

SUBMTC/1 CUMDE

SECTION A 40 marks

$$3x^2 - px - 1 = 0$$

$$x^2 - \frac{p}{3}x - \frac{1}{3} = 0$$

$$(\alpha + \beta) = \frac{p}{3} \quad \text{--- (1)}$$

$$\alpha\beta = -\frac{1}{3} \quad \text{--- (ii)}$$

$$(\alpha + \beta)^2 = \frac{p^2}{9}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{p^2}{9}$$

$$\frac{10}{9} - \frac{2}{3} = \frac{p^2}{9}$$

$$\frac{4}{9} = \frac{p^2}{9}$$

$$p = \frac{12}{9} = \frac{4}{3}$$

M1

M1 both
errors

M1

M1

A7

(05)

2.

$$5+c, 7+c, 11+c.$$

$$\frac{7+c}{5+c} = \frac{11+c}{7+c}$$

$$(7+c)(7+c) = (5+c)(11+c)$$

$$49 + 14c + c^2 = 55 + 16c + c^2$$

$$-2c = 6$$

$$c = -3.$$

$$\text{Common ratio} = \frac{7-3}{5-3} = \frac{4}{2} = 2.$$

M1

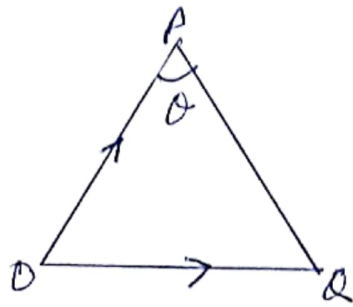
M1

A7

M1 A7

(05)

3



$$\vec{PO} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$$

$$|\vec{PO}| = \sqrt{(-5)^2 + 12^2} = 13$$

$$\vec{PQ} = \begin{pmatrix} -6 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} -11 \\ 20 \end{pmatrix}$$

$$|\vec{PQ}| = \sqrt{(-11)^2 + 20^2} = \sqrt{521}$$

$$\begin{pmatrix} -5 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} -11 \\ 20 \end{pmatrix} = 13\sqrt{521} \cos \theta$$

$$295 = 13\sqrt{521} \cos \theta$$

$$\cos \theta = \frac{295}{13\sqrt{521}}$$

$$\theta = 6.2^\circ$$

M₁M₁M₁M₁A₁

(05)

4.

SOLOMON

arrangement without any
conditions

$$\frac{7!}{3!} = 840$$

when the 30's are together

$$A \uparrow S \uparrow L \uparrow M \uparrow N \uparrow = 5! = 120$$

M₁B₁M₁

$$\begin{aligned}
 3 \text{ O's not together} &= \frac{7!}{3!} - 5! \\
 &= 840 - 120 \\
 &= 720
 \end{aligned}$$

-3-

M₁

A₇
(05)

5

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) \\
 &= \frac{3}{5} + \frac{1}{4} \\
 &= \frac{17}{20} \\
 P(A \cap B) &= \frac{3}{5} + \frac{1}{4} - \frac{3 \cdot 1}{5 \cdot 4} \\
 &= \frac{3}{5} + \frac{1}{4} - \frac{3}{20} \\
 &= \frac{7}{10}
 \end{aligned}$$

M₁

A₇

B₁ (independ)

M₁

A₇

(05)

6.

	A	B	C	D	E	F
	3	1	4	2	5	6
	4	6	3	5	2	1
d	-1	-5	1	-3	3	5
d ²	1	25	1	9	9	25
Σd^2	= 70.					

$$\rho = 1 - \frac{6 \times 70}{6(6^2 - 1)} = -1$$

Judges had nothing in common.

A₇

B₇

M₁

A₇

B₁

(05)

7.

F	W	S
150,000	108,000	120,000

$$\text{Index for F} = \frac{150,000}{120,000} \times 100 = 125$$

$$W = \frac{108,000}{120,000} \times 100 = 90$$

$$S = \frac{120,000}{120,000} \times 100 = 100$$

Index	125	90	100
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Weight	2	2	1
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$$\text{Weighted price index} = \frac{(125 \times 2) + (90 \times 2) + (100 \times 1)}{2 + 2 + 1}$$

$$= 106$$

Cost of living index increased by 6%

B₂ for all price indices

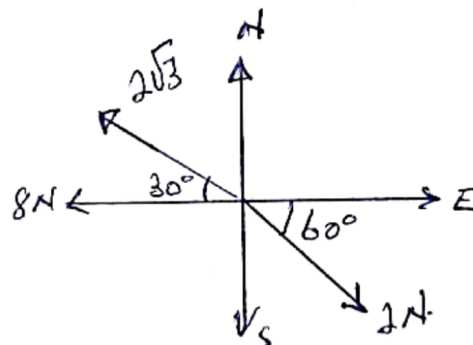
B₁

A₁

(OS)

B₁

8.



$$\rightarrow -8 - 2\sqrt{3} \cos 30^\circ + 2 \cos 60^\circ = -10N \text{ MI}$$

$$\uparrow 2\sqrt{3} \sin 30^\circ - 2 \sin 60^\circ = 0 \text{ MI}$$

$$R = \sqrt{(-10)^2 + 0^2} = 10N$$

M₁ A₁ (OS)

-9-

f

11

10

9

8

7

6

5

4

3

2

1

0

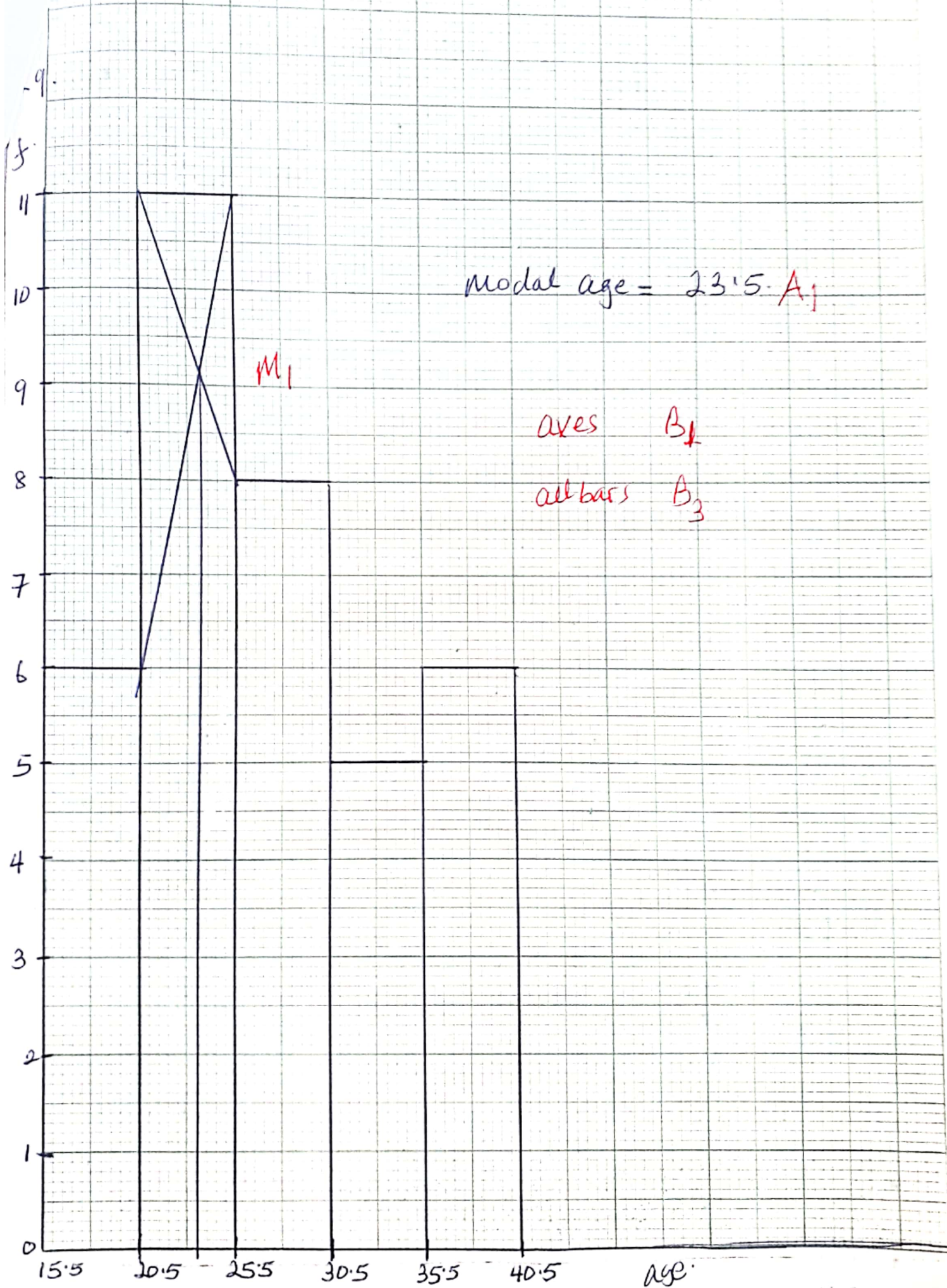
modal age = 23.5 A_1

M_1

aves B_1

all bars B_3

15.5 20.5 25.5 30.5 35.5 40.5 age



9.

class.	f	x	fx	fx ²
16-20	6	18	108	1944
21-25	11	23	253	5819
26-30	8	28	224	6272
31-35	5	33	165	5445
36-40	6	38	228	8664

$$B_1 \quad \Sigma f = 36 \quad B_1 \quad \Sigma fx = 978 \quad B_1 \quad \Sigma fx^2 = 28144 \quad B_1$$

$$\bar{x} = \frac{978}{36} \quad M_1$$

$$= 27.2 \quad A_1$$

$$\sigma = \sqrt{\frac{28144}{36} - \left(\frac{978}{36}\right)^2} \quad M_1 \quad M_1$$

$$= 6.6.$$

A₁

10

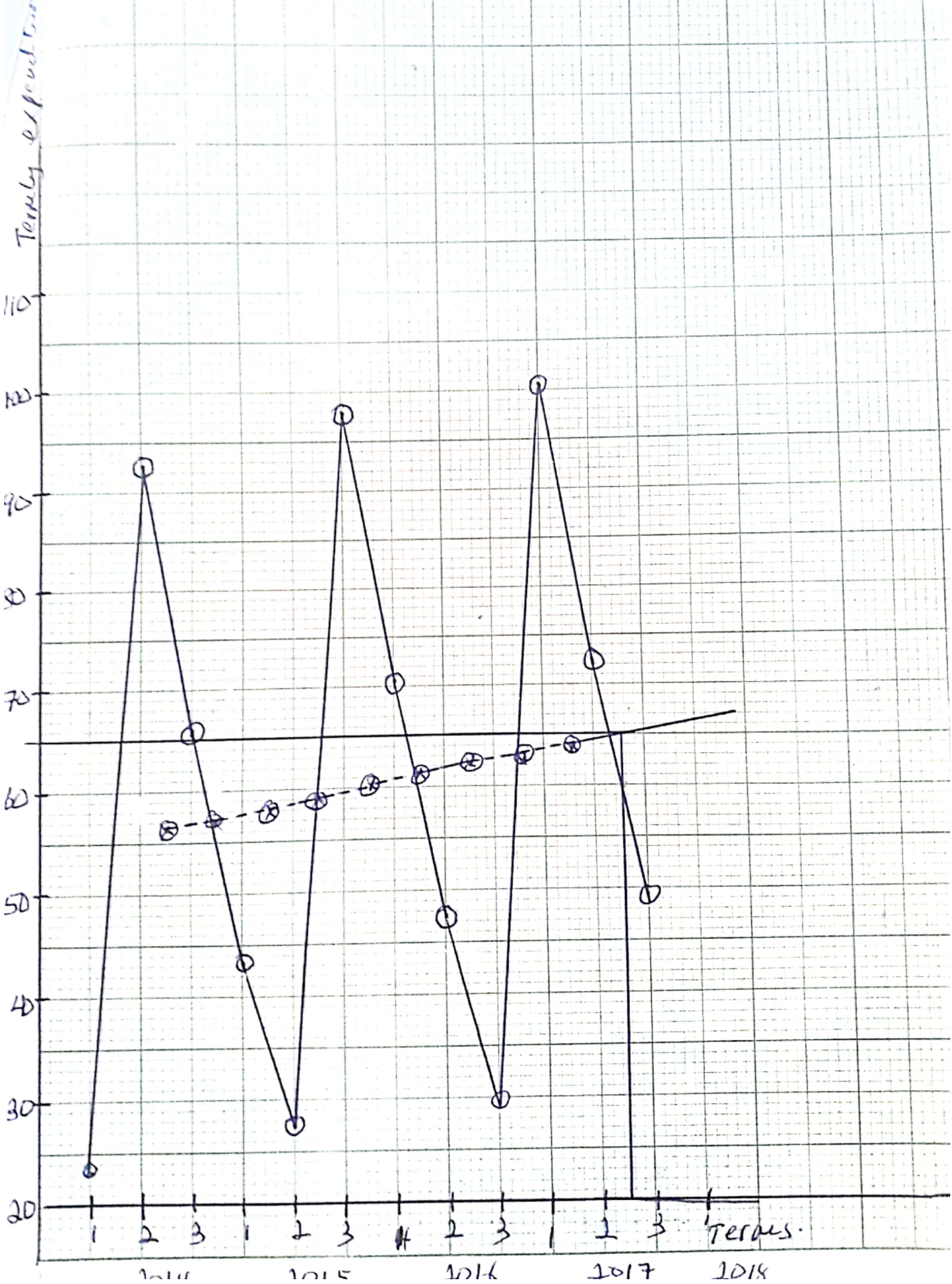
- 7 -

Year	Term	OD	MT	MA
2014	1	23.4		
	2	92.6	224.4	56.1
	3	65.3	228.5	57.125
2015	1	43.1	233.7	58.425
	2	27.5	238.8	59.7
	3	97.8	242.9	60.725
2016	1	70.4	245	61.25
	2	47.2	247.5	61.875
	3	29.6	249.9	62.475
2017	1	100.3	252.5	63.125
	2	72.8		
	3	49.8		

2018 1 X

$$\frac{X + 49.8 + 72.8 + 100.3}{4} = 65$$

$$X = 37.1$$



11 (a) $X \sim N(20, 25) \Rightarrow \mu = 20 \text{ and } \sigma = 5$

(i) $P(X < 24.3) = P\left(\frac{24.3 - 20}{5}\right)$

$= P(Z < 0.86)$

$= 0.5 + \Phi(0.86)$

$= 0.8051$

(ii) $P(X < 18.5) = P\left(Z < \frac{18.5 - 20}{5}\right)$

$= P(Z < -0.3)$

$= 0.5 - \Phi(-0.3)$

$= 0.3821$

(iii) $P(14.4 < X < 25.9) = P\left(\frac{14.4 - 20}{5} < Z < \frac{25.9 - 20}{5}\right)$

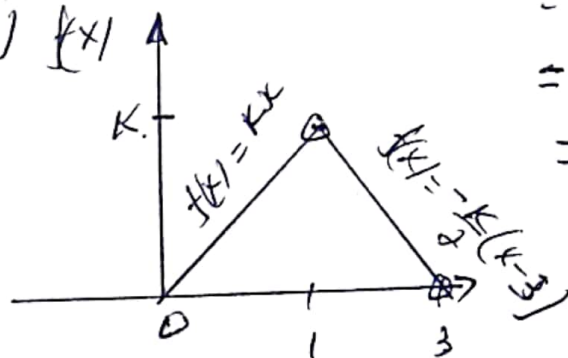
$= P(-1.12 < Z < 1.18)$

$= \Phi(-1.12) + \Phi(1.18)$

$= 0.3686 + 0.3810$

$= 0.7496$

(b)(i)



$\frac{1}{2} \times K \times 3 = 1$

$K = \frac{2}{3}$

$$\begin{aligned}
 \text{(ii)} \quad P(X > \frac{1}{2}) &= \frac{2}{3} \int_1^1 x dx + \int_1^3 -\frac{1}{3}(x-3) dx \\
 &= \frac{2}{3} \left[\frac{x^2}{2} \right]_{\frac{1}{2}}^1 + \left[-\frac{1}{3} \left(\frac{x^2}{2} - 3x \right) \right]_1^3 \\
 &= \frac{2}{3} \left(\frac{1}{2} - \frac{1}{8} \right) + -\frac{1}{3} \left[\left(\frac{9}{2} - 9 \right) - \left(\frac{1}{2} - 3 \right) \right] \\
 &= \frac{1}{4} + \frac{2}{3} \\
 &= \frac{11}{12}
 \end{aligned}$$

M1

M1

A1

(15)

$$12 \text{ (a) } A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 9 & 6 & 2 \\ 0 & 7 & -6 \\ 0 & -9 & 10 \end{pmatrix}$$

M1

$$A^3 = \begin{pmatrix} 9 & 6 & 2 \\ 0 & 7 & -6 \\ 0 & -9 & 10 \end{pmatrix} \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 27 & 0 & 26 \\ 0 & -25 & 26 \\ 0 & 39 & -38 \end{pmatrix}$$

M1 A1

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A1

$$12 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 13 \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix} - \begin{pmatrix} 17 & 0 & 26 \\ 0 & -25 & 26 \\ 0 & 39 & -38 \end{pmatrix} \quad \text{M1}$$

$$\begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix} = \begin{pmatrix} 39 & 0 & 26 \\ 0 & -13 & 26 \\ 0 & 39 & -38 \end{pmatrix} - \begin{pmatrix} 27 & 0 & 26 \\ 0 & -25 & 26 \\ 0 & 39 & -38 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix} \quad \text{M1}$$

$$\therefore \text{L.H.S} = \text{R.H.S.} \quad \text{B1}$$

(b)

$$15x - 6y = 3$$

$$6x + 9y = 24$$

$$\begin{pmatrix} 15 & -6 \\ 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 24 \end{pmatrix} \quad \text{M1}$$

$$\begin{pmatrix} 9 & 6 \\ -6 & 15 \end{pmatrix} \begin{pmatrix} 15 & -6 \\ 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 & 6 \\ -6 & 15 \end{pmatrix} \begin{pmatrix} 3 \\ 24 \end{pmatrix} \quad \text{M1}$$

$$\begin{pmatrix} 171 & 0 \\ 0 & 171 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 171 \\ 342 \end{pmatrix}$$

$$171x = 171$$

$$x = 1$$

$$171y = 342$$

$$y = 2.$$

13 (a) $\frac{dy}{dx} = 3 - 2x$

$$\int dy = \int 3 - 2x \, dx$$

$$y = 3x - x^2 + C$$

$$4 = 0 - 0 + C$$

$$\therefore C = 4$$

$$y = 3x - x^2 + 4$$

for turning point $\frac{dy}{dx} = 0$

$$\therefore 3 - 2x = 0$$

$$x = \frac{3}{2}$$

$$y = 3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2 + 4$$

$$= \frac{25}{4}$$

M1

M1

A1

M1

A1

(15)

M1

M1

A1

A1

M1

A1

M1

A1

$$\frac{dy}{dx} = 3 - 2x$$

-13-

$$\frac{d^2y}{dx^2} < 0 \Rightarrow \left(\frac{3}{2}, \frac{25}{4}\right)_{\text{max.}}$$

(b)

x-intercept is given by $y=0$

$$\therefore 3x - x^2 + 4 = 0$$

$$x^2 - 3x - 4 = 0$$

$$x^2 - 4x + x - 4 = 0$$

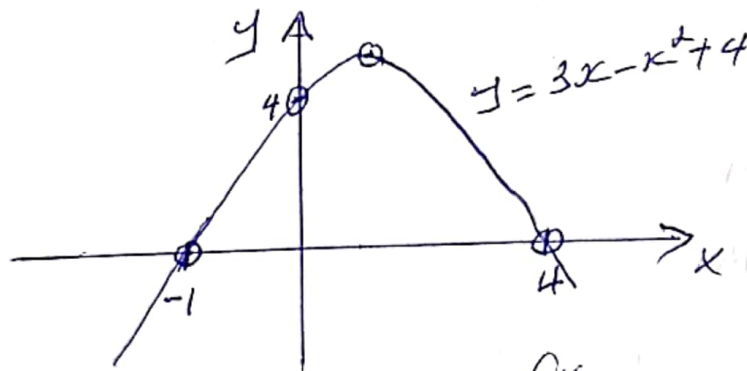
$$x(x-4) + 1(x-4) = 0$$

$$(x+1)(x-4) = 0$$

$$\therefore x = -1 \text{ or } x = 4$$

points are $(-1, 0), (4, 0)$.

y-intercept is $(0, 4)$.



(c)

$$\begin{aligned} \text{Required area} &= \int_{-1}^4 (3x - x^2 + 4) dx \\ &= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3 + 4x \right]_{-1}^4 \\ &= \left(24 - \frac{64}{3} + 16 \right) - \left(\frac{3}{2} + \frac{1}{3} - 4 \right) \\ &= 125/6. \end{aligned}$$

14 (a) let R be the reaction.

1st stage $R - mg = ma$.

$$\therefore R = 150 \times 9.8 + 150 \times 3$$

$$= 1920 \text{ N}$$

2nd stage $R - mg = ma$.

$$R - 150 \times 9.8 = 150 \times 0$$

$$\therefore R = 1470 \text{ N}$$

3rd stage. $Mg - R = ma$.

$$150 \times 9.8 - R = 150 \times 2.4$$

$$R = (150 \times 9.8) - (150 \times 2.4)$$

$$= 1110 \text{ N}$$

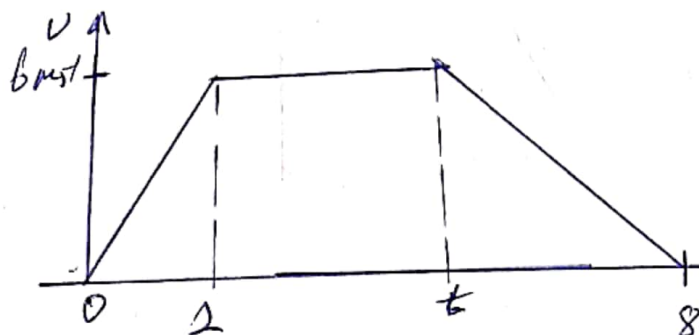
(b)

$$v = u + at$$

$$= 0 + 3 \times 2$$

$$= 6 \text{ ms}^{-1}$$

(c)



$$v = u + at$$

$$0 = 60 - 24(8 - t)$$

$$(8 - t) = 6$$

$$8 - t = \frac{60}{24} \Rightarrow t = 8 - \frac{60}{24} = 5.5$$

time of steady speed = 3.5