S.5 PURE MATHEMATICS

TIME: 3 HOURS

INSTRUCTIONS: Attempt **all** questions in **section A** and **any five** in **section B**.

SECTION A

1. Three consecutive terms of an A.P have the sum of 36 and a product of 1428. Find the three terms.
2. Determine the equation of the tangent and the normal to the curve $y = (x + 1)(2x + 3)$ at a point $(2,21)$
3. If the roots of the equation $ax^2 + bx + c = 0$ differ by 3. Show that $b^2 = 9a^2 + 4ac$.
4. Differentiate from the first principles $f(x) = 2x^2 + 5x - 3$. Hence find $f^1(2)$
5. Solve the equation $\tan 2x = \cot 3x$ for $0^0 \leq x \leq 180^0$
6. Find the equation of the circle whose end diameter is the line joining the points $A(1,3)$ and $(-2,5)$
7. A container is in the form of an inverted right circular cone. Its height is 100cm and base radius is 40cm. The container is full of water and has a small hole at 1B vertex. Water is flowing through the hole at a rate of 100cm^{3}s^{-1}. Find the rate at which the water level in the container is falling when the height of water in the container is halved.
8. A point P moves such that its distance from the two points $A(2,0)$ and $B(8,6)$ are in the ratio $AP: PB = 3: 2$. Show that the focus of P is a circle.

SECTION B

9. (a) Differentiate $\frac{x^2}{\sqrt{(1-2x^2)}}$ with respect to x.

   (b) Given that $x = \frac{t^2}{1+t^3}$ and $y = \frac{t^3}{1+t^3}$ find $\frac{d^2y}{d_{x^2}}$

10. (a) Solve the equations $\cos 2x = 4\cos^2 x - 2\sin^2 x$ for $0^0 \leq x \leq 180^0$

    (b) Show that if $\sin(x + \alpha) = p\sin(x - \alpha)$ then $\tan = (\frac{p+1}{p-1})\tan \alpha$

    Hence solve the equation $\sin(x + 20^0) = 2\sin(x - 20)$ for $0^0 \leq x \leq 180^0$

11. The function $f(x) = b + ax - 4x^2 + 8x^3$ gives a remainder of -19 when divided by $(x + 1)$ and a remainder of 2 when divided by $(2x - 1)$. Find the value of a and b

    (b) The roots of the equation $x^2 - 4x + 2 = 0$ are α and β for the equation whose roots are $(\alpha + 2\beta)$ and $(\beta + 2\alpha)$.

12. (a) Differentiate $\cos(x^2 e^x)$ with respect to x.

    (b) Given that $y = Ae^{3x} + Be^{-2x}$ show that $\frac{d^2y}{d_{x^2}} - \frac{dy}{dx} - 6y = 0$

    (c) Find the equation of the normal to the curve $x^{2y} + 3y^2 - 4x - 12 = 0$ at the point $(1,2)$

13. The parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ represents a curve.
    (i) Find the Cartesian equation of the curve.
    (ii) Determine the turning points of the curve and the nature.
    (iii) State the asymptotes and intercepts of the curve.
    (iv) Hence sketch the curve.

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14. (a) Determine the maximum and minimum value of the expression $6\sin x - 3\cos x$

(b) Prove that $\frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0} = \tan 56$

(c) Prove by induction that $\sum_{r=1}^{n} r^2(r+1) = \frac{n}{12}(n+1)(n+2)(3n+1)$ where n is a whole number.

END

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