UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A (40 MARKS)

*Answer **all** the questions in this section.*

1. Prove by induction that $\sum_{r=1}^{n} r^2 = \dfrac{n(n+1)(2n+1)}{6}$.   *(05 marks)*

2. If a line $y = mx + c$ is a tangent to the curve $4x^2 + 3y^2 = 12$, show that
   $c^2 = 4 + 3m^2$.   *(05 marks)*

3. Given that $y = e^x \cos 3x$, show that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 10y = 0$.   *(05 marks)*

4. Find the angle between the line $r = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$ and the

   plane $-x + 2y + 2z - 66 = 0$.   *(05 marks)*

5. Solve the inequality $\dfrac{7 - 2x}{(x+1)(x-2)} > 0$.   *(05 marks)*

6. Evaluate $\displaystyle\int_{0}^{\pi/3} (1 + \cos 3y)^2 \, dy$.   *(05 marks)*

7. Express $2\sin\theta + 3\cos\theta$ in the form $R \sin(\theta + \alpha)$.

        (05 marks)

8. Use Maclaurin's theorem to expand $\ln(2 + x)$, in ascending powers of x as
   far as the term in x^2.

        (05 marks)

2

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) Solve the equation $Z^3 - 7Z^2 + 19Z - 13 = 0$. *(06 marks)*

   (b) Find the fourth roots of $8(-\sqrt{3}+i)$. *(06 marks)*

10. Express $f(x) = \dfrac{3x^3 + 2x^2 - 3x + 1}{x(1-x)}$ in partial fractions.

    Hence find $\displaystyle\int f(x)\, dx$. *(12 marks)*

11. A point E has coordinates $(2, 0, -1)$. A line through E and parallel to the line whose equation is $\dfrac{x}{-2} = y = \dfrac{z+1}{2}$, meets a plane $x + 2y - 2z = 8$ at a point B.

    A perpendicular line from E meets the plane at a point C.

    Determine the coordinates of;

    (a) B. *(07 marks)*

    (b) C. *(05 marks)*

12. (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? *(02 marks)*

    (b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days? *(03 marks)*

    (c) Given that $^{20}C_r = {}^{20}C_{r-2}$, find the value of r. *(07 marks)*

13. (a) A curve is given by the parametric equations $x = t^2 - 3$, $y = t(t^2 - 3)$.

    Find the Cartesian equation of the curve. *(04 marks)*

    (b) A point P is such that its distance from the origin is five times its distance from $(12, 0)$.

    (i) Show that the locus of P is a circle.

    (ii) Determine the coordinates of the centre of the circle and its radius. *(08 marks)*

14. Given the curve $y = \dfrac{1}{4x^2 - 1}$, determine the;

    (a)   coordinates of the turning points of the curve.   *(03 marks)*

    (b)   equation of the asymptotes.
           Hence sketch the curve.   *(09 marks)*

15.   (a)   Show that $\tan 3\theta = \dfrac{\tan\theta\left(3 - \tan^2\theta\right)}{\left(1 - 3\tan^2\theta\right)}$.   *(05 marks)*

    (b)   Solve the equation $\cos 4x + \cos 6x + \cos 2x = 0$ for $0° \le x \le 180°$.
           (07 marks)

16.   The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature 25 °C. After 6 minutes the temperature of the body dropped from 90 °C to 60 °C.

    (a)   Form a differential equation for the rate of cooling of the body.
           (07 marks)

    (b)   Find the time it takes for the body to cool from 40 °C to 30 °C.
           (05 marks)

END