P425/1
PURE MATHEMATICS
Paper 1
Oct/Nov. 2022
3 hours

PRE-UNEB SET 4

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and only **five** in section **B**.*

*Any additional question(s) answered will **not** be marked.*

*Each question in section **A** carries **5** marks while each question in section **B** carries **12** marks.*

*All working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Silent , non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Sketch the locus of $|z-2-3i|=4$ given $z=x+yi$. State the greatest value of $|z+1+i|$. *(05 marks)*

2. When the polynomial x^3+4x^2+ax+b is divided by $(x+1)^2+3$, the remainder is $2x-4$. Determine the values of a and b. *(05 marks)*

3. A parabola in polar form is given as $r=4\cot\theta\cos ec\theta$. Find the Cartesian equation of this parabola. Hence state the directrix and focus. *(05 marks)*

4. Solve the equation: $2tanx = 3\tan\,(45^0 - x)$ for $-180^0 \leq x \leq 180^0$ *(05 marks)*

5. Find $\int_0^{\sqrt{\pi}} x\cos^2(x^2)dx$ correct to 4 significant figures. *(05 marks)*

6. Find the volume generated by rotating the area enclosed by the curve $y = 1 + x^2$ and line $y=1$ about the x-axis from $x=0$ to $x=2$. Leave π in your solution. *(05 marks)*

7. Solve the differential equation ; $\dfrac{dy}{dx}=\dfrac{2x-1}{1-x}$ given that $y(1)=2$ *(05 marks)*

8. Show that the lines with vector equations $\boldsymbol{r}=2\lambda\boldsymbol{i}-3\boldsymbol{j}+(\lambda-2)\boldsymbol{k}$ and $\boldsymbol{r}=(\mu+1)\boldsymbol{i}+(2-\mu)\boldsymbol{j}+(2\mu-5)\boldsymbol{k}$ are skew. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt only **five** questions from this section.*

9. (a) Show that the lines $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z}{4}$ and $r=\begin{pmatrix}3\\-1\\4\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix}$ are parallel to each other. Hence find the Cartesian equation of the plane containing the two lines. *(08 marks)*

(b) Find the angle between the line $\frac{x+2}{2}=5-y=\frac{z-5}{-2}$ and the plane $3x-4y+6z=4$. *(04 marks)*

10.(a) Expand $\sqrt[3]{1-5x}$ as far as the term in x^3 and state the range of values of x for which the expansion is valid. Hence estimate $\sqrt[3]{22}$ correct to 4 decimal places. *(07 marks)*

(b) Solve the equation: $log_8 x^3 = log_2 32 + log_x 64$ *(05 marks)*

11.Express $\frac{x^4-6x^2+3}{x^3+2x^2+x}$ in partial fractions.

Hence evaluate $\int_1^2 \frac{x^4-6x^2+3}{x^3+2x^2+x}\,dx$ *(12 marks)*

12.(a) Solve the inequality $\frac{x^2-1}{x^2-4}\geq\frac{1}{5}$ *(06 marks)*

(b) Solve the equation: $\frac{x^2+4x}{3}+\frac{84}{x^2+4x}=11$ *(06 marks)*

13.(a) Solve the equation $\tan 2x\cos x+\sin x=3\sin 3x$ for $0^0 \leq x \leq 180^0$.

(06 marks)

(b) Show that $\cos ec4\theta-\cot 4\theta=\tan 2\theta$. Hence or otherwise solve $2\sin 4\theta=3\cot 2\theta$ for $0^0 \leq\theta\leq 90^0$. *(06 marks)*

14.(a) Differentiate $\frac{x^2+1}{\sqrt[3]{x^2-1}}$ and simplify to the simplest form possible.

(05 marks)

(b) Water runs into a conical vessel fixed with its vertex downwards at the rate $3\pi cm^3 s^{-1}$, filling the vessel to a depth of 15cm in one minute. Find the rate at which the depth of water in the vessel is increasing when the water has been running for $7\frac{1}{2}$ seconds. *(07 marks)*

15.(a) Find the equation of the normal to the parabola to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$. The normal line meets the directrix at point Q. Find the equation of locus of N the midpoint of PQ. *(08 marks)*

(b) Find the equation of locus of a point P which is twice as far from the origin as it is from the point (9,12). *(04 marks)*

16.(a) If $y = \frac{A}{x} + Bx$ where A and B are constants. Form a differential equation independent of constants A and B. *(04 marks)*

(b) According to Newton's law of cooling, the rate at which a body cools is directly proportional to the difference between temperature θ of the body and the temperature θ_0 of the surrounding air (assumed to be constant). If a body cools from 100^0 to 80^0 in 10 minutes and from 80^0 to 65^0 in another 10 minutes.

(i) form a differential equation connecting θ and t. *(01 mark)*

(ii) By solving the differential equation, find the value of θ_0. *(07 marks)*

GOOD LUCK