A MUST-HAVE

A-LEVEL

Physics

PAPER 2

Revised Edition

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A MUST HAVE A-LEVEL

PHYSICS PAPER 2

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REVISED EDITION

PREFACE

The new A-Level syllabus stipulated by the NCDC covers so much ground that many teachers find it difficult to take their classes through all the material while still leaving time for practical work.

I have written this book to allow students cover parts of the syllabus on their own as they seek assistance from their teachers. This will reduce the amount of note making which needs to be done in class and will release time for discussion, reinforcement and practical work.

This book is designed to help both those students who are already achieving success in their physics and those who have less experience and little confidence in the subject.
It takes into account the development in the subject itself, its position in the curriculum and its application in other areas. It covers in great detail the current syllabus for A-level physics paper 2 and it is therefore aimed at alleviating the problem of the scarcity of well-arranged publications based on the current teaching syllabus for advanced level physics.

I have on each chapter given comprehensive notes, a good selection of worked examples, self-evaluation and examination past paper questions. These will help the students to widen the experience and improve their confidence.

The number of worked examples has been greatly increased in this revised edition. Many of these are easier than was previously considered necessary. Definitions and fundamental points have been highlighted.

To avoid making the book very bulky, which in turn makes the cost high, I have reduced the font size slightly to 11. I have not compromised on the content as the book has more explanation, worked examples and self-evaluation exercises.

It is therefore my sincere hope that students, teachers as well as general readers will find this book a good, reliable and an indispensable guide to physics and I am therefore optimistic that it will continue to meet the needs of readers at all levels.

Finally, all misfortunes, if any in this book are purely my responsibility because it is difficult to claim perfection. I will be glad for any comments or compliments that will be directed to me. For no one is a monopolist of knowledge and no scientific theory is born in vacuum. Each scientist builds on the work of his predecessors.

This edition gives me the opportunity to thank all those people who have suggested ways in which the book might be improved. I am particularly grateful to all the individuals who undertook the laborious task of assisting with proof reading and for the invaluable suggestions made throughout the preparation of this edition.

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CONTENTS

Examination tips

Go through the paper quickly reading or scanning it to get an idea of what has been asked.

Work your way through the paper. If you find any question particularly formidable, do not carry on with it. Come back to it later. You will lose valuable time being bogged down with one question. Get some easy marks in the bag first.

Sometimes the physics concepts just come in one ear and leave out the other at the speed of light therefore it's good to do continuous revision and remind yourself of the definitions, derivations, experiments and calculations therein. The exams will always test your knowledge and understanding of the physics concepts and principles.

When you are stuck, read the question again and ask yourself:

- Does the question fit what I have been trying to do?
- Is there a diagram I could draw?
- Have I pictured the situation described by the question?
- Have I missed an equation that is needed?
- Are there words in the question that I have disregarded, perhaps 'in series, steady, in parallel, etc.' in a question?

Always be careful with details such as units. Some questions ask you to 'explain' or 'describe' something. Read your answers through and see if they make sense to someone who does not already know the answer.

When you have finished do not sit arms folded looking up at the ceiling. You will not have scored 100%. There are some marks still to be had. Spend every last minute going through the paper carefully looking for errors. Trust me, there will be some.

If you run out of time or if you are running out of time. Look for a question with a lot of marks and with a broad answer range like an essay or experiment. Maximize marks.

It will also be important for you to understand the following requirements that might appear in different questions. Some of them include the following;

Define: It requires you to "state precisely the meaning of terms or words". A formal statement is required and must be precise.

State: A concise answer with no supporting argument. Just like definitions, statement of physical laws must be precise.

What is meant by ……or What do you understand by…… Normally implies that a definition should be given, together with some relevant comment on the context of terms concerned.

Explain: Give a detailed account of something. Use basic physics principles to justify or clarify the explanation. A graph or diagram will often be useful.

Describe: Give a detailed account or representation of something in words, always accompany your explanations with a detailed well labeled diagram.

Identify: Recognize or prove something as being a certain.

Name: Write down, mention or state.

Determine: Ascertain something after some observation, solve a problem or find out.

Discuss: Give points and general description in writing either for or against a statement.

There only remains for me to say good luck. But exams have little to do with luck. Luck goes to a prepared mind. If you have done the work and revised thoroughly, you will undoubtedly do well. The fact that you are reading, this shows your intent. So, don't look at exams as impossible hurdles to jump. Look at them as opportunities for you to shine and show everyone just what you can do.

"If I have seen further than most, it's because I have stood on the shoulders of giants"

Sir Isaac Newton

"Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things"

Sir Isaac Newton

"Education is what remains after one has forgotten what one has learned in school"

Albert Einstein

"The true sign of intelligence is not knowledge but imagination"

Albert Einstein

"Learning never exhausts the mind"

Leonardo da Vinci

"Nature is the source of all true knowledge. She has her own logic, her own laws, she has no effect without cause nor invention without necessity"

Leonardo da Vinci

"We cannot teach people anything; we can only help them discover it within themselves"

Galileo Galilei

"I have never met a man so ignorant that I couldn't learn something from him"

Galileo Galilei

To my beloved son Mutumba Fahad Ali

INTRODUCTION

GENERAL

In the present century, we find that science is held in high esteem. Although the word "science" has its origin in a Latin verb meaning “to know”, science has come to mean not merely knowledge but a body of knowledge of natural world in an organised and rational way. The scientific developments have exerted influence on every phase of human activity. The intelligent man recognises that the material progress of technology is a by-product of the discoveries of science. In all fairness, honest credit is being given to science for its contribution to knowledge, comfort, efficiency and to the general well-being of mankind.

Physics is the fundamental science. The main goal of physics is give correct and precise description of the material universe. The study of physics is an adventure. It is challenging, sometimes frustrating, occasionally painful and often richly rewarding and satisfying. Although physics has earned the reputation as a difficult subject, yet it is inherently simple. It is because there are only a few fundamental laws that you need to lodge in your memory bank. If you really understand these laws, you can readily use them to deduce how nature behaves in a variety of situations. As you read the text and work out physics problems, ask yourself how each problem you approach is really similar to other problems and to the text examples. You will find that similarity because most of the problems and examples really involve only a few underlying laws. So, physics is simple provided you understand its fundamental laws and have the experience in reasoning out how these laws apply to the world about you.

PHYSICS

The branch of science which deals with correct and precise description material universe is known as physics. The word physics comes from a Greek word meaning “nature”. Physics attempts to describe the fundamental nature of the universe and how it works. For example, physics explains why the sky is blue, why rainbows have colours, why steel ships float, what atoms and nuclei are made of, etc. To all these and many more questions, physics explains using as few laws as possible, revealing their underlying simplicity and beauty. The amount of knowledge available on physics is so vast that as a matter of convenience, it has been divided into two classes:

(i) Classical (old) physics
(ii) Modern physics

Classical physics: The physics prior to 1890 is known as classical physics. By 1890, the various laws governing the material universe were discovered and it appeared that knowledge of this subject was complete. Newton's laws of motion and gravitation had provided a solid foundation on which science of mechanics had been raised to impressive heights. The laws of thermodynamics and kinetic theory of matter gave a satisfactory account of the entire subject of heat. Maxwell developed the theory of electromagnetic radiation and explained all electrical, magnetic and optical phenomena. It appeared that the laws and theories of classical physics would hold the field unchallenged. However, startling discoveries after 1890 gave a death blow to the so-called classical physics.

Modern physics: The physics after 1890 is known as modern physics. After 1890, there had been enormous advances in physics which marked the beginning of revolutionary concepts. The thrilling discoveries posed new problems unknown to classical physics. In 1895, Roentgen discovered **X-rays** and a few weeks later, Henry Becquerel announced the discovery of **radioactivity**. In the year 1897, J.J. Thomson discovered **electrons**, a fundamental constituent of all matter. In 1901, Max Planck developed quantum theory according to which energy is not radiated continuously but in discrete packets of energy called quanta. These quanta eventually came to be known as photons. The **special theory of relativity** proposed by Albert Einstein in 1905 gave new dimensions to atomic and nuclear physics. In short, the rapid startling discoveries after 1890 gave a new shape to the entire concept of physics. The study of these concepts has been placed under the head of modern physics.

The reader may wonder about the difference between classical physics and modem physics. The answer is that classical physics does not work for microscopic particles (e.g., atoms) or for objects travelling at very high speed. In fact, these two aspects gave birth to what is now called modern physics. Indeed, modern physics may be considered as mere “fine-tuning” of classical physics.

Except for the microscopic particles and for motion at speeds near the speed of light, classical physics correctly and precisely describes the behaviour of the physical world. Although modern physics has made many important contributions to technology, great bulk of technical knowledge and skill is still based squarely on classical physics. It is through the applications of classical physics that we have been able to exploit natural resources successfully.

SOME SCIENTIFIC NOTIONS

Some important scientific notions are; (i) scientific law (ii) theory (iii) model (iv) scientific method.

Scientific law: The general statement of happenings in nature and after experiments is known as a scientific law.
A scientific law is a concise statement how nature behaves. It is actually the essence of a large number of observations and experiments. For example, the law of gravitation states that the two objects attract each other. Similarly, “unlike charges attract each other and like charges repel” is also a law and is based on observations and experiments. A law has no theoretical or other basis. In other words, we cannot ask “why like charges repel and not attract? Why two bodies attract each other and not repel?” The only answer to such questions is that this is how nature behaves and the matter ends there. No “buts” and “ifs”.
Physical laws are usually expressed as mathematical equations which are then used to make predictions about other phenomena. For example, the law of gravitation states that the two objects attract each other with a force which is proportional to the mass of each object and inversely proportional to the square of the distance between them. It is written in the mathematical form as $F = \frac{Gm_1m_2}{r^2}$. Because this expresses the law in the form of an equation, it is far preferable to the qualitative statement. It may be noted that a scientific law has only experimental proof

Theory: Speculation on the basis of determined laws is known as “theory”.
When several laws are determined, then one may make a guess about the way the universe must be constituted in order that such things happen. This guess is called theory. It may be noted that there is a basic difference between a theory and a law. Theory is given by man and hence may be wrong while law is made but by nature and always right.
Theories are never derived from observations - they are created to explain observations. They are inspirations that come from the minds of intelligent people. For example, the idea that matter is made up of atoms (atomic theory) was certainly not arrived at because someone observed atoms. Rather the idea was conceived by a creative mind. The theory of relativity and the electromagnetic theory were likewise the result of inspiration. Nevertheless, the history of physics tells us that theories come and go, that long-held theories are replaced by new ones. However, a new theory is accepted only if it explains a greater range of phenomena than does the older one.

Model: This is a visual picture of the phenomenon under study.
When scientists are trying to understand a particular phenomenon, they often make use of a model. The purpose of a model is to give us a visual picture – something to hold on to when we cannot see what actually is happening. One example is the wave model of light. We cannot see waves of light as we can see water waves. But it is valuable to think of light as if it were made up of waves because experiments on light show that it behaves in many aspects as water waves do.

Scientific method: To understand any natural process by a combination of logical reasoning and controlled experimentation is called the scientific method.
Prior to about 1600, questions of truth and falsehood were most often determined by political or religious dictates. But great scientists like Isaac Newton introduced scientific methods to the world. The two main tools of scientific method are logic and experimentation.
The result is that the experimental results are reproducible. That is, the same set of circumstances will always produce the same observed results in the same experiment, no matter who the observer is.
Notes: (i) Scientific laws are different from political laws in that the latter tell us how we must behave. Scientific laws do not say how nature behaves but rather describe how nature must behave.
(ii) To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term “principle” is often used (such as Archimedes’ principle). Where to draw the line between laws and principles is, of course, arbitrary and there is not always complete consistency.

SCOPE AND EXCITEMENT OF PHYSICS

The scope of physics is very wide. It covers an immense range of natural phenomena. Distances extend from incredibly small dimensions of subatomic particles to thousands of million metres that separate galaxies of the universe. For example, the radius of atomic nucleus is about 10^{-14} m while the radius of the universe is about $10^{25}\ m$. The time intervals encountered in the physical world vary over an extremely wide range. For instance, the time taken by light to cross distance of nuclear size is about $10^{-22}\ s$ while the life of the sun is about $10^{18}\ s$. Similarly, the mass of an electron is approximately 10^{-30} kg whereas the mass of the universe is about 10^{55} kg. All these wide range of measurements are in the domain of physics.

The subject of physics has wide applications. It includes medical physics, computers, meteorology, material science, geophysics, engineering communication, environmental physics etc. Physics is a very exciting subject It provides answers to exciting questions such as:

- Why is the sky blue?
- Why is sunset red?
- Why is glass transparent?
- Why are atoms held together?

These are only a few exciting questions. In fact, there are a very large number of such questions to which physics provides answers.

FIELDS OF PHYSICS

Physics is the physical science which deals with matter and energy and their transformations. Knowledge of physics will help you understand both natural phenomena and the technologies that increasingly prevade our lives. There are numerous problems that physics faces yet we can describe and understand any problem with the help of the following five fields of physics i.e.

Classical mechanics: It deals with objects from molecules to galaxies that are moving at speeds small compared with the speed of light. It deals with such ideas as inertia, motion, force and energy. Mechanics also includes the properties and laws of both solids and fluids. You use the laws of classical mechanics (also known as Newtonian mechanics) when you drive a car, ride a skateboard, build a skyscraper etc.

Indeed, many physics problems can be solved through the application of laws of classical mechanics. You will see that classical mechanics is but an approximation to a more comprehensive set of physical laws that includes Einstein's theory of relativity and the theory of quantum mechanics.

Thermodynamics: It deals with the study of heat and its interaction with matter. It includes effects of temperature on the properties of materials, heat flow and transformations involving heat and work.

Electromagnetism: It deals with electricity, magnetism and electromagnetic radiation. Today electromagnetic technology dominates our civilization as evidenced by radio, television, tape recorders, microwave ovens and computers.

Relativity: In 1905, Albert Einstein gave the theory of relativity. This theory has changed our notions of space and time and of mass and energy. Space and time are seen to be intimately connected with time being the fourth dimension in addition to space's three dimensions. Although theory of relativity provides a more correct description of physical reality than does classical mechanics, the two theories differ significantly only at speeds approaching that of light. In our everyday existence, we need not worry about relativity.

Quantum mechanics: It deals with the behaviour of objects on the extremely small scale of atomic dimensions. Indeed, classical physics fails to explain the behaviour of such microscopic particles. The quantum mechanics touches every aspect of modern physics and most of classical physics. Indeed, one requires an understanding and the application of quantum mechanics for a satisfactory explanation of most of the phenomena.

THE SIMPLICITY OF PHYSICS

There is a common notion that physics is incomprehensible to the people of average ability. If you think that, you are missing the point because it is so fundamental, physics is inherently simple. There are only a few basic laws to learn. If you really understand those laws, you can readily apply them in a wide variety of situations. The laws and theories of physics describe the working of the universe at the most basic level. For that reason, physics is extremely powerful. The same laws describe the behaviour of molecules, aeroplanes and galaxies. Scientists believe

that it would be possible to describe the operation of a living cell or organism using only the fundamental laws of physics. So, physics is simple – challenging too but with an underlying simplicity that reflects the scope and power of this fundamental science.
A physical science is one which primarily deals with non-living things. On the other hand, biological science deals with living things.

PHYSICS IS BASIC SCIENCE

Physics is the basic science. It is about the nature of basic things such as motion, force, energy, matter, heat, sound, light and the inside of atoms. An understanding of science begins with an understanding of physics. There is physics in everything you see, hear, smell, taste and touch.

Physics and Mathematics: The laws of physics are usually expressed as mathematical equations which are then used to make predictions about other phenomena and to test the range of validity of the laws. In order to understand physics on any level beyond qualitative description, you require a considerable knowledge of mathematics. The mathematical description of the laws and theories of physics gives new insight to the subject. Indeed, the primary goal of mathematics is to aid physics.

Physics and Chemistry: Chemistry is about how matter is put together, how atoms combine to form molecules and how molecules combine to make up the many kinds of matter around us. In order to explain these aspects, we have to apply the laws and theories of physics. So, underneath chemistry is physics.

Physics and Biology: Biology is more complex and involves matter that is alive. So, underneath biology is chemistry and underneath chemistry is physics. That is why physics is the most basic science.

Physics and other sciences: Since physics is a fundamental science, its laws and theories are applied to various physical sciences. The knowledge of physics is required in many fields and the list below shows some of these:

- Medical physics
- Meteorology
- Geophysics
- Environmental physics
- Communication and material science.

PHYSICS AND TECHNOLOGY

Physics and technology are different from each other. Physics is the knowledge of material universe for own sake whereas technology transfers the knowledge of physics into practical shape for the general well-being of human race. In other words, physicists provide the knowledge of material universe whereas the engineers provide the tools, techniques and procedures for putting this knowledge to practical use. For example, physics has provided the knowledge of semiconductors but technology made of them transistor radios. Many technologies have been developed on the basis of the knowledge of physics and a few of them are mentioned below by way of illustration.

(a) The discovery of Faraday's laws of electromagnetic induction enabled the engineers to develop the electric generator, electric motor, transformer and a host of other equipment.
(b) The discovery of the laws of thermodynamics led to the development of many types of heat engines.
(c) The discovery of electromagnetic radiation, vacuum tubes and semiconductors led to wireless communication e.g. radio and television etc.
(d) The discovery of nuclear fission led to the development of nuclear power plants which are generating huge amounts of electric power all over the world.

The above short list indicates how important the knowledge of physics is in developing new technologies. Indeed, technology may be regarded as **applied physics**.
When laws of physics are expressed in mathematical form, they do not have the double meanings that so often confuse the discussion of the ideas expressed in common language. When laws are expressed mathematically, they are easier to verify or disprove by experiments.

PHYSICS AND SOCIETY

The advancement of technology as a result of discoveries of knowledge of physics has made a significant impact on human lives. We travel faster than ever before in aeroplanes. We enjoy colour television. We use electricity to operate a host of electrical appliances. We can solve complex problems at amazing speed with the help of computers. We send satellites into orbit to learn about the conditions in outer space. The development of radio and television has led to instant communication. Indeed, the uses of physics have exerted considerable influence on every phase of human activity.

We are all familiar with the abuses of applied physics or technology. Many people blame technology itself for widespread pollution, resource depletion and even social decay in general. It is much wiser to combat the misuse of technology with knowledge than with ignorance. Wise applications of physics can lead to a better world.

BASIC FORCES IN NATURE

There are four basic forces that operate in nature. These are (i) gravitational force (ii) electromagnetic force (iii) the strong nuclear force and (iv) the weak nuclear force.

CONSERVATION LAWS

In physics, there are some physical quantities which remain the same before and after an event or interaction. Such quantities are called conserved quantities. For example, electric charge is a conserved quantity. In every event, the principle of conservation of charge has always been found to apply. Thus, when we produce a free negative charge by tearing an electron loose from an atom, an equal positive charge is left behind, thereby causing no change in total charge of the system. The laws relating to conserved quantities are called conservation laws.

Those laws of nature that state that some physical quantity is the same before and that after an event or interaction are called conservation laws.

Some important, physical quantities that are conserved in an event or interaction are:

(i) Electric charge: Law of conservation of electric charge

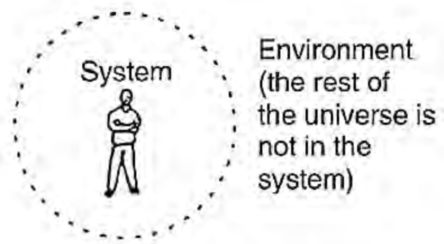
(ii) Energy: Law of conservation of energy

(iii) Momentum: Law of conservation of momentum

We do not know the reason for it. But scientists have performed a very large number of experiments over the years to test the conserved nature of the above physical quantities. In all the experiments, the conserved nature of the above physical quantities is proved beyond any doubt. Nevertheless, conservation laws reflect one of the most basic ways of describing nature. Because conservation laws allow us to consider quantities, such as energy, that do not change during an event, we do not need to know the details of the interaction, we simply deal with the value of the conserved quantity before and after the interaction or event. Moreover, the fact that some physical quantities are conserved and some are not, tells us something about nature itself.

System and Environment: A system is some part of the universe in which we have a special interest. All the rest of the universe outside the system is called environment. The system can be any part of the universe we choose e.g. a human body, the whole earth, a single atom etc.

The figure below shows the dashed line around a person. The part of the universe within the dashed line is the system. All the rest of the universe outside our chosen system is the environment. The dashed line can be an actual boundary between the system and the environment, or it can be an imaginary boundary that we use to help identify the system.



Isolated system: The system is said to be isolated if nothing (energy, matter, light) crosses the boundary of the system to or from the environment and if the net sum of all the external forces on the system is zero.

Within an isolated system, the amount of conserved quantity (e.g. electric charge, energy, momentum, etc.) will not change during an event or interaction. Often, we can consider a system approximately isolated by simply choosing its boundaries in an appropriate manner. Then we can apply conservation laws to learn more about elements within the system. For instance, when two cars collide, we can choose the system to be the region of space that includes both cars. The interaction of each car with the other is much greater than any other force affecting the cars. As a result, we consider the system as being approximately isolated. Therefore, we can use conservation laws to learn more about the motions of the cars during their collision

SECTION A:

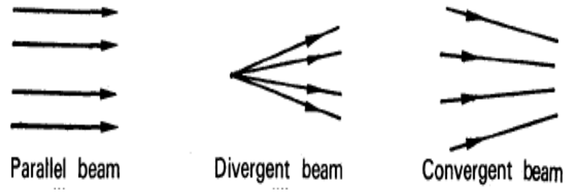
GEOMETRICAL OPTICS

- Introduction
- Propagation of light
- Reversibility of light
- Reflection at plane surfaces
- Laws of reflection of light
- Regular and irregular reflection
- Applications of regular and irregular reflection
- Deviation of light at: plane mirror, mirrors inclined at an angle
- Rotation of a mirror
- Optical lever in light beam galvanometer
- Principal of sextant
- Images in a plane mirror
- Formation of multiple images
- Reflection of light at curved surfaces
- Types of curved surfaces
- Optical properties of curved mirrors
- Caustic surfaces
- Parabolic mirrors
- Relationship between r and f
- Formation of images by curved mirrors
- Images in curved mirrors
- Linear magnification
- The mirror equation
- Experiments to measure the focal length of concave and convex mirrors
- Applications of curved mirrors
- Spherical aberration
- Refraction of light at plane surfaces
- Laws of refraction of light
- Verification
- Refractive index, determination
- Real and apparent depth
- Critical angle
- Total internal reflection, thick plane mirrors, mirage
- Air cell
- Applications of refraction of light
- Refraction of light through triangular prisms
- Refracting angle
- Minimum deviation
- Spectrometer
- Grazing incidence and grazing emergence
- Limiting angle of prism
- Deviation by thin prisms
- Dispersion of white light by prisms
- Applications of prisms (prism binoculars, periscopes)
- Refraction through a thin lens
- Types of lenses
- Optical properties of lenses
- Power of a lens
- Formation of images by lenses
- Thin lens formula
- Conjugate foci
- Displacement method for measuring focal length
- Newton's relation
- Focal length and refractive index
- Determination of focal length and radius of curvature of a lens
- Chromatic and spherical aberration
- Optical instruments
- Near point, far point and accommodation
- Visual angle
- Magnifying power of an optical instrument
- The simple microscope
- Images formed by a simple microscope
- Compound microscope
- Angular magnification
- Refracting telescopes
- Astronomical and Galileo's telescopes
- Reflecting telescopes
- The eye-ring
- Prism binoculars
- Projection lantern
- Simple lens camera
- Measuring the speed of light

Introduction

The branch of physics which deals with light and its behavior is called **optics**. Under ordinary situations, light travels in straight lines. This reasonable assumption has led to the ray model of light. The straight-line paths followed by light are called **light rays**. The ray model is found very successful in explaining phenomena like reflection, refraction and formation of images by mirrors and lenses. Since these explanations involve straight line rays at various angles, it is called **geometrical optics** or **ray optics**.

A collection (bundle) of light rays is called a **beam** of light and can be parallel, convergent or divergent.

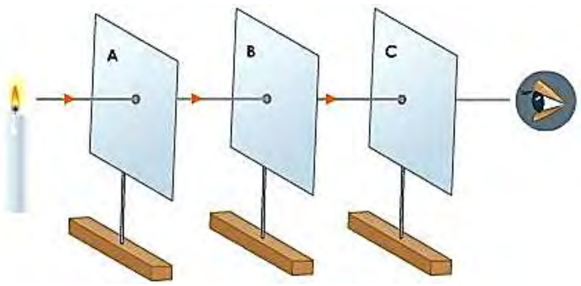


Propagation of light

Light travels in a straight line which is a property called rectilinear propagation of light.

Experiment to verify rectilinear propagation of light

Three cardboards A, B and C with holes are put in a straight line as shown below



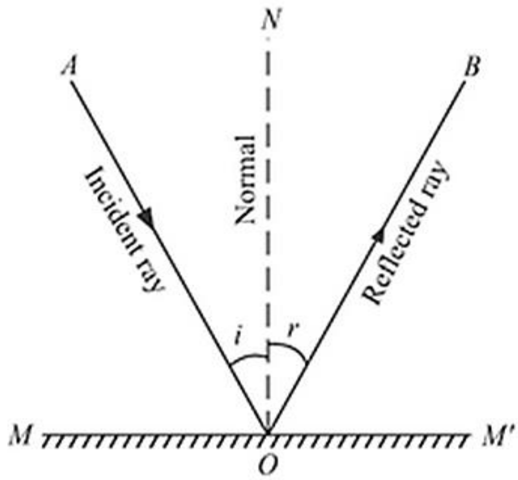
A lighted candle is placed in front of cardboard A and viewed from behind cardboard C.

Observation

When observing from cardboard C, the light from the candle can be seen. On displacing one of the cardboards, the light from the candle can no longer be seen as the holes are no longer in a straight line. This therefore confirms that light travels in a straight line.

Reflection of light

When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon is called reflection of light.



Note:

After reflection, velocity, wavelength and frequency of light remains the same but intensity decreases.

If a light ray is incident normally on a surface, after reflection it retraces the original path.

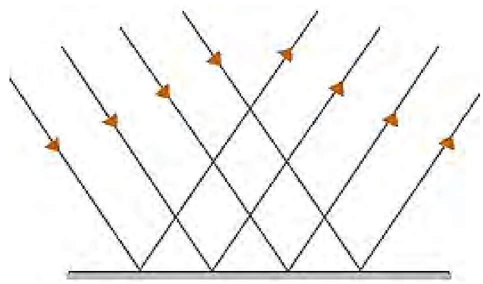
Regular and diffuse reflection

Reflections are obtained from hard and highly polished surfaces such as mirrors and sheets of glass than from rougher surfaces. Surfaces can be classified into two that is;

1. Highly polished surface for example mirror, polished cooking utensils and silvered iron sheets.
2. Rough surface for example unpolished wooden table, sheet of paper and cobblestone road.

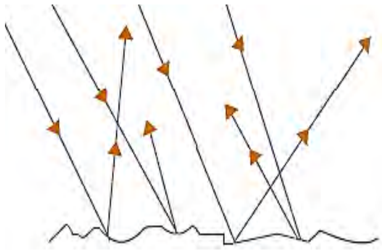
Regular reflection

This is the type of reflection whereby an incident parallel beam is reflected as a parallel beam e.g. on a plane mirror.



Diffuse/irregular reflection

This is the type of reflection in which an incident parallel beam is not reflected as a parallel beam e.g. on a sheet of paper.



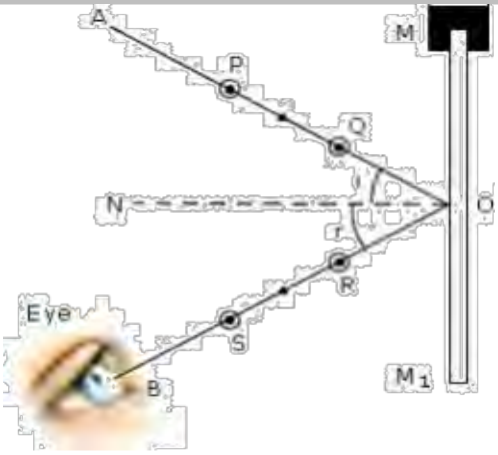
At each point on the paper, the laws of reflection are obeyed though the angle incidence varies.

Laws of reflection of light

Law 1: The incident ray, the reflected ray and the normal to the mirror at the point of incidence, all lie in the same plane.

Law 2: The angle of incidence is equal to the angle of reflection.

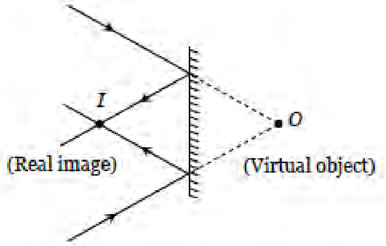
Experiment to verify the laws of reflection of light

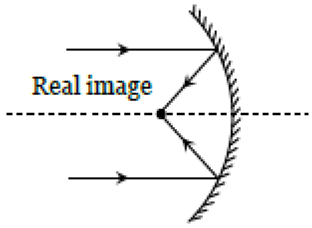


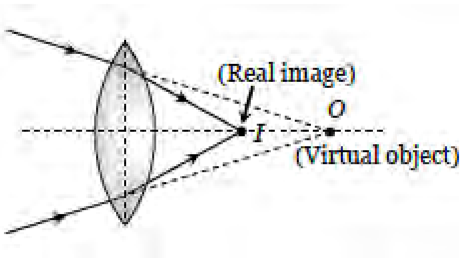
- Pin a white sheet of paper firmly on a drawing board. Place the mirror on it and trace its outline.
- Remove the mirror and draw a normal and place the mirror again on the outline.
- Place two pins P and Q in a straight line on one side of the normal on the white sheet of paper.
- Place two pins R and S on the other side of the normal in such a way that these two pins are in a straight line with the reflection of P and Q.
- Remove the mirror and the pins and join the pin marks to the normal.
- It will be seen that the angles AON and BON are equal.

Real and virtual images

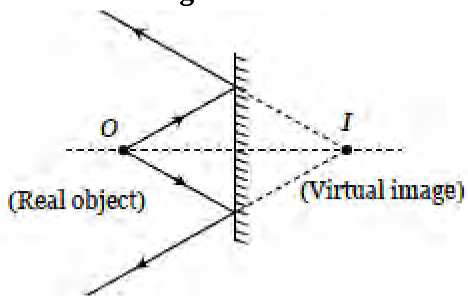
If light rays, after reflection or refraction, actually meet at a point, then a **real image** is formed.

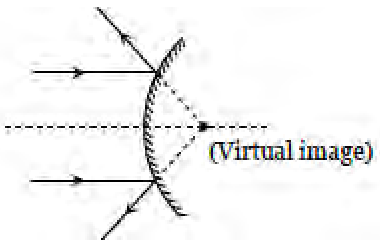


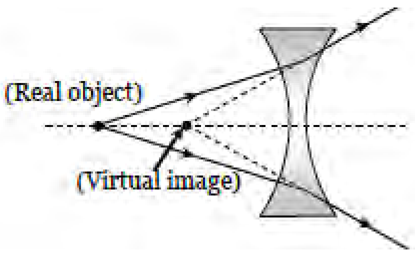




If the light rays, after reflection or refraction appear to meet, then a **virtual image** is formed







Therefore, 5 images of the point object held in-between M_1 and M_2 will be formed.

(ii) If $360°/\theta$ is an odd integer, then the number n of images formed is given by

$$n = \frac{360°}{\theta}$$

For example, if $\theta = 40°$, then $360°/\theta$ is an odd integer so that

$$n = \frac{360°}{40°} = 9$$

Therefore, 9 images will be formed.

(iii) If the mirrors are parallel, $\theta = 0°$.

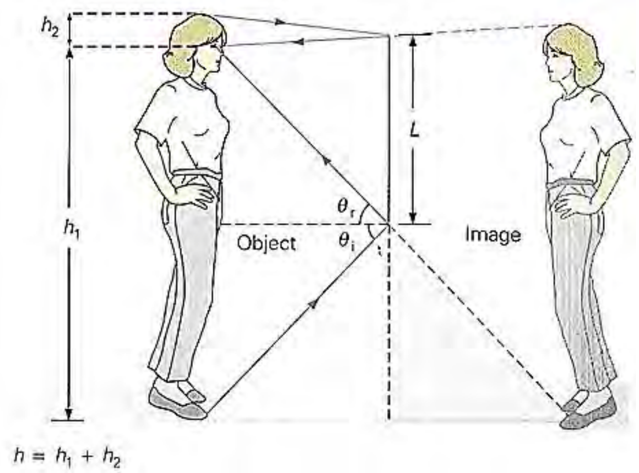
Thus $n = \frac{360°}{0} = \infty$.

Thus, infinite number of images are formed.

Minimum mirror length (All of me)

There is a minimum vertical length of a plane mirror needed for a person to be able to see a complete (head to toe) image

Consider the situation shown below. With a mirror of minimum length, a ray from the top of the head would be reflected at the top of the mirror and the ray from the feet would be reflected at the bottom of the mirror.



The length L of the mirror is then the distance between the dotted lines perpendicular to the mirror at its top and bottom.

However, these lines are also normals for the ray reflections. By the law of reflection, they bisect the angles between incident and reflected rays, i.e. $\theta_i = \theta_r$.

$$L = \frac{h_1}{2} + \frac{h_2}{2} = \frac{h_1 + h_2}{2} = \frac{h}{2}$$

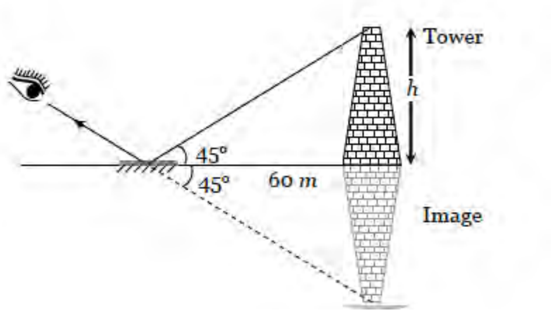
where h is the person's total height

Hence, for a person to see his or her complete image in a plane mirror, the minimum vertical height or vertical length of the mirror must be a half the height of the person.

Example 1

When a plane mirror is placed horizontally on a level ground at a distance of 60 m from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90° at the eye. Calculate the height of the tower

Solution



From the figure, it is clear that

$$\frac{h}{60} = \tan 45°$$

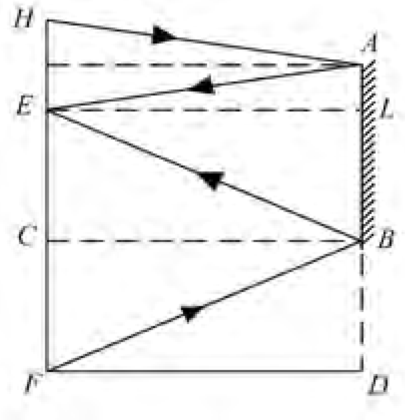
$$h = 60\ m$$

Example 2

A man 2 m tall stands in front of a vertical plane mirror. What is the minimum height of the mirror and how high must its lower edge be above the floor if he is able to see his whole body? Assume his eyes are 10 cm below the top of his head

Solution

The figure shows the conditions of the problem. The man is represented by HF where H is head and F is his feet. Further, E represents his eye.



$HE = 10\ cm = 0.1\ m$; $HF = 2\ m$

Since the man sees his head H, a ray HA from H to the top A of the mirror is reflected to E. Since angle of reflection is equal to angle of incidence, point A lies on the perpendicular bisector of HE

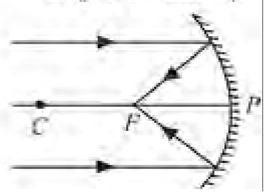
$$AL = \frac{1}{2}HE$$

Here L is a point on the mirror at the same levels as E

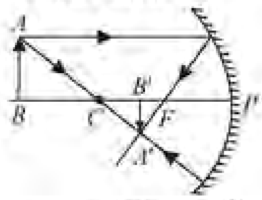
Position, size and nature of the image formed by a spherical mirror

Mirror	Object position	Image position	Image size	Nature
Concave	At infinity, $u = \infty$	At focus i.e. $v = f$	Diminished	Real, inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$ i.e $f < v < 2f$	Diminished	Real, inverted
	At centre of curvature, $u = 2f$	At centre of curvature	Same size as that of object	Real, inverted
	Between centre of curvature and focus i.e. $F < u < 2f$	Away from the centre of curvature i.e. $v > 2f$	magnified	Real, inverted
	At focus, i.e. $u = f$	At infinity i.e. $v = \infty$	Magnified	Real, inverted
	Between pole and focus ($u < f$)	$v > u$	Magnified	Virtual, erect
Convex	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	Diminished	Virtual, erect
	Anywhere between infinity and pole	Between pole and focus	diminished	Virtual, erect

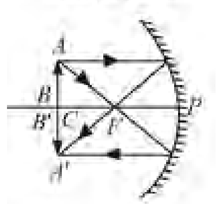
Object at infinity



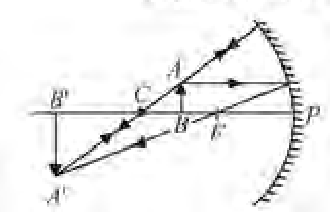
Object beyond C



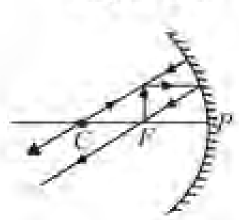
Object at C

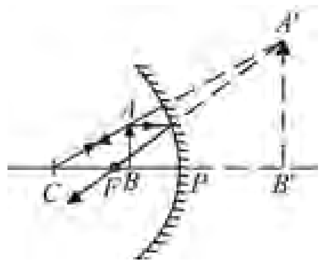


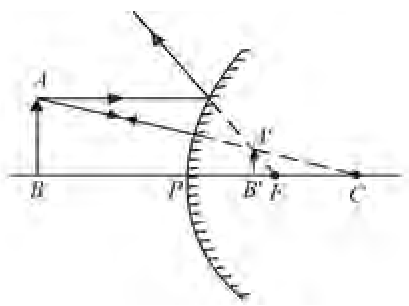
Object between F and C



Object at F







(a) Show that for two plane mirrors placed together at an angle α, a light ray successively reflected from both mirrors is deflected through an angle 2α irrespective of the incident angle, where 2α is the obtuse angle between the incident and reflected rays.

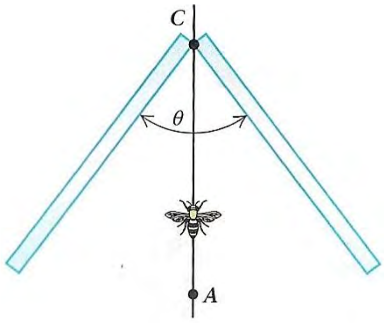
(b) If the angle α between the mirrors is 70° and the angle of incidence θ_{i_1} of light incident on M_1 is 35°, what is the angle of reflection θ_{r_2} from M_2?

(c) If $\alpha = 115°$ and $\theta_{i_1} = 60°$, what is θ_{r_2}?

[Ans: (b) 35° (c) 55°]

9. A dance studio has plane mirrors on opposite walls and multiple images are observed when a person stands between them. If a dancer stands 3.0 m from the mirror on the north wall and 5.0 m from the mirror on the south wall, what are the image distances for the first two images in both mirrors?
[Ans: 3.0 m and 13.0 m behind north mirror; 5 m and 11 m behind south mirror]

10. Two flat mirrors held together at one edge make an angle θ. A bee sits near the intersection of mirrors along the line bisecting the angle θ.



A student at A looking into the mirror sees five bees, the real one plus four images equally spaced around the vertical axis at C. What is the value of angle θ?

[Ans: 72°]

11. An object is placed 10 cm in front of a concave mirror of focal length 15 cm. Find the image position and the magnification.

[Ans: -30 cm, 3]

12. The image of an object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the object position and the magnification.

[Ans: u = 6 cm, $m = {}^2/_3$]

13. An object 3.0 cm tall is placed 20 cm from the front of a concave mirror with a radius of curvature of 30 cm. Where is the image formed and how tall is it?

[Ans: $v = 60\ cm$, $h_i = 9.0\ cm$]

14. An erect image, three times the size of the object, is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?

[Ans: 30 cm]

15. A pill bottle 4.5 cm tall is placed 12 cm from the front of a mirror. An upright image 9.0 cm tall is formed. What kind of mirror is it and what is the radius of curvature?

[Ans: concave, 48 cm]

16. If an object is 15 cm in front of a convex mirror that has a focal length of 30 cm, how far behind the mirror will the image appear to an observer, and how tall will it be?

[Ans: $v = -10\ cm$, $h_i = \left(\frac{2}{3}\right) h_o$]

17. Show that a concave spherical mirror can produce a focused image of an object when certain conditions are observed, and prove the relationship between the object and image distances.

18. A linear object, 10 cm long, lies along the axis of a concave mirror whose radius of curvature is 30 cm, the near end of the object lying 18 cm from the mirror. Find the magnification of the image?

[Ans: 5.8]

19. A convex mirror of radius of curvature 40 cm forma an image which is half the height of the object. Find the object and image positions.

[Ans: 20 cm; 10 cm behind mirror]

20. A concave mirror forms, on a screen, a real image of twice the linear dimensions of the object. Object and screen are then moved until the image is three times the size of the object. If the shift of the screen is 25 cm, determine the shift of the object and the focal length of the mirror

[Ans: 25 cm, $4\frac{1}{6}$ cm]

21. If a concave mirror has a focal length of 10 cm, find the two positions where an object can be placed to give in each case, to give an image twice the height of the object.

[Ans: 15 cm, 5.0 cm]

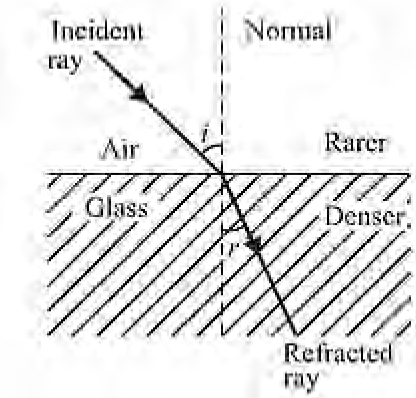
22. A converging lens of focal length 30 cm is 20 cm away from a diverging lens of focal length 5 cm. An object is placed 6 m distant from the former lens (which is the nearer to it) and on the common axis of the system. Determine the position, magnification and nature of the image formed.

[Ans: v = −8.8 cm, $m = \frac{1}{25}$]

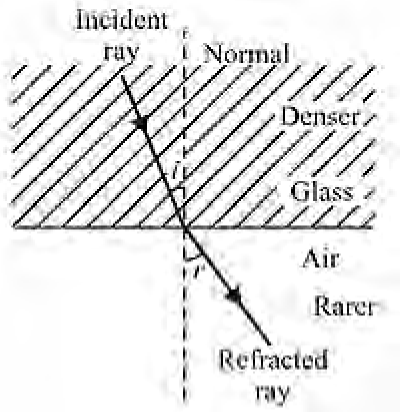
REFRACTION OF LIGHT AT PLANE SURFACES

Refraction is the bending of light rays as they travel from one medium to another of different optical densities. The deviation in the path of light is due to the change in speed of light when it passes from one medium to another.

When a ray of light goes from a rarer medium (where speed of light is more) to a denser medium (where the speed of light is less), it bends towards the normal as shown below. Clearly, in this case, angle of refraction (r) is less than the angle of incidence i.e. $\angle r < \angle i$.



When a ray of light goes from a denser medium to a rarer medium, it bends away from the normal as shown below. Clearly, in this case, $\angle r > \angle i$



Note:

When light goes from one medium to another, the frequency of light does not change. However, the speed of light and wavelength of light change.

The intensity of the refracted ray is less than that of the incident ray. It is because there is partial reflection and absorption of light at the surface

Laws of refraction of light

Law 1:

The incident ray, refracted ray and the normal to the surface separating the two media at the point of incidence, all lie on the same plane.

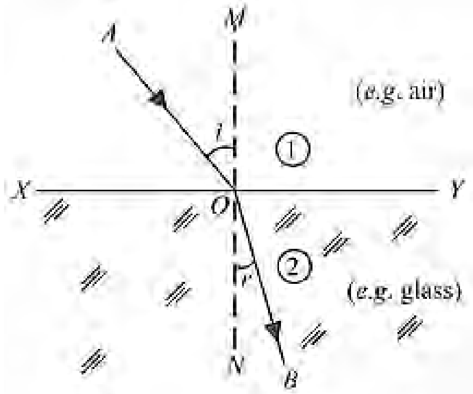
Law 2:

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media. This law is known as **Snell's law**.

$$\frac{\sin i}{\sin r} = n$$

where n is a constant called the refractive index

If medium 1 has refractive index n_1 and medium 2 has refractive index n_2 as shown below



$${}^{1}n_2 = \frac{n_2}{n_1}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

$$\boldsymbol{n \sin i = constant}$$

Example

A ray of light is incident on a water-glass boundary at 30°. Given that the refractive index of water is 1.33 and that of glass is 1.5, calculate the angle of refraction.

Solution

$$n_{i_w} \sin i_w = n_g \sin i_g$$

$$1.33 \sin 30° = 1.5 \sin i_g$$

$$i_g = \sin^{-1}\left(\frac{1.33 \sin 30°}{1.5}\right) = 26°$$

Refractive index of a medium

Refractive index of a medium is the ratio of the velocity of light in air or vacuum to its velocity in a given medium. It is known as **absolute refractive index**.

$$n = \frac{\text{velocity in air or vacuum (c)}}{\text{velocity in a given medium } (v)} \Rightarrow n = \frac{c}{v}$$

To find the speed of light in water, we have

$$n = 1.33, c = 3.0 \times 10^8\ ms^{-1}$$

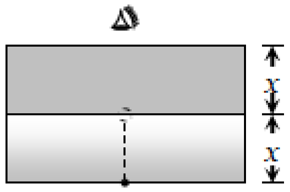
Thus, $v = \frac{c}{n} = \frac{3.0\times10^8}{1.33} = 2.26 \times 10^8\ ms^{-1}$

It follows that speed of light in water is about 75% that of light in a vacuum.

Example 8

A rectangular slab of refractive index n is placed over another slab of refractive index 3, both slabs being identical in dimensions. If a coin is placed below the lower slab, for what value of n will the coin appear to be placed at the interface between the slabs when viewed from the top?

Solution



Apparent depth of coin as seen from top,

$$x = \frac{x}{n_1} + \frac{x}{n_2}$$
$$\frac{1}{n_1} + \frac{1}{n_2} = 1$$
$$\frac{1}{3} + \frac{1}{n} = 1$$
$$3n = 3 + n$$
$$n = 1.5$$

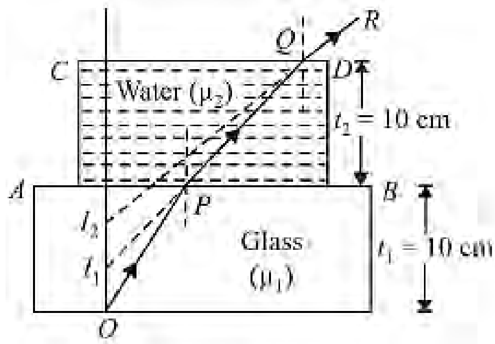
Example 9

A rectangular glass block of thickness $10\,cm$ and refractive index 1.5 is placed over a small coin. A beaker is filled with water of refractive index $4/3$ to a height of $10\,cm$ and is placed over the block.

(a) Find the apparent position of the object when it is viewed normally

(b) If the eye is slowly moved away from the normal, then at a certain position, the object is found to disappear due to total internal reflection. At what surface does it happen and why

Solution



(a) Apparent normal shift is

$$OI_2 = t_1\left(1 - \frac{1}{n_1}\right) + t_2\left(1 - \frac{1}{n_2}\right)$$
$$= 10\left(1 - \frac{1}{1.5}\right) + 10\left(1 - \frac{3}{4}\right)$$
$$= 3.33 + 2.5 = 5.83\,cm$$

Apparent depth of coin

$$= 20 - 5.83 = 14.17\,cm$$

(b) The image of the coin disappears when a ray suffers total internal reflection at either reflecting surfaces AB and CD.

The critical angle c_1 at glass-water interface AB is

$$\sin c_1 = \frac{1}{{}^{w}n_g} = \frac{n_w}{n_g} = \frac{4/3}{1.5} = \frac{8}{9}$$
$$c_1 = \sin^{-1}\left(\frac{8}{9}\right) = 62.7°$$

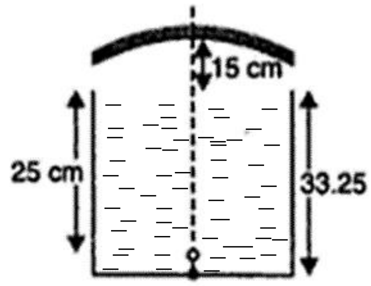
The critical angle c_2 at water-air interface CD is

$$\sin c_2 = \frac{1}{{}^{a}n_w} = \frac{n_a}{n_w} = \frac{1}{4/3} = \frac{3}{4}$$
$$c_2 = \sin^{-1}\left(\frac{3}{4}\right) = 48.59°$$

The object will disappear at the water-air interface since the critical angle for this interface is less than that for the glass-water interface.

Example 10

A tank of height $33.25\,cm$ is completely filled with a liquid of refractive index 1.33. An object is placed at the bottom of the tank on the axis of the concave mirror as shown below.



If the image of the object is formed $25\,cm$ below the surface of the liquid, calculate the focal length of the mirror.

Solution

$$n = \frac{RD}{AD}$$
$$AD = \frac{RD}{n} = \frac{33.25}{1.33} = 25\,cm$$

For concave mirror,

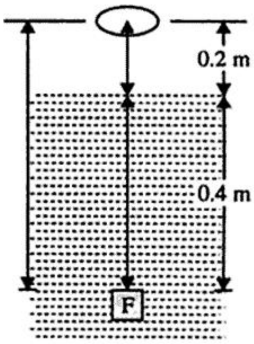
$u = 25 + 15 = 40\,cm$, $v = 40\,cm$, $f = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{40} + \frac{1}{40} = \frac{1}{f}$$
$$\frac{2}{40} = \frac{1}{f}$$
$$f = 20\,cm$$

Example 11

A small fish, $0.4\,m$ below the surface of a lake, is viewed through a simple converging lens of focal length $3\,m$. The lens is kept at $0.2\,m$ above the water surface such that the fish lies on the optical axis of the lens. Find the position of the fish as seen by the observer if the refractive index of water is $\frac{4}{3}$.

Solution



Object distance, $u = \left(0.2 + 0.4 \times \frac{4}{3}\right) m = \frac{11}{15}\ m$

$f = 3\ m$, $v = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{15}{11} + \frac{1}{v} = \frac{1}{3}$$

$$\frac{1}{v} = \frac{1}{3} - \frac{15}{11} = -\frac{34}{33}$$

$$v = -\frac{33}{34}$$

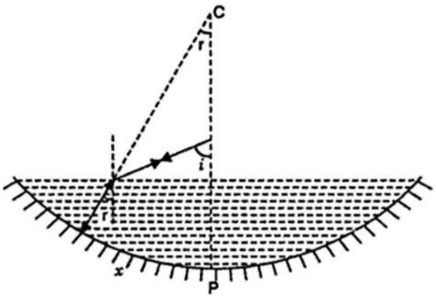
The negative sign indicates that the image is formed on the same side as the fish

Distance in water $= \left(\frac{33}{34} - \frac{1}{5}\right) \times \frac{3}{4} = 0.578\ m$

Example 12

The image of a luminous point object O placed on the principal axis of a concave mirror distant $24\ cm$ coincides with itself. When a few drops of liquid are placed on the mirror, the object has to be placed $15\ cm$ from the mirror to make the image coincide with it. Calculate the refractive index of the liquid.

Solution

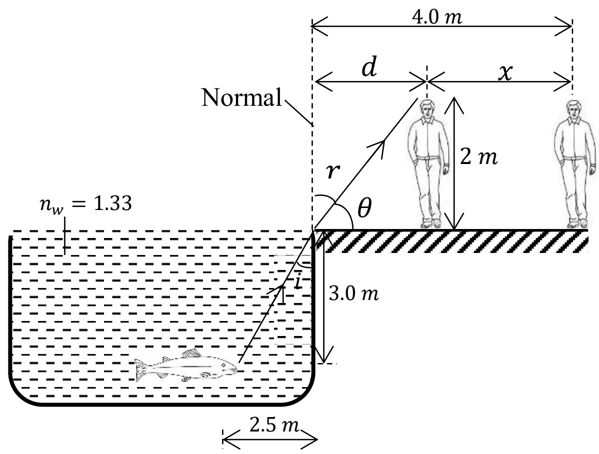


$$n = \frac{\sin i}{\sin r} = \frac{\tan i}{\tan r} = \frac{x/15}{x/24} = \frac{24}{15} = 1.6$$

Example 12

A small fish is 3.0 m below the surface of the pond and 2.5 m from the bank. A man 2.0 m tall stands 4.0 m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. (Refractive index of water = 1.33).

Solution



From the diagram,

$$\tan i = \frac{2.5}{3}$$

$$i = \tan^{-1}\frac{2.5}{3} = 39.8°$$

Applying Snell's law at the edge of the pond,

$$n_w \sin i_w = n_a \sin i_a$$

$$1.33 \sin 39.8° = 1 \times \sin r$$

$$r = 58.4°$$

$$\theta = 90° - 58.4° = 31.6°$$

From the diagram,

$$\tan\theta = \frac{2}{d}$$

$$d = \frac{2}{\tan 31.6°} = 3.2\ m$$

Required distance, $x = 4 - d$

$$x = 4 - 3.2 = 0.8\ m$$

Therefore, the man should move $0.8\ m$ towards the edge of the pond for his movement to become visible to the fish.

TOTAL INTERNAL REFLECTION

This is the complete reflection of light within the dense medium when the angle of incidence exceeds the critical angle of the medium at an interface with a less dense medium.

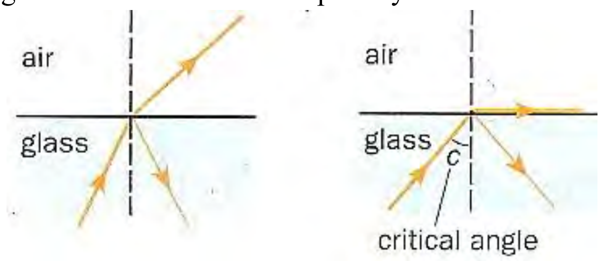
Conditions for total internal reflection

- ✓ The light rays should be traveling from a denser medium to a less dense medium
- ✓ The angle of incidence in the denser medium should be greater than the critical angle.

Critical angle

For small angles of incidence, a ray of light travelling from one medium to another of smaller refractive index, say from glass to air, is refracted away from the normal. A weak internally reflected ray is also formed. Increasing the angle of incidence increases the angle of refraction and at a certain angle of incidence, c, called the **critical angle**, the refracted ray just emerges along the surface of the glass and the angle of refraction is 90°.

At this stage, the internally reflected ray is still weak but just as c is exceeded, it suddenly becomes bright and the refracted ray disappears since all the incident light is reflected inside the optically denser medium.



Applying Snell's law in the form $n_1 \sin i_1 = n_2 \sin i_2$ to the critical ray at a glass-air boundary, we have

$$n_g \sin c = 1 \sin 90°$$

$$n_g = \frac{1}{\sin c}$$

Critical angle is the angle of incidence in a dense medium for which the angle of refraction in the less dense medium is 90°

Example 1

The refractive index of glass is 1.5. Find its critical angle

<u>Solution</u>

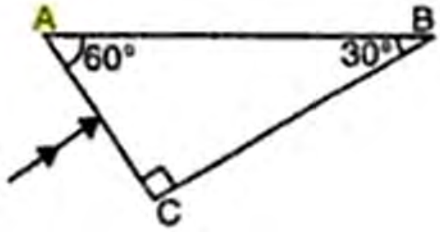
$$\text{From } n = \frac{1}{\sin c}$$

$$1.5 = \frac{1}{\sin c}$$

$$\Rightarrow \sin c = \frac{1}{1.5} = 0.666$$

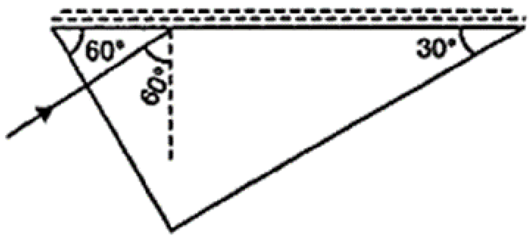
$$c = \sin^{-1}(0.666) = 42°$$

Example 2



ACB is a right-angled prism with other angles as 60° and 30°. The refractive index of the prism is 1.5. AB has a thin layer of liquid on it as shown above. Light falls normally on the face AC. Calculate the maximum refractive index of the liquid for total internal reflection

<u>Solution</u>



Clearly, $c \leq 60°$

So maximum possible value of $c = 60°$

$$_l n_g = \frac{1}{\sin c}$$

$$\frac{n_g}{n_l} = \frac{1}{\sin c}$$

$$n_l = n_g \sin c = 1.5 \sin 60°$$

$$n_l = 1.3$$

Measurement of the critical angle for an air-glass boundary

Two glass slides are cemented together to form an air cell. The air cell is dipped into water in a parallel-sided transparent vessel.

A narrow beam of monochromatic light is directed onto the air cell and is viewed from the opposite side.

The air cell is then turned in one direction until light just disappears. This position is noted.

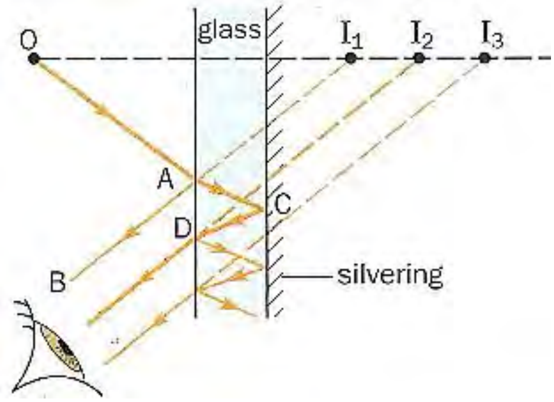
The air cell is then turned in the opposite direction until light disappears again. This position is noted again.

The angle θ between the two positions is noted.

The critical angle will be given by

$$c = \frac{\theta}{2}$$

Multiple images in thick mirrors



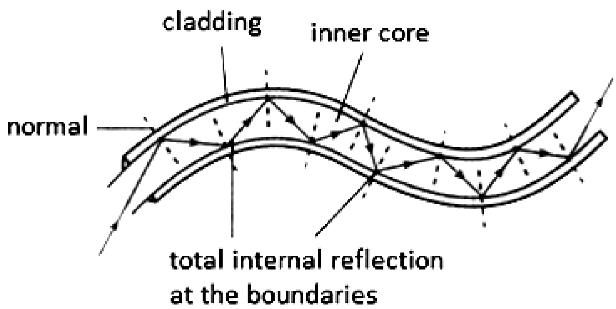
Several images are seen when an object is viewed obliquely in a thick glass mirror with silvering on the back surface.

I_1 is a faint image of the object O formed by the weak reflected ray AB from the front surface of the mirror.

I_2, the main image, is bright and is due to the refracted ray AC being reflected at the back silvered surface and again reflected at the front surface.

The net effect of these multiple reflections and refractions is to reduce the sharpness of the primary image I_2.

Optical fibres



An optical cable is made of a transparent material coated with another of less optical activity.

Light entering the pipe strikes the boundary of the media at an angle of incidence greater than the critical angle. Total internal reflection thus occurs.

This takes place repeatedly in the pipe until the light beam emerges from the pipe

Uses of optical fibres

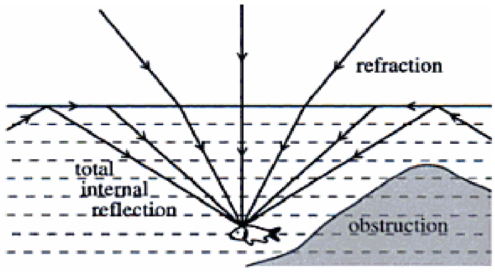
1. Communication

Optical fibres are used to carry telephone, television and computer signals as pulses of light over long distances.

2. Optical fibres are used as **endoscopes** or **fibre scopes**. An endoscope is a device used to examine internal organs of a human body by physicians.

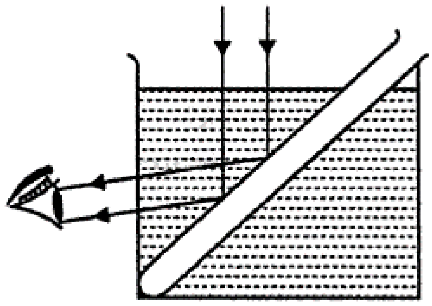
Fish's eye view

The fish is able to see objects above the water and inside the water. Light from above the water is refracted into the water.



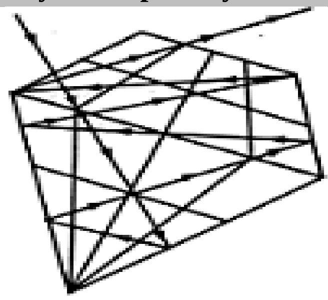
Total internal reflection and refraction enables the fish to see objects on the other side of an obstruction.

Glittering of a test tube



Consider an empty glass test tube held obliquely in a beaker containing water. Light falling on the test tube will suffer total internal reflection as shown above. The test tube will glitter as though it is full of mercury.

Sparkling of crystals especially diamond



Diamonds are cut into sharp angles. This gives rise to a number of refracting surfaces. Light rays entering diamond suffer total internal reflection. Light rays strike the diamond-air interface at an angle greater than the critical angle and thus the rays get trapped. These trapped light rays make the diamond to sparkle.

Totally reflecting prisms

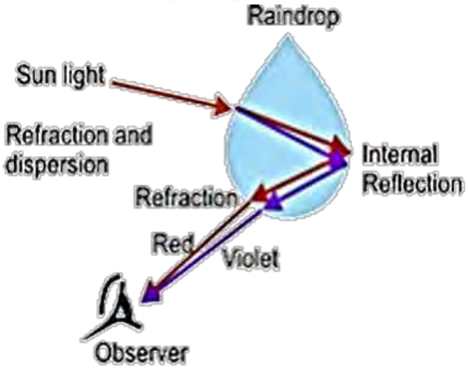
The disadvantages of multiple reflections in plane mirrors silvered on the back surface can be overcome by using right-angled isosceles prisms as reflectors.

A ray OA incident normally on face PQ suffers total internal reflection at face PR since the angle of

The spectrum is a series of monochromatic images of the slit and the narrower the slit, the purer the spectrum.

Rainbow

This is a beautiful natural spectrum appearing in the sky after a rain shower. It is due to the dispersion of sunlight by water droplets present in the atmosphere.

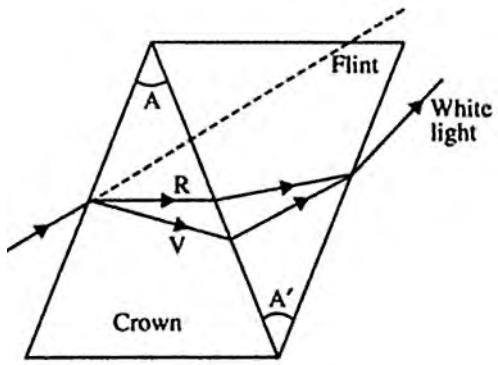


The water droplets act like small prisms. They refract and disperse the sunlight then reflect it internally and finally refract it again when it comes out of the rain drops. Due to the dispersion of sunlight and internal reflection by the water droplets, we can see the rainbow colours.

Combination of prisms

Although the dispersion of white light is useful when we want to look at the spectrum of the light, it is a real problem in optical instruments such as telescopes. The lenses in these instruments disperse different colours by different amounts and so bring the different colours to different foci. The images formed are coloured and blurred. It is therefore necessary to deviate the light without dispersing it, and prisms and lenses that do this are called **achromatic**.

Deviation without dispersion (achromatic combination)



For the combination of prisms shown above, if there is to be no angular dispersion, then

$$(d_v - d_r) + (d'_v - d'_r) = 0$$

$$(n_v - n_r)A + (n'_v - n'_r)A' = 0$$

$$(n_v - n_r)A = -(n'_v - n'_r)A'$$

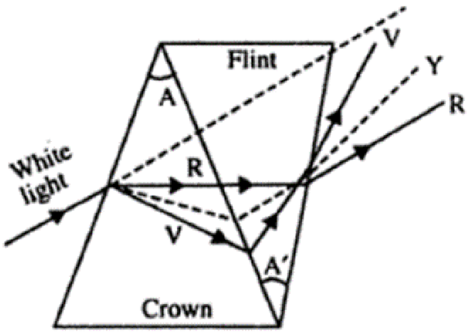
$$\frac{A'}{A} = -\frac{n_v - n_r}{n'_v - n'_r}$$

This is the condition for achromatism i.e. the condition for no dispersion

Dispersion without deviation

Consider a crown glass prism combined with a flint glass prism in opposition as shown below. Let A and A' be the angles of crown glass prism and flint glass respectively. Let n_v, n and n_r be the refractive indices of crown glass for violet, yellow and red colours respectively. Let n'_v , n' and n'_r be the corresponding values for the flint glass prism.

Let d and d' be the deviations suffered by yellow light through crown glass prism and flint glass prism respectively



If the combination does not produce any deviation, then

$$d + d' = 0$$

$$(n - 1)A + (n' - 1)A' = 0$$

$$(n' - 1)A' = -(n - 1)A$$

$$\frac{A'}{A} = -\frac{n - 1}{n' - 1}$$

This is the condition for no deviation. The negative sign indicates that the two prisms are placed in opposition.

Since $(n' - 1) > (n - 1)$ therefore the angle of crown glass prism should be greater than the angle for the flint glass prism.

Net angular dispersion $= (d_v - d_r) + (d'_v - d'_r)$

$$= (n_v - n_r)A + (n'_v - n'_r)A'$$

$$= A\left[(n_v - n_r) + (n'_v - n'_r)\frac{A'}{A}\right]$$

$$= A[\,(n_v - n_r) - \left(\frac{n - 1}{n' - 1}\right)(n'_v - n'_r)$$

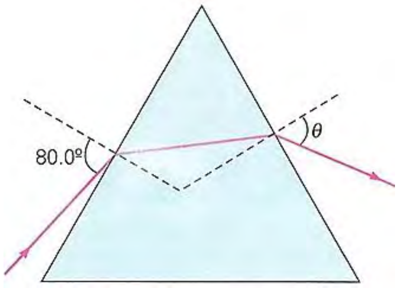
$$= (n - 1)A\left[\frac{n_v - n_r}{n - 1} - \frac{n'_v - n'_r}{n' - 1}\right]$$

$$= d(\omega - \omega')$$

for red light is 1.4925, what is the refractive index for blue light?

[Ans: 1.4980]

21. A beam of red light is incident on an equilateral prism as shown below.



(a) If the refractive index for red light is 1.400, at what angle θ does the beam emerge from the other face of the prism?
(b) Suppose the incident beam was white light, what would be the angular separation of the red and blue components in the emergent beam if the refractive index for blue light is 1.403?

[Ans: (a) 21.7° (b) 0.22°]

22. Light travels from a material whose refractive index is $n_1 = 2.0$ onto another material whose refractive index is $n_2 = 5/3$. The opposite parallel surface of the second material is exposed to the air.
(a) Find the critical angle at which light will be reflected at the interface of the materials
(b) Find the angle at which light will pass through the interface and then be reflected at the opposite surface of the second material

[Ans: (a) 56° (b) 30°]

23. Light passes from medium A into medium B at an angle of incidence of 30°. The refractive index of A is 1.5 times that of B.
(a) What is the angle of refraction?
(b) What is the ratio of the speed of light in B to the speed of light in A?
(c) What is the ratio of the frequency of the light in B to the frequency of light in A?
(d) At what angle of incidence would the light be internally reflected?

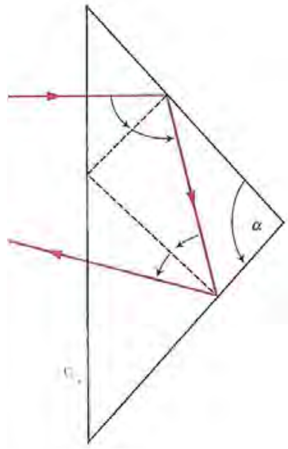
[Ans: (a) 49° (b) 1.5 (c) 1 (d) 42°]

24. By how much does an ink dot appears to be raised when covered by a glass plate 4.5 cm thick if the velocity of light in glass is $2 \times 10^{10}\ cm\ s^{-1}$ and in air $3 \times 10^{10}\ cm\ s^{-1}$? [Ans: 1.5 cm]

25. A boy is looking vertically downwards in a tank of water and his eyes are 1 m above the water surface. He sees a fish at a depth 3 m from the water surface. If $_a n_w = 4/3$, calculate the real depth of the fish from the water surface.

[Ans: 4 m]

26. Light is incident normally upon the long face of a symmetric prism of refractive index 1.55 shown below.



What is the range of values of α that will permit total internal reflection from both rear faces of glass?

[Ans: $86.8° \leq \alpha \leq 99.6°$]

27. A travelling microscope is focused on a scratch mark on the bottom of a beaker. On pouring a liquid into the beaker to a depth of 6.0 cm it is found that the microscope has to be raised a vertical distance of 1.52 cm for the scratch mark to be again in clear focus. Calculate the refractive index of the liquid and prove any formula used in the calculation.

[Ans: 1.34]

28. Describe and explain a total reflection method for determining the refractive index of a liquid.
If the refractive indices from air to glass and from air to water are 1.50 and 1.33 respectively, calculate a value for the critical angle for a water-glass surface.

[Ans: 62.73°]

29. A few drops of a liquid are squashed into a thin film below the base of a glass cube one vertical face of which is illuminated with sodium light. An eye looking into the cube through the opposite vertical face notices that the light ceases to be internally reflected from the base when the position of the eye is such that the emergent ray makes an angle with the normal greater than 48°. If the refractive index of the glass is 1.52, calculate the refractive index of the liquid for sodium light.

[Ans: 1.33]

30. Two glass prisms are placed together as shown below.

Images formed by a diverging lens

For any distance of the object from the diverging lens the image formed is upright, virtual and smaller than the object (diminished).

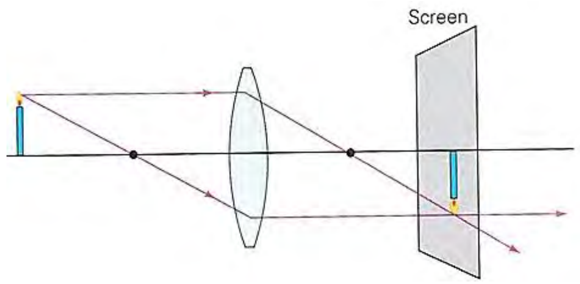
Sign convention

(i) If the object is real, then the value of u is positive. If the image is real, then the value of v is negative. The reverse of the above is true.

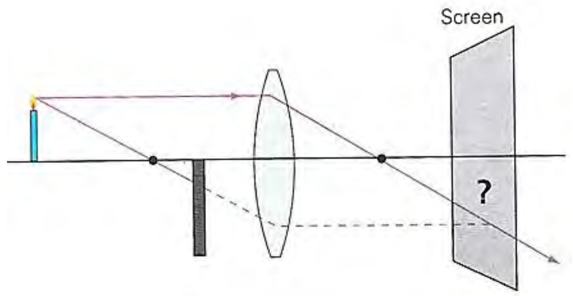
(ii) The focal length of a converging lens is positive while the focal length of a diverging lens is negative.

Blocking half of a lens

A converging lens forms an image on the screen as shown in the diagram below.



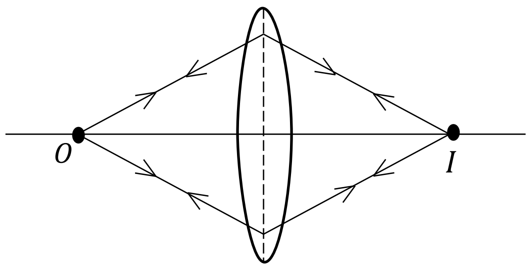
Then the lower half of the lens is blocked off.



At first thought, one might imagine that blocking off half of the lens would eliminate half of the image. However, rays from every point on the object pass through all parts of the lens. Thus, the upper half of the lens can form a total image (as could the lower half).

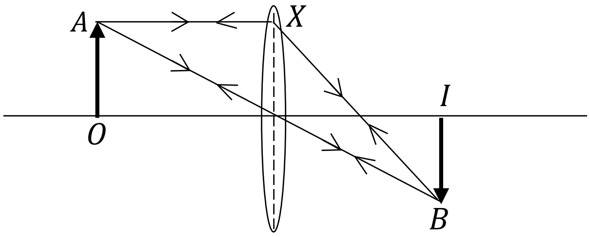
Blocking off half the lens would cut the amount of light focused at the image plane in half, so the resulting image would be less bright.

Conjugate foci (points) of a lens



Conjugate foci are points such that if an object is placed at one of them, an image will be formed at the other. O and I are conjugate points.

Principle of reversibility of light as applied to a lens



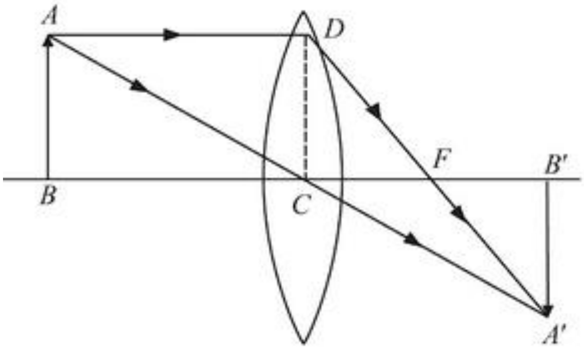
Consider an object A in front of a convex lens. Rays of light from it are refracted to meet at B. B is the image of A. When the object is shifted to B, light from it would follow the path BXA and BA forming a real image at A. This is reversibility of light.

THE LENS FORMULA

Convex lens

(a) Finite object

Consider a real, inverted image formed with respect to the object on the other side of the lens as shown below



Triangles $A'B'C$ and ABC are similar

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \ldots\ldots (i)$$

Also, triangles $A'B'F$ and CDF are similar

$$\frac{A'B'}{CD} = \frac{FB'}{CF}$$

Since $CD = AB$, the above equation becomes

$$\frac{A'B'}{AB} = \frac{FB'}{CF} \ldots\ldots (ii)$$

From (i) and (ii);

$$\frac{CB'}{CB} = \frac{FB'}{CF} = \frac{CB' - CF}{CF}$$

$CB = u$, $CB' = v$ and $CF = f$

$$\frac{v}{u} = \frac{v - f}{f}$$

$$fv = uv - uf$$

$$fv + fu = uv$$

Solution

Using $\frac{1}{m} = \frac{u}{f} - 1$

$$m = 4, f = 20, u = ?$$

$$\frac{1}{4} = \frac{u}{20} - 1$$

$$u = 25\ cm$$

Example 3

An object 5 cm high is held 25 cm away from a converging lens of focal length 10 cm. Find the position and size of the image.

Solution

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$u = +25\ cm, f = +10\ cm$$

$$\frac{1}{25} + \frac{1}{v} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{25} = \frac{3}{50}$$

$$v = \frac{50}{3} = 16.67\ cm$$

Magnification,

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

$$h_i = \frac{v}{u} \times h_o = \frac{16.67}{25} \times 5 = 3.33$$

Example 4

An object is placed 10 cm in front of a lens. The lens forms a real image three times magnified. Where is the image formed and what is the focal length of the lens?

Solution

Since the image is real, it is formed on the opposite side of the lens as the object.

$$m = \frac{v}{u}$$

$$3 = \frac{v}{10}$$

$$v = 30\ cm$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$u = +10\ cm, v = +30\ cm$$

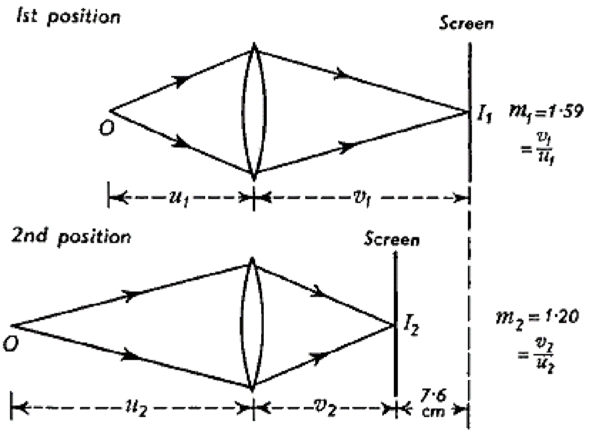
$$\frac{1}{f} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$$f = \frac{30}{4} = 7.5\ cm$$

Example 6

A small linear object is placed perpendicular to the axis of a thin converging lens and a sharp image is produced on a screen. The magnification is 1.59. The screen is moved a distance 7.6 cm towards the lens and the position of the object is adjusted so that a sharp image is again produced on the screen. The magnification is now 1.20. Determine the focal length of the lens.

Solution



From the lens formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

We have,

$$m = \frac{v}{u} = \frac{v}{f} - 1$$

Hence for first position,

$$1.59 = \frac{v_1}{f} - 1$$

$$v_1 = 2.59f$$

and for 2nd position,

$$1.20 = \frac{v_2}{f} - 1$$

$$v_2 = 2.20\ f$$

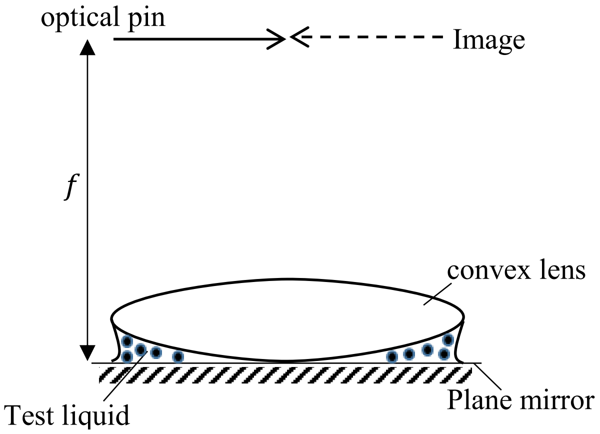
But $\quad v_2 = v_1 - 7.6$

$$2.20f = 2.59f - 7.6$$

$$f = \frac{7.6}{0.39} = 19.5\ cm$$

Note that the sign conventions still work in this case i.e. if f = negative, then the combination is a diverging one (concave) and if f = positive, the combination is a converging one (convex)

Measurement of the refractive index of a small quantity of liquid



A plane mirror is placed horizontally on a bench/table with its reflecting surface upwards.

A small quantity of liquid whose refractive index is required is poured on the mirror.

A biconvex lens of known focal length f_1 and known radius of curvature r is placed on the liquid such that it is sandwiched between the plane mirror and the lens.

An optical pin is then clamped horizontally above the lens such that its tip lies along the principal axis of the lens.

While observing from above, the pin is moved up and down until it coincides with its image by no parallax.

The distance f of the pin above the plane mirror is measured.

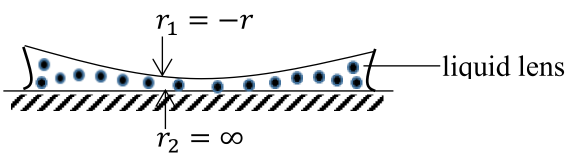
The focal length of f_2 of the liquid is then calculated from

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The refractive index n of the liquid is then calculated from

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{-r}\right)$$

Explanation



The convex and the liquid lens form a lens liquid combination and when the object coincides with its image, the rays from the object strike the plane mirror normally after refraction in the combination.

The rays from the image at the principal focus of the lens combination. Hence f is the focal length of the combined lens where $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

The liquid lens is a plano concave type of lens whose focal length is f_2.

The spherical surface of the liquid lens has the radius of curvature numerically equal to that of the convex lens surface in contact with it.

Hence from

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{-r_1} + \frac{1}{\infty}\right)$$

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{-r_1}\right)$$

Example 1

A converging lens of 5.0 cm focal length is in contact with a diverging lens of focal length 10 cm. Find the combined focal length.

<u>Solution</u>

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$f_1 = +5.0\ cm,\ f_2 = -10\ cm$$

$$\frac{1}{f} = \frac{1}{+5} + \frac{1}{-10} = \frac{1}{5} - \frac{1}{10}$$

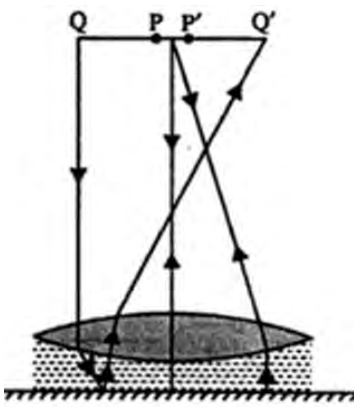
$$\frac{1}{f} = \frac{1}{10}$$

$$f = +10\ cm$$

This is a converging combination

Example 2

The figure below shows an equiconvex lens of refractive index 1.50 in contact with a liquid layer on top of a plane mirror.



A small needle with its tip on the principal axis is moved along the axis until its inverted image is found

This means that $L_1I = 30\ cm$

Now, the image I' formed by the convex lens L_1 will serve as the object for the concave lens L_2.

According to the ray diagram, the image formed by the concave lens is at infinity and this means that the virtual object I' is at its focus.

Focal length of concave lens is given by

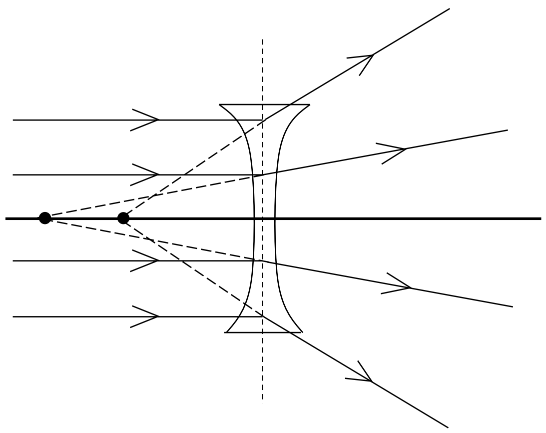
$$L_2I' = L_1I' - L_1L_2$$
$$= 30 - 10 = 20\ cm$$

Chromatic and spherical aberration in lenses

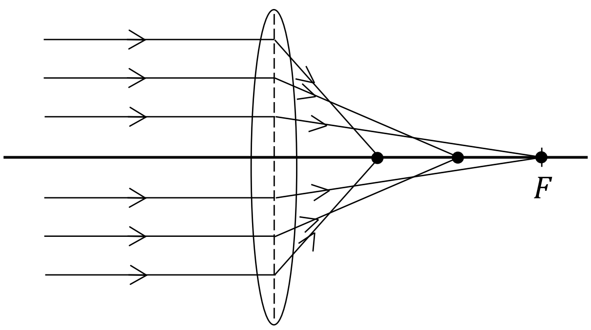
Spherical aberration

Spherical aberration is the formation of blurred but non coloured images when a wide beam of light is incident on a lens. This is because the lens has different foci for marginal rays and paraxial rays.

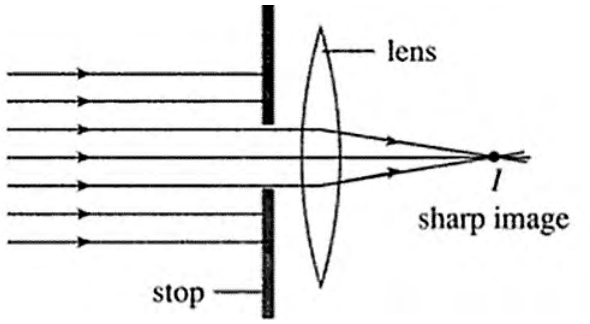
In the case of a concave lens, when a wide beam of light falls on the lens, marginal rays are deviated more than the paraxial rays. They therefore appear to come from different points on the principal axis. The image formed is distorted.



In case of a convex lens, when a wide beam of light falls on the lens, the central rays (paraxial rays) are refracted and converge far away from the lens. The rays which are far from the principal axis (marginal rays) are refracted and converge near the lens. The image formed is a circular blur due to a series of images of the same object.



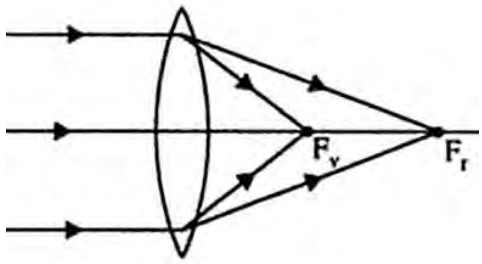
Spherical aberration is minimized by using an opaque disc with a central hole covering the lens or using a stop to reduce the aperture of the lens to a small central portion.

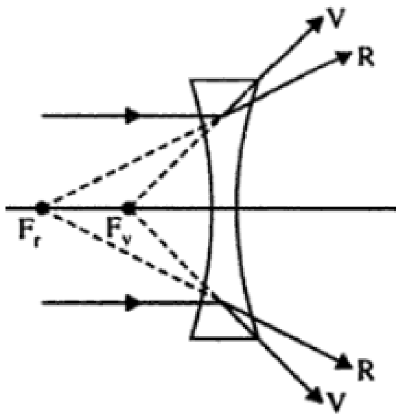


This reduces the brightness of the image. If the aperture is sufficiently small, diffraction effects can reduce the observable details.

Chromatic aberration

Chromatic aberration is the formation of blurred images with coloured edges when white light is incident on a lens due to the fact that the lens has different focal length for different constituent colours.





We know that $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Clearly, the focal length of the lens depends upon the refractive index of the material of the lens, which further depends upon the wavelength of light. In the visible region, the focal length is maximum for red and minimum for violet. Thus, if we use white light, each colour forms a separate image of the object.

The violet rays are deviated more than red rays. So, the image formed by violet rays is closer to the lens than the image formed by the red rays.

If light is incident on the lens from left to right, the violet image is to the left of the red image in the case of convex lens. However, it is to the right of the red image in the case of concave lens.

In the case of convex lens, the chromatic aberration is positive and negative in the case of concave lens. Thus, a suitable combination of convex and concave lenses can produce zero chromatic aberration. Such combination is called **achromatic combination**.

The defect of chromatic aberration can thus be reduced by using an **achromatic doublet**.

This consists of a converging lens of crown glass combined with a diverging lens of flint glass. One surface of each lens has the same radius of curvature to allow them be cemented together with Canada balsam and thereby reduce light loss by reflection.

The flint glass of the diverging lens produces the same dispersion as the crown glass of the converging lens but in the opposite direction and with less deviation of light, so that overall combination is converging.

Properties of lenses in achromatic combination

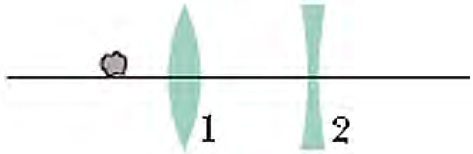
- One lens should be concave and the other convex
- The lenses should be of different materials
- The dispersion caused by the convex lens should be completely cancelled by the concave lens. The dispersive power of materials should be $\frac{f_1}{f_2} = \frac{-\omega_2}{\omega_1}$
- The radii of curvature of the concave and convex lens should be numerically equal.

Applications of lenses

- Lenses are used in a number of optical instruments such as hand lens, microscope, telescope, lens camera and projection lanterns
- They are used in spectacles to correct eye defects

Self-Evaluation exercise

1. What is meant by principal axis and principal focus as applied to a converging lens?
2. Describe how the focal length of a converging lens can be obtained using a plane mirror and the non-parallax method.
3. Draw a ray diagram to show the formation of an image by a diverging lens.
4. Describe an experiment to determine the focal length of a diverging lens.
5. What is meant by the terms focal length, conjugate foci and power of a lens.
6. Explain the term chromatic aberration as applied to a converging lens.
7. When an object is placed 50 cm from a thin converging lens, a real image is formed at a distance of $33\frac{1}{3}$ cm from the lens. When a thin diverging lens is placed in contact with the first lens, the image distance increases to 50 cm. What is the focal length of the diverging lens?
   [Ans: 100 cm]
8. A slide projector is required to produce a real image 684 mm wide from an object 36 mm wide. If the distance of the object from the screen is to be 2000 mm, calculate
   (i) the distance of the lens from the object,
   (ii) the focal length of the lens required.
   [Ans: (c) (i) 100 mm (ii) 95 mm]
9. An object is placed 0.15 m in front of a converging lens of focal length 0.10 m. It is required to produce an image on a screen 0.4 m from the lens on the opposite side to the object. This is to be achieved by placing a second lens mid-way between the first lens and the screen. What type and of what focal length should this lens be? Sketch a diagram showing two rays from a non-axial point on the object to the final image.
   [Ans: (c) 0.2 m (Diverging)]
10. An illuminated object is placed 48.0 cm from a screen. A lens is to be placed between the object and screen in order to produce a real image on the screen.
    (a) If the image is to be the same size as the object, what kind of the lens and what focal length would be needed?
    (b) If the image is to be twice the size of the object, what kind of the lens and what focal length would be needed?
    [Ans: (a) 12.0 cm (b) 10.7 cm]



(a) What is the distance between lens 2 and the image it produces of the sand grain?
(b) What are the characteristics of the image formed?

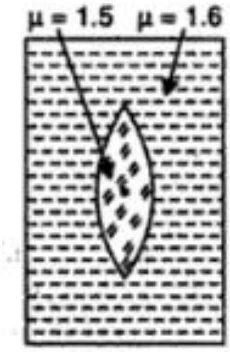
[Ans: (a) 3.33 cm (b) virtual, upright]

23. A luminous object and a screen are fixed at distance D apart.
(a) Show that a converging lens of focal length f, placed between object and screen, will form a real image on the screen for two lens positions that are separated by a distance $d = \sqrt{D(D-4f)}$
(b) Show that $\left(\frac{D-d}{D+d}\right)^2$ gives the ratio of the two image sizes for these two positions of the lens.

24. A lens is made of glass having an index of refraction of 1.5. One side of the lens is flat, and the other one is convex with a radius of curvature 20 cm.
(a) Find the focal length of the lens
(b) If an object is placed 40 cm in front of the lens, where is the image?

[Ans: (a) $+40$ cm (b) ∞]

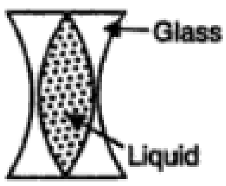
25. Shown in the diagram below is a convergent lens placed inside a cell filled with a liquid.



26. The lens has a focal length of $+20$ cm when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, calculate the focal length of the system.

[Ans: -100 cm]

27. A liquid of refractive index 1.6 is contained in the cavity of a glass specimen of refractive index 1.5 as shown below.



If each of the curved surfaces has a radius of curvature of 0.20 m, how does the arrangement behave?

[Ans: diverging lens of focal length 0.25 m]

28. Show that, in general, there are two positions which a convex lens can occupy when producing a real image on a screen of an illuminated object a fixed distance away.
If the ratio of the image sizes so produced is 9, and the distance between the two positions of the lens is 20 cm, calculate the focal length of the lens and the distance between the object and screen.

[Ans: 7.5 cm, 40 cm]

29. Describe the 'displacement method' of finding the focal length of a converging lens. What are the special merits of this method?
A converging lens is mounted in an inaccessible position inside a tube which is supported with its axis horizontal between an illuminated aperture and a screen 1 m away. It is found that clearly focused images of the aperture are formed on the screen when the end of the tube nearer the screen is 53 cm from it, and again when 33 cm from it. Find the focal length of the lens and its position in the tube.
[Ans: 24 cm; lens is 7 cm from the end of the tube nearer the screen]

30. An eye positioned 15 cm from a convex lens sees an image of itself by parallel rays when looking through the lens towards a plane mirror placed 20 cm behind the lens. Calculate possible focal lengths for the lens and give the corresponding ray diagrams.

[Ans: 10 cm or 60 cm]

31. Prove the formula $xx' = f^2$ for a thin lens where x and x' are the distances of the object and image from their corresponding focal points, and f is the focal length of the lens.
An illuminated linear object, 5 cm long, is positioned in front of an inaccessible lens, a real image of the object being received on a screen placed behind the lens. A movement of the object necessitates a displacement of the screen 10 cm further away from the lens, and it is observed that the image size has increased by 2 cm. Calculate the focal length of the lens and the original position of the object.

[Ans: 25 cm; $45\frac{1}{3}$ cm]

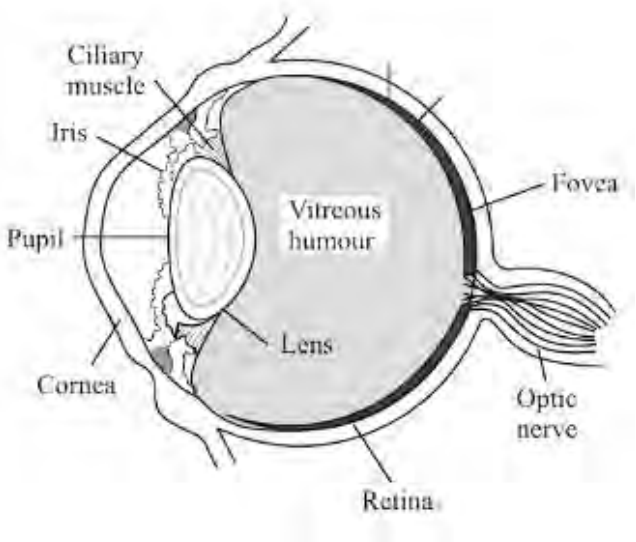
32. A convex lens of focal length 15 cm is placed on a plane mirror at the bottom of a gas-jar into which

OPTICAL INSTRUMENTS

The human eye is a remarkable optical instrument. However, it has its own short comings. For example, the human eye cannot see distinctly very tiny objects or very far off objects. Many optical instruments are designed to compensate for the shortcomings of the human eye. The essential components of optical instruments are combinations of lenses but sometimes they include mirrors or prisms. The images are formed by refraction or reflection and may look bigger or smaller, closer or farther away, erect or inverted with respect to the object.

HUMAN EYE

Human eye is a remarkable optical instrument which acts as a natural camera. The human eye is about 2.5 cm in diameter and its nearly spherical shape is maintained by the pressure of the fluid with in it. The figure below shows the essential parts.



Cornea: The front of the eye is covered by a transparent membrane called cornea. The light enters the eye through the cornea

Iris: It is a circular diaphragm behind the cornea and has a central hole. The hole is called pupil. Through muscular action, the iris can change the area of the pupil so as to control the amount of light entering the eye. The pupil becomes small in bright light and it becomes wide in dim light.

Crystalline lens: Behind the Iris is a double convex lens made of transparent and flexible tissues. The eye lens is held in place by **ciliary muscles**. The shape (curvature) and hence the focal length of the eye lens is changed by the ciliary muscles. When these muscles contract, the focal length of the eye lens decreases because curvature of the lens increases. However, when the ciliary muscles contract, the focal length of the eye lens decreases because curvature of the lens increases. However, when the ciliary muscles expand, the focal length of the eye lens increases.

Retina: It is a light-sensitive membrane at the back-interior wall of the eye-ball. The retina acts as a screen and the image of the external object is formed on it.

Optic nerve: The nerve which conveys the light signals from the retina to the brain is called optic nerve. The image of the object formed on the retina is real and inverted w.r.t the object. However, our brain interprets it as an erect image w.r.t object.

Note:

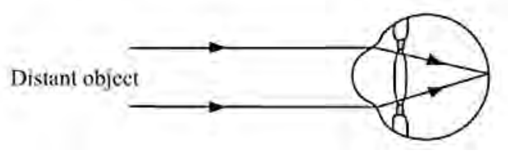
- When the eye is fully relaxed (i.e. there is no tension in the ciliary muscles), the focal length of the eye lens is about 2.5 cm (maximum value). However, when the ciliary muscles contract, the focal length of the eye lens is less than this value.
- The ability of the eye lens to change its focal length through ciliary muscles enables us to see clearly the objects at different distances from the eye.
- If your eyes become tired after reading for many hours, it is because of the tension in the ciliary muscles.

Accommodation of the eye

The eye lens has the ability to change its focal length slightly through ciliary muscles. This permits us to focus the sharp image of the object at different distances from the eye at the retina. This property of the eye is called accommodation of the eye.

The property of the eye to adjust automatically the focal length of the eye lens through ciliary muscles so as to form sharp images of objects at different distances on the retina is called accommodation of the eye.

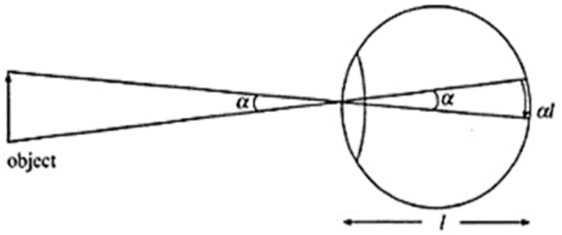
When the eye looks at distant objects, the ciliary muscles attached to the lens of the eye relax and the lens becomes less curved as shown below. When less curved, the focal length increases and an image is formed at the retina.



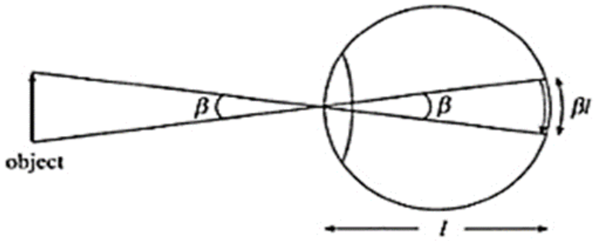
Visual angle

This is the angle subtended at the eye by the object.

The apparent size of an object as seen by the eye is determined by the length of the image formed on the retina. If l is the length of the eyeball, and the visual angle is α in radians, then the length of the image is αl.



For a distant object, the angle α is small and the object seems small. The same object when placed nearer to the eye has a larger visual angle β and the image on the retina is bigger hence the object appears bigger.



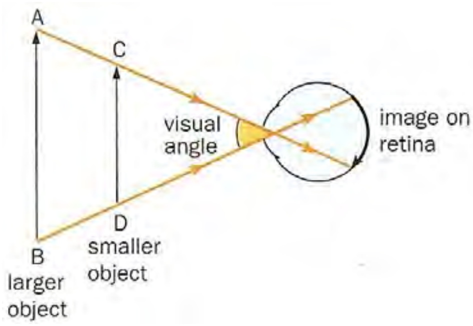
Note:

The visual angle is limited by the least distance of vision, if the object is nearer than the near point, the image becomes blurred.

Optical instruments such as the magnifying glass magnify by forming an image which subtends a greater visual angle.

Magnifying power of an optical instrument (angular magnification)

The apparent size of an object depends on the size of its image on the retina and this depends not so much on the actual size of the object on the angle it subtends at the eye (visual angle).



Thus, AB is larger than CD, but because it subtends the same visual angle as CD, it appears to be of equal size.

$$\boldsymbol{M = \frac{\beta}{\alpha}}$$

where

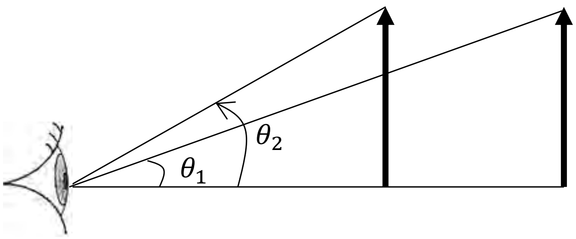
β = angle subtended at the eye by the image formed when using instrument

α = angle subtended at the unaided eye by the object at the distance of most distinct vision.

The magnifying power of an optical instrument is defined as the ratio of the visual angle subtended by the image formed by the instrument at eye to the visual angle subtended by the object at the unaided eye.

Appearance of a furthest pole in line with others of the same height

The furthest vertical pole which is line with others of the same height appears shorter. Why is this so?



The visual angle θ_1 subtended at the eye by the furthest pole is smaller than the visual angle θ_2 subtended by the nearest pole of the same height.

Since the apparent size/height is proportional to the visual angle, the furthest pole looks shorter.

MICROSCOPES

A microscope is an optical instrument which forms a magnified image of very small objects held close to the eye.

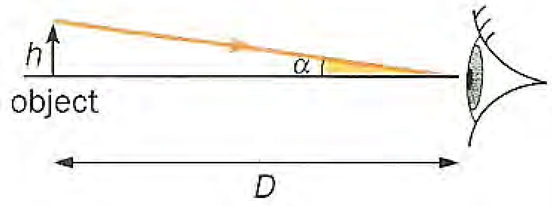
Very small objects subtend small visual angles at the naked eye due to their smallness. We can increase the visual angle by bringing these objects closer to the eye. But the object cannot be brought closer than D otherwise the image will be blurred. However, if we place a suitable converging lens close to the eye, we can move the object closer than D.The virtual image of the object formed by the lens is far from the eye and thus can be seen comfortably. This virtual image subtends a large visual angle at the eye and hence the tiny objects appear large. A microscope is based on this principle. We shall discuss two microscopes i.e. the simple microscope and the compound microscope.

Simple microscope/ Magnifying glass

A simple microscope consists of a convex lens of small focal length and is used to see magnified images of tiny objects placed close to the eye.

$$\beta = \frac{h_2}{D}$$

When the object is placed at the near point i.e.



$$\alpha = \frac{h}{D}$$

$$M = \frac{\beta}{\alpha} = \frac{h_2/D}{h/D} = \frac{h_2}{h_1}$$

$$M = \frac{h_2}{h_1} \times \frac{h_1}{h}$$

$\frac{h_2}{h_1}$ is the linear magnification, m_e produced by the eyepiece and $\frac{h_1}{h}$ is the linear magnification, m_o due to the objective

$$\boldsymbol{M = m_e \times m_o}$$

Also, from $m = \frac{v}{f} - 1$

$$m_o = \frac{v_o}{f_o} - 1$$

$$m_e = \frac{-D}{f_e} - 1$$

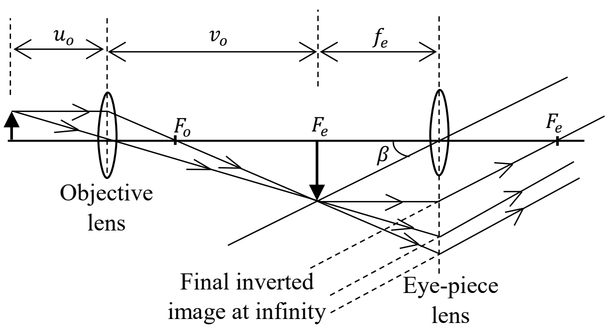
$$M = \left(\frac{-D}{f_e} - 1\right)\left(\frac{v_o}{f_o} - 1\right)$$

$$\boldsymbol{M = -\left(\frac{D}{f_e} + 1\right)\left(\frac{v_o}{f_o} - 1\right)}$$

It follows that M will be large if f_o and f_e are small

Image at infinity (relaxed eye)

This arrangement produces a smaller angular magnification than when the image is at near point, but has the advantage that the eye is relaxed (unaccommodated). The separation of the lens is such that the intermediate image is formed at the focal point of the eyepiece lens.

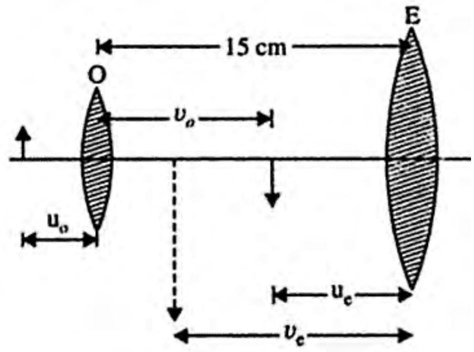


Example 1

A compound microscope consists of an objective lens of focal length 2.0 cm and an eye-piece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image

(a) at the least distance of distinct vision (25 cm)?
(b) at infinity?

<u>Solution</u>



(a) $f_o = 2\ cm$, $f_e = 6.25\ cm$, $u_o = ?$

For eyepiece, $v_e = -25\ cm$, $u_e = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e} = \frac{1}{6.25} + \frac{1}{25} = \frac{1}{5}$$

$$u_e = 5\ cm$$

Now, $v_o = 15 - u_e = 15 - 5 = 10\ cm$

$$\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

$$\frac{1}{u_o} = \frac{1}{f_o} - \frac{1}{v_o} = \frac{1}{2} - \frac{1}{10} = \frac{8}{10}$$

$$u_o = 1.25\ cm$$

(b) For eye-piece, $v_e = \infty$, $u_e = f_e = 6.25\ cm$

Now, $v_o = 15 - 6.25 = 8.75\ cm$

$$\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

$$\frac{1}{u_o} = \frac{1}{f_o} - \frac{1}{v_o} = \frac{1}{2} - \frac{1}{8.75} = \frac{6.75}{17.5}$$

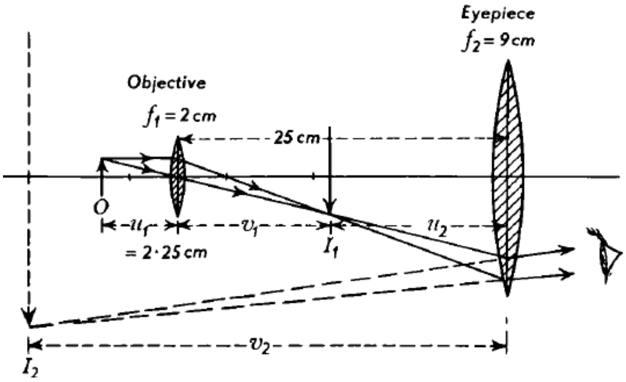
$$u_o = \frac{17.5}{6.75} = 2.59\ cm$$

Example 2

Thin converging lenses of focal lengths 2 cm and 9 cm are used respectively as the objective and eyepiece of a microscope, the centres of the lenses being 25 cm apart. If an object is placed at a distance of 2.25 cm from the objective, what will be the position and magnification of the final image?

<u>Solution</u>

Objective lens: O is the object, I, the real image produced by it.



$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1}$$

$$\frac{1}{2.25} + \frac{1}{v_1} = \frac{1}{2}$$

$$\frac{1}{v_1} = \frac{1}{2} - \frac{4}{9}$$

$$v_1 = 18\ cm$$

Eyepiece: I_1 acts as object for the eyepiece which produces a virtual image at I_2. Now the object distance u_2 is clearly $25 - 18 = 7\ cm$

Hence, since

$$\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2}$$

$$\frac{1}{7} + \frac{1}{v_2} = \frac{1}{9}$$

$$\frac{1}{v_2} = \frac{1}{9} - \frac{1}{7}$$

$$v_2 = -31.5\ cm$$

Thus, the final image is a virtual image (indicated by the negative sign) 31.5 cm in front of the eye-lens.

Magnification. The linear size of I_1 is $\frac{v_1}{u_1}$ times that of the object,

$$I_1 = \frac{18}{2.25} \times O = 8 \times O$$

The linear size of I_2 is $\frac{v_2}{u_2}$ times that of I_1

$$I_2 = \frac{31.5}{7} \times I_1 = 4.5 \times 8 \times O = 36\ O$$

Hence the magnification of the final image

$$= \frac{I_2}{O} = 36$$

TELESCOPES

A telescope is an optical instrument which enables us to see distant objects clearly by increasing the visual angle.

Very distant objects (such as the moon), although very big in size, appear very small because they subtend very small visual angle at the eye. To see them bigger, we cannot decrease their distance. However, if we place a suitable converging lens (convex lens) close to the eye, the image of the distant object can be brought close to the eye. The image will subtend a large visual angle at the eye and the object will appear bigger than when seen with naked eye. A telescope is based on this principle.

In general, telescopes are divided into two categories i.e. refracting telescopes and reflecting telescopes.

Refracting telescopes

These telescopes make use of converging lenses to distant objects clearly. Since lenses form images of objects by refraction of light, these are called refracting telescopes. These are of two types i.e. astronomical telescopes and terrestrial telescopes.

An astronomical telescope is used to see clearly heavenly bodies like moon, sun, stars, etc while a terrestrial telescope is used to see distant objects clearly on the surface of the earth.

Astronomical telescope (refracting type)

An astronomical telescope is an optical instrument used to see heavenly bodies like moon, sun, stars, etc. The image of the heavenly body formed by the telescope subtends a large visual angle at the eye so that the object appears bigger to the eye.

Construction

It consists of two convex lenses mounted coaxially at the outer ends of two sliding metallic tubes. The lens facing the object is called the objective lens and has a large focal length and large aperture. The other lens through which the image of the distant object is observed is called the eyepiece and has a small focal length and small aperture. The distance between the two lenses can be adjusted by using rack and pinion arrangement.

Final image at infinity (normal adjustment)

The eyepiece L_2 acts as a magnifying glass and forms a magnified, virtual image of I_1 and when in normal adjustment, this image is at infinity. I_1 must therefore

$$\frac{1}{f_o + f_e} + \frac{1}{v} = \frac{1}{f_e}$$

Multiplying through by $(f_o + f_e)$ gives;

$$1 + \frac{f_o + f_e}{v} = \frac{f_o + f_e}{f_e}$$

From (i);

$$1 + \frac{D_o}{D_e} = \frac{f_o + f_e}{f_e}$$

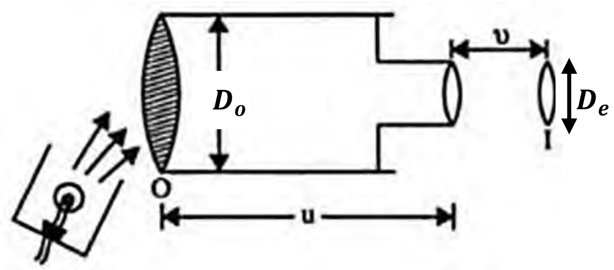
$$1 + \frac{D_o}{D_e} = \frac{f_o}{f_e} + 1$$

$$\frac{D_o}{D_e} = \frac{f_o}{f_e} = M$$

In normal adjustment,

$$\boldsymbol{M = \frac{D_o}{D_e} = \frac{\text{Diameter of objective lens}}{\text{Diameter of eye} - \text{ring}}}$$

Alternatively;



The objective lens of the telescope is illuminated from the side. An image of the lens is formed by the eyepiece slightly behind the eye-piece, at I. The diameter of this image (eye-ring) and the diameter of the objective lens are measured with a travelling microscope.

Let D_e be the diameter of the eye-ring (image) and D_o the diameter of the objective. If u is the distance of the objective from the eye-piece and v the distance of the image, then

$$\frac{D_o}{D_e} = \frac{u}{v} \text{ and } u = f_o + f_e$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f_e} - \frac{1}{u} = \frac{u - f_e}{uf_e}$$

$$\frac{D_o}{D_e} = \frac{u}{v} = \frac{u - f_e}{f_e} = \frac{f_o + f_e - f_e}{f_e} = \frac{f_o}{f_e} = M$$

So, the magnifying power of an astronomical telescope in normal adjustment is equal to the ratio of the diameter of its objective lens to the diameter of the image (eye-ring) formed of this lens by the eye-piece

Note: The above can act as a method for determining the magnifying power of a telescope

Advantage/significance of placing the eye at the eye-ring

When the eye is placed at the eye ring, all the rays passing through the objective enter it and the image viewed is brightest.

In other words, the brightest image is observed and it gives the widest field of view.

Differences between compound microscope and astronomical telescope

Compound microscope	**Astronomical telescope**
Views near objects	Views distant objects
Objective has smaller focal length	Objective has larger focal length
In normal adjustment, final image is at near point	In normal adjustment, final image is at infinity
Has greater resolving power	Has smaller resolving power

Example 1

An astronomical telescope has an objective lens of focal length 100 cm and an eye piece of focal length 5.0 cm. Calculate the angular magnification and the separation of the lenses when the telescope is in normal adjustment.

<u>Solution</u>

$f_o = +100\ cm$ and $f_e = +5.0\ cm$

In normal adjustment, $M = \frac{f_o}{f_e} = \frac{100}{5} = 20$

Separation of the lenses $= f_o + f_e$

$= 100 + 5 = 105\ cm$

Example 2

An astronomical telescope consists of two thin converging lenses. When it is in normal adjustment, the lenses are 650 mm apart and the angular magnification is 12.0. Calculate the focal length of the objective lens and the eyepiece lens.

<u>Solution</u>

$M = 12$, separation$= 650\ mm$

In normal adjustment,

$$M = \frac{f_o}{f_e} \Rightarrow f_o = 12f_e \quad \ldots\ldots\ldots\ldots(\text{i})$$

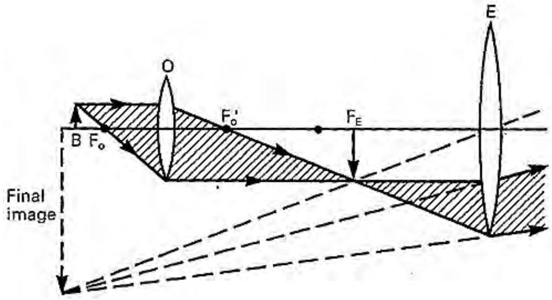
$$\text{Also } f_o + f_e = 650 \quad \ldots\ldots\ldots\ldots(\text{ii})$$

$$12f_e + f_e = 650$$

$$f_e = 50\ mm$$

$$f_o = 12f_e = 12 \times 50 = 600\ mm$$

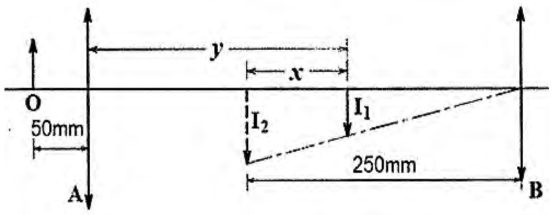
31. A slide projector is required to produce a real image 684 mm wide from an object 36 mm wide. If the distance of the object from the screen is to be 2000 mm, calculate:
    (a) the distance of the lens from the object
    (b) the focal length of the lens required
    [Ans: (a) 100 mm (b) 95 mm]
32. A camera is fitted with a lens of focal length 50 mm. If a distant building subtends an angle of 0.1 rad at the camera, what is the size of the image on the film?
    [Ans: 5 mm]
33. The diagram shows the paths of two rays of light from the tip of an object B through the objective, O, and the eye lens, E, of a compound microscope. The final image is at the near point of an observer's eye when the eye is close to E. F_o and $F_o{}'$ are principal foci of O and F_E is one of the principal foci of E.



    (a) Explain why:
        (i) the object is placed just to the left of F_o
        (ii) the eyepiece is adjusted so that the intermediate image is to the right of F_E
    (b) In this arrangement, the focal lengths of O and E are 10 mm and 60 mm respectively. If B is 12 mm from O and the final image is 300 mm from E, calculate the distance apart of O and E.
    [Ans: (b) 110 mm]
34. A refracting telescope has an objective of focal length 1.0 m and an eyepiece of focal length 2.0 cm. A real image of the Sun, 10 cm in diameter, is formed on the screen 24 cm from the eyepiece. What angle does the Sun subtend at the objective?
    [Ans: 9.1×10^{-3} rad]
35. The objective of an astronomical telescope in normal adjustment has a diameter of 150 mm and a focal length of 4.0 m. The eyepiece has a focal length of 25.0 mm. Calculate
    (a) the magnifying power of the telescope
    (b) the position of the eye ring
    (c) the diameter of the eye ring
    [Ans: (a) 160 (b) 25.2 mm (c) 0.938 mm]
36. Draw a ray diagram to show how the eye views the image formed by a convex lens when used as a magnifying lens.
    Define magnifying power of a magnifying lens, and derive an expression for it when the image, seen by the eye, is at infinity.

EXAMINATION QUESTIONS

1. (a) Define the following as applied to a telescope
        (i) Eye-ring
        (ii) Magnifying power
    (b) What is the significance of the eye-ring of an astronomical telescope?
    (c) State **two** advantages of a reflecting telescope over a refracting telescope.
    (d) The figure below shows an optical system consisting of two thin converging lenses arranged coaxially. Lens A has a focal length of 40 mm and lens B has a focal length of 375 mm. An object O of height 5 mm is placed 50 mm from A. I_1 is the real image of O in A and I_2 is the virtual image of I_1 in B and is 250 mm from B.



        (i) Determine the value of distance, y of image I_1 from lens A.
        (ii) Calculate the distance, x between the images I_1 and I_2
        (iii) Find the linear magnification produced by the lens system
    (e) Name **one** defect of images formed by a lens and explain how the defect is minimized in practice
    (f) Explain the following
        (i) total internal reflection
        (ii) formation of mirages
    [2018, No. 1; Ans: (d) (i) 20 cm (ii) 10 cm (iii) $20/3$]
2. (a) State the **laws of refraction of light**
    (b) Derive an expression for the refractive index of a prism in terms of the refracting angle A, and the angle of minimum deviation, D.
    (c) A ray of light is refracted through a prism in a plane perpendicular to its edge. The angle of incidence is 30° and the refractive index of the

prism is 1.50. Calculate the angle of the prism such that the ray does not emerge when it strikes the second face.

(d) (i) Describe with the aid of a labelled diagram, the structure and operation of a projection lantern.

(ii) A projector produces an image of area $2\,m^2$ on a screen placed $5\,m$ from the projection lens. If the area of the object slide is $8\,cm^2$, calculate the focal length of the projection lens.

[2018, No. 2; Ans: (c) $61.28°$ (d) (ii) 9.8 cm]

3. (a) (i) State **two** differences between **real** and **virtual** images

(ii) Explain with the aid of a diagram why a thick plane mirror forms multiple images

(b) A concave mirror forms a real image which is three times the linear size of the object. When the object is displaced through a distance y, the real image formed is four times the linear size of the object. If the distance between the two image positions is 20.0 cm, find the

(i) focal length of the mirror

(ii) distance y

(c) Use a geometrical ray diagram to derive the relation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ for a concave mirror

(d) Explain how a mirage is formed

[**2017, No. 1; Ans: (b)(i) 20 cm (ii) 1.67 cm**]

4. (a) Define the following as applied to a converging lens

(i) Principal focus

(ii) Centre of curvature

(b) Find the power of a lens of focal length 15 cm

(c) Derive an expression for the focal length of a lens in terms of the radii of curvature of its surfaces and its refractive index.

(d) Describe an experiment to determine the focal length of a thin converging lens mounted inside a short cylindrical tube

(e) A compound microscope consists of two thin lenses, an objective of focal length 1.0 cm and the eye-piece of focal length 5.0 cm. The objective forms an image of an object in front of it at a point 16.0 cm away. If the final image is formed at the near point of the eye, calculate the

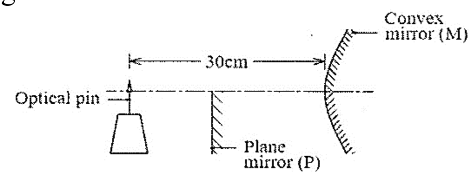
(i) separation of the lenses

(ii) magnifying power of the instrument

[**2017, No. 2;Ans: (b) 6.67 D (e)(i) 20.2 cm (ii) -90**]

5. (a) (i) Describe how the focal length of a convex mirror can be measured using a convex lens of a known focal length

(ii) The plane mirror, P in the figure below is adjusted to a position 20 cm from the optical pin, the image of the pin in P coincides with its image in M.



Calculate the focal length of the convex mirror

(b) A pin is clamped horizontally above a concave mirror with its tip along the principal axis. When the pin is adjusted, it coincides with its image at a distance R from the mirror. When a small amount of liquid of refractive index, n, is put in the mirror, the pin coincides again with its image at a distance R from the mirror. Show that the refractive index, n, is given by

$$n=\frac{R}{R'}$$

(c) (i) Explain the term **eye-ring** as applied to a telescope

(ii) Draw a ray diagram to show the formation of a final image in a Galilean telescope in normal adjustment.

(iii) Explain two advantages and one disadvantage of the telescope in (c) (ii) above.

[**2016, No. 1; Ans: (a) (ii) 15 cm**]

6. (a) (i) When does light pass through a prism symmetrically?

(ii) Find the angle of incidence, i, on an equilateral prism of refractive index 1.5 placed in air, when light passes through it symmetrically

(iii) Describe what happens to the deviation of light passing through the prism in (a)(ii) when the angle of incidence is increased from a value less than i to a value greater than i.

(b) Describe how the refractive of a prism can be determined using optical pins.

(c) (i) Draw a sketch ray diagram showing formation of the image of a finite size real object by a concave lens

(ii) A concave lens of focal length 15.0 cm is arranged coaxially with a concave mirror of focal length $10.0\,cm$, a distance of $4.0\,cm$ apart. An object is placed $20.0\,cm$ in front of the lens, on the side remote from the mirror Find the distance of the final image from the lens.

SECTION B:

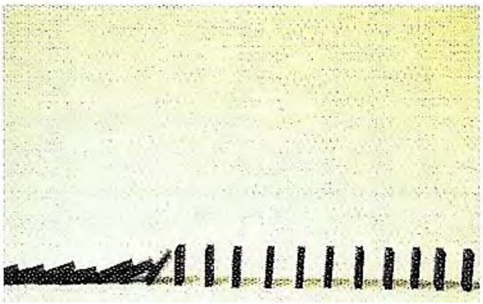
WAVES

- Introduction
- Basic properties of waves
- Terminologies used in waves
- Relationship between period and frequency
- Wavelength, frequency and velocity
- Transverse and longitudinal waves
- Progressive waves, motion, equation
- Phases of vibrations
- Superimposing of two progressive waves
- Stationary waves
- Formation of stationary waves
- Characteristics of stationary waves
- Stationary wave equation
- Stretched strings
- Factors affecting the pitch of a note from a stretched string
- Stationary waves in pipes
- End-correction
- Harmonics in strings and pipes
- Advantages and disadvantages of open and closed pipes
- Frequency of a stretched string
- Sonometer
- Resonance tube
- Kundt's tube
- Applications of stationary waves in pipes and strings
- Resonance
- Free and damped oscillations
- Applications of free and damped oscillations
- Causes of damping
- Forced oscillations
- Applications of resonance
- Resonance in strings and pipes
- Sound waves
- Production of sound
- Propagation of sound
- Tone, fundamental note, overtones, harmonics
- Speed of sound in air
- Beats
- Beat frequency
- Applications of beats
- Doppler effect
- Apparent frequency
- Wave theory
- Wave front, ray and optical path
- Types of wave fronts
- Reflection and refractions of waves
- Huygens' postulate
- Interference of waves
- Coherent sources and path difference
- Energy of a wave
- Division of amplitude and wave fronts
- Constructive and destructive interference
- Principle of Young's double slit experiment
- Separation of fringes
- Appearance of fringes
- Measurement of wavelength, slit separation, fringe separation
- Thin films, thin wedges, Lloyd's mirror, Fresnel's bi-prism and Newton's rings
- Applications of interference
- Diffraction of waves
- Diffraction equation
- Applications of diffraction
- Polarisation of waves
- Plane polarized light and non-plane polarized light
- Polarization of light
- Polaroid
- Reflection
- Double refraction
- Selective absorption
- Scattering
- Brewster's law
- Applications of polarisation

Introduction

The world is full of waves of various types – some examples are water waves, shock waves, sound waves, waves generated by earth quakes, light waves only to mention but a few. Any type of wave results from a disturbance.

When a medium is disturbed, energy is imparted to it. Suppose that energy is added to a material mechanically, such as by blow or (in the case of a gas) by compression. This sets some of the particles vibrating. Because the particles are linked by intermolecular forces, the oscillation of each particle affects that of its neighbors. The added energy propagates, or spreads by means of interactions between the particles of the medium. An analogy for this process is shown in the figure below where the particles are dominoes.



As each domino falls, it topples the one next to it. Thus, energy is transferred from domino to domino and the disturbances propagates through the medium.

In this case, there is no restoring force between the dominoes, so they do not oscillate as do particles in a continuous material medium. Therefore the disturbance moves in space, but it does not repeat itself in time at any one location.

In a continuous material medium, particles interact with their neighbors and restoring forces cause them to oscillate when they are disturbed. Thus any disturbance not only propagates through space but may be repeated over and over in time at each place along the way. Such a regular, rhythmic disturbance in both time and space is called a **wave**, and the transfer of energy is said to take place by means of **wave motion**.

Definition

A wave is a periodic disturbance that allows energy to be transferred from one point to another some distance away without any particles of the medium travelling between the two points.

Basically, wave motion in a material medium involves (a) the propagation of the disturbance (b) interparticle interactions and (c) the transfer of energy.

Electromagnetic waves

These are waves which do not require a material medium for their transmission. They can travel through vacuum also. The light waves, X-rays, etc. are electromagnetic waves.

The speed of electromagnetic waves in air/vacuum is $3 \times 10^8\ ms^{-1}$ and the transfer of energy by electromagnetic waves is due to the motion of electromagnetic fields.

Mechanical waves

These are waves which require a material medium for their transmission.

Examples include, sound waves, ripples on water surface, vibrations in a string, etc. Clearly, they cannot travel through vacuum.

The speed of mechanical waves is very small compared to electromagnetic waves. For example, the speed of sound waves in air is about $340\ ms^{-1}$.

A mechanical wave is a form of disturbance that travels through a medium due to the repeated motion of the particles of the medium about their mean positions, the motion being transferred from particle to particle without transfer of matter.

In order to produce a mechanical wave, we need a source which produces disturbance and an elastic medium. When a particle of a medium is disturbed by some source, it begins to vibrate about its mean position. The disturbance of this particle is transferred to the next particle due to the elasticity of the medium. The second particle also starts vibrating about its mean position and transfers its disturbance to the third particle and so on. In this way, the disturbance (i.e. wave) is transferred from particle to particle while the particles continue to vibrate about their mean positions.

Comparison of mechanical and electromagnetic waves

Mechanical waves	Electromagnetic waves
Need material medium for their propagation	Can propagate in a vacuum
Propagate at relatively low speed	Propagate at high speeds
Longer wavelengths	Shorter wavelengths
Are due to vibration or oscillations of particles in the transmitting medium	Are due to vibrations in electric and magnetic fields.

Types of waves

Waves may be divided into two types based on the direction of the particle's oscillations relative to the wave velocity. They are

- Transverse waves
- Longitudinal wave

Transverse waves

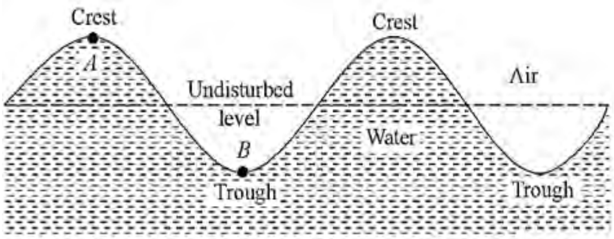
The transverse waves can only be set up in solids or at the surface of liquids. However, longitudinal waves can be set up in all types of media i.e. solids, liquids as well as gases.

Transverse waves

In a transverse wave, the particles of the medium vibrate about their mean positions at right angles to the direction of propagation of the wave. Examples include

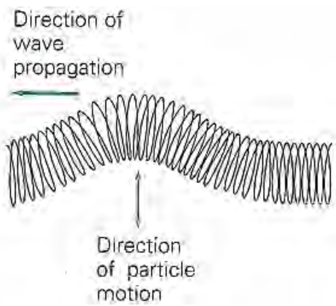
- Ripples produced on a water surface when a stone is dropped into it
- Waves produced in a rope fixed at one end when the free end is moved up and down rapidly
- Waves in stretched strings of musical instruments when the string is plucked.

A transverse wave (e.g. water wave) travels through a medium in the form of **crests** and **troughs** as shown below



When the particles of the medium move up from their mean positions, the medium is raised and a crest is formed. On the other hand, when the particles move down from their mean positions, the medium is depressed and a trough is formed.

In a stretched spring, a transverse wave can be set up as shown below



Note: A transverse wave is sometimes called a **shear wave** because the disturbance supplies a force that tends to shear the medium.

Shear waves can propagate only in solids, since a liquid cannot support a shear. That is, a liquid or a gas does not have sufficient restoring forces between its particles to propagate a transverse wave.

Why Liquids and gases can not propagate transverse waves

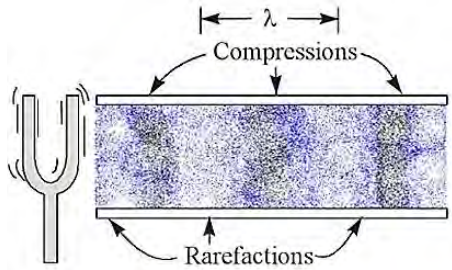
Transverse waves travel in the form of crests and troughs involving change in shape of the medium. Since liquids and gases do not possess elasticity of shape, mechanical transverse waves cannot popafgate through liquids and gases

Longitudinal waves

In a longitudinal wave, the particles of the medium vibrate about their mean positions along the direction of propagation of the wave. Examples include

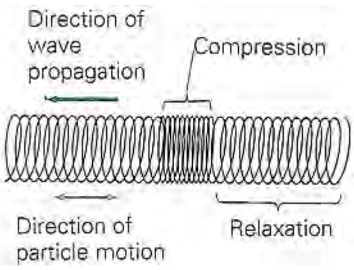
- Travel of sound waves through air
- Alternately expanding and stretching a spring
- Vibration of air column in organ pipes

A longitudinal wave (e.g. sound waves in air) travels through a medium in the form of **compressions** and **rarefactions** as shown below



When the particles of the medium move longitudinally towards their mean positions, the medium is compressed and a compression is formed. Here the medium is air and at the site of compression, the volume of air decreases and the its density increases. When the particles of the medium (air) move longitudinally away from their mean positions, the medium is rarefied and a rarefaction is formed. In the region of rarefaction, the volume of air increases and its density decreases.

Note: A longitudinal wave is sometimes called a compressional wave. A longitudinal wave may be produced by in a stretched spring by moving coils back and forth along the spring axis. Alternating pulses of compressions and relaxations move along the spring.



Longitudinal waves can propagate in solids, liquids and gases. All phases of matter can be compressed to some extent.

Water waves

Water waves are a combination of longitudinal and transverse motions. At the surface, the water particles move is circles but their motions become more longitudinal with depth.

The sinusoidal profile of water waves might make you think that these are transverse waves. Actually, they reflect a combination of longitudinal and transverse wave motions. The particle motion may be nearly circular at the surface but becomes more elliptical with depth, eventually becoming longitudinal.

A hundred of metres or so below the surface of large water body, the wave disturbances have little effect. For example, a submarine at these depths is undisturbed by large waves on the ocean's surface. As the wave approaches shallower water near the shore, the water particles have difficulty completing their elliptical paths. Finally, when the water becomes too shallow, the particles can no longer move through the bottom parts of the paths and the wave breaks. Its crest falls forward to form a surf.

Earthquakes

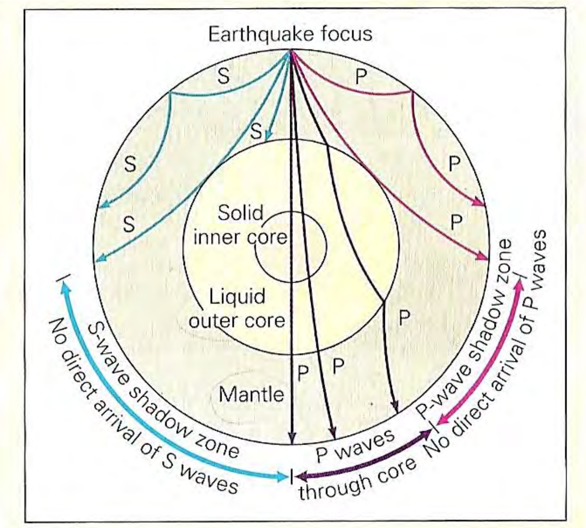
Earthquakes are caused by sudden release of built up stresses along cracks and faults. The geological theory of plate tectonics views the outer layer of the Earth as being a series of rigid plates or huge slabs of rock, that are in very slow motion relative to one another. Stresses are continually being build up, particularly along the boundaries between the plates.

The energy from a stress-relieving disturbance propagates outward (seismic) waves. These are of two general types: surface waves and body waves. The surface waves, which move along the Earth's surface, account for most earthquake damage. Body waves as the name implies, travel through the earth. These are both longitudinal and transverse vibrations. The compressional (longitudinal) waves are called P waves and shear (transverse) waves are called S waves. The P and S stand for primary and secondary and indicate the waves' relative speeds (actually, their arrival times at monitoring stations).

Primary waves travel through materials faster than secondary waves and are detected first.

Longitudinal waves can travel through solids and liquids but transverse waves can only travel through solids. When an earthquake occurs at a particular location, P waves are detected on the other side of the earth and S waves are not. The absence of S waves in a shadow zone leads to a conclusion that the Earth must have a region near its centre that is in the liquid phase. The region is a highly viscous metallic liquid, but definitely a liquid since it does not support a shear (transverse waves are not propagated).

When the transmitted P waves enter and leave the liquid region, they are refracted (bent). This gives rise to a P wave shadow zone which indicates that only the outer part of the core is liquid.



Comparison of Transverse and Longitudinal waves

Transverse waves	Longitudinal waves
Particles of the medium vibrate perpendicular to the direction of wave travel	Particles of the medium vibrate in the same direction in which the wave travels
Formed of crests and troughs	Formed of a series of compressions and rarefactions
Can propagate only in solids and at the surface of liquids	Can propagate in all types of media
There is no pressure variation	Pressure and density are maximum at compression and minimum at rarefactions

The energy that travels through unit area per unit time is that contained in a cylinder of cross-sectional area $1\ m^2$ and of length equal to the speed v of the wave.

Hence, $I = 2\pi^2 \rho v f^2 A^2$ .... (i)

But $f^2 = \frac{\omega^2}{4\pi^2}$

Therefore, $I = \frac{1}{2}\rho v \omega^2 A^2$ ..... (ii)

From equations (i) and (ii), the intensity of a wave is directly proportional to the square of the amplitude and the square of the frequency.

Why amplitude of a wave decreases as the wave spreads from the source

As the wave progresses, some energy is absorbed by the transmitting medium for the case of circular waves. As waves spread out, the energy is spread out over a wide area.

Therefore, intensity reduces with increasing distance d, given by the inverse square law, $I = \frac{1}{d^2}$,

but $I \propto A^2$

$$\therefore A^2 \propto I \propto \frac{1}{d^2}$$

Superposition of waves

The principle of superposition may be applied to waves whenever two (or more) waves travelling through the same medium at the same time. The waves pass through each other without being disturbed. The net displacement of the medium at any point in space or time, is simply the sum of the individual wave displacements

Applications of superposition of waves

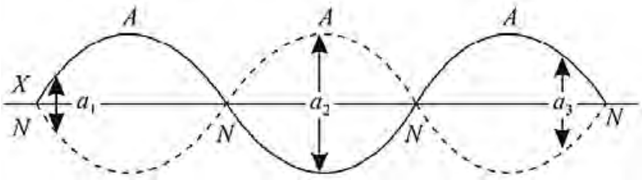
(i) When two waves of the same frequency and having the same amplitude moving with the same speed in opposite directions superpose in each other, they give rise to **stationary** or **standing waves**

(ii) When two waves of slightly different frequency moving with the same speed in the same direction superpose on each other, they give rise to **beats**

Stationary waves or standing waves

When two progressive waves of the same wavelength and amplitude travel with the same speed along the same straight line in opposite directions, these waves interfere to produces stationary waves or standing waves.

In a stationary or standing wave, the amplitude varies from place to place along the wave. There are points where the amplitude of the particles of the medium is zero. These points are called **nodes (N)**. Midway between these nodes, there are points where the amplitude of the particles of the medium is maximum. These points are called **antinodes (A)**.



Node is the region of destructive superposition when the waves always meet out of phase by π radians.

Antinode is the region of constructive superposition where the waves meet always in phase. Hence a particle vibrates with maximum amplitude.

Separation of adjacent nodes is $\frac{\lambda}{2}$

Separation of adjacent antinodes is $\frac{\lambda}{2}$

Conditions for formation of stationary waves

- Two waves must be travelling in opposite directions along the same line of travel and in the same plane.
- The waves must have the same speed and same frequency.
- The waves must have approximately the same amplitudes.

Types of stationary waves

Like progressive waves, stationary waves may be (i) transverse stationary waves (ii) longitudinal stationary waves

Transverse stationary waves

There are formed due to the superposition of two identical transverse waves travelling in the opposite directions. For example, the stationary waves produced on the vibrating string of a sonometer are transverse stationary waves

Longitudinal stationary waves

The longitudinal stationary waves are formed due to the superposition of two identical longitudinal waves travelling in the opposite directions. For example, the stationary waves in the organ pipes and in the air column of a resonance tube apparatus are longitudinal stationary waves

The stationary wave equation

Consider two progressive waves A and B travelling in opposite directions. Their displacements y_A and y_B are given by;

$$y_A = a\sin(2\pi ft - kx)$$
$$y_B = a\sin(2\pi ft + kx)$$
$$\text{where } k = \frac{2\pi}{\lambda}$$

When two waves are superposed, the resultant displacement, y is given by;

$$y = y_A + y_B$$
$$y = a\sin(2\pi ft - kx) + a\sin(2\pi ft + kx)$$

From trigonometry;

$$\sin(A - B) + \sin(A + B) = 2\sin A\cos B$$
$$If\ A = 2\pi ft\ and\ B = kx, \text{ then}$$
$$y = 2a\sin(2\pi ft)\cos(kx)$$
$$y = (2a\cos kx)(\sin 2\pi ft)$$
$$If\ A = 2a\cos kx$$
$$y = A\sin 2\pi ft$$

A is the vertical displacement of the resultant wave

At a node, $A = 0$ and at an antinode, $A = \pm 2a$

Thus, the maximum amplitude of the resultant wave is $2a$ i.e. when $\sin(2\pi ft) = 1$

Differences between stationary waves and progressive waves

Stationary waves	Progressive waves
Amplitude varies from maximum at the antinodes to zero at the nodes	Amplitude is the same for all the particles in the wave
The wavelength is twice the distance between a pair of adjacent nodes or antinodes	The wavelength is the distance between two consecutive points on a wave that are in phase
Particles in the same segments are in phase and particles in adjacent segments are anti-phase	All particles within one wavelength have different phases
No energy is transported by the wave	Energy is transported in the direction of the wave travel.

Self-Evaluation exercise

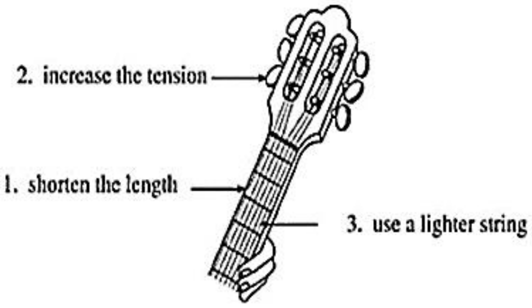
1. Explain what is meant by wavelength, the frequency and the speed of a sinusoidal travelling wave and derive the relation between them
2. Explain how energy is transmitted in wave motion
3. Distinguish between a progressive wave and a stationary wave
4. Explain the conditions necessary for the creation of stationary waves in air
5. A sound wave in air is reflected by a solid wall placed at right angles to the direction of travel of the wave, so that the incident and reflected waves are *superimposed.* A system of *stationary waves* is set up and as a result there are regularly spaced displacement *nodes* and *antinodes*
   (a) Explain the meanings of the words in italics giving diagrams where appropriate
   (b) If the sound wave is produced by a source of frequency $500\ Hz$ and the distance between adjacent nodes is 34.0 cm, calculate the speed of the sound wave

   [Ans: $340\ ms^{-1}$]
6. Distinguish between transverse and longitudinal waves. To which of these categories do (i) sound waves in air (ii) light waves belong?
7. A plane progressive wave travelling in the x-direction is represented by the equation $y = 0.25\sin 5\pi\left(20t - \frac{x}{15}\right)$ where t is time in seconds, y is the displacement of the wave in meters while x is in meters, Determine the
   (i) periodic time
   (ii) wave length
   (iii) speed of the wave

   [Ans: (i) $0.02\ s$ (ii) $6\ m$ (iii) $300\ ms^{-1}$]
8. A plane progressive wave in the x −direction has its displacement, y defined by the equation $y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$, where v is the velocity, t is the time and λ is the wave length.
   (i) Write down a corresponding wave that results into a standing wave and state its direction of travel.
   (ii) Derive the wave equation of the resulting standing wave. State its amplitude and the condition for maximum amplitude.
9. A progressive wave has amplitude $0.40\ m$ and wavelength $2.0\ m$. At a given time the displacement $y = 0\ at\ x = 0$. Calculate
   (i) the displacement at $x = 0.50\ m\ and\ 1.4\ m$
   (ii) the phase angles at $x = 0.50\ m\ and\ 0.80\ m$
   (iii) the phase difference between any two points which are $0.30\ m$ apart on the wave.

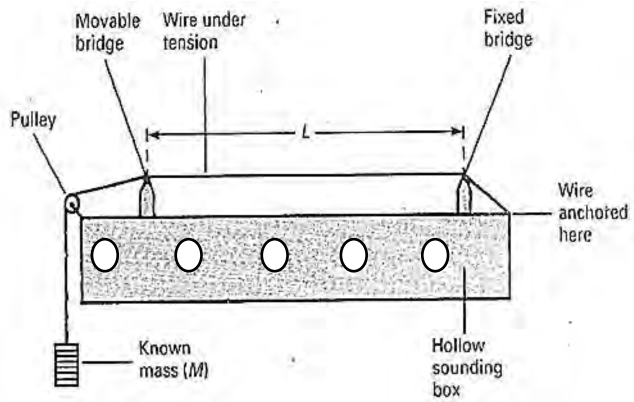
[Ans: (b)(i)$0.40\ m$, $-0.38\ m$ (ii) 0.5π rad, 0.8π rad (iii) 0.3π rad]

10. The equation $y = A\sin(\omega t + Rx)$ represents a progressive wave.



Experimental verification of $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

The laws of vibrations of stretched strings can be verified experimentally using a Sonometer



Variation with length, *L*

A suitable value of tension T in the string is set up by putting masses on a mass hanger.

The position of the movable bridge is altered so that the vibrating length L of the wire is such that when the wire is plucked in the middle, it produces the same note as the tuning fork of known frequency f.

The procedure is repeated using tuning forks of other known frequencies and the results are tabulated including values of f and $\frac{1}{L}$.

A graph of f against $\frac{1}{L}$ is plotted and a straight linear graph passing through the origin is obtained. This implies that $f \propto \frac{1}{L}$

Variation with tension in the wire, *T*

The length L between the bridges is kept constant. A suitable mass M is attached to the mass hanger so that when the wire is plucked in the middle, it produces the same note as a tuning fork of known frequency f. The mass M and the frequency f are recorded.

The procedure is repeated with forks of other known values of f. The results are tabulated including values of f^2

A graph of f^2 against M is plotted which gives a straight-line graph passing through the origin. This implies that $f^2 \propto M$ but since $T = Mg$, then $T \propto M$

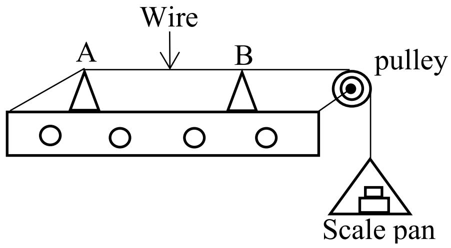
Therefore $f^2 \propto T \Rightarrow f \propto \sqrt{T}$ where T is the tension

Variation with mass per unit length, μ

The mass per unit length, μ of the wire is first determined by weighing. The length, L of the wire is then adjusted so that when the wire is plucked it produces the same note as one of the tuning forks. The procedure is repeated using wires of different mass per unit length. Each wire must be under the same tension as the first wire, and in each case the length is adjusted until the wire vibrates at the same frequency as the tuning fork that was used with the first wire.

A graph of L against $\frac{1}{\sqrt{\mu}}$ is plotted and it is a straight line graph through the origin verifying that $f \propto 1/\sqrt{\mu}$

Experiment to show that a wire under tension can vibrate with more than one frequency



The stretched wire AB is plucked in the middle. Tuning forks of different frequencies are struck and brought one at a time near the wire.

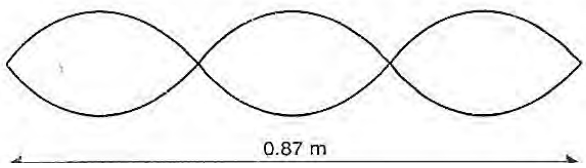
The one which produces the loudest sound with the wire is identified.

The procedure is repeated when the wire is plucked at $\frac{1}{4}$ way and $\frac{1}{8}$ way from B.

It is found that the tuning forks resonating with the wire in each of the cases is different hence the wire under tension resonates with more than one frequency.

Example 1

The figure below shows a standing wave set up on a wire of length 0.87 m. The wire is vibrated at a frequency of 120 Hz.



(a) Calculate the speed of the transverse wave along the wire.

18. A stretched string has a mass per unit length of 5.00 g/cm and a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the negative direction of an x axis. If the wave equation is of the form $y(x,t) = y_m \sin(kx \pm \omega t)$, what are
    (a) y_m
    (b) k,
    (c) ω,
    (d) the correct choice of sign in front of ω?

[Ans: (a) $0.12\ mm$ (b) $141\ m^{-1}$ (c) $628\ s^{-1}$ (d) +]

19. The linear density of a string is 1.6×10^{-4} kg/m. A transverse wave on the string is described by the equation $y = 0.021 \sin[2.0x + 30t]$ where x is in metres and t is in seconds. What are (a) the wave speed and (b) the tension in the string?

[Ans: (a) $15\ ms^{-1}$ (b) $0.036\ N$]

20. A uniform rope of mass m and length L hangs from a ceiling.
    (a) Show that the speed of the transverse wave on the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$
    (b) Show that the time a transverse wave takes to travel the length of the rope is given by
$$t = 2\sqrt{L/g}$$
21. A string fixed at both ends is 8.40 m long and has a mass of 0.120 kg. It is subjected to a tension of 96.0 N and set oscillating.
    (a) What is the speed of the waves on the string?
    (b) What is the longest possible wavelength for a standing wave?
    (c) Give the frequency of that wave.

[Ans: (a) $82\ ms^{-1}$ (b) $16.8\ m$ (c) $4.88\ Hz$]

22. What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

[Ans: (a) $7.91\ Hz$ (b) $15.8\ Hz$ (c) $23.7\ Hz$]

23. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz, with no intermediate resonant frequencies. What are (a) the lowest resonant frequency and (b) the wave speed?

[Ans: (a) $105\ Hz$ (b) $158\ ms^{-1}$]

24. One of the harmonic frequencies for a particular string under tension is 325 Hz. The next higher harmonic frequency is 390 Hz. What harmonic frequency is next higher after the harmonic frequency 195 Hz?

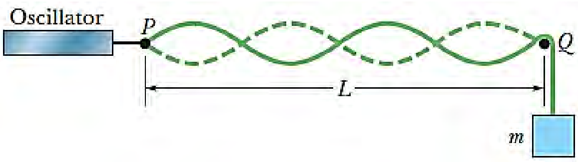
[Ans: $260\ Hz$]

25. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown below

Calculate the
    (a) speed,
    (b) wavelength, and
    (c) frequency of the travelling waves whose superposition gives this standing wave

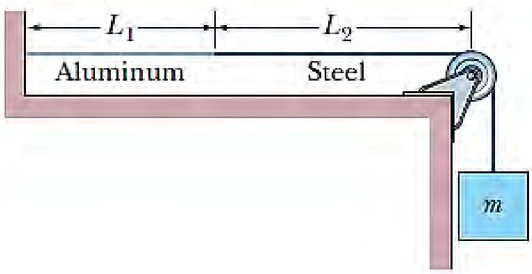
[Ans: (a) $144\ ms^{-1}$ (b) $60.0\ cm$ (c) $241\ Hz$]

26. In the figure below, a string, tied to a sinusoidal oscillator at P and running over a support at Q, is stretched by a block of mass m. Separation L = 1.20 m, linear density μ = 1.6 g/m, and the oscillator frequency f = 120 Hz. The amplitude of the motion at P is small enough for that point to be considered a node. A node also exists at Q.



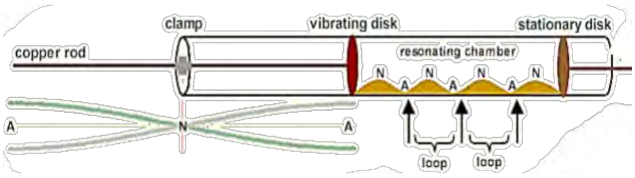
    (a) What mass m allows the oscillator to set up the fourth harmonic on the string?
    (b) What standing wave mode, if any, can be set up if m = 1.00 kg?

27. In the figure below, an aluminum wire, of length L_1 = 60.0 cm, cross-sectional area $1.0 \times 10^{-2}\ cm^2$, and density 2.60 g/cm^3, is joined to a steel wire, of density 7.80 g/cm3 and the same cross-sectional area. The compound wire, loaded with a block of mass m = 10.0 kg, is arranged so that the distance L_2 from the joint to the supporting pulley is 86.6 cm.



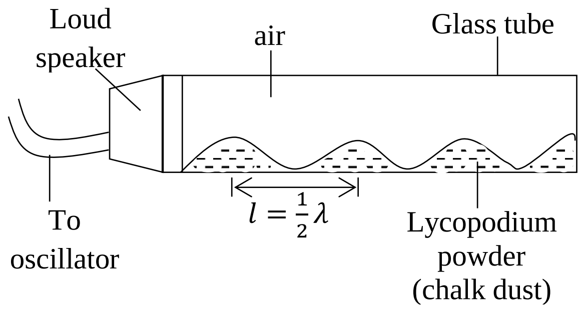
KUNDT'S TUBE APPARATUS

This is a device that can make longitudinal standing waves visible in air. It consists of a glass tube, closed at one end by a vibrating membrane (e.g. loud speaker or vibrating rod) and a movable piston that closes the other end. The bottom of the tube is covered with lycopodium powder. The membrane excites the air column in the tube into vibration and the length of the column may be controlled by the position of the piston.



The waves are reflected at the piston surface (free end) hence standing waves may arise for appropriate tube lengths.
At positions of vibration nodes, the powder remains at rest while it spreads perpendicular to the tube axis at the positions of antinodes.
By shifting the piston, the length of the air column may be varied and thus the resonance condition for the formation of standing waves may be observed.
The Kundt's tube can be used to measure the speed of sound in air and also to compare the speed of sound in different gases.

Measurement of speed of sound in air by Kundt's tube



A long glass tube is placed horizontally with ether chalk dust or lycopodium powder inside it.
The open end is fitted with a loud speaker which is connected to the oscillator. When the oscillator is switched on, sound is produced and a stationary wave is formed in the glass tube which makes the powder to settle into well-spaced heaps.
Measure the distance l between two consecutive heaps.
The heaps are formed at points where there is no vibration (i.e. nodes)

$$\Rightarrow l = \frac{1}{2}\lambda$$
$$\lambda = 2l$$

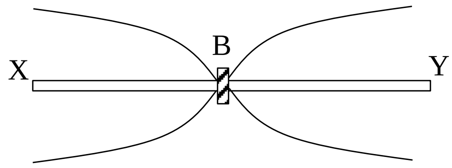
From, $v = \lambda f$
Speed of sound, $v = 2lf$
where f is the frequency of the oscillator

Note: When the velocity of sound in a liquid is to be determined, then iron fillings can be used instead of the lycopodium powder in the tube. This is because iron fillings are heavy and can settle in heaps in a liquid.

Errors of this method

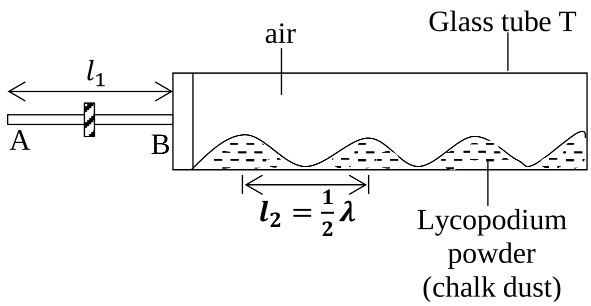
- Measurement of l from outside the tube may not be accurate
- The waves are damped by the sides of the tube

Measurement of velocity of sound in a metal rod



The rod XY is fixed by a clamp B at its midpoint.
The rod is then stroked by a compact cloth. A stationary wave is then formed with a node at its midpoint and antinodes at the free ends X and Y producing a high pitched note.
Measure the length l of the rod.
The wavelength λ of the wave generated is given by $\lambda = 2l$
The speed of sound in the rod is given by $v = 2lf$ where f is the frequency of the rod which can be determined by the method of beats.

Measurement of velocity of sound in a rod using Kundt's dust tube

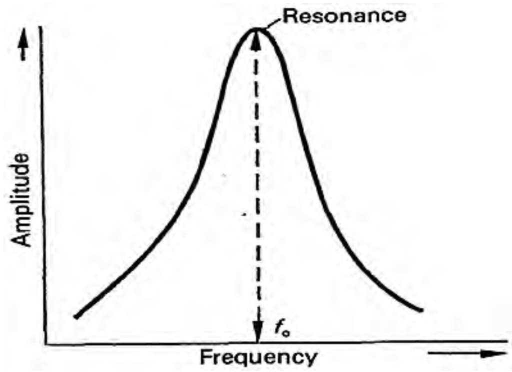


Sprinkle some chalk dust along tube T.
Clamp the rod AB at its midpoint but with one end projecting into the tube T.
A disc is connected to this end so that it just clears the sides of the tube.
Strike the rod with a compact cloth in the direction AB until the powder in the tube settles into heaps. Measure

Examples include; oscillations of a pendulum, electromagnetic damping in galvanometer

Forced oscillations
The oscillations in which a body oscillates under the influence of an external periodic force are known as forced oscillations.
The amplitude of oscillator decreases due to damping forces but on account of the energy gained from the external source it remains constant.
An example is the sound boards of stringed instruments

Resonance
This is situation that occurs when a particular body or system is set in oscillation at its own natural frequency due to impulses received from a nearby source of the same frequency.



Implications and uses of resonance
Implications
- Soldiers must ‘break step’ when crossing wooden bridges.
- Cars / aircraft / rockets are carefully designed so that parts do not resonate producing unwanted noises / dangerous vibrations.
- Electrical audio circuits are subject to ‘feedback’. This is the loud howling sound produced when a microphone is too close to a loud speaker and the amplifier gain is too high.
- A driver on a spring board builds up the amplitude of oscillation of the board by bouncing on it at its natural frequency.
- Opera singers can shatter wine glasses by forcing them to vibrate at their natural frequencies.

Uses
- Clocks and watches: In quartz crystals resonate producing accurate timing frequencies.
- Standing waves in pipes: These can be important when measuring the speed of sound in air.
- Ultrasonic cleaning: Dirt particles resonate with the applied frequency and are dislodged.
- Crystal radios: Circuits resonate at the same frequency as a radio station.
- Radio antennas (aerials): They resonate when they interact with radio waves.

SOUND WAVES
Sound waves are longitudinal waves that can travel through any material medium (i.e. solids, liquids or gases) with a speed that depends on the properties of the medium. As sound travels through a medium, the particles of the medium vibrate along the direction of motion of the wave. This is contrast to a transverse wave where the particle motion is perpendicular to the direction of wave motion. The displacements that occur.as a result of sound waves involve the longitudinal displacements of individual molecules from their mean or equilibrium positions. This results in a series of high and low-pressure regions called compressions and rarefactions respectively

Note: Sound cannot travel through a vacuum because in a vacuum there is no material to transmit this mechanical wave

Audible waves: Audible waves are sound waves that human ear can hear. The range of human hearing is $20\ Hz$ to $20\ kHz$. In other words, we cannot hear waves of frequency below $20\ Hz$ or above $20\ kHz$. The audible waves can be generated in a variety of ways such as by musical instruments, human vocal cords and loud speakers.

Inaudible waves: Those waves which human ear cannot hear are called inaudible waves. There are two types of inaudible waves i.e. **infrasonics** and **ultrasonics**. Infrasonics are longitudinal waves with frequencies below $20\ Hz$. Earthquake waves are an example. **Ultrasonic waves** are longitudinal waves with frequencies above $20\ kHz$. The human ear cannot hear ultrasonics.

Uses of ultrasonics

- Used in cleaning of water.
- Used in medical imaging (giving the image of a baby inside the womb).

Why propagation of sound in air is an adiabatic process

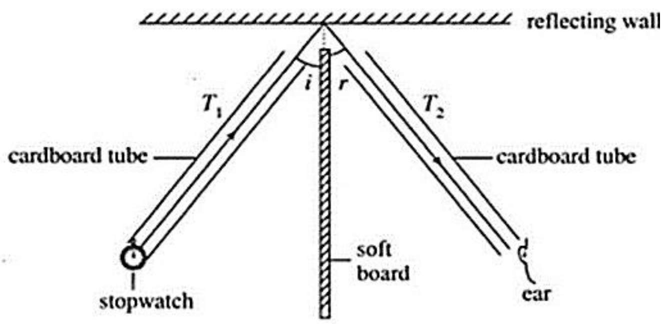
When sound passes through air, it makes a wave consisting of compression and rarefactions at a frequency of about 500Hz and of very short amplitude. This process of compression and rarefaction is fast enough not to allow significant transfer of energy from the system. The air through which the sound moves is so vast that it's considered to be a well-insulated system. Also, the compressions and rarefactions cause changes in pressure, temperature and volume. Thus, transmission of sound in air is an adiabatic process.

Properties of sound waves

Reflection

Sound waves obey the rules of reflection just like any other type of wave.

Verification



A cardboard tube T_1 with a ticking watch in it is arranged so that sound is incident at an angle i to the reflecting wall. Another cardboard T_2 is adjusted until the ticking of the watch could be heard distinctly. When this occurs, the angle of reflection r is found to be equal to the angle of incidence i.

A smooth bare wall is a good reflector of sound waves because;

(i) most of the incident sound wave energy is reflected from a smooth bare wall.
(ii) a smooth wall reflects the sound waves evenly. A rough wall would scatter the reflected waves.

Reflection of sound waves results into echoes which are a faint sound resembling the original sound.

An echo is reflected sound. When sound waves are incident on a hard plane surface, they are reflected and they obey the same laws which govern reflection of light.

Reverberation

The persistance or prolongation of sound in a room or hall (due to successive reflections from surfaces) after the original sound has ceased is called **reverberation**.

When sound is produced in an open space, it is heard by the listener only once, as the waves travel across him. However, when sound is produced in a hall or room, it is observed that sound persists even after the original sound has ceased. This is due to the repeated reflection of sound from the walls, floor, ceiling, etc. The reflected sound waves go on traversing the room (or hall) a number of times before they become too weak to be heard. This persistance of sound after the original sound has ceased is called reverberation. Reverberation tends to prolong the sound and considerably impairs its distinctness or clarity

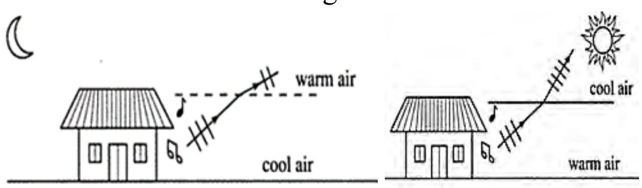
Methods of reducing reverberation

The effect of reverberation in a room or hall can be decreased by increasing the absorption of sound in it. This can be done by having

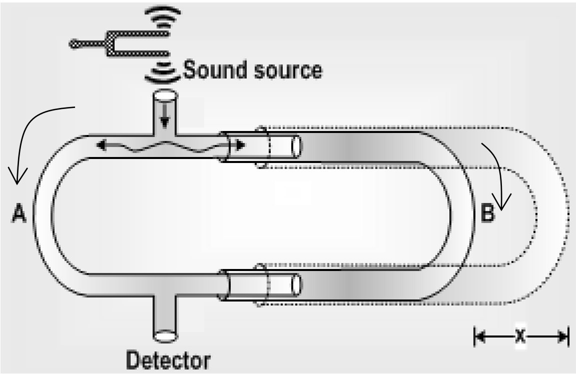
(i) a few open windows in the room or hall
(ii) sound absorbing materials in the room or hall e.g. perforated cardboards, heavy curtains, pictures and maps on the walls, carpet on the floor etc.
(iii) a good number of audience in the room or hall. Each member of the audience absorbs some sound
(iv) cushioned seats. Such seats not only provide comfort but also act as good absorbers of sound
(v) no large concave, spherical or cylindrical surfaces on the walls or ceiling of the hall or auditorium. This will avoid excesssive reflections of sound

Refraction

In air, sound travels faster where the temperature is higher. At night, air temperatures near the ground are often lower than in the air higher up and this causes sound to travel further at night.



Measurement of speed of sound using Quincke's tube



The two tubes are connected as shown above.
Sound from the source has two possible paths A and B, to the detector. When path A and B are equal in length, the loudest sound is recorded by the detector.
The movable tube is pulled slowly outwards and the sound intensity will decrease and afterwards it increases.
When the path difference is an odd number of half the wavelength, compressions from A meet rarefactions from B at the detector and minimum sound is recorded by the detector.
The movable tube is moved to find two positions of minimum distance.
The distance x between the two positions is measured.

Distance $x = \frac{1}{2}\lambda \Rightarrow \lambda = 2x$

speed of sound, $v = \lambda f = 2xf$

where f is the frequency of the source of sound

Factors affecting velocity of sound in a gas

The speed of sound in a gas is given by the equation,

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

where $\gamma = \frac{c_p}{c_v}$ the ratio of the molar heat capacities of a gas.

p = pressure of the gas and ρ = density of the gas.

Effect of density

If the value of γ for any two gases be the same, the speed of sound be v_1 and v_2 at the same temperature and pressure and the densities of the gases be ρ_1 and ρ_2 at this temperature and pressure, then

$$v_1 = \sqrt{\frac{\gamma p}{\rho_1}} \text{ and } v_2 = \sqrt{\frac{\gamma p}{\rho_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

If the molecular weights of the gases are M_1 and M_2 respectively, then

$$v_1 = \sqrt{\frac{\gamma RT}{M_1}} \text{ and } v_2 = \sqrt{\frac{\gamma RT}{M_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{M_1}{M_2}}$$

Thus, the speed of sound in a gas is inversely proportional to the square root of the density or the molecular weight of the gas.

Effect of temperature

For one mole of a gas, $pV = RT$. If M is the molecular weight of the gas, then,

$$\rho = \frac{M}{V} \text{ or } V = \frac{M}{\rho}$$

Substituting the value of V in the gas equation, we have,

$$p\frac{M}{\rho} = RT$$

$$\frac{p}{\rho} = \frac{RT}{M}$$

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

$$v \propto \sqrt{T}$$

Hence the velocity of sound in a gas is directly proportional to the square root of its absolute temperature. Molecules at higher temperatures have more energy thus they can vibrate faster and since the molecules vibrate faster, the sound waves can travel more quickly.

Note:

The speed of sound in air for a particular temperature is given by

$$v = (331 + 0.6T_C)\ m/s$$

where T_C is the air temperature in degrees Celsius.

Effect of pressure

The velocity of sound in a gas is independent of gas pressure provided the temperature remains constant.
At a constant temperature,

$$pV = \text{constant}$$

$$p\frac{M}{\rho} = \text{constant}$$

$$\frac{p}{\rho} = \frac{\text{constant}}{\text{M}} = \text{constant}$$

Thus, if the temperature of a gas remains constant, a change in pressure of the gas changes its density in the same ratio so that $\frac{p}{\rho}$ remains unchanged. Consequently, the velocity of sound is independent of the pressure of the gas provided temperature remains constant.

Effect of humidity

Since the density of water vapour is less than that of dry air, the presence of moisture tends to decrease the density of the atmospheres. Hence the velocity of sound in moist air is greater than in dry air. It is for this reason that the sirens and the whistle of trains are heard up to longer distances in the rainy season as compared to the dry season

Effect of wind

If air is blowing, the speed of sound changes. If the wind blows in the direction in which the sound travels, the velocity of sound increases. If the wind blows in the opposite direction, the velocity of sound decreases.

Musical sound and noise

All sounds which produce sensation of heraing may be roughly divided into two classes namely (i) musical sound and (ii) noise. The distinction between musical sound and noise is subjective i.e. it depends upon the senses of a person. A sound which is musical to someone may be noise to others

Musical sound: It is a pleasant, continous and uniform sound produced by regular and periodic vibratons e.g. sound produced by a tuning fork, flue, piano, etc.

Noise: It is an unpleasant, discontinous and non-uniform sound produced by irregular disturabances. All sounds other than musical notes are noises e.g. sound produced by a falling brick

Intensity of a sound note

This is the rate of flow of energy through an area of $1\,m^2$ perpendicular to the direction of flow of the sound wave.

Characteristics of a musical sound

There are three fundamental characteristics of a musical sound namely;

1. Pitch 2. Loundness 3. Quality

One musical sound can be distinguished from another by one or more of the above characteristics.

Pitch

It is a characteristic of musical sound by which we can distinguish a sharp/shrill sound from a grave/flat sound. The pitch of sound depends upon frequency (or wavelength). If the frequency of a sound is high, its pitch is also high and the sound is said to be sharp. If the frequency of sound is low, its pitch s low and the sound is said to be grave or flat.

Examples

- The voice of children and ladies is shrill because of higher pitch
- The voice of an old man is flat because of low pitch
- The ound produced by a mosquito is of high pitch and therefore shrill.

Loudness

It is the characteristic of a musical sound by which a loud sound can be distinguished from a faint sound even though the two have the same pitch. Loudness is a sensation which determines the degree of sound power produced in the ear. It depends upon the intensity of the sound near the ear. If the intensity of the sound near the ear is high, loudness will be more.

The intensity and hence loudness of sound depends upon the following factors

(i) Amplitude of the vibrating source: The greater the amplitude of the source, the greater the intensity (and hence loudness) of sound and vice-versa.

(ii) Motion of the medium: If wind is blowing in the direction of propagation of sound, loudness is increased. On the other hand, if wind is blowing in a direction opposite to the propagation of sound, loudness is decreased.

(iii) Presence of other bodies: The loudness of sound is increased due to the presence of other bodies near the source of sound. For example, sound appears to be much louder inside a hall than in open air. It is because the walls, roof, floor, etc. reflect sound. Consequently, loudness is increased.

(iv) Surface area of vibratiing body: The greater the surface area of the vibrating body, the greater is the loudness of sound.

Quality or Timbre

It is the characteristic of a musical sound which enables us to distinguish between two sounds of the same pitch and loudness produced by two different sources. It depends upon the waveform of the sound. The sound from an instrument or any other source does

DOPPLER EFFECT

The siren of an ambulance or a police car appears to change its pitch as it passes you. As the source of the sound approaches you, the pitch is higher and as it moves away, the pitch is lower.

The apparent change in the observed frequency of a wave due to the relative motion between the source of waves and the observer is known as Doppler effect.

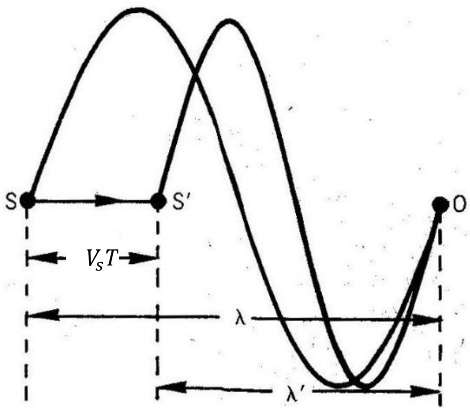
The frequency of the sound received by the listener is greater when the source approaches him as he receives more waves per second than if the source is stationary. However, when the source is moving away from the listener, fewer waves are received per second and consequently the frequency is smaller.

Proof of the formulae for doppler effect

Moving source with speed V_s

Consider a source S moving at a velocity V_s towards an observer O exactly one wavelength away. Let the source emit a continuous series of waves of wavelength λ, frequency f and velocity V.

In the time T that the wave travels from S to O, the source travels from S to S', a distance V_sT



Therefore, the observed wavelength λ' is

$$\lambda' = \lambda - V_sT$$

But $T = \frac{\lambda}{V}$

$$\lambda' = \lambda - \frac{V_s\lambda}{V}$$

$$\boldsymbol{\lambda' = \lambda\left(1 - \frac{V_s}{V}\right)}$$

If the source is receding, we have

$$\lambda' = \lambda + V_sT$$

$$\boldsymbol{\lambda' = \lambda\left(1 + \frac{V_s}{V}\right)}$$

A useful result is that the wavelength change $\Delta\lambda$ is given by

$$\Delta\lambda = \lambda' - \lambda = \lambda\left(1 + \frac{V_s}{V}\right) - \lambda$$

$$\boldsymbol{\Delta\lambda = \frac{\lambda V_s}{V}}$$

where V is the speed of sound in air and V_s is the speed of the source of sound.

Similarly, for apparent wavelength change in light,

$$\boldsymbol{\Delta\lambda = \frac{\lambda V_s}{c}}$$

where c is the speed of light and V_s is the speed of the source of light.

Alternatively, we can consider a source emitting f waves per second. In one second, the waves will move forward a distance V and the source will move forward a distance, V_s. Therefore f waves are now contained in a distance $(V - V_s)$.

Therefore, the new wavelength is given by

$$\boldsymbol{\lambda' = \frac{V - V_s}{f}}$$

If the source is moving away from the observer, f waves will be contained in a distance $(V + V_s)$ in one second.

Therefore, the new wavelength is given by

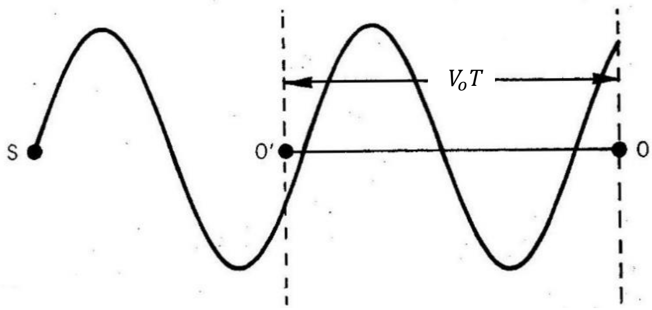
$$\boldsymbol{\lambda' = \frac{V - V_s}{f}}$$

Both the approaches show an apparent reduction in frequency and an increase in wavelength for a receding source and otherwise for an approaching source.

Moving observer with speed V_o

In one second, f waves will pass a stationary observer, but an observer who is moving towards the source will receive more waves per second and thus the observed frequency will increase.

Let the observer move from O to O' in one second.



Now $OO' = V_o$ with $T = 1$, and so the extra number of waves received per second is $\frac{V_o}{\lambda}$.

Therefore, the new frequency f' is given by

$$\boldsymbol{f' = f\left(1 + \frac{V_o}{V}\right)}$$

$$\boldsymbol{f' = \frac{(V + V_o)}{V}f}$$

For a receding observer, we have

Example 15

A SONAR system fixed in a submarine operates at a frequency $40.0\,kHz$. An enemy submarine moves towards the SONAR with a speed of $360\,km\,h^{-1}$. What is the frequency of the sound reflected by the submarine? (take the speed of sound in water to be $1450\,m\,s^{-1}$)

Solution

$$f = 40.0\,kHz;\, V = 1450\,ms^{-1};$$
$$u_0 = 360\,km\,h^{-1} = 100\,ms^{-1}$$

The frequency of the waves from the SONAR will undergo a change in frequency in two steps.

Step 1: Velocity of sound relative to enemy submarine

$$V' = V + V_o = 1450 + 100 = 1550\,ms^{-1}$$

Wavelength of the waves reaching energy submarine

$$\lambda' = \frac{V}{f}$$

Apparent frequency, $f' = \frac{V'}{\lambda'} = \frac{V+V_o}{V/f} = \frac{V+V_o}{V} \times f$

$$= \frac{1550}{1450} \times 40.0 = 42.758\,kHz$$

Step 2: The enemy submarine will reflect waves of frequency $42.758\,kHz$ and will act as a source of waves. Now the source (enemy system) is moving with a speed $V_s = 100\,ms^{-1}$ toward the SONAR system (observer).

Velocity of sound relative to SONAR system is

$$V'' = V$$

Wavelength of waves reaching the SONAR system is

$$\lambda'' = \frac{V - V_s}{f'}$$

Apparent frequency, $f'' = \frac{V''}{\lambda''} = \frac{V}{V-V_s} \times f'$

$$= \frac{1450}{1450 - 100} \times 42.758 = 45.92\,kHz$$

Example 16

An observer, standing by a railway track, notices that the pitch of an engine's whistle changes in the ratio 5 : 4 on passing him. What is the speed of the engine? (Velocity of sound in air = $340\,ms^{-1}$.)

Solution

Let f be the true pitch of the engine whistle and v_s be the speed of the engine.

The observed frequency f_1 as the engine approaches is given by

$$f_1 = \left(\frac{v}{v - v_s}\right) f$$

The observed frequency f_2 after the engine has passed is given by

$$f_2 = \left(\frac{v}{v + v_s}\right) f$$

where v is the velocity of sound in air

$$\frac{f_1}{f_2} = \frac{\left(\frac{v}{v - v_s}\right) f}{\left(\frac{v}{v + v_s}\right) f} = \frac{v + v_s}{v - v_s} = \frac{5}{4}$$

$$4(v + v_s) = 5(v - v_s)$$
$$9v_s = v$$
$$v_s = \frac{v}{9} = \frac{340}{9} = 37.8\,ms^{-1}$$

Example 17

Two stationary observers, A and B, situated at places some distance from each other along a straight railway track, are listening to a train's whistle which is being continuously sounded as a train travels at a speed of $90\,kmh^{-1}$ from A to B. If the whistle has a true frequency of $1000\,Hz$, and if the component of the wind velocity along AB is $30\,kmh^{-1}$ in the direction from A, calculate the apparent frequencies of the notes heard at A and B. Velocity of sound = $340\,ms^{-1}$

Solution

Velocity of the source, $V_s = \frac{90000}{3600}\,ms^{-1} = 25\,ms^{-1}$

Wind velocity, $V_w = \frac{30000}{3600}\,ms^{-1} = 8.3\,ms^{-1}$

$f = 1000\,Hz$

Since the wind velocity component is in the same direction as in which the source of sound is moving, velocity of sound relative to the ground is

$$V' = V + V_w = 340 + 8.3 = 348.3\,ms^{-1}$$

For A; The source of sound is moving away from the observer

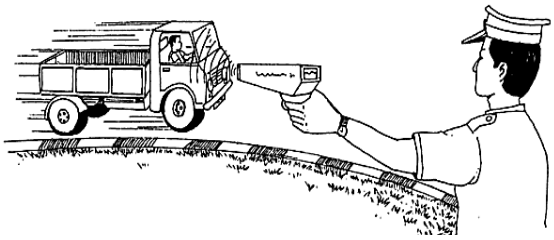
$$f' = \left(\frac{V'}{V' + V_s}\right) f$$
$$= \frac{348.3}{348.3 + 25} \times 1000$$
$$= 933\,Hz$$

For B; The source of sound is moving towards the observer

$$f' = \left(\frac{V'}{V' - V_s}\right) f$$
$$= \frac{348.3}{348.3 - 25} \times 1000$$
$$= 1077\,Hz$$

Applications of the Doppler effect

Radar speed traps



The speed of an approaching car or one speeding away can be found from the change in frequency of microwaves reflected from it that are emitted from a hand held speed gun in short bursts.

Consider a car moving with a speed, V, towards a stationary source of microwaves of frequency, f. The car acts as an observer towards a stationary source and the waves as received by the car have a frequency f'

$$V_s = 0 \text{ and } V_o = 0$$

$$f' = \left(\frac{c+V}{c}\right)f \text{ ……………(i)}$$

Waves of frequency, f' are reflected back to the source so that the car is now acting as a source moving with velocity, V and the radar set is acting as a stationary observer forward of the source. The waves on reaching the radar set have a frequency, f''.

$$V_s = V \qquad and \quad V_O = 0$$

$$f'' = \left(\frac{c}{c-V}\right)f' \text{ ……………(ii)}$$

Substituting for f' in equation (ii) gives;

$$f'' = \left(\frac{c}{c-V}\right)\left(\frac{c+V}{c}\right)f$$

$$f'' = \left(\frac{c+V}{c-V}\right)f$$

Fractional change in frequency,

$$\frac{\Delta f}{f} = \frac{f''-f}{f} = \frac{f''}{f} - 1$$

$$\frac{\Delta f}{f} = \left(\frac{c+V}{c-V}\right) - 1$$

$$\frac{\Delta f}{f} = \frac{2V}{c-V}$$

Since $V \lll c, \quad c - V \approx c$

$$\frac{\Delta f}{f} = \frac{2V}{c}$$

$$\boldsymbol{V = \frac{c\Delta f}{2f}}$$

Δf is the beat frequency of the waves transmitted and received by the radar set.

Example 18

In the determination of the speed of motor vehicles on a high way, a radar signal of frequency $2.43 \times 10^9\ Hz$ is sent out along the high way from a stationary transmitter. The signal is reflected from a vehicle receding from the transmitter and is detected by a stationary receiver close to the transmitter. If the speed of the vehicle is $100\ kmhr^{-1}$, deduce the apparent change in the wave length of the transmitter signal.

<u>Solution</u>

From $\boldsymbol{V = \frac{c\Delta f}{2f}}$

$$\Delta f = \frac{2Vf}{c} = \frac{2 \times 100 \times 10^3 \times 2.43 \times 10^9}{60 \times 60 \times 3.0 \times 10^8} = 450\ Hz$$

Measurement of plasma temperature (The Doppler broadening of spectral lines)

In a plasma (a gas at a very high temperature), the atoms move at very high velocities. The light emitted by the atoms will therefore be doppler-shifted. Since some atoms will be moving towards an observer and others away, the result will be that each line in the spectrum will be broadened.

The width $\Delta\lambda$ of a given line of wavelength λ is given by

$$\Delta\lambda = \frac{2v\lambda}{c}$$

where v is the r.m.s velocity of the atoms.

From the gas laws, we have

$$v = \sqrt{\frac{3RT}{M}}$$

where M is the molar mass of the gas, R the gas constant and T the absolute temperature of the gas.

Combining the two equations thus gives us a means of finding the temperature of the gas from the doppler broadening of the spectral lines.

Measurement of the speed of a star

When a distant light source such as a star moves away from us, the frequency of light from it is lowered. That is, the light is shifted toward the red (longer wavelength) end of the spectrum, an effect known as a Doppler red shift.

Similarly, the frequency of light from an object approaching us is increased i.e. the light is shifted toward the blue (shorter wavelength) end of the spectrum, producing a Doppler blue shift. The magnitude of the frequency shift is related to the speed of the source.

$$\text{Speed of star, } v = \frac{\Delta\lambda}{\lambda} \times c$$

Explanation

When the star is receding with a velocity v, the apparent wavelength λ' to the observer in line with the star's motion is

[Ans: $155\ Hz$]

21. A trumpet player on a moving railroad flatcar moves toward a second trumpet player standing alongside the track while both play a 440 Hz note. The sound waves heard by a stationary observer between the two players have a beat frequency of 4.0 beats/s. What is the flatcar's speed?
22. A siren emitting a sound of frequency 1000 Hz moves away from you toward the face of a cliff at a speed of 10 m/s. Take the speed of sound in air as 330 m/s.
    (a) What is the frequency of the sound you hear coming directly from the siren?
    (b) What is the frequency of the sound you hear reflected off the cliff?
    (c) What is the beat frequency between the two sounds?

    [Ans: (a) $970\ Hz$ (b) $1.0\ kHz$ (c) $60\ Hz$]
23. Two identical tuning forks can oscillate at 440 Hz. A person is located somewhere on the line between them. Calculate the beat frequency as measured by this individual if (a) she is standing still and the tuning forks move in the same direction along the line at 3.00 m/s, and (b) the tuning forks are stationary and the listener moves along the line at 3.00 m/s.

    [Ans: (a) $7.70\ Hz$ (b) $7.70\ Hz$]
24. A police car is chasing a speeding Porsche. Assume that the Porsche's maximum speed is 80.0 m/s and the police car's is 54.0 m/s. At the moment both cars reach their maximum speed, what frequency will the Porsche driver hear if the frequency of the police car's siren is 440 Hz? Take the speed of sound in air to be 340 m/s.

    [Ans: $400\ Hz$]
25. A listener at rest (with respect to the air and the ground) hears a signal of frequency f_1 from a source moving toward him with a velocity of 15 m/s, due east. If the listener then moves toward the approaching source with a velocity of 25 m/s, due west, he hears a frequency f_2 that differs from f_1 by 37 Hz. What is the frequency of the source? (Take the speed of sound in air to be 340 m/s.)

    [Ans: $4.8 \times 10^2\ Hz$]
26. A train approaching a hill at $36\ kmh^{-1}$ sounds a whistle of $580\ Hz$. Wind is blowing at $72\ kmh^{-1}$ in the direction of motion of the train. Calculate the frequency of the whistle as heard by an observer on the hill. (Speed of sound in air is $340\ ms^{-1}$)

    [Ans: $596.6\ Hz$]
27. A police radar set emits a parallel beam of electromagnetic radiation at wavelength λ_o and velocity c, which falls on a motor-car moving directly towards the set with a velocity u.
    (a) Derive an expression for the time it takes a wave front of the radiation initially a distance λ_o from the car, to reach the car.
    (b) Derive an expression for the wavelength λ of the radiation reflected from the car.
    (c) If $\lambda_o = 0.10\ m$, $c = 3.0 \times 10^8\ ms^{-1}$, calculate the change in wavelength of the radiation received at the set after reflection from the car.

    [Ans: (c) $2.2 \times 10^{-8}\ m$]
28. A star is moving away from the earth at a speed of $4.12 \times 10^5\ ms^{-1}$ relative to the earth. Calculate the wavelength shift observed on the earth for light of wavelength $5.15 \times 10^{-7} m$ emitted by the star. ($c = 2.998 \times 10^8\ ms^{-1}$)

    [Ans: $7.1 \times 10^{-10}\ m$]
29. The sun rotates about its centre. Observation of the light from the two edges at the ends of a diameter shows a Doppler shift of about $0.008\ nm$ for a wavelength $600\ nm$. Estimate the angular velocity of the sun about its centre, given that its radius is $7.0 \times 10^8\ m$ and $c = 3.0 \times 10^8\ ms^{-1}$.

    [Ans: $2.9 \times 10^{-6}\ rad\ s^{-1}$]
30. A source is emitting a note of constant frequency 512 Hz. What will be the apparent frequency of the note heard by an observer (a) if he is stationary and the source is approaching him with a velocity of 50 kmh^{-1}, (b) if the source is stationary and the observer is moving towards it with a velocity of 75 kmh^{-1} a nd (c) if both source and observer are moving towards each other with velocities of 50 kmh^{-1} and 75 kmh^{-1} respectively ? Develop all formulae used. (Velocity of sound = 340 ms^{-1}.)

    [Ans: (a) 600 Hz (b) 625 Hz (c) 733 Hz]
31. An engine, A, is moving away from a stationary engine, B, and meantime the two engines are continuously sounding their whistles, both of pitch 1000 Hz. If the driver of engine A hears beats of frequency 8 s^{-1}, what is the speed of his engine? Are the beats heard by the driver of engine B of the same frequency as those heard by the driver of engine A? (Velocity of sound in air = 349 ms^{-1})

    [Ans: 2.72 ms^{-1}; no; B hears beats of frequency 7.8 s^{-1}]

WAVE THEORY

In geometric optics, we used the ray model of light to explain the macroscopic phenomena like reflection of light, refraction of light, etc. and we largely ignored the wave properties of light and assumed that light travels in straight lines. This assumption is reasonable because the wavelength of light is extremely small and its wave properties are detected only under special circumstances.

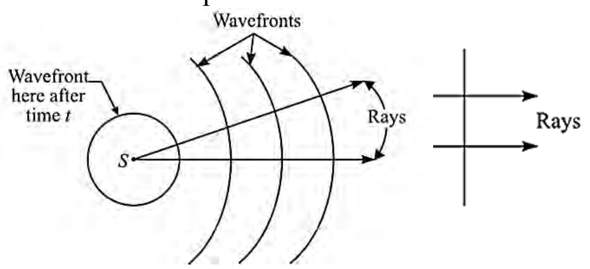
The microscopic phenomena of light like interference, diffraction and polarization are exhibited only by waves and cannot be explained by ray optics. It is now well known that light waves are electromagnetic waves which travel with different speeds in different media.

Wave front

A source of light sends out disturbance in all directions. In a homogeneous medium, the velocity of light is the same in all directions. Therefore, all the particles of the medium at the same distance from the source of light vibrate in the same phase. The locus of all such particles vibrating in the same phase is called the wave front

A wave front is a surface or line in the path of wave motion on which all the particles are in phase.

The figure below shows a point source of light **S** sending out spherical waves concentric with the source. Each arc represents a surface over which the phase of the wave is constant because each point on the surface is at the same distance from the source. Such a surface of constant phase is the wave front.



A ray is a line which represents the direction of travel of the wave and it is at right angles to the wave fronts

At large distance from the source **S**, the rays are nearly parallel and the wave fronts are substantially plane. Light from the sun reaches the earth in plane wave fronts because sun is very far off.

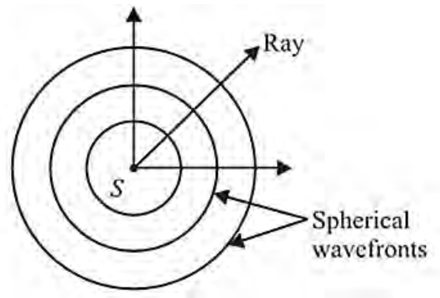
Types of wave fronts

Depending upon the shape of the source of light, the wave front can be of three types namely

(i) spherical wave front
(ii) cylindrical wave front
(iii) plane wave front

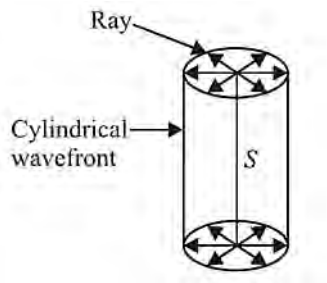
Spherical wave front

When the source of light is a point source **S,** the wave front is spherical with centre at the source as shown below. It is because the locus of all such points which are equidistant from the point source **S** is a sphere.



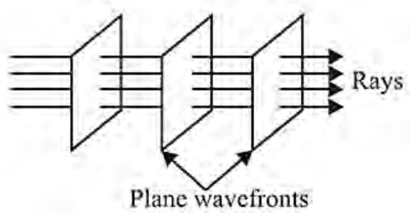
Cylindrical wave front

When the source of light **S** is linear in shape (such as slit), the wave front is cylindrical in shape as shown below. It is because the locus of all such points which are equidistant from the linear source lie on the surface of a cylinder.



Plane wave front

When a spherical wave front from a point source or a cylindrical wave front from a linear source advances, the curvature of the wave front decreases. Therefore, a small portion of the spherical wave front or cylindrical wave front at a large distance from the source appears to be plane. Such a wave front is called plane wave front.



Optical path

When light travels through a medium of refractive index n, and the distance travelled is L, the optical path is given by nL.

Optical path is the product of the refractive index of the medium and the geometrical path length in air or vacuum.

From $\tan\theta = \frac{t}{L}$

$t = L\tan\theta$

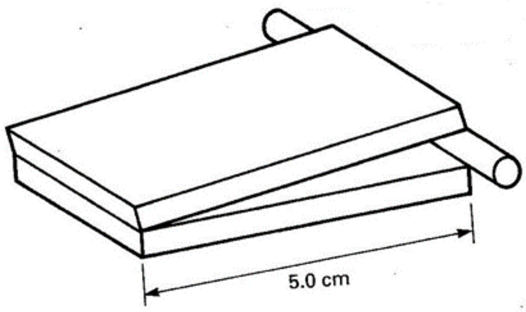
If θ is in radians, $t = L\theta$

Note

1. The angle, θ of the wedge has to be small such that the fringes are sufficiently far apart to be observable.
2. The fringes are clearer at the apex than at the thick end of the wedge. This is because the path differences near the thick end are many wavelengths in length.
3. The air wedge can be used in testing of optical components. For example, in making of optical flats, the plate under test is made to form an air wedge with a standard plane glass surface. Any uneven parts of the surface which require more grinding will show up as irregularities in what should be a parallel equally spaced, straight set of fringes.

Example 1

A wedge shaped film of air is formed between two thin parallel-sided optically flat glass plates as shown in the diagram below.



When the film is illuminated with monochromatic light of wavelength 550 nm, a series of bright and dark fringes is observed.

(a) Calculate the distance between adjacent bright fringes when viewed from above for a wire of 0.05 mm diameter.

(b) If the wedge shaped space between the plates is now filled with a transparent liquid of refractive index 1.2, what would be the new distance between alternate bright fringes?

<u>Solution</u>

(a) Diameter of the wire = $t = 0.05\times10^{-3}m$

$L = 5.0\,cm = 5\times10^{-2}m, \lambda = 550\times10^{-9}m$

$t = L\theta$

But $\theta = \frac{\lambda}{2d}$

Thus, $t = \frac{L\lambda}{2d} \Rightarrow d = \frac{L\lambda}{2t}$

$$d = \frac{550\times10^{-9}\times5\times10^{-2}}{2\times0.05\times10^{-3}} = 2.75\times10^{-4}m$$

(b) From $2\mu t = n\lambda$; $\mu = 1.2$

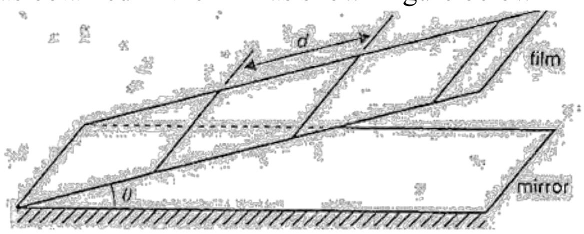
$\Rightarrow n = \frac{2\mu t}{\lambda} = \frac{2\times1.2\times0.0\times10^{-3}}{550\times10^{-9}} = 218.2$ lines

Separation of the fringes,

$d = \frac{L}{n} = \frac{5.0\times10^{-2}}{218.2} = 2.29\times10^{-4}m$

Example 2

In a certain experiment to demonstrate the occurrence of antinodes, a thin photographic film was placed at a very small angle θ to the mirror. A pattern of dark lines was obtained in the film as shown figure below



The wave length of the light used is 450 nm and the film is set at an angle of 4.3×10^{-3} degrees to the mirror.

(i) Calculate the distance, d between the adjacent dark lines on the processes film.

(ii) Describe what would happen to the pattern of lines if the angle between the film and the mirror was increased.

<u>Solution</u>

(i) Given $\lambda = 450\times10^{-9}m, \theta = 4.3\times10^{-3}$ deg

From $d = \frac{\lambda}{2\,tan\,\theta} = \frac{450\times10^{-9}}{2\times\tan(4.3\times10^{-3})}$

$= 2.998\times10^{-3}m$

(ii) Increase in the angle would reduce the distance between the pattern lines. They would become invisible as they come closer apart. ($d \propto \frac{1}{\tan\theta}$)

Example 3

An air wedge is formed by placing a sheet of foil between the edges of two glass plates $75\,mm$ from their point of contact. When the wedge is illuminated with light of wavelength $5.8\times10^{-7}\,m$, the fringes are $1.30\,mm$ apart. Calculate the thickness of the foil.

Solution

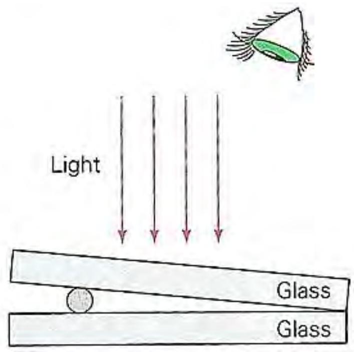
Number of bright bands in air wedge = $\frac{75}{1.30} = 57.7$

Change in vertical height from one fringe to the next

$= \frac{\lambda}{2}$

interference fringes using a double slit and any other essential apparatus

4. In a Young's double slit experiment, sodium light of wavelength $0.59 \times 10^{-6}\ m$ was used to illuminate a double slit with separation $0.36\ mm$. If the fringes are observed at a distance of $30\ cm$ from the double slits, calculate the fringe separation.
[Ans: $0.49\ mm$]
5. An interference pattern is formed when light whose wavelength is $550\ nm$ is incident on two parallel slits $50\ \mu m$ apart. The second-order bright fringe is $3.0\ cm$ from the centre of the central maximum. How far from the slits is the screen on which the pattern is formed?
[Ans: $1.4\ m$]
6. A beam of monochromatic light of wavelength $6.0 \times 10^{-7} m$ in air passes into glass of refractive index 1.5. If the speed of light in air is $3.0 \times 10^{8}\ ms^{-1}$, calculate
(a) the speed of light in glass
(b) the frequency of the light
(c) the wavelength of the light in glass
[Ans: (a) $2.0 \times 10^{8}\ ms^{-1}$ (b) $5.0 \times 10^{14}\ Hz$ (c) $4 \times 10^{-7}\ m$]
7. Monochromatic light illuminates a narrow slit which is $4.0\ m$ away from a screen. Two very narrow parallel slits $0.50\ mm$ apart are placed midway between the single slit and the screen so that interference fringes are obtained. If the spacing of five fringes is $10\ mm$, calculate the wave length of light.
[Ans: $5.0 \times 10^{-7}\ m$]
8. Explain what is meant by the term path difference with reference to the interference of two-wave motions
Why is it not possible to see interference where the light beams from the head lamps of a car overlap?
9. In Young's slits method, the wavelength of the light being used is $6.0 \times 10^{-5}\ cm$. When a film of material $3.6 \times 10^{-3}\ cm$ thick was placed over one of the slits, the fringe pattern was displaced by a distance equal to 30 times that between two adjacent fringes. Calculate the refractive index of the material. To which side are the fringes displaced?
[Ans: 1.5; to side of covered slit]
10. Blue light with a wavelength of $440\ nm$ is used in a double slit experiment where the slits are $0.50\ mm$ apart. If the screen is $1.5\ m$ from the slits, what is the angular separation of the first-order and third-order bright fringes?
[Ans: 1.8×10^{-3} rad]
11. Light of two different wavelengths is used in a double slit experiment. The location of the third bright fringe for yellow light of wavelength $600\ nm$ in air coincides with the location of the fourth bright fringe for the other light. What is the wavelength of the other light?
[Ans: $450\ nm$]
12. A lens of refractive index 1.6 is to be coated with a material of refractive index 1.4 that will make it nonreflecting for red light of wavelength $700\ nm$ normally incident on the lens. What is the minimum required thickness of the coating?
[Ans: $1.3 \times 10^{-7}\ m$]
13. Two microscope slides, each $7.5\ cm$ in length are placed with two of their faces in contact. A cover glass is then inserted between them at one end to enclose a wedge-shaped layer of air. When illuminated normally by sodium light and reflected light viewed, parallel interference bands are seen at a uniform distance apart of $0.11\ mm$. Explain the formation of these bands and calculate the thickness of the cover glass. (Wavelength of sodium light $= 589\ nm$)
[Ans: $0.20\ mm$]
14. An air wedge is shown in the diagram below whose top glass is illuminated with monochromatic light.



(a) Describe the observed interference pattern
(b) Express the locations of the bright interference fringes in terms of wedge thicknesses measured from the apex to the wedge
(c) Show that the number of dark fringes is given by
$$n = \frac{2t}{\lambda_{air}}$$
where t is the maximum thickness of wedge
(d) The glass plates are separated by a thin, round filament. When the top plate is illuminated

POLARIZATION OF WAVES

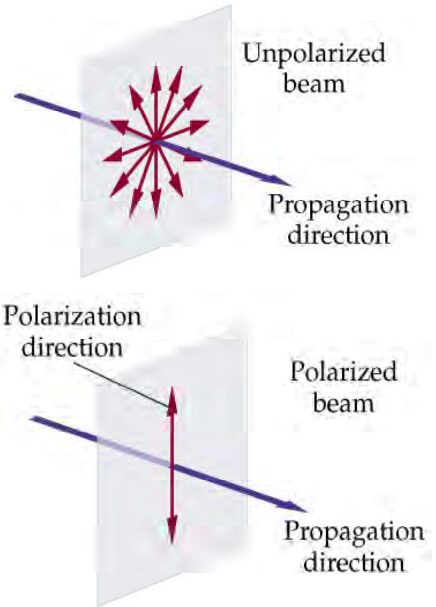
Polarization is the restriction of the vibrations in a wave so that the vibrations occur in a single plane. It is a property exhibited by transverse waves only and does not occur for longitudinal waves such as sound waves. The fact that light can be polarized shows that light waves are transverse waves.

Unpolarized and plane polarized wave

An **unpolarized wave** is a wave where the oscillation is not limited to directions perpendicular to the direction of propagation.

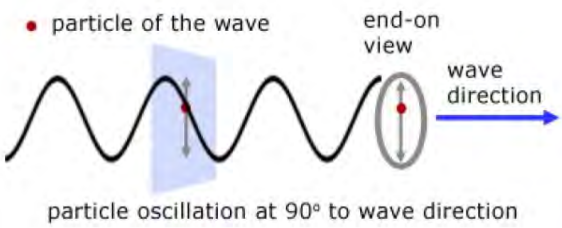
A **plane polarized wave** is a transverse wave whose vibrations are in a single plane which contains the direction of propagation of the wave.

A **plane of polarization** of light is one in which the electric vector of the polarized light varies or vibrates.

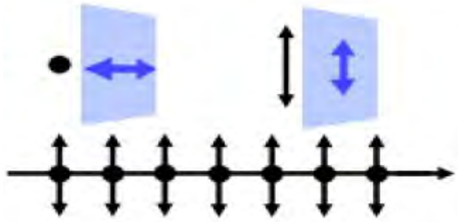


Polarization of light waves

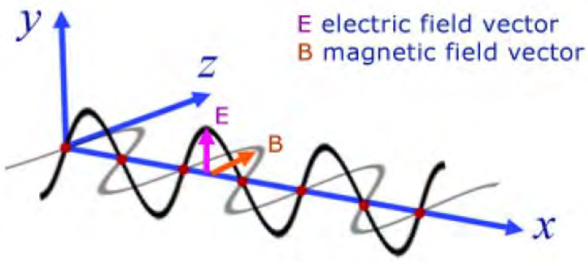
Light waves are transverse waves where particles of the wave oscillate in a line at right angles to the direction of travel.



In a light beam, there are many waves with lines of oscillation set at random angles. Light waves can be represented in the simplest form i.e. only vertical and horizontal waves are represented when explaining various phenomena around polarization.



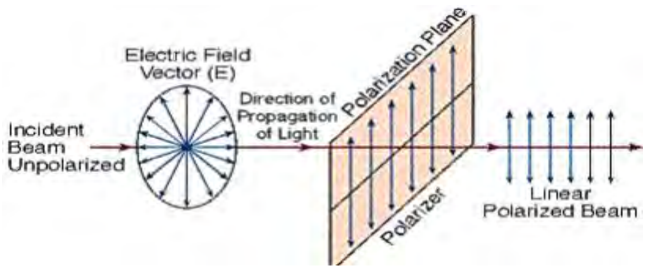
A light wave is actually two waves i.e. in phase and oriented at $90^o$to each other. One wave is E, the electric field vector and the other is B, the magnetic field vector.



When interacting with mater, the E wave is more important than the B wave. Thus for simplicity the B wave is ignored.

POLAROID

A Polaroid is a material, usually plastic which allows light to pass but only where the wave oscillates in one particular orientation. This orientation is called the reference direction.



A Polaroid strongly absorbs light in one plane while easily allowing light to pass through in another plane at right angles to the first.

Polaroid is used in sunglasses where the horizontal vibrations are absorbed as light are normally reflected from horizontal surfaces.

Polarization by reflection

When light is reflected from the surface of water or a piece of glass is observed through a piece of Polaroid which is rotated about the axis of vision, the intensity of the transmitted light is seen to vary. This shows that reflected light is plane polarized.

The effect is greatest when the angle of incidence i at the reflecting surface satisfies the equation.

$\tan \boldsymbol{i} = \boldsymbol{n}$, the refraction index of the material

This is known as **Brewster's law**.

EXAMINATION QUESTIONS

1. (a) (i) Define the following as applied to wave motion
   - Amplitude
   - Frequency
   - Wavelength

   (ii) Derive the relationship between velocity, wavelength and frequency of a wave

   (b) The displacement y, of a progressive wave is given as

$$y = 2\cos\pi\left(t - \frac{x}{20}\right)$$

   where x is the horizontal distance in metres and t is the time in seconds.
   Determine the:
   (i) velocity of the wave
   (ii) maximum velocity of the particles in the medium

   (c) (i) What is meant by **Doppler effect?**
   (ii) A source of sound moving with a velocity u, approaches an observer moving with velocity u_0 in the same direction. Derive the expression for the frequency of the sound heard by the observer.

   (d) Two whistles are sounded simultaneously. The wave lengths of the sounds emitted are $5.5\ m$ and $6.0\ m$ respectively. Find the beat frequency if the speed of sound in air is $330\ ms^{-1}$.

[2018, No.3; Ans: (b) (i) $20\ ms^{-1}$ (ii) 2π rad s^{-1} (d) $5\ Hz$]

2. (a) What is meant by the following as applied to light waves
   (i) Diffraction
   (ii) Polarization

   (b) A diffraction grating of spacing d, is illuminated normally with light of wavelength λ
   (i) Derive the condition for occurrence of diffraction maxima
   (ii) Describe briefly the intensity distribution on a screen placed beyond the grating.
   (iii) What is the effect on the diffraction pattern when a grating with a larger number of lines is used?

   (c) Light of wavelength $5.8 \times 10^{-7}\ m$ is incident on a diffraction grating of 500 lines per mm. Find the:
   (i) diffraction angle for the 2nd order image
   (ii) maximum number of images formed.

   (d) (i) Describe how polarised light can be produced by reflection
   (ii) List any **four** uses of polarized light

[2018, No.4; Ans: (c) (i) $35.45°$ (ii) 3]

3. (a) Define the following as applied to a wave
   (i) Amplitude
   (ii) Wavelength

   (b) (i) State the conditions necessary for the formation of a standing wave
   (ii) A string fixed at both ends is made to vibrate in two different modes. If the frequencies of the n^{th} harmonic and the fundamental note are f_n and f_1 respectively, show that

$$f_n = nf_1$$

   (c) The mass of a vibrating length of a sonometer wire is 1.20 g. A note of frequency $512\ Hz$ is produced when the wire is sounding its second overtone. If the tension in the wire is $100\ N$, calculate the vibrating length of the wire.

   (d) Explain why the quality of a note from an open pipe is preferred to that given by a closed pipe.

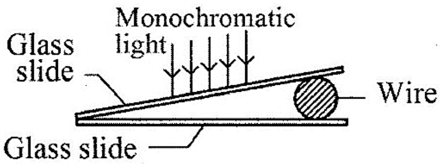
   (e) Describe an experiment to investigate the variation of frequency with length of a vibrating wire.

[$2017, No. 3$; Ans: (c) $71.5\ cm$]

4. (a) Define optical path

   (b) With reference to Young's double slit experiment,
   (i) explain how an interference pattern is formed
   (ii) state what happens to the fringes when the source is moved nearer to the slits
   (iii) state what happens to the fringes when separation of the slits is changed
   (iv) describe the appearance of the fringes when white light is used
   (v) calculate the separation of the slits if the distance from the slits to the screen is $800\ mm$ and the 8^{th} bright fringe is formed $5\ mm$ away from the centre of the fringe system, given that the wavelength of light is $6.2 \times 10^{-7}\ m$

   (c) An air wedge is formed by placing two glass slides of length $5.0\ cm$ in contact at one end and a wire at the other end as shown below



SECTION C: MAGNETISM & A.C CIRCUITS

Magnetism

- Magnetic field and magnetic field lines
- Comparing the strength of two magnetic fields
- Earth as a magnet
- Magnetic field patterns
- Neutral point
- Molecular/domain theory of magnetism
- Magnetic saturation
- Magnetic flux and magnetic flux density
- The Tesla
- Magnetic effect of an electric current
- Dependence of field at centre of a circular coil on N and R
- Direction of magnetic field due to current
- Magnetic field patterns around a current carrying conductor, circular coil and solenoid
- Magnetic flux linkage and flux density
- Measurement of flux density
- Expressions of B
- Force on a current carrying conductor
- Direction of the force (Fleming's left-hand rule)
- Expression of force
- Force between two long parallel current carrying conductors
- The ampere
- Current balance (absolute determination of current)
- Force on a charge moving in a magnetic field
- The hall effect
- Expression of the hall voltage
- Torque on a coil in a magnetic field
- The moving coil galvanometer
- Modifications of moving coil galvanometer to ballistic galvanometer
- Sensitivity of moving coil galvanometer
- Moving coil loudspeaker
- D.C motor
- Electromagnetic induction
- Electricity from magnetism
- Fleming's right hand rule (direction of induced e.m.f)
- Laws of electromagnetic induction (Faraday's and Lenz's laws)
- Experiment to demonstrate the laws
- Induced e.m.f
- A.C and D.C generators
- Production of back e.m.f in a motor coil
- Relationship between speed of rotation of the coil and the current flowing in the coil
- Eddy currents
- Production and applications of eddy currents
- Self and mutual induction
- The transformer
- Energy losses in a transformer
- Absolute determination of resistance
- Power transmission

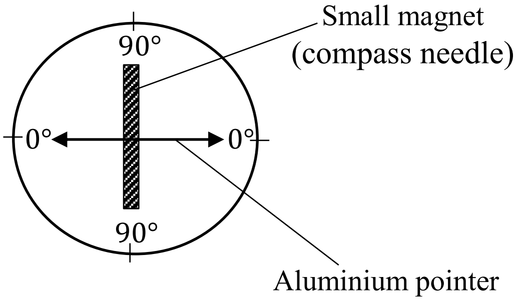
A.C circuits

- Terminologies of A.C
- Measurement of A.C
- Hot wire ammeter(thermal)
- Moving iron meter (attractive and repulsive)
- Rectifier type of ammeter
- Root mean square (r.m.s) value of an A.C
- Peak value
- A.C through a resistor, inductor and capacitor
- Lead and lag
- L, R and C circuits
- Condition for resonance
- Impedance
- Energy and power in A.C circuits
- Sparking in inductive circuits
- Average power in inductor, capacitor and resistor
- Applications of capacitors, inductors in A.C circuits

The Deflection magnetometer

It consists of a small compass needle (small magnet) which is pivoted on a vertical axis and carries a light aluminium pointer.

The pointer can rotate over a circular scale.



The deflection magnetometer is used to compare two magnetic field flux densities, one being the horizontal component of the earth's magnetic field B_H.

The two fields i.e. B_H and any other field to be compared are arranged at right angles to each other.

The compass needle sets itself at an angle θ to its initial direction when it was in field of B_H alone.

The needle now points in the direction of the resultant field of B_H and B.

The angles of deflection θ_1 and θ_2 of the needle are measured.

The average deflection $\theta = \frac{\theta_1+\theta_2}{2}$ is determined.

The ratio $\frac{B}{B_H} = \tan\theta$ is obtained.

Investigation of the variation of *B* at the centre of a circular coil with *I* using a deflection magnetometer

The circular coil is placed with its plane in the earth's magnetic meridian.

A deflection magnetometer is mounted on the vertical axis at the centre of the circular coil, with its pointers initially at zero marks.

The coil is then connected through a reversing switch to a battery, rheostat and an ammeter.

The rheostat is adjusted to read a suitable value of I.

The deflections θ_1 and θ_2 are noted. The current I is then reversed and the deflections θ_3 and θ_4 are noted.

The procedure is repeated for other suitable values of I and results tabulated including values of $\theta = \frac{\theta_1+\theta_2+\theta_3+\theta_4}{4}$ and $\tan\theta$

A graph of $\tan\theta$ against I is plotted, and it is linear. This shows that that $\tan\theta$ is proportional to I.

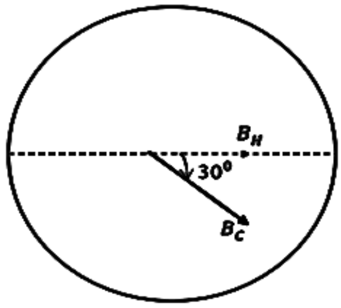
Since, $B = B_H\tan\theta$, where B_H is the horizontal component of the earth's magnetic flux density.

This implies that $B \propto I$

Example 1

A circular coil of 50 turns and mean radius 70 cm is set with its plane in the magnetic meridian. There is a small pivoted magnetic needle at its centre. When a current of 0.022 A is passed through the coil, it is found that the coil must be rotated through 30° before the magnet is once again in the plane of the coil. Calculate the horizontal component of the earth's magnetic field.

Solution



$$B_C = \frac{\mu NI}{2R} = \frac{4\pi\times10^{-7}\times 50\times0.023}{2\times0.7} = 1.03\times10^{-6}\ T$$

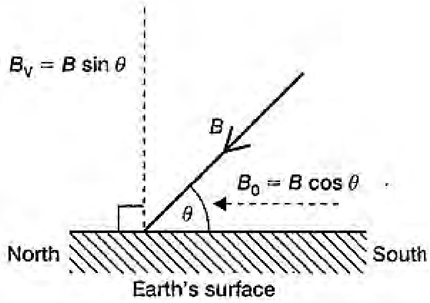
$$\cos 30° = \frac{B_H}{B_C} \Rightarrow B_H = B_C\cos 30°$$

$$B_H = 1.03\times10^{-6}\times\cos 30° = 8.9\times10^{-7}\ T$$

Example 2

Calculate the magnitude and direction of the force per metre length on a straight, horizontal wire lying with 2.0 A flowing through it in direction north to south if the Earth's horizontal field component is $1.6\times10^{-5}\ T$ and the angle of dip is 70°.

Solution



$$B_V (= B\sin\theta) = \frac{B_0\sin\theta}{\cos\theta} = B_0\tan\theta$$

$$B_V = 1.6\times10^{-5}\times\tan 70° = 4.4\times10^{-5}\ T$$

$$F = B_V IL = 4.4\times10^{-5}\times 20\times 1$$

$$= 8.8\times10^{-4}\ N$$

By the left-hand rule, with B_V downwards, F is eastwards.

Example 3

A coil of 50 turns and radius 4cm is placed with its plane within the earth's magnetic meridian. A compass needle is placed at the centre of the coil. When a

MAGNETIC EFFECT OF AN ELECTRIC CURRENT

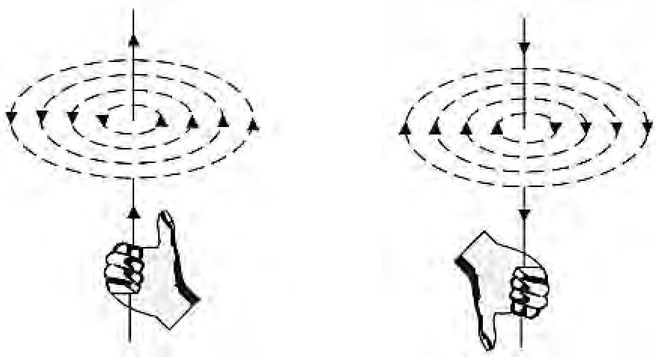
Magnetism can be caused by movement of electric charge hence an electric current being a flow of charge produces a magnetic field. If the current is flowing in a wire, the shape of the magnetic field is dependent on the configuration.

1. Current in a straight wire or conductor

For a long straight conductor/wire carrying current, the magnetic field lines of force are concentric circles with the conductor as the centre and lying in a plane perpendicular to the straight conductor. The direction of the magnetic field lines due to a straight carrying conductor can be found by one of the following two rules

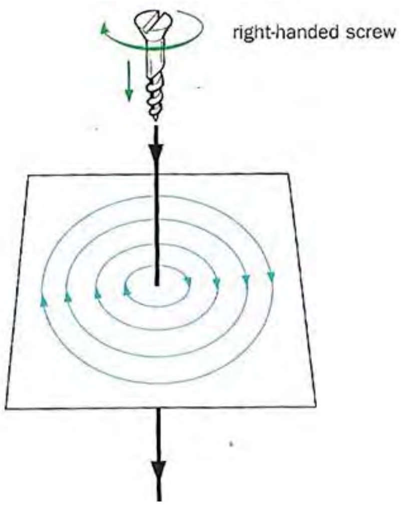
a. Right-hand grip rule

Grip the straight conductor with your right hand with the thumb pointing in the direction of current. Then curled fingers point in the direction of the magnetic lines of force.



b. Right-hand screw rule

If a right-handed screw moves forward in the direction of current (conventional), then the direction of rotation of the screw gives the direction of the magnetic field lines.

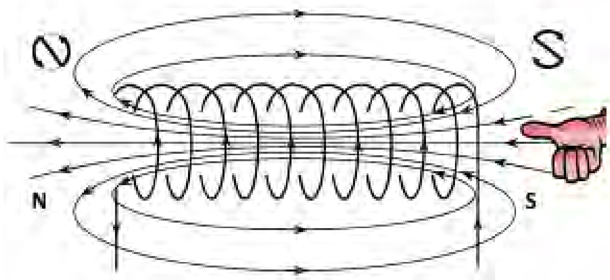


Note

- The magnetic field is strong in the region around the wire and weakens with increasing distance.
- The larger the current, the stronger is the magnetic field.

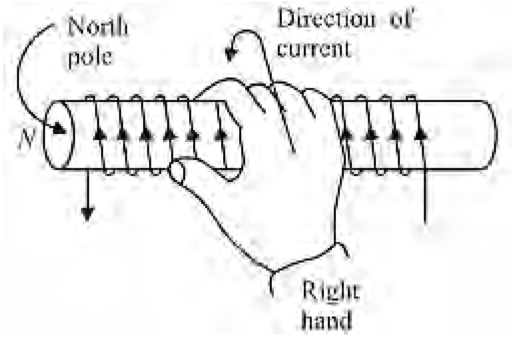
2. Current in a solenoid

A solenoid consists of a length of insulated wire coiled into a cylinder shape.



Current in the solenoid produces a stronger magnetic field inside the solenoid than outside. The field lines in this region are parallel and closely spaced showing that the field is highly uniform in strength and direction.
Field lines outside the solenoid are similar to those of a bar magnet and it behaves in a similar way as if it has a north pole at one end and a south pole at the other end. Strength of the field diminishes with distance from the solenoid.

Direction of the magnetic field



Imagine to grasp the solenoid with right hand so that the fingers are curled in the direction of current. The thumb stretched parallel to the axis of the solenoid will point towards the N-pole end of the solenoid

Variation of the magnetic field along the axis of the solenoid

A search coil connected to a ballistic galvanometer is inserted in the solenoid and moved from one end of the solenoid to the other along its axis.
The average deflection, θ on the galvanometer at various points is noted when the current is reversed in the solenoid.

Example 6

An electron revolves in a circular orbit of radius $4.0 \times 10^{-10}\ m$ at a frequency of 7.5×10^{15} revolutions per second. Calculate the magnetic flux density at the centre of the orbit.

<u>Solution</u>

The circulation of the electron constitutes a current.

$$I = \frac{ne}{t} = \frac{e}{t} \text{ as } n = 1$$

$$I = e\left(\frac{1}{t}\right) = ef$$

Assume that the magnetic field provided by the revolving electron is in relation with that at the centre of a circular coil of 1 turn

$$B = \frac{\mu_o NI}{2R}$$

$$B = \frac{\mu_0 ef}{2R}$$

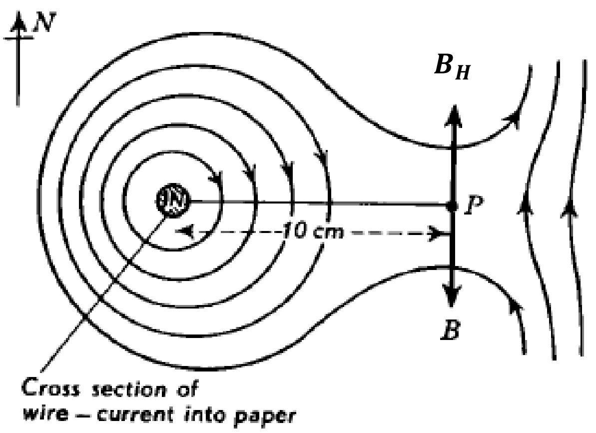
$$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 7.5 \times 10^{15}}{2 \times 4.0 \times 10^{-10}}$$

$$= 1.9\ T$$

Example 7

A current is passing down a long vertical wire situated in the Earth's magnetic field. If a neutral point is located at a perpendicular distance of 10 cm from the wire, what is the value of the current? (horizontal component of the earth's magnetic field $= 1.8 \times 10^{-5}\ T$)

<u>Solution</u>



The diagram shows a horizontal section of the wire and the con-figuration of the flux flow due to the current and the Earth's field. A point of zero flux intensity (neutral point) is shown at P, 10 cm due east of the wire at which position the flux intensities due to the wire (B) and the Earth's horizontal field (B_H) are equal and opposite. Thus,

$$B = B_H = 1.8 \times 10^{-5}\ T$$

Now the flux intensity B at a distance d from a long wire carrying a current I is given by

$$B = \frac{\mu_o I}{2\pi d} = \frac{4\pi \times 10^{-7} I}{2\pi \times 0.1} = 2I \times 10^{-6}$$

$$2I \times 10^{-6} = 1.8 \times 10^{-5}$$

$$I = 9\ A$$

Self-Evaluation exercise

1. What current in a long, straight wire will produce a magnetic field with a magnitude $10.0\ \mu T$ at a perpendicular distance of 40.0 cm from the wire?
[Ans:]
2. The magnetic field at the centre of a 50-turn coil of radius 15 cm is $0.60\ mT$. Find the current in the coil
[Ans: 2.9 A]
3. A solenoid 50 cm long with 1000 turns per metre has a central magnetic field of $1.5\ mT$. Find the current in the coil.
[Ans: 1.2 A]
4. A 2000 turn solenoid of length 40 cm and resistance $16\ \Omega$ is connected to a $20\ V$ supply. What is the flux density at the mid-point of the axis of the solenoid?
[Ans: $7.9 \times 10^{-3}\ T$]
5. A long, straight wire has a resistance of $2.0\ \Omega$. What potential difference between its ends will produce a magnetic field of $1.0 \times 10^{-4}\ T$ at a distance of 4 cm from the wire?
[Ans: 40 V]
6. How many turns should a solenoid 30 cm long have to produce a magnetic field of $5.0\ mT$ at its axis when operating with a current of $5.0\ A$?
[Ans: 240]
7. A solenoid 10 cm long has 3000 turns of wire and carries a current of 5.0 A. A 2000 turn coil of wire of the same length of the solenoid surrounds the solenoid and is concentric with it. If the outer coil carries a current of 10 A in the same direction as that of the current in the solenoid, find the magnetic field at the centre of both coils.
[Ans:]
8. A solenoid having 200 turns per metre and carrying a current of $0.050\ A$ lies with its axis east-west. Well inside the solenoid is a small compass whose needle points 37° west of north. Calculate the horizontal component of the earths's magnetic field.
[Ans: $1.7 \times 10^{-5}\ T$]
9. A circular coil of 100 turns has a radius of 10 cm and carries a current of 5 A. Determine the magnetic field at the centre of the coil.

$$\frac{F}{L} = IB\sin 0° = 0$$

Example 5

A horizontal straight wire 5 cm long of mass 1.2 gm^{-1} is placed perpendicular to a uniform magnetic field of flux density 0.6 T. If the resistance of the wire is 3.8 Ωm^{-1}, calculate the p.d that has to be applied between the ends of the wire to make it just self-supporting.

Solution

The current, I in the wire is to be in such a direction that the magnetic force acts on it vertically upward. To make the wire self-supporting, its weight should be equal to the upward magnetic force i.e.

$$BIL = mg$$
$$I = \frac{mg}{BL}$$
$$m = 1.2 \times 10^{-3}\,L,\ B = 0.6\,T$$
$$I = \frac{(1.2 \times 10^{-3}L) \times 9.8}{0.6 \times L} = 19.6 \times 10^{-3}\,A$$

Resistance of wire, $R = 0.05 \times 3.8 = 0.19\,\Omega$

Required p.d, $V = IR = (19.6 \times 10^{-3}) \times 0.19$
$= 3.7 \times 10^{-3}\,V$

Example 6

A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?

Solution

$m = 200\text{ g} = 0.2\,kg$, $I = 2\,A$, $L = 1.5\,m$, $B = ?$

As the wire is suspended in mid-air by magnetic field,

$$mg = BIL$$
$$B = \frac{mg}{IL} = \frac{(0.2)(9.81)}{2(1.5)} = 0.65\,T$$

Example 7

A wire of radius 0.20 cm and density 2000 kgm^{-3} is placed in a magnetic field. If the wire can carry a current of 500 A, what is the strength of the magnetic field that it must be placed in so that it can float by itself?

Solution

For the wire to be in equilibrium, weight of the wire = force of magnetic field

$$mg = BIL$$
$$B = \frac{mg}{IL} = \frac{\rho V g}{IL} = \frac{\rho A L g}{IL} = \frac{\rho \pi r^2 g}{I}$$
$$B = \frac{2000\pi(0.002)^2(9.81)}{500} = 4.93 \times 10^{-4}T$$

Example 8

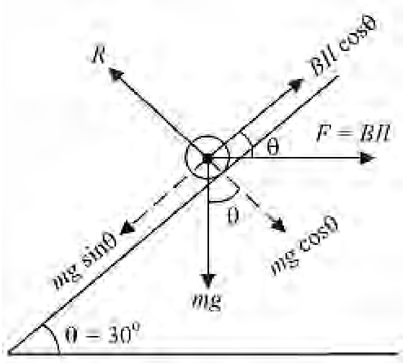
On a smooth inclined plane inclined at 30° with the horizontal, a thin current-carrying metal rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0.15 T in a vertical direction. For what value of current can the rod remain stationary given that its mass per unit length is 0.30 kg/m.

Solution

The forces acting on the current-carrying metallic rod are

(i) weight of the rod mg acting vertically downward

(ii)the magnetic force BIl

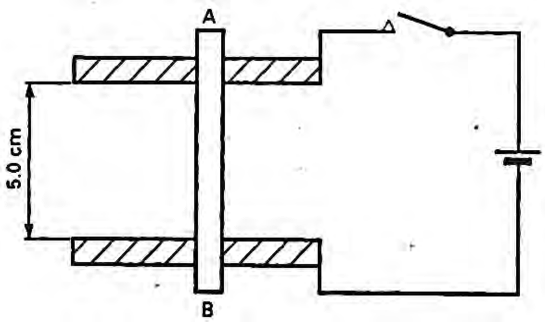


The component $mg\sin\theta$ tends to roll the rod down the plane while the component $BIl\cos\theta$ tends to move it up along the plane. For the rod to be stationary, these two components must be equal.

$$BIl\cos\theta = mg\sin\theta$$
$$I = \frac{mg\tan\theta}{Il}$$
$$m = 0.3 \times l\text{ kg},\ \theta = 30°,\ B = 0.15\,T$$
$$I = \frac{(0.3l) \times 9.81 \times \tan 30°}{0.15l} = 11.3\,A$$

Example 9

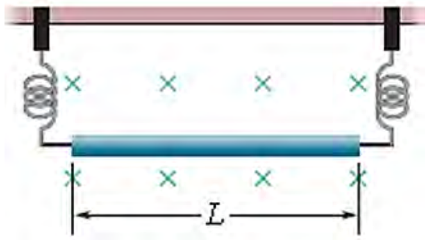
The diagram represents a cylindrical aluminum bar AB resting on two horizontal aluminum rails which can be connected to a battery to drive a current through AB.



A magnetic field of flux density 0.10 T acts perpendicularly to the paper and on to it

(a) In what direction will AB move if the current flows from A to B?

8. A 13.0 g wire of length $L = 62.0\ cm$ is suspended by a pair of flexible leads in a uniform magnetic field of magnitude $0.440\ T$.



What are the
   (a) magnitude and
   (b) direction of the current required to remove the tension in the supporting leads?

[Ans: (a) $467\ mA$ (b) right]

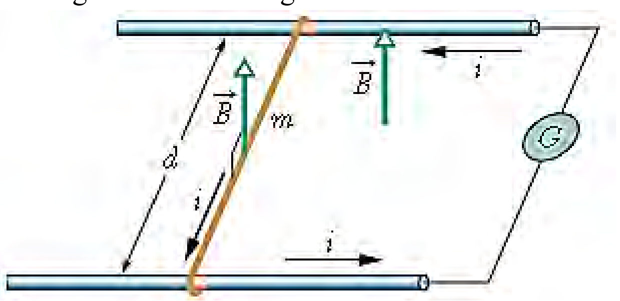
9. What is the force on a wire of length $4\ cm$ placed inside a solenoid near its centre making an angle of $60°$ with the axis? The wire carries a current of $12\ A$ and the magnetic field due to the solenoid has a magnitude of $0.25\ T$

[Ans: $0.1\ N$]

10. A straight horizontal conducting rod of length $0.45\ m$ and mass 60 g is suspended by two vertical wires at its ends. A current of $5\ A$ is set up in the rod through the wires
   (a) What magnetic field should be set up normal to the conductor in order that tension in the wires is zero?
   (b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field the same as before? Neglect the mass of the wires

[Ans: (a) $0.26\ T$ (b) $1.176\ N$]

11. In the figure below, a metal wire of mass $m = 24.1\ mg$ can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56\ cm$. The track lies in a vertical uniform magnetic field of magnitude $56.3\ mT$.



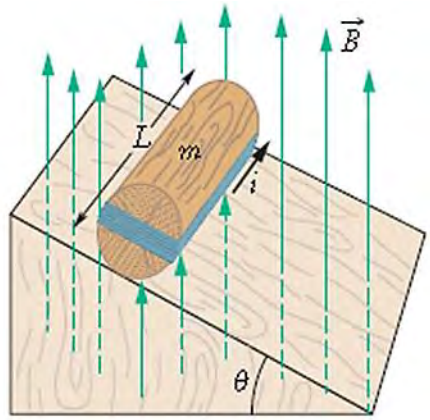
At time $t = 0$, device G is connected to the rails, producing a constant current $i = 91.3\ mA$ in the wire and rails. At $t = 61.1\ ms$, what are the wire's
   (a) speed and
   (b) direction of motion?

[Ans: (a)   (b)   ]

12. A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the
   (a) magnitude and
   (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

[Ans: (a) $0.10\ T$ (b) $31°$]

13. The figure below shows a wood cylinder of mass $m = 0.250\ kg$ and length $L = 0.100\ m$, $N = 10$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the inclined plane.



If there is a vertical uniform field of magnitude $0.50\ T$, what is the least current i through the coil that keeps the cylinder from rolling down the plane?

[Ans: $2.45\ A$]

MAGNETIC EFFECT OF AN ELECTRIC CURRENT

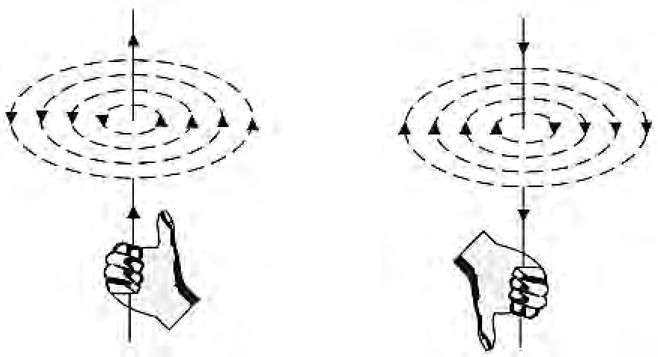
Magnetism can be caused by movement of electric charge hence an electric current being a flow of charge produces a magnetic field. If the current is flowing in a wire, the shape of the magnetic field is dependent on the configuration.

1. Current in a straight wire or conductor

For a long straight conductor/wire carrying current, the magnetic field lines of force are concentric circles with the conductor as the centre and lying in a plane perpendicular to the straight conductor. The direction of the magnetic field lines due to a straight carrying conductor can be found by one of the following two rules

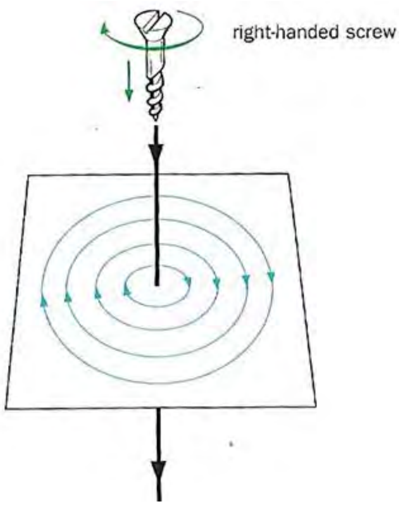
a. Right-hand grip rule

Grip the straight conductor with your right hand with the thumb pointing in the direction of current. Then curled fingers point in the direction of the magnetic lines of force.



b. Right-hand screw rule

If a right-handed screw moves forward in the direction of current (conventional), then the direction of rotation of the screw gives the direction of the magnetic field lines.

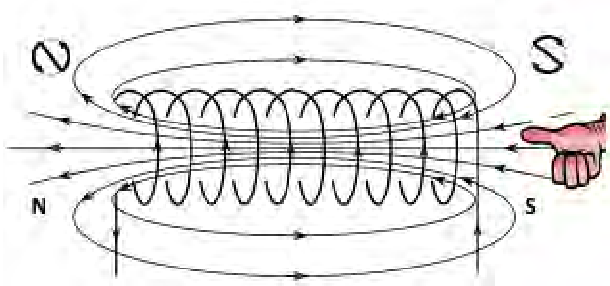


Note

- The magnetic field is strong in the region around the wire and weakens with increasing distance.
- The larger the current, the stronger is the magnetic field.

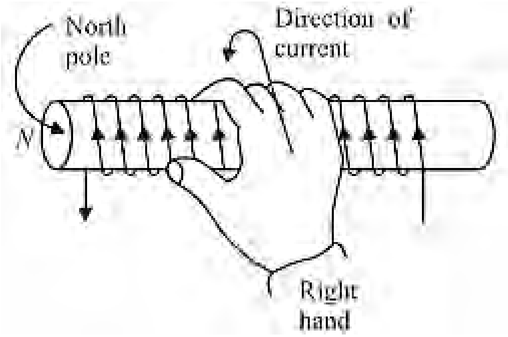
2. Current in a solenoid

A solenoid consists of a length of insulated wire coiled into a cylinder shape.



Current in the solenoid produces a stronger magnetic field inside the solenoid than outside. The field lines in this region are parallel and closely spaced showing that the field is highly uniform in strength and direction.
Field lines outside the solenoid are similar to those of a bar magnet and it behaves in a similar way as if it has a north pole at one end and a south pole at the other end. Strength of the field diminishes with distance from the solenoid.

Direction of the magnetic field



Imagine to grasp the solenoid with right hand so that the fingers are curled in the direction of current. The thumb stretched parallel to the axis of the solenoid will point towards the N-pole end of the solenoid

Variation of the magnetic field along the axis of the solenoid

A search coil connected to a ballistic galvanometer is inserted in the solenoid and moved from one end of the solenoid to the other along its axis.
The average deflection, θ on the galvanometer at various points is noted when the current is reversed in the solenoid.

Example 6

An electron revolves in a circular orbit of radius $4.0 \times 10^{-10}\, m$ at a frequency of 7.5×10^{15} revolutions per second. Calculate the magnetic flux density at the centre of the orbit.

Solution

The circulation of the electron constitutes a current.

$$I = \frac{ne}{t} = \frac{e}{t} \text{ as } n = 1$$

$$I = e\left(\frac{1}{t}\right) = ef$$

Assume that the magnetic field provided by the revolving electron is in relation with that at the centre of a circular coil of 1 turn

$$B = \frac{\mu_o NI}{2R}$$

$$B = \frac{\mu_0 ef}{2R}$$

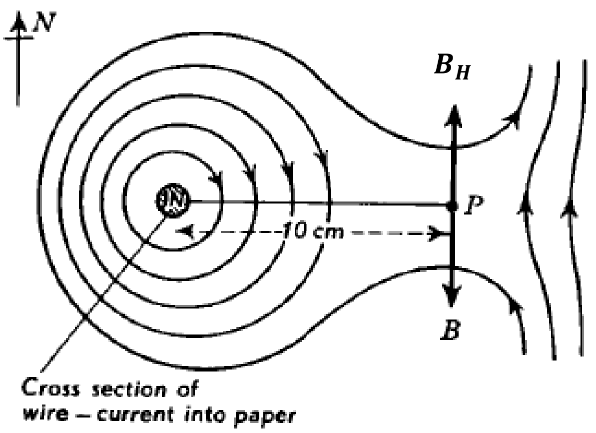
$$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 7.5 \times 10^{15}}{2 \times 4.0 \times 10^{-10}}$$

$$= 1.9\, T$$

Example 7

A current is passing down a long vertical wire situated in the Earth's magnetic field. If a neutral point is located at a perpendicular distance of 10 cm from the wire, what is the value of the current? (horizontal component of the earth's magnetic field $= 1.8 \times 10^{-5}\, T$)

Solution



The diagram shows a horizontal section of the wire and the con-figuration of the flux flow due to the current and the Earth's field. A point of zero flux intensity (neutral point) is shown at P, 10 cm due east of the wire at which position the flux intensities due to the wire (B) and the Earth's horizontal field (B_H) are equal and opposite. Thus,

$$B = B_H = 1.8 \times 10^{-5}\, T$$

Now the flux intensity B at a distance d from a long wire carrying a current I is given by

$$B = \frac{\mu_o I}{2\pi d} = \frac{4\pi \times 10^{-7} I}{2\pi \times 0.1} = 2I \times 10^{-6}$$

$$2I \times 10^{-6} = 1.8 \times 10^{-5}$$

$$I = 9\, A$$

Self-Evaluation exercise

1. What current in a long, straight wire will produce a magnetic field with a magnitude $10.0\, \mu T$ at a perpendicular distance of 40.0 cm from the wire?
   [Ans:]
2. The magnetic field at the centre of a 50-turn coil of radius 15 cm is $0.60\, mT$. Find the current in the coil
   [Ans: $2.9\, A$]
3. A solenoid 50 cm long with 1000 turns per metre has a central magnetic field of $1.5\, mT$. Find the current in the coil.
   [Ans: $1.2\, A$]
4. A 2000 turn solenoid of length 40 cm and resistance $16\, \Omega$ is connected to a $20\, V$ supply. What is the flux density at the mid-point of the axis of the solenoid?
   [Ans: $7.9 \times 10^{-3}\, T$]
5. A long, straight wire has a resistance of $2.0\, \Omega$. What potential difference between its ends will produce a magnetic field of $1.0 \times 10^{-4}\, T$ at a distance of 4 cm from the wire?
   [Ans: $40\, V$]
6. How many turns should a solenoid 30 cm long have to produce a magnetic field of $5.0\, mT$ at its axis when operating with a current of $5.0\, A$?
   [Ans: 240]
7. A solenoid 10 cm long has 3000 turns of wire and carries a current of 5.0 A. A 2000 turn coil of wire of the same length of the solenoid surrounds the solenoid and is concentric with it. If the outer coil carries a current of 10 A in the same direction as that of the current in the solenoid, find the magnetic field at the centre of both coils.
   [Ans:]
8. A solenoid having 200 turns per metre and carrying a current of $0.050\, A$ lies with its axis east-west. Well inside the solenoid is a small compass whose needle points 37° west of north. Calculate the horizontal component of the earths's magnetic field.
   [Ans: $1.7 \times 10^{-5}\, T$]
9. A circular coil of 100 turns has a radius of 10 cm and carries a current of $5\, A$. Determine the magnetic field at the centre of the coil.

$$\frac{F}{L} = IB\sin 0° = 0$$

Example 5

A horizontal straight wire 5 cm long of mass 1.2 gm^{-1} is placed perpendicular to a uniform magnetic field of flux density 0.6 T. If the resistance of the wire is 3.8 Ωm^{-1}, calculate the p.d that has to be applied between the ends of the wire to make it just self-supporting.

Solution

The current, I in the wire is to be in such a direction that the magnetic force acts on it vertically upward. To make the wire self-supporting, its weight should be equal to the upward magnetic force i.e.

$$BIL = mg$$

$$I = \frac{mg}{BL}$$

$$m = 1.2 \times 10^{-3}\ L,\ B = 0.6\ T$$

$$I = \frac{(1.2 \times 10^{-3}L) \times 9.8}{0.6 \times L} = 19.6 \times 10^{-3}\ A$$

Resistance of wire, $R = 0.05 \times 3.8 = 0.19\ \Omega$

Required p.d, $V = IR = (19.6 \times 10^{-3}) \times 0.19$

$= 3.7 \times 10^{-3}\ V$

Example 6

A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?

Solution

$m = 200\text{ g} = 0.2\ kg$, $I = 2\ A$, $L = 1.5\ m$, $B = ?$

As the wire is suspended in mid-air by magnetic field,

$$mg = BIL$$

$$B = \frac{mg}{IL} = \frac{(0.2)(9.81)}{2(1.5)} = 0.65\ T$$

Example 7

A wire of radius 0.20 cm and density 2000 kgm^{-3} is placed in a magnetic field. If the wire can carry a current of 500 A, what is the strength of the magnetic field that it must be placed in so that it can float by itself?

Solution

For the wire to be in equilibrium, weight of the wire = force of magnetic field

$$mg = BIL$$

$$B = \frac{mg}{IL} = \frac{\rho V g}{IL} = \frac{\rho A L g}{IL} = \frac{\rho \pi r^2 g}{I}$$

$$B = \frac{2000\pi(0.002)^2(9.81)}{500} = 4.93 \times 10^{-4} T$$

Example 8

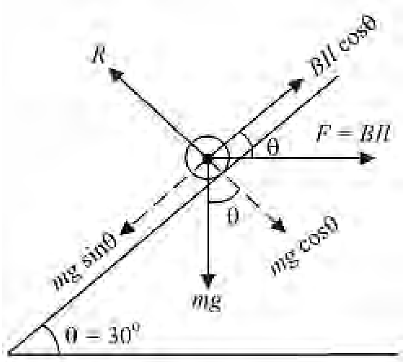
On a smooth inclined plane inclined at 30° with the horizontal, a thin current-carrying metal rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0.15 T in a vertical direction. For what value of current can the rod remain stationary given that its mass per unit length is 0.30 kg/m.

Solution

The forces acting on the current-carrying metallic rod are

(i) weight of the rod mg acting vertically downward

(ii)the magnetic force BIl



The component $mg\sin\theta$ tends to roll the rod down the plane while the component $BIl\cos\theta$ tends to move it up along the plane. For the rod to be stationary, these two components must be equal.

$$BIl\cos\theta = mg\sin\theta$$

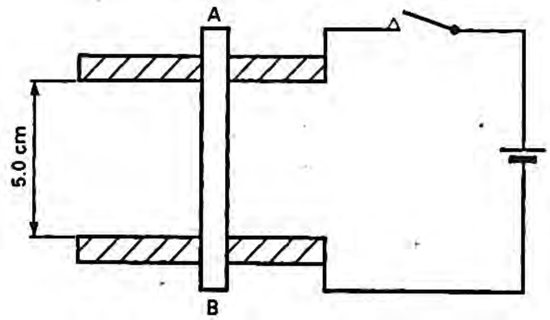
$$I = \frac{mg\tan\theta}{Il}$$

$$m = 0.3 \times l\ \text{kg},\ \theta = 30°,\ B = 0.15\ T$$

$$I = \frac{(0.3l) \times 9.81 \times \tan 30°}{0.15l} = 11.3\ A$$

Example 9

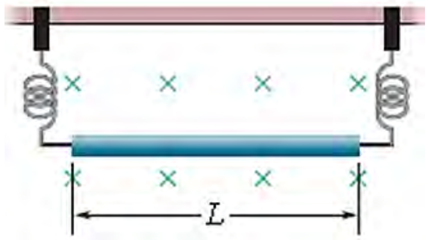
The diagram represents a cylindrical aluminum bar AB resting on two horizontal aluminum rails which can be connected to a battery to drive a current through AB.



A magnetic field of flux density 0.10 T acts perpendicularly to the paper and on to it

(a) In what direction will AB move if the current flows from A to B?

8. A 13.0 g wire of length $L = 62.0\ cm$ is suspended by a pair of flexible leads in a uniform magnetic field of magnitude $0.440\ T$.



What are the
   (a) magnitude and
   (b) direction of the current required to remove the tension in the supporting leads?

[Ans: (a) $467\ mA$ (b) right]

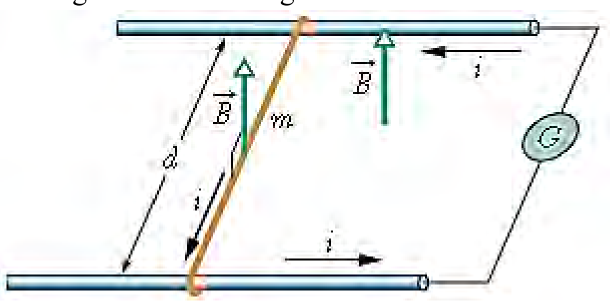
9. What is the force on a wire of length $4\ cm$ placed inside a solenoid near its centre making an angle of 60° with the axis? The wire carries a current of $12\ A$ and the magnetic field due to the solenoid has a magnitude of $0.25\ T$

[Ans: $0.1\ N$]

10. A straight horizontal conducting rod of length $0.45\ m$ and mass 60 g is suspended by two vertical wires at its ends. A current of $5\ A$ is set up in the rod through the wires
   (a) What magnetic field should be set up normal to the conductor in order that tension in the wires is zero?
   (b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field the same as before? Neglect the mass of the wires

[Ans: (a) $0.26\ T$ (b) $1.176\ N$]

11. In the figure below, a metal wire of mass $m = 24.1\ mg$ can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56\ cm$. The track lies in a vertical uniform magnetic field of magnitude $56.3\ mT$.



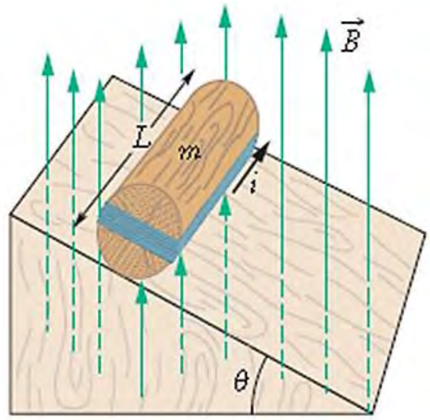
At time $t = 0$, device G is connected to the rails, producing a constant current $i = 91.3\ mA$ in the wire and rails. At $t = 61.1\ ms$, what are the wire's
   (a) speed and
   (b) direction of motion?

[Ans: (a)   (b)   ]

12. A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the
   (a) magnitude and
   (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

[Ans: (a) $0.10\ T$ (b) 31°]

13. The figure below shows a wood cylinder of mass $m = 0.250\ kg$ and length $L = 0.100\ m$, $N = 10$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the inclined plane.

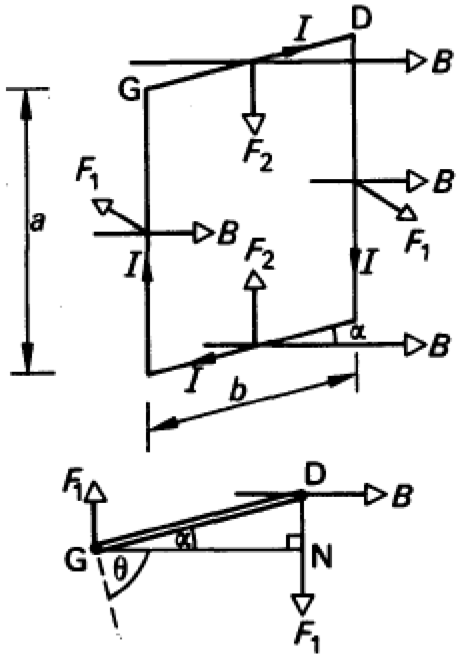


If there is a vertical uniform field of magnitude $0.50\ T$, what is the least current i through the coil that keeps the cylinder from rolling down the plane?

[Ans: $2.45\ A$]

TORQUE ON A COIL IN A MAGNETIC FIELD

Consider a rectangular coil of one turn, whose plane makes an angle α with a uniform magnetic field B. A current of I passes through the coil.



Forces, on the vertical sides, $F_1 = BIL = BIa$ where a = length of the coil

Forces on the horizontal sides, $F_2 = BIb\sin\alpha$ where b = width of the coil

The forces, F_2 are equal and opposite and thus cancel out.

The forces, F_1 set up a couple whose moment is given by;

$$\tau = F_1 \times \perp\ distance$$
$$= F_1 \times \overline{GN}$$
$$= F_1 b\cos\alpha$$
$$= BIab\cos\alpha$$
$$= BIA\cos\alpha$$

where A is the area of the coil.

If the coil has N turns, $\tau = BANI\cos\alpha$

If θ is the angle between the field and the normal to the plane of the coil, then $\theta = 90° - \alpha$

$$\tau = BANI\cos(90° - \theta)$$

$$\boldsymbol{\tau = BANI\sin\theta}$$

Electromagnetic moment of a coil

From the above expression, we see that τ depends on, among other things, I, A and N. It is convenient to write

$$m = IAN$$

where m is a property of the coil and the current it carries and is called the electromagnetic moment of the coil.

$$\tau = Bm\sin\theta$$

$$\boldsymbol{m = \frac{\tau}{B\sin\theta}}$$

If $B = 1\ T$ and $\theta = 90°$, then $m = \tau$

Electromagnetic moment of a coil can be defined as the torque of the couple acting on it when it lies with its plane parallel to a uniform magnetic field of flux density $1\ T$.

Example 1

A rectangular coil of area $3.0 \times 10^{-4}\ m^2$ is placed in a uniform magnetic field of strength 55 mT with the coil axis perpendicular to the field lines. A steady current of 0.85 A is passed through the coil windings. Calculate the couple exerted on the coil when it is

(a) positioned with its plane parallel to the field lines.

(b) positioned with its plane at 60° to the field lines.

<u>Solution</u>

(a) With plane parallel to the field, $\alpha = 0°$

$$\tau = BANI\cos 0° = BINA$$
$$= 55 \times 10^{-3} \times 0.85 \times 100 \times 3.0 \times 10^{-4}$$
$$= 1.403 \times 10^{-3} Nm$$

(b) With plane at 60° to the field, $\alpha = 60°$

$$\tau = BANI\cos 60°$$
$$= 55 \times 10^{-3} \times 0.85 \times 100 \times 3.0 \times 10^{-4}\cos 60°$$
$$= 7.013 \times 10^{-4}\ Nm$$

Example 2

A small circular coil of 10 turns and mean radius 2.5 cm is maintained at the centre of a long solenoid of 750 turns per metre when its axis is at right angles to the axis of the solenoid. The current in the solenoid is 2 A. Calculate the initial torque on the circular coil when a current of 1 A is passed through it.

<u>Solution</u>

Torque on the circular coil is due to the magnetic field inside the solenoid.

Inside the solenoid, $B = \mu_o nI$

$$= 4\pi \times 10^{-7} \times 750 \times 2 = 1.88 \times 10^{-3} T$$

Area of circular coil = $\pi r^2 = \pi \times 0.025^2$

$$= 1.96 \times 10^{-3} m^2$$

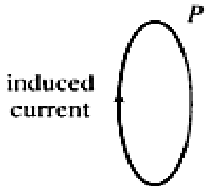
Torque on circular coil = $BINA$

$$= 1.88 \times 10^{-3} \times 1 \times 10 \times 1.96 \times 10^{-3}$$
$$= 3.68 \times 10^{-5} Nm$$

Example 3

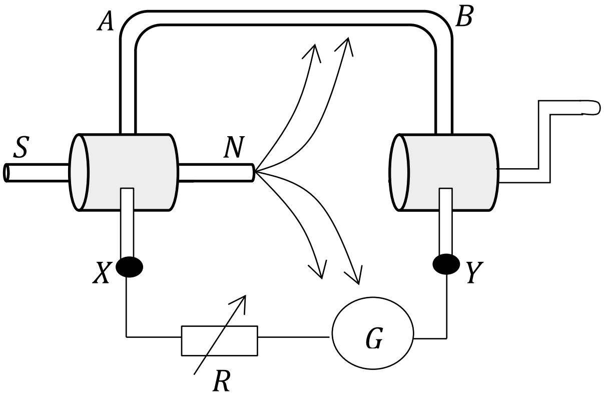
A small 10 turn coil of radius 5 cm is inserted into a solenoid so that the plane of the coil is at 45° relative to the axis of the solenoid. The solenoid is wound with 1000 turns per metre and carries a current of 25 A. If

(iv)



When the sliding contact of the rheostat is moved to the right, the rheostat resistances. The current through coil Q decreases. To oppose the decrease in the magnetic flux, the current induced in P must be in the same direction as the initial current through Q.

Experiment to verify Faraday's law



The apparatus is set up as shown above.

AB is a metal frame which is free to rotate around the magnet.

X and Y are metal contacts connecting the galvanometer G to the metal frame AB.

The frame is rotated at a constant speed so that G shows a steady deflection, θ.

The time for n revolutions is taken and the frequency f is calculated.

The experiment is repeated and the frequency f is recorded in the table with the corresponding steady deflection θ.

A graph of θ against f is plotted and a straight-line graph passing through the origin is obtained.

This shows that $\theta \propto f$ however $\theta \propto I \propto e.m.f$

$$\frac{d\Phi}{dt} \propto f \ \Rightarrow E \propto \frac{d\Phi}{dt}$$

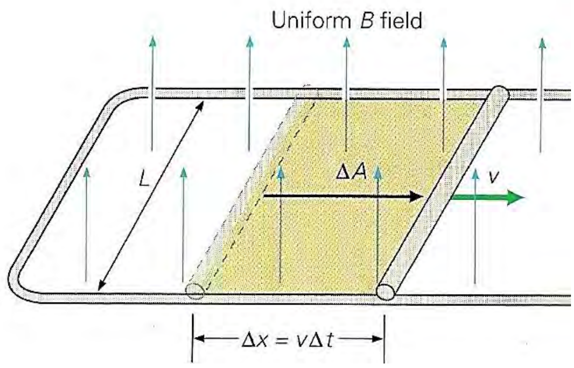
where $\frac{d\Phi}{dt}$ is the rate of change of magnetic flux.

INDUCED EMF IN A STRAIGHT CONDUCTOR

The e.m.f induced in a straight conductor can be derived in two ways i.e.

1. **Faraday's law**

   Consider a metal rod of length L pulled along a metal frame at constant velocity in a uniform magnetic field of flux density B normal to the plane of the frame



The flux through the loop formed by the frame and the rod increases because its area increases.

In a time Δt, the rod move a distance Δx given by

$$\Delta x = v\Delta t$$

Change in the area of the loop is

$$\Delta A = L\Delta x = Lv\Delta t$$

Rate of change (increase) of the area, $\frac{\Delta A}{\Delta t} = Lv$

From Faraday's law, induced e.m.f is given by

$$E = \text{rate of change of flux}$$

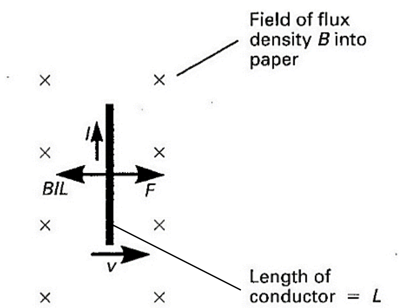
$$E = \frac{\Delta\Phi}{\Delta t} = B\frac{\Delta A}{\Delta t} = BLv$$

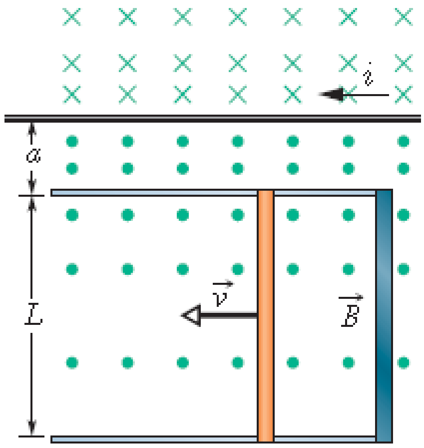
$$\boldsymbol{E = BLv}$$

2. **Law of conservation of energy**

   Consider a conductor that forms part of a closed circuit and that the induced e.m.f causes a current I to flow through it in a direction given by Fleming's right-hand rule.

   Since the conductor is carrying a current and is in a magnetic field, there will be a force, BIL acting on it whose direction is in accordance to the Fleming's left-hand rule. This force opposes the applied force





(a) Find the
  (i) e.m.f,
  (ii) current, induced in the loop
(b) At what rate is thermal energy generated in the rod?
(c) What is the magnitude of the force that must be applied to the rod to make it move at a constant speed?
(d) At what rate does this force do work on the rod?

[Ans: (a) (i) $240\,\mu V$ (ii) $0.60\,mA$ (b) $0.144\,\mu W$ (c) $2.87 \times 10^{-8}\,N$ (d) $0.144\,\mu W$]

22. A flat-bed truck travels due south at $120\,km/h$ in a location where the vertical component of the earth's magnetic field is $5 \times 10^{6}\,T$. What is the e.m.f induced in a $1.0\,m$ long copper bar held horizontally above the bed of the truck and perpendicular to its direction of motion?

[Ans: $0.17\,mV$]

23. A conducting rod of resistance R is moved along horizontal rails joined at one end to make a loop similar to the one in question 19. A uniform magnetic field B is perpendicular to the plane of the loop and extends over the region in which the rod moves. Assume that the rod moves with constant speed v and that the rails have negligible resistance and friction. Show that the force required to keep the rod moving with constant velocity is $\frac{B^2l^2v}{R}$

24. A $1.0\,m$ long aluminium bar is held horizontally in the east-west direction and dropped from a height of $10\,m$ at a place where the horizontal component of the earth's magnetic field is $3.8 \times 10^{-5}\,T$. What is the e.m.f between the ends of the bar just before it strikes the ground?

[Ans: $0.53\,mV$]

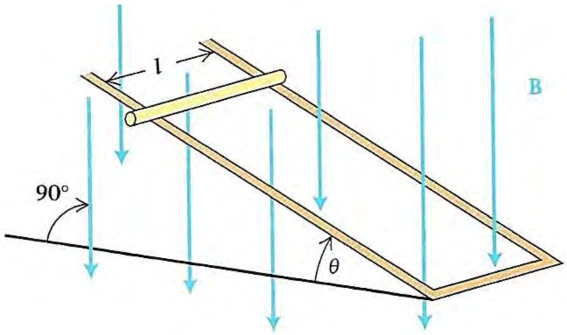
25. What is the e.m.f between the ends of a $1.0\,m$ long light metal chain if you swing it in a horizontal circle about your head at 2.0 revolutions per second at a location where the vertical component of the earth's magnetic field is $2.5 \times 10^{-5}\,T$?

[Ans: $0.16\,mV$]

26. A rectangular loop with an area $120\,cm^2$ is placed in a magnetic field of $0.013\,T$ and rotated about its long axis at $5.0\,Hz$. What is the instantaneous e.m.f generated at the instant when the normal to the loop makes an angle of $0.27°$ with the magnetic field?

[Ans: $2.3 \times 10^{-5}\,V$]

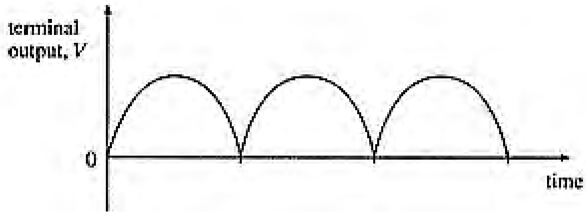
27. A rod of length l and mass m slides down parallel conducting rails making an angle θ with the horizontal. The rails have negligible resistance and are joined at the bottom to form a loop.



A uniform magnetic field B is directed vertically downwards in the region of the rails. What is the terminal velocity of the rod if it has an electrical resistance R? Ignore friction between rod and rails.

In this position, the e.m.f induced in the coil reverses and so one brush is always positive and the other always negative.

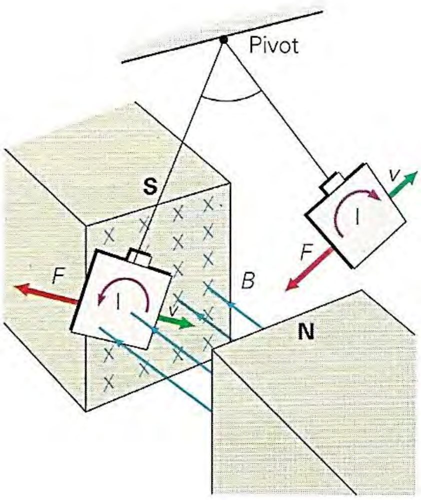
The variation of the output from the terminals with time is as shown below



EDDY CURRENTS

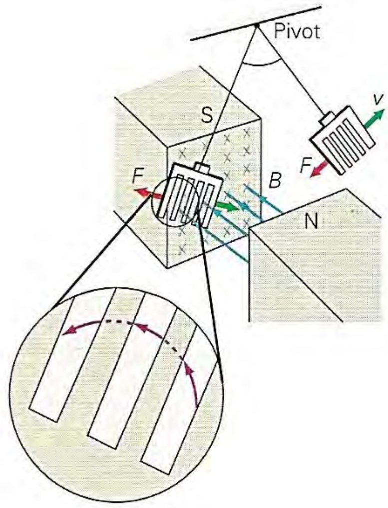
When a piece of metal moves in a magnetic field, a force acts on the delocalized electrons in accordance with Fleming's left-hand rule. The electron deficient atoms also attract electrons from other atoms. This movement of the electrons causes current loops that are called **eddy currents**.

An effect of eddy currents may be demonstrated by swinging a plate made of a nonmagnetic material such as aluminium or copper through a magnetic field as shown below.



Eddy currents are set up in the plate as a result of its motion in the field and the changing magnetic flux. By Lenz's law, the eddy currents set up opposing fluxes or effectively opposite magnetic poles on the plate. This gives rise to a repulsive force that retards the swinging motion and brings the plate quickly to rest.

The breaking up of eddy currents may be demonstrated using the plate with slits (sliced plate). When this plate swings between the magnetic poles, it swings relatively freely. This is because the eddy currents are much reduced with gaps (slits).



For the case of insulators such as glass plate, no current is induced in it and hence there is no resistance to it motion thus it swings freely.

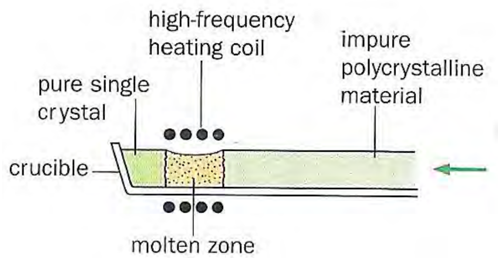
Applications of Eddy Currents

Dead beat galvanometer

When current is passed through a galvanometer, the coil oscillates about its mean position before it comes to rest. To bring the coil to rest immediately, the coil is wound on a metallic frame. Now, when the coil oscillates, eddy currents are set up in the metallic frame, which opposes further oscillations of the coil. This in turn enables the coil to attain its equilibrium position almost instantly. Since the oscillations of the coil die out instantaneously, the galvanometer is called dead beat galvanometer.

Induction furnaces

During metal refining, a narrow crucible containing the material is passed away slowly through the heating coil that is fed with very high alternating current.



The rapidly changing magnetic flux linking the material induces eddy currents in it. The internal energy induced causes the material to melt. So the impurities tend to collect in the molten zone which moves to one end of the crucible.

Car speedometers

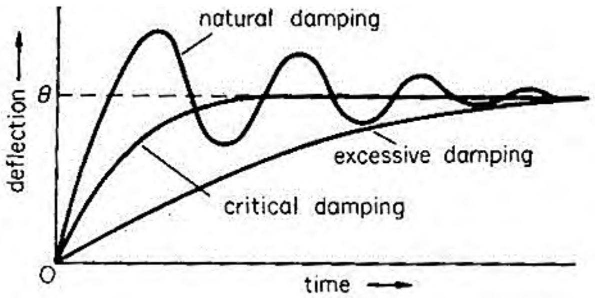
A rotating magnet driven by a flexible cable sets up eddy currents in a pivoted aluminium disc. The

magnetic effect of eddy currents sets up a couple on the disc that varies with the speed of the car. Therefore, the disc rotates until the electromagnetic couple is balanced by the opposing couple set up by the hair spring.
A pointer attached to the disc spindle shows the speed of the car on a carefully calibrated scale.

Induction motors
Eddy currents are produced in a metallic cylinder called rotor, when it is placed in a rotating magnetic field. The eddy current initially tries to decrease the relative motion between the cylinder and the rotating magnetic field. As the magnetic field continues to rotate, the metallic cylinder is set into rotation. These motors are used in fans.

Electromagnetic brakes
A metallic drum is coupled to the wheels of a train. The drum rotates along with the wheel when the train is in motion. When the brake is applied, a strong magnetic field is developed and hence, eddy currents are produced in the drum which oppose the motion of the drum. Hence, the train comes to rest.

Damping of moving-coil meters
Sometimes eddy currents can be made of use as we mentioned in damping of a meter.
When a current is passed through the coil of an ammeter, a couple acts on the coil which sets it swinging. If the swings are opposed only by the friction of the air, they decay very slowly and are said to be naturally damped. The pointer takes a long time to come to its final deflection θ.



To bring the pointer more rapidly to rest, the damping must be increased.
One way of increasing the damping is to wind the coil on a metal frame or former made of aluminium. Then, as the coil swings, the field of the permanent magnet induces eddy currents in the former, and these, by Lenz's law, oppose the motion. They therefore slow down the turning of the coil towards the equilibrium position and also stop its swings about that position. So in the end the deflected coil comes to rest sooner than if it were not damped.
Galvanometers coils which are wound on insulating formers can be damped by short-circuiting a few of their turns or by joining the galvanometer terminals with connecting wire so that the whole coil is short circuited.

Note:
Damping in a ballistic galvanometer should be as small as possible. This is because when the oscillation is ballistic, the deflection is not proportional to the charge.

Induced charge and flux change
When the flux linking a complete circuit changes, an e.m.f is induced in it and a current flows. A simple connection exists between the flux change and the total change circulation that constitutes the current.

Consider a coil of N turns in a circuit of total resistance R in which the flux linking each turn is changing and has value Φ at time t. The magnitude of the induced e.m.f at t is

$$E = \frac{d}{dt}(N\Phi)$$

Induced current, I at time t is

$$I = \frac{E}{R} = \frac{1}{R}.\frac{d}{dt}(N\Phi)$$

Since $I =$ rate of flow of charge $= \frac{dQ}{dt}$

$$\frac{dQ}{dt} = \frac{1}{R}.\frac{d}{dt}(N\Phi) = \frac{N}{R}\frac{d\Phi}{dt}$$

If the flux changes from Φ_1 to Φ_2, the total charge Q that passes is

$$Q = \int_0^Q dQ = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi$$

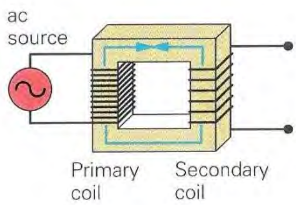
$$\boldsymbol{Q = \frac{N(\Phi_2 - \Phi_1)}{R}}$$

Or

$$\boldsymbol{Q = \frac{\text{flux linkage change}}{R}}$$

Note:
From the above expression, we can observe that Q does not depend on the time taken for the flux change.

Example
A search coil of an average cross-sectional area $3.0\ cm^2$ has 400 turns and is in a circuit of total resistance $200\ \Omega$. It is inserted into a magnetic field of flux density $2.5 \times 10^{-3}\ T$ so as to produce the



The current in the secondary is greater than the current in the primary.

Note:
When a load is connected to the secondary winding, a current flows in it. The current flows in such a direction as to reduce the flux in the core. This reduces the back e.m.f in the primary coil, hence the current increases.

Efficiency of a transformer

Efficiency of a transformer $= \frac{\text{Power output}}{\text{power input}} \times 100$

$$= \frac{V_s I_s}{V_p I_p} \times 100$$

For an ideal transformer (100% efficient), the power input in the primary coil is equal to the power output in the secondary coil. Therefore

$$V_p I_p = V_s I_s$$

$$\Rightarrow \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Energy losses in a transformer

Although practical transformers are almost 100% efficient, they do have power losses due to

1. **Resistance in the coil windings (joule heating in the wires)**
   This can be minimized by using thicker wires.
2. **Magnetic flux leakage**: The flux produced by the primary coil may not be the flux linked to the secondary coil.
   This can be minimized by the use of a soft iron core.
3. **Eddy currents:** Due to mutual induction, opposing eddy currents are induced with in the soft iron core. This can be minimized by having a laminated iron core to reduce the flow of charges with in the core.
4. **Hysteresis loss**: The core material offers some resistance to the changing strength and the direction of the magnetic field (called hysteresis loss). The resistance manifests itself as heat with in the core.
   This is minimized by making the core of specialist metal (e.g. perm alloy) where hysteresis loss is minimal.

Example 1

A transformer has 2000 turns in the primary coil. The primary coil is connected to a $240\,V$ mains. A $12V, 36\,W$ lamp is connected to the secondary coil. If the efficiency of the transformer is 90%. Determine the

(i) number of turns in the secondary coil
(ii) current flowing in the primary coil.

Solution

(i) From $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$\Rightarrow N_s = \frac{V_s}{V_p} \times N_p = \frac{12}{240} \times 2000$$

$$= 100\ turns$$

(ii) Since efficiency is 90%

$$I_s V_s = \frac{90}{100} \times I_p V_p$$

Power output, $I_s V_s = 36W$

$$\Rightarrow I_p = \frac{I_s V_s \times 100}{V_p \times 90} = \frac{36 \times 100}{240 \times 90} = 0.167\,A$$

Example 2

(a) What turns ratio would be needed for an ideal transformer to provide $12\,V$ r.m.s when connected to a $240\,V$ r.m.s mains supply.
(b) If the transformer was loaded with a non-inductive $12V, 60W$ heater, what current would flow in the mains supply lead?
(c) If the transformer in practice gives $11.8V$ r.m.s and $4.5\,A$ r.m.s when the primary current is $0.25\,A$ r.m.s, what is the efficiency of energy conversion by the transformer.

Solution

(a) From $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$V_s = 12, V_p = 240 \Rightarrow \frac{N_s}{N_p} = \frac{12}{240} = \frac{1}{20}$$

This means that the primary has greater number of turns

(b) From $\frac{N_s}{N_p} = \frac{I_p}{I_s}$

Power output $= V_s I_s$

$$60 = 12 I_s$$

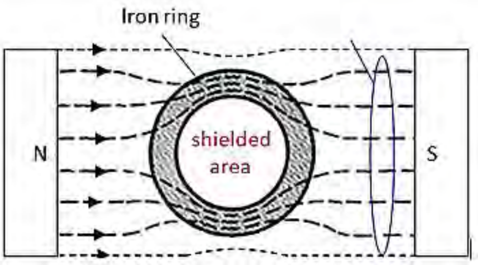
$$I_s = 5\,A$$

But $\frac{I_p}{I_s} = \frac{1}{20} \Rightarrow I_p = \frac{1}{20} \times 5 = 0.25A$

(c) The input power is $V_p I_p = 240 \times 0.25 = 60W$

Magnetic shielding

Magnetic shielding is done to prevent surrounding magnetic field lines from reaching the magnetic sensitive equipment whose operation may be affected by the magnetic fields.



Magnetic materials like iron concentrate the magnetic field lines and divert them from the ends.

POWER TRANSMISSION

The type of electricity transmitted over long distances is predominantly AC as it can easily be stepped up or stepped down using a transformer.
There are two main types of energy loss that occur in transmission lines i.e.

(i) those resulting from the resistance of the wires
(ii) those resulting from induction of eddy currents

1. **Resistive energy losses**
   Heat is generated in the transmission lines because of the resistance of the wires. Power loss in the transmission lines is given by $P = I^2R$. As the resistance of the conductor remains constant, power loss is mostly affected by the size of current.
   Energy losses are kept minimum by transmitting electricity at the highest practicable voltage with the lowest practicable current.
   Also, by using thick copper wires or aluminum wires that have a low resistivity, the energy losses can be minimized.

2. **Inductive energy losses**
   Energy is also lost through induction of eddy currents in the iron core of transformers. Transformer cores are usually made of laminated iron, consisting of many thin layers of iron sandwiched together with thin insulating layers separating them to reduce eddy currents.

MAGNETIC MATERIALS

Relative permeability (μ_r)

The flux density in a coil increases many times when it has a ferromagnetic core because the core becomes magnetized and contributes flux. The relative permeability μ_r is defined by the equation

$$\mu_r = \frac{B}{B_0}$$

where B_0 is the flux density in a current conductor containing air (vacuum) and B is the flux density when the same conductor is filled with the material

Classification of magnetic materials

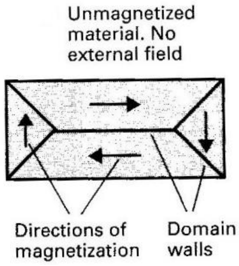
All substances are affected by magnetic fields and are classified as being diamagnetic, paramagnetic or ferromagnetic according to how they are affected.
For **diamagnetic** materials, μ_r is very slightly less than 1 and for **paramagnetic** materials it is slightly greater than 1. Thus paramagnetism and diamagnetism are very weak forms of magnetism.
Those materials which exhibit very strong magnetic effects are said to be **ferromagnetic** and have very large values of μ_r (typically 10^4). There is a strong linkage between neighbouring atoms to form **magnetic domains**.
The only elements which are ferromagnetic are iron, nickel, cobalt, gadolinium and dysprosium. A number of oxides and alloys are also ferromagnetic. For ferromagnetic materials, μ_r depends on

(i) the (magnetic) history of the sample and the strength of the magnetizing field i.e. on B_0
(ii) the temperature of the material. The value of μ_r decreases with increasing temperature, and at a critical temperature which is known as **Curie temperature** a ferromagnetic material becomes paramagnetic. It is about 770°C for iron.

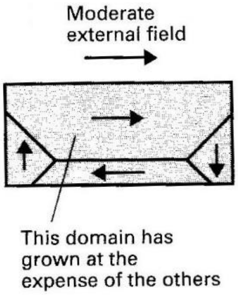
Domain theory of ferromagnetism

Paramagnetic and ferromagnetic materials contain atoms which have magnetic fields resulting from the motions of the electrons within the atoms. The atoms of diamagnetic materials, on the other hand, have no such fields because their electronic configurations are such that the fields of the individual electrons cancel each other. In paramagnetic materials, the fields of the individual atoms are oriented randomly, owing to their thermal agitation and there is no overall magnetization. In ferromagnetic materials, there are very strong interactions between neighbouring atoms which cause the magnetic fields of groups of them to line up in the same direction. These groups are called **domains**.

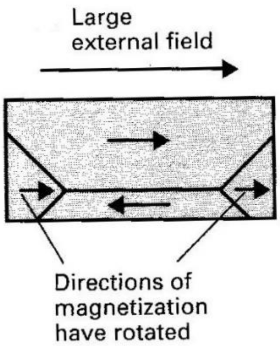
The direction of magnetization varies from one domain to another and in an unmagnetized sample of ferromagnetic material, the fields of the various domains cancel.



When an external field is applied to the sample, those domains whose directions of magnetization are in (or close to) the direction of the field grow at the expense of others and there is a general movement of domain walls. The sample now has a resultant magnetization in the direction of the external field.



If the strength of the field is increased, the extent to which the walls move increases. Furher increases in the field cause more and more domains to rotate and eventually all the domains are in line with the applied field. The sample is now saturated.

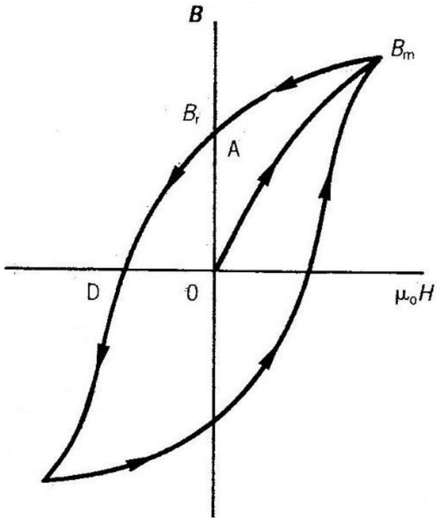


Note:

The wall movements which occur in weak fields are small and (almost completely) reversible i.e. the sample loses its magnetism when the external field is removed. The larger wall movements and rotations produced by stronger fields are mainly irreversible.

Hysteresis

Hysteresis comes from the Greek work meaning 'delay' or 'lagging behind'. It describes the relation between the magnetizing field and the magnetization produced within a specimen.



There is a maximum flux B_m that can be produced within a given specimen. This is known as **saturation**.

If a specimen is fully magnetized and then demagnetized, it will not return to a condition where both the magnetizing field and the magnetization produced in the specimen are both zero.

When the magnetizing field is reduced to zero, there will still be a small amount of magnetization left in the specimen. This is known as **remanent flux** and the effect is **remanence** (B_r). It is sown by length OA.

The reverse field needed to reduce the magnetization in the specimen to zero is known as **coercivity ($\boldsymbol{H}$)** of the specimen and is shown as OD.

The loop produced when the magnetizing field is taken through a full cycle is called **hysteresis loop**. The area within the loop represents loss of energy within the specimen when it is magnetized and demagnetized. This energy is lost as heat within the specimen and the larger the area within the loop, the more energy is lost in magnetizing and demagnetizing the specimen.

Properties and uses of magnetic materials

Ferromagnetic materials can be classified into two broad groups i.e. soft and hard.

Soft magnetic materials are easily magnetized and demagnetized.

Hard magnetic materials require large magnetizing fields and retain their magnetism. Typical hysteresis loops for soft and hard magnetic materials are shown below.

Additional questions

1. An overhead cable, running in an $E-W$ direction, carries a current of 100 A. What is the force per metre length of this cable produced on it by the earth's magnetic field if, at the position in question, the angle of dip is 60° and the horizontal component of the Earth's field is $1.8 \times 10^{-5}\ T$?

   [Ans: $36\ N$]

2. Two long straight wires A (carrying $2\ A$) and B (carrying $3\ A$ in the same direction as the current in A) run in parallel directions 10 cm apart. Considering a cross-section of the arrangement of the wires, find the position at which the field intensity due to the current-bearing conductors is zero. Where will this position be relative to the wires if B's current is reversed? (Neglect the effect of the Earth's field.)

   [Ans: At a point between the wires distant $4\ cm$ from wire A on the line joining A to B. At a point distant $40\ cm$ from A on the side remote from B along the line joining A to B]

3. A long straight wire is tightly stretched between two terminal blocks so that it lies along the magnetic meridian and parallel to the bench surface. A deflection magnetometer placed with its needle 10 cm vertically below the mid-point of the wire records a deflection of 30° when a current of 5.2 A is passed along the wire. From these observations obtain a value for the Earth's horizontal magnetic field.

   [Ans: $1.8 \times 10^{-5}\ T$]

4. A very long wire A carrying a current of $5\ A$ lies parallel with, and distant 2 cm from, a shorter wire B of length 10 cm and carrying a current of $2\ A$ in the same direction as A's current. Calculate the force exerted by A on B. Find also the work done in separating B a further 2 cm from A.

   [Ans: $10^{-5}\ N$; $1.387 \times 10^{-5}\ J$]

5. Two square-shaped single wire coils each carry a current of 2 A circulating in the same direction through each coil. Calculate the force required to hold the two coils at a distance of 0.4 cm apart if the length of the side of the square of each coil is 5 cm.

   [Ans: $4 \times 10^{-5}\ N$]

6. The 'force element' of a current balance consists of a 4 cm long horizontal copper wire piece which is supported at the end of a lever arm 20 cm long through which the current passes to the element. The balance is set up so that the element is positioned between the pole pieces of a large electromagnet and it is found that, when a current of 2 A is passed through the element a 2 g rider mass has to be positioned 15 cm from the pivot along the counterpoising arm of the balance. Calculate a value for the field intensity between the pole pieces of the magnet.

   [Ans: $0.184\ T$]

7. A circular coil having 20 turns of mean radius 10 cm is capable of rotation about a vertical diametral axis. Describe how you would position the coil and determine the current through it if it required to neutralize the Earth's horizontal component (of strength $1.8 \times 10^{-5}\ T$) in the plane of the coil.

   [Ans: Plane of the coil perpendicular to the meridian with a current of $0.143\ A$ circulating in the anticlockwise direction on looking along the meridian northwards to the face of the coil]

8. A flat circular coil of 20 turns of mean radius 10 cm is mounted with the plane of its face vertical and with its axis running along the $E-W$ line. At a point 20 cm along the axis from the centre of the coil is positioned the needle of a deflection magnetometer. Find what current passing round the coil will yield a deflection of 45° of the magnetometer needle. (Earth's horizontal component $= 1.8 \times 10^{-5}\ T$)

   [Ans: $1.6\ A$]

9. Two circular current-bearing coils are mounted symmetrically one inside the other and in such a way that the inner one can be rotated about a vertical diametral axis. The outer coil has 50 turns of mean radius 7.5 cm whilst the inner coil has 100 turns of mean radius 5 cm. The coils are wired in series and carry a current of 2 A. What is the magnitude and direction of the resultant magnetic field at the common centre of the coils (a) when their magnetic axes are coincident, (b) when the inner coil is reversed from its position in (a), (c) when the axis of the inner coil makes 30° with its original position? Ignore effects due to the Earth's field.

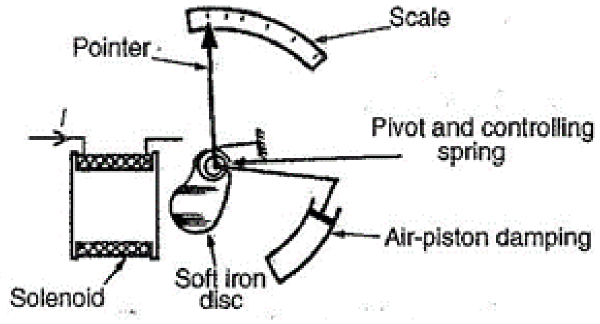
   [Ans: (a) $3.35 \times 10^{-4}\ T$ (b) $1.675 \times 10^{-4}\ T$ in reverse direction from (a) (c) $3.263 \times 10^{-4}\ T$]

10. Two similar coils, each of 20 turns of wire of radius 10 cm are arranged co-axially with their planes in the meridian and with their centres 10 cm apart. The coils are wound in series and when a current passes through them a magnetic needle, suspended mid-way between the coils, is deflected

Moving-iron instruments (iron meters)

Unlike the moving coil instruments like the moving coil galvanometer, moving iron instruments are based on the principle of the motion of the iron in a magnetic field and can be used to measure both a.c and d.c.

Attraction type



The current I to be measured is passed through the solenoid and its sets up a magnetic field which attracts the pivoted soft iron disc.

The soft iron will be attracted in whichever direction the current flows.

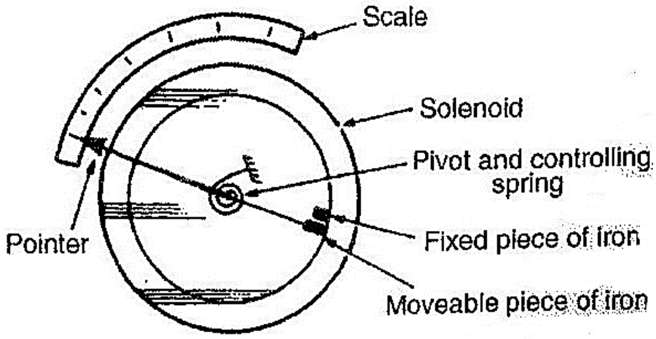
This consequently causes the pointer to rotate about the pivot. The control couple is provided by the spring.

The attraction force is reasonably proportional to the square of the average value of current I i.e.

$$\theta \propto I^2 \geq {I_{rms}}^2$$

The scale is however non-linear.

Repulsion type



Two pieces of iron are placed inside the solenoid, one being fixed and the other attached to the spindle carrying the pointer.

When current passes through the solenoid, the two pieces of iron are magnetized in the same direction and therefore repel each other.

The pointer thus moves across the scale.

The force moving the pointer is proportional to I^2 and because of this, the direction of current does not matter.

Since the deflection, $\theta \propto I^2$, the scale is non-linear.

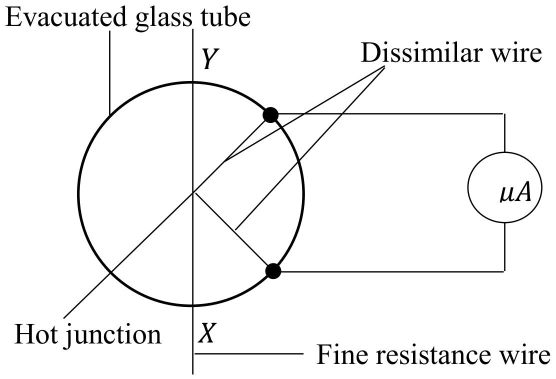
Comparison of moving-coil and moving-iron instruments

Type of instrument	**Moving-coil**	**Moving-iron**
Suitable for measuring	Direct current and voltage	Direct and alternating currents and voltages (reading in rms value
Scale	Linear	Non-linear
Method of control	Hairsprings	Hairsprings
Method of damping	Eddy currents	Air
Advantages	1. Linear scale 2. High sensitivity 3. Well shielded from stray magnetic fields 4. Low power consumption	1. Robust construction 2. Relatively cheap 3. Measures both ac and dc 4. Accurate
Disadvantages	1. Only suitable for dc 2. More expensive than moving iron type 3. Easily damaged	1. Non-linear scale 2. Affected by stray magnetic fields 3. Hysteresis errors in dc circuits 4. Liable to temperature errors 5. Due to the inductance of the solenoid, readings can be affected by variation of frequency

Why a moving coil meter cannot measure a.c

When an a.c is passed through a moving coil ammeter, the direction of the couple changes every time the current reverses. With a high frequency, the pointer only vibrates about the zero position hence the value of current cannot be read.

The thermocouple meter



Current to be measured is passed through the fine XY. The wire heats up due to its resistance.

Thermoelectric e.m.f therefore develops between the hot and cold junctions giving rise to a direct current through the micrometer.

The deflection is therefore proportional to the square of the average current.

Note:

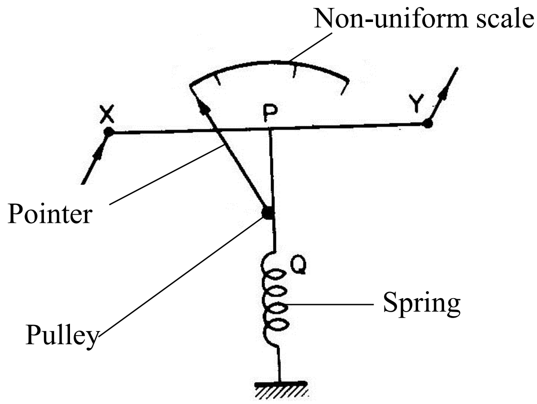
The meter is very sensitive because it has low inductance and capacitance. Thus, it can be used to measure a.c of frequency up to several megahertz.

Precautionary measure in the design of a thermocouple meter

The fine wire is enclosed in an evacuated glass bulb to shield it from draughts.

If the wire was in the open, some heat would be lost to the surroundings so that the temperature difference between the hot and cold junctions would not be proportional to the power dissipated in the wire.

The hot wire meter



Mode of operation

The current to be measured is passed through a fine resistance wire XY. The wire heats up, expands and sags.

The sag is taken up by a second fine wire PQ held taut by a spring.

Wire PQ passes round a pulley attached to the pointer which deflects over the scale as the pulley rotates when PQ moves down.

The deflection of the pointer is proportional to the square of the average current

Root mean square value of an alternating current

The root mean square value (r.m.s) of an alternating current is the steady current which dissipates electrical energy (heat) in a given resistor as the same rate as the alternating current.

Expression for r.m.s value

The instantaneous power dissipated, $P = I^2R$

Consider a sinusoidal then,

$$I = I_o \sin\omega t$$

$$\Rightarrow P = I_o^2 R sin^2\omega t$$

Average power,

$$< P > = I_o{}^2R < sin^2\omega t > = \frac{I_o{}^2R\int_0^{\frac{2\pi}{\omega}} \sin^2\omega t\, dt}{2\pi/\omega}$$

$$\text{But } \int_0^{\frac{2\pi}{\omega}} \sin^2\omega t\, dt = \frac{1}{2}\int_0^{\frac{2\pi}{\omega}}[1 - \cos 2\omega t]\, dt$$

$$= \frac{1}{2}[t + 2\omega \sin 2\omega t]_0^{2\pi/\omega} = \frac{1}{2}\left[\frac{2\pi}{\omega} + 0\right] = \frac{\pi}{\omega}$$

$$\Rightarrow < P > = \frac{I_0^2R(\pi/\omega)}{2\pi/\omega} = \frac{I_o{}^2R}{2}$$

Power dissipated by direct current,

$$P = I_d^2R = \frac{I_o{}^2R}{2}$$

$$I_d{}^2 = \frac{I_0{}^2}{2}$$

$$\therefore \boldsymbol{I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\, I_0}$$

Root mean square value of an alternating voltage

The root mean square value of an alternating voltage is the value of the steady voltage that dissipates electrical energy (heat) in a given resistor at the same rate as the alternating voltage.

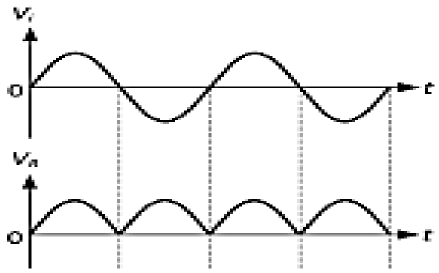
Similarly, the r.m.s value p.d V_{rms}, is related to the peak value V_0 by the equation;

$$\boldsymbol{V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0}$$

Note: Knowledge of the integration of $\sin^2\omega t$ is important in this situation to obtain the average power. The reader should also note that $\frac{2\pi}{\omega}$ is the period from $\omega = 2\pi f = \frac{2\pi}{T}$

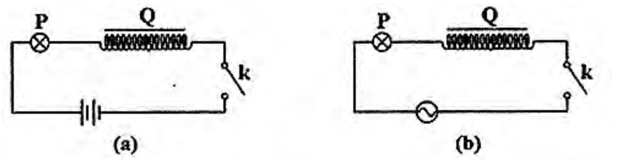
Once again current flow through R is unidirectional during both half-cycles of input p.d and a d.c output is obtained.

In this way, both halves of the a.c voltage are rectified and a full wave rectified voltage across the resistive load, R is achieved.



EXAMINATION QUESTIONS

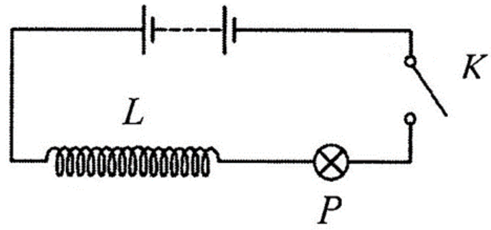
1. (a) (i) What is meant by **capacitive reactance**?
   (ii) Define **peak value** of an alternating voltage
   (iii) Show that the r.m.s value of an alternating voltage is
   $$V_{rms} = \frac{V_0}{\sqrt{2}}$$
   where V_0 is the peak voltage
   (b) Distinguish between mutual and **self** induction
   (c) (i) Describe with the aid of a diagram, the structure and working of the a.c transformer
   (ii) Explain the steps taken to minimize power losses in electric power transmission
   (d) In the figure below, P is a 6 V lamp and Q is a coil of negligible resistance.



   (i) Explain which of the lamps in the figure above will be brighter when k is closed.
   (ii) Explain what happens when a soft iron core is introduced in the coil in each of the circuits

[2018, No. 7]

2. (a) Define **root mean square** value of an alternating current
   (b) (i) Write down the expression for the e.m.f generated by a dynamo and use it to identify the factors which determine the maximum e.m.f.
   (ii) Explain the structural modifications needed to convert an a.c generator into a d.c generator.
   (c) An iron-cored coil having a low resistance and high inductance is connected in series with a filament lamp P. The coil and the lamp are then connected across a d.c supply as shown below.



   Explain what is observed when switch K is closed and then opened.
   (d) An alternating voltage $V = V_o \cos \omega t$ is connected across an inductor of inductance L
   (i) Derive the expression for the reactance of the inductor, X_L
   (ii) Sketch using the same axes the variation of applied voltage and current through the inductor with time.
   (e) Describe how a thermocouple ammeter is used to measure an alternating current.

[2017, No. 7]

3. (a) Define **root mean square (rms)** value of an alternating voltage.
   (b) A resistor of $100\,\Omega$ is connected across an alternating voltage $V = 20 \sin 120\pi t$
   (i) Find the frequency of the alternating voltage
   (ii) Calculate the mean power dissipated in the resistor
   (c) (i) Show that when an inductor is connected to an a.c supply voltage of $V = V_o \sin 2\pi f t$, the resulting current lags the voltage by 90°
   (ii) Sketch on the same axes the variation with time of the voltage and current if a capacitor is connected to the voltage supply in (c) (i)
   (d) (i) Describe how a thermocouple meter works
   (ii) Explain any precautionary measure taken in the design of the thermocouple meter.

[2016, No. 7; Ans: (b) (i) 60 Hz (ii) 2 W]

4. (a) (i) Define **root mean square (rms)** current of an a.c
   (ii) Derive an expression for capacitive reactance.
   (iii) Sketch on the same axes, the graphs showing variation of applied p.d and the current when an inductor is connected to an a.c supply.

SECTION D:

ELECTRICITY,

ELECTROSTATICS

&

CAPACITORS

Current electricity

- The Coulomb
- Concept of current electricity
- Potential difference
- The volt
- Electromotive force (E.m.f)
- Resistance, the Ohm
- Ohm's law
- Verification of Ohm's law
- Ohmic and non-ohmic conductors
- Factors determining resistance (Their effects)
- Resistivity
- Temperature coefficient of resistance
- Heating effect of current
- Arrangement of resistors in a circuit (parallel and series)
- Internal resistance and its cause
- Measurement of internal resistance
- Energy and power in an Ohmic circuit
- Maximum power out put
- Efficiency
- Kirchhoff's laws
- Potential divider
- Ammeters and voltmeters in circuits
- Shunts and multipliers
- Slide wire potentiometer
- Comparison of e.m.fs, internal resistance of a cell, measurement of current, calibration of an ammeter and voltmeter, measurement of resistance, comparison of resistances, thermoelectric e.m.fs
- Wheatstone bridge
- Slide wire meter bridge

Electrostatics

- Types of charges
- Charging by friction, induction, conduction
- Gold leaf electroscope
- Distribution of charge
- Corona discharge
- Van der Graff generator, lightning conductor
- Applications of electrostatics
- Hazards of electrostatics
- Electrostatic shielding
- Electric field, field lines, patterns
- Coulomb's law of electrostatics
- Electric field intensity
- Electric potential
- Potential difference
- Relationship between electric field intensity and electric potential
- Electric potential energy, electron volt

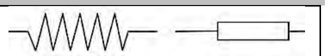
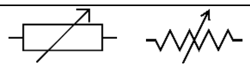
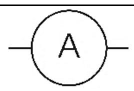
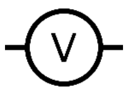
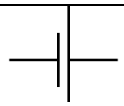
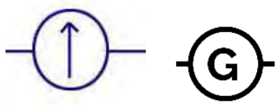
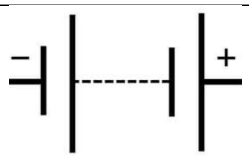
Capacitors

- Concept of capacitance
- Charging and discharging a capacitor
- Parallel plate capacitor
- Action of dielectric, dielectric constant
- Comparison of capacitances
- Arrangement of capacitors in series and parallel
- Energy stored in a charged capacitor
- Energy loss in connected capacitors
- Applications of capacitors

CURRENT ELECTRICITY

In the previous chapters, we dealt with electrostatics i.e. we studied the behaviour of charges at rest. In this chapter, we shall discuss current electricity i.e. charges in motion. We know that force acts on a charged particle placed in an electric field. If the charge on the particle is positive, it will move in the direction of electric field. On the other hand, if charge on the particle is negative, it will move in the direction opposite to the electric field. This flow of charge in a definite direction is called electric current and it is important in many ways. For example, it is the electric current by means of which electrical energy is transferred from one place to another for utilization. The operation of all electrical appliances we use such as heater, electric iron, incandescent lamp, etc., depends upon the flow of electric current through them. In this chapter, we shall deal with various aspects of current electricity.

Common symbols

Resistor	
Variable resistor/rheostat	
Ammeter	
Voltmeter	
Cell	
Galvanometer	
Battery	
a.c supply	

Definitions

Current (I)

This is the rate of flow of charge through a conductor. For steady current, $I = \frac{Q}{t}$ and for instantaneous current, $I = \frac{dQ}{dt}$

Current is measured in Amperes (A).

An ampere is the current flowing through a conductor when one coulomb of charge flows in one second.

Example 1

Each second 10^{17} electrons flow from right to left across a cross-section of a wire attached to the two terminals of a battery. Calculate the magnitude and direction of current in the wire.

Solution

Electric current, $I = \frac{q}{t} = \frac{ne}{t}$

$n = 10^{17}, e = 1.6 \times 10^{-19}\, C, t = 1\, s$

$$I = \frac{(10^{17}) \times (1.6 \times 10^{-19})}{1} = 1.6 \times 10^{-2}\, A$$

The direction of current is from left to right i.e. opposite to the direction of electron flow.

Example 2

A $60\, W$ light bulb has a current of $0.5\, A$ passing through it. Calculate (i) the number of electrons passing a cross-section of the bulb (ii) the number of electrons that pass that cross-section in one hour.

Solution

(i) $I = \frac{q}{t} = \frac{ne}{t}$

$$n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = 3.1 \times 10^{18} \text{ electrons/s}$$

(ii) Charge passing the cross-section in one hour is

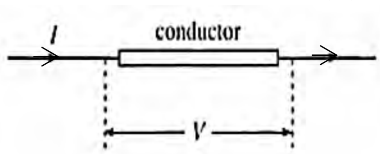
$$q = It = 0.5 \times (60 \times 60) = 1800\, C$$

Now $q = ne$

$$n = \frac{q}{e} = \frac{1800}{1.6 \times 10^{-19}} = 1.1 \times 10^{22} \text{ electrons/hour}$$

Resistance (R)

This is the opposition to the flow of charge or current in a conductor. It is measured in ohms (Ω)



Ohm is the resistance of a conductor through which a current of 1 A is flowing when the p.d across it is 1 volt.

be increased by a further 12 Ω in order to reduce the galvanometer deflection to 20 divisions. Compare the e.m.f.s of the two cells and explain your method.

[Ans: 1.25: 1]

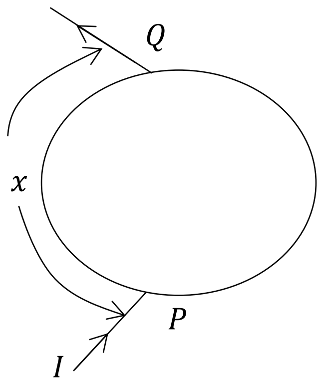
20. Define resistivity.
Taking the resistivity of copper as 1.6×10^{-8} Ωm, calculate the resistance of a cubic centimetre of copper (a) when in the form of a wire of diameter 0.02 cm, (b) when in the form of a sheet 2.5 mm thick, the current passing through the sheet perpendicularly to its faces.

[Ans: (a) 16.2 Ω (b) 10^{-7} Ω]

21. Two wires, A and B, have lengths which are in the ratio of 4 ∶ 5, diameters which are in the ratio 2 ∶ 1, and are made of materials with resistivities in the ratio 3 ∶ 2. If the wires are arranged in parallel, and a current of 1 ampere enters the arrangement, find the current in each branch.

[Ans: Current through $A = 0.77\ A$, $B = 0.23\ A$]

22. The diagram below shows a loop of copper wire of length l and diameter d. Current I enters the loop at point P and leaves the loop at point Q.



Show that the effective resistance of the loop is given by

$$\frac{4\rho x(l-x)}{\pi l d^2}$$

Super conductors

Super conductors are materials that at temperatures near absolute zero, exhibit no electrical resistance.

Applications of superconductors

(i) Superconductors form the basis of energy saving power systems, namely the superconducting generators, which are smaller in size and weight, in comparison with conventional generators.
(ii) Superconducting magnets have been used to levitate trains above its rails. They can be driven at high speed with minimal expenditure of energy.
(iii) Superconducting magnetic propulsion systems may be used to launch satellites into orbits directly from the earth without the use of rockets.
(iv) High efficiency ore separating machines may be built using superconducting magnets which can be used to separate tumor cells from healthy cells by high gradient magnetic separation method.
(v) Since the current in a superconducting wire can flow without any change in magnitude, it can be used for transmission lines.
(vi) Superconductors can be used as memory or storage elements in computers

The thermistor

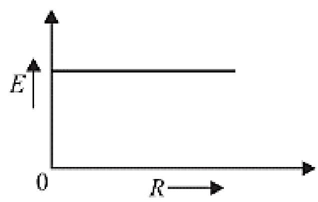
The thermistor is a bipolar semiconductor circuit element and is a temperature dependent resistor. When the thermistor is hot, its resistance is low. As the temperature decreases, the resistance of the thermistor increases.

When a current is passed through the thermistor, heat is generated. As the temperature increases, the loosely bound electrons are released for conduction. Hence increase in the current reduces the resistance of the thermistor.

Note: The thermistor can be used in potential divider circuits to monitor and control temperatures.

Variation of emf, *E* with resistance

The value of emf E of the cell does not depend upon R. Therefore, the plot of E versus R is a straight line parallel to R axis as shown below.



Variation of terminal p.d with resistance

Terminal p.d, $V = IR$

but $I = \frac{E}{R+r}$

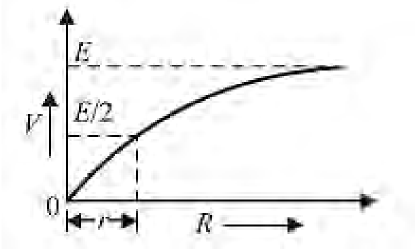
Multiplying R.H.S by $\frac{1}{R}$ both at numerator and denominator.

$$V = \frac{E/R}{1+\frac{r}{R}}$$

When $R = 0, \Rightarrow V = 0$

When $R = r, V = \frac{E}{2}$

When $R \gg r, V \longrightarrow E$



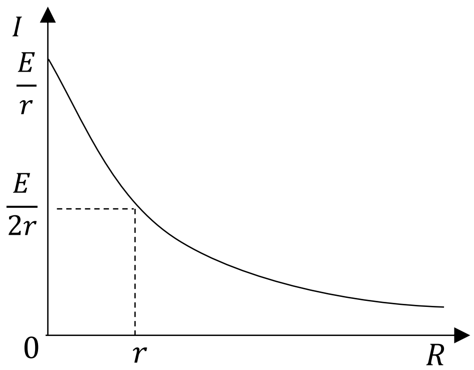
Variation of current with load resistance

From $I = \frac{E}{R+r}$

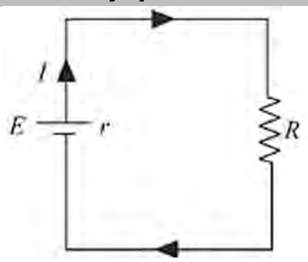
when $R = 0, I = \frac{E}{r}$

when $R = r, I = \frac{E}{2r}$

when $R \gg r,\ I \longrightarrow 0$



Variation of efficiency η with load resistance (R)



Power delivered to the load = power output = VI

Power supplied by the cell = power in put = EI

Efficiency of circuit (η) = $\frac{P_{out}}{P_{in}} \times 100$

$$\eta = \frac{VI}{EI} \times 100\%$$

$$\eta = \frac{V}{E} \times 100\%$$

But $V = IR$ and $E = I(R+r)$

$$\eta = \frac{IR}{I(R+r)} \times 100\%$$

$$\eta = \frac{R}{R+r} \times 100\%$$

Multiply by $\frac{1}{R}$ (Both numerator and denominator)

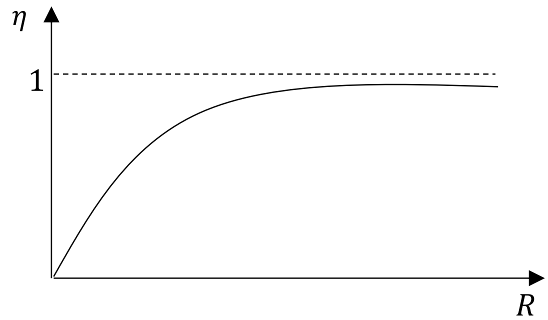
$$\eta = \frac{1}{1+\frac{r}{R}} \times 100\%$$

when $R = 0, \eta = 0\%$

when $R = r, \eta = 50\%$

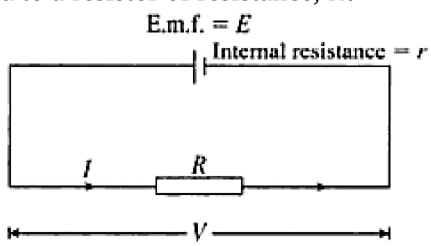
when $R \gg r, \eta \longrightarrow 100\%$

A graph of efficiency against load resistance



Variation of power with load resistance

Consider a cell of e.m.f, E and internal resistance, r connected to a resistor of resistance, R.



Power dissipated in R, $P = IV = I^2R = \left(\frac{E}{R+r}\right)^2 R$

To determine the value of R which would produced the maximum power in R, we differentiate P with respect to R.

$$\frac{dP}{dR} = \frac{E^2(R+r)^2 - 2E^2R(R+r)}{(R+r)^4}$$

For maximum power P, $\frac{dP}{dR} = 0$,

$$E^2(R+r)^2 - 2E^2R(R+r) = 0$$

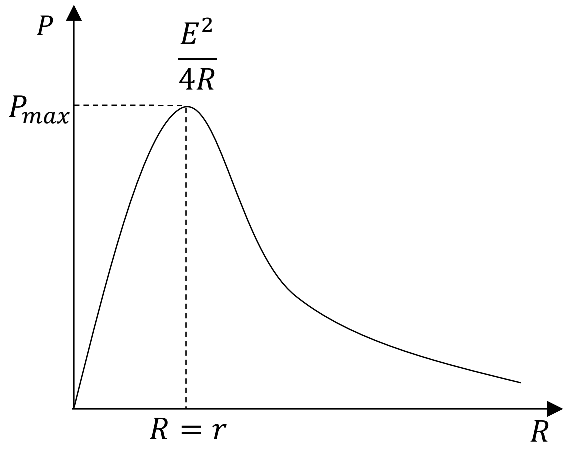
$$(R+r) - 2R = 0$$

$$\Rightarrow R = r$$

Hence the power dissipated in the external resistance, R is maximum when $R = r$

Maximum power, $\boldsymbol{P = \left(\frac{E}{2R}\right)^2 R = \frac{E^2}{4R}}$

A graph of power against load resistance



Example

A battery of emf $18.0\,V$ and internal resistance $3.0\ \Omega$ is connected to a resistor of resistance $8.0\ \Omega$. Calculate the

(a) power generated

(b) efficiency

Solution

$$E = 18.0\,V, r = 3.0\,\Omega, R = 8\,\Omega$$

(a) $P = EI, I = \frac{E}{R+r}$

$$= \frac{18}{8+3} = \frac{18}{11}\,A$$

$$P = 18 \times \frac{18}{11} = 29.45\,W$$

(b) Efficiency $= \frac{P_{out}}{P_{in}} \times 100$

$$= \frac{VI}{EI} \times 100$$

$$= \frac{V}{E} \times 100$$

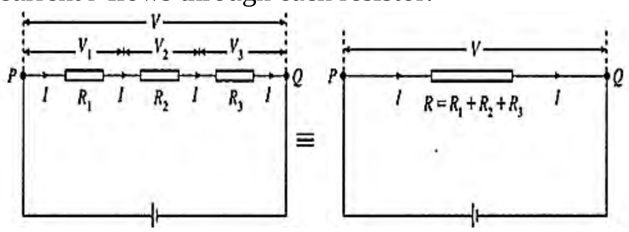
$$= \frac{IR}{I(R+r)} \times 100$$

$$= \frac{8}{8+3} \times 100 = 72.73\%$$

ARRANGEMENT OR RESISTORS

1. Series arrangement

When the resistors are connected in series, the same current I flows through each resistor.



$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$$

$$V = V_1 + V_2 + V_3$$

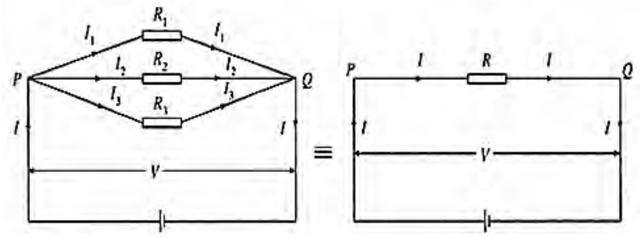
$$V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3)$$

$$\frac{V}{I} = R_1 + R_2 + R_3$$

$$\boldsymbol{R = R_1 + R_2 + R_3}$$

2. Parallel arrangement



The potential differences across the resistors are the same

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3$$

$$V = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{V}{I} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boldsymbol{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Note:

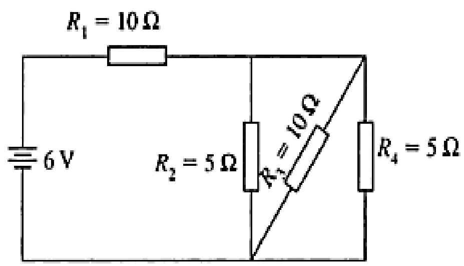
For two resistors in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\Rightarrow R = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{product}}{\text{sum}}$$

Example 1

In the circuit below, the e.m.f of the battery is $60\,V$ and its internal resistance is negligible



(a) What is the effective resistance in the circuit?

(b) What is the current flowing through each resistor?

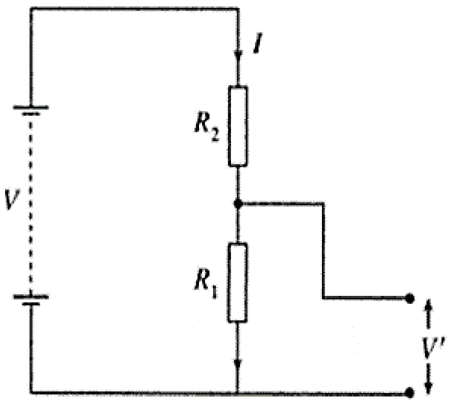
Solution

(a)

POTENTIAL DIVIDER

A potential divider is an arrangement of resistors which is used to obtain a fraction of the potential differences by a voltage supply.

Consider a simple divider which consists of two known resistors R_1 and R_2 connected in series to voltage supply V. The potential difference V' is then connected to a load such as a lamp.



The current I flowing is given by;

$$I = \frac{V}{R_1 + R_2}$$

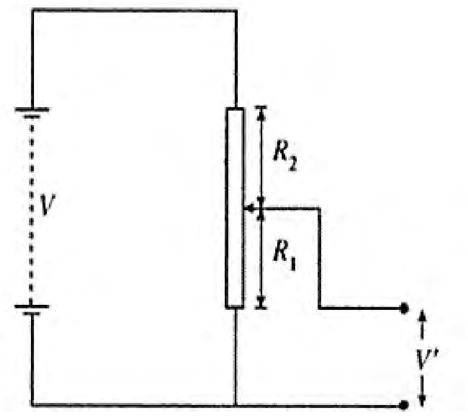
Potential difference across R_1, $V' = IR_1$

$$V' = \left(\frac{R_1}{R_1 + R_2}\right)V$$

which is a fraction of a voltage of supply.

Note:

To provide a continuously variable potential difference from zero to the full voltage of supply V, a resistor with a sliding contact is used.



KIRCHHOFF'S LAWS

To solve complicated circuits, Kirchhoff gave two simple laws, called Kirchhoff's laws. These laws are simply the applications of the laws of conservation of charge and energy.

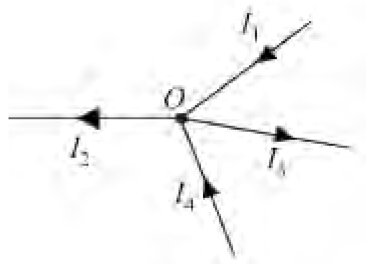
1. Kirchhoff's current law
2. Kirchhoff's voltage law

Kirchhoff's current law Kirchhoff's Junction rule

This law is based on the conservation of charge and stated as

The algebraic sum of the currents meeting at a junction in an electric circuit is zero.

An algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents I_1, I_2, I_3 and I_4 and meeting at point O as shown below.



If we take the signs of currents flowing towards point O as positive, then currents flowing away from point O will be assigned negative sign. Thus, applying Kirchhoff's current law to the junction O, we have

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

$$\text{Or } I_1 + I_4 = I_2 + I_3$$

i.e. sum of incoming currents = sum of outgoing currents

Hence, Kirchhoff's may be stated as;

The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction.

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge.

Kirchhoff's voltage law or Kirchhoff's loop rule.

This law is based on the conservation of energy and may be stated as;

In any closed electrical circuit or loop, the algebraic sum of all the electromotive forces (emfs) and voltage drops in resistors is equal to zero.

i.e., in any closed circuit or loop,

Algebraic sum of emfs + Algebraic sum of voltage drops = 0

Note that a closed electrical circuit is a **loop**.

galvanometer shows a small deflection. Explain why this occurs.

d) If the direction of the current is from X to Y, deduce which of the wires A or B has the greater resistance.

Solution

(a) If V = Potential difference across MN, and since resistance of A = resistance of B,

Potential difference across MX = p.d across $XN = \frac{V}{2}$

Since resistance of P = resistance of Q

Potential difference across MY = p.d across $YN = \frac{V}{2}$

Hence potential of X = potential of Y

Since there is no potential difference across XY, no current flows through the galvanometer

(b) Since $R_P = R_Q$, where the galvanometer is balanced.

$$R_A = R_B$$

$$\rho_1\left(\frac{l_1}{\pi d_1^2}\right) = \rho_2\left(\frac{4l_1}{\pi d_1^2}\right)$$

$$\rho_2 = \rho_1\left(\frac{l_1}{l_2}\right)\left(\frac{d_2^2}{d_1^2}\right)$$

$$= (4.0 \times 10^{-7}) \times 2 \times \left(\frac{1}{1.5}\right)^2$$

$$= 3.6 \times 10^{-7}\Omega m$$

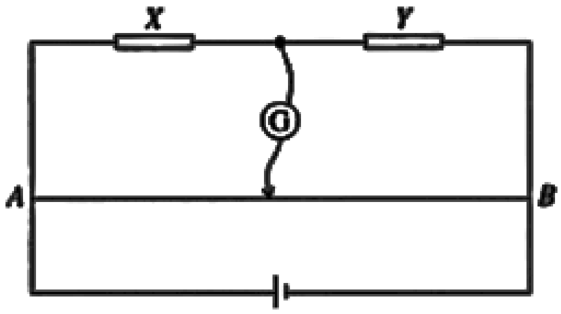
(c) With a cell of larger e.m.f, the current flowing through A and B is larger. The larger current produces a larger increase in resistance. Since A and B are made of different materials, the new resistance of A is different from that of B. The potential differences across A and B are not equal and a p.d exists between the points X and Y. Hence a current now flows through the galvanometer.

(d) Since the current flows from X to Y, the electric potential at X, V_X is greater than the electric potential at Y, V_Y. Hence potential difference across B > p.d across A

:. Resistance of B > resistance of A

Example 3

In the circuit shown below, the resistances of X and Y are 5 Ω and 3 Ω respectively and the length of $AB = 1.00\ m$.



When a shunt is connected in parallel to X, the balanced length is $0.527\ m$ from A. What is the resistance of the shunt?

Solution

Let R = effective resistance of shunt and 5 Ω resistor in parallel.

$Using;\ \frac{R}{S} = \frac{l_1}{l_2}$

$$R = \frac{0.527}{0.473} \times 3 = 3.342\ \Omega$$

Let R_s = resistance of the shunt

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{R_s}$$

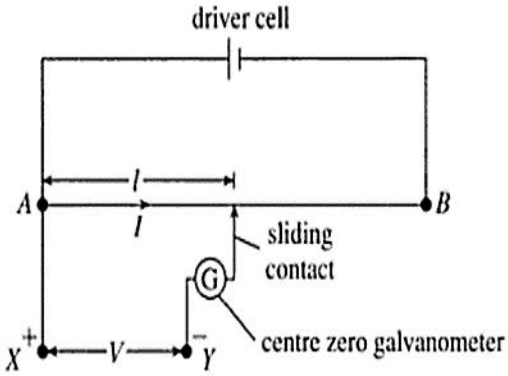
$$\frac{1}{R_s} = \frac{1}{3.342} - \frac{1}{5}$$

$$R_s = 10.08\ \Omega$$

THE SLIDE WIRE POTENTIOMETER

The potentiometer is used to measure the potential differences and e.m.f to a high degree of accuracy.

The slide wire potentiometer consists of a uniform resistance wire AB usually of length 1m. A cell known as the driver cell is used to maintain a constant potential difference across the ends of the wire.



Since the wire is uniform, the p.d across each centimeter length of the wire is the same all along the wire. Hence the potential difference V between the end A of the wire and the sliding contact is proportional to the length of the wire between two points.

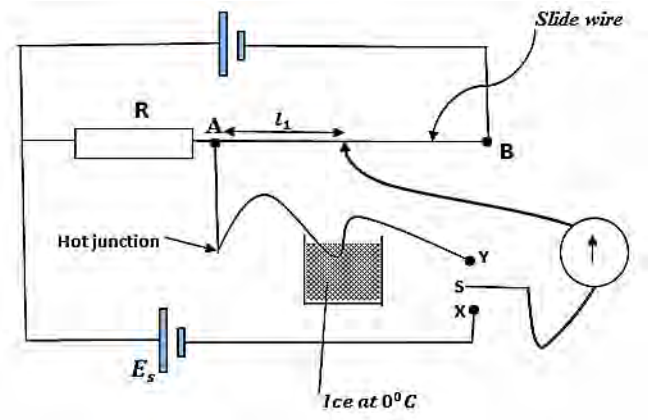
$V \propto l \Rightarrow V = kl$ where k = constant.

Thus, the unknown p.d can be compared with the p.d across the slide wire of the potentiometer

$$\frac{R}{S} \times \frac{S+12}{12} = \frac{51.9}{48.1}$$
$$\frac{33.7}{66.3} \times \frac{S+12}{12} = \frac{51.9}{48.1}$$
$$S = 13.1\ \Omega$$

From (i); $R = S \times \frac{33.7}{66.3} = 13.1 \times \frac{33.7}{66.3} = 6.64\ \Omega$

3. Measurement of small e.m.fs - the thermocouple



The circuit is connected as shown above with an appropriately high resistance R connected in series with the potentiometer wire. This is done to obtain an appreciable balance length since the e.m.f of a thermocouple is in order of millivolts.
With the switch S at position X, the sliding contact is moved along AB until the galvanometer shows no deflection. The balance length l_1 is recorded.
Witch S connected at position Y, the sliding contact is moved along AB until the galvanometer shows no deflection. The balance length l_2 is recorded.
The e.m.f E_T of the thermocouple is given by;

$$E_T = Ikl_2$$

while $E_s = I(kl_1 + R)$

$$E_T = \frac{E_s kl_2}{R + kl_1}$$

where k is resistance per cm of the uniform resistance slide wire.

Sensitivity of potentiometer

The sensitivity of a potentiometer indicates the smallest potential difference that can be measured with it. The sensitivity of a potentiometer depends upon the potential gradient of the potentiometer wire (i.e. fall in potential per unit length along the potentiometer wire). The smaller the potential gradient, the more is the sensitivity of the potentiometer. Therefore, the sensitivity of a potentiometer can be increased by decreasing the potential gradient i.e.

(i) by increasing the length of the potentiometer wire
(ii) by reducing the current in the potentiometer wire with a resistance in series with potentiometer wire. This is equivalent to increasing the length of the potentiometer wire.

Potentiometer versus voltmeter

We now discuss advantages and disadvantages of a potentiometer compared to a voltmeter.

Advantages

(i) It is a null-deflection method and therefore, the balance condition can be found with a high degree of sensitivity
(ii) No current is drawn from the circuit under test. Therefore, it can measure the emf of a cell accurately
(iii) The scale can be made long for maximum accuracy
(iv) Results are dependent only on measurements of lengths and the values of standard resistances and standard emfs.
(v) The potentiometer is more accurate than the moving-coil voltmeter for measuring the emf. The moving coil voltmeter has finite resistance and while connected across a cell, the voltmeter draws some current. Therefore, the voltmeter measures the emf of the cell approximately.
(vi) A potentiometer is a voltmeter with infinite resistance.
(vii) Apart from measuring potential difference, a potentiometer can be used for various purposes e.g. to measure the internal resistance of a cell, etc. However, voltmeter can measure only potential difference.

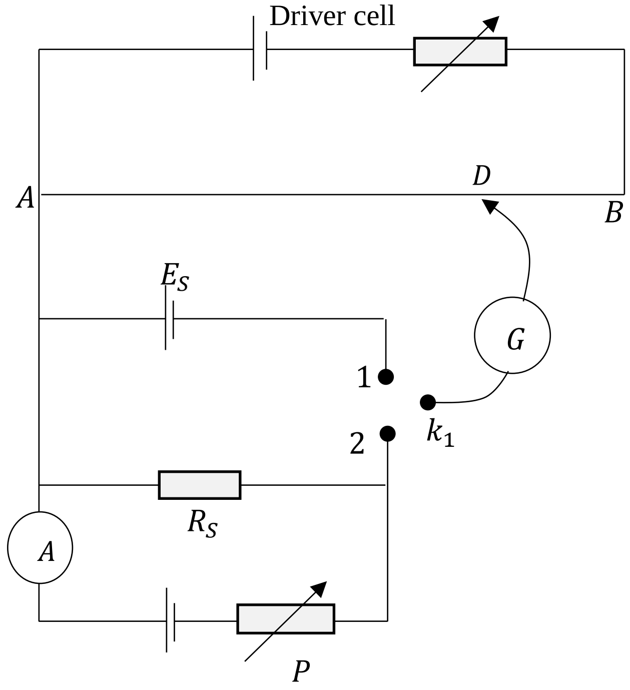
Disadvantages

(i) It is slow in operation
(ii) The potentiometer wire must be of uniform thickness
(iii) The temperature of the potentiometer wire must remain constant
(iv) There may be end errors

Why a potentiometer is preferred to a voltmeter in measuring voltages

1. It does not draw any current and thus gives accurate measurements
2. It can measure a wider range of potential differences since the length of the wire can be adjusted.
3. The results are accurate since they depend only on measurement of length, standard resistance and standard emf
4. They are more durable

6. Calibration of an ammeter



E_s and R_s are standard emf and standard resistance respectively

Switch k is connected to position 1 and the jockey moved along the wire AB to locate position D where the galvanometer shows no deflection

Measure and record the balance length l_s from A to D.

Disconnect k from 1 and connect it to 2 and then rheostat P adjusted to obtain a suitable reading I_r of the ammeter A.

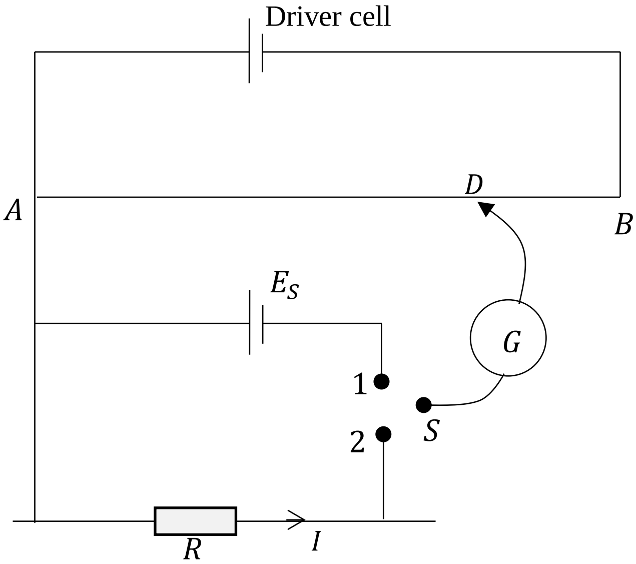
The balance point is then located along the wire AB and the balance length l is measured and recorded together with ammeter reading I_r.

The procedure is repeated for different settings of P and hence different ammeter readings I_r.

The results are tabulated including values of $I_a = \frac{E_s}{R_s l_s} l$

Plot a graph of I_a against I_r and it constitutes the calibration curve for the ammeter.

7. **Measurement of current**



Connect the circuit as above

Contact S is connected to 1 and the jockey is moved along the slide wire AB to a point D where the galvanometer shows no deflection

Measure and record the balance length l_s

The p.d per cm k of the wire is given by

$$k = \frac{E_s}{l_s}$$

Contact s is disconnected from 1 and then connected to 2.

The current I to be measured is then passed through the shunt R.

The new balance point is located and the balance length l is measured and recorded.

$$\text{p.d across } R = V_{AD}$$

$$IR = kl_2$$

$$IR = \frac{E_s l}{l_s}$$

$$I = \frac{E_s l}{R l_s}$$

where R and E_s are known resistance and emf

Conceptual questions

Why is the Wheatstone bridge not suitable for measuring very high resistances?

Ans: For high sensitivity of the bridge, all resistances should have high value. This will reduce the value of current through the galvanometer which would become insensitive.

What is the advantage of using a greater length of potentiometer wire?

Ans: The greater the length of the potentiometer wire, the smaller is the potential gradient along the wire.

Hence, the distance of the null-position will be increased which can be measured more accurately.

A potentiometer is equivalent to a voltmeter of infinite resistance. Discuss.

When the emf of a cell is measured by a potentiometer, then at null point, the current through the cell is zero, i.e. the cell is open circuit. Therefore, we get the actual value of emf of the cell. If the above condition is to be realized with a voltmeter, it should have infinite resistance.

A potentiometer measures exact p.d while voltmeter does not. Explain

Ans: The potentiometer method is a null method. At null point, no current flows in the circuit under measurement. Hence, potentiometer measures the exact p.d. On the other hand, when a voltmeter is connected to measure p.d across a component (say a resistor) a part of current is drawn by the voltmeter. As a result, current through the component decreases. Hence, p.d being measured by the voltmeter is slightly less than the actual value.

Sometimes balance point is not obtained on potentiometer wire. Why?

Ans: It is because the p.d under measurement is greater than the potential drop across the potentiometer wire. In that case, the driver battery of large emf should be used.

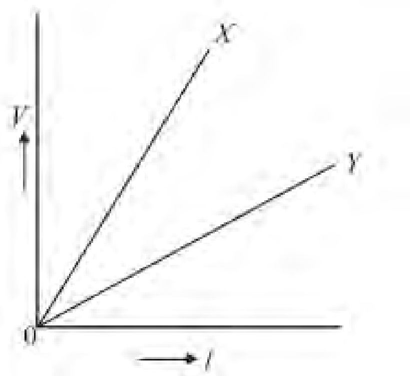
For measuring p.d by a voltmeter, deflection of the instrument is noted. When measuring p.d by a potentiometer, null position on the potentiometer wire is noted. In which case will the error of measurement be less?

Ans: The error will be less in case of a potentiometer. Let us explain this point. When p.d is measured by a voltmeter, there can be some error in reading the deflection. However, in reading the null-point position in a potentiometer, there can be a maximum error of 1 mm. Suppose the potentiometer wire is 2 m long and is connected to a cell of emf 2 V. Then potential gradient along the wire per mm will be 2/2000 = 0.001 V. Thus, the maximum error in measuring the p.d will be 0.001 V. Thus, the maximum error in measuring the p.d will be 0.001 V. This error can be further reduced by using a longer potentiometer wire.

Why should the cross-section of potentiometer wire be uniform?

Ans: If the cross-section of potentiometer wire is not uniform, then potential gradient will not be the same at all places on the potentiometer wire. Consequently, the measured value of the potential difference will not be correct.

The variation of potential difference V with length l in case of two potentiometers X and Y is shown below.



Which one of these two will you prefer for comparing emfs of two cells and why?

Ans: A potentiometer is said to be more sensitive if fall of potential per unit length along the potentiometer wire is small i.e. potential gradient (dV/dl) is small. Since dV/dl is the slope of $V - l$ graph, the slope for the potentiometer Y is less than that of potentiometer X. Therefore, potentiometer Y will be preferred for comparing emfs of two cells.

Why should not current be passed through a potentiometer wire for a long time?

Ans: If a current is passed through a potentiometer wire for a long time, it gets heated and its resistance increases. This will change the potential gradient along the wire.

What is the advantage of measuring unknown resistance with a Wheatstone bridge?

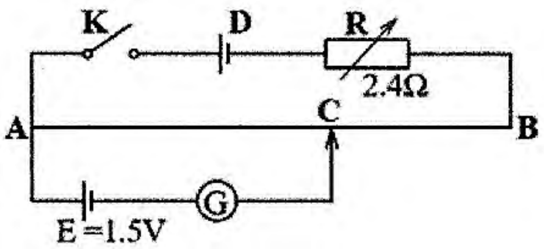
Ans: Although value of unknown resistance can be measured by such methods as voltmeter-ammeter method, potentiometer method, etc. yet Wheatstone bridge method is preferred. It is because this method is independent of the fluctuations and variations in the supply voltage. If you look at the equation of resistances at null point, you will find that battery voltage does not appear in the equation.

Why is Wheatstone bridge not suitable for measuring very low resistances?

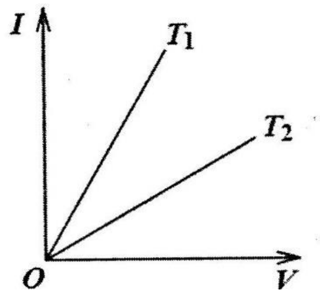
Ans: The sensitivity of a Wheatstone bridge is very high when all the four resistances are nearly equal. This means that to measure a low resistance, the resistances of other resistors should also be low. This means that a galvanometer of very low resistance should be used which itself would be insensitive.

EXAMINATION QUESTIONS

1. (a) Define the following as applied to a battery
   (i) Electromotive force
   (ii) Internal resistance
   (b) A battery of e.m.f E and internal resistance r, is connected across a load of resistance, R. Derive an expression for the maximum power delivered to the load
   (c) Describe how the resistance of a resistor may be determined using a slide wire potentiometer
   (d) Explain why the potentiometer is more suitable for measuring small resistances than a Wheatstone bridge
   (e) In the figure below, AB is a uniform resistance wire of $2.0\ \Omega$, and length $100\ cm$. E is a cell of e.m.f $1.5\ V$ and D is a driver cell of negligible internal resistance.
   When switch K is closed the balance length AC is $82.5\ cm$.



   (i) Find the e.m.f of cell D
   (ii) If cell E and the galvanometer are replaced by a voltmeter of resistance $20.0\ \Omega$, find the reading of the voltmeter when contact C is placed at the mid-point of AB and the value of R is $1.0\ \Omega$.
   [2018, No. 8; Ans: (e) (i) 4 V (ii) $1.29\ V$]
2. (a) (i) State the law of conservation of current at a junction in an electric circuit
   (ii) Explain why current from a battery is greater when bulbs are connected in parallel than when they are in series across the battery
   (b) A conductor of length l and cross-sectional area A has n free electrons per unit volume. The average drift velocity of the electrons is v and each electron carries a charge e. Derive an expression for the current which flows.
   (c) A battery with an e.m.f of $12\ V$ and internal resistance $2\ \Omega$ is connected to a wire of resistance $10\ \Omega$.
   (i) Calculate the p.d across the wire
   (ii) What will the p.d across the wire become if a $15\ \Omega$ resistor is connected in parallel with it?
   (d) (i) Define **electrical resistivity** and its unit
   (ii) Describe an experiment to determine the resistivity of the material of a wire using an ammeter, a meter rule and a voltmeter
   [2017, No. 8; Ans: (c) (i) $10\ V$ (ii) 9 V]
3. (a) (i) Define **temperature coefficient** of resistance and state its unit
   (ii) Explain why temperature coefficient of resistance is positive for metals
   (b) (i) Derive the condition for balance of a metre bridge
   (ii) Explain why the metre bridge is unsuitable for comparison of low resistances
   (c) A standard resistor is connected across the right-hand gap of a metre bridge and a coil X across the left-hand gap of the bridge.
   When the coil X is heated up to a temperature of $40°C$, the balance length is $525\ mm$ from the left end of the bridge.
   When the temperature of X is raised to $100°C$, the balance point is $546\ mm$ from the left end.
   (i) Calculate the temperature coefficient of resistance of coil X
   (ii) Why are standard resistors made of alloys such as constantan and manganin.
   [2017, No. 9; Ans: (c) (i) $1.57 \times 10^{-3}\ K^{-1}$]
4. (a) (i) Define **electrical resistivity**
   (ii) Explain how length and temperature of a conductor affect its resistance
   (iii) The figure below shows the current-voltage graphs for a metallic wire at two different temperatures T_1 and T_2



   State which one of the two temperatures is greater and explain your answer.
   (b) (i) Derive the balance condition when using a metre bridge to measure resistance.
   (ii) State **two** precautions taken to achieve an accurate measurement

ELECTROSTATICS

This area of physics is called, electrostatics, because all the charges remain at rest.

Electric charge

Like mass, electric charge is a fundamental property of matter. Electric charge is fundamentally associated with atomic particles, the electron and the proton.
An important distinction between the gravitational and electric forces is that apparently there is one type of mass and this gives rise to only attractive gravitational forces.
Electric charge, on the other hand, comes in two types, i.e. positive and negative charges. Different combinations of the two kinds of charges can produce either attractive or repulsive forces.
The directions of the electric forces when charges interact with one another are given by a simple law.

Law of electrostatics

Like charges repel and unlike charges attract.

That is, two negatively charged particles or two positively charged particles experience mutually repulsive forces, whereas particles with opposite sign are mutually attracted. These forces are equal and opposite and act on different objects in accordance with Newton’s third law.

Quantization of charge

The charge on the electron, e is taken as the fundamental unit of charge. Thus, the electric charge, q on an object is an integral multiple of fundamental charges i.e.

$$\boldsymbol{q = ne}$$

where n is an integer

This is known as quantization of charge which means that it occurs only in integral multiples of the fundamental electronic charge.

Electrostatic charging

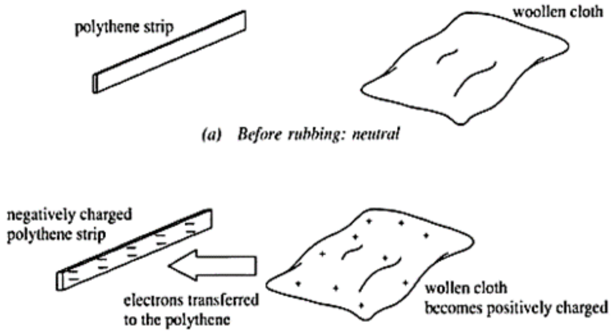
In general, electrostatic charging is a process by which an insulator or an insulated conductor receives a net charge. There are three processes by which a body can be charged i.e. contact, friction and induction.

Charging by friction

The process of rubbing two different materials together is known as charging by friction and it is the simplest way of giving something a charge.

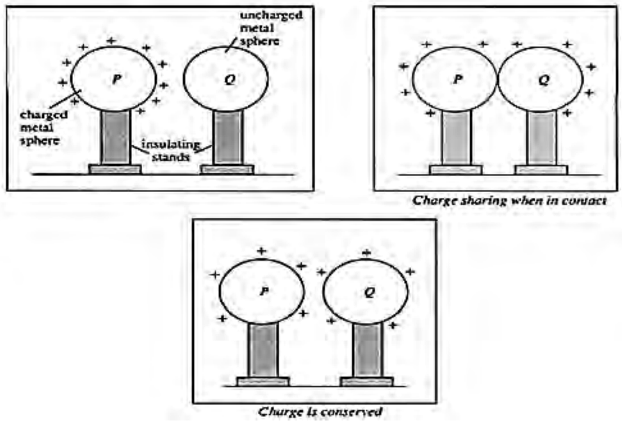
How a body acquires charge by friction

Rubber has a much greater attraction for electrons than animal fur. When these two materials are rubbed together, the atoms of rubber pull electrons from the atoms of animal fur, leaving both objects with an imbalance of charge. The rubber has an excess of electrons and the animal fur has a shortage of electrons. The rubber becomes negatively charged while fur becomes positively charged.
The two bodies will have equal and opposite charges since the amount of electrons lost by one body is equal to the amount of electrons gained by the other body.



Charging by conduction

A charged object, when brought into physical contact with a second object, may share some or all of its charges with the second uncharged object. The second uncharged gets the same type of charge as the first object originally had. Charge is therefore conserved between the two conductors.



Why a charged body attracts a neutral body

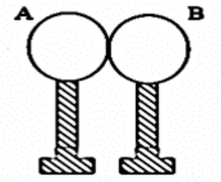
When a neutral metal is brought near a charged material, opposite charge is induced on the near side of the metal and charge similar to that of the charged body on the far side. Since opposite charges are now closer to each other, the attraction force between the material is greater than the repulsion force. Hence the metal body is attracted.

Electrostatic induction

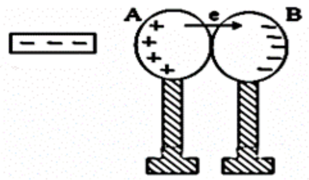
This is a process of providing an object with a charge without touching it.

Charging a two-sphere system using a negatively charged object

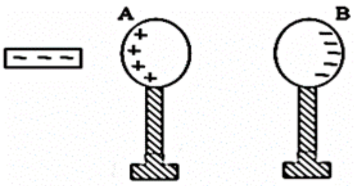
The two metal spheres are mounted on insulating stands and then brought in contact with each other.



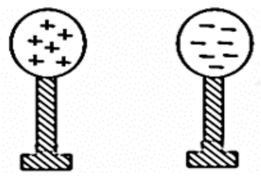
When the negatively charged object is brought close to the metal sphere, the electrons are induced to move from sphere A to B. The two-sphere system is polarized.



Sphere B is then separated from sphere A using an insulating stand.

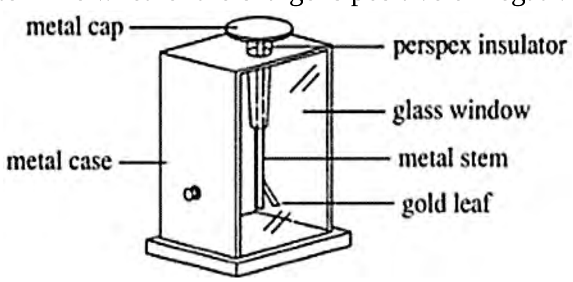


The excess charge then uniformly distributes itself over the surface of the spheres.



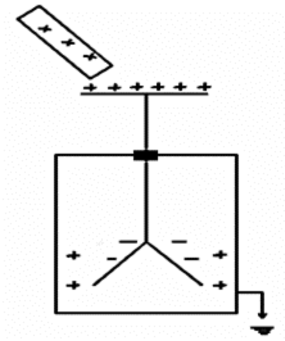
THE GOLD-LEAF ELECTROSCOPE

This is a device which measures electric potential but can also be used to detect the presence of charge and determine whether the charge is positive or negative.



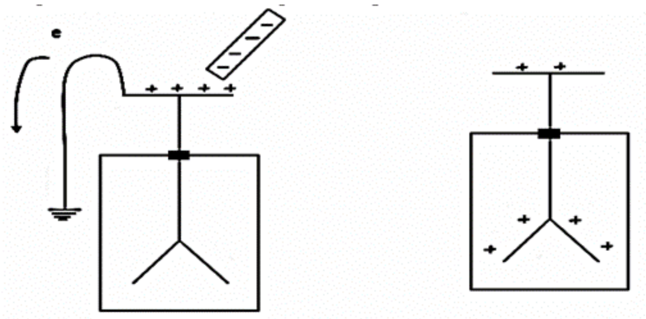
Charging an electroscope by induction

A negatively charged body is brought near to the cap of the gold leaf electroscope. A positive charge is induced on the cap and the leaves acquire a negative charge.



The negative charge on the leaves induces a positive charge on the case by causing the electrons to flow to it from the earth.

The positive charge on the case attracts the negative charge on the leaves, and makes them diverge further.



By momentarily earthing the cap while holding an inducing charge near it, the cap can acquire a permanent charge. In this way, the gold-leaf electroscope becomes positively charged.

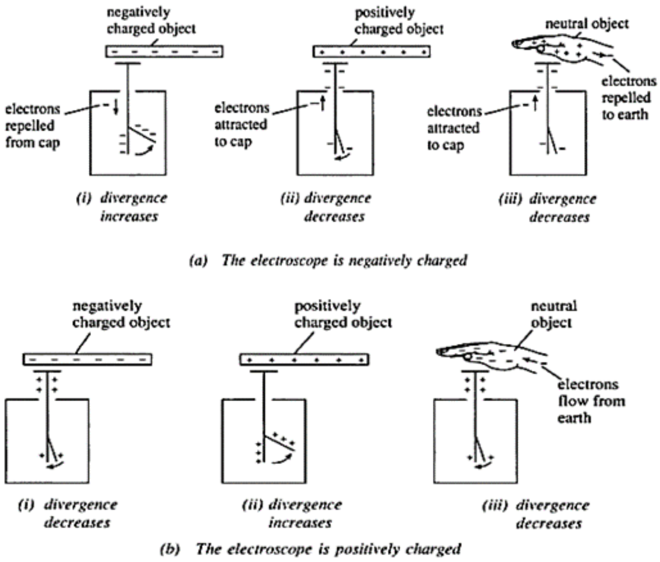
Determining the sign of charge

In order to identify the charge of a body, a charged electroscope is used.

If the electroscope is negatively charged and an unknown charged body is bought near its cap, the charge will be negative if the leaves diverge further.

If the leaves contract, the body may have a positive charge or may have no charge.

In order to confirm the positive charge, the uncharged body is brought near the cap of a positively charged electroscope and if the leaves diverge further, then the body will have a positive charge.

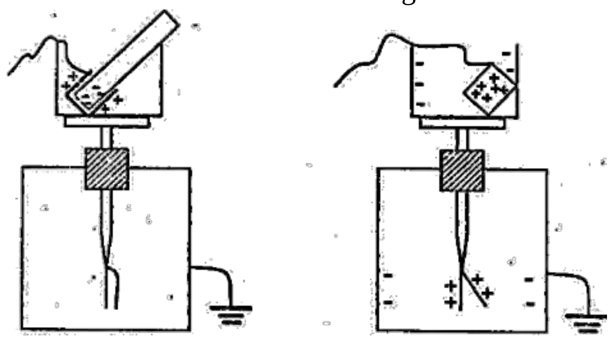


Experiment to verify that two insulating bodies rubbed together acquire equal and opposite charges

An ebonite rod is first discharged completely by drawing it quickly through air above a gas flame, then it is fitted with a fur cap that has silk thread attached to it.

The rod is held in one hand as the cap is rotated several times round the rod using the silk thread. This causes both the rod and cap to be charged by friction.

The capped end of the rod is now placed inside a metal can standing on the cap of the gold leaf electroscope and it is observed that no leaf divergence occurs.

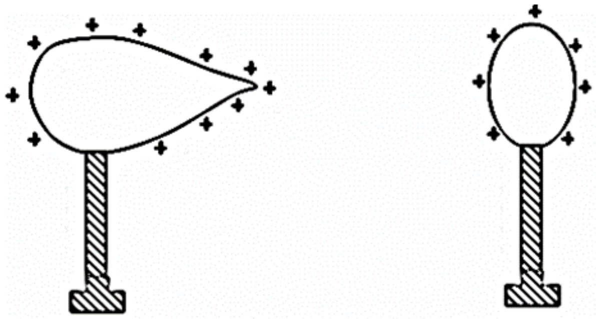


This implies that either there is no charge on both the rod and the cap or the rod and cap have equal but opposite charges.

The rod is removed and fur cap is left inside the can. This causes the leaf to diverge immediately indicating that when both the fur and the cap were in the can, their opposite charges exactly canceled each other because they were of equal magnitude.

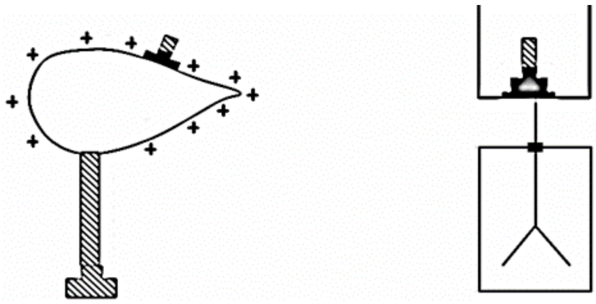
Distribution of charge

Since the charges repel, the charge on a hollow or solid metal sphere will move away from the centre as far as it will go. Thus, the charge on a conductor of any shape is always found on the surface and never inside the conductor. A charged metal sphere has a uniform charge density allover its surface however a pear-shaped conductor has a high charge density at the pointed part.



Investigation of charge distribution over the surface of a conductor

The conductor is charged by touching it with an electrophorus. A proof plane (a small metal disc on an insulating handle) is placed in contact with a section of the surface of a conductor.



The proof plane acquires a charge proportional to the charge density on the section of the conductor.

The proof plane is then removed and placed in contact with the inner surface of the metal can standing on the cap of the electroscope.

The leaves diverge by an amount proportional to the charge on the proof plane.

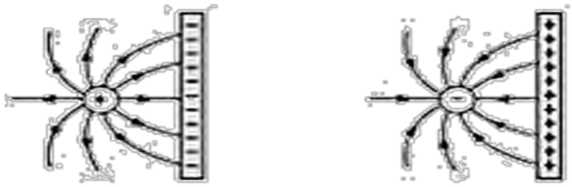
The proof plane can be used to test other parts of the conductor as it loses all its charge when placed inside the can.

It is observed that the charge density is greatest when the surface is highly curved.

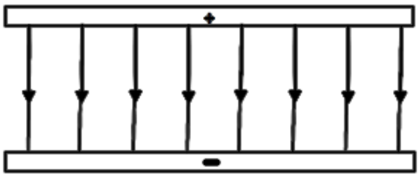
Corona discharge (action at points)

Corona discharge is an electrical discharge brought by the ionization of a fluid or a gas surrounding a conductor that is electrically charged.

(iii) Between a point charge and charged plate

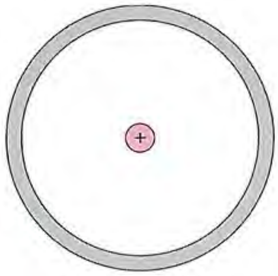


(iv) Between two charged plates.



Example

A positive charge is inside an isolated metal sphere as shown in the diagram below.



Metal conductor

Describe the situations in terms of the electric field and the charge on the sphere. How would the situation change if the charge were negative?

Solution

Negative charge on the inside surface of the sphere and equal amount of positive charge on the outside of the sphere. The excess like charges will be distributed evenly over the surface of the sphere.

The electric field outside the sphere behaves as though all the excess charge on the sphere were concentrated at its centre and the electric field is zero inside the sphere.

If the charge was negative, the field lines would all point towards the charge, the field inside would still be zero and the field outside would behave as though all the excess charge were concentrated at the centre, even though the excess charge is located on the surface.

COULOMB'S LAW OF ELECTROSTATICS

The magnitude of the force between two electrically charged bodies is directly proportional to the product of their charges and inversely proportional to the square of their mean distance of separation.

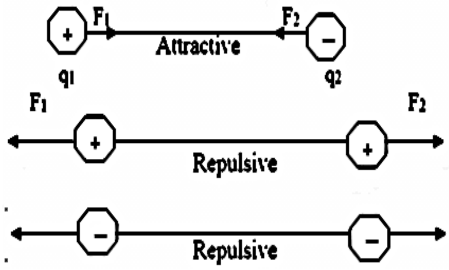
$$F = \frac{kQ_1Q_2}{r^2} \quad or \quad F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

where $k = 9.0 \times 10^9\ Nm^2C^{-2}$

The constant k is also written as $\frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space and has a value $8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Note:

The direction of the forces on each particle depends on the sign of the charges as illustrated below.



Comparison of the electrostatic force and the gravitational force

We can compare the electrostatic repulsive force between two electrons with the attractive force due to gravitation by finding the ratio of the magnitudes of the forces.

The electrostatic force is given by Coulomb's law and the gravitational force is given by Newton's law of gravitation. Both of these forces depend on $1/r^2$. The ratio of the magnitude of these forces is then independent of the separation r between the electrons.

The magnitude of the electrical force is

$$F_e = k\frac{q^2}{r^2} \text{ (repulsive)}$$

The magnitude of the gravitational force is

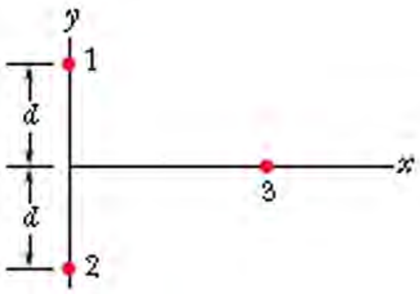
$$F_g = G\frac{m^2}{r^2} \text{ (attractive)}$$

The ratio of the two forces is

$$\frac{F_e}{F_g} = \frac{k}{G}\cdot\frac{q^2}{m^2}$$

$$\frac{F_e}{F_g} = \frac{9.0 \times 10^9}{6.67 \times 10^{-11}}\cdot\frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$\frac{F_e}{F_g} = 4.2 \times 10^{42}$$



(a) At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be

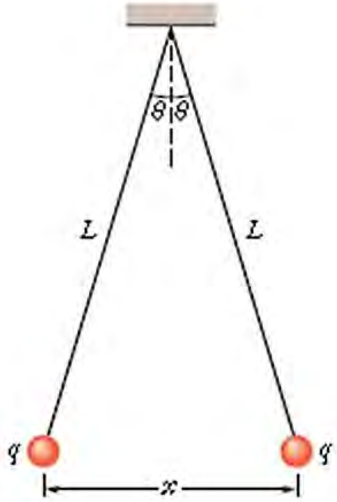
(i) minimum

(ii) maximum

(b) What are the minimum and maximum magnitudes?

[Ans: (a) (i) 0 (ii) 12 cm (b) $0, 4.9 \times 10^{-26}\ N$]

6. The figure below, two tiny conducting balls of identical mas m and identical charge q hang from non-conducting threads of length L. Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$.



(a) Show that $x = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$ gives the equilibrium separation x of the balls

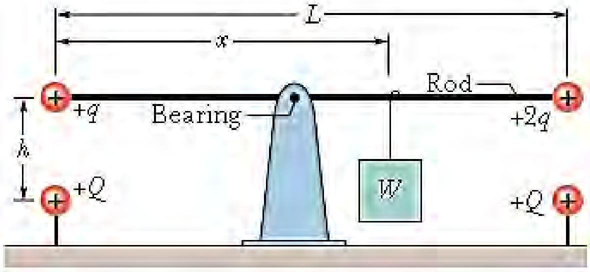
(b) If $L = 120\ cm$, $m = 10$ g and $x = 5.0\ cm$, what is the magnitude of q?

(c) Explain what happens to the balls if one of them is discharged (loses its charge q to say, the ground)

(d) Find the new equilibrium separation x, using the given values of L and m and the computed value of q

[Ans: (b) (d) 3.1 cm]

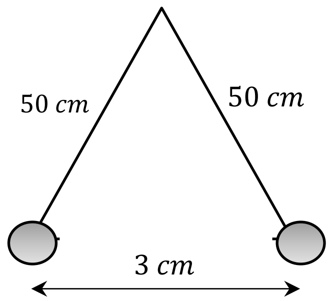
7. The figure below shows a long, non-conducting, massless rod of length L, pivoted at its centre and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q.



(a) Find the distance x when the rod is horizontal and balanced

(b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?

8.



Two light conducting spheres, each 6 mm in diameter and having a mass of 10 mg, are suspended from the same point by fine insulating fibres 50 cm long. Due to electrostatic repulsion, the spheres are in equilibrium when 3 cm apart. What is

(i) the force of repulsion between the sphere?

(ii) the charge of each sphere?

(iii) the potential of each sphere?

[Ans:(a) (i) $3.0 \times 10^{-6}\ N$ (ii) $5.5 \times 10^{-10} C$ (ii) $1.8 \times 10^3\ V$]

9. (a) Calculate the force between two small metal spheres with charges $+1.0 \times 10^{-9} C$ and $+9.0 \times 10^{-9} C$ whose centres are 30 cm apart in air, for which the permittivity is $8.9 \times 10^{-12} Fm^{-1}$. Is the force attractive or repulsive?

[Ans: 0.89 μN]

10. Three charges $q_1 = 1\ \mu$C, $q_2 = -2\ \mu C$ and $q_3 = 3\ \mu$C are placed on the vertices of an equilateral triangle of side 1.0 m. Find the net electric force acting on charge q_1.

Electrostatic potential and flux

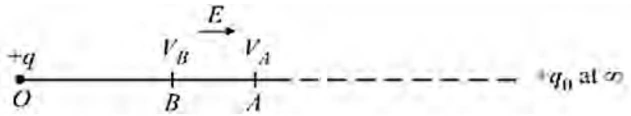
When a body is held at a height above the earth, it is said to have gravitational potential energy. The potential energy of the body depends upon its position in the gravitational field, being zero on the earth's surface.

Electric potential is analogous to gravitational potential. Every charge, q has electric field which theoretically extends up to infinity. If a small test charge q_0 is placed in this electric field, charge q_0 will have electric potential energy. The potential energy of q_0 depends upon its position in the electric field, being zero, if q_0 is situated at infinity.

Electrostatic potential energy

Just as mass has potential energy in the gravitational field, similarly electric charge has electrostatic potential energy in the electrostatic field.

Consider a point $+q$ placed at a point O as shown in the figure below. The charge $+q$ sets up electrostatic field which theoretically extends up to infinity.



If a small positive test charge $+q_0$ is placed at infinity, then potential energy of $+q_0$ is zero.

As the test charge $+q_0$ at infinity is moved towards $+q$ (with uniform velocity), work will have to be done against electrostatic force of repulsion. This work done is stored in $+q_0$ in the form of potential energy

Potential energy of $+q_0$ at point A is

$U_A = W_{\infty A} =$ work done in bringing $+q_0$ from infinity to point A.

Similarly, potential energy of $+q_0$ at point B is

$U_B = W_{\infty B} =$ work done in bringing $+q_0$ from infinity to point B.

Clearly, $U_B > U_A$. It means that potential energy of $+q_0$ at B is greater than its potential energy at point A.

Electrostatic potential

Just as electric field intensity is the force per unit charge, similarly electrostatic potential or electric potential is potential energy per unit charge.

Electric potential at A,

$$V_A = \frac{W_{\infty A}}{q_0} = \frac{U_A}{q_0}$$

Electric potential at B,

$$V_B = \frac{W_{\infty B}}{q_0} = \frac{U_B}{q_0}$$

Clearly, $V_B > V_A$. Thus if $+q_0 = 1$, then $V_A = W_{\infty A}$ and $V_B = W_{\infty B}$. Hence

Electric potential at a point in an electric field is the amount of work done in moving a positive charge of $1\,C$ from infinity to that point against the electrostatic force.

The following points should be noted clearly,

(i) Electric potential, $V = \frac{\text{Work done}}{\text{Charge}}$
Since work done and charge are scalars, electric potential is a scalar quantity.

(ii) The potential energy of charge q at a point where potential is V is given by
P.E of $q = qV$

(iii) Where as E describes electric field in terms of a vector, electric potential V describes it in terms of a scalar quantity.

(iv) Since electric field can be described in can be described terms of either vector E or scalar V, there must be a definite relation between them.

SI unit of electric potential

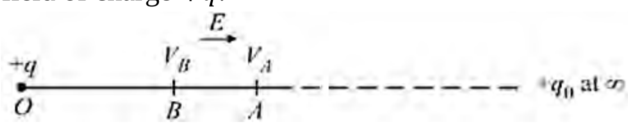
Electric potential, $V = \frac{\text{Work done}}{\text{Charge}} = \frac{J}{C}$

S.I unit of electric potential is JC^{-1} or V.

The electric potential at a point in an electric field is $1\,V$ if 1 joule of work is done in bringing a positive charge of $1\,C$ from infinity to that point against the electrostatic force.

Electric potential difference

Consider a point charge $+q$ placed at a point O in space as shown below. The points A and B are in the electric field of charge $+q$.



Electric potential at A, $V_A = \frac{W_{\infty A}}{q_0}$

Electric potential at B, $V_B = \frac{W_{\infty B}}{q_0}$

$$V_B - V_A = \frac{W_{\infty B}}{q_0} - \frac{W_{\infty A}}{q_0} = \frac{W_{\infty B} - W_{\infty A}}{q_0} = \frac{W_{AB}}{q_0}$$

$V_B - V_A =$ work done/C in moving $+q_0$ from A to B

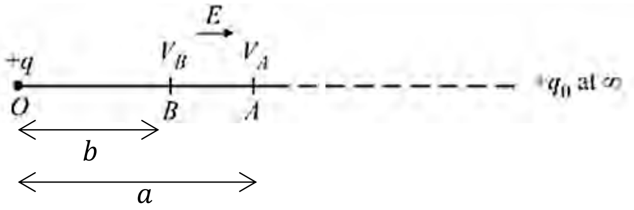
Clearly, point B is at a higher electric potential than A, $V_B > V_A$. Hence,

Electric potential difference between two points in an electric field is the amount of work done in bringing a charge of $1\,C$ from the point of lower potential to the point of higher potential.

SI unit of potential difference is volt, V.

Derivation of an expression for electric potential difference

Consider a test charge q_0 moved from A to Binto the electric field of charge $+q$



Work done = Force × Distance

The small change in work done dW to move the test charge q_0 through a distance dx is given by

$$dW = -F.dx$$

$$W_{AB} = -\int_a^b F\,dx$$

The negative sign indicates that the work is done against the electric field of $+q$

$$\text{But } F = \frac{q_0 q}{4\pi\varepsilon_0 x^2} = \frac{q}{4\pi\varepsilon_0 x^2} \text{ since } q_0 = +1\,C$$

$$W_{AB} = -\int_a^b \frac{q}{4\pi\varepsilon_0 x^2}\,dx$$

$$W_{AB} = -\frac{q}{4\pi\varepsilon_0}\int_a^b \frac{1}{x^2}dx$$

$$W_{AB} = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^b = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{b} - -\frac{1}{a}\right)$$

$$W_{AB} = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{b} + \frac{1}{a}\right)$$

$$W_{AB} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

From the definition, this work done is stored as electric potential difference.

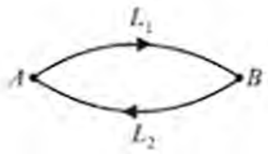
Therefore,

$$\boldsymbol{V_{AB} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)}$$

Electrostatic force as a conservative force

Like gravitational force, electrostatic force is also conservative in nature i.e. work done in moving a unit positive charge over a closed path in an electrostatic field is zero.

To prove this, consider a closed path AL_1BL_2A in an electric field as shown below.



Work done in carrying unit positive charge from A to B along path AL_1B is

$$\frac{W_{AB}}{q_0} = V_B - V_A \quad \ldots\ldots \text{(i)}$$

Work done in carrying unit positive charge from B to A along path BL_2A is

$$\frac{W_{BA}}{q_0} = V_A - V_B \quad \ldots\ldots \text{(ii)}$$

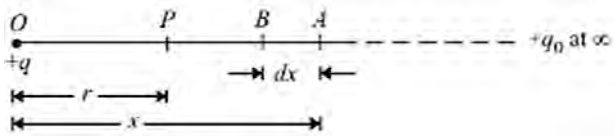
Adding (i) and (ii);

$$\frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0} = (V_B - V_A) + (V_A - V_B) = 0$$

Therefore, no work is done in moving a unit positive charge over a closed path in an electric field. Hence electrostatic force is conservative in nature.

Electric point due to a point charge

Consider a point charge $+q$ placed at point O in free space/air as shown below. It is desired to find electric potential at P due to charge $+q$. Let r be the distance of point P from O i.e. $OP = r$



At point A at a distance x from charge $+q$, electric field intensity is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

Small amount of work done in moving unit positive charge from A to B (where $AB = dx$) is

$$dW = -E\,dx$$

The negative sign is taken because dx is measured along the negative direction of x. (Or work is done against electrostatic force of repulsion)

Total amount of work done in bringing $+1\,C$ charge from infinity to r is

$$W = \int_\infty^r -E\,dx = \int_\infty^r -\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{x^2}dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_\infty^r$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r} + \frac{1}{\infty}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}$$

This is by definition electric potential at point P

Electric potential at P,

$$\boldsymbol{V_P = \frac{q}{4\pi\varepsilon_0 r}}$$

If q is positive, then potential at P is positive. On the other hand, if q is negative, then potential at P is negative.

$$\frac{1}{r_f} = \frac{1}{r_i} + \frac{\frac{1}{2}m_\alpha v_\alpha^2}{kq_\alpha q_{sphere}}$$

$$\frac{1}{r_f} = \frac{1}{1.0} + \frac{\frac{1}{2}(6.62 \times 10^{-27})(5.4 \times 10^5)^2}{(9 \times 10^9)(3.2 \times 10^{-19})(6.5 \times 10^{-6})}$$

$$\frac{1}{r_f} = 1.0 + 0.0516$$

$$\frac{1}{r_f} = 1.052$$

$$r_f = 0.951\ m$$

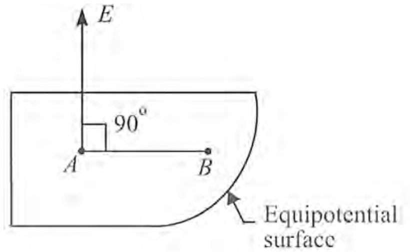
Self-Evaluation exercise

1. Two point charges $+40\ \mu C$ each are placed $20\ cm$ apart. What is the potential at the mid-point?
[Ans: $7.2 \times 10^6\ V$]
2. Two charges $+10\ \mu C$ and $-5\ \mu C$ are $10\ cm$. Determine the potential at point P at a distance of $1\ m$ from either charge.
[Ans: $4.5 \times 10^4\ V$]
3. A point charge A of $5 \times 10^{-9}\ C$ is placed at a distance of $6\ cm$ from another point charge B of $3 \times 10^{-9}\ C$. The charge B is brought towards charge A from $6\ cm$ to $5\ cm$. Calculate the work done.
[Ans: $4.5 \times 10^{-7}\ J$]
4. Two point charges of $0.12\ \mu C$ and $-0.06\ \mu C$ are situated at a situated at a distance of $3\ m$ from each other. What is the potential at a point mid-way between them? How much work will be done in bringing a charge $0.2\ \mu C$ from infinity to mid-point?
[Ans: $360\ V$, $7.2 \times 10^{-5} J$]
5. A square $ABCD$ has each side $1\ m$. Four charges $+0.02\ \mu C$, $+0.04\ \mu C$, $+0.06\ \mu C$ and $+0.02\ \mu C$ are placed at A, B, C and D respectively. Find the potential at the centre of the square.
[Ans: $1000\ V$]
6. Equal charges of $+6\mu C$ are positioned at the corners of a square of sides $4\ cm$ long, what is the potential at the centre of the square?
[Ans: $7.64 \times 10^6\ V$]

Equipotential surfaces

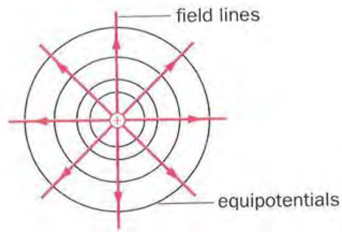
An equipotential surface is a surface which has the same electric potential at every point.

In other words, the potential difference between any two points of an equipotential surface is zero. For example, consider two points A and B on an equipotential surface as shown below



Now $V_A - V_B = 0$ or $V_A = V_B$

It may be noted that an equipotential surface may be surface of a material body or a surface in space. For example, the surface of a charged conductor is an equipotential surface.



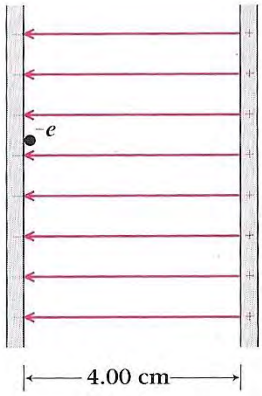
Properties of equipotential surfaces

1. No work is done in moving a test charge over an equipotential surface.
If a test charge, q_0 is moved from A to B, then work done will be
Work done = charge × potential difference
$$W_{AB} = q_0 \times (V_B - V_A)$$
Since the surface is equipotential, $V_B - V_A = 0$
$$W_{AB} = 0$$
2. Electric field is always normal to the equipotential surface at every point.
3. The spacing between equipotential surfaces enables us to identify regions of strong and weak fields
We know that $E = -\frac{dV}{dr}$
For a given dV (i.e. constant dV), $dr \propto 1/E$. This means that where the equipotential surfaces are crowded, the electric field intensity is greater and vice-versa.
4. Two equipotential surfaces can never intersect. If two equipotential surfaces could intersect, then at the point of intersection, there would be two values of electric potential which is not possible.

What is the charge q on the ball when it is deflected by 15°?

[Ans: 3.29 μC]

18. An electron of mass 9.1×10^{-31} kg and charge $-1.6 \times 10^{-19}\ C$ is initially at rest just to the right of the leftmost plate in the figure below. The separation between the two plates is 4.0 cm.



A uniform electric field of $5000\ NC^{-1}$ directed to the left is suddenly applied between the plates.

(a) What is the force on the electron?

(b) What kinetic energy will the electron have when it strikes the other plate?

19. A, B, C, D are the four corners of a square of side 10 cm. Point charges of magnitudes 1, −2, and 3 nC are situated at the corners A, B and C respectively. Find the electrostatic potential at D.

[Ans: 233 V]

20. Define potential at a point in an electrostatic field. An isolated conducting sphere of radius 5 cm is given a positive charge of $4 \times 10^{-9}\ C$. Calculate the potential at points 2, 5 and 15 cm from the centre of the sphere.

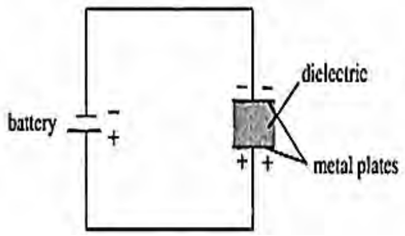
[Ans: 0, 720 V, 240 V]

CAPACITORS

A capacitor is a device for storing electric charge. Basically, a capacitor consists of two parallel metal plates with an insulator known as dielectric present between the plates.
Examples of dielectrics are air, paper, wax, ceramic and mica

Charging a capacitor

A battery can charge the capacitor as shown in the figure below



The electrons move from the negative terminal of the battery and accumulate on one plate of the capacitor and equal amount of positive charges is induced on the other plate.
The process of charging a capacitor continues until the potential difference across the capacitor equals the e.m.f of the battery.
If the resistance in charging the circuit is negligible, the charging is almost instantaneous and the result is that charge is stored in the capacitor.
The charge remains even after the battery is disconnected.

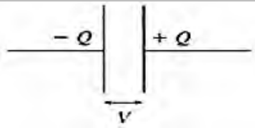
Discharging a capacitor

When the battery is disconnected and the plates are joined together by a wire, electrons flow back from the negative plate to the positive plate until the positive charge on it is completely neutralized.
A current thus flows for a time in the wire and at the end of the time, the charges on the plates become zero.

Applications of capacitors

- Capacitors are used to store electric charge.
- Capacitors are used in filter circuits to absorb undesirable radio frequency interference.
- Capacitors are used in signal coupling i.e. they are used to classify AC and DC signal components.
- Capacitors are used in a motor starting capacitor to start its rotational motion.

Capacitance of the capacitor



The capacitance of a capacitor, C, is defined by the equation;

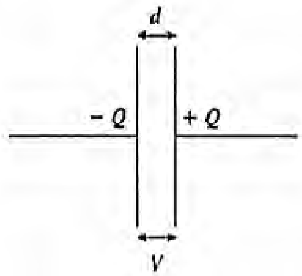
$$\text{Capacitance}, C = \frac{\text{change on any one plate of capacitor}}{\text{P.d across the capacitor}}$$

$$C = \frac{Q}{V}$$

Unit of capacitance is the Farad (F)

A farad is the capacitance of capacitor which stores one coulomb of charge when the potential difference across the capacitor is one volt.

Parallel plate capacitors



The capacitance of a parallel plate capacitor depends on a number of factors i.e.

(i) The area, A of each plate.
(ii) The distance, d between the plates.
(iii) The type of dielectric material present.

Suppose that when a parallel plate capacitor is charged to a p.d V, charge on each plate is Q.
The electric field intensity E between the plates is given by;

$$Flux = EA = \frac{Q}{\varepsilon}$$

where ε is the permittivity of the dielectric.

$$\Rightarrow E = \frac{Q}{\varepsilon A}$$

Also $E = \frac{V}{d}$ where d =separation of the plates

$$\therefore \frac{Q}{\varepsilon A} = \frac{V}{d}$$

$$\frac{Q}{V} = \frac{\varepsilon A}{d}$$

But $\quad C = \frac{Q}{V}$

$$\Rightarrow C = \frac{\varepsilon A}{d}$$

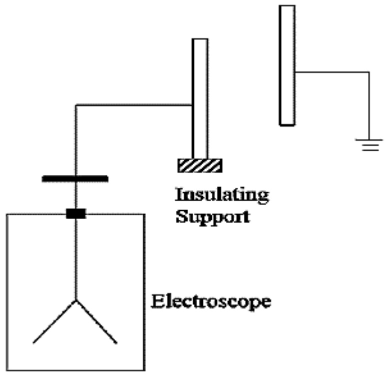
A capacitor with parallel plates separated by an air gap has it capacitance given by;

$$C = \frac{\varepsilon_0 A}{d}$$

Since increase in divergence means increase of potential difference then d ∝ V but C ∝ $\frac{1}{d}$.

Therefore $C \propto \frac{1}{d}$ and hence the capacitance of a capacitor increases with reduced separation between the plates.

2. Overlapping area of the capacitor's plates

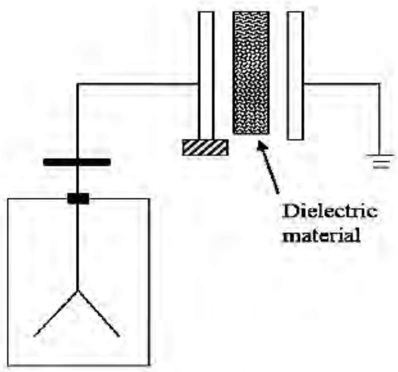


The insulated plate connected to the gold leaf electroscope is charged by means of an electrophorus. The deflection of the gold leaf electroscope is noted. The plates are slid relative to each other while keeping the plate separation constant.

It is observed that divergence increases if the overlapping area decreases and the divergence decreases when the overlapping area increases.

Since the increase in divergence means increase of potential difference, then area $A \propto \frac{1}{V}$ but $C \propto \frac{1}{V}$. Therefore $C \propto$ A and hence the capacitance a capacitor increases with increase in the area of overlap of the plates.

3. Dielectric between its plates

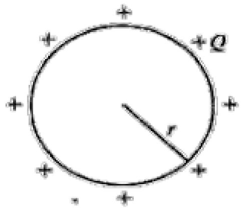


The insulated plate connected to a gold leaf electroscope is charged by means of an electrophorus. The divergence of the gold leaf electroscope is noted.

An insulator is inserted between the plates of the capacitor while keeping the plate separation and their overlapping area constant.

It is observed that divergence decreases on insertion of the dielectric. This implies that $\varepsilon \propto \frac{1}{V}$ but $C \propto \frac{1}{V}$. Therefore $C \propto \varepsilon$ and hence the capacitance of a capacitor increases with the existence of a dielectric between its plates.

Capacitance of an isolated charged sphere



When an isolated conducting sphere is given a charge Q, the charge is distributed uniformly on its surface.

If r is the radius of the sphere, the electric potential on the surface of the sphere.

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Capacitance, C of the sphere is given by;

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 r$$

Example

Calculate the capacitance of charged isolated metal sphere of radius 21 cm placed in air.

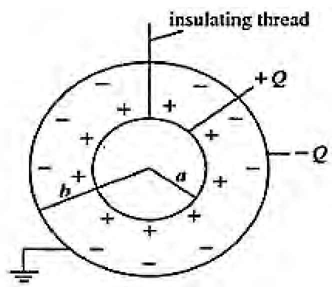
<u>Solution</u>

For an isolated charged sphere,

$$C = 4\pi\varepsilon_0 r$$
$$= 4\pi \times 8.85 \times 10^{-12} \times 2 \times 10^{-2}$$
$$= 2.34 \times 10^{-11}\ F$$

Capacitance of concentric spheres

The figure below shows two concentric conducting spheres of radii a and b respectively. The outer sphere is earthed.



Example 2

Calculate the magnitude of charge stored on a $4\,\mu F$ capacitor filled with a dielectric of constant 1.5 when charged by a battery of $20\,V$.

Solution

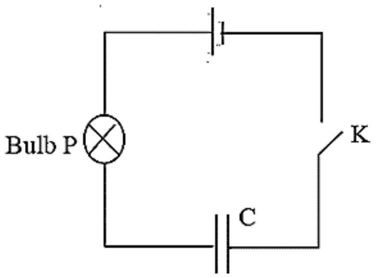
$$Q = CV$$

But $C = \varepsilon_r C_0$

$$Q = \varepsilon_r C_0 V = 1.5 \times 4 \times 10^{-6} \times 20$$
$$= 120 \times 10^{-6}\, C \text{ or } 120\mu C$$

Example 3

Bulb P and capacitor C are connected in series with a switch K and d.c source as shown below.



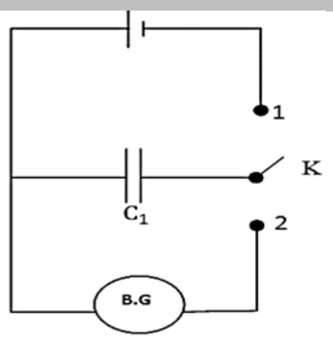
Explain what happens when the
- (i) switch K is closed
- (ii) d.c source is replaced with a connecting wire

Solution

(i) When the switch is closed, the bulb will light up as current flows from the d.c source to the capacitor to charge it up. The bulb will progressively become dimmer and finally goes off once the capacitor gets fully charged. This is because current stops flowing.

(ii) If the d.c source is replaced with a connecting wire, the bulb will light up as current flows from one plate of the capacitor to the other. The bulb will progressively become dimmer as the capacitor discharges. Finally, the bulb goes off once the capacitor gets fully discharged.

Comparison of capacitances using a ballistic galvanometer



A capacitor of capacitance C_1 is connected in the circuit as shown in the diagram above.

Switch K is closed at position 1 to charge the capacitor and later to 2 to discharge it through the ballistic galvanometer. The deflection θ_1 of the galvanometer is noted.

The capacitor is then replaced by another one of capacitance C_2. Switch K is again closed at position 1 and position 2 successively. The new deflection θ_2 of the galvanometer is noted.

The capacitances are proportional to the deflections of the galvanometer.

Thus $\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$

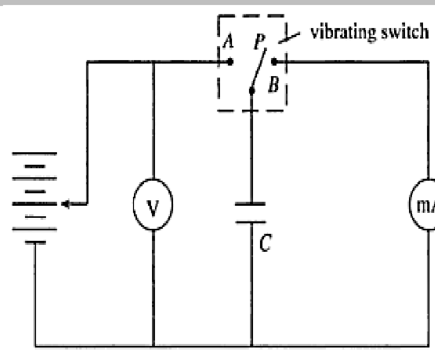
Note:

This same method can be used to find the unknown capacitance of a capacitor. This time however, a capacitance of unknown capacitance C is compared with a standard capacitor of capacitance C_s.

The unknown capacitance of the capacitor can then be calculated from the expression;

$$C = \frac{\theta}{\theta_s} C_s$$

Measurement of capacitance using a vibrating reed circuit method



The circuit is connected as shown above. The capacitor is repeatedly charged and then discharged at a frequency f by means of a reed switch vibrating between the points A and B. The voltmeter and ammeter readings V and I are respectively noted.

The required capacitance is calculated from the relation

$$C = \frac{I}{fV}$$

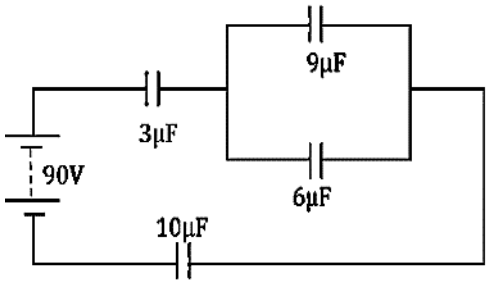
Note: The experiment can be repeated by applying different values of p.d across the capacitor. The corresponding values of I and V are recorded and a graph of I against V is plotted.

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(1.40 \times 10^{-2})^2}{(80 \times 10^{-6})} = 1.225\,J$$

(e) The energy stored in a capacitor has increased. The increase in energy comes from the work done in withdrawing the mica piece.

Example 4

The figure below a network of capacitors connected to a d.c supply of 90 V.



Calculate the

(i) total energy stored in the network
(ii) energy stored in the 10 μF capacitor
(iii) energy stored in the 6 μF capacitor

Solution

(i) Let $C_1 = 3\,\mu F$, $C_2 = 6\,\mu F$, $C_3 = 9\,\mu F$, and $C_4 = 10\,\mu F$

$C_2 = 6\,\mu F$, $C_3 = 9\,\mu F$ are in parallel;

Their net capacitance,

$$C_{23} = C_2 + C_3 = 6 + 9 = 15\,\mu F$$

$C_{23} = 15\,\mu F$, $C_1 = 3\,\mu F$ and $C_4 = 10\,\mu F$ are in series

Net circuit capacitance,

$$\frac{1}{C} = \frac{1}{C_{23}} + \frac{1}{C_1} + \frac{1}{C_4} = \frac{1}{15} + \frac{1}{3} + \frac{1}{10}$$

$$\frac{1}{C} = \frac{2 + 10 + 3}{30} = \frac{15}{30}$$

$$C = \frac{30}{15} = 2\,\mu F$$

Total energy stored in the network,

$E_T = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 90^2 = 8.1 \times 10^3$J

(ii) Charge on each capacitor in series,

$$Q = CV = 2 \times 10^{-6} \times 90 = 180 \times 10^{-6} C$$

p.d across $C_4 = 10\,\mu F$, $V_4 = \frac{Q}{C_4} = \frac{180 \times 10^{-6}}{10 \times 10^{-6}} = 18V$

Energy stored in the 10 μF capacitor.

$$E_4 = \frac{1}{2}C_4 V_4^2 = \frac{1}{2} \times 10 \times 10^{-6} \times 18^2$$
$$= 1.62 \times 10^{-3}\text{J}$$

(iii) p.d across the parallel combination,

$$V_p = \frac{Q}{C_{23}} = \frac{180 \times 10^{-6}}{15 \times 10^{-6}} = 12\,V$$

p.d across $C_2 = 6\,\mu F$, $V_2 = V_p = 12\,V$

Energy stored in the 6 μF capacitor,

$$E_2 = \frac{1}{2}C_2{V_2}^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 122$$
$$= 4.32 \times 10^{-4}\,J$$

Example 5

A 160 μF capacitor is charged to a potential difference of 200 V. It is then connected to a discharge tube which conducts until the potential difference across it falls to 100 V. How much energy is dissipated in the tube?

Solution

Initial energy stored in capacitor when p.d V_0 is 200 V is given by;

$$E_i = \frac{1}{2}CV_0^2$$

Final energy stored in capacitor when p.d V_1 is 100 V is given by;

$$E_f = \frac{1}{2}CV_1^2$$

Energy dissipated in discharged tube,

$$E_i - E_f = \frac{1}{2}CV_0^2 - \frac{1}{2}CV_1^2$$

$$= \frac{1}{2}(160 \times 10^{-6})(200^2 - 100^2) = 2.40\,J$$

CONNECTING TWO CHARGED CAPACITORS

When two charged capacitors are connected

(i) The total charge is conserved.

⟹Total charge before connection = Total charge after connection.

(ii) The p.d across both capacitors is the same

(iii) Loss of energy occurs. This is due to the transfer of electrical energy in to heat in the connecting wires.

Example 1

A 2 μF capacitor charged to a p.d of 50 V is connected across a 3 μF capacitor charged to a p.d of 100 V. Calculate the

(i) final p.d across the combination
(ii) the difference in the initial and final energies stored in the capacitors.
Comment on the difference.

Solution

(i) Total charge before connection = Total charge after connection

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

where V is the p.d across the combination

$$(2 \times 10^{-6} \times 50) + (3 \times 10^{-6} \times 100)$$
$$= (2 + 3) \times 10^{-6}V$$

Uncharged capacitors of 2 μF, 3 μF and 6 μF capacitance values are connected in series across 120 V d.c. mains. Find

(a) the quantity of charge drawn from the mains,

(b) the potential difference across each capacitor,

(c) the energy stored in the 3 μF capacitor.

[Ans: (a) $1.2 \times 10^{-4}\ C$ (b) $60\ V$, $40\ V$, $20\ V$ (c) $4.8 \times 10^{-3}\ J$]

38. The plates of a parallel-plate air capacitor, with a plate area of 100 cm^2, are connected across a battery with a terminal p.d. of 100 V.
What is the energy of the charged capacitor if the distance between the plates is 1 cm? How is the energy changed if the distance between the plates is increased to 2 cm (i) with the capacitor plates remaining connected to the battery, (ii) if the plates are disconnected before being moved?
[Ans: $4.42 \times 10^{-8}\ J$; (i) energy is halved (ii) energy is doubled]

39. Two capacitors of capacitance 3 μF and 2 μF respectively are connected in series, the free plate of the smaller being connected to earth. If the free plate of the larger is charged to a potential of 200 volt, determine the potential difference across the smaller capacitor and the energy stored in it.
[Ans: $120\ V$ $0.0144\ J$]

40. Derive an expression for the energy stored in a charged capacitor.
Two capacitors, of 2 μF and 3 μF, are joined (a) in series, (b) in parallel and then connected across a battery of 100 V. What is the energy stored in the 2 μF capacitor in each case?
[Ans: $3.6 \times 10^{-3} J$; $0.01\ J$]

41. Show that when two capacitors share their charges there is always a resulting energy loss. Account for this loss.
Two capacitors, of capacitances 2 μF and 5 μF each receive a charge of 100 μC. They are then connected together. Calculate,
(a) the common potential of the capacitors,
(b) the energy loss after connection.
[Ans: (a) $28\frac{4}{7}\ V$ (b) $0.643^{-3}\ J$]

42. A parallel plate air capacitor has its plates situate 3 cm apart. One of the plates is earthed and the other connected to a sensitive electroscope the deflection of which is observed when the capacitor is charged. A slab of glass 1.5 cm thick, and large enough to cover the plate area of the capacitor, is now placed between the capacitor plates when it is noticed that the deflection of the electroscope falls. Explain this and calculate a value for the relative permittivity of the glass if, to restore the deflection, it is necessary to increase the residual air gap between the plates by 1.25 cm.
[Ans: 6]

43. What do you understand by the term relative permittivity?
The charged plate of a parallel plate air capacitor is connected to a sensitive electroscope which records a deflection of 50 divisions. If the air space between the plates is now completely filled with another insulating medium, the deflection of the electroscope falls by 30 divisions. What is the relative permittivity of the new medium?
[Ans: 2.5]

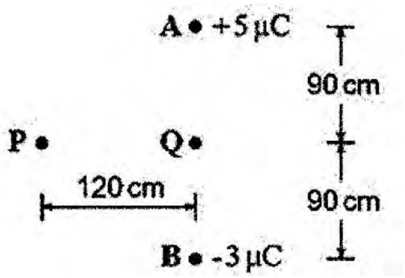
44. A capacitor A having a capacitance of 4 μF is charged from a cell and when discharged through a ballistic galvanometer produces a scale deflection of 30 divisions. The capacitor is again charged up from the same cell and, through a suitable switch, a second capacitor B is successively charged from A and subsequently discharged. After five such steps of drawing charge from A, the residual charge on it produces a scale deflection of 10 divisions when discharged through the ballistic galvanometer. Sketch a suitable circuit for this exercise and calculate the capacitance of B.
[Ans: 0.98 μF]

45. A parallel plate capacitor of capacitance C_1 is charged so that the potential difference across its plates is V_o. The plate separation of the capacitor is now increased to n times its original value when it is connected to an uncharged capacitor of capacitance C_2. What must be the value of C_2 for the potential difference across the capacitors still to be V_o? [Ans: $\left(\frac{n-1}{n}\right) C_1$]

46. A d.c. pulse generator of calibrated frequency, a reed (or vibrator) switch, a battery of known e.m.f., and a sensitive milliammeter are provided to make a determination of the capacitance of a given capacitor.
Draw a diagram of the necessary circuit arrangements and calculate the appropriate capacitance value if the battery used has an e.m.f. of 25 volt and a steady current of 8.0 mA is recorded by the ammeter when the reed makes 200 vibrations per second.
[Ans: 1.6 μF]

EXAMINATION QUESTIONS

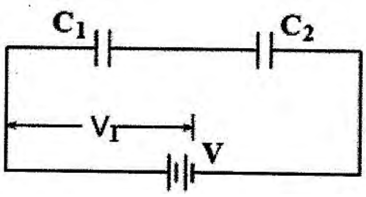
1. (a) (i) Define **electric field intensity** and **potential difference**
   (ii) Derive an expression for the electric potential difference between two points a and b in the field of a straight single point charge $+Q$
   (b) Describe an experiment to show that when two dissimilar dielectrics are rubbed together, they acquire equal but opposite charges
   (c) Two point charges of $+5\ \mu C$ and $-3\ \mu C$ are placed at points A and B shown in the figure below.



   Calculate the:
   (i) electric potential at P
   (ii) work done in moving a charge of $-3\ \mu C$ from P to Q
   (d) State any **two** characteristics of an equipotential

   [2018, No. 9; Ans: (c) (i) 12 kV (ii) $-0.024\ J$]

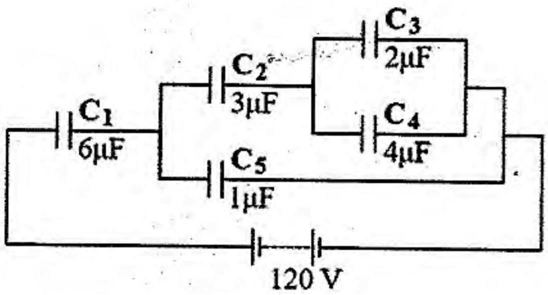
2. (a) Define the following
   (i) Capacitance
   (ii) Relative permittivity
   (b) Two capacitors of capacitances C_1 and C_2 are connected in series with a battery of e.m.f V as shown below



   If the p.d across the capacitor of capacitance C_1 is V_1, show that

$$\frac{1}{V_1} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{C_1}{V}$$

   (c) Describe an experiment to determine the capacitance of a capacitor using a vibrating reed circuit.
   (d) A battery of e.m.f $120\ V$ is connected to a network of capacitors as shown below.



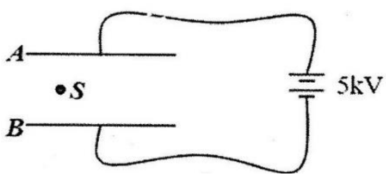
   Calculate the:
   (i) charge on C_1
   (ii) energy stored in C_5
   (e) Describe how the effect of a dielectric medium on the capacitance of a capacitor may be determined.

   [2018, No. 10; Ans: (d) (i) $120\ \mu F$ (ii) $3.2 \times 10^{-3}\ J$]

3. (a) Derive an expression for the energy stored in a capacitor of capacitance C, charged to a voltage V.
   (b) A parallel plate capacitor with plate area of $2 \times 10^{-2}\ m^2$ and plate separation of $5.0 \times 10^{-3}\ m$ is connected to a $500\ V$ supply.
   (i) Calculate the energy stored in the capacitor
   (ii) If the space between the plates is completely filled will oil and the total charge in the capacitor becomes $4.42 \times 10^{-8}\ C$, find the dielectric constant of the oil.
   (c) Explain how a lightning conductor may protect a building from damage by lightning.
   (d) Describe an experiment to show that charge on a hollow conductor resides on the outer surface.

   [2017, No. 10; Ans: (b) (i) $4.4 \times 10^{6}\ J$ (ii) 2.5]

4. (a) (i) Explain an **equipotential surface**
   (ii) Give an example of an equipotential surface
   (b) (i) State **Coulomb's law**
   (ii) With the aid of a sketch diagram, explain the variation of electric potential with distance from the centre of a charged metal sphere
   (iii) Two metal plates $\boldsymbol{A}$ and $\boldsymbol{B}$, $30\ cm$ apart are connected to a $5\ kV$ d.c. supply as shown below.



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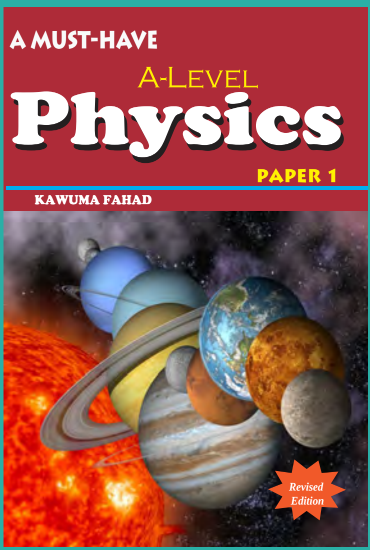

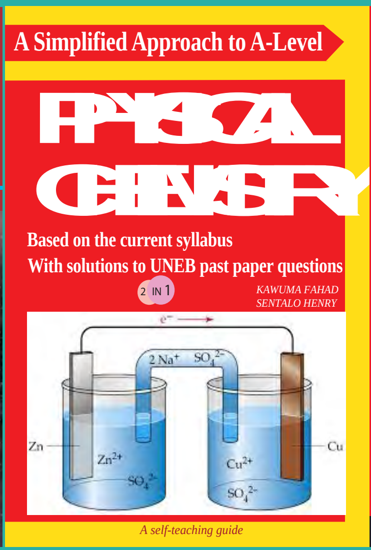

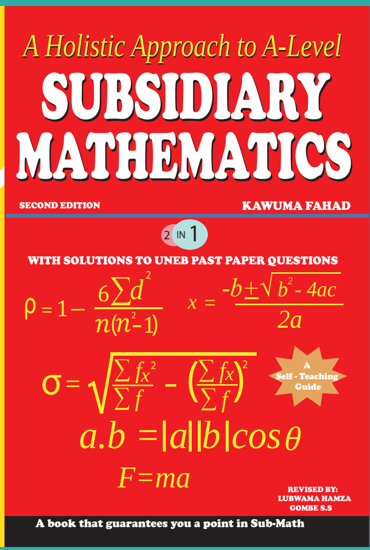

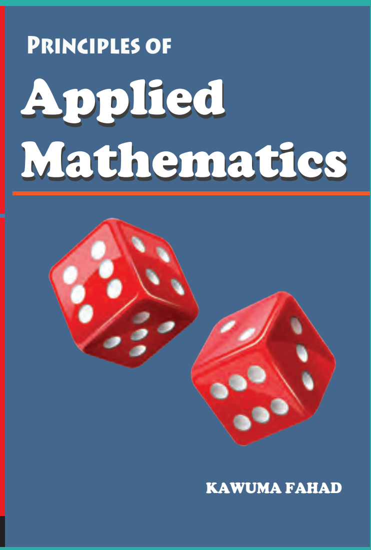

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