P425/1
PURE
MATHEMATICS
Paper 1
August, 2022
3 hrs

UNNASE
Community

UNNASE MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in Section A and any **five** from section B.*

Additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1. If $p^x = q^y = r^z$ and $p^3 = q^2r$, show that $z = \frac{xy}{3y-2x}$ (05 marks)

2. A rectangle is twice as long as it is broad. Find the rate of change of perimeter when the width is $1\ m$ and the area of the rectangle is changing at a rate of $18\ cm^2s^{-1}$. (05 marks)

3. If $2\ sin\theta + cos\theta = 1$, use $t - formulae$ to find the values of θ in the interval $0^o \leq \theta \leq 180^o$. (05 marks)

4. Convert the following into polar form;
   (i) the coordinates $(1, -1)$.
   (ii) equation $x^2 + y^2 = 4x$. (05 marks)

5. Evaluate $\frac{d}{dt}\left[\sqrt[t]{(2t+1)}\right]$ when $t = 2$ correct to 4 significant figures. (05 marks)

6. The points $A, B\ and\ C$ have position vectors $2\,\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$, $3\,\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ and $-\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ respectively. Determine the size of the angle $A\hat{B}C$. (05 marks)

7. When the expression $x^5 + 2x^2 + ax + b$ is divided by $x^2 - 4$, the remainder is $3x + 1$. Find the values of a and b. (05 marks)

8. Use the substitution $u = 2^x$ to evaluate; $\int_0^3 \frac{2^x}{\sqrt{2^x+1}}\,dx$. (05marks)

SECTION B (60 MARKS)

9. a) Use partial fractions to find; $\int \frac{x^3}{x^2-1}\,dx$ (07 marks)
   b) Find the volume of the solid generated when the region in the first quadrant bounded by the curve $y^2 - x + 1 = 0$ and the lines $x = 2$, $y = 0$ is revolved about the $y - axis$. (05 marks)

10. a) Show that the line $3x + 4y = 13$ is a tangent to the circle $x^2 + y^2 - 2x - 3 = 0$
    b) (i) Find the equation of the tangent to the curve $4ay = x^2$ at the point $P(2at, at^2)$.
    (ii) The tangent in (i) above meets the $x - axis$ at point Q. Given that S is the point $(0, a)$, prove that PQ is perpendicular to SQ.

11. a) Given the complex number $z = \frac{(1+2i)^2}{7-i}$.

(i) Express z in the form $a + ib$ and represent it on the argand diagram.

(ii) Determine the modulus and argument of z and hence write z in modulus- argument form. (07 marks)

b) The complex number $z = i + 2$ is a root of the polynomial $z^4 - 6z^3 + 16z^2 - 22z + 15 = 0$. Find the remaining roots. (05 marks)

12. a) Show that; $\frac{sin2A+\cos 2A+1}{sin2A+\cos 2A-1} = \frac{tan(45^o+A)}{\tan A}$. (05marks)

b) Find all values of x in the range 0^o to 360^o which satisfy the equation $sin3xsinx = 2cos2x + 1$ (07 marks)

13. a) Differentiate $tan^3 4x$ with respect to x. (03 marks)

b) A curve is defined parametrically by the equations; $x = 2t + \frac{1}{2}t^2$, $y = \frac{1}{3}t^2 - 3t$. Find;

(i) $\frac{d^2y}{dx^2}$

(ii) the coordinates of the points of inflexion of the curve. (09 marks)

14. The points A, B and C on a plane have position vectors $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{c} = 6\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ respectively. Find the;

(i) vector equation of the line through points B and C.

(ii) position vector of point R on line BC such that $\overline{AR}$ is perpendicular to $\overline{BC}$.

(iii) Coordinates of the point, P which divides AB externally in the ratio $1 : 2$.

15. a) Find the sum of all the terms of the series;

$5 + 10 + 20 + 40 + \cdots + 20480$ (06 marks)

b) Solve for n given that $\binom{n}{1} + \binom{n}{2} = 21$ (06 marks)

16. a) By use of the substitution $y = vx$, solve the differential equation $(2y - x)\frac{dy}{dx} = y + 2x$ (06 marks)

b) The acceleration of a particle after t seconds is given by $5 + cos\frac{1}{2}t$. Initially the particle is moving at $1\ ms^{-1}$. Find the velocity of the particle after 2π seconds. (06 marks)

END