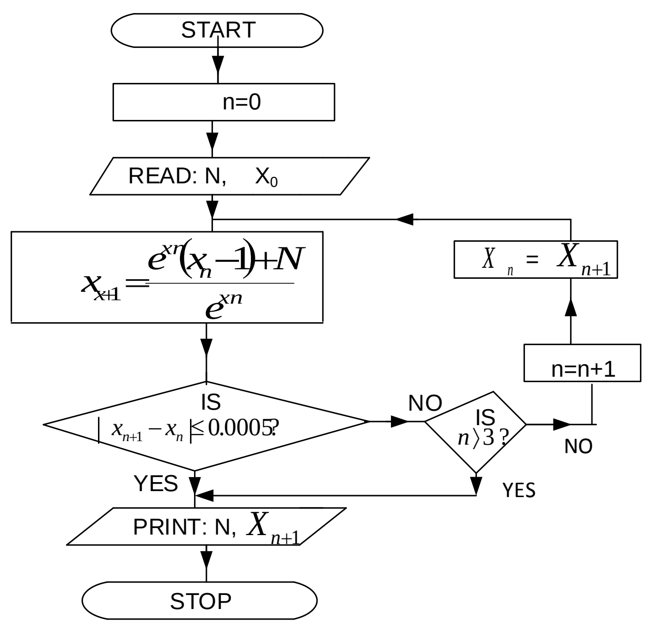
$$f(x) = \text{e}^x - N, \qquad f^1(x) = \text{e}^x$$

$$x_{n+1} = x_n - \left(\frac{\text{e}^{x_n} - N}{\text{e}^{x_n}}\right)$$

$$x_{n+1} = \frac{x_n \text{e}^{x_n} - \text{e}^{x_n} + N}{\text{e}^{x_n}}$$

$$x_{n+1} = \frac{\text{e}^{x_n}(x_n - 1) + N}{\text{e}^{x_n}} \qquad n = 0,1,2,3 \ldots\ldots$$

$9(b)$



$9(c)$

n	X_n	X_{n+1}	$\lvert X_{n+1} - X_n \rvert$
0	2.0	2.3533	0.3533
1	2.3533	2.3039	0.0494
2	2.3039	2.3026	0.0013
3	2.3026	2.3026	0.0000

Root =2.303

TEST 2

1. The table below shows the values of;

x	5	10	15
t	13	24.1	38.7

Use linear interpolation or extrapolation to find,

(i) t when x=8 (ii) x when t=44

(5marks)

2. Given that $A = 2.5, B = 1.71$, C=16.01, state the maximum possible error in A, B and C, hence find the limits within which the following are expected to lie

(i) $\frac{A+C}{B}$ (ii) $\frac{AB}{C}$

Give your answer to 2 decimal places

(5marks)

3. Derive Newton Raphson formula for approximating the root of the equation

$$20cosx - x = 0$$

Hence using the formula above and $x_0 = \frac{\pi}{2}$ as the initial approximation to the root of the equation, show that the second approximation is $\frac{10\pi}{21}$

(5marks)

4. (a) Show that the iterative formula based on Newton Raphson' s method for finding the fourth root of a number N is given by

$$x_{n+1} = 3\left(\frac{x_n}{4} + \frac{N}{12x_n^3}\right) \quad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots$$

(b) Draw a flow chart that;

(i) Reads N and the initial approximation x_0 of the root

(ii) Computes and prints the fourth root and N after three iterations and gives the root to two decimal places

(c) Taking $N = 99.1$, $x_0 = 3$, perform a dry run for the flow chart, give your root correct to two decimal places

(12marks)

5. (a) Show that the equation $e^{x} = 4x - x^3$ has a positive real root between $x = 1$ and $x = 1.5$

(b) Use Newton Raphson formula to find the root of the equation in (a) above, to two decimal places.

(12marks)

6. (a) By sketching graphs of $y = -x$ and $y = tanx$, show that the equation $x + tanx = 0$ has one root between 1 and 2

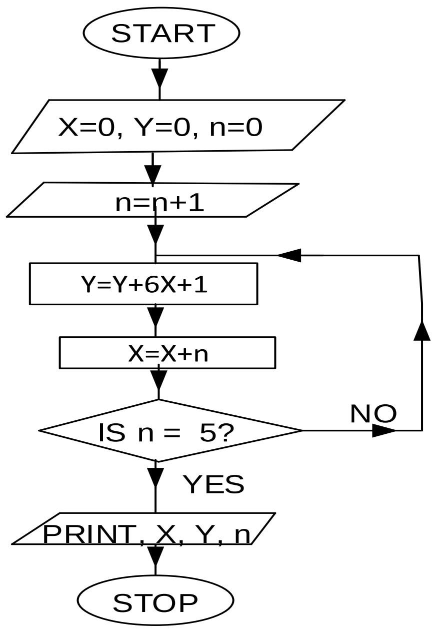
(b) Show that the Newton Raphson formula for approximating the root of the equation $x + tanx = 0$ is given by

$$x_{n+1} = \frac{2x_n - \sin 2x_n}{2(1 + \cos^2 x_n)}$$

(c) Use the formula above, to find the root of the given equation above after two iterations correct to 2 decimal places.

(12marks)

7. Study the flow chart below



a. Perform a dry run for the flow chart
b. State the relation between n and Y

(5marks)

9. The numbers M and N are approximated with possible errors of e_1 and e_2 respectively.

(a) Show that the maximum absolute relative error in the quotient MN is given by

$$\frac{|e_1|\mathrm{N} + |e_2|\mathrm{M}}{MN}$$

(b) Given that M=6.43, N=37.2, write down the maximum possible error in M and N. Hence find the interval in which

(i) Product MN lies

(ii) The quotient $\frac{M}{N}$ lies

(12marks)

GUIDE

1.

x	5	8	10
t	13	t_0	24.1

$$\frac{t_0 - 13}{8-5} = \frac{24.1 - 13}{10-5}$$

$$t_0 = 19.66$$

x	10	15	x_0
t	24.1	38.7	44

$$\frac{x_0 - 15}{44 - 38.7} = \frac{15-10}{38.7-24.1}$$

$$x_0 = 16.82$$

2.

$|e_A| = 0.05,\quad |e_B| = 0.005,\quad |e_C| = 0.005$

b. $\left(\frac{A+C}{B}\right)_{max} = \frac{2.45+16.005}{1.715} = 10.76$

$$\left(\frac{A+C}{B}\right)_{min} = \frac{2.55+16.015}{1.705} = 10.89$$

c. $\left(\frac{AB}{C}\right)_{min} = \frac{2.45x1.705}{16.015} = 0.26$

$$\left(\frac{AB}{C}\right)_{max} = \frac{2.55x1.715}{16.005} = 0.27$$

3.

$$f(x) = 20\cos x - x$$

$$f^1(x) = -20\text{sinx} - 1$$

$$x_{n+1} = x_n - \left(\frac{20cosx_n - x_n}{-20\ sinx_n - 1}\right)$$

$$x_{n+1} = \frac{-20\ x_n sinx_n - 20cosx_n}{-20\ sinx_n - 1}$$

$$x_0 = \frac{\pi}{2}$$

$$x_1 = \frac{-20\ \left(\frac{\pi}{2}\right) sin\frac{\pi}{2} - 20cos\frac{\pi}{2}}{-20\ sin\frac{\pi}{2} - 1}$$

$$= \frac{-20\ \left(\frac{\pi}{2}\right)}{-20\ -1}$$

$$= \frac{-20\ \left(\frac{\pi}{2}\right)}{-21} = \frac{10\pi}{21}$$

4.

$$x = N^{1/4} \qquad \therefore x^4 = N$$

$$x^4 - N = 0$$

$$f(x) = x^4 - N,\quad f^1(x) = 4x^3$$

$$x_{n+1} = x_n - \left(\frac{{x_n}^4 - N}{4{x_n}^3}\right)$$

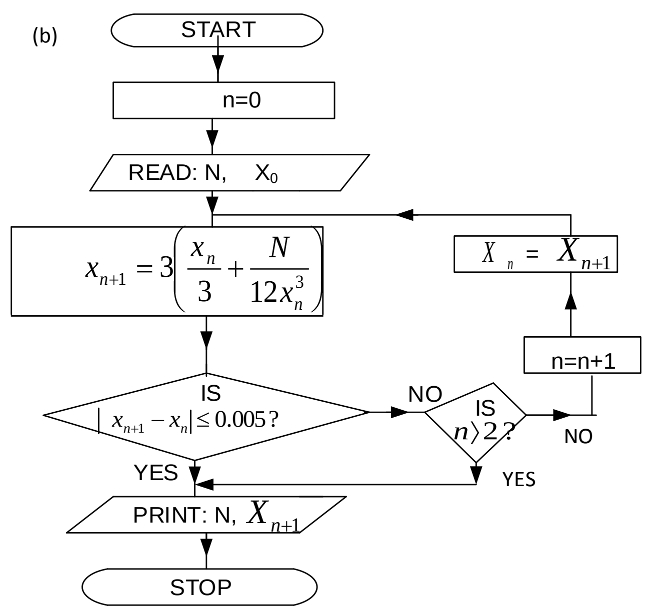
$$x_{n+1} = \frac{x_n(4{x_n}^3) - ({x_n}^4 - N)}{4{x_n}^3}$$

$$= \frac{3{x_n}^4 + N}{4{x_n}^3}$$

$$x_{n+1} = \frac{3x_n}{4} + \frac{N}{4x_n^3}$$

$$x_{n+1} = 3\left(\frac{x_n}{4} + \frac{N}{12x_n^3}\right)$$

(b)



(c)

n	X_n	X_{n+1}	$\lvert X_{n+1} - X_n \rvert$
0	3	3.168	0.168
1	3.168	3.155	0.013
2	3.155	3.155	0.000

Root =3.16

5.

$f(x) = e^{x} + x^3 - 4x$

$f(1) = e^1 + 1^3 - 4x1 = -0.2817$

$f(1.5) = e^{1.5} + 1.5^3 - 4x1.5 = 3.8568$

$f(1)\,f(1.5) < 0$

then 1 <root< 1.5

$f^1(x) = e^{x} + 3x^2 - 4$

$$x_{n+1} = x_n - \left(\frac{e^{x_n} + x^3{}_n - 4x_n}{e^{x_n} + 3x^2{}_n - 4}\right)$$

$$x_{n+1} = \frac{x_n^2 e^{x_n}(x_n - 1) + 2x_n^2}{e^{x_n} + 3x^2{}_n - 4}$$

$$x_0 = 1$$

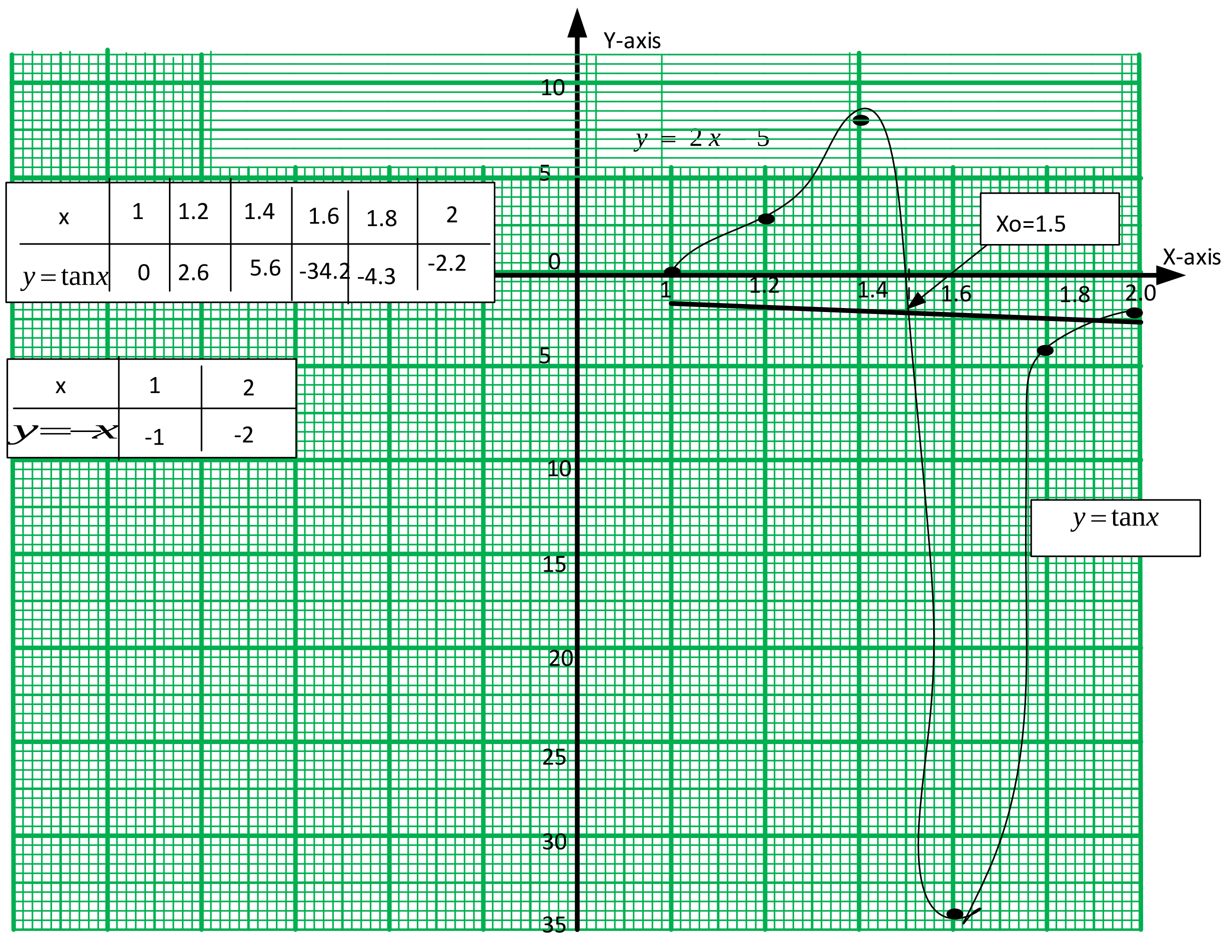
$$x_1 = \frac{1^2e^1(1-1) + 2x1^2}{e^1 + 3x1^2 - 4} = 1.164$$

$$x_2 = \frac{1.164^2e^1(1.164-1) + 2x1.164^2}{e^{1.164} + 3x1.164^2 - 4} = 1.126$$

$$x_3 = \frac{1.126^2e^1(1.126-1) + 2x1.126^2}{e^{1.126} + 3x1.126^2 - 4} = 1.124$$

Root= 1.12

6.



x	1	1.2	1.4	1.6	1.8	2
$y = \tan x$	0	2.6	5.6	-34.2	-4.3	-2.2

x	1	2
$y = -x$	-1	-2

$f(x) = x + tanx$

$f^1(x) = 1 + \sec^2 x$

$$x_{n+1} = x_n - \left(\frac{x_n + tanx_n}{1 + \sec^2 x_n}\right)$$

$$x_{n+1} = \frac{x_n(1 + \sec^2 x_n) - x_n - tanx_n}{1 + \sec^2 x_n}$$

$$x_{n+1} = \frac{x_n\left(1 + \frac{1}{\cos^2 x_n}\right) - x_n - \frac{sinx_n}{cosx_n}}{1 + \frac{1}{\cos^2 x_n}}$$

$$= \frac{\frac{x_n}{\cos^2 x_n} - \frac{sinx_n}{cosx_n}}{1 + \frac{1}{\cos^2 x_n}}$$

$$= \frac{x_n - cosx_n sinx_n}{1 + \cos^2 x_n}$$

$$x_0 = 1.5$$

$$x_1 = \frac{2\text{x}1.5 - \sin 2\text{x}1.5}{2(1 + \cos^2 1.5)} = 1.422$$

$$x_2 = \frac{2\text{x}1.422 - \sin 2\text{x}1.422}{2(1 + \cos^2 1.422)} = 1.248$$

Root= 1.25

7.

n	x	y
0	0	0
1	1	1
2	3	8
3	6	27
4	10	64
5	10	125

$$y = n^3$$

9. (a)

$$e_{MN} = (M + e_1)(N + e_2) - \text{MN}$$

$$e_{\text{MN}} = MN + Ne_1 + Me_2 + e_1 e_2 - MN$$

Since $e_1$and $e_2$are very small, then

$$e_1 e_2 \approx 0$$

$$e_{MN} = Ne_1 + Me_2$$

$$|e_{MN}| = |Ne_1 + Me_2|$$

$$|e_{\text{MN}}| \leq |Ne_1| + |Me_2|$$

$$e_{max} = |Ne_1| + |Me_2|$$

$$R.e_{max} = \frac{|Ne_1| + |Me_2|}{MN}$$

9. (b)

$M = 6.43, \qquad e_1 = 0.005$

$N = 37.2, \qquad e_2 = 0.05$

$$e_{max} = |Ne_1| + |Me_2|$$

$$e_{max} = |37.2x0.005| + |6.43x0.05|$$

$$= 0.5075$$

$MN = 6.43x37.2 = 239.196$

$= 239.196 \pm 0.5075$

Interval $[238.69, 239.70]$

$$e_{max} = \frac{|Me_2| + |Ne_1|}{|N^2|}$$

$$e_{max} = \frac{|6.43x0.05| + |37.2x0.005|}{|(37.2)^2|} = 0.000367$$

$t = 0.1728 \pm 0.000367$

Interval $[0.172, 0.173]$

ERRORS

1. An P_{lower}=673.09, P_{lupper}=673.23

2. An$(|e| = 0.175, [873.985, 873.335]$

3. An$((i)|\Delta T| = 0.05, |\Delta M| = 0.5, (ii)[29.65, 29.75], [47.5, 48.5]$

4. An$((i)[7.0,9.0],$

5. An$((i)|\Delta l| = 0.05, |\Delta w| = 0.05$ (ii) A_{lower}=7.99, A_{lupper}=8.58

6. An($(ii)[8.16, 8.28]$ $(iii) = 0.73\%$

7. An $[25.93, 29.79], or\ [25.99, 29.86]$

8. An($(i) = 1.739$, $(ii) = 2.823, (iii) = -0.393$

9. An($(i) = 50\ acres$, $(ii) = 2050, (iii) = 1750$

10. An $[0225, 0.233],$

11. An$(|\Delta X| = 0.00005, |\Delta Y| = 0.005,$

 $(i)[6.560, 6.570], (ii)[-1.0397, -1.0296](iii)[0.727, 0.729](iv)[0.623,0.626](v)[8.479, 8.514]$

12. An$(|\Delta X| = 0.005, |\Delta Y| = 0.05, (i)[6.560, 6.570],$

13. An$[-1.675, -1.607],$

14. An $\mathbf{7.15x10^{-5}}$

15. An $\mathbf{2.25}\%$

16. An$((i)|\Delta a| = 0.05, |\Delta b| = 0.05, (ii) = 0.211, (iii)\ limit = 4.0382, 4.4618$

17. An$[-41.097, -40.443],$

18. An $(i)limits$=10.76, 10.89, (ii) limits=0.26, 0.27

19. An$((i)|\Delta A| = 0.005, |\Delta B| = 0.0005, (ii) = 0.00347, (iii)\ limit = 2.572, 2.579$

20. An$(|\Delta A| = 0.00005, |\Delta B| = 0.0005, (i) = 0.053, (ii)\ limit = 5.916, 6.022$

21. An$(|\Delta M| = 0.005, |\Delta N| = 0.05, (i) = [238.69, 239.70], (ii) = [0.172, 0.173]$

INTERPOLATION

1. An$(i) = 2.9, (ii) = 2.333(iii) = 1.733$ $(iv) = 14$
2. An$(i) = 47, (ii) = 3.89$
3. An$(i) = 725/=$ $(ii) = 7km$
4. An$(i)x = 1.4$ $(ii)y = 23.2$
5. An $(iii) = 1.3m$ $(iv) = 22.8s$
6. An$(i)Q = 3.3125$ $(ii)P = 10.5$

7. An$(i)t = 30.6\ (ii)X = 6.33$
8. An$(i)Y = 30.6\ (ii)X = 6.33$
9. An$(i) = 0.1782\ (ii)y = 10°4^1$
10. An$(i) = 0.1633\ (ii)X = 17.93$
11. An$= 1.7657$
12. An$(i) = 12.36N, (ii) = 17.5(iii) = 1.61N$
13. An$(i) = 0.2249, (ii) = 1.043(iii) = 0.152$
14. An(i) $D = 73.8, (ii)T = 4.46(iii) = 125$
15. An $(\boldsymbol{i})Q = 64.25, (ii)T = 297.6\ (iii)Q = 40\ \ (iv)\ T = 750$
16. An $(\boldsymbol{i})D = 3.25min, (ii) = 240℃\ (iii) = 400℃$
17. An $(i) = 3875/=, (ii) = 5.93kg$
18. An $(i) = 371.01gm\ (ii) = 24.46mm$
19. An $(i) = 19.66\ (ii) = 16.82$
20. An$(i) = 216s, (ii) = 62℃\ (iii) = 46.67℃\ \ (iv) = 720s$

Trapezium rule

1. An(3.07,3.14,2%)
2. An 0.784
3. An(2.9418, 0.098%)
4. Ans (i)=7.712, ii)7.4560, iii)3.43%,
5. An (i)=5.483, (ii)=5.461, (iii)=0.403%
6. An =3.98
7. An= 0.74
8. An =0.237
9. An=0.7
10. An$(\boldsymbol{a}) = \mathbf{0.704}$ (b) (i)=0.693, (ii)= 1.587%
11. An(1.013)
12. An(1.105)
13. An(2.559)
14. (i)=0.917, (ii)=0.896 (iii)=0.021
15. An(1.6617)
16. An(0.75, 4.17%)
17. Ans [0.0593]
18. Ans (a)=4.98, (ii)=0.14
19. An(ii)=2.23%
20. An=41.038

GRAPH WORK

1. An$= 0.7$
2. An$= -2.2$
3. An$= \mathbf{1.6}$
4. An$= 0.35$
5. An$= 0.44$

6. An= 0.56

NRM

1. An($\boldsymbol{d}$) = 3.44
2. An($\boldsymbol{b}$) = 0.258, (c) = 0.258
3. An($\boldsymbol{c}$) = $\mathbf{1.90}$
4. An($\boldsymbol{c}$) = 0.85
5. An($\boldsymbol{c}$) = 3.90
6. An($\boldsymbol{b}$) = 0.58
7. An($\boldsymbol{b}$) = 0.766
8. An($\boldsymbol{b}$) = 0.567
9. An($\boldsymbol{b}$) = 2.41
10. An($\boldsymbol{b}$) = 0.79
11. b=1.07
12. An=1.6
13. An=0.58
14. An=1.12
15. An=2.14
16. An=2.65
17. An=0.97

ITERATIVE

2. An=4.56
3. An=1.33
4. An=1.26
5. An= $\mathbf{4.123}$
6. An= $\mathbf{2.289}$
7. An= $\mathbf{2.19}$