P425/2
Applied Mathematics
Principal Subject

3 Hrs

June 2006

Instructions:

Attempt all questions in Section A and not more than Five in Section B.
Calculators may be used
Mathematical tables, formulae and squared paper should be used where appropriate.

Section A: 40 Mks

1. Use trapezium rule with five subintervals to estimate $\int_0^1 e^2 x^2 \, d.c.$ Give your Answer correct to 4 significant figures. (5Mks)

2. A sample of 9 bottles of soda drawn at random from a normally distributed population of standard deviation 0.05ml has the following values, 297.5, 298.7, 296.5, 300.0, 297.4, 296.5, 297.5, 300.5, 300.0ml. Determine a 99% confidence interval for the mean fill of all bottles produced. (5Mks)

3. A and B are two events in the sample space such that P(A)= 8/15, P(A ∩B) =⅓ and P(A/B)=4/7,
Find (i) P(B)
(ii) P(B/A) (5Mks)

4. In a survey in a certain village it was found that the age distribution of the inhabitants was as follows:

Age(Years):<20	20-<30	30-<40	40-<50	50-<60	60-<80
Frequency: 40	30	25	18	14	10

Construct a histogram to illustrate the data. Hence estimate the modal age. (5Mks)

5. Two masses 3Kg and 5Kg connected by a light inextensible string 1.5m long lie on a smooth table 0.75m high. The string being straight and perpendicular to the edge of the table. The lighter mass is drawn gently just over the edge and released. Find the time that elapses before the first mass strikes the floor. (5Mks)

6. A particle starts at a point (6,-11) with initial velocity i+3j and constant acceleration –2i+j. Show that the particle will pass through the point (-6,9) and find the time when it passes through the point. (5Mks)

7. A uniform rod AB of weight W which is smoothly hinged at A is maintained in equilibrium by a horizontal force PN acting at B. Given that the rod is inclined at 30^0 to the horizontal with B below A. Find;
(i) An expression for P in terms of W.
(ii) The magnitude and direction of the reaction at the hinge. (5Mks)

8. Ten students took two examination papers in the same subject and the marks scored are as shown in the table below.

Paper1(x)	65	73	42	52	84	60	70	79	60	80
Paper2(y)	78	88	60	73	92	77	84	89	70	99

Determine the rank correlation coefficient between the two papers. Comment on the relationship.

SECTION B (60 MARKS)

9. Two uniform rods AB and BC of equal lengths are smoothly jointed together at B. The mass of AB is 6Kg and be 8Kg. The end C is freely hinged to a rough horizontal surface and the end A rests on the same surface. The coefficient of friction of the surface is μ. The points A, B and C lie in the same vertical plane and angle $BAC=30^0$.If A is on the point of slipping, find;

   (a) the horizontal and vertical components of the reactions at B and C.
   (b) the value of μ.

10. (a) Mukasa can swim across a river which is 500m wide from a point A to B, being directly opposite to B. The river is flowing at 2ms-1. If Mukasa can swim at a speed of $4ms^{-1}$ in still water, find;

   (i) the resultant speed,
   (ii) the time taken by Mukasa to cross the river as quickly as possible,

   (b) A bullet traveling at 60kmh-1 penetrates 12cm into a stationary wooden block before coming to rest. Find the velocity of the bullet in ms-1 when it penetrated 8cm of the block.

11. Two equal particles are projected at the same instant from points A and B on horizontal ground. The particle at A is projected with speed U at an angle of elevation α and from B with speed V at angle of elevation β. If the particles collide directly when they are moving horizontally in opposite directions; (a) Find
   (i) V in terms of U, α and β.
   (ii) Show that $AB = \frac{U^2 \sin\alpha \sin(\alpha+\beta)}{g\sin\beta}$
   (c) If the particles coalesce on impact, find in the simplified form in terms of U, α and β the speed of the combined particle immediately after collision.

12. (a) An elastic string of natural length 2m has its upper end p fixed and a body of mass 1.2Kg attached to its other end Q. If the modulus of elasticity is 6N and the string is stretched until the extending force is of magnitude 10.5N.

(i) Find the extension and work done in extending the string.
(ii) If the end Q is pulled vertically downwards to R, where QR=20cm, find the initial acceleration of the body when it is released from this position.

(b) A car of mass 1500Kg tows another car of mass 1000Kg up a hill of incline 1 in 10. If the resistance to the motion of the cars is 0.5N per kg and the power of the towing car is 150Kw, determine the tension in the tow rope at the instant when the speed is $10ms^{-1}$

13. A random variable X has a pdf given by

$$F(x) \begin{cases} Kx \ , \ 0 < x < 1 \\ 1/2\ k\ (3-x),\ 1<x<3 \\ 0, \quad \text{otherwise} \end{cases}$$

Find (i) the value of the constant k.

(i)E(x) and Var(x)
(ii)the cdf F(x)
(iii)$P(|x-1| < \frac{1}{2})$

14. In a given community, the probability of finding a person with secondary level of education is 2 in 5.

(a) If a random sample of 6 persons is taken, find the probability that
(i) exactly 3,
(ii)atmost 4 persons, have had secondary education.
(b) If a second sample of 100 persons is taken randomly, find the probability that;
(i) at least half have had secondary education,
(ii) between 30 and 40 persons have had secondary education.

15. The table below shows the distribution of marks obtained by a class of 30 students of a certain school;

Marks	Frequency
15-19	10
20-24	6
25-29	5
30-34	4
35-39	5

Calculate; (i) the mean and variance
(ii)median and modal mark
(iii) The percentage of students who scored less than 33 marks.

16. (a) Two numbers X and Y are each rounded off with maximum possible errors δx and δy. Show that the maximum possible error in the product xy^2 is given by

$$XY\left[\left|\frac{\delta x}{X}\right| + 2\left|\frac{\delta y}{Y}\right|\right]$$

(b) Show that the Newton-Raphson's iterative formula for finding the k^{th} root of a number N is given by

$$X_{n+1} = \frac{x_n^k (k-1) + N,\ n=0,1,2\ldots.}{K\, x_n^{k-1}}$$

Draw a flow chart that,

(i) needs in initial approximation x_0 and N.
(ii) Computes and prints the k^{th} root correct to 3 decimal places.

(c) Hence perform a dry run for your flow chart for $\sqrt{10}$, taking $x_0=3$.

END