KITABI SEMINARY

INTERNAL MOCK EXAMINATIONS 2023

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer ***all*** *the* ***eight*** *questions in section A and only* ***five*** *questions from section* ***B.***

Any additional question(s) answered will not be marked

All necessary working ***must*** *be shown clearly*

Begin each answer on a fresh sheet of paper

Graph paper is provided

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2}.

Turn Over

SECTION A

1. A and B are two events such that $P(B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{12}$ and $P(B/A) = \frac{1}{3}$. Find

   (i) $P(A \cup \overline{B})$ (ii) $P(\overline{A}/B)$

   (05 marks)

2. ABCDEF is a regular hexagon of side 2m. Forces of 2N, 3N, 4N, and 5N act along AC, AE, AF, and ED respectively. Taking AB as the horizontal, find the magnitude of the resultant force. *(05 marks)*

3. The table below shows an extract for $\tan\theta$:

θ	$10'$	$12'$	$14'$	$16'$
$\tan 40^0$	0.8441	0.8451	0.8461	0.8471

   Use linear interpolation or extrapolation to estimate the value of:

   (i) $\tan 40^0 13'$ (ii) $\tan^{-1} 0.8561$.

   (05 marks)

shape/subintervals
ordinates
strips
subintervals

4. Forces $F_1 = \binom{3}{-5}N$ and $F_2 = \binom{-3}{5}N$ act at points (6, 1) and (4, 1) respectively. Show that the forces reduce to a couple and find the moment of the couple. *(05 marks)*

5. A continuous random variable X is uniformly distributed over the interval $\alpha \le x \le \beta$. Given that $E(X) = 2$ and $P(X \le 3) = \frac{5}{8}$. Find the;

   (a) values of α and β *(04 marks)*

   (b) $p.d.f$ of X *(01 mark)*

   d-1

6. A particle with a position vector $2\boldsymbol{i} + 3\boldsymbol{k}$ at $t = 0$ moves with a constant speed of $6ms^{-1}$ in the direction $\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$. Find the position vector of the particle after 2s. *(05 marks)*

7. Use the trapezium rule with 6 ordinates to estimate $\int_1^2 tan^{-1} xdx$ correct to 4 decimal places. *(05 marks)*

8. The data below shows the ages X of patients and number of days taken, Y to recover from a particular disease.

X	55	51	62	66	72	59	78	55	62	70
Y	34	44	49	49	48	43	51	41	46	51

   (a) Calculate the rank correlation coefficient for the data *(04 marks)*

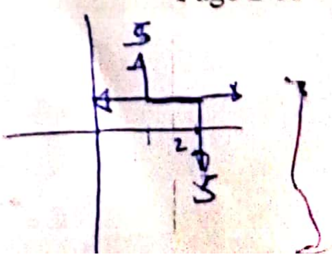
   (b) Comment on the significance of the age on the number of days taken by the patient to recover fully at 1% level of significance. *(01 mark)*

$|\hat{v}| = \frac{v}{|v|}$

0.8461

$\begin{pmatrix} 6 & 3 \\ 1 & -5 \end{pmatrix} \pm \begin{pmatrix} 4 & -3 \\ 1 & 5 \end{pmatrix}$

$-30-3 + 20+3$



SECTION B

9\. a) Numbers X and Y were estimated by x and y with maximum relative errors of E_x and E_y respectively. Show that the maximum relative percentage error in the xy is given by: $\left(\left|\frac{E_x}{x}\right| + \left|\frac{E_y}{y}\right|\right) \times 100$. *(06 marks)*

b) Given that $a = 1.50$, $b = 13.3$ and $c = 9.1000$ are rounded off to the given number of decimal place, find the range of values within which the exact value of $\frac{a-c}{b^2}$ is expected to lie. Give your answer to four decimal places. *(06 marks)*

10\. A ball is projected from a point A and falls at a point B which is in level with A and at a distance of 160m from A. The greatest height of the ball attained is 40m. Find the:

(a) Angle and velocity at which the ball is projected. *(09 marks)*

(b) Time taken for the ball to attain its greatest height. *(03 marks)*

11\. The table below shows the frequency distribution of marks obtained in a mathematics test by a group of students.

Marks(%)	10 –	20 –	30 –	40 –	50 –	60 –	70 –	80 – 90
Frequency	18	34	58	42	24	10	6	8

a) Calculate the:

i) mean mark, *(02 marks)*
ii) standard deviation, *(02 marks)*
iii) number of students who scored above 54%. *(02 marks)*

b) Draw a cumulative frequency curve and use it to estimate the:

i) 5th decile,
ii) number of students who passed the test if the pass mark was 40%,
iii) least mark for a distinction if 10% of the students scored a distinction. *(06 marks)*

12\. (a) A body of mass 4kg lies on a rough plane which is inclined at 30° to the horizontal. The angle of friction between the plane and the body is 15°. Find the magnitude of the least horizontal force that must be applied to the body to prevent motion down the plane. *(06 marks)*

(b) A mass of 12 kg rests on a smooth inclined plane which is 6 m long and 1 m high. The mass is connected by a light inextensible string, which passes over a smooth

Range $\frac{u^2 \sin 2\theta}{g}$

time to max $= \frac{u \sin\theta}{g}$

pulley fixed at the top of the plane, to a mass of 4 kg which is hanging freely. With the string taut, the system is released from rest. Find the acceleration of the system.

(06 marks)

13. A continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} \frac{kx^2}{2} & : 0 \le x < 2 \\ k(4-x) & : 2 < x \le 4 \\ 0 & : otherwise \end{cases}$$

a) Sketch the graph of $f(x)$. (02 marks)

b) Determine the value of the constant, k. (03 marks)

c) Find the (i) median, (04 marks)

(ii) $P(|x-1| < \frac{1}{2})$. (03 marks)

14. a) Show that the simplest iterative formula based on Newton Raphson formula for solving the equation $2x^3 + 5x = 8$ is

$$x_{n+1} = \frac{4x_n^3 + 8}{6x_n^2 + 5}, n = 0, 1, 2, \ldots$$ (03 marks)

b) Draw a flow chart that:

(i) Reads the initial approximation,

(ii) computes and prints the root correct to three decimal places together with the number of iterations.

(05 marks)

c) Taking 1.5 as the initial approximation, perform a dry of the flow chart. (04 marks)

15. A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontally in equilibrium by a string which has one end attached to B and the other end attached to a point C on the wall, 4m above A. Find the magnitude and direction of the reaction at A.

(12 marks)

16. The number of goats owned by residents in a village is assumed to be normally distribution. 15% of the residents have less than 60 goats, 5% of the residents have over 90 goats.

a) Determine the values of the mean and the standard deviation of the ~~cows~~ goats. (08 marks)

b) If there are 200 residents, find how many have more than 80 goats. (04 marks)

END

$\frac{30}{45}$

$\frac{1}{3} + \frac{2}{3}$ $\frac{1}{8}x^2$

$\frac{1}{3} + x + \frac{x^2}{8}$

$4 +$

MARKING GUIDE.

No 1.

$P(B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{12}$, $P(B/A) = \frac{1}{3}$.

(i) $P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B})$.

But from $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{3}$.

$P(A) = 3P(B \cap A) = 3 \times \frac{1}{12} = \frac{1}{4}$.

Also $P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{1}{4} - \frac{1}{12} = \frac{3-1}{12} = \frac{1}{6}$.

$\Rightarrow P(A \cup \bar{B}) = \frac{1}{4} + \left(1 - \frac{5}{6}\right) - \frac{1}{6}$.

$= \frac{1}{4} + \frac{1}{6} - \frac{1}{6} = \frac{1}{4}$

OR

$P(A \cup \bar{B}) = 1 - P(\bar{A} \cap B)$.

$= 1 - [P(B) - P(A \cap B)]$.

$= 1 - \left[\frac{5}{6} - \frac{1}{12}\right]$.

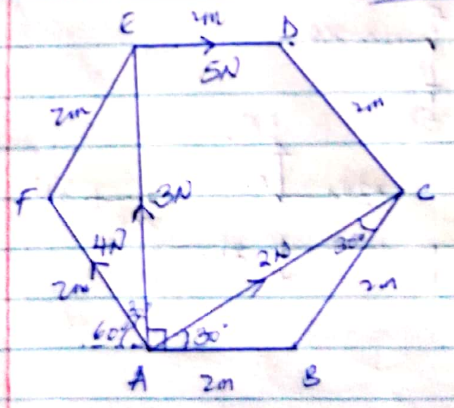
$= 1 - \left[\frac{9}{12}\right] = 1 - \frac{3}{4}$.

$= \frac{1}{4}$.

(ii) $P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{3/4}{5/6} = \frac{3}{4} \times \frac{6}{5} = \frac{9}{10}$

$= 0.9$

No 2.



$\rightarrow R\Sigma F_x = 5 + 2\cos 30° - 4\cos 60°$

$= 4.7321N$.

$\uparrow \Sigma F_y = 3 + 4\cos 30° + 2\cos 60°$

$= 7.4641N$.

Resultant force $R = (4.7321\underset{\sim}{i} + 7.4641\underset{\sim}{j})N$

$|R| = \sqrt{4.7321^2 + 7.4641^2}$

$= \sqrt{78.106}$

$= 8.8377N$.

No 3.

θ	$12'$	13	$14'$
$\tan 40°$	0.8451	y	0.8461

(i) $$\frac{14-12}{0.8461-0.8451} = \frac{14-13}{0.8461-y}$$

$$\frac{2}{0.001} = \frac{1}{0.8461-y}$$

$$1.6922 - 2y = 0.001$$

$$2y = 1.6912$$

$$y = 0.8456$$

(ii)

θ	$14'$	$16'$	x
$\tan 40°$	0.8461	0.8471	0.8561

$$\frac{x-16}{0.8561-0.8471} = \frac{16-14}{0.8471-0.8461}$$

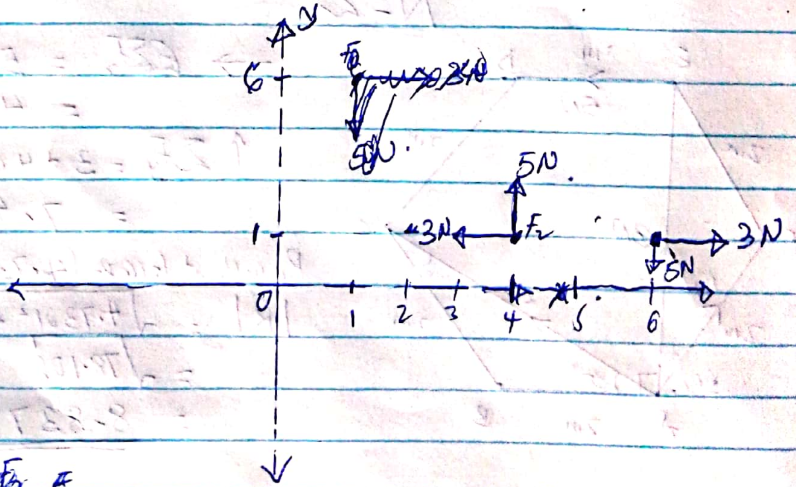
$$\frac{x-16}{0.009} = \frac{2}{0.001}$$

$$0.001x - 0.016 = 0.018$$

$$x = \frac{0.034}{0.001} = 34$$

$$\Rightarrow \tan^{-1} 0.8561 = \underline{\underline{40°34'}}$$

No 4



Suming the forces, $F_1 + F_2 = \begin{pmatrix} 3 \\ -5 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Hence the forces are in eq=m or reduce to a couple.

taking moments about origin,

$\overset{\circlearrowleft}{O},\ G = 5\times4 + 3\times1 - 3\times1 - 5\times6$

$= 20 + 3 - 3 - 30$

$= -10Nm$ anticlockwise.

$= 10Nm$ clockwise.

OR

$$\overset{\circlearrowright}{O}\ G = \begin{array}{cc} f & P_n \\ \end{array}\begin{vmatrix} 3 & 6 \\ 5 & 1 \end{vmatrix} + \begin{array}{cc} f & P_n \\ \end{array}\begin{vmatrix} -3 & 4 \\ 5 & 1 \end{vmatrix}.$$

$= 3\times1 - {}^{-}5\times6 + -3\times1 - 5\times4.$

$= 3 + 30 - 3 - 20.$

$= 10Nm$ clockwise.

OR $\overset{\circlearrowleft}{O},\ G = \begin{array}{cc} P_n & F \end{array}\begin{vmatrix} 6 & 3 \\ 1 & -5 \end{vmatrix} + \begin{array}{cc} P_n & F \end{array}\begin{vmatrix} 4 & -3 \\ 1 & 5 \end{vmatrix}$

$G = -30 - 3 + 20 + 3.$

$= -10Nm$ anticlockwise

$= 10Nm$ clockwise.

No 5.

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha \le x \le \beta. \\ 0, & \text{elsewhere.} \end{cases}$$

$E(X) = 2 \Rightarrow \frac{\beta+\alpha}{2} = 2.$

$\beta = 4 - \alpha$ — (i).

Also $P(X \le 3) = \frac{5}{8}.$

$\Rightarrow \int_{\alpha}^{3} \frac{1}{\beta-\alpha}\,dx = \frac{5}{8}.$

$\frac{1}{\beta-\alpha}\left[x\right]_{\alpha}^{3} = \frac{5}{8}.$

$\frac{1}{\beta-\alpha}\left[3-\alpha\right] = \frac{5}{8}.$

$24 - 8\alpha = 5\beta - 5\alpha.$

$5\beta = 24 - 3\alpha$ — (ii).

sub β in (ii),

$5(4-\alpha) = 24 - 3\alpha.$

$20 - 5\alpha = 24 - 3\alpha.$

$20 - 24 = -3\alpha + 5\alpha.$

$-4 = 2\alpha$

$\alpha = -2.$

$\Rightarrow \beta = 4 - -2 = \underline{\underline{6}}$

(b) p.d.f of X, f(x),

$$f(x) = \begin{cases} \frac{1}{9+7} & , -2 \le x \le 6. \\ 0 & , \text{elsewhere}. \end{cases}$$

No 6.

Position vector, $r_{(t)} = r_0 + Vt$.

$t = 2s$,

but $r_0 = 2i + 3k$,

$V = |V|\hat{V}$

$= 6 \times \frac{d}{|d|}$.

$= 6 \times \frac{(i - 2j + 2k)}{\sqrt{1^2 + (-2)^2 + 2^2}}$.

$= 6 \times \frac{(i - 2j + 2k)}{\sqrt{9}}$.

$= \frac{\cancel{6}^{2}}{\cancel{3}_{1}}(i - 2j + 2k) = (2i - 4j + 4k)\,m/s.$

$\Rightarrow r_{(t=2)} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \times 2$.

$= \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ 11 \end{pmatrix}$

$= (6\underset{\sim}{i} - 8\underset{\sim}{j} + 11\underset{\sim}{k})\,m$.

No 7.

$$\int_1^2 \tan^{-1} x \, dx$$

$$d = \frac{2-1}{6-1} = \frac{1}{5}$$

x	$y = \tan^{-1} x$	
1	0.78540	
6/5		0.87606
7/5		0.95055
8/5		1.01220
9/5		1.06370
2	1.10715	
	1.89255	3.90251

$$\int_1^2 \tan^{-1} x \, dx \approx \frac{d}{2}\left[(y_0 + y_5) + 2(y_1 + \dots + y_4)\right].$$

$$\approx \frac{1/5}{2}\left[1.89255 + 2 \times 3.90251\right].$$

$$\approx \frac{1}{10}\left[9.69757\right]$$

$$\approx 0.969757.$$

$$\approx 0.9698 \cdot (4 d.p).$$

No 8.

(a)

R_x	R_y	d	d^2
8.5	10	-1.5	2.25
10	7	3	9
5.5	3.5	2	4
4	3.5	0.5	0.25
2	5	-3	9
7	8	1	1
1	1.5	-0.5	0.25
8.5	9	-0.5	0.25
5.5	6	-0.5	0.25
3	1.5	1.5	2.25
			$\Sigma d^2 = 28.5$

$$\rho = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}.$$

$$= 1 - \frac{6 \times 28.5}{10(10^2-1)}.$$

$$= 1 - \frac{6 \times 28.5}{10 \times 99}$$

$$= 0.827272727$$

$$\approx 0.8273.$$

(b) $\rho_T = 0.79.$

Since $\rho_{cal} = 0.8273 > \rho_T = 0.79$, there is significant correlation b/th the ages of Patients and the no of days taken to recover from a particular [illegible].

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№ 9

(a) $X = x + \epsilon_x$, $Y = y + \epsilon_y$.

$$XY = (x+\epsilon_x)(y+\epsilon_y).$$
$$= xy + x\epsilon_y + y\epsilon_x + \epsilon_x\epsilon_y.$$
$$XY - xy = x\epsilon_y + y\epsilon_x + \epsilon_x\epsilon_y.$$

$\epsilon_x\epsilon_y$ is assumed to be very small and therefore neglected,

$\Rightarrow$

$$\epsilon_{xy} \approx x\epsilon_y + y\epsilon_x.$$

Relative error, $\dfrac{\epsilon_{xy}}{xy} = \dfrac{x\epsilon_y + y\epsilon_x}{xy}$

$$= \frac{\epsilon_y}{y} + \frac{\epsilon_x}{x}.$$

Absolute ~~Max~~ R.E is $\left|\dfrac{\epsilon_{xy}}{xy}\right|_{\cancel{max}} = \left|\dfrac{\epsilon_y}{y} + \dfrac{\epsilon_x}{x}\right|.$

$$\leq \left|\frac{\epsilon_y}{y}\right| + \left|\frac{\epsilon_x}{x}\right|.$$

max. R.E $\left|\dfrac{\epsilon_{xy}}{xy}\right|_{max} = \left|\dfrac{\epsilon_y}{y}\right| + \left|\dfrac{\epsilon_x}{x}\right|.$

maximum percentage relative error $= \left[\left|\dfrac{e_y}{y}\right| + \left|\dfrac{e_x}{x}\right|\right] \times 100.$

(b) $a = 1.50$, $e_a = \pm 0.005$, $b = 13.3$, $e_b = \pm 0.05$,
$c = 9.1000$, $e_c = \pm 0.00005$.

let $m = \dfrac{a-c}{b^2}$. Working Value $m = \dfrac{1.5 - 9.1000}{13.3^2} = -ve$.

$$m_{max} = \frac{(a-c)_{max}}{b_{min}^2} = \cancel{a_{max} - c_{min}}$$

$$= \frac{-[c-a]_{max}}{(b_{min})^2} = \frac{-[C_{max} - a_{min}]}{(b_{min})^2}.$$

$$= \frac{-[9.10005 - (1.50 - 0.005)]}{(13.3 - 0.05)^2} = \frac{-(9.10005 - 1.495)}{13.25^2} = \frac{7.60505}{13.25^2}$$

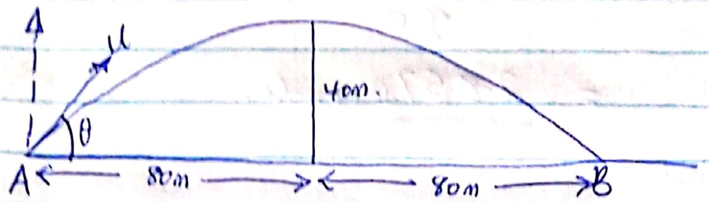
$$\approx -0.0433$$

$$M_{min} = \frac{-[C_{min} - a_{max}]}{(b_{max})^2} = \frac{-[(9.1000\cancel{5} - 0.00005) - (1.505)]}{13.35^2}.$$

$$\approx -0.0426. \qquad \text{or } -0.0427$$

Range $= [-0.0426, -0.0433]$.

No 10.



Vertical motion,

using $y = u\sin\theta t - \frac{1}{2}gt^2$.

at a time of landing at B, $y = 0$,

$\Rightarrow 0 = (u\sin\theta - \frac{1}{2}gt)t$.

either $t = 0$ or $t = \frac{2u\sin\theta}{g}$

$\Rightarrow$ Time to reach max. height $= \frac{u\sin\theta}{g}$.

Also at max. height,

using $y = u\sin\theta t - \frac{1}{2}gt^2$.

$40 = u\sin\theta \times \frac{u\sin\theta}{g} - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$

$40 = \frac{u^2\sin^2\theta}{g} - \frac{u\sin^2\theta}{2g}$.

$40 = \frac{1}{2}\frac{u^2\sin^2\theta}{g}$.

$u^2\sin^2\theta = 80g$. — (#)

OR.

At max height $V_y = 0$.

using $V_y^2 = U_y^2 - 2gS_y$

$0^2 = (u\sin\theta)^2 - 2g \times 40$

$\Rightarrow$

$u^2\sin^2\theta = 80g$. — (#)

Horizontal motion,

at greatest height, using $S_x = U_x t$.

$80 = u\cos\theta \times \frac{u\sin\theta}{g}$.

$80g = u^2\cos\theta\sin\theta$. — (##)

(#) ÷ (##) $\frac{u^2\sin^2\theta}{u^2\sin\theta\cos\theta} = \frac{80g}{80g}$

$\tan\theta = 1$.

$\theta = \tan^{-1}1 = 45°$.

Sub θ in —(1).

$u^2(\sin 45°)^2 = 80g$.

$u^2\left(\frac{1}{\sqrt{2}}\right)^2 = 80g$.

$\frac{u^2}{2} = 80g$

$u = \sqrt{160g} = \sqrt{160 \times 9.8}$

$= 39.59797975$

$\approx 39.60\,m/s$.

(b) Time, $t = \frac{u \sin\theta}{g}$

$= \frac{39.6 \times \sin 45°}{9.8}$

$= 2.857$ seconds.

№ 11.

marks	f	x	fx	fx^2	c.f
10 — 20	18	15	270	4050	18
20 — 30	34	25	850	21,250	52
30 — 40	58	35	2,030	71,050	110
40 — 50	42	45	1,890	85,050	152
50 — 60	24	55	1320	72,600 ~~85050~~	176
60 — 70	10	65	650	42,250	186
70 — 80	6	75	450	33,750	192
80 — 90	8	85	680	57800	200
	$\Sigma f = 200$		$\Sigma fx = 8140$	$\Sigma fx^2 =$ ~~400250~~ 387,800	

(i) Mean mark $= \frac{\Sigma fx}{\Sigma f} = \frac{8140}{200} = 40.7$.

(ii) Standard deviation, $\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{387,800}{200} - 40.7^2}$

$= 16.808$.

(iii)

mark	50	54	60
students (c.f)	152	y	176.

$\frac{60-50}{176-152} = \frac{54-50}{y-152}$.

$\frac{10}{24} = \frac{4}{y-152}$.

$10y - 1520 = 96$.

$y = \frac{1616}{10} = 161.6$.

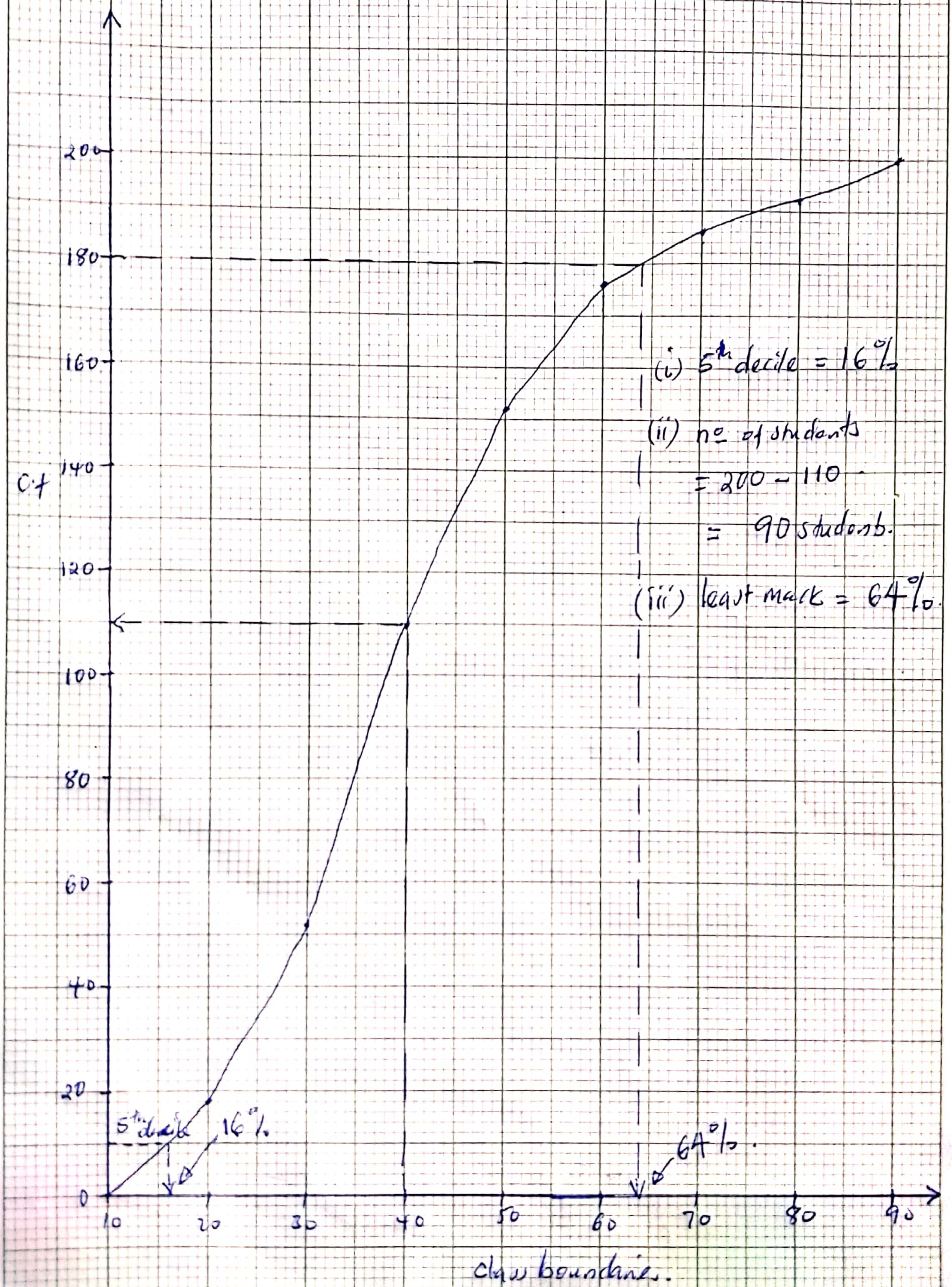
No of students who scored above 54% $= 200 - y$.

$= 200 - 161.6$

$= 38.4$ students.

≈ 38 students.

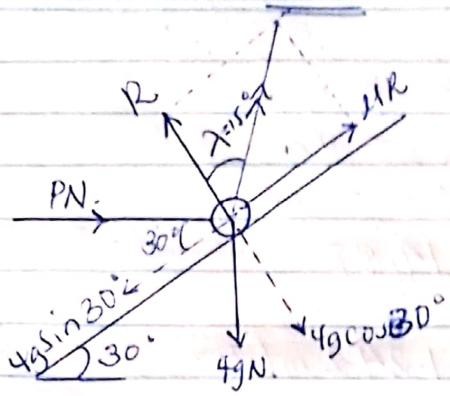
11 (b)



c.f
200
180
160
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70
80
90
class boundaries.
5th decile
16%
64%
(i) 5th decile = 16%
(ii) no of students
= 200 − 110
= 90 students.
(iii) least mark = 64%.

No 12

(a)



$\lambda = 15°$

$\tan\lambda = \frac{\mu R}{R}$

$\tan 15° = \mu$

At equilibrium

$R + \cancel{P} = P\sin 30° + 4g\cos 30°$

$R = P \times \frac{1}{2} + 4g \times \frac{\sqrt{3}}{2}$

$R = \frac{P}{2} + 2g\sqrt{3}$ — (I)

$\mu R + P\cos 30° = 4g\sin 30°$ — (II)

Sub (I) in (II)

$\tan 15° \times \left(\frac{P}{2} + 2g\sqrt{3}\right) + P \times \frac{\sqrt{3}}{2} = 4g \times \frac{1}{2}$

$0.2679 \times \frac{P}{2} + 0.2679 \times 2g\sqrt{3} + \frac{P\sqrt{3}}{2} = \cancel{4}\, 2g$

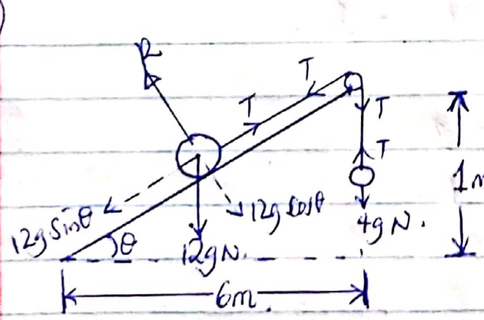
$0.13395P + 0.2679 \times 2 \times 9.8 \times \sqrt{3} + 0.866P = 2 \times 9.8$

$0.99995P = 19.6 + 9.09472$

$P = \frac{28.69472}{0.99995}$

$P = 28.696\,N$

(b)



For 12 kg mass, using $F = ma$,

$T - 12g\sin\theta = 12a$

$T - 12 \times 9.8 \times \sin 9.46° = 12a$

$T - 19.33 = 12a$ — (I)

For 4 kg mass, using $F = ma$,

$4g - T = 4a$

$4 \times 9.8 - T = 4a$

$39.2 - T = 4a$ — (II)

$\theta = \tan^{-1}\frac{1}{6}$

$\theta = 9.46°$

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$-\text{I} + -\text{II},$

$$\begin{array}{rl} & T - 19.33 = 12a \\ + & 39.2 - T = 4a \\ \hline & 39.2 - 19.33 = 16a. \end{array}$$

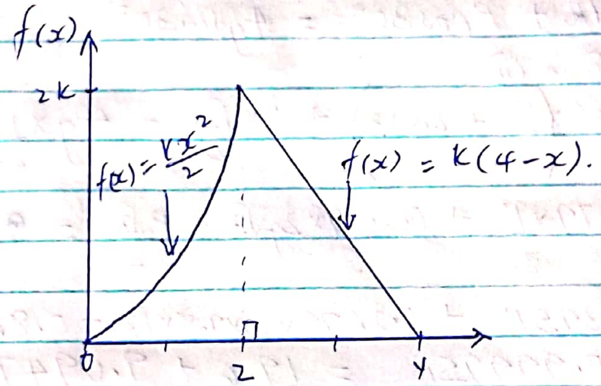
$$a = \frac{19.87}{16}.$$

$$a = 1.242\ m/s^2.$$

No 13.

$$f(x) = \begin{cases} \frac{kx^2}{2} & ; 0 \le x < 2. \\ k(4-x) & : 2 < x \le 4. \\ 0 & : \text{otherwise}. \end{cases}$$

(a)



(b) Area under the graph = 1.

$$\int_0^2 \frac{kx^2}{2}\,dx + \frac{1}{2}bh = 1.$$

$$\frac{k}{2\times3}\left[x^3\right]_0^2 + \frac{1}{2}\times2\times2k = 1.$$

$$\frac{k}{6}\left[2^3\right] + 2k = 1.$$

$$\frac{8k}{6} + 2k = 1.$$

$$8k + 12k = 6.$$

$$20k = 6$$

$$k = \frac{6}{20} = \frac{3}{10}.$$

(c) (i) $\int_m^4 k(4-x)\,dx = \frac{1}{2}$.

$k\left[4x - \frac{x^2}{2}\right]_m^4 = \frac{1}{2}$.

$k\left[\left(4\times4 - \frac{4^2}{2}\right) - \left(4m - \frac{m^2}{2}\right)\right] = \frac{1}{2}$.

$\frac{3}{10}\left[(16-8) - \left(4m - \frac{m^2}{2}\right)\right] = \frac{1}{2}$.

$8 - \left(4m - \frac{m^2}{2}\right) = \frac{10^5}{2\times3}$.

$8 - \frac{5}{3} = 4m - \frac{m^2}{2}$.

$\frac{19}{3}\times6 = \left(4m - \frac{m^2}{2}\right)\times6$.

$38 = 24m - 3m^2$.

$3m^2 - 24m + 38 = 0$.

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$m = \frac{24 \pm \sqrt{24^2 - 4\times3\times38}}{2\times3}$

$m = 2.174$ or $m = 5.826$.

$\therefore m = 2.174$.

(ii) $P(|x-1| < \frac{1}{2}) = P(-\frac{1}{2} < x-1 < \frac{1}{2})$.

$= P(-\frac{1}{2}+1 < x < \frac{1}{2}+1)$.

$= P(\frac{1}{2} < x < 1.5)$.

$= P(0.5 < x < 1.5)$.

$= \int_{0.5}^{1.5} \frac{kx^2}{2}\,dx$

$= \frac{k}{2\times3}\left[x^3\right]_{0.5}^{1.5} = \frac{3}{10\times6}\left[1.5^3 - 0.5^3\right]$.

$= \frac{13}{80}$

$= \underline{\underline{0.1625}}$.

No 14.

(a) $f(x) = 2x^3 + 5x - 8$

$f'(x) = 6x^2 + 5$.

Using NRF,

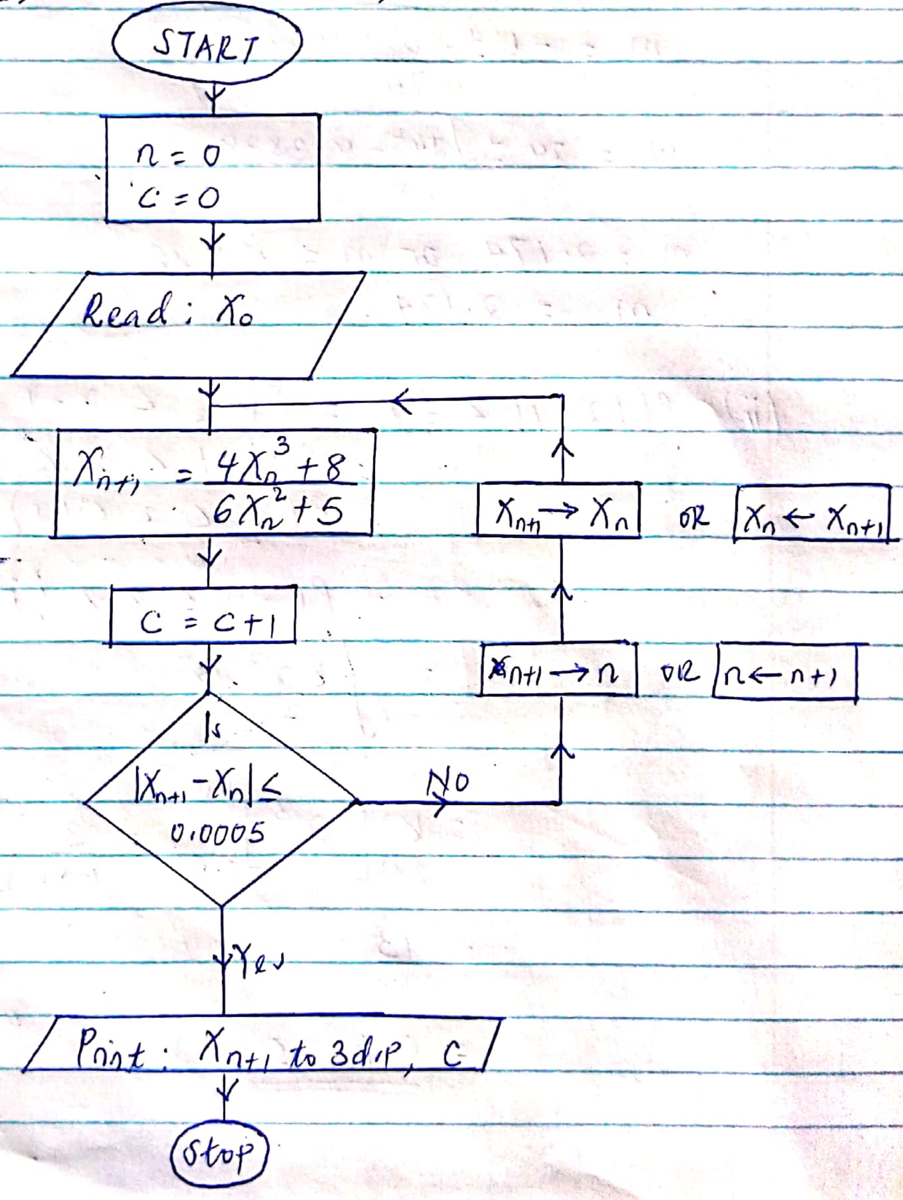
$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}.$$

$$X_{n+1} = X_n - \frac{2X_n^3 + 5X_n - 8}{6X_n^2 + 5}$$

$$= \frac{6X_n^3 + 5X_n - 2X_n^3 - 5X_n + 8}{6X_n^2 + 5}$$

$$= \frac{4X_n^3 + 8}{6X_n^2 + 5}$$

(b) let c = number of iterations.

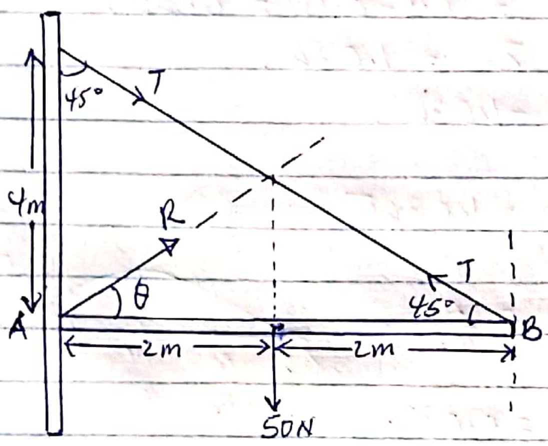


CS CamScanner

(c)

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$	Is $\lvert x_{n+1} - x_n \rvert < 0.0005$	C.
0	1.5	1.1622	0.3378	No	1
1	1.1622	1.0897	0.00725	No	2
2	1.0897	1.0867	0.0030	No	3
3	1.0867	1.0867	0.0000	Yes	4

No. 15.



& taking moments about A,

For eq = m,

$T\sin 45° \times 4 = 50g \times 2$.

$T \times \frac{\sqrt{2}}{2} \times 4 = 50 \times 9.8 \times 2$.

$T = \frac{50 \times 9.8}{\sqrt{2}} = 346.482N$.

$\rightarrow R\cos\theta = T\cos 45°$.

$R\cos\theta = 346.482 \times \cos 45°$.

$R\cos\theta = 245.0$ — (I)

$\uparrow R\sin\theta + T\sin 45° = 50$

$R\sin\theta = 50 - 346.482 \times \sin 45°$.

$R\sin\theta = -195.0$ — (II)

(II) ÷ (I).

$\tan\theta = \frac{-195.0}{245.0}$ or

$\theta = \tan^{-1} 0.7959$

$= -38.52°$. or $\theta =$ NE $38.52°N$

Sub θ in — (I).

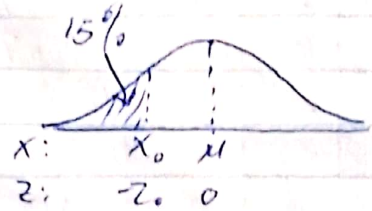
$R = \frac{245}{\cos -38.52°}$.

$R = 313.129N$.

No 16.

Let X be r.v, "the no goats owned by residents in a village".

$X \sim N(\mu, \sigma^2)$.



$P(X < 60) = \frac{15}{100}$.

$P\left(Z < \frac{60-\mu}{\sigma}\right) = 0.15$.

Let $\frac{60-\mu}{\sigma} = -Z_0$.

$P(Z < -Z_0) = 0.15$.

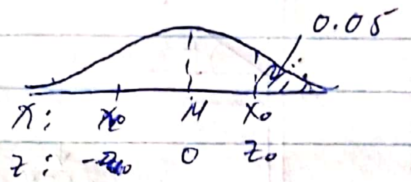
$P(0 < Z < Z_0) = 0.5 - 0.15 = 0.35$.

From tables, $Z_0 = 1.036$.

$\Rightarrow \frac{60-\mu}{\sigma} = -1.036$.

$60 - \mu = -1.036\sigma$ —— (I).

Also



$P(X > 90) = \frac{5}{100}$.

$P\left(Z > \frac{90-\mu}{\sigma}\right) = 0.05$.

$P(Z > Z_0) = 0.05$.

$P(0 < Z < Z_0) = 0.5 - 0.05 = 0.45$.

From tables, $Z_0 = 1.645$.

$\Rightarrow \frac{90-\mu}{\sigma} = 1.645$.

$90 - \mu = 1.645\sigma$ —— (II).

(II) − (I),

$$\begin{array}{rl} 90 - \mu &= 1.645\sigma \\ -\quad 60 - \mu &= -1.036\sigma \\ \hline 30 &= 2.681\sigma \end{array}$$

$\sigma = 11.18985453 \approx \underline{11.20}$. or 11.19.

Sub σ in (I).

$\mu = 60 + 1.036 \times 11.2 = \underline{71.6032}$.

(b) $P(X > 80) = P\left(Z > \frac{80-\mu}{\sigma}\right)$

$= P\left(Z > \frac{80 - 71.6032}{11.2}\right)$

$= P(Z > 0.7497) = 0.22672$ (calc).

No of goats $= 200 \times 0.2267$

$= 45.344$

$\approx \underline{45 \text{ residents}}$.