UGANDA MARTYRS' S.S NAMUGNOG
U.A.C.E PRE MOCK EXAMINATIONS 22004
P425/1 (PURE MATHS)
MATHEMATICS PAPER 1
3 HOURS

INSTRUCTIONS:
Attempt all questions in section A; and not more than five from section B.

SECTION A: (40 MARKS)

1. Solve the simultaneous equations:
   $x^2 - y = 9$
   $x + \sqrt{y} = 9$ (5 marks)

2. The line $y = mx - 5$ is a tangent to the curve $y = x^2 - 2mx + 4$. Find the possible values of m; and in each case determine the co-ordinates of the points of contact. (5 marks)

3. Given the circles c_1: $x^2 + y^2 - 6x + 4y = 12$
   c_2: $x^2 + y^2 + x - y = 7$
   (i) Show that c_1 passes through the centre of c_2; and
   (ii) Determine the co-ordinates of the other point at which the line joining the centres of the circles meets c_1.

4. Evaluate $\int_{o}^{\pi/4} \frac{1}{1 + \sin x} dx$, leaving surds in your answer.

5. Solve the equation Cot θ+ tan$\theta = 4/\sqrt{3}$; for $0^o \leq \theta \leq 180^o$.

6. The n^{th} term of a G.P is A_n. Given that $A_1 = 1$, $A_2 = 1 + x$, $A_3 = 5+x^2$, $A_4 = a+x^4$, find a and x; hence determine $\sum_{n=1}^{10} A_n$ (5 marks)

7. A is the point on the curve $y = \ln\left(\frac{xe^x}{\sqrt{2 - x^2}}\right)$ at which x = 1.
   Find:
   (i) the y – co-ordinate of B
   (ii) the value of the gradient of the curve at B (5 marks)

8. The normal from the point P(2, 1, -4) meets the plane $x - 2y + 2z = 1$ in point N. Find the equation of PN; and deduce the co-ordinates of N.

SECTION B (60 MARKS)

9. (a) Show that $\sqrt{(1 + 4ax)} = 1 + 2ax - 2a^2x^2 + 4a^3x^3 + \ldots$ and deduce the expansion of $\sqrt{(1 - 4x^2)}$ up to the term in x^6. Letting $x = {}^1/_{10}$; evaluate $\sqrt{6}$ to 5 dpls (6 marks)

(b) Given that $y(x) = e^x \sin x$; prove that $y^{11} = 2(y^1 - y)$ Hence find the first four non vanishing terms of the Maclaurin's expansion of y. (6 marks)

10. (a) Solve $\cos 3\theta = \cos(2\theta + 45^o)$ for $0^o \leq \theta \leq 180^o$

(b) Prove that, for any triangle ABC; $\sin\frac{1}{2}(A - B) = \frac{(a - b)}{c}\cos\frac{1}{2}C$; hence solve the triangle for $C = 60^o$, $a = 8$, $b = 5$. (7 marks)

11. Given the planes:

$$P_1: 2x + y - z = 4$$
$$P_2: x + 2y + z = 11$$

(i) Find the equation of L_1, the line of intersection of P_1 and P_2

(ii) L_1, is perpendicular to a plane P_3 containing the point $(-1, -3, 5)$; Find the equation of P_3 (12 marks)

(iii) Find the angle between P_2 and P_3

12. (a) One of the roots of $z^3 - 8z^2 + 22z - 20 = 0$ is $z = 2$; find the other roots (5 marks)

(b) Given that $\text{Arg}(Z) = {}^{\pi}/_4$ where Z is a variable complex number; find the locus of the point representing the number $z + \frac{2}{Z}$. Sketch this locus. (7 marks)

13. (a) Show that the general solution to the equation $y\frac{dy}{dx} + x = 2y$ is of the form

$$\ln\left(\frac{A}{y - x}\right) = \frac{x}{y - x}$$

where A is a constant (6 marks)

(b)A sports car is being tested on a Stretch of 10 km. It is found that the rate of increase of velocity v with distance covered x, is proportional to the velocity and (proportional to) the distance remaining. Given that its initial speed, and acceleration are 5kmh^{-1} and 25kmh^{-2} respectively; find its speed, and acceleration at the end of the stretch; and mid-way through the stretch respectively. (6 marks)

14. (a) Find $\int \sin^2 2x \tan x \, dx$ (5 marks)

(b) Use the substitution $x = \tan\theta$ to solve

$$\int \left(\frac{1 - x^2}{1 + x^2} \right) dx$$

(7 marks)

15. Find the equation of the tangent to the curve $y^2 = 4ax$ at the point $(at^2, 2at)$. The tangents from point $A(-6a, a)$ meet the curve in points B and C. The line through A parallel to the x – axis meets the chord BC in point T.
    - (i) Find the co-ordinates of B and C; and determine the angle between the tangents from A.
    - (ii) Show that BC is a normal to the curve.
    - (iii) Show that AT is bisected by the curve; and that it bisects the chord BC

(12 marks)

16. Given that $y = \dfrac{(x-2)^2}{x^2 + 4x}$

Find the range of values of y within which the curve cannot lie, hence determine the co-ordinates of the turning points. Determine the equations of the asymptotes; hence sketch the curve. (12 marks)

End