WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
UACE August
Mathematics P425/2
July/August 2023

$\frac{4}{7}+2.\frac{4}{7}\left(\frac{3}{2}-x\right)$

$\frac{4}{7}+\frac{21}{7}-\frac{8}{7}$

1.	$P(B)=\frac{1}{6}$, $P(AnB)=\frac{1}{12}$, $P(B/A)=\frac{1}{3}$ From $P(B/A)=\frac{P(BnA)}{P(A)}$ (a) (i) $P(A)=\frac{P(BnA)}{P(B/A)}$ $=\frac{1}{12}\div\frac{1}{3}$ $=\frac{1}{4}$	M_1 A_1	
	(ii) $P(A/B^1)=\frac{P(AuB^1)}{P(B^1)}$ $=P(A)=\frac{P(AuB)}{P(B^1)}$ $=\frac{\frac{1}{4}-\frac{1}{12}}{\frac{5}{6}}$ $=\frac{1}{5}$	M_1 A_1	
	(iii) For independence $\frac{1}{12}=\frac{1}{4}.\frac{1}{6}$ $\frac{1}{12}\neq\frac{1}{24}$ A and B are not independent.	B_1	
			05 marks
2.	(i) T(s) | 240 | 300 | 360 θ^0c | 75 | θ_1 | 69 $\frac{69-75}{360-240}=\frac{\theta_1-75}{300-240}$ $\frac{-6}{120}=\frac{\theta_1-75}{60}$ $\theta_1=75+\left(\frac{-6\times60}{120}\right)$ $=70^0c$ $\theta_1=72^0c$.	B_1 M_1 A_1	

	(ii) T(s) | 480 | 600 | T_1 θ^0c | 56 | 48 | 42 $\frac{42-56}{T_1-480}=\frac{48-56}{600-480}$ $\frac{-14}{T_1-480}=\frac{-8}{120}$ $T_1=480+\frac{14\times120}{8}$ $=690s$ T = 675s.	M_1 A_1	
			5 marks
3.	$u=72\times\frac{1000ms^{-1}}{3600}$ $=20ms^{-1}$ $v=36\times\frac{1000}{3600}$ $=10ms^{-1}$ From $v^2=u^2+2as$ $10^2=20^2+2a\times800$ $100=400+1600a$ $a=\frac{-3}{16}ms^{-2}$ or $-0.1875ms^{-2}$ From $v^2=u^2+at$ $10=20-\frac{3}{16}t$ $t=\frac{160}{3}$ second or 53.3 seconds	B_1 M_1 A_1 M_1 A_1	for both $20ms^{-1}$ and $10ms^{-1}$ with correct units. ≥1D
			5 marks
4.	$\frac{P2021}{P2000}=\frac{90}{100},\frac{P2022}{P2021}=\frac{120}{100}$ $\frac{P2022}{P2000}=\frac{P2022}{P2021}\times\frac{P2021}{P2000}$ $=\frac{120}{100}\times\frac{90}{100}$ $=\frac{27}{25}$ $\therefore P2022=\frac{27}{25}\times200,000$ $=216000/-$	B_1 M_1 A_1 M_1 A_1	
			5 marks

5. $d = \frac{2-1}{5} = 0.2$ or $\frac{1}{5}$ — B_1

x	$y = x\sin x$	$\sin x$
1.0	0.8415	
1.2		1.1184
1.4		1.3796
1.6		1.5993
1.8		1.7529
2.0	1.8186	
sum	2.6601	5.8502

B_1 – All values of x **B_1 all values y**

Using the trapezium rule for 6 ordinates. — M_1

$= \frac{1}{2} \times 0.2 \times [2.6601 + 2 \times 5.8502]$

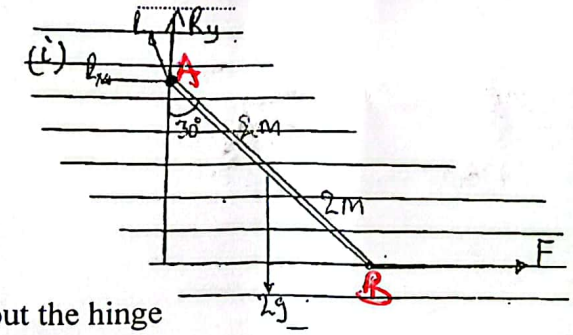
$= 0.1 \times 13.8205$ 14.3605

$= 1.38205$ ≃ 1.43605

$I \cong 1.382$ ≃ 1.436 . (3dp). — A_1

5 marks

6. (i)



B_1

Taking moments about the hinge — M_1

$4F\cos 30^0 = 2\sin 30 \times 2g$

$F = \frac{2 \times \sin 30 \times 2 \times 9.8}{4 \times \cos 30}$ — A_1

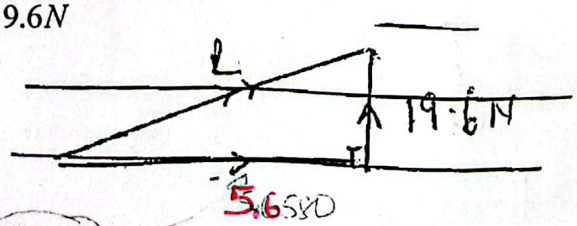
$= 5.6580N$

(ii) Resolving forces

$= Ry = 2g$ $R_x = F$

$= 2 \times 9.8$ $R_x = 5.6580N$

$= 19.6N$

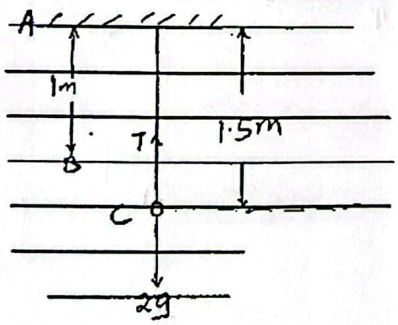


M_1

$R = \sqrt{5.6580^2 + 19.6^2}$ (5.6580)

$\therefore |R| = 20.4003N$ — A_1

5 marks

7.	(a) Let J: Jane, M: Mary, A: Alice $P(J) = P(J^1nM^1nA) = P(J^1) \cdot P(M^1) \cdot P(A)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$ $= \frac{25}{216}$ *or* 0.1157 (4dp)	M_1 A_1	
	(b) 1st time $P(A) = \frac{1}{6}$ 2nd time $P(A) = \frac{1}{6} \times \left(\frac{5}{6}\right)^3$ 3rd time $P(A) = \frac{1}{6} \times \left(\frac{5}{6}\right)^6$ $P(wining) = \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^3 + \frac{1}{6}\left(\frac{5}{6}\right)^6 + \ldots\ldots$ $a = \frac{1}{6}$, $r = \left(\frac{5}{6}\right)^3$ $S_\infty = \frac{a}{1-r}$ $= \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3}$ $= \frac{36}{91}$ or 0.3956 (4dp)	M_1 A_1	
			5 marks
8.	 From point A to B Loss in P.E = gain in K. E = 2 x 9.8 x 1 = 19.6 J From B to C Loss in P.E and K.E = elastic P.E stored $19.6 + 2x9.8x = \frac{\lambda x^2}{2lo}$ $19.6 + 2x9.8(1.5-1) = \frac{\lambda(1.5-1)^2}{2\times1}$ $\therefore \lambda = 235.2N$	B_1 A_1 M_1 M_1 A_1	
			5 marks

Height	Freq	x	fx	fx^2	C.f	cb
120 - 124	5	122	610	74420	5	
125 – 129	17	127	2159	274193	22	
130 – 134	20	132	2640	348480	42	
135 – 139	25	137	3425	469225	67	
140 – 144	15	142	2130	302460	82	
145 – 149	6	147	882	129654	88	
150 - 154	2	152	304	46208	90	
	$\Sigma f = 90$		$\Sigma fx = 12150$ B_1	$\Sigma fx^2 = 1644640$ B_1		

(a) Mean $= \frac{\Sigma fx}{\Sigma f}$ M_1

$= \frac{12150}{90}$

$= 135cm$ A_1

$SD = \sqrt{\frac{1644640 - (135)^2}{90}}$

$= 6.9841$

M_1 A_1

7/20

(b) On graph paper at the back.

(c) (i) Median $= \frac{50}{100} \times 90$

$= 45$

$= 135cm$ ±0.5

B_1

with location seen on graph.

(ii) 20^{th} percentile $= \frac{20}{100} \times 90$

$= 18$

$= 128.5$cm (graph)

80^{th} percentile $= \frac{80}{100} \times 90$ ±0.5

$= 72$

$= 141$ cm (graph)

Range $= 141 - 128.5$

$= 12.5$cm

B_1 A_1

for both values with location seen on graph.

10 (a) $x = \sqrt[4]{N}$

$x^4 - N = 0$

$f(x) = x^4 - N$

$f^1(x) = 4x^3$

$f^1(x_n) = 4x^3$

B_1 B_1

$f(x_n) = x^4_n - N$

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^4 - N}{4x_n^3} \quad M_1$$

$$= \frac{4x_n^4 - x^4_n + N}{4x_n^3}$$

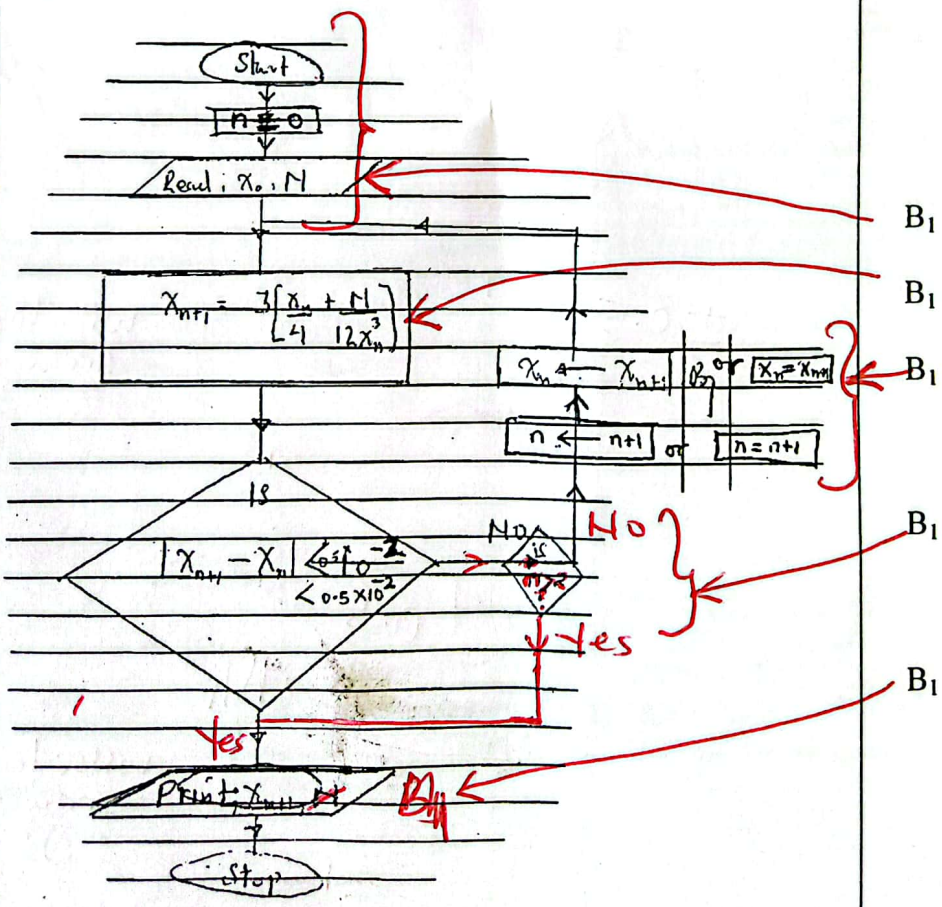
$$= \frac{3x_n^4 + N}{4x_n^3}$$

$$= \frac{3x^4}{4x_n^3} + \frac{N}{4x_n^3}$$

$$= 3\left[\frac{x_n}{4} + \frac{N}{3.4x_n^3}\right]$$

$$= 3\left[\frac{x_n}{4} + \frac{N}{12x_n^3}\right] \text{ as required} \quad B_1$$

(b) (i) & (ii)



B1 B1 B1 B1 B1

$= -2kx + 4k$

(c)

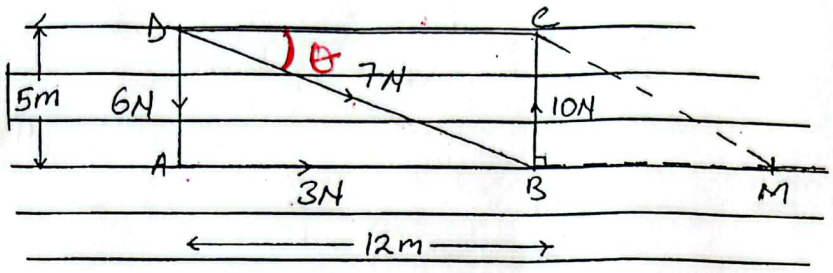
n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	3	3.1667	0.1667
1	3.1667	3.1544	0.0123
2	3.1544	3.1543	0.0001

$x = 3.15$, $N = 99$

B_1 → Correct x_n.
B_1 → Correct $|x_{n+1} - x_n|$
B_1 → Correct output.

12 marks

11 (a)



B_1

$$\underset{\sim}{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \end{pmatrix} + \begin{pmatrix} +7 \times \frac{12}{13} \\ -7 \times \frac{5}{13} \end{pmatrix}$$

M_1

Accept θ to ≥ 1D

to 1D of θ, i.e. 22°

$$= \begin{pmatrix} -3.4615 \\ 1.3077 \end{pmatrix} N$$

9.4615

A_1

$\underset{\sim}{R} = (9.4615$

$R^2 = 9.4615^2 + 1.3077^2$

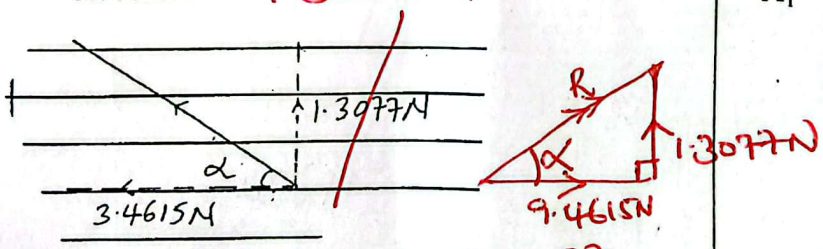
B_1

$$\left|\underset{\sim}{R}\right| = \sqrt{(-3.4615)^2 + (1.3077)^2}$$

M_1

$= 3.7002N$ → 9.551414

A_1 ≥ 2D.



$$\alpha = \tan^{-1}\left(\frac{1.3077}{3.4615}\right)$$

$\alpha = \tan^{-1} \frac{1.3077}{9.4615}$

$= 20.6958^0$

$\alpha = 7.87°$ → N82.13°E

→ with AB or with the horizontal

The resultant is 3.7002N in the direction N69.30°W

A_1 — Stating direction.

(b) A $G = 10 \times 12 - \left(7 \times 12 \times \frac{5}{13}\right)$

B_1

Correct value of G with units

$= 87.6923$ Nm.

$\begin{vmatrix} x & -3.4615 \\ y & 1.3077 \end{vmatrix} = 87.6923$ → $\begin{vmatrix} x & 9.4615 \\ y & 1.3077 \end{vmatrix} = 87.6923$

$1.3077x - 3.4615y = 87.6923$

B_1 {eqn 3.

When $y = 0, x = \frac{87.6923}{1.3077}$

M_1 (Subst. $y = 0$)

$= 67.0584m$

$\overline{BM} = 67.0584 - 12$

$= 55.0584$

$\overline{MC} = \sqrt{5^2 + 550584^2}$

$= 55.285m$

B_1

M_1

A_1

12 (i) Area, $\frac{1}{2}k\left(2+\frac{3}{2}\right)=1$

$\frac{7}{4}k = 1$

$k = \frac{4}{7}$

M_1

A_1

(ii) for $0 \le x \le \frac{3}{2}$, $f(x) = \frac{4}{7}$

B_1

for $\frac{3}{2} \le x \le 2$, $\frac{0-\frac{4}{7}}{2-\frac{3}{2}} = \frac{y-\frac{4}{7}}{x-\frac{3}{2}}$

M_1

$\frac{-\frac{4}{7}}{\frac{1}{2}} = \frac{y-\frac{4}{7}}{x-\frac{3}{2}}$

$y - \frac{4}{7} = \frac{-8}{7}\left(x-\frac{3}{2}\right)$

$y = \frac{-8x}{7} + \frac{16}{7}$

$f(x) = \frac{8}{7}(2-x)$

A_1

$f(x) = \begin{cases} \frac{4}{7}, & 0 \le x \le \frac{3}{2} \\ \frac{8}{7}(2-x), & \frac{3}{2} \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

B_1

(iii) $P\left(\frac{1}{2} \le x \le \frac{7}{4}\right) = \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{4}{7}\,dx + \int_{\frac{3}{2}}^{\frac{7}{4}} \frac{8}{7}(2-x)\,dx$

$= \left[\frac{4x}{7}\right]_{\frac{1}{2}}^{\frac{3}{2}} + \frac{8}{7}\left[2x - \frac{1}{2}x^2\right]_{\frac{3}{2}}^{\frac{7}{4}}$

$= \frac{4}{7} + \frac{8}{7}\left[\left(\frac{7}{2} - \frac{49}{32}\right) - \left(3 - \frac{9}{8}\right)\right]$

$= \frac{4}{7} + \frac{8}{7}\left(\frac{63}{32} - \frac{15}{8}\right)$

$= \frac{4}{7} + \frac{3}{28}$

$= \frac{19}{28}$

M_1 — Correct integrand with limits seen

M_1 — Substitution of limits

A_1

(iv) $E(X) = \int_0^{\frac{1}{2}} x.\frac{4}{7}\,dx + \int_{\frac{1}{2}}^{2} x.\frac{8}{7}(2-x)\,dx$

$= \frac{4}{7}\left(\frac{1}{2}x^2\right)\Big|_0^{\frac{1}{2}} + \frac{8}{7}\left[x^2 - \frac{1}{3}x^3\right]_{\frac{1}{2}}^{2}$

$= \frac{2}{7}.\frac{9}{4} + \frac{8}{7}\left(\frac{4}{3} - \frac{9}{8}\right)$

$= \frac{9}{14} + \frac{8}{7}\left(\frac{5}{24}\right)$

$= \frac{37}{42}$

M_1 Integration with limits

M_1 Substn of limits

A_1

12 marks

13 (a) $y_1 = x_1 + e_1,\ y_2 = x_2 + e_2$

$ey_1y_2 = y_1y_2 - x_1x_2$

$= (x_1 + e_2)(x_2 + e_2) - x_1x_2$

$= x_1x_2 + x_1e_2 + x_1e_2 + e_1e_2 - x_1x_2$

$= x_1e_2 + x_2e_1 + e_1e_2$

As e_1e_2 becomes two small then $e_1e_2 \approx 0$

$ey_1y_2 = x_1e_2 + x_2e_1$

$\left|\frac{ey_1y_2}{y_1y_2}\right| = \left|\frac{x_1e_2 + x_1e_2}{x_1x_2}\right|$

$= \left|\frac{e_2}{x_2} + \frac{e_1}{x_1}\right|$

$\leq \left|\frac{e_2}{x_2}\right| + \left|\frac{e_1}{x_1}\right|$

$\left|\frac{ey_1y_2}{y_1y_2}\right|_{Max} = \left|\frac{e_1}{x_1}\right| + \left|\frac{e_2}{x_2}\right|$ as required.

M_1 defn of error

B_1 assumption

B_1

M_1 RE

B_1 simplification

B_1 triangular inequality

A_1

(b) Let a = 2.675, b=4.800, c= 15.2

$e_a = 0.5 \times 10 = 3,\ e_b = 0.5 \times 10^{-3}\ e_c = 0.5 \times 10^{-1}$

$d = 0.92 \qquad e_d = 0.5 \times 10^{-2}$

$Max\left(2.675\left(4.800 - \frac{15.2}{0.92}\right)\right)$

$= 2.6755\left[4.8005 - \frac{15.15}{0.925}\right]$

$= -30.9766$

$Min\left[2.675\left(4.800 - \frac{15.2}{0.92}\right)\right]$

$= 2.6745\left[4.7955 - \frac{15.25}{0.915}\right]$

$= -31.7508$

$Range[-31.7508, -30.9766]$

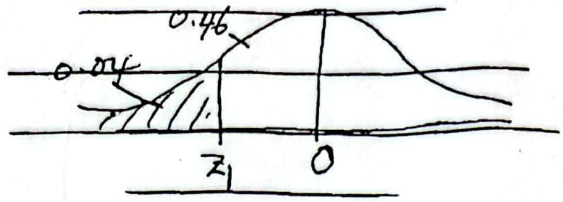
M_1
A_1

M_1

A_1
B_1

14	(a) $\underset{\sim}{r}(t) = 4\sin 3t\underset{\sim}{i} + 8\cos 3t\underset{\sim}{j}$ at $t = 0$, $r(o) = 4\sin o\underset{\sim}{i} + 8\cos o\underset{\sim}{j}$ $= 8\underset{\sim}{im}$ $8\underset{\sim}{j}$ m $v = \frac{d(r(t))}{dt} = 12\cos 3t\underset{\sim}{i} - 24\sin 3t\underset{\sim}{j}$ at $t = 0$, $v(o) = 12\cos oi + 8\cos oi - 24\sin o\underset{\sim}{j}$ $= 12\underset{\sim}{i}$ ms^{-1} $\underset{\sim}{a} = \frac{dv}{dt} = -36\sin 3t\underset{\sim}{i} - 72\cos 3t\underset{\sim}{j}$ but $\underset{\sim}{F} = m\underset{\sim}{a}$ $\underset{\sim}{F} = 3(-36\sin 3t\ \underset{\sim}{i} - 72\cos 3t\ \underset{\sim}{j})$ $= 3(-9)(4\sin 3t\ \underset{\sim}{i} - 8\cos 3t\underset{\sim}{j})$ $= -27(4\sin 3t\ \underset{\sim}{i} + 8\cos 3t\underset{\sim}{j})$ $= -27\underset{\sim}{r}$	B_1 M_1 M_1 A_1 B1 M_1 M_1 A_1	
	(b) Speed $= 10ms^{-1}$ cross section area $= 5cm^2$ $h = 4m$ Volume of water $= 10 \times \frac{5}{100^2} m^3$ $= \frac{50}{100^2} m^3$ Mass of water raised and issued per second. $= \frac{50}{100^2} \times 1000kg(density)$ $= 5kg$ $PE = mgh$ $= 5(9.8)(4)$ $= 196J$ $KE = \frac{1}{2}mv^2$ $= \frac{1}{2}(s)(10)^2$ $= 250J$ Work done per second by the pump. = PE + K.E = (96 + 250) J = 446J = 446W	B_1 B_1 M_1 A_1	
15	(a) Let x be a .r.v for "the marks scored" n = 350 $P(X < 40) = \frac{14}{350}$ $P(X > 60) = \frac{21}{350}$		

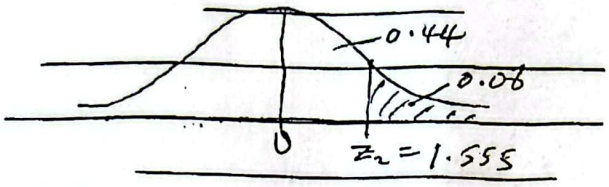
$$P(X<40)=P\left(z<\frac{40-\mu}{\sigma}\right)$$



$\frac{40-\mu}{\sigma}=-1.751$ B1

$40-\mu=-1.751\sigma$ .....(i)

$P(X>60)=P(z<)\frac{60-\mu}{\sigma}$



$\frac{60-\mu}{\sigma}=1.555$ B1

$60-\mu=1.555\sigma$ .....(ii)

$\mu-1.751\sigma=40$

$\mu+1.555\sigma=60$

$-3.30688=-20$ M1

$\sigma=6.050$ (3dp) A1 ≥ 3D

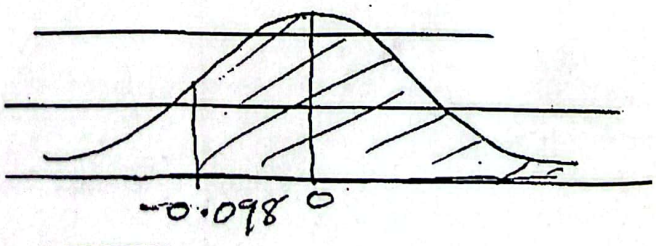
From $\mu=40+1.751\sigma$

$=40+1.751\,(6.050)$ M1

$=50.594$ (3dp) A1 ≥ 3D

(b) $P(X>50)=P\left(Z>\frac{50-50.594}{6.050}\right)$

$=P(Z>-0.098)$ M1 ≥ 3D



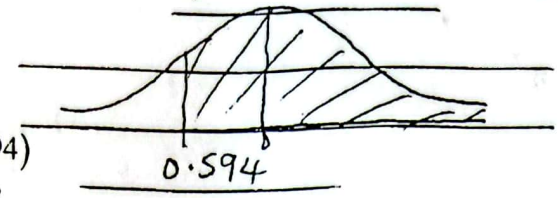
$=0.5+\phi(0.098)$

$=0.5+0.0391$ M1

$=0.5391$ A1

(c) $p(X > 47) = P\left(Z > \frac{47 - 50.594}{6.050}\right)$

$= P(Z > -0.594)$



$= 0.5 + \phi(0.594)$ M1

$= 0.5 + 0.2238$

$= 0.7238$ A1

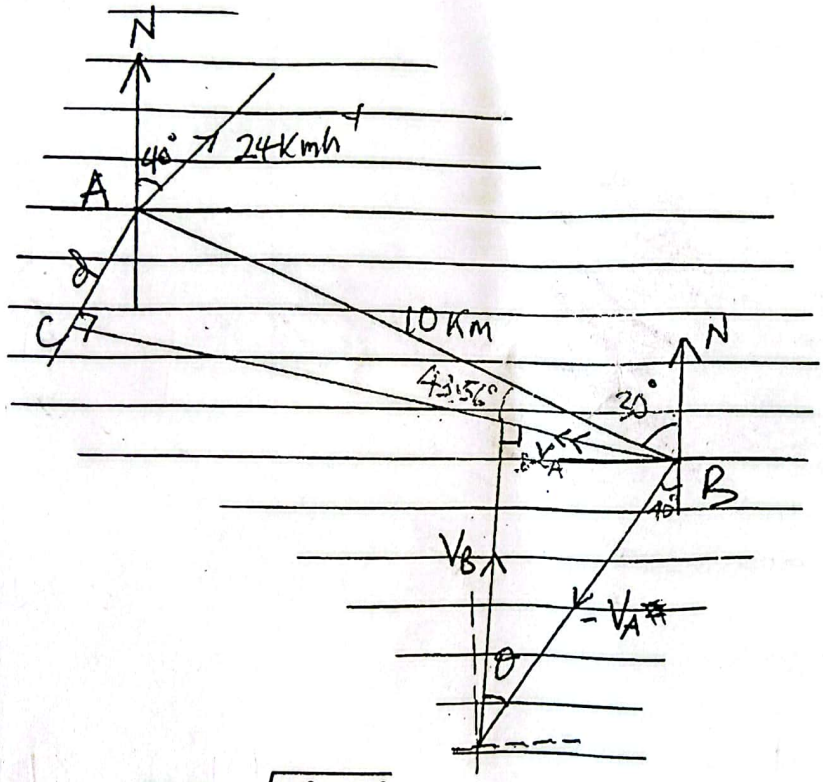
Number of students = 350 (0.7238 – 0.5391) M1

= 350 (0.1847)

= 64.6 A1

≈ 65 *students*

16 (a)



B1 Location of position

B1 correct vector diagrams

BVA $= \sqrt{24^2 - 22^2}$ M1

$= 9.5917 kmh^{-1}$ B1

$\text{Cos}\ \theta = \frac{22}{24}, \theta = Cos^{-1}\left(\frac{22}{24}\right), \theta = 23.56^0$ M1B1

Course is N16.44^{0}E or Bearing is 016.44^0 A1

(b) d = 10 sin 43.56^0

= 6.8911km M1 A1

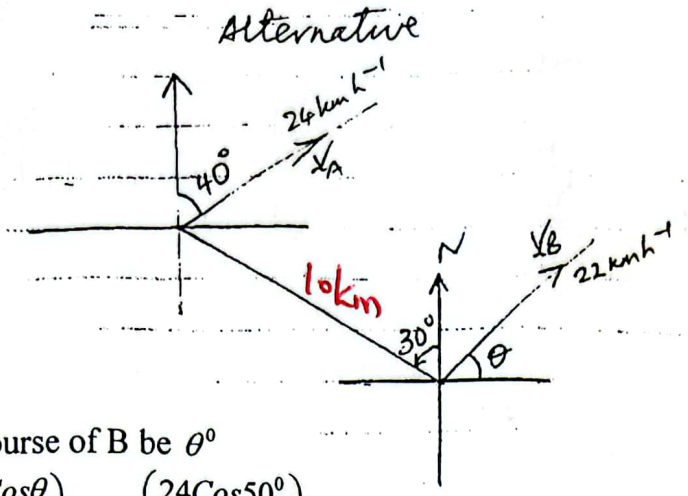
(c) Time $= \frac{BC}{|BVA|} = \frac{10Cos43.56^0}{9.5917}$ M1

$= 0.7555 hrs \times 60$ B1

$= 45 \text{ min} utes.$ A1

12marks

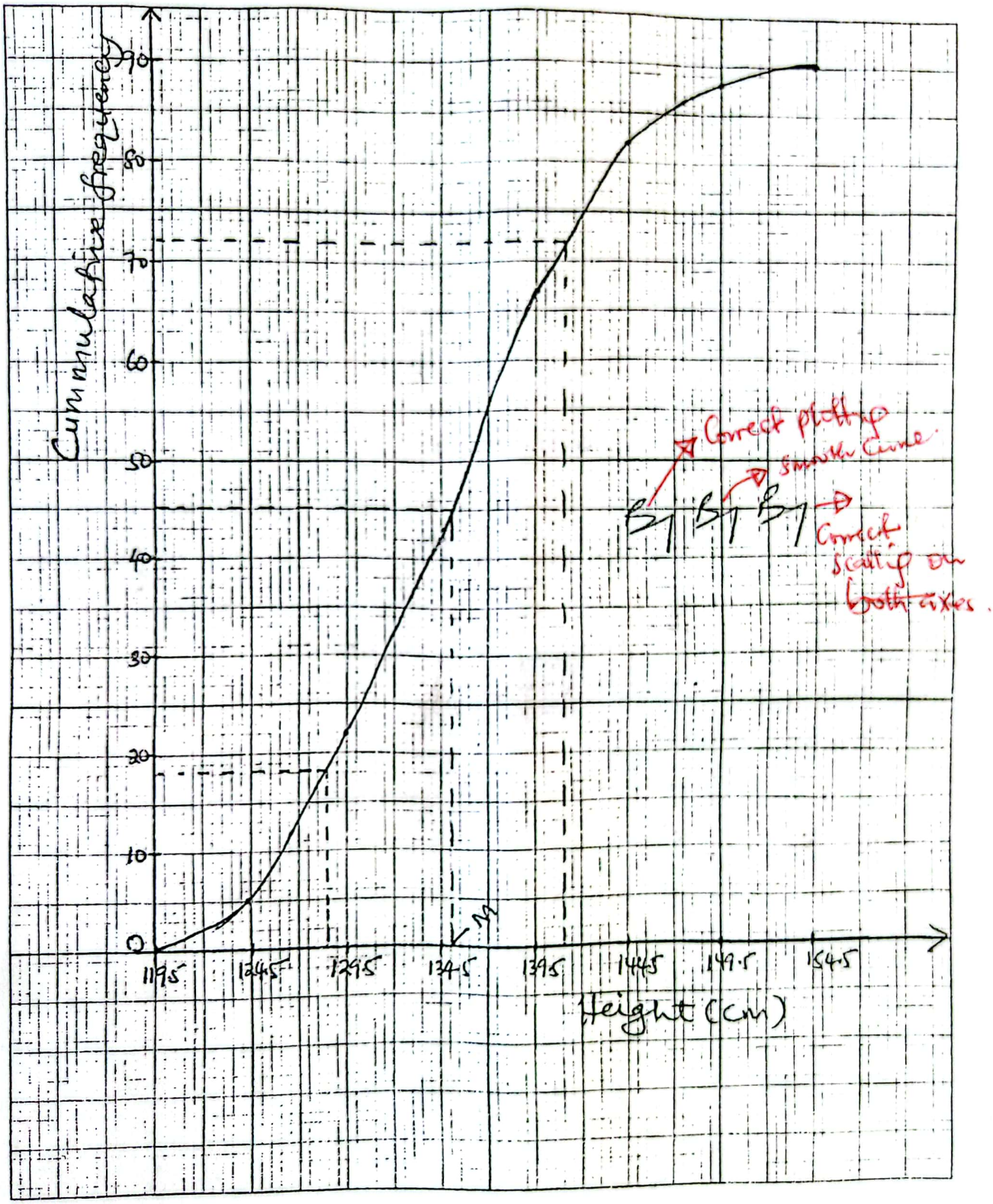
(a)



	B_1	Correct vector diagrams
Let the course of B be θ^0		
$V_B = \begin{pmatrix} 22Cos\theta \\ 22Sin\theta \end{pmatrix}, V_A = \begin{pmatrix} 24Cos50^0 \\ 24Sin50^0 \end{pmatrix}$	B_1	for both
${}_BV_A = \begin{pmatrix} 22Cos\theta \\ 22Sin\theta \end{pmatrix}, V_A = \begin{pmatrix} 24Cos50^0 \\ 24Sin50^0 \end{pmatrix}$		
$= \begin{pmatrix} 22Cos\theta - 15.4269 \\ 22Sin\theta - 18.3851 \end{pmatrix}$	B_1	
$V_B \cdot {}_BV_A = 0 = \begin{pmatrix} 22Cos\theta \\ 22Sin\theta \end{pmatrix} \cdot \begin{pmatrix} 22Cos\theta - 15.4269 \\ 24Cos\theta - 183851 \end{pmatrix} = 0$	M_1	
$339.3918Cos\theta + 404.4722Sin\theta = 484$	M_1	
$\sqrt{339.3918^2 + 404.4722^2}\,Cos(\theta - 50^0) = 484$		
$Cos(\theta - 50^0) = \frac{484}{528.0005}$		
$\theta - 50^0 = 23.56^0$		
$\theta = 73.56^0$	B_1	
The course must be on bearing 016.44^0 or N 16.44^0E	A_1	

(b) $r_B(o) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} km \quad r_A(o) = \begin{pmatrix} -10Cos60^0 \\ +10Sin60^0 \end{pmatrix}$	B_1	for both
$r_B(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 22Cos73.56^0 \\ 22Sin73.56^0 \end{pmatrix} t$		
$r_A(t) = \begin{pmatrix} -10Cos60^0 \\ 10\sin 60^0 \end{pmatrix} + \begin{pmatrix} 24Cos50^0 \\ 24Sin50^0 \end{pmatrix} t$		
$B_A(t) = \begin{pmatrix} 6.2262t \\ 21.8006t \end{pmatrix} - \begin{pmatrix} -5+15.4269t \\ 8.6603+18.3851t \end{pmatrix}$	B_1	
$B_A(t) = \begin{pmatrix} 5-9.227t \\ -8.6603+2.7155t \end{pmatrix} \ldots\ldots\ldots\ldots\otimes$		
$B_A . B_A = 0$		
$\begin{pmatrix} 5-9.227t \\ -8.6603+2.7155t \end{pmatrix} \cdot \begin{pmatrix} -9.2007 \\ 2.7155 \end{pmatrix} = 0$	M_1	
-46.0035 + 84.6529t – 23.5170 + 7.3739t =0		
92.0268t = 69.5205		
T = 0.7554 hours x 60		
Or t = 45 minutes	A_1	
Distance		
$B_A(0.7554) = \begin{pmatrix} -1.9502 \\ -6.6090 \end{pmatrix} km$		
$\lvert B_A(0.7554) \rvert = \sqrt{(-1.9502)^2 + (-6.6090)^2}$	A_1	
$= 6.8907 km$		
alternative ii (from)		
$B^vA(t) = \begin{pmatrix} 5-9.2007t \\ -8.6603+2.7155t \end{pmatrix}$	B_1	
$\frac{d}{dt}\lvert B^rA(t)^2 = 0 \rvert$		
$\frac{d}{dt}\left((5-9.2007t)^2 + (-8.6603+2.7155)^2\right) = 0$	M_1	
t = 0.7554 x 60		
= 45 minutes	A_1	
$\lvert B^rA(t) = 0.7554 \rvert = \left\lvert \begin{pmatrix} -1.9502 \\ -6.6090 \end{pmatrix} \right\rvert$	M_1	
$= 6.8907 km$	A_1	

9. (b)



END