P425/1
Pure Mathematics
July/ August 2023
3 hours

UNNASE MOCK EXAMINATIONS

UGANDA ADVANCED CERTIFICATE OF EDUCATION
MOCK EXAMINATIONS 2023
Pure Mathematics
PAPER 1
3 hours

INSTRUCTIONS

- *Attempt all the **eight** questions in section **A** and any five questions from section **B**.*
- *All working must be shown clearly*
- *Mathematical tables with list of formulae and squared paper are provided.*
- *Silent non- programmable scientific calculators may be used.*

SECTION A

1. The second term of a geometric series is -12 and its sum to infinity is 16. Find the common ratio (5 marks)

2. Evaluate $\int_2^4 \frac{(x+1)^2}{x^2}\, dx$ (5 marks)

3. The line $4y = x + c$ is a tangent to the curve $y^2 = x + 3$ at point P. find the;
   (i) value of constant c
   (ii) coordinates of P (5 marks)

4. A plane, P containing the point with position vector $\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ is parallel to plane Q whose equation is $2x - 3y + 6z = 16$. Find the:
   (i) Cartesian equation of plane Q
   (ii) Perpendicular distance between planes P and Q. (5 marks)

5. The gradient of the normal to the curve at the point $P(x, y)$ is $\frac{x}{x+1}$. If the curve passes through the point$(1, 4)$. Find the equation of the curve. (5 marks)

6. Find all the solutions of the equation $4cosx - 6sinx = 5$ which lie between 0^o and 360^0 (5 marks)

7. (i) Express the complex number $z = \frac{1}{i-\sqrt{3}}$ in the form $x + iy$.

   (ii) Represent z in the argand diagram hence find argument of z. (5 marks)

8. Given that $y = \frac{\sqrt{x^2+1}}{(2x-1)^3}$, show that $\frac{dy}{dx} = -\frac{2x^2+x+4}{(2x-1)^3\sqrt{x^2+1}}$ (5 marks)

SECTION B

9. A line, L has equation $\boldsymbol{r} = 5\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k} + \mu(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ and a plane, P has equation $(\boldsymbol{r} - \boldsymbol{i} - 2\boldsymbol{j}) \cdot (3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) = 0$. The line L intersects the plane P at point A. Find the;
   (i) position vector of A
   (ii) acute angle between L and P.
   (iii) equation of the line which lies in P and intersects L at right angles. (12 marks)

10. (a) Given that $y = xcos2x$. Find $\frac{dy}{dx}$ and hence evaluate $\int xsin2x\, dx$. (6 marks)

    (b) By use of the substitution $x = 2\cos\theta$, evaluate $\int_1^2 \frac{1}{x^2\sqrt{4-x^2}}\, dx$ (6 marks)

11. Express $(x) = \frac{x(6-x)}{(2+x)(4+x^2)}$. Hence obtain the expression of $f(x)$ in ascending powers up to x^2. (12 marks)

12. Sketch the curve $y = \frac{3x-9}{x^2-x-2}$. (12 marks)

13. (a) Eliminate A from the equations;

$cosA + sinA = a$ and $cos2A = ab$ (5 marks)

(b) Given that $tan\theta\ tan(\theta + \alpha) = k$, show that $cos(2\theta + \alpha) = \frac{1-k}{1+k}\ cos\alpha$. Hence solve the equation $tan\theta\ tan\left(\theta + \frac{\pi}{3}\right) = 2$ (7 marks)

14. The tangent at $P(3t^2, 2t^3)$ on the curve $4x^3 = 27y^2$ meets the x-axis Q. If the normal at P meets the y- axis at R, find the locus of the mid- point of QR. (12 marks)

15. (a) Solve the simultaneous equations

$$d - 2e + 3f = 4$$
$$5d + 6e - 7f = 8$$
$$7d - 5e + 6f = 4$$

(5 marks)

(b)The polynomial $f(x) = x^3 + ax^2 + bx - 24$ is divisible by $x - 2$ and leaves a remainder of -20 when divided by $x - 1$.

(i) Find the values of a and b.
(ii) Solve $f(x) = 0$ (7 marks)

16. (i) Find the solution of the differential question $\frac{dy}{dx} + 5\,y = e^{-7x}$ for which $y = 0$ when $x = 0$ (5 marks)

(ii)A liquid is being heated in an oven maintained at a constant temperature of $180^o\ C$. Given that the rate of increase of temperature is proportional the positive difference between temperature of the oven and that of the liquid. If the temperature of the liquid rises from $0^oC\ to\ 120^oC$ in 5 minutes, find the temperature of the liquid after a further 5 minutes. (7 minutes)

END