P425/2
APPLIED MATHEMATICS
Paper 2
Nov. /Dec. 2022
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms^{-2}.

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SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. A rough plane is inclined at an angle of $\sin^{-1}(3/5)$ to the horizontal. A car of mass 1500 kg moves up the plane with a steady speed. The coefficient of friction between the car and the plane is $1/4$. Calculate the driving force exerted by the engine of the car. *(05 marks)*

2. The table below shows the number of people in each of the 100 cars that passed through a check point on a certain day.

Number of people	1	2	3	4	5
Number of cars	41	33	18	6	2

Find:

(a) the mean number of people in each car. *(02 marks)*

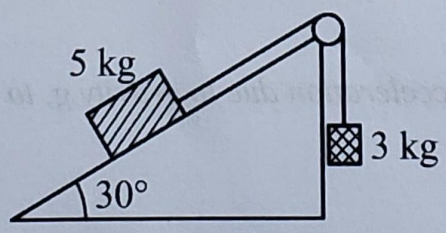
(b) the variance of the data. *(03 marks)*

3. Use the trapezium rule with **seven** (7) ordinates to estimate

$$\int_0^2 \frac{1}{3+4x^2}\,dx$$

correct to **three** decimal places. *(05 marks)*

4. The diagram below shows a mass of 5 kg lying on a rough plane inclined at 30° to the horizontal. From the 5 kg mass, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley carrying a mass of 3 kg hanging freely. The tension in the string is just sufficient to prevent the 5 kg mass from sliding down the slope.



Determine the coefficient of friction between the 5 kg mass and the rough plane. *(05 marks)*

5. Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(A' \cap B) = \frac{1}{2}$. Find $P(B' \cap A)$. *(05 marks)*

6. The table below shows the amount of Euros equivalent to US dollars.

US dollars	79	97	105
Euros	64	78	85

Using linear interpolation/extrapolation, calculate the amount of:

(a) dollars equivalent to 92 Euros. *(03 marks)*

(b) Euros equivalent to 85 dollars. *(02 marks)*

7. A river which is 50 m wide flows with a speed of 3 ms^{-1}. A boat must be set to cross the river to a point directly opposite the starting point. The speed of the boat relative to the river is 4 ms^{-1}. Determine the;

(a) velocity with which the boat sets off to cross the river. *(03 marks)*

(b) time taken by the boat to cross the river. *(02 marks)*

8. Bag A contains 7 red balls and 4 blue balls. Bag B contains 5 red balls and 8 blue balls. A ball is randomly transferred from A to B. A ball is then randomly removed from bag B. Find the probability that:

(a) a red ball was removed from bag B. *(03 marks)*

(b) the transferred ball is blue given that a red ball is removed from B. *(02 marks)*

SECTION B (60 MARKS)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

9. The table below shows the marks of eight students (A to H) in two tests.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Test 1	67	73	50	75	42	44	75	40
Test 2	82	70	64	78	52	64	80	64

(a) (i) Plot a scatter diagram for the given data.

(ii) Draw a line of best fit on the scatter diagram.

(iii) Use your scatter diagram to obtain a mark in **Test 2** which corresponds to a mark of 63 in **Test 1**. *(06 marks)*

(b) (i) Calculate the rank correlation coefficient for the data.

(ii) Comment on your result in b (i). *(06 marks)*

10. A particle of mass 4 kg is initially at rest at the point $A\ (2, -2, 3)$ metres. The particle is acted upon by a force of $\boldsymbol{F} = \begin{pmatrix} 4t \\ t^2 \\ 5 \end{pmatrix}$ Newtons, where t is time in seconds. Find the;

(a) acceleration at time t. *(02 marks)*

(b) speed of the particle after 3 seconds. *(05 marks)*

(c) displacement of the particle after 3 seconds. *(05 marks)*

11. (a) Two numbers x and y are approximated by X and Y with errors Δx and Δy respectively. Show that the relative error in approximating $\frac{x}{y}$ by $\frac{X}{Y}$ is $\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$ *(06 marks)*

(b) The values in the expression $T = \frac{673.16}{40.345}$ are rounded off correct to the given decimal places. Determine the interval within which the exact value of T can be expected to lie. *(06 marks)*

12. A discrete random variable X has a probability distribution given by

$$f(x) = \begin{cases} kx^2, & x = 1, 2, 3 \\ k(7-x)^2, & x = 4, 5, 6 \\ 0, & \textit{elsewhere} \end{cases}$$

Find the:

(a) (i) value of the constant k.

(ii) mean of X.

(iii) variance of X. *(09 marks)*

(b) (i) Determine the cumulative distribution function, $F(x)$. *(01 mark)*

(ii) Sketch the graph of $F(x)$. *(02 marks)*

13. Forces of magnitude 2N, 2N, 3N, 4N, $2\sqrt{2}$ N and $\sqrt{2}$ N act along sides $\overrightarrow{AB}$, $\overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{AD}, \overrightarrow{AC}$ and $\overrightarrow{DB}$ respectively. Where $ABCD$ is a square of side 2 m. Find the:

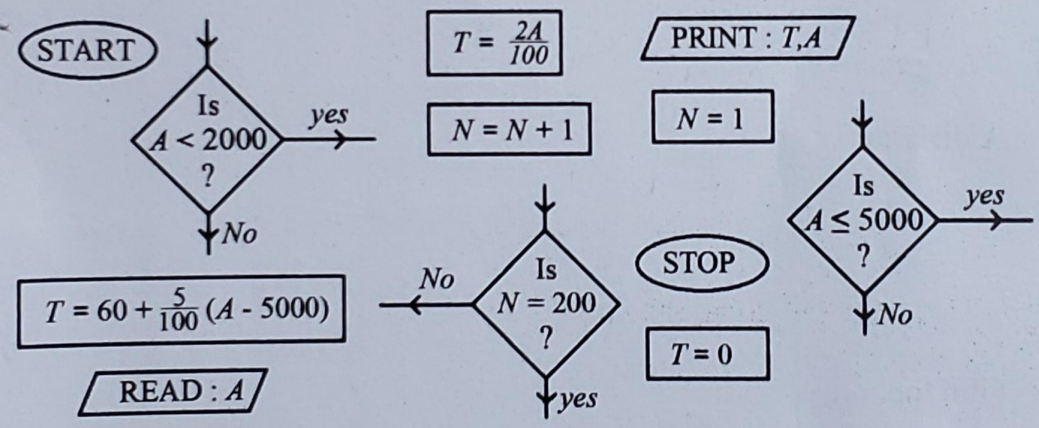
(a) resultant force. *(09 marks)*

(b) equation of the line of action of the resultant force. *(03 marks)*

14. The taxation system in a company is such that the tax T based on the amount A earned falls in three categories as shown in the table below:

AMOUNT EARNED A (DOLLARS)	**TAX PAID T (DOLLARS)**
$A < 2000$	$T = 0$
$2000 \leq A \leq 5000$	$T = \frac{2A}{100}$
$A > 5000$	$T = 60 + \frac{5}{100}(A\text{-}5000)$

Parts of a flow chart for computing the tax are given below.



(a) (i) Arrange the given parts to form a complete logical flow chart. *(08 marks)*

(ii) State the purpose of the flow chart. *(01 mark)*

(b) Perform a dry run for the flow chart and complete the table below: *(03 marks)*

N	A	T
1	1500	–
2	3500	–
3	9000	–

15. The masses of packets of tea leaves are normally distributed with mean mass 600 g and standard deviation 20 g.

(a) If a packet of tea leaves is chosen at random, find the mass exceeded by 7% of the packets. *(05 marks)*

(b) If 1,000 packets of tea leaves were sold, find the number of packets that weighed less than 545 g. *(07 marks)*

16. Two points A and B are on the same horizontal ground. A particle P is projected from point A towards point B with an initial speed of 49 ms^{-1} at an angle of 30^{o} to the horizontal. Two seconds later, another particle Q is projected from point B towards A with the same speed and at the same angle of elevation as P.

If the particles collide at a height h m above the ground, determine the:

(a) value of h. *(06 marks)*

(b) distance between the points A and B. *(06 marks)*

END