P425/1
PURE MATHEMATICS
Paper 1
Mid – Term III 2022
3 hours

INSTRUCTIONS:
Attempt **all** *the* **eight** *questions in Section* **A** *and any* **five** *from Section* **B**.
Begin each answer on a fresh page.
Any additional question(s) answered will **not** *be marked*
All necessary working **must** *be shown clearly.*
Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A; (40 MARKS)

Attempt **all** *questions in this section.*

1. Given that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are roots of the equation $2x^2 - 11x + 15 = 0$ without solving the equation, find the possible value of $\boldsymbol{\alpha} - \boldsymbol{\beta}$, hence form a quadratic equation with roots $\boldsymbol{\alpha}$ and $-\boldsymbol{\beta}$ $(\boldsymbol{\alpha} > \boldsymbol{\beta})$. (*05 marks*)

2. Solve the equation $3\sin x + \cos 2x = 2$, for $0 \leq x \leq 2\pi$. (*05 marks*)

3. The first, fourth and eighth terms of an A.P form a G.P. if the first term is 9, find the common ration of the GP and the common difference of an A.P. (*05 marks*)

4. Find the equation of a circle with diameter $\boldsymbol{AB}$, where $\boldsymbol{A}(-1, 6), \boldsymbol{B}(1, 12)$. (*05 marks*)

5. The function $y = \boldsymbol{a}x^3 + \boldsymbol{b}x^2 + \boldsymbol{c}$ has turning points at $(0, 4)$ and $(-1, 5)$. Find the values of $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$. (*05 marks*)

6. Find the equation of a line which is perpendicular to $3x + 2y = 1$ and passes through the point of intersection of the lines. $x + 2y - 1 = 0$ and $2x - y + 8 = 0$. (*05 marks*)

7. Given the parametric equation $y = \tan\theta, x = \sec^2\theta$,
Prove that $\frac{d^2y}{dx^2} = \frac{-1}{4}\cot^3\theta$. (*05 marks*)

8. Given that the complex number $\boldsymbol{Z}$ and its complex $\boldsymbol{X}$ conjugate $\bar{\boldsymbol{z}}$ satisfy $3\boldsymbol{Z}\bar{\boldsymbol{z}} + 2i\bar{\boldsymbol{z}} = 11 + \frac{10}{3}i$ find the possible values of $\boldsymbol{Z}$. (05 marks)

SECTION B; (60 MARKS)

Attempt any **five** *questions from this section.*

9. (a) The first term of an arithmetic progression is -11, the last term is 44 and the sum of the terms of the progression is 198. Find;

(i) The number of terms in the progression

(ii) The common difference (*06 marks*)

(b) John deposits shs. 3,000,000 at the beginning of every year in a macro – finance bank starting 2015, how much would he collect at the each of 2020 if the bank offers compound interest of 12.5% per annum and the no withdrawal is made within the period. (*06 marks*)

10. (a) Represent the following complex numbers on the same argand diagram.

$z_1 = 3 + 4i$

$z_2 = -2 + 3i$

$z_3 = -4 - 2i$

$z_4 = 3 - 4i$

Hence find the principle argument of each. (*08 marks*)

(b) Use Demoivre's theorem to show that;

$\frac{\cos 5\theta}{\cos \theta} = 1 - 12\sin^2 \theta + 16\sin^4 \theta.$ (*04 marks*)

11. Given that $y = \frac{\sin x - 2\sin 2x + \sin}{\sin x + 2\sin 2x + \sin 3}$

(a) Prove that $y + \tan^2\left(\frac{x}{2}\right) = 0$, and hence express the exact value of $\tan^2 15°$ in the form $p + q\sqrt{r}$ where p, q and r are integers.

(b) Hence find the value of x between $0°$ and $360°$ for which

$2y + \sec^2\left(\frac{x}{2}\right) = 0.$ (*12 marks*)

12. (a) Show that the circles,

$x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 40$ are orthogonal. (*06 marks*)

(b) Sketch the parabola $(x - 3)^2 = 16y$. State its;

(i) vertex

(ii) focus (*06 marks*)

13. (a) Given that $\frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{\boldsymbol{c}}{\boldsymbol{d}} = \boldsymbol{k}$, show that $\boldsymbol{k} = \frac{\boldsymbol{a+c}}{\boldsymbol{b+d}}$. Hence solve the equation

$\frac{x+4z}{4} = \frac{y+z}{6} = \frac{3x+y}{5}$ and $4x + 2y + 5z = 30$ (*07 marks*)

(b) Prove that $\log_c ab = \log_c a + \log_c b$. Hence solve the equation

$\log_3(x - 2) + \log_3(x + 3) = 3.$ (*05 marks*)

14. (a) Given that $y = \sqrt{\frac{1+\cos}{1-\cos}}$, show that $\frac{dy}{dx} = \frac{-1}{1-\cos}$ (*06 marks*)

(b) Given that $f(x) = 4x^2 - 8x + 13$. Express $f(x)$ in the form $a + b(x + c)^2$, hence find the minimum value of $f(x)$, starting the value of x which it occurs. (*06 marks*)

15. (a) A curve is given parametrically by $y = 3\left(\frac{1}{p^2} + \frac{2}{p} + 1\right)$ and $y = 6\left(\frac{1+p}{p}\right)$ show that the curve is a parabola and find its focus.

(*05 marks*)

(b) (i) Find the equation of the tangent to the parabola $y^2 = 4ax$ at the $T(at^2, 2at)$

(ii) The tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at R, find the coordinates of R.

(*07 marks*)

END