P425/1

PURE MATHS

Paper 1

$2^1/_4$ Hours

END OF YEAR EXAMINATIONS 2023

Uganda Advanced Certificate of Education

S.5 PURE MATHEMATICS

Paper 1

2 Hour 15 Minutes

INSTRUCTIONS:

- *Answer **all** the questions in section **A** and any **four (04)** questions from section **B**.*
- *Any additional question(s) answered shall **not** be marked.*
- *All the necessary working **should** be clearly shown.*
- *Begin each answer in section **B** on a **fresh** sheet of paper.*
- *Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *Attach the question paper on your answer sheets.*

SECTION A						SECTION B						TOTAL
1	2	3	4	5	6	7	8	9	10	11	12	

SECTION A (30 MARKS)

Answer **all** questions in this section.

1. Solve the equation $4^x - 2^{x+1} - 15 = 0$.

(05 marks)

2. Prove that $\frac{sinx+sin3x+sin5x}{cosx+cos3x+cos5x} = tan3x$.

(05marks)

3. Find the shortest distance from the point (4,3,5) to the plane $6x - y + 2z = 14$.

(05 marks)

4. Differentiate $\frac{\cos x}{1+\sin x}$ with respect to x.

(05 marks)

5. The vertices of the triangle *ABC* are *A*(1,2,-1), *B*(1,3,2) and *C*(0,2,1). Find the area of the triangle.

(05 marks)

6. If $y = Ax^k$, where A and k are non-zero constants. Find the value of k such that $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$. **(05 marks)**

SECTION B (48 MARKS)

Attempt only **four (04)** questions from this section. All questions carry equal marks.

7. A line is given by the equation $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{-1}$ and the plane is given by the equation $x + y + z = 12$.
   (a) Determine the point of intersection of the line and the plane **(06 marks)**
   (b) What is the angle between the line and the plane? **(06 marks)**

8. (a) Find all the angles of θ for the range $0° \leq \theta \leq 360°$ which satisfy the equation $sin^2\theta - 2sin\theta cos\theta - 3cos^2\theta = 0$. **(05 marks)**
   (b) Show that $\frac{cos\theta}{1+sin\theta} = \cot\left(\frac{\theta}{2} + 45°\right)$ hence solve the equation that $\frac{cos\theta}{1+sin\theta} = \frac{1}{2}$ for the range $0° \leq \theta \leq 360°$. **(07 marks)**

9. (a) When the quadratic expression $ap^2 + bp + c$ is divided by $p - 1$, $p - 2$ and $p + 1$, the remainders are 1, 1 and 25. Determine the factors of the quadratic expression.
   (b) Express $2x^3 + 5x^2 - 4x - 3$ in the form $(x^2 + x - 2)Q(x) + Ax + B$, where $Q(x)$ is a polynomial in *x*. Determine the;
   (i) value of the constants *A* and *B*.
   (ii) expression *Q(x)*.

(12 marks)

10. (a) In the expansion of $(9 + ax)^n$, the first three terms are $27 - 9x + \frac{1}{2}x^2$. Find the value of x and n. **(06 marks)**

(b) The first three terms of a geometric progression are $3 - \frac{1}{3} + \frac{1}{27}$. Find the least value of n for which the sum of the first n terms of this progression differs from the sum to infinity by less than 10^{-2}. **(06 marks)**

11. (a) Given that $y = \tan\left(\frac{x+1}{2}\right)$, show that $\frac{d^2y}{dx^2} = y\frac{dy}{dx}$. **(05 marks)**

(b) Partialise $f(x) = \frac{x^2-4}{(x+1)^2(x-5)}$ hence find $f'(x)$. **(07 marks)**

12. A piece of wire $48cm$ long is divided into two parts. One part is formed into a circle of radius $r\ cm$ and the other into a square of side $x\ cm$.

(i) Show that $r = \frac{24-2x}{\pi}$. **(06 marks)**

(ii) Find the expression in terms of x for the total area A of the two shapes and hence calculate the minimum value of A. **(06 marks)**

***** END *****