

P425/2
APPLIED MATHEMATICS
Paper 2
Sept. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS
(PRINCIPAL SUBJECT) SET 6

Paper 2
TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **Eight** questions in Section **A** and **Five** questions from Section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2} .

SECTION A: (40 MARKS)

Answer **all** the questions in this section.

1. If A and B are events such that $P(B) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{12}$, and $P(B/A) = \frac{1}{3}$. Find the;
- (i) $P(A)$ and $P(A/B)$ (ii) $P(A/\bar{B})$

2. The probability distribution of a discrete random variable X is given in the table below

| | | | | | |
|------|-----|-----|------|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 |
| F(x) | 0.1 | 0.4 | 0.55 | 0.8 | 1.0 |

Find;

- (i) $E(X)$ (ii) $\text{Var}(X)$
3. A train is timed between successive stages A, B and C each 1km apart. If it takes 100s to travel from A to B and 150s from B to C, calculate the;
- (a) Retardation (b) Initial velocity
4. A manufacturer makes a drink from three different fruits. The table below shows the cost per kg of these fruits in the year 2019 and 2021

| Item | Cost per kg (Shs) | | Weight |
|------|-------------------|-------|--------|
| | 2019 | 2021 | |
| A | 5,000 | 6,250 | 1 |
| B | 2,000 | 2,250 | 2 |
| C | 1,600 | 2,400 | 7 |

- (a) Calculate the average weighted index number for the cost of the juice in 2021 and comment on your results
- (b) If the cost of the juice in 2019 was 9,500. Find the cost in 2021, using the index in (a) above.
5. Given that $x = 2.876$, $y = 2.31$ and $z = 8.6$ are round off to the given number of decimal places. Find the maximum value and minimum value hence state the interval within which the exact value of $\left(x - \frac{y}{z}\right)$ lies correct to 3 decimal places
6. A particle begins to ascend a slope from the lowest point at a speed of 7ms^{-1} . The slope is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{5}{13}$. The coefficient of friction between the particle and the slope is $\frac{5}{8}$. Find the;

- (i) Deceleration
- (ii) Total distance the particle travels up the plane before coming to rest

7. Use the trapezium rule with six ordinates to estimate $\int_1^2 x e^{-2x} dx$, correct to 3 decimal places.

8. A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 2.3N inclined at an angle of 60° to horizontal. If the particle does not move, find the minimum value of the coefficient of friction

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. A random variable X has probability density function

$$f(x) = \begin{cases} kx; & 0 < x < 3 \\ 3k(4 - x); & 3 < x < 4 \\ 0; & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$, hence find the value of the constant k

(b) Find the;

(i) Mean, $E(X)$

(ii) Cumulative distribution function $F(x)$, hence find $P(X < 3.5)$

10. The table below represents marks for ten students in mathematics and English of a certain school

| | | | | | | | | | | |
|-------------|----|----|----|----|----|----|----|----|----|----|
| English | 40 | 90 | 54 | 32 | 80 | 65 | 55 | 48 | 55 | 30 |
| Mathematics | 68 | 40 | 47 | 64 | 55 | 41 | 62 | 76 | 74 | 80 |

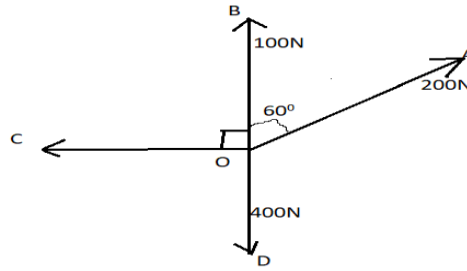
(a) Plot a scatter diagram for the data, draw a line of best fit. Comment on the relationship between the two subjects

(b) Estimate the English mark, if the mathematics marks was 60

(c) Calculate the rank correlation coefficient and comment at 5% level of significance

11.(a) Four athletes of masses 50kg, 60kg, 80kg and 60kg run a constant speed of 3ms^{-1} in the direction OA, OB, OC and OD respectively. Calculate their resultant momentum

(b) The diagram below shows forces acting on a particle in the directions below



Find the Magnitude of the resultant force and its direction

12. The table below shows the marks obtained by students of principal mathematics.

| Marks | Number of students |
|------------|--------------------|
| 20 - < 40 | 5 |
| 40 - < 50 | 15 |
| 50 - < 55 | 10 |
| 55 - < 60 | 10 |
| 60 - < 70 | 30 |
| 70 - < 90 | 25 |
| 90 - < 100 | 5 |

(a) Construct a histogram and use it to estimate the modal mark

(b) Calculate the;

- (i) Mean and variance
- (ii) Median

13.(a) Use the trapezium rule with 6 ordinates to find the approximate value of

$$\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4 \right) dx, \text{ correct to 3 decimal places}$$

(b) Calculate the exact value, hence find the absolute, relative and percentage error. How may the error be reduced?

14.(a) Show graphically that the curve of $y = x^3$ and the line $y = 2x - 3$ have a root between -2 and -1 correct it to 1 decimal place.

(b) Use x_0 , the first approximation in (a) above and the Newton Raphson method to find the root of the equation correct to 3 decimal places

15.(a) A car of mass 900kg moves on a level road at a maximum speed of 48m/s against a constant resistance of 350N.

- (i) Find the rate at which the engine is working
- (ii) With the resistance and the rate of working unchanged, find the maximum speed the car can ascend a slope of inclination 1 in 18

(b) A bullet of mass 20g travelling horizontally at 210ms^{-1} strikes a stationary block of wood of thickness 0.1m and emerges from the block travelling at 50ms^{-1} . Calculate the average resistance of the block to the motion of the bullet.

16. ABCDEF is a regular hexagon of side 2m. Forces of magnitude 2N, 3N, 4N, and 5N act along the line AC, AE, AF and ED respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, Find the;

- (i) Magnitude and direction of the resultant force
- (ii) Point where the line of action of the resultant cuts AB