P425/2
APPLIED MATHEMATICS
Paper 2
Oct. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) Set 9

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g $to\ be$ $9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Events A and B are such that P(A)= $\frac{3}{5}$, P($^{B}/_{A'}$)= $\frac{1}{3}$, and P(AnB) = $\frac{7}{10}$. Find the;
   (i) P(B) (ii) P($A \cup \bar{B}$)

2. A biker starts from rest at a point, O and moves in a straight line with acceleration of $4ms^{-2}$ until he reaches point A with speed $10ms^{-1}$. If he then moves with a constant acceleration of $3ms^{-2}$ for 6 seconds until he reaches point B. Find;
   (i) The speed of the biker at B (ii) The distance OB

3. Show that the equation $\frac{1}{2}x - sinx = 0$ has a root between 0.5 and 2.0. Hence use linear interpolation to find the first approximation to the root correct to 3 decimal places.

4. A random variable X has a probability function

$$f(x) = \begin{cases} \frac{2^x}{k}, & x = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

   Find the:
   (i) Values of k (ii) Mean of X

5. To a man moving at $80kmh^{-1}$ in the direction N450W, an aircraft appears to fly from the west at $30kmh^{-1}$. Find the true velocity of the aircraft

6. Given that x, y and z are rounded off to $4.2, 16.02$ and 25 are rounded off with corresponding percentage errors of $0.5, 0.45$ and 0.02, calculate absolute, relative and percentage errors in $\frac{xy}{z}$.

7. The forces $\begin{pmatrix} -1 \\ -5 \end{pmatrix} N$, $\begin{pmatrix} -2 \\ -3 \end{pmatrix} N$, $\begin{pmatrix} 1 \\ 4 \end{pmatrix} N$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix} N$ act at points with position vectors $\begin{pmatrix} 0 \\ -4 \end{pmatrix} m$, $\begin{pmatrix} 2 \\ 1 \end{pmatrix} m$, $\begin{pmatrix} 1 \\ 3 \end{pmatrix} m$ and $\begin{pmatrix} -4 \\ -2 \end{pmatrix} m$ respectively. Show that the resultant is a couple and find its moment.

8. The table below shows marks obtained by eighty students who sat for a test in senior six

Marks	$-<45$	$-<50$	$-<55$	$-<65$	$-<75$	$-<95$
Number of students	20	32	45	55	65	80

Draw a cumulative frequency curve and use it to find the:

(i) Median

(ii) Number of students who scored 72% and below

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. (a) Use the trapezium rule with 6 ordinates to estimate $\int_1^2 \frac{1}{1+e^{-x}}\, dx$, correct to 3 significant figures

(b) Find the range of values within which the exact value of $2.6954\left(4.6006 - \frac{16.175}{0.82}\right)$ lies, if the numbers are rounded off to the given number of decimal place.

10. ABCD is a rectangle with AB=2m as the positive x-axis and AD=3m as the positive y-axis. Forces of magnitude 4N, 5N, 10N and 2N act along the sides AB, AD, AC and BD respectively. Find the:

(a) Magnitude of the resultant force and its direction

(b) Line of action and where the resultant cuts AB, by taking moments about A

11. A random variable X of a continuous p.d.f is given by

$$f(x) = \begin{cases} \beta, & 0 \le x \le 2 \\ \beta(3-x), & 2 \le x \le 3 \\ 0, & else\ where \end{cases}$$

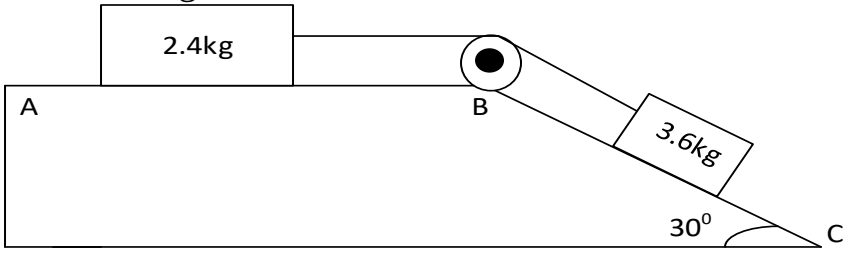
(a) Find the

(i) Value of β

(ii) Mean, μ and Standard deviation, σ

(b) Obtain the cumulative distribution function, F(x) and hence determine $P(X < \mu - \sigma)$

12. A particle of mass 2.4kg is held at rest on a rough horizontal surface AB with coefficient of friction of 0.5, it is connected by a light inextensible string passing over a smooth fixed pulley at B to a particle of mass 3.6kg. the slopping face BC is smooth and makes an angle of 30^0 to the horizontal



If the system is released from rest. Find the;

(i) Common acceleration of the particles and Tension in the string
(ii) Force on the pulley at B
(iii) Velocity of the 3k.6 kg mass after 2 seconds

13.(a) Show that the Newton Raphson formula for solving the equation $x^3 - 5x + 3 = 0$ is given by;

$$x_n = \left(\frac{2x_n^3 - 3}{3x_n^2 - 5}\right) \qquad n = 0,1,2 \ldots$$

(b) Show that the equation $x^3 - 5x + 3 = 0$ has a positive root between 1.8 and 2.1 hence find the root using the formula (a) above

14.The masses of 10 students in the table below

Height (cm)	156	151	152	146	160	157	149	142	158	141
Mass (kg)	62	58	63	58	70	60	55	57	68	56

(a) Plot a scatter diagram for the data.
(b) Draw a line of best fit for scatter diagram. Hence find mass corresponding to a height of 145cm
(c) Calculate the rank correlation coefficient for the data. Comment on your result at 1% level of significance

15.(a) A box contains 3 red pens and 6 black pens. Three pens are randomly drawn one after the other without replacement. Find the probability that;
(i) 3 red pens
(ii)Two red pens and one black pen
(iii) More than one black pen

(b) A student is to travel to a school for an interview. The probability that he will be in time for the interview when he travels taxi and boda respectively are 0.1 and 0.2 respectively. The probability that he will travel by taxi and boda are 0.6 and 0.4 respectively
(i) Find the probability that he will be on time
(ii)Given that he is not time, what is the probability that he traveled by boda

16.(a) A stone is thrown with initial speed of $12ms^{-1}$ at an angle of elevation 30^0 to the horizontal from the top of a tower 36m high. Determine the time the stone spends in air.
(b) A ball is projected from a level ground towards a vertical wall, 4m high and 30m away from the point of projection. It just passes the wall after one second. Find;
(i) its initial speed and the angle of projection
(ii) The distance beyond the wall where the ball lands