**P425/2**
**PURE**
**MATHEMATICS**
**PAPER 1**
**August. 2017**
3 hour

**NDEJJE SENIOR SECONDARY SCHOOL**
**Uganda Advanced Certificate of Education**
**MOCK SET 4 EXAMINATIONS 2017**
PURE MATHEMATICS
**Paper 1**
3 hours

**INSRUCTIONS TO CANDIDATES:**

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Additional question(s) answered will **not** be marked.

**All** working **must** be shown clearly.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

## SECTION A (40 MARKS)

(Answer **all** questions in this section.)

**Qn 1:** The sum of the second and third terms of a Geometric Progrssion (G.P) is 48. The sum of the fifth and sixth terms is 1296. Find the common ratio, the first term and the sum of the first 12 terms of the G.P. [5]

**Qn 2:** Use De Moivre's theorem to prove that
$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. [5]

**Qn 3:** When a polynomial $P(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $P(x)$ is divided by:
(i). $x - 7$,
(ii). $x + 2$. [5]

**Qn 4:** OAB is a triangle in which $= \underset{\sim}{\boldsymbol{a}}$ , $OB = \underset{\sim}{\boldsymbol{b}}$ . $C$ is a point on AB such that $AC\!:CB = 3\!:1$. D is the midpoint of $OA$. $DC$ and $OB$, both produced meet in point $T$. Find vector $OT$ in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$. [5]

**Qn 5:** Find the integral $\int x\cos^2 x\,dx$. [5]

**Qn 6:** Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point $(2, 4)$. Find the values of the constants $a$, $b$ and $c$. [5]

**Qn 7:** Find the volume of the solid of revolution generated when the area under $y = \frac{1}{x-2}$ from $x = 3$ to $x = 4$ is rotated through four right angles about the x-axis. [5]

**Qn 8:** In triangle ABC, $AB = x - y$, $BC = x + y$ and $CA = x$, show that $\cos A = \frac{x-4y}{2(x-y)}$. [5]

## SECTION B (60 MARKS)

*Answer any* **five** *questions from this section.* **All** *questions carry equal marks.*

**Question 9:**

(a). Find the centroid of the triangle whose sides are given by the equations $x + y = 11, y = x - 1$ and $3y = x - 3$. [5]

(b). ABCD is a rhombus such that the coordinates $A(-3, -4)$ and $C(5, 4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2, obtain the coordinates of B and D, prove that the area of the rhombus is $21\frac{1}{3}$ square units. [7]

**Question 10:**

Show that $\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)} dx = \frac{\pi}{20}$ . [12]

**Question 11:**

(a). Using Maclaurin's theorem, expand $e^{-x} \sin x$ upto the term in $x^3$. Hence evaluate $e^{-\frac{\pi}{3}} \sin \frac{\pi}{3}$ to four significant figures. [5]

(b). The curve $y = x^3 + 8$ cuts the $x$ and $y$ axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B and C hence find the area enclosed between the curve and the line. [7]

**Question 12:**

(a). The position vectors of the points P and Q are $4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 5\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$ respectively. Find the coordinates of the point $R$ such that $PQ: PR = 2: 1$. [4]

(b). If the vector $5\underset{\sim}{\boldsymbol{i}} - \lambda\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}$ is perpendicular to the line $\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{i}} - 4\underset{\sim}{\boldsymbol{j}} + t(2\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} - 4\underset{\sim}{\boldsymbol{k}})$. Find the value of $\lambda$. [3]

(c). Obtain the equation of the plane that passes through $(1, -2, 2)$ and it's perpendicular to the line $\frac{x-9}{4} = \frac{y-6}{-1} = \frac{z-8}{1}$. [5]

**Question 13:**

The parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ represent a curve.

(i). Find the cartesian equation of the curve. [4]

(ii). Determine the turning points of the curve and their nature. [3]

(iii). State the asymptotes and intercepts of the curve. [3]

(iv). Hence sketch the curve. [2]

**Question 14:**

(a). Determine the maximum value of the expression $6\sin x - 3\cos x$. [3]

(b). Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$ . [3]

(c). In a triangle ABC, prove that $\sin B + \sin C - \sin A = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.
[6]

**Question 15:**

(a). Simplify $(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$ expressing your answer in the form $a + bi$. [5]

(b). If $z = x + yi$ , where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Show that the locus of $Arg\left(\frac{z-1}{z-i}\right) = \frac{\pi}{3}$ is a circle. Find its centre and radius. [7]

**Question 16:**

(a). Using the substitution $y = ux$, solve the differential equation
$x^2\frac{dy}{dx} = x^2 + xy + y^2$. [4]

(b). The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20 cm in 1 hour and by 19 cm in the next hour. Find the depth at which the leak is located.
[8]

*****END*****