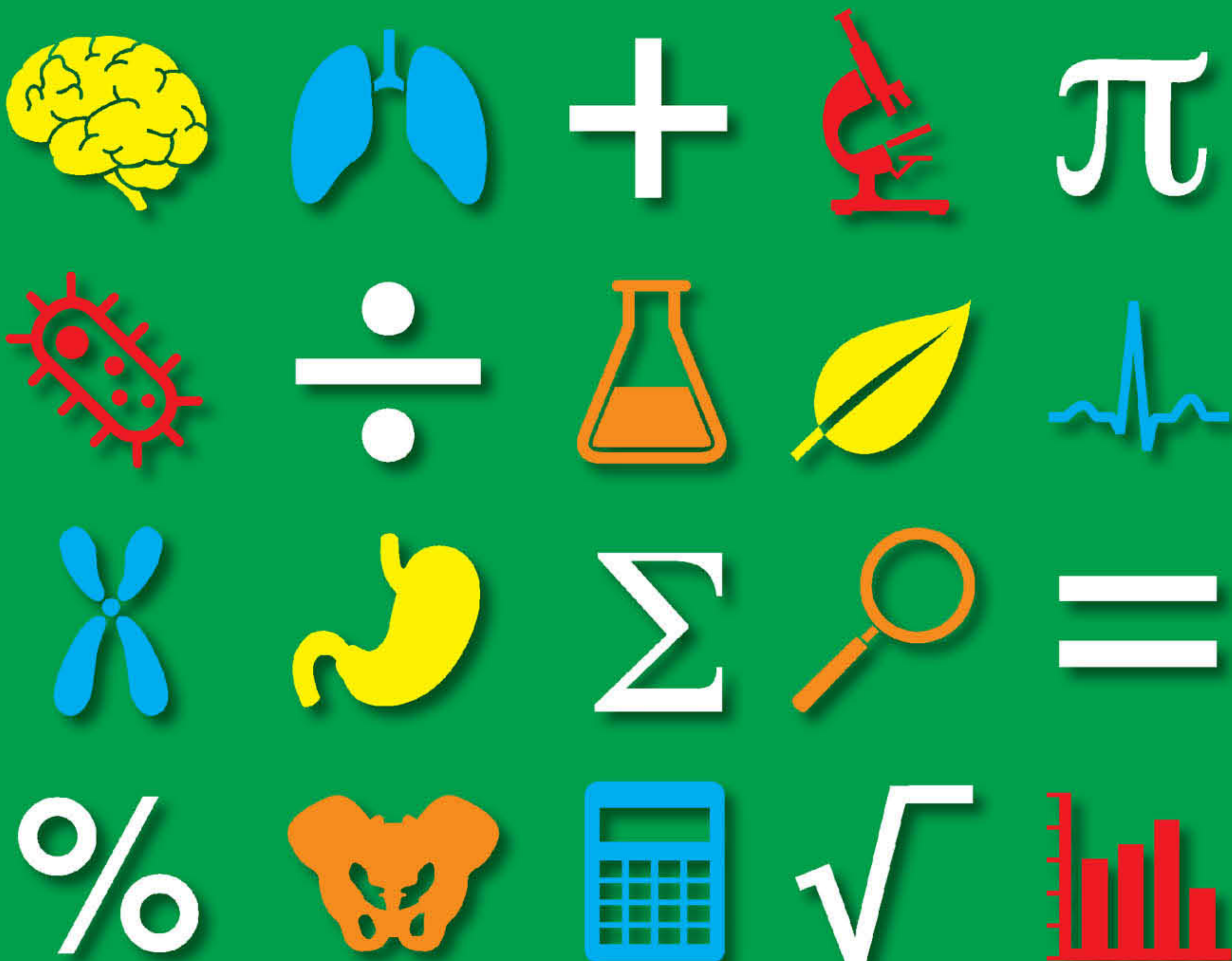
Essential Maths Skills

for AS/A-level

Biology

Dan Foulder



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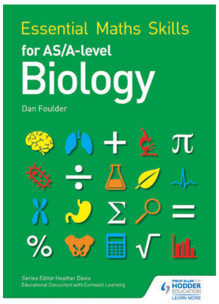

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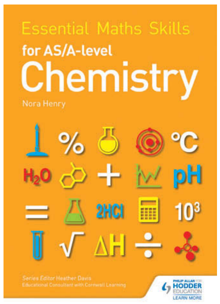

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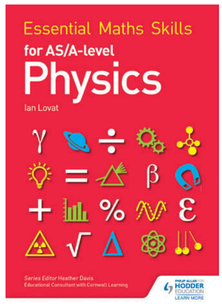

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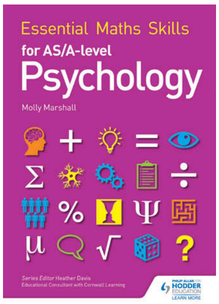

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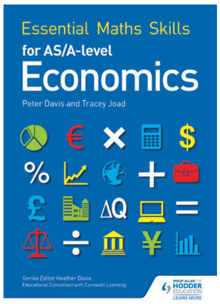

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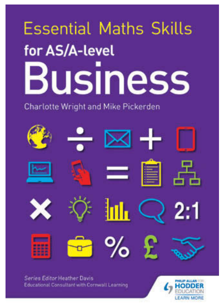

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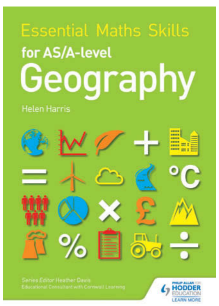

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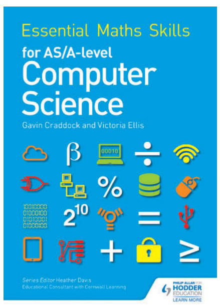

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Biology

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The content listed in bold is only specified to be assessed at A-level.

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Introduction

Mathematical skills are vitally important in biology. Not only will they make up at least 10% of the marks in your biology exams, but they are also essential if you wish to continue your biological studies beyond A-level. This book covers all the maths skills you need to know for your A-level biology exams. No prior knowledge is assumed, and all concepts are explained thoroughly. The questions in each section start off easier and then become more challenging.

You should use this book to practise applying any mathematical concepts that you come across in lessons or in your main textbook, especially those you find challenging. The key to improving your mathematical skills is practice! All of the questions in this book relate to the biology you're studying, so not only are you practising maths skills but you're developing your subject knowledge at the same time. There are also exam-style questions at the end of the book, which integrate different mathematical skills and biological subject knowledge.

The book units and sections are organised around the main maths skills set out by exam boards. While some areas of the specifications are more likely to include certain maths skills, it is important to remember that any of the skills could appear in questions on any topic area. The skill 'Translate information between graphical, numerical and algebraic forms' does not have its own specific unit, as this skill is covered across many of the other units in the book.

Full worked solutions to the guided and practice questions and exam-style questions can be found online at www.hoddereducation.co.uk/essentialmathsanswers.

1 Arithmetic and numerical computation

Appropriate units in calculations

Units are very important in biology. Without them numerical values are often meaningless, and leaving them out will cost you marks in the exam. You should ensure that you use appropriate units across all calculations and data handling and that you can convert between different units. You also need to be able to work out appropriate units in certain situations, for example, in a rate calculation.

A huge range of units is used in biology. Whenever possible you should use the internationally recognised SI units. Table 1.1 shows how prefixes can be used to obtain bigger and smaller versions of the SI units for length (m), mass (g), area (m^2) and volume (m^3).

Table 1.1 SI units

Prefix	Factor	Examples
kilo (k)	1×1000	kg, km, km^2
deci (d)	$\frac{1}{10}$	dm^3
centi (c)	$\frac{1}{100}$	cm, cm^2, cm^3
milli (m)	$\frac{1}{1000}$	mm, mm^2, mm^3, mg
micro (µ)	$\frac{1}{1\,000\,000}$	µm, µg
nano (n)	$\frac{1}{1\,000\,000\,000}$	nm, ng

For example, using the SI unit for length:

$$0.001\,\text{km} = 1\,\text{m} = 100\,\text{cm} = 1000\,\text{mm} = 1\,000\,000\,\mu\text{m} = 1\,000\,000\,000\,\text{nm}$$

It is important that you select the appropriate unit for each situation. Clearly it would be inappropriate to give the length of an organism in kilometres and, similarly, only the smallest organisms or structures would have their lengths measured in micrometres.

In some cases it is not appropriate to append a unit to a numerical value, for example, the absorbance measured by a colourimeter. In such cases arbitrary units (sometimes abbreviated AU) can be used. These units are not standard, cannot be interconverted with any other units and apply only to the results of that particular piece of equipment.

A Worked examples

a What is the length in mm of a bacteria that is 50 µm long?

There are 1000 µm in 1 mm, so you simply divide the length in µm by 1000:

$$\frac{50}{1000} = 0.05\,\text{mm}$$

b The total surface area of alveoli in a human lung is estimated to be 35 m^2. What is this area in cm^2? Show your working.

- Areas are given by lengths squared (e.g. m^2, cm^2, mm^2).
- 1 m contains 100 cm, so 1 m^2 = 100 cm × 100 cm = 10 000 cm^2.
- Similarly, 1 cm^2 = 10 mm × 10 mm = 100 mm^2.

1 m^2 = 10 000 cm^2, so multiply 35 by 10 000:

35 × 10 000

= 350 000 cm^2

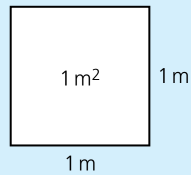

Figure 1.1

c The human body contains 4700 cm^3 of blood. What is this volume in m^3? Show your working.

- Volumes are given by lengths cubed (e.g. m^3, cm^3, mm^3).
- 1 m contains 100 cm, so 1 m^3 = 100 cm × 100 cm × 100 cm = 1 000 000 cm^3.
- Similarly, 1 cm^3 = 10 mm × 10 mm × 10 mm = 1000 mm^3.

1 m^3 = 1 000 000 cm^3, so divide 4700 by 1 000 000:

$$\frac{4700}{1\,000\,000}$$

= 0.0047 m^3

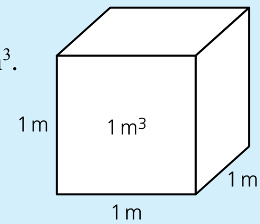

Figure 1.2

You can also get the correct answer by multiplication: 4700 × 0.000001.
This is because 1 cm^3 = 0.01 m × 0.01 m × 0.01 m = 0.000001 m^3.

d In an enzyme investigation, 10 g of product was produced in 30 minutes. What unit would be most appropriate to describe the rate of this reaction? Calculate the rate of the reaction.

- When measuring mass produced over time, the unit needs to include both mass and time.
- When calculating a rate, time always goes second in the unit, e.g. mass per time.
- In this case it seems most appropriate to use grams per minute, which should be written as g min^{-1}.

To calculate the rate of the reaction, divide the mass by the time.

$$\text{rate} = \frac{\text{mass}}{\text{time}} = \frac{10}{30} = 0.33$$

So the rate of the reaction is 0.33 g min^{-1}.

This can be converted into grams per second (g s^{-1}) by dividing by 60, which gives a rate of 0.0056 g s^{-1}.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **What is the mass in grams of a 15 kg soil sample?**

There are 1000 g in 1 kg, so multiply the mass in kg by 1000:

$15 \times 1000 =$ ____________ g

2 **The volume of solution in an experiment was given as 650 mm^3. What is the volume of this solution in cm^3?**

There are 1000 mm^3 in 1 cm^3.

3 **A student is carrying out an investigation into the rate of change of the volume of air in the lungs during a period of exercise. What would be the most appropriate unit to use in this investigation?**

The student is investigating a rate, so the unit will require a volume component and a time component.

C Practice questions

4 A sample of water from a lake exhibiting signs of eutrophication had a volume of 3.6 dm^3. What is the volume of this sample in cm^3?

5 In an A-level biological investigation it would be unusual to see volumes given in m^3. Explain why.

6 A measuring cylinder would not be an appropriate piece of apparatus to measure a volume in mm^3. Explain why.

Note: question 7 is for A-level candidates only.

7 a An ecologist is comparing the flow of energy in a small woodland and a larger area of grassland over several years. When drawing pyramids of energy to compare these two areas, what would be the most suitable unit to use?

b This same unit would not be suitable for studying energy flow in aquatic ecosystems. Explain why.

Expressions in decimal and standard form

In biology we often use very large numbers or very small numbers. Rather than writing these numbers with many zeros before or after the decimal point, we can use standard form (also known as scientific notation) to present the numbers more compactly. Here are some examples of standard form:

- $10\,000 = 1 \times 10^4$
- $1000 = 1 \times 10^3$
- $100 = 1 \times 10^2$
- $10 = 1 \times 10^1$
- $0.1 = 1 \times 10^{-1}$
- $0.01 = 1 \times 10^{-2}$
- $0.001 = 1 \times 10^{-3}$
- $0.0001 = 1 \times 10^{-4}$

As you can see, the number of zeros translates into a power of 10 when each number is written in standard form.

A positive power (or exponent) means you multiply by that power of 10. You can think of this as multiplying by 10 the same number of times as the power. For example,

$$1 \times 1000 = 1 \times 10^3 = 1 \times 10 \times 10 \times 10$$

When representing numbers that are smaller than 1 in standard form, you get negative powers. You can think of this as dividing by 10 the same number of times as in the power. For example,

$$0.1 = \frac{1}{10} = 1 \times 10^{-1}$$

and

$$0.001 = \frac{1}{1000} = 1 \times 10^{-3}$$

A Worked examples

a **A capillary has a width of 0.006 mm. Write this width in standard form.**

We know that $0.001 = 1 \times 10^{-3}$, so
$0.006\,\text{mm} = 6 \times 10^{-3}\,\text{mm}$

b **A temperate forest sample site was 2×10^3 m long and 1×10^3 m wide. What is the total area of this sample site?**

When multiplying numbers in standard form, add the powers together and multiply the other numbers.

$$2 \times 10^3 \times 1 \times 10^3$$

$$= 2 \times 1 \times 10^{3+3}$$

$$= 2 \times 10^6\,\text{m}^2$$

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **In an enzyme-catalysed reaction, 4×10^5 grams of product were produced in 2×10^2 seconds. What is the rate of reaction in g s^{-1}?**

- To find the rate of reaction, divide the mass of product by the time.
- When dividing numbers in standard form, subtract the powers of 10 and divide the other numbers.

$$\frac{4 \times 10^5}{2 \times 10^2}$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\ \text{g s}^{-1}$$

2 **Figure 1.3 shows a sample being serially diluted. What was the final dilution factor?**

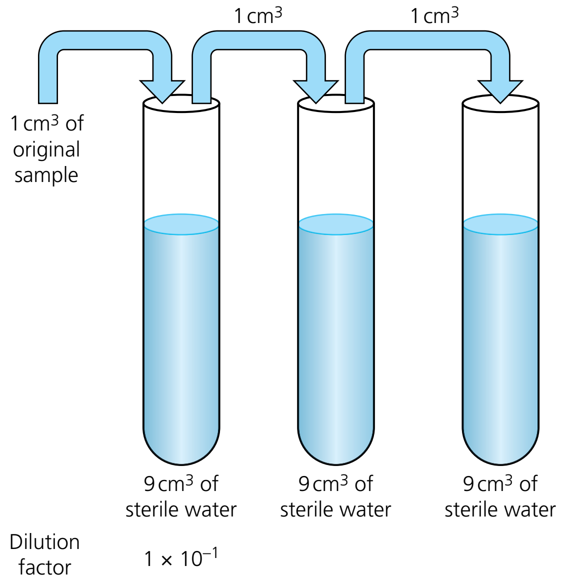

Figure 1.3

Step 1: after the first dilution, the volume has changed from 1 cm^3 to 10 cm^3, so the concentration becomes one-tenth of the original.

Dilution factor = $\frac{1}{10} = 0.1 = 10^{-1}$

Step 2: after the second dilution, the concentration becomes ______________ of the original.

Dilution factor = ______________

Step 3: after the third dilution, work out the concentration relative to the original sample.

C Practice questions

3 A ribosome has a length of 0.00003 mm. Write this length in standard form.

4 The body length of a blue whale is 25 m. What is this length in mm in standard form?

5 How much larger is the whale in question 4 than the ribosome in question 3? Give your answer to the nearest power of 10.

6 During an analysis of the number of bacteria in a sample of river water, 40 000 bacteria were found in 1 cm^3 of the river water. How many bacteria would there be in 100 cm^3 of the river water? Give your answer in standard form.

7 Figure 1.4 shows the serial dilution of a sample of bacteria. Find the final dilution factor. What was the number of bacteria in 1 cm^3 of the original sample?

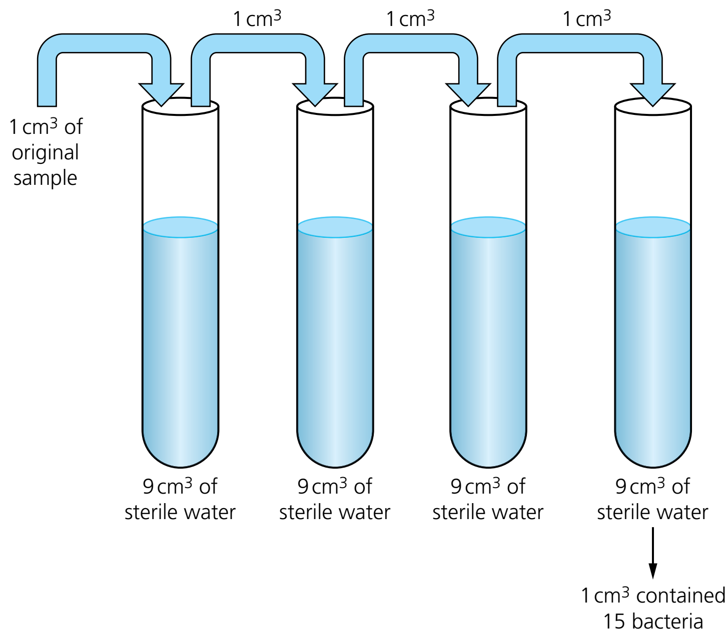

Figure 1.4

Ratios, fractions and percentages

Fractions

A fraction is part of a whole, and is expressed as a whole number divided by another whole number. For example, one-half is written as $\frac{1}{2}$, but can also be represented by $\frac{4}{8}$ or $\frac{5}{10}$. When using fractions, it is good practice to write each fraction in its simplest form.
To find the simplest form of a fraction, divide both numbers in the fraction (top and bottom) by the same whole number (a common factor), and carry on doing this until you are left with top and bottom numbers that cannot be divided further to give whole numbers.

For example:

- In $\frac{4}{16}$, both top and bottom can be divided by 4 to give whole numbers, so $\frac{4}{16} = \frac{1}{4}$.
- In $\frac{6}{9}$, both top and bottom can be divided by 3 to give whole numbers, so $\frac{6}{9} = \frac{2}{3}$.

  Note that 2 and 3 cannot be further divided by the same number to give whole numbers, so $\frac{2}{3}$ is the simplest way of writing this fraction.

Percentages

Like fractions, percentages are part of a whole, but they are expressed in the form of a number followed by the percentage symbol %, which means 'divided by 100' or 'out of 100'.

For example: $\frac{1}{4} = \frac{25}{100} = 25\%$.

To convert a fraction into a percentage, divide the top number by the bottom number and multiply by 100. In biology, common questions involving percentages include calculating the loss as a percentage or a finding the percentage yield of a reaction.

Ratios

A ratio expresses a relationship between quantities. It shows how many of one thing you have relative to how many of one or more other things.

For example, suppose that when two plants of a certain species are crossed, six offspring with blue flowers are produced for every four offspring with yellow flowers. Then the ratio of blue to yellow offspring is 6 blue : 4 yellow.

Just like with fractions, we should always try to represent ratios in their simplest form. To do this, divide all the numbers in the ratio (separated by a colon :) by the same number (a common factor), and continue doing this until you are left with an expression that cannot be divided further to give whole numbers.

In the example above, both sides of the ratio can be divided by 2 to give whole numbers. The ratio then becomes 3 blue : 2 yellow.

The numbers 3 and 2 cannot be divided by the same number to give whole numbers, so this is the simplest form of the ratio.

At A-level you may be asked to use a ratio to determine the expected number of offspring of each phenotype in a genetic cross. See Worked example **d** below.

A Worked examples

a **In an immobilised enzyme investigation the relative amount of product produced was given as the fraction $\frac{28}{84}$. What is the simplest form of this fraction?**

Step 1: divide both numbers in the fraction by the same number to get whole numbers. Both 28 and 84 are even and so have 2 as a factor. Divide them both by 2 to get $\frac{14}{42}$.

Step 2: both 14 and 42 are even as well, so divide them both by 2 again to get $\frac{7}{21}$.

Step 3: clearly the top number 7 has itself as a factor. Notice that the bottom number 21 is also divisible by 7. So divide both numbers by 7 to get $\frac{1}{3}$.

It is not possible to further divide top and bottom by the same number to get whole numbers, so $\frac{1}{3}$ is the simplest form of the fraction $\frac{28}{84}$.

If you recognise from the outset that 28 is a factor of 84, a quicker way to do the calculation would be to divide both top and bottom by their common factor 28:

$$\frac{28}{84} = \frac{28 \div 28}{84 \div 28} = \frac{1}{3}$$

b **In an osmosis investigation, a potato sample lost 2 g of its total mass of 10 g. What percentage of its mass did the potato lose?**

Step 1: in this case the ‘whole’ is 10 and the ‘part’ is 2, so the potato lost $\frac{2}{10}$ of its mass.

Step 2: to convert this fraction to a percentage, divide the top number 2 by the bottom number 10 and multiply by 100:

$$\frac{2}{10} = 2 \div 10 \times 100\% = 20\%$$

c i **A survey of birds in a garden gave the following results:**

- **2 crows**
- **3 pigeons**

What was the ratio of crows to pigeons?

In this case

number of crows : number of pigeons = 2 : 3

The numbers 2 and 3 do not have a common factor other than 1, so the simplest form of the ratio is 2 : 3.

ii **On another day the following results were recorded in the same garden:**

- **3 robins**
- **9 crows**
- **6 pigeons**

What was the ratio of bird species in the garden on this day?

The ratio of robins to crows to pigeons was

3 : 9 : 6

In this case, each of the three numbers in the ratio is divisible by 3 (a common factor), so the ratio can be simplified to

1 : 3 : 2

These numbers can no longer be divided by a common factor to give whole numbers, so the simplest form of the ratio is 1 : 3 : 2.

Note: Worked example d is for A-level candidates only.

d **In a genetic cross the predicted ratio of offspring is 4 red : 1 white. If there were 50 offspring, how many would be expected to be red and how many white?**

Step 1: add the numbers in the ratio together.

$4 + 1 = 5$

Step 2: divide the total number of offspring by the number found in Step 1.

$50 \div 5 = 10$

This is how many each '1' in the ratio represents.

Step 3: multiply each number in the ratio by the value found in Step 2.

Therefore, we expect there to be:

- **4 × 10 = 40 red offspring**
- **1 × 10 = 10 white offspring**

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **In an investigation into the effect of the surface area to volume ratio on the diffusion rate, a gelatin cube was used which had a surface area of 24 cm^2 and a volume of 8 cm^3. What is the surface area to volume ratio of this cube?**

Step 1: when writing a surface area to volume ratio, the surface area goes before the colon and the volume goes after the colon.
So the surface area to volume ratio of this cube is ____________

Step 2: if the ratio in Step 1 is not in its simplest form, divide the two numbers by a common factor until they can no longer be divided further to give whole numbers.
The highest common factor of the two numbers is ____________, so divide both numbers by this factor to get the simplified ratio ____________

2 **What is the percentage yield of an enzyme-catalysed reaction that produces 45 g of useful product from 134 g of substrate?**

To calculate percentage yield of a reaction, use the original amount of substrate as the 'whole' and the amount of product as the 'part'. Then express this fraction as a percentage.

C Practice questions

3 An enzyme-catalysed reaction produced 6 g of product from 17 g of substrate. What was the percentage yield?

4 An organism has a surface area of 16 cm^2 and a volume of 12 cm^3. What is the surface area to volume ratio of this organism?

5 Which of the following surface area to volume ratios is the largest?

A 2 : 3
B 15 : 8
C 7 : 2

6 The four bases in DNA are adenine, thymine, guanine and cytosine. Adenine always pairs with thymine, and cytosine always pairs with guanine. In a given sample of DNA, 24% of the bases are thymine. What percentage of the bases are guanine?

Note: questions 7 and 8 are for A-level candidates only.

7 Pea plants can be either tall (dominant) or short (recessive). If a heterozygous tall pea plant is crossed with a short pea plant, what would be the expected phenotype ratio?

8 In a certain species of fish, a larger dorsal fin is dominant to a smaller dorsal fin, and yellow scales are dominant to blue scales. If two heterozygotes with yellow scales and larger dorsal fins were to breed, what would be the expected phenotype ratio?

Estimating results

When carrying out calculations it is often useful to estimate the answer first. Then, if you enter the wrong number on your calculator, or divide instead of multiply etc., you can see that the answer you have obtained is clearly wrong. You can then check the calculation and correct your mistake.

When estimating, you want to make the calculations easier, so try to round each given value to the nearest ten, hundred or other convenient whole number.

While estimating is an useful skill that helps you check if a calculation result is correct, in an exam it is important to use your calculator to find the value precisely and write this as the answer.

A Worked example

What is the area of a woodland that is 15.65 km long by 2.15 km wide?

Step 1: for a quick estimation of the area, round both given values to the nearest whole number.

- **15.65 km rounds up to 16 km**
- **2.15 km rounds down to 2 km**

Step 2: perform the calculation with the rounded values. This gives an estimated area of

$16\,\text{km} \times 2\,\text{km} = 32\,\text{km}^2$

Using a calculator, the actual value is $15.65\,\text{km} \times 2.15\,\text{km} = 33.65\,\text{km}^2$.

Clearly the estimated and actual values are different, and $32\,\text{km}^2$ is not the correct answer and would not score a mark in an exam question. However, it is close enough to the correct value that if we had made a mistake in our calculation, we could spot it by comparing the answer with 32.

For example:

- **If we had pressed ÷ instead of × on the calculator, we would get $7.28\,\text{km}^2$ as the answer.**
- **If we had put the decimal point of the width in the wrong place, we might get $15.65 \times 21.5 = 336.48\,\text{km}^2$.**

In both of these cases, our estimate of 32 would tell us that something has gone wrong, and then we could go back and correct the mistake.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **An enzyme-catalysed reaction produces 478 g of product in 18 seconds. Estimate the rate of this reaction.**

Step 1: round the given values.

478 g rounded to the nearest hundred is ______________

18 s rounded to the nearest ten is ______________

Step 2: perform the calculation with the rounded values.

2 **A cube has a side length of 3.7 mm. Estimate the volume of the cube.**

3.7 mm rounded to the nearest whole number is ______________

Step 2: perform the calculation with the rounded value.

C Practice questions

3 A sample of solution contained 5 438 229 bacterial cells, and another solution contained 6 936 393 bacterial cells. Estimate the total number of bacterial cells in the two solutions.

4 In an ecological sampling activity, 429 of an endangered flower were found in a $9\,km^2$ area. Estimate the number of this type of flower per km^2.

5 The probability of a certain species of rabbit having a white coat is 0.53. Out of a population of 170 such rabbits, estimate how many are expected to have white coats.

6 The length of one side of a cube is 4.54 cm. Estimate the volume of the cube.

7 In a biological investigation, a sample of plant tissue lost 16 g of mass out of a total of 53 g. To estimate the percentage change in mass, a student did the following calculation:

$$\frac{10}{50} \times 100\% = 20\%$$

Why is this not the best estimate the student could have made?

Power, exponential and logarithmic functions

Note: this topic is assessed at A-level only.

It is important that you can correctly use your scientific calculator to find power, exponential and logarithmic functions. Different models of calculators may have different ways of accessing these functions, so make sure that you're familiar with how your own calculator works, or else consult the instruction manual.

A Worked examples

a **In a sampling activity, a 82 m by 82 m section of forest is sampled. What is the total surface area of this section of forest?**

$$\text{Total area} = 82\,\text{m} \times 82\,\text{m} = 82^2\,\text{m}^2$$

Rather than multiplying out 82^2, you could type 82 on your calculator and press the 'square' button (usually represented by x^2) to get the answer:

$$82^2 = 6724\,\text{m}^2$$

b **A cube of plant tissue used in an experiment has sides of length 4 cm. What is the volume of the cube?**

$$\text{Volume} = 4\,\text{cm} \times 4\,\text{cm} \times 4\,\text{cm} = 4^3\,\text{cm}^3$$

As in the previous example, rather than multiplying out 4^3 you could type 4 on your calculator and press the cube button (usually represented by x^3) to get the answer:

$$4^3 = 64\,\text{cm}^3$$

c **The number of zooplankton in a sample of sea water was estimated to be 4^5. Find the total number of zooplankton.**

Most calculators have a square button and a cube button, but for other powers you need to use the button labelled x^y or y^x.

In this case type 4 then x^y (or y^x) then 5 to get

$4^5 = 1024$

d **In a solution of stomach acid, the concentration of hydrogen ions, [H^+], was 0.0030 M. Using the formula pH = −log[H^+], calculate the pH of this solution.**

Step 1: substitute the given concentration of hydrogen ions into the formula.

pH = −log[0.0030]

Step 2: using the log button on your calculator, find this value by pressing ± then log then 0.003. This gives the answer

pH = 2.52

e **Exponential bacterial growth can be modelled using the formula**

$\boldsymbol{A = Pe^{rt}}$

Where

- ***A*** **is the number of bacteria in the population at any given time** ***t***
- ***P*** **is the number of bacteria in the initial population**
- ***r*** **is the growth constant**
- ***t*** **is time**

If there were five bacteria initially and the growth constant is 2 h^{-1}, how many bacteria would the population contain after seven hours? Give your answer in standard form and to two significant figures.

We need to substitute the given information into the formula.

Step 1: calculate the value of the power, *rt*.

In this example, $r = 2$ and $t = 7$, so

$rt = 2 \times 7 = 14$

Step 2: the initial population was five cells, so $P = 5$. Therefore the total population after seven hours is given by

$A = 5e^{14}$

Step 3: to work out this value on your calculator, it is best to find e^{14} first. Depending on what calculator model you have, either type 14 and then press the e^x key or press e^x first and then type 14. Finally, multiply the value obtained by 5. Thus, to the nearest whole number,

$A = 6\,013\,021$

Step 4: express the value of A in standard form and to two significant figures.

$A = 6.0 \times 10^6$ cells

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **The volume of a bird's throat pouch was estimated as 15^3 cm^3. Calculate this volume.**

15^3 = _____________ cm^3

2 **An investigation into catalase activity used hydrogen peroxide with a hydrogen ion concentration of 3×10^{-13} M. Using the formula pH = −log[H$^+$], find the pH of the hydrogen peroxide.**

In this case [H$^+$] = _____________

So pH = _____________

C Practice questions

3 An area of beach used in a sampling investigation was given as 365^2 m^2. Calculate the total area of the beach.

4 The approximate number of zooplankton in a sample of sea water was 3^9. Find the total number of zooplankton in the sample.

5 The population of a bacterial colony can be estimated using the equation

$$A = Pe^{rt}$$

where

- A is the number of bacteria in the population at any given time t
- P is the number of bacteria in the initial population
- r is the growth constant
- t is time

If the colony began with one bacterium and has a growth constant of 0.5 h^{-1}, how many bacteria would be in the colony after 20 hours?

6 Using the formula pH = −log[H$^+$], find the pH of a solution with a concentration of hydrogen ions of 0.1 M.

7 The number of different combinations of chromosomes in a gamete due purely to independent assortment is 2^n. For example, in a human the number of different combinations is 2^{23}.

a What does n represent?

b A fruit fly has diploid number 8. How many different combinations of chromosomes could be produced by crossing over in a fruit fly?

c Write down a formula for the possible number of different combinations of chromosomes due to independent assortment following random fertilisation of two gametes.

2 Handling data

Significant figures

Significant figures are a complex topic, and there are exceptions even to the rules outlined below. In general, all digits are significant figures **except**:

- **leading zeros**, i.e. zeros before a non-zero digit.
  For example, 0.07 has two leading zeros, and these are not significant figures. So 0.07 only has one significant figure, and the zeros are written there to make the place value correct.
- **zeros after a non-zero digit if they are due to rounding or used to indicate place value**.
  For example, a value rounded to 600 g to the nearest hundred has two trailing zeros, which are not significant, and so it has only one significant figure. A value that is exactly 600 g, however, would have three significant figures, and the zeros in this case would be significant.
- **spurious digits**, i.e. digits which make a calculated value appear more precise than the original data used in the calculation.
  For example, suppose that one side of a square was measured with a ruler to be 13.1 cm long. Using this measurement to calculate the area of the square gives a value of 171.61 cm^2 (13.1×13.1). The ruler measured only to three significant figures, while the answer appears to contain five significant figures. But the last two digits (6 and 1) are spurious and should not be included in the final answer. Therefore the result should be given rounded to 172 cm^2 (three significant figures as in the original measurement). When two or more different measuring apparatuses are used, calculated results should be reported to the limits of the least accurate measurement.

A Worked examples

a How many significant figures does the number 0.005601 have?

- Identify the first non-zero digit from the left. This is 5.
- The three zeros to the left of 5 are leading zeros and therefore not significant.
- The other four digits (5, 6, 0 and 1) are significant.

So this number has four significant figures.

b A nerve cell was measured as being 1302 mm long. Write this length to two significant figures.

- Identify the first non-zero digit from the left. This is 1, and there are no leading zeros to its left.
- The first significant digit is 1, and the digit immediately to its right, 3, is the second significant digit. These are all the significant digits we need to keep.
- Look at the digit immediately to the right of 3, and use it to decide whether to round up or down. In this case the digit is 0, so the number rounds down to 1300. The two trailing zeros are not significant.

Therefore, to two significant figures, the length is 1300 mm.

This could also be written as 1.3×10^3 mm in standard form.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Write the number 0.01068 to two significant figures.**

- Identify the first non-zero digit from the left: ____________
  Any leading zeros to its left are not significant.
- Identify the second significant figure that you need to keep: ____________
- Look at the digit immediately to the right of the second significant figure, and use it to decide whether to round up or down.

2 **A mass was given as 890 g to the nearest ten. How many significant figures has this number?**

As this mass is rounded to the nearest ten, the trailing zero is not significant.

C Practice questions

3 Write 1865 cm^3 to two significant figures.

4 Write 0.09076 M to three significant figures.

5 How many significant figures are in the number 2.809×10^5?

6 A ruler was used to measure the side of a cube, giving the length as 22.1 cm. The volume of the cube was then calculated to be 10 793.861 cm^3. Give this volume to the correct number of significant figures.

7 The mass of a fruit, as measured by a balance, was 200.32 g. The fruit was then placed into a column of immobilised enzymes and produced 54 cm^3 of juice. Calculate the volume of juice produced per gram of fruit, ensuring that your answer contains the correct number of significant figures.

Arithmetic means

The mean (often denoted by $\bar{x}$) is an 'average' that is calculated by adding all the individual data values together and dividing by the total number of data points. It is the most commonly used average in biology and the one you will normally use in biological practical investigations.

The mean does have some disadvantages, however. For instance, it can be skewed by extreme results. If such 'outlier' values are present in a set of data, it might be more appropriate to use another type of average (such as the median or mode — see pages 36–38) or discard the outlier data before calculating the mean.

A Worked examples

a **Table 2.1 shows the mitotic index of five samples. Calculate the mean mitotic index.**

Table 2.1

Sample	Mitotic index
1	66
2	48
3	64
4	53
5	59

Step 1: add all the data values together.

$$66 + 48 + 64 + 53 + 59 = 290$$

Step 2: divide by the total number of data points, which is 5 in this case.

$$\frac{290}{5} = 58$$

Hence, the mean mitotic index is 58.

b **Table 2.2 shows the volume of water absorbed by a plant stem over time. Three volume measurements were taken at each time point. Calculate the mean volume of water absorbed at each time point.**

Table 2.2

Time (min)	Volume of water absorbed (cm^3)		
	1	2	3
0	0.0	0.0	0.0
10	0.2	0.4	0.2
20	0.5	0.4	0.6
30	0.9	0.9	1.0
40	1.5	1.3	1.4
50	1.7	1.5	1.7
60	1.7	1.6	1.3

Step 1: at each time point, add the three volume measurements together.
Append a column to Table 2.2 showing this sum.

Table 2.3

Time (min)	Volume of water absorbed (cm^3)			
	1	2	3	Sum
0	0.0	0.0	0.0	0.0
10	0.2	0.4	0.2	0.8
20	0.5	0.4	0.6	1.5
30	0.9	0.9	1.0	2.8
40	1.5	1.3	1.4	4.2
50	1.7	1.5	1.7	4.9
60	1.7	1.6	1.3	4.6

Step 2: divide the sum at each time point by 3 (the number of repeat readings taken).

Table 2.4

Time (min)	Mean volume of water absorbed (cm^3)
0	0.0
10	0.3
20	0.5
30	0.9
40	1.4
50	1.6
60	1.5

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **A bamboo plant grew by 139 cm in one week. What was the mean growth per day?**

- Although we do not know the amount of growth each day, we do know that the total growth over 7 days (one week) was 139 cm.
- To find the mean growth per day, divide the total growth by the number of days.

2 **An investigation was carried out into the variation of heart rate over a 12-hour period. The heart rate was measured once every hour, and the results are shown in Table 2.5. What was the mean heart rate over this 12-hour period?**

Table 2.5

Time	Heart rate (beats per minute)
01:00	60
02:00	62
03:00	65
04:00	63
05:00	61
06:00	64
07:00	70
08:00	75
09:00	80
10:00	85
11:00	79
12:00	82

The total of all the measurements is ______________

To find the mean, divide the total by ______________ to get ______________

C Practice questions

3 In an investigation, the heights of five plants were measured to be 16 cm, 32 cm, 21 cm, 29 cm and 19 cm. What was the mean height of the plants?

4 a Table 2.6 shows the rainfall per day over a ten-day period on an area of grassland. What is the mean daily rainfall for this period?

Table 2.6

Day	1	2	3	4	5	6	7	8	9	10
Rainfall (mm)	2	0	1	4	3	0	0	1	3	2

b Following the period in part **a**, it then did not rain for ten days. What would the mean daily rainfall now be for this 20-day period?

5 In an investigation, potato cylinders were placed in different external solute potentials and the changes in their mass were recorded. Three measurements were taken at each solute potential. Find the mean percentage change in mass at each of the different solute potentials.

Table 2.7

External solute potential (kPa)	Change in mass (%)		
	1	2	3
0	10	9	11
−100	8	9	9
−200	6	6	6
−300	4	4	2
−400	2	1	−1
−500	0	1	0
−600	0	−3	−2
−700	−7	−2	−4
−800	−6	−5	−6
−900	−9	−8	−9
−1000	−9	−10	−12

6 An investigation into the action of catalase at different hydrogen peroxide concentrations was carried out, and the results are shown in Table 2.8. Calculate the mean time taken for the catalase-soaked disc to rise to the surface at each hydrogen peroxide concentration.

Table 2.8

Hydrogen peroxide concentration (% stock solution)	Time taken for catalase-soaked disc to rise to surface (seconds)		
	1	2	3
20	95	60	39
40	25	41	32
60	19	15	12
80	6	3	10
100	3	2	5

7 In an investigation into the effect of temperature on the cell membranes of beetroot, solutions containing discs of beetroot were placed in water baths of differing temperatures. The beetroot discs were then removed and a colourimeter was used to measure the transmission of green light of the solutions. Five measurements were taken at each temperature. Table 2.9 shows the results.

Table 2.9

Temperature (°C)	Transmission of green light (%)				
	1	2	3	4	5
20	98.6	99.0	100.0	97.0	4.0
30	87.5	89.8	79.0	85.6	89.2
40	75.4	67.0	78.2	76.5	33.0
50	46.4	53.5	49.3	45.0	47.4
60	18.4	26.0	22.4	21.0	23.2

a Calculate the mean percentage transmission of green light at each temperature.

b What do you notice about the mean results?

c What could have caused the effect you noticed in part **b**? What can you do to ensure that this does not adversely affect your calculation of the mean?

Interpreting tables and diagrams

This section covers several skills, such as correctly presenting a table of data and interpreting tables and a variety of diagrams.

When presenting results in a table, it is important to use informative headings, including units, and a consistent number of decimal places in the entries.

The examples in this section deal with some of the types of data representation that could come up in exam questions. Others are covered in the 'Graphs' unit later in this guide.

A Worked examples

a **The raw data below was collected in an investigation into the activity of an enzyme.**

Concentration of substrate (% of stock suspension)

Time taken for reaction to occur: 20%: 90, 81, 87 seconds. 40%: 40, 55, 72 seconds. 60%: 15, 25, 18 seconds. 80%: 11, 13, 15 seconds. 100%: 5, 3, 4 seconds.

Represent the data in a table, including mean values for the reaction time.

Step 1: construct the table. First set up the table headings. Usually the independent variable is in the leftmost column and measurements of the dependent variable are shown in columns to the right. Because you are asked for the mean values, include a column for the mean.

Table 2.10a

Concentration of substrate (% of stock suspension)	Time taken for reaction to occur (seconds)			
	1	2	3	Mean

Step 2: enter the data into the table. At each concentration, calculate the mean reaction time by adding up the three measured times and then dividing by 3.

Table 2.10b

Concentration of substrate (% of stock suspension)	Time taken for reaction to occur (seconds)			
	1	2	3	Mean
20	90	81	87	86
40	40	55	72	56
60	15	25	18	19
80	11	13	15	13
100	5	3	4	4

Remember the following points when constructing a table:

- Units should be shown in the column headings, not in the body of the table.
- Repeat readings of the dependent variable and the mean value should all be included under the dependent variable heading. A common mistake is to create a separate column for the mean with no units shown.
- The number of decimal places should be consistent throughout the table.

b **An ECG trace of a person is shown in Figure 2.1. Use it to calculate the heart rate of this person in beats per minute and the time between atrial and ventricular contractions.**

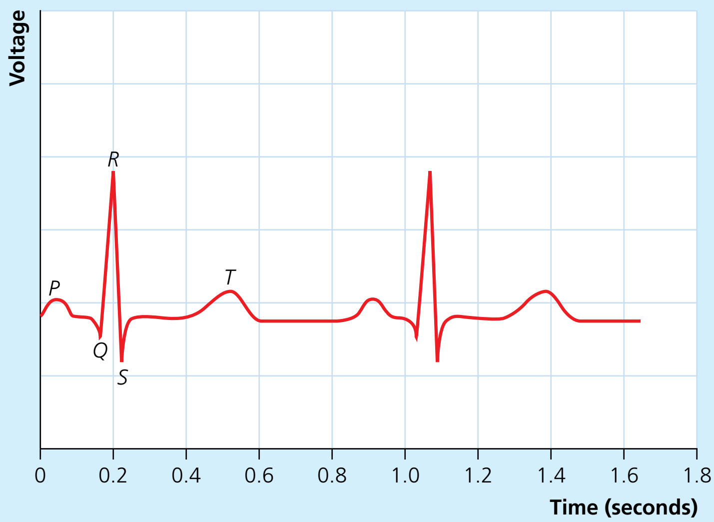

Figure 2.1

- The ECG trace is approximately a repeating pattern. Each cycle of this repeating pattern corresponds to one 'beat' of the heart.
- To calculate the heart rate, first find the length of one cycle of the pattern. This is the time between any two of the same points in the cycle. The 'R' peak is a good point to use, as it is the tallest and most obvious peak.

In this example, the time between two successive R peaks is approximately

$$1.06 - 0.20 = 0.86$$

This means that one cycle takes about 0.86 seconds.

So in one second there are $\frac{1}{0.86}$ cycles.

In one minute, i.e. 60 seconds, there are

$$60 \times \frac{1}{0.86} = 69.8 \text{ cycles}$$

Therefore the heart rate is 69.8 beats per minute.

- The time between atrial and ventricular contractions is the time between the P and R points on the ECG:

  P–R interval = 0.2 – 0.04 = 0.16 seconds

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Table 2.11 shows the vital capacities (maximum volume of air that can be expelled from the lungs after a maximum inhalation) of males of different age groups. Three measurements were taken for each age group.**

Table 2.11

Age (years)	Vital capacity (litres)		
	1	2	3
6–11	2.1	2.0	2.1
12–19	4.4	4.3	4.5
20–39	5.3	4.9	5.2
40–59	4.1	4.6	4.8
60–79	3.8	3.9	4.1

a **Which age group has the highest vital capacity?**

To summarise the data for each age group, you could calculate the mean of each group.

Look for the group with the highest value. This could be the highest value overall or the highest mean value.

b **Comment on the overall trend shown by the data.**

Start with the youngest group and look carefully at how the vital capacities change as you move down through the different age groups.

c **Which age group shows the greatest variation of vital capacities?**

Look for the group with the biggest differences between the values.

2 **From the ECG in Figure 2.2, calculate the person's heart rate in beats per minute and the time between atrial and ventricular contractions.**

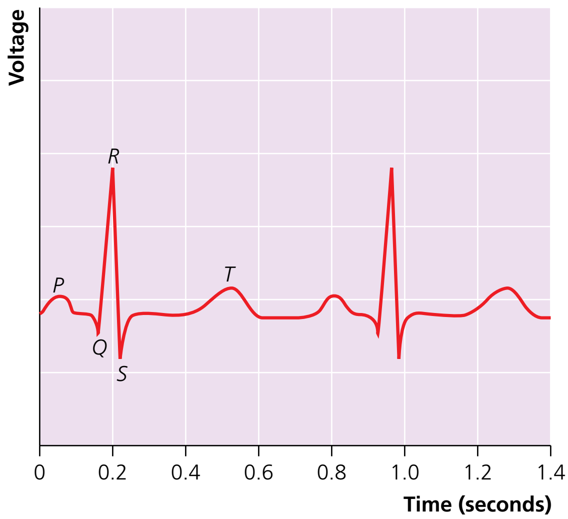

Figure 2.2

Following Worked example **b**, first find the length of one cycle of the pattern.

The time between two successive R peaks is approximately ______________ seconds.

In one second there are ______________ cycles.

So in one minute there are ______________ cycles.

Heart rate = ______________ beats per minute.

The time between atrial and ventricular contractions is the time between the P and R points.

C Practice questions

3 Represent the following raw data in a table and calculate the mean reaction times at the different pH values.

Time taken for reaction to occur, in seconds: pH 2: 67, 56, 59. pH 4: 33, 44, 39. pH 6: 12, 22, 19. pH 8: 18, 17, 16. pH 10: 31, 41, 38.

Note: question 4 is for A-level candidates only.

4 A student produced Table 2.12 during a practical investigation into the effect of distance from a lamp on the rate of photosynthesis in algal balls. The indicator in which the algal balls were suspended becomes darker (more absorbent) the more photosynthesis occurs. What mistakes has the student made?

Table 2.12

Distance from lamp	Absorbance of indicator (550 nm)			Mean
	1	2	3	
250 cm	0.80	0.79	0.83	0.807
350 cm	0.77	0.75	0.7	0.74
500 cm	0.58	0.56	0.52	0.5
750 cm	0.38	0.35	0.39	0.37
1200 cm	0.20	0.23	0.21	0.21

5 Describe how the ECG in Figure 2.3 differs from the ECG associated with a normal resting heart rate.

Figure 2.3

Note: question 6 is for A-level candidates only.

6 Table 2.13 shows the glomerular filtration rates (GFR) characterising five stages of kidney disease.

Table 2.13

Stage of kidney disease	GFR (mL/min/1.73 m^2)
1: Kidney damage with normal or raised GFR	≥ 90
2: Kidney damage with a small decrease in GFR	60–89
3: Moderate decrease in GFR	30–59
4: Severe decrease in GFR	15–29
5: Kidney failure requiring dialysis	< 15

Use the table to answer the following questions.

a At what stage would a person be who had a GFR of 95 mL/min/1.73 m^2?
b At what stage would a person be who had a GFR of 48 mL/min/1.73 m^2?
c At what stage would a person be who had a GFR of 14 mL/min/1.73 m^2?
d A new treatment was found to raise the GFR by 40 mL/min/1.73 m^2. What is the lowest GFR a patient could have, and which stage would they currently be in, if after being treated with the drug their GFR would then move up to stage 2?

Simple probability

Probability is a very important topic for biology in general, but at A-level you will encounter it mainly in the context of genetic crosses.

Probabilities are usually expressed as decimals or fractions, and sometimes as percentages. If something is certain to happen, it has a probability of 1 (100%). If something is certain not to happen, it has a probability of 0.

The sum of the probabilities of all possible outcomes of an experiment is 1. Therefore, if the probability of an event occurring is 0.25, then the probability of that event not occurring is 0.75 (because 0.25 + 0.75 = 1).

A Worked examples

a **The coat colour of a certain species of sheep is either black or white. The probability of a sheep having a white coat is 0.75. What is the probability that a sheep of this species has a black coat?**

- Black and white are the only possible coat colours.
- As the probabilities of all possible outcomes must add up to 1,

  probability of black coat + probability of white coat = 1
- Rearranging gives

  probability of black coat = 1 − probability of white coat
  = 1 – 0.75
  = 0.25

So the probability of a particular sheep having a black coat is 0.25.

Note: Worked example b is for A-level candidates only.

b **In a certain type of plant, green fruit is dominant to red fruit. What is the probability that two heterozygous green plants will produce a plant which has red fruit?**

To answer this question, use a genetic cross.

- Denote the dominant allele for green fruit by G.
- Both of the heterozygous parents have genotype Gg.
- The possible gametes from each parent are G, g.

Table 2.14 Genetic cross

	G	g
G	GG	Gg
g	Gg	gg

- Offspring genotypes: 25% GG, 50% Gg, 25% gg
- Offspring phenotypes: 75% green fruit, 25% red fruit

  Therefore, the probability of a red-fruited offspring being produced is 25% or 0.25.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

Note: questions 1 and 2 are for A-level candidates only.

1 **A species of flower can have red petals, white petals or pink petals. The heterozygotes have pink petals.**

A red-petalled flower was crossed with a pink-petalled flower and 32 plants were produced. How many flowers of each different phenotype would be expected among these 32 plants?

Step 1: assign labels to the alleles involved. In this case flower colour is controlled by codominant alleles. Let us use R to represent the allele for red petals and W to represent the allele for white petals.

- The parents are of genotypes RR and RW.
- The possible gametes are R, R from the red parent and R, W from the pink parent.

Step 2: carry out a genetic cross to work out the probabilities.

Table 2.15 Genetic cross

	R	R
R	RR	RR
W	RW	RW

- Offspring genotypes: _______ % RR, _______ % RW
- Offspring phenotypes: _______ % red, _______ % pink, _______ % white

Step 3: apply the probabilities obtained above to the total number of offspring flowers.

2 **An investigation was carried out into inheritance in a butterfly species. The butterflies' wings can be either orange or yellow and can have white tips or black tips. A dihybrid cross produced an expected ratio of**

9 orange, white-tipped : 3 orange, black-tipped : 3 yellow, white-tipped : 1 yellow, black-tipped

Given this ratio, what would be the expected number of each of the four phenotypes if 128 offspring were produced?

Step 1: add together the numbers in the ratio.

$9 + 3 + 3 + 1 =$ _____________

Step 2: divide the total number of offspring by the number obtained in step 1.

$128 \div$ _____________ $=$ _____________

This tells us the expected value that each '1' in the ratio represents.

Step 3: to calculate the expected number of each phenotype, multiply each of the numbers in the ratio by the value obtained in Step 2. It is easiest to organise this in a table.

Table 2.16

Phenotype	Expected number

C Practice questions

3 In humans the probability of having a boy is 50%. How many boys would be expected in a family with six children?

4 The seeds of a certain plant species can be either yellow or orange. The probability of a seed being yellow is 0.54. In a sample of 763 seeds of this plant, how many would you expect to be orange?

Note: questions 5 and 6 are for A-level candidates only.

5 a A genetic cross carried out by a dog breeder had the following expected genotype ratio for the offspring:

1 homozygous dominant : 2 heterozygous : 1 homozygous recessive

If 12 dogs were produced, what would be the expected number of each genotype?

b The numbers of the different genotypes actually produced from the cross are

4 homozygous dominant, 4 heterozygous, 4 homozygous recessive

Comment on this result.

> As probabilities are concerned with chance, there is always a possibility that the observed results do not match the expected results. Statistical tests (covered in a later section on pages 45–54) can be used to decide whether the difference between observed and expected results is 'significant'.

6 Gregor Mendel investigated inheritance in pea plants. The peas could be either green or yellow in colour, and could be either round or wrinkled in shape. When two heterozygotes of both characteristics were crossed, the offspring were found to have the following phenotype ratio:

9 yellow and round : 3 green and round : 3 yellow and wrinkled : 1 green and wrinkled

If 64 pea plants were produced, how many would you expect of each phenotype?

7 The probability of a certain breed of cat having long fur is 0.37. Two parent cats already have two offspring with long fur. What is the probability that their next kitten will have long fur?

8 Table 2.17 shows the number of individuals per year who suffer from heart attacks (myocardial infarction) in different countries. Based on this data, would it be correct to say that the probability of a random person in the UK suffering from a heart attack is 1.9 per thousand? Explain your answer.

Table 2.17

Country	Heart attacks per year, per 1000 population
USA	2.3
UK	1.9
Australia	2.4
Canada	2

Principles of sampling

Sampling is very wide-ranging practical topic. In this section we look at the mathematics involved in some of the ecological sampling techniques that may be used in your AS or A-level biology course.

Quadrats

Quadrats are tools for assessing the abundance of non-mobile organisms such as plants. Quadrats can be used to estimate:

- **species frequency** — the number of individuals of a certain species found in the sample area. This is often expressed as a percentage of the quadrats which contained organisms of that species. The raw frequency data can also be used to calculate Simpson's diversity index (see Practice question 6 in this section).
- **species density** — the number of individuals of a certain species per unit area.
- **percentage cover** — the percentage of the quadrat area that is occupied by individuals of a particular species. This measure is particularly useful for very numerous species such as grasses.

Mark and recapture

This sampling technique allows us to estimate the number of mobile organisms in a particular area. First, a number of organisms of a certain species are caught from a defined area and marked. These organisms are then released. After a period of time the same area is sampled again, and the marked individuals among this second sample are counted. The total population size can then be estimated using the Lincoln index equation:

$$\text{population size} = \frac{\text{total number in first sample} \times \text{total number in second sample}}{\text{number marked in second sample}}$$

The Lincoln index assumes that between the two sampling times, no individuals:

- die
- are born into the population
- immigrate into the population
- emigrate out of the population

Worked examples

a **Table 2.18 shows part of the results of an investigation into the plants growing in a field. The sampling was carried out using 0.25 m² quadrats.**

Table 2.18

Quadrat	1	2	3	4	5
Number of *Bellis perennis* plants	2	0	3	1	4

i **Using this data, estimate the species frequency of *B. perennis*.**

Step 1: to estimate the species frequency, first count how many of the quadrats contained *B. perennis*. The number is 4.

Step 2: divide the number of quadrats containing *B. perennis* by the total number of quadrats, and multiply by 100 to get a percentage:

$$\frac{4}{5} \times 100\% = 80\%$$

Therefore the species frequency of *B. perennis* is 80%.

ii **Calculate the species density of *B. perennis*.**

Step 1: to calculate the species density, first add together the numbers of *B. perennis* in the different quadrats:

$$2 + 0 + 3 + 1 + 4 = 10$$

Step 2: find the total area of the quadrats:

$$0.25\,\text{m}^2 \times 5 = 1.25\,\text{m}^2$$

Step 3: divide the total number of *B. perennis* plants found in the quadrats by the total area of the quadrats:

$$\frac{10}{1.25} = 8$$

Hence the species density is 8 plants per square metre.

iii **Each of the 0.25 m² quadrats was divided into 25 equal squares. In quadrat 1, grass filled 21 of the smaller squares. What was the percentage cover of grass in this quadrat?**

To find the percentage cover, take the area of the quadrat that is covered by the organism and divide by the total area of the quadrat:

$$\frac{21}{25} \times 100\%$$

$$= 0.84 \times 100\%$$

$$= 84\%$$

So the percentage cover of grass in this quadrat is 84%.

b **During a sampling activity, 34 woodlice were caught, marked and then released. The sampling was repeated a week later, and of the 40 woodlice caught this time, 15 were found to be marked. Calculate the size of the woodlouse population.**

Use the Lincoln index equation to calculate the size of the woodlouse population.

Substitute the given values into the formula:

$$\text{population size} = \frac{\text{total number in first sample} \times \text{total number in second sample}}{\text{number marked in second sample}}$$

$$= \frac{34 \times 40}{15} = \frac{1360}{15}$$

$$= 91 \text{ (to the nearest whole number)}$$

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Table 2.19 shows the number of *Urtica dioica* found in a sample area. The area of each quadrat is 0.25 m².**

Table 2.19

Quadrat	1	2	3	4	5	6	7	8	9	10
Number of *Urtica dioica*	1	3	0	0	0	0	2	0	0	1

a **What was the species frequency of *U. dioica*?**

Step 1: count how many of the quadrats contained *U. dioica.* The number is

____

Step 2: divide the number of quadrats containing *U. dioica* by the total number of quadrats, and multiply by 100 to get a percentage.

b **What was the species density of *U. dioica*?**

Step 1: find the total number of *U. dioica* in the different quadrats.

Step 2: calculate the total area of quadrats used.

Step 3: divide the total number of *U. dioica* by the total area of the quadrats.

2 **During a survey of a seal population, 26 seals were caught, tagged and then released. Several months later 30 seals were caught, and of these 21 had been tagged. Use the Lincoln index to calculate the size of the seal population.**

Substitute the given values into the Lincoln index equation

$$\text{population size} = \frac{\text{total number in first sample} \times \text{total number in second sample}}{\text{number marked in second sample}}$$

Number in first sample = ______________

Number in second sample = ______________

Number marked in second sample = ______________

Ⓒ Practice questions

3 Table 2.20 shows part of the results of a forest survey using 10 m^2 quadrats created with tape measures.

Table 2.20

Quadrat	1	2	3	4	5	6	7	8
Number of *Fraxinus excelsior*	2	6	0	1	0	0	8	0

a What was the species frequency of *F. excelsior*?
b What was the species density of *F. excelsior*?

4 a A 0.25 m^2 quadrat used in an investigation was divided into 25 equal squares. In one sample, 13 of the small squares were covered by *Hedera helix*. What was the percentage cover of *H. helix*?
b The same quadrat showed a 36% coverage by *Rumex magellanicus*. How many squares did *R. magellanicus* cover?

5 a During an investigation into a cockroach population, 63 cockroaches were caught and marked. A week later, 72 cockroaches were caught and five were found to be marked. Use the Lincoln index to calculate the size of the cockroach population.
b From a separate investigation that didn't rely on mark and recapture, it was found that the cockroach population was actually 2500. Give a possible explanation for the discrepancy between these results.

6 The equation for Simpson's diversity index is

$$D = 1 - \sum \left(\frac{n}{N}\right)^2$$

where

- n = number of organisms of a particular species in a certain area
- N = total number of individual organisms in that area

and Σ means 'sum' or 'add together' over the different species found in that area.

Table 2.21 shows the results of an investigation into rainforest insects using pitfall traps. Calculate the diversity index from this data.

Table 2.21

Species	Number (n)
Titanus giganteus	30
Theraphosa blondi	93
Damon diadema	34

Mean, median and mode

The three types of average that you'll come across in biology exam questions are:

- **mean** — this is covered in the 'Arithmetic means' section on pages 20–24.
  Add up all the data values and divide by the total number of data points.
- **median** — this is the middle value in the data set.
  To find the median, arrange all the data points in order and pick out the middle value in the sequence. If there are an even number of data points, take the two in the middle and calculate their mean (i.e. add them together and divide by 2).
- **mode** — this is the most common value in the data set.

The most appropriate type of average to use depends on the context. As mentioned earlier, the most commonly used average is the mean. However, the median is more useful than the mean if there are exceptionally high or low values (outliers) in the data, which would skew the mean. The mode is suitable for use with non-numerical data or when the data points cannot be put in a linear order.

A Worked examples

a **Table 2.22 shows the ages at which the females of a group of elephants in captivity first became pregnant. What was the median age at which the elephants became pregnant?**

Table 2.22

Age of first pregnancy
14
13
15
18
12
22
16

- To find the median, first put the data values in order.
  In ascending order, the ages are
  12, 13, 14, 15, 16, 18, 22
- Select the middle value.
  In this case there are 7 data points, so the middle one is the 4th value, which is 15.
  Therefore the median age of pregnancy is 15.

b **During an ecological investigation, the following daytime temperatures were recorded:**

21 °C, 20 °C, 20 °C, 19 °C, 22 °C, 18 °C, 19 °C, 18 °C, 20 °C, 21 °C

What was the modal temperature?

The mode is the value that occurs most frequently in a data set.

As 20°C occurs more often than any other value in this data set, the modal temperature is 20°C.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **What is the median value of the following set of data on plant heights?**

20 cm, 24 cm, 35 cm, 45 cm, 19 cm, 23 cm

Step 1: put the data values in order.

19 cm, _______, _______, _______, _______, _______

Step 2: select the middle value. As there are an even number of data points, find the two middle values, add them together and divide by 2.

The middle two values are _____________ and _____________

2 **Find the mode of the following set of seed masses:**

4 g, 3 g, 3 g, 3 g, 3 g, 2 g, 2 g, 4 g, 3 g, 3 g, 2 g, 4 g, 3 g, 3 g, 2 g, 3 g, 2 g, 4 g, 3 g, 3 g, 2 g, 3 g, 2 g, 4 g, 3 g, 3 g, 3 g, 2 g, 2 g, 4 g, 5 g

When there are a large number of data points in a data set, it is convenient to use a tally chart to keep track of how frequently each value occurs.

Table 2.23

Mass of seed (g)	Tally
2	
3	
4	
5	

C Practice questions

3 Table 2.24 shows the red blood cell counts of seven hospital patients. What is the median red blood cell count of this group of patients?

Table 2.24

Patient	1	2	3	4	5	6	7
Red blood cell count (million cells per microlitre)	4.7	4.1	5.9	5.4	4.9	5.2	6.1

4 An investigation was carried out into the resting heart rates of a group of people. The data is shown in Table 2.25. What was the median resting heart rate of this group of people?

Table 2.25

Subject	1	2	3	4	5	6
Heart rate (beats per minute)	75	80	99	62	89	76

5 Table 2.26 shows the numbers of different types of ants caught in a pitfall trap during a week-long study.

Table 2.26

Organism	Number caught
Pheidole	16
Crematogaster	53
Polyrhaclis	21
Tetraponera	1

What was the modal organism caught?

6 What is the mode of the following set of DNA fragment lengths?

53 kb, 43 kb, 39 kb, 76 kb, 11 kb, 93 kb

If a data set does not contain any values that occur more than once, then it does not have a mode. This is not the same as saying the mode is zero — the mode would only be zero if 0 was the most common value in the data set.

7 The blood groups of a sample of people are shown below:

A, B, AB, AB, O, A, B, AB, O

a What is the modal blood group?
b Explain why the mode is the only average that could be used for this data.

Scatter diagrams

When data points are plotted on a scatter diagram, it may be possible to identify **correlations** in the data. A correlation can be either positive or negative. Positive correlation means that as one variable increases, the other variable also tends to increase. Negative correlation means that as one variable increases, the other variable shows a decreasing trend.

In some situations it will be clear that correlation exists, but this is not always the case — some data will show no correlation. In exam questions on this skill, the correlation (positive, negative or none) should be evident from the scatter diagram. However, in biological research, statistical tests should always be used to determine whether any observed correlation is significant or not (see the section 'Statistical tests' on pages 45–54).

A Worked examples

a **The data plotted in Figure 2.4 shows the number of eggs laid by birds of different masses. Describe the relationship suggested by this data.**

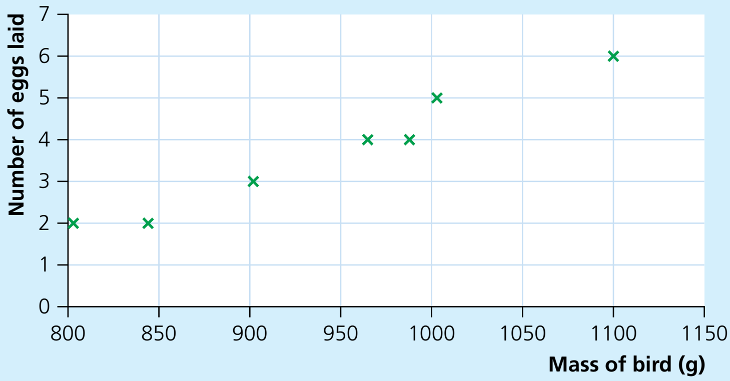

Figure 2.4

To see whether there is positive, negative or no correlation between the mass of a bird and the number of eggs laid, look for a general trend in the distribution of the points on the scatter diagram: as you move from left to right in the diagram, do the data points seem to be getting higher or lower?

In this case it appears that as the mass of a bird increases, the number of eggs produced also increases: birds with greater mass tend to lay a larger number of eggs, while birds with smaller mass tend to lay a smaller number of eggs.

Not every data point fits the pattern exactly, but this should not be surprising, as data involving living organisms will always show variation.

Therefore, the scatter diagram indicates a **positive correlation** between the mass of a bird and the number of eggs laid.

b **A study was carried out into the effect of exercise on resting heart rate. The results are plotted in Figure 2.5. Describe the relationship shown by this data and give a possible explanation for it.**

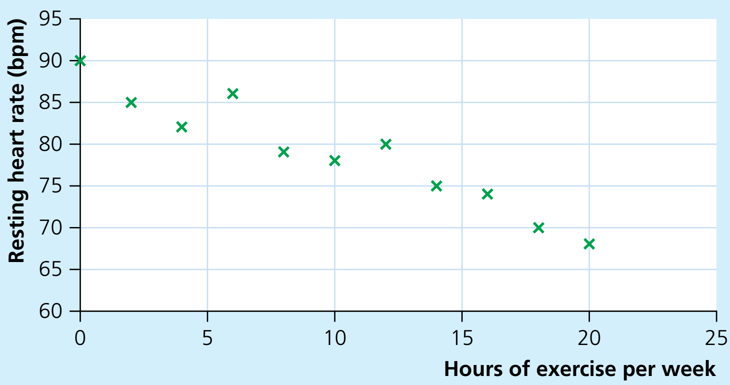

Figure 2.5

From the scatter diagram, as the number of hours of exercise per week increases, the resting heart rate tends to decrease.

So the data indicates a **negative correlation** between the number of hours of exercise per week and resting heart rate.

A possible reason for this is that the cardiac muscles become more developed with regular exercise, and so are able to produce a greater pressure per heart beat. Therefore, in a fixed interval of time, fewer beats are needed to pump blood throughout the body.

c **Figure 2.6 shows data on the number of iguanas found in areas with different amounts of canopy cover in a rainforest in the Philippines. Describe the pattern shown by this data.**

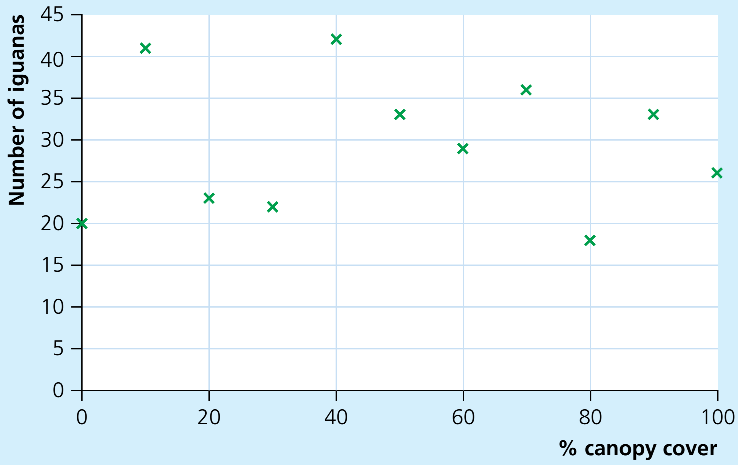

Figure 2.6

From the scatter diagram, there does not seem to be any obvious relationship between the percentage canopy cover and the number of iguanas found.

So there is no apparent correlation in the data.

To confirm that there is no correlation, a statistical test should be carried out.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **An investigation was carried out into the shell heights of limpets on a rocky shore. The data is plotted in Figure 2.7. What pattern does it show?**

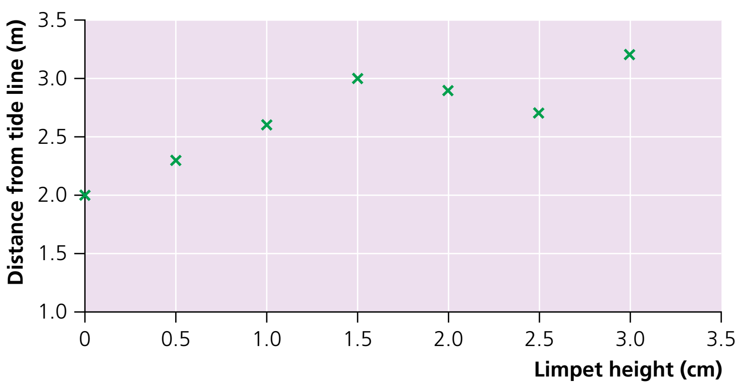

Figure 2.7

As you move from left to right in the scatter diagram, do the data points tend to get higher or lower?

This data shows ______________ correlation between limpet shell height and distance from the tide line.

2 **An investigation was carried out to measure the effect of different wavelengths of light on plant growth. The results are plotted in Figure 2.8. Describe the correlation shown by this data.**

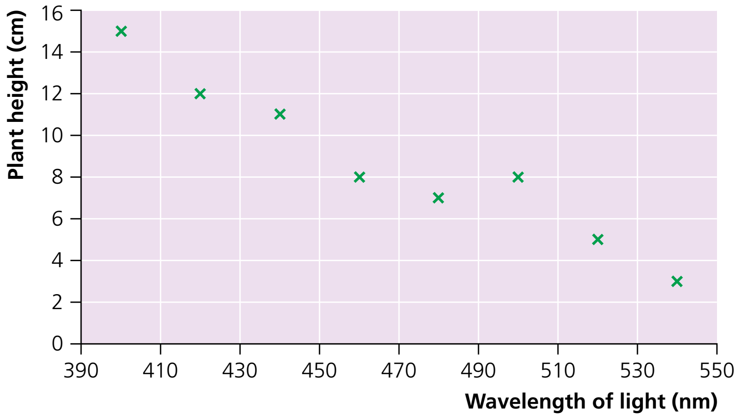

Figure 2.8

As the wavelength of light increases, the plant height tends to ______________

The data shows ______________ correlation between wavelength of light and plant height.

C Practice questions

3 Figure 2.9 shows the results of an investigation into the number of cigarettes smoked per day and the incidence of lung cancer. Describe the relationship shown by the results.

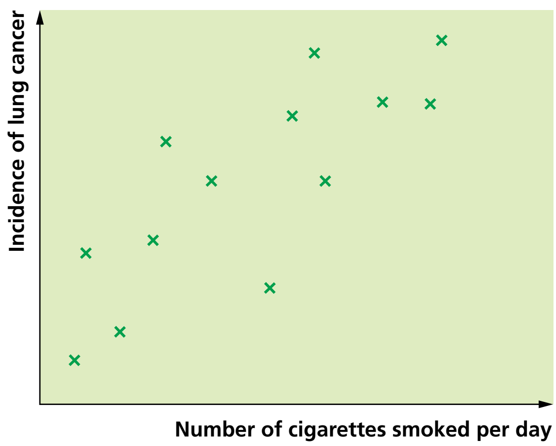

Figure 2.9

4 Figure 2.10 shows the results of an investigation into the effectiveness of a new anti-allergy drug. Describe the pattern of the results.

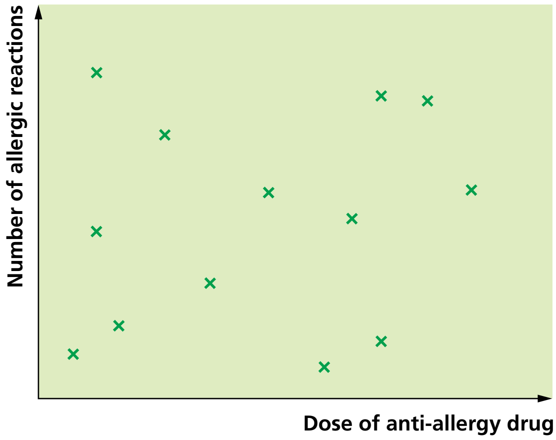

Figure 2.10

5 The oxygen dissociation curve for adult haemoglobin is shown in Figure 2.11. Would it be correct to say that there is a positive correlation between partial pressure of oxygen and the oxygen saturation of haemoglobin?

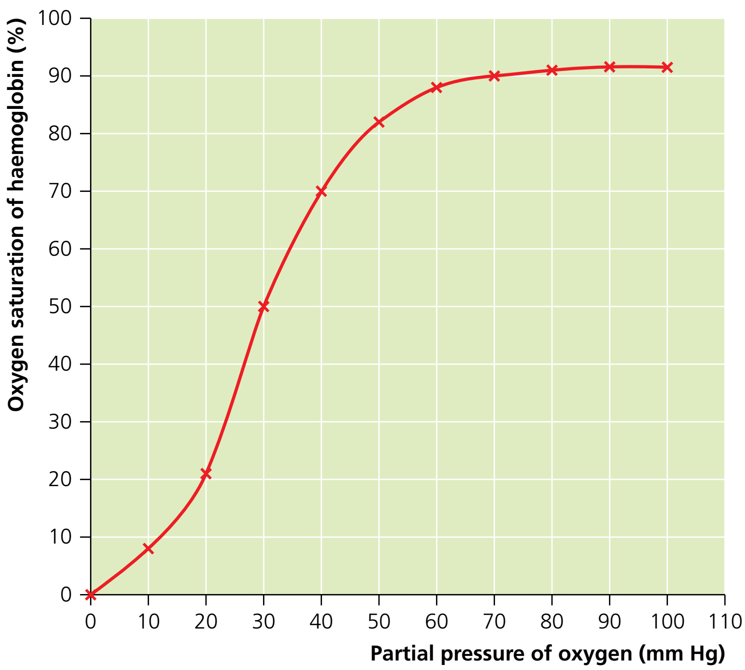

Figure 2.11

6 In Figure 2.12 the number of cases of asthma is plotted against the number of cars on roads in the UK. Describe the pattern you see in this data, and explain whether it shows conclusively that the rise in the number of cars has led to an increase in the incidence of asthma in the UK.

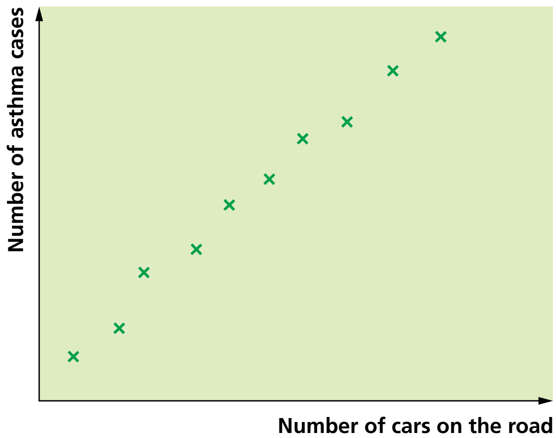

Figure 2.12

REMEMBER

Although a correlation between two variables may suggest a link between them, it does not prove that one factor causes the other. Correlation does not imply causation.

Order of magnitude

In AS and A-level biology, you will mainly be doing order-of-magnitude calculations with the magnification equation.

Biology often involves studying organisms and structures that are incredibly small. The magnification equation makes it possible to determine the actual dimensions of these organisms and structures from micrographs and scale drawings. It also helps you to create your own scale drawings.

The magnification equation is

$$\text{magnification} = \frac{\text{image size}}{\text{object size}}$$

You should be confident using and rearranging this equation to calculate:

- the magnification given an image size and an object size
- the image size given a magnification and an object size
- the object size given a magnification and an image size

An exam question on this skill might involve measuring part of a diagram or electron micrograph, so make sure you can do this accurately.

A Worked examples

a **An electron micrograph of a mitochondrion showed it as being 5 cm long. The actual length of the mitochondrion was labelled as 8 µm. What was the magnification?**

Step 1: convert both lengths to the same unit.

You may find it useful to refer to the section 'Appropriate units in calculations' on pages 6–8.

$$5\,\text{cm} = 50\,000\,\mu\text{m} = 5 \times 10^4\,\mu\text{m}$$

Step 2: substitute the values into the magnification formula.

$$\begin{aligned} \text{magnification} &= \frac{\text{image size}}{\text{object size}} \\ &= \frac{5 \times 10^4}{8} \\ &= 6250 \end{aligned}$$

Hence the magnification is 6250×.

b **In a diagram of a prokaryotic cell, its width was 2.6 cm. The magnification was given as 520×. What was the actual width of the cell?**

In this question you need to find the object size given the magnification and the image size.

Step 1: rearrange the magnification formula to make 'object size' the subject of the equation.

You may find it useful to refer to the section 'Changing the subject of an equation' on pages 64–67.

$$\text{magnification} = \frac{\text{image size}}{\text{object size}}$$

$$\text{magnification} \times \text{object size} = \text{image size}$$

$$\text{object size} = \frac{\text{image size}}{\text{magnification}}$$

Step 2: substitute the given numbers into the rearranged formula.

$$\begin{aligned} \text{object size} &= \frac{\text{image size}}{\text{magnification}} \\ &= \frac{2.6\,\text{cm}}{520} \\ &= 0.005\,\text{cm} \\ &= 50\,\mu\text{m} \end{aligned}$$

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **A student used a microscope to make a drawing of a transverse section of an artery. The student drew the lumen of the artery as 12 cm wide and measured the lumen to be 500 µm wide on the microscope slide. What magnification was the student using to observe the artery section?**

Step 1: convert both given lengths to the same unit.

12 cm = ______________ µm

Step 2: substitute the values into the magnification formula.

$$\text{magnification} = \frac{\text{image size}}{\text{object size}}$$

2 **The pollen sac of an anther was 0.4 cm wide. It was drawn with a magnification of 40×. What was the width of the pollen sac in the drawing?**

In this question you need to find the image size given the magnification and the object size.

Step 1: rearrange the magnification formula to make 'image size' the subject of the equation.

Step 2: substitute the given numbers into the rearranged formula.

C Practice questions

3 On an electron micrograph, a phospholipid bilayer was observed to be 8 nm wide. A drawing of the bilayer showed it as 4 cm wide. What was the magnification of the drawing?

4 A maggot is shown in a diagram as 9 cm long, with a magnification of 4.5×. What is the actual length of the maggot?

5 An epithelial cell had a length of 70 µm, and was magnified 190 times in a drawing. What was the length of the drawing in centimetres?

6 A virus was 300 nm long. In a diagram it was magnified 1×10^5 times. How long, in cm, was the virus in the drawing?

7 What would be indicated by a drawing that had a magnification less than 1×?

Statistical tests

Statistical tests are used to determine whether or not a set of data differs significantly from outcomes that might be produced by chance. Biological investigations often generate data that contains apparent anomalies or doesn't exactly fit expected trends, so statistical tests are of vital importance in analysing and drawing conclusions from the results.

In AS and A-level biology you are expected to use the following three statistical tests:

- **chi-squared test** — this is a statistical test normally used to compare observed data with the results expected from a specific hypothesis, for example, in genetic crosses.

- **t-test** — this test compares the means of two groups of data to decide if there is a significant difference between them. This then allows conclusions to be drawn about factors affecting the two groups which could have led to the difference between the means.
- **Spearman's rank correlation test** — this test is based on one of the most common correlation coefficients used in biology. It provides a way of measuring the strength of a relationship between paired data.

A Worked examples

Note: Worked example a is for A-level candidates only.

a A fish breeder wanted to produce fish with orange spots (which is a dominant trait). A genetic cross produces an expected ratio of 3 : 1 between offspring with orange spots and offspring with brown spots.

The actual breeding programme involving the same parents as in the genetic cross produced 60 orange-spotted fish and 40 brown-spotted fish. Are these results significantly different from the expected results?

In this example you want to determine if there is a significant difference between the observed and expected results of an investigation. The **chi-squared test** is the most suitable test for such a situation, and in A-level biology it is most often used to test whether the observed results of a genetic cross are significantly different from the expected results.

The chi-squared formula is

$$\chi^2 = \sum \frac{(o-e)^2}{e}$$

where

- o = observed value
- e = expected value

and Σ means sum of the data points.

Usually the calculation of the χ^2 value is presented in a table with the following headings:

o	e	o – e	$(o-e)^2$	$\frac{(o-e)^2}{e}$

The calculated chi-squared value is then compared to a critical value listed in a table. An excerpt from such a list of critical values is shown in Table 2.27. The relevant portion of the chi-squared table will always be given to you in an exam.

Table 2.27 Critical values for chi-squared test

Degrees of freedom	Probability of a larger value of χ^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21

If the calculated chi-squared value is greater than the critical value, then there is considered to be a significant difference between the observed and expected results. Otherwise, the difference is considered insignificant.

Step 1: the first stage of a statistical test is to write a null hypothesis and an alternative hypothesis. The test will then determine which hypothesis to accept and which to reject.

- **Null hypothesis:** there is **no significant difference** between the observed and expected results of the genetic cross.
- **Alternative hypothesis:** there is a **significant difference** between the observed and expected results of the genetic cross.

Step 2: to apply the chi-squared test, we need the observed and expected values. In this case, the observed values are the actual results obtained from the breeding programme. To find the expected results, apply the expected ratio to the total number of offspring.

The total number of offspring fish is

$$60 + 40 = 100$$

Splitting 100 in a 3 : 1 ratio gives 75 orange-spotted fish and 25 brown-spotted fish. These are the expected results.

Step 3: calculate the χ^2 value by filling in the table.

Table 2.28

	o	e	o – e	$(o - e)^2$	$\frac{(o-e)^2}{e}$
Orange spots	60	75	–15	225	3
Brown spots	40	25	15	225	9

Sum the final column to obtain the chi-squared value:

$$\chi^2 = \sum \frac{(o-e)}{e} = 3 + 9 = 12$$

Step 4: before we compare this value with the critical value from the chi-squared table, we need to determine the 'degrees of freedom'. For chi-squared tests this is the number of categories of data minus 1.

In this example there are two categories of data (orange spots and brown spots), so the degrees of freedom is 2 − 1 = 1.

Step 5: read off the critical value from Table 2.27. In biology the 0.05 level of significance is normally used.

Looking at the row for 1 degree of freedom and the column for significance level 0.05 gives a critical value of 3.84.

Step 6: compare the calculated chi-squared value with the critical value.

- If the calculated chi-squared value is **lower** than the critical value, then **accept the null** hypothesis and **reject the alternative** hypothesis.
- If the calculated chi-squared value is **higher** than the critical value, then **reject the null** hypothesis and **accept the alternative** hypothesis.

In this case

$12 > 3.87$

So accept the alternative hypothesis and reject the null hypothesis.
We conclude that there is a significant difference between the observed results and the expected results of the genetic cross.
This tells us that there may be some other factor affecting the inheritance of the alleles, rather than just the Mendelian genetics we assumed.

b **The numbers of seagulls feeding at two different sites (site A and site B) were recorded over a period of time. Use the t-test to determine if there was a significant difference between the numbers of seagulls feeding at the two sites.**

Table 2.29

Site A	Site B
12	11
9	25
22	28
13	29
15	15
18	22
10	19
8	21
16	23
17	18

The **t-test** can be used to determine whether there is a significant difference between two sets of data by comparing their **means**.

Step 1: as in the chi squared test, start by writing a null hypothesis and an alternative hypothesis.

- **Null hypothesis:** there is **no significant difference** between the numbers of seagulls visiting the two sites.
- **Alternative hypothesis:** there is a **significant difference** between the numbers of seagulls visiting the two sites.

Step 2: to apply the t-test, we need the mean ($\bar{x}$) and variance (s^2) of each data set.

$$\text{mean of site A} = \frac{12 + 9 + 22 + 13 + 15 + 18 + 10 + 8 + 16 + 17}{10} = 14$$

$$\text{mean of site B} = \frac{11 + 25 + 28 + 29 + 15 + 22 + 19 + 21 + 23 + 18}{10} = 21.1$$

The variance is the sum of the squared deviations from the mean, divided by the number of data points. Again, it is clearest to present these calculations in a table.

Table 2.30 Site A

Number of gulls x	Deviation from mean $(x-\bar{x})$	Squared deviation from mean $(x-\bar{x})^2$
12	–2	4
9	–5	25
22	8	64
13	–1	1
15	1	1
18	4	16
10	–4	16
8	–6	36
16	2	4
17	3	9

A good way of checking if the deviations from the mean are correct is to sum them. This sum should always equal zero.

Then add together the squared deviations from the mean (the values in the last column of Table 2.30) and divide by the number of data points to obtain the variance.

$$\text{variance of site A} = \frac{4+25+64+1+1+16+16+36+4+9}{10} = 17.6$$

Table 2.31 Site B

Number of gulls x	Deviation from mean $(x-\bar{x})$	Squared deviation from mean $(x-\bar{x})^2$
11	–10.1	102.01
25	3.9	15.21
28	6.9	47.61
29	7.9	62.41
15	–6.1	37.21
22	0.9	0.81
19	–2.1	4.41
21	–0.1	0.01
23	1.9	3.61
18	–3.1	9.61

$$\text{variance of site B} = \frac{102.01+15.21+47.61+62.41+37.21+0.81+4.41+0.01+3.61+9.61}{10} = 28.29$$

Step 3: use the t-test formula

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where

- $\bar{x}_1$ = higher mean
- $\bar{x}_2$ = lower mean
- s_1^2 = variance of data set with the higher mean
- s_2^2 = variance of data set with the lower mean
- n_1 = number of values in data set with the higher mean
- n_2 = number of values in data set with the lower mean

Substituting the values found in Step 2:

$$t = \frac{21.1 - 14}{\sqrt{\frac{28.29}{10} + \frac{17.6}{10}}}$$

$$= \frac{7.1}{\sqrt{2.829 + 1.76}}$$

$$= 3.314$$

Step 4: again, before we compare this value with the critical value from a table, we need to determine the degrees of freedom. For t-tests this is obtained by subtracting 1 from the number of data points in each set and adding them together.

In this example, each data set has 10 values, so the degrees of freedom is

$(10 - 1) + (10 - 1) = 18$

Step 5: read off the critical value from Table 2.32. In an exam you will always be given the relevant portion of table. The 0.05 level of significance is normally used.

Table 2.32 Critical values for t-test

Degrees of freedom	Significance level *p*					
	0.1	0.05	0.02	0.01	0.002	0.001
14	1.761	2.145	2.624	2.977	3.787	4.140
15	1.753	2.131	2.602	2.947	3.733	4.073
16	1.746	2.120	2.583	2.921	3.686	4.015
17	1.740	2.110	2.567	2.898	3.646	3.965
18	1.734	2.101	2.552	2.878	3.610	3.922
19	1.729	2.093	2.539	2.861	3.579	3.883

Looking at the row for 18 degrees of freedom and the column for significance level 0.05 gives a critical value of 2.101.

Step 6: compare the calculated t value with the critical value.

- If the calculated t value is **lower** than the critical value, then **accept the null** hypothesis and **reject the alternative** hypothesis.
- If the calculated t value is **higher** than the critical value, then **reject the null** hypothesis and **accept the alternative** hypothesis.

In this case,

$3.314 > 2.101$

So accept the alternative hypothesis and reject the null hypothesis. We conclude that there is a significant difference between the numbers of gulls visiting sites A and B.

c **A study was done on the body mass index (BMI) and average daily calorie intake of ten adults. From this data, is there a correlation between average daily calorie intake and BMI?**

Table 2.33

Average calorie intake per day	BMI
1500	23
2100	24
1700	28
2500	31
1800	27
1600	25
2800	33
2000	26
2050	29
2400	30

To decide whether there is significant correlation between two variables, we can use **Spearman's rank correlation test**.

Step 1: as with the other statistical tests, first write down the null and alternative hypotheses.

- Null hypothesis: there is no significant correlation between daily calorie intake and BMI.
- Alternative hypothesis: there is a significant correlation between daily calorie intake and BMI.

Step 2: rank order the values of the independent variable (average calorie intake), making sure that the data values remain linked to the corresponding values of the dependent variable.

Make a table for this, and create a column showing the rank order R_1 of each value of the independent variable.

Table 2.34

Average calorie intake per day	R_1	BMI
1500	1	23
1600	2	25
1700	3	28
1800	4	27
2000	5	26
2050	6	29
2100	7	24
2400	8	30
2500	9	31
2800	10	33

Step 3: now rank order the values of the dependent variable (BMI). Again make sure that the ranked values remain linked to the corresponding values of the independent variable, and create a new column showing the rank order R_2 of each value of the dependent variable.

Table 2.35

Average calorie intake per day	R_1	BMI	R_2
1500	1	23	1
1600	2	25	3
1700	3	28	6
1800	4	27	5
2000	5	26	4
2050	6	29	7
2100	7	24	2
2400	8	30	8
2500	9	31	9
2800	10	33	10

Step 4: for each row, calculate the difference d between the ranks ($d = R_1 - R_2$) and enter the results in a fifth column. A good way of checking if you have calculated the differences correctly is to sum the values in the d column. The sum should be zero, and if it is not then you should go back and check your working.

Then, square each d and enter the result (d^2) in a sixth column.

Table 2.36

Average calorie intake per day	R_1	BMI	R_2	Difference between ranks $d = R_1 - R_2$	d^2
1500	1	23	1	0	0
1600	2	25	3	–1	1
1700	3	28	6	–3	9
1800	4	27	5	–1	1
2000	5	26	4	1	1
2050	6	29	7	–1	1
2100	7	24	2	5	25
2400	8	30	8	0	0
2500	9	31	9	0	0
2800	10	33	10	0	0

Step 5: calculate Spearman's rank correlation coefficient

$$r_S = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where n is the number of pairs of data values.

Sum the values in the d^2 column to get $\sum d^2 = 38$.

Then substitute this into the formula:

$$r_S = 1 - \frac{6 \times 38}{10(10^2 - 1)}$$

$$= 1 - \frac{228}{990} = 0.77$$

Step 6: interpret this value. Spearman's rank correlation coefficient can tell us two things.

- **Direction** of correlation: if the value is positive, there is a positive correlation between the two variables. If the value is negative, there is a negative correlation between the two variables.
  As 0.77 is a positive value, there is a positive correlation between average daily calorie intake and BMI.
- **Strength** of correlation: by comparing r_S with the critical value from a table, we can test whether the correlation is significant or not.
  There are two types of test, one-tailed and two-tailed. If the direction of the correlation was stated in the alternative hypothesis (i.e. we took it as known), then use a one-tailed test. If the direction of the correlation was not stated in the alternative hypothesis (i.e. we did not know it beforehand), then use a two-tailed test.
  Table 2.37 is an extract of critical values for a two-tailed Spearman's rank correlation test.

Table 2.37

Number of pairs of observations	Significance level	
	0.1	0.05
8	0.643	0.783
9	0.600	0.700
10	0.564	0.648

  - If r_S is lower than the critical value, then accept the null hypothesis and reject the alternative hypothesis.
  - If r_S is higher than the critical value, then reject the null hypothesis and accept the alternative hypothesis.

In this example, the number of pairs of observations is 10, so at the 0.05 significance level typically used in biology, the critical value is 0.648.

Because

$$r_S = 0.77 > 0.648$$

we conclude that there is a significant, positive correlation between average daily calorie intake and BMI.

B Practice questions

Note: question 1 is for A-level candidates only.

1 In a genetic cross, the expected ratio of red, purple and blue flowers was 1 : 2 : 1. Of the offspring, 125 had purple flowers, 75 had blue flowers, and 60 had red flowers. Does this data differ significantly from the expected results?

2 An investigation was carried out into the numbers of zooplankton at two different sample sites, one with high-salinity water and one with low-salinity water. The results are shown in Table 2.38. Use the t-test to determine if there is a significant difference between the numbers of zooplankton at the two sample sites.

Table 2.38

Number of zooplankton in 100 cm^3 of high-salinity water	Number of zooplankton in 100 cm^3 of low-salinity water
68	54
72	45
83	67
91	61
102	59
85	49
92	52
77	65

Measures of dispersion

In scientific investigations, besides finding the average of a set of data, it is very important to get an idea of the spread, or dispersion, of the data. The reliability of an average depends greatly on the dispersion of the data.

Here are two measures of dispersion that you need to know for AS and A-level biology:

- **range** — the difference between the largest and smallest data values. A larger range indicates a greater spread of data.
- **standard deviation** — the square root of the variance s^2 (see page 48, where the variance appeared in the t-test formula), which is the sum of the squared deviations of the data values from the mean, divided by the number of data values:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

A Worked examples

a **An investigation was carried out into the effect of different nutrient deficiencies on the growth of a plant species. The results are shown in Table 2.39. Calculate the range of the heights of plants grown under each of the nutrient-deficient conditions.**

Table 2.39

Nutrient deficiency	Plant height (cm)		
Lack of nitrates	4	6	5
Lack of phosphates	2	6	8
Lack of magnesium	12	13	12
Control	15	17	20

For each type of nutrient deficiency, subtract the smallest of the three plant heights from the largest. This gives the following ranges:

- Lack of nitrates: 6 − 4 = 2 cm
- Lack of phosphates: 8 − 2 = 6 cm
- Lack of magnesium: 13 − 12 = 1 cm
- Control: 20 − 15 = 5 cm

b **Table 2.40 shows the results of an investigation into the masses of sea urchins. Calculate the standard deviation of this data.**

Table 2.40

Mass of sea urchin (g)	436	527	628	344	738	567

As with the calculations for the statistical tests in the previous section, it is often easier to calculate the standard deviation using a table.

Step 1: find the mean value, $\bar{x}$.

$$\bar{x} = \frac{436+527+628+344+738+567}{6} = 540$$

Step 2: find the difference of each data value from the mean, $x-\bar{x}$. Then square each of these differences to get $(x-\bar{x})^2$.

Table 2.41

Mass of sea urchin (g)	436	527	628	344	738	567
Deviation from mean $(x-\bar{x})$	−104	−13	88	−196	198	27
Squared deviation from mean $(x-\bar{x})^2$	10 816	169	7 744	38 416	39 204	729

A good way of checking if the deviations from the mean are correct is to sum them. This sum should always equal zero.

Step 3: find the average squared difference, i.e. the variance s^2, by adding up the squared deviations and dividing the sum by the total number of data points.

$$\sum (x-\bar{x})^2 = 97\,078$$

$$s^2 = \frac{\sum (x-\bar{x})^2}{6} = \frac{97\,078}{6} = 16\,179.67$$

Step 4: calculate the square root of the variance to find the standard deviation.

$$s = \sqrt{16\,179.67} = 127.2$$

A standard deviation closer to zero indicates that the data points don't deviate much from the mean, which suggests that the data is more consistent. In this example, the standard deviation is fairly large, because some of the data points were quite far from the mean. In other words, there is a large degree of variation in the data.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Table 2.42 shows the wing span of a sample of birds caught in a mist net. Find the range of this data.**

Table 2.42

Wing span (mm)	156	178	191	135	127	181

Step 1: find the highest and lowest values in the data set.

Highest value is ____________ mm

Lowest value is ____________ mm

Step 2: subtract the lowest value from the highest value.

2 **An experiment involving an enzyme-catalysed reaction produced the results shown in Table 2.43. For each time, find the standard deviation of the concentrations of product formed. Comment on the values you arrive at.**

Table 2.43

Time (minutes)	Product concentration ($g\,dm^{-3}$)		
	1	2	3
0	0	0	0
1	1	3	2
2	5	6	5
3	7	8	7
4	9	10	10
5	11	12	20

Step 1: find the mean value at each time, i.e. add up the product concentrations at each time and divide by 3.

Table 2.44

Time (minutes)	Mean $\bar{x}$
0	0
1	
2	5.33
3	
4	
5	

Step 2: at each time, find the difference of each measurement from the mean, $x - \bar{x}$.

Table 2.45

Time (minutes)	Deviation from mean $(x-\bar{x})$		
	1	2	3
0	0	0	0
1			
2	−0.33	0.67	−0.33
3			
4			
5			

Step 3: square each of these differences to get $(x-\bar{x})^2$.

Table 2.46

Time (minutes)	Squared deviation from mean $(x-\bar{x})^2$		
	1	2	3
0	0	0	0
1			
2	0.11	0.44	0.11
3			
4			
5			

Step 4: sum each row and divide by the number of readings (3) to get the variance at each temperature.

Table 2.47

Time (minutes)	Squared deviation from mean $(x-\bar{x})^2$			$\sum(x-\bar{x})^2$	Variance s^2
0	0	0	0	0	0
1					
2	0.11	0.44	0.11	0.66	0.22
3					
4					
5					

Step 5: calculate the square root of the variance at each time to obtain the standard deviation.

C Practice questions

3 Table 2.48 shows the birth masses of a group of penguin chicks. What is the range of the data?

Table 2.48

Penguin chick mass (kg)	1.5	0.6	1.2	1.4	0.9

4 An investigation was carried out into the heights of trees in a national park. The results are shown in Table 2.49. Calculate the range of the data. Comment on this range.

Table 2.49

Tree height (m)	101	95	52	86	78	99

5 Table 2.50 below shows the number of rabbits in an area over several years. Calculate the standard deviation of this data.

Table 2.50

Year	2001	2002	2003	2004	2005	2006	2007
Number of rabbits	53	41	77	32	49	62	43

Uncertainties in measurements

Uncertainties can arise from inaccuracies in the process of measuring and the equipment used. They can also be due to laboratory conditions or other sources of error. There are two main ways of quantifying uncertainties, i.e. giving them a numerical value.

- An uncertainty can be attached to each piece of equipment used for measurement in a practical investigation. This uncertainty is usually given in the form of ± <a value> or stated as 'accurate to <a value>'.
- If an expected or theoretical result of an experiment is known, then the value actually achieved in a real experiment can be compared to this theoretical value. The discrepancy is usually expressed as a percentage error.

In biology you need to be comfortable with calculating and using both of these types of error in the context of practical examples.

A Worked examples

a In an investigation into the effect of placing different plant tissues into glucose solutions, a balance accurate to 0.1 g was used. The initial mass of a sample of plant tissue was 5 g. What is the percentage error in using the balance to measure this plant tissue?

$$\text{apparatus percentage error} = \frac{\text{apparatus margin of error}}{\text{quantity measured}} \times 100\%$$

The margin of error is how accurate the balance is, which in this case is 0.1 g.

The quantity measured is 5 g.

Therefore

$$\text{apparatus percentage error} = \frac{0.1}{5} \times 100\%$$

$$= 2\%$$

b **In an experiment on amylase activity, 5.1 g of product was obtained. The expected, theoretical yield should have been 5.4 g of product. What was the percentage error in this investigation?**

Use the formula

$$\text{percentage error} = \frac{\text{observed value} - \text{expected value}}{\text{expected value}} \times 100\%$$

In this case the observed value was 5.1 g and the expected value was 5.4 g.

$$\frac{5.1 - 5.4}{5.4} \times 100\%$$

$$= \frac{-0.3}{5.4} \times 100\%$$

$$= -5.6\%$$

So the percentage error was 5.6%.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **In a transpiration investigation, the potometer used had an uncertainty of ±0.05 cm^3. If it gave a reading of 10.1 cm^3 water lost over a certain period of time, what is the percentage error of this result?**

Substitute the given values into the formula

$$\text{apparatus percentage error} = \frac{\text{apparatus margin of error}}{\text{quantity measured}} \times 100\%$$

2 **For an investigation into the action of lipase, the theoretical yield is 19 g of fatty acids and glycerol. In the actual investigation 15 g of fatty acids and glycerol were produced. Find the percentage error of this result.**

Substitute the given values into the formula

$$\text{percentage error} = \frac{\text{observed value} - \text{expected value}}{\text{expected value}} \times 100\%$$

C Practice questions

3 An investigation into the action of protease had a theoretical yield of 54 g of product. The actual investigation produced 48 g of product. Calculate the percentage error of this result.

4 A pipette has an uncertainty of ±0.1 cm^3. What would be the percentage error if this pipette was used to measure a volume of 7.2 cm^3?

5 The same pipette used in question 4 is now used to measure a volume of 15.6 cm^3. Will the percentage error be larger or smaller? Explain your answer.

6 A 100 cm^3 gas syringe used in an investigation into the rate of photosynthesis had an uncertainty of ±0.5 cm^3. In the investigation it gave a measurement of 2.3 cm^3 of gas produced. Calculate the uncertainty of this measurement and comment on the value. Do you think this piece of apparatus is suitable for this investigation?

7 Suggest why percentage error calculations that involve comparing an actual yield with a theoretical yield usually give negative results.

3 Algebra

Commonly used symbols in algebra

The following symbols are often used in algebra, and you may encounter them in biology exam questions.

Table 3.1

Symbol	Meaning
$=$	equals
$>$	greater than
$\geq$	greater than or equal to
$<$	less than
$\leq$	less than or equal to
$\propto$	proportional to
$\sim$ or $\approx$	is approximately

Examples:

- 5.854 is a bigger number than 5.601, so
  $5.854 > 5.601$
- 67 is smaller than 98, so
  $67 < 98$
- In an enzyme-catalysed reaction where the substrate is in excess, the rate of reaction is proportional to the enzyme concentration. This can be written as
  rate of reaction $\propto$ enzyme concentration
- The mass 3.98 g is approximately 4 g, so
  $3.98\,\text{g} \sim 4\,\text{g}$

Algebraic equations

This unit covers two important mathematical skills for dealing with the different kinds of equations that you may encounter in biological investigations or exam questions. The first skill is to substitute numbers for the letters or terms in an equation and thus calculate the value of an unknown quantity.

To solve an equation successfully, the key is to work carefully and logically through the steps, ensuring that all substitutions are done correctly and all the functions in the equation are evaluated accurately. It is particularly important to double-check all your working, as it is very easy to make mistakes in algebra.

The equations featured in this section are some particular examples taken from the biology specifications, but you should practise the skills covered in this section so that you become confident applying them to any equation given in biology exam questions.

A Worked examples

a **The equation below gives the pulmonary ventilation rate (PVR) in litres per minute:**

PVR = tidal volume × breathing rate

where the tidal volume is in litres and the breathing rate is the number of breaths per minute.

If the tidal volume is 0.5 litre and the breathing rate is 13 breaths per minute, what is the PVR?

Step 1: in the formula, replace 'tidal volume' and 'breathing rate' with the given values. This gives

PVR = 0.5 × 13

Step 2: work out the multiplication to get

PVR = 6.5

Therefore the PVR is $6.5\,\mathrm{L\,min^{-1}}$.

b **The following equation describes the results of an investigation into the effect of enzyme concentration on the rate of reaction:**

$y = mx + c$

where y is the rate of reaction and *x* is the enzyme concentration.

Calculate the rate of reaction if *m* = 0.5, *c* = 0.1 and the enzyme concentration is 3.

Step 1: replace the letters in the equation with their values given in the question. In this case, substitute 0.5 for m, 3 for x and 0.1 for c to get

$y = 0.5 \times 3 + 0.1$

Step 2: now do the calculations in the equation to get

$y = 1.5 + 0.1 = 1.6$

Therefore the rate of reaction is 1.6.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **The respiratory quotient (RQ) is given by the equation**

$$\mathrm{RQ} = \frac{CO_2 \text{ produced}}{O_2 \text{ consumed}}$$

If 27 units of O_2 were consumed and 19 units of CO_2 were produced, find the respiratory quotient.

Replace the quantities in the right-hand side of the equation with their given values.

2 **The equation for Simpson's diversity index is**

$$D = 1 - \sum\left(\frac{n}{N}\right)^2$$

where

- ***n* = number of organisms of a particular species in a certain area**
- ***N* = total number of individual organisms in that area**

and Σ means sum of the different species found in that area.

Table 3.2 shows the results of a sampling activity. Calculate Simpson's diversity index from this data.

Table 3.2

Species	Number (n)
Daisy	34
Bramble	9
Buttercup	25
Total	**68**

Step 1: in the equation, we need to calculate $\left(\frac{n}{N}\right)^2$ for each species. It is easiest to do this by appending an extra column to the table.

Table 3.3

Species	Number (n)	$\left(\frac{n}{N}\right)^2$
Daisy	34	$\left(\frac{34}{68}\right)^2 = 0.25$
Bramble	9	
Buttercup	25	
Total	**68 = *N***	

Step 2: according to the equation, we need to add together the values in the last column to get $\sum\left(\frac{n}{N}\right)^2$.

$0.25 +$ ____________ $+$ ____________ $=$ ____________

Step 3: substitute the sum from Step 2 into the equation for the diversity index.

C Practice questions

3 The stroke volume of a person was 65 cm^3 per beat, and their heart rate was 80 beats per minute. Use the equation below to calculate the person's cardiac output.

Cardiac output = stroke volume × heart rate

4 The following equation can be used to calculate the water potential of a cell:

$$\Psi = \Psi_s + \Psi_p$$

where Ψ_s is the solute potential and Ψ_p is the pressure potential.

At incipient plasmolysis, the pressure potential is zero while the solute potential inside the cell is −200 kPa. What is the water potential in the cell?

5 The mitotic index is calculated by the formula

$$\text{mitotic index} = \frac{\text{number of cells undergoing mitosis}}{\text{total number of cells in sample}}$$

A student observed 14 out of 35 cells undergoing mitosis. What is the mitotic index?

6 The proportion of polymorphic gene loci is given by the equation

$$\text{proportion of polymorphic gene loci} = \frac{\text{number of polymorphic gene loci}}{\text{total number of loci}}$$

If there are four polymorphic gene loci out of a total of 48 loci, what is the proportion of polymorphic gene loci?

7 Another version of Simpson's diversity index is given by the following equation:

$$d = \frac{N(N-1)}{\sum n(n-1)}$$

where

- n = number of organisms of a particular species in a certain area
- N = total number of individual organisms in that area

and Σ means sum of the different species found in that area.

Calculate the diversity index of the following data on mammal species in a woodland.

Table 3.4

Species	Number of individuals
Deer	19
Rabbits	65
Shrews	8

Changing the subject of an equation

The second useful mathematical skill is to change the subject of an equation. As in the previous section, ensure that you check your working carefully — it is very easy to make a mistake not only when substituting values but also when rearranging the terms in an equation.

A Worked examples

a The water potential in a cell is −189 kPa, and the solute potential is −256 kPa. What is the pressure potential in the cell?

Step 1: write down the equation that you need to use. The water potential equation (see Practice question 4 in the previous section) is

$$\Psi = \Psi_s + \Psi_p$$

where Ψ_s is the solute potential and Ψ_p is the pressure potential.

Step 2: since we want to find the pressure potential, we need to make it the subject of the equation. To do this, subtract Ψ_s from both sides of the equation:

$$\Psi - \Psi_s = \Psi_s + \Psi_p - \Psi_s$$

which gives $\Psi_p = \Psi - \Psi_s$

Step 3: substitute the given values into the equation.

$$\begin{aligned}\Psi_p &= \Psi - \Psi_s \\ &= -189 - (-256) \\ &= -189 + 256 \\ &= 67\end{aligned}$$

So the pressure potential is 67 kPa.

b **A spider leg is 0.5 cm long. The spider is drawn with a magnification of 15×. What is the length of the spider leg in the drawing?**

Step 1: write down the equation to use. To answer this question you need the magnification equation

$$\text{magnification} = \frac{\text{image size}}{\text{object size}}$$

Step 2: since we want to find the length in the drawing, i.e. the image size, we need to make it the subject of the equation. To do this, multiply both sides of the equation by the object size.

$$\text{magnification} \times \text{object size} = \frac{\text{image size}}{\text{object size}} \times \text{object size}$$

which leads to

$$\text{image size} = \text{magnification} \times \text{object size}$$

Step 3: substitute the given numbers into the equation.

$$\begin{aligned}\text{image size} &= 15 \times 0.5\,\text{cm} \\ &= 7.5\,\text{cm}\end{aligned}$$

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **A nucleus is 6 µm wide. If it is drawn with a magnification of 2×10^4, what is the width of the nucleus in the drawing?**

Step 1: write down the equation to use. To answer this question you need the magnification equation

$$\text{magnification} = \frac{\text{image size}}{\text{object size}}$$

Step 2: the question asks for the width in the drawing, i.e. the image size, so we need to make it the subject of the equation. To do this, multiply both sides of the equation by the object size to get

image size = ______________ × ______________

Step 3: substitute the given numbers into the equation.

Note: question 2 is for A-level candidates only.

2 **The following equation can be used to calculate the photosynthetic efficiency of a plant:**

$$\textbf{photosynthetic efficiency} = \frac{\textbf{energy incorporated into products of photosynthesis}}{\textbf{energy falling on plant}}$$

What is the energy falling on the plant when the photosynthetic efficiency is 0.0125 and the energy incorporated into the products of photosynthesis is $2.2 \times 10^4\,kJ\,m^{-2}\,yr^{-1}$?

Step 1: we need to make 'energy falling on plant' the subject of the equation. First multiply both sides of the equation by 'energy falling on plant'. This gives

photosynthetic efficiency × energy falling on plant = ______________

Step 2: to get 'energy falling on the plant' by itself on one side of the equation, divide both sides by 'photosynthetic efficiency' to get

energy falling on plant = ______________

Step 3: substitute the given numbers into the equation.

3 **In an investigation into amylase activity, starch was mixed into agar. The agar then had amylase placed onto it. After a set time, the agar was flooded with iodine and regions were formed which did not turn blue or black. The area of one of these regions is given by the formula for the area of a circle (πr^2). If the area is $78\,cm^2$, find the radius r.**

First write down the equation:

$$\text{Area} = \pi r^2$$

Think about the steps you would need to take to make r the subject of the equation.

C Practice questions

Note: question 4 is for A-level candidates only.

4 The following is the equation for net primary productivity:

$$\text{net primary productivity (NPP)} = \text{gross primary productivity (GPP)} - \text{respiration}$$

What is the value for respiration when the GPP is $2.6 \times 10^4\,kJ\,m^{-2}\,yr^{-1}$ and the NPP is $1.2 \times 10^4\,kJ\,m^{-2}\,yr^{-1}$?

5 The equation below gives cardiac output:

$$\text{cardiac output} = \text{stroke volume} \times \text{heart rate}$$

If a person's cardiac output is $4300\,cm^3\,min^{-1}$ and their heart rate is 70 beats per minute, find the stroke volume.

6 The following equation is used to calculate Rf values during chromatography investigations:

$$\text{Rf} = \frac{\text{distance moved by substance}}{\text{distance moved by solvent front}}$$

In an investigation, the solvent front moved 7 cm and the Rf value was 0.43. How far did the substance move?

7 In an investigation into enzyme action over time, the following equation relates the rate of reaction to the concentration of product and the elapsed time:

$$\text{rate of reaction} = \frac{\text{concentration of product}}{\text{time}}$$

If the concentration of product is 60.5 $cm^3\,dm^{-3}$ and the rate of reaction is 6.7 $cm^3\,dm^{-3}\,s^{-1}$, what is the time taken?

Note: question 8 is for A-level candidates only.

8 In a population of birds, the allele for long beaks is dominant and the allele for short beaks is recessive. Use the Hardy–Weinberg equation (shown below) to calculate the frequency of birds who are heterozygotes if the short-beak allele frequency is 0.25.

$$p^2 + 2pq + q^2 = 1$$

where p and q are the frequencies of the dominant and recessive alleles.

Logarithms

Note: this topic is assessed at A-level only.

A logarithmic scale is a non-linear scale that is useful for representing data when the range of values is very large (spanning several orders of magnitude). Logarithmic scales usually use logarithms to base 10 (also known as common logarithms), so that powers of 10 are marked along the vertical axis of the graph.

In biology, logarithmic scales are mainly used for plotting microbial growth curves. At A-level you will not be asked to plot a graph with a logarithmic scale, but you should be comfortable using and interpreting such graphs.

Here are some important features of graphs plotted on a logarithmic scale:

- A straight line represents exponential growth (if the line is rising from left to right) or exponential decay (if the line is falling).
- A rising curve that bends downwards represents growth that is slower than exponential.
- A rising curve that bends upwards represents growth that is faster than exponential.

A Worked examples

a **The graph in Figure 3.1 shows the growth of a bacteria population over a period of time. Describe the growth of the bacteria population.**

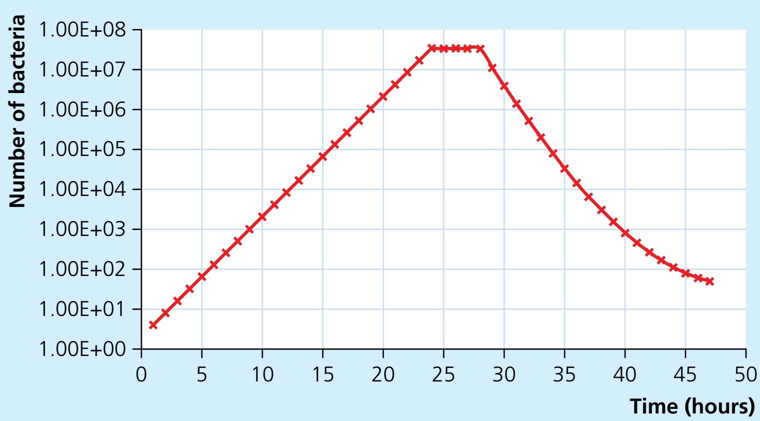

Figure 3.1

Initially, up to about 24 hours, the graph is a straight line, so the population was growing at an exponential rate. The graph then plateaus for a while, which means that there was no growth during those few hours. Following the plateau, the graph falls, indicating that the population was declining.

b **In the graph shown in Figure 3.2, during which time period is growth exponential? Explain how you arrived at your answer.**

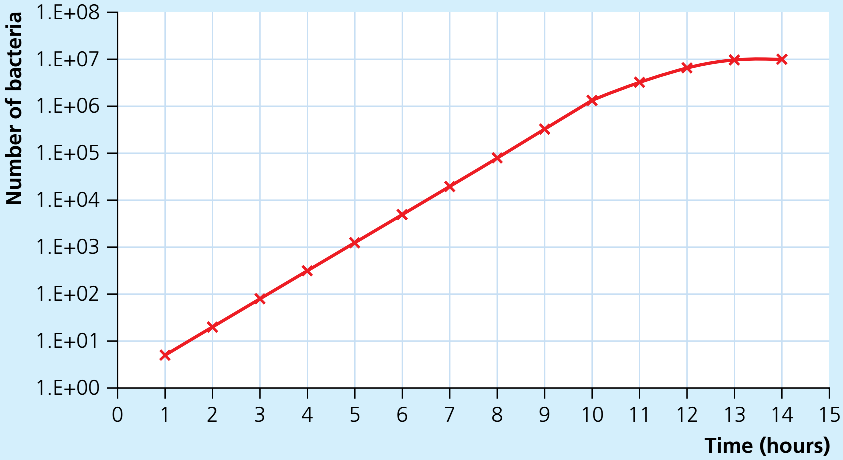

Figure 3.2

Between 1 and 10 hours the graph is a straight line, so during this time period there is exponential growth.

After 10 hours the graph is still rising but bends downwards, indicating that the population is still growing but at a rate slower than exponential.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **The graph in Figure 3.3 shows the hydrogen ion concentration at pH values of 1 to 10. Describe the relationship between pH and hydrogen ion concentration.**

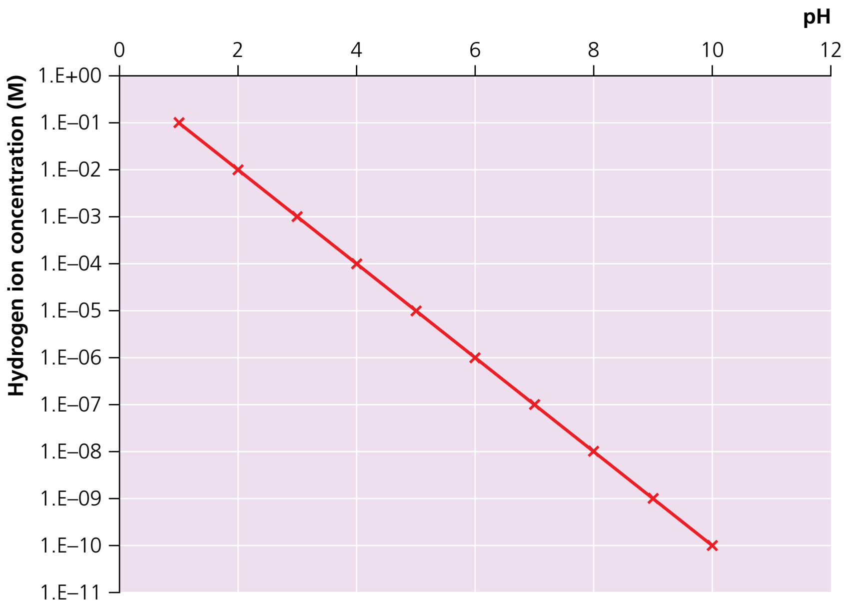

Figure 3.3

As the pH increases, the concentration of hydrogen ions ____________

What does the shape of the graph tell you?

2 During the stationary phase of population growth, the production rate equals the death rate. From Figure 3.4, find the time at which the stationary phase begins.

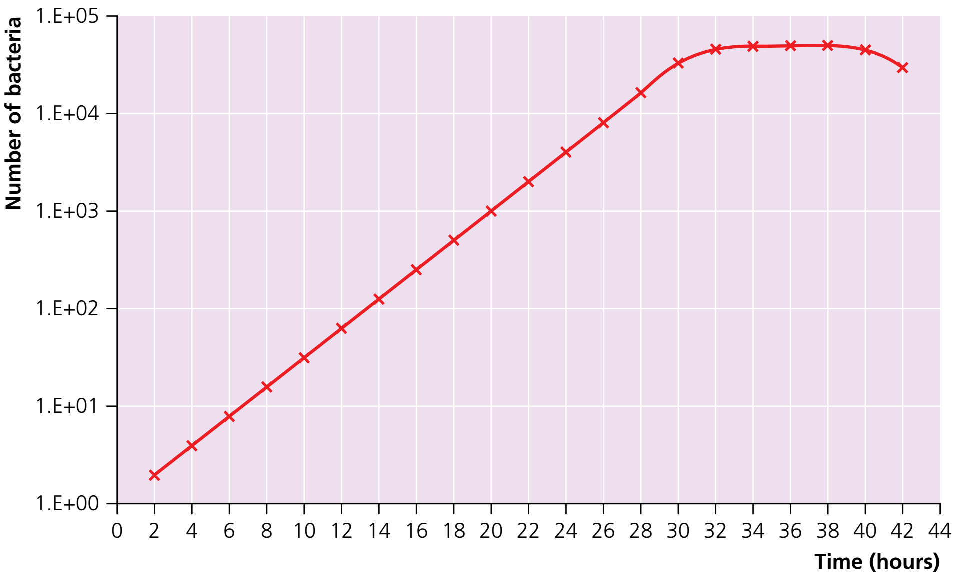

Figure 3.4

When production rate = death rate, the graph should plateau.

C Practice questions

3 Why are microbial growth curves often represented by graphs with logarithmic scales?

4 **a** From the graph in Figure 3.5, estimate the highest total population.

b At which point was the growth fastest? Explain your answer.

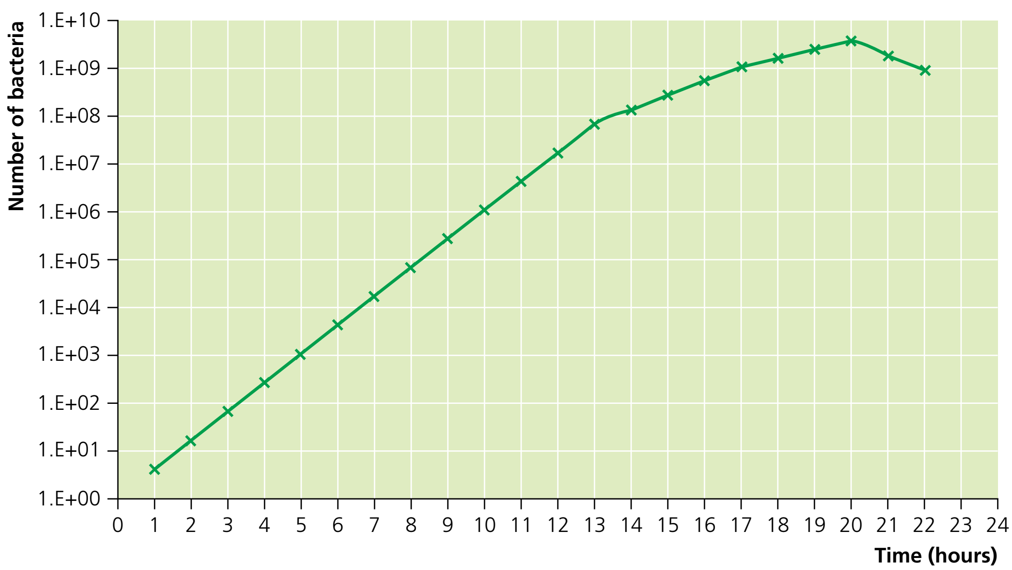

Figure 3.5

5 Figure 3.6 shows a graph plotted on a linear scale, below which are three logarithmic graphs labelled A, B and C. Which of the logarithmic graphs best represents the relationship shown in graph with the linear scale?

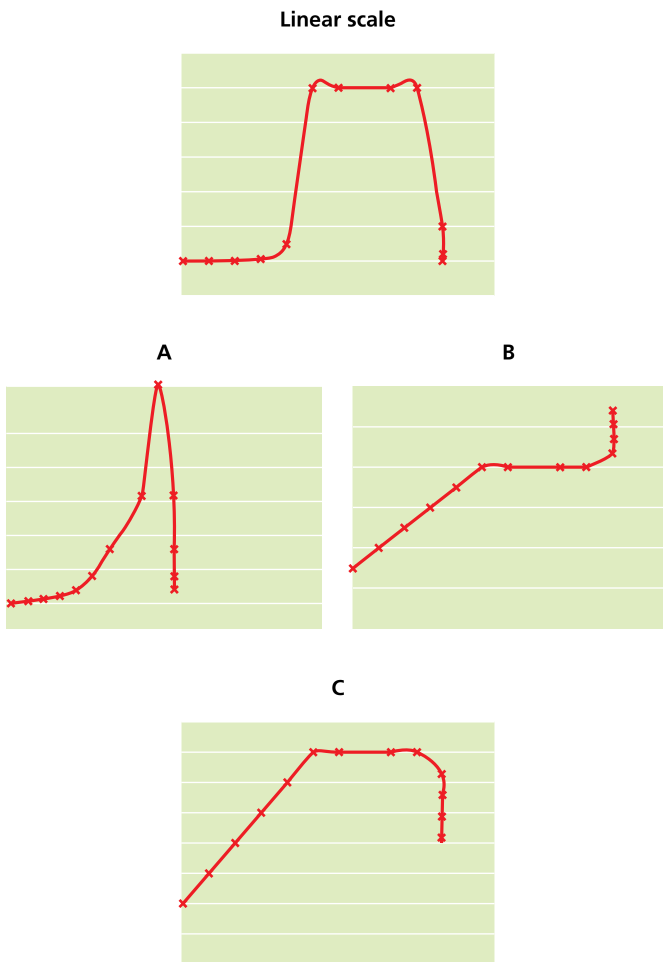

Figure 3.6

4 Graphs

Bar charts, histograms and line graphs

This unit should be studied alongside the section 'Interpreting tables and diagrams' (pages 24–28). Make sure you are confident in drawing and analysing the following types of graphs, which are often used to represent data in AS and A-level biology:

- bar charts
- histograms
- scatter diagrams
- line graphs.

A Worked examples

a **Table 4.1 shows the numbers of flowers of different colours in an investigation into variation in a plant species. Draw a graph of this data.**

Table 4.1

Colour	Number of flowers
Red	13
Blue	15
Pink	3
Yellow	8
Violet	9

Step 1: determine what type of data it is, and hence choose the most appropriate type of graph to represent the data.

In this example the data is categorical, as each flower colour is a distinct category and there is no overlap between the different categories. The best way to present such data is in a **bar chart**.

Step 2: decide what information to show along each axis of the graph, and what scale to use for each axis.

For a bar chart, the horizontal axis would normally show the different categories of data (the colours in this case), and the vertical axis would show the values of the categories. The scale on the vertical axis should be linear (i.e. increase by the same value between successive graduations) and have an origin (a start point). The scale could start at zero but doesn't have to.

Step 3: plot the data on the graph.

In a bar chart, for each category we draw a bar extending from the horizontal axis up to the value on the vertical axis associated with that category. The bars in a bar chart do not touch each other, indicating that the data is in separate, non-overlapping categories.

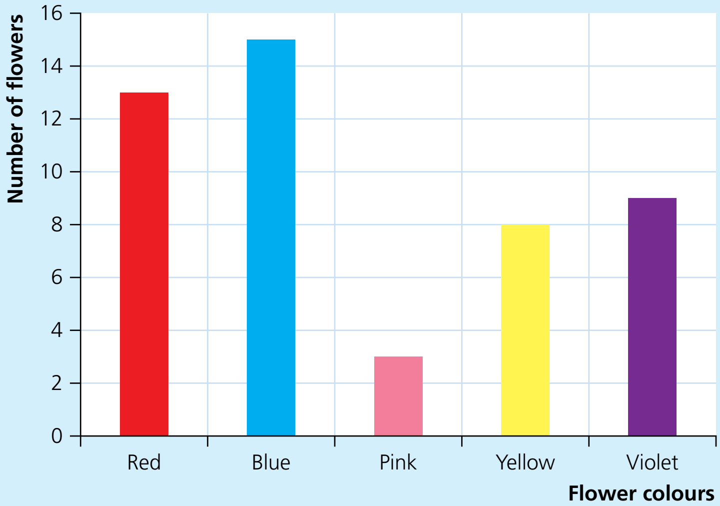

Figure 4.1

b **Data collected on the lengths of monitor lizards is shown in Table 4.2. Draw a graph to represent this data.**

Table 4.2

Length of lizard (cm)	Frequency
55–59	1
60–64	2
65–69	3
70–74	3
75–79	1

Step 1: determine what type of data it is, and hence choose the most appropriate type of graph to represent the data.

The independent variable here, length, is a continuous variable, which can take any value in a continuous range. Although the numbers in the left column of the table are whole numbers, they represent lengths that have been rounded to the nearest centimetre, so the group 55–59 stands for ‘all lengths that are at least 54.5 cm and less than 59.5 cm’, the group 60–64 stands for ‘all lengths that are at least 59.5 cm and less than 64.5 cm’ etc.

The most suitable kind of graph for this type of data is a **histogram**.

Step 2: decide what information to show along each axis of the graph, and what scale to use for each axis.

The histogram will have the groups of lizard lengths shown along its horizontal axis, and the vertical axis will show the frequency of each of these groups.

Step 3: plot the data on the graph.

A histogram looks similar to a bar chart, but the bars of a histogram touch each other, because there are no ‘gaps’ between adjacent groups of continuous data values (for example, the groups 55–59 and 60–64 ‘touch’ at 59.5).

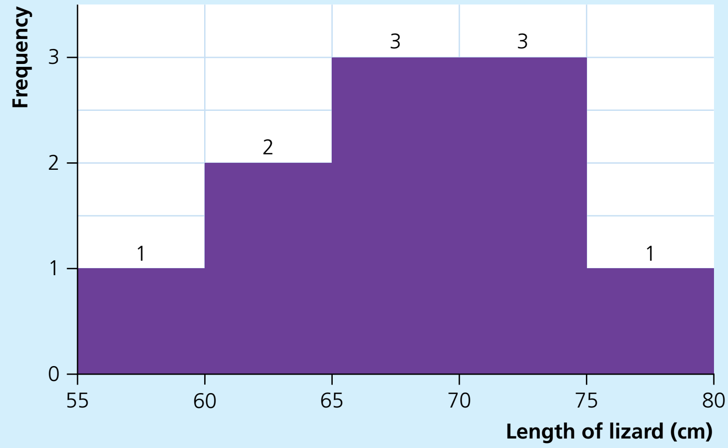

Figure 4.2

c **A student carried out an investigation into the effect of pH on an enzyme-controlled reaction. Plot a graph of the data.**

Table 4.3

pH	Rate of reaction ($g\,cm^{-3}\,min^{-1}$)
4	0
5	1
6	1
7	3
8	6
9	5
10	2
11	0

Step 1: determine what type of data it is, and hence choose the most appropriate type of graph to represent the data.

Unlike the previous examples, the dependent variable in this case, rate of reaction, is also a continuous variable. A **line graph** is therefore the most suitable way of representing this data.

Step 2: decide what information to show along each axis of the graph, and what scale to use for each axis.

Plot the independent variable (pH) on the horizontal axis and the dependent variable (rate of reaction) on the vertical axis. Ensure that each axis has a continuous scale and an origin. The origin does not have to be zero or be the same for both axes.

Step 3: plot the data on the graph.

The points can be joined by ruled straight lines, or a curve of best fit can be drawn through them.

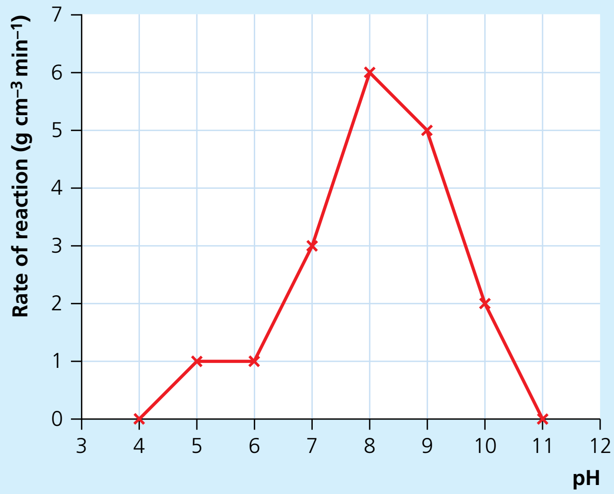

Figure 4.3

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Table 4.4 shows the heights of a group of rhinos. Plot the data on a histogram.**

Table 4.4

Height (cm)	165	172	181	176	184	143	155	144	167	162

Step 1: in order to plot the data on a histogram, we first have to group the data into classes. Here it would be appropriate to use groups of size 10 cm, giving five classes. Make a frequency table for the data.

Table 4.5

Height (cm)	Frequency
140–149	2
150–159	
160–169	
170–179	
180–189	

Step 2: choose appropriate origins and scales for the graph axes.

Step 3: plot the data.

2 **A long-term study into penguin populations on an island produced the data in Table 4.6. Plot the data on a line graph.**

Table 4.6

Year	Number of penguins
1982	490
1983	560
1985	700
1986	580
1989	400
1992	410

A common mistake in answering this question is to have a scale on the horizontal axis that isn't linear (i.e. doesn't increase by the same value from each marked number to the next). The 'year' data has gaps in it, but the axis should nevertheless have a continuous, linear scale.

C Practice questions

3 Table 4.7 shows the number of people in a sample with different blood groups. Plot the data on an appropriate graph.

Table 4.7

Blood group	Number of people
A	5
B	12
AB	3
O	25

4 The data in Table 4.8 was collected during an investigation into the effect of temperature on rate of respiration in a yeast cell. Draw a graph to show the results.

Table 4.8

Temperature (°C)	Time taken for methylene blue to decolourise (seconds)			
	1	2	3	Mean
15	65	90	44	66
25	41	42	43	42
35	12	17	19	16
45	5	6	7	6
55	23	26	30	26
65	55	72	71	66

5 Table 4.9 shows the dry mass of a selection of seeds. Draw an appropriate graph to show the data.

Table 4.9

Dry mass of seeds (g)	Frequency
0.11–0.30	100
0.31–0.50	65
0.51–0.70	46
0.71–0.90	32
0.91–1.10	12

6 From the results of a practical investigation, a student drew the following graph. What mistakes have they made?

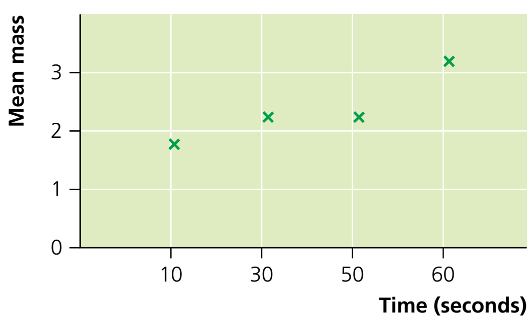

Figure 4.4

$y = mx + c$

The graph of a linear relationship plotted on x- and y-axes is a straight line and can be represented by the equation

$$y = mx + c$$

where

- m = gradient of the line
- c = y-intercept (the point where the line crosses the y-axis)

In biology exams you could be asked to sketch a graph of a linear relationship. As the graph is a straight line, only two points are needed to draw the line, although it is recommended that you use a third point to check whether the line is correct.

If a set of axes is given in the question, it does not particularly matter which values on the x-axis you use to construct the line, as long as their corresponding y values are within the range shown on the y-axis. However, using points fairly far apart could make the line easier to draw accurately.

If the question leaves it up to you to draw the axes, make sure that the scale you choose for each axis covers the range of values in the given data. Do this by finding the maximum and minimum y values before starting to draw the axes.

A Worked examples

a On the axes below, sketch a graph of the equation $y = 2x$.

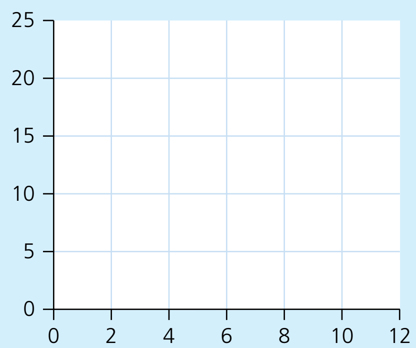

Figure 4.5

Step 1: identity the values of m and c in the linear equation.

This is an equation of the form $y = mx + c$ with $m = 2$ and $c = 0$.

Therefore the graph is a straight line with a gradient of 2 and a y-intercept at 0.

Step 2: to construct the graph, choose two x values (within the range shown on the given x-axis) and calculate their corresponding y values using the equation.

Using the points $x = 4$ and $x = 8$:

- At $x = 4$, $y = 2 \times 4 + 0 = 8$
- At $x = 8$, $y = 2 \times 8 + 0 = 16$

Step 3: plot these points on the set of axes and draw a straight line through them.

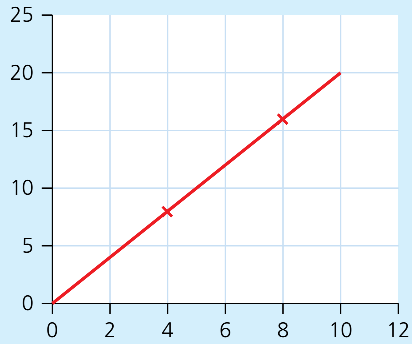

Figure 4.6

b **A biological relationship is predicted by the equation $y = -2x + 10$. Sketch the graph of this relationship on the axes provided.**

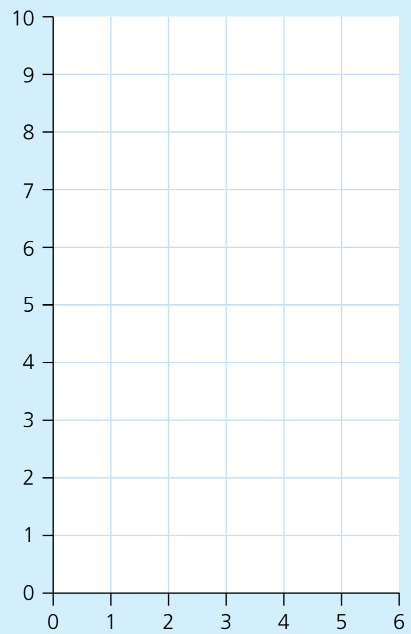

Figure 4.7

Step 1: identity the values of m and c in the linear equation.

The given equation is of the form $y = mx + c$ with $m = -2$ and $c = 10$.

As the gradient (m) is negative, the graph is a straight line with negative gradient, i.e. it will slope downwards from left to right, as opposed to the previous example where the line sloped upwards.

Step 2: choose two x values (within the range shown on the given x-axis) and calculate their corresponding y values:

- At $x = 1$, $y = -2 \times 1 + 10 = 8$
- At $x = 5$, $y = -2 \times 5 + 10 = 0$

Step 3: plot these points on the set of axes and draw a straight line through them.

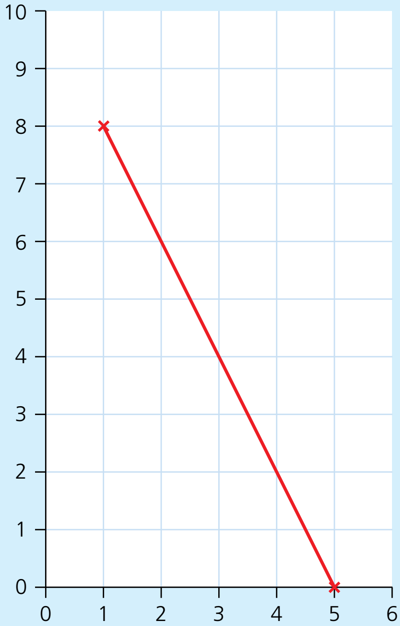

Figure 4.8

This line could be extended leftwards to intersect the y-axis at 10 (this is the y-intercept 'c').

c **Sketch the graph of $y = 6x + 20$ for values of x from 0 to 100.**

Step 1: in this question the axes are not given, so first you need to draw them. As the question asks for values of x from 0 to 100, the x-axis should run from 0 to 100. To work out the scale on the y-axis, calculate the y values corresponding to the highest and lowest x values:

- At $x = 0$, $y = 6 \times 0 + 20 = 20$
- At $x = 100$, $y = 6 \times 100 + 20 = 620$

Therefore the y-axis should span at least the range 20–620.

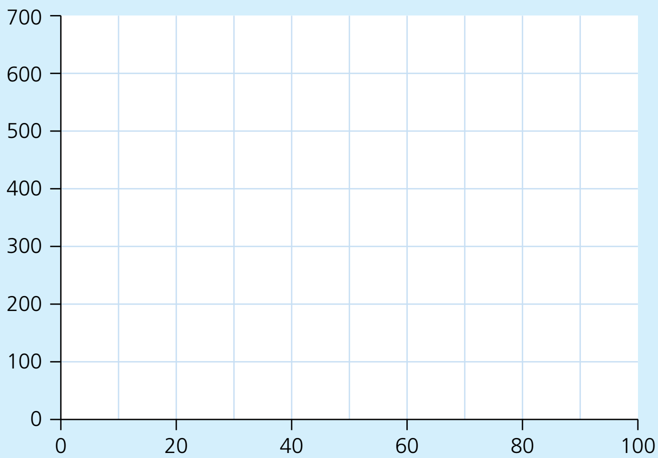

Figure 4.9

Step 2: plot (at least) two points and draw a straight line through them.

In this case we already have the two points calculated in Step 1. We can add another point just to make sure:

- At $x = 50$, $y = 6 \times 50 + 20 = 320$

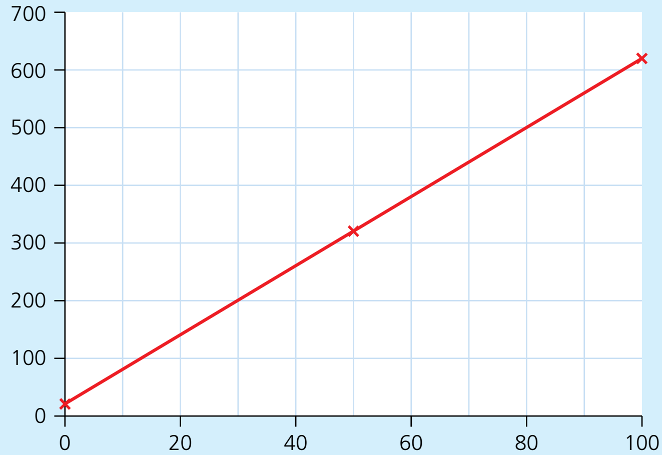

Figure 4.10

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Sketch the graph of $y = -0.5x + 9$ on the axes provided.**

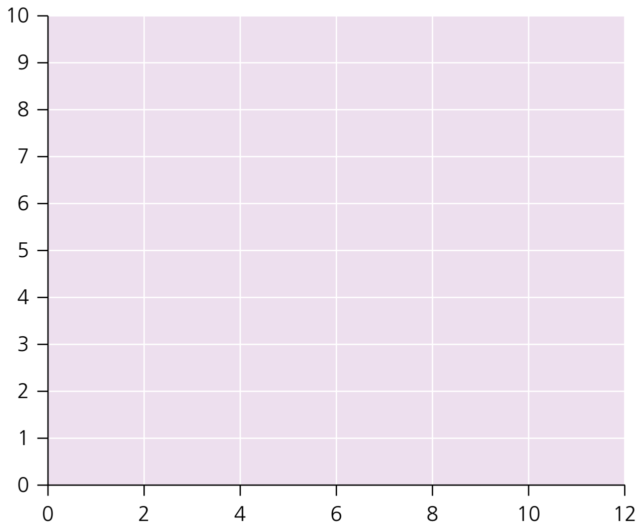

Figure 4.11

Step 1: identity the values of m and c in the linear equation.

In this equation, $m =$ _____________ and $c =$ _____________

What does this tell you about the shape of graph to expect?

Step 2: choose two x values (within the range shown on the given x-axis) and calculate their corresponding y values.

- At $x = 0$, $y =$ ______________
- At $x = 10$, $y =$ ______________

Step 3: plot these points on the set of axes and draw a straight line through them.

2 Sketch the graph of $y = 3x + 10$ for values of x from 0 to 20.

Step 1: in this question the axes are not given, so first you need to draw them. As the question asks for values of x from 0 to 20, the x-axis should run from 0 to 20. To work out the scale on the y-axis, calculate the y values corresponding to the highest and lowest x values:

- At $x = 0$, $y =$ ______________
- At $x = 20$, $y =$ ______________

Step 2: plot (at least) two points and draw a straight line through them.

C Practice questions

3 Use the axes below to draw the line represented by the equation $y = 2x + 3$.

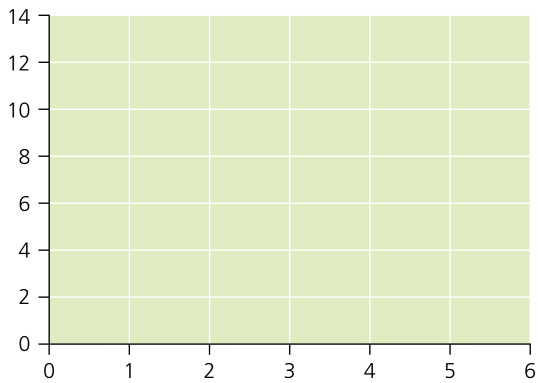

Figure 4.12

4 Sketch a graph to represent the equation $y = x + 9$ for values of x from 5 to 10.

5 The graph of a relationship between two variables is a straight line with a y-intercept at 15 and a gradient of 20. Give the equation that represents this line and sketch the line on the axes below.

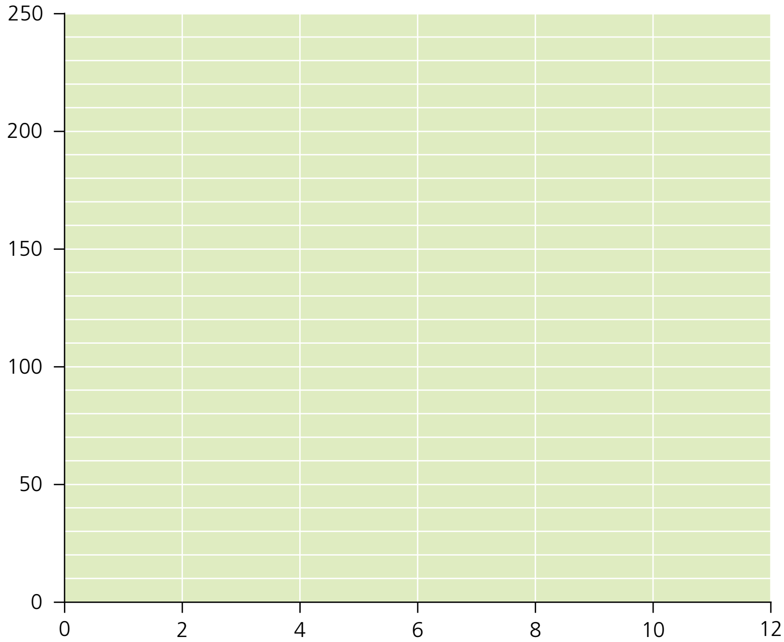

Figure 4.13

6 The three lines shown in Figure 4.14 represent the following equations:

- $y = 2x + 3$
- $y = 3x + 1$
- $y = 4x + 3$

Match the graphs A, B and C with the correct equations, giving a reason for your answers.

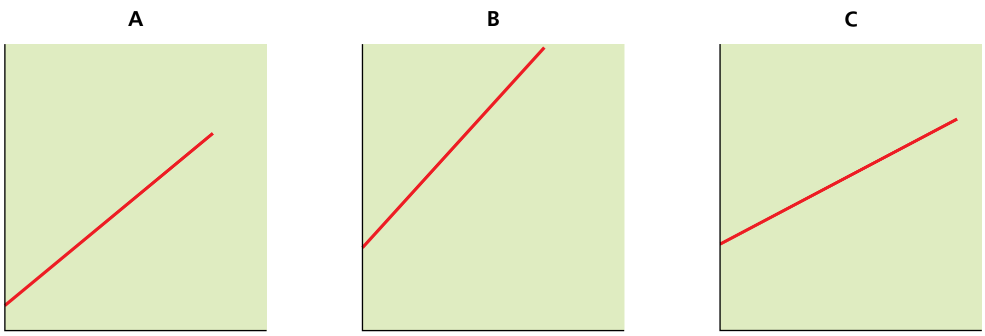

Figure 4.14

Determining the intercept of a graph

Note: this topic is assessed at A-level only.

In biological applications, you may need to find an intercept of a graph, i.e. the point where the graph crosses one of the axes. In exam questions you would normally be asked to find the intercept on the *x*-axis. To do this, either read off the *x* value at which the line or curve crosses the *x*-axis or, if the crossing point is not shown on the graph, you may be able to extrapolate from the line to find where it intersects the axis.

Areas in which this skill is applied most often are problems involving osmosis and water potential or photosynthesis compensation points. Both of these topics are covered in the examples and questions in this section.

A Worked examples

a **The graph in Figure 4.15 shows the change in mass of a potato sample placed in different concentrations of sucrose solution. At what concentration of sucrose would there be no change in mass?**

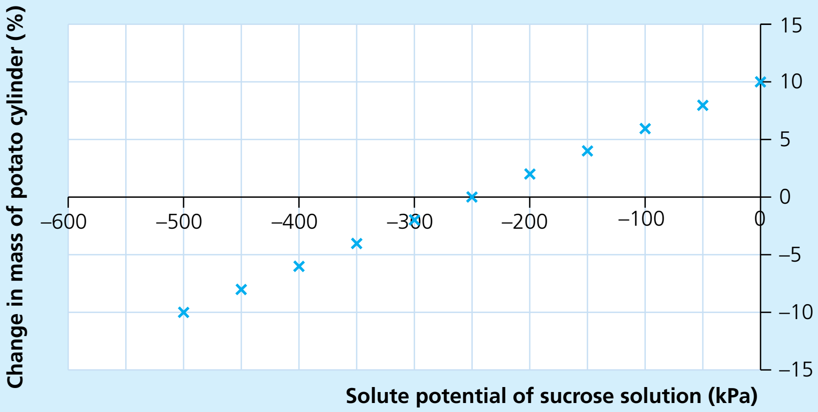

Figure 4.15

We need to find the value of *x* (concentration of sucrose) at which the *y* value (change in mass) is zero; this is the *x*-axis intercept.

In this case the intercept is straightforward to work out: simply draw a straight line through the data points and look for where it crosses the *x*-axis.

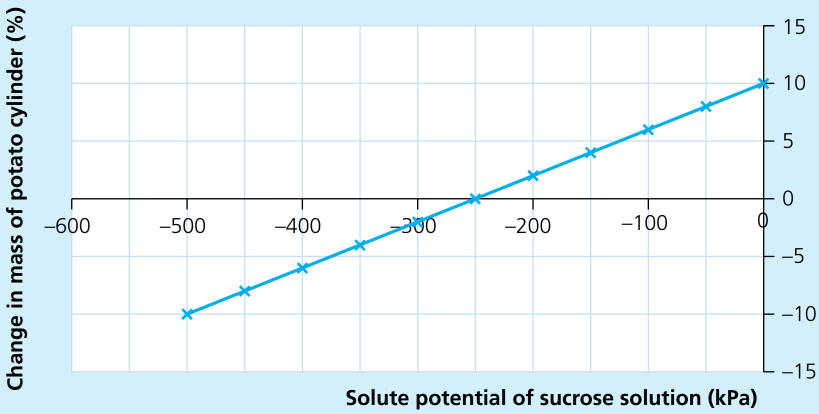

Figure 4.16

Reading off the *x* value where the line crosses the *x*-axis, we find that at −250 kPa sucrose solution there is 0% change in mass.

b The graph in Figure 4.17 shows the effect of light intensity on the carbon dioxide uptake of a plant. Find the compensation point.

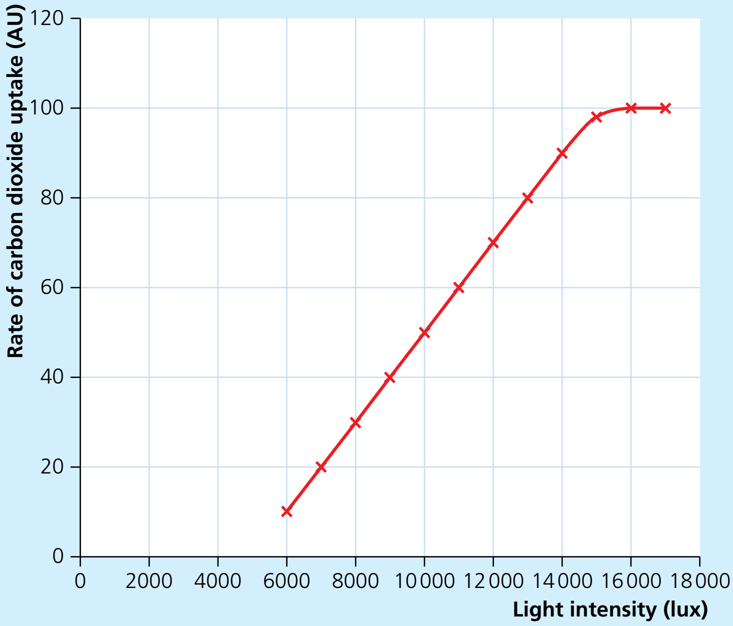

Figure 4.17

In photosynthesis the compensation point is the point at which the rate of photosynthesis equals the rate of respiration. At this point no carbon dioxide is taken up by the plant and no oxygen is released.

Since the y-axis in Figure 4.17 is rate of carbon dioxide uptake, we are looking for the point on the graph where the y value is zero, i.e. the x-axis intercept.

Unlike in the previous example, we cannot simply read off the x value of this point, because the graph does not actually reach the x-axis. So we have to work backwards (i.e. extrapolate) from the existing data points to extend the graph to the x-axis.

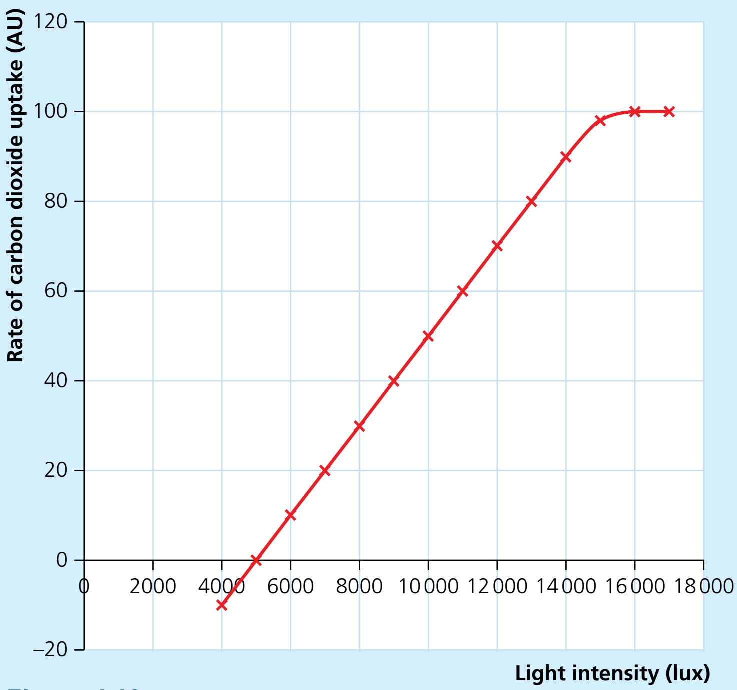

Figure 4.18

From this extended graph we see that the x-intercept is approximately at 5000. Therefore the compensation point is 5000 lux.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Figure 4.19 shows the change in mass of onion tissue placed in different concentrations of sodium chloride. Predict at what concentration there will no change in mass.**

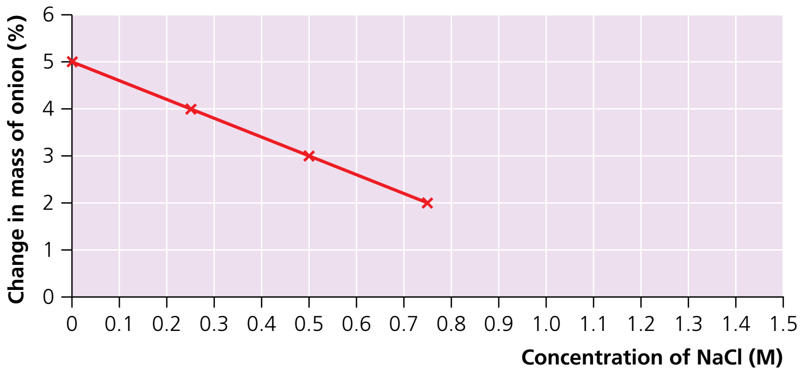

Figure 4.19

As change in mass is shown on the y-axis, no change in mass means $y = 0$, so we are looking for the line's intercept with the x-axis.

Because the line in Figure 4.19 does not actually reach the x-axis, first extend it to the x-axis. Then read off the x value where it meets the axis.

2 **The data in Table 4.10 was recorded during an investigation into the effect of light intensity on the rate of oxygen production in a plant. Find the light intensity which gives the compensation point of this plant.**

Table 4.10

Light intensity (lux)	Rate of oxygen production ($cm^3 min^{-1}$)
3500	−0.2
4500	0.3
5500	0.4
6500	0.6
7500	0.8
8500	0.9
9500	0.9
10500	0.9

Step 1: plot the data on a scatter diagram.

Ensure that the scale on the y-axis (rate of oxygen production) includes zero, to make it easier to find the x-axis intercept later.

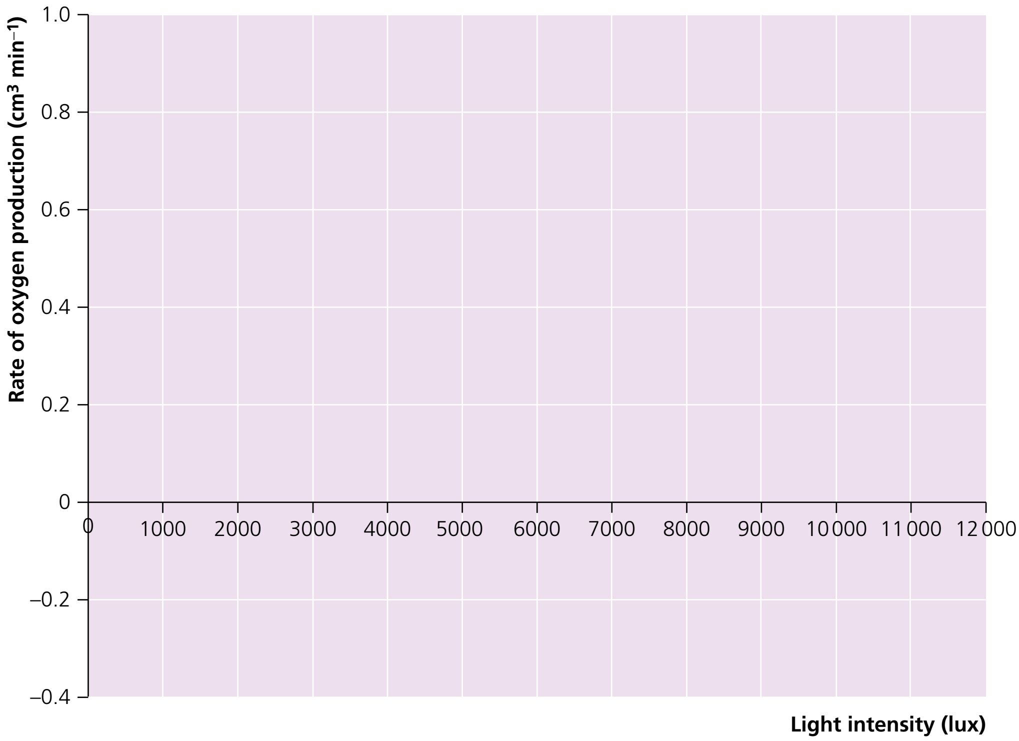

Figure 4.20

Step 2: once the points are plotted, connect them with straight lines to get a line graph.

Step 3: the compensation point is the point at which no oxygen is produced, i.e. $y = 0$, so it corresponds to the graph's x-intercept. Look for where the line graph crosses the x-axis.

C Practice questions

3 The graph in Figure 4.21 shows the effect of light intensity on the oxygen production of a plant. What is the compensation point of this reaction?

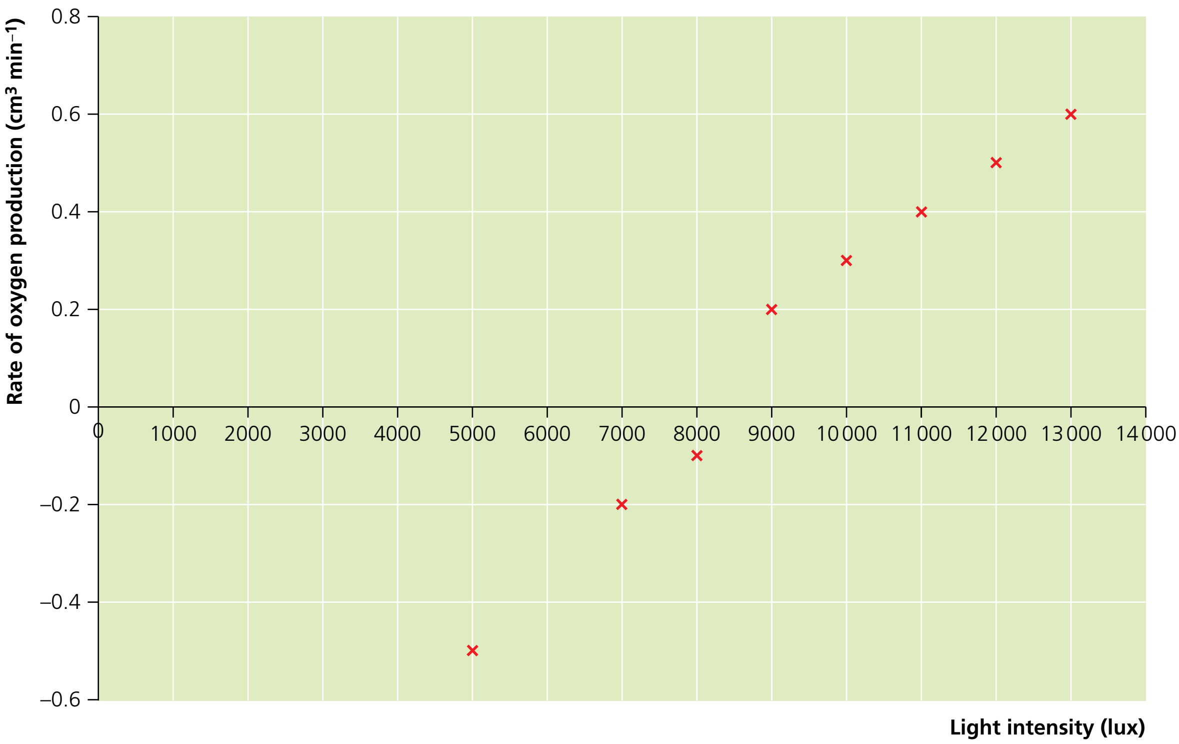

Figure 4.21

4 The graph in Figure 4.22 came from another investigation into photosynthesis. Can the compensation point be determined from this graph? Explain your answer.

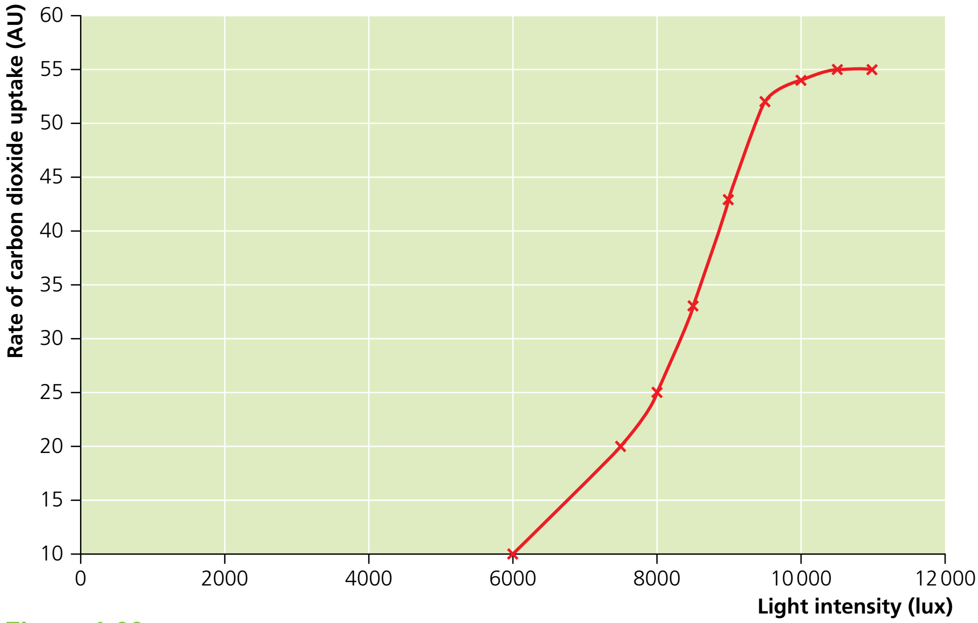

Figure 4.22

5 Is extrapolating backwards to determine an axis intercept always accurate? Explain your answer.

6 Table 4.11 shows data on the change in mass of an onion sample at different solute potentials. Plot the data on the axes below, and from it determine the point at which the water potential inside the cell was equal to the water potential outside the cell.

Table 4.11

Solute potential (kPa)	Change in mass of sample (%)
–1000	–24
–900	–21
–700	–15
–500	–9
–300	–3
–100	3
0	6

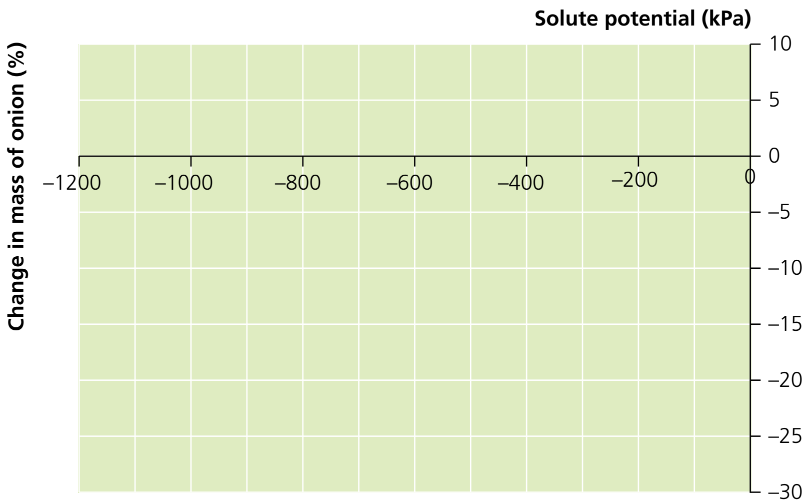

Figure 4.23

Calculating the rate of change for a linear relationship

We have seen that a linear relationship can be represented by a straight line on a graph, or by the equation $y = mx + c$. The rate of change of the linear relationship is the gradient of the line graph, or the value of m in the equation.

When provided with a graph showing a linear relationship, you can calculate the rate of change by finding the gradient of the line. To do this, divide the change in the variable on the y-axis by the corresponding change in the variable on the x-axis. It is easiest to determine the amount of change in each variable by drawing a right-angled triangle with its hypotenuse along the line (see the Worked examples which follow). Because the gradient is the same at all points on a line, it does not matter where on the line you place this triangle.

In an exam question you will be expected to give units for the rate of change you have calculated. This is obtained by dividing the unit of the variable on the y-axis by the unit of the variable on the x-axis.

A Worked examples

a **The graph in Figure 4.24 shows the variation in Na^+ concentration inside a cell over time. What is the rate of change of the Na^+ concentration?**

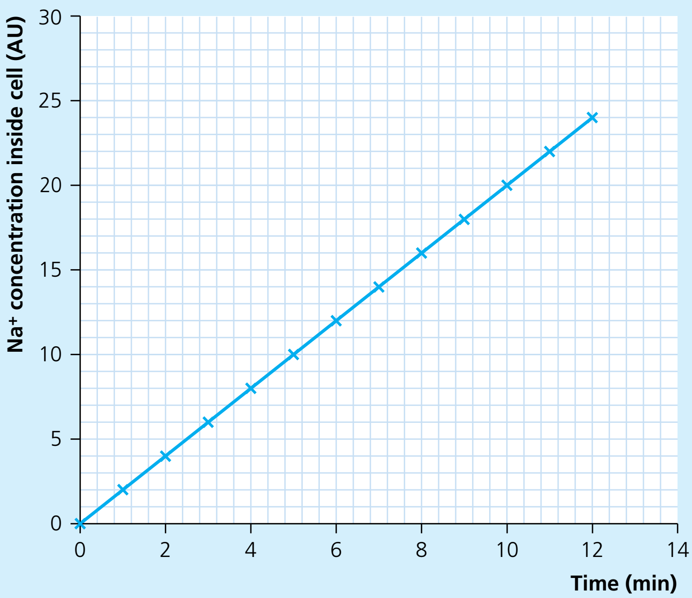

Figure 4.24

The rate of change is the gradient of the line. To find the gradient, draw a right-angled triangle as shown in Figure 4.25. The triangle has a vertical edge and a horizontal edge, and its hypotenuse (slanted edge) lies along the line graph.

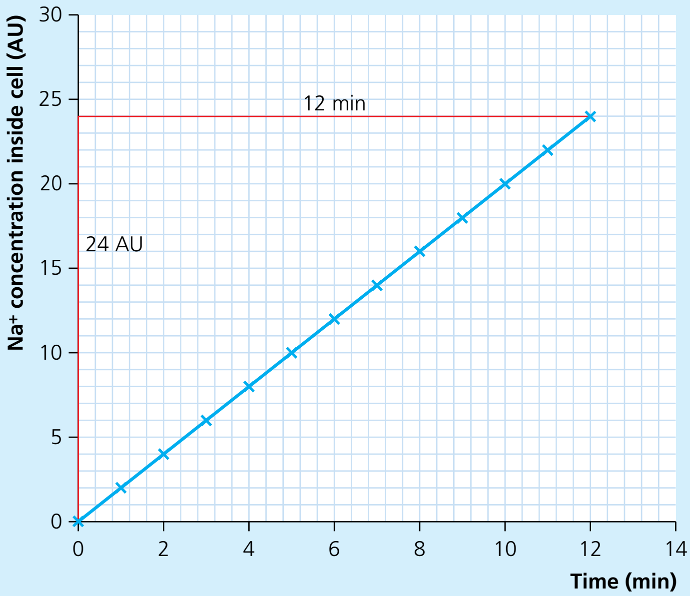

Figure 4.25

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{length of vertical edge of triangle}}{\text{length of horizontal edge of triangle}}$$

$$= \frac{24-0}{12-0} = 2$$

The unit of the rate of change is the concentration unit (AU) divided by the time unit (min), i.e. AU min^{-1}.

So the rate of change of Na^+ concentration is 2 AU min^{-1}.

b **The graph in Figure 4.26 shows the decrease in nitrate levels in soil over time. What is the rate of change of the concentration of nitrates?**

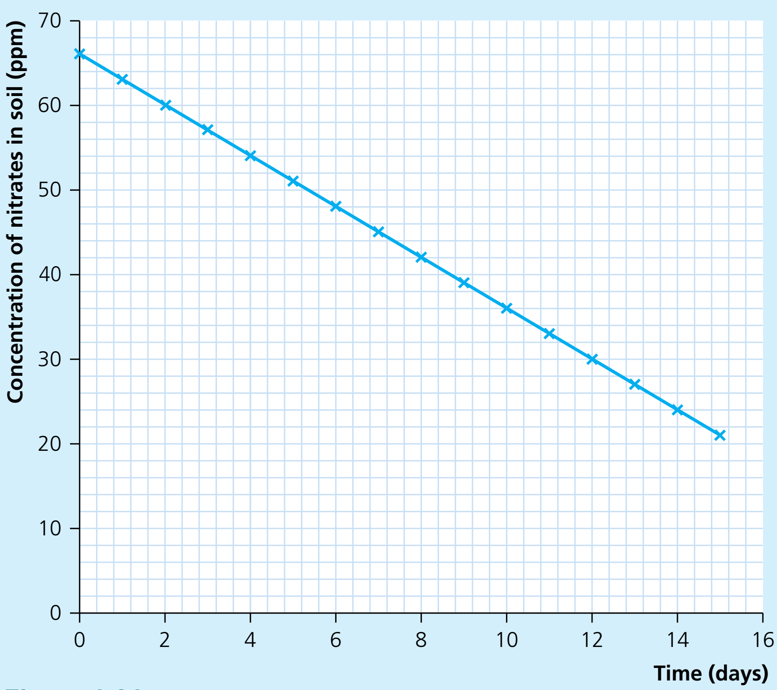

Figure 4.26

To find the rate of change, i.e. the gradient of the line, draw a right-angled triangle as shown in Figure 4.27.

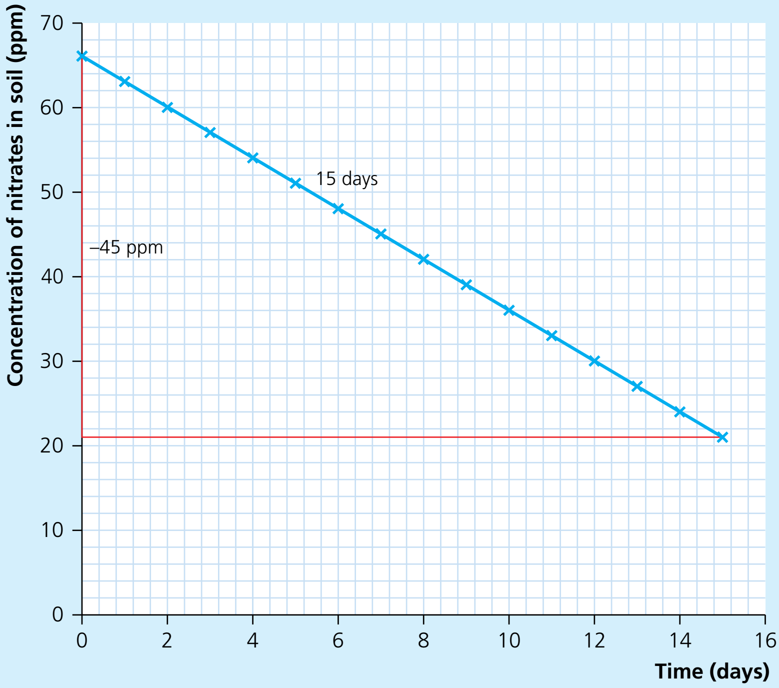

Figure 4.27

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{21-66}{15-0} = \frac{-45}{15} = -3$$

The unit of the rate of change is the concentration unit (ppm) divided by the time unit (day), i.e. ppm day^{-1}.

So the rate of change of nitrate concentration is −3 ppm day^{-1}.

Notice that because the line slopes downwards, it has a negative gradient and therefore a negative rate of change.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 The graph in Figure 4.28 shows the growth of a tapeworm. Calculate the rate of the tapeworm's growth.

Step 1: draw a right-angled triangle with hypotenuse on the line.

Step 2: read off the change in y by looking at the vertical edge of the triangle and the change in x by looking at the horizontal edge of the triangle.

Step 3: substitute into the formula

$$\text{gradient of line} = \frac{\text{change in } y}{\text{change in } x}$$

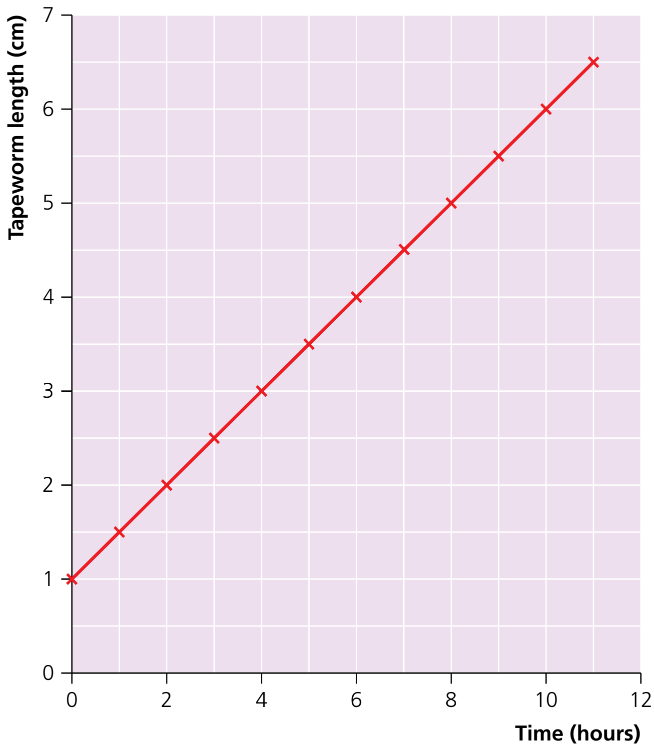

Figure 4.28

2 **The graph in Figure 4.29 shows the relationship between maximum flight speed and body mass of a certain species of bird. Calculate the rate of decrease of maximum flight speed with body mass.**

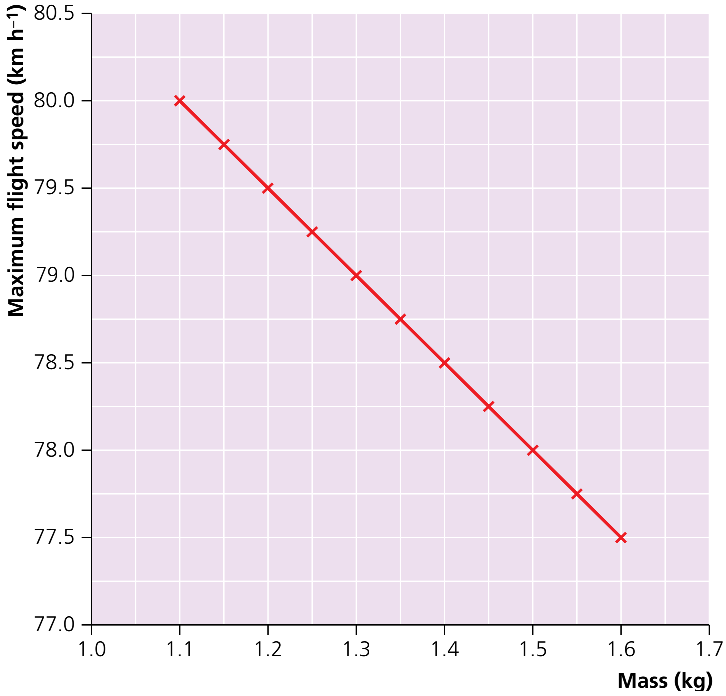

Figure 4.29

In this question the rate of change is not with respect to time, as in the previous examples, but with respect to mass. This just means that mass is the variable plotted on the x-axis. Calculation of the rate of change proceeds in the same way: find the gradient of the line by drawing a right-angled triangle and calculating

$$\frac{\text{change in } y}{\text{change in } x}$$

C Practice questions

3 Figure 4.30 shows the results of an investigation into the effect of substrate concentration on the rate of an enzyme reaction. Calculate the rate of change from the graph.

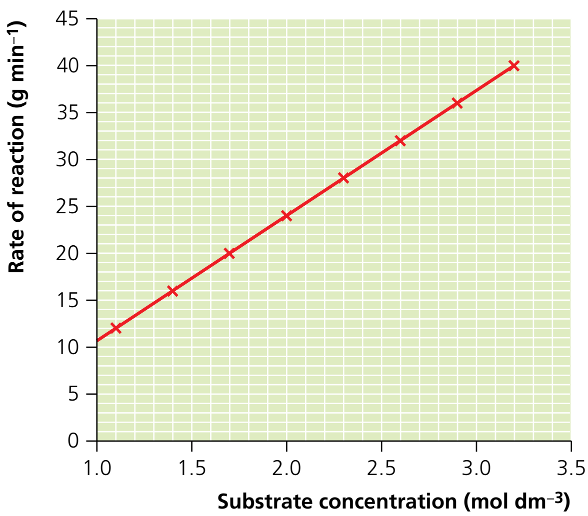

Figure 4.30

4 In an investigation into osmosis, the graph in Figure 4.31 was produced. Calculate the rate of change of the percentage change in onion mass with respect to the external solute potential.

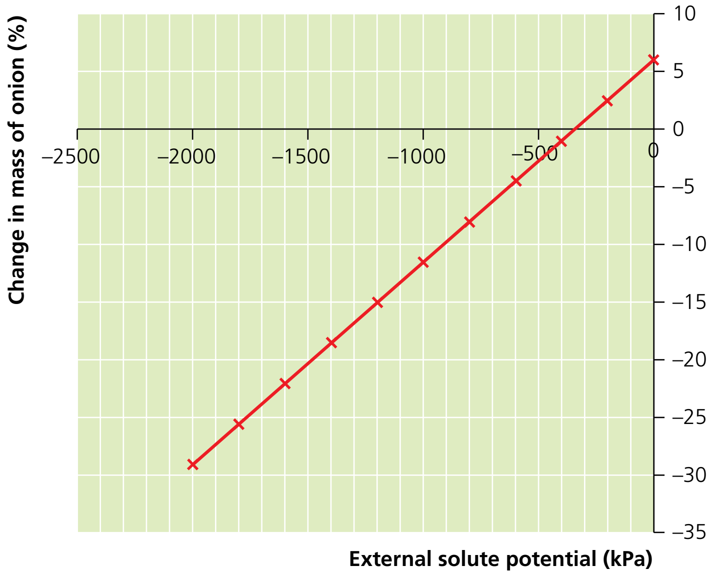

Figure 4.31

5 The graph in Figure 4.32 was produced during an investigation into transpiration.

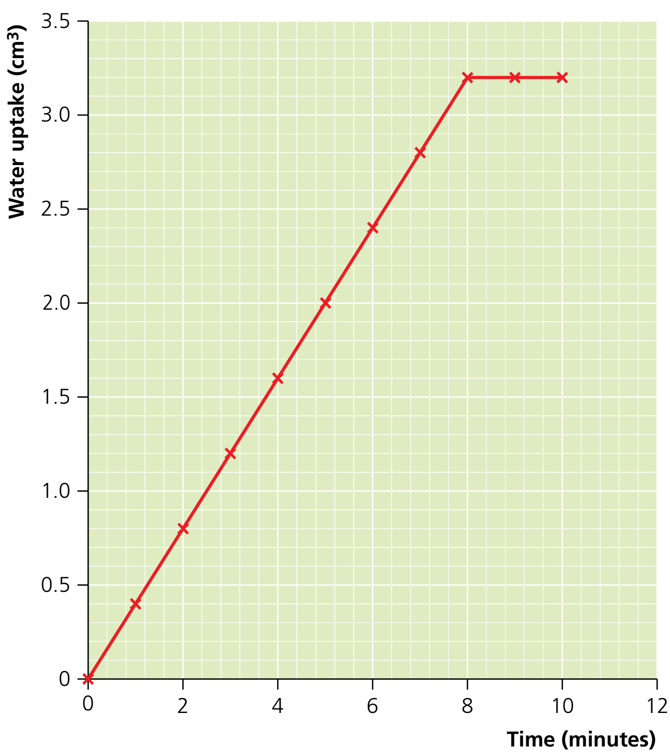

Figure 4.32

a Calculate the rate of water uptake during the following time intervals:

  i 0–8 minutes

  ii 8–10 minutes

b The experiment was repeated at a warmer temperature. How would you expect the rate you calculated in part **ai** to change?

Measuring the rate of change at a point on a curve

If a graph is curved rather than straight, the gradient will be different at different points on the graph. To measure the rate of change at a particular point, draw a **tangent** to the curve at that point and find the gradient of this tangent line.

A tangent is a straight line that touches the graph at a single point. By forming a right-angled triangle with hypotenuse on this tangent (as in the previous section dealing with linear relationships), the gradient and hence the rate of change can then be calculated.

A Worked examples

a **The graph in Figure 4.33 shows the change in the mass of product in a reaction over time. Calculate the rate of product formation at 13 minutes.**

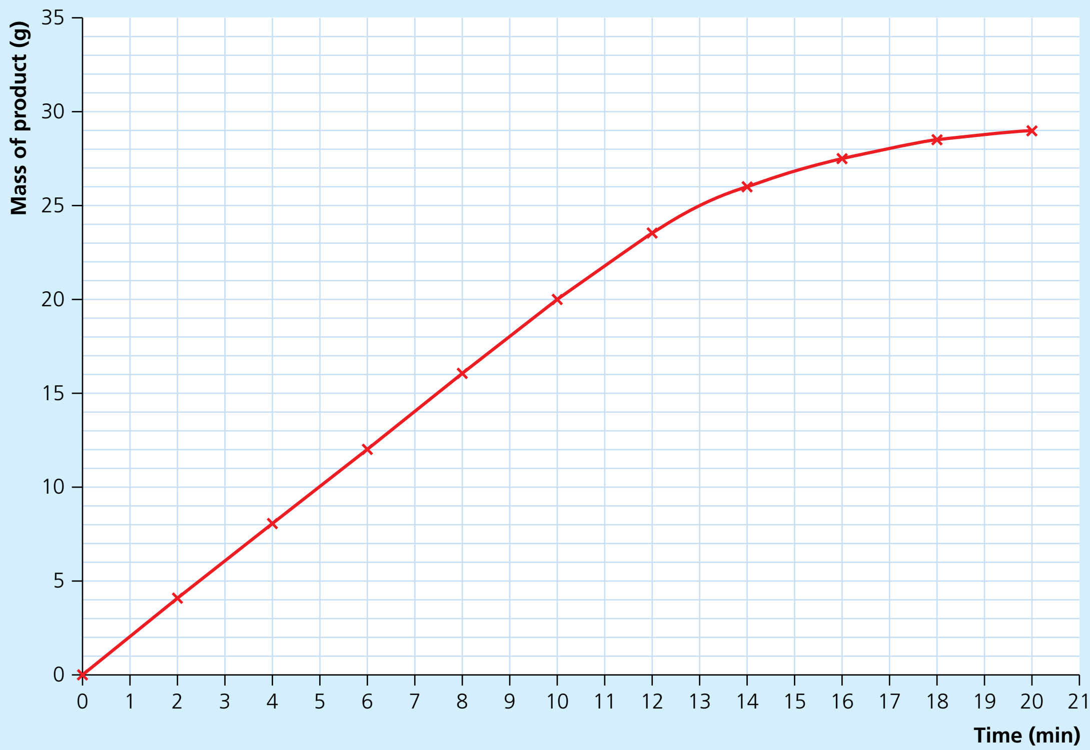

Figure 4.33

Step 1: to find the rate of change at 13 minutes, draw a tangent to the graph at 13 minutes. This is a straight line that touches the curve at the x value 13.

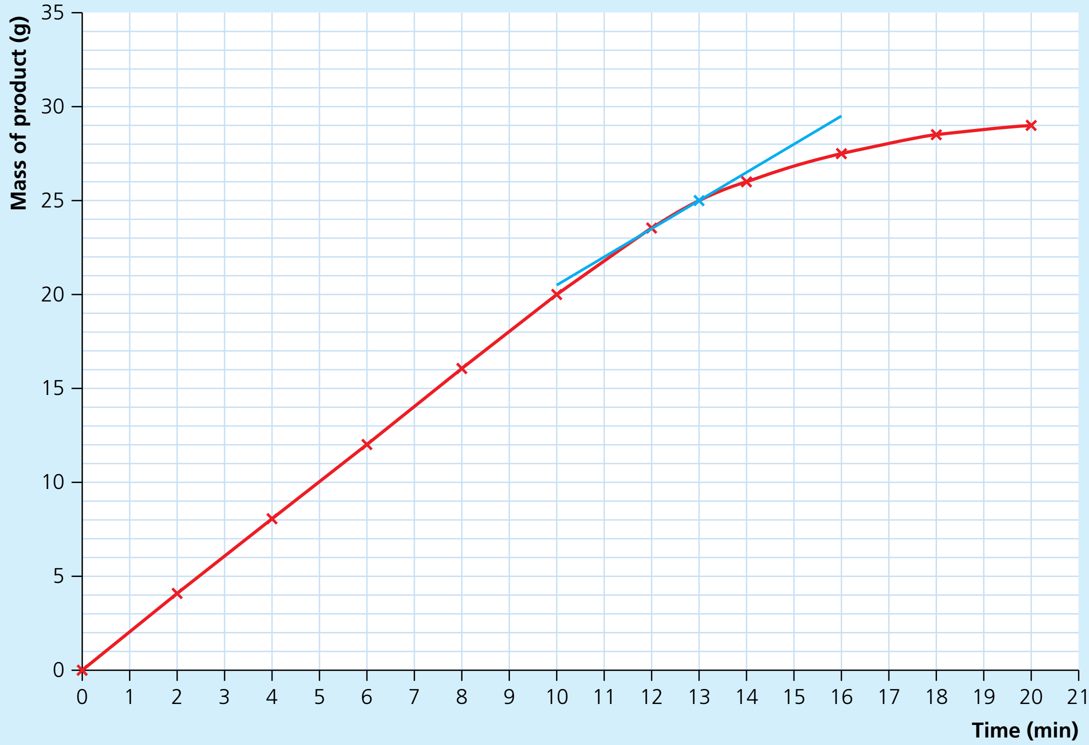

Figure 4.34

Step 2: form a right-angled triangle with hypotenuse on the tangent line. Determine the change in y by looking at the vertical edge of the triangle and the change in x by looking at the horizontal edge of the triangle.

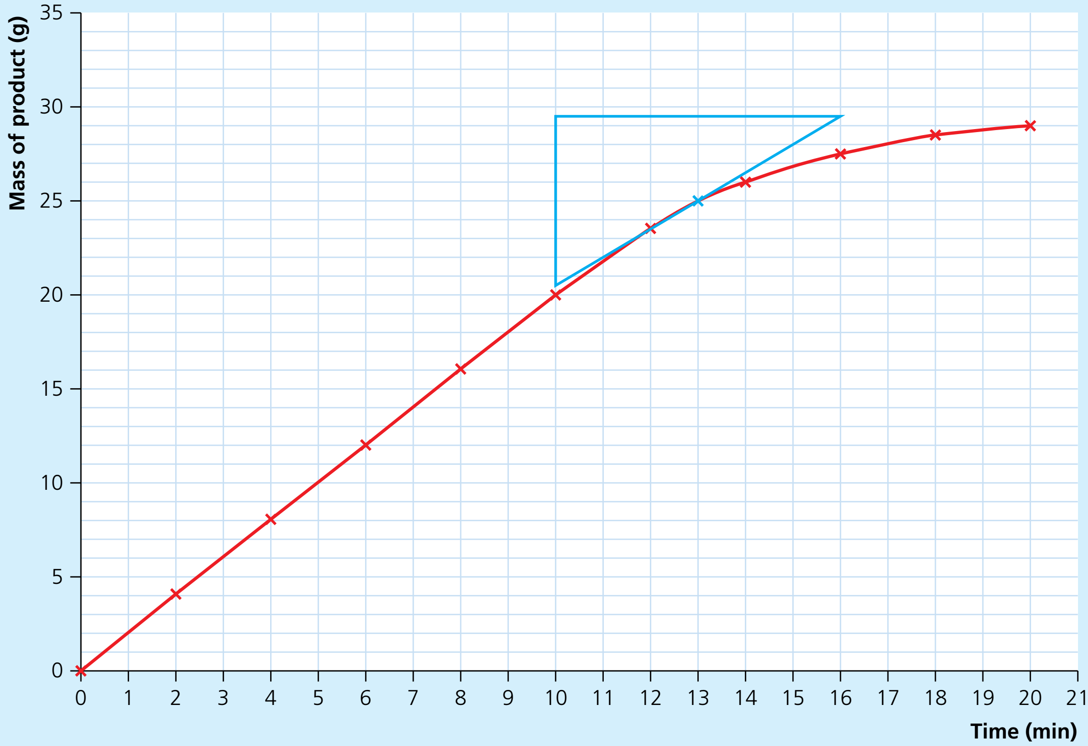

Figure 4.35

Step 3: divide the change in y by the change in x to find the gradient of the tangent.

$$\text{gradient of tangent} = \frac{\text{change in } y}{\text{change in } x} = \frac{29-21}{16-10} = \frac{8}{6} = 1.33$$

Therefore the rate of product formation at 13 minutes is $1.33\,\text{g}\,\text{min}^{-1}$.

b **Figure 4.36 shows the change in mass of substrate over time in an enzyme-catalysed reaction. Calculate the rate of decrease in substrate mass at 3 minutes.**

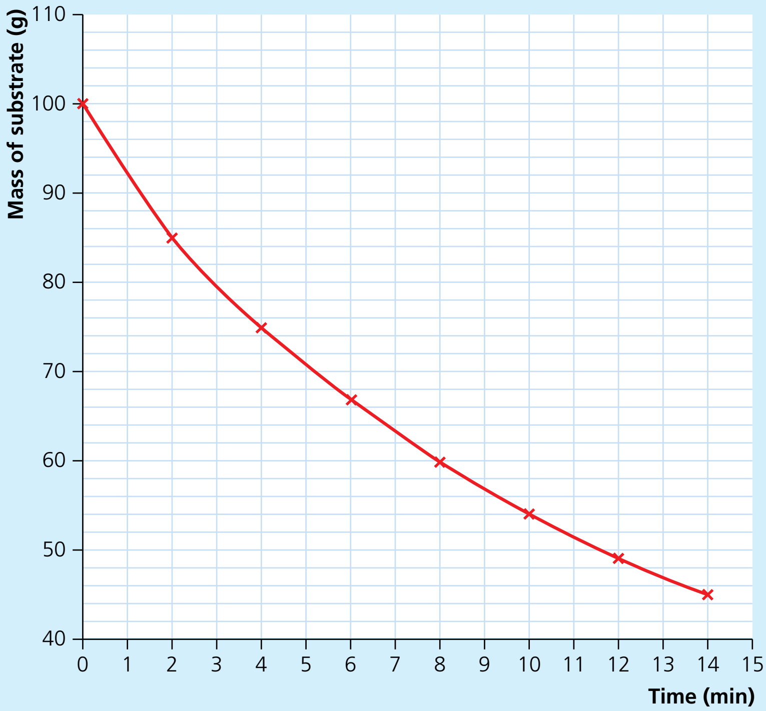

Figure 4.36

Step 1: following the same method as in previous example, draw a tangent at 3 minutes, form a right-angled triangle, and use the triangle to work out the change in x and the change in y.

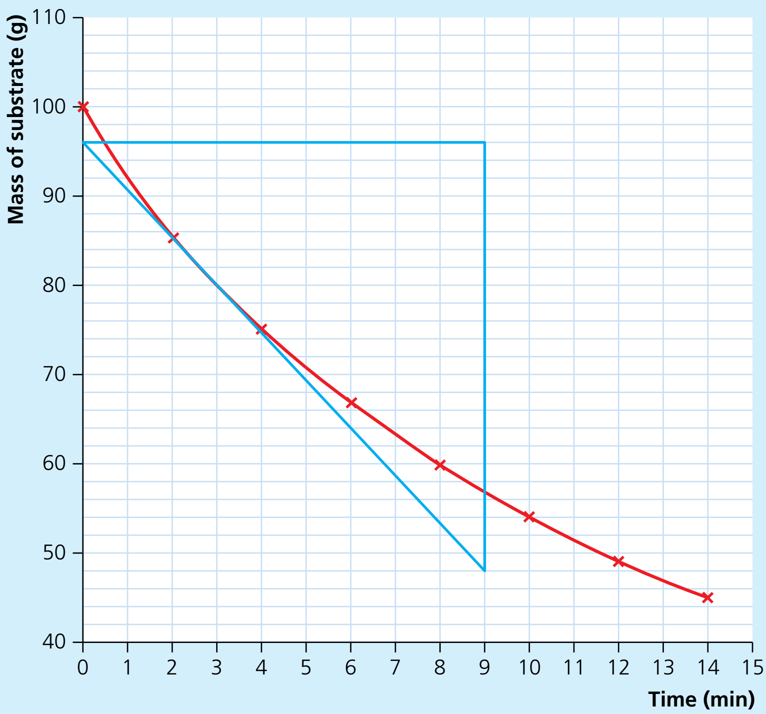

Figure 4.37

Step 2: divide the change in y by the change in x to find the gradient of the tangent.

$$\text{gradient of tangent} = \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{48 - 98}{9 - 0}$$
$$= \frac{-50}{9} = -5.6$$

Therefore the rate of decrease in substrate mass at 3 minutes is 5.6 g min^{-1}.

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 **Figure 4.38 shows the change in concentration of a substance over time during a biological investigation. What is the rate of change at 44 hours?**

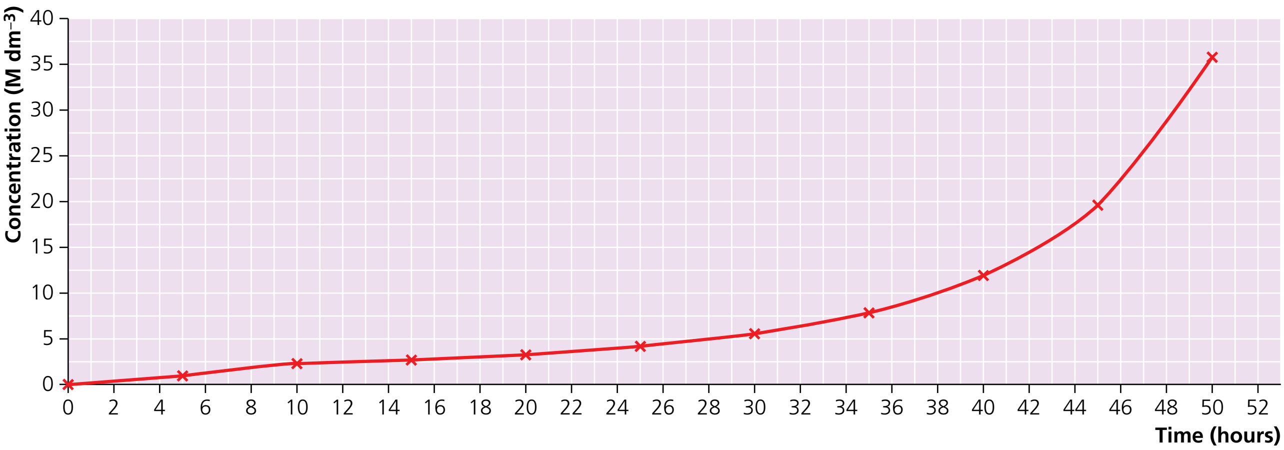

Figure 4.38

Step 1: draw a tangent at 44 hours, and form a right-angled triangle.

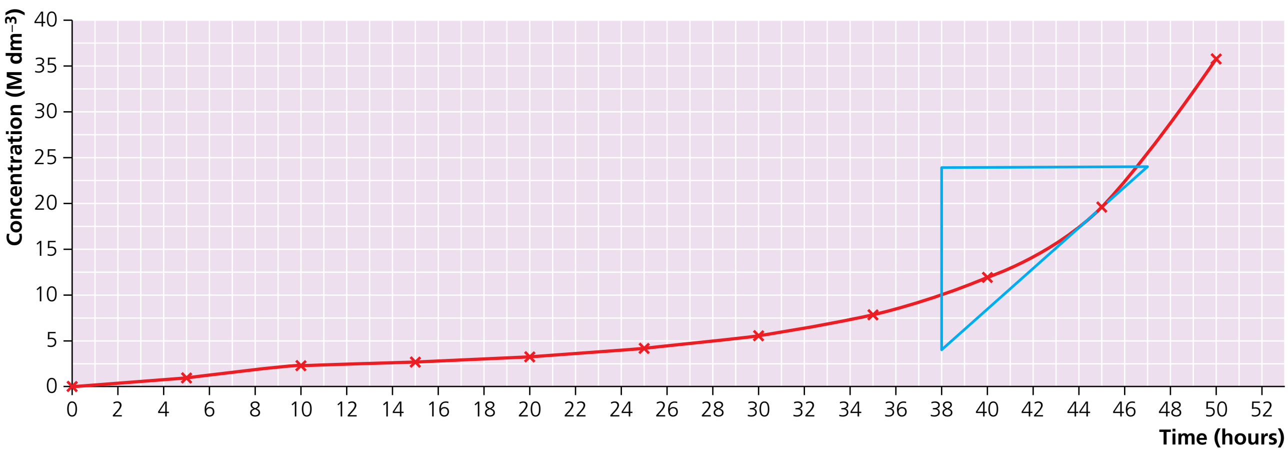

Figure 4.39

Step 2: divide the change in y by the change in x to find the gradient of the tangent.

2 **The oxygen dissociation curve for human haemoglobin is shown in Figure 4.40. What is the rate of change of percentage oxygen saturation at 44 mm Hg?**

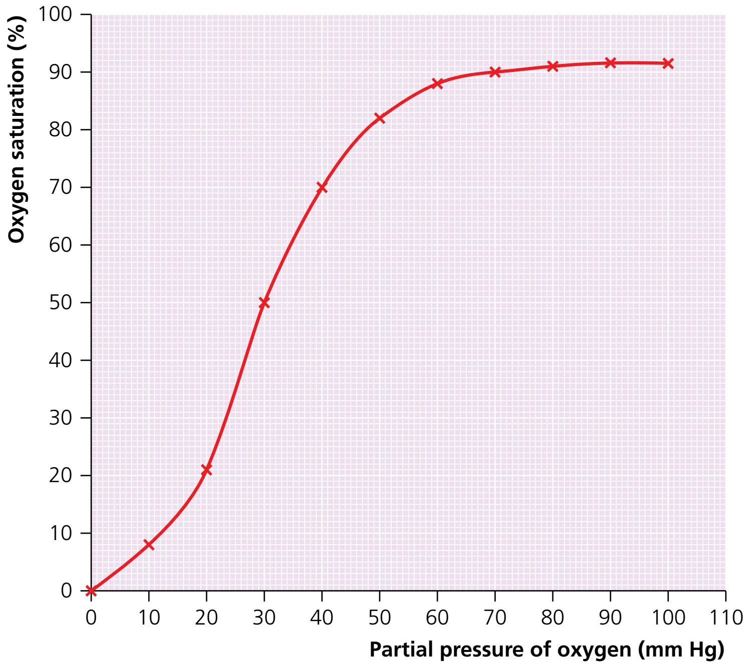

Figure 4.40

Step 1: draw a tangent at 44 mm Hg, and form a right-angled triangle.

Step 2: divide the change in y by the change in x to find the gradient of the tangent.

C Practice questions

3 Figure 4.41 shows the change in mass of product over time during an enzyme-catalysed reaction. What is the rate of reaction at 7 minutes?

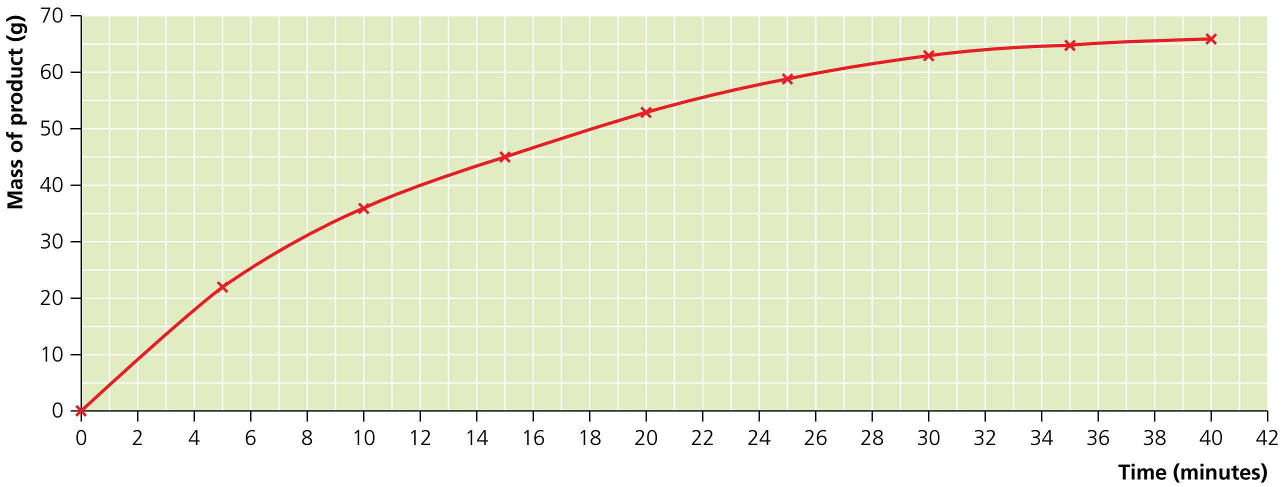

Figure 4.41

4 Figure 4.42 shows the change in population over time of a seabird colony. What was the rate of change of the population in the year 1990?

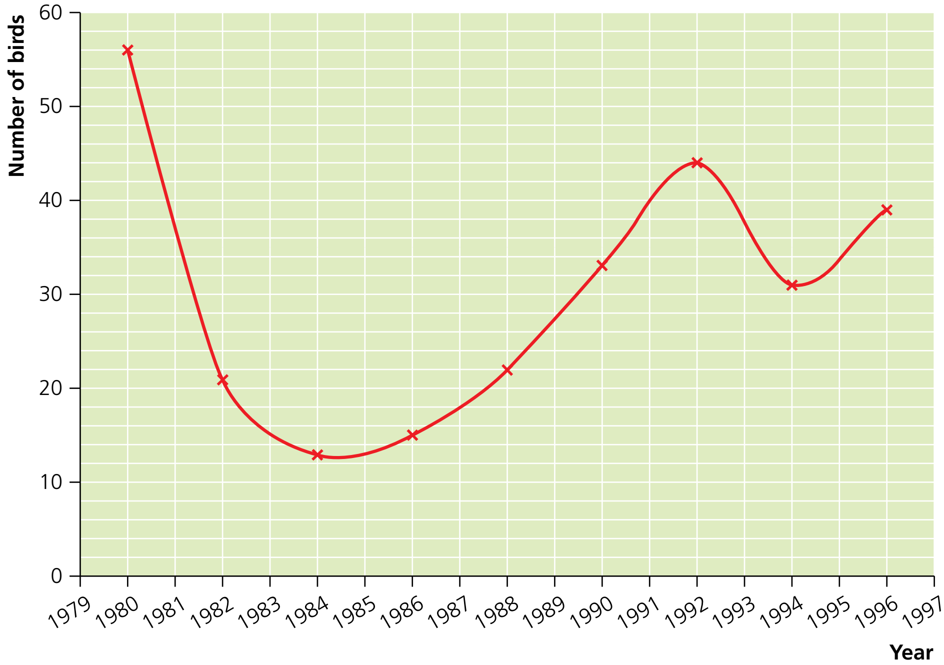

Figure 4.42

5 a The oxygen dissociation curves for human and llama haemoglobin are shown in Figure 4.43. The blue curve is for human haemoglobin and the red curve for llama haemoglobin. Calculate the rate of change in oxygen saturation at 28 mm Hg in both humans and llamas.

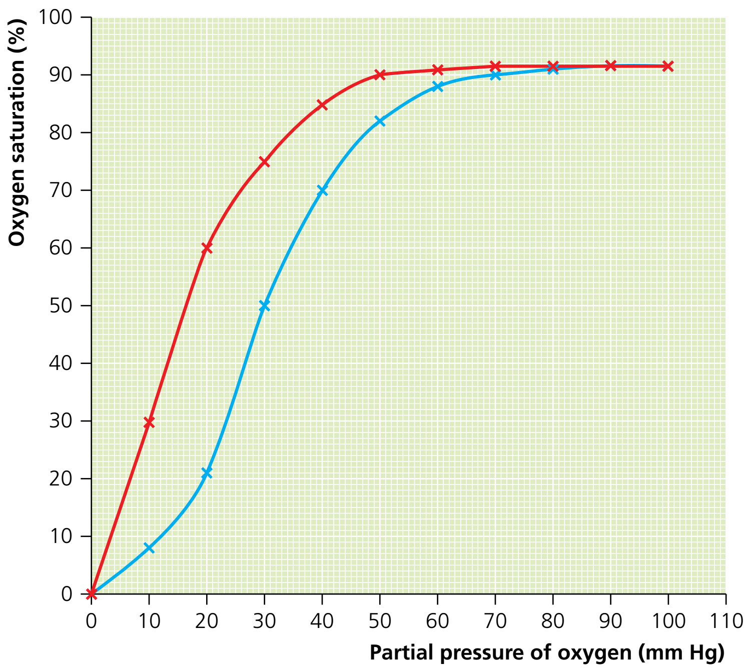

Figure 4.43

b Llamas live at high altitude where the partial pressure of oxygen is low. Use this information to suggest an explanation for the values calculated in part **a**.

Exam questions may ask you to explain differences in rates of change that you have calculated. Think carefully about how to apply your biological knowledge to give the most detailed explanations possible.

5 Geometry and trigonometry

Circumferences, surface areas and volumes of regular shapes

Biological organisms and structures are often represented as simple shapes such as cubes, rectangular prisms and spheres. Therefore you need to be able to calculate:

- the circumference and area of a circle
- the surface area and volume of a rectangular prism, cylindrical prism or sphere.

Exam questions covering this skill may involve calculation of ratios, such as surface area to volume ratios. See the 'Ratios, fractions and percentages' section on pages 11–14 for more information and examples. You may also find it helpful to refer to the section 'Appropriate units in calculations' on pages 6–8 when working through the examples and questions in this section.

A Worked examples

a A cube has side length 5 cm. Find the surface area and volume of this cube.

As all sides of a cube are the same length, you only need to know the length of one side to calculate the cube's volume and surface area.

- Volume of cube = length × width × height
  So the volume of this cube is
  $5\,\text{cm} \times 5\,\text{cm} \times 5\,\text{cm} = 125\,\text{cm}^3$
- Surface area of cube = area of one face × number of faces
  = side length × side length × number of faces

  So the surface area of this cube is
  $5\,\text{cm} \times 5\,\text{cm} \times 6 = 150\,\text{cm}^2$

b A circular paper disc used in an investigation into the action of catalase had a diameter of 0.5 cm. Calculate the disc's circumference and area.

- The formulas for the circumference and area of a circle are given in terms of the radius r, so first find the radius. We are given the diameter, so the radius is half of this:
  $r = 0.5\,\text{cm} \div 2 = 0.25\,\text{cm}$
- Circumference of circle = $2\pi r$
  So the circumference of this disc is
  $2 \times \pi \times 0.25\,\text{cm} = 1.57\,\text{cm}$
- Area of circle = πr^2
  So the area of this disc is
  $\pi \times 0.25^2 = 0.196\,\text{cm}^2$

c Figure 5.1 shows a rectangular prism. Find its surface area and volume

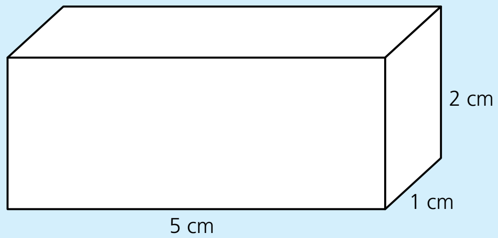

Figure 5.1

- A rectangular prism has six faces, consisting of three pairs of identical faces. To find the surface area of the prism in Figure 5.1, calculate the area of each pair of identical faces, then add these areas together.
  - The smallest faces on the left and right ends each have area
    $2\,\text{cm} \times 1\,\text{cm} = 2\,\text{cm}^2$
  - The front and back faces, with sides 5 cm and 2 cm, each have area
    $5\,\text{cm} \times 2\,\text{cm} = 10\,\text{cm}^2$
  - The top and bottom faces, with sides 5 cm and 1 cm, each have area
    $5\,\text{cm} \times 1\,\text{cm} = 5\,\text{cm}^2$
  - Therefore the total surface area is
    $2 \times 2\,\text{cm}^2 + 2 \times 10\,\text{cm}^2 + 2 \times 5\,\text{cm}^2 = 34\,\text{cm}^2$
- Calculating the volume of a rectangular prism is much easier. Simply find the area of one face and then multiply it by the third dimension.
  - Area of smallest face on the right end $= 2\,\text{cm}^2$
  - Length $= 5\,\text{cm}$
  - Therefore the volume is
    $2\,\text{cm}^2 \times 5\,\text{cm} = 10\,\text{cm}^3$

  Alternatively, the volume can be calculated using the formula

  volume of rectangular prism = length × width × depth

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 An earthworm has a diameter of 0.6 cm and a length of 9 cm. Treating it as a cylinder, find its surface area and volume.

To work out the surface area of a cylinder (cylindrical prism), imagine slitting open the curved side lengthwise and rolling the cylinder out to form a rectangle:

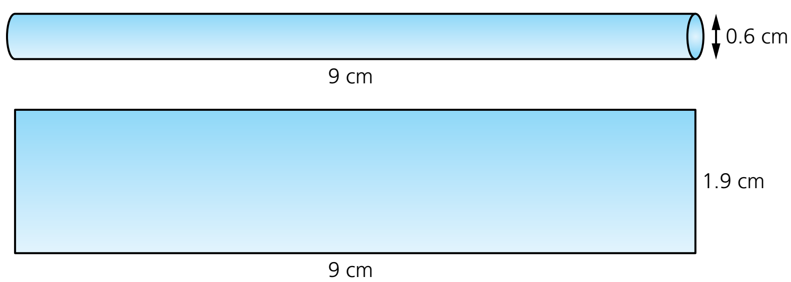

Figure 5.2 The area of the rectangle is the surface area of the curved side of the cylinder.

Step 1: the 'height' of the rectangle is the circumference of the circular end of the cylinder, which is $2\pi r$ where r is the radius of the circle. So

height of rectangle = ______________

Step 2: the length of the rectangle is the length of the cylinder. Therefore

surface area of curved side of cylinder = area of rectangle

= length of rectangle × height of rectangle

= ______________

Step 3: to obtain the total surface area of the cylinder, add the area of the two circular ends of the cylinder to the result in Step 2.

Area of each circular end = πr^2 = ______________

Area of both ends = ______________

Total surface area of cylinder = ____________ + ____________ = ____________

Step 4: the volume is much simpler to calculate.

volume of cylinder = area of one circular end × length

= ______________ × ______________

2 **A lipid droplet is in the shape of a sphere with diameter 60 µm. Calculate the volume and surface area of this lipid droplet.**

- The volume of a sphere is given by the formula

$$\text{volume of sphere} = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

- The surface area of a sphere is given by

$$\text{surface area of sphere} = 4\pi r^2$$

Find r and substitute its value into these formulas.

3 **Cube A has side lengths of 5 cm and cube B has side lengths of 3 cm. Which of the two cubes has the greater surface area to volume ratio?**

Step 1: calculate the surface area and volume of cube A.

- Surface area of cube A = 5 × 5 × 6 = ______________
- Volume of cube A = 5 × 5 × 5 = ______________

Step 2: write down and simplify the surface area to volume ratio for cube A.

Cube A surface area : volume = _________ : _________ = _________ : _________

Step 3: calculate the surface area and volume of cube B.

- Surface area of cube B = ______________
- Volume of cube B = ______________

Step 4: write down and simplify the surface area to volume ratio for cube B.

Cube B surface area : volume = _________ : _________ = _________ : _________

Step 5: compare these two ratios by expressing both in the form <a value> : 1.

C Practice questions

4 Cell Z is in the shape of a cube with side length 50 µm. Cell Y has the shape of a rectangular prism of length 60 µm, depth 10 µm and width 30 µm. Which cell has the larger surface area?

5 In an investigation into the antimicrobial effects of different substances, a clear ring of diameter 3 cm was created around a disc of diameter 0.5 cm. What is the total surface area of the clear region (not including the disc)?

6 A section of aorta has a radius of 1.5 cm and a length of 7 cm. What is the total volume of this section of aorta?

7 How do species such as *Planaria* (flatworms) ensure that their surface area to volume ratio is large enough to not need a specialised gas exchange or circulatory system?

8 During emulsification by bile, a lipid droplet of diameter 15 µm was split into five smaller droplets of diameter 8 µm. How does the total surface area of the new droplets compare to the surface area of the original droplet? Suggest the biological implications of the effect you have described.

Exam-style questions

AS and A-level questions

1 An investigation into immobilised enzymes was carried out using a column of alginate beads with immobilised enzymes on their surface.

a The alginate beads are spheres with a diameter of 2 cm.

i What is the surface area of one bead? **(2)**

ii What is the volume of one bead? **(2)**

iii What is the surface area to volume ratio of one bead? **(1)**

iv There are approximately 54 beads in the investigation. What would be a reasonable estimate of the total surface area of these beads? **(2)**

b The relationship between the rate of flow of substrate through the apparatus ($cm^3\,min^{-1}$) and the diameter of the beads (cm) is represented by the equation $y = 5x + 4$. Sketch the graph of this relationship for bead sizes x from 0.5 cm to 3 cm. **(3)**

c The theoretical yield for this reaction was 15 cm^3 of apple juice. In the actual investigation 13 cm^3 was produced. What is the percentage error of this investigation? **(2)**

2 An investigation was carried out into the lengths of mitochondria in an animal cell. The results are shown in the following table.

Table E.1

Mitochondrion	A	B	C	D	E	F
Length of mitochondrion (μm)	1.1	1.2	0.9	5	1	1.1

a **i** Calculate the mean length of the mitochondria. **(1)**

ii Calculate the range and standard deviation of the mitochondrial lengths. Comment on the values you obtained. **(4)**

iii Given your answers to **i** and **ii,** is the mean the most suitable average to use in this investigation? Explain your answer and suggest an improvement. **(3)**

b If mitochondrion C was drawn with a length of 4 cm, what would be the magnification of the drawing? **(2)**

3 The table below shows some data on the dimensions of two different organisms.

Table E.2

Organism	Dimensions
Flatworm	Length 14 mm, width 3 mm, height 1 mm
Leech	Diameter 2 cm, length 6.5 cm

a Assuming that the organisms roughly conform to standard shapes, calculate the surface area to volume ratio of each of these organisms. **(6)**

b Which organism has the greater surface area to volume ratio? **(1)**

c The leech has a specialised circulatory system while the flatworm does not. Use your answer to **b** to explain why. **(2)**

4 a A red blood cell has a diameter of 7 µm and a depth of 2 µm. Treating a red blood cell as a cylinder, calculate the volume of one red blood cell. **(2)**

b Each red blood cell contains 2.8×10^8 haemoglobin molecules. How many oxygen molecules can each red blood cell carry? **(2)**

c i A human has 25 000 000 000 000 red blood cells. Write this number in standard form. **(1)**

ii What is the theoretical maximum number of oxygen molecules that can be carried at any one time in a human body? **(2)**

iii Explain why this theoretical value would never be actually reached. **(2)**

d i The average eukaryote nucleus has a diameter of 6 µm. Assuming that the nucleus is a sphere, calculate the volume of a typical nucleus. **(2)**

ii Use this value to explain why red blood cells do not have a nucleus. **(2)**

A-level only questions

1 The graph below shows the growth curve of a bacteria population.

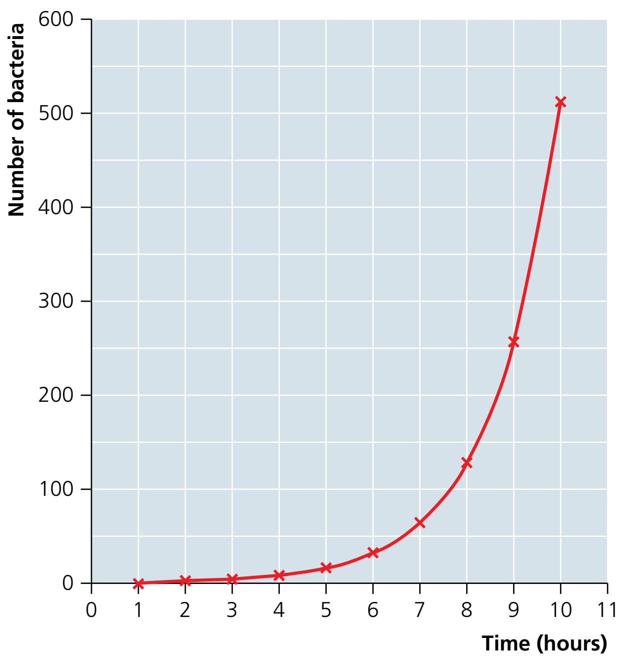

Figure E.1

a If this graph had been plotted with a logarithmic scale, how would its shape differ? **(1)**

b The growth of another bacterial population can be estimated using the equation

$$A = Pe^{rt}$$

where

- A is the number of bacteria in the population at any given time t
- P is the number of bacteria in the initial population
- r is the growth constant
- t is time

The initial population of bacteria was 10 cells, and the growth constant is 1 h^{-1}. After 50 hours how many bacteria would there be in the sample? Give your answer to two significant figures and in standard form. **(2)**

2 **a** A certain species of flower has three different petal colours: red, yellow and orange. A red-flowered plant was crossed with a yellow-flowered plant and produced 100% orange-flowered plants. When these offspring plants were bred together, they produced the following:

- 102 orange-flowered plants
- 55 yellow-flowered plants
- 47 red-flowered plants

Are these results significantly different from the expected results? **(8)**

b A sample of the same species of plant in the wild found a percentage cover of 60% of a 0.25 m^2 quadrat. What was the area covered in square metres? **(2)**

3 In an investigation into the effect of different solute potentials on plant tissue, potato cores were placed in solutions of various solute potentials for a set period of time, and their initial and final masses were recorded.

a The potato cores were shaped like cylinders with a length of 3 cm and a diameter of 0.7 cm. Find the surface area and volume of each of the cylinders. **(3)**

b The table below shows the data collected on the masses of the potato cylinders in different solute potentials.

Table E.3

Solute potential of external solution (kPa)	Initial mass (g)	Final mass (g)
0	2.1	2.310
−200	1.9	2.014
−400	2.0	2.040
−600	1.8	1.764
−800	2.1	1.974
−1000	1.8	1.620

i Calculate the percentage change in mass at each of the concentrations. **(2)**

ii Why is the percentage change in mass a more appropriate variable to use in this investigation than the change in mass in grams? **(1)**

iii Plot the results from **i** on an appropriate graph. **(3)**

iv Determine the solute potential at which no change in mass occurred. Explain how you arrived at your answer. **(2)**

v At what solute potentials would the potato cells be:

- turgid
- plasmolysed? **(2)**

vi Explain why samples of animal tissue would be unsuitable for use in this investigation. **(2)**

vii At what points on the graph would the water potential of the cells be equal to the solute potential? Explain your answer. **(2)**

4 In a study, the effect of a faulty allele of a gene was tested using 'knockout' mice. The allele was intentionally deactivated in some of the mice, mimicking the effect of the faulty allele. In a population of 86 mice, 15 showed a recessive trait caused by the deactivated allele of the gene.

a Use the Hardy–Weinberg equation, $p^2 + 2pq + q^2 = 1$, to determine the frequency of heterozygotes in the population. **(3)**

b Genetic testing showed that 44 of the mice were heterozygotes. Comment on this value. **(3)**

c The diagram below shows the effect of a drug that overcomes the negative effect of the deactivated allele. What conclusion can be drawn from this data? **(3)**

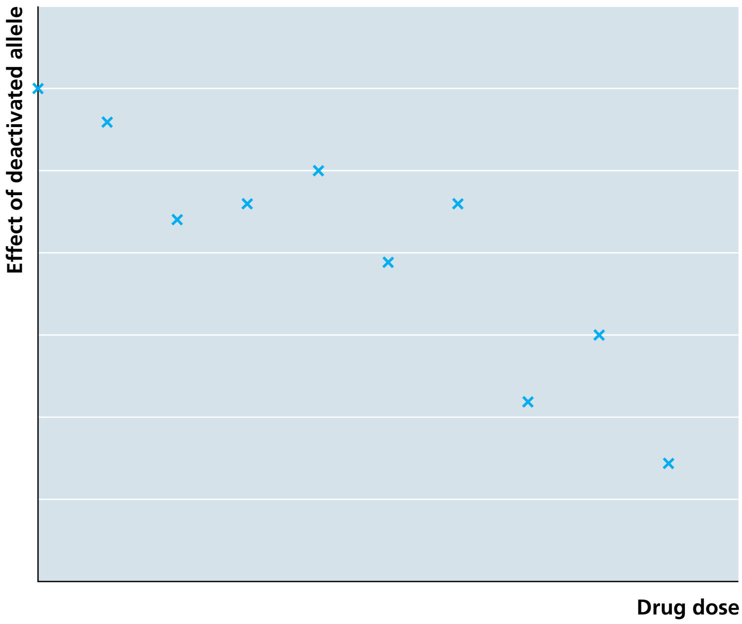

Figure E.2

5 The diagram below shows the flow of energy through an ecosystem.

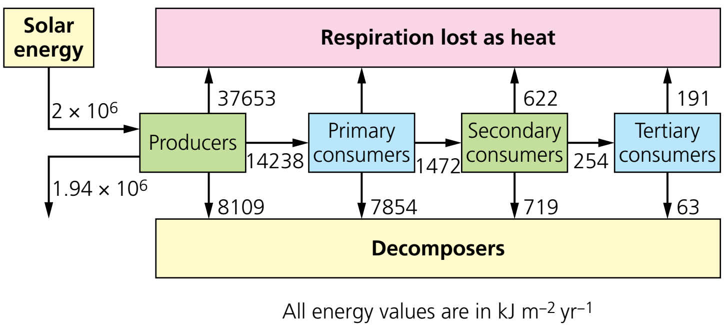

Figure E.3

a Calculate:

i the energy lost from the second trophic level by respiration. **(1)**

ii the gross primary productivity of the producers. **(1)**

iii the photosynthetic efficiency of the producers. **(2)**

iv the trophic efficiency of the primary consumers. **(1)**

v the trophic efficiency of the secondary consumers. **(1)**

b Give an explanation for the difference in the values you calculated in **iv** and **v**. **(2)**

Appendix

All the mathematical skills covered in this book are relevant for the following UK exam boards: AQA, Edexcel, OCR, WJEC/Eduqas and CCEA. The table below shows which mathematical skills are examined at A-level only (rather than AS) and also indicates the A-level only biological topics appearing in the book, which AS students need not study. The information is intended as a guide only. You should refer to your specification for full details of the topics you need to know.

Topic	A-level only maths skill	A-level only biology topic
Appropriate units in calculations		Energy flow: Practice Q7 p.8
Expressions in decimal and standard form		
Ratios, fractions and percentages		Ratios in genetic crosses: Worked example **d** p.13 Practice Q7 p.14 Practice Q8 p.14
Estimating results		
Power, exponential and logarithmic functions	✓	
Significant figures		
Arithmetic means		
Interpreting tables and diagrams		Photosynthesis: Practice Q4 p.27 Kidney function: Practice Q6 p.28
Simple probability		Genetic crosses: Worked example **b** p.29 Guided Q1 pp.29–30 Ratios in genetic crosses: Guided Q2 p.30 Practice Q5 p.31 Practice Q6 p.31
Principles of sampling		
Mean, median and mode		
Scatter diagrams		
Order of magnitude		
Statistical tests		Chi-squared analysis of genetic crosses: Worked example **a** pp.46–48 Practice Q1 p.53
Measures of dispersion		
Uncertainties in measurements		
Algebraic equations		
Changing the subject of an equation		Photosynthetic efficiency: Guided Q2 p.66 Net primary productivity: Practice Q4 p.66 Hardy–Weinberg equation: Practice Q8 p.67
Logarithms	✓	
Bar charts, histograms and line graphs		
$y = mx + c$		

Determining the intercept of a graph	✓	
Calculating the rate of change for a linear relationship		
Measuring the rate of change at a point on a curve		
Circumferences, surface areas and volumes of regular shapes		