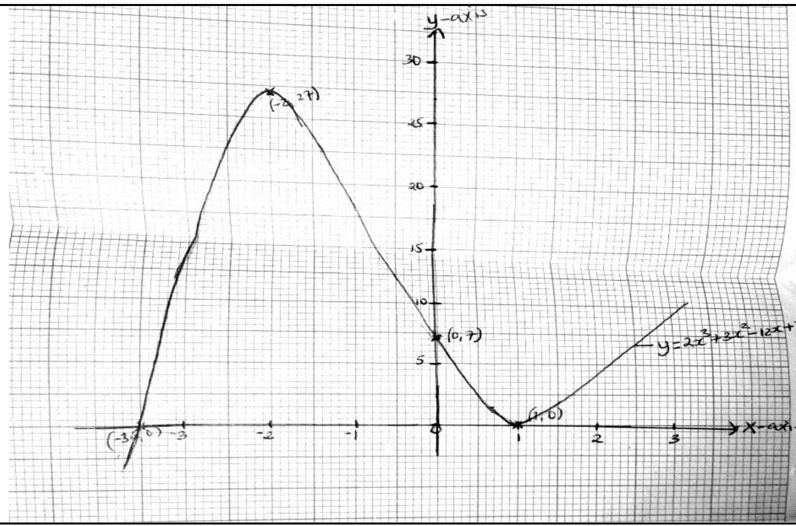
OUR LADY OF AFRICA S.S NAMILYANGO (OLAN) SOLUTIONS TO A LEVEL SUBSIDIARY MATHEMATICS S475 SEMINAR QUESTIONS 2022.

1(a)	$U_n = a + (n-1)d,\ u_2 = a + (2-1)d = 1,\quad a + d = 15\ \ldots\ldots\ldots(1)$ $u_5 = a + (5-1)d = 21,\quad a + 4d = 21\ldots\ldots\ldots(ii)$ $Solving\ (i) and\ (ii) simultaneousl\ a = 13,\ d = 2$ $s_n = \frac{n}{2}[2a + (n-1)d],\ s_{10} = \frac{10}{2}[2(13) + (10-1)2] = 220$ $Therefore\ the\ first\ term\ is\ 13, common\ difference\ is\ 2$ $and\ the\ sum\ of\ the\ first\ ten\ terms\ is\ 220.$
(b)	$n-2, n,\ n+3\quad \frac{n}{n-2} = \frac{n+3}{n}$ On cross multiplying, $\mathrm{n}^2 = (\mathrm{n}-2)(\mathrm{n}+3),\ \ \mathrm{n}^2 = \mathrm{n}^2 + \mathrm{n} - 6,\ \ \mathrm{n} = 6$ $\mathrm{n}-2\quad \mathrm{n}\quad \mathrm{n}+3\quad \mathrm{y}$ $\mathrm{a} + \mathrm{ar} + \mathrm{ar}^2 + \mathrm{ar}^3$ $4 + 6 + 9\quad \mathrm{r} = \frac{\mathrm{ar}}{\mathrm{a}} = \frac{6}{4} = 1.5$ $ar^3 = 4(1.5)^3 = 13.5,$ $Therefore\ the\ next\ term\ is\ 13.5$
(c)	$P(1 + \frac{r}{100})^n > A\ ,\ 800000(1 + \frac{15}{100})^n > 8000000$ $1.15^n > 10$ Introduce log_{10} on both sides $\log_{10} 1.15^n > \log_{10} 10$ $n\frac{\log_{10} 1.15}{\log_{10} 1.15} > \frac{\log_{10} 10}{\log_{10} 1.15}$ $n > 16.4751$ $n = 17 years$
2(a) (i)	$\sqrt{8}+\sqrt{18}-2\sqrt{2} = \sqrt{4X2}+\sqrt{9X2}-2\sqrt{2}$ $= 2\sqrt{2} + 3\sqrt{2} - 2\sqrt{2}$ $= 3\sqrt{2}$
(ii)	$\sqrt{24}-3\sqrt{6} - \sqrt{216} + \sqrt{294} = \sqrt{4X6} - 3\sqrt{6} - \sqrt{36X6} + \sqrt{49X6}$ $= 2\sqrt{6} - 3\sqrt{6} - 6\sqrt{6} + 7\sqrt{6}$ $= 0\sqrt{6}$ $= 0$
(b)	$\frac{2}{(3-\sqrt{2})}, Rationalizing\ the\ denominator, \frac{2}{(3-\sqrt{2})}\frac{3+\sqrt{2}}{(3+\sqrt{2})} = \frac{6+2\sqrt{2}}{9-2}$ $= \frac{6}{7} + \frac{2\sqrt{2}}{7}\ in\ the\ form\ A + B\sqrt{C}$ Where; $A = \frac{6}{7}, B = \frac{2}{7}, C = 2.$
3(a)	$x^2 - px + 8 = 0, sum = p, product = 8$

	$\propto + \propto + 2 = p, 2 \propto + 2 = p \ldots\ldots\ldots\ldots (i)$ $\propto (\propto + 2) = 8, \quad \propto^2 + 2 \propto - 8 = 0, \quad (\propto - 2)(\propto + 4) = 0$ $either, \propto = 2 \ or \ \propto = -4$ $for \ \propto = 2, from\ (i), 2 \propto + 2 = p, \quad 2(2) + 2 = p, \quad p = 6$ $for \ \propto = -4, from\ (i), 2 \propto + 2 = p, 2(-4) + 2 = p, \quad p = -6$ $the\ two\ possible\ values\ of\ p\ are - 6\ and\ 6$
(b)	let one of the roots be $\propto$ *and the other* $\propto -1$ $sum = \propto + \propto - 1 = 2 \propto - 1, \ product = \propto (\propto - 1)$ $equation\ given, ax^2 + bx + c = 0$ $sum = \frac{-b}{a}, \quad product = \frac{c}{a}, \quad 2 \propto - 1 = \frac{-b}{a}, \quad 2 \propto = 1 + \frac{b}{a}$ $\propto = \frac{a-b}{2a}, substitute \propto in \ \propto (\propto - 1) = \frac{c}{a}, \quad \left(\frac{a-b}{2a}\right)^2 - \left(\frac{a-b}{2a}\right) = \frac{c}{a}$ $\frac{(a-b)^2}{4a^2} - \frac{a-b}{2a} = \frac{c}{a}, \quad \frac{(a-b)^2 - 2a(a-b)}{4a^2} = \frac{c}{a}$ $(a-b)^2 - 2a(a-b) = 4ac, \quad a^2 - 2ab + b^2 - 2a^2 + 2ab = 4ac$ $b^2 - a^2 = 4ac, \ a^2 = b^2 - 4ac \quad as\ required$
(c)	Using the remainder theorem $f(x) = (x-a)Q(x) + R(x), \ ax^4 + bx^3 - 8x + 6 = (x^2 - 1)Q(x) + 2x + 1$ $when\ x = 1, a(1)^4 + b(1)^3 - 8(1) + 6 = (1-1)Q(x) + 2(1) + 1$ $a + b = 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$ $when\ x = -1, \quad a(-1)^4 + b(-1)^3 - 8(-1) + 6 = 2(-1) + 1$ $a - b - 8 + 6 = -1, \ a - b = 1 \ldots\ldots\ldots\ldots\ldots.. (ii)$ $solving\ (i)\ and\ (ii) simultaneously, a = 3, \quad b = 2$
4(a)	$log_2^{(x+y)} = log^{100}, \ log_2^{(x+y)} = log^{10^2}, \ log_2^{(x+y)} = 2log^{10}$ $log_2^{(x+y)} = 2 \ , \ x + y = 4 \ldots\ldots\ldots\ldots. (i)$ $log_2^{(2x-y)} = log^{10}, \quad log_2^{(2x-y)} = log^{10^1}, \ log_2^{(2x-y)} = 1$ $2x - y = 2 \ldots\ldots\ldots\ldots\ldots\ldots.. (ii)$ $solving\ (i) and\ (ii)\ simultaneaously;\ x = 2\ and\ y = 2$
b(i)	$by\ completing\ squares, t^2 - 4t - 8 = 0$ $t^2 - 4t + (-2)^2 = 8 + (-2)^2, \ (t-2)^2 = 8 = 4$ $(t-2)^2 = 12, \quad (t-2) = \pm\sqrt{12}, \quad t = 2 \pm \sqrt{12}$ $either\ t = 5.4641 \quad or \quad t = -1.4641$
b(ii)	$by\ completing\ squares, \ 2t^2 - 6t + 4 = 0$ $t^2 - 3t + 2 = 0, \quad t^2 - 3t + \left(-\frac{3}{2}\right)^2 = -2 + \left(-\frac{3}{2}\right)^2$ $(t - \frac{3}{2})^2 = -2 + \frac{9}{4}, \quad \left(t - \frac{3}{2}\right)^2 = \frac{1}{4}, \quad t - \frac{3}{2} = \pm\sqrt{\frac{1}{4}}$ $t - \frac{3}{2} = \pm\frac{1}{2}$ either $t = \frac{3}{2} - \frac{1}{2} = 1 \quad or\ t = \frac{3}{2} + \frac{1}{2} = 2$
(c)	**)** $e^{3x} - 2e^{2x} - e^x + 2 = 0 \ , e^{x.3} - 2e^{x.2} - e^x + 2 = 0$, let $e^x = y$

	$y^3-2y^2-y+2=0, using\ inspection\ with\ y=1$ $1^3-2(1)^2-1+2=0, so\ (y-1) is\ a\ factor.$ $by\ longdivision\ or\ factorisation, y^3-2y^2-y+2=\ (y-1)(y^2-y-2)$ $(y-1)(y^2-y-2)=0,\quad (y-1)(y+1)(y-2)=0$ Either, $y=1,\quad y=-1,\quad y=2$ For $y=1,\quad e^x=1,\quad lne^x=ln1,\quad x=0$ $for\ y=-1,\quad e^x=-1,\quad lne^x=ln-1,\quad x=DNE$ $y=2,\quad e^x=2,\quad lne^x=ln2,\quad x=0.693$
(d)	**Method 1.** $f(x)$=$2+x-3x^2\ writing\ it\ in\ the\ form\ a-b(x+c)^2$ $expanding\ a-b(x+c)^2, a-b[(x+c)(x+c)]\ a-bx^2-2bxc-bc^2\qquad 2+x-3x^2=a-bc^2-2bxc-bx^2$ Comparing the coefficients, 2= $a-bc^2, 1=-2bc\ , 3=b,\ \ b=3, c=\frac{-1}{6},\ a=\frac{25}{12}$ $2+x-3x^2=\frac{25}{12}$-$3(x-\frac{1}{6})^2$ **Method 2** $f(x)=-3\left(x^2-\frac{x}{3}-\frac{2}{3}\right), -3\left[(x-\frac{1}{6})^2-(\frac{1}{6})^2-\frac{2}{3}\right]$ -3$\left[(x-\frac{1}{6})^2-\frac{25}{36}\right]\quad f(x)=\frac{25}{12}$-$3(x-\frac{1}{6})^2$ where $a=\frac{25}{12}, b=3, c=\frac{-1}{6}$ $f(x)_{min}=\frac{25}{12}$ and it occurs when $(x-\frac{1}{6})^2=0, x=\frac{1}{6}$
5(a)(i)	$(x^2+3x)^7$, Let $y=(x^2+3x)^7$ Let $u=x^2+3x\ , \frac{du}{dx}=2x+3, let\ y=u^7, \frac{dy}{du}=7u^6$ $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}=7u^6.2x+3=7(2x+3)(x^2+3x)^6$
(ii)	$(x+3)\sqrt{(1-x^3)}$ Let $u=x+3, \frac{du}{dx}=1\ ,\ v=\sqrt{(1-x^3)}\quad \frac{dv}{dx}$=$\frac{1}{2}(1-x^3)^{\frac{-1}{2}}-3x^2$ $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\ ,\ \ \frac{(x+3)3x}{2(1-x^3)^{\frac{1}{2}}}+(1-x^3)^{\frac{1}{2}}.1$ $=\frac{-3x(x+3)+2(1-x^3)}{2(1-x^3)^{\frac{1}{2}}}=\frac{2-9x-3x^2-2x^3}{2(1-x^3)^{\frac{1}{2}}}$
(iii)	$(\frac{1+x^2}{x})^{\frac{1}{2}}$ **Method 1.** Let $y=(\frac{1+x^2}{x})^{\frac{1}{2}}\ y^2=\left(\frac{1+x^2}{x}\right), introducing \ln\ both\ sides, ln\ \ y^2=[\ln(1+x^2)-lnx]$ $\frac{2}{y}\frac{dy}{dx}=\frac{2x}{1+x^2}-\frac{1}{x}\ ,\qquad \frac{dy}{dx}$=$\frac{2x^2-1-x^2}{2x(1+x^2)}.y\qquad =\frac{x^2-1}{2x(1+x^2)}(\frac{1+x^2}{x})^{\frac{1}{2}}\qquad =\frac{x^2-1}{2x\sqrt{x((1+x^2)}}$ **Method 2.** Let $y=(\frac{1+x^2}{x})^{\frac{1}{2}}=\frac{\sqrt{1+x^2}}{\sqrt{x}}\quad let\ u=\sqrt{1+x^2}\quad \frac{du}{dx}=\frac{1}{2}(1+x^2)^{\frac{-1}{2}}2x=\frac{x}{\sqrt{1+x^2}},$

	let $v=\sqrt{x}$ $\frac{dv}{dx}=\frac{1}{2\sqrt{x}}$ by quotient rule, $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{\sqrt{x}\frac{x}{\sqrt{1+x^2}}-\sqrt{1+x^2}\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$ $=\frac{2x^2-(1+x^2)}{2x\sqrt{x}\sqrt{1+x^2}}=\frac{x^2-1}{2x\sqrt{x((1+x^2)}}$
(iv)	$y=3\sqrt[3]{x}+2\sqrt{x}$ $\frac{dy}{dx}=3\frac{d(x)^{3/2}}{dx}+2\frac{d(x)^{1/2}}{dx}$ $=3.\frac{3}{2}(x)^{1/2}+2.\frac{1}{2}(x)^{\frac{-1}{2}}$ $=\frac{9\sqrt{x}}{2}+\frac{1}{\sqrt{x}}$ $=\frac{9x+2}{2\sqrt{x}}$
(v)	$y=3x^2cosx$ let $u=x^2$,$\frac{du}{dx}$=2x $v=cosx,$ $\frac{dv}{dx}=-sinx$ $\frac{dy}{dx}=3\left[u\frac{dv}{dx}+v\frac{du}{dx}\right]$ $=3.2xcosx-3x^2sinx,$ $=6xcosx-3x^2sinx.$
(b)	$x^2-3xy+y^2-2y+4x=0,$ $2x-3\left[x\frac{dy}{dx}+y\right]+2y\frac{dy}{dx}-2\frac{dy}{dx}+4=0$……………………………….(i) $(2y-2-3x)\frac{dy}{dx}=3y-4-2x,$ $\frac{dy}{dx}=\frac{3y-4-2x}{(2y-2-3x)}$ $from\ equation\ (i), 2-3\left[\frac{dy}{dx}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}\right]+2\left[(\frac{dy}{dx})^2+y\frac{d^2y}{dx^2}\right]-2\frac{d^2y}{dx^2}=0$ $collecting\ like\ terms\ together, (2y-2-3x)\frac{d^2y}{dx^2}-6\frac{dy}{dx}+2(\frac{dy}{dx})^2+2=0$ $\frac{d^2y}{dx^2}=\frac{2y-2-3x}{6\frac{dy}{dx}-2\frac{dy}{dx})^2-2},$ $=\frac{(2y-2-3x)}{6\frac{3y-4-2x}{(2y-2-3x)}-2(\frac{3y-4-2x}{(2y-2-3x)})^2+2}$ $=\frac{(2y-2-3x)^3}{2(2y-2-3x)(11y-10-9x)-2(3y-4-2x)^2}$ $y=2x^3+3x^2-12x+7$ $\frac{dy}{dx}=6x^2+6x-12, For\ turning\ points;$ $\frac{dy}{dx}=0.$ $6x^2+6x-12=0, x^2+x-2=0,$ $x^2-x+2x-2=0, x(x-1)+2(x-1)=0, (x+2)(x-1)=0,$ $x=1, or\ x=-2.$ $when\ x=1, y=0$ $(1,0) and\ when\ x=-2, y=27(-2,27)$ $to\ distinguish\ between\ the\ points,$ $\frac{d^2y}{dx^2}=12x+6$ $\frac{d^2y}{dx^2}_{x=1}>0$ and $\frac{d^2y}{dx^2}_{x=-2}<0$ $Therefore\ (1,0) is\ a\ minimum\ point\ and\ (-2,27) is\ a\ maximum\ point.$ $intercepts; for\ x=0, y=7\ (0,7).$ $for\ y=0, x_1=1, x_2=-3.5\ (1,0), (-3.5,0)$

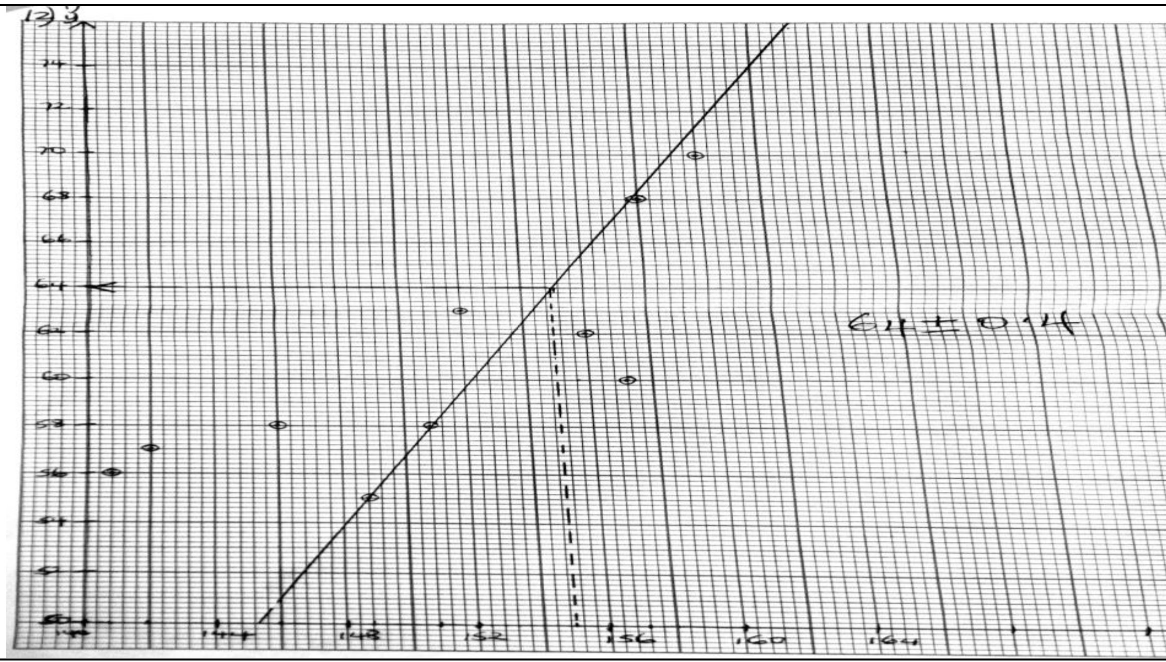
	
(d)	$Let\ the\ profit\ be\ P,\quad P = revenue - cost,\quad Px - cost$ P=$\left(\frac{375-5x}{3}\right)x - (500 + 13x + \frac{x^2}{5})\quad = 125x - \frac{5}{3}x^2 - 500 - 13x - \frac{x^2}{5}$ =-500+112$x - \frac{28}{15}x^2\quad \frac{dp}{dx} = \frac{d}{dx}$(-500+112$x - \frac{28}{15}x^2$) =112 - $\frac{56x}{15}$ At $minimum\ or\ maximum\ profit,\quad \frac{dp}{dx} = 0,\ 112 - \frac{56x}{15} = 0,\ x = 30.$ $\frac{d^2p}{dx^2} = \frac{d}{dx}(112 - \frac{56x}{15}) = \frac{-56}{15} < 0$ $Hence\ profit\ is\ maximum\ when\ x = 30.$ $P_{max} = -500 + 112(30) - \frac{28}{15}(30)^2\quad = shs. 1180$
6(a)(i)	$\int 3\sqrt{x} - \sqrt[3]{x}\,dx\quad = \int 3x^{\frac{1}{2}}dx \int x^{\frac{3}{2}}d = \frac{3.2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$
(ii)	$\int \frac{x^2}{3} + \frac{3}{x^2}dx\quad = \int \frac{x^2}{3} + 3x^{-2})dx\quad = \frac{x^3}{9} - \frac{3}{x} + c$
(iii)	$\int \frac{x^2 sec^2 x + x^5}{x^2}dx\quad = \int sec^2 x dx + \int x^3 dx\ = tanx + \frac{x^4}{4} + c$
b(i)	$\int_0^6 2x(x^2+3)dx\quad let\ u = x^2 + 3\ \frac{du}{dx} = 2x\ \int 2x.u.\frac{du}{2x}, \int udu = \left[\frac{u^2}{2}\right] + c, \left[\frac{x^2+3)^2}{2}\right] = \frac{[(6^2+3)^2]}{2} - \frac{[(0^2+3)^2]}{2}\quad = 756$
(ii)	$\int_1^2 \frac{(x^2+1)^2}{x^2}dx\quad \int_1^2 \frac{x^4+2x^2+1}{x^2}dx\quad \int_1^2 x^2 + 2 + x^{-2})dx\ = \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]$ $= \frac{37}{6} - \frac{4}{3}\quad = \frac{29}{6}$
(iii)	$\int_0^{\frac{\pi}{3}} \frac{3x^4+2x^2cosx}{x^2}dx\ = \int_0^{\frac{\pi}{3}} 3x^2 + 2cosx\,dx\ = \left[\frac{3x^3}{3} + 2sinx\right]\ = (\pi/3)^3 + 2sin\pi/3) - (0^3 + 2sin0)\quad = 2.880$
(c)(i)	$\frac{dy}{dx} = kx,\quad from\ \theta = \tan^{-1}6.$Gradient of the tangent m=6.$At\ (2,3) the\ point\ of\ contact\ of\ the\ tangent\ and\ the\ curve, kx = 6,\ 2k = 6,\ k = 3$

(ii)	Equation of the curve $\frac{dy}{dx} = kx$, $\int dy = \int 3xdx$ $y = \frac{3x^2}{2} + c$ $At\ (2{,}3)$ $3 = \frac{3x4}{2} + c$, $c = -3$, $y = \frac{3x^2}{2} - 3$
(iii)	Gradient of the tangent m=6, gradient of the normal =-1/6. Using $y = mx + c\ At\ (2{,}3)\ 3 = \frac{-1}{6}(2) + c, c = \frac{10}{3}$ $y = -\frac{1}{6}x + \frac{10}{3}$
(d)(i)	$\frac{dA}{dt} = -3\sqrt{t}$ *by separating variables,* $dA = -3\sqrt{t}\mathrm{dt}$ $\int dA = \int -3\sqrt{t}\mathrm{dt}$ A=-3($\frac{2}{3}t^{3/2}$)+c At $t = 0, A = 16$ $16 = -2(0)^{3/2}$+c, c=16. $A = -2t^{3/2} + 16.$
(ii)(a)	$Reduce\ to\ 14mm^2$, $A = 14, t = ?$ $14 = -2t^{3/2} + 16.$ $-2 = -2t^{\frac{3}{2}}, 1 = t^{\frac{3}{2}}$, $t = 1\ day.$
(ii)(b)	To $heal\ completely$, $A = 0\ t = ?$ $0 = -2t^{\frac{3}{2}} + 16$, $-16 = -2t^{\frac{3}{2}}, 2^3 = t^{\frac{3}{2}}$ $t = 4days$
7(a)	$2sin^2\theta + cos\theta + 1 = 0,$ $2(1 - cos^2\theta) + cos\theta + 1 = 0, -2cos^2\theta + cos\theta + 3 = 0$ $let\ y = cos\theta,$ $-2y^2 + y + 3 = 0$ $y = \frac{-1 \pm \sqrt{1^2 - 4(-2)(3)}}{2(-2)}$ $y_1 = 1, y_2 = 1.5\ cos = -1,$ $\cos^{-1}(-1) =\ 180$ $cos\theta = 1.5,$ $\theta = \cos^{-1}1.5,,, DNE.\ The\ reflex\ angle\ \theta = 180^0$
(b)	$2sin2\theta = 3cos\theta,$ $2(2sin\theta cos\theta) = 3cos\theta, 4sin\theta cos\theta - 3cos\theta = 0$, $cos\theta(4sin\theta - 3) = 0, either\ cos\theta = 0,$ $\theta = \cos^{-1}(0) = 90^0, 270^0$ Or 4sinθ $= 3, sin\theta = \frac{3}{4}, \theta = \sin^{-1}(\frac{3}{4})$ =48.6,131.4. $\therefore \theta = 48.4^0, 90^0, 131.4^0, 270^0.$
8(i)	$\frac{sin\theta + tan\theta}{1 + cos\theta} = tan\theta. considering\ the\ LHS, \frac{sin\theta}{1 + cos\theta} + \frac{sin\theta}{cos\theta(1 + cos\theta)}$ $\frac{sin\theta(1 - cos\theta)}{1 - cos^2\theta} + \frac{sin\theta(1 - cos\theta)}{cos\theta(1 - cos^2\theta)}, \frac{cos\theta sin\theta - sin\theta cos^2\theta + sin\theta - cos\theta sin\theta}{cos\theta(1 - cos^2\theta)}$ $\frac{sin\theta((1 - cos^2\theta)}{cos\theta(1 - cos^2\theta)}$, $=tan\theta$ *As required.*
(ii)	$\frac{1 + sinx + cosx}{1 + sinx - cosx} = \cot\frac{x}{2}$, use t $= tan\frac{x}{2}$ substitution, to obtain $\frac{1}{t}$
(iii)	$\sin(x + y)\sin(x - y) = sin^2x - sin^2y,$ $(sinxcosy + sinycosx)(sinxcosy - sinycosx),$ $sin^2xcos^2y - sinxcosysinycosx + sinxcosysinycosx - sin^2ycos^2x$ $sin^2xcos^2y - sin^2ycos^2x,$ $sin^2x(1 - sin^2y) - sin^2y(1 - sin^2x)$ $sin^2x - sin^2xsin^2y - sin^2y + sin^2ysin^2x, =\ sin^2x - sin^2y\ As\ required.$
b(i)	$x = acos\theta, y = bsin\theta, from\ x^2 = a^2cos^2\theta,$ $a^2(1 - sin^2\theta), a^2(1 - (y/b)^2),$ $x^2 = a^2 - (a^2(y/b)^2), b^2x^2 + a^2y^2 = a^2b^2.$ ***ALTERNATIVELY***; $\frac{x}{a} = cos\theta,$ $\frac{y}{b} = sin\theta, from\ sin^2\theta + cos^2\theta = 1, (\frac{x}{a})^2 + (\frac{y}{b})^2 = 1,$

	$b^2x^2 + a^2y^2 = a^2b^2.$
(ii)	$x = sin\theta + cos\theta, \dots\dots..(1) \quad y = sin\theta - cos\theta \dots\dots\dots.(2),$ $(1) + (2) gives \ \frac{(x+y)}{2} = sin\theta$ $(1) - (2) gives \ \frac{(x-y)}{2} = cos\theta.$ $from \ sin^2\theta + cos^2\theta = 1; \quad \left(\frac{(x+y)}{2}\right)^2 + \left(\frac{(x-y)}{2}\right)^2 = 1$
9a(i)	$\overrightarrow{OA} = 2i + 2j, \quad \overrightarrow{BA} = 7i - j, \quad \overrightarrow{OB} = ?, \quad \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB},$ $\begin{pmatrix} 7i \\ -j \end{pmatrix} = \begin{pmatrix} 2i \\ 2j \end{pmatrix} - \overrightarrow{OB}, \quad \overrightarrow{OB} = \begin{pmatrix} -5i \\ 3j \end{pmatrix}$
(ii)	$from \ \overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}, \quad \overrightarrow{AM} = \frac{1}{2}(-\overrightarrow{BA}) = -\frac{1}{2}\begin{pmatrix} 7i \\ -j \end{pmatrix} = -\frac{7}{2}i + \frac{1}{2}j$ $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}, \left(-\frac{7}{2}i + \frac{1}{2}j\right) = \overrightarrow{OM} - (2i + 2j), \overrightarrow{OM} = \frac{-3}{2}i + \frac{5}{2}j$
b(i)	$a = 3i - 4j, \quad b = -5i + 12j,$ $(3a + b).b = 3(3i - 4j) + (-5i + 12j) = 4i.$ $4i.(-5i + 12j) = -20 + 0 = -20$
(ii)	Angle between **a** and **b**, $\cos\theta = \frac{a.b}{\lvert a\rvert \lvert b\rvert}$ a.b=$(3i - 4j).(-5i + 12j), -15 - 48 = -63, \lvert a\rvert = \sqrt{3^2 + (-4)^2} = 5, \lvert b\rvert = \sqrt{(-5)^2 + 12^2} = 13, \lvert a\rvert\lvert b\rvert = 65, \cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert} = \frac{-63}{65}, \theta = \cos^{-1}\left(\frac{-63}{65}\right), 165.8^0, 194.3^0$
c(i)	$\lvert a + 2b - 3c\rvert = \lvert(-2i + 4j) + 2(-5i + 10j) - 3(3i + 4j)\rvert$ $= \lvert -21i + 12j\rvert$ $= \sqrt{(-21)^2 + (12)^2} = 24.2 Units.$
(ii)	$\boldsymbol{r} = \boldsymbol{c} + \lambda(\boldsymbol{a} - \boldsymbol{b})$ $\boldsymbol{r} = (3i + 4j) + \lambda((-2i + 4j) - (-5i + 10j))$ $= (3 + 3\lambda)i + (4 - 6\lambda)j$ $\lvert r\rvert = 10$ $\sqrt{(3 + 3\lambda)^2 + (4 - 6\lambda)^2} = 10$ $9\lambda^2 + 6\lambda - 15 = 0$ $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a = 9, b = 6, c = -15$ $\lambda_1 = 1, \lambda_2 = 5/3$
10a(i)	$ABC = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} -9 & 15 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} -3 & 66 & 39 \\ 4 & 2 & -2 \end{pmatrix}$

(ii)	(A+B)C= $\left[\begin{pmatrix}5 & 1\\0 & 2\end{pmatrix}+\begin{pmatrix}-2 & 3\\1 & 0\end{pmatrix}\right]\begin{pmatrix}2 & 1 & -1\\1 & 5 & 2\end{pmatrix}$ $=\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}\begin{pmatrix}2 & 1 & -1\\1 & 5 & 2\end{pmatrix}$ $=\begin{pmatrix}6+4 & 3+20 & -3+8\\2+2 & 1+10 & -1+4\end{pmatrix}$ $=\begin{pmatrix}10 & 23 & 5\\4 & 11 & 3\end{pmatrix}$
b(i)	$A=\begin{pmatrix}2 & 4\\-1 & 3\end{pmatrix}$ $A^2+xA+yI=0$ $\begin{pmatrix}2 & 4\\-1 & 3\end{pmatrix}\begin{pmatrix}2 & 4\\-1 & 3\end{pmatrix}+x\begin{pmatrix}2 & 4\\-1 & 3\end{pmatrix}+y\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ $\begin{pmatrix}0 & 20\\-5 & 5\end{pmatrix}+\begin{pmatrix}2x & 4x\\-x & 3x\end{pmatrix}+\begin{pmatrix}y & 0\\0 & y\end{pmatrix}=\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ $2x+y=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$ $20+4x=0\ldots\ldots\ldots\ldots\ldots\ldots(2)$ $from\ (2)\ x=-5, y=1$
(c)	$3x+4y=8$ $x+2y=3$ $using\ matrix\ method, \begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}8\\3\end{pmatrix}$ $\begin{pmatrix}3 & 4\\1 & 2\end{pmatrix}\begin{pmatrix}2 & -4\\-1 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2 & -4\\-1 & 3\end{pmatrix}\begin{pmatrix}8\\3\end{pmatrix}$ $\begin{pmatrix}6-4 & -12+12\\2-2 & -4+6\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}16-12\\-8+9\end{pmatrix}$ $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\1\end{pmatrix}$ $2x=4,\quad x=2.\qquad 2y=1,\quad y=\frac{1}{2}$
(i)	2, 4, 7, 3,5, 6, 3, 6,10, 7, 8, 9, 3,4,3. $Mode=3$ (the number that appears most)
(ii)	$Median$, 2,3,3,3,3,3,4,4,5,6,6,7,7,8,9,10, $median$ is 5.
(iii)	Mean=$\frac{\sum f}{n}=\frac{(2\times1)+(3\times4)+(4\times2)+(5\times1)+(6\times2)+(8\times1)+(9\times1)+(10\times1)}{15}=\frac{80}{15}=5.33$
(iv)	Standard deviation=$\sqrt{\frac{(X-\overline{X})^2}{N}}=\frac{175.0137}{15}=11.668$

12(i)



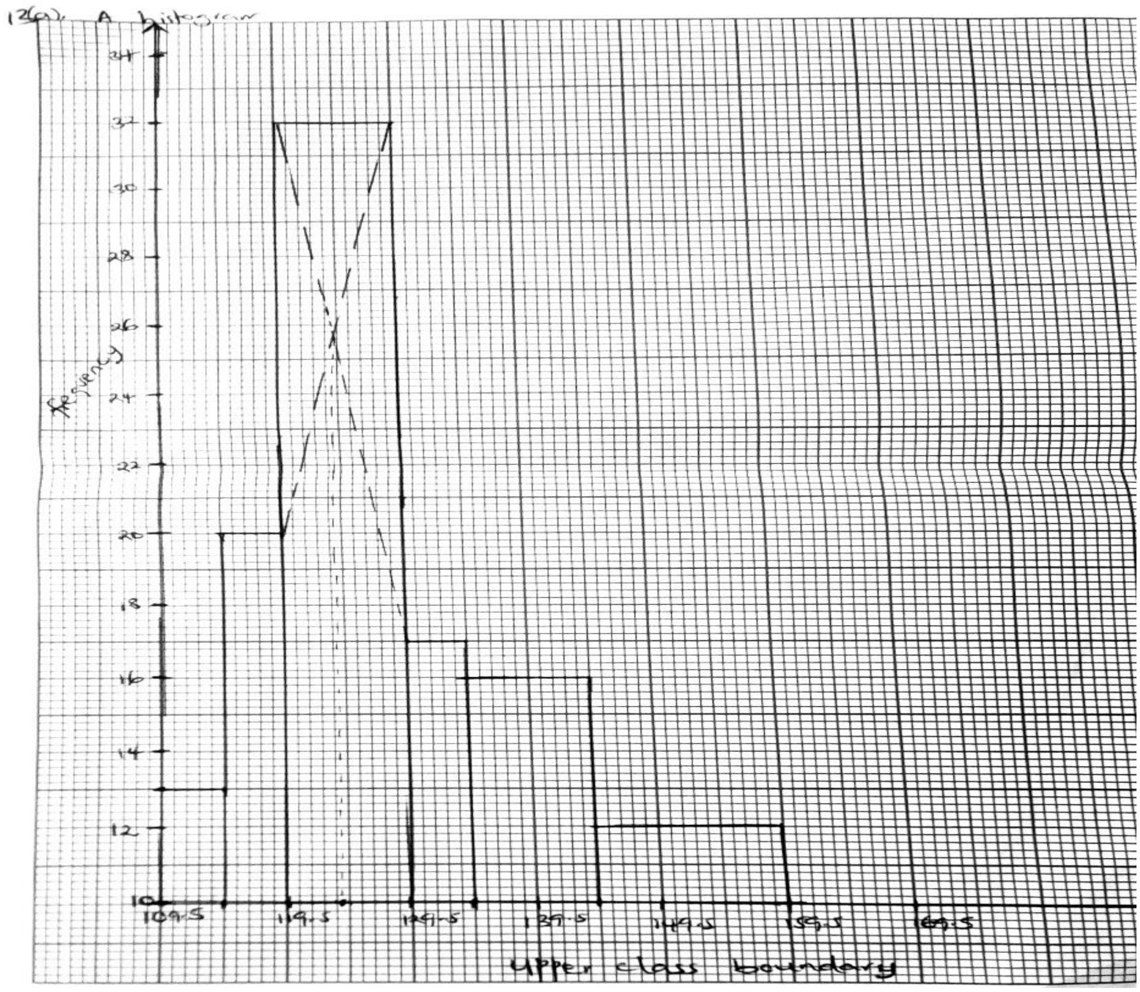
(ii)

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6X21.5}{10(100-1)}$ =0.8697.

The correlation is significant at 5% level of significance

13(a)

Amount	f	$f.d$	cf	x	fx	x^2	fx^2	Class boundaries
110-114	13	2.6	13	112	1456	12544	163072	109.5-114.5
115-119	20	4	33	117	2340	13689	273780	114.5-119.5
120-129	32	3.2	65	124.	3984	15500.2	496008	119.5-129.5
130-134	17	3.4	82	132	2244	17424	296208	129.5-134.5
135-144	16	1.6	98	139.	2232	19460.2	311364	134.5-144.5
145-159	12	0.8	110	152	1824	23104	277248	144.5-159.5
	110				1408		181768	



The modal allowance is 124 ± 1 shillings.

b(i)

Median$=L_1+\left(\frac{\frac{N}{2}-cf_b}{f_m}\right)i,$

$$where L_1 = 119.5, \quad \frac{N}{2} = 55, \quad cf_b = 33, f_m = 32, i = 10.$$

$$Median\ allowances = 126.375X1000 = 126375shs$$

(ii)

Mean allowances$=\frac{\sum fx}{\sum f} = \frac{14080}{110} = 128X1000 = 128000shs.$

(iii)

Standard deviation$=\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{1817680}{110} - (128)^2}$

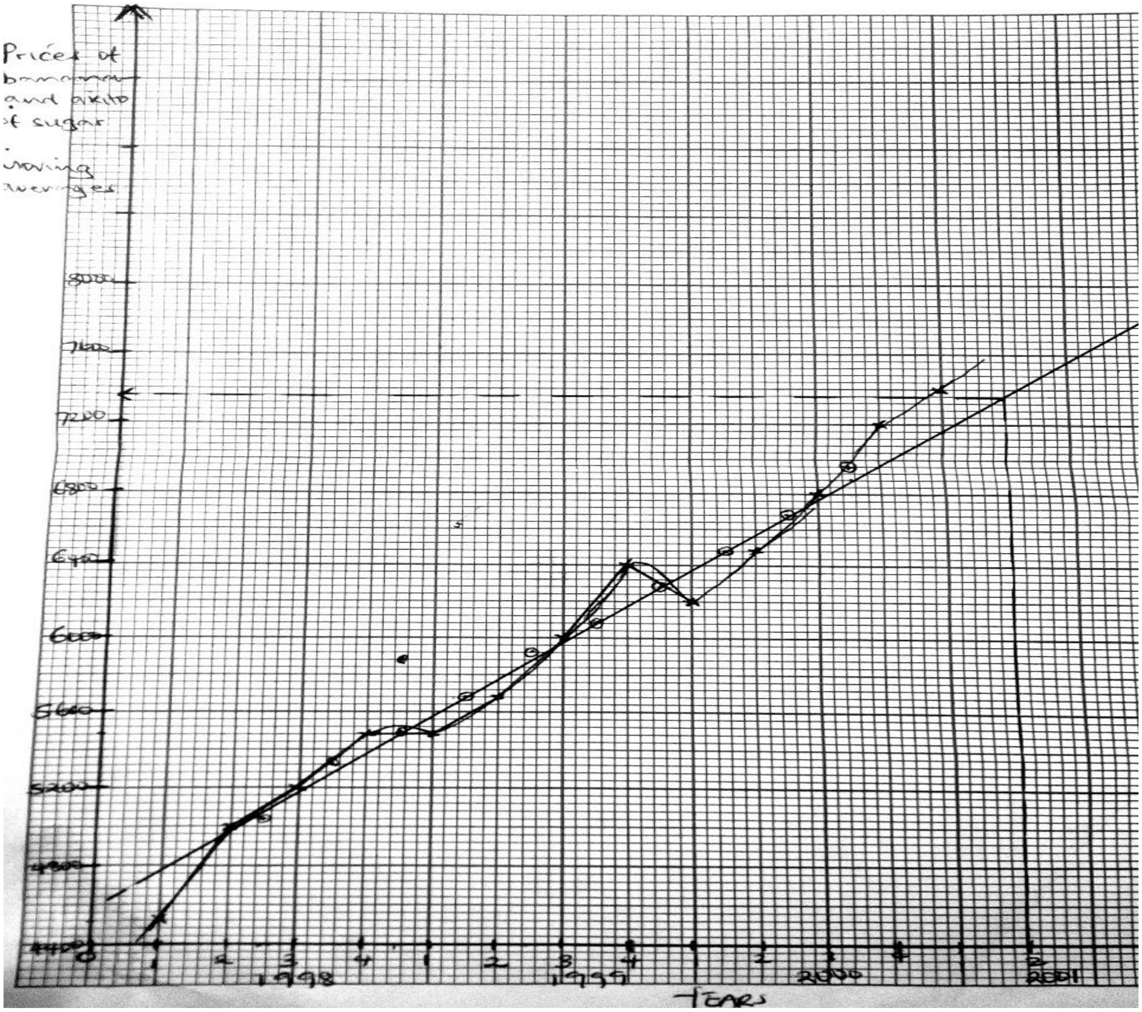
$= 11.8475X1000$=11847.5shs.

14.

	Q_0	P_0	Q_1	P_1	P_0Q_1	P_1Q_1	P_0Q_0	P_1Q_0
A	41.2	3.85	42.5	4.60	163.63	199.33	158.62	193.23
B	55.0	3.70	60.0	4.35	222.00	261.00	203.50	239.25
C	83.0	2.60	78.0	2.90	202.80	226.20	215.80	240.70
					588.43	686.53	577.20	673.18

(i)	Paasche's Aggregate price index=$\frac{\sum P_1 Q_1}{\sum P_0 Q_1}\ X100$ $= \frac{686.53}{588.43} X100 = 116.7$ This indicates that the prices increased by 16.7%
(ii)	Lespeyre's aggregate=$\frac{\sum P_1 Q_0}{\sum P_0 Q_0} X100$ $= \frac{673.18}{577.20} X100 = 116.6$ This indicates that the prices increased by 16.6%.
(iii)	$Value\ index = price\ index\ \times\ Quantity\ index;$ $= \frac{\sum P_1}{\sum P_0} . \frac{\sum Q_1}{\sum Q_0} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0} = \frac{686.53}{577.20} X100 = 118.9.$ This shows that there is an increase of 18.9% *in the value of goods traded in* 2018.
15	

Year	Quarter	Av. price o banana	Moving totals	Moving averages
	1	4500		
	2	5000		
1998			20200	5050
	3	5200		
			21200	5300
	4	5500		
			21900	5475
	1	5500		
			22700	5675
1999	2	5700		
			23600	5900
	3	6000		
			24300	6075
	4	6400		
			25100	6275
	1	6200		
			25900	6475
2000	2	6500		
			26700	
	3	6800		
			27900	
	4	7200		
2001	1	7400		
	2	Y		



We use moving averages to "Smooth" data so that we can try to predict future values.

There is a general increase in the prices of the two commodities.

$$\frac{6800+7200+7400+Y}{4} = 7360, Y = 8040 Shillings$$

16

x	1	2	3	4	5
$P(X=x)$	0.10	P	0.20	q	0.30
$xP(X=x)$	0.10	2p	0.60	4q	1.5
$x^2\mathrm{P}(X=x)$	0.10	4p	1.8	16q	7.5

(i)

$$\sum P(X=x) = 1, 0.10 + p + 0.20 + q + 0.3 = 1, p + q = 0.4 \ldots\ldots (i)$$

$$E(X) = 3.5;$$

$$0.10 + 2p + 0.60 + 4q + 1.5 = 3.5,$$

$$2p + 4q = 1.3 \ldots . (ii)$$

$$solving\ (i) and\ (ii) simultaneously,$$

$$p = 0.15, q = 0.25$$

(ii)

$$Var(x) = \left[\sum x^2\ P(X=x)\right] - (xP(X=x)^2 = 14 - 3.5^2 = 1.75$$

$$Standard\ deviation;\ \sigma = \sqrt{Var(x)} = \sqrt{1.75} = 1.3115$$

(iii)	$Var(3-2x) = Var(3) - 2Var(x),$ $but\ Var(x) = 1.75$ $Var(3-2x) = 0 - 2(1.75) = -3.5.$
(iv)	P(x$\geq 2/x \leq 4) = \frac{P(x\geq 2 \cap x \leq 4)}{P(x\leq 4)}, = \frac{P(x=2,3,4)}{P(1,2,3,4)}, = 0.6/0.7, = 6/7$
17(a) (i)	$P(A) = \frac{2}{5}, P(B) = \frac{1}{2},$ $P(AnB) = 0, Since\ they\ are\ Mutually\ Exclusive\ events.$
(ii)	$P(An\bar{B}) = P(A) = \frac{2}{5}. OR. P(AnB) + P(An\bar{B}) = P(A), P(An\bar{B}) = \frac{2}{5}$
(iii)	P$(\bar{A}n\bar{B}) + P(An\bar{B}) = P(\bar{B})$ $P(\bar{B}) = 1 - P(B) = \frac{1}{2},$ P$(\bar{A}n\bar{B}) = \frac{1}{2} - \frac{2}{5} = 1/10$
b(i)	Let Isuzu make be x, super tyre make be y, tata make be z Let R represent cars with radios $P(x) = \frac{60}{100} = \frac{3}{5},$ $P(y) = \frac{25}{100} = \frac{1}{4}$ $P(z) = \frac{15}{100} = \frac{3}{20}$ $P(x/R) = \frac{50}{100} = \frac{1}{2}$ $P\left(y/R\right) = \frac{5}{100} = \frac{1}{20}$ $P(z/R) = \frac{1}{100}.$ $P(R) = P(x).P\left(R/x\right) + P(y).P\left(R/y\right) + P(z).P\left(R/z\right)$ $= \left(\frac{3}{5} \times \frac{1}{2}\right) + \left(\frac{1}{4} \times \frac{1}{20}\right) + \left(\frac{3}{20} \times \frac{1}{100}\right) = \frac{157}{500}$
(ii)	P$(y/R) = \frac{P(ynR)}{P(R)} = \frac{\frac{1}{80}}{\frac{157}{500}} = 0.0398$
18(a) (i)	$\int_{-\infty}^{+\infty} f(x)dx = 1; \int_0^1 kxdx + \int_1^2 \frac{k}{2} xdx = 1,\ k\left[\frac{x^2}{2}\right]_0^1 + \frac{k}{2}\left[\frac{x^2}{2}\right]_1^2 = 1,$ $\frac{k}{2}[(1^2 - 0^2)] + \frac{k}{4}[(2^2 - 1^2)] = 1,$ $\frac{k}{2} + \frac{3k}{4} = 1, k = \frac{4}{5}$
(ii)	E(x)=$\int xf(x)dx$

	$\frac{4}{5}\int_0^1 x^2dx + \frac{2}{5}\int_1^2 x^2dx = \frac{4}{5}\left[\frac{x^3}{3}\right]\begin{matrix}1\\0\end{matrix} + \frac{2}{5}\left[\frac{x^3}{3}\right]\begin{matrix}2\\1\end{matrix}$ $= \frac{4}{15}[1^3 - 0^3] + \frac{2}{15}[2^3 - 1^3] = \frac{4}{15} + \frac{14}{15} = \frac{18}{15}.$
(iii)	Median; $k\int_0^1 xdx \geq \frac{1}{2}, \frac{4}{5}\int_0^1 xdx \geq \frac{1}{2}$ $\frac{4}{5}\left[\frac{x^2}{2}\right]\begin{matrix}1\\0\end{matrix} \geq \frac{1}{2}$ $\frac{4}{10}(1^2 - 0^2) \geq \frac{1}{2}$ $\frac{2}{5} \leq \frac{1}{2}$ $Since \frac{2}{5} \leq \frac{1}{2}, the\ median\ lies\ in\ the\ second\ interval.$ $\frac{2}{5} + \frac{2}{5}\int_1^m xdx = \frac{1}{2}, \frac{2}{5}\left[\frac{x^2}{2}\right]\begin{matrix}m\\1\end{matrix} = \frac{1}{10}, \frac{1}{5}(m^2 - 1^2) = \frac{1}{10}, m^2 = \frac{3}{2}, m = \mp 1.225,$ $Median = 1.225.$
(b)	P$\left(\left(\frac{1}{2} \leq x \leq 1\frac{1}{2}\right) =\right.$ $\int_{\frac{1}{2}}^1 kxdx + \frac{k}{2}\int_1^{\frac{3}{2}} xdx, \frac{4}{5}\left[\frac{x^2}{2}\right]\begin{matrix}1\\\frac{1}{2}\end{matrix} + \frac{2}{5}\left[\frac{x^2}{2}\right]\begin{matrix}1.5\\1\end{matrix} \frac{2}{5}(1^2 - 0.5^2) + \frac{1}{5}(1.5^2 - 0.5^2)$ $= 0.3 + 0.25 = 0.55$
19a(i)	$\mu = 50, \sigma = 10, let\ x\ represent\ marks, P(x > 70) = P\left(Z - \frac{70-50}{10}\right) = P(Z > 2)$ $= 0.5 - P(0 < Z < 2), 0.5 - 0.4772, = 0.02275$
(ii)	Between 40 and 60. P(40< $x < 60) = P\left(\frac{40-50}{10} < Z < \frac{60-50}{10}\right)$ $= P(-1 < Z < 1), By\ symmetry, 2(0.3413) = 0.6826.$
b(i)	n=10000 $\sigma = 10$), $\mu = 50$, $P(Z > 65), P\left(Z > \frac{65-50}{10}\right), P(Z > 1.5) = 0.5 -$ $P(0 < Z < 1.5), 0.5 - 0.4332 = 0.0668.$ $Number\ of\ candidates\ are\ 10000X0.0668 = 668\ Students.$
(ii)	$P(Z < 45) = P\left(Z < \frac{45-50}{10}\right), P(Z < -0.5) = P(Z > 0.5) = 0.5 - P(0 < z < 0.5),$ $0.5 - 0.1915 = 0.3085. Number\ of\ students\ is\ 10000X0.3085 = 3085.$
20(a) (i)	Let A represent hot breakfast represent hot lunch. $P(A) = 0.1, P(B) =$ $0.2, P(AnB) = 0.25.$ $P(AnB) = 0.25$
(ii)	P$(^B/_A) = \frac{P(BnA)}{P(A)} = \frac{0.25}{0.1} = 2.5$
(b)	$Box\ A, 4Red\ and\ 3White.\ Box\ B\ ,3Red\ and\ 4White.$ $P(Red\ ball\ is\ drawn) = P(AnR) + P(BnR), = \left(\frac{1}{2} \times \frac{4}{7}\right) + \left(\frac{1}{2} \times \frac{3}{7}\right)$ $= \frac{4}{14} + \frac{3}{14} = \frac{1}{2}$
21(a)	$\overrightarrow{AC} = \overrightarrow{OC}\text{-}\overrightarrow{OA}, = \begin{pmatrix}0\\6\end{pmatrix} - \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}-4\\6\end{pmatrix}$

	$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}, = \begin{pmatrix}-10\\-5\end{pmatrix} - \begin{pmatrix}14\\11\end{pmatrix} = \begin{pmatrix}-26\\-16\end{pmatrix}$ To meet at right angles, $\overrightarrow{AC}.\overrightarrow{BD} = 0,$ $\begin{pmatrix}-4\\6\end{pmatrix}.\begin{pmatrix}-26\\-16\end{pmatrix} = 96 - 96 = 0. Since\ \overrightarrow{AC}.\overrightarrow{BD} =$ $0\ then\ the\ diagonals\ meet\ at\ right\ angles.$ **Similarly,** Gradient for AC;$m_1 = \frac{6-0}{0-4} = \frac{6}{-4},$ $Gradient\ for\ BD; m_2 = \frac{-5-11}{-10-14} = \frac{2}{3},$ $For\ perpendicular\ lines\ m_1 \times\ m_2 = -1,$ $\frac{6}{-4} \times \frac{2}{3} = -1.\ Therefore\ the\ two\ diagonals\ meet\ at\ right\ angles.$ $\lvert\overrightarrow{BD}\rvert = 4\lvert\overrightarrow{AC}\rvert,\ \lvert\overrightarrow{BD}\rvert = \sqrt{(-10-14)^2 + (-5-11)^2} = \sqrt{832}$ $\lvert\overrightarrow{AC}\rvert = \sqrt{(4-0)^2 + (0-6)^2} = \sqrt{52},$ $\frac{\lvert\overrightarrow{BD}\rvert}{\lvert\overrightarrow{AC}\rvert} = \frac{\sqrt{832}}{\sqrt{52}} = 4,$ $Hence\ \lvert\overrightarrow{BD}\rvert = 4\lvert\overrightarrow{AC}\rvert$
(b)	By resolving forces; Horizontally;$\sum x = 1 + 6\cos 30 - 7 = -0.8038.$ Vertically; $\sum y = 5 - 6sin30 - 3 = -1,$ $\lvert R\rvert = \sqrt{(-1)^2 + (-0.8038)^2} = 1.283N.$ $Hence\ the\ magnitude\ of\ the\ resultant\ force\ is\ 1.283N.$ $tan\theta = \frac{-1}{-0.804}, \theta = 51.2^0,$ $The\ direction\ of\ the\ resultant\ force\ is\ 51.2^0\ with\ AB\ produced\ downwards.$
22(a) (i)	$Work\ done = Force\ X\ Distance, 50X20 = 1000J.$
(ii)	$Rate\ at\ which\ foce\ is\ acting = \frac{work\ done}{time} = \frac{1000}{15} = 66.67W$
(b)	$m = 0.6Kg, u = 3.5ms^{-1}, s = 5m, g - 10ms^{-2},$ Using $v^2 = u^2 + 2gs, = (3.5)^2 + 2\ X9.8\ X5, v^2 = 110.25, v = 10.5ms^{-1}.$ $The\ body\ reaches\ O\ with\ a\ speed\ of\ 10.5ms^{-1}$

END

THANK YOU SO MUCH FOR ATTENDING MAY GOD BLESS YOU ABUNDANTLY