Section A.

Attempt **All** questions in this section.

1. Solve the simultaneous equations.

$x - 2y + 3z = 6$

$3x + 4y - z = 3$

$4x + 6y - 5z = 0$

2. Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ for $0 \le \theta \le 2\pi$.

3. Differentiate $x(10)^{\sin x}$ with respect to x.

4. Show that $\log_8 x = \frac{2}{3}\log_4 x$. Hence without using tables or calculator, evaluate $\log_8 6$ correct to 3 d.p, if $\log_4 3 = 0{\cdot}7925$

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,dx$

6. Show that the line $x - 2y + 10 = 0$ is a tangent to the ellipse $9x^2 + 64y^2 = 676$.

7. The rate of growth of bacteria in a culture is proportional to the population present at time **t**. The population doubles everyday. Given that the initial population $\mathbf{P_o}$ is one million, determine the day when the population will be one hundred million.

8. Show that the equation of the line through points (1, 2, 1) and (4, −2, 2) is given by $\frac{x-1}{3} = \frac{y-2}{-4} = z-1$.

Section B.

Attempt **only Five** questions in this section.

All questions carry **Equal** marks.

9(a). The n^{th} term of a series is $U_n = \mathbf{a}3^n + \mathbf{b}n + \mathbf{c}$. Given that $U_1 = 4$, $U_2 = 13$ and $U_3 = 46$. Find a, b and c.

(b). If α and β are roots of $x^2 - px + q = 0$. Find the equation whose roots are $\frac{\alpha^3 - 1}{\alpha}$ and $\frac{\beta^3 - 1}{\beta}$.

10. Expand $\sqrt{\dfrac{1+3x}{2-x}}$ up to the third term , hence by putting $x=\dfrac{1}{5}$,

approximate $\sqrt{8}$ (correct to 4 s.f).

(b). Solve the equation $\sqrt{(3-x)} - \sqrt{(7+x)} = \sqrt{(16+2x)}$.

11(a). Point P is twice as far from line $x+y=5$ as from point (3,0). Find the locus of P.

(b). T is a variable point given by the parametric equations ;

$x=\dfrac{1}{2}a\left(t+\dfrac{1}{t}\right)$ and $y=\dfrac{1}{2}b\left(t-\dfrac{1}{t}\right)$. Show that the locus of T is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}=1$.

12(a). Show that equation of a plane through point with position vector $-2\mathbf{i}+4\mathbf{k}$ perpendicular to the vector $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ is

$x+3y-2z+10=0$.

b(i). Show that the vector $2\mathbf{i}-5\mathbf{j}+3{\cdot}5\mathbf{k}$ is perpendicular to line

$\mathbf{r}=2\mathbf{i}-\mathbf{j}+\lambda(4\mathbf{i}+3\mathbf{j}+2\mathbf{k})$.

(ii). Calculate the angle between vector $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and the line in b(i) above .

13(a). Solve $\cot^2\phi=5(\operatorname{cosec}\phi+1)$ for $0^\circ \le \phi \le 360^\circ$.

(b). Use the substitution of $\tan\dfrac{\theta}{2}=t$ to solve $5\cos\theta-2\sin\theta=2$

for values of θ from 0° up to 360° .

14. Express $f(x) = \dfrac{6x}{(x-2)(x+4)^2}$ as partial fractions .

Hence evaluate $\int f(x)\,dx$.

15. Show that the tangent to the curve $4-2x-2x^2$ at points $(-1,4)$ and $\left(\dfrac{1}{2}, 2\dfrac{1}{2}\right)$ respectively pass through the point $\left(-\dfrac{1}{4}, 5\dfrac{1}{2}\right)$.

Calculate the area of the curve enclosed between the curve and the x-axis .

16(a). An inverted cone with vertical angle of 60° is collecting water leaking from a tap at a rate of $0{\cdot}2\ \text{cm}^3\text{s}^{-1}$. If the height of the water collected in the cone is 10cm, find the rate at which the surface area of water is increasing.

(b). Given that $y = e^{\tan x}$, show that $\frac{d^2y}{dx^2} - (2\tan x + \sec^2 x)\frac{dy}{dx} = 0$

" **Behold I stand ...and knock , if any man...opens...,I will come in......**" [**Revelation** 3:20]

END: djco Exams @ 21