Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Mock Excel

Monday 16th October 2023 – Morning

Paper reference **P425/ 1**

Mathematics

Advanced

PAPER 1 : Pure

Time: 3hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

INSTRUCTIONS

- Answer **all** the **eight (8)** questions in section **A** and any **five (5)** from section **B**.
- Any additional question(s) answered will not be marked.
- All working must be shown clearly.
- Begin each answer on a fresh sheet of paper.
- Silent, non-programable scientific calculators and mathematical tables with a list of formulae may be used.

A	
B	

SECTION A (40 MARKS)

1. The council of Broxbourne undertook a housing development scheme which started in the year **2001** and is to finish in the year **2025**. Under this scheme the council will build **760** houses in **2012** and **240** houses in **2025**. The number of houses the council builds every year, forms an arithmetic sequence.
   a) Determine the number of houses built in **2001**.
   b) Calculate the total number of houses that will be built under this scheme. **(5)**
2. Solve the two pairs of simultaneous equations.
$$x^3 + 9x^2y = -28$$
$$y^3 + xy^2 = 1$$ **(5)**
3. The straight line l passes through the point **P(4,5)** and has gradient **3** .The point **Q** also lies on l so that the distance **PQ** is $3\sqrt{10}$. Determine the coordinates of the two possible positions of **Q**. **(5)**
4. Show that:
$$\int_0^2 \frac{4x^3 - 12x^2 - 22x - 3}{(4-x)(2x+1)}\,dx = \frac{1}{2}In\left(\frac{5}{64}\right) - 6$$
5. Use **De Moivre's theorem** to show that:
   $\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$, hence solve.
   $\sin 5\theta = 10\cos\theta\sin 2\theta - 11\sin\theta$, for $0 < \theta < \pi$. **(5)**
6. A committee of **3 people** is to be picked from **9 individuals**, of which **4 are women** and **5 are men**. One of the **4 women** is married to one of the **5 men**. The selection rules state that the committee must have at least a member from each gender and no married couple can serve together in a committee. Determine the **number of possible committees** which can be picked from these 9 individuals. **(5)**
7. A circle has centre at the origin and radius **R**. This circle fits wholly inside the circle with equation: $x^2 + y^2 - 10x - 24y = 231$.
   Determine the range of possible values of **R**. **(5)**
8. The surface area **A**, of a metallic cube of side length **x**, is increasing at the constant rate of $0.4521 cms^{-1}$. Find the rate at which the volume of the cube is increasing, when the cube's side length is **8 cm.** **(5)**

SECTION B (60)

9. Solve the system of simultaneous equations is given below

$x + y + z = 1$

$x^2 + y^2 + z^2 = 21$

$x^3 + y^3 + z^3 = 55$. **(12)**

10. A scientist is investigating the population growth of farm mice. The number of farm mice *N*, *t* months since the start of the investigation, is modelled by the equation

$$N = \frac{600}{1 + 5e^{-0.25t}}$$

a) State the number of farm mice at the start of the investigation.

b) Calculate the number of months that it will take the population of farm mice to reach 455.

c) Show clearly that:

$$\frac{dN}{dt} = \frac{1}{4}N - \frac{1}{2400}N^2$$

d) Find the value of *t* when the rate of growth of the population of these farm mice is largest. **(12)**

11. $2x^2 + kx + 1 = 0.$

The roots of the above equation are a and b, where k is a non-zero real constant. Given further that the following two expressions;

$$\frac{\alpha}{\beta(1 + \alpha^2 + \beta^2)} \text{ and } \frac{\beta}{\alpha(1 + \alpha^2 + \beta^2)}$$

are real, finite and distinct, determine the range of the possible values of k. **(12)**

12. Max is revising for an exam by practicing papers. He takes **3 hours and 20 minutes** to complete the first paper and **3 hours and 15 minutes** to complete the second paper.

It is assumed that the times Max takes to complete each successive paper are consecutive terms of a geometric progression.

a) **Assuming** this model, shows that Max will take approximately …

i. … **176 minutes** to complete the sixth paper.

ii. … **35 hours** to complete the first 12 papers.

Max aims to be able to complete a paper in under two hours.

b) Determine, by using logarithms, the minimum number of papers he needs to practice in order to achieve his **target** according to this model.

(12)

13. The points P and Q are two distinct points which lie on the curve with equation

$$\boldsymbol{y = \frac{1}{x}\ , x \in \mathbf{R}, x \neq 0}$$

P and Q are free to move on the curve so that the straight-line segment PQ is a normal to the curve at P. The tangents to the curve at P and Q meet at the point R.

Show that R is moving on the curve with Cartesian equation

$$\boldsymbol{(y^2 - x^2)^2 + 4xy = 0}$$ **(12)**

14. Relative to a fixed origin *O*, the straight-line *l* passes through the points $A(a, -3, 6)$, $B(2, b, 2)$ and $C(3,3,0)$, where *a* and *b* are constants.

a) Find the value of *a* and the value of *b,* and hence find a vector equation of *l.* The points *P* and *Q* lie on the *l* so that $|\boldsymbol{OP}| = |\boldsymbol{OQ}|$ and $POQ = 90°$.

b) Find the coordinates of *P* and the coordinates of *Q*. **(12)**

15. Find, in exact surd form, the only real solution of the following trigonometric equation

$$\boldsymbol{\sin^{-1}(2x - 1) - \cos^{-1} x = \frac{\pi}{6}}$$

The rejection of any additional solutions must be fully justified. **(12)**

16. The curve C has equation

$$y = \frac{1}{x^3 - 9x^2 + 24x}$$

Sketch the graph of C. **(12)**