P425/1

PURE MATHEMATICS

Paper 1

Oct/Nov. 2022

3 hours

PRE-UNEB SET 3

Uganda Advanced Certificate of Education

PURE MATHEMATICS

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3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Express $5 + 4x - x^2$ in the form $a + b(x - c)^2$. Hence find the range of values of x for which $5 + 4x - x^2 \geq 0$. *(05 marks)*
2. The points A,B and C have position vectors $\boldsymbol{a} = 5\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{c} = 7\boldsymbol{i} - 3\boldsymbol{j} + 10\boldsymbol{k}$ respectively, show that ABC is a triangle. *(05 marks)*
3. Solve for θ if $2sin2\theta = tan\theta$ for $0 < \theta < 2\pi$. *(05 marks)*
4. Evaluate: $\int_0^{\pi/12} cos^2 3x dx$. *(05 marks)*
5. Show that $1 + i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$. Hence find other roots. *(05 marks)*
6. Find the acute angle between the straight lines $3x - y + 2 = 0$ and $x - 2y - 1 = 0$. Hence find the gradient of the line that bisects this angle correct to 3dps. (05 marks)
7. Find the equation of the normal to the curve with parametric equations $x = 3t + 5, y = 1 - t^2$, which is parallel to the line $4y + 3x = 12$. *(05 marks)*
8. Given that $y = Ae^{-x} + Be^{3x}$, form a differential equation independent of A and B. State the order of this differential equation. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt **only five** questions in this section.*

9. (a)The points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ with respect to an origin O. Show that the area of triangle OAB is given by $\frac{1}{2}\sqrt{a^2b^2 - (\boldsymbol{a}\,.\,\boldsymbol{b})^2}$ where a and b are lengths of A and B from the origin respectively. *(04 marks)*

   (b) Given the plane with equation $5x - y + 7z = 9$ and the line $\boldsymbol{r} = (2\boldsymbol{i} - \boldsymbol{k}) + t(\boldsymbol{i} + 3\boldsymbol{j})$, find

   (i) the coordinates of the point of intersection of the line and the plane. *(04 marks)*

   (ii) the angle between the line and the plane. *(04 marks)*

10.(a) The sum of the first seven terms of a G.P. is 7 and the sum of the next seven terms is 896. Find the common ratio of the progression. If the k^{th} term is the first term of the GP greater than 1, find k. *(07 marks)*

(b) A team of four members is to be chosen at random from 5 girls and 6 boys. Find the number of ways the team can be chosen if there must be more boys than girls. *(05 marks)*

11.(a) Given that k and h are non zero constants, prove that if the equations $x^2 + hx + k = 0$ and $4x^2 - hx + 6b = 0$ have a common root, then $35h^2 + 4k = 0$. *(05 marks)*

   (b) The sum of the squares of the roots of the equation $x^2 + px + q = 0$ is 56 and the sum of the reciprocals is 2. Find the values of p and q. Hence solve the quadratic equation. *(07 marks)*

12.(a)Solve the equation $cos3x + sin2x = cosx$ for $0^0 \leq x \leq 180^0$ *(06 marks)*

   (b) Solve the equation $4cos2\theta + 2sin\theta = 1$ for $-180^0 \leq \theta \leq 180^0$.

(06 marks)

13. Given the curve $y = 1 + 2x - x^2$.

(a) By obtaining the turning point and intercepts, sketch the curve.

(04 marks)

(b) Calculate the area between the curve and the line $y = 1$.

(04 marks)

(c) Find the volume generated by rotating completely about the line $y = 1$, the area in (b) above. *(04 marks)*

14.(a) $\int_0^{\pi/2} x^2 sinxdx$ *(06 marks)*

(b) A man wishes to fence a rectangular enclosure of area $200m^2$. One side of the enclosure is formed by part of the wall already in position. Find the least possible length of fencing required for the other three sides.

(06 marks)

15.(a) A circle has the points (-7,3) and (1,9) as the ends of the diameter. Find the length of the tangent from the point (9,1) to the circle.

(06 marks)

(b) The tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ meets the y-axis at L. Find the equation of locus of M the midpoint of PL as P moves on the parabola. *(06 marks)*

16.(a) solve the differential equation $x\frac{dy}{dx} = xy + y; y = 1, x = 1$.

(05 marks)

(b) The rate of increase of the temperature of a liquid being heated in an oven is proportional to the excess temperature of the oven over that of the liquid. The temperature of the oven is maintained at 200^0C. The temperature of the liquid rises from 0^0C to 100^0C in 5 minutes. Find the time it takes the temperature of the liquid to rise to 160^0C. *(07 marks)*

END