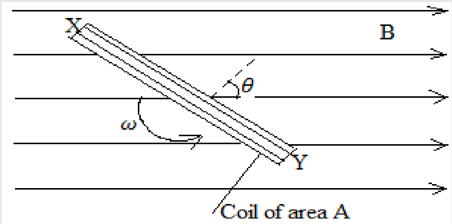
Continuation of electromagnetism (should be hand written in the notes book)

2. emf Induced due to rotation of a coil in a magnetic field ($emf\ in\ a\ dynamo$)

Consider a rectangular coil XY of N turns, each of area A being rotated with a constant angular velocity ω in a uniform magnetic field of flux density B about an axis which is perpendicular to the paper.



When the normal to the coil is at an angle θ to the field, then the flux through each turn of the coil is given by;

$\emptyset = AB\cos\theta$ But $\theta = \omega t$

$\rightarrow \emptyset = AB\cos\omega t$ .For N turns, the flux linkage $N\emptyset = NAB\cos\omega t$

But $E = -\frac{d}{dt}(N\emptyset)$ $\rightarrow E = -\frac{d}{dt}(NAB\cos\omega t) = -NAB\frac{d}{dt}(\cos\omega t)$

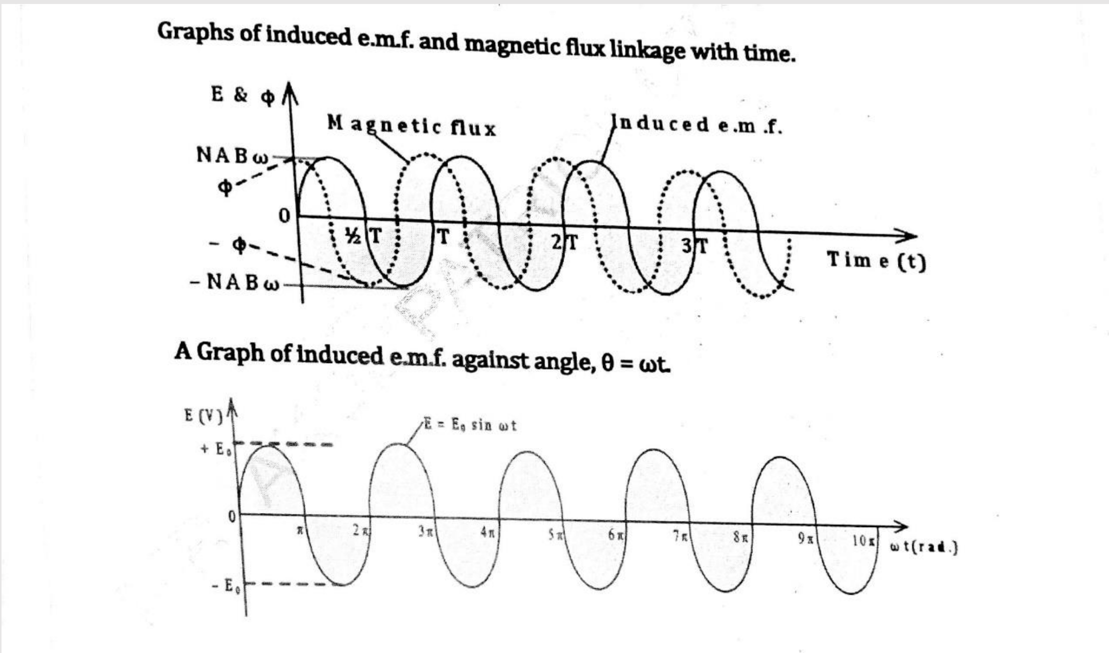
$E = -NAB(-\omega\sin\omega t)$ $\therefore \boldsymbol{E = NAB\omega\sin\omega t}$ $\boldsymbol{OR}$ $\boldsymbol{E = 2\pi fNAB\sin 2\pi ft}$

Maximum emf

The emf is maximum when $\theta = 90°$

$\therefore \boldsymbol{E_0 = NAB\omega}$ $\boldsymbol{OR}$ $\boldsymbol{E_0 = 2\pi fNAB}$

Therefore, the emf induced is given by $\boldsymbol{E = E_0\sin\omega t}$



Examples

1. A rectangular coil of 100 turns and area 400cm^2 is rotated about a horizontal axis at a constant rate of 50 revolutions per seconds in a horizontal magnetic field of 0.21T to the axis. Calculate

(i) The maximum emf induced in the coil
(ii) Induced emf when the coil is $30^0$to the horizontal

Solution

(i) $E_0 = NAB\omega \;\; = E_0 = NAB2\pi f = 100 \times 400 \times 10^{-4} \times 0.21 \times 2 \times 3.14 \times 50$
$E_0 = 263.76V$

(ii) $E = E_0 \sin\theta \; = 263.76 \sin(90 - 30) \quad \therefore E = 228.4V$

2. A flat circular coil with 2000 turns each of radius 40cm is rotated at a uniform rate of 480 revolutions per minute about its diameter at right angle to a uniform magnetic field of flux density$5 \times 10^{-4}T$. Calculate

(i) The peak value of emf induced in the coil
(ii) The instantaneous value of induced emf when the plane of the coil makes an angle of $20°$ with the field direction.

Solution

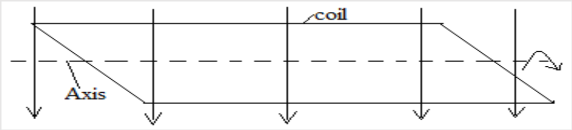
(i) $E_0 = NAB\omega \;\; = E_0 = NAB2\pi f = 2000 \times \frac{22}{7} \times (40 \times 10^{-2})^2 \times 5 \times 10^{-4} \times 2 \times \frac{22}{7} \times \frac{480}{60}$

$E_0 = 2.53V$

(ii) $E = E_0 \sin\theta = 25.3\sin(90-20) \quad \therefore E = 23.77V$

EXERCISE:

1. A coil of 100 turns and cross sectional area $2 \times 10^{-2} m^2$ lies in a magnetic field of flux density $3 \times 10^{-3} T$ and rotates uniformly at 100 revolutions per second about an axis perpendicular to the magnetic field as shown below.



Calculate

(i) The emf induced when the plane of the coil makes 60^0 with the field $\boldsymbol{ANS}\ (\mathbf{1.88V})$

(ii) The amplitude of the induced emf $\boldsymbol{ANS}\ (\mathbf{3.77V})$

2. A flat circular coil of 500 turns each of radius 10cm is rotated at a frequency of 200 revolutions per minute about its diameter at right angle to a uniform magnetic field of flux density 0.18T. Calculate the;

(i) Maximum magnetic flux linking the coil $\boldsymbol{ANS}\ (\mathbf{2.83Wb})$

(ii) emf Induced in the coil when the normal to the coil makes an angle of 30^0 with the magnetic field. $\boldsymbol{ANS}\ (\mathbf{51.3V})$

(iii) Root mean square value of the emf induced in the coil. $\boldsymbol{ANS}\ (\mathbf{41.9V})$

3. A current $I = 120\pi t$ is passed through a solenoid of 1000 turns per metre. A small circular coil of 500 turns of radius 3.5cm has an axis through its centre being parallel to that of the solenoid.

(i) Determine the peak value of the emf induced in the circular coil when the current is flowing in the solenoid $\boldsymbol{ANS}\ (\mathbf{9.12V})$.

(ii) Sketch a graph of emf with time

4. A metallic disc of diameter 20cm rotates at a constant speed of 60 ervmin^{-1} about an axis through its centre and perpendicular to a uniform magnetic field of flux density $5 \times 10^{-3} T$ established parallel to the axis of rotation. Calculate the emf developed between the axis and the rim of the disc. $\boldsymbol{Ans}(\mathbf{1.57mV})$**.**

5. A square coil of side 20cm and 100 turns is arranged to rotate at 240 revolutions per minute about a vertical axis perpendicular to the coil placed in a horizontal uniform magnetic field of flux density 1.5T, the axis of rotation passes through the mid points of a pair of opposite sides of the coil. Calculate the emf induced in the coil if the plane of the coil makes an angle of 30^0 with the field. $\boldsymbol{Ans}(\mathbf{130.6V})$**.**

6. A horizontal straight wire 5cm long has a mass of 1.2gm^{-1} and is placed perpendicular to a uniform horizontal magnetic field of flux density 0.6T. The resistance of the wire is 3.8Ω in the magnetic field. Find the potential difference that must be applied between the ends of the wire to make it self-supporting $\boldsymbol{Ans}(\mathbf{29.8V})$**.**

7. A rectangular coil of 200 turns and dimensions 5cm by 4cm is placed with its plane normal to a uniform magnetic field of flux density 0.008T is rotated about an axis through the centre of the shorter sides perpendicular to the magnetic field at 420 revolutions per minutes. Determine the peak value of the emf induced in the coil. $\boldsymbol{Ans}\ (\mathbf{1.41\times 10^{-1}}\boldsymbol{V})$
8. A coil of 300 turns and with area of $0.05m^2$ is rotated 20 times per second in a magnetic field of flux density 2.2T. calculate the;
   i. Maximum emf induced in the coil $\boldsymbol{Ans}\ (\mathbf{3.77\times 10^{2}}\boldsymbol{V})$
   ii. Torque required to maintain this rate of rotation if the current in the coil is 0.8A when the emf generated is maximum $\boldsymbol{Ans}\ (\mathbf{2.4}\boldsymbol{Nm})$