P425/1
PURE MATHEMATICS
Paper 1
Nov./Dec. 2023
3 hours

# UGANDA NATIONAL EXAMINATIONS BOARD

## Uganda Advanced Certificate of Education

PURE MATHEMATICS

**Paper 1**

3 hours

### INSTRUCTIONS TO CANDIDATES:

*Answer* **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

*Any additional question(s) answered will* **not** *be marked.*

**All** *necessary working* **must** *be shown clearly.*

*Begin each answer on a fresh sheet of paper.*

*Graph paper is provided.*

*Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

## SECTION A (40 MARKS)

*Answer* **all** *the questions in this section.*

1. Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. *(05 marks)*

2. If a line $y = mx + c$ is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $c^2 = 4 + 3m^2$. *(05 marks)*

3. Given that $y = e^x \cos 3x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$. *(05 marks)*

4. Find the angle between the line $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$ and the plane $-x + 2y + 2z - 66 = 0$. *(05 marks)*

5. Solve the inequality $\frac{7-2x}{(x+1)(x-2)} > 0$. *(05 marks)*

6. Evaluate $\int_0^{\pi/3} (1+\cos 3y)^2 \, dy$. *(05 marks)*

7. Express $2\sin\theta + 3\cos\theta$ in the form $R \sin(\theta + \alpha)$. *(05 marks)*

8. Use Maclaurin's theorem to expand $\ln(2+x)$, in ascending powers of $x$ as far as the term in $x^2$. *(05 marks)*

**SECTION B (60 MARKS)**

*Answer any* **five** *questions from this section.* **All** *questions carry equal marks.*

**9.** (a) Solve the equation $Z^3 - 7Z^2 + 19Z - 13 = 0$. *(06 marks)*

(b) Find the fourth roots of $8(-\sqrt{3}+i)$. *(06 marks)*

**10.** Express $f(x) = \dfrac{3x^3 + 2x^2 - 3x + 1}{x(1-x)}$ in partial fractions.

Hence find $\int f(x)\, dx$. *(12 marks)*

**11.** A point $E$ has coordinates (2, 0, –1). A line through $E$ and parallel to the line whose equation is $\dfrac{x}{-2} = y = \dfrac{z+1}{2}$, meets a plane $x + 2y - 2z = 8$ at a point $B$.

A perpendicular line from $E$ meets the plane at a point $C$.

Determine the coordinates of;

(a) $B$. *(07 marks)*

(b) $C$. *(05 marks)*

**12.** (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? *(02 marks)*

(b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days? *(03 marks)*

(c) Given that ${}^{20}C_r = {}^{20}C_{r-2}$, find the value of $r$. *(07 marks)*

**13.** (a) A curve is given by the parametric equations $x = t^2 - 3,\ y = t\,(t^2 - 3)$.

Find the Cartesian equation of the curve. *(04 marks)*

(b) A point $P$ is such that its distance from the origin is five times its distance from (12, 0).

(i) Show that the locus of $P$ is a circle.

(ii) Determine the coordinates of the centre of the circle and its radius. *(08 marks)*



Turn Over

14. Given the curve $y = \frac{1}{4x^2 - 1}$, determine the;

(a) coordinates of the turning points of the curve. *(03 marks)*

(b) equation of the asymptotes.
Hence sketch the curve. *(09 marks)*

15. (a) Show that $\tan 3\theta = \frac{\tan\theta\left(3 - \tan^2\theta\right)}{\left(1 - 3\tan^2\theta\right)}$. *(05 marks)*

(b) Solve the equation $\cos 4x + \cos 6x + \cos 2x = 0$ for $0^\circ \le x \le 180^\circ$. *(07 marks)*

16. The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature 25 °C. After 6 minutes the temperature of the body dropped from 90 °C to 60 °C.

(a) Form a differential equation for the rate of cooling of the body. *(07 marks)*

(b) Find the time it takes for the body to cool from 40 °C to 30 °C. *(05 marks)*



END

P425/2
APPLIED MATHEMATICS
Paper 2
Nov./Dec. 2023
3 hours

# UGANDA NATIONAL EXAMINATIONS BOARD

## Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

**Paper 2**

3 hours

**INSTRUCTIONS TO CANDIDATES:**

*Answer* **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

*Any additional question(s) answered will* **not** *be marked.*

**All** *necessary working* **must** *be shown clearly.*

*Begin each answer on a fresh sheet of paper.*

*Graph paper is provided.*

*Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

*In numerical work, take acceleration due to gravity g, to be 9.8 $ms^{-2}$.*

 **Turn Over**

## SECTION A (40 MARKS)

*Answer* **all** *the questions in this section.*

**1.** A coin is biased such that when it is tossed the head is twice as likely to occur as the tail. Find the probability that in seven tosses, there will be exactly two tails. *(05 marks)*

**2.** Two bodies $A$ and $B$ of masses 6 kg and 2 kg moving along a straight line with velocities 4 $ms^{-1}$ and 2 $ms^{-1}$ respectively, collide head on. After collision, $A$ moves with a velocity of 2.6 $ms^{-1}$ in the same direction.

Calculate the;

(a) velocity of $B$ after collision. *(02 marks)*

(b) loss in kinetic energy. *(03 marks)*

**3.** The values of a function $f(x)$ are given in the table below.

| $x$ | 0 | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.1003 | 0.0391 | 0.0801 | 0.0602 | 0.0649 | 0.0380 | 0.0327 |

Use the trapezium rule to estimate the value of

$$\int_0^3 f(x)\,dx$$

correct to **three** decimal places. *(05 marks)*

**4.** A ball of mass 1 kg rolls from rest down a rough plane inclined at 30° to a horizontal ground. The ball rolls for 4 m before it reaches the ground. The coefficient of friction between the ball and the plane is $\frac{1}{4}$. Find the velocity with which the ball reaches the ground. *(05 marks)*

**5.** The table below shows the age distribution of a population of a certain town in a census.

| AGE (years) | NUMBER ('000) |
|---|---|
| Under 10 | 15 |
| 10 and under 20 | 19 |
| 20 and under 30 | 16 |
| 30 and under 40 | 18 |
| 40 and under 60 | 30 |
| 60 and under 80 | 6 |
| 80 and under 90 | 1 |

(a) Draw a histogram for the data. *(03 marks)*

(b) Use the histogram to estimate the modal age of the population.

*(02 marks)*

6. The numbers $x = 6.45$, $y = 0.00215$ and $z = 2.7$ are each rounded off to the given number of decimal places.

Determine the interval in which $w = \frac{x+z^3}{\sqrt{y}}$ lies. *(05 marks)*

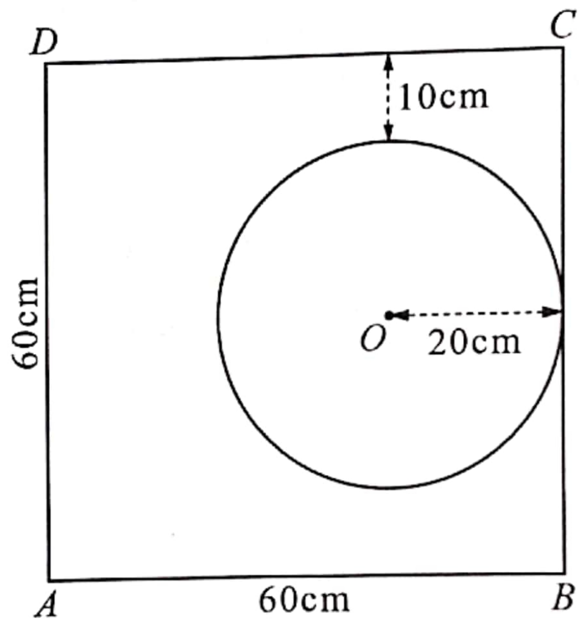
7. Two independent events $R$ and $S$ are such that $P(R) = {}^3/_4$ and $P(S) = P(S' \cap R')$.

Find;

(a) $P(S)$. *(03 marks)*

(b) $P(S' \cap R)$. *(02 marks)*

8. A uniform lamina in form of a square with side 60 cm has a circular hole of radius 20 cm made in it as shown in the diagram below.




Find the position of the centre of gravity of the lamina from side $AD$.

*(05 marks)*

## SECTION B (60 MARKS)

*Answer any* **five** *questions from this section.* **All** *questions carry equal marks.*

**9.** The table below shows the scores of 10 candidates in Biology and Economics.

| CANDIDATE | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| BIOLOGY | 45 | 63 | 56 | 61 | 75 | 83 | 73 | 50 | 77 | 70 |
| ECONOMICS | 90 | 64 | 76 | 70 | 55 | 53 | 62 | 85 | 53 | 62 |

(a) (i) Plot a scatter diagram for the data.

(ii) Draw a line of best fit on the scatter diagram.

(iii) Use your line of best fit to estimate the Biology mark for a candidate who scored 57 in Economics. *(06 marks)*

(b) Calculate a rank correlation coefficient between the candidates' performance in the two subjects.
Comment on your result. *(06 marks)*

**10.** Two points $A$ and $B$ are 526 m apart along a straight road. A car moving along the road passes point $A$ with a constant speed of 25 $\text{ms}^{-1}$. The car maintains this speed for 10 seconds and then decelerates uniformly for 8 seconds until it attains a speed of $V$ $\text{ms}^{-1}$. The car maintains this speed until it passes point $B$. The total time taken by the car to move from point $A$ to $B$ is 30 seconds.

(a) Sketch a Velocity – Time graph for the motion of the car. *(04 marks)*

(b) Determine the;

(i) value of $V$. *(05 marks)*

(ii) deceleration of the car. *(03 marks)*

**11.** Given that $f(x) = xe^x + 5x - 10$;

(a) (i) Evaluate $f(1)$ and $f(2)$, correct to **four** decimal places.

(ii) Deduce that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$. *(04 marks)*

(b) Use linear interpolation **twice** to obtain the root of the equation

$$xe^x + 5x - 10 = 0,$$

correct to **three** decimal places. *(08 marks)*

12. A continuous random variable $X$ has a cumulative distribution function given by

$$F(x) = \begin{cases} \dfrac{1+x}{6}, & -1 \le x \le 0 \\ \dfrac{1+2x}{6}, & 0 \le x \le 2 \\ \dfrac{4+3x}{12}, & 2 \le x \le \dfrac{8}{3} \\ 1, & x \ge \dfrac{8}{3} \end{cases}$$

Find;

(a) the median. (*03 marks*)

(b) the probability density function $f(x)$. (*03 marks*)

(c) $P(1 \le X \le 2.5)$. (*03 marks*)

(d) the mean of $X$. (*03 marks*)

13. Three forces $\boldsymbol{F}_1 = (2\boldsymbol{i} - 3\boldsymbol{j})$ N, $\boldsymbol{F}_2 = (5\boldsymbol{i} + 2\boldsymbol{j})$ N and $\boldsymbol{F}_3 = (-2\boldsymbol{i} - 11\boldsymbol{j})$ N act at points (2, 3), (–2, 3) and (3, –2) respectively.

Determine the;

(a) magnitude of their resultant force. (*03 marks*)

(b) equation of the line of action of the resultant force. (*05 marks*)

(c) distance from the origin at which the resultant cuts the $x$-axis. (*02 marks*)

(d) force that should be added to form a couple. (*02 marks*)

14. (a) Show that the formula based on Newton Raphson method for approximating the $k^{\text{th}}$ root of a number $N$ is given by

$$x_{n+1} = \frac{(k-1)x_n^{\,k} + N}{kx_n^{\,k-1}} \qquad n = 0, 1, 2, 3, \ldots$$

(*04 marks*)

(b) Construct a flow chart that;

(i) reads in the initial approximation $x_0$, $k$ and $N$,

(ii) computes and prints $N$ and its $k^{\text{th}}$ root correct to **three** decimal places. (*05 marks*)

(c) Perform a dry run for your flow chart when $N = 13$, $x_0 = 1.6$ and $k = 4$. (*03 marks*)

**15.** (a) A woman travelling to work by a car goes through three police check points $A$, $B$ and $C$. The probabilities that she is delayed at $A$, at $B$ and at $C$ are 0.3, 0.5 and 0.7 respectively.

Determine the probability that she is delayed at;

(i) only one check point. *(03 marks)*

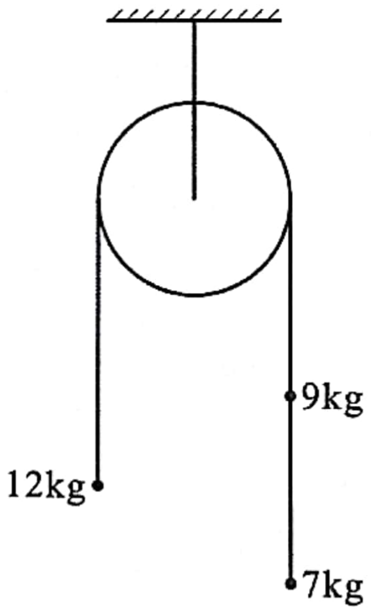
(ii) two or more check points. *(03 marks)*

(b) A man goes to work by route $P$ or route $Q$. The probability that he takes route $P$ is 0.6. The probability that he is late given that he goes through $P$ is $^2/_3$ and through $Q$ is $^1/_3$.

(i) Find the probability that he is late for work on a certain day. *(03 marks)*

(ii) Given that he is **not** late, determine the probability that he went through $P$. *(03 marks)*

**16.** The diagram below shows three masses of 12 kg, 9 kg and 7 kg connected by light inelastic strings. The string connecting the 12 kg and 9 kg masses passes over a smooth fixed pulley. The other string connects the 9 kg and 7 kg masses.




The system is released from rest and the 12 kg mass accelerates upwards.

(a) Calculate the;

(i) acceleration of the system.

(ii) tensions in the strings. *(10 marks)*

(b) Determine the velocity of the 12 kg mass after 1.5 seconds. *(02 marks)*



**END**