SECTION A
MECHANICS

SECTION B
HEAT

SECTION C
MODERN PHYSICS

Go through the paper quickly reading or scanning it to get an idea of what has been asked.

Work your way through the paper. If you find any question particularly formidable, do not carry on with it. Come back to it later. You will lose valuable time being bogged down with one question. Get some easy marks in the bag first.

Sometimes the physics concepts just come in one ear and leave out the other at the speed of light therefore it's good to do continuous revision and remind yourself of the definitions, derivations, experiments and calculations therein. The exams will always test your knowledge and understanding of the physics concepts and principles.

When you are stuck, read the question again and ask yourself:

- Does the question fit what I have been trying to do?
- Is there a diagram I could draw?
- Have I pictured the situation described by the question?
- Have I missed an equation that is needed?
- Are there words in the question that I have disregarded, perhaps 'in series, steady, in parallel, etc.' in a question?

Always be careful with details such as units. Some questions ask you to 'explain' or 'describe' something. Read your answers through and see if they make sense to someone who does not already know the answer.

When you have finished do not sit arms folded looking up at the ceiling. You will not have scored 100%. There are some marks still to be had. Spend every last minute going through the paper carefully looking for errors. Trust me, there will be some.

If you run out of time or if you are running out of time. Look for a question with a lot of marks and with a

Maximize marks.

There only remains for me to say good luck. But exam have little to do with luck. Luck goes to a prepare mind. If you have done the work and revise thoroughly, you will undoubtedly do well. The fa that you are reading, this shows your intent. So, don look at exams as impossible hurdles to jump. Look them as opportunities for you to shine and sho everyone just what you can do.

Dedication

To my beloved son Fahad Ali

In the present century, we find that science is held in high esteem. Although the word "science" has its origin in a Latin verb meaning "to know", science has come to mean not merely knowledge but a body of knowledge of natural world in an organised and rational way. The scientific developments have exerted influence on every phase of human activity. The intelligent man recognises that the material progress of technology is a by-product of the discoveries of science. In all fairness, honest credit is being given to science for its contribution to knowledge, comfort, efficiency and to the general well-being of mankind.

Physics is the fundamental science. The main goal of physics is give correct and precise description of the material universe. The study of physics is an adventure. It is challenging, sometimes frustrating, occasionally painful and often richly rewarding and satisfying. Although physics has earned the reputation as a difficult subject, yet it is inherently simple. It is because there are only a few fundamental laws that you need to lodge in your memory bank. If you really understand these laws, you can readily use them to deduce how nature behaves in a variety of situations. As you read the text and work out physics problems, ask yourself how each problem you approach is really similar to other problems and to the text examples. You will find that similarity because most of the problems and examples really involve only a few underlying laws. So, physics is simple provided you understand its fundamental laws and have the experience in reasoning out how these laws apply to the world about you.

PHYSICS

The branch of science which deals with correct and precise description material universe is known as physics. The word physics comes from a Greek word meaning "nature". Physics attempts to describe the fundamental nature of the universe and how it works. For example, physics explains why the sky is blue, why rainbows have colours, why steel ships float, what atoms and nuclei are made of, etc. To all these and many more questions, physics explains using as few laws as possible, revealing their underlying simplicity and beauty. The amount of knowledge available on physics is so vast that as a matter of convenience, it has been divided into two classes:

(i) Classical (old) physics
(ii) Modern physics

as classical physics. By 1890, the various laws governing the material universe were discovered and it appeared that knowledge of this subject was complete. Newton's laws of motion and gravitation had provided a solid foundation on which science of mechanics had been raised to impressive heights. The laws of thermodynamics and kinetic theory of matter gave a satisfactory account of the entire subject of heat. Maxwell developed the theory of electromagnetic radiation and explained all electrical, magnetic and optical phenomena. It appeared that the laws and theories of classical physics would hold the field unchallenged. However, startling discoveries after 1890 gave a death blow to the so-called classical physics.

Modern physics: The physics after 1890 is known as modern physics. After 1890, there had been enormous advances in physics which marked the beginning of revolutionary concepts. The thrilling discoveries posed new problems unknown to classical physics. In 1895, Roentgen discovered **X-rays** and a few weeks later, Henry Becquerel announced the discovery of **radioactivity**. In the year 1897, J.J. Thomson discovered **electrons**, a fundamental constituent of all matter. In 1901, Max Planck developed quantum theory according to which energy is not radiated continuously but in discrete packets of energy called quanta. These quanta eventually came to be known as photons. The **special theory of relativity** proposed by Albert Einstein in 1905 gave new dimensions to atomic and nuclear physics. In short, the rapid startling discoveries after 1890 gave a new shape to the entire concept of physics. The study of these concepts has been placed under the head of modern physics.

The reader may wonder about the difference between classical physics and modem physics. The answer is that classical physics does not work for microscopic particles (e.g., atoms) or for objects travelling at very high speed. In fact, these two aspects gave birth to what is now called modern physics. Indeed, modern physics may be considered as mere "fine-tuning" of classical physics.

Except for the microscopic particles and for motion at speeds near the speed of light, classical physics correctly and precisely describes the behaviour of the physical world. Although modern physics has made many important contributions to technology, great bulk of technical knowledge and skill is still based

[...]ly on classical physics. It is through the applications of classical physics that we have been able to exploit natural resources successfully.

SCIENTIFIC NOTIONS

Some important scientific notions are; (i) scientific law (ii) theory (iii) model (iv) scientific method.

Scientific law: The general statement of happenings in nature and after experiments is known as a scientific law.

A scientific law is a concise statement how nature behaves. It is actually the essence of a large number of observations and experiments. For example, the law of gravitation states that the two objects attract each other. Similarly, "unlike charges attract each other and like charges repel" is also a law and is based on observations and experiments. A law has no theoretical or other basis. In other words, we cannot ask "why like charges repel and not attract? Why two bodies attract each other and not repel?" The only answer to such questions is that this is how nature behaves and the matter ends there. No "buts" and "ifs".

Physical laws are usually expressed as mathematical equations which are then used to make predictions about other phenomena. For example, the law of gravitation states that the two objects attract each other with a force which is proportional to the mass of each object and inversely proportional to the square of the distance between them. It is written in the mathematical form as $F = \frac{Gm_1m_2}{r^2}$. Because this expresses the law in the form of an equation, it is far preferable to the qualitative statement. It may be noted that a scientific law has only experimental proof

Theory: Speculation on the basis of determined laws is known as "theory".

When several laws are determined, then one may make a guess about the way the universe must be constituted in order that such things happen. This guess is called theory. It may be noted that there is a basic difference between a theory and a law. Theory is given by man and hence may be wrong while law is made but by nature and always right.

Theories are never derived from observations - they are created to explain observations. They are inspirations that come from the minds of intelligent people. For example, the idea that matter is made up of atoms (atomic theory) was certainly not arrived at because someone observed atoms. Rather the idea was conceived by a creative mind. The theory of relativity and the electromagnetic theory were likewise the result of inspiration. Nevertheless, the history of physics tells us that theories come and go, that long-held theories are replaced by new ones. However, a new theory is accepted only if it explains a greater range of phenomena than does the older one.

Model: This is a visual picture of the phenomenon under study.

When scientists are trying to understand a particular phenomenon, they often make use of a model. The purpose of a model is to give us a visual picture – something to hold on to when we cannot see what actually is happening. One example is the wave model of light. We cannot see waves of light as we can see water waves. But it is valuable to think of light as if it were made up of waves because experiments on light show that it behaves in many aspects as water waves do.

Scientific method: To understand any natural process by a combination of logical reasoning and controlled experimentation is called the scientific method.

Prior to about 1600, questions of truth and falsehood were most often determined by political or religious dictates. But great scientists like Isaac Newton introduced scientific methods to the world. The two main tools of scientific method are logic and experimentation.

The result is that the experimental results are reproducible. That is, the same set of circumstances will always produce the same observed results in the same experiment, no matter who the observer is.

Notes: (i) Scientific laws are different from political laws in that the latter tell us how we must behave. Scientific laws do not say how nature behaves but rather describe how nature must behave.

(ii) To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term "principle" is often used (such as Archimedes' principle). Where to draw the line between laws and principles is, of course, arbitrary and there is not always complete consistency.

SCOPE AND EXCITEMENT OF PHYSICS

The scope of physics is very wide. It covers an immense range of natural phenomena. Distances extend from incredibly small dimensions of subatomic particles to thousands of million metres that separate galaxies of the universe. For example, the radius of atomic nucleus is about 10^{-14} m while the radius of the universe is about $10^{25}\,m$. The time intervals encountered in the physical world vary over an extremely wide range. For instance, the time taken by light to cross distance of nuclear size is about $10^{-22}\,s$ while the life of the sun is about $10^{18}\,s$. Similarly, the mass of an electron is approximately 10^{-30} kg whereas the mass of the universe is about 10^{55} kg. All these wide range of measurements are in the domain of physics.

The subject of physics has wide applications. It includes medical physics, computers, meteorology, material science, geophysics, engineering communication, environmental physics etc. Physics is a very exciting subject It provides answers to exciting questions such as:

- Why is the sky blue?
- Why is sunset red?
- Why is glass transparent?
- Why are atoms held together?

These are only a few exciting questions. In fact, there are a very large number of such questions to which physics provides answers.

FIELDS OF PHYSICS

Physics is the physical science which deals with matter and energy and their transformations. Knowledge of physics will help you understand both natural phenomena and the technologies that increasingly prevade our lives. There are numerous problems that physics faces yet we can describe and understand any problem with the help of the following five fields of physics i.e.

Classical mechanics: It deals with objects from molecules to galaxies that are moving at speeds small compared with the speed of light. It deals with such ideas as inertia, motion, force and energy. Mechanics also includes the properties and laws of both solids and fluids. You use the laws of classical mechanics (also known as Newtonian mechanics) when you drive a car, ride a skateboard, build a skyscraper etc.

Indeed, many physics problems can be solved through the application of laws of classical mechanics. You will see that classical mechanics is but an approximation to a more comprehensive set of physical laws that includes Einstein's theory of relativity and the theory of quantum mechanics.

Thermodynamics: It deals with the study of heat and its interaction with matter. It includes effects of temperature on the properties of materials, heat flow and transformations involving heat and work.

Electromagnetism: It deals with electricity, magnetism and electromagnetic radiation. Today electromagnetic technology dominates our civilization as evidenced by radio, television, tape recorders, microwave ovens and computers.

Relativity: In 1905, Albert Einstein gave the theory of relativity. This theory has changed our notions of space and time and of mass and energy. Space and time are seen to be intimately connected with time being the fourth dimension in addition to space's three dimensions. Although theory of relativity provides a more correct description of physical reality than does classical mechanics, the two theories differ significantly only at speeds approaching that of light. In our everyday existence, we need not worry about relativity.

Quantum mechanics: It deals with the behaviour of objects on the extremely small scale of atomic dimensions. Indeed, classical physics fails to explain the behaviour of such microscopic particles. The quantum mechanics touches every aspect of modern physics and most of classical physics. Indeed, one requires an understanding and the application of quantum mechanics for a satisfactory explanation of most of the phenomena.

THE SIMPLICITY OF PHYSICS

There is a common notion that physics is incomprehensible to the people of average ability. If you think that, you are missing the point because it is so fundamental, physics is inherently simple. There are only a few basic laws to learn. If you really understand those laws, you can readily apply them in a wide variety of situations. The laws and theories of physics describe the working of the universe at the most basic level. For that reason, physics is extremely powerful. The same laws describe the behaviour of molecules, aeroplanes and galaxies. Scientists believe that it would be possible to describe the operation of a living cell or organism using only the fundamental laws of physics. So, physics is simple – challenging too but with an underlying simplicity that reflects the scope and power of this fundamental science.

A physical science is one which primarily deals with non-living things. On the other hand, biological science deals with living things.

PHYSICS IS BASIC SCIENCE

Physics is the basic science. It is about the nature of basic things such as motion, force, energy, matter, heat, sound, light and the inside of atoms. An understanding of science begins with an understanding of physics. There is physics in everything you see, hear, smell, taste and touch.

Physics and Mathematics: The laws of physics are usually expressed as mathematical equations which are then used to make predictions about other phenomena and to test the range of validity of the laws. In order to understand physics on any level beyond qualitative description, you require a considerable knowledge of mathematics. The mathematical description of the laws and theories of physics gives new insight to the subject. Indeed, the primary goal of mathematics is to aid physics.

Physics and Chemistry: Chemistry is about how matter is put together, how atoms combine to form molecules and how molecules combine to make up the many kinds of matter around us. In order to explain these aspects, we have to apply the laws and theories of physics. So, underneath chemistry is physics.

Physics and Biology: Biology is more complex and involves matter that is alive. So, underneath biology is chemistry and underneath chemistry is physics. That is why physics is the most basic science.

Physics and other sciences: Since physics is a fundamental science, its laws and theories are applied to various physical sciences. The knowledge of physics is required in many fields and the list below shows some of these:

- Medical physics
- Meteorology
- Geophysics
- Environmental physics
- Communication and material science.

PHYSICS AND TECHNOLOGY

Physics and technology are different from each other. Physics is the knowledge of material universe for own sake whereas technology transfers the knowledge of physics into practical shape for the general well-being of human race. In other words, physicists provide the knowledge of material universe whereas the engineers provide the tools, techniques and procedures for putting this knowledge to practical use. For example, physics has provided the knowledge of semiconductors but technology made of them transistor radios. Many technologies have been developed on the basis of the knowledge of physics and a few of them are mentioned below by way of illustration.

(a) The discovery of Faraday's laws of electromagnetic induction enabled the engineers to develop the electric generator, electric motor, transformer and a host of other equipment.
(b) The discovery of the laws of thermodynamics led to the development of many types of heat engines.
(c) The discovery of electromagnetic radiation, vacuum tubes and semiconductors led to wireless communication e.g. radio and television etc.
(d) The discovery of nuclear fission led to the development of nuclear power plants which are generating huge amounts of electric power all over the world.

The above short list indicates how important the knowledge of physics is in developing new technologies. Indeed, technology may be regarded as **applied physics**.

When laws of physics are expressed in mathematical form, they do not have the double meanings that so often confuse the discussion of the ideas expressed in common language. When laws are expressed mathematically, they are easier to verify or disprove by experiments.

PHYSICS AND SOCIETY

The advancement of technology as a result of discoveries of knowledge of physics has made a significant impact on human lives. We travel faster than ever before in aeroplanes. We enjoy colour television. We use electricity to operate a host of electrical appliances. We can solve complex problems at amazing speed with the help of computers. We send satellites into orbit to learn about the conditions in outer space. The development of radio and television has led to instant communication. Indeed, the uses of phase of human activity.

We are all familiar with the abuses of applied physics or technology. Many people blame technology itself for widespread pollution, resource depletion and even social decay in general. It is much wiser to combat the misuse of technology with knowledge than with ignorance. Wise applications of physics can lead to a better world.

BASIC FORCES IN NATURE

There are four basic forces that operate in nature. These are (i) gravitational force (ii) electromagnetic force (iii) the strong nuclear force and (iv) the weak nuclear force.

CONSERVATION LAWS

In physics, there are some physical quantities which remain the same before and after an event or interaction. Such quantities are called conserved quantities. For example, electric charge is a conserved quantity. In every event, the principle of conservation of charge has always been found to apply. Thus, when we produce a free negative charge by tearing an electron loose from an atom, an equal positive charge is left behind, thereby causing no change in total charge of the system. The laws relating to conserved quantities are called conservation laws.

Those laws of nature that state that some physical quantity is the same before and that after an event or interaction are called conservation laws.

Some important, physical quantities that are conserved in an event or interaction are:

(i) Electric charge: Law of conservation of electric charge

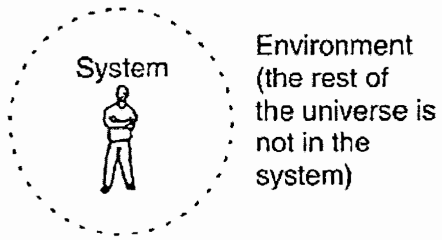
(ii) Energy: Law of conservation of energy

(iii) Momentum: Law of conservation of momentum

We do not know the reason for it. But scientists have performed a very large number of experiments over the years to test the conserved nature of the above physical quantities. In all the experiments, the conserved nature of the above physical quantities is proved beyond any doubt. Nevertheless, conservation laws reflect one of the most basic ways of describing nature. Because conservation laws allow us to consider quantities, such as energy, that do not change during an event, we do not need to know the details of the interaction, we simply deal with the value of the conserved quantity before and after the interaction or event. Moreover, the some are not, tells us something about nature itself.

System and Environment: A system is some part of the universe in which we have a special interest. All the rest of the universe outside the system is called environment. The system can be any part of the universe we choose e.g. a human body, the whole earth, a single atom etc.

The figure below shows the dashed line around a person. The part of the universe within the dashed line is the system. All the rest of the universe outside our chosen system is the environment. The dashed line can be an actual boundary between the system and the environment, or it can be an imaginary boundary that we use to help identify the system.



Isolated system: The system is said to be isolated if nothing (energy, matter, light) crosses the boundary of the system to or from the environment and if the net sum of all the external forces on the system is zero.

Within an isolated system, the amount of conserved quantity (e.g. electric charge, energy, momentum, etc.) will not change during an event or interaction. Often we can consider a system approximately isolated by simply choosing its boundaries in an appropriate manner. Then we can apply conservation laws to learn more about elements within the system. For instance, when two cars collide, we can choose the system to be the region of space that includes both cars. The interaction of each car with the other is much greater than any other force affecting the cars. As a result, we consider the system as being approximately isolated. Therefore, we can use conservation laws to learn more about the motions of the cars during their collision

SECTION A:

MECHANICS

Physical quantities and units

Physical quantities, fundamental quantities, dimensions of physical quantities, uses of dimensions, derivations of equations, checking the consistency of equations.

Kinematics

Motion in a straight line, displacement, speed, velocity and acceleration, equations of motion, motion graphs, Free fall under gravity, projectiles, time of flight, maximum height, range, vectors and scalars, relative velocity

Newton's laws of motion

Mass and inertia, linear momentum and its units, principle of conservation of momentum, Newton's laws of motion, resultant force $F = ma$, impulse, elastic and inelastic collisions, conveyer belt, recoil velocity of a rifle.

Solid friction

Factors that affect solid friction, laws of solid friction, static and kinetic friction, coefficients of static and kinetic friction, explanation of the laws of solid friction using the molecular theory, applications of solid friction.

Work, energy and power

Concept of work and energy, work-energy theorem, types of energy, transformational energy, conservational energy, conservative and non-conservative forces, concept of power, inclined planes.

Static equilibrium

Parallel forces, moment of a force, principle of moments, applications of principle of moments, couple, Torque (τ), work done by a couple, centre of gravity, coplanar forces, fluids in static equilibrium, density and relative density measurement, pressure, Archimedes' principle, verification of Archimedes' principle, law of floatation, applications of Archimedes' principle.

Fluid flow

Stream line and turbulent flow, equation of continuity, Bernoulli's principle, applications of Bernoulli's principle, derivation, measurement of fluid velocity, design of aerodynamic shapes, effect of viscosity on motion in a fluid, velocity gradient and coefficient of viscosity, Stokes' formula, Poisuelle's formula, coefficient of viscosity of a liquid, effect of temperature on viscosity of liquids and gases.

Mechanical properties of matter

Stress-strain curves for a stretched wire or spring, work hardening, Hooke's law, features of a stress strain curve for a ductile material, investigation of ductility, brittleness, stiffness and strength, stress, strain, Young's modulus, measurement of stress, strain and Young's modulus, work done during an extension or compressional force (area under a stress strain curve, Elastic potential energy, applications.

Surface tension

Molecular theory, common surface tension phenomena, pressure difference across a spherical surface, angle of contact, capillary rise, measurement of surface tension, factors that affect surface tension, effects of surface tension, applications of surface tension.

Uniform motion in a circle

Angular velocity, expression for angular velocity, acceleration and force in circular motion, motion of a bicycle rider, car around a circular track, conditions for skidding and slipping, banked tracks (with or without friction), the conical pendulum, applications of circular motion.

Gravitation

Kepler's laws, Newton's law of gravitation, gravitational field intensity, laboratory determination of gravitational constant G, dimensions of G, derivation of Kepler's third law, masses of the earth and the moon, variation of acceleration due to gravity with distance from the centre of the earth and with latitude, weightless mass in a satellite, orbits around the earth, parking orbits, expression for the period of the satellite in a parking object, concept of gravitational potential and velocity of escape, mechanical energy of a satellite, total energy in an orbit, effects of friction orbits of satellites, communication satellites.

Simple harmonic motion

Expressions for acceleration, velocity and displacement in SHM, graphical representation of SHM, examples of SHM-simple pendulum, floating cylinder, liquid in U-tube, mass at the end of a vertical spring or horizontal spring on a smooth surface, mass between two coupled springs on a smooth horizontal surface, mass at the end of two coupled vertical springs, mass at one end of two parallel vertical springs of the same spring constant, expression for period in SHM, determination of acceleration due to gravity using SHM methods, energy in SHM, graphical representation of energy in SHM, applications of SHM.

PHYSICAL QUANTITIES AND UNITS

Physical quantities are quantities that can be measured. Examples include mass, length, time, weight, electric current, force, etc.

When stating a particular physical quantity, a standard size of that quantity is required. This standard size is known as the unit of that particular physical quantity.

Sometimes prefixes are used for multiples of the S.I units (System International) as in the table below.

Prefix	Multiplying factor	Symbol
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
kilo	10^{3}	k
mega	10^{6}	M
giga	10^{9}	G
tera	10^{2}	T

Fundamental quantities
These are physical quantities which cannot be expressed in terms of any other quantities. Examples include mass, length and time.

Derived quantities
These are quantities which can be expressed in terms of the fundamental physical quantities of mass, length and time. Examples include velocity, force, acceleration, work, power, etc.

Dimensionless quantities
These are certain quantities which do not possess dimensions. Examples include strain, angle, relative density, etc. They are dimensionless because they are ratios of the quantities having the same dimension.

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised. Square brackets are used to indicate 'the dimensions of' and the symbols M, L and T are used to denote mass, length and time.

$$[\text{velocity}] = \frac{[\text{displacement}]}{[\text{time}]} = \frac{L}{T} = LT^{-1}$$
$$= M^0L^1T^{-1}$$

The velocity has zero dimensions in mass, one dimension in length and -1 dimension in time.

Note:
The dimensions of a unit must be the same as those of the quantity to which it applies. So, in the place of
$$[F] = MLT^{-2}$$
we can write
$$[N] = [kg]\,[m]\,[s]^{-2}$$
and $kg\,ms^{-2}$ is a suitable unit for any force

Dimensional formulae of some derived quantities

Physical quantity	Expression	Dimensional formula
Area	length × width	L^2
Density	mass/volume	ML^{-3}
Acceleration	velocity/time	LT^{-2}
Momentum	mass × velocity	MLT^{-1}
Force	mass × acceleration	MLT^{-2}
Work	Force × distance	ML^2T^{-2}
Power	work/time	ML^2T^{-3}
Energy	Work	ML^2T^{-2}
Impulse	Force × time	MLT^{-2}
Pressure	Force/area	$ML^{-1}T^{-2}$
Surface tension	Force/length	MT^{-2}
Frequency	1/time period	T^{-1}
Moment of force (torque)	Force × distance	ML^2T^{-2}
Stress	Force/area	$ML^{-1}T^{-2}$
Heat	Energy	ML^2T^{-2}

Uses of dimensional analysis

(i) Convert a physical quantity from one system units to another.
(ii) Check the dimensional correctness of a given expression or equation.
(iii) Establish a relationship between different quantities in an equation.

Example 1
Let us find an expression for the time period, T of a simple pendulum. The time period, T may depend upon mass, m of the bob, length, l of the pendulum and acceleration due to gravity, g at the place where the pendulum is suspended.

$$T \propto m^x l^y g^z$$
$$T = km^x l^y g^z$$

where k is a dimensionless constant of proportionality.

$$[T] = T, [m] = M, [l] = L, [g] = LT^{-2}$$
$$T = M^x L^y (LT^{-2})^z$$
$$M^0L^0T^1 = M^x L^{y+z} T^{-2z}$$

Comparing the power of M, L, and T on both sides;

$$x = 0, y + z = 0 \text{ and } -2z = 1$$

Solving for x, y and z; $x = 0, y = \frac{1}{2}$ and $z = -\frac{1}{2}$

$$T = km^0 l^{1/2} g^{-1/2}$$
$$T = k(l/g)^{1/2}$$

Experimentally, the value of k is determined to be 2π

$$T = 2\pi\sqrt{l/g}$$

Example 2

The viscous drag F between two layers of liquid with surface area of contact A in a region of velocity of gradient $\frac{dv}{dx}$ is given by;

$$F = \eta A \frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. What is the dimension of η? Hence write the unit for η in terms of the base units in S.I

Solution

$$F = \eta A \frac{dv}{dx}$$
$$\eta = \frac{F}{A\frac{dv}{dx}}$$
$$[\eta] = \frac{[F]}{[A]\left[\frac{dv}{dx}\right]} = \frac{MLT^{-2}}{L^2 \times \left[\frac{LT^{-1}}{L}\right]} = ML^{-1}T^{-1}$$

Unit for $\eta = kg\,m^{-1}s^{-1}$

Example 3

The speed of sound v in a medium depends on its wavelength, λ, the Young's modulus E and the density ρ of the medium. Derive the expression for the speed of sound v in a medium.

Solution

$$v \propto \lambda^x E^y \rho^z$$
$$v = k\lambda^x E^y \rho^z$$
$$[v] = k[\lambda]^x [E]^y [\rho]^z$$
$$LT^{-1} = k(L)^x (ML^{-1}T^{-2})^y (ML^{-3})^z$$
$$M^0LT^{-1} = kL^{x-y-3z}T^{-2y}M^{y+z}$$
$$-1 = -2y \Rightarrow y = \frac{1}{2}$$
$$0 = y + z \Rightarrow z = -y = -\frac{1}{2}$$
$$1 = x - y - 3z$$
$$1 = x - \frac{1}{2} + \frac{3}{2}$$
$$x = 0$$
$$v = kE^{\frac{1}{2}}\rho^{-\frac{1}{2}}$$
$$\therefore v = k\sqrt{\frac{E}{\rho}}$$

Example 4

Poiseuille assumed that the rate of flow of a liquid through a horizontal tube under streamline flow depends on

(i) the radius, a of the tube
(ii) viscosity, η of the liquid and
(iii) the pressure gradient, p/l along the tube where $p =$ pressure difference across the length of the tube and $l =$ length of the tube

Using Poiseuille's assumption, derive an expression for the rate of flow of a liquid through a horizontal tube in terms of a, l, p and η

Solution

$$\text{Rate of flow, } \frac{dV}{dt} = ka^x\left(\frac{p}{l}\right)^y \eta^z$$

where k, x, y and z are dimensionless constants.

$$\left[\frac{dV}{dt}\right] = \left[ka^x\left(\frac{p}{l}\right)^y \eta^z\right]$$
$$L^3T^{-1} = k(L)^x\left(\frac{MLT^{-2}}{L^2} \times \frac{1}{L}\right)^y (ML^{-1}T^{-1})^z$$
$$M^0L^3T^{-1} = kM^{y+z}L^{x-2y-z}T^{-2y-z}$$

Equating indices of M, L and T;

$$0 = y + z \quad \ldots\ldots\ldots\text{(i)}$$
$$-1 = -2y - z \ldots\ldots\text{(ii)}$$
$$3 = x - 2y - z \ldots\ldots\ldots\text{(iii)}$$

(i)+(ii); $\quad -1 = -y \Rightarrow y = 1$

From (ii); $z = -y = -1$

From (iii); $x = 3 + 2y + z = 3 + 2 - 1 = 4$

$\therefore$ Rate of flow; $\frac{dV}{dt} = \frac{ka^4p}{\eta l}$

Example 5

Test the following equations for dimensional consistency.

(i) $\quad s = ut + \frac{1}{2}at^2$

Solution

$[LHS] = L$

$[RHS] = (LT^{-1} \times T) + (LT^{-2} \times T^{-2}) = L + L$

Since dimensions on the RHS = dimensions on the LHS, the equation is dimensionally consistent.

(ii) $\quad v^2 = 8u^2 + 4as$

Solution

$$[LHS] = (LT^{-1})^2 = L^2T^{-2}$$
$$[RHS] = L^2T^{-2} + L^2T^{-2}$$

Therefore, the dimensions on the RHS are consistent with those on the left hand side.

(iii) $V = \sqrt{\frac{Tl}{M}}$ T is the tension in the string of length l and mass m and V is the velocity

Solution

$$[LHS] = LT^{-1}$$
$$[RHS] = \sqrt{\frac{(MLT^{-2}) \times L}{M}}$$

Since [LHS] = [RHS], the equation is dimensionally consistent

(iv) $v = u^2 + at$

Solution

$$[LHS] = LT^{-1}$$
$$[RHS] = L^2T^{-2} + LT^{-1}$$
$$\therefore [LHS] \neq [RHS]$$

The equation is dimensionally inconsistent

Limitations of dimension analysis

(i) The value of dimensionless constants cannot be determined

(ii) This method cannot be applied to equations involving exponential and trigonometric functions

(iii) It cannot be applied to an equation involving more than three physical quantities

(iv) It can check only whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct.
For example, applying this technique, $s = ut + \frac{1}{4}at^2$ is dimensionally correct whereas the correct relation is $s = ut + \frac{1}{2}at^2$

Principle of homogeneity of dimensions

An equation is dimensionally correct if the dimensions of the various terms on either sides of the equation are the same. The principle is based on the fact that two quantities of the same dimensions only can be added up, the resulting quantity also possessing the same dimensions.

The equation $A + B = C$ is valid only if the dimensions of A, B and C are the same.

Thus different terms can only be added or subtracted together if they have the same dimension i.e. you cannot add length to a volume or subtract mass from velocity.

Example 6

In the gas equation $\left(\boldsymbol{p} + \frac{\boldsymbol{a}}{\boldsymbol{V^2}}\right)(\boldsymbol{V} - \boldsymbol{b}) = \boldsymbol{RT}$ where p = pressure, V = Volume, T = absolute temperature and R = gas constant. What are the dimensions of the constants $\boldsymbol{a}$ and $\boldsymbol{b}$?

Solution

$$[p] = \left[\frac{a}{V^2}\right]$$
$$[p] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$
$$\Rightarrow \left[\frac{a}{V^2}\right] = ML^{-1}T^{-2}$$
$$\frac{[a]}{[V]^2} = ML^{-1}T^{-2}$$
$$\frac{[a]}{(L^3)^2} = ML^{-1}T^{-2}$$
$$[a] = ML^{-1}T^{-2} \times L^6 = ML^5T^{-2}$$
$$\therefore [a] = ML^5T^{-2}$$

$$[V] = [b]$$
$$\therefore [b] = L^3$$

Self-Evaluation exercise

1. (a) Explain what is meant by dimensions of a physical quantity
(b) In an experiment to determine the viscosity η of a liquid of density ρ, the time, t is measured for the level of the liquid fall in a vessel to fall to a measured distance is recorded. The viscosity is given by the equation $\boldsymbol{\eta} = \boldsymbol{A\rho t} - \frac{\boldsymbol{B\rho}}{\boldsymbol{t}}$
where A and B are constants to be determined.
(i) The units of η is Nsm^{-2}. Show that this unit can be expressed as $kg\,m^{-1}s^{-1}$
(ii) If the dimensions of η is $ML^{-1}T^{-1}$, find the dimensions of A and B.
2. Evaluate α and β in the equation $\boldsymbol{E} = \boldsymbol{Cm^{\alpha}v^{\beta}}$ where E is the kinetic energy, m is the mass, v is the velocity and C is a dimensionless constant.
[Ans: $\alpha = 1, \beta = 2$]
3. The force of attraction F between two particles of masses m_1 and m_2 situated a distance d apart is given by $\boldsymbol{F} = \frac{\boldsymbol{Gm_1m_2}}{\boldsymbol{d^2}}$. Show that the dimensions of G are $M^{-1}L^3T^{-2}$.
4. For streamline flow of a non-viscous, incompressible fluid, the pressure P at any point is

related to the height h and the velocity v by the equation:

$(P - a) = \rho g(h - b) + \frac{1}{2}\rho(v^2 - d)$, where a, b and d are constants, ρ is the density of the fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent, find the dimensions of a, b and d; and hence their units.

[Ans: $[a] = ML^{-1}T^{-2},\ [b] = L,\ [d] = L^2T^{-2}$]

5. Assuming that the mass m of the largest stone that can be moved by a flowing river depends on the velocity v of the water, its density ρ and acceleration due to gravity g, show that $m = \frac{k\rho v^6}{g^3}$ where k is a dimensionless constant.
6. Experiments show that the frequency, f of a tuning fork depends on the length, l, of the prong, the density, ρ and Young's modulus, Y. Using dimensional analysis, show that

   $f = \frac{k}{l}\left(\frac{Y}{\rho}\right)^{\frac{1}{2}}$ where k is a dimensionless constant.
7. The minimum velocity needed for a body to escape from the earth is given by $v = \sqrt{\frac{2GM}{R}}$ where M is the mass of the earth and R is the radius. Show that the equation is dimensionally correct if the dimensions of G are $M^{-1}L^3T^{-2}$.
8. Obtain by dimensional analysis, an expression for the surface tension of a liquid rising in a capillary tube. Assume that surface tension T depends on mass, m of the liquid, pressure, P of the liquid and the radius, r of the capillary tube. (Take the constant $k = \frac{1}{2}$)

   [Ans: $T = \frac{1}{2}Pr$]
9. The force, F acting on a body moving in a circular path depends on the mass, m of the body, velocity, v and radius, r of the circular path. Obtain an expression for the force. Take value of k to be 1.

   [Ans: $F = \frac{mv^2}{r}$]
10. When a body is moving through a resisting medium such as air, it experiences a drag force D which opposes the motion. D is given by

    $$D = \frac{1}{2}C\rho Av^2$$

    where ρ is the density of the resisting medium, A is the effective cross-sectional area of the body, i.e. that area perpendicular to its velocity v. C is called the drag coefficient. Show that C has no dimensions.
11. Coulomb's law for the force between charges q_1 and q_2, separated by a distance r, is given by $F = \frac{kq_1q_2}{r^2}$ where k is a constant.

    If the force, charge and distance are expressed in Newton N, Coulomb C, and metre m respectively, deduce a unit for k in terms of N, C and m.

    [Ans: Nm^2C^{-2}]

SCALAR AND VECTOR QUANTITIES

Scalar quantities

A scalar quantity is one which can be described fully by stating its magnitude only. Examples include: mass, time, length, temperature, density, speed, energy, volume, power, etc.

Vector quantities

A vector quantity is one which can be fully described by both magnitude and direction. Examples include displacement, acceleration, velocity, impulse, force, momentum, magnetic flux density, electric field intensity, etc.

Addition and subtraction of vectors

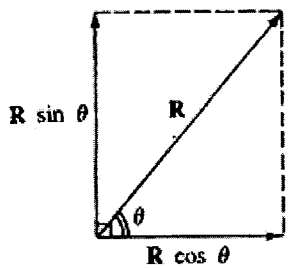
The addition of two vectors P and Q yields another vector which is known as the resultant or sum of the two vectors and is written as $P + Q$.

The subtraction of two vectors P and Q can be treated as the addition of the negative vector

$$P - Q = P + (-Q)$$

Resolving a vector

It is useful to find the component of a vector in the mutually perpendicular directions. This process is known as the resolving of a vector into components.



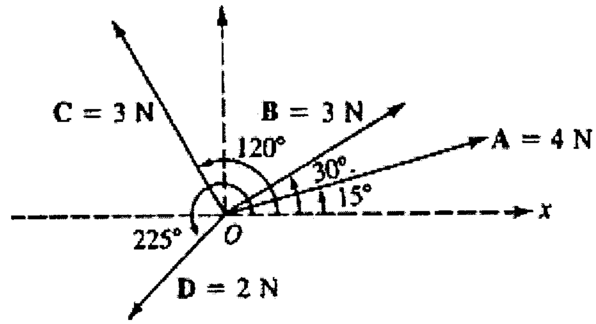
The magnitude of the component can be written in the form $R\cos\theta$ and $R\sin\theta$. The advantage is that very often a force, for example, the force due to gravity only acts along one direction (vertical) and not the other direction (horizontal). Then the two directions can be considered independent.

A mother pushing a pram, applies force F which can be resolved into two components.

Vertical component, $Y = F\sin\theta$

Example 1

The forces acting on a point O are shown in the figure below.



Find the magnitude and direction of the resultant force.

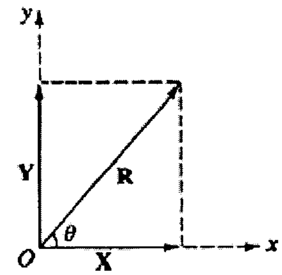
Solution

Algebraic sum of components of forces along O_x

$$X = A\cos 15° + B\cos 30° - C\cos 60° - D\cos 45°$$
$$= 4\cos 15° + 3\cos 30° - 3\cos 60° - 2\cos 45°$$
$$= 3.55\ N$$

Algebraic sum of components of forces along O_y

$$Y = 4\sin 15° + 3\sin 30° + 3\sin 60° - 2\sin 45°$$
$$= 3.72\ N$$



Magnitude of the resultant, $R = \sqrt{X^2 + Y^2}$

$$= \sqrt{3.55^2 + 3.72^2} = 5.14N$$

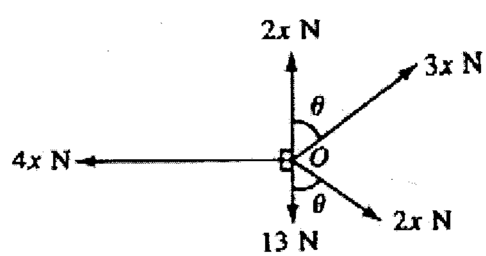
$$\tan\theta = \frac{Y}{X} = \frac{3.72}{3.55}$$

$$\theta = 46.3°$$

The resultant is $5.14\ N$ at an angle $46.3°$ to the direction O_x

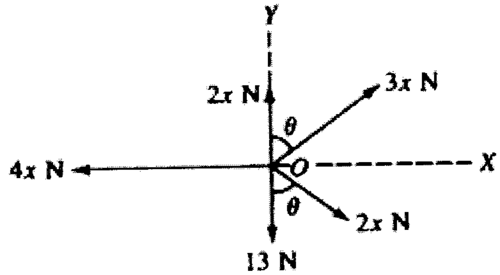
Example 2

The figure below shows a point O in equilibrium under the action of 5 coplanar forces.



Calculate the value for θ and x .

Solution



Since the forces are in equilibrium, the algebraic sum of the components of the forces in any direction is zero.

Algebraic sum of the components of forces along $OX = O$

$$3x\sin\theta + 2x\sin\theta - 4x = 0$$
$$5x\sin\theta - 4x = 0$$
$$\sin\theta = \frac{4}{5}$$
$$\theta = 53.13°$$

Algebraic sum of the components of forces along $OY = O$

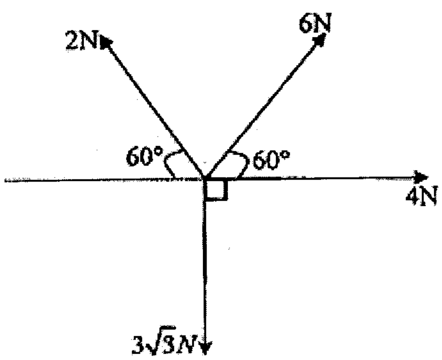
$$3x\cos\theta + 2x - 2x\cos\theta - 13 = 0$$
$$2x + x\cos\theta = 13$$

If $\sin\theta = \frac{4}{5}$, then $\cos\theta = \frac{3}{5}$

$$2x + x\left(\frac{3}{5}\right) = 13$$
$$\frac{13}{5}x = 13$$
$$x = 5$$

Example 3

Find the resultant of the forces shown in the figure below



Solution

Resolving,

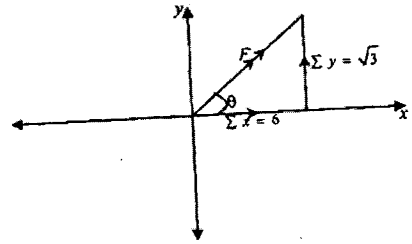
Force, F	Horizontally(→)	vertically (↑)
4	4 cos 0° = 4	4 sin 0° = 0
6	6 cos 60° = 3	6 sin 60° = $3\sqrt{3}$
2	2 cos 60° = 1	2 sin 60° = $\sqrt{3}$
3	$3\sqrt{3}$ cos 90° = 0	$-3\sqrt{3}$ sin 90° = $-3\sqrt{3}$
	$\sum_x = 6$	$\sum_y = \sqrt{3}$

$\sum x$ and $\sum y$ are the summations of the horizontal and vertical components of the resultant force.

If F is the resultant force, then,

$$F = \sqrt{x^2 + y^2} = \sqrt{6^2 + \left(\sqrt{3}\right)^2} = 6.24\ N$$

Since force is a vector quantity, we also have to find its direction

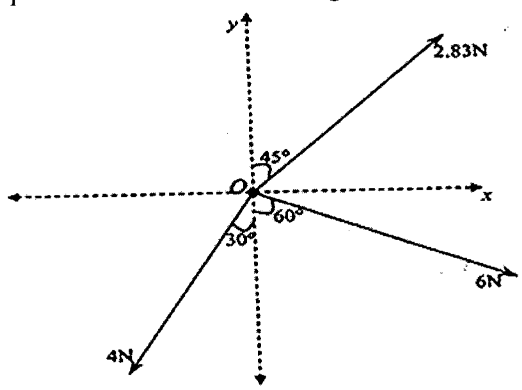


$$\tan\theta = \frac{\sum y}{x} = \frac{\sqrt{3}}{6} \Rightarrow \theta = 16.1°$$

Therefore, the resultant force is 6.24 N and makes an angle of 16.1° to the horizontal

Example 4

In the figure forces of 4 N, 6 N and 2.83 N act on a particle O. Find the resultant force and the acceleration of the particle if it has a mass 2 kg

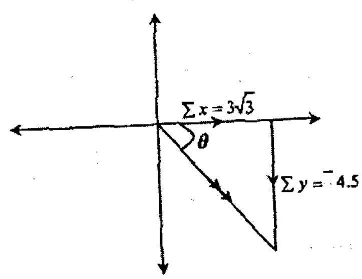


Solution

Resolving,

Force, F	Horizontally(→)	vertically (↑)
4	- 4 cos 60° = -2	- 4 sin 60° = - 3.5
2.83	2.83 cos 45° = 2	2.83sin 45° = 2
6	6 cos 30° = $3\sqrt{3}$	3 sin 30° = -3
	$\sum_x = 3\sqrt{3}$	$\sum_y = -4.5$

$$R = \sqrt{\sum x^2 + \sum y^2} = \sqrt{\left(3\sqrt{3}\right)^2 + (-4.5)^2} = 6.88\ N$$



$$\tan\theta = \frac{\sum y}{x} = \frac{4.5}{3\sqrt{3}} \Rightarrow \theta = 40.9°$$

Therefore the resultant force is 6.88 N and is 40.9° below the horizontal

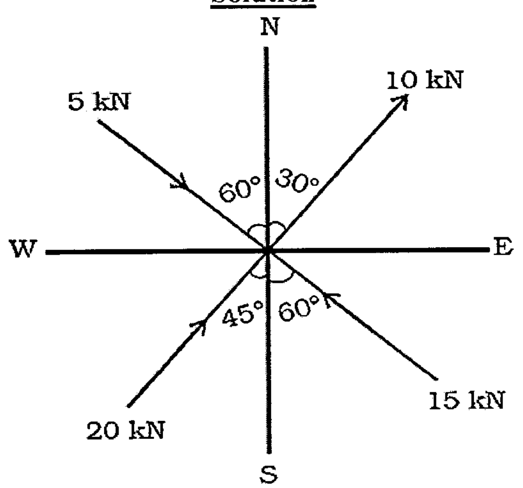
$$\text{Force} = \text{mass} \times \text{acceleration}$$
$$\Rightarrow 6.88 = 2a$$
$$\therefore a = 3.44\ ms^{-2}$$

Example 5

Determine the magnitude and direction of the resultant of the following four forces acting at a point

(i) 10 kN pull $N\ 30°\ E$
(ii) 20 kN push $S\ 45°\ W$
(iii) 5 kN push $N 60°\ W$
(iv) 15 kN push $S\ 60°\ E$

Solution



Resolving horizontally:

$$E_x = 10 \sin 30° + 5 \sin 60° + 20 \sin 45° - 15 \sin 60°$$
$$= 10.48\ kN$$

Resolving vertically:

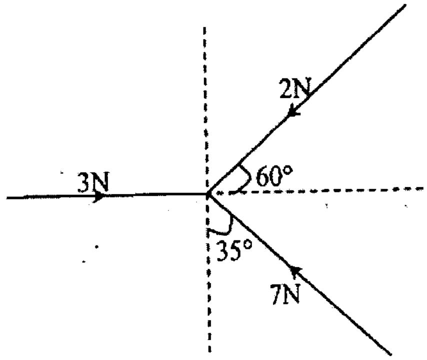
$$E_y = 10 \cos 30° - 5 \cos 60° + 20 \cos 45° + 15 \cos 60°$$
$$= 27.8\ kN$$

$$R = \sqrt{E_x^2 + E_y^2} = \sqrt{10.48^2 + 27.8^2} = 29.7\ kN$$

$$\tan\theta = \frac{E_y}{E_x} = \frac{27.8}{10.48} = 2.65$$
$$\theta = 69.34°$$

Example 6

Find the resultant of the forces in the figure below



Resolving,

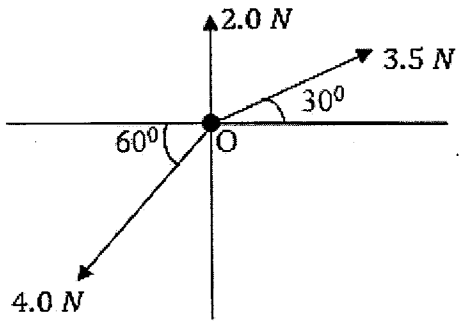
$Force, F$	Horizontally(→)	vertically (↑)
2	- 2 cos 60° = -1	- 2 sin 60° = - $\sqrt{3}$
3	3 cos 0° = 3	3 sin 0° = 0
7	-7 cos 55° = -4.02	7 sin 55° = -3
	$\sum_x = -2.02$	$\sum_y = 4$

$$\text{Resultant} = \sqrt{\sum x^2 + \sum y^2} = \sqrt{(-2.02)^2 + (4)^2}$$
$$= 4.481\ N$$
$$\tan\theta = \frac{\sum y}{\sum x} = \frac{4}{2.02}$$
$$\theta = 63.21°$$

The resultant is 4.48 N at 63.21° below the horizontal

Self-Evaluation exercise

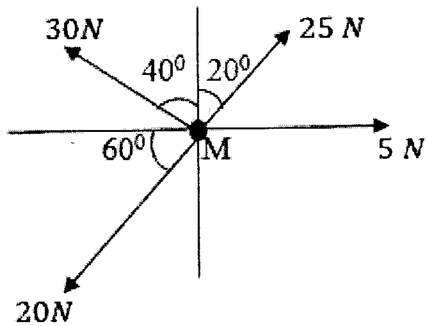
1.



Three forces of 3.5 N, 4.0 N and 2.0 N act at a point O shown in the figure. Find the resultant force

[Ans: 1.07 N at 15.5° to the horizontal]

2.



A body, M of mass 6 kg is acted on by forces of 5 N, 20 N, 25 N and 30 N as shown. Find the acceleration of M

[Ans: 5.52 ms^{-2}]

RELATIVE MOTION

This is the observed motion of one moving object with respect to another.

For a person in a moving vehicle, trees and buildings near the road appear to be moving in the opposite direction, yet those that are far appear to be moving in the same direction.

If a car B moving with velocity V_B overtakes another car A moving with a velocity V_A, the passenger in car A sees car B apparently moving towards him. However, the person in car B sees a gradual catching up. In this case, car A appears to be stationary as B overtakes it .

The velocity car B appears to have to an observer in car A is called the velocity of B relative to A.

Therefore the velocity of B relative to A is the resultant velocity of B when A appears stationary.

One way of making a moving body stationary is by reversing its velocity. The velocity of B relative to A is written as;

$$_BV_A \text{ or } V_{BA} = V_B - V_A$$

Similarly

$$_AV_B = V_A - V_B \text{ is the velocity of A relative to B}$$

$$\Rightarrow {_AV_B} = -\,{_BV_A}$$

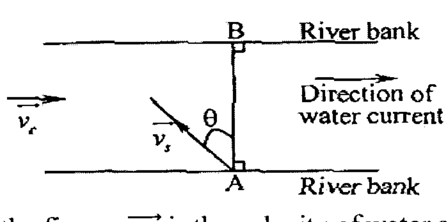
Resultant velocity

Since velocity is a vector quantity with both magnitude and direction, two velocities can be combined to form a single velocity. Such velocities are usually met in problems of crossing a river by a boat or swimmer.

Consider a problem of crossing a river from a point on one bank to a point on the other bank. Assuming that the banks are parallel, we shall consider two cases.

Case 1:

If one is to cross from point A on one bank to a point B directly opposite to A on the other bank, course set by the boat or swimmer must be upstream.



In the figure, $\vec{v_c}$ is the velocity of water current and $\vec{v_s}$ the velocity of the boat or swimmer in still water.

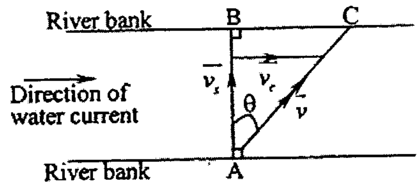
Resolving velocities; $v_s \sin\theta = v_c$

The resultant velocity in crossing the river is along AB and is given; $v_s \cos\theta$

It is instructive to note that in this case the boat or swimmer crosses the river by the shortest distance.

Case 2:

If the river is to be crossed by the shortest time possible (as quickly as possible), the course of the boat or swimmer is directly across (perpendicular) to the river bank, such that the water current carries the boat or swimmer down stream.



Resultant velocity,

$$v = \sqrt{v_s^2 + v_c^2}$$

This resultant velocity is at an angle θ to the horizontal, where $\tan\theta = \frac{v_c}{v_s}$

Time taken to cross the river $= \frac{\overline{AB}}{v_s} = \frac{\overline{AC}}{v}$

The distance $\overline{BC}$, which the boat or swimmer moves down stream $= v_c \times t$

Example 1

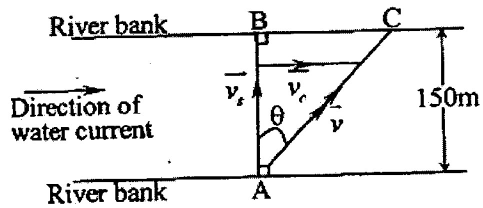
A boy who can swim at 2 ms^{-1} in still water wishes to swim across a river, 150 m wide. If the river flows at 1.5 ms^{-1}, find

(i) the time the boy takes for the crossing and how far downstream he travels, if he is to cross the river as quickly as possible.

(ii) the course that he must set in order to cross to a point exactly opposite the starting point, and the time taken for the crossing

Solution

(i)



$v_c = 1.5$ ms^{-1}, $v_s = 2$ ms^{-1}

Time taken $= \frac{\overline{AB}}{v_s} = \frac{150}{2} = 75\ s$

Distance down stream $= v_c \times t = 1.5 \times 75$

$= 112.5\ m$

(ii) Let the course be set at an angle θ

$$v_s \sin\theta = v_c \Rightarrow \sin\theta = \frac{1.5}{3}$$

$$\therefore \theta = 48.6°$$

The course set is therefore upstream at an angle of $(90 - 48.6) = 41.4°$ to the river bank

Speed across the river $= v_s \cos\theta = 2\cos 48.6°$

Time taken $= \frac{150}{2\cos 48.6} = 113.41\ s$

Example 2

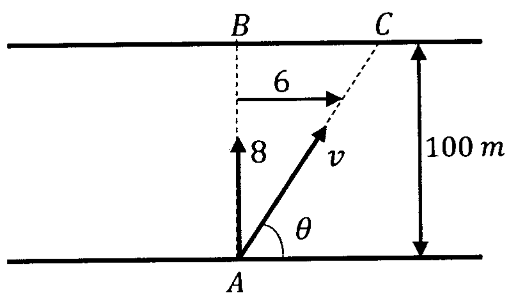
A boat is rowed with a velocity of 8 kmh^{-1} straight across a river which is flowing at 6 kmh^{-1}.

(a) Find the magnitude and direction of the resultant velocity of the boat

(b) If the river is 100 m wide, find how far down the river the boat will reach the opposite end

Solution

The component velocities of the boat are 8 kmh^{-1} and 6 kmh^{-1} at right angles.



(a)

Let v = resultant velocity

$$v = \sqrt{6^2 + 8^2} = \sqrt{100} = 10\ kmh^{-1}$$

Let θ be the direction this velocity makes with the bank.

$$\tan\theta = \frac{8}{6}$$

$$\theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°$$

(b) If A is the point from which the boat starts and B is the point directly opposite on the other bank, C will be the point where the boat reaches the opposite bank.

$$\tan\theta = \frac{BA}{BC}$$

$$BC = \frac{BA}{\tan\theta} = \frac{100}{\frac{8}{6}} = 75\ m$$

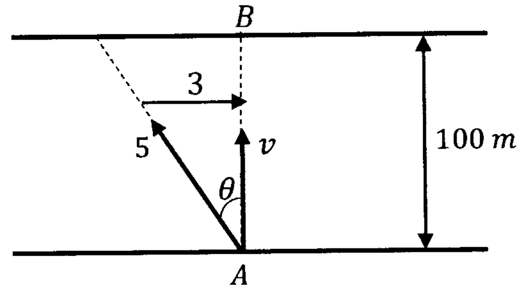
Hence the boat will be carried downstream a distance of 75 m.

Example 3

A stream is running at 3 kmh^{-1} and its width is 100 m. If a man can row a boat at 5 kmh^{-1}, find the direction in which he must row in order to go straight across the stream and the time it takes him to cross.

Solution

Let A be the point from which the man starts and AB is perpendicular to the banks.



Resultant velocity is along AB,

$$v = \sqrt{5^2 - 3^2} = 4\ kmh^{-1}$$

$$\cos\theta = \frac{4}{5}$$

$$\theta = \cos^{-1}\frac{4}{5} = 36.87°$$

Direction is 36.87° to AB

$$\text{Time taken to cross} = \frac{\text{distance}}{\text{velocity}}$$

$$\text{velocity} = 4\ kmh^{-1} = \frac{4000}{3600} = \frac{10}{9}\ ms^{-1}$$

$$\text{Time taken} = \frac{100}{\frac{10}{9}} = 90\ s$$

Self-Evaluation exercise

1. The banks of a river are parallel and 50 m apart and a current flowsat 8 ms^{-1}. A boat with a speed of 10 ms^{-1} in still water sails from a point A at one bank to point B directly opposite on the other bank.
   (i) Find the direction in which the boat is steered, and the time it takes to cross the river
   (ii) Explain why the shortest time of crossing is achieved when the boat is steered in a direction perpendicular to the banks, and find where the boat reaches on the opposite bank.
   [Ans: $\theta = 36.9°, 8.33\ s,\ 40\ m$]
2. A boat is rowed with a velocity of 5 kmh^{-1} and directed straight across a river flowing at 3 kmh^{-1}. If the river is 120 m wide, find how far down the river the boat will reach the opposite bank.
   [Ans: 72 m]
3. A man who swims at 5 kmh^{-1} in still water wishes to cross a river 150 m wide, flowing at 8 kmh^{-1}. Indicate the direction in which he should swim in order to reach the opposite bank.
   (i) as soon as possible
   (ii) as little downstream as possible

   How long will he take to cross and how far will he be carried downstream in each case?

   [Ans: (i) 1.8 minutes, 240 m (ii) $\cos^{-1}\frac{5}{8}$ with bank upstream, 2.3 minutes, 187 m]

KINEMATICS

Kinematics is the study of the relationship between displacement, velocity, acceleration and time of a given motion without considering the forces that cause the motion.

Kinetics deals with the relationship between the motion of the bodies and forces acting on them.

Fundamental definitions

Displacement: This is the distance covered by a particle in a specified direction.

Velocity: This is the rate of change of displacement of a particle.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{time taken}}$$

Uniform velocity: A body is said to move with uniform velocity if it moves along a fixed direction and covers equal displacements in equal intervals of time however small these intervals of time may be.

Non uniform velocity or variable velocity:
The velocity is variable (non-uniform) if it covers unequal displacements in equal time intervals or if the direction of motion changes or if both the rate of motion and direction change.

$$\textbf{Average velocity} = \frac{\text{change in displacement}}{\text{change in time}}$$

Instantaneous velocity: This is the velocity at any given instant of time or at any given point of its path.

Acceleration: This is the rate of change of velocity. If the magnitude or the direction or both of the velocity changes with respect to time, the particle is said to be under acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

If u is the initial velocity and v, the final velocity of the particle after a time t, then the acceleration,

$$a = \frac{v-u}{t}$$

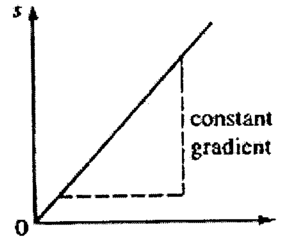
Uniform acceleration: If the velocity changes by an equal amount in equal intervals of time however small these intervals of time may be, the acceleration is said to be uniform

Retardation or deceleration: If the velocity decreases with time, the acceleration is negative. The negative acceleration is called deceleration or retardation.

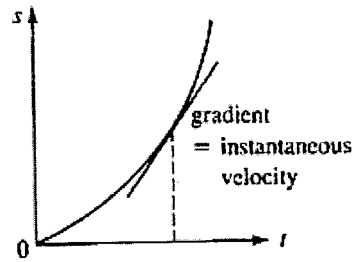
Uniform motion: A particle is in uniform motion when it moves with a constant velocity i.e. zero acceleration.

Graphical representation

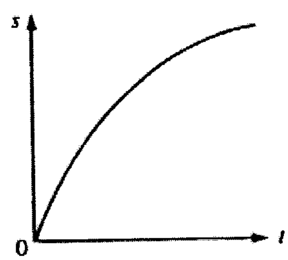
Displacement time graphs



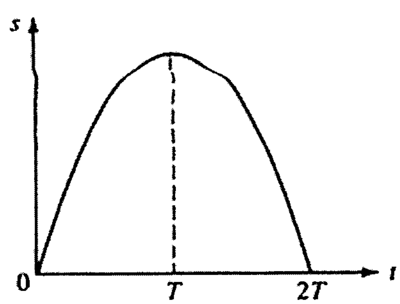
Motion under constant velocity, displacement = velocity × time.



Motion under constant acceleration.



Motion under constant deceleration.

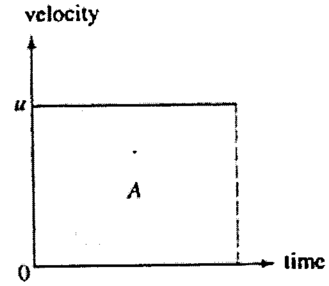


Motion of a particle projected vertically and then return to the point of projection.

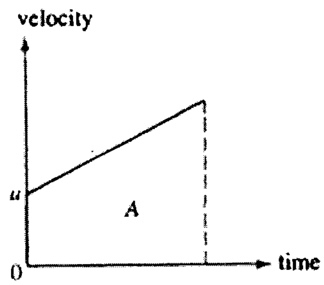
$t = T$, particle at highest point.

$t = 2T$, particle returns to the point of projection.

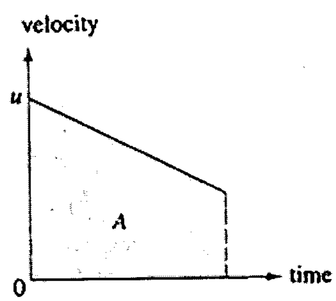
Velocity time graphs



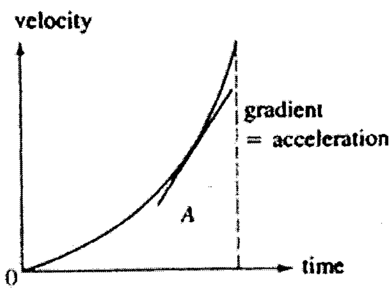
Constant velocity



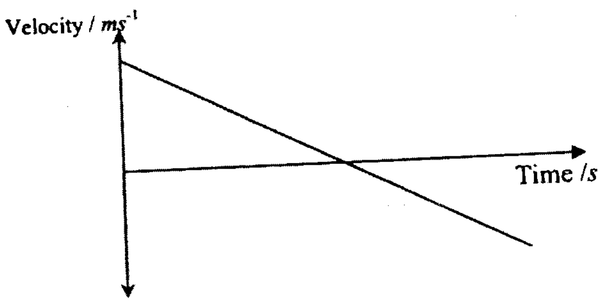
Uniform acceleration
Initial velocity $u \neq 0$.



Uniform retardation



Increasing acceleration



Body thrown vertically upwards

Equations of uniformly accelerated motion

If a body changes its velocity from u to v, its acceleration, $a = \frac{v-u}{t}$

$$\boldsymbol{v = u + at} \quad \text{...... [1]}$$

Since the acceleration is uniform, the average velocity during the time t is given by

Average velocity $= \frac{1}{2}$(initial velocity + final velocity)

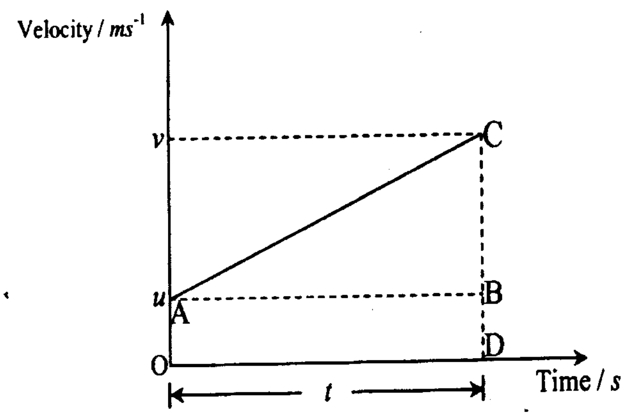
$$= \frac{1}{2}(u + v)$$

The displacement during the time t is

$$s = \text{average velocity} \times \text{time}$$
$$= \frac{1}{2}(u + v) \times t$$
$$= \frac{1}{2}(u + u + at) \times t$$
$$\boldsymbol{s = ut + \frac{1}{2}at^2} \quad \text{...... [2]}$$

Alternatively;

Consider a body with initial velocity u and final velocity v as it moves from A to B.



Displacement = total area under velocity time graph
= Area of OABD + Area of ACB

Area of OABD = ut

Area of ABC = $\frac{1}{2} \times t \times (v - u)$ but $v = u + at$

$\Rightarrow$Area of ABC = $\frac{1}{2}at^2$

$\therefore$Total area = $ut + \frac{1}{2}at^2$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

Now, from $v = u + at$, $t = \frac{v-u}{a}$

But $s = \left(\frac{u+t}{2}\right) \times t \Rightarrow s = \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right)$

$$s = \frac{uv - uv - u^2 + v^2}{2a} \Rightarrow s = \frac{v^2 - u^2}{2a}$$

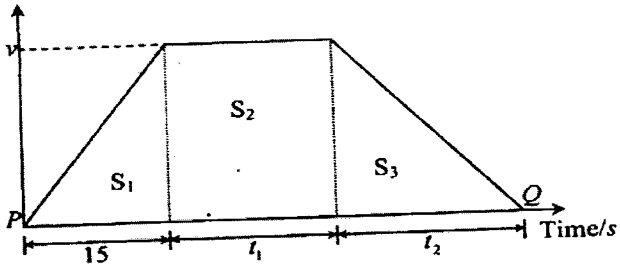
$$\boldsymbol{v^2 = u^2 + 2as} \quad \text{...... [3]}$$

Example 1

A train stops at two stations P and Q, which are 2 km apart. It accelerates uniformly from P at $1\ ms^{-2}$ for $15\ s$, and maintains a constant speed for a time before decelerating uniformly to rest at Q. If the deceleration is 0.5 ms^{-2}, find the time for which the train is travelling at a constant speed.

Solution

Let the time for which the train moves with constant velocity be t_1 and that for deceleration be t_2.



$$s_1 + s_2 + s_3 = 2000 \ldots\ldots\ldots(i)$$

From $v = u + at$

$$v = 0 + (1 \times 15) = 15\ \text{ms}^{-1}$$

For the deceleration,

From $v = u + at,\ \ 0 = 15 + -0.5t_2$

$$\Rightarrow t_2 = 30\ s$$

$$s_1 = \frac{1}{2} \times 15 \times 15 = 112.5\ m$$

$$s_2 = 15 \times t_1 = 15t_1$$

$$s_3 = \frac{1}{2} \times 30 \times 15 = 225\ m$$

Substituting for s_1, s_2 and s_3 in equation (i) gives;

$$112.5 + 225 + 15t_1 = 2000$$

$$\therefore 15t_1 = 1662.5$$

$$\Rightarrow t_1 = 110.83\ s$$

$\therefore$ The time for which the train is travelling at a constant speed is 110.83 s

Example 2

The driver of a car travelling at $72\ km/hr$ observes the light $300\ m$ ahead of him turning read. The traffic light is timed to remain red for $20\ s$ before it turns green. If the motorist wishes to pass the light without stopping to wait for it to turn green, determine

(i) the required uniform acceleration of the car

(ii) the speed with which the motorist crosses the traffic light.

Solution

$$u = \frac{72km}{1\ hr} = \frac{72 \times 1000}{3600} = 20\ ms^{-1}$$

$$s = 300\ m,\ \ t = 20\ s,\ a = ?,\ \ v = ?$$

(i) $s = ut + \frac{1}{2}at^2$

$$300 = 20 \times 20 + \frac{1}{2}a(20)^2$$

$$a = -0.5\ ms^{-2}$$

(ii) $v = u + at$

$$v = 20 - 0.5 \times 20 = 10\ ms^{-1}$$

Example 3

A car is being driven along a road at a steady speed $25\ ms^{-1}$ when the driver suddenly notices that there is a fallen tree blocking the road 65 m ahead. The driver immediately applies the brakes giving the car a constant retardation of 5 ms^{-2}.

(a) How far in front of the tree does the car come to rest?

(b) If the driver had not reached immediately and the brakes were applied one second later, with what speed would the car have hit the tree?

Solution

(a) $u = 25\ ms^{-1}, v = 0, a = -5\ ms^{-2}$

$$v^2 = u^2 + 2as$$

$$0 = 25^2 - 2 \times 5 \times s$$

$$s = 62.5\ m$$

Distance in front of the tree $= 65 - 62.5 = 2.5\ m$

(b) Since the car is at steady speed, $a = 0$

$$s = ut$$

In 1 s, distance travelled by car $= 25 \times 1 = 25\ m$

Distance to reach tree when the driver applies breaks (starts to decelerate) $= 65 - 25 = 40\ m$

$$s = 40\ m, u = 25\ ms^{-1}, v = ?, a = -5\ ms^{-1}$$

$$v^2 = u^2 + 2as$$

$$v^2 = 25^2 - 2 \times 5 \times 40$$

$$v^2 = 22$$

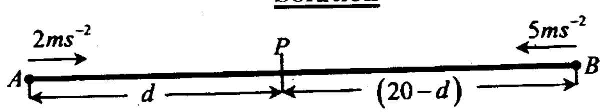
$$v = 15\ ms^{-1}$$

$\therefore$ The car hits the tree with a velocity of $15\ ms^{-1}$

Example 4

Two particles are travelling along a straight-line AB of length 20 m. At the instant when one particle starts from rest at A and travels towards B with a constant acceleration of 2 ms^{-2}, the other starts from rest at B and travels towards A with a constant acceleration of $5\ ms^{-2}$. Find the time after which the two bodies collide and how far from A they collide.

Solution



Let the particles collide at a point P, a distance d from A

$$s = ut + \frac{1}{2}at^2$$

Particle from A:

$$u = 0,\ \ s = d, a = 2$$

$$d = \frac{1}{2} \times 2 \times t^2$$

$$d = t^2 \ldots\ldots\ldots(i)$$

Particle from B:

$$u = 0, s = 20 - d, a = 5$$

$$20 - d = \frac{1}{2} \times 5 \times t^2$$

$$20 - d = 2.5t^2 \text{ ....(ii)}$$

Substituting for d in (ii) gives:

$$20 - t^2 = 2.5t^2$$

$$\Rightarrow 20 = 3.5t^2$$

$$\therefore t = 2.39\ s$$

$$d = 2.39^2 = 5.714\ m$$

$\therefore$The particles collide $5.714\ m$ from A after $2.39\ s$

Example 5

A bus travelling steadily at $30\ ms^{-1}$ along a straight road passes a stationary car which 5 s later begins to move with uniform acceleration of $2\ ms^{-2}$ in the same direction as the bus.

(a) How long does it take the car to acquire the same speed as bus?

(b) How far has the car travelled when it is level with the bus?

Solution

Let the total time taken by the car to reach the bus be t. In this time, both the car and the bus cover the same distance, d. The bus takes a time of $(t+5)s$ to cover this distance.

(a) $u = 0, a = 2\ ms^{-2}, v = 30\ ms^{-1}, t = ?$

$$v = u + at$$

$$30 = 0 + 2t$$

$$t = 15s$$

It takes the car 15 s to acquire the speed of $30\ ms^{-1}$ as that of the bus.

(b)

Bus:

Since velocity is steady, $a = 0$

$$s = ut$$

$$d = 30(t+5) \qquad \text{........(i)}$$

Car:

$u = 0, a = 2ms^{-2}, t = t, s = d$

$$d = \tfrac{1}{2} \times 2t^2$$

$$d = t^2 \quad \text{..................(ii)}$$

Solving equations (i) and (ii) simultaneously;

$$30(t+5) = t^2$$

$$t^2 - 30t - 150 = 0$$

$$t = 34.36 \text{ or } t = -4.36$$

$$\therefore t = 34.36\ s$$

Distance travelled by the bus,

$$d = 30(34.36 + 5) = 1180.8\ m$$

Example 6

A car A, travelling at a constant velocity of 25 ms^{-1}, overtakes a stationery car B. Two seconds later, car B acceleration 6 ms^{-2}. How far does B travel before catching up with A?

Solution

Let the time taken by car A to travel distance d be t. Car B will cover the same distance catching up with A after a time $(t-2)s$.

Car A	**Car B**
$u = 25, t = t, a = 0$	$u = 0, t = t-2, a = 6$
$s = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}at^2$
$d = 25t$ ....(i)	$d = 3(t-2)^2$........(ii)

Solving equations (i) and (ii) gives:

$$25t = 3(t-2)^2$$

$$\Rightarrow 3t^2 - 37t + 12 = 0$$

$$\therefore (t-12)(3t-1) = 0$$

$$\Rightarrow \text{either } t = 12 \text{ or } t = \frac{1}{3}$$

Note that $t = \frac{1}{3}$ cannot work, since the time taken by B will be $\left(\frac{1}{3} - 2\right)$ which gives a negative.

Therefore, time taken by A before B catches it is 12 s.

From (ii), $d = 3(t-2)^2 = 3(12-2)^2 = 300\ m$

From (i), $d = 25t = 25 \times 12 = 300\ m$

Distance travelled in the n^{th} second

Let a body move with an initial velocity u and travel along a straight line with uniform acceleration, a

Distance travelled in the n^{th} second is;

$$s_n = \begin{pmatrix} \text{distance covered} \\ \text{during the first} \\ n \text{ seconds} \end{pmatrix} - \begin{pmatrix} \text{distance covered} \\ \text{during} \\ n-1 \text{ seconds} \end{pmatrix}$$

Distance travelled in n seconds, $D_n = un + \frac{1}{2}an^2$

Distance travelled in $n-1$ seconds,

$$D_{n-1} = u(n-1) + \tfrac{1}{2}a(n-1)^2$$

Distance travelled in the n^{th} second $= D_n - D_{n-1}$

$$s_n = \left(un + \tfrac{1}{2}an^2\right) - \left[u(n-1) + \tfrac{1}{2}a(n-1)^2\right]$$

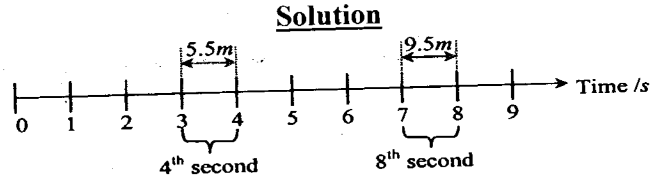
$$s_n = u + a\left(n - \tfrac{1}{2}\right)$$

$$s_n = u + \tfrac{1}{2}a(2n-1)$$

Example 7

A motorcar moving with a uniform acceleration covers 5.5 m in its 4th second and 9.5 m in its 8th second of its motion. Find its acceleration and initial velocity.

Solution



Distance covered in 4th second

= (distance after 4 s) – (distance after 3 s)

$\Rightarrow 5.5 = \left(4u + \frac{1}{2} \times a \times 4^2\right) - \left(3u + \frac{1}{2} \times a \times 3^2\right)$

Multiplying through by 2 gives;

$$11 = 8u + 16a - 6u - 9a$$
$$11 = 2u + 7a \ldots\ldots\ldots\ldots\text{(i)}$$

Distance covered in 8th second

= (distance after 8s) – (distance after 7s)

$\Rightarrow 9.5 = \left(8u + \frac{1}{2} \times a \times 8^2\right) - \left(7u + \frac{1}{2} \times a \times 7^2\right)$

Multiplying through by 2 gives;

$$19 = 16u + 64a - 14u - 49a$$
$$19 = 2u + 15a \ldots\ldots\ldots\ldots\text{(ii)}$$

Subtracting equations (ii) and (i) gives:

$$8 = 8a$$
$$a = 1\ ms^{-2}$$

Substituting for a in equation (ii) gives;

$$2u = 19 - 15(1)$$
$$2u = 4$$
$$u = 2\ ms^{-1}$$

Self-Evaluation exercise

1. A body starting from rest travels in a straight line with a constant acceleration, a for a time, t.
   (i) Sketch a velocity time graph for the body
   (ii) Deduce from the graph, that the distance, s travelled by the body in a time t is given by $s = \frac{1}{2}at^2$
2. A, B and C are three points which lie in that order on a straight road with $\overline{AB} = 95\ m$ and $\overline{BC} = 80\ m$. A car is travelling along the road in the direction $\overrightarrow{ABC}$ with a constant acceleration a ms^{-2}. The car passes through A with a speed u ms^{-1}, reaches B five seconds later, and C two seconds after that. Calculate the values of u and a
   [Ans: $u = 4$ ms^{-1}, $a = 6$ ms^{-2}]
3. A train starts from rest at station A and accelerates at 1.25 ms^{-2} until it reaches a speed of 20 ms^{-1}. It then travels at this steady speed for a distance of 1.56 km and then decelerates at 2 ms^{-2} to come to rest at B. Find the distance from A to B. [Ans:1.82 km]
4. Two cars A and B are traveling along a straight path. The cars are observed to be side by side when they are at point P of the path and again at another point Q. Assuming that A and B moved with a uniform acceleration a_1 and a_2, prove that if their velocities are u_1 and u_2 respectively, the distance PQ is given by; $\frac{2(u_1-u_2)(u_2a_1-u_1a_2)}{(a_1-a_2)^2}$
5. A particle moving in a straight line covers distances of 90 m and 240 m in successive times of 2 seconds and 4 seconds. Find the constant acceleration of the particle. [Ans: $5\ ms^{-2}$]
6. A train which is moving with uniform acceleration is observed to take 20 s and 30 s to travel successive 400 m. How much further will it travel before coming to rest if the acceleration remains uniform. [Ans: 163.3 m]
7. A train is timed between successive points A, B and C each 2 km apart. If it takes 100 s to travel from A to B and 150 s from B to C, find the retardation of the train, assuming that it remains uniform after the point A. Find also how far beyond C the train travels before it stops.
   [Ans: $0.053\ ms^{-2}$, $0.82\ km$]
8. A particle moving in a straight line covers 28 m in the fifth second of its motion and 52 m in the eleventh second. Find the distance covered in 10 s from the beginning of the motion [Ans: 300 m]
9. Two particles P and Q move in the same straight line being initially at rest and Q being 18 m in front of P. Q starts from rest with an acceleration of $3\ ms^{-2}$ and P starts in pursuit with a velocity of $10\ ms^{-1}$ and an acceleration of $2\ ms^{-2}$. Prove that P will overtake and pass Q after an interval of $2\ s$ and that Q will in turn overtake P after a further interval of 16 s.
10. A particle starts from a point O with an initial velocity of $2\ ms^{-1}$ and travels along a straight line with a constant acceleration of $2\ ms^{-2}$. Two seconds later, a second particle starts from rest at O and travels along the same line with an acceleration of $6\ ms^{-2}$. Find how far from O the second particle overtakes the first. [Ans: 48 m]
11. A particle P which is moving along a straight line with a constant acceleration of $0.3\ ms^{-2}$ passes a point A on the line with a velocity of $20\ ms^{-1}$. At the time when P passes A, a second particle Q is 20 m behind A and is moving with a constant velocity of $30\ ms^{-1}$. Prove that the particles collide.
12. A particle starts from rest at a point O in a straight line and moves along the line with a constant acceleration of $2\ ms^{-2}$. Three seconds later, a second particle starts from rest at O and moves along the line with constant acceleration of $4\ ms^{-2}$. Find when the second particle overtakes the first particle. [Ans: $10.24\ s$]

MOTION UNDER GRAVITY

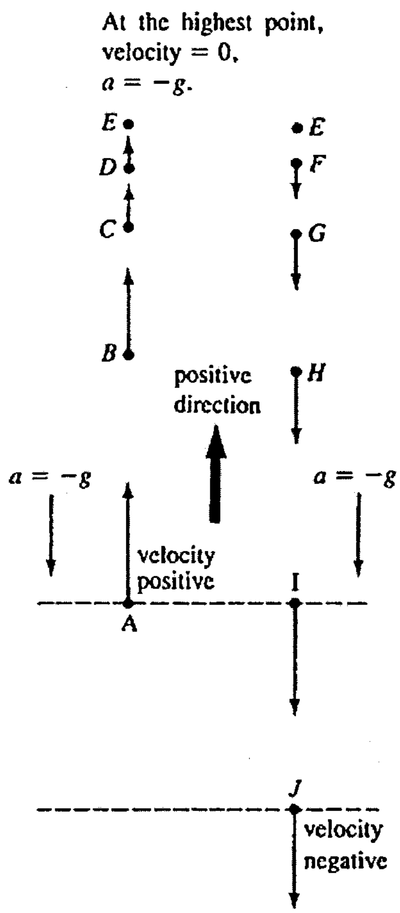
If there is no resistance, all objects irrespective of mass, shape and size fall towards the earth with the same acceleration known as acceleration of free fall or acceleration due to gravity, g. The value of g is assumed to be 9.81 ms^{-2}.

Free falling body:
For a freely falling body, $a = g$ and $u = 0$ since it starts from rest.

Upward motion:
For a particle moving upwards, $a = -g$ since the particle moves against gravity.

Downward motion:
For a body moving downwards, $a = g$ since the body moves in the direction of gravity.

Special case:
Consider a particle projected vertically upwards from **A**. It travels to **E** before falling downwards.
If the **positive direction** is assumed to be upward, then for the upward motion, acceleration, $a = -g$ because the direction of acceleration is constant and always downwards (the negative direction).



During the particle's upward motion
- the velocity is positive (upwards)
- the displacement is positive (above A)
- acceleration is negative

During the particle's downward motion
- the velocity is negative (downwards)
- the displacement is positive (above A) for H, G, F and E
- the displacement is negative (below A) for J
- acceleration is negative

Example 1
An object is dropped from a height of 45 m. Calculate
(a) the time taken to reach the ground
(b) the velocity with which it hits the ground

Solution

(a) $u = 0, a = g, s = 45\ m$

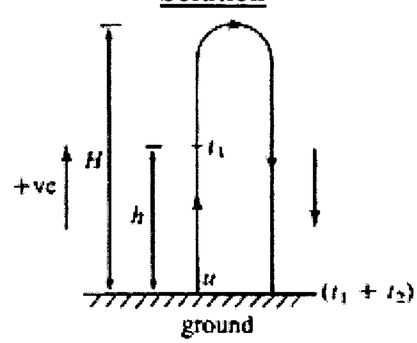
$$s = ut + \frac{1}{2}at^2$$
$$45 = \frac{1}{2} \times 9.81t^2$$
$$t = 3.03\ s$$

(b) $v = u + at$

$$v = 0 + 3.03 \times 9.81$$
$$v = 29.72\ ms^{-1}$$

Example 2
A ball is thrown vertically upwards from the ground with an initial velocity u. The time taken by the ball to reach a height h is t_1. The ball then takes a further time of t_2 to return to the ground. Find in terms of t_1, t_2 and g,
(a) the initial velocity, u
(b) the height, h and
(c) the maximum height, H reached by the ball

Solution



(a) acceleration, $a = -g$
Total time taken for the ball to return to the ground, $t = (t_1 + t_2)$.
When the ball returns to the ground, displacement $s = 0$.

$$s = ut + \frac{1}{2}at^2$$

$$0 = u(t_1 + t_2) - \frac{1}{2}g(t_1 + t_2)^2$$

$$u(t_1 + t_2) = \frac{1}{2}g(t_1 + t_2)^2$$

$$u = \frac{1}{2}g(t_1 + t_2)$$

(b) When the ball is at height h

$$s = h, t = t_1$$

Using; $s = ut + \frac{1}{2}at^2$

$$h = ut_1 - \frac{1}{2}gt_1^2$$

Substituting for u in the equation

$$h = \frac{1}{2}g(t_1 + t_2)t_1 - \frac{1}{2}gt_1^2$$

$$h = \frac{1}{2}gt_1t_2$$

(c) At the maximum height, $s = H, v = 0$

Using $v^2 = u^2 + 2as$

$$0 = u^2 - 2gH$$

$$H = \frac{u^2}{2g}$$

$$H = \frac{1}{2g}\left[\frac{1}{2}g(t_1 + t_2)\right]^2$$

$$H = \frac{1}{8}g(t_1 + t_2)^2$$

Example 3

A cricket ball is thrown vertically upwards with a velocity of 20 ms^{-1}. Calculate

(a) the maximum height reached

(b) the time taken to return to the ground

Solution

Taking the upward motion to be positive

(a) $u = 20\,ms^{-1}, v = 0, a = -g, H = ?$

$$v^2 = u^2 - 2gH$$

$$0^2 = 20^2 - 2 \times 9.81H$$

$$H = 20.39\,m$$

(b) On its return to the ground, after time t, we have $s = 0$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 20 \times t - \frac{1}{2} \times 9.81t^2$$

$$4.905t^2 - 20t = 0$$

$$t(4.905t - 20) = 0$$

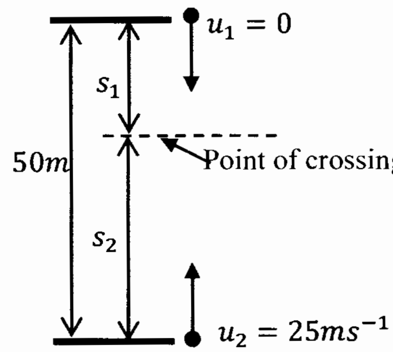
$$t = 4.08\,s$$

Note that $t = 0$ is also, obviously, a solution when $s = 0$

Example 4

A stone is dropped from the top of the tower 50 m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25 ms^{-1}. At what distance from the top and after how much time do the stones cross each other.

Solution



Height of tower = $50m$, $u_1 = 0$; $u_2 = 25ms^{-1}$

Let s_1 and s_2 be the distance travelled the stones at the time of crossing, t

Thus; $s_1 + s_2 = 50m$

For stone 1: $s_1 = \frac{1}{2}gt^2$

For stone 2: $s_2 = u_2t - \frac{1}{2}gt^2$

$$s_2 = 25t - \frac{1}{2}gt^2$$

Thus $s_1 + s_2 = 50$

$$\frac{1}{2}gt^2 + 25t - \frac{1}{2}gt^2 = 50$$

$$25t = 50$$

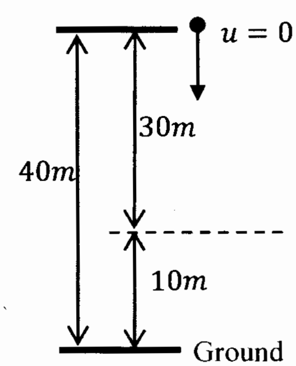
$$\Rightarrow t = 2 \text{ seconds}$$

$$s_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \times 2^2 = 19.62\,m$$

Example 5

A stone is dropped from the top of a tower of height 40 m. The stone falls from rest and air resistance is negligible. How long does it take for the stone to fall the last 10 m to the ground?

Solution



Time taken to reach the ground = t_2

$$s = ut_2 + \frac{1}{2}gt_2^2$$

$$40 = 0 + \frac{1}{2} \times 9.81 t_2^2$$
$$t_2 = 2.84\ s$$

Time taken to travel 30 m from top of the tower (10 m from the ground) = t_1

$$s = ut_1 + \frac{1}{2}gt_1^2$$
$$40 = 0 + \frac{1}{2} \times 9.81 t_1^2$$
$$t_1 = 2.46\ s$$

Time to fall the last 10 m to the ground = $t_2 - t_1$
$$= 2.84 - 2.46 = 0.38\ s$$

Example 6

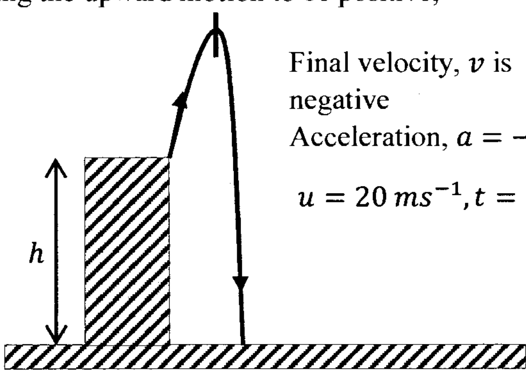
A stone is thrown vertically upwards with a speed of 20 ms^{-1} from a point at a height h above the ground level. If the stone hits the ground 5 s later, find the

(i) velocity with which it hits the ground

(ii) the value of h

Solution

Taking the upward motion to be positive,



Final velocity, v is negative

Acceleration, $a = -$g

$u = 20\ ms^{-1}, t = 5\ s$

(i) From $v = u + at$
$$-v = u - gt$$
$$-v = 20 - 9.81 \times 5$$
$$v = 29.05\ ms^{-1}$$

(ii) Displacement, h below point of projection is negative.

From $s = ut + \frac{1}{2}at^2$
$$-h = ut - \frac{1}{2}gt^2$$
$$-h = 20 \times 5 - \frac{1}{2} \times 9.81 \times 5^2$$
$$h = 22.63\ m$$

Alternatively; using $v^2 = u^2 + 2as$
$$(-v)^2 = u^2 + 2(-g)(-h)$$
$$v^2 = u^2 + 2gh$$
$$29.05^2 = 20^2 + 2 \times 9.81 \times h$$
$$h = \frac{29.05^2 - 20^2}{19.62} = 22.63\ m$$

Example 7

A stone is thrown vertically upwards from the top of a tower and hits the ground 10 s later, with a speed of 51 $m\,s^{-1}$. Find the initial velocity and height of the tower.

Solution

Considering the upward motion to be positive;

Final velocity, v is negative

Acceleration, $a = -$g

Height, h of the tower is below the point of projection, hence negative.

From $v = u + at$
$$-v = u - gt$$
$$-51 = u - 9.81 \times 10$$
$$u = 98.1 - 51 = 47.1\ ms^{-1}$$

From $s = ut + \frac{1}{2}at^2$
$$-h = ut - \frac{1}{2}gt^2$$
$$-h = 47.1 \times 10 - \frac{1}{2} \times 9.81 \times 10^2$$
$$h = 490.5 - 471 = 19.5m$$

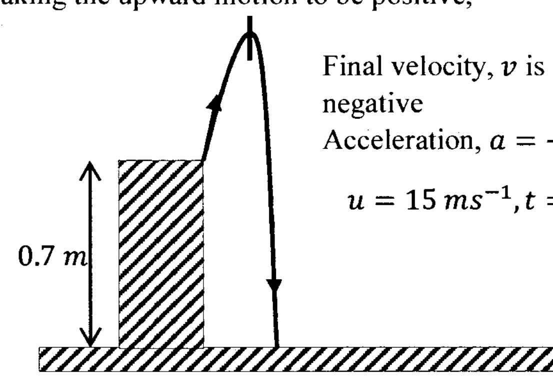
Alternatively; using $v^2 = u^2 + 2as$
$$(-v)^2 = u^2 + 2(-g)(-h)$$
$$v^2 = u^2 + 2gh$$
$$51^2 = 47.1^2 + 2 \times 9.81h$$
$$19.62h = 382.59$$
$$h = 19.5\ m$$

Example 8

A ball is thrown vertically upwards with a speed of 15 ms^{-1}, from a point which is 0.7 m above the ground. Find the speed with which the ball hits the ground, and the time taken.

Solution

Taking the upward motion to be positive,



Final velocity, v is negative

Acceleration, $a = -$g

$u = 15\ ms^{-1}, t = ?$

From $v^2 = u^2 + 2as$
$$(-v)^2 = u^2 + 2(-g)(-h)$$
$$v^2 = u^2 + 2gh$$
$$v^2 = 15^2 + 2 \times 9.81 \times 0.7$$
$$v = 15.4\ ms^{-1}$$

Also, from $v = u + at$
$$-v = u - gt$$
$$-15.4 = 15 - 9.81t$$
$$t = 3.1\ s$$

Self-Evaluation exercise

1. A stone is dropped vertically from the top of an overlapping cliff, and hits the sea 3 seconds later. The acceleration due to gravity at that location is 10 ms^{-2}. Find the speed of the stone as it hits the sea, and the height of the cliff.
[Ans: $v = 30\ ms^{-1}$, $h = 45$ m]
2. A ball is projected from a point 2 m above the ground with an upward speed of 3 ms^{-1}. Assuming that the acceleration due to gravity is 10 ms^{-2}, find the;
   (i) time taken for the ball to reach its greatest height above the ground
   (ii) maximum height reached
   (iii) speed of the ball when it first strikes the ground
   [Ans: (i) 0.3 m (ii) 0.45 m (iii) 7 ms^{-1}]
3. Two stones are thrown from the same point at the same time, one vertically upwards with a speed of 40 ms^{-1}, and the other vertically downwards at 40 ms^{-1}. Find how far apart the stones are after two seconds.
[Ans: 160 m]
4. A ball is projected vertically upwards with a speed of 50 ms^{-1}. On return it passes the point of projection and falls 78 m below. Calculate the total time taken
[Ans: 11.57 s]
5. A ball is projected from a cliff and takes 3.0 s to reach the beach below. Calculate
   (a) the height of the cliff
   (b) the velocity acquired by the ball
   [Ans: $44.55\ m$, $29.56\ ms^{-1}$]
6. With what velocity must a ball be thrown upwards to reach a height of 15 m?
[Ans: $17.16\ ms^{-1}$]
7. A man stands on the edge of a cliff and throws a stone vertically upwards at $15\ ms^{-1}$. After what time will the stone hit the ground 20 m below?
[Ans: $4.03\ s$]
8. A cricketer throws a ball vertically upwards and catches it 30 s later. Neglecting air resistance, find
   (i) the speed with which the ball leaves his hands
   (ii) the maximum height to which it rises
   Draw a sketch graph showing how the velocity of the ball depends on time during its flight.
   [Ans: (i) $14.72\ ms^{-1}$ (ii) $11.04\ m$]
9. A pebble is dropped from rest at the top of a cliff 125 m high.
   (a) How long does it take to reach the foot of the cliff and with what speed does it strike the ground?
   (b) With what speed must a second pebble be thrown vertically downwards from the cliff top if it is to reach the bottom in 4 s?
   [Ans: (a) $5\ s, 50\ ms^{-1}$ (b) $11.25\ ms^{-1}$]
10. A ball is thrown vertically upwards from a point 0.5 m above the ground level with a speed of $7\ ms^{-1}$. Find the height above this point reached by the ball and the speed with which it hits the ground.
[Ans: $2.5\ m$, $7.66\ ms^{-1}$]
11. A stone is dropped from the top of a building and at the same time, a second stone is thrown vertically upwards from the bottom of the building with a speed of $20\ ms^{-1}$. They pass each other 3 second later. Find the height of the building.
[Ans: 60 m]
12. A body is projected vertically upwards with a velocity of $21\ ms^{-1}$. How long will it take to reach a point 280 m below the point of projection?
[Ans: 10 s]
13. A body is projected vertically upwards with a velocity of $35\ ms^{-1}$. Find
   (i) how long it takes to reach the highest point
   (ii) the distance it ascends during the third second of its motion
   [Ans: (i) 3.57 s (ii) 10.5 m]
14. A body falls from rest from the top of a tower and during the last second, its falls $\frac{9}{25}$ of the whole distance. Find the height of the tower.
[Ans: 122.5 m]

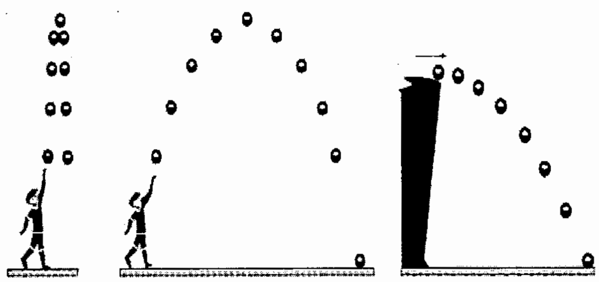
PROJECTILE MOTION

A body thrown with some initial velocity and then allowed to move under the action of gravity alone is known as a projectile. Examples include

(i) A bomb thrown from an areophane.
(ii) A javelin or shotput thrown by an athlete.
(iii) Motion of a ball hit by a cricket bat, etc.

Types of projectiles

The different types of projectiles are shown below.



A body can be projected in the following ways;

(i) It can be projected horizontally from a certain height.
(ii) It can be thrown from the ground in a direction inclined to it.
(iii) It can be projected from a point at a certain height above the horizontal and inclined at an angle to the horizontal

Note: In the study of projectile motion, it is assumed that the air resistance is negligible and the acceleration due to gravity remains constant.

Fundamental definitions

Angle of projection

This is the angle between the initial direction of projection and the horizontal direction through the point of projection.

Velocity of projection

This is the velocity with which the body is projected.

Range

This is the horizontal distance between the point of projection and the point where the projectile hits the ground.

Trajectory

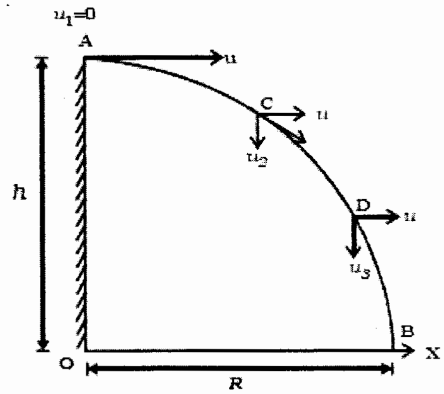
This is the path described by the projectile.

Time of flight

This is the total time taken by the projectile from the instant of projectile till it strikes the ground.

Motion of a projectile thrown horizontally

Consider an object thrown horizontally with a velocity u, from a point A, which is at a height h from the horizontal plane OX



The object acquires the following motions simultaneously;

(i) Uniform velocity with which it is projected in the horizontal direction OX
(ii) Vertical velocity, which is non-uniform due to the acceleration due to gravity

Path of the projectile

Let the time taken by the object to reach C from $A = t$

Vertical distance travelled by the object in time t is;

$$s = y$$

Initial vertical velocity, $u_1 = 0$

From the equation of motion, $s = ut + \frac{1}{2}at^2$

$$y = (0)t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \ldots\ldots\ldots\text{(i)}$$

At A, initial velocity in the horizontal direction is u

Horizontal distance travelled by object in time t is x

$$x = ut \text{ or } t = \frac{x}{u} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Substituting for t in equation (i);

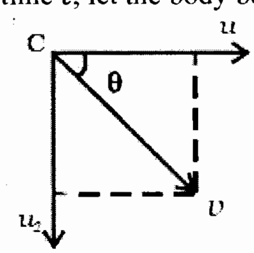
$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \frac{1}{2}g\frac{x^2}{u^2}$$

$$y = kx^2 \text{ where } k = \frac{g}{2u^2} \text{ is a constant.}$$

The above equation is the equation of a parabola. Thus, the path taken by the projectile is a parabola.

Resultant velocity at C

At an instant of time t, let the body be at C



At A, initial vertical velocity, $u_1 = 0$

At C, the horizontal velocity, $u_x = u$

Vertical velocity $= u_2$

From the equation of motion, $u_2 = u_1 + gt$

$$u_2 = gt$$

Resultant velocity at C is $v = \sqrt{u_x^2 + u_2^2}$

$$= \sqrt{u^2 + g^2t^2}$$

The direction of v is given by $\tan\theta = \frac{u_2}{u_x} = \frac{gt}{u}$

where, θ is the angle made by v with the x-axis

Time of flight and range

The distance $OB = R$, is the range of the projectile

Range = horizontal velocity × time taken to reach the ground

$$R = ut \;\ldots\ldots\text{(i)}$$

where, t is the time of flight

Vertical distance travelled by the object in time t,

$$= s_y = h$$

From $s = ut + \frac{1}{2}gt^2$

$$h = (0)t + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

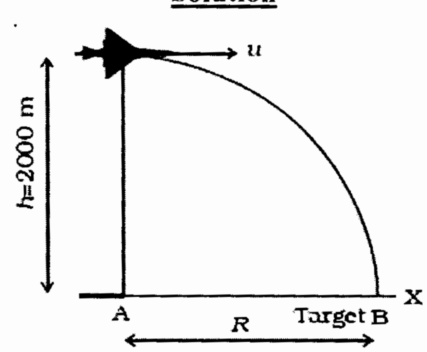
Substituting for t in (i);

$$\text{Range, } R = u\sqrt{\frac{2h}{g}}$$

Example 1

The pilot of an aero plane flying horizontally at a height of $200\,m$ with a constant speed of $540\,km/hr$ wishes to hit a target on the ground. At what distance from the target should the pilot release the bomb to hit the target?

Solution



Initial velocity of the bomb in the horizontal is the same as that of the bomb

Initial velocity of bomb in horizontal direction

$$= 540 \times \frac{1000}{3600} = 150\,ms^{-1}$$

Initial velocity of bomb in vertical direction, $u = 0$

Vertical distance, $s = 2000\,m$

Time of flight, $t = ?$

From $s = ut + \frac{1}{2}at^2$

$$2000 = 0 \times t + \frac{1}{2} \times 9.81t^2$$

$$t^2 = \frac{2000 \times 2}{9.81} = 407.7472$$

$$t = 20.19\,s$$

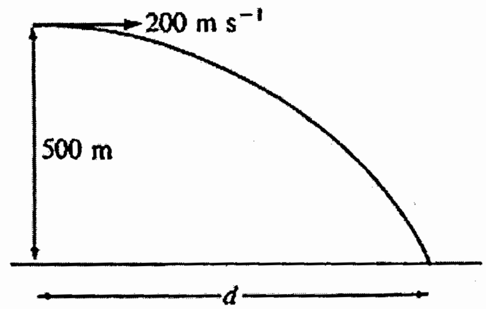
Horizontal range $= 150 \times 20.19$

$$= 3028.9\,m$$

Example 2

An aeroplane, flying in a straight line at a constant height of 500 m with a speed of $200\,ms^{-1}$ drops an object. The object takes a time t to reach the ground and travels a horizontal distance d in doing so. Ignoring air resistances, find the values of t and d.

Solution



When the object is released from the plane, its horizontal component of velocity is the same as the velocity of the plane.

Horizontal velocity $= 200\,ms^{-1}$

For vertical motion of the object,

$u = 0, a = g = 9.81\,ms^{-2}, s = 500\,m$

$$s = ut + \frac{1}{2}at^2$$

$$500 = 0 + \frac{1}{2} \times 9.81t^2$$

$$t^2 = \frac{100}{9.81}$$

$$t = 10.1\,s$$

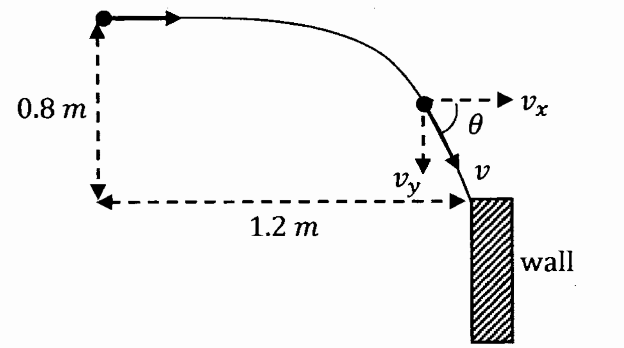
For horizontal motion, distance $d = 200 \times t$

$$= 200 \times 10.1 = 2020\,m$$

Example 3

A small ball is projected horizontally towards a vertical wall $1.2\,m$ away and hits the wall $0.8\,m$ below its initial horizontal level. Find the speed and direction of the ball when it hits the wall.

Solution



$$y = \frac{1}{2}gt^2$$

$$t^2 = \frac{2\times0.8}{9.81} = 0.1631$$

$$t = 0.4039\ s$$

$$x = ut$$

$$1.2 = 0.4039u$$

$$u = 2.971\ ms^{-1}$$

$$v_x = u = 2.971\ ms^{-1}$$

$$v_y = u_y + gt = 0 + 9.81 \times 0.4039$$

$$v_y = 3.962\ ms^{-1}$$

Speed of ball, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{2.971^2 + 3.692^2}$

$$= 4.95\ ms^{-1}$$

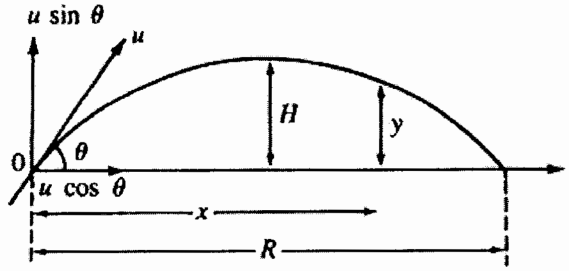
Direction of the ball,

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{3.962}{2.971} = 53.13°$$

Motion of a projectile projected at an angle with the horizontal

The motion of a projectile can be studied by looking at the vertical component and horizontal component of the motion individually. If the object is projected with a velocity u at an angle θ to the horizontal, the horizontal component of the velocity remains unchanged as $u\cos\theta$.

The initial vertical component of velocity is $u\sin\theta$



Considering the vertical component of motion:

Initial velocity $= u\sin\theta$, acceleration $= -g$

Let $H =$ maximum height reached. At maximum height, vertical component of velocity $= 0$

Using $v^2 = u^2 + 2as$

$$0 = (u\sin\theta)^2 - 2gH$$

$$2gH = u^2\sin^2\theta$$

Maximum height, $\boldsymbol{H = \frac{u^2\sin^2\theta}{2g}}$

Let the time taken to reach the maximum height be t_1

Using $v = u + at$

$$0 = u\sin\theta - gt_1$$

$$t_1 = \frac{u\sin\theta}{g}$$

When the object landed on the ground, the vertical displacement is zero. Let $T =$ time of flight

Using $s = ut + \frac{1}{2}at^2$

$$0 = (u\sin\theta)T - \frac{1}{2}gT^2$$

$$\frac{1}{2}gT^2 = (u\sin\theta)T$$

$\therefore$ **Time of flight,** $\boldsymbol{T = \frac{2u\sin\theta}{g}}$

Note: Time of flight $\boldsymbol{T = 2\left(\frac{u\sin\theta}{g}\right) = 2t_1}$

This implies that the time taken for the object to go to its maximum height is the same as the time taken by it to move from maximum height to the ground.

Considering the horizontal component of motion:

Horizontal component of velocity

$$= u\cos\theta = \text{constant}$$

At any time t, the horizontal displacement

$$x = (u\cos\theta)t$$

In particular, the range R of the projectile is

$$R = (u\cos\theta)T$$

$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$R = \frac{(2\sin\theta\cos\theta)(u^2)}{g}$$

$$\boldsymbol{R = \frac{u^2\sin2\theta}{g}}$$

Maximum range

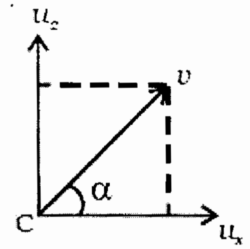
It is seen that for the given velocity of projection, the horizontal range depends on the angle of projection only. The range is maximum only if the value of $\sin2\theta$ is maximum i.e. when $\sin2\theta = 1$ or when $\theta = 45°$

$$R_{max} = \frac{u^2\times1}{g}$$

$$\therefore \boldsymbol{R_{max} = \frac{u^2}{g}}$$

Resultant velocity of the projectile at any instant t_1

At C, the velocity along the horizontal direction is $u_x = u\cos\theta$ and the velocity along the vertical direction is $u_y = u_2$



From $v = u + at$

$$u_2 = u_1 - gt_1$$

$$u_2 = u\sin\theta - gt_1$$

The resultant velocity at C is $v = \sqrt{u_x^2 + u_2^2}$

$$v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt_1^2)}$$

$$= \sqrt{u^2 + g^2t^2 - 2ut_1 g\sin\theta}$$

The direction of v is given by;

$$\tan\alpha = \frac{u_2}{u_x} = \frac{u\sin\theta - gt_1}{u\cos\theta}$$

$$\text{Or } \alpha = \tan^{-1}\left[\frac{u\sin\theta - gt_1}{u\cos\theta}\right]$$

Note: α is the angle which the direction of motion of the particle makes with the horizontal at that time, t. If $\tan\alpha$ is positive, then α is above the horizontal, and hence the particle has not yet reached the maximum point. If $\tan\alpha$ is zero, then the particle is at the maximum point. If $\tan\alpha$ is negative, then α is below the horizontal, and hence the particle is falling after passing the maximum point.

Equation of the trajectory

At any time t, the height (vertical displacement), y of the object is given by

$$s = ut + \frac{1}{2}at^2$$

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad \text{...... (i)}$$

The horizontal displacement, x in this time is given by

$$x = (u\cos\theta)t \quad \text{............ (ii)}$$

From (ii), $t = \frac{x}{u\cos\theta}$

Substituting for t in (i);

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \text{....... (iii)}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta \quad \text{....... (iv)}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta) \quad \text{....... (v)}$$

Equations (iii), (iv) and (v) are equations of a parabola. They are called trajectory equations and hence a projectile describes a parabolic path.

Example 4

A ball is thrown with an initial velocity of 15 ms^{-1} from the horizontal ground at an angle of 60° to the horizontal. Given that the air resistance can be neglected, determine

(i) the maximum height to which the ball rises
(ii) the time of flight
(iii) the horizontal distance between the point from which the ball was thrown and the point where it strikes the ground.

Solution

(i) Let H be the maximum height to which the ball rises

At maximum height, vertical speed, $v_y = 0$

$$v_y^2 = (u\sin\theta)^2 - 2gH$$

$$0 = (15\sin 60°)^2 - 2(9.81)H$$

$$H = 8.6\, m$$

(ii) Let T be the time of flight

At $t = T, s = 0$

From $s = ut + \frac{1}{2}at^2$

$$0 = (u\sin\theta)T - \frac{1}{2}gT^2$$

$$T = \frac{2\times 15\sin 60°}{9.81} = 2.65\, s$$

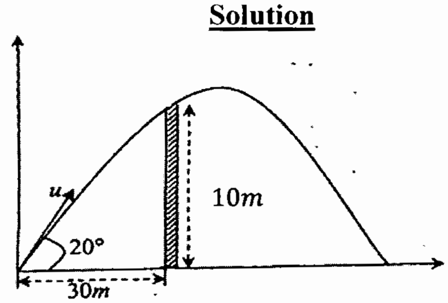
(iii) Horizontal range, $R = (u\cos\theta)T$

$$= (15\cos 60°)(2.65) = 19.9\, m$$

Example 5

A particle is projected at 20° to the horizontal and just clears a wall 10 m high and 30 m away from the point of projection. Find the speed of the projection, velocity of the projectile when it strikes the building and the time taken to reach the building.

Solution



Initial velocity

From $y = x\tan\theta - \frac{gx^2}{2u^2cos^2\theta}$

$$10 = 30\times\tan 20° - \frac{9.81\times 30^2}{2u^2cos^2 20°}$$

$$\Rightarrow 10 - 10.92 = \frac{8829}{1.766u^2}$$

$$\therefore u = 73.76\, ms^{-1}$$

Time taken

From $x = ut\cos\theta$

$$30 = 73.76\times t\times\cos 20°$$

$$\Rightarrow t = \frac{30}{73.76\cos 20°} = 0.43\ s$$

Velocity

At the point when the particle strikes the building,

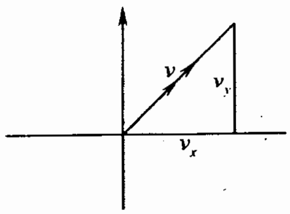
$$v_x = u\cos\theta = 73.76\ \cos 20° = 69.3\ ms^{-1}$$

$$v_y = u\sin\theta - gt = (73.76\sin 20° - 9.81 \times 0.43)$$

$$= 21\ ms^{-1}$$

$$v = \sqrt{{v_y}^2 + {v_x}^2} = \sqrt{69.3^2 + 21^2} = \ 72.4\ ms^{-1}$$

Since velocity is a vector quantity, we need to also find its direction



$$\tan\theta = \frac{v_y}{v_x} = \frac{21}{69.3}$$

$$\Rightarrow \theta = 16.68°$$

Example 6

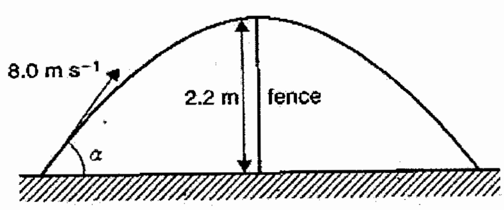
A ball is kicked so that at the highest point of its path, it just clears a fence a few metres away. The ground is level and the fence is 2.2 m high. The ball is kicked with an initial velocity of 8.0 ms^{-1} at an angle of projection α to the horizontal.

(a) Calculate the angle of projection α if the ball just clears the fence

(b) Find the horizontal velocity of the ball as it passes over the fence

(c) Calculate the total time for which the ball is in the air from the instant it is kicked until it reaches the ground.

Solution



(a) Initial vertical velocity, $u_y = 8\sin\alpha$

Final vertical velocity as it clears the fence, $v_y = 0$

From $v = u + at$

$$0 = 8\sin\alpha - gt$$

$$t = \frac{8\sin\alpha}{g}$$

Maximum vertical distance travelled in this time;

$$s = ut + \frac{1}{2}at^2$$

$$2.2 = (8\sin\alpha)\left(\frac{8\sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{8\sin\alpha}{g}\right)^2$$

$$2.2 = \frac{64\sin^2\alpha}{g} - \frac{32\sin^2\alpha}{g}$$

$$2.2 = \frac{32\sin^2\alpha}{g}$$

$$\sin\alpha = \sqrt{\frac{2.2\times 9.81}{32}}$$

$$\alpha = 55.2°$$

(b) Horizontal velocity $= u\cos\alpha$

$$= 8\cos 55.2° = 4.57\ ms^{-1}$$

(c) Time taken to reach maximum height,

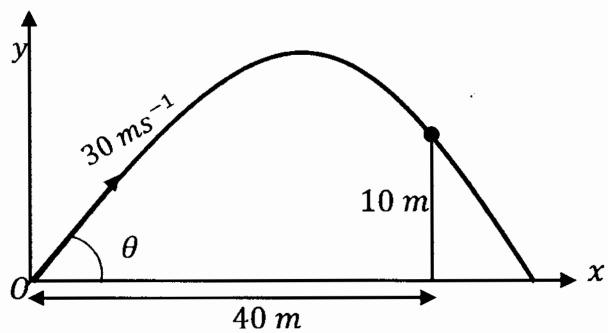
$$t = \frac{8\sin\alpha}{g} = \frac{8\sin 55.2°}{9.81} = 0.67\ s$$

Time to reach ground $= 2 \times 0.67 = 1.34\ s$

Example 7

A particle is projected from a point O with an initial speed of 30 ms^{-1} to pass through a point which is 40 m from O horizontally and 10 m above O. Show that there are two possible angles of projection for which this is possible. Hence find the angles.

Solution



Let the angle of projection be θ. The path of the particle has to pass through the point where $x = 40$ and $y = 10$.

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$10 = 40\tan\theta - \frac{9.81\times 40^2}{2\times 30^2}(1 + \tan^2\theta)$$

$$10 = 40\tan\theta - 8.72(1 + \tan^2\theta)$$

$$8.72\tan^2\theta - 40\tan\theta + 18.72 = 0$$

This is a quadratic equation in $\tan\theta$ with two positive roots. Therefore there are two posiible angles of projection.

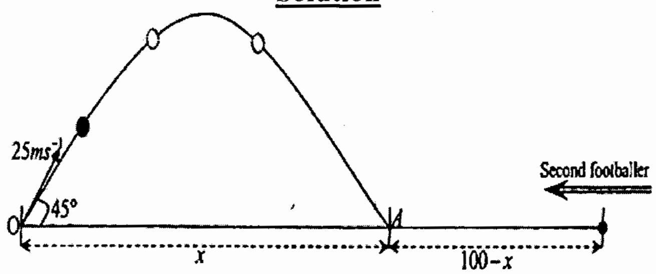
$$\tan\theta = 4.058 \quad \text{or } \tan\theta = 0.529$$

$$\theta = 76.16° \quad \text{or} \quad \theta = 27.88°$$

Example 8

Two footballers 100 m apart, stand facing each other. One of them kicks the ball from the ground such that the ball takes off at a velocity of 25 ms^{-1} at 45° to the horizontal. Find the speed at which the second footballer should run towards the first baller in order to trap the ball as it touches the ground, if he starts running at the instant the ball is kicked.

Solution



Considering the ball;

At the point A, the vertical displacement, $y = 0$

$$\therefore \text{From} \quad y = ut\sin\theta - \frac{1}{2}gt^2$$

$$0 = 25t\sin 45° - \frac{1}{2} \times 9.81 \times t^2$$

Either $t = 0$ i.e. at O or $25\sin 45° - \frac{1}{2} \times 9.81t = 0$

$$\therefore 4.905t = 25 \times \sin 45°$$

$$t = 3.604\ s$$

From $x = ut\cos\theta$

$$x = 25 \times 3.604 \times \cos 45° = 63.71\ m$$

Alternatively; as the ball touches the ground, it has travelled a distance equal to the range.

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$x = \frac{25^2 \sin 90°}{9.81} = 63.71\ m$$

Consider the second footballer;

He is supposed to travel a distance of

$$(100 - x) = 100 - 63.71 = 36.29\ m$$

Since the second footballer starts running at the instant the ball is kicked and is supposed to run so as to trap the ball as it falls, he should take the same time as that taken by the ball to land.

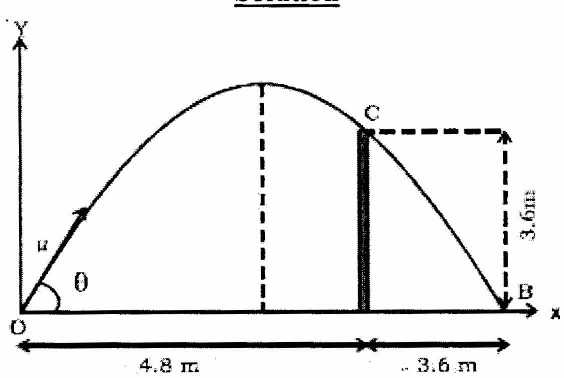
$\Rightarrow$ He should take 3.604 s

$$\therefore \text{speed} = \frac{\text{Distance}}{\text{time}} = \frac{36.29}{3.604} = 10.07\ ms^{-1}$$

Example 9

A boy throws a ball so that it may just clear a wall 3.6 m high. The boy is at a distance of 4.8 m from the wall. The ball was found to hit the ground at a distance of 3.6 m on the other side of the wall. Find the velocity with which the ball can be thrown.

Solution



Range of the ball $= 4.8 + 3.6 = 8.4\ m$

Height of the wall $= 3.6m$

$u = ?, \qquad \theta = ?$

The top of the wall C must lie on the path of the projectile.

Equation of projectile is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

The point $C(x = 4.8m, y = 3.6m)$ lies on the trajectory

$$3.6 = 4.8\tan\theta - \frac{g(4.8)^2}{2u^2\cos^2\theta} \quad \text{.................(i)}$$

Range of projectile $R = \frac{u^2\sin 2\theta}{g} = 8.4$

$$\frac{u^2}{g} = \frac{8.4}{\sin 2\theta} \quad \text{.................(ii)}$$

$$\Rightarrow \frac{g}{u^2} = \frac{\sin 2\theta}{8.4}$$

Substituting $\frac{g}{u^2}$ in (i);

$$3.6 = 4.8\tan\theta - \frac{(4.8)^2}{2\cos^2\theta} \times \frac{\sin 2\theta}{8.4}$$

$$3.6 = 4.8\tan\theta - \frac{(4.8)^2}{2\cos^2\theta} \times \frac{2\sin\theta\cos\theta}{8.4}$$

$$3.6 = 4.8\tan\theta - 2.7429\tan\theta$$

$$2.0571\tan\theta = 3.6$$

$$\theta = \tan^{-1}\left(\frac{3.6}{2.0571}\right) = 60.26°$$

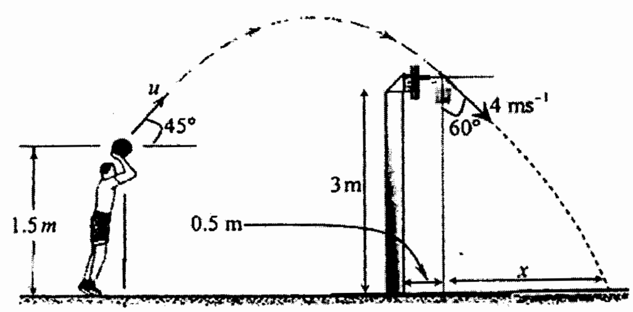
Substituting for θ in (ii)

$$u^2 = \frac{8.4 \times g}{\sin 2\theta} = \frac{8.4 \times 9.81}{\sin 2(60.26)} = 95.6571$$

$$u = 9.78\ ms^{-1}$$

Example 10

In the following figure, a ball is projected with a speed u at an angle of 45° to the horizontal from a point 1.5 m above the ground as shown above. It passses through a horizontal ringwith a velocity of 4 ms^{-1}, at an angle of 60° to the axis of the ring. If the axis of the ring is 0.5 m and that the height of the ring is 3 m, calculate:



(i) the time taken by the ball to hit the ground from the instant it passes through the ring,

(ii) the distance between the pole and the point where the ball hits the ground

(iii) the initial speed of projection

Solution

(i) Starting from the ring;

velocity of the ball at the ring $v = 4\ ms^{-1}$

$$\text{From} \quad y = ut\sin\theta - \frac{1}{2}gt^2$$

$$3 = 4t\ \sin 30° - \frac{1}{2} \times 9.81 \times t^2$$

$$4.905t^2 + 2t - 3 = 0$$

$$\text{Using } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = 0.6\ or\ t = -1.01$$

$$\therefore t = 0.6\ s$$

(ii)
From $x = ut\cos\theta$

$$x = 4 \times 0.6 \times \cos 30° = 2.08\,m$$

Required distance $= 0.5 + x$

$$= 0.5 + 2.08 = 2.58\,m$$

(iii) Since the horizontal velocity of the ball is the same through out its motion

Then $u\cos 45° = v\cos 30°$

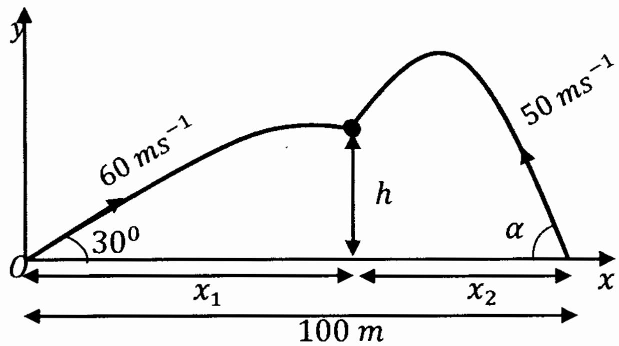
$$\therefore u = \frac{v\cos 30°}{\cos 45°} = \frac{4\cos 30°}{\cos 45°}$$

$$u = 4.89\,ms^{-1}$$

Example 11

A particle P is projected from a point O with an initial velocity of $60\,ms^{-1}$ at an angle of $30°$ to the horizontal . At the same instant, a second particle Q is projected in the opposite direction with an initial speed $50\,ms^{-1}$ from a point level with O and $100\,m$ from O. If the particles collide, find the angle of projection of Q and when the collision occurs.

Solution



If the particles collide, they must be at the same point at the same time. So as time is an important consideration, we do not use the trajectory equation.

Let t be the time interval from projection to collision

For P;

$$x_1 = (60\cos 30°)t$$
$$h = (60\sin 30°)t - \frac{1}{2}gt^2$$

For Q;

$$x_2 = (50\cos\alpha)t$$
$$h = (50\sin\alpha)t - \frac{1}{2}gt^2$$
$$x_1 + x_2 = 100$$

$\Rightarrow (60\cos 30° + 50\cos\alpha)t = 100$ .... (i)

Also $(60\sin 30°)t - \frac{1}{2}gt^2 = (50\sin\alpha)t - \frac{1}{2}gt^2$

$$\sin\alpha = \frac{30}{50}$$
$$\alpha = \sin^{-1}\frac{3}{5} = 36.9°$$

Q is projected at 36.9^0 to the horizontal

From (i), $t = \frac{100}{(60\cos 30° + 50\cos\alpha)}$

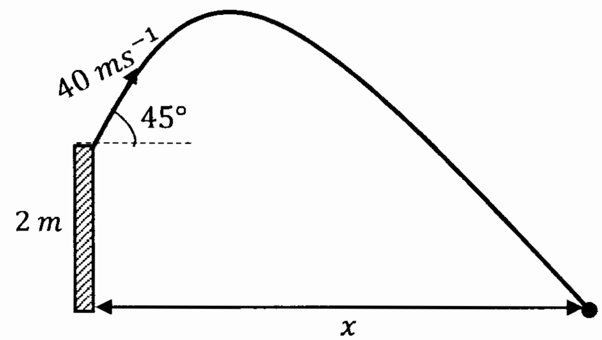
$$t = \frac{100}{(60\cos 30° + 50\cos 36.9°)} = 1.09$$

The particles collide $1.09\,s$ after projection.

Example 12

A particle is projected from a point which is $2\,m$ above the ground level with a velocity of $40\,ms^{-1}$ at an angle of 45° to the horizontal. Find its horizontal distance from the point of projection.

Solution



$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

We require the value of x when $y = -2$

$$-2 = x\tan 45° - \frac{gx^2(1+\tan^2 45°)}{2(40)^2}$$
$$-2 = x - \frac{gx^2}{1600}$$
$$49x^2 - 8000x - 16000 = 0$$
$$x = \frac{8000 \pm \sqrt{(-8000)^2 - 4(49)(-16000)}}{2(49)}$$
$$x = 165.24\,m$$

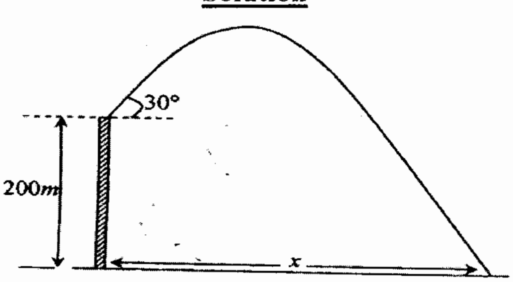
Therefore the horizontal distance of the particle from the point when it his the ground is $165.24\,m$

Example 13

A shot is fired from the top of a cliff 200 m high with a velocity of 500 ms^{-1} at an elevation of 30°. Find the

(i) distance from the point where the shot strikes the ground to the bottom of the cliff
(ii) time taken
(iii) distance from the ground to the highest point reached

Solution



(i) From $y = ut\sin\theta - \frac{1}{2}gt^2$

Considering upward motion:

$$-200 = 500t\sin 30° - \frac{1}{2} \times 9.81 \times t^2$$
$$\Rightarrow 4.905t^2 - 250t - 200 = 0$$
$$t = \frac{250 \pm \sqrt{250^2 - (4 \times 4.905 \times (-200))}}{2 \times 4.905}$$
$$t = 57.76 \text{ or } t = -0.79$$
$$\therefore t = 57.76\,ms^{-1}$$

(ii) From $x = ut\cos\theta$,

$x = 500 \times 51.76 \times \cos 30°$
$= 22412.1\, m$

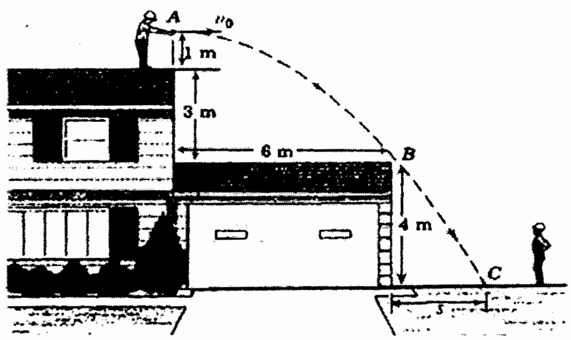
(iii) From $H = \frac{u^2 sin^2\theta}{2g}$

$H = \frac{500^2 \sin^2 30°}{2 \times 9.81} = 3185.5\, m$

Required distance $= 200 + 3185.5$
$= 3385.5\, m$

Self-Evaluation exercise

1.



In the figure above, a roofer tosses a small tool towards a coworker on the ground. What is the minimum horizontal velocity v_0 necessary so that the tool clears point B? Calculate the distance s of the point of impact for the tool.

[Ans: $v_0 = 6.64\, ms^{-1}, s = 2.49\, m$]

2. Calculate the range of a projectile which is fired at an angle of 45° to the horizontal with a speed of 20 ms^{-1}.

[Ans: 40.77 m]

3. A projectile is fired horizontally from the top of the cliff 250 m high. The projectile lands $1.414 \times 10^3 m$ from the bottom of the cliff. Find the
   (i) initial speed of the projectile
   (ii) velocity of the projectile just before it hits the ground

[Ans: (i) 198 ms^{-1} (ii) 210 ms^{-1} at 19.5°]

4. An object A is projected upwards from a height $60\, m$ above the ground with a velocity of 20 ms^{-1} at 30° to the horizontal. At the same time, another object B is projected from the ground upwards towards A at 30° to the horizontal. A and B collide at height of 60 m above the horizontal ground, when they are both moving downwards, Find the
   (i) the speed of projection of B
   (ii) the horizontal distance between the points of projection

[Ans: (i) $78.8\, ms^{-1}$ (ii) $174.58\, m$]

5. Two particles are projected simultaneously from two points A and B on level ground and a distance of 150 m apart. The first particle is projected vertically upwards from A with an initial speed of $u\, ms^{-1}$ and the second particle is projected from B towards A with an angle of projection α. If the particles collide when they are both at their greatest height above the level of AB, prove that

$$\tan\alpha = \frac{u^2}{150g}$$

6. Two particles A and B are projected simultaneously, A from the top of a vertical cliff, and B from the base. Particle A is projected with a speed of $3u$, while B is projected at an angle θ above the horizontal with a speed of $5u$. The two particles collide after 2 s. If the cliff is 56 m high, find the
   (i) values of u and θ
   (ii) horizontal and vertical distances from the base of the cliff to the point of collision of the two particles.

[Ans: (i) $u = 7\, ms^{-1}$ and $\theta = 53.13°$ (ii) $42\, m$]

7. A bullet is fired out to sea in a horizontal direction from a gun situated on the top of a cliff $78.4\, m$ high. Calculate the distance out to sea at which the bullet will strike the water, given that the initial velocity of the bullet is $240\, ms^{-1}$. Calculate also the inclination to the horizontal at which the bullet will strike the surface of the water.

[Ans: $960\, m$, 9.3°]

8. A particle is projected with a velocity of $30\, ms^{-1}$ at an elevation of 30°. Find
   (i) the greatest height reached
   (ii)the time of flight and the horizontal range
   (iii) the velocity and direction of motion at a height of 4 m

[Ans: (i) $11.5\, m$ (ii)$3.06\, s$, $79.5\, m$ (ii) $28.66\, ms^{-1}$ at $\tan^{-1} 0.466$ to the horizontal]

9. A projectile is fired horizontally from a point $60\, m$ above a horizontal plane with a velocity of $600\, ms^{-1}$. How far will it be horizontally from the point of projection when it reaches the plane?

[Ans: $2100\, m$]

10. A shot is fired from a gun on the top of a vertical cliff, 160 m high, with a velocity of $180\, ms^{-1}$, at an elevation of 30°. Find the horizontal distance from the foot of the cliff of the point where the shot strikes the water.

[Ans: $3117.69\, m$]

11. Find the velocity and direction of projection of a particle which passes in a horizontal direction just over the top of a wall which is $32\, m$ distant and $12\, m$ high.

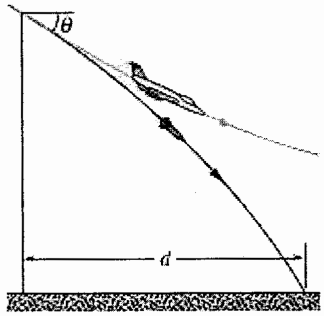
[Ans: $25.6\, ms^{-1}$ at $\tan^{-1}\frac{3}{4}$ to the horizontal]

12. A particle is projected horizontally at 20 ms^{-1} from a point 78.4 m above the horizontal surface. Find

the time taken for the particle to reach the surface and the horizontal distance travelled in that time.

[Ans: 4 s, 80 m]

13.



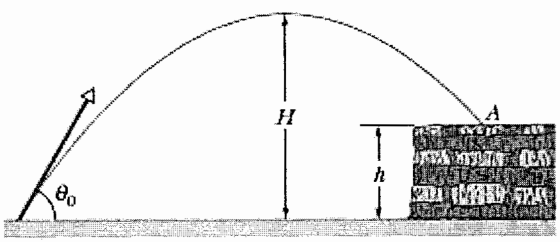
A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m.

(a) How long is the decoy in the air?

(b) How high was the release point?

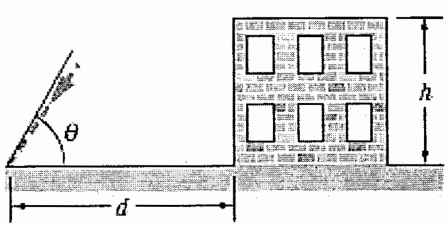
[Ans: (a) 10 s (b) 897 m]

14. A stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0°$ above the horizontal. The stone strikes at A, 5.50 s after launching.



Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.

15. A ball is thrown leftward from the left edge of the roof, at height h above the ground. The ball hits the ground 1.50 s later, at distance $d = 25.0$ m from the building and at angle $\theta = 60.0°$ with the horizontal.



Find

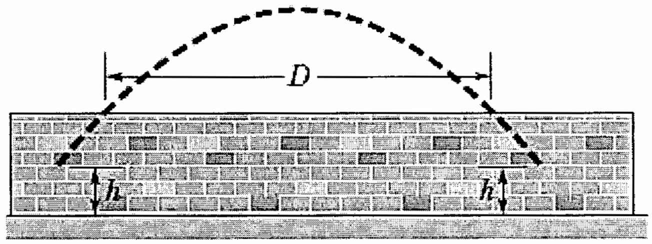
(a) h.

(b) the magnitude and angle relative to the horizontal of the velocity at which the ball is thrown

(c) Is the angle above or below the horizontal?

[Ans: (a) 32.3 m (b) 21.8 $m\,s^{-1}$, 40.4° (c) below]

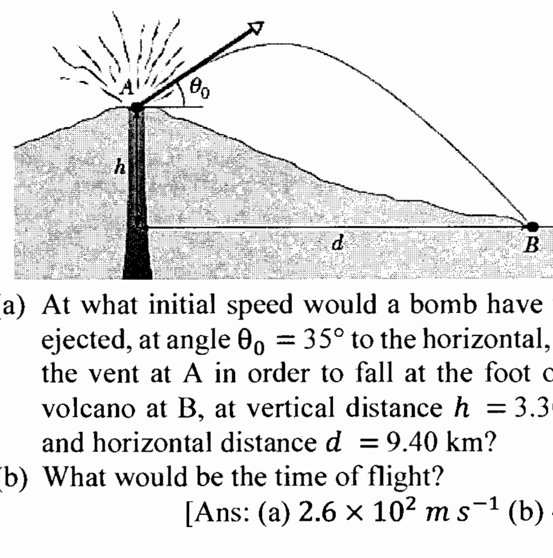
16. A baseball is hit at a height $h = 1.00$ m and then caught at the same height. It travels alongside a wall, moving up past the top of the wall 1.00 s after it is hit and then down past the top of the wall 4.00 s later, at distance $D = 50.0$ m farther along the wall.



(a) What horizontal distance is traveled by the ball from hit to catch? What are the (b) magnitude and (c) angle (relative to the horizontal) of the ball's velocity just after being hit? (d) How high is the wall?

[Ans: (a)75.0 m (b) 31.9 $m\,s^{-1}$ (c) 66.9° (d) 25.5 m]

17. During volcanic eruptions, chunks of solid rock can be blasted out of the volcano; these projectiles are called volcanic bombs.



(a) At what initial speed would a bomb have to be ejected, at angle $\theta_0 = 35°$ to the horizontal, from the vent at A in order to fall at the foot of the volcano at B, at vertical distance $h = 3.30$ km and horizontal distance $d = 9.40$ km?

(b) What would be the time of flight?

[Ans: (a) $2.6 \times 10^2\ m\,s^{-1}$ (b) 45 s]

NEWTON'S LAWS OF MOTION

First law

It states that everybody continues in its state of rest or uniform motion along a straight line unless it is compelled by an external force.

An external force can either be a push or pull or other dissipative forces such as friction, air resistance, etc.

A **force** is a push or pull which produces or tends to produce, stops or tends to stop motion in a body.

This law is based on inertia.

Inertia

This is the tendency of a body to maintain its state of rest or of uniform motion in a straight line unless acted upon by some net external force.

Inertia is of three types i.e.

(i) Inertia of rest
(ii) Inertia of motion
(iii) Inertia of direction

Inertia of rest

It is the inability of the body to change its state of rest by itself.

Examples

- A person standing in a bus falls backwards when the bus suddenly starts moving. This because the person who is initially at rest continues to be at rest even after the bus has started moving.
- A book lying on the table will remain at rest until it is moved by some external agencies.

Inertia of motion

Inertia of motion is the inability of the body to change its state of motion by itself.

Examples

- When a passenger gets down from a moving bus, he falls down in the direction of motion of the bus
- A passenger sitting in a moving car falls forward when the car stops suddenly
- An athlete running in a race will continue to run even after reaching the finishing point.

Inertia of direction

It is the inability of the body to change its direction of motion by itself.

Example

When a bus moving along a straight line takes a turn to the right, the passengers are thrown towards left. This is due to inertia which makes the passenger travel along the same straight line, even though the bus has turned towards the right.

Examples of the law of inertia

In everyday life, there are a large number of examples of Newton's first law of motion. Some include the following.

(i) When a train suddenly starts, the passengers standing in the compartment tend to fall backward. It is because the lower part of the body of the passenger, which is contact with the train, comes in motion but the upper part tends to be at rest due to inertia. Consequently, the passengers tend to fall backward.

(ii)When a moving bus is suddenly stopped, the passengers tend to fall forward. It is because the lower part of the body of the passenger which is in contact with the bus comes to rest but the upper part tends to be in motion due to inertia. As a result, the passengers tend to fall forward.

(iii) When we beat a carpet with a stick, dust particles are removed. It is because the carpet is sudden set into motion but the dust particles tend to remain at rest due to inertia. Therefore, dust particles get removed from the carpet.

(iv) An athlete runs some distance before executing a jump. It is because by running some distance, velocity acquired due to inertia is added to the velocity of the athlete at the time of the jump. Consequently, the athlete jumps through a longer distance.

(v) The mud from the wheels of a moving vehicle flies off tangentially. As the mud leaves the wheel, there is no net external force acting on it. Therefore, it follows a straight line path tangent to the wheel at the point of leaving. As a result, mud from the wheels of a moving vehicle flies off tangentially.

Second law

It states that the rate of change of momentum of a body is directly proportional to the external force applied on it and takes place in the direction of force.

Momentum of a body

It is observed experimentally that the force required to stop a moving object depends on two factors. i.e. mass of the body and its velocity. A body in motion thus has momentum.

The momentum of a body is defined as the product of its mass and velocity.

If m is the mass of the body and u is its velocity, then linear momentum of the body is given by mu.
Momentum has both magnitude and direction and it is therefore a vector quantity. The momentum is measured in terms of kg ms^{-1} and its dimensional formula is MLT^{-1}.
If p is the momentum of a body and F is the external force acting on it, then according to Newton's second law of motion

$$F \propto \frac{dp}{dt} \quad or\ F = \frac{kdp}{dt}$$

where k is the constant of proportionality.
If the body of mass m is moving with velocity, v then the momentum is given by $p = mv$

$$F = k\frac{d(mv)}{dt} = km\frac{dv}{dt}$$

when $F = 1\,N, m = 1\,kg$ and $\frac{dv}{dt} = 1\,ms^{-2}$,

constant $k = 1$

$F = m\frac{dv}{dt} = ma$ where $a = \frac{dv}{dt}$ is the acceleration produced in the motion of the body.

$$\therefore F = ma$$

The force acting on a body is measured by the product of mass of the body and acceleration produced by the force acting on the body. Thus, the second law of motion gives a measure of the force.

Impulsive force and impulse of a force

Impulsive force

An impulsive force is a very great force acting for a short time on the body so that the change in position of the body during the time the force acts on it may be neglected.

Impulse of a force

The impulse, j of a constant force F acting for a time, t is defined as a product of force and time.

$$\text{impulse} = \text{Force} \times \text{time}$$

$$j = F \times t$$

When a variable force is acting for a short interval of time, then the impulse is given by

$$j = F_{average} \times \Delta t$$

Relationship between impulse and momentum

$$\text{Impulse of force} = F \times t = (ma)t$$

If u and v are the initial and final velocities of the body, then

$$a = \frac{v-u}{t}$$

$$\text{Impulse of the force} = m\left(\frac{v-u}{t}\right) \times t = m(v-u)$$

$$= mv - mu$$

mu = initial momentum of the body

mv = final momentum of the body

∴ Impulse of force = change in momentum

Note

Whenever you wish the force of impact to be small, extend the time of impact. On the other hand, if the time of impact is small, the impact force will be large.

Examples

- ✓ A cricket player while catching the ball lowers his hands in the direction of the ball.
  If the change in momentum is brought about in a very short of time, the average force is very large according to the equation $F = \frac{mv-mu}{t}$
  By increasing the time interval, the average force is increased. It is for this reason that a cricket player while catching the ball increases the time of contact. The player should lower his hand in the direction of the ball so that he is not hurt.
- ✓ When a batsman strikes a ball, he follows through in order to keep the bat in contact with the ball for a long time as possible. It follows from the equation $Ft = mv - mu$ that this increases impulse and therefore produces a larger momentum change and so increases the speed at which the ball leaves the bat.
- ✓ A person jumping from an elevated position on a floor bends his knees upon making contact. This extends the time of impact. Therefore, the force of impact is reduced.
  Note: Similarly, for the same reason, a goal keeper draws his hands backwards when catching a fast-moving hard ball.
- ✓ A person falling on a cemented floor gets injured more whereas a person falling on a sand floor does not get hurt. For this reason, in wrestling, high jump, long jump, etc., soft ground is provided.
- ✓ The vehicles are fitted with springs and shock absorbers to reduce jerks while moving on uneven or navy roads.
- ✓ China wares are wrapped in straw or paper before packing so that when they receive jerks during transportation, the time of impact is increased. As a result, the average force exerted on the glasswares is small and chances of their breaking reduce

Example 1

A 100 kg man jumps into a swimming pool from a height of 5 m. It takes 0.4 s for the water to reduce his velocity to zero. What is the average force exerted by the water on the man?

Solution

Let v be the velocity of the man just before entering the water

$$v^2 = u^2 + 2gh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 5} = 9.905\ m\,s^{-1}$$

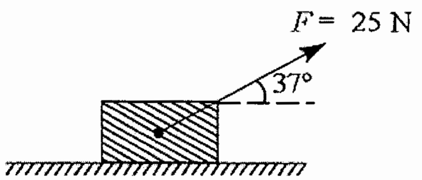
$$F = \frac{mv - 0}{t} = \frac{mv}{t} = \frac{100 \times 9.905}{0.4}$$

$$F = 2476\ N$$

Example 2

A girl pulls a box of mass 10.4 kg with a force of 25 N inclined to the horizontal at an angle of 37°. As a result, the box accelerates horizontally. What is the magnitude of the acceleration?

Solution



The horizontal component of force accelerates the box in the horizontal direction

Horizontal component of force, $F_x = F\cos 37°$

$$= 25\cos 37° = 19.97\ N$$

Acceleration, $a = \frac{F_x}{m} = \frac{19.97}{10.4} = 1.92\ m\,s^{-2}$

Example 3

A bullet of mass 0.04 kg moving with a speed of 90 $m\,s^{-1}$ enters a heavy wooden block and it is stopped after a distance of 60 cm. Calculate the average resistive force of the block

Solution

Let the initial speed of the bullet be u. As the bullet enters the wooden block, its velocity decreases to zero after covering a distance of 60 cm

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 90^2}{2 \times 0.6} = -6750\ m\,s^{-2}$$

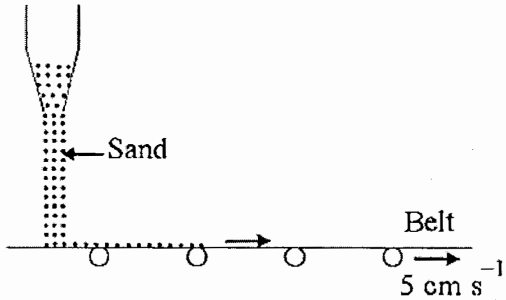
Average retarding force,

$$F = ma = 0.04 \times 6750 = 270\ N$$

Example 4

Sand drops vertically at a rate of 100 g s^{-1} on to a horizontal conveyer belt moving at a steady velocity of 5 $cm\,s^{-1}$. Calculate the force required to keep the belt moving.

Solution



The initial horizontal velocity of the sand, $u = 0$

The final horizontal velocity of sand, $v = 5\ cm\,s^{-1}$

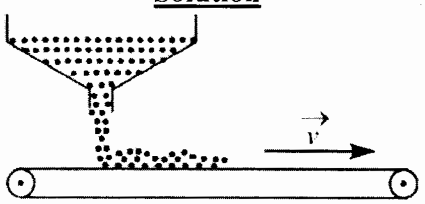
Force required to keep belt moving = Rate of increase in horizontal linear momentum

$$F = \frac{mv - mu}{t} = \frac{0.1 \times 0.05 - 0}{1} = 5 \times 10^{-3}\ N$$

Example 5

A hopper drops sand onto a conveyer belt at a rate of 75 kg s^{-1}. If the belt moves at a constant speed of 2.20 $m\,s^{-1}$, what force is needed to keep the conveyer belt moving?

Solution



$$F = m\frac{dv}{dt} = v\frac{dm}{dt}$$

$$= 2.2 \times 75 = 165\ N$$

Example 6

A block of metal of mass 2 kg is resting on a frictionless floor. It is struck by a jet releasing water at the rate of 1 $kg\,s^{-1}$ and at a speed of 5 $m\,s^{-1}$. Calculate the initial acceleration of the block

Solution

Force exerted on the block by jet of water is

$$F = \frac{d}{dt}(mv) = v\frac{dm}{dt}$$

$v = 5\ m\,s^{-1}$, $\frac{dm}{dt} = 1\ kg\,s^{-1}$

$$F = 5 \times 1 = 5\ N$$

Now $F = ma$

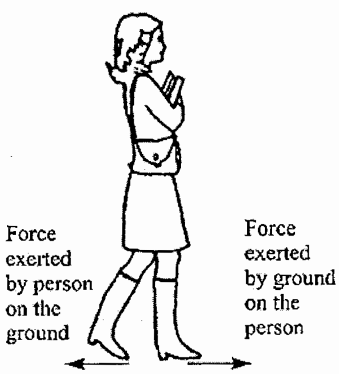
$$a = \frac{F}{m} = \frac{5}{2} = 2.5\ m\,s^{-2}$$

Third law

It states that for every action, there is an equal and opposite reaction i.e. whenever one body exerts a certain force on a second body, the second body exerts an equal and opposite force on the first.

The effect of third law of motion can be observed in many activities in our everyday life. Examples include;

- ✓ When a bullet is fired from a gun with a certain force (action), there is an equal and opposite force exerted on the gun in the backward direction. (reaction)
- ✓ When a man jumps from a boat to the shore, the boat moves away from him. The force he exerts on the boat (action) is responsible for its motion and his motion to the shore is due to the force of reaction exerted by the boat on him.
- ✓ The swimmer pushes the water in the backward direction with a certain force (action) and the water pushes the swimmer in the forward direction with an equal and opposite force (reaction)
- ✓ When a person walks on the ground, he or she exerts a force on the ground and in turn the ground exerts an equal force on the person. It is this force on the person which moves him or her forward.



Similarly, when a man jumps, he exerts a downward force on the ground. The ground exerts an equal upward force on the man

It is thus hard to walk on a muddy ground because we are not able to exert action force in backward direction and so the ground does not exert sufficient reaction force in the forward direction needed for walking.

- ✓ A bird flies with the help of its wings. The wings of a bird push air downwards (action). In turn, the air reacts by pushing the bird upwards (reaction)
- ✓ When a force is exerted directly on the wall by pushing the palm of our hand against it (action), the palm is distorted a little because the wall exerts an equal force on the hand (reaction).

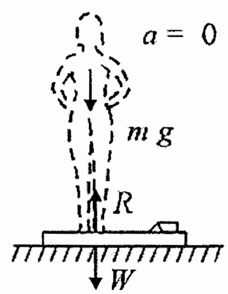
Applications of Newton's third law

1. Apparent loss of weight in a lift

Consider a man of mass m standing on a weighing machine placed inside a lift. The actual weight of the man $= mg$. This weight (action) is measured by the weighing machine and in turn, the machine offers a reaction, R. This reaction offered by the surface of contact on the man is the apparent weight of the man.

Case(i): When the lift is at rest

Acceleration of the man $= 0$



net force acting on the man $= 0$

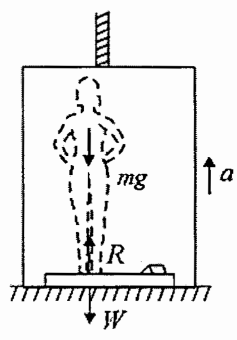
$$R - mg = 0$$
$$R = mg$$

That is, the apparent weight of the man is equal to the actual weight.

Case (ii): When the lift is moving uniformly in the upwards or downward direction.

For uniform motion, the acceleration of the man is zero. Hence in this case, also the apparent weight of the man is equal to the actual weight.

Case (iii): When the lift is accelerating upwards.



Let a be the upward acceleration of the man in the lift, then the net upward force of the man is $F = ma$.

$$\text{Net force, } F = R - mg$$
$$ma = R - mg$$
$$R = m(g + a)$$

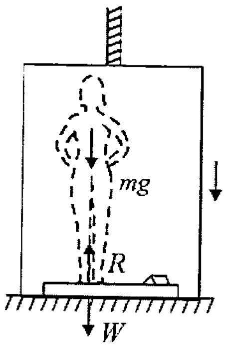
Therefore, the apparent weight of the man is greater than the actual weight.

Case (iv): When the lift is accelerating downwards. Let a be the downward acceleration of the man in the

Example 1

A 10n '

...d force on the man is $F =$



Net force, $F = mg - R$

$$ma = mg - R$$

$$R = m(g - a)$$

The apparent weight of the man in less than the actual weight.

Note: When the downward acceleration of the man is equal to the acceleration due to gravity i.e. $a = g$

$$R = m(g - g) = 0$$

Hence, the apparent weight of the man becomes zero. This known as the **weightlessness** of the body.

Example 7

The mass of an elevator is $500\ kg$. Calculate the tension in the cable of the elevator when the elevator is

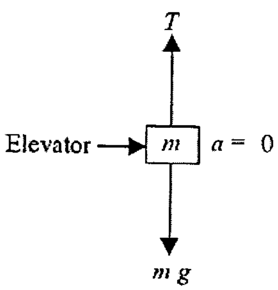
(a) stationary

(b) accelerating with an acceleration of $2\ m\ s^{-2}$

(c) descending with the same acceleration

Solution

(a) When the elevator is stationary, its acceleration is zero, therefore the net force is zero

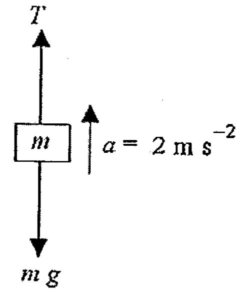


$$\text{Net force} = T - mg$$

$$0 = T - mg$$

$$T = mg = 500 \times 9.81 = 4905\ \text{N}$$

(b) When the elevator is ascending, the net force on it is upward. Therefore, tension T in the cable will be greater than gravity force mg



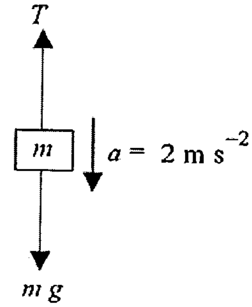
$$\text{Net force} = T - mg$$

$$ma = T - mg$$

$$T = mg + ma = m(g + a)$$

$$T = 500(9.81 + 2) = 5905\ N$$

(c) When the elevator is descending, the net force is in the downward direction. Therefore, tension, T in the cable will be less than mg



$$\text{Net force} = mg - T$$

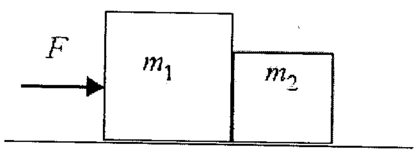
$$ma = mg - T$$

$$T = mg - ma = m(g - a)$$

$$T = 500(9.81 - 2) = 3905\ N$$

2. Connected particles

(i) Consider two blocks of masses m_1 and m_2 in contact on a smooth horizontal surface as shown below



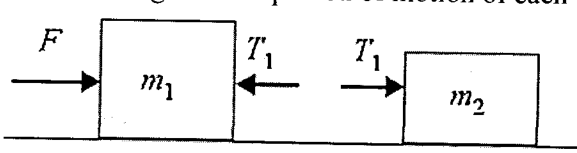
When a force F is applied on m_1, two blocks move with a common acceleration a given by

Common acceleration, $a = \frac{F}{m_1+m_2}$

Force on m_1, $F_1 = F$

Force on m_2, $F_2 = m_2 a = m_2 \times \frac{F}{m_1+m_2}$

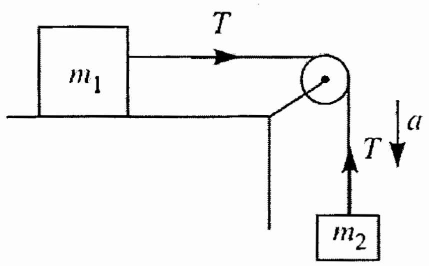
Let us investigate the equation of motion of each body



For the first body, $F - T_1 = m_1 a$

For the second body, $T_1 = m_2 a$

(ii) Consider a mass m_1 on a smooth horizontal table and connected to a mass m_2 by a light string passing over a frictionless pulley of negligible mass.



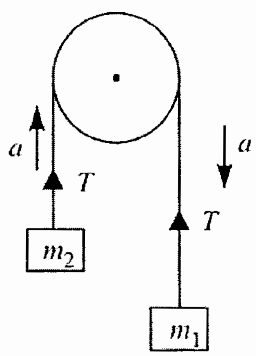
If a is the acceleration of the system in the direction shown, then equations of motion for the two masses are

$$T = m_1 a \quad ...... \text{(i)}$$
$$m_2 g - T = m_2 a \quad ...... \text{(ii)}$$

On solving (i) and (ii);

$$a = \frac{m_2}{(m_1 + m_2)} \times g$$
$$T = \frac{m_1 m_2}{m_1 + m_2} \times g$$

(iii) Consider two masses m_1 and m_2 connected to opposite ends of a light string passing over a frictionless and light pulley. Suppose $m_1 > m_2$. As a result, mass m_1 moves downward while mass m_2 moves upward. Let the common acceleration of the system be a



The equations of motion are

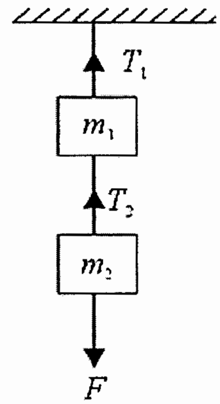
$$m_1 g - T_1 = m_1 a \quad .... \text{(i)}$$
$$T - m_2 g = m_2 a \quad .... \text{(ii)}$$

On solving (i) and (ii);

$$a = \frac{m_1 - m_2}{m_1 + m_2} \times g$$
$$T = \frac{2m_1 m_2}{m_1 + m_2} \times g$$

Note that when $m_1 = m_2$, then $a = 0$ and $T = m_1 g$

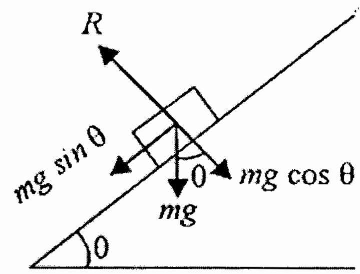
(iv) Consider two masses m_1 and m_2 suspended vertically from a rigid support with the help of strings.



If mass m_2 is pulled down with a force F, then tensions T_1 and T_2 are given by

$$T_2 = F + m_2 g$$
$$T_1 = F + (m_1 + m_2)g$$

(v) Consider a block of mass m sliding down a smooth inclined plane making an angle θ with the horizontal.



Force on the block down the plane is given by

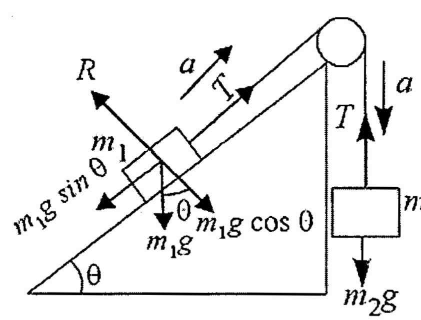
$$F = mg\sin\theta$$

Acceleration of the block down the plane is given by

$$a = \frac{F}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$

Note that gravity is the accelerating force yet the acceleration is not vertical. Further, acceleration a is independent of the mass of the block. It depends only on the angle of inclination θ and g

(vi) Consider two masses m_1 and m_2 attached to the opposite ends of a string passing over a frictionless pulley at the edge of a smooth inclined plane.



If the system moves up the inclined plane with an acceleration a, then the equations of motion of the two bodies are

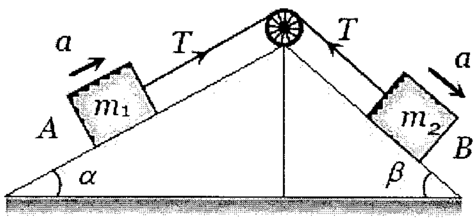
$$T_1 - m_1 g \sin\theta = m_1 a \text{ ...... (i)}$$
$$m_2 g - T = m_2 a \text{ .... (ii)}$$

On solving equations (i) and (ii);

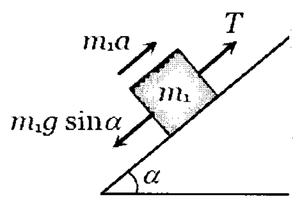
$$a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$$
$$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2}$$

If m_2 is greater than $m_1 \sin\theta$, the acceleration of m_1 is up the inclined plane. If $m_1 \sin\theta$ exceeds m_2, the acceleration of m_1 is down the inclined plane

(vii) Consider two blocks A and B of masses m_1 and m_2 on two incined planes joined together. Let $m_2 > m_1$

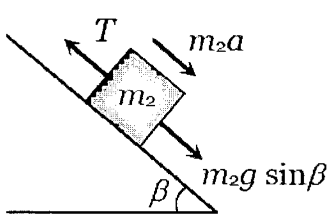


The motion of the two blocks can be split into two i.e.
For block A;



$$T - m_1 g \sin\alpha = m_1 a \text{ ...... (i)}$$

For block B;

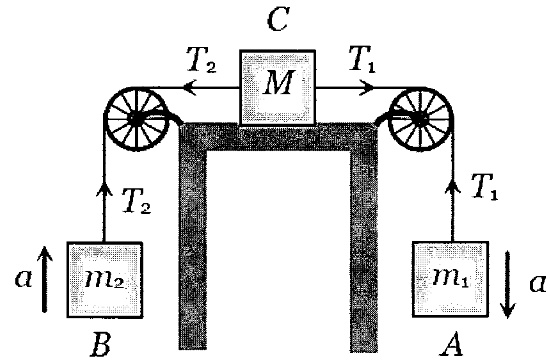


$$m_2 g \sin\beta - T = m_2 a \text{ .... (ii)}$$

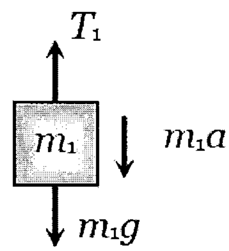
Solving (i) and (i);

$$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2}$$
$$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$$

(viii) Consider three blocks connected as shown below. Block C of mass M rests on a smooth horizontal table and the blocks A and B of masses m_1 and m_2 hung on the sides of the table with strings passing over frictionless pulleys. The tensions in the strings connecting A to C and B to C are different and are T_1 and T_2 respectively. The acceleration is the same for all the blocks and let it be a.

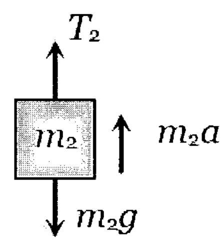


Considering the blocks seperately
Block A;



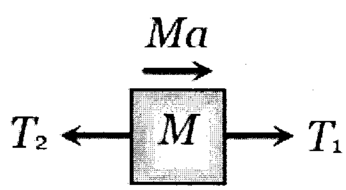
$$m_1 a = m_1 g - T_1 \text{ ...... (i)}$$

Block B;



$$m_2 a = T_2 - m_2 g \text{ ...... (ii)}$$

Block C;



$$T_1 - T_2 = Ma \text{ .... (iii)}$$

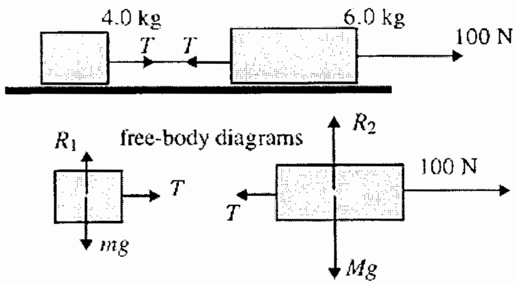
Solving equations (i), (ii) and (iii);

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g$$
$$T_1 = \frac{m_1 (2m_2 + M)}{(m_1 + m_2 + M)} g$$
$$T_2 = \frac{m_2 (2m_2 + M)}{(m_1 + m_2 + M)} g$$

Example 8

Two blocks of mass 4.0 kg and 6.0 kg are joined by a string and rest on a frictionless horizontal table. If a force of 100 N is applied on one of the blocks, find the acceleration of each block and the tension in the string

Solution



The net force on the 6.0 kg mass is $100 - T$ and on the 4.0 kg mass just T.

Thus, applying Newton's second law separately on each mass.

$$100 - T = 6a \quad ..... \text{(i)}$$

$$T = 4a \quad ..... \text{(ii)}$$

(i) + (ii);

$$100 = 10a$$

$$a = 10\, m\, s^{-2}$$

From (ii); $T = 4a = 4 \times 10 = 40\, N$

Alternatively taking the combination of masses;

$$100 = 10a$$

$$a = 10\, m\, s^{-2}$$

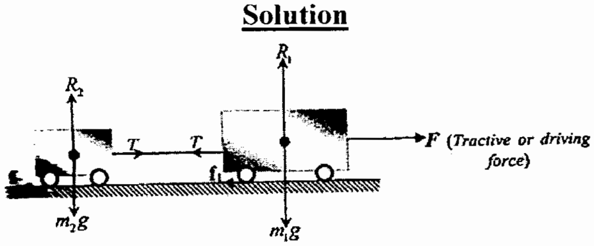
Example 9

A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find

(a) the acceleration of the car and trailer,

(b) the magnitude of the tension in the tow-bar.

Solution



Considering the whole system

$$F - (f_1 + f_2) = (m_1 + m_2)a$$

$$1200 - (400 + 200) = 1000 \times 0.6$$

$$a = 0.6\, m\, s^{-2}$$

For car; $F - (f_1 + T) = m_1 a$

$$1200 - (400 + T) = 800a$$

$$T = 320\, N$$

Or for trailer;

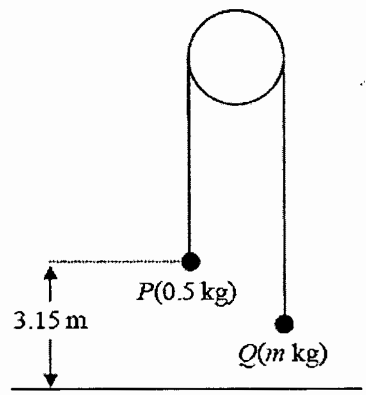
$$T - f_2 = m_2 a$$

$$T - 200 = 200 \times 0.6$$

$$T = 320\, N$$

Example 10

Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in the diagram below. After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.



Find the

(a) acceleration of P as it descends

(b) tension in the string

(c) value of m

Solution

(a) $u = 0, a = ?, t = 1.5\, s, s = 3.15\, m$

$$s = ut + \frac{1}{2}at^2$$

$$3.15 = \frac{1}{2}a(1.5)^2$$

$$a = 2.8\, m\, s^{-2}$$

(b) Using $F = ma$

$$0.5g - T = 0.5a$$

$$T = 0.5(g - a)$$

$$= 0.5(9.81 - 2.8) = 3.51\, N$$

(c) $T - mg = ma$

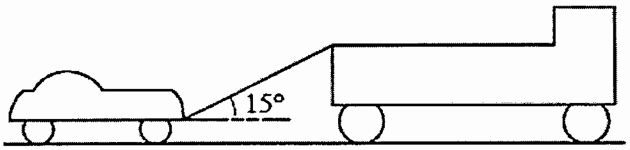
$$3.51 - 9.81m = 2.8m$$

$$12.61\, m = 3.51$$

$$m = 0.278\, kg$$

Example 11

The following figure shows a lorry of mass 1600 kg towing a car of mass 900 kg along a straight horizontal road. The two vehicles are joined by a light tow-bar which is at an angle of 15° to the road. The lorry and the car experience constant resistances to motion of magnitude 600 N and 300 N respectively. The lorry's engine produces a constant horizontal force on the lorry of magnitude 1500 N.

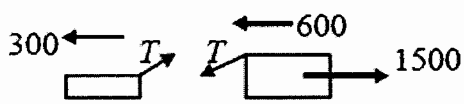


Find

(a) the acceleration of the lorry and the car,
(b) the tension in the tow-bar.

Solution

(a)



Lorry + car

$$1500 - (300 + 600) = 2500a$$
$$a = 0.24\, m\, s^{-2}$$

(b) For car;

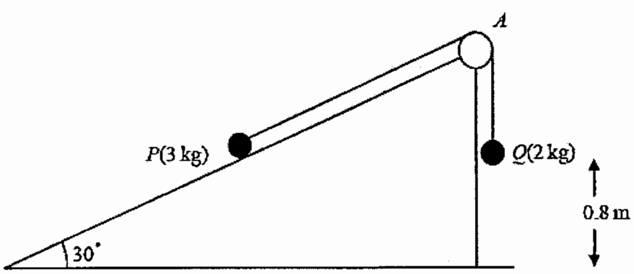
$$T\cos 15° - 300 = 900a$$
$$T\cos 15° = 900(0.24) + 300$$
$$T = 534\, N$$

Or for lorry;

$$1500 - (T\cos 15° + 600) = 1600a$$
$$T\cos 15° = 1500 - 1600(0.24) - 600$$
$$T = 534\, N$$

Example 12

The diagram below shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.



Find

(a) the acceleration of the masses and the tension in the string
(b) the speed of Q as it reaches the ground

Solution

(a) Using Newton's second law of motion

$$2g - T = 2a \;\; \ldots\ldots\text{(i)}$$
$$T - 3g\sin 30° = 3a \;\; \ldots\ldots\text{(ii)}$$

(i) + (ii);

$$2g - 3g\sin 30° = 5a$$
$$a = 0.981\, m\, s^{-2}$$

From (i);

$$T = 2(g - a)$$
$$= 2(9.81 - .981) = 17.66\, N$$

(b) $u = 0,\ v = ?,\ s = 0.8,\ a = 0.981$

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2(0.981)(0.8)$$
$$v = 1.253\, m\, s^{-1}$$

Law of conservation of linear momentum

It states that for a system of colliding bodies, the total momentum in a given direction remains constant provided no external forces act.

From impulse of force, $j = mv - mu$

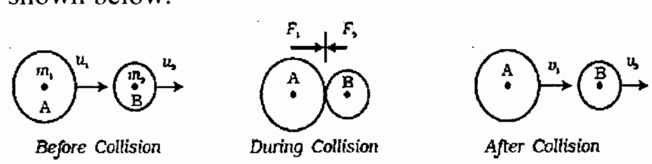
If $j = 0$, then $mv - mu = 0$

$$\Rightarrow mv = mu$$

Final momentum = Initial momentum

Proof:

Consider a body A of mass m_1 moving with a velocity u_1 colliding head-on with another body B of mass m_2 moving in the same direction as A with velocity u_2 as shown below.



After collision, let the velocities of the bodies be changed to v_1 and v_2 respectively and both move in the same direction. During collision, each body experiences a force.

The force acting on one body is equal in magnitude and opposite in direction to the force acting on the other body. Both forces act for the same interval of time.

Let F_1 be force exerted by A on B (action), F_2 be the force exerted by B on A (reaction) and t be the time of contact of the two bodies during collision.

Now, F_1 acting on the body B for a time t, changes its velocity from u_2 to v_2.

$$F_1 = \text{mass of B} \times \text{acceleration of B}$$
$$F_1 = m_2 \frac{(v_2 - u_2)}{t}$$

Similarly, F_2 acting on the body A for the same time, t changes its velocity from u_1 to v_1

$$F_2 = \text{mass of A} \times \text{acceleration of A}$$
$$F_2 = m_1 \frac{(v_1 - u_1)}{t}$$

Then by Newton's third law of motion; $F_1 = -F_2$

$$m_2\frac{(v_2-u_2)}{t} = -m_1\frac{(v_1-u_1)}{t}$$

$$m_2(v_2-u_2) = -m_1(v_1-u_1)$$

$$m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$$

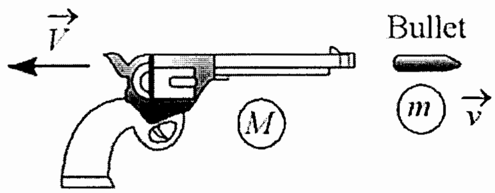
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Total momentum before impact = Total momentum after impact.

∴Total momentum of the system is a constant.

Examples of conservation of linear momentum

(i) Recoil velocity of a gun



Consider a gun and bullet of mass M and m respectively. The gun and the bullet form a single system. Before the bullet is fired, both the gun and the bullet are at rest. Therefore, the velocities of the gun and the bullet are zero. Hence total momentum of the system before firing is

$$M(0) + m(0) = 0$$

When the gun is fired, the bullet moves forward and the gun recoils backwards. If v and V are their respective velocities, the total momentum of the bullet-gun system after firing is

$$mv + MV$$

According to law of conservation of momentum, total momentum before firing = total momentum after firing.

$$mv + MV = 0$$

$$V = -\frac{mv}{M}$$

The minus sign shows that the direction of the recoil of the gun is opposite to that of the bullet. Since the velocity has a much larger mass, its velocity is much less than that of the bullet.

Note: The initial kinetic energy of the system is zero because both the gun and bullet are at rest. However, the final kinetic energy $= \frac{1}{2}MV^2 + \frac{1}{2}mv^2$ which is greater than zero. This kinetic energy is created by the explosion of the gun powder i.e. chemical energy is converted into kinetic energy.

Example 13

A hunter fires a bullet of mass 10 g with a velocity of 400 $m\,s^{-1}$ from a gun of mass 5 kg. What will be the ecoil velocity of the gun?

Solution

Mass of gun = 5 kg

Velocity of gun, V =?

Mass of bullet, m = 10 g = 0.01 kg

Velocity of bullet, v = 400 ms^{-1}

$$MV + mv = 0$$

$$V = -\frac{mv}{M} = -\frac{0.01\times400}{5} = -0.8\,ms^{-1}$$

The recoil velocity of the gun is 0.8 ms^{-1} in the direction opposite to that of the bullet.

(ii) When the bullet is fired, the gun is always held close to the shoulder. Otherwise the shoulder may get hurt due to the recoil velocity of the gun. If the gun is held close to the shoulder, the total mass that recoils is equal to the masses of the man and the gun. Consequently, the recoil velocity is very much decreased and the man's shoulder will not get hurt.

(iii) Rocket propulsion

The motion of a rocket is based on the principle of conservation of linear momentum. The rocket and fuel form an isolated system. Therefore, the total linear momentum of the system remains constant. Before the rocket is fired, the total linear momentum of the system is zero because the rocket is at rest. Therefore, the total linear momentum of the rocket and exhaust gases should remain zero after firing of the rocket. When the rocket is fired, fuel is burnt and very hot gases are formed. As the hot gases gain linear momentum to the rear on leaving the rocket, the rocket acquires equal linear momentum in the forward (i.e. opposite) direction because linear momentum is conserved.

Collisions

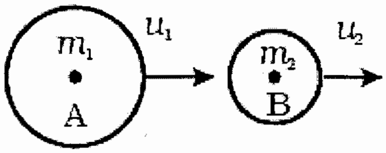
A collision between two particles is said to occur if they physically strike against each other or if the path of the motion one is influenced by the other.

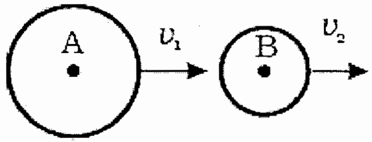
Collisions are divided into two types i.e. (i) elastic collision (ii) inelastic collision.

Elastic collision

In elastic collision, the linear momentum and kinetic energy of the system are conserved. The collision between subatomic particles is generally elastic. The collision between two steel or glass balls is nearly elastic.

Consider two bodies A and B of masses m_1 and m_2 moving along the same straight in the same direction with velocities u_1 and u_2 respectively as shown below.





Let us assume that u_1 is greater than u_2. The bodies A and B suffer a head on collision when they strike and continue to move along the same straight line with velocities v_1 and v_2 respectively.

From the law of conservation of linear momentum,

Total momentum before collision

= Total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{..............(i)}$$

Since the kinetic energy of the bodies is also conserved during the collision,

Total kinetic energy before collision

= Total kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$\frac{m_1}{m_2} = \frac{v_2^2 - u_2^2}{u_1^2 - v_1^2} \quad \text{.....................(ii)}$$

From equation (i)

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{............*}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{v_2 - u_2}{u_1 - v_1}$$

Substituting for $\frac{m_1}{m_2}$ in equation (ii)

$$\frac{v_2 - u_2}{v_1 - u_1} = \frac{(v_2 - u_2)(v_2 + u_2)}{(u_1 - v_1)(u_1 + v_1)}$$

Thus $u_1 + v_1 = u_2 + v_2$

$$u_1 - u_2 = v_2 - v_1 \quad \text{..................(iii)}$$

Equation (iii) shows than in an elastic collision, the relative velocity with which two bodies approach each other before collision is equal to the relative velocity with which they recede from each other after collision.

From equation (iii) $v_2 = u_1 - u_2 + v_1$

Substituting for v_2 in the equation * gives

$$m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$$

$$m_1u_1 - m_1v_1 = m_2u_1 - 2m_2u_2 + m_2v_1$$

$$(m_1 + m_2)v_1 = m_1u_1 - m_2u_1 + 2m_2u_2$$

$$(m_1 + m_2)v_1 = u_1(m_1 - m_2) + 2m_2u_2$$

$$v_1 = \frac{2m_2u_2}{(m_1 + m_2)} + \frac{u_1(m_1 - m_2)}{(m_1 + m_2)}$$

Similarly;

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{u_2(m_2 - m_1)}{(m_1 + m_2)}$$

Special cases:

Case (i): If the masses of colliding bodies are equal, i.e. $m_1 = m_2$ then $v_1 = u_2$ and $v_2 = u_1$

After head on collision, the velocities of the colliding bodies are mutually interchanged.

Case (ii): If the particle B is initially at rest, i.e. $u_2 = 0$ then

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

Inelastic collision

During a collision between two bodies, if there is a loss of kinetic energy, then the collision is said to be an inelastic collision.

Since there is always some loss of kinetic energy in any collision, collisions are generally inelastic. In an inelastic collision, the linear momentum is conserved but the energy is not conserved. If two bodies stick together after colliding, the collision is perfectly inelastic.

Consider a situation in which two bodies of masses m_A and m_B collide. Let their velocities be u_A *and* u_B respectively. After collision, both bodies stick together and move with common velocity, v.

By law of conservation of momentum;

$$m_Au_A + m_Bu_B = (m_A + m_B)v$$

$$v = \frac{m_Au_A + m_Bu_B}{m_A + m_B}$$

Thus, knowing the masses of the two bodies and their velocities before collision, the common velocity of the system after collision can be calculated.

If the second particle is at rest, i.e. $u_B = 0$,then

$$v = \frac{m_Au_A}{m_A + m_B}$$

Kinetic energy of the system before collision,

$$E_1 = \frac{1}{2}m_Au_A^2$$

Kinetic energy of the system after collision,

$$E_1 = \frac{1}{2}(m_A + m_B)v^2$$

$$\frac{E_2}{E_1} = \frac{\text{kinetic energy after collision}}{\text{kinetic energy before collision}}$$

$$\frac{E_2}{E_1} = \frac{(m_A + m_B)v^2}{m_Au_A^2}$$

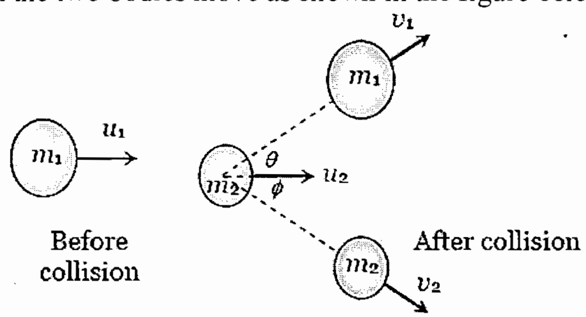
Substituting for v gives: $\frac{E_2}{E_1} = \frac{m_A + m_B}{m_Au_A^2} \times \frac{m_A^2u_A^2}{(m_A + m_B)^2}$

$$\frac{E_2}{E_1} = \frac{m_A}{m_A + m_B} \Rightarrow \frac{E_2}{E_1} < 1$$

It's is clear from the above equation that in a perfectly inelastic collision, the kinetic energy after collision is less than the kinetic energy before impact. The loss in kinetic energy may appear as heat energy or sound energy.

Perfectly elastic oblique collision

Let the two bodies move as shown in the figure below



By the law of conservation of momentum;

Along the x-axis;

$$m_1u_1 + m_2u_2 = m_1v_1 \cos\theta + m_2v_2 \cos\phi \quad \ldots \text{(i)}$$

Along the y-axis;

$$0 = m_1v_1 \sin\theta - m_2v_2 \sin\phi \quad \ldots\ldots\ldots \text{(ii)}$$

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \ldots\ldots \text{(iii)}$$

Special condition

If $m_1 = m_2$ and $u_2 = 0$; substituting these values in (i), (ii) and (iii) we get

$$u_1 = v_1 \cos\theta + v_2 \cos\phi \quad \ldots\ldots\ldots \text{(iv)}$$

$$0 = v_1 \sin\theta - v_2 \sin\phi \quad \ldots\ldots\ldots \text{(v)}$$

$$u_1^2 = v_1^2 + v_2^2 \quad \ldots\ldots\ldots \text{(vi)}$$

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2 \cos(\theta + \phi) \quad \ldots\ldots \text{(vii)}$$

Using (vi) and (vii) we get

$$\cos(\theta + \phi) = 0$$

$$\theta + \phi = 90°$$

Therefore, after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90°

Example 14

Two billiard balls each of mass $0.05\ kg$ moving in opposite directions with speed $6\ m\ s^{-1}$ collide and rebound with the same speed. What is the impulse imparted on each ball due to the other?

Solution

Let the two billiard balls be A and B.

Initial linear momentum of A

$$= 0.05 \times 6 = 0.3 \text{ kg ms}^{-1}$$

Final linear momentum of A

$$= -0.05 \times 6 = -0.3 \text{ kg ms}^{-1}$$

Impulse received by A = change in momentum

$$= 0.3 - (-0.3)$$

$$= 0.6 \text{ kg ms}^{-1}$$

The body B will also receive an equal impulse.

Example 15

A hunter has a machine gun that can fire 50 g bullets with a velocity of $150\ m\ s^{-1}$. A 60 kg tiger springs at him with a velocity of $10\ m\ s^{-1}$. How many bullets must the hunter fire into the tiger to stop him in his track?

Solution

Let n bullets be fired per second at the tiger to stop him in his track. Then,

Linear momentum of bullets = Linear momentum of tiger

$$n(50 \times 10^{-3}) \times 150 = 60 \times 10$$

$$n = \frac{600}{50 \times 10^{-3} \times 150} = 80$$

Example 16

A car of mass $400\ kg$ and travelling at $72\ m\ s^{-1}$ crashes into a truck of mass $4000\ kg$ and travelling at $9\ m\ s^{-1}$, in the same direction. The car bounces back at a speed of $18\ m\ s^{-1}$. Calculate the speed of the truck after impact.

Solution

By the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

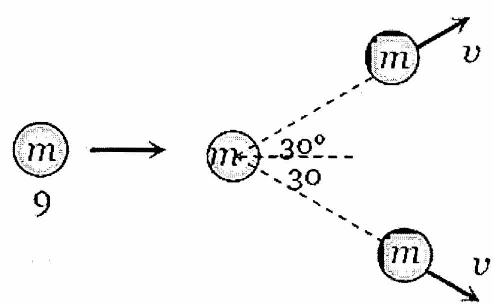
$$400 \times 72 + 4000 \times 9 = 400 \times (-18) + 4000v_2$$

$$v_2 = 18\ m\ s^{-1}$$

Example 17

A ball moving with a velocity of $9\ m\ s^{-1}$ collides with another similar stationary ball. After the collision both the balls move in directions at an angle of 30° with the initial direction. Calculate their speed after collision

Solution



Initial horizontal momentum of the system

$$= 9m$$

Final horizontal momentum of the system

$= 2\,mv\cos 30°$

According to the law of conservation of momentum,

$$9m = 2\,mv\cos 30°$$
$$v = 5.2\ m\,s^{-1}$$

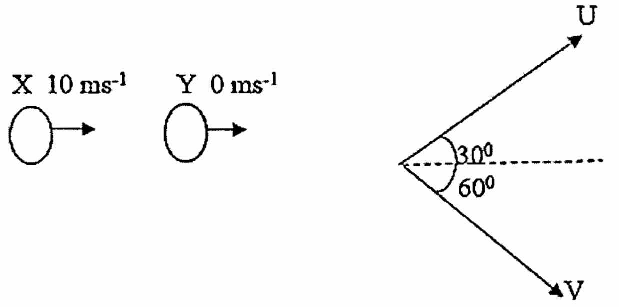
Example 18

An object X of mass M, moving with a velocity of 10 ms^{-1} collides with a stationary object Y of equal mass. After collision, X moves with a speed U, at an angle of 30° to its initial direction, while Y moves with a speed V at an angle of 90° to the new direction.

(a) Calculate the speeds U and V

(b) Determine whether the collision is elastic or not

Solution



(a) Applying the law of conservation of horizontal momentum;

$$M \times 10 = MU\cos 30° + MV\cos 60°$$
$$10 = U\frac{\sqrt{3}}{2} + \frac{V}{2}$$
$$\Rightarrow 20 = U\sqrt{3} + V \ \ldots\ldots\ldots\ldots\ (i)$$

Applying the law of conservation of vertical momentum;

$$M \times 0 = MU\sin 30° - MV\sin 60°$$
$$0 = \frac{U}{2} - V\frac{\sqrt{3}}{2}$$
$$\Rightarrow U = V\sqrt{3} \ \ldots\ldots\ldots\ldots\ (ii)$$

Substituting for U in equation (i)

$$\Rightarrow 20 = 3V + V$$
$$\therefore V = 5\ ms^{-1}$$

From equation (ii) $U = 5 \times \sqrt{3} = 8.66\ ms^{-1}$

(b) Total kinetic energy before collision

$$= \frac{1}{2} \times M \times 10^2 = 50M\ J$$

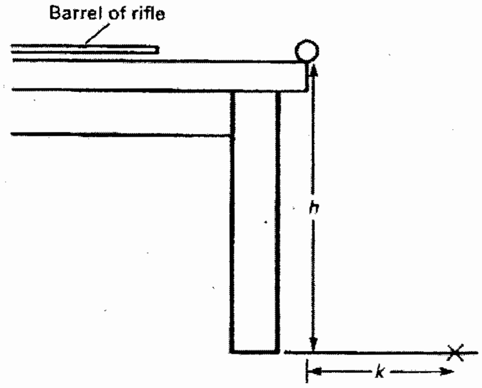
Total kinetic energy after collision

$$= \frac{1}{2}M \times 5^2 + \frac{1}{2}M\left(5\sqrt{3}\right)^2 = \frac{25M}{2} + \frac{75M}{2} = 50M\ J$$

Kinetic energy is conserved hence it is an elastic collision

Example 19

A student devises the following experiment to determine the velocity of a pellet from an air rifle.



A piece of plasticine of mass M is balanced on the edge of the table such that it just fails to fall off. A pellet of mass m is fired horizontally into the plasticine and remains embedded in it. As a result, the plasticine reaches the floor a horizontal distance k away. The height of the table is h.

(a) Show that the horizontal velocity of the plasticine with pellet embedded is $k\left(\frac{g}{2h}\right)^{1/2}$

(b) Obtain an expression for the velocity of the pellet just before impact

Solution

(a) Let the horizontal velocity of the pellet and plasticine be v

From $s = ut + \frac{1}{2}at^2$

Considering horizontal motion;

$$k = vt \ \ldots\ldots\ (i)$$

Considering vertical motion;

$$h = \frac{1}{2}gt^2 \ \ldots\ldots\ (ii)$$

From (i); $t = \frac{k}{v}$

Substituting for t in (ii);

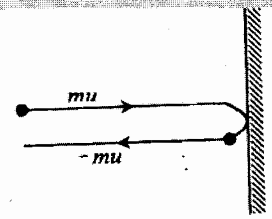
$$h = \frac{1}{2}g\left(\frac{k}{v}\right)^2$$
$$\frac{2h}{g} = \frac{k^2}{v^2}$$
$$v^2 = \frac{k^2 g}{2h}$$
$$v = k\left(\frac{g}{2h}\right)^{1/2}$$

(b) Let the velocity of the pellet before the collision be u

From the principle of conservation of linear momentum,

$$mu = (m+M)v$$
$$u = \frac{(m+M)v}{m}$$
$$u = \left(\frac{m+M}{m}\right)k\left(\frac{g}{2h}\right)^{1/2}$$
$$u = k\left(\frac{m+M}{m}\right)\sqrt{\frac{g}{2h}}$$

ce on a surface



hen a particle of mass m is traveling horizontally at peed, u strikes a wall, it rebounds with an equal but posite momentum.

$$\text{Change in momentum} = mu - (-mu) = 2mu$$

$$\text{But impulse} = \text{change in momentum}$$

$$\Rightarrow Ft = 2mu$$

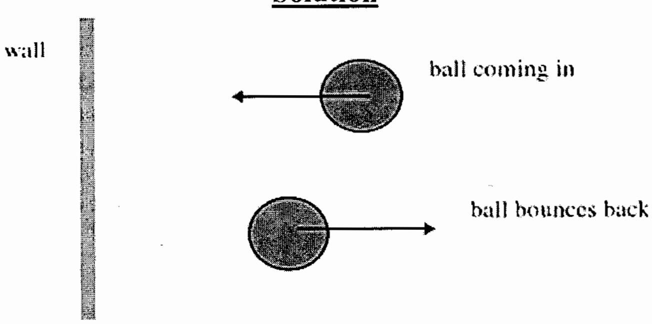
$$\therefore F = \frac{2mu}{t}$$

where t is the time for which the body is in contact with the surface

Example 20

A $0.10\,kg$ ball moving at $5\,m\,s^{-1}$ bounces off a vertical wall without change in its speed. If the collision with the wall lasted for $0.1\,s$, what was the force exerted on the wall

Solution



Initial momentum of ball $= -0.1 \times 5 = 0.5\,kg\,\text{ms}^{-1}$

Final momentum of the ball $= 0.1 \times 5 = 0.5\,kg\,\text{ms}^{-1}$

$$\text{Force on wall} = \frac{\text{momentum change}}{\text{time}}$$

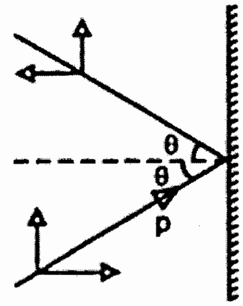
$$= \frac{0.5 - (-0.5)}{0.1} = 10\,N$$

This is also the force exerted by the wall on the ball according to Newton's third law

Example 21

A jet of water with a cross sectional area $6\,cm^2$ strikes a wall at angle of $60°$ to the normal and rebounds elastically from the wall without change in velocity. Find the force acting on the wall if the velocity of water in the jet is $12\,m\,s^{-1}$

Solution



Mass of water flowing per second $= A\rho v$

Momentum of water per second, $p = A\rho v^2$

Incident normal momentum per second $= A\rho v^2 \cos\theta$

Reflected normal momentum per second

$$= -A\rho v^2 \cos\theta$$

Momentum change per second

$$= A\rho v^2 \cos\theta - (-A\rho v^2 \cos\theta) = 2A\rho v^2 \cos\theta$$

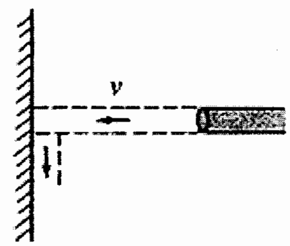
Force = momentum change per second

$$= 2 \times 6 \times 10^{-4} \times 1000 \times \cos 60$$

$$= 86.4\,N$$

Pressure exerted by a jet of liquid

When a jet of liquid hits a wall, a pressure is exerted on the wall. The figure below shows a liquid of density ρ issued from a hose and hits a vertical wall normally



Suppose that v is the velocity of the liquid and assuming that the liquid flows down along the wall after hitting it, the force on the wall is given by

$$F = \text{rate of change of momentum}$$

$$F = \frac{d}{dt}(mv) = v\frac{dm}{dt} \quad (v = \text{constant})$$

$\frac{dm}{dt}$ = mass of liquid that hits the wall per second

$$\text{Mass} = \text{density} \times \text{volume} = \rho Ah$$

$$\frac{dm}{dt} = \frac{\rho Ah}{t} = \rho A\left(\frac{h}{t}\right) = \rho Av$$

where A = cross sectional area of jet of liquid

$$\text{Therefore, } F = v\frac{dm}{dt}$$

$$= v(\rho Av) = A\rho v^2$$

$$\text{Pressure on the wall, } P = \frac{F}{A} = \rho v^2$$

Example 22

The outboard motor of a small boat has a propeller which sends back a column of water of cross-sectional area $0.03\,m^2$ at a speed of $8.0\,ms^{-1}$. Assuming the boat is released at rest, calculate

(i) the rate (in kgs^{-1}) at which water is propelled backwards

(ii) the rate of change of momentum of the water assuming that it was originally at rest

(iii) the force exerted by the motor on the boat

Assume density of water $= 1.0 \times 10^3\ kgm^{-3}$

Solution

(a) volume of water sent back per second = area of cross-section × speed

$$= 0.03 \times 8.0 = 0.24\ m^3s^{-1}$$

Mass of water sent back per second = volume per second × density

$$= 0.24 \times 1.0 \times 10^3 = 0.24 \times 10^3\ kgs^{-1}$$

(b) Rate of change of momentum $= \frac{m(v-u)}{t}$

$$= \frac{m}{t}(v-u)$$
$$= 0.24 \times 10^3 \times (8.0 - 0)$$
$$= 1920\ kg\,ms^{-2}$$

(c) Force = rate of change of momentum

$$= 1920\ N$$

The force exerted by the motor, via the propeller, on the boat arises as a reaction to the force needed to change the momentum of the water.

Example 23

A jet of water travelling at a velocity of $15\ ms^{-1}$ hits a wall normally. Find the pressure on the wall if the water doesn't rebound. (water density = $1000\ kgm^{-3}$)

Solution

Let A = cross sectional area of jet of water

$$\text{Force } F = \frac{d}{dt}(mv) = v\frac{dm}{dt} = v(A\rho v)$$

$$\therefore \text{Pressure } P = \frac{F}{A} = \rho v^2$$
$$= 1000 \times 15^2 = 2.25 \times 10^5 Nm^{-2}$$

Example 24

In a typical rainstorm, 40 mm of rain fell in one hour. Assuming that the drops struck on adequately drained roof normally with an average speed of $10\ ms^{-1}$, find the pressure exerted on the roof by the rain.

Solution

Let A = surface area of roof

Mass of water falling on the roof per hour

$$= (40 \times 10^{-3})A\rho$$

Rate of water falling on the roof

$$\frac{dm}{dt} = \frac{(40\times10^{-3})A\rho}{60\times60}$$

$$\text{Force on roof, } F = \frac{d}{dt}(mv) = v\frac{dm}{dt}$$

$$\therefore \text{Pressure, } P = \frac{F}{A} = \frac{10(40\times10^{-3})A\times(1\times10^3)}{60\times60\times A} = 0.11\ Pa$$

Self-Evaluation exercise

1. A train of mass $1.4 \times 10^5\ kg$ accelerates uniformly from rest along a level track. It travels $100\ m$ in the first $26\ s$. Calculate
   (a) the acceleration of the train
   (b) the speed reached after $26\ s$
   (c) the resultant force required to produce this acceleration
   (d) the average power required

[Ans: (a) $0.296\ m\,s^{-2}$ (b) $7.69\ m\,s^{-1}$ (c) $41.4\ kN$ (d) $159\ kW$]

2. A stationary ball of mass $6.0 \times 10^{-2}\ kg$ is hit horizontally with a tennis racket for $30\ ms$ and leaves the racket with a speed of $27\ m\,s^{-1}$.
   (a) Calculate
      (i) the change in momentum of the ball
      (ii) the average force which the racket exerts on the ball
   (b) Calculate the horizontal distance travelled by the ball before it hits the ground, if it leaves the racket at a vertical height of $2.5\ m$
   (c) (i) Explain what is meant by an inelastic collision
      (ii) Suggest a reason why the collision between the ball and the racket is inelastic

   [Ans: (a)(i) $1.62\ kg\,ms^{-1}$ (ii) $54\ N$ (b) $19.3\ m$]

3. (a) An empty railway truck of mass 10 tonnes is travelling horizontally at a speed of $0.50\ m\,s^{-1}$. Calculate its
      (i) momentum
      (ii) kinetic energy
   (b) Sand falls vertically into the truck at a constant rate of $40\ kg\,s^{-1}$. Calculate the additional horizontal force which must be applied to the truck if it is to maintain a steady speed of $0.50\ m\,s^{-1}$

   [Ans: (a) (i) $5 \times 10^3\ kg\,ms^{-1}$ (ii). $25\ kJ$ (b)$20N$]

4. A moving ball of mass M and speed v collides head-on with a stationary ball of different mass. After the collision, the first ball is stationary and 10% of the kinetic energy is lost. Show that the mass of the second ball is $\frac{10M}{9}$.

5. A stationary atomic nucleus disintegrates into an α-particle of mass 4 units and a daughter nucleus of mass 234 units. Calculate the ratio

$$\frac{K.E \text{ of } \alpha \text{ particle}}{\text{K.E of daughter nucleus}}$$

[Ans: 58.5]

6. A ball of mass 0.12 kg strikes a stationary cricket bat with a speed of 18 $m\,s^{-1}$. The ball is in contact with the bat for 0.14 s and returns along its original path with a speed of 15 $m\,s^{-1}$. Calculate
   (a) the average force acting on the ball during contact with the bat
   (b) the kinetic energy lost by the ball as the result of the collision

   [Ans: (a)28 N (b) 5.9 J]

7. (a) A bullet of mass 5.0 g takes 2.0 ms to accelerate uniformly from rest along the 0.60 m length of a rifle barrel.
   (i) Calculate the speed with which the bullet leaves the barrel
   (ii)The rifle recoils against the shoulder of the person firing it. Calculate the magnitude of the recoil force

   (b) A jet of water is directed at a vertical rigid wall with a horizontal velocity of 15 $m\,s^{-1}$. After the jet strikes the wall, the motion of the water is parallel to the wall. Calculate the magnitude of the force on the wall due to the jet. (Assume density of water = 1000 $kg\,m^{-3}$)

   [Ans: (a)(i)0.60 $km\,s^{-1}$ (ii)1.5 kN (b)0.135 kN]

8. A hose with a nozzle 80 mm in diameter ejects a horizontal stream of water at a rate of 0.044 $m^3\,s^{-1}$
   (a) With what velocity will the water leave the nozzle?
   (b) What will be the force exerted on a vertical wall situated close to the nozzle at right angles to the stream of the water, if, after hitting the wall,
   (i) the water falls of vertically to the ground,
   (ii)the water rebounds horizontally?

   [Ans: (a) 8.75 $m\,s^{-1}$ (b) (i) 385 N (ii) 770 N]

9. Sand is poured at a steady rate of 5.0 $g\,s^{-1}$ onto the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.20 m onto the pan and it does not bounce off the pan, then neglecting any motion of the pan, calculate the reading on the balance 10 s after the sand first hits the pan.

   [Ans: 0.051 kg]

10. A sphere of mass 3 kg moving with a velocity 4 $m\,s^{-1}$ collides head-on with a stationary sphere of mass 2 kg and imparts to it a velocity of 4.5 $m\,s^{-1}$. Calculate the velocity of the 3 kg sphere after the collision and the amount of energy lost by the moving bodies in the collision

    [Ans: 1 $m\,s^{-1}$; 2.25 J]

11. A railway truck of mass 40 tonnes moving with a velocity of 3 $m\,s^{-1}$ collides with another truck of mass 20 tonnes which is at rest. The couplings join and the trucks move off together.
    (a) What fraction of the first truck's initial kinetic energy remains as the kinetic energy of the two trucks after collision?
    (b) Is the energy conserved in a collision such as this? Explain your answer briefly

    [Ans: (a) 2/3]

12. A bullet of mass 0.020 kg is fired horizontally at 150 $m\,s^{-1}$ at a wooden block of mass 2.0 kg resting on a smooth horizontal plane. The bullet passes through the block and emerges undeviated with a velocity of 90 $m\,s^{-1}$. Calculate
    (a) the velocity acquired by the block
    (b) the total kinetic energy before and after penetration and account for the difference

    [Ans: (a) 0.6 $m\,s^{-1}$ (b) 225 J ; 81.4 J]

13. A body of mass m makes a head-on, perfectly elastic collision with a body of mass M, initially at rest. Show that

$$\frac{\Delta E}{E_0} = \frac{4(M/m)}{\left(1 + \frac{M}{m}\right)^2}$$

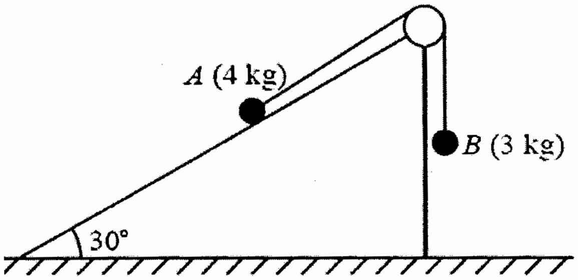
    where E_0 is the original kinetic energy of the mass m and ΔE is the kinetic energy it loses in the collision.

14. The rotary blades of a helicopter sweep out a cross sectional area A. The motion of the blades helps the helicopter to hover by giving a downward velocity, v, to a cylinder of air of density, ρ. The cylinder of air has the same cross-sectional area as that swept out by the rotor blades.
    (a) Derive an expression for the a mass of air flowing downwards per second
    (b) Derive an expression for the momentum given per second to this air
    (c) Hence show that the motion of the air results in an upward force, F on the helicopter given by

$$F = \rho A v^2$$

15. A particle A of mass 4 kg moves on the inclined face of a smooth wedge. This face is inclined at 30° to the horizontal. The wedge is fixed on horizontal ground. Particle A is connected to a particle B, of mass 3 kg, by a light inextensible string. The string passes over a small light smooth pulley which is fixed at the top of the plane. The section of the string from A to the pulley lies in a

line of greatest slope of the wedge. The particle B hangs freely below the pulley, as shown in the diagram below



The system is released from rest with the string taut. For the motion before A reaches the pulley and before B hits the ground, find the tension in the string

[Ans: 25.2 N]

16. A car which has run out of petrol is being towed by a breakdown truck along a straight horizontal road. The truck has mass 1200 kg and the car has mass 800 kg. The truck is connected to the car by a horizontal rope which is modelled as light and inextensible. The truck's engine provides a constant driving force of 2400 N. The resistances to motion of the truck and the car are modelled as constant and of magnitude 600 N and 400 N respectively. Find
    (a) the acceleration of the truck and the car,
    (b) the tension in the rope.

[Ans: (a) 0.7 $m\,s^{-2}$ (b) 960 N]

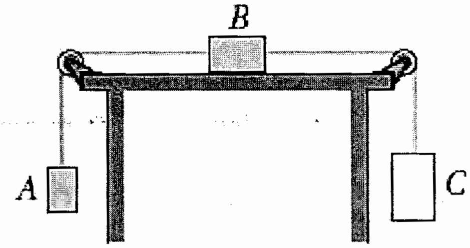
17. A batsman deflects a ball of mass 0.15 kg by an angle of 45° without changing its initial speed of 54 $km\,h^{-1}$. What is the impulse imparted to the ball.?

[Ans: 4.16 $kg\,m^{-1}$]

18. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string

[Ans: 1.962 $m\,s^{-2}$; 94.48 N]

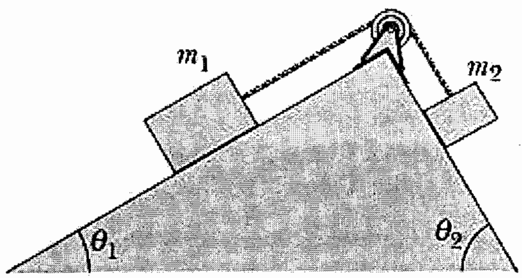
19. The figure below shows three blocks attached by cords that loop over frictionless pulleys. Block B lies on a frictionless table; the masses are m_A = 6.00 kg, m_B = 8.00 kg, and m_C = 10.0 kg.



When the blocks are released, what is the tension in the cord at the right?

[Ans: 81.7 N]

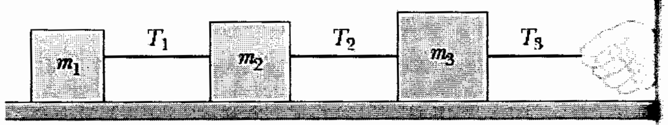
20. The figure below shows a box of mass m_1 = 3.0 kg on a frictionless plane inclined at an angle $\theta_1 = 30°$. The box is connected via a cord of negligible mass to another box of mass m_2 = 2.0 kg on a frictionless plane inclined at an angle $\theta_2 = 60°$. The pulley is frictionless and has negligible mass.



Calculate the tension in the cord

[Ans: 16 N]

21. In the figure below, three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude T_3 = 65.0 N.



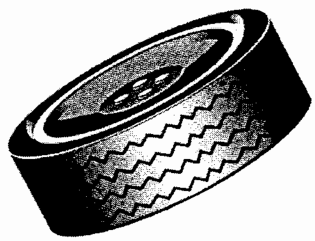
If m_1 = 12.0 kg, m_2 = 24.0 kg, and m_3 = 31.0 kg, calculate
    (a) the magnitude of the system's acceleration,
    (b) the tension T_1,
    (c) the tension T_2.

[Ans: (a)0.97 $m\,s^{-2}$ (b)11.6 N (c)34.9 N]

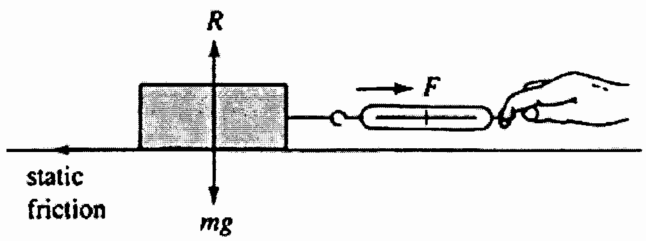
FRICTION

Friction is a force that always acts to oppose motion. In some circumstances, friction is regarded as undesirable as it causes wear and tear and energy wastages. In other circumstances, friction is necessary. For example, a car will not move if there is no friction between the wheels and the road and friction is also essential for the car to slow down.

If the road is wet, the wheels may lose their grip. Hence it is important to have sufficient gaps between the threads on a car tyre. The gaps help to drain away water from the area of contact between the tyre and the road. A film of water between the tyre and the road will be disastrous when the brakes are applied.



To study friction between two solid surfaces, a wooden block is pulled along a table using a spring balance



When the wooden block is pulled with an increasing force F, friction oppose the applied force.

Before motion occurs,

Friction = applied force, F

but in the opposite direction.

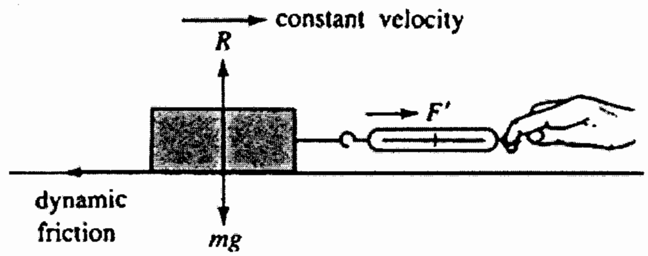
The friction that acts before motion occurs is known as **static friction**. The maximum static frictional force is known as the **limiting static friction** and is dependent on

(i) the nature of the surfaces in contact

(ii) the forces that press the surfaces together

The limiting static friction is independent of the surface area of contact.

It is often mistaken that the wider tyres produce more friction. The friction is independent of the contact area. The larger contact area reduces pressure and thus reduces wear, besides heat can be conducted away faster.



When the wooden block is pulled with a constant velocity, frictional force just balances the applied force F'. The frictional force acting when there is relative motion between the surfaces is known as **dynamic** or **kinetic friction.**

dynamic friction = F', but in opposite direction

Dynamic friction also depends on the nature of the surfaces in contact and the forces that press the two surfaces together. It is independent of the surface area of contact or the relative velocity between the two surfaces.

Laws of solid friction

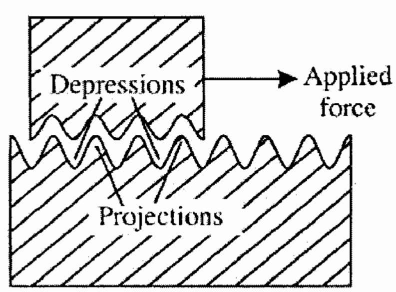
Law 1: Friction force always acts parallel to the surfaces in contact and in a direction such that it opposes their relative motion or attempted motion.

Law 2: The frictional force depends upon the nature of the surfaces in contact but it is independent of the area of contact provided the normal reaction remains constant.

Law 3: If two bodies in contact have relative motion, the sliding frictional force is directly proportional to the normal reaction and is independent of the relative velocity of the surfaces.

Explanation using molecular theory

On a microscopic level, even the most polished surface is far from plain i.e. every surface has roughness. Therefore, when two surfaces are put together, the actual area of contact is very much less than the apparent area of contact.



The pressure at the contact points is very high causing the molecules to be pushed into close proximity that the attractive forces between them weld the surfaces together at contact points.

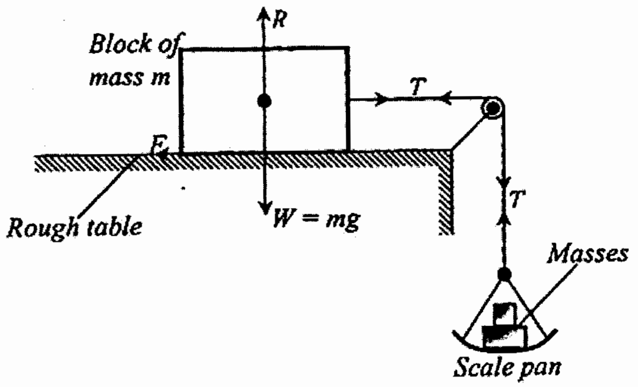
(i) The tiny welds formed have to be broken before one surface can move over the other. This creates an opposing force regardless of the direction of motion. This explains the first law.

(ii) When the apparent area of contact of the body is reduced, the number of contact points is decreased. Since the weight of the body is not changed, there is increased pressure at the contact points. As a result, contact points flatten so that that the total contact area and pressure return to their original values. This explains law 2.

(iii) An increase in weight (and hence normal reaction) of the upper surface increases the pressure at welded joints. This leads to a greater degree of interlocking and hence a bigger force is required to cause motion. This explains the third law.

Experiment to determine the coefficient of static friction

Method 1



Procedure

Known masses are added onto the scale pan in bits until the block of known mass m is just about to slide, when the maximum frictional force is reached. The total mass, M of the scale pan together with the added masses is determined.

Either:

At the point of impending motion,

Limiting frictional force, F = weight of the scale pan together with the added masses

$$\mu_s R = Mg \quad \text{but } R = mg$$

$$\therefore \mu_s = \frac{Mg}{mg}$$

$$\Rightarrow \mu_s = \frac{M}{m}$$

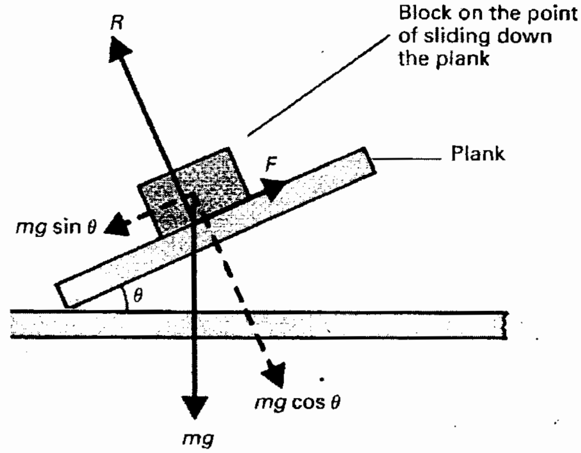
$$\mu = \frac{\text{weight of scale pan together with added masses}}{\text{weight of wooden block}}$$

Alternatively:

The process is repeated for different values of m (obtained by adding known masses to the block), and the corresponding values of M determined.

A graph M against m is plotted, and it is a straight line through the origin whose slope gives the value of the coefficient of static friction μ_s.

Method 2



A block is placed on a plank which is initially in horizontal position

One end of the plank is raised gradually until the block is on the point of sliding

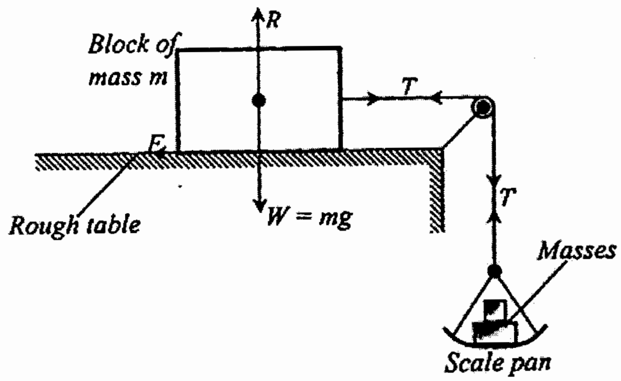
The value of θ is measured.

When the block is about to slide, $\mu = \frac{F}{R}$

Since $mg\sin\theta = F$ and $mg\cos\theta = R$

$$\mu = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

Experiment to determine the coefficient of dynamic/kinetic/sliding friction



Procedure

Known masses are added to the scale pan, and each time a mass is added, the block of mass m is given a slight push. A certain time comes when the block continues to move with a constant velocity after being pushed. The mass M of the scale pan together with its contents is determined.

Either:

For motion,

Kinetic friction, F = weight of the scale pan together with the added masses

$$\mu_k R = Mg \quad \text{but } R = mg$$

$$\therefore \ \mu_k = \frac{Mg}{mg}$$

$$\Rightarrow \mu_k = \frac{M}{m}$$

$$\mu = \frac{\text{weight of scale pan together with added masses}}{\text{weight of wooden block}}$$

Alternatively:

The process is repeated for different values of m (obtained by adding known masses to the block), and the corresponding values of M determined.

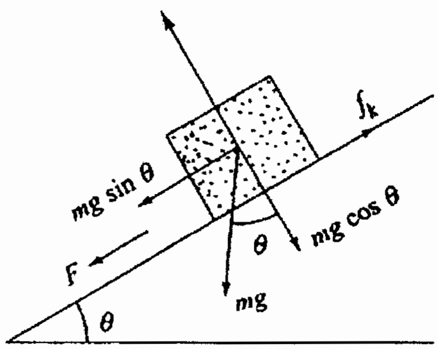
A graph M against m is plotted, and it is a straight line through the origin, whose slope gives the value of the coefficient of kinetic friction, μ_k

Angle of repose

Angle of repose is the angle of the inclined plane with the horizontal such that a body placed on it just begins to slide.

Acceleration of a body sliding down a rough inclined plane

Suppose a plane is inclined to the horizontal at angle θ greater than the angle of repose. When a block of mass m is placed on this inclined plane, it begins to slide down with an acceleration a.



The forces acting on this sliding block are

(i) weight mg of the block acting vertically downward

(ii) normal reaction R of the plane

(iii) kinetic friction f_k parallel to the plane in the upward direction

Let us revolve the weight of the block into two components i.e. along and normal to the plane

The component parallel to the plane is $mg\sin\theta$ whereas the component normal to the plane is $mg\cos\theta$.

In equilibrium, when the block is just about to slide, the normal reaction balances the normal component i.e.

$$R = mg\cos\theta \quad \ldots\ldots \text{(i)}$$

Net force on the body sliding down on the inclined plane

$$F = mg\sin\theta - f_k$$

Since a is the acceleration of the block during the downward sliding motion

$$F = ma$$

$$ma = mg\sin\theta - \mu_k R \quad \ldots\ldots \text{(ii)}$$

Substituting for R in equation (ii);

$$ma = mg\sin\theta - \mu_k\, mg\cos\theta$$

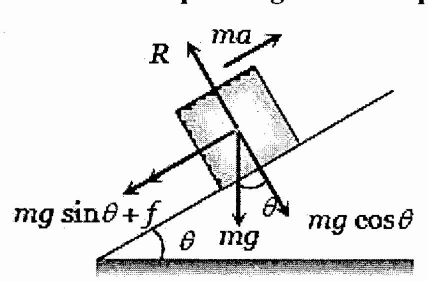
$$a = g(\sin\theta - \mu_k\cos\theta) \quad \ldots\ldots\ \text{(iii)}$$

This equation represents the acceleration of the block sliding down a rough inclined plane

Conclusions

1. From equation (iii), it is clear that $a < g$ i.e. the acceleration of the body sliding down a rough inclined plane is less than acceleration due to gravity, g
2. If the angle of inclination of the plane is small that the block does not slide down, then minimum force required to move the body up the inclined plane will be
$$F_1 = mg\sin\theta + f = mg(\sin\theta + \mu\cos\theta)$$
3. The minimum force required to push the body down the inclined plane is
$$F_2 = f - mg\sin\theta = mg(\mu\cos\theta - \sin\theta)$$
where f is the force of friction

Retardation of block up a rough inclined plane



When the angle of an inclined plane is less than the angle of repose, then for the upward motion

$$ma = mg\sin\theta + f$$

$$ma = mg\sin\theta + \mu\, mg\cos\theta$$

$$a = g(\sin\theta + \mu\cos\theta)$$

Example 1

Explain the following

(a) Kinetic friction is less than the limiting friction
(b) Tyres of the car are made wider yet friction is independent of the area of contact

Solution

(a) The maximum static friction must be overcome to cause the object to start moving. Before this stage is reached, the interlocking between the irregularities of the surfaces resist sliding. However, once sliding has begun, the surfaces do not have time to settle down on each other completely. As a result, less force is required to keep the object moving than to start its motion
(b) The friction between the tyre and the road is the same whether the tyre is wide or narrow. The wider tyre simply spreads the weight of the car over more surface area to reduce heating and wear.

Example 2

A heavy box of mass $20\ kg$ is placed on a horizontal surface. If the coefficient of kinetic friction between the box and the horizontal surface is 0.25, calculate the

(a) force of kinetic friction.
(b) acceleration produced under a force of $98\ N$ applied horizontally

Solution

$$m = 20\ kg, \mu = 0.25, f_k = ?$$
$$F = 98\ N, a = ?$$

(a) $f_k = \mu R = \mu\, mg$

$$= 0.25 \times 20 \times 9.81$$
$$= 49.2\ N$$

(b) Resultant force that produces acceleration

$$f = F - f_k = 98 - 49.2 = 48.8\ N$$
$$a = \frac{f}{m} = \frac{48.8}{20} = 2.44\ m\,s^{-2}$$

Example 3

Find the distance travelled by a body before coming to rest if it is moving with a velocity of $15\ m\,s^{-1}$ and the coefficient of friction between the ground and the body is 0.45.

Solution

Let a be the retardation of the body

$$\mu = \frac{F}{R} = \frac{ma}{mg} = \frac{a}{g}$$

Retardation, $a = \mu \times g = 0.45 \times 9.81$

$$= 4.4145\ \mathrm{m\,s^{-2}}$$

From $v^2 = u^2 + 2as$

$$0^2 = 15^2 - 2(4.4145)s$$
$$s = 25.48\ m$$

Example 4

A car moves over a horizontal road with a velocity of $72\ km\,h^{-1}$. After its engine is disengaged and brakes are applied, it stops having covered a distance of $50\ m$. Find the coefficient of friction between the tyres of the car and the road.

Solution

The brakes stop the wheels of the car but it is the frictional force between the tyres and the road that stops the car. Let the force of friction be f, then

$$f = \mu R = \mu\, mg$$

Retardation due to friction is

$$a = \frac{f}{m} = \mu\, g$$

If the distance covered before coming to rest is s, then from $v^2 = u^2 + sas$

$$0 = u^2 - 2as$$
$$u^2 = 2as = 2\mu\, gs$$
$$\mu = \frac{u^2}{2g\,s}$$
$$u = 72\ km\,h^{-1} = 72 \times \frac{1000}{3600} = 20\ m\,s^{-1}$$
$$s = 50\ m, g = 9.81\ \mathrm{m\,s^{-2}}$$
$$\mu = \frac{20^2}{2(9.81)(50)} = 0.408$$

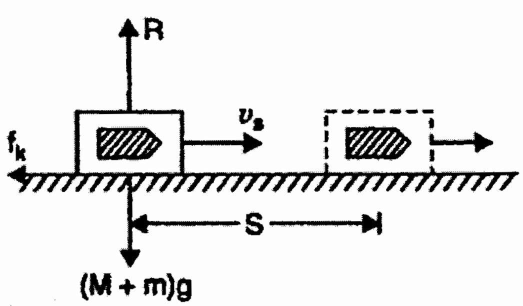
Example 5

A bullet of mass $0.01\ kg$ is fired horizontally into a $4\ kg$ wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.25. The bullet gets embedded in the block and the combination moves $20\ m$ before coming to rest. With what speed did the bullet strike the block?

Solution

Mass of bullet, $m = 0.1\ kg$

Mass of block, $M = 4\ kg$



When the bullet hits the block, the block-bullet system gets retarded due to friction between this system and the surface. If a is the retardation produced in the block-bullet system, then

$$a = \frac{F}{M+m} = \frac{\mu R}{M+m} = \frac{\mu(M+m)g}{M+m} = \mu g$$
$$a = 0.25 \times 9.81 = 2.2545\, m\, s^{-2}$$

Let V be the velocity of the block-bullet system after the bullet hits the block. The system comes to rest after covering a distance of $20\, m$

$$0^2 = V^2 + 2as$$

$a = -2.4545\, m\, s^{-2}$; $s = 20\, m$, $V = ?$

$$V^2 = 2(2.4545)(20)$$
$$V = 9.905\, m\, s^{-1}$$

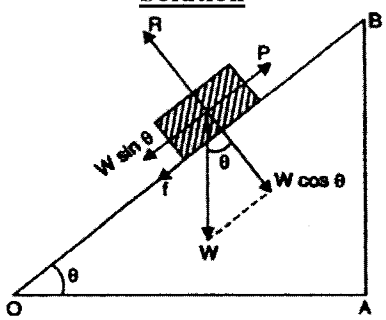
Let v be the velocity of the bullet before it gets embedded into the block. By the law of conservation of momentum

$$mv = (M+m)V$$
$$v = \frac{(M+m)V}{m} = \frac{4.01}{0.01} \times 9.905$$
$$v = 3971.91\, m\, s^{-1}$$

Example 6

When an automobile moving with a speed of $36\, km\, h^{-1}$ reaches an upward inclined road of angle $30°$, its engine is switched off. If the coefficient of friction involved is 0.1, how much distance will the automobile move before coming to rest?

Solution



Force against which work is to be done,

$$P = f + W\sin\theta$$

But $f = \mu R = \mu W\cos\theta = \mu\, mg\cos\theta$

where m is the mass of the automobile

Thus, $P = \mu\, mg\cos\theta + mg\sin\theta$

Retardation, $a = \frac{P}{m} = \frac{\mu\, mg\cos\theta + mg\sin\theta}{m}$

$$= \mu\, g\cos\theta + g\sin\theta$$
$$= (0.1 \times 9.81\cos 30° + 9.81\sin 30°)$$
$$= 5.755\, m\, s^{-2}$$

$u = 36\, km\, h^{-1} = \frac{36\times1000}{3600} = 10\, m\, s^{-1}$

$v = 0$, $a = -5.755\, m\, s^{-2}$, $s = ?$

Using $v^2 = u^2 + 2as$

$$0^2 = 10^2 - 2(5.755)s$$
$$s = 8.69\, m$$

Example 7

A body slides down along an inclined plane with a slope of 30°. Find its velocity at the end of $4\, s$ from the beginning of sliding if the coefficient of friction is 0.3.

Solution

Normal reaction, $R = mg\cos\theta$

Net force down the plane, $F = mg\sin\theta - f_k$

$$= mg\sin\theta - \mu\, mg\cos\theta$$
$$= mg(\sin\theta - \mu\cos\theta)$$
$$a = \frac{F}{m} = g(\sin\theta - \mu\cos\theta)$$
$$= 9.81(\sin 30° - 0.3\cos 30°)$$
$$= 3.4335\, m\, s^{-2}$$

$u = 0$, $t = 4$, $a = 3.4335$

From $v = u + at$

$$v = 0 + 3.4335 \times 4 = 13.734\, m\, s^{-1}$$

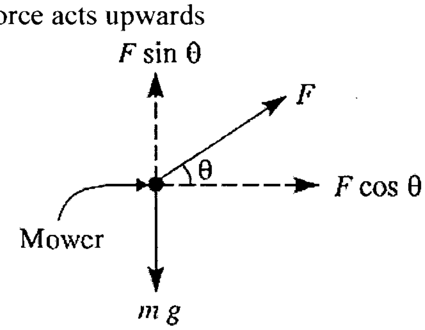
Example 8

Explain why it is easier to pull a lawn mower than to push it

Solution

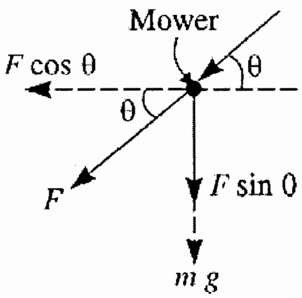
When we move a body shorter than us, (a lawn mower in this case) the force applied is not horizontal

In pushing of the lawn mower, the vertical component of the force acts upwards



Normal reaction $= mg - F\sin\theta$

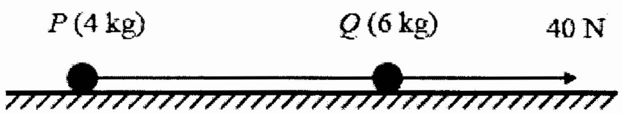
In case of pushing the lawn mower, the vertical component of the force acts downwards and adds up to the weight of the mower



Normal reaction $= mg + F\sin\theta$

Since frictional force exerted by the ground on the lawn mower is directly proportional to the normal reaction, it is easier to pull the lawn mower than to push it

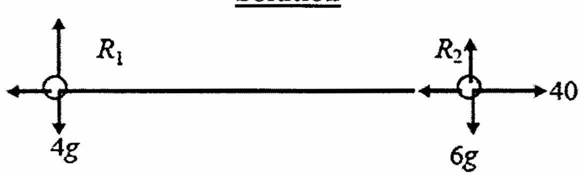
Example 9



Two particles P and Q, of mass 4 kg and 6 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. The coefficient of friction between each particle and the plane is $\frac{2}{7}$. A constant force of magnitude 40 N is then applied to Q in the direction PQ, as shown in the diagram above. Calculate the

(a) acceleration of the particles
(b) tension in the string when the system is moving.

Solution



(a) $F_1 = \mu R_1 = \frac{2}{7}\times 4g = 11.21\ N$

$F_2 = \mu R_2 = \frac{2}{7}\times 6g = 16.82$ N

Using, $F = ma$ for the whole system;

$$40 - (11.21 + 16.82) = 10a$$
$$a = 1.2\ m\,s^{-2}$$

(b) For P;

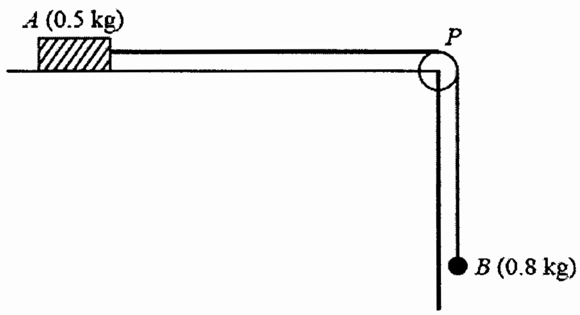
$$T - 11.21 = 4\times 1.2$$
$$T = 16\ N$$

Or For Q;

$$40 - (T + 16.82) = 6\times 1.2$$
$$T = 16\ N$$

Example 10

A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the diagram below. The coefficient of friction between A and the table is μ.



The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Calculate

(a) the acceleration of B,
(b) the tension in the string,
(c) the value of μ.

Solution

(a) $s = 0.4\ m,\ t = 0.5\ s,\ u = 0$

$$s = ut + \frac{1}{2}at^2$$
$$0.4 = 0 + \frac{1}{2}a(0.5)^2$$
$$a = 3.2\ m\,s^{-2}$$

(b) For B;

$$0.8g - T = 0.8a$$
$$T = 0.8(9.81) - 0.8(3.2)$$
$$T = 5.288\ N$$

(c) Let the frictional force be F

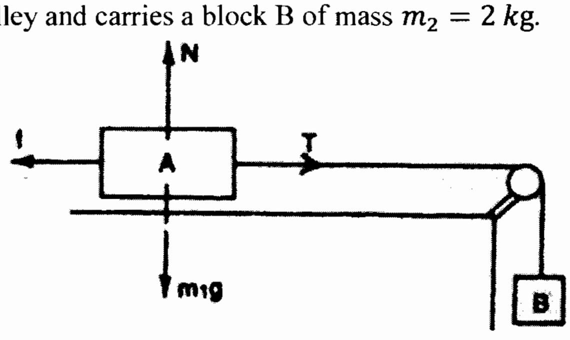
$$F = \mu R$$

For mass A;

$$T - F = 0.5a$$
$$F = T - 0.5a = 5.288 - 1.6$$
$$F = 3.688\ N$$
$$\mu = \frac{F}{R} = \frac{3.688}{0.5g} = 0.752$$

Example 11

A block A of mass $m_1 = 1\ kg$ is kept on a table. A string is connected to it which passes over a frictionless pulley and carries a block B of mass $m_2 = 2\ kg$.



If the coefficient of friction between block A and the table is 0.2, calculate

(a) the acceleration of the system when the block B is released

(b) the force of friction on block A

Solution

(a) The forces acting on the block A are; tension T, normal reaction N and the weight m_1g. Since there is motion in the vertical dircetion,

$$N = m_1 g$$

$$f = \mu N$$

$$f = \mu m_1 g \text{ .... (i)}$$

If the acceleration of the system is a, then

$$\text{For } m_1;\ T - f = m_1 a \quad \text{... (ii)}$$

$$\text{For } m_2;\ m_2 g - T = m_2 a \quad \text{... (iii)}$$

Adding (i) and (iii);

$$m_2 g - f = (m_1 + m_2)a$$

From (i);

$$m_2 g - \mu m_1 g = (m_1 + m_2)a$$

$$a = \frac{(m_2 - \mu m_1)g}{(m_1 + m_2)a}$$

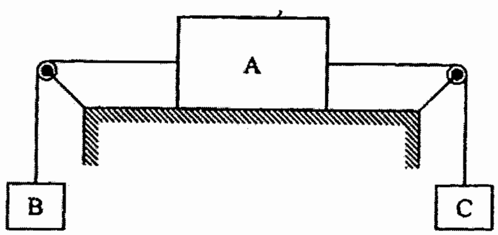
$$= \frac{(2 - 0.2(1)) \times 9.81}{2 + 1} = 5.886\, m\, s^{-2}$$

(b) As the block A is in motion, the force of friction on A is kinetic friction hence

$$f = \mu mg = 0.2 \times 1 \times 0.81 = 1.962 \text{ N}$$

Example 12

Three blocks A, B and C of masses 5 kg, 3 kg and 2 kg respectively are arranged as shown below, and the system is released from rest.

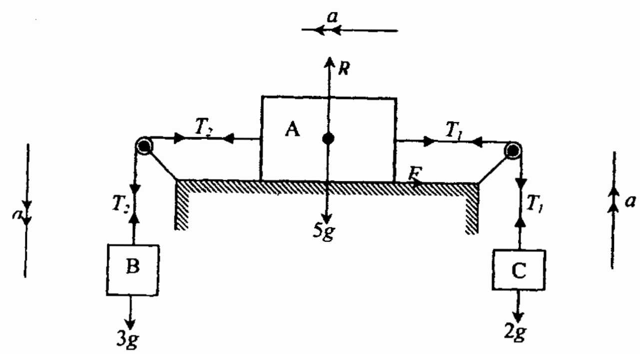


If the coefficient of sliding friction between the block A and the table is $\frac{1}{7}$, calculate;

(i) the acceleration of the system

(ii) the tensions in the strings

Solution



(i) $F = \mu_k R$ but $\mu_k = \frac{1}{7}$ and $R = 5\,g$

$$\Rightarrow F = \frac{1}{7} \times 5\,g = \frac{5}{7} g$$

Block A:

Net force $= T_2 - T_1 - F$

$$\Rightarrow 5a = T_2 - T_1 - \frac{5}{7} g \text{ ....(i)}$$

Block B:

Net force $= 3g - T_2$

$$\Rightarrow 3a = 3g - T_2 \text{ ....(ii)}$$

Block C:

Net force $= T_1 - 2g$

$$\Rightarrow 2a = T_1 - 2g \text{ ....(iii)}$$

Adding the three equations i.e. (i) + (ii) + (iii) gives;

$$5a + 3a + 2a = T_2 - T - \frac{5}{7} g + 3g - T_2 + T_1 - 2g$$

$$\Rightarrow 10a = \frac{2}{7} g$$

$$\therefore a = \frac{2}{70} g = \frac{2}{70} \times 9.81 = 0.28 \text{ ms}^{-2}$$

(ii) Substituting for a in equations (ii) and (iii) gives;

$$3 \times 0.28 = 3 \times 9.81 - T_2$$

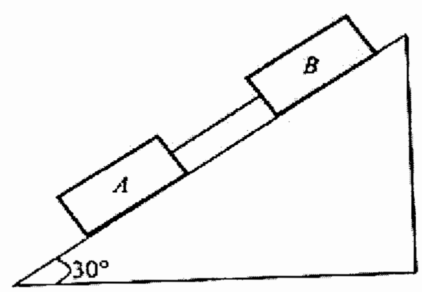
$$\Rightarrow T_2 = 28.6\,N$$

$$2 \times 0.28 = T_1 - 2 \times 9.81$$

$$\Rightarrow T_1 = 20.18\,N$$

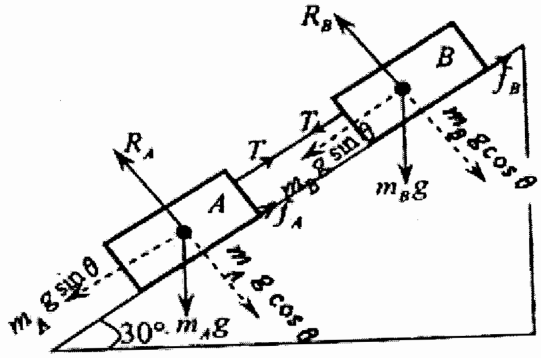
Example 13

In the figure below, blocks A and B have masses 8 kg and 16 kg respectively, and are connected by a light rigid rod. The blocks slide down the plane, and the coefficient of kinetic friction between block A and the plane is 0.25, while that between block B and the plane is 0.5.



Calculate the acceleration of each block, and the tension in the rod.

Solution



Consider block A:

$$\mu_A = 0.25, R_A = m_A g\cos 30 = 8g\frac{\sqrt{3}}{2}$$

$$f_A = \mu_A R_A = 0.25 \times 4g\sqrt{3} = 16.99 \text{ N}$$

$$\text{Net force} = 8g\sin\theta - T - f_A$$

$$\Rightarrow 8a = 8 \times 9.81 \times \sin 30 - T - 16.99$$

$$8a = 22.25 - T \ldots\ldots\ldots\text{(i)}$$

Consider block B:

$$\mu_B = 0.5, R_B = m_B g\cos 30 = 16g\frac{\sqrt{3}}{2}$$

$$f_B = \mu_B R_B = 0.5 \times 8g\sqrt{3}$$

$$= 67.97 \text{ N}$$

$$\text{Net force} = T - f_B + 16g\cos\theta$$

$$\Rightarrow 16a = T - 67.97 + 78.48$$

$$16a = T - 10.51 \ldots\ldots\ldots\text{(ii)}$$

Adding the two equations gives;

$$24a = 32.76$$

$$\therefore a = 1.36 \text{ ms}^{-2}$$

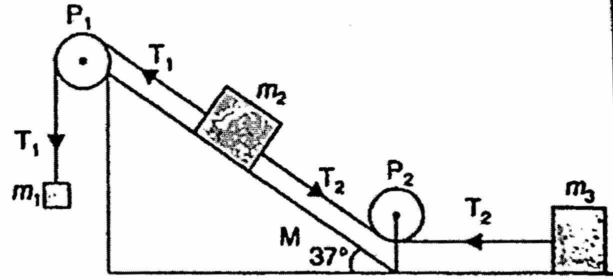
Substituting for a in equation (i) gives;

$$8 \times 1.36 + T = 22.25$$

$$\therefore T = 11.4\ N$$

Example 14

Masses m_1, m_2 and m_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown below. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and m_3 is horizontal. The masses m_2 and m_3 are 4 kg each. The coefficient of kinetic friction between the masses and the surface is 0.25. The inclined plane makes an angle of 37° with the horizontal. Mass m_1 moves downwards with uniform velocity. Calculate the value of mass m_1 and the tension in the horizontal part of the string



Solution

Let, T_1 = tension in the string between masses m_1 an m_2

T_2 = tension in the string between masses m_2 and m_3

μ = coefficient of kinetic friction between masses an the surface

Since mass m_1 moves with constant velocity, th acceleration of the system is zero

Now, For mass m_3;

$$T_2 - \mu m_3 g = 0$$

$$T_2 = \mu m_3 g$$

$$= 0.25 \times 4 \times 9.81 = 9.81\ N$$

For the mass m_2;

$$T_1 - (T_2 + \mu\, m_2 g\cos\theta) = m_2 g\sin\theta \quad \ldots\text{(i)}$$

For the mass m_1;

$$T_1 = m_1 g \ \ldots\ldots\text{(ii)}$$

Substituting for T_1 in (i);

$$m_1 g - T_2 - \mu\, mg\cos\theta = m_2 g\sin\theta$$

$$m_1 = \frac{m_2 g\sin\theta + T_2 + \mu\, mg\cos\theta}{g}$$

$$= \frac{4 \times 9.81\sin 37° + 9.81 + 0.25 \times 4 \times 9.81\cos 37°}{9.81}$$

$$= 4.21\ kg$$

Advantages and disadvantages of friction

Friction always opposes motion. It has advantages and disadvantages

Advantages

- We would not be able to walk if there had been no friction between the soles of our shoes and the ground. While walking, we push on the ground and the force of friction acts in the opposite direction. In the absence of friction, the person's shoes would slip when placed on the groud.
- The frictional forces are very beneficial for cars and other moving vehicles. Without friction between tyres and the road, the car would not stop, start or turn corners
- The brakes of a car or any moving device depend upon friction. When we apply brakes, the car is

stopped due to the force of friction between the brake lining and the drum of on the wheel

Disadvantages

- Much of the energy is wasted in overcoming frictional forces
- The nergy used to overcome the frictional forces is converted into heat. This raises the tempertaure. In almost every machinery, the generation of heat causes several problems such as bursting of car tyres
- Since some energy is converted into heat, friction lowers the efficiency of machinery
- Friction creates unnecessary noise
- It leads to wear and tear of machinery such as shoe soles, car tyres, piston rings, etc.

Methods of reducing friction

Use of lubricants: When machinery is working, many moving solid parts come in direct contact. By using a lubricant (e.g. oil or grease), a thin layer of the lubricant is formed between the solid parts. Since friction between two liquid layers is much smaller than that between solid surfaces, the use of lubricants reduces friction considerably

Polishing: Polishing means to deposit a fine layer of the material over the surface of the body. This layer covers the irregularities of the surfaces. Consequently, friction is greatly reduced

Use of ball bearings: In rotating machinery, the shafts are mounted on ball bearings to reduce friction

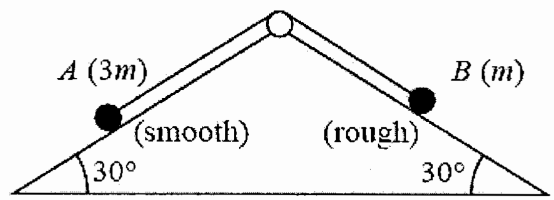
Proper selection of materials: Friction depends upon the nature of material. Usage of materials with low friction combinations such as tyres made of rubber because rubber-road forms a low friction combination

Streamlining: In order to reduce frictional forces in air, moving objects are given special shapes called streamline shapes. By streamlining, the power necessary to drive aeroplanes or automobiles is reduced to a great extent

Self-Evaluation exercise

1. A body of mass $2\,kg$ slides down, with constant velocity down an inclined plane inclined at 30° with the horizontal.
   (a) Show in a diagram the forces acting on the body and find the coefficient of kinetic friction between the body and the plane
   (b) If the plane were now tilted so as to make an angle of 30° with the horizontal, with what acceleration would the body slide down the plane
   (c) What force parallel to the plane in (b) would be required to cause the body to move up the plane with a constant velocity?

   [Ans: (a) 0.58 (b) $5.66\,m\,s^{-2}$ (c) $22.7\,N$]

2. 

A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass $3m$ and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{10}g$

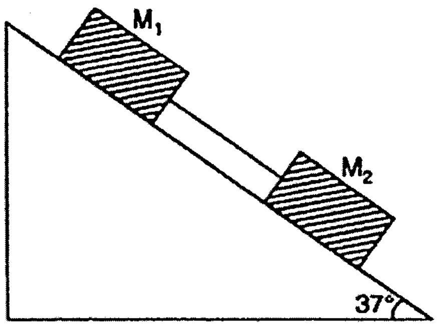
   (a) By considering the motion of A, find, in terms of m and g, the tension in the string.
   (b) By considering the motion of B, find the value of μ.

   [Ans: (a) $\frac{6}{5}mg$ (b) 0.693]

3. A block of mass 2 kg slides on an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.
   (a) What force should be applied to the block so that it moves down without any acceleration?
   (b) What force should be applied to the block so that it moves up without any acceleration?

   [Ans: (a) 4.905 N(b) 24.525 N]

4. Two blocks connected by a massless string slide down an inclined plane having an angle of inclination 37°. The masses of the two blocks are $M_1 = 4\,kg$ and $M_2 = 2$ kg and the coefficients of friction of M_1 and M_2 with the inclined plane are 0.75 and 0.25 respectively.



Assuming the string to be taut, find

(a) the common acceleration of the masses

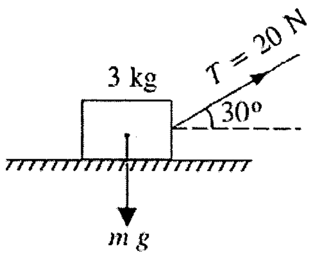
(b) the tension in the string

[Ans: (a) $1.3\ m\ s^{-2}$ (b) $5.2\ N$]

5. How long will it take for a wooden block of mass $2\ kg$ to slide from rest a distance of $3\ m$ down a rough plane inclined at 30° to the horizontal? Given $\mu = 0.20$

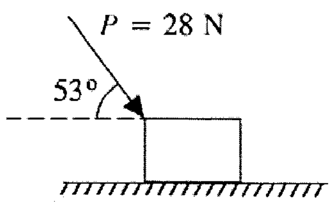
[Ans: $1.37\ s$]

6. Find the normal reaction



[Ans: $19.4\ N$]

7. Find the normal reaction if the weight of the block shown below is $47\ N$



[Ans: $69.4\ N$]

8. A 5 g bullet is fired horizontally into a 2.995 kg wooden block resting on a horizontal surface whose coefficient of friction is 0.2. The bullet remains embedded in the block which is observed to slide 25 cm along the surface before coming to rest. Find the velocity of the bullet just before collision.

[Ans: $594\ m\ s^{-1}$]

9. A body of mass $2\ kg$ initially at rest moves under the action of an applied horizontal force of $7\ N$ on a table with coefficient of kinetic friction 0.1. Calculate the

(a) work done by the net force on the body in $10\ s$

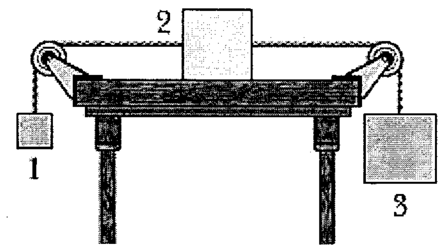
(b) change in kinetic energy of the body in $10\ s$ and interprete your results

[Ans: (a) $635.04\ J$ (b) $634.04\ J$]

10. When an automobile moving with a speed of $36\ km\ h^{-1}$ reaches an upward inclined road of an angle 30°, its engine is switched off. If the automobile moves a distance of $8.5\ m$ before coming to rest, what is the coefficient of friction between the road and the automobile?

[Ans: 0.1]

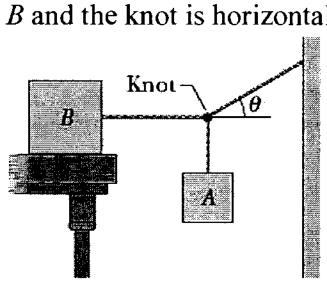
11.



When the three blocks in the figure above are released from rest, they accelerate with a magnitude of 0.500 $m\ s^{-2}$. Block 1 has mass M, block 2 has $2M$, and block 3 has $2M$. What is the coefficient of kinetic friction between block 2 and the table?

[Ans: 0.37]

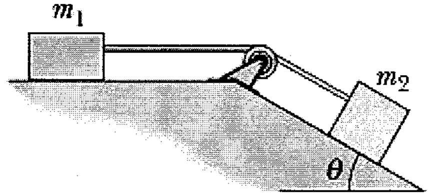
12. Block B in the figure below weighs 711 N. The coefficient of static friction between block and table is 0.25; angle θ is 30°; assume that the cord between B and the knot is horizontal.



Find the maximum weight of block A for which the system will be stationary

[Ans: $100\ N$]

13. In the figure below, block 1 of mass $m_1 = 2.0$ kg and block 2 of mass $m_3 = 3.0$ kg are connected by a string of negligible mass and are initially held in place. Block 2 is on a frictionless surface tilted at $\theta = 30°$.



The coefficient of kinetic friction between block 1 and the horizontal surface is 0.25. The pulley has negligible mass and friction. Once they are released, the blocks move. What is the tension in the string?

WORK, ENERGY AND POWER

WORK

Work is only done when the force acting on an object produces displacement in it in the direction of the component of the force. Therefore, for work to be done by a force on an object, the following two conditions must be met.

(i) There must be displacement of the object

(ii) There must be a component of force in the direction of displacement

Let us apply these conditions to some practical situations.

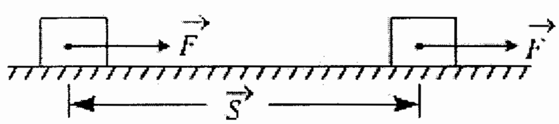
(a) Suppose you are pushing the wall very hard but the wall does not move. In this case, you are doing zero work on the wall because its displacement is zero. You may feel tired after pressing hard against the wall but the work done will be zero.

(b) Suppose a man holding a bucket of water is walking on a horizontal road. According to the definition of work, the man is doing no work. In this case, the man is applying an upward force equal to the weight of the bucket. The direction of the force he applies is perpendicular to the horizontal motion of the bucket. Hence work done by a man on the bucket is zero.

Work done by a constant force

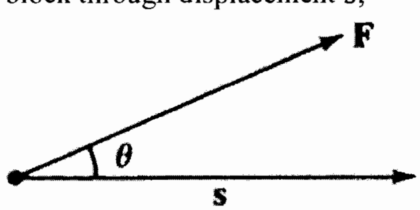
The work done on an object by a constant force is defined as the product of the component of the force in the direction of the displacement and the magnitude of the displacement.

If the force acts in the direction of displacement as shown below



then work done, $W = F \times S$

If the force is making an angle θ to the horizontal and pulls the block through displacement s,



then work done, $W = (F\cos\theta)s = Fs\cos\theta$

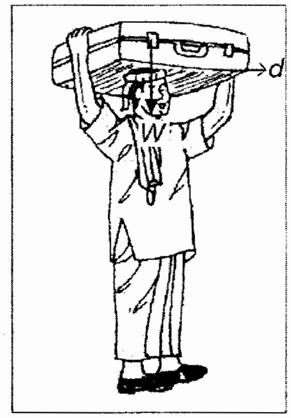
Zero work

Under three conditions, the work done becomes zero

$$W = F.s\cos\theta = 0$$

1. If the force is perpendicular to the displacement ($F \perp s$)

(a) When a porter travels on a horizontal platform with a load on his head, work done against gravity is zero

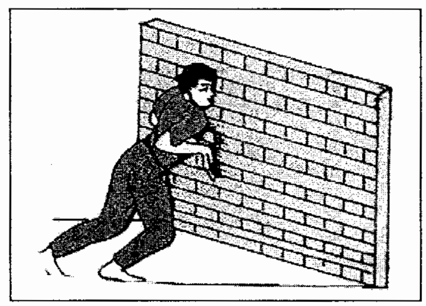


(b) When a body moves in a circle, the work done by the centripetal force is zero

(c) In case of motion of a charged particle in a magnetic field, as force is always perpendicular to the direction of motion, work done by this force is zero

2. If there is no displacement ($s = 0$)

(a) When a person tries to displace a wall or heavy stone by applying a force and it does not move, the work done is zero



(b) A weight lifter does work in lifting the weight off the ground but does not work in holding it up

3. If there is no force acting ($F = 0$)

For example, motion of an isolated body in free space.

Unit of work

The SI unit of force is 1 N and that of displacement is 1 m.

SI unit of work $= 1\,N \times 1\,m = 1\,Nm$ or Joule

Hence 1 Nm or 1 J is the amount of work done when a force of 1 N moves the body through a displacement of 1 m in the direction of the force.

Example 1

A body of mass 0.3 kg is taken up an incline of length 10 m and a height 5 m and then allowed to slide down again. The coefficient of friction between the body and the plane is 0.15. Find the

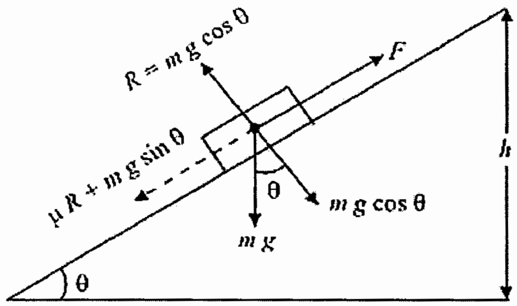
(i) work done by the gravitational force round the trip

(ii) work done by the applied force over the upward journey

(iii) work done by the frictional force over the trip

(iv) kinetic energy of body at the end of the trip

Solution



$m = 0.3\,kg$, $\mu = 0.15$, $\sin\theta = \frac{5}{10} = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

(i) Work done by gravitational force to move body up the inclined plane is

$$W_1 = F.S = (mg\sin\theta)S$$
$$= 0.3 \times 9.81 \times \frac{1}{2} \times 10 = -14.72\,J$$

Work done by gravitational force to move body down the inclined plane is

$$W_2 = F.S = (mg\sin\theta)S$$
$$= 0.3 \times 9.81 \times \frac{1}{2} \times 10 = 14.72\,J$$

Work done by gravitational force round the trip

$$= -14.72 + 14.72 = 0$$

This is expected because gravitational force is a conservative force

(ii) Frictional force $= \mu R = \mu mg\cos\theta$

For upward journey, the frictional force acts along the plane downward. Therefore, force required to move the body along the plane upwards is

$$F = mg\sin\theta + \mu mg\cos\theta$$
$$= mg(\sin\theta + \mu\cos\theta)$$
$$= 0.3 \times 9.81\left(\frac{1}{2} + \frac{0.15\sqrt{3}}{2}\right) = 1.85\,N$$

Work done by the applied force for the upward journey is

$$W_3 = F \times S = 1.85 \times 10 = 18.5\,J$$

(iii) Work done by the frictional force is negative. Therefore, work done by the frictional force round trip is

$$W_4 = -\mu mg\cos\theta\,.S - \mu mg\cos\theta\,.S$$
$$= -2\mu mg\cos\theta\,.S$$
$$= -2 \times 0.15 \times 0.3 \times 9.81 \times \frac{\sqrt{3}}{2} = -7.6\,J$$

(iv) Kinetic energy of the body at the end of the trip = Total work done

$$K.E = W_1 + W_2 + W_3 + W_4$$
$$= -14.72 + 14.72 + 18.5 - 7.6 = 10.9\,J$$

Efficiency

Any machine that does work needs energy. It uses energy of one form and converts it into useful work and other forms of energy.

Efficiency is the percentage of the useful work done compared to the input energy.

$$\text{Efficiency} = \frac{\text{Work output}}{\text{Energy used}} \times 100\%$$

The efficiency of most machines is less than 100% because some of the energy supplied is used in moving the machine parts so that useful work can be done and also to overcome friction

POWER

This is the rate of doing work

$$\text{Power, } P = \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t} = \frac{F \times s}{t} = F \times \frac{s}{t} = F.v$$

Unit of power is $J\,s^{-1}$ or W

Example 2

Oil is pumped from the tanks of a ship to a storage tank on land 45 m higher in elevation. What is the power required to pump 20,000 litres of oil per hour given that 1 litre of oil has a mass of 0.8 kg?

Solution

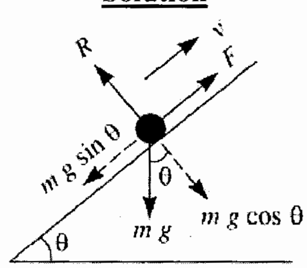
Work done $= mgh$, Power, $P = \frac{mgh}{t}$

$$m = 20{,}000 \times 0.8 = 16{,}000\,kg$$
$$h = 45\,m$$
$$t = 1\text{ hour} = 60 \times 60\,s$$
$$P = \frac{16000 \times 9.81 \times 45}{60 \times 60} = 1962\,W$$

Example 3

A man cycles up a hill whose slope is 1 in 20 at the rate of 12 km/hr. The mass of the man and bicycle is 150 kg. Find the power of the man.

Solution



Force exerted by man, $F = mg\cos\theta$

$$F = 150 \times 9.81 \times \frac{1}{20} = 73.575\ N$$

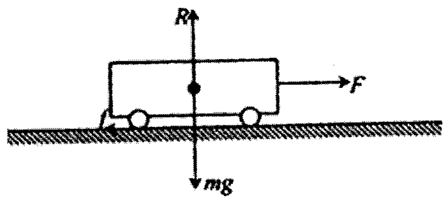
Velocity of man, $v = 12\ km/h = \frac{12\times1000}{3600} = \frac{10}{3}\ ms^{-1}$

Power, $P = F \times v = 73.575 \times \frac{10}{3} = 245.25\ W$

Example 4

A car of mass 750 kg resting on a level road is uniformly accelerated for 10 seconds, until the speed is 18 kmh^{-1}. If the resistance to motion is 5g N, calculate the power of the car 10 seconds after starting the motion.

Solution



$m = 750, v = 18\ kmh^{-1} = \frac{18\times1000}{3600}$ ms^{-1}= 5 ms^{-1}

$u = 0, t = 10$

From $v = u + at$

$5 = 0 + 10a \Rightarrow a = 0.5$ ms^{-2}

Net force = $F - f$

$\Rightarrow ma = F - f$

$750 \times 0.5 = F - 5 \times 9.81$

$\therefore F = 424.05\ N$

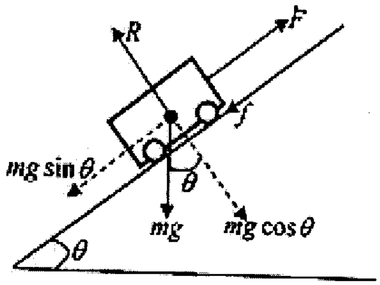
But P = Fv

$\Rightarrow P = 424.05 \times 5 = 2120.25\ W$

Example 5

A train of mass 500 tonnes has a maximum speed of 90 kmh^{-1} while moving up an inline of $\sin^{-1}\frac{1}{50}$ against frictional resistance of 100,000 N. Find the maximum power of the engine.

Solution



$\sin\theta = \frac{1}{50}, f = 100000, m = 500000$

$v_{max} = 90$ kmh^{-1} $= \frac{90\times1000}{3600} = 25$ ms^{-1}

When velocity is maximum, acceleration is zero

$\therefore$Net force = $F - f - mg\sin\theta$

$\Rightarrow 500000 \times 0 = F - f - mg\sin\theta$

$F = f + mg\sin\theta$

$= 100000 + 500000 \times 9.81 \times \frac{1}{50}$

$= 198100\ N$

$$Power = Fv$$

$\Rightarrow$ When speed is maximum, power is also maximum.

$$P_{max} = Fv_{max}$$

$$P_{max} = 198100 \times 25 = 4952500\ W$$

ENERGY

Energy of a body is defined as the ability or capacity of the body to do work.

Energy of the body is the work stored in it and may appear in various forms such as mechanical energy, heat energy, sound energy, light energy, etc.

Mechanical energy

The energy associated with the motion and position of mechanical systems is called mechanical energy. The mechanical energy is of two types i.e. kinetic energy and potential energy

Kinetic energy

This is the energy possessed by a body by virtue of its motion.

Potential energy

This is the energy possessed by a body by virtue of its position or configuration (shape or size). There are two important types of potential energy i.e. gravitational potential energy and elastic potential energy.

Gravitational potential energy: This is the energy possessed by a body by virtue of its position above the surface of the earth.

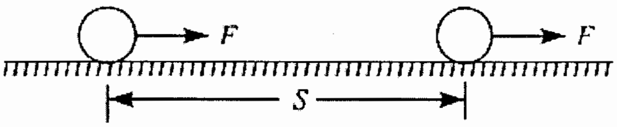
The work done against the gravitational force is stored in the form of gravitational potential energy. When a heavy brick is lifted from the ground to some height, work is done against the gravitational force. This work done is stored in the brick in the form of gravitational potential energy.

Elastic potential energy: The potential energy associated with elastic materials is called elastic potential energy. For example, when a spring is compressed or stretched, work will have to be done against the spring force. This work done is stored in the spring in the form of elastic potential energy.

Expression for kinetic energy

Method 1

Consider a body of mass m moving in a straight line with velocity v. The kinetic energy of the body must be equal to the work done in imparting it to a velocity v. Suppose initially the body is at rest, let a constant horizontal force be applied to the body till it attains the velocity v.



If S is the displacement, then work done by the force is given by

$$W = F \times S$$
$$W = maS$$

From $v^2 = u^2 + 2aS$

$$v^2 = 0 + 2aS$$
$$S = \frac{v^2}{2a}$$
$$W = ma\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$$

Since W = kinetic energy gained by the body

Kinetic energy, $K.E = \frac{1}{2}mv^2$

Method 2

Suppose a body of mass m is initially at rest. If a force F applied on the body increases its velocity from zero to v, then total work done W is given by;

$$W = \int_0^v F\, dS$$

According to Newton's second law, $F = m\frac{dv}{dt}$

$$W = \int_0^v m\frac{dv}{dt}dS = \int_0^v m\frac{dS}{dt}\,dv = \int_0^v mv\,dv$$
$$W = \left[\frac{mv^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

Kinetic energy, $K.E = \frac{1}{2}mv^2$

Work-energy theorem

It states that the work done by the net force acting on a body is equal to the change in the kinetic energy of the body.

Proof:

Method 1

Consider a body moving with a velocity u whose velocity changes to v under the action of a constant force F after moving a distance s

Work done on body, $W = F \times s = mas$

From $v^2 = u^2 + 2as$

$$as = \frac{v^2 - u^2}{2}$$
$$W = m\left(\frac{v^2 - u^2}{2}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Work done = Change in kinetic energy

Method 2

Let m = mass of the body

u = initial velocity of the body

v = final velocity of the body

F = force applied on the body in direction of motion

The small amount of work done dW by the force causing a small displacement ds in the direction of F is $dW = F.ds$

$$dW = ma\,ds = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{ds}{dt}\right)dv$$
$$dW = mv\,dv$$

The total work done by the force in increasing the velocity of the body from u to v is

$$W = \int_u^v mv\,dv = \left[\frac{1}{2}mv^2\right]_u^v$$
$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Net work done = Change in kinetic energy

Example 6

A cycle of mass 200 kg traveling at 144 kmh^{-1} on horizontal road is brought to rest in a distance of 80 by action of brakes and frictional forces. Find the

(i) average stopping force

(ii) time taken to stop the cycle

Solution

(i) $m = 200kg, u = 144\frac{km}{h} = \frac{144\times1000}{3600} = 40ms^{-1}$

$$s = 80\,m, v = 0$$

Initial kinetic energy $= \frac{1}{2}mu^2 = \frac{1}{2}\times 200\times 40^2$

$= 160000$ J

Final kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}\times 200 \times 0 = 0$

Change in K.E $= 160000 - 0 = 160000$

Work done = change in kinetic energy

$$\Rightarrow F \times s = 160000$$
$$\therefore F = \frac{160000}{80} = 2000\,N$$

(ii) From $F = ma$, $a = -\frac{F}{m}$ the negative sign means *that the car is decelerating or retarding*

$$\therefore a = -\frac{2000}{200} = -10\text{ ms}^{-2}$$

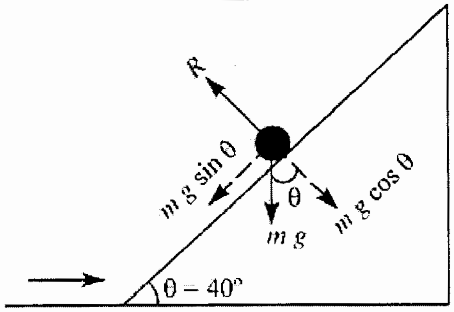
From $v = u + at$

$$0 = 40 + (-10)t$$
$$\therefore t = 4\,s$$

Example 7

An automobile moving horizontally at a speed of $54\,km\,h^{-1}$ reaches the foot of an inclined smooth plane and the engine is switched off. How much distance does the automobile go up the incline before coming to rest if the inclination of the plane is 40°.

Solution



Initial velocity of car = 54 km/h = 15 ms^{-1}

Final velocity, $v = 0$

Suppose distance covered by car before coming to rest is s Work done by opposing force

$$W = -Fs = -(mg\sin\theta)s$$

According to the work energy theorem, net work done on the car is equal to the change in kinetic energy.

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 0 - \frac{1}{2}mu^2 = -\frac{1}{2}mu^2$$

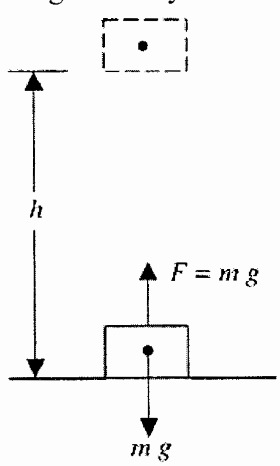
$$-Fs = -\frac{1}{2}mu^2$$

$$(mg\sin\theta)s = \frac{1}{2}mu^2$$

$$s = \frac{u^2}{2g\sin\theta} = \frac{15^2}{2\times9.81\times\sin 40°} = 17.86\ m$$

Expression for gravitational potential energy

Consider a body of mass m lying at rest on the surface of the earth as shown below. The gravitational force on the body is mg acting vertically downward.



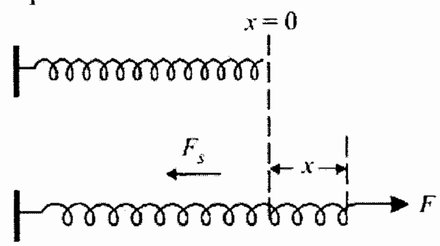
In order to lift the body upward at a constant velocity (zero acceleration), we require an upward force $F = mg$. If the body is raised through a height h, work done by the lifting force is

$$W = F \cdot s = mgh$$

Gravitational potential energy = mgh

Elastic potential energy stored in a spring

Consider a spring having one end attached to a rigid support and the other end free as shown below. If no stretching force is applied to the free end of the spring, the extension of the spring is zero i.e. $x = 0$. However, when an external stretching force F is applied to the spring, there is an extension x of the spring from its equilibrium position.



From Hooke's law, $F = kx$ where k is the spring constant

If the extension is increased by Δx where Δx is so small that F can be considered constant, then small work ΔW done by the force is

$$\Delta W = F\,\Delta x \text{ or } \Delta W = kx\,\Delta x$$

The total work done in increasing the extension from 0 to x is given by

$$W = \int_0^x kx\,dx = k\int_0^x x\,dx$$

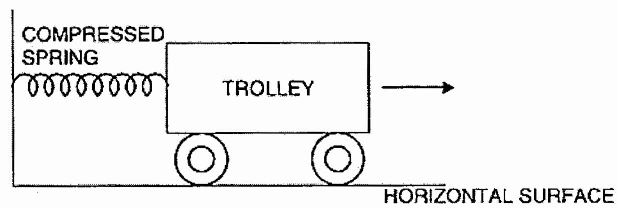
$$= k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

$$W = \frac{1}{2}kx^2$$

Elastic potential energy of spring = $\frac{1}{2}kx^2$

Example 8

A spring is kept compressed by a small trolley of mass 0.5 kg lying on a smooth horizontal surface as shown below.



When the trolley is released, it is found to move at a speed of 2 ms^{-1} . What elastic potential energy did the spring possess when compressed?

Solution

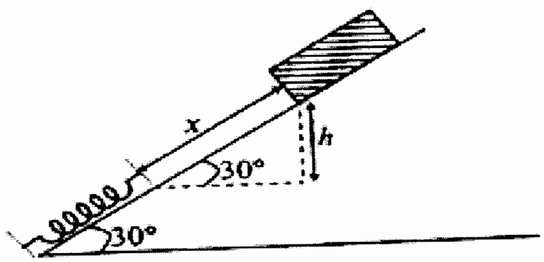
E.P.E of spring = K.E of the trolley

$$= \frac{1}{2}mv^2 = \frac{1}{2}\times 0.5\times 2^2 = 1\,J$$

Example 9

A block of mass 0.2 kg is released from rest at the top of a smooth plane inclined at 30° to the horizontal. The block compresses a spring placed at the bottom of the plane by 10 cm before it momentarily comes to rest. If the force constant of the spring is 20 Nm^{-1}, determine the distance the block has travelled down the incline before it comes to rest and its speed just before it reaches the spring.

Solution



Elastic potential energy $= \frac{1}{2}kx^2 = \frac{1}{2} \times 20 \times 0.1^2$

$= 0.1$ J

Gravitational potential energy $= mgh$

but $h = x\sin 30$

G.P.E $= mg \times \frac{1}{2}x = 0.2 \times 9.81 \times 0.5x = 0.981x$

But from the conservation of energy,

Gravitational P.E = Elastic P.E

$\Rightarrow 0.981x = 0.1$

$\therefore x = 0.1\ m$

Kinetic energy = potential energy

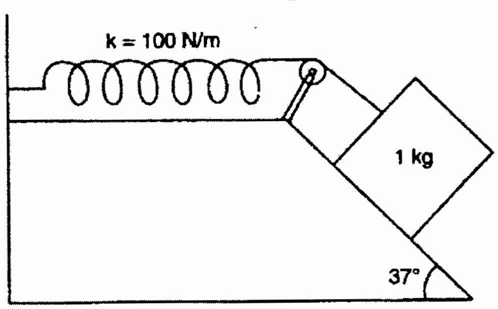
$\frac{1}{2}mv^2 = 0.1$

$\therefore \frac{1}{2} \times 0.2 \times v^2 = 0.1$

$v = 1$ ms^{-1}

Example 10

A 1 kg block situated on a rough incline is connected to a spring with spring constant $100\ N\ m^{-1}$ as shown below. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest.



Assuming that the spring has negligible mass and the pulley is frictionless, find the coefficient of friction between the block and the incline

Solution

Force, $F = mg\sin\theta - \mu\, mg\cos\theta$

$= mg\,(\sin\theta - \mu\cos\theta)$

Distance moved, $x = 10\ cm = 0.1\ m$

Work done = Potential energy of stretched spring

$$mg\,(\sin\theta - \mu\cos\theta)x = \frac{1}{2}kx^2$$

$$mg\,(\sin\theta - \mu\cos\theta) = \frac{1}{2}kx$$

$$\mu = \frac{2mg\sin\theta - kx}{mg\cos\theta}$$

$$= \frac{2 \times 9.81\sin 37° - 100 \times 0.1}{2 \times 9.81\cos 37°}$$

$$= 0.115$$

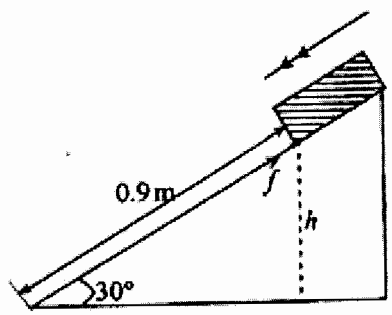
Example 11

A 12 kg block is released from rest from a rough pla inclined at 30° to the horizontal from a point 0.9 from the base. The coefficient of kinetic frictio between the block and the inclined plane is 0.25.

(i) With what speed will the block reach the botto of the incline?

(ii) If the block is projected up the incline with speed of 20 ms^{-1}, how far up the incline will th block travel?

Solution



$h = 0.9\sin 30° = 0.45\ m$

(i) Potential energy lost by the block = Kineti energy gained + work done against friction in moving down the incline

Potential energy = mgh,

Kinetic energy = $\frac{1}{2}mv^2$

Work done $= f \times d$

where v is the velocity of the bottom of the incline, f the frictional force, and d the distance travelled by the block along the plane.

$\Rightarrow mgh = \frac{1}{2}mv^2 + f \times d$ ....(i)

$f = \mu R$ where $R = m\,g\cos\theta$

$f = \mu\, m\, g\cos\theta = 0.25 \times 12 \times 9.81 \times \cos 30°$

$= 25.49\ N$

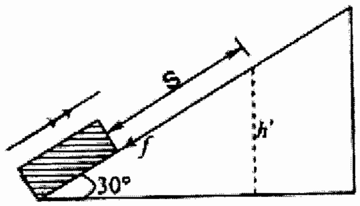
Thus, from equation (i);

$12 \times 9.81 \times 0.45 = \frac{1}{2} \times 12 \times v^2 + 0.9 \times 25.49$

$52.974 = 6v^2 + 22.941$

$\therefore v = 2.24$ ms^{-1}

(ii)



$$h' = S\sin 30° \Rightarrow h = \frac{S}{2}$$

$$u = 20, v = 0, f = 25.49\ N$$

Kinetic energy lost = Potential energy gained + work done against friction

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh' + f \times S$$

$$\frac{1}{2} \times 12 \times 20^2 = 12 \times 9.81 \times \frac{S}{2} + 25.49 \times S$$

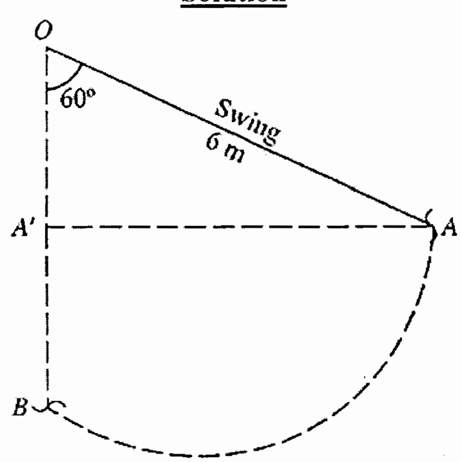
$$2400 = 58.86S + 25.49s$$

$$S = 28.45\ m$$

Example 12

A body of mass 40 kg sits in a swing by a rope 6 m long. A person pulls the swing to a side so that the rope makes an angle of 60° with the downward vertical. What is the gain in potential energy of the body?

Solution



$$A'B = OB - OA' = OA - OA\cos 60°$$

$$= OA(1 - \cos 60°)$$

$$= 6(1 - \cos 60°) = 3\ m$$

Potential energy gained by the body $= 40 \times 9.81 \times 3$

$$= 1177.2\ J$$

Pump raising and ejecting water

Consider a pump which is used to raise water from a source (well or tank) and then eject it at a given speed. The total work done is the sum of the potential energy in raising the water, and the kinetic energy given to the water. The work done per second gives the rate (power) at which the pump is working.

Work done per second

= P.E given to the water per second + K.E given to water per second

Note: Some power is wasted in terms of work done in overcoming friction as the water goes through the pipe and some is converted into sound energy.

Example 13

A pump on the ground floor of a building can pump up water to fill a tank of volume $30\ m^3$ in 15 minutes. If the tank is 40 m above the ground and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Solution

Mass of water pumped, m = volume × density

$$= 30 \times 1000 = 30{,}000\ kg$$

Output power, $P_o = \frac{\text{Work done}}{\text{time}} = \frac{mgh}{t}$

$$= \frac{30000 \times 9.81 \times 40}{15 \times 60} = 13080\ J$$

Input power $= \frac{P_o}{\eta} = \frac{13080}{0.3} = 43600\ W\ or\ 43.6\ kW$

Example 14

A pump draws 3.6 m^3 of water of density 1000 kgm^{-3} from a well 5 m below the ground in every minute, and issues it at ground level through a pipe of cross sectional area 40 cm^2. Find

(i) the speed with which water leaves the pipe,

(ii) the rate at which the pump is working

(iii) if the pump is only 80% efficient, find the rate at which it must work.

Solution

(i) Volume $= 3.6\ m^3$ and time $= 60\ s$

$\Rightarrow$volume per second $= \frac{3.6}{60}$

But volume per second = Area × velocity $= \frac{40}{10000}v$

$$\therefore \frac{40}{10000}v = \frac{3.6}{60}$$

$$v = 15\ ms^{-1}$$

(ii) Mass per second

= (volume per second) × (density)

$= \frac{3.6}{60} \times 1000 = 60$ kgs^{-1}

Kinetic energy per second $= \frac{1}{2}(mass\ per\ second)v^2$

$= \frac{1}{2} \times 60 \times 15^2 = 6750$ Js^{-1}

Potential energy per second $= (mass\ per\ second)gh$

$= 60 \times 9.81 \times 5 = 2943$ Js^{-1}

$$\text{Power} = 6750 + 2943 = 9693\ W$$

(iii)

Power output $= 9693\ W$, Power input = ?

Efficiency = 80%

$$\text{Efficiency} = \frac{\text{Power output}}{\text{power input}} \times 100$$

$$\therefore 80 = \frac{9693}{P_{in}} \times 100$$

$$\Rightarrow P_{in} = 12116.25\ W$$

Example 15

The blades of a windmill sweep out a circle of A.

(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?

(b) What is the kinetic energy of the air?

(c) Assuming that the windmill converts 25% of the winds energy into electrical energy, and that $A = 30\ m^2$, $v = 36\ km\ h^{-1}$ and the density of air is $1.2\ kg\ m^{-3}$, what is the electric power produced?

Solution

(a) Volume of wind flowing per second $= Av$

Mass of wind flowing per second $= Av\rho$

Mass of air passing in time $t = Av\rho t$

(b) Kinetic energy of air $= \frac{1}{2}mv^2$

$$= \frac{1}{2}(Av\rho t)v^2 = \frac{1}{2}Av^3\rho t$$

(c) Electrical energy produced

$$= \frac{25}{100} \times \frac{1}{2}Av^3\rho t = \frac{1}{8}Av^3\rho t$$

Electrical power $= \frac{1}{8}Av^3\rho t \div t = \frac{Av^3\rho}{8}$

$A = 30\ m^2$, $v = 10\ m\ s^{-1}$, $\rho = 1.2\ kg\ m^{-3}$

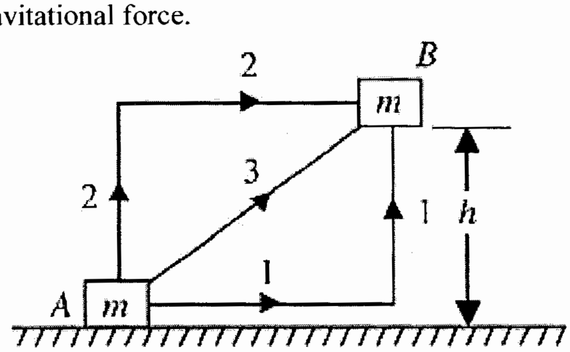
Electrical power $= \frac{30\times10^3\times1.2}{8} = 4500\ W$

Conservative and non-conservative forces

Conservative Force

A force is said to be conservative if the amount of work done in moving an object against the force is independent of how the object moves from the initial position to the final position.

One important example of conservative force is the gravitational force.



It means that the amount of work done in moving a body against gravity from location A to location B is the same whichever path you may follow in going from A to B.

Whether the body follows paths 1 or 2 or 3, the work done against gravity is mgh. The work done against gravity is path independent, it only depends upon the mass of the body and on the initial and final positions of the body.

A force is **conservative** if the total work it does on an object is zero when the object moves around any closed path.

Work done by a conservative force is recoverable i.e. we shall have to do mgh amount of work in taking the body from A to B however, when the body is released at B, we recover mgh amount of work.

Common examples of conservative forces are

(i) gravitational force

(ii) force exerted by a spring

(iii) electrostatic force between two charges

Non-conservative force

A force is non-conservative if the work done by that force on an object moving between two points depends on the path taken between the two points. Work done in moving a body in a closed path is not zero.

The work done by a non-conservative force is not recoverable. Examples include

(i) friction force

(ii) viscous drag

(iii) air resistance

Different forms of energy

Energy has many forms and may be transformed from one form to another. Energy, in all its forms is measured in the same units as work. Some of the forms of energy are given below

1. Mechanical energy: The mechanical energy of a body is the sum of the kinetic and potential energy of the body
2. Heat energy: It is the energy possessed by a body due to the kinetic energy of its molecules. We can produce heat by converting mechanical energy into it.
3. Electrical energy: The energy associated with electric current is called electrical energy. In order to move electric charges, work will have to be done. This work done appears as electrical energy.
4. Internal energy: The internal energy of a body is the sum of kinetic energy and potential energy of its molecules. The kinetic energy of the molecules results from their random motions. The potential energy of molecules results from their bonds and interactions with each other.
5. Chemical energy: A body such as a chemical compound possesses chemical energy because of the chemical bonding of its atoms. A chemical

compound has less energy than the parts of which it is made. The difference in energy is called the chemical energy.

6. Nuclear energy: The energy available from an atomic nucleus is called nuclear energy. We can obtain nuclear energy in two ways i.e. nuclear fusion and nuclear fission

Transformation of energy

In all physical processes, energy is transformed from one form to another. For example

(i) A stone held high in the air has potential energy. As it falls, it loses potential energy and at the same time it gains kinetic energy. In this case, potential energy is being transferred into kinetic energy

(ii) Water at the top of the dam has potential energy which is transformed into kinetic energy as the water falls

(iii) At the base of the dam, kinetic energy of water can be transferred to turbine blades and further transformed into electric energy.

(iv) The potential energy stored in a bent bow can be transferred into kinetic energy of an arrow.

Note: Work is done whenever energy is transferred from one object to another.

Law of conservation of energy

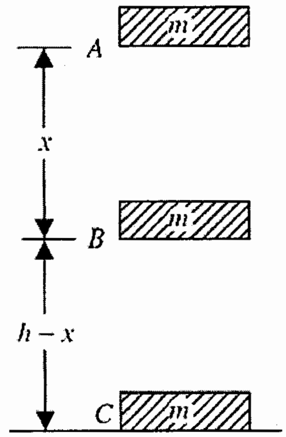
The total energy neither increases nor decreases in any process. Energy can be transformed from one form to another and transformed from one body to another but the total amount remains the same.

Law of conservation of mechanical energy

It states that if a body is under the action of a conservative force, the total mechanical energy remains constant.

Proof:

Let a body of mass m fall from a height h above the ground. We shall calculate the total mechanical energy of the body at any point during its downward journey.



At point A: The body starts its downward journey from point A, so that its initial velocity is zero.

$$K.E = 0, P.E = mgh$$

Total mechanical energy $= K.E + P.E = 0 + mgh$

$$= mgh$$

At point B: Suppose at point B, the body has a velocity v_B

$$v_B^2 = 0^2 + 2gx$$

$$v_B^2 = 2gx$$

$$\text{K.E} = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2gx) = mgx$$

$$\text{P.E} = mg(h - x)$$

Total mechanical energy $= K.E + P.E$

$$= mg(h - x) + mgx = mgh$$

At point C: Suppose at point C (ground), the body has a velocity v_C

$$v_C^2 = 0^2 + 2gh$$

$$v_C^2 = 2gh$$

$$\text{K.E} = \frac{1}{2}mv_C^2 = \frac{1}{2}m(2gh) = mgh$$

$$\text{P.E} = 0 \quad (h = 0)$$

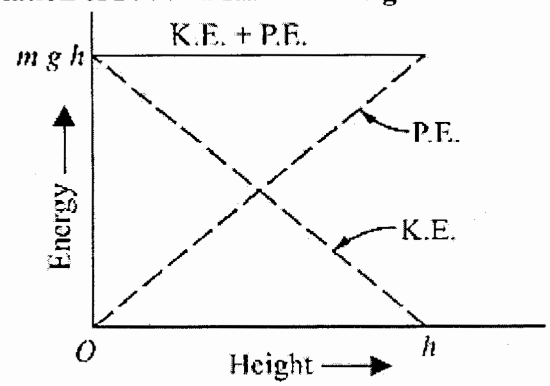
Total mechanical energy $= K.E + P.E$

$$= mgh + 0 = mgh$$

Thus the total mechanical energy of the body remains the same at all points during the downward journey.

Note: Considering a body projected from the ground with a certain velocity will yield the same result. Try it out.

Variation of P.E and K.E with height



Example 16

A body is thrown vertically upwards from the ground with a velocity of 39.2 ms^{-1}. At what height will kinetic energy be reduced to one-fourth of its original kinetic energy?

Solution

When the body is thrown up, its velocity decreases and hence potential energy increases. Let h be the height at which the potential energy is reduced to one-fourth of its initial value.

Loss in kinetic energy = Gain in potential energy

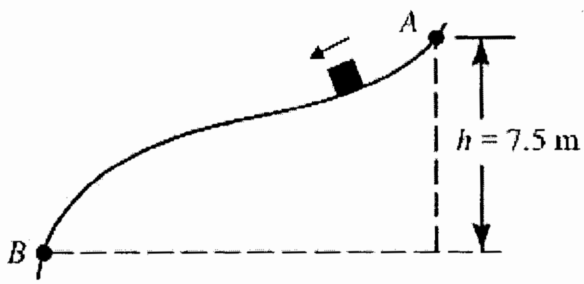
$$\frac{3}{4}\times\frac{1}{2}mv^2 = mgh$$

$$\frac{3}{4}\times\frac{1}{2}(39.2)^2 = 9.81h$$

$$h = 58.8\,m$$

Example 17

A small block is released from rest and slides down a smooth curved track as shown below. Calculate the velocity of the block when it reaches B, a vertical distance of 7.5 m below A.



Solution

Assuming there are no frictional forces, according to the energy conservation principle,

K.E gained = P.E lost

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2\times 9.81\times 7.5} = 12.12\,ms^{-1}$$

Example 18

A ball is dropped from rest at a height of 12 m. If it loses 25% of its kinetic energy on striking the ground, what is the height to which it bounces?

Solution

We shall use the energy-conservation principle to solve this problem

At a height $h = 12\,m$, P.E = mgh and K.E = 0. As the ball is dropped, its P.E starts converting into K.E.

On reaching the ground, K.E = mgh, P.E = 0.

Since the ball loses 25% of K.E on striking the ground,

Reduced energy = $0.75\,mgh$

If on striking the ground, it bounces to a height h', its potential energy is mgh'

$$mgh' = 0.75\,mgh$$

$$h' = 0.75\,h = 0.75\times 12 = 9\,m$$

Example 19

A ball falls under gravity from a height of 10 m with an initial downward velocity v_0. It collides with the ground, loses 50% of energy in collision and then raises to the same height. Find the initial velocity v_0.

Solution

Let v be the velocity of the ball when it just touches the ground.

Increase in K.E = Decrease in P.E

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh$$

$$v^2 - v_0^2 = 2gh \,\ldots\ldots \text{(i)}$$

Now, when the ball hits the ground, 50% of K.E is lost and the ball rises to same height.

Decrease in K.E = Increase in P.E

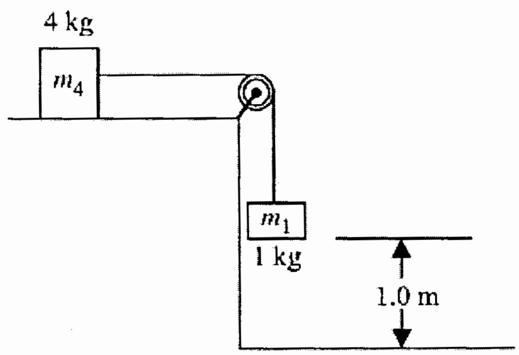
$$\frac{50}{100}\times\frac{1}{2}mv^2 = mgh$$

From (i); $4gh - v_0^2 = 2gh$

$$v_0 = \sqrt{2gh} = \sqrt{2\times 9.81\times 10} = 14\,ms^{-1}$$

Example 20

A 4 kg block (m_4) is on a smooth horizontal table. The block is connected to a second block of mass (m_1) by a massless flexible taut chord that passes over a frictionless pulley. The 1 kg block is 1 m above the floor. The two blocks are released from rest. With what speed does the 1 kg mass hit the ground?



Solution

Let us make use of conservation of mechanical energy

Initial K.E of two blocks, K.E = 0 (at rest)

Initial $P.E$ of two blocks, $P.E = m_1gh$

Let v be the speed of both m_1 and m_4 when m_1 hits the floor.

Final K.E of two blocks, $K.E = \frac{1}{2}m_1v^2 + \frac{1}{2}m_4v^2$

Final P.E of two blocks, $P.E = 0$

From the law of conservation of M.E,

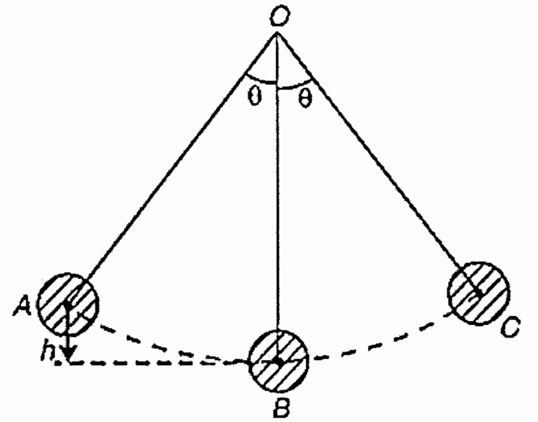
$$0 + m_1gh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_4v^2$$

$$v = \sqrt{\frac{2m_1gh}{m_1+m_4}} = \sqrt{\frac{2\times1\times9.81\times1}{1+4}} = 1.98\ ms^{-1}$$

Example 21

Draw a diagram to show the energy changes in an oscillating pendulum. Indicate on your diagram how the total mechanical energy in it remains constant during the oscillation

Solution



At extreme left position, A

$$K.E = 0,\ P.E = mgh = \text{maximum}$$

At the mean position, B

$$\text{speed} = \text{maximum},\ K.E = max,\ P.E = 0$$

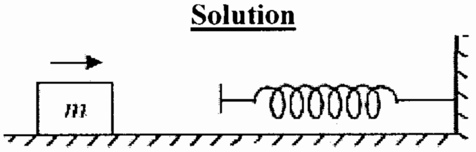
At extreme right position, C

$$K.E = 0,\ P.E = mgh = \text{maximum}$$

Example 22

A 1 kg block collides with a horizontal weightless ;pring of force constant 2 Nm^{-1}. The block presses he spring 4 m from the rest position. If the coefficient ıf friction between the block and the horizontal surface ; 0.25, what was the speed of the block at the instant f collision?

Solution



:t v be the speed of the block at the instant of llision with the spring. By law of conservation of E, initial K.E of the block is equal to the elastic ergy stored in the spring plus energy spent in doing ork against friction.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx$$

$$m = 1\ kg,\ x = 4\ m,\ k = 2\ Nm^{-1}$$

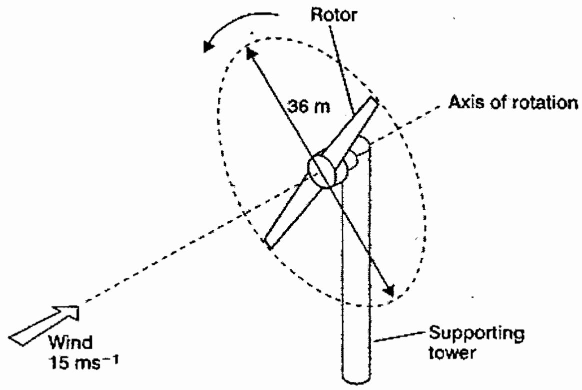
$$\frac{1}{2}\times 1\times v^2 = \frac{1}{2}\times 2\times 4^2 + 0.25\times 1\times 9.81\times 4$$

$$v = \sqrt{51.6} = 7.18\ ms^{-1}$$

Self-Evaluation exercise

1. A force of 0.35 kN is needed to move a vehicle of mass $1.50 \times 10^3\ kg$ at a constant speed along a horizontal road. Calculate the work done, against the frictional forces, in travelling a distance of 0.30 km along the road. [Ans: $1.05 \times 10^5 J$]
2. A car of mass $1.0 \times 10^3\ kg$ travelling at $20\ m\ s^{-1}$ on a horizontal road is brought to rest by the action of its brakes in a distance of 25 m.
   (a) Find the average retarding force
   (b) If the same car travels up an incline of 1 in 20 at a constant speed of $20\ m\ s^{-1}$, what power does the engine develop if the frictional resistance is 100 N
   [Ans: (a) 8.0 kN(b)12.0 kW]
3. Starting with the definition of work, deduce the change in the gravitational potential energy of a mass m, when moved a distance h against a gravitational field of strength g
4. By using the equations of motion, show that the kinetic energy E_K of an object of mass m travelling with speed v is given by
   $$E_K = \frac{1}{2}mv^2$$
5. A cyclist together with his bicycle has a total mass of 90 kg and is travelling with a constant speed of $15\ m\ s^{-1}$ on a flat raised road. He then goes down a small slope to a point on another level ground descending 4.0 m. Calculate
   (a) the initial kinetic energy
   (b) the loss in potential energy
   (c) the speed at the level ground assuming that all the lost potential energy is converted into kinetic energy of the cyclist and the bicycle
   [Ans: (i) 10125 J (b) 3600 J (c) $17.5\ m\ s^{-1}$]
6. A vehicle has a mass of 600 kg. Its engine exerts a tractive force of 1500 N, but motion is resisted by a constant frictional force of 300 N. Calculate
   (a) the acceleration of the vehicle
   (b) its momentum 10 s after starting to move
   (c) its kinetic energy 15 s after starting to move
   [Ans: (a) $2.0\ m\ s^{-2}$ (b) 1.2×10^4 kg ms^{-1} (c) $2.7 \times 10^5\ J$]
7. A car of mass $1.0 \times 10^3 kg$ travelling at $72\ km\ h^{-1}$ on a horizontal road is brought to rest in a distance of 40 m by the action of brakes and frictional force. Find
   (a) the average stopping force
   (b) the time taken to stop the car
   [Ans: (a) 5000 N (b) 4.0 s]

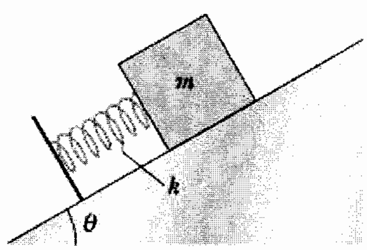
8. The diameter of the rotor of a wind turbine is 36 m. The rotor rotates about a horizontal axis as shown below



The axis points directly into a wind which is blowing at 15 $m\,s^{-1}$. Assume that the air emerges from the rotor at a mean axial speed of 13 $m\,s^{-1}$. Take the density of air to be 1.2 kg m^{-3}.
   (a) Calculate the mass of air incident in one second on the circle swept by the rotor
   (b) Calculate the kinetic energy lost by the air
   (c) What is the horizontal force exerted by the air on the rotor in a direction parallel to its axis of rotation?
   (d) The turbine converts the kinetic energy lost by the air into electrical energy with an efficiency of 40%. Calculate how many such turbines would be needed to provide the output of a conventional 500 MW power station

   [Ans: (a) $1.83 \times 10^4\,kg$ (b) $5.12 \times 10^5\,J$ (c)37 kN (d) about 2450]

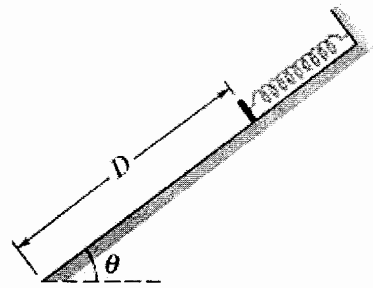
9. A block with mass m = 2.00 kg is placed against a spring on a frictionless incline with angle 30°. (The block is not attached to the spring.) The spring, with spring constant k = 19.6 N/cm, is compressed 20.0 cm and then released.



   (a) What is the elastic potential energy of the compressed spring?
   (b) What is the change in the gravitational potential energy of the block–earth system as the block moves from the release point to its highest point on the incline?
   (c) How far along the incline is the highest point from the release point?

   [Ans: (a)39.2 J (b) 39.2 J (c)4.0 m]

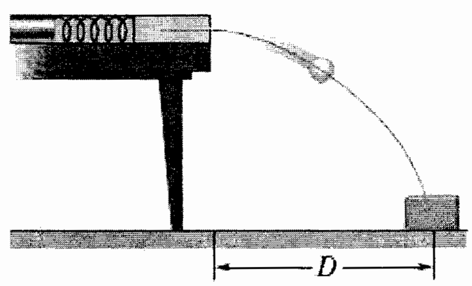
10. In the figure below, a spring with k = 170 $N\,m^{-1}$ is at the top of a frictionless incline of angle 37.0°. The lower end of the incline is distance D = 1.00 m from the end of the spring, which is at its relaxed length. A 2.00 kg canister is pushed against the spring until the spring is compressed 0.200 m and released from rest.



   (a) What is the speed of the canister at the instant the spring returns to its relaxed length (which is when the canister loses contact with the spring)?
   (b) What is the speed of the canister when it reaches the lower end of the incline?

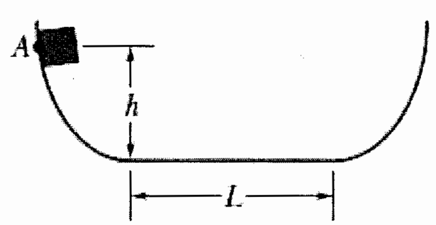
   [Ans: (a)2.40 $m\,s^{-1}$ (b) 4.19 $m\,s^{-1}$]

11. Two children are playing a game in which they try to hit a small box on the floor with a marble fired from a spring-loaded gun that is mounted on a table. The target box is horizontal distance D = 2.20 m from the edge of the table. Bobby compresses the spring 1.10 cm, but the center of the marble falls 27.0 cm short of the center of the box.



How far should Rhoda compress the spring to score a direct hit? Assume that neither the spring nor the ball encounters friction in the gun.

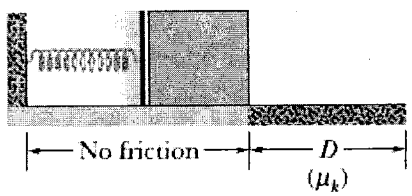
12. A particle can slide along a track with elevated ends and a flat central part, as shown in the figure below.



The flat part has length $L = 40$ cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.25$. The particle is released from rest at point A, which is at height $h = L/2$. How far from the left edge of the flat part does the particle finally stop?

[Ans: $20\ cm$]

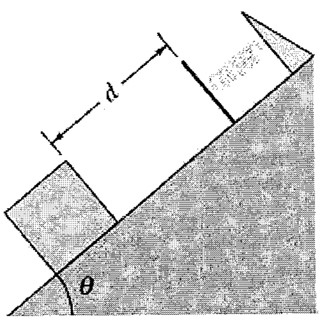
13. In the figure below, a 3.5 kg block is accelerated from rest by a compressed spring of spring constant $640\ N\ m^{-1}$. The block leaves the spring at the spring's relaxed length and then travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance $D = 7.8$ m.



What are (a) the increase in the thermal energy of the block–floor system, (b) the maximum kinetic energy of the block, and (c) the original compression distance of the spring?

[Ans: (a) $67\ J$ (b) $67\ J$ (c) $46\ cm$]

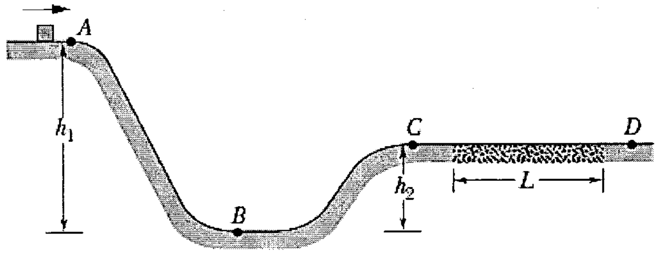
14. A spring ($k = 200$ N/m) is fixed at the top of a frictionless plane inclined at angle 40°. A 1.0 kg block is projected up the plane, from an initial position that is distance $d = 0.60\ m$ from the end of the relaxed spring, with an initial kinetic energy of 16 J.



(a) What is the kinetic energy of the block at the instant it has compressed the spring 0.20 m?
(b) With what kinetic energy must the block be projected up the plane if it is to stop momentarily when it has compressed the spring by 0.40 m?

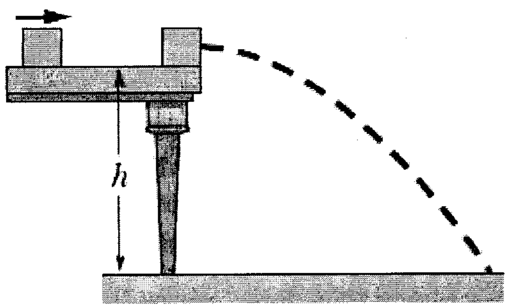
[Ans: (a) $7.0\ J$ (b) $22\ J$]

15. In the figure below, a small block is sent through point A with a speed of 7.0 m/s. Its path is without friction until it reaches the section of length $L = 12$ m, where the coefficient of kinetic friction is 0.70. The indicated heights are $h_1 = 6.0$ m and $h_2 = 2.0\ m$.



What are the speeds of the block at (a) point B and (b) point C? (c) Does the block reach point D? If so, what is its speed there; if not, how far through the section of friction does it travel?

16. In the figure below, a 3.2 kg box of running shoes slides on a horizontal frictionless table and collides with a 2.0 kg box of ballet slippers initially at rest on the edge of the table, at height $h = 0.40$ m.

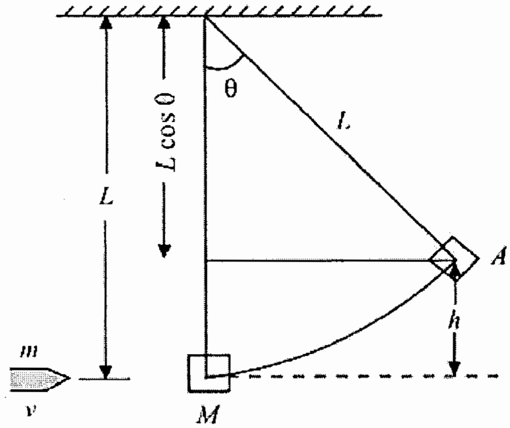


The speed of the 3.2 kg box is 3.0 m/s just before the collision. If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

[Ans: $29\ J$]

THE BALLISTIC PENDULUM

This a device used to determine the speed of a bullet. Consider a bullet of mass m moving horizontally with a velocity v. It strikes a stationary block of mass M suspended by a sting of length L. The combination moves with a velocity V after collision and comes to rest at a height h above the original position of the wooden block.



Let θ be the maximum angle made by the string.

From the law of conservation of M.E

$$mv = (m + M)V$$

$$V = \frac{mv}{m+M} \quad \text{..........(i)}$$

By law of conservation of M.E,

K.E of combined mass = P.E of combined mass at A

$$\frac{1}{2}(m + M)V^2 = (m + M)gh$$

$$V^2 = 2gh$$

But $h = L - L\cos\theta = L(1 - \cos\theta)$

$$V^2 = 2gL(1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{V^2}{2gL} \cdots \quad \text{(ii)}$$

Substituting for V in (ii);

$$\cos\theta = 1 - \frac{m^2v^2}{2gL(m+M)^2}$$

$$\theta = \cos^{-1}\left[1 - \frac{m^2v^2}{2gL(m+M)^2}\right]$$

Example 1

A 10 g bullet is fired from a riffle horizontally into a 5 kg block of wood suspended by a string and the bullet gets embedded into the block. The impact causes the block to swing to a height of 5 cm above its initial level. Calculate the initial velocity of the bullet.

Solution

Mass of the bullet, $m = 10\text{ g} = 0.01\ kg$

Mass of wooden block, $M = 5\ kg$

Initial velocity of the bullet before impact $= v$

Initial velocity of the block before impact $= 0$

Final velocity of the bullet and block $= V$

By the law of conservation of linear momentum,

$$0.01v + 5 \times 0 = (0.01 + 5)V$$

$$V = \left(\frac{0.01}{5.01}\right)v = \frac{v}{501} \ldots\ldots\ldots\ldots\text{(i)}$$

Applying the law of conservation of mechanical energy

K.E of the combined mass = P.E at the highest point.

$$\frac{1}{2}(m + M)v^2 = (m + M)gh$$

$$v^2 = 2gh \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Substituting for v in equation (ii);

$$\frac{v^2}{(501)^2} = 2gh$$

$$v^2 = 501^2 \times 2 \times 9.81 \times 0.05$$

$$v = 350.88\ ms^{-1}$$

Example 2

A bullet of mass 20 g moving with a velocity v is embedded into a block of mass 10 kg. As a result of this collision, the block and the bullet rise to a height of 25 cm from the equilibrium position. Find the original velocity of the bullet.

Solution

Mass of bullet, $m = 20\text{ g} = 20 \times 10^{-3} kg$

Mass of block, $M = 10\ kg$

Initial momentum of bullet-block system

$$= mv + M \times 0 = mv$$

Let v' be the final velocity of block when the bullet is embedded in it.

Final momentum of bullet- block system = $(m + M)v'$

$$(m + M)v' = mv$$

$$v' = \frac{mv}{m+M}$$

After collision, the bullet-block system acquires K.E

$$K.E = \frac{1}{2}(m + M)v'^2$$

The K.E is converted to P.E when the bullet rises to a height, h

$$P.E = (m + M)gh$$

$$\frac{1}{2}(m + M)v'^2 = (m + M)gh$$

$$v' = \sqrt{2gh}$$

$$\sqrt{2gh} = \frac{mv}{m+M}$$

$$v = \frac{m+M}{m} \times \sqrt{2gh}$$

$$v = \frac{20\times10^{-3}+10}{20\times10^{-3}} \times \sqrt{2 \times 9.81 \times 0.25}$$

$$v = 1109\ ms^{-1}$$

Example 3

A bullet of mass 0.01 kg and travelling horizontally at a speed of $500\ ms^{-1}$ strikes a block of mass 2 kg which is suspended by a sting of length 5 m. The centre of gravity of the block is found to rise a vertical distance of 0.1 m. What is the speed of the bullet after it emerges from the block?

Solution

Mass of bullet, $m = 0.01\ kg$

Initial velocity of bullet, $v = 500\ ms^{-1}$

Mass of block $= 2\ kg$

When it is struck by the bullet, let its velocity be V. As a result, the block rises to a height h, converting $K.E$ to $P.E$

$$K.E \text{ lost} = P.E \text{ gained}$$

$$\frac{1}{2}MV^2 = Mgh$$

$$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.1} = 1.4\ ms^{-1}$$

Suppose on emerging from the block, the speed of the bullet is v'. Then by law of conservation of linear momentum,

Initial momentum of bullet-block system

= Final momentum of bullet-block

$$mv + M \times 0 = mV + mv'$$

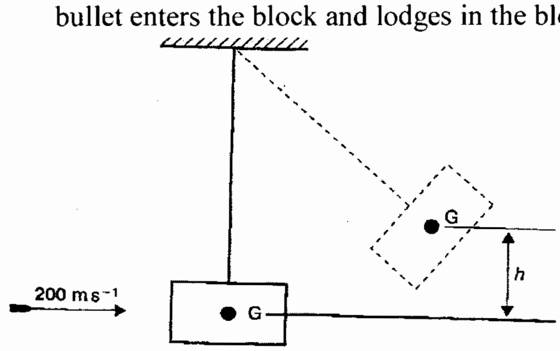
$$(0.01 \times 500) + 0 = (2 \times 1.4) + (0.01 \times v')$$

$$v' = 220\ ms^{-1}$$

Self-Evaluation exercise

1. (a) State the principle of conservation of linear momentum for two colliding bodies

(b) A bullet of mass $0.10\ kg$ travelling at a speed of $200\ ms^{-1}$ strikes a block of wood of mass $0.390\ kg$ hanging at rest from a string. The bullet enters the block and lodges in the block



Calculate

(i) the linear momentum of the bullet before it strikes the block

(ii) the speed with which the block first moves from rest after the bullet strikes it

(c) During the collision of the bullet and the block, kinetic energy is converted into internal energy which results in a temperature rise

(i) Show that the kinetic energy of the bullet before it strikes the block is $200\ J$

(ii) Show that the kinetic energy of the combined block and bullet immediately after the bullet has lodged is $5.0\ J$

(iii) The material from which the bullet is made has a specific heat capacity of $250\ J\ kg^{-1}\ K^{-1}$. Assuming that all the lost energy becomes internal energy in the bullet, calculate its temperature rise during collision

(d) The bullet lodges at the centre of mass G of the block. Calculate the vertical height h through which the block rises after the collision

[Ans: (b) (i) $2.0\ kg\,m\,s^{-1}$ (ii) $5.0\ m\,s^{-1}$ (c) (iii) $78\ K$ (d) $1.3\ m$]

2. A block of wood of mass $1.0\ kg$ is suspended freely by a thread. A bullet of mass $10\ g$ is fired horizontally at the block and becomes embedded in it. The block swings to one side, rising a vertical distance of $50\ cm$. With what speed did the bullet hit the block?

[Ans: $319.4\ m\,s^{-1}$]

3. A bullet of mass $10\ g$ travelling horizontally at a speed of $100\ m\,s^{-1}$ embeds itself in a block of wood of mass $990\ g$ suspended by strings so that it can swing freely. Find

(a) the vertical height through which the block rises

(b) how much of the bullet's energy becomes its internal energy

[Ans: (a) $0.051\ m$ (b) $49.5\ J$]

4. A bullet of mass $0.012\ kg$ and horizontal speed $70\ m\,s^{-1}$ strikes a block of wood of mass $0.4\ kg$ and instantly comes to rest with respect to the block. The block is suspended from the ceiling by a thin wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

[Ans: $0.212\ m$; $28.54\ J$]

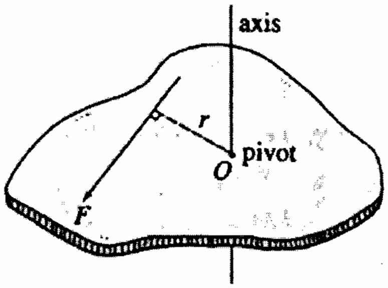
STATIC EQUILIBRIUM

We discussed the equilibrium of a body acted upon by a number of forces which pass through a common point. In this chapter, we shall discuss the equilibrium of a rigid body under the action of a number of coplanar forces which may not be concurrent (not acting at the same point)

A rigid body is one whose shape would not change when acted upon by a force.

Moment of a force (Torque)

Consider a rigid body pivoted at O and free to rotate about an axis through O. When a force F acts on the object, the effect of the force F is to rotate the rigid object about an axis through O



The turning effect of a force about an axis is called moment of force about that axis.

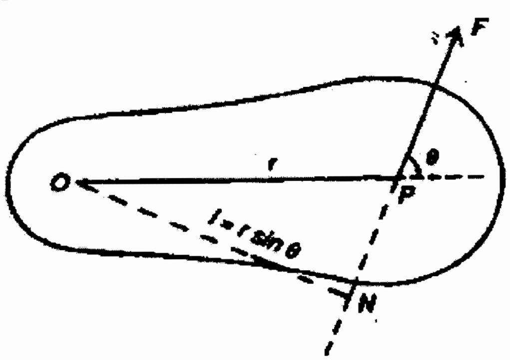
The moment of force F about an axis is given by the product of the force F and the perpendicular distance r of F from the axis.

$$\text{Moment of force} = F \times r$$

If the force F passes through the point O, its perpendicular distance from the axis is zero and hence the moment of F about an axis through O is zero.

In the figure above, the force F tends to rotate the rigid object in the anti-clockwise direction. If the moment in the anticlockwise direction is assumed to be positive, then the moment in the clockwise direction will be negative, and vice versa.

When the force applied is not perpendicular to the line through O i.e.



In this case, the turning effect is decided by the product of the force F and the perpendicular distance ON of the force from the axis of rotation

The torque is given by

$$\tau = lF = (r\sin\theta)F$$

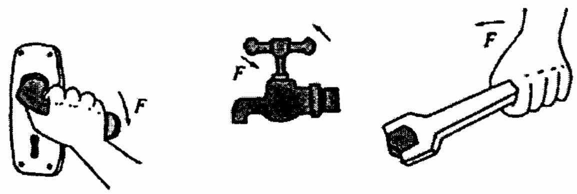
Moment of force is a vector quantity since it has both magnitude and direction.

The moment of the force i.e. torque about a point is defined as the product of the force and the perpendicular distance of the force from the axis of rotation

SI unit for moment of force or torque is Nm

Applications of torque

We use a lot of torque in our everyday life. You apply a torque when you turn a door knob, turn on the tap, use a wrench to tighten a nut, etc.



Some implications of torque

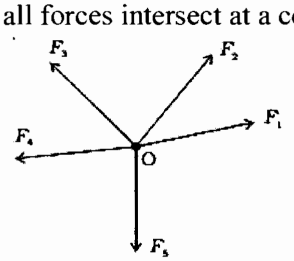
Handle on door: The handle to open the door is placed as far away as possible from its hinge to enable a small force to produce a large turning effect

Long spanner: To open a tight nut, a long spanner is needed. By increasing the length of the spanner, the moment arm gets increased and a small force causes a large turning effect.

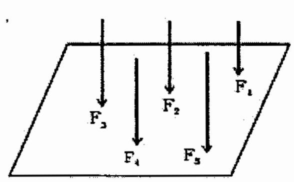
Lever: A lever is a rigid bar which is free to turn about a pivot called fulcrum. A force applied at one end of the lever can lift the load placed at the other end

Concurrent and coplanar forces

A force system is said to be concurrent, if the lines of all forces intersect at a common point



Concurrent forces

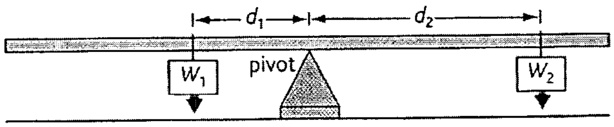


Coplanar forces

A force system is said to be coplanar, if the lines of the action of all forces lie in one plane

Principle of moments

When an object is in equilibrium, the sum of the anticlockwise moments about a turning point is equal to the sum of the clockwise moments.



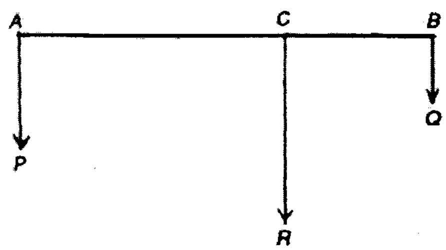
$$W_1 d_1 = W_2 d_2$$

Applications of principle of moments

- Principle of physical balance
- Platform balance
- Gyroscopes

Parallel forces

Consider an object acted upon by two parallel forces P and Q which do not give a zero resultant. Let us find the resultant R of such forces



Like forces

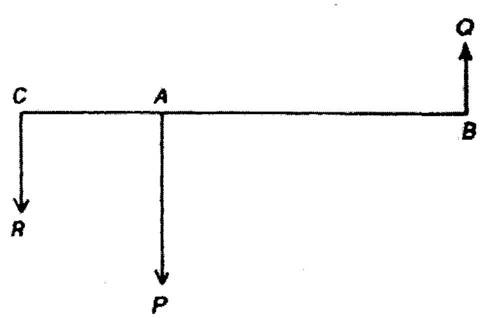
If the two parallel forces P and Q act in the same direction, then their resultant R is given by

$$R = P + Q$$

and the resultant R acts at C such that

$$P \times AC = Q \times BC$$

Unlike forces



If the two parallel forces P and Q act in opposite directions, then their resultant R is given by

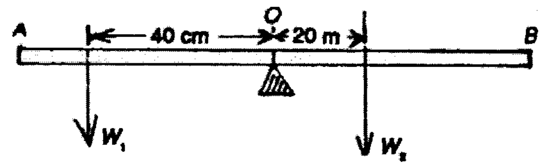
$$R = P - Q$$

and the principle of moments shows that this must act at C such that

$$P \times AC = Q \times BC$$

Example 1

The figure below shows a uniform metre rule weighing 100 g pivoted at its centre O. Two weights $W_1 = 150$ g and $W_2 = 250$ g hang from the metre rule as shown in the figure.



Calculate

(a) total anticlockwise moments about O

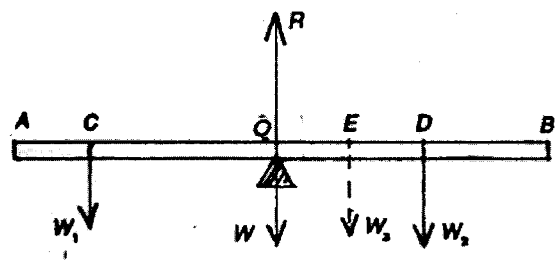
(b) total clockwise moments about O

(c) resultant moment about O and discuss its effects

(d) the distance from O where a weight $W_3 = 100$ g should be suspended to balance the metre rule

<u>Solution</u>

Let AB be the metre rule. It is pivoted at its centre O from where its own weight W acts. Weights W_1 and W_2 are hanging from the points C and D respectively



(a) The anticlockwise moment is due to the weight W_1 and is given by

ACW moment $= W_1 \times CO$

$= 1.50 \times 0.4 = 0.6\ Nm$

(b) The clockwise moment is due to the weight W_2 and is given by

CW moment $= W_2 \times DO$

$= 2.50 \times 0.2 = -0.5\ Nm$

(c) Resultant moment

$= 0.6 + (-0.5) = 0.1\ Nm\ (ACW)$

The beam will therefore tend to turn anticlockwise

(d) To keep the beam balanced, CW moment equal to 0.1 Nm is required. If a weight $W_3 = 100$ g is placed at the point E towards right of O, then for balance,

$$W_3 \times EO = 0.1$$
$$1 \times EO = 0.1$$
$$EO = 0.1\ m = 10\ cm$$

Equilibrium of a rigid body

A rigid body under the action of a number of coplanar forces is in equilibrium if

1. the resultant force is zero
2. the algebraic sum of the moments of the forces about any axis is zero

Therefore, a rigid body under the action of coplanar forces is not in equilibrium if

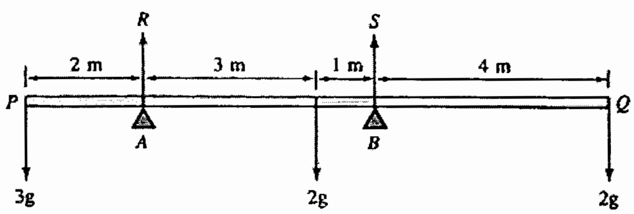
1. there is a resultant force acting on it
2. the algebraic sum of the moments of the forces about any axis is not zero

Example 2

A uniform rod PQ of length 10 m and mass 2 kg is supported horizontally at A, 2 m from one end and at B, 4 m from the other end. A mass of 3 kg is placed at P and another mass of 2 kg at Q. What are the normal reactions at the supports A and B?

Solution

Let R and S be the normal reactions at A and B respectively



Since the rod is in equilibrium, the resultant force in any direction is zero. Therefore, in the direction vertically upwards

$$R + S - 3g - 2g - 2g = 0$$
$$R + S = 7g$$

Also, the algebraic sum of moments about any axis is zero. Therefore, taking moments about the horizontal axis that passes through A

$$-(2g \times 3) + (S \times 4) - (2g \times 8) + (3g \times 2) = 0$$
$$4S = 16g$$
$$S = 4g = 4 \times 9.81 = 39.24\ N$$
$$R = 7g - 4g = 3g = 29.43\ N$$

Example 3

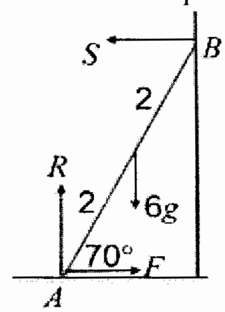
A uniform ladder of length 4 m and weight 6 kg rests on a horizontal ground against a smooth vertical wall. The ladder is inclined at an angle of 70° to the vertical when it is on the point of slipping. Calculate the coefficient of friction between the ladder and the ground

Solution

We first need to draw a diagram showing the forces acting on the ladder. Since the ladder is uniform, the weight will act through its midpoint

There is a normal contact force from the smooth wall

The total reaction force on the ladder from the rough ground has normal friction components, R and F



Resolving vertically gives: $R = 6\,g$

Resolving horizontally gives: $F = S$

Taking moments about A:

$$6g \times 2\cos 70° = S \times 4\sin 70°$$
$$S = \frac{12g\ \cos 70°}{4\sin 70°} = 3g\cot 70°$$

The ladder is on the point of slipping, so $F = \mu R$

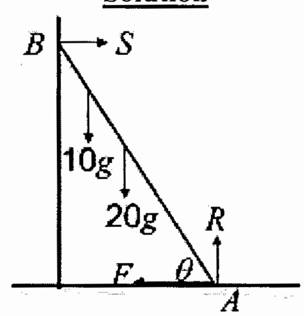
Since $F = S$ and $R = 6\,g$,

$$3g\cot 70° = 6\mu g$$
$$\mu = 0.182$$

Example 4

A uniform ladder AB of mass 20 kg stands on a rough horizontal ground and leans against a smooth vertical wall. A mass of 10 kg is attached to the ladder $\frac{3}{4}$ of the way up. The coefficient of friction between the ladder and the ground is $\frac{1}{2}$. If the ladder is on the point of slipping, find the angle it makes with the ground.

Solution



Resolving vertically, $R = 30\,g$

Resolving horizontally, $S = F$

At the point of slipping, $F = \mu R$

$$F = \frac{1}{2} \times 30g = 15\,g$$

Let the length of the ladder be l

Taking moments about A;

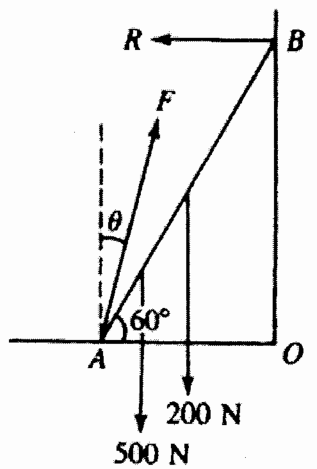
$$20\,g \times \frac{1}{2} l\cos\theta + 10\,g\ \times \frac{3}{4} l\cos\theta = S \times l\sin\theta$$

$$17.5\,g\cos\theta = 15\,g\sin\theta$$
$$\frac{\sin\theta}{\cos\theta} = \frac{17.5g}{15g}$$
$$\tan\theta = \frac{7}{6}$$
$$\theta = 49.4°$$

Example 5

A ladder of length 10 m and weight 200 N leans against a smooth wall such that it is at an angle 60° to the horizontal. A boy of weight 500 N stands on the ladder $\frac{1}{4}$ of the way from its lower end. Calculate the normal reaction at the wall and the magnitude and direction of the resultant force acting on the lower end of the ladder.

Solution



Since the wall is smooth, the reaction R is normal to the wall.

The forces acting on the lower end A of the ladder consists of the normal reaction vertically upwards and the friction in the direction AO. Therefore, the resultant force F at A is in the direction as shown.

Taking moments about A (so that the force F does not appear in the equation);

$$R \times 10\sin 60° = 200 \times 5\cos 60° + 500 \times 2.5\cos 60°$$
$$10\,R\sin 60° = 2250\cos 60°$$
$$R = 130\,N$$

Resolving vertically (↑);

$$F\cos\theta = 500 + 200$$
$$F\cos\theta = 700 \quad .....(i)$$

Resolving horizontally (→);

$$F\sin\theta = R$$
$$F\sin\theta = 130 \quad ....(ii)$$

$$(i)^2 + (ii)^2;\ F^2\cos^2\theta + F^2\sin^2\theta = 700^2 + 130^2$$
$$F^2(\cos^2\theta + \sin^2\theta) = 700^2 + 130^2$$
$$F = \sqrt{700^2 + 130^2}$$
$$F = 712\,N$$

$$(ii) \div (i);\ \tan\theta = \frac{130}{700}$$
$$\theta = 10.52°$$

Example 6

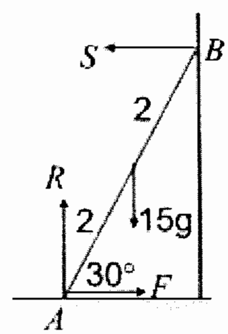
A uniform ladder AB of mass 15 kg and length 4 m is resting on a rough horizontal ground and leaning against a smooth vertical wall.

(a) When the ladder is inclined at an angle of 30° to the horizontal, it is on the point of slipping. Find the coefficient of friction between the ladder and the ground

(b) The ladder is moved so that it now makes an angle of 40° with the horizontal. A boy of mass 40 kg climbs the ladder. How far can he climb up the ladder before it starts to slip?

Solution

(a)



Resolving ↑: $R = 15g$

Resolving →: $S = F$

Taking moments about A:

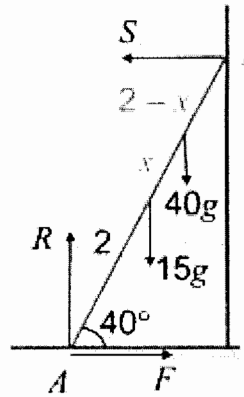
$$S \times 4\sin 30° = 15g \times 2\cos 30°$$
$$S = \frac{30g\cos 30°}{4\sin 30°}$$
$$S = 127.31\,N$$
$$\Rightarrow F = 127.31\,N$$

When the ladder is on the point of slipping, $F = \mu R$

$$127.31 = \mu(15\,g)$$
$$\mu = 0.866$$

(b)

Let x be the distance from the midpoint of AB to the point at which the boy makes the ladder slip



Resolving ↑: $R = 55\text{ g}$

Resolving →: $S = F$

At the point of slipping, $F = \mu R$

$$F = 0.866 \times 55\text{g} = 466.79\ N$$

Taking moments about A:

$$15\text{g} \times 2\cos 40° + 40\text{g} \times (2 + x)\cos 40° = S \times 4\sin 40°$$

$$30\text{g}\cos 40° + 80\text{g}\cos 40° + 40\text{g}x\cos 40° = 466.79 \times 4\sin 40°$$

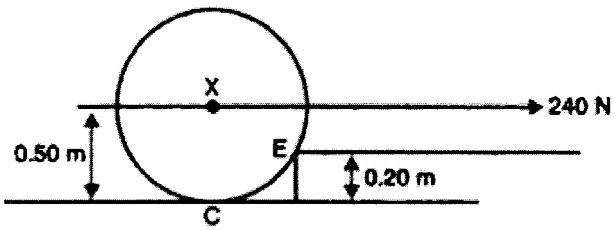
$$x(40\text{g}\cos 40°) = 1967.2\sin 40° - 110\text{g}\cos 40°$$

$$x = 1.25\ m$$

The boy can climb 1.25 m past the midpoint of the ladder. Therefore, he can climb 3.25 m up the ladder before it starts to slip.

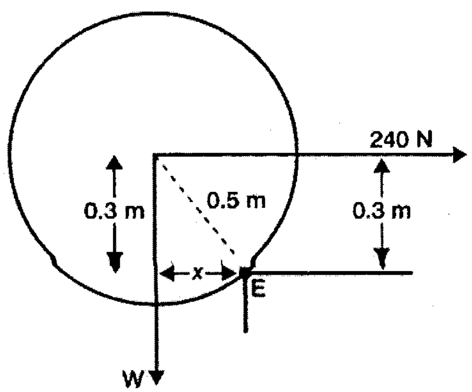
Example 7

A wheel of radius 0.50 m rests on a level road at a point C and makes contact with the edge E at a kerb of height 0.20 m, as shown below



A horizontal force of 240 N, applied through the axle of the wheel at X, is required to move the wheel over the kerb. Show that the weight of the kerb is 180 N.

Solution



$$x^2 + 0.3^2 = 0.5^2$$

$$x^2 = 0.5^2 - 0.3^2$$

$$x = 0.4\ m$$

Taking moments about E;

$$W \times x = 240 \times 0.3$$

$$W = \frac{240 \times 0.3}{0.4} = 180\ N$$

Example 8

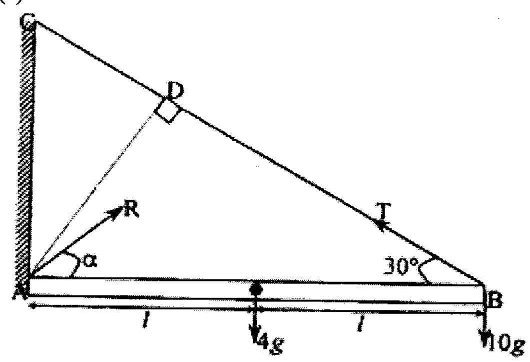
A uniform beam AB of mass 4 kg is hinged to a wall at end A and held horizontally by a wire joining B to a point C which is on the wall vertically above A. If a stone of mass 10 kg is hinged at B, and given that $\angle ABC$ is 30°, find the force

(i) in the wire connecting B to C

(ii) exerted by the beam on the hinge

Solution

(i)



Let $AB = 2l$

$$\Rightarrow AD = 2l\sin 30° = 2l \times \frac{1}{2} = l$$

Taking moments about point A,

T is anticlockwise, 4 g and 10 g are clockwise, and R does not have a moment about A

$$4\text{ g} \times l + 10\text{ g} \times 2l = T \times l$$

$$24\text{ g}l = Tl$$

$$T = 24\text{ g} = 24 \times 9.81 = 235.44\text{ N}$$

(ii)

Resolving

Forces	(→)	(↑)
4 g	0	- 4g
10 g	0	-10g
T	-235.44 cos 30	235.44 sin 30
R	$R\cos\alpha$	$R\sin\alpha$
Σ	$R\cos\alpha - 203.9$	$R\sin\alpha - 19.62$

Since the body is in equilibrium,

then $\sum F_x = 0$ and $\sum F_y = 0$

$$\therefore R\cos\alpha - 203.9 = 0$$

$$R\sin\alpha - 19.62 = 0$$

$$R\cos\alpha = 203.9 \ldots\ldots (i)$$

$$R\sin\alpha = 19.62 \ldots\ldots (ii)$$

$$(ii) \div (i);\quad \frac{R\sin\alpha}{R\cos\alpha} = \frac{19.62}{203.9}$$

$$\tan\alpha = 0.09622$$

$$\Rightarrow \alpha = 5.5°$$

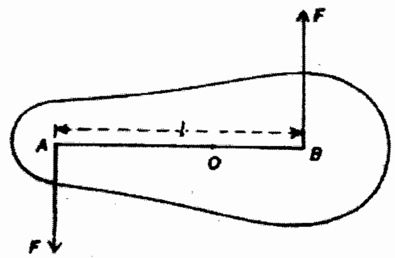
From equation (i), $R = \frac{203.9}{\cos 5.5°} = 204.8\ N$

Therefore, the force at the hinge is 204.8 N at an angle of 5.5° to AB

Couple

When two forces are applied simultaneously on a body which are equal in magnitude, opposite in direction and have different lines of action, they are said to constitute a couple. These two forces do not produce any bodily movement, but they tend to turn the body in the same sense.

Let two forces act on the body as shown below. The body is capable of turning about an axis through O



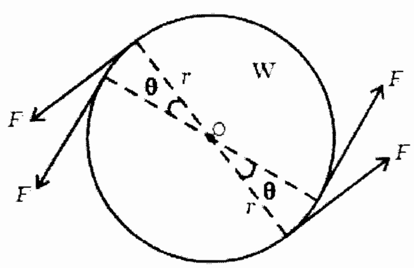
Moment of a couple = moment of F acting at A
+ moment of F acting at B

$$= AO \times F + OB \times F$$
$$= F(AO + OB)$$
$$= Fl$$

where l is the perpendicular distance between the two forces

Work done by a couple

Suppose two equal and opposite forces F act tangentially to a wheel, W and rotate it through an angle θ as shown below



Work done by each force = Force × distance

Distance moved by a point on the rim $= r\theta$

Work done by each force $= Fr\theta$

Total work done $= Fr\theta + Fr\theta = 2Fr\theta$

But torque, $\tau = F \times 2r = 2Fr$

Work done by the couple, $\boldsymbol{W = \tau\theta}$

Applications of a couple

Opening a water tap: In order to open a water tap, forces are applied by fingers. These forces constitute a couple.

Turning a key in the lock: In order to turn a key in the keyhole of a lock, forces are applied by fingers. These forces form a couple

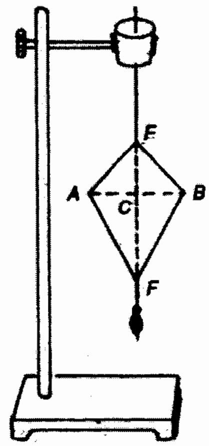
Removing a cap: For removing a cap of a water bottle, we apply a couple with the help of our fingers

Steering a motor car wheel: For turning a steering wheel, the driver of the car applies a couple by his two hands

Centre of gravity

The centre of gravity G of an object is the point where the line of action of the weight of the object passes. It is also the point where the weight of the object can be assumed to act

Experimental determination of centre of gravity



To find the centre of gravity of a lamina, it is suspended by a thread from E.

A vertical line EF passing through E is drawn with the help of a plumb line.

The lamina is then again suspended from the other corner A. A vertical line AB is drawn with the help of a plumb line.

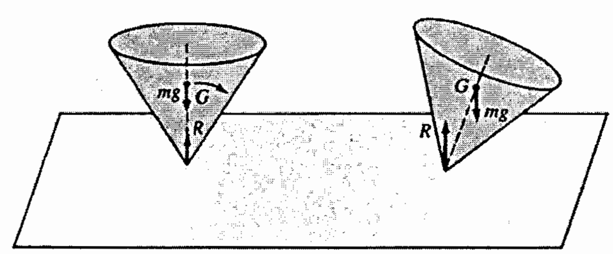
The centre of gravity will lie on the point of intersection of the lines AB and EF

Stability

We learn to balance things, including ourselves, at a very early age. We know that standing on both legs is more stable than standing on one leg, a tricycle is more stable than a bicycle. What makes some objects more stable than others?

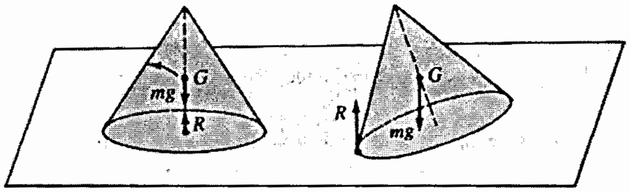
Unstable equilibrium

Consider a solid cone standing on its tip.



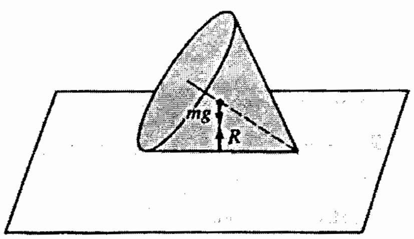
When the cone is displaced slightly, it falls over. Its centre of gravity is lowered. The cone is said to be in unstable equilibrium.
An object is said to be in unstable equilibrium if any displacement lowers the centre of gravity

Stable equilibrium



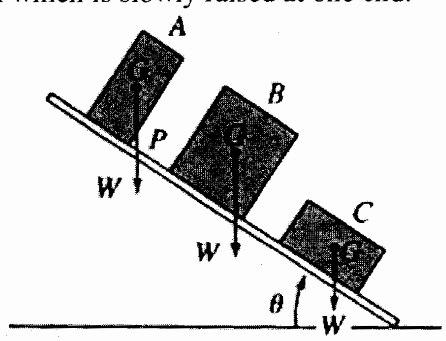
When the cone resting on its base is displaced, its centre of gravity is raised. On releasing it, the cone falls back to its initial position, due to the clockwise moment provided by its weight. The cone is said to be in stable equilibrium.
A body is said to be in stable equilibrium if it returns to its initial position when slightly displaced.

Neutral equilibrium



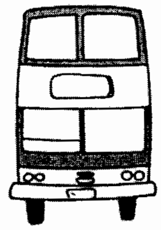
When the cone is on its side, its centre of gravity is neither raised nor lowered when displaced horizontally. It is possible to roll the cone to a new position and when released, the cone will not roll further or roll back to its initial position. The cone is said to be in neutral equilibrium

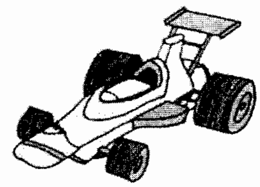
Applications of stability principle
The figure below shows three objects A, B and C on a platform which is slowly raised at one end.



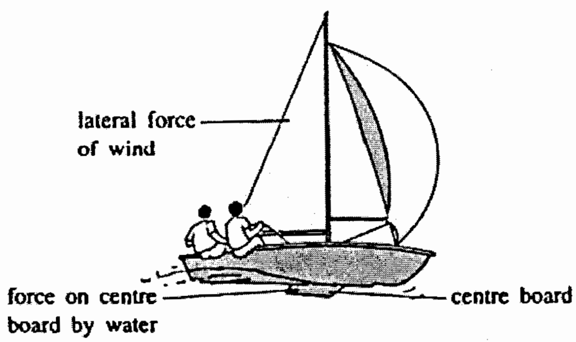
In the position as shown, object A topples over because the line of action of its weight acts outside the corner of its base. The weight W has a clockwise moment about the corner P which causes it to topple over.

Object B has a wider base than object A. The line of action of the weight passes within the base and it does not topple.
The centre of gravity of object C is much lower and again the line of action of its weight W acts within its base.
Hence an object is more stable if
(i) it has a larger base, and
(ii) its centre of gravity is lower
The idea is used in the design of the double decker bus and a racing car.

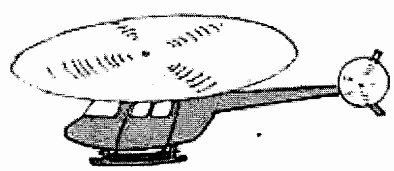




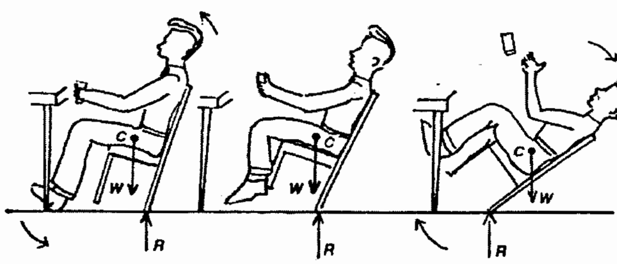
To make the bus more stable, light weight material such as aluminium is used on the upper deck and the engine is mounted as low as possible. Passengers are not allowed to stand on the upper deck.
The wide base of a racing car and its low centre of gravity enhances its stability.



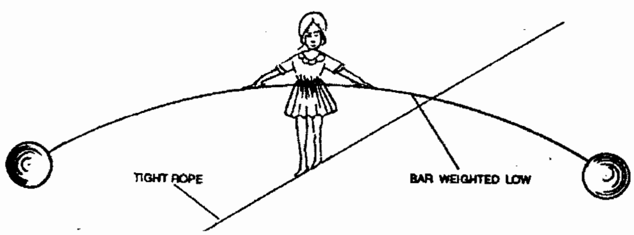
In sailing a boat, the lateral force acting on the sails produces a moment which tends to topple the boat. A moment in the opposite direction due to a force acting on the centre board opposes the moment due to the force on the sail, and thus stabilizes the boat. Stability is also maintained with the crews leaning backwards from the side of the boat.



A helicopter is essentially a rotating wing craft. When the main rotor rotates, the body will tend to rotate the other way. To prevent this from occurring, a small propeller is fitted to rotate about a horizontal axis on the tail. This rear rotor produces a torque to encounter that exerted by the main rotor.



For a boy leaning back on a chair, the chair will return to its equilibrium position for as long as the centre of gravity is within the base. When the centre of gravity is just over the fulcrum, the boy just balances. This a situation of unstable equilibrium. When he leans back too far, the centre of gravity goes out of the base and he falls down.



A person who performs a tight rope walking in a circus carries a pole which is loaded at its both ends. This helps the person to keep his centre of gravity low and vertically above the rope. Whenever the person bends and is likely to fall, he or she tilts the pole in the opposite side, so as to keep the vertical line passing through the rope.

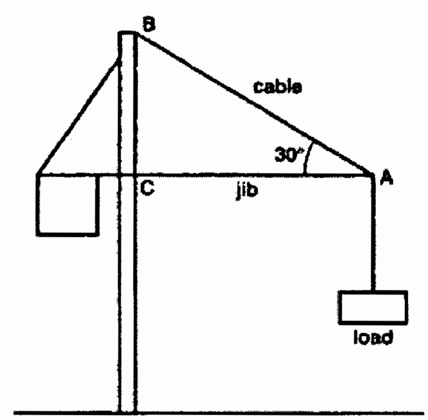
Sometimes, the dancer who walks on a tight rope in circus carries an umbrella in her hand which helps her to keep the centre of gravity vertically above the rope.

A man carrying heavy load in one hand bends his body towards the other hand so that the centre of gravity is properly placed.

In order to keep the centre of gravity within the base, a person climbing up a mountain has to bend forward.

Self-Evaluation exercise

1. Does the C.O.G. rise when a body is displaced from stable equilibrium?
2. What happens to the C.O.G. when a body is displaced from unstable equilibrium?
3. Why do wine glasses have wide and thick bases?
4. Why is a heavy load not allowed on the top of a bus?
5. State and explain the principle of moments.
6. Distinguish clearly between moment and momentum.
7. Give reasons
   a) The passengers in a bus are not allowed to stand.
   b) A man carrying a bucket in one hand, stretches his other hand.
   c) A rope dancer holds an umbrella in her hand.
   d) A man climbing up a mountain bends forward.
8. Why is it easier to pull down the branch of a tree from its free end than from anywhere else?
9. Explain clearly the difference between the moment of a force and moment of a couple. Give one example of each.
10. Some heavy boxes are to be loaded on a truck together with some empty boxes. Which boxes should be put first?
11. What are the two essential conditions of equilibrium? Explain them.
12. Define stable, unstable and neutral equilibrium. Give illustrations.
13. Define centre of gravity of a body. Explain how you would find the C.G. of an irregular laminar experimentally.
14. The figure below illustrates a crane. Assume that the jib AC has a negligible weight.



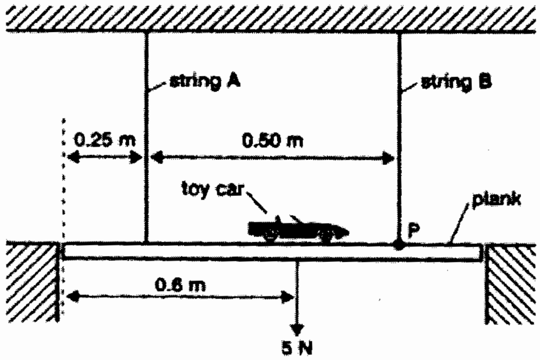
AB is a cable which makes an angle of 30° with the jib, which is horizontal. The jib carries a load of 2000 N. The load is in equilibrium.

(i) Calculate the tension in the cable AB

(ii) Calculate the compression force in the jib AC

[Ans: (i) 4.0 kN (ii) 3.46 kN]

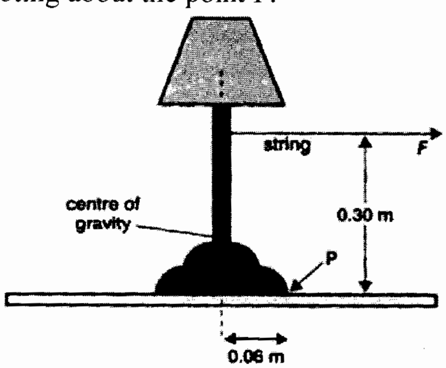
15. The figure below shows a model bridge consisting of a uniform plank of wood. The plank is 1.0 m long and weighs 10 N. A toy car of weight 5 N is placed on it. The bridge is suspended from a rigid support by two strings and is in equilibrium. The plank does not touch the shaded blocks.



(i) Show and label the forces acting on the bridge
(ii) By taking moments about point P, calculate the tension in string A
(iii) Calculate the tension in string B

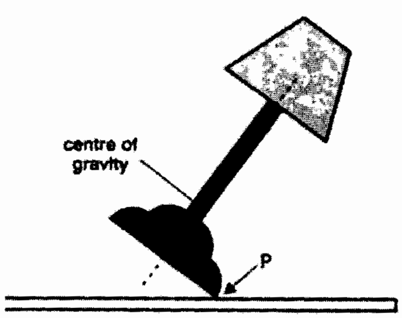
[Ans: (ii) 6.5 N (iii) 8.5 N]

16. Some tests are carried out on the stability of a table lamp.
(a) A string is attached to the lamp, as shown, and pulled with a steadily increasing force F. When F reaches 7.2 N, the lamp is about to tilt, pivoting about the point P.



(i) Calculate the moment(torque) of F about P when F = 7.2 N
(ii) By considering when the lamp is about to tilt, calculate its weight. Its centre of gravity is shown on the diagram.

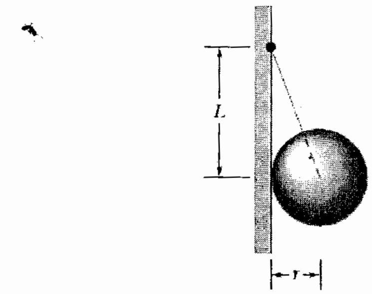
(b) The lamp is now tilted as shown below and released.



Explain, in terms of moments, whether it wi fall over or return to the upright. Feel free t add to the diagram
(c) State two ways in which the lamp could b redesigned to make it more stable

[Ans: (a) (i) 2.16 Nm (ii) 36 N]

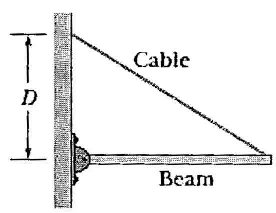
17. A uniform sphere of mass $m = 0.85$ kg and radius $r = 4.2$ cm is held in place by a massless rope attached to a frictionless wall a distance $L = 8.0$ cm above the center of the sphere.



Find (a) the tension in the rope and (b) the force on the sphere from the wall.

[Ans: (a) 9.4 N (b) 4.4 N]

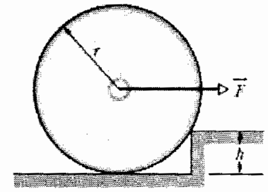
18. A uniform beam of weight 500 N and length 3.0 m is suspended horizontally. On the left it is hinged to a wall; on the right it is supported by a cable bolted to the wall at distance D above the beam. The least tension that will snap the cable is 1200 N.



(a) What value of D corresponds to that tension?
(b) To prevent the cable from snapping, should D be increased or decreased from that value?

[Ans: (a) 0.64 m (b) increased]

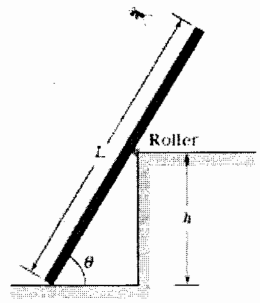
19. What magnitude of (constant) force applied horizontally at the axle of the wheel is necessary to raise the wheel over a step obstacle of height $h = 3.00$ cm?



The wheel's radius is $r = 6.00$ cm, and its mass is $m = 0.800$ kg.

[Ans: 13.6 N]

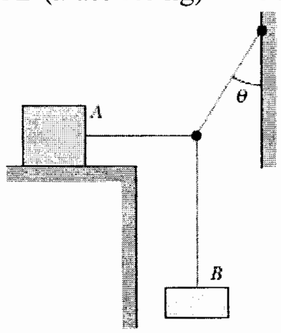
20. A uniform plank, with a length L of 6.10 m and a weight of 445 N, rests on the ground and against a frictionless roller at the top of a wall of height h = 3.05 m.



The plank remains in equilibrium for any value of $\theta \geq 70°$ but slips if $\theta < 70°$. Find the coefficient of static friction between the plank and the ground.

[Ans: 0.34]

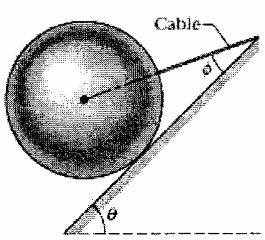
21. Block A (mass 10 kg) is in equilibrium, but it would slip if block B (mass 5.0 kg) were any heavier.



For angle $\theta = 30°$, what is the coefficient of static friction between block A and the surface below it?

[Ans: 0.29]

22. A 10 kg sphere is supported on a frictionless plane inclined at angle $\theta = 45°$ from the horizontal. Angle ϕ is 25°.



Calculate the tension in the cable.

[Ans: 76 N]

23. A rigid rod PQ has length 2 m. A body of mass 12 kg hangs from P and another body of mass 8 kg hangs from Q. The system is suspended from a point A of the rod, where A is $x\ m$ from Q, and is in equilibrium with the rod horizontal. Find the value of x

[Ans: $x = 1.2\ m$]

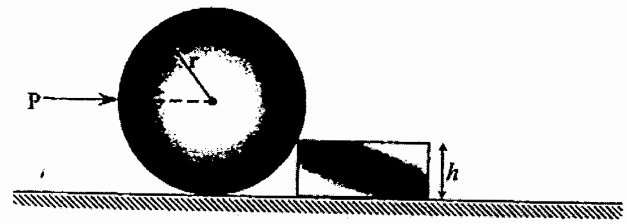
24. A uniform ladder of mass 40 kg and length 5 m, rests with its upper end against a smooth vertical wall and with its lower end at 3 m from the wall on a rough ground. Find the magnitude and the direction of the force exerted at the bottom of the ladder.

[Ans: 418.7 N at 69.4° to the horizontal]

25. A mass of 5 kg is suspended from end A of a uniform beam of mass 1.0 kg and length 1.0 m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C, in the wall at a height 0.75 m above B.
    (i) Draw a sketch diagram to show the forces acting on the beam
    (ii) Calculate the tension in the rope
    (iii) What is the force exerted by the hinge on the beam?

    [Ans: T = 89 N, R = 72.0 N at 3.9° above the horizontal]

26. In the figure below, if a horizontal force P is required to begin rolling the uniform cylinder of mass m over the obstruction of height h,

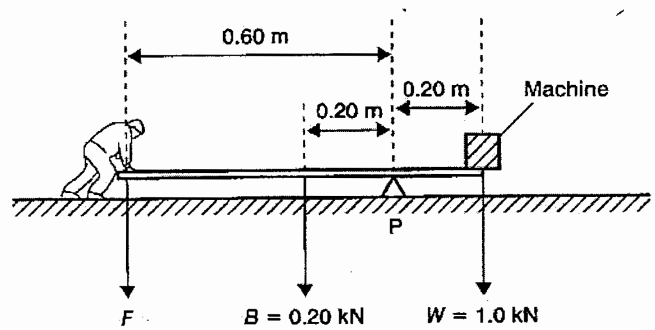


show that $P = \frac{mg\sqrt{2rh-h^2}}{r-h}$

27. If an oil drum of diameter 70 cm and mass 80 kg rests on a stone like in the diagram in question 26, find the least horizontal force applied through the centre of the drum which will cause the drum to roll over the stone of height 10 cm.

[Ans: 471.6 N]

28. The figure below shows a man attempting to lift a piece of machinery of weight $W = 1.0\ kN$ using a uniform iron bar of weight $B = 0.20\ kN$. He uses a pivot P placed as shown

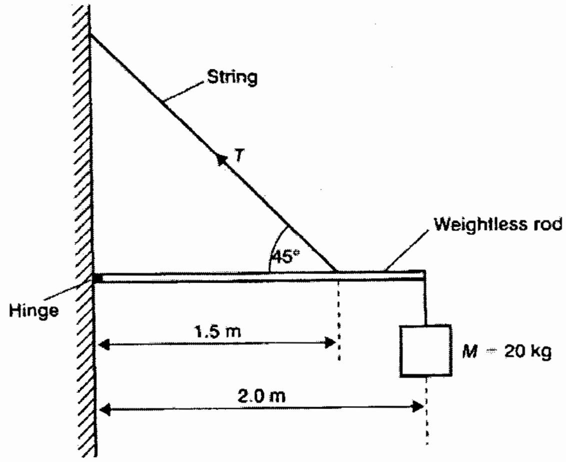


Calculate
    (i) the magnitude of the force F which he must apply downwards if he is to lift the machinery

(ii) the reaction force provided by the pivot

[Ans: (i) $0.27\ kN$ (ii) $1.5\ kN$]

29. An object M of mass $20\ kg$ is supported by a hinged weightless rod and string as shown below



Calculate

(a) the tension T iin the string

(b) the horizontal force acting on the hinge

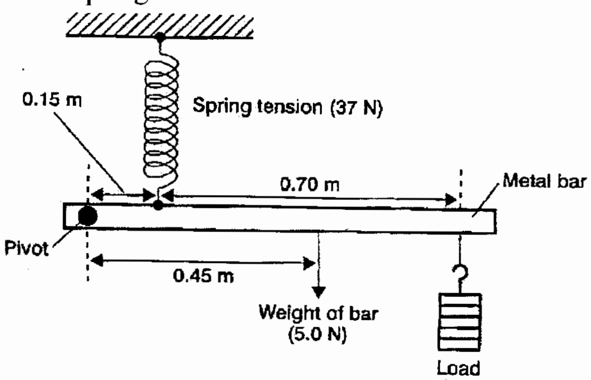
(c) the maximum additional mass which may be added to mass M prior to the string breaking if the maximum tension which the string can withstand is $500\ N$

(assume $g = 10\ m\ s^{-2}$)

[Ans: (a) $0.38\ kN$ (b) $0.27\ kg$ (c) $6.5\ kg$]

30. (a) State the principle of moments

(b) To increase the extension of a stiff spring for a given load, a student set up the system shown below. The weight of the metal bar was $5.0\ N$ and the tension the student achieved in the spring was $37\ N$.



(i) Apply the principle of moments to calculate the mass of the load that the student used

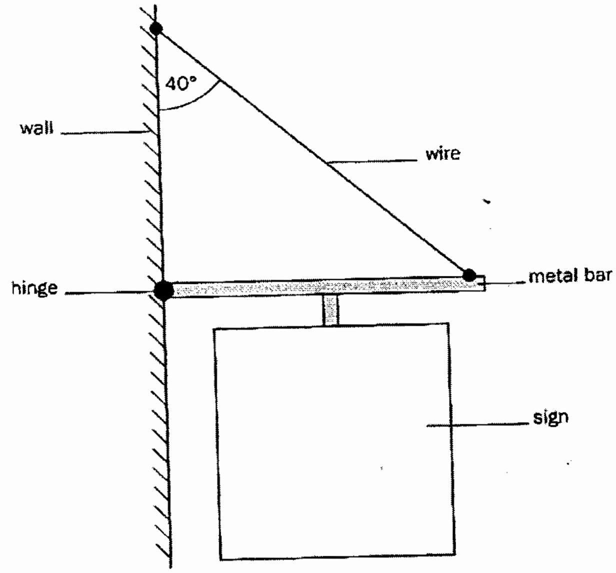
(ii) Calculate the magnitude of the force exerted on the metal bar at the pivot

(iii) Draw on the diagram an arrow to show the direction of the force calculated above

(c) The spring stiffness k of the spring was $550\ N\ m^{-1}$. Calculate the energy stored in the spring

[Ans: (b) (i) $0.40\ kg$ (ii) $28\ N$ (iii) downwards (c) $1.24\ J$]

31. A public house sign is fixed to a vertical wall as shown in the figure below



A uniform metal bar $0.75\ m$ long is fixed to the wall by a hinged joint that allows free movement in the vertical plane only. The wire is fixed to the wall directly above the hinge and to the free end of the horizontal metal bar. The wire makes an angle of 40° with the wall. A single support holds the sign and is mounted at the mid-point of the metal bar so that the weight of the sign acts through that point. If the combined mass of the metal bar and the sign is $12\ kg$ and the mass of the wire is negligible, calculate the tension in the wire.

[Ans: $7.8\ N$]

32. The foot of a uniform ladder of mass 40 kg is on a horizontal ground and the top rests against a smooth vertical wall. A man of mass 80 kg stands on the ladder one-quarter of its length from the bottom. If the inclination of the ladder to the horizontal is 30°, find the reaction at the wall and the total force at the ground.

[Ans: $676.89\ N$, $1363.59\ N$]

FLUIDS AT REST

Liquids and gases can flow and are called fluids. An important concept in connection with fluids is that of pressure. The pressure in a fluid depends on its density.

Density

The density ρ of a sample of a substance of mass m and volume V is defined by the equation

$$\rho = \frac{m}{V}$$

In other words, density is the mass per unit volume. The density of water is $1.00\, g\, cm^{-3}$ or in SI units $1.00 \times 10^3\, kgm^{-3}$.

The density of mercury at room temperature is about $13.6\, g\, cm^{-3}$ or $13.6 \times 10^3\, kg\, m^{-3}$.

Relative density (R.D)

The term relative density is sometimes used and is given by

$$\text{R.D} = \frac{\text{Density of object}}{\text{Density of water}}$$

$$= \frac{\text{mass of object}}{\text{mass of an equal volume of water}}$$

$$= \frac{\text{weight of object}}{\text{weight of an equal volume of water}}$$

Any of the above formulas can be used to find the R.D of both liquids and solids.

To find the R.D of solids only

$$\text{R.D} = \frac{\text{mass of object in air}}{\text{Apparent loss of mass of object in water}}$$

$$= \frac{\text{weight of object in air}}{\text{Apparent loss of weight of object in water}}$$

To find the R.D of liquids only

$$\text{R.D} = \frac{\text{apparent loss in mass of object in the liquid}}{\text{apparent loss in mass of object in water}}$$

$$\text{R.D} = \frac{\text{Apparent loss in weight of object in the liquid}}{\text{Apparent loss in weight of object in water}}$$

Experiment to determine the density of an irregular solid which floats in water

A thread is tied to the irregular solid and its weight in air W_1 determined.

A sinker such as a stone is attached to the irregular solid, and the weight W_2 of the two in air is determined. The sinker and the solid are completely immersed in water of known density and their weight W_3 is determined by a spring balance.

The sinker is detached from the solid and the weight W_4 of the sinker only when in water is determined.

Weight of irregular solid in water = $W_3 - W_4$

$\Rightarrow$ Relative density of irregular solid

$$= \frac{W_1}{W_1 - (W_3 - W_4)}$$

From Density = Relative density × Density of water, the density of the irregular solid can be determined

Example 1

An object is weighed with a spring balance, first in air and then while totally immersed in a liquid. The readings on the balance are 0.48 N and 0.36 N respectively. Calculate the density of the object. (density of water = $1000\, kg\, m^{-3}$)

Solution

$$\text{R.D} = \frac{\text{weight of object in air}}{\text{Apparent loss of weight of object in water}}$$

$$= \frac{0.48}{0.48 - 0.36} = 4$$

Also, $\text{R.D} = \frac{\text{Density of substance}}{\text{Density of water}}$

Density of object = R.D × density of water

$= 4 \times 1000$

$= 4000\, kg\, m^{-3}$

Example 2

A block of mass 0.1 kg is suspended from a spring balance. When the block is immersed in water of density 1000 kgm^{-3}, the spring balance reads 0.63 N. When the block is immersed in a liquid of unknown density, the spring balance reads 0.70 N. Find the

(i) density of the block

(ii) density of the liquid.

Solution

(i) Mass of solid in air = 0.1 kg

Weight of solid in air = $0.1 \times 9.81 = 0.981$ N

Weight of solid in water = 0.63 kg

Apparent loss of weight of the solid in water

$= 0.981 - 0.63 = 0.351$ N

$$\text{R.D} = \frac{\text{weight of solid in air}}{\text{Apparent loss of weight of solid in water}}$$

$$= \frac{0.981}{0.351} = 2.795$$

But $\text{R.D} = \frac{\text{Density of solid}}{\text{Density of water}}$

$$\Rightarrow 2.795 = \frac{\rho_s}{1000}$$

Density of solid = $2.795 \times 1000 = 2795$ kgm^{-3}

(ii) Apparent loss of weight of solid in liquid

$= 0.981 - 0.7 = 0.281$ N

R.D of liquid

$$= \frac{\text{apparent loss of weight of a solid in liquid}}{\text{Apparent loss of weight of the solid in water}}$$

$$= \frac{0.281}{0.351} = 0.8$$

∴ Density of liquid = $0.8 \times 1000 = 800$ kgm^{-3}

Example 3

A specimen of an alloy of silver and gold, whose densities are 10.5 gcm^{-3} and 18.9 gcm^{-3} respectively, weighs 35.2 g in air and 33.13 g in water. Find the composition by mass of the alloy, assuming that there has been no volume change in the process of producing the alloy. Assume that the density of water is 1 gcm^{-3}

Solution

$$\rho_s = 10.5\, g\, cm^{-3}, \rho_g = 18.9\, g\, cm^{-3}$$

Let the volume of silver be v_s and that of gold be v_g,

Also, let the mass of silver be m_s and that of gold be m_g.

Mass of alloy in air, $m_s = 35.2$ g .

Mass of alloy in water, $m_g = 33.13$ g

$\Rightarrow m_s + m_g = 35.2$ .... (i)

R.D of alloy $= \frac{\text{Mass of alloy in air}}{\text{Apparent loss of mass in water}}$

$= \frac{35.2}{(35.2-33.13)} = 17$

Density of alloy = R.D × density of water

$= 17 \times 1 = 17\, gcm^{-3}$

Volume of alloy $= \frac{35.2}{17} = 2.07\, cm^3$

Volume of alloy $= v_s + v_g = \frac{m_s}{\rho_s} + \frac{m_g}{\rho_s}$

$= \frac{m_s}{10.5} + \frac{m_g}{18.9}$

$2.07 = \frac{m_s}{10.5} - \frac{m_g}{18.9}$ .... (ii)

But from equation (i), $m_g = 35.2 - m_s$

Substituting for m_g in equation (ii) gives:

$2.07 = \frac{m_s}{10.5} - \frac{35.2 - m_s}{18.9}$

$\therefore 410.79 = 369.6 - 10.5m_s + 18.9m_s$

$\Rightarrow m_s = 4.9$ g

Substituting for m_s in equation (i) gives;

$m_g = 35.2 - 4.9$

$\Rightarrow m_g = 30.3$ g

Example 4

An alloy contains two metals X and Y of densities $3.0 \times 10^3\, kgm^{-3}$ and $5.0 \times 10^3\, kgm^{-3}$ respectively. Calculate the density of the alloy if

(i) the volume of X is twice that of Y,

(ii) the mass of X is twice that of Y.

Solution

(i) Let the volume of Y be v.

$\Rightarrow$ Volume of X is $2v$.

Also let the mass of X be x, and that of Y be y

From density $= \frac{\text{mass}}{\text{volume}}$

$3000 = \frac{x}{2v}$

$\Rightarrow x = 6000v$ .... (i)

$5000 = \frac{y}{v}$

$\Rightarrow v = \frac{y}{5000}$ .... (ii)

Substituting for v in equation (i) gives:

$x = 6000 \times (\frac{y}{5000})$

$\Rightarrow y = \frac{5}{6}x$

Therefore, total mass of alloy $= x + y$

$= x + \frac{5}{6}x = \left(\frac{11}{6}x\right) kg$

Also, volume of alloy $= 2v + v = 3v$

But density $= \frac{mass}{volume} = \frac{(\frac{11}{6}x)}{3v} = \frac{11x}{18v}$

From equation (i), $x = 6000v$

$\Rightarrow Density = \frac{11 \times (6000v)}{18v}$

$= 3.7 \times 10^3\, kg\, m^{-3}$

(ii) Let the mass of Y be m.

$\Rightarrow$ mass of X is $2m$

Let the volume of X be v_x, and that of Y be v_y

$3000 = \frac{2m}{v_x}$

$\Rightarrow v_x = \frac{m}{1500}$ ....(i)

$5000 = \frac{m}{v_y}$

$\Rightarrow m = 5000v_y$ ....(ii)

Substituting for m in equation (i) gives:

$v_x = \frac{5000v_y}{1500}$

$\Rightarrow v_y = \frac{3}{10}v_x$

Total volume of alloy $= v_x + v_y$

$= v_x + \frac{3}{10}v_x = \frac{13}{10}v_x\, m^3$

Also, total mass of alloy $= 2m + m = 3m$ kg

From density $= \frac{\text{mass}}{\text{volume}}$

Density of alloy $= \frac{3m}{(\frac{13}{10}v_x)} = \frac{30m}{13v_x}$

From equation (i), $v_x = \frac{m}{1500}$

$\therefore$ Density of alloy $= \frac{30m}{13 \times (\frac{m}{1500})}$

$= 3.5 \times 10^3\, kg\, m^{-3}$

Density rod

This is an instrument used to measure the relative density of liquids.

Procedure

Let the cross-sectional area of the density rod be A.
The rod is submerged in a liquid of density ρ_1 and its length, s_1 submerged in the liquid is noted.
The rod is then submerged in water of density ρ_2, and its length, s_2 submerged in the water is noted
When the rod is floating in the liquid,

volume of liquid displaced $= As_1$

mass of liquid displaced $= A\rho_1 s_1$

weight of liquid displaced $= A\rho_1 s_1 g$

From the principle of floatation,

weight of liquid displaced is equal to the weight of the floating rod.

$\therefore$ Weight of floating rod $= A\rho_1 s_1 g$ ... ... (i)

When the rod is floating in water,

volume of liquid displaced $= As_2$

mass of liquid displaced $= A\rho_2 s_2$

weight of liquid displaced $= A\rho_2 s_2 g$

Similarly,

weight of floating rod $= A\rho_2 s_2 g$ ....(ii)

$$\Rightarrow A\rho_1 s_1 g = A\rho_2 s_2 g$$

$$\therefore \frac{\rho_1}{\rho_2} = \frac{s_2}{s_1}$$

But, $\frac{\text{Density of a substance } (\rho_1)}{\text{Density of water } (\rho_2)} = \text{Relative density}$

$$\Rightarrow \text{Relative density} = \frac{s_2}{s_1}$$

Note: This method of determining relative density has an advantage that the relative density can be got even if the densities of the substances are not known.

Example 5

A hydrometer floats in water with 72% of its volume submerged. The hydrometer floats in another liquid with 80% of its volume submerged. Find the relative density of the liquid.

Solution

Let the volume of the hydrometer be v, the density of water be ρ_w, and that of the liquid be ρ_l.

While in water: volume of water displaced $= \frac{72}{100} v$

mass of water displaced $= \frac{72}{100}\rho_w v$

weight of water displaced $= \frac{72}{100}\rho_w vg$

From the law of floatation,

weight of hydrometer $= \frac{72}{100}\rho_w vg$ .... (i)

While in the liquid:

volume of liquid displaced $= \frac{80}{100} v$

mass of liquid displaced $= \frac{80}{100}\rho_1 v$

Weight of liquid displaced $= \frac{80}{100}\rho_1 vg$

Similarly,

weight of hydrometer $= \frac{80}{100}\rho_1 vg$ ... ... (ii)

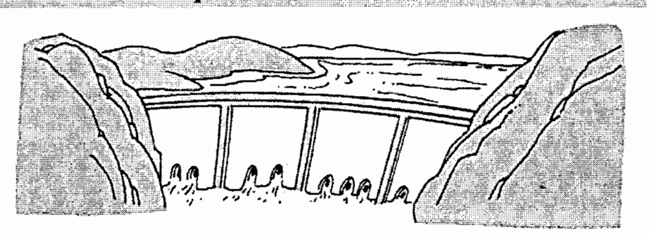
Equating equations (i) and (ii) gives:

$$\frac{72}{100}\rho_w vg = \frac{80}{100}\rho_l vg$$

$$\Rightarrow \frac{\rho_l}{\rho_w} = \frac{72}{80}$$

$\therefore$Relative density $= 0.9$

Pressure in a liquid



In designing a dam, like that shown above, the engineer has to know, among other things, about the size and point of action of the resultant force exerted on the dam by the water behind it. This involves making calculations based on the expression of liquid pressure.

The force acting normally on a unit surface area is known as pressure.

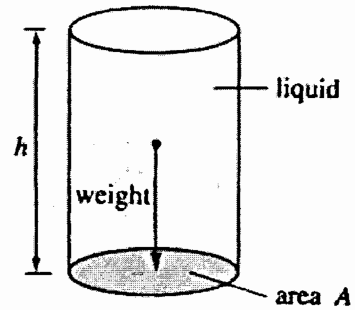
$$\text{Pressure} = \frac{\text{Normal Force}}{\text{Area}} = \frac{F}{A}$$

SI unit of pressure is Nm^{-2} or Pascals (Pa)

$$1\,Nm^{-2} = 1\,Pa$$

Expression for liquid pressure

Consider a column of liquid of height h and base area A as shown below.



Volume of liquid column $= Ah$

If $\rho =$ density of the liquid,

mass of liquid column, $m = Ah\rho$

weight of liquid column $= mg$

$= Ah\rho g$

The weight of the liquid exerts a force on the base area A. Therefore, the pressure at the base of the liquid column is given by

$$\text{Pressure, } P = \frac{\text{normal force}}{\text{area}}$$

$$= \frac{Ah\rho g}{A} = h\rho g$$

Note: Pressure in fluids

(a) increases with depth

(b) acts equally in all directions

Example 6

(a) What is the pressure on the base of a tank of uniform cross-sectional area $4.0\ m^2$ when the tank is filled with water to a depth of $5.0\ m$?

(b) What is the new pressure on the base of the tank when a wood block of volume $1.0\ m^3$ floats on the water in the tank?

(density of water $= 1 \times 10^3 kg\, m^{-3}$, density of wood $= 0.6 \times 10^3\ kg\, m^{-3}$, atmospheric pressure $= 1 \times 10^5\ Pa$)

Solution

(a) Pressure on the base of the tank

$= \text{atmospheric pressure} + h\rho g$

$= 1 \times 10^5 + 5.0 \times 1 \times 10^3 \times 9.81$

$= 1.49 \times 10^5\ Pa$

(b) Total weight of the wood block and water,

$F = (1.0 \times 0.6 \times 10^3 + 4.0 \times 5.0 \times 10^3) \times 9.81$

$= 2.021 \times 10^5\ N$

New pressure $=$ atmospheric pressure $+ \frac{F}{A}$

$= \left(1 \times 10^5 + \frac{2.021 \times 10^5}{4}\right)$

$= 1.505 \times 10^5\ Pa$

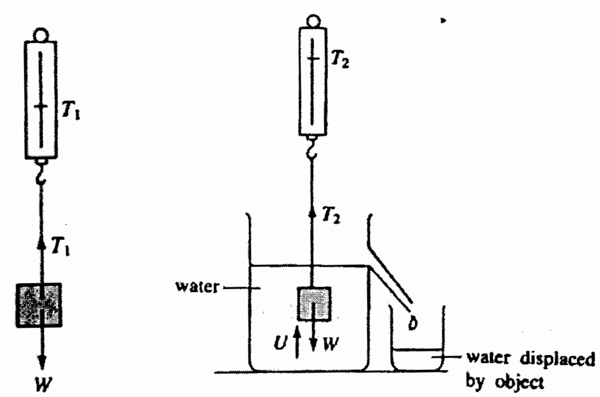
Example 7

Explain why the surface of a liquid at rest is plane and horizontal

Solution

If the surface of a liquid is not plane and horizontal, some parts of the liquid will be at higher pressures than other parts. Since the liquids can flow, the difference in pressure in the liquid will cause those parts of the liquid at higher pressure to flow until the pressure on the liquid surface is the same. Therefore, the liquid surface becomes plane and horizontal

Archimedes' principle



When an object is first weighed in air and then in water, its weight T_2 in water is less than its weight T_1 in air. This is because when in water, there is buoyancy force or upthrust acting upwards on th object.

In air, spring balance reading, $T_1 = W$, weight o object

In water, spring balance reading, $T_2 = W - U$, wher $U =$ upthrust or buoyancy force

Therefore, $T_2 < T_1$ i.e. there is an apparent loss i weight when an object is weighed in water.

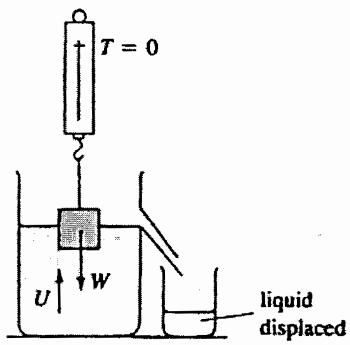
The apparent loss in weight, $T_1 - T_2 = W - (W - U$

$= U$, upthrust

Archimedes' principle states that when an object immersed completely or partially in a fluid, th upthrust is equal to the weight of fluid displaced

Since the upthrust is also equal to the apparent loss i weight, then

Apparent loss in weight = weight of liquid displaced



For an object that floats on a liquid, its appare weight in the liquid, $T = 0$. Since the object is i equilibrium on the liquid surface,

Upthrust, $U = W$, weight of the object

According to Archimedes' principle,

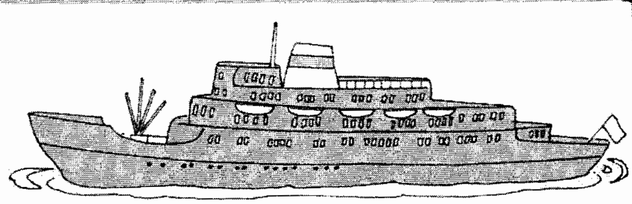
Upthrust, U = weight of fluid displaced

Therefore, for an object floating on liquid,

Weight of object = weight of liquid displaced

The above statement is known as the principle o floatation.

The principle of floatation states that a floati object displaces its own weight of fluid in which floats



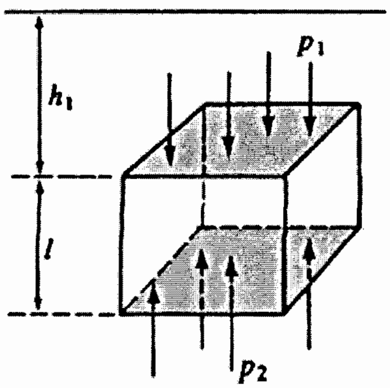
A ship is able to float because it is designed to displac a large volume of water, thus providing a big upthrus

A block of metal however, sinks because it displaces a small volume of liquid and hence the small upthrust is insufficient to balance the weight.

Proof of Archimedes' principle

The buoyancy force or upthrust acting on an object in liquid is due to the difference in pressure acting on the upper and lower surfaces of the object when it is in a liquid. The forces acting horizontally against the sides cancel out one another.

Consider an object of uniform cross-sectional area A, and length l in a liquid of density ρ



The pressure on the top surface, $p_1 = h_1\rho g$

$\Rightarrow$ Force from the top $= h_1\rho gA$

The pressure on the lower surface, $p_2 = (h_1 + l)\rho g$

$\Rightarrow$ Force from the bottom $= (h_1 + l)\rho gA$

Upthrust $= (h_1 + l)\rho gA - h_1\rho gA$

$= \rho gAl$

Al is the volume of the object which is equal to the volume of liquid displaced.

Weight of liquid displaced $= mg = \rho Vg = \rho(Al)g$

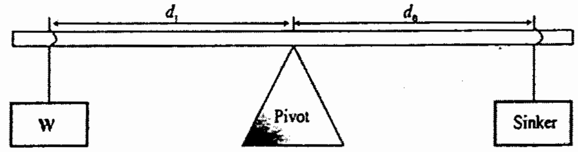
$= \rho gAl$

Therefore, the upthrust is equal to the weight of fluid displaced

Experimental verification of Archimedes' principle

- Suspend a glass stopper from a spring balance to obtain the weight of the stopper in air
- Gently lower the stopper into a displacement can filled to the spout with water.
- The difference between the two spring balance readings is the upthrust on the stopper
- Collect the water that runs out of the can in a previously weighed beaker.
- Weigh the beaker with the water in it to find the weight of the water displaced by the stopper.
- It is found that the weight of the water displaced is equal to the upthrust, hence verification of Archimedes' principle

Experiment to determine the relative density of a liquid using the principle of moments

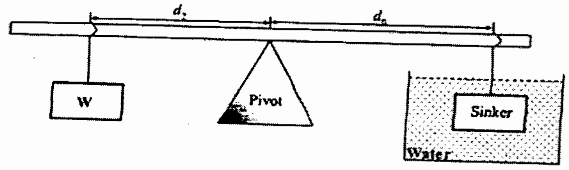


While in air, the sinker (solid) and weight W are attached to the meter rule as shown above. The weight is adjusted until the meter rule balances horizontally. The distances d_1 and d_0 are measured and recorded.

If W_1 is the weight of the sinker in air, then taking moments about the pivot gives:

$$W_1 d_0 = W d_1$$

$$\Rightarrow W_1 = W\frac{d_1}{d_0} \text{ .... (i)}$$

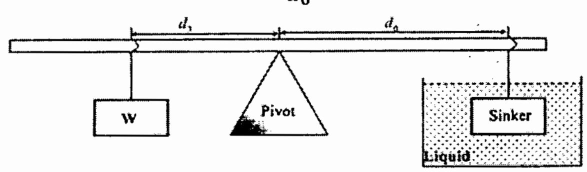


The sinker is then immersed in water in a beaker while keeping d_0 constant. The position of the weight W is adjusted until balance is restored. The distance d_2 is measured.

If W_2 is the weight of the sinker in water, then taking moments about the pivot gives:

$$W_2 d_0 = W d_2$$

$$\Rightarrow W_2 = W\frac{d_2}{d_0} \text{ .... (ii)}$$



The sinker is then immersed in a liquid in a beaker while keeping d_0 constant. The position of the weight W is adjusted until balance is restored. The distance d_3 is measured. If W_3 is the weight of the sinker in water, then taking moments about the pivot gives:

$$W_3 d_0 = W d_3$$

$$\Rightarrow W_3 = W\frac{d_3}{d_0} \text{ .... (iii)}$$

$$\text{R.D} = \frac{\text{apparent loss of weight of the sinker in liqiuid}}{\text{apparent loss of weight of the sinker in water}}$$

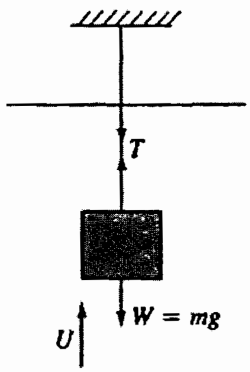
$$= \frac{\left(W\frac{d_1}{d_0} - W\frac{d_3}{d_0}\right)}{\left(W\frac{d_1}{d_0} - W\frac{d_2}{d_0}\right)} = \frac{d_1 - d_3}{d_1 - d_2}$$

Note: The advantage of this method is that relative density can be determined even when the weights are not known.

Example 8

A copper block of mass 0.5 kg is hung from the end of a thread and put in water. If the density of copper is 9.0×10^3 kg m^{-3} and the density of water is 1×10^3 kg m^{-3}, calculate the tension in the thread

Solution



Volume of the block, $V = \frac{\text{mass}}{\text{density}}$

$= \frac{0.5}{9 \times 10^3} = 5.556 \times 10^{-5}$ m^3

= volume of water displaced

By Archimedes' principle,

Upthrust, U = weight of liquid displaced

$= (5.556 \times 10^{-5}) \times (1 \times 10^3) \times 9.81$

$= 0.5450$ N

W (weight of block) = T (tension) + U (upthrust)

$T = W - U$

$= 0.5 \times 9.81 - 0.5450$

$= 4.36$ N

Example 9

What fraction of the volume of an iceberg is above water, if the density of iceberg $\rho_i = 920$ kg m^{-3}, density of sea water $\rho_a = 1030$ kg m^{-3}?

Solution

Let V_i = volume of iceberg

Weight of iceberg $= V_i\rho_i g$

If V_a = volume of iceberg in water, using Archimedes' principle,

Weight of sea water displaced $= V_a\rho_a g$

Since the iceberg floats on water,

Weight of iceberg = weight of sea water displaced

$$V_i\rho_i g = V_a\rho_a g$$

$$\frac{V_a}{V_i} = \frac{\rho_i}{\rho_a}$$

$$= \frac{920}{1030} = 0.8932$$

Fraction of iceberg above water $= 1 - 0.8932$

$= 0.1068$

Example 10

A certain volume V of an iron sphere (density = 7.8 10^{-3} kgm^{-3}) floats in a pool of mercury (density 13.6×10^3 kg m^{-3}). What fraction of the iron submerged under the surface of the mercury?

Solution

Let V′ be the volume of the submerged iron sphere

According to Archimedes' principle,

Weight of sphere = weight of mercury displaced

$$\rho_{Fe} gV = \rho_{Hg} gV'$$

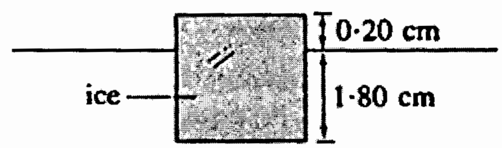
$$\frac{V'}{V} = \frac{\rho_{Fe}}{\rho_{Hg}}$$

$$= \frac{7.8 \times 10^{-3}}{13.6 \times 10^3} = 0.57$$

Example 11

An ice cube of sides 2.0 cm floats in a cup of tea. O of its faces is 0.20 cm above the surface of the tea i cup. Calculate the density of the tea if the density ice is 920 kg m^{-3}.

Solution



By Archimedes' principle, when an object floats on liquid,

weight of object = weight of liquid displaced

$$V_{ice}\rho_{ice} g = V_{tea}\rho_{tea} g$$

$$\rho_{tea} = \frac{V_{ice}}{V_{tea}} \times \rho_{ice}$$

$$= \frac{(2 \times 2 \times 2)}{(2 \times 2 \times 1.8)} \times 920$$

$$= 1.022 \times 10^3 \text{ kg m}^{-3}$$

Example 12

A meteorological balloon of mass 5 kg has a volum of 10 m^3 and is filled with 2 kg of helium. Assume th the density of air is 1.3 kg m^{-3}

(a) Calculate the buoyant force on the balloon

(b) What is the upward acceleration of the balloon?

(c) Discuss qualitatively how high the balloon wil rise

Solution

(a) According to Archimedes' principle, the upward buoyant force on the balloon is equal to the weight W_a of the air displaced

$$W_a = \rho_a gV$$

$$= 1.3 \times 9.81 \times 10$$

$$= 127.53 \text{ N}$$

(b) The total weight of helium and the balloon

$$= (m_1 + m_2)g$$
$$= (5 + 2) \times 9.81 = 68.67 \text{ N}$$

Net upward force $= 127.53 - 68.67 = 58.86$ N

From, $F = ma$, $a = \frac{F}{m} = \frac{58.86}{7} = 8.41 \text{ m s}^{-2}$

) Because of the fact that the density of the air decreases with elevation, the weight of the displaced air also decreases gradually as the balloon rises. Eventually, the balloon reaches an elevation where the weight of the displaced air is 68.67 N and the balloon stops ascending

xample 13

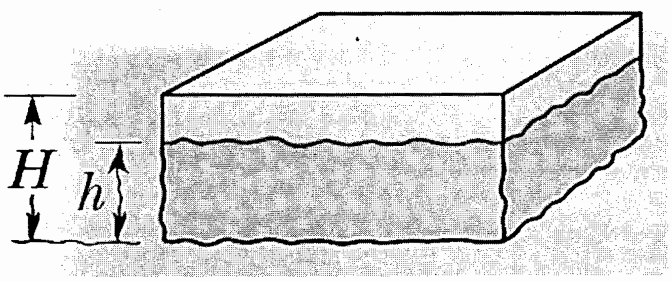
block of density 800 kg m^{-3} floats face down in a uid of density $1200 \, kg \, \text{m}^{-3}$. The block has a height f 6.0 cm

(a) By what depth is the block submerged?

(b) If the block is held fully submerged and then released, what is the magnitude of its acceleration?

Solution

a)



Upthrust = weight of fluid displaced

$$\rho_b AHg = \rho_f Ahg$$
$$h = \frac{\rho_b}{\rho_f} H$$
$$= \frac{800}{1200} \times 6$$
$$= 4 \, cm$$

(b) The weight of the block is the same but with the block fully submerged, the volume of the displaced fluid will be bigger giving rise to a bigger buoyancy force. The block will nolonger remain stationary but accelerate upwards.

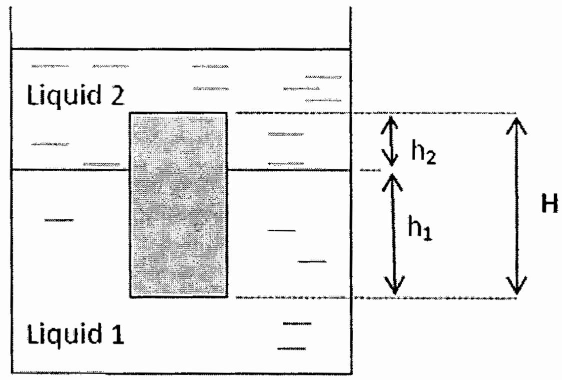
From Newton's second law;

$$U - W = ma$$
$$\rho_f AHg - \rho_b AHg = \rho_b AHa$$
$$a = \left(\frac{\rho_f}{\rho_b} - 1\right) g$$
$$= \left(\frac{1200}{800} - 1\right) \times 9.81$$
$$= 4.905 \, m \, s^{-2}$$

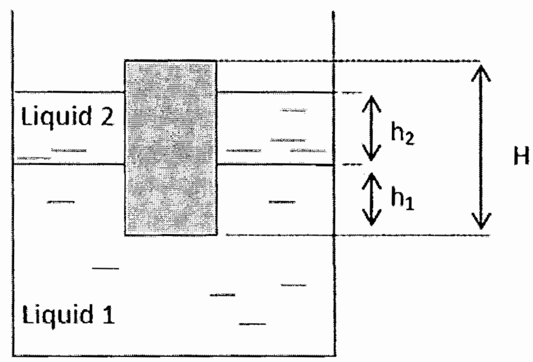
Floating between two liquids

Consider the hydrostatic equilibrium of a body of height H, area of the cross-section A, mean density ρ_s situated in a container with two immiscible liquids of density, respectively ρ_1 and ρ_2, such that $\rho_1 > \rho_s > \rho_2$. The hydrostatic equilibrium of the solid "between two liquids" may be like one of the two cases shown below

Case 1: The floating object is covered by liquid 2



Case 2: The floating object is not covered by liquid 2, then body is in contact with the air



According to Archimedes' principle, upthrusts due to each liquid and the weight of the body balance out.

$$\rho_1 h_1 Ag + \rho_2 h_2 Ag = \rho_s AHg$$
$$\rho_1 h_1 + \rho_2 h_2 = \rho_s H \quad \ldots\ldots\ldots \text{(i)}$$
$$h_1 = \frac{\rho_s H - \rho_2 h_2}{\rho_1}$$
$$\text{Or } h_2 = \frac{\rho_s H - \rho_1 h_1}{\rho_2}$$

These expressions give the height of the object immersed in each liquid for both cases.

Notes:

1. For case 1: $h_1 + h_2 = H$

$$h_2 = H - h_1$$

Substituting for h_2 in (i);

$$\rho_1 h_1 + \rho_2 (H - h_1) = \rho_s H$$
$$\rho_1 h_1 + \rho_2 H - \rho_2 h_1 = \rho_s H$$
$$\rho_1 h_1 - \rho_2 h_1 = \rho_s H - \rho_2 H$$

$$h_1 = \left(\frac{\rho_s - \rho_2}{\rho_1 - \rho_2}\right)H$$

If $\rho_s = \rho_2$, $h_1 = 0$ i.e. the object will be submerged in liquid 2. This is similar to replacing the body with the same volume of liquid 2

Now substituting for H in (i);

$$\rho_1 h_1 + \rho_2 h_2 = \rho_s(h_1 + h_2)$$
$$\rho_1 h_1 + \rho_2 h_2 = \rho_s h_1 + \rho_s h_2$$
$$\rho_1 h_1 - \rho_s h_1 = \rho_s h_2 - \rho_2 h_2$$
$$h_1(\rho_1 - \rho_s) = h_2(\rho_s - \rho_2)$$
$$h_1 = \left(\frac{\rho_s - \rho_2}{\rho_1 - \rho_s}\right)h_2$$
$$\text{Or } h_2 = \left(\frac{\rho_1 - \rho_s}{\rho_s - \rho_2}\right)h_1$$

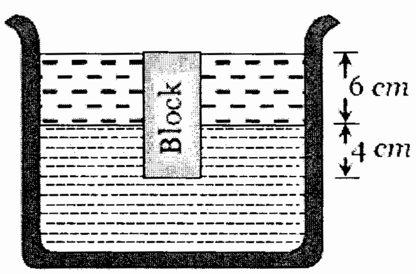
It is clear that if $\rho_1 = \rho_s$, $h_2 = 0$ i.e. the object will be submerged in liquid 1.

2. For case 2; $h_1 + h_2 < H$
   Neglecting the effects of buoyancy due to air, the height, h of the body above liquid 2 will be given by
   $$h = H - (h_1 + h_2)$$

Example 14

A cubical block of wood 10 cm on a side floats at the interface between oil and water with its lower surface horizontal and 4 cm below the interface. The density of oil is 0.6 g cm^{-3} . Calculate the mass of the block

Solution



From Archimedes' principle,

Weight of block = upthrust due to both liquids

$$mg = \rho_o V_o g + \rho_w V_w g$$
$$m = \rho_o A h_o + \rho_w A h_w$$
$$m = A(\rho_o h_o + \rho_w h_w)$$

$A = 10 \times 10 = 100\, cm^2$, $h_o = 6\, cm$, $h_w = 4\, cm$

$\rho_o = 0.6$ g cm^{-3}, $\rho_w = 1$ g cm^{-3}

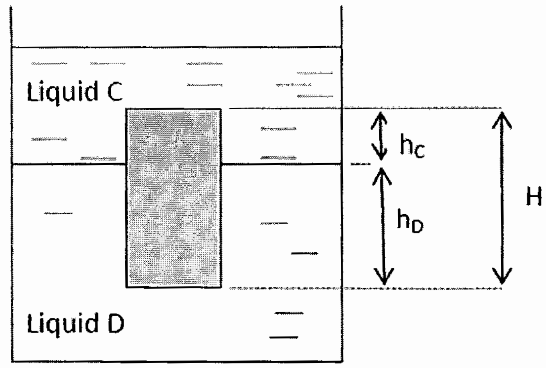
$$\therefore m = 100(0.6 \times 6 + 1 \times 4)$$
$$= 760 \text{ g or } 0.760\, kg$$

Example 15

Liquid C of density ρ_C floats on liquid D of density ρ_D without mixing. A solid object of density ρ floats with part of its volume in liquid D and the remainder in liquid C. Prove that the fraction of the volume of the object immersed in liquid D is given by the expression $\frac{\rho - \rho_C}{\rho_D - \rho_C}$

Solution

Let the object be of uniform cross-sectional area A and height H



Weight of object = upthrust due to C + Upthrust due to D

$$\rho A H g = \rho_C A h_C g + \rho_D A h_D g$$
$$\rho H = \rho_C h_C + \rho_D h_D$$

But $H = h_C + h_D$

$$\rho(h_C + h_D) = \rho_C h_C + \rho_D h_D$$
$$\rho h_C + \rho h_D = \rho_C h_C + \rho_D h_D$$
$$\rho h_C - \rho_C h_C = \rho_D h_D - \rho h_D$$
$$h_C = \left(\frac{\rho_D - \rho}{\rho - \rho_C}\right)h_D$$

$$\text{Volume fraction in D} = \frac{V_D}{V_C + V_D} = \frac{A h_D}{A h_C + A h_D}$$
$$= \frac{h_D}{h_C + h_D}$$

Substituting for h_C;

$$\text{Volume fraction in D} = \frac{h_D}{\left(\frac{\rho_D - \rho}{\rho - \rho_C}\right)h_D + h_D} = \frac{h_D}{h_D\left[\frac{\rho_D - \rho}{\rho - \rho_C} + 1\right]}$$
$$= \frac{1}{\left[\frac{\rho_D - \rho_C + \rho - \rho_C}{\rho - \rho_C}\right]}$$
$$= \frac{\rho - \rho_C}{\rho_D - \rho_C}$$

Self-Evaluation exercise

1. A solid weighs 237.5 g in air and 212.5 g whe totally immersed in a liquid of density 0.9 g/cc Calculate the
   (i) density of the solid,
   (ii) density of a liquid in which the solid would float with $\frac{1}{5}$ of its volume exposed above the liquid surface.
   [Ans. (i) $\rho = 9500\, kgm^{-3}$ (ii) $\rho = 1190\, kgm^{-3}$]
2. A body has a weight of 160 N when weighed in air and a weight of 120 N when totally immersed in a

liquid of relative density 0.8. What is the relative density of the body? [Ans: 3.2]

3. A ball with volume $32\ cm^3$ floats on water with exactly half of the ball below the surface. What is the mass of the ball? (Density of water = $1000\ kg\,m^{-3}$) [Ans: 16 g]

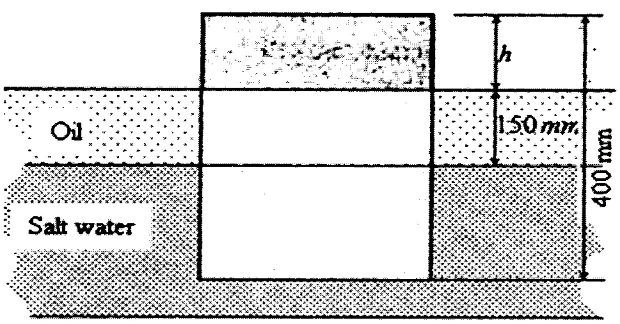
4. An object floats in a liquid of density $1.2 \times 10^3\ kg\,m^{-3}$ with one quarter of its volume above the liquid surface. What is the density of the object? [Ans: $900\ kg\,m^{-3}$]

5. An object with a volume of $1.0 \times 10^{-5}\ m^3$ and density $400\ kg\,m^{-3}$ floats on water in a tank of cross-sectional area $1.0 \times 10^{-3}\ m^2$.
   (a) By how much does the water level drop when the object is removed?
   (b) Show that this decrease in water level reduces the force on the base of the tank by an amount equal to the weight of the object.
   [Ans: (a) $4.0\ mm$]

6. A block of wood of density ρ floats at the interface between immiscible liquids of densities ρ_1 and ρ_2, such that three quarters of the block's volume is in the liquid of density ρ_1. If the whole block is covered by the liquids, and given that $\rho_2 > \rho_1$, show that

$$\frac{\rho_2 - \rho_1}{\rho - \rho_1} = 4$$

7.



In the figure above, a water proof block of wood in form of cube of side 400 mm is floating in a tank of salt water with a 150 mm layer of oil floating on the water. Given that the density of oil is 900 kgm^{-3}, that of salt water 1030 kgm^{-3} and that of wood 800kgm^{-3}, calculate the height h of the block above the surface of the oil. [Ans $h = 70.4\ mm$]

8. A block of wood floats in water of density $1000\ kg\,m^{-3}$ with $\frac{2}{3}$ of its volume submerged. In oil it has $\frac{9}{10}$ of its volume submerged. Find the densities of wood and oil.
   [Ans: $\rho_{oil} = 740.74, \rho_{water} = 666.67\ kgm^{-3}$]

9. Define pressure and state its SI unit
   Derive an expression for the pressure at a point at a depth h of a liquid of density ρ. Does it also hold for gas?
   What force is exerted on the bottom of a tank of uniform cross-section area $2.0\ m^2$ by water which fills it to a depth of $0.50\ m$? (density of water = $1000\ kg\,m^{-3}$, $g = 10\ m\,s^{-2}$)
   Find the extra force on the bottom of the tank when a block of wood of volume $0.1\ m^3$ and relative density 0.50 floats on the surface
   [Ans: 10 kN, 500 N]

10. a) State Archimedes' principle
    b) A string supports a solid copper block of mass 1 kg (density $9 \times 10^3\ kg\,m^{-3}$) which is completely immersed in water (density $1 \times 10^3\ kg\,m^{-3}$). Calculate the tension in the string. [Ans: 9 N]

11. A tank contains a liquid of density $1.2 \times 10^3\ kg\,m^{-3}$. A body of volume $5.0 \times 10^{-3}\ m^3$ and density $900\ kg\,m^{-3}$ is totally immersed in the liquid and is attached by a thread to the bottom of the tank. What is the tension in the thread?
    [Ans: 15 N]

12. a) Define density
    b) Copper has density $8930\ kg\,m^{-3}$ and zinc has a density $7140\ kg\,m^{-3}$. Brass is an alloy consisting of 70% copper and 30% zinc by volume. Assume that the volume of the alloy is equal to the some of the volumes of the zinc and copper used to calculate the density of brass.
    [Ans: $8394\ kg\,m^{-3}$]

13. A simple hydrometer, consisting of a loaded glass bulb fixed at the bottom of the glass stem of uniform section, sinks in water of density $1.0\ g\,cm^{-3}$ so that a certain mark X on the stem is 4.0 cm below the surface. It sinks in a liquid of density $0.90\ g\,cm^{-3}$ until X is 6.0 cm below the surface. It is then placed in a liquid of density $1.1\ g\,cm^{-3}$. How far below the surface will X be?
    [Ans: 2.4 cm]

14. A physicist inflates a balloon with air to a volume of 1.5 litres and seals it. The density of the surrounding air at the time of the experiment was 1.30 g per litre.
    (a) Calculate the upthrust on the balloon after the inflation process.
    (b) Given that the mass of the air in the balloon is 2.15 g and the mass of the balloon fabric is

4.10 g, calculate the density of the air in the balloon

(c) Why is the density of the air in the balloon greater than the density of air outside the balloon?

[Ans: (a) 0.0191 N (b) 1.43 $g\,l^{-1}$]

15. A hot air balloon has a volume of $500\ m^3$. The balloon moves upwards at a constant speed in air of density $1.2\ kg\,m^{-3}$ when the density of the hot air inside it is $0.80\ kg\,m^{-3}$.
    (a) What is the combined mass of the balloon and the air inside it?
    (b) What is the upward acceleration of the balloon when the temperature of the air inside it has been increased so that its density is $0.7\ kg\,m^{-3}$?

[Ans: (a) 600 kg (b) $0.91\ m\,s^{-2}$]

16. A block of wood floats in fresh water with two thirds of its volume V submerged and in oil with 0.90 V submerged. Find the density of
    (a) the wood
    (b) the oil

[Ans: (a) $6.7 \times 10^2\ kg\,m^{-3}$(b) $7.4 \times 10^2\ kg\,m^{-3}$]

17. An iron anchor of density $7870\ kgm^{-3}$ appears 200 N lighter in water than in air.
    (a) What is the volume of the anchor?
    (b) How much does it weigh in air?

[Ans: (a) $2.04\ \times 10^{-2}\ m^3$(b) 1.57 kN]

18. A hollow spherical iron shell floats almost completely submerged in water. The outer diameter is 60.0 cm and the density of iron is $7.87\ g\,cm^{-3}$. Find the inner diameter.

[Ans:57.3 cm]

19. A hollow sphere of inner radius 8.0 cm and outer radius 9.0 cm floats half submerged in a liquid of density $800\ kg\,m^{-3}$.
    (a) What is the mass of the sphere?
    (b) Calculate the density of the material of which the sphere is made

[Ans: (a) 1.2 kg (b) $1.3 \times 10^3\ kg\,m^{-3}$]

20. An object hangs from a spring balance. The balance registers 30 N in air, 20 N when this object is immersed in water, and 24 N when the object is immersed in another liquid of unknown density. What is the density of that other liquid?

[Ans: $600\ kg\,m^{-3}$]

21. An alloy of mass 588 g and volume $100\ cm^3$ is made of iron of relative density 8.0 and aluminium of relative density 2.7. Calculate the proportion (i) by volume (ii) by mass of the constituents of t alloy

[Ans: (i) 3: 2 (ii) 40:

22. A string supports a solid iron object of mass 18(totally immersed in a liquid of dens $800\ kg\,m^{-3}$. Calculate the tension in the string the density of iron is $8000\ kg\,m^{-3}$

[Ans: 1.59

23. A hydrometer floats in water with 6.0 cm of graduated stem immersed, and in oil of relati density 0.8 with 4.0 cm of the stem immers What is the length of stem immersed when t hydrometer is placed in a liquid of relative dens 0.9?

[Ans: 5.1 c

24. An alloy of mass 170 g has an apparent weight 0.932 N in a liquid of density $1.5\ g\,cm^{-3}$. If tl two constituents of the alloy have relative densiti of 4.0 and 3.0 respectively, calculate tl proportion by volume of the constituents in tl alloy.

[Ans: 2 :

25. A solid weighs 237.5 g in air and 12.5 g whe totally immersed in a liquid of relative density 0.9 Calculate the relative density of a liquid in whicl the solid would float with one-fifth of its volume exposed above the liquid surface.

[Ans: 1.19]

FLUID FLOW

Fluid flow is a branch of fluid mechanics that deals with fluids in motion. A fluid is a substance that can flow when an external force is applied on it. The term fluids includes both liquids and gases. Gases are compressible while liquids are nearly incompressible.

Two types of fluid flow exist i.e. streamline flow (also known as orderly, steady flow or uniform flow) and turbulent flow (also known as disorderly flow)

Streamline flow

The flow of a fluid is said to be steady, streamline or uniform if all the fluid particles that pass any given point follow the same path at the same speed.

Turbulent flow

This is a type of fluid flow where the speed and direction of the fluid particles passing any point vary with time

Line of flow

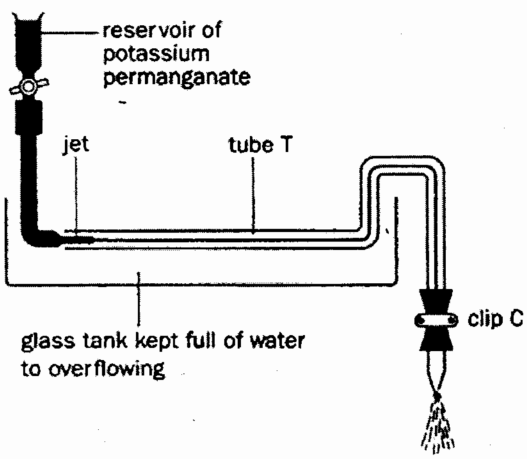
This is the path followed by a particle of the fluid.

Streamline

A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particle at that point. Streamlines never cross.

Demonstration of steady and turbulent flow

When the velocity of flow exceeds a particular critical value, the motion becomes turbulent, the liquid is churned up and the streamlines are no longer parallel and straight. The apparatus shown below can be used to study the change from steady to turbulent flow



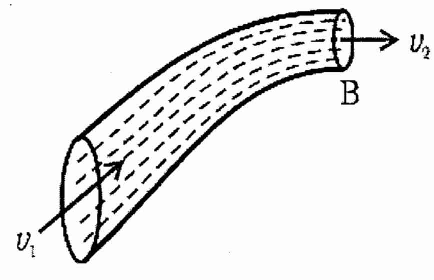
The flow of water along the tube T is controlled by the clip C.

Potassium permanganate solution from the reservoir is fed into the water flowing through *T* by a fine jet.

At low flow velocities, a fine coloured stream is observed along the centre of *T*, but as the rate of flow increase, it starts to break up and the colour rapidly spreads out throughout *T* indicating the onset of turbulence.

Equation of continuity

Consider a non-viscous liquid in streamline flow through a tube AB of varying cross-section as shown below. Let A_1 and A_2 be the area of cross-section, v_1 and v_2 be velocity of flow of the liquid at A and B respectively.



Volume of liquid entering per second at A

$$= A_1 v_1$$

If ρ is the density of the liquid, then mass of liquid entering per second at A, $m_A = A_1 v_1 \rho$

Mass of liquid leaving per second at B

$$m_B = A_2 V_2 \rho$$

If there is no loss of liquid in the tube and the flow is steady, then $m_A = m_B$

$$A_1 v_1 \rho = A_2 v_2 \rho$$

$$A_1 v_1 = A_2 v_2$$

$$\boldsymbol{Av = constant}$$

This is called the equation of continuity and from this equation, $v \propto \frac{1}{A}$ which means the larger the area of cross-section, the smaller will be the velocity of flow of liquid and vice versa.

Example 1

A cylindrical tube of a spare pump has a cross sectional area of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the fluid flow inside the tube is 1.3 $m\ \text{min}^{-1}$, what is the speed of ejection of the liquid through the holes

Solution

Total cross-sectional area of 40 holes,

$$A_2 = 40 \times \pi \frac{(1 \times 10^{-3})^2}{4} = 7.85 \times 10^{-7}\ m^2$$

Cross-sectional area, $A_1 = 8 \times 10^{-4}\ m^2$

Speed inside the tube, $v_1 = \frac{1.5\ m}{min} = \frac{1.5}{60}\ m\ s^{-1}$

Speed of ejection, $v_2 = ?$

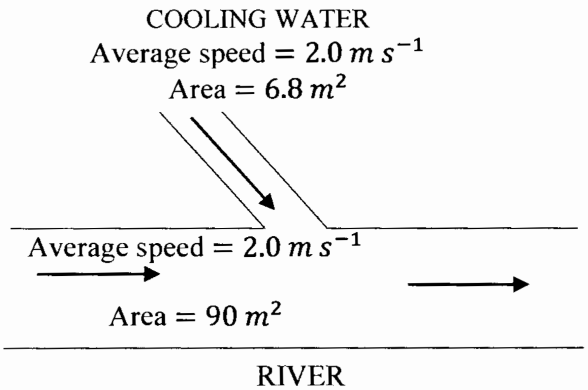
From $A_1 v_1 = A_2 v_2$;

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{(8 \times 10^{-4}) \times \frac{1.5}{60}}{7.85 \times 10^{-7}}$$

$$v_2 = 0.64\ m\ s^{-1}$$

Example 2

The figure below shows the cooling water from a power station joining a river. The areas given on the diagram are the cross-sectional areas of the respective flow regions



(a) Calculate the volume flow rate of the water in the river before cooling water joins it

(b) Calculate the river velocity after the cooling water has joined it

Solution

(a) Volume flow rate $= Av$

$= 90 \times 2 = 180\ \text{m}^3\ \text{s}^{-1}$

(b) Initial volume per second of river + volume per second of cooling water = final volume per second of river

Or $A_1v_1 + A_2v_2 = A_3v_3$

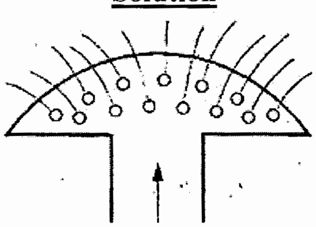
$180 + 6.8 \times 2 = 90\ v$

$v = 2.15\ m\,s^{-1}$

Example 3

A garden sprinkler has 150 small holes, each 2.0 mm^2 in area. If water is supplied at the rate of $3.0 \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$, what is the average velocity of the spray?

Solution



Using the continuity equation, $A_1v_1 = A_2v_2$

Volume supplied per second from sprinkler = volume per second released from sprinkler holes

Total area of sprinkler holes $= 150 \times 2 \times 10^{-6}\ \text{m}^2$

$= 3 \times 10^{-4}\ \text{m}^2$

Let the average velocity of spray be v

$3 \times 10^{-4} v = 3.0 \times 10^{-3}$

$v = 10\ \text{m}\,\text{s}^{-1}$

Bernoulli's principle

Daniel Bernoulli proposed a theorem for the streamline flow of a fluid based on the law of conservation of energy

It states that for the streamline flow of a non-viscous incompressible fluid, the sum of the pressure, kinetic energy and potential energy per unit volume is a constant.

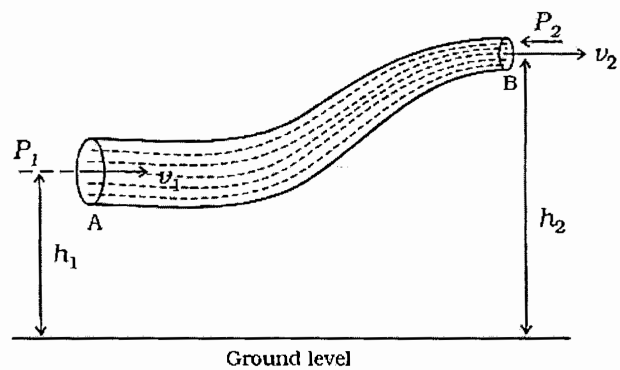
$$\boldsymbol{P + \frac{1}{2}\rho v^2 + \rho g h = constant}$$

This is known as Bernoulli's equation

Derivation

Consider streamline flow of a non-viscous incompressible fluid of density ρ through a pipe AB of varying cross-section. Let P_1 and P_2 be the pressures, A_1 and A_2, the cross-sectional areas at A and B respectively. The fluid enters A normally with velocity v_1 and leaves B normally with velocity v_2.

The fluid is accelerated against the force of gravity while flowing from A to B because the height of B is greater than that of A



Force acting on the fluid at A, $F_A = P_1A_1$

Force acting on the fluid at B, $F_B = P_2A_2$

If F_A moves the fluid a distance Δx and the fluid moves a distance Δy against the force F_B, then

Work done on the fluid at A

$= P_1A_1 \times \Delta x = P_1A_1\Delta x = P_1V$

Work done by the fluid at B

$= P_1A_2 \times \Delta y = P_2A_2\Delta y = P_2V$

Net work done on the fluid $= P_1V - P_2V$

Gain in potential energy $= mgh_2 - mgh_1$

Gain in kinetic energy $= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Work done = Gain in P.E + Gain in K.E

$P_1V - P_2V = mgh_2 - mgh_1 + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$P_1\frac{m}{\rho} - P_2\frac{m}{\rho} = mgh_2 - mgh_1 + \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$

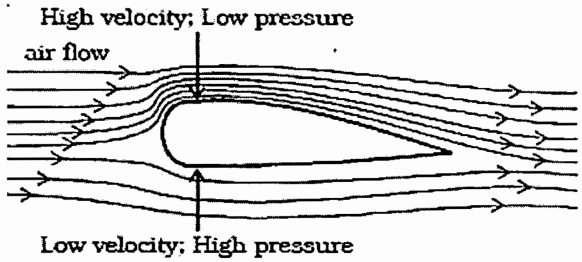
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$\boldsymbol{P + \frac{1}{2}\rho v^2 + \rho g h = constant}$$

Note: Bernoulli's equation cannot apply exactly. Real fluids are viscous and gases are easily compressed.

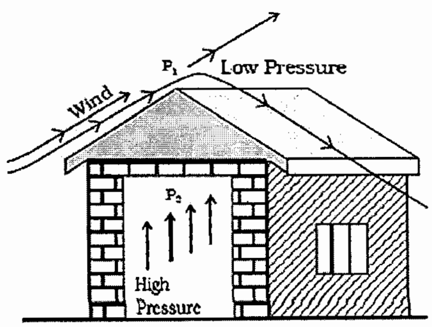
Applications of Bernoulli's principle

1. **Aerofoil**
   This is a device which is shaped so that the relative motion between it and a fluid produces a force perpendicular to the flow. Examples of aerofoils are aircraft wings, turbine blades and propellers



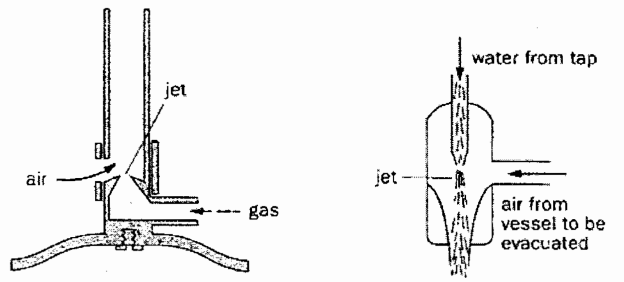
   The shape of the air craft wing is such that fluid flows faster over the top surface than over the bottom i.e. the streamlines are closer above than below the wing.
   By Bernoulli's principle, it follows that the pressure underneath is increased and that above reduced. A resultant upward force is thus created, normal to the flow, and it is this force which provides most of the lift of an aeroplane.

2. **Blowing of roofs**



   During storms, the roofs or huts or tinned roofs are blown off without any damage to other parts of the hut. The blowing wind creates a low pressure P_1 on top of the roof. The pressure P_2 under the roof is however greater than P_1. Due to this pressure difference, the roof is lifted and blown off with the wind.

3. **Jets and nozzles**
   Bernoulli's equation suggests that for fluid flow where the potential energy change $h\rho g$ is very small or zero, as in a horizontal pipe, the pressure falls when the velocity rises. The velocity increases at a constriction (a slow stream of water from a tap can be converted into a fast jet by narrowing the exit with a finger) and the greater the change in cross-sectional area, the greater is the increase of velocity and so the greater is the pressure drop. Several devices with jets and nozzles use this effect e.g. Bunsen burners, filter pumps, paint sprays, etc.

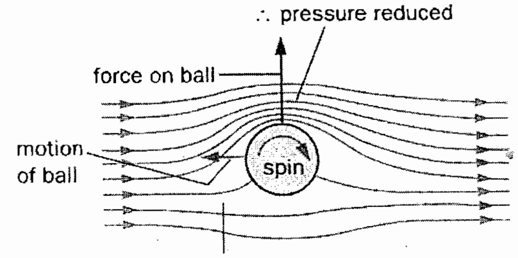


(a) Bunsen burner (b) Filter pump

4. **Motion of two parallel boats**
   When two boats separated by a small distance row parallel to each other along the same direction, the velocity of water between the two boats becomes very large compared to that on the outer sides. Because of this, the pressure in between the two boats gets reduced. The high pressure on the outer sides pushes the boats inwards. As a result of this, the boats come closer and may collide

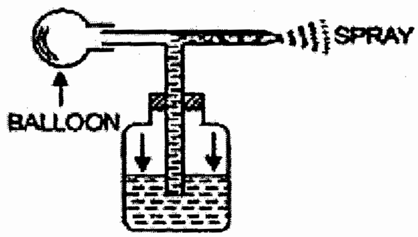
5. **A person standing close to a railway line**
   The air between the person and a moving train has a higher velocity than that behind him. It therefore follows that the pressure in front of the man is less than that behind him. This causes a pull on the man towards the train

6. **Spinning ball**
   If a tennis ball is cut or a golf ball is sliced, it spins as it travels through the air and experiences a sideways force which causes it to curve in flight. This is due to air being dragged round by the spinning ball, thereby increasing the air flow on one side and decreasing it on the other. A pressure difference is thus created



7. **Atomiser or sprayer**

It is based on Bernoulli's principle. It is used to spray liquid. It is generally used in perfumes and deodorant bottles.



When the rubber balloon is pressed, the air in the horizontal tube passes with a large velocity. According to Bernoulli's theorem, the pressure in the tube will be reduced. But the pressure in the container is equal to atmospheric pressure. This pressure difference makes the liquid rise in a vertical tube. On the top of the vertical tube, the liquid is blown away through the nozzle in the form of a fine spray

Example 4

A particular aircraft design calls for a dynamic lift of 2.4×10^4 N on each square metere of the wing when the speed of the air craft through the air is $80\ ms^{-1}$. Assuming that the air flows past the wing with streamline line flow and that the flow past the lower surface is equal to the speed of the air craft, what is the required speed of the air over the upper surface of the wing? (Assume that density of air is 1.29 kgm^{-3})

Solution

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Where, P_1 is the pressure on upper part, P_2 pressure on lower part, v_1 the velocity of air on upper part, v_2 velocity of air on lower surface and A = 1 m² the area of the wing

$$h_1 = h_2 = h$$

$$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= \frac{1}{2} \times 1.29(v_1^2 - 80^2)$$

force of dynamic lift = (pressure difference) × (surface area of wing)

$$\therefore 24000 = \left[\frac{1}{2} \times 1.29(v_1^2 - 6400)\right] \times 1$$

$$v_1 = 208.8\ ms^{-1}$$

Example 5

In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surface of the wing are $70\ ms^{-1}$ and $63\ ms^{-1}$ respectively. What is the lift on the wing if its area is $2.5\ m^2$? Take the density of air to be $1.3\ kg\, m^{-3}$

Solution

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Force on wing, $F = (P_1 - P_2)A$

where A is area of wing

$$= \frac{1}{2}\rho(v_2^2 - v_1^2)A$$

$$= \frac{1}{2} \times 1.3(70^2 - 63^2) \times 2.5$$

$$= 1512.875\ N$$

Example 6

Calculate the minimum pressure required to force the blood from the heart to the top of the head (a vertical distance of $0.5\ m$) given that the density of blood = $1040\ kg\, m^{-3}$. Neglect friction

Solution

$h_2 - h_1 = 0.5\ m$, $\rho = 1040\ kg\, m^{-3}$

According to Bernoulli's theorem

$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

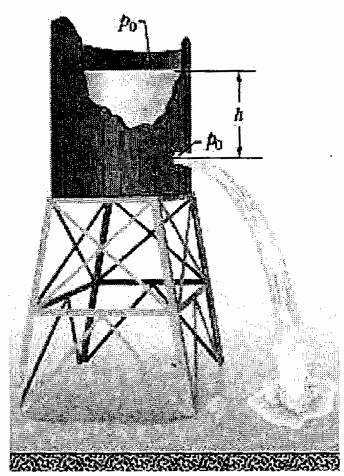
If $v_2 = v_1$, then $P_1 - P_2 = \rho g(h_2 - h_1)$

$$P_1 - P_2 = 1040 \times 9.81 \times 0.5$$

$$P_1 - P_2 = 5.096 \times 10^3\ N\, m^{-2}$$

Bernoulli's principle for a leaky water tank

Water pours through a hole in a water tank, at a distance h below the water surface. The pressure at the water surface and at the hole is atmospheric pressure p_0.



At the top, $h_1 = h, v_1 = 0, P_1 = p_0$
At the bottom, $P_2 = p_0, h_2 = 0, v_2 = v$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$p_0 + \frac{1}{2} \times \rho(0)^2 + \rho g h = p_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

$$\Rightarrow \frac{1}{2}v^2 = gh$$

$$\therefore v = \sqrt{2gh}$$

This is the same speed that an object would have when falling a height h from rest.

Example 7

Obtain an estimate for the velocity of emergence of a liquid from a hole in the side of a wide vessel 10 cm below the liquid surface

Solution

$$h\rho g = \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.81 \times 0.1} = 1.4\, m\, s^{-1}$$

Example 8

An open tank holds water 1.25 m deep. As small hole of cross sectional area 3 cm^2 is made at the bottom of the tank. Assuming that the density of water is 1000 kgm^{-3}, calculate the mass of water per second initially flowing out of the hole.

Solution

Mass per second = (volume per second) × (density)
∴ Volume per second = (area) ×(velocity)
⟹ Mass per second = (area) × (velocity) × (density)

$$\text{Velocity} = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.81 \times 1.25}$$

$$= 4.95\, m\, s^{-1}$$

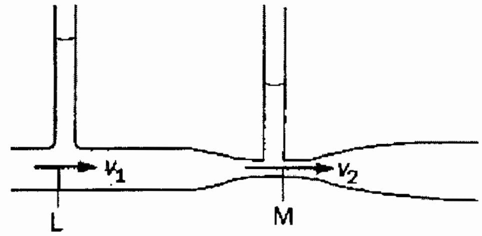
∴ Mass per second = $(3 \times 10^{-4}) \times 4.95 \times 1000$
$= 1.49\, kg\, s^{-1}$

FLOWMETERS

Flowmeters measure the rate of flow of a fluid through a pipe.

Venturi meter

This consists of a horizontal tube with a constriction and replaces part of the piping system. The two vertical tubes record the pressures (above atmospheric) in the fluid flowing in the normal part of the tube and in the constriction



If p_1 and p_2 are the pressures and v_1 and v_2 the velocities of the fluid (density ρ) at L and M on the same horizontal level. Assuming Bernoulli's equation holds,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \; (h_1 = h_2)$$

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

If A_1 and A_2 are the cross-sectional areas at L and M respectively and the fluid is incompressible, the same volume passes each section of the tube per second

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

$$p_1 - p_2 = \frac{1}{2}\rho\left(\left(\frac{A_1}{A_2}v_1\right)^2 - v_1^2\right)$$

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

Knowing A_1, A_2, ρ and $p_1 - p_2$, v_1 can be found and so the rate of flow $A_1 v_1$

Example 9

Water flows along a horizontal pipe of cross-sectional area $48\, cm^2$ which has a constriction of cross-sectional area 12 cm^2 at one part. If the speed of the water at the constriction is 4 ms^{-1},

(i) Calculate the speed of water at the wider section.
(ii) Given that the pressure at the wider section is 1.0×10^5 Pa, and that the density of water is 1000 kgm^{-3}, calculate the pressure at the constriction.

Solution

(i) $A_1 v_1 = A_2 v_2$

$$v_1 = \frac{A_2}{A_1} v_2 = \frac{12}{48} \times 4 = 1 ms^{-1}$$

(ii) From Bernoulli's principle,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

But since the tube is horizontal is horizontal, $h_1 = h_2$

$$\therefore P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

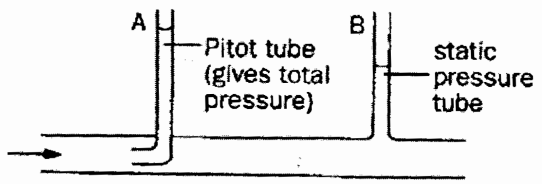
$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$100000 - P_2 = \frac{1}{2} \times 1000 \times (4^2 - 1^2)$$

$$P_2 = 9.25 \times 10^4\, Pa$$

Pitot-static tube

The pressure exerted by a moving fluid, called the total pressure, can be regarded as having two components, the static component, which the fluid would have if it were at rest, and the dynamic component which is the pressure equivalent of its velocity. A pitot tube measures the total pressure and in essence is a manometer with one limb parallel to the flow and open to the oncoming fluid, A. The fluid at the open end is at rest and a stagnant region exists there. The total pressure is called the stagnation pressure. The static component is measured by a manometer connected at right angles to the pipe, B.



From Bernoulli's principle,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

$$\text{Static pressure} = P + \rho g h$$

$$\text{Dynamic pressure} = \frac{1}{2}\rho v^2$$

Static pressure + Dynamic pressure = Total pressure

Dynamic pressure = Total pressure − Static pressure

$$\therefore \frac{1}{2}\rho v^2 = \text{Total pressure} - \text{Static pressure}$$

$$v = \sqrt{\frac{2}{\rho} \times (\text{Total pressure} - \text{Static pressure})}$$

Example 10

Water flows steadily along a uniform flow tube of cross sectional area 30 cm^2. The static pressure is $1.2 \times 10^5\ Pa$ and the total pressure is $1.28 \times 10^5\ Pa$. Assuming that the density of water is 1000 kgm^{-3}, calculate the

(i) flow velocity,

(ii) volume flux,

(iii) mass of water passing through a section of the tube per second.

Solution

(i) $v = \sqrt{\frac{2}{\rho} \times (\text{Total pressure} - \text{Static pressure})}$

$$v = \sqrt{\frac{2}{1000} \times (1.28 - 1.20) \times 10^5}$$

$$v = 4\ ms^{-1}$$

(ii) Volume flux = volume per second

= (area) × (velocity)

$= (30 \times 10^{-4}) \times 4$

$= 0.012\ m^3\ s^{-1}$

(iii) Mass per second

= (volume per second) × (density)

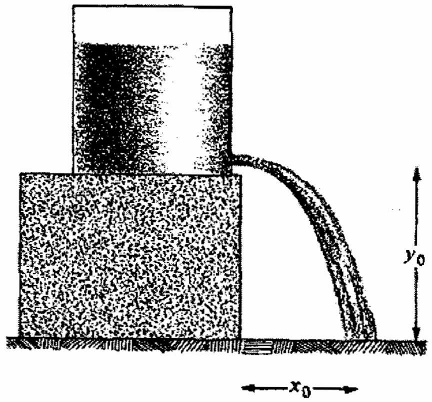
$= 0.012 \times 1000$

$= 12\ kgs^{-1}$

Self-Evaluation exercise

1. What is the fluid velocity associated with the flo of water that issues from a hose of radius 1.1 cm a rate of 50 $cm^3\ s^{-1}$? [Ans: 13 $cm\ s^{-1}$
2. The depth of water in a tank of large cross-section area is maintained at 20 cm and water emerges in continuous stream out of a hole 5 mm in diamet in the base. Calculate
   (a) the speed of efflux of water from the hole
   (b) the rate of mass flow of water from the hole
   (density of water = $1.0 \times 10^3\ kg\ m^{-3}$)
   [Ans: (a) 2.0 $m\ s^{-1}$ (b) $3.9 \times 10^{-2}\ kg\ s^{-1}$
3. The static pressure in a horizontal pipe is 4.3 × $10^4\ Pa$, the total pressure is $4.7 \times 10^4\ Pa$, and th area of cross-section is 20 cm^2. The fluid may b considered to be incompressible and non-viscou and has a density of $10^3 kg\ m^{-3}$. Calculate
   (a) the flow velocity in the pipeline
   (b) the volume flow rate in the pipeline
   [Ans: (a) 2.8 $m\ s^{-1}$ (b) $5.7 \times 10^{-3}\ m^3\ s^{-1}$
4. Air flows over the upper surfaces of the wings of a aeroplane at a speed of 120.0 $m\ s^{-1}$ and past th lower surfaces of the wings at 110.0 $m\ s^{-1}$ Calculate the lift force on the aeroplane if it has total wing area of 20.0 m^2. (density of air = 1.29 $kg\ m^{-3}$)
   [Ans: $2.97 \times 10^4\ N$
5. The velocity of the air on top of an aeroplane wing is 100 ms^{-1} and that on the bottom is 80 ms^{-1}. Assuming that the density of air is 1 $kg\ m^{-3}$.
   (a) What is the difference in pressure on the two sides of the wing?
   (b) Assuming the plane has a mass of $2 \times 10^3 kg$ what must be the minimum area of the wing so that the plane flies?
6. A large tank contains water to a depth of 1.0 m Water emerges from a small hole in the side of the tank 20 cm below the level of the surface. Calculate
   (a) the speed at which the water emerges from the hole
   (b) the distance from the base of the tank at which the water strikes the floor on which the tank is standing
   [Ans: (a) 2.0 $m\ s^{-1}$ (b) 0.8 m]

7. A rain barrel is standing on a platform of height y_0. If a small hole is punched in the bottom of the barrel, it is found that the resultant stream of water strikes the ground at a distance x_0.

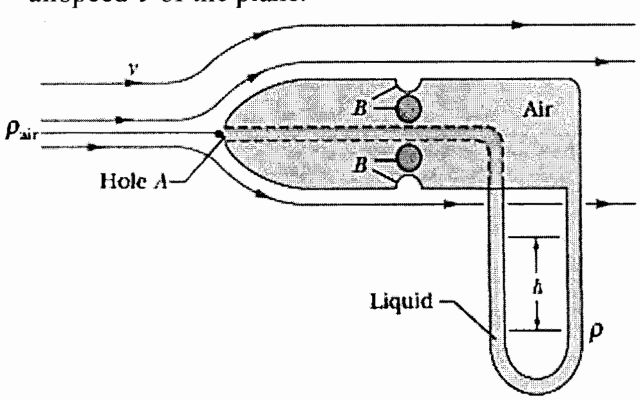


(a) Show that the velocity v_0 of the water as it comes out of the hole is

$$v_0 = \left[\frac{g x_0^2}{2\, y_0}\right]^{\frac{1}{2}}$$

(b) Calculate the height h of the water level above that of the small opening in the bottom.

8. A pitot tube shown below is used to determine the air speed of an aeroplane. It consists of an outer tube with a number of small holes B (four are shown) that allow air into the tube; that tube is connected to one arm of a U-tube. The other arm of the U-tube is connected to hole A at the front end of the device, which points in the direction the plane is headed. At A the air becomes stagnant so that $v_A = 0$. At B, however, the speed of the air presumably equals the airspeed v of the plane.



Use Bernoulli's equation to show that

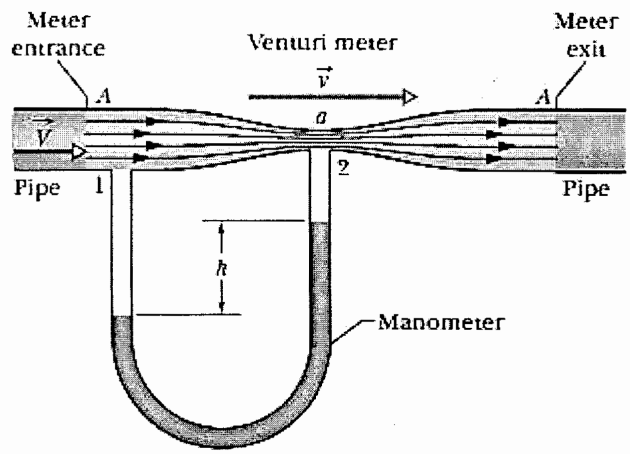
$$v = \sqrt{\frac{2\rho g h}{\rho_{air}}}$$

where ρ is the density of the liquid in the U-tube and h is the difference in the liquid levels in that tube.

9. A garden hose with an internal diameter of 1.9 cm is connected to a stationary lawn sprinkler that's consists merely of a container with 24 holes, each 0.13 cm in diameter. If the water in the hose has a speed of 0.91 $m\,s^{-1}$, at what speed does it leave the sprinkler holes?

[Ans: 8.1 $m\,s^{-1}$]

10. A Venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections of the pipe. The cross-sectional area A of the entrance and exit of the meter matches the pipe's cross-sectional area. Between the entrance and exit, the fluid flows from the pipe with speed V and then through a narrow "throat" of cross-sectional area a with speed v. A manometer connects the wider portion of the meter to the narrower portion. The change in the fluid's speed is accompanied by a change Δp in the fluid's pressure, which causes a height difference h of the liquid in the two arms of the manometer.



(a) By applying Bernoulli's equation and the equation of continuity to points 1 and 2 show that

$$V = \sqrt{\frac{2a^2\Delta p}{\rho(a^2 - A^2)}}$$

where ρ is the density of the fluid.

(b) Suppose that the fluid is fresh water, the cross-sectional areas are 64 cm^2 in the pipe and 32 cm^2 in the throat, and that the pressure is 55 kPa in the pipe and 41 kPa in the throat. What is the rate of water flow in cubic meters per second?

[Ans: $2.0 \times 10^{-2}\ m^3\,s^{-1}$]

11. If the radius at point 1 is 10 cm and at point 2 is 6 cm and difference in the level of the mercury in the two columns of the manometer is 10 cm (refer to the diagram in question 10), calculate

(a) the velocity V

(b) the velocity v

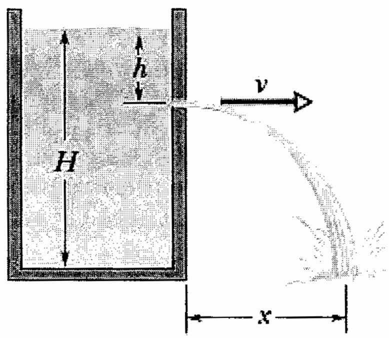
(c) the flow rate

[Ans: (a) 1.9 ms^{-1} (b) 5.3 m s^{-1} (c) 0.06 $m^3\,s^{-1}$]

12. A liquid of density 900 $kg\,m^{-3}$ flows through a horizontal pipe that has a cross-sectional area of $1.90 \times 10^{-2}\ m^2$ in region A and a cross-sectional area of $9.50 \times 10^{-2}\ m^2$ in region B. The pressure difference between the two regions is $7.20 \times 10^3\ Pa$. Calculate the
    (a) volume flow rate
    (b) mass flow rate

[Ans: (a) $0.0776\ m^3\,s^{-1}$ (b) $69.8\ kg\,s^{-1}$]

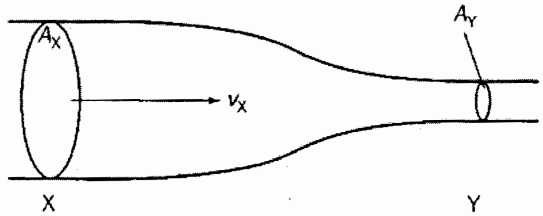
13. The figure below shows a stream of water flowing through a hole at depth $h = 10\ cm$ in a tank holding water to height $H = 40\ cm$.



At what distance x does the stream strike the floor?

[Ans: $34.64\ cm$]

14. An ideal, incompressible fluid of density ρ flows through a horizontal tube which narrows between X, where the area of cross-section is A_X, and Y, where the area of cross section is A_Y as shown below



(a) The liquid has speed V_X at X. Deduce an expression for V_Y at Y.

(b) Consider a mass m of liquid moving from X to Y
    (i) Deduce an expression for the increase in kinetic energy of this mass
    (ii) Apply the principle of conservation of energy to show that the pressure in the fluid decreases as it passes from X to Y
    (iii) Hence show that the drop in pressure is given by

$$\Delta p = \frac{1}{2}\rho(V_Y^2 - V_X^2)$$

15. A horizontal pipe of diameter $36.0\ cm$ tapers to a diameter of $18.0\ cm$ at a constriction. An ideal gas at a pressure of $2.0 \times 10^5\ Pa$ is moving along the wider part of the pipe at a speed of $30.0\ m\,s^{-1}$ The pressure of the gas at the constriction i $1.80 \times 10^5\ Pa$. Assuming that the temperature o the gas remains constant, calculate the speed of th gas at the constriction.

[Ans: $133\ m\,s^{-1}$

16. A simple garden syringe used to produce a jet c water consists of a piston of area $4.0\ cm^2$ whic moves in a horizontal cylinder which has a sma hole of area $4.0\ mm^2$ at its end. If the force on tł piston is $50\ N$, calculate the speed at which tł water is forced out of the small hole, assuming tł speed of the piston is negligible. (density of wat $= 1.0 \times 10^3\ kg\,m^{-3}$)
    Explain why the speed of the piston may l ignored

[Ans: $15.8\ m\,s^{-1}$

17. Water flows through a horizontal pipe of non uniform cross-section. The pressure is $0.01\ m$ of mercury where the velocity of flow is $035\ m\,s^{-1}$. Find the pressure at a point where the velocity is $0.65\ m\,s^{-1}$.

[Ans: $8.9 \times 10^{-3}\ m$ of Hg]

18. A pipe is running full of water. At a certain point P, it tapers from $0.60\ m$ diameter to $0.20\ m$ diameter at the point Q. The pressure difference between P and Q is $1\ m$ of water column. Find the rate of flow of water through the pipe if the pipe is horizontal.

[Ans: $0.14\ m^3s^{-1}$]

19. A pitot tube is fixed in a main pipe of diameter $0.20\ m$ and the difference of pressure indicated by the gauge is $0.05\ m$ water column. Find the volume of water passing through the main pipe in one minute

[Ans: $1.87\ m^3$]

20. Water flows steadily along a uniform flow-tube of cross-section $30 \times 10^{-4}\ m^2$. The static pressure is $1.20 \times 10^5\ Pa$ and the total pressure is $1.28 \times 10^5\ Pa$. Calculate the flow velocity and the mass of the water per second flowing past a section of the tube.

[Ans: $4\ m\,s^{-1}, 12\ kg\,s^{-1}$]

21. Water is maintained at a height of $10\ m$ in a tank. Calculate the diameter of a circular aperture needed at the base of a tank to discharge water at a rate of $26.4\ m^3\,min^{-1}$.

[Ans: $0.2\ m$]

VISCOSITY

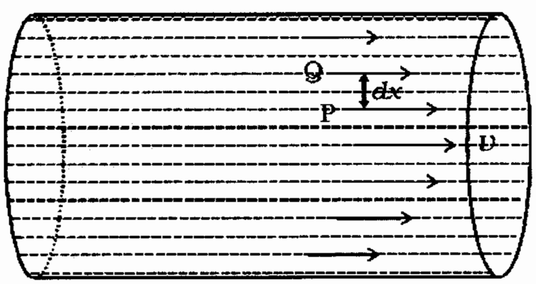
On pouring equal amounts of water and oil in two identical funnels, it is observed that water flows out of the funnel very quickly whereas the flow of oil is very slow. This is because of the frictional force acting within the liquid. This force offered by the adjacent liquid layers is known as viscous force and the phenomenon is called viscosity.
Viscosity is the property of the fluid by virtue of which it opposes relative motion between its different layers. Both liquids and gases exhibit viscosity but liquids are much more viscous than gases.

Origin of viscosity

For a flowing fluid, molecular layers in contact with the sides of the tube are practically stationary because of the attraction between the molecules of the tube and those of the fluid. (adhesive forces). The successive layers towards the center must therefore slide over one another against the attraction between the molecules of the individual layers (cohesive forces). This effect results into layers towards the center of the tube moving faster than those towards the sides of the tube. Since the velocities of the neighboring layers are different, a frictional force occurs between the various layers of the fluid.

Coefficient of viscosity

Consider a liquid flowing steadily through a pipe as shown below



The layers of the liquid which are in contact with walls of the pipe have zero velocity. As we move towards the axis, the velocity of the liquid layer increases and the centre layer has the maximum velocity v.
Consider any two layers P and Q separated by a distance dx. Let dv be the difference in velocity between the two layers.
The viscous force F acting tangentially between the two layers of the liquid is proportional to

(i) area A of the layers in contact

(ii) velocity gradient $\frac{dv}{dx}$ perpendicular to the flow of the liquid

$$F \propto A\frac{dv}{dx}$$
$$F = \eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid
This is known as the Newton's law of viscous flow in fluids

$$\eta = \frac{F}{A\frac{dv}{dx}}$$

If $A = 1\ m^2$, $\frac{dv}{dx} = 1\ s^{-1}$, then $\eta = F$
The coefficient of viscosity of a fluid is the viscous force acting tangentially between two layers of a fluid of area $1\ m^2$ and velocity gradient $1\ s^{-1}$ normal to the direction of flow of the liquid.
SI unit of η is $N\ s\ m^{-2}$. Its dimension is $M\ L^{-1}\ T^{-1}$

Effect of temperature on the viscosity of fluids

Gases

Viscosity in gases increases with increase in temperature.

Explanation

Viscosity in gases is due to momentum transfer between the neighbouring layers of gases. The viscosity in gases is directly proportional to the average speed of the gas molecules and since the average speed of the gas molecules increases with increasing temperature, viscosity in gases increases with increasing temperature.

Liquids

Viscosity in liquids decreases with increase in temperature.

Explanation

Viscosity in liquids is due to molecular attraction between molecules of neighbouring layers. As the temperature increases, the intermolecular forces are broken down and so the molecules travel faster and further apart. This decreases the viscosity.

Note: This is why oil is ineffective as a lubricant when a motor vehicle has just been started.

Stokes' law

When a body falls through a highly viscous liquid, it drags a layer of the liquid immediately in contact with it. This results in relative motion between the different layers of the liquid. As a result of this, the falling body experiences a viscous force, F. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force, F acting on a spherical body depends on

(i) coefficient of viscosity, η
(ii) radius, r of the body
(iii) velocity, v of the spherical body

$$\text{i.e. } F \propto \eta^x v^y r^z$$
$$\Rightarrow F = k\eta^x v^y r^z$$

where k is a constant of proportionality

Using dimensions;

$$[F] = [\eta]^x \times [v]^y \times [r]^z$$
$$MLT^{-2} = (ML^{-1}T^{-1})^x \times (LT^{-1})^y \times (L)^z$$
$$MLT^{-2} = M^x\, L^{-x+y+z}\, T^{y+z}$$

Equating corresponding indices on both sides;

$$x = 1$$
$$1 = -x + y + z$$
$$2 = y + z$$

Solving the three equations simultaneously gives;

$$x = 1,\ \ y = 1\ \ and\ z = 1$$

Other experiments show that $k = 6\pi$

$$\boldsymbol{F = 6\pi\eta v r}$$

This is known as Stokes' law

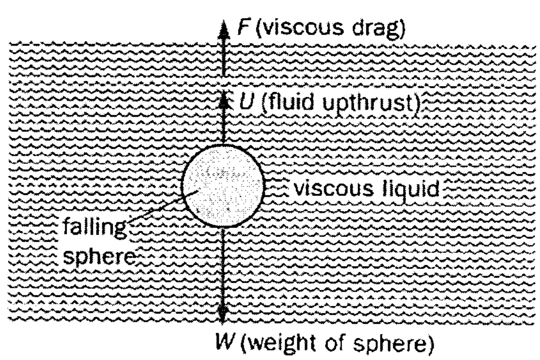
Note: Stokes' law only holds for steady motion in a fluid of infinite extent.

Terminal velocity

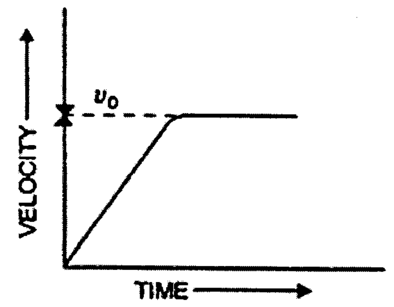
This the maximum velocity acquired by a body while flowing through a viscous liquid

Consider a metallic sphere of radius r and density σ falling under gravity in a liquid of density ρ. Three forces act on it i.e.

- its weight, W acting downwards
- the upthrust, U due to the weight of fluid displaced, acting upwards
- the viscous drag, F acting upwards



The viscous force acting on the metallic sphe
increases as its velocity increases. A stage is reach
when the weight, W of the sphere becomes equal to t
viscous force and the upthrust, U due to buoyanc
Now there is no net force acting on the sphere and
moves down with a constant velocity (termin
velocity)



Expression for terminal velocity

At terminal velocity v_0, $W = F + U$

$$\text{Then, } F = 6\pi\eta\, v_0 r$$
$$W = mg = \rho V g = \frac{4}{3}\pi r^3 \rho g$$

Upthrust, U = weight of fluid displaced = $\frac{4}{3}\pi r^3 \sigma$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g = 6\pi\eta\, v_0 r + \frac{4}{3}\pi r^3 \sigma g$$
$$\frac{4}{3}\pi r^3 (\rho - \sigma) g = 6\pi\eta\, v_0 r$$
$$\frac{4}{3} r^2 (\rho - \sigma) g = 6\eta\, v_0$$
$$\eta = \frac{2gr^2}{9v_0}(\rho - \sigma)$$
$$v_0 = \frac{2gr^2}{9\eta}(\rho - \sigma)$$

Example 1

A spherical ball of radius $1 \times 10^{-4}\, m$ and densit
$10^4\, kg\, m^{-3}$ falls freely under gravity through
distance h before entering a tank of water. If, afte
entering the water, the velocity of the ball does no
change, find h. (coefficient of viscosity of water i
$9.8 \times 10^{-6}\, N\, s\, m^{-2}$)

Solution

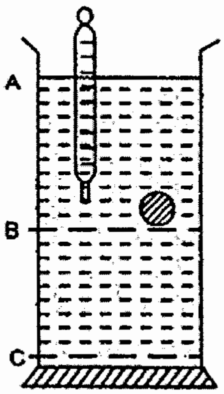
$$\text{From } v^2 = u^2 + 2as$$
$$v = \sqrt{2gh}$$

The velocity $\sqrt{2gh}$ attained by the sphere in falling freely through a height h becomes the terminal velocity of the sphere in water.

$$\sqrt{2gh} = \frac{2gr^2}{9\eta}(\rho - \sigma)$$
$$h = \left[\frac{2gr^2}{9\eta}(\rho - \sigma)\right]^2 \frac{1}{2g}$$

$$h = \left[\frac{2 \times 9.81 \times (10^{-4})^2(10^4 - 10^3)}{9 \times 9.8 \times 10^{-6}}\right] \times \frac{1}{2(9.81)}$$
$$= 20.4\ m$$

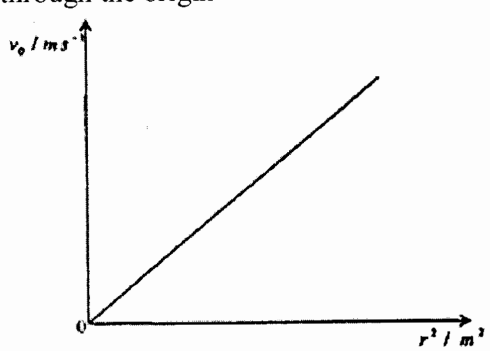
Experimental determination of viscosity of highly viscous fluids – Stokes' method



- The experimental liquid is put into a tall wide jar. Two markings B and C are made as shown with B some distance below the free surface of the liquid.
- A solid sphere of known density, ρ and radius r (measured by a micrometer screw gauge) is gently dropped in the jar.
- When the ball crosses B, a stop watch is switched on and the time taken, t to reach C is noted.
- If BC = s, then terminal velocity, $v_0 = \frac{s}{t}$

From the expression $\eta = \frac{2gr^2}{9v_0}(\rho - \sigma)$, if the values of r, ρ and σ, the coefficient of viscosity of the fluid can be obtained

- When different solid spheres of different radii are used, their corresponding terminal velocities are determined.
- The results are tabulated including values of r^2
- A graph of v against r^2 is plotted and it is a straight line through the origin



From $v_0 = \frac{2g}{9\eta}(\rho - \sigma)r^2$

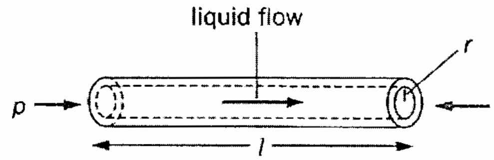
Comparing with the equation, y = mx

$$m = \frac{2g}{9\eta}(\rho - \sigma)$$

$$\eta = \frac{2g(\rho - \sigma)}{9m}$$

where m is the gradient or slope of the graph

Poiseuille's equation



Poiseuille investigated the steady flow of a liquid through a capillary tube. He derived that the volume of liquid flowing out of the tube per second depends on

(i) the coefficient of viscosity, η
(ii) the radius of the tube, r
(iii) the pressure gradient, $\frac{p}{l}$

i.e. volume per second $\propto \eta a \frac{p}{l}$

$$\frac{V}{t} = k\eta^x r^y \left(\frac{p}{l}\right)^z$$

where k is a constant of proportionality

Using dimensions;

$$\left[\frac{V}{t}\right] = [\eta]^x \times [r]^y \times \left[\frac{p}{l}\right]^z$$

$$L^3T^{-1} = [ML^{-1}T^{-1}]^x \times [L]^y \times [ML^{-2}T^{-2}]^z$$

Equating corresponding indices gives:

$$3 = -x + y - 2z$$
$$-1 = -x - 2z$$
$$0 = x + z$$

Solving simultaneously gives:

$$z = 1,\ x = -1,\ \text{and}\ y = 4$$

Therefore, Volume per second $= k\eta^{-1}r^4\left(\frac{p}{l}\right)^1 = \frac{kr^4p}{\eta l}$

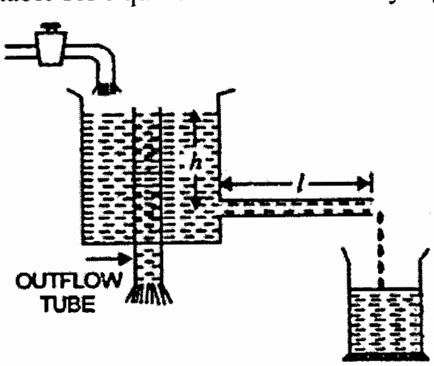
Other experiments show that $k = \frac{\pi}{8}$

$$\frac{V}{t} = \frac{\pi r^4 p}{8\eta l}$$

This is known as Poiseuille's formula for fluid flow

Experiment to determine the coefficient of viscosity of liquid using Poiseuille's formula

This method makes use of the apparatus shown below and is suitable for liquids which flow easily e.g. water



- The liquid, of known density, ρ under test is made to flow steadily through the capillary tube from a constant head device.
- The volume V of the liquid which emerges in a known time t is measured.

$$\text{Volume per second} = \frac{V}{t}$$

- The pressure difference between the ends of the capillary tube is hρg
- The volume per second is measured at different values of h
- A graph of $\frac{V}{t}$ against h is plotted and it is a straight line through the origin

$$\frac{V}{t} = \frac{\pi r^4 p}{8\eta l} = \frac{\pi r^4 h\rho g}{8\eta l}$$

$$\frac{V}{t} = \left(\frac{\pi r^4 \rho g}{8\eta l}\right)h$$

$$\text{Gradient} = \frac{\pi r^4 \rho g}{8\eta l}$$

- The mean radius, r of the tube can be found by measuring the length and mass of a mercury thread introduced in the tube

Example 2

Calculate the mass of water flowing in 10 minutes through a tube of radius $0.01\,m$, $1\,m$ in length and having a constant pressure head of $0.20\,m$ of water. Coefficent of viscosity $= 9 \times 10^{-4}\,N\,s\,m^{-2}$

Solution

time, $t = 10$ minute $= 600\,s$

radius, $r = 0.01\,m$,

length, $l = 1\,m$

pressure difference,

$$p = h\rho g = 0.20 \times 1000 \times 9.81 = 1962\,N\,m^{-2}$$

volume of water collected per second, $\frac{V}{t} = \frac{\pi p r^4}{8\eta l}$

$$= \frac{3.142 \times 1962 \times (0.01)^4}{8 \times 9 \times 10^{-4} \times 1}$$

$$= 8.55 \times 10^{-3}\,m^3\,s^{-1}$$

Volume of water collected in $600\,s$

$$= 8.55 \times 10^{-3} \times 600 = 5.13\,m^3$$

Mass of water colected in $600\,s$,

$$= 5.13 \times 1000 = 5.13 \times 10^3\,kg$$

Example 3

A liquid flows through a pipe of $10^{-3}\,m$ radius and $0.1\,m$ length under a pressure of $10^3\,N\,m^{-2}$. Calculate the rate of flow and the speed of the liquid coming out of the pipe. The coefficient of viscosity of the liquid is $1.25 \times 10^{-3}\,N\,s\,m^{-2}$

Solution

$r = 10^{-3}\,m,\ l = 0.1\,m,\ p = 10^3\,N\,m^{-2}$

$$\text{Rate of flow, } \frac{V}{t} = \frac{\pi p r^4}{8\eta l}$$

$$= \frac{\pi \times 10^3 \times (10^{-3})^4}{8 \times 1.25 \times 10^{-3} \times 0.1}$$

$= \pi \times 10^{-6}\,m^3\,s^{-1}$ or $3.142 \times 10^{-6}\,m^3\,s^{-1}$

From flow rate = cross sectional area × velocity

$$\text{Velocity, } v = \frac{\text{flow rate}}{\text{cross sectional area}}$$

$$= \frac{\pi \times 10^{-6}}{\pi(10^{-3})^2} = 1\,m\,s^{-1}$$

Self-Evaluation exercise

1. Draw diagrams to show the forces acting on an object falling through a viscous liquid
   (i) at the instant of release
   (ii) when it has reached its terminal velocity
   Write down an equation for the forces acting on the object in (ii)
2. Describe briefly how the terminal velocity of a small sphere falling through motor oil can be measured
3. A sphere has a radius r and it is made from a material of density ρ. It is dropped into oil. Write down an expression for the weight, W of the sphere
   The sphere experiences an upthrust force $U = \frac{4}{3}\pi r^3 \sigma g$ where σ is the density of oil. Explain briefly the cause of this upthrust force on the sphere.
   Explain why, when the sphere is falling with terminal velocity, F = W − U where F is the Stokes' law force
   Hence show that the viscosity η can be calculated from the expression
   $$\eta = \frac{2r^2 g(\rho - \sigma)}{9v}$$
   where v is the velocity of the sphere
4. In an experiment to determine the coefficient of viscosity of motor oil, the following measurements are made
   Mass of glass sphere $= 1.2 \times 10^{-4}$ kg
   Diameter of sphere $= 4.0 \times 10^{-3}$ m
   Terminal velocity of sphere $= 5.4 \times 10^{-2}$ m s^{-1}
   Density of oil $= 860$ kg m^{-3}
   Calculate the coefficient of viscosity of the oil
   [Ans: 0.45 N s m^{-2}]
5. A tank has a light lubricating oil. The oil flows out of the tank through a horizontal pipe of length 0.10 m and internal diameter 4.0 mm. Calculate the volume of oil which flows out of the pipe in one

minute when the level of oil in the tank is 1.2 m above the pipe and does not significantly alter during this time. (density of oil = $900\ \text{kg m}^{-3}$, coefficient of viscosity of oil $= 8.4 \times 10^{-2}\ \text{N s m}^{-2}$)

[Ans: $5.0 \times 10^{-4}\ \text{m}^3$]

Water flows steadily through a horizontal tube which consists of two parts joined end to end, one part is 21 cm long and has a diameter 0.225 cm and the other is 7.0 cm long and has a diameter of 0.075 cm. If the pressure difference between the ends of the tube is 14 cm of water, find the pressure difference between the end of each part.

[Ans: 0.5 cm, 13.5 cm of water]

(a) A sphere of radius a moving through a fluid of density ρ with high velocity V experiences a retarding force F given by $F = k\, a^x \rho^y V^z$, where k is a non-dimensional coefficient. Use the method of dimensions to find the values of x, y and z.

(b) A sphere of radius 2 cm and mass 100 g, falling vertically through air of density $1.2\ kg\, m^{-3}$, t a place where the acceleration due to gravity is $9.81\ ms^{-2}$, attains a stead speed of $30\ m\, s^{-1}$. Explain why a constant velocity is reached and use the data to find the value of k in this case

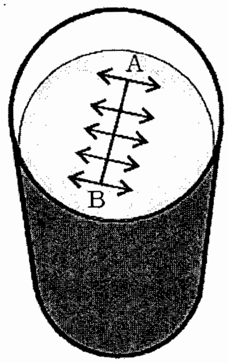
[Ans: $x = 2, y = 1, z = 2$, $k = 2.3$]

Castor oil at 20°C has a coefficient of viscosity $2.42\ N\, s\, m^{-2}$ and a density $940\ kg\, m^{-3}$. Calculate the terminal velocity of a steel ball of radius 2.0 mm falling under gravity in the oil, taking the density of steel as $7800\ kg\, m^{-3}$.

[Ans: $0.025\ m\, s^{-1}$]

SURFACE TENSION

Surface tension is the property of the free surface of a liquid at rest to behave like a stretched membrane in order to acquire minimum surface area.



Imagine a line AB in the free surface of a liquid at rest as shown above. The force of surface tension is measured as the force acting per unit length on either side of this imaginary line AB. The force is perpendicular to the line and tangential to the liquid surface.

If F is the force acting on the length l of the line AB, then the surface tension is given by

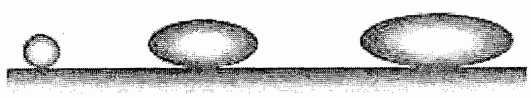
$$F = \frac{\gamma}{l}$$

Surface tension is defined as the force per metre length acting perpendicular on an imaginary line drawn on the liquid surface.

Its unit is N m^{-1} and dimensions M T^{-2}

Experiments to demonstrate surface tension

(i) Small drops of mercury are spherical but large ones are flat

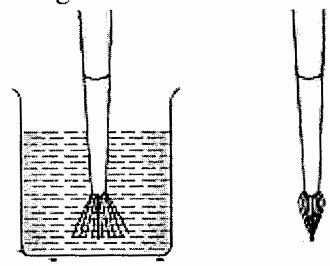


The shape of a drop is governed by two factors – surface tension and gravitational force. Gravity tends to spread the liquid on the solid surface so that centre of gravity is at the lowest level while the surface tension tries to collect it in form of sphere so that it may have minimum surface area

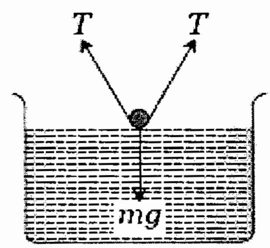
The force of gravity is proportional to the volume and therefore to r^3 whereas surface tension effect is proportional to the surface area and hence r^2

In case of large drop, the effect of gravity predominates whereas in case of small drops, surface tension has more important role to play. The large drops will be flat but rounded near the edges. This explains why a smaller drop is spherical but a large drop is flat

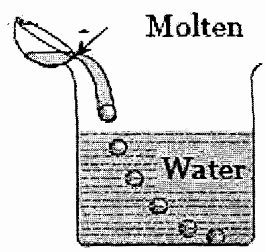
(ii) Hair of shaving brush/painting brush when dipp in water spread out, but as soon as it is taken o its hair stick together.



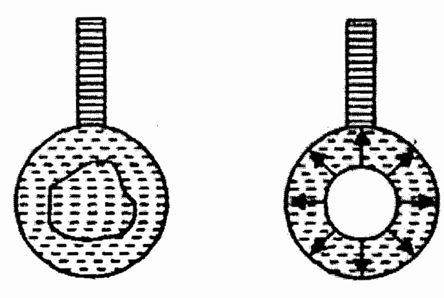
(iii) When a greased iron needle is placed gently o the surface of water at rest, so that it does no prick the water, the needle floats on the surface o water despite it being heavier because the weigh of needle is balanced by the vertical componen of the forces of surface tension. If the wate surface is picked by one end of the needle, th needle sinks down.



(iv) When a molten metal is poured into water from a suitable height, the falling stream of metal breaks up and the detached portion of the liquid in small quantity acquire the spherical shape



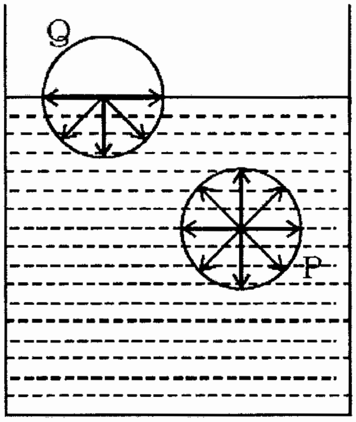
(v) Take a circular wire frame with a handle. Dip the frame in soap solution so as to form a thin soap film. Take a cotton thread loop and place it in an irregular shape on the film gently. Now prick the film within the loop by a sharp needle. The film in the loop disappears and the loop of the thread takes a circular shape. Thus, the soap film possesses minimum surface area



(vi) Rain drops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of sphere is minimum.

Molecular theory of surface tension

Consider two molecules P and Q as shown below.



The molecule P is attracted in all directions equally by the neighbouring molecules. Therefore, the net force acting on P is zero.

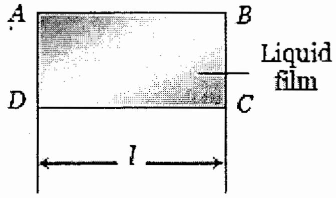
The molecule Q is on the free surface of the liquid. It experiences a net downward force because the number of molecules in the lower half of the sphere is more and the upper half is completely outside the surface of the liquid. Therefore, all molecules lying on the surface experience only a net downward force.

If a molecule from the interior is to be brought to the surface of the liquid, work must be done on the molecule against this downward force. This work done is stored as potential energy.

For equilibrium, a system must possess minimum potential energy. The free surface of a liquid tends to assume minimum surface area by contracting and remains in a state of tension like a stretched elastic membrane.

Example 1

A liquid film is formed over a frame *ABCD* as shown in figure below. Wire *CD* can slide without friction.



Calculate the mass to be hung from *CD* to keep it in equilibrium.

Solution

Weight of the body hung from the wire

= upward force due to surface tension

$$mg = 2l\gamma$$

$$m = \frac{2l\gamma}{g}$$

Example 2

A $10\,cm$ long wire is placed horizontally on the surface of water and is gently pulled up with a force of $2 \times 10^{-2}\,N$ to keep the wire in equilibrium. Calculate the surface tension of water.

Solution

Force on wire due to surface tension, $F = \gamma \times 2l$

$$\gamma = \frac{F}{2l}$$

$$= \frac{2 \times 10^{-2}}{2 \times 0.10} = 0.1\,N\,m^{-1}$$

Example 3

A wire ring of $0.03\,m$ radius is rested on the surface of a liquid and then raised. The pull required is $0.03\,kg$ weight more than before the film breaks than it is after. Find the surface tension of liquid

Solution

The additional pull F of $0.003\,kg$ weight is equal to the force due to surface tension

Force due to surface tension, $F = \gamma \times$ length of ring in contact wih liquid

$$= \gamma \times 2 \times 2\pi r = 4\pi\gamma r$$

Thus, $mg = 4\pi\gamma r$

$$\gamma = \frac{mg}{4\pi r} = \frac{0.003 \times 9.81}{4 \times \pi \times 0.03} = 0.078\,N\,m^{-1}$$

Example 4

A soap film is formed on a rectangular frame of length $0.03\,m$ dipping in soap solution. The frame hangs from the arm of a balance. An extra mass of $2.20 \times 10^{-4}\,kg$ must be placed in the other pan to balance the pull of the film. Calculate the surface tension of the soap solution

Solution

Force acting on the frame due to surface tension,

$$F = \gamma l$$

where l is the length of the frame in contact with the liquid

Since the soap film has two surfaces,

$$l = 2 \times 0.03 = 0.06\,m$$

$$F = 0.06\gamma$$

This must be equal to the extra weight

$$0.06\gamma = 2.20 \times 10^{-4} \times 9.81$$

$$\gamma = 0.036\,N\,m^{-1}$$

Surface energy

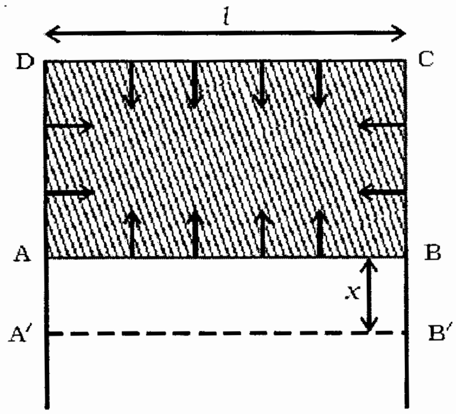
Surface energy is the potential energy per m^2 of the surface film.

Or

Surface is energy is the work done to increase the area of a surface film by 1 m^2

$$\text{Surface energy} = \frac{\text{work done in increasing surface area}}{\text{increase in surface area}}$$

Consider a metal frame ABCD in which AB is movable dipped in a soap solution. A film is formed which pulls AB inwards due to surface tension.



If γ is the surface tension of the film and l is the length of the wire AB, this inward force is given by $2 \times \gamma l$ since there are two free surfaces of the film.

If AB is moved through a distance x to the position $A'B'$, then

$$\text{Work done, } W = 2\gamma l x$$

$$\text{New surface area} = 2 \times lx = 2lx$$

$$\text{Surface energy} = \frac{W}{A}$$

$$= \frac{2\gamma lx}{2lx} = \gamma$$

∴ Surface energy is numerically equal to surface tension.

Surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

Work done in blowing a liquid drop or soap bubble

(i) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$$W = \gamma \times \text{increase in surface area}$$

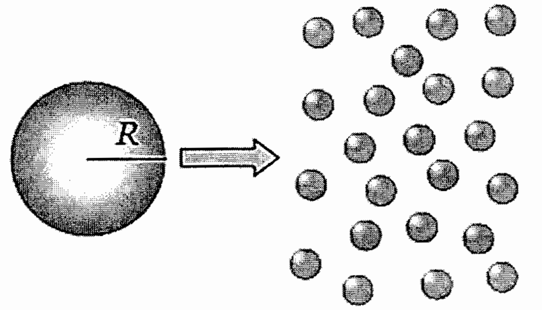
Since drop has one free surface

$$W = \gamma \times 4\pi[r_2^2 - r_1^2]$$

(ii) In case of soap bubble, it has two free surfaces

$$W = \gamma \times 8\pi[r_2^2 - r_1^2]$$

Splitting of a bigger drop



When a drop of radius R splits into n smaller drop each of radius r, then the surface area of the liqu increases. Hence work has to be done against surfac tension

Since the volume of the liquid remains constant,

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$

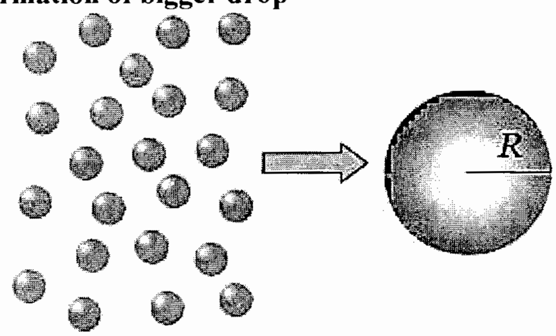
$$\therefore R^3 = nr^3$$

Work done = $\gamma \times \Delta A$

= γ[surface area of n drops – surface area of big drop

$$= \gamma[n4\pi r^2 - 4\pi R^2]$$

Formation of bigger drop



If n small drops of radius r coalesce to form a big dro of radius R, then surface area of the liquid decreases.

Amount of surface energy released

= Initial surface energy – final surface energy

$$= n(4\pi r^2)\gamma - 4\pi R^2\gamma$$

Example 5

Calculate the work done in blowing a soap bubble of 10 cm given that the surface tension of the soap solution is 0.03 $N\,m^{-1}$

Solution

$$W = 8\pi R^2\gamma$$

$$= 8\pi(10 \times 10^{-2})^2 \times 0.03$$

$$= 7.536 \times 10^{-3}\,J$$

Example 6

A drop of mercury of radius 2 mm is split into 8 identical droplets. Find the increase in surface energy if the surface tension of mercury is 0.465 $N\,m^{-1}$

Solution

Increase in surface energy = $\gamma[n(4\pi r^2) - 4\pi R^2]$

$R = 2 \times 10^{-3}\,m$, $r = ?$, $n = 8$, $\gamma = 0.465\,N\,m^{-1}$

Since the volume of the liquid remains constant,

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$

$$R^3 = nr^3$$

$$r^3 = \frac{R^3}{n} = \frac{(2\times10^{-3})^3}{8}$$

$$r = 1\times10^{-3}\ m$$

Increase in surface area

$$= 0.465[8(4\pi(1\times10^{-3})^2 - 4\pi(2\times10^{-3})^2]$$

$$= 23.6\ \times 10^{-6}\ J$$

Example 7

The work done in increasing the size of a soap film from $10\ cm\ \times 6\ cm$ to $10\ cm\ \times 11\ cm$ is $3\times10^{-4}J$. Calculate the surface tension of the film

Solution

$$A_1 = 10\times6 = 60\ cm^3 = 60\times10^{-4}\ m^2$$

$$A_2 = 10\times11 = 110\ cm^2 = 110\times10^{-4}\ m^2$$

Increase in surface area,

$$\Delta A = (110-60)\times10^{-4} = 50\times10^{-4}\ m^2$$

As the soap film has two surfaces,

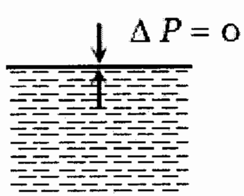
$$W = \gamma\times2\Delta A$$

$$\gamma = \frac{W}{2\Delta A} = \frac{3\times10^{-4}}{2\times50\times10^{-4}} = 0.03\ N\ m^{-1}$$

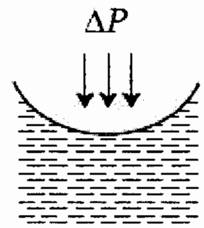
Excess pressure

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So, in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure. In case of a drop excess pressure is provided by hydrostatic pressure of the liquid within the drop while in case of bubble the gauge pressure of the gas confined in the bubble provides it.

At a plane surface

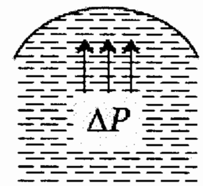


Concave surface



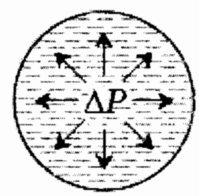
$$\Delta P = \frac{2\gamma}{R}$$

Convex surface



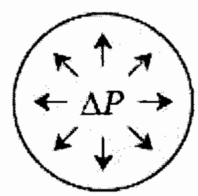
$$\Delta P = \frac{2\gamma}{R}$$

Liquid drop



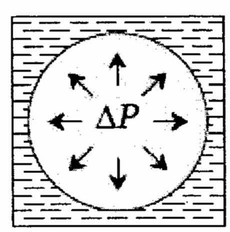
$$\Delta P = \frac{2\gamma}{R}$$

Bubble in air



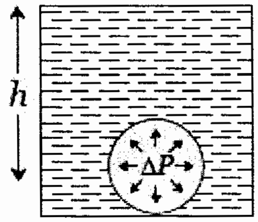
$$\Delta P = \frac{4\gamma}{R}$$

Bubble in liquid



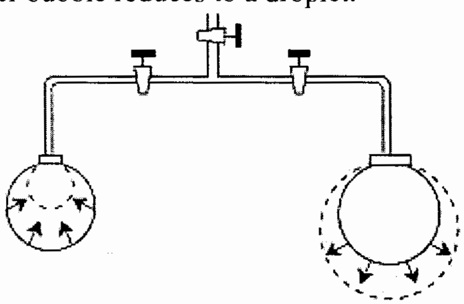
$$\Delta P = \frac{2\gamma}{R}$$

Bubble at depth h below the free surface of liquid of density ρ



$$\Delta P = \frac{2\gamma}{R} + h\rho g$$

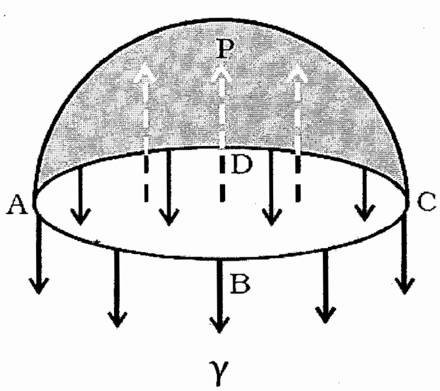
Note: Excess pressure is inversely proportional to the radius of bubble (or drop), i.e. pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop). This is why when two bubbles of different sizes are put in communication with each other, the air will rush from smaller to larger bubble, so that the smaller will shrink while the larger will expand till the smaller bubble reduces to a droplet.



The pressure needed to form a very small bubble is high. This explains why one needs to blow hard to start a balloon growing. Once the balloon has grown, less air pressure is needed to expand it more.

Excess pressure inside a liquid drop

Imagine the drop to be divided into two halves.



Considering the equilibrium of the upper hemisphere, upward force on the plane ABCD due to excess pressure ΔP is given by

$$F_1 = \Delta P \times \pi r^2$$

Force due to surface tension acting downward, along the circumference of the circle ABCD,

$$F_2 = \gamma \times 2\pi r$$

At equilibrium, $F_1 = F_2$

$$\Delta P\, \pi r^2 = 2\pi r\, \gamma$$

$$\mathbf{\Delta P = \frac{2\gamma}{r}}$$

Excess pressure inside a soap bubble

A soap bubble has two liquid surfaces in contact \ air, one inside the bubble and the other outside bubble.

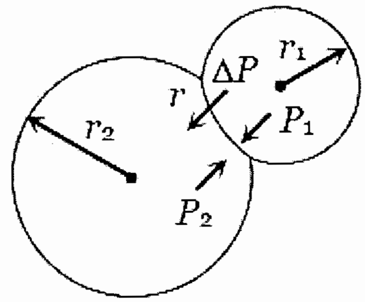
Force due to surface tension $= 2 \times 2\pi r\gamma = 4\pi r\gamma$

Force due to excess pressure $= \Delta P\, \pi r^2$

At equilibrium, $P\, \pi r^2 = 4\pi r\gamma$

$$\mathbf{\Delta P = \frac{4\gamma}{r}}$$

Formation of a double soap bubble



If r_1 and r_2 are the radii of smaller and larger but and P_0 is the atmospheric pressure, then the press inside them will be

$$P_1 = P_0 + \frac{4\gamma}{r_1} \text{ and } P_2 = P_0 + \frac{4\gamma}{r_2}$$

Since $r_1 < r_2$, $P_1 > P_2$

So for interface, $\Delta P = P_1 - P_2$

$$\Delta P = 4\gamma\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \text{.........(i)}$$

As excess pressure acts from concave to convex si the interface will be concave towards the sma bubble and convex towards the larger bubble.

Let r be the radius of the interface

$$\Delta P = \frac{4\gamma}{r} \text{ ...... (ii)}$$

From (i) and (ii),

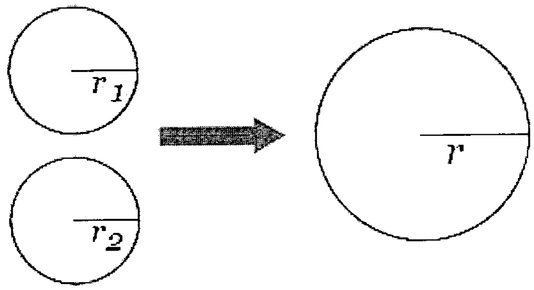
$$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

Radius of interface, $r = \frac{r_1 r_2}{r_2 - r_1}$

Thus, $\Delta P = \frac{4\gamma(r_2 - r_1)}{r_1 r_2}$

Formation of a single bubble

Under isothermal conditions, two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r.



For bubble 1: surface energy = $2 \times [\,(4\pi r_1^2) \times \gamma\,]$

For bubble 2: Surface energy $= 2 \times [\,(4\pi r_2^2) \times \gamma\,]$

When the drops are combined,

surface energy = $2 \times [(4\pi r^2) \times \gamma\,]$

By conservation of energy,

$$8\pi r_1^2\gamma + 8\pi r_2^2\gamma = 8\pi r^2\gamma$$

$$r_1^2 + r_2^2 = r^2$$

$$r = \sqrt{r_1^2 + r_2^2}$$

$\therefore$ Pressure difference, $\Delta P = \frac{4\gamma}{r} = \frac{4\gamma}{\sqrt{r_1^2 + r_2^2}}$

Example 8

Calculate the pressure inside a small air bubble of radius 0.1 mm situated just below the surface of water. (surface tension of water = 0.07 Nm^{-1}, atmospheric pressure = $1.013 \times 10^5\ Nm^{-2}$)

Solution

Pressure inside a bubble when it is in liquid = $P_0 + \frac{2\gamma}{r}$

$$= 1.013 \times 10^5 + \frac{2 \times 0.07}{0.1 \times 10^{-3}}$$

$$= 1.027 \times 10^5\ Pa$$

Example 9

Two spherical soap bubbles combine. If V is the change in volume of the contained air, A is the change in total surface area then show that $3P_aV + 4AT = 0$, where T is the surface tension and P_a is the atmospheric pressure.

Solution

Let R_1 and R_2 be the radii of the two bubbles before combination and R be the radius of the bubble after combination.

Total pressure inside the first bubble is

$$P_1 = P_a + \frac{4T}{R_1}$$

Total pressure inside the second bubble is

$$P_2 = P_a + \frac{4T}{R_2}$$

Total pressure inside the resultant bubble is

$$P_3 = P_a + \frac{4T}{R}$$

By Boyle's law,

$$P_1V_1 + P_2V_2 = P_3V_3$$

$$\left(P_a + \frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3 + \left(P_a + \frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3 = \left(P_a + \frac{4T}{R}\right)\frac{4}{3}\pi R^3$$

$$P_a\left(\frac{4}{3}\pi R_1^3 + \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R^3\right) + \frac{4T}{3}(4\pi R_1^2 + 4\pi R_2^2 - 4\pi R^2) = 0$$

But $\frac{4}{3}\pi R_1^3 + \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R^3 = V$

and $4\pi R_1^2 + 4\pi R_2^2 - 4\pi R^2 = A$

$$P_aV + \frac{4T}{3} \times A = 0$$

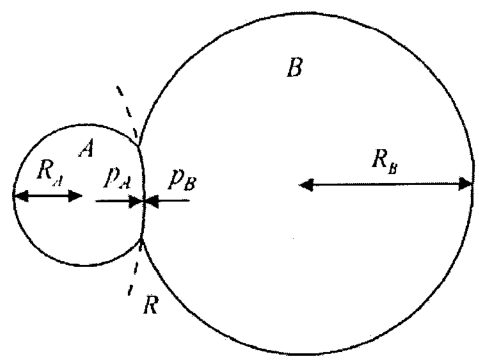
$$3P_aV + 4AT = 0$$

Example 10

Two separate air bubbles of radii 0.002 m and 0.004 m formed of the same liquid of surface tension 0.07 $N\,m^{-1}$ come together to form a double bubble. Find the radius of curvature of the internal film surface common to both bubbles

Solution

Let R_A and R_B be the radii of the two bubbles A and B. Let R be the radius of curvature of the common interface



Excess pressure in A, $p_A = \frac{4\gamma}{R_A}$

Excess pressure in B, $p_B = \frac{4\gamma}{R_B}$

Therefore, in the double bubble, the pressure difference between A and B on either side of the common surface is

$$p_A - p_B = \frac{4\gamma}{R_A} - \frac{4\gamma}{R_B} = 4\gamma\left[\frac{1}{R_A} - \frac{1}{R_B}\right]$$

This pressure difference will be equal to $\frac{4\gamma}{R}$

$$4\gamma\left[\frac{1}{R_A} - \frac{1}{R_B}\right] = \frac{4\gamma}{R}$$

$$R = \frac{R_AR_B}{R_B - R_A} = \frac{0.002 \times 0.004}{0.004 - 0.002} = 0.004\ \text{m}$$

Intermolecular forces

The force between two molecules of a substance is called intermolecular force. This intermolecular force is basically electric in nature and can either be attractive or repulsive.

Two types of intermolecular forces exist i.e. cohesive and adhesive forces

Cohesive force

This is the force of attraction between molecules of the same substance. The cohesive force is very strong in solids, weak in liquids and extremely weak in gases.

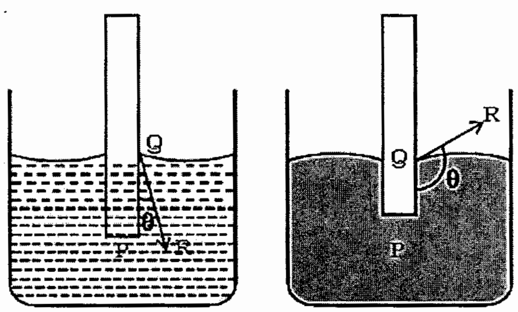
Adhesive force

This is the force of attraction between molecules of two different substances. Due to adhesive force, ink sticks on paper while writing, fevicol, gum, etc. exhibit strong adhesive property.

Water wets glass because the cohesive force between water molecules is less than the adhesive force between water and glass molecules whereas mercury does not wet glass because the cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules.

Angle of contact

When the free surface of a liquid comes in contact with a solid, it becomes curved at the point of contact.



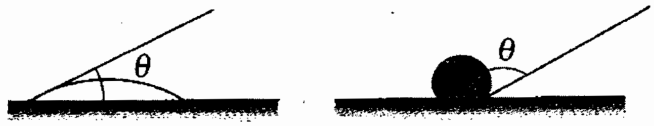
water mercury

The angle of contact θ is defined as the angle between the solid surface and the tangent plane to the liquid surface, measured through the liquid.

QR is the tangent drawn at the point of contact Q. The angle PQR is called the angle of contact. When the liquid has a concave meniscus, the angle of contact is acute($< 90°$). When it has a convex meniscus, the angle of contact is obtuse ($> 90°$)

The angle of contact depends on the nature of liquid and solid in contact. For water and glass, $\theta \approx 10°$, for pure water and clean glass, θ is very small and it is taken as zero. The angle of contact of mercury with glass is 140°

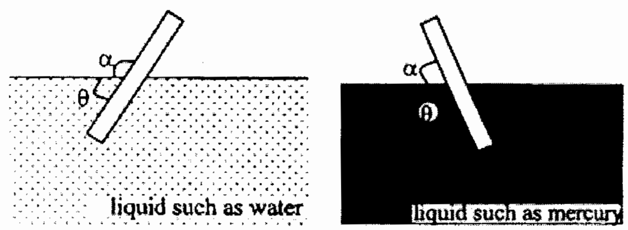
Liquids with acute angles of contact are said to wet the surface, those with obtuse angles of contact do not wet it.



Note:

A large force is required to draw apart normally two glass plates enclosing a thin water film because the thin water film formed between the two glass plates will have a concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

Experiment to determine angle of contact

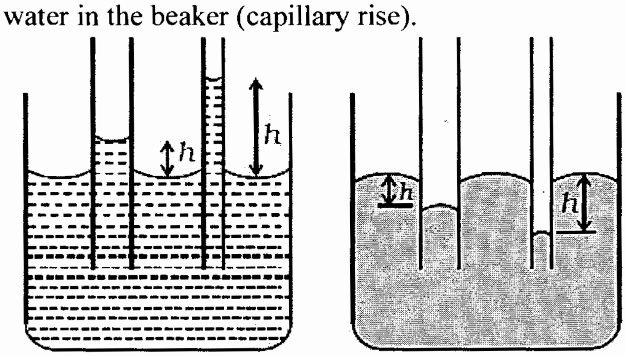


A clean flat plate is dipped into the liquid and tilted until the liquid surface on one side of the plate is horizontal up to the line of contact.

The angle α between the flat plate and the liquid surface is measured by means of a protractor. suitably placed against the edge of the plate. The angle of contact $\theta = 180 - \alpha$

CAPILLARITY

Surface tension gives rise to a phenomenon of practical importance called capillarity. When a capillary tube is dipped in water, the water rises up in the tube. The level of water in the tube is above the free surface of water in the beaker (capillary rise).



water mercury

When a capillary tube is dipped in mercury, mercury also rises in the tube. But the level of mercury is depressed below the free surface of mercury in the beaker (capillary fall).

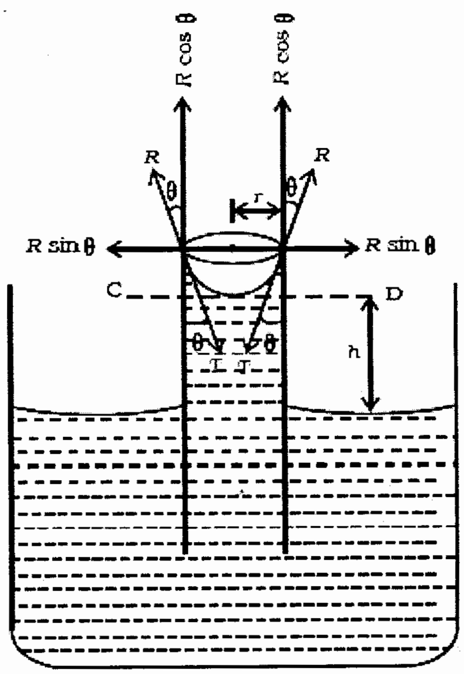
The rise of a liquid in a capillary tube is known as capillarity. The height h indicates the capillary rise (for water) or capillary fall (for mercury).

Illustrations of capillarity

- A blotting paper absorbs ink by capillary action. The pores in the blotting paper act as capillaries.
- The oil in a lamp rises up the wick through the narrow spaces between the threads of the wick.
- A sponge retains water due to capillary action.
- Walls get damped in rainy season due to absorption of water by bricks.

Derivation of h

Let us consider a capillary tube of uniform bore dipped vertically in a beaker containing water. Due to surface tension, water rises to a height h in the capillary tube as shown below. The surface tension γ of the water acts inwards and the reaction of the tube R outwards. **R** is equal to γ in magnitude but opposite in direction. This reaction R can be resolved into two components.



The horizontal component acting all along the circumference of the tube cancel each other whereas the vertical component balances the weight of water column in the tube.

Total upward force = $R\cos\theta \times$ *circumference,* $2\pi r$

$$F = 2\pi r\,\gamma\cos\theta$$

Weight of liquid column, $W = \pi r^2 h\rho g$

As the water column is in equilibrium, $F = W$

$$\pi r^2 h\rho g = 2\pi r\,\gamma\cos\theta$$

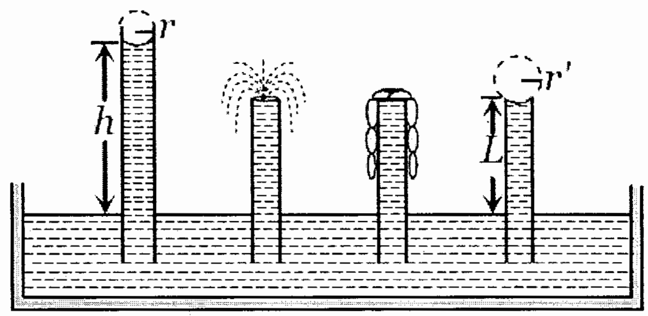
$$\boldsymbol{h = \frac{2\gamma\cos\theta}{r\rho g}}$$

If θ is greater than 90°, the meniscus is convex upwards, $\cos\theta$ is negative and h will be negative. This means the liquid falls in the capillary tube below the level of the surrounding liquid.

Notes:

(i) For a given liquid and solid at a given place, $h \propto \frac{1}{r}$ where γ, θ, ρ and g are constant. This means that the lesser the radius of the capillary tube, the greater will be the rise and vice versa

(ii) In case of a capillary tube of insufficient length, i.e. $L < h$, the liquid will neither overflow from the upper end like a fountain nor will it tickle along the vertical sides of the tube.

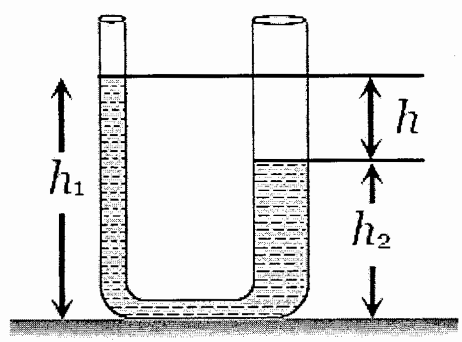


The liquid after reaching the upper end will increase the radius of its meniscus without changing its nature such that

$$hr = Lr'$$

$$L < r \therefore r' > r$$

***U* – tube**

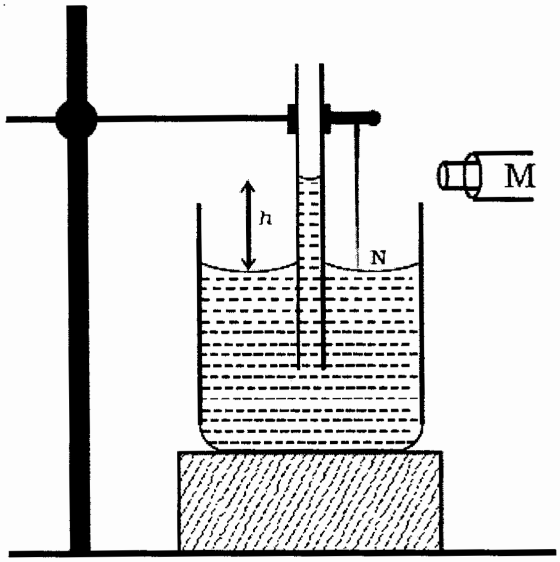


The difference of levels of liquid column in two limbs of U- tube of unequal radii r_1 and r_2 is given by

$$h = h_1 - h_2 = \frac{2\gamma\cos\theta}{r_1\rho g} - \frac{2\gamma\cos\theta}{r_2\rho g}$$

$$\boldsymbol{\therefore h = \frac{2\gamma\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

Experimental determination of surface tension of a liquid



- A clean capillary tube of uniform bore is fixed vertically with its lower end dipping into the liquid in a beaker.
- A needle N is also fixed with the capillary tube. The tube is adjusted until the tip of the needle just touches the liquid surface.
- A travelling microscope M is focused on the meniscus of the liquid in the capillary tube. The reading h_1 corresponding to the lower meniscus is noted.
- The microscope is lowered and focused on the tip of the needle and the corresponding reading is taken as h_2.
- The difference between h_1 and h_2 gives the capillary rise h.
- To find r, the tube is broken at the meniscus level and the average reading of two diameters at right angles taken with the travelling microscope.
- If ρ is the density of the liquid, then its surface tension is given by

$$\gamma = \frac{hr\rho g}{2\cos\theta}$$

where g is the acceleration due to gravity.

Assumptions made

- Temperature is constant
- The weight of the small quantity of liquid in the meniscus is negligible

Example 11

Calculate the capillary rise in a tube of diameter 1 mm if the surface tension of water is $0.06\ N\,m^{-1}$. (assume that the angle of contact is $0°$)

Solution

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$
$$= \frac{2\times0.06\times\cos 0°}{0.5\times10^{-3}\times1000\times9.81}$$
$$= 0.0244\ m$$

Example 12

Water rises to a height of $10\ cm$ in a capillary tube and mercury falls to a depth of $3.5\ cm$ in the same capillary tube. If the density of mercury is $13.6\ g\,cm^{-3}$ and its angle of contact $134°$ and the density of water is $1\ g\,cm^{-3}$ and its angle of contact is $0°$, calculate the ratio of surface tensions of the two liquids

Solution

From $h = \frac{2\gamma\cos\theta}{r\rho g}, \gamma = \frac{hr\rho g}{2\cos\theta}$

$$\gamma_w = \frac{h_w r\rho_w g}{2\cos\theta_w} \text{ and } \gamma_m = \frac{h_m r\rho_m g}{2\cos\theta_m}$$
$$\frac{\gamma_w}{\gamma_m} = \frac{h_w\rho_w\cos\theta_m}{h_m\rho_m\cos\theta_w} = \frac{10\times1\times\cos 134°}{3.5\times13.6\times\cos 0°}$$
$$= -\frac{5}{34}$$

Example 13

Two capillaries made of the same material but of different radii are dipped in a liquid. The rise of liquid in one capillary is $2.2\ cm$ and that in the other is $6.6\ cm$. Calculate the ratio of their radii

Solution

As $h \propto \frac{1}{r}, \frac{h_1}{h_2} = \frac{r_2}{r_1}$

Thus, $\frac{r_1}{r_2} = \frac{h_2}{h_1} = \frac{6.6}{2.2} = \frac{3}{1}$

Example 14

The internal diameter of the tube of a mercury barometer is $3.00\ mm$. Find the correct reading of the barometer after allowing for the error due to surface tension if the observed reading is $76.56\ cm$. (surface tension of mercury $= 4.40\times10^{-1}\ N\,m^{-1}$, angle of contact $= 130°$)

Solution

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$
$$= \frac{2(4.80\times10^{-1})(\cos 130°)}{(1.5\times10^{-3})(13.6\times10^{3})(9.81)}$$
$$= -0.0031\ m$$
$$= -0.31\ cm$$

The negative sign indicates that the reading of the barometer is lower than the correct value by $0.31\ cm$

Correct value $= 76.56 + 0.31 = 76.87\ cm$

Example 15

A U-tube with limbs of diameters 5.0 mm and 2.0 mm contains water of surface tension $7.0 \times 10^{-2}\ N\,m^{-1}$, angle of contact zero and density $1.0 \times 10^{3}\ kg\,m^{-3}$. Find the difference in levels

Solution

$$h\rho g = 2\gamma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$r_1 = 1\ mm = 10^{-3}\ m, r_2 = 2.5\ mm = 2.5 \times 10^{-3}\ m$

$\gamma = 7.0 \times 10^{-2}\ N\,m^{-1}, \rho = 1000\ kg\,m^{-3}$

$$h = \frac{2\gamma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{2 \times 7.0 \times 10^{-2}}{1000 \times 9.81}\left(\frac{1}{10^{-3}} - \frac{1}{2.5 \times 10^{-3}}\right)$$

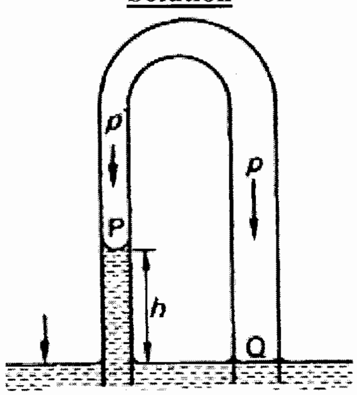
$$= 8.4 \times 10^{-3}\ m$$

$$= 8.4\ mm$$

Example 16

A glass U-tube is inverted with the open ends of the straight limbs, of diameters respectively 0.50 mm and 1.00 mm, below the surface of water in a beaker. The air pressure in the upper part is increased until the meniscus in one limb is level with water outside. Find the height of water in the other limb. (density of water $= 1000\ kg\,m^{-3}$)

Solution



Let p be the air pressure inside the U-tube when the meniscus Q is level with the water outside and P is the other meniscus at height h

Pressure difference at P, $\Delta P = (p + h\rho g) - H$ where H is the atmospheric pressure

But $\Delta P = \frac{2\gamma}{r}$

$$(p + h\rho g) - H = \frac{2\gamma}{r_1} \text{......(i)}$$

Pressure difference at Q,

$$p - H = \frac{2\gamma}{r_2} \text{........(ii)}$$

(i) – (ii); $h\rho g = \frac{2\gamma}{r_1} - \frac{2\gamma}{r_2}$

$$h = \frac{2\gamma}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 0.075}{1000 \times 9.81}\left[\frac{1}{0.25 \times 10^{-3}} - \frac{1}{0.5 \times 10^{-3}}\right]$$

$$= 0.031\ m$$

Factors affecting surface tension

- Impurities present in a liquid appreciably affect surface tension. A highly soluble substance like salt increases the surface tension whereas sparingly soluble substances like soap decreases the surface tension.
- The surface tension decreases with rise in temperature. The temperature at which the surface tension of a liquid becomes zero is called critical temperature of the liquid.

Applications of surface tension

- Motion of insects on water
- Umbrellas and other water proof objects
- Bubble machines
- In soldering, a good joint is formed only if the molten solder (a tin-lead alloy) wets and spreads over the metal involved. Spreading will occur most readily if the liquid solder has a small surface tension. The use of a flux (e.g. resin) with the solder cleans the metal surface and acts a wetting agent which assists spreading.
- Wetting agents play a key role in painting and spraying where the paint must not form drops but remain in layer once spread out.
- During stormy weather, oil is poured into the sea around the ship. As the surface tension of oil is less than that of water, it spreads on water surface. Due to the decrease in surface tension, the velocity of the waves decreases. This reduces the wrath of the waves on the ship.
- Lubricating oils spread easily to all parts (axles, bearings, etc.) because of their low surface tension.
- Dirty clothes cannot be washed with water unless some detergent is added to water. When detergent is added to water, one end of the hair pin shaped molecules of the detergent gets attracted to water and the other end, to molecules of the dirt. Thus, the dirt is suspended surrounded by detergent molecules and this can be easily removed. This detergent action is due to the reduction of surface tension of water when soap or detergent is added to water.

- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for the sweat.

Self-Evaluation exercise

1. (a) Explain, using simple molecular theory, why the surface of a liquid behaves in a different manner from the bulk of the liquid
   (b) Giving the necessary theory, explain how the rise of water in the capillary tube may be used to determine the surface tension of water
   (c) A microscope slide measures $6.0\ \text{cm} \times 1.5\ \text{cm} \times 0.2\ \text{cm}$. It is suspended with its face vertical and with its longest side horizontal and is lowered into water until it is half immersed. Its apparent weight is then found to be the same as its weight in air. Calculate the surface tension of water, assuming the angle of contact to be zero

   [Ans: $7.1 \times 10^{-2}\ \text{Nm}^{-1}$]
2. A clean glass capillary tube of internal diameter 0.60 mm is held vertically with its lower end in water and with 80 mm of the tube above the surface.
   (i) How high does the water rise in the tube?
   (ii) If the tube is now lowered until only 30 mm of its length is above the surface, what happens?
   (surface tension of water = $7.2 \times 10^{-2}\ \text{N m}^{-1}$)
   [Ans: (i) 49 mm (ii) water does not overflow but remains at the top of the tube with angle of contact 52.5°]
3. (a) Describe and explain two experiments of a different nature to illustrate the phenomenon of surface tension
   (b) Define surface tension and explain what is meant by the angle of contact
   (c) The internal diameter of the tube of a mercury barometer is 3.00 mm. Find the corrected reading of the barometer after allowing for the error due to surface tension, if the observed reading is 76.56 cm. (Surface tension of mercury = $0.48\ \text{Nm}^{-1}$, angle of contact of mercury with glass = 140°, density of mercury = $13600\ \text{kg m}^{-3}$)

   [Ans: (c) 76.93 cm]
4. Two spherical soap bubbles of radii 30 mm and 10 mm coalesce so that they have a common surface. If they are made from the same solution and if the radii of the bubbles stay the same after they join together, calculate the radius of curvature of the common surface

   [Ans: 60 mm]
5. A spherical drop of mercury of radius 2 mm falls to the ground and breaks into 10 smaller drops of equal size. Calculate the amount of work that has to be done. (surface tension of mercury = $0.472\ \text{N m}^{-1}$)
   What is the minimum speed with which the original drop could have hit the ground?

   [Ans: 2.74×10^{-5} J, $0.35\ \text{m s}^{-1}$]
6. Two soap bubbles are of radii of 3 cm and 4 cm. The bubbles are in a vacuum and they combine to form a single larger bubble. Calculate the radius of this bubble. Assume that the surface tension of the soap solution is constant throughout.

   [Ans: 5 cm]
7. A glass barometer tube has an internal radius of 3 mm. Calculate the actual atmospheric pressure on a day when the height of the mercury column is 760.2 mm.(Surface tension of mercury = $0.472\ \text{N m}^{-1}$, angle of contact of mercury with glass = 137°, density of mercury = $1.36 \times 10^4\ \text{kg m}^{-3}$)

   [Ans: 761.9 mmHg]
8. A soap bubble whose radius is 12 mm becomes attached to one of radius 20 mm. Calculate the radius of curvature of the common interface.

   [Ans: 30 mm]
9. The end of a clean glass capillary tube, having internal diameter 0.6 mm, is dipped into a beaker containing water, which rises up the tube to a vertical height of 5.0 cm above the water surface in the beaker. Calculate the surface tension of water. (density of water = $1000\ \text{kg m}^{-3}$)
   What would be the difference if the tube were not perfectly clean, so that the water did not wet it, but had an angle of contact of 30° with the tube surface?

   [Ans: $0.075\ \text{N m}^{-1}$; water would rise 4.3 cm]
10. (a) Define surface tension
    (b) Give a concise explanation of the origin of surface tension in terms of intermolecular forces
    (c) Derive an expression for the height of the liquid column in a vertical, uniform capillary tube. (Assume the angle of contact is zero and neglect any correction for the mass of the meniscus)

(d) Describe the experimental determination of the surface tension of water by the capillary rise method

(e) The two vertical arms of a manometer containing water, have different internal radii of 10^{-3} m and 2×10^{-3} m respectively. Determine the difference in height of the two liquid levels when the arms are open to the atmosphere. (Surface tension of water $= 0.07\ N\ m^{-1}$, density of water $= 10^3\ kg\ m^{-3}$)

[Ans: (b) 7 mm]

11. There is a soap film on a restangular frame of wire of area $4\ cm \times 4cm$. If the area of the frame is increased to $4\ cm \times 5\ cm$, find the work done in the process. (surface tension of soap film $= 0.03\ N\ m^{-1}$)

[Ans: $2.4 \times 10^{-5}\ J$]

12. Assume that 64 water droplets combine to form a large drop. Determine the ratio of the total surface energy of 64 droplets to that of large drop given that the surface tension of water is $0.072\ N\ m^{-1}$.

[Ans: 4]

13. Calculate the energy spent when a mercury drop of 1 cm radius is sprayed into 10^5 drops of equal size given that surface tension of mercury $= 35 \times 10^{-3}\ N\ m^{-1}$.

[Ans: $4.356 \times 10^{-3}\ J$]

14. A capillary tube of $0.4\ mm$ diameter is placed vertically inside (i) water of surface tension $6.5 \times 10^{-2}\ N\ m^{-1}$ and zero angle of contact, (ii) a liquid of density $800\ kg\ m^{-3}$, surface tension $5.0 \times 10^{-2}\ N\ m^{-1}$ and angle 30°. Calculate the height to which the liquid rises in the capillary tube in each case.

[Ans: (i) 6.6 cm (ii) 5.5 cm]

15. A capillary tube is immersed in water of surface tension $0.07\ N\ m^{-1}$ and rises 6.2 cm. By what depth will mercury be depressed if the same capillary tube is immersed in it? (surface tension of mercury $= 0.54\ N\ m^{-1}$, angle of contact between mercury and glass = 140°, density of mercury $= 13600\ kg\ m^{-3}$)

[Ans: 2.7 cm]

16. A soap bubble has a diameter of 4 mm. Calculate the pressure inside it if the atmospheric pressure is $10^5\ N\ m^{-2}$. (surface tension of soap solution $= 2.8 \times 10^{-2}\ N\ m^{-1}$)

[Ans: $1.00056 \times 10^5\ N\ m^{-2}$]

17. Calculate the radius of a bubble formed in water if the pressure outside it is $1.000 \times 10^5\ N\ m^{-2}$ and the pressure inside it is $1.001 \times 10^5\ N\ m^{-2}$. (surface tension of water $= 7.0 \times 10^{-2}\ N\ m^{-1}$)

[Ans: 0.14 cm]

18. A glass tube whose inside diameter is 1 mm is dipped vertically into a vessel containing mercury with lower end 1 cm below the surface. To what height will the mercury rise in the tube if the air pressure inside it is $3 \times 10^3\ N\ m^{-2}$ below atmospheric pressure.? (surface tension of mercury $= 0.5\ N\ m^{-1}$, angle of contact with glass = 180°, density of mercury $= 13600\ kg\ m^{-3}$)

[Ans: 0.75 cm]

19. The diameters of the arms of a U-tube are respectively 1 cm and 1 mm. A liquid of surface tension $7.0 \times 10^{-2}\ N\ m^{-1}$ is poured into the tube which is placed vertically. Find the difference in levels in the two arms. (density of water is $1000\ kg\ m^{-3}$ and the contact angle zero)

[Ans: 2.6 cm]

20. A clean glass capillary tube, of internal diameter 0.04 cm, is held vertically with its lower end below the surface of clean water in a beaker and with 10 cm of the tube above the surface.
   (i) To what height will the water rise in the tube?
   (ii) What will happen if the tube is now depressed until only 5 cm of its length is above the surface?

   (surface tension of water is $0.072\ N\ m^{-1}$)

   [Ans: (i) 7.35 cm (ii) angle of contact now 47°]

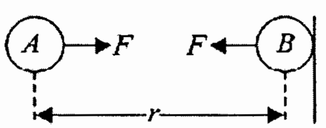
MECHANICAL PROPERTIES OF MATTER

Most of the substances that exist in the world can be classified into one of three phases: solid, liquid or gas. These three phases of matter are governed by interatomic and intermolecular forces under ordinary conditions of temperature and pressure. When acted on by outside forces, solids tend to keep their volume and shape, liquids tend to keep their volume but not their shape and gases tend to keep neither their volume nor their shape. To properly describe a substance, we should provide a description of motion of each atom of which it is composed. Such a description would be worthless for most purposes. It would be far too complicated and detailed for the everyday uses to which the materials are put. The engineer who wishes to use a certain type of steel in construction neither wants nor requires an atomic description of the material. We are generally interested in the overall properties of a material rather than its atomic description.

Interatomic and intermolecular forces

The forces which bind two or more atoms together to make a molecule are called interatomic forces. When molecules are formed as a result of interatomic forces between the atoms, there must be some intermolecular forces which bind the molecules together. The interatomic and intermolecular forces are electrical in nature, arising from interactions of the electrically charged particles that make up the atoms and molecules. The gravitational forces between atoms/molecules are so weak compared with the electrical forces that they are completely negligible.

Consider two atoms or molecules exerting forces of attraction on each other as shown below.



If the force F on A moves it a small distance Δr to the right, then work done ΔW on A is given by

$$\Delta W = F\Delta r$$

If ΔU is the resulting change in potential energy, then

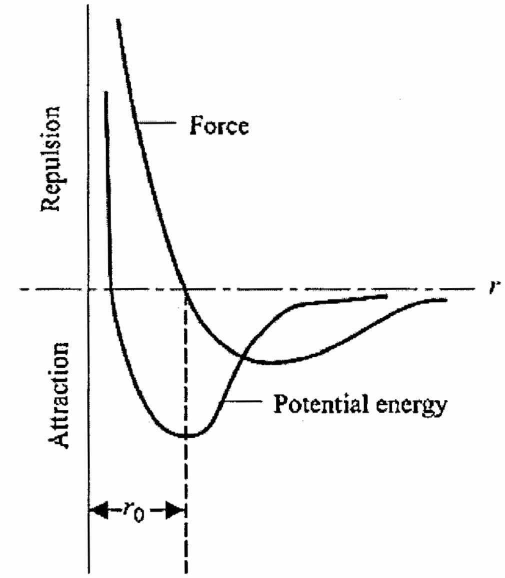
$$\Delta U = -\Delta W$$

$$\Delta U = -F\Delta r$$

In the limit, $F = -\frac{dU}{dr}$

An atom has positively charged nucleus and negatively charged electrons. The total positive charge of the nucleus is equal to the total negative charge of electrons. Therefore, an atom is electrically neutral. However, when two atoms are brought close together, there is electrical interaction between the electrons and the nuclei of the atoms. As a result, the force between the atoms and the potential energy of the system undergo change.

Force and potential energy variation with separation of atoms/molecules



At large distances, the force is small and attractive and the potential energy of the system is negative. As the separation between the atoms decreases, the potential energy decreases (more negative) and the force of attraction becomes larger. The attractive force becomes maximum and then decreases to zero at an equilibrium separation r_0. At $r = r_0$, there is not net force between the atoms and their potential energy has a **minimum value**. When the separation is less than r_0, the force becomes repulsive and increases quite rapidly.

If the two atoms have a separation of r_0, they are in their equilibrium separation. Any increase or decrease in their separation would require energy, since work would have to be done against the net attraction or the net repulsion respectively. The equilibrium is stable because an increase in r leads to an attractive force which restores r to r_0. Similarly, a decrease in r produces a repulsive force which again restores r to r_0. For a small displacement from equilibrium, the force curve is linear and resembles the force exerted by a spring when compressed or extended.

When we try to deform the solid by pushing or pulling it in a bid to change the separation distance between atoms, the strong forces between the atoms resist compression or extension

Elasticity

When an external force is applied on a body, which is not free to move, there will be a relative displacement of the particles. Due to the property of elasticity, the particles tend to regain their original position. The external forces may produce change in length, volume and shape of the body. This external force which produces these changes in the body is called **deforming force**. A body which experiences such a force is called **deformed body**. When the deforming force is removed, the body regains its original state due to the force developed within the body. This force is called **restoring force**.

Elasticity is the property of a material by virtue of which it tries to regain its original shape and size when deforming forces are removed. The bodies which possess this property are called **elastic bodies**. Bodies which do not exhibit the property of elasticity are called **plastic**. The study of mechanical properties helps us to select the material for specific purposes. For example, springs are made of steel because steel is highly elastic.

Stress and strain

In a deformed body, restoring force is set up within the body which tends to bring the body back to the normal position. The magnitude of this restoring force depends upon the deformation caused. Stress is the restoring force per unit area of a deformed body.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

SI unit of stress is N m^{-2}

Due to the application of deforming force, length or shape of a body changes. Or in other words, the body is said to be strained. Thus, strain produced in a body is defined as the ratio of change in length of a body to the original length.

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

Strain is a ratio of two similar quantities therefore it has no units

Elastic limit

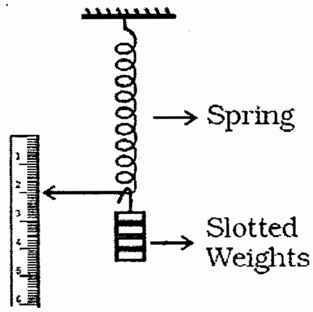
If an elastic material is stretched or compressed beyond a certain limit, it will not regain its original state and will remain deformed. The limit beyond which permanent deformation occurs is called the elastic limit.

Hooke's law

Hooke's law states that the extension of a wire is directly proportional to the applied force provided that the elastic limit is not exceeded.

i.e. $F \propto e \Rightarrow F = ke$

Experimental verification of Hooke's law



A spring is suspended from a rigid support as shown above.

A weight hanger and a light pointer is attached at its lower end such that the pointer can slide over a graduated scale.

The initial reading on the scale is noted.

A slotted weight of m kg is added to the weight hanger and the pointer position is noted. The same procedure is repeated with every additional m kg weight.

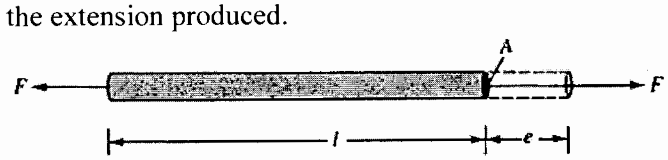
A graph of m against e is plotted and it is a straight line through the origin

This verifies Hooke's law.

Young's modulus of elasticity

Young's modulus of the material of the wire is defined as the ratio of tensile stress to tensile strain.

Consider a wire of length l and cross-sectional area A stretched by a force F acting along its length. Let e be the extension produced.



Tensile stress $= \frac{F}{A}$

Tensile strain $= \frac{\text{change in length}}{\text{original length}} = \frac{e}{l}$

Young's modulus, $E = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{e/l}$

SI unit of Young's modulus is $N\,m^{-2}$

Note:

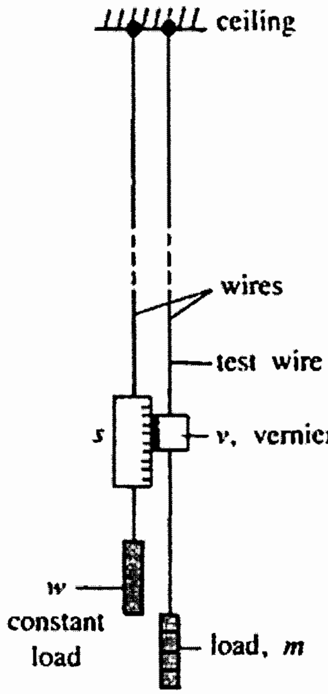
From $E = \frac{F/A}{e/l}$

$$F = \left(\frac{AE}{l}\right)e$$

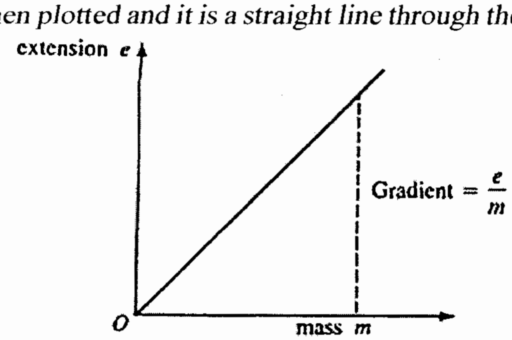
$$F \propto e$$

This is consistent with Hooke's law

Experimental determination of Young's modulus



Two identical long thin wires of the same material are hung from the ceiling.
Initial weights are hung from the lower ends of the wires to remove any kinks (make the wires taut)
The original length l is measure from the ceiling to the Vernier scale
Various loads are added to the test wire and the corresponding extensions are noted.
To check whether the elastic limit had not been exceeded, the extension is also measured when the loads are slowly removed. The extension during loading and unloading should be the same
A graph of extension e against the mass m of the load *is then plotted and it is a straight line through the origin*



$$\text{Young's modulus} = \frac{F/A}{e/l} = \frac{Fl}{Ae}$$

$$\text{But } F = mg \text{ and } A = \frac{\pi d^2}{4}$$

$$\text{Young's modulus} = \frac{mgl}{\left(\frac{\pi d^2}{4}\right)e}$$

$$= \frac{4gl}{\pi d^2\left(\frac{e}{m}\right)}$$

$$= \frac{4gl}{\pi d^2(\text{gradient})}$$

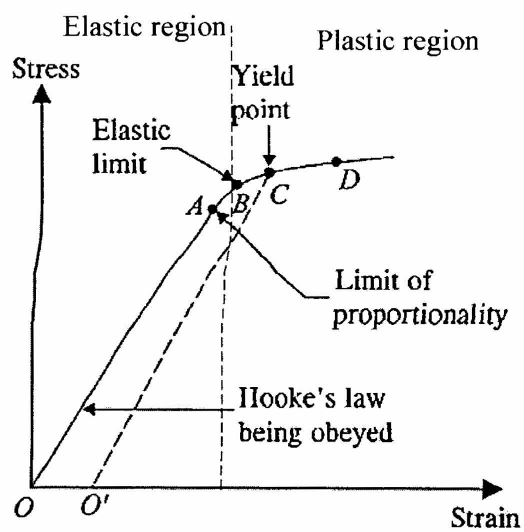
The initial length l of the wire is measured using a metre rule and the diameter d of the wire measured at different points along the wire using a micrometer screw gauge to obtain an average value.

Note

- The wires used are thin and long in order that a larger or measurable extension is obtained. A larger extension gives a higher degree of accuracy
- By using two identical wires of the same material, any error due to thermal expansion or the support yielding can be eliminated
- To prevent kinks along the wires, weights are hung from the lower ends of the wires. Any kinks in the wires will produce errors in the measurements of the extensions

Stress-strain curve for an elastic material

Suppose a metallic wire of uniform cross-section is suspended from a rigid support. When the load on the other end is gradually increased, the length of the wire goes on increasing. If a graph of stress against strain is plotted, it will appear as shown below



Portion OA

The portion OA of the graph is a straight line upto point A. Strain produced in the wire is directly proportional to the stress i.e. strain ∝ stress. In this portion, the material obeys Hooke's law. The point A is called the **proportionality limit**. The proportionality limit is the greatest stress a material can sustain without the departure from a linear stress-strain relation.

If the applied force is removed at any point between O and A, the wire regains its original length

Portion AB

The portion AB of the graph is not a straight line showing that in this region, strain is not proportional to the stress. The slope of the graph decreases which means that strain increases more rapidly with stress. If the load is removed between O and B, the wire will return to its original length. The point B is called the **elastic limit.**

Beyond the elastic limit, plastic deformation occurs.

Portion BC

If the stress is increased beyond the elastic limit, a point C is reached at which there is a marked increase in extension. This point is called **yield point**. Between B and C, the material becomes plastic i.e. if the wire is unloaded between B and C, it does not return to its original length.

Portion CD

If the stress is increased beyond point C, the wire lengthens rapidly until a point D at the top of the curve. The point D is called the **ultimate strength** or **breaking stress.** Beyond point D, even a stress smaller than at C may continue to stretch the wire until it breaks

Strength of a material

This relates to the maximum force which can be applied to a material without it breaking

Stiffness

This relates to the resistance offered by a material to having its size or shape changed

Ductility

A ductile material is one which can be permanently stretched

Brittleness

A brittle material cannot be permanently stretched. It breaks soon after the elastic limit has been reached. Brittle materials are often very strong in compression

Work hardening

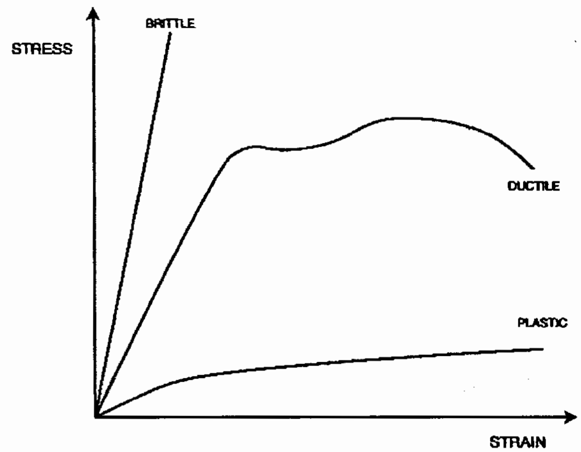
When a metal is deformed by bending it repeatedly, it becomes harder and brittle, and its resistance to plastic deformation increases. The increase in deformation leads to an increase in the density of dislocations. This process is called work hardening of metals.

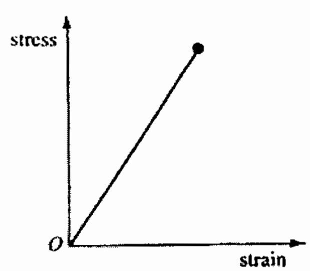
Plastic deformation

Metals have certain planes which are rich in atoms. These planes are called atomic or crystal planes. During plastic deformation, some atomic planes of the material slide over each other. Movement of dislocations takes place, and on removing the stress, the material does not recover its original length and shape. There is also a corresponding loss in mechanical energy.

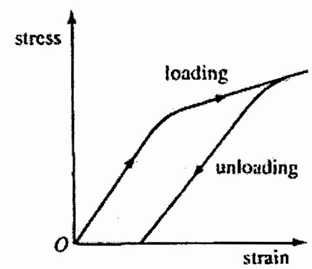
Materials that undergo plastic deformation before breaking are said to be ductile.

Substances like glass which do not show plastic behaviour are said to be brittle.

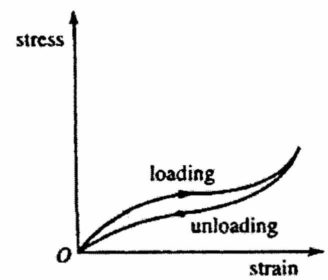




Glass



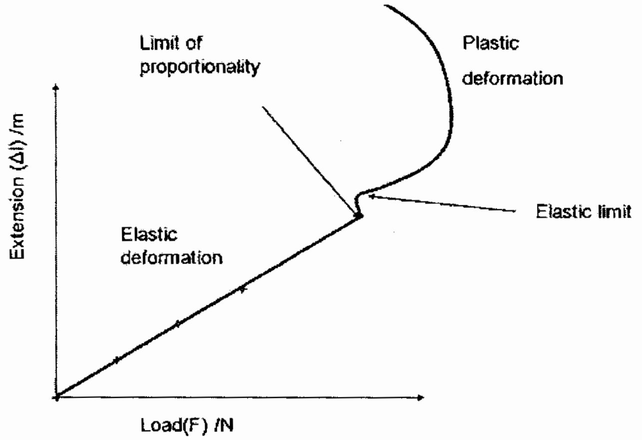
Copper



Rubber

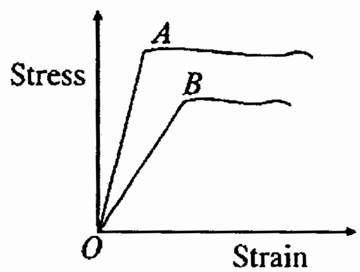
The stress strain curve for rubber departs from Hooke's law but rubber behaves elastically, regaining its original shape. When rubber is stretched by an increasing force and then the force is slowly decreased, the strain during unloading is always higher than the that during loading.

A graph of extension against force



Example 1

The figure below sows the stress-strain relationship of two materials A and B.



State with reasons which material is

(i) more ductile

(ii) more brittle

Solution

(i) The ductility of a material is the extent of plastic deformation. Clearly it is greater for material A

(ii) The plastic region for material B is small. Therefore, material B is more brittle

Example 2

(a) Why are springs made of steel and not of copper?

(b) Why is work done in stretching a wire?

(c) What happens to the work done during stretching a wire?

Solution

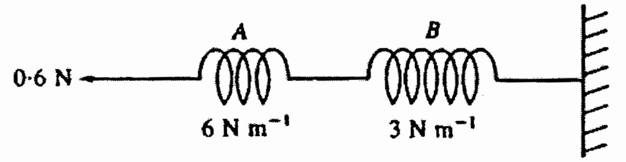
(a) It is because Young's modulus of steel is more than that of copper. Therefore, for a given deforming force steel spring is stretched lesser than copper spring and regains its original state quickly on the removal of the deforming force

(b) When a wire is stretched, the interatomic forces oppose the increase in length of the wire. Therefore, work has to be done against these forces

(c) The work done in stretching the wire is stored in it in the form of elastic potential energy

Example 3

The force constant k of a spring is the constant of proportionality in the Hooke's law relation $T = ke$ between tension T and extension e.



A spring A of force constant $6\ N\ m^{-1}$ is connected in series with a spring B of force constant $3\ N\ m^{-1}$ as shown above. One end of the combination is securely anchored and a force of $0.6\ N$ is applied to the other end.

(a) By how much does each spring extend?

(b) What is the force constant of the combination?

Solution

(a) By Hooke's law, $F = ke$

For spring A, extension, $e = \frac{F}{k}$

$$= \frac{0.6}{6} = 0.10\,m$$

For spring B, extension $= \frac{0.6}{3} = 0.20\ m$

Total extension $= 0.10 + 0.20 = 0.30\,m$

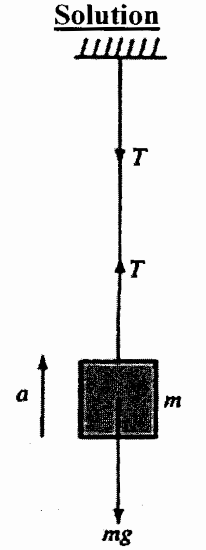
(b) Force constant of combination $= \frac{F}{e}$

$$= \frac{0.6}{0.30} = 2\ N\ m^{-1}$$

Example 4

A lift of mass 1000 kg hangs from one end of a steel cable. The maximum acceleration upwards of the lift is $1.2\ m\ s^{-2}$. If the maximum safe stress for steel is $1.1 \times 10^8\ N\ m^{-2}$, calculate the diameter of the cable

Solution



When the lift accelerates upwards with acceleration a

Using $F = ma$

$$T - mg = ma$$

$$T = m(g + a)$$

Maximum tension, $T_{max} = m(g + a_{max})$

$$= 1000(9.81 + 1.20)$$

$$= 11010\ N$$

Maximum stress $= \frac{T_{max}}{\text{minimum cross-sectional area}}$

$$\pi\, d_{min}^2 = \frac{11010}{1.1 \times 10^8}$$

$$d_{min} = 1.13 \times 10^{-2}\ m$$

Example 5

A steel wire and a brass wire of the same cross-sectional area of $1.00\ mm^2$ and length $4.00\ m$ are suspended from the same point. The lower ends of the wires are joined together. What is the common extension of the wires when a mass of $10.0\ kg$ is hung from the lower end of the composite wire? (Young's modulus of steel $= 2.00 \times 10^{11}\ Pa$, brass $= 9.0 \times 10^{10}\ Pa$)

Solution

Let $x =$ common extension

Tension in steel wire, $T_1 = \frac{E_1 A_1}{l} x$

Tension in brass wire, $T_2 = \frac{E_2 A}{l} x$

$$T_1 + T_2 = mg$$

$$\frac{E_1 A_1}{l} x + \frac{E_2 A}{l} x = mg$$

$$x = \frac{mgl}{A(E_1 + E_2)}$$

$$= \frac{10 \times 9.81 \times 4.00}{(1.00 \times 10^{-6})(2 + 0.9) \times 10^{11}}$$

$$= 1.35 \times 10^{-3}\ m$$

Example 6

Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is $1\ cm$, find how much each wire is elongated. (Young's modulus of steel $= 20 \times 10^{10}\ N\, m^{-2}$, for copper $= 12 \times 10^{10}\ N\, m^{-2}$)

Solution

Let l and A be the length and area of X-section of each wire respectively

For steel, $E_s = \frac{Fl}{Ae_s}$

For copper, $E_c = \frac{Fl}{Ae_c}$

$$\frac{E_s}{E_c} = \frac{e_c}{e_s}$$

$$\frac{e_c}{e_s} = \frac{E_s}{E_c} = \frac{20 \times 10^{10}}{12 \times 10^{10}} = \frac{5}{3}$$

$$e_c = \frac{5}{3} e_s \ \ldots\ldots \text{(i)}$$

Also $e_s + e_c = 1$ .....(ii)

Substituting for e_c in (ii) gives;

$$e_s + \frac{5}{3} e_s = 1$$

$$\frac{8e_s}{3} = 1$$

$$e_s = \frac{3}{8} = 0.375\ cm$$

$$e_c = 1 - 0.375 = 0.675\ cm$$

Example 7

A composite wire of diameter $1\ cm$ consists of copper and steel wires of lengths $2.2\ m$ and $2\ m$ respectively. Total extension of the wire when stretched by a force is $1.2\ mm$. Calculate the force given that Young's modulus for copper is $1.1 \times 10^{11}\ Pa$ and for steel is $2 \times 10^{11}\ Pa$

Solution

The values of F and A are the same in the two cases

For copper wire: $E_c = \frac{Fe_c}{Al_c}$

For steel wire: $E_s = \frac{Fe_s}{Al_s}$

$$\frac{E_c}{E_s} = \frac{l_c e_s}{l_s e_c}$$

$$\frac{e_s}{e_c} = \frac{E_c l_s}{E_s l_c} = \frac{1.1 \times 10^{11} \times 2.0}{2.0 \times 10^{11} \times 2.2} = \frac{1}{2}$$

$$e_s = \frac{1}{2} e_c \ \ldots\ldots \text{(i)}$$

$$\text{But } e_s + e_c = 1.2 \ \ldots \text{(ii)}$$

$$\Rightarrow 0.5\, e_c + e_c = 1.2$$

$$e_c = \frac{1.2}{1.5} = 0.8\ mm$$

Area of cross-section of copper wire

$$= \frac{\pi d^2}{4} = \frac{\pi(1 \times 10^{-2})^2}{4} = 0.785 \times 10^{-4}\ m^2$$

From $E_c = \frac{Fe_c}{Al_c}$,

$$F = \frac{AE_c}{l} e_c$$

$$= \frac{0.785 \times 10^{-4} \times 1.1 \times 10^{11}}{2.2} \times 0.8 \times 10^{-3}$$

$$= 3.14 \times 10^3\ N$$

Example 8

A light rod of length $20\ cm$ is supported horizontally from the ends of two vertical wires attached to its ends. The first wire is made of steel of diameter $1.0\ mm$ and the second wire of copper of diameter $2.0\ mm$. Initially the wires are of the same length. Find the point where a load must be hung so that the rod remains horizontal. (Young's modulus of steel $= 2.0 \times 10^{11}\ Pa$, copper $= 1.2 \times 10^{11}\ Pa$)

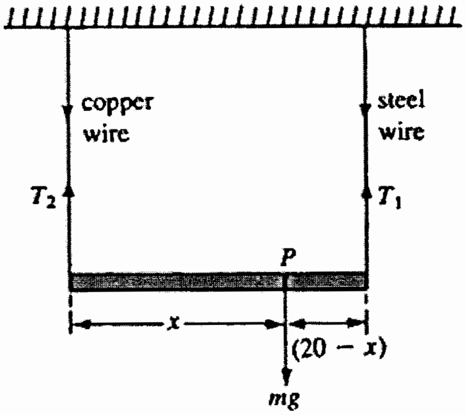
Solution

In order for the rod to remain horizontal, the extensions in both wires must be the same. Let e = common extension

Tension in steel wire, $T_1 = \frac{E_1A_1e}{l}$

Tension in copper wire, $T_2 = \frac{E_1A_2e}{l}$

Let the point P, where the rod is loaded, be at a distance x from the copper wire.



Taking moments about P.

Clockwise moment = anticlockwise moment

$$T_2x = T_1(20 - x)$$

$$\frac{E_2A_2e}{l}x = \frac{E_1A_1e}{l}(20 - x)$$

$$E_2A_2x = 20E_1A_1 - E_1A_1x$$

$$E_2A_2x + E_1A_1x = 20E_1A_1$$

$$x = \frac{20E_1A_1}{E_1A_1 + E_2A_2}$$

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi(1.0\times10^{-3})^2}{4} = 7.85\times10^{-7}\,m^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi(2.0\times10^{-3})^2}{4} = 3.14\times10^{-6}\,m^2$$

$$x = \frac{20\times2.0\times10^{11}\times7.85\times10^{-7}}{2.0\times10^{11}\times7.85\times10^{-7} + 1.2\times10^{11}\times3.14\times10^{-6}}$$

$= 5.88\,cm$ from the copper wire

Example 9

A light rod of length $200\,cm$ is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its end. One of the wires is made of steel and is of cross-section $0.1\,cm^2$ and the other is of copper of cross-section $0.2\,cm^2$. Find out the position along the rod at which a weight may be hung to produce

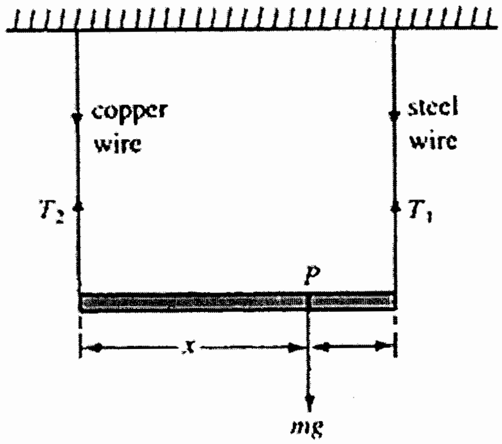
(a) equal stresses in both wires

(b) equal strains in both wires

(Young's modulus of copper and steel are $1.0\times10^{11}\,Pa$ and $2.0\times10^{11}\,Pa$ respectively)

Solution

(a)



let the weight be applied at P at a distance of x from the copper wire.

If T_1 and T_2 are the tensions in the steel and copper wires and the areas of cross-section are A_1 and A_2, then for equal stresses,

$$\frac{T_1}{A_1} = \frac{T_2}{A_2}$$

$$\frac{T_1}{T_2} = \frac{A_1}{A_2}$$

Since $A_1 = 0.1$ and $A_2 = 0.2$

$$\frac{T_1}{T_2} = \frac{0.1}{0.2} = \frac{1}{2}$$

The light rod is in equilibrium under the action of forces T_1, T_2 and mg

Taking moments about P,

$$T_2x = T_1(2 - x)$$

$$x = \frac{T_1}{T_2}(2 - x)$$

$$x = \frac{1}{2}(2 - x)$$

$$3x = 2$$

$$x = \frac{2}{3} = 0.67\,m$$

Thus, the weight must be hung at $0.67\,m$ from the copper wire or $1.33\,m$ from the steel wire

(b) Let the weight mg be applied at P' at a distance x' from the copper wire. If the tensions in steel and copper wires are T_1' and T_2' respectively

From, strain $= \frac{stress}{E} = \frac{T/A}{E} = \frac{T}{AE}$

Strain in steel wire $= \frac{T_1'}{A_1E_1}$

Strain in copper wire $= \frac{T_2'}{A_2E_2}$

For equal strains, $\frac{T_1'}{A_1E_1} = \frac{T_2'}{A_2E_2}$

$$\frac{T_1'}{T_2'} = \frac{A_1}{A_2}\times\frac{E_1}{E_2}$$

$$= \frac{0.1}{0.2} \times \frac{2.0 \times 10^{11}}{1.0 \times 10^{11}} = 1$$

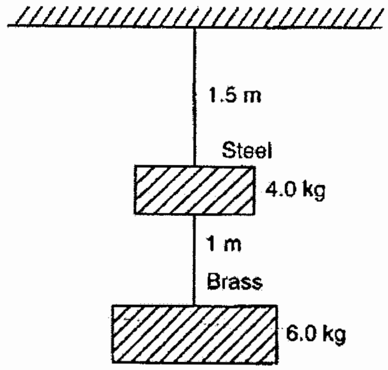
$$\therefore T_1' = T_2'$$

For producing equal tensions, the weight mg must be suspended at the middle of the rod

$$\therefore x' = 1.00\, m$$

Example 10

Two wires of diameter $0.25\, cm$ each, one made of steel and the other made of brass are loaded as shown in the figure below



The unloaded length of steel is $1.5\, m$ and that of brass wire is $1.0\, m$. Young's modulus of steel is $2.0 \times 10^{11}\, Pa$ and that of brass is $0.91 \times 10^{11}\, Pa$. Calculate the extensions in the steel and brass wires

Solution

For steel wire, total force on steel wire

$$F_1 = (4 + 6) \times 9.81 = 98.1\, N$$

$l_1 = 1.5\, m$, $e_1 = ?$,

$$r_1 = \frac{0.25}{2} cm = 0.125 \times 10^{-2}\, m$$

$$E_1 = 2.0 \times 10^{11}\, Pa$$

For brass wire, $F_2 = 6 \times 9.81 = 58.86\, N$

$$r_2 = 0.125 \times 10^{-2}\, m$$

$$E_2 = 0.91 \times 10^{11}\, pa$$

$l_2 = 1.0\, m$, $e_2 = ?$

$$E_1 = \frac{F_1 l_1}{A_1 e_1} = \frac{F_1 l_1}{\pi r_1^2 e_1}$$

$$e_1 = \frac{F_1 l_1}{\pi r_1^2 E_1} = \frac{98.1 \times 1.5}{\pi (0.125 \times 10^{-2})^2 (2 \times 10^{11})}$$

$$= 1.49 \times 10^{-4}\, m$$

$$e_2 = \frac{F_2 l_2}{\pi r_2^2 E_2} = \frac{58.86 \times 1}{\pi (0.125 \times 10^{-2})^2 (0.91 \times 10^{11})}$$

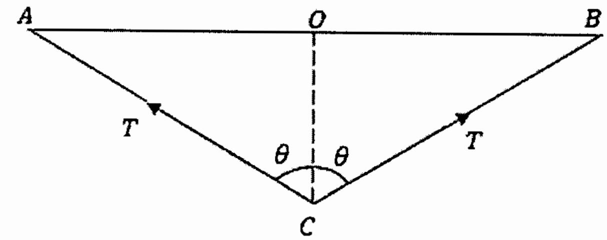
$$= 1.3 \times 10^{-4}\, m$$

Example 11

A steel wire of diameter $0.8\, mm$ and length $1\, m$ is clamped firmly at two points A and B which are $1\, m$ apart and in the same horizontal plane. A body is suspended from the middle point of the wire. If the middle point sags $1\, cm$ lower from the original position, calculate the mass of the body. (Young's modulus $= 2.0 \times 10^{11}\, Pa$)

Solution

Let the tension in the wire be T.



For the equilibrium of body of mass m,

$$2T \cos\theta = mg$$

$$m = \frac{2T\cos\theta}{g} \quad .....(i)$$

To find T, we use Hooke's law

$$E = \frac{T/A}{e/l}$$

$$T = EA\frac{e}{l} \quad ...... (ii)$$

Substituting for T in (i);

$$m = 2\left(EA\frac{e}{l}\right)\frac{\cos\theta}{g}$$

From geometry of the figure,

$$AC = \sqrt{AO^2 + OC^2} = \sqrt{50^2 + 1^2}$$

$$AC = 50.01\, cm$$

$$\frac{e}{l} = \frac{AC - AO}{AO} = \frac{50.01 - 50}{50} = \frac{0.01}{50}$$

$$\cos\theta = \frac{OC}{AC} = \frac{1}{50.01}$$

$$A = \pi(0.4 \times 10^{-3})^2 = 5.03 \times 10^{-7}\, m^2$$

Therefore, $m = 2\left(EA\frac{e}{l}\right)\frac{\cos\theta}{g} = \frac{2EA}{g}\left(\frac{e}{l}\right)\cos\theta$

$$= \frac{2 \times 2.0 \times 10^{11} \times 5.03 \times 10^{-7}}{9.81} \times \frac{0.01}{50} \times \frac{1}{50.01}$$

$$= 82 \times 10^{-3}\, kg$$

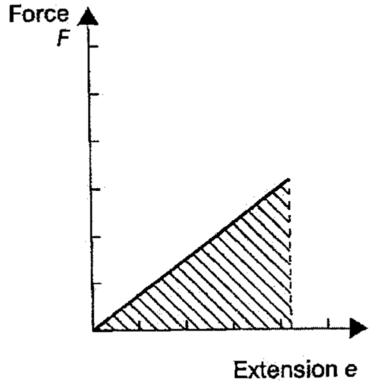
$$= 82\, g$$

Energy stored in a stretched wire

When a wire is stretched, the work done on the wire is stored as elastic potential energy. This elastic potential energy in a stretched wire can be obtained from a graph of extension against force.

Method 1

Let the wire be of unstretched length l and let a force F produce an extension e.



The work done by the force is Fs but in this case the force varies from 0 at the start to F at the end when the wire is stretched by an amount e. Thus

Work done on the wire during stretching

$$= \text{average force} \times \text{extension}$$
$$= \frac{0+F}{2} \times e$$
$$= \frac{1}{2}Fe$$

Method 2

When a wire is stretched by a small extension dx by a force F, the work done by the force

$$dW = F\,dx$$

From Hooke's law, $F = kx$

$$dW = kx\,dx$$

Total work done in extending the wire from 0 to e is given by

$$W = \int_0^e kx\,dx$$
$$= k\left[\frac{x^2}{2}\right]_0^e$$
$$= \frac{1}{2}ke^2 = \frac{1}{2}(ke)e = \frac{1}{2}Fe$$

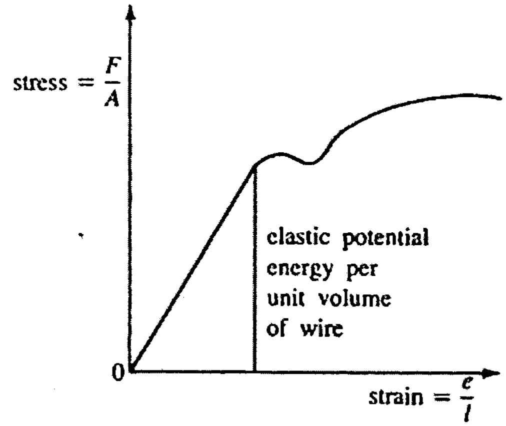
Work done by unit volume

Work done = elastic potential energy $= \frac{1}{2}Fe$

elastic potential energy per unit volume of wire

$$= \frac{\frac{1}{2}Fe}{\text{volume of wire}}$$
$$= \frac{\frac{1}{2}Fe}{Al}$$
$$= \frac{1}{2}\left(\frac{F}{A}\right) \times \left(\frac{e}{l}\right)$$
$$= \frac{1}{2}(\text{stress})(\text{strain})$$

= area under stress-strain graph



The work done per unit volume or the elastic potenti energy per unit volume is equal to area under th stress-strain graph within the proportional limit.

Example 12

A vertical steel wire of length $0.80\,m$ and radi $1.0\,mm$ has a mass of $20\,kg$ applied to its lower end Assuming that the proportional limit is not exceeded calculate

(a) the extension

(b) the energy stored per unit volume

(Young's modulus of steel $= 2.0 \times 10^{11}\,Pa$)

<u>Solution</u>

(a) From $E = \frac{F/A}{e/l}$

$$e = \frac{Fl}{EA}$$
$$F = 20\,g = 20 \times 9.81 = 196.2\,N$$
$$A = \pi r^2 = \pi(1.0 \times 10^{-3})^2 = \pi \times 10^{-6}\,m^2$$
$$l = 0.80\,m$$
$$E = 2.0 \times 10^{11}\,Pa$$
$$e = \frac{196.2 \times 0.80}{2.0 \times 10^{11} \times \pi \times 10^{-6}}$$
$$= 0.255 \times 10^{-3}\,m$$

(b) Stress $= \frac{F}{A} = \frac{196.2}{\pi \times 10^{-6}} = 6.25 \times 10^{7}\,Nm^{-2}$

Strain $= \frac{stress}{E} = \frac{6.25 \times 10^{7}}{2.0 \times 10^{11}} = 3.125 \times 10^{-4}$

Work done per unit volume $= \frac{1}{2}(stress)(strain)$

$$= \frac{1}{2} \times 6.25 \times 10^{7} \times 3.125 \times 10^{-4}$$
$$= 9.77 \times 10^{3}\,J\,m^{-3}$$

Example 13

A mass of $3.5\,kg$ is gradually applied to the lower end of a vertical wire and produces an extension of $0.80\,mm$. Calculate the

(a) energy stored in the wire

(b) loss in gravitational potential energy of the mass during loading

(c) Account for the difference between the two answers

Solution

(a) $F = mg = 3.5 \times 9.81 = 34.335\ N$

$$e = 0.80 \times 10^{-3}\ m$$

$$\text{Work done} = \frac{1}{2}Fe$$

$$= \frac{1}{2} \times 34.335 \times 0.80 \times 10^{-3}$$

$$= 13.73 \times 10^{-3}\ J$$

This is stored as elastic potential energy

(b) Loss in PE $= mgh$

$$= 3.5 \times 9.81 \times 0.80 \times 10^{-3}$$

$$= 27.47 \times 10^{-3}\ J$$

(c) The energy stored is only half the loss in gravitational potential energy because the wire needs a gradually increasing load from 0 to $34.335\ N$, to extend it. The remaining gravitational energy is given to the loading system (e.g. hand as it gradually attaches the load to the wire)

Example 14

A wire of natural length l and cross-sectional area A is fixed at one end. A mass m is attached to the lower end of the wire and lowered slowly so that it finally hangs in equilibrium.

(a) If E is the Young's modulus and the elastic limit is not exceeded, find the extension in the wire in terms of g, l, A, m and E

(b) What is the energy stored in the stretched wire?

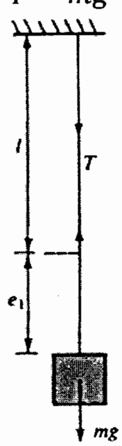
(c) What is the loss in potential energy of the mass? Account for any differences to answer in (b)

(d) Find the maximum extension in the wire if instead of slowly lowering the mass down, it is released when the length of the wire is l

Solution

(a) When the mass hangs in equilibrium,

$$T = mg$$



From the definition of Young's modulus,

$$E = \frac{F/A}{e/l}$$

$$E = \frac{mgl}{Ae_1}$$

$$e_1 = \frac{mgl}{AE}$$

(b) Energy stored in the stretched wire $= \frac{1}{2}Fe_1$

$$= \frac{1}{2}mg\left(\frac{mgl}{AE}\right)$$

$$= \frac{1}{2}\frac{m^2g^2l}{AE}$$

(c) Loss in potential energy of mass $= mge_1$

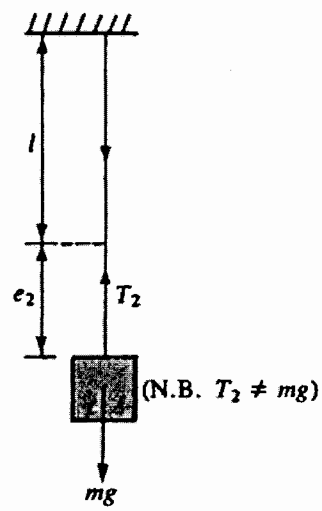
$$= mg\left(\frac{mgl}{AE}\right)$$

$$= \frac{m^2g^2l}{AE}$$

= twice the energy stored in a stretched wire

Half of the loss in potential energy of the mass is converted into elastic potential energy of the wire. The other half of the loss in the potential energy is used to work against the resistance provided by the hand when the mass in lowered by the hand.

(d) Let e_2 = maximum extension of wire when the mass is released.



By the principle of conservation of energy,

Loss in P.E = gain in elastic potential energy

$$mge_2 = \frac{1}{2}T_2e_2$$

$$\text{But } T_2 = \frac{EAe_2}{l}$$

$$mge_2 = \frac{1}{2}\left(\frac{EAe_2}{l}\right)e_2$$

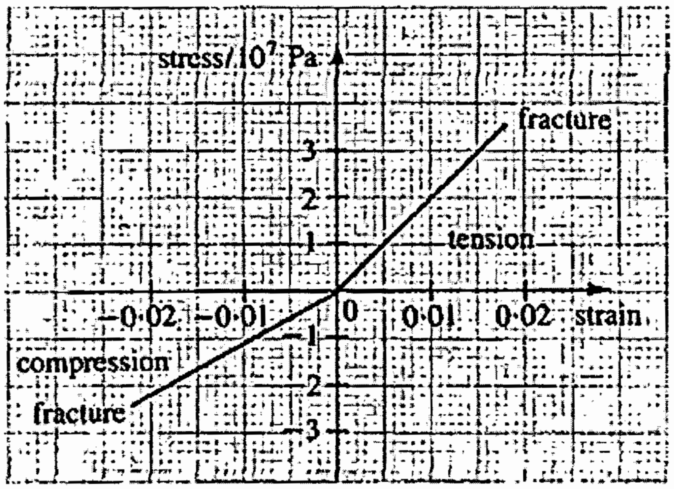
$$e_2 = \frac{2mgl}{EA}$$

$$= 2e_1$$

When the wire is extended by e_2, the mass m is not in equilibrium as $T_2 > mg$. Subsequently, the mass m will perform a vertical oscillation

Example 16

A stress-strain graph for bone is shown in the figure below. The left-hand portion of the graph is for compression while the right-hand portion is for tension



(a)(i) What feature of the graph enables you to conclude that bone is a brittle material?

(ii) Use your graph to determine a value for the Young's modulus of bone in compression. Determine the corresponding value for bone in tension

(b)The minimum cross-sectional area of a particular thigh bone is $6.0 \times 10^{-4}\ m^2$ and its length is $0.45\ m$

(i) Use the graph to determine the compressive load at which fracture occurs

(ii) By how much will the length of the bone have been reduced just before fracture?

(iii) Calculate the elastic energy stored in the bone just before fracture

(iv) State two assumptions made in these calculations

(c)Describe a situation which might give rise to a compressive fracture of the thigh. Explain the principle involved

Solution

(a)(i) The graph is a straight line that ends abruptly without curving before fracture occurs

(ii) In compression,

$$\text{Young's modulus} = \frac{stress}{strain}$$

$$= \text{gradient of graph}$$

$$= \frac{2.4 \times 10^7}{0.022} = 1.1 \times 10^9\ Pa$$

In tension,

$$\text{Young's modulus} = \frac{3.5\times10^7}{0.018}$$

$$= 1.9 \times 10^9\ Pa$$

(b)(i) Stress $= \frac{F}{A}$

Force, $F = stress \times A$

From the compression part of the graph, frac occurs when stress $= 2.4 \times 10^7\ Pa$

Compression load when fracture occurs,

$$F = (2.4 \times 10^7) \times (6.0 \times 10^{-4})$$

$$= 1.44 \times 10^4\ N$$

(ii) Using strain $= \frac{\text{compression}}{\text{original length}}$

compression = strain × original length

From the compression part of the graph, wh fracture occurs, strain = 0.022

compression $= 0.022 \times 0.45$

$$= 9.9 \times 10^{-3}\ m$$

(iii) Elastic energy stored $= \frac{1}{2} \times$ final force compression

$$= \frac{1}{2} \times (1.44 \times 10^4) \times (9.9 \times 10^{-3})$$

$$= 71\ J$$

(iv) Assumptions

Hooke's law is obeyed

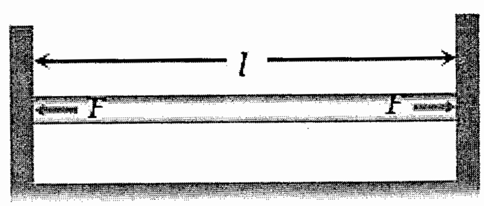
The bone is of uniform cross-section

(c) A person jumping from a height and landing with his legs upright.

A large force is exerted on his legs due to rapid change in momentum. This force together with the weight of the person exceeds the maximum compression force on the thigh bone

Temperature effects

If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This stress is called **thermal stress.**



$$\Delta l = \alpha l \Delta\theta$$

where Δl is the change in length, α is the linear expansivity, l is the original length and $\Delta\theta$ is the temperature change.

If during a temperature change, the rod is to be prevented from changing in length, large forces are often required.

Example 16

A steel wire 8 m long and 4 mm in diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls by 10°C. Given $\alpha = 12 \times 10^{-6}\ °C^{-1}$, $E = 2 \times 10^{11}\ N\,m^{-2}$.

Solution

Increase in length, $\Delta l = \alpha l \Delta\theta$

Strain $= \frac{\Delta l}{l} = \alpha\Delta\theta$

Stress $= E \times$ strain $= E\alpha\Delta\theta$

Increase in tension = stress × Area of cross-section

$$= (E\alpha\Delta\theta) \times \pi r^2$$

$$= (2 \times 10^{11} \times 12 \times\times 10^{-6} \times 10) \times \pi(2 \times 10^{-3})^2$$

$$= 301.7\ N$$

Example 17

A uniform rod of 2 mm^2 cross-section is heated from 0°C to 25°C. Find the force which must be exerted to prevent it from expanding. Given $\alpha = 12 \times 10^{-6}\ °C^{-1}$, $E = 2 \times 10^{11}\ N\,m^{-2}$.

Solution

Stress $= E\alpha\Delta\theta$

$$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 25$$

$$= 10^7\ N\,m^{-2}$$

Force required = stress × Area of cross-section

$$= 10^7 \times 2 \times 10^{-6} = 20\ N$$

Example 18

A solid copper rod is of cross-sectional area 15 mm^2 and length 2.0 m. Calculate

(a) its change in length when its temperature rises by 30°C

(b) the force needed to prevent it from expanding by the amount in (a).

Take the linear expansivity for copper as $20 \times 10^{-6}\ K^{-1}$ and Young's modulus as $1.2 \times 10^{11}\ N\,m^{-2}$

Solution

(a) $\Delta l = \alpha l \Delta\theta$

$$= 20 \times 10^{-6} \times 2 \times 30$$

$$= 12 \times 10^{-4}\ m$$

(b) A compressive force must be supplied which is insufficient to decrease the length by

$$\Delta l = 12 \times 10^{-4}\ m$$

$$F = \frac{EeA}{l}$$

$$= \frac{1.2 \times 10^{11} \times 12 \times 10^{-4} \times 15 \times 10^{-6}}{2.0}$$

$$= 1080\ N$$

Practical applications of elasticity

- The metallic parts of machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed.
- The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.
- The bridges are declared unsafe after long use because during its long use, a bridge undergoes quick alternating strains continuously. It results in the loss of elastic strength.
- A hollow shaft is stronger than a solid shaft made of same mass, length and material.

Self-Evaluation exercise

1. A wire of cross-sectional area $1.0 \times 10^{-6}\ m^2$ and length 5.0 m is stretched $4.0 \times 10^{-3}\ m$ by a load of 60 N. Calculate
   (a) stress
   (b) strain
   (c) Young's modulus

[Ans: (a) $6 \times 10^7\ Pa$ (b) 8×10^{-4} (c) $7.5 \times 10^{10}\ Pa$]

2. A wire of 1 mm diameter and 1 m long fixed at one end is stretched by 0.01 mm when a load of 10 kg is attached to its free end. Calculate the Young's modulus of elasticity

[Ans: $1.25 \times 10^3\ N\,m^{-2}$]

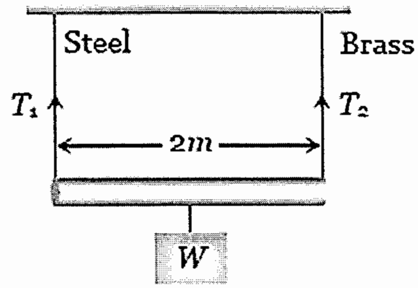
3. Calculate the percentage increase in length of a wire of diameter 2.5 mm stretched by a force equal to the weight of 100 kg given that Young' modulus for the wire is $12.5 \times 10^{10}\ N\,m^{-2}$

[Ans: 0.159%]

4. Two exactly similar wires, one of steel and the other of copper, are stretched by an equal force. If the total elongation is 2 cm, find how much each wire is elongated. (Young's modulus of steel = $2 \times 10^{11}\ N\,m^{-2}$, for copper = $12 \times 10^{10}\ N\,m^{-2}$)

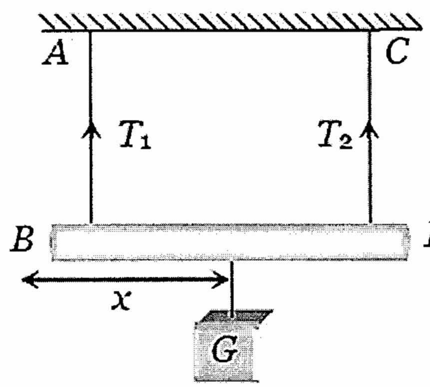
[Ans: steel: 0.75 cm; copper: 1.25 cm]

5. A two metre long rod is suspended with the help of two wires of equal length. One wire is of steel and its cross-sectional area is 0.1 cm^2 and another wire is of brass and its cross-sectional area is 0.2 cm^2.



If a load W is suspended from the rod and the stress produced in both wires is the same, show that $\frac{T_1}{T_2} = 0.5$

6. AB is an iron wire and CD is a copper wire of same length and same cross-section. BD is a rod of length $0.8\,m$. A mass of $2\,kg$ is suspended from the rod.



At what distance x from the point B should the load be placed for the rod to remain in a horizontal position? ($E_{cu} = 11.8 \times 10^{10} Nm^{-2}$,$E_{Fe} = 19.6 \times 10^{10} Nm^{-2}$)

[Ans: $0.3\,m$]

7. The maximum upward acceleration of a lift of total mass $2500\,kg$ is $0.5\,m\,s^{-2}$. The lift is supported by a steel cable, which has a maximum safe working stress of $1.0 \times 10^8\,Pa$. What minimum area of cross-section of cable should be used?

[Ans: $2.6 \times 10^{-4}\,m^2$]

8. A cylindrical copper wire and a cylindrical steel wire, each of length $1.000\,m$ and having equal diameters are joined at one end to form a composite wire $2.000\,m$ long. This composite wire is subjected to a tensile stress until the length becomes $2.002\,m$. Calculate the tensile stress applied to the wire.

(The Yong's modulus for copper $= 1.2 \times 10^{11}\,Pa$ and for steel $= 2.0 \times 10^{11}\,Pa$)

[Ans: $1.5 \times 10^8\,Pa$]

9. A heavy rigid bar is supported horizontally from a fixed support by two vertical wires A and B of the same initial length and which experience the same extension. If the ratio of the diameter of A to that of B is 2 and the ratio of Young's modulus of A to that of B is 2, calculate the ratio of the tension in A to that in B.

[Ans: 8 : 1]

10. The ends of a uniform wire of cross-sectional area $10^{-6}\,m^2$ and negligible mass are attached to two fixed points A and B which are $1\,m$ apart in the same horizontal plane. The wire is initially straight and unstretched. A mass of $0.5\,kg$ is attached to the mid-point of the wire and hangs in equilibrium with the mid-point at a distance $10\,mm$ below AB. Calculate the value of the Young's modulus of the wire.

[Ans: $6.25 \times 10^{11}\,N\,m^{-2}$]

11. A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass. Each wire is $2.00\,m$ long. The diameter of the steel wire is $0.60\,mm$ and the length of the bar is $0.20\,m$. When a mass of $10.0\,kg$ is suspended from the centre of the bar, it remains horizontal
(a) What is the tension in each wire?
(b) Calculate the extension of the steel wire and the energy stored in it
(c) Calculate the diameter of the brass wire
(d) If the brass wire were replaced by another brass wire of diameter $1.0\,mm$, where should the mass be suspended so that the bar remains horizontal?

(Young's modulus for steel $= 2.0 \times 10^{11}\,Pa$, for brass $= 1.0 \times 10^{11}\,Pa, g = 10\,ms^{-2}$)

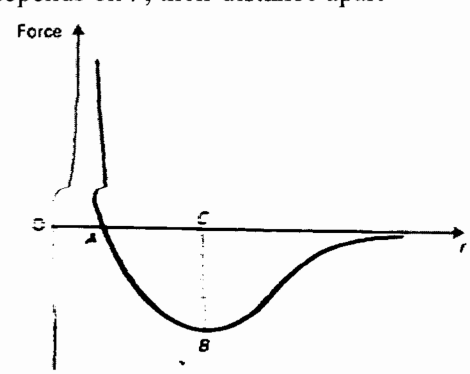
[Ans: (a) $50\,N$ (b) $1.8 \times 10^{-3}\,m$; $0.044\,J$ (c) $0.85\,mm$ (d) $0.084\,m$ from Brass wire]

12. A nylon guitar string $62.8\,cm$ long and $1\,mm$ diameter is tuned by stretching it $2.0\,cm$. Calculate
(a) the tension
(b) the elastic energy stored in the string

(Young's modulus of nylon $= 2 \times 10^9\,Pa$)

[Ans: (a) $50\,N$ (b) $0.5\,J$]

13. The sketch shows, approximately how the resultant force between adjacent atoms is a solid depends on r, their distance apart

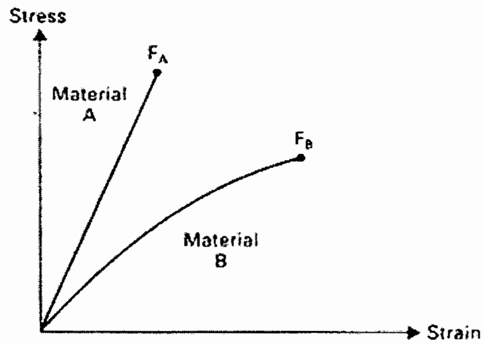


(a) Which distance on the graph represents the equilibrium separation of the atoms? Briefly justify your answer

(b) What is the significance of the shaded area?

(c) Use the graph to explain why you would expect the solid to obey Hooke's law for small extensions and compressions

14. The graphs represent stress-strain curves for two different materials, A and B. F_A and F_B are the respective points at which each material fractures



State, giving your reasons, which material, A or B,

(a) obeys Hooke's law up to the point of fracture

(b) is the weaker

(c) has the greater value of Young's modulus

15. A thin steel wire initially $1.5\,m$ long and of diameter $0.50\,mm$ is suspended from a rigid support. Calculate

(i) the final extension

(ii) the energy stored in the wire when a mass of $3.0\,kg$ is attached to the lower end. Assume that the material obeys Hooke's law

(Young's modules for steel $= 2.0 \times 10^{11}\,N\,m^{-2}$)

[Ans: (i) $1.1\,mm$ (ii) 0.017 J]

16. When materials are stretched their behaviour may be either elastic or plastic. Distinguish carefully between these terms

17. While stretching a length of thin copper wire, it is noticed that

(i) at first a fairly strong pull is needed to stretch it by a small amount and it stretches uniformly

(ii) beyond a certain point the wire extends by very much larger amount for no further increase in the pull

(iii) finally, the wire breaks

Sketch a force -extension graph to illustrate the behaviour of this wire. Mark on it the region where the behaviour is elastic and the region where it is plastic

18. (a) Distinguish between elastic and plastic deformation of a material

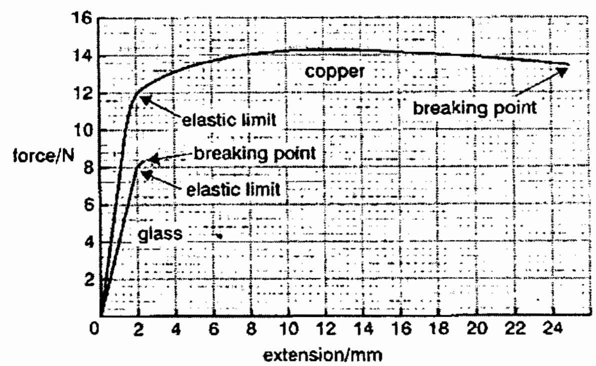
(b) Sketch a graph to show how the extension x of a copper wire varies with F, the applied load. Mark on your sketch the region where the wire obeys Hooke's law

(c) A force is required to cause an extension of a spring. Explain why this causes energy to be stored in the spring

(d) A spring of spring constant k undergoes an elastic change resulting in an extension x. Deduce that W, its strain energy is given by

$$W = \frac{1}{2}kx^2$$

19. A specific fibre of glass has the same dimensions as a specimen of copper wire. The length of each specimen is $1.60\,m$ and the radius of each is $0.18\,mm$. Force-extension graphs are shown in the diagram below



(a) State, giving reasons, which of the two materials is brittle

(b) Using the graphs and the data given, determine

(i) the area of cross-section of each specimen

(ii) the Young's modulus of the glass

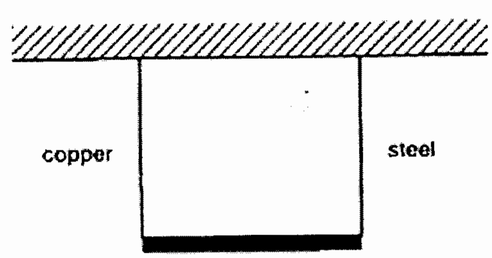
(iii) the ultimate tensile stress for copper

(iv) an appropriate value for the work done to stretch the copper wire to its breaking point

[Ans: (b) (i) $0.10 \times 10^{-6}\,m^2$ (ii) $63 \times 10^9\,Pa$ (iii) $1.4 \times 10^8\,Pa$ (iv) 0.32 J]

20. Describe, in detail, an experiment to determine the Young's modulus of copper. Mention two safety precautions which should be taken

21. A uniform rod of length $0.80\,m$ and weight $150\,N$ is suspended from a horizontal beam by two vertical wires as shown below



The wire at the left-hand end of the rod is copper, of original length $2.0\,m$ and area of cross-section $0.25\,mm^2$. That at the right-hand end is steel, of the same original length but of area of cross-section $0.090\,mm^2$. The Young's modulus of copper is $1.3 \times 10^{11}\,Pa$ and that of steel is $2.1 \times 10^{11}\,Pa$.

(a) Find the extension in each wire, assuming that the wires remain vertical and Hooke's law is obeyed

(b) Because the wires extend by different amounts, the suspended rod is not exactly horizontal. It is required to return the rod to the horizontal position by attaching a load to it. Find the minimum additional load required to do this, and state the point on the rod where this additional load should be attached

[Ans: (a) copper: $4.6\,mm$, steel: $7.9\,mm$ (b) $54\,N$, to the base of copper wire]

22. A steel wire of uniform cross-section of $1\,mm^2$ is heated to 70°C and stretched by tying its two ends rigidly. Calculate the change in the tension of the wire when the temperature falls from 70°C to 35°C. (coefficent of linear expansion of steel $= 1.1 \times 10^{-5}\,°C^{-1}$ and Young's modulus is $2.0 \times 10^{11}\,Pa$)

[Ans: $77\,N$]

23. A section of a railway track consists of a steel bar of length $15\,mm$ and cross-sectional area $80\,cm^2$. It is rigidly clamped at its ends on a day when the temperature is 20°C. If the temperature falls to 0°C, calculate

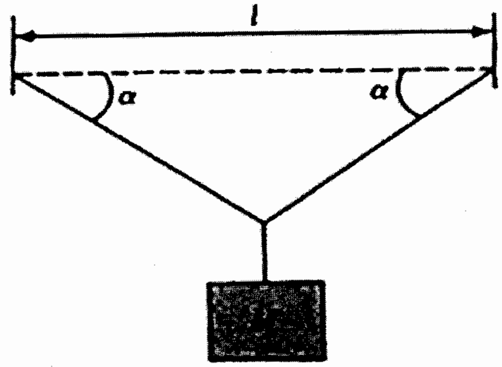
(a) the force the clamps must exert to stop the bar contracting

(b) the strain energy stored in the bar

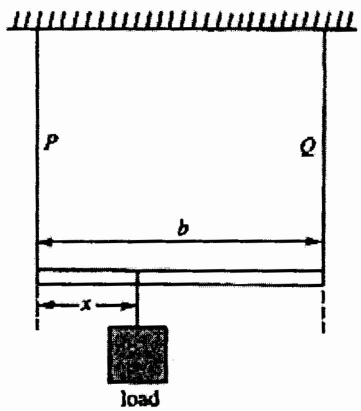
(Take α for steel as $12 \times 10^{-6}\,K^{-1}$, $E = 2.0 \times 10^{11}\,N\,m^{-2}$)

[Ans: (a) $3.8 \times 10^5\,N$ (b) $0.69\,kJ$]

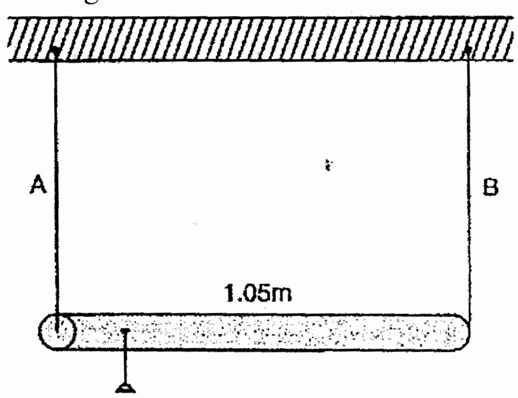
24. The figure below shows a mass M hanging from the mid-point of a wire whose original length is l and cross-sectional area A. What is the stress in the wire in terms of M, g, A and the angle α



25. The figure below shows a light rod of length b hanging from the lower ends of two vertical wires P and Q which are of the same natural length and diameter but have different Young moduli E_1 and E_2 respectively. A load is placed on the rod at a distance x from the wire P so that the rod remains horizontal. What is the value of x in terms of b, E_1 and E_2?



26. A rod of length $1.05\,m$ having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in the figure below



The cross-sectional areas of wire A and B are $1.0\,mm^2$ and $2.0\,mm^2$. At what point along the rod should a mass m be suspended in order to produce

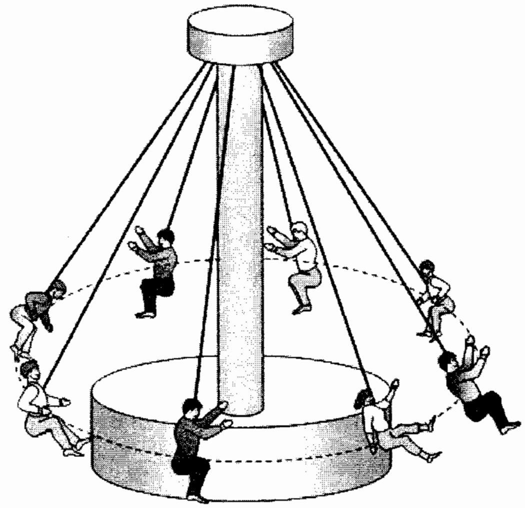
(a) equal stresses,

(b) equal strains; in both steel and aluminium wires

[Ans: (a) $70\,cm$ from A (b) $43.2\,cm$ from A]

CIRCULAR MOTION

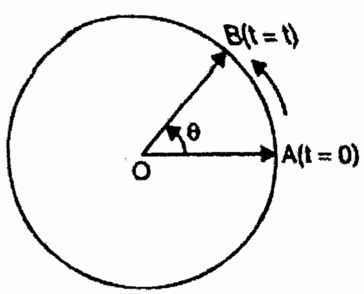
If you swing a pail of water at the end of a string overhead in a circle, why doesn't the water drop on you? What direction would the pail move if the string breaks? Why does a speed motor cyclist have to lean when he rounds a corner? In which direction must he lean: inwards or outwards? Why are roads banked at round corners? Why does an aeroplane bank its wings by an angle when changing course? All these are examples of circular motion in our daily life



Angular velocity

Angular velocity is the rate of change of the angle for an object moving in a circular or path about the centre. SI unit is radians per second ($rad\ s^{-1}$)

Consider a particle moving along a circular path in the anticlockwise direction. Let the rotating particle be at any instant, $t = 0$. Let the particle be at B after time t. Let the angle AOB described by the particle during this time be θ radian.



The magnitude of the angular velocity is given by

$$\omega = \frac{\theta}{t}$$

If the particle describes one complete revolution, then $\theta = 2\pi$ and $t = T$ (time period)

$$\omega = \frac{2\pi}{T}$$

If f is the frequency i.e. number of revolutions per second, then

$$\omega = 2\pi f$$

Uniform angular velocity

If the particle describes equal angles in equal intervals of time, then the angular velocity is said to be uniform

Instantaneous angular velocity is given by

$$\omega = \frac{d\theta}{dt}$$

Relationship between angular velocity and linear velocity

Let the particle travel from A to B along a circular path in time t with linear speed v

$$AB = vt \quad \dots \text{(i)}$$

$$\text{Also, } AB = r\theta \quad \dots\dots \text{(ii)}$$

where r is the radius of the circle along which the particle is moving

equating (i) and (ii);

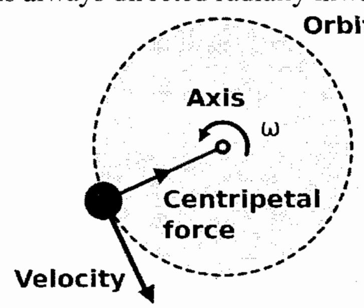
$$vt = r\theta$$

$$v = r\frac{\theta}{t}$$

$$\boldsymbol{v = \omega r}$$

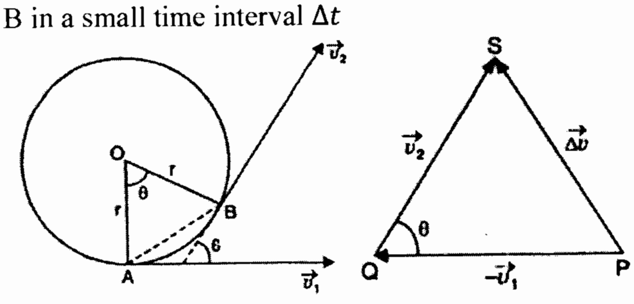
Centripetal force

Centripetal force is the external force which deflects a particle from its linear path to make it move along a circle and is always directed radially inwards



Expression for centripetal force

Consider a particle moving along a circle of radius r with a constant speed v. Suppose it moves from A to B in a small time interval Δt



Let $\vec{v_1}$ and $\vec{v_2}$ be the velocities of the particle at A and B respectively such that

$$|\vec{v_1}| = |\vec{v_2}| = v$$

However, the two velocities have different directions.

If θ is the angle between the directions of $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$, then change in the velocity $\Delta\vec{v}$ due to change in direction can be obtained

Applying triangle law of vectors to the vector triangle PQS,

$$\overrightarrow{PQ} + \overrightarrow{QS} = \overrightarrow{PS}$$
$$-\overrightarrow{v_1} + \overrightarrow{v_2} = \overrightarrow{\Delta v}$$
$$\overrightarrow{\Delta v} = \overrightarrow{v_2} - \overrightarrow{v_1}$$

Triangles AOB and PQS are similar,

$$\frac{PS}{AB} = \frac{QS}{OB}$$
$$\frac{\Delta v}{AB} = \frac{v}{r}$$

Since Δt is very small,

Chord AB = Arc AB = $v\,\Delta t$

Thus, $\frac{\Delta v}{v\,\Delta t} = \frac{v}{r}$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Acceleration, $a = \lim\limits_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

$$\boldsymbol{a = \frac{v^2}{r}}$$

This equation gives the magnitude of the centripetal acceleration. This acceleration is along the radius of the circle and is directed towards the centre of the circle

If m is the mass of the particle, then the centripetal force,

$$F = ma$$
$$\boldsymbol{F = \frac{mv^2}{r}}$$

Since $v = \omega r$,

$$\boldsymbol{F = m\omega^2 r}$$

Examples of centripetal force

1. When a stone tied to a string is revolved in a circle, the tension in the string supplies the necessary centripetal force
2. In the case of the motion of the earth around the sun, the gravitational force of attraction between the sun and the earth provides the necessary centripetal force
3. For an electron revolving around the nucleus, the centripetal force is provided by the electric force of attraction between the nucleus and the electron

Note

- When the centripetal force ceases to act, the particle would move in a straight line along the tangent to the circular path at the point where the force has ceased to act
- In the absence of the centripetal force, particle has a natural tendency to move straight line in accordance with Newton's law of motion
- When a car rounds a curve, the passengers in car are thrown outward not because of so outward force but because there is no centrip force to hold the passengers in circular motio

Work done by centripetal force

The work done by the centripetal force is always ze as it is perpendicular to velocity and hen instantaneous displacement.

Work done = change in kinetic energy of revolvi body

But change in $K.E = 0$

$\therefore$ Work done = 0

Also, $W = F \cdot S = F \cdot S\cos\theta$

$= F \cdot S\cos 90° = 0$

Examples

- When an electron revolves around the nucleus i a hydrogen atom in a particular orbit, it neithe absorbs nor emits any energy means its energ remains constant.
- When a satellite established once in an orbi around the earth and it starts revolving wit particular speed, then no fuel is required for i circular motion.

Centrifugal force

Suppose a body is rotating in circular path. Let the centripetal force suddenly vanish. Now, the body would leave the circular path. For an observer standing outside the circular path, the body appears to fly off tangentially at the point of release

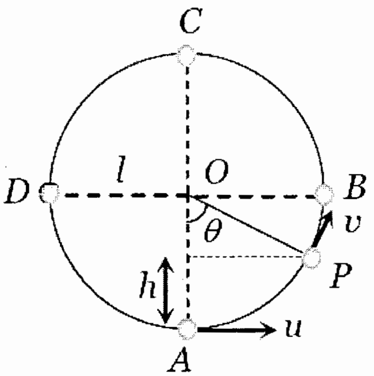
For an observer rotating with the body with the same velocity, the body appears to be stationary before it is released. It appears to the observer as if it is thrown off along the radius away from the centre by the same force. This force is called centrifugal force. Its magnitude is the same as that of the centripetal force i.e. $\frac{mv^2}{r}$

Note: Centrifugal force is not a force of reaction. It is a fictitious force which has a concept only in a rotating frame of reference

ı in a vertical circle

motion, the body is under the influence of ı of earth. When the body moves from lowest o highest point, its speed decreases and becomes ıum at highest point. Total mechanical energy of ›dy remains conserved and *KE* converts into *PE* ice versa.

:ity at any point, *P* on a vertical loop



u is the initial velocity imparted to the body at its west point, then velocity of a body at a height h is ven by

$$v = \sqrt{u^2 - 2gh}$$

ɜut $h = l - l\cos\theta = l(1 - \cos\theta)$

Thus, $v = \sqrt{u^2 - 2gl(1 - \cos\theta)}$

where l is the length of the string

At A, $\theta = 0°$,

$$v = \sqrt{u^2 - 2gl(1 - \cos 0°)}$$

$$v = u$$

At B, $\theta = 90°$,

$$v = \sqrt{u^2 - 2gl(1 - \cos 90)}$$

$$v = v = \sqrt{u^2 - 2gl}$$

At C, $\theta = 180°$,

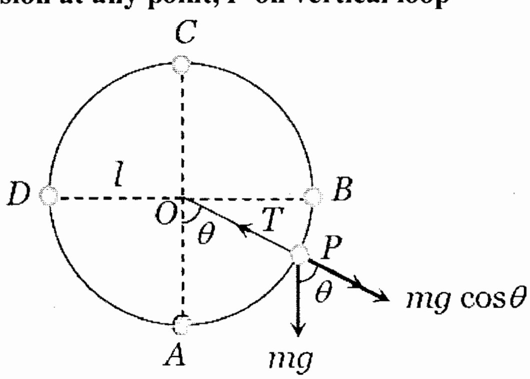
$$v = \sqrt{u^2 - 2gl(1 - \cos 180°)}$$

$$v = \sqrt{u^2 - 4gl}$$

At D, $\theta = 270°$, $v = \sqrt{u^2 - 2gl(1 - \cos 270°)}$

$$v = \sqrt{u^2 - 2gl}$$

Tension at any point, *P* on vertical loop



According to Newton's second law of motion,

Net force towards centre = centripetal force

$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$T = mg\cos\theta + \frac{mv^2}{l}$$

But $v = \sqrt{2gh} = \sqrt{u^2 - 2gl(1 - \cos\theta)}$

Thus, $T = mg\cos\theta + \frac{m}{l}(u^2 - 2gl(1 - \cos\theta)$

$$T = mg\cos\theta + \frac{mu^2}{l} - 2mg + 2mg\cos\theta$$

$$T = \frac{mu^2}{l} + 3mg\cos\theta - 2mg$$

At A, $\theta = 0°$, $T_A = \frac{mu^2}{l} + 3m\cos 0° - 2mg$

$$T_A = \frac{mu^2}{l} + mg$$

At B, $\theta = 90°$, $T_B = \frac{mu^2}{l} + 3m\cos 90° - 2mg$

$$T_B = \frac{mu^2}{l} - 2mg$$

At C, $\theta = 180°$, $T_C = \frac{mu^2}{l} + 3m\cos 180° - 2mg$

$$T_C = \frac{mu^2}{l} - 5mg$$

At D, $\theta = 270°$, $T_D = \frac{mu^2}{l} + 3m\cos 270° - 2mg$

$$T_D = \frac{mu^2}{l} - 2mg$$

It is clear that $T_A > T_B > T_C$ and $T_B = T_D$. Therefore, the tension is maximum at the lowest point A and minimum at the highest point C.

The string is therefore most likely to break at point A

Critical condition for vertical looping

If the tension at C is zero, then the body will just complete revolution in the vertical circle.

$$T_C = \frac{mu^2}{l} - 5\,mg$$

$$\frac{mu^2}{l} - 5mg = 0$$

$$u = \sqrt{5gl}$$

It means that to complete the vertical circle, the body must be projected with minimum velocity of $\sqrt{5gl}$ at the lowest point.

Various conditions for vertical motion

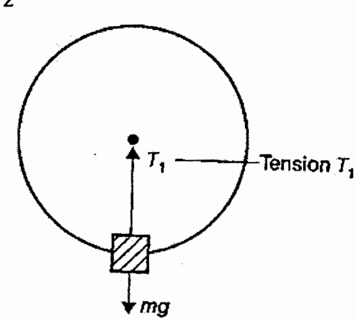
Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any point and body will continue the circulation
$u_A = \sqrt{5gl}$	Tension at highest point C will be zero and body will just complete the circle
$\sqrt{2gl} < u_A < \sqrt{5gl}$	Particle will not follow circular motion. Tension will become zero somewhere between points B and C whereas velocity remains positive. Particle leaves circular path and follow a parabolic trajectory
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between A and B and the particle will oscillate along semi-circular path
$u_A < \sqrt{2gl}$	Velocity of particle becomes zero between A and B but tension will not be zero and the particle will oscillate about the point A

Example 1

An object of mass $4.0\ kg$ is whirled round in a vertical circle of radius $2.0\ m$ with a speed of $5.0\ ms^{-1}$. Calculate the maximum and minimum tension in the string connecting the object to the centre of the circle.

Solution

Let the maximum tension be T_1 and the minimum tension be T_2

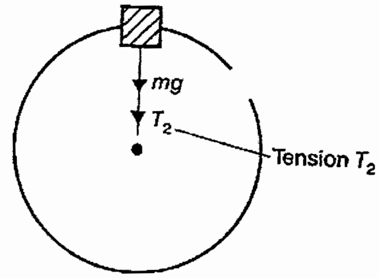


$$T_1 - mg = \frac{mv^2}{r}$$

$$T_1 = \frac{mv^2}{r} + mg$$

$$T_1 = m\left(\frac{v^2}{r} + g\right)$$

$$= 4.0\left(\frac{5^2}{2} + 9.81\right) = 89.24\ N$$



$$T_2 + mg = \frac{mv^2}{r}$$

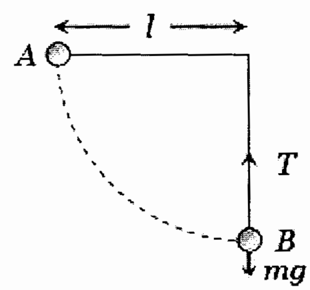
$$T_2 = m\left(\frac{v^2}{r} - g\right)$$

$$= 4.0\left(\frac{5^2}{2} - 9.81\right) = 10.76\ N$$

Example 2

The mass of the bob of a simple pendulum of length l is m. If the bob is left from its horizontal position, calculate the speed of the bob and the tension in the thread in the lowest position of the bob.

Solution



By the conservation of energy;

Potential energy at point A = Kinetic energy at point B

$$mgl = \frac{1}{2}mv^2$$

$$v = \sqrt{2gl}$$

$$T - mg = \frac{mv^2}{l}$$

$$T = \frac{mv^2}{l} + mg$$

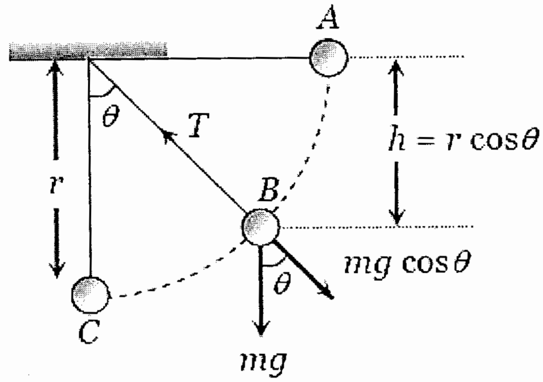
$$T = \frac{m(2gl)}{l} + mg$$

$$T = 3\ mg$$

Example 3

In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. Show at the string breaks when it makes an angle of $\cos^{-1}\frac{2}{3}$ the vertical

Solution



Let the string break at point B

$$T - mg\cos\theta = \frac{mv_B^2}{r}$$

T is the breaking strength, $T = 2\,mg$

$$mg\cos\theta + \frac{mv_B^2}{r} = 2\,mg\ \text{..... (i)}$$

If the bob is released from rest at point A, then the velocity acquired at point B,

$$v_B = \sqrt{2gh} = \sqrt{2g\,r\cos\theta}$$

Substituting for v_B in equation (i);

$$mg\cos\theta + \frac{m}{r}(2g\,r\cos\theta) = 2\,mg$$

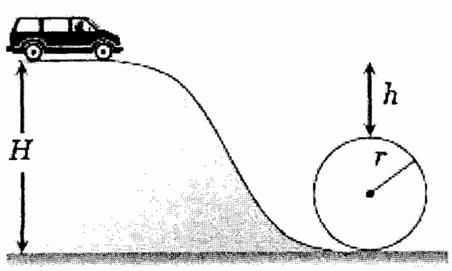
$$3\,mg\cos\theta = 2\,mg$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\frac{2}{3}$$

Example 4

A toy car rolls down an inclined plane as shown below.



If it goes round the loop at the bottom, find the relationship between H and h.

Solution

Let the velocity acquired at the lowest point be v

$u = 0, v = ?, a = g, s = H$

From $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2gH$$

$$v = \sqrt{2gH}$$

For looping of the loop, $v = \sqrt{5gr}$

$$\sqrt{2gH} = \sqrt{5gr}$$

$$H = \frac{5r}{2}$$

From the figure, $H = h + 2r$

$$\Rightarrow r = \frac{H-h}{2}$$

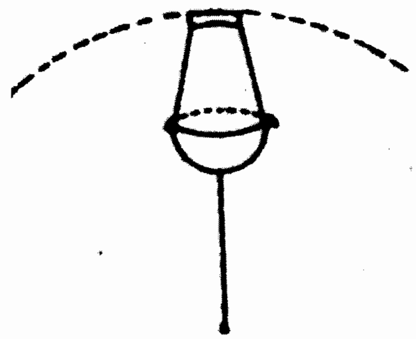
Therefore, $H = \frac{5}{2}\frac{(H-h)}{2}$

$$4H = 5H - 5h$$

$$H = 5h$$

Example 5

A small bucket of water is rotated rapidly in a vertical circular plane using a thin rope attached to its handle



Explain why the water in the bucket does not spill when the bucket is in vertically downward position as shown above.

Solution

The weight mg of water and the normal reaction R of the bottom of the can provide the necessary centripetal force i.e.

$$R + mg = \frac{mv^2}{r}$$

$$R = \frac{mv^2}{r} - mg$$

For the water to remain in the can, $R > 0$

$$\frac{mv^2}{r} - mg > 0$$

$$v > \sqrt{gr}$$

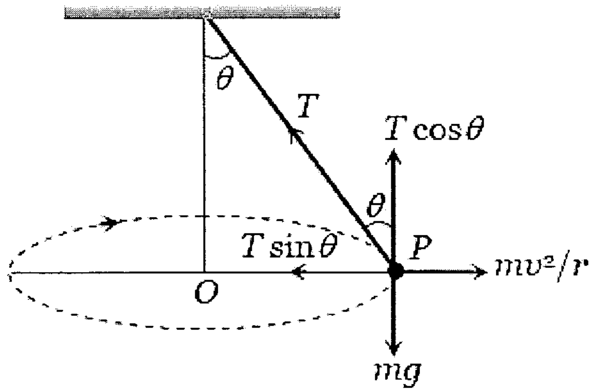
If the can is swung with a velocity greater than this value, the water will not spill

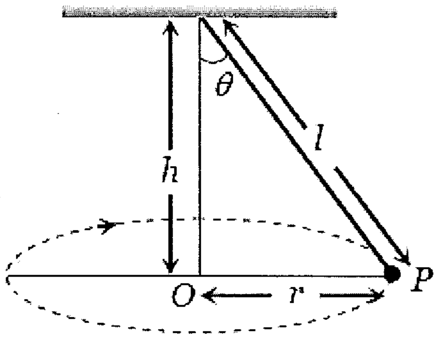
Conical pendulum

This is the example of uniform circular motion in horizontal plane.

A bob of mass m attached to a light and inextensible string rotates in a horizontal circle of radius r with constant angular speed ω about the vertical. The string makes angle θ with vertical and appears tracing the surface of a cone. So, this arrangement is called conical pendulum.

The forces acting on the bob are tension and weight of the bob.





$$T\sin\theta = \frac{mv^2}{r} \quad \ldots\ldots \text{(i)}$$

$$T\cos\theta = mg \quad \ldots\ldots \text{(ii)}$$

(i)÷ (ii);

$$\boldsymbol{\tan\theta = \frac{v^2}{rg}}$$

$$T = \frac{mg}{\cos\theta}$$

But $\cos\theta = \frac{h}{l} = \frac{\sqrt{l^2-r^2}}{l}$

$$T = \frac{mgl}{\sqrt{l^2-r^2}}$$

Angle of the string from the vertical, $\theta = \tan^{-1}\frac{v^2}{rg}$

Linear velocity of the bob, $v = \sqrt{rg\tan\theta}$

Angular velocity of bob,

$$\omega = \frac{v}{r} = \sqrt{\frac{g}{r}\tan\theta} = \sqrt{\frac{g}{r}\times\frac{r}{h}} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{l\cos\theta}}$$

Time period of revolution,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{l^2-r^2}{g}}$$

$$= 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

Frequency,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{h}} = \frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}} = \frac{1}{2\pi}\sqrt{\frac{g}{l^2-r^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g\tan\theta}{r}}$$

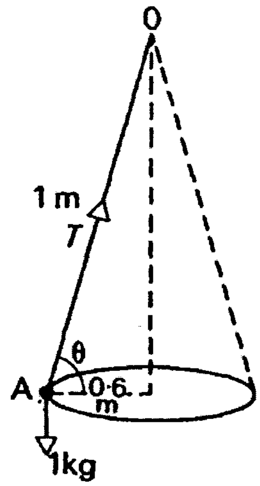
Example 6

A pendulum bob of mass 1 kg is attached to a string 1 m long and made to revolve in a horizontal circle of radius 60 cm. Find the period of motion and the tension in the string

Solution

Let A be the bob, OA is the string, T be the tension and θ be the angle of inclination to the horizontal

$$r = 60\ cm = 0.6\ m$$



$$T\cos\theta = \frac{mv^2}{r} \quad \ldots.. \text{(i)}$$

$$T\sin\theta = mg \quad \ldots\ldots \text{(ii)}$$

$$\cos\theta = \frac{60}{100} = \frac{3}{5} \Rightarrow \sin\theta = \frac{4}{5}$$

From (ii);

$$T = \frac{mg}{\sin\theta} = \frac{1\times 9.81}{4/5} = 12.25\ N$$

From (i);

$$v = \sqrt{\frac{rT\cos\theta}{m}}$$

$$= \sqrt{\frac{0.6\times 12.25\times\left(\frac{3}{5}\right)}{1}} = 2.1\ m\ s^{-1}$$

angular velocity, $\omega = \frac{v}{r} = \frac{2.1}{0.6} = 3.5\ rad\ s^{-1}$

Period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3.5} = 1.8\ s$

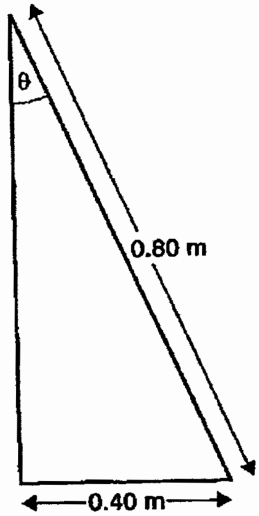
Example 7

A conical pendulum consists of a small bob of mass 0.20 kg attached to an extensible string of length 0.80 m. The bob rotates in a horizontal circle of radius 0.40 m, of which the centre is vertically below the point of suspension. Calculate

(a) the linear speed of the bob in $m\ s^{-1}$
(b) the period of rotation of the bob
(c) the tension in the string

Solution

$m = 0.20,\ r = 0.40,\ l = 0.80$



$$\sin\theta = \frac{0.40}{0.80} = 0.5$$
$$\theta = \sin^{-1} 0.5 = 30°$$

(i) $T\sin\theta = \frac{mv^2}{r}$ ...... (i)

$T\cos\theta = mg$ ...... (ii)

(i)÷ (ii);

$$\tan\theta = \frac{v^2}{rg}$$
$$v = \sqrt{rg\tan\theta}$$
$$v = \sqrt{0.4 \times 9.81 \times \tan 30°}$$
$$v = 1.51\, m\, s^{-1}$$

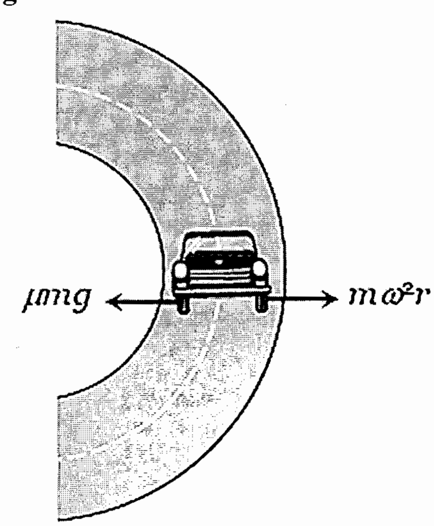
(ii) Periodic time, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{v}{r}} = \frac{2\pi r}{v}$

$$T = \frac{2\pi \times 0.4}{1.51} = 1.66\, s$$

(iii) From (ii);

$$T = \frac{mg}{\cos\theta} = \frac{0.2 \times 9.81}{\cos 30°}$$
$$T = 2.27\, N$$

Skidding of vehicle on a level road



When a vehicle turns on a circular path it requires centripetal force. If friction provides this centripetal force, then the vehicle can move in circular path safely if

Friction force ≥ Required centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$
$$\mu rg \geq v^2$$
$$v^2 \leq \mu rg$$
$$v_{safe} \leq \sqrt{\mu rg}$$

This is the maximum speed by which a vehicle can turn in a circular path of radius r, whose coefficient of friction between the road and tyres is μ.

v_{safe} can be increased by

(i) increasing r

(ii) increasing μ (make the road rougher)

Example 8

A crate is placed on the floor of a truck. The coefficient of friction between the crate and the floor is 0.6. With what maximum speed can the truck go around a curve of radius 200 m without the crate sliding?

Solution

For an observer inside the truck, which is moving with acceleration, a pseudo force $\frac{mv^2}{r}$ acts on the crate which tries to slide it in the outward direction. However the crate will not slide unless the pseudo force becomes larger than the force of friction μmg. The condition for the crate not to slide is

$$\mu mg \geq \frac{mv^2}{r}$$
$$v^2 \leq \mu gr$$
$$v_{max} = \sqrt{0.6 \times 9.81 \times 200}$$
$$= 34.3\, m\, s^{-1}$$

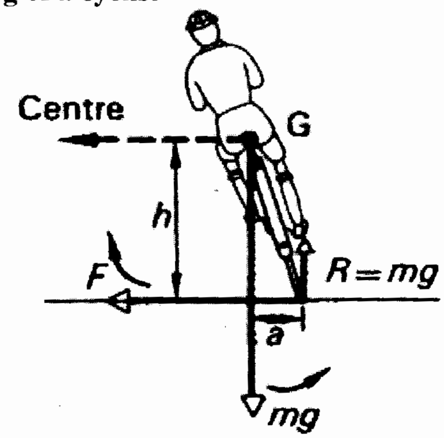
Example 9

Find the maximum velocity for overturn for a car moved on a circular track of radius 100 m if the coefficient of friction between the road and the tyres is 0.2

Solution

$$v_{max} = \sqrt{\mu rg}$$
$$= \sqrt{0.2 \times 100 \times 9.81}$$
$$= 14.01\, m\, s^{-1}$$

Bending of a cyclist



When a person on a bicycle rides round a circular racing track, the frictional force F at the ground provides the centripetal force. This produces a moment about his centre of gravity, G which is counter balanced when he leans inwards by the moment of the normal reaction R. Thus, provided no skidding occurs,

$$F.h = R.a$$
$$F.h = mg.a$$
$$\frac{a}{h} = \frac{F}{mg}$$
$$\tan\theta = \frac{a}{h} = \frac{F}{mg}$$
$$F = mg\tan\theta$$

where θ is the angle of inclination to the vertical

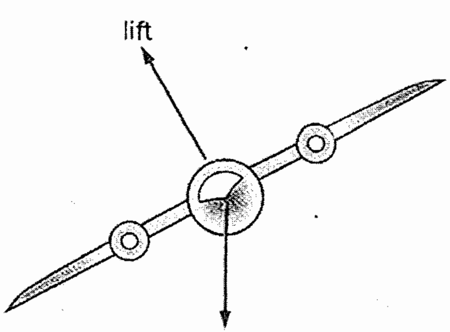
$$F = \frac{mv^2}{r}$$
$$\therefore \tan\theta = \frac{mv^2}{r}$$

When F is greater than the limiting friction, skidding occurs. In this case $F > \mu mg$

$$mg\tan\theta > \mu\, mg$$
$$\tan\theta > \mu$$

Note

For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.



An aircraft in straight, level flight experiences a lift force at right angles to the surface of its wings, which balances its weight. To turn, the ailerons are o| so that the aircraft banks and the horizontal com of the lift supplies the necessary centripetal forc aircraft's weight is now opposed only by the v component of the lift, and height will be lost unl lift is increased by for example, increasing the s

Example 10

A boy on a cycle pedals around a circle of 20 r radius at a speed of 20 $m\,s^{-1}$. The combined m the boy and the cycle is 90 kg. Calculate the ang cycle makes with the vertical so that it may not f

Solution

$$\theta = \tan^{-1}\frac{v^2}{rg}$$
$$= \tan^{-1}\left(\frac{20^2}{4\times 9.81}\right)$$
$$= 63.9°$$

Example 11

If a cyclist moving with a speed of 4.9 $m\,s^{-1}$ on a lev road can take a sharp circular turn of radius 4 m calculate the coefficient of friction between the cycl tyres and the road.

Solution

$$\mu mg = \frac{mv^2}{r}$$
$$\mu = \frac{v^2}{rg} = \frac{4.9^2}{4\times 9.81} = 0.61$$

Example 12

A pilot banks the wings of an air craft so as to travel at a speed of 540 kmh^{-1} along a horizontal circular paths of radius 8 km. Calculate

(i) the centripetal force

(ii) angle the pilot should bank the air craft

Solution

$$v = 540\ kmhr^{-1} = 540\times\frac{1000}{3600} = 150\ ms^{-1}$$
$$r = 8000\,m$$

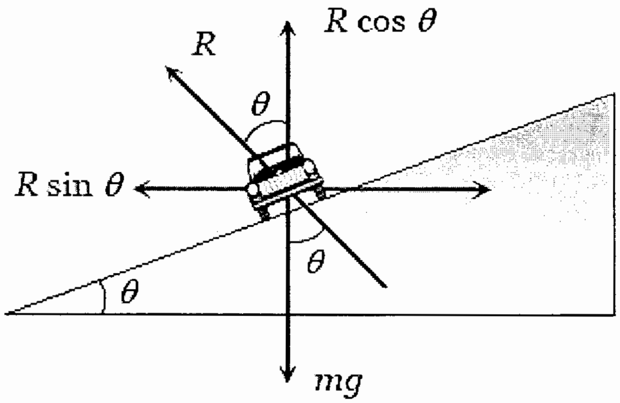
(i) $a = \frac{v^2}{r} = \frac{150^2}{8000} = 2.81\ ms^{-2}$

(ii) $\tan\theta = \frac{v^2}{rg} = \frac{150^2}{8000\times 9.81} = 0.2867$

$$\Rightarrow \theta = 16°$$

Banking of a road

For getting a centripetal force, cyclists bend towards the centre of circular path but it is not possible in case of four wheelers. Therefore, the outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.



The reaction R is resolved into two components, the component $R\cos\theta$ balances weight of the vehicle

$$R\cos\theta = mg \;\ldots\ldots\;(\text{i})$$

The horizontal component $R\sin\theta$ provides the necessary centripetal force as it is directed towards the centre of the desired circle

$$R\sin\theta = \frac{mv^2}{r} \;\ldots(\text{ii})$$

(ii) ÷ (i);

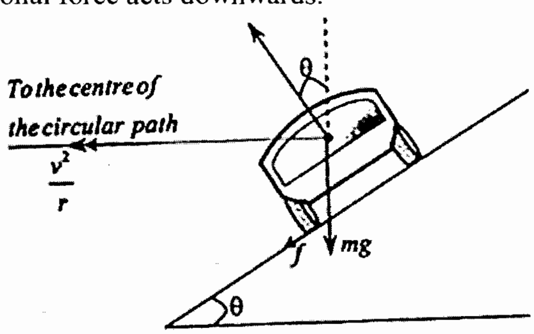
$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

Banking of a road with tendency to skid

1. When speed is maximum

When a car is moving as fast as possible, the maximum frictional force μR acts in such a way to prevent the car from slipping up the plane. This implies that the frictional force acts downwards.



Resolving:

$$\leftarrow R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r} \;\ldots\ldots\ldots\ldots(\text{i})$$

$$\uparrow R\cos\theta = mg + \mu R\sin\theta$$

$$\Rightarrow R\cos\theta - \mu R\sin\theta = mg \ldots\ldots\ldots\ldots(\text{ii})$$

Dividing eqn (i) by eqn (ii) gives:

$$\frac{R\sin\theta + \mu R\cos\theta}{R\cos\theta - \mu R\sin\theta} = \frac{v^2}{rg}$$

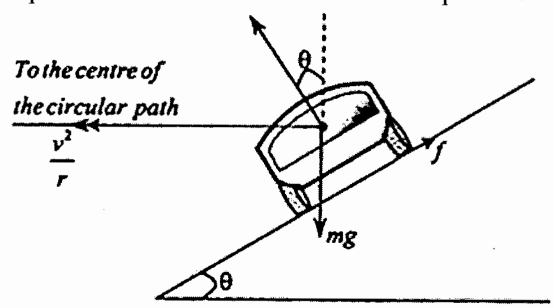
Dividing every term by $R\cos\theta$ gives;

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

$$\text{Maximum speed, } v = \sqrt{\frac{rg(\tan\theta+\mu)}{1-\mu\tan\theta}}$$

2. When speed is minimum

When the car is moving as slowly as possible, the maximum frictional force μR acts in such a way to prevent the car from slipping down the plane. This implies that the frictional force acts upwards.



Resolving:

$$\leftarrow R\sin\theta - \mu R\cos\theta = \frac{mv^2}{r} \;\ldots\ldots\ldots\ldots(\text{i})$$

$$\uparrow R\cos\theta + \mu R\sin\theta = mg \;\ldots\ldots\ldots\ldots(\text{ii})$$

Dividing eqn (i) by eqn (ii) gives:

$$\frac{R\sin\theta - \mu R\cos\theta}{R\cos\theta + \mu R\sin\theta} = \frac{v^2}{rg}$$

Dividing every term by $R\cos\theta$ gives;

$$\frac{\tan\theta - \mu}{1 + \mu\tan\theta} = \frac{v^2}{rg}$$

$$\text{Minimum speed, } v = \sqrt{\frac{rg(\tan\theta-\mu)}{1+\mu\tan\theta}}$$

Example 13

For traffic moving at 60 $km\,h^{-1}$ along s circular track of radius 0.1 km, what is the correct angle of banking?

Solution

$$v = 60\frac{km}{h} = 60 \times \frac{1000}{3600} = \frac{50}{3}\,m\,s^{-1}$$

$$r = 0.1\,km = 100\,m$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\frac{v^2}{rg}$$

$$= \tan^{-1}\left[\frac{\left(\frac{50}{3}\right)^2}{100 \times 9.81}\right] = 15.8°$$

Example 14

The radius of curvature of a road at a certain turn is $50\,m$. The width of the road is $10\,m$ and its outer edge is $1.5\,m$ higher than the inner edge. What is the safe speed for such an inclination?

Solution

$v =?, h = 1.5\,m, l = 10\,m$

$$\tan\theta = \frac{v^2}{rg}$$

$$\frac{h}{l} = \frac{v^2}{rg}$$

$$v = \sqrt{\frac{hrg}{l}} = \sqrt{\frac{1.5 \times 50 \times 9.81}{10}}$$

$$= 8.6\,m\,s^{-1}$$

Example 15

The maximum speed at which a car can negotiate a curve of radius $60\,m$ is $36\,\text{km}\,\text{h}^{-1}$. What is the coefficient of friction? What should be the angle of banking if we do not want to depend on friction?

Solution

Let the mass of the car be m

For a level road,

$$R = mg \quad .... (i)$$

$$f_{max} = \frac{mv_{max}^2}{r} \quad .... (ii)$$

where f is the force of friction

(ii) ÷ (i);

$$\frac{f_{max}}{R} = \frac{v^2}{rg}$$

$$r = 60\,m, v = \frac{36000}{3600} = 10\,m\,s^{-1}$$

$$\mu = \frac{10^2}{60 \times 9.81} = 0.17$$

If there is no friction, and the road is banked at an angle θ, then

$$\tan\theta = \frac{v^2}{rg} = \frac{10^2}{(60)(9.8)} = 0.17$$

$$\theta = \tan^{-1} 0.17 = 9.65°$$

Example 16

A circular race track of radius $300\,m$ is banked at an angle of $15°$. If the coefficient of friction between the car wheels of a race car and the road is 0.2, calculate the

(i) optimum speed of the race car to avoid wear and tear on its tyres

(ii) maximum permissible speed to avoid skidding

Solution

(i) On a banked road, the horizontal component of the normal reaction and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without skidding. At the optimum speed, the component of the normal reaction is enough to provide the required centripetal force. In this case, the frictional force is not required.

$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{300 \times 9.81 \tan 15°} = 28.1\,m\,s^{-1}$$

(ii) The maximum permissible speed is given by

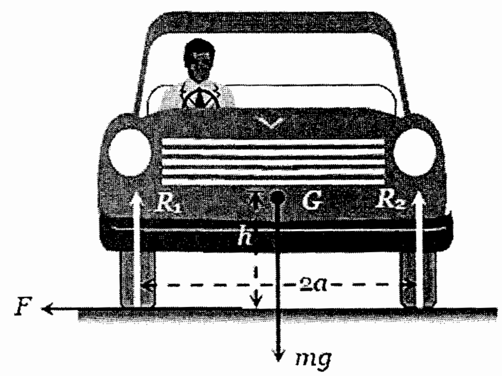
$$v_{max} = \sqrt{\left(\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}\right)}$$

$$v_{max} = \sqrt{\frac{300 \times 9.81(\tan 15° + 0.5)}{1 - 0.5\tan 15°}}$$

$$= 38.1\,m\,s^{-1}$$

Overturning of vehicle

When a car moves in a circular path with a speed more than the maximum speed, then it overturns and its inner wheels leave the ground first



Weight of the car $= mg$

Speed of the car $= v$

Radius of the circular path $= r$

Distance between the centre of wheels of the car $= 2a$

Height of the centre of gravity (G) of the car from the road level $= h$

Reaction on the inner wheels by the ground $= R_1$

Reaction on the outer wheels by the ground $= R_2$

When a car moves in a circular path, horizontal frictional force F provides the required centripetal force

$$F = \frac{mv^2}{r} \quad .... (i)$$

By taking moments of forces R_1, R_2 and F about G

$$Fh + R_1 a = R_2 a \quad .... (ii)$$

As there is no vertical motion,

$$R_1 + R_2 = mg \quad .... (iii)$$

$a \times (iii) - (ii)$;

$$R_1 a + R_2 a = mga$$

$$\underline{-Fh + R_1a = R_2a}$$
$$\underline{R_2a - Fh = mga - R_2a}$$
$$2R_2a = mga + Fh$$
$$2R_2a = mga + \frac{mv^2}{r}h$$
$$R_2 = \frac{1}{2}m\left[g + \frac{v^2h}{ra}\right]$$

From (iii),

$$R_1 = mg - R_2$$
$$R_1 = mg - \frac{1}{2}m\left[g + \frac{v^2h}{ra}\right]$$
$$R_1 = \frac{1}{2}m\left[g - \frac{v^2h}{ra}\right]$$

For overturning, $R_1 = 0$

$$\frac{v^2h}{ra} = g$$
$$v = \sqrt{\frac{gra}{h}}$$

This is the maximum speed of a car without overturning on a flat road

If the car is driven at a velocity $v = \sqrt{\frac{gra}{h}}$, then its at a point of toppling/overturning/upsetting.

Therefore, for no toppling/overturning/upsetting, then $v < \sqrt{\frac{gra}{h}}$. If $v > \sqrt{\frac{gra}{h}}$, then the car overturns/topples/upsets

It can therefore be noted that for upsetting/toppling/overturning;

- h should be large
- r should be small
- a should be small

Example 17

The distance between two rails is $1.5\ m$. The centre of gravity of the train is at a height of $2\ m$ from the ground. Calculate the maximum speed of the train on a circular path of radius $120\ m$

Solution

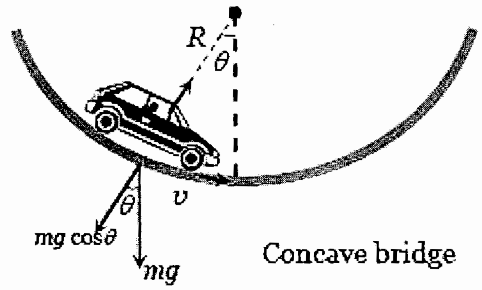
Distance between rails, $2a = 1.5$

$a = 0.75, h = 2\ m, r = 120\ m$

$$v_{max} = \sqrt{\frac{gra}{h}}$$
$$= \sqrt{\frac{9.81 \times 120 \times 0.75}{2}}$$
$$= 21.2\ m\ s^{-1}$$

Reaction of road on a car

1. Concave bridge



Concave bridge

When a car moves on a concave bridge, then

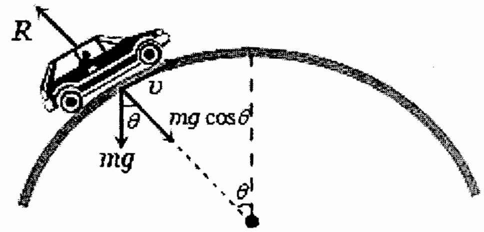
Centripetal force $= R - mg\cos\theta = \frac{mv^2}{r}$

$$R = mg\cos\theta + \frac{mv^2}{r}$$

At bottom of the bridge(lowest point), $\theta = 0°$

Therfore, $R = mg + \frac{mv^2}{r}$

2. Convex bridge



Convex bridge

When a car moves on a convex bridge,

Centripetal force $= mg\cos\theta - R = \frac{mv^2}{r}$

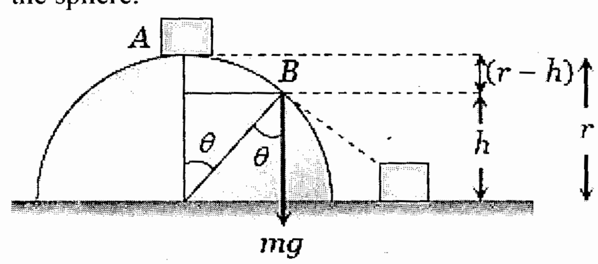
$$R = mg\cos\theta - \frac{mv^2}{r}$$

At top of bridge, (highest point), $\theta = 0°$,

Therfore, $R = mg - \frac{mv^2}{r}$

Motion of a block on frictionless hemisphere

A small block of mass m slides down from the top of a frictionless hemisphere of radius r. The component of the force of gravity ($mg\cos\theta$) provides required centripetal force but at point B its circular motion ceases and the block loses contact with the surface of the sphere.



$$mg\cos\theta - R = \frac{mv^2}{r}$$

On losing contact at B, $R = 0$

$$\Rightarrow mg\cos\theta = \frac{mv^2}{r} \text{ ..... (i)}$$

By law of conservation of energy,

Total energy at point A = Total energy at point B

$$K.E_A + P.E_A = K.E_B + P.E_B$$

$$0 + mgr = \frac{1}{2}mv^2 + mgh$$

$$v = \sqrt{2g(r-h)} \text{ ... (ii)}$$

From the figure,

$$h = r\cos\theta$$

$$\Rightarrow \cos\theta = \frac{h}{r} \text{ ..... (iii)}$$

By substituting v from (ii) and h from (iii) into (i);

$$mg\left(\frac{h}{r}\right) = \frac{m}{r}\left(\sqrt{2g(r-h)}\right)^2$$

$$gh = 2g(r-h)$$

$$h = 2(r-h)$$

$$h = 2r - 2h$$

$$h = \frac{2}{3}r$$

The block will lose contact at the height of $\frac{2}{3}r$ from the ground

Angle from the vertical is given by;

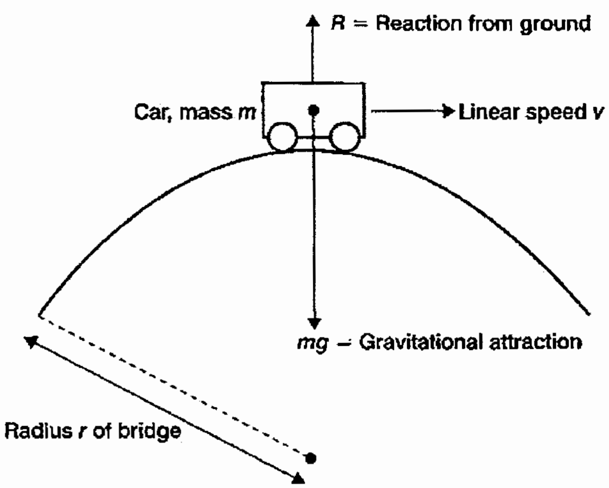
$$\cos\theta = \frac{h}{r} = \frac{2}{3}r \div r = \frac{2}{3}$$

$$\therefore \theta = \cos^{-1}\frac{2}{3}$$

Example 18

The road way bridge over a canal is in the form of an arc of a circle of radius $20\ m$. What is the maximum speed with which a car can cross the bridge without leaving contact with the ground at the highest point?

Solution



At the highest point, $mg - R = \frac{mv^2}{r}$

$$\Rightarrow R = mg - \frac{mv^2}{r}$$

To keep on ground, $R \geq 0$

$$mg - \frac{mv^2}{r} \geq 0$$

$$mg \geq \frac{mv^2}{r}$$

$$rg \geq v^2$$

$$v^2 \leq rg$$

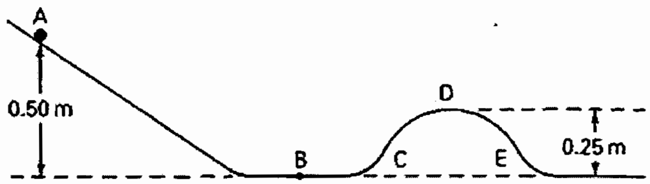
$$v \leq \sqrt{rg}$$

$$v \leq \sqrt{20 \times 9.81} \leq 14.01\ m\ s^{-1}$$

Therefore, the maximum speed is $14.01\ m\ s^{-1}$

Example 19

The diagram below shows a section of a curtain track in a vertical plane. The curved section, CDE, forms a circular arc of radius of curvature $0.75\ m$ and the point D is $0.25\ m$ higher than B. A ball-bearing of mass $0.060\ kg$ is released from A, which is $0.50\ m$ higher than B.



(a) Calculate the speed of the ball bearing at
  (i) B
  (ii) D

(b) Calculate the reaction between the track and the ball-bearing at D

Solution

(a) Using $v^2 = u^2 + 2as$

(i) $u = 0, a = g = 9.81, s = 0.50$

$$v_B^2 = 0^2 + 2 \times 9.81 \times 0.50$$

$$v_B = 3.13\ m\ s^{-1}$$

(ii) Using the principle of conservation of energy

Total energy at B = Total energy at D

$$K.E_B + P.E_B = K.E_D + P.E_D$$

$$\frac{1}{2}mv_B^2 + 0 = \frac{1}{2}mv_D^2 + mgh$$

$$V_D^2 = v_B^2 - 2gh$$

$$V_D = \sqrt{v_B^2 - 2gh}$$

$$= \sqrt{3.13^2 - 2 \times 9.81 \times 0.25}$$

$$= 2.21\ m\ s^{-1}$$

(b) Since, the ball-bearing remains on track throughout the motion, a net force always acts towards the centre of the arc CDE

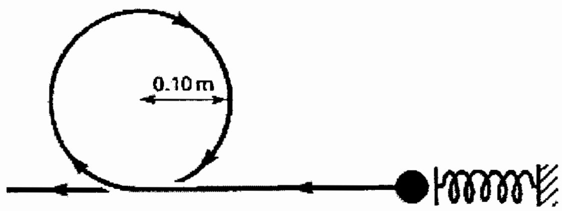
$$mg - R = \frac{mv_D^2}{r}$$

$$R = mg - \frac{mv_D^2}{r}$$

$$R = m\left(g - \frac{v_D^2}{r}\right)$$
$$= 0.06\left(9.81 - \frac{2.21^2}{0.75}\right)$$
$$= 0.198\,N$$

Example 20

A compressed spring is used to propel a ball bearing along a track which contains a circular loop of radius $0.10\,m$ in a vertical plane. The spring obeys Hooke's law and requires a force of $0.20\,N$ to compress it $1.0\,mm$.



(a) The spring is compressed by $30\,mm$. Calculate the energy stored in the spring

(b) A ball bearing of mass $0.025\,kg$ is placed against the end of the spring which is then released. Calculate

(i) the speed with which the ball-bearing leaves the spring

(ii) the speed of the ball at the top of the loop

(iii) the force exerted on the ball by the track at the top of the loop

Assume that effects of friction can be ignored

Solution

(a) Elastic potential energy $= \frac{1}{2}kx^2$

$$k = \frac{0.20}{1.0 \times 10^{-3}} = 200\,N\,m^{-1}$$
$$x = 30\,mm = 30 \times 10^{-3}\,m$$
$$E.P.E = \frac{1}{2} \times 200 \times (30 \times 10^{-3})^2$$
$$= 0.09\,J$$

(b)

(i) Energy stored in the spring is converted into kinetic energy of the ball-bearing

$$\frac{1}{2}mv^2 = 0.09$$
$$v = \sqrt{\frac{2 \times 0.09}{0.025}} = 2.68\,m\,s^{-1}$$

(ii) Let the velocity at the top of the loop be V

Using $v^2 = u^2 + 2as$

$u = 2.68\,ms^{-1}$, $s = 0.2\,m$, $a = -g$

$$V^2 = 2.68^2 - 2 \times 9.81 \times 0.2$$
$$V = 1.81\,ms^{-1}$$

(iii) At the top of the loop,

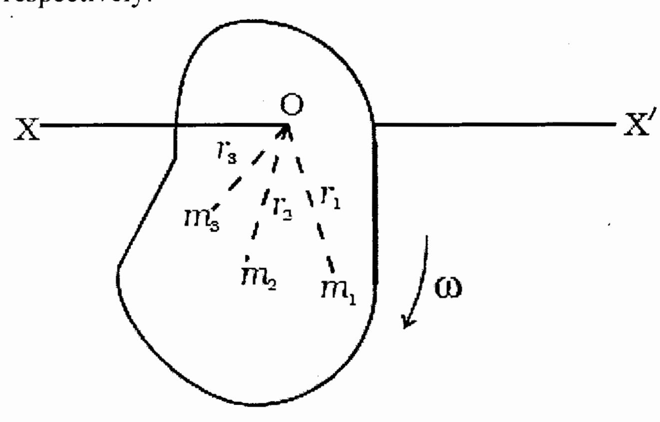
$$R + mg = \frac{mV^2}{r}$$
$$R = \frac{mV^2}{r} - mg$$
$$R = m\left(\frac{V^2}{r} - g\right)$$
$$= 0.025\left(\frac{1.81^2}{0.1} - 9.81\right)$$
$$= 0.57\,N$$

Rotational kinetic energy and moment of inertia

When a body rotates, it possesses energy which is due to the rotation. Since it has this energy because of its motion rather than its position, it is kinetic energy. It is distinct from the additional kinetic energy that it would have if it were also undergoing translational motion and will be referred to as **rotational kinetic energy**. The wheels on a moving car rotate as they are moving along and therefore have both types of kinetic energy.

Consider a rigid body rotating with angular velocity ω about an axis XOX'. Consider the particles of masses $m_1, m_2, m_3...$ situated at distances $r_1, r_2, r_3...$ respectively from the axis of rotation. The angular velocity of all the particles is the same but the particles rotate with different linear velocities.

Let the linear velocities of the particles be $v_1, v_2, v_3 \ldots$ respectively.



Kinetic energy of first particle $= \frac{1}{2}m_1v_1^2$

But $v = r\omega$

Kinetic energy of first particle $= \frac{1}{2}m_1r_1^2\omega^2$

Kinetic energy of second particle $= \frac{1}{2}m_2r_2^2\omega^2$

Kinetic energy of third particle $= \frac{1}{2}m_3r_3^2\omega^2$ and so on

The kinetic energy of a rotating body is equal to the sum of the kinetic energies of all the particles

Rotational kinetic energy

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 + \cdots$$

$$= \frac{1}{2}\omega^2(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots)$$

$$= \frac{1}{2}\omega^2 \sum_{i=1}^{n} m_i r_i^2$$

For a given body $\sum_{i=1}^{n} m_i r_i^2$ is a constant and is called the moment of inertia, I, of the body about that axis.

Thus, Rotational $KE = \frac{1}{2}I\omega^2$

where ω is the angular velocity ($rad\ s^{-1}$)

In translational motion, $KE = \frac{1}{2}mv^2$

The **moment of inertia** of a body is a measure of the way in which its mass is distributed in relation to the axis about which it is rotating. As such it depends on the mass of the body, its size, its shape, and which axis is being considered.

Moment of inertia = mass × $(distance)^2$

SI unit is $kg\,m^2$

Note:

Work done = $T \times \theta$

where T is the torque and θ is the angular displacement

If a couple of torque T about a certain axis acts on a body of moment of inertia I through an angle θ about the same axis and its angular velocity increases from 0 to ω, then

Work done by the couple = kinetic energy of rotation

$$T\theta = \frac{1}{2}I\omega^2$$

Example 21

Calculate the rotational kinetic energy of a flywheel of moment of inertia $5.0\ kg\,m^2$ rotating at $120\ rev\ min^{-1}$.

Solution

$$\omega = 120\frac{rev}{min} = 2\ \frac{rev}{s} = 2 \times 2\pi\ rad\ s^{-1}$$

Rotational $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 5 \times (4\pi)^2$

$= 395\ J$

Example 22

A car engine is quoted as having an output power of $28.0\ kW$ at a torque of $110\ Nm$. Calculate the rate of rotation of the output shaft of the engine in revolutions per minute.

Solution

Work done = Torque × θ

Work done per second = Torque × $\frac{\theta}{s}$

$$\frac{\theta}{s} = \frac{Power}{Torque}$$

$$= \frac{28000}{110} = 245.5\ rad\ s^{-1}$$

$$= 245.5 \times 60\ rad\ min^{-1}$$

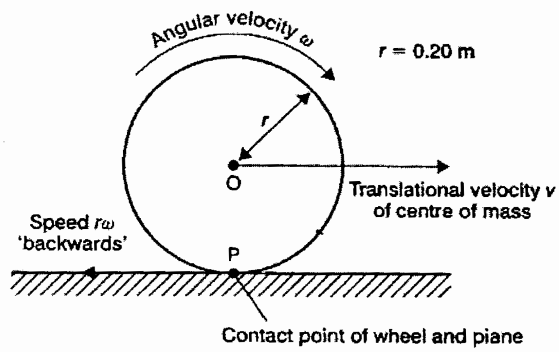
$$= 245.5 \times 60 \times \frac{1}{2\pi}\ rev\ min^{-1}$$

$$= 2.43\ \times 10^3\ rev\ min^{-1}$$

Example 23

Calculate the total kinetic energy of a cylinder of mass $12\ kg$ and radius $0.20\ m$ if it is rolling along a plane with translational velocity of $0.30\ m\ s^{-1}$. The moment of inertia of the cylinder is $0.24\ kg\,m^2$.

Solution



The translational velocity v (forwards) must be cancelled by speed $r\omega$ in the opposite direction due to the rotation of the wheel, for there to be no sliding between P on the wheel and the plane it touches i.e. P must be stationary at the instant of contact.

Total KE = Translational KE + Rotational KE

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\omega = \frac{v}{r} = \frac{0.3}{0.2} = 1.5$$

Therefore, Total KE $= \frac{1}{2}(12)(0.3)^2 + \frac{1}{2}(0.24)(1.5)^2$

$= 0.81\ J$

Self-Evaluation exercise

1. A turntable makes 33 revolutions per minute. Calculate
   (i) its angular velocity in $rad\ s^{-1}$
   (ii) the linear velocity of a point $0.12\ m$ from the centre
   [Ans: (i) $1.1\pi\ rad\ s^{-1}$ (ii) $0.13\ \pi\ m\ s^{-1}$]

2. (a) What is meant by a centripetal force?
   Why does such a force do no work in a circular orbit
   Describe and explain one example where such a force exists
   (b) An object of mass $0.50\ kg$ on the end of a string is whirled round a horizontal circle of radius $2.0\ m$ with a constant speed of $1.0\ ms^{-1}$. Find its angular velocity and tension in the string
   (c) If the same object is now whirled in a vertical circle of the same radius with the same speed, what are the maximum and minimum tensions in the string
   [Ans: (b) $5.0\ rad\ s^{-1}$; $25\ N$ (c) $30\ N$; $20\ N$]

3. A car is travelling round a bend in a road at a constant speed of $22\ ms^{-1}$, the driver moves along a circular path of radius $25\ m$. Calculate the magnitude of the driver's acceleration. Suggest what provides a force which a force to cause this acceleration
   [Ans: $19.4\ m\ s^{-2}$; friction]

4. A car travels on a humpback bridge of radius of curvature $45\ m$. Calculate the maximum speed of the car if its road wheels are to stay in contact with the bridge.
   [Ans: $21.01\ m\ s^{-1}$]

5. A car of mass $1000\ kg$ travels over a humpback bridge of radius of curvature $50\ m$ at a constant speed of $50\ m\ s^{-1}$. Calculate the magnitude and direction of the force exerted by the car on the road when it is at the top of the bridge.
   [Ans: $5.5 \times 10^3\ N$, downwards]

6. A racing car goes around a circular curve as fast as it can without skidding. The radius of the curve is $50\ m$ and the road is banked at $20°$ to allow faster speeds. The coefficient of static friction between the road and car tyres is 0.80. Determine the maximum speed which the car can have.
   [Ans: $28.4\ m\ s^{-1}$]

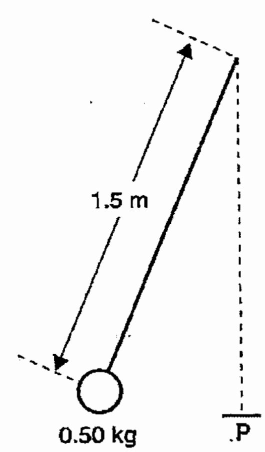
7. A conical pendulum consists of a bob of mass $0.50\ kg$ attached to a string of length $1.0\ m$. The bob rotates in a horizontal circle such that the angle the string makes with the vertical is $30°$. Calculate
   (i) the period of the motion
   (ii) the tension in the string
   [Ans: (i) $1.8\ s$ (ii) $5.8\ N$]

8. A car of mass $1.0 \times 10^3\ kg$ is moving at $30\ m\ s^{-1}$ around a bed of radius $0.60\ km$ on a horizontal track. What centripetal force is required to keep the car moving around the bend, and where does this force come from?
   [Ans: $1.5\ kN$; friction]

9. An object of mass $6.0\ kg$ is whirled in a vertical circle of radius $2.0\ m$ with a speed of $8.0\ m\ s^{-1}$. Calculate the maximum and minimum tension in the string connecting the object to the centre of the circle If the string breaks when the tension in it exceeds $360\ N$, calculate the maximum speed of rotation, in $m\ s^{-1}$ and state where the object will be when the string breaks
   [Ans: $252\ N$, $132\ N$, $10\ m\ s^{-1}$ at bottom]

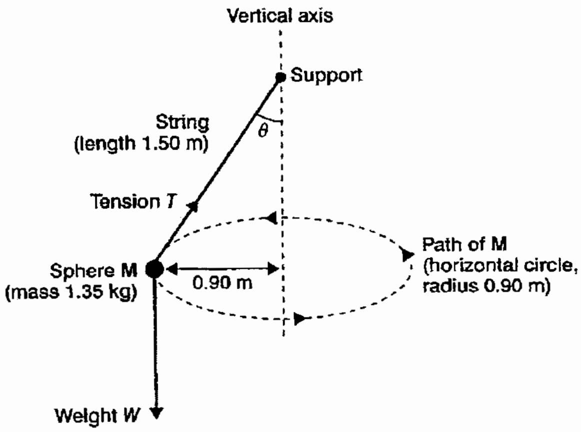
10. A car travels over a humpback bridge at a speed of $30\ m\ s^{-1}$. Calculate the minimum radius of the bridge if the car road wheels are to remain in contact with the bridge. What happens if the radius is less than the limiting value?
   [Ans: $90\ m$; leaves road]

11. The diagram below shows a simple pendulum with a length of $1.5\ m$ and a bob of mass $0.50\ kg$. When it passes through the lowest point P it has a speed of $2.0\ m\ s^{-1}$.



Calculate the tension in the string as the bob passes through point P
   [Ans: $6.3\ N$]

12. A metal sphere M of mass $1.35\ kg$ is suspended from a rigid support by a light string of length $1.50\ m$. The sphere is made to move in a horizontal circle of radius $0.90\ m$ as shown below



(a) Calculate the magnitude of the tension in the string
(b) Calculate the linear speed of the sphere as it moves in a horizontal plane
(c) Calculate the time required for the sphere to make one complete revolution of its horizontal motion

[Ans: (a) 16.8 N (b) 2.60 ms^{-1} (c) 2.18 s]

13. Explain why there must be a force acting on a particle which is moving with uniform circular speed in a circular path. Write down an expression for its magnitude
14. A conical pendulum consists of a small massive bob hung from a light string of length 1 m and rotating in a horizontal circle of radius 30 cm. Calculate the speed of rotation of the bob in revolutions per minute

[Ans: 31 rev $\min^{-1}$]

15. A small mass of 5 g is attached to one end of a light inextensible string of length 20 cm and the other end of the string is fixed. The string is held taut and horizontal and the mass is released. When the string reaches the vertical position, what are the magnitudes of
(a) the kinetic energy of the mass
(b) the velocity of the mass
(c) the acceleration of the mass
(d) the tension in the string

[Ans: (a) 0.10J (b) 2 $m\,s^{-1}$ (c) 20 $m\,s^{-2}$ (d) 0.15 N]

16. A boy ties a string around a stone and then whirls the stone so that it moves in a horizontal circle at a constant speed.
(a) The mass of the stone is 0.15 kg and the length of string between the stone and the boy's hand is 0.50 m. The period of rotation of the stone is 0.40 s. Calculate the tension in the string
(b) The boy now whirls the stone in a vertical circle, but the string breaks when it is horizontal. At this instant, the stone is 1.0 m above the ground and rising at a speed of 15 $m\,s^{-1}$. Describe the subsequent motion of the stone until it hits the ground and calculate its maximum height

[Ans: (a) 18.5 N (b) 12.25 m]

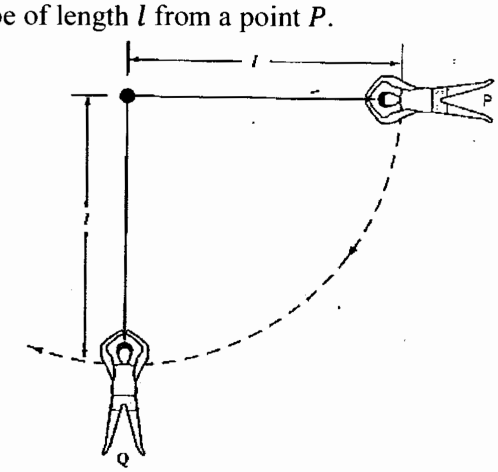
17. Derive an expression for the magnitude of the acceleration of a particle moving with speed v in a circle of radius r.
18. A particle of mass m moves in a circle in a vertical plane, being attached to a fixed point A by a string of length r. The motion of the mass is such that the string is just fully extended at the highest point. Determine
(a) the minimum speed v at the highest point for this to happen
(b) the speed V of the particle, and the tension in the string at its lowest point
19. A stone of mass 500 g is attached to a string of length 50 cm which will break if the tension in it exceeds 20 N. The stone is whirled in a vertical circle, the axis of rotation being at a height of 100 cm above the ground. The angular speed is very slowly increased until the string breaks. In what position is this break most likely to occur, and at what angular speed? Where will the stone hit the ground?

[Ans: 7.7 $rad\,s^{-1}$; 122 cm from point below suspension]

20. A 2 tonne car has to go over a turn whose radius is 750 m and the angle of slope is 5°. The coefficient of friction between the car wheels and the road is 0.5. What should be the maximum speed of the car so that it may go over the turn without slipping?

[Ans: 67.2 $m\,s^{-1}$]

21. A gymnast of mass m swings from rest on a light rope of length l from a point P.



Considering the resistances to forward motion to be negligible, derive expressions for

(a) the gymnast's speed on passing point Q

(b) the angular velocity of the rope at this moment

(c) the tension in the rope as the gymnast passes Q

Rotational motion

22. A couple of torque 5 $N\,m$ is applied to a flywheel initially at rest. Calculate its kinetic energy after it has completed 5 revolutions. Ignore friction

[Ans: 157 J]

23. (a) (i) Explain what is moment of inertia of a body

(ii) Why is there no unique value for the moment of inertia of a given body?

(iii) A rigid body rotates about an axis with an angular velocity ω. If the relevant moment of inertia of the body is I, show that its rotational kinetic energy is $\frac{1}{2}I\omega^2$

(b) A motor car is designed to runoff the rotational kinetic energy stored in a flywheel in the car. The flywheel is to be accelerated up to some maximum rotational speed by electric motors placed at various stations along the route. If the flywheel has a moment of inertia of 300 $kg\,\mathrm{m}^2$ and is accelerated to 4200 revolutions per minute at a station, calculate the kinetic energy stored in the flywheel.

[Ans: (b) $2.9 \times 10^7\ J$]

24. A wheel possesses 200 J of rotational kinetic energy and has a moment of inertia of 0.80 $kg\,\mathrm{m}^{-2}$. Calculate its rate of rotation

(a) in $rad\ s^{-1}$

(b) in $rev\ min^{-1}$

[Ans:(a) 22 $rad\ s^{-1}$ (b) 214 $rev\ min^{-1}$]

25. The wheels of a car rotate 8.0 times each second. Each wheel has a mass 15 kg, radius 0.30 m and moment of inertia 0.27 $kg\,\mathrm{m}^2$. Calculate

(a) the translational speed of the car

(b) the total KE of the four wheels

[Ans: (a) $4.8\pi\ m\ s^{-1}$ (b) 8.2 kJ]

26. Calculate the moment of inertia of a flywheel which has rotational kinetic energy of 50.0 kJ when it is rotating at 20.0 $rad\ s^{-1}$.

[Ans: 250 $kg\,\mathrm{m}^2$]

27. The drum of a spin drier has a moment of inertia of 0.24 $kg\,\mathrm{m}^{-2}$ when it is loaded with wet clothes. During operation it rotates with an angular velocity of 250 $rad\ s^{-1}$. Calculate the rotational kinetic energy

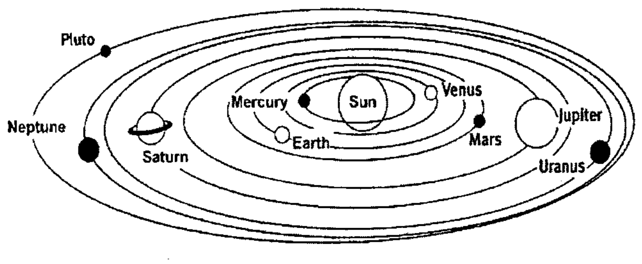
[Ans: 7.5 kJ]

28. A flywheel is initially at rest and a torque of 8.0 Nm is applied to it. Calculate its rotational kinetic energy after it has completed 6.0 revolutions. Ignore effects of friction

[Ans: 300 kJ]

GRAVITATION

Have you ever wondered what holds up bodies like the sun, moon and stars in space? What makes the moon revolve round the earth and planets round the sun? The answer is gravity or gravitational force. The same force that makes an object fall to the ground is responsible for the motion of the moon round the earth. In fact, the moon is constantly falling towards the earth.



Kepler's laws

1.The law of orbits

Each planet moves in an elliptical orbit with sun at one focus

2.The law of areas

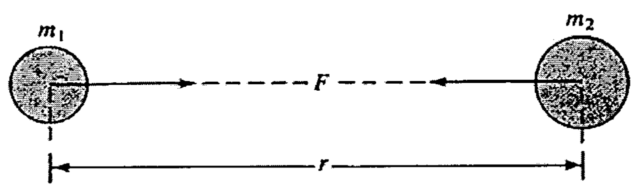
The imaginary line joining the sun and the planet sweeps out equal areas in equal time intervals

3.The law of periods

The square of the period of revolution of a planet round the sun is directly proportional to the cube of the mean distance between the planet and the sun

Newton's law of gravitation

Newton's law of gravitation states that the force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.



The direction of the force is along the line joining the particles.

Thus, the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant

$$G = 6.67 \times 10^{-11}\ kg^{-1}\,m^3\,s^{-2}$$

Dimensions of G

$$[G] = \frac{[F] \times [r^2]}{[M] \times [m]} = \frac{MLT^{-2} \times L^2}{M \times M} = L^3T^{-2}M^{-1}$$

Special features of the law

(i) The gravitational forces of attraction between two bodies form an action-reaction pair as in Newton's third law of motion. The mass m_1 is attracted towards m_2 with a force F towards the right. At the same time, the mass m_2 is attracted by the mass m_1 with an equal force in the opposite direction.

(ii) The low value of G implies that gravitational force is a very weak force. For a pair of 1 kg mass separated by a distance of 1 m, the gravitational force is only $6.67 \times 10^{-11}\ N$, too small to be measured by any ordinary means.
However, the forces of attraction between you and the earth is quite appreciable and can be measured. It is actually your weight

Gravitational field strength

The gravitational field strength at a point in a gravitational field is defined as the force per unit mass acting on a mass placed at that point.

$$g = \frac{F}{m}$$

where

$g =$ gravitational field strength ($N\,kg^{-1} = m\,s^{-2}$)

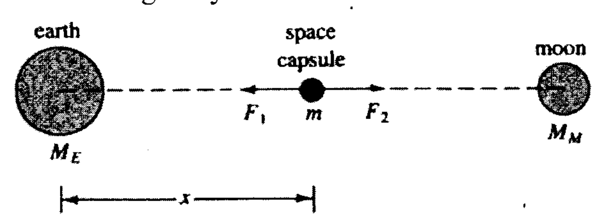
$F =$ force acting on mass m

Example 1

A space capsule travels from the earth towards the moon. Calculate its distance from the earth where it is under zero gravity (Mass of earth $= 6.0 \times 10^{24}$ kg, mass of moon $= 7.4 \times 10^{22}\ kg$, distance between centre of earth and centre of moon $= 3.8 \times 10^{8}\ m$)

<u>Solution</u>

Let $x =$ distance of space capsule from earth where it is under zero gravity



Zero gravity occurs when $F_1 = F_2$ or $F_1 - F_2 = 0$

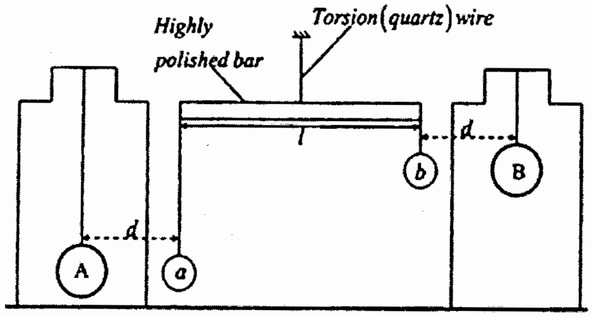
$$\frac{GmM_E}{x^2} = \frac{GmM_M}{(3.8 \times 10^8 - x)^2}$$

$$\left(\frac{x}{3.8 \times 10^8 - x}\right)^2 = \frac{M_E}{M_M}$$

$$\frac{x}{3.8 \times 10^8 - x} = \sqrt{\frac{6.0 \times 10^{24}}{7.4 \times 10^{22}}}$$

$$x = 3.42 \times 10^8\ m$$

Measurement of G



Two identical small gold spheres a and b of known masses m each are suspended from the ends of a highly polished bar of known length l.

The bar is in turn suspended by a long, fine torsion wire of known torsion constant c.

Two identical large lead spheres A and B of masses M each are then respectively brought near the small gold spheres a and b.

Because of the attraction between the two pairs of spheres near each other, a couple is set up, such that two equal but opposite and parallel forces F act at the ends of a polished bar.

The bar is deflected through an angle θ (in radians) which can be measured by the lamp and scale method.

If d is the measured distance between the large and small spheres, then $F = \frac{GMm}{d^2}$

But, Moment of a couple = one of the forces, $F\ \times$ Perpendicular distance between the forces

$$= \frac{GMm}{d^2} \times l = \frac{GMml}{d^2}$$

Also resisting or opposing torque T in the torsion wire is given by; $T = c\theta$, where c is the torsional constant of the wire.

However, for the polished bar to stop rotating, then the opposing torque T produced by the torsion wire should be equal to the deflecting torque.

$$\frac{GMml}{d^2} = c\theta$$

$$\therefore G = \frac{c\theta d^2}{Mml}$$

On substitution of m, M, c, d, l and θ, the value of G can be obtained

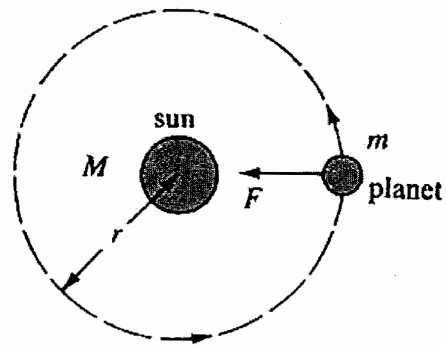
Note

The torsion wire should be very sensitive such that big enough deflections which can accurately be measured by the lamp and scale method are obtained.

The whole setup of the apparatus should be small such that it can easily be screened from air convectional currents.

Consistency of Kepler's third law with Newton's law of gravitation

Suppose that a planet of mass m is in a circular orbit of radius r around the sun



The centripetal force is provided by the gravitational attraction on the planet by the sun

$$F = \frac{GMm}{r^2}$$

where M = mass of the sun

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r^2} = \omega^2 r$$

But $\omega = \frac{2\pi}{T}$ where T = period

$$\frac{GM}{r^2} = \left(\frac{2\pi}{T}\right)^2 r$$

$$T^2 GM = 4\pi^2 r^3$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

where $\frac{4\pi^2}{GM}$ = constant

$$\boldsymbol{T^2 \propto r^3}$$

This is Kepler's third law of gravitation

Example 2

An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. Calculate the period of revolution of S in seconds.

Solution

Orbit radius of C = r_c, time period T_c

Orbital radius of S, $r_s = 4r_c$; time period = T_s

From Kepler's third law, $T^2 \propto r^3$

$$T_c^2 = kr_c^3 \quad \text{.... (i)}$$

$$T_s^2 = kr_s^3 \quad \text{...... (ii)}$$

(ii) ÷ (i);

$$\left(\frac{T_s}{T_c}\right)^2 = \left(\frac{r_s}{r_c}\right)^3 = \left(\frac{4r_c}{r_c}\right)^3$$

$$\frac{T_s}{T_c} = 4^{\frac{3}{2}}$$

$$T_s = 8T_c$$

But $T_c = 24$ hours $= 24 \times 3600\, s = 86400\, s$

$$T_s = 8 \times 86400 = 691200\, s$$

$$= 6.912 \times 10^5\, s$$

Example 3

A Saturn year is 29.5 times the Earth year. How far is Saturn from the sun if the earth is $1.50 \times 10^8\, km$ away from the sun?

Solution

Let r_s = distance of Saturn from sun

T_s = time period of Saturn about sun

r_e = distance of earth from the sun

T_e = time period of earth about sun

From Kepler's third law;

$$\frac{T_s^2}{T_e^2} = \frac{r_s^3}{r_e^3}$$

$$\frac{r_s}{r_e} = \left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

$$r_s = r_e \times \left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

But $T_s = 29.5\, T_e$

$$\Rightarrow r_s = r_e \times \left(\frac{29.5T_e}{T_e}\right)^{\frac{2}{3}}$$

$$r_s = 1.5 \times 10^{11} \times (29.5)^{\frac{2}{3}} = 14.32 \times 10^{11}\, m$$

Mass of the sun

Suppose that the planet is the earth, then distance of the from the sun (orbital radius),

$$r_0 = 1.5 \times 10^{11}\, m$$

Period, T of revolution of the earth around the sun is 1 year

1 year $= 365 \times 24 \times 60 \times 60 = 3.15 \times 10^7\, s$

From $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$

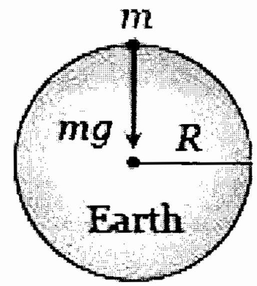
Mass of the sun, $M = \frac{4\pi^2 r^3}{GT^2}$

$$= \frac{4\pi^2(1.5 \times 10^{11})^3}{(6.67 \times 10^{-11})(3.15 \times 10^7)^2}$$

$$= 2.0 \times 10^{30}\, kg$$

Mass and density of the earth

Considering a body of mass m on the surface of the earth, its weight is provided by the gravitational attraction of the earth.



$$mg = \frac{GMm}{R^2}$$

$$M = \frac{gR^2}{G}$$

Radius of earth, $R = 6.4 \times 10^6\, m$, $g = 9.81\, \text{ms}^{-2}$

Mass of the earth, $M = \frac{9.81\,(6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$

$$= 6.02 \times 10^{24}\, kg$$

Alternatively;

Considering a moon of mass m_m moving with a speed v_m round the earth of mass m_e in a circular orbit of radius r_m

$$G\frac{m_e m_m}{r_m^2} = \frac{m_m v_m^2}{r_m}$$

$$m_e = \frac{v_m^2 r_m}{G} = \frac{4\pi^2}{G} \cdot \frac{r_m^3}{T_m^2}$$

Period, T_m of the moon around the earth is 27.3 days

$$T_m = 2.4 \times 10^6\, s$$

Radius, r_m of moon's orbit about the earth

$$= 4.0 \times 10^8\, m$$

$$m_e = \frac{4\pi^2(4.0 \times 10^8)^3}{6.67 \times 10^{-11}(2.4 \times 10^6)^2}$$

$$= 6.0 \times 10^{24}\, kg$$

Density of earth, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{gR^2}{G} \div \frac{4}{3}\pi R^3$

$$= \frac{3g}{4\pi GR}$$

$$= \frac{3 \times 9.81}{4\pi \times 6.67 \times 10^{-11} \times 6.4 \times 10^6}$$

$$= 5478.4\, kg\, \text{m}^{-3}$$

Example 4

One of the satellites of Jupiter has an orbital period of 1.769 days and the radius of the orbit is $4.22 \times 10^8\, m$. Calculate the mass of Jupiter

Solution

From $\frac{GMm}{r} = \frac{mv^2}{r}$

The mass of Jupiter is given by

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 \times (4.22 \times 10^8)^3}{6.67 \times 10^{-11} \times (1.769 \times 24 \times 60 \times 60)^2}$$

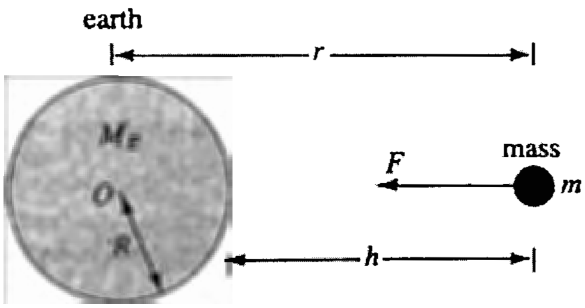
$$= 1.9 \times 10^{27}\ kg$$

Acceleration due to gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity. We know that when a force acts on a body, it produces an acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity and it is denoted by g.

Variation of g with the distance from the centre of the earth



For a mass m at distance $r \geq R$ from the centre of the earth, the gravitational force

$$F = \frac{GmM_E}{r^2}$$

where M_E = mass of the earth

The acceleration due to gravity, g′ at a distance r from the centre of the earth

$$g' = \frac{F}{m}$$

$$g' = \frac{GmM_E}{r^2} \div m$$

$$g' = \frac{GM_E}{r^2} \text{ ...... (i)}$$

If the mass is on the earth's surface,

$$g = \frac{GM_E}{R^2} \text{ .... (ii)}$$

(i) ÷ (ii);

$$\frac{g'}{g} = \frac{R^2}{r^2}$$

$$g' = \frac{R^2 g}{r^2}$$

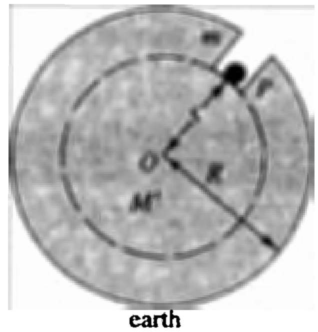
$$g' \propto \frac{1}{r^2} \text{ (if } r \geq R)$$

At a height h from the earth's surface, $r = R + h$

$$g' = \frac{R^2 g}{(R+h)^2}$$

Variation of g with distance below the earth's surface

The gravitational field of the earth exists both inside and outside the earth



Consider a mass m at a point P below the earth's surface at a distance r $(r < R)$ from the centre of the earth. The gravitational attraction on the mass m is only due to that part of the earth centre O and radius, r. Assuming the earth to be a uniform sphere of density ρ, the mass M' of the part of the earth of radius r is

$$M' = \frac{4}{3}\pi r^3 \rho$$

$$mg' = \frac{GmM'}{r^2}$$

where g′ = acceleration due to gravity at P

$$g' = \frac{G}{r^2}\left(\frac{4}{3}\pi r^3 \rho\right)$$

$$g' = 4\pi r \rho G \text{ .... (i)}$$

On the earth's surface,

$$g' = \frac{GM_E}{R^2}$$

where R = radius of the earth

$$g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

$$g = 4\pi R \rho G \text{ ...... (ii)}$$

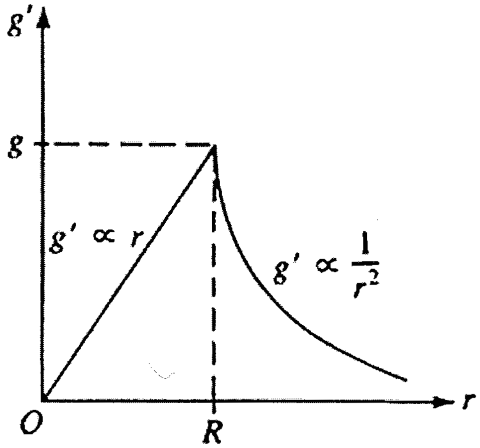
(i) ÷ (ii);

$$\frac{g'}{g} = \frac{r}{R}$$

$$g' = \frac{r}{R} g$$

$$g' \propto r \text{ (if } r < R)$$

Graph showing variation of g with distance from the centre of the earth

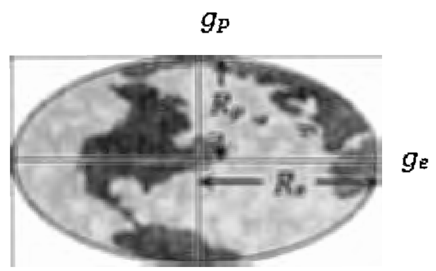


Variation of g with latitude

There are two reasons why the acceleration due to gravity varies with latitude.

1. Shape of the earth

The earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 km longer than the polar radius.



From $g = \frac{GM}{R^2}$

At equator, $g_e = \frac{GM}{R_e^2}$ ..... (i)

At poles, $g_p = \frac{GM}{R_p^2}$ ...... (ii)

(i) ÷ (ii);

$$\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$$

Since $R_{equator} > R_{pole}$

$$\Rightarrow g_{pole} > g_{equator}$$

Therefore, the weight of the body increases as it is taken from the equator to the poles

2.Rotation of the earth

As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force. Due to it, the apparent weight of the body decreases. Since the magnitude of centrifugal force varies with the latitude of the place, the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

Example 5

The mass of the earth is approximately 80 times the mass of the moon and the earth's radius is 3.7 times that of the moon. If the acceleration of free fall on the earth is 10 $m\,s^{-2}$, estimate its value on the moon

Solution

Let M = mass of moon, then mass of earth = 80 M

Let r = radius of moon, then radius of earth = $3.7r$

For a mass on the earth's surface,

$$\frac{Gm(80M)}{(3.7r)^2} = mg \text{ .... (i)}$$

For the same mass m on the moon's surface,

$$\frac{GmM}{r^2} = mg' \text{ .... (ii)}$$

where g' = acceleration of free fall on the moon

(i) ÷ (ii);

$$\frac{80}{3.7^2} = \frac{g}{g'}$$

$$g' = \frac{10 \times 3.7^2}{80} = 1.7\, m\,s^{-2}$$

Example 6

A body is raised to a height of 500 km. What is the acceleration due to gravity at this point? (acceleration dur to gravity at earth's surface = 9.81 $m\,s^{-2}$)

Solution

At the earth's surface, $g = \frac{GM}{R^2}$ .... (i)

At a height above earth's surface,

$$g' = \frac{GM}{(R+h)^2} \text{ .... (ii)}$$

(ii) ÷ (i);

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$g' = g \times \frac{R^2}{(R+h)^2}$$

$$= 9.81 \times \frac{(6.4 \times 10^6)^2}{(6.4 \times 10^6 + 5 \times 10^5)}$$

$$= 9.1\, m\,s^{-2}$$

Example 7

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth? (radius of the earth = 6400 km)

Solution

Weight of the body = mg = 63 N; $h = \frac{R}{2}$

The value of acceleration due to gravity g' at a height h is

$$g' = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{\left(R+\frac{R}{2}\right)^2} = \frac{gR^2}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9}g$$

Gravitational force on the body at height h is

$$F = mg' = m \times \frac{4}{9}g = \frac{4}{9}mg = \frac{4}{9} \times 63 = 28\ N$$

Example 8

The acceleration due to gravity on the moon is $\frac{1}{6}$ of the acceleration due to gravity on earth. If the ratio of the density of the earth to that of the moon is $\frac{5}{3}$, calculate the radius of the moon. (Radius of the earth $= 6.4 \times 10^6\ m$)

Solution

$$g_M = \frac{1}{6}g_E \Rightarrow \frac{g_M}{g_E} = \frac{1}{6}$$

If you consider a body of mass m placed on the earth's surface

$$mg_E = \frac{GM_Em}{R_E^2}$$

$$g_e = \frac{GM_E}{R_E^2} = \frac{G\left(\frac{4}{3}\pi R_E^3 \times \rho_E\right)}{R_E^2}$$

$$g_E = \frac{4}{3}\pi R_E\rho_E G \ \ ...... \text{(i)}$$

For a body on the surface of the moon,

$$g_M = \frac{4}{3}\pi R_M\rho_M G \ \ ...... \text{(ii)}$$

(ii) ÷ (i);

$$\frac{g_M}{g_E} = \frac{R_M\rho_M}{R_E\rho_E}$$

$$R_M = \frac{g_M}{g_E} \times \frac{\rho_E}{\rho_M} \times R_E$$

$$= \frac{1}{6} \times \frac{5}{3} \times 6.4 \times 10^6$$

$$= 1.78 \times 10^6\ m$$

Example 9

A planet has a mass $\frac{1}{10}$ of that of the earth while radius is $\frac{1}{3}$ that of the earth. If a person can throw a stone on the earth's surface to a height of $90\ m$, calculate the height to which he will be able to throw the stone on the planet

Solution

Acceleration due to gravity: $g = \frac{GM}{R^2}$

$$\frac{g_{planet}}{g_{earth}} = \frac{M_{planet}}{M_{earth}}\left(\frac{R_{earth}}{R_{planet}}\right)^2 = \frac{1}{10} \times \left(\frac{3}{1}\right)^2 = \frac{9}{10}$$

If a stone is thrown with velocity u from the surface of the planet, then the maximum height

$$H = \frac{u^2}{2g}$$

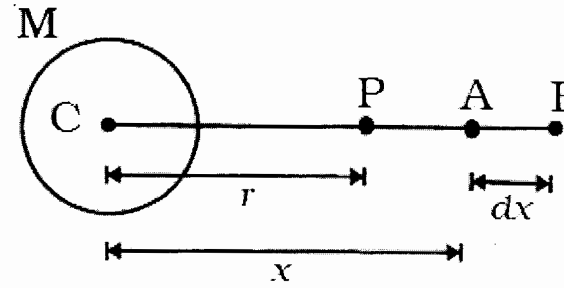
$$\frac{H_{planet}}{H_{earth}} = \frac{g_{earth}}{g_{planet}}$$

$$H_{planet} = \frac{10}{9} \times H_{earth} = \frac{10}{9} \times 90 = 100\ m$$

Gravitational potential

Gravitational potential at a point is defined as the amount of work done in moving a 1 kg mass from a point to infinity against the gravitational field.

Consider a body of mass M at the point C. Let P be a point at a distance r from C.



Gravitational field intensity at A, $g = \frac{GM}{x^2}$

The work done in moving a 1 kg mass from A to B through a small distance dx is

$$dW = dU = -g\,dx$$

Negative sign indicates that work is done against the gravitational field

$$dU = -\frac{GM}{x^2}dx$$

The total work done in moving a 1 kg mass from P to infinity

$$U = -\int_r^{\infty} \frac{GM}{x^2}dx$$

$$= -GM\left[-\frac{1}{x}\right]_r^{\infty}$$

$$= -GM\left[-\frac{1}{\infty} - \left(-\frac{1}{r}\right)\right]$$

$$\boldsymbol{U = -\frac{GM}{r}}$$

The gravitational potential is negative since work is done against the gravitational field which is always attractive.

Gravitational potential energy

The gravitational potential energy of a body at a point is defined as the amount of work done in moving the body from a point in the gravitational field to infinity

Consider a mass m at a distance x from another mass M, the gravitational force between them is given by

$$F = \frac{GMm}{x^2}$$

If the mass, m is moved through a small distance dx away from the field, then the small work done,

$$dW = -F.dx$$

Total work done in moving the mass from r to ∞,

$$W = -\int_r^{\infty} \frac{GMm}{x^2} dx$$

$$= -GMm\left[-\frac{1}{x}\right]_r^{\infty}$$

$$= -\frac{GMm}{r}$$

This work is stored inside the body as its gravitational potential energy

$$U = -\frac{GMm}{r}$$

Analogy between gravity and electricity

Gravitational quantity	Electrical quantity
$U = -G\frac{m}{r}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
$g = G\frac{m}{r^2}$	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
$F = G\frac{m_1 m_2}{r^2}$	$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$

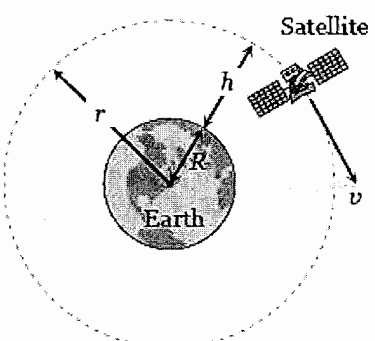
Note

The gravitational force, unlike the electrical force, is always attractive

The gravitational force between two masses does not depend on the medium in which they are situated. Electrostatic force depends on ε.

Orbital velocity of satellite

Satellites are natural or artificial bodies describing orbits around a planet under its gravitational attraction. Moon is a natural satellite of the earth and the earth is a natural satellite of the sun. The condition for establishment of artificial satellites is that the centre of orbit of satellite must coincide with centre of earth or the satellite must move around the great circle of the earth.



Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

As $GM = gR^2$ and $r = R + h$,

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

Note

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit i.e. satellites of different masses have same orbital velocity, if they are in the same orbit

(ii) Orbital velocity depends on the mass of the central body and radius of the orbit

(iii) For a given planet, the greater the radius of orbit the lesser will be the orbital velocity of the satellite

(iv) Orbital velocity when the satellite revolves to the surface of the planet i.e. $h = 0$

$$v = \sqrt{gR}$$

For the surface of the earth,

$$v = \sqrt{9.81 \times 6.4 \times 10^6} = 7.9\ km\ s^{-1}$$

Time period of satellite

It is the time taken by a satellite to go once around the earth.

$$T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$$

As $GM = gR^2$;

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}}$$

Since $r = R + h$,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

If the satellite is very close to the earth, i.e. $h = 0$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi\sqrt{\frac{6.4\times10^6}{9.81}} = 84.6 \text{ minutes}$$

Height of satellite

As we know, time period of the satellite,

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

By squaring and rearranging both sides,

$$\frac{g R^2 T^2}{4\pi^2} = (R+h)^3$$

$$h = \left(\frac{g R^2 T^2}{4\pi^2}\right)^{\frac{1}{3}} - R$$

By knowing the value of time period, we can calculate the height of satellite above the surface of the earth.

Geostationary satellite

The satellite which appears stationary relative to the earth is called geostationary or geosynchronous satellite or communication satellite.

A geostationary satellite always stays over the same place above the earth. Such a satellite is never at rest but appears stationary due to its zero relative velocity with respect to that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

A parking orbit is a path in space of a satellite which makes it appear to be in the same position relative to the observer at a point on the earth.

For a satellite to be in a parking orbit, its orbital period around the earth must be equal to the rotational period of the earth i.e. 24 hours. Its sense of rotation should be the same as that of the earth i.e. anticlockwise direction

Height of geostationary satellite

$T = 24 \times 3600 = 8.64 \times 10^4\ s$

From $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (8.64 \times 10^4)^2}{4\pi^2}$$

$$r = 42000\ km$$

But $R + h = r$

$$\therefore h = r - R = 42400 - 6400$$

$$= 36000\ km$$

Weightlessness

Television pictures show astronauts and objects floating in satellites orbiting the earth. The apparent weightlessness is sometimes explained wrongly as zero gravity condition.

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

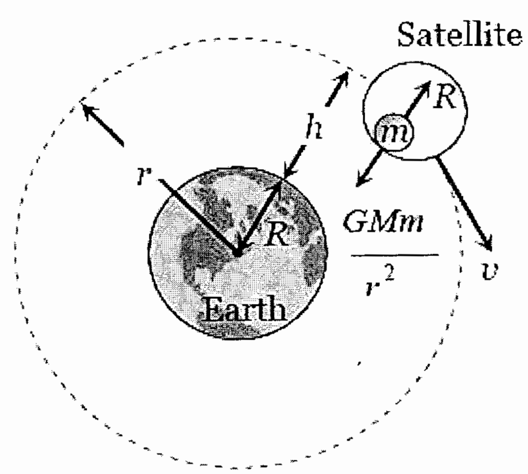
We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

- When objects fall freely under gravity. For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.
- When a satellite revolves in its orbit around the earth. Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

Weightlessness in a satellite

A satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of a satellite is $\frac{GM}{r^2}$ towards the centre of the earth



If a body of mass m is placed on a surface inside a satellite moving around the earth, then the forces on the body are

(i) the gravitational pull of earth $= \frac{GMm}{r^2}$

(ii) the reaction by the surface $= R$

By Newton's second law of motion,

$$\frac{GMm}{r^2} - R = ma$$
$$R = \frac{GMm}{r^2} - ma$$
$$R = \frac{GMm}{r^2} - m\left(\frac{GM}{r^2}\right) = 0$$

Thus, the surface does not exert any force on the body and hence its apparent weight is zero.
Such a state is called weightlessness.

Note

Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce its own gravity.
E.g. moon is a satellite of earth but due to its own weight it applies a gravitational force of attraction on the body placed on its surface and hence weight of the body will not be equal to zero at the surface of the moon.

Mechanical energy of a satellite

At a distance r from the centre of the earth,

Gravitational PE of satellite $= -\frac{GMm}{r}$

From $\frac{GMm}{r^2} = \frac{mv^2}{r}$;

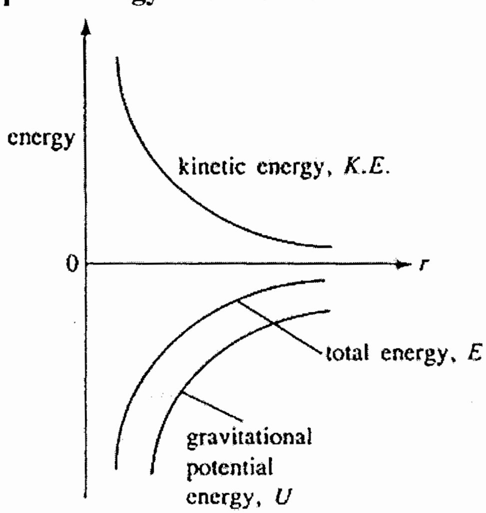
$$\frac{1}{2}\frac{GMm}{r} = \frac{1}{2}mv^2$$

KE of satellite $= \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$

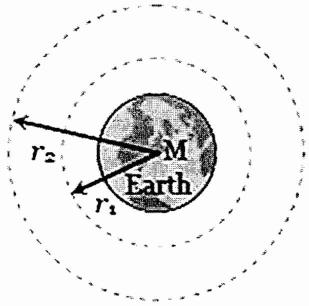
Total energy of satellite $= PE + KE$

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$
$$= -\frac{1}{2}\frac{GMm}{r}$$

A graph of energy of a satellite



Change in the orbit of a satellite



When a satellite is transferred to a higher orbit ($r_2 > r_1$), then

Work done in changing the orbit, $W = E_2 - E_1$

$$W = \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$
$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Note

When a satellite moves to a higher orbit (r increases). the following changes happen according to the equations

- Kinetic energy decreases
- Potential energy increases (less negative)
- Total energy increases (less negative)
- Orbital velocity decreases
- Time period increases

Effect of friction on orbits of satellites

If a satellite encountered air resistance, it would do work against friction and thus its total energy would decrease. It follows that the orbital radius would decrease (orbital decay) and the potential energy would decrease. Kinetic energy of the satellite increases and the satellite may strike the earth's surface.
If the total energy is decreased to an extent that it cannot overcome the work done against friction, the satellite may burn.

Work done against gravity

If a body of mass m is moved from the surface of the earth to a point at a distance h above the surface of the earth, then change in potential energy of work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

As $r_1 = R$ and $r_2 = R + h$,

$$W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$W = GMm\left[\frac{R+h-R}{R(R+h)}\right]$$

$$W = \frac{GMmh}{R(R+h)} = \frac{GMmh}{R^2(1+\frac{h}{R})}$$

As $GM = gR^2$,

$$W = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

If h is very small as compared to the radius of the earth, then $\frac{h}{R} \approx 0$

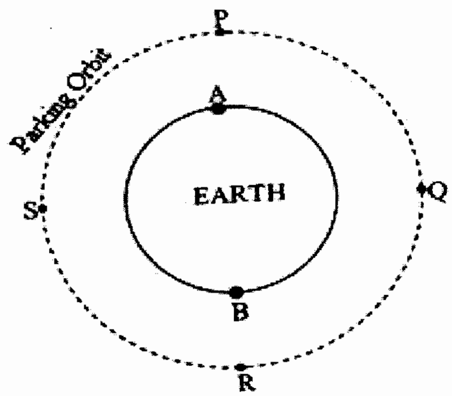
$$\Rightarrow W = mgh$$

If, $h = R$, then

$$W = \frac{1}{2}mgR$$

Satellite communication

Communication satellites are used to send radio and television signals over long distances. These satellites are fitted with devices which can receive signals from an earth station and transmit them in different directions



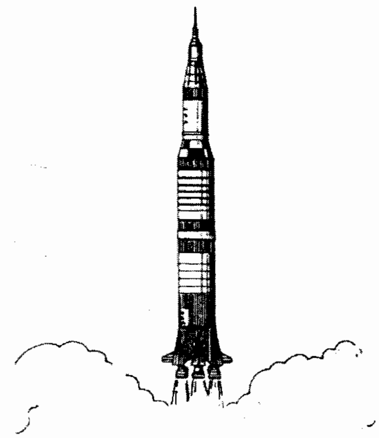
A set of three or more satellites are launched in a parking orbit as shown above. Assuming that it required that radio signals be transmitted from A to B, the signals can be transmitted from A to a geosynchronous satellite P, then re-transmitted from P to another geosynchronous satellite Q, then to R, and finally to B. The signals could also take the paths;

A→P→S→R→B

Note that communication can only occur provided that there is no obstruction between the transmitter, the satellite and the receiver. This is the disadvantage in communication using geostationary satellites.

Escape velocity

This is the minimum velocity with which a body must be projected so as to enable it to just overcome the gravitational pull of a planet.



The work done to displace a body from the surface of the earth ($r = R$) to infinity ($r = \infty$) is

$$W = -F\,dx$$

$$W = -\int_R^\infty \frac{GMm}{x^2}dx = -GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$= -\frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth

If v_e is the required escape velocity,

$$KE = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

But $GM = gR^2$

$$v_e = \sqrt{2gR}$$

At the earth, $g = 9.81\ m\,s^{-2}$, $R = 6.4 \times 10^6\ m$

$$v_e = \sqrt{2 \times 9.81 \times 6.4 \times 10^6}$$
$$= 11.2\ km\,s^{-1}$$

Note

1. If a body is projected with velocity less than the escape velocity ($v < v_e$), it will reach a certain maximum height and then may either move in an orbit around the planet or may fall back down to the planet

Maximum height attained by the body

Let a projection velocity of a body of mass m be v so that it attains a maximum height h. At maximum height, the velocity of the particle is zero, so kinetic energy is zero

By law of conservation of energy,

Total energy at surface = Total energy at height h

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$$\frac{v^2}{2} = GM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$
$$\frac{v^2}{2} = \frac{GMh}{R(R+h)}$$
$$\frac{2GM}{v^2R} = \frac{R+h}{h}$$
$$\frac{2GM}{v^2R} = \frac{R}{h} + 1$$
$$\frac{R}{h} = \frac{2GM}{v^2R} - 1$$
$$h = \frac{R}{\left[\left(\frac{2GM}{R}\right)\left(\frac{1}{v^2}\right) - 1\right]}$$

From $v_e = \sqrt{\frac{2GM}{R}}, \frac{2GM}{R} = v_e^2$

$$h = \frac{R}{\left(\frac{v_e^2}{v^2} - 1\right)}$$
$$h = R\left[\frac{v^2}{v_e^2 - v^2}\right]$$

2. If a body is projected with velocity greater than escape velocity ($v > v_e$), then by conservation of energy,

Total energy at surface = Total energy at infinity

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mV^2 + 0$$
$$V^2 = v^2 - \frac{2GM}{R}$$

As $\frac{2GM}{R} = v_e$,

$$V^2 = v^2 - v_e^2$$
$$V = \sqrt{v^2 - v_e^2}$$

Therefore, the body will move in interplanetary or interstellar space with velocity $\sqrt{v^2 - v_e^2}$

Atmosphere

There are two factors which determine whether the planets have atmospheres or not. They are

(i) acceleration due to gravity on its surface

(ii) surface temperature of the planet

A planet will have atmosphere if the velocity of molecules in its atmosphere i.e. $v_{rms} = \sqrt{\frac{3RT}{M}}$ is less than the escape velocity. The earth has atmosphere as $v_{rms} < v_e$ while moon has no atmosphere as $v_{rms} > v_e$

Why the moon has no atmosphere

At the moon, the value of g is very small ($\frac{1}{4}$ that of the earth). Consequently, the escape speed is very small i.e. $v \propto \sqrt{g}$. As the average velocity (r.m.s velocity) of the atmospheric air molecules at the surface temperature of the moon is greater than the escape velocity, the air molecules escape. Thus, the moon has no atmosphere

Conditions for life on any planet

- The planet must have a suitable living temperature range
- The planet must have a sufficient and right kind of atmosphere
- The planet must have a considerable amount of water

The above conditions are what makes our earth a better place to be otherwise as people dream to go abroad, other people dream to go to other planets. Who would not wish to go to the moon and spend a vacation there? For the sun, you cannot even go close, you would melt. We therefore have to appreciate the almighty God who put us on earth and made it the better place to be. Think about it.

Example 10

Assuming a geostationary satellite orbits the earth at a height of 36000 km from the surface of the earth. What is the potential due to the earth's gravity at the site of the satellite? (radius of earth = 6400 km, mass of the earth = $6.0 \times 10^{24}\ kg$)

Solution

Distance of the satellite from the centre of the earth is

$$r = R + h = 6400 + 36000 = 42400\ km$$

Gravitational potential due to gravity at the site of the satellite is

$$U = -\frac{GM}{r}$$
$$= -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{42400 \times 10^3}$$
$$= -9.44 \times 10^6\ J\ kg^{-1}$$

Example 11

Calculate the escape velocity from earth for a 5000 kg spacecraft and determine the kinetic energy it must have at earth's surface in order to escape the earth's field. (Mass of the earth, $M = 5.98 \times 10^{24}\ kg$ and radius of earth, $R = 6.37 \times 10^6\ m$)

Solution

The escape velocity v_e of a body on the earth's surface is $v_e = \sqrt{\frac{2GM}{R}}$

$$v_e = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{6.37 \times 10^6}}$$
$$= 1.12 \times 10^4\ ms^{-1}$$

The K.E of the spacecraft is given by;

$$K.E = \frac{1}{2}mv_e^2$$
$$= \frac{1}{2} \times 5000 \times (1.12 \times 10^4)^2$$
$$= 3.14 \times 10^{11}\ J$$

Example 12

A spaceship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull? (radius of earth = $6400\ km$)

Solution

Orbital velocity of spaceship in the circular orbit is

$$v = \sqrt{\frac{gR^2}{R+h}}$$

When the satellite orbits close to the earth, $h = 0$

$$v = \sqrt{gR}$$
$$= \sqrt{9.81 \times 6400 \times 1000} = 7.92\ km\ s^{-1}$$

Escape velocity, $v_e = \sqrt{2gR}$

$$= \sqrt{2 \times 9.81\ \times 6400 \times 1000} = 11.20\ km\ s^{-1}$$

Additional velocity required = $11.02 - 7.92$

$$= 3.28\ km\ s^{-1}$$

Example 13

Given that the radius of the moon is $1.7 \times 10^6\ m$ and its mass is $7.35 \times 10^{22}\ kg$. Compute

(i) acceleration due to gravity

(ii) escape velocity

Solution

From $g_m = \frac{GM_m}{R_m^2}$

$$g_m = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{(1.7 \times 10^6)^2}$$
$$g_m = 1.70\ m\ s^{-2}$$

The escape velocity is given by

$$v_e = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{2g_mR_m}$$
$$v_e = \sqrt{2 \times 1.70 \times (1.7 \times 10^6)}$$
$$= 2.4 \times 10^3\ m\ s^{-1} \text{ or } 2.4\ km\ s^{-1}$$

Example 14

A rocket is fired vertically upward with a speed of $5\ km\ s^{-1}$ from earth's surface. How far from the earth does the rocket go before returning to the earth? (mass of earth $= 6 \times 10^{24}\ kg$, radius of earth $= 6.4 \times 10^6\ m$)

Solution

$$v = 5\ km\ s^{-1} = 5000\ m\ s^{-1}$$

Suppose the rocket goes to a height h before returning to the earth. Clearly, at this height, the kinetic energy of the rocket is zero. According to principle of conservation of energy

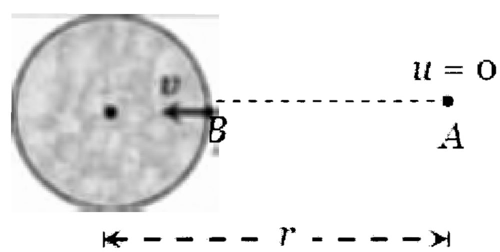
$(K.E + P.E)$ at earth = $(K.E + P.E)$ at height h

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 + \left(-\frac{GMm}{(R+h)}\right)$$
$$v^2 - \frac{2GM}{R} = -\frac{2GM}{R+h}$$
$$\frac{2GM}{R+h} = \frac{2GM}{R} - v^2 = \frac{2GM - v^2R}{R}$$
$$R + h = \frac{2GMR}{2GM - v^2R}$$
$$R + h = \frac{2 \times (6.67 \times 10^{-11})(6 \times 10^{24})(6.4 \times 10^6)}{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24} - (5000)^2 \times (6.4 \times 10^6)}$$
$$= 8 \times 10^6\ m$$
$$h = 8 \times 10^6 - R = 8 \times 10^6 - 6.4\ \times 10^6$$
$$= 1.6 \times 10^6\ m$$

Example 15

A satellite is revolving round the earth with orbital speed v_0. If it stops suddenly, prove that the speed with which it will strike the surface of the earth will be given by the expression $\sqrt{v_e^2 - 2v_0}$ where v_e is the escape speed at the earth's surface

Solution



Applying the law of conservation of energy at A and B

$$-\frac{GMm}{r} = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$
$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{r}$$
$$v^2 = \frac{2GM}{R} - \frac{2GM}{r}$$

Escape speed, $v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 = \frac{2GM}{R}$

Orbital velocity, $v_0 = \sqrt{\frac{GM}{r}} \Rightarrow v_0^2 = \frac{GM}{r}$

$$v^2 = v_e^2 - 2v_0^2$$
$$v = \sqrt{v_e^2 - 2v_0^2}$$

Example 16

A rocket is fired vertically from the surface of Mars with a speed of $2\ km\ s^{-1}$. If 20% of its initial energy is lost due to the Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it? (Mass of Mars $= 6.4 \times 10^{23}\ kg$, radius of Mars $= 3395\ km$)

Solution

Let m be the mass of the rocket

Initial K.E of the rocket

$$= \frac{1}{2}mv^2 = \frac{1}{2}m \times (2 \times 10^3)^2 = 2 \times 10^6 m\ J$$

Since 20% of the energy is lost in Martian atmosphere,

Net K.E of the rocket

$$= \frac{80}{100} \times 2 \times 10^6\ m\ J = 1.6 \times 10^6\ m\ J$$

As the rocket rises, its K.E decreases and P.E increases. Suppose at distance R' from the centre of Mars, K.E of the rocket becomes zero. The change in K.E of the rocket will appear as increase in its P.E

Increase in P.E of rocket

$$= \left(-\frac{GMm}{R'}\right) - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R'}\right)$$

According to the law of conservation of energy,

$$GMm\left(\frac{1}{R} - \frac{1}{R'}\right) = 1.6 \times 10^6 m$$

$$\frac{GM}{R}\left(1 - \frac{R}{R'}\right) = 1.6 \times 10^6$$

$$\frac{R}{R'} = 1 - \frac{R \times 1.6 \times 10^6}{GM}$$

$R = 3395\ km = 3395 \times 10^3\ m, M = 6.4 \times 10^{23} kg$

$$\frac{R}{R'} = 1 - \frac{(3395 \times 10^3) \times (1.6 \times 10^6)}{(6.67 \times 10^{-11}) \times (6.4 \times 10^{23})}$$

$$\frac{R}{R'} = 1 - 0.127 = 0.873$$

$$R' = \frac{R}{0.873} = \frac{3395}{0.873} = 3888.9\ km$$

From $R + h = R', h = R' - R = 3888.9 - 3395$

$$= 493.5\ km$$

Therefore, the rocket will go up to a height of 493.5 km above the surface of Mars

Self-Evaluation exercise

1. The orbit of the moon is approximately a circle of radius 60 times the equatorial radius of the earth. Calculate the time taken for the moon to complete one orbit, neglecting the rotation of the earth. (acceleration of free fall at the poles of the earth $= 9.8\ m\ s^{-2}$, equatorial radius of earth $= 6.4 \times 10^6\ m$. 1 day $= 8.6 \times 10^4\ s$)

   [Ans: 27 days]

2. A satellite of mass 66 kg is in an orbit round the earth at a distance of 5.7 R above its surface where R is the value of the mean radius of the earth. If the gravitational field strength at the earth's surface is $9.8\ N\ kg^{-1}$, calculate the centripetal force acting on the satellite

   Assuming the earth's mean radius to be 6400 km. calculate the period of the satellite in orbit in hours

   [Ans: 14.4 N, 24.5 hrs]

3. A communication satellite is placed in an orbit such that it remains directly above a fixed point on the earth's surface at all times
   (a) What is the period of this satellite?
   (b) Explain why the satellite must be in orbit above the equator
   (c) Show that the correct height for the orbit does not depend upon the mass of the satellite

4. An artificial satellite travels in a circular orbit round the earth. Explain why its speed would have to be greater for an orbit of small radius than for one of large radius

5. (a) State the Kepler law of planetary motion which relates period to orbital radius

   Show that it is consistent with an inverse square law of force between massive bodies

   (b) When a space shuttle is in an orbit at a mean height $0.33 \times 10^6\ m$ above the surface of the earth, it requires 91 minutes to complete one orbit. Obtain a value for the mass of the earth

   (c) Explain why an astronaut inside the shuttle feels weightless even though the intensity of the earth's gravitational field at that height is approximately $9\ N\ kg^{-1}$

   [Ans: $6.0 \times 10^{24}\ kg$]

6. (a) State Newton's law of gravitation and derive the dimensions of the gravitational constant G

   (b) If a planet is assumed to move round the sun in a circular orbit of radius r with periodic time T. derive an expression for T in terms of r and other relevant quantities

(c) Describe the circumstances under which a body can be said to be weightless

7.(a) Explain what is meant by gravitational potential and gravitational potential energy

(b) Use your explanations to show that the difference in potential energy between a point on the earth's surface and one at a height, h above it is, approximately mgh where m is the mass of the body under consideration and g is the gravitational field strength at the earth's surface

(c) Derive the relationship between G, the universal gravitational constant and g, the acceleration of free fall

(d) Draw a graph showing g varies with distance from the earth's centre

(e) Explain why the rotation of the earth about its axis affects the value of g at the equator

8.A communications satellite occupies an orbit such that its period of revolution is $24\ hr$. Explain the significance of this period and show that the radius, R_0 of the orbit is given by

$$R_0 = \sqrt{\frac{GMT^2}{4\pi^2}}$$

M = mass of the earth, T = period of revolution of satellite

Calculate the least kinetic energy which must be given to a mass of $2000\ kg$ at the earth's surface for the mass to reach a point distance R_0 from the centre of the earth.

[Ans: $1.07 \times 10^{11}\ J$]

9.(a) Explain why a force is required for a mass to travel at a constant speed in a circular path. State the direction of this force and give an equation for its magnitude defining all the terms used

(b) State how this force is provided in case of a satellite orbiting the earth

(c) Show that the speed of a satellite in orbit close to earth is given by $(gR)^{\frac{1}{2}}$ where g is the acceleration of free fall and R is the radius of the earth

(d) Calculate the speed of the satellite and the period of the orbit given that $g = 9.8\ \text{m s}^{-2}$ and $R = 6.4\ \times 10^3\ km$

[Ans: (d) $7.9 \times 10^3\ m\,s^{-1}$; $5.1\ \times 10^3\ s$]

10. The mass of the earth is $6.0 \times 10^{24}\ kg$ and that of the moon is $7.4\ \times 10^{22}\ kg$. If the distance between their centres is $3.8 \times 10^8\ m$, calculate at what point on the line joining their centres is no gravitational force. Neglect the effect of other planets and the sun

[Ans: $3.4 \times 10^8\ m$ from earth]

11. The acceleration due to gravity at the Earth's surface is $9.8\ ms^{-2}$. Calculate the acceleration due to gravity on a planet which has

(a) the same mass and twice the radius

(b) the same radius and twice the density

(c) half the radius and twice the density

[Ans: (a) $2.45\ ms^{-2}$ (b) $19.6\ ms^{-2}$ (c) $9.8\ m\,s^{-2}$]

12. The moon has a mass $7.7\ \times 10^{22}\ kg$ and radius $1.7\ \times 10^6\ m$. Calculate

(a) the gravitational potential at its surface

(b) the work needed to remove a $1.5\ \times 10^3\ kg$ space craft from its surface in the outer space. Neglect the effect of other planets

[Ans: (a) $3.0 \times 10^6\ J\ kg^{-1}$ (b) $4.5 \times 10^9\ J$]

13. (a) Define

(i) gravitational potential at a point

(ii) velocity of escape

(b) Use the data below to show that the radius of the orbit of a geostationary satellite is about $4.2\ \times 10^7\ m$.

Mass of the earth = $6.0\ \times 10^{24} kg$

Gravitational constant = $6.7 \times 10^{-11}\ N\,m^2 kg^{-1}$

(c) Show that the speed v of a particle in a circular orbit of radius r around a planet of mass M is given by the expression

$$v = \sqrt{\frac{GM}{r}}$$

(d) The escape speed is independent of the mass of the object being launched. Explain why it is nevertheless desirable to keep the mass of the space probe as small as possible

14. The lowest orbiting satellites have an orbital period of about 90 minutes

(a) Show that the radius at which they orbit the earth is about $6.7 \times 10^6\ m$

(b) Show that the orbital speed is about $7.8 \times 10^3\ m\,s^{-1}$

(c) Show that the kinetic energy of a $1000\ kg$ satellite in this orbit is about $3.0 \times 10^{10}\ J$

15. (a) Satellites used for telecommunications are frequently placed in a geostationary orbit. State two features of the motion of a satellite in a geostationary orbit

(b) The planet Mars has a radius $3.39 \times 10^6\ m$ and mass $6.50 \times 10^{23} kg$. The length of a day on Mars is $8.86\ \times 10^4\ s$

(i) A stellite is to be placed in geostationary orbit about Mars. At what height above the surface of Mars should the satellite be placed? Show clearly how you obtain your answer

(ii) Calculate the acceleration of free fall on the surface of Mars

(c) Mars has two moons, Phobos and Deimos, which move in circular orbits about the planet. The radii of these orbits are $9.38 \times 10^3\ km$ and $23.5 \times 10^3\ km$ respectively. The orbital period of Phobos is 0.319 days. Calculate the orbital period of Deimos.

[Ans: (b)(i) $17.1 \times 10^6\ m$ (ii) $3.77\ m\,s^{-2}$ (c) 1.26 days]

16. The mass of the earth is $5.98\ \times 10^{24}\ kg$ and the gravitational constant is $6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-1}$. Assuming the earth is a uniform sphere of radius $6.37 \times 10^6 m$, find the gravitational force on a mass of $1.0\ kg$ at the earth's surface

[Ans: 9.83 N]

17. State the period of the earth about the sun. Use this value to calculate the angular speed of the earth about the sun in $rad\ s^{-1}$.
The mass of the earth is $5.98 \times 10^{24} kg$ and the average distance from the sun is $1.50 \times 10^{11}\ m$. Calculate the centripetal force acting on the earth. What provides this centripetal force?
[Ans: 365 days; $199 \times 10^{-9}\ rad\ s^{-1}$; $35.5 \times 10^{21}\ N$; gravitational attraction]

18. (a) State Newton's law of gravitation.
(b) If the acceleration of free fall, g_m, at the moon's surface is $1.70\ m\,s^{-2}$ and its radius is $1.74 \times 10^6\ m$, calculate the mass of the moon
(c) To what height would a signal rocket rise on the moon, if an identical one fired on the earth could reach 200 m.
(d) What is meant by weightlessness experienced by an astronaut orbiting the earth and how is it caused?

[Ans: (b) $7.71\ \times 10^{22}\ kg$ (c) $1.15 \times 10^3\ m$]

19. (a) A white dwarf star has a mass of $1.4 \times 10^{30}\ kg$ and radius of $1.2\ \times 10^6\ m$. Show that the gravitational field strength at its surface is $6.5 \times 10^7\ N\ kg^{-1}$.
(b) Calculate the escape speed for the white dwarf star described above

[Ans: (b) $12.5\ \times 10^6\ m\,s^{-1}$]

20. Ganymede orbits Jupiter once every 7.16 days and the radius of its orbit is $1.07\ \times 10^9\ m$. Calculate the mass of Jupiter.

[Ans: $1.89 \times 10^{27}\ kg$]

SIMPLE HARMONIC MOTION (SHM)

Simple harmonic motion is a periodic motion whose acceleration is directly proportional to the displacement from a fixed point, and it is directed towards that point.

Characteristics of SHM

- It is periodic
- Its acceleration is directly proportional to the displacement from a fixed point
- Its acceleration is always directed towards a fixed point in the line of motion.
- Mechanical energy is always conserved

Examples of SHM

- Horizontal and vertical oscillations of a loaded spring.
- Vertical oscillation of liquid in a U–tube
- Oscillations of a floating cylinder
- Oscillations of a simple pendulum
- Vibrations of the prongs of a tuning fork.
- Vibrations of air molecules when sound waves travel through air

Important definitions

Time period, T

It is the time taken for the body to complete one oscillation. S.I. unit is seconds.

Frequency, *f*

It is defined as the number of oscillations executed by the body per second. S.I unit is hertz (Hz)

Amplitude, *a*

This is the maximum displacement of the particle from the equilibrium position

Phase

Phase of a vibrating particle at any instant is a physical quantity, which completely expresses the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion, the phase of a vibrating particle is the argument of *sine* or *cosine* function involved to represent the generalised equation of motion of the vibrating particle.

$$y = a\sin\theta = a\sin(\omega t + \phi_0)$$

where $\theta = \omega t + \phi_0$ = phase of the vibrating particle

(i) **Initial phase/epoch**: It is the phase of the vibrating particle at $t = 0$

In $\theta = \omega t + \phi_0$, when $t = 0$; $\theta = \phi_0$

Here ϕ_0 is the angle of epoch

(ii) **Same phase:** Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π or path difference is an even multiple of $\frac{\lambda}{2}$ or time interval is an even multiple of $\frac{T}{2}$ because 1 time period is equivalent to 2π rad or 1 wavelength (λ)

(iii) **Opposite phase:** When two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two particles is 180°

Opposite phase means the phase difference between the particles is an odd multiple of π (say $\pi, 3\pi, 5\pi, 7\pi$........) or the path difference is an odd multiple of $\frac{\lambda}{2}$ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$, ...) or the time period is an odd multiple of $\frac{T}{2}$

(iv) **Phase difference:** If two particles perform SHM and their equations are

$y_1 = a\sin(\omega t + \phi_1)$ and $y_2 = a\sin(\omega t + \phi_2)$

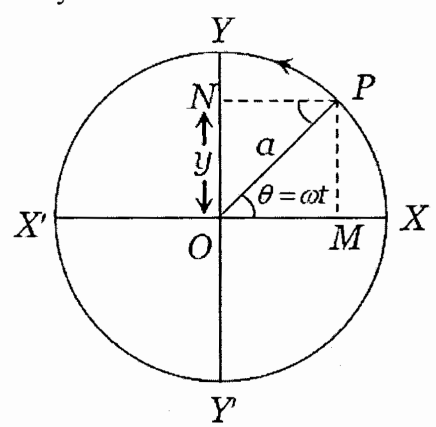
then phase difference,

$$\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$$

Displacement in SHM

The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

Consider the projection of a body P in uniform circular motion on any diameter of circle of reference.



If the projection of P is taken on y-axis, then from the figure

$$y = a\sin\omega t$$

$$y = a\sin\frac{2\pi}{T}t$$

$$y = a\sin 2\pi f t$$

$$y = a\sin(\omega t \pm \phi)$$

where $OY = OY' = a =$ amplitude

If the projection of P is taken on the x-axis, then the equations of SHM can be given as

$$x = a\cos(\omega t \pm \phi)$$

$$x = a\cos\left(\frac{2\pi}{T}t \pm \phi\right)$$

$$x = a\cos(2\pi f \pm \phi)$$

where $OX = OX' = a =$ amplitude

Graphical representation of SHM

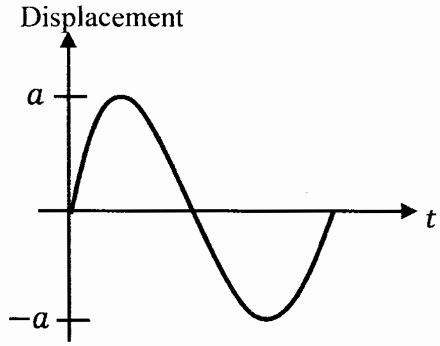
Considering deflection on the y-axis,

$$y = a\sin\frac{2\pi}{T}t$$

at $t = 0$, $y = 0$

at $t = \frac{T}{2}$, $y = a$

at $t = \frac{3T}{2}$, $y = -a$



Velocity in SHM

Velocity of the particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant. When the body is considered from the equilibrium position,

$$y = a\sin\omega t$$

So $v = \frac{dy}{dt} = \frac{d}{dt}a\sin\omega t$

$$v = a\omega\cos\omega t$$

$$v = a\omega\sqrt{1 - \sin^2\omega t}$$

$$\sin\omega t = \frac{y}{a}$$

$$\Rightarrow v = a\omega\sqrt{1 - \frac{y^2}{a^2}}$$

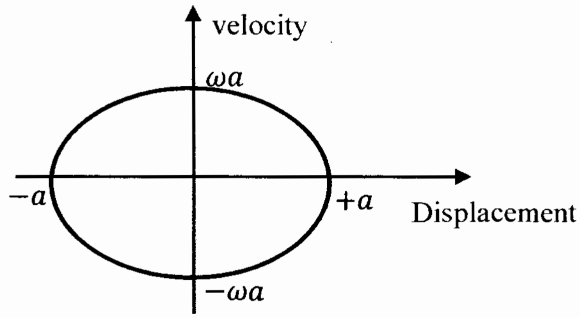
$$v = a\omega\sqrt{\frac{a^2 - y^2}{a^2}}$$

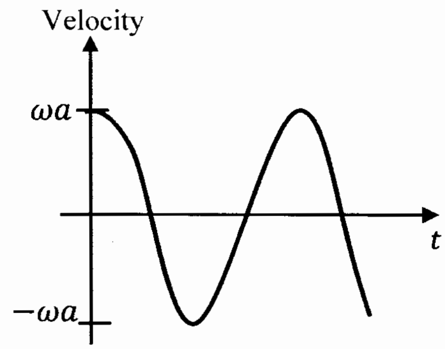
$$\boldsymbol{v = \omega\sqrt{a^2 - y^2}}$$

Graphical representation of velocity in SHM

When $y = 0$, $v = \pm\omega a$

When $y = \pm a$, $v = 0$





Velocity is maximum at the equilibrium position and minimum(zero) at maximum displacement

Acceleration in SHM

The acceleration of the particle executing SHM at any instant is defined as the rate of change of its velocity at that instant.

Acceleration, $A = \frac{dv}{dt}$

From, $v = a\omega\cos\omega t$

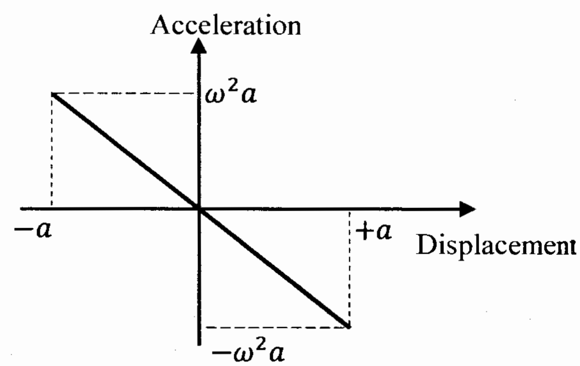
$$\Rightarrow A = \frac{d}{dt}(a\omega\cos\omega t)$$

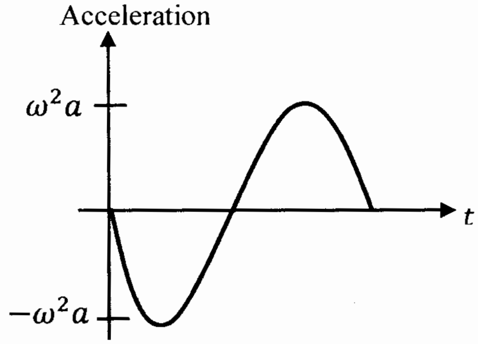
$$A = -\omega^2 a\sin\omega t$$

As $a\sin\omega t = y$,

$$A = -\omega^2 y$$

Graphical representation of acceleration in SHM





Acceleration is maximum at the extreme position i.e. $|A_{max}| = \omega^2 a$ when $\sin\omega t$ = maximum = 1 or $\omega t = \frac{\pi}{2}$

Acceleration is minimum at the mean position i.e.

$$A_{min} = 0 \text{ when } \sin\omega t = 0 \text{ or } \omega t = \pi$$

$$A_{min} = 0 \text{ when } y = 0$$

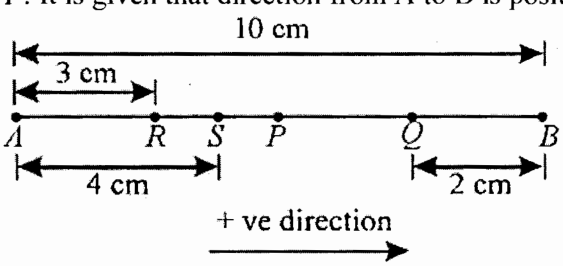
Example 1

A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from B going towards A,
(f) At 4 cm away from A going towards A.

Solution

For a particle executing SHM, the acceleration and force are always directed towards the mean position i.e. P. It is given that direction from A to B is positive



(a) At the extreme end A, velocity is zero. The acceleration and force are directed towards the mean position P i.e. along AP – positive direction. Hence acceleration as well as force is positive
(b) At the extreme end B, velocity is zero. The acceleration and force are directed towards the mean position i.e. along BP – the negative direction. Hence acceleration and force are negative.
(c) The particle is at mean position P and going towards A i.e. it has a tendency to move along PA i.e. negative direction. Hence velocity is negative. Both acceleration and force are zero.
(d) The particle is t point Q. Since the motion is directed along BP (negative direction), velocity, acceleration and force are negative.
(e) The particle is at point R. Since the motion is directed along AP, velocity, acceleration and force are positive
(f) The particle is at point S. The particle is moving along SA, the velocity is negative. However, acceleration and force are directed towards the mean position P i.e. along SP. Hence acceleration and force are positive.

Example 2

A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is

(a) 5 cm (b) 3 cm and (c) 0 cm

Solution

Here a = 5 cm; T = 0.2 s;

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi\, rad\, s^{-1}$$

If a particle executing SHM, has displacement x, then

Acceleration, $A = -\omega^2 x$; Velocity, $v = \omega\sqrt{a^2 - x^2}$

(a) When x = 5 cm = 0.05 m,

$$A = -(10\pi)^2 \times 0.05 = -5\pi^2\, ms^{-2}$$

$$v = 10\pi\sqrt{0.05^2 - 0.05^2} = 0$$

(b) When x = 3 cm = 0.03 m

$$A = -(10\pi)^2 \times 0.03 = -3\pi^2\, ms^{-2}$$

$$v = 10\pi\sqrt{0.05^2 - 0.03^2} = 0.4\pi\, ms^{-1}$$

(c) When $x = 0\, cm$,

$$A = -(10\pi)^2 \times 0 = 0$$

$$v = 10\pi\sqrt{0.05^2 - 0^2} = 0.5\pi\, ms^{-1}$$

Example 3

The equation of a particle executing SHM is given by

$$y = 5\sin\left(\pi t + \frac{\pi}{3}\right) \text{ where } y \text{ in metres}$$

Calculate the
(i) amplitude
(ii) period
(iii) maximum velocity
(iv) velocity after 1 second

Solution

The equation of SHM is $y = a\sin(\omega t + \phi)$

Comparing the equations

(i) Amplitude, $a = 5\,m$

(ii) Period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\,s$

(iii) $v_{max} = a\omega = 5\pi = 15.7\,m\,s^{-1}$

(iv) Velocity, $v = a\omega\cos(\omega t + \phi)$

$$\text{Velocity after } 1\,s = 15.7\cos\left(\pi(1) + \frac{\pi}{3}\right)$$
$$= 7.85\,m\,s^{-1}$$

Example 4

The velocities of a particle executing SHM are $4\,cm\,s^{-1}$ and $3\,cm\,s^{-1}$ when its distance from the mean position is $2\,cm$ and $3\,cm$ respectively. Calculate its amplitude and period

Solution

$v_1 = 4\,cm\,s^{-1} = 0.04\,m\,s^{-1}$, $y_1 = 2\,cm = 0.02\,m$

$v_2 = 2\,cm\,s^{-1} = 0.02\,m\,s^{-1}$, $y_1 = 3\,cm = 0.03\,m$

$$v_1 = \omega\sqrt{a^2 - y_1^2} \quad \text{......... (i)}$$
$$v_2 = \omega\sqrt{a^2 - y_2^2} \quad \text{......... (ii)}$$

Squaring and dividing the equations;

$$\frac{v_1^2}{v_2^2} = \frac{a^2 - y_1^2}{a^2 - y_2^2}$$
$$v_1^2(a^2 - y_2^2) = v_2^2(a^2 - y_1^2)$$
$$v_1^2a^2 - v_1^2y_2^2 = v_2^2a^2 - v_2^2y_1^2$$
$$v_1^2a^2 - v_2^2a^2 = v_1^2y_2^2 - v_2^2y_1^2$$
$$a^2 = \frac{v_1^2y_2^2 - v_2^2y_1^2}{v_1^2 - v_2^2}$$
$$a^2 = \frac{(0.04 \times 0.03)^2 - (0.03 \times 0.02)^2}{0.04^2 - 0.03^2}$$
$$a = \sqrt{1.543 \times 10^{-3}} = 0.03928\,m$$

From (i); $\omega = \frac{v_1^2}{\sqrt{a^2 - y_1^2}} = \frac{0.04^2}{\sqrt{1.543 \times 10^{-3} - 0.02^2}}$

$$\omega = 0.0473\,rad\,s^{-1}$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.0473} = 132.76\,s$$

Dynamics of harmonic oscillations

The oscillations of a physical system results from two basic properties namely elasticity and inertia. Let us consider a body displaced from a mean position. The restoring force brings the body to the mean position.

(i) At extreme position when the displacement is maximum, velocity is zero. The acceleration becomes maximum and directed towards the mean position.

(ii)Under the influence of restoring force, the body comes back to the mean position and overshoots because of negative velocity gained at the mean position.

(iii) When the displacement is negative maximum the velocity becomes zero and the acceleration is maximum in the positive direction. Hence the body moves towards the mean position. Again when the displacement is zero in the mean position velocity becomes positive.

(iv) Due to inertia the body overshoots the mean position once again. This process repeats itself periodically. Hence the system oscillates.

The restoring force is directly proportional to the displacement and directed towards the mean position i.e. $F \propto x$

$$F = -kx$$

Where k is the force constant in $N\,m^{-1}$

From Newton's second law, $F = ma$

$$ma = -kx$$
$$a = -\frac{k}{m}x$$

From definition, $a = -\omega^2 x$

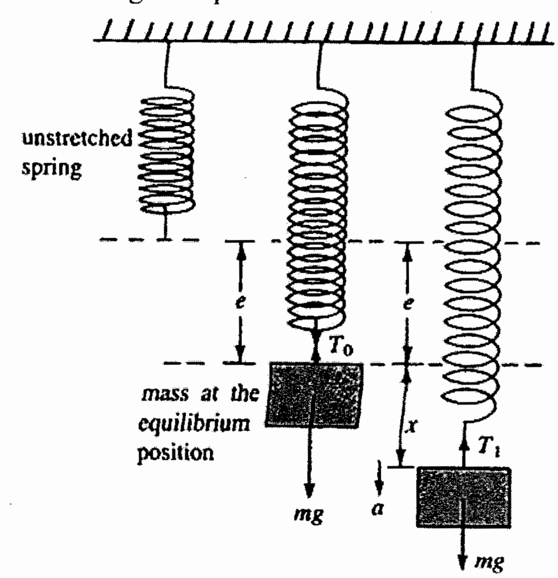
$$\omega = \sqrt{\frac{k}{m}}$$

Period of SHM, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{spring factor}}}$$

Vertical oscillations in a spring

Consider a spring being stretched by an amount e when a mass m hangs in equilibrium from the end of it



When in equilibrium, tension in the spring

$$T_0 = mg$$

If the string is not stretched beyond its elstic limit, it obeys Hooke's law

$$T_0 = ke$$
$$mg = ke \quad \text{........ (i)}$$

where k is the spring constant
The mass m is now pulled a further distance x and then released.
From Newton's second law, $F = ma$

$$mg - T_1 = ma$$

Using Hooke's law, $T_1 = k(x + e)$

$$mg - k(x + e) = ma$$

But from (i), $mg = ke$

$$\text{Thus } ke - k(x + e) = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m}x$$

$$a \propto x$$

Therefore, the mass moves with SHM

$$\omega = \sqrt{\frac{k}{m}}$$

$$\text{Period, } T = 2\pi\sqrt{\frac{m}{k}}$$

since $mg = ke$

$$\frac{m}{k} = \frac{e}{g}$$

$$\text{Thus, period } T = 2\pi\sqrt{\frac{e}{g}}$$

Assumptions made

- Hooke's law is obeyed (the spring is not strectched beyond the elastic limit)
- The mass of the spring is negligible

Note: If the mass of the spring is not to be neglected, then the period

$$T = 2\pi\sqrt{\frac{m + s}{k}}$$

where s is the mass of the spring

Example 5

A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

<u>Solution</u>

The conditions of the problem suggest that when a mass of mass 50 kg is suspended from the spring balance, the extension produced is 20 cm.

Force constant of the spring is

$$k = \frac{F}{y} = \frac{mg}{y} = \frac{50 \times 9.81}{0.2} = 2452.5\ N\ m^{-1}$$

Let M be the mass of the suspended body. The period of oscillations is given by

$$T = 2\pi\sqrt{\frac{M}{k}}$$

$$0.60 = 2\pi\sqrt{\frac{M}{2452.5}}$$

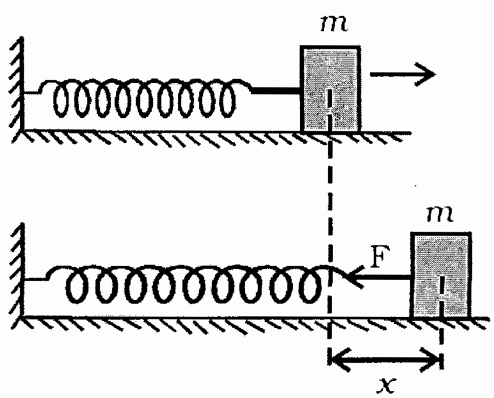
$$M = 22.36\ kg$$

Weight of the body $= Mg$

$$= 22.36 \times 9.81 = 219.4\ \text{N}$$

Horizontal oscillations of a spring

Consider a mass m attached to an end of a spiral spring (which obeys Hooke's law) whose other end is fixed to a support. The body is placed on a smooth horizontal surface.



Let the body be displaced through a distance x towards the right and released. It will oscillate about its mean position. The restoring force acts in the opposite direction and is proportional to the displacement.

$$\text{Restoring force, } F = -kx$$

From Newton's second law, $F = ma$

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

$$a \propto x$$

The body executes SHM

comparing with $a = -\omega^2 x$

$$\omega^2 = \frac{k}{m}, \quad \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Example 6

A mass of 0.2 kg is attached to the lower end of a light helical spring and produces an extension of 5.0 cm. The mass is now pulled down a further distance of 2.0 cm and released. Calculate

(i) the time period of the subsequent oscillations

(ii) the maximum value of the acceleration during this motion

Solution

(i) We assume that the spring obeys Hooke's law

$$F = ke$$

$$0.2 \times 9.81 = k \times 0.05$$

$$k = 39.24\ N\ m^{-1}$$

From period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{0.2}{39.24}} = 0.449\ s$$

(ii) $\omega^2 = \frac{k}{m} = \frac{39.24}{0.2}$

$$a = -\omega^2 a$$

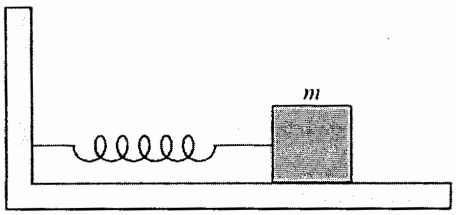
$a = 2\ cm = 0.02\ m$

$$a = -\frac{39.24}{0.2} \times 0.02 = -3.924\ m\ s^{-2}$$

The negative sign indicates the direction

Example 7

A spring having a spring constant $1200\ N\ m^{-1}$ is mounted on a horizontal table as shown below. A mass of $3\ kg$ is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.



Determine the
(i) frequency of oscillations
(ii) maximum acceleration of the mass
(iii) maximum speed of the mass

Solution

$k = 1200\ Nm^{-1}$; $m = 3.0\ kg$; $a = 2.0\ \text{cm} = 0.02\ \text{m}$

(i) The frequency of the mass is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1200}{3.0}} = 3.18\ s^{-1}$$

(ii) Angular velocity, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20\ rad\ s^{-1}$

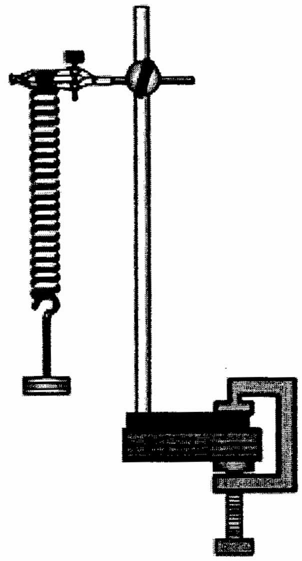
Maximum acceleration of the mass is

$$A_{max} = \omega^2 a = 20^2 \times 0.02 = 8.0\ ms^{-2}$$

(iii) Maximum speed of the mass is

$$V_{max} = \omega a = 20 \times 0.2 = 0.40\ ms^{-1}$$

Measurement of the mass of the spring



The apparatus is setup as shown above

A mass m is hang on the spring and it is displaced slightly downwards and left to oscillate freely

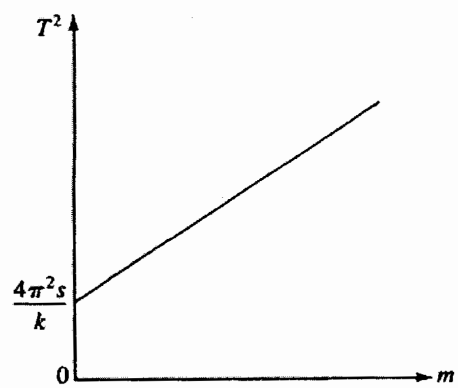
The time t for 20 oscillations is noted using a stop clock

The period, T for one complete oscillation is calculated

The procedure is repeated for different values of m and the corresponding values of T determined

The results are tabulated including values of T^2

A graph of T^2 against m is plotted



From $T = 2\pi\sqrt{\frac{m+s}{k}}$

$$T^2 = \frac{4\pi^2}{k}m + \frac{4\pi^2 s}{k}$$

Gradient of graph $= \frac{4\pi^2}{k} \Rightarrow k = \frac{4\pi^2}{\text{gradient}}$

The mass of s of the spring can thus be obtained from the intercept

$$\text{Intercept} = \frac{4\pi^2 s}{k}$$

$$s = \frac{k \times \text{Intercept}}{4\pi^2} = \frac{4\pi^2}{\text{gradient}} \times \frac{\text{Intercept}}{4\pi^2}$$

$$s = \frac{\text{Intercept}}{\text{gradient}}$$

Measurement of g

Suspend the spring from a retort stand. Attach a pointer to the spring so that it is horizontal and note its position.

A mass m is suspended from the end of the spring and the extension e it produces is noted.

The mass is then slightly pulled down and the periodic time T for the oscillations is noted.

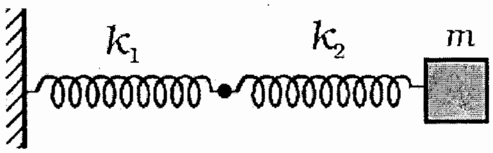
The procedure is repeated for other masses in steps and the results recorded in a table including values of T^2.

A graph of T^2 against e is plotted and its slope s is obtained

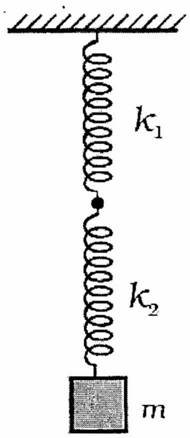
$$\text{The slope, } s = \frac{4\pi^2}{g} \Rightarrow g = \frac{4\pi^2}{s}$$

Mass at the end of two coupled springs

a) Horizontal



b) Vertical



In the above systems, when the combination of the two springs is displaced to a distance y, it produces extensions y_1 and y_2 in the springs of spring constants k_1 and k_2 respectively

The restoring force F is the same for both springs

$$F = -k_1y_1 \Rightarrow y_1 = -\frac{F}{k_1}$$

$$F = -k_2y_2 \Rightarrow y_2 = -\frac{F}{k_2}$$

$$\text{Total extension, } y = y_1 + y_2 = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\text{But } F = -ky \Rightarrow y = -\frac{F}{k}$$

$$-\frac{F}{k} = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$k = \frac{k_1k_2}{k_1 + k_2}$$

$$ma = -\left(\frac{k_1k_2}{k_1 + k_2}\right)y$$

$$a = -\frac{1}{m}\left(\frac{k_1k_2}{k_1 + k_2}\right)y$$

$$\omega^2 = \frac{1}{m}\left(\frac{k_1k_2}{k_1 + k_2}\right)$$

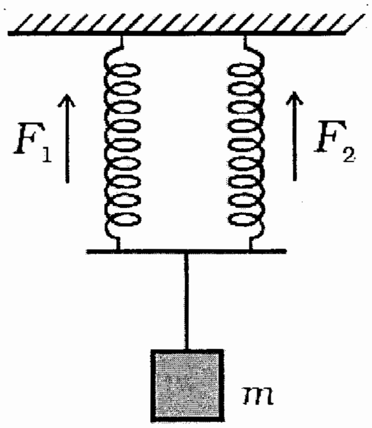
$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$$

If both springs have the same spring constant, i.e. $k_1 = k_2 = k$, then

$$T = 2\pi\sqrt{\frac{2m}{k}}$$

Mass at end of two vertical parallel springs

Two springs of spring factors k_1 and k_2 are suspended from a rigid support as shown below. A load m is attached to the combination



Let the load be pulled downwards through a distance y from its equilibrium position

The extensions in both springs will be y however the restoring forces will be different

$$F_1 = -k_1y \text{ and } F_2 = -k_2y$$

$$\text{Total restoring force, } F = F_1 + F_2$$
$$= -(k_1 + k_2)y$$

From Newton's second law, $F = ma$

$$ma = -(k_1 + k_2)y$$

$$a = -\frac{k_1 + k_2}{m}y$$

$$a \propto y$$

$$\omega^2 = \frac{k_1 + k_2}{m}$$

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

$$\text{Period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

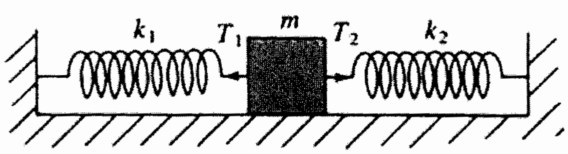
$$\text{Frequency, } f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

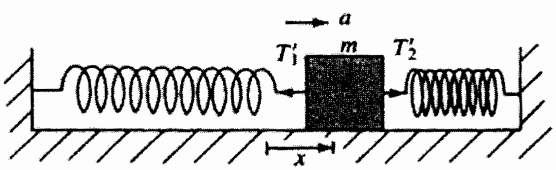
If $k_1 = k_2 = k$, then

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

Mass between two coupled strings on a smooth horizontal surface

Consider a mass m in equilibrium on a smooth table between two springs of force constants k_1 and k_2





$$T_1 = T_2$$

$$k_1e_1 = k_2e_2$$

where e_1, e_2 are the respective extensions in the springs

The mass m is then given a displacement x to the right and then released. It oscillates between the two springs

The net force in the direction of increasing x,

$$F = T_2' - T_1'$$

$$= k_2(e_2 - x) - k_1(e_1 + x)$$

$$= k_2e_2 - k_2x - k_1e_1 - k_1x$$

$$= (k_2e_2 - k_1e_1) - (k_1 + k_2)x$$

Since $k_2e_2 = k_1e_1$,

$$F = -(k_1 + k_2)x$$

From Newton's second law;

$$F = ma$$

$$a = -(k_1 + k_2)x$$

$$a = -\frac{k_1 + k_2}{m}x$$

$$a \propto y$$

$$\omega^2 = \frac{k_1 + k_2}{m}$$

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

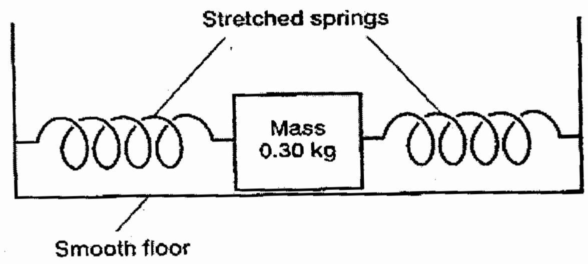
$$\text{Period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

$$\text{Frequency, } f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

If $k_1 = k_2 = k$, then

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

Example 8



A 0.30 kg mass is tethered by two identical springs force constant 2.5 $N\,m^{-1}$. If the mass is now displac by 20 mm to the left of its equilibrium position a then released, calculate the

(i) time period and frequency of subseque oscillations

(ii) acceleration at the centre and extremities of th oscillation

<u>Solution</u>

(i) From, Period, $T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$

$$T = 2\pi\sqrt{\frac{m}{2k}}\ (k_1 = k_2 = k)$$

$$T = 2\pi\sqrt{\frac{0.30}{2(2.5)}} = 1.54\ s$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{1}{1.54} = 0.65\ Hz$$

(ii) Acceleration, $A = -\omega^2 x$

At the centre, $x = 0$

Therefore, $A = 0$

At the extreme positions, $x = \pm a$;

Thus, $A = \pm\omega^2 a$

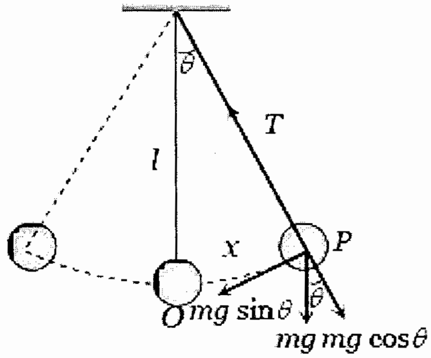
$$\omega^2 = \frac{2k}{m} = \frac{2 \times 2.5}{0.30} = \frac{50}{3}$$

$$a = 20 \times 10^{-3}\ m$$

$$\therefore A = \pm\frac{50}{3} \times 20 \times 10^{-3} = 0.33\ m\,s^{-2}$$

Simple pendulum

A simple pendulum consists of a heavy point mass body suspended by a light, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.



Let mass of the bob = m

Length of simple pendulum = l

Displacement of mass from mean position (OP) = x

When the bob is displaced to position P, through a small angle θ from the vertical. Restoring force acting on the bob

$$F = -mg\sin\theta$$

When θ is small, $\sin\theta \approx \theta = \frac{\text{Arc}}{\text{Length}} = \frac{OP}{l} = \frac{x}{l}$

$$F = -mg\frac{x}{l}$$

From Newton's second law, $F = ma$

$$ma = -mg\,\frac{x}{l}$$

$$a = -\frac{g}{l}\,x$$

$$a \propto x$$

The negative sign indicates that the direction of acceleration is opposite to displacement, Hence the motion of a simple pendulum is simple harmonic

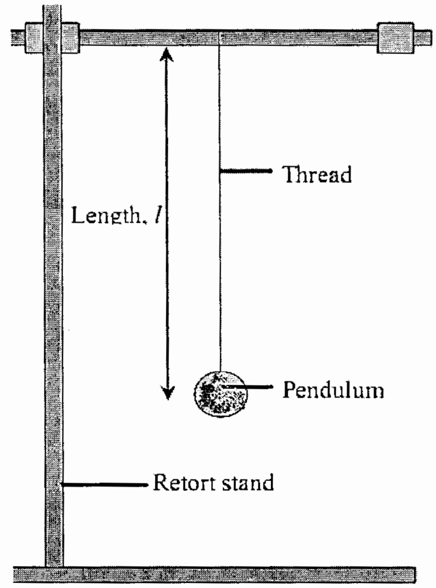
Comparing with $a = -\omega^2 x$

$$\omega^2 = \frac{g}{l}$$

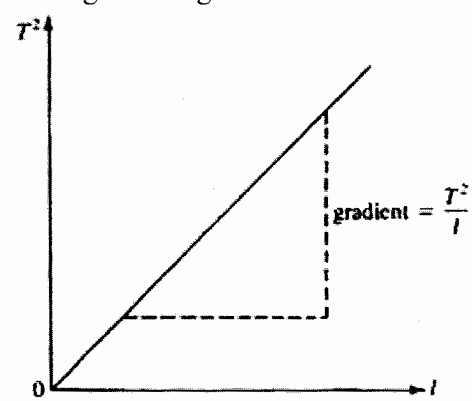
$$\Rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$\text{Period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

Measurement of g



- The apparatus is set up as shown above
- The length l of the string is measured
- The pendulum bob is drawn aside slightly and released to oscillate.
- The time t for 20 oscillations is noted.
- The period T for one oscillation is calculated
- The procedures are repeated using different values of the length of the string, l
- The results are tabulated including values of T^2
- A graph of T^2 against l is plotted and it is a straight line through the origin



From the equation, period $T = 2\pi\sqrt{\frac{l}{g}}$

$$T^2 = 4\pi^2\frac{l}{g}$$

$$g = 4\pi^2\frac{l}{T^2} = \frac{4\pi^2}{\text{gradient}}$$

By calculating the gradient of the graph, the value of g can be obtained

Example 9

Calculate the frequency of oscillation of a simple pendulum of length 80 cm

Solution

From $T = 2\pi\sqrt{\frac{l}{g}}$

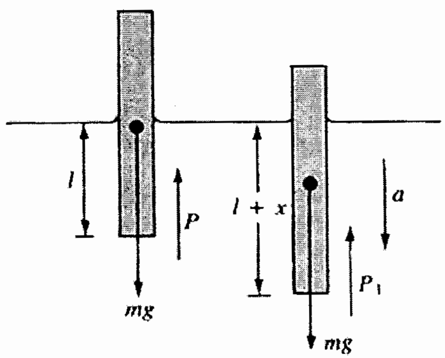
$$T = 2\pi\sqrt{\frac{0.80}{9.81}} = 1.794\ s$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{1}{1.794} = 0.557\ Hz$$

Vertical oscillation of a cylinder

Case 1

Consider a cylinder of uniform cross-sectional area A, floating with a length l immersed in a liquid of density ρ



Since the cylinder is floating in equilibrium,

Upthrust, P = weight of the cylinder

According to Archimedes Principle,

Upthrust = weight of liquid displaced

$= lA\rho g$

$\therefore lA\rho g = mg$ ...... (i)

If the cylinder is depressed into the liquid so that its total length immerse is $l + x$ and then released, the cylinder oscillates

New upthrust, $P_1 = (l + x)A\rho g$

From Newton's second law; $F = ma$

$$mg - P_1 = ma$$

Substituting for mg from (i) and for P_1;

$$lA\rho g - (l + x)A\rho g = ma$$

$$-xA\rho g = ma$$

$$-xA\rho g = (lA\rho)a$$

$$a = -\frac{g}{l}x$$

$$a \propto x$$

Therefore, the motion of the cylinder is simple harmonic

Comparing with $a = -\omega^2 x$

$$\omega^2 = \frac{g}{l}$$

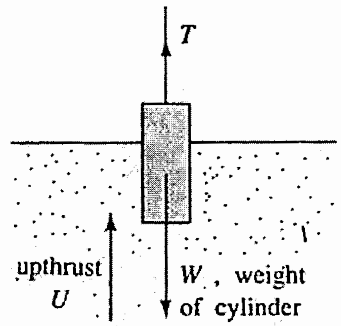
$$\Rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$\text{Period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the cylinder in the liquid at equilibrium

Case 2

Consider a uniform cylinder of length l and mass M having a cross sectional area A suspended with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density ρ at equilibrium position.



Let the cylinder be given a small downward push through a distance x and then released.

The restoring force comes into play and has two components i.e. upthrust, U and spring force, T

Restoring force, $F = -$ (upthrust + spring force)

$$F = -(\rho Axg + kx)$$

But $F = ma$

$$Ma = -(\rho Ag + k)x$$

$$a = -\frac{(\rho Ag + k)}{M}x$$

$$a \propto x$$

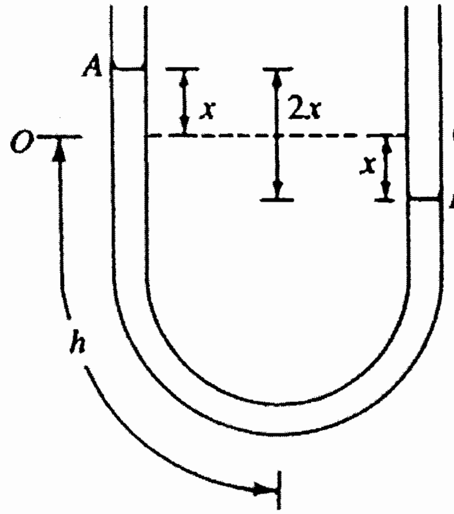
Therefore, the cylinder oscillates with SHM

$$\omega = \sqrt{\frac{\rho Ag + k}{M}}$$

$$\text{Period, } T = 2\pi\sqrt{\frac{M}{\rho Ag + k}}$$

Oscillation of a liquid in a U-tube

The liquid level in one limb of the tube can be depressed slightly by blowing into the limb. The liquid in the tube then oscillates about the equilibrium level OC



Suppose that at any instant, the liquid level at A is at a height x from OC and at B is at a distance x below OC

The excess pressure on the liquid due to the restoring force is

$$\text{Pressure} = 2x\rho g$$

Force on the liquid , $F = \text{Pressure} \times \text{Area}$

$$F = -2x\rho gA$$

The negative sign denotes that the force F is in the direction of decreasing x

Mass of liquid column, $m = \text{volume} \times \text{density}$

$$= 2 \times Ah\rho = 2Ah\rho$$

From, $F = ma$

$$-2x\rho gA = (2Ah\rho)a$$

$$a = -\frac{g}{h}x$$

$$a \propto x$$

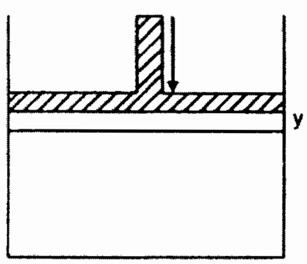
The motion is simple harmonic

$$\omega = \sqrt{\frac{g}{h}}$$

$$\text{Period, } T = 2\pi\sqrt{\frac{h}{g}}$$

Piston in a cylinder

Consider an ideal gas enclosed in a vertical cylindrical container and supports a freely moving (frictionless) piston of mass M. The piston and the cylinder have the same area of cross-section A. The piston is slightly forced downwards, and then released.



Let P and V be respectively the pressure and volume of the gas enclosed in the cylinder-piston system at equilibrium.

Let the piston be depressed through a depth y. This will cause an increase in pressure and a decrease in volume.

Let the new pressure and volume be $P + dP$ and $V - dV$ respectively

Assuming Boyle's law i.e. $PV = \text{constant}$

$$PV = (P + dP)(V - dV)$$

$$PV = PV - PdV + VdP - dPdV$$

Assuming dV and dP are very small, $dPdV \approx 0$

$$\text{Thus, } V\,dP = P\,dV$$

$$dP = \frac{P\,dV}{V}$$

$$\text{But } dV = A \times y$$

$$dP = \frac{PAy}{V}$$

The restoring force, $F = -dP \times A$

$$= -\frac{PA^2y}{V}$$

From $F = ma$,

$$Ma = -\frac{PA^2y}{V}$$

$$a = -\left(\frac{PA^2}{MV}\right)y$$

$$a \propto x$$

Therefore, the piston executes simple harmonic motion

$$\omega = \sqrt{\frac{PA^2}{MV}} = A\sqrt{\frac{P}{MV}}$$

$$\text{Period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{A}\sqrt{\frac{MV}{P}}$$

Note: If the system is completely isolated, we can assume adiabatic conditions ($PV^{\gamma} = \text{constant}$) and differentiate both sides to yield $dP = -\frac{\gamma PA}{V}$ and period $T = \frac{2\pi}{A}\sqrt{\frac{MV}{\gamma P}}$

Energy in SHM

The total energy, E of an oscillating particle is equal to the sum of its kinetic energy and potential energy if conservative forces act on it.

The velocity of a particle executing SHM where its displacement is y from the mean/equilibrium position is $v = \omega\sqrt{a^2 - y^2}$

Kinetic energy

$\text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega(a^2 - y^2)$

Potential energy

From definition of SHM, $F = -ky$. The work done by the force during a small displacement dy is

$$dW = -F\,dy = -(-ky)dy = ky\,dy$$

Total work done for the displacement y,

$$W = \int_0^y ky\,dy$$

$$W = k\left[\frac{y^2}{2}\right]_0^y$$

$$W = \frac{1}{2}ky^2$$

But $k = m\omega^2$

$$W = \frac{1}{2}m\omega^2 y^2$$

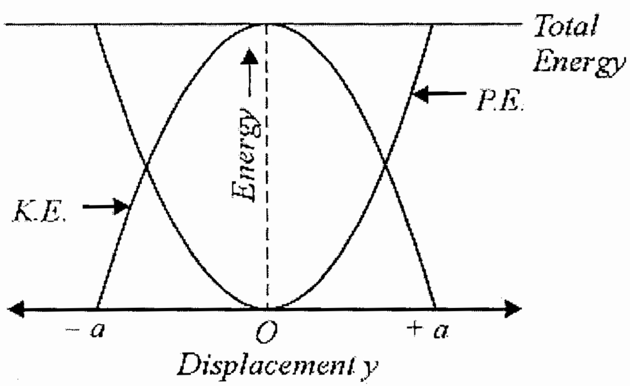
This energy is stored in the body as potential energy

Total energy, $E = KE + PE$

$$= \frac{1}{2}m\omega(a^2 - y^2) + \frac{1}{2}m\omega^2 y^2$$

$$= \frac{1}{2}m\omega^2 a^2$$

Graphical representation of energy in SHM



Note:

When the particle is at the mean position $y = 0$, the kinetic energy is maximum and the potential energy is zero. Hence the total energy is wholly kinetic

$$E = K.E_{max} = \frac{1}{2}m\omega^2 a^2$$

When the particle is at the extreme position $y = +a$, the kinetic energy is zero and the potential energy is maximum. Hence the total energy is wholly potential.

$$E = P.E_{max} = \frac{1}{2}m\omega^2 a^2$$

At any other position, the energy is partly kinetic and partly potential

Variation of p.e and k.e with time

$$K.E = \frac{1}{2}mv^2$$

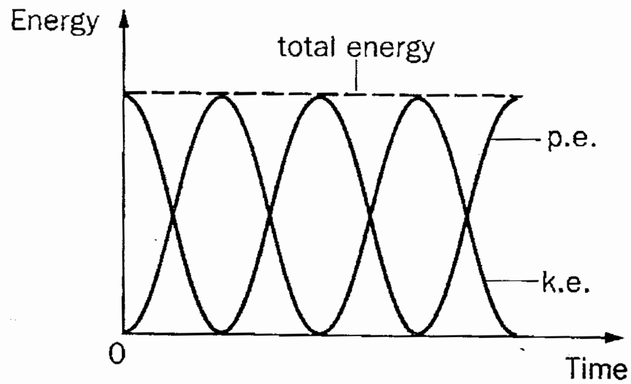
But $v = -\omega r \sin\omega t$

$$K.E = \frac{1}{2}m\omega^2 r^2 \sin^2\omega t$$

$$P.E = \frac{1}{2}\,m\omega^2 y^2$$

But $y = r\cos\omega t$

$$P.E = \frac{1}{2}m\omega^2 r^2 \cos^2\omega t$$



Example 10

A body of mass $0.10\,kg$ oscillates in SHM with an amplitude of $5.0\,cm$ and with a frequency of $0.50\,Hz$. Calculate the

(a) maximum value of its kinetic energy

(b) minimum value of its kinetic energy

State where these occur

Solution

(a) The maximum KE is at the centre of the motion

$$\omega = 2\pi f = 2\pi \times 0.50 = \pi \text{ rad } s^{-1}$$

$$a = 0.05\,m$$

$$m = 0.10\,kg$$

$$K.E = \frac{1}{2}m\omega^2 a^2$$

$$= \frac{1}{2} \times 0.1 \times \pi^2 \times 0.05^2$$

$$= 12 \times 10^{-4}\,J$$

(b) The minimum value of KE is at the extremities of the motion. Since the velocity v is zero, here the KE is zero

Example 11

A point particle of mass 0.1 kg is executing simple harmonic motion of amplitude of $0.1\,m$. When the particle passes through the mean position, its kinetic energy is $8 \times 10^{-3}\,J$. Obtain the equation of motion of this particle if the initial phase is 45°

Solution

Let the equation be $y = a\sin(\omega t + \phi)$

Velocity, $\frac{dy}{dt} = a\omega\cos(\omega t + \phi)$

When the particle passes through the mean position, its velocity is maximum and is equal to ωa

$$KE = \frac{1}{2}m\omega^2 a^2$$

$$\frac{1}{2} \times 0.1 \times \omega^2 \times 0.1^2 = 8 \times 10^{-3}$$

$$\omega = \pm 4$$

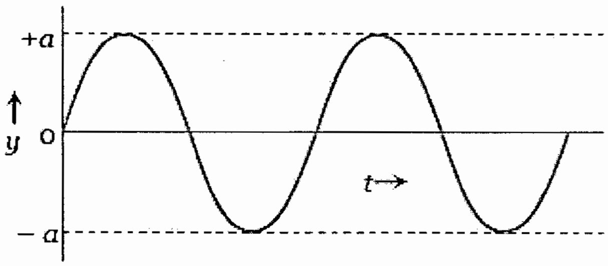
$$\phi = 45° = \frac{45}{180}\pi = \frac{\pi}{4}$$

Required equation is

$$y = 0.1\sin\left(\pm 4t + \frac{\pi}{4}\right)$$

Free, damped and forced oscillations

Free oscillations



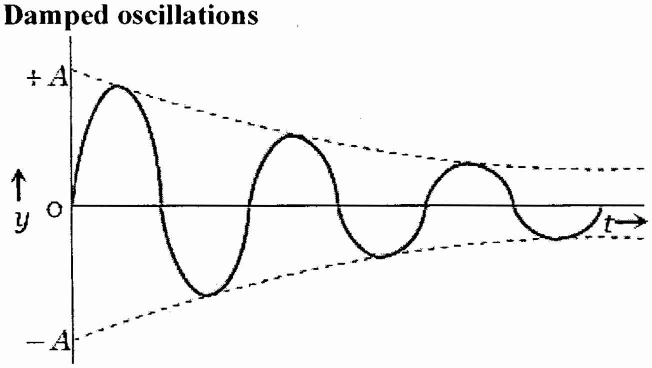
The oscillations of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

The amplitude, frequency and energy of oscillation remains constant

Frequency of free oscillations is called natural frequency because it depends upon the nature and structure of the body.

Examples include: vibrations of a tuning fork, vibrations in a stretched string, etc.

Damped oscillations

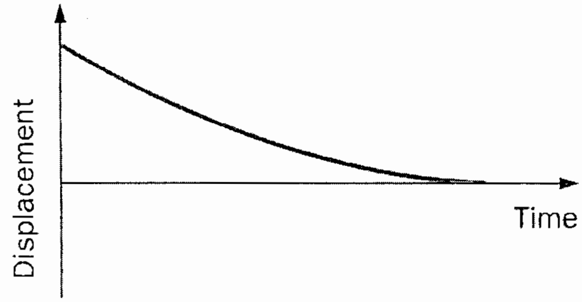


The oscillations of a body whose amplitude goes on decreasing with time are defined as damped oscillation

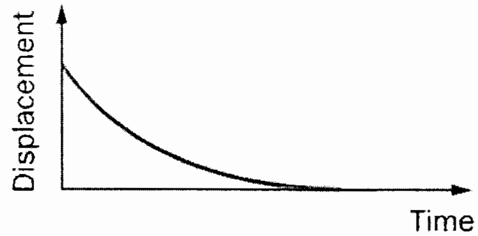
In these oscillations the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hysteresis etc.

Due to decrease in amplitude, the energy of the oscillator also goes on decreasing exponentially

When **heavily damped**, no oscillations occur and the system returns very slowly to its equilibrium position



When the time taken for the displacement to become zero is minimum, the system is said to be **critically damped**.



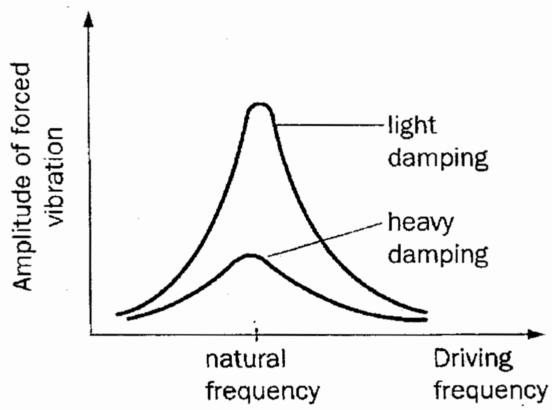
Examples include; oscillations of a pendulum, electromagnetic damping in galvanometer, etc.

Forced oscillations

The oscillations in which a body oscillates under the influence of an external periodic force are known as forced oscillations.

The amplitude of oscillator decreases due to damping forces but on account of the energy gained from the external source it remains constant.

An example is the sound boards of stringed instruments



Resonance

This is a state when the frequency of the external force is equal to the natural frequency of the oscillator.

Applications

1. Shock absorbers in vehicles are designed to introduce damping forces to the oscillation of vehicles.
2. Electrical meters are designed to be critically damped by eddy currents so that the pointer moves quickly to the correct position without oscillation.

Self-Evaluation exercise

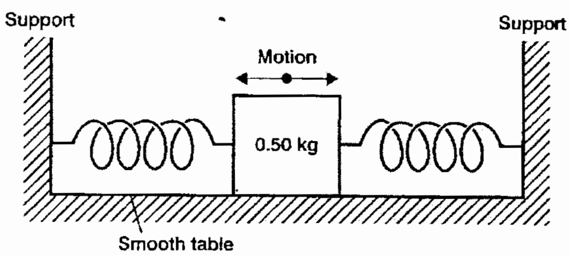
1. (a) Write a short account of simple harmonic motion explaining the terms amplitude, time period and frequency
   (b) A particle of mass m moves such that its displacement from the equilibrium position is given by $y = a\sin\omega t$ where a and ω are constants. Derive an expression for the kinetic energy of the particle at a time t and show that its value is maximum as the particle passes through the equilibrium position
   (c) A steel strip clamped one end vibrates with a frequency of $30\,Hz$ and an amplitude of $4.0\,mm$ at the free end. Find
   (i) the velocity of the free end as it passes through the equilibrium position
   (ii) the acceleration at maximum displacement
   [Ans: (i) $75\,cm\,s^{-1}$ (ii) $1.4\times10^{4}\,cm\,s^{-2}$]
2. A simple pendulum has a period of $2.0\,s$ and oscillates with an amplitude of $10\,cm$.
   (a) What is the frequency of the oscillations?
   (b) At what points of the swing is the speed of the bob maximum? Calculate this maximum speed
   (c) At what points of swing is the acceleration of the pendulum bob a maximum? Calculate this acceleration
   [Ans: (a) $0.5\,Hz$ (b) $0.314\,m\,s^{-1}$ (c) $0.99\,ms^{-2}$]
3. A body of mass 200 g is executing SHM with an amplitude of 20 mm. The maximum force which acts on it is $0.064\,N$. Calculate
   (i) its maximum velocity
   (ii) its period of oscillation
   [Ans: (i) $0.08\,m\,s^{-1}$ (ii) $\frac{\pi}{2}\,s$]
4. A body of mass $0.30\,kg$ executes SHM with a period of $2.5\,s$ and an amplitude of $4.0\times10^{-2}\,m$. Determine
   (i) the maximum velocity of the body
   (ii) the maximum acceleration of the body
   (iii) the energy associated with the body
   [Ans: (i)$0.10\,m\,s^{-1}$ (ii) $0.25\,m\,s^{-2}$ (iii)$1.5\times10^{-3}\,J$]
5. A body is moving with simple harmonic motion and has a velocity v and acceleration a from its mean displacement x. Sketch graphs of a against x, v against x
6. The displacement x, in m, from the equilibrium position of a particle moving with SHM is given by

$$x = 0.05\sin 6t$$

   where t is the time in s, measured from an instant when $x = 0$
   (i) State the amplitude of the oscillation
   (ii) Calculate the time period of the oscillation and the maximum acceleration of the particle
   [Ans: (i) $0.05\,m$ (ii) $1.0\,s$; $1.8\,m\,s^{-2}$]
7. A mass hanging from a spring suspended vertically is displaced by a small amount and released. By considering the forces on the mass at the instant when the mass is released, show that the motion is simple harmonic and derive an expression for the time period. Assume that the spring obeys Hooke's law
8. A mass m hangs on a string of length l from a rigid support. The mass is pulled aside, so that the string makes an angle θ to the vertical and then released
   (i) Show that the mass executes SHM, stating any assumptions made
   (ii) Prove that the period T of this SHM is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

9. A mass hangs from a light spring. The mass is pulled down $30\,mm$ from its equilibrium position and then released from rest. The frequency of the oscillation is $0.50\,Hz$. Calculate
   (i) the angular frequency ω of the oscillation
   (ii) the magnitude of the acceleration at the instant it is released from rest
   [Ans: (i) $3.1\,rad\,s^{-1}$ (ii) $0.30\,m\,s^{-2}$]
10. The figure below shows a mass of $0.50\,kg$ which is in contact with a smooth horizontal table. It is attached to two light springs to two fixed supports as shown.



   If the mass moves in linear simple harmonic motion with a period of $2.0\,s$ and an amplitude of $4.0\,cm$, calculate the energy associated with this motion
   [Ans: $3.9\times10^{-3}\,J$]

Examination questions

1. (a) (i) State **Newton's laws of motion**
   (ii) A molecule of gas contained in a cube of side l strikes the wall of the cube repeatedly with a velocity u. Show that the average force F on the wall is given by
   (b) (i) Define **linear momentum** and state the **law of conservation of linear momentum.**
   (ii) A body of mass m_1 moving with another body of mass m_2 at rest. If they stick together after collision, find the common velocity with which they will move.
   (c) A bullet of mass 10 g is fired horizontally with a velocity of 300 ms^{-1} into a block of wood of mass 290 g which rests on a rough horizontal floor. After impact, the block and bullet move together and come to rest when the block has travelled a distance of 15 m. Calculate the coefficient of sliding friction between the block and the floor.

[2017, No. 1]

2. (a) State **Kepler's laws of planetary motion**
   (a) Use Newton's law of gravitation to derive the dimension of the universal gravitational constant
   (b) A satellite is revolving at a height h above the surface of the earth with a period, T.
   (i) Show that the acceleration due to gravity g on the earth's surface is given by

$$g = \frac{4\pi^2(r_e + h)^3}{T^2 r_e^2}$$

   where r_e is the radius of the earth
   (ii) What is meant by **parking orbit**?
   (c) A satellite revolves in a circular orbit at a height of 600 km above the earth's surface. Calculate the
   (i) speed of the satellite
   (ii) periodic time of the satellite

[2017, No. 2]

3. (a) (i) Define **simple harmonic motion**
   (ii) Sketch a displacement-time graph for a body performing simple harmonic motion.
   (b) A uniform cylindrical rod of length 16 cm and density 920 kgm^{-3} floats vertically in a liquid of density 1000 kgm^{-3}. The rod is depressed through a distance of 7 mm and then released.
   (i) Show that the rod performs simple harmonic motion
   (ii) Find the frequency of the resultant oscillations
   (iii) Find the velocity of the rod when it is at a distance of 5 mm above the equilibrium position.
   (c) What is meant by **potential energy**?
   (d) Describe the energy changes which occur when
   (i) ball is thrown upwards in air.
   (ii) loud speaker is vibrating

[2017, No. 3]

4. (a) (i) Define **elastic deformation** and **plastic deformation**
   (ii) Explain what is meant by **work hardening**
   (b) (i) Sketch using the same axes, stress-strain curves for a ductile material and for rubber.
   (ii) Explain the features of the curve for rubber
   (c) A capillary tube is held in a vertical position with one end dipping in a liquid of surface tension γ and density ρ. If the liquid rises to a height, h, derive an expression for h in terms of γ, ρ and radius r of the tube assuming the angle of contact is zero.
   (d) A mercury drop of radius 2.0 mm falls vertically and on hitting the ground, it splits into two drops each of radius 0.5 mm. Calculate the change in surface energy given that surface tension of mercury is 0.52 Nm^{-1}.
   (e) State the effect of temperature on surface tension of a liquid.

[2017, No. 4]

5. (a)(i) Define dimensions of a physical quantity.
   (ii) In the gas equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

   where p = pressure, V = volume, T = absolute temperature and R = gas constant, what are the dimensions of the constants a and b?
   (b) A particle is projected from a point on a horizontal plane with a velocity, u, at an angle, θ, above the horizontal. Show that the maximum horizontal range R_{max} is given by

$$R_{max} = \frac{u^2}{g}$$

   where, g, is acceleration due to gravity.
   (c) (i) Define **elastic limit** of a material.
   (ii) Describe an experiment to determine Young's Modulus of a steel wire.
   (d) Explain why tyres of a vehicle travelling on a hard surfaced road may burst.

[2016, No. 1]

6. (a) (i) What is meant by efficiency of a machine?
   (ii) A car of mass 1.2×10^3 kg moves up an incline at a steady velocity of 15 ms^{-1} against a frictional force of $6.0 \times 10^3\ N$. The incline is such that car rises 1.0 m for every 10 m along the incline. Calculate the output power of the car engine.
   (b) (i) Define the **impulse** and **momentum**
   (ii) An engine pumps water such that the velocity of the water leaving the nozzle is 15 ms^{-1}. If the water jet is directed perpendicularly onto a wall and comes to a stop at the wall, calculate the pressure exerted on the wall.
   (c) (i) Define **inertia**
   (ii) Explain why a body placed on a rough plane will slide when the angle of inclination is increased.
   (d) (i) State the conditions for a body to be in equilibrium under action of coplanar forces.
   (ii) Briefly explain the **three** states of equilibrium.

[2016, No. 2; Ans: (a)(ii) $1.08 \times 10^5\ W$ (b)(ii) $2.25 \times 10^5 Nm^{-2}$]

7. (a) (i) What is meant by a **conservative force**?
   (ii) Give **two** examples of conservative force.
   (b) Explain the following;
   (i) Damped oscillations.
   (ii) Forced oscillations.
   (c) (i) State **Newton's law of gravitation**.
   (ii) Show that Newton's law of gravitation is consistent with Kepler's third law.
   (d) If the earth takes 365 days to make one revolution around the sun, calculate the mass of the sun.
   (e) Explain briefly how satellites are used in world-wide radio or television communication.

[2016, No. 3; Ans: (d) $2.0 \times 10^{30}\ kg$]

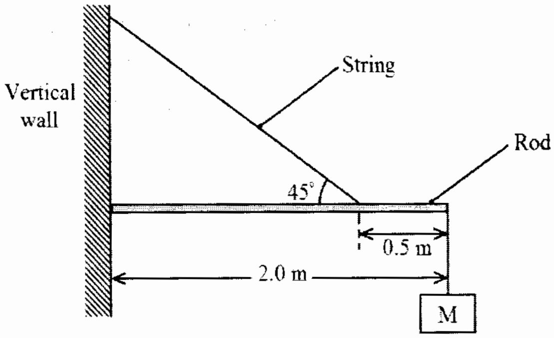
8. (a) (i) What is meant by **fluid element** and a **flow line** as applied of fluid flow?
   (ii) Explain why some fluids flow more easily than others.
   (b) (i) State **Bernoulli's Principle**.
   (ii) Explain how a Pitot-static tube works.
   (c) Air flowing over the surface of an air craft's wings causes a lift force of 6.4×10^3 N. The air flows under the wings at a speed of $120\ ms^{-1}$ over an area of $28\ m^2$. Find the speed of air flow over an equal area of the upper surface of the air craft's wings. (Assume density of air = 1.2 kgm^{-3}).
   (d) (i) What is meant by surface tension and angle of contact of a liquid?
   (ii) A water drop of radius 0.5 cm is broken up into other drops of water each of radius 1 mm. Assuming isothermal conditions, find the total work done to break up the water drop.

[2016, No. 4; (c) $121.6\ ms^{-1}$ (d)(ii) $8.8 \times 10^{-5}\ J$]

9. (a)(i) What is meant by a conservative force?
   (ii) Give two examples of a conservative force
   (b) (i) State the law of conservation of mechanical energy
   (ii) A body of mass, m, is projected vertically upwards with speed, u show that the law of conservation of mechanical energy is obeyed throughout its motion.
   (iii) Sketch a graph showing the variation of kinetic energy of the body with time.
   (c) (i) Describe an experiment to measure the coefficient of static friction.
   (ii) State two disadvantages of friction
   (d) A bullet of mass 20 g moving horizontally strike and gets embedded in a wooden block of mass 500 g resting on a horizontal table. The block slides through a distance of 2.3 m before coming to rest. If the coefficient of kinetic friction between the block and the table is 0.3 calculate the
   (i) friction force between the block and the table
   (ii) velocity of the bullet just before it strikes the block

[2015, No. 1; Ans: (d)(i) $1.53\ N$ (ii) $95.69\ ms^{-1}$]

10. (a)(i) State the **principle of moments**
   (ii) Define the terms **centre of gravity** and **uniform body**
   (b) The figure below shows a body, M of mass 20 kg supported by a rod of negligible mass horizontally hinged to a vertical wall supported by a string fixed 0.5 m from the other end of the rod



Calculate the
- (i) tension in the string
- (ii) reaction of the hinge
- (iii) maximum additional mass which can be added to the mass of 20 kg before the string can break given that the string cannot support a tension of more than 500 N

(c) (i) Define Young's modulus
(ii) Explain the precautions taken in the determination of Young's modulus of a wire.
(iii) Explain why a piece of rubber stretches much more than be metal wire of the same length and cross sectional area.

[2015, No. 2; Ans: (b)(i) 370 N (ii) 270 N (iii) 7 kg]

11. (a) State Kepler's laws of planetary motion
(b) (i) What is a parking orbit?
(ii) Derive an expression for the period T of a satellite in circular orbit of radius r, above the earth in terms of the mass of the earth m, gravitational constant G and r.
(c) (i) A satellite of mass 200 kg is launched in a circular orbit at a height of $3.59 \times 10^7\ m$ above the earth's surface. Find the mechanical energy of the satellite
(ii) Explain what will happen to the satellite if its mechanical energy was reduced
(d) Describe a laboratory method for determining the universal gravitational constant, G

[2015, No. 3; Ans: (c)(ii) $-9.41 \times 10^8\ J$]

12. (a) (i) Distinguish between surface tension and surface energy
(ii) Show that the surface energy and surface tension are numerically equal.
(a) Explain why water dripping out of a tap does so in spherical shapes.
(b) Two soap bubbles of radii 2.0 cm and 4.0 cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5 \times 10^{-2}\ Nm^{-1}$, calculate the excess pressure inside the resulting soap bubble.
(c) (i) State Bernoulli's principle
(ii) Explain how and at a high speed over the roof of a building can cause the roof to be ripped off the building
(d) An aeroplane has a mass of 8,000 kg and total using area of 8.0 m^2 . When moving through still air, the ratio of its velocity to that of the air at its lower surface is 10, whereas the ratio of its velocity to that of the air above its wings is 0.25. At what velocity will the aeroplane be able to just lift off the ground? (density of air = 1.3 $kg\,m^{-3}$

[2015, No. 4; Ans: (b) $2.24\ Pa$ (c)(iii) $31.72\ ms^{-1}$]

13. (a) (i) What is a projection motion?
(ii) A bomb is dropped from an aeroplane when it is directly above a target at a height of 1402.5 m, the aeroplane is moving horizontally with a speed of $500\ kmh^{-1}$. Determine whether the bomb will hit the target
(b) (i) Define angular velocity
(ii) A satellite is revolving around the earth in a circular orbit at an altitude of $6 \times 10^5\ m$ where the acceleration due to gravity is $9.4\ ms^{-2}$. Assuming that the earth is spherical, calculate the period of the satellite.
(c) (i) State Newton's laws of motion
(ii) Explain how a rocket is kept in motion
(iii) Explain why passengers in a bus are thrown backwards when the bus suddenly starts moving.

[2014, No. 1, Ans: (a) (ii) no (b) (ii) $5.419 \times 10^3\ s$]

14. (a) (i) What is meant by Young's modulus?
(ii) State Hooke's law
(iii) Derive an expression for the energy released in a unit volume of a stretched wire in terms of stress and strain
(b) A steel wire of length 0.6 m and cross sectional area $1.5 \times 10^{-6}\ m$ is attached at B to a copper wire BC of length $0.39\ m$ and cross sectional area $3.0 \times 10^{-6}\ m^2$. The combination is suspended vertically from a fixed point at A and supports a weight of $250\ N$ at C. Find the extension in each of the wires given that Young's modulus for steel is $2.0 \times 10^{11}\ Nm^{-2}$ and that for copper is $1.3 \times 10^{11}\ Nm^{-2}$

(c) With the aid of a labeled diagram, describe an experiment to determine the Young's modulus of a steel wire.

(d) Explain the term plastic deformation in metals.

[2014, No. 2; Ans; (b) $2.5 \times 10^{-4}\ m$]

15. (a) Define work and energy

(b) Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road

(c) A pump discharges water through a nozzle of diameter 4.5 cm with a speed of 62 ms^{-1} into a tank above the intake .

(i) Calculate the work done per second by the pump in raising the water if the pump is ideal.

(ii) Find the power wasted if the efficiency of the pump is 73%

(iii) Account for the power lost in (c)(ii)

(d) (i) State the work energy theorem for a body moving with constant acceleration

(ii) Prove the work energy theorem for a body moving with constant acceleration.

(e) Explain briefly what is meant by internal energy of a substance.

[2014, No. 3, Ans: (b)(i) $2.05 \times 10^5\ J$ (ii) 75.8 kW]

16. (a) Define coefficient of viscosity and state its units

(b) Explain the origin of viscosity in air and account for the effect of temperature on it.

(c) Describe stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stokes' law

(d) A steel ball bearing of diameter 80 mm falls steadily through oil and covers a vertical height of 20.0 cm in 0.50 s. if the density of steel is 7800 kgm^{-3} and that of oil is 900 kgm^{-3}, calculate the

(i) up thrust on the ball

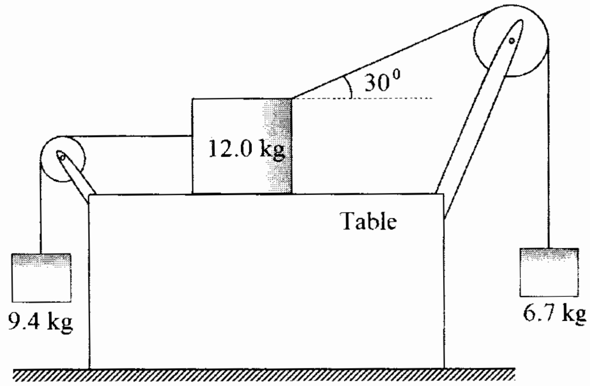
(ii) viscosity of the oil

[2014, No. 4, Ans: (d)(i) $2.37 \times 10^{-3}\ N$ N (ii) $0.674\ Nsm^{-2}$]

17. (a) Using the molecular theory, explain the laws of friction between solid surfaces

(b) With the aid of a labelled diagram, describe how the coefficient of static friction for an interface between a rectangular block of wood and a plane surface can be determined

(c) The diagram below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 12.0 kg is 0.25.



If the system is released from rest, determine the

(i) acceleration of the 12.0 kg mass

(ii) tension in the string

[2013, No. 1, Ans: (b)(i) $0.537\ ms^{-2}$(ii)$69.3\ N$, $87.2\ N$]

18. (a) Define **terminal velocity**

(b) Explain **laminar** and **turbulent** flow.

(c) Explain an experiment to measure the coefficient of velocity of water using Poisuelle's formula.

(d) (i) State **Bernoulli's principle**

(ii) Explain why a person standing near a railway line is sucked towards the railway line when a fast moving train passes.

(d) A horizontal pipe of cross sectional area 0.4 m^2, tapers to a cross sectional area of 0.2 m^2. The pressure at the large section of the pipe is $8.0 \times 10^4\ Nm^{-2}$ and the velocity of water through the pipe is 1.2 ms^{-1}. If the atmospheric pressure is $1.01 \times 10^5\ Nm^{-2}$, find the pressure at the small section of the pipe.

[2013, No. 2 Ans: $7.784 \times 10^4\ Nm^{-2}$]

19. (a)(i) State the law of conservation of linear momentum

(ii) A body explodes and produces two fragments of masses m and M. if the velocities of the fragments are u and v respectively, show that the ratio of the kinetic energies of the fragments is $\frac{E_1}{E_2} = \frac{M}{m}$ where E_1 is the kinetic energy of m and E_2 is the kinetic energy of M

(b) Show that the centripetal acceleration of an object moving with constant velocity, v in a circle of radius r, is $\frac{v^2}{r}$

(c) A car of mass 1000 kg moves round a banked track at a constant speed of 108 kmh^{-1}.

Assuming the total reaction at the wheels is normal to the track and the radius of curvature of the track is 100 m, calculate the

(i) angle of inclination of the track to the horizontal

(ii) reaction of the wheels

(d) (i) Define uniformly accelerated motion

(i) A train starts from rest at station A and accelerates at $1.25\, ms^{-2}$ until it reaches a speed of $20\, ms^{-1}$. It then travels at this steady speed for a distance of 1.56 km and then decelerates at $2\, ms^{-2}$ to come to rest at station B. Find the distance from A to B

[2013 No. 3, Ans: (i) 42.5 (ii) $1.33 \times 10^4\, N$(d) (ii) $1820\, m$]

20. (a) (i) State Kepler's laws of planetary motion

(ii) Estimate the mass of the sun of the orbit of the earth round the sun is circular.

(b) Explain Brownian motion.

(c) Explain the energy changes which occur when a pendulum is set into motion.

(d) A simple pendulum of length 1cm has a bob of mass 100 g. It is displaced from its mean position A to position B so that the string makes an angle of 45° with the vertical. Calculate the

(i) maximum potential energy of the bob

(ii) velocity of the bob when the string makes an angle of 30° with the vertical (neglect air resistance)

[2013, No. 4, Ans: (d) $0.287\, J$(ii) $1.766\, ms^{-1}$]

21. (a) State Hooke's law

(b) A copper wire is stretched until it breaks

(i) Sketch a stress-strain graph for the wire and explain the main features of the graph

(ii) Explain what happens to the energy used to stretch the wire at each stage.

(iii) Derive the expression for the work done to stretch a spring of force constant, k by distance, e

(c) (i) Define Young's modulus.

(ii) Two identical steel bars A and B of radius 2.0 mm are suspended from the coiling. A mass of 2.0 kg is attached to the free end of the bas A. Calculate the temperature to which B should be raised so that the bars are of equal length.

(Young's modulus of steel = $1.0 \times 10^{11}\, Nm^{-2}$. Linear expansivity of steel = $1.2 \times 10^{-5}\, K^{-1}$)

(d) Why does an iron roof make cracking sound at night?

[2012, No. 1; Ans: (c)(ii) 1.3 K]

22. (a) Define the following terms as applied to oscillatory motion

(i) amplitude

(ii) period

(b) State four characteristics of simple harmonic motion

(c) A mass, m is suspended from a rigid support by a string of length, L. the mass is pulled aside so that the string makes an angle θ with the vertical and then released.

(i) Show that the mass executes simple harmonic motion with a period,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(ii) Explain why this mass comes to stop after a short time.

(d) A piston in a car engine performs a simple harmonic motion of frequency 12.5 Hz. If the mass of the piston is 0.5 kg and its amplitude of vibration is 45 mm, find the maximum fore on the piston.

(e) Describe an experiment to determine the acceleration due to gravity, of using a spiral spring, of known force constant.

[2012, No. 2; Ans: (d) 138.79 N]

23. (a) What is meant by centripetal force?

(b) (i) Derive an expression for the centripetal force acting on a body of mass, m moving in a circular path of radius r.

(ii) A body moving in a circular path of radius $0.5\, m$ makes 40 revolutions per second. Find the centripetal force if the mass is $1\, kg$.

(c) Explain the following

(i) A mass attached to a string rotating at constant speed in a horizontal circle will fly off at a tangent if the string breaks.

(ii) A cosmonaut in a satellite which is in a free circular orbit around the earth experiences the sensation of weightlessness even though there is influence of gravitational field of earth.

(d) (i) Derive an expression for the maximum horizontal distance travelled by a projectile in terms of its initial speed, u and the angle of projection, θ to the horizontal.

(ii) Sketch a graph to show the relationship between kinetic energy and the height above the ground in a projectile.

[2012, No.3 (b)(ii) 3.158×10^4 *N*]

24. (a)(i) What is meant by the terms steady flow and viscosity?

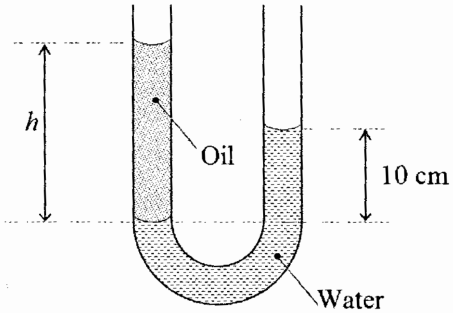
(b) Explain the effect of increase in temperature on the viscosity of a liquid

(c) (i) Show that the pressure, P, exested at a depth, h, below the free surface of a liguid of density, f, is given by;

$$P = h\rho g$$

(ii) Define relative density

(iii) A U-tube whose ends are open to the atmosphere, contains water and oil as shown below



Given that density if oil is 800 kgm^{-3}, find the value of h

(d) A metal ball of diameter 10 mm is timed as it falls through oil at a steady speed. It takes 0.5 s to fall through a vertical distance of 0.30 m. Assuming that the density of the metal is 750 kgm^{-3}, find

(i) the weight of the ball

(ii) the up thrust on the ball

(iii) the coefficient of viscosity of oil

(Assume the viscous force = $6\pi\mu v_0$ where μ is the coefficient viscosity, r is the radius of the ball an v_0 is the terminal velocity).

[2012, No. 4, Ans: (b) 12.5 cm (c)(i) 0.0385 N (i) 0.00462 N (iii) 0.599 Nsm^{-2}]

25. (a) Define the following terms.

(i) uniform acceleration

(ii) angular velocity

(b) (i) What is meant by banking of a track?

(ii) Derive an expression for the angle of banking θ for a car of mass, m, moving at aspeed, v around a banked track of radius, r

(c) A bob of mass, m is is tied to an inelastic thread of length, l and whirled with constant speed in a vertical circle.

(i) With the aid of a sketch diagram, explain the variation of tension in the string along the circle.

(ii) If the string breaks at one point along the circle, state the most likely position and explain the subsequent motion of the bob.

(d) A body of mass 15 kg is moved from the earth's surface to a point $1.8 \times 10^6\ m$ above the earth. If the radius of the earth is $1.8 \times 10^6\ m$ and its mass is $6.0 \times 10^{24}\ kg$, calculate the work done in taking the body to that point.

[2011, No. 1, Ans: (d) $2.06 \times 10^8 J$]

26. (a) State Newton's laws of motion

(b) Use Newton's laws of motion to show that when two bodies collide, their momentum is conserved

(c) Two balls P and Q travelling in the same line inapposite directions with speeds $6\ ms^{-1}$ and $15\ ms^{-1}$ respectively make a perfect inelastic collisions. If the masses of P and Q are 8 kg and 5 kg respectively, find the

(i) final velocity of P

(ii) change in kinetic energy

(d) (i) What is an impulse of force?

(ii) Explain why a long jumper should normally land on sand.

[2011, No. 2, Ans: (c) (i) $2.08\ ms^{-1}$ (ii) $678.38 J$]

27. (a) (i) What is meant by viscosity?

(ii) Explain the effect of temperature on the viscosity of liquid.

(b) Derive an expression for the terminal velocity of a sphere of radius, a, falling in a liquid of viscosity, η

(c) Explain why velocity of a liquid at a wide part of a tube is less than that at a narrow part.

(d) A solid weighs 237.5 g in air and 12.5 g when totally immersed in a fluid of density 9.0×10^2 kgm^{-3}, calculate the density of the liquid in which the solid would float with one fifth of its volume exposed above the liquid surface.

(e) Describe an experiment to measure the coefficient of static friction between a rectangular block of wood and a plane surface.

[2011, No. 3; Ans: (d) $1.188 \times 10^3 kg\ m^{-3}$]

28. (a) (i) What is meant by simple harmonic motion?

(ii) State two practical examples of simple harmonic motion

(iii) Using graphical illustration, distinguish between under damped and critically damped oscillations.

(b) (i) Describe an experiment to measure acceleration due to gravity using a spiral spring.

(ii) State two limitations of the value obtained in (b)(i)

(c) A horizontal spring of force constant $200\ Nm^{-1}$ fixed at one end has a mass of 2 kg attached to the free end and resting on a smooth horizontal surface. The mass is pulled through a distance of 4.0 cm and released. Calculate the

(i) angular speed

(ii) maximum velocity attained by the vibrating body.

(iii) acceleration when the body is half way towards the centre from its initial position.

[2011, No 4; Ans: (c)(i) $10\,rads^{-1}$ (ii) $0.4\,ms^{-1}$ (iii) $2\ ms^{-2}$]

29. (a) (i) State the laws of conservation of linear momentum

(ii) Use Newton's law to derive the law of (a)(i)

(b) Distinguish between elastic and inelastic collisions.

(c) An object X of mass M, moving with a velocity 10 ms^{-1} collides with a stationary object Y of equal mass. After collision X moves with speed U, at an angle of 30° to its initial direction while Y, moves with a speed of V at an angle of 90° to the new direction.

(i) Calculate the speeds U and V

(ii) Determine whether the collision is elastic or inelastic

(d) (i) Define uniform acceleration.

(ii) With the aid of a velocity-time graph, describe the motion of a body projected vertically upwards.

(iii) Calculate the range of a projected which is fired at an angle of 45° to the horizontal with a speed of 20 ms^{-1}

[2010, No. 1, Ans: (c)(i) 5 ms^{-1}, 8.66 ms^{-1} (d) (iii) 40.77 m]

30. (a)(i) State Archimedes' principle

(ii) A solid weighs 20.0 g in air, 15.0 g in water and 16.0 g in a liquid, R. Find the relative density of liquid R.

(b) (i) What is meant by simple harmonic motion?

(ii) Distinguish between damped and forced oscillations.

(c) A cylinder of length, l cross sectional area, A, and density, σ. The cylinder is pushed down slightly and released.

(i) Show that it performs simple harmonic motion.

(ii) Derive the expression for the period of oscillation.

(d) A spring of force constant 40 Nm^{-1} is suspended vertically. A mass of 0.1 kg suspended from the spring is pulled down a distance of 5 mm and released. Find the

(i) period of oscillation

(ii) maximum oscillation of the mass

(iii) net force acting on the mass when it is $2\ mm$ below the centre of oscillation.

[2010, No. 2, Ans:(i) 0.314 s (ii) 2 ms^{-1} (iii) 0.08 N]

31. (a) Define viscosity of a fluid.

(b) (i) Derive an expression for the terminal velocity attained by a sphere of density, σ, and radius, a falling through a fluid of density, ρ, and viscosity η.

(ii) Explain the variation of the viscosity of a liquid with temperature.

(c) (i) State the laws of solid friction

(ii) With the aid of a well labeled diagram, describe an experiment to determine the coefficient of kinetic friction between two surface.

(d) A body slides down a rough plane inclined at 30° the horizontal. If the coefficient of kinetic friction between the body and the plane is 0.4, find the velocity after it has travelled 6 m along the plane.

[2010, No. 3; Ans: (d) 4.25 ms^{-1}]

32. (a) (i) Describe the terms tensile stress and tensile strain as applied to a stretched wire.

(ii) Distinguish between elastic limit and proportional limit

(b) With the aid of a labelled diagram, describe an experiment to investigate the relationship between tensile stress and tensile strain of a steel wire.

(c) (i) A load of 60 N is applied to a steel wire of length 2.5 m and cross sectional area of 0.22 mm^2. If Young's modulus for steel is 210 GPa, find the expansion produced.

(ii) If the temperature rise of 1 K causes a fractional in areas of 0.001%, find the change in the length of a steel wire of length 2.5 mm when the temperature increases by 4 K.

(d) The velocity, V of a wave in a material of Young's modulus, E and density, ρ is given by $V = \sqrt{\frac{E}{\rho}}$ show that the relationship is dimensionally correct.

[2010, No. 4, Ans: (c)(i) $3.247 \times 10^{-3}\ m$ (ii) $1.0 \times 10^{-4}\ m$]

33. (a) (i) Define the term impulse

(ii) State Newton's laws of motion

(b) A bullet of mass 10 g travelling horizontally at a speed of $100\ ms^{-1}$ strikes a block of wood of mass 900 kg suspended by a light vertical string and is embedded in the block which subsequently swings freely. Find the

(i) vertical length through which the block rises.

(ii) kinetic energy lost by the bullet

(c) Explain the terms time of flight and range as applied to projectile motion

(d) A stone is projected at an angle of 20° to the horizontal and just clears a wall which is $10\ m$ high and $30\ m$ from the point of projection. Find the

(i) speed of projection

(ii) angle which the stone makes with the horizontal as it clears the wall

[2009, No. 1. Ans: (b) (i) $6.2 \times 10^{-2} m$ (ii) $49.99\ J$ (d)(i) $73.78\ ms^{-1}$, $16.9°$]

34. (a) Define the following terms

(i) velocity

(ii) moment of a force

(b) A ball is projected vertically upwards with a speed of $50\ ms^{-1}$. On return, it passes the point of projection and falls and falls $78\ m$ below.

(i) Calculate the total time taken.

(ii) State the energy changes that occurred during the motion of the ball in (b)(i) above.

(c) (i) State the conditions necessary for mechanical equilibrium to be attained.

(ii) A uniform ladder of mass $40\ kg$ and length $5\ m$ rests with its upper end against a smooth vertical wall and with its lower end at $3\ m$ from the wall on a rough ground. Find the magnitude and the direction of the force exerted at the bottom of the ladder.

(d) State four instances where increasing friction is useful.

[2009, No. 2, Ans: (b) (i) $11.57\ s$ (c) (ii) $418.7\ N, 69.4°$]

35. (a) What is meant by simple harmonic motion?

(b) (i) A cylindrical vessel of cross sectional area, A contains air of volume, V, at a pressure, P trapped by a frictionless tight piston of mass M. If the piston oscillates with simple harmonic motion, show that the frequency is given by

$$f = \frac{A}{2\pi}\sqrt{\frac{P}{MV}}$$

(ii) Show that the expression for f in (b)(i) is dimensionally correct.

(c) A particle executing simple harmonic motion vibrates in a straight line. Given that the speeds of the particle are $4\ ms^{-1}$ and $2\ ms^{-1}$ when the particle is $3\ cm$ and $6\ cm$ respectively from the equilibrium, Calculate the

(i) amplitude of oscillation

(ii) frequency of the particle

(d) Given two examples of oscillatory motion which approximate the simple harmonic motion and state the assumption made in each case.

[2009, No. 3; Ans: (c)(i) $0.067\ m$ (ii) $10.68\ Hz$]

36. (a) (i) State Archimedes principle

(ii) Use Archimedes principle to derive an expression for the resultant force a body of weight, W, and density, σ, totally immersed in a fluid of density, ρ.

(b) A tube of uniform cross section area of $4 \times 10^{-3} m^2$ and mass of $0.2\ kg$ is separately floated vertically in water of density $1.0 \times 10^3 kgm^{-3}$ and in oil of density $8.0 \times 10^2\ kgm^{-3}$. Calculate the difference in the lengths immersed.

(c) (i) Define surface tension in terms of work.

(ii) Use the molecular theory to account for the surface tension of a liquid.

(iii) Explain the effect of the increasing temperature of a liquid on its surface tension.

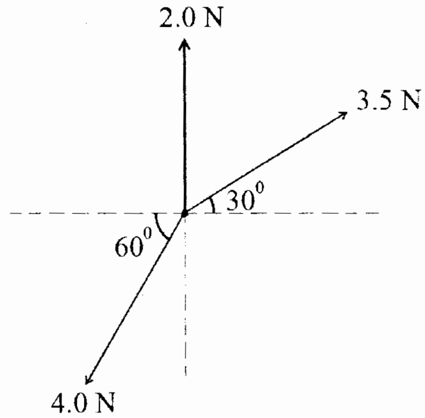
(iv) Calculate the excess pressure inside a soap bubble of diameter $3.0\ cm$ if the surface tension of the soap solution is $2.5 \times 10^{-2} Nm^{-1}$.

[2009, No. 4 (b) $1.25 \times 10^{-2} m$ (c)(iv) $6.67\ Pa$]

37. (a) (i) Define the terms velocity and displacement

(ii) Sketch a graph of velocity against time for an object thrown vertically upwards.

(b)



Three forces of 3.5 N, 4.0 N and 2.0 N act at appoint O as shown as shown above. Find the resultant force.

(c) (i) What is meant by saying that a body is moving with velocity, v relative to another

(ii) A ship, A is moving due north at 20 kmh^{-1} and ship B is travelling due east at 15 $kmhr^{-1}$. Find the velocity of A relative to B.

(iii) If the ship B in (c)(ii) is 10 km due west of A at noon, find the shortest distance apart and when this occurs.

(d) (i) What is meant by a couple in mechanics

(ii) State the conditions for equilibrium of a system of coplanar forces.

[2008, No. 1; Ans: (b) $1.07\ N, 15.5°$ (c) (ii) $N36.9°W$. (iv) 0.24 hours]

38. (a) (i) State the laws of friction between solid surfaces

(ii) Explain the origin of friction force between two solid surfaces in contact.

(iii) Describe an expression to measure the coefficient of kinetic friction between two solid surfaces.

(b) (i) A car of mass 1000 kg moves along a straight surface of speed of 20 ms^{-1}. When brakes are applied steadily, the car comes to rest after travelling 50 m. Calculate the coefficient of friction between the surface and the tyre.

(ii) State the energy charges which occur from the time the brakes are applied to the time the car comes to rest.

(c) (i) State two disadvantages of friction

(ii) Give on method of reducing friction between solid surfaces.

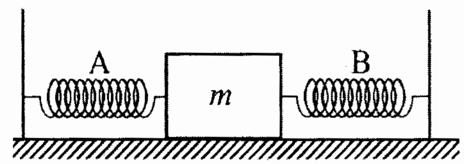
(d) Explain what happens when a small steel ball is dropped centrally in a tall jar containing oil.

[2008, No. 2; Ans: (b)(i) 0.408]

39. (a) (i)Define simple harmonic motion

(ii) A particle of mass on executes simple harmonic motion between two points A and B about equilibrium position, O. Sketch a graph of the restoring force acting on the particle as a function of distance, r moved by the particle.

(b)



Two springs A and B of spring constants K_A and $K_B$respectively are connected to a mass m as shown in the figure above. The surface on which the mass slides is frictionless.

(i) Show that when the mass is displaced slightly, it oscillates with simple harmonic motion of frequency, f , given by

$$f = \frac{1}{2\pi}\sqrt{\frac{K_A + K_B}{m}}$$

(ii) If the two springs are identical such that $K_A = K_B = 5.0\ Nm^{-1}$ and mass $m = 50$ g, calculate the period of the oscillation.

(c) (i)With the aid of a diagram, describe an experiment to determine the universal gravitational constant, G.

(ii) If the moon moves round the earth in a circular orbit of radius $= 4.0 \times 10^{8} m$ and takes exactly 27.3 days to go round once, calculate the value of acceleration due to gravity, g, at the earth's surface.

[2008, No. 3; Ans: (b)(ii) $0.44\ s$ (c) (ii) $11.08\ ms^{-2}$]

40. (a) State

(i) Newton's laws of motion

(ii) the principle of conservation of momentum.

(b) A body of mass m_1 moves with velocity u_1 and collides head on elastically with another body B of mass, m_2 which is at rest. If the velocities of A and B are v_1 and v_2 respectively and given that $x = \frac{m_1}{m_2}$, show that

(i) $\frac{u_1}{v_1} = \frac{x+1}{x-1}$

(ii) $\frac{v_2}{v_1} = \frac{2x}{x-1}$

(c) Distinguish between conservation and non-conservative forces

(d) A bullet of mass 40 g is fired from a gun $200\ ms^{-1}$ and hits a block of wood of mass $2\ kg$ which is suspended by a light vertical string $2\ m$ long. If the bullet gets embedded in the wooden block,

(i) calculate the maximum angle the string makes with the vertical.

(ii) state a factor on which the angle of swing depends.

[2008, No. 4, Ans (d)(i) $52.4°$]

41. (a) Define simple harmonic motion (S.H.M)

(b) Sketch a graph of

(i) velocity against displacement

(ii) acceleration against displacement, for a body executing S.H.M

(c) A glass U-tube containing a liquid is tilted slightly and then released

(i) Show that the liquid oscillates with simple harmonic motion

(ii)Explain why the oscillations ultimately come to rest.

(d) Explain why the maximum speed of a car on a banked road is higher than that on unbanked road.

(e) A small bob of mass $0.20\ kg$ is suspended by an inextensible string of length $0.80\ m$. The bob is then rotated in a horizontal circle of radius $0.40\ m$. Find the

(i) linear speed of the bob

(ii) tension in the string

[2007, No. 1; Ans: (c)(i) $1.51\ ms^{-1}$ (ii) $2.28\ N$]

42. (a) State Kepler's laws of planetary motion.

(b) (i) A satellite moves in a circular orbit of radius, R about a planet of mass M, with period, T. Show that $R^3 = \frac{GMT^2}{4\pi^2}$ where G is the universal gravitational constant

(ii) The period of the moon around the earth is 27.3 days. If the distance of the moon from the earth is $3.83 \times 10^8\ km$, calculate the acceleration due to gravity at the surface of the earth.

(iii) Explain why any resistance to th forward motion of an artificial satelli results into an increase in its speed.

(c) (i) What is meant by weightlessness?

(ii)Why does acceleration due to gravi vary with location on the surface of th earth?

[2007, No. 2; Ans: (b)(ii) $9.72\ ms^{-2}$

43. (a)(i) State the laws of solid friction

(ii)Using the molecular theory, explain the law stated in (a)(i)

(b) Describe an experiment to determine th coefficient of static friction for an interfac between a rectangular block of wood and plane surface.

(c) (i) State the difference between conservativ and non-conservative forces, giving in example of each

(ii) State the work energy theory

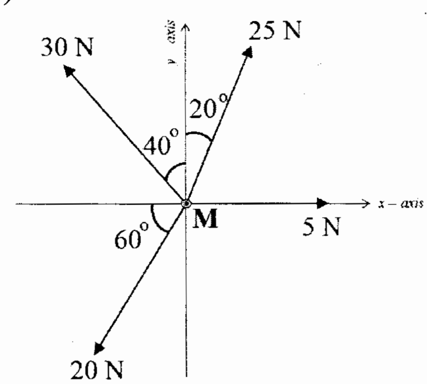
(iii) A block of mass $6.0\ kg$ is projected with velocity of $12\ ms^{-1}$ up a rough plane inclined at 45° to the horizontal. If it travels $5.0\ m$ up the plane, find the frictional force.

(d) Explain the effect of temperature on the viscosity of a liquid.

[2007, No. 3; Ans: (c) (iii) $44.8\ N$

44. (a)(i) Define vector and scalar quantities and give one example of each.

(ii)



A body, M of mass 6 kg is acted on by forces of $5\ N, 20\ N, 25\ N$ and $30\ N$ as shown in the figure above. Find the acceleration of M.

(b) (i) What is meant by acceleration due to gravity?

(ii) Describe how you would use a spiral spring, a retort stand with a clamp, a pointer, seven 50 g masses, a metre rule and a stop clock to determine the acceleration due to gravity.

(iii) State any two sources of error in the experiment (b)(ii) above.

(iv) A body of mass 1 kg moving with simple harmonic motion has speed of 5 ms^{-1} and 3 ms^{-1} when it is at distances of 0.10 m and 0.20 m respectively from the equilibrium point. Find the amplitude of motion.

[2007, No. 4; Ans (a)(ii) $5.5\ ms^{-1}$ (b)(iv) $0.24\ m$]

45. (a) (i) What is meant by uniformly accelerated motion?

(ii) Sketch the speed against time graph for a uniformly accelerated body.

(b) Derive the expression, $s = ut + \frac{1}{2}at^2$ for the distance, s moved by a body which is travelling with speed, u and is uniformly accelerated for time, t.

(c) A projectile is fired horizontally from the top of a cliff 250 m high. The projectile lands $1.414 \times 10^3\ m$ from the bottom of the cliff. Find the

(i) initial speed of the projection

(ii) velocity of the projectile just before it hits the ground.

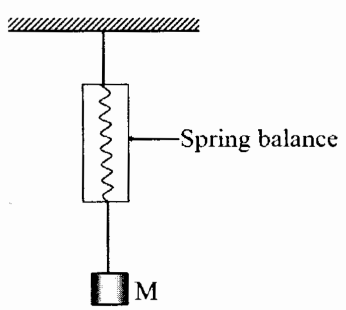
(d) Describe an experiment to determine the centre of gravity of a plane sheet of material having an irregular shape.

[2006, No. 1; Ans: (c)(i) $198\ ms^{-1}$ (ii) $210\ ms^{-1}$, $19.5°$]

46. (a) (i) Define force and power

(ii) Explain why more energy is required to push a wheelbarrow uphill than on a level ground.

(b)



A mass, M, is suspended from a spring balance as shown above. Explain what happens to the reading on the spring balance when the setup is raised up slowly to a very high height above the ground.

(c) (i) State the work energy theorem.

(ii) A bullet of mass 0.1 kg moving horizontally with a speed of 420 ms^{-1} strikes a block of mass of 2.0 kg at rest on a smooth table and becomes embedded in it. Find the kinetic energy lost if they move together.

(d) State the condition for equilibrium of a rigid body under the action of coplanar forces.

(e) A 3 m long ladder rests at angle of 60° to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5 kg and its centre of gravity is one-third from the bottom of the ladder.

(i) Draw a sketch diagram to show the forces acting on the ladder.

(ii) Find the reaction of the ground on the ladder.

[2006, No. 2; Ans: (c) (ii) 8400 J (c)(i) 49.95 N]

47. (a)(i) Define stress and strain

(ii) Determine the dimensions of Young's modulus.

(b) Sketch a graph of stress versus strain for a ductile material and explain its features

(c) A steel wire of cross-section area 1 mm^2 is cooled from a temperature of 60°C to 15°C. Find the

(i) strain

(ii) force needed to prevent it from contracting

(Young's modulus = $2.0 \times 10^{11} Pa$, coefficient of linear expansion for steel = $1.1 \times 10^{-5} K^{-1}$)

[2006, No. 3; Ans (c)(i) 4.95×10^{-4} (ii)99 N]

48. (a)(i) State Archimedes' principle

(ii) Describe an experiment to determine the relative density of an irregular solid which floats in water.

(iii) A block of wood floats at an interface between water and oil with 0.25 of its submerged in the oil. If the density of water is $7.3 \times 10^2 kgm^{-3}$, find the density of the oil.

(b) (i) State Bernoulli's principle

(ii) Explain the origin of the lift force on the wings of an airplane at takeoff.

(c) Water flowing in a pipe on the ground with a velocity of 8 ms^{-1} and at a gauge pressure of $2.0 \times 10^5 Pa$ is pumped into a water tank 10 m above the ground. The water enters the tank at a pressure of $1.0 \times 10^5\ Pa$. Calculate the velocity with which the water enters the tank.

(d) Describe how terminal velocity can be measured in a liquid.

[2006, No. 4; Ans: (c) $16.36\ ms^{-1}$]

49. (a) Distinguish between scalar and vector quantities giving two examples of each

(b) The equation for the volume, V, of a liquid flowing through a pipe in time, t, under a steady flow, is given by

$$\frac{V}{t} = \frac{\pi r^4 p}{8\eta l}, \text{ where}$$

r = radius of the pipe

p = pressure difference between the ends of the pipe

l = length of the pipe

η = coefficient of viscosity of the liquid

If the dimensions of η are $ML^{-1}T^{-1}$, show that the above equation is dimensionally consistent.

(c) (i) Define linear momentum

(ii) State the law of conservation of linear momentum

(iii) Show that the law in (c)(ii) above follows from Newton's law of motion

(iv) Explain, why when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.

(d) A car of mass $1000\ kg$ travelling at a uniform velocity of $20\ ms^{-1}$collides perfectly inelastically with a stationary car of mass $1500\ kg$. Calculate the loss in the kinetic energy of the car as a result of the collision.

(e) (i) What is meant by conservation of energy?

(ii) Explain how conservation of energy applies to an object falling from rest in a vacuum.

[2005, No. 1; Ans: (d) $1.2 \times 10^5\ J$]

50. (a) Explain the terms

(i) ductility

(ii) stiffness

(b) A copper wire and steel wire each of length $1.0\ m$ and diameter $1.0\ mm$ are joined end to end to form a composite wire $2.0\ m$ long. Find the strain in each wire when the composite wire stretched by $2.0 \times 10^{-3}\ m$

(Young's moduli for copper and steel are $1.2 \times 10^{11}\ Pa$ and $2.0 \times 10^{11}\ Pa$ respectively)

(c) (i) Define centre of gravity

(ii) Describe an experiment to find the centre of gravity of a flat irregular piece of cardboard

(d) Explain the laws of solid friction using the molecular theory

[2005, No. 2; Ans: (b) 1.25×10^{-3}]

51. (a) What is meant by the following terms

(ii) Velocity gradient

(iii) Coefficient of viscosity

(b) Derive an expression for the terminal velocity of a steel-ball bearing of radius, r and density ρ, falling through a liquid of density σ and coefficient of viscosity, η

(c) (i) Define surface tension

(ii) Explain the origin of surface tension

(iii) Describe an experiment to measure the surface tension of a liquid by the capillarity method

(d) Explain, with the aid of a diagram why air flow over the wings of an aircraft at take-off causes a lift.

[2005, No. 3]

52. (a) (i) Define angular velocity

(ii) Derive an expression for the force F, on a particle of mass, m moving with angular velocity ω, in a circle of radius r

(b) A stone of mass $0.5\ kg$ is attached to a string which will break if the tension in it exceeds $20\ N$. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of $1.0\ m$ above the ground. The angular speed is gradually increased until the string breaks.

(i) In what position is the string most likely to break? Explain

(ii) At what angular speed will the string break?

(iii) Find the position where the stone hits the ground when the string breaks.

(c) Explain briefly the action of a centrifuge

(d) Describe how the acceleration due to gravity can be measured using a helical spring of unknown force constant and other relevant apparatus.

[2005, No. 4; Ans: (b)(ii)$3.885\ ms^{-1}$ (ii) $1.26\ m$]

SECTION B:

HEAT

Thermometry

- Thermometric properties
- Thermometric quantities
- Temperature scales, Fixed points on a thermometer, the absolute zero,
- Conversion of temperature from one scale to another,
- Types of thermometers - Liquid-in-glass, constant volume gas, electrical resistance thermometers, thermoelectric, pyrometers (optical and total radiation).
- Comparison of temperatures measured using different thermometers
- Advantages and disadvantages of different types of thermometers

Specific heat

- Heat capacity and specific heat capacity
- Units of heat
- Measurement of specific heat capacity by
  - method of mixtures, electrical method, continuous flow method
- Comparison of the different methods of measuring S.H.C.
- Heat leakages
- Newton's law of cooling
- Factors that affect the rate of cooling
- Verification of Newton's law

Change of state

- Melting, boiling and evaporation
- Latent heat
- Measurement of latent heat

Gas laws

- Boyle's, Charles' and Pressure laws
- Verification of gas laws
- Graphical representation of the gas laws
- Equation of state
- Pressure and volume coefficients of expansion
- Proof of equality of α_V and α_P

Vapours

- Vapours and gases
- Saturated and unsaturated vapours
- Explanation using kinetic theory
- Saturated vapour pressure (SVP)
- Variation of SVP with temperature
- Linkage between boiling, SVP and external pressure
- Boiling point
- Applications

Thermodynamics

- Internal energy
- Factors which affect the internal energy of a gas
- Isobaric and isovolumetric processes
- Work done by an expanding gas
- Principal specific heat capacities
- Expressions for work in isovolumetric and isobaric processes using the principal specific heat capacities
- First law of thermodynamics
- Relationship between c_p and c_v
- Isothermal and adiabatic changes
- Reversible isothermal and reversible adiabatic processes.
- Conditions necessary for reversible adiabatic processes
- Work done during isothermal and adiabatic processes
- Applications of isothermal and adiabatic processes

Heat transfer

- Heat conduction - Mechanism
- Rate of heat conduction
- Coefficient of thermal conductivity and its measurement
- Measurement of thermal conductivity (for good and bad conductors).
- Convection of heat
- Radiation of heat
- Detection (thermopile, bolometer)
- Good and bad radiators / absorbers
- Black body radiation
- Examples of black bodies
- Energy distribution in the spectrum of black body radiation
- Stefan's law
- Wien's displacement law
- Provost's theory of exchanges
- Temperature of the sun and other black bodies
- The electromagnetic spectrum
- Properties and uses of each of the components in the electromagnetic spectrum
- Methods of detection
- Applications of heat transfer
- Cooling correction

Survey of energy

- Concept of energy and energy transfer
- Forms of energy
- Energy sources and resources
- Primary and secondary sources of energy
- Renewable and non-renewable energy resources
- Conservation of energy
- Energy converters
- Energy use
- Energy degradation

THERMOMETRY

Temperature
The temperature of a body is its degree of hotness or coldness.

Heat
Heat is a form of energy that is transferred from one object to another due to a temperature difference between the two objects.

Quantity of heat
The quantity of heat is measure in **Joules, J** since it is a form of energy
The rate at which heat is gained or lost by a body is expressed in joules per second or **watts, *W***.

Thermal equilibrium
When objects or substances are in contact such that heat is able to flow from one object or substance to another, the objects or substances are said to be in **thermal contact.**
When two bodies are in thermal contact and a condition is reached where there is no net transfer of heat from one body to another, then the two bodies are said to be in **thermal equilibrium.**

Thermometry
This is the study of the principles used in the measurement of temperature

Thermometric property
A thermometric property is a physical quantity whose values change continuously with temperature.

Examples of thermometric properties
- Length of liquid in a capillary tube
- Pressure of a fixed mass of a gas at constant volume
- Resistance of a coil of wire
- E.m.f of a thermocouple

Examples of thermometers and the thermometric properties they measure are as shown in the table below

Thermometric property	Thermometer
Length of liquid in a capillary tube	Liquid in glass thermometer e.g. mercury thermometer
Pressure of a fixed mass of a gas	Constant volume gas thermometer
Resistance of a coil of wire	Resistance thermometer
E.m.f of a thermocouple	Thermoelectric thermometer

Desirable features of a thermometric property
- *It must be accurately measurable over a wide range* of temperatures
- A small change in temperature should produce a considerable change in the thermometric property
- It must change linearly with temperature
- Should quickly come to thermal equilibrium with other systems

Temperature scale
A scale of temperature needs the following
- Thermometric property
- Fixed points
- Numerical scale

Scales of temperature may not agree because of the different thermometric properties used.

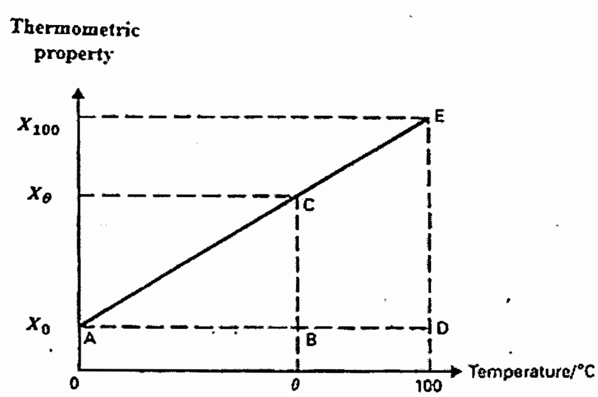
Fixed points
A fixed point is a temperature where all the thermometers show the same reading.

For a Centigrade (Celsius) scale, the ice point (0°C) is the **lower fixed point** while the steam point (100°C) is the **upper fixed point**.

Ice point is the temperature at which pure ice can exist in equilibrium with water at standard atmospheric pressure

Steam point is the temperature at which pure water exists in equilibrium with its vapour at standard atmospheric pressure

Establishing a Celsius scale or empirical scale of temperature
- A thermometric property X is selected and the value of the property at steam point and at ice point is obtained.
- The value of the property is measured at an unknown temperature
- A graph of the thermometric property versus temperature is a straight line as below.



$X_0 =$ value of X at the ice point
$X_{100} =$ value of X at steam point
$X_\theta =$ value of X at θ°C, the temperature to be measured

Since ΔABC and ΔADE are similar,

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{\theta}{100} = \frac{X_\theta - X_0}{X_{100} - X_0}$$

$$\theta = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100\ °\text{C}$$

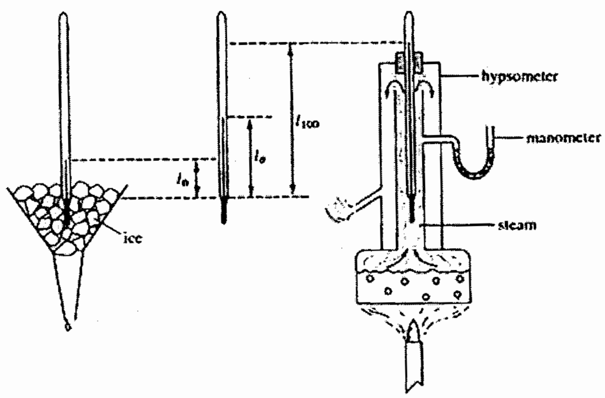
Liquid in glass thermometers

This type of thermometer uses the thermometric property of length of the liquid column. The most preferred liquid used in these thermometers is mercury.

Desirable features of mercury to other liquids

- It is opaque hence can easily be seen
- It does not wet the glass walls
- Being a good conductor, it can rapidly take up the temperature of the surroundings

Calibration of liquid in glass thermometers



- The bulb of the thermometer is placed in pure melting ice in a funnel. When the mercury level in the thermometer is stable, its level is marked on the stem and the length l_0 measured.
- The thermometer is then placed in a hypsometer in which pure water and steam are in equilibrium at atmospheric pressure.
- If the liquid levels in the manometer are the same and the mercury level in the thermometer is stable, the level is marked and the length l_{100} measured.
- To measure a temperature of θ °C when the length of mercury thread is l_θ, θ °C is given by

$$\theta\ °\text{C} = \frac{l_\theta - l_0}{l_{100} - l_0} \times 100\ °\text{C}$$

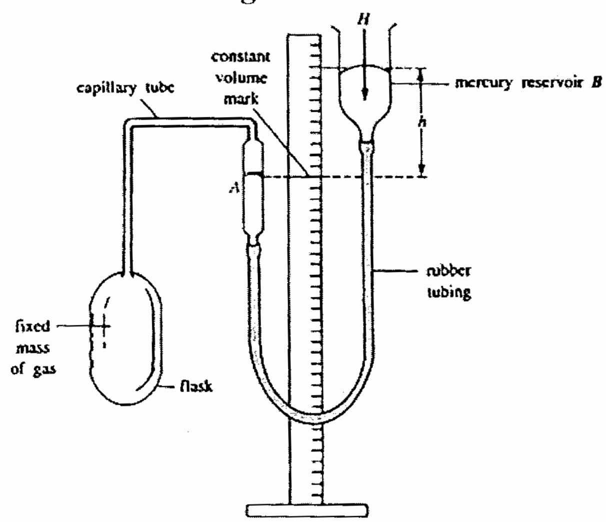
Advantages of liquid in glass thermometers

- Cheap and easy to produce
- Light or portable
- Temperature is read directly from the thermometer

Limitations of Liquid in glass thermometers in providing accurate measurements

- Its bore is non-uniform
- Accuracy of calibration depends on whether the thermometer is upright or not and how much of the stem is exposed.
- Parallax errors prevent the scale from being read accurately
- Glass expands and contracts and can take a long time to reach its correct size. This spoils calibration.

Constant volume gas thermometer



It consists of a flask containing dry air or nitrogen connected by a capillary tube to a manometer. The mark A is the constant volume mark.

To measure the pressure of the gas in the flask, the height of the mercury reservoir B is adjusted until the level of mercury in the capillary tube reaches the level of A, the constant volume mark.

If $h/cm =$ difference in height between mercury meniscus in the manometer when thermal equilibrium is reached.

and $H/cm =$ height of mercury barometer (atmospheric pressure),

then the pressure of the gas in the flask

$$p = (H + h)\ cm \text{ mercury}$$

If $h = h_0$ at ice point
$h = h_{100}$ at the steam point
$h = h_\theta$ at the temperature to be measured θ

$$\text{then } \theta\ °\text{C} = \frac{p_\theta - p_0}{p_{100} - p_0} \times 100\ °\text{C}$$

$$= \frac{(H + h_\theta) - (H - h_0)}{(H + h_{100}) - (H - h_0)} \times 100\ °\text{C}$$

$$= \frac{h_\theta - h_0}{h_{100} - h_0} \times 100\ °\text{C}$$

Sources of error

- The bulb expands
- Air is not an ideal gas
- The air in the capillary tube is not at the temperature being measured.

Advantages of constant volume gas thermometer

- Can be used over wide range of temperatures
- Very accurate and can be used as a standard to calibrate other thermometers
- Very sensitive and can thus measure very low temperatures

Disadvantages

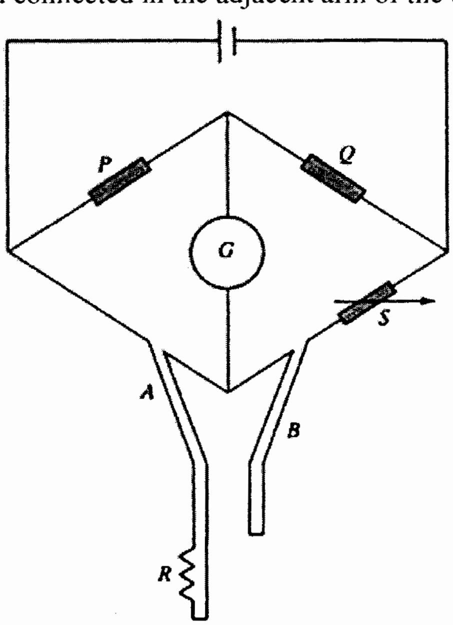
- It is bulky
- Slow to respond to temperature changes. It can thus not be used to measure **rapidly changing temperatures**
- Cannot be used to measure temperature at a point.

Platinum resistance thermometer

The thermometric property used in the platinum resistance thermometer is the resistance of a platinum wire. The platinum wire is wound non-inductively on a strip of mica as shown below.

The ends of the coil are attached to a pair of leads A, for connecting them to a Wheatstone bridge. The dummy leads compensate for any resistance change in the leads to the platinum coil when the temperature changes.

A similar pair of leads B is near to the leads from the coil and connected in the adjacent arm of the bridge.



The resistance, R of the platinum coil is measured using the Wheatstone bridge circuit shown above. P and Q are standard resistors.

The variable resistance S is varied until the galvanometer is balanced, then

$$\frac{R}{S} = \frac{P}{Q}$$

$$R = \left(\frac{P}{Q}\right)S$$

The resistance scale of temperature is then defined by the equation

$$\theta\ °\text{C} = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100\ °\text{C}$$

Advantages of platinum resistances thermometer

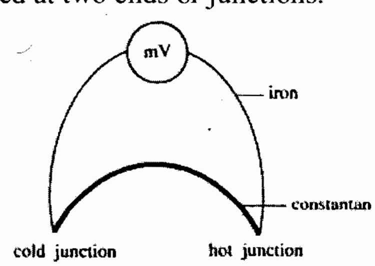
- It is easier to use than that of a gas thermometer
- It gives a precise measurement of temperature because the resistance of the wire can be measured with high accuracy
- It has a wide range
- It has a high sensitivity

Disadvantages

- Has a large thermal capacity so that it takes appreciable heat from the experimental body. Hence the measured temperature is slightly lower than the actual temperature of the body.
- The tube of the thermometer has a low thermal conductivity. Hence temperature cannot be measured quickly.
- At higher temperatures (1200 °C), platinum begins to vaporize resulting in a considerable change in resistance of the wire.

Thermocouple

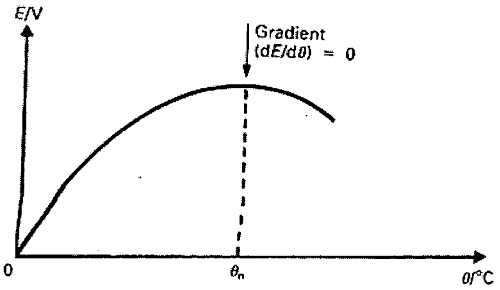
A thermocouple consists of two wires of different metals joined at two ends or junctions.



If there is a temperature difference between the two junctions, a small e.m.f is set up. The e.m.f increases with temperature for temperatures up to about 1200 °C.

The e.m.f generated can be measured using a voltmeter of a potentiometer for accuracy.

Thermocouple e.m.f as a function of temperature



Advantages of thermocouple thermometer

- Power source is not required
- Robust, easy to use and cheap
- Can be used to measure up to 1200 °C

Disadvantages

- Not very accurate
- Require accurate reference temperature
- Not suitable for very low temperatures because the expansion of metals tends to be similar, so the device becomes a rather insensitive thermometer.

Examples

1. A particular resistance thermometer has a resistance of $30.0\,\Omega$ at the ice point, $41.58\,\Omega$ at the steam point and $34.59\,\Omega$ when immersed in a boiling liquid. A constant-volume gas thermometer gives readings of $1.333 \times 10^5\ Pa$, $1.821 \times 10^5 Pa$ and $1.528 \times 10^5\ Pa$ at the same three temperatures. Calculate the temperature at which the liquid is boiling
   (a) on the scale of the gas thermometer
   (b) on the scale of the resistance thermometer

Solution

(a) on the gas thermometer

$$\theta = \frac{p_\theta - p_0}{p_{100} - p_0} \times 100\ °C$$
$$= \frac{1.528\times10^5 - 1.333\times10^5}{1.821\times10^5 - 1.333\times10^5} \times 100$$
$$= \frac{0.195}{0.488} \times 100$$
$$= 39.96\ °C$$

(b) on the resistance thermometer

$$\theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100\ °C$$
$$= \frac{34.59 - 30.0}{41.58 - 30.0} \times 100$$
$$= \frac{4.59}{11.58} \times 100$$
$$= 39.64\ °C$$

2. The resistance R_θ of a particular resistance thermometer at a Celsius temperature θ as measured by a constant volume-gas thermometer is given by

$$R_\theta = 50.0 + 0.170\theta + 3.0 \times 10^{-4}\theta^2$$

Calculate the temperature as measured on the scale of the resistance thermometer which corresponds to a temperature of 60 °C on the gas thermometer.

Solution

$$\theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100\ °C$$
$$\theta = \frac{R_{60} - R_0}{R_{100} - R_0} \times 100\ °C$$
$$R_\theta = 50.0 + 0.170\theta + 3.0 \times 10^{-4}\theta^2$$
$$R_{60} = 50.0 + 0.170(60) + 3.0 \times 10^{-4}(60)^2$$
$$= 61.28\,\Omega$$
$$R_{100} = 50.0 + 0.170(100) + 3.0 \times 10^{-4}(100)^2$$
$$= 70.0\,\Omega$$
$$R_0 = 50.0 = 50.0\,\Omega$$
$$\theta = \frac{R_{60} - R_0}{R_{100} - R_0} \times 100\ °C$$
$$= \frac{61.28 - 50.0}{70.0 - 50.0} \times 100$$
$$= 56.40\ °C$$

3. (a) Explain how a Celsius temperature scale is defined, illustrating your answer by reference to a platinum resistance thermometer.
(b) The resistance R_t of a platinum wire at temperature t °C, measured on the gas scale, is given by $R_t = R_0(1 + at + bt^2)$, where $a = 3.8 \times 10^{-3}$ and $b = -5.6 \times 10^{-7}$. What temperature will the platinum thermometer indicate when the temperature on the gas scale is 200°C?

Solution

(a) The temperature θ_p in °C on a resistance thermometer scale is given by the relation

$$\theta_p = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100$$

where R_θ, R_0, R_{100} are the respective resistances at the temperature concerned, at 0°C, and at 100°C

(b) $R_t = R_0(1 + at + bt^2)$

$$R_{200} = R_0(1 + 200a + 200^2 b)$$
$$R_{100} = R_0(1 + 100a + 100^2 b)$$
$$\theta_p = \frac{R_{200} - R_0}{R_{100} - R_0} \times 100$$
$$= \frac{R_0(1 + 200a + 200^2 b) - R_0}{R_0(1 + 100a + 100^2 b) - R_0} \times 100$$
$$= \frac{200a + 200^2 b}{a + 100b}$$
$$= \frac{200(a + 200b)}{a + 100b}$$
$$= \frac{200(3.8\times10^{-3} - 11.2\times10^{-5})}{3.8\times10^{-3} - 5.6\times10^{-5}}$$
$$= \frac{200 \times 0.003688}{0.003744} = 197°C$$

Thermodynamic scale of temperature

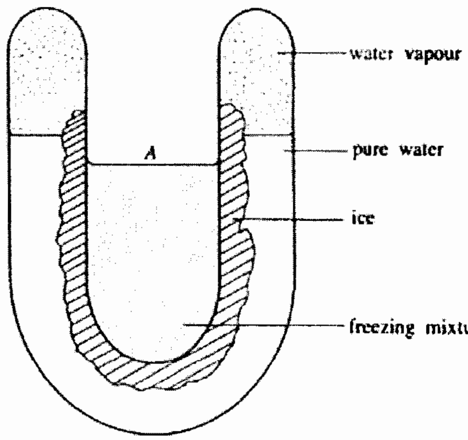
This is a scale of temperature that is not based on any thermometric property or experimental results. The thermodynamic scale of temperature is based on the properties of an ideal gas. An ideal gas only exists in theory. The property of an ideal gas used to set up the thermodynamic scale of temperature is the product of the pressure, p and volume, V of a fixed mass of a gas i.e. pV

The two fixed points in the thermodynamic scale are:

Absolute zero which is the temperature at which the pressure of an ideal gas becomes zero and is given the value 0 K (zero kelvin)

Triple point of water which is the temperature at which ice, water and water vapour are in thermal equilibrium.

The triple point of water can be obtained by using a triple point cell shown below.



The triple point cell consists of a double-walled glass container with pure water in the space between the walls. All the air is evacuated from the water enclosure.

A freezing mixture is placed in A and the water cooled until ice is formed. When equilibrium is attained, the pressure in the cell is 0.61 kPa and the temperature is fixed as 273.16 K. The triple point temperature is fixed at 273.16 K to make the temperature difference between the ice point and steam point exactly 100 K.

The triple point of water is preferred to the ice point because the **triple point temperature can be obtained more accurately**. At triple point, the three phases of water are in equilibrium, whereas only two phases of water are in equilibrium at the ice point. Since the temperature of triple point of water is fixed as 273.16 K, a temperature interval of one kelvin (1 K) is defined as

$$1\,K = \frac{1}{273.16} \times \text{(temperature of triple point of water)}$$

The thermodynamic temperature scale is defined by the equation

$$T = \frac{X_T}{X_{tr}} \times 273.16\text{ K}$$

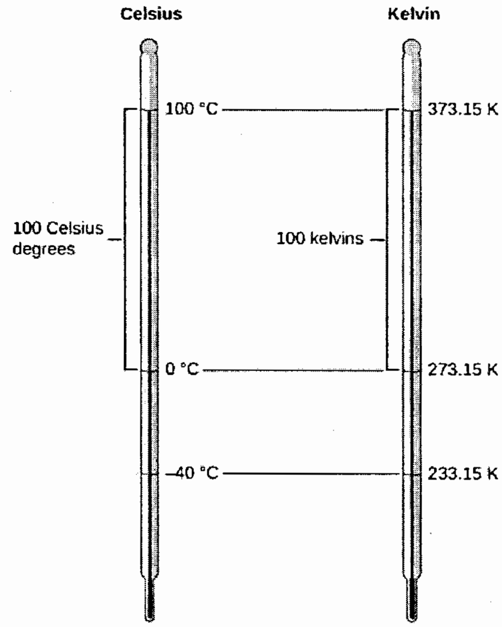
where X_T = value of X at a temperature T to be determined.

X_{tr} = value of X at the triple point of water

The Celsius scale of temperature θ°C is defined in terms of the thermodynamic temperature by the relationship

$$\theta/°\text{C} = T/K - 273.15$$

Therefore, the ice point on the thermodynamic scale is 273.15 K.



Examples

1. At the triple point of water, the pressure of a fixed mass of gas is 2680 Pa. The temperature is changed to T while the volume of the gas is kept constant. The pressure is then 4870 Pa.
   (i) Find the value of T
   (ii) What is the advantage of making this determination at such a low pressure?

Solution

(i) $p_{tr} = 2680\,Pa,\ p_T = 4870\,Pa$

Using $T = 273.16\frac{p_T}{p_{tr}}$

$$= 273.16 \times \frac{4870}{2680} = 496.38\,K$$

(ii) At low pressure, real gas behaves like ideal gas and the value of $\frac{p_T}{p_{tr}}$ does not depend on the type of gas. The advantage is that all real gases would give the same value for $\frac{p_T}{p_{tr}}$.

2. (a)Distinguish between an empirical scale of temperature and thermodynamic temperature
   (b)Describe how an empirical Celsius scale of temperature based on expansion of a liquid may be set up.
   (c) Thermodynamic temperatures may be found from measurements with a constant-volume gas thermometer. Give a labelled sketch of such a thermometer and explain how the readings are taken.

(d)When the bulb of a constant-volume gas thermometer was placed in a liquid bath, the pressure in the thermometer was $1.95 \times 10^5 Pa$. When the bulb was maintained at the triple point of water, the pressure was $1.74 \times 10^5\ Pa$. Find a value for the temperature of the liquid.
The experiment was then repeated with a small mass of gas in the thermometer and the corresponding pressure readings were $5.26 \times 10^4\ Pa$ and $4.71 \times 10^4\ Pa$. Why do these values lead to a different value for the temperature? What procedure should be followed to obtain the thermodynamic temperature of the path?

Solution

(a) An empirical scale of temperature is based on experimental results using a thermometric property. A thermodynamic scale of temperature is not based on any physical property or experimental results but uses the properties of an ideal gas i.e. the variation of pV with temperature

(b) Let V_0 = volume of liquid at ice point

V_{100} = volume of liquid at steam point

V_θ = volume of liquid at unknown temperature θ

$$\theta = \frac{V_\theta - V_0}{V_{100} - V_0} \times 100\ °C$$

(c) Constant-volume gas thermometer - refer to notes

(d) Temperature of liquid,

$$T = 273.16\frac{p_T}{p_{tr}}$$

$$= 273.16 \times \frac{1.95 \times 10^5}{1.74 \times 10^5}$$

$$= 306.13\ K$$

With a smaller mass of gas

Temperature of liquid,

$$T_1 = 273.16 \times \frac{5.26 \times 10^5}{4.71 \times 10^5}$$

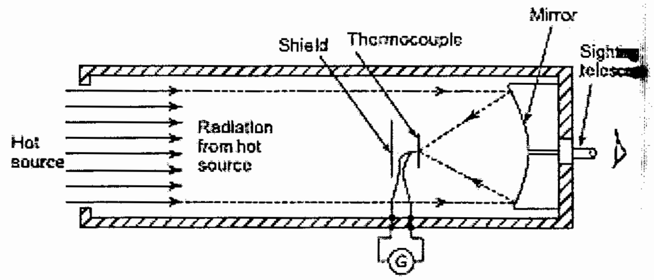
$$= 305.06\ K$$

Measurement of very high temperatures

The platinum resistance and gas thermometers are standard instruments which can be used for measuring high temperatures. These instruments cannot be used to measure temperatures above 1500 °C as the bulb containing the wire or gas would melt if it came into contact with the very hot object.

The laws of radiation are used in pyrometers for measuring very high temperatures. Pyrometers are used at glass works, at kilns for making bricks, and at steel and iron works. Here manufacturing processes must be regulated at certain known high temperatures.

Total radiation pyrometer



- Radiant energy from a hot source, such as a furnac is focused on to the hot junction of a thermocoupl after reflection from a concave mirror.
- The temperature rise recorded by the thermocou depends on the amount of radiant energy receiv which in turn depends on the temperature of the source.
- The galvanometer G shown connected to thermocouple records the current which res from the e.m.f. developed and may be calibrate give a direct reading of the temperature of the source.
- The thermocouple is protected from direct radia by a shield as shown and the hot source may viewed through the sighting telescope.
- For greater sensitivity, a thermopile may be use thermopile being a number of thermocoup connected in series.

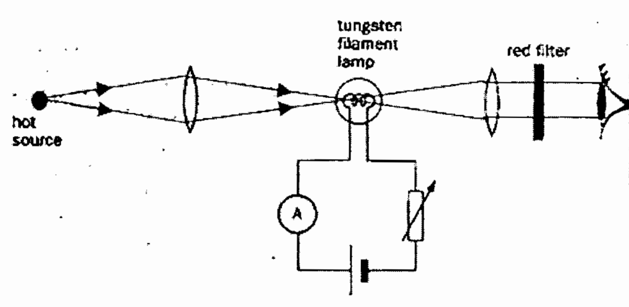
Note: Total radiation pyrometers are used to meas temperature in the range 700 °C to 2000 °C.

Optical pyrometer

When the temperature of an object is rais sufficiently, two visual effects occur i.e. the obje appears brighter and there is a change in colour of th light emitted.

These effects are used in the optical pyrometer whe a comparison or matching is made between the brigh ness of the glowing hot source and the light from filament of known temperature.

The most frequently used optical pyrometer is th disappearing filament pyrometer as shown below.



- A filament lamp is built into a telescope arrangement which receives radiation from a hot source, an image of which is seen through an eyepiece. A red filter is incorporated as a protection to the eye
- The current flowing through the lamp is controlled by a variable resistor.
- As the current is increased, the temperature of the filament increases and its colour changes.
- When viewed through the eyepiece, the filament of the lamp appears superimposed on the image of the radiant energy from the hot source.
- The current is varied until the filament glows as brightly as the background.
- It will then merge into the background and seem to disappear.
- The current required to achieve this is a measure of the temperature of the hot source and the ammeter can be calibrated to read the temperature directly.

Note: Optical pyrometers may be used to measure temperatures up to, and even in excess of 3000 °C.

Advantages of pyrometers

- There is no practical limit to the temperature that a pyrometer can measure.
- A pyrometer need not be brought directly into the hot zone and so is free from the effects of heat and chemical attack that can often cause other measuring devices to deteriorate in use.
- Very fast rates of change of temperature can be followed by a pyrometer.
- The temperature of moving bodies can be measured.
- The lens system makes the pyrometer virtually independent of its distance from the source.

Disadvantages of pyrometers

- A pyrometer is often more expensive than other temperature measuring devices.
- A direct view of the heat process is necessary.
- Manual adjustment is necessary.
- A reasonable amount of skill and care is required in calibrating and using a pyrometer. For each new measuring situation, the pyrometer must be re-calibrated.
- The temperature of the surroundings may affect the reading of the pyrometer and such errors are difficult to eliminate.

Self-Evaluation exercise

1. (a) Explain what is meant by a scale of temperature and how a temperature is defined in terms of a specified property.
   (b) When a particular temperature is measured on scales based on different thermometric properties, it has a different numerical value on each scale except at certain points. Explain why this is so and state
   (i) at what points the values agree, and
   (ii) what scale of temperature is used as standard
   (c) Explain the principles of two different types of thermometer, one of which is suitable for measuring a rapidly varying temperature and the other for measuring a steady temperature whose value is required to a high degree of accuracy. Give reasons for your choice of thermometer in each case.
2. The pressure in a constant-volume gas thermometer at the triple point of water, 273.16 K, is 720 mmHg. Calculate the temperatures on the thermodynamic temperature scale when the pressures are respectively (i) 770 mmHg, (ii) 40 mmHg.
   [Ans: (i) 292.1 K (ii) 15.2 K]
3. A platinum resistance thermometer has a resistance of 2.50 ohms at the triple point of water. What is the temperature on the thermodynamic scale when the resistance is (i) 2.45 ohms, (ii) 2.85 ohms?
   [Ans: (i) 267.7 (ii) 311.4 K]
4. A thermocouple thermometer has an e.m.f. of 2.0 millivolts at the lower fixed point of the Celsius scale and an e.m.f. of 2.2 millivolts at the upper fixed point. Calculate the temperature on the Celsius scale when the e.m.f. is (i) 1.8 millivolts, (ii) 2.3 millivolts.
   [Ans: (i) −100 °C (ii) 150 °C]
5. Explain how a Celsius scale of temperature may be defined in terms of a property of a substance.
   Discuss the desirable features of a thermometric property, illustrating your answer by reference to (a)the expansion of alcohol in a glass tube, (b) the resistance of platinum.
   Describe how you would calibrate a constant-volume air thermometer and compare its reading with that of a mercury-in-glass thermometer at about 50 °C.
6. A bath of oil is maintained at a steady temperature of about 180 °C which is measured both with a platinum resistance thermometer and a mercury-in-glass thermometer. Explain why you would expect the temperatures indicated by the two thermometers to be different. At what temperatures would the two thermometers show the same value?

7. Distinguish between heat and temperature and explain what is meant by the statement that the temperature of a body is t °C on the scale of a certain thermometer. Explain how you would use a piece of resistance wire as a thermometer, and describe how you would determine the readings corresponding to the fixed points of its scale.

8. Explain the principle underlying the establishment of a centigrade temperature scale in terms of some suitable physical property. What type of thermometer would you choose for use in experiments involving (a) the plotting of a cooling curve for naphthalene in the region of its melting-point, (b) finding the boiling-point of oxygen, (c) the measurement of the thermal conductivity of a small crystal? In each instance give reasons for your choice.
If the resistance R_t of the element of a resistance thermometer at a temperature of t °C on the ideal gas scale is given by $R_t = R_0(1 + At + Bt^2)$, where R_0 is the resistance at 0 °C and A and B are constants such that $A = -6.50 \times 10^3\ B$, what will be the temperature on the scale of the resistance thermometer when $t = 50$ °C? [Ans: 50.4 °C]

9. On what evidence do you accept the statement that there is an absolute zero of temperature at about −273 °C?
In a special type of thermometer, a fixed mass of gas has a volume of 100.0 cm³ and a pressure of 81.6 cm of mercury at the ice point, and volume 124.0 units with pressure 90.0 units at the steam point. What is the temperature when its volume is 120.0 units and pressure 85.0 units, and what value does the scale of this thermometer give for absolute zero? Explain the principle of your calculation.
[Ans: 68 °C, −272 °C]

10. Give a brief account of the principles underlying the establishment of a scale of temperature and explain precisely what is meant by the statements that the temperature of a certain body is (a) t °C on the constant-volume air scale, (b) t_p°C on the platinum resistance scale, and (c) t_T °C on the Cu-Fe thermocouple scale. Why are these three temperatures usually different?
Describe an optical pyrometer and explain how it is used to measure the temperature

11. Explain what is meant by a change in temperature of 1 °C on the scale of a platinum resistance thermometer.
Draw and label a diagram of a platinum resistance thermometer together with a circuit in which it is used.
Give two advantages of this thermometer a[nd] explain why, in its normal form, it is unsuitable f[or] measurement of varying temperatures.
The resistance R_t of platinum varies with th[e] temperature 1 °C as measured by a constant-volu[me] gas thermometer according to the equation
$$R_t = R_0(1 + 8000\alpha t - \alpha t^2)$$
where α is a constant. Calculate the temperature o[n] the platinum scale corresponding t400 °C on this ga[s] scale.
[Ans: 385 °C]

12. A platinum resistance thermometer is calibrated i[n] ice and steam. It is then used to find the temperatur[e] of an oil bath. The values of its resistance are
at ice point: 25.60 Ω
at steam point: 35.60 Ω
at temperature of oil bath: 45.35 Ω
What is the temperature of the oil bath in °C?
[Ans: 177.5 °C]

13. A resistance thermometer has a resistance o[f] 9.97 Ω at the ice point and 14.04 Ω at the steam point. Find the temperature on the centigrade scale of this thermometer when its resistance is 11.51 Ω. If the least detectable change of resistance is 0.01 Ω, what is the least change of temperature that can be detected by this thermometer?
[Ans: 37.8 °C, 0.246 °C]

14. Explain how a centigrade scale of temperature is defined
Describe the working principle of a resistance thermometer. Explain how the readings on the thermometer are obtained at the fixed points of the centigrade temperature scale
The resistance R_t of an aluminium wire at a temperature measured by a mercury-in-glass thermometer is given by
$$R_t = R_0(1 + \alpha t + \beta t^2)$$
where $\alpha = 4.46 \times 10^{-3}$ °C^{-1} and $\beta = 1.8 \times 10^{-6}$ °C^{-1}. The wire is used in a resistance thermometer. Find the temperature recorded by this thermometer when the reading of the mercury thermometer is 250 °C
[Ans: 265 °C]

15. The value of the property X of a certain substance is given by
$$X_t = X_0 + 0.50t + (2 \times 10^{-4})t^2$$
where t is the temperature in degrees Celsius measured on a gas thermometer scale. What would be the Celsius temperature defined by the property X which corresponds to a temperature of 50 °C on the gas thermometer scale? [Ans: 49.04 °C]

16. A thermocouple has the cold junction immersed in an ice-water mix at 0 °C. When the hot junction is in

boiling water, the e.m.f. is 1.65 mV. Estimate the temperature of the hot junction when the e.m.f. is 1.47 mV. [Ans: 89.1 °C]

17. The pressure recorded in a certain constant-volume gas thermometer at the triple point of water and at the boiling point of liquid were 600 $mmHg$ and 800 $mmHg$ respectively. What is the apparent temperature of the boiling point? However, it was found that the volume of the thermometer increased by 1% between the two temperatures. Obtain a more accurate value of the boiling point.
[Ans: 364.21 K, 367.86 K]

18. The resistance of the element in a platinum resistance thermometer is 6.75 Ω at the triple point of water and 7.166 Ω at room temperature. What is the temperature of the room on the scale of the resistance thermometer? The triple point of water is 273.16 K. State one assumption you have made.
[Ans: 290 K]

19. The resistance of a certain platinum resistance thermometer is found to be 2.56 Ω at 0 °C, 3.56 Ω at 100 °C and 6.78 Ω at 444.5 °C, the boiling point of Sulphur on the gas scale.
   (i) Calculate the boiling point of Sulphur on the platinum resistance thermometer
   (ii) The thermometer is immersed in a given liquid and its resistance is observed to be 5.05 Ω. Determine the temperature of the liquid on the platinum resistance thermometer.
[Ans: (i) 422 °C (ii) 249 °C]

20. A resistance R_θ of a platinum resistance thermometer is given by

$$R_\theta = R_0(1 + a\theta + b\theta^2)$$

where $a = 1.3 \times 10^{-2}\,K^{-1}$, $b = 1.33 \times 10^{-6}\,K^{-1}$, R_0 being the resistance at 0 °C. Calculate the temperature of the thermometer when the temperature on the constant volume gas thermometer is 300 °C. [Ans: 306.2 °C]

21. The pressure recorded by a constant volume gas thermometer at kelvin temperature is $4.8 \times 10^{-4}\,Nm^{-2}$. Calculate T if the pressure at triple point of water is $4.24 \times 10^{4}\,Nm^{-2}$.
[Ans: 312.2 K]

22. A resistance thermometer has a resistance of 21.42 Ω at the ice point. 29.01 Ω at the steam point and 28.11 Ω at some unknown temperature θ. Find θ. [Ans: 87 °C]

23. (a) A fixed mass of a gas at constant pressure has a volume of 200 cm^3 at a temperature of pure melting ice and 273.2 cm^3 at the temperature of boiling water at standard pressure. Calculate the temperature which corresponds to 525.1 cm^3 in the same thermometer.
(b) The resistance of a platinum resistance wire is 2 Ω at the ice point and 2.73 Ω at steam point. What temperature on this thermometer corresponds to a resistance of 8.43 Ω?
[Ans: (a) 444.13 K (b) 881 °C]

24. The table below refers to observations of a particular room temperature using two types of thermometer.

	Resistance of resistance thermometer	Pressure recorded by constant volume gas thermometer
Steam point	75.0 Ω	$1.10 \times 10^{7}\,Nm^{-2}$
Ice point	63.0 Ω	$8.00 \times 10^{6}\,Nm^{-2}$
Room temperature	64.992 Ω	$8.51 \times 10^{6}\,Nm^{-2}$

Calculate the room temperature on the scale of the resistance thermometer and on the scale of the constant volume gas thermometer.
Why do these values differ slightly?
[Ans: 16.6 °C, 17.0 °C]

25. A certain gas thermometer has a bulb of volume 50 cm^3 connected by a capillary tube of negligible volume to a pressure gauge of volume 5.0 cm^3. When the bulb is immersed in a mixture of ice and water at 0°C, with the pressure gauge at room temperature (17°C), the gas pressure is 700 mmHg. What will be the pressure when the bulb is raised to a temperature of 50°C if the gauge is maintained at room temperature? You may assume that the gas is ideal and that the expansion of the bulb can be neglected. [Ans: 815 mmHg]

26. If the resistance of a platinum thermometer is 1.50 ohms at 0°C, 2.060 ohms at 100°C and 1.788 ohms at 50°C on the gas scale, what is the difference between the numerical values of the latter temperature on the two scales? [Ans: 0.89 °C]

27. The resistance R_t of a platinum wire at temperature t °C, measured on the gas scale, is given by $R_0(1 + at + bt^2)$, where $a = 4.0 \times 10^{-3}$ and $b = -6.0 \times 10^{-7}$. What temperature will the platinum thermometer indicate when the temperature on the gas scale is 300°C? [Ans: 309 °C]

HEAT CAPACITY AND LATENT HEAT

Different substances have different capacities of storing internal energy. If you heat a beaker of water with a Bunsen flame, it may take 10 minutes to bring it to boil but an equal mass of iron in the same flame would rise to the same temperature in less than 2 minutes. Therefore, we find that different materials require different quantities of heat to raise the temperature of a given mass of the material by a certain number of degrees centigrade. A term used in relationship to the quantity of heat mentioned is the specific heat capacity of the material.

Heat capacity (C)

The heat capacity C of a body is the quantity of heat required to raise the temperature of the body by 1 kelvin.

Specific heat capacity (c)

The specific heat capacity of a substance is the quantity of heat required to raise the temperature of 1 kg mass of the substance by 1 kelvin

S.I unit for specific heat capacity is $J\ kg^{-1}K^{-1}$

The specific heat capacity of water is relatively high compared to other common substances. A relatively small amount of water absorbs a large amount of heat for a corresponding small temperature rise.

This property of water is the reason for its use as a cooling agent, such as in the cooling system of a car and other engines. Water also takes a long time to cool. This accounts for the creation of sea breeze during the night. The climate of coastal states is affected by the water around it.

Relationship between C and c

$$C = mc$$

where m is the mass of the body

When the temperature of a body of mass m and specific heat capacity c increases by $\Delta\theta$, the heat absorbed

$$Q = mc\ \Delta\theta$$

On the other hand, if the temperature of a body decreases by $\Delta\theta$, the heat lost

$$Q = mc\ \Delta\theta$$

Electrical heating

The heat supplied per second by an electric heater is given by VI where V is the p.d across the heater and I is the current through it. This results in the transfer of thermal energy into the surrounding material at a rate referred to as the "power dissipated"

$$\text{Power} = V \times I$$

In time t seconds, the energy Q transferred by the heater is

$$Q = IVt$$

Examples

1. A solid copper block of mass 5.0 kg is heated for minutes exactly by an electric heater embedded in the block. A potential difference of 25 V is applied across the heater and the current is recorded as 2.0 A. If the temperature of the block rises by 10 K calculate the specific heat capacity of copper assuming that no heat escapes from the apparatus and that the heat capacity of the heater itself is negligible.

Solution

Heat supplied by heater = heat gained by copper

$$IVt = mc\Delta\theta$$

$$25 \times 2.0 \times 7 \times 60 = 5.0 \times c \times 10$$

$$c = 420\,J\,kg^{-1}\,K^{-1}$$

2. 50 g of water at 12 °C is placed in a copper calorimeter which weighs 0.10 kg. An electric heater coil of negligible thermal capacity is immersed in the water. With 7.0 V across the heater producing a steady current of 1.0 A for exactly 6 minutes, a final temperature of 22 °C was obtained. If the heat loss to the surroundings is negligible, what is the value of the specific heat capacity for water? (SHC of copper $= 420\,J\,kg^{-1}\,K^{-1}$)

Solution

Heat supplied by heater, $Q = IVt$

$$= 7.0 \times 1.0 \times 6.0 \times 60 = 2520\,J$$

Heat absorbed by water, $Q_w = mc_w\Delta\theta$

$$= 50 \times 10^{-3} \times c_w(22 - 12)$$

$$= 0.5c_w$$

Heat absorbed by calorimeter, $Q_c = 0.1 \times 420 \times 10$

$$= 420$$

Assuming no heat losses to the surroundings, then

$$Q = Q_w + Q_c$$

$$2520 = 0.5c_w + 420$$

$$c_w = \frac{2100}{0.5} = 4200\,J\,kg^{-1}\,K^{-1}$$

3. A block of copper of mass 0.50 kg at an initial temperature of 77 °C is placed in 0.40 kg of water at 30 °C. When thermal equilibrium is attained, the temperature of the mixture is 35 °C. How much heat is lost by the copper block and how much heat is absorbed by the water? Comment on your answers. (specific heat capacities: water $= 4200\,J\,kg^{-1}K^{-1}$. copper $= 400\,J\,kg^{-1}K^{-1}$)

Solution

Heat lost from the copper $= mc\ \Delta\theta$

$$= 0.50 \times 400 \times (77 - 35)$$

HEAT CAPACITY AND LATENT HEAT

Different substances have different capacities of storing internal energy. If you heat a beaker of water with a Bunsen flame, it may take 10 minutes to bring it to boil but an equal mass of iron in the same flame would rise to the same temperature in less than 2 minutes. Therefore, we find that different materials require different quantities of heat to raise the temperature of a given mass of the material by a certain number of degrees centigrade. A term used in relationship to the quantity of heat mentioned is the specific heat capacity of the material.

Heat capacity (C)

The heat capacity C of a body is the quantity of heat required to raise the temperature of the body by 1 kelvin.

Specific heat capacity (c)

The specific heat capacity of a substance is the quantity of heat required to raise the temperature of 1 kg mass of the substance by 1 kelvin

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This property of water is the reason for its use as a cooling agent, such as in the cooling system of a car and other engines. Water also takes a long time to cool. This accounts for the creation of sea breeze during the night. The climate of coastal states is affected by the water around it.

Relationship between C and c

$$C = mc$$

where m is the mass of the body

When the temperature of a body of mass m and specific heat capacity c increases by $\Delta\theta$, the heat absorbed

$$Q = mc\,\Delta\theta$$

On the other hand, if the temperature of a body decreases by $\Delta\theta$, the heat lost

$$Q = mc\,\Delta\theta$$

Electrical heating

The heat supplied per second by an electric heater is given by VI where V is the p.d across the heater and I is the current through it. This results in the transfer of thermal energy into the surrounding material at a rate referred to as the "power dissipated"

$$\text{Power} = V \times I$$

In time t seconds, the energy Q transferred by the heater is

$$Q = IVt$$

Examples

1. A solid copper block of mass 5.0 kg is heated for 7 minutes exactly by an electric heater embedded in the block. A potential difference of 25 V is applied across the heater and the current is recorded as 2.0 *A*. If the temperature of the block rises by 10 K, calculate the specific heat capacity of copper, assuming that no heat escapes from the apparatus and that the heat capacity of the heater itself is negligible.

Solution

Heat supplied by heater = heat gained by copper

$$IVt = mc\Delta\theta$$

$$25 \times 2.0 \times 7 \times 60 = 5.0 \times c \times 10$$

$$c = 420\,J\,kg^{-1}\,K^{-1}$$

2. 50 g of water at 12 °C is placed in a copper calorimeter which weighs 0.10 kg. An electric heater coil of negligible thermal capacity is immersed in the water. With 7.0 V across the heater producing a steady current of 1.0 A for exactly 6 minutes, a final temperature of 22 °C was obtained. If the heat loss to the surroundings is negligible, what is the value of the specific heat capacity for water? (SHC of copper $= 420\,J\,kg^{-1}\,K^{-1}$)

Solution

Heat supplied by heater, $Q = IVt$

$$= 7.0 \times 1.0 \times 6.0 \times 60 = 2520\,J$$

Heat absorbed by water, $Q_w = mc_w\Delta\theta$

$$= 50 \times 10^{-3} \times c_w(22 - 12)$$

$$= 0.5c_w$$

Heat absorbed by calorimeter, $Q_c = 0.1 \times 420 \times 10$

$$= 420$$

Assuming no heat losses to the surroundings, then

$$Q = Q_w + Q_c$$

$$2520 = 0.5c_w + 420$$

$$c_w = \frac{2100}{0.5} = 4200\,J\,kg^{-1}\,K^{-1}$$

3. A block of copper of mass 0.50 kg at an initial temperature of 77 °C is placed in 0.40 kg of water at 30 °C. When thermal equilibrium is attained, the temperature of the mixture is 35 °C. How much heat is lost by the copper block and how much heat is absorbed by the water? Comment on your answers. (specific heat capacities: water $= 4200\,J\,kg^{-1}K^{-1}$, copper $= 400\,J\,kg^{-1}K^{-1}$)

Solution

Heat lost from the copper $= mc\,\Delta\theta$

$$= 0.50 \times 400 \times (77 - 35)$$

$= 8400\,J$

Heat gained by water $= mc\,\Delta\theta$

$= 0.40 \times 4200 \times (35 - 30)$

$= 8400\,J$

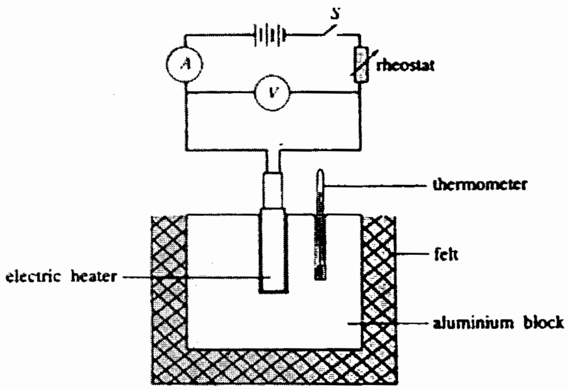
$\therefore$ Heat gained = heat lost

This is consistent with the principle of conservation of energy.

Determination of specific heat capacity – Electrical methods

1. Solids

The specific heat capacity of a solid such as aluminium can be determined using an electrical method as shown below



- The solid is in the form of aluminium cylinder with a hole in the middle where an electric heater exactly fits.
- Another hole in the cylinder is for the thermometer.
- The mass m of the aluminium block is first measured and recorded.
- The aluminium block is lagged with felt to reduce heat loss to the surrounding.
- The initial temperature θ_1 of the block is measured and recorded
- The current through the heater is switched on for a measured time and the voltmeter and ammeter readings are obtained.
- After the measured time, the current is switched off and the highest temperature θ_2 of the aluminium block is recorded.

If $I =$ current

$V =$ potential difference across the heater

$t =$ heating time in seconds

$c =$ specific heat capacity of the block

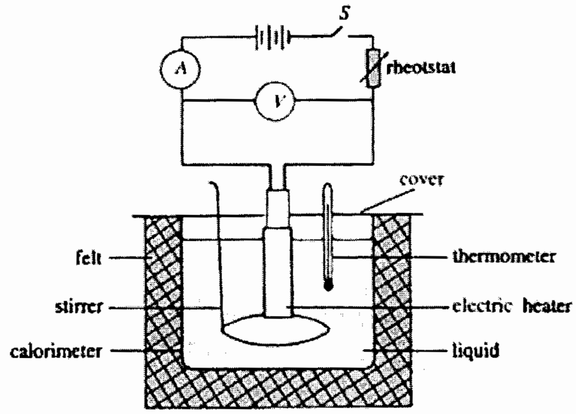
Electrical energy supplied by heater $= IVt$

Heat gained by the block $= mc(\theta_2 - \theta_1)$

Hence $\quad mc(\theta_2 - \theta_1) = IVt$

$$c = \frac{IVt}{mc(\theta_2 - \theta_1)}$$

2. Liquids



- A calorimeter, usually made of copper or aluminium of known specific heat capacity is used to contain the liquid.
- The mass of the calorimeter when empty is first measured.
- Switch S is closed
- A measured mass m of the liquid of initial temperature θ_1 is then heated using an electric heater.
- The rheostat is adjusted to maintain a constant current while the liquid is stirred.
- The voltmeter reading V and ammeter reading I are recorded.
- After a measured time t the heater is switched off and the highest temperature θ_2 of the liquid recorded.

Heat supplied heater $= IVt$

Heat gained by liquid and calorimeter

$= mc_l(\theta_2 - \theta_1) + Mc_c(\theta_2 - \theta_1)$

where $c_l =$ specific heat capacity of liquid

$c_c =$ specific heat capacity of calorimeter

Heat gained by liquid and calorimeter

= heat supplied by heater

$$mc_l(\theta_2 - \theta_1) + Mc_c(\theta_2 - \theta_1) = IVt$$

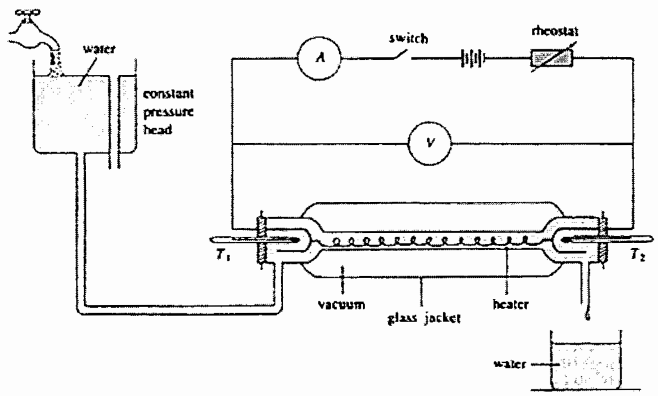
$$c_l = \frac{IVt}{m(\theta_2 - \theta_1)} - \frac{Mc_c}{m}$$

Assumption made

Heat loss to the surrounding is negligible

Continuous flow method

This method is used to measure the specific heat capacity of water and liquids that are readily available in large quantities.



- Water from a constant pressure head flows at a constant rate through an electric heater.
- The switch is closed
- The inlet temperature of the water is measured by the thermometer T_1.
- As the water flows through the heater which is surrounded by a vacuum in a glass jacket, it is heated and the temperature of the water at the outlet is measured by the thermometer T_2.
- After the current has flowed for a few minutes, a steady state is reached when the inlet and outlet temperatures remain steady at θ_1 and θ_2 respectively.
- During the steady state, the temperature of the apparatus is constant and thus the electrical energy supplied is used for heating the water and some heat is lost to the surrounding.
- No heat is used to heat up the apparatus further.
- In this steady state, the temperatures θ_1 and θ_2, ammeter reading I and voltmeter reading V are recorded.
- The rate of flow of water m is determined by collecting and weighing the mass M from the outlet in a measured time t. Therefore, $m = \frac{M}{t}$
- Electrical power supplied by heater = rate of heating the water + rate of heat loss to the surrounding

  $$IV = mc(\theta_2 - \theta_1) + h \text{ ...... (i)}$$

  where c = specific heat capacity of water
- The rate of heat loss h is directly proportional to the temperature difference between the temperature of the apparatus and the temperature of the surrounding.
- To eliminate h, the experiment is repeated for a different flow of water, but the current and potential difference across the heater are adjusted so that the temperature of the water at the inlet θ_1 and at the outlet θ_2 remains unchanged.

If m' = new rate of flow

I' = new ammeter reading

V' = new voltmeter reading

then $I'V' = m'c(\theta_2 - \theta_1) + h$ ...... (ii)

Equation (i) – (ii);

$$IV - I'V' = (m - m')c(\theta_2 - \theta_1)$$

$$\therefore c = \frac{IV - I'V'}{(m - m')(\theta_2 - \theta_1)}$$

Advantages of continuous flow method

1. The difference in temperature between the ends can be measured accurately since temperatures are only recorded when the steady state is attained.
2. The heat capacity of the calorimeter need not to be used in the calculation
3. For the same reason, more accurate thermometers such as resistance thermometers can be used
4. The loss of heat to the surrounding is taken into consideration and eliminated from the calculation by repeating the experiment but maintaining the inlet temperature θ_1 and outlet temperature θ_2 unchanged.
5. The cooling correction is negligible except for very accurate work.
6. The apparatus can be used for determination of the specific heat capacity of the liquid at different temperatures

Disadvantage of this method

- A large amount of liquid is required

Example

In a continuous-flow calorimeter, the readings were: 6.0 V, 2.1 A, $\theta_1 = 17.0$ °C, $\theta_2 = 22.0$ °C, 35 gmin^{-1} followed by 4.0 V, 1.4 A, $\theta_1 = 17.0$ °C, $\theta_2 = 22.0$ °C 15 g min^{-1}.

Obtain a value for the specific heat capacity of the liquid and the rate of loss of heat to the surroundings.

Solution

Using $IV = mc(\theta_2 - \theta_1) + h$

$$6.0 \times 2.1 = \left(\frac{35}{60} \times c \times 5.0\right) + h \text{ .... (i)}$$

using $I'V' = m'c(\theta_2 - \theta_1) + h$

$$4.0 \times 1.4 = \left(\frac{15}{60} \times c \times 5.0\right) + h \text{ .... (ii)}$$

(i) – (ii)

$$12.6 - 5.6 = \frac{20}{60} \times c \times 5.0$$

$$c = \frac{7.0}{5.0} \times \frac{60}{20} = 4.2\,J\,g^{-1}\,K^{-1}$$

Substituting the value of c in (i)

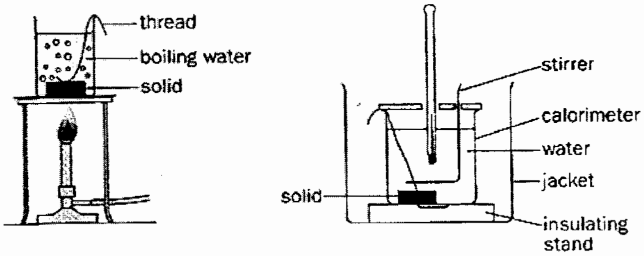
$$12.6 = \left(\frac{35}{60} \times 4.2 \times 5.0\right) + h$$

$$h = 12.6 - 12.25 = 0.35\,W$$

Determination of specific heat capacity – Method of mixtures

1.Solids

If a solid is relatively small, the heat capacity and the specific heat capacity its material can be found by a method of mixtures.



- A solid whose specific heat capacity c_s is to determined is weighed and its mass m_s is measured and recorded.
- The solid is heated to a temperature θ_3 by boiling water or steam for 10 minutes
- A known mass of cold water, m_w is poured into a calorimeter whose mass m_c and specific heat capacity c_c are known.
- The temperature θ_1 of water and calorimeter is measured and recorded.
- The heated solid is quickly transferred to the water in the calorimeter.
- The mixture is stirred and the final temperature θ_2 of the solid, water and calorimeter is measured and recorded.

Assuming no heat loss to the surrounding;

Heat lost by solid = heat gained by water and calorimeter

$$m_s c_s(\theta_3 - \theta_2) = m_w c_w(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

$$c_s = \frac{m_w c_w(\theta_2-\theta_1)+m_c c_c(\theta_2-\theta_1)}{m_s(\theta_3-\theta_2)}$$

$$\text{Or } c_s = \frac{(m_w c_w+m_c c_c)(\theta_2-\theta_1)}{m_s(\theta_3-\theta_2)}$$

Thus, the specific heat capacity of the solid can be obtained.

2. Liquids

Here, the same procedure is used as in for solids, but the water is replaced by the liquid whose specific heat capacity is to be determined.

- The mass m_l of a liquid whose specific heat capacity is to be determined is measured and recorded.
- The liquid is then transferred to a metal calorimeter of known mass m_c and specific heat capacity c_c
- The initial temperature θ_1 of the calorimeter and the liquid is measured and recorded
- A solid block of known mass m_s and specific heat capacity c_s is heated to a temperature θ_3 by boiling water or steam.
- The solid block is quickly transferred to the liquid and the mixture is stirred
- The final temperature θ_2 of the mixture is measured and recorded.

Assuming no heat loss to the surrounding

Heat lost by solid = heat gained by liquid and calorimeter

$$m_s c_s(\theta_3 - \theta_2) = m_l c_l(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

$$c_l = \frac{m_s c_s(\theta_3-\theta_2)-m_c c_c(\theta_2-\theta_1)}{m_l(\theta_2-\theta_1)}$$

The specific heat capacity of the liquid can thus be defined.

Note:

1. In place of the solid, water whose specific heat capacity is known can be heated to a certain temperature and then transferred to the calorimeter containing liquid whose specific heat capacity is to be determined.
2. For inflammable liquids, ice is dropped into the liquid and the final temperature is observed after all the ice has melted.

Examples

1. 200 g of copper of specific heat capacity $0.4\ kJ\ kg^{-1}\ K^{-1}$ is heated to 100 °C. It is then quickly transferred to 100 g of water at 10 °C inside a calorimeter of heat capacity $100\ J\ K^{-1}$. Calculate the final water temperature ($c_w = 4.2\ kJ\ kg^{-1}\ K^{-1}$)

<u>**Solution**</u>

Let θ be the final temperature

Heat lost by copper = $0.2 \times 400(100 - \theta)$

Heat gained by water and calorimeter

$$= (0.08 \times 4200(\theta - 10) + 100\ (\theta - 10)$$

$$8000 - 80\,\theta = 336\,\theta - 3360 + 100\,\theta - 1000$$

$$516\,\theta = 12360$$

$$\theta = 23.95\ ℃$$

2.21.0 g of a liquid at 60.0 °C is mixed into 100 g of water at 12.5 ℃ which is already in a metal calorimeter of mass 70.0 g and specific heat capacity $400\ J\ kg^{-1}\ K^{-1}$. If heat escape to the surroundings maybe neglected, calculate the expected new temperature of the water given that the specific heat capacity is $4200\ J\ kg^{-1}\ K^{-1}$ for water and $4000\ J\ kg^{-1}\ K^{-1}$ for the liquid

<u>**Solution**</u>

Let the new temperature of water be θ °C

The liquid cools from 60 ℃ to the final temperature θ ℃ while the calorimeter and water start at 12.5 ℃ and rise to θ ℃

Heat given out by liquid = $\frac{21}{1000} \times 4000(60 - \theta)$

Heat absorbed by water and calorimeter

$$= \frac{100}{1000} \times 4200(\theta - 12.5) + \frac{70}{100} \times 400(\theta - 12.5)$$
$$84(60 - \theta) = 420(\theta - 12.5) + 28(\theta - 12.5)$$
$$5040 - 84\theta = 420\theta - 5250 + 28\theta - 350$$
$$\theta = \frac{5040+5250+350}{420+28+84}$$
$$\theta = 20\ °C$$

3. 200 g of water and an equal volume of another liquid of mass 250 g are placed in turn in the same calorimeter of mass 100 g and specific heat capacity 0.4 $kJ\ kg^{-1}\ K^{-1}$. The liquids which are constantly stirred are found to cool from 60 °C to 40 °C in 180 s and 140 s respectively. Find the specific heat capacity of the liquid.

Solution

The average heat lost per second by the liquid and calorimeter while the temperature falls from 60 °C to 40 °C is equal to the average heat lost per second by the water and calorimeter from 60 °C to 40 °C

Average rate of loss of heat of liquid and calorimeter

$$= \frac{(0.25c+0.1\times400)(60-40)}{140} = \frac{(0.25c+40)(20)}{140}$$

Average loss of heat of water and calorimeter

$$= \frac{(0.2\times4200+0.1\times400)(60-40)}{180} = \frac{880\times20}{180}$$

Thus $\frac{(0.25c+40)(20)}{140} = \frac{880\times20}{180}$

$$45c + 7200 = 123200$$
$$c = 2577.8\ J\ kg^{-1}\ K^{-1}$$

Cooling correction

When a hot solid is dropped into a liquid inside a calorimeter, as in the method of mixtures, the temperature of the liquid begins to rise above that of the surroundings.

The liquid and calorimeter then lose heat to the surroundings.

It therefore follows that the observed final temperature is less than the final temperature had no heat been lost.

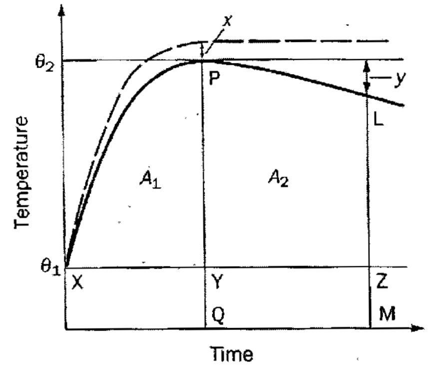
The temperature to be added to the observed final temperature to compensate for loss of heat is known as the "cooling correction"

Cooling correction is the extra temperature added to the observed maximum temperature of the mixture to compensate for the heat losses to the surroundings during temperature rise

A cooling correction is necessary for accuracy in the method of mixtures.

Estimation of cooling correction

- The temperature of the solid is measured at half-minute intervals just before the solid is added to the calorimeter and ending when the temperature has fallen by at least 1 K from its observed maximum value.
- A graph of temperature against time is plotted
- θ_1 is the intial temperature of the calorimeter and its contents while θ_2 is the observed maximum temperature.
- The dotted line shows how the temperature might have risen if there were no heat lost



- The cooling correction required is x. To obtain it, PQ is drawn through the top of the curve parallel to the temperature axis and similarly LM further along the curves so that y is 1 K.
- XYZ is then drawn through θ_1 parallel to the time axis
- The areas A_1 and A_2 are found by counting the squares on the graph paper
- The cooling correction is given by

$$x = \frac{A_1}{A_2} \times y$$

- The estimated maximum temperature is then $\theta_2 + x$

Newton's law of cooling

This applies when a body is cooling under conditions of **forced convection.**

It states that the rate of heat loss of a body is proportional to the difference in temperature between the body and its surroundings

$$\frac{dQ}{dt} = k(\theta - \theta_0)$$

where $\frac{dQ}{dt}$ = rate of heat loss to the surrounding

θ = temperature of the body

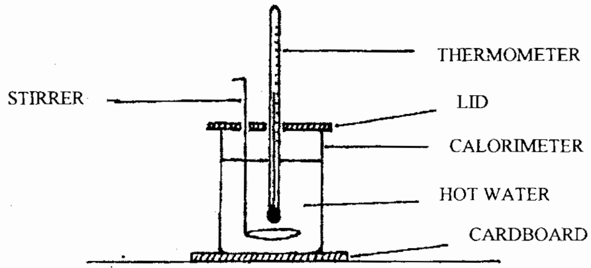
θ_0 = temperature of the surrounding

k = constant of proportionality whose value depends on both the nature and the area of the body's surface

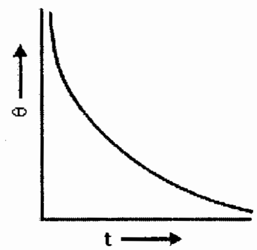
Note:

The law can be a good approximation for cooling under conditions of natural convection i.e. a body cooling in still air provided the excess temperature is not greater than 30 °C

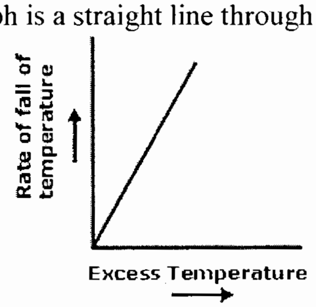
Experimental verification of Newton's law of cooling



- A calorimeter containing water heated to boiling point is placed on an insulating surface e.g. cardboard
- The temperature θ of the water is measured at one-minute intervals using a thermometer
- The water should be stirred gently prior to each temperature measurement.
- A graph of temperature against time is plotted



- The gradient of this graph at any temperature θ is the rate of fall of temperature at that value of θ and it is measured by constructing tangents to the curve at various values of θ
- The rate of fall of temperature is plotted against the excess temperature $\theta - \theta_0$
- The graph is a straight line through the origin



- This verifies Newton's law of cooling

Precautions

- Double walled enclosure should be used to maintain the surrounding at constant temperature
- Stirring should remain continuous for uniform cooling

Source of error

- Surrounding temperature may change

Examples

1. A metal cube takes 5 minutes to cool from 60 °C to 52 °C. How much time will it take to cool to 44 °C, if the temperature of the surroundings is 32 °C?

Solution

While cooling from 60 °C to 52 °C,

Rate of cooling $= \frac{60-52}{5} = 1.6\ °C/\text{ minute}$

$= \frac{1.6}{60}\ °C/second$

Average temperature excess $= 56 - 32 = 24°C$

According to Newton's law of cooling

$$\text{Rate of cooling} \propto \text{temperature excess}$$

$$\text{Rate of cooling} = k \times \text{temperature excess}$$

$$\frac{1.6}{60} = k \times 24$$

$$k = \frac{1}{900}$$

Let the time taken by the cube to cool from 52 °C to 44 °C be t

$$\text{Rate of cooling} = \frac{52-44}{t} = \frac{8}{t}$$

Average temperature while cooling $= \frac{52+44}{2} = 48°C$

Average temperature excess $= 48 - 32 = 16$ °C

From Newton's law of cooling,

$$\text{Rate of cooling} = k \times \text{temperature excess}$$

$$\frac{8}{t} = k \times 16$$

$$\frac{8}{t} = \frac{1}{900} \times 16$$

$$t = 450\ s$$

2. A copper calorimeter of mass 100 g containing 150 cm^3 of liquid of specific heat capacity 2.5 $kJ\ kg^{-1}\ K^{-1}$ and relative density 1.2 is found to cool at the rate of 2 °C per minute when its temperature is 50 °C above that of its surroundings. If the liquid is emptied out and 150 cm^3 of a liquid of specific heat capacity 1.7 $kJ\ kg^{-1}\ K^{-1}$ and relative density 0.9 are substituted, what will be the rate of cooling when the temperature is 40 °C above that of the surroundings? (specific heat capacity of copper = 0.4 $kJ\ kg^{-1}\ K^{-1}$)

Solution

Since density of water = 1 g cm^{-3}, density of liquid = 1.2 g cm^{-3}.

Mass of liquid $= 150 \times 1.2 = 180$ g $= 0.18\ kg$, specific heat capacity = 2500 J $kg^{-1}\ K^{-1}$

For calorimeter, mass = 100 g = 0.1 kg, specific heat capacity = 400 J $kg^{-1}\ K^{-1}$

Total heat lost per minute at 50 °C above the surroundings $= (0.18 \times 2500 + 0.1 \times 400)(2)$

$= 980\ J$

For second liquid,

Mass = 150 × 0.9 = 135 g = 0.135 kg, specific heat capacity = 1700 J kg^{-1} K^{-1}.

Let x °C per minute be the rate of cooling at 40 °C above surroundings.

Total heat lost per minute

$$= (0.135 \times 1700 + 0.1 \times 400)x$$
$$= 269.5x$$

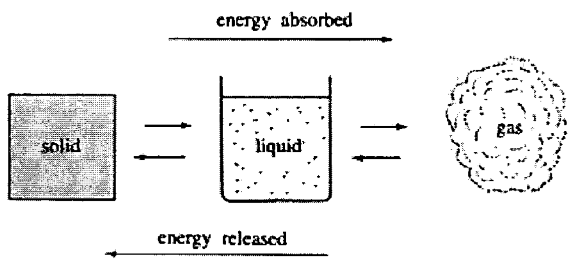
According to Newton's law of cooling;

$$\frac{269.5x}{980} = \frac{40}{50}$$
$$x = 2.91\ ℃\ \mathrm{min}^{-1}$$

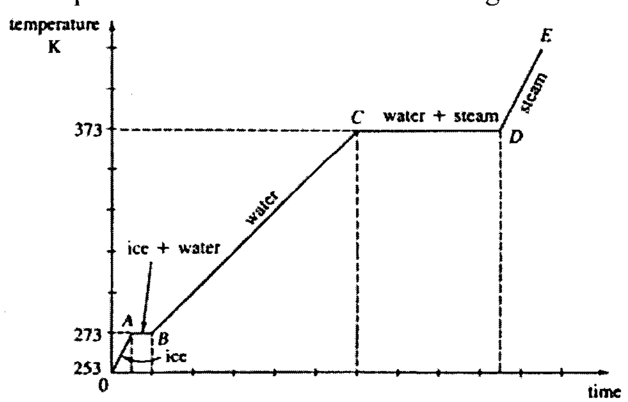
Change of state

There are three states of matter i.e. solid, liquid and gaseous. Matter can be changed from one state to another.

When heated, the rigid molecular structure in a solid breaks down to the liquid state. Further heating changes the liquid to the gaseous state. Conversely, when energy is extracted from a substance, it changes from a gas to liquid to a solid.



When a solid such as ice is heated at a constant late, its temperature varies as shown in the diagram below.



Along OA, the temperature of ice increases up to 273 K (0 °C). At 273 K, the temperature stops increasing even though heat is continuously supplied. Change of phase occurs along AB. The ice melts and is in thermal equilibrium with water.

At B, all the ice has melted. Further heating raises the temperature of water along BC.

At 373 K (100 °C), the temperature remains constant while more and more of the water becomes steam.

Any further heating of steam results in its temperature increasing, along DE.

Note:

Temperature remains constant during change of phase because during change of phase, the transfer of heat does not cause a change in the kinetic energy of the molecules.

During melting, the heat absorbed is used to break up the bonds between the particles. The particles are freed from their fixed positions and are able to vibrate and move.

During boiling, the heat absorbed is used to break the bonds between the particles and to do work against the atmospheric pressure when gaseous vapour enters into the atmosphere.

Melting

In a solid the strong attractions between the particles hold them tightly packed together. When a solid is heated, the particles gain kinetic energy and start to vibrate faster and faster. Further heating provides more energy until the particles start to break free of the structure. Although the particles are still loosely connected they are able to move around. At this point the solid is melting to form a liquid. The particles in the liquid are the same as in the solid but they have more energy.

To melt a solid, energy is required to overcome the attractions between the particles and to pull them apart. The energy is provided when the solid is heated up. The temperature at which something melts is called its **"melting point"**.

Evaporation

Within a liquid some particles have more energy than others. These "more energetic particles" may have sufficient energy to escape from the surface of the liquid as gas or vapour. Evaporation takes place at room temperature which is often well below the boiling point of the liquid.

Evaporation happens from the surface of the liquid. As the temperature increases, the rate of evaporation increases.

Evaporation is also assisted by windy conditions which help to remove the vapour particles from the liquid so that more escape.

Water in an open container evaporates. The water that disappears becomes water vapour in the air.

Evaporation is a change of state from liquid to gas that takes place at the surface of a liquid at any

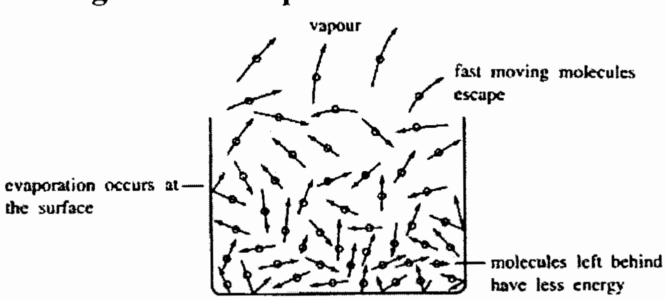
temperature. The rate of evaporation increases with temperature.

Boiling

If a liquid is heated, the particles are given more energy and move faster and faster expanding the liquid. The most energetic particles at the surface escape from the surface of the liquid as vapour as it gets warmer. Liquids evaporate faster as they heat up and more particles have enough energy to break away. The particles need energy to overcome the attractions between them. At normal atmospheric pressure, all materials have a specific temperature at which boiling occurs. This is called the "**boiling point**".

When a gas turns to a liquid (condenses) or a liquid turns to a solid (solidifies), the particles lose energy to the surroundings

Cooling effect of evaporation



In a liquid, the molecules move in all directions at different speeds. They collide with each other and transfer of energy takes place. Some molecules gain energy and others lose energy.

The fast-moving molecules have more kinetic energy. If a very fast molecule happens to be moving upwards when it is near the surface, it will have enough energy to escape from the force of attraction of the liquid molecules. It will leave the liquid and become a vapour molecule.

Since the more energetic molecules escape, molecules that remain in the liquid have lower kinetic energy. A liquid with molecules of less kinetic energy has a lower temperature. Thus evaporation causes cooling.

Latent heat, Q

This is the heat required to change the state of a substance from one form to another.

$$Q = ml$$

where

m = mass of the substance

l = specific latent heat of the body

Specific latent heat of fusion

This is the heat required to change 1 kg mass of a solid to a liquid at its melting point. S.I unit is $J\ kg^{-1}$

The specific latent heat of fusion of ice is $3.36 \times 10^5\ J\ kg^{-1}$

Specific latent heat of vaporization

This is the heat required to change 1 kg mass of a liquid to gaseous state at constant temperature.

S.I unit is $J\ kg^{-1}$

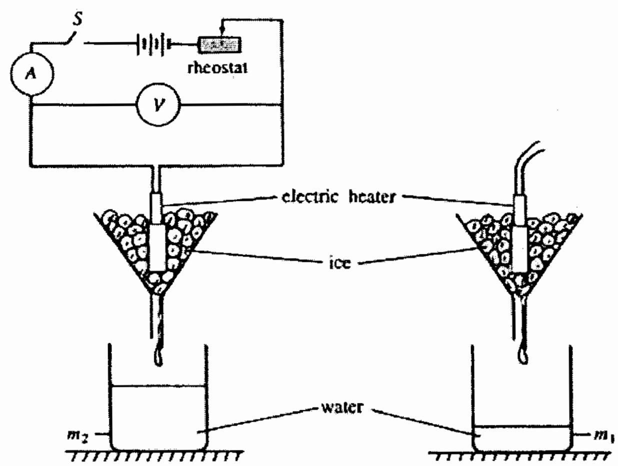
The specific latent heat of vaporization of water is $2.26 \times 10^6\ J\ kg^{-1}$

Why specific latent heat of vaporization is greater than that of fusion

The latent heat supplied to a solid is used to increase the potential energy of the molecules in fusion and the intermolecular forces are thus broken. In vaporization, more work is done against the external pressure (atmospheric pressure) when the liquid expands to become vapour. Thus, for the molecules to be completely broken, a larger amount of heat is required.

Measurement of specific latent heat of fusion of ice

1.Electrical method



- An electric heater is placed in a funnel containing ice.
- At the instant the switch S is closed, a beaker is placed below the funnel to collect the water from the melting ice.
- After a measured time t, the beaker is removed from below the funnel and the mass m_2 of water collected is measured.
- To take into consideration the mass of ice melted due to absorption of heat from the surrounding, the same amount of ice is placed in another funnel

containing an electric heater which is not connected to the circuit.

- The mass m_1 of the ice melted after the measured time t is obtained by collecting the water with a beaker.

mass of ice melted by electric heater = $m_2 - m_1$

If I = current

V = potential difference across heater

t = time heater is on

l = latent heat of fusion of ice

Electrical energy supplied by heater

= heat required to melt ice

$$IVt = (m_2 - m_1)l$$

$\therefore$ latent heat of fusion of ice, $l = \frac{IVt}{m_2 - m_1}$

2. Method of mixtures

Pieces of ice are dried and dropped into a calorimeter containing warm water and the final temperature is noted after all the ice has melted.

Suppose the following measurements were taken;

Mass of copper calorimeter = 47.3 g, specific heat capacity = $400\, J\, kg^{-1}\, K^{-1}$, mass of water = 75.7 g, specific heat capacity of water = $4200\, J\, kg^{-1}\, K^{-1}$, mass of ice = 11.0 g.

Initial temperature = 18.4 °C; final temperature = 6.4 °C

Heat lost by calorimeter = $\frac{47.3}{1000} \times 400(18.4 - 6.4)$

Heat lost by water = $\frac{75.7}{1000} \times 4200(18.4 - 6.4)$

Heat gained by ice in changing to water at 6.4 °C

$$= \frac{11}{1000} \times l + \frac{11}{1000} \times 4200(6.4 - 0)$$

Assuming no heat loss;

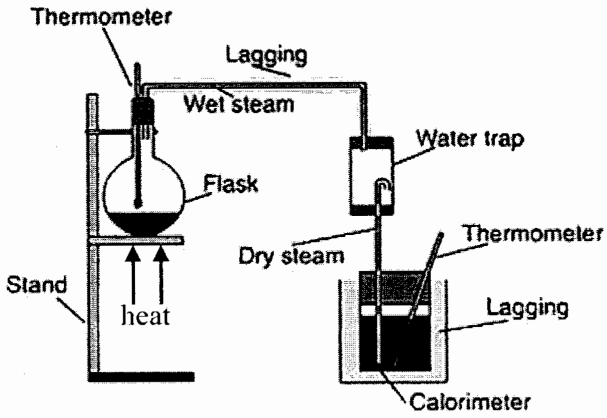
$$\frac{11}{1000} l + 295.7 = (47.3 \times 0.4 + 75.7 \times 4.2)(12)$$

$$l = 340 \times 10^3\, J\, \text{kg}^{-1} = 340\, kJ\, \text{kg}^{-1}$$

Determination of specific latent heat of vaporization

1. Method of mixtures

The specific latent heat of steam can be obtained by the method of mixtures.



- Some water is boiled in a flask and using a trap for drops of water, steam alone is obtained issuing from the tap
- The weights and initial temperature of water and the calorimeter are measured and recorded.
- The steam is passed into water placed inside the calorimeter and the final temperature of water is observed.
- The mass of steam condensed is obtained by re-weighing the calorimeter and its contents.

Example

Assuming the following measurements are taken;

Mass of calorimeter = 47.5 g, specific heat capacity = 400 J $\text{kg}^{-1} K^{-1}$

Mass of water = 122.6 g, specific heat capacity = 4200 J $\text{kg}^{-1}\, K^{-1}$

Mass of steam = 4.1 g, temperature = 100 °C

Initial temperature of water = 15.0 °C, final temperature = 33.0 °C

Heat gained by water and calorimeter

$$= \left(\frac{122.6}{1000} \times 4200 + \frac{47.5}{1000} \times 400\right)(33.0 - 15.0)$$

$$= 9610.56\, J$$

Heat given up by steam in condensing to water at 33.0 °C

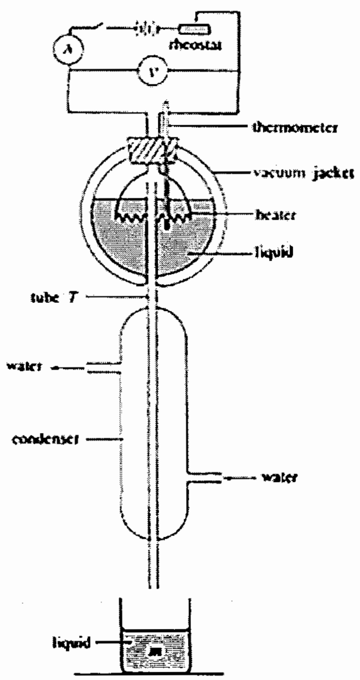
$$= \frac{4.1}{1000} l + \frac{4.1}{1000} \times 4200(100 - 33)$$

$$= 4.1 \times 10^{-3} l + 1153.74$$

$$4.1 \times 10^{-3} l + 1153.74 = 9610.56$$

$$l = 2060 \times 10^3\, J\, kg^{-1} = 2060\, kJ\, kg^{-1}$$

2. Electrical method



- With the switch closed, the liquid in a flask which is surrounded by a vacuum jacket is heated by an electric heater.
- When the liquid is boiling, a steady state is reached.
- Heat from the heater is used to evaporate the liquid and some heat is lost to the surrounding.
- The mass m of liquid evaporated in a measured time t is obtained by collecting the liquid condensed by the condenser using a beaker.
- The current I and potential difference V across the heater are measured and recorded.

$$IV = \frac{m}{t}l + h \quad .... \quad (i)$$

where h = rate of heat loss to the surrounding

- To eliminate h from the equation, the experiment is repeated with different power input. If I' and V' are the new ammeter and voltmeter readings respectively and m' the mass of liquid evaporated in a time t, then

$$I'V' = \frac{m'}{t}l + h \quad .... (ii)$$

(i) – (ii) ; $IV - I'V' = (m - m')\frac{l}{t}$

∴ specific latent heat of vaporization, $l = \frac{(IV - I'V')t}{(m-m')}$

Examples

1. (a) (i) Describe briefly what happens to the heat required to vaporize a liquid, and hence explain why it is called latent heat
   (ii) Making reference to the kinetic theory of matter, explain why the specific latent heat of vaporization of a substance is greater than its specific latent heat of fusion.
   (b) In an electrical method to determine the specific latent heat of vaporization of a liquid, the liquid was heated by means of a heating coil and steady state quantities measured under two different operating conditions are given in the table below.

Current in the coil	2.5 A	3.0 A
p.d cross coil	15.0 V	18.0 V
Mass of liquid evaporated in 250 s	38.5 g	59.6 g

   (i) State one advantage of this method over the method of mixtures
   (ii) Calculate the specific latent heat of vaporization of the liquid
   (iii) Calculate the rate of heat loss from the calorimeter.

Solution

(a)(i) During vaporization, the heat supplied is used to break the bonds between molecules, thus increasing the potential energy of the molecules and do work against the external pressure.

The heat is called latent heat because although heat is supplied to the liquid, there is no increase in temperature.

(ii) When a solid changes into a liquid, the molecules in the vapour are changed into weaker short-range forces between the molecules in the liquid.

When a liquid changes to vapour, the molecules in the vapour are separated far apart until the forces between the molecules are negligible and work is done against the external pressure. More energy is required when evaporation occurs compared to during melting. Hence specific latent heat of vaporization is greater than that of fusion.

(b) (i) Heat loss to the surrounding is accounted for and subsequently eliminated in the calculation.

(ii) $IV = \frac{m}{t}l + h \quad .... \quad (i)$

$$I'V' = \frac{m'}{t}l + h \quad .... (ii)$$

From (i) – (ii);

$$l = \frac{(IV - I'V')t}{(m-m')} = \frac{(3.0\times18.0 - 2.5\times15.0)250}{(59.6-38.5)\times10^{-3}}$$

$$= 1.95 \times 10^5\ Jkg^{-1}$$

(iii) From equation (i)

Rate of heat loss, $h = IV - \frac{ml}{t}$

$$= 3.0 \times 18.0 - \frac{(59.6\times10^{-3})(1.95\times10^5)}{250}$$

$$= 7.51\ W$$

2. Water in a vacuum flask is boiled steadily by passing an electric current through a coil of wire immersed in the water. When the potential difference across the coil is 5.25 V and the current through it is 2.58 A, 6.85 g of water evaporate in 20 minutes. When the potential difference and current are maintained at 3.20 V and 1.57 A respectively, 2.38 g of water evaporate in 20 minutes, all other conditions being the same. Calculate the specific latent heat of vaporization of water.

Solution

$$IV = \frac{m}{t}l + h \quad .... \quad (i)$$

$$I'V' = \frac{m'}{t}l + h \quad .... (ii)$$

From (i) – (ii);

$$l = \frac{(IV - I'V')t}{(m-m')} = \frac{1200(2.58\times5.25 - 1.57\times3.20)}{(6.85-2.38)\times10^{-3}}$$

$$= 2290 \times 10^3\ J\ kg^{-1}$$

$$= 2290\ kJ\ kg^{-1}$$

3. A copper calorimeter of mass 120 g contains 150 g of a liquid at 26 °C. 25 g of ice at 0 °C are added and the minimum temperature of the mixture is recorded. At this temperature, vapour from the same boiling liquid is introduced into the mixture until the final temperature of the calorimeter and new content is 30 °C. What mass of vapour is condensed? (specific heat capacity of copper is 0.4 kJ $kg^{-1}\,K^{-1}$, specific heat capacity of liquid is 2.4 kJ $kg^{-1}\,K^{-1}$, the boiling point of the liquid is 64.7 °C, the specific latent heat of vaporization of the liquid is 1100 kJ kg^{-1}, specific latent heat of fusion of ice is 320 kJ kg^{-1})

Solution

Let θ °C = the minimum temperature of the mixture

Heat lost by liquid and calorimeter

= heat gained by ice and water formed

$$(0.15 \times 2400 + 0.12 \times 400)(26 - \theta)$$
$$= 0.025 \times 320 \times 10^3 + 0.025 \times 4200(\theta - 0)$$
$$\theta = 5.1\ °C$$

Let m = mass in kg of vapour condensed

Heat given out by condensed vapour and liquid formed

= Heat gained by liquid, water and calorimeter

$$1100 \times 10^3 m + m \times 2400(64.7 - 30.0)$$
$$= (0.15 \times 2400 + 0.025 \times 4200 + 0.12 \times 400)(30 - 5.1)$$
$$1183 \times 10^3 m = 12774$$
$$m = 10.8 \times 10^{-3} kg = 10.8\ g$$

4. A calorimeter of heat capacity of 80 $J\,°C^{-1}$ contains 50 g of water at 40 °C. What mass of ice at 0 °C needs to be added in order to reduce the temperature to 10 °C? Assume no heat is lost to the surroundings. (specific heat capacity of water = 4200 $J\,kg^{-1}\,°C^{-1}$, specific latent heat of ice = $3.4 \times 10^5\,J\,kg^{-1}$)

Solution

Heat lost by calorimeter cooling to 10 °C

$$= 80(40 - 10) = 2400\,J$$

Heat lost by water cooling to 10 °C

$$= \frac{50}{1000} \times 4200(40 - 10) = 6300\,J$$

Total heat lost = $2400 + 6300 = 8700\,J$

Let the mass of ice to be added = m

Heat used to melt ice at 0 °C = $3.4 \times 10^5\ m$

Heat used to increase temperature of melted ice to 10 °C $= m \times 4200(10 - 0) = 42000m$

Total heat absorbed = $3.4 \times 10^5\ m + 42000m$

Since no heat is lost to the surrounding

$$3.4 \times 10^5\ m + 42000m = 8700$$
$$m = 0.0228\ kg$$

Mass of ice required = 22.8 g

5. 10.0 g of steam at 100 °C is added to 50.0 g of ice at 0°C. Find the final temperature after all the ice has melted (l for water = $2.26 \times 10^6\,J\,kg^{-1}$, l for ice = $3.33 \times 10^5\,J\,kg^{-1}$)

Solution

Let the final temperature of the mixture be θ

Heat lost by the steam = heat gained by ice

10.0 g of steam at 100 °C converts to 10.0 g water at 100 °C, then this water melts 50.0 g ice at 0 °C to 50.0 g water at 0 °C and then raises its temperature to θ while that of 10.0 g of water drops from 100 °C to θ

$$m_w l_v + m_w c_w(100 - \theta) = m_i l_f + m_i c_w(\theta - 0)$$
$$\frac{10}{1000} \times 2.26 \times 10^6 + \frac{10}{1000} \times 4200(100 - \theta)$$
$$= \frac{50}{1000} \times 3.33 \times 10^5 + \frac{50}{100} \times 4200\,\theta$$
$$22600 + 4200 - 42\,\theta = 16650 + 210\,\theta$$
$$252\,\theta = 10150$$
$$\theta = 40.28\ °C$$

Sources of errors in calorimetric experiments

- Heat losses to the surrounding by convection, radiation or conduction
- The substances in the calorimeter may not be well mixed or stirred

Precautions to minimize heat losses

- Polishing the calorimeter to reduce radiation loss
- Enclosing the calorimeter with lagging of a poor heat conductor to reduce convection and conduction loss
- Continuously stirring the mixture
- Supporting on an insulating stand

Self-Evaluation exercise

1. Calculate (i) the temperature rise of 200 g of copper, $c = 0.4\,kJ\,kg^{-1}\,K^{-1}$, when 2.0 kJ of heat is given to it, (ii) the heat gained by 250 g of water when its temperature rises from 20 °C to 60 °C.
   [Ans: (i) 25 °C (ii) 42 kJ]
2. A calorimeter has a mass of 200 g and a specific heat capacity 0.4 $kJ\,kg^{-1}\,K^{-1}$. What is its heat capacity? If the calorimeter contains 100 g of water at 10 °C, find the new temperature when 10 kJ is supplied to the water and calorimeter by a burner.
   [Ans: 0.08 $kJ\,kg^{-1}\,K^{-1}$, 30 °C]
3. A lead bullet, mass 1.5 g, has a speed of 20 m s^{-1} and is brought to rest in a target. If 20% of the heat energy is gained by the bullet, calculate its temperature rise. (Specific heat capacity of lead = 0.12 $kJ\,kg^{-1}\,K^{-1}$) [Ans: $\frac{1}{3}$°C]

4. An electric current of 2 A flows in a resistance wire whose p.d. is 50 V. Calculate the heat produced in 7 min. If the wire is fully immersed in 250 g of water initially at 20 °C, how long will the water take to reach boiling-point under normal atmospheric pressure? [Ans: 42 kJ, 14 min]
5. (i) 250 g of water at 15 °C is heated until it all evaporates, the atmospheric pressure being 760 mmHg. Calculate the heat supplied
(ii) 3 g of steam of 100 °C is condensed and the final temperature of the water formed is 35 °C. Find the total heat given up ($l = 2268\ kJ\ kg^{-1}$)
[Ans: (i) 656 kJ (ii) 7.6 kJ]
6. (i) In melting to water at 8 °C, 5 g of ice at 0 °C requires 1800 J of heat. Calculate the specific latent heat of fusion of ice.
(ii) What mass of iron at 17 °C, dropped into liquid oxygen at its boiling point −183 °C, will cause 2 g of oxygen to evaporate. (l for oxygen = 210 $kJ\ kg^{-1}$, specific heat capacity of iron = 0.4 $kJ\ kg^{-1}\ K^{-1}$)
[Ans: (i) 326 $kJ\ kg^{-1}$ (ii) 15.25 g]
7. 5.8 g of steam at 100 °C is passed into a copper calorimeter containing 85 g of water at 16 °C. If the final temperature is 50 °C, calculate the mass of the calorimeter. (l for steam = 210 $kJ\ kg^{-1}$, specific heat capacity of copper = 0.4 $kJ\ kg^{-1}\ K^{-1}$)
[Ans: 164 g]
8. 100 cm^3 of alcohol takes 6 min to cool in a draught from 60 °C to 20 °C. An equal volume of water in the same calorimeter takes 18 min to cool from 60 °C to 20 °C in a draught. If the calorimeter heat capacity is 42 $J\ K^{-1}$, calculate a value for the specific heat capacity of alcohol, whose density is 0.8 gcm^{-3}. [Ans: 1.4 kJ kg^{-1} K^{-1}]
9. Give a labelled diagram of a continuous-flow apparatus which could be used to determine the specific heat of water.

In such an experiment, the following readings were taken:

Current in heating coil	2.0 A	1.5 A
Potential difference across coil	6.0 V	4.5 V
Mass of water collected	42.3 g	70.2 g
Time of flow	60 s	180 s
Inlet temperature	38.0°C	38.0°C
Outlet temperature	42.0°C	42.0°C

10. Explain how each reading would be taken, and use the figures to obtain a value for the specific heat capacity of water in J kg^{-1} K^{-1}.
[Ans: 4170 J $kg^{-1}\ K^{-1}$]
11. 380 g of a liquid at 12 °C in a copper calorimeter weighing 90 g is heated at a rate of 20 watt for exactly 3 minutes to produce a temperature of 17 °C. If the specific heat capacity of copper is 400 $J\ kg^{-1}\ K^{-1}$, the thermal heat capacity of the heater is negligible and there is negligible heat loss to the surroundings, obtain the value for the specific heat capacity of the liquid. [Ans: 1800 $J\ kg^{-1}\ K^{-1}$]
12. Explain why the temperature of a boiling liquid remains constant although heat energy is supplied to it. Describe briefly how you would measure the latent heat of vaporization of a liquid and explain the precautions that must be taken to achieve an accurate result.
13. 100 g of water are placed in a calorimeter fitted with an electrical heater. When the heater dissipates 10.6 watts, the temperature of the water rises to a steady value to a steady value below its prevailing boiling point. With the heater switched off, the rate of fall of temperature is found to be 1.34 °C min^{-1}. The experiment is repeated with 100 g of another liquid in the same calorimeter. With a dissipation of 6.0 watts, a steady temperature below the boiling point of this liquid is attained and at this temperature with the heater switched off, the rate of fall of temperature is 1.0 °C min^{-1}. Calculate the specific heat capacity of the liquid. (specific heat capacity of water = 4200 $J\ kg^{-1}\ K^{-1}$) [Ans: 3.05 $kJ\ kg^{-1}\ K^{-1}$]
14. Describe, with the aid of a labelled diagram, how you would find the specific heat capacity of a liquid by the method of continuous flow.
Discuss the advantages and disadvantages of the method compared with the method of mixtures.
The temperature of 50 g of a liquid contained in a calorimeter is raised from 15.0°C (room temperature) to 45.0 °C in 530 seconds by an electric heater dissipating 10.0 watts. When 100 g of liquid is used and the same change in temperature occurs in the same time, the power of the heater is 16.1 watts. Calculate the specific heat capacity of the liquid.
[Ans: 2.2 $kJ\ kg\ K^{-1}$]
15. An experiment was performed to determine the specific latent heat of vaporization of a liquid at its boiling point. The following table summarizes the results

Voltage(V)	Current (A)	Mass(g) evaporated in 400 s
10.0	2.00	14.6
15.0	2.50	30.6

Calculate (a) the specific latent heat of vaporization of the liquid (b) the energy loss to the surroundings in 400 s (c) the rate of evaporation of the liquid when a 30.0 W rate of heating is used.

[Ans: (i) 438 $kJ\ kg^{-1}$ (ii) 1.61 kJ (iii) 59.4 $mg\ s^{-1}$]

16. Ethyl alcohol has a boiling point of 78.0 °C, a freezing point of −114 °C, a heat of vaporization of 879 $kJ\ kg^{-1}$, a heat of fusion of 109 $kJ\ kg^{-1}$ and a specific heat capacity of 2.43 $kJ\ kg^{-1}K^{-1}$. How much energy must be removed from 0.510 kg of ethyl alcohol that is initially a gas at 78.0 °C so that it becomes a solid at −114 °C? [Ans: 742 kJ]

17. Two 50 g ice cubes are dropped into 200 g of water in a thermally insulated container. If the water is initially at 25 °C and the ice comes directly from a freezer at −15 °C, (a) what is the final temperature at thermal equilibrium? (b) what is the final temperature if only one ice cube is used?

[Ans: (a) 0 °C (b) 2.5 °C]

18. A 50.0 g copper calorimeter contains 250 g of water at 20.0 °C. How much steam at 100 °C must be condensed into the water if the final temperature of the system is to reach 50.0 °C? [Ans: 13 g]

19. Water flows at the rate of 150.0 g min^{-1} through a tube and is heated by a heater dissipating 25.3 W. The inflow and outflow water temperatures are 15.2°C and 17.4°C respectively. When the rate of flow is increased to 231.8 g min^{-1} and the rate of heating to 37.8 W, the inflow and outflow temperatures are unaltered. Find (i) the specific heat capacity of water, (ii) the rate of loss of heat from the tube. [Ans: (i) 4.2 $Jg^{-1}K^{-1}$ (ii) 2.1 Js^{-1}]

20. A copper calorimeter of mass 70.5 g contains 100.0 g of water at 19.5°C. Naphthalene (M.P.79.9°C) is melted in a test tube, cooled to 80.0°C, and then poured into the calorimeter. If the highest temperature reached by the water after stirring is 28.7°C and the final mass of the calorimeter and its contents is 188.3 g, calculate the latent heat of fusion of naphthalene. (Specific heat capacity of copper 0.4, of naphthalene 1.3 kJ kg^{-1} K^{-1}.) [Ans: 164 kJ kg^{-1}]

21. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted, the temperature of the flask and contents is 40°C.When a further 80 g of ice has been added and has all melted, the temperature of the whole becomes 10°C. Calculate the specific latent heat of fusion of ice, neglecting any heat lost to the surroundings.

22. In the above experiment the flask is well shaken before taking each temperature reading. Why is this necessary? [Ans: 378 kJ kg^{-1}]

23. A 600 watt electric heater is used to raise th temperature of a certain mass of water from roo temperature to 80°C. Alternatively, by passin steam from a boiler into the same initial mass o water at the initial temperature, the sam temperature rise is obtained in the same time. I 16 g of water were evaporated every minute in th boiler, find the specific latent heat of steam assuming that there were no heat losses.

[Ans: 2230 kJ kg^{-1}]

24. Oil at 15.6°C enters a long glass tube containin an electrically heated platinum wire and leaves i at 17.4 °C, the rate of flow being 25 cm^3 per mi and the rate of supply of energy 1.34 watts. O changing the rate of flow to 15 cm^3 per min an the power to 0.76 watt, the temperature again rise from 15.6 °C to 17.4 °C. Calculate the mea specific heat capacity of the oil between these temperatures. Assume that the density of oil is 870 $kg\ m^{-3}$. [Ans: 2.22 $kJ\ kg^{-1}$]

25. Describe a continuous flow method of measuring the specific heat capacity of a liquid. Explain the advantages of the method.

Use the following data to calculate the specific heat capacity of the liquid flowing through a continuous flow calorimeter.

Experiment I: current 2.0 A, applied p.d 3.0 V, rate of flow of liquid 30 g min^{-1}, rise in temperature of liquid 2.7°C. Experiment II: Current 2.5 A applied p.d 3.75 V, rate of flow of liquid 48 g min^{-1}, rise in temperature of liquid 2.7°C [Ans: 4.2 $kJ\ kg^{-1}\ K^{-1}$]

GAS LAWS

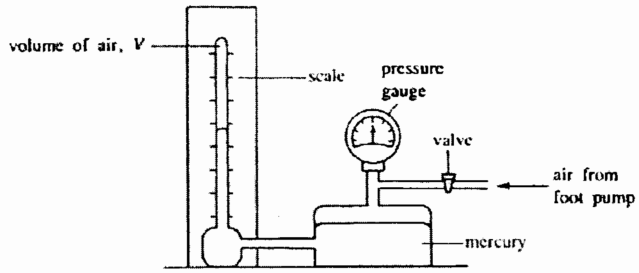
Boyle's law

It states that for a fixed mass of a gas at constant temperature, its pressure is inversely proportional to its volume.

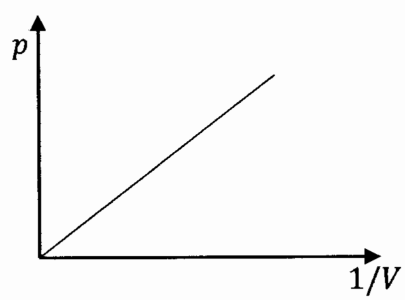
i.e. pressure, $p \propto \frac{1}{V}$ if temperature = constant

$$pV = constant$$

Experiment to verify Boyle's law

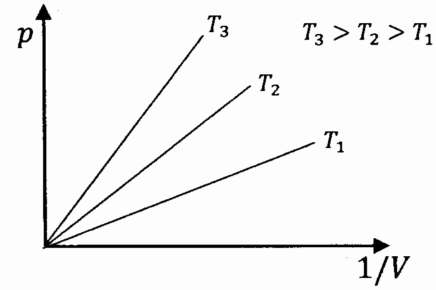


- A fixed mass of air is trapped at the top of a uniform vertical tube as shown above.
- To compress the air, the valve is opened and air is pumped from a foot pump.
- When the gas is in thermal equilibrium with the surrounding, the volume V of the air is measured and the pressure p recorded from the pressure gauge.
- Enough time is left in between taking one reading and the next and in this way, the temperature of the gas is maintained at the temperature of the surroundings which is assumed to be constant during the experiment.
- The pressure is increased in steps and at each step, the corresponding volume is measured.
- A graph of pressure p against $\frac{1}{Volume,\ V}$ is plotted

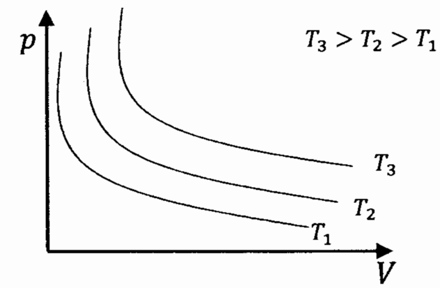


- The graph is a straight line passing through the origin.
- This verifies Boyle's law

Note: If the experiment is repeated for other fixed temperatures of the gas, the gradient of each straight line increases with temperature as shown in the following figure.

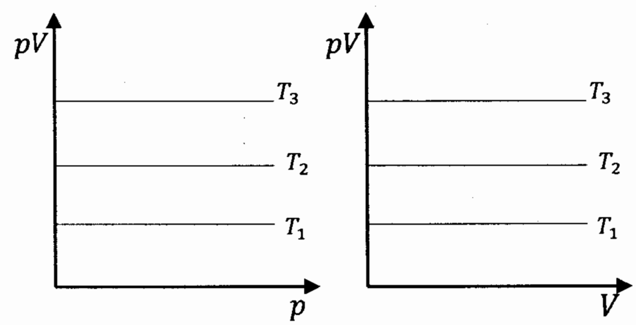


A graph of pressure p against volume V will appear as shown below.



The constant temperature curves are known as **isothermals**

From the equation pV = constant, if temperature is constant, Boyle's law can also be illustrated in the graph of pV against p or pV against V.



Note:

Boyle's law is not obeyed for all gases or at all temperatures and pressures. Most gases obey Boyle's law at low pressures. At high pressures, most gases deviate from Boyle's law. A gas that obeys Boyle's law for all pressures and temperatures is known as an ideal gas. None of the real gases such as hydrogen, oxygen, nitrogen or neon are ideal.

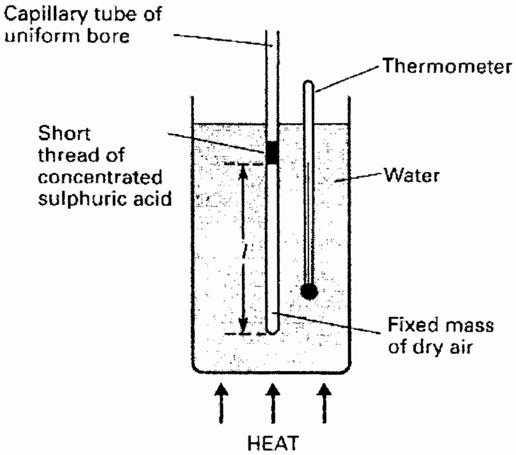
Charles' law

It states that the volume of a fixed mass of a given gas at constant pressure is directly proportional to its temperature in kelvins.

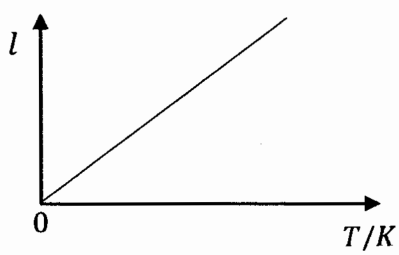
$V \propto T$ if p = constant

$$\frac{V}{T} = \text{constant}$$

Experiment to verify Charles' law

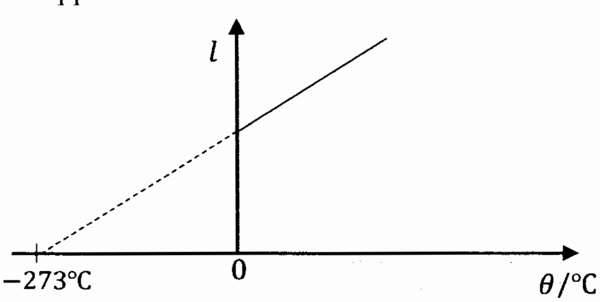


- A column of dry air is trapped inside a capillary tube of a uniform bore by a short thread of concentrated sulphuric acid. Sulphuric acid absorbs any water that might be in the air.
- The water is then heated slowly and stirred to allow the dry air reach the temperature of water.
- The volume of the air trapped is determined by measuring the length of the air column since the volume of the trapped air is proportional to the length of the air column.
- The length l of the air column is measured for a different number of temperatures T in kelvin
- A graph of l against kelvin temperature T is plotted and it is a straight-line graph through the origin



- This verifies Charles' law

Note: A graph of l against Celsius temperature θ will appear as shown below



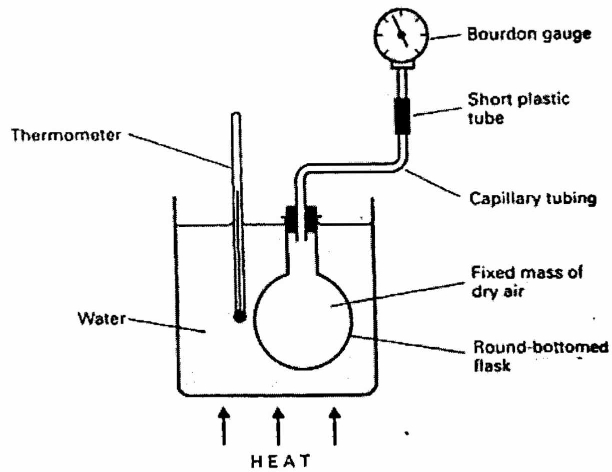
Pressure law

It states that the pressure of a fixed mass of a gas at constant volume is directly proportional to its absolute temperature.

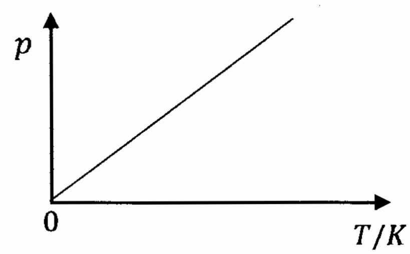
$$p \propto T \text{ if } V = \text{constant}$$

$$\frac{p}{T} = \text{constant}$$

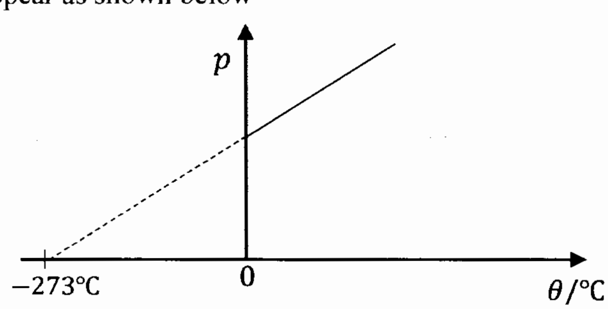
Experiment to verify the pressure law



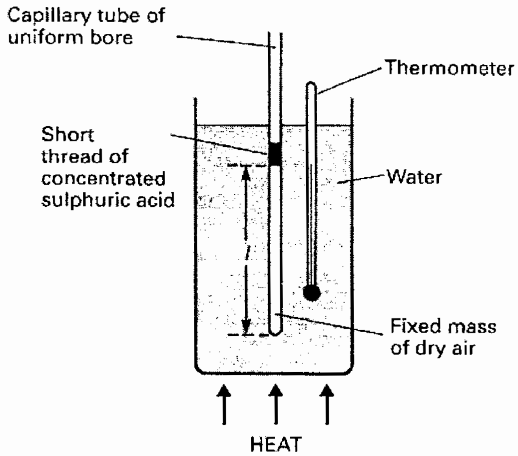
- The apparatus is setup as shown above.
- The water is heated slowly and stirred to allow the air in the flask reach the temperature of water.
- The temperature of the water is measured from the thermometer and the corresponding pressure is read off from the pressure gauge
- The water is heated at a number of different temperatures T and the corresponding pressures p are measured and recorded.
- A graph of p against kelvin temperature T is plotted
- The graph is a straight line through the origin which verifies the pressure law.



Note: A graph of p against Celsius temperature θ will appear as shown below



Experiment to verify Charles' law



- A column of dry air is trapped inside a capillary tube of a uniform bore by a short thread of concentrated sulphuric acid. Sulphuric acid absorbs any water that might be in the air.
- The water is then heated slowly and stirred to allow the dry air reach the temperature of water.
- The volume of the air trapped is determined by measuring the length of the air column since the volume of the trapped air is proportional to the length of the air column.
- The length l of the air column is measured for a different number of temperatures T in kelvin
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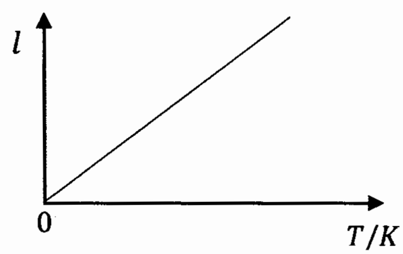
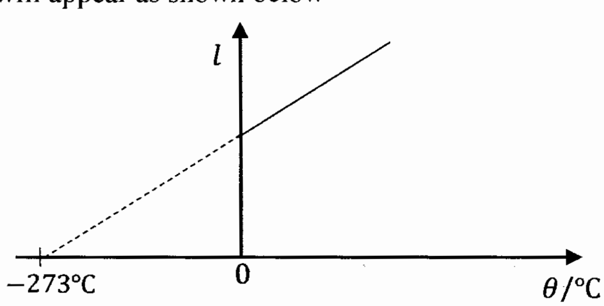


- This verifies Charles' law

Note: A graph of l against Celsius temperature θ will appear as shown below



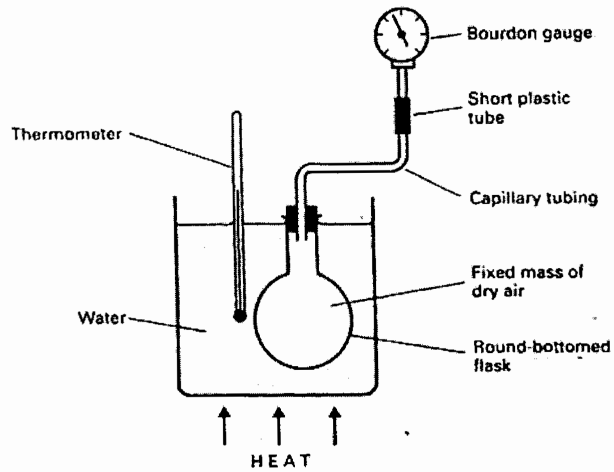
Pressure law

It states that the pressure of a fixed mass of a gas at constant volume is directly proportional to its absolute temperature.

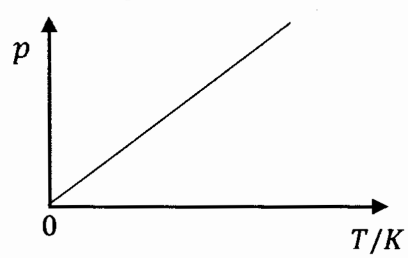
$$p \propto T \text{ if } V = \text{constant}$$

$$\frac{p}{T} = \text{constant}$$

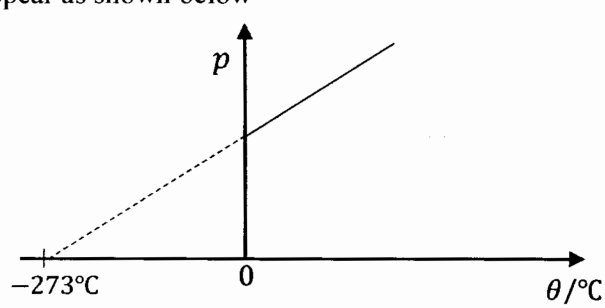
Experiment to verify the pressure law



- The apparatus is setup as shown above.
- The water is heated slowly and stirred to allow the air in the flask reach the temperature of water
- The temperature of the water is measured from the thermometer and the corresponding pressure is read off from the pressure gauge
- The water is heated at a number of different temperatures T and the corresponding pressures p are measured and recorded.
- A graph of p against kelvin temperature T is plotted
- The graph is a straight line through the origin which verifies the pressure law.



Note: A graph of p against Celsius temperature θ will appear as shown below



Ideal gases

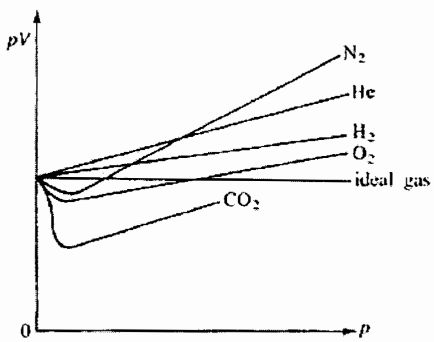
An ideal gas is a gas which perfectly obeys all the gas laws under all conditions and its intermolecular forces are negligible.

Behaviour of an ideal gas

- It obeys gas laws at all temperatures.
- There are no intermolecular forces between molecules of an ideal gas.
- The molecules move with a constant velocity in between collisions
- The molecules make perfectly elastic collisions.
- The molecules occupy negligible volumes compared to the volume of the container.

The Ideal Gas equation

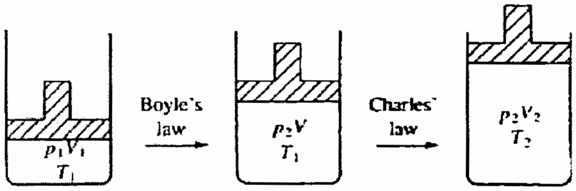
An ideal gas obeys Boyle's law and Charles' law perfectly. In practice, an ideal gas does not exist at all. Nevertheless real gases like hydrogen, helium, nitrogen, oxygen and carbon dioxide behave almost like an ideal gas at low pressure and high temperature. At higher pressures, the real gases deviate from the behaviour of an ideal gas as shown below.



The state of an ideal gas is determined by three parameters, pressure p, volume V and temperature T. These three parameters p, V and T are related by an equation known as the ideal gas equation or the equation of state of an ideal gas

To derive the ideal gas equation, we assume a fixed mass of ideal gas in a cylinder fitted with a smooth light piston as shown below.

Let the initial pressure, volume and temperature of the gas be p_1, V_1 and T_1 respectively.



If the pressure of the gas is changed to p_2 at constant temperature, according to Boyle's law, the volume V of the gas is given by

$$p_2V = p_1V_1 \text{ .... (i)}$$

If the temperature of the gas is then changed to T_2 at constant pressure, according to Charles' law, the new volume V_2 is given by

$$\frac{V_2}{T_2} = \frac{V}{T_1} \text{ .... (ii)}$$

(i) × (ii)

$$p_2V \times \frac{V_2}{T_2} = p_1V_1 \times \frac{V}{T_1}$$

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

$$\boldsymbol{\frac{pV}{T} = constant}$$

The value of this constant depends on the mass and type of gas.

For a mole of a gas, the value of the constant is the same for all gases and is known as the **molar gas constant**. The symbol for the molar gas constant is R.

Thus $\frac{pV_m}{T} = R$

$$pV_m = RT$$

Where V_m = volume of 1 mole of gas.

At s.t.p, the volume of 1 mole of a gas is 22.4 dm^3 i.e. when $T = 273\ K, p = 101.3\ kPa,\ V = 22.4\ dm^3 = 22.4\ \times 10^{-3}\ m^3$

$$pV_m = RT$$

$$R = \frac{pV_m}{T} = \frac{(101.3 \times 10^3)\times(22.4 \times 10^{-3})}{273}$$

$$= 8.31\ J\ K^{-1}\ mol^{-1}$$

If the volume of n mole of an ideal gas is V, then volume of 1 mole, $V_m = \frac{V}{n}$

From $pV_m = RT$

$$p\frac{V}{n} = RT$$

$$\boldsymbol{pV = nRT}$$

If the mass of a gas of volume V is m and M is the mass of 1 mole of the gas, then the number of moles

$$n = \frac{m}{M}$$

From $pV = nRT$

$$\boldsymbol{pV = \left(\frac{m}{M}\right)RT}$$

Also, if N = number of molecules in the volume V and N_A = Avogadro's number, number of molecules in a mole of a gas, then the number of moles $n = \frac{N}{N_A}$

$$\boldsymbol{pV = \left(\frac{N}{N_A}\right)RT}$$

Examples

1. The density of argon is 1.60 $kg\,m^{-3}$ at 27 °C and a pressure of 750 mmHg. What is the mass of argon in an argon-filled electric lamp bulb of volume 100 cm^3 if the pressure inside is 750 mmHg when the average temperature of the gas is 120 °C?

Solution

The volume of a fixed mass of gas at a constant pressure is proportional to its absolute temperature. The pressure is constant at 750 mmHg

Let the volume at 27 °C be V

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\frac{V}{100} = \frac{27+273}{120+273}$$

$$V = \frac{300}{393} \times 100 = 76.3\ cm^3$$

Mass of argon at 27 °C = density × volume at 27 °C

$= 1.60\ \times 76.3\ \times 10^{-6}$

$= 1.22\ \times 10^{-4}\ kg$

2. A gas cylinder of volume 40 litres contains oxygen at temperatures of 15 °C and pressure of $2.5 \times 10^6\ Nm^{-2}$. Calculate the
   (i) equivalent volume of oxygen at standard temperature and pressure (s.t.p)
   (ii) the mass of oxygen in the cylinder given that the density of oxygen is 1.4 $kg\,m^{-3}$ *at* s.t.p

Solution

Standard temperature and pressure are 0 °C and $1.01\ \times 10^5\ Nm^{-2}$ respectively

(i) $p_1 = 2.5\ \times 10^6\ Nm^{-2}, V = 4.0\ \times 10^3\ m^3$

$T_1 = 273 + 15 = 288\ K$

$p_2 = 1.01\ \times 10^5\ Nm^{-2}, V_2 = ?$

$T_2 = 273 + 0 = 273\ K$

From $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

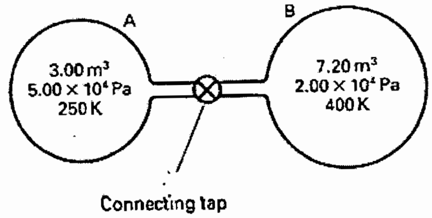
$$V_2 = \frac{p_1V_1T_2}{T_1p_2} = \frac{2.5\times10^6\times4.0\times10^3\times273}{288\times1.01\times10^5}$$

$= 93.86\ \times 10^{-3}\ m^3$

(ii) Mass = density × volume

$= 93.86\ \times 10^{-3} \times\ 1.4 = 0.131\ kg$

3. Initially in the figure below, A contains 3.00 m^3 of an ideal gas at a temperature of 250 K and a pressure of $5.0 \times 10^4\ Pa$ while B contains 7.20 m^3 of the same gas at 400 K and $2.0 \times 10^4\ Pa$. Find the pressure after the connecting tap has been opened and the system has reached equilibrium, assuming that A is kept at 250 K and B is kept at 400 K.



Solution

On opening the tap, some gas moves from A to B, reducing the pressure in A and increasing it in B. This continues until, at equilibrium, the pressure in A is equal to that in B. The trick is to recognize that the total mass of gas and therefore the total number of moles is the same after the tap is opened as it was before.

From $pV = nRT, n = \frac{pV}{RT}$

Number of moles initially in A $= \frac{5.0\times10^4\times3.0}{250R} = \frac{600}{R}$

Number of moles initially in B $= \frac{2.0\times10^4\times7.2}{400R} = \frac{360}{R}$

Total number of moles initially $= \frac{600}{R} + \frac{360}{R} = \frac{960}{R}$

Let the final pressure be p

Number of moles finally in A $= \frac{p\times3.0}{250R} = \frac{0.012p}{R}$

Number of moles finally in B $= \frac{p\times7.2}{400R} = \frac{0.018p}{R}$

Total number of moles finally

$$= \frac{0.012p}{R} + \frac{0.018p}{R} = \frac{0.03p}{R}$$

The total number of moles does not change.

Therefore $\frac{0.03p}{R} = \frac{960}{R}$

$$p = 3.2\ \times 10^4\ Pa$$

4. A cylinder of volume $2.0\ \times 10^{-3}\ m^3$ contains a gas at a pressure of $1.50\ \times 10^6\ Nm^{-2}$ and a temperature of 300 K. Calculate the
   (i) number of moles of the gas
   (ii) number of molecules of the gas
   (iii) mass of the gas if its molar mass is 32.0×10^{-3} kg
   (iv) mass of one molecule of the gas

Assume that the universal gas constant $R = 8.31\,J\,K^{-1}\,mol^{-1}$ and Avogadro constant $= 6.02 \times 10^{23}\ mol^{-1}$

Solution

(i) $p = 1.5\ \times 10^6\ Pa, V = 2\ \times 10^{-3}$, $R = 8.31$ and $T = 300$

From $pV = nRT$

$$n = \frac{pV}{RT} = \frac{1.5\times10^6\times2\times10^{-3}}{8.31\times300} = 1.20$$

(ii) One mole contains $6.02\ \times 10^{23}$ molecules

1.2 *mole* contain $1.02\ \times 6.02\ \times 10^{23}$

$= 7.22\ \times 10^{23}$ molecules

(iii) $M = 32 \times 10^{-3}, n = 1.2, m = ?$

$m = nM = 1.2 \times 32 \times 10^{-3} = 38.4 \times 10^{-3}$ kg

(iv) From $n = \frac{m}{M} = \frac{N}{N_A}$

$m = \frac{NM}{N_A}$

Since $N = 1$, $m = \frac{M}{N_A} = \frac{32 \times 10^{-3}}{6.02 \times 10^{23}}$

$= 5.32 \times 10^{-26}$ kg

5. A cylinder contains 2.0 kg of nitrogen at a pressure of $3.0 \times 10^6\ Nm^{-2}$ and at a temperature of 17 °C. What mass of nitrogen would a cylinder of the same volume contain at s.t.p?

Solution

At $p_1 = 3.0 \times 10^6\ Nm^{-2}$, $T_1 = 273 + 17 = 290\ K$, $m = 2.0$ kg

From $pV = nRT$

$$pV = \left(\frac{m}{M}\right)RT$$

$$pV = m\left(\frac{R}{M}\right)T$$

$$3.0 \times 10^6 V = 2\left(\frac{R}{M}\right) \times 290 \quad \text{....... (i)}$$

At s.t.p, $p_2 = 1.01 \times 10^5\ Pa$, $T_2 = 273, m = ?$

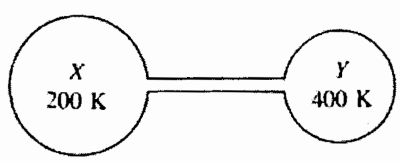
$$1.01 \times 10^5 V = m\left(\frac{R}{M}\right) \times 273 \quad \text{..... (ii)}$$

Dividing equation (i) by (ii) gives;

$$\frac{3.0 \times 10^6}{1.01 \times 10^5} = \frac{2 \times 290}{m \times 273}$$

$$m = 0.072 \text{ kg}$$

6. In the diagram below, the volume of flask X is twice that of flask Y. The system is filled with an ideal gas and a steady state is established with the flasks held at 200 K and 400 K respectively. If the mass of the gas in X is m, what is the mass of gas in Y



Solution

During the steady state, the pressure in X and Y are the same

If V = volume of Y and M = mass of 1 mole of a gas

Using $pV = \left(\frac{m}{M}\right)RT$

For Y: $pV = \left(\frac{m'}{M}\right)R \times 400$ ....(i)

For X: $p(2V) = \left(\frac{m}{M}\right)R \times 200$ ....(ii)

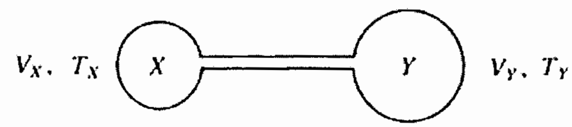
(i) ÷ (ii)

$$\frac{1}{2} = 2\frac{m'}{m}$$

$$m' = \frac{m}{4}$$

7. Two vessels X and Y of volumes V_X and V_Y connected by a tube of negligible volume and kept at temperatures T_X and T_Y respectively contain the same ideal gas. What is the value of the ratio $\frac{\text{number of molecules in X}}{\text{number of molecules in Y}}$?

Solution



Pressures in X and Y are the same

Using $pV = nRT$ where n = number of moles

For X: $pV_X = n_X RT_X$ ...... (i)

For Y: $pV_Y = n_Y RT_Y$ ...... (ii)

(i) ÷ (ii);

$$\frac{V_X}{V_Y} = \frac{n_X T_X}{n_Y T_Y}$$

$$\frac{n_X}{n_Y} = \frac{V_X T_Y}{V_Y T_X}$$

But $n_X = \frac{N_X}{N_A}$

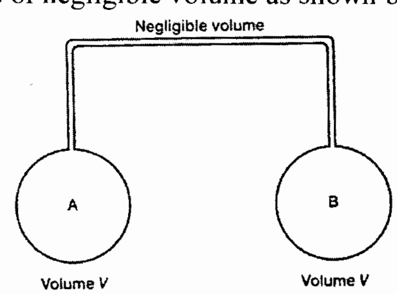
where N_X = number of molecules in X

N_A = Avogadro's number

Similarly, $n_Y = \frac{N_Y}{N_A}$

Hence $\frac{n_X}{n_Y} = \frac{N_X}{N_Y} = \frac{V_X T_Y}{V_Y T_X}$

8. Vessels A and B, of equal volume are connected by a tube of negligible volume as shown below.



The vessels contain a total mass of 2.50×10^{-3} kg of air and initially both vessels are at 27 °C when the pressure is $1.01 \times 10^5\ Nm^{-2}$. Vessel A is now cooled to 0 °C and vessel B heated to 100 °C. Calculate the

(a) mass of gas now in each vessel

(b) pressure in the vessels

Solution

(a) Let the volume of each vessel be V (we assume this does not change)

Since the vessels are connected, the pressure is equal in the two vessels.

Vessel A:

$$pV = m_A\left(\frac{R}{M}\right) \times 273 \quad \text{...... (i)}$$

Vessel B:

$$pV = m_B\left(\frac{R}{M}\right) \times 373 \quad \text{........ (ii)}$$

Comparing equations (i) and (ii) gives;

$$273\, m_A = 373\, m_B \quad \text{....... (iii)}$$

But total mass of gas, $m_A + m_B = 2.5 \times 10^{-3}$

From (iii); $m_A = \frac{373}{273} m_B$

$$\frac{373}{273} m_B + m_B = 2.5 \times 10^{-3}$$

$$\frac{646}{273} m_B = 2.5 \times 10^{-3}$$

$$m_B = 1.06 \times 10^{-3} \text{ kg}$$

$$m_A = \frac{373}{273} \times 1.06 \times 10^{-3} = 1.44 \times 10^{-3} \text{ kg}$$

(b) For the original whole system,

$T = 273 + 27 = 300\ K$

$p = 1.01 \times 10^5\ Nm^{-2}$

Volume $= V + V = 2V$

$m = 2.5 \times 10^{-3}$ kg

$$1.01 \times 10^5 \times 2V = 2.5 \times 10^{-3}\left(\frac{R}{M}\right) \times 300 \text{ ..(iv)}$$

To find the final pressure, p, we make use of (i) in which $m_A = 1.44 \times 10^{-3}$ kg

$$pV = 1.44 \times 10^{-3}\left(\frac{R}{M}\right) \times 273 \quad \text{...... (i)}$$

Dividing (ii) by (iv) gives;

$$\frac{p}{1.01 \times 10^5 \times 2} = \frac{1.44 \times 10^{-3} \times 273}{2.5 \times 10^{-3} \times 300}$$

$$p = 1.06 \times 10^5\ Nm^{-2}$$

Note: Using (ii) for which $m_B = 1.06 \times 10^{-3}$ kg should give the same answer for p. Try this as a check.

9. Oxygen stored in a cylinder has a mass of 4.0 kg at a pressure of $8.0 \times 10^{15}\ Pa$ and a temperature of 27 °C. When the temperature rises, some oxygen escapes from the cylinder. Calculate the mass of the gas that escaped if the temperature of the remaining oxygen in the cylinder is 47 °C and the pressure is $8.2 \times 10^5\ Pa$.

Solution

Initially, $p_1V_1 = n_1RT_1$

$$p_1V_1 = \left(\frac{m}{M}\right) RT_1$$

$$V_1 = \frac{mRT_1}{p_1M}$$

$$\Rightarrow V_1 = \frac{4.0 \times 8.31 \times 300}{8.0 \times 10^5 \times 0.032} = 0.3895\ m^3$$

From $n = \frac{m}{M} = \frac{4}{0.032} = 125$

When oxygen has escaped;

$$p_2V_2 = n_2RT_2$$

$$n_2 = \frac{p_2V_2}{RT_2}$$

But $V_1 = V_2$ (same vessel/cylinder)

$$n_2 = \frac{8.2 \times 10^5 \times 0.3895}{8.31 \times 320} = 120$$

Moles escaped, $n = n_1 - n_2 = 125 - 120 = 5$

Mass escaped $= nM = 5 \times 0.032 = 0.16$ kg

Volume coefficient of expansion (α_p)

The volume coefficient (expansivity) of a gas is defined as the increase in volume per m^3 volume of the gas at 0 °C per °C temperature rise at constant pressure.

$$\alpha_p = \frac{\textit{increase in volume from } 0\ °C}{\textit{volume at } 0°C \times \textit{temperature rise}}$$

$$\alpha_p = \frac{V - V_0}{V_0\theta}$$

$$V = V_0(1 + \alpha_p\theta)$$

where V_0 = volume at 0 °C

V = volume at θ °C

Thus the volume of a given mass of a gas at θ °C when its volume at 0 °C and its expansivity α_p are both known.

Pressure coefficient of a gas (α_V)

The pressure coefficient of a given mass of gas at constant volume is defined as the fractional increase in pressure to its pressure at 0 °C per degree Celsius temperature rise.

$$\alpha_V = \frac{\textit{increase in pressure}}{\textit{pressure at } 0°C \times \textit{temperature rise}}$$

$$\alpha_V = \frac{p - p_0}{p_0\theta}$$

$$p = p_0(1 + \alpha_V\theta)$$

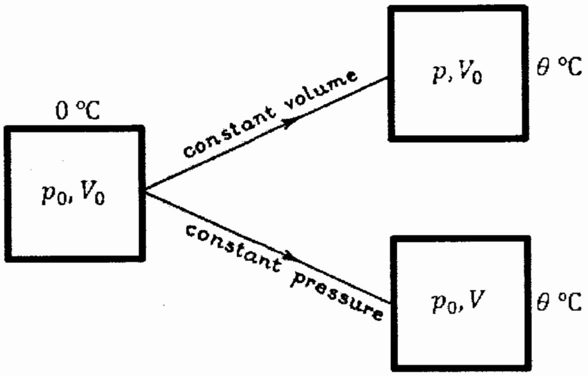
where p_0 = pressure at 0 °C

p = pressure at θ °C

To show that volume coefficient and pressure coefficient are equal

The equality of α_V and α_p can be shown to follow if the gas obeys Boyle's law and Charles' law.

Suppose that a given mass of a gas at 0 °C has a pressure p_0 and volume V_0 and it heated to a temperature θ °C. This change of temperature can be accomplished either at constant volume, when the pressure changes to a value p or at constant pressure when the volume changes to V.



From Boyle's law;

$$p_0V = pV_0$$

$$\frac{V}{V_0} = \frac{p}{p_0} \quad \ldots\ldots \text{(i)}$$

But $V = V_0(1 + \alpha_p\theta)$

$$\Rightarrow \frac{V}{V_0} = 1 + \alpha_p\theta$$

Also $p = p_0(1 + \alpha_V\theta)$

$$\Rightarrow \frac{p}{p_0} = 1 + \alpha_V\theta$$

From (i) $1 + \alpha_p\theta = 1 + \alpha_V\theta$

$$\therefore\ \alpha_p = \alpha_V$$

Self-Evaluation exercise

1. The molar mass of nitrogen is $28.0 \times 10^{-3}\ kg$. A sample of the gas contains 6.02×10^{22} molecules. Calculate (a) the number of moles of the gas, (b) the mass of the gas and (c) the volume occupied by the gas at a pressure of $0.110\ MNm^{-2}$ and a temperature of $290\ K$.
   [Ans: (a) 0.10 (b) $0.0028\ kg$ (c) $0.00219\ m^3$]
2. Define the expansivity (volume coefficient) of a gas. At constant pressure, a fixed mass of gas increases from a volume of 350 cm^3 at 0°C to 360.25 cm^3 at 8°C. Calculate the volume coefficient of the gas, and its volume at 100 °C.
   [Ans: $3.66 \times 10^{-3}\ K^{-1}$, $478\ cm^3$]
3. A fixed mass of air has a pressure of 100 cm of mercury at 100 °C. Find its pressure (i) at 20 °C, (ii) at 0 °C, if the volume of the gas remains constant.
   [Ans: (i) 78.5 (ii) 73.2 cmHg]
4. The volume of a fixed mass of gas at 50°C is 80 cm^3. At what temperature does the volume become 100 cm^3, the pressure remaining constant?
   [Ans: 131 °C]
5. An oxygen cylinder contains 0.50 kg of a gas at a pressure of $0.50\ M\ Nm^{-2}$ and a temperature of 7 °C. What mass of oxygen must be pumped into the cylinder to raise its pressure to $3.0\ M\ Nm^{-2}$ at a temperature of 27 °C? If the molar mass of oxygen is $32 \times 10^{-3}\ kg$, calculate the volume of the cylinder.
   [Ans: 2.30 kg. $0.073\ m^3$]
6. What is the relation between the pressure, volume, and temperature of a fixed mass of gas? A quantity of oxygen gas has a volume of 250 cm^3 at 15°C and 774 mmHg pressure. Calculate its volume at s.t.p. (0 °C and 760 mmHg). [Ans: $241\ cm^3$]
7. Two vessels, one having three times the volume of the other are connected by a narrow tube of negligible volume. Initially the whole system is filled with a gas at a pressure of $1.05 \times 10^5\ Pa$ and a temperature of $290\ K$. The smaller vessel is now cooled to $250\ K$ and the larger heated to $400\ K$. Find the final pressure of the system.
   [Ans: $1.26 \times 10^5\ Nm^{-2}$]
8. The density of air is 1.29 kg m^{-3} at s.t.p. Calculate the gas constant of air for a mass of (i) 1 kg, (ii) 10 kg. What mass of air has a volume of 850 cm^3 at 27 °C and 750 mmHg pressure?
   [Ans: (i) 0.29 (ii) $2.9\ kJ\ kg^{-1}$; 0.99 g]
9. A gas expands from 80 cm^3 to 200 cm^3 under a constant external pressure of one atmosphere, assumed as 10^5 N m^{-2}. Find the work done by the gas against this pressure. [Ans: 12 J]
10. State Boyle's law and Charles' law and show how they may be combined to give the equation of state of an ideal gas.
    Two glass bulbs of equal volume are joined by a narrow tube and are filled with a gas at s.t.p. When one bulb is kept in melting ice and the other is placed in a hot bath, the new pressure is 877.6 mm mercury. Calculate the temperature of the bath.
    [Ans: 100 °C]
11. Give brief accounts of experiments which illustrate the relationship between the volume of a fixed mass of gas and (a) the pressure it exerts at a fixed temperature, (b) the temperature on a Celsius mercury thermometer at a fixed pressure. State the two "laws" which summarize the results.
    A gas cylinder contains 6400 g of oxygen at a pressure of 5 atmospheres. An exactly similar cylinder contains 4200 g of nitrogen at the same temperature. What is the pressure on the nitrogen? (Molar masses: oxygen = 32, nitrogen = 28; assume that each behaves as a perfect gas.) [Ans: 3.75 atm]
12. State Boyle's law and Charles' law, and show how they lead to the gas equation $PV = RT$. What volume of liquid oxygen (density 1140 kg m^{-3}) may be made by liquefying completely the contents of a cylinder of gaseous oxygen containing 100 litres of oxygen at 120 atmospheres pressure and 20 °C?

Assume that oxygen behaves as an ideal gas in this latter region of pressure and temperature.

[1 atmosphere = $1.01 \times 10^5\ Nm^{-2}$; gas constant = 8.31 J mol^{-1} K^{-1}; molar mass of oxygen = 32.0.]

[Ans: $0.014\ m^3$]

13. Two gas containers A and B, have equal volumes and contain different gases at the same temperature and pressure. Use the ideal gas equation to show that there are equal number of molecules in the two containers

14. The formula $pv = mrT$ is often used to describe the relationship between the pressure p, volume v, and temperature T of a mass m of a gas, r being a constant. Referring in particular to the experimental evidence how do you justify (a) the use of this formula, (b) the usual method of calculating T from the temperature t of the gas on the centigrade (Celsius) scale?

Two vessels each of capacity 1.00 litre are connected by a tube of negligible volume. Together they contain 0.342 g of helium at a pressure of 800 mm of mercury and temperature 27 °C. Calculate (i) a value for the constant r for helium, (ii) the pressure developed in the apparatus if one vessel is cooled to 0 °C and the other heated to 100 °C, assuming that the capacity of each vessel is unchanged.

[Ans: (i) $2.08\ kJ\ kg^{-1}\ K^{-1}$ (ii) 842 mmHg]

15. State Boyle's law.

Two glass vessels of equal volume are joined by a tube, the volume of which may be neglected. The whole is sealed and contains air at s.t.p. If one vessel is placed in boiling water at 100 °C and the other is placed in melting ice, what will be the resultant pressure of the air? [Ans: 878 mmHg]

16. (a) State two quantities which increase when the temperature of a given gas is increased at constant volume.

(b) A car tyre of volume $1.0 \times 10^{-2}\ m^3$contains air at a pressure of 300 kPa and a temperature of $290\ K$. The mass of one mole of air is 2.9×10^{-2} kg. Assuming that the air behaves as an ideal gas, calculate

(i) the amount in mol of air

(ii) the mass of the air

(iii) the density of the air

(c) Air contains oxygen and nitrogen molecules. State, with a reason, whether the following are the same for oxygen and nitrogen molecules in air at a given temperature.

(i) the average kinetic energy per molecule

(ii) the r.m.s speed

[Ans: (a) r.m.s speed of molecules and pressure (b) (i) 1.24 (ii) 0.036 kg (iii) $3.6\ kgm^{-3}$ (c)(i) same – temperature is same (ii) different – mass of molecules is different]

17. A rigid gas-tight container holds $150\ cm^3$ of air at a temperature of 100 °C and a pressure of $1.0 \times 10^5\ Pa$. The temperature of the air is raised to 150 °C. Calculate the new pressure.

[Ans: $1.13 \times 10^5\ Pa$]

18. A bottle of gas has a pressure of 303 kPa above atmospheric pressure at a temperature of 0 °C. The bottle is left outside on a very sunny day and the temperature rises to 35 °C. Given that atmospheric pressure is 101 kPa, calculate the new pressure of the gas inside the bottle. [Ans: 456 kPa]

19. A cylinder containing 19 kg of compressed air at a pressure 9.5 times that of the atmosphere is kept in a store at 7 °C. When it is moved to a workshop where the temperature is 27 °C, a safety valve on the cylinder operates releasing some of the air. If the valve allows air to escape when its pressure exceeds 10 times that of the atmosphere, calculate the mass of air that escapes. [Ans: 0.33 kg]

20. Two vessels A and B of equal volume are connected by a narrow tube of negligible internal volume. Initially, the whole system is filled with 3 g of dry air at a pressure of $10^5\ Pa$ and a temperature $300\ K$. The temperature of the vessel B is now raised to 600 K, the temperature of A remaining $300\ K$. Calculate

(i) the new pressure in the system

(ii) the mass of air in A and in B

[Ans: (i) $1.33 \times 10^5 Pa$ (ii) 2 g in A, 1 g in B]

21. Explain what is meant by the room mean square velocity of the molecules of a gas. Use the concepts of the elementary kinetic theory of gases to derive an expression for the root mean square velocity of the molecules in terms of the pressure and density of the gas.

Assuming the density of nitrogen at s.t.p.to be 1.251 kgm^{-3}, find the root mean square velocity of nitrogen molecules at 127°C. [Ans:$597\ ms^{-1}$]

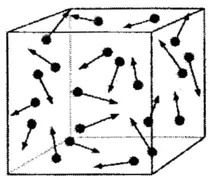
22. State the postulates on which the simple kinetic theory of gases is based. What modifications are made to the postulates in dealing with real gases? How are these modifications represented in van der Waals' equation?

KINETIC THEORY OF GASES

Gases are made up of small particles called molecules which are held by very weak intermolecular forces of attraction. The molecules are in continuous random motion and continuously collide with themselves and the walls of the container making perfect collisions.

The kinetic theory of gases relates the macroscopic properties of gases (e.g. pressure and temperature) to the microscopic properties of gas molecules e.g. speed and kinetic energy.

Why a gas exerts pressure



The molecules are continually colliding with each other and will the walls of the container. When a molecule collides with the wall, it exerts a small force on the wall. The pressure exerted by the gas is due to the sum of all these collision forces. The more the particles that hit the walls, the higher the pressure.

If a gas is heated up, its particles move around more quickly. They hit the walls of the container harder and more often. This increases the pressure. Sometimes the pressure gets so great that the container bursts.

This is why balloons and car tyres burst if you blow them up too much. It is for the same reason that deodorant spray cans carry warning signs to tell you not to leave them in the sunshine. If they get too hot, they explode.

Absolute zero temperature

Absolute zero is defined as the temperature at which the molecules have their lowest possible energy.

According to kinetic theory of gases, molecules of a gas have an average speed which increases with temperature. As a gas is cooled, the speed of molecules decreases and hence kinetic energy also decreases. A point is reached when the molecules come to rest and their kinetic energy becomes zero and at this point, the gas has the lowest possible temperature called absolute zero.

Evidence to support random motion

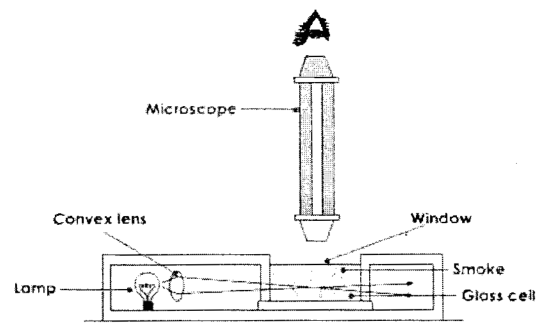
The striking evidence of the molecular agitation of matter comes from physical phenomena such as diffusion, evaporation, brownian motion. etc.

Brownian motion

This is the random motion of suspended molecules of a gas or liquid

Experiment to demonstrate brownian motion

Brownian motion can be demonstrated simply by releasing some smoke particles from burning cord into a small glass container and to put a cover plate to seal the smoke and air into the cell.



The microscope is adjusted until very bright specks are seen.

The particles of smoke reflect the light shining on them and so appear as bright points of light darting about in a random or erratic motion.

Note that the smoke particles are much larger than the air molecules. The particles can be seen by the light they scatter but the molecules themselves are too small to be seen.

Conclusion

The irregular movement of the visible particles of smoke is due to an uneven bombardment of the particles by the invisible molecules of air. It is due to Brownian motion. The lighter the particles, the faster the motion and the denser the particles, the slower the motion.

The kinetic theory of gases consists of a set of basic assumptions for the properties of gas molecules. The assumptions are:

1. A gas consists of a large number of molecules
2. The gas molecules are constantly in rapid, free random motion
3. Gas molecules collide elastically with one another and the walls of the container
4. There are no intermolecular forces except during collision
5. The volume of the gas molecules is negligible compared to the volume of the container which is also the volume of the gas
6. The duration of collision is negligible compared to the time between collisions

Explanation of the kinetic theory of gases

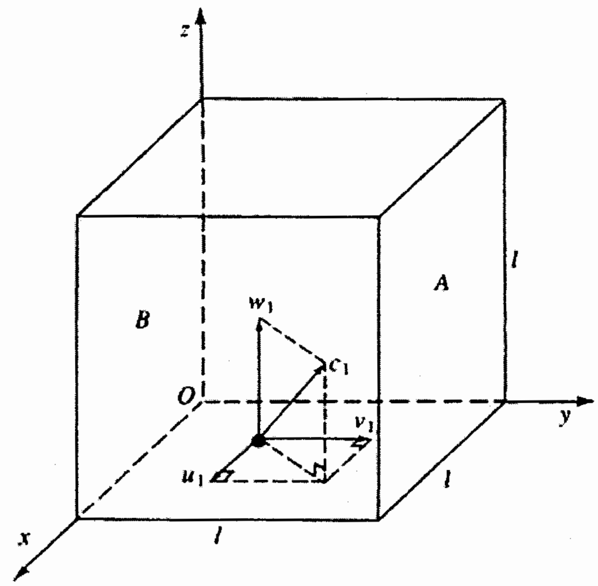
- Gases consist of large number of identical particles (atoms or molecules) that are so small and so far apart on the average that the actual volume of the molecules is negligible in comparison to the empty space between them. They are considered as **point masses**. This assumption explains the great compressibility of gases.
- There is **no force of attraction** between the particles of a gas at ordinary temperature and pressure. The support for this assumption comes from the fact that gases expand and occupy all the space available to them.
- Particles of a gas are always in **constant and random motion**. If the particles were at rest and occupied fixed positions, then a gas would have had a fixed shape which is not observed.
- Particles of a gas move in all possible directions in **straight lines**. During their random motion, they collide with each other and with the walls of the container. Pressure is exerted by the gas as a result of collision of the particles with the walls of the container.
- Collisions of gas molecules are **perfectly elastic**. This means that total energy of molecules before and after the collision remains same. There may be exchange of energy between colliding molecules, their individual energies may change, but the sum of their energies remains constant. If there were loss of kinetic energy, the motion of molecules will stop and gases will settle down. This is contrary to what is actually observed.
- At any particular time, different particles in the gas have **different speeds** and hence **different kinetic energies**. This assumption is reasonable because as the particles collide, we expect their speed to change. Even if initial speed of all the particles was same, the molecular collisions will disrupt this uniformity. Consequently, the particles must have different speeds, which go on changing constantly. It is possible to show that though the individual speeds are changing, the distribution of speeds remains constant at a particular temperature.
- If a molecule has variable speed, then it must have a variable kinetic energy. Under these circumstances, we can only consider **average kinetic energy**. In kinetic theory, it is assumed that average kinetic energy of the gas molecules is directly proportional to the absolute temperature. On heating a gas at constant volume, the pressure increases since kinetic energy of the particles increases and these strike the walls of the container more frequently thus exerting more pressure.

Kinetic theory of gases allows us to **deri** **theoretically**, all the gas laws studied previously.

Derivation of an expression for the pressure of a ideal gas from the kinetic theory of gases

A gas consists of many molecules in constant rando free motion, colliding elastically with each other an the walls of the container. The pressure of the gas i due to the collision of the gas molecules with the wal of the container.

To obtain this pressure, consider one of the ga molecules in a cubic container of length l.

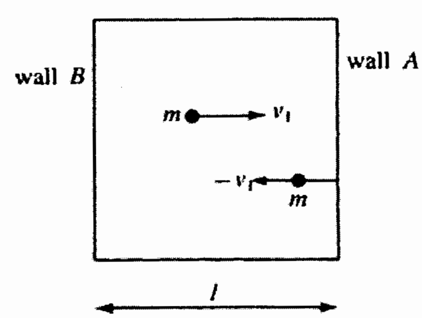


Suppose that the mass of the molecule is m and it is moving with velocity c_1. The velocity c_1 of the molecule can be resolved into the x, y *and* z directions as u_1, v_1 and w_1 respectively.

Using Pythagoras theorem, c_1 is related to its components by the equation.

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$

Consider the component of velocity v_1 along the y-axis.



After colliding with the wall A, the molecule rebounds with a velocity of $-v_1$ since the collision is elastic.

Momentum before collision $= mv_1$

Momentum after collision $= -mv_1$

Change of momentum $= mv_1 - (-mv_1) = 2mv_1$

Assuming that the molecule does not collide with other molecules, the time interval between a collision of the molecule with the wall A and its next collision with the same wall is the same as the time the molecule takes to move from wall A to wall B and back

$$t = \frac{2l}{v_1}$$

Number of collisions with wall A per second,

$$= \frac{n}{t} = \frac{1}{\left(\frac{2l}{v_1}\right)} = \frac{v_1}{2l}$$

By Newton's second law of motion,
Force on the wall due to this molecule

$$= \text{rate of change of momentum}$$

$$= 2mv_1 \times \frac{v_1}{2l} = \frac{mv_1^2}{l}$$

If there are N molecules, each with the component of velocity along the y-axis as $v_1, v_2, v_3, \ldots\ldots v_N$ respectively, then the total force on the wall A due to the collision of these N molecules is

$$F = \frac{mv_1^2}{l} + \frac{mv_2^2}{l} + \frac{mv_3^2}{l} + \cdots + \frac{mv_N^2}{l}$$

Pressure on wall A

$$p = \frac{F}{Area} = \frac{F}{l^2} = \frac{m}{l^3}[v_1^2 + v_2^2 + v_3^2 + \cdots + v_N^2]$$

If $\overline{v^2}$ is the average or mean of the squares of the velocities of N molecules,

$$\overline{v^2} = \frac{v_1^2+v_2^2+v_3^2+\cdots+v_N^2}{N}$$

$$v_1^2 + v_2^2 + v_3^2 + \cdots + v_N^2 = N\overline{v^2}$$

$$\text{Thus pressure, } p = \frac{m}{l^3}(N\overline{v^2})$$

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Since the motion of the gas molecules is random in nature and the number of molecules is large,

$$\overline{u^2} = \overline{v^2} = \overline{w^2}$$

Thus $\overline{c^2} = \overline{v^2} + \overline{v^2} + \overline{v^2}$

$$\overline{c^2} = 3\overline{v^2}$$

$$\overline{v^2} = \frac{1}{3}\overline{c^2}$$

$$p = \frac{m}{l^3} \times N \times \frac{1}{3}\overline{v^2} = \frac{1}{3}\frac{Nm}{V}\overline{c^2}$$

But Nm = mass of gas inside container and $\frac{Nm}{V} = \rho$, density of gas

$$\therefore p = \frac{1}{3}\rho\overline{c^2}$$

Where $\overline{c^2}$ is the mean square velocity
The root mean square velocity

$$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \cdots + v_N^2}{N}}$$

r.m.s velocity of gases. Earth's atmosphere

The r.m.s velocity of hydrogen molecules can be calculated.

The density of hydrogen at 0 °C and $1.01 \times 10^5\ Nm^{-2}$ pressure is about $0.09\ kg\,m^{-3}$

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3\times1.01\times10^5}{0.09}} = 1840\ ms^{-1}$$

The r.m.s velocity of hydrogen molecules is about 2 km per second. The r.m.s velocities of molecules of other gases are less than that of hydrogen. Since r.m.s velocity $\propto \frac{1}{\sqrt{\rho}}$. Oxygen gas, which is about 16 times as dense as hydrogen has $\frac{1}{\sqrt{16}}$ of the r.m.s velocity of hydrogen. Air has a density of $1.29\ kg\,m^{-3}$ about 14.4 times that of hydrogen.

The escape velocity of an object launched from the earth, the least velocity which can overcome completely the earth's gravitational attraction, is about $11\ km\,s^{-1}$. The earth's atmosphere stays round the earth because most air molecules have speeds less than the escape velocity of this planet. Only small quantities of helium are found close to the earth because it is an extremely light gas and diffuses upwards. The moon has little or no atmosphere. The gravitational attraction is about one-sixth that of the earth, so the escape velocity is low.

Examples

1. The speeds of nine particles are distributed as follows

Speed/ms^{-1}	1.0	2.0	3.0	4.0	5.0	6.0
No. of particles	1	1	4	1	1	1

What is the root-mean-square speed?

Solution

Root-mean-square speed $= \sqrt{\frac{v_1^2+v_2^2+\ldots\ldots+v_N^2}{N}}$

$$= \sqrt{\frac{1.0^2+2.0^2+4(3.0)^2+4.0^2+5.0^2+6.0^2}{9}}$$

$$= 3.6\ ms^{-1}$$

2. Calculate the root-mean-square speed of air molecules at 273 K and one atmospheric pressure $1.01 \times 10^5\ Pa$, if the density of air under these conditions is $1.29\ kg\,m^{-3}$

Solution

$$p = \frac{1}{3}\rho\,\overline{c^2}$$

$$c_{r.m.s} = \sqrt{\overline{c^2}} = \sqrt{\frac{3p}{\rho}}$$

$$= \sqrt{\frac{3\times1.01\times10^5}{1.29}} = 485\ ms^{-1}$$

3. Calculate the temperature at which the r.m.s speed of oxygen molecules is twice as great as their r.m.s speed at 27 °C.

Solution

$$27\,°C = 27 + 273\,K = 300\,K$$

From $\frac{1}{2}m\overline{c^2} = \frac{3}{2}KT$

$$\overline{c^2} \propto T$$

$$\frac{c_{r.m.s}\ at\ T}{c_{r.m.s}\ at\ 300} = \frac{\sqrt{T}}{\sqrt{300}}$$

$$2 = \frac{\sqrt{T}}{\sqrt{300}}$$

Squaring both sides

$$T = 4 \times 300 = 1200\,K$$

$$\therefore T = 927\,°C$$

4. A fixed mass of gas at constant pressure occupies a volume V. The gas undergoes a rise in temperature so that the root-mean-square velocity of its molecules is doubled. What is the new volume?

Solution

Let M = mass of gas and V = volume of gas

Then $\rho = \frac{M}{V}$

Using $p = \frac{1}{3}\rho\,\overline{c^2}$

$$p = \frac{1}{3}\frac{M}{V}\overline{c^2} \quad \text{.........}\ (i)$$

When the r.m.s is doubled,

$$p = \frac{1}{3}\frac{M}{V_1}(4\overline{c^2}) \quad \text{......}\ (ii)$$

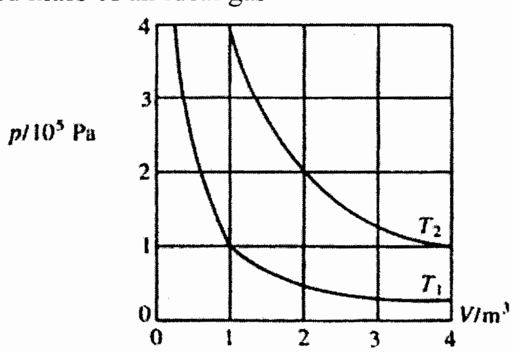
(i) = (ii)

$$\frac{1}{3}\frac{M}{V}\overline{c^2} = \frac{1}{3}\frac{M}{V_1}(4\overline{c^2})$$

$$\frac{1}{V} = \frac{4}{V_1}$$

$$\therefore V_1 = 4V$$

5. The two curves shown below are isotherms for a fixed mass of an ideal gas



What is the ratio $\frac{r.m.s\ speed\ of\ the\ molecules\ at\ temperature\ T_2}{r.m.s\ speed\ of\ the\ molecules\ at\ temperature\ T_1}$?

Solution

$$p = \frac{1}{3}\rho\,\overline{c^2}$$

$$p = \frac{1}{3}\frac{M}{V}\,\overline{c^2}$$

where M = mass of gas, V = volume of gas

$$pV = \frac{1}{3}M\,\overline{c^2}$$

For isotherm T_1, when $V = 1\,m^3, p = 1 \times 10^5 Pa$

$$1 \times 10^5 \times 1 = \frac{1}{3}M\overline{c_1^2} \quad \text{..........}\ (i)$$

For isotherm T_2, when $V = 2\,m^3, p = 2 \times 10^5 Pa$

$$2 \times 10^5 \times 2 = \frac{1}{3}M\overline{c_2^2} \quad \text{..........}\ (ii)$$

(ii) ÷ (i);

$$\frac{\overline{c_2^2}}{\overline{c_1^2}} = \frac{4 \times 10^5}{1 \times 10^5}$$

$$\frac{\overline{c_2^2}}{\overline{c_1^2}} = 4$$

$$\frac{c_2}{c_1} = 2$$

Mean kinetic energy and temperature

Using the kinetic theory of gases, the relation between pressure p, density ρ and mean square velocity $\overline{c^2}$ is given by

$$p = \frac{1}{3}\rho\overline{c^2}$$

$$p = \frac{1}{3}\frac{Nm}{V}\overline{c^2}$$

$$pV = \frac{1}{3}Nm\overline{c^2}$$

But $pV = nRT$

$$\frac{1}{3}Nm\overline{c^2} = nRT$$

But $N = nN_A$

$$\frac{1}{3}nN_Am\overline{c^2} = nRT$$

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

The ratio $\frac{R}{N_A}$ is known as the Boltzmann's constant

$$\therefore \quad \boldsymbol{\frac{1}{2}m\overline{c^2} = \frac{3}{2}KT}$$

Boltzmann's constant, $K = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}$

We assume that the mean kinetic energy of a given mass of the gas depends only on its temperature in accordance to the theory that heat is a form of energy. Thus, the mean translational kinetic energy of a gas is directly proportional to its absolute temperature.

$$(\frac{1}{2}m\overline{c^2} \propto T)$$

Similarly, $\overline{c^2} \propto T$ since the mass m is constant.

Applications of the kinetic theory of gases

1. **Elastic collisions between gas molecules**

From the equation $p = \frac{1}{3}\rho\overline{c^2}$

$$= \frac{1}{3}\frac{Nm}{V}\overline{c^2}$$

Where N = number of molecules in the gas of volume V

m = mass of each molecule

Pressure, $p = \frac{2}{3V} \times N\left(\frac{1}{2}m\overline{c^2}\right)$

$$\boldsymbol{p = \frac{2}{3V} \times \text{total kinetic energy}}$$

Therefore for a gas of volume V in its equilibrium state, if its pressure p does not change with time, then the total energy of the gas molecules is constant. Since collisions between gas molecules occur and the total kinetic energy is constant, the collisions must be elastic.

2. **Avogadro's hypothesis**

Avogadro's hypothesis states that equal volumes of different gases at the same temperature and pressure contain equal number of molecules

Using the equation $p = \frac{1}{3}\rho\overline{c^2}$

$$= \frac{1}{3}\frac{Nm}{V}\overline{c^2}$$

$$pV = \frac{1}{3}Nm\overline{c^2} \quad \text{........(i)}$$

For the first gas, let $N = N_1$, $m = m_1$ and $c = c_1$

then $pV = \frac{1}{3}N_1m_1\overline{c_1^2}$

For the second gas, $N = N_2, m = m_2$ and $c = c_2$

then $pV = \frac{1}{3}N_2m_2\overline{c_2^2}$

pV is the same for both gases, since they are the same pressure and have the same volume.

$$\text{Thus } \frac{1}{3}N_1m_1\overline{c_1^2} = \frac{1}{3}N_2m_2\overline{c_2^2} \quad \text{..... (ii)}$$

Since the two gases are at the same temperature,

$$\frac{1}{2}m_1\overline{c_1^2} = \frac{3}{2}KT \text{ and } \frac{1}{2}m_2\overline{c_2^2} = \frac{3}{2}KT$$

$$m_1\overline{c_1^2} = m_2\overline{c_2^2}$$

Hence from equation (ii);

$$\frac{N_1}{N_2} = \frac{m_2\overline{c_2^2}}{m_1\overline{c_1^2}} = 1$$

$$\therefore \boldsymbol{N_1 = N_2}$$

In other words, the number of molecules in two different gases having the same pressure p, volume V and temperature T are the same.

Note: Avogadro's law can be established using the ideal gas equation

Consider two gases one with n_1 moles, volume V_1 at a temperature T_1, pressure p_1 and another with n_2 moles, volume V_2 at a temperature T_2, pressure p_2

$$p_1V_1 = n_1RT_1 \quad \text{and } p_2V_2 = n_2RT_2$$

For equal volumes at the same pressure,

$$p_1V_1 = p_2V_2$$

$$\text{Thus, } n_1RT_1 = n_2RT_2$$

If the gases are at the same temperature, $T_1 = T_2$

$$\therefore n_1 = n_2$$

3. **Dalton's law of partial pressures**

The law states that when two or more gases which do not react with one another are in the same container, then the total pressure is the sum of the pressures which each gas would exert by itself

Suppose that two gases are mixed in a container of volume V, then using the equation below

$$pV = \frac{1}{3}Nm\overline{c^2}$$

For the first gas: $p_1V = \frac{1}{3}N_1m_1\overline{c_1^2}$ .... (i)

For the second gas: $p_2V = \frac{1}{3}N_2m_2\overline{c_2^2}$ ....(ii)

The two gases in the same container are at the same temperature, thus the mean translational kinetic energies must be the same

$$\frac{1}{2}m_1\overline{c_1^2} = \frac{1}{2}m_2\overline{c_2^2}$$

(i) ÷ (ii)

$$\frac{p_1}{p_2} = \frac{N_1}{N_2}$$

$$p_1 \propto N_1 \text{ and } p_2 \propto N_2$$

i.e. pressure is directly proportional to the number of molecules

The total number of molecules in the container is $(N_1 + N_2)$, therefore the total pressure

$$p \propto (N_1 + N_2)$$

$$\boldsymbol{p = p_1 + p_2}$$

The total pressure is equal to the sum of the pressures exerted by each gas

Suppose that volume V contains n_1 moles of a gas whose partial pressure is p_1 and n_2 moles of a gas whose partial pressure is p_2. If the equation of the gases is T, then

$$p_1V = n_1RT \quad \text{and } p_2V = n_2RT$$

Dividing gives;

$$\frac{p_1}{p_2} = \frac{n_1}{n_2} \quad \text{..... (i)}$$

From Dalton's law, the total pressure is given by

$$p = p_1 + p_2$$

From (i), $p_1 = \frac{n_1}{n_2}p_2$

Substituting for p_2

$$p_1 = \frac{n_1}{n_2}(p - p_1)$$

$$n_2p_1 = n_1p - n_1p_1$$

$$n_1p_1 + n_2p_1 = n_1p$$

$$p_1 = \left(\frac{n_1}{n_1+n_2}\right)p$$

Similarly, substituting for p_1 gives

$$p_2 = \left(\frac{n_2}{n_1+n_2}\right)p$$

4. **Graham's law of diffusion**

It states that the rate of diffusion of a gas through a porous partition is inversely proportional to the square root of its density

From $p = \frac{1}{3}\rho\overline{c^2}$

where p is the pressure and ρ is the density. Thus for a given gas pressure,

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3p}{\rho}}$$

$$\text{Rate of diffusion} \propto \sqrt{\overline{c^2}}$$

$$\therefore \text{rate of diffusion} \propto \frac{1}{\sqrt{\rho}}$$

5. **Charles' law**

From $p = \frac{1}{3}\frac{Nm}{V}\overline{c^2}$

$$p = \frac{2N}{3V}\left(\frac{1}{2}m\overline{c^2}\right)$$

$$p = \frac{2N}{3V}\left(\frac{3}{2}KT\right)$$

$$p = \frac{NKT}{V} \quad \text{...... (i)}$$

$$\frac{V}{T} = \frac{NK}{p}$$

If p = constant, $\frac{V}{T}$ = constant

6. **Pressure law**

From (i); $p = \frac{NKT}{V}$

$$\frac{p}{T} = \frac{NK}{V}$$

If V = constant, $\frac{P}{T}$ = constant

7. **Boyle's law**

From (i); $p = \frac{NKT}{V}$

$$pV = NKT$$

If T = constant, PV =constant

8. **Ideal gas equation**

From $\frac{1}{2}m\overline{c^2} = \frac{3}{2}nRT$ .... (i)

also $p = \frac{1}{3}\rho\overline{c^2}$ ...... (ii)

$$\overline{c^2} = \frac{3p}{\rho}$$

substituting for $\overline{c^2}$ in (i);

$$\frac{1}{2}m\left(\frac{3p}{\rho}\right) = \frac{3}{2}nRT$$

$$\frac{m}{\rho}p = nRT$$

$$pV = nRT$$

Examples

1. In order to achieve a fusion reaction between deuterium nuclei, temperatures of the order of $1 \times 10^7\ K$ must be attained. Estimate the mean speed of deuterium nuclei at this temperature. [Take the Boltzmann's constant, K as $1.4 \times 10^{-23}\ J\ K^{-1}$ and the mass of deuterium nucleus as 3.3×10^{-27}kg]

Solution

From the equation $\frac{1}{2}m\overline{c^2} = \frac{3}{2}KT$

$$c_{r.m.s} = \sqrt{\frac{3KT}{m}}$$

$$= \sqrt{\frac{3\times(1.4\times10^{-23})\times(1\times10^{7})}{3.3\times10^{-27}}} = 3.6 \times 10^5\ ms^{-1}$$

2. Calculate the root-mean-square speed of the molecules of hydrogen at (a) 273 K (b) 373 K Density of hydrogen at s.t.p = $0.09\ kg\ m^{-3}$ and 1 standard atmosphere = $1.01 \times 10^5\ Pa$

Solution

(a) At 273 K

From $p = \frac{1}{3}\rho\overline{c^2}$

$$c_{r.m.s} = \sqrt{\frac{3\times1.01\times10^5}{0.09}}$$

$$= 1.84 \times 10^3\ ms^{-1}$$

(b) At 373 K

$$\overline{c^2} \propto T$$

$$\frac{c_{r.m.s}\ at\ 373}{c_{r.m.s}\ at\ 273} = \sqrt{\frac{373}{273}}$$

$$c_{r.m.s}\ at\ 373 = 1.84 \times 10^3\sqrt{\frac{373}{273}}$$

$$c_{r.m.s}\ at\ 373 = 2.15 \times 10^3\ ms^{-1}$$

Real gases – Non-ideal behaviour

In consideration of the ideal gases, a number of assumptions are made however two of them do not hold good i.e.

- The volume taken up by the imaginary ideal gas molecules is ignored.
- Gas molecules do not attract or repel each other

However, in real life, gases are made up of atoms and molecules that actually take up a finite volume and we know that atoms and molecules interact with each other through intermolecular forces.

1. At high pressure, the gas molecules get more crowded and the amount of empty space between the molecules is reduced. Thus, for a given pressure, the real gas will end up taking a greater volume than predicted by the ideal gas law.
2. At low temperatures, attractive forces between molecules will pull them closer which slows down the molecules before they hit the container walls. This results in decrease in volume if the pressure is constant compared to what is expected based on the ideal gas equation.
   The effect of intermolecular forces is much more prominent at low temperatures because the molecules have less kinetic energy to overcome the intermolecular attractions

Conditions under which real gases behave ideally

- High temperatures (intermolecular forces are overcome)
- Low pressures (molecules move further away apart reducing their volume)

Differences between real gases and ideal gases

Ideal gases	Real gases
Obey Boyle's law	Deviate from Boyle's law
Negligible intermolecular forces	Intermolecular forces are significant
Volume of molecules is negligible	Volume of molecules is significant
Velocity of molecules is constant	Velocity of molecules varies

Van der Waal's equation

The Van der Waal's equation basically incorporates the effect of gas molecule volume and intermolecular forces into the ideal gas equation.

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

where p = measured pressure
V = volume of container
R = gas constant
T = temperature in kelvin

For n moles of a gas, the equation becomes

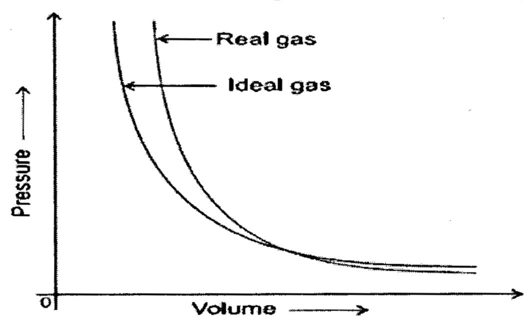
$$\left(p+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

The term "$\frac{a}{V^2}$" is a correction to the pressure which accounts for the measured pressure being lower due to attraction between gas molecules.

The term "b" is a correction to the volume which accounts for the volume of the gas molecules.

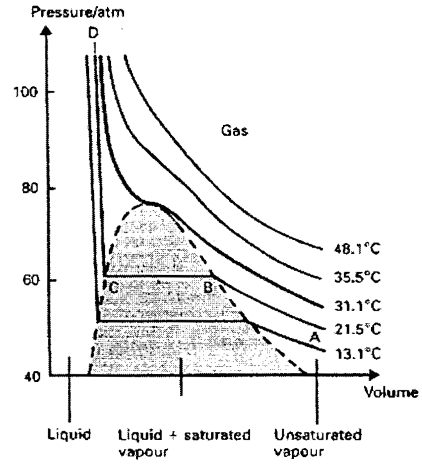
Note: a and b are measured constants for a specific gas and they might have a slight or negligible temperature and pressure dependence.

Variation of volume with pressure for a real gas



Liquefaction of gases

First complete data on pressure - volume - temperature relations of a substance in both gaseous and liquid state was obtained by Thomas Andrews on carbon dioxide. He plotted isotherms of carbon dioxide at various temperatures. Later on, it was found that real gases behave in the same manner as carbon dioxide. Andrews noticed that at high temperatures isotherms look like that of an ideal gas and the gas cannot be liquefied even at very high pressure. As the temperature is lowered, shape of the curve changes and data shows considerable deviation from ideal behaviour.



The graph shows the critical nature of the 31.1 °C isothermal. Above 31.1 °C, the carbon dioxide exists as a gas no matter how high the pressure and the curve appears as if it is an ideal gas.

Below 31.1 °C, the carbon dioxide can exist in both the gaseous state (as a vapour) and the liquid state. Consider the carbon dioxide to be in the state of pressure, volume and temperature that is represented by the point A on the 21.5 °C isothermal. In this state, the carbon dioxide is an unsaturated vapour and if it is compressed, the p-V curve is almost hyperbolic until pressure at point B.

At B, the carbon dioxide begins to liquefy.

Between B and C, the volume decreases with no increase in pressure. The decrease in volume is due to the fact that in moving from B to C more and more liquid forms, so that at C the carbon dioxide is entirely liquid.

From C to D and beyond, large increases in pressure produce very little decrease in volume (as might be expected) since liquids are virtually incompressible.

Terminologies

Critical temperature (T_c) is the temperature above which gas cannot be liquified however great the pressure may be

Critical pressure (p_c) is the minimum pressure that will cause liquefaction of a gas at its critical temperature

Critical volume (V_c) is the volume occupied by 1 kg of a gas at its critical temperature and critical pressure

Significance of critical temperature

The term **fluid** is used for either a liquid or a gas. Thus, a liquid can be viewed as a very dense gas.
Liquid and gas can be **distinguished** only when the fluid is **below its critical temperature** when the liquid and gas are in equilibrium and a surface separating the two phases is visible.
Gas is the term applied to a substance which is in the gaseous phase and it is **above** its critical temperature.
Vapour is the term applied to a substance which is in the gaseous phase and is below its critical temperature. Thus, a vapour can be liquefied by simply increasing the pressure on it, a gas cannot.

Self-Evaluation exercise

1. A small dust particle suspended in a gas is struck successively by five gas molecules whose speeds are $300\,ms^{-1}$, $500\,ms^{-1}$, $400\,ms^{-1}$, $600\,ms^{-1}$ and $200\,ms^{-1}$. Calculate the root mean square speed of these five molecules. [Ans: $424\,ms^{-1}$]
2. (a) State the assumptions of the kinetic theory of gases
   (b) The kinetic theory predicts that the pressure exerted by an ideal gas is given by
   $$p = \frac{1}{3}\,nm\overline{c^2}$$
   (i) State the physical quantity represented by each term in the equation
   (ii) Use the equation to obtain an expression for the root-mean-square speed of the atoms of a gas in terms of T, the temperature of the gas, M its molar mass, and R the molar gas constant
   (iii) Calculate the root-mean-square speed of hydrogen molecules at a temperature of $-60°C$
   (Molar mass of hydrogen = 2.0×10^{-3} kg mol^{-1})
3. Show how the elementary kinetic theory of gases accounts for Boyle's law. Calculate the root mean square velocity of the molecules of hydrogen at 0 °C and at 100 °C. [Density of hydrogen at 0°C and 760 mm mercury pressure = 0.09 kg m^{-3}]
   [Ans: 1840, $2150\,ms^{-1}$]
4. State the basic postulates of the kinetic theory of gases.
   It can be shown that the pressure p exerted by a gas on the walls of its container is equal to $\frac{1}{3}\rho c^2$ where ρ is the density, and c is the root mean square velocity of the molecules. Explain why the root mean square velocity occurs in this expression. How is the concept of temperature introduced into the kinetic theory?
   Show that the kinetic theory is consistent with the following: (a) Graham's law of diffusion; (b) Dalton's law of partial pressures; (c) Avogadro's hypothesis.
   Suggest an explanation of the following observation.
   A mixture of hydrogen and nitrogen is kept under pressure in a cylinder which has a leaky valve. Relatively more hydrogen escapes than nitrogen.
5. Derive an expression connecting the pressure of a gas with its density and the mean square of the velocity of its molecules. State the assumptions that are required in the derivation of this expression.
   Show that this expression is consistent with the gas equation pV/T = constant, provided a certain assumption is made. State this assumption.
6. Explain what is meant by the root mean square velocity of the molecules of a gas. Use the concepts of the elementary kinetic theory of gases to derive an expression for the root mean square velocity of the molecules in terms of the pressure and density of the gas.
   Assuming the density of nitrogen at s.t.p. to be 1.251 kg m^{-3}, find the root mean square velocity of nitrogen molecules at 127 °C. [Ans: $597\,ms^{-1}$]
7. A balloon has a volume $5.50 \times 10^{-2}\,m^3$. It is filled with helium to a pressure of $1.10 \times 10^5\,Pa$ at a temperature of 20 °C. Calculate
   (a) the number of moles inside the balloon
   (b) the number of helium atoms inside the balloon
   (c) the net force acting on $1\,cm^2$ of the material of the balloon if the atmospheric pressure is $1.01 \times 10^5\,Pa$
   [Ans: (a) 2.48 (b) 15.0×10^{23} (c) 0.90 N]
8. Give non-mathematical explanations, in terms of molecules, for the following
   (i) A gas exerts a pressure on the walls of its container
   (ii) The gas pressure increases as the temperature increases
   (b) A cylinder of volume $30 \times 10^{-3}\,m^3$ contains 0.20 kg of oxygen gas at a temperature of 300 K. Calculate
   (i) the number of molecules of gas in the container
   (ii) the pressure exerted by the gas
   (iii) the root-mean-square speed of the molecules
   (mass of 1 mole of oxygen = 0.032 kg]

[Ans:(b)(i) 38×10^{23} (ii) $5.2 \times 10^5 Pa$ (iii) $0.48\ km\,s^{-1}$]

9. If the density of nitrogen at s.t.p is $1.25\ kgm^{-3}$, calculate the root-mean square speed of nitrogen molecules at 227 °C. [Ans: $663\ ms^{-1}$]
10. State the assumptions that are made in the kinetic theory of gases and derive an expression for the pressure exerted by a gas which conforms to these assumptions, in terms of its density ρ and the mean square velocity c^2 of its molecules.
    Show (a) how temperature may be interpreted in terms of the theory, (b) how the theory accounts for Dalton's law of partial pressures.
11. A volume of $71200\ cm^3$ of a certain ideal gas contains 1.03×10^{24} atoms. The gas has a density $0.800\ kg\ m^{-3}$
    (a) Calculate the mass of one atom of the gas
    (b) The pressure exerted by this gas is measured and is found to be 80.0 kPa
       (i) Calculate the root-mean-square speed of the atoms of the gas
       (ii) Calculate the temperature of the gas
    [Ans: (a) $5.53 \times 10^{-26}\ kg$ (b)(i) $1.6\ ms^{-1}$ (ii) 401 K]
12. Two moles of argon have a mass of 0.036 kg and occupy a rigid container of volume $0.04\ m^3$ at a pressure of $1.0 \times 10^5 Pa$. Calculate
    (i) the root-mean-square speed of an argon atom
    (ii) the temperature of the argon gas
    (iii) the total internal energy of the gas atoms
    (iv) The safety valve in the container will open if the pressure of the gas inside exceeds 1.5×10^5 pa. If the gas is now heated, calculate the temperature at which the safety valve will open.
    [Ans: (i) $0.58\ kms^{-1}$ (ii) $241\ K$ (iii) $6.0\ kJ$ (iv) $361\ K$]
13. Calculate the pressure in mm of mercury exerted by hydrogen gas if the number of molecules per cm^3 is 6.80×10^{15} and the root mean square speed of the molecules is $1.90 \times 10^3\ ms^{-1}$. Comment on the effect of a pressure of this magnitude (a) above the mercury in a barometer tube;(b) in a cathode ray tube, (Avogadro constant $= 6.02 \times 10^{23}$. Molecular weight of hydrogen = 2.02)
    [Ans: 0.21 mmHg]
14. Use a simple treatment of the kinetic theory of gases, stating any assumptions you make, to derive an expression for the pressure exerted by a gas on the walls of its container. Hence deduce a value for the root mean square speed of thermal agitation of the molecules of helium in a vessel at 0 °C. (Density of helium at s.t.p $= 0.1785\ kgm^{-3}$; 1 atmosphere $= 1.013 \times 10^{15}\ Nm^{-2}$)
    If the total translational kinetic energy of all the molecules of helium in the vessel is 5×10^{-6} joule, what is the temperature in another vessel which contains twice the mass of helium and in which the total kinetic energy is 10^{-5} joule? (Assume that helium behaves as a perfect gas)
    [Ans: $1305\ ms^{-1}$, 0 °C]
15. (a) Describe how the concept of absolute zero is explained in terms of the kinetic theory of gases and ideal gases.
    (b) A flask of volume $2.0 \times 10^{-3}\ m^3$ containing an ideal gas at a temperature of 290 K and pressure 100 kPa is sealed with a rubber stopper. Calculate the number of gas molecules in the flask
    (c) On heating the flask in (b), the rubber stopper is forced out when the temperature exceeds 400 K.
       (i) The area of the lower surface of the stopper is $4.0 \times 10^{-4}\ m^2$. Calculate the force exerted on this area at $400\ K$.
       (ii) Calculate, for ideal gas molecules, the ratio $\frac{r.m.s\ speed\ at\ 400\ K}{r.m.s\ speed\ at\ 290\ K}$
    [Ans: (b) 5.0×10^{22} (c)(i) 55 N (ii) 1.38]
16. Suggest reasons why the molecules in a gas at constant temperature are not all moving with the same speed.
    A vessel of volume 500 cm^3 contains hydrogen at a pressure of 10^{-3} mmHg and at a temperature of 17°C. Estimate (a) the number of molecules in the vessel, (b) their distance apart, on the average, (c) their root-mean-square speed, (d) the number of impacts they make per second on an area of 1 cm^2 of the wall of the vessel. (R = 8.3 J mol^{-1} K^{-1}; number of molecules in a mole $= 6.02 \times 10^{23}$; molecular weight of hydrogen = 2.0, density of mercury = 13600 kg m^{-3})
    [Ans: (a) $1.7 \times 10^{16}\ moles$ (b) $3 \times 10^{-5}\ cm$ (c) $1.9 \times 10^3\ ms^{-1}$ (d) $10^{18}\ cm^{-2}s^{-1}$]
17. A vessel of volume $1.0 \times 10^{-3}\ m^3$ contains helium gas at a pressure of $2.0 \times 10^5 Pa$ when the temperature is 300 K.
    (a) What is the mass of helium in the vessel?
    (b) How many helium atoms are there in the vessel?
    (c) Calculate the r.m.s speed of the helium atoms (relative atomic mass of helium = 4)
    [Ans: (a) 0.32 g (b) 4.8×10^{22} (c) $1.4 \times 10^3\ ms^{-1}$]

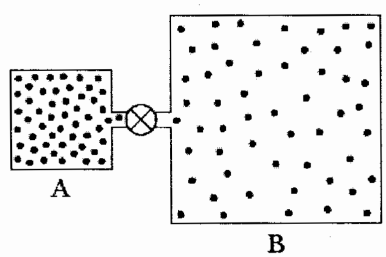
18. A cubical container of volume 0.10 m^3contains uranium hexafluoride gas at a pressure of 1.0×10^6 Pa and a temperature of 300 K.
   (a) Assuming that the gas is ideal, determine
      (i) the number of moles of gas present
      (ii) the mass of gas present given that its relative molecular mass is 352
      (iii) the density of the gas
      (iv) the r.m.s speed of the molecules
   (b) A student suggests that since the molecules are so massive, the density of the gas at the bottom of the container would be significantly greater than the density at the top. Explain whether you agree or disagree.
   [Ans: (a)(i) 40.2 (ii) 14.1 kg (iii) 141 kgm^{-3}]

19. (a) List the basic assumptions of the kinetic model of a gas
   (i) List the simplifying assumptions usually made to obtain the relation $p = \frac{1}{3}\rho\overline{c^2}$ between the pressure and density of an ideal gas
   (ii) Explain the meaning of the term $\overline{c^2}$. Give the equation relating $\rho\overline{c^2}$to temperature.
   (iii) Show that, for a mixture of gases that do not react, the total pressure exerted by the mixture at a given temperature is the sum of the pressures which would be exerted if each component alone filled the vessel.
   1.0 mol of liquid dinitrogen tetroxide (N_2O_4) is introduced into an evacuated vessel of volume 5.0×10^{-3} m^3. The temperature is increased to 27 °C, by which the temperature all liquid has evaporated. The vapour consists of a mixture of N_2O_4 and NO_2 molecules in equilibrium, the NO_2 molecules being produced by the dissociation of N_2O_4; two NO_2 molecules are obtained from each N_2O_4 molecule that dissociates. At 27 °C, the total pressure of in the vessel is 6.5×10^5 Pa
   (iv) If at 27 °C a fraction x of the N_2O_4 molecules originally present has been dissociated, state in terms of x how many moles of (i) N_2O_4 (ii) NO_2, are then in the vessel
   (v) Write down expressions in terms of x for the partial pressures of the vapours of
      (i) N_2O_4
      (ii) NO_2, in the vessel at 27 °C
   (vi) Find the fraction x
   (Assume that both vapours behave as ideal gases, Molar gas constant = 8.3 $J\,K^{-1}\,mol^{-1}$)

20. The temperature T of an ideal gas at a pressure p is defined by the equation $p = nkT$
   (a) Identify the quantities n and k in this equation.
   (b) Write down another equation from kinetic theory involving $p, n,$ the mean square speed $\overline{c^2}$ of the molecules and the mass m of the molecule.
   (c) Hence find an equation for the root-mean-square speed of the molecules in terms of k, T and m
   (d) A certain plasma contains hydrogen ions (protons) and electrons in thermal equilibrium. Both protons and electrons can be assumed to behave as the molecules of an ideal gas. The root mean square speed of the electrons in the plasma is estimated to be 3×10^6 ms^{-1}
      (i) What is the root-mean-square speed of the hydrogen ions?
      (ii) Estimate the temperature of the plasma

21. A gas cylinder contains 6400 g of oxygen at a pressure of 5 atmospheres. An exactly similar cylinder contains 4200 g of nitrogen at the same temperature. What is the pressure on the nitrogen? (Molecular weights: oxygen = 32, nitrogen = 28, assume that each behaves as a perfect gas.
   [Ans: 3.75 atm]

22. Container A in the figure below holds an ideal gas at a pressure of 5×10^5 Pa and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A.



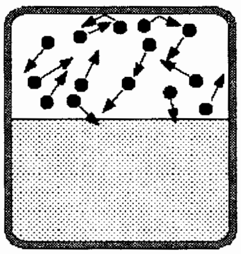
Container B holds the same ideal gas at a pressure of 1.0×10^5 Pa and a temperature of 400 K. The valve is opened to allow the pressures to equalize, but the temperature of each container is maintained. What then is the pressure? [Ans: 2×10^{-5} Pa]

VAPOURS

A gas is a substance in the gaseous phase above its critical temperature while vapour is a substance in the gaseous phase but below its critical temperature.
If the vapour is in a closed vessel above the liquid, it exerts a pressure on the liquid. At dynamic equilibrium, this will be saturated vapour pressure otherwise it is unsaturated.

A saturated vapour is a vapour which is in equilibrium with its own liquid while an unsaturated vapour is a vapour which is not in equilibrium with its own liquid.

Occurrence of saturated vapour pressure

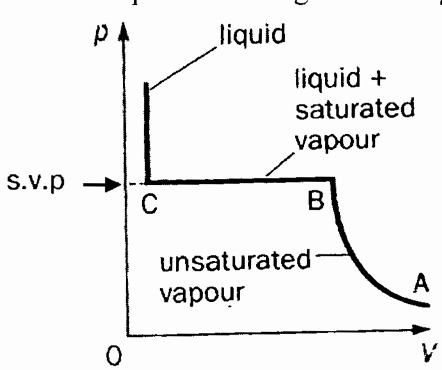


According to the kinetic theory, the most energetic molecules overcome attraction by other molecules when a liquid is heated in a closed vessel. These molecules leave the surface of the liquid to become vapour molecules (evaporation).
The vapour molecules collide with the walls of the container creating vapour pressure. These molecules lose their energy on collision and re-enter the liquid (condensation). If the rate at which the molecules enter the vapour is equal to the rate at which they return to the liquid, dynamic equilibrium is said to be established.
The region above the liquid is then said to be saturated with vapour and the pressure exerted by this vapour is the saturated vapour pressure.

The saturated vapour pressure of a substance is the pressure exerted by the vapour in equilibrium with the liquid

Vapours and the gas laws

Unsaturated vapours obey Boyle's law roughly up to near saturation point i.e. along AB in the p-V graph

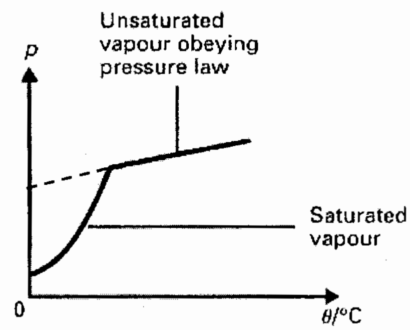


At B, condensation of the vapour starts, liquid and saturated vapour exist together along BC and since the mass of the vapour is changing, Boyle's law is no longer relevant.
The pressure along BC is the saturated vapour pressure and does not change as the volume of saturated vapour decreases.

Note: If the temperature of such a system is increased, there are two distinct consequences

(i) The kinetic energy of the vapour molecules increases
(ii) The rate of evaporation increases and therefore there is an increase in the number of molecules in the vapour phase

If the volume of the system is constant, each of these effects produces an increase in pressure.
The effect of (i) alone would give a pressure increase of the form predicted by the pressure law. The additional effect of (ii) means that the increase in pressure with increasing temperature is much more rapid than this.



If the temperature is increased at constant pressure, the volume increases but because of (ii) it increases much more rapidly than required by Charles' law

Kinetic theory explanations

In an unsaturated vapour, the rate at which molecules leave the liquid surface exceeds that at which they enter it from vapour i.e. evaporation occurs more rapidly than condensation. In a saturated vapour, the rates are equal and dynamic equilibrium exists. The rate of leaving depends on the average kinetic energy of the molecules and so on the temperature. The rate of entering is determined by the temperature and density of the vapour

The saturated vapour pressure increases when the temperature of the liquid is raised.
This is because as the average kinetic energy of the molecules increases, more are able to escape from the surface. The rate of evaporation becomes greater thereby increasing the density of the vapour and so also the rate of condensation. Eventually equilibrium and

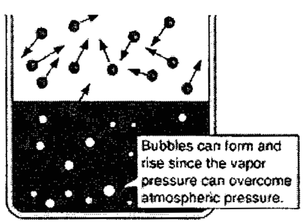
saturation are re-established at a greater saturated vapour pressure than before.

The saturated vapour pressure is not affected by changes of volume (at constant temperature)
If the volume available to the vapour decreases, its density momentarily increases and more molecules return to the liquid in the given time than previously. The rate at which molecules leave the surface remains steady, however, and so the rate of condensation now exceeds the rate of evaporation until the density of the vapour falls to its original value and dynamic equilibrium is restored once more, with the saturated vapour pressure having its initial value. What happens if the volume of the vapour increases?

Evaporation
Evaporation is the process by which a liquid becomes vapour. It can take place at all temperatures but occurs at the greatest rate when the liquid is at its boiling point.
The rate of evaporation can be increased by
- increasing the area of the liquid surface
- increasing the temperature of the liquid.
- causing a draught to remove the vapour molecules before they get a chance to return to the liquid
- reducing the air pressure above the liquid

When a liquid evaporates it loses those of its molecules which have the greatest kinetic energies and therefore **when a liquid evaporates, it cools.**

Boiling



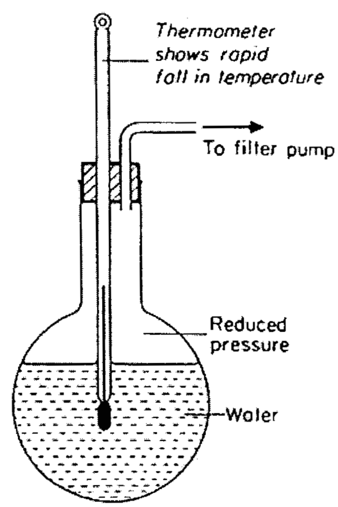
Whereas evaporation occurs from the surface of the liquid at all temperatures, boiling takes place at a temperature determined by the external pressure and consists in the formation of bubbles of vapour throughout the liquid. The pressure inside such bubbles must at least equal the pressure in the surrounding liquid.
The pressure inside is the saturated vapour pressure at the temperature of the boiling point since the vapour is in contact with liquid.
A liquid thus boils when its saturated vapour pressure equals external pressure.

The boiling point of a liquid is that temperature at which its saturated vapour pressure is equal to the external pressure

Note: Observations show that water boils at a much lower temperature than 100 °C at the top of a mountain. Since the external pressure is the atmospheric pressure, it decreases with increase in altitude which in turn reduces the boiling point of the liquid.
Thus, reduced external pressure on a liquid leads to a reduction in the boiling point

Dew-point
In the early morning, when the temperature is low, dew may be observed on grass, showing that the air near the grass has become saturated with water-vapour. The temperature at which the air becomes saturated with the water-vapour present in it is known as the dew-point, and the latter can be determined by progressively cooling a bright metal surface in the air. At some point the surface becomes misty, showing that water has condensed on it, and the dew-point is the corresponding temperature of the cooled metal surface. The dew-point is utilized in one of the standard methods of measuring relative humidity

Boiling under reduced pressure



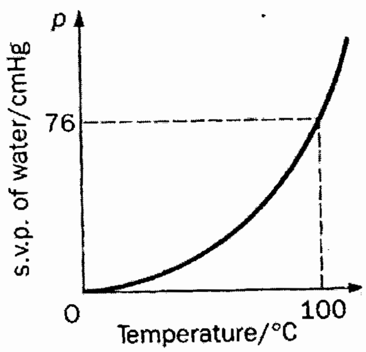
Water can be made to boil without heating it simply by reducing the atmospheric pressure above it to a value less than saturated vapour pressure. This may be done with aid of a filter pump.
A stout, round-bottomed flask containing warm water is fitted with a two-holed rubber bung through which passed a thermometer and a short glass tube. When the flask is connected to a good filter pump the water begins to boil as soon as the pressure becomes less than the saturated vapour corresponding to the temperature

of the water at the time. Since no heat is being supplied from outside, the necessary latent heat of vaporization has to come from the water itself. It therefore cools and the temperature indicated by the thermometer drops rapidly.

Variation of saturated vapour pressure of water with temperature

If the pressure above a boiling liquid is increased, it stops boiling because the external pressure is now greater than the saturated vapour pressure. If the temperature of the liquid is increased, its saturated vapour pressure rises and eventually becomes equal to the new external pressure.

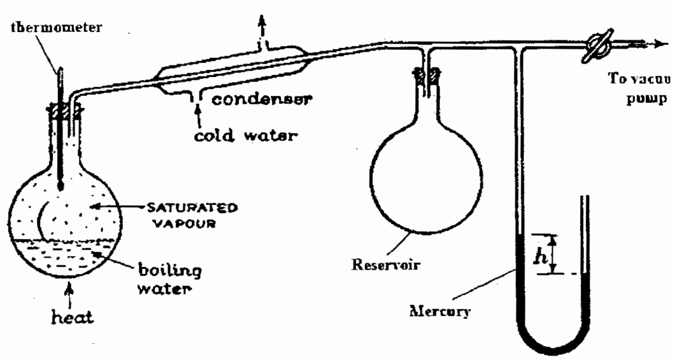
Thus the boiling point of a liquid increases with pressure.



Differences between boiling and evaporation

- Boiling occurs throughout the volume of a liquid whereas evaporation occurs only at surface
- For any given external pressure, a liquid boils at a single temperature only whereas evaporation takes place at all temperatures

Experimental determination of saturated vapour pressure



- The apparatus is setup as shown above
- Pressure above the liquid is reduced to some desired value below atmospheric pressure by means of a vacuum pump.
- The liquid is then heated gently and it starts to boil at a temperature which is determined by the pressure inside the apparatus
- The vapour is condensed and returned to the round-bottomed flask thereby preventing a pressure build up with in the apparatus.
- After a few minutes, the liquid boils at a temperature indicated by the thermometer.
- The corresponding pressure obtained from the manometer is the saturated vapour pressure of the liquid at this temperature
- If h is the height in mmHg, then the saturated vapour pressure $p = H - h$ where H is the barometric height

To determine saturated vapour pressures above atmospheric pressure, the vacuum pump is replaced by the bicycle pump

Mixture of gas and saturated water-vapour

Suppose that some water and air are present in the closed space at the top of a column of mercury, the temperature being 10 °C. If the s.v.p of water at 10 °C is 8 mm of mercury, the pressure of the water vapour and air is 700 mm of mercury.

From Dalton's law,

$$\text{Pressure of air} = 700 - 8 = 692\ mm$$

Suppose that the volume of the mixture is 30 cm^3 and that the volume is reduced to 25 cm^3, the temperature remaining constant. Then if p is the new total pressure of the mixture, the pressure of the air is now $(p - 8)\ mm$, as the s.v.p of water depends only on its temperature. Further, the volume originally occupied by the air was 30 cm^3 and 25 cm^3 when the volume is altered.

To find the magnitude of p, we apply Boyle's law to the given mass of air since the temperature is unchanged.

$$pV = constant$$

$$(p - 8) \times 25 = 692 \times 30$$

$$\therefore p = 838\ mm\text{Hg}$$

Examples

1.(a) A vessel contains a mixture of air and water-vapour in contact with excess of the liquid. How will the pressure in the vessel change, (i) if the volume is changed at constant temperature, (ii) if the temperature changes at constant volume?

(b) A closed vessel contains a mixture of air and water-vapour in contact with excess of water. The pressures in the vessel at 27 °C and 60 °C are respectively 777 mm and 981 mm of mercury. If the vapour of water at 27 °C is 27 mm of mercury, what is the vapour pressure at 60 °C?

Solution

(a) (i) If the volume is reduced at constant temperature, the pressure of the air increases, by Boyle's law. The s.v.p. of the water remains unchanged, however, and the total pressure of the mixture thus increases. If the volume is increased, similar reasoning shows that the total pressure decreases.

(ii) If the temperature is increased at constant volume, the pressure of the air and the s.v.p. of water both increase; hence the total pressure increases. Similar reasoning shows that the total pressure decreases if the temperature decreases.

(b) The pressure of the air at 27 °C

$$= 777 - 27 = 750\, mmHg$$

The pressure of the air at 60 °C = $981 - p$

where p is the s.v.p. of water at 60 °C.

Since the pressure of a gas at constant volume is proportional to its absolute temperature, for the constant mass of air in the mixture,

$$\frac{981 - p}{750} = \frac{273 + 60}{273 + 27}$$

$$981 - p = \frac{333}{300} \times 750$$

$$p = 149\, mmHg$$

2. A closed vessel contains air saturated with water vapour at 77 °C. The total pressure in the vessel is 1000 mmHg. Calculate the new pressure in the vessel if the temperature is reduced to 27 °C. (the s.v.p of water at $77\,°C = 314\, mmHg$; s.v.p of water at $27\,°C = 27\, mmHg$)

Solution

$$77\,°C = 77 + 273 = 350\, K$$

$$27\,°C = 27 + 273 = 300\, K$$

By Dalton's law of partial pressures, the pressure of the air at $350\, K = 1000 - 314 = 686\, mmHg$

Treating the air as an ideal gas and assuming that its volume is V and is constant, its pressure p at $300\, K$ is given by

$$\frac{680 \times V}{350} = \frac{pV}{300}$$

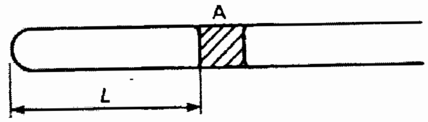
$$p = 588\, mmHg$$

Pressure of the water at 27 °C = $27\, mmHg$

Total pressure at 27 °C = $588 + 27$

$$= 615\, mmHg$$

3. A long uniform horizontal capillary tube, sealed at one end, and open to the air at the other, contains air trapped a short column of water A.



The length L of the trapped air column at temperature $300\, K$ and $360\, K$ is 10 cm and 30 cm respectively. Given that the vapour pressures of water at the same temperatures are $4\, kPa$ and $63\, kPa$ respectively, calculate the atmospheric pressure.

Solution

The capillary tube traps a mixture of air and saturated water vapour.

Let $H =$ atmospheric pressure and $A =$ cross-sectional area.

Atmospheric pressure = air pressure + s.v.p

Air pressure = atmospheric pressure – s.v.p

Volume $= AL$

At 300 K,

air pressure, $p_1 = (H - 4000)Pa$, $V_1 = 0.1\, A\, m^3$

At 360 K,

air pressure, $p_2 = (H - 62000)Pa$, $V_2 = 0.3\, A\, m^3$

Assuming that the air in the capillary tube behaves as an ideal gas,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\frac{(H-4000)(0.1\,A)}{300} = \frac{(H-62000)(0.3\,A)}{360}$$

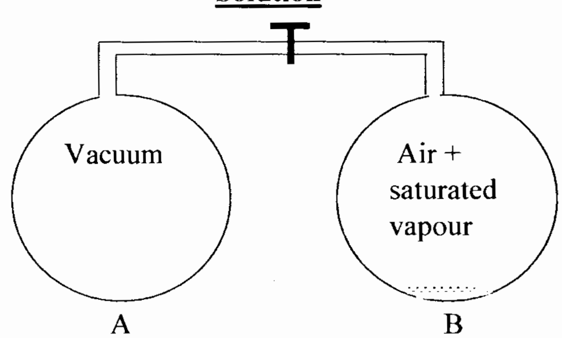
$$36H - 144000 = 90H - 5580000$$

$$54H = 5436000$$

$$H = 1.006 \times 10^5\, Pa$$

4. Two one-litre flasks are joined by a closed tap and the whole is held at a constant temperature of 50 °C. One flask is evacuated and the other contains air, water-vapour, and a small quantity of liquid water. The total pressure in the latter flask is 200 mmHg. The tap is then opened and the system is allowed to reach equilibrium, when some liquid water remains. Assuming that air obeys Boyle's law, find the final pressure in the flasks. [Vapour pressure of water at 50 °C = 93 mmHg.]

Solution



When the tap is closed, A does not contain air, while B contains a mixture of air and saturated vapour.

When the tap is open, Both A and B will contain a mixture of air and saturated vapour at the same pressure at equilibrium.

With tap closed,

Total pressure in B = s.v.p + air pressure in B

Air pressure in B, $p_B = 200 - 93 = 107$ mmHg

Volume of air in B, $V_B = 1\ m^3$

Air pressure in A, $p_A = ?$

Volume of air in $A, V_A = 0$

With tap open,

Final air volume, $V = 2 \times 1 = 2\ m^3$

Final pressure $= p_t$

Air pressure, $p = p_t - 93$

Total number of moles = moles in A + moles in B

$$n = n_A + n_B$$

$$\frac{pV}{RT} = \frac{p_A V_A}{RT} + \frac{p_B V_B}{RT}$$

Since temperature is constant, $pV = p_A V_A + p_B V_B$

$$(p_t - 93) \times 2 = p_A \times 0 + 107 \times 1$$

$$p_t - 93 = 53.5$$

$$p_t = 146.5\ mmHg$$

The final pressure in the flasks is 146.5 mmHg

Differences between saturated vapours and unsaturated vapours

Saturated vapours	Unsaturated vapours
Deviates from gas laws	Obeys gas laws
Exists at fixed temperature	Exists at any temperature
Exists only when the liquid is present	Does not presence of liquid
In dynamic equilibrium with its own liquid	Not in dynamic equilibrium with its own liquid

Self-Evaluation exercise

1. Distinguish between a saturated and an unsaturated vapour.
   Describe how the variation of the saturation vapour pressure of water-vapour with temperature may be investigated.
2. A closed vessel of fixed volume contains air and water. The pressures in the vessel at 20°C and 75°C are respectively 737.5 mm and 1144 mm of mercury, and some of the water remains liquid at 75°C. If the saturation vapour pressure of water at 20 °C is 17.5 mm of mercury, find its value at 75°C. [Ans: 855 mmHg]
3. Distinguish between evaporation and boiling.
4. State Boyle's law and Dalton's law of partial pressures.
   A mixture of air and unsaturated alcohol vapour in the presence of liquid alcohol exerts a pressure of 12.8 mm of mercury at 20 °C. When the mixture is heated at constant volume to the boiling-point of alcohol at standard pressure (i.e. 78 °C), the vapour remaining saturated, the pressure becomes 86.0 mm of mercury. Find the saturation vapour pressure of alcohol at 20 °C. [Ans: 44.5 mmHg]
5. What is meant by saturation vapour pressure? Describe an experiment to investigate the variation of the saturation pressure of water-vapour with temperature.
6. Use the simple kinetic theory of matter to answer the following questions:
   (a) How do gases conduct heat?
   (b) Why does a liquid tend to cool when it evaporates?
   (c) Why does the boiling-point of a liquid depend upon the external pressure?
7. A column of air was sealed into a horizontal uniform-bore capillary tube by a water index. When the atmospheric pressure was 762.5 mm of Hg and the temperature was 20 °C, the air-column was 15.6 cm long; with the tube immersed in a water bath at 50 °C it was 19.1 cm long, the atmospheric pressure remaining the same. If the s.v.p. of water at 20 °C is 17.5 mmHg, deduce its value at 50 °C. [Ans: 91.7 mmHg]
8. The saturation vapour pressure of ether vapour at 0 °C is 185 mm of mercury and at 20 °C it is 440 mm. The bulb of a constant-volume gas thermometer contains dry air and sufficient ether for saturation. If the observed pressure in the bulb is 1000 mm at 20 °C, what will it be at 0 °C? [Ans: 707 mmHg]
9. The saturation vapour pressure of water is $6 \times 10^4\ Nm^{-2}$ at temperature $360\ K$ and $0.3 \times 10^4\ Nm^{-2}$ at temperature $300\ K$. A vessel contains only water vapour at a temperature of $360\ K$ and pressure $2 \times 10^4\ Nm^{-2}$. It may be assumed that unsaturated water vapour behaves like an ideal gas. If the vapour were to remain unsaturated, what would be the pressure in the vessel at $300\ K$? What is the actual pressure at this temperature and what fraction, if any of the vapour has condensed? [Ans: $1.7 \times 10^4\ Nm^{-2}$, $3000\ Nm^{-2}$, 82%]
10. Two flasks A and B, each of volume 1.5 litres are joined by a closed tap and are placed in a constant temperature bath at 60 °C. A contains a vacuum while B contains air and saturated vapour. The total pressure in B is 200 mmHg. The tap is opened and the system is allowed to reach equilibrium with the water vapour remaining saturated. If the final pressure in the cylinder is 150 mmHg, calculate the saturated vapour pressure of water at 60 °C . [Ans: 100 mmHg]

THERMODYNAMICS

Thermodynamics is the study of the relationship between the energy transformation in a system and other physical quantities such as temperature, pressure and volume. The word "system" refers to a definite group of molecules for example the molecules in a container of a gas.

We learned that the state of an ideal gas is given by the ideal gas equation or the equation of state $pV = nRT$. This equation relates the pressure p, volume V and temperature T of the system. Another important quantity is called the internal energy U of the system

Internal energy

According to the molecular theory of matter, matter is made up many molecules. The molecules are in constant motion and hence have kinetic energy and attraction between them. Hence the molecules also have potential energy.

The internal energy U of a system is the sum of all the microscopic kinetic and potential energies of the molecules in the system.

In an ideal gas, it is assumed that there is no force of attraction (negligible) between molecules. Hence the potential energy is zero. This means that the internal energy of an ideal gas is just the kinetic energy of the molecules. Since the kinetic energy of the molecules is directly proportional to the absolute temperature, the internal energy U of an ideal gas is directly proportional to T, its dynamic temperature.

First law of thermodynamics

The first law of thermodynamics is based on the law of conservation of energy

When a quantity of heat ΔQ is supplied to a fixed mass of gas, its temperature may increase causing an increase in internal energy ΔU. The gas may expand and if the external work done by the gas, then applying the law of conservation of energy

$$\Delta Q = \Delta U + \Delta W$$

The heat supplied to a system equals the sum of the increase in internal energy of the system and the external work done by the system.

Sign convention

	Positive (+)	Negative (−)
ΔQ	Heat supplied to the system	Heat loss by system
ΔU	Increase in internal energy	Decrease in internal energy
ΔW	Work done by system	Work done on system

Example

The molar heat capacity at constant volume of an ideal gas is $20.5\,J\,mol^{-1}\,K^{-1}$ and may be assumed to be constant. Find the internal energy of 5 moles of the gas at 27 °C

Solution

The internal energy of an ideal gas at 0 K is zero. When the temperature of the gas is raised from 0 K to 27 °C(330 K), the heat supplied is used to increase the internal energy of the gas.

Internal energy of gas at 300 K = $nC_v\Delta T$

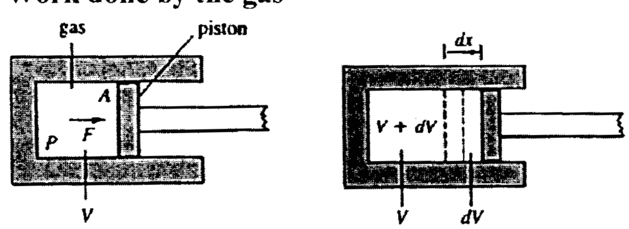
$$= 5 \times 20.5 \times (300 - 0)$$
$$= 3.075 \times 10^4\,J$$

Isobaric and Isovolumetric processes

An isobaric process is a process which takes place when a gas is heated or cooled at constant pressure

An isovolumetric process is the process which takes place when a gas is heated or cooled at constant volume

Work done by the gas



When a gas expands, the forces due to its pressure push back the piston. Therefore the gas does external work when it expands.

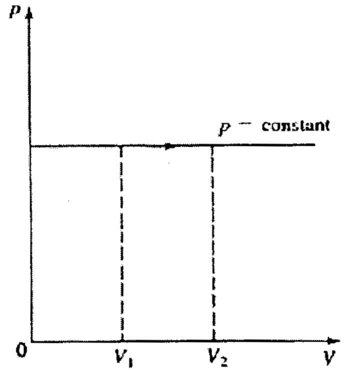
Suppose that the pressure of the gas in the cylinder is p, the piston is of area A and the gas expands by pushing the piston a small distance dx, then the external work done by the gas

$$\Delta W = F\,dx$$
$$= (pA)dx$$
$$= p(A\,dx)$$
$$\Delta W = p\,dV$$

For a finite expansion from volume V_1 to V_2, the external work done

$$W = \int_{V_1}^{V_2} p\, dV$$
$$= \text{area under } p\text{-}V \text{ curve}$$

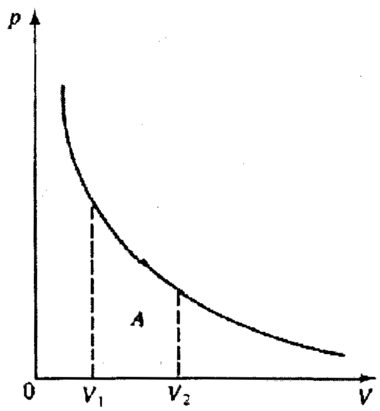
External work done by a gas when it expands is considered as positive work



If the pressure of the gas remains constant as shown above,

Work done by the gas $= \int_{V_1}^{V_2} p\, dV$

$$= p\int_{V_1}^{V_2} dV$$
$$= p\,[V]_{V_1}^{V_2} = p(V_2 - V_1)$$



If the pressure of the gas decreases when it expands as shown above,

the work done by the gas $= \int_{V_1}^{V_2} p\, dV$

$$= \text{area under } p\text{-}V \text{ curve}$$

When a gas is compressed, work is done on the gas and it is considered as negative work

Molar heat capacities of a gas

The heat required to increase the temperature of one mole of an ideal gas by 1 K depends on the manner in which the gas is heated i.e. whether it is allowed to expand or not.

There are two principal molar heat capacities of a gas i.e. C_v the molar heat capacity at constant volume and C_p the molar heat capacity at constant pressure.

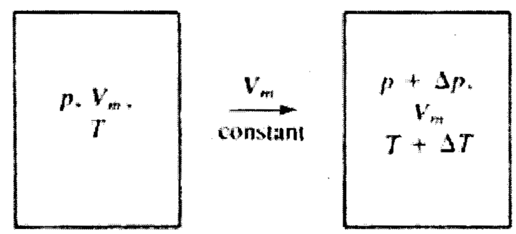
The **molar heat capacity at constant volume, C_v** of an ideal gas is the heat required to raise the temperature of one mole of the gas by 1 K at constant volume.

The **molar heat capacity at constant pressure, C_p** of an ideal gas is the heat required to raise the temperature of one mole of the gas by 1 K at constant pressure.

When a mole of an ideal gas is heated at constant volume so that its temperature rises by 1 K, heat is required only to increase the internal energy of the gas.

On the other hand, when a mole of an ideal gas is heated at constant pressure so that its temperature rises by 1 K, heat is required to increase the internal energy of the gas and to supply energy for the gas to do external work since the volume of the gas increases. **This explains why C_p is greater than C_v**

Relationship between C_v and C_p



Suppose that the volume of one mole of an ideal gas at a pressure of p and temperature T is V_m. When the gas is heated at constant volume to a temperature of $(T + \Delta T)$, from the definition of C_v,

Heat supplied, $\Delta Q = C_v \Delta T$

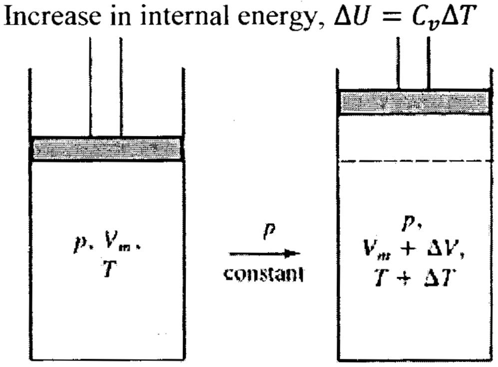
Since the gas does not expand, external work done by the gas,

$$\Delta W = p\Delta V$$
$$= p(0) = 0$$

Using the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
$$C_v \Delta T = \Delta U + 0$$

Increase in internal energy, $\Delta U = C_v \Delta T$



When the gas is heated at constant pressure, the volume of the gas increases from V_m to $V_m + \Delta V$ when the temperature increases by ΔT. From the definition of C_p,

$$\text{Heat supplied}, \Delta Q = C_p\Delta T$$

Since the temperature increases by ΔT, as when the gas is heated at constant volume and because the internal energy of the gas depends only on its temperature,

$$\Delta U = C_v\Delta T$$

Work done by the gas, $\Delta W = p\Delta V$

From the ideal gas equation,

$$pV_m = RT \quad .... \text{(i)}$$

Whwn temperature $= T + \Delta T$, volume $V_m + \Delta V$ and pressure $= p\ (constant)$

$$p(V_m + \Delta V) = R(T + \Delta T) \quad .... \text{(ii)}$$

(ii) – (i);

$$p\Delta V = R\Delta T$$

∴Work done by the gas, $\Delta W = p\Delta V = R\Delta T$

Using the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$C_p\Delta T = C_v\Delta T + R\Delta T$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

Ratio of specific heat capacities

The ratio of the principal molar heat capacities is denoted by the symbol γ.

$$\gamma = \frac{C_p}{C_v}$$

At normal temperatures, the value of γ for monoatomic gases is the same i.e $\frac{5}{3}$ or 1.67; for diatomic gases, $\gamma = \frac{7}{5} = 1.40$ and for polyatomic gases, the value of γ is taken as $\frac{4}{3} = 1.33$

Examples

1. At atmospheric pressure, the specific heat capacity at constant pressure of dry air is 1.0 $kJ\ kg^{-1}\ K^{-1}$. If the density of air at 0 °C and $10^5\ Nm^{-2}$ pressure is 1.29 $kg\ m^{-3}$, calculate a value for C_v

Solution

The gas constant per kg,

$$R = \frac{pV}{T} = \frac{10^5 \times \left(\frac{1}{1.29}\right)}{273}\ J\ kg^{-1}\ K^{-1}$$

$$= 0.28\ kJ\ kg^{-1}\ K^{-1}$$

From $C_p - C_v = R$

$$C_v = C_p - R$$

$$= 1.00 - 0.28 = 0.72\ kJ\ kg^{-1}\ K^{-1}$$

2. Heat is supplied to one mole of an ideal gas in a cylinder fitted with a light smooth piston.
   (a) Explain qualitatively why the amount of heat supplied to raise the temperature of the gas by 1 K depends on whether the piston is allowed to move or not
   (b) If the piston is allowed to move freely so that the gas expands at constant pressure,
      (i) derive an expression in terms of ΔT, C_p and R for the increase in internal energy U for the gas if the temperature of the gas increases by ΔT and C_p is the molar heat capacity at constant pressure of the gas,
      (ii) calculate the increase in internal energy of the gas if its temperature increases from 300 K to 350 K
   (c) If the piston is allowed to move and the gas is heated from 300 K by the total amount of heat required in (b)(ii), calculate the final temperature of the gas ($C_v = 20.5\ J\ mol^{-1}\ K^{-1}$)

Solution

(a) If the piston is not allowed to move when the gas is heated, the volume of the gas remains unchanged and no external work is done by the gas. Hence the heat supplied is only required to increase the internal energy of the gas

If the piston is allowed to move, the volume of the gas increases when heated. Therefore, heat is required not only to increase the internal energy of the gas but also for the gas to do external work.

Since the increase in internal energy does not depend on the volume but on the increase in temperature, for the temperature of the gas to increase by 1 K in both cases, more heat would be required if the piston is able to move.

(b) (i) If the piston is allowed to move, to raise the temperature of 1 mole of gas by ΔT, heat required

$$\Delta Q = C_p\Delta T$$

From $\quad pV_m = RT$ for 1 mole of ideal gas

$$p\Delta V = R\Delta T$$

Work done by the gas, $\Delta W = p\Delta V = R\Delta T$

Using the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$C_p\Delta T = \Delta U + R\Delta T$$

$$\Delta U = C_p\Delta T - R\Delta T$$

∴ Increase in internal energy, $\Delta U = (C_p - R)\Delta T$

(ii) Increase in internal energy.

$$\Delta U = C_v \Delta T$$
$$= 20.5 \times (350 - 300)$$
$$= 1025\,J$$

(c) When the piston is allowed to move,

Heat required, $\Delta Q = \Delta U + R\Delta T$

$$= 1025 + 8.31(350 - 300)$$
$$= 1441\,J$$

If the gas is supplied with 1441 J of heat and the piston is not allowed to move, using

$$\Delta Q = C_v \Delta T$$
$$\Delta T = \frac{1441}{20.5} = 70.3\,K$$

Final temperature of gas = 300 + 70.3 = 370.3 K

Isothermal and adiabatic processes

An isothermal process is a process that takes place at constant temperature.

An isothermal change is the change in pressure and volume of a fixed mass of a gas at constant temperature.

Examples of isothermal changes include

- Condensation, evaporation, melting of ice, stretching a rubber band quickly, etc.

An adiabatic process is a process which takes place in such a way that no heat enters or leaves the gas.

An adiabatic change is the change in temperature, pressure and volume of a fixed mass of a gas when no heat is allowed to enter or leave the gas.

Or

An adiabatic change is a change in the state of a gas where no heat enters or leaves the system

Examples of adiabatic changes include

- Sound transmission in air
- Inflating a tyre or a ball
- Compression stroke of a four-stroke engine cycle
- Bursting tyre or ball

Reversible and irreversible processes

A reversible process is one in which the pressure of the gas differs only by an infinitesimal amount from the external pressure at every stage.

In this process, any change in volume of the gas takes place very slowly, without dissipative effects due to friction or viscosity. The smallest change conditions in the opposite direction is needed to make the process retrace exactly its previous values of p and V

Conditions for achieving a reversible isothermal process

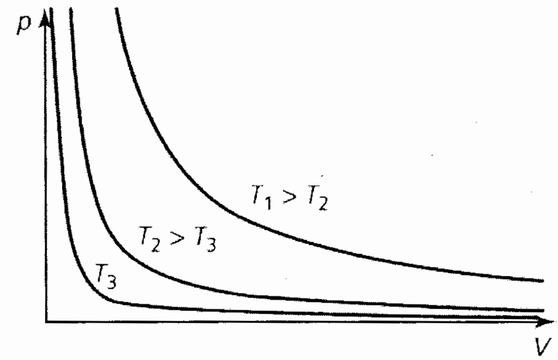
- Gas container should be thin
- Process must be carried out slowly
- A light frictionless piston should be used
- Gas container should be surrounded by a constant temperature bath

Conditions for achieving a reversible adiabatic process

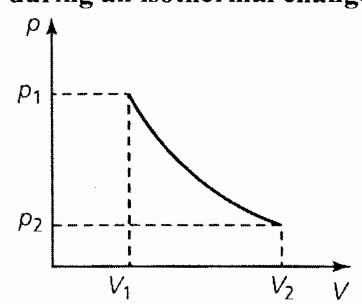
- Gas container must be thick
- Process should be carried out rapidly
- The piston used must be light, frictionless and thick

Isothermal curves

At different temperatures, isothermals which progressively move away from the origin as the temperature T increases are obtained.



Work done during an isothermal change



When the gas expands isothermally from V_1 to V_2, the work done by the gas

$$W = \int_{V_1}^{V_2} p\,dV$$

But $p = \frac{nRT}{V}$

$$\therefore W = \int_{V_1}^{V_2} \frac{nRT}{V} dV$$
$$= nRT \int_{V_1}^{V_2} \frac{1}{V} dV$$
$$= nRT[\ln V]_{V_1}^{V_2}$$
$$= nRT(\ln V_2 - \ln V_1)$$

$$= nRT \ln\left(\frac{V_2}{V_1}\right)$$

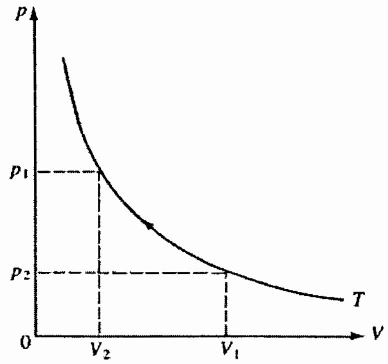
Since $p_1V_1 = p_2V_2 = nRT$

$$W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = p_2V_2 \ln\left(\frac{V_2}{V_1}\right)$$

Also, since the temperature of the ideal gas remains unchanged, there is no change in the internal energy of the gas i.e. $\Delta U = 0$.

$$\text{From, } \Delta Q = \Delta U + \Delta W$$
$$= 0 + \Delta W$$
$$\therefore \Delta Q = \Delta W$$

This implies that the energy required by the gas to do external work comes from the heat supplied.



Similarly, when a gas is compressed isothermally as shown above,

Work done on the gas $= \int_{V_1}^{V_2} p\,dV$

$$= \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right)$$

Since $V_2 < V_1$, the value of $\ln\frac{V_2}{V_1}$ is negative and the work done on the gas has a negative value. This is consistent with the sign convention for the first law of thermodynamics.

Thus whatever work done on the gas escapes from the gas in form of heat

Example

How much heat is given off when 4 g of hydrogen gas is compressed isothermally to half its original volume? (The temperature of the gas is maintained at 27°C)

Solution

Mass of 1 mole of hydrogen gas = 2g

Number of moles $n = \frac{4}{2} = 2$

In an isothermal compression, the heat given off is the equal to the work done on the gas

Work done on the gas $= nRT \ln\frac{V_2}{V_1}$

$$= 2 \times 8.31 \times 300 \ln\frac{0.5V_1}{V_1}$$

$$= -3.46 \times 10^3\,J$$

$\therefore$ Heat given off $= 3.46 \times 10^3\,J$

Adiabatic expansion

Consider a gas contained inside a cylinder fitted with a piston, and that the outside of the cylinder has been surrounded by insulating material such as cotton wool and the piston is also made of insulating matreial.

If the piston is depressed, work is done on the gas and an equivalent amount of heat is produced. No heat escapes from the gas because the container and piston are insulated and the temperature of the gas rises. This is an example of adiabatic contraction

If the piston is raised, work is done on the gas and an equivalent heat is taken from the gas itself, which is therefore **cooled**. No heat, however enters or leaves the gas while it expands and this is an example of an adiabatic expansion

A sudden compression or expansion of a gas is initially adiabatic because there is then no time for heat to enter or leave the system.

Also the rapid expansion and contraction of air when sound waves pass through it is a near adiabatic process. Thus γ is involved in the equation for the speed of sound in a gas.

$$c = \sqrt{\frac{\gamma p}{\rho}}$$

Reversible adiabatic pressure-volume changes

When a gas undergoes a reversible adiabatic change, the pressure-volume changes obey the law

$$pV^{\gamma} = \text{constant}$$

where γ is the ratio of the molar heat capacities

From this expression, the new pressures and volumes of gases can be calculated where reversible adiabatic changes occur.

Proof of $pV^{\gamma} = constant$

Suppose that an ideal gas expands adiabatically and reversibly. Then no heat enters or leaves the gas.

From the first law of thermodynamics,

$$\delta Q = \delta U + \delta W$$
$$= \delta U + \delta \Delta V$$
$$\delta U + p\delta V = 0 \text{ since } \Delta Q = 0$$

But $\delta U = C_v \delta T$ where ΔT is the small change in temperature of unit mass of the gas

$$C_v\delta T + p\delta V = 0$$

$$\delta T = -\frac{p\delta V}{C_v} \qquad \ldots\ldots \text{(i)}$$

For one mole of an ideal gas,

$$pV = RT$$

Differentaiting both sides of the equation with respect to T,

$$p\delta V + V\delta p = R\delta T \quad \text{(product rule)}$$

But $C_p - C_v = R$

$$p\delta V + V\delta p = (C_p - C_v)\Delta T \quad \text{..... (ii)}$$

Substututing for δT from (i) in (ii);

$$p\delta V + V\delta p = \frac{(C_p - C_v)p\delta V}{C_v}$$

$$p\delta V + V\delta p = -\frac{C_p}{C_v}p\delta V + p\delta V$$

$$V\delta p = -\frac{C_p}{C_v}p\delta V$$

$$V\delta p = -\gamma p\delta V$$

$$\frac{\delta p}{p} = -\gamma\frac{\delta V}{V}$$

This relation holds between small changes of p and V in a reversible adiabatic change. To find the relation between actual values of p and V, this relation must be integrated.

$$\int\frac{dp}{p} = -\gamma\int\frac{dV}{V}$$

$$\ln p = -\gamma\ln V + C$$

where C is a constant

$$\ln p + \gamma\ln V = C$$

$$\ln pV^{\gamma} = C$$

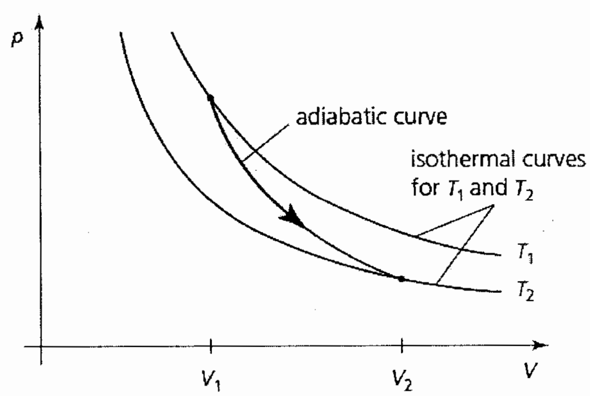
$$\log_e pV^{\gamma} = C$$

$$pV^{\gamma} = e^{C}$$

But e^{C} is a constant,

$$pV^{\gamma} = \text{constant}$$

Adiabatic curve



On the isothermal for T_1, $p_1V_1 = RT_1$

On the isothermal for T_2, $p_2V_2 = RT_2$

$$\therefore \quad \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \text{......... (i)}$$

But Q and S are both on the adiabatic curve OQS, which has the equation

$$pV^{\gamma} = \text{constant}$$

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \quad \text{......... (ii)}$$

To eliminate p_1 and p_2, we divide (ii) ÷ (i)

Hence $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

Thus, in general, for an adiabatic change,

$$TV^{\gamma-1} = constant$$

Eliminating V from $pV = RT$ and $pV^{\gamma} = \text{constant}$

$$\frac{p^{\gamma-1}}{T^{\gamma}} = constant$$

Examples

1. A mass of air at a temperature of 40 °C and a pressure 70 cm of mercury is compressed adiabatically and reversibly until its volume is halved. Calculate its new absolute temperature and new pressure of the gas assuming that γ is 1.4.

Solution

From $TV^{\gamma-1} = constant$

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$T_1 = 40 + 273 = 313\,K,\ V_1 = V$$

$$T_2 = ?,\ V_2 = \frac{1}{2}V$$

$$T_2 \times \left(\frac{1}{2}V\right)^{\gamma-1} = 313 \times V^{\gamma-1}$$

$$T_2 = 313 \times \left(\frac{V}{\frac{1}{2}V}\right)^{\gamma-1}$$

$$= 313 \times 2^{\gamma-1}$$

$$= 313 \times 2^{1.4-1} = 413\,K$$

New temperature of gas $= 413 - 273 = 140$ °C

From $pV^{\gamma} = constant$

$$p\left(\frac{1}{2}V\right)^{1.4} = 70(V)^{1.4}$$

$$p = 70 \times \left(\frac{V}{\frac{1}{2}V}\right)^{1.4}$$

$$= 70 \times 2^{1.4}$$

$= 185$ cm of mercury

2. (a) How are the pressure and volume of a fixed mass of a gas related during reversible isothermal expansion and during reversible adiabatic expansion?

   (b) 22.4 litres of air at 15 °C and 76 cm of mercury pressure, weighing 27.3 g, expand adiabatically until the volume has increased by 50% . What will be the final pressure and temperature?

   (c) By considering the internal energy of the gas, find how much work is done against external pressure during the expansion. [$C_v = 0.71$; $C_p = 1.00\ kJ\ kg^{-1}\ K^{-1}$]

Solution

(a) In an isothermal expansion, $pV = constant$

In an adiabatic expansion, $pV^{\gamma} = constant$

(b) 22.4 litres is the original volume,

the final volume $= 22.4 + \frac{50}{100} \times 22.4$

$= 33.6$ litres

$$\gamma = \frac{C_p}{C_v} = \frac{1.00}{0.71} = 1.4$$

Applying $pV^{\gamma} = constant$

$$p \times 33.6^{\gamma} = 76 \times 22.4^{\gamma}$$

$$p = 76\left(\frac{22.4}{33.6}\right)^{1.4}$$

$p = 43.1\ cm$ of mercury

$$T = 15\ °C = 15 + 273 = 288\ K$$

Using $TV^{\gamma-1} = constant$

$$T(33.6)^{1.4-1} = 288(22.4)^{1.4-1}$$

where T is the final pressure

$$T = 288 \times \left(\frac{22.4}{33.6}\right)^{0.4}$$

$$= 288 \times \left(\frac{2}{3}\right)^{0.4}$$

$$= 245\ K$$

Temperature in °C = $245 - 273 = -28\ °C$

(c) Since no external heat is supplied to a gas in an adiabatic expansion, the change in the internal energy of the gas is equal to the external work done

Change in internal energy

= mass of gas × C_v × temperature change

Internal energy change

$$= 27.3 \times 0.71 \times [15 - (-28)]$$

$$= 833\ J$$

3. A gas occupies a volume of 25 m^3 and is at a temperature of 273 K. The gas has a value of γ of 1.40. The gas is now compressed adiabatically to 10 m^3. Calculate its new temperature.

Solution

Using the equation,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$273 \times 25^{1.4-1} = T_2 \times 10^{1.4-1}$$

$$T_2 = 25 \times \left(\frac{25}{10}\right)^{0.4}$$

$$= 393\ K$$

The new temperature is 393 K

4. When you are pumping up a tyre on a bicycle, you will have noticed that the bicycle pump becomes warmer. Use the law of thermodynamics to explain this effect.

Solution

$$Q = \Delta U + W$$

No energy is supplied to the air as heat, so $Q = 0$, as the pumping motion is adiabatic. Work is done on the air in the tube, so W is negative. The gain in internal energy is therefore equal to the amount of work done on the gas. The increased temperature of the gas is a result of the gain in internal energy of the air in the wall of the hand pump, making it feel warm to the touch.

A gas whose γ is 1.4 is initially at a pressure of 40 Nm^{-2} and occupies a volume of 10 m^3.

(a) If it is compressed isothermally to 2 m^3, calculate its new pressure.

(b) If, instead, its compressed adiabatically to 2 m^3, calculate its new pressure.

(c) Sketch a p-V graph for the changes to show the effect of the two types of compression.

Solution

(a) $p_1V_1 = p_2V_2$

$$40 \times 10 = p_2 \times 2$$

$$p_2 = 200$$

The new pressure is 200 Nm^{-2}

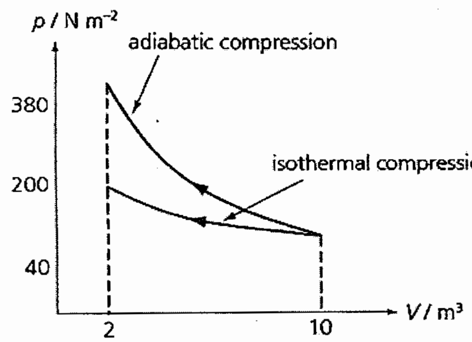
(b) $p_1V_1^{\gamma} = p_2V_2^{\gamma}$

$$40 \times 10^{1.4} = p_2 \times 2^{1.4}$$

$$p_2 = 40\left(\frac{10}{2}\right)^{1.4}$$

$$= 381\ Nm^{-2}$$

(c)



5. A gas at a pressure of 20 Nm^{-2} occupies a volume of 5 m^3. The gas has a value of $\gamma = 1.40$. If the gas is compressed adiabatically to 2 m^3, calculate its new pressure.

Solution

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$20 \times 5^{1.4} = p_2 \times 2^{1.4}$$

$$20 \times 9.52 = p_2 \times 2.64$$

$$p_2 = 72.1\ Nm^{-2}$$

6. A gas at pressure of 40 Nm^{-2} occupies a volume of 50 cm^3. The gas has $\gamma = 1.33$. If the gas is allowed to expand very quickly and its pressure drops to 10 Nm^{-2}, calculate the new volume.

Solution

In this situation, the very quickly implies an adiabatic expansion, so we use

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$
$$40 \times 50^{1.33} = 10 \times V_2^{1.33}$$
$$40 \times 182 = 10 \times V_2^{1.33}$$
$$728 = V_2^{1.33}$$
$$V_2 = 142\ cm^3$$

Self-Evaluation exercise

1. What is the first law of thermodynamics? When a fixed mass of gas is compressed, what quantitative relation does the law establish between the external work done, the internal energy of the gas, and the heat which leaves the system? When can the conditions of the compression be described as (a) isothermal and (b) adiabatic? Show how the law leads to the relation pV^{γ} = constant for reversible adiabatic changes in an ideal gas.
   Explain qualitatively why the work done in the reversible compression of a given mass of an ideal gas from an initial volume V_1 to a final volume V_2 is greater if the change takes place adiabatically than if it takes place isothermally, the initial temperature in both cases being the same.
2. What is meant by (a) an isothermal change, (b) an adiabatic change in a gas? Explain how you would attempt to achieve each type of change experimentally. Three litres of an ideal gas at atmospheric pressure and 27°C are compressed adiabatically to a volume of 1 litre. Calculate the resultant pressure. At its new temperature, the gas is expanded isothermally to its original volume. Calculate the final pressure and temperature. Assume that the ratio of the specific heat capacities of the gas is 1.67.
   Illustrate the whole process on a pV diagram. By referring to the diagram, explain which is greater - the work done on the gas, or the work done by the gas. [Ans: $6.26, 2.09\ atm$; 353 °C]
3. What do you understand by the internal energy of a system? A rigid metal vessel initially contains a mixture of hydrogen and oxygen at room temperature. The mixture is ignited by a spark (which itself produces negligible heat) and the temperature of the vessel and its contents is allowed to return to room temperature. Has the internal energy of the contents of the vessel increased or decreased? Give reasons for your answer and explain briefly how you would measure the change.
4. A certain mass of a perfect gas undergoes a change from an initial pressure P_1 and volume V_1 to a final pressure P_2 and volume V_2 such that $P_2V_2 > P_1V_1$. Find the change in internal energy of the gas by considering that the change is effected in two stages: an initial heating at constant volume followed by further heating at constant pressure.
   [Ans: $\frac{c_v(P_2V_2-P_1V_2)}{R}$]
5. What do you understand by the internal energy of a gas?
   53 g of an ideal gas initially at a pressure of 32 atmospheres and at 27 °C occupy a volume of 1 litre. If the gas is allowed to expand reversibly and adiabatically to a volume of 8 litres, calculate (a) the final pressure, (b) the final temperature, (c) the work done by the gas during the expansion.
   The gas in its initial state is now taken to the same final state in two stages, namely:
   (i) at constant pressure until the volume is 8 litres, followed by (ii) cooling at constant volume. Illustrate these changesr on a pV diagram and calculate the temperature at the end of stage (i). Also discuss the energy changes which occur during the two stages. (Specific heat capacity at constant volume and constant pressure = $0.30, 0.50\ kJ\ kg^{-1}\ K^{-1}$ respectively; 1 atmosphere pressure = $10^5\ Nm^{-2}$).
   [Ans: (a) $10^5\ Nm^{-2}$ (b) $75\ K$ (c) $3600\ J$, 2400 K]
6. What is meant by an adiabatic change and internal energy?
   A certain mass of an ideal gas is changed from an initial state of pressure p_1 and volume v_1 to a final state p_2, v_2 ($p_2 > p_1$) by two different methods: (i) reversible adiabatic compression from p_2, v_2 direct to p_2, v_2 and (ii) isothermal compression from a volume v_1 to a volume v_2 and then a change of pressure at constant volume v_2 to the final pressure p_2. Show these changes on a pv diagram. State whether (a) the change of internal energy, and (b) the work done on the gas is the same, or different, in the two cases, and justify your answers.
   [Ans: (a) same (b) different]
7. Distinguish between an isothermal and an adiabatic compression of a gas. Explain the precautions necessary to ensure that an actual compression approximates to each of these conditions.
8. A cylinder fitted with a frictionless piston contains 1.0 g of oxygen at a pressure of 760 mmHg and at a temperature of 27 °C. The following operations are performed:

(a) the oxygen is heated at constant pressure to 127 °C and then

(b) the oxygen is compressed isothermally to its original volume; and finally

(c) the oxygen is cooled at constant volume to its original temperature.

9. Illustrate these changes on a p-V diagram drawn to scale. What is the heat input to the cylinder in stage (a)? How much work does the oxygen do in pushing back the piston during this stage? How much work is done on the oxygen in stage (b)? How much heat must be extracted from the oxygen in stage (c)? (Specific heat capacity of oxygen at constant volume = $0.65\ kJ\ kg^{-1}$, density of oxygen at s.t.p. = 1.43 kgm^{-3}; molecular weight of oxygen = 32.)

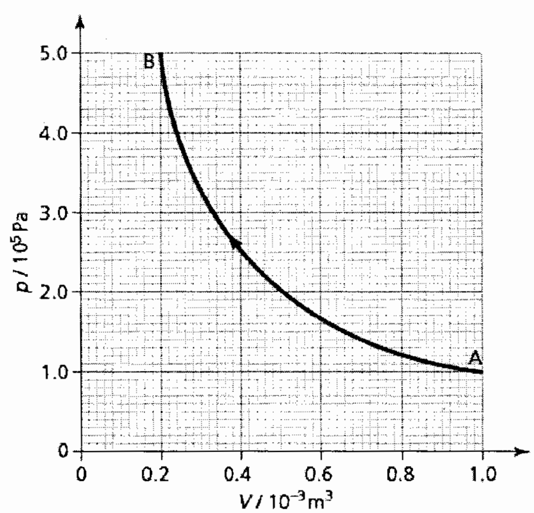
[Ans: (a) 92 J, 26 J (b) 30 J (c) 67 J]

10. A gas at a pressure of 15 Nm^{-2} occupies a volume of 5.0 m^3. If it is compressed isothermally to 2.0 m^3, calculate its new pressure. [Ans: 37.5 Nm^{-2}]

11. A gas at a pressure of 78 Nm^{-2} occupies a volume of 50 cm^3. It is kept at constant temperature. If the pressure applied to the gas decreases to 43 Nm^{-2}, calculate the new volume it occupies.

[Ans: 90.7 cm^3]

12. The curve in the figure below shows the isothermal compression for 0.12 mol of an ideal gas.



(a) By choosing three points on the curve, verify that the curve is isothermal.

(b) Calculate the temperature of the gas.

[Ans: (b) 100 K]

13. A gas is initially at temperature of 40 °C and occupies 450 cm^3. The gas has $\gamma = 1.33$. The gas expands very quickly and its temperature drops to 10 °C. Assuming no heat leaves the system, calculate its final volume. [Ans: 611 cm^3]

14. A gas at pressure of 60 Nm^{-2} occupies a volume of 250 cm^3. The gas has $\gamma = 1.67$. If the gas is pressurized very quickly and its pressure increased to 150 Nm^{-2}, calculate the new volume. Assume no heat leaves the system. [Ans: 144 cm^3]

15. Explain why, when quoting the specific heat of a gas, it is necessary to specify the conditions under which the change of temperature occurs. What conditions are normally specified?

16. A vessel of capacity 10 litres contains 130 g of a gas at 20°C and 10 atmospheres pressure. 8000 joule of heat energy are suddenly released in the gas and raise the pressure to 14 atmospheres. Assuming no loss of heat to the vessel, and ideal gas behavior, calculate the specific heat of the gas under these conditions. [Ans: 053 $kJ\ kg^{-1}$]

17. A litre of air, initially at 20°C and at 76.0 cm of mercury pressure, is heated at constant pressure until its volume is doubled. Find (a) the final temperature, (b) the external work done by the air in expanding, (c) the quantity of the heat supplied. [Assume that the density of air at s.t.p is 1.293 kgm^{-3} and that the specific heat capacity of air at constant volume is 0.714 $kJ\ kg^{-1}\ K^{-1}$]

[Ans: (a) 586 K (b) 101.2 J (c) 355 J]

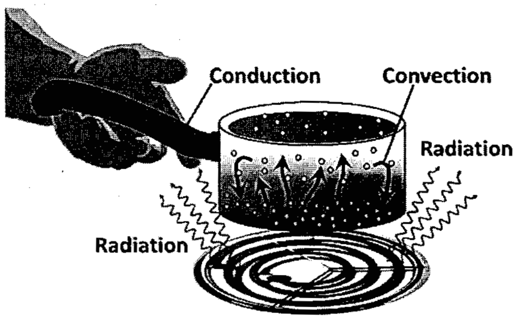
18. Distinguish between an isothermal change and an adiabatic change. In each instance state, for a reversible change of an ideal gas, the relation between pressure and volume.

19. A mass of air occupying initially a volume 2000 cm^3 at a pressure of 76.0 cm of mercury and a temperature of 20.0°C is expended adiabatically and reversibly to twice is volume, and then compressed isothermally and reversibly to a volume of 3000 cm^3. Find the final temperature and pressure, assuming the ratio of the specific heat capacities of air to be 1.40.

[Ans: 222 K, 38.4 cmHg]

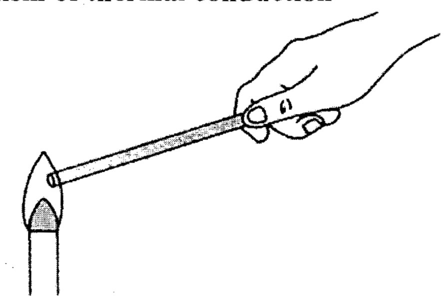
20. Air initially at 27°C and 75 cm of mercury pressure is compressed isothermally until its volume is halved. It is then expanded adiabatically until its original volume is recovered. Assuming the changes to be reversible, find the final pressure and temperature. [Take the ratio of the specific heat capacities of air as 1.40.] [Ans: 56.8 cmHg, 227 K]

TRANSFER OF HEAT



Heat transfer deals with passage of heat energy from one body at higher temperature to another body at lower temperature.
There are three ways in which heat energy may get transferred from one place to another. These are conduction, convection and radiation.

Mechanism of thermal conduction



If you hold an iron rod at one end and heat the other end in a Bunsen burner, before long the rod becomes too hot to hold. Heat has been transferred from the flame to your hand by conduction. For conduction to take place, a material medium is required. Heat flows through the medium without the medium moving.

Conduction is the process of heat transfer which requires a material medium without movement of the material itself.

The mechanism of heat conduction can be explained by the behaviour of atoms within the material. There are two ways in which heat is transferred.

1. In a crystalline solids or non-metals, atoms are arranged in an orderly manner in the crystal lattice. When one end of the solid is heated, atoms in the crystal lattice at the heated end vibrate more vigorously. These atoms vibrate against the neighbouring atoms, which pass on the energy to other atoms. The **lattice vibrations** in solids produce waves which are propagated in the solid.
2. In metals, there are many free electrons. When one end of the metal is heated, the atoms in the heated part vibrate more. The free electrons that collide with these atoms gain kinetic energy and move faster, diffusing into the colder part of the metal where collision with other free electrons and atoms in the lattice result in transfer of energy.

Thermal conductivity

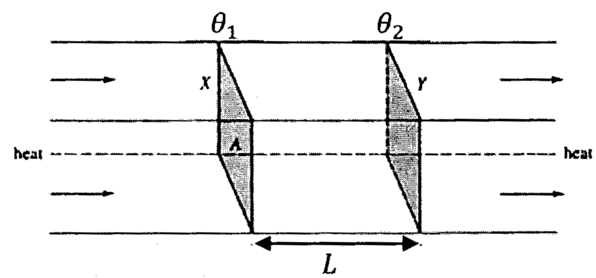
The conduction of heat varies in different types of solids. Metals are, in general, good conductors and non-metals poor conductors. Silver is the best conductor followed by copper among the common metals, aluminium and iron are next in order of conductivity. Wood, paper, cork and polystyrene are poor conductors of heat and are called good insulators.

The metal knob of a door feels colder to touch than the wooden door. Is the metal knob **really** colder? The door and metal knob are in thermal equilibrium and therefore have the same temperature. But the metal knob feels colder because it is a better conductor. Heat from your hand is easily conducted by the metal where as little heat is transferred from your hand to the wood as heat is conducted very slowly by the wood.

Liquids and gases are in general good insulators. Air trapped in materials such as expanded polystyrene, fiberglass, wool, fur, feathers and snowflakes makes these materials good insulators.

If a metal rod is placed with one end kept at a high constant temperature and the other end kept in contact with another object at a lower temperature, the rate of heat flow through each section of rod depends on

(i) the temperature gradient, which is the change in temperature per metre distance along the bar



$$\text{Temperature gradient} = \frac{\text{change in temperature}}{\text{length}}$$
$$= \frac{\theta_1 - \theta_2}{L}$$

(ii) the cross-sectional area A. A larger cross-sectional area will allow a greater rate of heat flow.

(iii) the material of the rod

These can be summarized by the equation:

$$\textbf{Rate of heat flow, } \frac{Q}{t} = kA\frac{(\theta_1 - \theta_2)}{L}$$

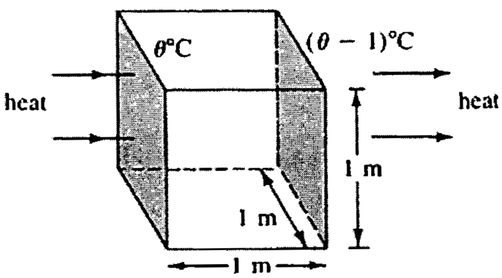
where k is a constant known as the thermal conductivity of the material.

Rearranging the equation,

$$\text{thermal conductivity, } k = \frac{\frac{Q}{t}}{A\frac{\theta_1-\theta_2}{L}}$$

this can be used to obtain a definition of thermal conductivity.

The thermal conductivity k of a material is the rate of heat flow through two opposite faces of a cube of the material of length 1 m when the temperature difference between the two faces is 1 K

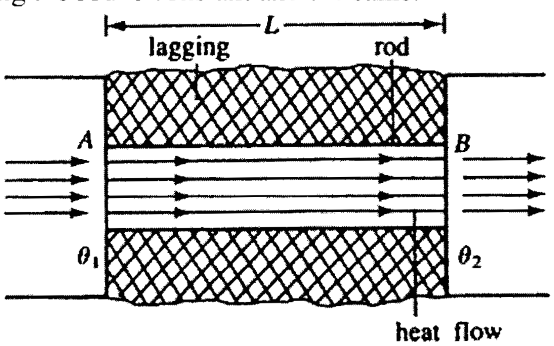


The unit for thermal conductivity is $W\ m^{-1}\ K^{-1}$.

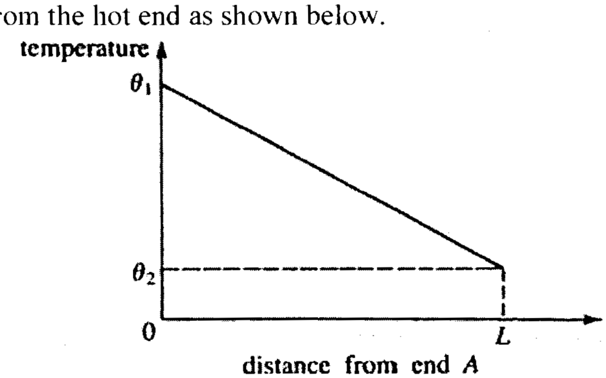
Insulated rod and non-insulated rod

Insulated rod

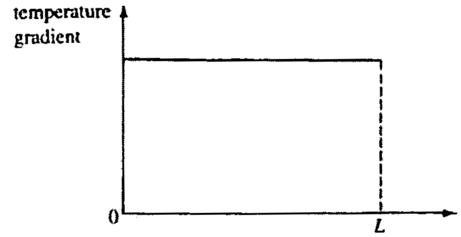
A rod of length L and uniform cross-sectional area A is perfectly lagged. The temperatures at the hot end and cold end are at steady state θ_1 and θ_2 respectively. Since the rod is perfectly lagged, no heat will escape from the sides of the rod. Hence the rate of heat flow along the rod is constant and the same.



The temperature decreases linearly with the distance from the hot end as shown below.

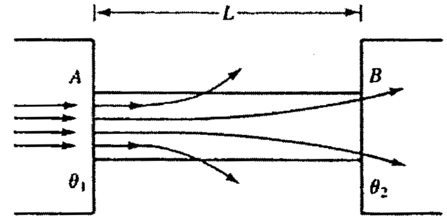


Temperature gradient-distance graph

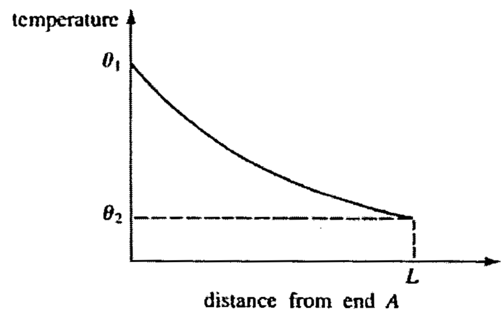


Non-insulated rod

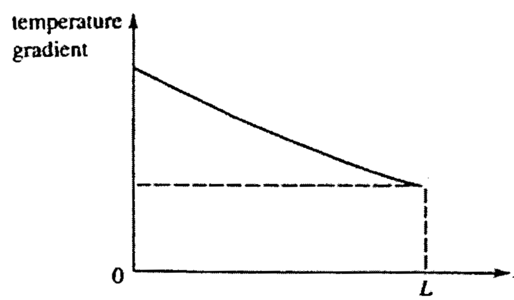
If the rod is not lagged, heat escapes from the sides of the rod. Therefore the rate of heat flow decreases as the distance from the hot end increases.



The graph of temperature against distance from the hot end is a curve as shown below



The temperature gradient decreases as the distance from the hot end increases.



Examples

1. The table below gives approximate values of the thermal conductivity k of a number of different materials at room temperature. The values may be taken as typical of metals, insulators, liquids and gases

Material	$k\ /\ W\ m^{-1}\ K^{-1}$
Aluminium	2×10^{2}
Rubber(insulator)	2×10^{-1}
Methanol (liquid)	2×10^{-1}
Air (gas)	2×10^{-2}

(a) Metals are good conductors of both heat and electricity, thermal insulators are also electrical insulating. Comment on this

(b) Suggest a reason why the thermal conductivities of typical liquids and typical insulating solids should be similar

(c) Suggest why gases have thermal conductivities which are generally less than those of liquids

Solution

(a) Metals have many free electrons. Electrical and thermal conduction are both due to the motion of free electrons. Therefore good conductors of heat are also good conductors of electricity.
Thermal insulators do not have free electrons. Therefore thermal insulators are also electrically insulating.

(b) The mechanism of thermal conductivity in typical liquids and typical insulating solids are similar i.e. by vibration of atoms.
Separation between molecules in solids and liquids are about the same hence their thermal conductivities are the same.

(c) The number of molecules per unit volume in a gas is less than in a liquid. In gases, the distance between the molecules is very much larger than in liquids. Energy is transferred when molecules collide with each other. In gases, collision between molecules is not as often or frequent as in liquids.

2. The total surface area of windows in a room is $2.5\,m^2$ and the glass thickness is 3 mm. Calculate the lost heat per hour through the windows when the temperature inside the room is 20 °C and the temperature outside is 10 °C. Assume $k = 1\,W\,m^{-1}K^{-1}$ for glass.

Solution

Temperature gradient $= \frac{20-10}{3\times10^{-3}}\,K\,m^{-1}$

$\frac{Q}{t} = kA \times$ temperature gradient

$= 1 \times 2.5 \times \frac{20-10}{3\times10^{-3}}\,Js^{-1}$

Q per hour $= 1 \times 2.5 \times \frac{20-10}{3\times10^{-3}} \times 3600\,J$

$= 3 \times 10^7\,J$

3. The temperature inside a boiler is 105 °C. The wall of the boiler is 2 cm thick and is lagged with 4 cm thickness of a material whose thermal conductivity is $\frac{1}{9}$ that of the boiler. When in the steady state, the temperature of the outside surface of the lagging in contact with the air is 10 °C. What is the temperature of the common surface of the boiler and the lagging?

Solution

Let θ = temperature in °C of the common surface of the boiler and the lagging.

Temperature gradient across boiler wall

$$= \frac{105-\theta}{0.02}\,Km^{-1}$$

Temperature gradient across lagging

$$= \frac{\theta-10}{0.04}\,K\,m^{-1}$$

Let the area of the lagging material and boiler wall be A and k be the thermal conductivity of boiler.

Heat passing through the boiler wall per second

$$= kA\frac{(105-\theta)}{0.02}$$

Heat passing through the lagging per second

$$= \frac{k}{9}A\frac{(\theta-10)}{0.04}$$

In steady state, the quantity of heat passing per second through the wall is equal to the quantity of heat passing per second through the lagging.

$$kA\frac{(105-\theta)}{0.02} = \frac{k}{9}A\frac{(\theta-10)}{0.04}$$

$$\frac{105-\theta}{2} = \frac{\theta-10}{36}$$

$$19\theta = 1900$$

$$\theta = 100\text{ °C}$$

4. One room in a house has a floor made entirely of concrete which is 200 mm thick. The lower surface of the concrete in contact with the ground, has a temperature of 10 °C and the upper surface in contact with the living area has a temperature of 15.0 °C. The floor is a square and of sides $10\,m \times 10\,m$

(a) Calculate the rate at which thermal energy is conducted through the concrete. Assume the thermal conductivity of concrete is $0.750\,W\,m^{-1}\,K^{-1}$

(b) The house owner decides to cover the concrete with carpet of thickness 15.0 mm. Calculate

(i) the temperature at the carpet/concrete interface

(ii) the rate at which thermal energy is conducted through the two layers

Assume that the carpet has thermal conductivity $= 0.060\,Wm^{-1}\,K^{-1}$. Assume also that the temperature of the upper surface of the carpet is 15 °C and that the temperature of the lower surface of the concrete remains at 10.0 °C

Solution

(a) For concrete floor, $A = 10 \times 10 = 100\,m^2$

$L = 200\,mm = 0.200\,m$

$\theta_1 = 15.0\,°C, \theta_2 = 10.0\,°C$

$\theta_1 - \theta_2 = 15.0 - 10.0 = 5.0\,K$

Rate of heat flow $= kA\,\frac{\theta_1-\theta_2}{L}$

$= \frac{0.75 \times 100 \times 5.0}{0.200}$

(b) (i) The rate of conduction of thermal energy through the carpet and then through the concrete floor is the same since the two conductors are in series.

Let θ_3 be the temperature of the carpet/concrete interface.

Rate of heat flow through carpet $= k_1 A \frac{\theta_1-\theta_3}{L_1}$

$= 0.06 \times 100 \times \frac{(15.0-\theta_3)}{0.015}$

$= 400(15.0 - \theta_3)$

Rate of heat flow through concrete $= k_2 A \frac{\theta_1-\theta_3}{L_2}$

$= 0.75 \times 100 \times \frac{(\theta_3-10.0)}{0.200}$

$= 375(\theta_3 - 10.0)$

$400(15.0 - \theta_3) = 375(\theta_3 - 10.0)$

$\theta_3 = \frac{9750}{775} = 12.58\,°C$

(ii) We can use θ_3 with either the expression of the above expression for carpet or concrete.

For carpet,

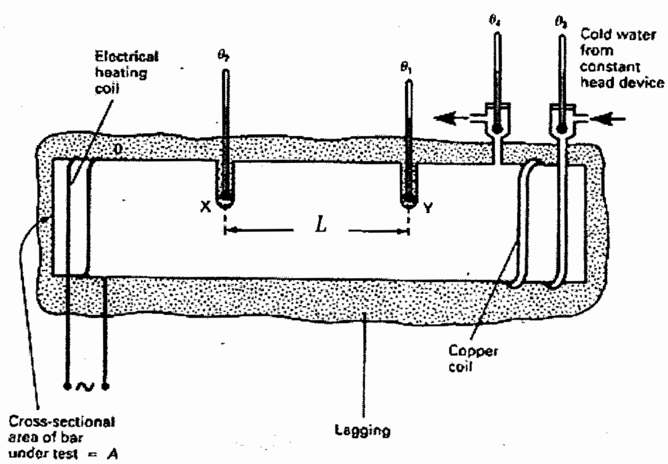
$\frac{Q}{t} = 400(15.0 - 12.58)$

$= 968\,W$

Measurement of thermal conductivity

Good conductor – Searle's apparatus

Searle designed an apparatus for measuring the thermal conductivity of a good conductor such as a metal



- A think cylindrical bar of the of the specimen, heated electrically at one end A by the coil/heater is well lagged by felt and enclosed.
- Two holes are drilled at X and Y with a distance of L between them.
- They are then filled with mercury so that the thermometers placed in the holes are in good thermal contact with the bar.
- A copper coil, soldered round the bar has a steady flow of water and the inlet and outlet temperatures θ_3 and θ_4 are measured by the thermometers.
- When the electrical heating coil is switched on, the temperatures of the thermometers begin to rise. After a certain time, steady state is reached when the temperature readings on the thermometers become constant.
- The temperature readings $\theta_1, \theta_2, \theta_3, \theta_4$ are noted
- The steady flow of water through the copper coil is measured by means of a beaker and a stop clock. The mass m of water collected in a time t is noted.
- The rate of heat flow between X and Y,

$$\frac{Q}{t} = kA\,\frac{\theta_2 - \theta_1}{L}$$

where $A = \frac{\pi d^2}{4}$ (where d is the diameter of the bar measured using a Vernier caliper)

- The rate of heat flow through the water

$$\frac{Q}{t} = \frac{m}{t}c(\theta_4 - \theta_3)$$

where c is the specific heat capacity of water

$$kA\,\frac{\theta_2 - \theta_1}{L} = \frac{m}{t}c(\theta_4 - \theta_3)$$

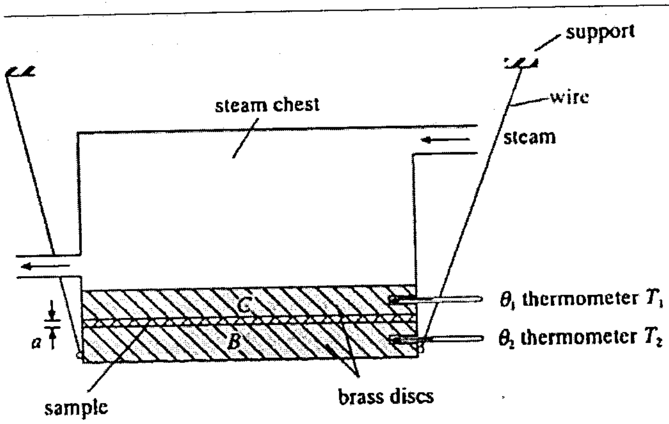
Hence, the thermal conductivity k of the conductor can be obtained.

Precautions

1. The quantity of heat per second through the bar is proportional to its cross-sectional area. A thick bar is required to provide a measurable quantity of heat in a reasonable time since the metal is a good conductor
2. The bar should be long enough to provide a reasonable temperature gradient for the metal when one end is heated electrically or by steam.
3. Lagging is essential since the bar is long, otherwise heat would escape through the sides of the bar by radiation and the heat flow would not be linear.

Poor conductors – Lees' Method

Lees devised a method of measuring the thermal conductivity of a bad conductor in form of a disc. This method is applicable to materials such as cardboard or ebonite.



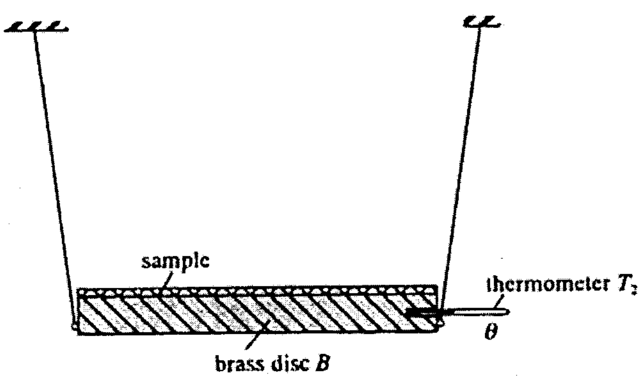
- It consists of a brass disc B which is suspended in air using fine wires. On top of this disc is placed the sample which is in form of a thin disc with a large cross-section. The Steam chest which has a thick brass base C is then placed on top of the sample.
- Steam is passed into the steam chest and when the steady state is attained, the readings θ_1 and θ_2 of the thermometers T_1 and T_2 respectively are noted.
- The thickness a of the sample is measured using a micrometer screw gauge and the diameter, D measured using a metre rule

  Temperature gradient $= \frac{\theta_2-\theta_1}{a}$

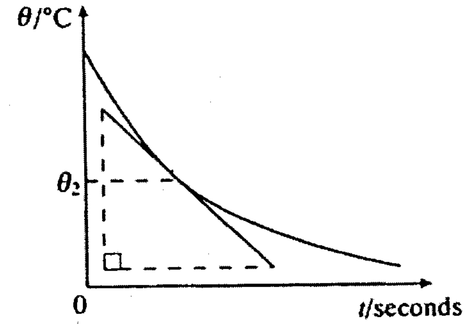
  Cross-sectional area $A = \frac{\pi D^2}{4}$
- When steady state is attained, the rate of heat flow through the base of the steam chest C, the sample and the brass disc B are the same. The heat then escapes from the from the bottom of the disc.

  Rate of heat flow through sample $= kA\ \frac{\theta_2-\theta_1}{a}$

  where k is the thermal conductivity of the sample
- The sample is removed so that B comes into direct contact with C and is heated by it.
- When the temperature of B has risen by a few degrees (or 10 °C) above θ_2, C is removed and the sample is put back on top of B



- B cools and the reading θ of the thermometer T_2 is recorded every half minute until the temperature is a few degrees (or 10 °C) below θ_2.
- A graph of θ against time t is plotted



From the graph, the gradient $\frac{\Delta\theta}{\Delta t}$ at θ_2 is calculated

Let m = mss of disc B found by weighing

c = specific heat capacity of brass

Rate of heat loss of disc B $= mc\frac{\Delta\theta}{\Delta t}$

$$kA\ \frac{\theta_2-\theta_1}{a} = mc\frac{\Delta\theta}{\Delta t}$$

$\therefore$ Thermal conductivity, $k = \frac{mc\frac{\Delta\theta}{\Delta t}}{A\left(\frac{\theta_2-\theta_1}{a}\right)}$

Precautions

- The upper and lower surfaces of the sample should be smeared with petroleum jelly or Vaseline to give good thermal contact with B and C
- The cross-sectional area of the sample should be large so that the flow of heat through it is large and heat loss from the sides can be neglected even though the sample is not lagged.
- The sample is thin because the temperature gradient along a poor conductor is high and for steady state to be attained fast, the disc has to be thin.

Applications of conduction of heat

- Determination of U-values during construction

  U-value of a structure is the heat transferred per second through $1\,m^2$ of the structure when temperature difference across it is $1\ K\ m^{-1}$

  These values provide architects and building engineers with a simple means of estimating heat losses from buildings.
- The houses of Eskimos are made up of double walled locks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the coldest surroundings.
- Birds often swell their feathers in winter to enclose air between their body and the feathers. Air prevents the loss of heat from the body of the bird to the cold surroundings.
- Mercury is used in thermometers to absorb heat
- Aluminium is used for making frying pans to absorb heat quickly.

- Motor vehicle engines are made of iron to conduct away heat
- Car radiators are made of iron to conduct away heat
- Refrigerators have copper pipes at the back for conducting away heat from coolant.
- The iron plate of an electric iron is made of steel to absorb heat quickly

Self-Evaluation exercise

1. The opposite sides of a solid cube of side 5 cm are maintained at temperatures of (i) 100 °C and 15 °C, (ii) 20 °C and −5 °C. Calculate the temperature gradient in each case in $K\,m^{-1}$ (°C m^{-1})
   [Ans: (i) $1700\ K\,m^{-1}$ (ii) $500\ K\,m^{-1}$]
2. Write down the expression for the quantity of heat per second flowing through a substance in the steady state. Two opposite faces of a solid copper cube of side 20 cm are maintained at temperatures of 85 °C and 5 °C respectively. Calculate the heat flowing through the cube in the steady state in 5 min. (Assume $k = 400\ W\,m^{-1}\,K^{-1}$ for copper.)
   [Ans: 192×10^4 J]
3. Show that $W\,m^{-1}\,K^{-1}$ is the unit of thermal conductivity.
   The thickness of a glass window is 2 mm, and it has an area of 0.8 m^2. If one side of the glass has a temperature of 16 °C and the other has a temperature of −4 °C, calculate the quantity of heat flowing through the glass in 10 seconds. (Assume k for glass = $10\ W\,m^{-1}\,K^{-1}$.) [Ans: 8×10^4 J]
4. The ends of a long cylindrical bar are maintained at a constant difference of temperature. Draw a sketch of the temperature variation of the different sections of the bar in the steady state if it is (i) unlagged, (ii) lagged. Explain briefly the reason for the temperature variation in both cases.
5. A domestic refrigerator can be thought of as a rectangular box of dimensions $0.90\,m \times 0.50\,m \times 0.50\,m$ and is lined throughout with a layer of insulation which is 4.0 mm thick and of thermal conductivity $0.40\,W\,m^{-1}\,K^{-1}$. If the room temperature is 24 °C and the temperature inside the refrigerator is maintained at 4 °C, calculate the rate at which heat flows into the refrigerator from the room. [Ans: 460 W]
6. On a very cold day, the air temperature is −5.0 °C. A pond has a layer of ice of thickness 50 mm and the temperature of the water in the pond is uniform at 0 °C. Calculate the
   (a) magnitude of the temperature gradient across the ice layer
   (b) rate of transfer of thermal energy per m^2 through the ice layer (thermal conductivity of ice = $2.3\,W\,m^{-1}\,K^{-1}$)
   [Ans: (a) $100\,K\,m^{-1}$ (b) $0.23\,kW$]
7. How is the quantity of heat flowing per second through the bar in the steady state measured in Searle's method of determining thermal conductivity? What is the advantage of lagging the bar?
8. Define thermal conductivity.
   Describe a method of measuring this quantity for a metal.
   Assuming that the thermal insulation provided by a woolen glove is equivalent to a layer of quiescent air 3 mm thick, determine the heat loss per minute from a man's hand, surface area 200 cm^2 on a winter's day when the atmospheric air temperature is −3 °C. The skin temperature is to be taken as 34 °C and the thermal conductivity of air as $24 \times 10^{-3}\ W\,m^{-1}\,K^{-1}$. [Ans: 355 J]
9. Give a critical account of an experiment to determine the thermal conductivity of a material of low thermal conductivity such as cork.
   Why is it that most cellular materials, such as cotton wool, felt, etc., all have approximately the same thermal conductivity?
10. A greenhouse, which may be assumed to be made entirely of glass, needs a 3.00 kW heater to maintain its steady temperature. The glass is 3.0 mm thick and has a total area of $5.0\,m^2$ and thermal conductivity of glass is $1.20\,W\,m^{-1}\,K^{-1}$. Calculate the temperature difference across the glass.
    [Ans: 1.50 K]
11. One face of a sheet of cork, 3 mm thick, is placed in contact with one face of a sheet of glass 5 mm thick, both sheets being 20 cm square. The outer faces of this square composite sheet are maintained at 100 °C and 20°C, the glass being at the higher mean temperature. Find (a) the temperature of the glass-cork interface, and (b) the rate at which heat is conducted across the sheet, neglecting edge effects. (Thermal conductivity of cork = $6.3 \times 10^{-2}\ W\,m^{-1}\,K^{-1}$, thermal conductivity of glass = $7.0 \times 10^{-1}\ W\,m^{-1}\,K^{-1}$)
    [Ans: (a) 90 °C (b) 58.4 W]
12. (a) Discuss the suitability of air as a material for thermal insulation. (b) State the factors which affect the rate of rise of temperature of one end of a metal bar which is heated at the other end.
13. Define thermal conductivity.
    Describe briefly the mechanism responsible for the conduction of heat in a non-metallic solid.

What is a possible reason for the fact that metals generally have a higher thermal conductivity than non-metals?

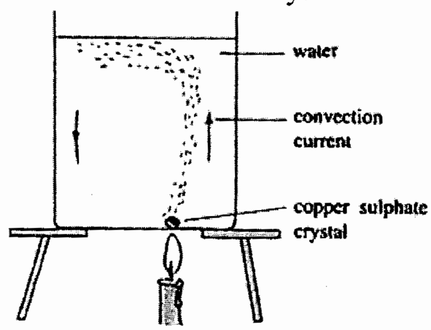
14. A large sheet of rubber rests on the flat surface of the ice on a frozen pond on a day when the air temperature is below 0 °C and the sky is clear. Radiation from the sun falls on the upper surface of the sheet of rubber. Explain what happens to the radiant energy and discuss what factors determine the temperature distribution in the rubber. What conditions are satisfied when the temperature distribution becomes steady? Hence indicate what factors determine whether or not the ice under the sheet of rubber starts to melt.
15. Define thermal conductivity.
Describe in detail a method of determining the thermal conductivity of cork in the form of a thin sheet.
The base and the vertical walls of an open thin-walled metal tank, filled with water maintained at 35 °C, are lagged with a layer of cork of superficial area 2.00 m^2 and 1.00 cm thick and the water surface is exposed. Heat is supplied electrically to the water at the rate of 250 watts. Find the mass of water that will evaporate per day, if the outside surface of the cork is at 15 °C. (Assume that the thermal conductivity of cork is 5.0×10^{-2} W m^{-1}K^{-1} and that the specific latent heat of vaporization of water at 35 °C is 2520 kJ kg^{-1}) [Ans: 1.7 kg]
16. A large hot-water tank has four steel legs in the form of cylindrical rods 2.5 cm in diameter and 15 cm long. The lower ends of the legs are in good thermal contact with the floor, which is at 20 °C, and their upper ends can be taken to be at the temperature of the water in the tank. The tank and the legs are well lagged so that the only heat lost is through the legs. It is found that 22 watts are needed to maintain the tank at 60 °C. What is the thermal conductivity of steel?
When a sheet of asbestos 1.5 mm thick is placed between the lower end of each leg and the floor only 5 watts are needed to maintain the tank at 60 °C. What is the thermal conductivity of asbestos?
[Ans: 42 $W\,m^{-1}\,K^{-1}$, 0.12 $W\,m^{-1}\,K^{-1}$]
17. Describe in detail how you would find the thermal conductivity of a piece of cardboard in the form of a thin disc. Give reasons for the choice of shape of the specimen used in the experiment.
18. In an experiment using a calorimeter, a quantity of liquid, which is thoroughly stirred, is maintained at a constant temperature by a heater made from a length of resistance wire of uniform circular cross-section covered with a layer of insulation 0.050 cm thick. The potential gradient along the wire is 4.0 V cm^{-1} and the wire is carrying a current of 1.5 A. Calculate the temperature gradient at a point in the insulation 0.025 cm from the surface of the wire. The diameter of the wire is 0.020 cm and the thermal conductivity of the insulating material is 2.0 $W\,m^{-1}\,K^{-1}$. [Ans: $1.4 \times 10^5\ Km^{-1}$]
19. The surface temperatures of the glass in a window are 20 °C for the side facing the room and 5 °C for the outside. Compare the rate of flow of heat through (i) a window consisting of a single sheet of glass 5.0 mm thick, and (ii) a double-glazed window of the same area consisting of two sheets of glass each 2.5 mm thick separated by a layer of still air 5.0 mm thick. It may be assumed that the steady state has been attained. (Use the following values of coefficient of thermal conductivity: glass, 1.0 $W\,m^{-1}\,K^{-1}$, air $2.5 \times 10^{-2}\ W\,m^{-1}\,K^{-1}$)
[Ans: 41 : 1]
20. An ideally lagged compound bar 25 cm long consists of a copper bar 15 cm long joined to an aluminium bar 10 cm long and of equal cross-sectional area. The free end of the copper is maintained at 100 °C and the free end of aluminium at 0 °C. Calculate the temperature gradient in each bar when steady state conditions have been reached. (Thermal conductivity of copper = 390 $W\,m^{-1}\,K^{-1}$, Thermal conductivity of aluminium = 210 $W\,m^{-1}\,K^{-1}$)
[Ans: 300 Km^{-1}- copper, 550 Km^{-1} - aluminium]
21. An iron pan containing water boiling steadily at 100 °C stands on a hot-plate and heat conducted through the base of the pan evaporates 0.090 kg of water per minute. If the pan has an area of 0.04 m^2 and a uniform thickness of $2.0 \times 10^{-3}\ m$, calculate the surface temperature of the underside of the pan. (thermal conductivity of iron = 66 $W\,m^{-1}\,K^{-1}$, specific latent heat of vaporization of water at 100 °C = $2.2 \times 10^6\ J\,kg^{-1}$)
[Ans: 102.5 °C]
22. A window pane consists of a sheet of glass of area 2.0 m^2 and thickness 5.0 mm. If the surface temperatures are maintained at 0 °C and 20 °C, calculate the rate of flow of heat through the pane assuming a steady state is maintained.
The window is now double glazing by adding a similar sheet of glass so that a layer of air 10 mm thick is trapped between the two panes. Assuming that the air is still, calculate the ratio of flow of heat through the window in the first case to that in the second. (conductivity of glass = 0.80 $W\,m^{-1}\,K^{-1}$, conductivity of air = 0.025 $W\,m^{-1}\,K^{-1}$)
[Ans: 6400 W, 66 : 1]

23. Explain why, in an experiment to determine the thermal conductivity of copper using a Searle's bar arrangement, it is necessary (a) that the bar should be thick, of uniform cross-section and have its sides well lagged, (b) that the temperatures used in the calculation should be the steady values finally registered by the thermometer.
24. Ice is forming on the surface of a pond. When it $4.6\ cm$ thick, the temperature of the surface of the ice in contact with air is $260\ K$ while the surface in contact with the water is at a temperature $273\ K$. Calculate the rate of heat loss per unit area from the water. Hence determine the rate at which the thickness of the ice is increasing. (Thermal conductivity of ice = $2.3\ W\ m^{-1}\ K^{-1}$, density of water = $1000\ kgm^{-3}$, specific latent heat of fusion of ice = $3.25\ \times 10^{5}\ J\ kg^{-1}$)
[Ans: $650\ Js^{-1}m^{-2}$, $2.0\ \times\ 10^{-3}\ mms^{-1}$]
25. (a) Calculate the heat passing per second through $1\ m^{2}$ of glass of thickness 2 mm when its faces are maintained at 20 °C and 5 °C respectively. (Thermal conductivity of glass = $1.2\ W\ m^{-1}\ K^{-1}$)
(b) Two such sheets of glass are now placed 4 mm apart and sealed so as to trap air in the space between them forming a 'sandwich' of thickness $8\ mm$. Given that the thermal conductivity of air is $0.024\ W\ m^{-1}\ K^{-1}$, calculate the rate of heat conduction per m^{2} when the outside faces of glass are again maintained at 20 °C and 5 °C respectively.
[Ans: (a) $9.0\ kW$ (b) $88\ W$]

Convection

It is a phenomenon of transfer of heat in a fluid wi the actual movement of the particles of the fluid.
When a fluid is heated, the hot part expands an becomes less dense. It rises and upper colder pa replaces it. This again gets heated, rises up and i replaced by the colder part of the fluid. This proces goes on.
This mode of heat transfer is different from conductio where energy transfer takes place without the actua movement of the molecules or material medium.
Convection occurs in all fluids, whether liquids o gases.

To illustrate convection, a piece of copper sulphate crystal is carefully placed in a beaker of water and a candle flame is held below the crystal.



Warm coloured water is seen to rise and cold water from the top moves down forming a convection current.

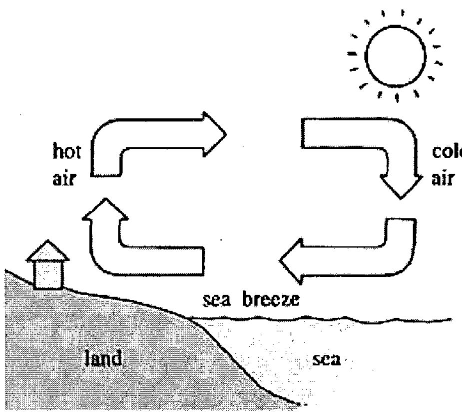
Because of convection, it is not possible to hold the hand above a candle flame for a long time. Heat travels upwards by convection. On the other hand, you can hold your finger besides the candle flame because air is a poor conductor of heat.

Types of convection

There are two subcategories in convection namely Natural convection and Forced Convection.
Natural convection includes the heating of the earth and atmosphere through the sun's rays.
Forced convection involves pumps, motors or any machines or systems that get forced to other fluid. Home cooling air conditioners through fans comes under Forced Convection.

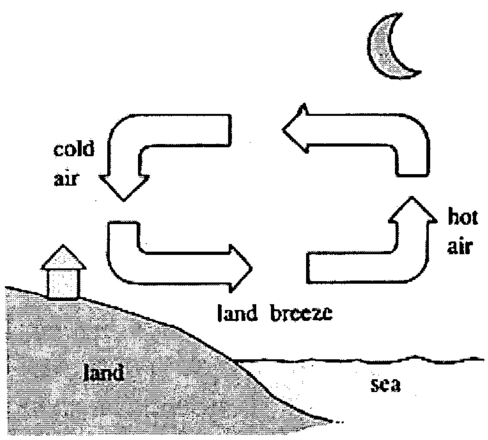
Applications of convection

1. Sea breeze



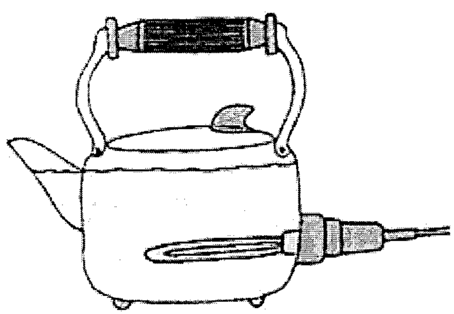
During the day, the land becomes hotter than sea because the land is better conductor of heat than sea. The air above the land gets heated becomes less dense and rises. Cooler air from the sea moves towards the land, causing a sea breeze.

2. Land breeze



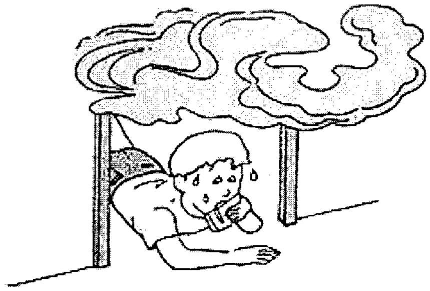
At night, the sea is warmer than the land because the land loses heat faster than the sea. The air above the sea is less dense and rises. Cooler air from the land moves towards the sea, causing a land breeze.

3. The heating element of an electric kettle is placed at the bottom of the kettle. Water heated at the bottom rises to the top and cold water from the top moves down to be heated.

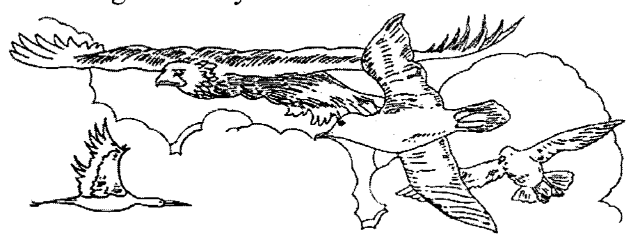


4. The ventilation in the house depends on convection.
Cool fresh air enters the house through windows and doors. Warm, stale air which is less dense leaves the house through ventilators situated near the roof of house.

5. One should crawl out of a smoke-filled room as the air further up is hot and contains more smoke.



6. Birds are able to fly for hours without flapping their wings and hang gliders are able to rise by riding on **thermals** which are streams of hot air rising in the sky.



Radiation

This is the process of heat transfer where the medium does not take part in the transfer

The energy emitted by a body in the form of radiation on account of its temperature is called thermal radiation. It depends on the

(i) temperature of the body,
(ii) nature of the radiating body
(iii) surface area of the body

Thermal radiation is emitted by all objects whose temperature is above 0 K. The thermal radiation consists of a mixture of wavelengths.

Objects at low temperature emit waves of long wavelengths while higher temperature objects emit waves of shorter wavelengths.

Objects such as a lighted filament lamp, a coal fire or a human body emit waves mostly in the long wavelengths at the end of the infrared region. Thus when we speak of heat radiation, we are mainly referring to infrared radiation.

Electromagnetic waves

Electromagnetic waves are transverse waves due to electric and magnetic vibrations which travel together

They produce effects according to their frequency of vibration or wavelength. Generally, the speed of any electromagnetic wave in a vacuum is $3.0 \times 10^8\ ms^{-1}$

Radio waves, infra-red rays, visible light, ultra-violet rays, X-rays and gamma rays are all electromagnetic waves. Their wavelengths vary from radio waves, which may have long wavelengths of the order of 1000 m or short wavelengths of the order of a few centimetres, to gamma rays, which have the shortest wavelengths of the order of $10^{-11}\ m$.

Visible light has wavelengths in a narrow band, roughly in the range $4.0 \times 10^{-7}\ m$ to $7.5 \times 10^{-7}\ m$.

Ultra-violet rays have shorter wavelengths than $4.0 \times 10^{-7}\ m$.

X-rays have much shorter wavelengths than ultra-violet rays and may approach the wavelengths of γ-rays.

Infra-red rays are invisible rays of wavelengths longer than the red wavelength, $7.5 \times 10^{-7}\ m$. Most of the heat radiation from hot objects are infra-red rays, roughly in the range $8.0 \times 10^{-7} m$ to $4 \times 10^{-4} m$. Radiators at very high temperatures, however, such as some stars, may have a good deal of their heat radiation in the visible spectrum

Although we experience different sensations from heat radiation and light, the only difference between them is their difference in wavelength (or frequency). Both are forms of energy which travel in space as electromagnetic waves. Glass is opaque to infra-red rays from fires and may therefore be used as a fire screen. Quartz and rock salt are transparent to such rays.

Properties of electromagnetic waves

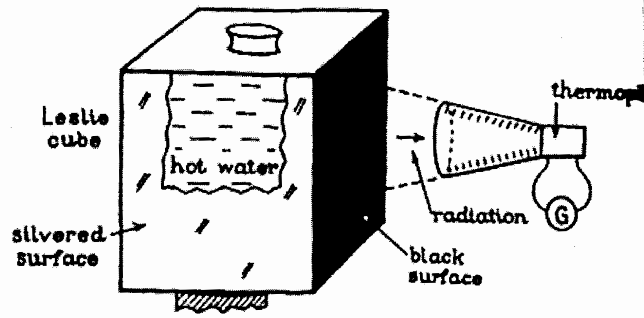
1. They travel at the same speed in a vacuum
2. They are unaffected by electric and magnetic fields
3. They travel in straight lines
4. They can be polarized as they are transverse waves
5. They can be caused to produce interference effects
6. They affect photographic films

Applications of infrared radiations

- Remote sensing which is used in weather forecasting
- Detection and communication systems
- Environment monitoring
- Astronomy – detection of objects such as planets
- Military acquisitions such as target acquisition, surveillance, night vision, tracking, etc.
- Radio photography
- Dry paint on cars during manufacture
- The fact that people are emitters of infrared radiation is used in a wide variety of anti-intruder devices such as burglar alarms
- Automatic light switches

Comparison of energy radiated by different surfaces

This is done by use of a Leslie cube (a cubical tank with sides of different finishes e.g. dull black surface and highly polished or silvered surface)



The tank has boiling water so it is at a constant temperature.

A thermopile fitted with a blackened conical mouth piece is made to face different faces of the cube in turn. The deflection of the galvanometer is detected in each case. The surface that radiates most will give the greatest deflection of the galvanometer.

Note: The radiating powers of the surfaces are as follows: (1) lamp-blackened (2) roughened (3) white (4) polished.

Thus, in order to minimize loss of heat by radiation, a body should be brightly polished.

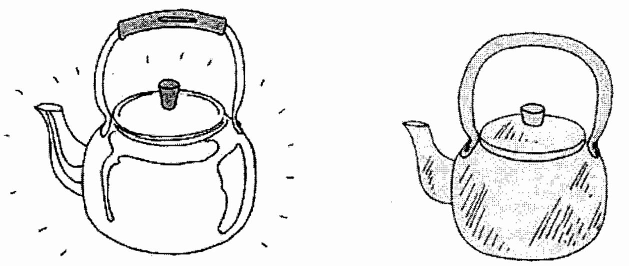
Absorption and emission of radiant energy

When thermal radiation falls on an object, it will be partly absorbed and partly reflected. A shiny silvered surface is the best reflector where as a dull black surface is the best absorber.

Surfaces which appear black in day light do not reflect any light so they absorb all the light which falls on them. Such surfaces are also good absorbers of thermal radiation. Therefore a good absorber of thermal radiation is dull black whereas a poor absorber is shiny. This explains why you feel warmer if you wear dark coloured clothes.

Good absorbers are also good emitters and poor absorbers are poor emitters

Hot water in a kettle covered with soot cools faster than water in a similar kettle but with its surface shinny like a mirror.



If the two kettles are each filled with cold water, the waterstays cold longer in the shiny kettle as it is a poor absorber.

In general, dark-skinned longer distance runners are able to perform better than fair skinned runners. This is because heat is emitted at a faster rate from a dark surface than from a fair one. Hence the body temperature of the dark-skinned runner is always below the optimum temperature before the human body efficiency decreases.

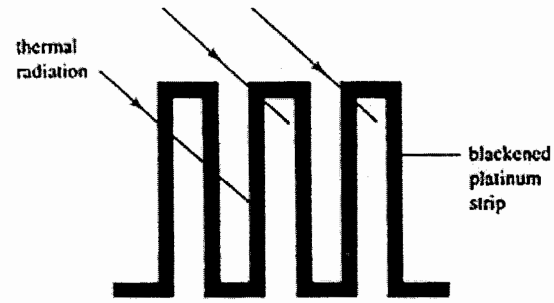
Buildings are painted in light colours so as to keep cool in summer because of their poor absorption property. In winter these buildings, being poor emitters, retain the heat better.

Whether a body plays the role of an emitter or absorber depends on whether its temperature is above or below the surroundings. If the surface is hotter than the surroundings, it is a net emitter and will become cooler. If the surface is colder than the surroundings, it becomes a net absorber and will become warmer.

Detectors of thermal radiation

The human skin is quite sensitive to thermal radiation. A thermometer with a bulb blackened with lamp soot is able to detect infrared radiation. If the blackened bulb of the thermometer is held just outside the red end of the visible spectrum produced using a glass prism, the mercury level in the thermometer rises.

Bolometer

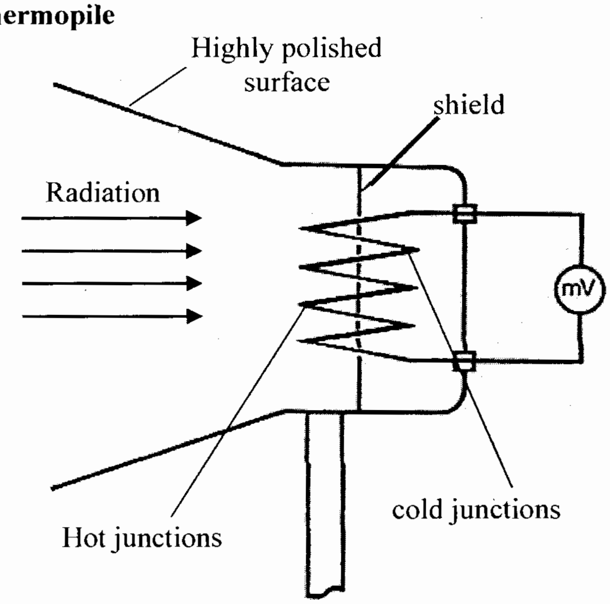


It consists of a narrow strip of platinum with its surface blackened.

Thermal radiation falling on the bolometer is absorbed. The temperature of the platinum strip rises and so does the resistance of the strip.

The increase is measured using an electrical in resistance is measured using an electrical circuit such as Wheatstone bridge where the galvanometer will show a deflection.

Thermopile



It consists of a number of thermocouples connected in series.

One set of thermocouple junctions is exposed to the radiation and is heated by it. These junctions exposed to the radiations are coated with lamp-black (blackened) to enhance the efficiency with which the radiation is absorbed.

The other set of thermocouple junctions is shielded from the radiation.

A highly polished metal cone concentrates the radiation on the exposed junctions.

When thermal radiation falls on the blackened tin pieces, they become hot and a thermo e.m.f is produced.

The emf produced is measured using a millivoltmeter.

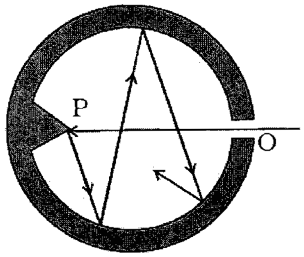
Black body radiation

This is the radiation from a black body

A black body is the body which absorbs all the radiation which is incident on it

A black body radiator is one which emits radiation which is characteristic of its temperature and does not depend on the nature of its surfaces

Approximation of a black body



A small hole O is made in an enclosure e.g. a hollow cylinder or sphere whose inner walls are painted black. When the radiation enters into the cylinder through a small hole, it undergoes multiple reflections.

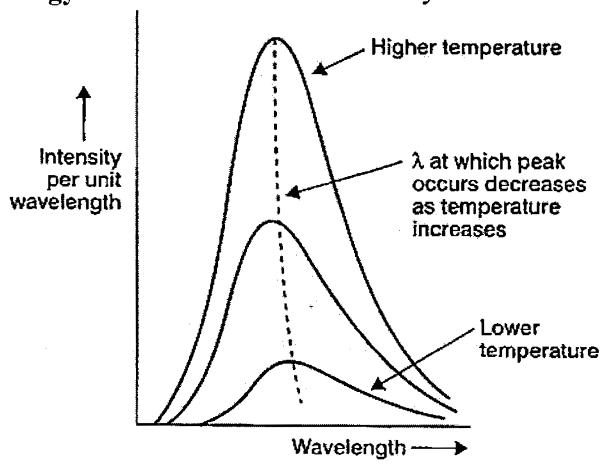
At each reflection, a certain amount of radiation is absorbed until all the radiation is absorbed by the black walls of the inner sphere or cylinder.

The projection P helps in avoiding any direct reflections. When this body is placed in a bath at fixed temperature, the heat radiations come out of the hole. The opening O thus acts as a black body radiator.

Examples of black bodies

- The sun and stars
- Light bulbs
- Heat lamps
- Electric heaters
- A candle's flame
- Warm blooded animals

Energy distribution of a black body



The energy of thermal radiations from a black body is not distributed evenly in the spectrum.

At lower temperatures i.e. 500 K, only infrared radiation is emitted, no light is emitted at all. The object does not glow.

At moderate temperatures i.e. 1000K or 1500 K most of the wavelengths emitted consist of infrared radiations and a little from the visible end of the spectrum. Since light is from the red end of the visible spectrum, a body at 1000 K appears red.

When the temperature is higher, the intensity for each wavelength increases. The intensity of the shorter wavelengths increases more than that of the longer wavelengths.

At higher temperatures, all wavelengths in the visible spectrum are emitted together with infrared radiation. Hence a body now appears white. This explains why a white hot object is hotter than a red hot object.

Ultraviolet radiation is also emitted as the spectrum extends beyond the violet end of the visible spectrum.

Note:

The area under the intensity-wavelength curve represents the total energy emitted per second per m^2 of the black body.

Wien's displacement law

The wavelength, λ_{max}, at which the intensity is at peak decreases as temperature increases and is related to the temperature T of the black body by the relationship

$$\lambda_{max}T = constant = k$$

This is known as Wien's displacement law and k is the Wien's constant typically with a value of $2.90 \times 10^{-3}\ mK$

Stefan's law

It states that the total energy radiated per m^2 surface area per second of a black body is proportional to the fourth power of the absolute temperature of the body.

$$E \propto T^4$$

$$E = \sigma T^4$$

where σ = Stefan's constant

$$E = \frac{P}{A}$$

$$P = A\sigma T^4$$

ssive power

total emissive power e of the surface of a body is ned as the total energy per second per unit area ted by it. It is expressed in watt $re^{-2}(Wm^{-2})$.

nagnitude depends on the nature and temperature ne surface.

total emissivity, ε, of a surface is defined as the ɔ of its emissive power e to that E of a black body ne same temperature

$$e = \varepsilon E$$

ere E is the total emissive power of a black body

$$e = \varepsilon\sigma T^4$$

$$\text{Power radiated } = \varepsilon\sigma A T^4$$

evost's theory of heat exchanges

evost applied the idea of thermal equilibrium to liation. According to him the rate at which a body liates or absorbs heat depends on the nature of its rface, its temperature and the temperature of the rroundings. The total amount of heat radiated by a ody increases as its temperature rises. A body at a gher temperature radiates more heat energy to the urroundings than it receives from the surroundings. hat is why we feel warm when we stand before the urnace. Similarly, a body at a lower temperature eceives more heat energy than it loses to the urroundings. That is why we feel cold when we tand before an ice block.

Thus, the rise or fall of temperature is due to the exchange of heat radiation. When the temperature of the body is the same as that of surroundings, the exchanges of heat do not stop. In such a case, the amount of heat energy radiated by the body is equal to the amount of heat energy absorbed by it.

A body will stop emitting radiation only when it is at absolute zero i.e. 0 K. At this temperature, the kinetic energy of the molecules is zero.

revost's theory states that a body emits radiation at a ate determined only by the nature of its surface and its emperature and absorbs radiation at a rate determined by the nature of its surface and the temperature of its urroundings.

Examples

1. The Wien constant is $2.90 \times 10^{-3}\ mK$ and the Stefan constant is $5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$. Given that the sun has a radius of approximately $7.0 \times 10^{8}\ ms^{-1}$ and its surface temperature is $5800\ K$, calculate
   (i) the wavelength at which the sun's radiation peaks
   (ii) the energy emitted per second from the sun's surface

Solution

(i) The Wien's law states that,

$$\lambda_{max} T = constant$$

$$\lambda_{max} \times 5800 = 2.90 \times 10^{-3}$$

$$\lambda_{max} = \frac{2.90 \times 10^{-3}}{5800} = 5.0 \times 10^{-7}\ m$$

(ii) The energy per second is the power, P

$$P = \sigma A T^4$$

$$P = 5.67 \times 10^{-8} \times 4\pi(7.0 \times 10^{8})^2 \times 5800^4$$

$$= 3.95 \times 10^{26}\ watt$$

2. The element of a radiant electric heater which can be assumed to be a black body that radiates all the energy supplied to it, attains a steady temperature of 1000 K for a certain power input. What temperature would it attain if the power was halved? (Neglect any radiation received from the surrounding)

Solution

Let A = surface area of the heating element

P = power of heater at 1000 K

Using Stefan's law, $P = \sigma A T^4$

$$P = \sigma A (1000)^4 \quad \text{......... (i)}$$

Let T_1 = temperature of the heater when power = $\frac{1}{2}P$

$$\frac{1}{2}P = \sigma A T^4 \quad \text{....... (ii)}$$

(ii) ÷ (i);

$$\frac{1}{2} = \frac{T^4}{1000^4}$$

$$T = 841\ K$$

3. If each square centimeter of the sun's surface radiates energy at the rate of $6300\ W\,cm^{-2}$ and Stefan's constant is $5.7 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$, calculate the temperature of the sun's surface in °C, assuming Stefan's law applies to the radiation

Solution

$$P = \sigma A T^4$$

$$P = 6300\ W\,cm^{-2} = 6300 \times 10^4\ W\,m^{-2}$$

$$5.7 \times 10^{-8} T^4 = 6300 \times 10^4$$

$$T = \left[\frac{6300 \times 10^4}{5.7 \times 10^{-8}}\right]^{\frac{1}{4}} = 5765\ K$$

$$T = 5765 - 273 = 5492\ °C$$

4. A metal sphere of radius 1.5 cm is suspended within an evacuated enclosure whose walls are at 320 K. The emissivity of the metal is 0.40. Find

the power input required to maintain the sphere at a temperature of 320 K, if the heat conduction along the supports is negligible

Solution

$$r = 1.5\ cm = 0.015\ m$$

Area of sphere, A= $4\pi r^2 = 4\pi(0.015)^2$

$$= 2.83 \times 10^{-3}\ m^2$$

Emissivity, $\varepsilon = 0.40,\ T = 320\ K$

To maintain the sphere at 320 K, the power input to the sphere = power radiated by the sphere

Using Stefan's law,

Power radiated by sphere = $\delta\sigma AT^4$

$$= 0.4 \times 5.67 \times 10^{-8} \times 2.83 \times 10^{-3} \times 320^4$$

$$= 0.673\ W$$

5. Calculate the radius of a star with an absolute surface temperature of 6000 K and a power output of $3.5 \times 10^{34}\ W$.

Solution

$$P = A\sigma T^4$$

$$P = 4\pi R^2\sigma T^4$$

$$R^2 = \frac{P}{4\pi\sigma T^4}$$

$$R = \sqrt{\frac{P}{4\pi\sigma T^4}}$$

$$= \sqrt{\frac{3.5 \times 10^{34}}{4\pi(5.67 \times 10^{-8})(6000)^4}}$$

$$= 6.16 \times 10^{12}\ m$$

6. If the mean equilibrium temperature of the Earth's of the earth's surface is T and the total rate of energy emission by the sun is E, show that

$$T^4 = \frac{E}{16\sigma\pi R^2}$$

where σ is the Stefan constant and R is the radius of the earth's orbit around the sun (Assume the sun behaves as a black body)

Solution

Intensity of energy received by the earth per $m^2 = \frac{E}{4\pi R^2}$

Assuming that the earth is in form of a uniform circular disc to this radiation of radius R_E, then

area of the earth = πR_E^2

total energy received by the earth per second = $\frac{E}{4\pi R^2} \times \pi R_E^2$

Assuming the earth to be a black body,

Power received by the earth = power radiated by the earth

From Stefan's law,

Power radiated by the earth = $A\sigma T^4 = 4\pi R_E^2\sigma T^4$

$$\therefore \frac{E}{4\pi R^2} \times \pi R_E^2 = 4\pi R_E^2\sigma T^4$$

$$T^4 = \frac{E}{16\sigma\pi R^2}$$

7. The sun is a black body of surface temperature 6000 K.
   (i) Calculate the amount of radiant energy approaching the earth
   (ii) Why would the actual energy be less than that obtained above in (i)
   (iii) Calculate the earth's temperature

(Stefan's constant = $5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$, radius of sun = $7 \times 10^8\ m$, radius of earth = $6.4 \times 10^6\ m$, radius of earth's orbit about the sun = $1.5 \times 10^{11}\ m$)

Solution

(i) Radiant power of sun, $P = \sigma AT^4$

Area of sun = $4\pi R_S^2 = 4\pi(7 \times 10^8)^2$

$$= 6.158 \times 10^{18}\ m^2$$

$$P = 5.67 \times 10^{-8} \times 6.158 \times 10^{18} \times 6000^4$$

$$= 4.525 \times 10^{26}\ W$$

Intensity of radiation received by the earth per m^2

$$= \frac{P}{4\pi r_0^2}$$

$$I = \frac{4.525 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} = 1600.39\ Wm^{-2}$$

The earth is assumed to be in the form of a circular disc to this radiation,

Area of earth = $\pi R_E^2 = \pi(6.4 \times 10^6)^2$

$$= 1.287 \times 10^{14}\ m^2$$

Total energy received by the earth = Intensity × Area of the earth

$$= 1600.39\ Wm^{-2} \times 1.287 \times 10^{14}\ m^2$$

$$= 2.06 \times 10^{17}\ W$$

(ii) This is because solar radiation incident on the atmosphere is gradually is partially absorbed by atmospheric gases and partially scattered. This accounts for the difference

(iii) A portion of the sun's radiated power/energy is received by the earth in accordance to the inverse square law.

Power received by the earth

$$= \frac{P}{4\pi r_0^2} \times \text{Area of earth}$$

$$= \frac{A_S\sigma T_S^4}{4\pi r_0^2} \times \pi R_E^2$$

Power received by the earth = power emitted by the earth

Power emitted by the earth

$$= A_e \sigma T^4 = 4\pi R_E^2 \sigma T_e^4$$

$$\frac{A_s \sigma T_s^4}{4\pi r_0^2} \times \pi R_E^2 = 4\pi R_E^2 \sigma T_e^4$$

$$\frac{4\pi R_s^2 \sigma T_s^4}{4\pi r_0^2} \times \pi R_E^2 = 4\pi R_E^2 \sigma T_e^4$$

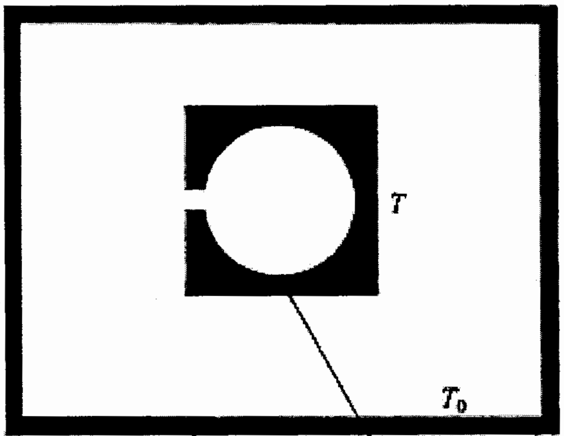
$$T_e^4 = T_s^4 \frac{R_s^2}{4r_0^2}$$

$$T_e = T_s \sqrt{\frac{R_s}{2r_0}}$$

$$= 6000\sqrt{\frac{7\times10^8}{2\times1.5\times10^{11}}} = 289.83\ K$$

Heat lost by radiation

Suppose a black body radiatior is initially at an absolute temperature T K and is placed inside a black body enclosure at a lower temperature T_0 K.



At the same times, the black body is emitting at a rate of σT^4 per second per unit area. It absorbs radiation emitted from the constant temperature enclosure at a rate of σT_0^4.

If T is greater than T_0, the rate of thermal energy radiated from the black body is greater than the rate of absorption. Hence the net energy radiated per second per unit area of the black body is

$$E_{net} = \sigma T^4 - \sigma T_0^4$$
$$= \sigma(T^4 - T_0^4)$$

This would result in the temperature of the black body falling until thermal equilibrium is attained and the temperature of the black body is equal to the temperature of the constant temperature enclosure i.e. T_0

Note: In case of a non-black body,

$$E_{net} = \varepsilon = \sigma(T^4 - T_0^4)$$

where ε is the total emissivity of the body

Examples

1. A metal sphere with an electric heater in it is hung in an evacuated enclosure. The enclosure is made of the same metal as the sphere. The walls of the enclosure are maintained at 20 °C. The power supplied to the heater to maintain the temperature of the sphere at 40 °C is 50 W. What is the power required to maintain the temperature of the sphere at 100 °C?

Solution

$$E_{net} = \sigma T^4 - \sigma T_0^4$$

When $T_0 = 20\ °C = 293\ K, T = 40\ °C = 313\ K$

$$E_{net} = 50\ W$$

$$50 = \sigma(313^4 - 293^4) \ldots\ldots\ldots (i)$$

When $T = 100\ °C = 373\ K, T_0 = 273\ K$

$$E_{net} = \sigma(373^4 - 293^4) \ldots\ldots\ldots (ii)$$

$(ii) \div (i)$

$$\frac{E_{net}}{50} = \frac{(373^4 - 293^4)}{(313^4 - 293^4)}$$

$$E_{net} = 269\ W$$

2. A diode valve consists of two long axial cylinders The inner cathode cylinder, of radius 0.5 mm, radiates heat like a black body. The radius of the anode cylinder is large compared with that of the cathode. The cathode heater element dissipates one watt per centimeter length. If the steady anode temperature is 227°C, estimate the temperature of the cathode. Neglect end effects (Stefan's constant $= 5.74 \times 10^{-8} W\,m^{-2}\,K^{-4}$)

Solution

Energy radiated by cathode per m length = 100 watt

Since the cathode is effectively in an enclosure of 227 °C (500 K),

energy radiated per m

$$= \sigma(2\pi \times 0.5 \times 10^{-3})(T^4 - 500^4)$$

$$= 5.74 \times 10^{-8} \times (2\pi \times 0.5 \times 10^{-3})(T^4 - 500^4)$$

For dynamic equilibrium,

$$100 = 1.8 \times 10^{-10}(T^4 - 500^4)$$

$$T^4 = 500^4 + \frac{100}{1.8\times10^{-10}}$$

$$T = 886.3\ K$$

3. A solid copper sphere of diameter 10 mm and a temperature of 150 K is placed in an enclosure maintained at a temperature of 290 K. Calculate stating any assumptions made, the initial rate of rise of temperature of the sphere (density of copper = $8.93 \times 10^3\ kgm^{-3}$, S.H.C of copper $370\ Jkg^{-1}K^{-1}$)

Solution

Net power absorbed $= \sigma(T^4 - T_0^4)$

Energy absorbed by the sphere $= mc\theta$

Power absorbed $= \frac{d}{dt}(\rho Vc\theta) = \rho Vc\frac{d\theta}{dt}$

$$\rho Vc\frac{d\theta}{dt} = \sigma(T^4 - T_0^4)$$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5 \times 10^{-3})^3 =$

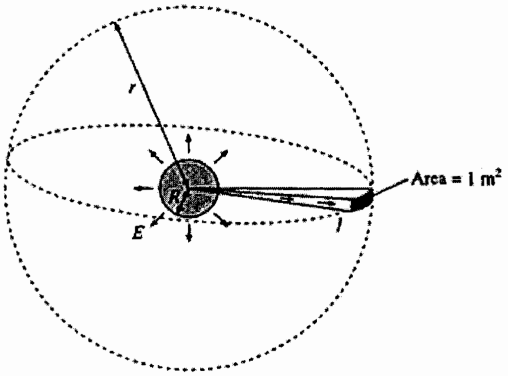
$8.93 \times 10^3 \times \times 370\frac{d\theta}{dt}$

$= 5.67 \times 10^{-8}(290^4 - 150^4)$

$$\frac{d\theta}{dt} = 372.32\ °C\ s^{-1}$$

The inverse square law

The intensity of the thermal radiation falling on a surface decreases as the distance from the source increases



If E = total energy emitted per second per unit area of the sphere,

then total energy emitted per second by the sphere

$$= E(4\pi R^2)$$

where $4\pi R^2$ = surface area of sphere

To determine the intensity of the thermal radiation at a distance r $(r > R)$ from the sphere, we assume that the sphere is enclosed by a concentric sphere of radius r. Then all the thermal energy radiated from the sphere of radius R would fall on the inner surface of the large sphere of radius r.

If I = intensity of energy received per second per unit area at a distance r,

then total energy emitted by sphere of radius R

= total energy received by sphere of radius r

$$E(4\pi R^2) = I(4\pi r^2)$$

$$I = \left(\frac{R^2}{r^2}\right)E$$

$$I \propto \frac{1}{r^2}$$

The intensity is inversely proportional to the square of the distance from the emitter

Examples

1. The total power radiated by the Sun is about 4.0×10^{26} watt. Calculate the energy received per second by the Earth from the Sun. Take the Earth's radius as $6.4 \times 10^6\ m$ and the distance of the earth from the sun as $1.5 \times 10^{11}\ m$.

Solution

The intensity received is given by $I = \frac{P}{4\pi r^2}$ and is the energy received per m^2 per second

$$I = \frac{4.0 \times 10^{26}}{4 \times \pi \times (1.5 \times 10^{11})^2} = 1.4 \times 10^3\ W\ m^{-2}$$

Assuming that the earth appears as a disc to this radiation,

Area of the earth $= \pi r_E^2$

$= \pi \times (6.4 \times 10^6)^2$

$= 1.287 \times 10^{14}\ m^2$

Total energy received per second,

$= 1.4 \times 10^3\ W\ m^{-2} \times 1.287 \times 10^{14}\ m^2$

$= 1.8 \times 10^{17}\ W$

Temperature on the surface of the sun

The temperature on the surface of the sun may be estimated using Stefan's law and the inverse square law. On the earth's surface, on the average, the energy received per second by an area of $1\ m^2$ which is held normally to the radiation from the sun is $S = 1.34 \times 10^3\ Wm^{-2}$. This value is known as the solar constant

By the inverse square law, if E is the total energy emitted per second per unit surface area of the sun, then

$$\text{Solar constant, } S = \left(\frac{R^2}{r^2}\right)E$$

$$E = \frac{r^2}{R^2}S$$

where r = average distance of the earth from the sun $= 1.5 \times 10^{11}\ m$

$$R = radius\ of\ the\ sun = 7.0 \times 10^8\ m$$

If the sun is assumed to be a black body with surface temperature T, then using Stefan's law,

$$E = \sigma T^4$$

$$\sigma T^4 = \frac{r^2}{R^2}S$$

$$T^4 = \frac{r^2}{R^2}S$$

$$T^4 = \frac{r^2}{R^2}\frac{S}{\sigma}$$

$$= \frac{(1.5 \times 10^{11})^2 \times (1.34 \times 10^3)}{(7.0 \times 10^8)^2 \times (5.67 \times 10^{-8})}$$

$$T = 5700\ K$$

Note:

When the sun radiates heat, its total mass decreases and the decrease in mass is known as mass defect (m). The expression relating mass defect and the total energy radiated by the sun is given by;

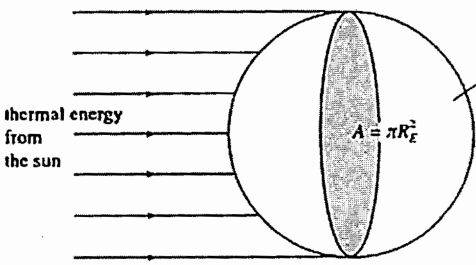
$$\Delta E = \Delta m\, c^2$$
$$\frac{dE}{dt} = c^2 \frac{dm}{dt}$$

where c is the velocity of light in a vacuum and has a value $3 \times 10^8\ ms^{-1}$.

Therefore, the total rate of loss of mass by the sun is given by;

$$\frac{dm}{dt} = \frac{\frac{dE}{dt}}{c^2}$$

Equilibrium temperature of the Earth's surface



At any instant, only half of the earth's surface is facing the sun and the radiation from the sun does not fall normally on every part of the earth's surface. The earth is considered to be a circular disc of radius R_E.

Energy received by the earth from the sun = $\pi R_E^2 S$

At the same instant that the earth absorbs energy from the sun, it is emitting radiation. If the earth is assumed to be a black body at a temperature T, then using Stefan's law,

Energy emitted per second per unit area of the earth's surface is

$$E = \sigma T^4$$

Total energy emitted per second by the earth

$$= E(4\pi R_E^2) = 4\pi R_E^2 \sigma T^4$$

where $4\pi R_E^2$ is the surface area of the earth

When the earth is in thermal equilibrium

Energy emitted per second = energy absorbed per second

$$4\pi R_E^2 \sigma T^4 = \pi R_E^2 S$$

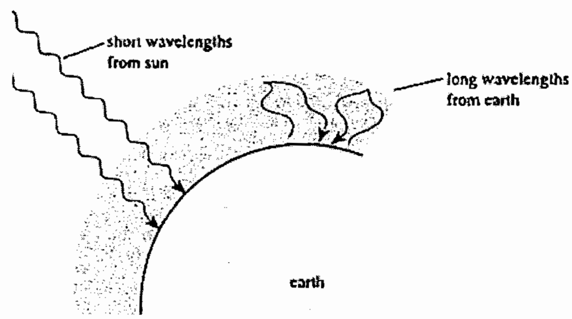
$$T^4 = \frac{S}{4\pi\sigma} = \frac{1.34 \times 10^3}{4\pi \times (5.67 \times 10^{-8})}$$

Equilibrium temperature of the earth's surface, $T = 208\ K$

At present, the average temperature of the earth is about 300 K which is higher than the equilibrium temperature calculated due to a number of factors

The energy received from the sun is mainly in the range of ultraviolet, visible light and short wavelength infrared because of the high temperature of the sun, about 6000 K. These short wavelength pass through freely to the earth's atmosphere without being absorbed. Subsequently the radiation from the sun is absorbed by the earth's surface

On the other hand, the earth being a cooler source emits radiation which is mainly in the longer wavelengths i.e. infrared. These wavelengths are absorbed by water vapour and carbon dioxide in the earth's atmosphere. The atmosphere radiates most of this energy back to the earth, resulting in the temperature of the earth being higher than it would be otherwise



Fumes emitted from factories and motor vehicles have increased greatly the amount of carbon dioxide in the atmosphere. This has resulted in a steady rise in the temperature of the earth due to **greenhouse effect**

Self-Evaluation exercise

1. Define a black-body radiator. How can such a radiator be realized experimentally? Draw sketches showing roughly how the energy in a black-body radiator varies with the different wavelengths for (i) 700 °C, (ii) 1000 °C, (iii) 1500 °C.
2. State Stefan's law of radiation. A hot metal sphere at 1000 °C cools to 900 °C. If the temperature of the room containing the sphere is constant at 20 °C, compare the rate of cooling of the sphere at these two temperatures, assuming it acts as a blackbody radiator. Compare also the rates of cooling when the sphere reaches a temperature of 60 °C and 40 °C respectively.

[Ans: (i) 1:5:1 (ii) 2:2:1]

3. Define Stefan's constant and state its units. The temperature of a furnace is 1727 °C. Calculate the

heat radiated per cm^2 per minute by the furnace, assuming black-body radiation (Stefan's constant $\sigma = 5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$). [Ans: $5470\,J$]

4. The temperature of a furnace, which is not glowing, is 500 °C. Name the pyrometer you would use to determine its temperature, and draw a labelled sketch of its principal features. What other type of thermometer could also be used?

5. Explain the statement: Stefan's radiation constant is $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$. Sketch a curve showing the energy distribution in the spectrum of a black-body radiator at a particular temperature. How may the curve be used to obtain a value for Stefan's constant?

6. Estimate the filament temperature attained by a 75W electric lamp, given that Stefan's constant is $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$. The filament of the lamp has a surface area of 0.80 cm^2 and you may assume that it radiates as a black body. State one further assumption you make in your calculation.
Explain whether the actual temperature would be greater or less than your estimate because of each of these assumptions. [Ans: 2014 K]

7. Discuss the nature of the processes by which a hot body may lose heat to its surroundings.
A blackened platinum strip of area 0.20 cm^2 is placed at a distance of 200 cm from a white-hot iron sphere of diameter 1.0 cm, so that the radiation causes the temperature, and hence the resistance, of the platinum to increase. It is found that the same increase in resistance can be produced under similar conditions, but in the absence of radiation, when a current of 3.0 mA is passed through the platinum strip, the potential difference between its ends being 24 mV. Estimate the temperature of the iron sphere. (Stefan's constant = $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$).
[Ans: 1780 K]

8. What do you understand by a black body? In what respects does the radiation from a black body at 2000 K differ from that from a black body at 1000 K? How would you devise a black body to radiate at 1000 K? A blackened sphere, of radius 2.0 cm, is contained within a hollow evacuated enclosure, the walls of which are maintained at 27 °C. Assuming that the sphere radiates like a black body and that Stefan's constant is $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$, calculate the rate at which the sphere loses heat when its temperature is 227 °C.
[Ans: 15.6 W]

9. Explain what is meant by a black body. How does the total energy radiated by a black body and its distribution among the wavelengths in the spectrum depend upon the temperature of the radiator?
Describe the structure of an optical pyrometer and explain how it is used to measure the temperature of a furnace.

10. How can the temperature of a furnace be determined from observations on the radiation emitted?
Calculate the apparent temperature of the sun from the following information:
Sun's radius: $7.04 \times 10^5\, km$
Distance from earth: $14.72 \times 10^7\, km$
Solar constant: 014 watt per cm^2.
Stefan's constant: $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$
[Ans: 5450 °C]

11. Give an account of Stefan's law of radiation explaining the character of the radiating body to which it applies and how such a body can be experimentally realized.
If each square centimetre of the sun's surface radiates energy at the rate of $6.3 \times 10^3\, Js^{-1}\, cm^{-2}$ and Stefan's constant is $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$, calculate the temperature of the sun's surface in degrees Celsius, assuming Stefan's law applies to the radiation.
[Ans: 5490 °C]

12. What is black-body radiation?
Using the same axes sketch graphs, one in each instance, to illustrate the distribution of energy in the spectrum of radiation emanating from (a) a black body at 1000 K, (b) a black body at 2000 K and (c) a source other than a black body at 1000 K. Point out any special features of the graphs. Indicate briefly how the relative intensities needed to draw one of these graphs could be determined.

13. Explain what is meant by (a) a black body, (b) black-body radiation.
State Stefan's law and draw a diagram to show how the energy is distributed against wavelength in the spectrum of a black body for two different temperatures. Indicate which temperature is higher.
A roof measures 20 m × 50 m and is blackened. If the temperature of the sun's surface is 6000 K, Stefan's constant = $5.7 \times 10^{-8}\, Wm^{-2}K^{-4}$, the radius of the sun is 7.5×10^{10} cm, and the distance of the sun from the earth is 1.5×10^{13} cm, calculate how much solar energy is incident on the roof per minute, assuming that half is lost in

passing through the earth's atmosphere, the roof being normal to the sun's rays.
[Ans: $5.6 \times 10^{17} J$]

14. Explain what is meant by Stefan's constant, defining any symbols used.
A sphere of radius 2.0 cm with a black surface is cooled and then suspended in a large evacuated enclosure the black walls of which are maintained at 27 °C. If the rate of change of thermal energy of the sphere is $1.848 Js^{-1}$ when its temperature is -73 °C, calculate a value for Stefan's constant.
[Ans: $5.7 \times 10^{-8} Wm^{-2}K^{-4}$]

15. What do you understand by the term radiation of heat?
Describe an experiment to show that radiant heat (a) travels in straight lines (b) can be reflected by a polished metal surface (c) is emitted to a greater extent by a dull black surface than by a polished silver surface at the same temperature.

16. How would you compare experimentally the radiating powers of different surfaces?
What steps would you take to make the radiating power of a surface (a) as small as possible (b) as large as possible.

17. What is Prevost's Theory of Exchanges? Describe some phenomenon of theoretical or practical importance to which it applies.
A metal sphere of 1 cm diameter, whose surface acts as a black body, is placed at the focus of a concave mirror with aperture of diameter 60 cm directed towards the sun. If the solar radiation falling normally on the earth is at the rate of $0.14 W cm^{-2}$, Stefan's constant is $6 \times 10^{-8} Wm^{-2}K^{-4}$ and the mean temperature of the surroundings is 27 °C, calculate the maximum temperature which the sphere could theoretically attain, stating any assumptions you make.
[Ans: 2140 K]

18. Calculate the rate of loss of heat energy of a blackbody of area 40 m^2 at a temperature of 70°C if the radiation it receives from the sun is equivalent to a temperature in space of -200°C.
[Ans: 3.133×10^4W]

19. A metal sphere with a blackbody surface and radius 30 mm is cooled to -73°C and is placed inside an enclosure at a temperature of 27°C. Assuming that the density of metal is 800 kgm^{-3} and that its specific heat capacity is 400 kJ $kg^{-1} K^{-1}$, calculate the initial rate of temperature rise of the sphere. [Ans: 0. 115 Ks^{-1}]

20. A black hemisphere of radius 2 cm is contained in a hollow evacuated enclosure which is maintained at 27°C. Assuming that the sphere radiates likes a blackbody, calculate the rate at which the sphere loses heat when it is at 227°C. [Ans: 15.5 W]

21. The normal flux of radiant energy from the sun at the earth surface is 1360 Wm^{-2}. Calculate
(i) the total power emitted by the sun.
(ii) the temperature of the sun.
(iii) the rate of loss of mass by the sun.
[Ans: 3.845×10^{26} W, 5760.72 K, 4.27×10^9 kgs^{-1}]

22. A solid copper sphere of diameter 10 mm and temperature 150 K is placed in an enclosure maintained at a temperature of 290 K. Assuming that density of copper is 8.93×10^3 kgm^{-3} and that its specific heat capacity is 3.7×10^2 J $kg^{-1}K^{-1}$, calculate the initial rate of temperature rise of the sphere. [Ans: 0.069 K s^{-1}]

Greenhouse

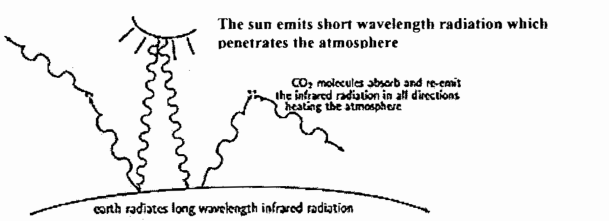
A green house is normally made out of glass or a black wire gauze net. Very hot bodies especially from the sun emit most of their radiation in form of light and infrared rays of short wavelength and high frequency which can easily penetrate glass without being absorbed.

The objects inside a greenhouse absorb this heat and in turn raise the temperature by convection and conduction processes. The warm air being enclosed in the greenhouse cannot escape through the glass.

Warm objects inside the green house also re-radiate this energy but due to their comparatively low temperature, they emit infrared rays of long wavelength and low frequency which cannot penetrate glass to escape. Hence the green house is warmer on the inside than outside.

Global warming and greenhouse effect

The sun emits radiation (light) over a range of wavelengths which are mainly in the visible part of the spectrum. Radiation at these wavelengths passes through the gases of the atmosphere to warm the land and the oceans below. The warm earth then radiates this heat at longer infrared wavelengths. Carbon-dioxide (one of the main greenhouse gases) in the atmosphere has energy levels which correspond to the infrared wavelengths which allow it to absorb the infrared radiation. It then also emits at infrared wavelengths in all directions. **This effect stops a large amount of the infrared radiation getting out of the atmosphere which causes the atmosphere and the earth to heat up**. More radiation is coming in than is getting back out.



Therefore, increasing the amount of greenhouse gases in the atmosphere increases the amount of trapped infrared radiation and therefore the overall temperature of the earth. The earth is a very sensitive and complicated system upon which life depends and changing the delicate balances of temperature and atmospheric gas content may have disastrous consequences.

Effects of global warming

- Leads to world climate changes and temperature rise (change in wind and weather patterns)
- Rise in sea average water levels leaving small islands and coastal areas at a greater risk

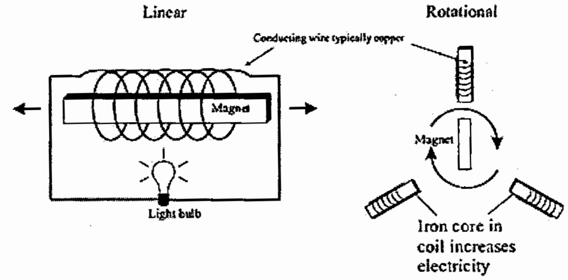
Energy and Electricity

Why do we need energy? On a broad scale it stimulates economic growth, etc. but on a personal level it allows us to lead a comfortable lifestyle e.g.

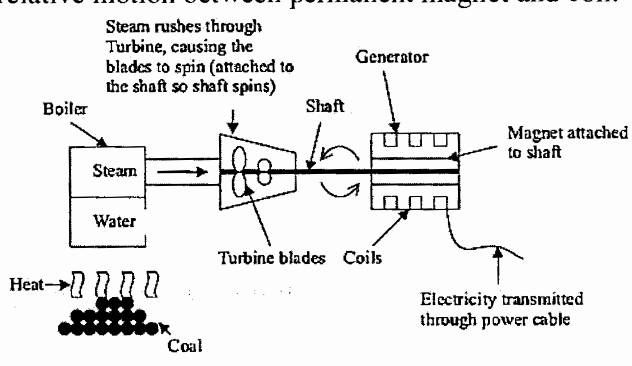
- Flick a switch and
- Heat for cooking
- Entertainment such as television and radio
- Heat for water and interior of house
- Ironing
- Electronic and electrical devices such as alarms, garage doors, etc.

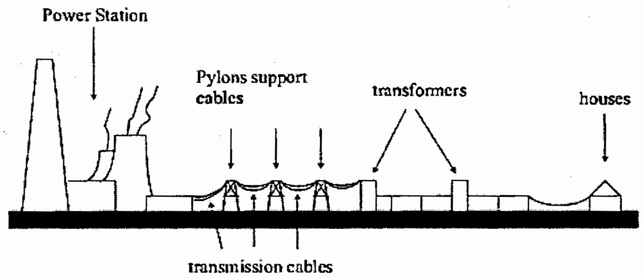
In a modern household, this energy is provided in the form of electricity which is powered via fossil fuels or nuclear.

How is electricity made? In a nutshell: By moving a magnet through or near a set of conducting coils.



Most power stations produce steam through heat (nuclear reaction or burning fossil fuels), the steam drives a turbine which moves a magnet relative to a coil (the generator - like the above but on a much larger scale i.e. bigger magnets, bigger coils, etc.) which produces electricity that is transmitted via a power network to our homes. Gas fired plants burn gas directly in a gas turbine to produce the same desired relative motion between permanent magnet and coil.





Fossil fuels

Coal, oil and gas are fossil fuels. Fossil fuels were created by decomposing organic (plant and animal) matter a long, long time ago and are typically found underground. Different temperatures and pressures resulted in the organic matter transforming into coal, oil or gas.

Shortcomings of fossil fuels

1. Fossil fuel power is bad news in the long run. It pollutes and contributes to the greenhouse effect (global warming resulting in melting polar ice caps, floods, droughts, diseases, etc.).
2. It is not going to last forever (nonrenewable)

Renewable energy

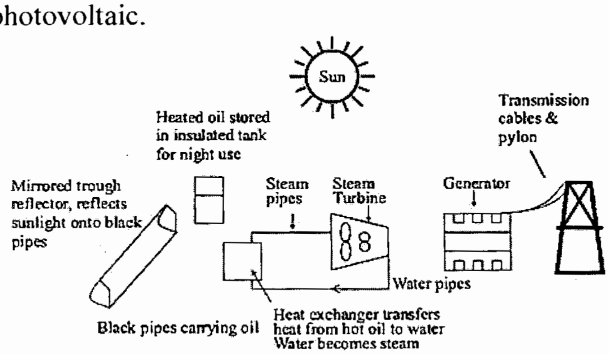
Renewable energy, as the name suggests it lasts 'forever'. Solar (sun), wind, geothermal, wave, hydro and biomass (organic) are all sources of energy that will last until the sun eventually explodes many millions of years from now.

Generally, the principal of renewable electricity generation is similar to fossil fuel electricity generation in that electricity is generated by moving a magnet relative to a conducting coil.

Renewable energy technologies

1. Solar

There are different types of solar electricity technologies, the main ones being solar thermal and photovoltaic.



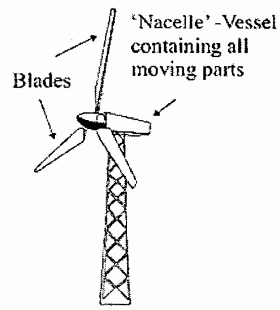
Solar thermal uses the heat of the sun to produce electricity. Sun is concentrated using mirrors. This heat either creates steam which drives a turbine which in turn drives a generator (as per fossil fuel generation), or drives an air engine (engine that uses expanding air to obtain motion) that drives a generator.

Photovoltaic panels convert sunlight directly into electricity. The benefit of photovoltaic panels is that there are no moving parts, and is therefore relatively maintenance free. They are however very expensive.

Solar water heaters could save up to 30% of the total electricity used in a house.

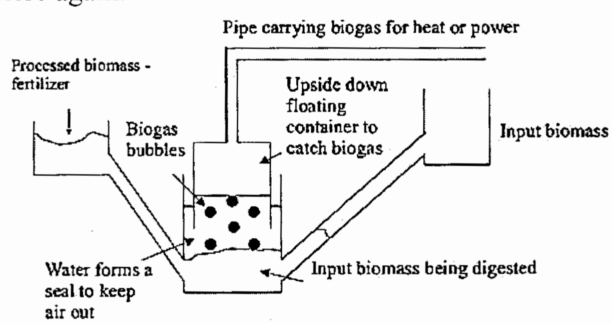
2. Wind

Wind turbines catch wind that spins the blades. The blades are connected to a shaft that spins because of the wind. This spinning shaft spins another shaft that turns a permanent magnet relative to conducting coils.



3. Biomass

Biomass is anything organic i.e. plant or animal matter. It can be used in the place of coal as per a normal coal fired plant and is renewable as long as the biomass e.g. wood; is handled in a sustainable manner. By sustainable I mean that suitable farming practices are used so that the land is not 'over farmed' which will result in the soil becoming barren and nothing growing there again.



Biomass can also be processed using anaerobic digestion to produce a gas that can be burned for heat or electricity. This biogas is made up of a number of other gases that are similar to those found in fossil fuel natural gas

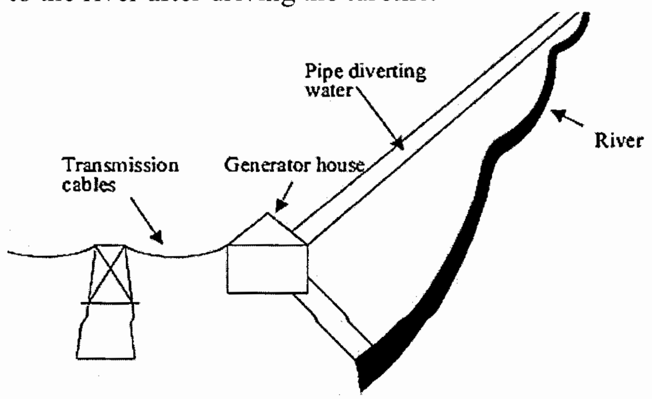
Anaerobic digestion: Anaerobic means 'no air'. Therefore, anaerobic digestion means to digest in the absence of air. Bacteria that naturally exist in organic matter will convert organic matter to biogas and fertilizer when all the air is removed.

Thousands of anaerobic digesters have been installed in rural areas where cow dung, human waste and chicken litter (faeces) are all processed using anaerobic digestion to produce gas that can be burned in the home for cooking and heating. The leftover is used as fertilizer.

4. Geothermal Energy

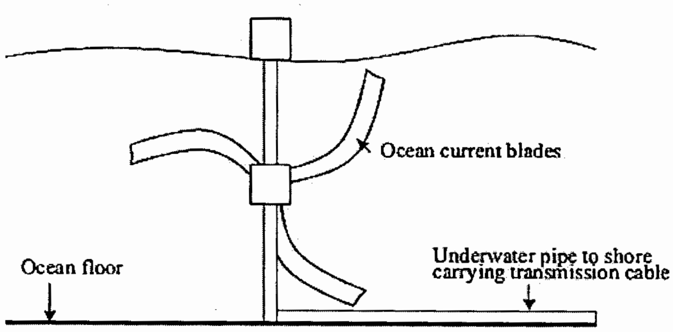
In some places on earth, the earth's crust is thinner than others. As a result, the heat from the earth's core escapes. The heat can be captured by converting water to steam, and using the steam to drive a steam generator as discussed above.

Hydroelectric power Water from a river is diverted to turn a water turbine to create electricity similar to the principles of steam generation. The water is returned to the river after driving the turbine.



5. Wave Energy

Some wave energy generators work similarly to wind turbines except that underwater ocean currents turns the blades instead of wind and most of the structure is under water



Note:

Liquid fuels are used mainly for transportation. Petrol and diesel are the most common liquid fuels and are obtained from oil.

However, as discussed above coal, gas and oil are fossil fuels and are not renewable. Petrol and diesel are obtained from fossil fuels and therefore pollute and contribute to the greenhouse effect (global warming).

Alternatives

1. Biodiesel

Oil can be extracted from plants such as the soya bean, sunflower and rapeseed by pressing it through a filter. This oil if mixed correctly with either methanol or dry ethanol and Sodium Hydroxide will separate the plant oil into biodiesel, glycerol and fertilizer.

The biodiesel can be used as produced in a conventional diesel engine with little or no modifications required.

The glycerol can be refined a bit further for pharmaceutical companies to use, or can be used to make soap.

2. Ethanol

Corn, maize and sugar cane (fibre) can be used to make ethanol as a fuel substitute for petrol. It is made by the same fermentation process used to make alcohol. Enzymes are often used to speed up the process.

In ethanol from sugar cane production, the leftover 'bagasse' (the fibre part of the sugar cane) can be burned in a biomass power station to produce electricity.

3. Hydrogen

Through the process of electrolysis, electricity (hopefully clean, renewable electricity) can split water into hydrogen and oxygen. The stored hydrogen can be used in a fuel cell to create electricity in a process that is opposite to electrolysis to drive electric motors in a car.

The hydrogen can also be burned directly in a modified internal combustion engine. In both cases the waste product is water.

Additional questions

1. (a) Define the specific heat capacity of a material

(b) It is required to determine the specific heat capacity of copper, using an electrical method.

(c) A block of material of mass 1.75 kg is heated by 120 W heater for 5.0 minutes. The block is completely lagged. The initial temperature of the block is 18.0 °C. The initial temperature of the block is $435\, J\, kg^{-1}\, °C^{-1}$.

(i) Calculate the final temperature of the block

(ii) What is the purpose of having the block completely lagged.

(d) The following data refer to a dishwasher

Power of heating element	2.5 kW
Time used to heat water	360 s
Mass of water used	3.0 kg
Initial temperature of water	20 °C
Final temperature of water	60 °C

Taking the specific heat capacity of water to be $4200\, J\, kg^{-1}\, K^{-1}$, calculate

(i) the energy provided by the heating element

(ii) the energy required to heat the water

(e) Give two reasons why your answers in part (d) differ from each other.

[Ans: (c)(i) 65.3 °C (d)(i) 0.90 MJ (ii) 0.504 MJ]

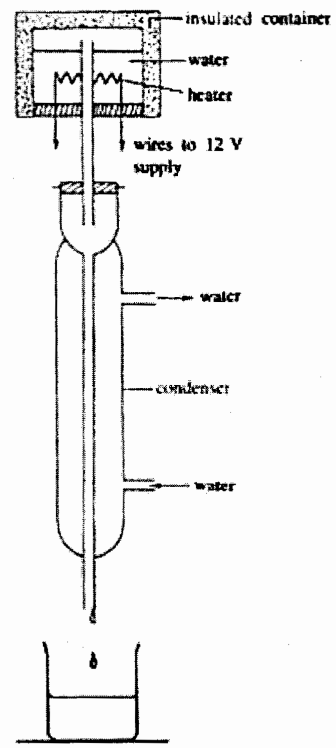
2. (a) Explain why the temperature of a liquid that is boiling does not increase even though heat is constantly supplied.

(b) Describe an electrical method for measuring the specific latent heat of vaporization of steam

(c) Ice cubes at a temperature of -10 °C and mass 50 g are put into thermos flask containing water at 0 °C. If the final temperature remains at 100 °C, what is the mass of water that has frozen? Steam at 100 °C is passed into the flask. What mass of steam is required to just melt ice?

(specific heat of ice = $2100\, J\, kg^{-1}\, °C^{-1}$, specific latent heat of fusion of ice = $3.34 \times 10^5\, J\, kg^{-1}$, specific latent heat of vaporization of water = $2.26 \times 10^6\, J\, kg^{-1}$)

3. The figure below shows an apparatus used to measure the specific latent heat of vaporization of water. The electric heater is labelled 12 V/ 50 W. When the water is boiling, the steam produced goes into the condenser through the vertical tube. The condensed water is collected in a beaker. When conditions are steady, 2.0 g of condensed water are collected in 100 s.



(a) What is meant by "labelled 12 V/50 W?

(b) Why is it necessary to wait until steady conditions are attained before collecting the condensed water?

(c) Calculate the specific latent heat of vaporization of water from the results of the experiment. Discuss whether the value obtained is bigger or smaller than the actual value.

(d) To obtain a more accurate value for the specific latent heat, the experiment is repeated using lower power input. The voltage is reduced to 10.5 V and the current through the heater is 3.7 A. When steady conditions are attained, 1.5 g of water are collected in 100 s. Use both sets of results to obtain a more accurate value for the specific latent heat of vaporization of water.

4. Describe the structure of a constant-volume gas thermometer.

Compare this thermometer as a means of measuring temperature with (a) a mercury in-glass thermometer, (b) a thermoelectric thermometer

5. Give an account of a method of determining the specific latent heat of evaporation of water, pointing out the ways in which the method you describe achieves, or fails to achieve, high accuracy.

A 600-watt electric heater is used to raise the temperature of a certain mass of water from room temperature to 80 °C. Alternatively, by passing steam from a boiler into the same initial mass of water at the same initial temperature, the same temperature rise is obtained in the same time. If 16 g of water were being evaporated every minute in the boiler, find the specific latent heat of steam, assuming that there were no heat losses. [Ans: 2230 $kJ\ kg^{-1}$]

6. Explain how the kinetic theory of gases accounts for the pressure of a gas. Obtain an expression for the pressure of a perfect gas, stating clearly what assumptions are made.

7. Discuss the nature of the processes by which a hot body may lose heat to its surroundings.

8. Distinguish between reversible isothermal and adiabatic changes. An ideal gas is compressed isothermally until its volume is reduced to one-tenth of its initial value, and is then allowed to expand adiabatically to its original volume. Finally, the pressure is altered at constant volume until the original state is restored. Represent these changes on a pressure-volume diagram and state whether, on the whole, work has been done on or by the gas. If the initial pressure is 76 cmHg, what is the change of pressure in the last stage of the process? Assume that γ = 1.40. [on the gas, 46 cmHg]

9. Compare the properties of saturated and unsaturated vapours. By means of diagrams show how the pressure of (a) a gas, and (b) a vapour, vary with change (i) of volume at constant temperature, and (ii) of temperature at constant volume.

The saturation vapour pressure of ether vapour at 0 °C is 185 mm mercury and at 20 °C is 440 mm. The bulb of a constant-volume gas thermometer contains dry air and sufficient ether for saturation. If the observed pressure in the bulb is 1000 mm at 20 °C, what will it be at 0 °C? [Ans: 707 mmHg]

10. Describe how you would determine the thermal conductivity of a good conductor.

The ends of a bar are maintained respectively at 0 °C and 100 °C. Discuss, with the help of diagrams, the temperature distribution along the bar (a) when it is well lagged, (b) when there is a considerable escape of heat from its side faces.

A copper bar of diameter 1.5 cm has an electric heating coil inserted half-way along its length. The bar is well lagged, except at the ends, which are exposed to the air. When the power supplied to the heater under equilibrium conditions is 12 watts, the steady temperature gradient on each side is 1 °C/cm. Calculate the thermal conductivity of copper.

State briefly the factors that determine the steady temperatures of the exposed faces.

[Ans: 340 $Wm^{-1}K^{-1}$]

11. Indicate how the concepts of a black body and of black-body radiation are derived from Prevost's theory of exchanges. Draw curves to illustrate how the energy radiated at different wavelengths varies with the black-body temperature.

The cathode of a diode valve consists of a cylinder 2.0 cm long and 0.10 cm diameter, and is surrounded by a coaxial anode of diameter large compared with that of the cathode. The anode remains at 127 °C when 4 watts is dissipated in heating the cathode. Estimate the temperature of the cathode. List the assumptions you have made in arriving at your estimate. Discuss whether the estimate is realistic. (Stefan's constant = $5.74 \times 10^{-8}\ Wm^{-2}K^{-4}$.) [Ans: 1030 K]

12. State what is meant by a temperature on the centigrade (Celsius) scale of a platinum resistance thermometer.

Point out the relative merits of (a) a platinum resistance thermometer, and (b) a thermoelectric thermometer for measuring (i) the rise in temperature of the water flowing through a continuous-flow calorimeter, and (ii) the temperature of a small crystal as it is being heated rapidly.

13. What do you understand by the specific heat capacity of a substance? Describe how you would measure the specific heat capacity of a sample of rock, describing the precautions that you would take to obtain an accurate result.

A room is heated during the day by a 1 kW electric fire. The fire is to be replaced by an electric storage heater consisting of a cube of concrete which is heated over night and is allowed to cool during the day, giving up its heat to the room. Estimate the length of an edge of the cube if the heat it gives out in cooling from 70 °C to 30 °C is the same as that given out by the electric fire in 8 hours.

[Density of concrete = 2700 kg m^{-3}; specific heat capacity of concrete = 0.85 $kJ\ kg^{-1}\ K^{-1}$]

[Ans: 0.68 m]

14. State the laws of gases usually associated with the names of Boyle, Charles, Dalton, and Graham. Two gas containers with volumes of 100 cm^3 and 1000 cm^3 respectively are connected by a tube of negligible volume, and contain air at a pressure of 1000 mm of mercury. If the temperature of both vessels is originally 0 °C, how much air will pass through the connecting tube when the temperature of the smaller is raised to 100 °C? Give your answer in cm^3 measured at 0 °C and 760 mm of mercury. [Ans: 33 cm^3]

15. State the factors which determine the rate at which a body loses heat by radiation. Discuss one method by which the temperature of a radiating body can be determined, and mention the assumptions involved.
A flat surface disc 2 cm thick is held so that the sun's radiation falls normally upon one face. Given that Stefan's constant is $5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$, the radius of the sun is 7×10^{5} km, the mean distance from earth to sun is $1.5 \times 10^{8}\ km$, and assuming that the sun radiates as a black body at a temperature of 6000 K, calculate in watts per cm^2, the rate of incidence of solar energy on the disc.
If the steady temperatures reached by the two faces of the disc are 110 °C and 86 °C while the surroundings are at 7 °C, calculate the thermal conductivity of the material of the disc on the supposition that the surface facing the sun can be treated as a perfectly black body and that no heat escapes from the edge.

[Ans: $0.16\ W\ cm^{-2}, 0.61\ Wm^{-2}K^{-4}$]

16. "The two specific heat capacities in kJ units for argon are 0.521 and 0.313 and for air are 1.012 and 0.722." Explain these statements and discuss their significance in relation to (a) the atomicity of the molecules of the two gases, (b) the relative values of the adiabatic elasticities of argon and air.

17. Without deriving any formulae, use the kinetic theory of gases to explain (a) how a gas exerts a pressure, (b) why the temperature of a gas rises when the gas is compressed, (c) what happens when a quantity of liquid is introduced into a closed vessel. How are the differences in the behaviour of real and ideal gases explained by the kinetic theory? If there are 2.7×10^{19} molecules in a cubic centimetre of gas at 0 °C and 760 mm mercury pressure, what is the number per cubic centimetre (i) at 0 °C and 10.6 mm pressure, (ii) at 39 °C and 10^{-6} mm pressure?

[Ans: (i) 3.6×10^{11} (ii) 3.1×10^{11}]

18. Define coefficient of thermal conductivity, and describe how you would determine the conductivity of a bad conductor, such as cork.
A thin-walled metal hot-water tank, of effective surface area $6\ m^2$, is covered with a layer of insulation 3 cm thick. The coefficient of thermal conductivity of the insulating material is $10^{-1}\ Wm^{-1}K^{-1}$. Find the power which must be supplied by an electric immersion heater to maintain the water at 60 °C when the outer face of the insulation is at 30 °C.
Taking the air temperature to be constant at 15 °C, and assuming that the temperature difference between the outer face of the lagging and the air is proportional to the power supplied, find the temperature of the water in the tank when the power supply is raised to 867 watts. [Ans: $400\ W, 80°C$]

19. Discuss the nature of the processes by which a hot body may lose heat to the surroundings.
A blackened strip of area 0.20 cm^2 is placed at a distance of 200 cm from a white-hot iron sphere of diameter 1.0 cm, so that the radiation falls normally on the strip. The radiation causes the temperature, and hence the resistance, of the platinum to increase. It is found that the same increase in resistance can be produced under similar conditions, but in the absence of radiation, when a current of 3.0 mA is passed through the platinum strip, the potential difference between its ends being 24 mV. Estimate the temperature of the iron sphere. (Stefan's constant = $5.7 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$) [Ans: $1780\ K$]

20. Three types of thermometer in common use are based on (a) the expansion of a fluid, (b) the production of an electromotive force, (c) the variation of electrical resistance. Describe briefly one example of each and the way in which it is used. In each case, state how a value of the temperature on a centigrade scale is deduced from the quantities actually measured.
If all three of the thermometers you have described were used to measure the temperature of the same object, would they give the same result? Give reasons for your answer.

21.3 g of a certain metal of density about 10 g cm^{-3} is available in the form of a coarse powder, together with a calorimeter of heat capacity about $33.6\ J\ K^{-1}$ and volume about 160 cm^3, and a 50 °C thermometer reading to 3 deg.
Using this and other necessary apparatus, how would you verify, by the method of mixtures, that the specific heat of the metal is $0.13\ kJ\ kg^{-1}\ K^{-1}$?
In the experiment you describe why is it (a) unnecessary to apply a correction for heat exchange with the surroundings, (b) necessary to decide on a suitable maximum temperature of the mixture? How would you ensure that such a temperature is realized?

22. The bulb of a simple constant-volume air thermometer has a volume of 120 cm^3, and it is connected to the manometer by tubing of volume 10 cm^3, which remains at room temperature (15 °C) throughout an experiment. When the bulb is immersed in an ice-water mixture at 0 °C, the gas in the bulb is at a pressure of 880 mm mercury.
To what pressure will it be subjected when the bulb is immersed in steam at 100 °C? [Ans: $1170\ mmHg$]

23. Distinguish between an isothermal and an adiabatic compression of a gas. Explain the precautions necessary to ensure that an actual compression approximates to each of these conditions.
Sketch curves to show the relationship between pressure and volume, determined under isothermal conditions at a number of temperatures between about 10 °C and 40 °C for (a) carbon dioxide, (b) hydrogen. Explain the form of the curves.

24. Define the coefficient of thermal conductivity. Describe a method of measuring the thermal conductivity of a solid which is a bad conductor of heat, and explain carefully why it is not possible to employ the same method as that used for a good conductor.
A pond is 40 cm deep. If the air temperature above the water is −5.0 °C and the temperature of the water at the bottom of the pond is maintained at 4.0 °C, find the thickness of the ice that will eventually be formed, (Thermal conductivity of ice = $2.3\ W\,m^{-1}\,K^{-1}$ of water = $0.56\ W\,m^{-1}\,K^{-1}$) [Ans: $33.5\ cm$]

25. What do you understand by an ideal gas? Discuss the extent to which the behaviour of (a) real gases, and (b) saturated vapours can be represented by relations derived for an ideal gas.
Describe an experiment to investigate the relation between the pressure and temperature of a sample of air maintained at constant volume over the temperature range from 0 °C to 100 °C.

26. Explain what is meant by the coefficient of thermal conductivity of a substance, and describe an experiment to determine this quantity for copper.
A copper kettle has a circular base of radius 10 cm and thickness 3.0 mm. The upper surface of the base is covered with a uniform layer of scale 1.0 mm thick. The kettle contains water which is brought to the boil over an electric heater. In the steady state 5.0 g of steam is produced each minute. What is the temperature of the lower surface of the base, assuming that conduction of heat up the sides of the kettle can be neglected? (Values of thermal conductivity: copper, 382 W m^{-1} K^{-1}; scale, 1.34 W m^{-1} K^{-1}. Specific latent heat of vaporization of water = 2268 kJ kg^{-1}.)
[Ans: 104.5 °C]

27. Give an account of the transmission of heat energy by radiation.
Explain how the distribution of energy between the various wavelengths in the spectrum of a black body, and also the total radiation it emits per unit area per second, depend on the absolute temperature.
A heating panel of area 2.5 m^2 is supplied with energy at a steady rate of 1.5 kilowatt. Calculate the surface temperature of the panel when the surroundings are at 20 °C, assuming that heat exchange takes place only by radiation and that both the panel and the surroundings behave as perfectly black bodies. (Take Stefan's constant to be $5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$)
[Ans: 93 °C]

28. Explain the principle of a constant-volume gas thermometer and describe a simple instrument suitable for measurements in the range 0° to 100 °C. What factors determine (a) the sensitivity and (b) the accuracy of the instrument you describe?
A certain gas thermometer has a bulb of volume 50 cm^3 connected by a capillary tube of negligible volume to a pressure gauge of volume 5.0 cm^3. When the bulb is immersed in a mixture of ice and water at 0 °C, with the pressure gauge at room temperature (17 °C), the gas pressure is 700 mmHg. What will be the pressure when the bulb is raised to a temperature of 50 °C if the gauge is maintained at room temperature? You may assume that the gas is ideal and that the expansion of the bulb can be neglected. [Ans: 815 mmHg]

29. Give an account of an electrical method of finding the specific latent heat of vaporization of a liquid boiling at about 60 °C. Point out any causes of inaccuracy and explain how to reduce their effect.
Ice at 0 °C is added to 200 g of water initially at 70 °C in a vacuum flask. When 50 g of ice has been added and has all melted, the temperature of the flask and contents is 40 °C. When a further 80 g of ice has been added and has all melted, the temperature of the whole becomes 10 °C. Calculate the specific latent heat of fusion of ice, neglecting any heat lost to the surroundings.
In the above experiment the flask is well shaken before taking each temperature reading. Why is this necessary? [Ans: $378\ kJ\ kg^{-1}$]

30. What are the main assumptions of the kinetic theory of gases? Explain fully how the ideal gas equation $pV = RT$ is derived from the kinetic theory.
The density of air is 1.3 kg m^{-3} at 0 °C and 10^5 N m^{-2} pressure. Calculate the r.m.s. velocity of air molecules at 27 °C and 10^5 N m^{-2} pressure. [Ans $500\ ms^{-1}$]

31. Explain what is meant by a reversible adiabatic change. Use the concepts of simple kinetic theory to explain why the temperature of a gas enclosed in a cylinder by a piston will rise while the piston moves so as to reduce the volume.

Examination past paper questions

1. (a) (i) State the thermometric property used in the constant-volume gas thermometer
   (ii) Give **two** characteristics of a good thermometric property
   (b) (i) Describe the steps taken to set up a Celcius scale of temperature for a mercury-in-glass thermometer
   (ii) State **four** disadvantages of mercury-in-glass thermometer
   (c) Describe with the aid of a labelled diagram of an optical pyrometer
   (d) When oxygen is withdrawn from a tank of volume 50 l, the reading of a pressure gauge attached to the tank drops from $2.14 \times 10^5\ Pa$ to $7.8 \times 10^5\ Pa$. If the temperature of a gas remaining in the tank falls from 30°C to 10°C, calculate the mass of oxygen withdrawn.
   [2017, No. 5]
2. (a) (i) What is meant by **boiling point**?
   (ii) Explain why boiling point of a liquid increases with increase in the external pressure
   (b) Explain how the pressure of a fixed mass of a gas can be increased at
   - constant temperature
   - constant volume

   (c) (i) Sketch a pressure versus volume curve for a real gas undergoing compression
   (ii) Explain the main features of the curve in (c)(i) above
   (d) The cylinder of an exhaust pump has a volume of 25 cm^3. If it is connected through a valve to a flask of volume 225 cm^3 containing air at a pressure of 75 $cmHg$, calculate the pressure of the air in the flask after two strokes of the pump, assuming that the temperature of the air remains constant
   [2017, No.6]
3. (a) (i) Define **thermal conductivity**
   (ii) Explain the mechanism of heat transfer by convection
   (b) (i) State **Newton's law of cooling**
   (ii) Describe briefly an experiment to verify Newton's law of cooling
   (c) A wall is constructed using two types of bricks. The temperatures of the inner and outer surfaces of the wall are 29°C and 21°C respectively. The value of the thermal conductivity for the inner brick is $0.4\ Wm^{-1}K^{-1}$ and that of the outer brick is $0.8\ Wm^{-1}K^{-1}$
   (i) Explain why in a steady state the rate of thermal energy transfer is the same in both layers
   (ii) If each layer is 12.0 cm thick, find the temperature at the interface between the layers.
   (d) Explain the greenhouse effect and how it leads to rise of earth temperatures.
   [2017, No. 7]
4. (a) (i) Define **specific latent heat of fusion**
   (ii) State the effect of impurities on melting point
   (b) Explain why there is no change in temperature when a substance is melting
   (c) With the aid of a labelled, describe the continuous flow method of measuring the specific heat capacity of a liquid
   (d) In an experiment to determine the specific latent heat of fusion of ice, a heating coil is placed in a filter funnel and surrounded by lumps of ice. The following sets of readings were obtained.

$V(V)$	4.0	6.0
$I(A)$	2.0	3.0
Mass of water m(g) collected in 500 s	14.9	29.8

   Calculate the;
   (i) specific latent heat of fusion of ice
   (ii) energy gained in the course of obtaining the first set of readings
   (e) Why are two sets of readings necessary in (d) above?
   [2016, No. 5; Ans: (d)(i) $3.36 \times 10^5\ Jkg^{-1}$ (ii)1 kJ]
5. (a) (i) State **Dalton's law of partial pressures**
   (ii) Using the expression $p = \frac{1}{3}\rho\bar{c}^2$, where p is the pressure of a gas density ρ and mean square speed $\bar{c}^2$, derive Dalton's law of partial pressures
   (b) (i) What is meant by **isothermal** process and **adiabatic** process
   (ii) Explain why adiabatic expansion of a gas causes cooling
   (c) A gas at a temperature of 17°C and pressure $1.0 \times 10^5\ Pa$ is compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume
   (i) Sketch on a P-V curve the above processes

(ii) If the specific heat capacity at constant pressure is $2100\,J\,mol^{-1}K^{-1}$ and at constant volume is $1500\,J\,mol^{-1}K^{-1}$, find the final temperature of the gas

(d) (i) What is meant by a **saturated vapour**?

(ii) Explain briefly the effect of altitude on the boiling point of the liquid.

[2016, No. 6; Ans: (e) (ii) $219.8\,K$]

6. (a) (i) Define a **black body**

(ii) Sketch and explain graphs of intensity versus wavelength for three different temperatures of a black body

(b) Describe with the aid of a labelled diagram how an optical radiation pyrometer is used to measure temperature

(c) (i) State **Prevost's theory** of heat exchanges

(ii)A metal sphere of radius $1.5\,cm$ is suspended within an evacuated enclosure whose walls are at $320\,K$. The emissivity of the metal is 0.40. Find the power input required to maintain the sphere at a temperature of $320\,K$, if heat conduction along the supports is negligible

(d) A metal boiler is $1.5\,cm$ thick. Find the difference in temperature between the inner and outer surfaces if $40\,kg$ of water evaporate from the boiler per metre squared per hour. (Latent heat of vaporization of water $= 2268\,kJ\,kg^{-1}$, Thermal conductivity of the metal of the boiler $= 63\,Wm^{-1}\,K^{-1}$)

[2016, No. 7; Ans: (d) (i) 0 (ii) 6.0 K]

7. (a) (i) State four desirable properties a material must have to be used as a thermometric substance

(ii) State why scales of temperature based on different thermometric properties may not agree

(b) With the aid of a diagram, explain how a bolometer is used to detect thermal radiation

(c) Describe, with the aid of a diagram an experiment to determine specific latent heat of steam using the method of mixtures

(d) A 600 W electric heater is used to raise the temperature of a certain mass of water in a thermos flask from room temperature to 80°C. The same temperature rise is obtained when steam from a boiler is passed into an equal mass of water at room temperature in the same time. If 16 g of water were being evaporated every minute in the boiler, find the specific latent heat of vaporization of steam, assuming no heat losses.

[2015, No. 5; Ans: (d) $2.25 \times 10^6 Jkg^{-1}$]

8. (a) Define the following

(i) absolute zero

(ii) cooling correction

(b) (i) State Dalton's law of partial pressures

(ii) The kinetic theory expression for the pressure p of an ideal gas of density ρ and mean square speed $\bar{c}^2$ is $p = \frac{1}{3}\rho\bar{c}^2$. Use the expression to deduce Dalton's law.

(b) Explain clearly the steps taken to determine the cooling correction when measuring the specific heat capacity of a poor conductor by the method of mixtures

(c) The density of air at 0°C and pressure 101 kPa is $1.29\,kgm^{-3}$. Calculate pressure of 200 kPa.

[2015, No. 6]

9. (a) Define thermal conductivity of a material and state its unit.

(b) Describe an experiment to determine the thermal conductivity of copper

(c) A double glazed window has two glass sheets each of thickness 4.0 mm, separated by a layer of air of thickness 1.5 mm. If the two inner air-glass surfaces have steady temperatures of 20°C and 4°C respectively, find the

(i) temperatures of the outer air glass surfaces

(ii) amount of heat that flows across an area of the window of $2\,m^2$ in 2 hours.

(conductivity of glass $= 0.72\,Wm^{-1}K^{-1}$ and that of air $= 0.025\,Wm^{-1}K^{-1}$)

(d) (i) What is a black body?

(ii) Explain how a welder can prevent can protect the eyes from the damage

(iii) Calculate the wavelength of the radiation emitted by a black body at 6000 K (Wien's displacement constant $= 2.9 \times 10^{-3} mK$)

[2015, No. 7; Ans: (c)(i) 21.48°C, 2.52°C(ii) $3.84 \times 10^6\,J$ (d)(iii) $4.83 \times 10^{-7} m$]

10. (a)(i) State two differences between saturated and unsaturated vapours

(ii) Sketch graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0°C.

(b) The specific heat capacity of oxygen at constant volume is $719\,Jkg^{-1}K^{-1}$ and its density at standard temperature and pressure is

1.429 kgm^{-3}. Calculate the specific heat capacity of oxygen at constant pressure.

(c) (i) With the aid of a labelled diagram, describe an experiment to determine the saturated vapour of water.

(ii) State how the experimental setup in (c)(i) may be modified to determine the saturated vapour pressure above atmospheric pressure.

(d) (i) Define an ideal gas

(ii) State and explain the conditions under which real gases behave as ideal gases.

[2014, No. 5; Ans: (b) $978.67\, Jkg^{-1}K^{-1}$]

11. (a)(i) What is a black body?

(ii) Explain with the aid of a diagram how a black body may be approximated

(iii) With the aid of sketch graphs, explain the salient features of the spectral distribution of a black body radiation.

(b) Give four properties of ultraviolet radiations

(c) Describe an experiment to compare the energy radiated by two surfaces of different nature

(d) (i) State Stefan's law

(ii) The earth receives energy from the sun at a rate of $1.4 \times 10^3 Wm^{-2}$. If the earth's orbit to the sun's radius is 216, calculate the surface temperature of the sun.

[2014, No. 6; Ans: (d)(ii) 5825.9 K]

12. (a) Define specific latent heat of vaporization

(b) With the aid of a labelled diagram, describe an experiment to measure the specific latent heat of vaporization of a liquid using an electrical method

(c) Explain the effect of pressure on boiling point of a liquid

(d) A liquid of specific heat capacity $2.8 \times 10^3\, Jkg^{-1}K^{-1}$ and latent heat of vaporization $9.0 \times 10^5\, JK^{-1}$ is contained in a flask of heat capacity $800\, JK^{-1}$ at a temperature of 32°C. An electric heater rated 1 kW is immersed in 2.5 kg of the liquid and switched on for 12 minutes. Calculate the amount of liquid that boils off given that the boiling point of the liquid is 80°C.

(e) (i) Two thermometers are used to measure the temperature of a body. Explain why the temperature values may be different.

(ii) A platinum resistance thermometer has a resistance of 5.42 Ω at triple point of water. Calculate the resistance at a temperature of 50.0°C.

[2014, No.7; Ans: (d) 0.3841 kg (e)(ii) 6.41 Ω]

13. (a) Define

(i) specific heat capacity

(ii) specific latent heat of vaporization of a liquid

(b) With the aid of a labelled diagram, describe an electrical method of determining the specific heat capacity of a solid.

(c) An electrical heater rated 48 W, 12 V is placed in a well-insulated metal of mass 1.0 kg at a temperature of 18°C. When the power is switched on for 5 minutes, the temperature of the metal rises to 34°C. Find the specific heat capacity of the metal.

(d) (i) State Newton's law of cooling

(ii) Use the Newton's law of cooling to show that $\frac{d\theta}{dt} = -k(\theta - \theta_R)$ where $\frac{d\theta}{dt}$ is the rate of fall of temperature and θ_R is the temperature of the surroundings.

(e) Explain why evaporation causes cooling

[2013, No. 5; Ans: (c) $900\, Jkg^{-1}K^{-1}$]

14. (a) The pressure, p of an ideal gas is given by $p = \frac{1}{3}\rho\bar{c}^2$ where ρ is the density of the gas and $\bar{c}^2$ is the mean square speed.

(i) Show clearly the steps taken to derive this expression

(ii) State the assumptions made in deriving this expression

(b) Sketch the pressure versus volume curve for a real gas for a temperature above and below the critical temperature.

(c) For one mole of a real gas, the equation of state is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Explain the significance of the terms $\frac{a}{V^2}$ and b.

(d) A balloon of volume $5.5 \times 10^{-2}\, m^3$ is filled with helium to a pressure of $1.10 \times 10^5\, Nm^{-2}$ at a temperature of 20°C. Calculate the

(i) number of helium atoms in the balloon

(ii) net force acting on the square metre of material of the balloon if the atmospheric pressure is $1.01 \times 10^5 Nm^{-2}$.

[2013, No.6; Ans: (d)(i) 1.49×10^{24} (ii) $9.0 \times 10^3 N$]

15. (a)(i) Define thermal conductivity of a material

(ii) Describe an experiment to determine the thermal conductivity of copper

(b) (i) What is meant by a black body?

(ii) Describe how infrared radiation can be detected using a bolometer

(iii) Give one characteristic property of infrared radiation

(c) (i) A spherical black body of radius 2.0 cm at $-73°C$ is suspended in an evacuated enclosure whose walls are maintained at 27°C. If the rate of exchange of thermal energy is equal to $1.85\,Js^{-1}$, find the value of Stefan's constant, σ

(ii) Calculate the wavelength at which the radiation emitted by the enclosure has maximum intensity.

[2013, No.7; Ans: (c)(i) $5.66 \times 10^{-8} Wm^{-2}K^{-4}$ (ii) $9.7 \times 10^{-6} m$]

16. (a)(i) Define the terms specific heat capacity and specific latent heat of fusion

(ii) Explain the changes that take place in the molecular structure of substances during fusion and vaporization

(b) With the aid of a labelled diagram, describe an experiment to determine the specific heat capacity of a liquid using the continuous flow method

(c) Steam at 100°C is passed into a copper calorimeter of mass 150 g containing 340 g of water at 15°C. This is done until the temperature of the calorimeter and its contents is 71°C. If the mass of the calorimeter and its contents is found to be 525 g, calculate the specific latent heat of vaporization of water.

[2012, No. 5; Ans: (c) $2.259 \times 10^{6} Jkg^{-1}$]

17. (a)(i) Define saturated vapour pressure

(ii) Describe with the aid of a diagram how saturated vapour pressure of a liquid can be determined at a given temperature.

(b) Use the kinetic theory to explain the following observations

(i) Saturated vapour pressure of a liquid increases with temperature

(ii) Saturated vapour pressure is not affected by a decrease in volume at constant pressure

(c) When hydrogen gas is collected over water, the pressures in the tube at 15°C and 75°C are 65.5 cm and 105.6 cm of mercury respectively. If the saturated vapour pressure at 15°C is 14.2 cm of mercury, find its value at 75°C

(d) Explain why the molar heat capacity of an ideal gas at constant pressure differs from the molar heat capacity at constant volume.

[2012, No. 6; Ans: (c) 28.17 cmHg]

18. (a)(i) Define thermal conductivity

(ii) Compare the mechanisms of heat transfer in poor and good solid conductors

(b) Describe, with the aid of a diagram, how you would measure the thermal conductivity of a poor conductor, stating the necessary precautions

(c) A cylindrical iron vessel with a base diameter 15 cm and thickness 0.30 cm has its base coated with a thin film of soot of thickness 0.10 cm. It is then filled with water at 100°C and placed on a large block of ice at 0°C. Calculate the initial rate at which the ice will melt. (Thermal conductivity of soot = $0.12\,Wm^{-1}K^{-1}$)

[2012, No.7; Ans: (c) $6.44 \times 10^{-4}\,kgs^{-1}$]

19. (a)(i) State Boyle's law

(ii) Describe an experiment to verify Boyle's law

(iii) Explain why the pressure of a fixed mass of a gas rises if its temperature increases

(b) (i) Define the term thermometric property

(ii) State two qualities of a good thermometric property

(c) (i) With reference to a liquid in glass thermometer, describe the steps involved in setting up a Kelvin scale of temperature.

(ii) State one advantage and one disadvantage of the resistance thermometer

(d) A resistance thermometer has a resistance of 21.42 Ω at the ice point, 21.90 Ω at the steam point and 28.11 Ω at some unknown temperature θ. Calculate θ on the scale of this thermometer.

[2011, No. 5; Ans: (d) 87.11°C]

20. (a) Define specific heat capacity of a substance and state its unit

(b) (i) Describe how specific heat capacity of a liquid can be obtained by the continuous flow method

(ii) State one disadvantage of this method

(c) An electric kettle rated 1000 W, 240V is used on 220 V mains to boil 0.52 kg of water. If the heat capacity of the kettle is $400\,JK^{-1}$ and the initial temperature of the water is 20°C, how long will the water take to boil?

(d) (i) Distinguish between isothermal and adiabatic change

(ii) An ideal gas at 18°C is compressed adiabatically until the volume is halved. Calculate the final temperature of the gas. (Assume specific heat capacities of the gas at constant pressure and volume are $2100\,Jkg^{-1}K^{-1}$ and $1500\,Jkg^{-1}K^{-1}$ respectively)

[2011, No. 5; Ans: (c) 246 s (d)(ii) 383.98 K]

21. (a) State Stefan's law of black body radiation
(b) Briefly describe how a thermocouple can be used to detect thermal radiation
(c) Explain the temperature distribution along
(i) a perfectly lagged metal bar
(ii) an unlagged metal bar
(d) The wall of a furnace is constructed with two layers. The inner layer is made of bricks of thickness 10.0 cm and thermal conductivity $0.8\,Wm^{-1}K^{-1}$ and the outer layer is made of material of thickness 10.0 cm and thermal conductivity $1.6\,Wm^{-1}K^{-1}$. The temperatures of the inner and outer surfaces are 600°C and 460°C respectively.
(i) Explain why in steady state, the rate of thermal energy transfer must be the same in both layers
(ii) Calculate the rate of heat flow per square metre through the wall
(e) Explain the greenhouse effect and how it is related to global warming

[2011, No.7; Ans: (d)(ii) 746.4 Wm^{-2}]

22. (a)(i) Define the terms specific heat capacity, internal energy and state their units
(ii) Why is the distribution between the specific heat capacity at constant pressure and that at constant volume important for gases but less important for solids and liquids
(b) Explain why the temperature of a liquid does not change when the liquid is boiling
(c) One kilogram of water is converted to steam at a temperature of 100°C and a pressure of $1.0 \times 10^5 Pa$. If the density of steam is $0.58\,kgm^{-3}$ and the specific latent heat of vaporization of water is $2.3 \times 10^6\,Jkg^{-1}$, calculate the
(i) external work done
(ii) internal energy
(d) Explain why the specific latent heat of fusion and specific latent heat of vaporization of a substance at the same pressure are different.

[2010, No. 5; Ans: (c)(i) $1.723 \times 10^5\,J$ (ii) 2.128×10^6]

23. (a)(i) State the difference between isothermal and adiabatic expansion of a gas
(ii) Using the same axes and points, sketch the graphs of pressure versus volume for a fixed mass of a gas undergoing isothermal and adiabatic change.
(b) Show that the work, W, done by a gas which expands reversibly from V_0 to V_1 is given by $W = \int_{V_1}^{V_2} PdV$
(c) (i) State two differences between real and ideal gases
(ii) Draw a labelled diagram showing $P - V$ isothermals for a real gas above and below critical temperature.
(d) Ten moles of a gas, initially at 27°C are heated at a constant pressure $1.0 \times 10^5 Pa$ and the volume increased from $0.250\,m^3$ to $0.375\,m^3$. Calculate the increase in the internal energy (Assume: $c_p = 28.5\,Jmol^{-1}K^{-1}$)

[2010, No. 6; Ans: (d) $3.013 \times 10^4\,J$]

24. (a) What is meant by the following
(i) Conduction
(ii) Convection
(iii) Greenhouse effect
(b) One end of a long copper bar is in steam chest and the other kept cool by a current of circulating water. Explain with the aid of sketch graphs, the variation of temperature along the bar, when steady state has been attained if the bar is
(i) lagged
(ii) exposed to the surroundings
(c) (i) State Prevost's theory of heat exchanges
(ii) Sketch the variation with wavelength, the intensity of radiation emitted by a black body at two different temperatures
(d) A cube of side 1cm has a grey surface that emits 50% of radiation emitted by a black body at the same temperature. If the cube's temperature is 700°C, calculate the power radiated by the cube.

[2010, No.7; Ans: (c) 15.25 W]

25. (a)(i) Define the term thermometric property
(ii) State two thermometric properties
(iii) With the aid of a labelled diagram, describe how the room temperature can be measured using uncalibrated resistance thermometer.

(b) (i) Define specific heat capacity of a substance

(ii) Hot water at 85°C and cold water at 10°C are run into a bath at a rate of $3 \times 10^{-2}\ m^3 min^{-1}$ and V respectively. At the point of filling the bath, the temperature of the mixture of water was 40°C, calculate the time taken to fill the bath of its capacity in 1.5 m^3

(c) The specific latent heat of fusion of a substance is significantly different from its latent heat of vaporization at the same pressure. Explain how the difference arises

(d) Explain in terms of specific heat capacity why water is used in a car radiator than any other liquid.

[2009, No. 5; Ans: (b)(ii) 20 minutes]

26. (a)(i) State Boyle's law

(ii) Describe an experiment that can be used to verify Boyle's law

(b) Explain the following observations using the kinetic theory

(i) A gas fills any container in which it is placed and exerts pressure on its walls

(ii) The pressure of a fixed mass of a gas rises when its temperature is increased at constant volume

(c) (i) What is meant by a reversible process?

(ii) State the conditions necessary for isothermal and adiabatic processes to occur

(d) A mass of an ideal gas of volume 200 m^3 at 144 K expands adiabatically to a temperature of 137 K. Calculate its new volume (Take $\gamma = 1.40$)

[2009, No. 6; Ans: (d) 226.47cm^3]

27. (a) (i) Define thermal conductivity

(ii) Explain the mechanism of thermal conduction in non-metallic solids

(b) Why are metals better thermal conductors than non-metallic solids?

(c) With the aid of a diagram, describe an experiment to determine the thermal conductivity of a poor conductor.

(d) (i) What is meant by a black body?

(ii) Sketch curves showing the spectral distribution of energy radiated by a black body at three different temperatures

(iii) Describe the main features of the curves you have drawn in (d)(ii)

(e) A small blackened solid copper sphere of radius 2 cm is placed in an evacuated enclosure whose walls are kept at 100°C. Find the rate at which energy must be supplied to the sphere to keep its temperature constant at 127°C.

[2009, No.7; Ans: (e) 1.78 W]

28. (a) Define the following terms

(i) specific latent heat of vaporization of a liquid

(ii)coefficient of thermal conductivity

(b) Describe an experiment to measure the specifi latent heat of vaporization of water by a electrical method

(c) An appliance rated 240 V, 200 W evaporates 2 g of water in 5 minutes. Find the heat loss if th specific latent heat of vaporization is 2.26 $10^6\ Jkg^{-1}$

(d) Explain why at a given external pressure liquid boils at a constant temperature

(e) With the aid of suitable sketch graphs, expla the temperature distributions along lagged a unlagged metal rods heated at one end.

[2008, No. 5; Ans: (c) 14800

29. (a) Describe an experiment to verify Newton's la of cooling

(b) (i) Distinguish between a real and an ideal g

(ii) Derive the expression $p = \frac{1}{3}\rho\bar{c}^2$ for pressure of an ideal gas of density, ρ mean square speed $\bar{c}^2$

(c) (i) Explain why the pressure of a fixed mass a gas in a closed container increases when temperature of the container is raised.

(ii)Nitrogen gas is trapped in a container b movable piston. If the temperature of the is raised from 0°C to 50°C at a cons pressure of $4.0 \times 10^5 Pa$ and the total added is $3.0 \times 10^4 J$, calculate the work d by the gas. (molar heat capacity of nitroge constant pressure is 29.1 $Jmol^{-1}K^{-1}$ $\frac{c_p}{c_v} = 1.4$)

[2008, No. 6; Ans: (c)(ii) 8.57×1

30. (a)(i) State the laws of black body radiation

(ii)Sketch the variation of intensity wavelength in a black body for three diff temperatures.

(b) (i) What is a perfectly black body?

(ii) How can a perfectly black body approximated in reality?

(c) The energy intensity received by a spherical planet from a star is $1.4 \times 10^3\ Wm^{-2}$. The star is of radius $7.0 \times 10^5 km$ and is 14.0×10^7 km from the planet.
   (i) Calculate the surface temperature of the star
   (ii) State any assumptions you made in (c)(i) above

(d) (i) What is convection?
   (ii) Explain the occurrence of land and sea breeze.

[2008, No. 7; Ans: (c)(i) 5605.98 K]

31. (a)(i) Define latent heat
   (ii) Explain the significance of latent heat in regulation of body temperature
   (b) (i) Using kinetic theory, explain boiling of a liquid
   (ii) Describe how you would determine the specific latent heat of vaporization of water using a method of mixtures.
   (iii) Explain why latent heat of vaporization is always greater than that of fusion

[2007, No. 6]

32. (a)(i) Define a thermometric property and give its examples
   (ii) When is the temperature 0 K attained?
   (b) (i) With reference to a constant-volume gas thermometer, define temperature on the Celsius scale.
   (ii) State two advantages and two disadvantages of the constant volume gas thermometer
   (c) (i) Define the triple point of water
   (ii) Describe how you would measure the temperature of a body on a thermodynamic scale using a thermocouple.
   (d) The resistance R_θ of a platinum wire varies with the temperature θ°C as measured by the constant volume gas thermometer according to the equation

$$R_\theta = 50.0 + 0.17\theta + 3.0 \times 10^{-4}\theta^2$$

   (i) Calculate the temperature on the platinum scale corresponding to 60°C on the gas scale
   (ii) Account for the difference between the two values and state the temperatures at which they agree.

[2007, No. 5; Ans: (d)(i) 56.4°C]

33. (a) Show that the work done, W by a gas when it expands from V_1 to V_2 is given by $W = \int_{V_1}^{V_2} PdV$
   (b) State the first law of thermodynamics and use it to distinguish between isothermal and adiabatic changes in a gas.
   (c) The temperature of one mole of helium gas at a pressure of $1.0 \times 10^5 Pa$ increases from 20°C to 100°C when the gas is compressed adiabatically. Find the final pressure of the gas. (Take $\gamma = 1.67$)
   (d) With the aid of a $P - V$ diagram, explain what happens when a real gas is compressed at different temperatures.
   (e) The root mean square speed of the molecules of a gas is $44.72\ ms^{-1}$. Find the temperature of the gas if its density is $9.0 \times 10^{-2} kgm^{-3}$ and the volume is $42.0\ m^3$.

[2007, No.7; Ans: (c) $1.83 \times 10^5 Pa$ (e) $\frac{303.2}{n} K$]

34. (a) Define saturated vapour pressure (S.V.P)
   (b) Use the kinetic theory of matter to explain the following observations
      (i) Saturated vapour pressure of a liquid increases with temperature
      (ii) Saturated vapour pressure is not affected by a decrease in volume at constant temperature
   (c) Describe how the saturated vapour pressure of a liquid at various temperatures can be determined.
   (d) (i) State Dalton's law of partial pressures
      (ii) A horizontal tube of uniform bore, closed at one end has some air trapped by a small quantity of water. The length of the enclosed air column is $20\ cm$ at 12°C. Find, stating any assumptions made, the length of the air column when the temperature is raised to 38°C
      (S.V.P of water at 12°C and 38°C are 105 mmHg and 45 mmHg respectively, atmospheric pressure = $75.0\ cmHg$)

[2006, No. 5; Ans: (d)(ii) 23.04 cm]

35. (a)(i) Define specific heat capacity of a substance
   (ii) State three advantages of the continuous flow method over the method of mixtures in the determination of specific heat capacity of a liquid.
   (b) In a continuous flow experiment, a steady difference of temperature of 1.5°C is maintained when the rate of liquid flow is $45\ gs^{-1}$ and the rate of electrical heating is $60.5\ W$. On reducing the liquid flow rate to $15\ gs^{-1}$, 36.5 W is required to maintain the same temperature difference. Calculate the
      (i) specific heat capacity of the liquid
      (ii) rate of heat loss to the surrounding

(c) (i) Describe an electrical method for the determination of the specific heat capacity of a metal

(ii) State the assumptions made in the above experiment

(iii) Comment about the accuracy of the results of the experiment in (c)(i) above

[2006, No. 6;Ans:(b)(i) $533.3\, Jkg^{-1}K^{-1}$(ii) 24.5 W]

36. (a)(i) Define thermal conductivity

(ii) Explain the mechanism of heat transfer in metals

(b) Two brick walls each of thickness 10cm are separated by an air gap of thickness 10cm. The outer faces of the brick walls are maintained at 20°C and 5°C respectively.

(i) Calculate the temperatures of the inner surfaces of the walls.

(ii) Compare the rate of heat loss through the layer of air with that through a single brick wall

(Thermal conductivity of air is $0.02\, Wm^{-1}K^{-1}$ and that of brick is $0.6\, Wm^{-1}K^{-1}$)

(c) (i) State Stefan's law of black body radiation

(ii) The average distance of Pluto from the sun is about 40 times that of the Earth from the sun. If the sun radiates as a black body at 6000 K and is $1.5 \times 10^{11} m$ from the earth, calculate the surface temperature of Pluto.

[2006, No. 7; Ans: (b)(i) 5.5°C, 19.5°C (ii) 1:32.1 (c)(ii) 45.8 K]

SECTION C:

MORDERN PHYSICS

Nuclear physics

- The atomic nucleus- the nuclide
- Constituents of the nucleus
- Atomic number and mass number
- Scientific representation of a nuclide
- Isotopes
- Examples of isotopes
- Unified atomic mass unit (U)
- The equivalent of the atomic mass unit in electron-volts
- Einstein's mass-energy relation.
- Binding energy and mass defect
- Binding energy per nucleon
- Variation of binding energy per nucleon with mass number
- Significance of binding energy per nucleon
- Nuclear fission and fusion.
- Balanced equations

Radioactivity

- Radiations emitted during radioactivity
- Decay equations
- Properties of radiations emitted during radioactivity
- Detection of ionising radiation
- Background radiation
- Sourcesof background Radiation
- Law of radioactivity
- Decay constant
- The expression
- Decay curve
- Half life
- The expression
- Artificial (induced) radioactivity
- Applications of radioisotopes (biological and industrial)
- Hazards of radiation
- Safety precautions

Charged particles

- Discharge tube phenomena
- Cathode rays – production
- Cathode rays – properties
- Positive rays – production
- Positive rays – properties
- Motion of cathode rays and ion beams in electric and magnetic fields
- Specific charge
- Thomson's experiment
- Mass spectrometer
- The Millikan's oil drop experiment.
- The mole, Avogadro's number and Faraday's constant.

Quantum theory

- Photoelectric effect
- Characteristics of photoelectric emission
- Quantization of electromagnetic (e/m) radiation
- Work function and threshold frequency
- Einstein's photoelectric equation
- Experiment to verify Einstein's equation and measure Plank's constant
- Stopping potential
- Graphs to show the variation of stopping potential with frequency of radiation for various metals.
- Applications of photoelectric emission
- Rutherford's scattering experiment
- Rutherford's atomic model
- Bohr's atom
- Stable electron energy levels.
- Emission and absorption spectra
- Wave particle treatment of the electron
- Ground, excited and ionisation states
- X-ray tube
- X-rays
- Properties of X-rays
- X-ray radiation
- Characteristics of X-ray radiation
- Continuous(background) radiation, line X-ray spectra and cut off wavelength
- Uses of X-rays
- Diffraction of X-rays
- Bragg's law
- Hazards of X-rays
- Safety precautions of X-rays.

Electronic devices

- The CRO
- Comparison of CRO with moving coil instruments
- Transistors
- The p-n junction (intrinsic, extrinsic conductors, doping)
- The junction diode
- Rectification
- Applications of a p-n junction diode
- Transistor characteristics
- Two-transistor amplifier
- Logic gates and their combinations
- Boolean algebra
- Solution to logic questions, Truth tables.

THE NUCLEUS OF AN ATOM

The nucleus of the atom consists of the elementary particles, protons and neutrons which are known as **nucleons.**

Electrons revolve round the nucleus in circular orbits.

A proton has a positive charge of the same magnitude as that of the electron and its mass is about 1836 times the mass of an electron.

A neutron is electrically neutral whose mass is almost equal to the mass of the proton. The nucleons inside the nucleus are held together by strong attractive forces called **nuclear forces.**

Nuclide

A nuclide of an element is represented as ^A_ZX where X is the chemical symbol of the element, Z represents the atomic number which is equal to the number of protons and A is the mass number which is equal to the total number of protons and neutrons.

The number of neutrons is represented as N which is equal to $A - Z$. For example, the chlorine nucleus is represented as $^{35}_{17}Cl$. It contains 17 protons and 18 neutrons.

Isotopes

Isotopes are atoms of the same element with the same atomic number but different mass number.

The nuclei $^1_1H, ^2_1H, ^3_1H$ are the isotopes of hydrogen. In other words, isotopes of an element contain the same number of protons but different number of neutrons.

Radioisotopes

These are radioactive elements with the same atomic number but different mass numbers.

Nuclear charge

The charge of a nucleus is due to the protons present in it. Each proton has a positive charge equal to $1.6 \times 10^{-19}C$.

Nuclear charge $= Ze$, where Z is the atomic number.

Avogadro's number(N_A)

This is the number of constituent particles (atoms or molecules) that are contained in one mole of a substance. It has a value of $6.02 \times 10^{23}\, mol^{-1}$

Unified atomic mass unit (U)

It is convenient to express the mass of a nucleus in atomic mass unit though the unit of mass is kg.

Unified atomic mass unit is a twelfth of the mass of one atom of carbon $^{12}_{6}C$

Its symbol is U

Carbon of atomic number 6 and mass number 12 has mass equal to 12 g

$$6.02 \times 10^{23} \text{ atoms contain } 12\text{ g}$$

$$1 \text{ atom contains } \frac{12}{6.02 \times 10^3} \times 10^{-3}\, kg$$

Unified mass atomic unit,

$$1\,U = \frac{1}{12} \times \frac{12}{6.02 \times 10^3} \times 10^{-3} kg$$

$$= 1.66 \times 10^{-27} kg$$

Einstein's mass-energy relation

The energy equivalence of $1\,U$ can be calculated in electron-volt.

Einstein's mass energy relation is, $E = mc^2$

$$1\,U = 1.66 \times 10^{-27} kg$$

$$c = 3 \times 10^8\, ms^{-1}$$

$$E = 1.66 \times 10^{-27} \times (3 \times 10^8)^2\text{ J}$$

Electron volt (eV)

An electron-volt is the energy of an electron when it is accelerated through a potential difference of 1 volt.

$$1\,eV = 1.6 \times 10^{-19} C \times 1\, volt$$

$$1eV = 1.6 \times 10^{-19} J$$

$$E = \frac{1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} eV = 931\, \times 10^6 eV$$

Energy equivalent of $1\,U = 931\, MeV$

Nuclear mass

As the mass of the nucleus contains protons and neutrons, the mass of the nucleus is assumed to be the mass of its constituents.

Assumed nuclear mass $= Zm_p + Nm_n$ where m_p and m_n are the masses of proton and neutron respectively.

When the mass of the electron is given, the nuclear mass is given by

Mass of nucleons + mass of electrons

However, from measurement of mass by the mass spectrometer, it is found that the mass of a stable nucleus, m is less than the total mass of the nucleons.

i.e. mass of nucleus, $m < (Zm_p + Nm_n)$

$$\Delta m = Zm_p + Nm_n - m$$

where Δm is the mass defect.

Thus, the difference in the total mass of the nucleons and the actual mass of the nucleus is known as **mass defect.**

Mass defect is the mass equivalent of energy required to split up the nucleus into its constituent nucleons

Binding energy

Binding energy of a nucleus is the energy required to split a nucleus into its constituent nucleons.

The binding energy of a nucleus determines its stability against disintegration. If the binding energy is large, the nucleus is stable and vice versa.

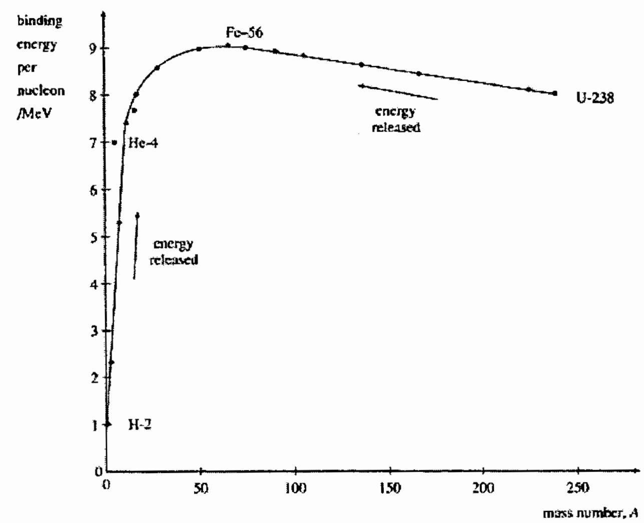
Binding energy per nucleon

This is the ratio of the energy required to split a nucleus into its individual nucleons to the number of nucleons in the nucleus.

Binding energy per nucleon $= \dfrac{\textbf{Binding energy}}{\textbf{Mass number}}$

It is found that the binding energy per nucleon varies from element to element.

Graph of binding energy per nucleon against mass number



The binding energy per nucleon of very large and very small nuclides is low. The maximum binding energy per nucleon occurs at a mass number of approximately 56.

There are peaks for small nuclides where the number of protons equals the number of neutrons.

Explanation of nuclear fusion and nuclear fission

During fission, a heavy nucleus splits to form two lighter nuclei having greater binding energy per nucleon. But the total mass of the two daughter nuclei is less than the mass of the parent nucleus. The difference in mass is accounted for by the energy released.

During fusion, two light nuclei combine to form heavier nucleus that has greater binding energy p nucleon. However, the mass of the heavier nucleus less than the sum of the two light nuclei. The ma difference is accounted for by the energy released.

RADIOACTIVITY

This is the spontaneous disintegration of unstab radioactive element with emission of alpha particle beta particles or gamma rays.

The radioactivity phenomenon is spontaneous and unaffected by an external agent like temperatur pressure, magnetic fields, etc. It cannot be slowe down nor speeded up by any physical or chemica process.

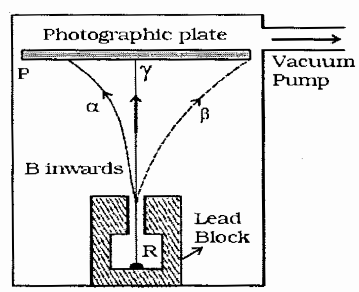
Alpha particles, Beta particles and Gamma rays

The existence of three different types of radiations α, β and γ-rays can be easily found by the following experiment.

Procedure

A small amount of Radium (R) is placed at the bottom of a small hole drilled in a lead block which is kept in an evacuated chamber.

A photographic plate is placed at a short distance above the lead block. A strong magnetic field is applied at right angles to the plane of the paper and acting inwards.



Three distinct traces can be seen on the photographic plate when it is developed. The trace towards left is due to positively charged particles (α-particles). The trace towards the right is due to negatively charged particles (β-particles). The undeviated trace is due to neutral radiations which are called γ-rays.

If an electric field is applied, the α-rays are deflected towards the negative plate, β-rays towards the positive plate and γ-rays are not deflected.

Properties of α-particles

- An alpha particle is a helium nucleus consisting of 2 protons and 2 neutrons. Its symbol is ${}^{4}_{2}He$
- Move along straight lines with high velocities.
- Deflected by electric and magnetic fields
- Produce intense ionisation
- Scattered by heavy elements like Gold
- Produce fluorescence when they fall on substances like zinc sulphide

Properties of β-particles

- β-particles carry one unit of negative charge and mass equal to that of electron. Therefore, they are nothing but electrons. Its symbol is ${}^{0}_{-1}e$
- Deflected by electric and magnetic fields
- Comparatively low ionisation power
- Affect photographic plates
- Penetrate through thin metal foils and their penetrating power is greater than that of α-particles

Properties of γ-rays

- They are electromagnetic waves of short wavelength
- Not deflected by electric and magnetic fields
- Travel with the velocity of light
- Produce very less ionisation
- Affect photographic plates
- Have a very high penetrating power, greater than that of β-particles
- Produce fluorescence
- Diffracted by crystals in the same way X-rays are diffracted.

α-decay

When a radioactive nucleus disintegrates by emitting a α-particle, the atomic number decreases by 2 and mass number decreases by 4. The α-decay ca be expressed as

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He $$

Example: Radium ${}^{226}_{88}Ra$ is converted to radon ${}^{222}_{86}Rn$ due to α-decay

$$ {}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He $$

β-decay

When a radioactive nucleus disintegrates by emitting a β-particles, the atomic number increases by 1 and the mass number remains the same. β-decay can be expressed as

$$ {}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}X + {}^{0}_{-1}e $$

Example: Thorium $({}^{234}_{90}Th)$ is converted to Protactinium $({}^{234}_{91}Pa)$ due to β-decay.

$$ {}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e $$

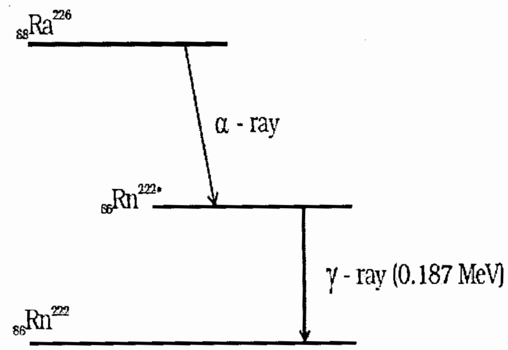
At a time, either α or β particle is emitted. Both α and β particles are not emitted during a single decay.

γ-decay

When a radioactive nucleus emits γ-rays, only the energy level of the nucleus changes and the atomic number and muss number remain the same.

During α or β-decay, the daughter nucleus is mostly in the excited state. It comes to the ground state with the emission of γ-rays.

Example: During the radioactive disintegration of radium, ${}^{226}_{88}Ra$ into radon, ${}^{222}_{86}Rn$, gamma ray of energy $0.187\ MeV$ is emitted when radon returns from the excited state to the ground state.



Background radiation

This is a radiation in the atmosphere caused by natural radioactivity in the ground, rocks and air. This radiation is detected even when there is no source. Background radiation is due to the following

- Objects in the atmosphere contain radioactive elements which decay and the resulting radiation enters the atmosphere
- Cosmic radiation. This is a radiation in the atmosphere from the outer space. It has a high ionizing effect.
- Ultraviolet radiation from the sun also ionizes the air in the atmosphere.

Decay law

The rate of disintegration at any instant is directly proportional to the number of atoms of the element present at the instant.

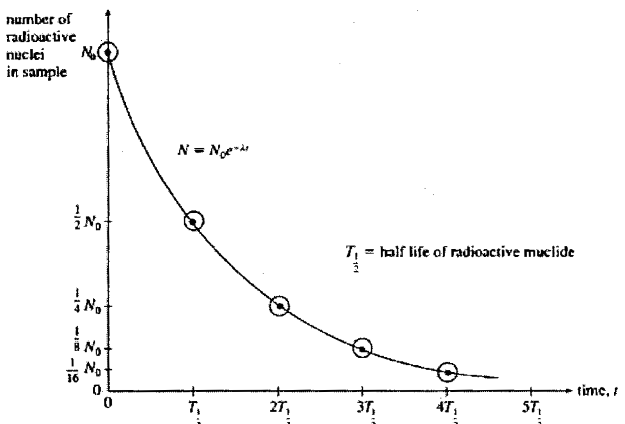
Let N_0 be the number of radioactive atoms present initially and N, the number of atoms at a given instant t.

$$-\frac{dN}{dt} \propto N$$
$$\frac{dN}{dt} = -\lambda N$$

where λ is the decay constant. The negative sign indicates that N decreases with increase in time.

$$\frac{dN}{N} = -\lambda dt$$
$$\int \frac{dN}{N} = \int -\lambda dt$$
$$\ln N = -\lambda t + c$$
$$\text{At } t = 0,\ N = N_0$$
$$c = \ln N_0$$
$$\ln N = -\lambda t + \ln N_0$$
$$\ln \frac{N}{N_0} = -\lambda t$$
$$\frac{N}{N_0} = e^{-\lambda t}$$
$$N = N_0 e^{-\lambda t}$$

The above equation shows that the number of atoms of a radioactive substance decreases exponentially with increase in time.



Initially the disintegration takes place at a faster rate. As time increases, N gradually decreases exponentially. Theoretically an infinite time is required for the complete disintegration of all the atoms.

Decay constant

This is the fractional number of disintegrations per second.

Activity

This is the rate of disintegration or decay.

If N is the number of atoms present at a certain t the activity A is given by

$$A = -\frac{dN}{dt}$$
$$\text{but } -\frac{dN}{dt} = \lambda N$$
$$\text{Therefore, } A = \lambda N$$

The unit of activity is becquerel (Bq)

1 becquerel = 1 disintegration per second

It is also expressed in Curie where 1 curie = 10^{10} disintegrations per second.

Half-life period

Since all the radioactive elements have infinit period, in order to distinguish the activity of element with another, half-life period is introduc

Half-life of a radioactive element is the time t for half of the radioactive element to und disintegration

From the decay law, $N = N_0 e^{-\lambda t}$

Let $T_{\frac{1}{2}}$ be the half-life period

$$\text{At } t = T_{\frac{1}{2}},\ N = \frac{1}{2}N_0$$
$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$
$$\ln 2 = \lambda T_{\frac{1}{2}}$$
$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Artificial radioactivity

This is the process by which elements are n radioactive by artificial or induced methods.

When lighter elements such as boron and alumin are bombarded with α-particles, there is a contin emission of radioactive radiations even the α- so has been removed. The radiation is due to emissio a particle carrying one unit positive charge with e mass to that of an electron **(positron)**

Note:

Artificial radioactive elements emit electrons, posit and γ-rays.

Production of artificial radioisotopes

Artificial radioisotopes are produced by placing target element in the nuclear reactor where plenty neutrons are available or bombarding the tar element with particles from particle accelerators l cyclotron.

$$^{31}_{15}P + ^{1}_{0}n \rightarrow ^{32}_{15}P + \gamma$$
$$^{23}_{11}Na + ^{1}_{0}n \rightarrow ^{24}_{11}Na + \gamma$$

Applications of radio-isotopes

The radioisotopes have wide applications in medicine, agriculture, industry and research. A radioactive element is added to a particular system and the course of the isotope is studied to understand the system.

Medical applications

- Radioisotopes are used both in diagnosis and therapy. Radio cobalt ^{60}Co emitting γ-rays is used in the treatment of cancer.
- Radio sodium ^{24}Na is used to detect the presence of blocks in blood vessels, to check the effective functioning of the heart in pumping blood and maintaining circulation.
- Radio iodine ^{56}Fe is used to diagnose anaemia. An anaemic patient retains iron longer in the blood than a normal patient
- Radio Phosphorus ^{32}P is used in the treatment of skin diseases.
- Radioisotopes can also be used in sterilizing pharmaceutical and surgical instruments.
- Determination of volume and concentration of blood in mammals.

  Radioactive Na is mixed with a small volume of blood taken from a mammal and then injected into its blood stream. After a given time, a small volume of blood is removed from the mammal and the activity of Na in the blood sample determined. High activity implies less volume of blood and low activity implies large blood quantity.

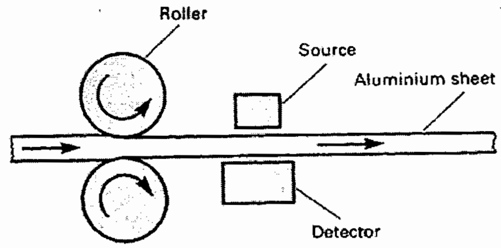
Agriculture

In agriculture, radioisotopes help to increase the crop yields.

- Radio-phosphorus (^{32}P) incorporated with phosphate fertilizer is added to the soil. The plant and soil are tested from time to time. Phosphorus is taken by the plant for its growth and radio phosphorus is found to increase the yield.
- Sprouting and spoilage of onions, potatoes, etc. are prevented by exposure to a very small amount of radiation. Certain perishable cereals remain fresh beyond their normal life span when exposed to radiation.

Industry

- In industry, the lubricating oil containing radioisotopes is used to study the rate of wear and tear of machinery.
- The thickness of metal sheet can be monitored during manufacture by passing it between a gamma ray source and a suitable detector. The thicker the sheet, the greater the absorption of gamma rays i.e.



- The exact position of an underground pipe can be located if a small quantity of radioactive liquid is added to the liquid being carried by the pipe. This also allows leaks to be detected. The soil close to the leak becomes radioactive.
- Can be used to examine the accuracy of welded joints.

Radio-carbon dating

In the upper atmosphere, ^{14}C is continually formed from ^{14}N due to the bombardment by neutrons produced from X-rays

Carbon dioxide in the atmosphere contains a small portion of ^{14}C. Living things take ^{14}C from food and air however with death, the intake of ^{14}C stops and the ^{14}C that is already present begins to decay.

Since the half-life of ^{14}C is 5570 years, the life time or age of the sample can be estimated using the equation

$$A = A_0 e^{-\lambda t}$$

Biological hazards of nuclear radiations

- They can cause damage to blood cells, organisms, skin disorder and loss of hair
- Too much exposure may cause diseases like leukemia (death of red blood cells in the blood) or cancer and may eventually lead to death
- The radiations cause injury to genes in the reproductive cells. This gives rise to mutations which pass on from generation to generation

Safety precautions when working with radiations

- Radioactive materials are kept in thick-walled lead containers.
- Lead aprons and lead gloves are used while working in hazardous areas.
- All radioactive samples must be handled by a remote-control process
- A small micro-film badge must always be worn by the person and should periodically be checked for the safety limit of radiation

Nuclear fission

This is the process of breaking up of a nucleus of a heavier atom into two fragments with the release of a large amount of energy.

The fission reaction is effected with the bombardment of neutron with the target atom. A large amount of energy is released.

Neutrons are preferred as they are neutral and there is no electrostatic repulsion between them and the target atom which would influence the collision. They can therefore penetrate deep into the nucleus.

Conditions for nuclear fission

The nucleus must be heavy and unstable.

The neutron must be at high speeds.

The fission reactions with ${}^{235}_{92}U$ are represented as

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Energy$$

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n + Energy$$

Such reactions where one neutron leads to production of more neutrons are called chain reactions and are used in the production of **atomic bombs**.

Energy released in fission

Let us calculate the amount of energy released during fission of ${}^{235}_{92}U$ with a neutron. The fission reaction is

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Energy$$

Mass of ${}^{235}_{92}U$ = 235.045733 U

Mass of ${}^{1}_{0}n$ = 1.008665 U

Mass of ${}^{141}_{56}Ba$ = 140.9177 U

Mass of ${}^{92}_{36}Kr$ = 91.8854 U

Solution

Total mass of reactants = 235.045733 + 1.008665

= 236.054398 U

Total mass of products

= 140.9177 + 91.8854 + 3(1.008665)

= 235.829095 U

Mass defect = 236.054398 − 235.829095

= $0.225303\,U$

1 U = 931 MeV

Energy released = 0.225303 × 931 = $209.8\,MeV$

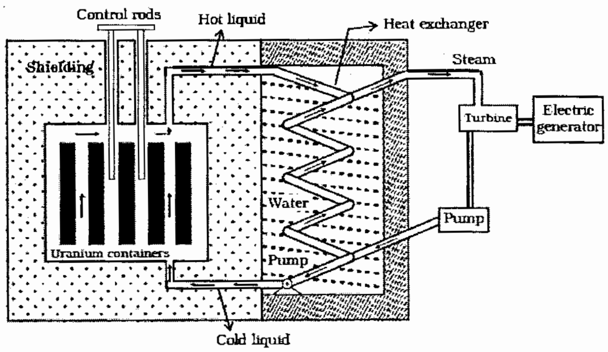
Nuclear reactor

A nuclear reactor is a device in which the nuclear fission reaction takes place in a sustained and controlled manner.

Depending on the purpose for which the reactors are used, they may be classified into research reactors, production reactors and power reactors.

- Research reactors are used primarily to supply neutrons for research purpose and for production of radio-isotopes
- Production reactors convert fertile (non-fissile but abundant) material into fissile material.
- Power reactors convert nuclear fission energy into electric power. The power reactors can be further classified into boiling water reactor, pressurized water reactor, etc. depending upon the choice of the moderator and the coolant used.

Schematic diagram of a nuclear reactor



Uses of nuclear reactors

- Power production because of the large amount of energy evolved with fission
- Production of radio-isotopes
- Source of neutrons, hence used in scientific research.

Nuclear fusion

This is the process by which two or more light nuclei combine to form a heavier nucleus with a release of energy.

The mass of the product nucleus is always less than the sum of the masses of the lighter nuclei. The difference in mass is converted into energy.

$${}^{3}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + Energy$$

Conditions for nuclear fusion

- The fusion process is carried out at extremely high temperature such that the nuclei gain enough kinetic energy to overcome electrostatic repulsion. Blast furnaces kept at very high temperatures can effect nuclear fusion reactions.
- The nuclei must be light or small

Natural occurrence of nuclear fusion

Fusion occurs naturally in the sun's core and it is the main source of the sun's energy.

Similarity between nuclear fusion and nuclear fission

In both nuclear fusion and nuclear fission, energy is released.

Examples

1. Calculate the binding energy and the binding energy per nucleon of the $^{56}_{26}Fe$ given;
   Mass of $^{56}_{26}Fe$ nucleus = 55.9349 U
   Mass of 1 proton = 1.007825 U
   Mass of 1 neutron = 1.008665 U

Solution

Number of protons = 26, Number of neutrons = 30
Mass of 26 protons = 26 × 1.007825 = 26.20345 U
Mass of 30 neutrons = 30 × 1.008665 = 30.25995 U
Total mass of nucleons = 26.20345 + 30.25995
= 56.46340 U
Actual mass of the $^{56}_{26}Fe$ nucleus = 55.9349 U
Mass defect = 56.46340 − 55.9349 = 0.5285 U
But 1 U = 931 MeV
Binding energy = $0.5285 \times 931 = 492.0335\ MeV$
Binding energy per nucleon = $\frac{492.0335}{56} = 8.786\ MeV$

2. The binding energy per nucleon for $^{12}_{6}C$ nucleus is 7.68 MeV and that for $^{13}_{6}C$ is 7.47 MeV. Calculate the energy required to remove a neutron from $^{13}_{6}C$ nucleus.

Solution

$$^{13}_{6}C \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$$

Binding energy of $^{13}_{6}C = 13 \times 7.47 = 97.11\ MeV$
Binding energy of $^{12}_{6}C = 12 \times 7.68 = 92.16\ MeV$
Binding energy of reactants = binding energy of products
Binding energy of neutron = $97.11 - 92.16$
$= 4.95\ MeV$

3. Calculate the energy released in the reaction

$$^{27}_{13}Al + {}^{2}_{1}H \rightarrow {}^{25}_{12}Mg + {}^{4}_{2}He$$

Given;
Mass of $^{27}_{13}Al$ = 26.981535 U
Mass of $^{2}_{1}H$ = 2.014102 U
Mass of $^{25}_{12}Mg$ = 24.98584 U
Mass of $^{4}_{2}He$ = 4.002604 U

Solution

Mass of the reactants = 26.981535 + 2.014102
= 28.995637 U
Mass of the products = 24.98584 + 4.002604
= 28.98444 U
Mass defect = 28.995637 − 28.98444
= $0.007193\ U$
Energy released in the reaction = 0.007193×931
$= 6.697\ MeV$

4. Calculate the energy released when 1 kg of $^{235}_{92}U$ undergoes nuclear fission. Assume energy per fission is 200 MeV and Avogadro's number = $6.02 \times 10^{23}\ mol^{-1}$

Solution

According to Avogadro's hypothesis,
Number of atoms in 235g of Uranium = 6.02×10^{23}
Number of atoms in 1kg of uranium
$= \frac{6.02 \times 10^{23} \times 1000}{235}$
Energy produced by 1 kg of uranium during fission,

$$E = \frac{6.02 \times 10^{23} \times 1000}{235} \times 200\ MeV$$
$$= 5.126 \times 10^{26}\ MeV$$

5. Calculate the time taken for 60% of a sample of radon to undergo decay if decay if the half-life of radon is 3.8 days.

Solution

Amount of sample disintegrated = 60%
Amount of sample present = 100 – 60 = 40%
Let N_0 be the original amount of the sample present

$$\lambda = \frac{0.693}{3.8}$$

From $N = N_0 e^{-\lambda t}$

$$\frac{40}{100} N_0 = N_0 e^{-\lambda t}$$
$$e^{\lambda t} = \frac{10}{4}$$
$$\lambda t = \ln 2.5$$
$$t = \frac{\ln 2.5}{\left(\frac{0.693}{3.8}\right)} = 5.02 \text{ days}$$

6. Determine the amount of ^{210}Po required to provide a source of α-particles of activity 5 milli curie given that the half-life of polonium is 138 days and 1 curie $= 3.7 \times 10^{10}$dis/sec.

Solution

Activity, $A = 5 \times 10^{-3} \times 3.7 \times 10^{10} = 1.85 \times 10^{8}$ dis/sec

$$\lambda = \frac{0.693}{138\times24\times60\times60} = 5.812 \times 10^{-8} \text{ per sec}$$

But $A = \lambda N$

$$N = \frac{A}{\lambda} = \frac{1.85\times10^{8}}{5.812\times10^{-8}} = 3.1825 \times 10^{15} \text{ atoms}$$

According to Avogadro's principle,

6.02×10^{23} atoms are contained in 210 g of ^{210}Po

3.1825×10^{15} atoms are contained in

$$\frac{3.1825\times10^{15}}{6.02\times10^{23}} \times 210 \text{ g}$$

$$= 1.11 \times 10^{-6} \text{ g}$$

Amount required is 1.11×10^{-6} g

7. A piece of bone from an archeological site is found to give a count rate of 15 counts per minute. A similar sample of fresh bone gives a count rate of 19 counts per minute. Calculate the age of the specimen given that the half-life of ^{14}C is 5570 years.

Solution

Count rate of fresh sample, $N_0 = 19$

Count rate of bone, $N = 15$

$$T_{\frac{1}{2}} = 5570 \text{ years}$$

$$N = N_0 e^{-\lambda t}$$

$$\lambda = \frac{0.693}{5570}$$

$$15 = 19e^{-\lambda t}$$

$$e^{\lambda t} = \frac{19}{15}$$

$$\lambda t = \ln\frac{19}{15}$$

$$t = \ln\frac{19}{15} \times \frac{1}{\lambda} = \ln\frac{19}{15} \times \frac{5570}{0.693}$$

$$t = 1899 \text{ years}$$

8. Initially, 1.0 grams of Sr-90 are present. If 0.953 grams remain after 2 years,
(a) what is the half-life of strontium - 90?
(b)how much Strontium-90 will remain after 5 years?

Solution

$$N_0 = 1.0 \text{ g},\ N_t = 0.953\text{g} \ \ and \ \ t = 2$$

(a)

$$\text{Using } \ln\left(\frac{N_t}{N_0}\right) = -\lambda t$$

$$\ln\left(\frac{0.953}{N_t}\right) = -\lambda(2)$$

$$\lambda = 0.0241\ yr^{-1}$$

From $t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{0.0241} = 28.79\ yrs$

(b) $N_t = N_0 e^{-\lambda t}$

$after\ t = 5$

$$N_t = 1.0 \times e^{(0.0241\times5)} = 0.886 \text{ g}$$

9. The half-life of radium is 1590 years. How long will it take for a sample of radium to decay to 10% of its original radioactivity?

Solution

$$\text{Using } t_{\frac{1}{2}} = \frac{0.693}{\lambda}, \lambda = \frac{0.693}{1590} = 4.36 \times 10^{-4}\ yr^{-1}$$

$$Using\ \lambda t = -\ln\frac{N_t}{N_0}$$

$$4.36 \times 10^{-4} \times t = -\ln\frac{10\%}{100\%}$$

$$t = 5280\ years$$

10. Consider the following nuclear reaction.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{133}_{x}A + ^{y}_{40}R + 6^{1}_{0}n$$

(i). Determine the values of x and y.
(ii). What is the importance of this reaction?

Solution

(i) $235 + 1 = 133 + y + 6$

$$y = 6$$

$$92 + 0 = x + 40 + 0$$

$$x = 52$$

(ii) The reaction is for nuclear fission. Therefore, energy is released during the process which can be changed to electricity at power stations and used in the manufacture of atomic bombs.

11. A steel piston ring contains 15g of radioactive iron $^{54}_{26}Fe$. The activity of $^{54}_{26}Fe$ is 3.7×10^{5} disintegrations per second. After 100 days of continuous use, the crank case oil was found to have a total activity of 1.23×10^{3} disintegrations per second. Find the
(i) half-life of $^{54}_{26}Fe$
(ii) average mass of iron worn off the ring per day assuming that all the metal removed from the ring accumulates in the oil.

Solution

54 g of $^{54}_{26}Fe$ contains $6.02 \times 10^{23} atoms$

15 g of iron contains $\frac{6.02\times10^{23}}{54} \times 15$

$$= 1.67 \times 10^{23} atoms$$

But $A = \lambda N$

$$3.7 \times 10^{5} = \frac{0.693\times1.67\times10^{23}}{T_{1/2}}$$

$$T_{1/2} = 3.625 \times 10^{12}\ days$$

Activity expected after time t is given by;

$$A = A_0e^{-\lambda t}$$

$$A = 3.7 \times 10^5 e^{-(1.912\times10^{-11})}$$

$$= 3.7 \times 10^5 \text{dis/sec}$$

mass worn off

$$= \frac{\text{Activity of oil}}{\text{Activity expected after time t}} \times \text{ total ring mass}$$

$$= \frac{1.23\times10^3}{3.7\times10^5} \times 15 = 0.04986g \approx 0.05g$$

$$\text{Average mass worn off per day} = \frac{\text{mass worn off}}{\text{time of use}}$$

$$= \frac{0.05}{100} = 5 \times 10^{-4} g/day$$

12. A small volume of a solution which contains a radioactive isotope of sodium had an activity of 1200 disintegrations per minute when it was first introduced in the blood stream of the patient. After 30 minutes, the activity of 1 cm^3 of the blood was 0.5 disintegrations per minute. If the half-life of sodium is 15 minutes. Estimate the volume of blood in the patient.

Solution

$$A_0 = 1200\ dis\ /\ min, T_{1/2} = 15\ min$$

1cm^3 of blood has 0.5 dis/min after 30 min

$$\lambda = \frac{0.693}{15} = 0.0462\ min^{-1}$$

$$A = A_0e^{-\lambda t} = 1200e^{-(0.0462\times30)} = 300.84 \text{ dis/ min}$$

$$\text{Volume of blood} = \frac{300.84}{0.5} = 601.68\ cm^3$$

13. A radioisotope has a half-life of 2 hours and activity of 2.88×10^7 disintegrations per hour is injected in a cow. $2\ cm^3$ of blood is then removed from the cow 16 hours later and was found to have an activity of 3 disintegrations per hour. Find the volume of blood in the cow.

Solution

$A_0 = 2.88 \times 10^7$ dis/hr

Activity after 16 hours, $A = A_0e^{-\lambda t}$

$$A = 2.88 \times 10^7 e^{-\frac{0.693}{2}\times16} = 112500\ dis/hr$$

$3\ dis/hr$ are contained in $2\ cm^3$

$112500\ dis/hr$ are contained in $\frac{2}{3}\times\ 112500$

$$= 75000\ cm^3$$

Self-Evaluation exercise

1. The half-life of $^{218}_{84}Po$ is 3 minutes. What percentage of the sample has decayed in 15 minutes. [Ans: 96.875%]
2. Find the energy released when two ^2_1H nuclei fuse together to form a single ^4_2He nucleus. Given, the binding energy per nucleon of ^2_1H and ^4_2He nucleus are 1.1 MeV and 7.0 MeV respectively. [Ans: 23.6 MeV]
3. Calculate the binding energy and binding energy per nucleon of $^{40}_{20}Ca$ nucleus. Given mass of 1 proton = 1.007825 U, mass of 1 neutron = 1.008665 U, mass of $^{40}_{20}Ca$ nucleus = 39.96259 U. [Ans: 341.8725 MeV; 8.5468 MeV]
4. Calculate the energy released in the following reaction.

$$^6_3Li + ^1_0n \to ^4_2He + ^3_1H$$

Given mass of ^6_3Li nucleus = 6.015126 U

Mass of ^3_1H nucleus = 3.016049 U

Mass of ^4_2He nucleus = 4.002604 U

Mass of ^1_0n = 1.008665 U

[Ans: 4.783 MeV]
5. Show that the mass of radium ($^{226}_{88}Ra$) with an activity of 1 curie is almost a gram given half-life of radium is 1600 years. (1 curie = 3.7×10^{10} dis/sec). [Ans: 1.0107g]
6. A carbon specimen found in a cave contained a fraction of $\frac{1}{8}$ of ^{14}C to that present in a living system. Calculate the approximate age of the specimen given that $T_{\frac{1}{2}}$ for ^{14}C = 5560 years. [Ans: 16681 years]
7. The radioactive isotope $^{214}_{84}Po$ undergoes a successive disintegration of two α –decays and two $\beta-$ decays. Find the atomic number and mass number of the resulting isotope. [Ans: 82, 206]
8. If 50% of a radioactive sample decays in 5 days, how much of the original sample will be left over after 30 days? [Ans: 6.25%]
9. Calculate the activity of $2.0\mu g$ of $^{64}_{29}Cu$ given that the half-life of $^{64}_{29}Cu$ is 13 hours. [Ans: $2.8 \times 10^{11} Bq$]
10. The radioactive isotope of iodine ^{131}I has a half-life of 8.0 days and is used as a tracer in medicine. Calculate
   (i) the number of atoms of ^{131}I which must be present in the patient when she is tested to give disintegration of $6.0 \times 10^5 s^{-1}$
   (ii) the number of atoms of ^{131}I which must have been present in a dose prepared 24 hours before
   [Ans: (i) 6.01×10^{11} (ii) 6.5×10^{11}]
11. A steel piston ring of mass 16g was irradiated with neutrons until its activity due to formation of this isotope was 10 microcurie. 10 days after the irradiation the ring was installed in an engine and

after 80 days continuous use, the crankcase oil was found to have a total activity of 1.85×10^3 disintegrations per second. Determine the average mass of iron worn off the ring per day assuming that all the metal removed from the ring accumulated in the oil and that 1 curie is equivalent to 3.7×10^{10} disintegrations per second.

[Ans: 4.0 mg per day]

12. A small volume of a solution which contained a radioactive isotope of sodium had an activity of 12000 disintegrations per minute when it was injected into the blood stream of a patient. After 30 hours, the activity of $1.0\ cm^3$ of the blood was found to be 0.50 disintegrations per minute. If the half-life of the sodium isotope is taken as 15 hours, estimate the volume of blood in the patient.

[Ans: $6000\ cm^3$]

13. At the start of an experiment a mixture of radioactive materials contains $20.0\ \mu g$ of a radioisotope A, which has a half-life of 70 s, and $40.0\ \mu g$ of radioisotope B, which has a half-life of 35 s.
   (i) After what period of time will the mixture contain equal masses of each isotope? What is the mass of each isotope at this time?
   (ii) Calculate the rate at which the atoms of isotope A are decaying when the masses are the same.

(Molar mass of isotope A = 234 g, Avogadro constant = $6.02 \times 10^{23}\ mol^{-1}$)

[Ans: (i) 70 s, $10.0\ \mu g$ (ii) $2.5 \times 10^{14}\ s^{-1}$]

14. A typical fission reaction is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{\ 0}_{-1}e$$

Calculate the total energy released by 1 g of $^{235}_{92}U$ undergoing fission by this reaction, neglecting the masses of the electrons.

Mass of neutron = 1.009 U, Mass of $^{95}_{42}Mo$ = 94.906 U

Mass of $^{139}_{57}La$=138.906 U, Mass of $^{235}_{92}U$ =235.044 U

[Ans: $8.53 \times 10^{10} J$]

15. A radioactive source contains $1.0 \times 10^{-6}\ g$ of plutonium-239. It is estimated that this source emits 2300 α-particles per second. Calculate the half-life of plutonium. [Ans: $7.59 \times 10^{11}\ s$]

16. A small volume of a solution which contained the radioactive isotope sodium had an activity of 2000 Bq when it was injected into the blood stream of a patient. After 30 hours, $1.0\ cm^3$ of blood was drawn from the patient and its activity was found to be 0.08 Bq. If the half-life of sodium – 24 is 15 hours, estimate the volume of blood in the patient.

[Ans: $17.7 \times 10^3\ cm^3$]

17. In a particular use of radioactivity in medicine, a source of initial activity $3.90 \times 10^3\ Bq$ is required. The nuclide selected has a half-life of $1.80 \times 10^5\ s$ and is prepared in a radiation centre one week before the start of the treatment. What should be the activity of the source when it is being prepared?

[Ans: $4.01 \times 10^4\ Bq$]

18. In a test engine, a radioisotope of iron which has a half-life of 45 days is added to the steel used in making of the piston ring. At the start of the experiment, a piston ring of mass 16 g was found to have an activity of 3.7×10^4 Bq. 30 days after the engine had been kept running, the crank case oil was found to have an activity of 6.0×10^2 Bq. Assuming that the metal worn off from the ring had accumulated in the oil, estimate the average mass of iron that wore off from the ring per day.

[Ans: 0.412 g]

19. When a nitrogen nucleus $^{14}_{7}N$ is bombarded with an α-particle of a certain energy, it transmutes to an oxygen nucleus $^{17}_{8}O$ and a proton.
(a) Write an equation for the nuclear reaction
(b) What is the minimum energy of the α- particle to make the reaction possible?

[Mass of $^{14}_{7}N = 2.32530 \times 10^{-26}\ kg$, mass of $^{17}_{8}O$ = $2.82282 \times 10^{-26}\ kg$, mass of a proton = $0.16735 \times 10^{-26}\ kg$, mass of α-particle = $0.666466 \times 10^{-26}\ kg$ [Ans: $1.89 \times 10^{-13} J$]

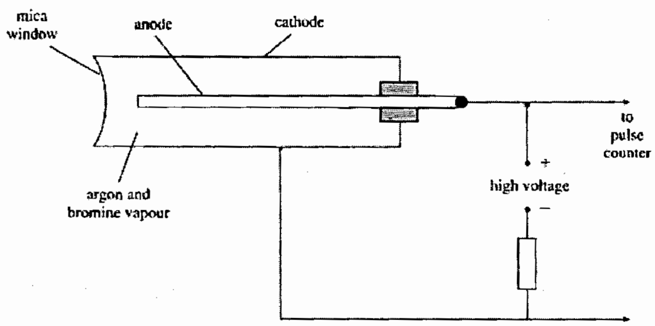
20. (a) A randon ($^{222}_{86}Rn$) nucleus of mass $3.6 \times 10^{-25}\ kg$ decays by emission of an α-particle of mass $6.7 \times 10^{-27}\ kg$ and energy $8.8 \times 10^{-13}\ J$
   (i) Write down the values of the mass number and atomic number of the resulting nucleus.
   (ii) Calculate the momentum of the emitted α-particle
   (iii) Find the velocity of recoil of the resulting nucleus.

(b) The sun obtains its radiant energy from a thermonuclear fusion process. The mass of the sun is $2 \times 10^{30}\ kg$ and it radiates $4 \times 10^{23}\ kW$ at a constant rate. Estimate the lifetime of the sun in years if 0.7% of its mass is converted into radiation during the fusion process and it loses energy only by radiation. (1 year = $3 \times 10^7 s$).

[Ans: (a)(i) 218, 84 (ii) $1.08 \times 10^{-19}\ Ns$ (iii) $3.1 \times 10^5\ ms^{-1}$ (b) 1×10^{11} years]

DETECTORS OF RADIATIONS

Geiger Muller tube



- Radioactive particles enter through the thin mica window and ionize the argon gas molecules.
- The electrons move at very high speeds to the anode because of the high p.d between the anode and the cathode.
- These fast-moving electrons collide with other argon gas molecules producing an avalanche of electrons.
- On reaching the anode, a pulse of current is discharged through the resistor.
- The resulting voltage pulse is amplified and measured by a scalar or ratemeter
- The positive ions move towards the cathode where bromine vapour absorbs their kinetic energy to avoid secondary emission of electrons.

Note: The G.M tube is efficient for β-particles and less efficient for α-particles and γ-rays. This is because the penetrating power of α-particles is weak and some α-particles may not penetrate the mica window of the G.M tube.
γ-rays give rise to secondary pulses of current and hence the count rate may be inaccurate.

Functions of some parts of the G.M tube

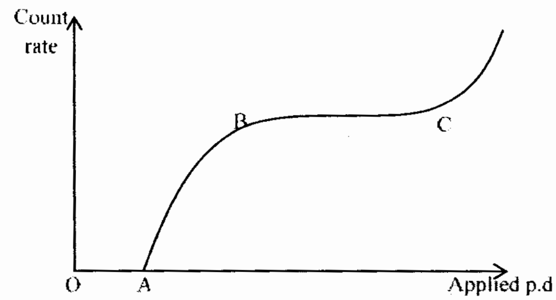
1.Thin mica window
It allows the ionizing particles to easily enter the G.M tube

2. Argon gas at low pressure
It enables ion pairs to be formed when the particles collide with the neutral atoms. These electron and positive ions formed do move to the electrodes with little interference

3. Halogen mixed with argon gas
This forms a quenching agent so as to prevent secondary electrons to be emitted from the cathode by the positive ions bombarding it.

4.Anode in form of a wire
This produces an intense electric field since electric field strength is inversely proportional to the radius of the wire.

Count-rate voltage characteristics

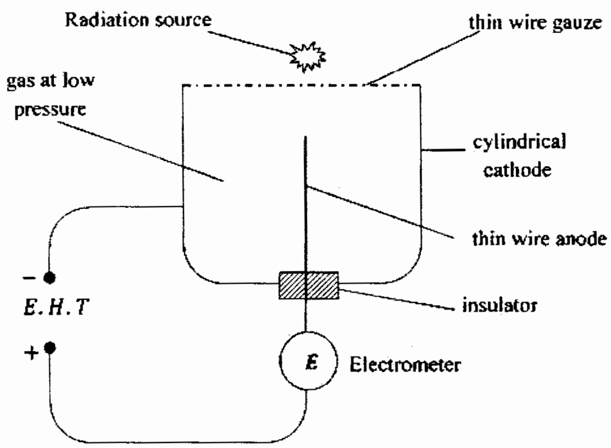


- Along OA, the applied p.d is less than the threshold voltage and there is insufficient gas amplification so no detectable pulses are produced.
- Along AB, the proportional region, the magnitude of any particular pulse depends on the strength of the initial ionisation.
- In the plateau region BC, all the pulses have the same amplitude regardless of the strength of the initial ionisation. Also, in this region, every particle that produces ionisation is detected.
- Beyond C, the quenching process becomes less and less effective and eventually a continuous discharge occurs.

Note: The suitable range for operating the tube is along BC because within this region, every particle that produces ionisation is detected.

Threshold p.d
This is the minimum p.d below which no current pulse can be detected.

Ionisation chamber



- Radiations enter through the thin wire gauze and ionize the gas molecules.
- The positive ions move towards the cathode while the electrons move towards the anode constituting a current pulse which is detected by the electrometer.
- The pulse per second (count rate) gives a measure of the intensity of the radiation.

Note:

$$\boldsymbol{I \propto n}$$

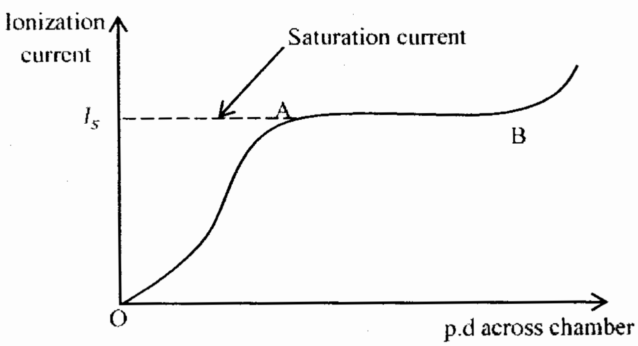
$$\boldsymbol{I = ne}$$

Where I = ionizing current

n = number of ion pairs per second

e = electronic charge

The variation of the ionisation current with the p.d is as indicated below.



Along OA, the p.d is no high enough to take all the electrons and positive ions to their positive electrodes before recombination occurs.

Along AB, the p.d is large enough to prevent recombination but it is not high enough to cause secondary ionisation. So the current reaches the saturation value.

Beyond B, there is secondary ionisation and a point is reached when the gas grows uncontrollably.

Dead time

This is the time taken by the ions to travel towards the cathode before the electric field at the anode returns to a level large enough for an avalanche to start.

Avalanche

This is the production of a large number of moving ion pairs as a result of violent collisions between electrons and atoms as the former is accelerated towards the anode. This secondary ionization causes large amounts of electrons to spread around the anode wire that absorbs them to produce a large pulse of the anode current.

Examples

1. A source of α-particles has an activity of 2×10^3 disintegrations per second. When α-particles enter an ionisation chamber, a saturation current of $2 \times 10^{-9}\,A$ is obtained. If the energy required to produce an ion pair is 32 eV, determine the energy of one α-particle.

<u>Solution</u>

$$I = ne$$

$$2 \times 10^{-9} = n \times 1.6 \times 10^{-19}$$

$n = 1.25 \times 10^{10}$ ion pairs per second

Total energy of $\alpha -$ particles = Total energy of ion pairs

$$2 \times 10^3 E_\alpha = 1.25 \times 10^{10} \times 32\,eV$$

But 1 eV = $1.6 \times 10^{-19}\,J$

$$E_\alpha = \frac{1.25 \times 10^{10} \times 32 \times 1.6 \times 10^{-19}}{2 \times 10^3}$$

$$E_\alpha = 3.2 \times 10^{-11}\,J$$

2. A radioactive source emits 2×10^5 α-particles per second. The particles produce a saturation current of $1.1 \times 10^{-8}\,A$ in an ionisation chamber. If the energy required to produce an ion pair is 32 eV, determine the energy in MeV of an α-particle emitted by the source.

<u>Solution</u>

Activity, $A = 2 \times 10^5\,s^{-1}$, $E_{ion} = 32\,eV$

$$I = ne$$

$$1.1 \times 10^{-8} = n \times 1.6 \times 10^{-19}$$

$n = 6.875 \times 10^{10}$ ion pairs per second

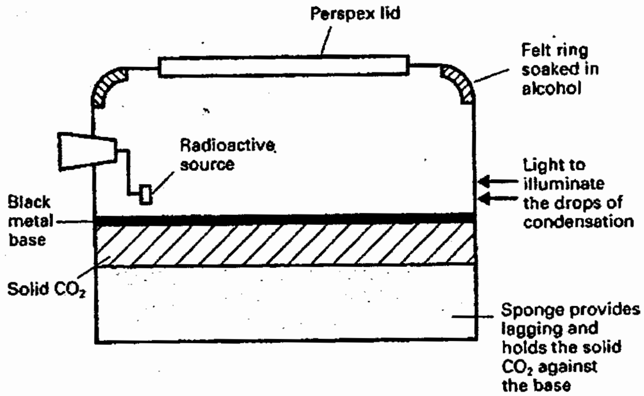
Total energy of α-particles = Total energy of ion pairs

$$2 \times 10^5 E_\alpha = 6.875 \times 10^{10} \times 32\,eV$$

$$E_\alpha = 1.1 \times 10^7\,eV$$

$$E_\alpha = \frac{1.1 \times 10^7}{10^6}\,MeV = 11\,MeV$$

Diffusion cloud chamber

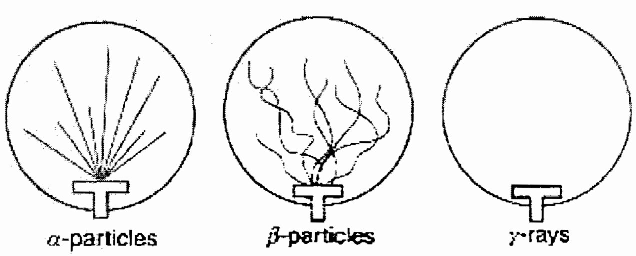


- The base of the chamber is maintained at a very low temperature by solid carbon dioxide while the top of the chamber is at room temperature.
- Air above the chamber is at room temperature while that close to the metal base is supersaturated. The alcohol vapour steadily diffuses from the top to the bottom where it becomes supersaturated because of the temperature gradient.
- Ionizing radiations from the source ionize the air molecules giving off ion pairs. The ions formed diffuse to the bottom where they act as condensation centres on which the supersaturated alcohol vapour condenses.
- A string of liquid droplets known as chamber tracks is obtained along the path where ionisation has occurred.
- These droplets are viewed through a microscope or photographed by a camera and appear white on a dark background.
- The length, thickness and nature of the tracks shows the extent of the ionisation.

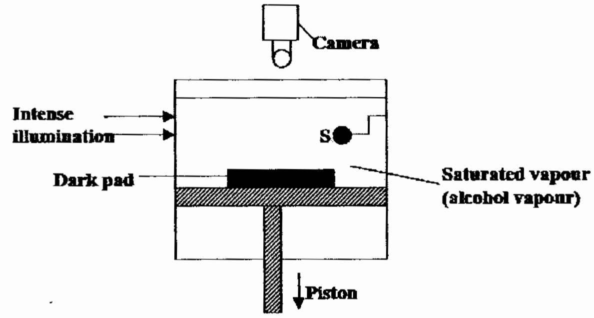
Alpha particles leave thick, straight tracks since they are heavy and cause intense ionisation.

Beta particles leave thin, irregular tracks since they are light and cause less ionisation.

Gamma rays leave very thin, disjointed and irregular tracks as they cause very little ionisation and are constantly absorbed by the vapour.

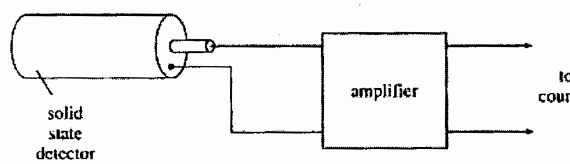


Expansion cloud chamber



- The vapour in the chamber is cooled until it reaches saturation.
- The piston is then rapidly withdrawn such that the gas inside expands adiabatically, cools and the vapour becomes supersaturated.
- The shield on the radioactive source is removed and radioactive particles ionize the gas molecules.
- The ions formed act as condensation centres on which the supersaturated alcohol vapour condenses.
- Viewing through the microscope, the length, size and nature of the chamber tracks is used to indicate the extent of ionisation. These tracks appear as white water droplets on a black background.

The solid-state detector



The solid-state detector is useful as an $\alpha -$ particle detector. It consists of a p-n junction diode which is reverse biased.

When an ionizing particle hits the diode, more electron hole pairs are created and a pulse of current flows round the circuit.

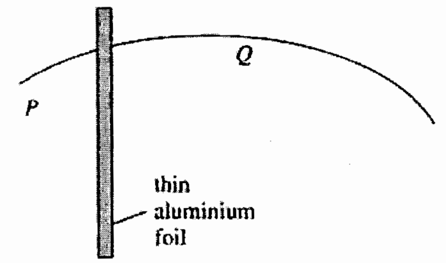
Examples

1. The track of a charged particle in a cloud chamber looks like a fine white line. What constitutes of the line and how is it formed?

Solution

The fine white line consists of condensed alcohol. The charged particle ionized the air in the cloud chamber where the air is supersaturated with vapour. The vapour then condensed on the ions forming the white line

2. The following figure shows the track of a charged particle that penetrated a thin aluminium foil. A uniform magnetic field acts vertically downwards in the region shown and an α-particle moves in the plane of this paper.



(i) Does the α-particle move from P to Q or from Q to P? Give reasons for your answer

(ii) Is the particle positively charged or negatively charged? Explain your answer

(iii) How do you differentiate the α-particle tracks and β-particle tracks if there is no electric or magnetic field applied.

Solution

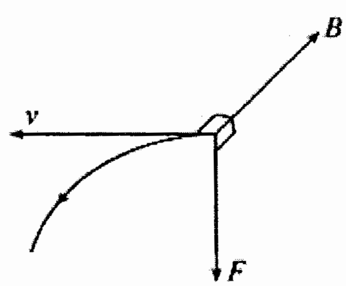
(i) The α-particle moves from Q to P

From the equation $\frac{mv^2}{r} = Bqv$

$$r = \frac{mv}{Bq}$$

The radius r of the track is directly proportional to the speed. After penetrating the thin aluminum foil, the speed v of the particle is reduced thus P, is the track after penetration as it has a smaller radius.

(ii) From the direction of v, B and F, using Fleming's left-hand rule



The particle must be positively charged

(iii) The tracks of α-particles are thick, straight and short while those of β-particles are thin, wavy and long.

THERMIONIC EMISSION

All metals contain some electrons which are free to move about within the lattice. Even though the attractive forces exerted on these electrons by the atomic nuclei are not strong enough to bind them to particular atoms, they do prevent them from leaving the surface.

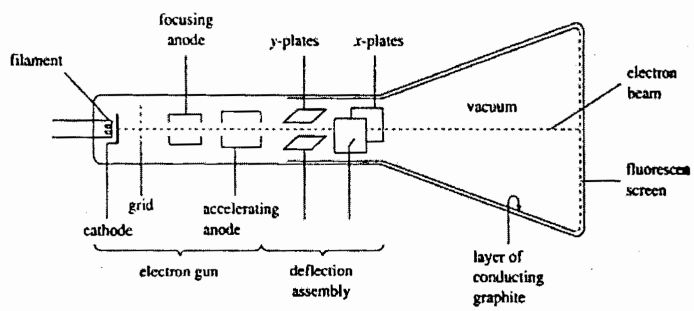
When a metal is heated, the energies of its electrons increase and some of them acquire sufficient energy to escape from the surface. This process is called thermionic emission.

The rate at which the electrons are emitted increases rapidly with temperature.

THE CATHODE RAY OSCILLOSCOPE (C.R.O)

This device is the most useful electronic instrument. It gives visual representation of electrical quantities such as voltage and frequency in any electronic circuit. It makes use of cathode rays which are deflected in electric and magnetic fields and produces scintillation on the fluorescent scree.

Main features of the C.R.O



1. The electron gun

The electron gun in the C.R.O consists of

a. The filament

When current flows through the filament, it glows and heat from the filament heats the cathode

b. The cathode

Electrons are emitted from the cathode is heated. The electrons form a cloud of electrons known as the space charge close to the cathode.

c. The grid

It is usually at a potential slightly negative relative to the cathode. It controls the rate of electrons that finally reach the screen.

By adjusting the grid potential, the brightness of the spot of light on the screen can be varied.

d. The focusing anode

Its potential is positive relative to the cathode. The electrons that pass though this anode are focused into a fine beam.

e. The accelerating anode

It is at a positive potential relative to the cathode. The potential difference between this anode and cathode accelerates the electron to a high velocity.

2. The deflection assembly

It consists of two pairs of plates, the x-plates and the y-plates.

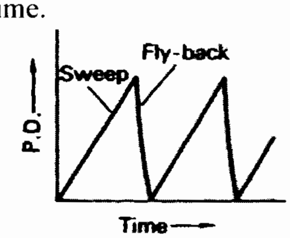
The electric field between the y-plates deflects the electron beam vertically while that between the x-plates deflects the electron beam horizontally.

3. The fluorescent screen

This is the wide end of the tube and its inside is coated with zinc sulphide or graphite that glows when hit by energetic electrons. Also, the screen has a graphite coating that provides a path for collecting secondary electrons to the earth. It also shields the electron beam from external electric fields by providing an equipotential surface.

Time base of the C.R.O

The time base is connected to the X – plates and provides a saw tooth p.d that sweeps the electron spot from the left to the right of the screen at a steady speed. Consequently, it helps in studying the variation of a quantity with time.



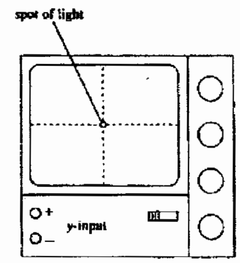
The time taken for this right to left sweep is called the **fly-back time**.

Uses of the CRO

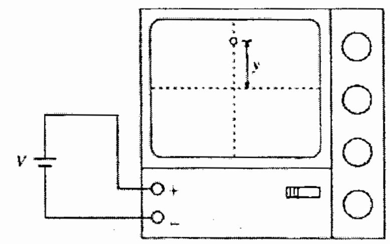
- To display waveforms
- Measure voltages, both a.c and d.c
- Measure frequencies
- Measure phase differences
- Measure small time intervals

Measurement of voltage

When a C.R.O is switched on, a spot of light appears at the centre of the screen if no voltage is connected to either of the deflection plates.

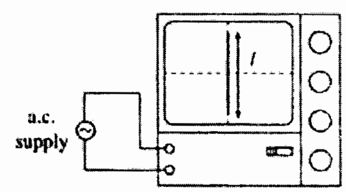


When a d.c voltage is connected to the y – plates of the C.R.O through the y – input, the spot of light is deflected as shown below. The deflection y of the spot is directly proportional to the voltage applied.



If the sensitivity of the y – plates is known in $V\ mm^{-1}$, the voltage can be determined after measuring the deflection y.

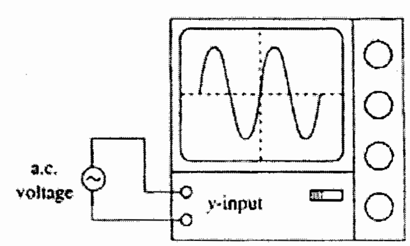
For a.c voltage supply, the voltage varies from $+V_0$ to $-V_0$ is the amplitude of the alternating voltage. The spot of light moves up and down with a frequency equal to the frequency of the a.c supply. If the frequency is high, due to persistence of vision, a vertical line is seen on the screen.



The length l of the line represents $2V_0$ where V_0 is the amplitude of the alternating voltage

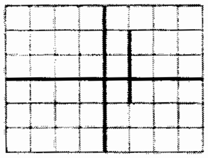
r.m.s voltage, $V_{rms} = \frac{V_0}{\sqrt{2}}$

If the time base is switched on and an a.c voltage is applied across the y – plates, the trace of the spot is as shown below



Examples

1. The figure below shows the trace on a C.R.O when a sinusoidal potential difference is applied to the $y -$ plates. The y – sensitivity is 4.00 V per division. What is the root mean square value of the applied potential difference?



Solution

Length of trace = 3 divisions

If V_0 = peak voltage, then

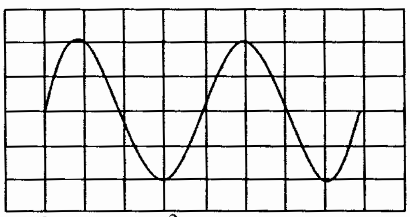
$2V_0 = 3 \times 4.00$

$V_0 = 6.0\ V$

Root mean square voltage = $\frac{V_0}{\sqrt{2}}$

$= \frac{6.0}{\sqrt{2}} = 4.24\ V$

2. A cathode ray oscilloscope with time base set at $5.0 \times 10^{-3}\ scm^{-1}$ with a voltage gain of 5 Vcm^{-1} is connected to a power source using its y – plates and the wave displayed is as shown below



Each square is of area 1 cm^2

(i) Identify the type of the signal generated by the power source.

(ii) Determine the amplitude of the signal and its V_{rms}

(iii) Calculate the frequency of the source

Solution

(i) Alternating signal

(ii) Amplitude = 5 × 2 = 10 V

$\therefore V_{rms} = \frac{20}{\sqrt{2}} = 7.07$ V

(iii) Period T = 4 × 5 × $10^{-3} = 0.02\ s$

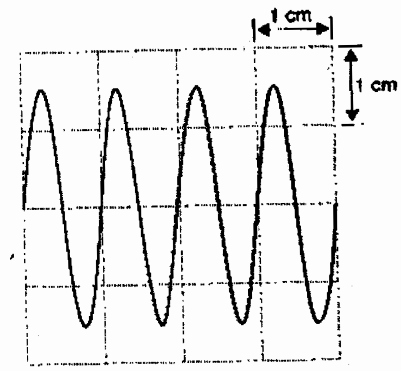
$\therefore$ Frequency f = $\frac{1}{T} = \frac{1}{2.0 \times 10^{-2}}$ = 50 Hz

Advantages of CRO over voltmeter in measuring voltage

- It can be used for both a.c and d.c
- It has no coil to burn out
- It has instantaneous response
- It has nearly infinite resistance to d.c and very high impedance to a.c and therefor it draws very little current.

Self-evaluative exercise

1. The screen of a cathode ray oscilloscope displays the trace shown below.



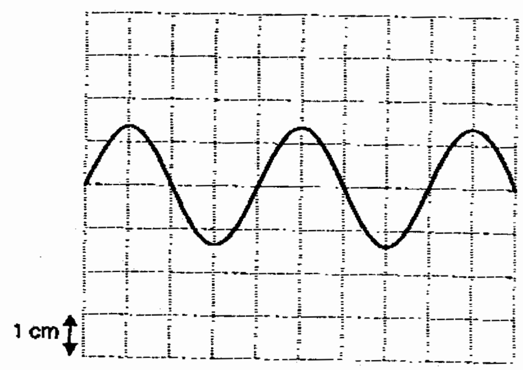
The Y-sensitivity is set at 10 mV/cm and the time base is set at 0.20 ms/cm. Obtain values for

(a) the peak voltage and

(b) the frequency of the alternating signal

[Ans: (a) 15 mV (b) 5.0 kHz]

2. A cathode ray oscilloscope has its amplifier sensitivity control set at 10 Vcm^{-1}. An a.c voltage of frequency 10 kHz is applied to the input of the amplifier. Below is the trace obtained on the screen.



(i) Calculate the amplitude of the input signal

(ii) What is the setting of the time base control?

[Ans: (i) 13 V (ii) 25 μs per cm]

3. The gain control of an oscilloscope is set on 1 Vcm^{-1}. What is

(i) the peak value, and

(ii) the r.m.s value of alternating p.d that provides a vertical line trace 2 cm long when the time base is off? [Ans: (i) 1 V (ii) 0.7 V]

4. (a) Sketch and explain the forms of the traces seen on an oscilloscope screen when a p.d alternating at 50 Hz is connected across the Y-plates if the time base is linear and has a frequency of

(i) 10 Hz (ii) 100 Hz

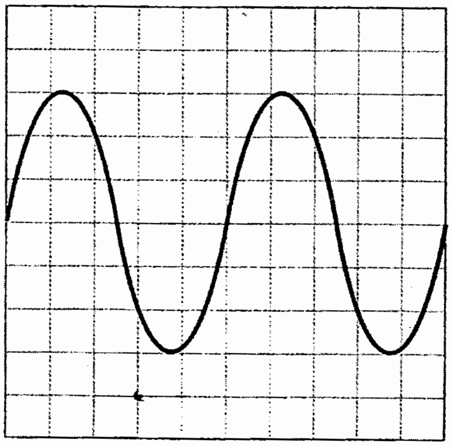
(b) What is the frequency of an alternating p.d applied to the Y-plates of an oscilloscope and produces five complete waves on a 10 cm length of the screen when the time base setting is $10\ ms\ cm^{-1}$?

[Ans: (a)(i) 5 waves (ii) ½ wave (b) 50 Hz]

5. A C.R.O has its Y-sensitivity set at $10\ Vcm^{-1}$. A sinusoidal input is suitably applied to give a steady trace with the time base set so that the electron beam takes 0.01 s to traverse the screen. If the trace screen has a total peak to peak height of 4.0 cm and contains 2 complete cycles, what is the r.m.s voltage and the frequency of the input signal?

[Ans: 14.1 V, 200 Hz]

6. A sinusoidally varying p.d of frequency 250 Hz and r.m.s value 12.0 V is connected to an oscilloscope giving the trace shown below.



Estimate

(i) the p.d represented by 1.0 division in the Y-direction

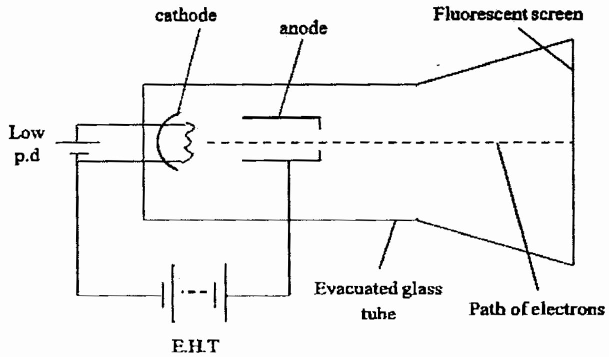
(ii) the time represented by 1.0 division in the X-direction

[Ans: (i) $5.6\ V\ div^{-1}$ (ii) $0.8\ ms$]

CATHODE RAYS

This is a beam of fast moving electrons.

Production of cathode rays



When the cathode inside an evacuated glass tube is heated by a low voltage supply, electrons are produced by thermionic emission

The electrons are accelerated by a positive high voltage applied between the cathode and the anode

The electrons travel without loss of energy across the vacuum via the anode and produces a glow when they strike the fluorescent screen.

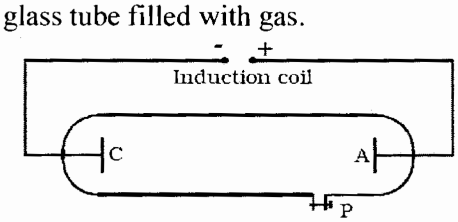
It is this beam of fast moving electrons from the cathode which constitute cathode rays.

Properties of cathode rays

- They travel in straight lines
- They possess momentum and kinetic energy
- Cathode rays produce heat when allowed to fall on matter
- They produce fluorescence when they strike a number of crystals, minerals and salts
- They are deflected by electric and magnetic fields.
- When they strike a substance of high atomic weight, X-rays are produced
- They ionize the gas through which they pass
- They affect photographic plates
- Cathode rays comprises of electrons which are fundamental constituents of all atoms.

Discharge of electricity through gases at low pressure (discovery of electrons)

A discharge tube is an arrangement to study the conduction of electricity through gases. It is a closed, strong glass tube filled with gas.



Two metal electrodes C and A are fitted inside the tube at the ends. The side tube P is connected to a high vacuum pump and a low-pressure gauge.

The electrodes C and A are connected to the secondary of a powerful induction coil which maintains a high potential difference.

Electrode C is the cathode while electrode A is the anode.

When the pressure of the gas inside the discharge tube is reduced to about 110 mmHg by the vacuum pump, no discharge occurs.

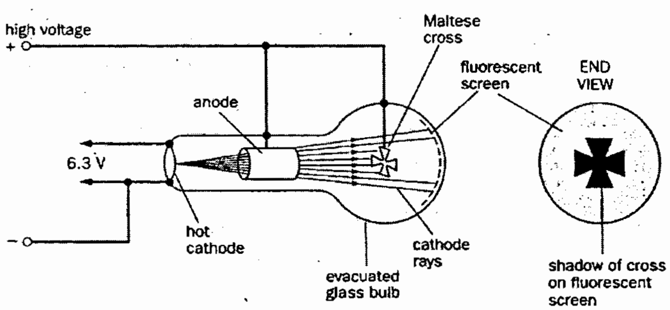
At a pressure of 110 mmHg, the discharge of electricity through the gas begins and irregular streaks of light appear accompanied by a cracking sound.

As the pressure is reduced to about 10 mmHg, the irregular streaks broaden out into a luminous column (positive column) extending from the anode almost up to the cathode.

With further reduction in pressure to about 0.01 mmHg, the positive column disappears and Crooke's dark space fills the whole tube.

At this stage, the walls of the glass tube fluorescence with green colour. The greenish glow is found to be a fluorescence of the glass produced by some invisible rays emanating from the cathode. These rays are called cathode rays and are found to be electrons.

Experiment to show that cathode rays travel in a straight line



Electrons from the hot cathode are accelerated by the positive high voltage connected between the cathode and the anode.

A dark shadow of the maltese cross appears on the fluorescent screen.

This suggests that cathode rays travel in straight lines from the cathode.

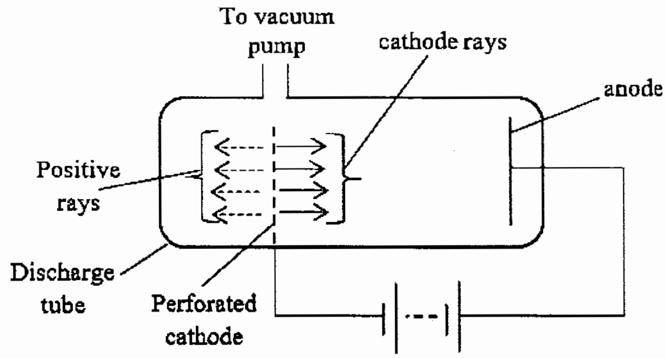
Those not intercepted by the maltese cross cause fluorescence on the screen when they strike it.

Note: When the cathode rays strike the maltese cross, it rotates. This means that the cathode rays possess kinetic energy (mechanical energy)

POSITIVE RAYS

These are positive gas ions produced when cathode rays from the cathode collide with the gas atoms in the discharge tube.

Production of positive rays



At low pressure and high voltage in the discharge tube, cathode rays from the perforated cathode collide with gas atoms in the tube.

The positive gas ions produced and accelerated to high energies are the positive rays.

They form a luminous beam of light in the space beyond the cathode.

Properties of positive rays

- They are a stream of positive ions of the gas enclosed in the discharge tube. The mass of each ion is nearly equal to the mass of the atom.
- They are deflected by electric and magnetic fields. Their deflection is opposite to cathode rays
- They travel in straight lines
- The affect photographic plates
- They ionize the gas through which they pass
- These rays can cause fluorescence

Differences between cathode rays and positive rays

Cathode rays	Positive rays
Negatively charged	Positively charged
Deflected more in electric and magnetic fields	Deflected less in electric and magnetic fields
Travel with the same velocity	Travel with a range of velocities
Not related to the gas in the tube	Related to the gas in the tube
Produce X-rays on striking matter	Do not produce X-rays on striking matter

Applications of discharge tubes

- Fluorescent tube to give light
- TV monitors

Disadvantages of the discharge tube in the production of positive and cathode rays

- A gas is needed at very low pressure which may not be easy to achieve practically
- A high voltage/ p.d is needed across the tube which may not be easy to achieve practically.
- X-rays may be produced which are very dangerous to handle.

Electron movement in electric and magnetic fields

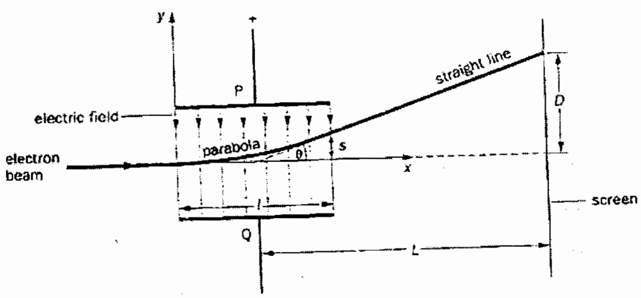
Electrons are usually projected into an electric filed or magnetic field after they have been accelerated by a potential difference, V_a applied to the anode. This potential provides the electron with the kinetic energy with which they will move in the field.

$$eV_a = \frac{1}{2}mv^2$$

where m is the mass of the electron

Deflection of electrons in an electric field

A stream of fast moving electrons is called cathode rays. These are usually projected midway between two parallel oppositely charged plates. Electric field, E is usually provided by a potential difference, V applied between the two parallel plates separated by a distance d.



$$E = \frac{V}{d}$$

Electrostatic force acts perpendicularly between the plates just like gravitational force does. For any charge Q of mass m moving with a uniform velocity v in an electric field, the force acting on it is given by

$$F = QE = \frac{QV}{d}$$

For electrons of charge, e of mass m moving with a uniform velocity v in an electric field, the force acting is given by

$$F = eE = \frac{eV}{d}$$

This force will cause vertical acceleration but horizontal motion will remain constant.

Considering horizontal motion:

Initial horizontal velocity $= u$

Horizontal displacement $= x$

Acceleration $= 0$

From $s = ut + \frac{1}{2}at^2$

$$x = ut$$

$$t = \frac{x}{u}$$

Considering vertical motion:

$$F = ma$$

$$\frac{eV}{d} = ma$$

$$a = \frac{eV}{md}$$

Initial vertical velocity $= 0$

Vertical displacement $= y$

From $s = ut + \frac{1}{2}at^2$

$$y = 0 + \frac{1}{2}\left(\frac{eV}{md}\right)t^2$$

$$y = \frac{1}{2}\left(\frac{eV}{md}\right)\frac{x^2}{u^2}$$

Or $y = \left(\frac{eV}{2mdu^2}\right)x^2$

This is an equation in the form of a parabola. This means that the path of electrons or charged particles is like those of a projectile in a gravitational field.

The electrons gain vertical velocity which can be determined

$$u = 0, v = v_y, a = \frac{eV}{md}$$

From $v = u + at$

$$v_y = 0 + \frac{eV}{md} \times \frac{x}{u}$$

$$v_y = \frac{eVx}{mdu}$$

The actual velocity, v of the electrons at any within the field is given by the resultant of v_x and v_y

$$v = \sqrt{v_x^2 + v_y^2}$$

At the edge of the parallel plates, $x = l$, the length of the plates. At this point

$$v_y = \frac{eVl}{mdu}$$

From this point onwards, the electrons will travel in a straight line to the screen with a uniform velocity which is the resultant v. When the straight line is extended backwards, it will cut the horizontal central line midway the length l. The distance to the screen is measured from this point

$$\tan\theta = \frac{v_y}{v_x} = \frac{s}{\frac{l}{2}} = \frac{D}{L}$$

Examples

1. An electron starts from rest and moves freely in an electric field whose intensity is $2.5 \times 10^3\ Vm^{-1}$. Find
   (i) the electric force on the electron
   (ii) the acceleration of the electric field
   (iii) the velocity in moving through a p.d of 90 V

Solution

(i) $F = eE$

$F = 1.6 \times 10^{-19} \times 2.5 \times 10^3 = 4.0 \times 10^{-16}\ N$

(ii) $a = \frac{F}{m} = \frac{4.0\times10^{-16}}{9.11\times10^{-31}} = 4.40 \times 10^{14}\ ms^{-2}$

(iii) Work done = gain in K.E

$eV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2eV}{m}}$

$v = \sqrt{\frac{2\times1.6\times10^{-19}\times90}{9.11\times10^{-31}}} = 5.62 \times 10^6\ ms^{-1}$

2. An electron is projected with a speed of $3.0 \times 10^7\ ms^{-1}$ in a direction of a uniform magnetic field. After travelling a distance of 40 cm, the electrons reverse direction.
   (a) Why does the electron reverse direction?
   (b) Calculate the magnitude of the electric field

Solution

(a) The electrons reverse direction because they are moving against the field i.e. towards the negative plate.

(b) This part is like a projectile projected vertically upward against gravity.

$u = 3.0 \times 10^7\ ms^{-1},\ s = 0.4\ m, a = ?$

$v^2 = u^2 + 2as$

$a = \frac{v^2-u^2}{2s} = \frac{0-(3.0\times10^7)^2}{2\times0.4} = -1.125 \times 10^{15}\ ms^{-1}$

From $F = ma$

$eE = ma$

$E = \frac{ma}{e} = \frac{9.11\times10^{-31}\times-1.125\times10^{15}}{1.6\times10^{-19}}$

$E = -6.41 \times 10^3\ NC^{-1}$

3. A beam of electrons is accelerated through a p.d of $2.0 \times 10^3\ V$ and is directed mid-way between two horizontal metal plates of length $5.0\ cm$ and separation $2.0\ cm$. The p.d between the plates is $80\ V$. Calculate
   (a) the speed of the electrons as they enter the region between the plates
   (b) the speed of the electrons as they emerge from the region between the plates.
   (c) the angle between the initial and final direction

Solution

V_a = accelerating p.d

V = p.d between the plates

u = speed of the electrons as they enter

v = velocity of electrons as they emerge out of the

(a) $eV_a = \frac{1}{2}mu^2$

$u = \sqrt{\frac{2eV_a}{m}}$

$u = \sqrt{\frac{2\times1.6\times10^{-19}\times2.0\times10^3}{9.11\times10^{-31}}} = 2.652 \times 10^7\ ms^{-1}$

(b) $ma = eE = \frac{eV}{d}$

$a = \frac{eV}{md} = \frac{1.6\times10^{-19}\times80}{9.11\times10^{-31}\times2\times10^{-2}}$

$a = 7.03 \times 10^{14}\ ms^{-1}$

The time, t taken by the electron in the field between the plates is given by

$t = \frac{l}{u}$

$t = \frac{5\times10^{-2}}{2.652\times10^7} = 1.88 \times 10^{-4}\ s$

$v = u + at$

$v_y = 0 + 7.03 \times 10^{14} \times 1.88 \times 10^{-4}$

$v_y = 1.325 \times 10^6\ ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2}$

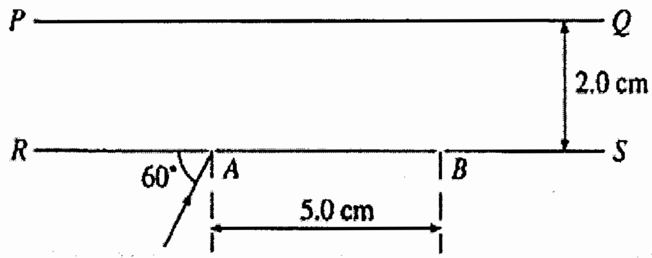
$v = \sqrt{(2.652 \times 10^7)^2 + (1.325 \times 10^8)^2}$

$v = 2.655 \times 10^7\ ms^{-1}$

(c) $\tan\theta = \frac{v_x}{v_y} = \frac{1.325\times10^6}{2.652\times10^7}$

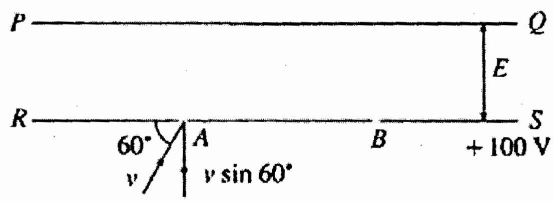
$\theta = \tan^{-1}\left(\frac{1.325\times10^6}{2.652\times10^7}\right) = 2.861^0$

4. The figure below shows two plane parallel metal plates PQ and RS in an evacuated enclosure. The separation of the plats is 2.0 cm and RS is maintained at a potential +100 V relative to PQ. A and B are two slits in the plate RS separated by $5.0\ cm$. A collimated beam of electrons of different kinetic energies is directed at A at an angle of 60^0 to the plate as shown.



   (a) Find the kinetic energy of the electron which just reach plate PQ
   (b) Find the velocity of the electrons that emerge from B

Solution



(a) Let $v =$ velocity of electrons that reach PQ, the component of the electric field between the plates PQ and RS is $v \sin 60^0$

In order to reach PQ, the energy of the electrons must be equal to the work done against the field

$$\frac{1}{2}m(v\sin 60^0)^2 = eV$$

K.E of electrons, $\frac{1}{2}mv^2 = \frac{eV}{(\sin 60^0)^2}$

$$= \frac{(1.6\times10^{-19})\times100}{(\sin 60^0)^2} = 2.1\times10^{-17}\,J$$

(b) Let $v_1 =$ velocity of electrons that enter at A and manage to emerge at B. The component of v_1 along the direction AB is $v_1 \cos 60^0$

Time taken for the electron to move from A to B

is $\quad t = \frac{5.0\times10^{-2}}{v_1\cos 60^0}$

Considering the motion perpendicular to AB,

Initial velocity, $u = v_1 \sin 60^0$

Acceleration, $a = -\frac{eE}{m}$

When the electrons emerge from B,

displacement $s = 0$

and time $t = \frac{5.0\times10^{-2}}{v_1\cos 60^0}$

Using the equation

$$s = ut + \frac{1}{2}at^2$$

$$0 = ut - \frac{1}{2}\left(\frac{eE}{m}\right)t^2$$

$$u = \frac{1}{2}\left(\frac{eE}{m}\right)t$$

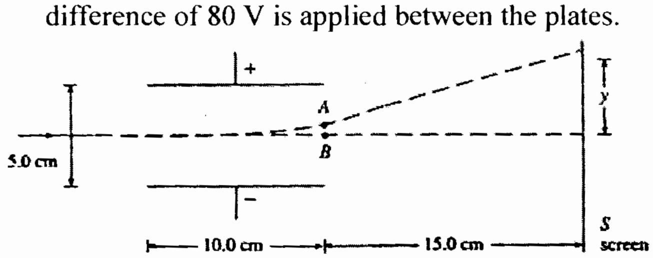
$$v_1 \sin 60^0 = \frac{1}{2}\left(\frac{eV}{md}\right)\left(\frac{5.0\times10^{-2}}{v_1\cos 60^0}\right)$$

$$v_1^2 = \frac{1}{2}\left[\frac{(1.6\times\times10^{-19})\times100\times(5\times10^{-2})}{9.11\times10^{-31}\times(2.0\times10^{-2})\sin 60^0\cos 60^0}\right]$$

$$v_1 = 7.12\times10^6\,ms^{-1}$$

Self -Evaluation exercise

1. A beam of electrons travelling at $1.2\times10^7\,ms^{-1}$ enters the region between the y-plates of an oscilloscope as shown below. A potential difference of 80 V is applied between the plates.



Calculate

(i) the time an electron in the beam takes to move through the region between the plates

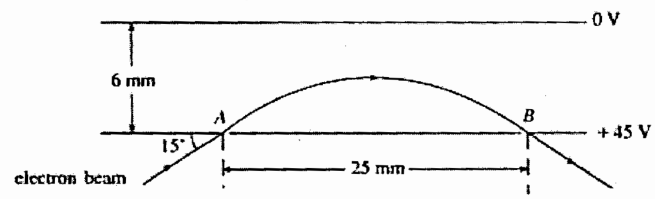
(ii) the vertical acceleration of the electron

(iii) the vertical distance AB of the electron beam as it leaves the region between the plates

(iv) the vertical displacement y of the electron beam on the screen S which is at a distance of $15.0\,cm$ from the end of the plates.

2. The figure below shows the principle used for a type of velocity selector for electrons. Two parallel metal plates are separated by a distance of $6.0\,mm$ in a vacuum. The lower plate is at a potential of $+45\,V$ relative to the upper plate and has two slits A and B which are 25 mm apart.

A collimated electron beam containing electrons of various speeds enter A at an angle of 15^0 to the lower plate. Electrons that emerge from B, all have the same velocity.



(i) What is the magnitude and direction of the acceleration of an electron when it is in between the plates (neglect the effects of gravity)?

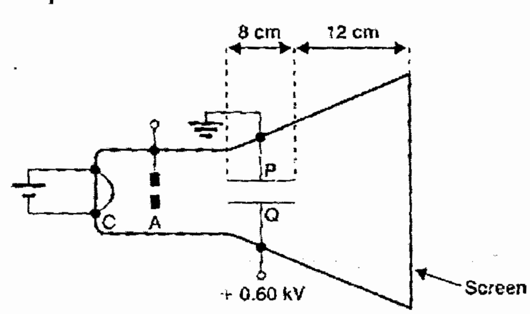
(ii) Calculate the speed of electrons that emerge from slit B

[Ans: (i) $1.32\times10^{15}\,ms^{-2}$ vertically downwards (ii) $8.12\times10^6\,ms^{-1}$]

3. In the cathode ray tube below, electrons are accelerated by a potential difference of 1.8 kV between cathode C and the anode A

(a) (i) Calculate the kinetic energy in J of the electrons after they have passed the anode.

(ii) Calculate the velocity of the electrons after they have passed the anode



(b) The plates P and Q are 8.0 cm long and are separated by a gap of 4.0 cm

(i) Calculate the force acting on the electron when it is between P and Q and state the direction of the force

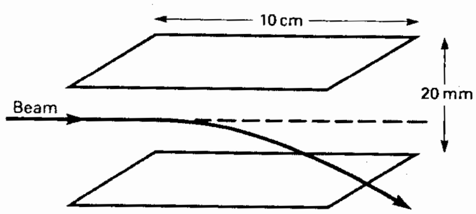
(ii) Calculate the time taken for an electron to pass between the plates

(iii) Calculate the vertical component of the velocity at the time the electron leaves the electric field between P and Q

(iv) Calculate the additional vertical displacement of the electron between the time it leaves the electric field intensity between P and Q and when it reaches the screen.

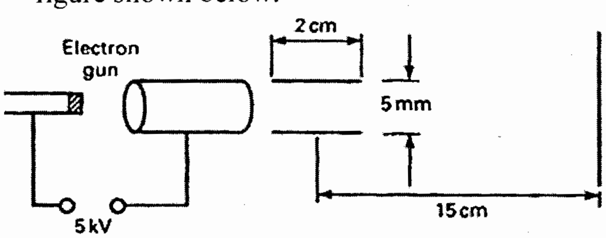
[Ans: (a)(i) 2.9 $\times$ $10^{-16}\,J$ (ii) 2.5 $\times$ $10^{-7}\,ms^{-1}$ (b)(i) 0.24 $\times 10^{-14}\,N$ (ii) 3.2 $\times$ $10^{-9}\,s$ (iii) 8.4 $\times$ $10^{6}\,ms^{-1}$ (iv) 4.0 cm]

4. Two parallel metal sheets of length 10 cm are separated by 20 mm in a vacuum. A narrow beam of electrons enters symmetrically between them as shown below.



When a p.d of 1000 V is applied between the plates the electron beam just misses one of the plates as it emerges. Calculate the speed of the electrons as they enter the gap. [Ans: 6.7 $\times$ $10^{7}\,ms^{-1}$]

5. Calculate the deflection sensitivity (deflection of spot in mm per volt potential difference) of a cathode ray tube from the following data from the figure shown below.



[Ans: 6 $\times$ $10^{-2}\,mmV^{-1}$]

6. In an evacuated tube, electrons are accelerated through a potential difference of 500 V. Calculate their final speed and consider whether this depends on the accelerating field being uniform.

After this acceleration, the electrons pass through a uniform electric field which is perpendicular to the direction of travel of the electrons as they enter the field. This electric field is produced by by applying a p.d of 10 V to two parallel plates which are 0.06 m long and 0.02 m apart. Determine the angular deflection of the electron beam. [Ans: 1.33 $\times$ $10^{7}\,ms^{-1}$, 1.71^{0}]

7. In a cathode ray tube, the electrons are accelerated through a potential difference of 500 V and then pass between deflecting plates which are 0.05 m long.

(i) Calculate the time it takes an electron to pass between the plates

(ii) If the p.d across the plates is 10 V and the plates are 1 cm apart, calculate the angle through which the electrons are deflected.

[Ans: (i) 3.8 $\times$ $10^{-9}\,s$ (ii) 2.9^{0}]

8. A potential difference of 600 V is maintained between two identical horizontal metal plates placed 4.0 cm apart one above the other in an evacuated vessel. Particles each with mass 9.1 $\times 10^{-31}\,kg$ and electric charge 1.6 $\times 10^{-19}\,C$ are emitted with negligible velocity from the plate at a lower potential. Calculate

(a) its acceleration

(b) the kinetic energy it acquires on reaching the other plate (Assume g = 10 ms^{-2})

[Ans: (a) 2.6 $\times 10^{13}\,ms^{-1}$ (b) 9.6 $\times$ $10^{-17}\,J$]

9. A beam of electrons is accelerated through a p.d of 500 V and then enters midway into a uniform electric field of strength 3.0 $\times 10^{3}\,Vm^{-1}$ created by two parallel plates each of length 2.00 $\times 10^{-2}\,m$. Calculate

(i) the speed of the electrons as they enter the field

(ii) the time that each electron spends in the field

(iii) the angle through which the electrons have been deflected by the time they emerge from the field.

[Ans: (i) 1.33 $\times 10^{7} ms^{-1}$ (ii) 1.51 $\times 10^{-9}\,s$ (iii) 3.4°]

10. A beam of electrons, moving with a velocity of 1.0 $\times 10^{7}\,ms^{-1}$ enters midway between two horizontal plates of length 5 cm and 2 cm apart. Calculate the p.d between the plates if the beam is deflected so that it just grazes the edge of the plate. (e/m = 1.8 $\times 10^{11}\,Ckg^{-1}$). [Ans: 89 V]

11. Two plane metal plates 4.0 cm long are held horizontally 3.0 cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300 V and the lower is earthed. Electrons having a velocity of 1.0 $\times 10^{-7}\,ms^{-1}$ are injected horizontally midway between the plates in a direction parallel to the 4.0 cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates. (e/m of the electron = 1.8 $\times 10^{11}\,Ckg^{-1}$)

[Ans: 14.4 $\times$ $10^{-3}\,m$]

Deflection of electrons in a magnetic field

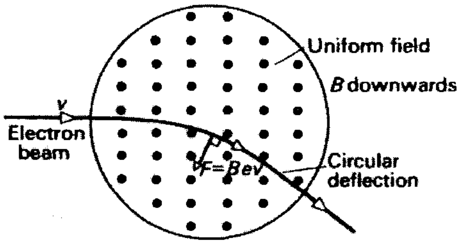
For a charged particle of mass m and charge Q projected at a velocity v in a uniform magnetic field of flux density B, the force F is given by

$$F = BQv$$

For a stream of electrons of mass m and charge e moving at the same velocity v, the force on them is

$$F = Bev$$

The direction of the force can be provided by Fleming's left-hand rule.



Unlike in the electric field, the force on a charged particle in a magnetic field is always perpendicular to the direction of motion at that particular time. This force will therefore not affect the magnitude of velocity but only its direction.

The force therefore will provide centripetal acceleration making the path to be circular.

$$F = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = Bev$$

$$mv = Ber$$

This equation can be used to obtain values of the variables v, B or r. It can also be used to determine the charge to mass ratio (specific charge) of the electron using a magnetic field.

$$\frac{e}{m} = \frac{v}{Br}$$

Examples

1. A beam of electrons moving with a uniform speed of $4 \times 10^7\ ms^{-1}$ is projected normal to the uniform magnetic field of flux density $1 \times 10^{-3}\ T$. What is the path of the beam in the magnetic field?

Solution

Since the electrons are released normal to the field, the electrons travel in a circular path

$$BeV = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} = \frac{9.11\times10^{-31}\times4\times10^7}{1\times10^{-3}\times1.6\times10^{-19}}$$

$$r = 0.2275\ m$$

2. An electron accelerated by a potential difference of $5.0\ kV$ enters a uniform magnetic field of $2.0 \times 10^{-2}\ T$ perpendicular to its direction of motion. Determine the radius of the path of electrons,

Solution

$$eV_a = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV_a}{m}} = \sqrt{\frac{2\times5.0\times10^3\times1.6\times10^{-19}}{9.11\times10^{-31}}}$$

$$v = 4.2 \times 10^7\ ms^{-1}$$

$$F_m = F_c$$

$$Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} = \frac{9.11\times10^{-31}\times4.2\times10^7}{2.0\times10^{-2}\times1.6\times10^{-19}}$$

$$r = 1.2 \times 10^{-2}\ m$$

3. A particle of charge $3.2 \times 10^{-19}\ C$ is accelerated through a p.d of $1.0 \times 10^4\ V$ and enters into a region of uniform magnetic field of flux density $0.5\ T$. The particle describes a circular path of radius $8.94\ cm$. Find
   (a) the kinetic energy of the particle on entering the magnetic field
   (b) the mass of the particle

Solution

(a) K.E $= qV$

$$= 3.2 \times 10^{-19} \times 1.0 \times 10^4$$

$$= 3.2 \times 10^{-15}\ J$$

(b) Magnetic force Bqv provides the centripetal force $\frac{mv^2}{r}$

$$Bev = \frac{mv^2}{r}$$

$$v = \frac{Ber}{m}$$

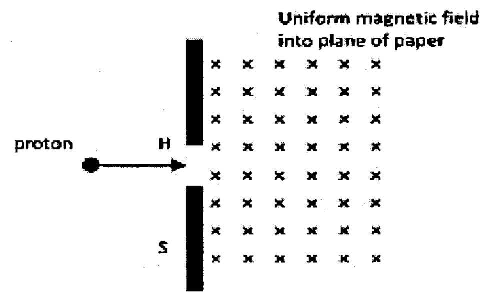
$$\text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ber}{m}\right)^2 = \frac{(Ber)^2}{2m}$$

$$m = \frac{(Ber)^2}{2\times K.E}$$

$$m = \frac{\left(0.5\times3.2\times10^{-19}\times8.94\times10^{-2}\right)^2}{2\times3.2\times10^{-15}}$$

$$m = 3.2 \times 10^{-26}\ kg$$

4. (a) Sketch the path of the proton in the figure below. Indicate the magnetic force acting on the proton at an arbitrary point on the path.



(b) Derive the speed of the proton when it is at the hole, H

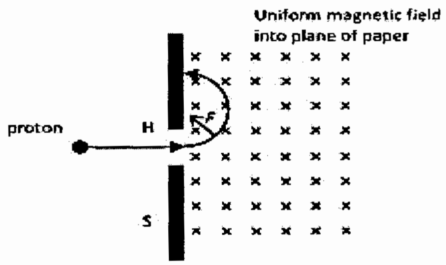
(c) Hence or otherwise, prove that the distance d from the hole H to the point where the proton hits the screen is given by the expression. $d = \sqrt{\frac{2V_o m}{qB^2}}$

(d) Explain why the proton did not gain kinetic as it moved through the magnetic field.

(e) Show that the time spend by the proton in the magnetic field is independent of its initial speed at H and distance d

Solution

(a) By Fleming's left-hand rule, the particle will experience a force towards the left of its path. i.e.



(b) Since the electron is accelerated from rest by the electric field of p.d V_o, the gain in the kinetic energy of the proton is equal to the work done by the electric potential energy.

$$\frac{1}{2}mv^2 = qV_o$$

$$v = \sqrt{\frac{2qV_o}{m}}$$

(c) In the magnetic field, the centripetal acceleration is provided by the magnetic force.

Magnetic force = centripetal force

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq} = \frac{m}{Bq}\sqrt{\frac{2qV_o}{m}} = \sqrt{\frac{m^2}{B^2q^2} \times \frac{2qV_o}{m}} = \sqrt{\frac{2V_o}{qB^2}}$$

$$\text{Point of impact} = d = 2r = 2\sqrt{\frac{2V_o}{qB^2}}$$

(d) For the proton to gain kinetic energy, work must be done on the proton. Since the magnetic force on the proton is always perpendicular to its displacement, there is no work done and hence its kinetic energy remains constant. The magnetic field provides the energy /force to change the direction of the proton but does not increase its speed.

(e) Let the time spent by the proton be t

$$t = \frac{\text{distance travelled}}{\text{speed of proton}} = \frac{\frac{1}{2}\times \text{circmference}}{\text{speed of proton}}$$

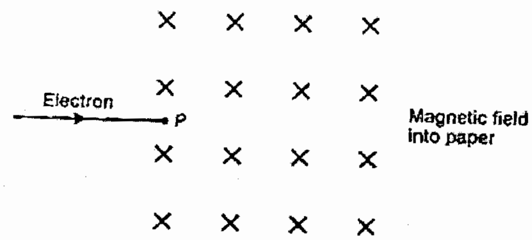
$$t = \frac{\pi d/2}{V} = \frac{\pi\sqrt{2V_o m/qB^2}}{\sqrt{2qV_o/m}} = \frac{\pi m}{qB}$$

The above expression is not a function of the initial speed.

Self-Evaluation exercise

1. An electron is accelerated through a p.d of 3600 V. It enters normally into a magnetic field and describes a circular arc of radius $0.1\ m$. Calculate
   (a) the speed of the electron
   (b) the magnitude of the magnetic field intensity

2. An electron travelling at $8.0 \times 10^6\ ms^{-1}$ in a vacuum enters a region of uniform magnetic field of flux density 30 mT as shown below.



   (i) Mark the direction of the force on the electron when it enters the magnetic field at P
   (ii) Calculate the magnitude of the force on the electron
   (iii) Explain why, when the electron is moving in the magnetic field, it follows part of a circular path
   (iv) Calculate the radius of this circular path
   [Ans: (ii) $3.8 \times 10^{-14}\ N$ (iv) $1.5\ mm$]

3. A beam of protons is accelerated from rest through a potential difference of 2000 V and then enters a uniform magnetic field which is perpendicular to the direction of the proton beam.
   If the flux density is 0.2 T, calculate the radius of the path which the beam describes. (Proton mass $= 1.7 \times 10^{-27}\ kg$) [Ans: 3.3 cm]

4. (a) An electron of mass m, charge e travels with speed v in a circle of radius r in a plane perpendicular to a uniform magnetic field of flux density B.
   (i) Write down an equation relating the centripetal and electromagnetic forces acting on the electron
   (ii) Show that the time for one orbit of the electron is given by
   $$T = \frac{2\pi m}{Be}$$
   (b) If the speed of the electron changed to $2v$, what effect, if any would this change on
   (i) the orbital radius, r
   (ii) the orbital period, T

5. A beam of electrons travelling with speed $1.2 \times 10^7\ ms^{-1}$ in an evacuated tube is made to move in a circular path of radius $0.048\ m$ by a uniform magnetic field of flux density $1.4\ mT$. Calculate,

in electron volts, the kinetic energy of the electron beam. [Ans: 400 eV]

6. When low energy electrons are moving at right angles to a uniform magnetic field of flux density $10^{-3}\ Wbm^{-2}$, they describe circular orbits 2.82×10^{7} times per second. Deduce the value of e/m.

   [Ans: $1.8 \times 10^{11}\ Ckg^{-1}$]

7. In the ionosphere electrons execute 1.4×10^{6} revolutions in a second. Find the strength of the magnetic field induction B in this region.

   [Ans: $5 \times 10^{-5}\ T$]

8. An ion, for which the charge per unit mass is $4.40 \times 10^{7}\ Ckg^{-1}$ has a velocity of $3.52 \times 10^{7}\ cms^{-1}$ and moves in a circular orbit in a magnetic field of induction $0.4\ Wbm^{-2}$. What will be the radius of the orbit? [Ans: 2 cm]

9. Photons, with a charge to mass ratio of $1.0 \times 10^{8}\ Ckg^{-1}$ are rotated in a circular orbit of radius r when they enter a uniform magnetic field of $0.5\ T$. Show that the number of revolutions per second, f is independent of r hence calculate f.

   [Ans: $8 \times 10^{6}\ revs^{-1}$]

10. An electron travelling at $1.0 \times 10^{7}\ ms^{-1}$ to the right enters a uniform magnetic field of flux density $5.0 \times 10^{-4}\ T$ directed into the paper. Calculate the radius r of the described path by electrons.

Crossed field

A crossed field is one where a uniform electric field applied to charged particles moving in the field would cause them to deflect in one direction.

When a uniform magnetic field is instead applied to the same charged particles, the particles would deflect in the opposite direction.

If both fields are applied at the same time such that the effect of one field is exactly cancelled out by the other field, then we have a crossed field. The charged particles will proceed in a straight line as if the fields are not there.

The electric force F_e will be equal to the magnetic force F_m.

$$eE = Bev$$

$$v = \frac{E}{B}$$

Only particles with a specific velocity will proceed in a straight line and thus a crossed field acts as a velocity selector.

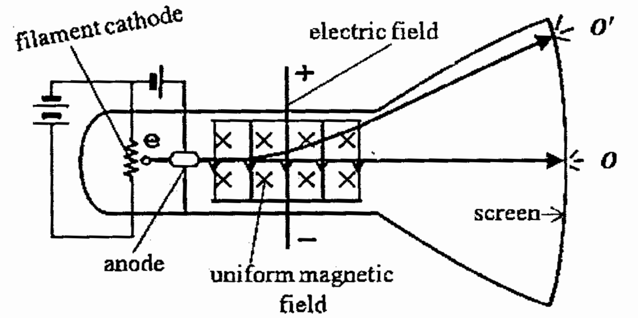
Specific charge

Specific charge of an electron is the ratio of charge to mass of an electron.

$$\frac{e}{m} = \frac{1.6\times10^{-19}}{9.11\times10^{-31}} = 1.76 \times 10^{11}\ Ckg^{-1}$$

Measurement of specific charge (e/m ratio)

J.J Thomson method



- Electrons are emitted from the filament cathode thermionically and are accelerated to the anode.
- With both the electric and magnetic fields switched off, the electrons will strike point O on the screen. Position O is noted.
- The magnetic field of known flux density B is applied to deflect the electron beam at O'.
- Electric field is also simultaneously applied and adjust until the electron beam goes back to O.
- The p.d across the plates V, plate separation, d and accelerating voltage V_a are noted.
- If u is the velocity of the electrons,

$$F_e = F_m$$

$$eE = Beu$$

$$u = \frac{E}{B} = \frac{V}{dB}$$

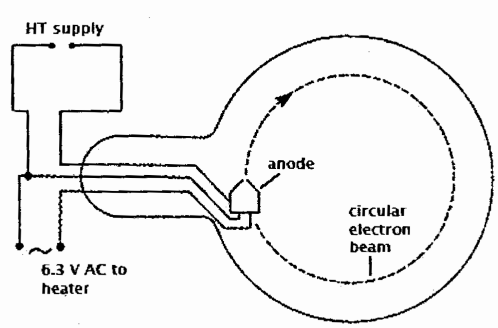
Work done = change in K.E

$$eV_a = \frac{1}{2}mu^2$$

$$\frac{e}{m} = \frac{u^2}{2V_a} = \frac{V^2}{2d^2B^2V_a}$$

Since the values V, d, B and V_a are known, the specific charge can be obtained.

Fine beam method



Electrons are ejected from the cathode and accelerated by a potential V_a at a conical anode having a small hole through which electrons pass.

The electrons are deflected by a uniform magnetic field in a circle of radius r back to the cathode

The path of the electrons is made visible by presence of hydrogen gas at low pressure in the tube and it emits visible light when hit by the fast-moving electrons.

$$Bev = \frac{mv^2}{r}$$

$$v = Br\left(\frac{e}{m}\right)$$

The anode potential V_a provides the kinetic energy required.

$$eV_a = \frac{1}{2}mv^2$$

$$v^2 = 2V_a\left(\frac{e}{m}\right)$$

$$B^2r^2\left(\frac{e}{m}\right)^2 = 2V_a\left(\frac{e}{m}\right)$$

$$\frac{e}{m} = \frac{2V_a^2}{B^2r^2}$$

Examples

1. In Thomson's experiment for measurement of specific charge of an electron, the beam remains undeflected when the electric field is $10^5\ Vm^{-1}$ and the magnetic field is $10^{-2}\ T$. The beam was originally accelerated through a potential difference of 285 V. Calculate the value of the specific charge of the electron.

Solution

Work done against electric field = change in kinetic energy of the electron

$$eV_a = \frac{1}{2}mv^2$$

$$\frac{e}{m} = \frac{v^2}{2V_a}$$

But under the action of crossed electric and magnetic fields

$$Bev = eE \Rightarrow v = \frac{E}{B}$$

$$\frac{e}{m} = \frac{E^2}{2B^2V} = \frac{(10^5)^2}{2\times(10^{-2})^2\times 285}$$

$$= 1.754\times 10^{11}\ C\ kg^{-1}$$

2. Electrons are accelerated from rest by a p.d of 100 V. The electron beam now enters normally a uniform electric field of intensity $10^5\ Vm^{-1}$.
 (i) Calculate the velocity of the electrons as they enter the field
 (ii) A magnetic field of flux density B is applied perpendicular to electric field and the path of electrons is unchanged from its original direction. Calculate B ($\frac{e}{m} = 1.8\times 10^{-11}\ Ckg^{-1}$)

Solution

(i) Work done = change in kinetic energy

$$eV_a = \frac{1}{2}mu^2$$

$$u = \sqrt{\frac{2eV_a}{m}} = \sqrt{2\left(\frac{e}{m}\right)V_a}$$

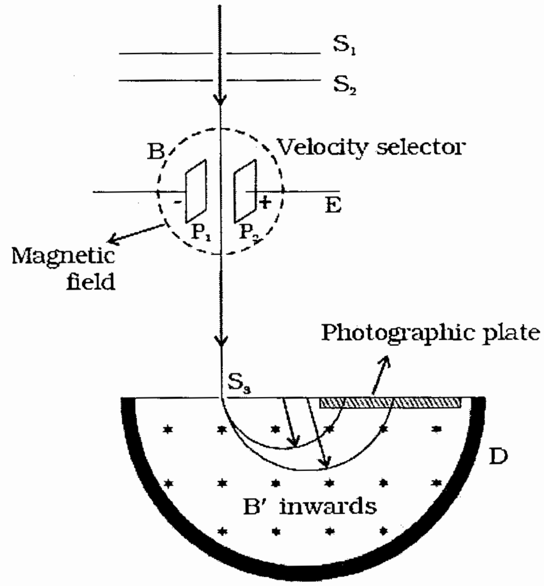
$$u = \sqrt{2\times 1.8\times 10^{11}\times 100} = 6\times 10^6\ ms^{-1}$$

(ii) $Ee = Beu$

$$B = \frac{E}{u} = \frac{10^5}{6\times 10^6} = 0.0167\ T$$

Bain bridge mass spectrometer (Determination of isotopic masses of nuclei)

Bainbridge mass spectrometer is an instrument used for the accurate determination of atomic masses



- A beam of positive of positive ions produced in a discharge tube is collimated into a fine beam by two narrow slits S_1 and S_2. This fine beam enters into the velocity selector.
- In the velocity selector, the force exerted by the electric field is equal to the force exerted by the magnetic field.

$$qE = Bqv$$

$$v = \frac{E}{B}$$

- Only those ions having the velocity v pass out of the velocity selector undeflected and then through slit S_3 to enter the evacuated deflection chamber D.
- These positive ions having the same velocity are subjected to another strong magnetic field of flux density B' at right angles to the plane of the paper acting inwards.

- These ions are deflected along circular paths of radius r and strike the photographic plate.
- The force due to the magnetic field provides the centripetal force.

$$B'qv = \frac{mv^2}{r}$$

$$m = \frac{B'qr}{v} = \frac{BB'qr}{E}$$

- Ions with different masses trace semi-circular paths of different radii and produce dark lines on the plate.
- The distance between the opening of the chamber and the position of the dark line gives the diameter, $2r$.
- Since q, B, B', E and r are known, the masses of the positive ions and hence the isotopic masses can be calculated.

Examples

1. In an experiment with a mass spectrometer, a singly charged positive ion is accelerated through a p.d of 1000 V. It enters a uniform magnetic field of flux density 0.1 T and moves in a circular orbit of radius $0.223\ m$. Calculate

(a) the speed of the ion
(b) the mass of the ion
(c) the mass number of the ion

Solution

(a) Work done on the ion = gain in k.e of the ion

$$eV = \frac{1}{2}mv^2$$

$$m = \frac{2eV}{v^2} \quad \text{....... (i)}$$

The magnetic force provides the centripetal force

$$Bev = \frac{mv^2}{r}$$

$$m = \frac{Ber}{v} \quad \text{........ (ii)}$$

Combining equations (i) and (ii);

$$\frac{2eV}{v^2} = \frac{Ber}{v}$$

$$v = \frac{2V}{Br} = \frac{2\times1000}{0.1\times0.223}$$

$$v = 8.97 \times 10^4\ ms^{-1}$$

(b) From (ii),

$$m = \frac{Ber}{v}$$

$$m = \frac{0.1\times1.6\times10^{-19}\times0.223}{8.97\times10^4}$$

$$m = 3.978 \times 10^{-26}\ kg$$

(c) 1 mole = 6.02×10^{23} ions

Mass number = mass of one mole in g

Mass of one mole

$= 3.978 \times 10^{-26} \times 6.02 \times 10^{23} \times 1000 = 24$ g

Or:

1 atomic mass unit $(u) = 1.66 \times 10^{-27}\ kg$

Mass, m of the ion $= \frac{3.978\times10^{-26}}{1.66\times10^{-27}} = 24$ g

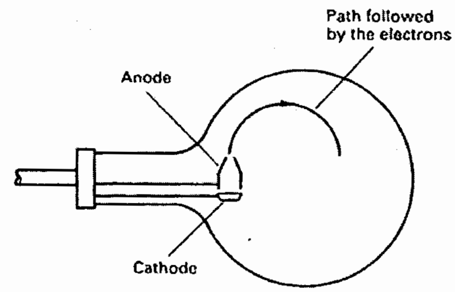
Self-Evaluation exercise

1. (a) A beam of singly ionized carbon atom is directed into a region where a magnetic and electric field are perpendicular to each other and to the beam. The fields have intensities 0.10 T and $1.0 \times 10^4 NC^{-1}$ respectively. If the beam is able to pass undeviated through this region, what is the velocity of the ions?

(b)The beam then enters a region where a magnetic field alone is acting. As a result, the beam describes an arc of radius 0.75 m. Calculate the flux density of this magnetic field.(mass of carbon atom = $2.0 \times 10^{-26} kg$)

[Ans: (a) $1.0 \times 10^5\ ms^{-1}$ (b) $0.017\ T$]

2. The diagram shows a type of cathode ray tube containing a small quantity of gas. Electrons from a hot cathode emerge from small hole in a conical shaped anode and the path subsequently followed is made visible by the gas in the tube



(a) the accelerating voltage is 5.0 kV. Calculate the speed of the electrons as they emerge from the anode.

(b) The apparatus is situated in a uniform magnetic field acting into the plane of the diagram. Explain why the path followed by the beam is circular. Calculate the radius of the path for a flux density of $2.0 \times 10^{-3}\ T$.

(c) Suggest a possible process by which the gas in the tube might make the path of the beam visible.

[Ans: (a) $4.2 \times 10^7\ ms^{-1}$ (b) 0.12 m]

3. A narrow beam passes undeviated through an electric field $E = 3 \times 10^4\ V/m$ and an overlapping magnetic field $B = 2 \times 10^{-3}\ Wb/m^2$. The electron motion, electric field and magnetic field are mutually perpendicular. Calculate the speed of the electron.

[Ans: $1.5 \times 10^7\ ms^{-1}$]

4. Electrons starting from rest and passed through a potential difference of 1000 V are found to acquire

a velocity of $1.88 \times 10^7\ ms^{-1}$. Calculate the ratio of the charge to mass of the electron

[Ans: $1.8 \times 10^{11}\ C\ kg^{-1}$]

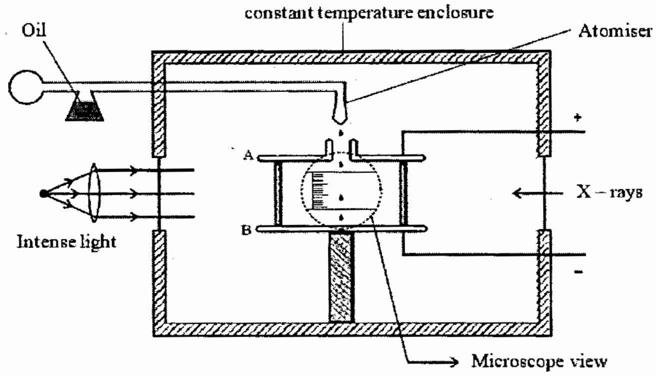
5. Singly ionized magnesium atoms enter into the velocity selector of a Bainbridge mass spectrometer having electric and magnetic field of 30 kV/m and 0.1 T respectively. Calculate the radii of the path followed by the three isotopes of mass numbers 24, 25 and 26 when the deflecting magnetic field is $0.5\,T$. (mass of a nucleon $= 1.67 \times 10^{-27}\ kg$, charge on the ion, $e = 1.6 \times 10^{-19}\ C$) [Ans: $R_1 = 0.1503\ m, R_2 = 0.1565\ m, R_3 = 0.1628\ m$]
6. An electron emitted from a hot cathode in an evacuated tube is accelerated by a p.d of 1000 V.
   (i) Calculate the speed acquired by the electron
   (ii) The electron now enters at right angles a uniform magnetic field of flux density $1.0 \times 10^{-3}\ T$. Determine its path
   (iii) Find the intensity of the uniform field which when applied perpendicular, the electron passes undeflected.

   [Ans: (i) $1.8 \times 10^7\ ms^{-1}$ (ii) $0.10\ m$ (iii) $1.8 \times 10^{-4}\ Vm^{-1}$]
7. (a) Describe the method for measuring the ratio of charge to mass (e/m) of an electron.
   (b) Calculate
   (i) the speed achieved by an electron accelerated in a vacuum through a p.d of $2.0 \times 10^3\ V$ and
   (ii) the magnetic flux density required to make an electron travelling with speed $8.0 \times 10^6\ ms^{-1}$ traverse a circular path of diameter $10.0 \times 10^{-2}\ m$.

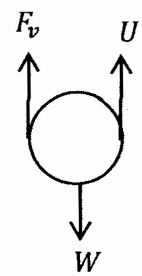
   [Ans: (b) (i) $2.65 \times 10^7\ ms^{-1}$ (ii) $9.09 \times 10^{-4}\ T$]
8. The mass of singly charged neon isotope ${}^{20}_{10}Ne^+$ is $3.3 \times 10^{-26} kg$. A beam of these ions enters a uniform transverse magnetic field of 0.3 T and describes a circular orbit of radius 0.22 m. What is
   (i) the velocity of the ions
   (ii) the potential difference which has been used to accelerate them to this velocity?

   [Ans: (i) $3.2 \times 10^5\ ms^{-1}$ (ii) 10.6 kV]

Millikan's oil drop experiment (measurement of electronic charge)



- Oil drops are sprayed through a hole in the upper o the two parallel plates A, B separated by a distance d
- These drops acquire charge by friction or by X-rays.
- With the electric field switched off, a suitable drop is selected and its terminal velocity v_0 is determined by measuring the distance it falls through a measured time.



$$W = F_v + U$$

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_0 + \frac{4}{3}\pi r^3 \sigma$$

$$\frac{4}{3}\pi r^3(\rho - \sigma)g = 6\pi\eta r v_0 \quad \text{..... (i)}$$

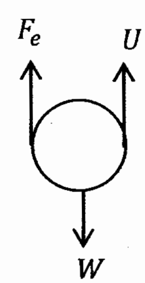
where ρ = density of oil

σ = density of air

η = coefficient of viscosity of air

r = radius of oil drop

- The electric field between the plates is switched on and the p.d V is adjusted until the oil drop remains stationary
- P.d is noted



$$W = U + F_e$$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma + qE$$

$$\frac{4}{3}\pi r^3 g(\rho - \sigma) = \frac{V}{d}q \quad ..... \text{(ii)}$$

Equating equations (i) and (ii);

$$6\pi r\eta r v_0 = \frac{Vq}{d}$$

$$q = \frac{6\pi r\eta v_0 d}{V}$$

- The experiment is repeated for several oil drops and the highest common multiple of the results gives the electronic charge.

Note: The value of q is found to be an integral multiple of the electronic charge e.

$$q = ne$$

The number of electron charges can be determined.

Precautions

- A non-volatile or low vapour pressure oil should be used to prevent evaporation which would change the mass of the oil drops
- A constant temperature enclosure is used to prevent convection currents.

Quantization of charge

Charge quantization means that charge cannot take any arbitrary values but only values that are integral multiples of the fundamental charge (charge of proton/electron). Any charge can be expressed as ne, where n is an integer and e is the fundamental unit of charge.

Examples

1. In Millikan's experiment, an oil drop of mass $4.9 \times 10^{-14}\ kg$ is balanced by applying a potential difference of $2\ kV$ between the two plates which are 8 mm apart. Calculate the number of elementary charges on the drop.

<u>**Solution**</u>

At balance, the electric force is equal to the weight of the oil drop.

$$qE = mg$$

$$q\frac{V}{d} = mg$$

$$q = \frac{mgd}{V} = \frac{4.9\times10^{-14}\times9.81\times8\times10^{-3}}{2\times10^3}$$

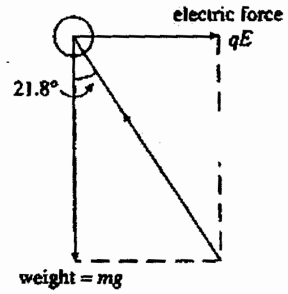
$$q = 1.92 \times 10^{-18}\ C$$

But $q = ne$

$$n = \frac{q}{e} = \frac{1.92\times10^{-18}}{1.6\times10^{-19}} = 12$$

2. An oil drop of mass $2 \times 10^{-15}\ kg$ falls with its terminal velocity between a pair of parallel vertical plates. When a potential gradient of $5.0 \times 10^4\ Vm^{-1}$ is applied between the plates, the direction of fall becomes inclined at an angle of 21.8^0 to the vertical. Calculate the charge on the oil drop

<u>**Solution**</u>



$$\tan 21.8^0 = \frac{qE}{mg}$$

$$q = \frac{mg\tan 21.8^0}{E}$$

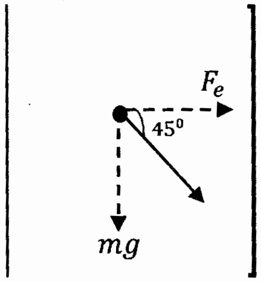
$$= \frac{(2.0\times10^{-15})\times9.81\times\tan 21.8^0}{5.0\times10^4} = 1.57 \times 10^{-19}\ C$$

3. An oil drop of mass $3.25 \times 10^{-15}\ kg$ falls vertically with a uniform velocity between vertical plates which are 2 cm apart. When a p.d of 1000 V is applied between the plates, the drop moves towards the negatively charged plate, its path being inclined at 45° to the vertical.
   (a) Explain why the vertical component of velocity remains constant
   (b) Calculate the charge on the oil drop and the number of electrons attached to it
   (c) If the path of the drop suddenly changes to 26.5° to the vertical then to the vertical, what conclusion can you draw

<u>**Solution**</u>

(a) This is because the applied electric force has no component in the vertical direction

(b)



$$\tan 45° = \frac{F_e}{mg} = \frac{qE}{mg}$$

$$\tan 45° = \frac{qV}{mgd}$$

$$q = \frac{mg\tan 45°}{V}$$

$$= \frac{3.25\times10^{-15}\times9.81\times2\times10^{-2}\tan 45°}{1000}$$

$$= 6.3765 \times 10^{-19}\ C$$

$$q = ne$$
$$6.3765 \times 10^{-19} = n \times 1.6 \times 10^{-19}$$
$$n = 4$$

(c) If $\theta = 26.5°, q = \frac{mg \tan 26.5°}{V}$

$$= \frac{3.25{\times}10^{-15}{\times}9.81{\times}2{\times}10^{-2} \tan 26.5°}{1000}$$
$$= 3.1792 \times 10^{-19}\ C$$
$$n = 2$$
$$q = 2e$$

If $\theta = 37°$, $q = \frac{mg \tan 37°}{V}$

$$= \frac{3.25{\times}10^{-15}{\times}9.81{\times}2{\times}10^{-2} \tan 37°}{1000}$$
$$q = 4.805 \times 10^{-19}\ C$$
$n = 3$ thus $q = 3e$

The charge changes from 4e to 2e to 3e.

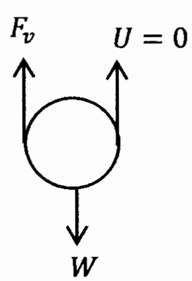
As the angle was changing, the number of electrons attached to the oil drop changed from 4, 2 to 3. The charge of the drop changed from 4e, 2e to 3e.

4. (a) Calculate the radius of a drop of oil, density $900\ kg\,m^{-3}$ which falls with a terminal velocity of $2.9 \times 10^{-2}\ cms^{-1}$ through air of viscosity $1.8 \times 10^{-5}\ Nsm^{-2}$. Ignore the density of air.

(b) If the charge on the drop is $-3e$, what p.d must be applied between two plates 5 cm apart for the drop to be held stationary between them?

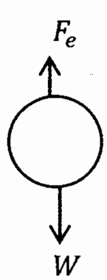
Solution

(a)



$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$
$$r = \sqrt{\frac{9\eta v}{2\rho g}} = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 2.9 \times 10^{-4}}{2 \times 900 \times 9.81}}$$
$$= 1.63 \times 10^{-6}\ m$$

(b)



$$qE = \frac{4}{3}\pi r^3 \rho g$$
$$3e \times \frac{V}{d} = \frac{4}{3}\pi r^3 \rho g$$
$$V = \frac{4\pi r^3 \rho g d}{9e}$$
$$= \frac{4\pi \times (1.6 \times 10^{-6})^3 \times 900 \times 9.81 \times 5 \times 10^{-2}}{9 \times 1.6 \times 10^{-19}}$$
$$= 5779.3\ V$$

5. In a Millikan's oil drop experiment, a charged oil drop of radius $9.2 \times 10^{-7}\ m$ and density 800 $kg\,m^{-3}$ is held stationary in an electric field of intensity $4.0 \times 10^4\ Vm^{-1}$.

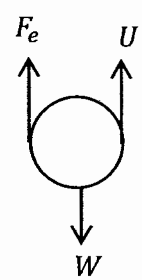
(i) How many charges are on the drop?

(ii) Find the electric field intensity that can be applied vertically to move the drop with a velocity $0.005\ ms^{-1}$ upwards.

[density of air is $1.29\ kg\,m^{-3}$, coefficient of viscosity of air is $1.8 \times 10^{-5}\ Nsm^{-1}$]

Solution

(i) With the electric field

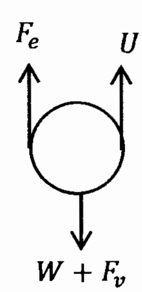


$$F_e + U = W$$
$$qE + \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 \rho g$$
$$qE = \frac{4}{3}\pi r^3 g(\rho - \sigma)$$
$$q \times 4 \times 10^4 = \frac{4}{3}\pi \times (9.2 \times 10^{-7})^3 \times 9.81(800 - 1.29)$$
$$q = 6.38925 \times 10^{-19}\ C$$

But $q = ne$

$$n = \frac{6.38925{\times}10^{-19}}{1.6{\times}10^{-19}} = 3.993$$
$$n = 4$$

(ii) When the drop moves upwards,



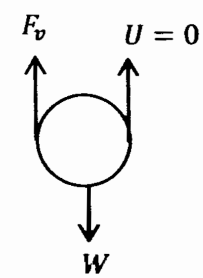
$$F_e + U = F_v + W$$
$$qE + \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \rho$$

$6.38925 \times 10^{-19}E + \frac{4}{3}\pi(9.2 \times 10^{-7})^3 \times 1.29$
$= 6\pi \times 1.8 \times 10^{-5} \times 0.05 \times 9.2 \times 10^{-7} +$
$\frac{4}{3}\pi(9.2 \times 10^{-7})^3 \times 800$
$6.38925 \times 10^{-19}E = 1.5633 \times 10^{-12}$
$E = 2.447 \times 10^{6}\ Vm^{-1}$

6. In the measurement of electronic charge by Millikan's apparatus, a p.d of 1.6 kV is applied between two horizontal plates 14 mm apart. With the p.d off, the oil drop is observed to fall with a constant velocity of $4 \times 10^{-2}\ cm\,s^{-1}$. When the p.d is switched on, the drop rises with a velocity of $8 \times 10^{-3}\ cm\,s^{-1}$. If the mass of the oil drop is $1 \times 10^{-14}\ kg$, find the number of electron charges on the drop. (Assume that air resistance is proportional to the velocity of the oil drop and neglect the upward up thrust due to air)

Solution

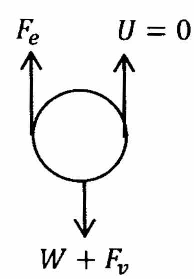
1st case: With p.d off



$F_v \propto v$
$F_v = kv$
$F_v = mg$
$kv_1 = mg$

$k = \frac{mg}{v_1} = \frac{1\times10^{14}\times9.81}{4\times10^{-4}} = 2.45 \times 10^{-10}\ Nsm^{-1}$

2nd case: With p.d on



$F_v = kv_2$
$F_e = F_v + W$
$qE = mg + kv_2$
$\frac{qV}{d} = mg + kv_2$

$\frac{1600}{14\times10^{-3}}q$
$= 1 \times 10^{-14} \times 9.81 + 2.45 \times 10^{-10} \times 8 \times 10^{-5}$
$q = 1.03 \times 10^{-18}\ C$

$q = ne$
$n = \frac{1.03\times10^{-18}}{1.6\times10^{-19}} = 6.4378 \approx 6$

Self-Evaluation exercise

1. (a) Explain what is meant by quantization of charge
   (b) A cloud of oil droplets is formed between two horizontal parallel metal plates. Explain the following observations
   (i) In the absence of an electric field between the plates, all the oil droplets fall slowly at uniform speeds.
   (ii) On applying a vertical electric field, some droplet speeds are unaltered, some are increased downwards whereas some droplets move upwards.
2. (a) A charged oil drop falls at constant speed in the Millikan oil drop experiment when there is no p.d between the plates. Explain this.
   (b) Such an oil drop of mass $4.0 \times 10^{-15}\ kg$ is held stationary when an electric field is applied between the two horizontal plates. If the drop carries 6 electric charges, calculate the value of the electric field strength. [Ans: 45 kVm^{-1}]
3. In a measurement of the electron charge by Millikan's method, a potential difference of 1.5 kV can be applied between horizontal parallel metal plates 12 mm apart. With the field switched off, a drop of oil of mass $10^{-14}\ kg$ is observed to fall with constant velocity 400 μms^{-1}. When the field is switched on, the drop rises with constant velocity 80 μms^{-1}. How many electron charges are there on the drop? (You may assume that the air resistance is proportional to the velocity of the drop and that the air buoyancy may be neglected)
   [Ans: 6]
4. A small oil drop, carrying a negative electric charge, is falling in air with a uniform speed of $8.0 \times 10^{-5}ms^{-1}$ between two horizontal parallel plates. The upper plate is maintained at a positive potential relative to the lower one.
   (a) Draw a diagram showing all the forces acting on the drop, stating the cause of each force
   (b) Use the following data to determine the charge on the oil drop
   Radius of drop = $1.60 \times 10^{-6}\ m$
   Density of oil = $800\ kg\,m^{-3}$
   Density of air = $1.30\ kg\,m^{-3}$
   Viscosity of air = $1.80 \times 10^{-5}\ Nsm^{-2}$
   Distance between plates = $1.00 \times 10^{-2}\ m$

P.d between plate = 2000 V

[Ans: 4.68 $\times\ 10^{-19}\ C$]

5. Calculate the potential difference in volts necessary to be maintained between two horizontal conducting plates, one 0.50 cm above the other, so that a small oil drop, of mass 1.31 $\times\ 10^{-11}$ g with two electrons attached to it, remains in equilibrium between them. Take the electronic charge to be $-1.6\times 10^{-19}\ C$. Which plate would be at the positive potential?

[Ans: 2.0006 V]

6. In a Millikan-type apparatus, the horizontal plates are 1.5 cm apart. With the electric field switched off, an oil drop is observed to fall with a steady velocity of 2.5 $\times\ 10^{-2}\ cm\ s^{-1}$. When the field is switched on the upper plate being positive, the drop just remains stationary when the p.d between the two plates is 1500 V.
   (a) Calculate the radius of the drop
   (b) How many electronic charges does it carry?
   (c) If the p.d between the two plates remains unchanged, with what velocity will the drop move when it has collected two more electrons as a result of exposure to ionizing radiation?

   [Ans: (a) 1.5 $\times 10^{-6}\ m$ (b) 1 (c) 5.0 $\times$ $10^{-4}\ ms^{-1}$]

7. An oil drop of radius 1.0 $\times 10^{-3}\ cm$ falls freely in air midway between two vertical parallel metal plates of large extent, which are 0.50 cm apart and its terminal velocity is 1.066 cms^{-1}. When a potential difference of 3000 V is applied between the plates, the path of the drop becomes a straight line inclined at an angle of $31^0\ 36'$ to the vertical. Find the charge on the drop. (Assume the viscosity of air to be 1.816 $\times 10^{-5}\ kg\,m^{-1}s^{-1}$)

   [Ans: 3.72 $\times 10^{-17}\ C$]

8. In Millikan's experiment, an oil drop of mass 1.92 $\times\ 10^{-14}\ kg$ is stationary in the space between two horizontal plates which are 0.02 m apart, the upper plate being earthed and the lower one at a potential of $-6000\ V$.
   (a) State, with the reason, the sign of the electric charge on the drop
   (b) Neglecting the buoyancy of the air, calculate the magnitude of the charge
   (c) With no change in the potentials of the plates, the drop suddenly moves upwards and attains a uniform velocity. Explain why
      (i) the drop moves
      (ii) the velocity becomes uniform

   [Ans: 6.4 $\times\ 10^{-19}\ C$]

X-RAYS

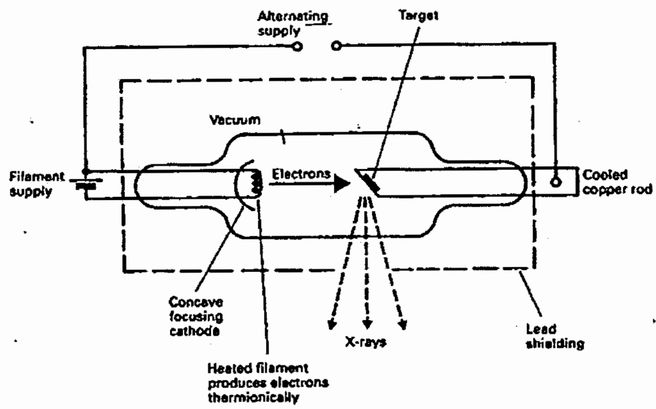
X-rays are electromagnetic waves of short wavelength

Production of X-rays

X-rays are produced when fast moving electrons strike a metal target of suitable material

the basic requirements for the production of X-rays are
(i) source of electrons
(ii) effective means of accelerating electrons
(iii) target of suitable material of high melting point

Modern X-ray tube



It consists of a highly evacuated glass tube containing a cathode and anode target.

The cathode is heated by passing current through from a low-tension battery. The electrons are emitted by the process of thermionic emission from the cathode.

The target consists of a copper block in which a piece of tungsten is fixed. The anode should have the following characteristics
(i) High atomic weight – to produce hard X-rays
(ii) High melting point – so that it is not melted due to the bombardment of fast moving electrons, which cause a lot of heat generation.
(iii) High thermal conductivity to carry away the heat generated.

A high p.d is applied between the filament and the target. Due to this high potential difference, the electrons emitted from the filament are accelerated.

When these accelerated electrons strike the target, they give up their kinetic energy as heat and thereby produce X-rays.

The heat is carried away by the cooling fins or water running through the channels.

Energy conversion in an X-ray tube

Electrical energy (a.c voltage) → Heat energy (filament) → Kinetic energy (moving electrons) → Heat (target)

Intensity of X-rays

The intensity of X-rays depends upon the number of electrons striking the target i.e. the rate of emission of electrons from the filament. This can be controlled by varying the filament current.

Types of X-rays

There are two types of X-rays with different penetrating power which depend on the kinetic energy on impact with the target i.e. soft X-rays with low penetrating power and hard X-rays with high penetrating power.

Soft X-rays

These are produced using a low accelerating p.d which results into electrons of small kinetic energy giving soft X-rays of long wavelength and low penetrating power.

Hard X-rays

They are produced using a high p.d which results into electrons of high kinetic energy giving hard X-rays of short wavelength and high penetrating power

Properties of X-rays

- X-rays are electromagnetic waves of very short wavelength. They travel in straight lines with the velocity of light and are invisible to eyes.
- They undergo reflection, refraction, interference, diffraction and polarization
- They are not deflected by electric and magnetic fields. This indicates that X-rays do not have charged particles.
- They ionize the gas through which they pass
- They affect photographic plates
- X-rays can penetrate the substances which are opaque to ordinary light e.g. wood, flesh, thick paper, thin sheets of metals.
- When X-rays fall on certain materials, they liberate photoelectrons (photoelectric effect)
- X-rays have destructive effect on living tissue
- X-rays do not pass through heavy metals such as lead and bones. If such objects are placed in their path, they cast their shadow.

Detection of X-rays

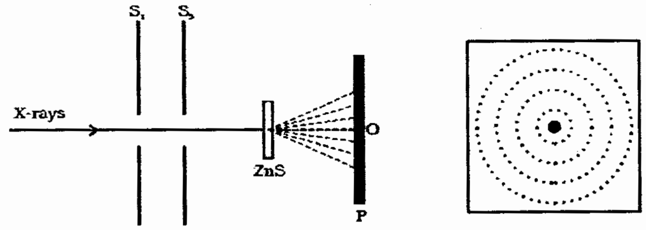
The basic properties which are generally used for detection of X-rays are

(i) darkening of a photographic plate.

(ii) ionization produced by X-rays in a gas or vapour. An ionization chamber which utilizes the property of ionization is generally used to detected and measure the intensity of X-rays

Diffraction of X-rays

Diffraction is the spreading of waves after passing through openings or obstacles. A crystal can act as a three-dimensional grating for an X-ray beam. The experimental arrangement to produce diffraction in X-rays is shown below.



X-rays from the X-ray tube are collimated into a fine beam by two slits s_1 and s_2.

The beam is now allowed to pass through a zinc sulphide crystal.

The emergent rays are made to fall on a photographic plate P.

The diffraction pattern so obtained consists of a central spot at O and a series of spots arranged in a definite pattern about O.

The central spot is due to the direct beam whereas the regularly arranged spots are due to the diffraction pattern from the atoms of the various crystal planes.

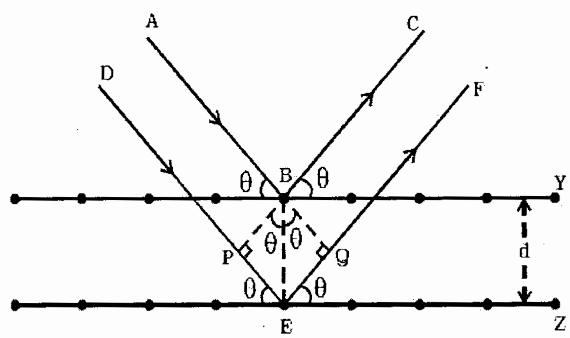
Note: This experiment was established following two important facts.

(i) X-rays are electromagnetic waves of extremely short wavelength

(ii) The atoms in a crystal are arranged in a regular three-dimensional lattice.

Bragg's law

Consider a homogeneous x-ray of wavelength λ incident on a crystal at a glancing angle θ. The incident rays AB and DE after reflection from the lattice planes Y and Z travel along BC and EF respectively as shown below.



Let the crystal lattice spacing between the planes be d. Path difference between two waves ABC and DEF

$= PE + EQ$

From $\Delta PBE, \sin\theta = \frac{PE}{BE}$

$\Rightarrow PE = BE\sin\theta = d\sin\theta$

From $\Delta QBE, \sin\theta = \frac{EQ}{BE}$

$\Rightarrow EQ = BE\sin\theta = d\sin\theta$

Path difference $= PE + EQ$

$= d\sin\theta + d\sin\theta = 2d\sin\theta$

For the scattered waves to be in phase (constructive interference), the path difference must be an integral multiple of the wavelength.

$2d\sin\theta = n\lambda$ where $n = 1, 2, 3$ ....

n is the order of diffraction

This is known as Bragg's law

Density of crystals

Density of crystal $= \frac{\text{Mass}}{\text{Volume}}$

Mass of crystal (in kg) $= \frac{\text{Molecular weight}}{1000} = \frac{M}{1000}$

Volume of crystal $= N_A \times d^3$ where d is the atomic spacing or length of crystal and N_A is the Avogadro's number.

$$\rho = \frac{M}{1000N_A d^3}$$

From Bragg's law, $2d\sin\theta = n\lambda$

$$d = \frac{n\lambda}{2\sin\theta}$$

$$\rho = \frac{M}{1000N_A\left(\frac{n\lambda}{2\sin\theta}\right)^3}$$

$$\rho = \frac{8M\sin^3\theta}{1000N_A(n\lambda)^3}$$

$$\rho = \frac{M\sin^3\theta}{125N_A(n\lambda)^3}$$

Examples

1. Calculate the longest wavelength that can be analysed by a rock salt crystal of spacing 2.82×10^{-10} m in the first order.

Solution

From $2d\sin\theta = n\lambda$, $\lambda = \frac{2d\sin\theta}{n}$

For longest wavelength, $\lambda_{max} = \frac{2d(\sin\theta)_{max}}{1}$

$(\sin\theta)_{max} = 1$

$\lambda_{max} = 2 \times 2.82 \times 10^{-10} \times 1$

$\lambda_{max} = 5.64 \times 10^{-10}\,m$

2. A second order diffraction image is obtained by reflection of x-rays at atomic planes of a crystal at a glancing angle of $11°24'$. Calculate the atomic spacing if the wavelength of X-rays is $4 \times 10^{-11}\,m$.

Solution

$\theta = 11°24' = \left(11 + \frac{24}{60}\right)^{\circ} = 11.4°$

$n = 2$ and $\lambda = 4 \times 10^{-11}\,m$

$2d\sin\theta = n\lambda$

$2d\sin 11.4 = 2 \times 4 \times 10^{-11}$

$d = 2.024 \times 10^{-10}\,m$

3. In Bragg's spectrometer, the glancing angle for th first order spectrum was observed to be 8 Calculate the wavelength of X-ray if $d = 2.82$ $10^{-10}m$. At what angle will the second maximu occur?

Solution

For $n = 1, \theta_1 = 8°, d = 2.82 \times 10^{-10}\,m, \lambda = ?$

For $n = 1, \theta_1 = ?$

When $n = 1$, $2d\sin\theta = 1 \times \lambda$

$\lambda = 2 \times 2.82 \times 10^{-10}\sin 8°$

$\lambda = 7.849 \times 10^{-11}\,m$

When $n = 2$, $2d\sin\theta_2 = 2\lambda$

$\sin\theta_2 = \frac{\lambda}{d} = \frac{7.849\times10^{-11}}{2.82\times10^{-10}} = 0.2783$

$\theta_2 = \sin^{-1} 0.2783 = 16.16°$

4. X-rays of wavelength $2 \times 10^{-10}\,m$ are incident c a set cubic planes of a crystal whose molar mass 45 g. First order diffraction maxima occurs at glancing angle of 15.4°. Calculate the density (the crystal.

Solution

$\lambda = 2 \times 10^{-10}\,m$, $m = 45$ g, $n = 1$, $\theta = 15.4°$

$2d\sin\theta = n\lambda$

$2d\sin 15.4^0 = 1 \times 2 \times 10^{-10}$

$d = 3.76 \times 10^{-10}\,m$

Volume of 1 atom $= d^3$

$= (3.76 \times 10^{-10})^3 = 5.32 \times 10^{-29}\,m^3$

6.02×10^{23} atoms have a mass $= 45 \times 10^{-3}\,kg$

Mass of 1 atom $= \frac{45\times10^{-3}}{6.02\times10^{23}} = 7.48 \times 10^{-26}\,kg$

Density of crystal $= \frac{\text{mass of 1 atom}}{\text{volume of 1 atom}}$

$= \frac{7.48\times10^{-26}}{5.32\times10^{-29}} = 1.41 \times 10^3\,kg\,m^{-3}$

OR

Volume of 1 mole $= 5.32 \times 10^{-29} \times 6.02 \times 10^{23}$

$= 3.203 \times 10^{-5}\,m^3$

Density of crystal $= \frac{\text{mass of 1 mole}}{\text{volume of 1 mole}}$

$= \frac{45\times10^{-3}}{3.203\times10^{-5}}$

$= 1.405 \times 10^3\,kg\,m^{-3}$

5. The density of sodium chloride (NaCl) $2.17\,g\,cm^{-3}$ and the mass of one of it is 58.5 g. the solid state, each molecule of NaCl consists two ions, one Na^+ and one Cl^-, which are arrang alternately in a cubic manner.

(a) How many ions are there in a crystal of NaCl in the form of a cube of side 1.0 cm?

(b) What is the separation between adjacent ions?

(c) Find the smallest angle formed with a plane of the ions that will allow X-rays of wavelength $1.54 \times 10^{10}\ m$ to be reflected by a crystal of NaCl

(d) How many orders of reflection are obtained from these planes?

Solution

(a) Number of NaCl molecules in one mole

$$= 6.02 \times 10^{23}$$

Number of ions in 58.5 g NaCl = $2 \times 6.02 \times 10^{23}$

Hence number of ions in a cube of side 1.0 cm and having a mass of 2.17 g

$$= \frac{2.17}{58.5} \times 2 \times 6.02 \times 10^{23} = 4.47 \times 10^{22}$$

(b) Number of ions on each side of length 1.0 cm of the cube

$$= (4.47 \times 10^{22})^{\frac{1}{3}} = 3.55 \times 10^{7}$$

Distance between two adjacent ions,

$$d = \frac{1.0\times10^{-2}}{3.55\times10^{7}} = 2.82 \times 10^{-10}\ m$$

(c) Using Bragg's equation,

$$2d \sin\theta = n\lambda$$

Smallest Bragg's angle is when $n = 1$.

$$\sin\theta = \frac{1\times\lambda}{2d} = \frac{1.54\times10^{-10}}{2\times2.82\times10^{-10}}$$

$$\theta = 15.83°$$

(d) From $2d \sin\theta = n\lambda$

Maximum value of $\theta = 90°$

$$n = \frac{2d}{\lambda} = \frac{2\times2.82\times10^{-10}}{1.54\times10^{-10}} = 3.66$$

$$n = 4$$

4 orders of reflection are possible

6. The potential difference between the cathode and the anode of an X-ray tube is 5.0×10^{-4} V. If only 0.4% of the kinetic energy of the electrons is converted into X-rays and the rest is dissipated as heat in the target at a rate of 600W, find the
   (i) current that flows
   (ii) speed of the electrons striking the target

Solution

(i) Percentage of heat generated per second

$$= 100 - 0.4 = 99.6\%$$

Heat per second at target $= \frac{99.6}{100} \times IV$

$$600 = \frac{99.6}{100} \times I \times 5.0 \times 10^{-4}$$

$$I = 1.21 \times 10^{6}\ A$$

(ii) Energy of incident electrons $= \frac{1}{2}mu^2$

$$\frac{1}{2}mu^2 = eV$$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times5.0\times10^{-4}}{9.11\times10^{-31}}}$$

$$= 1.33 \times 10^{4}\ ms^{-1}$$

7. An X-ray tube is operated at 20 kV with an electron current of 16 mA in the tube. Estimate the
   (i) number of electrons hitting the target per second
   (ii) rate of production of heat, assuming 99.5% of the kinetic energy of electron is converted to heat.

Solution

(i) $$I = \frac{Q}{t} = \frac{ne}{t}$$

$$\frac{n}{t} = \frac{I}{e} = \frac{1.6\times10^{-3}}{1.6\times10^{-19}}$$

$$= 1.0 \times 10^{17} \text{ electrons per second.}$$

(ii) Energy of an electron $= eV$

For 1.0×10^{17} electrons per second,

energy per second

$$= 1.0 \times 10^{17} \times 1.6 \times 10^{-19} \times 20 \times 10^{3}$$

$$= 320\ W$$

Rate of heat production $= \frac{99.5}{100} \times 320 = 318.4\ W$

8. The current in a water-cooled X-ray tube operating at 60 kV is 30 mA. 99% of the energy supplied to the tube is converted into heat at the target which has water flowing in it at $0.06\ kgs^{-1}$. Calculate the
   (i) rate at which energy is supplied to the tube
   (ii) increase in temperature of the cooling water of S.H.C $4200\ J\ kg^{-1}K^{-1}$
   (iii) minimum wavelength of the X-rays radiated.

Solution

$$I = 30\ mA, V = 60\ kV$$

$$\frac{99}{100}E = Heat, \quad \frac{m}{t} = 0.06\ kg\,s^{-1}$$

(i) $$\frac{E}{t} = P = IV = 60 \times 10^{3} \times 30 \times 10^{-3}$$

$$= 1800\ W$$

(ii) Heat generated per second $= \frac{99}{100} \times P = \frac{m}{t}c\theta$

$$\frac{99}{100} \times 1800 = 0.06 \times 4200\ \theta$$

$$\theta = 7.07\ °C$$

(iii) $$E = hf = eV$$

$$\frac{hc}{\lambda_{min}} = eV$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times60\times10^{3}}$$

$$\lambda_{min} = 2.07 \times 10^{-11}\ m$$

9. An X-ray tube is operated at 50 kV and 20 mA. If 1% of the total energy supplied is emitted as X-ray radiation, calculate the

(i) maximum frequency of the emitted radiation

(ii) rate at which heat must be removed from the target in order to keep at a steady temperature

Solution

(i) The most energetic X-rays are those produced by electrons that lose all their kinetic energy on impact with the target.

$$hf_{max} = eV$$

$$f_{max} = \frac{eV}{h} = \frac{1.6\times10^{-19}\times50\times10^{3}}{6.6\times10^{-34}} = 1.21 \times 10^{19}\ Hz$$

(ii) Power supplied = IV

Power supplied = $20 \times 10^{-3} \times 50 \times 10^{3}$

$= 1000\ W$

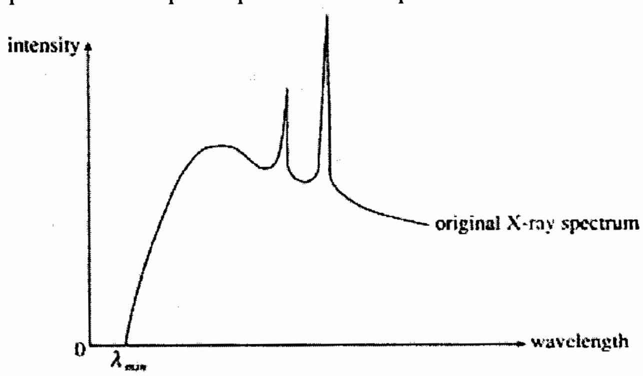
Power converted to heat = $\frac{99}{100} \times 1000 = 990\ W$

X-ray spectra

There are two types of X-ray spectra i.e.

- Continuous spectrum which is independent of the target material
- Line spectrum which is dependent on the target material

It consists of a continuous spectrum/background of definite cut-off wavelength. On top of the continuous spectrum is superimposed a line spectrum.



Line spectrum

It is formed when a highly energetic electron knocks out of the inner most K or L shells. This puts the atom in excited state.

To restore stability, an electron from the outer shell moves to fill the vacancies left. Electron transition to the vacancies left results in the emission of X-rays of definite wavelength hence a line spectrum.

The frequency of the radiation depends upon the characteristics of the target material and increasing the voltage increases the number of lines produced.

Continuous spectrum

It is a result of multiple collisions of the energetic electrons with target atoms. Different amounts of energy are lost during these collisions giving off X-rays of different wavelengths ranging from a certain minimum to infinity.

Cut off wavelength (minimum wavelength)

This is the minimum wavelength obtained when an energetic electron loses all its kinetic energy in a single collision with the target atom.

X-ray photon given off has maximum energy with the shortest possible wavelength.

$$E = hf = eV$$

$$\frac{hc}{\lambda_{min}} = eV$$

$$\lambda_{min} = \frac{hc}{eV}$$

Note: The majority of the electrons lose their kinetic energy too gradually for X-rays to be emitted and therefore nearly emit infrared radiations which are absorbed by the target and converted into heat at the anode.

Applications of X-rays

Medical applications

- X-rays are being widely used for detecting fractures, tumors, the presence of foreign matter like bullets, etc. in the human body.
- X-rays are used for diagnosis of tuberculosis, stones in kidneys, gall bladder, etc.
- Many types of skin diseases, malignant sores, cancer and tumors are cured by controlled exposure of X-rays of suitable quality
- Hard X-rays are used to destroy tumors very deep inside the body
- They are used to investigate broken bones

Industrial applications

- X-rays are used to detect the defects or laws within a material
- X-rays can be used for testing the homogeneity of welded joints, insulating materials, etc.
- X-rays are used to analyse the structure of alloys and other composite bodies
- X-rays are used to study the structure of materials like rubber, cellulose, plastic fibres, etc.

Scientific research

- X-rays are used for studying the structure of crystalline solids and alloys.
- X-rays are used for identification of chemical elements including determination of their atomic numbers

- X-rays can be used for analyzing the structure of complex molecules by examining their X-ray diffraction pattern.

Dangers of X-rays

- They damage body cells
- They cause barrenness/ deformed off springs when sprayed on reproductive areas
- Cause mutations leading to abnormalities
- They cause and accelerate skin and blood cancer

Precautions when handling X-rays

- Too much unnecessary exposure to body must be avoided
- Personnel handling X-rays must wear protective clothing coated with a layer of lead
- X-ray equipment must be stored in thick lead containers

Self-Evaluation exercise

1. Calculate the maximum frequency of X-rays emitted by an X-ray tube using an accelerating voltage of 33.0 kV. [Ans: 8.0 $\times 10^{18}$ Hz]
2. A certain X-ray tube operates at 110 kV. Calculate the shortest wavelength of X-rays produced.
[Ans: 1.1 $\times$ 10^{-11} m]
3. An X-ray tube has an electron beam current 10 mA and an accelerating voltage is 50 kV. If only 0.5% of the power is converted into X-rays, calculate the
(i) the input power
(ii) the power lost in tube as heat
(iii) minimum wavelength of the X-rays produced
[Ans: (i) 500 W (ii) 497.5 W (iii) 0.025 nm]
4. A beam of X-rays of wavelength 0.154 nm is diffracted by a crystal. For first-order diffraction by a certain set of planes, it is found that the X-ray beam is deviated by 32.0^0
(a) What angle does the incident X-ray beam make with these planes?
(b) Find the spacing of the planes
[Ans: (i) 16.0^0 (ii) 0.279 nm]
5. An X-ray tube works at a d.c potential difference of 50 kV. Only 0.4% of the energy of the cathode rays is converted into X-radiation and heat is generated in the target at a rate of 600 W. Estimate
(i) the current passed through the tube
(ii) the velocity of the electrons striking the target.
[Ans: (i) 12 mA (ii) 1.33 $\times$ 10^8 ms^{-1}]
6. A 900 W X-ray tube operates at a d.c potential difference of 30 kV.
(i) Calculate the minimum wavelength of the X-rays produced
(ii) Calculate the current through the tube
(iii) If 99% of the power is dissipated as heat, estimate the number of X-ray photons produced per second.
[Ans:(i) 4.1 $\times 10^{-11}$ m (ii) 30 mA (iii) 1.9 $\times$ 10^{15} s^{-1}]
7. Explain how the radiation from an evacuated X-ray tube is affected by changing
(i) the filament current
(ii) the filament-target p.d
(iii) the target material
8. An X-ray diffraction of a crystal gave the closest line at an angle of $6^0 27'$. If the wavelength of the X-ray is 5.8 $\times$ 10^{-9} m, find the distance between two cleavage planes.
[Ans: 2.581 $\times 10^{-8}$ m]
9. A stream of electrons accelerated through a p.d of 12 kV is directed against the target of an X-ray tube. Taking the charge on an electron to be -1.6×10^{-19} C and the Planck's constant to be 6.6×10^{-34} Js, estimate
(i) the kinetic energy of each electron
(ii) the minimum wavelength of the X-rays which could be emitted by the target.
[Ans: (i) 1.9 $\times 10^{-15}$ J (ii) 1.0 $\times 10^{-10}$ m]
10. An X-ray tube operates with a p.d of 100 kV between the anode and cathode. The tube current is 20 mA. Calculate
(i) the rate at which energy is transformed in the target of the X-ray tube
(ii) the number of electrons which reach the target per second
(iii) the maximum energy of an X-ray photon produced.
[Ans: (i) 2 kW (ii) 1.25 $\times$ 10^{17} (iii) 10^5 eV]
11. An X-ray tube operates at 30 kV and the current through it is 30 mA. Calculate
(i) the electric power input.
(ii) the number of electrons striking the target per second
(iii) the speed of the electrons when they hit the target.
(iv) the longer wavelength limit of the X-rays emitted.
[Ans: (i) 60 W (ii) 1.3 $\times 10^{16}$ s^{-1} (iii) 1.0 $\times$ 10^8 ms^{-1} (iv) 0.41 $\times 10^{-10}$ m]
12. The potential difference between the target and cathode of an X-ray tube is 20 kV and the current is 20 mA. Only 0.5% of the total energy supplied is emitted as X-rays.

(a) What is the minimum wavelength of the emitted X-rays?
(b) At what rate must heat be removed from the target in order to keep it at a steady temperature?
[Ans: (a) $6.19 \times 10^{-11} m$ (b) $398\, W$]

13. Electrons are accelerated from rest through a p.d of 10000 V in an X-ray tube. Calculate
(i) the resultant energy of the electrons in eV
(ii) the wavelength of the associated electron waves
(iii) the maximum energy and minimum wavelength of the X-radiation generated.
[Ans: (i) $10^4\, eV$ (ii) $1.23 \times 10^{-11} m$ (iii) $1.6 \times 10^{-15} J, 1.24 \times 10^{-10} m$]

14. A parallel beam of electrons moving with a velocity v is incident normally on a thin graphite film of atomic spacing $1.2 \times 10^{-10} m$. The beam is diffracted through an angle θ of 11^0 where $2d \sin\theta = \lambda$. Calculate
(i) the wavelength
(ii) the velocity, v
(iii) the accelerating voltage needed to produce this velocity
[Ans: (i)$4.6 \times 10^{-11} m$ (ii)$1.4 \times 10^7 ms^{-1}$ (iii)450 V]

15. An X-ray tube operated at a d.c potential difference of 40 kV produces heat at a target at the rate of 720 W. Assuming 0.5% of the energy of the incident electrons is converted into X-radiation, calculate
(i) the number of electrons striking the target
(ii) the velocity of the incident electrons
[Ans: (i) 1.1×10^{17} (ii) $1.2 \times 10^8 ms^{-1}$]

WAVE PARTICLE DUALITY

Electromagnetic waves such as light exhibit a dual nature as they possess both wave properties and particle like properties. Conversely, particles like electrons have wave-like properties as well as particle-like properties. This is referred to as the wave particle duality and it forms the basis of quantum theory.
The wave particle duality was extended to particles as matter by Louis de Broglie. His theoretical study on the nature of particles and waves led to the invention of new mechanics of particles called quantum mechanics.

Quantum theory

Einstein presented the idea of light energy consisting of packets of electromagnetic energy like bullets fired from a machine gun rather than the water flowing from a running tap. This was an extension of the idea put forward by Max Planck to explain the emission and absorption of energy from a black surface.
According to Planck, the energy emitted or absorbed by the atoms in a black surface is in the form of individual packets which he called **quanta**. The energy in each quantum is proportional to the frequency f of the incident radiation.

$$E = hf$$

where h = Planck's constant = $6.63 \times 10^{-34} Js$
Einstein extended Planck's idea to light. He put forward the idea that the energy in a light beam is quantized i.e. comes in packets or quanta and only a whole number of quanta can exist. The quanta of light or electromagnetic radiation are known as photons. The energy of a photon is $E = hf$

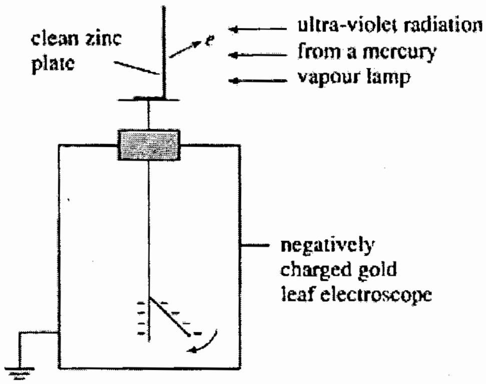
Photoelectric effect

Photoelectric emission is the emission of electrons from a metal surface when electromagnetic radiations fall on it.

The photoelectric effect is identical to thermionic emission of electrons. In photoelectric emission, the electrons in the metal escape from the metal surface using energy from the incident radiation whereas in thermionic emission, energy in the form of heat is absorbed by the electrons.

Experiments to study/demonstrate photoelectric effect

Experiment I



A freshly clean zinc plate is placed on the metal cap of a gold leaf electroscope which is then charged negatively. Ultra violet radiation from a mercury vapour lamp is allowed to fall onto the zinc plate.

The divergence of the gold leaf of the electroscope decreases. This shows that the electroscope loses its negative charge through the emission of electrons.

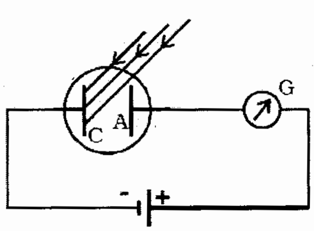
If the electroscope is recharged and a glass plate placed between the zinc plate and the mercury vapour lamp, the divergence of the gold leaf remains unchanged when the light from the lamp falls on the zinc plate. Ultra-violet radiation is absorbed by the glass plate and only light whose frequency is lower than that of the ultra violet radiation is incident on the plate.

Since the divergence of the gold leaf remains unchanged, it implies that no electrons are emitted from the zinc surface. This shows that only radiation of sufficiently high frequency is able to eject electrons from the zinc surface.

If a positively charged electroscope is used and ultra violet radiation allowed to fall on it, the divergence of the leaf remains unchanged. This is because any electrons emitted from the zinc surface are immediately attracted back by the positive charges on the electroscope.

Experiment II

The experimental setup consists of an evacuated bulb with two zinc plates, cathode C and anode A.



The plates are connected to a battery and a sensitive galvanometer.

In the absence of any radiation incident on the plates, there is no flow of current and hence there is no deflection in the galvanometer.

When an electromagnetic radiation like U.V light is allowed to fall on the plate C connected to the negative terminal of the battery, a current begins to flow indicated by a deflection in the galvanometer G.

When U.V light is allowed to fall on plate A, there is no deflection in the galvanometer.

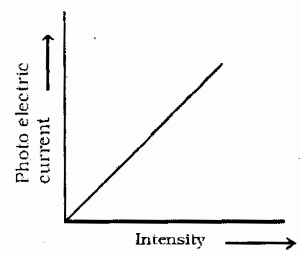
Conclusion

The observations reveal that the particles emitted by the plate C due to the photoelectric effect are negatively charged. These particles were found to be electrons.

The current observed known as **photoelectric current** is due to the flow of electrons.

Effect of intensity of incident radiation on photoelectric current

Keeping the frequency of the incident radiation and the potential difference between the cathode and anode constant, the intensity of the incident radiation is varied. The corresponding photoelectric current is measured.

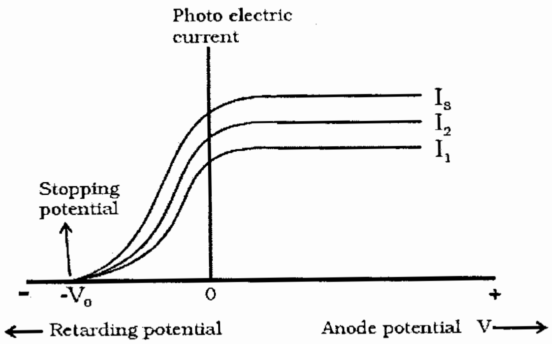


It is found that the photocurrent increases linearly with the intensity of the incident radiation.

Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, it implies that the number of photoelectrons emitted per second is proportional to the intensity of the incident radiation.

Effect of potential difference on photoelectric current

For a given metallic surface, keeping the intensity and frequency of the incident radiation constant, the effect of potential difference between the plates on the photoelectric current can be studied.



When the positive potential of A is increased, the photoelectric current is also increased. However, if the positive potential is further increased such that it is large enough to collect all the photoelectrons emitted from the plate C, the photoelectric current reaches a maximum value known as saturation current.

If the potential of plate A is made negative, the photocurrent does not immediately drop to zero but flows in the same direction as for positive potential.

If the negative or retarding potential is further increased, the photocurrent decreases and finally becomes zero at a particular value called **cut-off or stopping potential.**

Stopping potential is the minimum potential which reduces the photoelectric current to zero

If m is the mass of the photoelectron emitted with a velocity v_{max}, then its kinetic energy is $\frac{1}{2}mv_{max}^2$

Since at stopping potential V_0, the fastest electron emitted is just prevented from reaching the plate A, work done in bringing the fastest electron to rest = kinetic energy of the fastest electron.

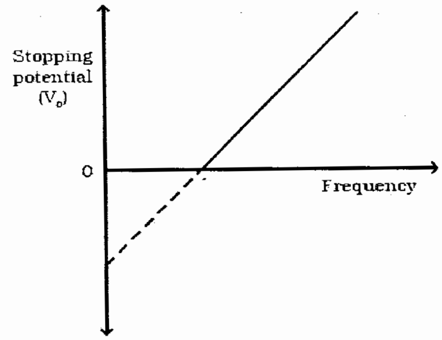
$$eV_0 = \frac{1}{2}mv_{max}^2$$

The experiment is repeated with the incident radiation of the same frequency but higher intensities I_1 and I_2. It is found that the saturation currents are proportional to the intensities of the radiation. The stopping potential remains the same for all the intensities.

Thus, for a given frequency of the incident radiation, the stopping potential is independent of its intensity.

Variation of stopping potential with frequency of incident radiation

If the frequency of the incident radiation is plotted against the corresponding stopping potential, a straight line is obtained as shown below.



From the graph, it is found that at a frequency f_0, the value of the stopping potential is zero. This frequency is known as the **threshold frequency** for the metal used. The photoelectric effect occurs above this frequency and ceases below it.

Threshold frequency is the minimum frequency of the incident radiation below which photoelectric emission is completely impossible however high the intensity of the incident radiation may be. The threshold frequency is different for different metals.

Work function (W_0)

This is the minimum amount of energy that is required to liberate an electron from the metal surface.

Einstein's equation of photoelectric effect

Energy of the incident photon = work function + kinetic energy of electron.

$$hf = W_0 + \frac{1}{2}mv^2$$

$$W_0 = hf_0 = \frac{hc}{\lambda_0}$$

Examples

1. The work function of zinc is $6.8 \times 10^{-19}\,J$. What is threshold frequency for emission of photoelectrons from zinc?

Solution

Work function, $W_0 = hf_0$

$$f_0 = \frac{W_0}{h} = \frac{6.8\times10^{-19}}{6.6\times10^{-34}} = 1.03 \times 10^{15}\,Hz$$

2. A metallic surface when illuminated with light of wavelength $3.33 \times 10^{-7}\,m$ emits electrons with energies up to $0.6\,eV$. Calculate the work function of the metal.

Solution

Work function, $W_0 = hf_0 - K.E_{max}$

$$= \frac{hc}{\lambda} - K.E_{max}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{3.33\times10^{-7}} - 0.6 \times 1.9 \times 10^{-19}$$

$$= 5 \times 10^{-19}\,J$$

3. Lithium has a work function of 2.3 eV. It is exposed to light of wavelength $4.8 \times 10^{-7}\,m$. Find the maximum kinetic energy with which the electron leaves the surface. What is the longest wavelength which can produce the photoelectrons?

Solution

$$\text{K.E} = hf - W_0$$

$$hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{4.8 \times 10^{-7}}$$

$$= 4.125 \times 10^{-19}\,J$$

$$K.E = 4.125 \times 10^{-19} - 2.3 \times 1.6 \times 10^{-19}$$

$$= 4.45 \times 10^{-20}\,J$$

Work function, $W_0 = hf_0 = \frac{hc}{\lambda_0}$

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2.3 \times 1.6 \times 10^{-19}} = 5.40 \times 10^{-7}\,m$$

4. When light of frequency $5.4 \times 10^{14}\,Hz$ is incident on the metal surface, the maximum energy of the electrons emitted is $1.2 \times 10^{-19}\,J$. If the same metal surface is illuminated with light of frequency $6.6 \times 10^{14}\,Hz$, the maximum energy of the electrons is $2.0 \times 10^{-19} J$. Using the data, calculate the
   (i) value of Planck's constant h
   (ii) work function W_0
   (iii) threshold frequency and threshold wavelength

Solution

(i) $$hf = W_0 + \frac{1}{2}mv^2$$

$$5.4 \times 10^{14} f = W_0 + 1.2 \times 10^{-19} \text{ .... (i)}$$

$$6.6 \times 10^{14} f = W_0 + 2.0 \times 10^{-19} \text{ ....(ii)}$$

$$(ii) - (i)$$

$$1.2 \times 10^{14} h = 0.8 \times 10^{-19}$$

$$h = 6.67 \times 10^{-34}\,Js$$

(ii) $5.4 \times 10^{14} \times 6.67 \times 10^{-34} = W_0 + 1.2 \times 10^{-19}$

$$W_0 = 2.4018 \times 10^{-19}\,J$$

(iii) $W_0 = hf_0$

$$f_0 = \frac{W_0}{h} = \frac{2.4018 \times 10^{-19}}{6.67 \times 10^{-34}} = 3.6 \times 10^{14}\,Hz$$

(iv) $f_0 = \frac{c}{\lambda_0}$

$$\lambda_0 = \frac{c}{f_0} = \frac{3 \times 10^{8}}{3.6 \times 10^{14}} = 8.3 \times 10^{-7}\,m$$

5. The value of $W_0 = 1.35\,eV$
   (a) What is the longest wavelength
   (b) What is the maximum velocity of photoelectrons which will be emitted from caesium surface if illuminated with light of wavelength $4 \times 10^{-7}\,m$?
   (c) What p.d would just prevent the current from passing through the caesium photocell illuminated with light of wavelength $4 \times 10^{-7}\,m$.

Solution

(a) $$W_0 = 1.35\,eV = 1.35 \times 1.6 \times 10^{-19}$$

$$= 2.16 \times 10^{-19}\,J$$

$$W_0 = hf_0 = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.16 \times 10^{-19}}$$

$$\lambda_0 = 9.208 \times 10^{-7}\,m$$

(b) $\lambda = 4 \times 10^{-7}\,m, v_{max} = ?$

$$hf = W_0 + K.E_{max}$$

$$\frac{hc}{\lambda} = W_0 + K.E_{max}$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}} = 2.16 \times 10^{-19} + K.E_{max}$$

$$K.E_{max} = 2.8125 \times 10^{-19}\,J$$

$$K.E_{max} = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{2\,K.E_{max}}{m}}$$

$$= \sqrt{\frac{2 \times 2.8125 \times 10^{-19}}{9.11 \times 10^{-31}}} = 7.86 \times 10^{5}\,ms^{-1}$$

(c) $$K.E_{max} = eV_0$$

$$V_0 = \frac{K.E_{max}}{e} = \frac{2.8125 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.76\,V$$

6. A monochromatic source emits a narrow, parallel beam of light of wavelength 546 nm, the power in the beam being 0.080 W.
   (a) How many photons leave the source per second?
   (b) If this beam falls on the cathode of a photocell, what is the photocell current, assuming that 1.5 % of the photons incident on the cathode liberate electrons?

Solution

(a) Energy of photon, $E = \frac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{546 \times 10^{-9}} = 3.626 \times 10^{-19}\,J$$

Number of photons per second $= \frac{\text{Joules per second}}{\text{photon energy}}$

$$= \frac{0.08}{3.626 \times 10^{-19}} = 2.2 \times 10^{17}$$

(b) Number of electrons liberated per second

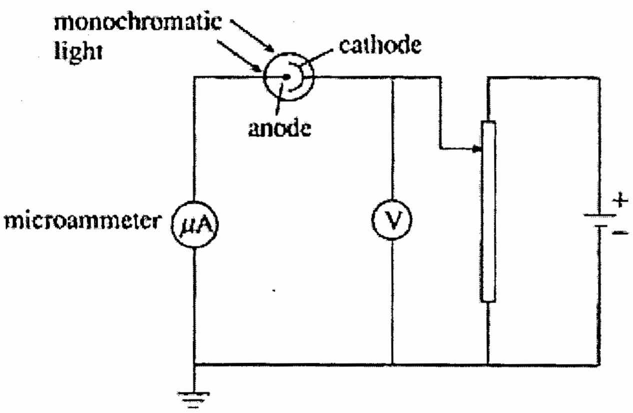
$$= \frac{1.5}{100} \times 2.2 \times 10^{7} = 3.3 \times 10^{15}$$

Current = Electrons per second × e or $I = ne$

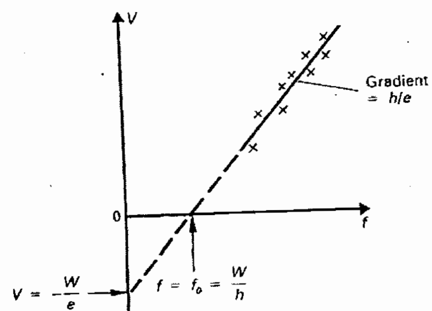
$$I = 3.3 \times 10^{15} \times 1.6 \times 10^{-19}$$

$$= 5.28 \times 10^{-4}\,A$$

Experimental verification of Einstein's equation



- The anode is made more negative in potential relative to the cathode. The electrons from the cathode experience a retarding p.d
- The p.d V is increased until the current becomes zero and the potential V_0 is read from the voltmeter.
- Different values of V_0 are obtained using incident light of light of different frequency.
- A graph of V_0 is plotted against f
- A straight-line graph is obtained.



From Einstein's equation

$$eV_0 = hf - W_0$$
$$V_0 = \left(\frac{h}{e}\right)f - \frac{W_0}{e}$$

$\frac{h}{e}$ is the gradient of the line and $-\frac{W_0}{e}$ is the intercept which verifies Einstein's equation.

Experimental observations/ laws/ characteristics/ features of photoelectric emission

- For any metal, there is a minimum frequency of the incident radiation below which no photoelectric emission takes place irrespective of the intensity of the incident radiation
- The photocurrent or number of electrons emitted per second is directly proportional to the intensity of the incident radiation.
- The kinetic of photoelectrons ranges from 0 to maximum and the maximum kinetic energy of the photoelectrons emitted is directly proportional to frequency but independent of the intensity of the incident radiation.
- There is no time lag between irradiation and emission i.e. photoelectric emission is instantaneous.

Classical theory (wave theory)

According to the classical theory, electromagnetic radiations are considered to be waves.

According to the quantum theory, electromagnetic radiation is made up packets of energy called photons/quanta.

The two theories do not contradict but play a complementary rule. Certain properties of the radiations can only be explained by the quantum theory e.g. photoelectric emission and black body radiation. Other properties like reflection, refraction, diffraction, interference and polarization can only be explained by the wave/classical theory.

Failure of the classical wave theory to explain photoelectric emission

- From the wave theory, radiation energy is uniformly spread over the whole wave front. It is noted that the electron absorbs only a fraction of the total energy.
- The theory predicts or allows continuous absorption and accumulation of energy by an electron. Whatever the frequency of the radiation, the electron should eventually be able to accumulate enough energy to be free. There should be no threshold frequency. Therefore, the theory fails to account for f_0.
- The theory states that the energy from the incident radiation would be continuously supplied to an electron. The electron would take some time to accumulate sufficient energy that would enable it to escape from the metal surface. Thus, the emission of the photoelectrons would not be instantaneous i.e. there is a time lag.
- Increasing the intensity would mean more energy and hence greater values of maximum kinetic energy of electrons. However maximum kinetic energy depends on the frequency of the radiation and not intensity.

Explanation using the quantum theory

According to the quantum theory, radiation is emitted and absorbed in separate packets of energy called photons. Energy of a photon $E = hf$

When a single photon interacts with an electron on the metal surface, it gives all or none of its energy. This implies that only one electron absorbs energy of one photon.

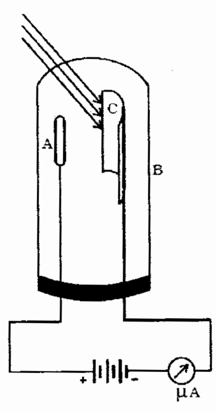
- The higher the intensity of the radiation, the higher the number of photons. Therefore, the number of photoelectrons/photocurrent is proportional to the number of incident photons(intensity)
- Of the photons energy hf, part is used to overcome attraction of the electron by the metal surface and the rest appears as kinetic energy of the emitted electron. Minimum energy required to emit an electron $W_0 = hf_0$. Thus below f_0, no photoelectric emission occurs.
- Increasing frequency of the incident radiation increases energy of the photons so maximum kinetic energy increases with frequency.
- Increasing intensity only increases the number of photons of the radiation but the energy of each photon remains the same. Therefore, the same amount of energy will be available for each electron so maximum kinetic energy is independent of the intensity of the radiation.

Photoelectric cells and their applications

The photoelectric cell is a device which converts light energy to electrical energy. The photoelectric cells are of three types.

(i) Photo emissive cell
(ii) Photo voltaic cell
(iii) Photo conductive cell

A simple photo emissive cell is shown below.



It consists of a highly evacuated bulb B made of glass or quartz. A semi cylindrical metal plate C connected to the negative terminal of a battery acts as cathode. This plate is coated with a low work function material such as caesium oxide in order to get a large number of photoelectrons.

A thin platinum wire A is connected to the positive terminal of the battery and kept along the axis of the metal plate C and this serves as the anode.

When a light of suitable wavelength falls on the cathode, photoelectrons are emitted which are attracted by the anode.

The resulting current is measured by a microammeter. The current produced by this type of cell is proportional to the intensity of the incident light for a given frequency.

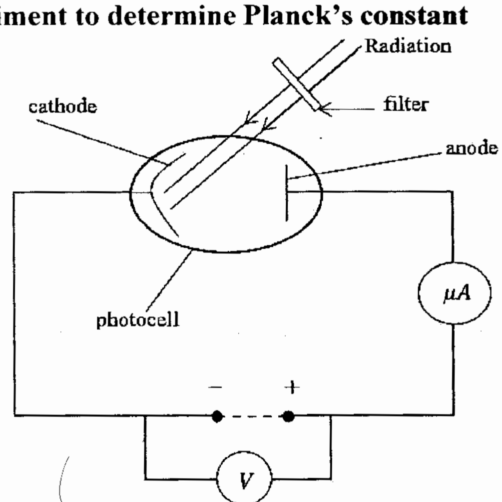
Applications of photoelectric cells

- Photoelectric cells are used for reproducing sound in cinematography.
- They are used for controlling the temperature of furnaces
- Photoelectric cells are used for automatic switching off and switching on traffic lights
- They are used in the study of temperature and spectra of stars
- They are used in automatic opening and closing of doors
- Photoelectric cells are used in burglar alarms and fire alarms

  In burglar alarm, ultraviolet light is continuously made to fall on the photocell installed at the door way. A person entering the door interrupts the beam falling on the photocell. The abrupt change in the photocurrent is used to start an electric bell ringing.

  In fire alarm, a number of photocells are installed at suitable places in a building. In the event of breaking out of fire, light radiations fall upon the photocell. This completes the electric circuit through an electric bell or siren which starts operating as a warning signal.

Experiment to determine Planck's constant



A radiation of known frequency $f > f_0$ is incident on the metal cathode of a photocell

A p.d V is varied from zero until the photocurrent measured by the microammeter is zero.

The p.d at this point is recorded as stopping potential V_0 and it is measured by a voltmeter.

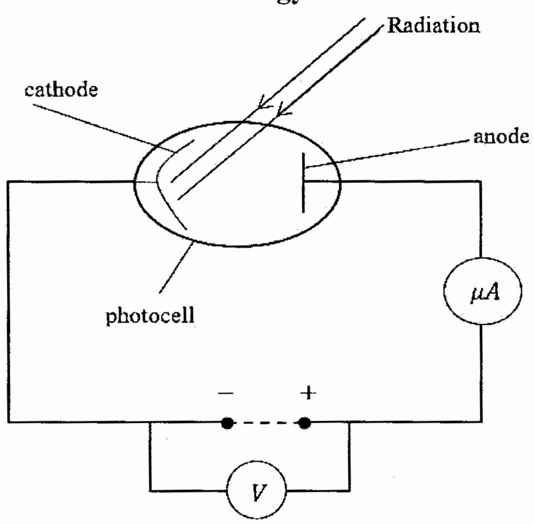
The procedure is repeated with other known frequencies f of the radiations.

The results are tabulated and a graph of V_0 against f is plotted.

The slope S is calculated.

Planck's constant $h = Se$ where e is the electronic charge.

Experiment to determine the stopping potential and maximum kinetic energy



A radiation of known frequency $f > f_0$ is incident on the metal cathode of a photocell

A p.d V is varied from zero until the photocurrent measured by the microammeter is zero.

The p.d at this point is recorded as stopping potential V_0 and it is measured by a voltmeter.

$K.E_{max} = eV_0$ where e is the electronic charge

Matter waves

The radiant energy has dual aspects of particle and wave hence a natural question arises; if radiation has dual nature, why not the matter?

Louis de Broglie put forward the hypothesis that moving particles should possess wave-like properties under suitable conditions.

de Broglie's wavelength of matter waves

de Broglie equated the energy equations of Planck (wave) and Einstein (particle).

For a wave of frequency f, the energy associated with each photon is given by Planck's equation

$$E = hf$$

According to Einstein's mass energy relation, a mass m is equivalent to energy.

$$E = mc^2$$

If $\quad hf = mc^2$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc}$$

For a particle moving with a velocity, v, if $c = v$, then

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

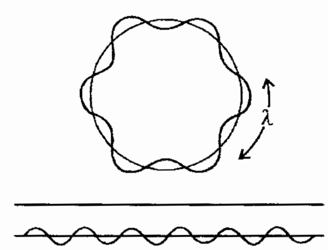
where $p = mv$, the momentum of the particle

Wave mechanical concept of the atom

According to the de Broglie's hypothesis, an electron of mass m in motion with velocity v is associated with a wave whose wavelength λ is given by

$$\lambda = \frac{h}{mv} \text{ where h is Planck's constant}$$

On the basis of de Broglie's hypothesis, an atom model was proposed in which the stationary orbits of Bohr's model were retained but with difference that an electron in various orbits behaves as a wave.



It was suggested that stationary orbits are those in which the orbital circumference $(2\pi r)$ is an integral multiple of the de Broglie wavelength.

$$2\pi r = n\lambda$$

where $n = 1, 2, 3, ...$ and r is the radius of the circular orbit.

$$2\pi r = n\left(\frac{h}{mv}\right)$$

$$mvr = \frac{nh}{2\pi}$$

From the above equation, it is seen that the total angular momentum of the moving electron is an integral multiple of $\frac{h}{2\pi}$.

Thus, de Broglie's concept confirms Bohr's postulate

Self-Evaluation exercise

1. A metal surface is illuminated with monochromatic light and it becomes charged to a positive potential of 1.0 V relative to its surroundings. The work function energy of the metal surface is 3.0 eV. Calculate the frequency of light.

[Ans: $0.97 \times 10^{15}\ Hz$]

2. A clean surface is irradiated with the light of wavelength $5.5 \times 10^{-7}\, m$ and electrons are just able to escape from the surface. When light of wavelength $5.0 \times 10^{-7}\, m$ is used, electrons emerge with energies of up to $3.6 \times 10^{-20}\, J$. Find the value of the Planck's constant.

[Ans: $6.6 \times 10^{-34}\, Js$]

3. Ultra violet light of wavelength 12.2 nm is shone onto a metal surface. The work function of the metal is 6.20 eV.

(a) Calculate the maximum kinetic energy of the emitted photoelectrons

(b) Show that the maximum speed of these photoelectrons is $6.0 \times 10^{6}\, ms^{-1}$

[Ans: (a) $1.53 \times 10^{-17}\, J$]

4. Electromagnetic radiation of frequency $0.88 \times 10^{15}\, Hz$ falls upon a surface whose work function is 2.5 V.

(a) Calculate the maximum kinetic energy of photoelectrons released from the surface

(b) If a nearby electrode is made negative with respect to the first surface using a p.d V, what value of V is just sufficient to stop the photoelectrons from reaching the negative electrode?

[Ans: (a) 1.125 eV (b) 1.125 V]

5. Light of frequency $5.0 \times 10^{14}\, Hz$ liberates electrons with energy $2.31 \times 10^{-19}\, J$ from a certain metallic surface. What is the wavelength of ultra-violet light which liberates electrons of energy $8.93 \times 10^{-19}\, J$ from the same surface?

[Ans: $2.0 \times 10^{-7}\, m$]

6. In an experiment on the photoelectric effect using radiation of wavelength $4.0 \times 10^{-7}\, m$, the maximum electron energy was observed to be $1.40 \times 10^{-19}\, J$. With radiation of wavelength $3.0 \times 10^{-7}\, m$, the maximum energy was $3.06 \times 10^{-19}\, J$. Calculate the value of Planck's constant.
[Ans: $6.64 \times 10^{-34}\, Js$]

7. An α-particle accelerated between a pair of parallel plates in a vacuum tube acquires a kinetic energy of $10^{3}\, eV$. What is the potential difference between the plates? [Ans: 500 V]

8. The maximum kinetic energy of photoelectrons ejected from a tungsten surface by monochromatic light of wavelength 248 nm was found to be $8.6 \times 10^{-20}\, J$. Find the work function of the target.

[Ans: 4.45 eV]

9. An argon laser emits a beam of light of wavelength $4.88 \times 10^{-7}\, m$, the power in the beam being $100\, mW$.

(i) How many photons per second are emitted by the laser?

(ii) If the beam falls on the caesium cathode of a photocell, what photoelectric current would be observed assuming 10% of the photons are able to eject an electron

(iii) Given that the limiting frequency of caesium is $5.2 \times 10^{14}\, Hz$, what reverse potential difference between the cell electrodes is needed to suppress the photocell current?

[Ans: (i) $2.5 \times 10^{17}\, s^{-1}$ (ii) 3.9 mA (iii) 0.39 V]

10. Caesium has a work function of $1.9\, eV$. Find the

(i) threshold wavelength

(ii) maximum energy of the liberated electrons

(iii) the stopping p.d

[Ans: (i) $6.5 \times 10^{-7}\, m$ (ii) $1.4 \times 10^{-19}\, J$ (iii) $0.9\, V$]

11. Sodium has a work function of 2.0 eV. Calculate the maximum energy and speed of the emitted electrons when sodium is illuminated by radiation of wavelength 150 nm. What is the least frequency of radiation for which electrons are emitted?

[Ans: $13.2 \times 10^{-19}\, J$, $1.5 \times 10^{-6}\, ms^{-1}$, $4.8 \times 10^{14}\, Hz$]

12. If a photo emissive surface has a threshold wavelength $0.65\, \mu m$, calculate

(i) its threshold frequency

(ii) work function in eV

(iii) maximum speed of the electrons emitted by violet light of wavelength $0.40\, \mu m$.

[Ans: (i) $4.6 \times 10^{14}\, Hz$ (ii) $1.9\, eV$ (iii) $6.5 \times 10^{5}\, ms^{-1}$]

13. Light of photon energy 3.5 eV is incident on a plane photocathode of work function 2.5 V. Parallel and close to the cathode is a plane collecting electrode. The cathode and collector are mounted in an evacuated tube.

(a) Find the maximum kinetic energy of photoelectrons emitted from the cathode.

(b) Find the minimum value of the potential difference which should be applied between the collector and the cathode in order to prevent electrons of maximum energy from reaching the collector for electrons emitted

(i) normal to the cathode

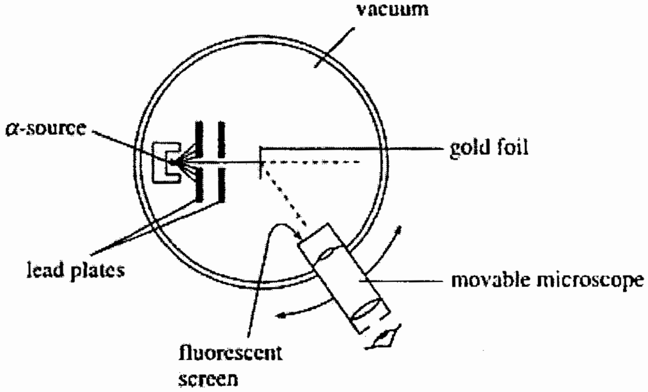
(ii) at an angle of 60^{0} to the cathode.

[Ans: (a) 1 eV (b)(i) 1 V (ii) 0.75 V]

ATOMIC STRUCTURE

Rutherford's α-particle scattering experiment

A fine beam of α-particles was obtained from a radioactive material like radium or radon by placing it in a lead box with a narrow opening.



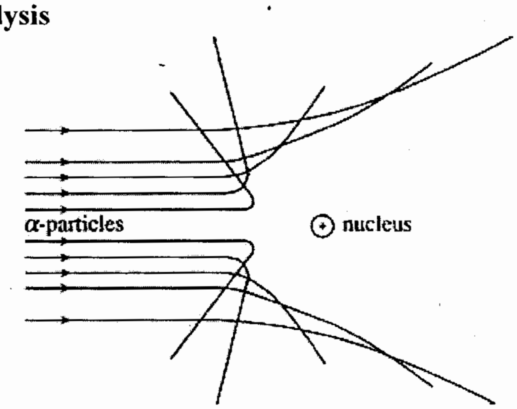
The α-particles are emitted from the source in all possible directions but only a narrow beam emerges from the lead box. The remaining particles are absorbed in the lead box.

After passing through the lead plates, a narrow beam of α-particles incident on a gold foil is scattered through different angles. The scattered α-particles strike a fluorescent screen coated with zinc sulphide. When the α-particles strike the screen, tiny flashes of light are produced. The observations can be made with the help of a low power microscope.

Note:

The experiment is carried out in a vacuum because the range of α-particles in air is limited so the vacuum allows the α-particles to reach the gold foil and the screen beyond the foil.

Analysis



Observations

- Most α-particles passed through the thin gold foil undeflected. This is because most space in the atom is empty
- Few α-particles are deflected or scattered through small angles $< 90^0$. This is due to repulsion by the positive charge in the nucleus.
- Very few α-particles are scattered through large angles $> 90^0$. This is because the nucleus occupies a small portion of the available space of the atom. So the chance of a head on collision is very small.

Conclusion

- The positive charge of the atom and nearly all the mass are concentrated in a very small volume at the centre.
- Electrons were moving very fast and hence their effect on α-particles was negligible
- Electrons are in motion in spheres around the nucleus and the volume of the atom is accounted for by this electron cloud.

Rutherford' model of the atom

An atom consists of a positive charge confined to the centre where most of the mass is concentrated. Electrons round the nucleus in circular orbits. The electron cloud accounts for the volume of the atom.

Distance of closest approach

An α-particle directed towards the centre of the nucleus will move up to a distance r_0 where its kinetic energy will appear as electrostatic potential energy. After this, the α-particle begins to retrace its path.

Let m and v be the mass and velocity of the α-particle directed towards the centre of the nucleus.

$$K.E \text{ of the } \alpha\text{-particle} = \frac{1}{2}mv^2$$

$$\text{Charge of } \alpha\text{-particle} = 2e$$

$$\text{Charge of nucleus} = Ze$$

$$\text{Electrostatic potential energy} = \frac{(2e)(Ze)}{4\pi\varepsilon_0 r_0}$$

where Z is the atomic number of the atom and ε_0, the permeability of free space.

On reaching the distance of closest approach, r_0, the kinetic energy of the α-particle appears as its potential energy.

$$K.E = E.P.E$$

$$\frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Ze^2}{mv^2}$$

Examples

1. α-particles of mass $4u$ moving with a velocity of $2 \times 10^7\ ms^{-1}$ are accelerated towards a gold nucleus of atomic number 79. Find the distance of closest approach of the α-particle towards the nucleus.

Solution

$m = 4u = 4 \times 1.66 \times 10^{-27}\ kg$

$v = 2 \times 10^7\ ms^{-1},\ z = 79$

$$\frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Ze^2}{mv^2}$$

$$r_0 = \frac{9\times10^9\times4\times79\times(1.6\times10^{-19})^2}{4\times1.66\times10^{-27}}$$

$$r_0 = 2.74 \times 10^{-14}\ m$$

2. An alpha particle is projected with an energy of 4 MeV directly towards a gold nucleus. Calculate the distance of its closest approach given that the atomic number of Gold is 79.

Solution

Energy of α-particle = 4 MeV

$$= 4 \times 10^6 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-13}\ J$$

$$Z = 79, r_0 = ?$$

$$K.E = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K.E}$$

$$r_0 = \frac{9\times10^9\times2\times79\times(1.6\times10^{-19})^2}{6.4\times10^{-13}} = 5.688 \times 10^{-14}\ m$$

Self-Evaluation exercise

1. α-particles travelling at a speed of $3.0 \times 10^6\ m\ s^{-1}$ strike a block of gold. By assuming that the gold atoms are fixed in the block, calculate the nearest distance that an α-particle came to a gold nucleus. (mass of α-particle = $6.8 \times 10^{-27}\ kg$, atomic number of gold = 79, permittivity of free space = $8.9 \times 10^{-12}\ Fm^{-1}$) [Ans: $1.2 \times 10^{-12}\ m$]
2. In the Rutherford's α-particle scattering experiment, α-particles of mass $7 \times 10^{-27} kg$ and speed $2 \times 10^7\ ms^{-1}$ were fired at a gold foil. What was the closest distance of approach between an α-particle and a gold nucleus? (Atomic number of gold = 79) [Ans: $2.6 \times 10^{-14}\ m$]
3. Calculate the distance of closest approach of α-particles to the copper nucleus when α-particles of 5 MeV are scattered back by a thin sheet of copper. (Atomic number of copper = 29) [Ans: $1.67 \times 10^{-14}\ m$]

Rutherford's failures/ Draw backs

Rutherford's atomic model offered serious difficulties as regards the stability of the atom. The following are the draw backs of Rutherford's model.

(i) An orbiting electron is constantly changing its direction and therefore has acceleration due to electric charges

(ii) In classical physics, charges undergoing acceleration emit electromagnetic radiations and therefore they would lose energy. This implies the electron would move (spiral) towards the nucleus and the atom would collapse or cease within a short time yet the atom is in a stable structure. Thus the atom cannot be stable.

(iii) Since the electrons are continuously accelerating around the nucleus, continuous emission spectrum should be emitted by the atom. However, experimental observations show that it is line emission which occurs.

Bohr's atomic model

Bohr modified Rutherford's atomic model in order to explain the stability of the atom and the emission of sharp spectral lines. He proposed the following postulates

(i) An electron cannot revolve round the nucleus in all possible orbits. The electrons can revolve round the nucleus only in those allowed or permissible orbits for which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$ where h is Planck's constant

These electrons are called stationary orbits or non-radiating orbits and an electron revolving in these orbits does not radiate any energy.

If m and v are the mass and velocity of the electron in a permitted orbit of radius r, then

$$\text{Angular momentum} = mvr = \frac{nh}{2\pi}$$

n is called the principal quantum number and has integral values 1, 2, 3, ...

(ii) An atom radiates energy only when an electron jumps from a stationary orbit of higher energy to an orbit of lower energy. If an electron jumps from an orbit of energy E_2 to an orbit of energy E_1, a photon of energy $hf = E_2 - E_1$ is emitted

Radius of the n^{th} orbit (r_n)

Consider a Bohr atom whose nucleus has a positive charge e. Let an electron revolve around the nucleus in the n^{th} orbit of radius r_n

Electrostatic force of attraction between the nucleus and electron $= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(e)(e)}{r_n^2}$

Since the electron revolves in a circular orbit, it experiences a centripetal force.

$$F = \frac{mv_n^2}{r_n} = mr_n\omega_n^2$$

where m is the mass of the electron, v_n and ω_n are the linear and angular velocity of the electron in the n^{th} orbit respectively.

The necessary centripetal force is provided by the electrostatic force of attraction.

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r_n^2} = mr_n\omega_n^2$$

$$\omega_n^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n^3} \quad \text{.........} \quad \text{(i)}$$

angular momentum of an electron in the n^{th} orbit

$$= mv_n r_n = mr_n^2\omega_n$$

By Bohr's postulate, $mr_n^2\omega_n = \frac{nh}{2\pi}$

$$\omega_n = \frac{nh}{2\pi m r_n^2}$$

Squaring both sides; $\omega_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^4}$ .... (ii)

From equations (i) and (ii);

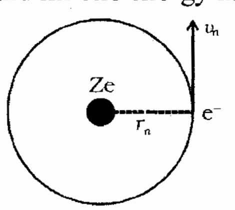
$$\frac{e^2}{4\pi\varepsilon_0 m r_n^3} = \frac{n^2h^2}{4\pi^2m^2r_n^4}$$

$$\boldsymbol{r_n = \frac{n^2h^2\varepsilon_0}{\pi m e^2}}$$

Note: It is seen that the radius of the n^{th} orbit is proportional to the square of the principal quantum number i.e. $r_n \propto n^2$. Therefore the radii of the orbits are in the ratio 1 : 4 : 9 ....

Energy of an electron in the n^{th} orbit

The total energy of the electron is the sum of its potential energy and kinetic energy in its orbit.



For hydrogen atom, $Z = 1$

Potential energy, $E_P = \frac{(Ze)(-e)}{4\pi\varepsilon_0 r_n} = -\frac{e^2}{4\pi\varepsilon_0 r_n}$

Kinetic energy, $E_K = \frac{1}{2}mv_n^2$

From $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r_n} = mv_n^2$

$$\frac{1}{2}mv_n^2 = \frac{1}{8\pi\varepsilon_0} \cdot \frac{Ze^2}{r_n}$$

$$E_K = \frac{e^2}{8\pi\varepsilon_0 r_n}$$

Total energy of an electron in the n^{th} orbit is

$$E_n = E_P + E_K = -\frac{e^2}{4\pi\varepsilon_0 r_n} + \frac{e^2}{8\pi\varepsilon_0 r_n}$$

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n}$$

Substituting for r_n gives;

$$E_n = -\frac{e^2}{8\pi\varepsilon_0\left(\frac{n^2h^2\varepsilon_0}{\pi m e^2}\right)}$$

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

Substituting the known values and calculating in electron volt, $E_n = -\frac{13.6}{n^2}\ eV$

As there is a negative sign, it is seen that the energy of the electron in its orbit increases as n increases.

Electrons are bound to the nucleus of the atom so work has to be done against the attraction binding the electrons in the atom.

Bohr atom

This is an atom with a small central positive nucleus with electrons revolving round it only in certain allowed circular orbits and while in these orbits, do not emit radiations but when an electron makes transition from an orbit of higher energy to one of lower energy, an electromagnetic radiation is emitted of frequency given by $\Delta E = hf$

Shortcomings of the Bohr's theory or model of the atom

Bohr's theory was able to explain a number of experimental observations and correctly predicted the spectral lines of hydrogen atom. However, it fails in the following aspects.

- The theory could not account for the spectra of atoms more complex than hydrogen
- The theory does not give any information regarding the distribution of electrons in the atom
- It does not explain the experimentally observed variations in intensity of the spectral lines of the element.
- Bohr's theory failed to account for the fine structure of the spectral lines of Hydrogen.
- Bohr said the electron orbits are circular but were discovered by Sommerfeld to be elliptical.
- It could not explain the Zeeman and Stark effect.

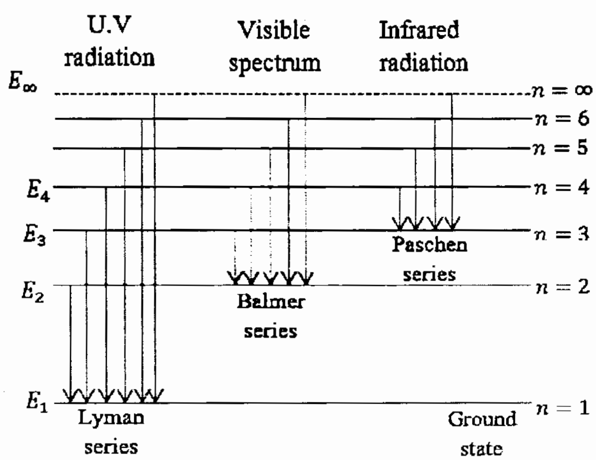
Spectral series of hydrogen atoms

Whenever an electron in a hydrogen atom jumps from higher energy level to lower energy level, the differences in energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line.

As the wavelength of the spectral line depends upon the two orbits (energy levels) between which the transition of electron takes place, various spectral lines are obtained.

The different wavelengths constitute spectral lines which are characteristic of the atoms emitting them. The spectral series of the hydrogen atom include the Lyman series, Balmer series, Paschen series, Brackett series and Pfund series.

Energy level diagram



Energy associated with the n^{th} orbit of the hydrogen atom is given by $E_n = -\frac{13.6}{n^2}\ eV$

Energy associated with the first orbit of the hydrogen atom is

$$E_1 = -\frac{13.6}{1^2} eV = -13.6\ eV$$

It is called the ground state of the hydrogen atom

when $n = \infty, E_\infty = -\frac{13.6}{\infty^2} = 0$

Lyman series

These correspond to a series of lines with different frequency or wavelength emitted by an electron from excited states to the ground state. They correspond to ultra violet part of the spectrum

Balmer series

These correspond to a series of lines with different frequency or wavelength emitted by an electron jumping from excited states to the first excited state ($n = 2$). They correspond to the visible part of the spectrum.

Paschen's series

These correspond to a series of lines with different frequency or wavelength emitted by an electron jumping from excited states to the second excited state ($n = 3$). They correspond to the infrared part of the spectrum.

Ionisation energy

This is the energy required to remove an electron completely from an atom in its ground state to infinity.

$$\text{Ionisation energy } = E_\infty - E_1$$
$$= 0 - (-13.6) = 13.6\ eV$$
$$\text{Ionisation potential} = 13.6\ \text{V}$$

Both ionisation energy and ionisation potential have the same numerical values but different units

Examples

1. Hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength $9.706 \times 10^{-8}\ m$.
   (a) How many different transitions are possible in the resulting emission spectrum?
   (b) Find the longest wavelength among these

(Ionisation energy of hydrogen in its ground state is 13.6 eV, take $h = 6.6 \times 10^{-34}\ Js$)

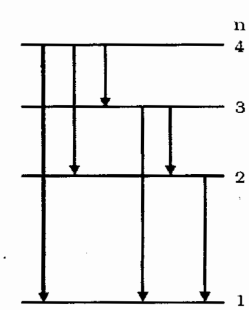
Solution

(a)

Energy of the excited state, $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{9.706\times10^{-8}}$

$$E = 2.04 \times 10^{-18}\ J$$
$$E = \frac{2.04\times10^{-18}}{1.6\times10^{-19}} eV = 12.75\ eV$$
$$12.75 = E_n - (-13.6)$$
$$E_n = 12.75 - 13.6 = -0.85\ eV$$
$$\text{But } E_n = -\frac{13.6}{n^2}$$
$$n^2 = -\frac{13.6}{E_n} = -\frac{13.6}{-0.85} = 16$$
$$n = 4$$

The number of possible transitions in going to the lower state and hence the number of different wavelengths in the spectrum will be six as shown below.



(b) The longest wavelength corresponds to minimum energy difference (i.e. for transition 4 → 3)

$$E_3 = -\frac{13.6}{3^2} = -1.51 \text{ eV}$$

$$\frac{hc}{\lambda_{max}} = E_4 - E_3$$

$$\frac{hc}{\lambda_{max}} = (-0.85 - -1.51)$$

$$\lambda_{max} = \frac{6.6\times10^{-34}\times3\times10^{8}}{(1.51-0.85)\times1.6\times10^{-19}}$$

$$\lambda_{max} = 1.875 \times 10^{-6}\ m$$

Line emission spectrum

It is emitted when atoms of elements are excited in some form of heat from electricity.

When electrons make a transition to higher energy levels, the atoms become unstable since energy is increased. Electron transition may occur to a vacancy left in the lower energy level and an electromagnetic radiation is emitted and lines formed on the spectrum. The lines appear bright against a dark background. These lines are separated and discontinuous and this gives evidence that energy levels of an atom are separate.

Line spectrum are discontinuous lines produced by excited atoms and ions as they fall back to the lower energy levels

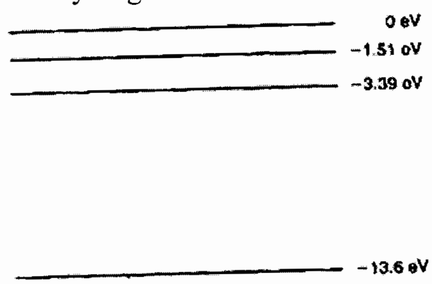
Line absorption spectrum

An atom's energy can change only by discrete amounts. If a photon of energy hf is just enough to excite the atoms, an electron can jump to one of the higher energy levels and the photon will be absorbed. The intensity of the incident radiation is reduced since it has lost a photon.

A dark line on a white background is observed whose wavelength or frequency is that of the absorbed photon. This forms the line absorption spectrum.

Self-Evaluation exercise

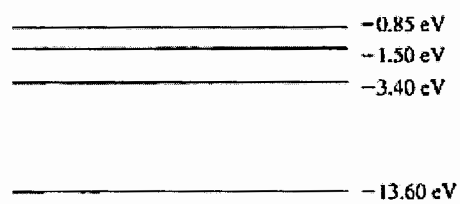
1. Explain the results of Rutherford α-particle scattering experiment.
2. What are the drawbacks of Rutherford atom model?
3. State the postulates of Bohr atom model
4. Obtain the expression for the radius of the n^{th} orbit of an electron based on Bohr's theory.
5. Prove that the energy of an electron for hydrogen atom in the n^{th} orbit of an electron based on Bohr's theory is $E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$
6. Explain the spectral series of hydrogen atom
7. What is meant by energy level diagram?
8. The diagram below shows some of the energy levels for atomic hydrogen



Identify the transition which would result in the emission of light of wavelength 660 nm.

[Ans: from $-3.39\ to - 1.51\ eV$]

9. The figure below represents the four lowest energy levels of the hydrogen atom



(a) Calculate the longest wavelength of the spectral lines which might be emitted

(b) Determine the total number of different spectral lines which might be detected in the emission spectrum of atomic hydrogen due to transitions between these four energy levels

(c) Explain briefly what is meant by ground first excited state and ionisation energy

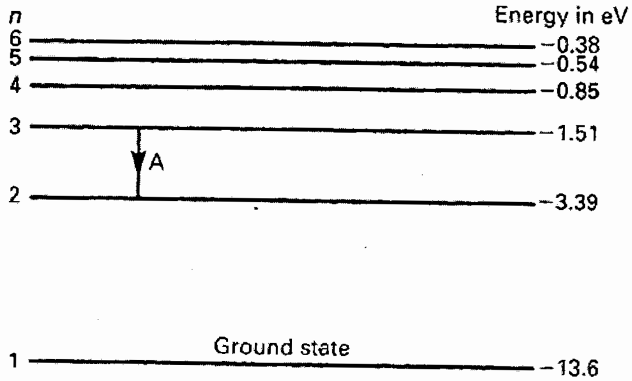
(d) When two hydrogen atoms collide, one or both of them may be boosted

[Ans: (a) $1.9 \times 10^{-6}\ m$ (b) 6]

10. (a) Explain what is meant by electron energy levels in an atom.

(b) How does this concept account for the characteristic emission spectrum of an element?

(c) The diagram below represents the lowest energy levels of the electron in the hydrogen atom, giving the principal quantum number n associated with each level and the corresponding values of the energy.



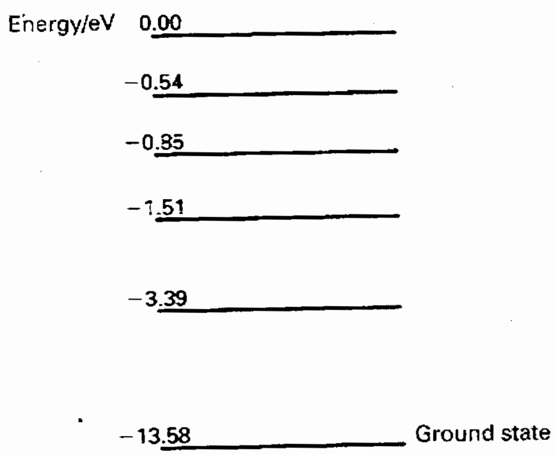
(i) Why are the energies quoted with negative values?

(ii) Calculate the wavelength of the line arising from the transition A, indicating in which region of the electromagnetic spectrum this occurs.

(iii) What happens when 13.6 eV of energy is absorbed by a hydrogen atom in ground state?

[Ans: (c) (ii) $6.6 \times 10^{-7}\ m$]

11. Some of the energy levels of the hydrogen atom are shown in the diagram



$1\ eV = 1.6 \times 10^{-19}\ J$

(a) State which transition will result in the emission of radiation of wavelength 487 nm

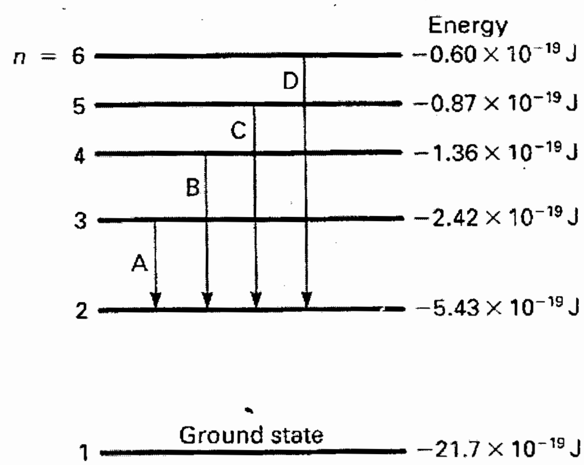
(b) What is likely to happen to a beam of photons of energy

(i) 12.07 eV

(ii) 5.25 eV, when passed through a vapour of atomic hydrogen?

12. (a) What are the chief characteristics of a line spectrum?

(b) The figure below representing the lowest energy levels of the electron in the hydrogen atom, gives the principal quantum number n associated with each, and corresponding value of the energy, measured in joules



(i) Calculate the wavelength of the lines arising from the transitions marked A, B, C, D on the figure

(ii) Show that the other transitions that can occur give rise to lines which are either in the ultraviolet or the infrared regions of the spectrum

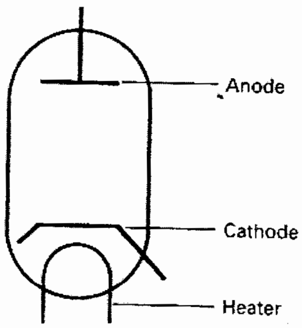
[Ans: (i) 661 nm, 489 nm, 436 nm, 412 nm]

ELECTRONIC DEVICES

Electronic devices are components for controlling the flow of electrical currents for the purpose of information processing and system control. Prominent examples include transistors and diodes. Electronic devices are usually small and can be grouped together into packages called integrated circuits

Thermionic diode

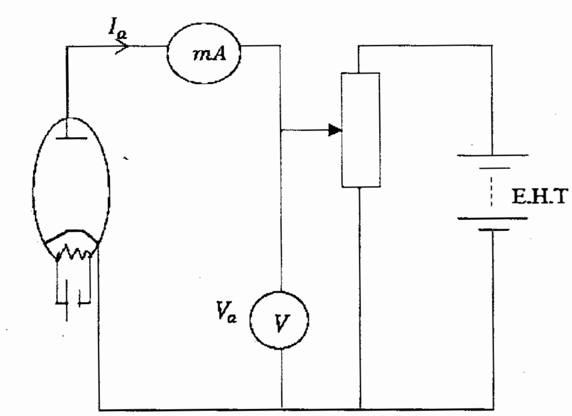
The thermionic diode is used to rectify alternating currents. It has two electrodes i.e. the anode and the cathode.



The cathode is heated and emits electrons by the process of thermionic emission. When the anode is positive with respect to the cathode, the electrons emitted by the cathode are drawn to the anode and current flows. When the anode is negative with respect to the cathode, the electrons are unable to reach the anode and there is no current flow.

The device is often referred to as a diode valve because it allows current to pass in one direction only.

Diode characteristics

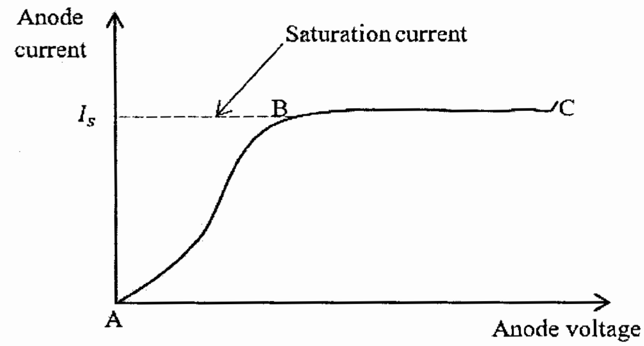


When the cathode filament is heated with current from low voltage supply, electrons are emitted thermionically. If the anode is kept at a positive potential V_a relative to the cathode by using a variable voltage from the E.H.T supply, some electrons move from cathode to anode due to an attractive effect on them and therefore the diode conducts.

However, if the anode is at a negative potential relative to the cathode, no electrons reach the anode and the diode does not conduct due to a repulsive effect on the electrons.

Anode current, I_a is read from the milliammeter and the voltmeter reading gives the anode potential V_a

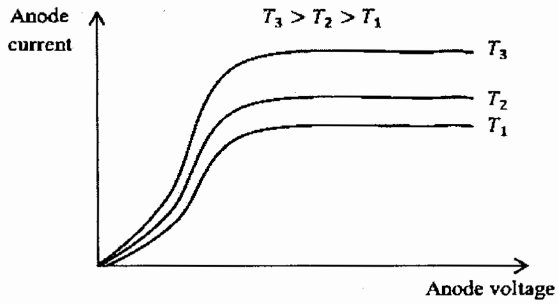
A graph of I_a against V_a



- At A, V_a is zero since electrons are emitted by the cathode is a range of velocities. Some of the electrons have sufficient energy to reach the anode and a very small current is registered. The majority of the electrons with low kinetic energy gather around the cathode to form an electron cloud which has a **space charge**.
- Along AB, as V_a increases, the space charge decreases and an increased number of electrons reach the anode hence anode current increasing. This current is said to be **space charge limited**. In this case, the number of electrons collected by the anode is less than the number of electrons emitted by the cathode.
- Along BC, when the p.d is sufficiently large, all electrons emitted by the cathode reach the anode. The space charge ceases to exist and the current reaches a maximum value called **saturation current**. This current is said to be **temperature limited**. In this case, the number of electrons emitted by the cathode is equal to the number of electrons collected by the anode.

Note:

1. Space charge is the negative charge of the electron cloud.
2. Anode resistance $R_a = \frac{\Delta V_a}{\Delta I_a}$
3. At higher temperature, saturation current increases therefore more electrons are emitted from the cathode at higher temperature as shown below



Transistors

Transistors are electronic devices made from semiconductors. They are used as automatic switches and amplifiers in circuits.

Semi-conductors

Certain materials like germanium, silicon, carbon, etc. have resistivity between good conductors like copper and insulators like glass. These are known as semiconductors.

The resistivity of a semiconductor lies approximately between $0.01\,\Omega\text{m}$ and $10000\,\Omega m$ at room temperature. The resistance of a semiconductor decreases with increase in temperature over a particular temperature range. This behavior is contrary to that of a metallic conductor for which the resistance increases with increase in temperature.

Germanium and silicon are mostly widely used as semiconductors.

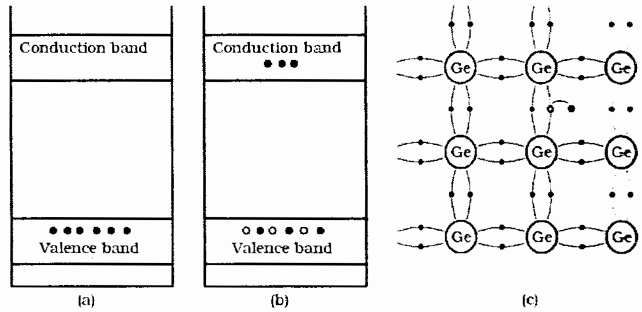
Electrons and holes in semiconductors

Figures (a) and (b) represent the charge carriers at absolute zero temperature and at room temperature respectively.

The electrons in an intrinsic semiconductor which move into the conduction band at high temperatures are called intrinsic carriers.

In the valence band, a vacancy is created at the place where the electron was present before it had moved into the conduction band. This vacancy is called **hole**.

Figure (c) helps in understanding the creation of a hole. Consider the case of pure germanium crystal. It has four electrons in its outer or valence orbit. These electrons are known as valence electrons. When two atoms of germanium are brought close to each other, a covalent bond is formed between the atoms. If some energy is received, one of the electrons contributing to the covalent bond breaks and it is free to move in the crystal lattice.



While coming out of the bond, a hole is said to be created at its place which is usually represented by an open circle. Since an electron has a unit negative charge, the hole is associated with a unit positive charge.

The importance of a hole is that it may serve as a carrier of electricity in the same manner as the free electron but in the opposite direction.

Intrinsic semiconductor

A semiconductor which is pure and contains no impurity is known as an intrinsic semiconductor. In an intrinsic semiconductor, the number of free electrons and holes equal.

Common examples of intrinsic semiconductors are pure germanium and silicon.

Doping a semiconductor

Electrons and holes can be generated in a semiconductor crystal with heat or light energy but in these cases, the conductivity remains low. The efficient and convenient method of generating free electrons and holes is to add a very small amount of selected impurity inside the crystal.

The process of addition of a very small amount of impurity into an intrinsic semiconductor is called doping. The impurity atoms are called dopants.

Methods of doping a semiconductor

1. The impurity atoms are added to the semiconductor in its molten state.
2. The pure semiconductor is bombarded by ions of impurity atoms
3. When the semiconductor crystal containing the impurity atoms is heated, the impurity atoms diffuse into the hot crystal.

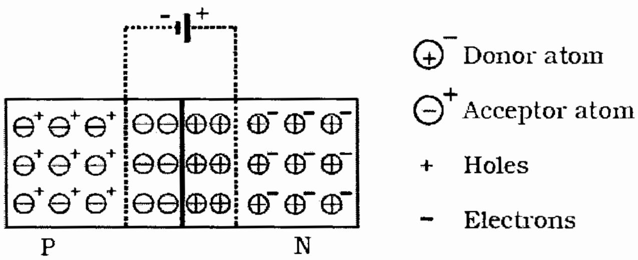
Extrinsic semiconductor

An extrinsic semiconductor is one in which an impurity with a valence higher or lower than the valence of the pure semiconductor is added so as to increase the electrical conductivity of the semiconductor.

Depending on the type of impurity atoms added, an extrinsic semiconductor can be classified as N – type or P – type.

PN junction diode

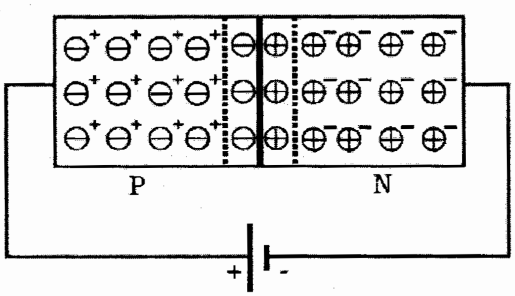
If one side of a single crystal of pure semiconductor is doped with acceptor impurity atoms, a PN junction is formed as shown below.



P region has a high concentration of holes and N region contains a large number of electrons.

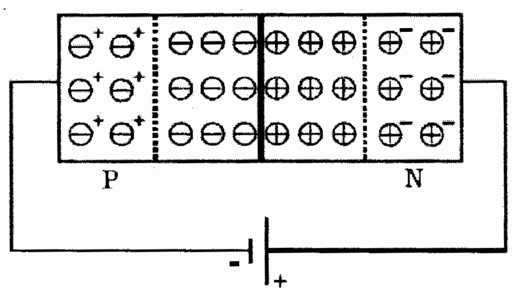
Forward biased PN junction diode

When the positive terminal of the battery is connected to the P – side and the negative terminal to the N – side, so that the potential difference acts in opposite direction, then the PN junction diode is said to be forward biased.



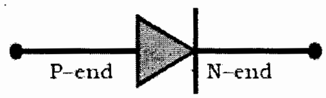
Reverse biased PN junction diode

When the positive terminal of the battery is connected to the N – side and the negative terminal to the P – side, so that the applied potential difference is in the same direction as that of barrier potential, the junction diode is said to be reverse biased.



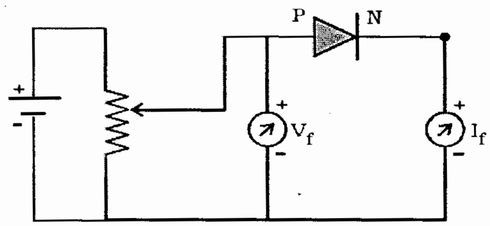
Symbol for a semiconductor diode

The diode symbol is shown in the figure below. The arrow on the diode points in the direction of convection current.

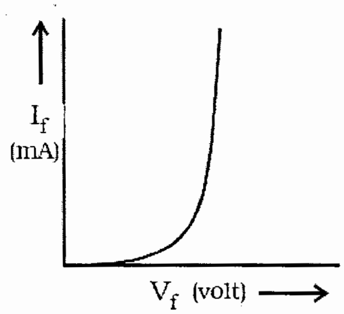


Forward bias characteristics

The circuit for the study of forward bias characteristics of PN junction diode is shown below. The voltage between the P – end and N – end is increased from zero in suitable equal steps and the corresponding currents noted down.



(a) Diode circuit–Forward bias



(b) Forward characteristics

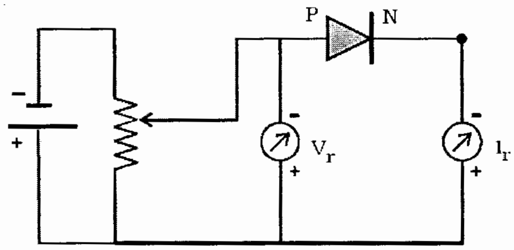
Figure (b) shows the forward bias characteristic curve of the diode.

Voltage is the independent variable thus plotted along the X – axis. The following conclusions can be made

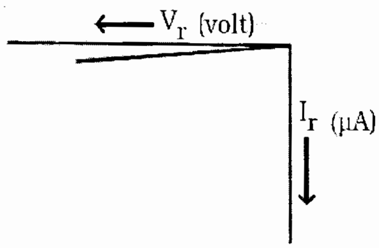
- The forward characteristic is not a straight line hence the ratio $\frac{V}{I}$ is not constant i.e. the diode does not obey ohm's law. This implies that the semiconductor diode is a non – linear conductor of electricity.
- Initially, the current is very small. This is because the diode will start conducting, only when the external voltage overcomes the barrier potential. The voltage at which the current starts to increase rapidly is known as cut – in voltage or knee voltage of the diode.

Reverse bias characteristics

The circuit for the study of reverse bias characteristics of PN junction diode is shown in the figure (a). The voltage is increase from zero in suitable steps. For each voltage, the corresponding current readings are noted down. Figure (b) shows the reverse bias characteristic curve of the diode.



(a) Diode circuit-Reverse bias



(b) Reverse characteristics

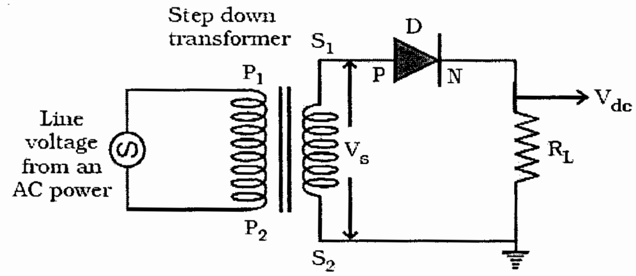
From the characteristic curve, it can be concluded that as voltage is increased from zero, reverse current (in the order of micoamperes) increases and reaches the maximum value at a small value of the reverse voltage. When the voltage is further increased, the current is almost independent of the reverse voltage up to a certain critical value. This reverse current is known as the reverse saturation current or leakage current. This current is due to the minority charge carriers which depends on the junction temperature.

PN junction diode as rectifier

The process in which alternating voltage or alternating current is converted into direct voltage or direct current is known as rectification. The device used for this process is called a **rectifier**. The junction diode has the property of offering low resistance and allowing current to flow through it in the forward biased condition. This process is used in the process of rectification.

Half wave rectifier

A circuit which rectifies half of the a.c wave is called half wave rectifier.



The a.c voltage (V_S) to be rectified is obtained across the secondary ends S_1 and S_2 of the transformer.

The P – end of the diode D is connected to S_1 of the secondary coil of the transformer.

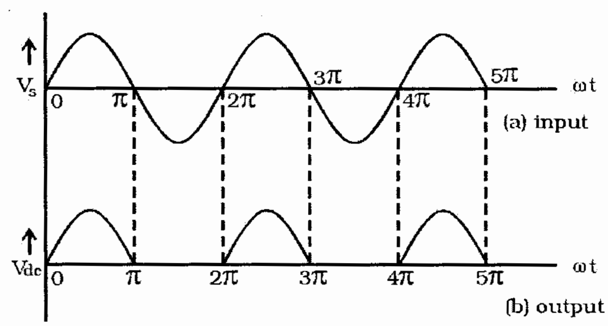
The N – end of the diode is connected to the other end S_2 of the secondary coil of the transformer through a load resistance R_L.

The rectified output voltage V_{dc} appears across the load resistance R_L.

During the positive half cycle of the input a.c voltage V_S, S_1 will be positive and the diode is forward biased and hence it conducts. Therefore, current flows through the circuit and there is a voltage drop across R_L.

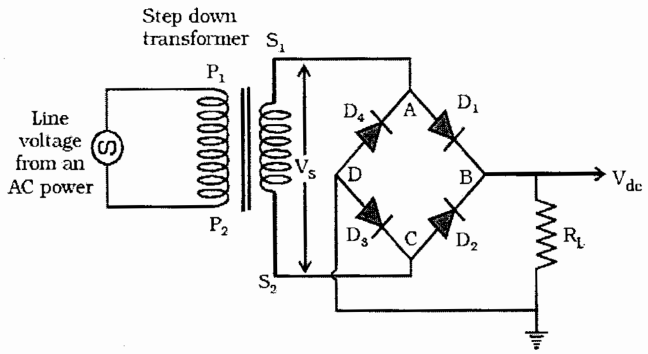
During the negative half cycle of the input a.c voltage V_S, S_1 will be negative and the diode is reverse biased hence does not conduct.

No current flows through the circuit and the voltage drop across R_L will be zero. Hence no output voltage is obtained.

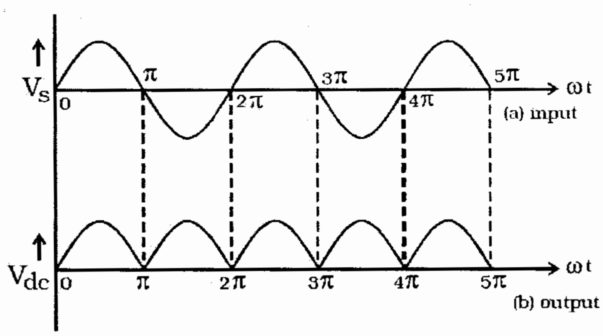


Full wave rectification (Bridge rectifier)

A bridge rectifier is shown in the figure below. There are four diodes D_1, D_2, D_3 and D_4 used in the circuit which are connected to form a network. The input ends A and C of the network are connected to the secondary ends S_1 and S_2 of the transformer. The output ends B and D are connected to the load resistance R_L.

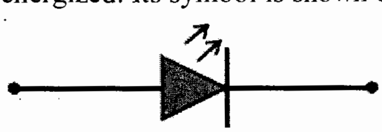


During positive input half cycle a.c voltage, the point A is positive with respect to C. The diodes D_1 and D_3 are forward biased and conduct whereas the diodes D_2 and D_4 are reverse biased and do not conduct. Hence current flows along S_1ABDCS_2 through R_L.
During the negative half cycle, the point C is positive with respect to A. The diodes D_2 and D_4 are forward biased and conduct whereas diodes D_1 and D_3 are reverse biased and they do not conduct. Hence current flows along S_2CBDAS_1 through R_L. The same process is repeated for subsequent half cycles.
It can be seen that current flows through R_L in the same direction during both cycles of the input a.c signals. The output signal corresponding to the input signal corresponding to the input signal as shown below.



Light emitting diode (LED)

A light emitting diode (LED) is a forward biased PN junction diode which emits visible light when energized. Its symbol is shown below.



LEDs are used for instrument displays, calculators and digital watches.

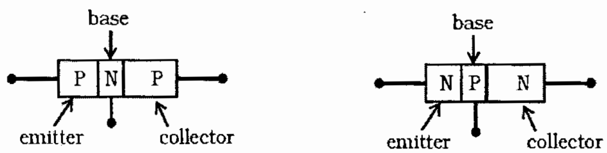
Junction transistor

A junction transistor is a solid-state device. It consists of silicon or germanium crystal containing two PN junctions. The two PN junctions are formed between three layers called base, emitter and collector.

(i) Base (B) layer: It is a very thin layer and the central region of the transistor.
(ii) Emitter (E) and collector (C) layers: The two layers on the opposite sides of the B layer are emitter and collector layers. They are of the same type of semiconductor.

An ohmic contact is made to each of these layers. The junction between emitter and base is called emitter junction. The junction between collector and the base is called collector junction

The construction of PNP and NPN transistors are shown below.

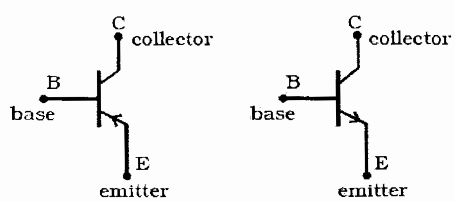


For a transistor to work, the biasing to be given are as follows

(i) The emitter-base junction is forward biased so that majority charge carriers are repelled from the emitter and the junction offers very low resistance to the current.
(ii) The collector-base junction is reverse biased so that it attracts majority charge carriers and this junction offers a high resistance to the current.

Transistor circuit symbols

The circuit symbols for a PNP and NPN transistors are shown below.



(a) PNP (b) NPN

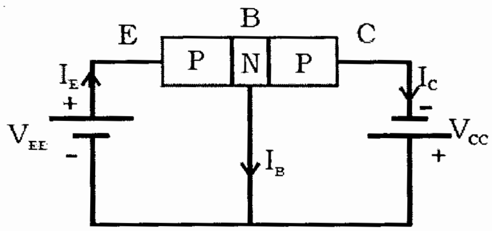
The arrow on the emitter lead pointing towards the base represents a PNP transistor. When the emitter base junction of a NPN transistor is forward biased, the direction of the convectional current flow is from emitter base.

NPN transistor is represented by arrow on the emitter lead pointing away from the base. When the emitter base junction of a NPN transistor is forward biased, the direction of the convectional current is from base to emitter.

Working of a PNP transistor

A PNP transistor is like two PN junction diodes which are placed back to back. At each junction, there is a depletion region which gives rise to a potential barrier. The external biasing of the junction is provided by the batteries V_{EE} and V_{CC}.

The emitter-base junction is forward biased and the collector base junction is reverse biased.



Applying Kirchhoff's current law to circuit, the emitter current is the sum of the collector current and base current i.e.

$$I_E = I_B + I_C$$

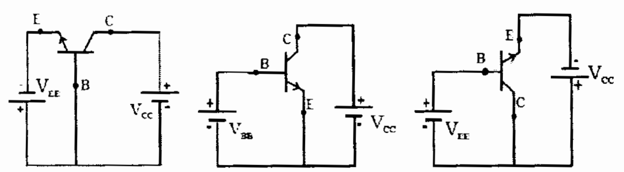
This equation is the fundamental relation between the currents in a transistor circuit and is true regardless of transistor type or transistor configuration.

The action of a NPN transistor is similar to that of a PNP transistor.

Transistor circuit configurations

There are three types of circuit connections called configurations or modes for operating a transistor. They are (i) common base (CB) mode (ii) common emitter (CE) mode and (iii) commo collector (CC) mode.

The term common is used to denote the lead that is common to the input and output circuits. The different modes are shown below for NPN transistor.



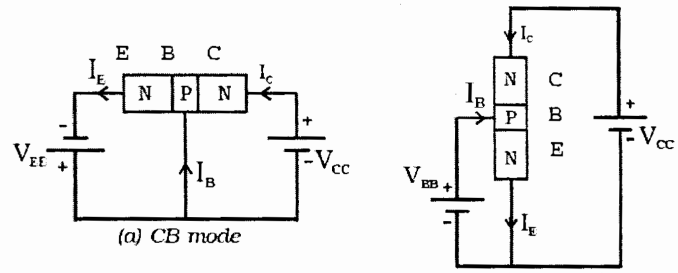
(a) CB mode (b) CE mode (c) CC mode

In a similar way, three configurations can be drawn for PNP transistors.

Current amplification factors α and β and the relation between themThe current amplification factor or current gain of a transistor is the ratio of the output current to the input current

In common base mode, current gain $\alpha = \frac{I_C}{I_E}$

In common emitter mode, current gain $\beta = \frac{I_C}{I_B}$



(a) CB mode (b) CE mode

$$\alpha = \frac{I_C}{I_E} = \frac{I_C}{I_B+I_C} \text{ since } I_E = I_B + I_C$$

$$\frac{1}{\alpha} = \frac{I_B+I_C}{I_C} = \frac{I_B}{I_C} + 1$$

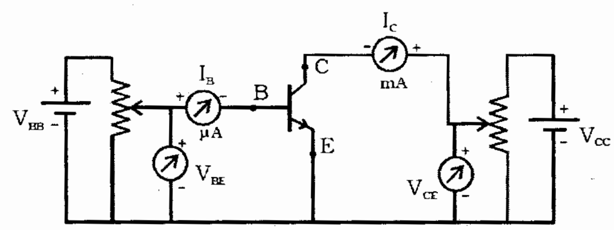
$$\frac{1}{\alpha} - 1 = \frac{1}{\beta}$$

$$\therefore \beta = \frac{\alpha}{1-\alpha}$$

Characteristics of a NPN transistor in common emitter configuration

The three important characteristics of a transistor in any mode are (i) input characteristics (ii) output characteristics (iii) transfer characteristics

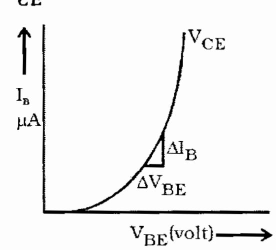
The circuit to study the characteristic curves of NPN transistor in common emitter mode is shown below.



(i) **Input characteristics**

Input characteristic curve is drawn between the base current (I_B) and voltage between the base and emitter (V_{BE}) when the voltage between collector and emitter (V_{CE}) is kept constant at a particular value.

V_{BE} is increased in suitable equal steps and the corresponding base current is noted. The procedure is repeated for different values of V_{CE} and I_B values are plotted against V_{BE} for constant V_{CE}



The input impedance of the transistor is defined as the ratio of the base-emitter voltage to the base current at a given V_{CE}

Input impedance $= \frac{V_{BE}}{I_B}$

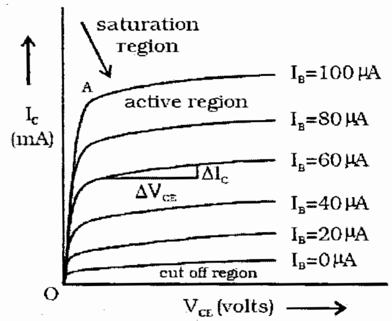
The input impedance of the transistor in CE mode is very high.

(ii) Output characteristics

Output characteristic curves are drawn between I_C and V_{CE} when I_B is kept constant at a particular value.

The base current I_B is kept at a constant value by adjusting the base emitter voltage V_{BE}. V_{CE} is increase in suitable equal steps and the corresponding collector current is noted. The procedure is repeated for different values of I_B.

Now I_C versus V_{CE} curves are drawn for different values of I_B. The output characteristics thus obtained are represented below.



Saturation region

The initial part of the curve (ohmic region, OA) is called the saturation region i.e. the region in between the origin and knee point. Knee point is the point where I_C is about to become a constant.

Cut off region

There is very small collector current in the transistor, even when the base current is zero ($I_B = 0$). In the output characteristics, the region below the curve $I_B = 0$ is called the cut off region. Below the cut-off region, the transistor does not function.

Active region

The central region of the curves is called the active region. In the active region, the curves are uniform. In this region, $E - B$ junction is forward biased and $C - B$ junction is reverse biased.

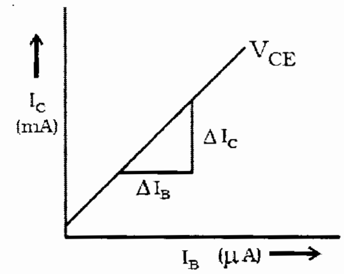
The output independence is defined as the ratio of the collector-emitter voltage to the collector current at a constant base current in the active region of the transistor.

Output independence $= \frac{V_{CE}}{I_C}$

The output impedance of a transistor in CE mode i low.

(iii) Transfer characteristics

The transfer characteristic curve is drawı between I_C and I_B when V_{CE} is kept constant at ; particular value. The base current I_B is increase(in suitable steps and the collector current I_C i: noted down for each value of I_B. The transfe: characteristic curve is shown below

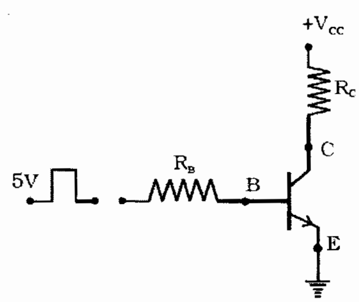


Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B}$

The common emitter configuration has a high input impedance, low output impedance and higher current gain when compared with common base configuration.

Transistor as a switch

Transistors are widely used in switching operations. In the figure below, NPN transistor is connected in a common emitter configuration and a resistor R_B is connected in series with the base. The load resistance R_C is connected in series with the collector.



A pulse type waveform is applied as the input to the transistor through R_B. When the input is high, base emitter junction is forward biased and current flows through R_B into the base. The values of R_B and R_C are chosen in such a manner that the base current flowing is enough to saturate the transistor.

When the transistor is saturated, it is to be **ON.** (maximum current)

When the input is low i.e. at $0\,V$, the base-emitter junction is not forward biased. So no base current flows hence the transistor is said to be **OFF.**

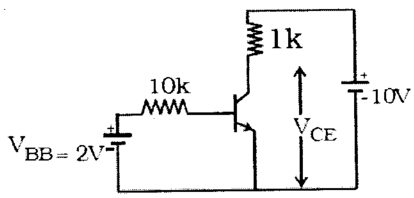
Transistor as amplifier

The important function of a transistor is the amplification. An amplifier is a circuit capable of magnifying the amplitude of weak signals.

The important parameters of an amplifier are input impedance, output impedance, current gain and voltage gain. A good design of an amplifier circuit must possess high input impedance, low output impedance and a high current gain.

Examples

1. The current gain β of the silicon transistor used in the circuit is 50.



If the barrier potential for silicon is 0.69 V, find

(i) I_B (ii) I_C (iii) I_E (iv) V_{CE}

Solution

(i) $V_{BB} = 2\,V, V_{CC} = 10\,V,\ \beta = 50$

$R_B = 10\,k\Omega,\ R_C = 1\,k\Omega$

Barrier potential, $V_{BE} = 0.69\,V$

$V_{BB} = I_B R_B + V_{BE}$

$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{2 - 0.69}{10 \times 10^3} = 131\,\mu A$

(ii) Current gain, $\beta = \frac{I_C}{I_B}$

$I_C = \beta I_B = 50 \times 131 \times 10^{-6} = 6.5\,mA$

(iii) Emitter current, $I_E = I_C + I_B$

$= 6.5\,mA + 131\,\mu A$

$= 6.5\,mA + 0.131\,mA = 6.631\,mA$

(iv) $V_{CC} = V_{CE} + I_C R_C$

$V_{CE} = V_{CC} - I_C R_C$

$= 10 - (6.5 \times 10^{-3} \times 1 \times 10^3)$

$= 3.5\,V$

2. A transistor is connected in CE configuration. The voltage drop across the load resistance (R_C) of 3 $k\Omega$ is 6 V. Find the base current. The current gain α of the transistor is 0.97.

Solution

Voltage across the collector resistance $= I_C R_C = 6\,V$

$I_C = \frac{6}{R_C} = \frac{6}{3 \times 10^{-3}} = 2\,mA$

Current gain, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.97}{1-0.97} = 32.33$

$I_B = \frac{I_C}{\beta} = \frac{2 \times 10^{-3}}{32.33} = 61.86\,\mu A$

LOGIC GATES

There are circuits which are used to process digital signals. They are binary in nature.

A gate is a digital circuit with one or more inputs but with only one output. The output appears only for certain combination of input logic levels.

Logic gates are the basic building blocks from which most of the digital systems are built up.

The numbers 0 and 1 represent the two possible states of a logic circuit. The two states can also be referred to as ON and OFF or HIGH and LOW or TRUE and FALSE.

Basic logic gates using discrete components

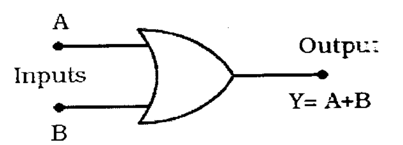
The basic elements that make up a digital system are OR, AND and NOT gates. These three gates are called basic logic gates.

All the possible inputs and outputs of a logic circuit are represented in a table called TRUTH TABLE. The functions of the basic gates are explained below with circuits or truth tables.

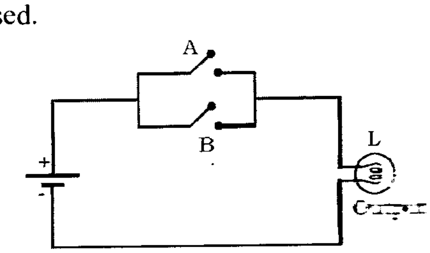
(i) OR gate

An OR gate has two or more inputs but only one output. It is known as OR gate because the output is high if any or all of the inputs are high. The logic symbol of the two input OR gate is shown below.

The Boolean expression to represent OR gate is given by $Y = A + B$ (+ symbol should be read as OR)

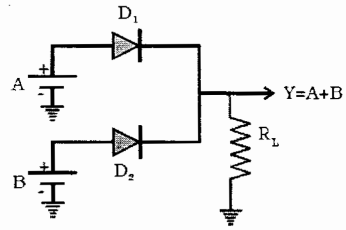


The OR gate can be thought of like an electrical circuit shown below where the switches are connected parallel with each other. The lamp will glow if both the inputs are close or if one of them is closed.



Diode OR gate

The figure below shows a simple circuit using diodes to build a two input OR gate.



The working of the circuit can be explained as follows:

Case (i): A = 0 and B = 0

When both A and B are at zero level i.e. low, the output voltages will be low because the diodes are non – conducting.

Case (ii) : A = 0 and B = 1

When A is low and B is high, diode D_2 is forward biased so that current flows through R_L and output is high.

Case (iii) : A = 1 and B = 0

When A is high and B is low, diode D_1 conducts and the output is high.

Case (iv): A = 1 and B = 1

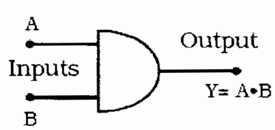
When A and B are both high, both diodes D_1 and D_2 are conducting and the output is high. Therefore Y is high.

Truth table of OR gate

Inputs		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

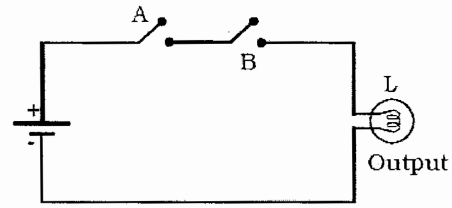
(ii) AND gate

An AND gate has two or more inputs but only one output. It is known as AND gate because the output is high only when all the inputs are high. The logic symbol of a two input AND gate is shown below.



The Boolean expression to represent AND gate is given by $Y = A \cdot B$ ($\cdot$ should be read as AND)

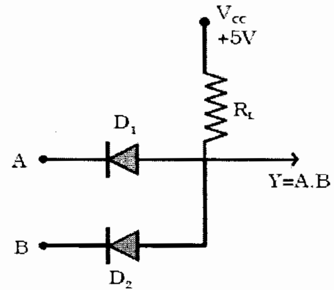
AND gate may be thought of as an electrical circuit shown below in which the switches are connected in series.



Only if A and B are closed, the lamp will glov and the output is high.

Diode AND gate

The figure below shows a simple circuit using tw diodes to build a two input AND gate. The working o the circuit can be explained as follows



Case (i): A = 0 and B = 0

When A and B are O, both diodes are in the forward bias condition and they conduct and hence the output will be zero because the supply V_{CC} will be dropped across R_L only. Therefore $Y = 0$

Case (ii): A = 0 and B = 1

When A = 0 and B is high, diode D_1 is forward biase and diode D_2 is reverse biased. The diode D_1 will now conduct due to forward biasing. Therefore outpu $Y = 0$.

Case (iii): A = 1 and B = 0

In this case, diode D_2 will be conducting and hence output $Y = 0$

Case (iv): A = 1 and B = 1

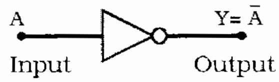
In this case both the diodes are not conducting. Since D_1 and D_2 are in OFF condition, no current flows through R_L. The output is equal to the supply voltage. Therefore $Y = 1$

Thus, the output will be high only when the inputs A and B are high. The table below summarizes the function of an AND gate.

Inputs		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

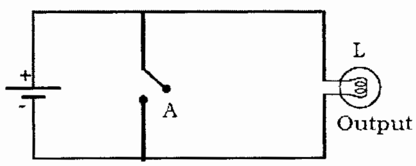
(iii) NOT gate (inverter)

The NOT gate is a gate with only one input and one output. It is so called because its output is complement to the input. It is also known as inverter.

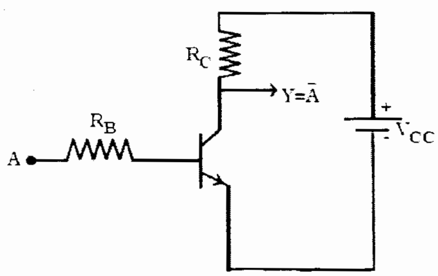


The Boolean expression to represent NOT operation is $Y = \bar{A}$.

The NOT gate can be thought of like an electrical circuit as shown below.



When switch A is closed, input is high and the bulb will not glow i.e. the output is low and vice versa. For a transistor in CE mode which is used as a NOT gate shown below



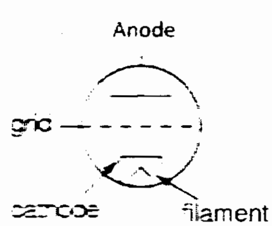
When the input A is high, the transistor is driven into saturation and hence the output Y is low. If A is low, the transistor is cut off and hence the output Y is high. Hence it is seen that whenever input is high, the output is low and vice versa.

Truth table of NOT gate

Inputs	Output
A	$\bar{A}$
0	1
1	0

The Triode

The triode works on the same principle as the thermionic diode though it has a third electrode called the grid between the anode and cathode as shown below.



The grid is used to regulate the space charge i.e. when it is slightly more positive than the filament current, space charge is minimized and the anode current increases(amplified) and when it slightly more negative relative to the cathode, space charge increases and current flow reduces. The triode is therefore used as an amplifier.

Space charge

This is the large number of electrons that gather close to the cathode as an almost stationary cloud of negative charge due to lack of sufficient energy to enable them reach the anode.

Space charge limitation

When the anode potential is not sufficiently high to attract all the electrons emitted by the cathode, a space charge is formed. The space charge exerts an electrostatic repulsive force on the electrons being emitted by the cathode. They are prevented from reaching the anode thereby decreasing the anode current. The anode current is said to be space charge limited for the low potentials.

Saturation

It occurs when the anode potential has a value such that all the electrons that are emitted per second by the cathode do reach the anode.

Amplification

This is the process by which an input signal is increased by some factor i.e. $V_0 > V_i$. The output signal is made bigger than the input signal.

Anode resistance, R_a

This is the ratio of the change in anode voltage V_a to the change in anode current I_a at a constant grid voltage V_g

$$R_a = \left(\frac{\Delta V_a}{\Delta I_a}\right)_{V_g}$$

Mutual conductance, g_m

This is the ratio of the change in the anode current I_a to the change in grid voltage V_g at a constant anode voltage V_a

$$g_m = \left(\frac{\Delta I_a}{\Delta V_g}\right)_{V_a}$$

Amplification factor, μ

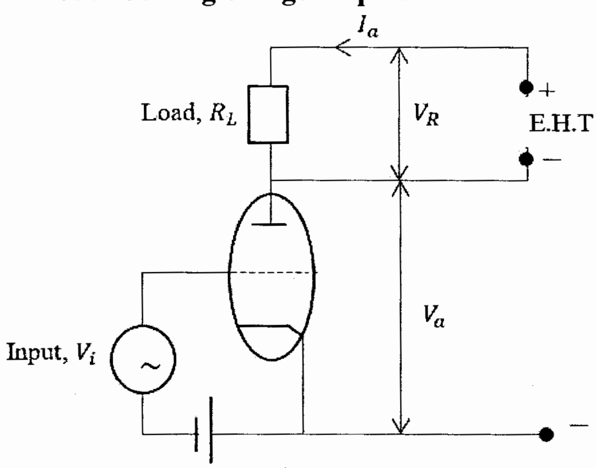
This is the ratio of the change in the anode voltage V_a to the change in grid voltage V_g at a constant anode current I_a.

$$\mu = \left(\frac{\Delta V_a}{\Delta V_g}\right)_{I_a}$$

Also, $\mu = \frac{\Delta V_a}{\Delta V_g} = \frac{\Delta V_a}{\Delta I_a} \times \frac{\Delta I_a}{\Delta V_g} = R_a \times g_m$

Thus $\mu = R_a g_m$

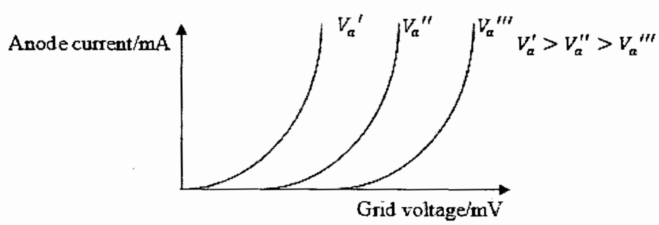
A triode as a single stage amplifier



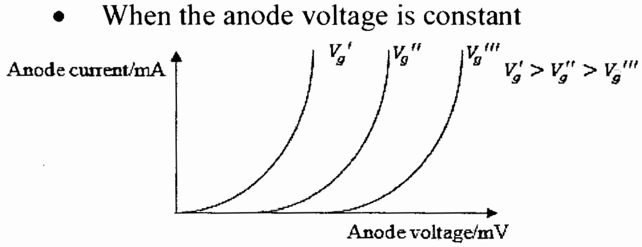
An alternating input signal results into changes in the grid voltage V_g making it highly positive such that space charge is eliminated making anode current to increase thus amplified output signal V_0.

Current – voltage characteristics of a triode

- When the grid voltage is constant



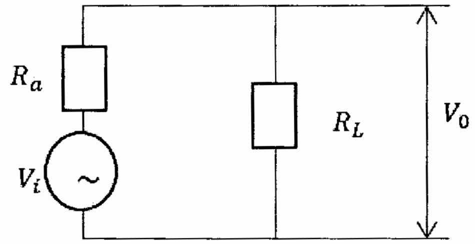
- When the anode voltage is constant



Voltage gain, A

This is the ratio of the output voltage, V_0 to the input voltage V_i

$$A = \frac{V_0}{V_i}$$



Current through the load $= \frac{E.m.f}{Total\ resistance}$

E.m.f = amplification factor × input voltage

$= \mu V_i$

$\Rightarrow I_a = \frac{\mu V_i}{R_a + R_L}$

Output voltage, $V_0 = IR_L$

$\Rightarrow V_0 = \frac{\mu V_i R_L}{R_a + R_L}$

Voltage gain, $A = \frac{V_0}{V_i} = \frac{\mu R_L}{R_a + R_L}$

Examples

1. A triode valve passes an anode current of 5 mA at anode voltage of 150 V and grid voltage – 2 V. When the grid voltage is reduced to – 3.5 V, the triode passes an anode current of 3.2 mA when anode voltage is maintained at 150 V. When the anode voltage is reduced to 128 V, maintaining grid voltage at – 2 V, anode current reduces to 2.6 mA. Calculate
   (i) the mutual conductance
   (ii) the anode resistance
   (iii) amplification factor

Solution

(i) Anode resistance $= \left(\frac{\Delta V_a}{\Delta I_a}\right)_{V_g} = \left(\frac{150-128}{(5-2.6)\times10^{-3}}\right)$

$= 9166.7\ \Omega$

(ii) Mutual conductance

$= \left(\frac{\Delta I_a}{\Delta V_g}\right)_{V_a} = \left(\frac{(5.0-3.2)\times10^{-3}}{-2.0+3.5}\right) = 0.0012\ \Omega$

(iii) Amplification factor

$$\mu = \left(\frac{\Delta V_a}{\Delta I_a}\right)_{V_g} \times \left(\frac{\Delta I_a}{\Delta V_g}\right)_{V_a} = \left(\frac{\Delta V_a}{\Delta V_g}\right)_{I_a}$$

$$\mu = 9166.7 \times 0.0012 = 11.0$$

2. A single stage triode amplifier has an anode load resistance of 15 kΩ and anode resistance of 10 kΩ. Calculate the amplifier gain when mutual conductance is 0.003 Ω^{-1}.

Solution

Voltage gain A $= \frac{\Delta V_o}{\Delta V_i} = \frac{\mu R}{R + R_a}$

But $\mu = g_m R_a$

$\Rightarrow A = \frac{R_a g_m R}{R+R_a} = \frac{0.003 \times 15000 \times 10000}{25000} = 18.0$

3. A triode with mutual conductance of 4.0×10^{-3} AV^{-1} and anode resistance 5 kΩ is connected in series with a load resistance of 10 kΩ. Calculate the peak value of an output alternating voltage with V_{rms} is 30 mV (assume the triode to operate at optimum conditions).

Solution

Peak value of input signal = $30\times10^{-3}\times\sqrt{2}$

$= 42.43\,mV$

Since voltage gain, $A = \frac{\Delta V_o}{\Delta V_i} = \frac{R_a g_m R}{R+R_a}$

$V_o = \left(\frac{4.0\times10^{-3}\times5000\times10000}{15000}\right)\times 42.43\times10^{-3}$

$= 0.57\,V$

4. A triode value of anode resistance 3000 Ω is used as an amplifier to an alternating signal of amplitude 0.5V. Calculate the V_{rms} of the output signal and the voltage gain across a load resistor of 50 kΩ given that amplification factor is 15.

Solution

Amplification factor "μ" = $\left(\frac{\Delta V_a}{\Delta V_g}\right)_{I_a}$

$\Rightarrow V_0 = 15 \times 0.5 = 7.5\,V$

Since $(R_a + R_L) = \frac{V_a}{I_a}$

$I_a = \left(\frac{7.5}{3000+50000}\right) = 1.42\times10^{-4}\,A$

$V_0 = I_a R_L = 1.42\times10^{-4}\times50000 = 7.08\,V$

∴ The V_{rms} across $R_L = \frac{7.08}{\sqrt{2}} = 5.003\,V$

The voltage gain $A = \frac{V_0}{V_i} = \frac{7.08}{0.5} = 14.16$

Advantages of a transistor as an amplifier as compared to the triode

- A transistor needs low voltage as compared to the triode
- No heat is required to produce electrons (current carriers are available in the semiconductor)
- Transistor has no vacuum which may deteriorate like in the triode causing unnecessary ionisation.

Self-Evaluation exercise

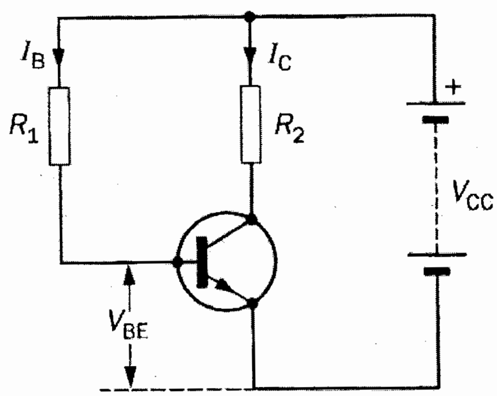
1. Explain what is meant by p-type and n-type semiconductors. Describe a p-n junction diode. Draw a graph which shows the variation of the current through such a diode with the potential difference across it and explain why the diode behaves differently when the potential difference across it is reversed.

   Describe the junction transistor. Sketch curves to show the variation of the collector current with the collector-base voltage for various values of the emitter current and explain their form.
2. Draw a sketch of a p-n-p transistor used in (i) common-base (CB) and (ii) common-emitter (CE) arrangement showing clearly the polarities of the batteries. Explain why the common-emitter arrangement is preferred in an alternating frequency amplifier circuit.
3. Sketch graphs using the same axes showing how the current through a thermionic diode varies with the d.c potential difference applied between the anode and filament for two different temperatures. Explain three special features of the graphs.
4. What is meant by
   (a) half wave rectification
   (b) full wave rectification?
   Explain with the aid of labelled circuit diagrams how each of these may be achieved using thermionic diodes
5. (a) Describe the structure of a diode
   (b) Explain how a triode differs in structure and operation from a diode
   (c) Explain how a triode may be used to amplify small alternating potential differences
6. For a triode, sketch curves to show
   (i) the form of anode current/grid voltage characteristics
   (ii) the form of the anode current/grid voltage characteristics.

   How may the amplification factor of the valve be deduced from these curves?
7. Give a brief description of the construction of a high vacuum diode. Draw a graph which shows the variation of the current through such a diode with the potential difference across it, and account for the main features of the curve.

8. The base current of the transistor is $50\,\mu A$ and collector current is $25\,mA$. Determine the values of β and α. [Ans: $500,\ 0.998$]
9. A triode valve is to be used to amplify a direct current of $10^{-7}\,A$ flowing in a circuit incorporating a resistance of $10^5\,\Omega$. The valve has a mutual conductance of $2\,mAV^{-1}$ and an anode resistance (impedance) of $10^6\,\Omega$. Draw a diagram of a suitable circuit and calculate the current amplification.

[Ans: 200]
10. In the junction transistor voltage amplifier circuit shown below, if $R_1 = 100\,k\Omega$, $R_2 = 1\,k\Omega$, $V_{CC} = 6.0\,V$ and $V_{BE} = 0.60\,V$. Calculate
    (i) voltage across R_1
    (ii) I_B
    (iii) I_C if current gain is 60
    (iv) voltage across R_2
    (v) voltage across the collector emitter.



[Ans: (i) 5.4 V (ii) $54\,\mu A$ (iii) $3.2\,mA$ (iv) $3.2\,V$ (v) $2.8\,V$]

Additional questions

1. (a) (i) Draw a labelled diagram to illustrate the arrangement of the apparatus used in Millikan's oil drop experiment to the determine the magnitude of the charge of an electron.
(ii) Why must the arrangement be maintained at a constant temperature?
(iii) How are the oil droplets charged?
(b) What are the measurements and how are these measurements used to deduce the magnitude of the charge on the drop?
(c) An oil drop of density $800\,kgm^{-3}$ falls uniformly through a distance of 4.00 mm in 16.0 s between two horizontal metal plates in the absence of the electric field.
    (i) If the viscosity of air is $1.80 \times 10^{-5}\,Nsm^{-2}$, determine the radius of the oil drop
    (ii) If the oil drop carries a charge of one electron and the electric field between the plates is $2.0 \times 10^5\,Vm^{-1}$, calculate the ratio of the electric force on the oil drop to its weight.
    (iii) Describe what happens if the value of the ratio in (c) (ii) above is one
(d) In a Millikan's oil drop experiment, experiment, the terminal velocity of an oil drop in a constant electric field was measured repeatedly and it was found to change greatly. Explain this observation.

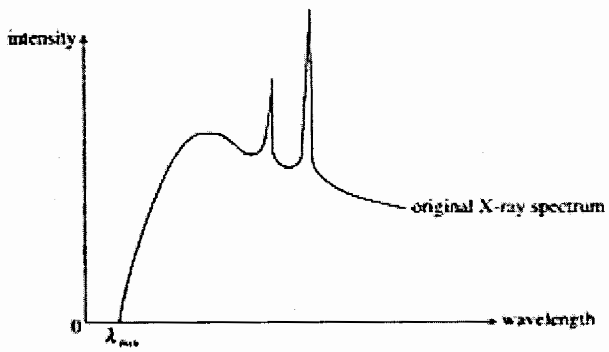
2. (a) Differentiate between soft X-rays and Hard X-rays
(b) (i) Sketch graphs to show the variation of intensity with wavelength in an X-ray spectra obtained at two different voltages V_1 and V_2 ($V_2 > V_1$) across an X-ray tube.
(ii) Explain how the continuous background spectrum and the characteristic line spectrum are produced.
(c) A beam of X-rays incident on a crystal of potassium chloride gives a first order diffraction image at a glancing angle of $8.58°$. If the density of the crystal is $1984\,kg\,m^{-3}$ and its relative molecular weight is 74.55,
    (i) estimate the separation between its atomic planes and deduce the wavelength of the X-rays used
    (ii) determine the maximum order of diffraction obtainable

(iii) explain what happens if X-rays of wavelength $8.12 \times 10^{-10}\,m$ are used

(d) Explain why the emission of X-rays may be considered as the reverse effect of photoelectric emission

3. (a) X-rays can easily penetrate matter. What conclusion can be derived about the interaction between X-rays and a single atom?
(b) Derive Bragg's equation $n\lambda = 2d\sin\theta$ for diffraction of X-rays by a crystal
(c) The separation between a set of atomic planes in a nickel crystal is 0.215 nm. Find the maximum order of diffraction that can be obtained from these planes if X-rays of wavelength 0.154 nm are used.

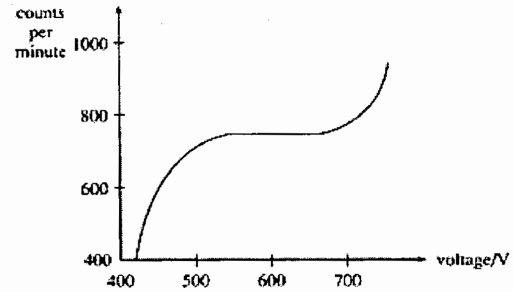
4. (a) State two properties of X-rays
(b) The graph below shows the original X-ray spectrum produced by an X-ray tube



Copy the graph and sketch on the same axes, the new X-ray spectrum for each of the following cases.
(i) The filament current in the tube is increased
(ii) The potential difference across the tube is increased
(iii) The target element in the tube is replaced by an element of higher atomic number

Give reason for the change to the X-ray spectrum in each of the cases above.

5. (a) Give a labelled diagram of a Geiger-Muller tube and explain its working principle
(b) The response of a Geiger-Muller tube at a steady source of $\beta -$ particles is determined as a function of the voltage applied to the tube and the characteristic curve is as shown below. Explain briefly the shape of the curve.



(c) Suggest a suitable voltage range for the working of the tube
(d) Explain why the tube is an efficient detector of β- particles but less efficient for α-particles and γ-rays.
(e) A Geiger-Muller tube was used to examine the radioactivity decay of a source. The corrected count rate N at time t is as follows.

$t/minute$	0	50	150	300	450
$N/minute^{-1}$	7150	4450	1790	446	112

Estimate the half-life of the source

6. In a model of the hydrogen atom, an electron of mass m and charge $-e$ moves in a circular orbit of radius of radius r about a stationary proton of charge $+e$.
(a) If the centripetal force is provided by the electrostatic force between the electron and the proton, derive an expression for the angular velocity ω of the electron in terms of e, r, m and ε_0, the permittivity of free space.
(b) Hence show that the angular momentum L of the electron in its orbit is $\left(\frac{mre^2}{4\pi\varepsilon_0}\right)^{\frac{1}{2}}$
(c) Show that the total energy of the electron is $-\frac{L^2}{2mr^2}$
(d) If the angular momentum of the electron can only take discrete values $\frac{nh}{2\pi}$, where n is an integer and h is the Planck's constant, find an expression for the radius r_0 of the orbit of lowest possible angular momentum in terms of e, m, ε_0 and h. If $r_0 = 5.3 \times 10^{-11}\,m$, calculate the energy required to ionize the atom.

7. (a) When electromagnetic radiation falls on a metal surface, electrons may be emitted. This is photoelectric effect.
(i) State Einstein's equation photoelectric equation, explaining the meaning of each term
(ii) Explain why, for a particular metal, electrons are emitted only when the frequency of the incident radiation is greater than a certain value.

(iii) Explain why the maximum speed of the emitted electrons is independent of the incident radiation

(b) A source emits monochromatic light of frequency $5.5 \times 10^{14}\ Hz$ at a rate of 0.10 W. Of the photons given out, 0.15% fall on the cathode of the photocell which gives a current of $6.0\ \mu A$ in an external circuit. You may assume that this current consists of the photoelectrons emitted. Calculate

(i) the energy of a photon

(ii) the number of photons leaving the source per second

(iii) the percentage of the photons falling on the cathode which produce photoelectrons

(c) (i) Calculate the wavelength associated with electrons which have been accelerated from rest through 3000 V.

(ii) Indicate one situation you would expect electrons of about this energy to behave as waves. Give a reason for your answer

8. (a)(i) Explain what is meant by photoelectric emission

(ii) Briefly describe a simple experiment to demonstrate this effect qualitatively

(b) Describe and explain the effect of increasing the intensity of the incident radiation.

(c) In an experiment in photoelectricity, the maximum kinetic energy of the photoelectrons was determined for different wavelengths of the incident radiation. The following results were obtained.

Wavelength/nm	300	375	500
Maximum kinetic energy	2.03	1.20	0.36

Use the results to determine

(i) the work function of the metal

(ii) a value for Planck's constant

(d) (i) Describe how a photocell functions

(ii) Describe one application of a photocell

9. (a) List the important experimental facts relating to the photoelectric effect and explain how Einstein's equation accounts for them

(b) A clean surface of potassium in a vacuum is irradiated with light of wavelength $5.5 \times 10^{-7}\ m$ and electrons are found just to emerge, but when light of wavelength $5 \times 10^{-7}\ m$ is incident, electrons emerge each with energy $3.62 \times 10^{-20}\ J$. Estimate the value of Planck's constant h

(c) Deduce the effect of irradiating in vacuum

(a) a copper surface

(ii) a caesium surface, with light of wavelength $5 \times 10^{-7}\ m$, given that the work functions of copper and caesium are respectively $6.4 \times 10^{-19}\ J$ and $3.2 \times 10^{-19}\ J$.

(d) Describe an experiment to verify the equation for the kinetic energy of the photoelectrons and show how the work function of the surface and Planck's constant can be obtained

(e) Light of frequency $6.0 \times 10^{14}\ Hz$ incident on a metal surface ejects photoelectrons having a kinetic energy of $2.0 \times 10^{-19}\ J$

10. (a) Use quantum theory to explain the experimental observations of photoelectric effect

(b) Explain how the wave theory fails to account for these of observations

(c) In an experiment with a vacuum photocell the maximum kinetic energy of the electrons emitted was measured for different wavelengths of the illuminating radiation. The following results were obtained.

Maximum kinetic energy/$10^{-19}\ J$	Wavelength/ $10^{-7}\ m$
3.26	3.00
2.56	3.33
1.92	3.75
1.25	4.29
0.58	5.00

Use these results to plot a linear graph and derive a value for Planck's constant

(d) If the experiment were repeated with radiation of wavelength (i) $7.5 \times 10^{-7}\ m$ (ii) $2.8 \times 10^{-7}\ m$, would photoelectrons be emitted and if so, what would be their maximum kinetic energy?

(e) Describe and explain how the graph might change if a different metal were used for the surface of the photo-cathode.

Answers

1. (a) (ii) so that viscosity of air remains unchanged

(iii) due to friction of the air or X-rays

(c) (i) $1.61 \times 10^{-6} m$

(ii) 0.234

(iii) Oil drop remains stationary

(d) Due to the oil drop gaining or losing electrons

2. (a) Soft X-rays: longer λ, less penetrating

Hard X-rays: shorter λ, more penetrating

(b) (ii) Continuous spectrum: The energy of retarding electrons is converted to X-rays on striking the target atoms.

Line spectrum: The bombarding electron ejects an electron in the target atom from a lower energy level to a higher energy level. When the vacancy in the lower energy level is filled by an electron from an outer energy level, the difference in the energy level is radiated as a photon of characteristic X-ray.

(c) (i) $3.15 \times 10^{-10}\,m$, $\lambda = 9.40 \times 10^{-11}\,m$

(ii) $n_{max} = 6$

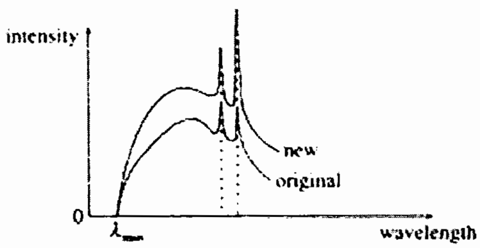
(d) Photoelectric emission

- Electromagnetic radiation incident on a metal surface ejects electrons
- X-rays are produced when fast electrons bombard a heavy metal surface.

3. (a) Interaction between X-rays and atoms is negligible

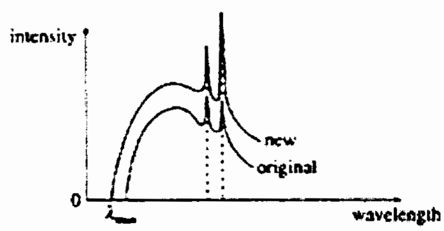
(c) Maximum order = 2

4. (b)(i) When the filament current is increased, more electrons bombard the target every second. The intensity for all λ increases but λ_{min} remains unchanged.



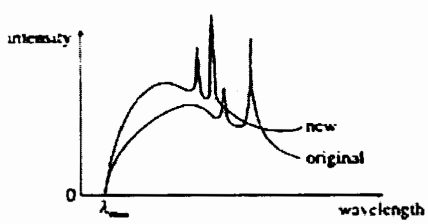
(ii) When a p.d increases, λ_{min} decreases, since $eV = \frac{hc}{\lambda_{min}}$.

Line X-rays remain at the same λ since it is characteristic of the target.



(iii) With a target made of an element of a higher atomic number, the wavelengths of characteristic X-rays are shorter and the intensity of all λ increases as frequency increases.

λ_{min} remains unchanged as V is unchanged.



5. (c) 550 V to 650 V

(d) penetrating power of α-particles is weak and some α-particles may not penetrate the mica window of a G-M tube.

The γ-rays give rise to secondary pulses of current and hence the count may be inaccurate.

(e) $T_{\frac{1}{2}} = 80$ minutes

6. (a) $\omega = \frac{e}{\sqrt{4\pi\varepsilon_0 m r^3}}$

(d) $r_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$

7. (b)(i) $3.6 \times 10^{-19}\,J$ (ii) $2.8 \times 10^{17}\,s^{-1}$ (iii) 9.1%

(c)(i) 2.2×10^{-11}

8. (c)(i) $3.4 \times 10^{-19}\,J$ (ii) $6.68 \times 10^{-34}\,Js$

9. (b) $6.64 \times 10^{-34}\,Js$ (e) $1.96 \times 10^{-19}\,J$

10. (c) $6.7 \times 10^{-34}\,Js$ (d)(i) no emission (ii) electrons emitted with KE of $3.74 \times 10^{-19}\,J$

Examination questions

1. (a) What is meant by the following
   - (i) Radioactivity,
   - (ii) Isotopes?

   (b) (i) Define **mass defect**

   (ii) State the condition for a heavy nucleus of an atom to be unstable

   (iii) Explain your answer in (b)(ii)

   (c) A sample of $^{226}_{88}Ra$ emits both α-particles and γ-rays. A mass defect of 0.0053u occurs in the decay

   (i) Calculate the energy released in joules

   (ii) If the sample decays by emission of α-particles, each of energy 4.60 MeV and γ-rays, find the frequency of the γ-rays emitted

   (d) (i) Sketch a graph showing the variation of binding energy per nucleon with mass number, clearly showing the fusion and fission regions

   (ii) Use the sketch in (d)(i) to explain how energy is released in each of the processes of fusion and fission.

   (e) State two
   - (i) applications of radioisotopes
   - (ii) health hazards of radioisotopes

   [2017, No. 8]

2. (a) What are **X-rays**?

   (b) (i) With the aid of a diagram explain how X-rays are produced in an X-ray tube

   (ii) State the energy changes that take place in the production of X-rays in an X-ray tube

   (c) In an X-ray tube, the electrons strike the target with a velocity of $3.75 \times 10^7\ ms^{-1}$ after travelling a distance of $5.0\ cm$ from the cathode. If a current of $10\ mA$ flows through the tube, find the

   (i) tube voltage

   (ii) number of electrons striking the target per second

   (iii) number of electrons within a space of $1\ cm$ between the anode and cathode

   (d) Briefly explain **one** application of X-rays

   [2017, No. 9]

3. (a) State Bohr's postulate of the atom

   (a) Explain the occurrence of the emission and absorption line spectra

   (b) Explain the main observations in **Rutherford's** α-particles scattering experiment.

   (c) A beam of alpha particles of energy 3.5 MeV is incident normal to a gold foil

   (i) Calculate the least distance of approach the nucleus of the gold atom given that i atomic number is 79.

   (ii) State the significance of the value of tl least distance of approach

   [2017, No. 1(

4. (a) (i) Distinguish between **mass defect** an **binding energy** of an atomic nucleus

   (ii) Sketch a graph of nuclear binding energ per nucleon versus mass number fc naturally occurring isotopes and use it t distinguish between nuclear fission an fusion.

   (b) Describe with the aid of a labelled diagram Millikan's oil drop experiment to determin the charge on an oil drop

   (c) (i) Explain briefly diffraction of X-rays by crystals and derive **Bragg's law**

   (ii) A second order diffraction image is obtained by reflection of X-rays at atomic planes of a crystal for a glancing angle of $11°24'$. Calculate the atomic spacing if the wavelength of X-rays is $4.0 \times 10^{-11}\ m$.

   [2016, No. 8; (c) (ii) $2.02 \times 10^{-10}\ J$]

5. (a) State **Bohr's model** of an atom

   (b) An electron of mass m and charge $-e$, is considered to move in a circular orbit about a proton.

   (i) Write down the expression for the electric force on the electron

   (ii) Derive an expression for the total energy of the electron given that the angular momentum of the electron is equal to $\frac{nh}{2\pi}$ where n is an integer and h is Planck's constant

   (c) With the aid of a labelled diagram, describe the operation of a diffusion cloud chamber

   (d) The energy levels of an atom have values

$$E_1 = -21.4\ eV$$
$$E_2 = -4.87\ eV$$
$$E_3 = -2.77\ eV$$
$$E_4 = -0.81\ eV$$
$$E_\infty = 0.00\ eV$$

   (i) Calculate the wavelength of radiation emitted when an electron makes transition from E_3 and E_2

   (ii) State the region of the electromagnetic spectrum where the radiation lies

   [2016, No. 9; Ans: (c) (ii) $5.97 \times 10^{-7}\ m$]

6. (a) Describe how positive rays are produced
   (b) Describe how a Bainbridge spectrometer can be used to detect isotopes
   (c) (i) What is a **time base** as applied to a Cathode Ray Oscilloscope?
   (ii) Draw a sketch graph showing the variation of time-base voltage with time
   (d) An alternating p.d applied to the Y-plates of an oscilloscope produces five complete waves on a 10 cm length of the screen when the time base setting is $10\,ms\,cm^{-1}$. Find the frequency of the alternating voltage
   (e) (i) Explain the motion of an electron projected perpendicular into a uniform magnetic field
   (ii) An electron accelerated from rest by a p.d of $100\,V$, enters perpendicularly into a uniform electric field of intensity $10^5\,Vm^{-1}$. Find the magnetic flux density, B which must be applied perpendicularly to the electric field so that the electron passes undeflected through the fields.

[2016, No. 10: Ans: (d) $50\,Hz$ (e)(ii)$0.0169\,T$]

7. (a) (i) **Define Avogadro's constant** and Faraday's constant
   (ii) Show how the charge carried by a monovalent ion is 1.6×10^{-19} C
   (b) With the aid of labeled diagram, describe **Millikan's oil drop** experiment for determination of the charge of an electron.
   (c) A beam of positive ions moving with a velocity $\vec{V}$ enters a region of uniform magnetic field of density $\vec{B}$ with the velocity at right angles to the field $\vec{B}$. By use of a diagram, describe the motion of the ions.
   (d) A charged oil drop of density $880\ kgm^{-3}$ is held stationary between two parallel plates 6.0 mm apart held at a potential difference of 103 V. When the electric field is switched off, the drop is observed to fall a distance of 2.0 mm in 35.7 s. (Velocity of air $= 1.8 \times 10^{-5} Nsm^{-2}$, Density of air$= 1.29\ kgm^{-3}$)
   (i) Calculate the radius of the drop
   (ii) Estimate the number of excess electrons on the drop

[2015, No. 8; Ans: (d)(i) $7.254 \times 10^{-7} m$ (ii) 5]

8. (a) (i) State the laws of photoelectric emission
   (ii) Explain briefly one application of photo electric effect
   (b) In a photoelectric setup, a point source of light of power 3.2×10^{-3} W emits mono-energetic photons of energy 5.0 eV. The source is located at a distance of 0.8mfrom the Centre of a stationary metallic spheres of work function 3.0 eV and radius 8.0×10^{-3}m. The efficiency of photo electron emission is one in every 10^6 incident photons. Calculate the
   (i) number of photoelectrons emitted per second
   (ii) maximum kinetic energy in joules, of the electrons
   (c) (i) State the Bragg's law of X-ray diffraction
   (ii) Show that density ρ, of a crystal can be given by

$$\rho = \frac{M\, sin^3\, \theta}{125 N_A (n\lambda)^3}$$

where θ is the glancing angle, n is the order of diffraction, λ is the X-ray wavelength and M is the molecular weight of the crystal.

[2015, No. 9; (c)(i) 1.0×10^5 (ii)$3.2 \times 10^{-19}\,J$]

9. (a) With reference to a Geiger Muller tube, define the following
   (i) Quenching agent
   (ii) Background count rate
   (b) (i) With the aid of a labelled diagram, describe the operation of a Geiger-Muller (GM) tube
   (ii) Explain how half-life of a short lived radioactive source can be obtained by use of a Geiger-Muller tube
   (c) A radioactive isotope $^{32}_{15}P$ which has a half-life of 14.3 days disintegrates to form a stable product. A sample of the isotope is prepared with an initial activity of $2.0 \times 106\,s^{-1}$. Calculate the number of ^{32}P atoms after 30 days. [Assume $N = N_0 e^{-\lambda t}$]

[2015, No. 10; Ans: (c) (i) 3.78×10^8 (ii) $49.5\,s^{-1}$ (iii) 18.83×10^7]

10. (a) What is photo electric emission?
   (b) (i) Describe a simple experiment to demonstrate photoelectric effect
   (ii) When a clean surface of a metal in vacuum is irradiated with light of wavelength 5×10^{-7}m is incident on a metal surface, electrons are emitted each with energy 3.62×10^{-20}J. Find the value of the plank's constant

(c) (i) With the aid of a labelled diagram, describe an X-ray tube and how X-rays are produced

(ii) Describe how intensity and quality of X-rays is controlled in an X-ray tube

(d) An X-ray tube operates at 1.5×10^{-3} V and and the current through it is 1.0×10^{-3}A. Find the

(i) number of electrons crossing the tube per second

(ii) kinetic energy gained by electron traversing the tube.

[2014, No. 9; Ans: (b)(ii) 6.637×10^{-34}Js (d)(i)6.25×10^{15} (ii) 2.4×10^{-22}J]

11. (a) (i) What is specific charge?

(ii) State the unit of specific charge

(iii) Describe with the aid of a diagram how the specific charge of positive ion can be determined using a mass spectrometer

(b) A beam of singly ionized carbon atoms pass undeflected through the region of cross magnetic and electric field of 0.10 T and 1.0×10^4 NC^{-1} respectively. When it enters a region of uniform magnetic field, it is deflected through an arc of radius 0.75 m. Calculate the magnetic flux density of this magnetic field (mass of carbon atom = 1.0×10^{-26} kg)

(c) (i) Draw a graph to illustrate the variation of ionization current and p.d across an ionization chamber and explain it main features

(ii) Explain how ionisation chamber can be used to detect ionizing radiation.

[2014, No. 10; Ans: (b) 1.667×10^{-2} T]

12. (a) Explain briefly how positive rays are produced

(b) An electron charge, e and mass, m is emitted from a hot cathode and then accelerated by an electric field towards the anode. If the potential difference between the cathode and the anode is V. Show that the speed of the electron, u is given by

$$u = \sqrt{\left(\frac{2eV}{m}\right)}$$

(c) An electron starts from rest and moves in an electric field intensity of 2.4×10^3 Vm^{-1}. Find the

(i) force of the electron

(ii) acceleration of the electron

(iii) velocity required in moving through a p.d of 90 V

(d) A beam of electrons each of mass. m and charge, e is directed horizontally with speed, u into an electric field between two horizontal metal plates separated by a distance, d.

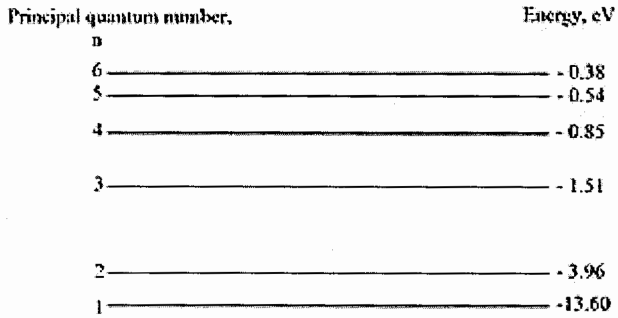
(i) If the p.d between the plates is V, show that the deflection of the beam is given by

$$y = \frac{1}{2m}\left(\frac{eV}{du^2}\right)x^2$$

(ii) Explain the path of the electron beam as it emerges onto of the electric field.

[2013, No. 8; Ans (c) (i) 3.84×10^{-16} N (ii) 4.22×10^{14} ms^{-2} (iii) 5.62×10^6 ms^{-1}]

13. (a) The figure below shows some of the energy levels of a hydrogen atom



(i) Why are the energies for the different levels negative?

(ii) Calculate the wavelength of the lines arising from a transition from the third to the second energy level

(iii) Calculate the ionization energy in joules of hydrogen

(b) Explain the physical processes in an X-ray tube that account for

(i) cut off wavelength

(ii) characteristic lines

(c) Calculate maximum frequency of radiation emitted by an X-ray tube using an accelerating voltage of 33.0 kV

(d) Derive Bragg's law of X-ray diffraction in crystals

[2013, No. 9, Ans: (a) (ii) 6.58×10^{-7} (iii) 2.18×10^{-18}J (c) 8.0×10^{18}Hz]

14. (a) A beam of α-particles is directed normally to a thin metal foil. Explain why

(i) most of the α-particles passed straight through the foil

(ii) few α-particles are deflected through angles more than 90°

(b) Calculate the least distance of approach of a 3.5 MeV α-particle to the nucleus of a gold atom (Atomic number of gold = 79)

(c) (i) Define space charge as applied to thermionic diodes
(ii) Draw anode-anode voltage curves of a thermionic diode for two different filament currents and explain their main features
(d) (i) What is a decay constant
(ii) A sample from fresh wood of certain species of tree has an activity of 16.0 counts per minute per gram. However, the activity of 5 g of a dead wood of the same species of tree is 10.0 counts per minute. Calculate the age of the dead wood. (assume half-life of 5730 years)

[2013, No. 10; Ans (b) 6.49×10^{-14}m (d) (ii) 1.719×10^4 years]

15. (a) (i) What are cathode rays?
(ii) With the aid of a diagram, describe an experiment to show that cathode rays travel in straight lines.
(b) A beam of electrons is accelerated through a potential difference of 500 V. The beam enters midway between parallel plates of length 10 cm and are 3 cm apart. If the potential difference across the plates is 600 V, find the velocity of the electron as it leaves the region between the plates.
(c) State the laws of photoelectric emission
(d) Explain how line emission spectra are produced

[2012 No. 8; Ans: (b) 2.93×10^7ms^{-1} , 63.3°]

16. (a) (i) What is meant by the term radioactive decay, half-life and decay constant?
(ii) Show that the half-life $t_{\frac{1}{2}}$ of a radioisotope is given by

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

where λ is the decay constant
(b) With the aid of a labelled diagram, describe the structure and action of a cloud chambers
(c) A radioactive isotope $^{99}_{43}X$ decays by emission of a gamma ray. The half-life of the isotope if 360 minutes. What is the activity of the 1 mg of the isotope?
(d) Explain the term avalanche as applied to the ionization chamber

[2012, No. 9; Ans: 1.95×10^{14}s^{-1}]

17. (a) Define the terms below as applied to the diode
(i) Space charge
(ii) Amplification factor
(iii) Mutual conductance
(b) Derive an expression for the amplification factor in terms of anode resistance, R_a and mutual conductance, g_m of a triode value.
(c) A triode with mutual conductance 3 mAV^{-1} and anode resistance of 10 kΩ is connected to a load resistance of 20 kΩ. Calculate the amplitude of output signal, if the amplitude of the input signal is 25 mV
(d) (i) Sketch the output characteristic of a transistor
(ii) Identify on the sketch in (d)(i), the region over which the transistor can be used as an amplifier

[2012, No. 10; Ans: (d) 0.5V]

18. (a) (i) Describe with aid of a well labelled diagram, the structure and mode of operation of a C.R.O
(ii) State advantages of C.R.O over a moving coil galvanometer
(b) In the determination of the electron charge by Millikan's method, potential difference of 1.5 kV is applied between horizontal metal plates, 12 mm apart. With the field switched off, a drop of oil mass 1.0×10^{-24} kg is observed to fall with constant velocity 4.0×10^{-24} kg between two metal plates 12 mm. When a potential difference of 1.5 kV is applied across the plates, the drop rises with constant velocity of 8.0×10^{-5}ms^{-1}. How many electron charges ae there on the drop? (Assume air resistance is proportional to the velocity of the drop and neglect air buoyancy)
(c) Explain why
(i) the apparatus in Millikan's experiment is surrounded by a constant temperature enclosure.
(ii) low vapour-pressure oil is used
(d) In Millikan's experiment, the radius, r of the drop is calculated from

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

where η is the viscosity of air and ρ is the density of the oil
Identify the symbol v and describe how it is measured

[2011; No. 8; Ans: (b) 6]

19. (a) (i) Explain how X-rays are produced in an X-ray tube
(ii) Explain the emission of X-ray characteristic spectra

(iii) Derive Bragg's X-ray diffraction equation

(iv) Under what conditions does X-ray diffraction occur?

(b) With the aid of a labelled diagram, describe how a Bain bridge mass spectrometer is used to measure specific charge

[2011, No. 9]

20. (a) What is meant by unified atomic mass unit?

(b) (i) Distinguish between nuclear fusion and nuclear fission

(ii) State the conditions necessary for each of the nuclear reactions in (b)(i) to occur

(c) (i) With the aid of a labelled diagram, describe the operation of an ionization chamber

(ii) Sketch the curve of ionization current against applied p.d and explain its main features

(d) A typical nuclear reaction is given by

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}M_0 + ^{139}_{57}La + 2^{1}_{0}n + 7^{\ 0}_{-1}e$$

Calculate the total energy released by 1 g of uranium.

Mass of

$$^{1}_{0}n = 1.009\mu$$
$$^{\ 0}_{-1}e = 0.00055\mu$$
$$^{95}_{42}M_o = 94.906\mu$$
$$^{139}_{57}La = 138.906\mu$$
$$^{235}_{92}U = 235.044\mu$$
$$1\mu = 1.66 \times 10^{-27}kg$$

[2011, No. 10: Ans; (d) 8.837×10^{10}J]

21. (a) (i) With the aid of a labelled diagram, describe what is observed when a high-tension voltage is applied across a gas tube in which pressure is gradually reduced to low values

(ii) Give two applications of discharge tubes

(b) Describe Thompson's experiment to determine specific charge of an electron

(c) In Millikan's oil drop experiment, a charged oil drop of radius 9.2×10^{-7}m and density 800 kgm^{-3} is held stationary in an electric field of intensity 4.0×10^4 Vm^{-1}

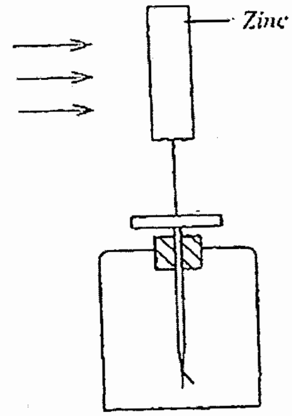
(i) How many electron charges are on the drop?

(ii) Find the electric field intensity that can be applied to move the drop with a velocity 0.005 ms^{-1} upwards. (Density of air = 1.29 kgm^{-3}, coefficient of viscosity of air = 1.8×10^5 Nsm^{-1})

[2010, No. 8; Ans: (c) (i) 4 (ii) 2.48×10^6Vm^{-1}]

22. (a) Explain what is meant by photoelectric effect?

(b)



Ultraviolet and infrared radiation are directed in turns into a zinc plate which is attached to a gold leaf electroscope as shown in the figure above. Explain what happens when

(i) Ultra violet radiation falls on the zinc plate

(ii) Infrared radiation falls on the zinc plate

(iii) The intensity of each radiation is increased

(c) An X-ray of wavelength 10^{10} m is required for the study of its diffraction in a crystal. Find the least accelerating voltage to be applied to an X-ray tube in order to produce these X-rays.

(d) Sodium has a work function of 2.0 eV and is illuminated by radiation of wavelength 150 mm. Calculate the maximum speed of the emitted electrons.

(e) With the aid of a well labelled diagram, describe how stopping potential of a metal can be measured.

[2010, No. 9: Ans: (c) 12375V (d) 3.2×10^{-19}J]

23. (a) (i) What is meant by mass defect?

(ii) Sketch a graph showing how binding energy per nucleon varies with mass number and explain its main features

(iii) Find the binding energy per nucleon of $^{56}_{26}$Fe given that

mass of 1 proton = 1.007825u, mass of 1 neutron = 1.008665u, 1u = 931 MeV

(b) With aid of a diagram, explain how an ionization chamber works

(c) (i) Show that when an alpha particle collides ahead with an atom of atomic number z, the closest distance of approach to the nucleus, x_0 is given by

$$x = \frac{ze^2}{\pi\varepsilon_0 mv^2}$$

where e is the electron charge, ε_0 is the permittivity of free space, m is the mass of the alpha particle and v is the initial speed of the alpha particle.

(ii)In a head on collision between alpha particle and a gold nucleus, the minimum distance of approach is 5×10^{-14}m. Calculate the energy of alpha particle (in MeV). (Atomic number of gold = 79)

[2010, No. 10; Ans: (b) (iii) 7.704 MeV (d) (i) 4.55MeV]

24. (a) State four differences between cathode rays and positive rays

(b) An electron having energy of 4.5×10^2 eV moves at right angles to uniform magnetic field of flux density 1.5×10^{-3} T. Find the

(i) radius of the path followed by the electron

(ii) period of motion

(c) (i) Define the terms Avogadro's constant and Faraday constant

(ii) Use the Avogadro constant and Faraday constant to calculate charge on an ion of a monovalent element.

(d) Explain the meaning of the following terms as applied to a GM-tube

(i) threshold potential difference

(ii) dead time

(iii) a quenching agent

[2009, No. 8; Ans: (b) (i) 4.78×10^{-2}m (ii) 2.38×10^{-8}s]

25. (a) State the laws of Photoelectric effect

(b) Describe an experiment to determine the stopping potential of a metal surface

(c) A 100 mW beam of light of wavelength 4.0×10^{-7}m falls on caesium surface of a photocell.

(i) How many photons strike the caesium surface per second?

(ii) If 65% of the photons emit photoelectrons, find the resulting photo current.

(iii) Calculate the kinetic energy of each of photon if the work function of caesium is 2.20 eV

(d) Distinguish between continuous and line spectra in an X-ray tube.

[2009, No. 9; Ans (c) (i) 2.02×10^{17}photons (ii) 2×10^{-2}A (iii) 1.43×10^{-19}J]

26. (a) (i) Explain the observations made in the Rutherford's particle scattering experiment

(ii)Why is a vacuum necessary in this experiment?

(b) Distinguish between excitation and ionization energies of an atom

(c) Draw a labelled diagram showing the main components of an X-ray tube

(d) An X-ray is operated at 50 kV and 20 mA. If 1% of the total energy supplied is emitted as radiation, calculate the

(i) maximum frequency of the emitted radiation

(ii) rate at which heat must be removed from the target in order to keep it at a steady temperature.

(e) A beam of X-rays of wavelength 0.20 nm is incident on a crystal at a glancing angle of 30°. If the interplanes separation is 0.20 nm, find the order of diffraction.

[2009, No. 10; Ans: (d) (i) 1.21×10^{19}Hz (ii) 1990 W (e) $n = 1$]

27. (a) What is meant by a line spectrum?

(b) Explain how line spectrum accounts for the existence of discrete energy levels in atoms

(c) The energy levels in a mercury atom are -10.4eV, -5.5eV, -3.7eV and -1.6eV

(i) Find the ionization energy of mercury joules

(ii) What is likely to happen to happen if a mercury atom in an excited state is bombarded in an electron of energy 4.0eV, 6.7eV or 11.0eV?

(d) An X-ray tube is operated at 20 kV with an electron current of 16 mA in the tube. Estimate the

(i) number of electrons heating the target per second

(ii) rate of production of heat assuming 99.5% of the kinetic energy of electrons is converted into heat ($e = 1.6 \times 10^{-19}$C)

[2008, No. 8, Ans: (e)(i) 1×10^{17} (ii) 318.4 W]

28. (a) (i) Define the term binding energy

(ii) Sketch a graph showing the variation of the binding energy per nucleon with mass number

(iii) Use the sketch graph you have drawn in (a) (ii) to explain how energy is released during fusion and fission.

(b) Explain why a high temperature is required during fusion of nuclides

(c) The isotope $^{238}_{92}U$ emits an alpha particle and forms an isotope of thorium (Th), while the isotope $^{235}_{92}U$ when bombarded by a neutron, forms $^{144}_{56}Ba$, $^{235}_{36}Kr$ and neutrons.

(i) Write the nuclear equation for the reactions of $^{238}_{92}U$ and $^{235}_{92}U$

(ii) How does the reaction of $^{235}_{92}U$ differ from that of $^{238}_{92}U$?

(d) A steel piston ring contains 15 g radioactive, $^{54}_{26}Fe$. The activity of $^{54}_{26}Fe$ is 3.7×10^5 disintegrations per second. After 100 days of continuous use, the crank case oil was found to have an activity of 1.23×10^3 disintegrations per second. Find the

(i) half-life of $^{54}_{26}Fe$

(ii) average mass of iron worn off the ring per day, assuming that all the metal removed from the ring accumulates in the oil

[2008, No. 9, Ans (d) (i) 3.13×10^{17}s (ii) 4.9×10^{-14}g]

29. (a) Describe the mechanism of thermionic emission

(b) Explain the following as applied to a vacuum diode

(i) space charge limitation

(ii) saturation

(iii) Rectification

(c) Sketch the current potential difference characteristic of a thermionic diode for two different operating temperatures and explain their main features.

(d) (i) A triode value with an anode resistance of $3.0 \times 10^3\,\Omega$ is used as an amplifier. A sinusoidal alternating signal of amplitude 0.5 V is applied to the grid of the value. Find the r.m.s value of the output voltage if the amplification factor is 15 and the anode load is 50 kΩ

(ii) Draw an equivalent circuit of a triode a single stage amplifier.

[2008, No 10; Ans: (d)(ii) 5.003 V]

30. (a) Describe briefly the mechanism of thermionic emission

(b) (i) Draw a labelled circuit to show a triode being used as a single stage voltage amplifier

(ii) With the aid of an equivalent circuit of the triode as an amplifier, obtain an expression for the voltage gain.

(iii) A triode with mutual conductance of $3.0 \times 10^3\,AV^{-1}$ and anode resistance of $1 \times 10^4\,\Omega$ is used as a single stage amplifier. If the load resistance is $3 \times 10^4\,\Omega$, calculate the voltage of the amplifier.

(c) (i) Describe the structure of a junction transistor

(ii) Sketch and describe the collector current against the collector emitter voltage characteristic of a junction transistor.

[2007; No. 8, Ans: (b) (iii) 22.5]

31. (a) What are isotopes?

(b) With the aid of a diagram, describe the operation of a Bain bridge spectrometer in determining the specific charge of ions.

(c) Explain the purpose of each of the following in a Geiger-Muller tube.

(i) a thin mica window

(ii) argon gas at low pressure

(iii) halogen gas mixed with argon gas

(iv) an anode in form of a wire

(d) (i) What is meant by binding energy per nucleon of a nucleus?

(ii) Sketch a graph of binding energy per nucleon against mass number for naturally occurring nuclides

(iii) State one similarity between nuclear fusion and nuclear fission

(e) (i) At a certain time, an α-particle detector registers a count rate of $32\,s^{-1}$. Exactly 10 days later, the count rate dropped to $8\,s^{-1}$. Find the decay constant.

(ii) State any two uses of radioactivity and two health hazards

[2007, No. 9; Ans: (e) 0.139 per day]

32. (a) (i) Describe with the aid of a diagram, the production of cathode rays

(ii) State and justify two properties of cathode rays

(b) Explain each of the following terms as applied to photo electric emission

(i) stopping potential

(ii) threshold frequency

(c) Explain X-ray diffraction by crystal and derive Bragg's law

(d) The potential difference between the cathode and anode of X-ray tube is 5.0×10^{-4} V. If only 0.4% of the kinetic energy of the electrons is converted into X-rays and the rest is dissipated as heat in the target at a rate of 600 W, find the

(i) current that flows

(ii) speed of the electrons striking the target

[2007, No. 10; Ans (d) (i) 1.21×10^6 A (ii) 1.33×10^4 ms^{-1}]

33. (a)(i) What is a photon?
  (ii) Explain, using quantum theory, the experimental observations on the photoelectric effects.
  (iii) When light of wavelength 450 nm falls on a certain metal, electrons of maximum kinetic energy 0.76 eV are emitted. Find the threshold frequency for the metal
  (b) Explain, using suitable sketch graphs, how X-ray spectra in an X-ray tube are formed.
  (c) A beam of X-rays of wavelength 8.42×10^{-11}m is incident on sodium chloride crystal of interplanal separation 2.82×10^{-10}m. Calculate the first order diffraction angle.

[2006, No. 8; Ans: (a)(iii) 4.83×10^{14} Hz (c) $8.6°$]

34. (a) (i) A beam of electrons, having a common velocity of enters a uniform magnetic field in a direction normal to the field. Describe and explain the subsequent path of the electrons.
  (ii) Explain whether a similar path would be followed if a uniform electric field were substituted for the magnetic field.
  (b) Describe an experiment to measure the ratio of the charge to mass of an electron.
  (c) Electrodes are mounted at opposite ends of low pressure discharge tube and a potential difference of 1.20 kV applied between then. Assuming that the electrons are accelerated from the rest, calculate the maximum velocity which they could acquire (specific charge of electron $= -1.76 \times 10^{11}$ Ckg^{-1})
  (d) (i) Give an account of the stages observed when an electric discharge passes through a gas at pressure varying from atmospheric to about 0.01 mm Hg as air is pumped out when the p.d across the tube is maintained at extra high tension.
  (ii) State two disadvantage of discharge tubes when used to study cathode rays

[2006, No. 9; Ans: (c) 2.06×10^7ms^{-1}]

35. (a)(i) What is meant by half-life of a radioactive material?
  (ii) Given that radioactive law, $N_t = N_0 e^{-\lambda t}$, obtain the relation between λ and half-life $T_{\frac{1}{2}}$
  (iii) What are radioisotopes.
  (iv) The radioisotope $^{90}_{38}Sr$ decays by emission of β-particle. The half-life of the radioisotope is 28.8 days. Determine the activity of 1 g of the isotope.
  (b) (i) With the aid of the diagram, describe the structure and action of a Geiger-Muller tube
  (ii) Sketch the count rate-voltage characteristic of the Geiger-Muller tube and explain its main features.
  (iii) Identify, giving reasons, the suitable range in (b)(ii) of operation of the tube.

[2006, No. 10, Ans: (iv) 5.1×10^{12}s^{-1}]

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