UACE COUHIEA MOCK EXAMS
P425/1 PURE MATHEMATICS
MARKING GUIDE
2023

QN	SOLUTION	MKS	REMARKS
1.	$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \\ -2 & 3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$		
	$\begin{array}{l} \\ R_2-2R_1 \\ R_3+2R_1 \end{array}\begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -3 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	M1	For both R_2 and R_3 Correct
	$\begin{array}{l} \\ \\ 5R_3-R_2 \end{array}\begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -3 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}$	M1	For R_3 Correct
	$\Rightarrow 3z = -6$; $z = -2$	A1	for z
	$5y - 3z = 1$; $y = -1$	A1	for y
	$x - y + 2z = 1$; $x = 4$	A1	for x
		05	

QN	SOLUTION	MKS	REMARKS
2	$x+y=7$ —— (i) $2x-y=5$ —— (ii)		
	(i)+(ii) $3x=12$	M1	Correct use of a given method
	$x=4$		
	From (i), $y=3$		
	Point of Concurrency $(4,3)$	A1	Correct point
	Gradient of the line $4x-y=7$ is 4 Required gradient $=-\frac{1}{4}$	B1	Stating the gradient of the line L
	Required equation: $\frac{y-3}{x-4}=\frac{-1}{4}$	M1	
	$\Rightarrow 4y+x=16$	A1	
		05	
3.	$\frac{3x^2-2x-11}{x^2-4x+3}\leq 3$		
	$\frac{3x^2-2x-11}{x^2-4x+3}-3\leq 0$	M1	Subtracting 3 from both sides
	$\frac{3x^2-2x-11-3x^2+12x-9}{(x-1)(x-3)}\leq 0$	M1	Multiplying by the LCM

QN	SOLUTION	MKS	REMARKS
	$\dfrac{10x-20}{(x-1)(x-3)} \leq 0$	M1	Simplification

	$x<1$	$1<x<2$	$2<x<3$	$x>3$
$10x-20$	$-$	$-$	$+$	$+$
$(x-1)(x-3)$	$+$	$-$	$-$	$+$
Quotient	$-$	$+$	$-$	$+$

B1 — Correct table

Solution set $\{x: x<1,\ 2\leq x<3\}$ — A1 — Both Inequalities Correct

05

4

Let $y = \tan x + \sec x$

$$y+\Delta y = \frac{\sin(x+\Delta x)}{\cos(x+\Delta x)} + \frac{1}{\cos(x+\Delta x)}$$

M1 — Introducing small increments

$$\Delta y = \frac{\sin(x+\Delta x)\cos x - \cos(x+\Delta x)\sin x + \cos x - \cos(x+\Delta x)}{\cos(x+\Delta x)\cos x}$$

$$\Delta y = \frac{\sin(\Delta x) + 2\sin\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\cos(x+\Delta x)\cos x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\dfrac{\sin(\Delta x)}{\Delta x} + \dfrac{\sin\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}}{\cos(x+\Delta x)\cos x}$$

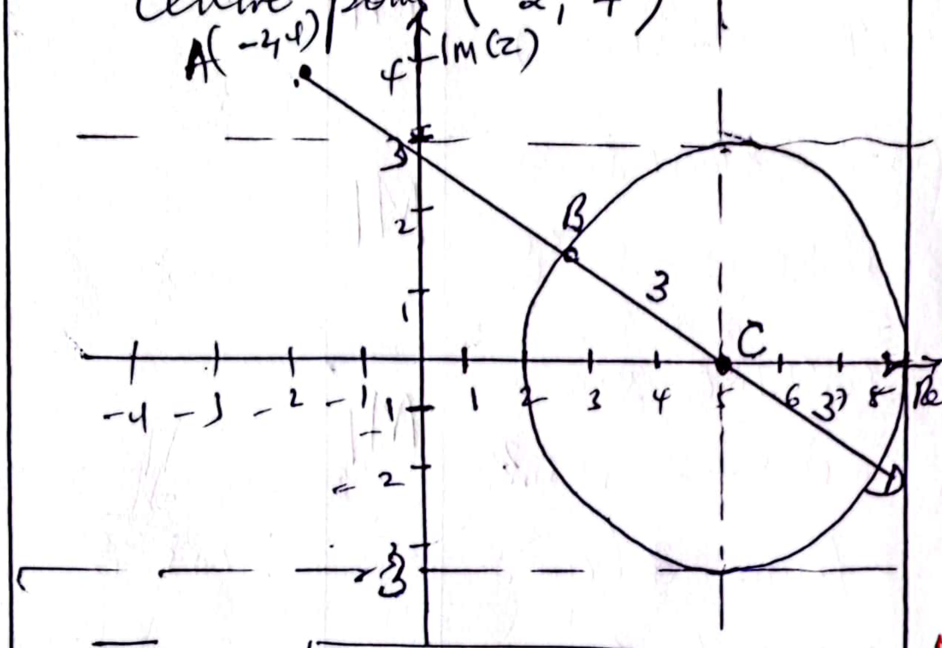
M1 — obtaining the gradient of the chord correctly

$$\frac{dy}{dx} = \frac{1+\sin x}{\cos x\cos x}$$

QN	SOLUTION	MKS	REMARKS
1	As $\Delta x \to 0$; $\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$; $\frac{\sin \Delta x}{\Delta x} \to 1$; $\frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x/2} \to 1$	B1	
	$\Rightarrow \frac{dy}{dx} = \frac{1+\sin x}{\cos^2 x}$		
	$\frac{dy}{dx} = \frac{1+\sin x}{1-\sin^2 x}$	M1	
	$\frac{dy}{dx} = \frac{(1+\sin x)}{(1-\sin x)(1+\sin x)}$		
	$\frac{dy}{dx} = \frac{1}{1-\sin x}$		
	Thus, $\frac{d}{dx}(\tan x + \sec x) = \frac{1}{1-\sin x}$	A1	
		05	
5	$\overrightarrow{QP} = \begin{pmatrix}5\\3\\5\end{pmatrix}$; $\overrightarrow{QR} = \begin{pmatrix}7\\-5\\3\end{pmatrix}$		
	$\overrightarrow{QP}\cdot\overrightarrow{QR} = \begin{pmatrix}5\\3\\5\end{pmatrix}\cdot\begin{pmatrix}7\\-5\\3\end{pmatrix} = 35$	M1	
	$\lvert\overrightarrow{QP}\rvert = \sqrt{25+9+25} = \sqrt{59}$	M1	
	$\lvert\overrightarrow{QR}\rvert = \sqrt{49+25+9} = \sqrt{83}$	M1	

QN	SOLUTION	MKS	REMARKS
	Angle PQR $= \cos^{-1}\left(\frac{35}{\sqrt{59}\times\sqrt{83}}\right)$	M1	
	$= 59.99^\circ$	A1	
		05	
6	Let $u = \ln x$; $du = \frac{dx}{x}$	B1	
	$dv = x^2$; $v = \frac{1}{3}x^3$	B1	
	$\Rightarrow \int x^2 \ln x\, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}dx$	M1	Correct Substitution
	$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$		
	$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\cdot\frac{1}{3}x^3 + c$	M1	Correct integration
	$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$	A1	
		05	
7.	$4\cos x + 3\sin x \equiv R\cos(x-\alpha)$		
	$R = \sqrt{4^2+3^2} = 5$	M1	
	$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$	M1	
	$\Rightarrow 4\cos x + 3\sin x = 5\cos(x - 36.87^\circ)$	A1	
	Max Value $= \frac{2}{-5+10} = \frac{2}{5}$	A1	
	Smallest value of $x = 216.87^\circ$	A1	
		05	

QN	SOLUTION	MKS	REMARKS
8	$y = \cos^2(x^2)$		
	$\frac{dy}{dx} = -2\cos(x^2)\cdot\sin(x^2)\cdot 2x$	M1	first derivative
	$\frac{dy}{dx} = -2x\sin 2x^2$		
	$\frac{d^2y}{dx^2} = -2\sin 2x^2 - 8x^2\cos 2x^2$	M1	Second derivative
	$\frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx} - 8x^2(2\cos^2 x^2 - 1)$	M1	Correct use of double angle
	$\frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx} - 16x^2y + 8x^2$	M1	Correct substitution
	$x\frac{d^2y}{dx^2} = \frac{dy}{dx} - 16x^3y + 8x^3$		
	$\Rightarrow x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 16x^3y = 8x^3$ #	A1	Complete proof
		05	

QN	SOLUTION	MKS	REMARK
9.	a) $\|Z-5\| = 3$		
	Centre $(5,0)$, radius $= 3$		
	$\|z+2-4i\| = \|z-(-2+4i)\|$		
	Centre point $(-2, 4)$		
		B1	
	$\overline{AC} = \sqrt{(5+2)^2+(0-4)^2} = 8.0623$	M1	
	Greatest value $= \overline{AC} + CD$		
	$= 8.0623 + 3$	M1	
	$= 11.0623$ units	A1	
	Least value $= \overline{AC} - \overline{BC}$		
	$= 8.0623 - 3$		
	$= 5.0623$ units	A1	

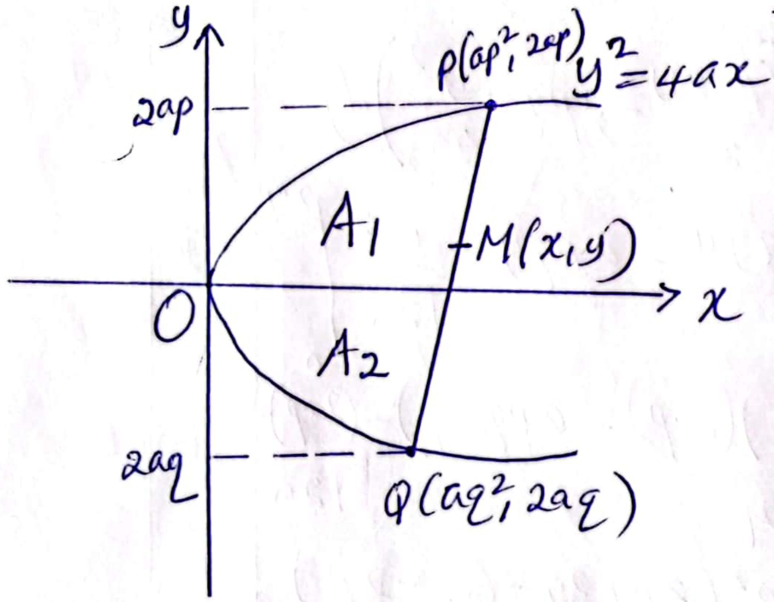
QN	SOLUTION	MKS	REMARKS
9b)	$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^2$		
	$= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$		
	$= \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$	M1	
	$\Rightarrow \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$		
	$\Rightarrow \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$	M1	
	$\tan 3\theta = \dfrac{3\cos^2\theta\sin\theta - \sin^3\theta}{\cos^3\theta - 3\cos\theta\sin^2\theta}$	M1	
	$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$		$\div \cos^3\theta$
	$\tan 3\theta = \dfrac{3t - t^3}{1 - 3t^2}$; $t = \tan\theta$	A1	
	Hence		
	$\tan 3\theta = 1 \Rightarrow \theta = \frac{1}{3}\left(K\pi + \frac{\pi}{4}\right)$	M1	
	Taking $K = 0, 1, 2$		
	$K=0$; $\theta = \frac{\pi}{12} \Rightarrow t = \tan\frac{\pi}{12} = 0.268$	M1	Correct substitution
	$K=1$, $\theta = \frac{5\pi}{12} \Rightarrow t = \tan\frac{5\pi}{12} = 3.73$	A1	for all correct
	$K=2$, $\theta = \frac{9}{12}\pi \Rightarrow t = \tan\frac{9}{12}\pi = -1.00$		

QN	SOLUTION	MKS	REMARKS
10	a) $x+y-2z=2$ —— (i)		
	$2x+y-z=0$ —— (ii)		
	(i)−(ii), $-x-z=2$	M1	
	$x=-2-z$		
	let $z=\lambda$, $x=-2-\lambda \Rightarrow \lambda=\frac{x+2}{-1}$	B1	
	From (i), $-2-\lambda+y-2\lambda=2$		
	$y=4+3\lambda$		
	$\Rightarrow \lambda=\frac{y-4}{3}$		
	Line of Intersection:		
	$\frac{x+2}{-1}=\frac{y-4}{3}=z$	A1	
	Direction vector; $\underset{\sim}{d_1}=\begin{pmatrix}-1\\3\\1\end{pmatrix}$		
	Direction vector of a given line,		
	$\underset{\sim}{d_2}=\begin{pmatrix}+1\\-3\\-1\end{pmatrix}$		
	Let $\underset{\sim}{d_1}=k\underset{\sim}{d_2}$		or equivalent
	$\begin{pmatrix}-1\\3\\1\end{pmatrix}=k\begin{pmatrix}1\\-3\\-1\end{pmatrix}$	M1	Accept cross product $\underset{\sim}{d_1}\times\underset{\sim}{d_2}=\underset{\sim}{0}$
	$-1=k$		

QN	SOLUTION	MKS	REM
	$3 = -3K, \quad K = -1$	A1	
	$1 = -K \; ; -K = -1$		
	Since the value of K is consistent, then the lines are parallel.	B1	
b)	General point on the line is		
	$Q(\lambda+1, 1-\lambda, \lambda)$, $P(1, 0, 2)$		
	$\overrightarrow{PQ} = \begin{pmatrix} \lambda+1-1 \\ 1-\lambda-0 \\ \lambda-2 \end{pmatrix}$		
	$\overrightarrow{PQ} = \begin{pmatrix} \lambda \\ 1-\lambda \\ \lambda-2 \end{pmatrix}$	M1	
	$\Rightarrow \begin{pmatrix} \lambda \\ 1-\lambda \\ \lambda-2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0$	M1	
	$\lambda - 1 + \lambda + \lambda - 2 = 0$		
	$3\lambda - 3 = 0$		
	$\lambda = 1$	A1	
	$\Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$	B1	

QN	SOLUTION	MKS	REMARKS
	$\lvert\overrightarrow{PQ}\rvert = \sqrt{1^2+1^2}$	M1	
	$= \sqrt{2}$	A1	
	OR		
	Point on the line $Q(1,1,0)$	B1	
	direction vector, $\underset{\sim}{d} = \underset{\sim}{\hat{i}} - \underset{\sim}{\hat{j}} + \underset{\sim}{\hat{k}}$		
	$\Rightarrow \overrightarrow{PQ} = \begin{pmatrix}1\\1\\0\end{pmatrix} - \begin{pmatrix}1\\0\\2\end{pmatrix} = \begin{pmatrix}0\\1\\-2\end{pmatrix}$	M1	
	$\overrightarrow{PQ} \times \underset{\sim}{d} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -2\\ 1 & -1 & 1\end{vmatrix}$	M1	
	$= -\underset{\sim}{\hat{i}} - 2\underset{\sim}{\hat{j}} - \underset{\sim}{\hat{k}}$		
	$\lvert\overrightarrow{PQ} \times \underset{\sim}{d}\rvert = \sqrt{1+1+4} = \sqrt{6}$ $\lvert\underset{\sim}{d}\rvert = \sqrt{1+1+1} = \sqrt{3}$	M1	
	$\perp$lar distance $= \dfrac{\sqrt{6}}{\sqrt{3}}$	M1	
	$= \sqrt{2}$	A1	
		12	

QN	SOLUTION	MKS	REMARKS
11.	Let $\frac{2x^2+3x+5}{(x+1)(x^2+3)} \equiv \frac{A}{x+1} + \frac{Bx+C}{x^2+3}$	M1	
	$2x^2+3x+5 \equiv A(x^2+3)+(Bx+C)(x+1)$		
	$x=-1,\ 4 = 4A \Rightarrow A = 1$	A1	
	$x=0,\ 5 = 3A+C \Rightarrow C = 2$	A1	
	$x=1,\ 10 = 4A+2B+2C$		
	$10 = 4(1)+2B+2(2)$		
	$2B = 2$	A1	
	$B = 1$		
	$\Rightarrow \frac{2x^2+3x+5}{(x+1)(x^2+3)} = \frac{1}{x+1} + \frac{x+2}{x^2+3}$	A1	
	a) $f(x) = \frac{1}{x+1} + \frac{x+2}{x^2+3}$		
	$f(x) = (x+1)^{-1} + (x+2)(x^2+3)^{-1}$		
	$f'(x) = -1(x+1)^{-2} + (x^2+3)^{-1} - \frac{2x(x+2)}{(x^2+3)^2}$	M1	
	$f'(x) = -\frac{1}{(x+1)^2} + \frac{1}{(x^2+3)} - \frac{2x(x+2)}{(x^2+3)^2}$		
	$f'(0) = -1 + \frac{1}{3} = -\frac{2}{3}$	M1A1	

QN	SOLUTION	MKS	REMARKS
11b)	$\int_0^{\sqrt{3}} f(x)\,dx = \int_0^{\sqrt{3}} \frac{dx}{x+1} + \int_0^{\sqrt{3}} \frac{x+2}{x^2+3}\,dx$	M1	
	$= \left[\ln\lvert x+1\rvert + \frac{1}{2}\ln\lvert x^2+3\rvert + \frac{2}{3}\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}\right]_0^{\sqrt{3}}$	M1	Correct integration
	$= \ln\lvert\sqrt{3}+1\rvert + \frac{1}{2}\ln 6 + \frac{2}{3}\sqrt{3}\tan^{-1}1 - \frac{1}{2}\ln 3$	M1	Correct substitution
	$= 2.2585.$	A1	
		12	
12		B1	Correct sketch
	a) Gradient of $\overline{PQ} = \frac{2aq-2ap}{aq^2-ap^2}$		
	$= \frac{2}{p+q}$		
	Equation of $\overline{PQ}$;		
	$\frac{y-2ap}{x-ap^2} = \frac{2}{p+q}$		

QN	SOLUTION	MKS	REMARKS
	$(p+q)y - 2apq = 2x$		
	$\Rightarrow x_1 = \frac{(p+q)y}{2} - apq$ ✓	M1	
	From eqn of parabola;		
	$x_2 = \frac{y^2}{4a}$		
	$\text{Area}(A_1) = \int_0^{2ap} (x_1 - x_2)\,dy$		
	$A_1 = \int_0^{2ap} \left(\frac{(p+q)y}{2} - apq - \frac{y^2}{4a}\right) dy$		
	$A_1 = \left[\frac{(p+q)y^2}{4} - apqy - \frac{y^3}{12a}\right]_0^{2ap}$		
	$A_1 = \frac{(p+q)(2ap)^2}{4} - apq(2ap) - \frac{(2ap)^3}{12a}$		
	$A_1 = \frac{1}{3}a^2p^2(3p+3q-6q-2p)$		
	$A_1 = \frac{1}{3}a^2p^2(p-3q)$ ✓	M1	
	$A_2 = \left[\left(\frac{p+q}{4}\right)y^2 - apqy - \frac{y^3}{12a}\right]_{2aq}^{0}$		
	$A_2 = -\left[a^2q^2(p+q) - 2a^2pq^2 - \frac{2a^2q^3}{3}\right]$		
	$A_2 = -\frac{1}{3}a^2q^2(q-3p)$ ✓	M1	

QN	SOLUTION	MKS	REMARKS
	Required Area $(A) = A_1 + A_2$		
	$\Rightarrow A = \frac{1}{3}a^2p^2(p-3q) - \frac{1}{3}a^2q^2(q-3p)$	M1	
	$A = \frac{1}{3}a^2\left(p^3 - 3p^2q - q^3 + 3pq^2\right)$		
	$3A = a^2(p-q)^3$		
	$9A^2 = a^4(p-q)^6$ as required	A1	
b)	Midpoint $M(x,y)$;		
	$x = \frac{ap^2 + aq^2}{2}$		
	$x = \frac{a\left(p^2 + (p-4)^2\right)}{2}$		
	$x = a(p^2 - 4a + 8)$	M1	Correct x-coordinate
	$y = \frac{2ap + 2aq}{2}$		
	$y = a(p + p - 4)$		
	$y = a(2p-4)$	M1	Correct y-coordinates
	$\therefore M\left(a(p^2-4a+8); a(2p-4)\right)$	A1	Mid point

QN	SOLUTION	MKS	REMARKS
	$y = a(2p-4)$		
	$\Rightarrow \quad p = \dfrac{y+4a}{2a}$	B1	
	Then, $x = a\left[\left(\dfrac{y+4a}{2a}\right)^2 - 4\left(\dfrac{y+4a}{2a}\right) + 8\right]$	M1	
	$x = \dfrac{1}{4a}\left(y^2 + 8ay + 16a^2 - 8ay - 32a^2 + 32a^2\right)$		
	$4ax = y^2 + 16a^2$		
	$\Rightarrow \quad y^2 = 4a(x-4a)$	A1	
		12	

QN	SOLUTION	MKS	REMARKS
13	$y = \dfrac{x^2+x-2}{x^3-7x^2+14x-8} = \dfrac{(x+2)(x-1)}{(x-1)(x-2)(x-4)}$		
	$\Rightarrow y = \dfrac{x+2}{(x-2)(x-4)}$		
	a) $x-1=0$		
	$x=1$	M1	
	$y = \dfrac{1+2}{(1-2)(1-4)} = \dfrac{3}{3} = 1$		
	$\therefore$ Coordinates of the hole: $(1,1)$	A1	
	b) <u>Vertical asymptotes</u>		
	As $y \to \infty$, $(x-2)(x-4) \to 0$		
	$\Rightarrow (x-2)(x-4)=0$		
	$x=2$ and $x=4$	B1	Both asymptotes correct
	<u>Horizontal asymptote</u>		
	$y = \dfrac{x+2}{x^2-6x+8}$		
	$yx^2-(6y+1)x+8y-2=0$		

QN	SOLUTION	MKS	REMARKS
	$x = \frac{6y+1 \pm \sqrt{(6y+1)^2 - 4y(8y-2)}}{2y}$		
	As $x \to \infty$, $2y \to 0$ $\Rightarrow y = 0$.	B1	for horizontal asymptote
	c) $y = \frac{x+2}{(x-2)(x-4)}$		
	$\frac{dy}{dx} = \frac{(x^2-6x+8) - (x+2)(2x-6)}{(x^2-6x+8)^2}$	M1	Correct differentiation.
	$\frac{dy}{dx} = \frac{20-4x-x^2}{(x^2-6x+8)^2}$		
	$\Rightarrow 20-4x-x^2 = 0$ $x^2+4x-20 = 0$ $x = 2.9$, $x = -6.9$		
	$x = 2.9$, $y = -4.9$; $(2.9, -4.9)$ $x = -6.9$, $y = -0.1$; $(-6.9, -0.1)$	A1	for both points correct

QN	SOLUTION	MKS	REMARKS

	2.5	2.9	3	-7	-6.9	-6.5
dy/dx	+	0	-	-	0	+

max min

$\therefore (2.9, -4.9)$ maximum
$(-6.9, -0.1)$ minimum — B1 — for both correct

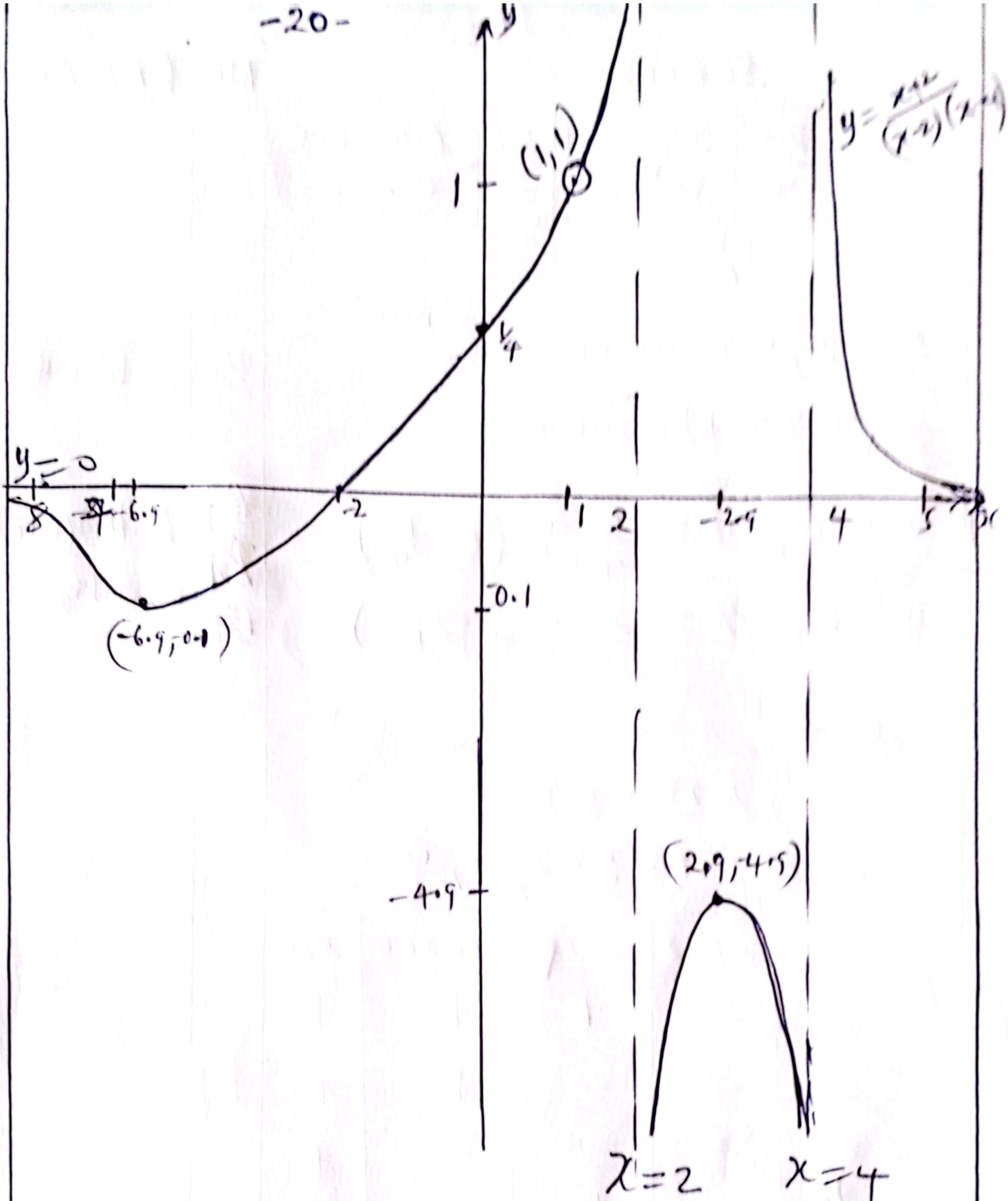
d) $x = 0,\ y = \frac{1}{4}$; $(0, \frac{1}{4})$ — B1 — Both intercepts correct
$y = 0,\ x = -2$; $(-2, 0)$

$$y = \frac{x+2}{(x-2)(x-4)}$$

Critical values: $-2, 2, 4$

	$x<-2$	$-2<x<2$	$2<x<4$	$x>4$
$x+2$	—	+	+	+
$x-2$	—	—	+	+
$x-4$	—	—	—	+
y	—	+	—	+

B1

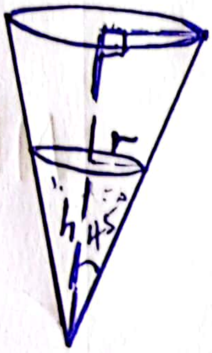


B1 for axes and asymptotes correctly drawn

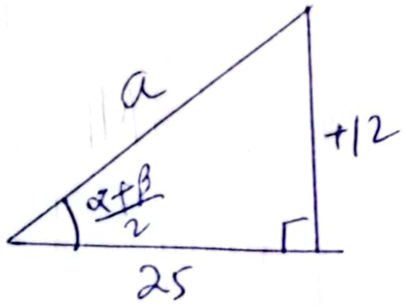
B1 for turning points and intercept correctly drawn

B1 for the correct curve drawn

QN	SOLUTION	MKS	REMARKS
14	a) Let x and $l-x$ be the two portions.		
	Volume of the first cube: $V_1 = \frac{x^3}{1728}$	M1	
	Volume of the second cube; $V_2 = \frac{(l-x)^2}{1728}$		
	$V_2 = \frac{l^3 - 3l^2x + 3l^2x - x^3}{1728}$	M1	
	Sum of the two volumes (V)		
	$V = V_1 + V_2$		
	$V = \frac{x^3}{1728} + \frac{l^3 - 3l^2x + 3lx^2 - x^3}{1728}$	M1	
	$V = \frac{l^3 - 3l^2x + 3lx^2}{1728}$	A1	
	$\frac{dv}{dx} = \frac{-3l^2 + 6lx}{1728}$		
	$\Rightarrow -3l^2 + 6lx = 0$		
	$x = \frac{1}{2}l$	M1	

QN	SOLUTION	MKS	REMARKS
	$\Rightarrow V_{min} = \dfrac{l^3 - 3l^2 \cdot \frac{l}{2} + \frac{3l \cdot l^2}{4}}{1728}$		
	$= \dfrac{l^3}{6912}$	A1	
b)			
	$\tan 45° = \dfrac{r}{h}$		
	$h = r$	B1	for $h=r$
	$\dfrac{dV}{dt} = 30\,cm^3/s.$		
i)	The volume, V, of the water in the cone: $V = \frac{1}{3}\pi r^2 h$		
	$V = \frac{1}{3}\pi h^3$		
	$\dfrac{dV}{dh} = \pi h^2$		
	$\dfrac{dV}{dh} = 60^2\pi$	M1	
	$\Rightarrow \dfrac{dh}{dt} = \dfrac{dh}{dV} \cdot \dfrac{dV}{dt}$		
	$= \dfrac{30}{60^2\pi}\,cm/s$		

QN	SOLUTION	MKS	REMARKS
	$\frac{dh}{dt} = 0.002653\,cm/s$	A1	
(ii)	$A = \pi r^2 = \pi h^2$		$r = h.$
	$\frac{dA}{dh} = 2\pi h = 2\pi h$	M1	
	$\frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dt}$		
	$\frac{dA}{dt} = 2\pi h \cdot \frac{30}{\pi h^2}$	M1	
	$\frac{dA}{dt} = \frac{60}{h}$		
	$\frac{dA}{dt} = \frac{60}{60}$		
	$\frac{dA}{dt} = 1\,cm^2/s$	A1	
		12	

QN	SOLUTION	MKS	REMARKS
15.	a) $\cos\alpha - \cos\beta = \frac{2}{5}$		
	$-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = \frac{2}{5}$ —— (i)		
	$\sin\alpha - \sin\beta = \frac{5}{6}$	M1	for both (i) and (ii) correct
	$2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = \frac{5}{6}$ —— (ii)		
	(i) $\div$ (ii); $\tan\frac{\alpha+\beta}{2} = -\frac{12}{25}$	A1	for $\tan\left(\frac{A+B}{2}\right)$ correct
	i) 		
	$a^2 = 12^2 + 25^2$	M1	
	$a = \sqrt{769}$		
	$\Rightarrow \sin\frac{\alpha+\beta}{2} = \frac{12}{\sqrt{769}} = 0.4327$	A1	Accept $\sin\frac{A+B}{2} = \frac{-12}{\sqrt{769}}$
	(ii) $\cos(\alpha+\beta) = 1 - 2\sin^2\frac{A+B}{2}$		
	$= 1 - 2\left(\frac{144}{769}\right)$	M1	Correct substitution
	$= \frac{481}{769}$	A1	
	$= 0.62{:}55$		

QN	SOLUTION	MKS	REMARKS
15	(b)		
	$LHS = \sin^2 A + \sin^2 B + \sin^2 C$		
	$= \frac{1}{2}(1-\cos 2A) + \frac{1}{2}(1-\cos 2B) + \frac{1}{2}(1-\cos 2C)$	B1	Transformation to double angle (All correct)
	$= \frac{1}{2}\left[3-(\cos 2A + \cos 2B + \cos 2C)\right]$		
	$= \frac{1}{2}\left[3-(2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1)\right]$	M1	Correct use of factor formular
	$= \frac{1}{2}\left[4 + 2\cos C\cos(A-B) - 2\cos^2 C\right]$		
	$= \frac{1}{2}\left[4 + 2\cos C\left(\cos(A-B) - \cos C\right)\right]$	M1	factorisation
	$= \frac{1}{2}\left[4 + 2\cos C\left(\cos(A-B) + \cos(A+B)\right)\right]$		
	$= \frac{1}{2}\left[4 + 2\cos C \cdot 2\cos A\cos B\right]$	M1	Simplification
	$= 2 + 2\cos A\cos B\cos C$	A1	
	$= RHS$	B1	
		12	

QN	SOLUTION	MKS	REMARKS
16.	a) $(x^2+1)\frac{dy}{dx} + 4xy = 12x^3$		
	$\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{12x^3}{x^2+1}$		
	Integrating factor $= e^{\int \frac{4x}{x^2+1}dx}$		
	$= e^{2\int \frac{2x}{x^2+1}dx}$		
	$= e^{2\ln(x^2+1)}$		
	$= (x^2+1)^2$	M1	
	$\Rightarrow \frac{d}{dx}(x^2+1)^2 y = \frac{12x^3}{(x^2+1)}\cdot(x^2+1)^2$	M1	
	$\int \frac{d}{dx}\left((x^2+1)^2 y\right) = \int (12x^5 + 12x^3)dx$		
	$(x^2+1)^2 y = 2x^6 + 3x^4 + c$	A1	
	$y=1, x=1, \quad 4 = 5 + c$	M1	
	$-1 = c$		
	$\therefore (x^2+1)^2 y = 2x^6 + 3x^4 - 1$	A1	

QN	SOLUTION	MKS	REMARKS
b)	$\frac{dT}{dt} \propto (T - T_0)$		
	$\frac{dT}{dt} = -K(T-20)$	B1	
	$\int \frac{dT}{T-20} = \int -K\,dt$		
	$\ln\lvert T-20\rvert = -Kt + c$	M1	
	$\ln\lvert 100-20\rvert = c$		
	$\Rightarrow \ln\lvert T-20\rvert = -Kt + \ln\lvert 80\rvert$	M1	$T=100^\circ c,\ t=0$
	$T = 60^\circ c,\ t = 20\text{ min}$,		
	$\ln\lvert 60-20\rvert = -K(20) + \ln\lvert 80\rvert$		
	$K = \frac{1}{20}\ln 2 = 0.034657$	M1	
	$\Rightarrow \ln\lvert T-20\rvert = -0.034657t + \ln 80$		
	$T = 30,\ t = ?$		
	$\Rightarrow \ln\lvert 30-20\rvert = -0.034657t + \ln 80$	M1	
	$\ln 10 = -0.034657t + \ln 80$		
	$t = \frac{\ln 8}{0.034657} = 60\text{ min}$	A1	
	Thus, it takes a body a further 40min to cool to $30^\circ c$.	A1	$(60-20)$ min
			END
		12	