P425/2
APPLIED MATHEMATICS
Paper 2
Oct. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) Set 11

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g *to be* $9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section*

1. The probability that events A and B occur are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability that only one of them occurs is $\frac{5}{12}$, find the;
   (i) $P(AnB)$ (ii) P(A'nB')

2. A particle of mass 2 kg resting on a rough horizontal plane is pulled by a force of magnitude of 11.3N inclined at an angle of 60^0 to the horizontal. If the particle does not move, find the minimum value of the coefficient of friction between the particle and the plane

3. Given that for a function f(x), $f(0.9) = 0.2661$, $f(1.0) = 0.2420$ and $f(1.1) = 0.2179$. Use linear interpolation or extrapolation to estimate;
   (i) $f(0.96)$ (ii) $f^{-1}(0.2082)$

4. A biased coin is tossed six times. The coin is such that the ratio of showing a tail to that of a head is 3:1, find the probability of getting;
   (i) At least 4 heads (ii) Between 2 and 4 tails

5. A bullet is fired vertically upwards at a speed of 150ms^{-1}. Find the length of time for which the bullet is at least 900m above the ground level.

6. Given that $X = 2.79$, $Y = 1.375$ and $z = 1.4$, find the limits within which $X - \frac{Y}{Z}$ lies

7. A random variable X has a probability density function

$$f(x) = \begin{cases} \frac{2^x}{k}, & x = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

   Find the:
   (a) Values of k (b) Mean of X

8. A particle moves such that its displacement at any time (t) is given by $r = t^2(\mathbf{2\hat{i} - 4\hat{k}}) - t^3(\mathbf{3\hat{j} - 2\hat{i}}) + \mathbf{2}sin(\mathbf{2t})\hat{\mathbf{k}}\, metres$. Find the magnitude of the acceleration of the particle at $t = 3\ seconds$

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows while 90% of the residents have less than 100 cows
   (a) Determine the value of the mean μ, and standard deviation σ of the cows
   (b) If there are 200 residents, find how many have more than 80 cows

10. (a) Use the trapezium rule with 6 ordinates to find the approximate value of $\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4\right)\, dx$, correct to 4 significant figures
   (b) Calculate the exact value, hence find the absolute, relative and percentage error in your estimation in (i). Suggest how the error may be reduced

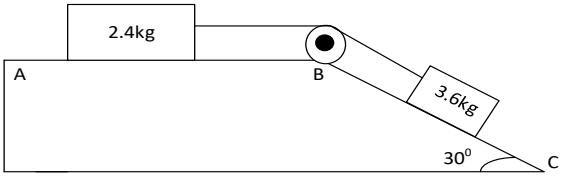
11. Two particles A and B move with velocities $(\boldsymbol{\lambda i + 3j + 30k})$ms^{-1} and $(\boldsymbol{4i - 2j - 15k})$ms^{-1} respectively where λ is a constant. At t=0, the particles are located at points $(\mathbf{2, 1, -15})\boldsymbol{m}$ and $(\mathbf{1, 4, 12})\mathbf{m}$ respectively.
   (a) Find the value λ such that A and B will colide and find the value of t when collision occurs
   (b) When $\lambda = 2$, find the time after which the particles will be nearest to each other

12. The masses (x) in kg of 50 students were as follows.

$Masses\ (x)$	$Number\ of\ students$
$40 \leq x < 45$	3
$45 \leq x < 50$	2
$50 \leq x < 60$	20
$60 \leq x < 65$	18
$65 \leq x < 70$	3
$70 \leq x < 75$	4

   (a) Calculate the Mean and standard deviation
   (b) Construct an ogive and use it to estimate
      (i) Median
      (ii) Percentage of students who have a mass of 55kg and above

13. A particle of mass 2.4kg is held at rest on a rough horizontal surface AB with coefficient of friction of 0.5, it is connected by a light inextensible string passing over a smooth fixed pulley at B to a particle of mass 3.6kg. The slopping face BC is smooth and makes an angle of 30^0 to the horizontal



If the system is released from rest. Find the;

(i) acceleration of the system and Tension in the string

(ii) Force exerted on the pulley at B

(iii) Velocity of the 3.6 kg mass after 2 seconds

14. A random variable X has probability density function

$$f(x)=\begin{cases} kx; & 0 < x \leq 3 \\ 3k(4-x); & 3 < x \leq 4 \\ 0; & otherwise \end{cases}$$

(a) Sketch f(x), hence find the value of the constant k

(b) Find the;

(i) Mean, E(X)

(ii) Cumulative distribution function F(x), hence find P(X< 3.5)

15.(a) Use graphical method to show that the equation $e^x + x - 4 = 0$ has a root between 1 and 2, correct to 1 decimal place

(b) Show that the Newton Raphson formula for finding the root of the equation (i) above

$$x_{n+1} = \frac{e^{x_n}(x_n-1)+4}{1+e^{x_n}}; \qquad n = 0,1,2, \ldots$$

(c) Hence use the initial approximation x_o obtained in (a) above to find the root of the equation correct to 3 decimal places

16. A uniform plank of length 8m and mass 100kg rests in limiting equilibrium with the end A on a horizontal ground and the end B against vertical wall. If the coefficient of friction at each end of the plank is 0.3. Find the;

(a) Angle the plank makes with the vertical

(b) The plank is now placed at an angle of β to the horizontal where $\tan\beta = 2$ and a body of mass Mkg is attached to the plank at B causing the plank to slip. Find the maximum value of M and the magnitude of the corresponding normal reaction at A.