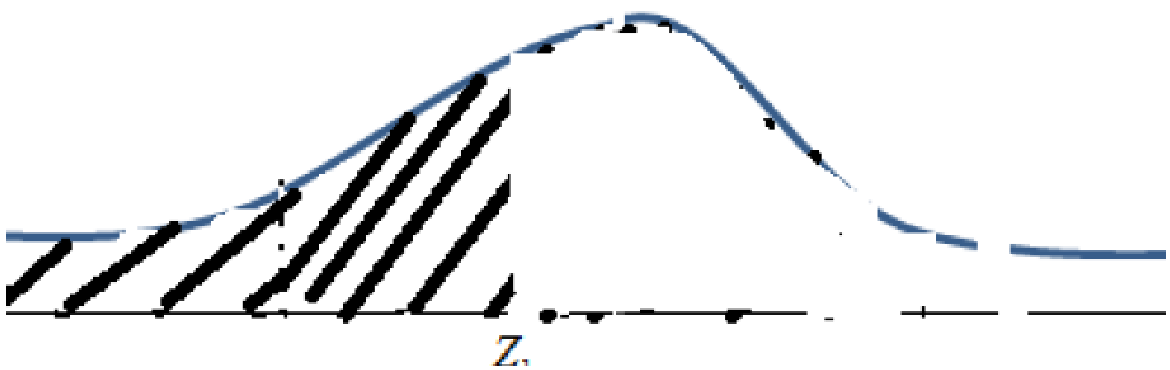
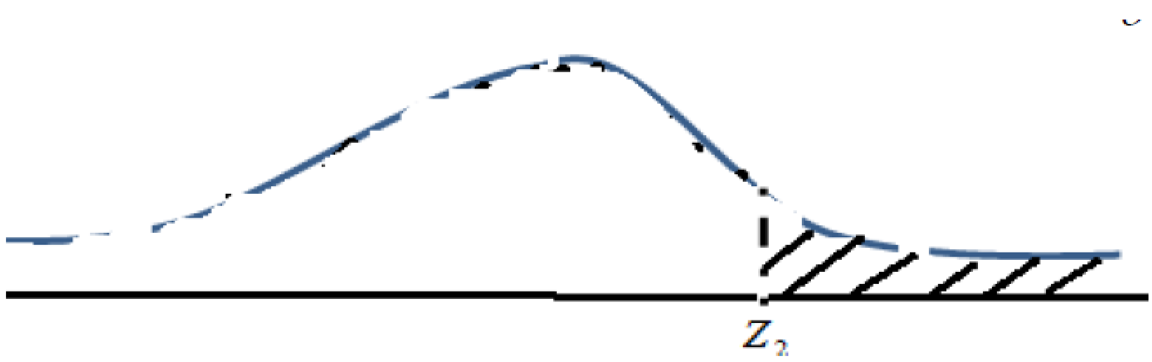
A LEVEL APPLIED MATHEMATICS SEMINAR SOLUTIONS 2022

1.(a)(i)	$P(both\ occurs) = P(AnB) = P(B).P(A/B) = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$
(ii)	$P(only\ one\ occurs) = P(AUB) - P(AnB)$ $but, P(AUB) = P(A) + P(B) - P(AnB) = \frac{8}{15} + \frac{1}{3} - \frac{1}{15} = \frac{4}{5}$ $P(only\ one\ occurs) = P(AUB) - P(AnB) = \frac{4}{5} - \frac{1}{15} = \frac{11}{15}$
(iii)	$P(\text{Neither of the events occurs}) = \text{P}(A'nB') = 1 - P(AuB)$ $= 1 - \frac{4}{5} = \frac{1}{5}$
(b)(i)	$P(all\ green) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$ $P(first\ two\ pink\ and\ third\ is\ green) = \frac{4}{10} \times \frac{3}{9} \times \frac{3}{8} = \frac{1}{20}$ $P(first\ two\ yellow\ and\ third\ is\ green) = \frac{3}{10} \times \frac{2}{9} \times \frac{3}{8} = \frac{1}{40}$ $P(first\ two\ same\ colour\ and\ third\ is\ green) = \frac{1}{120} + \frac{1}{20} + \frac{1}{40} = \frac{1}{12}$
(ii)	$let\ x\ denote\ number\ of\ pink\ counters.$

x	$P(X = x)$	$xP(X = x)$
0	$\frac{4C_0 \times 6C_3}{10C_3} = \frac{1 \times 20}{120} = \frac{20}{120} = \frac{1}{6}$	0
1	$\frac{4C_1 \times 6C_2}{10C_3} = \frac{4 \times 15}{120} = \frac{60}{120} = \frac{1}{2}$	$\frac{1}{2}$
2	$\frac{4C_2 \times 6C_1}{10C_3} = \frac{6 \times 6}{120} = \frac{36}{120} = \frac{3}{10}$	$\frac{3}{5}$
3	$\frac{4C_3 \times 6C_0}{10C_3} = \frac{4 \times 1}{120} = \frac{4}{120} = \frac{1}{30}$	$\frac{1}{10}$
$TOTAL$	1	$\frac{6}{5}$

	$E(x) = \frac{6}{5} \approx 1\ pink\ counter.$
2(a)	*Let X be the r.v* "marks scored by the students $X \sim N(\mu, \sigma^2)$ $P(X < 84) = \frac{30}{100} = 0.3$ $P\left(Z < \frac{84-\mu}{\sigma}\right) = 0.3$ $P(Z < Z_1) = 0.3\ ; where\ Z_1 = \frac{84-\mu}{\sigma}$  $0.5 - P(-Z_1) = 0.3$ $P(-Z_1) = 0.2$ $From\ Critical\ points\ table, Z_1 = -0.842$ $-0.842 = \frac{84-\mu}{\sigma};\quad \mu - 0.842 = 84 \ldots\ldots\ldots\ldots\ldots (i)$ $Also\ P(X > 154) = 0.1;\ P(Z > Z_2) = 0.1\ ; where\ Z_2 = \frac{154-\mu}{\sigma}$  $from\ Critical\ point\ table;\ Z_2 = 1.282$ $Hence\ 1.282 = \frac{154-\mu}{\sigma};\quad \mu + 1.282 = 154 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

	Solving equations (i) and (ii) simultaneously gives; $\mu = 111.75\ and\ \sigma = 32.96$ $\therefore P(X > 155) = P\left(Z > \frac{155-111.75}{32.96}\right) = P(Z > 1.312)$  $P(Z > 1.312) = 0.5 - P(1.312) = 0.5 - 0.4052 = 0.0948$
(b)	$Let\ X\ be\ a\ r.v$ nmber of loan applicants $Hence\ X \sim B(450, 0.2)$ $Since\ n > 20\ then\ X \sim N(np, npq)$ $Mean, \mu = np = 450 \times 0.2 = 90\ and\ variance, \sigma^2 = npq = 450 \times 0.2 \times 0.8 = 72$ $P(X = 90) = P(89.5 < X < 90.5) = P\left(\frac{89.5-90}{\sqrt{72}} < Z < \frac{90.5-90}{\sqrt{72}}\right)$ $= P(-0.059 < Z < 0.059) = 2P(Z > 0.059) = 2 \times 0.0235 = 0.0470$ $Hence\ the\ percentage\ is$ $0.047 \times 100 = 4.7$

3(a)	

B \ A	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Let x be the random variable that faces show the same colour.

x	1	2	3	4
$P(X=x)$	k	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{4}$

For a discrete pdf;

$$\sum P(X=x)=1$$

$$k+\frac{k}{2}+\frac{k}{3}+\frac{k}{4}=1,\quad k=\frac{12}{25}$$

$$P(faces\ show\ same\ number)=P(1,1)+P(2,2)+P(3,3)+P(4,4)$$

$$=P(1n1)+P(2n2)+P(3n3)+P(4n4)$$

$$=\left(\frac{12}{25}\right)^2+\left(\frac{6}{25}\right)^2+\left(\frac{4}{25}\right)^2+\left(\frac{3}{25}\right)^2=\frac{41}{125}=0.328$$

(b)(i)

$$let\ s=event\ that\ Sir\ Fred\ goes\ to\ play\ football,$$

$$B=event\ that\ Bob\ goes\ to\ play\ football$$

$$F=event\ that\ it\ is\ a\ fine\ day.$$

Now;$P(S/F)=\frac{9}{10},\quad P(B/F)=\frac{3}{4},\ \ P(S/F')=\frac{1}{2},\ \ P(B/F')=\frac{1}{4}$

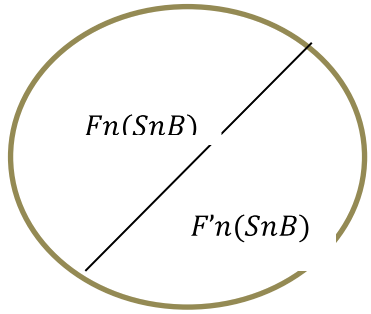
Let $P(F')=x\ \ ,\ \ then\ \ P(F)=2x$

$$Now\ \ P(F')+P(F)=1\ \ ;\ \ x+2x=1\ \ ;\ \ 3x=1\ \ ;\ \ x=\frac{1}{3}$$

$$P(F')=\frac{1}{3}\ \ ;\ \ P(F)=\frac{2}{3}$$

$$Now\ both\ will\ go\ to\ play\ when\ it\ is\ either\ fine\ or\ not\ fine;$$

$$Let\ D=Event\ that\ both\ go\ to\ play$$



$$P(D) = P(FnSnB) + P(F'nSnB) = P(F) \times P(SnB/F) + P(F') \times P(SnB/F')$$
$$= P(F) \times P(S/F) \times P(B/F) + P(F') \times P(S/F') \times P(B/F')$$
$$= \left(\frac{2}{3} \times \frac{9}{10} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}\right) = \frac{54}{120} + \frac{1}{24} = \frac{54}{120}$$

(ii)

$$P(F/D) = \frac{P(FnD)}{P(D)} = \frac{P(F) \times P(D/F)}{P(D)} = \frac{P(F) \times P((SnB)/F)}{P(D)}$$
$$= \left(\frac{2}{3} \times \frac{9}{10} \times \frac{3}{4}\right) \div \frac{59}{120} = \frac{54}{59}$$

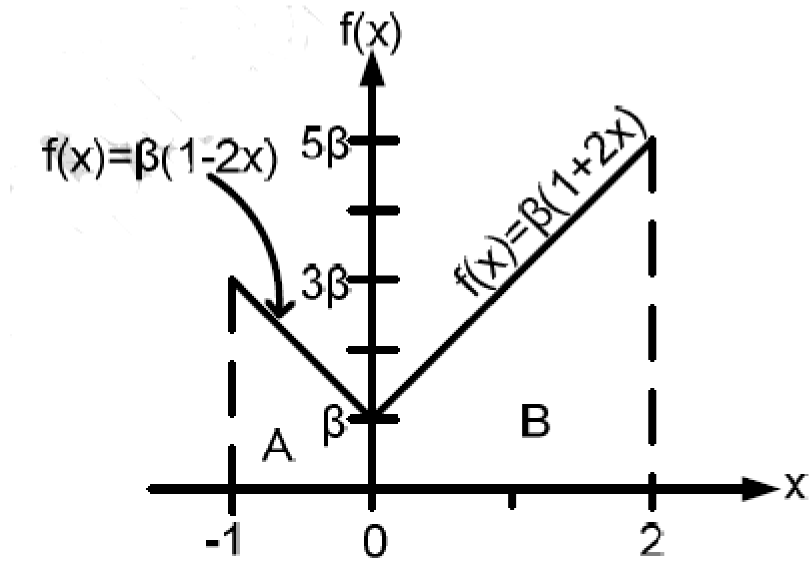
The probability that they both go to play given it is a fine day is $\frac{54}{59}$

4(a)(i)

x	$P(X = x)$	$xP(X = x)$	$x^2P(X = x)$
1	k	K	K
2	2k	4k	8k
3	3k	9k	27k
.	.	.	.
.	.	.	.
.	.	.	.
40	40k	1600k	64000k

$$\sum_{allx} P(X = x) = 1$$
$$k(1 + 2 + 3 + \cdots + 40 = 1$$
$$k \sum_{n=1}^{40} \frac{n}{2}(n + 1) = 1$$

	$$k\left(\frac{40}{2}\right)(40+1)=1$$ $$820k=1;\ k=\frac{1}{820}$$
(ii)	$$E(X)=k+4k+9k+\cdots+1600k$$ $$=k(1^2+2^2+\cdots+40^2)$$ $$=k\sum_{n=1}^{40}\frac{n}{6}(n+1)(2n+1)$$ $$=\frac{1}{820}\left(\frac{40}{2}\right)(40+1)(80+1)=27$$ $$E(X^2)=k+8k+27k+\cdots+64000k$$ $$=k(1^3+2^3+\cdots+40^3)$$ $$=k\sum_{n=1}^{40}\frac{n^2}{4}(n+1)^2$$ $$=\frac{1}{820}\left(\frac{40^2}{4}\times 41^2\right)=820$$ $$Var(X)=E(X^2)-\left(E(X)\right)^2=820-27^2=91$$ $S.D=\sqrt{Var(x)}=\sqrt{91}=9.5394$ (4 dps)
(iii)	$$P(x<35/x>20)=\frac{P(x<35\ n\ x>20)}{P(x>20)}$$ $$P(x<35\ n\ x>20)=k\sum_{r=21}^{34}\frac{n}{2}(n+1)=\frac{1}{820}\left[\left(\frac{34}{2}(34+1)\right)-\left(\frac{21}{2}(21+1)\right)\right]$$ $$\frac{119}{164}-\frac{231}{820}=\frac{91}{205}=0.4439$$ $$P(x>20)=k\sum_{r=21}^{40}\frac{n}{2}(n+1)=\frac{1}{820}\left[\left(\frac{40}{2}(40+1)\right)-\left(\frac{21}{2}(21+1)\right)\right]$$ $$1-\frac{231}{820}=\frac{589}{820}=0.7183$$ $$P(x<35/x>20)=\frac{P(x<35\ n\ x>20)}{P(x>20)}=\frac{0.4439}{0.7183}=\frac{4439}{7183}=0.6180$$

b(i)	$-1 \le x \le 0;\ f(-1) = \beta(1+2) = 3\beta;\ f(0) = \beta(1-0) = \beta$ $0 \le x \le 2;\ f(0) = \beta(1+0) = \beta;\ \ f(2) = \beta(1+4) = 5\beta$ 
(ii)	Total area under the graph= $area\ A + area\ B$ $$1 = \frac{1}{2} \times (0+1)(3\beta+\beta) + \frac{1}{2} \times (2-0)(\beta+5\beta)$$ $$1 = 2\beta + 6\beta; \quad \beta = \frac{1}{8}$$
(ii)	$$f(x) = \begin{cases} \frac{1}{8}(1-2x)\ ; -1 \le x \le 0 \\ \frac{1}{8}(1+2x); \quad 0 \le x \le 2 \\ 0\ ; \qquad elsewhere \end{cases}$$ $$E(X) = \int xf(x)dx$$ $$= \frac{1}{8}\int_{-1}^{0}(x-2x^2)dx + \frac{1}{8}\int_{0}^{2}(x+2x^2)dx$$ $$= \frac{1}{8}\left[\frac{1}{2}x^2 - \frac{2}{3}x^3\right]_{-1}^{0} + \frac{1}{8}\left[\frac{1}{2}x^2 + \frac{2}{3}x^3\right]_{0}^{2}$$ $$\frac{1}{8}\left\{0 - \left(\frac{1}{2}+\frac{2}{3}\right)\right\} + \frac{1}{8}\left\{\left(2+\frac{16}{3}\right) - 0\right\}$$ $$= \frac{7}{48} + \frac{11}{12} = \frac{17}{16} = 1.0625$$

(iii)	$$Let\ p\ be\ the\ 60^{th}\ percentile;\ \int_{-1}^{p} f(x)dx = 0.6$$ $$but, \int_{-1}^{0} \frac{1}{8}(1-2x)dx = 2\beta = 2 \times \frac{1}{8} = 0.25$$ $$\int_{-1}^{p} f(x)dx = \int_{-1}^{0} \frac{1}{8}(1-2x)dx + \int_{0}^{p} \frac{1}{8}(1+2x)dx$$ $$0.6 = 0.25 + \frac{1}{8}[x+x^2]_0^p$$ $$0.35 = \frac{1}{8}\left((p+p^2)-0\right)$$ $$2.8 = p + p^2$$ $$p = \frac{-1 \pm \sqrt{1^2 - 4\times 1 \times (-2.8)}}{2\times 1}$$ $$p = 1.246, \quad or\ ,\ p = -2.246$$ For the interval; $0 \le x \le 2, p \neq -2.246$. Thus $60^{th}\ percentile = 1.246$
5(a)(i)	Mean; $\bar{x} = \frac{\sum f(x)}{\sum f} = \frac{563}{20} = 28.15$
(ii)	Standard deviation, $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{16143}{20} - (28.15)^2}$ $$= \sqrt{14.7275} \approx 3.8376$$
b(i)	Mean; $\bar{x} = \frac{\sum f(x)}{\sum f} = \frac{(0\times m)+(1\times n)}{m+n} = \frac{n}{m+n}$
(ii)	$$Variance, \sigma^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 = \frac{(0^2\times m)+(1^2\times n)}{m+n} - \left(\frac{n}{m+n}\right)^2$$ $$= \frac{n(m+n)-n^2}{(m+n)^2} = \frac{mn+n^2-n^2}{(m+n)^2} = \frac{mn}{(m+n)^2}$$ Standard deviation; $\sigma = \sqrt{Variance} = \sqrt{\frac{mn}{(m+n)^2}} = \frac{\sqrt{mn}}{m+n}$
(c)	

		X	x^2
		29.3	858.49
		30.9	954.81
		31.8	1011.24
		33.5	1122.25
		34.5	1190.25
		35.2	1239.04
		$\sum x = 195.2$	$\sum x^2 = 6376.08$

(i) Mean time;$\bar{x} = \frac{\sum x}{n} = \frac{195.2}{6} = 32.5333\ (4dps)$

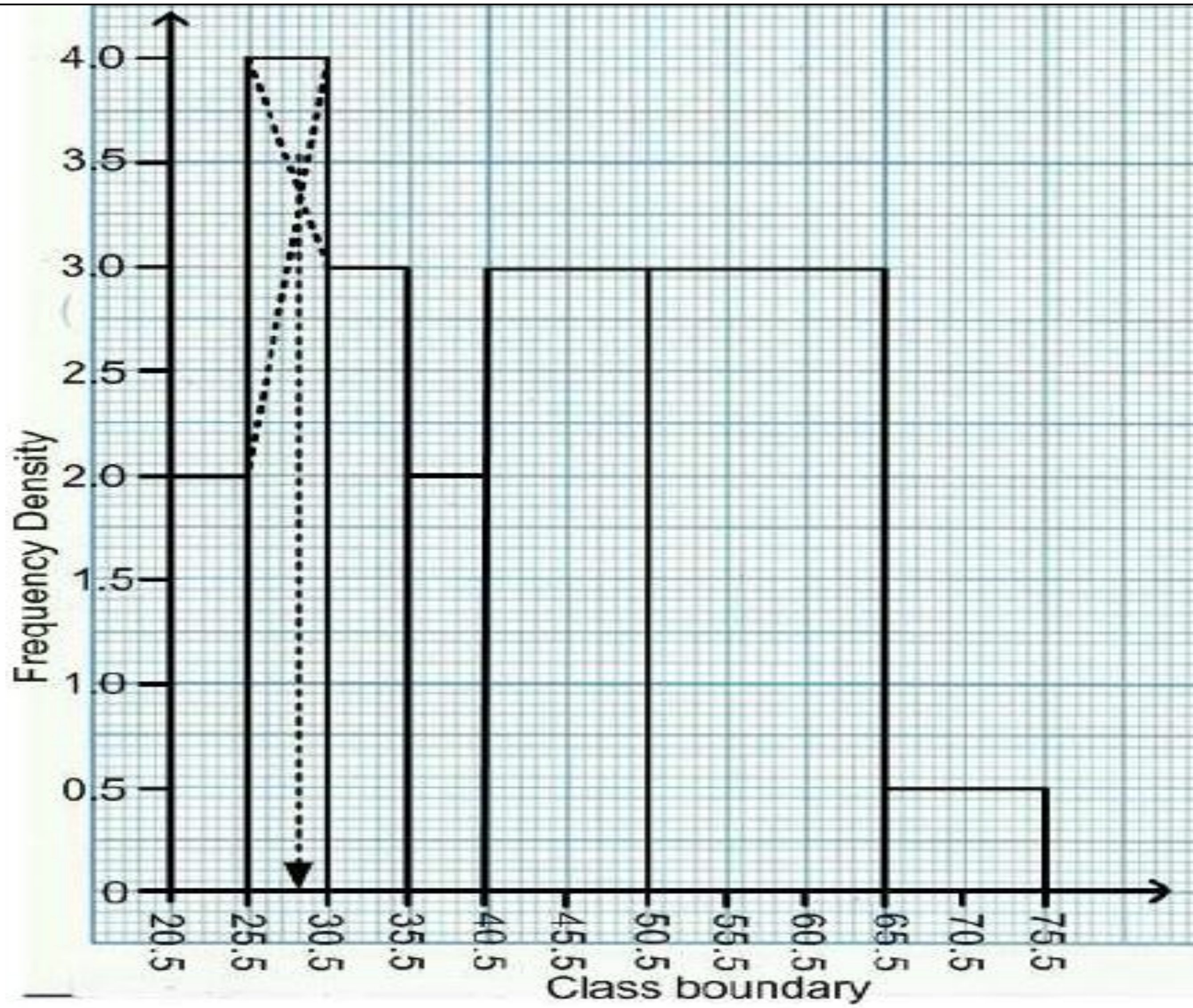
Hence the mean time is 32.5333 seconds

(ii) Standard deviation $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{6376.08}{6} - (32.5333)^2} = 2.0645\ (4\ dps)$

6(a)

Class	f	Class boundary	C	$f/_c$	C.F
21-25	10	20.5-25.5	5	2	10
26-30	20	25.5-30.5	5	4	30
31-35	15	30.5-35.5	5	3	45
36-40	10	35.5-40.5	5	2	55
41-50	30	40.5-50.5	10	3	85
51-65	45	50.5-65.5	15	3	130
66-75	5	65.5-75.5	10	0.5	135

(i)



From the histogram, the estimated mode is 27.5

(ii)

$N = 135;\ {}^{N}/_{2} = 67.5, \quad L_m = 40.5, \quad C.F_b = 55,\ f_m = 30, \quad c = 10$

Median$= L_m + \left(\frac{{}^{N}/_{2} - C.F_b}{f_m}\right) c = 40.5 + \left(\frac{67.5-55}{30}\right) \times 10 = 40.5 + 4.167 = 44.6667$

6(b)(i)

3, 3, 5, 6, 8, 9, 12, 14, 19, 20, 24

Upper quartile position$= \frac{3}{4}(N+1) = \frac{3}{4} \times 12 = 9^{th}$

$\therefore\ upper\ quartile, q_3 = 19$

(ii)

Lower quartile position$= \frac{1}{4}(N+1) = \frac{1}{4} \times 12 = 3^{rd}$

$\therefore\ Lower\ quartile, q_1 = 5$

(iii)

Median$=\frac{1}{2}(N+1)=\frac{1}{2}\times 12=6^{th}$

Median = 9

(iv)

$$\sum X = 3+3+5+6+8+9+12+14+19+20+24=123$$

$$\sum X^2 = 3^2+3^2+5^2+6^2+8^2+9^2+12^2+14^2+19^2+20^2+24^2=1901$$

$$Variance, \sigma^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2 = \frac{1901}{11} - \left(\frac{123}{11}\right)^2 = 47.7851$$

(v)

Standard deviation $\sigma = \sqrt{variance} = \sqrt{47.7851} = 6.9127$

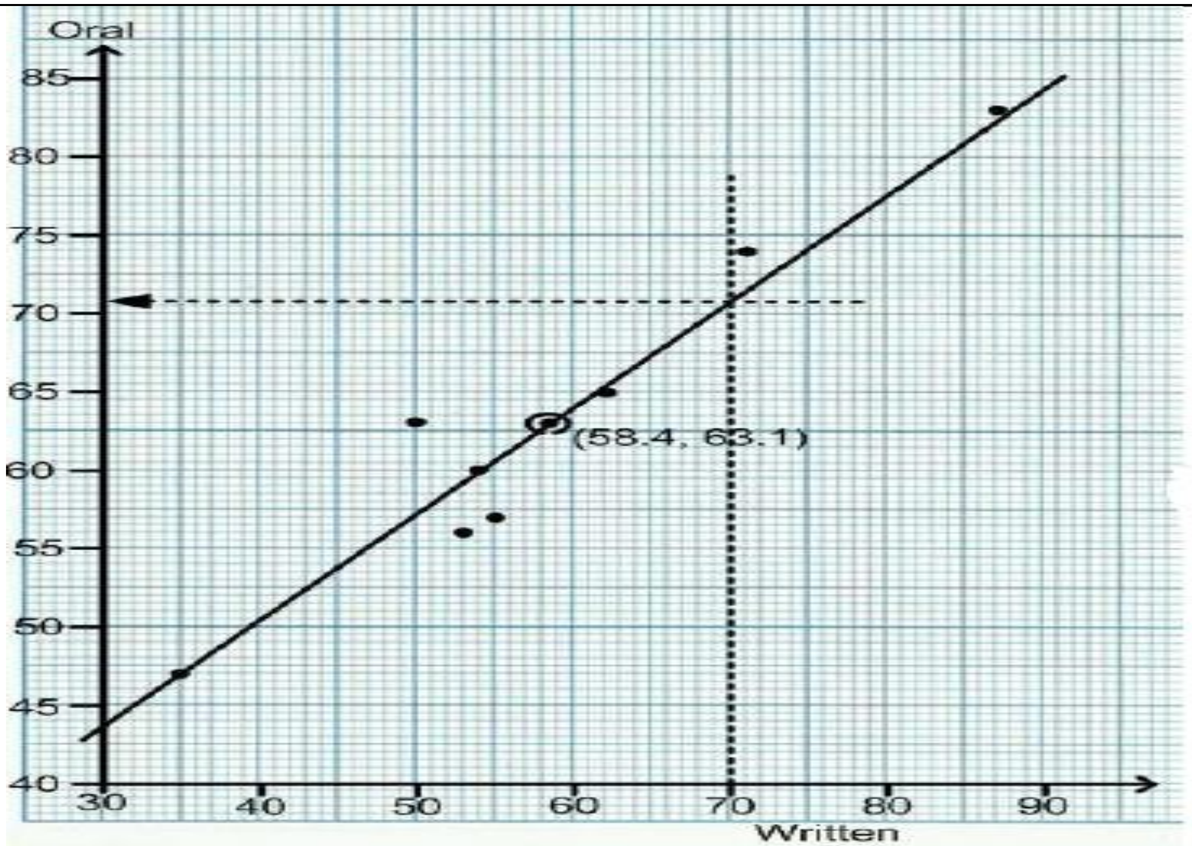
7(a)

(i),(ii)

x	y	R_x	R_y	d	d^2
55	57	4	6	-2	4
54	60	5	5	0	0
35	47	8	8	0	0
62	65	3	3	0	0
87	83	1	1	0	0
53	56	6	7	-1	1
71	74	2	2	0	0
50	□3	7	4	3	9
467	505				$\sum d^2 = 14$

$$\overline{x} = \frac{467}{8} = 58.375 \approx 58.4, \quad \overline{y} = \frac{505}{8} = 63.125 \approx 63.1$$

$$mean\ point, (\overline{x}, \overline{y}) = (58.4, 63.1)$$



(iii)

$$Y = 70.75 \; when \; X = 70$$

(iv)

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 14}{8(8^2-1)} = 0.8333$$

There is a very high positive correlation between the score of written and oral tests.

8(a)

$$Price \; relative = \frac{P_{2016}}{P_{2014}}$$

Item	Price Relative
Milk(Per litre)	$= \frac{1300}{1000} = 1.3$
Eggs (per tray)	$= \frac{8300}{6500} = 1.2769$
Sugar (per kg)	$= \frac{3800}{3000} = 1.2667$
Blue ban	$= \frac{9000}{7000} = 1.2857$

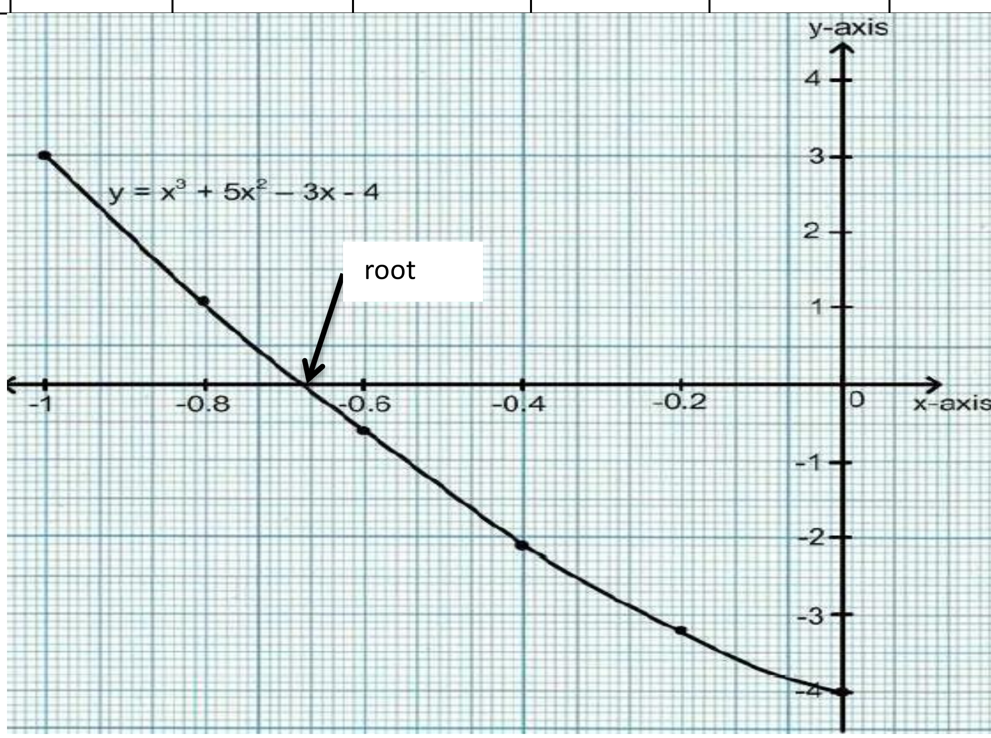
	NOTE; Accept : $Price\ relative = \frac{P_{2016}}{P_{2014}} \times 100$
(b)	$$Simple\ aggregate\ price\ index = \frac{\sum P_{2016}}{\sum P_{2014}} \times 100$$ $$= \frac{1300 + 8300 + 3800 + 9000}{1000 + 6500 + 3000 + 7000} \times 100 = \frac{22400}{17500} \times 100 = 128$$ **NOTE**; Accept: $S.A.P.I = \frac{\sum P_{2016}}{\sum P_{2014}}$
(c)	Weighted aggregate price index= $\frac{(\sum P_{2016} \times W)}{(\sum P_{2014} \times W)} \times 100$ $$= \frac{(1300 \times 0.5) + (8300 \times 1) + (3800 \times 2) + (9000 \times 1)}{(1000 \times 0.5) + (6500 \times 1) + (3000 \times 2) + (7000 \times 1)} \times 100$$ $$= \frac{25550}{20000} \times 100 = 127.75$$ The prices increased by 27.75% between 2014 and 2016 NOTE; Accept: $W.A.P.I = \frac{(\sum P_{2016} \times W)}{(\sum P_{2014} \times W)}$
(d)	$$I = \frac{P_{2016}}{P_{2014}} \times 100, \quad 127.75 = \frac{45000}{P_{2014}} \times 100, \quad P_{2014} = shs.35225.048$$
9(a)	$$y_n = 5^{2x_n}, \quad h = \frac{1-0}{5} = \frac{1}{5} = 0.2$$

n	x_n	y_0, y_5	$y_1 \dots y_1$
0	0	1	
1	0.2		1.90365
2	0.4		3.62390
3	0.6		6.89865
4	0.8		13.13264
5	1	25	
Totals		26	25.55884

$$\int_0^1 5^{2x} dx \approx \frac{1}{2} h[(y_0 + y_4) + 2(y_1 + \cdots + y_3] \approx \frac{1}{2} \times \frac{1}{5} [26 + 2 \times 25.55884]$$

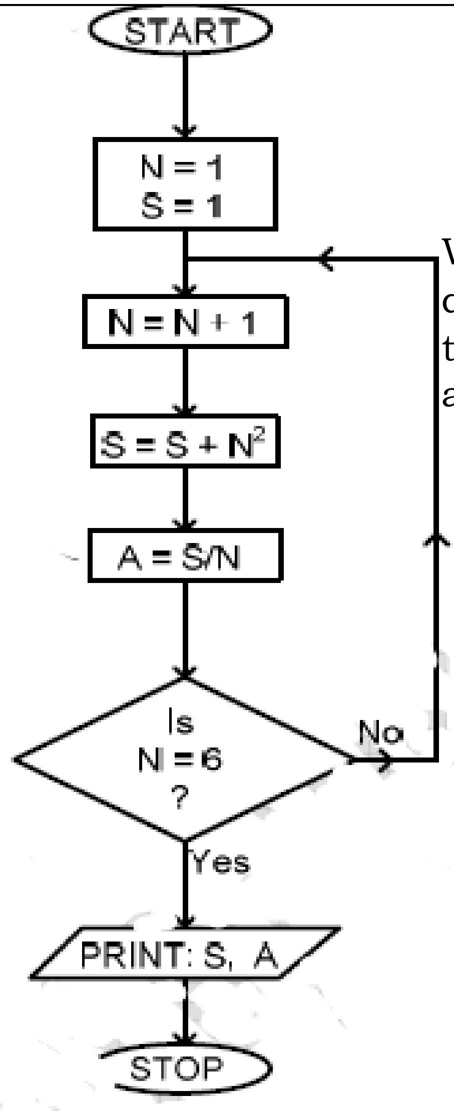
	$\approx 7.712\ (3dps)$
(b)	$let\ y = 5^{2x},\ \ lny = 2xln5,\ \ \frac{1}{y}\frac{dy}{dx} = 2ln5,\ \ \frac{dy}{dx} = (2ln5)(5^{2x})$ $\frac{d(5^{2x})}{dx} = (2ln5)(5^{2x}),\ \ \int 5^{2x}dx = \frac{5^{2x}}{2ln5} + c$ $\int_0^1 5^{2x}dx = \left[\frac{5^{2x}}{2ln5}\right]_0^1 = \frac{25}{2ln5} - \frac{1}{2ln5} = \frac{12}{ln5} \approx 7.456(3dps)$
(c)	$Absolute\ error = \lvert exact\ value - estimated\ value \rvert$ $= \lvert 7.456 - 7.712 \rvert = 0.256$ $Percentage\ error = \frac{absolute\ error}{exact\ value} \times 100 = \frac{0.256}{7.456} \times 100 = 3.433$
(d)	By increasing the number of sub- intervals
10(a)	Let $y = x^3 + 5x^2 - 3x - 4$

X	-1.0	-0.8	-0.6	-0.4	-0.2	0
Y	3	1.1	-0.6	-2.1	-3.2	-4



From the graph, the root $x_0 = -0.68$

(b)	Let $f(x) = x^3 + 5x^2 - 3x - 4$; $f'(x) = 3x^2 - 10x - 3$ $f(x_n) = {x_n}^3 + 5{x_n}^2 - 3x_n - 4$; $f'(x_n) = 3{x_n}^2 - 10x_n - 3$ Using N.R.M; $X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$ $$X_{n+1} = X_n - \frac{{x_n}^3 + 5{x_n}^2 - 3x_n - 4}{3{x_n}^2 - 10x_n - 3}$$ From the graph, $x_0 = -0.68$ $$X_1 = (-0.68) - \frac{(-0.68)^3 + 5(-0.68)^2 - 3(-0.68) - 4}{3(-0.68)^2 - 10(-0.68) - 3} = -0.6755(4dps)$$ $$Error = \lvert -0.6755 - (-0.68) \rvert = 0.005(3\ dps)$$ $$X_2 = (-0.6755) - \frac{(-0.6755)^3 + 5(-0.6755)^2 - 3(-0.6755) - 4}{3(-0.6755)^2 - 10(-0.6755) - 3} = -0.6755$$ $$\lvert X_2 - X_1 \rvert = \lvert -0.6755 - (-0.6755) \rvert = 0 < 0.005$$ Hence the root is -0.68 (2dps)
11(a)	$$y = sec(45^0 \pm 10\%);\ y = \sec(45^0 \pm 0.1)$$ $$Lower\ limit = y_{min} = sec44.9^0 = 1.4118$$ $$Upper\ limit = y_{max} = sec45.1^0 = 1.4167$$
(b)	**Flowchart;**

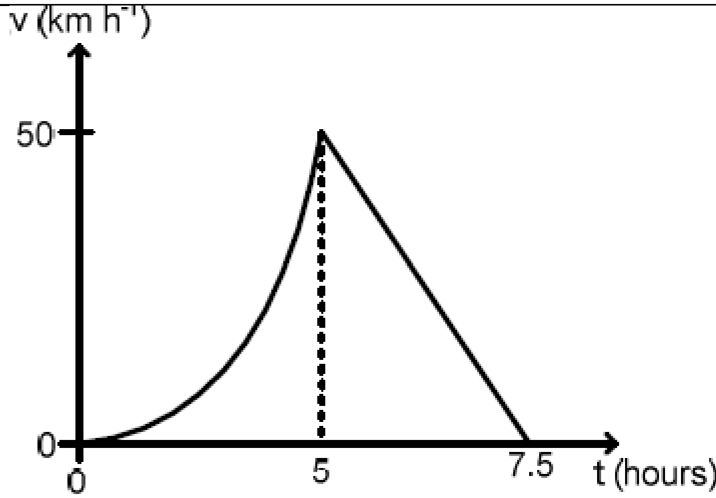
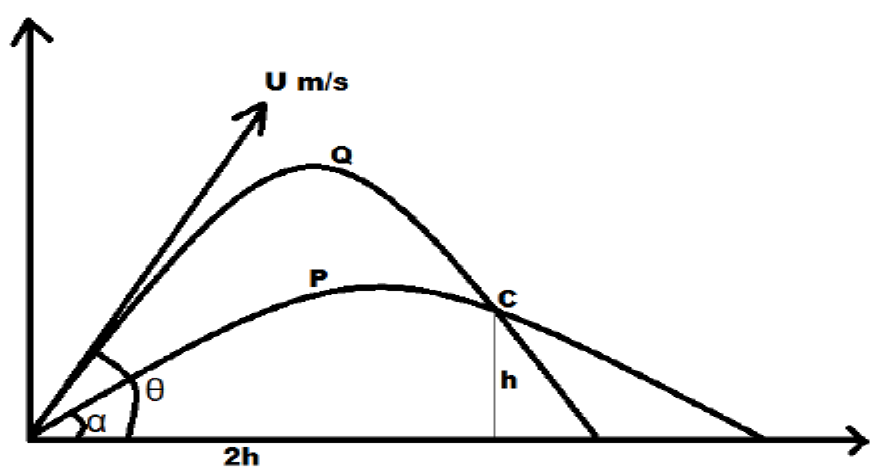


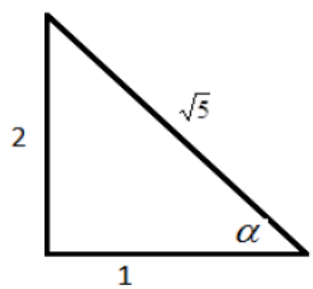
Where N, S and A denote number of terms, sum and average respectively

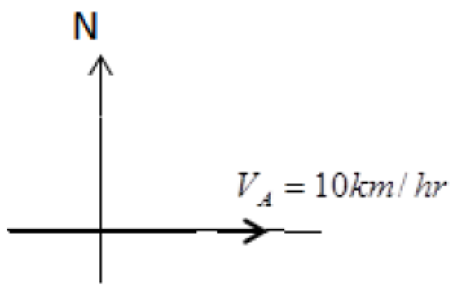
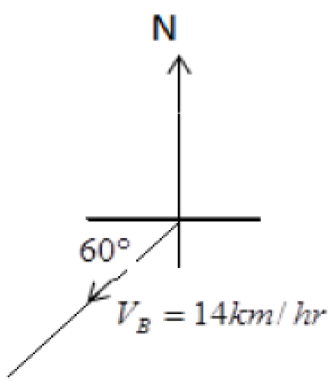
Dry run;

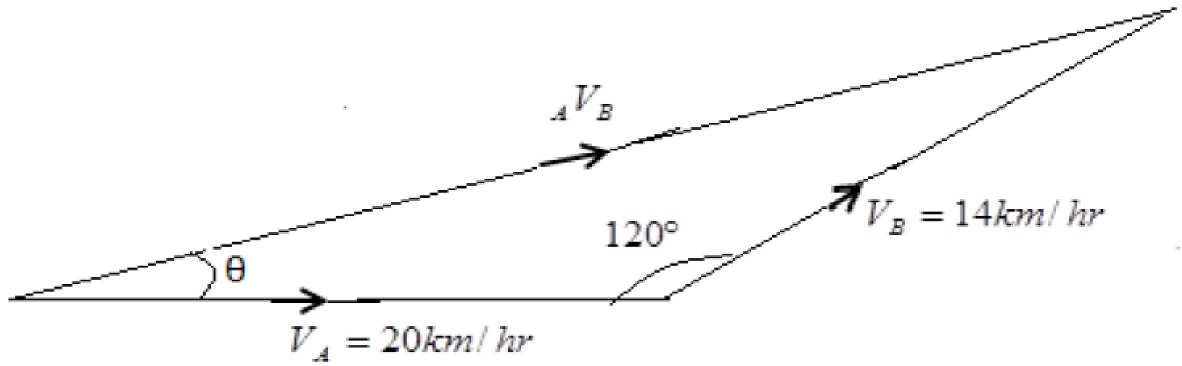
N	S	A
1	1	1
2	5	2.5
3	14	$^{14}/_{3}$
4	30	7.5
5	55	11
6	91	$^{91}/_{6}$

12(a)	$Let\ exact\ value\ be\ Z = X^2Y\ and\ approximate\ value\ be\ z = x^2y$ $Z = X^2Y$ $z + \Delta z = (x + \Delta x)^2(y + \Delta y) = (x^2 + 2x\Delta x + (\Delta x)^2)(y + \Delta y)$ $\Delta z = x^2y + y(\Delta x)^2 + 2xy\Delta x + (\Delta x)^2\Delta y + 2x\Delta x\Delta y - x^2y$ Assumption; $if\ \Delta x <<< x\ and\ \Delta y <<<< y\ are\ very\ small, then;\ \Delta x\Delta y \approx 0, (\Delta x)^2 \approx 0;$ $\Delta z = x^2\Delta y + 2xy\Delta x$ Absolute error, $\lvert\Delta z\rvert = \lvert x^2\Delta y + 2xy\Delta x\rvert \leq \lvert x^2\Delta y\rvert + \lvert 2xy\Delta x\rvert$ Hence maximum absolute error is $\Delta z = \lvert x^2\Delta y\rvert + \lvert 2xy\Delta x\rvert$
	Maximum absolute error in $x^2y\ is\ \lvert\Delta z\rvert = (2.8^2 \times 0.008) + (2 \times 1.44 \times 2.8 \times 0.016) = 0.1917$ $Exact\ value, Z = 2.8^2 \times 1.44 = 11.2896$ $Upper\ limit = 11.2896 + 0.1917 = 11.4813$ $Lower\ limit = 11.2896 - 0.1917 = 11.0979$
(b)	Let $p = \frac{x}{y}$ $p_{min} = \frac{2.425}{3.8155} = 0.6356$ $p_{max} = \frac{2.435}{3.8145} = 0.6384$ $Absolute\ error, \lvert\Delta p\rvert = \frac{p_{max-p_{min}}}{2} = \frac{0.6384 - 0.6356}{2} = 0.001$ $Least\ value = 0.6356\ and\ Greatest\ value = 0.6384$
13(a)(i)	$v = \int 4tdt = 2t^2 + c\ ;\ \ when\ t = 0, v = 0;\ \ c = 0\ ; v = 2t^2$ $when\ t = 5;\ \ v = 2 \times 5^2 = 50kmh^{-1}$ $During\ retardation, u = 50kmh^{-1}, v = 0kmh^{-1}; a = -20kmh^{-2}$ $v = u + at, \quad t = \frac{v-u}{a} = \frac{0-50}{-20} = 2.5\ hours$ $Total\ time\ taken, T = 5 + 2.5 = 7.5 hours$ $In\ 24 hours\ clock; 0800 hours + 0730\ hours = 1530 hours$ The bus reaches stage B at 3:30pm.

(ii)	From $t = 0\ to\ t = 5 hours$; $$s_1 = \int_0^5 vdt = \int_0^5 2t^2 dt = \left[\frac{2}{3}t^2\right]_0^5 = \frac{2}{3} \times 5^2 - 0 = \frac{250}{3} \approx 83.333km$$ $From\ t = 5\ to\ t = 7.5 hours;$ $$u = 50kmh^{-1}, v = 0kmh^{-1}, a = -20kmh^{-2}$$ $v^2 = u^2 + 2as_2; s_2 = \frac{v^2 - u^2}{2a} = \frac{0^2 - 50^2}{-2\times 20} = 62.5km$ Total distance; $s = s_1 + s_2 = \frac{250}{3} + 62.5 = \frac{875}{6} \approx 145.83km$
(b)	
14(a)	 $$let\ \alpha = tan^{-1}(2); \quad tan\alpha = 2$$ $$Using\ equation\ of\ trajectory\ on\ particle\ P;\ y = xtan\alpha - \frac{gx^2(1 + tan^2\alpha)}{2u^2}$$

	$h = 2h(2) - \frac{g(2h)^2(1+2^2)}{2u^2}$; $1 = 4 - \frac{10gh}{u^2}$; $u^2 = \frac{10gh}{3}$; $u = \sqrt{\frac{10gh}{3}}$
(ii)	Using equation of trajectory on particle; $y = xtan\alpha - \frac{gx^2(1+tan^2\alpha)}{2u^2}$ $$h = 2htan\theta - \frac{g(2h)^2(1+tan^2\theta)}{2\left(\frac{10gh}{3}\right)} ; \quad 1 = 2tan\theta - \frac{3(1+tan^2\theta}{5}$$ $$3tan^2\theta - 10tan\theta + 8 = 0$$ $$3tan^2\theta - 6tan\theta - 4tan\theta + 8 = 0$$ $$3tan\theta(tan\theta - 2) - 4(tan\theta - 2) = 0$$ $$(3tan\theta - 4)(tan\theta - 2) = 0$$ $$tan\theta = \frac{4}{3} \ or \ tan\theta = 2(ignore)$$ $$hence \ \theta = tan^{-1}\left(\frac{4}{3}\right)$$
(c)	Horizontal displacement; $x = (ucos\beta)t$ $$t = \frac{x}{ucos\beta} \ where \ \beta \ is \ the \ angle \ the \ particle \ makes \ with \ the \ horizontal$$ $$Particle \ P; \ t_p = \frac{2h}{\sqrt{\left(\frac{10gh}{3}\right)} \ cos\alpha}$$  $$cos\alpha = \frac{1}{\sqrt{5}}$$ Hence $t_p = \frac{2h}{\sqrt{\left(\frac{10gh}{3}\right)} \times \frac{1}{\sqrt{5}}} = \frac{2h}{\sqrt{\left(\frac{2gh}{3}\right)}} = \frac{2h\sqrt{3}}{\sqrt{2gh}} = \frac{\sqrt{12h^2}}{\sqrt{2gh}} = \sqrt{\frac{12h^2}{2gh}} = \sqrt{\frac{6h}{g}} = \sqrt{\frac{18h}{3g}} = 3\sqrt{\frac{2h}{3g}}$

	$$Particle\ Q; \quad t_Q = \frac{2h}{\sqrt{\left(\frac{10gh}{3}\right)}\ cos\alpha} \quad but\ cos\theta = \frac{3}{5}$$ $$t_Q = \frac{2h}{\sqrt{\left(\frac{10gh}{3}\right)}} \times \left(\frac{5}{3}\right) = \frac{10h\sqrt{3}}{3\sqrt{10gh}} = \sqrt{\frac{100h^2}{9 \times 10gh}} = \sqrt{\frac{10h}{3g}} = \sqrt{5}\sqrt{\frac{2h}{3g}}$$ $$Therefore\ the\ time\ interval\ between\ arrival\ is\ (t_p - t_Q = (3-\sqrt{5})\sqrt{\frac{2h}{3g}}$$
15(a)	$ship\ A;$  $V_A = \begin{pmatrix} 10 \\ 0 \end{pmatrix} kmhr^{-1}$ Ship B;  $V_B = \begin{pmatrix} -14cos60^0 \\ -14sin60^0 \end{pmatrix} = \begin{pmatrix} -7 \\ -7\sqrt{3} \end{pmatrix} kmhr^{-1}$ Velocity of Ship A relative to Ship B, ${}_AV_B = V_A - V_B$ Using Geometrical method;



Using Cosine rule;

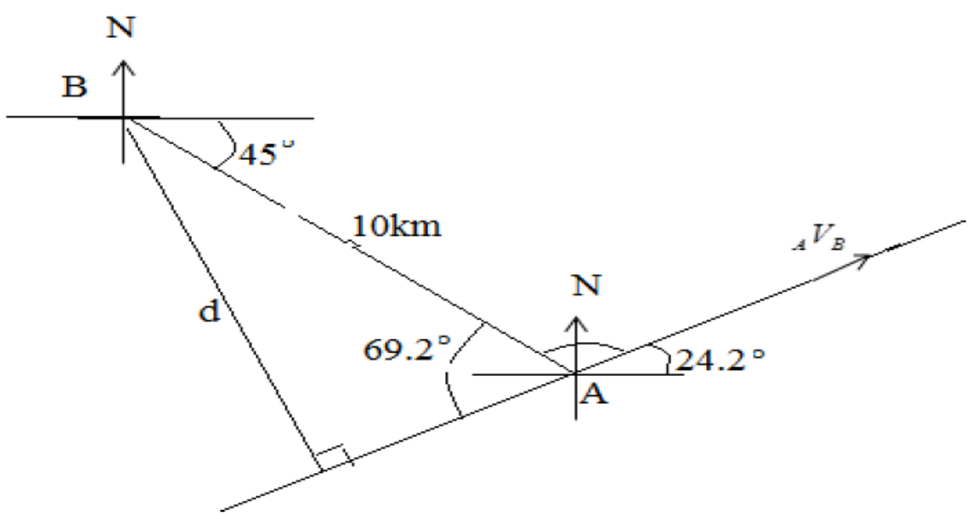
$$\left|{}_AV_B\right|^2 = 20^2 + 14^2 - 2x14x20Cos120$$

$$\left|{}_AV_B\right| = \sqrt{876} \approx 29.597km/hr$$

Using Sine rule;

$$\frac{Sin\theta}{14} = \frac{Sin120^0}{29.597}; \quad \theta = Sin^{-1}\left(\frac{14Sin120^0}{29.597}\right) = 24.2^0$$

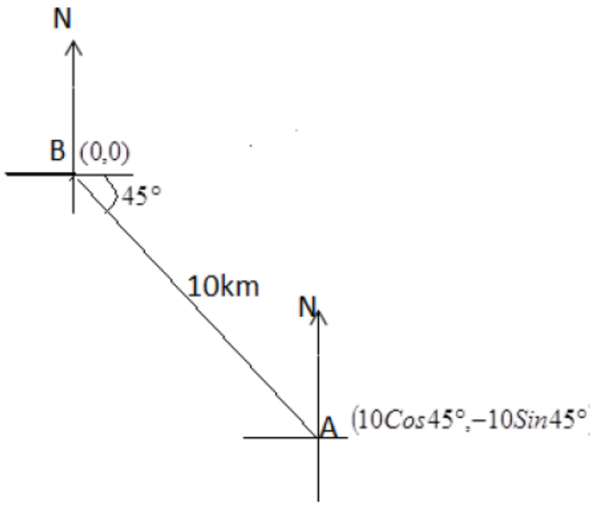
Initial Condition;

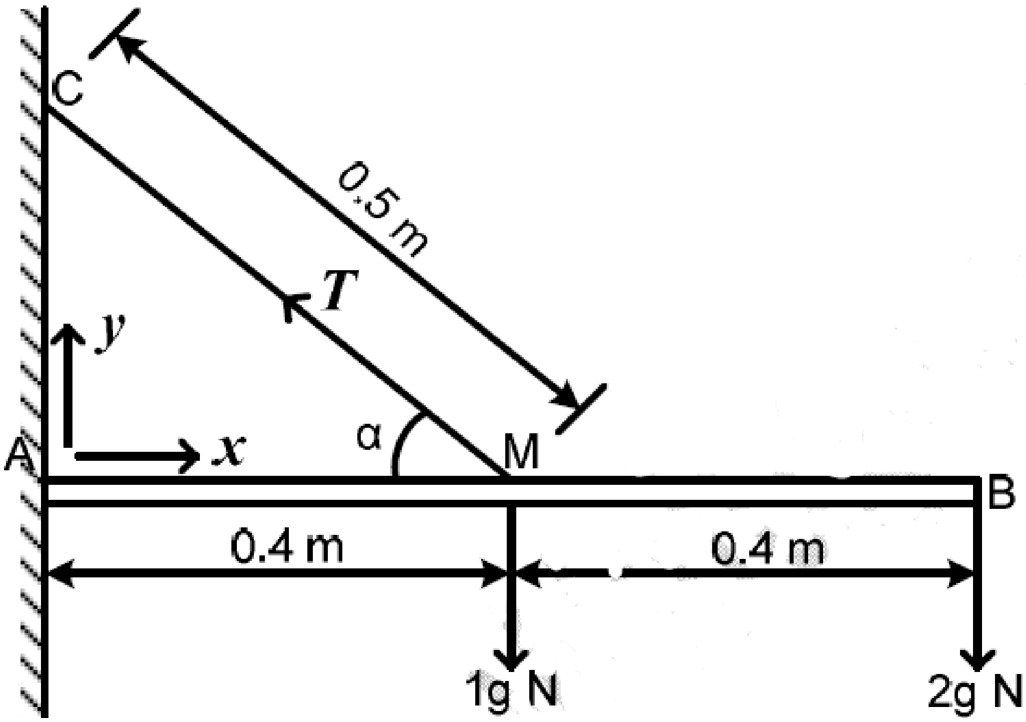


Distance of closest approach; $d = 10sin69.2^0 = 9.35km$

Let time be t; $\left|{}_AV_B\right|.t = 10Cos69.2°$

$$t = \frac{10Cos69.2^0}{29.597} \approx 0.11998 \times 60 \approx 7min$$

	(a) Hence the ships are closest at 10:07 am (b)Distance of closest approach= 9.35km
(c)	 From $r(t) = r(0) + vt$ Displacement at any time; $r_A = \begin{pmatrix} 10Cos45^0 \\ -10Sin45^0 \end{pmatrix} + \begin{pmatrix} 10 \\ 0 \end{pmatrix} t = \begin{pmatrix} 5\sqrt{2} + 10t \\ -5\sqrt{2} \end{pmatrix} km$ $r_B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -7 \\ -7\sqrt{3} \end{pmatrix} t = \begin{pmatrix} -7t \\ -7\sqrt{3}t \end{pmatrix} km$ ${}_A r_B = r_A - r_B = \begin{pmatrix} 5\sqrt{2} + 10t \\ -5\sqrt{2} \end{pmatrix} - \begin{pmatrix} -7t \\ -7\sqrt{3}t \end{pmatrix} = \begin{pmatrix} 5\sqrt{2} + 17t \\ -5\sqrt{2} + 7\sqrt{3}t \end{pmatrix} km$ $When\ t = 7\ minutes;$ ${}_A r_B = \begin{pmatrix} 5\sqrt{2} + 17 \times 7 \\ -5\sqrt{2} + 7\sqrt{3} \times 7 \end{pmatrix} = \begin{pmatrix} 126.07 \\ 77.80 \end{pmatrix}$ Let the angle be θ; $tan\theta = \frac{77.80}{126.07}$; $\theta = 31.7^0$ Hence bearing of A from B is 058.3^0

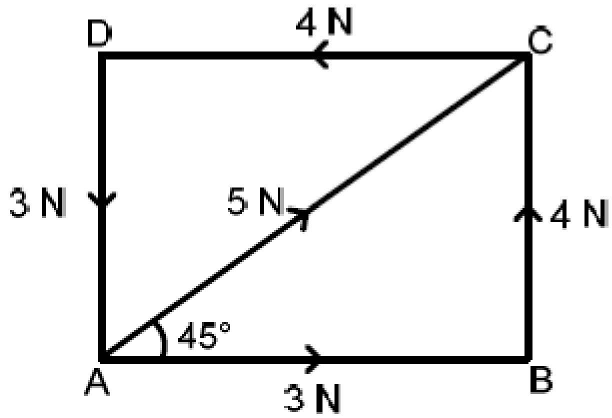
16(a)	 From triangle AMC; $\cos\alpha = \frac{0.4}{0.5} = 0.8, \quad \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-(0.8)^2} = 0.6$ Taking moments about A; $$T \times 0.4\sin\alpha = (1g \times 0.4) + (2g \times 0.8)$$ $$T \times 0.4 \times 0.6 = (9.8 \times 0.4) + (2 \times 9.8 \times 0.8)$$ $$0.24T = 19.6; \quad T = 81.6667N$$

(b)	Resolving Vertically; $$y + 81.6667sin\alpha = 1g + 2g$$ $$y + (81.6667 \times 0.6) = (3 \times 9.8)$$ $$y = 29.4 - 49.6$$ $$y = -19.6N$$ Resolving Horizontally; $$x = 81.6667cos\alpha$$ $$x = 81.6667 \times 0.8$$ $$x = 65.3334N$$ Let F be the force exerted by the hinge $$F = \sqrt{(65.3334)^2 + (19.6)^2} = 68.2101N$$ $$tan\theta = \frac{19.6}{65.3334};\ \theta = tan^{-1}\left(\frac{19.6}{65.3334}\right) = 16.6992^0$$ The force exerted at the hinge is $68.2101N$ in the direction 16.6992^0 below AB.
17(a)(i)	$$F = \begin{pmatrix} 3 \\ a - c \end{pmatrix} + \begin{pmatrix} 2a + 3c \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$$ $$\begin{pmatrix} 7 + 2a + 3c \\ 11 + a - c \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$$ $$7 + 2a + 3c = 10; \quad 2a + 3c = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$ $$11 + a - c = 12; \quad a = c + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$ Substituting (ii) into (i) gives; $$2(c + 1) + 3c = 3$$ $$2c + 2 + 3c = 3$$ $$5c = 1;\ c = \frac{1}{5}$$ From equation (ii), $a = \frac{1}{5} + 1 = \frac{6}{5} = 1.2$

(ii)

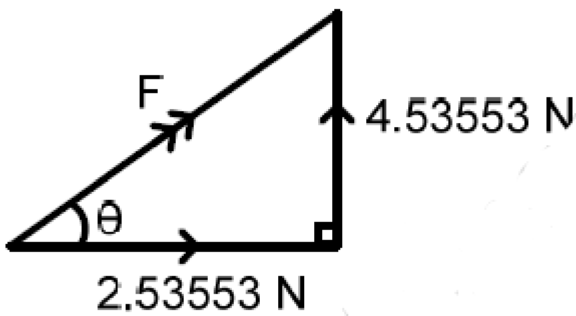
$$F_2 = \begin{pmatrix} (2 \times 1.2) + (3 \times 0.2) \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$|F_2| = \sqrt{3^2 + 5^2} = \sqrt{34} = 5.8310N$



$$F = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} 5cos45^0 \\ 5sin45^0 \end{pmatrix} = \begin{pmatrix} -1 + 2.5\sqrt{2} \\ 1 + 2.5\sqrt{2} \end{pmatrix} = \begin{pmatrix} 2.53553 \\ 4.53553 \end{pmatrix} N$$

Magnitude, $|F| = \sqrt{(2.53553)^2 + (4.53553)^2} \approx 5.19615N$



Direction, $\theta = tan^{-1}\left(\frac{4.53553}{2.53553}\right) = 60.7932^0$ to AB.

WISHING YOU GREAT SUCCESS IN YOUR UNEB EXAMINATIONS

END