$y = \sin^3 x$

$y + \Delta y = \sin^3(x + \Delta x)$

$\Delta y = \sin^3(x+\Delta x) - \sin^3 x$

from

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$\Delta y = (\sin(x+\Delta x) - \sin x)\left[\sin^2(x+\Delta x) + \sin(x+\Delta x)\sin x + \sin^2 x\right]$

(use ~~factor~~ ~~formula~~) $\Big\{$ As $\Delta x \to 0$ $\Big\}$

$\Delta y = 2\cos\left(x + \frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}\left[\sin^2 x + \sin x \cdot \sin x + \sin^2 x\right]$

As $\Delta x \to 0$

$\Delta y = 2\cos x \cdot \sin\frac{\Delta x}{2}\left[3\sin^2 x\right]$

$\frac{\Delta y}{\Delta x} = \left(2\cos x \cdot 3\sin^2 x \cdot \sin\frac{\Delta x}{2}\right) \times \frac{1}{\Delta x}$

As $\Delta x \to 0$ $\quad \sin\frac{\Delta x}{2} \to \frac{\Delta x}{2}$

$\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$

$\frac{dy}{dx} = \left(\cancel{2}\cos x \cdot 3\sin^2 x \cdot \frac{\cancel{\Delta x}}{\cancel{2}}\right) \times 1/\cancel{\Delta x}$

$\frac{dy}{dx} = 3\sin^2 x \cos x$

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If $y = \tan\left(2\tan^{-1}\frac{x}{2}\right)$, show

that $\frac{dy}{dx} = \frac{4(1+y^2)}{4+x^2}$

$y = \tan\left(2\tan^{-1}(x/2)\right)$

$u = 2\tan^{-1}(x/2)$

$\frac{u}{2} = \tan^{-1}(x/2)$

$\frac{x}{2} = \tan\left(\frac{u}{2}\right)$

$\frac{1}{2}dx = \frac{1}{2}\sec^2\frac{u}{2}\,du$

$\frac{dx}{du} = \sec^2\frac{u}{2} = 1 + \tan^2\frac{u}{2}$

$= 1 + \frac{x^2}{4}$

$= \frac{4+x^2}{4}$

$y = \tan u$

$\frac{dy}{du} = \sec^2 u = 1 + \tan^2 u$

$\frac{dy}{du} = 1 + y^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 1+y^2 \times \frac{4}{4+x^2}$

$= \frac{4(1+y^2)}{4+x^2}$

Scanned with CamScanner

Given that $x = \frac{3t-1}{t}$, and $y = \frac{t^2+4}{t}$

Show that $\frac{d^2y}{dx^2} = 2t^3$.

SECTION A: (40 marks)

Attempt **all** *questions in this section.*

1. Solve the simultaneous equations below;

$x + 3y + 2z = -13$

$2x - 6y + 3z = 32$

$3x - 4y - z = 12$

(05 marks)

2. A line $2x - y + 3 = 0$ touches a circle whose centre is $(-4, 5)$. Determine the equation of this circle. (05 marks)

3. Show that $sec2x - tan2x = tan\left(\frac{\pi}{4} - x\right)$. (05 marks)

4. Evaluate;

$\int_2^5 x\sqrt{(x-1)}\,dx.$ (05 marks)

5. Find the square root of $17 - 12\sqrt{2}$ and simplify it as far as possible. (05 marks)

6. Differentiate from first principles. tan x (05 marks)

7. Find the Cartesian equation of plane passing through the midpoint of AB with $A\,(-1, 2, -5)$ and $B\,(3, 0, -1)$ which is perpendicular to the line

$\frac{x-1}{2} = \frac{y+7}{-3} = \frac{6-Z}{8}.$

(05 marks)

SECTION A: (40 marks)

Attempt **all** *questions in this section.*

1. Solve the simultaneous equations below;

$$x + 3y + 2z = -13$$
$$2x - 6y + 3z = 32$$
$$3x - 4y - z = 12$$

(05 marks)

2. A line $2x - y + 3 = 0$ touches a circle whose centre is $(-4, 5)$. Determine the equation of this circle.

(05 marks)

3. Show that $sec2x - tan2x = tan\left(\frac{\pi}{4} - x\right)$.

(05 marks)

4. Evaluate;

$$\int_2^5 x\sqrt{(x-1)}\, dx.$$

(05 marks)

5. Find the square root of $17 - 12\sqrt{2}$ and simplify it as far as possible. (05 marks)

6. Differentiate [redacted] from first principles. tan x

(05 marks)

7. Find the Cartesian equation of plane passing through the midpoint of AB with $A(-1, 2, -5)$ and $B(3, 0, -1)$ which is perpendicular to the line

$$\frac{x-1}{2} = \frac{y+7}{-3} = \frac{6-Z}{8}.$$

(05 marks)

8. Solve the differential equation below;

$$\frac{dy}{dx} - y\,tanx = cosx, \text{ given that } y = 0 \text{ at } x = \frac{\pi}{2}.$$

(05 marks)

SECTION B: (60 marks)

Answer any **five** *questions from this section.* **All** *questions carry* **equal** *marks.*

9\. (a) The roots to aquadratic equations $ax^2 + bx + c = 0$ are in a ratio $(3b) : (ac)$, show that $3b^3 = (ac + 3b)^2$. (05 marks)

(b) The roots to aquadratic equation $2x^2 + x - 4 = 0$ are $\propto$ and β, form aquadratic equation whose roots are $\left(\frac{\propto^2}{\beta-4} + \frac{\beta^2}{\propto-4}\right)$ and $\left(\frac{\propto-1}{\propto^2\beta} + \frac{\beta-1}{\propto\beta^2}\right)$. (07 marks)

10\. (a) Use De – moivre's theory to show that $16\,Sin^5\theta = Sin5\theta - 5Sin3\theta + 10Sin\theta$. (05 marks)

(b) Prove that $3i + 2$ is a root to the equation $Z^4 - 5Z^3 + 18Z^2 - 17Z + 13 = 0$, and hence find all other roots to this equation. (07 marks)

11\. (a) P, Q and R are vertices of a triangle with position vectors $5\boldsymbol{i} + 7\boldsymbol{j} - 9\boldsymbol{k}$, $7\boldsymbol{i} + 6\boldsymbol{j} + 2\boldsymbol{k}$ and $11\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ respectively. Using vectors prove that PQR is right angled and hence find its area. (05 marks)

(b) A perpendicular from a point $Q\ (3, -2, 10)$ meets the line

$$r = \begin{pmatrix} 8 \\ -1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ --2 \end{pmatrix}$$ at N, find the;

(i) coordinates of N,

(ii) perpendicular distance of Q from the line above. (07 marks)

12\. (a) Solve for θ if $Cos\,\theta + Cos\,3\theta + Cos\,5\theta = 0$ and $0^o \le \theta \le 180^o$. (05 mar

(b) If A, B and C are angles of a triangle prove that;

$$\frac{1}{a}\,Cos^2\left(\frac{A}{2}\right) + \frac{1}{b}\,Cos^2\left(\frac{B}{2}\right) + \frac{1}{c}Cos^2\left(\frac{C}{2}\right) = \frac{(a+b+c)^2}{4abc}.$$ (07 marks

Turn Over

13. (a) Find $\int \sqrt{x(6-x)}\, dx$. (06 marks)

(b) Differentiate $\frac{Cos2x\, e^{5x}}{In(1-x)}$ with respect to x. (06 marks)

14. (a) Use maclaurin theory to expand $(1-3x+5x^2)^9$ upto the third non zero term. (05 marks)

(b) Expand $\sqrt{\left(\frac{1-2x}{1+2x}\right)}$ as far as to the term containing x^3 and hence using $x = \frac{1}{16}$, estimate $\sqrt{7}$ correct to three decimal places. (07 marks)

15. The parametric coordinate of a curve is $(4Cos\theta, 3\ Sin\theta)$.

(a) Show that the curve represents an ellipse and hence determine its eccentricity. (04 marks)

(b) Find the equations of tangents to the ellipse in (a) above which passes through a point $(-3, 3)$. (08 marks)

16. Water which has been heated to temperature of 99^oC cools in an experimental room which is at a constant temperature of 25^oC at a rate which is proportional to the excess temperature. It was placed in the room at 2 : 30Pm and its temperature at 2 : 31 Pm was 90^oC.

(a) Form a differential equation and solve it. (06 marks)

(b) Determine the temperature of water in the next two minutes. (03 marks)

(c) Find the time at which the temperature of water is 43^oC. (04 marks)

END

SECTION A (40 MARKS)

1. Solve the equation $\sqrt{2y-5}$ - $\sqrt{y-3} = 1$

(05 marks)

2. Differentiate $\cos^2 x$ from first principles

(05 marks)

3. The roots of equation $3x^2 + 2x - 5 = 0$ are α and β. Find the value of $\alpha^4 + \beta^4$. (05 marks)

4. Solve for x;

$$\tan^{-1}(x) + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{9}{7}\right)$$

(05 marks)

5. Evaluate $\int_0^4 \frac{dx}{x+\sqrt{x}}$

(05 marks)

6. The normal to the rectangular hyperbola $xy = c^2$ at P $\left(ct, \frac{c}{t}\right)$ meets the curve again at

$Q\left(cT, \frac{c}{T}\right)$ prove that $Tt^3 = -1$

(05marks)

7. Find the acute between the line $\frac{x-4}{2} = \frac{y+3}{-1} = \frac{z-1}{-2}$ and the place $6x + 2y - z = -4$.

(05marks)

8. Solve the differential equation;

$\frac{dy}{dx} = xy \ln x$ given $y = x = 1$

(05 marks)

SECTION B (60 MARKS)

9. (a) Show that $\tan 3\theta = \frac{3t-t^3}{1-3t^2}$ where $t = \tan\theta$ hence solve $t^3 - 3t^2 - 3t + 1 = 0$, correct to 3 significant figures.

(06 marks)

(b) Solve $2\,cosec^2\,\theta + 3\,cosec\,\theta = 2$ for $0° \leq \theta \leq 360°$ (06 marks)

10. (a) Given that $Z_1 = 3 + i$, $Z_2 = x + i$ and $Arg(Z_1Z_2) = \frac{\pi}{4}$, Find the value of x (05 marks)

(b) Solve: $Z^4 - 6Z^2 + 25 = 0$

(07marks)

11. (a) Evaluate

$$\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) dx$$

(03marks)

(b) Prove that the area enclosed by the two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. If this area is rotated through four right angles about the x –*axis*, show that the volume generated is $\frac{96}{5}\pi a^3$.

(09 marks)

12. (a) Given that $x = \frac{3t-1}{t}$ and $y = \frac{t^2+4}{t}$, show that $\frac{d^2y}{dx^2} = 2t^3$ (06marks)

(b) Determine the area of the largest rectangular piece of land that can be enclosed by 100m of fencing if part of an existing wall is used. (06 marks)

13. (a) Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$ (05 marks)

(b) If x is so small that is fourth and higher powers may be neglected, show that

$\sqrt[4]{1+x} + \sqrt[4]{1-x} = a - bx^2$, find the values of a and b,

Hence show that $\sqrt[4]{17} + \sqrt[4]{15} \approx 3.9985$. $\left(Take\ x = \frac{1}{6}\right)$ (07 marks)

SECTION A: (40 MARKS)

1. Solve the inequality; $\frac{x^2+x}{x+4} \geq 1$ *(05 marks)*

2. O (0, 0) and B (6, 0) are points on a circle whose centre lies in the first quadrant. If the chord OB subtends an angle $2\tan^{-1}\left(\frac{3}{4}\right)$ at the centre of the circle, find the equation of the circle. *(05 marks)*

3. Given that A, B and C are angles of a triangle, prove that; $\frac{\sin A+\sin(B-C)}{\cos(B-C)+\cos A} = \cot C$ *(05 marks)*

4. Given that $\frac{d}{dx}(7^x) = 7^x \ln 7$ show that; $\int \frac{7^x}{7^x+1}dx = \log_7 A(7x+1)$, where A is a constant. *(05 marks)*

5. The parametric equations of a line are; $x = 2\lambda + 1$, $y = \lambda - 2$ and $z = 3 - 2\lambda$, where λ is the parameter.
   (i) Find the Cartesian equation of the line.
   (ii) Determine the coordinates of the point of intersection of this line and the plane $x - y + z = 1$. *(05 marks)*

6. Find the principal argument of $\left(\frac{-\sqrt{3}+i}{1-i}\right)^2$ *(05 marks)*

7. Given that $x = \ln(xy)$, show that; $\frac{dy}{dx} = \left(\frac{x-1}{x^2}\right)e^x$ *(05 marks)*

8. The gradient function of a curve is $2x + \frac{16}{x^2}$. If the y – coordinate of the stationary point of the curve is 13, find the equation of the curve. *(05 marks)*

SECTION B: (60 MARKS)

9. Given that curve: $y = \frac{x^2-4}{x^2-1}$,
   (i) Find the equations of the asymptotes.

(ii) State the equation of the line of symmetry of the curve, hence or otherwise determine the coordinates of the stationary point.

(iii) Sketch the curve; hence deduce the range of values of y within which the curve does not lie. *(12 marks)*

10. (a) Evaluate; $(1+i)^8$ *(05 marks)*

(b) Find and sketch the locus $\text{Arg}\,(Z-1-2i) = \pm\frac{\pi}{4}$, where $Z = x + yi$. *(07 marks)*

11. (a) Evaluate; $\int_{\sqrt{2}}^{\sqrt{5}} \frac{x}{\sqrt{x^2-1}}\,dx$ *(05 marks)*

(b) Find; $\int \frac{d\theta}{4 + 5\cos\theta}$ *(07 marks)*

12. (a) Calculate the distance of the point P(1,2,3) from the line $\frac{x}{3} = \frac{y+3}{4} = z$. *(06 marks)*

(b) Find, in scalar product form, the equation of the plane containing the point P(1,2,3) and the line $\frac{x}{3} = \frac{y+3}{4} = z$ in (a) above. *(06 marks)*

13. (a) Express; $11 + 8x - 2x^2$ in the form; $a + b(x+c)^2$, where a, b and c are constants, hence deduce the minimum positive value of $\frac{1}{12 + 8x - 2x^2}$. *(06 marks)*

(b) Find the Maclaurin's expansion of the function $\ln(2+x)$ up to the term in x^4. State the range of values of x for which the expansion is valid hence evaluate $\ln 1.98$ to 3 decimal places. *(06 marks)*

14. (a) Use the mathematics of small changes to evaluate $\tan 44.6^\circ$ to 4 decimal places. *(06 marks)*

(b) The parametric equations of a curve are functions of t. Given that at any point (x, y) on the curve $\frac{dy}{dx} = 2t$ and $\frac{dx}{dt} = \frac{1}{t^2}$ and the gradient of the curve at $(2, 4)$ is -2, find the parametric equations. *(06 marks)*

15. (a) Solve the equations; $\sin 3\theta = 2\sin^3\theta$ for $0^\circ \leq \theta \leq 360^\circ$ *(06 marks)*

(b) Prove that; $\frac{\sin\theta}{1+\cos\theta} = \tan\frac{1}{2}\theta$, hence solve the equation; $\sin\theta - \cos\theta = 1$ for $-2\pi \leq \theta \leq \frac{\pi}{2}$ *(06 marks)*

16. The rate at which the price, P dollars, of one barrel of crude oil on the World market changed t months after the beginning of a certain year was found to be; $\frac{dP}{dt} = a + bt$, where a, b are constants. At the beginning of the year the price was 51 dollars and at that instant it was decreasing at 8 dollars per month; and after 2½ months the price was increasing at 2 dollars per month.

(a) Prove that the particular solution to this problems is; $P = 2t^2 - 8t + 51$ *(06 marks)*

(b) (i) Find how long it takes for the price to fall to its minimum value, and state the minimum price.

(ii) Calculate the price of 100 barrels of oil, in pounds, at the end of that year. (3 pounds = 4 dollars) *(06 marks)*

END

SECTION A: (40 MARKS)

1. Find the coordinates of the point C on the line joining the points A (-1, 2) and B (-9, 14) which divides AB internally in the ratio 1:3. Find also the equation of the line through C which is perpendicular to AB *05mks*

2. Solve the equation $\sin^2 x + \sin x \cos x = 0$, for $-180^0 \leq x \leq 180^0$ *05mks*

3. Solve the simultaneous equations

$$\log_2 x + 2\log_4 y = 4,$$
$$x + 12y = 52.$$
05mks

4. The area bounded by the curve $y = x^2 + 1$, the x-axis and the ordinates $x = -1$ and $x = 1$ is rotated through four right – angles about the x-axis to form a solid of revolution. Calculate the volume of the solid. *05mks*

5. Solve the differential equation $x\frac{dy}{dx} + 3 = y - 4\frac{dy}{dx}$; when $x = 1$, $y=13$. Give y in terms of x. Hence, find x when $y=17$. *05mks*

6. Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$ and find the perpendicular distance from the line to the plane *05mks*

7. Using the substitution $t = \log_e x$

Evaluate $\int_e^{e^3} \frac{dx}{x(\log_e x)^2}$ *05mks*

8. (i) One root of the equation $Z^2 + aZ + b = 0$, where a and b are real constants, is $2+3i$. Find the values of a and b.

(ii) If $Z_1 = 3+2i$, $Z_2 = 4-3i$ find Z_1Z_2 and $arg\left(\frac{Z_1}{Z_2}\right)$ *05mks*

SECTION B (60 marks)

*Answer only **5** questions from this section.*

***All** questions carry equal marks.*

9. (a) Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{(5x^3 - 12x + 4)}{\sqrt{(1-x^2)}}\, dx$ *05mks*

(b) By means of the substitution $t = \tan x$, prove that $\int_0^{\frac{\pi}{4}} \frac{dx}{1+\sin 2x} = \frac{1}{2}$ *07mks*

and find the value of $\int_0^{\frac{\pi}{4}} \frac{dx}{(1+\sin 2x)^2}$

10. (a) Find the equation of the normal to the curve $x = 2t$, $y = t^2$ at the point with parameter **t**. If this normal meets the x- and y- axes at the points A and B respectively, find the equation of locus of the mid-point of AB. *06mks*

(b) The tangent to the parabola $y^2 = 4ax$ at the point **P**$(at^2, 2at)$ meets the x- axis at **T**. The straight line through **P** parallel to the axis of the parabola meets the directrix at **Q**. If **S** is the focus of the parabola, show that **PQTS** is a rhombus. If **M** is the mid-point of **PT** and **N** is the mid-point of **PM**, Find the equation of the locus of *06mks*

(i) M (ii) N

11. The planes p and q are given by the equations $3x + 2y + z = 4$ and $2x + 3y + z = 5$ respectively. The plane π containing the point A(2,2,1) is perpendicular to each of the planes p and q find;

(a) the distance from the point A to the plane p,

(b) the angle between the planes p and q,

(c) a Cartesian equation for the plane π

(d) Cartesian equations for the line of intersection, l, of the planes p and q

12mks

12. (a) Use the substitution $y = vx$, where **v** is a function of x, or otherwise to solve the differential equation $\frac{xdy}{dx} = 2x - y$, stating

(i) the general solution and

(ii) the particular solution for which y=5 when $x = 1$ *06mks*

(b) The rate at which a body loses speed at any given instant as it travels through a resistive medium is given by **KV** ms^{-2} where **V** is the speed of the body at that instant and **K** is a positive constant. If it's initial speed is **U**ms^{-1} show that the time taken for the body to decrease it's speed to ½**u** is $\frac{1}{k}$**ln2** seconds *06mks*

13. (a) In the Argard diagram, the point **P** represents the complex number **Z**. Given that $|Z - 1 - i| = \sqrt{2}$, sketch the locus of **P**. Deduce the greatest and least values of $|Z|$ for points **P** lying on the locus. 06mks

(b) If Z_1 and Z_2 are complex numbers, solve the simultaneous equations

$4Z_1 + 3Z_2 = 23$,

$Z_1 + iZ_2 = 6+8i$, giving both answers in the form $x + iy$. 06mks

14. (a) If $x = \frac{3t}{t+3}$ and $y = \frac{4t+1}{t-1}$, find $\frac{d^2y}{dx^2}$ in terms of t. 06mks

(b) Obtain the first two non-zero terms of the McLaurin's expansion of

$f(x) = \frac{\ln(x+1)}{(1+x)}$ 06mks

15. (a) Express $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ in the form $a + b\cos(2x - \alpha)$ where a, b, α are independent of x. Hence, or otherwise, find the maximum and minimum values of $5\sin^2 x - 3\sin x \cos x + \cos^2 x$ as x varies. 06mks

(b) If A, B and C are angles of a triangle; prove that

$\text{Sin}(B+C-A) + \text{Sin}(C+A-B) + \text{Sin}(A+B-C) = 4\text{SinASinSsinC}$. 06mks

16. (a) When $(1 + bx)^n$ is expanded in ascending powers of x, the first three terms of the expansion are $1 - \frac{3}{5}x - \frac{27}{100}x^2$. Find the values of n and b. 06mks

(b) Prove by induction that $8^n - 7n + 6$ is divisible by 7 for all positive integral values of n.

06mks

P425/1
Pure Maths
$1\frac{1}{2}$ hrs.

Attempt All Questions.

Section A (40 Marks)

1. Differentiate $3x - \cos 2x$ from first principles. (5)

2. Differentiate: (i) $e^{-2x} \ln(x^2)$ (2)

(ii) $(2x)^x$ (3)

3. Find the angle between the vectors $-\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$. (5)

4. Find $\int \frac{4}{\sqrt{4 - x^2}}\, dx$ (5).

5. Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$. (5)

6. Expand $(1 - x/2)^{\frac{1}{2}}$ up to the term in x^2, hence evaluate $\sqrt{15}$ to 3dps. (5)

7. Given that $\log_3 m = x$, $\log_{15} m = y$, show that $\log_5 3 = \frac{y}{x - y}$. (5)

8. Sketch the curve $y = x^3 - 2x^2$. (5)

P.T.O.

(i) Show that the curve does not have turning points.(4marks)

(ii) Find the equations of the a symptotes. Hence sketch the curve.(4marks)

15. (a) Prove that $2\cot\frac{A}{2} + \tan A = \tan A \cot^2\frac{A}{2}$ (4marks)

(b) Solve $\sin\theta \quad \sin 2\theta = \sin 4\theta - \sin 3\theta$ for $0^0 \leq \theta \leq 360^0$ (4marks)

(c) If A, B and C are angles of a triangle , prove that:

$\text{Sin}^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \text{ Cos } B \text{ Cos } C.$ (4marks)

16. (a) Find the general solution of

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x(x+y)}$$

(b) The rate at which a radioactive material decays is proportional to the amount of such material present

Half of the original; mass **M** of the radioactive material undergoes disintegration in a period of 1500years

(i) What percentage of the original mass will remain after 3000 years ?

(ii) In how many years will one tenth of the original mass remain? (7marks)

END

Attempt any 10 questions

1. Prove that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$
2. Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{3-4x-4x^2}}\,dx$
3. Find the square root of $17-12\sqrt{2}$ and simplify it as far as possible
4. Differentiate tanx form firs principles
5. Use maclaurins' theory to expand $(1-3x+5x^2)^9$ up to the third non-zero term
6. Differentiate with respect to x
   (i) $2^{\cos x^2}$
   (ii) $\log_e\left(\frac{(1+x)e^{-2x}}{1-x}\right)^{\frac{1}{2}}$
7. *Show that* $\frac{d}{dx}(\tan^{-1}(x^x)) = \frac{x^x(inx+1)}{1+x^{2x}}$
8. solve for x : $\tan^{-1}(x) + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{9}{7}\right)$
9. Find $\int \frac{1}{x^2+6x+34}\,dx$
10. Given that $y = \text{Log}_3(5x)^{½}$, show that $\frac{dy}{dx} = \frac{1}{xin9}$
11. For the curve , $y^2 = \sin x \cos^3 x$ where $0 \leq x \leq \frac{\pi}{2}$, show that
    $\left(\frac{dy}{dx}\right)^2 = ¼ \cot x(\cos^2 x - 3\sin^2 x)^2$, provided $x \neq 0$
12. Find the maclaurins expansion of the function $In(1 + \sin x)$ up to the term in X^3
13. Solve for x in the equation
    $\tan^{-1}(x/2) + \tan^{-1}(x/3) = \frac{\pi}{4}$
14. Given that $y = \log_2 x + \text{Log}_x 2$, show that $\frac{dy}{dx} = \frac{(In2x)In\left(\frac{x}{2}\right)}{In2^x(Inx)^2}$
15. Given that $y = \frac{\sin x}{x}$, find the value of $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy$

P425/1
PURE MATHEMATICS
PAPER 1
OCTOBER 2016
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK A EXAMINATION 2016

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Attempt* ***ALL*** *the questions in section* ***A*** *and any* ***FIVE*** *from section* ***B.***
- *All working must be clearly shown.*

SECTION A (40 marks)

Attempt ***ALL*** *questions in this section*

1. Show that $1+2i$ is a root of the equation $2z^3 - z^2 + 4z + 15 = 0$, hence find the other roots. (5 marks)

2. Prove the identity: $\cos 2(\alpha+\beta) - \cos 2\alpha + \cos 2\beta - 1 = -4\sin(\alpha+\beta)\cos\alpha\sin\beta$. (5 marks)

3. The sum of the first $n-$ terms of a certain series is given by $S_n = \dfrac{n(n^2-1)}{3}$, find an expression for the $n-th$ term hence find the forth term. (5 marks)

4. Find the perpendicular distance from the point $A(1,\ 2,\ -4)$ to the plane which passes through the point $B(1,\ 4,\ 9)$ and is normal to the vector $3\mathbf{i} - \mathbf{k}$. (5 marks)

5. Evaluate: $\sqrt{9.09}$ using small increments. (5 marks)

6. Given that $y = \sqrt{\dfrac{1+\sin x}{1-\sin x}}$, show that $\dfrac{dy}{dx} = \dfrac{1}{1-\sin x}$. (5 marks)

7. The point P moves in such a way that its distance from the line $4x + 3y = 0$ is equal to its distance from the point $(-1, -2)$. Find the locus of P. (5 marks)

8. If $\dfrac{dy}{dx} + 2y\tan x = \sin x$, find y in terms of x if $y\left(\dfrac{\pi}{3}\right) = 0$. (5 marks)

SECTION B

*Attempt **ONLY** 5 questions from this section.*

9a) Prove that the circles $x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 16$ are orthogonal. (5 marks)

b)i) Find the points of intersection of the two circles, $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 - 2x - 4x - 6y + 14 = 0$.

ii) Show that the common area of intersection of the circles is given by $8\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}\right)$. (7 marks)

10a) Use de Moivre's theorem to show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. (6 marks)

b) Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\dfrac{z-3}{z-2i}\right) = \dfrac{\pi}{4}$. (6 marks)

11a) Evaluate: $\displaystyle\int_0^{\pi/3} \frac{\sin x}{(1+\cos x)^2}\,dx$ (5 marks)

b) Show that $\displaystyle\int_0^{\pi} x\sin^2 x\,dx = \frac{1}{4}\left(\pi^2 - 1\right)$. (7 marks)

12a) Prove that: $\frac{3\sin\theta + \sin 2\theta}{1 + 3\cos\theta + \cos 2\theta} = \tan\theta$, hence, solve the equation $\frac{3\sin\theta + \sin 2\theta}{1 + 3\cos\theta + \cos 2\theta} + \frac{1}{\cos^2\theta} = 2$, for $0^\circ \leq \theta \leq 360^\circ$.

(6 marks)

b) Prove that $\cos^2 2A + \cos^2 2B + \cos^2 2C - 1 = 2\cos 2A \cos 2B \cos 2C$, given that $A + B + C = 180^\circ$..

(6 marks)

13a) A container in the shape of a hollow cone of semi-vertical angle 30° is held with its vertex pointing downwards. Water is poured into the cone at the rate of $5\,cm^3\,s^{-1}$. Find the rate at which the depth of water in the cone is increasing when this depth is $10cm$.

(6 marks)

b) Given that $y = \log_2 x + \log_x 2$, show that $\frac{dy}{dx} = \frac{(In2x)In\frac{x}{2}}{In2^x(Inx)^2}$.

(6 marks)

14. Determine the turning points and asymptotes of the curve $y = \frac{4x^2 - 10x + 7}{(x-1)(x-2)}$

hence, sketch the curve.

15. A laboratory investigation was carried out to investigate the effect of a newly produced drug on a certain poultry bacteria. It was revealed that the rate at which the bacteria is killed is directly proportional to the population present at that time. Initially, the population was P_o and at t months, it was found to be P.

i) Obtain a differential equation connecting P and t and solve it.

ii) If the bacteria population reduced to two thirds of the initial population 6 months later, solve the equation in (i) above.

iii) Find how long it will take for only 10% of the original population to remain.

(12 marks)

SECTION A (*40 Marks*)

1. Given that the equations $x^2+ax+b=0$ and $4x^2-ax+6b=0$ have a common root where a and b are non-zero constants, prove that $35a^2+4b=0$. (*05mks*)
2. Find three numbers in a geometrical progression (G.P) such that their sum is 13 and their product is 27. (*05mks*)
3. The line $y=mx$ and the curve $y=x^2-2x$ meet at point O(0,0) and A. if P is the midpoint of OA, find the equation of locus of P as m varies. (*05mks*)
4. Prove that in any triangle *ABC*,

   $\sin\left(\frac{B-C}{2}\right)=\frac{b-c}{a}\cos\frac{A}{2}$ (*05mks*)
5. Evaluate $\int_0^{\frac{1}{\sqrt{2}}}\frac{x^2}{\sqrt{1-x^2}}\,dx$. (*05mks*)
6. The tangent at any point *P* on the curve in the first quadrant cuts the *x*-axis at *A*. Given that *OP*=*PA*, where *O* is the origin and that point (1,4) lies on the curve, shows that the equation of the curve is $xy=4$. (*05mks*)
7. Find the area bounded by the curves $y^2=4x$ and $y=\frac{1}{4}x^2$. (*05mks*)
8. Find the vector equation of the line that passes through the point with position vector $2i+3j$ and perpendicular to the line with equation $r=\begin{pmatrix}3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\end{pmatrix}$ (*05mks*)

SECTION B. (*60 marks*)

9. (a)Find the equation of the chord joining the points $\left(5t_1,\frac{5}{t_1}\right)$ and $\left(5t_2,\frac{5}{t_2}\right)$ on the hyperbola $xy=25$, hence deduce the equation of the tangent to the hyperbola at the point $(5t,\frac{5}{t})$. (*05mks*)

   (b)Find the equations of the tangents from the point (1,0) to the hyperbola with parametric co-ordinates $(2\sec\theta, 3\tan\theta)$ (*07mks*)
10. (a)Give that α and β are the roots of the equation $x^2-x-3=0$, show that

    $\alpha^3+\beta^3=10$. (*03mks*)

    (b)Given the equation $x^3+2x^2-11x-12=0$

    (i) show that $x={}^-1$ is a root.

    (ii)Deduce the values of $\alpha+\beta$ where α and β are the other roots of the equation, hence form a quadratic equation whose roots are α^3 and β^3. (*09mks*)

11. (a) Form a quadratic equation whose one of its roots is $2\text{-}3i$. *(04mks)*

(b)(i) With out using tables simplify.

$$\frac{\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)^4}{\left(\cos\frac{\pi}{9} - i\sin\frac{\pi}{9}\right)^5}$$

(ii) Express $Z_1 = \frac{7+4i}{3-2i}$ in the form $p+qi$ where p and q are real.

Sketch in an argand diagram the locus of the points representing *(08mks)*

such that $|z\text{-}z_1|=5$. *(06mks)*

12. (a) Find $\frac{d}{dx}\left\{ln(secx + tanx)\right\}$. hence show that $\int_0^{\frac{\pi}{6}} secx\, dx = \frac{1}{2}\, ln3.$ *(06mks)*

(b) Resolve $y = \frac{(1-x)(1+x)}{x^3+x}$ in partial fractions, hence find $\int ydx.$

13. (a) Given that $y = e^x\, sinx$, prove that *(05mks)*

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

(b) The time, T seconds taken for one complete swing of a pendulum,

length l metres is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where g is a constant.

If a 1% error is made in measuring the length of a pendulum, estimate the percentage *(07mks)*

error in the value of T.

14. (a) If $cosA = \frac{5}{7}$ and $sinB = \frac{1}{5}$ where A is acute and B is obtuse, show that

(i) $Sin(A - B) = \frac{-29}{35}$

(ii) $Cos(A - B) = \frac{-8\sqrt{6}}{35}$. *(06mks)*

(b) Solve the equation *(06mks)*

$$Sec\theta - 3\tan\theta = 2;\ 0^\circ \leq \theta \leq 360^\circ$$

15. (a) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. D is the midpoint of CB and OD meets AC at E. if $\overrightarrow{OE} = h\,\overrightarrow{OD}$ and $\overrightarrow{AE} = k\,\overrightarrow{AC}$, find

(i) OE in terms of h, **a** and **c**.
(ii) AE in terms of k, **a** and **c**
(iii) The values of h and k. *(07mks)*

(b) Find the perpendicular distance of the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{-3}$

from the plane $r.\begin{pmatrix}4\\-1\\3\end{pmatrix} = 4.$ *(05mks)*