P425/2 APPLIED MATHEMATICS Paper 2 Sept. 2023 3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS (PRINCIPAL SUBJECT) SET 2

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer all the Eight questions in Section A and Five questions from Section B. Any additional question(s) answered will not be marked.

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2} .

SECTION A: (40 MARKS)

Answer all the questions in this section

- 1. A good football striker is nursing his injury on the leg. The probability that his team will win the next match when he is playing is $\frac{4}{5}$, otherwise it is $\frac{2}{3}$. The probability that he will have recovered by the time of the match is $\frac{1}{4}$. Find the probability that;
 - (i) his team will win the match.
 - (ii) Will have recovered given that they win the match
- 2. A particle of mass 2kg is initially moving with a velocity i 4j, its acted upon by a force F = 4i + tj. Find the velocity of the particle at t = 2s.
- 3. Use the trapezium rule to estimate $\int_0^{\frac{\pi}{3}} e^x \cos x \, dx$, using six ordinates correct to 3 decimal places.
- 4. A discrete random variable x has cumulative distribution function F(x) as indicated below

X	1	2	3	4	5
F(x)	0.2	0.32	0.67	0.9	1

Determine the;

(i) Mean E(x)

- (ii) P(X > 2)
- 5. A 2kg body lies on a plane of inclination 60°. The coefficient of friction between the body and the plane is 0.25. find the least horizontal force which prevents the body from sliding down the plane.
- 6. The table below is an extract from tables of $sin\theta^{\circ}$

10°	0'	6'	12'	18'		
sin10°	0.1736	0.1754	0.1771	0.1788		

Use linear interpolation or extrapolation to estimate

(i) $sin 10^{\circ} 16'$

- (ii) θ° when $\sin \theta^{\circ} = 0.18$
- 7. Eight schools participated in athletics competition in 2019 in a certain district and obtained the following scores in 200m and 400m

200m	48	45	35	45	42	15	45	33
400m	68	54	48	58	75	40	60	57

Calculate the rank correlation coefficient and comment at 5% level of significance

- 8. A body of mass 16kg is released from rest on the surface of a smooth plane whose slope is 1 in 40, the resistance to motion being $\frac{1}{8}N$ per kg, acting up the plane. Determine the;
 - acceleration of the body down the plane (i)
 - time the body takes when velocity is 0.6ms⁻¹ after release. (ii)

SECTION B: (60 MARKS)

Answer any five questions in this section. All questions carry equal marks

- 9. A machine fills a large bag of sand, the weights of which are normally distributed with mean μ and variance σ^2 . It is known that 5% of bags are heavier than 115.515kg, while 33% are lighter than 96.92kg. Determine the;
 - (a) Value of mean, μ and standard deviation, σ
 - (b) Probability that the bags weigh less than 96kg
- 10. The position vectors of two particles at any time (t) are given by;

$$r_1 = (4i - 2j)t + (3i + j)t^2$$
 and $r_2 = (10i + 4j) + (5i - 2j)t$

- $r_1 = (4i 2j)t + (3i + j)t^2$ and $r_2 = (10i + 4j) + (5i 2j)t$. (a) Show that the two particles will collide, state the time of collision and find the position of collision
- (b) Determine the speeds at the time of collision
- 11.(a) By plotting graphs of $y = e^{-2x}$ and y = sinx on the same axes show that there is root between 0 and 1.4 correct the root (x_0) to one decimal place
 - (b) Hence using (x_0) the initial approximation and Newton Raphson method, find the root correct it to 3 decimal places
- 12. A continuous random variable x has probability density function below;

$$f(x) = \begin{cases} 2k(x+1); & -1 < x < 0 \\ k(2-x); & 0 < x < 2 \\ 0; & otherwise \end{cases}$$

where k is a constant. Determine:

- (a) Value of k
- (b) Mean, E(x) and standard deviation, δ
- (c) Cumulative distribution function F(x)

- 13.(a) Initially a particle is projected with a constant velocity of 2i ms⁻¹, from a point with position vector 10i + 90j. Find the position of the particle from origin after 4 seconds
 - (b) A man kicks a ball with a velocity of 52ms⁻¹ at an angle $\theta = tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. Determine the:
 - (i) times when the ball is 15m above the ground level
 - (ii) horizontal distance covered by the ball between the two times in (i) above
- 14.(a) The numbers x = 26.23, y = 13.18 and z = 5.1 are calculate with percentage errors 4, 3 and 2 respectively. Find the errors in x, y and z, hence the limits within which the exact value of the expression $xy \frac{y}{z}$ lies correct to 3 decimal places
 - (b) Hence find the absolute error, relative error and percentage error in the above expression
- 15. The table below shows the marks obtained by students of principal mathematics in a certain school

Marks	Frequency		
$0 < x \le 10$	3		
$10 < x \le 20$	6		
$20 < x \le 30$	9		
$30 < x \le 40$	10		
$40 < x \le 50$	12		
$50 < x \le 60$	18		
$60 < x \le 70$	14		
$70 < x \le 80$	11		
$80 < x \le 90$	7		

- (a) Calculate the;
 - (i) Mean mark

- (ii) Standard deviation
- (b) Construct an ogive and use it to estimate median mark
- 16. Forces of magnitude 7N, 6N, 10N, 13N, and 15N act along the lines BA, BC, DC, DA and AC respectively of a rectangle, where AB = 3a and AD = 4a, if AB and AD are the x and y axes respectively, find the:
 - (a) Magnitude and the direction of the resultant force
 - (b) Line of action and point where the resultant force crosses AB by taking moments about point A