Trapezium rule

It is used for estimating an integral area under a curve of continuous function over a given interval [a, b]

if y = f(x)

A = $\int_a^b ydx$

Using several strips between x = a and x = b of equal width, trapezium rule can be used to determine the area.

A $\approx \frac{1}{2}h[(first + last\ ordinates) + 2(sum\ of\ the\ middle\ ordinates)]$

where h = $\frac{b-a}{subintervals}$

Note

(i) sub-intervals, subdivision and strips are the same
(ii) subinterval = ordinates
(iii) when dealing with a trigonometric function, calculators must be in radian mode
(iv) when the final answer is required to a specific number of d.p'c, the working's should be done at least a d.p higher but the final answer rounded to the required d.p's

Example 1

Use the trapezium rule with four-intervals to estimate $\int_{0.2}^{1.0}\left(\frac{2x+1}{x^2+x}\right)dx$. Correct to two decimal places.

Let y = $\left(\frac{2x+1}{x^2+x}\right)$

h = $\frac{1.0-0.2}{4}$ = 0.2

x	y = $\frac{2x+1}{x^2+x}$	
0.2	5.8333	
0.4		3.2143
0.6		2.2917
0.8		1.8056
1.0	1.5000	
Sum	7.3333	7.3116

$\int_{0.2}^{1.0}\left(\frac{2x+1}{x^2+x}\right)dx = \frac{1}{2}\ x\ 0.2(7.3333 + 7.3116\ )$

= 2.1955

= 2.20 (2D)

Example 2

Use the trapezium rule with seven coordinates to estimate

$\int_0^3[(1.2)^x - 1]^{\frac{1}{2}}$dx correct to 2 decimal places (05marks)

Solution

For 7 ordinates, there are 6 subintervals

Width, h = $\frac{b-a}{subinterval} = \frac{3-0}{6} = 0.5$

Let y = $\sqrt{(1.2)^x - 1}$

x	y	
0	0	
0.5		0.309
1		0.447
1.5		0.561
2		0.663
2.5		0.760
3	0.853	
Sum	0.853	2.74

Using the trapezium rule

$\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}}$dx = $\frac{0.5}{2}[0.853 + 2(2.74)]$ = 1.58

Example 3

(a) Use the trapezium rule with 6-ordinated to estimate the value of $\int_0^{\frac{1}{2}}(x + sinx)dx$, correct to three decimal places.

$h = \frac{\frac{1}{2}-0}{5} = \frac{\pi}{10}$

x	y	
0	0	
$\frac{\pi}{10}$		0.6232
$\frac{2\pi}{10}$		1.2161
$\frac{3\pi}{10}$		1.7515
$\frac{4\pi}{10}$		2.2077
$\frac{\pi}{2}$	2.5708	
Sum	2.5708	5.7985

$\int_0^{\frac{1}{2}}(x + sinx)dx = \frac{1}{2}\ x\ \frac{\pi}{10}(2.5708 + 2\ x\ 5.7985)$

= 2.225

(b)(i) Evaluate $\int_0^{\frac{1}{2}}(x + sinx)dx$, correct to three decimal places

$\int_0^{\frac{1}{2}}(x + sinx)dx = \left|\frac{x^2}{2} - \cos x\right|_0^{\frac{1}{2}}$

= $\frac{1}{2}\left(\frac{\pi^2}{4} - 0\right) - (\cos\frac{\pi}{2} - \cos 0)$

= $\frac{\pi^2}{8} + 1$

= 2.234

(ii) Calculate the error in your estimation in (a) above

Error = $|2.234 - 1.225|$ = 0.009

(iii) Suggest how the error may be reduced (06marks)

Increasing on number of strips or subintervals

Example 4

A student used the trapezium rule with five sub-intervals to estimate

$\int_2^3 \frac{x}{(x^2-3)} dx$ correct to **three** decimal places

Determine;

(a) The value the student obtained (06marks)

h = $\frac{3-2}{5}$ =0.2

X	y_1, y_6	y_2 ……y_5
2.0	2.0	
2.2		1.1956
2.4		0.8696
2.6		0.6915
2.8		0.5785
3	0.5	
Sum	2.5	3.3352

$\int_2^3 \frac{x}{(x^2-3)} dx = \frac{1}{2}\ x\ 0.2[2.5 + 2(3.3352)]$

= 0.91704 = 0.917 (3D)

(b) The actual value of the integral (03marks)

$\int_2^3 \frac{x}{(x^2-3)} dx$ = $\left[\frac{1}{2} Inx^2 - 3\right]_2^3$

=$\frac{1}{2}(In\ 6 - In\ 1)$

= 0.896

(c) (i) the error the student made in the estimate

Error = $|0.896 - 0.917|$ = 0.021

(ii) how the student can reduce the error(03marks)

Increasing on the number of sub-intervals or ordinates or reducing the width of h

Example 5

Use trapezium rule with 4 subintervals to estimate to 3 decimal places $\int_0^{\frac{\pi}{2}} \cos xdx$

Solution

h = $\frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

x	f(x) = cos x	
0	1.0000	
$\frac{\pi}{8}$		0.9239
$\frac{2\pi}{8}$		0.7071
$\frac{3\pi}{8}$		0.3827
$\frac{4\pi}{8}$	0.0000	
sum	1.0000	2.0137

$\int_0^{\frac{\pi}{2}} \cos x dx = \frac{1}{2} x \ \frac{\pi}{8}[1 + 2\ x\ 2.0137]$

=0.987

Example 6

Use trapezium rule with 7 ordinates to estimate $\int_0^3 \frac{1}{1+x} dx$ correct to 3dp

Solution

h = $\frac{3-0}{7-1} = 0.5$

x	f(x) = cos x	
0	1.0000	
0.5		0.6667
1.0		0.5000
1.5		0.4000
2.0		0.3333
2.5		0.2757
3.0	0.2500	
sum	1.2500	2.157

$\int_0^3 \frac{1}{1+x} dx = \frac{1}{2} x\ 0.5[1.25 + 2\ x\ 2.1857]$

= 1.405

Example 7

(a) Use the trapezium rule to estimate the integral value of $\int_2^3 \frac{x}{1+x^2} dx$ using five subinterval and correct to 3d.p.

(b) (i) find the exact value of $\int_2^3 \frac{x}{1+x^2} dx$

(ii) suggest how the error may be reduced.

(a) h = $\frac{3-2}{5} = 0.2$

x	f(x) = $\frac{x}{1+x^2}$	
2.0	0.40000	
2.2		0.37671
2.4		0.35503
2.6		0.33505
2.8		0.31674
3.0	0.30000	
sum	0.70000	1.3353

$\int_2^3 \frac{x}{1+x^2} dx = \frac{1}{2} x\ 0.2[0.7 + 2\ x\ 1.38353]$

= 0.3467

(b)(i) $\int_2^3 \frac{x}{1+x^2}dx = \left[\frac{1}{2}In(1+x^2)\right]_2^3 = \frac{1}{2}(In10\ In5)$ = 0.3466

(ii) error = $|exact\ value - \ approximate\ value| = |0.3466 - 0.3467|$ = 0.0001

(iii) the error can be reduced by reducing h or increasing the number of sub-intervals.

Example 8

(a) Use trapezium rule to estimate the integral value of $\int_0^1 x^2e^x\,dx$

(b) (i) find exact value of $\int_0^1 x^2e^x\,dx$

(ii) determine the percentage error in your estimation

(a) h = $\frac{1-0}{5} = 0.2$

x	f(x) = x^2e^x	
0	0	
0.2		0.0489
0.4		0.2387
0.6.		0.6560
0.8		1.4243
1.0	2.7183	
sum	2.7183	2.3679

$\int_0^1 x^2e^x\,dx = \frac{1}{2}x\ 0.2[2.713 + 2\ x\ 2.3679]$

=0.74541≈ 0.745

(b)(i) $\int_0^1 x^2e^x\,dx = [x^2e^x - 2xe^x + 2e^x]_0^1$

= 0.718

(ii) error = $|0.718 - 0.745|$= 0.027

Percentage error = $\frac{error}{exact\ value}\ x\ 100\%$

= $\frac{0.027}{0.718}\ x\ 100$ = 3.8%

Revision Exercise

1. (a) Use trapezium rule with six strips to estimate $\int_0^{\pi} xsinxdx$ [3.069]

   (b) Determine the percentage error in your determination. [2.3%]
2. Use the trapezium rule to estimate the approximate value of $\int_0^1 \frac{1}{1+x^2dx}$ using 6 ordinates and correct to 3 decimal places. [0.784]
3. (a) Use trapezium rule with six strips to estimate $\int_2^4 \frac{10}{2x+1}dx$ correct 4dp. [2.9418]

   (b) Determine the percentage error in your estimation and suggest how this error may be reduce. [0.098%]
4. (a) Use trapezium rule to estimate the area of y = 3x between x-axis, x = 1 and x = 2, using five subintervals. Give your answer correct to four significant figures. [5.483]

   (b) Find the exact value of $\int_1^2 3^x dx$ [5.461]

   (c) Find the exact percentage error in calculations (a) and (b) above. [0.4028%]
5. Use trapezium rule with 7 ordinates to estimate $\int_0^3 \frac{1}{1+x}dx$, correct to 3 decimal places [1.405]
6. Use the trapezium rule with 6 ordinates to evaluate$\int_0^1 e^{-x^2}$ correct to 2 decimal place. [0.74]
7. Use the trapezium rule with 6 ordinates to estimate $\int_1^2 \frac{Inx}{x}dx$.Give your answer correct to 3 decimal places [0.237]
8. Find the approximate value to one decimal place of $\int_0^1 \frac{dx}{1+x}$' using the trapezium rule with five strips. [0.7]
9. (a) Use trapezium rule with five subintervals to estimate $\int_0^{\frac{\pi}{3}} \tan x\,dx$ correct to 3dp. [0.704]

(b) (i) Find the exact value of $\int_0^{\frac{\pi}{3}} \tan x \, dx$ to 3 d.p. [0.693]

(ii) Calculate the percentage error in your estimation in (a) above [1.587%]

(iii) Suggest how the percentage error in (b)(ii) may be reduced.

10. Use the trapezium rule with four subdivisions to estimate $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx} dx$. Give your answer correct to three decimal places. [1.013]
11. Find the approximate value of $\int_0^2 \frac{1}{1+x^2} dx$ using trapezium rule with 6 ordinates. Give your answer to 3 decimal places (05marks)[1.105]
12. Use the trapezium rule with five subintervals to estimate $\int_2^4 \frac{5}{(x+1)} dx$. Give your answer correct to 3 decimal places (05marks)[2.559]
13. A student used the trapezium rule with five sub-intervals to estimate $\int_2^3 \frac{x}{(x^2-3)} dx$ correct to **three** decimal places

    Determine;

    (a) The value the student obtained (06marks) [0.917]

    (b) The actual value of the integral (03marks) [0.896]

    (c) (i) the error the student made in the estimate [0.021]

    (ii) how the student can reduce the error(03marks)
14. (a) Use the trapezium rule with 6-ordinated to estimate the value of $\int_0^{\frac{1}{2}} (x + sinx) dx$, correct to three decimal places, [2.225]

    (b)(i) Evaluate $\int_0^{\frac{1}{2}} (x + sinx) dx$, correct to three decimal places [2.234]

    (ii) Calculate the error in your estimation in (a) above [0.009]

    (iii) suggest how the error may be reduced (06marks)

Thank you

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