

SMATA 4TH ANNUAL POST MOCK MATHS SEMINAR

S.6 APPLIED MATHEMATICS



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ST JOSEPH OF NAZARETH HIGH SCHOOL

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APPLIED MATHEMATICS QUESTIONS

P425/2

STATISTICS AND CORRELLATIONS

1. The table below shows the frequency distribution of Marks obtained by a group of students in a test.

Marks	$- < 10$	$- < 30$	$- < 40$	$- < 45$	$- < 60$	$- < 70$	$- < 80$	$- < 90$
Frequency	8	30	46	12	30	10	6	8

- a. Calculate i). the Mean Mark
ii). the Mode
iii). the Middle 70% Marks range.
iv). Number of students with marks between 42% and 55%
- b. Plot a cumulative frequency curve and use it to estimate
i). the median
ii). the semi-interquartile range.

PROBABILITY THEORY/RANDOM VARIABLES

2. Show that if two events A and B are independent so is A' and B'

The probabilities of three players A, B and C scoring in a net ball team game are $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{1}{3}$ respectively. , B and C take it in turns to score starting with A until when one scores.

Find the probability that

- i) A wins
 - ii) C wins
3. Factory has a machine shop in which three machines A, B and C produce 100cm aluminum tubes. An inspector is equally likely to sample tubes from A and B, and three times likely to select tubes from C as he is from B. The defective rates from the machines are 10%, 10% and 20% respectively.

What is the probability that the tube selected by the inspector is;

- i) From A
 - ii) Defective
 - iii) B given that it is defective.
4. A game consists of tossing 4 unbiased coins simultaneously. The total score is calculated by giving 3 points for each head and 1 point for each tail. The random variable Y represents the total score.
5. a). Show that the probability of $P(Y = 8) = \frac{3}{8}$.
- b). Copy and complete the table given below for the symmetrical probability distribution of Y.

y	4	6	8	10	12
$P(Y = y)$			$\frac{3}{8}$		

- i) Calculate the $\text{Var}(Y)$
- ii) $\text{Var}(4Y-3)$

6. The continuous random variable T has a probability density function

$f(t)$, where;

$$f(t) = \begin{cases} t/3 - 2/3 & ; 2 \leq t \leq 3 \\ \lambda & ; 3 \leq t \leq 5 \\ 2 - \mu t & ; 5 \leq t \leq 6 \\ 0 & ; \text{otherwise} \end{cases}$$

- a) Find the values of the constants λ and μ
- b) Sketch the graph of $f(t)$
- c) Find (i). the 60th percentile of distribution.
(ii). the cumulative distribution $F(t)$. Hence sketch it.
(iii). $P(2.5 \leq t \leq 4)$

NORMAL AND BINOMIAL DISTRIBUTION

7. The probability that a marksman hits a target with a single shot is 0.4. If the marksman is given 50 bullets, find the probability that he hits the target;
 - i) Exactly 24 times
 - ii) Between 18 and 27 times inclusive.
8. A study of annual rainfall, x cm to the nearest centimeter, over the last 20 years for a small town gave the following results.
 $\sum x = 1325$, $\sum x^2 = 90316$
 - (i) Find the unbiased estimates of the Mean and Variance of the annual rainfall for this town.
 - (ii) Calculate a symmetric 90% confidence interval for μ .
9. Metal rods produced by a machine have lengths that are normally distributed. 2% of the rods are rejected as being too short and 5% rejected as being too long.
 - a). Given that the least and greatest acceptable length of the rods are 6.32 and 7.52cm, calculate the mean and variance of the lengths of the rods.

- b). If ten rods are chosen at random from a batch produced by the machine, find the probability that exactly three of them are rejected as being too long.

INTERPOLATION AND EXTRAPOLATION

10. The distance between Nakasongola and Kampala is 80km, A motorist leaves Nakasongola at 8:00am and reaches a distance of 20km, 50km, 70km, at 10:00 am, 1:45pm and 8:15pm respectively. On that day at 2:00pm his tyre burst and had to hire a lorry to carry his car to Kampala. If the car charged shs.1000 per km.
Find how much he paid for carrying the car.

ERRORS

11. If numbers a and b are approximately with errors e_a and e_b , show that the maximum relative errors in the approximation of $\frac{a}{b^{1/2}}$ is given by $\left| \frac{e_a}{a} \right| + \frac{1}{2} \left| \frac{e_b}{b} \right|$. Hence, find the limits within which the true value of $\frac{a}{b^{1/2}}$ lies given that $a = 1.98 \pm 0.00238$ and $b = 18 \pm 0.378$.

TRAPEZIUM RULE

12. (a). Use the trapezium rule to find $\int_{0.5}^1 6^x dx$, using 6 strips, correct to three decimal places.
(b) Find the exact value of $\int_{0.5}^1 6^x dx$, hence find the percentage error made in the two calculations above.

LOCATION OF ROOTS

- 13a).i). Show that the equation $6 \cos x + x = 0$ has a root between -2 and -1.
ii). Use linear interpolation to find the root in (i) above correct to 2dps.

b). Derive an expression based on N.R.M that can be used to find the root of the equation $6 \cos x + x = 0$ hence, use the answer in a(ii) as the initial approximation to find the root correct to 3dps.

14.a.i). Show graphically that the equation $\log_e(x + 2) - 2x = 1$ has a root between -1 and 0. State the root to 2dps.

ii). Form an expression based on N.R.M that can be used to solve the root of the equation in (i) above.

b) Construct a flow chart that reads the initial approximation x_0 , computes and prints the root corrected to three decimal places with a maximum of three iterations.

c) Perform a dry run for the flow chart in (b) above using x_0 as obtained in a(i).

VECTORS AND MOTION BY VARIABLE ACCELERATION

15. Four forces; $(-i - 2j)N$, $(5i - 6j)N$; $(3i + 2bj)N$ and $(bi + (b - 1)j)N$

Act on a particle. The resultant of the four forces makes an angle of 45° with the horizontal positive x-axis upwards.

i). Calculate the value of b

ii). Determine the magnitude of the resultant force.

16. A particle with position vector $(4i + 10j + 20k)m$, moves with a constant speed of 18 Km/hr in the direction of the vector $4i + 7j + 4k$. Find its distance from the origin after 9 seconds.

17. In the gulf water's, a battle ship steaming Northwards at 16 km/hr is 5km south West of a Submarine. Find the two possible courses which the submarine could take in order to intercept the battleship if it's moving at a speed of 12km/hr.

18. A boat travelling at a speed of 18kmh^{-1} in the direction of $\text{N}30^\circ\text{E}$ in still water is blown by wind moving at a speed of 8m/s from the bearing of 150° . Calculate the speed and the course the boat will be steered.
19. The velocity vector of a particle of mass 0.4 kg at any time t seconds is given by $V = (3\sin 2t\mathbf{i} + 5e^{4t}\mathbf{j})\text{ms}^{-1}$
- Determine,
- Force acting on the particle at any time, t .
 - Hence the power of the force at $t = \frac{\pi}{4}$ seconds.

LINEAR MOTION AND FREE FALL

20. A particle projected with a speed of 12ms^{-1} to move in a straight line on a rough horizontal surface comes to rest in 5 seconds. Calculate the distance it covers in its last second of motion.
21. Two particles of mass 4kg and 9kg are connected by a light inextensible string passing over a smooth pulley at the edge of the table and under a smooth moveable pulley.
- Determine the accelerations of the 4kg and 9 kg masses.
22. A body is projected vertically upwards with velocity of 21ms^{-1} . Determine the time it takes to reach a point 280m below the point of projection.
23. An overloaded Bus travelling at a constant speed of 90 kmh^{-1} over takes a stationary traffic police car. Two seconds later, the police car sets off in pursuit of the Bus accelerating at a rate of 6ms^{-2} . How far does the traffic police car travel before catching up with the Bus?
24. A bullet travelling at a speed of 150 m/s penetrates into a fixed block of wood 8cm before coming to rest.
- Find the velocity of the bullet when it has penetrated 4cm of the block.

RELATIVE MOTION

25. To a motorist travelling due North at 40 km/hr, the wind appears to come from the direction N 60°E at 50km/hr.
- Find the true velocity of the wind.
 - If the wind velocity remains constant but the speed of the motorist is increasing; find his speed when the wind appears to be blowing from the direction N45°E.
26. At 12:00 noon, the position vector \mathbf{r} and velocity vector (\mathbf{v}) of two ships A and B are as follows.

Ship	Position vector (\mathbf{r})	Velocity vector (\mathbf{v})
A	$\mathbf{r}_A = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} km$	$\mathbf{V}_A = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} kmh^{-1}$
B	$\mathbf{r}_B = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} km$	$\mathbf{V}_B = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} kmh^{-1}$

- Determine the displacement of A relative to B.
 - If the two ships collide, determine the time and position of collision.
27. At noon ship A is sailing due East at a constant velocity of 20kmh⁻¹. At the same time ship B is sailing in the direction N60°E at a constant velocity of 15 kmh⁻¹. If they continue sailing with these velocities in these directions, Determine the;
- time at shortest distance
 - shortest distance.

EQUILIBRIUM AND RESOLUTION OF FORCES

28. A particle of weight 8N is attached to point Q of a light inextensible string RQ. It hangs in equilibrium with point R fixed and RQ at an angle of 30° to the downward vertical. A force of F N at Q acting at right angle to RQ, keeps the particle in equilibrium. Find the magnitude of force F and the tension in the string.
29. For forces of magnitudes 3N, 10N, 6N and 7N act along sides AB, BC, DA and DB respectively. The direction of the forces being indicated by the order of the letters, of a rectangle ABCD with sides $\overline{AB} = 12m$ and $\overline{BC} = 5m$.
- Taking AB and AD as x and y axes respectively; find the magnitude and direction of the resultant of the forces.
 - If the line of action of the resultant of the forces cuts AB produced at point M, find the length MC.

CONNECTED PARTICLES/FRICTION/WORK, POWER AND ENERGY

30. An object of mass 2000g is at rest on a plane inclined at 25° to the horizontal. The coefficient of friction between the object and the plane is $\frac{2}{5}$. What minimum force applied parallel to the plane would move the object up the plane?
31. A mass of 3kg is at rest on a smooth horizontal table. It is attached by a light inextensible string passing over a smooth fixed pulley at the edge of the table to another mass of 2000 grams which is hanging freely. The system is released from rest.
- Calculate the acceleration and tension of the system.

32. Wooden block of mass 112 Kg is dragged across a rough horizontal floor by a force F Newton's inclined at 30° above the floor at a constant speed. If the coefficient of friction between the block and floor is $\frac{2}{7}$.

Find the;

- i). Value of F
- ii). Work done by the dragging force in moving the block through 5.5m under the above condition.

c) A vehicle of mass 1200kg tows a trailer of mass 250 kg up along an incline of $\sin^{-1}\left(\frac{1}{49}\right)$ above the horizontal. If the engine of the car is working at a constant rate of 4.2 kN and that the resistance to motion of the car is four times that of the trailer find the:

- i) Resistance to motion of the car when it is moving with a steady speed of 12 ms^{-1} .
- ii). tension in the tow bar.

RIGID BODIES/COPLANAR FORCES

33. Uniform ladder of mass 25 Kg is placed with its base on a rough horizontal floor (angle of friction = $\tan^{-1}\left(\frac{1}{5}\right)$, and its top against a rough vertical wall (angle of friction = $\tan^{-1}\left(\frac{1}{3}\right)$, with the ladder making an angle of 61° with the floor.

Calculate the maximum horizontal force that could be applied at the base without slipping occurring.

SIMPLE HARMONIC MOTION, ELASTICITY AND C.O.G

34. An elastic string of natural length 120 cm and modulus of elasticity 8N is stretched until the extending force is 6N. Calculate the

- a). extension of the string
- b). work done by the string.

35. A particle moving with simple harmonic motion has speeds of 5m/s and 8m/s at distances 16m and 12m respectively from its equilibrium position. Find the amplitude and the period of the motion.
36. A particle executing SHM about point “O” has a velocity of $3\sqrt{3}\text{ms}^{-1}$ and 3ms^{-1} when at distances of 100cm and 26.8cm respectively, from the end point. Calculate the amplitude of the motion.
37. Strings AC and BC are both of natural length $5l$. String AC is inelastic and string BC is elastic with modulus of elasticity λN . A and B are attached to points in a horizontal line at a distance of $5l$ apart. A mass of 1kg is attached to point C and the system is in equilibrium in a vertical plane with BC of length $6l$.
- Calculate the tensions in AC and BC
 - Find the value of x .
38. ABCD is a square lamina of side “a” from which a triangle ADE is removed. E being a point on CD at a distance “t” from C.
- Show that the center of mass of the remaining lamina is at a distance of $\frac{a^2+at+t^2}{3(a+t)}$ from BC