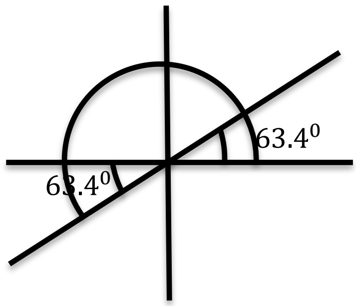
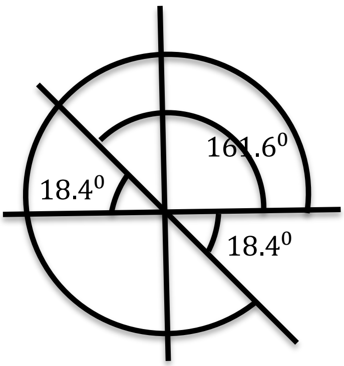
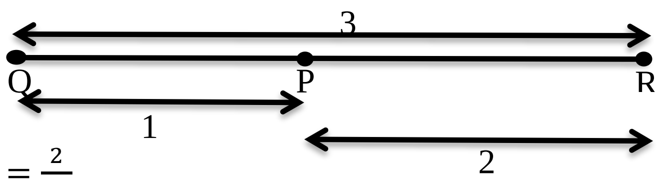
Proposed UACE guide mtc1 2022

SECTION A. (40 Marks)

NO.	ANSWERS	MARKS	COMMENT
1	From, $2log_{10}y = log_{10}2 + log_{10}x$ $log_{10}y^2 = log_{10}2x$ $y^2 = 2x$-------------------------------------(i) Also from, $2^y = 4^x$ $2^y = 2^{2x}$ $y = 2x$---------------------------------------(ii) Equating (i) and (ii) $y^2 = y$ $y^2 - y = 0$ $y(y-1) = 0$ Either, $y = 0$ Or, $(y-1) = 0$ ⇨ $y = 1$ For, $y = 0, 0 = 2x,\ x = 0$ For, $y = 1, 1 = 2x,\ x = \frac{1}{2}$	M1 M1 M1 M1 A1	5 Marks
2	$5tan^2A - 5tanA = 2(1 + tan^2A),\ sec^2A = 1 + tan^2A$ $3tan^2A - 5tanA - 2 = 0$ Let, $tanA = x$ $3x^2 - 5x - 2 = 0$ $x = \frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-2)}}{2(3)}$ $x = 2, x = -\frac{1}{3}$ For, $x = 2$, $tanA = 2$, $A = tan^{-1}(2) = 63.4^0$  $A = 63.4^0, 243.4^0$ For, $x = -\frac{1}{3}$, $tanA = -\frac{1}{3}$, $A = tan^{-1}\left(-\frac{1}{3}\right) = 161.6^0$  $A = 161.6^0, 341.6^0$	M1 M1 M1 M1 A1	5 Marks

3	 $\frac{PR}{RQ} = \frac{2}{-3}$ $-3PR = 2RQ$ $-3(0R - 0P) = 2(0Q - 0R)$, $-30R + 30P = 20Q - 20R$, $0R = 30P - 20Q$ $0Q = 3\begin{pmatrix}1\\-2\\1\end{pmatrix} - 2\begin{pmatrix}3\\-4\\6\end{pmatrix}$ $0Q = \begin{pmatrix}3\\-6\\3\end{pmatrix} - \begin{pmatrix}6\\-8\\12\end{pmatrix}$ $0Q = \begin{pmatrix}-3\\2\\9\end{pmatrix}$ Therefore the coordinates of R are $(-3,\ 2, -9)$	M1 M1 M1 M1 A1	5 Marks
4	$\frac{d}{dx}[x^3 + 2y^3 + 3xy = 0]$ $3x^2 + 6y^2\frac{dy}{dx} + 3\left(y + x\frac{dy}{dx}\right) = 0$ $(6y^2 + 3x)\frac{dy}{dx} = -(3x^2 + 3y)$ $\frac{dy}{dx} = \frac{-(3x^2+3y)}{(6y^2+3x)}$ At (2,-1) $\frac{dy}{dx} = \frac{-(3(2)^2+3(-1))}{(6(-1)^2+3(2))} = -\frac{3}{4}$ Therefore the gradient of the tangent at (2,-1) is; $-\frac{3}{4}$ From $y = mx + c$ $(-1) = \left(-\frac{3}{4}\right)(2) + c$ $c = \frac{1}{2}$ Therefore the equation of the tangent is,$y = \left(-\frac{3}{4}\right)x + \frac{1}{2}$	M1 M1 B1 M1 A1	5 Marks
5	$\frac{(5-4x)}{(1-x)} - 3 < 0$ $\frac{(5-4x)-3(1-x)}{(1-x)} < 0$ $\frac{(5-4x-3+3x)}{(1-x)} < 0$ $\frac{(2-x)}{(1-x)} < 0$, The critical values of x; $x - 1 = 0$, $x = 1$ $x - 2 = 0$	M1 M1 M1	

<table>
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$x = 2$

	$x < 1$	$1 < x < 2$	$x > 2$
$2 - x$	$+$	$+$	$-$
$1 - x$	$+$	$-$	$-$
$\frac{2-x}{1-x}$	$+$	$-$	$+$

The solution set is; $1 < x \leq 2$</td><td>B1

A1</td><td>

5 Marks</td></tr>
<tr><td>6</td><td>

$\frac{dv}{dt} = -2cm^3s^{-1}$
But , $v = \frac{1}{3}\pi x^3$
$\frac{dv}{dx} = \frac{d}{dx}\left[\frac{1}{3}\pi x^3\right] = \pi x^2$
But also, $\frac{dx}{dt} = \frac{dx}{dv} x \frac{dv}{dt}$
$\frac{dx}{dt} = \frac{1}{\pi x^2}(-2) = \frac{-2}{\pi x^2}$
$=> \frac{dx}{dt} = \frac{-2}{\pi(5)^2} = \frac{-2}{25\pi} cms^{-1}$</td><td>M1
M1
B1
M1 A1</td><td>

5 Marks</td></tr>
<tr><td>7</td><td>

$x - 3y - 4 = 0$ --------------------------------(i)
$3x + y - 2 = 0$---------------------------------(ii)
Solving (i) and (ii) simultaneously;
$3(ii) + (i)$
$10x = 10$
$x = 1$
Substituting $x = 1$ into (ii)
$3(1) + y = 2,$
$y = -1$
Point of intersection is (1,-1)
$4y + 3x = 0$
$y = \frac{-3}{4}x$
From $m_1 m_2 = -1,$
$m_2 = \frac{-1}{\frac{-3}{4}} = \frac{4}{3}$
From, $y = mx + c$
$(-1) = \left(\frac{4}{3}\right)(1) + c$
$c = \frac{-7}{3}$
$y = \frac{4}{3}x - \frac{7}{3}$</td><td>M1
B1
M1
M1
A1</td><td>

5 Marks</td></tr>
<tr><td>8</td><td>

Let, $y = vx$
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
$x\left(v + x\frac{dv}{dx}\right) = (2vx + x)$
$\left(v + x\frac{dv}{dx}\right) = 2v + 1$
$x\frac{dv}{dx} = v + 1$</td><td>M1
M1</td><td></td></tr>
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	$\int \frac{1}{v+x} dv = \int \frac{1}{x} dx$	M1	
	$\ln(v+1) = \ln x = c$	B1	
	But, $v = \frac{y}{x}$		
	$\ln\left(\frac{y}{x}+1\right) = \ln x + c$	A1	5 Marks

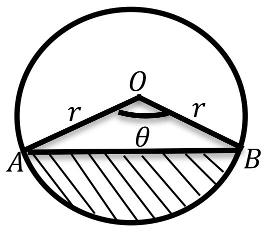
SECTION B. (60 Marks)

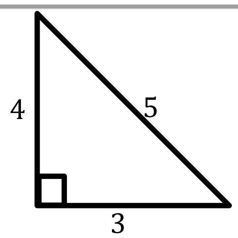
QN	ANSWER	MARKS	COMMENT
09 (a)	$a = 2$		
	$r = \frac{6}{2} = 3$		
	$n = 10$	M1	
	$S_n = a\frac{(r^n-1)}{r-1} = 2\frac{(3^{10}-1)}{3-1}$	M1B1	
	$S_n = 3^{10} - 1 = 59048$	A1	
(b)	$(a+4d) + (a+15d) = 44$		
	$2a + 19d = 44$---------------------------------(i)	M1B1	
	$\left(\frac{18}{2}\right)(2a+12d) = 3x\frac{10}{2}(2a+9d)$		
	$3(2a+12d) = 5(2a+9d)$	M1	
	$3d - 2a = 0$-------------------------------------(ii)		
	Solving (i) and (ii) simultaneously;	M1	
	$22d = 44$	A1	
	⇨ $d = 2$		
	The common difference is 2		
	$3(2) - 2a = 0$		
	⇨ $a = 3$		
	The first term is 3	A1	
	$S_{30} = \frac{30}{2}(2(3) + 29(2))$		
	$S_{30} = 15(6+58)$		
	$S_{30} = 960$	M1A1	
	TOTAL		**12 MARKS**
10	Let; $\frac{11x-1}{(1-x)^2(2+3x)} \equiv \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{C}{(2+3x)}$	M1	
	$11 - x \equiv A(1-x)(2+3x) + B(2+3x) +$ $C(1-x)^2$		
	For $x = 1$;	M1	
	$11 - 1 = 5B$		
	$=> B = 2$		
	For $x = -\frac{2}{3}$	M1	

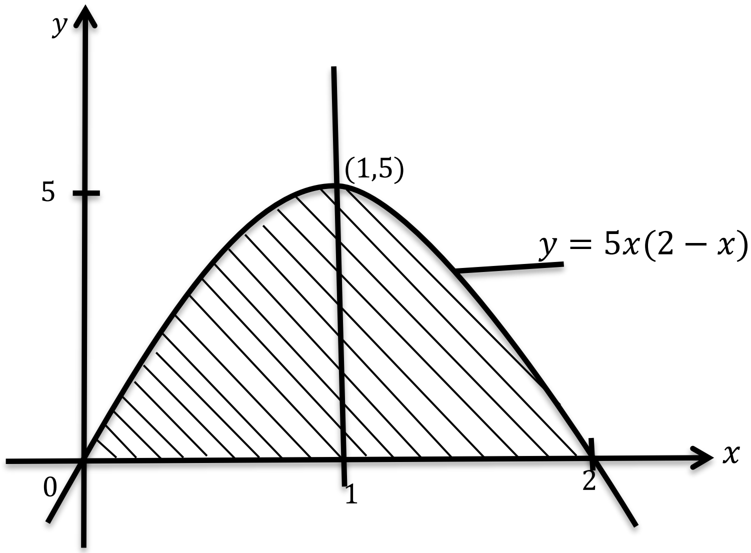
	$\left(-\frac{22}{3}\right)-1=C\left(\frac{5}{3}\right)^2$ $C=-3$ For $x=0$ $-1=A(1)(2)+B(2)+C$ $-1=2A+(2)(2)-3$ $2=2A+4$ $A=-1$ Therefore; $\frac{11x-1}{(1-x)^2(2+3x)}\equiv\frac{-1}{(1-x)}+\frac{2}{(1-x)^2}+\frac{-3}{(2+3x)}$ $\int_0^{\frac{1}{2}}\frac{11x-1}{(1-x)^2(2+3x)}dx=\int_0^{\frac{1}{2}}\frac{-1}{(1-x)}dx+\int_0^{\frac{1}{2}}\frac{2}{(1-x)^2}dx+\int_0^{\frac{1}{2}}\frac{-3}{(2+3x)}dx$ $=[\ln(1-x)]_0^{\frac{1}{2}}+\left[2\left(\frac{1}{1-x}\right)\right]_0^{\frac{1}{2}}-\frac{3}{3}[\ln(2+3x)]_0^{\frac{1}{2}}$ $=\left[\left[\ln(1-\frac{1}{2})\right]+\left[2\left(\frac{1}{1-\frac{1}{2}}\right)\right]-\frac{3}{3}\left[\ln(2+3\frac{1}{2})\right]\right]-\left[[\ln(1-0)]+\left[2\left(\frac{1}{1-0}\right)\right]-\frac{3}{3}[\ln(2+3(0))]\right]$ $=2+\ln\left(\frac{2}{7}\right)$	M1 M1 B1 M1 M1 M1B1 A1	
	TOTAL		**12 MARKS**
11 (a)	$\overrightarrow{b_1}=\begin{pmatrix}1\\1\\2\end{pmatrix},\overrightarrow{b_2}=\begin{pmatrix}2\\3\\1\end{pmatrix}$ $\vec{n}=\vec{b}_1x\vec{b}_2=\begin{pmatrix}1\\1\\2\end{pmatrix}x\begin{pmatrix}2\\3\\1\end{pmatrix}$ $\vec{n}=\begin{vmatrix}i&j&k\\1&1&2\\2&3&1\end{vmatrix}$ $\vec{n}=i\begin{vmatrix}1&2\\3&1\end{vmatrix}-j\begin{vmatrix}1&2\\2&1\end{vmatrix}+k\begin{vmatrix}1&1\\2&3\end{vmatrix}$ $\vec{n}=-5i+3j-4k=\begin{pmatrix}-5\\3\\1\end{pmatrix}$ $\vec{a}=\begin{pmatrix}-1\\2\\-4\end{pmatrix}$ From, $\vec{r}.\vec{n}=\vec{n}.\vec{a}$ $=>\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-5\\3\\1\end{pmatrix}=\begin{pmatrix}-5\\3\\1\end{pmatrix}.\begin{pmatrix}-1\\2\\-4\end{pmatrix}$	M1 M1 M1B1 M1 M1B1	

	$-5x+3y+z=5+6-4$ $-5x+3y+z=7$, This gives the equation of the plane.	A1	
(b)	Distance $=\left\lvert\frac{-5x+3y+z-7}{\sqrt{(-5)^2+(3)^2+(1)^2}}\right\rvert$ At origin, (0,0,0) Distance $=\left\lvert\frac{-5(0)+3(0)+(0)-7}{\sqrt{(-5)^2+(3)^2+(1)^2}}\right\rvert=\frac{1}{5}\sqrt{35}units$	M1M1 B1A1	
	TOTAL		**12 MARKS**
12	$\left(\frac{1+3x}{1-x}\right)^{\frac{1}{2}}=(1+3x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ From, $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3---$ For, $(1+3x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)(3x)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(3x)^2+\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(3x)^3---$ ⇨ $(1+3x)^{\frac{1}{2}}=1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3---$ For, $(1-x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(-x)^2+\frac{(-\frac{1}{2})(-\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-x)^3---$ ⇨ $(1-x)^{-\frac{1}{2}}=1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3---$ $(1+3x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}=\left(1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3---\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3---\right)$ ⇨ $\left(\frac{1+3x}{1-x}\right)^{\frac{1}{2}}=1+2x+2x^3$ For, $x=\frac{1}{5}$ $\left(\frac{1+3(\frac{1}{5})}{1-(\frac{1}{5})}\right)^{\frac{1}{2}}=1+2(\frac{1}{5})+2(\frac{1}{5})^3$ $\sqrt{8}=\frac{177}{125}=2.83\ (2\ dps)$	M1 M1M1 M1 M1M1 M1 M1B1 A1 M1 A1	
	TOTAL		**12 MARKS**
13	$y^2=4ax$ $2y\frac{dy}{dx}=4a$	M1	

	$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$		
	At $P(at^2, 2at)$		
	$\frac{dy}{dx} = \frac{2a}{(2at)} = \frac{1}{t}$	M1	
	Therefore the gradient of the tangent at P is $\frac{1}{t}$		
	From, $y = mx + c$		
	$(2at) = \left(\frac{1}{t}\right)(at^2) + c$		
	$c = at$	M1	
	$ty = x + at^2$------------------------(i)		
	This gives the equation of the tangent at P.		
	Since the chord and the tangent are parallel, they have the same gradient, $\frac{1}{t}$		
	From, $y = mx + c$		
	$(0) = \left(\frac{1}{t}\right)(0) + c$		
	$c = 0$	M1	
	$y = \left(\frac{1}{t}\right)x$, which gives the equation of the chord.		
	For points of intersection of the chord with the parabola,		
	From, $y^2 = 4ax$		
	But $x = yt$,	M1	
	$y^2 = 4a(yt)$		
	$y(y - 4at) = 0$		=>
	$y = 0$ *or* $y = 4at$	B1	
	For, $y = 0$,		
	$x = ((0)t),\ x = 0$		
	For, $y = 4at$,	M1	
	$x = (4at)t, x = 4at^2$		
	The coordinates of Q; $(4at^2, 4at)$	M1	
	From, $\frac{dy}{dx} = \frac{2a}{y}$		
	At $Q(4at^2, 4at)$		
	$\frac{dy}{dx} = \frac{2a}{(4at)} = \frac{1}{2t}$		
	Therefore the gradient of the tangent at Q is $\frac{1}{2t}$	M1	
	From, $y = mx + c$		
	$(4at) = \left(\frac{1}{2t}\right)(4at^2) + c$		
	$c = 2at$	B1	
	$2ty = x + 2at^2$----------------------(ii)		
	This which gives the equation of the tangent at Q.		

	Solving the equations (i) and (ii) simultaneously, $$\begin{array}{r} ty = x + at^2 \\ -\ 2ty = x + 4at^2 \\ \hline y = 3at \end{array}$$ $\Rightarrow\ x = t(3at) - at^2,\ x = 2at^2$ Therefore $R(2at^2, 3at)$	M1 A1	
	TOTAL		**12 MARKS**
14 (a)	$\frac{d}{dx}\left[\frac{(x^2+1)}{(x+1)^3}\right] = \frac{(x+1)^3(2x)-(x^2+1)(3(x+1)^2)}{[(x+1)^3]^2}$ $= \frac{(x+1)^2[(x+1)(2x)-3(x^2+1)]}{(x+1)^6}$ $= \frac{(x+1)^2[2x-x^2-3]}{(x+1)^6}$ $\frac{d}{dx}\left[\frac{(x^2+1)}{(x+1)^3}\right] = \frac{-x^2+2x-3}{(x+1)^4}$	M1 M1 M1 A1	
(b)	From, $x = \frac{3t}{t+3}$ $\frac{dx}{dt} = \frac{(t+3)(3)-3t(1)}{(t+3)^2}$ $\frac{dx}{dt} = \frac{9}{(t+3)^2}$ ---(i) Also, $\frac{dy}{dt} = \frac{(t-2)(4)-(4t+1)(1)}{(t-2)^2}$ $\frac{dy}{dt} = -\frac{9}{(t-2)^2}$ --(ii) But, $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \left(-\frac{9}{(t-2)^2}\right)\left(\frac{(t+3)^2}{9}\right)$ $\frac{dy}{dx} = -\left[\frac{(t+3)^2}{(t-2)^2}\right]$ ---------------------------------------(iii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$ $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left[-\left[\frac{(t+3)^2}{(t-2)^2}\right]\right]$ $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{(t-2)^2(2(t+3)-(t+3)^2(2(t-2))}{(t-2)^4} = \frac{10(t-2)(t+3)}{(t-2)^4}$ $\frac{d^2y}{dx^2} = \left(\frac{10(t-2)(t+3)}{(t-2)^4}\right)\left(\frac{(t+3)^2}{9}\right) = \frac{10}{9}\left(\frac{t+3}{t-2}\right)^3$	M1 M1 M1 B1 M1 M1 M1 A1	
	TOTAL		**12 MARKS**
15 (a)	 	B1	

	Area of sector, $AB = \frac{\theta}{360}\pi r^2 = \frac{\theta}{2\pi}\pi r^2 = \frac{\theta}{2}r^2$ Area of triangle, $OAB = \frac{1}{2}r.rsin\theta = \frac{r^2 sin\theta}{2}$ Area of the minor segment $AB = \frac{\theta}{2}r^2 - \frac{r^2 sin\theta}{2}$ $= \frac{r^2}{2}(\theta - sin\theta)$ But area of the circle $= \pi r^2$ But the area of the circle is three times the area of the minor segment. Therefore, $\pi r^2 = 3x\frac{r^2}{2}(\theta - sin\theta)$ $2\pi = 3\theta - 3sin\theta$ $3\theta = 3sin\theta + 2\pi$ as required	M1 M1 M1 A1	
(b) (i)	From, $tan\alpha = sec\alpha - \frac{1}{3}$ ---------------------------(i) Squaring both sides of (i), $tan^2\alpha = \left(sec\alpha - \frac{1}{3}\right)^2$ But $tan^2\alpha = (sec^2\alpha - 1)$ Therefore, $(sec^2\alpha - 1) = \left(sec^2\alpha - \frac{2}{3}sec\alpha + \frac{1}{9}\right)$ $-1 = -\frac{2}{3}sec\alpha + \frac{1}{9}$ $-\frac{10}{9} = -\frac{2}{3}sec\alpha$ $sec\alpha = \frac{5}{3}$ ⇨ $cos\alpha = \frac{3}{5} = 0.6$	M1 M1 B1 M1 A1	
(ii)	 From the figure, $tan\alpha = \frac{4}{3} = 1.3333$	B1 A1	
	TOTAL		**12 MARKS**
16 (a)	From, $y = 5x(2 - x)$ Intercept, When $x = 0, y = 0 => (0,0)$ When $y = 0, 0 = 5x(2 - x)$ $x = 0, => (0,0)$ $and\ x = 2 => (2,0)$ Turning points,	M1 B1	

	At turning points, $\frac{dy}{dx} = 0$ $\frac{d}{dx}[5x(2-x)] = 0$ $\frac{d}{dx}[10x - 5x^2] = 0$ $(10 - 10x) = 0$ $x = 1, y = 5$ Therefore the turning point is $(1,5)$  	M1 M1 B1 B1B1	
(b)	From, $volume = \pi \int_a^b y^2 dx$ $= \pi \int_0^2 (10x - 5x^2)^2 dx$ $= \pi \int_0^2 (100x^2 - 100x^3 + 25x^4)\, dx$ $= 25\pi \left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right]_0^5$ $= 25\pi \left[\left[\frac{4}{3}(5)^3 - (5)^4 + \frac{1}{5}(5)^5\right] - \left[\frac{4}{3}(0)^3 - (0)^4 + \frac{1}{5}(0)^5\right]\right]$ Volume $= 25\pi x \frac{16}{15} = \frac{80}{3}\pi\ units$	M1 M1 M1 A1	
	TOTAL		**12 MARKS**