P425/2
APPLIED MATHEMATICS
Paper 2
Nov./ Dec. 2020
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B.**

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms $^{-2}$.

SECTION A: (40 MARKS)

*Answer **all** the questions in this section.*

1. The position vector of a particle at any time t seconds is given by
$$r(t) = (t^2 + 4t)\,i + (3t - t^3)\,j \text{ metres.}$$
   Calculate the speed of the particle when $t = 3$ seconds. *(05 marks)*

2. The table below shows the values of x and the corresponding values of a function $f(x)$.

x	0.3	0.6	0.9	1.2
$f(x)$	3.00	3.22	3.69	4.06

   Use linear interpolation to find;
   (a) $f(x)$ when $x = 0.4$ *(03 marks)*
   (b) x when $f(x) = 3.82$ *(02 marks)*

3. The table below shows the price indices of beans, maize, rice and meat with the corresponding weights.

Item	Price index 2008 (2007 = 100%)	Weight
Beans	105	4
Maize	x	7
Rice	104	2
Meat	113	5

   Calculate the;
   (a) value of x given that the price indices of maize in 2007 and 2008 using 2006 as the base year are 112 and 130 respectively. *(02 marks)*
   (b) weighted price index for 2008 using 2007 as the base year. *(03 marks)*

4. A particle moves with simple harmonic motion (SHM) about a mean position O with periodic time of $\dfrac{2\pi}{3}$ seconds. When the particle is 0.8 m from one extreme end, its speed is 3.6 ms^{-1}. Determine the amplitude of the motion. *(05 marks)*

5. The numbers $X = 1.2$, $Y = 1.33$ and $Z = 2.245$ have been rounded off to the given decimal places. Find the maximum possible value of
$$\frac{Y}{Z - X}$$
   correct to **three** decimal places. *(05 marks)*

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6. Two events are such that $P(A) = 0.7$, $P(B) = 0.2$ and $P(A/B) = 0.1$.
Find:

(a) $P(A \cup B)$. (03 marks)

(b) $P(A \cap B')$. (02 marks)

7. A particle of weight 20 N is placed on a rough plane inclined at an angle of $40°$ to the horizontal. The coefficient of friction between the plane and the particle is ¼. When a horizontal force P is applied on the particle, it rests in equilibrium. Calculate the value of P. (05 marks)

8. A mobile phone dealer imports Nokia and Motorola phones. In a given consignment, 55% were Nokia and 45% were Motorola phones. The probability that a Nokia phone is defective is 4%. The probability that a Motorola phone is defective is 6%. A phone is picked at random from the consignment. Determine the probability that it is;

(a) defective. (03 marks)

(b) a Motorola given that it is defective. (02 marks)

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. The table below shows marks obtained by 100 students in a Mathematics test.

Marks	Number of students
$20 - <40$	5
$40 - <50$	15
$50 - <55$	10
$55 - <60$	15
$60 - <70$	25
$70 - <90$	25
$90 - <100$	5

(a) Calculate the mean mark. (05 marks)

(b) Draw a cumulative frequency curve (Ogive) and use it to find the;

(i) median mark.

(ii) range of the middle 40% of the marks.

(07 marks)

Turn Over

0. Two bodies A and B of masses 3 kg and 2 kg respectively are 7 m apart on a smooth horizontal surface. A is moving directly towards B with a speed of 2 ms^{-1} and an acceleration of 0.3 ms^{-2}. B is moving in the same direction as A with a speed of 5 ms^{-1} and a retardation of 0.2 ms^{-2}. If the bodies collide and coalesce, calculate the;

(a) time taken before collision occurs. *(08 marks)*

(b) common velocity immediately after the collision. *(04 marks)*

1. (a) Use the trapezium rule with 6-ordinates to estimate

$$\int_{0.1}^{0.5} \frac{1}{2x+1}\,dx$$

correct to **three** significant figures. *(06 marks)*

(b) Evaluate $$\int_{0.1}^{0.5} \frac{1}{2x+1}\,dx$$

correct to **three** significant figures. *(02 marks)*

(c) (i) Determine the percentage error in the estimation in (a) above, correct to **two** decimal places.

(ii) Suggest how the percentage error may be reduced. *(04 marks)*

2. A continuous random variable X has a cumulative distribution function given by

$$F(x) = \begin{cases} 0; & x \le 0 \\ \dfrac{k}{2}x^2; & 0 \le x \le 2 \\ k(6x - x^2 - 6); & 2 \le x \le 3 \\ 1; & x \ge 3 \end{cases}$$
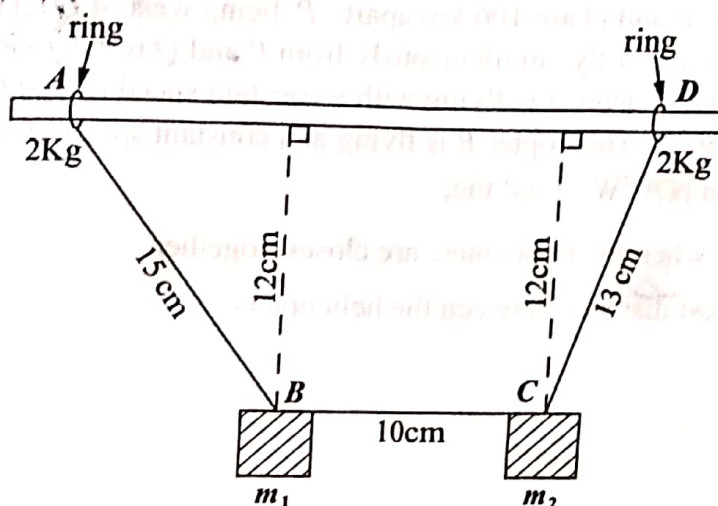
(a) Determine the value of k.
Hence sketch the graph of $F(x)$. *(08 marks)*

(b) Find the probability density function (pdf) of X. *(04 marks)*

$$\int \frac{1}{2x+1}\left[\frac{x^{1}+1}{2}\right]$$

1·0500
1·0900
1·1300
1·1700
1·2100
1·2500

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13. The diagram below shows three strings $AB = 15$ cm, $BC = 10$ cm and $CD = 13$ cm. A and D are fixed to small rings each of mass 2 kg which can slide on a rough horizontal rail AD. Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance of 12 cm below AD.

(a) Show that $9m_1 = 5m_2$. (07 marks)

(b) If the coefficient of friction between each ring and the rail is ¼ and the ring at A is on the point of slipping, determine the value of m_1. (05 marks)

14. (a) (i) Draw on the same axes the graphs of $y = x \sin x$ and $y = e^x - 2$ for $0.5 \leq x \leq 1.5$.

(ii) Use your graphs to find an approximate root of the equation $2 - e^x + x \sin x = 0$. (06 marks)

(b) Using Newton Raphson iterative formula and your approximate root in a (ii) above as the initial value, calculate the root of the given equation correct to **three** decimal places. (06 marks)

15. A certain football team has three matches to play. The probabilities of winning the first, second and third matches are $\frac{3}{5}, \frac{2}{5}$ and $\frac{1}{5}$ respectively.

(a) Find the probability that the team wins;

(i) exactly two matches.
(ii) all matches.
(iii) no match. (07 marks)

(b)   If a random variable X is defined as "the number of matches won",

      (i)   construct a probability distribution table for X.

      (ii)   calculate the expectation of X, $E(X)$.

                                    (05 marks)

16.   Two airstrips P and Q are 100 km apart, P being west of Q. Two helicopters A and B fly simultaneously from P and Q respectively, at 11.00 a.m. Helicopter A is flying with a constant speed of 400 kmh^{-1} in a direction N50°E. Helicopter B is flying at a constant speed of 500 kmh^{-1} in the direction N70°W. Find the;

(a)   time when the helicopters are closest together.   *(08 marks)*

(b)   closest distance between the helicopters.   *(04 marks)*

END