P425/1
PURE MATHEMATICS
Paper 1
Nov. /Dec. 2022
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. Solve the simultaneous equations:

$$2\ \log_{10} y = \log_{10} 2 + \log_{10} x$$

$$2^y = 4^x$$

(05 marks)

2. Solve $5\tan^2 A - 5\tan A = 2\sec^2 A$ for $0^\circ \le A \le 360^\circ$. *(05 marks)*

3. The position vectors of points P and Q are given by $\boldsymbol{OP} = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{OQ} = 3\boldsymbol{i} - 4\boldsymbol{j} + 6\boldsymbol{k}$ respectively. Point R divides the line $\overline{PQ}$ in the ratio $2 : -3$. Determine the coordinates of the point R. *(05 marks)*

4. Find the equation of the tangent to the curve $x^3 + 2y^3 + 3xy = 0$ at the point $(2, -1)$. *(05 marks)*

5. Solve the inequality $\dfrac{5-4x}{1-x} < 3$ *(05 marks)*

6. An inverted conical container has a hole at the bottom. A liquid is dripping through the hole at a rate of $2\ \text{cm}^3\text{s}^{-1}$. When the depth of the liquid in the container is x cm, its volume is $\frac{1}{3}\pi x^3\ \text{cm}^3$. Find the rate at which the level of the liquid is decreasing when x is 5 cm. *(05 marks)*

7. A line L passes through the point of intersection of the lines $x - 3y - 4 = 0$ and $y + 3x - 2 = 0$. If L is perpendicular to the line $4y + 3x = 0$, determine the equation of the line L. *(05 marks)*

8. Using the substitution $y = Vx$ or otherwise, solve the differential equation

$$x\frac{dy}{dx} = 2y + x$$

$\frac{dx}{d}$

(05 marks)

At $\frac{dx}{dt} = \frac{dx}{dv} \cdot \frac{dv}{d}$

SECTION B (60 MARKS)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

9. (a) Given the geometrical progression (G.P.) 2, 6, 18, 54, … find the sum of the first ten terms of the G.P. *(03 marks)*

(b) In an arithmetical progression (A.P.), the sum of the fifth and sixteenth terms is 44. The sum of the first 18 terms is three times the sum of the first ten terms. Determine the:

(i) value of the first term.

(ii) common difference of the A.P.

(iii) sum of the first 30 terms of the A.P. *(09 marks)*

10. Express $\dfrac{11x-1}{(1-x)^2(2+3x)}$ in partial fractions.

Hence evaluate $\displaystyle\int_0^{1/2} \frac{11x-1}{(1-x)^2(2+3x)}\,dx$ giving your answer in the form $k + \ln b$ where k is an integer and b is a fraction. *(12 marks)*

11. The equation of a line is $\mathbf{r} = (\lambda - 1)\,\mathbf{i} + (\lambda + 2)\mathbf{j} + (2\lambda - 4)\,\mathbf{k}$. The line is parallel to the direction vector $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

(a) Find the equation of the plane containing the line. *(10 marks)*

(b) Calculate the distance between the origin and the plane. *(02 marks)*

12. Expand $\left(\dfrac{1+3x}{1-x}\right)^{\frac{1}{2}}$ up to the term in x^3.

Hence substitute $x = 1/5$ to evaluate $\sqrt{8}$ correct to **two** decimal places. *(12 marks)*

13. The point $P(at^2, 2at)$ is on the parabola $y^2 = 4ax$. The chord OQ passes through the origin O. The tangent at P is parallel to the chord OQ. The tangents to the parabola at P and Q meet at a point R. Determine the coordinates of points Q and R in terms of a and t. *(12 marks)*

14. (a) Differentiate $\dfrac{(x^2+1)}{(x+1)^3}$ with respect to x. *(04 marks)*

(b) Given that $x = \dfrac{3t}{t+3}$ and $y = \dfrac{4t+1}{t-2}$, find $\dfrac{d^2y}{dx^2}$ in terms of t in the simplest form. *(08 marks)*

$\frac{15}{2}$

$12 \times 6r5 + 25x^{-1}$ $t.35$

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15. (a) A chord AB subtends an angle θ radians at the centre O of a circle of radius r. The area of the circle is three times the area of the minor segment AB. Show that $3\theta = 3\sin\theta + 2\pi$. *(06 marks)*

(b) Given that $\tan\alpha = \sec\alpha - \frac{1}{3}$, find the values of:

(i) $\cos\alpha$.

(ii) $\tan\alpha$. *(06 marks)*

16. (a) Sketch the curve $y = 5x(2 - x)$. *(08 marks)*

(b) The area bounded by the curve and the x-axis is rotated about the x-axis through one revolution. Determine the volume of the solid generated. *(04 marks)*

END