S.6 MOCK EXAMINATIONS 2005

P425/1 PURE MATHEMATICS

PAPER 1

TIME: 3 HOURS

Instructions:

Answer ***all*** questions in section A and ***NOT*** more than 5 in section B

SECTION A: (40 MARKS)

1. Differentiate $\log_e\left(\frac{(1+x)e^{-2x}}{1-x}\right)^{\frac{1}{2}}$

2. Find the equation of the line of intersection of the planes:
   $2x - y + 5Z = 7$ and $5x + 3y - Z = 4$

3. Find the foci of the ellipse
   $9x^2 + 25y^2 - 54x + 100y - 44 = 0$

4. Given that $Z_1 = 3 + i$ and $Z_2 = \mathbf{a} + i$ and $\arg(Z_1,Z_2) = {}^{\pi}/_{4}$ Find the value of **a** .

5. Using the substitution y = XZ, or otherwise, show that the solution of the equation
   $\frac{2dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ is given by $\frac{(y-x)^2}{xy^2} = C$, where C is a constant.

6. (a) How many different selections, taking any number of letters at a time, can be made from the letters of the word PARALLELOGRAM?

   (b) A group consists of 5 boys and 7 girls. In how many ways can a team of seven be selected if it is to contain at least 3 boys?

7. By reducing the system of equations to echelon form, solve:
   $3x - 2y + 4Z = -7$
   $x + y - 6Z = -10$
   $2x + 3y + 2Z = 3$

8. Solve the equation:

   $\sin^{-1}\left({}^{x}/_{2}\right) - \cos^{-1}\left({}^{x}/_{2}\right) = \sin^{-1}\left(1 - {}^{x}/_{4}\right)$

SECTION B: (60 MARKS)

9. (a) Prove that if f(x) has a repeated factor (x – a), then (x – a) is also a factor of
$f^1(x)$

(b) Given that $p(x) = 8x^3 - 12x^2 - 18x + K$, find the values of K such that the
equation $p(x) = 0$ has a repeated root.

(c) Prove that $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Hence solve $x^3 - 6x^2 - 3x + 2 = 0$ to 3 d.p

10.(a) Integrate

(i) $\dfrac{1}{4 - 5\sin\frac{1}{2}x}$

(ii) $\sqrt{x}\tan^{-1}\sqrt{x}$

(b) Evaluate $\displaystyle\int_0^{\sqrt{2}} \frac{x^5}{\sqrt{(16 - x^4)}}\,dx$

11. (a) Show that the lines $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \lambda(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$ and
$\mathbf{r} = -4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} + \mu(4\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$ Intersect.
State the coordinates of the point of intersection and determine the angle between the lines.

(b) Find the equation of the plane containing the line
$\begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and parallel to $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

Hence state the distance from the origin to this plane.

12. A body is placed in a room which is kept at a constant temperature. The temperature of the body falls at a rate $K\theta^{o}C$ per minute where K is a constant and θ is the difference between the temperature of the body and that of the room at the time t. Express this information in the form of a differential equation and hence show that $\theta = \theta_o e^{-Kt}$, where θ is the temperature at time t = 0.

    The temperature of the body falls $5^{o}C$ in the first minute and $4^{o}C$ in the second minute. Show that the fall of temperature in the third minute is $3.2^{o}C$.

13. Obtain the equation of the normal at P (at^2, at) on $y^2 = 4ax$. Prove that this normal meets the parabola again at a point Q whose parameter is $-t - {}^{2}/_{t}$. Find the coordinates of the midpoint M of PQ and show that the locus of M is $y^4 + 2a(2a - x)y^2 + 8a^2 = 0$

14. For the curve $y = \dfrac{5x - 25}{x^2 + 3x - 4}$ find the range of values of y where the curve does not exist, stating clearly any maximum or minimum points. Hence sketch the curve.

15. (a) Given that $Z(5 + 5i) = p(1 + 3i) + q(2 - i)$ where p and q are real and that $\arg Z = {}^{\pi}/_{2}$, $|Z| = 7$, Find the values of p and q

    (b)) By representing the complex numbers Z_1, Z_2 and $Z_1 + Z_2$ on an Argand diagram where $Z_1 = \dfrac{1 + i\sqrt{3}}{2}$, $Z_2 = i$ prove that $\tan\left(\dfrac{5\pi}{12}\right) = 2 + \sqrt{3}$

16. (a) Find the equation of the line through the intersection of the lines $3x - 4y + 6 = 0$ and $5x + y + 13 = 0$ which
    (i) passes through the point (2,4)
    (ii) makes an angle of 60^{o} with the x – axis.

    (b) The circle $ax^2 + ay^2 + 2gx + 2y + c = O$ cuts the x – axis at the points A and B. Find in terms of a, c and g, the distance between A and B
    A circle touches the y – axis at distance +4 from the origin and cuts off intercept B from the x – axis. Find the equation of this circle.

<u>END</u>

RM/CS/04