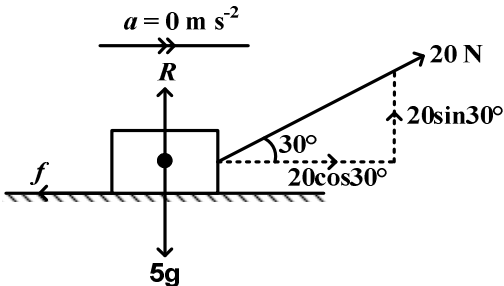
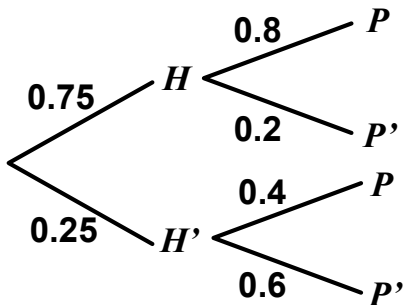


**ALEVEL APPLIED MATHEMATICS PAPER TWO MOCK MARKING GUIDE 2023**

SNo.	Working	Marks
1	$P(X < 4) = P(X > 4)$ $a = b + c$ $a - b = c \rightarrow (1)$ $P(X \leq 5) = 2P(X > 5)$ $a + b = 2c \rightarrow (2)$ <p>Equation (1) + (2) gives:</p> $2a = 3c$ $a = 1.5c \rightarrow (3)$ <p>Equation (2) - (1) gives:</p> $2b = c$ $b = 0.5c \rightarrow (4)$ <p>but,</p> $\sum_{all\ x} P(X = x) = 1$ $a + b + c = 1$ $1.5c + 0.5c + c = 1$ $3c = 1$ $c = \frac{1}{3}$ <p>From equation (3),</p> $a = 1.5 \times \frac{1}{3} = \frac{1}{2}$ <p>From equation (4),</p> $b = 0.5 \times \frac{1}{3} = \frac{1}{6}$	<p><b>B1</b>-eqn 1</p> <p><b>B1</b>-eqn 2</p> <p><b>M1</b>-solving to get <math>a</math> and <math>b</math> in terms of <math>c</math></p> <p><b>M1</b>-substitution and equating to 1</p> <p><b>A1</b>-all values of <math>a</math>, <math>b</math> and <math>c</math> correct</p>
		<b>05</b>
2	<p>For the first ball,</p> $u_1 = 0 \text{ m s}^{-1}, \quad t_1 = t, \quad s_1 = h$ $s_1 = u_1 t_1 + \frac{1}{2} g t_1^2$ $h = 0 + \frac{1}{2} \times 9.8 \times t^2$ $h = 4.9 t^2 \rightarrow (1)$ <p>For the second ball,</p> $u_2 = 14 \text{ m s}^{-1}, \quad t_2 = t - 1, \quad s_2 = h$ $s_2 = u_2 t_2 + \frac{1}{2} g t_2^2$	<p><b>M1</b>-eqn for motion of the first ball</p>

	$h = 14 \times (t - 1) + \frac{1}{2} \times 9.8 \times (t - 1)^2$ $4.9t^2 = 14t - 14 + 4.9 \times (t^2 - 2t + 1)$ $4.9t^2 = 14t - 14 + 4.9t^2 - 9.8t + 4.9$ $0 = 4.2t - 9.1$ $t = \frac{13}{6} \approx 2.1667 \text{ s}$ <p>From equation (1),</p> $h = 4.9 \times \left(\frac{13}{6}\right)^2 = \frac{8281}{360} \approx 23.0028 \text{ m}$	<b>M1</b> -eqn for motion of the 2 <sup>nd</sup> ball and subs. For $h$  <b>A1</b> -value of $t$  <b>M1 A1</b> -substitution and output																												
		<b>05</b>																												
3	$h = \frac{\frac{\pi}{2} - 0}{5 - 1} = \frac{\pi}{8}$ <table><tr><td><math>n</math></td><td><math>x_n</math></td><td><math>y_0, y_5</math></td><td><math>y_1, \dots y_4</math></td></tr><tr><td>0</td><td>0</td><td>0.000000</td><td></td></tr><tr><td>1</td><td><math>\frac{\pi}{8}</math></td><td></td><td>0.150279</td></tr><tr><td>2</td><td><math>\frac{2\pi}{8}</math></td><td></td><td>0.555360</td></tr><tr><td>3</td><td><math>\frac{3\pi}{8}</math></td><td></td><td>1.088420</td></tr><tr><td>4</td><td><math>\frac{\pi}{2}</math></td><td>1.570796</td><td></td></tr><tr><td>sums</td><td></td><td>1.570796</td><td>1.794059</td></tr></table> $\int_0^{\frac{\pi}{2}} x \sin x \, dx \approx \frac{1}{2} h [(y_0 + y_4) + 2(y_1 + \dots + y_3)]$ $\approx \frac{1}{2} \times \frac{\pi}{8} \times [1.570796 + 2 \times 1.794059] \approx 1.012950$ $\approx 1.0130 \text{ (4 d. p)}$	$n$	$x_n$	$y_0, y_5$	$y_1, \dots y_4$	0	0	0.000000		1	$\frac{\pi}{8}$		0.150279	2	$\frac{2\pi}{8}$		0.555360	3	$\frac{3\pi}{8}$		1.088420	4	$\frac{\pi}{2}$	1.570796		sums		1.570796	1.794059	<b>B1</b> -value of $h$  <b>B1</b> -values of $x_n$  <b>B1</b> -values of $y_n$  <b>M1</b> -substitution  <b>A1</b> -output to 4 d.p
$n$	$x_n$	$y_0, y_5$	$y_1, \dots y_4$																											
0	0	0.000000																												
1	$\frac{\pi}{8}$		0.150279																											
2	$\frac{2\pi}{8}$		0.555360																											
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4	$\frac{\pi}{2}$	1.570796																												
sums		1.570796	1.794059																											
		<b>05</b>																												
4																														

	<div></div> <p>Resolving horizontally,</p> $20 \cos 30^\circ - f = ma$ $10\sqrt{3} - f = 0$ $f = 10\sqrt{3} \approx 17.3205 \text{ N}$ <p>The friction force is 17.3205 N.</p> <p>Resolving vertically,</p> $R + 20 \sin 30^\circ = 5g$ $R + 10 = 5 \times 9.8$ $R = 39 \text{ N}$ <p>The normal reaction is 39 N.</p>	<p><b>B1</b>-force diagram</p> <p><b>M1</b>-resolving</p> <p><b>A1</b>-value of <math>f</math></p> <p><b>M1</b>-resolving and subs. For <math>g</math></p> <p><b>A1</b>-value of <math>R</math></p>																
		<b>05</b>																
5	<p>(i).</p> <table border="1"><tr><td><math>X</math></td><td>10.1</td><td><math>x_1</math></td><td>10.34</td></tr><tr><td><math>Y</math></td><td>0.8008</td><td>0.99</td><td>1.3003</td></tr></table> $\frac{x_1 - 10.1}{10.34 - 10.1} = \frac{0.99 - 0.8008}{1.3003 - 0.8008}$ $\frac{x_1 - 10.1}{0.24} = \frac{0.1892}{0.4995}$ $x_1 = 10.1 + \frac{0.1892}{0.4995} \times 0.24 = 10.1909$ <p>(ii).</p> <table border="1"><tr><td><math>X</math></td><td>9.9</td><td>10</td><td>10.1</td></tr><tr><td><math>Y</math></td><td><math>y_1</math></td><td>0.6484</td><td>0.8008</td></tr></table> $\frac{y_1 - 0.6484}{0.8008 - 0.6484} = \frac{9.9 - 10}{10.1 - 10}$ $\frac{y_1 - 0.6484}{0.1524} = \frac{-0.1}{0.1}$ $y_1 = 0.6484 - 1 \times 0.1524 = 0.496$	$X$	10.1	$x_1$	10.34	$Y$	0.8008	0.99	1.3003	$X$	9.9	10	10.1	$Y$	$y_1$	0.6484	0.8008	<p><b>B1</b>-extracting necessary values</p> <p><b>M1</b>-equation quotients</p> <p><b>A1</b>-output</p> <p><b>M1</b>-equating quotients</p> <p><b>A1</b>-output</p>
$X$	10.1	$x_1$	10.34															
$Y$	0.8008	0.99	1.3003															
$X$	9.9	10	10.1															
$Y$	$y_1$	0.6484	0.8008															
		<b>05</b>																
6	$\underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} = m \underset{\sim}{a}$																	

	$\binom{9}{3} + \binom{7}{3} + \binom{a}{b} = \frac{2000}{1000} \binom{10}{2}$ $\binom{16}{6} + \binom{a}{b} = \binom{20}{4}$ $\binom{a}{b} = \binom{20}{4} - \binom{16}{6}$ $\binom{a}{b} = \binom{4}{-2}$ $\therefore a = 4, \quad \text{and,} \quad b = -2$	<b>M1 M1 M1</b> -LHS, RHS, equating  <b>B1</b> -simplified output <b>B1</b> -stating both $a$ and $b$ .
		<b>05</b>
7	<p>(i).</p> $E(X) = np$ $2 = n \times \frac{1}{20}$ $n = 40$ <p>(ii).</p> $\text{Standard deviation} = \sqrt{npq}$ $= \sqrt{40 \times \frac{1}{20} \times \left(1 - \frac{1}{20}\right)} = \frac{\sqrt{190}}{10} \approx 1.3784$	<b>M1</b> -substitution  <b>A1</b> -value of $n$  <b>M1 M1 A1</b> -variance, square root, output
		<b>05</b>
8	<p>Let <math>H</math> denote doing homework and <math>P</math> denotes passing the examination.</p>  <p>(a).</p> $P(H' \cap P) = 0.25 \times 0.4 = 0.1$ <p>(b).</p> $P(P) = P(H \cap P) + P(H' \cap P)$ $= (0.75 \times 0.8) + (0.25 \times 0.4) = 0.6 + 0.1 = 0.7$	<b>B1</b> -tree diagram  <b>M1 A1</b> -subs. And output  <b>M1 A1</b> -substitution and output
		<b>05</b>

Vaccines	$f$	$x$	$fx$	$fx^2$	$c$	$f/c$
0 – 100	80	50	4,000	200,000	100	0.8
100 – 200	250	150	37,500	5,625,000	100	2.5
200 – 300	500	250	125,000	31,250,000	100	5
300 – 500	800	400	320,000	128,000,000	200	4
500 – 550	100	525	52,500	27,562,500	50	2
550 – 600	40	575	23,000	13,225,000	50	0.8
600 – 650	25	625	15,625	9,765,625	50	0.5
650 – 800	15	725	10,875	7,884,375	150	0.1
800 – 850	10	825	8,250	6,806,250	50	0.2
Total	1,820		596,750	230,318,750		

(a). (i).

$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f} = \frac{596,750}{1,820} = \frac{8525}{26} \approx 327.8846$$

(ii).

$$\begin{aligned} \text{Variance} &= \frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2 \\ &= \frac{230,318,750}{1,820} - \left( \frac{596,750}{1,820} \right)^2 = 19040.44273 \end{aligned}$$

**ALT:**

$$\text{Let } d = \frac{x-525}{100}$$

Vaccines	$f$	$x$	$d$	$fd$	$fd^2$	$c$	$f/c$
0 – 100	80	50	-14.75	-380	1805	100	0.8
100 – 200	250	150	-3.75	-937.5	3515.625	100	2.5
200 – 300	500	250	-2.75	-1375	3781.25	100	5
300 – 500	800	400	-1.25	-1000	1250	200	4
500 – 550	100	525	0	0	0	50	2
550 – 600	40	575	0.5	20	10	50	0.8
600 – 650	25	625	1	25	25	50	0.5
650 – 800	15	725	2	30	60	150	0.1
800 – 850	10	825	3	30	90	50	0.2
Total	1,820			-3,587.5	10,536.875		

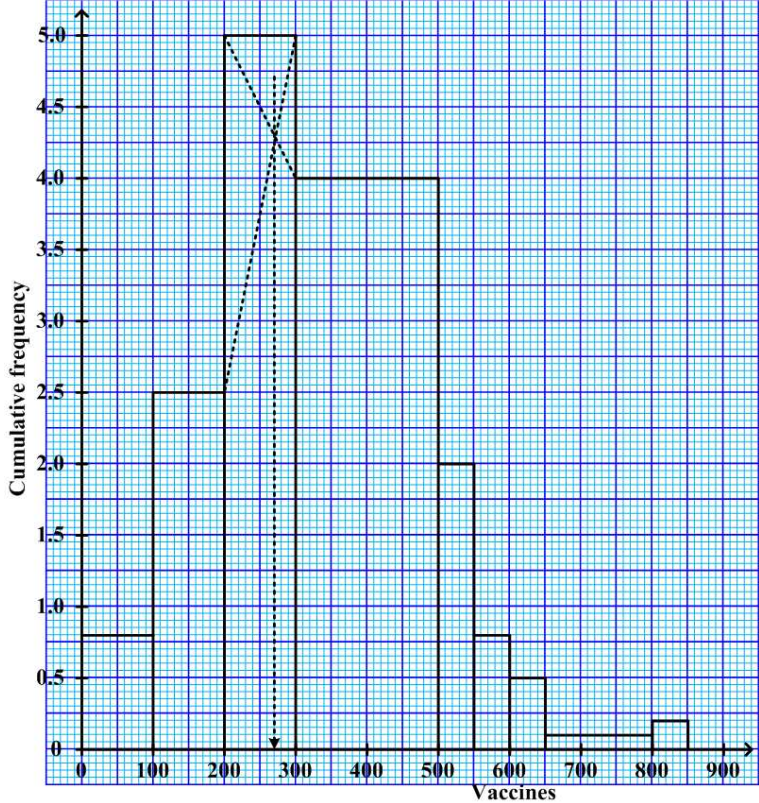
(a). (i).

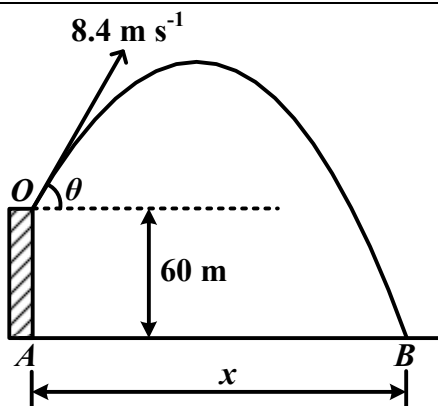
$$\begin{aligned} \text{Mean, } \bar{x} &= A + \frac{c \sum fd}{\sum f} \\ &= 525 + \frac{100 \times (-3,587.5)}{1,820} = \frac{8525}{26} \approx 327.8846 \end{aligned}$$

(ii).

$$\text{Variance} = c^2 \left[ \frac{\sum fd^2}{\sum f} - \left( \frac{\sum fd}{\sum f} \right)^2 \right]$$

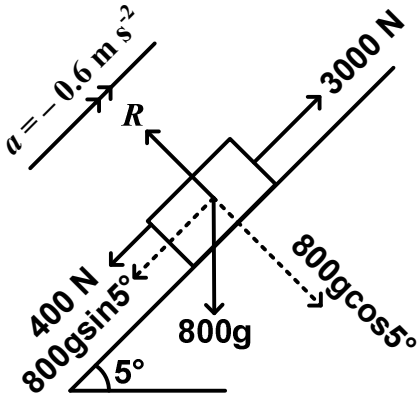
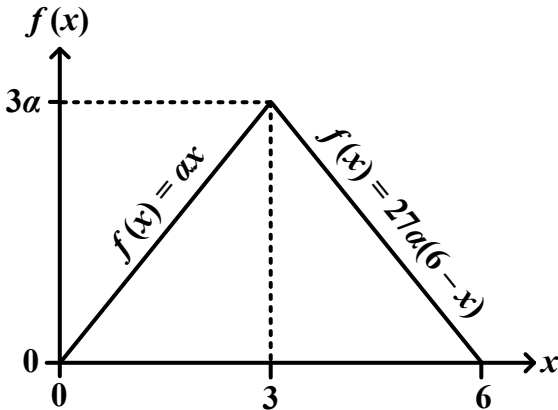
**B1-**  $fx$  values and summation**B1-**  $fx^2$  values and summation**B1-**  $f/c$  values**M1 A1-** substitutio and output**M1 A1-** substitution and output

	$= 100^2 \times \left[ \frac{10,536.875}{1,820} - \left( \frac{-3,587.5}{1,820} \right)^2 \right] = 19040.44273$ <p>(b).</p>  <p>From the histogram, mode is 360.</p>	<p><b>B1</b>-axes</p> <p><b>B1</b> <b>B1</b>-bars</p> <p><b>M1</b>-attempting to get mode</p> <p><b>B1</b>-mode</p>
10	<p>(a).</p> $y_{\max} = \frac{(u \sin 30^\circ)^2}{2g}$ $0.9 = \frac{u^2 \sin^2 30^\circ}{2 \times 9.8}$ $0.9 = \frac{5u^2}{392}$ $u^2 = \frac{1764}{25}$ $u = \frac{42}{5} = 8.4 \text{ m s}^{-1}$ <p>(b).</p>	<p><b>M1</b> <b>M1</b>-substitution</p> <p><b>M1</b> <b>A1</b>-square root and output</p>

	 <p>(i). Considering vertical motion,</p> $y = ut \sin \theta - \frac{1}{2}gt^2$ $-60 = 8.4t \sin 30^\circ - \frac{1}{2} \times 9.8t^2$ $-60 = 4.2t - 4.9t^2$ $4.9t^2 - 4.2t - 60 = 0$ $t = \frac{4.2 \pm \sqrt{(-4.2)^2 - 4 \times 4.9 \times (-60)}}{2 \times 4.9}$ $t = \frac{3 \pm \sqrt{609}}{7}$ $t = 3.9540, \quad \text{or}, \quad t = -3.0968$ <p>but, <math>t \neq -3.0968, \Rightarrow t = 3.9540 \text{ s}</math></p> <p>(ii). Considering horizontal motion,</p> $x = ut \cos \theta = 8.4 \times 3.9540 \cos 30^\circ = 28.7638 \text{ m}$	<p><b>M1 M1-</b> substitution <b>M1</b>-quadratic eqn <b>M1</b>-solving</p> <p><b>B1</b>-both values <b>B1</b>-conclusion</p> <p><b>M1 A1-</b> substitution and output</p>
		<b>12</b>
11	<p>(a).</p> $f(x) = -\frac{\cos x}{x}$ $f(1) = -\frac{\cos 1}{1} = -0.5403$ $f(2) = -\frac{\cos 2}{2} = 0.2081$ <p>Since <math>f(1) \cdot f(2) &lt; 0</math>, then a root exists between the <math>x</math> values of 1 and 2.</p> <p>(b).</p> $f(x) = \frac{-\cos x}{x}$	<p><b>B1</b>- <math>f(1)</math></p> <p><b>B1</b>- <math>f(2)</math></p> <p><b>B1</b>-conclusion</p>

	$f'(x) = \frac{x \times \sin x - (-\cos x) \times 1}{x^2} = \frac{x \sin x + \cos x}{x^2}$ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \dots$ $x_{n+1} = x_n - \left( \frac{-\cos x_n}{x_n} \div \frac{x_n \sin x_n + \cos x_n}{x_n^2} \right)$ $x_{n+1} = x_n + \left( \frac{x_n \cos x_n}{x_n \sin x_n + \cos x_n} \right)$ <table border="1"><tr><td><math>x</math></td><td>1</td><td><math>x_0</math></td><td>2</td></tr><tr><td><math>f(x)</math></td><td>-0.5403</td><td>0</td><td>0.2083</td></tr><tr><td></td><td></td><td></td><td>3</td></tr></table> $\frac{x_0 - 1}{2 - 1} = \frac{0 + 0.5403}{0.2083 + 0.5403}$ $\frac{x_0 - 1}{1} = \frac{0.5403}{0.7486}$ $x_0 = 1 + \frac{0.5403}{0.7486} \times 1 = 1.7217$ $x_1 = 1.7217 + \left( \frac{1.7217 \cos 1.7217}{1.7217 \sin 1.7217 + \cos 1.7217} \right) = 1.55491$ $x_2 = 1.55491 + \left( \frac{1.55491 \cos 1.55491}{1.55491 \sin 1.55491 + \cos 1.55491} \right) = 1.57064$ $x_3 = 1.57064 + \left( \frac{1.57064 \cos 1.57064}{1.57064 \sin 1.57064 + \cos 1.57064} \right) = 1.57080$ $\therefore \text{Root} = 1.571 \text{ (3 d.p.)}$	$x$	1	$x_0$	2	$f(x)$	-0.5403	0	0.2083				3	<b>M1</b> -derivative  <b>M1</b> -substitution         <b>B1</b> -initial approximate root <b>M1 B1</b> -substitution and output <b>M1 B1</b> -substitution and output <b>M1</b> -substitution and output <b>A1</b> -root to 3 d.p
$x$	1	$x_0$	2											
$f(x)$	-0.5403	0	0.2083											
			3											
		<b>12</b>												
12	(a). $v = 43.2 \text{ km h}^{-1} = \frac{43.2 \times 1000}{3600} = 12 \text{ m s}^{-1}$ <p>Forward force, <math>F = \frac{P}{v} = \frac{36 \times 1000}{12} = 3000 \text{ N}</math></p> <p>By Newton's second law:</p> $F - 400 = ma$ $3000 - 400 = 800 \times a$ $2600 = 800 \times a$ $a = 3.25 \text{ m s}^{-2}$ (b). (i).	<b>B1</b> -velocity in $\text{m s}^{-1}$ <b>M1 B1</b> -tractive force  <b>M1 M1</b> -LHS, RHS <b>A1</b> -acceleration												



	 <p>Resolving parallel to the plane gives,</p> $3000 - (400 + 800g \sin 5^\circ) = 800a$ $3000 - (400 + 800 \times 9.8 \sin 5^\circ) = 800a$ $1916.6990 = 800a$ $a = 2.3959 \text{ m s}^{-2}$ <p>(ii).  P.E = <math>mgh = 800 \times 9.8 \times 120 \sin 5^\circ = 81996.12278 \text{ J}</math>  <b>Note:</b> Part (b) is not so clear since initial velocity up the incline was not given.</p>	<p><b>B1</b>-force diagram</p> <p><b>M1 M1</b>-LHS, RHS  <b>A1</b>-acceleration</p> <p><b>M1 A1</b>-substitution and output</p>
		<b>12</b>
13	<p>(a).</p> $f(0) = \alpha \times 0 = 0, \quad f(3) = 3\alpha$ $f(3) = \frac{\beta}{9} \times 3 = \frac{\beta}{3}, \quad f(3) = \frac{\beta}{9} \times 0 = 0$ <p>but, <math>\frac{\beta}{3} = 3\alpha, \quad \Rightarrow \beta = 9\alpha</math></p>  <p>Total area = 1  <math>\frac{1}{2} \times 6 \times 3\alpha = 1</math></p>	<p><b>B1</b>-relation between <math>\beta</math> and <math>\alpha</math></p> <p><b>B1</b>-graph</p> <p><b>M1</b>-area and equating to 1</p>



	$f(t) = \frac{1}{9}(6-t), \quad \Rightarrow F(x) = F(3) + \frac{1}{9} \int_3^x (6-t) dt$ $= \frac{1}{2} + \frac{1}{9} \left[ 6t - \frac{1}{2}t^2 \right]_3^x$ $= \frac{1}{2} + \frac{1}{9} \left[ \left( 6x - \frac{1}{2}x^2 \right) - \left( 6 \times 3 - \frac{1}{2} \times 3^2 \right) \right]$ $= \frac{1}{2} + \frac{1}{9} \left( 6x - \frac{1}{2}x^2 - \frac{27}{2} \right) = \frac{1}{2} + \frac{1}{9} \left( 6x - \frac{1}{2}x^2 - \frac{27}{2} \right)$ $= \frac{1}{18} (9 + 12x - x^2 - 27) = \frac{1}{18} (12x - x^2 - 18)$ $\therefore F(6) = \frac{1}{18} (12 \times 6 - 6^2 - 18) = 1$ <p>For <math>x \geq 6</math>,</p> $f(t) = 0, \quad \Rightarrow F(x) = F(6) = 1$ $\therefore F(x) = \begin{cases} 0 & ; x \leq 0 \\ \frac{1}{18}x^2 & ; 0 \leq x \leq 3 \\ \frac{1}{18}(12x - x^2 - 18) & ; 3 \leq x \leq 6 \\ 1 & ; x \geq 6 \end{cases}$ <p><b>For the hence part:</b></p> $F(3) = \frac{1}{2}, \quad \Rightarrow \text{Median} = 3$ <p>(c).</p>	<p><b>M1-</b> <math>F(x)</math> for <math>3 \leq x \leq 6</math></p> <p><b>B1-</b> <math>F(6)</math> seen</p> <p><b>B1-</b> stating <math>F(x)</math></p> <p><b>B1-</b> stating median</p> <p><b>B1-</b> axes and necessary coordinates</p> <p><b>B1-</b> correct shape</p>
		<b>12</b>
14	(a).	

	<div data-bbox="354 309 1101 600" data-label="Diagram"> </div> <div data-bbox="478 672 981 761" data-label="Equation-Block"> <math display="block">\cos \beta = \frac{v_Q}{20} = \frac{18}{20}, \quad \Rightarrow \beta = 25.84^\circ</math> </div> <div data-bbox="558 784 877 1108" data-label="Diagram"> </div> <div data-bbox="534 1153 917 1209" data-label="Text"> <p><math>\therefore</math> Course set by Q = <math>034.16^\circ</math></p> </div> <div data-bbox="303 1198 359 1243" data-label="Text"> <p>(b).</p> </div> <div data-bbox="303 1243 542 1288" data-label="Text"> <p>For triangle QRS,</p> </div> <div data-bbox="486 1276 965 1321" data-label="Equation-Block"> <math display="block">\alpha = 90 - \beta = 90 - 25.84 = 64.16^\circ</math> </div> <div data-bbox="375 1310 1077 1355" data-label="Equation-Block"> <math display="block">\theta = 90 - (60 + \alpha) = 90 - (60 + 25.84) = -34.16^\circ</math> </div> <div data-bbox="510 1377 941 1691" data-label="Diagram"> </div> <div data-bbox="303 1736 1109 1825" data-label="Text"> <p>Shortest distance, <math>D_{\min} = 4 \sin 34.16^\circ = 2.2460 \text{ km}</math></p> </div> <div data-bbox="303 1780 359 1825" data-label="Text"> <p>(c).</p> </div>	<p><b>B1</b>-geometric diagram</p> <p><b>B1</b>-value of <math>\beta</math></p> <p><b>B1</b>-attempting to find course</p> <p><b>B1</b>-stating course in bearing form</p> <p><b>B1</b>-angle <math>\theta</math></p> <p><b>M1 A1</b>-substitution and output</p>
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	<p>For triangle <math>QRS</math>,</p> ${}_Qv_P = \sqrt{20^2 - 18^2} = 2\sqrt{19} \approx 8.7178 \text{ km h}^{-1}$ <p>Time for shortest distance,</p> $t = \frac{PS}{{}_Qv_P} = \frac{4 \cos 34.16^\circ}{8.7178} = 0.3797 \text{ hours}$ $= 0.3797 \times 60$ $= 22.7802 \text{ minutes} \approx 23 \text{ minutes}$ <p>The two motor boats are closer to each other at 2:23 pm.</p>	<p><b>M1 B1-</b> substitution and output <b>M1 B1-</b>division and output</p> <p><b>B1</b>-time to nearest minute</p>
		<b>12</b>
15	<p>(a).</p> $\text{Error} = (x + \Delta x)^2 \sin(\theta + \Delta\theta) - x^2 \sin \theta$ $= [x^2 + 2x \cdot \Delta x + (\Delta x)^2](\sin \theta \cos \Delta\theta + \cos \theta \sin \Delta\theta) - x^2 \sin \theta$ <p>Assuming <math>(\Delta x) \ll x</math>, and that <math>(\Delta\theta)</math> is a small angle measured in radians, <math>(\Delta x)^2 \approx 0</math>, <math>\sin \Delta\theta \approx \Delta\theta</math> and <math>\cos \Delta\theta \approx 1</math>.</p> $\text{Error} = (x^2 + 2x \cdot \Delta x)(\sin \theta + \Delta\theta \cdot \cos \theta) - x^2 \sin \theta$ $= x^2 \sin \theta + x^2 \cdot \Delta\theta \cdot \cos \theta + 2x \cdot \Delta x \cdot \sin \theta$ $+ 2x \cdot \Delta x \cdot \Delta\theta \cdot \cos \theta - x^2 \sin \theta$ <p>Assuming <math>(\Delta\theta) \ll \theta</math>, then <math>\Delta x \cdot \Delta\theta \approx 0</math>.</p> $\therefore \text{Error} = x^2 \cdot \Delta\theta \cdot \cos \theta + 2x \cdot \Delta x \cdot \sin \theta$ $\text{Absolute error} =  x^2 \cdot \Delta\theta \cdot \cos \theta + 2x \cdot \Delta x \cdot \sin \theta $ $\text{Relative error} = \left  \frac{x^2 \cdot \Delta\theta \cdot \cos \theta + 2x \cdot \Delta x \cdot \sin \theta}{x^2 \sin \theta} \right $ $= \left  \Delta\theta \cdot \cot \theta + \frac{2\Delta x}{x} \right  \leq  \Delta\theta \cdot \cot \theta  + \left  \frac{2\Delta x}{x} \right $ $\therefore \text{Maximum relative error} = 2 \left  \frac{\Delta x}{x} \right  +  \Delta\theta \cdot \cot \theta $ <p><b>For the hence part:</b></p> $x = 4.1, \quad \Rightarrow \Delta a = 0.05$ $\theta = 30^\circ = \frac{30\pi}{180} = \frac{\pi}{6}, \quad \Rightarrow \Delta\theta = 0.5^\circ = \frac{0.5\pi}{180} = \frac{\pi}{360}$ $\text{Percentage error} = \left( 2 \left  \frac{\Delta x}{x} \right  +  \Delta\theta \cdot \cot \theta  \right) \times 100$ $= \left( 2 \left  \frac{0.05}{4.1} \right  + \left  \frac{\pi}{360} \times \cot \frac{\pi}{6} \right  \right) \times 100 = 3.9505\%$ <p>(b).</p> $\left( \frac{1}{x} - \frac{1}{y} \right)_{\max} = \frac{1}{x_{\min}} - \frac{1}{x_{\max}} = \frac{1}{(0.479 - 0.0005)} - \frac{1}{(3.1 + 0.05)}$ $= \frac{1}{0.4785} - \frac{1}{3.15} = 1.772403841 \approx 1.77240 \text{ (5 d.p.)}$ $\left( \frac{1}{x} - \frac{1}{y} \right)_{\min} = \frac{1}{x_{\max}} - \frac{1}{x_{\min}} = \frac{1}{(0.479 + 0.0005)} - \frac{1}{(3.1 - 0.05)}$	<p><b>M1</b>-definition of error</p> <p><b>B1</b>-all assumptions correct</p> <p><b>M1</b>-simplifying error</p> <p><b>M1</b>-relative error <b>M1</b>-triangular inequality <b>B1</b>-max. R.E</p> <p><b>M1 A1</b>- substitution and output</p> <p><b>M1</b>-substitution</p> <p><b>A1</b>-output-5 d.p</p> <p><b>M1</b>-substitution</p>

	$= \frac{1}{0.4795} - \frac{1}{3.05} = 1.757636883 \approx 1.75764 \text{ (5 d.p.)}$	<b>A1-output-5 d.p</b>
		<b>12</b>
16	<p>(a). Let <math>B</math> denote Bazooka, <math>H</math> denote Hybrid, <math>L</math> denote Longe H, <math>T</math> denote Traditional, and <math>F</math> denote flowering.</p> <p>Total ratio = <math>4 + 3 + 2 + 1 = 10</math></p> $P(F) = P(F \cap B) + P(F \cap H) + P(F \cap L) + P(F \cap T)$ $= P(B).P(F/B) + P(H).P(F/H) + P(L).P(F/L) + P(T).P(F/T)$ $= \left(\frac{4}{10} \times \frac{30}{180}\right) + \left(\frac{3}{10} \times \frac{70}{180}\right) + \left(\frac{2}{10} \times \frac{60}{180}\right) + \left(\frac{1}{10} \times \frac{50}{180}\right)$ $= \frac{1}{15} + \frac{7}{60} + \frac{1}{15} + \frac{1}{36} = \frac{5}{18}$ <p>(b). Let <math>X \sim</math> be the number of seeds that will flower.</p> $n = 200, \quad p = \frac{5}{18}, \quad q = 1 - \frac{5}{18} = \frac{13}{18}$ $\mu = np = 200 \times \frac{5}{18} = \frac{500}{9} \approx 55.5556$ $\sigma = \sqrt{npq} = \sqrt{200 \times \frac{5}{18} \times \frac{13}{18}} = \sqrt{\frac{3250}{81}} \approx 6.3343$ <p><math>P(\text{less than 170 will flower}) = P(X &lt; 170) \rightarrow P(X &lt; 170.5)</math></p> $= P\left(Z < \frac{170.5 - 55.5556}{6.3343}\right) = P(Z < 18.1463) = 1$	<p><b>B1-total ratio</b></p> <p><b>M1 M1 M1 M1</b> <b>M1</b>-each bracket and addition <b>A1-output</b></p> <p><b>B1-mean</b></p> <p><b>B1-variance</b></p> <p><b>M1</b>-continuity correction <b>M1 A1</b>-standardising and probability</p>
		<b>12</b>

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