P425/I
PURE MATHEMATICS
PAPER 1
JUL/AUG 2022
3HOURS

ST.MARY'S COLLEGE – KITENDE

INTERNAL ASSESSMENT EXAMINATIONS
Uganda advanced certificate of education
PURE MATHEMATICS
PAPER 1
3HOURS

Instructions

- Attempt all the eight questions from section A and only five questions from section B.
- All your working must clearly be shown.
- Begin each answer on a fresh sheet of paper.
- Mathematical tables and graph papers are provided.
- Silent non-programmable calculators may be used.

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SECTION A (40 marks)

Answer all the questions from this section.

1. If X is sufficiently small enough to allow any terms in x^5 or higher powers of x to be neglected, show that $1 + x)^6 \ (1 - 2x^3)^{10} = 1 + 6x + 15x^2 - 105x^4$. (05 marks)

2. A straight line AB of length 10 units is free to move with its ends on the axes. Find the locus of a point P on the line at a distance of 3 units from the end on the axes.
(05 marks)

3. Find $\int \frac{ln\,(1+ln\,x)dx}{x}$
(05 marks)

4. Find the perpendicular distance from the point P (4, 6, 4) to the line passing through the points A (2, 2, 1) and B (4, 3, -1).
(05 marks)

5. Given that y = e^{2x}cos3x, show that $\frac{d2y}{dx}$ - 4$\frac{dy}{dx}$ + 13y = 0
(05 marks)

6. Solve the equation 2tanθ + sin2θsecθ = 1 + secθ in the range $0^0 \leq \theta \leq 360^0$.
(05 marks)

7. Prove by induction that $2^{4n} - 1$ is a multiple of 15 if n is a natural number.
(05 marks)

8. Water is poured into a vessel in the shape of a right circular cone of vertical angle 90^0 with the axis vertical, at the rate of $125cm^3s^{-1}$. At

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what rate is the level of water surface rising when the depth of the water is 10cm? (05 marks)

SECTION B (60 marks)

Answer only five questions from this section.

9. (a) Given Z = -10 + 9i as a complex number
   (i) Find the complex number w which satisfy the equation zw = -11 + 28i (04 marks)
   (ii) Verify that $|z + w| = \sqrt[8]{2}$.
   (02 marks)

   (b) Express that $\sqrt{3} + i$ in the modulus argument form. Hence find $(\sqrt{3} + i)^{10}$ in the form a + bi.
   (06 marks)

10. (a) Evaluate $\int_0^{\frac{\pi}{3}} (1 + cos3x)^2 \, dx$
    (05 marks)
    (b) Use the substitution $t = \tan\frac{x}{2}$ and find $\int \frac{\cos x}{1-\cos x} dx$
    (07 marks)

11. (a) Given that $x^3 + 5x^2 + ax + b$ is divisible by $x^2 + x - 2$, find
    (i) the values of a and b.
    (05 marks)
    (ii) the linear factor of the polynomial .
    (02 marks)
    (b) Find the number of arrangement of all the letters of the word MATHEMATICS in a row.
    (i) without restriction
    (02 marks)
    (ii) in which the A's are separated .
    (03 marks)

12. (a) Prove that in any triangle ABC $\frac{bc}{ab+ac} = \frac{cosec(B+C)}{cosecB+cosecC}$
    (06 marks)
    (b) Solve the equation $\tan^{-1}(2x+1) + \tan^{-1}(2-1) = \tan^{-1}2$
    (06 marks)

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13. (a) Differentiate $\frac{\sin x}{x^2+\cos x}$ with respect to x.

(b) Express $f(x) = \frac{1}{(x+2)(1+x)^2}$ into partial fractions and hence find the definite integral of $\int_0^1 f(x)dx$.

(08 marks)

14. The co-rdinates of the points A and B are (0,2,5) and (-1, 3, 1) respectively

and the equation of the line L is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

(i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.

(06 marks)

(ii) Find the position vector of the point of intersection of the line L and the plane in (i) above.

(06 marks)

15. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P(a $\cos\theta$, b$\sin\theta$) meets the minor axis at L. If the normal at P meets major axis t=at M. Find the locus of the midpoint of LM.

(12 marks)

16. In a certain type of chemical reaction a substance is continuously transformed

into a substance B throughout the reaction, the sum of the masses of A and B remains constant and equal to M. The mass of B present at time t after the commencement of the reaction is donated by x. At any time, the rate of increase of mass of A where k is constant.

(a) Write down a differential equation relating x and t.

(b) Solve this differential equation given that x= 0 and t= 0. Given also that x $=\frac{1}{2}$m where t = In 2, determine the value of k and show at time t x = m$(1 - e^{-t}$.Hence find

(i) the value of x interms of m when t = 3In2.

(ii) the value of t when x = $\frac{3}{4}$m.

(12marks)

THE END

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P425/2

APPLIED MATHEMATICS

3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION - 2022

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in Section **A** and any **five** from Section **B**.*
- *Any additional questions answered will **not** be marked.*
- ***All** working **must** be shown clearly.*
- *Graph paper is provided.*
- *Silent non-programmable **scientific calculators** and **mathematical tables** with a list of formulae may be used.*
- *In numerical work, take g to be $9.8ms^{-2}$*

SECTION: A (40marks)

1. From the top of a building 45m high, a stone is projected upwards with a speed Vms^{-1} at an angle of 30^0 to the horizontal. Two seconds later another stone is dropped from the same point. If the stones reach the ground at the same time. Find the value of V. (5marks)

2. Events A and B are such that $P\left(A/_B\right)=\frac{2}{3}, P\left(A/_{B^1}\right)=\frac{2}{5}$ and $P(B)=\frac{4}{7}$.Find the;
   (i) $P(A)$ (3marks)
   (ii) $P\left(B/_{A^1}\right)$ (2marks)

3. The table below shows values of $\tan\theta$.

θ	1.11	1.15	1.19	1.23
$\tan\theta$	2.0143	2.2345	2.4979	2.8198

   Use linear interpolation or extrapolation to find;
   (i) $\tan\theta$ when $\theta = 1.17$.
   (ii) θ when $\tan\theta = 2.923$ (5marks)

4. A body of mass 5kg initially at rest at a point with position vector $(2i - j)$m is acted upon by a force $(2i - 4j - 3k)N$. Find the;
   (i) Position of the body,
   (ii) Work done by the force after the 2 seconds. (5marks)

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5. The table below shows the expenditure of a certain family for months September and October in 2020.

ITEMS	EXPENDITURE(Shs.)		WEIGHT
	SEPTEMBER	OCTOBER	
Food	300,000	325,000	5
Accomodation	260,000	365,500	3
Electricity	150,000	160,000	1
Miscellaneous	620,000	725,000	2

Calculate the cost of living index for the month of October based on september in 2020. (5marks)

6. Use the trapezium rule with 6 ordinates to evaluate $\int_1^{1.2} x^2 \sin\left(\frac{1}{2}x\right)dx$ giving your answer correct to three decimal places. (5marks)

7. The distribution function of a continuous random variable X is as follows:

$$F(x)=\begin{cases} 0 & , \ x<1 \\ \frac{1}{4}(x-1)^2 & , \ 1 \le x \le 3 \\ 1 & , \ x>3 \end{cases}$$

Find: (i) $P(1.5 \le X \le 2)$. (3marks)

(ii) The p.d.f of X. (2marks)

8. A non-uniform ladder AB of weight 78.4N ad length 5m is freely suspended horizontally by two light inelastic strings AC and BD that make angles 30^0 and 40^0 respectively with the vertical; find the distance from A, where weight of the ladder acts. (5marks)

SECTION:B (60marks)

9. (a)The weight of a particular variety of mangoes is normally distributed with mean 205grams and standard deviation 25grams .Find the probability that a mango chosen at random from the variety is ;

(i) less than 250grams

(ii) between 200grams and 250grams. (6marks)

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from the instant they are called is approximately normally distributed. the value of X was recorded on a random sample of 50 Occassions on which the fire brigade was called and the results summarised below.

$\sum x = 286.5$, $\sum (x-\bar{x})^2 = 45.16$

Determine the 98.5% confidence interval for the mean time taken by the town fire brigade team to reach a fire scene for all the occasions in town from the instant they are called. (6marks)

10. (a) The numbers X and Y were estimated with maximum errors of ΔX and ΔY respectively. Show that the maximum possible relative error in the estimation of X^2Y is given by $2\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta y}{Y}\right|$. (5marks)

(b) Given that the numbers A= 7.4, B= 5.42 and C= 9.80 are rounded off with percentage errors 2, 3 and 1 respectively, calculate the relative error made in evaluating $\frac{B}{A-C}$, correct to two decimal places. (7 marks

11. (a)To an observer on a train travelling at $3kmh^{-1}$,a bird appears to fly due west at 4 kmh^{-1}.If the bird actually travels due North-West, find its speed. (5marks)

(b)At time $t = 0$, particles A and B are moving with constant velocities $(\mu i + 3j + 30k)ms^{-1}$ and $(4i - 2j - 15k)ms^{-1}$ are located at position vectors $(2i + j - 15k)m$ and $(-i + 4j + 12k)m$ respectively. Find:

(i) value of μ such that A and B will collide,

(ii)the value of t when this collision occurs (7marks)

12. The table below shows marks obtained by 8 candidates in physics and mathematics.

Candidate	A	B	C	D	E	F	G	H
Mathematics (X)	52	65	41	65	81	31	65	55
Physics (Y)	50	60	35	65	66	35	69	48

(a) calculate the rank correlation coefficient for the data and comment on the significance of mathematics on physics at 1% level. (5marks)

(b) (i) plot a scatter diagram for the scores in mathematics and physics

(ii) Draw a line of best fit hence find the marks scored in physics by a student who scored 75 marks in mathematics (7marks)

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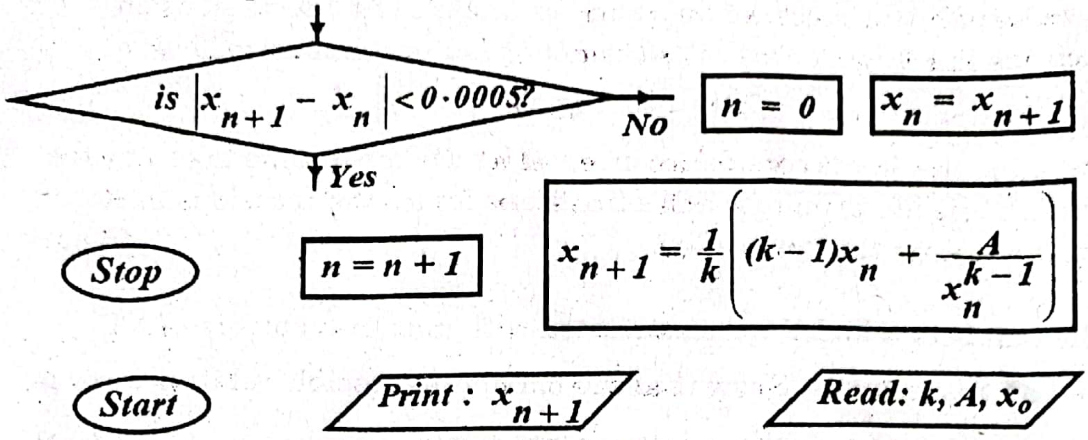
13. Given below are parts of the flow chart not arranged in order.



i) By re-arranging the given parts, draw a logical flow chart.

ii) Using $x_0 = 1.6, A = 28$ and $k = 6$, perform a dry run for the flow chart.

iii) State the purpose of the flow chart, basing on the values given in (ii) above. (12marks)

14. A body of mass $m$kg lies on a rough plane inclined at θ^0 to the horizontal. When a force of $\frac{mg}{2}N$ parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{mg}{4}N$ parallel to and down the plane is applied to the body, it just about to move down the plane.

Calculate the;

i) Value of θ.

ii) Coefficient of friction between the body and the plane. (12marks)

15. (a)In a certain school 15% of the students are left-handed. Determine the probability that in a random sample of 10 students ;

(i) exactly 3 are left-handed, (2marks)

(ii) at least 8 are left-handed (3marks)

(b)Tommy and her daughter Lisa, support their town soccer team. When their town soccer team plays, the probability that Tommy watches the game is 0.8.The probability that Lisa watches the game when her father watches the game 0.9 otherwise it is 0.4. Calculate the probability that;

(i)Neither Tommy nor Lisa watches a particular game (3marks)

(ii)Tommy watches a particular game when Lisa does not watch the game (4marks)

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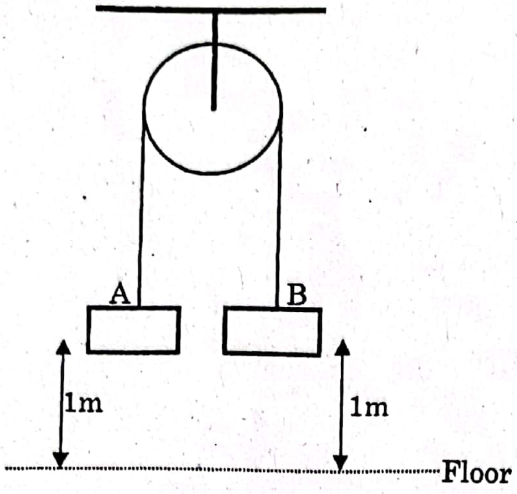
16.



Two particles A and B have mass0.4kg and 0.3kg respectively. The particles are attached to ends of a light inextensible string. The string passes over a light smooth pulley which is fixed above a horizontal floor. Both particles are held at rest, with the string taut, at a height 1m from above the floor as shown in the diagram above. The particles are released from rest and in the subsequent motion B does not reach the pulley.

(a) Find the tension in the string and the acceleration of the particles immediately after they are released. (6marks)

(b)when the particles have been moving for 0.5 seconds, the string breaks. Find the further time that elapses until B hits the floor. (6marks)

END

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. A force $(3i - 2j + 8k)$ N acts on a body of mass 4kg initially at the origin. If the velocity is $(2ti + 3j)$ ms^{-1}, find the work done after 4 seconds.
(05 marks)

2. The temperature (°C) of a liquid measured at an interval of 2 minutes were recorded as 55 and 52. If the initial temperature is 60, use linear interpolation or extrapolation to find;
   (i) Temperature after 5 minutes,
   (ii) Time taken if the temperature is 53.5°C.

3. A random sample of 200 people were asked the length of time they spent in the shower, the last time they took one. The results were as follows:
$\sum x = 909, \quad \sum x^2 = 4555.$
   (a) Calculate the unbiased estimate of the population variance. *(02 marks)*
   (b) Determine the 97.5% confidence limits for the mean time spent in the shower.
(03 marks)

4. To a dove flying eastwards at $3ms^{-1}$ an eagle appears to be flying North East wards at $4ms^{-1}$. Find the true velocity of the eagle. *(05 marks)*

5. The numbers $x = 4.2$, $y = 16.02$ and $z = 25$ are rounded off with corresponding percentage errors of 0.5, 0.45 and 0.02, Calculate the absolute error made in $\frac{xy}{z}$.
(05 marks)

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6. A and B are two independent events with A twice as likely to occur as B. if $P(A) = \frac{1}{2}$, find:

(i) P(A or B but not both), *(03 marks)*

(ii) $P(A/B')$. *(02 marks)*

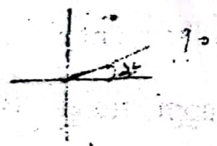
7. Forces of magnitude 90N and 60N act on a particle at angle of 35^0 to each other. Determine the magnitude and direction of the resultant force.

(05 marks)

8. The probability that a certain function starts early is $^4/_7$.
If the function starts early, the probability that it takes a longer time is $^2/_5$.
If the function starts late, the probability that it takes a shorter time is $^1/_3$. Find the probability that function;

(i) Takes a shorter time; *(03 marks)*

(ii) Starts early if it takes a shorter time *(02 marks)*



SECTION B: (60 MARKS)

Answer any **five** *questions from this section.*

All *questions carry equal marks*

9. The table below shows the marks obtained by students in Fine Art(x) and mathematics (y).

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4 5 1 7 6.5 10 8 3 6.5 2

Fine Art (x)	80	76	96	41	68	31	42	88	68	91
Mathematics (y)	43	32	27	64	65	64	65	32	64	43

6.5 8.5 10 3.5 1.5 3.5 1.5 8.5 3.5 6.5

(a) Draw a scatter diagram for the above data and on it draw a line of best fit. Use the line of best fit to estimate the mark of a student who scored;

(i) 61 in mathematics,

(ii) 25 in Fine Art. *(07 marks)*

(b) Calculate a rank correlation coefficient between the students' performance in the two subjects and comment on your result at 1% level of significance. *(05 marks)*

10. A particle is projected with a speed of $36ms^{-1}$ at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 meters on the horizontal plane from the point of projection. Find the;

(a) (i) time taken for the particle to reach the wall.

(ii) height of the wall. *(08 marks)*

(b) Maximum height reached by the particle from the point of projection. *(04 marks)*

11. (a) Use the trapezium rule with six ordinates to find the approximate value of $\int_{0.5}^{1.5}\left(\frac{3}{x}+x^4\right)dx$, correct to three significant figures, (05 marks)

(b) Evaluate $\int_{0.5}^{1.5}\left(\frac{3}{x}+x^4\right)dx$ correct to three significant figures.

(c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places. *(03 marks)*

(ii) Suggest how the percentage error may be reduced. *(01 mark)*

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12. The continuous random variable X has probability density function (p.d.f)

given by; $f(x) = \begin{cases} (4x - 4x^3); & 0 \le x \le 1 \\ 0 & ; otherwise \end{cases}$

Find the

(a) Mode *(03 marks)*

(b) Cumulative distribution function of x, *(03 marks)*

(c) $P(0.1 < x < 0.6)$ *(02 marks)*

(d) Median of x *(04 marks)*

13. Forces of magnitude 4N, 5N, 5N, 4N and 6N act along the lines **AB, BC, CD, DA** and **AC** respectively of the square **ABCD** whose side has a length of a units. The direction of the forces are indicated by the order of the letters.

(a) Find the magnitude and direction of the resultant force. (09 marks)

(b) If the line of action of the resultant force cuts **AB** produced at E, find the length **AE.** *(03 marks)*

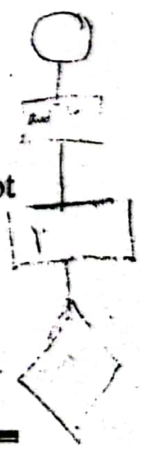
14. (a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = ln3$ it satisfies

$$x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}$$ *(04 marks)*

(b) (i) Construct a flow chart that;

- reads the initial approximation as x_0
- computes, using the iterative formula in (a), and prints the root of the equations $3x = in3, to$ 4 significant figures.

(ii) Perform a dry-run for your flow chart for $x_0 = {}^1/_3$ *(08 marks)*



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