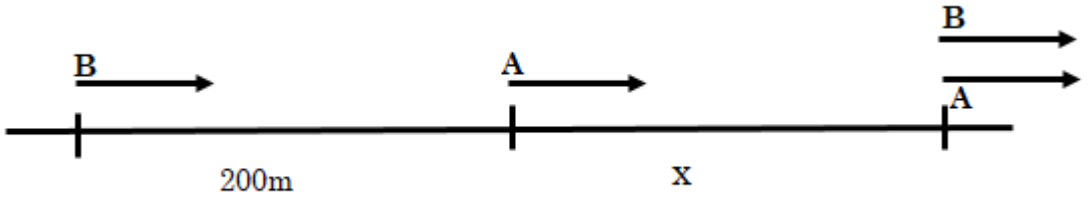
MATIGO EXAMINATIONS BOARD

UACE

APPLIED MATHEMATICS P425/2

MARKING GUIDE 2023

Qn	Answer	Mark	Part Marks
1	$i = class\ width$ $\begin{array}{\|c\|c\|c\|c\|}\hline \boldsymbol{Mass/g} & \boldsymbol{i} & \boldsymbol{fd} & \boldsymbol{f} \\ \hline 46-50 & 4 & 4 & 16 \\ \hline 50-52 & 2 & 8 & 16 \\ \hline 52-54 & 2 & 12 & 24 \\ \hline 54-56 & 2 & 10 & 20 \\ \hline 56-58 & 2 & 2 & 4 \\ \hline & & & \sum fx = 80 \\ \hline \end{array}$	**B1**	For the table with frequencies
1(a)	Number of people whose mass is less than $54g = 16 + 16 + 24 + = 56$ $\text{Probability} = \frac{56}{80}$ $= \frac{7}{10}$	**B1** **A1**	For number of ducklings whose mass is less than $54g$ For correct answer

(b)	Number of duckling whose mass is greater than $53g = 16 + 16 + \frac{53-52}{2} \times 24 = 44$ $\text{Probability} = \frac{44}{80}$ $= \frac{11}{20}$	**B1** **A1**	For number of ducklings whose mass $> 53g$ For the answer
3(a)	<table><tr><td>55</td><td>52</td><td>x</td></tr><tr><td>2</td><td>4</td><td>5</td></tr></table> *applying linear extrapolation* $\frac{x-52}{5-4} = \frac{52-55}{4-2}$ $x = 50.5$ The temperature after 5minutes is 50.5℃	**B1** **M1** **A1**	For the table Application of the method For the answer
(b)	<table><tr><td>55</td><td>53.5</td><td>52</td></tr><tr><td>2</td><td>y</td><td>4</td></tr></table> *applying linear interpolation* $\frac{y-2}{53.5-55} = \frac{4-2}{52-55}$ $y = 3$ Time for the temperature of 53.5℃ is 3 minutes	**M1** **A1**	Application of the method For the answer
2	 For car B, $U = 44\,ms^{-1}$, $a = 0.5ms^{-2}$,	**M1**	For Correct substitution

	$S = ut + \frac{1}{2}at^2$ $200 + x = 44t + \frac{1}{2} \times 0.5\, t^2$ $200 + x = 44t + 0.25\, t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$ For car A, $U = 35ms^{-1}, a = 0.4ms^{-2}$ $S = ut + \frac{1}{2}at^2$ $x = 35t + \frac{1}{2} \times 0.4t^2$ $x = 35t + 0.2t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$ Solving equations 1 and 2 $200 + 35t + 0.2t^2 = 44t + 0.25t^2$ $0.05t^2 + 9t - 200 = 0$ $t = \frac{-9 \pm \sqrt{9^2 - 4 \times 0.005 \times -200}}{2 \times 0.05}$ $t = 20\ and\ t = -200$ $\therefore t = 20s$	**M1** **B1** **B1** **A1**	For substitution For the quadratic equation in terms of t For values of t For the correct value of t
4	$C = 170mm, \Delta C = \frac{1}{2} \times 10^{-0} = 0.5$ $d = 54mm, \Delta d = \frac{1}{2} \times 10^{-0} = 0.5$ $\pi = \frac{C}{d}$ $\pi_{max} = \frac{C + \Delta C}{d - \Delta d}$ $\frac{170 + 0.5}{54 - 0.5}$ 3.186915888 $\pi_{min} = \frac{C - \Delta C}{d + \Delta d}$ $\frac{170 - 0.5}{54 + 0.5}$ 3.110091743	M1 M1 A1 M1 A1	For π_{max} method For substitution For the answer For substitution For the answer

7

x	y	R_x	R_y	$d = R_x - R_y$	d^2
7	5	8	7	1	1
18	3	7	8	-1	1
37	9	6	6	0	0
52	12	5	5	0	0
61	17	4	4	0	0
68	41	3	3	0	0
75	49	2	2	0	0
82	97	1	1	0	0
					$\sum d^2 = 2$

B1 For R_x
B1 For R_y

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

M1 For substitution

$$= 1 - \frac{6 \times 2}{8(8^2-1)}$$

A1 For the answer

$$0.9762$$

comment: it is significant at 5% level of significance

A1 For the comment

6

(a)

$$\sum P(X = x) = 1$$

$$k(1) + k(2) + k(3) + k(4) + k(5) = 1$$

$$15k = 1$$

$$k = \frac{1}{15}$$

B1 For k

$$P(X = x) = 3k$$

M1 For the working

$$3 \times \frac{1}{15}$$

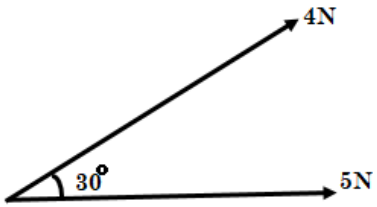
$$0.2$$

A1 For the answer

(b)

x	1	2	3	4	5
$P(X = x)$	$1/15$	$2/15$	$1/15$	$4/15$	$1/3$

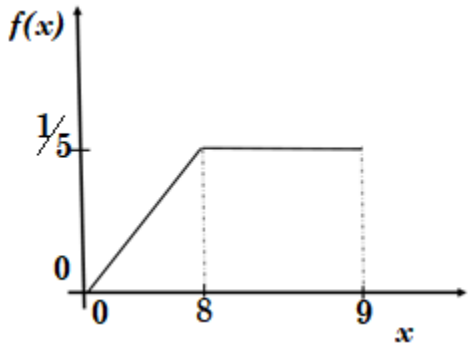
B2 For values of x *and* $P(X = x)$

5	$\Delta KE = -16$ $\frac{1}{2} m (v^2 - u^2) = -16$ $\frac{1}{2} 4 (v^2 - 5^2) = -16$ $v^2 = 17$ From: $v^2 = u^2 + 2as$ $17 = 5^2 + 2as$ $a = -0.1ms^{-2}$	**M1** **B1** **M1 M1** **A1**	For substitution For v^2 *or* v For the method and substitution For a
Alternative	$W.D = \Delta KE$ $W.D = -16$ $W.D = F \times S$ $-16 = F \times 40$ $F = -0.4N$ $F = ma$ $-0.4 = 4a$ $a = -0.1ms^{-2}$	**B1** **M1** **B1** **M1** **A1**	For $W.D = -16$ For method and substitution For F For substitution For a
8			
	$\rightleftarrows \quad F_x = 5 + 4cos30$ $8.4641N$		
	$\uparrow\downarrow \quad F_y = 4 \sin 30$ $2N$		
	$R = \sqrt{F_x^2 + F_y^2}$ $R = \sqrt{8.4641^2 + 2^2}$ $R = 8.69718N$		
	Direction	**B1** **B1**	For F_x For F_y

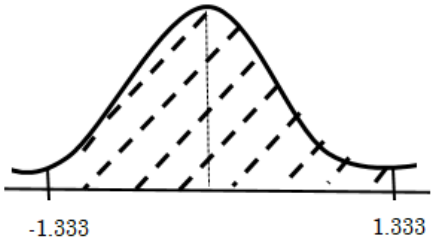
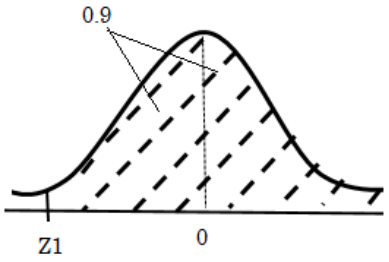
	$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right)$ $= tan^{-1}\left(\frac{2}{8.4641}\right)$ $= 13.3°$ The resultant is 8.69718N in the direction 13.3° above the $5N$ force	**A1** **B1** **A1**	For R For θ A1 for the conclusion
9(a)	$i = class\ width$	**B1** **B1** **B1**	For fx For fx^2 For fd Also recognize other symbols for class width such as c, w etc

Time (s)	f	x	fx	fx^2	i	fd	*Class boundaries*
10 – 19	20	14.5	290	4205	10	2.0	9.5 – 19.5
20 – 24	20	22	440	9680	5	4.0	19.5 – 24.5
25 – 29	15	27	405	10935	5	3.0	24.5 – 29.5
30	14	30	420	12600	1	14.0	29.5 – 30.5
31 – 34	16	33	528	17424	4	4.0	30.5 – 34.5
35 – 39	10	37	370	13690	5	2.0	34.5 – 39.5
40 – 59	10	49.9	495	24502.5	20	0.5	39.5 – 59.5
	$\sum 105$		$\sum fx = 2948$	$\sum fx^2 = 93036.5$			

(b)(i)	Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{2948}{105}$ $28.0762 seconds$	**M1** **A1**	For substitution For answer
(ii)	Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$ $= \sqrt{\frac{930365}{105} - \left(\frac{2948}{105}\right)^2}$ = 89.8462 seconds	**M1** **A1**	For substitution For answer

13	$f(x)=\begin{cases} kx & 0 \le x \le 8 \\ 8x & 8 \le x \le 9 \\ 0 & otherwise \end{cases}$		
(a)	$\int_0^8 kx dx + \int_8^9 8x dx = 1$ $k\left[\frac{x^2}{2}\right]_0^8 + 8k\left[\frac{x}{1}\right]_8^9 = 1$ $k\left[\frac{8^2}{2} - \frac{0^2}{2}\right] + 8k[9-8] = 1$ $40k = 1$ $k = \frac{1}{40}$	**M1** **M1** **M1** **A1**	For the method For integrating For substituting limits For k
(b)	For $0 \le x \le 8$ $f(x) = kx = \frac{x}{40}$ x: 0, 8 $f(x)$: 0, $^1/_5$ For $8 \le x \le 9$ x: 8, 9 $f(x)$: $^1/_5$, $^1/_5$  	**B1** **B1** **A2**	For the table For the table For the sketch of f(x)

(c)	$E(X) = \int_0^8 xkxdx + \int_8^9 x8kdx$ $= k\left[\frac{x^3}{3}\right]_0^8 + 8k\left[\frac{x^2}{2}\right]_8^9$ $= \left[\frac{8^3}{3} - \frac{0^3}{3}\right] + 8k\left[\frac{9^2}{2} - \frac{8^2}{2}\right]$ $= \frac{713k}{3}$ $= \frac{179}{30}$ $= 5.9667$	**M1** **M1** **M1** **A1**	For the method For integrating For substituting limits For $E(X)$
11(a)	$P(B) = 0.4, P\left(B^I/_C\right) = 0.64$ $0.24 + 0.07 + P = 0.4$ $P = 0.09$ $P\left(B^I/_C\right) = \frac{P(B^1nC)}{P(C)}$ $\frac{P(C) - P(BnC)}{P(C)}$ $0.64 = \frac{r}{0.09 + r}$ $r = 0.16$ A and B are independent $\rightarrow P(AnB) = P(A) \times P(B)$ $0.24 = (q + 0.16 + 0.24)0.4$ $q = 0.2$ $q + 0.16 + 0.24 + 0.07 + p + r + s = 1$ $0.2 + 0.16 + 0.24 + 0.07 + 0.09 + s = 1$ $s = 0.08$	**M1** **A1** **M1** **A1** **A1** **A1**	For the method For P For the method For r For q For s

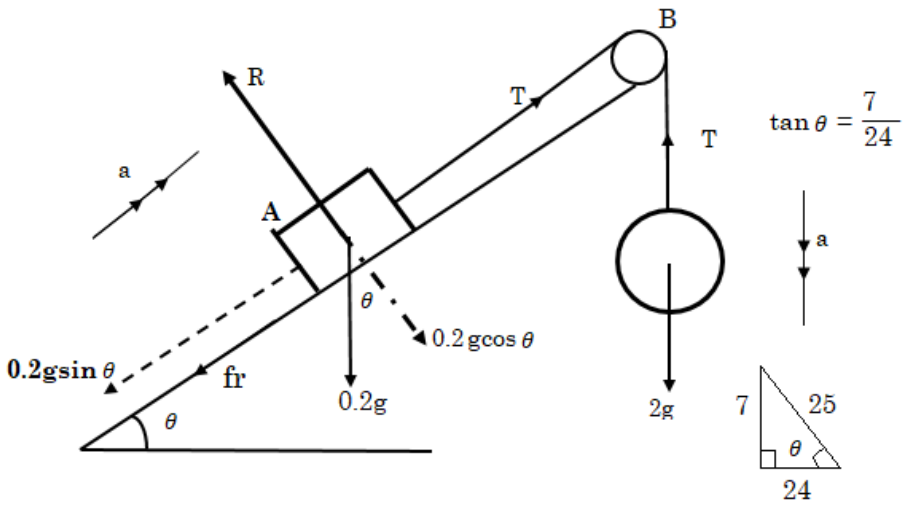
(b)	Let X be $r.v$ the life time of bulbs $X \sim N(8, 1.5^2)$		
(i)	$P(6 < X < 10) = P\left(\frac{6-8}{1.5} < Z < \frac{10-8}{1.5}\right)$ $= (-1.333 < Z < 1.333)$ $= 2P(0 < Z < 1.333)$ $= 2 \times 0.4087$ $= 0.8174$	**M1**	For standardizing
	 $= 0.4087 + 0.4087 = 0.8174$	**B1**	For the curve and adding probabilities
	$P(X > a) = {}^{90}/_{100} = 0.9$	**A1**	For the answer
	$P\left(Z > \frac{a-8}{1.5}\right) = 0.9$		
	$let\ Z_1 = \frac{a-8}{1.5}$		
(ii)	 $Z_1 = -1.282 = \frac{a-8}{1.5}$ $a = 6.077$ $\therefore$ *The life time exceeded by 90% of techno smartphones is 6.077hours*	**M1** **M1**	For the method For standardizing
		A1	For correct life time

14(a)	$let\ f(x) = X(^1/_2 - sinX)$ $f(^1/_2) = {}^1/_2\left(\frac{1}{2} - \sin\frac{1}{2}\right) = 0.0103$ $f(^3/_5) = {}^3/_5\left(\frac{1}{2} - \sin\frac{3}{5}\right) = -0.0388$ $since\ f(^1/_2) > 0\ and\ f(^3/_5) < 0, then\ there\ is\ a\ root\ between\ {}^1/_2\ and\ {}^3/_5$ $ALT: f\left(\frac{1}{2}\right).f\left(\frac{2}{5}\right) < 0\ then\ \frac{1}{2} < X_r < \frac{3}{5}$	**B1** **B1** **A1**	For $f(^1/_2)$ For $f(^3/_5)$ For conclusion
(b)	<table><tr><td>$^1/_2$</td><td>X_0</td><td>$^3/_5$</td></tr><tr><td>0.0103</td><td>0</td><td>−0.0388</td></tr></table> $\frac{X_0 - {}^3/_5}{0 - -0.0103} = \frac{{}^1/_2 - {}^3/_5}{0.0103 - -0.0388}$ $X_0 = 0.52098$ $X_0 \approx 0.521$	**B1** **M1** **A1**	For the table For the method For the answer
(c)	$f(x) = X(^1/_2 - sinX) = {}^X/_2 - XSinX$ $f^I(X) = {}^1/_2 - (XCosX + SinX) = {}^1/_2 - XCosX - SinX$ $X_{n+1} = X_n - \frac{f(X_n)}{f^I(X_n)}$ $X_{n+1} = X_n - \frac{\frac{X_n}{2} - X_nSinX_n}{^1/_2 - X_nCosX_n - SinX_n}; n = 0,1,2,....$ $n = 0 \qquad X_0 = 0.521$	**M1**	For $f^I(X)$

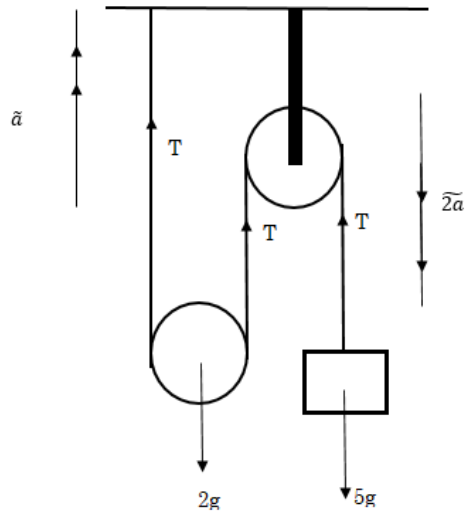
	$X_1 = 0.521 - \frac{\frac{0.521}{2} - 0.521Sin0.521}{1/2 - 0.521Cos0.521 - Sin0.521}, \lvert e\rvert = 0.0005$ $X_1 = 0.5236$ $e_1 \leq \lvert X_1 - X_0\rvert$ $\lvert X_1 - X_0\rvert = \lvert 0.5236 - 0.521\rvert$ $= 0.0026$ $n = 1$ $X_2 = 0.5236 - \frac{\frac{0.5236}{2} - 0.5236Sin0.5236}{1/2 - 0.5236Cos0.5236 - Sin0.5236}$ $X_2 = 0.5236$ $e_2 \leq \lvert X_2 - X_1\rvert$ $\lvert X_2 - X_1\rvert = \lvert 0.5236 - 0.5236\rvert$ $= 0.0000$ $since \lvert e\rvert = 0.0005 < 0.0000, the\ root\ X_r = 0.524$	**M1** **B1** **M1** **B1** **A1**	For substitution For X_1 For substitution For X_2 For the correct answer

15(a)	$h = \frac{2-1}{5}$ $= \frac{1}{5}$ $= 0.2$	**B1**	For h
		B1	For y_0, y_5 *and the sum*
		B1	For y_1, y_2, y_3, y_4 and the sum
	$\int_1^2 \frac{x}{7x^2-3}dx = {}^1/_2\, h\,[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$	**M1**	For the substitution
	$= {}^1/_2 \times 0.2[0.33 + 2(0.49876)]$ $= 0.132752$ ≈ 0.133	**A1**	For the answer
	$\int_1^2 \frac{x}{7x^2-3}dx = \frac{1}{14} In[7x^2-2]_1^2$	**M1M1**	For integration
	$= \frac{1}{14}\, In\,(7(2)^2-2) - \frac{1}{14}\, In\,(7(1)^2-2)$	**M1**	For substituting limits
	$= 0.11776133$ ≈ 0.118	**A1**	For correct answer
(ii)	$\%error = \frac{\lvert error \rvert}{\lvert exact\ value \rvert} \times 100$	**M1**	For substitution

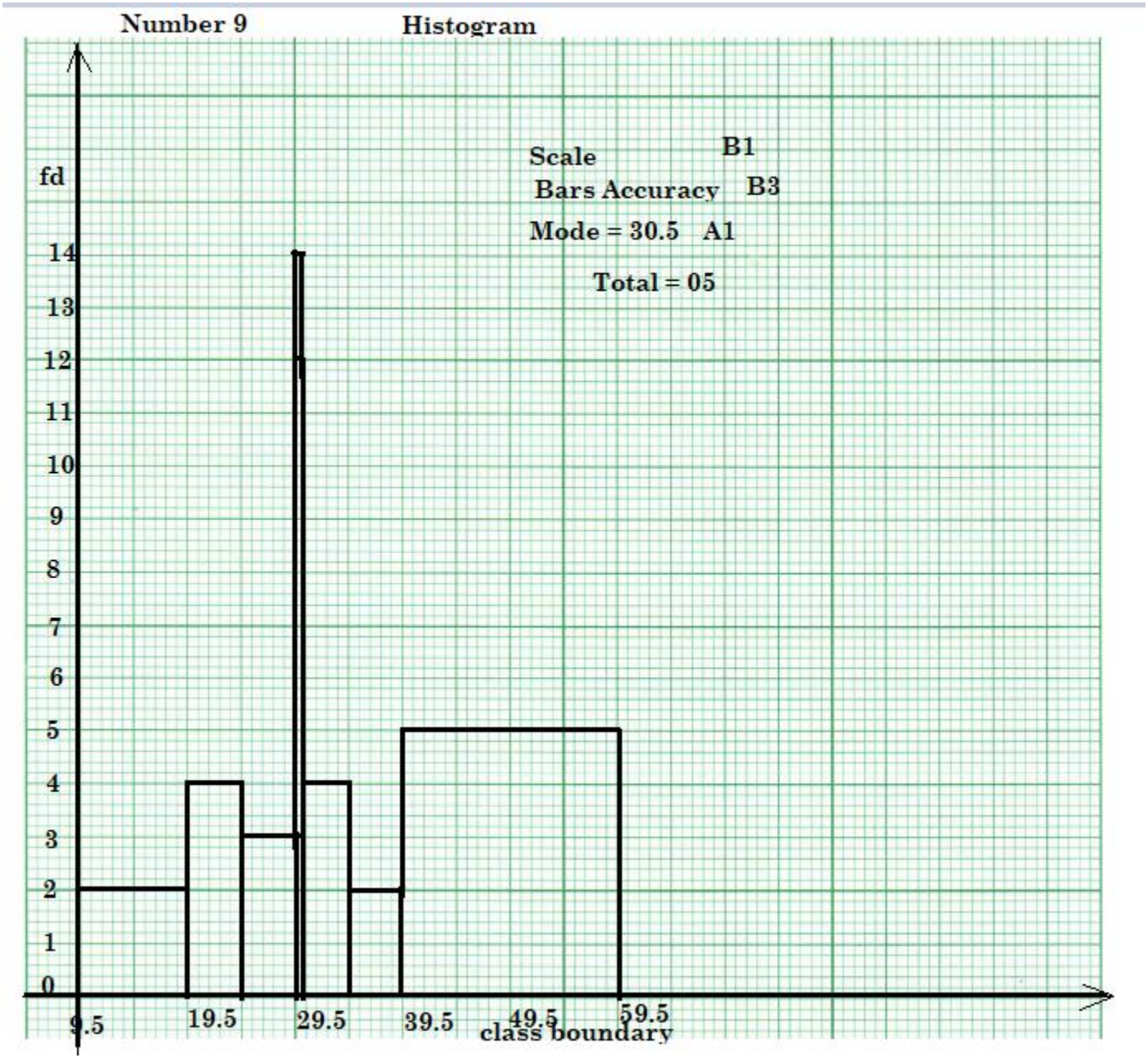
(iii)	$= \frac{\lvert 0.118 - 0.133 \rvert}{\lvert 0.118 \rvert} \times 100$ 12.7% *By increasing the number of sub − intervals*	**A1** **A1**	For correct answer For the answer
12(a)	*for P* $S_1 = \begin{pmatrix}1\\3\end{pmatrix} km$ $V_1 = \begin{pmatrix}1\\2\end{pmatrix} km/hr$ *for q* $S_2 = \begin{pmatrix}1\\2\end{pmatrix} km$ $V_2 = \begin{pmatrix}5\\6\end{pmatrix} km/hr$ $pVq = V_1 - V_2$ $= \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}5\\6\end{pmatrix}$ $= \begin{pmatrix}-4\\-4\end{pmatrix} km/hr$ $p\boldsymbol{r}q = [S_1 - S_2] + PVqt$ $= \begin{pmatrix}1\\3\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}-4\\-4\end{pmatrix}t$ $= \begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}-4\\-4\end{pmatrix} t = \begin{pmatrix}-4t\\1-4t\end{pmatrix}$ For shortest distance $p\boldsymbol{r}q \,.\, PVq = 0$	**B1** **M1** **B1** **M1**	For pVq For working out $p\boldsymbol{r}q$ For $p\boldsymbol{r}q$ For $p\boldsymbol{r}q.PVq$

	$\begin{pmatrix} -4t \\ 1-4t \end{pmatrix}\begin{pmatrix} -4 \\ -4 \end{pmatrix} = 0$ $16t - 4 + 16t = 0$ $32t = 4$ $t = 0.125hrs$	**M1** **A1**	For working out the dot product For t
(b)	Shortest distance = $\lvert \mathbf{pr_q} \rvert$ $\mathbf{pr_q}(0.125) = \begin{pmatrix} -4\times0.125 \\ 1-4\times0.125 \end{pmatrix}$ $= \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$ $\lvert \mathbf{pr_q} \rvert = \sqrt{(-0.5)^2 + (0.5)^2}$ $= \sqrt{0.5}$ $0.7071km$	**B1** **M1** **A1**	For $\mathbf{pr_q}$ For working out $\lvert \mathbf{pr_q} \rvert$ For correct answer
10(a)			

	$for\ the\ 0.2kg\ mass$ $T-(0.2gSin\theta+\mu R)=0.2a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$ $T-(0.2gSin\theta+\mu.0.2gCos\theta)=0.2a$ $T-0.2g\left(\frac{7}{25}\right)-\mu.0.2g\left(\frac{24}{25}\right)=0.2\left(\frac{543g}{625}\right)$ $T-\frac{24\mu}{12.5}=\frac{718g}{3125}$ $Sin\theta=\frac{7}{25}$ $Cos\theta=\frac{24}{25}$ $for\ the\ 2kg\ mass$ $2g-T=2a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$ $T=2g-2a$ $solving\ 1\ and\ 2$ $2g-2a-\frac{24}{125}\mu=\frac{718}{3125}g$ $2g-2\left(\frac{543g}{625}\right)-\frac{24}{125}\mu=\frac{718}{3125}g$ $\mu=0.17g$	**M1** **B1** **B1** **M1** **M1** **A1**	For equation 1 For $Sin\theta$ and $Cos\theta$ For substitution $of\ R = 0.2gCos\theta$ For equation 1 For solving 1 and 2 For μ

(b)	When the string breaks $0 - (0.2gSin\theta + \mu R) = 0.2a$ $a = \dfrac{-(0.2g \times \frac{7}{25} + 0.17g \times 0.2g \times \frac{24}{25}}{0.2}$ $a = -18.4177ms^{-2}$ $v^2 = u^2 + 2aS$ $0^2 = 0.5^2 + 2(-18.4177)S$ $S = 0.0068m$	**M1** **B1** **M1** **A1**	For the method For a For substitution For correct value of distance
16(i)	 For the $5kg$ mass $5g - T = 5(2a)$ $5g - T = 10a$ ……..1	**B2** **M1**	For the system well drawn

	For the $2kg$ mass $2T - 2g = 2a \ldots\ldots\ldots\ldots\ldots\ldots 2$ $10g - 2T = 20a$ $+ \quad 2T - 2g = 2a$ $8g = 22a$ $a = 3.5636ms^{-2}\ for\ the\ 2kg\ mass$ For the $5kg$, acceleration = 2×3.5636 $= 7.1272ms^{-2}$	**M1** **B1** **A1** **M1** **A1**	For equation 1 For equation 2 For solving 1 and 2 For a For substitution For 2a
(ii)	Using equation 1 $5g - T = 10(3.5636)$ $T = 13.364N$	**M1** **A1**	For substitution For T
(iii)	$S = ut + \frac{1}{2}at^2$ $t = 1.5$ $S = 0(t) + \frac{1}{2}at^2$ $S = 0(1.5) + \frac{1}{2}(3.5636)1.5^2$ $S = 4.00905m$	**M1** **A1**	For substitution For correct answer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Histogram
Scale
B1
Bars Accuracy
B3
Mode = 30.5
A1
Total = 05
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13
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11
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9
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9.5
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29.5
39.5
49.5
59.5
class boundary