1. $\log_5 x + \log_x 5 = 5/2$

$\log_5 x + \frac{1}{\log_5 x} = 5/2$ M1

let $y = \log_5 x$

$y + 1/y = 5/2$

$2y^2 + 2 = 5y$

$2y^2 - 5y + 2 = 0$ A1

$2y^2 - 4y - y + 2 = 0$

$2y(y-2) - 1(y-2) = 0$ M1

$(2y-1)(y-2) = 0$

$y = \frac{1}{2}$ or $y = 2$ A1

for $y = 2$

$\log_5 x = 2$

$x = 5^2 = 25$

for $y = \frac{1}{2}$

$\log_5 x = \frac{1}{2}$

$x = 5^{1/2} = \sqrt{5}$ B1 /05

2. $y = \ln\left(\frac{(1+2x)^{1/2}}{(1-x)^3}\right)$

$y = \ln(1+2x)^{1/2} - \ln(1-x)^3$

$y = \frac{1}{2}\ln(1+2x) - 3\ln(1-x)$ M1

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2}{1+2x} - 3 \cdot \frac{-1}{1-x}$ M1 M1

$$\frac{dy}{dx} = \frac{1}{1+2x} + \frac{3}{1-x}$$

$$\frac{dy}{dx} = \frac{1-x+3+6x}{(1+2x)(1-x)} \quad M_1$$

$$\frac{dy}{dx} = \frac{5x+4}{(1+2x)(1-x)} \quad A_1 \quad /05$$

3. $\sin(\theta-45^\circ) = 3\cos(\theta+45^\circ)$

$\sin\theta\cos45^\circ - \cos\theta\sin45^\circ = 3\cos\theta\cos45^\circ - 3\sin\theta\sin45^\circ$ M_1

$\sin\theta\cos45^\circ + 3\sin\theta\sin45^\circ = 3\cos\theta\cos45^\circ + \cos\theta\sin45^\circ$

$\sin\theta(\cos45^\circ + 3\sin45^\circ) = \cos\theta(3\cos45^\circ + \sin45^\circ)$ M_1

$$\tan\theta = \frac{3\cos45^\circ + \sin45^\circ}{\cos45^\circ + 3\sin45^\circ}$$

$$\tan\theta = \frac{3+\tan45^\circ}{1+3\tan45^\circ} = \frac{4}{4} = 1 \quad A_1$$

$\theta = 45^\circ, 225^\circ$ M_1

$\theta = \pi/4, \; 5\pi/4$ A_1 /05

4. $2x^2 - 4xy + 3y^2 - 8x = 2$

$4x - 4\left(x\frac{dy}{dx} + y\cdot1\right) + 6y\frac{dy}{dx} - 8 = 0$ $M_1 A_1$

$4x - 4x\frac{dy}{dx} - 4y + 6y\frac{dy}{dx} - 8 = 0$

$(6y-4x)\frac{dy}{dx} = 8+4y-4x$

$$\frac{dy}{dx} = \frac{8+4y-4x}{6y-4x} = \frac{4+2y-2x}{3y-2x} \quad B_1$$

At $P(1,-1)$

$$\frac{dy}{dx} = \frac{4 + 2x - 1 - 2(1)}{3(-1) - 2(1)}$$ M1

$$\frac{dy}{dx} = 0.$$ A1

/05

5. let the ages be

$\frac{a}{r}$, a and ar

$\frac{a}{r} \cdot a \cdot ar = 1728$

$a^3 = 1728$

$a = 12$ B1

$\frac{a}{r} + a + ar = 52$

$\frac{12}{r} + 12 + 12r = 52$ M1

$12 + 12r^2 = 40r$

$12r^2 - 40r + 12 = 0$

$3r^2 - 10r + 3 = 0$

$3r^2 - 9r - r + 3 = 0$

$3r(r-3) - 1(r-3) = 0$ M1

$(3r-1)(r-3) = 0$

$r = \frac{1}{3}$ or $r = 3$ A1

take $r = 3$.

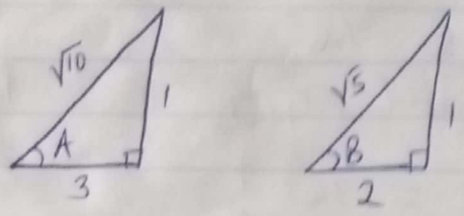
Age of mother $= ar = 12 \times 3 = 36$ years B1

05

6. $LHS = \tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

let $A = \tan^{-1}\left(\frac{1}{3}\right)$ & $B = \cancel{\tan}\ \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\tan A = \frac{1}{3}$, $\sin B = \frac{1}{\sqrt{5}}$ B1



$\tan B = \frac{1}{2}$ B1

$LHS = A + B$

$LHS = \tan^{-1}(\tan(A+B))$

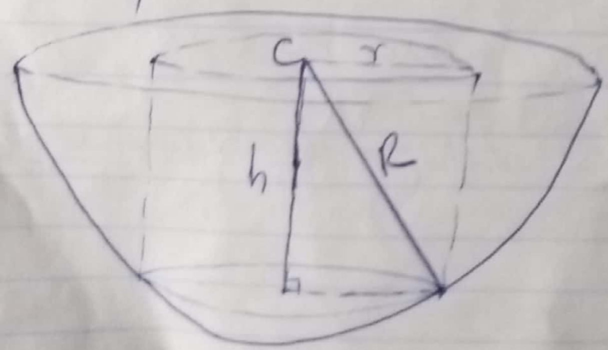
$LHS = \tan^{-1}\left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right)$ m1

$LHS = \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}}\right)$ m1

$LHS = \tan^{-1}(1) = 45^\circ$

$LHS = \frac{\pi}{4} = RHS$. A1 /05

7.



$r^2 + h^2 = R^2$

$r^2 = R^2 - h^2$ B1

$$V = \pi r^2 h$$

$$V = \pi (R^2 - h^2) h$$

$$V = \pi (R^2 h - h^3) \qquad B1$$

$$\frac{dv}{dh} = \pi (R^2 - 3h^2) \qquad B1$$

$$\frac{dv}{dh} = 0 \text{ for } V_{max}$$

$$\pi (R^2 - 3h^2) = 0$$

$$R^2 = 3h^2$$

$$h = \frac{1}{\sqrt{3}} R \qquad B1$$

$$V_{max} = \pi \left(R^2 \cdot \frac{1}{\sqrt{3}} R - \left(\frac{1}{\sqrt{3}} R\right)^3 \right)$$

$$= \pi \left(\frac{R^3}{\sqrt{3}} - \frac{R^3}{3\sqrt{3}} \right)$$

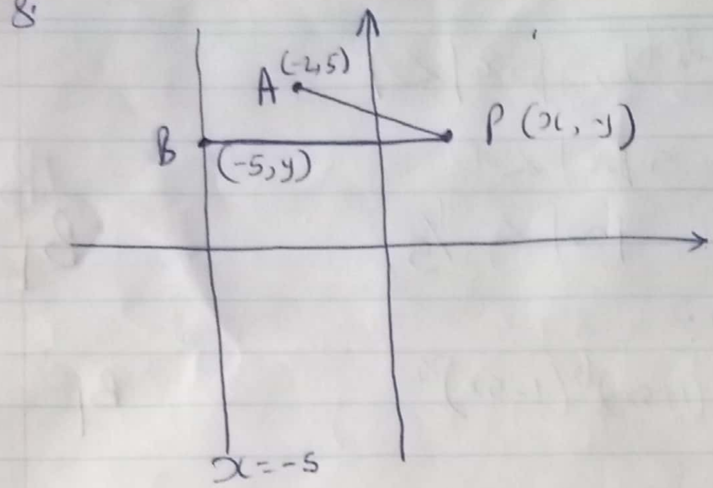
$$= \frac{\pi}{3\sqrt{3}} (3R^3 - R^3)$$

$$= \frac{\pi}{3\sqrt{3}} \cdot 2R^3$$

$$V_{max} = \frac{2\sqrt{3}}{9} \pi R^3 \qquad B1$$

05

8.



$\overline{PA} = \frac{1}{2}\overline{PB}$

$2\overline{PA} = \overline{PB}$

$4\overline{PA}^2 = \overline{PB}^2$ — M1

$4\left((x--2)^2+(y-5)^2\right) = (x--5)^2+(y-y)^2$ M1 M1

$4\left((x^2+4x+4)+y^2-10y+25\right) = x^2+10x+25$

$4x^2+16x+16+4y^2-40y+100 = x^2+10x+25$ M1

$3x^2+4y^2+6x-40y+91=0$ A1

05

SECTION B.

9(a) $\sqrt{\frac{1+5x}{1-5x}} = \sqrt{\frac{(1+5x)^2}{(1-5x)(1+5x)}}$ M1

$= (1+5x)(1-25x^2)^{-1/2}$ A1 B1

$= (1+5x)\left(1+-\frac{1}{2}(-25x^2)+--\right)$ M1

$=(1+5x)\left(1+\frac{25x^2}{2}---\right)$

$=1+\frac{25}{2}x^2+5x+\frac{125}{2}x^3$ M1

$\sqrt{\frac{1+5x}{1-5x}} = 1+5x+\frac{25}{2}x^2+\frac{125}{2}x^3+---$ A1 for validity, $|5x| < 1$

$$5|x| < 1$$

$$|x| < 1/5 \qquad B1$$

ALT:

$$\sqrt{\frac{1+5x}{1-5x}} = (1+5x)^{1/2}(1-5x)^{-1/2} \qquad B1$$

$$(1+5x)^{1/2} = 1 + \tfrac{1}{2}(5x) + \frac{\frac{1}{2}(\frac{1}{2}-1)(5x)^2}{2!} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(5x)^3}{3!} + \cdots \quad M1$$

$$(1+5x)^{1/2} = 1 + \frac{5x}{2} - \frac{25}{8}x^2 + \frac{125}{16}x^3 \qquad A1$$

$$(1-5x)^{-1/2} = 1 + -\tfrac{1}{2}(-5x) + \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-5x)^2}{2!} + \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)(-5x)^3}{3!} \quad M1$$

$$= 1 + \frac{5x}{2} + \frac{75}{8}x^2 + \frac{625}{16}x^3 \qquad A1$$

$$\sqrt{\frac{1+5x}{1-5x}} = \left(1 + \frac{5x}{2} - \frac{25}{8}x^2 + \frac{125}{16}x^3\right)\left(1 + \frac{5x}{2} + \frac{75}{8}x^2 + \frac{625}{16}x^3\right)$$

$$= 1 + \frac{5x}{2} + \frac{75}{8}x^2 + \frac{625}{16}x^3 + \frac{5x}{2} + \frac{25}{4}x^2 + \frac{375}{16}x^3 - \frac{25}{8}x^2 - \cancel{\frac{125}{16}x^3} + \cancel{\frac{125}{16}x^3}$$

$$\sqrt{\frac{1+5x}{1-5x}} = 1 + 5x + \frac{25}{2}x^2 + \frac{125}{2}x^3 + \cdots \quad A1$$

(b) $$\sqrt{\frac{1+5(1/9)}{1-5(1/9)}} = 1 + 5\left(\tfrac{1}{9}\right) + \frac{25}{2}\left(\tfrac{1}{9}\right)^2 + \frac{125}{2}\left(\tfrac{1}{9}\right)^3$$

M1 M1

$$\sqrt{\frac{14/9}{4/9}} = 1 + \frac{5}{9} + \frac{25}{162} + \frac{125}{1458} \qquad M1$$

$$\frac{\sqrt{14}}{2} = 1.79561 \qquad A1$$

$\sqrt{14} = 3.591$ (4 s.f) B_1 /12

10 (a) $R(4,2)$, $P(2,0)$, $Q(6,8)$

let the equation be $x^2+y^2+2gx+2fy+c=0$

At $(4,2)$

$4^2+2^2+2g(4)+2f(2)+c=0$

$8g+4f+c=-20$ ---(i) B_1

At $(2,0)$

$2^2+0^2+2g(2)+2f(0)+c=0$

$4g+c=-4$ ---(ii) B_1

At $(6,8)$

$6^2+8^2+2g(6)+2f(8)+c=0$

$12g+16f+c=-100$ ---(iii) B_1

(i) $-$ (ii)

$4g+4f=-16$

$g+f=-4$ ---(iv)

(iii) $-$ (ii)

$8g+16f=-96$

$g+2f=-12$ —

$g=-12-2f$ ---(v) m_1

Use (v) in (iv)

$-12-2f+f=-4$

$-f=8$

$f=-8$ A_1

$g=-12-2(-8)=4$

from (ii), $c=-4-4\times4=-20$ A_1

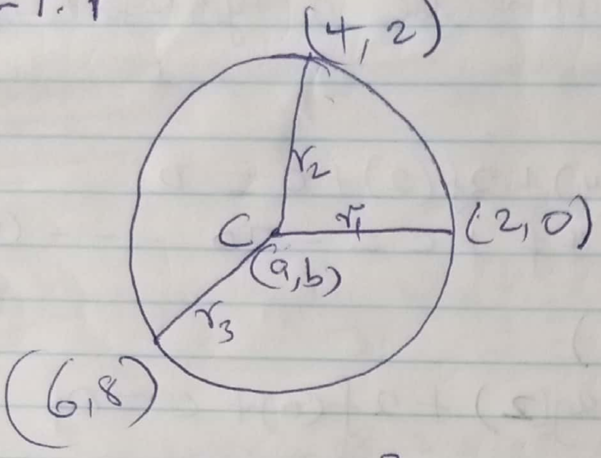
1.

$$x^2 + y^2 + 2(4)x + 2(-8)y + -20 = 0$$

$$x^2 + y^2 + 8x - 16y - 20 = 0 \quad . \qquad B_1$$

$\overline{07}$

ALT. 1



$$r_1^2 = r_2^2$$

$$(a-2)^2 + (b-0)^2 = (a-4)^2 + (b-2)^2 \qquad m_1$$

$$a^2 - 4a + 4 + b^2 = a^2 - 8a + 16 + b^2 - 4b + 4$$

$$4a + 4b = 16$$

$$a + b = 4$$

$$b = 4 - a \quad --- \quad (1) \qquad A_1$$

$$r_1^2 = r_3^2$$

$$(a-2)^2 + (b-0)^2 = (a-6)^2 + (b-8)^2 \qquad m_1$$

$$a^2 - 4a + 4 + b^2 = a^2 - 12a + 36 + b^2 - 16b + 64$$

$$8a + 16b = 96$$

$$a + 2b = 12 \quad --- \quad (11) \qquad A_1$$

Use (1) ~ (11)

$$a + 2(4 - a) = 12$$

$$a + 8 - 2a = 12$$

$$-a = 4$$

$$a = -4$$

$$b = 4 - -4 = 8$$

$$C(-4, 8) \qquad B_1$$

radius, $r^2 = (-4-2)^2 + (8-0)^2$

$r^2 = 100$ B1

from $(x-a)^2 + (y-b)^2 = r^2$

$(x - -4)^2 + (y-8)^2 = 100$

$x^2 + 8x + 16 + y^2 - 16y + 64 = 100$

$x^2 + y^2 + 8x - 16y - 20 = 0$. B1 /07

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1(b) from $x^2+y^2+8x-16y-20=0$

$2x+2y\frac{dy}{dx}+8-16\frac{dy}{dx}-0=0$ M1 A1

$2(y-8)\frac{dy}{dx}=-2x-8$

$\frac{dy}{dx}=\frac{-x-4}{y-8}$ A1

At $(4,2)$

$\frac{dy}{dx}=m_1=\frac{-4-4}{2-8}=\frac{-8}{-6}=\frac{4}{3}$ B1

eqn: $\frac{y-2}{x-4}=\frac{4}{3}$ M1

$3y-6=4x-16$

$3y=4x-10$ A1/05

$=12=$

11. Let $\dfrac{5x^2-3x+1}{(x-2)(x^2+1)} \equiv \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+1}$ B1

$5x^2-3x+1 \equiv A(x^2+1) + (Bx+C)(x-2)$ M1

put $x = 2$

$5(2)^2-6+1 = 5A$

$15 = 5A$

$A = 3$. A1

Put $x = 0$

$1 = A - 2C$

$1 = 3 - 2C$

$2C = 2$

$C = 1$ A1

Put $x = 1$

$5-3+1 = 2A - B - C$

$3 = 2(3) - B - 1$

$B = 6-3-1$

$B = 2$ A1

$\therefore \dfrac{5x^2-3x+1}{(x-2)(x^2+1)} = \dfrac{3}{x-2} + \dfrac{2x+1}{x^2+1}$ B1

$$\frac{d}{dx}\left[\frac{5x^2-3x+1}{(x-2)(x^2+1)}\right]=\frac{d}{dx}\left(\frac{3}{x-2}\right)+\frac{d}{dx}\left(\frac{2x+1}{x^2+1}\right)$$

$$=\frac{d}{dx}\left[3(x-2)^{-1}\right]+\frac{(x^2+1)2-(2x+1)2x}{(x^2+1)^2}$$ m m

$$=-3(x-2)^{-2}+\frac{2x^2+2-4x^2-2x}{(x^2+1)^2}$$ m

$$=\frac{-3}{(x-2)^2}+\frac{2-2x-2x^2}{(x^2+1)^2}$$ A

when $x=3$

$$\frac{d}{dx}\left(\frac{5x^2-3x+1}{(x-2)(x^2+1)^2}\right)=\frac{-3}{(3-2)^2}+\frac{2-2(3)-2(3)^2}{(3^2+1)^2}$$ m

$$=-3-\frac{22}{10}$$

$$=-\frac{26}{5}=-5.2$$ A

06

=12=

12 (a) $\tan 3\theta = \tan(2\theta + \theta)$

$$= \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$$ M1

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \frac{2\tan\theta}{1-\tan^2\theta} \cdot \tan\theta}$$ M1

$$= \frac{\frac{2\tan\theta + \tan\theta - \tan^3\theta}{1-\tan^2\theta}}{\frac{1-\tan^2\theta - 2\tan^2\theta}{1-\tan^2\theta}}$$ M1

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$ A1

$$\tan 3\theta = \frac{3t - t^3}{1-3t^2}$$ B1

/05

(b) $\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ M1

$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ M1

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. A1

Hence from $4x^2 - 3x - \frac{\sqrt{2}}{2} = 0$

$4x^3 - 3x = \frac{\sqrt{2}}{2}$

By comparison,

$x = \cos\theta$

$\cos 3\theta = \frac{\sqrt{2}}{2}$ B1

$3\theta = 45°, 315°, 405°$ m1

$\theta = 15°, 105°, 135°$ A1

$x = \cos\theta$

$x = \cos 15°, \cos 105°, \cos 135°$

$x = 0.9659, -0.2588, -0.7071$ A1

$\frac{07}{}$

13 (a) $\frac{x}{2+3i} - \frac{y}{3-2i} = \frac{6+2i}{8+i}$ =12=

$$\frac{x(2-3i)}{(2+3i)(2-3i)} - \frac{y(3+2i)}{(3-2i)(3+2i)} = \frac{(6+2i)(8-i)}{(8+i)(8-i)}$$ m1

$$\frac{2x-3ix}{2^2+3^2} - \frac{3y+2iy}{3^2+2^2} = \frac{48-6i+16i-2i^2}{8^2+1^2}$$ m1

$$\frac{2x-3ix}{13} - \frac{3y+2iy}{13} = \frac{50+10i}{65}$$ A1

$$\frac{2x-3y-i(3x+2y)}{13} = \cancel{50+} \frac{10+2i}{13}$$

$3|\ 2x-3y = 10$ ---(i)

$2|\ -3x+2y = 2$ ---(ii) B1

$6x - 9y = 30$

$+\ -6x + 4y = 4$

$-13y = 34$ m1

$y = -34/13 = -2.6154$ A1

$x = [10 + 3(\frac{-34}{13})] \div 2$ $x = -5 - 2$ $= 14/13 = 1.0769$ A1

07

(b) $z\bar{z} + 3\bar{z} = 34 + 12i$

Let $z = x + iy$, $\bar{z} = x - iy$

$(x+iy)(x-iy) + 3(x-iy) = 34 + 12i$ M1

$x^2 + y^2 + 3x - 3yi = 34 + 12i$

$-3y = 12$

$y = -4$ A1

$x^2 + y^2 + 3x = 34$

$x^2 + (-4)^2 + 3x - 34 = 0$ M1

$x^2 + 3x - 18 = 0$ B1

$x^2 + 6x - 3x - 18 = 0$

$x(x+6) - 3(x+6) = 0$

$(x-3)(x+6) = 0$

$x = 3$ or $x = -6$ A1

$\therefore z = 3 - 4i$ or $z = -6 - 4i$ B1

05

=12=

14 (a) $x = \frac{1+t^2}{1-t}$

$\frac{dx}{dt} = \frac{(1-t)(2t) - (1+t^2)(-1)}{(1-t)^2}$ M1

$\frac{dx}{dt} = \frac{2t - 2t^2 + 1 + t^2}{(1-t)^2}$

$\frac{dx}{dt} = \frac{1 + 2t - t^2}{(1-t)^2}$ A1

$$y = \frac{2t}{1-t}$$

$$\frac{dy}{dt} = \frac{(1-t)(2) - 2t(-1)}{(1-t)^2} \quad M1$$

$$\frac{dy}{dt} = \frac{2 - 2t + 2t}{(1-t)^2} = \frac{2}{(1-t)^2} \quad A1$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2}{(1-t)^2} \cdot \frac{(1-t)^2}{1+2t-t^2}$$

$$\frac{dy}{dx} = \frac{2}{1+2t-t^2} \quad B1$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(1+2t-t^2)0 - 2(2-2t)}{(1+2t-t^2)^2} \quad M1$$

$$= \frac{-4(1-t)}{(1+2t-t^2)^2} \quad A1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = -\frac{4(1-t)}{(1+2t-t^2)^2} \times \frac{(1-t)^2}{(1+2t-t^2)}$$

$$\frac{d^2y}{dx^2} = -4\left(\frac{1-t}{1+2t-t^2}\right)^3 \quad B1$$

/08

14 (b) $y = \sin 2x$

$y + \delta y = \sin(2x + 2\delta x)$

$\delta y = \sin(2x + 2\delta x) - \sin 2x$ m1

$\delta y = 2\cos\left(\frac{2x + 2\delta x + 2x}{2}\right)\sin\left(\frac{2x + 2\delta x - 2x}{2}\right)$ m1

$\delta y = 2\cos(2x + \delta x)\sin\delta x$

$\sin\delta x \simeq \delta x$

$\delta y = 2\cos(2x + \delta x)\cdot\delta x$ A1

$\frac{\delta y}{\delta x} = 2\cos(2x + \delta x)$

As $\delta x \to 0$, $\frac{\delta y}{\delta x} \simeq \frac{dy}{dx}$

$\frac{dy}{dx} = 2\cos 2x$. B1

/04

=12=

15 (a) $\sin x - \sin 4x = \sin 2x - \sin 3x$

$\sin 3x + \sin x = \sin 4x + \sin 2x$

$2\sin 2x\cos x = 2\sin 3x\cos x$ M1

$\sin 2x\cos x - \sin 3x\cos x = 0$

$\cos x(\sin 2x - \sin 3x) = 0$ m1

$2\cos x\cos\frac{5}{2}x\sin\left(-\frac{x}{2}\right) = 0$ m1

$-2\cos\frac{5}{2}x\cos x\sin\frac{x}{2} = 0$

$\cos\frac{5}{2}x = 0$ or $\cos x = 0$ or $\sin\left(\frac{x}{2}\right) = 0$ A1

For $\cos \frac{5}{2}x = 0$

$\frac{5}{2}x = \pm 90^\circ, \pm 270^\circ, \pm 450^\circ$

$x = \pm 36^\circ, \pm 108^\circ, \pm 180^\circ$ B1

For $\cos x = 0$

$x = \pm 90^\circ$

For $\sin(x/2) = 0$

$x/2 = 0^\circ$

$x = 0^\circ$ B1 /06

(b) $\cot(-15^\circ) = \frac{-1}{\tan 15^\circ} = \frac{-1}{\tan(60-45^\circ)}$ M1

$= \frac{-1}{\frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}}$ M1

$= \frac{-1}{\frac{\sqrt{3}-1}{1+\sqrt{3}}}$ A1

$= -\frac{(1+\sqrt{3})}{\sqrt{3}-1}$

$= \frac{-(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ M1

$= -\frac{(\sqrt{3}+1+3+\sqrt{3})}{(\sqrt{3})^2-(1)^2}$ M1

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$$= -\frac{(4+2\sqrt{3})}{2}$$

$\text{Cot}(-15°) = -2-\sqrt{3}$ A1 /06 =12=

16 (a) $y = x(x-1)(x-2)$

when $x=0$, $y=0$

pt $(0,0)$

when $y=0$

$x=0$, $x=1$, $x=2$

pts $(0,0)$, $(1,0)$, $(2,0)$ B1

Turning points

$y = (x^2-x)(x-2)$

$y = x^3-2x^2-x^2+2x$

$y = x^3-3x^2+2x$

$\frac{dy}{dx} = 3x^2-6x+2$ B1

$\frac{dy}{dx} = 0$ for turning points

$3x^2-6x+2=0$

$$x = \frac{--6 \pm \sqrt{(-6)^2-4(3)(2)}}{2\times3}$$

$x = 1.5774$ or $x = 0.4226$ B1

for $x = 1.5774$

$y = 1.5774^3-3(1.5774)^2+2\times1.5774$

$y = -0.3849$. pt $(1.5774, -0.3849)$ B1 for $x = 0.4226$

$y = 0.4226^3 - 3\times 0.4226^2 + 2(0.4226)$

$y = 0.3849$

pt $(0.4226, 0.3849)$ B1

Nature: $\frac{d^2y}{dx^2} = 6x - 6$

At $x = 1.5774$, $\frac{d^2y}{dx^2} = +ve$

$\therefore (1.5774, -0.3849)_{min}$

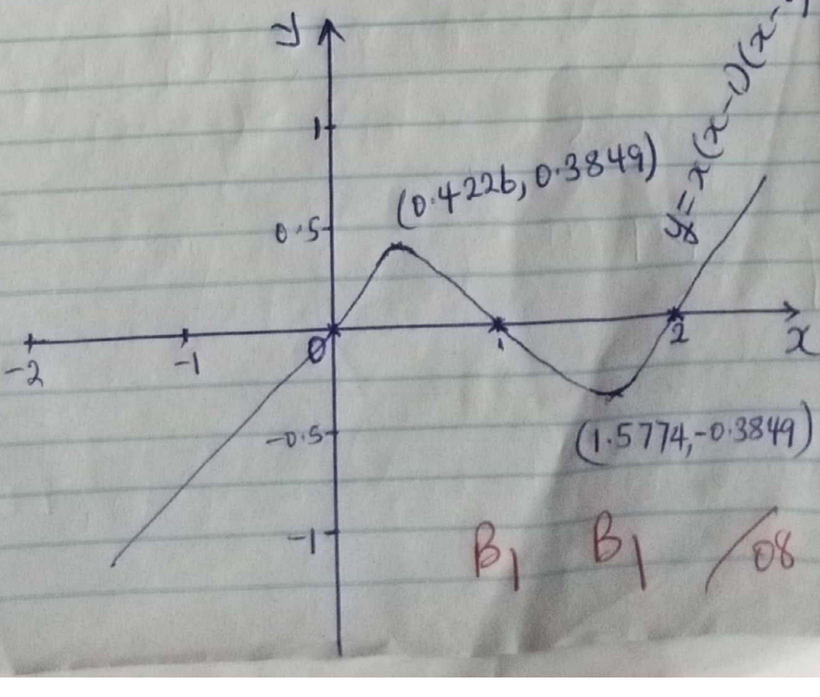
At $x = 0.4226$

$\frac{d^2y}{dx^2} = -ve$

$\therefore (0.4226, 0.3849)_{max}$ B1

from $y^3 = x^3 - 3x^2 + 2x$

As $x \to \pm\infty$, $y \to \pm\infty$



B1 B1 /08

16 (b) $\frac{dv}{dt} = 3\,cm^3 s^{-1}$

$V = x^3$, $x = \text{side}$, $\frac{dx}{dt} = ?$

$\frac{dv}{dx} = 3x^2$ B1

$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$3 = 3x^2 \frac{dx}{dt}$ m1

$\frac{dx}{dt} = \frac{1}{x^2}$ A1

when $x = 5\,cm$

$\frac{dx}{dt} = \frac{1}{5^2} = \frac{1}{25} = 0.04\,cm\,s^{-1}$ B1

$\overline{04}$

$= 12 =$

PURE MATHEMATICS

Paper 1

Nov. 2022

3 hours

ST. MARY'S COLLEGE, RUSHOROZA:

Uganda Advanced Certificate of Education

S.5 (P425/1) PURE MATHEMATICS END OF 3RD TERM EXAMS - 2022

TIME: 3 Hours

INSTRUCTIONS:

- Answer all the **eight** questions in Section **A** and **only five** questions from Section **B**.
- Any additional question(s) answered will **not** be marked.
- All necessary working must be shown clearly.
- Mathematical tables with a list of formulae are provided.

SECTION A: (40 marks)

1. Solve the equation

$$\log_5 x + \log_x 5 = 2.5$$

2. If $y = \ln\left[\frac{\sqrt{1+2x}}{(1-x)^3}\right]$, show that $\frac{dy}{dx} = \frac{5x+4}{(1+2x)(1-x)}$

3. Solve the equation: $\sin(\theta - 45^\circ) = 3\cos(\theta + 45^\circ)$ for $0 \le \theta \le 2\pi$.

4. Find the gradient of the curve $2x^3 - 4xy + 3y^2 - 8x = 2$ at the point $P(1, -1)$.

5. The current ages of a mother and her two daughters are in a geometrical progression. The sum of their ages is 52 years and the product of their ages is 1728 years. Determine the age of the mother.

6. Show that $\tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$.

7. A cylinder is to be inscribed inside a hemisphere of radius R. Show that the maximum volume of the cylinder is $V_{max} = \frac{2\pi\sqrt{3}}{9}R^3$.

8. Find the locus of a point $P(x, y)$ whose distance from $(-2, 5)$ is half its distance from the line $x = -5$.

SECTION B: (60 MARKS)

9. (a) Expand $\sqrt{\frac{1+5x}{1-5x}}$ in ascending powers of x as far as the term in x^3.

   State the values of x for which the expansion is valid. By substituting $x = \frac{1}{9}$.

   (b) Use your expansion to estimate $\sqrt{14}$ to your significant figures.

10. (a) Find the equation of the circle passing through the points R(4, 2), P(2, 0) Q(6, 8). (7mks)

    (b) Determine the equation of the tangent to the circle in (a) above at point R(4, 2). (5mks)

11. Express $\frac{5x^2-3x+1}{(x-2)(x^2+1)}$ in partial fractions hence determine $\frac{d}{dx}\left[\frac{5x^2-3x+1}{(x-2)(x^2+1)}\right]$ at $x = 3$. (12mks)

12. (a) Prove that $\tan 3\theta = \frac{3t-t^3}{1-3t^2}$ where $t = tan\theta$. (4mks)

    (b) Show that $cos3\theta = 4cos^3\theta - 3cos\theta$ hence solve the equation $4x^3 - 3x - \frac{\sqrt{2}}{2} = 0$. (8mks)

13. (a) Find the values of x and y in $\frac{x}{2+3i} - \frac{y}{3-2i} = \frac{6+2i}{8+i}$. (6mks)

    (b) Given that the complex number Z and its conjugate $\bar{Z}$ satisfy the equation $Z\bar{Z} + 3\bar{Z} = 34 + 12i$, find the values of Z. (6mks)

14. (a) If $x = \frac{1+t^2}{1-t}$ and $y = \frac{2t}{1-t}$, find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} = -4\left(\frac{1-t}{1+2t-t^2}\right)^3$. (7mks)

    (b) Differentiate $y = sin2x$ from first principles. (8mks)

15. (a) Solve $sinx - sin4x = sin2x - sin3x$ for $-180^o \leq x \leq 180^o$. (6mks)

    (b) Without using tables or calculator, find the value of cot (-15^o) leaving your answer in the simplest surd form. (6mks)

16. (a) Sketch the curve $y = x(x-1)(x-2)$. (8mks)

    (b) The volume of a cube is increasing at a rate of $3cm^3s^{-1}$.

    Find the rate of the change of side when its length is 5cm. (4mks)

= END =

MERRY CHRISTMAS AND HAPPY NEW YEAR 2023