## SECTION A

1. Show that $\frac{\sin\theta}{1-\cos\theta} = \cot\frac{1}{2}\theta$ Hence solve $\tan\frac{1}{2}\theta = \sqrt{3}\sin\theta$ for $0^0 \leq \theta \leq 180^0$.

   (5 marks)

2. In an Arithmetic progression the third term is -2, the sum of the first two terms is four times the second term and sum of the first term is -164. Find the value of n

   (5 marks)

3. Given that $y = 4^x \sin x$, find the value of $\frac{dx}{dy}$ when $x = \frac{\pi}{4}$.

   (5 marks)

4. The variable line $y = mx + \frac{1}{m}$ intersects the curve $y = x^2$ at points A and B. Find the locus of the mid point of AB

   (5 marks)

5. Find the equation of line through point $A(1, -2, 3)$ perpendicular to the line $\frac{x-5}{2} = \frac{y-2}{1} = \frac{z-1}{3}$

   (5 marks)

6. Twelve people are to travel by three cars each of which holds four. Find the number of ways in which they may be divided if two people refuse to travel in the same car.   (5 marks)

7. Evaluate $\int_1^2 \ln x^{x^2} dx$

   (5 marks)

8. Solve the differential equation
   $\frac{dy}{dx} + \frac{2xy}{x^2+1} - x = 0$

   (5 marks)

SECTION B

9. (a) One side of a rectangle is three times the other. If the perimeter increases by 0.5%, find the percentage increase in the area.

   (b) Find the coordinates of the points on the curve $y^2 - 3xy - 2x^2 + 17x + 1$ for which the tangents are perpendicular to the line $3x - 4y + 5 = 0$

   (7 marks)

10. (a) Given that one root of the equation $z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$ is $1 + i$. Find the others(06 marks)

    (b) If z is a general complex number on argand diagram, show the region given by $|z + 1 - 4i| > |z - 2 - i|$.

    (6 marks)

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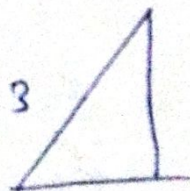
11. (a) Find the Cartesian equation of the line through points $A(1,6,-4)$ and $B(7,-2,3)$ (3 marks)

(b) The point $(6,-9,5)$ lies on the line $\frac{x-a}{3} = \frac{y-5}{b} = \frac{z-c}{-4}$ which is parallel to the plane $3x + y + 4z - 3 = 0$
   (i) values of a, b and c
   (ii) shortest distance between the line and plane.　　　　　　(9 marks)

12. (a) Show that $\frac{\cos 6\theta + \cos 10\theta}{\sin 10\theta - \sin 6\theta} = \cot 2\theta$ hence solve for θ if $\frac{\cos 6\theta + \cos 10\theta}{\sin 10\theta - \sin 6\theta} = \frac{1}{\sqrt{3}}$ for $-180 < \theta \le 360^0$

(b) Solve the equation $5\sin\theta\cos\theta - 6\cos 2\theta = 2$ for $0 < \theta \le 360^0$

13. A curve is given parametrically by the e2quations $x = 5\cos\theta$, $y = \sqrt{3}\sin\theta$ where θ is the parameter.
   (i) Show that the curve represents an ellipse.
   (ii) Find the foci of the ellipse
   (iii) Find the equation of the tangent to the ellipse at point$\left(5/2, 3/2\right)$ (12 marks)

14. Sketch the curve $y = 2x + \frac{12}{x-5}$ showing clearly the intercept and asymptotes. Hence find the area between the curve $x - axis$ $x = 0$ and $x = 3$.

15. (a) Determine the Binormial expansion of $\left(\frac{1}{1-\frac{x}{2}}\right)^{\frac{1}{3}}$ in ascending powers of x up to term in x^3,

hence find $\sqrt[3]{\frac{1}{7}}$ 　　　　　　(6 marks)

(b) The quadratic equation $5x^2 - 7x + 4 = 0$, has roots $\propto$ and β. Find the quadratic equation with integral coefficients whose roots are $\frac{\propto^2}{\beta} + \frac{\beta^2}{\propto}$ and $\frac{1}{\propto^2\beta} + \frac{1}{\beta^2\propto}$ 　　　　(6 marks)

16. (a) Solve the differential equation $x\frac{dy}{dx} - y = \frac{x}{x-1}$ 　　　　　　(4 marks)

(b) Bacteria in a culture increase at a rate proportional to the number of bacteria present. If the number increase from 600to 1200 in one hour
   (i) How many bacteria will be present after 1 ½ hours
   (ii) How long will it take for the number of bacterial is the culture to become 3000
　　　　　　(8 marks)