P425/1
PURE MATHEMATICS
Paper 1
December 2022
3 hours

(MEPSA) RESOURCEFULL ASSESSMENT

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B.**

Any additional question (s) answered will not be marked

All *necessary working* **must** *be shown clearly*

Begin each answer on a fresh sheet of paper

Squared paper is provided

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *questions in this section.*

1. Solve the equation: $\frac{16^x - 4^x}{4^x + 2^x} = 5(2^x) - 8$ (*05 marks*)

2. Find the equation of the tangent to the circle $(x-1)^2 + (y+2)^2 = 8$ at the point (3, – 4). (*05 marks*)

3. Show that the identity $\frac{\sin^2 5A - \sin^2 A}{\cos^2 A - \cos^2 3A} = 1 + 2\cos 4A$ (*05 marks*)

4. The gradient function of a curve is given by $2x + \frac{54}{x^2}$. If the $y-$ coordinate of the stationary point of the curve is 7, find the equation of the curve. (*05 marks*)

5. Prove by mathematical induction that $7^n + 4^n + 1$ is divisible by 6 for all positive integers n. (*05 marks*)

6. Evaluate $\int_4^9 \frac{dx}{3+\sqrt{x}}$ (*05 marks*)

7. If the parametric equations of a line are such that $x = 2\lambda + 1,\ y = \lambda + 3$ and $z = \lambda + 2$, where λ is a parameter, determine the;

(i) Cartesian equation of the line

(ii) Coordinates of the point where the line meets the plane $x - y + z = 4$

(*05 marks*)

8. Solve the differential equation $(x+2)\frac{dy}{dx} - 2y = 5$ given that $y = 0$ when $x = 0$ (*05 marks*)

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. Solve the equation $2\cos x + 3\sin x = 1$ for $0° \leq x \leq 360°$, hence find the minimum and maximum values of $\frac{1}{4 + 2\cos x + 3\sin x}$ distinguishing between them. (*12 marks*)

10. (a) The ages of a man and his three children are in a Geometric progression (G.P) whose common ratio is greater than one. The sum of their ages is 80. If the sum of the ages of the two younger children is 8 years. Find the age of the youngest child. (*05 marks*)

(b) Given the function $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$. Hence find the first three non-vanishing terms of the maclaurin's expansion of $e^x \sin x$. (*07 marks*)

11. The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C.

(a) Find the coordinates of A, B and C. (07 marks)

(b) If the area bounded by the chord BC and the curve is rotated completely about the $y-$ axis, calculate the volume generated. (05 marks)

12. (a) Given that $Z = 2\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{3}\right]$, express Z in modulus-argument form and then deduce the Cartesian form of Z^5. (*06 marks*)

(b) Find the equation of the locus $\left|\frac{Z-3}{Z-i}\right| = 1$, hence shade the region on the Argand diagram that represents $|Z-3| < |Z-i|$ (*06 marks*)

13. Evaluate the following integrals;

(a) $$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{x(1-x)}$$ (*06 marks*)

(b) $$\int_{0}^{1.5} x^3 e^{x^2} dx$$ (*06 marks*)

14. (a) Calculate the distance of the point P (1, 2, 3) from the line $\frac{x}{3} = \frac{y+3}{4} = z$ (*08 marks*)

(b) Find the scalar product equation of the plane containing the point P (1, 2, 3) and the line $\frac{x}{3} = \frac{y+3}{4} = z$ in (i) above. (*04 marks*)

15. The parametric equations of a curve are given as $y = \frac{3}{2}t^2 - 6t + 1$ and $x = t^2 + t + 1.$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $t.$ (*04 marks*)

(b) Determine the nature of the stationary point of the curve. (*04 marks*)

(c) Obtain the equation of the tangent to the curve at the point when $t = 1$ (*04 marks*)

16. (a) Show that the normal to the rectangular hyperbola $xy = c^2$ at the point T $\left(ct, \frac{c}{t}\right)$ is given by the equation $t^3x = ty + c(t^4 - 1)$ (*04 marks*)

(b) If the normal meets the hyperbola at S $\left(cs, \frac{c}{s}\right)$, show that $t^3s + 1 = 0.$ (*03 marks*)

(c) Given that $t = -2$ and $15c = 16$, determine the equation of a circle with TS as diameter (*05 marks*)

 End