P425/2
APPLIED MATHEMATICS
Paper 2
Sept. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) SET 4

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take $\mathrm{g}\ to\ be\ 9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Events A and B are independent such that 4P(A)= 3P(B), P(AnB)= $\frac{1}{6}$. Find the P(AuB)

2. In an experiment, two fair dice numbered from 1-6 are simultaneously tossed. Determine the probability that;
   (i) A sum of either a prime number or even number is obtained
   (ii) A sum of 5 or a prime number is obtained

3. A lorry covers distances of 25.6m and 32m in the fourth and eighth second of its uniform motion respectively. Find its;
   (i) Acceleration (ii) Initial velocity

4. Use the trapezium rule with 6 ordinates to estimate $\int_1^5 5e^{-2x}\,dx$, correct to 3 significant figures

5. A bullet leaves the barrel of a gun 1.5m above the ground level, travelling horizontally at 70ms^{-1}. If it experiences a downward acceleration of 10m^{-2}, find the;
   (i) Time taken for the bullet to hit the ground
   (ii) Distance it had travelled horizontally

6. The results for the time taken for a certain liquid to evaporate in an open container is as shown below.

Volume (m^3)	98.5	82.5	72.0	60.5
Time (s)	0.00	14.5	18.0	23.5

   Use either linear interpolation or extrapolation to estimate
   (i) Time when volume is $40.5m^3$.
   (ii) Volume when time is 16.5s.

7. The price relatives for four items with their respective weights are as follows

Items	A	B	C	D
Price relatives	110	140	130	118
Weights	x	$2x$	y	$y-6$

   Given that the sum of the weights is 40 and the weighted average price index for the items is 126.7, find the values of x and y

8. A particle of mass 2kg is acted upon by a force $F = (5 + 4t)N$. Given that initially the particle is moving at a speed of $5ms^{-1}$. Show that the speed of the particle after $t = 2s$ is $14ms^{-1}$.

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The data below shows the mathematics grades (y) and their respective intelligence scores (x) for 10 students of a certain school.

Intelligence test score (x)	78	66	73	75	84	66	89	84	67	77
Mathematics grades (y)	81	68	81	75	80	67	85	83	66	78

(a) Plot a scatter diagram for the data. Draw a line of best fit, comment on your graph
(b) Hence find y when x=70
(c) Calculate the rank correlation coefficient and comment on your result at 5% level of significance

10. Two hundred and forty army recruits have the following heights

Height (cm)	Number of recruits
165 -	10
170 -	37
175 -	60
180 -	65
185 -	48
190 -195	20

(a) Calculate the;
(i) Mean weight and standard deviation (ii) Interquartile range
(b) Construct a cumulative frequency curve for the data and use it to estimate the median and the 7^{th} decile

11. A bike starts from a point A and moves along a straight horizontal road with a constant acceleration of $2ms^{-2}$. At the same time a car moving with a speed of $5ms^{-1}$ and a constant acceleration of $3ms^{-2}$ is 5m behind point A and moving in the same direction as the bike. Find;
(i) distance from A the car over takes the bike
(ii) time at which at which over take occurs
(iii) the speed of the car when it over takes the bike

12.(a) Show graphically that the curve of $y = 2x^3$ and the line $y = 8 - 5x$ have a root between $x = 1$ and $x = 2$ correct it to 1 decimal place.

(b) Derive the simplest iterative formula based on Newton-Raphson method for solving equation $2x^3 + 5x - 8 = 0$ is given by $x_{n+1} = \frac{4{x_n}^3+8}{6{x_n}^2+5}$, n=0, 1, 2…

(c) Use x_0, the first approximation in (a) above and the Newton Raphson method in (b) to find the root of the equation correct to 3 decimal places

13.Given that $x = 5, y = 14\ and\ z = 8$ all corrected to the nearest integer. Find the maximum value and minimum value, absolute error and percentage error in

(i) $\frac{x+z}{yx}$, (ii) $\frac{xz}{y} - \frac{y}{zx}$

14.A random variable X takes integer values only and has probability density function given as;

$$P(X=x)=\begin{cases} ax; & x = 1,2,3,4 \\ a(10-x); & x = 5,6,7,8 \\ 0; & otherwise \end{cases}$$

Where a is a constant, determine the;

(i) Value of a

(ii) E(X) and Var (X)

(iii) Median and mode

(iv) $P\left(x \geq 6/x \leq 7\right)$

15.(a) A particle of mass 6kg is placed on a rough plane inclined at 45^0 to the horizontal, the coefficient of friction between the parcel and the plane is 0.5. Find the least horizontal force and reaction required to maintain the particle in equilibrium

(b) Two stations A and B are a distance 6x meters apart along a straight line. A car starts from rest at A and accelerates uniformly to a speed of V m/s covering a distance x meters, then maintains this speed until it has travelled a further 3x meters, it then retards uniformly to rest at B. By sketching a velocity-time graph for the motion prove that the time, T taken to travel from A to B is $T = \frac{9x}{V}$ seconds

16.ABCD is a rectangle with AB=3m as the positive x-axis and AD=4m as the positive y-axis. Forces of magnitude 6N, 4N, 10N and 15N act in the directions AB, AD, AC and BD respectively. Find the:

(a) Magnitude of the resultant force and its direction

(b) Line of action and point where the resultant cuts x-axis, by taking moments about A