


## Linear interpolation and extrapolation

### Linear interpolation

Deals with computations of values that lie within a given range

***Example 1***

The table below shows values of a function f(x)

| x | 1.8 | 2.0 | 2.2 | 2.4 |
|---|---|---|---|---|
| f(x) | 0.532 | 0.484 | 0.436 | 0.384 |

Find values of (i) f(1.88) (ii) x corresponding to f(x) = 0.4

Solution

| 1.8 | 1.88 | 2.0 |
|---|---|---|
| 0.532 | y | 0.484 |

$\frac{y-0.532}{1.88-1.8} = \frac{0.884-0.532}{2.0-1.8}$

y = 0.513

(ii)

| 2.2 | $x_0$ | 2.4 |
|---|---|---|
| 0.436 | 0.4 | 0.384 |

$\frac{x_0-2.2}{0.4-0.436} = \frac{2.4-2.2}{0.384-0.436}$

$x_0$ =2.34

***Example 2***

Given the table below

| x | 9 | 10 | 11 | 12 |
|---|---|---|---|---|
| f(x) | 2.66 | 2.42 | 2.18 | 1.92 |

Using linear interpolation find

(i) f(x) when x = 10.15 (ii) $f^{-1}$ (2.02)

Solution

| 10 | 10.15 | 11 |
|---|---|---|
| 2.42 | y | 2.18 |

$\frac{y-2.42}{10.15-10} = \frac{2.18-2.42}{11-10}$

y =2.384

| 11 | $x_0$ | 12 |
|---|---|---|
| 2.18 | 2.02 | 1.92 |

$\frac{x_0-11}{2.02-2.18} = \frac{12-11}{1.92-2.18}$

$x_0$ = 11.62

***Example 3***

Given table below

| $x^0$ | $40.0^0$ | $40.4^0$ | $40.8^0$ | $50.4^0$ |
|---|---|---|---|---|
| sin $x^0$ | 0.6428 | 0.6481 | 0.6534 | 0.7705 |

Find (i) sin $40.5^0$ (ii) $\sin^{-1}$ 0.6445

Solution

| $40.4^0$ | $40.5^0$ | $40.8^0$ |
|---|---|---|
| 0.6481 | y | 0.6534 |

$$\frac{y-0.6481}{40.5-40.4} = \frac{0.6534-0.6481}{40.8-40.4}$$

y = 0.6494

| $40.0^0$ | $x_0$ | $40.4^0$ |
|---|---|---|
| 0.6428 | 0.6445 | 0.6481 |

$$\frac{x_0-40.0}{0.6445-0.6428} = \frac{40.4-40.0}{0.6481-0.6428}$$

$x_0$ = 40.13

**Linear extrapolation**

This deals with computation of values that lie outside given values

***Example 4***

Given the table below

| x | 2.2 | 2.6 | 3.1 |
|---|---|---|---|
| $x^3$ | 10.648 | 17.576 | 29.791 |

Find $3.4^3$

| 2.6 | 3.1 | 3.4 |
|---|---|---|
| 17.576 | 29.791 | y |

$$\frac{y-29.791}{3.4-3.1} = \frac{29.791-17.576}{3.1-2.6}$$

y = 37.12

***Example 5***

The table below is an extract from table of sec x

| x = $60^0$ | 0’ | 12’ | 24’ | 36’ | 48’ |
|---|---|---|---|---|---|
| $sec\ x$ | 2.0000 | 2.0122 | 2.0245 | 2.0371 | 2.0498 |

Use linear interpolation to determine

(i) sec $60^0$15’

(ii) angle whose secant is 2.0436 [$60^0$42’]

Solution

(i)

| 12’ | 15’ | 24’ |
|---|---|---|
| 2.0122 | y | 2.0245 |

$$\frac{y-2.0122}{15-12} = \frac{2.0245-2.0122}{24-12}$$

y = 2.03065

(ii)

| 36’ | x | 48’ |
|---|---|---|
| 2.0371 | 2.0436 | 2.0498 |

$$\frac{x-36}{2.0436-2.0371} = \frac{48-36}{2.0498-2.0371}$$

x = 42’ hence angle = $60^0$42’

***Example 6***

The table below shows the values of a function f(x)

| x | 1.8 | 2.0 | 2.2 | 2.4 |
|---|---|---|---|---|
| f(x) | 0.532 | 0.484 | 0.436 | 0.384 |

Use linear interpolation to find the value of

(i) F(2.08)

| 1.8 | 2.08 | 2.0 |
|---|---|---|
| 0.532 | f(x) | 0.484 |

$$\frac{f(x)-0.436}{2.08-2.0} = \frac{0.436-0.484}{2.2-2.0}$$

$$\frac{f(x)-0.436}{0.08} = \frac{-0048}{0.2}$$

f(x) = 0.4648 or 0.465 (3D)

(ii) x corresponding to f(x) = 0.5 (05marks)

| 1.8 | x | 2.0 |
|---|---|---|
| 0.532 | 0.5 | 0.484 |

$$\frac{0.5-0.532}{x-1.8} = \frac{0.484-0.532}{2.0-18}$$

$$\frac{-0.032}{x-1.8} = \frac{-0.048}{0.2}$$

$x = 1.9333$ or 1.9 (1D)

**Example 7**

Given the table below,

| x | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| y | 6.6 | 2.9 | -0.1 | -2.9 |

Use linear interpolation to find

(a) y when x = 16

Extract

| x | 10 | 16 | 20 |
|---|---|---|---|
| y | 2.9 | $y_0$ | -0.1 |

$$\frac{y_0-2.9}{16-10} = \frac{-0.1-2.9}{20-10}$$

$$\frac{y_0-2.9}{6} - \frac{-3.0}{10}$$

$y_0 = 1.1$

hence when x = 16, y =1.1

(b) x when y = -1

Extract

| x | 20 | $x_0$ | 30 |
|---|---|---|---|
| y | -0.1 | -1 | -2.9 |

$$\frac{x_0-20}{-1-(-0.1)} = \frac{30-20}{-2.9-(-0.1)}$$

$x_0 = 23.2$

Hence when y = -1; x = 23.2

***Example 8***

The table below shows the values of a function f(x) for given values of x.

| x | 9 | 10 | 11 | 12 |
|---|---|---|---|---|
| f(x) | 2.66 | 2.42 | 2.18 | 1.92 |

Use linear interpolation or extrapolation to find

(a) f(10.4)

Extract

| | | |
|---|---|---|
| 10 | 10.4 | 11 |
| 2.42 | f(x) | 2.1 |

Using gradient approach

$\frac{2.18-f(x)}{11-10.4} = \frac{2.18-2.42}{11-10}$

$\frac{2.18-f(x)}{0.6} = \frac{-0.24}{1}$

f(x) = 2.18 + 0.24 x 0.6 = 2.324

(b) the value of x, corresponding to f(x) = 1.46 (05marks)

Extract

| | | |
|---|---|---|
| x | 12 | 11 |
| 1.46 | 1.92 | 2.18 |

Using gradient approach

$\frac{2.18-1.46}{11-x} = \frac{2.18-1.92}{11-12}$

$\frac{0.72}{11-x} = \frac{0.26}{-1}$

X = 11 - $\frac{-1\ x\ 0.72}{0.26}$ = 13.769

**Revision exercise**

1. Table below is an extract from the table of cos x

| x | $0^0$ | $10^0$ | $20^0$ | $30^0$ | $40^0$ | $50^0$ |
|---|---|---|---|---|---|---|
| Cos x | 0.1736 | 0.1708 | 0.1679 | 0.1650 | 0.1622 | 0.1593 |

Use linear interpolation to determine: (i) cos $80^0$ 36′ [0.1633] cos $^{-1}$(0.1685) [$80^0$18′]

2. The table below shows variation of temperature with time in a certain experiment.

| Time (s) | 0 | 120 | 240 | 360 | 480 | 600 |
|---|---|---|---|---|---|---|
| Temperature ($^0$C) | 100 | 80 | 76 | 65 | 50 | 48 |

Use linear interpolation to determine

(i) value of $^0$C corresponding to 400s [62$^0$C]

(ii) time at which the temperature is 77$^0$C [192s]

3. The table below shows the value of a function In(x) for given values of x

| x | 1.4 | 1.5 | 1.6 | 1.7 |
|---|---|---|---|---|
| In(x) | 0.3365 | 0.4055 | 0.4700 | 0.5306 |

Using linear interpolation or extrapolation, find

(i) In(1.66) [0.5064] (ii) find value of x corresponding to In(x) = 0.400 [1.492]

4. The table below shows variation of temperature with time in certain experiment.

| Time (s) | 0 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|
| Temperature ($^0$C) | 80 | 70.2 | 65.8 | 61.9 | 54.2 |

Use linear interpolation to determine

(i) value of $\theta^0$ corresponding to T= 18s [63.5$^0$C]

(ii) Time T at which the temperature $\theta^0$ = 60$^0$C [22.5s]

5. Given the table below

| x | -1.0 | -0.5 | -1.4 |
|---|---|---|---|
| y | -1.0 | -2.2 | -3.7 |

Using linear interpolation or linear extrapolation to find

(i) y when x = 0.5 [-4.6] (ii) x when y = -4.5 [ 0.458]

6. In an examination, scaling is done such that candidate A who originally scored 35% gets 50% and candidate B with 40% gets 65%, determine the original mark for candidate C whose new mark is 80% [45%]
7. The table below is an extract of $\log_{10} x$

| x | 80.00 | 80.20 | 80.50 | 80.80 |
|---|---|---|---|---|
| $\log_{10} x$ | 1.9031 | 1.9042 | 1.9058 | 1.9074 |

Using linear interpolation find

(i) $\log_{10} 80.759$[1.9072]

(ii) the number whose logarithm is 1.90388 [80.14]

8. The table below shows the values of a function f(x) for given values of x

| x | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $f(x)$ | 3.88 | 5.11 | 8.14 | 11.94 |

Use linear interpolation to determine

(i) f(2.15) [4.06]

(ii) the value of x corresponding to f(10.6) [4.68]

9. The table below shows distance in km a truck moves with a given amount of fuel in litres (l)

| Distance (km) | 20 | 28 | 33 | 42 |
|---|---|---|---|---|
| $Fuel\ (l)$ | 10 | 13 | 21 | 24 |

Use linear interpolation or extrapolation to find

(i) How far the truck can move on 27.5l of fuel [52.5km]

(ii) the amount of fuel required to cover29.8km [15.88l]

10. The table below shows the values of a continuous f(t) with respect to t

| t | 0 | 0.3 | 0.6 | 1.2 | 1.6 |
|---|---|---|---|---|---|
| $f(t)$ | 2.72 | 3.00 | 3.32 | 4.06 | 4.95 |

Use linear interpolation or extrapolation, find

(i) f(t) when t = 0.9 [3.69]

(ii) the value of t corresponding to f(t) = 4.48 [1.48]

11. The table below shows the delivery charges by courier company

| Mass (g) | 200 | 400 | 600 |
|---|---|---|---|
| $charges\ (shs.)$ | 700 | 1200 | 300 |

Use linear interpolation or extrapolation, find

(i) the delivery charge of a parcel weighing 352g [1080]

(ii) mass of a parcel whose delivery charge is shs. 3,300 [ 633.33kg]

12. The table below shows the cost y shillings for hiring a motor cycle for a distance x kilometres.

| Distance x (km) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Cost (shs.) | 2800 | 3600 | 4400 | 5200 |

Use linear interpolation or extrapolation, find

(i) the cost of hiring the motor cycle for distance of 45km [shs. 5600]

(ii) distance travelled if he paid shs. 4000 [ 25km]

13. The table below shows the values of a function f(x) for given values of x

| x | 0.4 | 0.6 | 0.8 |
|---|---|---|---|
| f(x) | -0.9613 | -0.5108 | -0.2231 |

Use linear interpolation to determine $f^{-1}$(-0.4308} correct to 2 decimal places [0.66]

14. The table below shows how T caries with S

| T | -2.9 | -0.1 | 2.9 | 3.1 |
|---|---|---|---|---|
| S | 30 | 20 | 12 | 9 |

Use linear interpolation/extrapolation to estimate values of

(a) T when S = 26 [-1.78] (ii) S when T = 3.4 [4.5]

15. The table below shows the commuter bus fare from stages A to B, C, D and E

| Stage | A | B | C | D | E |
|---|---|---|---|---|---|
| Distance (km) | 0 | 12 | 16 | 19 | 23 |
| Fare (shs) | 0 | 1300 | 1700 | 2200 | 2500 |

(a) Jane boarded from A and stopped at a place 2km after E. How much did she pay? (03marks) [shs. 2650]

(b) Okello paid shs 2000. How far from A did the bus leave him? (02marks) [17.km]

16. The table below shows the value of x and corresponding values of a function f(x)

The table below shows how T caries with S

| x | 0.3 | 0.6 | 0.9 | 1.2 |
|---|---|---|---|---|
| f(x) | 3.00 | 3.22 | 3.69 | 4.06 |

Use linear interpolation/extrapolation to estimate values of

(i) f(x) when x = 0.4 [3.0733] (ii) x when f(x) = 3.82 [ 1.0054]

Thank you

Dr. Bbosa Science