P425/1
Pure math
Paper 1
3 hours

BEGINNING OF TERM ONE 2023
UGANDA ADVANCED CERTIFICATE OF EDUCATION
PURE MATHEMATICS

INSTRUCTIONS TO CANDIDATES:

- Attempt **all** the eight questions in section A and any **five** questions from section B.
- Clearly show all the necessary working
- Begin each answer on a fresh sheet of paper
- Silent, simple non-programmable scientific calculators may be used.

SECTION A (40 MARKS)

1. Solve for x, given $\log_2(11-6x) = 2\log_2(x-1)+3$.

2. Differentiate $\dfrac{1}{\sqrt{\cos x}}$ from first principles.

3. Solve: $4\cos 2x + 3\sin 2x = 3$ for $0^o \le x \le 360^o$.

4. The lines L_1 and L_2 are given by the equations $\dfrac{x-3}{k} = y-4 = \dfrac{z-4}{-k}$ and $\dfrac{x-8}{1} = \dfrac{y-1}{3} = \dfrac{z-3}{3}$ respectively. Find the value of k for which L_1 and L_2 intersect and hence, find the point of intersection.

5. The length of a rectangular block is three times its width. If the total surface area of the block is $180\,cm^2$, find the maximum volume.

6. Prove by induction: $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$

7. Evaluate: $\displaystyle\int_2^6 \frac{\sqrt{x-2}}{x}\,dx$

8. Show that $3x^2+2y^2+6x-8y=7$ is an ellipse and determine the coordinates of its centre and foci.

SECTION B (60MARKS)

9a) The roots of the equation $3x^2-2x+24=0$ are 2α and 2β. Find an equation whose roots are α and β.

b) Find the term independent of x: $\left(3x-\frac{2}{x}\right)^9$

c) Evaluate: $\log_8 2+\frac{1}{2}\log_4 8$

10a) Find the equations of the tangent and normal to the curve $x^2+3y^2=2a^2$ at the point $\left(a,\ \frac{a}{\sqrt{3}}\right)$.

b) Show that the gradient of the curve $y=x(x-3)^2$ is zero at the point $P(1,\ 4)$ and sketch the curve. If the tangent at P cuts the curve again at Q, calculate the area contained between the chord PQ and the curve.

11a) Given that $z=5-2i$, find the modulus of $z^*-\frac{3}{z}$ where z^* is the conjugate of z.

b) Given that $\left|\frac{3z+1}{2z-i}\right|=\sqrt{2}$, find the locus of z and describe the locus.

12a) The line $\frac{x+1}{2}=\frac{y+3}{a}=\frac{z+2}{3}$ lies on the plane $x+2y+bz=3$, find the values of a and b.

b) A line and a plane are given by the equations $\frac{x+1}{2}=\frac{y-3}{5}=\frac{z+2}{-1}$ and $2x-y+3z=20$ respectively. Determine:

i) the point of intersection of the line and the plane.

ii) the acute angle between the line and the plane.

13a) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, hence, find the exact value of $\cos 3\theta$ if $\cos\theta = -\frac{2}{3}$.

b) Solve: $3\sin 2x + 4\cos^2 x = -1$ for $0^o \leq x \leq 180^o$.

14a) If $y = \frac{3\sin 2x + 4\cos 2x}{2x+1}$, show that $(2x+1)\frac{dy}{dx} + 2y = 10\cos(2x+\alpha)$.

b) Differentiate the following:

i) $y = x^2 \sin\left(\frac{1}{x}\right)$ ii) $y = xInx^3$

15a) The line $y = ax + 3$ is a tangent to the parabola $y^2 = 4ax$, find the value of a.

b) A tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2,\ 2ap)$ meet the directrix at point Q. Point R is the foot of the perpendicular from the vertex to the tangent.

i) Show that SP and SQ are perpendicular.

ii) Find the locus of the mid pointof OR.

16a) Solve the differential equation. $\frac{dy}{dx} = (y-3)(4x+3)$ given $x = -1$ and $y = 3\left(\frac{1}{e}+1\right)$.

b)

END