UACE MATHEMATICS PAPER 1 2017 guide

SECTION A (40 marks)

Answer all questions in this section

1. The coefficient of the first three terms of the expansion of $\left(1+\frac{x}{2}\right)^n$ are in arithmetic Progression (AP). Find the value of n. (05marks)
2. Solve the equation $3\tan^2\theta + 2\sec^2\theta = 2(5 - 3\tan\theta)$ for $0^0 < \theta < 180^0$ (05marks)
3. Differentiate $\left(\frac{1+2x}{1+x}\right)^2$ with respect to x. (05marks)
4. Solve for x in the $4^{2x} - 4^{x+1} + 4 = 0$
5. The vertices of a triangle are P(4, 3), Q(6, 4) and R(5, 8). Find angle RPQ using vectors. (05marks)
6. Show that $\int_2^4 xInxdx = 14In2 - 3$ (05matks)
7. The equation of the curve is given by $y^2 - 6y + 20x + 49 = 0$
   (a) Show that the curve is a parabola. (03marks)
   (b) Find the coordinates of the vertex. (02marks)
8. A container is in form of an inverted right angled circular cone. Its height is 100cm and base radius is 40cm. the container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of $10cm^3s^{-1}$. Find the rate at which the water level in the container is falling when the height of water in the container is halved. (05marks)

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. (a) Given that the complex number Z and its conjugate $\overline{Z}$ satisfy the equation $Z\overline{Z} - 2Z + 2\overline{Z} = 5 - 4i$. Find possible values of Z. (06marks)
   (c) Prove that if $\frac{Z-6i}{Z+8}$ is real, then the locus of the point representing the complex number Z is a straight line. (06marks)
10. A circle whose centre is in the first quadrant touches the x – and y –axes and the line 8x – 15y = 120. Find the
   (a) equation of the circle (10marks)
   (b) point at which the circle touches the x-axis. (02marks)
11. A curve whose equation is $x^2y + y^2 - 3x = 3$ passes through points A(1, 2)and B(-1, 0). The tangent at A and the normal at the curve at B intersect at point C. Determine;
   (a) equation of the tangent. (06marks)

(b) coordinates of C. (06marks)

12. (a) Express cos $(\theta + 30)^0$ – cos $(\theta + 48)^0$ in the form RsinPsinQ, where R is constant. Hence solve th3 equation
cos $(\theta + 30)^0$ – cos $(\theta + 48)^0$ = 0.2 (06marks)
(b) Prove that in any triangle ABC, $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{c^2}$ (06marks)

13. (a) solve the simultaneous equation
$(x - 4y)^2 = 1$
3x = 8y = 11 (06marks)
(b) Solve the inequality
$4x^2 + 2x < 3x + 6$ (06marks)

14. (a) The points A and B have position vectors a and b. A point C with vector position c lies on AB such that $\frac{AC}{AB} = \lambda$. Show that c = (1 – λ)a + λb. (04marks)
(b) the vector equation of two lines are;
r_1 = 2i + j + λ(i + j + 2k) and r_2 = 2i+ 2j + tk + $\mu(i + 2j + k)$
where i, j and k are unit vectors and λ, μ and t are constants. Given that the two lines intersect, find
(i) the value of t.
(ii) the coordinates of the point of intersection. (08marks)

15. (a) sketch the curve $y = x^3 - 8$ (08marks)
(b) The area enclosed by the curve in (a), the y-axis and x-axis is rotate about the line y = 0 through 3600. Determine the volume of the solid generated. (04 marks)

16. Solve the differential equation $\frac{dy}{dx} = (xy)^{\frac{1}{2}} Inx$, given that y = 1 when x = 1.
Hence find the value of y when x = 4 (12marks)

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. The coefficient of the first three terms of the expansion of $\left(1 + \frac{x}{2}\right)^n$ are in arithmetic Progression (AP). Find the value of n. (05marks)

The expansion of $\left(1 + \frac{x}{2}\right)^n$ is given by pinch

$\left(1 + \frac{x}{2}\right)^n = 1 + \frac{n}{2}x + \frac{\frac{n(n-1)}{4}x^2}{2!} + \cdots$

$= 1 + \frac{n}{2}x + \frac{n(n-1)x^2}{8} + \cdots$

$U_1 = 1$, $U_2 = \frac{n}{2}$; $U_3 = \frac{n(n-1)}{8}$

But 3rd term – 2nd term = 2nd term – 1st term

$\frac{n(n-1)}{8} - \frac{n}{2} = \frac{n}{2} - 1$

$\frac{n(n-1)}{8} = n - 1$

$n(n - 1) = 8n - 1$

$n^2 - 9n + 1 = 0$

(n -8)(n -1) = 0

n – 8 = 0

n= 8

2. Solve the equation $3\tan^2\theta + 2\sec^2\theta = 2(5 - 3\tan\theta)$ for $0^0 < \theta < 180^0$ (05marks)

   Let t = tan θ

   $3t^2 - 2(1 + t^2) = 2(5 - 3t)$

   $5t^2 + 6t - 8 = 0$

   $t = \frac{-6 \pm \sqrt{6^2 - 4(5)(-8)}}{2(5)} = \frac{-6 \pm 14}{10} = -2$ or $\frac{4}{5}$

   Taking t = -2; $\theta = \tan^{-1}(-2) = 116.57^0$

   Taking $t = \frac{4}{5}$; $\theta = \tan^{-1}(\frac{4}{5}) = 38.66^0$

   Hence $\theta = 38.66^0, 116.57^0$

3. Differentiate $\left(\frac{1+2x}{1+x}\right)^2$ with respect to x. (05marks)

   Let $y = \left(\frac{1+2x}{1+x}\right)^2$

   $\frac{dy}{dx} = \frac{4(1+2x)(1+x)^2 - 2(1+x)(1+2x)^2}{(1+x)^4}$

   $= \frac{2(1+2x)(1+x)[2+2x-1-2x]}{(1+x)^4}$

   $= \frac{2(1+2x)(1+x)(1)}{(1+x)^4}$

   $\frac{dy}{dx} = \frac{2(1+2x)}{(1+x)^3}$

4. Solve for x in the $4^{2x} - 4^{x+1} + 4 = 0$

   $(4^x)^2 - 4(4^x) + 4 = 0$

   Let $q = 4^x$

   $q^2 - 4q + 4 = 0$

   $(q - 2)^2 = 0$

   q = 2

   ⇨ $4^x = 2$

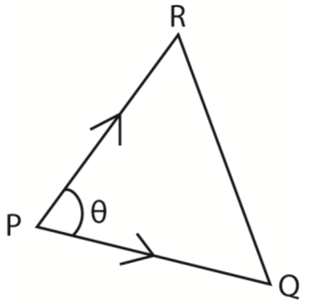
   $2^{2x} = 2^1$

   2x = 1

   $x = \frac{1}{2}$

5. The vertices of a triangle are P(4, 3), Q(6, 4) and R(5, 8). Find angle RPQ using vectors. (05marks)

   Let <RPQ = θ



   $\overline{PQ} = \binom{6}{4} - \binom{4}{3} = \binom{2}{1}$

   $\overline{PR} = \binom{5}{8} - \binom{4}{3} = \binom{1}{5}$

   $|\overline{PQ}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$|\overline{PR}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

$\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}1\\5\end{pmatrix} = \sqrt{5}\,.\sqrt{26}cos\theta$

2 + 5 = $\sqrt{130}cos\theta$

θ = $cos^{-1}\left(\frac{7}{\sqrt{130}}\right)$ = 52.13^0

6. Show that $\int_2^4 xInxdx = 14In2 - 3$ (05matks)

Let u = Inx and v^2 = x

⇨ u' = $\frac{1}{x}$ and v' = x

$\int_2^4 xInxdx = \left[\frac{x^2}{2}Inx\right]_2^4 - \frac{1}{2}\int_2^4 xdx$

$= \frac{1}{2}(16In4 - 4In2) - \frac{1}{4}[x^2]_2^4$

$= \frac{1}{2}(16In\,2^2 - 4In2) - \frac{1}{4}(16-4)$

$= \frac{1}{2}(32In\,2 - 4In2) - \frac{1}{4}(16-4)$

= 14In2 -3

7. The equation of the curve is given by $y^2 - 6y + 20x + 49 = 0$

(a) Show that the curve is a parabola. (03marks)

$y^2 - 6y + 20x + 49 = 0$

$(y-3)^2 - 9 + 20x + 49 = 0$

$(y-3)^2 = -20x - 40$

$(y-3)^2 = -20(x+2)$

(b) Find the coordinates of the vertex. (02marks)

V(-2,3)

8. A container is in form of an inverted right angled circular cone. Its height is 100cm and base radius is 40cm. the container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of $10cm^3s^{-1}$. Find the rate at which the water level in the container is falling when the height of water in the container is halved. (05marks)

$\frac{h}{100} = \frac{r}{40}$ =>r = $\frac{2}{5}h$

$v = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 = \frac{4}{75}\pi h^2$

$\frac{dv}{dt} = -10$

$\frac{dv}{dh} = \frac{4}{25}\pi h^2$

$\frac{dh}{dt} = \frac{dh}{dv} x \frac{dv}{dt} = \frac{25}{4\pi h^2}\ x - 10$

$= \frac{250}{4\pi(50)^2}$

$= 0.00796$

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. (a) Given that the complex number Z and its conjugate $\overline{Z}$ satisfy the equation

$Z\overline{Z} - 2Z + 2\overline{Z} = 5 - 4i$. Find possible values of Z. (06marks)

Z = x + yi, $\bar{Z} = (x - yi)$

$Z\overline{Z} - 2Z + 2\overline{Z} = 5 - 4i.$

(x + yi)(x – yi) – 2(x + yi) + 2(x – yi) = 5 – 4i

$x^2 + y^2 -2x -2yi + 2x - 2yi = 5 -4i$

$x^2 + y^2 - 4yi = 5 -4i$

equating imaginary part

$-4yi = -4$

$y = 1$

equating real parts

$x^2 + y^2 = 5$

$x^2 + 1^2 = 5$

$x = \pm 2$

$\therefore Z = \pm 2 + 1$

(b) Prove that if $\frac{Z-6i}{Z+8}$ is real, then the locus of the point representing the complex number Z is a straight line. (06marks)

$\frac{Z-6i}{Z+8} = \frac{x+yi-6i}{x+yi+8}$

$= \frac{x+(y-6)i}{x+8+yi}$

$= \frac{x+(y-6)i}{x+8+yi} \cdot \frac{x+8-yi}{x+8+yi}$

$= \frac{x^2+8x+y^2-6y+(xy-6x-48-xy)i}{(x+8)^2+y^2}$

$IM \frac{z-6i}{z+8} = 0$

$\frac{8y-6x-48}{(x+8)^2+y^2} = 0$

$8y - 6x - 48 = 0$

$4y - 3x - 24 = 0$

Or

$y = \frac{3}{4}x + 6$

10. A circle whose centre is in the first quadrant touches the x – and y –axes and the line $8x - 15y = 120$. Find the

(a) equation of the circle (10marks)

Radius $a = \frac{|8a-15a-120|}{\sqrt{8^2+(-15)^2}}$

$= \frac{|-7a+120|}{17}$

$17a = 7a + 120$

$10a = 120$

$a = 12$

Equation of the circle

$(x - 12)^2 + (y - 12)^2 = 12^2$

$X^2 + y^2 - 24x -24y + 144 = 0$

(b) point at which the circle touches the x-axis. (02marks)

$y = 0$

$(x - 12)^2 = 0$

$x = 12$

the point (12, 0)

11. A curve whose equation is $x^2y + y^2 -3x =3$ passes through points A(1, 2)and B(-1, 0). The tangent at A and the normal at the curve at B intersect at point C. Determine;

(a) equation of the tangent. (06marks)

$x^2y + y^2 -3x =3$

$2xy + 2\frac{dy}{dx} - 3 + 2y\frac{dy}{dx} = 0$

At (1,2)

$2(1)(2) + 2\frac{dy}{dx} - 3 + 2(2)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{1}{5}$

Tangent $y - y_1 = m(x - 1)$

$y - 2 = \frac{1}{5}(x-1)$

$y = -\frac{1}{5}x + \frac{11}{5}$

(b) coordinates of C. (06marks)

At B(-1, 0)

$2(-1)(0) + 2\frac{dy}{dx} - 3 + 2(0)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3$

$y - 0 = -\frac{1(x+1)}{3}$

$y = -\frac{1}{3}x - \frac{1}{3}$

At C

$-\frac{1}{5}x + \frac{11}{5} = -\frac{1}{3}x - \frac{1}{3}$

$-3x + 33 = -5x - 5$

$-2x = 38$

$x = -19$

$y = \frac{19}{3} - \frac{1}{3} = 6$

C(-19, 6)

12. (a) Express $\cos(\theta + 30)^0 - \cos(\theta + 48)^0$ in the form RsinPsinQ, where R is constant. Hence solve th3 equation
$\cos(\theta + 30)^0 - \cos(\theta + 48)^0 = 0.2$ (06marks)

$\cos(\theta + 30)^0 - \cos(\theta + 48)^0$

$= -2\sin\left(\frac{\theta + 30^0 + \theta + 48^0}{2}\right)\sin\left(\frac{\theta + 30^0 - \theta - 48^0}{2}\right)$

$= -2\sin(\theta + 39^0)\sin(-9^0)$

$\cos(\theta + 30)^0 - \cos(\theta + 48)^0 = 0.$

⇨ $-2\sin(\theta + 39^0)\sin(-9^0) = 0.2$

$\sin(\theta + 39^0) = 0.63925$

$\theta + 39^0 = 39.74^0$

$\theta = 0.74^0$

(b) Prove that in any triangle ABC, $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{c^2}$ (06marks)

$$\frac{a^2 - b^2}{c^2} = \frac{(2R\sin A)^2 - (2R\sin B)^2}{(2R\sin C)^2}$$

$$= \frac{4R^2(\sin^2 A - \sin^2 B}{4R^2 \sin^2 C}$$

$$= \frac{(\sin A + \sin B)(\sin A - \sin B)}{\sin^2[180^0 - (A+B]}$$

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{\sin^2(A+B)}$$

$$= \frac{\sin(A+B)\sin(A-B)}{\sin^2(A+B)}$$

$$= \frac{\sin(A-B)}{\sin(A+B)}$$

13. (a) solve the simultaneous equation

$(x - 4y)^2 = 1$

$3x = 8y = 11$ (06marks)

Solving equations

$(x - 4y) = 1$ ……………………. (i)

$3x = 8y = 11$……………………. (ii)

Eqn. (ii) - 3Eqn. (i)

$20y = 8$

$y = \frac{8}{20} = \frac{2}{5}$

From eqn. (i)

$x = 1 + 4\left(\frac{2}{5}\right) = \frac{13}{5}$

And

$(x - 4y) = -1$ ……………………. (i)

$3x = 8y = 11$……………………. (ii)

2(eqn (i)) + eqn. (ii)

$5x = 9$

$x = \frac{9}{5}$

From equation (i)

$4y = \frac{9}{5} + 1$

$y = \frac{7}{10}$

$\therefore$ (x,y) = $\left(\frac{13}{5}, \frac{2}{5}\right), \left(\frac{9}{5}, \frac{7}{10}\right)$

(c) Solve the inequality

$4x^2 + 2x < 3x + 6$ (06marks)

$4x^2 + 5x - 6 < 0$

Critical values

$x = \frac{-5 \pm \sqrt{5^2 - 4(4)(-6)}}{2(4)}$

$= \frac{-5 \pm \sqrt{121}}{8}$

$x = -2, \frac{3}{4}$

	$x < -2$	$-2 < x < \frac{3}{4}$	$x > \frac{3}{4}$
$4x^2 + 5x - 6$	+	-	+

$\therefore -2 < x < \frac{3}{4}$

14. (a) The points A and B have position vectors a and b. A point C with vector position c lies on AB such that $\frac{AC}{AB} = \lambda$. Show that c = (1 – λ)a + λb. (04marks)

$$\frac{\overline{AC}}{\overline{AB}} = \lambda$$
$$\overline{AC} = \lambda\overline{AB}$$
$$\overline{OC} - \overline{OA} = \lambda(\overline{OB} - \overline{OA}$$

c – a = λ(b – a)

c = a + λ(b – a)

= (1 – λ)a + λb

(b) the vector equation of two lines are;

r_1 = 2i + j + λ(i + j + 2k) and r_2 = 2i+ 2j + tk + $\mu(i + 2j + k)$

where i, j and k are unit vectors and λ, μ and t are constants. Given that the two lines intersect, find

(i) the value of t.

x = 2 + λ = 2+ μ ……………… (i)

y = 1 + λ= 2 + 2μ ……………..(ii)

z = 2λ = t + λ ………………….. (iii)

From eqn. (i)

2 + λ = 2+ μ

λ = μ

from eqn. (ii)

1 + λ= 2 + 2μ

1 + μ = 2 + 2μ

μ = λ = -1

from eqn. (iii)

2λ = t + λ

2(-1) = t – 1

t = -1

(ii) the coordinates of the point of intersection. (08marks)

x =2 + λ = 2 -1 = 1

y = 1 + λ = 1 -1 = 0

z = 2λ = 2(-1) = -2

∴ (x, y, z) = (1, 0, -2)

15. (a) sketch the curve y = x^3 – 8 (08marks)

y = x^3 – 8

Intercepts

When x = 0, y = -8

When y = 0, x = 2

(x, y) = (2, 0)

Turning point: $\frac{dy}{dx} = 3x^2$

$3x^2$ = 0

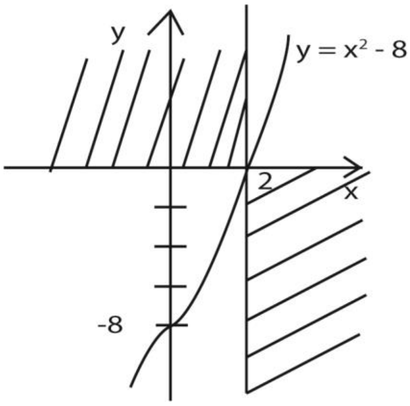
x = 0

$$\frac{d^2y}{dx^2} = 6x$$

$\frac{d^2y}{dx^2} = 0, x = 0$

Point of reflection= (0, 8)

	x < 2	x > 2
y	-	+



(b) The area enclosed by the curve in (a), the y-axis and x-axis is rotate about the line y = 0 through 360^0. Determine the volume of the solid generated. (04 marks)

V = $\pi \int_0^2 y^2 dx$

= $\pi \int_0^2 (x^3 - 8)^2 dx$

= $\pi \int_0^2 (x^6 - 16x^3 + 64) dx$

=$\pi \left[\frac{x^7}{7} - 4x^4 + 64x\right]_0^2$

= $\pi \left(\frac{128}{7} - 64 + 128\right)$

= $\frac{576\pi}{7}$ =250.5082 units^3

16. Solve the differential equation $\frac{dy}{dx} = (xy)^{\frac{1}{2}} Inx$, given that y = 1 when x = 1. Hence find the value of y when x = 4 (12marks)

$\frac{dy}{dx} = (xy)^{\frac{1}{2}} Inx = \frac{dy}{dx} = y^{\frac{1}{2}} x^{\frac{1}{2}} Inx$

$\int y^{-\frac{1}{2}} dy = \int x^{\frac{1}{2}} Inx\ dx$

$2\sqrt{y} = x^{\frac{1}{2}} Inx\ dx$

u = lnx, u' = $\frac{1}{x}$

v' = $x^{\frac{1}{2}}$, v = $\frac{2}{3} x^{\frac{1}{2}}$

$2\sqrt{y} = \frac{2}{3} x \sqrt{x} Inx - \frac{4}{9} x\sqrt{x} + c$

$2\sqrt{1} = \frac{2}{3}(1) \sqrt{(1)} In(1) - \frac{4}{9}(1)\sqrt{(1)} + c$

c = 2 + $\frac{4}{9} = \frac{22}{9}$

$2\sqrt{y} = \frac{2}{3} x \sqrt{x} Inx - \frac{4}{9} x\sqrt{x} + \frac{22}{9}$

$\sqrt{y} = \frac{1}{3} x \sqrt{x} Inx - \frac{2}{9} x\sqrt{x} + \frac{11}{9}$

Hence

$\sqrt{y} = \frac{1}{3}(4) \sqrt{(4)} In(4) - \frac{2}{9}(4)\sqrt{(4)} + \frac{11}{9}$

$y = 9.8673$