

P425/1  
PURE MATHEMATICS  
Paper 1  
Nov./Dec. 2023  
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD  
Uganda Advanced Certificate of Education  
PURE MATHEMATICS

Paper 1

3 hours

**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in section A and any **five** from section B.*
- Any additional question(s) answered will **not** be marked.*
- All necessary working **must** be shown clearly.*
- Begin each answer on a fresh sheet of paper.*
- Graph paper is provided.*
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

## SECTION A (40 MARKS)

Answer **all** the questions in this section.

1. Prove by induction that  $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$ . (05 marks)
2. If a line  $y = mx + c$  is a tangent to the curve  $4x^2 + 3y^2 = 12$ , show that  $c^2 = 4 + 3m^2$ . (05 marks)
3. Given that  $y = e^x \cos 3x$ , show that  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 10y = 0$ . (05 marks)
4. Find the angle between the line  $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$  and the plane  $-x + 2y + 2z - 66 = 0$ . (05 marks)
5. Solve the inequality  $\frac{7-2x}{(x+1)(x-2)} > 0$ . (05 marks)
6. Evaluate  $\int_0^{\pi/3} (1 + \cos 3y)^2 dy$ . (05 marks)
7. Express  $2\sin\theta + 3\cos\theta$  in the form  $R \sin(\theta + \alpha)$ . (05 marks)
8. Use Maclaurin's theorem to expand  $\ln(2+x)$ , in ascending powers of  $x$  as far as the term in  $x^2$ . (05 marks)

## SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks.

- 9✓ (a) Solve the equation  $Z^3 - 7Z^2 + 19Z - 13 = 0$ . (06 marks)
- (b) Find the fourth roots of  $8(-\sqrt{3}+i)$ . (06 marks)
10. Express  $f(x) = \frac{3x^3 + 2x^2 - 3x + 1}{x(1-x)}$  in partial fractions.  
Hence find  $\int f(x) dx$ . (12 marks)
11. A point  $E$  has coordinates  $(2, 0, -1)$ . A line through  $E$  and parallel to the line whose equation is  $\frac{x}{-2} = y = \frac{z+1}{2}$ , meets a plane  $x + 2y - 2z = 8$  at a point  $B$ . A perpendicular line from  $E$  meets the plane at a point  $C$ . Determine the coordinates of;
- (a)  $B$ . (07 marks)
- (b)  $C$ . (05 marks)
12. (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? (02 marks)
- (b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days? (03 marks)
- (c) Given that  ${}^{20}C_r = {}^{20}C_{r-2}$ , find the value of  $r$ . (07 marks)
- 13✓ (a) A curve is given by the parametric equations  $x = t^2 - 3$ ,  $y = t(t^2 - 3)$ . Find the Cartesian equation of the curve. (04 marks)
- (b) A point  $P$  is such that its distance from the origin is five times its distance from  $(12, 0)$ .
- (i) Show that the locus of  $P$  is a circle.
- (ii) Determine the coordinates of the centre of the circle and its radius. (08 marks)

14. Given the curve  $y = \frac{1}{4x^2 - 1}$ , determine the;

(a) coordinates of the turning points of the curve. (03 marks)

(b) equation of the asymptotes.  
Hence sketch the curve. (09 marks)

15. (a) Show that  $\tan 3\theta = \frac{\tan \theta (3 - \tan^2 \theta)}{(1 - 3 \tan^2 \theta)}$ . (05 marks)

(b) Solve the equation  $\cos 4x + \cos 6x + \cos 2x = 0$  for  $0^\circ \leq x \leq 180^\circ$ . (07 marks)

16. The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature  $25^\circ\text{C}$ . After 6 minutes the temperature of the body dropped from  $90^\circ\text{C}$  to  $60^\circ\text{C}$ .

(a) Form a differential equation for the rate of cooling of the body. (07 marks)

(b) Find the time it takes for the body to cool from  $40^\circ\text{C}$  to  $30^\circ\text{C}$ . (05 marks)