P425/2
APPLIED MATHEMATICS
Paper 2
July, 2023
3 hour

Uganda Advanced Certificate of Education

INTERNAL MOCK EXAMINATIONS, 2023

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Attempt all the **eight** questions in section* **A** *and* **five** *questions from section* **B**.

Any additional question(s) answered will not be marked.

All working **MUST** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used. In numerical work, take $g=9.8ms^{-2}$

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. A force $(3\boldsymbol{i} - 2\boldsymbol{j} + 8\boldsymbol{k})$ N acts on a body of mass 4kg initially at the origin. If the velocity is $(2t\boldsymbol{i} + 3\boldsymbol{j})$ ms^{-1}, find the work done after 4 seconds.

(05 marks)

2. The temperature (^{o}C) of a liquid measured at an interval of 2 minutes were Recorded as 55 and 52. If the initial temperature is 60, use linear Interpolation or extrapolation to find;

(i) Temperature after 5 minutes,

(ii) Time taken if the temperature is 53.5^{0}C.

3. A random sample of 200 people were asked the length of time they spent in the shower, the last time they took one. The results were as follows:

$\sum x = 909, \quad \sum x^2 = 4555.$

(a) Calculate the unbiased estimate of the population variance. *(02 marks)*

(b) Determine the 97.5% confidence limits for the mean time spent in the shower. *(03 marks)*

4. To a dove flying eastwards at 3ms^{-1} an eagle appears to be flying North East wards at 4ms^{-1}. Find the true velocity of the eagle. *(05 marks)*

5. The numbers $x = 4.2,\ y = 16.02$ and $z = 25$ are rounded off with Corresponding percentage errors of 0.5, 0.45 and 0.02, Calculate the absolute error made in $\frac{xy}{z}$. *(05 marks)*

6. A and B are two independent events with A twice as likely to occur as B. if P(A) = ½ , find:

(i) P(A or B but not both), *(03 marks)*

(ii) P(A/B'). *(02 marks)*

7. Forces of magnitude 90N and 60N act on a particle at an angle of 35^0 to each

Other. Determine the magnitude and direction of the resultant force.*(05 marks)*

8. The probability that a certain function starts early is $^4/_7$. If the function starts early, the probability that it takes a longer time is $^2/_5$. If the function starts late, the probability that it takes a shorter time is $^1/_3$. Find the probability that function;

(i) Takes a shorter time, *(03 marks)*

(ii) Starts early if it takes a shorter time *(02 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.*

All *questions carry equal marks*

9. The table below shows the marks obtained by students in Fine Art(x) and Mathematics (y).

Fine Art (x)	80	76	96	41	68	31	42	88	68	91
Mathematics (y)	43	32	27	64	65	64	65	32	64	43

(a) Draw a scatter diagram for the above data and on it draw a line of best fit. Use the line of best fit to estimate the mark of a student who scored;

(i) 61 in mathematics,

(ii) 25 in Fine Art *(07 marks)*

(b) Calculate a rank correlation coefficient between the students' performance in the two subjects and comment on your result at 1% level of significance. *(05 marks)*

10. A particle is projected with a speed of $36ms^{-1}$ at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 meters on the horizontal plane from the point of projection.

Find the;

(a) (i)Time taken for the particle to reach the wall.

(ii) Height of the wall. *(08 marks)*

(b) Maximum height reached by the particle from the point of projection. *(04 marks)*

11. (a) Use the trapezium rule with six ordinates to find the approximate value of $\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4\right) dx$, correct to three significant figures, (05 marks)

(b) Evaluate $\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4\right) dx$ correct to three significant figures.

(c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places. *(03 marks)*

(ii) Suggest how the percentage error may be reduced. *(01 mark)*

12. The continuous random variable X has probability density function (p.d.f) given by; $f(x) = \begin{cases} (4x - 4x^3); & 0 \leq x \leq 1 \\ 0 & ; otherwise \end{cases}$

Find the

(a) Mode *(03 marks)*

(b) Cumulative distribution function of *x*, *(03 marks)*

(c) P $(0.1 < x < 0.6)$ *(02 marks)*

(d) Median of *x* *(04 marks)*

13. Forces of magnitude 4N, 5N, 5N, 4N and 6N act along the lines **AB, BC**, **CD, DA** and **AC** respectively of the square **ABCD** whose side has a length of a units. The direction of the forces are indicated by the order of the letters.

(a) Find the magnitude and direction of the resultant force. (09 marks)

(b) If the line of action of the resultant force cuts **AB** produced at E, find the length **AE**. *(03 marks)*

14. (a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = ln3$ it satisfies

$$x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}, \text{ n = 0, 1, 2…} \quad \textit{(04 marks)}$$

(b) (i) Construct a flow chart that;

- reads the initial approximation as x_0
- Computes, using the iterative formula in (a), and prints the root of the equations $3x = in3, to$ 4 significant figures.

(ii) Perform a dry-run for your flow chart for $x_0 = {}^1/_3$ *(08 marks)*

15. (a) A machine cuts poles whose lengths are normally distributed with mean 4.2 m and standard deviation 1.2m. If a random sample of 100 poles is selected, find the probability that a pole selected at random has its mean length in the range 4.0m to 4.3m *(04 marks)*

(c) It is known that 30% of an apple crop plantation has been attacked by pests. A random sample of 150 apples is selected from the plantation; estimate the probability that;

(i) Less than 20% of the sampled apples are damaged, *(05 marks)*

(ii) Number of damaged apples is between 35 and 50 inclusive. *(03 marks)*

16. A force $\mathbf{F} = (2t\boldsymbol{i} + \boldsymbol{j} - 3t\boldsymbol{k})$ N acts on a particle of mass 2kg. The particle is initially at a point $(0,0,0)$ and moving with a velocity $(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ ms^{-1}.

Determine the;

(a) Magnitude of the acceleration of the particle after 2 seconds. *(04 marks)*

(b) Velocity of the particle after 2 seconds. *(04 marks)*

(c) Displacement of the particle after 2 seconds. *(04 marks)*

END