P425/1

PURE MATHEMATICS

JUNE/JULY 2023

UGANDA ADVANCED CERTIFICATE OF EDUCATION

EXTERNAL MOCK EXAMINATION 2023

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Answer all the **eight** questions in section **A** and only **five** questions in section **B**.
- Any additional question(s) will not be marked.
- **All** working **must** be shown clearly.
- Graph paper is provided.
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- Clearly indicate the questions you have attempted on the answer scripts as illustrated, **DO NOT** hand in the question paper.

Question		Mark
Section A		
Section B		
Total		

SECTION A (40 MARKS)

1. How many terms of the G.P $2+2\times(1.1)+2\times(1.1)^2+\ldots$ must be taken for the sum to exceed 1000? (5 marks)

2. Solve the equation: $6\cos\theta\tan\theta-3\cos\theta+4\tan\theta-2=0$ for $-180^o\le\theta\le180^o$. (5 marks)

3. Find the equation of a line parallel to $y-3x+5=0$ and passes through the point of intersection between the lines $2y+3x=5$ and $3y-2x=14$. (5 marks)

4. Find the equation of the tangent to the curve $y=\sqrt{(4x+5)}$ at the point $(1,\ 3)$. (5 marks)

5. Given that $\mathbf{OP}=4\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ and $\mathbf{OQ}=\mathbf{i}+4\mathbf{k}$, find the coordinates of point M which divides $\mathbf{PQ}$ externally in the ratio $1:2$. (5 marks)

6. Find the coefficient of x^2 in the expansion of $(2-3x+x^2)(1+2x)^4$. (5 marks)

7. Differentiate from first principles: $y=\cos2x$. (5 marks)

8. A container in the shape of a hollow cone of semi-vertical angle 30^o is held with its vertex pointing downwards. Water is poured into the cone at the rate of $5\ cm^3\ s^{-1}$. Find the rate at which the depth of water in the cone is increasing when this depth is $10cm$. (5 marks)

SECTION B: 60 MARKS

ATTEMPT ONLY 5 QUESTIONS IN THIS SECTION

9. The triangle ABC with vertices $A(1,\ -2)$, $B(7,\ 6)$ and $C(9,\ 2)$. Find the equations of the perpendicular bisectors of AB and BC, and the coordinates of the point of intersection of the perpendicular bisectors, hence find the equation of the circle passing through the three points. (12 marks)

10. A particle is moving in a straight line such that its distance from a fixed point O, t s after motion begins is $s = \cos t + \cos 2t \ \ m$, find:

i) the time when the particle first passes through O.

ii) the velocity of the particle at this instant.

iii) the acceleration when the velocity is zero. (12 marks)

11a) The profit y generated from the sale is given by the function $y = 72x + 3x^2 - 2x^3$. Calculate how many terms should be sold to receive maximum profit and determine the maximum profit. (5 marks)

b) Sketch the curve $y = x^2 + 4x$, and find the area enclosed between the curve $y = x^2 + 4x$, the $x-$axis from $x = -2$ to $x = 2$. (7 marks)

12a) Solve the equation $(2\cot 2x - 1)^2 = 3(\cos ec^2 2x - 2)$ for $0^o \le x \le 200^o$. (5 marks)

b) Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x + \alpha)$, hence or otherwise solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ in the range $0^o \le x \le 360^o$. (7 marks)

13a) Given the points $A(2,\ 13,\ -5)$, $B(3,\ \alpha,\ -3)$ and $C(6,\ -7,\ \beta)$ are collinear, find α and β. (6 marks)

b) The points A and B have position vectors $4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i} + t\mathbf{j}$, determine the values of t given that the angle $AOB = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$, O is the origin. (6 marks)

14a) Given that $2 - 3i$ is a root of the equation $z^3 + pz^2 + qz + 13 = 0$, find the other roots and the real value of the constants p and q. (6 marks)

b) Solve the equation: $z\bar{z} - 2z + 2\bar{z} = 5 - 4i$. (6 marks)

15a) Solve the equation: $x - x\log 5 = \log 6 - \log\left(1 + 2^x\right)$. (6 marks)

b) Solve the equation: $5^{3x} - 19\left(5^x\right) - 30 = 0$. (6 marks)

16a) A curve has an equation satisfied by $\frac{d^2y}{dx^2} = 24x^2 - 2$, the point $P(1,\ 4)$ lies on the curve and the gradient of the curve at P is 5, determine the equation of the curve. (5 marks)

b) The variables x and y are related by the differential equation $x\frac{dy}{dx} = 1 - y^2$. When $x = 2,\ y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. (7 marks)

GOOD LUCK