

P425/2
APPLIED MATHEMATICS
Paper 2
Oct. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS
(PRINCIPAL SUBJECT) Set 10

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **Eight** questions in Section A and **Five** questions from Section B.*

*Any additional question(s) answered will **not** be marked.*

*All necessary working **must** be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2} .

SECTION A: (40 MARKS)

Answer **all** the questions in this section.

1. The probability that two independent events A and B occur together is $\frac{1}{8}$. The probability that either or both events occur is $\frac{5}{8}$. Find the;
(i) $P(A)$ (ii) $P(B)$
2. A body of mass 4kg is moving with an initial velocity of 5ms^{-1} on a plane. The kinetic energy of the body is reduced by 16 Joules in a distance of 40m. find the;
(i) Final velocity (ii) deceleration
3. Use the trapezium rule with 5 strips to estimate the area enclosed by the curve $y = 3^x$ from $x=1$ to $x=2$ correct to 3 decimal places.
4. The data below shows the weights in Newtons (N) of potatoes picked from a certain farm: 8.5, 7.6, 8.9, 7.6, 8.9, 8.2, 9.1, 7.9 and 8.5, determine the;
(i) Mean weight of the potatoes
(ii) Standard deviation
5. A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 13N inclined at an angle of 50° to the horizontal, if the particle does not move, find the;
(a) Normal reaction (b) Coefficient of friction
6. The table below shows the value of x with corresponding values of y

| | | | | |
|---|-----|-----|-----|-----|
| x | 0 | 8 | 12 | 20 |
| y | 9.2 | 6.0 | 4.4 | 1.5 |

Use linear interpolation or extrapolation to estimate
(i) y when $x=15$ (ii) x when $y=9.8$
7. A biased coin is tossed six times. The coin is such that the ratio of showing a tail to that of a head is 2:1, find the probability of getting;
(i) At least 4 heads (ii) Between 2 and 4 tails
8. A particle starts from a point P with an initial velocity of 2ms^{-1} and travels along a straight line with a constant acceleration of 2ms^{-2} . Two seconds later a second particle Q starts from rest and travels along the same line with an acceleration of 6ms^{-2} , find the;

- (i) Time
- (ii) Distance at which overtake occurs

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The table below shows the percentage of sand (y) in the soil depth (x)

| | | | | | | | | | | |
|------------------------|----|----|----|----|----|----|----|----|----|----|
| Soil depth (x) | 35 | 65 | 55 | 25 | 45 | 75 | 20 | 90 | 51 | 60 |
| Percentage of sand (y) | 86 | 70 | 84 | 92 | 79 | 68 | 96 | 58 | 86 | 77 |

- (a) Plot a scatter diagram for the data, draw a line of best fit and comment.
- (b) Estimate y when x=31
- (c) Calculate the rank correlation coefficient and comment at 5% level of significance

10. A rectangle ABCD (3m x 4m) has forces of magnitude 5N, 10N, 15N, 20N and 25N act along the sides BA, CB, DC, AD and BCA respectively. If AB=3m as the positive x-axis and AD=4m as the positive y-axis. Find the:

- (a) Magnitude of the resultant force and its direction
- (b) Line of action and where the resultant cuts x-axis, by taking moments about A

11.(a) Show graphically on the same axes that $y = \frac{x}{2}$ and the line $y = x \sin x$ have a root between 0.2 and 0.8 correct to 1 decimal place.

(b) Use x_0 , the first approximation in (a) above and the Newton Raphson method to find the root of the equation correct to 3 decimal places

12. The continuous random variable X has the following p.d.f

$$f(x) = \begin{cases} \alpha x & ; 0 \leq x \leq 1 \\ \frac{\alpha}{2} (3 - x) & ; 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

where α is a constant, determine the;

- (a) values of α
- (b) expected value and variance
- (c) cumulative distribution function F(x), hence determine the mean

13. At time t=0, the position vectors and velocity vectors of two trains A and B are as follows.

| Trains | Position vector | Velocity vector |
|--------|---|---|
| A | $\mathbf{r}_A = (3\mathbf{i} + \mathbf{j} + 5\mathbf{k})\text{m}$ | $\mathbf{v}_A = (4\mathbf{i} + \mathbf{j} - 3\mathbf{k})\text{ms}^{-1}$ |
| B | $\mathbf{r}_B = (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})\text{m}$ | $\mathbf{v}_B = (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})\text{ms}^{-1}$ |

If they continue with the same velocities, find the

- (a) Position vector of B, relative to A at time t and
- (b) time when A and B are nearest to each other
- (c) Shortest distance between the trains

14.(a) The volume of the sphere increases by 2%. Find the corresponding percentage increase in the;

(i) radius

(ii) surface area

(b) The period of a simple pendulum period (T) and is given by $T = 2\pi \sqrt{\frac{l}{g}}$, where π and g are constants. If the percentage increase in the length (l) is 4%, find the corresponding percentage increase in T .

15. Given that X is a continuous random variable which is normally distributed with mean, μ and standard deviation, σ such that $P(X > 50) = 0.3$ and $P(X < 30) = 0.4$, find the:

(i) Value of μ and σ

(ii) Percentage that $P(X > 40)$ takes

16. A car of mass 1,200kg tows a van of mass 300kg up a hill inclined at 1 in 100 against a constant resistance of 0.2N per kg. given that the car moved at a constant speed of 1.5ms^{-1} for 5 minutes, calculate the;

(i) Tension in the tow bar

(ii) Work done by the engine of the car during this time

(iii) Total resistance, if the engine develops a power of 15kW at a maximum speed of 120kmh^{-1} on a level road