P425/1
PURE
MATHEMATICS
PAPER 1
June/July. 2023
3 hours

ACEITEKA JOINT MOCK EXAMINATIONS, 2023

Uganda Advanced Certificate of Education

Pure Mathematics

Paper 1

Time: 3 Hours

NAME:..INDEX No: ....

INSTRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Indicate the five questions attempted in section B in the table aside.

Additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

Qn 1: Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$. [5 Marks]

Qn 2: Find the angle $\alpha = \angle BAC$ of the triangle ABC whose vertices are $A(1,0,1), B(2,-1,1)$ and $C(-2,1,0)$. [5 Marks]

Qn 3: The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficients whose roots are p^{-6} and q^{-6}. [5Marks]

Qn 4: Use method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3 decimal places. [5 Marks]

Qn 5: Points S and S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y}{16} = 1$.

Find the coordinates of S and S'. [5 Marks]

Qn 6: Evaluate: $\int_0^1 \frac{8x-8}{(x+1)^3(x-3)^3}dx$. [5 Marks]

Qn 7: Given the function, $f(x) = \frac{3}{13 + 6\sin x - 5\cos x}$.

Use the substitution $t = \tan\left(\frac{x}{2}\right)$, to show that $f(x)$ can be written in the form: $\frac{3(1+t^2)}{2(3t+1)^2 + 6}$. [5 Marks]

Qn 8: Given that $y = \frac{\sin x}{x}$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$. [5 Marks]

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Prove by induction that for all positive integer $\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$ [6 Marks]

(b). Prove by induction that for all positive **odd** integers, n, $f(n) = 4^n + 5^n + 6^n$ is divisible by 15. [6 Marks]

Question 10:

A circle that passes through the points $A(3,4)$ and $B(6,1)$ and the equation of the tangent to this circle at A is the line $2y = x + 5$. Find:

(i). the coordinates of the centre of circle. **[9 Marks]**

(ii). the radius of the circle. **[2 Marks]**

(ii). the equation of the circle. **[1 Mark]**

Question 11:

(a). Given that $f(x) = \dfrac{64x^4 - 148x + 78}{(4x-5)^3}$. Express $f(x)$ into partial fractions.

(b). Hence evaluate $\int_4^6 f(x)dx$. **[12 Marks]**

Question 12:

(a). Use de Moivre's theorem to prove that: $\sin 5\theta = 5\sin\theta - 20\sin^2\theta + 16\sin^5\theta$.

(b). Hence or otherwise, find the distinct roots of the equation $2 + 10x - 40x^3 + 32x^5 = 0$ giving your answer to 3 decimal places where appropriate.

[12 Marks]

Question 13:

The planes P_1 and P_2 are respectively given by the equations:

$r = 2i + 4j - k + \lambda(i + 2j - 3k) + \mu(-i + 2j + k)$ and

$r.(2i - j + 3k) = 5$; where λ and μ are scalar parameters. Find:

(i). the Cartesian equation for plane, P_1.

(ii). to the nearest degree, the acute angle between P_1 and P_2 .

(iii). the coordinates of the point of intersection of the plane, P_1 , and the line $\dfrac{x-1}{5} = \dfrac{y-3}{-3} = \dfrac{z+2}{4}$. **[12 Marks]**

Question 14:

(a). Show that the volume of the solid generated by rotating the area enclosed by the curve $y = 2^x$, the lines $x = 0$ and $y = 2$ about the $x-$axis is $\dfrac{\pi}{\ln 4}(4\ln 4 - 3)$. **[8 Marks]**

(b). Evaluate $\int_0^{\frac{\pi}{4}} \dfrac{4}{1+\cos 2x}dx$. **[4 Marks]**

[4 Marks]

Question 15:

(a). Given that $\cot^2\theta + 3\operatorname{cosec}^2\theta = 7$, show that $\tan\theta = \pm 1$.

(b). (i). Express the function $y = 3\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ where R is a constant and $0 \le \alpha \le 2\pi$.

Hence find the coordinates of the minimum point of y.

(ii). State the values of x at which the curve cuts the $x-$axis . **[8 Marks]**

Question 16:

A sample of bacteria in a sealed container is being studied.

The number of bacteria, p, in thousands, is given by the differential equation:

$$(1+t)\frac{dp}{dt} + p = (1+t)\sqrt{t}$$

where t is the time in hours after the start of the study.

Initially, there are exactly 5,000 bacteria in the container.

(a). Determine, according to the differential equation, the number of bacteria in the container 8 hours after the start of the study.

(b). Find, according to the differential equation, the rate of change of the number of bacteria in the container 4 hours after the start of the study.

[12 Marks]