

P425/2
APPLIED MATHEMATICS
S6 POST MOCK 2005

1. The marks at a certain examination are approximately normally distributed with mean 60 and standard deviation 15. The top 9.2% of the students receive grade A whilst the bottom 8.1% fail. Find to the nearest integer the
- minimum mark required to obtain grade A,
 - maximum mark consistent with failure.
- (Give your answers to the nearest whole number)

2. The position vector of a moving particle of unit mass at time t s is given by $\mathbf{r} = (4 + 10t)\mathbf{i} + (15 + 20t - 5t^2)\mathbf{j} + (10t^3 - 20)\mathbf{k}$ metres. Find the work done by the resultant force acting on the particle in the time interval from $t = 0$ to $t = 2$ seconds.

3. A random variable x has probability density function given by

$$f(x) = \begin{cases} \lambda(1 - \cos x); & 0 \leq x \leq \pi/2 \\ \lambda \sin x & ; \pi/2 < x \leq \pi \\ 0 & ; \text{elsewhere} \end{cases}$$

Calculate the value of

- λ
 - $E(x)$
4. A particle is projected from a point A, whose position vector is $(2\mathbf{i} + 3\mathbf{j})\text{m}$ with initial velocity $20\mathbf{i} + 30\mathbf{j} + 20\mathbf{k}$, m/s where \mathbf{i} and \mathbf{k} are horizontal and \mathbf{j} is the unit vector in the direction of the upward vertical. Find the distance of the particle from the origin after 2s.
5. Show graphically that the equation $x - e^{-x} = 0$ has one real root which lies between 0 to 1. Estimate from your graph the root correct to two decimal places.
6. Using the change of sign search method, show that the equation $F(x) = 3x^3 - 5x^2 - 19x + 31$ has three roots in the interval $[-3, 3]$. Use linear interpolation to find the numerically smallest roots correct to two decimal places.
7. A particle is travelling in a straight line with constant acceleration. It covers 3.5m in the third second and 4.1m in the fourth second, find its acceleration and initial speed.
8. Three Iraq soldiers agreed that 20 years after the Gulf war, they would meet and celebrate the event when at least two of them are still alive. If their probabilities of living for 20 years are $^{11}/_{13}$, $^9/_{10}$ and $^{10}/_{12}$. What is the probability that they will carry out their agreement?

9. (a) A particle is projected down an inclined plane with a speed of 2m/s. If it goes 8m in 4s, find the coefficient of friction between the particle and the plane whose angle is 30° .

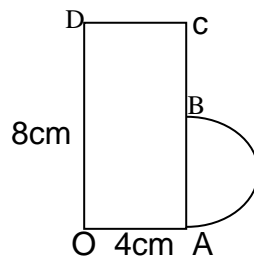
(b) A train of mass M tonnes running at 34.36kmh^{-1} is brought to rest on an upgrade of 1 in 120 by the brakes which exert a force of $M(\frac{1}{10} + \frac{t}{110})\text{N}$ where t is the time in seconds measured from the instant when the brakes are applied. Show that the particle is brought to rest in 30 seconds and find the distance travelled in the due course.

10. A uniform rod AB, of length $2a$ and weight w , rests in limiting equilibrium in a vertical plane with end A on a rough horizontal floor OX and B on a smooth vertical wall OY. A string of length a attached to O and the midpoint of AB, is in tension T . Prove that, if AB makes an angle θ with the vertical when A is on the point of slipping away from O, the

$$T = \frac{W(\sin\theta - 2\mu\cos\theta)}{2\mu\cos^2\theta}$$

and find the reactions at A and B, where μ is the coefficient of friction between the rod and the floor.

11. (a) Four particles of masses 2,3,4 and 6 kg are at points (0,4), (- 3,6), (2,0) and (- 1, - 4). Find the co-ordinates of the centre of mass



The figure above shows a uniform lamina OABCD in the form of letter b formed by a rectangle and a semi-circle. The diameter of the semi-circle is 4cm.

- (i) Find the position of the centre of gravity from OA and OD.
 - (ii) If the lamina be suspended by means of a thread through a hole drilled near D, find the inclination of OA to the vertical.
12. (a) The probability that a certain surgical operation is successful is 0.8. If the operation is performed on 120 people, find the probability that 90 or more operations are successful.
- (b) The lengths of tilapia in a certain lake are normally distributed with means of 7 inches and a standard deviation of 2.1 inches. If the fish and game department would like fishermen to keep only the largest 20% of the tilapia, what should the minimum size for “keepers” be? Determine the probability of there being a tilapia with a length of more than 7.75 inches.

13. (a) (i) Round off 0.00205785 to 3 significant figures.
 (ii) Truncate 238579 to 3 significant figures.

(b) Use the trapezium rule with five subinterval to find the value of

$$\int_0^1 \frac{1}{1+e^x} dx$$

Find the percentage error in your answer.

14. (a) Derive the Newton Raphson formula for finding the reciprocal of a number N. Using $x_0 = 0.02$, find the value of $\frac{1}{417}$ correct to 3 decimal places.

(b) The period of oscillation T of a simple pendulum depends on the length of the pendulum, l, acceleration due to gravity, g and 2π according to the equation

$$T = 2\pi \sqrt{\left[\frac{l}{g} \right]}$$

$$\pi = 3.14 \pm 0.005, g = 9.81 \pm 0.005 \text{ N/kg};$$

- (i) Write down an expression for the error in T, hence determine its value;
 - (ii) Obtain the working value for T;
 - (iii) State the range of values within which the exact value of T lies correct to 2 decimal places.
15. (a) A bird is 10m vertically above a man who throws a stone at an angle θ . The bird is flying with a uniform speed of 14m/s in a direction making 60° with the horizontal. Show that, for the stone to hit the bird, $\tan\theta \geq 2 + \sqrt{3}$
- (b) A particle is fired, from the top of a cliff of height 49m, with a speed of 14m/s at an angle of 45° with the horizontal.
 Find the ;
- (i) maximum height reached,
 - (ii) point where the particle enters the sea,
 - (iii) angle at which the particle hits the sea.
16. (a) A particle is moving with simple harmonic motion in a straight line. Its speed when moving through its mean position is 12m/s, and the magnitude of the acceleration at a point 3m from the mean position is 6ms^{-2} . Find the amplitude and periodic time.
- (b) A ship A is moving due east at 18kmh^{-1} and ship B is moving in a direction $\text{N}20^\circ\text{E}$ at 10kmh^{-1} . B is 12km due North of A and both ships continue with the same velocities. Find the shortest distance between the ships in the subsequent motion and the time taken for such a situation to occur.