P425/2
APPLIED MATHEMATICS
Paper 2
Oct. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) Set 13

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g *to be* $9.8 \ ms^{-2}$.

SECTION A: (40 MARKS)
*Answer **all** the questions in this section.*

1. The data below shows the length of 8 leaves picked for a biology practical: 4.4, 6.2, 9.4, 12.6, 10.0, 8.8, 3.8, and 13.6, determine the;
   (i) Mean length of the leaves
   (ii) Standard deviation   (05 marks)

2. Two particles A and B are moving along a straight path. B is ahead of A by 14m, the speed of B is 16ms⁻¹ and that of A is 25ms⁻¹. Given that A and B have constant retardations of 6ms⁻² and 4ms⁻² respectively. Find the two possible times of overtake.   (05 marks)

3. Show that the equation $x^3 - 3x - 12 = 0$ has a root between $x = 2$ and $x = 3$. Hence use linear interpolation to find the first approximation to the root correct to 2 decimal places.   (05 marks)

4. Events A and B are such that P(A)$= \frac{5}{12}$, P($^A/_B$)$= \frac{7}{12}$, and P(AnB) $= \frac{1}{8}$. Find the;
   (i) P(B)
   (ii) P(Au$\bar{B}$)   (05 marks)

5. One end of a light inextensible string of length 75cm is fixed to a point on a vertical. A particle of mass 1.2kg is attached to the other end of the string. The particle is kept in equilibrium 21cm away from the pole by a horizontal force, F Newtons. Find the;
   (i) Tension in the string
   (ii) Magnitude of **F**   (05 marks)

6. Use the trapezium rule with six ordinates to estimate $\int_0^{\frac{\pi}{2}} x\sin x \, dx$, correct to **three** decimal places.   (05 marks)

7. A continuous random variable X has probability density function
$$f(x) = \begin{cases} a + bx; & 0 \leq x \leq 1 \\ 0; & otherwise \end{cases}$$
   Given that $F(0.5) = \frac{3}{5}$, find the values of a and b.   (05 marks)

8. To an observer in a train moving due east at 40kmh⁻¹, an aircraft appears to fly due north at 75kmh⁻¹. Find the velocity and course of the aircraft.   (05 marks)

SECTION B: (60 MARKS)
*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The time (t) in seconds spent by each 100 customers at an automated Teller Machine (ATM) are summarized in the table below

Time (t) in seconds	Frequency Density
$10 \leq x \leq 15$	0.4
$15 \leq x \leq 25$	0.8
$25 \leq x \leq 30$	3.4
$30 \leq x \leq 35$	5.2
$35 \leq x \leq 45$	2.4
$45 \leq x \leq 50$	3.2
$50 \leq x \leq 60$	0.6
$60 \leq x \leq 80$	0.05

   (a) Calculate the mode

   (b) Construct an ogive and estimate;

      (i)   median.

      (ii)   Interquartile range   (12 marks)

10. A particle of mass 4 kg starts from a point with position vector (3, 4, -5) moving with a velocity $V = 3t^2\hat{\imath} + (4t^3 - 3t)\hat{\jmath} + 4t^3\hat{k}$ ms⁻¹, where t is time in seconds. Determine the,

        a. Magnitude of acceleration and hence the force when $t = 2s$

        b. Displacement and hence distance when time $t = 3s$.   (12 marks)

11. Given that $a = 5, b = 14.5$, and $c = 8.24$

   (a) State the maximum errors of a, b and c

   (b) Find the interval within which the exact value of $\frac{abc}{(b-a)}$ lies correct to 3 d.p

   (c) Find the absolute, relative and Percentage error in $\frac{abc}{(b-a)}$ (12 marks)

12. The table below shows the marks of 13 students in two separate tests

Aptitude test x	54	52	42	31	43	23	32	49	37	13	13	36	39
Statistics test y	84	68	71	37	79	58	33	60	47	60	44	64	49

   (a) Plot a scatter diagram for the data, draw the line of best fit and comment, hence estimate y when x=30

   (b) Calculate the rank correlation co-efficient and comment at 5% level of significance.   (12 marks)

13. The diagram below shows particles A, B and C of masses 15kg, 10kg and 5kg respectively connected by a light in extensible strings. The string connecting B and C passes over a smooth light pulley fixed at the top of the plane
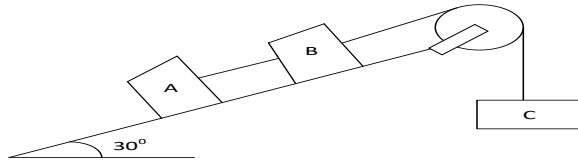
If the coefficient of friction between the plane and particles A and B are 0.2 and 0.3 respectively, calculate;

        (i)   Acceleration of the system
        (ii)   Tensions in the strings   (12marks)

14. (a) Show that the iterative formula base on Newton Raphson's method for the equation $e^{-x} = x - 1$ is given by

$$x_{n+1} = \frac{e^{-x_n}(x_n + 1) + 1}{1 + e^{-x_n}} , \qquad n = 0, 1, 2 \dots \dots \dots \dots \dots \dots$$

(b) Draw a flow chart that;

    (i) Reads the initial approximation x_0 of the root
    (ii) Computes and corrects the root to 3 decimal places
    (iii)   prints the root

(c) Taking, $x_0 = 1.2$, perform a dry run for the flow chart, give your root correct to three decimal places   (12marks)

15. Given that X is a continuous random variable which is normally distributed with mean, μ and standard deviation, δ. Such that $P(X > 70) = 0.2$ and $P(x < 50) = 0.3$. Find the;

    (a) Mean and standard deviation.
    (b) Percentage of those with $P(x > 60)$ (12 marks

16. A uniform plank of length 6m and mass 50kg rests in limiting equilibrium with the end A on a horizontal ground and end B against vertical wall. If the coefficient of friction at end A and B are 0.25 and 0.3 respectively and that a body of mass, m is attached at B. If the plank makes an angle of β to the horizontal, where $\tan \beta = 2$. Find the;

    (i) Reactions at A and B
    (ii) mass, m.   (12 marks)