ADVANCED LEVEL PRINCIPAL MATHEMATICS SEMINAR

TO BE HELD AT LUBIRI SS ON SUNDAY 18/09/2022

(A) P425/1

(a)Coordinate Geometry (02 questions in UNEB)

1.(a)find the equation of the line that passes through the point of intersection of the lines 3x-2y=4 and 2x+2y-6=0 and makes angle 135^0 with the horizontal.

(b)find the locus of the point p(x,y) which moves such its distance from a point A(1,2) is twice its distance from the line 2x-y=4.

2. (a)The normal to the parabola $y^2=4ax$ at a point $P(at^2,2at)$ meets the axis of the parabola at S.If SP is produced beyond P to R such that SP=PR,Show that the equation of locus of R is $y^2 =16ax+2a$. *

(b) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are two variable points on the parabola $y^2= 4ax$. If PQ subtends a right angle at the origin,

(i)prove that pq + 4 = 0,

(ii)Show that PQ passes through a fixed point on the axis of the parabola.

(iii) The tangents at P and Q meet at R. Find the equation of the locus of R.

3.Given parametric equations $x = 2 + 3 \cos\theta$, $y= 1 + 4 \sin\theta$,
(i) Show that the equation represents an ellipse.

(ii) Find the centre and length of the major axis.
4.(a) Express the circle $x^2+y^2=3x$ in polar form.
(b)The circle $x^2+y^2-4x-6y-12=0$ touches another circle whose centre is(2,-4),find (i)the equation of the second circle (ii)equation of common tangent(iii)point of contact of the two circles.
(c) Find the equation of the circle whose centre lies on the line $x - 2y + 2 = 0$ and which touches the positives axes.

(b)Trigonometry (02 questions in UNEB)

5. If $\sin A=\frac{12}{13}$ and $\cot B=\frac{3}{4}$, where A is acute and B is reflex,

find the value of $2\sec A\text{-}Cosec^2B$.

6. Solve for θ, $2\cos\theta\cos2\theta - \cos\theta + 1 = 0$ where $0 \le \theta \le \pi$.

7. In triangle *ABC*, prove that; $\frac{bc}{ab+ac} = \frac{Cosec(B+C)}{CosecB+Cosec\,C}$

8. (a)Show that $\tan\frac{\pi}{8} = -1 \pm \sqrt{2}$

(b) Solve for θ in $\sin 2\theta + \sin 4\theta + \sin 6\theta = 0$ for $-180 \le \theta \le 180$.

9. (i)Prove that $\sin^4\theta + \cos^4\theta = \frac{1}{4}(\cos 4\theta + 3)$

(ii)prove that $$\frac{\cos 2(\alpha+\beta) + \cos 2\alpha + \cos 2\beta + 1}{\cos 2(\alpha+\beta) - \cos 2\alpha - \cos 2\beta + 1} = \cot\alpha\cot\beta$$

(c)Vectors (02 questions in UNEB)

10. Points O,P and Q are vertices of a triangle OPQ where O is the origin and vectors OP=**p** and OQ=**q**.R lies on OP produced such that OR=3OP,S is the mid point of OQ and T divides the line PQ in the ratio 1:3.S is joined to T and to R.

(a) express in terms of vectors p and q

(i)PT (ii) ST (iii) SR(b)If nST=kTR, find the values of n and k

11. (a)Given that r and s are inclined at 60°, t is perpendicular to r + s and $|r| = 8, |s| = 5, |t| = 10$, find (i) $|r + s + t|$ (ii) $|r - s|$

(b)Find the shortest distance of the point C (10,-3,-2) from the line which satisfies the coordinates A(4,-1,2) and B(-2,2,-4)

12. (a)Given $AB = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $AC = \begin{pmatrix}4\\-1\\2\end{pmatrix}$ are vectors. Find a vector which is perpendicular to both AB and AC.

(b)Given that the line $\frac{4-x}{-2} = -2z = y = \lambda$ is perpendicular to a plane Π that passes a point (1,4,2).find the Cartesian equation of a plane Π.

13 The line L: $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{-1}$ and the plane P: $x + y + z = 12$, intersect at a point A on the plane, find the

a. coordinates of A
b. angle between L and P

(d)Algebra (04 questions in UNEB)

14.(a)Express the complex number z=-8i in polar form and hence evaluate $z^{\frac{2}{3}}$

(b)Show that the locus of a complex number w which moves such that $\text{Arg}\left(\frac{w+2i}{w+3}\right)=-135$,is a circle.

(c) Solve $z^3-1=0$. Synthetic formular.

(d)Given that $Z=\frac{(1-i)(2+3i)}{(3+4i)}$, determine:

(i) $\overline{Z}$ in the form x + iy and represent it on an Argand diagram

(ii)The magnitude and argument of $(\overline{Z})^2$.

15. (a)Coach Simon has 18 players and 3 goal keepers to choose from a foot ball team of Buddo s.s to play in East African games in Arusha Tanzania 2022. In how many ways can he select his team if(i)all players are fit(ii)one sticker must be on the team and one goal keeper has injury problems.

(b)in how many ways can all the letters of the word SEMINAR be arranged if(i)all letters are taken each time(ii)vowels are not together.

(c)A committee consisting of 4 persons is to be selected from 5 boys and 8 girls.in how mays can this be done for the committee to contain (i)2 boys and 2girls (ii)more boys than girls.

16.(a)Evaluate $\sum_{n=1}^{11}(1.2)^n+1.2n$

(b)Solve the simultaneous equations.

$y\log_2 8=x$

$2^x+8^y=8192$

17.(a)Given that one root of the equation $kx^2+hx+l=0$ is square the other, without solving the equation, prove that $l(k-h)^3=k(l-h)^3$.

(b)Show that if log a, log b *and* log c are consecutive terms of an AP, then a, b, c are in a geometric progression.

(c)Show that $3\log(a+b)=3\log a+\log(1+3\frac{b}{a}+3\frac{b^2}{a^2}+\frac{b^3}{a^3})$

18(a) Given that the first three terms in the expansion in ascending powers of x of $(1-8x)^{\frac{1}{4}}$ are the same as the first three terms in the expansion of

$\left(\frac{1+ax}{1+bx}\right)$, find the values of a and b. Hence, find an approximation to $(0.6)^{\frac{1}{4}}$

orm p/q.

valuate the square root of $5-\sqrt{3}$

19. Given the curve $y=\frac{x^2-7x+10}{x-6}$

Show that for real x, y cannot be between 1 and 9

(e) Analysis (06 questions in UNEB) (Calculations)

20.A vessel holding flowers is of the shape formed by the revolution of the curve $5y^2=2x$ about the y-axis a complete turn .if its height is 2cm.find its volume.

21. Water runs at constant rate of $6cm^3s^{-1}$ in a vessel whose volume is obtained by rotating the area bounded by the curve $4y=x^2$ about the y-axis y=0 to y=h cm

(i) Show that the volume of the vessel is $2\Pi h^2 cm^3$

(ii) Find the rate at which the water level is rising when the water has been flowing for 3s.

22. (a)Solve the differential equation:

$$\sin x\frac{dy}{dx}+2y\cos x=x, \text{ given } y=1 \text{ and } x=\pi/2.$$

(b) A liquid is being heated in an oven maintained at a constant temperature of $180^\circ C$. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180 - \theta)$, where $\theta^\circ C$ is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from $0^\circ C$ to $120^\circ C$ in 5 minutes, form a D.E and solve it

Find(i) the temperature of the liquid by the 11^{th} minute(ii)time taken for the temperature to rise from 120^0c to 150^0c.

23.(a)Given that y = In(1 + sin x), prove that $\frac{d^2y}{dx^2}+e^{-y}=0$

(b) Differentiate with respect to x ,$y-x^{\sin(x+y)}=0$

(c) If $y = \tan^{-1}\frac{2x}{1-x}$ show that $dy/dx = 2/1+x$

24. (a)Evaluate $\int_0^{\pi/2}(3\sin^2 x + 2\cos^2 x)\ dx$

(b)Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}\ dx$

25. Find (i) $\int \frac{2x+1}{(x-2)^5(x+3)^5}\ dx$ (ii) $\int \ln(x^2-16)dx$

(iii) Show that $\int_0^{\pi/3} 2\sin(\frac{x}{4})\sin(\frac{5x}{4})\mathrm{Cos}(\frac{x}{4})\ dx = \frac{\sqrt{3}}{4}$

26. Solve the Differential equations

(i) $(\mathrm{Sec}x)\frac{dy}{dx} = e^{(\sin x+\cos y)}$ (ii) $\frac{dy}{dx} - \frac{2}{x}y = x^3e^x$

(B) P425/2

PROBABILITY AND STATISTICS (06 questions in UNEB)

1. The table below shows marks given to 6 students in sub-maths WAKISHA mock examination by 3 different examiners,

Student	A	B	C	D	E	F
Examiner 1	60	35	52	38	70	65
Examiner 2	40	55	71	40	42	80
Chief examiner	55	60	41	63	73	76

(a) Calculate the rank correlation between Examiner 1and Examiner 2 with the chief examiner.

(b) State with a reason which of the examiners had a better correlation with the chief examiner.

2. Three events A, B and C are such that A and B are independent, A and C are mutually exclusive.
Given that P(A) = 0.4, P(B) = 0.2, P(C)=0.3 and P(CnB) = 0.1 ,find
(i) $P(AUB^I)$ (ii) P(A/BUC).

3. The data below shows masses of objects in kg obtained.

Mass(kg)	20-	25-	30-	35-	40-	50-	55-<65
Frequency density	0.4	1.2	1.4	2.2	1.8	1.6	1.2

Calculate the ;(i)modal mass (ii)mean mass.(iii)limits of middle 60%.

4. The diameters of a large consignment of pancakes are normally distributed with variance $0.09mm^2$. Find the smallest sample size needed to give 99% confidence interval of width less than 0.2mm.

5. Box A contains 3 red and 4green apples, Box B contains 5red and 7 green apples. Two apples are picked at random such that when the first apple comes from box A, the second comes from box B and vice versa.

Write down the probability distribution for the number of green apples picked and hence obtain the mean.

6.An examination question has two parts A and B. The probability of a student getting part A correct is $\frac{2}{3}$ .If she gets A correct, the probability that she gets part B correct is $\frac{3}{4}$, otherwise it is $\frac{1}{6}$.

(a) Find the probability of a student getting (i) both parts correct(ii)part A only correct(iii)part B only correct.

(b) There are three marks for a correct solution of part A, two marks for a correct solution of part B and a bonus mark if both parts are correct.

Find the probability distribution function for the student's total mark and hence find (i)the expected student's total marks and variance.

7. A random variable T has its p.d.f given by;

$$f(t) = \begin{cases} kt; & 0 \le t \le 2 \\ k4\text{-}t); & 2 \le t \le 4 \\ 0 & ; \text{otherwise} \end{cases}$$

(a)sketch the graph of f(t) and hence find(i)mode(ii)median(iii)E(T)(iv)value of k

(b) Obtain the C.D.F ,F(t) and hence or otherwise find the

(i) $P(T-\mu>1)$ (ii)$P(0<T<\frac{4}{T}>0)$.

8.The marks of all the candidates at lubiri ss in biology mock examination were normally distributed with mean 52% and standard deviation 16%.

(a) Determine the percentage of candidates who scored a Distinction if the lowest mark for a distinction was 68%

(b) Given that 20 candidates scored below 40%, estimate the number of candidates in the school.

(c) If a random sample of 16 candidates was picked from the school,

(i) find the probability that their mean score exceeded 58%.

(ii) Establish a 96.3% confidence interval for the mean .

9. The probability that a chameleon changes its color after walking a distance of 50m is 0.45, if it walks a total distance of 5km. Determine the probability that the chameleon changes its color (i) exactly 40 times(ii)atleast 42 but less than 48 times.

NUMERICAL METHODS (04 questions in UNEB)

10.(a)Given the formula $x_{n+1} = \sqrt[3]{3 - x_n^{-2}}$ .Find the equation whose root is saught. Hence show that the equation has three real roots .

(b)use linear interpolation to approximate the greatest root of the equation to 3 significant figures.

11.(a)Show that the iterative formula based on Newton Rapson's method for approximating the root of the equation $ax^2+bx+c=0$ is given by $x_{n+1}=\frac{ax^2{}_n-c}{2ax_n+b}$,n=0,1,2,...

(b) construct a flowchart that;

(i)reads the initial approximation as p and constants a,b,c.

(ii)computes and prints the root of the equation in (a) above to 4 decimal places.

(c)perform a dry for your flowchart by taking p=5.4,evaluate $\sqrt{29.8}$ to 4 decimal places.

12.(a)Show that the percentage error in x^2y is given by $(2\left|\frac{e_1}{x}\right|+\left|\frac{e_2}{y}\right|)$x100 where e_1 and e_2 are small compared to x and y.

(b)given that x=2.01rounded to 2 decimal places and y= -3.84 rounded with an error of 4%

(i) Obtain the accurate value of x^2y(ii)limits within which the exact value of x-y is expected to lie.

13. Use the trapezium rule with strips of width $\frac{\pi}{12}$ to determine the approximate value of $\int_0^{\pi/3} e^x cosxdx,$ *correct to four significant figures.*

MECHANICS (06 questions in UNEB)

14.(a)A ship A is travelling on a course of 060° at a speed of 30kmh-1 and a ship B is travelling at 20kmh-1 ,At noon B is 260km due east of A.

(i)Find the course B must take to come as close as possible to A.

(ii) Find the time when A and B are closest together and the shortest distance.

15(a) A pump draws water from a tank and issues it at a speed of $10ms^{-1}$ from the end of a hose of cross-sectional area $5\ cm^2$, situated $4m$ above the level ..m which the water is drawn. Find the rate at which the pump is working.

..ar of mass 800kg is towing a trailer of mass 150kg on a level road .the ..onal resistance to each vehicle amounts to 7N per kg of mass.

(i) Calculate the tension in the tow bar when the vehicles are travelling at a constant speed.

(ii)the car and trailer now climb a slope of inclination 1 in 20, and if the frictional resistances are the same as before and the power of the engine is 50kW, calculate the;

(a) Maximum speed up the slope.(b)Acceleration when the speed is $54kmh^{-1}$.

16. ABCD is a square of side 2m. Forces of magnitudes 3N, 5N, 7N AND 2N act a long sides DA, AB, BC and CD respectively. Calculate:

(i)The magnitude of the resultant of the forces and the angle made by the resultant with AD.
(ii)The sum of the moments of the forces about A
(iii)The distance from A of the point where the line of action of the resultant of the forces cuts DA produced.

17.A body of mass 4kg lies on a rough plane, which is inclined at 30^0 to the horizontal. The angle of friction between the plane and the body is 15^0. Find the least horizontal force that can be applied to the body without motion occurring.

18.Musoke of mass 75kg attempts to climb up a Uniform ladder of mass 25kg and length 5m, resting on a smooth vertical wall with its lower end on a rough ground, coefficient of friction $^2/_5$. The ladder is inclined at 60^0 to the horizontal. Find how far he can ascend the ladder without it slipping.

19.During a football match,Pogba shot a goal kick at an angle of elevation of 30^0 to the horizontal with a velocity of ums^{-1} and hit the ground through the point of projection a distance of 20m where Ronaldo was.when the ball hit the ground, it bounced and the horizontal component of the initial velocity remained the same but the vertical component of initial velocity reversed in direction and became quarter of the original component in magnitude and now Ronaldo kicked the ball when it was a distance of 2m away from where it landed.

Find (i)the value of u,initial speed of projection.

(ii)time between ball bouncing and Ronaldo kicking it again.

(iii)height above sea-level where Ronaldo kicked ball.

20.A ball was thrown vertically upwards from the ground level with a velocity of $20ms^{-1}$. A girl was leaning outside of a second floor window, 8m above the ground to catch it. When the ball was on its way up, she missed it but managed to catch it on its way down.

Find (i)How fast the ball was moving when it was caught?

(ii)The time of flight of the ball.

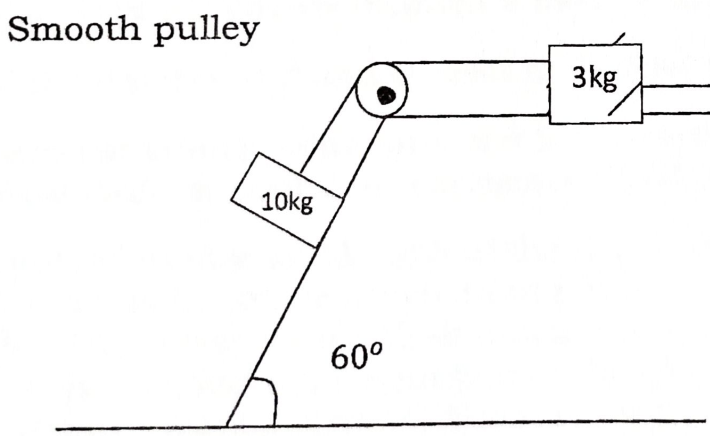
21. A particle of mass 2kg initially located at point (2,1)m at rest moves in a Cartesian plane with an acceleration of $(4e^{2t}\mathbf{i}+9\sin 3t\mathbf{j})ms^{-2}$.Find (i) speed at t=2s(ii) displacement at t=1s.

22.A boat is initially 30 km west of a ship travelling at 80 km/h due North. If the boat is travelling at 60 km/h due N30E,

a) Find the velocity of the boat relative to the ship

b) Show that collision will never occur, hence find the time when they nearest together.

23.The figure below shows a mass of 10kg placed on a smooth incline of inclination 60^o attached to a mass of 3kg placed on a rough horizontal table (μ=0.2) by means of an inelastic string and connected to a second mass of 2kg on the table by means of another string.



(a) Find the acceleration of each mass.

(b) Find the tension in each string and the reaction on the pulley

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