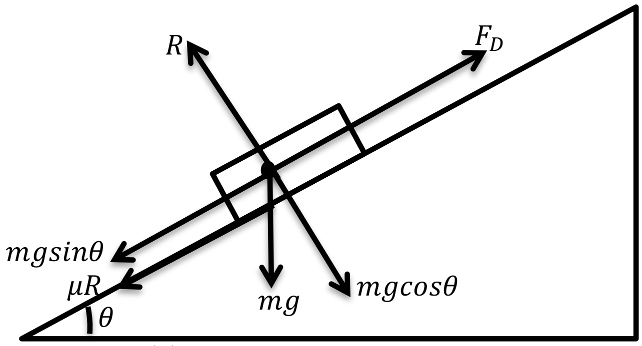
Proposed guide UACE Applied Mathematics 2022

Qns	Answers	Marks
	SECTION A. (40 marks)	

1.



From Newton's second law, $F = ma$

But $F = F_D - (mgsin\theta + \mu R)$

$ma = F_D - (1500x9.8xsin\theta + \frac{1}{4}x1500x9.8xcos\theta)$

But, $sin\theta = \frac{3}{5}, cos\theta = \frac{4}{5}$

At steady speed, acceleration, $a = 0ms^{-1}$

$F_D = 1500x9.8x\frac{3}{5} + \frac{1}{4}x1500x9.8x\frac{4}{5}$

$F_D = 11760N$

Therefore the driving force is 11760N

5 marks

2

x	f	fx	fx^2
1	41	41	41
2	33	66	132
3	18	54	162
4	6	24	96
5	2	10	50
	$\sum f = 100$	$\sum fx = 195$	$\sum fx^2 = 481$

(a) From mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$\bar{x} = \frac{195}{100}$

$\bar{x} = 1.95 \approx 2$ people

(b) From variance, $var(x) = \frac{\sum fx^2}{\sum f} - (\bar{x})^2$

$var(x) = \frac{481}{100} - (1.95)^2$

$var(x) = 1.0075$

5 marks

3

Since given is the number of ordinates, to get the number of sub-intervals we subtract a one.

$h = \frac{2-0}{6} = \frac{1}{3}$, and $f(x) = \frac{1}{3+4x^2}$

x	$f(x) = \frac{1}{3+4x^2}$	$f(x) = \frac{1}{3+4x^2}$
0	0.3333	
$\frac{1}{3}$		0.2903
$\frac{2}{3}$		0.2093
1		0.1429
$\frac{4}{3}$		0.0989
$\frac{5}{3}$		0.0707
2	0.0526	
sum	0.3859	0.8121

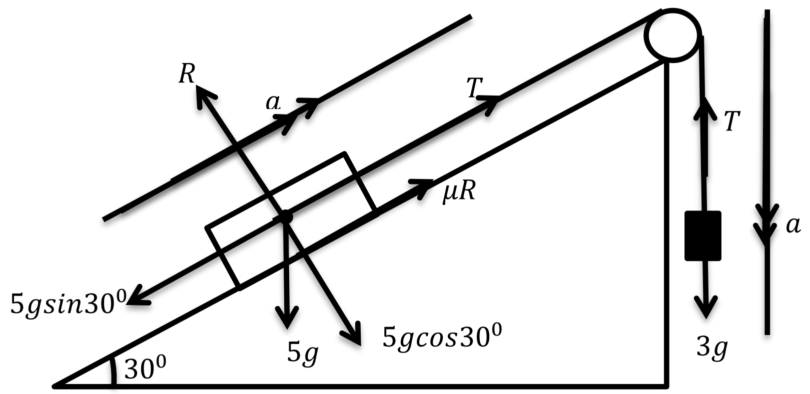
From $\int_0^2 \frac{1}{3+4x^2}dx \approx \frac{1}{2}h\left[\left(f(x)\right)+2\left(f(x)\right)\right]$

$\int_0^2 \frac{1}{3+4x^2}dx \approx \frac{1}{2}x\frac{1}{3}[(0.3859)+2(0.8121)]$

$\int_0^2 \frac{1}{3+4x^2}dx \approx 0.335\ (3dps)$

5 marks

4



$T = 3g = 3x9.8 = 29.4N$

$T = 3gsin30^0 - \mu R$

But, $R = mgcos30^0$

$T = 3gsin30^0 - \mu xmgcos30^0$

$29.4 = 3x9.8xsin30^0 - \mu x5x9.8xcos30^0$

$\mu = -0.3464$

Therefore the coefficient of friction between the two surfaces in contact is -0.3464

5 marks

5

$P(A) = \frac{1}{2}, P(B) = \frac{7}{12}, P(\bar{A}nB) = \frac{1}{2}$

$P(\bar{B}nA) = P(A) - P(AnB)$

But, $P(\bar{A}nB) = P(B) - P(AnB)$

$\frac{1}{2} = \frac{7}{12} - P(AnB)$

$P(AnB) = \frac{1}{2} - \frac{7}{12} = \frac{1}{12}$

⇨ Therefore, $P(\bar{B}nA) = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$

5 marks

6

Extract,

97	105	x
78	85	92

$\frac{x-97}{92-78} = \frac{108-97}{85-78}$

$x = 113$

Therefore 113dollars are equivalent to 92 Euros

Extract,

79	85	97
64	y	78

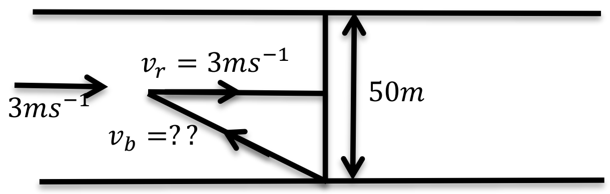
$\frac{y-64}{85-79} = \frac{78-64}{97-79}$

$y = 68.667$

Therefore 69 Euros are equivalent to 85 dollars

5 marks

7



(a) Velocity of the boat relative to the river,

${}_bv_r = v_b - v_r$

$4 = v_b - 3$

$v_b = 1ms^{-1}$

(b) $d = vxt$

$50 = 1xt$

$t = 50seconds$

5 marks

8

(a) $P(R\ removed\ from\ B) = P(R_1nR_2) + P(B_1nR_2)$

$= \frac{7}{11} x \frac{6}{14} + \frac{4}{11} x \frac{5}{14}$

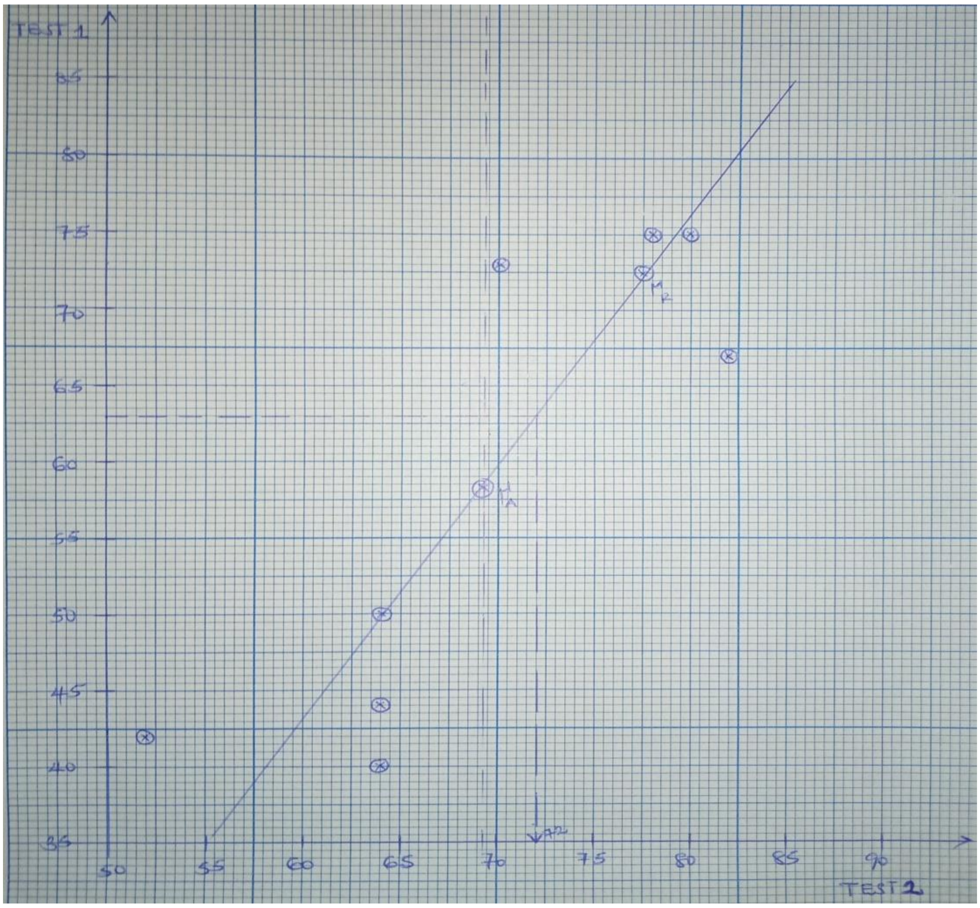
$P(R\ removed\ from\ B) = \frac{31}{77}$

(b) $P\left({}^{B_1}/_{R}\right) = \frac{P(B_1nR)}{P(R)} = \frac{\frac{4}{11}x\frac{5}{14}}{\frac{31}{77}} = \frac{10}{31}$

5 marks

SECTION B. (60 Marks)

9 **(a)**



63 in test 1 corresponds to 72 in test 2.

(b)(i)

R_{T_1}	R_{T_1}	d	d^2
4	1	3	9
3	4	-1	1
5	6	-1	1
1.5	3	-1.5	2.25
7	8	-1	1
6	6	0	0
1.5	2	-0.5	0.25
8	6	2	4
			$\sum d^2 = 18.5$

From, $\rho = 1 - \frac{6\sum d^2}{8(8^2-1)}$

$\rho = 1 - \frac{6(18.5)}{8(8^2-1)} = 0.7798$

(ii) Significant at 5% levels

Total **12 Marks**

10	**(a)**	$r_0 = (2i - 2j + 8k)m$ $F = (4ti + t^2j + 5k)$ From, $F = ma$ $(4ti + t^2j + 5k) = 4a$ $a = \frac{1}{4}(4ti + t^2j + 5k)$ $a = \left(ti + \frac{t^2}{4}j + \frac{5}{4}k\right)\ ms^{-2}$	
	(b)	From, $a = \frac{dv}{dt}$ $\int dv = \int adt$ $v = \int_0^3 adt$ $v = \int_0^3 \left(ti + \frac{t^2}{4}j + \frac{5}{4}k\right)dt$ $v = \left(\frac{t^2}{2}i + \frac{t^3}{12}j + \frac{5t}{4}k\right)\Big\vert_0^3$ $v = \left(\frac{3^2}{2}i + \frac{3^3}{12}j + \frac{5(3)}{4}k\right) - \left(\frac{0^2}{2}i + \frac{0^3}{12}j + \frac{5(0)}{4}k\right)$ $v = \left(\frac{9}{2}i + \frac{27}{12}j + \frac{15}{4}k\right)ms^{-1}$ $\lvert v\rvert = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{27}{12}\right)^2 + \left(\frac{15}{4}\right)^2} = 6.27495ms^{-1}$	
	(c)	From, $v = \frac{dr}{dt}$ $r = \int vdt$ $r_{(t)} = \int \left(\frac{t^2}{2}i + \frac{t^3}{12}j + \frac{5t}{4}k\right)dt$ $r_{(t)} = \left(\frac{t^3}{6}i + \frac{t^4}{48}j + \frac{5t^2}{8}k\right) + c$ where c is a constant of integration But; at $t = 0, r_o = 2i - 2j + 3k, c = 2i - 2j + 3k$ $r_{(t)} = \left(\frac{t^3}{6}i + \frac{t^4}{48}j + \frac{5t^2}{8}k\right) + (2i - 2j + 3k)$ $r_{(t)} = \left(\frac{t^3+2}{6}i + \frac{t^4-2}{48}j + \frac{5t^2+3}{8}k\right)\Big\vert_{t = 3}$ $r_{(t)} = \left(\frac{3^3+2}{6}i + \frac{3^4-2}{48}j + \frac{5(3)^2+3}{8}k\right)$ $r_{(t=3)} = (\frac{29}{6}i + \frac{79}{48}j + \frac{48}{8}k)m$	
Total			**12 Marks**
11	**(a)**	Let,$m = \frac{x}{y}$, if the M is used to approximate m with small change Δm then, $(M + \Delta m) = \frac{(X+\Delta x)}{(Y+\Delta y)}$ $\Delta m = \frac{X+\Delta x}{Y+\Delta y} - M$ $\Delta m = \frac{X+\Delta x}{Y+\Delta y} - \frac{X}{Y}$	

$\Delta m = \frac{Y(X+\Delta x)-X(Y+\Delta y)}{Y(Y+\Delta y)}$

$\Delta m = \frac{Y\Delta x - X\Delta y}{Y^2(1+\frac{\Delta y}{Y})}$

Since, $\Delta y \ll y$ then, $\frac{\Delta y}{Y} \approx 0$

$\Delta m = \frac{Y\Delta x - X\Delta y}{Y^2}$

$\frac{\Delta m}{M} = \frac{\left[\frac{Y\Delta x - X\Delta y}{Y^2(1+\frac{\Delta y}{Y})}\right]}{\frac{X}{Y}}$

$\frac{\Delta m}{M} = \frac{Y\Delta x - X\Delta y}{YX}$

$\frac{\Delta m}{M} = \frac{\Delta x}{X} - \frac{\Delta y}{Y}$

$\left|\frac{\Delta m}{M}\right| = \left|\frac{\Delta x}{X} - \frac{\Delta y}{Y}\right|$

$\left|\frac{\Delta m}{M}\right| \leq \left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$

Therefore the relative error in approximating $\frac{x}{y}$ is $\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$

(b)

From, $T = \frac{673.16}{40.345}$

Let $x = 673.16, y = 40.345$

then,

$\Delta x = 0.5x10^{-2} = 0.005,\ \Delta y = 0.5x10^{-3} = 0.0005$

$upper\ limit = \frac{673.16+0.005}{40.345-0.0005} = 16.6854$

$lower\ limit = \frac{673-0.005}{40.345+0.0005} = 16.6848$

Therefore the interval within which the exact value of T can be expected to lie is $[16.6848, 16.6854]$

Total — **12 Marks**

12 (a)

From $f(x) = \begin{cases} kx^2; & x = 1,2,3 \\ k(7-x)^2; & x = 4,5,6 \\ 0; & else\ where \end{cases}$

(i)

x	1	2	3	4	5	6
$P(X=x)$	k	$4k$	$9k$	$9k$	$4k$	k

From, $\sum_{all\ x} P(X=x) = 1$

$(k + 4k + 9k) + (9k + 4k + k) = 1$

$28k = 1$

$k = \frac{1}{28}$

(ii)

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{28}$	$\frac{4}{28}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{4}{28}$	$\frac{1}{28}$

From, $E(x) = \sum_{all\ x} xP(X = x)$

$$E(x) = 1\left(\frac{1}{28}\right) + 2\left(\frac{4}{28}\right) + 3\left(\frac{9}{28}\right) + 4\left(\frac{9}{28}\right) + 5\left(\frac{4}{28}\right) + 6\left(\frac{1}{28}\right) = 3.5$$

(iii)

From, $var(x) = E(x^2) - ((E(x)^2)$

But, $E(x^2) = \sum_{all\ x} x^2 P(X = x)$

$$E(x^2) = 1\left(\frac{1}{28}\right) + 4\left(\frac{4}{28}\right) + 9\left(\frac{9}{28}\right) + 16\left(\frac{9}{28}\right) + 25\left(\frac{4}{28}\right) + 36\left(\frac{1}{28}\right)$$

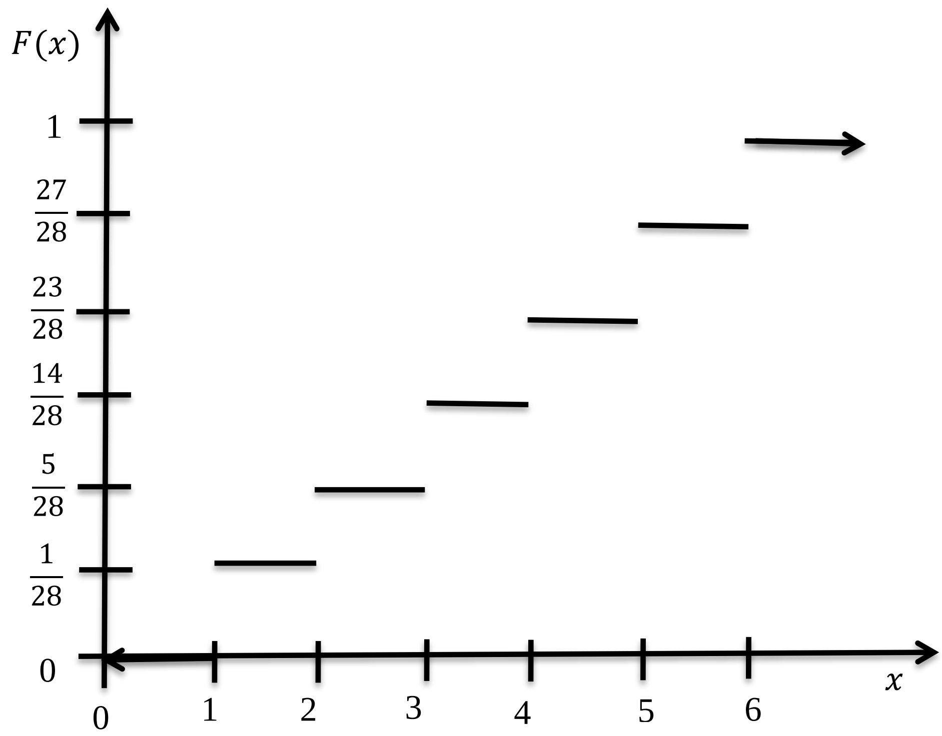
$E(x^2) = 13.5$

$var(x) = 13.5 - ((3.5^2)$

$var(x) = 1.25$

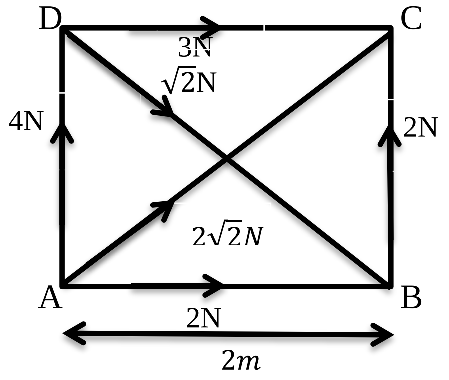
(b)

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{28}$	$\frac{4}{28}$	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{4}{28}$	$\frac{1}{28}$
$F(x) = P(X \leq x)$	$\frac{1}{28}$	$\frac{5}{28}$	$\frac{14}{28}$	$\frac{23}{28}$	$\frac{27}{28}$	1



Total — **12 Marks**

13 (a)



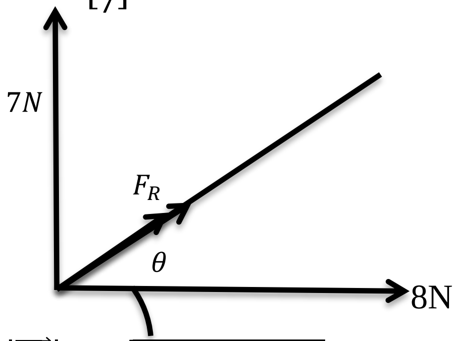
Resolving horizontally,

$F_x = 2 + 3 + 2\sqrt{2}cos45^0 + \sqrt{2}cos45^0 = 8N$

Resolving vertically,

$F_y = 4 + 2 + 2\sqrt{2}sin45^0 - \sqrt{2}sin45^0 = 7N$

$\vec{F}_R = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$



$|\vec{F_R}| = \sqrt{(F_x)^2 + (F_y)^2}$

$|\vec{F_R}| = \sqrt{(8)^2 + (7)^2} = 10.6301N$

From, $\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{7}{8}\right) = 36.9^0$

Therefore the resultant force is 10N and acts at 41.2^0 above the positive x-axis.

(b) From, $\begin{vmatrix} F_x & F_y \\ x & y \end{vmatrix} = G$

$\begin{vmatrix} 8 & 7 \\ x & y \end{vmatrix} = G$

Taking clockwise moments about A;

$G = 3x2 - 2x2 + (\sqrt{2})x\frac{\sqrt{8}}{2} = 4Nm$

$\begin{vmatrix} 8 & 7 \\ x & y \end{vmatrix} = 4$

$8y - 7x = 4$

Therefore the equation of line of action of the resultant force is;

$8y - 7x = 4$

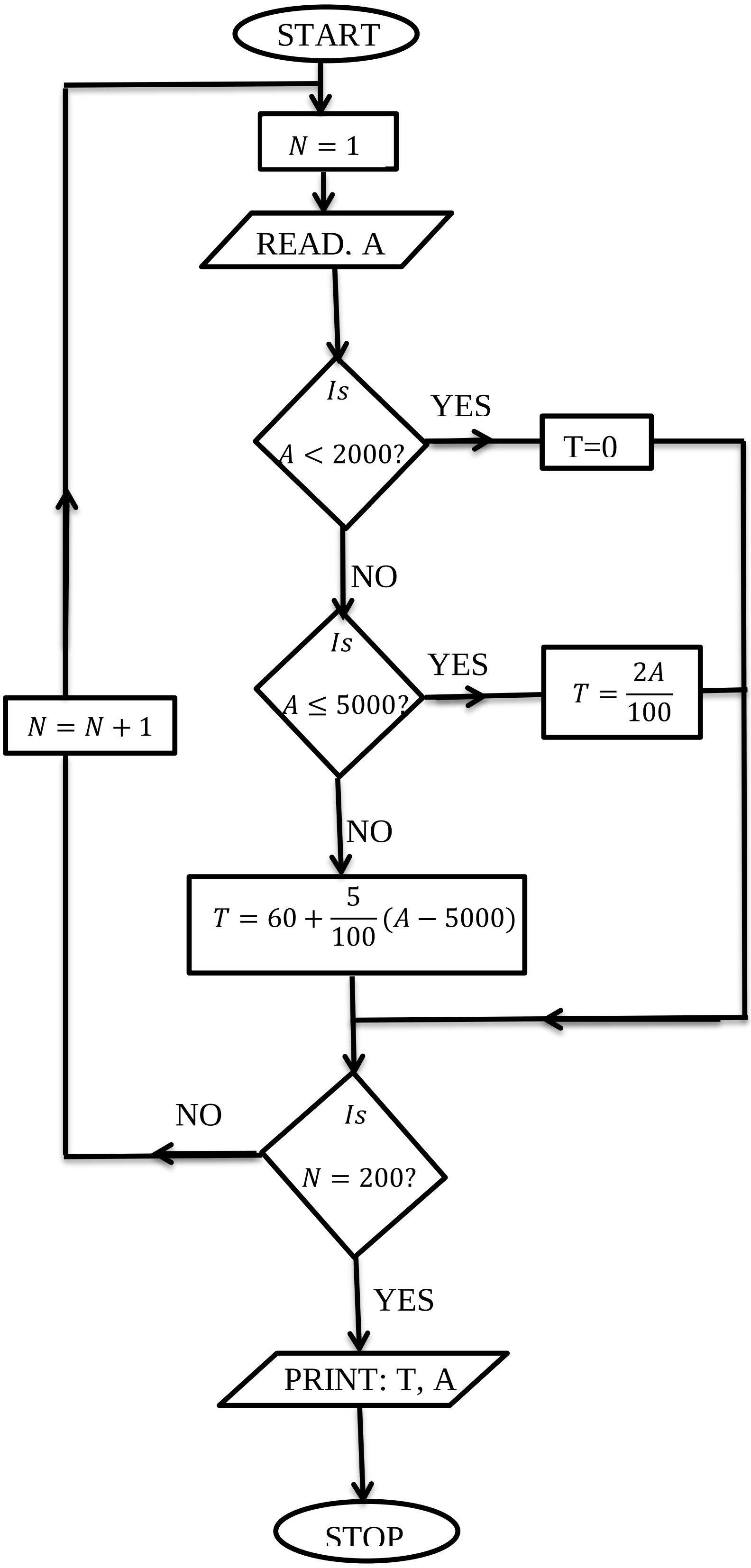
Total — **12 Marks**

14 (a) (ii) To calculate the tax paid (T) in dollars based on the amount (A) earned by 200 employees,

(b)

N	A	T
1	1500	0
2	3500	70
3	9000	260

(a) (i)



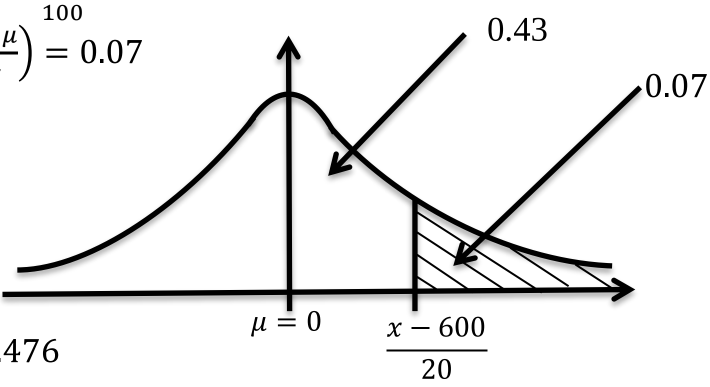
Total		**12 Marks**

15 (a)

$\mu = 600g, \sigma = 20g$

$P(X > x) = \frac{7}{100}$

$P\left(Z > \frac{x-\mu}{\sigma}\right) = 0.07$



$\frac{x-600}{20} = 1.476$

$x = 20x(1.476) + 600$

$x = 629.52g$

(b)

$n = 1000$

$P(X < 545)$

$P\left(Z < \frac{545-600}{20}\right)$

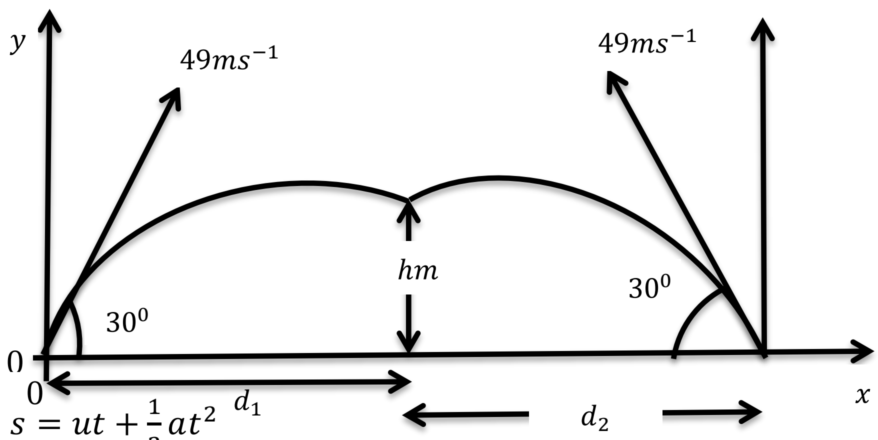
$P(Z < -2.75) = 2.98x10^{-3}$

Number of packets that weighed less than 545g is;

$2.98x10^{-3}x1000 = 2.98 \approx 3$packets

Total — **12 Marks**

16 (a)



From, $s = ut + \frac{1}{2}at^2$

For P, $s = (49sin30^0)(t) - \frac{1}{2}x9.8xt^2$

For Q, $s = (49sin30^0)(t-2) - \frac{1}{2}x9.8x(t-2)^2$

At the point they met, they had travelled the same distance, therefore;

$(49sin30^0)(t) - \frac{1}{2}x9.8xt^2 = (49sin30^0)(t-2) - \frac{1}{2}x9.8x(t-2)^2$

$68.6 = 19.6t$

$t = 3.5seconds$

$h = (49sin30^0)(3.5) - \frac{1}{2}x9.8x3.5^2 = 25.725m$

Therefore the two met at 25.725m from the start.

(b)	Distance between A and B is $d = d_1 + d_2$ From, $s = ut + \frac{1}{2}at^2$ Horizontally there is no acceleration. $d_1 = (49cos30^0)(3.5) = 148.5234m$ $d_2 = (49cos30^0)(3.5 - 2) = 63.6529m$ $d = 148.5234 + 63.6529 = m$ Therefore the distance between A and B is 212.1763m	
Total		**12 Marks**