Examination questions:

1. The r th term of an arithmetic series is $(2r - 5)$.

   (a) Write down the first three terms of this series and state the value of the common difference.

   (b) Show that $\sum_{r=1}^{n}(2r - 5) = n(n - 4)$.

2. (a) The terms of an arithmetic sequence are given by $u_n = (n+k)^2$, $n \geq 1$, where k is a positive constant. Given that $u_2 = 2u_1$, find the value of k and show that $u_3 = 11 + 6\sqrt{2}$.

   (b) Prove that;

   $\log a + \log ax + \log ax^2 + \cdots \dots \dots to\ n\ terms = n \log a + \frac{1}{2}n(n-1)\log x$.

3. The 5^{th} and 12^{th} terms of an AP are 2 and 23 respectively. Find:

   (a) The 9^{th} term.

   (b) The sum of the first 20 terms.

4. There are 20 terms of an AP. The sum of the first 10 terms is 55 and the sum of the last 10 terms is 355. Find the first term and the common difference.

5. If the 9^{th} term of an AP is 3 times the 3^{rd} term and the sum of the first 10 terms is 110, find the first term and the common difference.

6. In an AP the sum of the first three terms is 12 and their product is 28. Find the possible values of the first term and the common difference.

7. The numbers $x+3$, $5x+3$ and $11x+3$ $(x \neq 0)$ are three consecutive terms of a GP. Find the value of x and the common ratio.

8. (a) The sum of the first 8 terms of an arithmetic progression is 24 and the sum of the first 18 terms is 90. Calculate the value of the seventh term.

   (b) A geometric progression with a positive common ratio is such that the sum of the first 2 terms is $17\frac{1}{2}$ and the third term is $4\frac{2}{3}$. Calculate the value of the common ratio.

9. (a) An arithmetic progression contains 20 terms. Given that the 8^{th} term is 25 and the sum of the last 8 terms is 404, calculate the sum of the first 10 terms.

(c) The first term of a geometric progression exceeds the second term by 2, and the sum of the second and the third term is $\frac{4}{3}$.

   Calculate the possible values of the first term and the common ratio of the progression. Given further that all the terms of the progression are positive, calculate the sum to infinity.

10. (a) The first term of an arithmetic progression is 4. The sum of the first 10 terms is 100 and the sum of the whole series is 136.

    Calculate (i) the common difference, (ii) the number of terms, (iii) the last term.

    (b) Find the sum of the integers between 50 and 150 which are divisible by 8.

11. The fourth term of an arithmetic series is $3k$, where k is a constant , and the sum of the first six terms of the series is $7k+9$.

    (a) Show that the first term of the series is $9-8k$.

    (b) Find an expression for the common difference of the series in terms of k.

    Given that the seventh term of the series is 12, calculate:

(c) The value of k.

(d) The sum of the first 20 terms of the series.

12. (a) Evaluate:

$$\sum_{r=1}^{75}(1-2r^2).$$

(b) Show that $\sum_{r=1}^{n}\frac{r+3}{2}=kn(n+7)$ where k is a rational constant to be found.

13. The first two terms of an arithmetic series are $(x-2)$ and (x^2+4) respectively, where x is a positive constant.

(a) Given also that the third term of the series is 20, find the value of x.

(b) Given that the sum of n terms of the series is 277. Find the n

14. A geometric series has first term a and common ratio r. The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2-25r+4=0$ and find the two possible values of r and a.

(b) Show that the sum S_n of the first n terms of the series is given by $S_n=25(1-r^n)$.

(c) Given that r takes the larger of the two possible values, find the smallest value of n for which S_n exceeds 24.

15. The first three terms of a geometric series are $(p-2)$, $(p+6)$ and p^2 respectively.

(a) Show that p must be a solution of the equation $p^3-3p^2-12p-36=0$.

(b) Verify that $p=6$ is a solution of equation and show that there are no other real solutions.

(c) Using $p=6$, find the common ratio of the series, and the sum of the first six terms of the series.

16. The second and fifth terms of a geometric series are -48 and 6 respectively.

(a) Find the first term and the common ratio of the series.

(b) Find the sum to infinity of the series.

(c) Show that the difference between the sum of the first n terms of the series and its sum to infinity is given by 2^{6-n}.

17. The second and third terms of a geometric series are $\log_3 4$ and $\log_9 256$ respectively.

(a) Show that the common ratio of the series is 2.

(b) Show that the first term of the series is $\log_3 2$.

(c) Find, to 1 decimal place, the sum of the first five terms of the series.

18. The sum of the first three terms of a geometric series is 210. The sum to infinity of the series is 480.

(a) Find the two possible values of the common ratio and first term.

(b) Given that r is positive and that the sum of the first n terms of the series is greater than 300. Calculate the smallest possible value of n.

19. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is $\frac{1}{2}n[2a + (n-1)d]$.

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence. He repays \$149 in the first month, \$147 in the second month, \$145 in the third month, and so on.

(b) Find the amount Sean repays in the 21^{st} month.

Over the n months, he repays a total of \$5,000.

(c) Form an equation in n, and show that your equation may be written as $n^2 - 150n + 5000 = 0$. Find n.

(d) State, with a reason, which of the solutions to the equation in part (c) is not a sensible solution to the repayment problem.

20. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was \$500 and on each following birthday the allowance was increased by \$200.

(a) Show that, immediately after her 12^{th} birthday, the total of the allowance that Alice had received was \$1200.

(b) Find the total of the allowances that Alice had received up to an including her 18^{th} birthday.

(c) When the total of the allowances that Alice received reached \$32,000 the allowance stopped. Find how old Alice was when she received the last allowance.

21. An athlete prepares for a race by completing a practice run on each of the 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11^{th} day, and he runs a total of 77 km over the 11 day period. Find the value of a and the value of d.

22. (a) Prove that the sum of the first n terms of an arithmetic series with first term a and common difference d is given by $\frac{1}{2}n[2a + (n-1)d]$.

A novelist begins writing a new book. She plans to write 16 pages during the first week, 18 during the second and so on, with the number of pages increasing by 2 each week. Find, according to her plan:

(a) How many pages she will write in the fifth week.

(b) The total number of pages she will write in the first five weeks.

(c) Using algebra, find how long it will take her to write the book if it has 250 pages.

23. As part of a new training program, Habib decides to do sit-ups everyday. He plans to do 20 per day in the first week, 22 per day in the second week, 24 per day in the third week and so on, increasing the daily number of sit-ups by two at the start of each week.

(a) Find the number of sit-ups that Habib will do in the fifth week.

(b) Show that he will do a total of 1512 sit-ups during the first eight weeks.

(c) In the nth week of training, the number of sit-ups that Habib does is greater than 300 for the first time. Find the value of n.

24. (a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$

Mr. King will be paid a salary of \$35 000 in the year 2005. Mr. King's contract promises a 4% increase in salary every year, the first increase being in 2006, so that his annual salaries form a geometric sequence.

(b) Find to the nearest \$100, Mr. King's salary in the year 2008.

(c) Mr. King will receive a salary each year from 2005 until he retires at the end of 2024. Find to the nearest \$1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.

Examination Questions

1. (a) $(-3)+(-1)+1;\ d=2$

   (b) $\sum_{r=1}^{n}(2r-5)=n(n-4)$
2. (a) $k=\sqrt{2}$
3. (a) $U_9=14$ (b) $S_{20}=370$
4. $(a=-8, d=3)$
5. $(a=2, d=2)$
6. $(d=-3, a=7)$ *or* $(d=3, a=1)$
7. $\left(x=\frac{3}{7}, r=\frac{3}{2}\right)$
8. (a) $U_7=4$ (b) $r=\frac{2}{3}$
9. (a) $S_{10}=175$ (b) $\left(r=\frac{1}{3}, a=3\right)$ *or* $\left(r=-2, a=\frac{2}{3}\right);\ S_\infty=\frac{9}{2}$
10. (a) (i) $d=\frac{4}{3}$ (ii) $n=12$ (iii) $\frac{56}{3}$

    (b) 1200
11. (a) $a=9-8k$ (b) $d=\frac{1}{3}(11k-9)$ (c) $k=\frac{3}{2}$ (d) $S_{20}=415$
12. (a) -16725

    (b) $\frac{1}{4}n(n+7);\ k=\left(\frac{1}{4}\right)$
13. (a) $x=\frac{5}{2}$ (b) $n=8$
14. (a) $\left(r=\frac{1}{5}, a=20\right)$ *or* $\left(r=\frac{4}{5}, a=5\right.$ *b* $n=15$
15. (c) $r=3;\ S_6=1456$
16. (a) $a=96;\ r=-\frac{1}{2}$ (b) $S_\infty=64$
17. (c) 20.2
18. (a) $\left(r=-\frac{3}{4}, a=840\right)$ *or* $\left(r=\frac{3}{4}, a=120\right.$ *b* $n=4$
19. (b) \$109 (c) $n=100$ *or* $n=50$

    (d) $U_{100}<0;\ \therefore n=100$ *is not sensible.*
20. (b) \$9600 (c) 26 *yrs*
21. $a=5;\ d=\frac{2}{5}$
22. (a) 24 *pages* (b) 100 *pages* (c) 10 *weeks*
23. (a) 196 *sit ups* (c) $n=13$
24. (b) 39400 (c) 1,042,000