P425/2
APPLIED MATHEMATICS
Paper 2
Oct. 2023
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT) Set 8

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g $to\ be$ $9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section*

1. A box contains 6 blue pens and 4 red pens. 2 pens are picked from the box one by one without replacement. Find the probability that:
   (i) The second pen is blue
   (ii) The two pens have the same colour given that the second pen is blue

2. Four forces of $a\boldsymbol{i} + (a-1)\boldsymbol{j}$, $(3\boldsymbol{i} + 2a\boldsymbol{j})$, $(5\boldsymbol{i} - 6\boldsymbol{j})$ and $(-\boldsymbol{i} - 2\boldsymbol{j})\boldsymbol{N}$ act on a particle. The resultant force makes an angle of 45° with the horizontal. Determine the;
   (i) Value of a. (ii) resultant force.

3. Given that $y = 5^{2x}$, find the absolute error in y when x=0.21. Hence find the limit within which the exact value of y lies

4. Two events A and B are such that $P(AUB) = \frac{19}{25}$, $P(A/B) = \frac{5}{9}$ and $P(A) = \frac{14}{25}$. Find
   (i) $P(AnB)$ (ii) $P(A/B')$

5. A particle of mass 0.2kg moving with a velocity of $(\mathbf{5i} + \mathbf{7j})ms^{-1}$ collides with a particle of mass 0.3kg with a velocity $(\mathbf{2i} - \mathbf{3j})ms^{-1}$. If the particles couple, find the
   (i) Common velocity. (ii) Loss in kinetic energy.

6. The table below gives values of a continuous function f(x)

X	0.9	1.0	1.1	1.2
f (x)	0.2661	0.2420	0.2179	0.1928

   Estimate:
   (i) $f(0.96)$ (ii) $f^{-1}(0.2372)$

7. A particle of weight 10N is suspended by two strings. If these strings make angles of 30° and 40° to the horizontal, find the tension in the strings. (05 marks)

8. The score below are for 8 students in two mathematics tests

Test 1	52	75	41	60	81	31	65	52
Test 2	50	60	35	65	66	45	69	48

   Calculate the rank correlation co-efficient and comment at 1% level of significance

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. Use the trapezium rule with 6 ordinates to find the approximate value of $\int_{0.0}^{1.0}(x^2e^x)\,dx$, correct to 3 significant figures. Hence find the percentage error in your calculation.

10. The table below summarizes the distance to the nearest mile, travelled to school by a random sample of students

Distance (miles)	Number of students
$0-9$	15
$10-19$	38
$20-29$	22
$30-39$	15
$40-49$	8
$50-59$	2

(a) Draw a histogram for the data and use it to estimate the modal distance

(b) Calculate the;

(i) Mean mark (ii) Standard deviation

(iii) Limits within two standard deviations of the mean distance

11. (a) A brick of mass 0.8 kg slides 6 m down a plane inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal with an initial speed of 0.4ms^{-1}. If coefficient of friction along the inclined is 0.25, find the speed of the brick at the bottom

(b) A car of mass 750kg is travelling along a horizontal road at a maximum speed of 50ms^{-1}, and the engine is working at a constant rate of 12kW, find;

(i) Resistance to motion

(ii) Maximum velocity up hill of 1 in 25

12. A random variable X has probability density function

$$f(x) = \text{kx} \quad 0 \le x \le 3$$
$$f(x) = 3\text{k}(4-\text{x}) \quad 3 \le x \le 4$$
$$f(x) = 0 \quad \text{x} < 0 \text{ and } x > 4$$

(a) Sketch f(x) hence find the value of the constant k.

(b) Find $E(3X-1)$

(c) Obtain the Cumulative distribution function F(x), hence find P(X<3.5)

13. A two particles A and B move with velocities $(\boldsymbol{\lambda i + 3j + 30k})$ms^{-1} and $(\boldsymbol{4i - 2j - 15k})$ms^{-1} respectively where λ is a constant. At t=0, the particles are located at points $(\boldsymbol{2, 1, -15})\boldsymbol{m}$ and $(\boldsymbol{1, 4, 12})\mathbf{m}$ respectively.
    (a) Find the value λ such that A and B will colide and find the value of t when collision occurs
    (b) When $\lambda = 2$, find the time after which the particles will be nearest together

14. The numbers $a = 19.67, b = 4.934, c = 13.8822$ and $d = 47.1300$ are each rounded off to the given number of decimal places, calculate
    (a) Limit within which the exact value of $a - \frac{d}{(c-b)}$ lies correct to 3 decimal places
    (b) Percentage error in $ad - (c + b)$ correct to 3 decimal places

15. At a certain school, the heights of students are normally distributed. 10% are over 1.8 meters and 20% are below 1.6 meters
    (a) Find the mean height, μ and standard deviation, σ of the students
    (b) Find the inter quartile range

16. ABCD is a rectangle with $AB = 5m$, and $BC = 3m$. Forces of magnitude 2N, 4N, 3N and 11N act along the line AB, BC , DC and DA respectively in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, Find the;
    (i) Magnitude and direction of the resultant
    (ii) Equation of the line of action of the resultant
    (iii) Distance from A where the line of action of the resultant force cuts AB