P425/2
APPLIED MATHEMATICS
Paper 2
July /Aug. 2022
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in* **section** *A and any* **five** *from* **section B.**

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section **B** *will* **not** *be marked.*

SECTION A (40 MARKS)

1. A *and* B are events such that $P(A^1 \cup B) = \frac{2}{5}$; $P(A \cap B) = \frac{3}{10}$. Find $P(B/A)$. **(05 marks)**

2. A particle, accelerating uniformly, moves with an average velocity of $8ms^{-1}$ for 4 seconds. If its final velocity is $12ms^{-1}$. Calculate the;
   (i) distance covered
   (ii) acceleration of the particle. **(05 marks)**

3. Give that $x = 4.8\,(1dps)$; $y = 3.25\,(2dps)$ find the interval within which the exact value of $x - y$ lies. **(05 marks)**

4. Five people are picked at random from a group of 7 men and 3 women. Calculate the probability that at least 2 are men. **(05 marks)**

5. A particle is projected from a point O on a horizontal ground with a speed of $20ms^{-1}$ at an elevation $tan^{-1}\frac{4}{3}$. Calculate the;
   (i) time of flight,
   (ii) height risen in a $\frac{1}{3}$ *of the time of flight.* **(05 marks)**

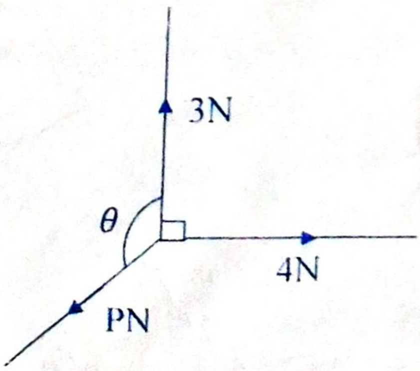
6.

Subject	Ranks						
Economics	1	2	3	4	5	6	7
Maths	2	4	3	5.5	1	5.5	7

Use the table to calculate the Spearman's rank correlation for the performance of the two subjects. Comment on your answer. **(05 marks)**

7. Use the ~~Neaton~~ Newton – Raphson method to compute $\sqrt{3}$ to 4 dps. **(05 marks)**

8.



The 3 forces are in equilibrium, find the values of **P** and **Q** **(05 marks)**

SECTION B (60 MARKS)

9. (a) Given that X *and* Y are approximate values with respective errors ΔX *and* ΔY, show that the maximum error in

$\frac{X}{Y}$ is $\left|\frac{X}{Y}\right| \left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta Y}{Y}\right|\right\}$ ***(07marks)***

(b) Given $X = 4.8$, $Y = 3.56$ corrected to the given number of decimal places; using the results in (a) above, or otherwise compute the maximum error in $\frac{X - Y}{X + Y}$. ***(05 marks)***

10. The table shows the distribution of heights of pupils in a school.

Height (cm)	$0-<50$	$50-<90$	$90-<100$	$100-<120$	$120-<160$
Frequency	8	16	20	32	4

(a) Construct a histogram for this data, and use it to find the mode. ***(07 marks)***

(b) Calculate the number of pupils whose heights lie between 80cm and 116cm. ***(05 marks)***

11. A boat is travelling north wards at $80km^{-1}$ when a wind starts to blow east wards at $60kmh^{-1}$. ***(05 marks)***

(a) Find the resultant velocity of the boat.

(b) Calculate the direction in which the boat must be steered so as to remain on its original course, and compute the resultant speed of the boat in this case. ***(07 marks)***

12. In a large group of patients 75% suffer from malaria.

(a) Ten patients are picked at random from the group, find the probability that between 4 and 9 are malaria patients. ***(05 marks)***

(b) Forty eight patients are picked at random, calculate the probability that:

(i) exactly 34, 34

(ii) at most ~~26~~ are malaria patients. ***(07 marks)***

32.

13. A body of mass 2.5kg is placed on a rough inclined plane of angle $tan^{-1}\frac{4}{3}$. calculate the;

(a) minimum force parallel to the plane that will keep the body at rest, the coefficient of friction being 0.5. ***(07 marks)***

Tech Calc

(b) the acceleration of the body if it is released to move down the plane. *(05 marks)*

14. (a) Show that the positive root of the equation $x^3 - 2x - 1 = 0$ lies between 1 and 2; use linear interpolation to find the first approximation of the root. to 4 dps. *(04 marks)*

(b) Construct a flow chart based on the Newton – Raphson algorithm for computing the root of the equation in (a) above. Perform a dry run of your flow chart. *(08 marks)*

15. A boat 100 km North East of a ferry, is travelling ~~North~~ West wards at 60kmh^{-1}. At that instant, the ferry is travelling at $45\sqrt{2}\, kmh^{-1}$ due North West. Calculate the;

(a) velocity of the ferry relative to the boat. *(06 marks)*

(b) shortest distance between the vessels. *(06 marks)*

16. The marks obtained by 2000 UNEB candidates in Maths Paper 2 of a certain year were normally distributed with a mean of 64. The records showed that 60% of the candidates scored above 50.

(a) Calculate the standard deviation of the candidates' marks. *(04 marks)*

(b) Find the pass mark, if 75% of the candidates passed the paper. *(04 marks)*

(c) Calculate the number of candidates that scored between 45 and 55 marks. *(04 marks)*

END

SOLUTIONS

	Comments
$P(A' \cup B) = P(A \cap B')'$ w.m	
i.e, $\frac{2}{5} = 1 - P(A \cap B')$ M1 $\therefore P(A \cap B') = \frac{3}{5}$ B1	
$P(A) = P(A \cap B) + P(A \cap B')$	
$= \frac{3}{10} + \frac{3}{5}$ M1	Explore other approaches.
$= \frac{9}{10}$	
$P(B/A) = \frac{P(A \cap B)}{P(A)}$ M1	
$= \frac{3/10}{9/10} = \frac{1}{3}$ A1	
Average vel. $8 = \frac{u+12}{2}$ M1 $\Rightarrow u = 4ms^{-1}$ A1	
(i) distance, S = Average speed x time	
$= 8 \times 4$	
$= 32 m$ A1	Allow other methods.
(ii) Acceleration $= \frac{v-u}{t}$	
$= \frac{12-4}{4}$ M1	
$= 2ms^{-2}$. A1	

SOLUTIONS

3. $x_{max} = 4.85$ | $y_{max} = 3.255$

$x_{min} = 4.75$ | $y_{min} = 3.245$ (B1)

$x = 4.8 \pm 0.05$
$y = 3.25 \pm 0.005$

$Min(x-y) = x_{min} - y_{max}$
$= 4.75 - 3.255$ (M1)
$= 1.495$ (B1)

$Max(x-y) = x_{max} - y_{min}$
$= 4.85 - 3.245$
$= 1.605$ (B1)

ACCEPT $1.495 \leq x-y \leq 1.$

The required interval is $[1.495, 1.605]$ (A1)

Alternatively: Approx. value $= 4.8 - 3.25$
$= 1.55$ (B1)

Max. error in $x-y = 0.05 + 0.005$ (M1)
$= 0.055$ (B1)

Interval $= 1.55 \pm 0.055$ (M1)
$= [1.495, 1.605]$ (A1)

SOLUTIONS

	Comments

Let X = the number of men picked.

$\sim B(5, 0.7)$

$P(X \geq 2) = 1 - P(X \leq 1)$ M1 B1

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \{ {}^5C_0 (0.7)^0 (0.3)^5 + {}^5C_1 (0.7)(0.3)^4 \}$ B1

$= 1 - (0.00243 + 0.02835)$ B1

$= 0.96922$ (CAL) A1

USE FIVE for a calculator

Also accept $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

Alternatively:

$P(X \geq 2)$ at $p = 0.7 \Leftrightarrow P(X \leq 3)$ at $p = 0.3$ M1 — Symmetry Property

$\Leftrightarrow 1 - P(X \geq 4)$ at $p = 0.3$ M1 B1

$= 1 - 0.0308$ B1

$= 0.9692$ (TAB) A1

(i) $T = \frac{2u\sin\theta}{g}$

$= \frac{2 \times 20 \times 4/5}{9.8}$ M1

$= \frac{160}{49}$ seconds

$\simeq 3.265306$ s. A1

(ii) $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$t = \frac{2u\sin\theta}{3g} \Rightarrow y = \frac{2u^2\sin^2\theta}{3g} - \frac{2u^2\sin^2\theta}{9g}$ M1

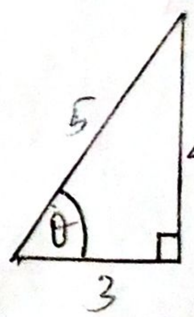
$v_y^2 = u^2 \sin^2\theta$

$v_y = u\sin\theta - gt$

$= \frac{4u^2\sin^2\theta}{9g}$ B1

$= \frac{4 \times 400 \times 16}{45 \times 49}$

$\simeq 11.6100$ m A1



$\sin\theta = 4/5$

$\alpha = 53.13$

$t = 3.265$ Accept

Other methods

SOLUTIONS

R_{ECON}	R_{MATHS}	d	d^2
1	2	−1	1
2	4	−2	4
3	3	0	0
4	5.5	−1.5	2.25
5	1	4	16
6	5.5	0.5	0.25
7	7	0 B1	0
			$\Sigma d^2 = 23.5$ B1

$$\rho = 1 - \frac{6 \times 23.5}{7 \times 48} \quad B_1$$

$= 0.58$; A1 ; correlation is moderate and positive.

Let $x = \sqrt{3} \Rightarrow x^2 - 3 = 0$ M1

$\Rightarrow f(x) = x^2 - 3 \Rightarrow f'(x) = 2x$

$$x_{n+1} = x_n - \frac{(x_n^2 - 3)}{2x_n} \quad B_1$$

$$= \frac{1}{2}\left(x_n + \frac{3}{x_n}\right);\ n = 0, 1, 2, \dots$$

Use $x_0 = 1.5$ B1 since $1 < \sqrt{3} < 2$

$$\Rightarrow x_1 = \frac{1}{2}\left(1.5 + \frac{3}{1.5}\right)$$

$= 1.75$; $|x_1 - x_0| = 0.25$

$$x_2 = \frac{1}{2}\left(1.75 + \frac{3}{1.75}\right) \quad M1$$

$|x_2 - x_1| = 0.017857$

$$x_3 = \frac{1}{2}\left(1.732143 + \frac{3}{1.732143}\right)$$

$\simeq 1.732051$

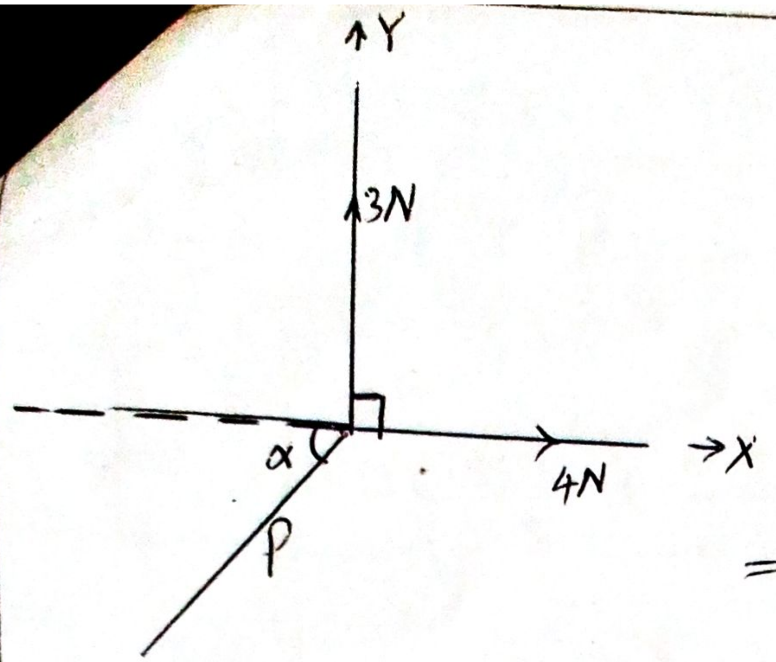
$|x_3 - x_2| = 0.000092$

$x_4 \simeq 1.73205$ A1

Thus $\sqrt{3} \simeq 1.7321$ (4 dps)

14
33

Comments



$(\rightarrow)\ P\cos\alpha = 4$ (M1)

$(\uparrow)\ P\sin\alpha = 3$

(M1)

Squaring: $P^2(\cos^2\alpha + \sin^2\alpha) = 4^2 + 3^2$

$\Rightarrow P^2 = 25 \therefore P = 5N$ (A1)

$\frac{P\sin\alpha}{P\cos\alpha} = \frac{3}{4} \Rightarrow \tan\alpha = 3/4$ (M1)

$\alpha \simeq \underline{36.87^\circ}$

Lami's Theorem can be used.

Thus, $\theta = 90^\circ + \alpha$
$= 126.87^\circ$ (A1)

SECTION B (60 marks)

1 (a) Let x, y be the exact values $\Rightarrow \Delta X = x - X \Rightarrow x = X + \Delta X$
$\Delta Y = y - Y \Rightarrow y = Y + \Delta Y$

Error in $\frac{X}{Y} = \frac{X+\Delta X}{Y+\Delta Y} - \frac{X}{Y}$ (M1)

$= \frac{XY + Y\Delta X - XY - X\Delta Y}{Y^2(1+\frac{\Delta Y}{Y})}$

$= \frac{Y\Delta X - X\Delta Y}{Y^2(1+\frac{\Delta Y}{Y})}$ (M1)

$= \frac{Y\Delta X - X\Delta Y}{Y^2}$ (B1)

$= \frac{\Delta X}{Y} - \frac{X\Delta Y}{Y^2}$

$= \frac{X}{Y}\left[\frac{\Delta X}{X} - \frac{\Delta Y}{Y}\right]$ (M1)

$\leq \left|\frac{X}{Y}\right|\left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{-\Delta Y}{Y}\right|\right\}$ (M1)

Assumption

$\Delta Y << Y$ (B1)

$\Rightarrow \frac{\Delta Y}{Y} \simeq 0$

Hence maximum error is

$\left|\frac{X}{Y}\right|\left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta Y}{Y}\right|\right\}$ (B1)

1 (b) Max error in $(X-Y) = |\Delta X| + |\Delta Y|$; $X-Y = 1.24$

$= 0.55$ (B1)

Max. error in $(X+Y) = 0.55$ (B1) ; $X+Y = 8.36$

Accept the Simple Interval Arithmetic Method

Hence maximum error $= \frac{1.24}{8.36}\left\{\frac{0.55}{1.24} + \frac{0.55}{8.36}\right\}$ M1B1

$\simeq 0.0755$ (4dps) A1

3

Height	0–50	50–90	90–100	100–120	120–160
freq.	8	16	20	32	4
f. density	0.16	0.4	2	1.6	0.1
c. frequency	8	24	44	76	80

B1 B1

(a) From the Histogram, mode $\simeq 98$ A (see graph)

(b)

Height	50	80	90
c.f.	8	n_1	24

$\Rightarrow \frac{n_1 - 8}{24-8} = \frac{80-50}{90-50}$ M1

$n_1 = 8 + \frac{16 \times 30}{40}$

$= 20$ (B1)

Height	100	116	120
c.f.	44	n_2	76

$\Rightarrow \frac{n_2 - 44}{116-100} = \frac{76-44}{120-100}$

$n_2 = 20 + 16$

$= 36$ (B1)

$\frac{n_2 - 44}{16} = \frac{32}{20}$

$20n_2 - 880 = 320$

$n_2 = 70$

The required no of pupils $= n_2 - n_1$ M1

$= 36 - 20$

$= 16.$ A1

$n_2 - n_1$

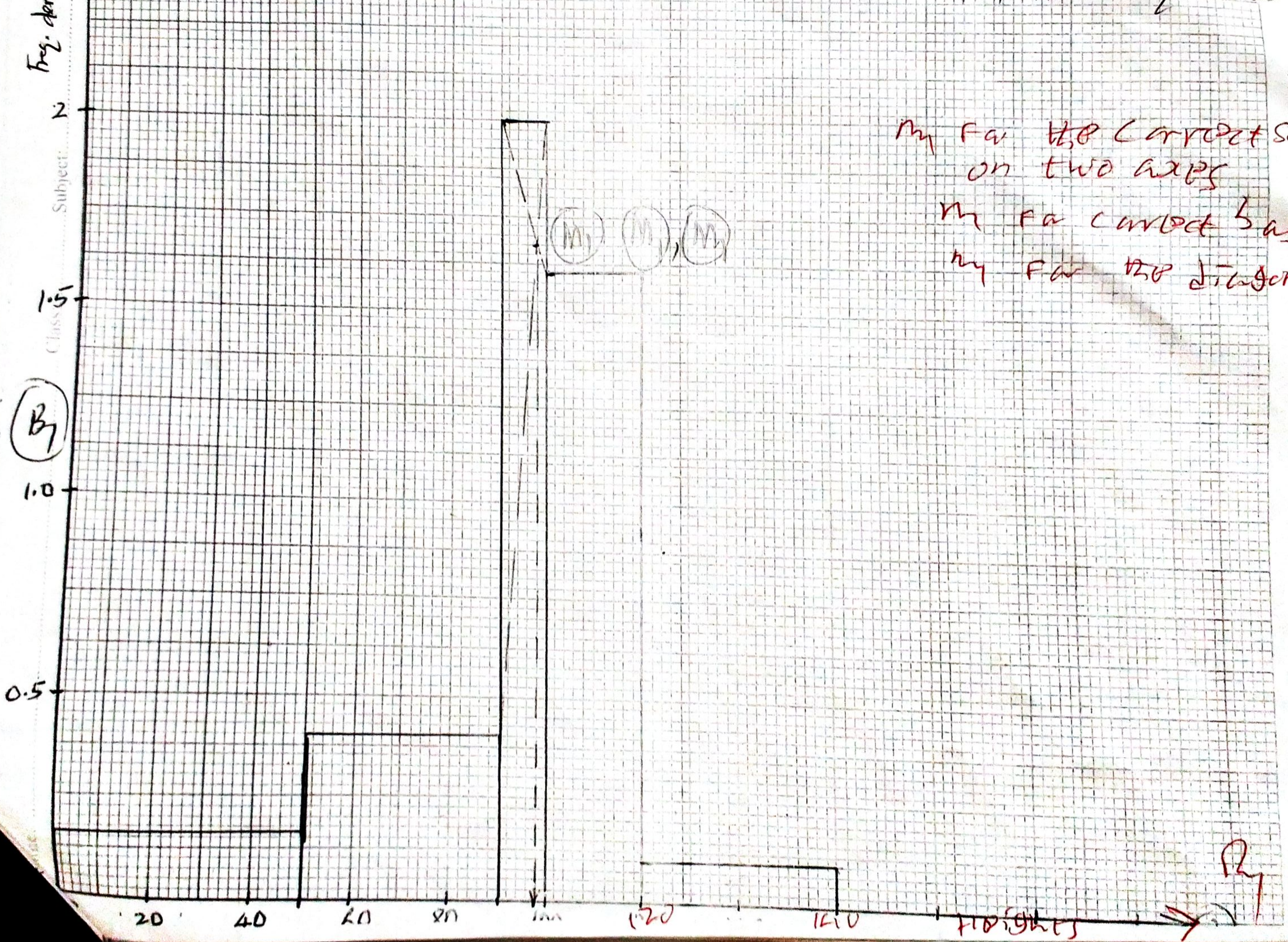
$70 - 20$

$= 50$ pupils

№ 10 (a)

HISTOGRAM

Vertical Axis: 1 small square ≡ 0.025

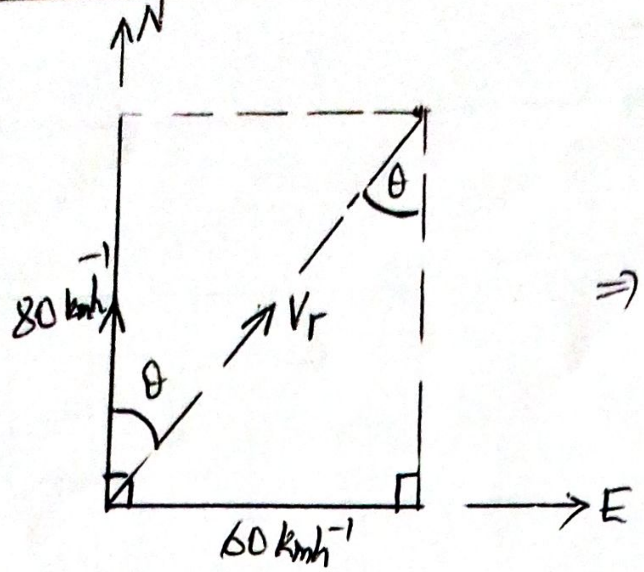


M1 For the correct scale on two axes

M1 For correct bars

M1 For the diagonal

1 (a)



$V_r^2 = 60^2 + 80^2$

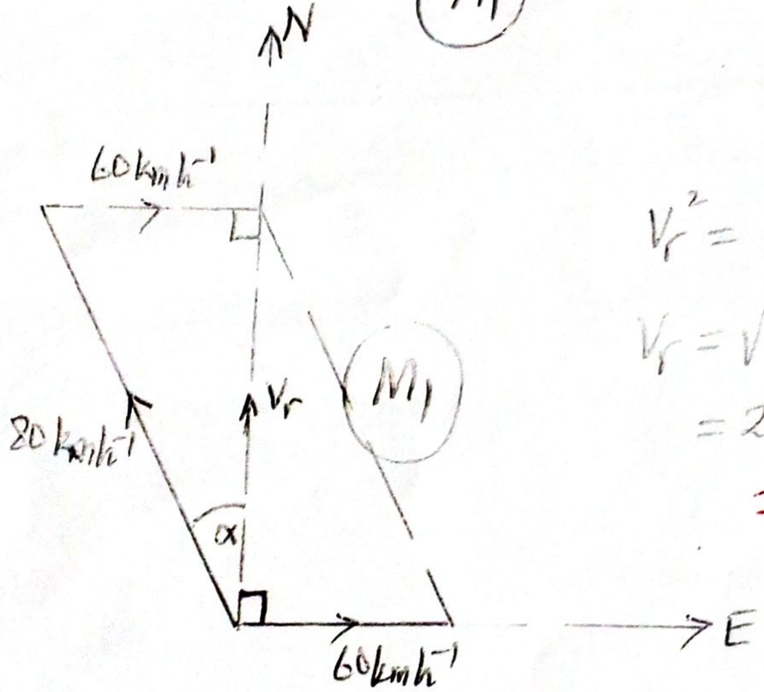
$\Rightarrow V_r = 100\ \text{km h}^{-1}$ M1

$\tan\theta = \frac{60}{80}$. M1

$\theta \simeq 36.87^\circ$ B1

The resultant vel. is $100\ \text{km h}^{-1}$ A1 due N36.87°E. A1

(b)



M1

$V_r^2 = 80^2 - 60^2$

$V_r = \sqrt{80^2 - 60^2}$ M1

$= 20\sqrt{7}\ \text{km h}^{-1}$ B1

$= 52.9150\ \text{km h}^{-1}$

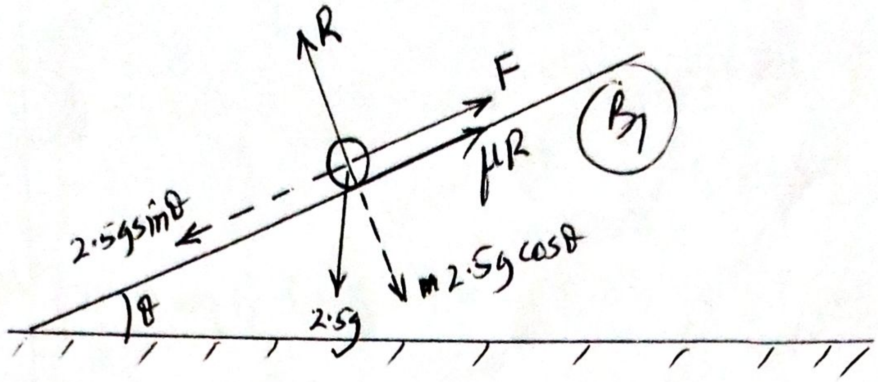
M1 $\sin\alpha = \frac{60}{80} \Rightarrow \alpha = 48.59^\circ$ B1

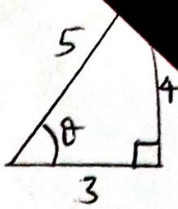
The required direction is N48.59°W A1, with a resultant speed of $20\sqrt{7}\ \text{km h}^{-1}$. A1

SOLUTIONS	Comments
(a) $X \sim$ no of malaria patients.	
$\sim B(10, 0.75)$ B1	
$P(4 < X < 9) = P(X \leq 8) - P(X \leq 4)$; $p = 0.75$ M1	
$= P(X \geq 2) - P(X \geq 6)$ M1 ; $p = 0.25$	Symmetry property.
$= 0.7560 - 0.0197$ B1	
$= 0.7363$ (TAB) A1	
(b) $X \sim B(48, 0.75)$; n is large B1	
$X \sim N(\mu, \sigma^2)$; $\mu = 48 \times 0.75 = 36$ B1 ; $\sigma = \sqrt{36 \times 0.25} = 3$ B1	
(i) $P(X = 4) \Rightarrow P(3.5 < X < 4.5)$	
$= P\left(\frac{3.5 - 36}{3} < Z < \frac{4.5 - 36}{3}\right)$ M1	
$= 0.0000$ (4 dps)	
(ii) $P(X \leq 26) = P(X \leq 26.5)$ M1	
$= P\left(Z \leq \frac{26.5 - 36}{3}\right)$	
$= P(Z \leq -3.167)$ B1	
$= \phi(3.167)$ B1	
$= 0.0000$ (4 dps)	

N | SOLUTIONS

3 (a)





$\tan\theta = 4/3$

$\Rightarrow \sin\theta = 4/5$ (B1)

$\cos\theta = 3/5$

Let F be the minimum force:

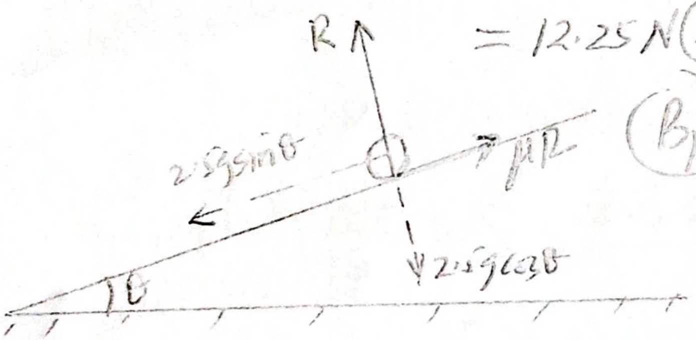
$R = 2\cdot5g\cos\theta$ (B1) and $F = 2\cdot5g\sin\theta - \mu R$ (M1)

$= 2\cdot5g\sin\theta - \frac{1}{2}\times 2\cdot5g\cos\theta$ (M1)

$= 2\cdot5\times9\cdot8\left(\frac{4}{5} - \frac{1}{2}\times\frac{3}{5}\right)$ (B1)

$= 12\cdot25\,N$ (A1)

(b)



$\Rightarrow R = 2\cdot5g\cos\theta$ (B1) ; resultant force $= 2\cdot5g\sin\theta - 2\cdot5\mu g\cos\theta$ (B1)

Acceleration $= \dfrac{2\cdot5g(\sin\theta - \mu\cos\theta)}{2\cdot5}$ (M1)

$= \dfrac{2\cdot5\times9\cdot8(0\cdot2 - 0\cdot3)}{2\cdot5}$

$= 4\cdot9\ ms^{-2}$ (A1)

Comments.

(a) $f(x) = x^3 - 2x - 1$

$f(1) = 1 - 2 - 1$
$= -2$ B1

$f(2) = 8 - 4 - 1$
$= 3$.

Since $f(1) < 0$ and $f(2) > 0$

$\Rightarrow 0 < x_r < 2$ B1

x	1	x_0	2
$f(x)$	-2	0	3

By linear interpolation: $\frac{x_0 - 1}{2 - 1} = \frac{0-(-2)}{3-(-2)}$ M1

$x_0 = 1 + \frac{2}{5}$
$= 1.4$ A1

$f(x) = x^3 - 2x - 1$

$f'(x) = 3x^2 - 2$

(b) $x_{n+1} = \frac{x_n}{1} - \frac{(x_n^3 - 2x_n - 1)}{3x_n^2 - 2}$

$\therefore x_{n+1} = \frac{2x_n^3 + 1}{3x_n^2 - 2}$; $n = 0, 1, 2, \ldots$

Dry-Run

$x_0 = 1.4$

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.4	1.6722	0.2722
1	1.6722	1.6203	0.0519
2	1.6203	1.6180 B1	0.0023 B1
3	1.6180	1.6180	0.0000

B1

DDry manes other method other than the dry run.

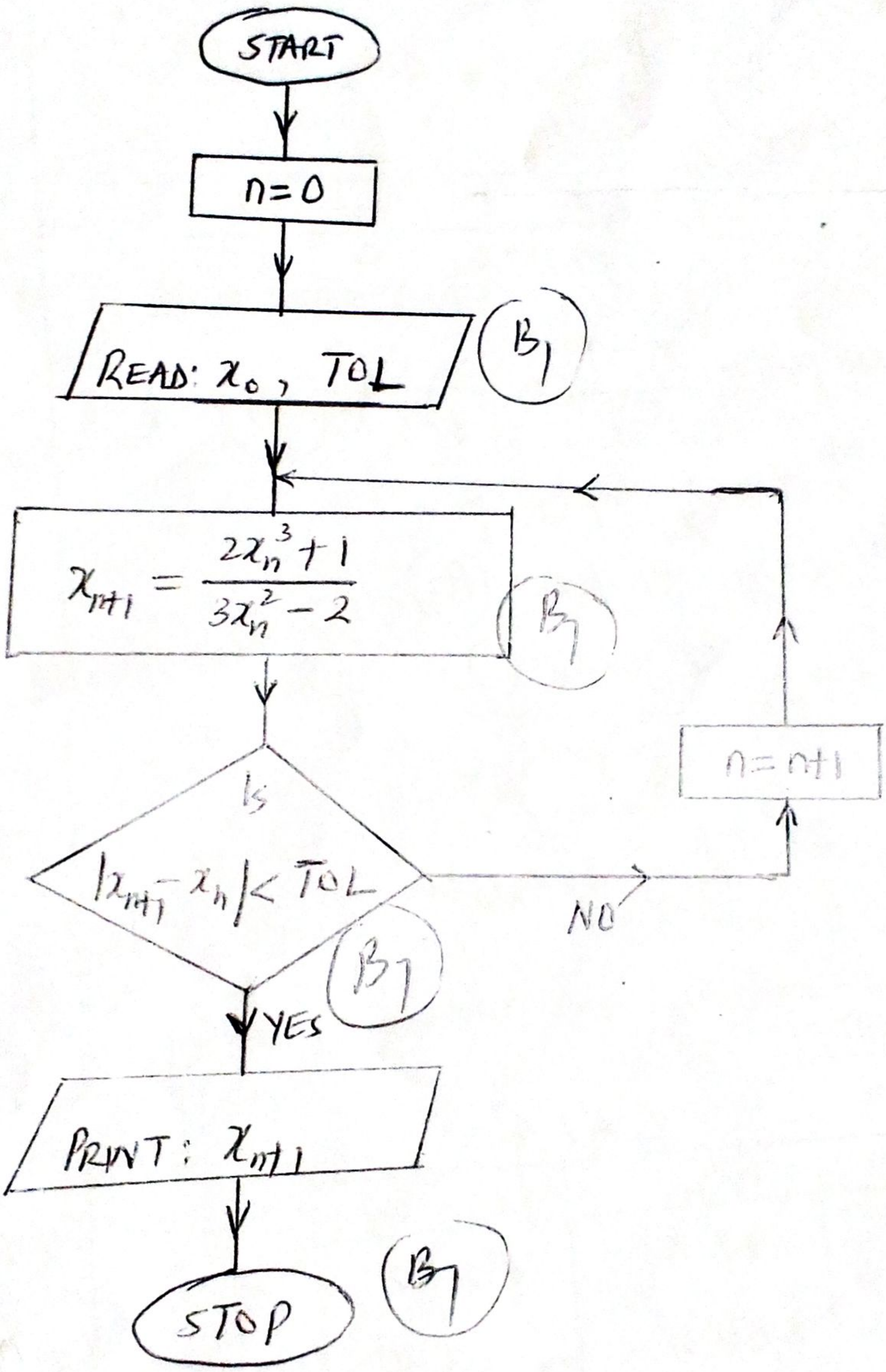
The root is 1.6180

$\simeq 1.618$ (3 dps) A1

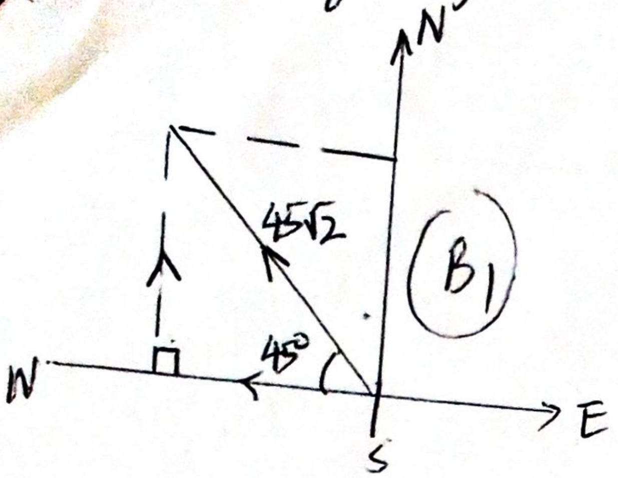
N SOLNS

7(b) cont'd.

Flow chart



(a) Velocity of Ferry



(B1)

$$\underset{\sim}{V}_F = \begin{pmatrix} -45\sqrt{2}\cos 45° \\ 45\sqrt{2}\sin 45° \end{pmatrix} \quad (M_1)$$

$$= \begin{pmatrix} -45 \\ 45 \end{pmatrix} km h^{-1} \quad (A_1)$$

Velocity of Boat

$$\underset{\sim}{V}_B = \begin{pmatrix} 0 \\ 60 \end{pmatrix} km h^{-1} \quad (B_1)$$

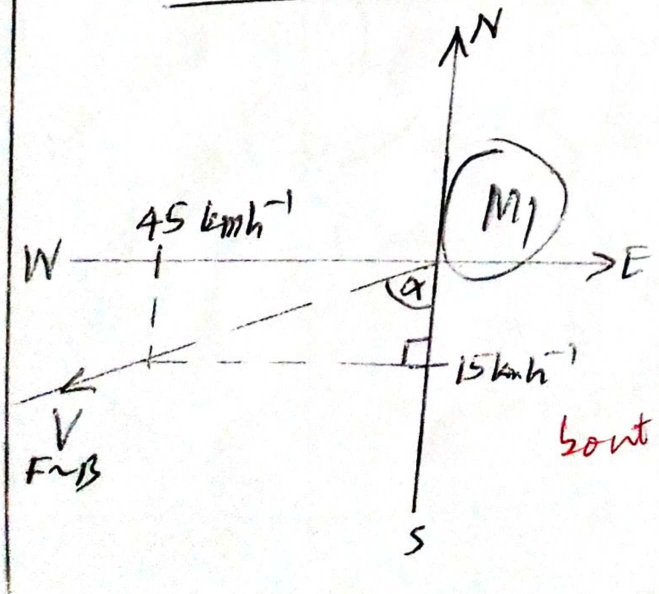
$$\Rightarrow {}_F\underset{\sim}{V}_B = \begin{pmatrix} -45 \\ 45 \end{pmatrix} - \begin{pmatrix} 0 \\ 60 \end{pmatrix} \quad (M_1)$$

$$= \begin{pmatrix} -45 \\ -15 \end{pmatrix} km h^{-1} \quad (B_1)$$

$$|{}_W V_B| = \sqrt{(-45)^2 + (-15)^2} \quad (M_1)$$

$$= 15\sqrt{10}\ km h^{-1} \quad (A_1)$$

Direction of ${}_F\underset{\sim}{V}_B$



(M1)

$$\tan\alpha = \frac{45}{15} = 3$$

$$\alpha \simeq 71.57° \quad (B_1)$$

Hence velocity of the Ferry relative to the boat is $15\sqrt{10}\ km h^{-1}$ due S 71.57° W

(A_1) (A_1)

47.4342 km/h

Comments

*** Typing error

** We did not see!..

The Ferry is travelling westwards so Part (b) is unworkable.

SOLUTIONS

COMMENTS

Let X be the marks obtained by a candidate

$\Rightarrow X \sim N(64, \sigma^2)$

(a) $P(X > 50) = 0.60$ (M1)

$\Rightarrow P(Z > z_0) = 0.60$; where $z_0 = \frac{50-\mu}{\sigma}$ (M1)

From tables: $z_0 = -0.253 \Rightarrow -0.253 = \frac{50-64}{\sigma}$ (B1)

$\therefore \sigma = \frac{14}{0.253}$

$\simeq 55$ (A1)

(b) Let x_0 be the pass mark

$\Rightarrow P(X \geq x_0) = 0.75$ (M1)

$\Rightarrow P(Z \geq z_0) = 0.75$, $z_0 = \frac{x_0 - 64}{55}$

(B1)

$\Rightarrow -0.674 = \frac{x_0 - 64}{55}$

$\Rightarrow x_0 = 64 - 0.674 \times 55$ (M1)

$\simeq 27$ (A1)

(c) $P(45 < X < 55) = P\left[\frac{45-64}{55} < Z < \frac{55-64}{55}\right]$ (M1)

$= P(-0.3455 < Z < -0.1636)$

$\Leftrightarrow P(0.1636 < Z < 0.3455)$

$= 0.1353 - 0.0652$

$= 0.0701$ (A1)

No required $= 2000 \times 0.070$ (M1)

$= 140$ (A1)

14