S.5

PURE MATHEMATICS

P425/1

Time allowed: 3 hours

Instructions

Attempt ALL questions in section A and any five questions in section B

SECTION A (40 MARKS)

1. Find the exact value of x in the equation $\sqrt{logx} + 3logx^{\frac{1}{2}} = 4$ (5 marks)
2. Solve the equation $3sin2x + 4cos2x = 2$ for $-180 < x < 180$ (5 marks)
3. If one root of the equation $px^2 + qx + r = 0$ is the square of the other, show that $q(r-q)^3 = r(q-p)^3$. (5 marks)
4. Find the equation of the tangent to the curve $y = x(10^{lnx})$ at the point $(1,1)$ (5 marks)
5. Find the coefficient of x^9 in the expansion of $\left(x^2 + \frac{2}{x}\right)^{12}$ (5 marks)
6. Find $\int \frac{x^2+2x+1}{\sqrt{3-x^2-2x}}dx$ (5 marks)
7. A curve is defined parametrically as $x = (1-t)^2 + 3; y = (t-1)^4$. Find the cartesian equation of the curve hence find $\frac{d^2y}{dx^2}$. (5 marks)
8. When a polynomial is divided by $x-1, x+1$ and $x-2$ the remainders are $-2, -4$ and 5 respectively, find the remainder when $(x-1)(x+1)(x-2)$ divides the polynomial. (5 marks)

SECTION B (60 MARKS)

9. Sketch the curve $y = x^3 - 3x + 2$ hence calculate the area bounded by the curve and the line $y = x + 2$.
10. (a) If $y = xe^{-x}$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ (6 marks)

    (b) Differentiate $tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ simplifying your answer as far as possible. (6 marks)

11. (a) Solve the simultaneous equations

$$\frac{x+y}{2} = \frac{2y-z}{4} = \frac{2z+x}{3} \quad ; \quad 2x + 3y + z = 18$$

    (b) Find the positive square root of $-4\sqrt{10} + 13$ hence evaluate $\sqrt{\frac{90}{13-4\sqrt{10}}}$ in the form $\sqrt{p} + \sqrt{q}$

12. (a) Show that in nay triangle ABC

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$$ (5 marks)

    (b) Prove that in any triangle ABC

$$\frac{b-c}{b+c} = cot\frac{1}{2}(B+C)tan\frac{1}{2}(B-C)$$

    Further, deduce that

$$tan\frac{1}{2}(B-C) = \left(\frac{b-c}{b+c}\right)cot\frac{1}{2}A$$

    Hence solve the triangle when $A = 46^0 44', b = 10.76, c = 21.70$ (7 marks)

13. (a) The ovals of the cassini is a curve defined as

$$(x^2+y^2)^2 - 4(x^2-y^2) + 3 = 0$$

    Find point(s) on this curve where there are horizontal tangent lines. (6 marks)

    (b) A window is in the shape of a rectangle surmounted by a semi-circle. If the perimeter is to be 18 feet, find in terms of π the dimensions of a semi-circle which maximizes its area. (6 marks)

END