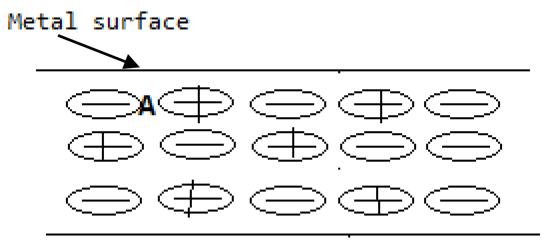
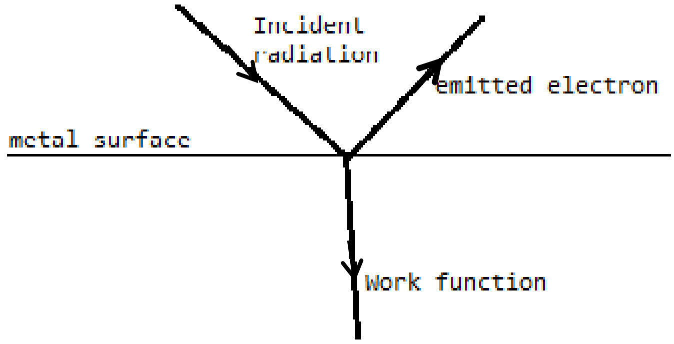
MODERN PHYSICS

PHOTOELECTRIC EFFECT

In metals, atoms exists as positive ions in a sea of electrons. An electron near the surface of the metal, say A, experiences an attractive inward force from the positive charges below it.





For such an electron to escape from the metal surface, a specific amount of work has to be done to overcome the forces which are inward.

Definitions

Photoelectric emission: this is the liberation of an electron from a metal surface by use of light of a suitable frequency.

Thermionic emission: this the liberation of an electron from a metal surface by application of heat.

N.B

- The light (radiation) supplies the electrons with an amount of energy equal or exceeding the energy that binds them to the surface.
- The liberated electrons are called **photo electrons.**
- Surfaces which are able to undergo electric emission are said to be photo emissive e.g. K, Na, Ca, etc. generally group I elements (*why?*).
- The occurrence of photoelectric effect can be demonstrated by using a gold leaf electroscope and a suitable metal e.g. zinc.

Laws of photoelectric emission

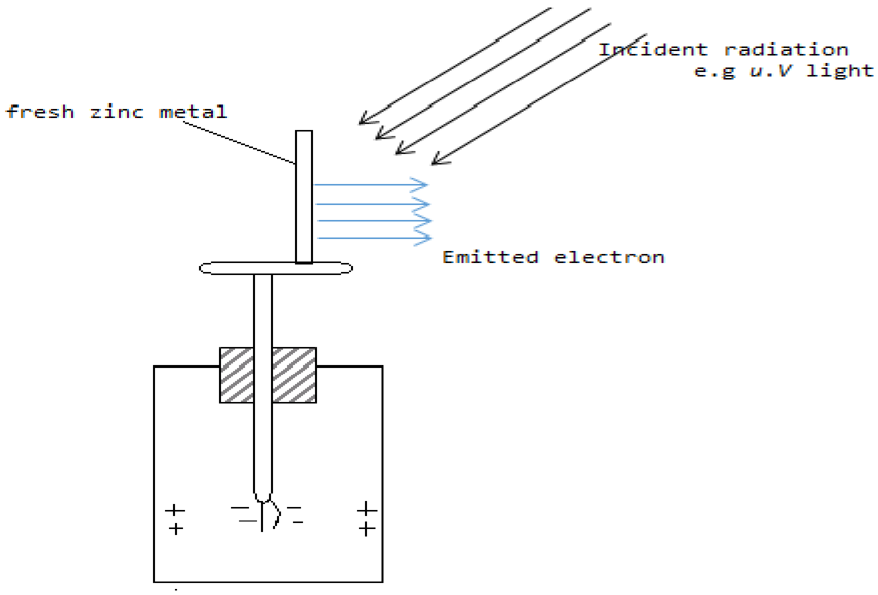
The laws (*characteristics or features*) are just a summary of experimental results on photoelectric effect.

1. The time lag between irradiation of the metal surface and emission of the electrons by the metal surface is negligible.

2. For a given metal, surface there is a minimum value of frequency of radiation called threshold frequency (f_0) below which no photo electrons are emitted from the metal however intense the incident radiation may be.
3. The velocity and K.E of the emitted photo electrons increase with increase in the frequency of the incident radiation.
4. the number of photoelectrons emitted from the surface per second is directly proportional to the intensity of incident radiation for a particular incident frequency
5. the K.E of the photoelectrons emitted is independent of the intensity of the incident radiation but depends only on its frequency

A simple experiment to demonstrate Photo electric effect

- A freshly cleaned Zinc plate is connected to the cap of a negatively charged gold leaf electroscope.
- Ultra violet radiations are allowed to fall on the zinc plate



Observations

- The leaf of the electroscope gradually falls
- This shows that both the zinc plate and the electroscope have lost charges.
- The lost charges are found to be electrons, hence photoelectric effect has occurred.

Note:

If a positively charged electroscope is used instead, there is no observable change in the divergence of the leaf.

Reasons:

This is because the emitted electrons are immediately attracted back by the positive charges on the cap of the electroscope hence restoring the charges.

Planks Quantum theory

States that the energy emitted from a body (*also called radiation*) is emitted or absorbed in discrete packets called **quanta.**

That's, we can have integral values such as 1, 2, 3 … n, but not fractional amount of energy

The energy E, contained in a quantum of radiation is proportional to the frequency *f*, of the radiation

i.e. $E \propto f$ => $\boxed{E = hf}$ where h = Planks constant (6.626 x 10^{-34}Js)

Dimensions of h

$$h = \frac{Energy}{frequency} = \frac{force \times distance}{frequency}$$

$$\Rightarrow [\mathrm{h}] = \frac{[force] \times [distance]}{[frequency]} = \frac{\mathrm{MLT^{-2} \times L}}{\mathrm{T^{-1}}}$$

$$\therefore [\mathrm{h}] = \mathrm{ML^2T^{-1}}$$

For an electromagnetic radiation of wavelength λ,

We have c = λ*f*,

$$\Rightarrow \boxed{E = \frac{hc}{\lambda}}$$

Thus $E \propto f$ and $E \propto \frac{1}{\lambda}$

Einstein's theory of photoelectric effect

- He considered a beam of light as consisting of several streams particles called photons

- Each photon carries (*or delivers*) a packet of energy or quanta given by *hf*. Where *f* is the frequency of the light/radiation.

- It is the photon that knocks off electrons from the metal surface.

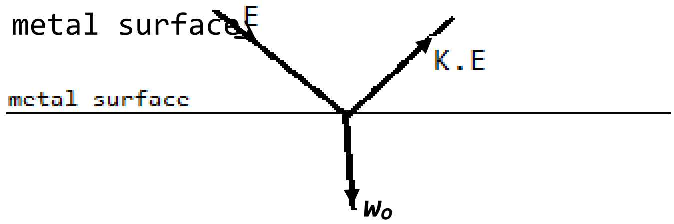
- When the photon (of energy *hf*) collides with an electron, it is either

  a) Reflected with no change in its energy or

b) Absorbed by the electron and the photon gives up all its energy to that single electron without sharing with other electrons

- To liberate/eject an electron from a particular metal surface, a quantity of energy called work function W_o (which is characteristic of the metal) has to be supplied by the incident radiation

- Thus a photon of energy E, (hf) causes an electron to be emitted from the metal surface



If the energy E, (hf) is greater than the work function (W_o) of the metal, the excess energy (hf - W_o) appears as the K.E of the emitted electron or photoelectron

I.e. $hf - W_o = \frac{1}{2}mv^2_{max}$

$\Rightarrow hf = W_o + \frac{1}{2}mv^2_{max}$ --------**(1)** called Einstein's photo-electric equation

The emitted electron escapes with a velocity having any value up to a maximum. The value of maximum velocity depends on:

i) The work function W_o of the metal and
ii) The frequency f of the incident radiation

From (1) $hf = W_o + \frac{1}{2}mv^2_{max}$

a) hf = energy of incident radiation of frequency f.
b) W_o = work function of the metal. It is defined as the minimum amount of energy required to liberate/release an electron from a metal surface.
c) $\frac{1}{2}mv^2$ = the maximum K.E of emitted electron

N.B the electrons are involved in collisions on their way out of the surface and therefore emerge with energy which is less than the maximum K.E

- If a photon has **just** enough energy to liberate the electron, the emitted electron gains no K.E and therefore floats on the surface of the metal.

Since the work function W_o is constant for a particular metal, there exists a minimum frequency (threshold frequency, f_o) given by $W_o = hf_o$

From (1) $hf = hf_o + \frac{1}{2}mv^2_{max}$ or $h(f - f_o) = \frac{1}{2}mv^2_{max}$

Where

f - The frequency of the incident radiation

f_o – the threshold frequency of the metal.

Note:

1) Threshold frequency is defined as the minimum frequency below which no electrons are emitted from the metal surface

   The associated threshold wavelength λ_o can be expressed as;

   $W_0 = hf_o$, but $c = f_o \lambda_o$ where c is the speed of light.

   Thus $w_o = \frac{hc}{\lambda_o}$

Task: *define threshold wavelength* λ_o

2) If an electron of charge e is accelerated by a voltage V volts, it gains K.E given by K.E = eV.

   Hence from above $h(f - f_o) = eV$

3) An electron volt (*ev*) is the K.E gained by an electron which has been accelerated through a p.d of one volt

$$\boxed{1eV = 1.6 \times 10^{-19}J}$$

Exercise

Where necessary assume: $h = 6.64 \times 10^{-34}Js$, $c = 3.0 \times 10^8m/s$, $e = 1.6 \times 10^{-19}c$

1. Calculate the energy of a photon of wavelength 3.0×10^{-7}m.

2. Calculate the speed of a proton which has been accelerated through a p.d of 400V. (Mass of a proton = 1.67×10^{-27}kg).

   Ans: 2.77×10^5m/s

3. U.v light of wavelength 0.4μm is incident on a metal surface of threshold wavelength 0.65μm. find the maximum speed of the emitted electron (mass of an electron = 9.11×10^{-31}kg).

   Ans: 6.48×10^5m/s

4. The threshold frequency of sodium metal is 5.6×10^{14}Hz. Calculate the velocity of photoelectrons emitted when sodium is illuminated by light of wavelength 5.0×10^{-7}m (Mass of an electron = 9.11×10^{-31}kg). Ans: 2.5×10^5m/s

5. A photo emissive has a threshold length 0.45μm. Calculate the maximum speed of the electron emitted by the surface when

irradiated with light of 0.35μm. Given mass of electron = 9.1×10^{-31} kg, speed of light, $c = 3 \times 10^{8}$ ms^{-1} and $h = 6.63 \times 10^{-34}$ Js.

6. Calcium has a work function of 2.7eV.
   a) What is the work function of calcium in Joules?
   b) What is the threshold frequency of calcium?
   c) What is the maximum wavelength that will cause emission from calcium metal? Ans: 4.3×10^{-19}J, 6.5×10^{14}Hz, 4.6×10^{7}m

7. Light of frequency 6×10^{14} Hz incident on a metal surface ejects photo electrons having kinetic energy of 2×10^{-19} J. calculate the energy needed to remove an electron from the metal surface given that Planck's constant , $h = 6.6 \times 10^{-34}$ Js.

8. Light of wavelength 0.5μm incident on a metal surface ejects electrons with kinetic energies up to a maximum value of 2×10^{-19} J. what energy is required to remove an electron from the metal.

9. The photo electric work function of potassium is 2.0eV.
   (i) What potential difference would have to be applied between the potassium surface and the collecting electrode in order to prevent the escape of electrons when the surface is illuminated with radiation of wavelength 350nm?
   (ii) What is the kinetic energy of the most energetic electrons emitted in this case?
   (iii) What is the speed of the most energetic electrons emitted?

10. The work function of caesium is 1.35eV.
    (a) What is the longest wave length that can cause photo electric emission from a caesium surface?
    (b) What is the maximum velocity with which photoelectrons will be emitted from a caesium surface illuminated with radiation of 400nm?
    (c) What p.d will prevent a current from passing through a caesium photocell illuminated by radiation of wavelength 400nm?

11. Monochromatic radiation of frequency 1.0×10^{15} Hz is incident on a clean magnesium surface for which the work function is 0.59×10^{-18}J. calculate

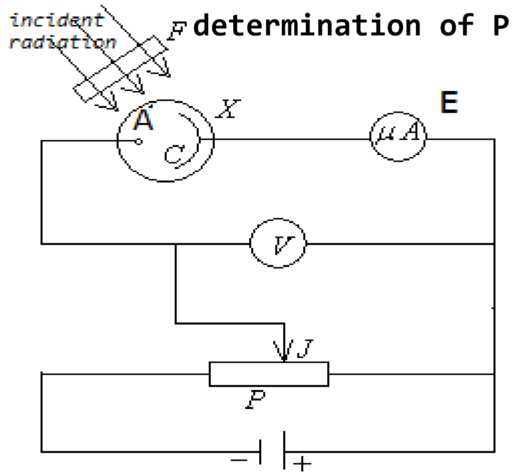
(i) the maximum kinetic energy of the emitted electrons

(ii) the potential to which the magnesium surface must be raised to prevent the escape of electrons

(iii) The cut-off wavelength.

Ans: $7.4x10^{-20}$J, 0.46V, $3.37x10^{-7}$m

Experiment to verify Einstein's photoelectric equation and determination of Planks constant h

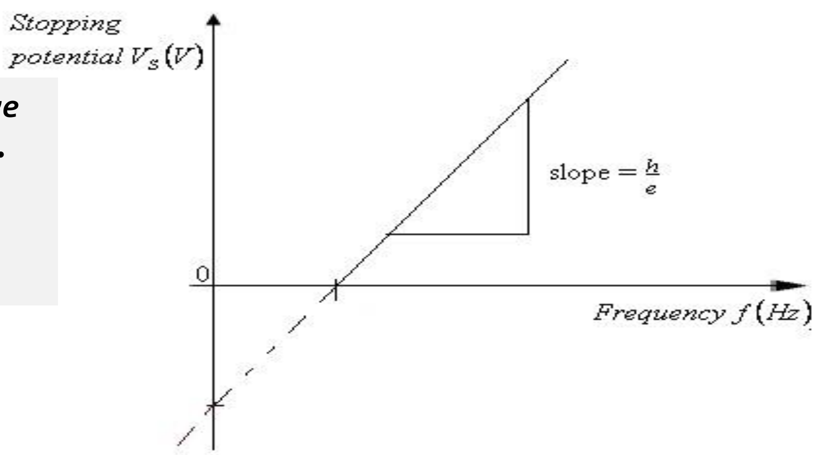


F = Filter
X = Photocell
A = Anode
C = Photo cathode
P = Variable resistor
J = Photo Jockey
E=Current Detector

- Incident radiations of different frequencies are filtered at F to fall on the cathode C.
- The anode is made negative with respect to the cathode by the potential divider circuit.
- The filtered frequency falling on the cathode causes emission of electrons.
- These electrons travel to the anode and cause a current to flow, detected at E.
- The p.d V is adjusted until the reading of E is zero (i.e. no current flows).
- The value of this p.d is the stopping potential (V_s) and is recorded from the voltmeter V.
- The procedure is repeated with light of different frequencies.
- A graph of stopping potential (V_s) against frequency (f) is plotted

The nature of the curve verifies the equation.

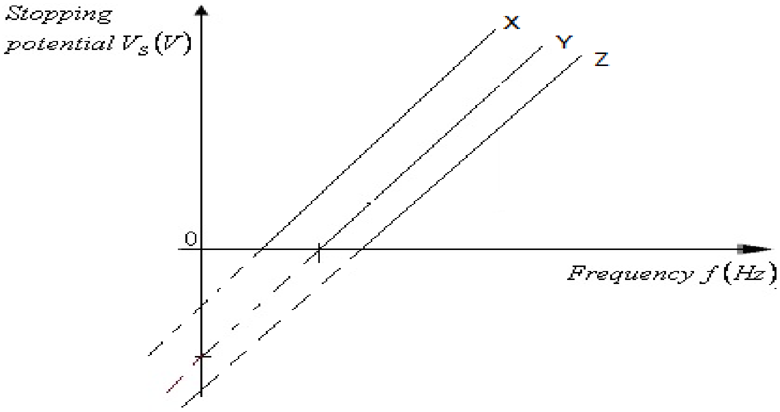
$$V_S = \left(\frac{h}{e}\right)f - \left(\frac{h}{e}\right)f_0 .$$



Determination of *h* from the graph.

- The slop s of the graph $S = \frac{h}{e}$ $\Rightarrow h = Se$, hence h can be calculated

NOTE: For all different types of metals, the slope of the graph of V_s against frequency f is constant (the same) $\left(\frac{h}{e}\right)$

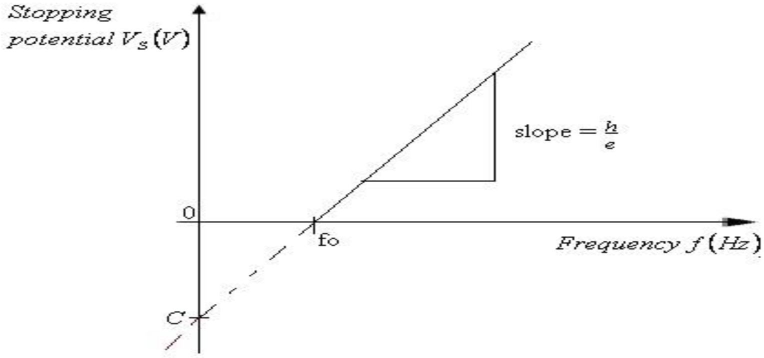


X, Y and Z are different metals with different W_o

The graphs are parallel implying they have the same gradient

Obtaining the work function W_o and threshold frequency f_o

The threshold frequency f_o and the work W_o function can be obtained from the intercepts



- The work function W_o

  The intercept C on the V_s axis is noted and $C = \frac{-W_0}{e} \quad \Rightarrow W_0 = -Ce$

  .

Stopping potential: the minimum potential between the cathode and the anode that prevents the most energetic electrons from reaching the anode

Task: *Document how to obtain the threshold wavelength (λ_o) and the stopping potential V_s.*

Exercise

Where necessary assume = 6.64 x 10^{-34}Js, c = 3.0 x 10^{8}m/s, e = 1.6 x 10^{-19}c, mass of an electron = 9.11 x 10^{-31}kg, 1eV = *1.6 x 10^{-19}J*

1. Calculate the stopping potential of platinum surface irradiated with U.v light of wavelength 1.2 x 10^{-7}m if the total work function of platinum is 6.3eV (Ans: 4.0V).

2 When light of wavelength 5.9x10^{-7}m is incident on a sodium metal, electrons of maximum K.E of 1.71x10^{-20}J are emitted. Calculate the maximum K.E of the electrons that will be emitted by sodium metal illuminated by light of wavelength 4.5x10^{-7}m. (Ans: 1.21x10^{-19}J).

3. In an experiment on photoelectric effect using radiations of wavelength 4.00x10^{-7}m, maximum K.E was observed to be 1.40x10^{-19}J. With radiations of wavelength 3.00x10^{-7}m, the maximum K.E was 3.06x10^{-19}J.Derive the value of planks Constant

   (Ans: 6.64x10^{-34}Js)

4. Gold has a work function of 4.9eV.
   (a) Calculate the maximum K.E in joules of the electrons emitted when Gold is illuminated with U.v of frequency 1.7 x 10^{15}Hz
   (b) What is the energy expressed in eV.
   (c) What is the stopping potential of the electrons?

   (Ans: (b) 2.1eV (c) 2.1eV)

5. A metal of work function 2.5eV is irradiated with light of unknown frequency. The maximum velocity of the photoelectrons emitted is 1.14x10^{6}m/s. Calculate the maximum wavelength of the incident radiation (Ans: 2.7x10^{-7}m)

6. The following results were obtained from an experiment on photoelectric effect of a particular metal surface.

Wavelength (nm)	stopping potential (V)
500	0.6
429	1.0
375	1.4
333	1.8
300	2.2

   (a) Plot a suitable graph and use the graph to determine;
      (i) Planks constant
      (ii) The work function of the metal

(iii) The threshold frequency of the metal surface

7. The following results were obtained from a sensitive plate of a photocell of different wavelengths.

Wavelength (nm)	Stopping potential (V)
404.7	1.13
435.8	0.93
491.6	0.62
546.1	0.36
579.1	0.24
607.3	0.15

Plot a graph of stopping potential against frequency. Use your graph to determine;

(a) The threshold frequency ($\approx 4.57 x 10^{14}$Hz)

(b) Planks constant ($\approx 6.53 x 10^{-34}$Js)

(c) The work function of the sensitive plate ($\approx 2.9 x 10^{-19}$J)

8. A metal of work function 3.50eV is irradiated with light of unknown frequency. The maximum velocity of the photoelectrons is $4.71 x 10^{6}$m/s. Calculate the maximum wavelength of the incident radiation (Ans: $1.7 x 10^{-8}$m)

9. Light of wavelength $6.0 x 10^{-7}$m is incident on zinc plate. The electrons are emitted with maximum K.E of the $2.2 x 10^{-20}$J. Calculate the maximum K.E of the electron emitted from the zinc plate with light of wavelength of $5.0 x 10^{-7}$m (Ans: $8.83 x 10^{-20}$J)

10. Sodium has a work function of 2.3eV. Calculate;

i) The threshold frequency of sodium

ii) The maximum velocity of the electrons produced when sodium is illuminated by light of wavelength $5 x 10^{-7}$m

iii) The stopping potential with light of this wavelength

(Ans: (i) $5.6 x 10^{14}$Hz (ii) $2.5 x 10^{5}$m/s (iii) 0.18V)

Failures of the wave theory (*classical theory*) to account for the photoelectric emission

1. **Existence of threshold frequency**

   According to the classical theory, the energy of the incident radiation depends on its intensity; the greater the intensity of illumination, the greater the supply of energy. This would imply that radiations of high enough intensity should cause emission even when the frequency is below the minimum value

   However as long as the incident radiation is below the threshold frequency, no photoelectrons are emitted however intense the incident radiation is

2. **Instantaneous emission of photoelectrons**

   Classical theory suggests that the energy of the incident radiation would be continuously absorbed by the electron. Implying that the electron would take some time to accumulate sufficient energy that would enable them escape from the metal surface. By this theory, emission of photoelectrons would not be instant

3. **Variation of K.E of the emitted photoelectrons**

   According to the classical theory, increasing the intensity of the incident radiation would mean more incident energy and a greater maximum K.E of the emitted photoelectrons

   But instead the maximum K.E of the photoelectrons emitted depend on the frequency of the incident radiation.

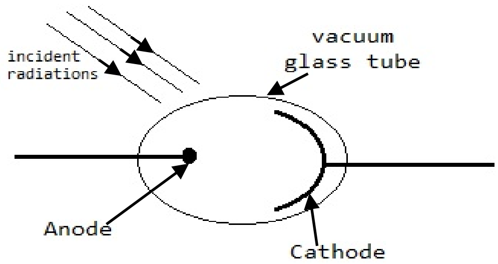
4. **Variation of photoelectric current with intensity**

   When the intensity of illumination is increased, the number of photons incident on the metal surface also increases. Hence more free electrons in the metal receive sufficient energy to escape. The rate of emission increases and therefore a large current flows.
   Thus the size of the photocurrent depends on the intensity of the incident radiation.

   However, According to classical theory, increase in the intensity would increase the K.E of the emitted electron and they would escape with greater speed instead, which is false.

The Photocell

- Photocells change radiation into electric current.
- In their construction, the anode is made thin so that it does not obstruct the incident radiation
- It's placed in vacuum because the metals are reactive

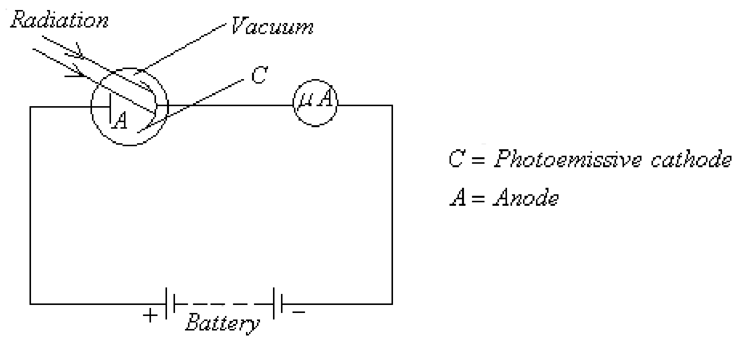


- When radiations fall on the cathode, electrons are emitted and these electrons are collected by the anode.
- If the anode is positive respect to the cathode, current flows in the circuit

Types of Photocells

(a) Photo emissive cells
(b) Photovoltaic cells
(c) Photoconductive cells.

Photo emissive Cell



- When radiation of frequency f greater than f_0 (threshold frequency) of the photo emissive cathode is incident on the cathode, electrons are emitted, they move to the anode and current flows in the external circuit.
- The size of the current increases with the intensity of the incident radiation.
- If the light beam is interrupted, the current stops flowing.
- When the device is connected to a suitable relay circuit, it can be used to open doors, act as a burglar alarm or as switching device.

Photovoltaic Cell

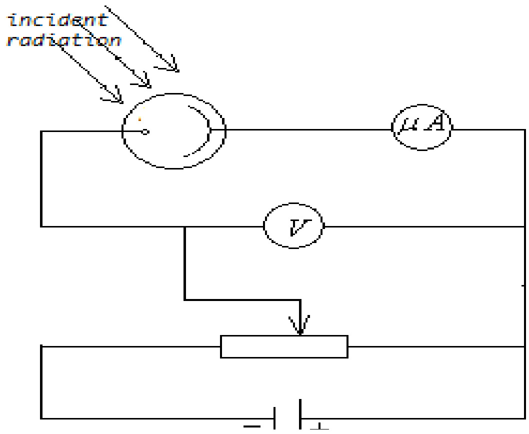
It generates an e.m.f dependent on the intensity of the incident radiation. Such cells are used in solar panels, solar calculators and for powering electronic watches.

Photoconductive Cell

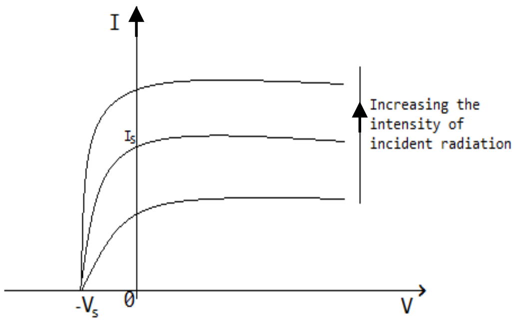
It consists of a plate of a material called photoconductor, whose resistance decreases when it is illuminated by light or infrared radiation, mounted in an evacuated glass bulb. An applied voltage causes current to flow which increases with the intensity of the radiation due to release of electrons in the photoconductor.

Variation of p.d *V* with current (I)

(*See diagram*)



- A monochromatic light i.e. constant frequency is used.
- The photocurrent (I) is measured for increasing values of V.
- For negative values of V, the polarity of the battery is reversed.
- A graph of photocurrent (I) against the p.d V is plotted. (*See graph*)



- The experiment is repeated by increasing the intensity of the radiations; by moving the light source closer to the photocell

  I_S = *saturation current at that intensity*

  V_S = *the stopping potential for the cathode*

N.B: The photocurrent is not zero even when the p.d is zero. This is because electrons are emitted with varying velocities (K.E), some of which are sufficient to overcome the repulsive electric field and reach the anode.

Example

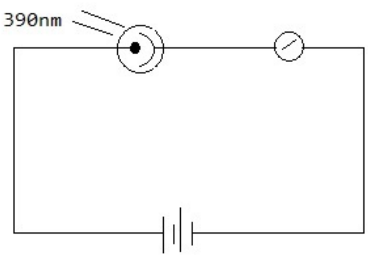


fig (i)

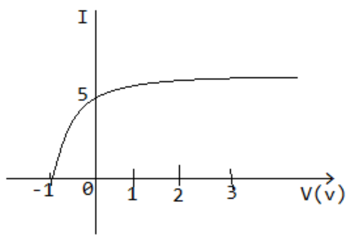


fig (ii)

A photocell is connected in the circuit as shown in fig (i). The cathode is illuminated with monochromatic light of wavelength 390nm and the current I in the circuit recorded for different p.d V applied between the anode and the cathode. The graph fig (ii) shows the results obtained.

(a) Find the maximum K.E of the photoelectrons
(b) What is the work function of the cathode in eV?
(c) If the experiment is repeated using monochromatic light of wavelength 310nm, where would the new graph cut the V-axis?

Solution:

(a) $\textit{from the graph } V_s = -1.0V$

$K.E_{\max} = eV_s \equiv 1.0\times1.6\times10^{-19}J$

(b) $\textit{from}$ Eintsteins $\textit{equation}$

$K.E_{\max} = \dfrac{hc}{\lambda} - W_0 \Rightarrow W_0 = 2.19eV$

$\textit{When } \lambda_2 = 310nm$

(b) $K.E_{\max} = \dfrac{hc}{\lambda} - W_0 \Rightarrow K.E_{\max} = 1.82V$

Hence the graph would cut the V-axis at -1.82V

Applications of Photocells (Photoelectric Emission)

(ii) A photocell can make doors open automatically in buildings when a light beam is interrupted by somebody/obstacle.
(iii) Intruder alarm systems. The intruder intercepts the infrared beam falling on a photocell, hence cutting off of current. This interruption therefore sets the alarm on.
(iv) Photovoltaic cells are used in solar panels, calculators and for powering electronic watches.
(v) Used as automatic devices for switching on light at night when it tries to darken or when the frequency of the light reduces.
(vi) Automatic counting machines in industries.
(vii) Production of sound from a film ****

DIFFERENCES BETWEEN X-RAY PRODUCTION AND PHOTOELECTRIC EFFECT:

PHOTOELECTRIC EFFECT	X-RAYS
• Electromagnetic radiation falls on metal surface and electrons are emitted.	• Fast moving electrons hit the metal target and x-rays (electromagnetic radiation) is produced.
• Little heat is generated	• A lot of heat is generated

QUESTIONS:

1. A $100mW$ beam of light of wave length $4.0\times10^{-7}m$ falls on a caesium surface of a photocell.
   (i) How many photons strike the caesium surface per second?
   (ii) If 80% of the photons emit photoelectrons. Find the resulting photocurrent.
   (iii) Calculate the kinetic energy of each photoelectron if the work function of caesium is $2.15eV$.

Solution:

$(i)\ Power = 100mW = 100mJs^{-1}$

$\lambda = 4.0\times10^{-7}m$

$Energy\ of\ each\ photon\ (one\ photon)\ E = hf$

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.0\times10^{-7}} = 4.97\times10^{-19}J$$

$$Number\ of\ photons\ n = \frac{Total\ energy}{Energy\ of\ one\ photon}$$

$$n = \frac{100\times10^{-3}}{4.97\times19^{-19}} = 2.02\times10^{17}s^{-1}$$

$(iii)\ K.e_{\max} = hf - W_0$

$$= \frac{hc}{\lambda} - W_0$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{4.0\times10^{-7}} - \left(2.15\times1.6\times10^{-19}\right)$$

$$= 4.9725\times10^{-19} - 3.44\times10^{-19}$$

$$= 1.53\times10^{-19}J$$

$$= 0.944eV$$

$(ii)\ 80\%\ of\ photons\ emit\ photoelectrons$

$Number\ of\ electrons\ emitted\ n = 2.02\times10^{17}\times0.8 = 1.616\times10^{17}s^{-1}$

$Photocurrent\ I = ne$

$I = 1.616\times10^{17}\times1.6\times10^{-19} = 2.57\times10^{-2}A.$

EXPERIMENTAL EVIDENCE FOR QUANTAM THEORY

(i) Photoelectric effect:

To liberate an electron from a metal surface, a quantum or packet of energy called the work function which is characteristic of the metal surface has to be supplied i.e. $hf - W_0 = \frac{1}{2}mv^2$ where W_0 is the work function.

(ii) Optical spectra:

A line in the optical emission spectrum indicates the presence of a particular frequency f of light and is considered to arise from loss of energy which occurs in an excited atom when an electron jumps directly or in steps from a higher energy level E_2 to lower energy level E_1.The frequency of the packet of energy emitted is given by $hf = E_2 - E_1$.

(iii) X-ray line spectra:

Electron transition from one shell to another leads to liberation of energy in packets characteristic of the target atom.

DIFFERENCES BETWEEN CLASSICAL THEORY AND QUANTUM THEORY:

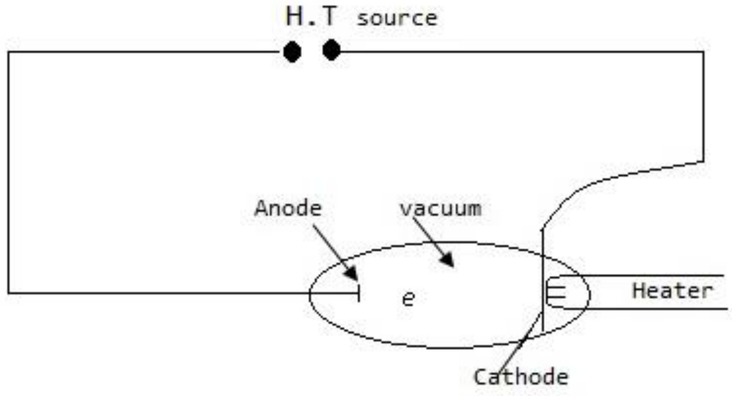
CLASSICAL(WAVE) THEORY	QUANTUM THEORY
• It allows continuous absorption and accumulation of energy.	• No continuous absorption allowed. The energy is either absorbed or rejected.
• Energy of radiation is evenly distributed over the wave front.	• Energy is radiated, propagated and absorbed in packets (quanta or photons).
• What matters is total energy of the incident radiation (beam).	• What matters is the energy of individual photon.

TASK

1. State the conditions under which photoelectric emission occurs
2. Explain how the photoelectric effect provides evidence for the quantum theory of light.
3. Explain why light whose frequency is less than the threshold frequency cannot cause photoelectric emission.

4. Explain why the classical theory (wave theory) of light fails to account for the photoelectric effect (emission).

Electron Dynamics



Consider an electron moving from cathode to Anode.

- Let the p.d between the cathode and anode be V. The electron will be accelerated by the electric field and hence it gains K.E.
- If the electron starts from the cathode with zero velocity and reaches the anode with velocity $u$ms^{-1}, then the K.E gained by the electron will be

$$\tfrac{1}{2}mu^2 = eV$$, where e = electric charge, 1.6 x 10^{-19}C

$$u = \sqrt{\left(\frac{2eV}{m}\right)} \qquad \textbf{OR} \qquad u = \sqrt{\left\{2\left(\frac{e}{m}\right)V\right\}}.$$

The quantity $\left(\frac{e}{m}\right)$ *is called the specific charge of the electron.*

Exercise:

1. Calculate the speed of a proton which has been accelerated through a p.d of 400V (mass of a proton = 1.67x10^{-27}kg).
   Ans: 2.77x10^{5}m/s

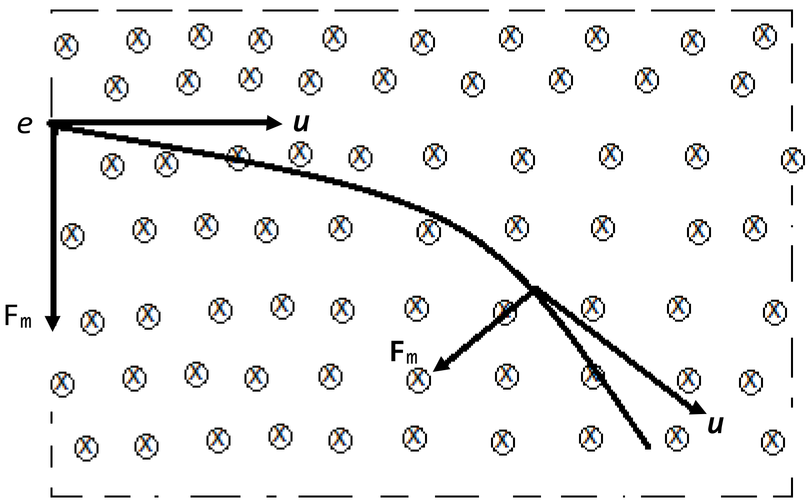
2. The K.E of an alpha particle from a radioactive source is 4.0MeV. What is its speed?
   (Take m_e = 9.11x10^{-31}kg, e = 1.6x10^{-19}C) Ans: 1.414x10^{7}m/s

3. An electron is accelerated from rest through a p.d of 1000V.What is: (a). its K.E in eV (b) its speed
   (Take m_e = 9.11x10^{-31}kg, 1eV = 1.6x10^{-19}J).
   Ans: 1.6x10^{-16}J, 1.87x10^{7}m/s

4. What value of the p.d between the cathode and anode that will accelerate an electron from the cathode to a speed of 2.91x10^{7}m/s. Ans: 2.4Kv

Deflection of an electron in a magnetic field

Consider an electron entering a uniform magnetic field of flux B, at right angles to its direction of motion with velocity *u*.



When the electron enters the field, the magnitude of its speed *u* does not change because the magnetic force is perpendicular to the direction of the electron, But instead its direction changes and the electron moves in a circular arc.

Let *r* be the radius of the circular arc (path)

The centripetal force on the electron $F = \frac{mu^2}{r}$ .... (1)

The force due to the magnetic field $F = Beu$ .... (2)

From (1) and (2) => $\frac{mu^2}{r} = Beu$ $\quad \therefore r = \frac{mu}{Be}$

Since the speed of the electron is constant, its K.E is also constant and expressed as Kinetic energy $= \frac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m}$.

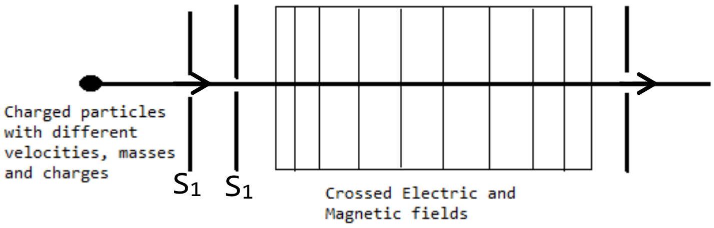
Example: An electron moves in a circular path at 3.0 x 10^6m/s in a uniform magnetic field of flux 2.0 x 10^{-4}T. Find the radius of the path

(mass of an electron m_e = 9.11 x 10^{-31}kg, *e = 1.6 x* 10^{-19}*c*).*Ans:8.5cm*

CROSSED FIELDS

Crossed fields are fields in which a uniform magnetic field and a uniform electric field are perpendicular to each other producing deflections opposite to each other.

If the magnetic force and electric force in the crossed fields are of the same magnitude, there is no deflection on charged particles that enter such fields.



The slits S_1 and S_2 confine the particles into a narrow beam as they enter the crossed fields.

The only particles that emerge at slit S_3 are those which are undeflected, and therefore they emerge with the same velocity U.

The electric force F_E due to the electric field = eE

The magnetic force F_m due to the magnetic field = BeU

For crossed fields $F_E = F_m$

$$eE = Beu$$

$$\therefore u = \frac{E}{B}$$ *This the velocity of the particle emerging at S3*

Therefore all particles that emerge at S_3 will have the same velocity $\therefore u = \frac{E}{B}$ regardless of their mass and charge.

The crossed fields can be used as a velocity selector of particles of a single velocity from a beam of particles of different velocities.

Example: An electron accelerated by a p.d of 1.5KV passes through an electric field crossed with a uniform magnetic field of flux density 0.45T. Calculate the value of the electric field needed for the electron to emerge undeflected.

Solution:

For an electron to pass through the crossed fields undeflected

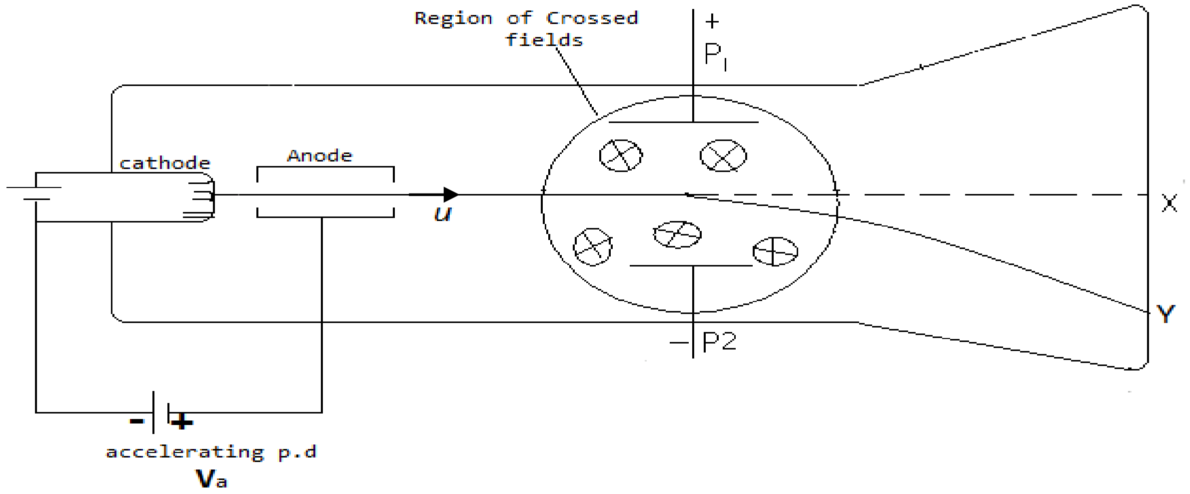
$$F_E = F_m \quad \Rightarrow E = Bu \;.................... (i)$$

$$But\ \tfrac{1}{2}mu^2 = eV \qquad \therefore u = \sqrt{\frac{2eV}{m}} = 2.295\times10^7\,ms^{-1}$$

$$from\ (1)\ \ E = 1.033\times10^7\,NC^{-1}$$

Determination of Specific Charge $\left(\frac{e}{m}\right)$ of an electron

(J.J Thomson's Method)



- The electrons are produced thermionically by a hot filament cathode and are accelerated towards a cylindrical anode and pass through it.
- The small hole on the anode confines the electrons to a narrow beam.
- When both the electric field and the magnetic field are off, the electrons reach the screen at X and cause fluorescence.
- If the velocity of the electrons on emerging from the anode is ***u*** then

$$eV_a = \frac{1}{2}mu^2 \quad \Rightarrow \quad \frac{e}{m} = \frac{u^2}{2Va} \quad \text{...(1)}$$

Where V_a is the accelerating voltage between the cathode and anode.

- The magnetic field is switched on and the beam is deflected to position Y.
- In order to bring the beam back to the original position X, the electric field is switched on and adjusted until the beam is at X again. This implies that

The magnetic force = the electric force

$$Beu = eE\text{. Hence} \quad u = \frac{E}{B}$$

Thus from (1) $\frac{e}{m} = \frac{E^2}{2B^2Va}$, *but* $E = \frac{V}{d}$

$\frac{e}{m} = \frac{V^2}{2B^2d^2Va}$, *where V is the P.d between the plates at a separation of d apart.*

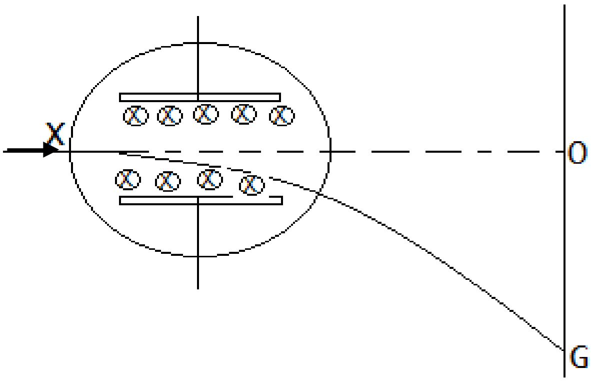
Exercise

1. A beam of protons is accelerated from rest through a potential difference of 200V and then enters a uniform magnetic field which is perpendicular to the deflection of the proton beam. If the flux density is 0.2T, calculate the radius of the path which the beam describes. (Proton mass = 1.7×10^{-27} Kg, e = 1.6×10^{-19} C).

2. A particle of charge 3.2×10^{-19}C is acceleration from rest through a p.d of 10.0KV. It enters a region of uniform magnetic field of flux density 0.5T. The particle describes a circular path of radius 8.94cm. Find;
   (i). The K.E of the particle on entering the magnetic field
   (ii). the mass of the particle.

   Ans: (i) 3.2×10^{-15}J (ii). 3.196×10^{-36}kg

3. In an experiment to determine the specific charge of an ion, the ion is projected horizontally along a region of uniform magnetic field of flux densitv 0.4T at **X** as shown below.

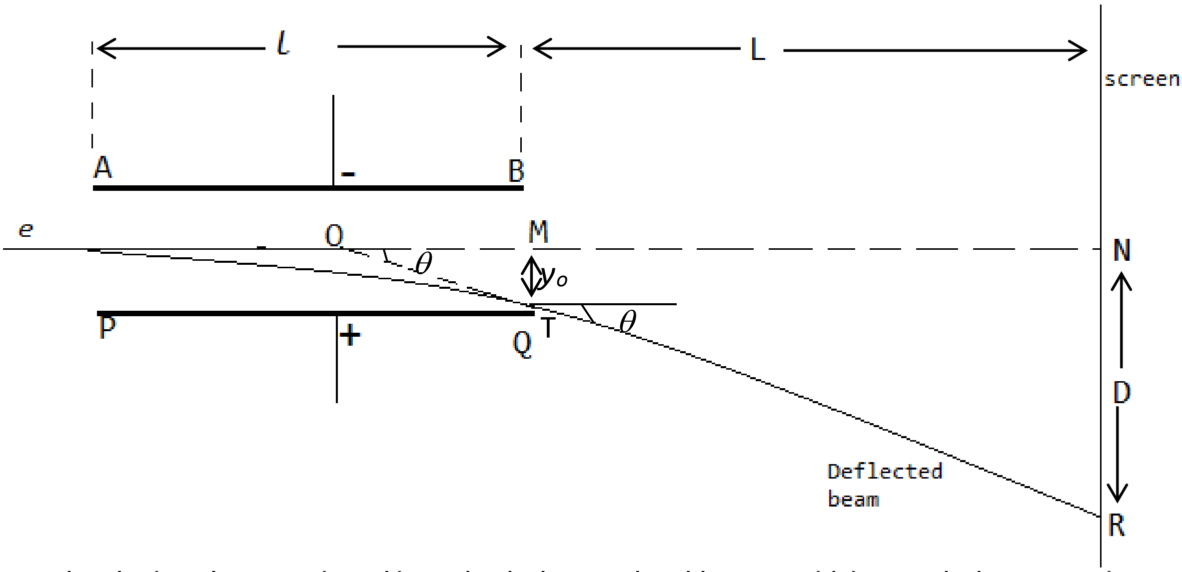


On leaving the magnetic field the ions strike the screen at G. When an electromagnetic field is applied perpendicularly to the magnetic field, the ion returns to O. If the path of the ion in the region of magnetic field is an arc of circle of radius 2.0cm and the electric field intensity is 1.408×10^{5}Vm^{-I}. Calculate the specific charge of the ion.

(Ans: 4.4×10^{7}Ckg^{-I})

Motion of an electron in an electric field

Consider two parallel plates AB and PQ such that AB is vertically above PQ and at a distance *d* apart. Let *l* be the length of the plates and *V* the p.d between the plates.



This electric force is directed towards the positive plate causing the deflection of the beam as shown above.

But for parallel plates $E = \frac{V}{d}$

Thus $F_E = \frac{eV}{d}$

Since the electric field intensity is vertical, there is no horizontal force acting on the electron. Hence the horizontal component of the velocity of the electron does not change

Let *u* be the horizontal component of the velocity of the electron entering the electric field.

Motion in the X-direction

$s = ut + \frac{1}{2}at^2$ but $s = x$ and $a = 0$

⇨ $\therefore t = \frac{x}{u}$ ....(1)

Motion in the y-direction

$s = ut + \frac{1}{2}at^2$ but $u_y = 0$, $s = y$ and $a_y = \frac{eE}{m}$ from $ma = eE$

Thus $y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{x}{u}\right)^2$

$y = \left(\frac{eE}{2mu^2}\right)x^2$, $\Rightarrow \therefore y \propto x^2$ or $y = kx^2$ which is a parabola

N.B: *Thus the motion of the electron while in the electric field is parabolic.*

NOTE:

1. If y_o is the vertical deflection of the electron just at the end of the plate, with E the electric field, then: $y = y_o$ and $x = l$

From $s = ut + \frac{1}{2}at^2 \Rightarrow y_o = \left(\frac{eE}{m}\right)\frac{l^2}{u^2}$ .... (2)

2. The time t_o taken for the electron to pass through the electric field (leave the plates)

When $t = t_o$, $x = l \Rightarrow t_o = \frac{l}{u}$ .... (3)

3. The velocity V_o with which electrons leave the plates

$$V_o = \sqrt{v_x^2 + v_y^2}$$

$\Rightarrow v_x = u$, and

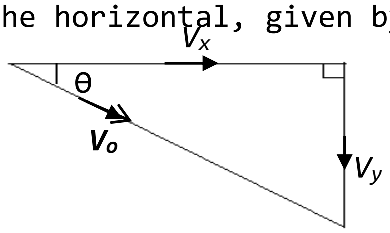
$\Rightarrow v_y = u_y + a_y t$, But $u_y = 0$ and $a_y = \frac{eE}{m}$

Thus $v_y = \left(\frac{eE}{m}\right)t_0$ also $t = t_o \Rightarrow t_o = \frac{l}{u}$

$v_y = \left(\frac{eE}{m}\right)\frac{l}{u}$, Where $E = \frac{V}{d}$

Substitute for v_y and v_x in $V_o = \sqrt{v_x^2 + v_y^2}$

4. **Direction (angle):** The electron emerges from the region between the plates at an angle θ to the horizontal, given by:



$$\tan\theta = \frac{v_y}{v_x}$$

$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mu^2}$ .... (4)

5. **The deflection D** of the electron on the screen placed at a distance **L** from the edge of the plates is obtained from:

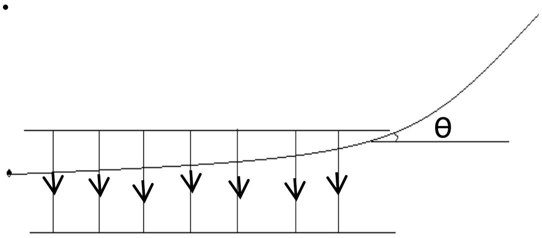
$\tan\theta = \left(\frac{\Delta}{L + \frac{1}{2}l}\right)$ but from (4)

$\left(\frac{eEl}{mu^2}\right) = \left(\frac{\Delta}{L + \frac{1}{2}l}\right)$, hence $\Delta = \frac{eEl}{mu^2}\left(L + \frac{1}{2}l\right)$ .... (5)

Exercise

1. A beam of electrons is accelerated through a p.d of 500V and enters a uniform electric field of strength 3.0×10^{3}V/m created by two parallel plates of length 2.0cm. Calculate:
   (a). the speed of the electrons as they enter the field.
   (b). the time that each electron spends in the field.
   (c). the angle from which the electrons have been deflected by the time they emerge from the field.

Solution:



(a) $From\ \frac{1}{2}mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 1.327\times10^{7}\,ms^{-1}$

(b) $From\ u = \frac{l}{t_0} \quad \therefore t_0 = \frac{2.0\times10^{-2}}{1.327\times10^{7}} = 1.51\times10^{-9}\,s$

(c) $u_x = u = 1.327\times10^{7}\,ms^{-1}$

$u_y = a_y t_0 \quad But\ a_y = \frac{eE}{m} \Leftrightarrow u_y = 7.957\times10^{5}\,ms^{-1}$

$Let\ \theta\ be\ the\ angle;\ \tan\theta = \frac{u_y}{u_x} \quad \therefore \theta = 3.43^{0}$

2. An electron gun operated at 3×10^{3}V is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm. Calculate the deflection of the electrons as they emerge from the region between the charged plates when the p.d is 1.0×10^{3}V.

Solution:

$From\ \frac{1}{2}mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 3.246\times10^{7}\,ms^{-1}$

$Also\ s = ut + at^2 \Leftrightarrow y_0 = a_y t_0^2\ \ but\ a_y = \frac{eE}{m}\ and\ t_0^2 = \frac{l}{u_x}$

$\Rightarrow y_0 = ½\left(\frac{eE}{m}\right)\left(\frac{l}{u_x}\right)^2 \quad and\ E = \frac{V}{d} = 2\times10^{4}\,Vm^{-1}$

$\therefore y_0 = 1.667\times10^{-2}\,m$

3. A beam of electrons is accelerated through a p.cd of 2000V and is directed mid-way between two horizontal parallel plates of length 5.0cm and separation of 2.0cm. The p.d across the plates is 80V.
   (a). Calculate the speed of the electrons as they enter the region of between the plates

(b). Find the speed of the electrons as they emerge from the region between the plates.

(c). Explain the motion of the electrons between the plates.

Solution:

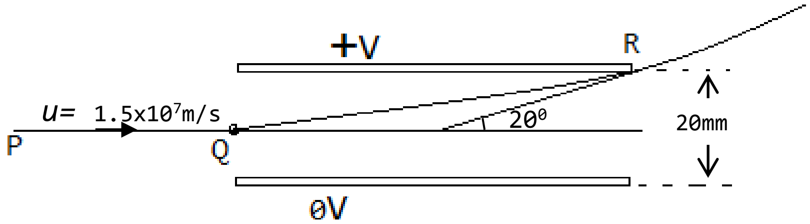
(a) $From\ \frac{1}{2}mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 2.652\times10^7\,ms^{-1}$

$$u_y = a_y t_0 \quad But\ a_y = \frac{eE}{m}, \quad E = \frac{V}{d}\ and\ t_0 = \frac{l}{u_x}$$

(b) $u_y = \left(\frac{eV}{md}\right)\left(\frac{l}{u_x}\right) = 1.325\times10^6\,ms^{-1}$

$$Now\ V = \sqrt{u_x^2 + u_y^2} \equiv 2.653\times10^7\,ms^{-1}$$

4. A beam of electrons moving at a uniform speed of 1.5 x 10^7m/s in a vacuum enters the space between two plane parallel deflecting plates along the line PQ as shown in the fig below.



PQ is along the middle of the space between the plates. The plates are 40mm long and separated by 20mm. The upper plate is at a positive potential **V** relative to the lower plate. The electron emerges from the plates at a point R with a speed of 1.60 x 10^7m/s and at an angle of 20^0 to its initial direction.

(a) How long does the electron take to move from the Q to R? Hence find the acceleration of the electron during the deflection.

(b) Find the value of the p.d V which produces the deflection

Solution:

$$From\ u = \frac{l}{t_0} \quad \therefore t_0 = \frac{40.0\times10^{-3}}{1.5\times10^7} = 2.67\times10^{-9}\,s$$

(a) $Hence:\ a_y = \frac{u_y}{t_0} \quad but\ u_y = \sqrt{V^2 - u_y^2} \equiv 5.6\times10^6\,ms^{-1}$

$$\therefore\ a_y = 2.1\times10^{15}\,ms^{-2}$$

(b) $a_y = \frac{eE}{m}, \quad E = \frac{V}{d}$

$$\Rightarrow V \equiv 239V$$

6. Two parallel plates 4cm long are held horizontally, 3cm apart in vacuum, one being vertically above the other. The upper plate is at a potential of 300V and the lower is earthed. Electrons having velocity 1.0x10^{7}m/s are ejected horizontally mid-way between the plates. Calculate the deflection of the electron beam as it emerges from the plates. ($\frac{e}{m} = 1.8\times10^{11}Ckg^{-1}$). Ans: 1.41cm

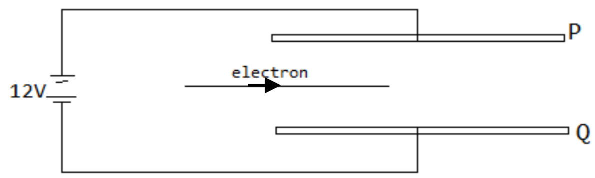
Solution:

$$From\ \ s = ut + at^2 \ \ \Leftrightarrow \ \ y_0 = a_y t_0^2 \ \ but\ a_y = \frac{eE}{m} and\ t_0^2 = \frac{l}{u_x}$$

$$\Rightarrow y_0 = \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{l}{u_x}\right)^2 \ \ and\ E \ = \ \frac{V}{d}$$

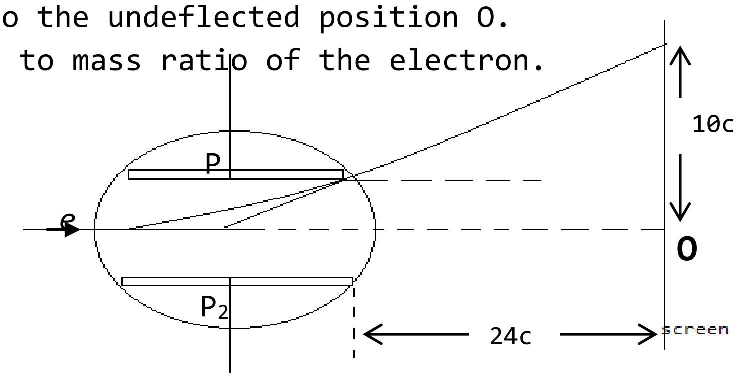
$$\therefore\ \ y_0 = 1.405\times10^{-2}m$$

6.



In the diagram above PQ are parallel plates of length 4.0cm and 4.0cm apart. A p.d of 12.0V is applied between the plates P and Q, the space between P and Q is in vacuum. A beam of electrons of speed 1.0x10^{6}m/s is directed mid – way between P and Q. Show that the electron beam emerges from the space between P and Q at an angle of 64.6^0 to the initial direction of the beam. (m_e = 9.11x10^{-31}kg, *e* = 1.6x10^{-19}C)

7. In the fig below, the metal plates P_1 and P_2 are metal plates each of length 2.0cm and are separated by a distance of 5.0mm, in a uniform magnetic field of 4.7x10^{-3}T. An electron beam incident mid – way between the plates is deflected by the magnetic field by a distance of 10.0cm on a screen placed a distance of 24cm from the end of the plates. When a p.d of 1.0Kv is applied between P_1 and P_2, the electron spot on the screen is restored to the undeflected position O.
Calculate the charge to mass ratio of the electron.



Solution:

$$\tan\theta = \frac{\Delta y}{L+\frac{1}{2}l} \equiv \frac{10}{25} \text{ .... } (i)$$

$$But\ also\ \tan\theta = \frac{u_y}{u_x} \text{ .... } (ii)$$

since the electron spot is restored on the screen

$$F_m = F_E \Rightarrow u = \frac{E}{B}\ but E = \frac{V}{d} = \frac{1\times10^3}{5\times10^{-3}} = 2\times10^5 Vm^{-1}$$

$$u_x = u = \frac{E}{B} = 4.255\times10^7 ms^{-1}$$

$$u_y = a_y t_0 \quad But\ a_y = \frac{eE}{m}$$

$$\Leftrightarrow u_y = \left(\frac{eE}{m}\right)\frac{l}{u_x} \equiv \left(\frac{e}{m}\right)\frac{El}{u_x}$$

$$from\ (ii)\ \tan\theta = \left(\frac{e}{m}\right)\frac{El}{u_x^2}$$

Combine (i) and (ii)

$$\left(\frac{e}{m}\right) = \left(\frac{10}{25}\right)\frac{u_x^2}{El} \equiv 1.81\times10^{11} Ckg^{-1}$$

8. An electron of energy 10KeV enters mid – way between two horizontal metal plates each of length 5cm and separated by a distance of 2cm. A p.d of 20V is applied across the plates. A fluorescent screen is placed 20cm beyond the plates. Calculate the vertical deflection of the electron on the screen. (Take m_e = 9.11x10^{-31}kg, *e* = 1.6x10^{-19}C) Ans: 5.63 x 10^{-4}m

9. A beam of cathode rays is directed mid – way between two parallel metal plates of length 4cm and separation 1cm. The beam is deflected through 10cm on a fluorescent screen placed 20cm beyond the nearest edge of the plates when a p.d of 200V is applied across the plates. If this deflection is annulled (*canceled*) by the magnetic field of flux density 1.14x10^{-3}T applied normal to the electric field between the plates. Find the ration of mass to the charge of the cathode rays. Ans: 1.75 x 10^{11}Ckg^{-I}

Solution:

$$since\ F_m = F_E \quad \therefore u = u_x = \frac{E}{B} \ and\ E = \frac{V}{d}$$

$$From\ \frac{\Delta y}{L+\frac{1}{2}l} = \frac{u_y}{u_x}\ and\ u_y = \left(\frac{eE}{m}\right)\left(\frac{l}{u_x}\right)$$

$$\Rightarrow \left(\frac{e}{m}\right) = 1.75\times10^{11} Ckg^{-1}$$

10. In a C.R.O an electron beam passes between the *y*-deflector plates each 5cm long and 0.5cm apart. The distance between the center of the *y*-plates and the screen is 20cm and the p.d between the anode and the electron gun is 250V. Determine the deflection in volts per meter of the electron beam on the screen of the C.R.O.

(Take m_e = 9.11x10^{-31}kg, *e* = 1.6x10^{-19}C) Ans: 1.0

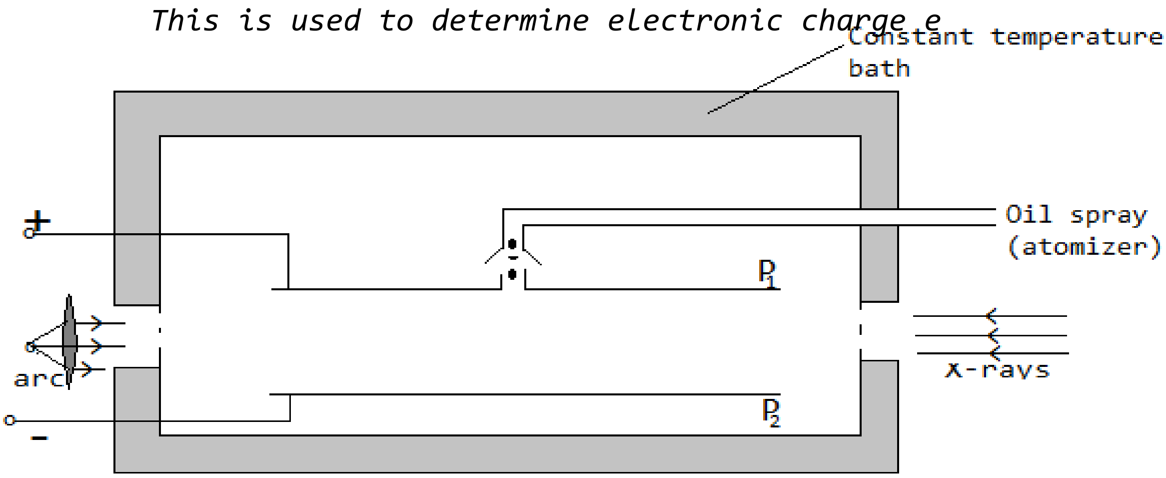
$$From\ \tfrac{1}{2}mu^2 = eV_1 \quad \therefore u = \sqrt{\frac{2eV_1}{m}} = 9.371\times10^6 ms^{-1}$$

$$u_y = a_y t_0 \quad But\ a_y = \frac{eE}{m}, \quad E = \frac{V_2}{d}\ and\ t_0 = \frac{l}{u_x}$$

$$But\ p.d\ V_1 = V_2 \quad \Rightarrow \Delta y \equiv 1.0m$$

nMillikan's Oil drop experiment

This is used to determine electronic charge e



Procedure

- Set up of the apparatus is as shown above
- Oil drops are introduced between the plates P_1 and P_2 by spraying using the atomizer.
- These oil drops are charged in the process of spraying by friction.
- The oil drops are strongly illuminated by an intense light from the arc lamp so that they appear as bright spots when observed through a low power microscope.
- With no electric field between the plates, record the time t_1 taken for drop to fall from P_1 to P_2.
- The electric field between the plates is turned on and adjusted so that the drop becomes stationary.

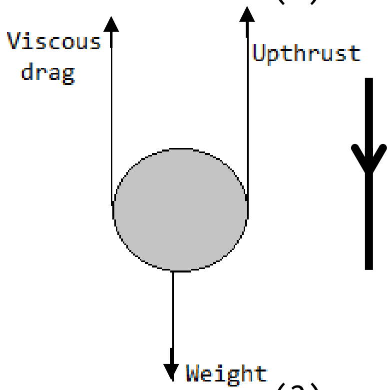
Theory:

<u>Case I</u>

With no electric field:

The oil drop falls with a uniform velocity (*called terminal velocity*) v_1 and has <u>no acceleration</u>.

Weight = Upthrust + Viscous drag ---------------------------------- (1)



$weight = \text{Volume of the drop} \times \text{density of oil} \times g$

$weight = \frac{4}{3}\pi r^3 \rho g$

$Upthrust = \text{weight of the air displaced by the drop}$

$weight = \text{Volume of the drop} \times \text{density of air} \times g$

$Upthrust = \frac{4}{3}\pi r^3 \sigma g$

$Viscous\ drag = 6\pi r \eta v_1$ (From Stokes' law).

From (1)

$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_1$ ---------------------------- (2)

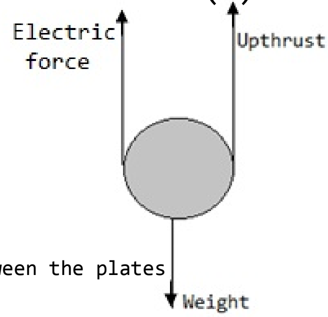
$r = \left[\dfrac{9\eta v_1}{2g(\rho-\sigma)}\right]^{\frac{1}{2}}$ *This is used to find the radius r of the oil drop*

<u>Case II</u>

When the electric field is applied so that the drop is stationary, the drop has <u>no velocity</u> and <u>no acceleration</u>.

Weight = Upthrust + Electric force

$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + qE$ ------------------------------ (3)



Combine equations (2) and (3)

$q = \dfrac{6\pi r \eta v_1}{E}$ Substitute for *r* from above.

$q = \dfrac{6\pi\eta}{E}\left(\dfrac{9\eta v_1}{2g(\rho-\sigma)}\right)^{\frac{1}{2}} v_1$, and $E = \dfrac{V}{d}$ where V is the p.d between the plates

Note: the density of air at room temperature is very small compared to that of oil and thus maybe assumed negligible (*when it's not given*) in calculations. This implies that the upthrust due to the displaced air is Zero. Thus $q = \dfrac{6\pi\eta}{E}\left(\dfrac{9\eta v_1}{2g\rho}\right)^{\frac{1}{2}} v_1$

Precautions

To improve on the accuracy of the experiment, the following precautions need to be taken into account

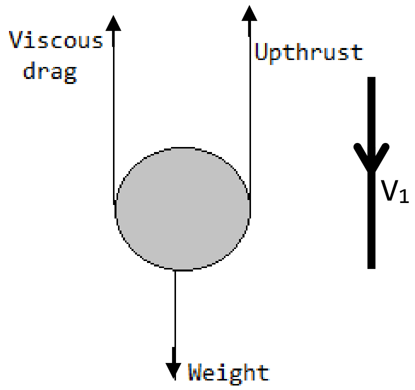
1) A non-volatile oil or low vapour pressure oil should be used to reduce evaporation. Evaporation would alter the mass of the drop
2) The experiment is enclosed in a constant temperature enclosure. This is to eliminate convection currents and changes in the viscosity of air as a result of temperature changes.
3) An X-ray tube is used to increase the charge of the oil drop.

The terminal velocity $v_1 = \dfrac{d}{t_1}$, where d is the separation between the plates $E = \dfrac{V}{d}$

Example 1:

Oil droplets are introduced into the space between two flat horizontal plates, set 5.0mm apart. The plate voltage is then adjusted to exactly 780V so that one of the droplets is held stationary. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given that the density of the oil used is 900kgm^{-3} and the viscosity of air is 1.8 X 10^{-5}Nsm-2, find;

i) The radius of the oil drop
ii) The charge of the droplets
iii) The number of electrons on the drop



Solution:

i) Weight = Upthrust + Viscous drag

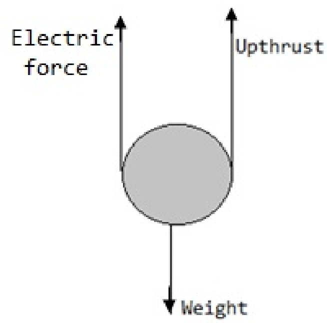
$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_1$$

But $\sigma = 0$ can be neglected

$$r = \left[\frac{9\eta v_1}{2g\rho}\right]^{\frac{1}{2}} \text{-------------------------------- (X)}$$

But the velocity $v_1 = \dfrac{distance\ (d)\ traveled}{time\ (t)\ taken} = \dfrac{1.5\times10^{-3}}{11.2} = 1.339\times10^{-4}ms^{-1}$

From (X), radius r = 1.1 x 10^{-6}m



ii) Weight = Upthrust + Electric force

$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi r^3 \sigma g + qE$$

But $\sigma = 0$ can be neglected

$$q = \frac{4\pi r^3 \rho g}{3E} \quad \text{But } E = \frac{V}{d} = \frac{780}{5\times10^{-3}} = 1.56\times10^5 Vm^{-1}$$

$\therefore\ q = 3.204\times10^{-19}C$ *This is the total charge on the drop*

iii) Let n be the number of electrons on the drop

$\Rightarrow\ q = ne$

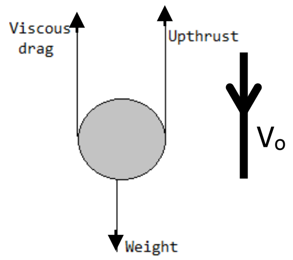
$n = \dfrac{q}{e} = \dfrac{3.204\times10^{-19}}{1.6\times10^{-19}} = 2$, thus the drop has acquired 2 electrons

Example 2:

a) Calculate the radius of a drop of oil of density 900kgm^{-3} which falls with a terminal velocity of 2.9 x 10^{-4}m/s through air of viscosity 1.8 X 10^{-5}Nsm-2.

b) If the charge on the drop is -3e, what p.d must be applied between the plates 5.0mm apart in order to keep the drop stationary?

Solution:

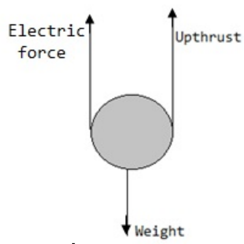


a) Weight = Upthrust + Viscous drag

$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_o$$

But $\sigma = 0$ can be neglected

$$r = \left[\frac{9\eta v_o}{2g\rho}\right]^{\frac{1}{2}} \quad \Rightarrow \quad r = 16.3mm$$



b) Weight = Upthrust + Electric force

$$\tfrac{4}{3}\pi r^3 \rho g = \tfrac{4}{3}\pi r^3 \sigma g + qE$$

But $\sigma = 0$ can be neglected

$$qE = 4\pi r^3 \rho g \qquad \text{But} \qquad q = 3e \qquad \text{and} \qquad E = \frac{V}{d}$$

$$\Rightarrow \quad V = 1665.8V$$

Example 3:

An oil drop carrying a charge of 3e falls under gravity in air with a velocity of 4.6x10^{-4}m/s between two parallel plates 5mm. when a p.d of 4.6Kv is applied between the plates, the drop rises steadily. Assuming the effect of air (buoyancy) on the drop is negligible. Calculate the velocity with which the oil drop rises.

Example 4:

In Millikan's oil drop experiment, an oil drop of density 890kgm^{-3} and radius 2.35 x 10^{-4}m has an excess charge of 10 electrons.

(i) Find the free fall terminal velocity with the electric field turned off

(ii) What values of the electric field intensity is needed to produce zero net force on the droplet.

(Assume: density and viscosity of are 1.0kgm^{-3} and 1.83 x 10^{-5}Nsm^{-2} respectively)

Solution:

Example 5:

In a Millikan type apparatus, the horizontal plates are 1.5cm apart. With the electric field is switched off, an oil drop is observed to fall with a steady velocity of 2.5x10-2cm/s. When the field is switched on, the upper plate being positive, the drop just remains stationary when the p.d between the plates is 1500V.

(a) Calculate the radius of the drop

(b) How many electronic charges does it carry?

(c) If the p.d between the plates remains unchanged, with what velocity will the drop move when it has collected two more electrons.

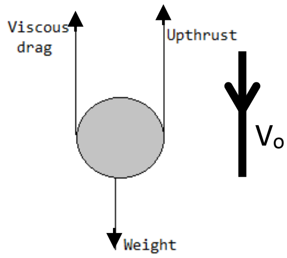
(Take density of oil as $900kgm^{-3}$ and viscosity of air as $1.83x10^{-5}Nsm^{-2}$)

Solution:

a) When the drop is falling steadily

Weight = Upthrust + Viscous drag

$$\tfrac{4}{3}\pi r^3 \rho g \quad = \quad \tfrac{4}{3}\pi r^3 \sigma g \quad + \quad 6\pi r \eta v_o$$



But the density of air $\sigma = 0$ can be neglected

$$r \quad = \quad \left[\frac{9\eta v_o}{2g\rho}\right]^{\frac{1}{2}} \qquad \Rightarrow \qquad r \quad = \quad 1.52\times10^{-6}m$$

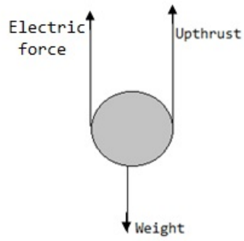
b) Let the number of electronic charges (electrons) on the oil drop be n.

$\therefore$ Total Charge on the drop $Q \quad = \quad ne$ ----------------- (1)

When the drop is held stationary, then

Weight = Upthrust + Electric force

$$\tfrac{4}{3}\pi r^3 \rho g \quad = \quad \tfrac{4}{3}\pi r^3 \sigma g \quad + \quad QE$$



But $\sigma = 0$ can be neglected

$$Q = \frac{4\pi r^3 \rho g}{3E} \quad \text{But} \quad E = \frac{V}{d} \quad \Leftrightarrow \quad Q = \frac{4\pi r^3 \rho g d}{3V}$$

$\therefore$ Q = *This is the total charge on the drop*

From (1), $n \quad = \quad \dfrac{Q}{e} \qquad n \quad = \quad 8$

c) When the drop has collected two more electrons, then

Total Charge Q = 10e.

ADD MORE EXAMPLES AND PROBLEMS

:::
::

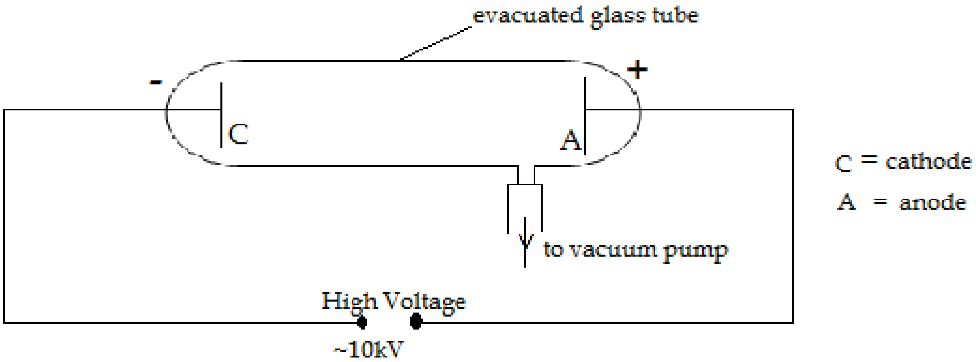
Thermionic emission and thermionic diode

Gaseous discharge in a discharge tube

A discharge tube is a hard glass tube connected to a vacuum pump the tube contains two electrodes which are connected to an external high voltage source.

The discharge tube contains air (*which is a poor conductor of electricity*) at very low pressure since it is evacuated.

Structure



Action of a discharge tube

- As the air pressure inside the tube decreases, the gas starts to ionize while the p.d inside the tube is constant.
- When one gas atom is ionized, the electron escaping from it also ionizes other gas atoms.
- Streams of positive ions and electrons are created which move to the cathode and anode respectively.
- Current is thus generated

Explanation:

- When pressure inside the tube is not very low, gas movement looks bluish <<>>>>>
- As the pressure decreases, the gas looks pinkish
- When the evacuated tube is subjected to high p.d, the inside of the tube will start looking black because there is no gas inside to conduct any current. This dark space is called *Faradays dark space*. A small glow can be observed at the anode region called *Crooke's dark space*.

  <<<<<<<<Diagrams>>>>>>>>>>>>>>> 3 of them
- As the pressure is further reduced, a greenish glow appears behind the anode. *Reason: some rays/particle from the cathode to the anode overshoot the anode and reach the inner surface of the tube which cause the glow.*
- *These rays are called cathode rays.*

Cathode Rays

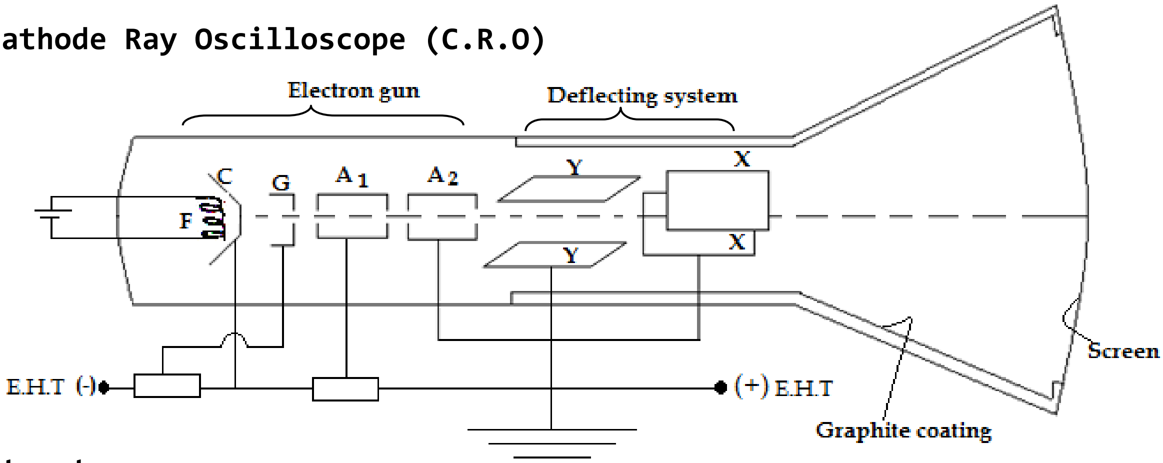
These are streams of fast moving electrons that travel from the cathode to the anode.

Properties of cathode rays

(i) They travel in straight lines.
(ii) They are negatively charged.
(iii) They are deflected by both electric and magnetic fields.
(iv) They cause fluorescence in some metals e.g. Zinc metal.
(v) They ionize gas atoms.
(vi) They affect photographic plates.
(vii) They produce X – rays when stopped by a metal target.
(viii) They possess momentum therefore they have mass.

N.B: *Read some experiments for properties, (i), (ii), (iii) and (viii).*

Cathode Ray Oscilloscope (C.R.O)



Structure

The C.R.O consists of three principal components or features, namely

1) *The electron gun*
2) *The deflecting system*
3) *The fluorescent screen*

Functions of the main parts of a C.R.O

The electron gun: this is made of the following;

- ***The Filament F:*** When a current flows through the filament, it glows and heat from the filament heats the cathode.

- ***The Cathode C:*** Emits electrons when heated by the filament.

- ***The Grid G:*** the grid is made at a negative potential with respect to the cathode. It controls the number of electrons entering the Anode, thus the Grid controls the brightness on the screen.

- ***Accelerating and focusing Anode A_1 and A_2:*** the anode is at a positive potential. It accelerates the electron beam to a high speed along the tube.

Deflecting system: this consists of the X – plates and Y – plates;
- ***X – Plates*** are placed vertically and form a horizontal electric field. Therefore it deflects the electron beam horizontally.
- ***Y – Plates*** are placed horizontally and form a vertical electric field. Therefore it deflects the electron beam vertically.

Fluorescent screen: This is coated with a fluorescent material such as Zinc sulphide or Phosphor. The fluorescent screen emits light (glows) when struck by fast moving electrons.

Graphite coating: This is used:
(i) to prevent the electron beam from the influence of electric field
(ii) It collects secondary electrons emitted by the screen to the earth.

Mode of action of a C.R.O
- The cathode is heated using a low voltage supply and produces electrons by the process of thermionic emission.
- The electrons are accelerated and focused into a fine beam by the anode to fall on the screen producing a bright spot.
- The brightness of the spot on the screen is controlled by the control grid

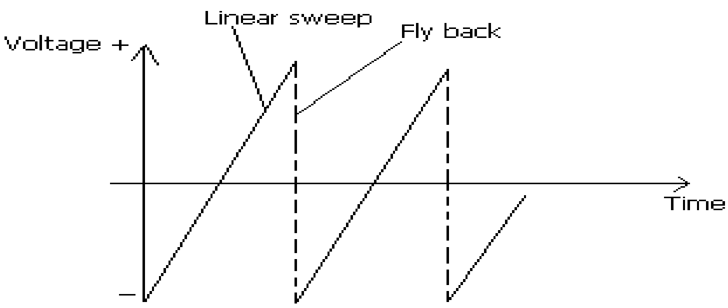
How the grid controls the brightness of the spot of a C.R.O

- The grid is connected to a negative potential.
- If the potential of the grid is more negative, only few electrons with high K.E (*or speed)* will pass through it. Thus the screen will be less bright.
- If the potential of the grid is less negative, many electrons will pass through it. Thus the screen will be brighter.

Time base of the C.R.O

The time base is an electrical circuit connected to the X-plates which generates a saw – tooth type of voltage.

- The time base causes the spot (electron beam) to move from left to right of the screen. This is known as *linear sweep*.

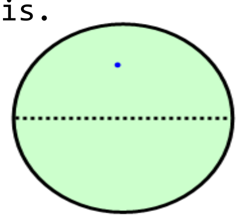


- The spot then returns to its initial position, the left of the screen before the next sweep. This is the *fly-back*. The fly-back time is very small (negligible)

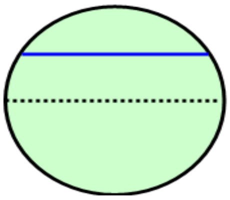
The beam is moved from the left-hand side of the screen to the right during the time that the voltage rises to a maximum, and then is returned rapidly to the left as the voltage returns to zero. This fly-back time should be as short as possible.

Wave forms on the screen of a C.R.O

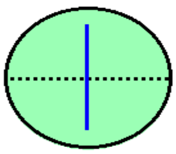
The speed of the time base will change what we see on the screen even if the input signal is kept the same. The following four diagrams show this.



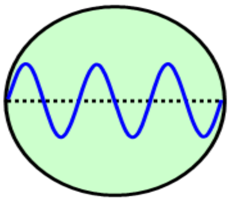
d.c. input with the time base off



d.c. input with the time base on



a.c. input with the time base off



a.c. input with the time base on

Because the deflection of the spot depends on the voltage connected to the Y-plates the C.R.O can be used as an accurate voltmeter. The oscilloscope is also used in hospitals to look at heartbeat or brain waves, as computer monitors, radar screens and is also the basis of the television receiver.

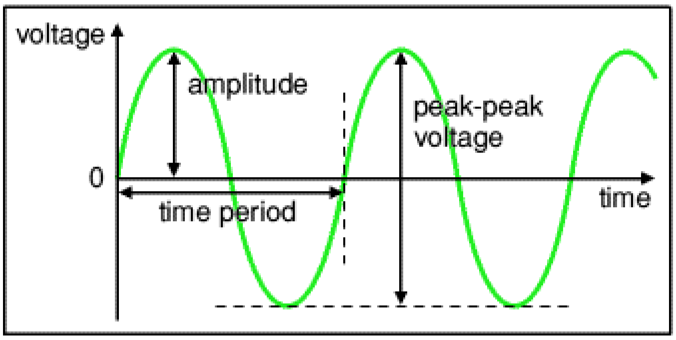
Comparison of CRO with a moving coil Voltmeter.

a) The C.R.O has very high impedance. It gives accurate voltages than a moving coil voltmeter.
b) A CRO can measure both d.c and a.c voltage. A moving coil voltmeter measures only D.C voltages unless a rectifier is used.
c) A CRO has negligible inertia as compared to a moving coil voltmeter. The C.R.O respond almost instantaneously.
d) CRO doesn’t give direct voltage readings.

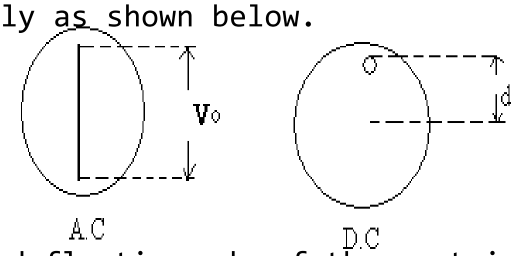
The CRO as a voltmeter.

The voltage to be measures is applied to the Y – plates and if the C.R.O.

If the time base is on, a sinusoidal alternating voltage produces a display of the form.



Voltage measurement is done when the time base is off, which enables centralizing the beam by the X – shift control. When this is done, then A.C and D.C signals appear as a vertical line and as a deflected sport respectively as shown below.



- For d.c Voltage, the deflection, d, of the spot is measurement.

  $V_{D.C}$ = voltage gain x deflection *d*

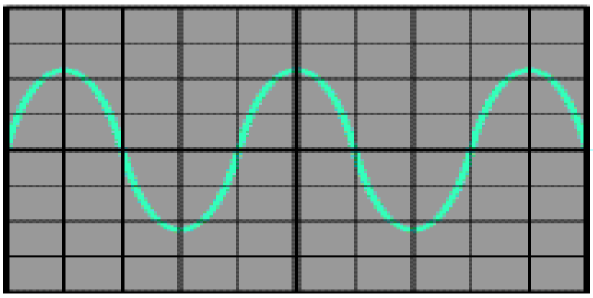
- For A.C the length, *l*, of the vertical trace is measured.

  $2V_o$ = voltage gain x length, where V_o is the peak value.

  ($2V_o$ *is called the peak to peak (peak – peak) voltage*)

$$V_{R.M.S} = \frac{Vo}{\sqrt{2}} = 0.707V_o .$$

Example



The trace of an AC signal

The signal shown was obtained from a C.R.O with the following settings
Y AMPLIFIER: 2V/cm
TIMEBASE: 5ms/cm

Example measurements:

peak-peak voltage = 8.4V
amplitude voltage = 4.2V

time period = 20ms
frequency = 50Hz

Voltage is shown on the **vertical y-axis** and the scale is determined by the

Y- AMPLIFIER (VOLTS/CM) control.

Usually **peak-peak voltage** is measured because it can be read correctly even if the position of 0V is not known.

The **amplitude** is half the peak-peak voltage.

If you wish to read the amplitude voltage directly you must check the position of 0V (normally halfway up the screen):

Voltage = distance in cm × volts/cm
Using the above figure: peak-peak voltage = 4.2cm × 2V/cm = 8.4V
amplitude (peak voltage) = ½ × peak-peak voltage = 4.2V

Period: Time is shown on the **horizontal x-axis** and the scale is determined by the ***time-base*** (TIME/CM) control. The **period** is the time for one cycle of the signal.

The frequency is the number of complete cycles per second,

$$frequency = \frac{1}{period}$$

Time = distance in cm × time/cm
Using the above figure: time period = 4.0cm × 5ms/cm = 20ms

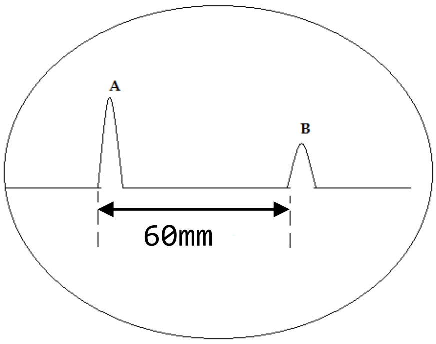
$$frequency = \frac{1}{period} = \frac{1}{20\times10^{-3}} = 50Hz$$

Uses of a C.R.O

1) Measures both a.c and d.c voltages
2) Measures frequencies or compare frequencies
3) Measures phase difference
4) Measure small time intervals (used as a clock)
5) Displays wave forms

C.R.O as a clock

Example: the figure shows the trace on a C.R.O of time base $10\mu s mm^{-1}$. A radar signal sent at **A** to a plane arrives back at **B,** 60mm from **A.** what is the distance between the radar and the plane.



Solution:

$$\text{time from A to B on the screen} = 10\times10^{-6}\times60$$
$$= 6.0\times10^{-4}s$$

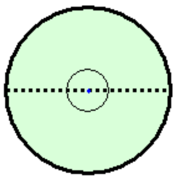
Assuming the radar signal is an electromagnetic radiation, then

$$\text{to and fro distance of the signal} = 3.0\times10^{8}\times6.0\times10^{-4}$$
$$= 1.8\times10^{5}m$$
$$\text{distance between the plane and radar} = \tfrac{1}{2}\times1.8\times10^{5}m$$
$$= 9.0\times10^{5}m \equiv 90km$$

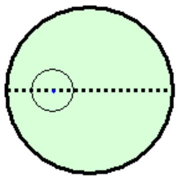
C.R.O Appendix

Display on C.R.O screen with time base off

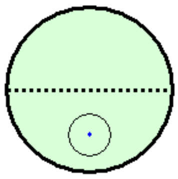
(*The small circles to help see the spot*)



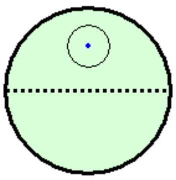
no input



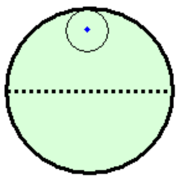
no input – spot adjusted left



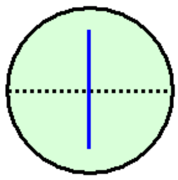
d.c input lower plate positive



d.c input upper plate positive

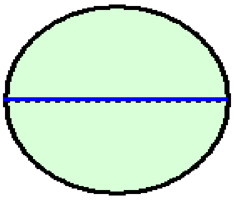


d.c input upper plate more positive

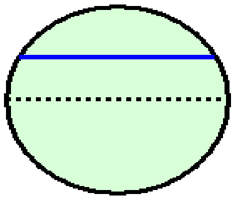


a.c input

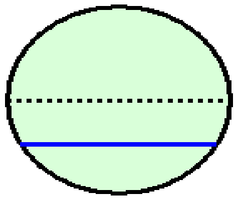
Display on C.R.O screen with time base off



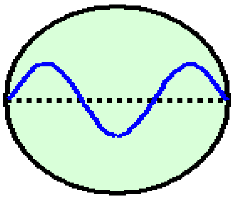
no input



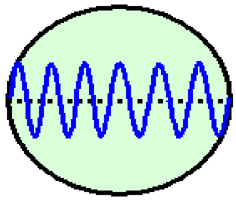
d.c input – upper plate positive



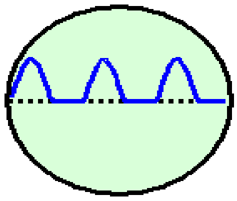
d.c input – lower plate positive



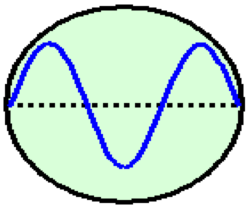
low frequency a.c input



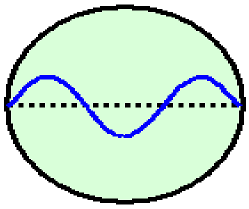
high frequency a.c. input



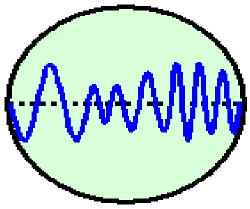
a.c input with a diode



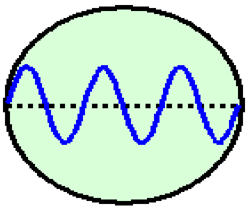
large amplitude a.c input



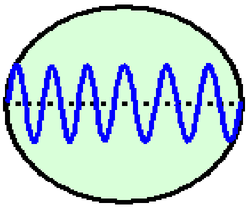
small amplitude a.c input



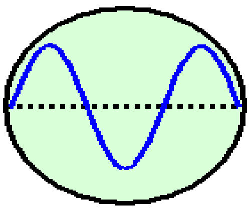
voice or music



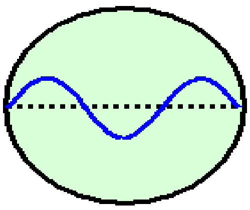
a.c. input with a slow time base



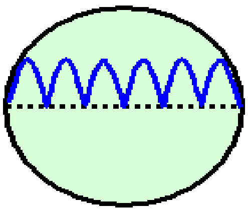
a.c. input with a fast time base



a.c input large Y gain



a.c input small Y gain



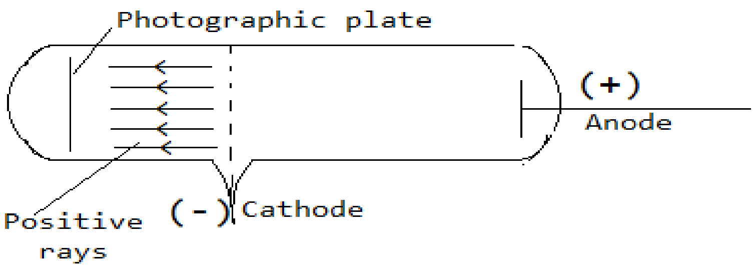
full wave rectification

Positive rays

- Positive rays are streams of positively charged particles.
- Positive rays are produced when gases at low pressure are ionized in the presence of a strong electric field, during which one or more electrons are removed from the atom or molecule, leaving the rest of the atom positively charged hence positive rays.

Production of positive rays in a discharge tube

- Positive rays are produced when p.d is applied to a perforated cathode of a discharge tube.
- The applied p.d causes a discharge in the residual gas and luminous appear in the space behind the cathode.
- These luminous streams are positive rays.



Properties of positive rays

1) They travel in straight lines
2) They are positively charged
3) They are more massive compared to cathode rays
4) They are reflected by strong magnetic fields
5) They have smaller specific charge () compared to the cathode rays (*this is because they are more massive than the cathode rays*)
6) They are deflected by strong electric fields in opposite direction to that of the cathode rays in the same field

The Bainbridge Mass Spectrometer

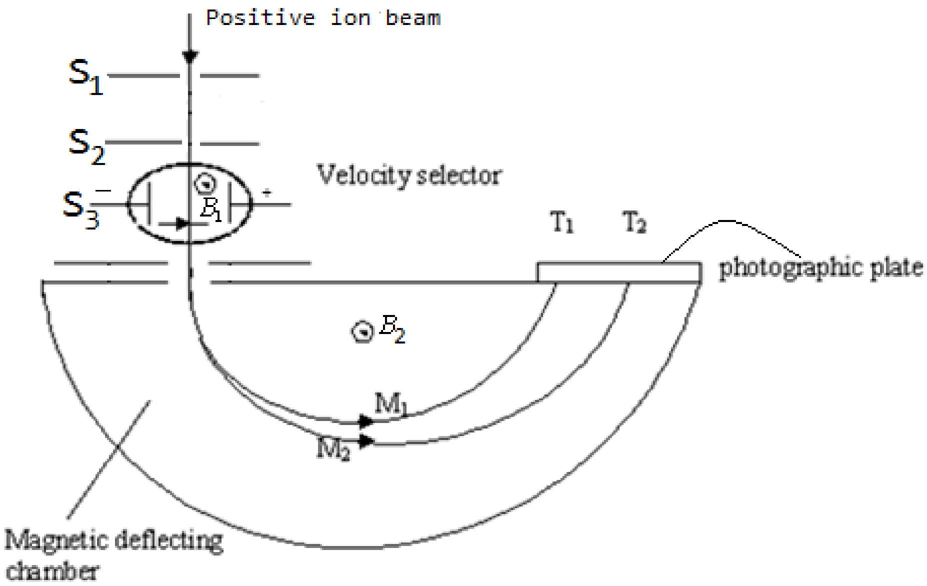
Consists of three main parts, namely

- Accelerating electric field
- Velocity selector
- Deflecting chamber

Uses of a Bainbridge mass spectrometer

1. To measure the specific charge of positive rays or ions
2. To separate isotopes of an element
3. To determine the atomic masses of the positive rays or ions

Determination of specific charge using a Bainbridge spectrograph



T_1 and T_2 are tracers on photographic plate, S_1, S_2 and S_3 are slits

Mode of Action

The positive ions are produced from the discharge tube with different velocity but having the same charge. These ions are then accelerated at a high speed using the accelerating electric field Ea and enter the velocity selector with different speeds.

The velocity selector consists of two fields; the electric field E and the magnetic field B_1 at right angles to each other. The ions experience two forces here.

The electric force $F_E = Eq$ and the magnetic force $F_1 = B_1qv$, where v is the velocity of the ions. The two fields are opposite each other. Only those ions which go un deflected from the velocity selector when $F_E = F_1$ merge through slit S_2 with a common velocity

$v = \frac{E}{B_1}$.

All ions enter the deflecting chamber with the same speed and continue moving with that speed. The magnetic force F_2 in this region provides the necessary centripetal force

$F_2 = B_2qv = \frac{mv^2}{r}$, where m is the mass of the ions and r is the radius of the path

$$B_2qv = \frac{mv^2}{r},\ but\ v = \frac{E}{B_1}$$

$$\frac{mv}{r} = B_2q$$

Thus $r = \frac{mv}{B_2q} = \frac{mE}{B_2B_1q}$

$$r = m\left(\frac{E}{B_2B_1q}\right)\ and\ \frac{E}{B_2B_1q}\ is\ a\ constant$$

$$Thus\ r \propto m$$

For the charge – mass ratio or specific charge

$$from\quad B_2qv = \frac{mv^2}{r}$$

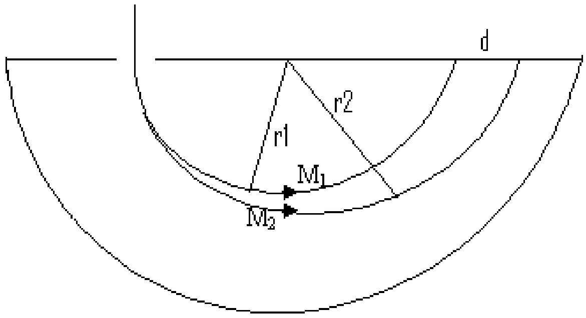
$$\frac{q}{m} = \frac{mv^2}{B_2vr} = \frac{mv}{B_2r},\ but\ v = \frac{E}{B_1}$$

$$Thus\quad \frac{q}{m} = \frac{E}{B_2B_1r}$$

But $r = \frac{x}{2}$ where x is the distance from S_2 to the point on the photographic plate where the ions of mass, m, lands. Hence

$$\frac{q}{m} = \frac{2Es}{xB_d Bs}$$

Separation of isotopes



If r2 > r1, then m2 > m1

For isotopes above, their separation is found out by finding the diameters of their paths

From $r = \frac{mE}{B_2 B_1 q}, d = 2r$

For ions of mass m1,

$$r_1 = \frac{m_1 Es}{B_2 B_1 q}, \quad d_1 = 2r_1 = \frac{2m_1 E}{B_2 B_1}$$

For ions of mass m2

$$r_2 = \frac{m_2 E}{B_2 q B_1}, \quad d_2 = 2r_2 = \frac{2m_2 E}{B_2 B_1}$$

$$Separation\, d = d_2 - d_1$$

$$d_2 - d1 = \frac{2m_2 E}{qB_1 B_2} - \frac{2m_1 E}{qB_1 B_2}$$

$$d_2 - d_1 = \frac{2E}{qB_1 B_2}(m_2 - m_1)$$

The difference in isotopic masses is proportion to the separation of the lines on the photographic plate.

Examples

The magnetic flux density in both fields is 0.4T and the electric field in the velocity selector is $2x10^4$Vm.

(i) What is the velocity of an ion which goes un deviated through the slit system

(ii) The source is set to produce singly charged ions of magnesium isotopes as Mg – 24 and Mg – 26. Find the distance between the images formed by the isotopes on the photographic plate, assuming the atomic masses of the isotopes are equal to their mass numbers numerically.

(iii) Calculate the ratio of the times the two isotopes take to complete a semi-circle.

{Assume 1U = $1.67x10^{-27}$kg and e = 1.60x10-19C}

Solution:

i) $B_1 = B_2 = B = 0.4T$ and $E = 2\times10^4 Vm$

For crossed fields in the velocity selector

$$B_1qu = Eq \qquad \therefore u = \frac{E}{B_1} \Rightarrow u = 5\times10^4 ms^{-1}$$

ii) In the deflection chamber, $F_m = F_c$

$$B_2qu = \frac{mu^2}{r} \quad r = \frac{mu}{B_2q} \quad d = \frac{2mu}{B_2q} \text{ since } r = \tfrac{1}{2}d$$

Also $m_1 = 24U$ *and* $m_2 = 26U$

$$d_1 = \frac{2m_1u}{B_2q} \text{ and } d_2 = \frac{2m_2u}{B_2q}$$

Separation of the images

$$d_2 - d_1 = \frac{2u}{B_2q}(m_2 - m_1)$$

$$d_2 - d_1 = \frac{2\times5\times10^5}{0.4\times1.6\times10^{-19}}(26-24)\times1.67\times10^{-27} \Rightarrow d_2 - d_1 = 52.2cm$$

iii) TO BE CONTINUED !!!!!!?????

Exercise.

1. A beam of ions passes through a velocity selector in a mass spectrometer that has an electric field of 1.4 x 10^5NC^{-1}.

(b) If the ions emerging from the selector have a velocity of 2 x 10^5m/s, what is the magnetic field in the selector?

(c) The magnetic field in the bending region is 1T, what is the radius of the path followed by a He^+ ion with charge e and mass 6.68 x 10^{-27} kg?

2. A mass spectrometer has an electric field of 1×10^5 N/C and a magnetic field of 0.6T in its velocity selector, and a magnetic field of 0.8T in its bending region.
   (a) What is the velocity of the ions passing through the velocity selector?
   (b) Find the spatial separation of singly ionized neon 20 and 22 isotopes with charges ^+e after they have been bent through a half circle
   Mass of ^{20}Ne = 20U, Mass of ^{22}Ne = 22U

3. (a) A beam of singly ionized chlorine atoms of different speeds leave an ion source, and enter a velocity selector through a small slit. A transverse magnetic field of 0.2T is applied to the chamber. What size of electric field would enable ions traveling at
   $\times 10^5$m/s to describe a linear trajectory, and so escape through the defining slit on
   the opposite side of the chamber?

   (b) the ions now pass into a deflecting chamber in which a second magnetic field of

   0.6T causes them to describe semi-circular paths which finish on a photographic

   Plate. Calculate

   (i) the radius of the path of the ^{35}Cl ion

   (ii) the linear separation of the traces on the plate, given by $^{35}Cl^1$ and ^{37}Cl.

4. A charged particle moving in a circle of radius ,r, in a plane perpendicular to a uniform magnetic field B. show that
   (j) the linear momentum of the particle is proportional to r
   (ii) the kinetic energy of the particle is directly proportional to r^2B^2

   (ii) The frequency of the particle depends on the particle's specific charge and the magnetic flux density B.
5. if V is the accelerating voltage, verify that the path of the ions is a circle.

6. show that the mass of the ions is given as $m = \dfrac{B^2qr^2}{2v}$

7. (a) An ion source emits a beam of charged particles of different masses but carrying the same charge with. With the aid of a suitable diagram explain how to select from the beam, particles having the same velocity

(b) Two isotopes from beam of positive ions each carries a charge of 1.6 x 10^{-19}C, moving with a speed of 15800 m/s. the ions enter perpendicularly into a region of uniform magnetic flux density of 0.1T which deflects the ions through 180^0 before they strike a photographic plate. Given that the separation between the two images on the plate caused by the ions is 6.56mm, and atomic mass of the lighter isotope is 240U(i) draw a sketch diagram to show the path of the ions

(ii) Find the atomic mass of the heavier isotope

(iii) Calculate the difference in the times spent by the two isotopes in the magnetic field

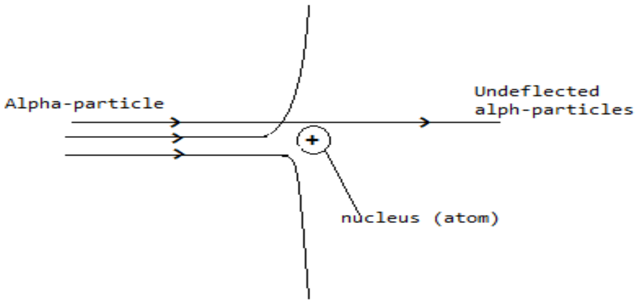
(iii) Find the average radius of the paths of the ions (1U = 1.66 x 10^{-27} kg)

THE ATOMIC NUCLEUS

Rutherford model of an atom

- Positively charged alpha particles were directed towards an atom

Observations: Some alpha - particles passed through the atom un-deflected and a few others deflected at various angles of deflection.



- A negligible number of alpha-particles are reflected back towards the source.

Conclusions:

- Nucleus is positively charged
- All the positive charge and nearly all the mass of the atom is concentrated in a very small volume (called the nucleus) at the center of the atom
- The large angle of deflection is due to the strong electrostatic repulsion between alpha-particles and the positive nucleus on approaching the nucleus

- Rutherford considered electrons to be on the outside of the nucleus at a relatively large distance apart so that the whole atom is neutral.
- The negative charge of the electrons does not act as a shield to the positive charge of the nucleus if the alpha-particle penetrates the atom.
- The electrons move in a circular orbit around the nucleus.
- The electrostatic attraction between the electrons and the positive nucleus provides the necessary centripetal force.

The Bohr Model of the Atom

Bohr's postulates

(i) Electrons in atoms exist only in certain discrete orbits and while in these orbits they don't radiate energy.

(ii) Whenever an electron makes a transition (jumps) from one orbit to another of lower energy, a quantum of electromagnetic radiation is given off.

The energy of the quantum of radiation emitted is given by:

$hf = E_i - E_f$ Where;

E_i is the energy of the electron from the initial orbit,

E_f is the energy of the electron in the final orbit,

f is the frequency of the radiation emitted and

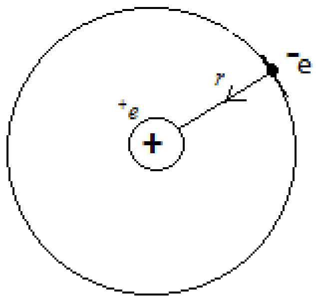
h is Planck's constant.

(iii) The angular momentum of an electron in its orbit in an atom is an integral multiple of $\frac{h}{2\pi}$.

Application of Bohr's Postulates to the Hydrogen Atom

Consider the electron in a hydrogen atom to be in a circular orbit of radius r about the nucleus.

An electron has K.E due to its motion round the nucleus and P.E in the electrostatic field of the nucleus. (*See diagram*)



Kinetic energy of electron in the orbit

Centripetal force on electron $= \frac{mv^2}{r}$

Electrostatic force on electron $= \frac{ee}{4\pi \in_0 r^2}$

Centripetal force = electrostatic force.

$$\frac{mu^2}{r} = \frac{e^2}{4\pi \in_0 r^2}$$

$$mu^2 = \frac{e^2}{4\pi \in_0 r} \text{ ---------------------------- (1)}$$ *Multiply* (1) by $\frac{1}{2}$

$$\tfrac{1}{2} mv^2 = \frac{e^2}{8\pi \in_0 r}$$ But $\tfrac{1}{2} mu^2 = K.E$

$$\therefore K.E = \frac{e^2}{8\pi \in_0 r} \text{ ------------------------ (2)}$$

Potential energy of electron in the orbit

The potential due to the charge e on the nucleus at a distance r is given by: $$v = \frac{^{+}e}{4\pi \in_0 r}$$

P.E of electron $= qv = \dfrac{^{-}e.e}{4\pi \in_0 r} = \dfrac{^{-}e^2}{4\pi \in_0 r}$ ------------- (3)

Total energy of electron in orbit E = P.E + K.E

$$E = \frac{^{-}e^2}{4\pi \in_0 r} + \frac{e^2}{4\pi \in_0 r}$$

$$E = \frac{^{-}e^2}{8\pi \in_0 r} \text{ ---------------------------------- (4)}$$

Angular momentum of the electron in an orbit of radius r is given by $(mu)\times r$.*i.e. the product of linear momentum and the radius*

Applying Bohr's postulates regarding angular momentum

$$mur = \frac{nh}{2\pi}$$ *Square this equation*

$$m^2u^2r^2 = \frac{n^2h^2}{4\pi^2}$$ *Combine this with (1)*

$$\Leftrightarrow \quad r = \frac{n^2h^2 \in_0}{m\pi e^2} \qquad \Rightarrow \quad r_n = \frac{n^2h^2 \in_0}{m\pi e^2} \qquad \text{Where} \qquad n = 1,2,3---$$

- This gives the radii of the allowed Bohr orbits.

The allowed electron energies are obtained by substituting for r_n

Substituting for r in equation (4)

$$\Rightarrow E_n = \frac{-me^4}{8\in_0^2 h^2n^2} \qquad n = 1,2,3----$$

Bohr's assumptions

1. Each electron moves in a circular orbit with the nucleus as its center.
2. The necessary centripetal force is provided by the electrostatic force of attraction between the positively charged nucleus and the negatively charged electron.

Bohr's failure

1. It can only explain spectra for simpler atoms with few electrons such as hydrogen
2. It cannot explain fine structure of spectral lines of hydrogen
3. His model assumes that the electron orbits are circular yet they are elliptical.

Note:

(i) The energy of the electron is always negative. This means that the work has to be done to remove the electron from infinity where it is considered to have zero energy. The electron is thus bound to the atom.

(ii) When r_n is increased, n also increases and the energy E_n becomes less negative.

(iii) The lowest energy level or energy state occurs when *n* = 1 and is referred to as the ***ground state.*** The other energy levels (*energy state*) are called excited energy states.

(iv) A transition of an electron from energy level n_1 to n_2 will lead to a radiation of energy hf, such that $E_1 - E_2 = hf$

$$hf = \frac{me^4}{8\in_0^2 h^2}\left(\frac{1}{n_2}-\frac{1}{n_1}\right)$$

$$f = \frac{me^4}{8\in_0^2 h^3}\left(\frac{1}{n_2}-\frac{1}{n_1}\right)$$

$$f = R\left(\frac{1}{n_2}-\frac{1}{n_1}\right)$$ Where R is the Rydberg's constant

(v) The energy of the stationary/ ground state with least energy $(n=1)$ is :

$$E_1 = \frac{-me^4}{8\in_0^2 h^2} = \frac{-9.1\times10^{-31}\left(1.6\times10^{-19}\right)^4}{8\left(8.85\times10^{-12}\right)^2\left(6.625\times10^{-34}\right)^2} = -2.18\times10^{-18}J = -13.6eV$$

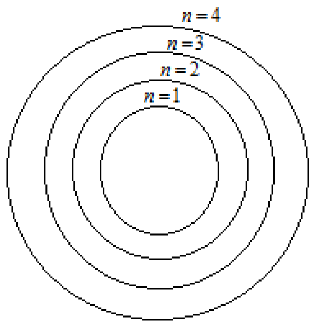
The energies of the other stationary states can be expressed as $E_n = -\frac{13.6}{n^2}eV$. ***Here are the approximate values***

$E_2 = -3.39,\ E_3 = -1.59,\ E_4 = -0.85,\ E_5 = -0.54,\ E_6 = -0.38eV$

(vi) **Bohr atom** is defined as one whose center is the nucleus surrounded by the electrons moving in definite orbits

Example:

The diagram below shows the possible electron orbits in the Bohr model for hydrogen. (*see diagram*)



(i) show that the total energy of the atom is $E_n = \frac{-me^4}{8\in_0^2 n^2h^2}$.

(ii) Calculate the wavelength of the radiation that will be emitted when the electron makes a transition from n=4 to n=3 orbits.

Solution:

If the radiation has frequency f and wavelength λ $E_4 - E_3 = hf = \frac{hc}{\lambda}$.

But $E_4 = \frac{-me^4}{8\epsilon_0^2 h^2 4^2}$ and $E_3 = \frac{-me^4}{8\epsilon_0^2 h^2 3^2}$

Hence $E_4 - E_3 = \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{-1}{4^2} - \frac{-1}{3^2}\right) = \frac{hc}{\lambda}$

$$\frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{7}{144}\right) = \frac{hc}{\lambda} \qquad \lambda = \frac{hc \times 144 \times 8 \times \epsilon_0^2 h^2}{7me^4}$$

$$\lambda = \frac{\left(6.63\times10^{-34}\right)^3 \times 3.0\times10^8 \times 144\times 8\times\left(8.85\times10^{-12}\right)^2}{7\times9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^4}$$

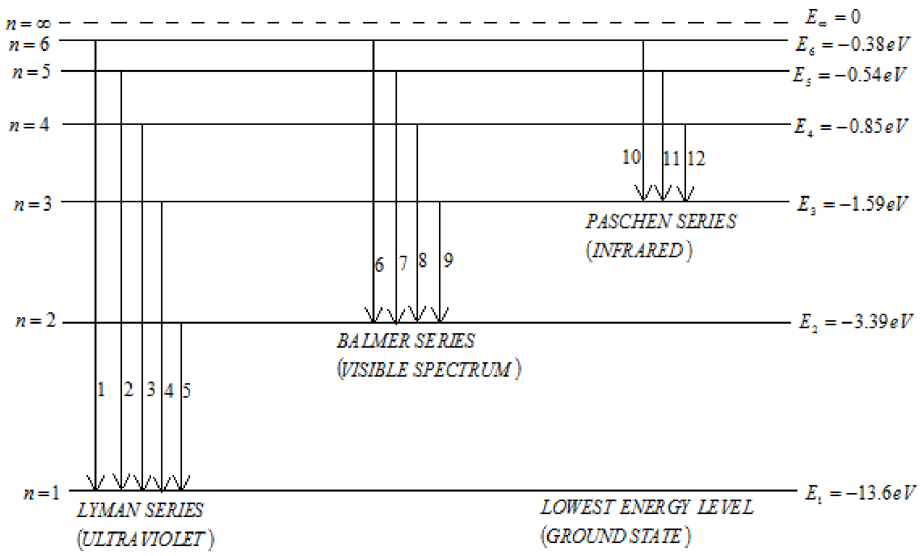
$$\lambda = 1.89\times10^{-6}\,m$$

Energy Level

- According to Bohr's model of an atom, electrons are arranged in permitted (*allowed*) orbits of definite amount of energy. These orbits are also known as energy levels of an atom
- The energies of the electrons in an atom have only certain values called energy levels of the atom.
- All atoms of a given element have the same set of energy levels and these are characteristic of the element i.e. they are different from those of other elements.
- Energy levels are calculated using Bohr model and expressed in eV.
- The lowest energy level of an atom is known as ground state.
- Electrons have least energy in the ground state.
- The atom becomes ionized when an electron receives energy to just exceed its highest energy level and leave the atom.
- The highest energy level given the value zero and lower energy levels are negative.

Energy level in the hydrogen atom

The spectrum of atomic hydrogen contains distinct groups of lines. The three major ones include; Lyman series, the Balmer series and the Paschen series



- The spectral lines of hydrogen are experimental evidence for the existence of discrete or separate energy levels of the hydrogen atom.
- Transition involving LYMAN SERIES involve high energy changes. Those in PASCHEN SERIES involve lowest energy changes.
- Lyman series involve the transition to the energy state n_1 and the resulting radiation emitted is the Ultra-Violet.
- Balmer series involve the transition to the energy state n_2 and the resulting radiation emitted is the visible spectrum.
- Paschen series involve the transition to the energy state n_3 and the resulting radiation emitted is the Infra-red.
- The energy required to remove the electron in the ground state to infinity is the ionization energy E_∞

  Thus $E_\infty = 0 - E_1$ $\qquad E_\infty = 0 - {}^{-}13.6eV = 13.6eV = 2.176\times10^{18} J$

Transitions

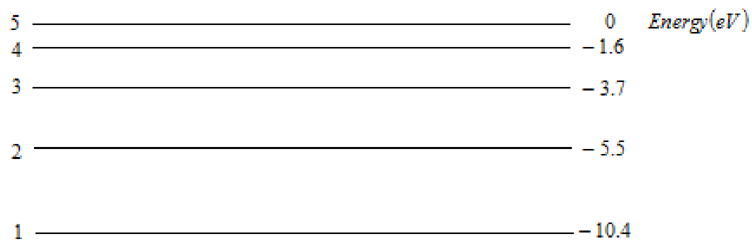
- A transition of an electron at a higher energy level E_i to a lower energy level E_f results in loss of energy given by: $\left(E_i - E_f\right) = hf$ Where *h* is Planck's constant, *f* is the frequency of the electromagnetic radiation.
- Sometimes an electron may pass through an intermediate state E_m to the final energy state. It emits frequencies f_1 and f_2 given by $E_i - E_m = hf_1$ and $E_m - E_f = hf_2$
- When an electron absorbs energy, it jumps from a lower energy level to a higher energy level. The atom is then said to be excited. Such an atom is unstable and to gain stability, the electron falls back to its original energy level while releasing the energy in a form of radiations (*light*).

Note:

1) **Excitation Energy:** This is the energy required to raise an atom from its ground state to an excited state.
2) **Ionization Potential:** This is the potential required to enable the electron to escape completely from the atom.
3) **Ionization Energy:** this the energy required by an electron to escape completely from the atom.

Examples 1:

Some of the energy levels of mercury are shown in the diagram above. Level 1 is the ground state level occupied by the electrons in an unexcited atom.



(i) Calculate the ionization energy of mercury atom in Joules

(ii) Calculate the wavelength of the radiation emitted when an electron moves from level 4 to level 2.

Solution:

(i) Ionization energy $E_I = E_\infty - E_1 = 0 - \left({}^{-}10.4eV\right) = 10.4eV$

But $1eV = 1.6 \times 10^{-19} J$

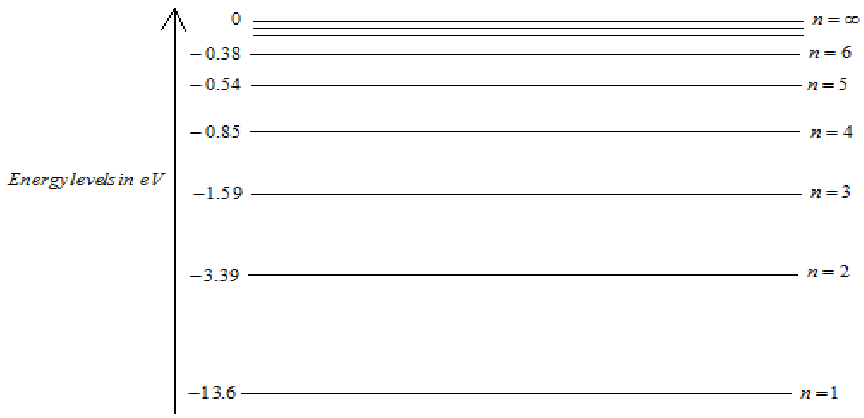
$E_I = 10.4 \times 1.6 \times 10^{-19} = 1.66 \times 10^{-18} J$

(ii) $E_4 - E_2 = hf = \dfrac{hc}{\lambda}$

$$\lambda = \frac{hc}{E_4 - E_2} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{-(1.6 - 5.5) \times 1.6 \times 10^{-19}} = 3.2 \times 10^{-7} M$$

$Wavelength = 3.2 \times 10^{-7} m.$

Example 2:



Using the above diagram, calculate

(i) The smallest frequency (longest wavelength) in the Lyman series.

(ii) The highest frequency (shortest wavelength) in the Paschen series.

Solution:

(i) Smallest frequency in any series corresponds to the change involving the smallest amount of energy. This involves transition from n=2 to n=1.

$$E = hf = \frac{hc}{\lambda}$$

$$E = E_2 - E_1 = \left({}^{-}3.4 - {}^{-}13.6\right)eV = 10.2\,eV$$

$$\lambda = \frac{hc}{E} = \frac{3 \times 10^{8} \times 6.6 \times 10^{-34}}{10.2 \times 1.6 \times 10^{-19}} = 1.2 \times 10^{-7} m$$

(ii) Highest frequency corresponds to the maximum energy change in any series. For Paschen series this is

$$E_\infty - E_3 = (0 - {}^{-}1.5)eV = 1.5eV$$

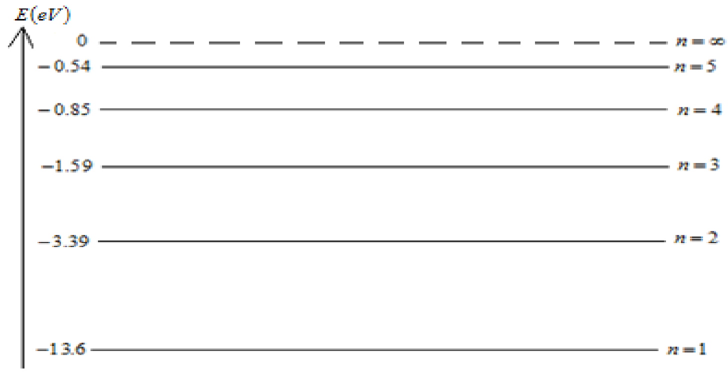
$$E = hf = 1.5 \times 1.6 \times 10^{-19} J = 2.4 \times 10^{-19} J.$$

Example 3:
Explain the following.

(i) Bohr atom

(ii) Electron volt

(b) The diagram below shows some energy levels of the hydrogen atom.



(i) Use the diagram to explain the emission spectrum of hydrogen.

(ii) Calculate the speed of an electron which could just ionize the hydrogen atom.

(iii) Calculate the minimum wavelength of the hydrogen spectrum and state the region of the electromagnetic spectrum in which it lies.

Solution:

(a) (i) The Bohr atom is one whose center, the nucleus, is surrounded by electrons moving in definite circular or elliptical orbits. While the electrons are in these stable orbits, the atom does not radiate. But when an electron makes a transition from an orbit to one of lower energy, electromagnetic radiation is emitted.

(ii) Electron volt is equal to the kinetic energy of an electron which has been accelerated from rest through a p.d of $1V$.

(b) (ii) $\frac{1}{2}mV^2 = -E_1$

$$\frac{1}{2}\left(9.11\times10^{-31}\right)V^2 = 13.6\times 6\times10^{-19}\,J$$

$$V = 2.19\times10^{6}\,ms^{-1}$$

(c) Minimum wavelength corresponds to the transition $E_\infty \; to \; E_1$

$$\frac{hc}{\lambda} = E_\infty - E_1$$

$$\lambda = \frac{hc}{E_\infty - E_1} = \frac{6.625\times10^{-34}\times3\times10^{8}}{13.6\times1.6\times10^{-19}}\,m = 9.13\times10^{-8}\,m$$

This lies in the ultraviolet of the electromagnetic spectrum.

Example:

The table below gives some of the energy levels of a hydrogen atom

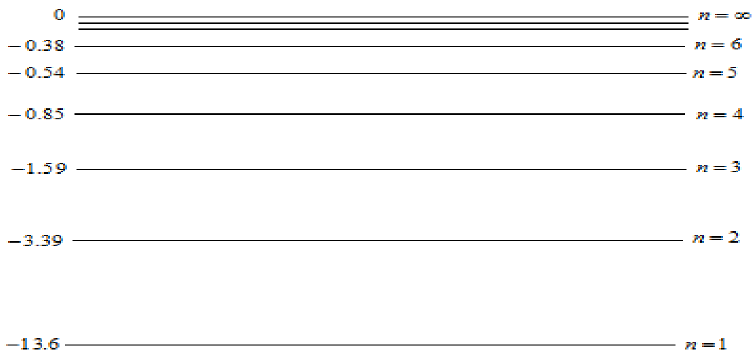
Principle quantum number *n*	**1**	**2**	**3**	**4**	$\propto$
Energy E_n (eV)	-13.58	-3.42	-1.51	-0.85	0

(a) Calculate the wave length of the radiation emitted in an electron transition from n = 3 to n = 2.

(b) Determine the highest energy level which a hydrogen atom, initially at ground state can be excited by absorption of energy of 12.73eV.

Example:

The figure below shows the energy level in a hydrogen atom



(a) Calculate the wavelength of the line arising from electron transition by the hydrogen atom from:

(i) n = 5 to n = 1

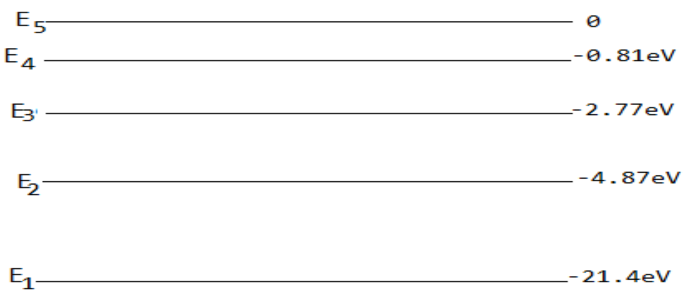
(ii) n = 4 to n = 2

(b) What happens when 13.6eV of energy is absorbed by the hydrogen atom?

Example:

The figure below shows some of the energy levels of a neon atom.

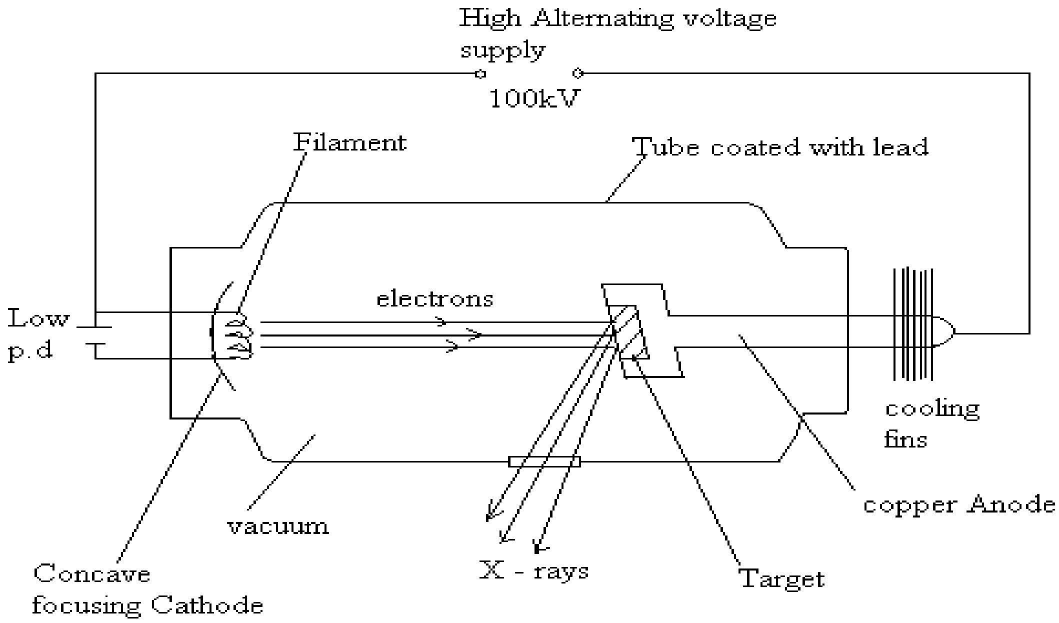
In what region of the electromagnetic spectrum does the radiation emitted in the transition from E_3 to E_2 lie?



X-Rays

X- Rays are electromagnetic radiations of short wavelength produced when fast moving electrons are stopped by heavy metal target.

Production of X – Rays



Mode of operation

- The filament is heated by a low voltage supply and the electrons are emitted by *thermionic emission.*
- The concave focusing cathode focuses the electrons from the filament onto the target.
- These electrons are accelerated towards the anode by the high voltage between the filament and the Anode.
- When the electrons (*cathode rays*) strike the metal target, about only 1% their kinetic energy is converted to X-rays and the 99% of their kinetic energy is converted to heat, which is conducted away by the cooling fins.

Note.

i) The target is made of a high melting point metal.

ii) The X-ray tube is covered by a lead shield with a small window for the X-rays to prevent the leakage of the X-rays.

Intensity of X-rays (Quantity or number of X-rays)

- The intensity of X- rays in an X – ray tube is proportional to the number of electrons colliding with the target.
- The number of electrons produced at the cathode depends on the filament current supply.

- The greater the heating current, the greater the number of electrons produced and hence more x- rays are produced.
- Therefore the intensity of X- rays is controlled by the filament current.

Penetration of X – rays (quality)

- Penetration power of X-rays depends on the kinetic energy of the electrons striking the target.
- The higher the accelerating voltage, the faster the electrons produced.
- Faster electrons possess higher kinetic energy and shorter wavelength x-rays of greater penetration power are produced.
- Hence penetrating power of X-rays is determined by the accelerating Voltage across the tube.

Types of X-rays

There are two types of X-rays, namely: *Hard X-rays and soft X- rays*

Hard X-rays: They are produced when a high p.d is applied across the tube.

- They have very short wave lengths
- They have a high penetrating power. This is because they have very short wavelengths.

Soft X-rays: are produced by electrons moving at relatively lower velocities than those produced by hard X –rays.

- They have longer wavelengths.
- They have a low penetration power compared to hard x-rays. This is because of their long wavelengths

Note:

- Hard X-rays can penetrate flesh but are absorbed by bones, they are therefore used to study bone fractures
- Soft X-rays are used to show malignant growths since they only penetrate soft flesh. They are absorbed by such growths.

Properties of X –rays

1) They travel in a straight line at a speed of light in vacuum
2) They are not deflected by both magnetic and electric fields. This indicates that they carry no charge.
3) They penetrate all matter to some extent. Penetration is least in materials with high density and atomic number e.g. lead.
4) They ionize gases through which they pass.
5) They affect photographic plates.
6) They cause fluorescence in some materials.
7) They cause photoelectric emission
8) They are diffracted by crystals leading to an interference pattern.

Uses of X-rays

1. Structural analysis, stresses, fractures in solids, castings and welded joints can be analyzed by examining X-ray photograph.
2. Crystallography; Orientation and identification of minerals by analysis of diffraction patterns using Bragg's law.
3. Medical uses;
   i) Analytical uses. These include location of fractures, cancer and tumor/defective tissue absorbs x-rays differently from normal tissue.
   (ii) Therapeutics use for destroying cancerous cells and tumors.
4. Detection of fire arms at international airports.

HEALTH HAZARDS CAUSED BY X-RAYS:

- Destroy living cells in our bodies especially hard X-rays.
- Cause Gene mutation (genetic changes in our bodies).
- Cause damage of our eye sight and blood.
- Produce deep skin burns.

NOTE: *Its highly important to remember that each time you are exposed to X-rays, your health is also at risk yet we cannot live without them.*

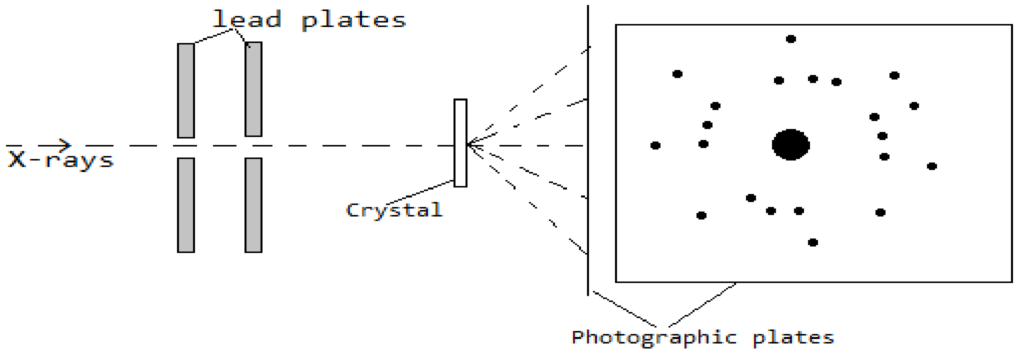
SAFETY PRECAUTIONS:

- Avoid unnecessary exposure to X-rays.
- When exposure is necessary, keep it as short as possible.
- X-ray beams should ONLY be restricted to the body part being investigated.
- A worker should wear a shielding jacket with a layer of Lead.

- Exposure should be avoided for unborn babies and very young children.

Diffraction of X-Rays by Crystals

1) Laue's experiment:



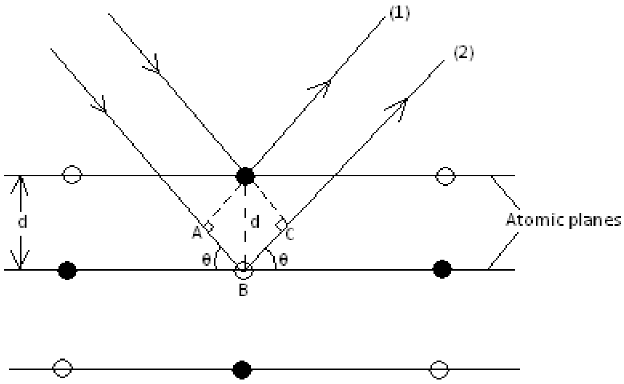
- After long exposure of the crystal to the x-rays, the photographic plate is developed and printed.
- A regular pattern of dark sports called Laue spots is observed around a central dark image.
- The pattern is due to the X-rays which have been scattered by interaction of the X-rays with the electrons in the atoms of the crystal.
- The regularity of the Laue spots implies that the atoms in a crystal are arranged in a regular pattern.

2) Bragg's Law

- When x-rays fall on a single plane of atoms in a crystal, each atom scatters a small fraction of the incident X-ray beam.
- The scattered X-rays from various planes interfere constructively in those directions for which the angle of incidence is equal to the angle of reflection (*i.e. the scattered rays are parallel*).

Derivation of Bragg's law

Consider a beam of monochromatic X-rays incident on a crystal of interplanar spacing ***d*** between consecutive planes.



The path difference between the X-rays diffracted/scattered along paths 1 and 2

= AB + BC
= dsinθ + dsinθ
= 2dsinθ

For constructive interference to occur, the path difference is an integral multiple of wavelengths ($n\lambda$),

That is when **2dsinθ = nλ.** Where n = 1, 2, 3...

For first, second, third, etc., order diffraction. This relation is called Bragg's law for X-ray diffraction.

- ✓ If n =1, 2dsinθ = λ.

Since sinθ ≤ 1, then λ ≤ 2d.

- ✓ If λ > 2d, no diffraction pattern will be observed.
- ✓ The maximum order, $n_{\max}$ is given by **2dsin**θ_n **=** **n**λ**,** but **sin**$\theta_n \leq 1$.
- ✓ $n_{\max}$ is the largest integer satisfying the inequality $n_{\max} \leq \frac{2d}{\lambda}$.

  θ is called a glancing angle.

Examples.

1. Calculate the glancing angle for 2nd diffraction image for an X-ray beam with wavelength of 0.2×10^{-10} m, incident on a crystal with spacing between atomic planes of the order of 1.0×10^{-10} m.

Solution

$$n = 2 \qquad \text{using } 2d\sin\theta = n\lambda$$

$$\lambda = 0.2 \; x \; 10^{-10} \; m \qquad \sin\theta = \frac{n\lambda}{2d} = \frac{2 \; x \; 0.2 \; x \; 10^{-10}}{2 \; x \; 1.0 \; x \; 10^{-10}} = 0.2$$

$$d = 1.0 \; x \; 10^{-10} \; m$$

$$\theta = 11.5^0$$

2. (a) Derive Bragg's law for X- ray diffraction.

(b) Calculate the smallest grazing angle at which X - rays of wavelength 0.7 x 10^{-10} m will be reflected from a quartz crystal which has an atomic spacing of 1.5 x 10^{-10} m. what is the highest reflected order that can be observed with this radiation?

Solution.

$$\theta = ?,\ \lambda = 0.7\ x\ 10^{-10}\ m,\ d = 1.5\ x\ 10^{-10}\ m,\ n = 1$$

$$\sin\theta = \frac{n\lambda}{2d} = \frac{0.7\ x\ 10^{-10}}{2\ x\ 1.5\ 10^{-10}} = 0.2333$$

$$\theta = 13.5^0$$

The highest order $n_{\max}$ is the largest integer satisfying

$$n_{\max} \le \frac{2d}{\lambda} = \frac{2\ x\ 1.5\ x\ 10^{-10}}{0.7\ x\ 10^{-10}} = 4.$$

3. A monochromatic beam of X-rays is incident on a set of parallel atomic planes of sodium chloride crystals. A first order diffraction maximum is observed at a glancing angle of $5^0 24^/$. When the same X-rays is incident on a potassium chloride crystals, a second order diffraction maximum is observed at a glancing angle of $10^0 11^/$. Compare the spacing between the respective atomic planes.

Solution:

$$\textit{From Bragg's law}:\quad 2d\sin\theta = n\lambda$$

For Sodium Chloride Crystals

$$2d_1 \sin 5.4^0 = \lambda,\ \text{since } n = 1\ (\textit{for first order difraction})$$

$$\Rightarrow\quad d_1 = \frac{\lambda}{2\sin 5.4^0}\ \text{- - - - - - - - - - - - - - *}$$

For potassium chloride crystals

$$2d_2 \sin 10.18^0 = 2\lambda,\ \text{since } n = 2\ (\textit{for second order difraction})$$

$$\Rightarrow\quad d_2 = \frac{\lambda}{\sin 10.18^0}\ \text{- - - - - - - - - - - - - - **}$$

divide (*) *by* (**)

$$\frac{d_1}{d_2} = \frac{\sin 10.18^0}{2\sin 5.4^0} = 0.939$$

4. A monochromatic beam of X-rays is incident on a set of planes on a certain crystals. At 0^0c first order diffraction at a glancing angle of 30.4^0. When the temperature is raised to 400^0c, the first order diffraction is observed at 30.0^0.

Calculate the mean coefficient of linear expansion of the crystal for the temperature range 0^0c to 400^0c.

Solution:

Let d_0 and d_{400} be the atomic spacing at 0^0c and 400^0c respectively with the respective temperatures θ_0 and θ_{400}.

$$Using\ Bragg's\ law: \quad 2d\sin\theta = n\lambda$$

$$At\ 0^0c: \qquad d_0 \sin\theta_0 = \lambda,\ since\ n = 1 \text{ - - - - - - - - - - - - - - } (i)$$

$$At\ 400^0c: \quad d_{400}\sin\theta_{400} = \lambda,\ since\ n = 1 \text{ - - - - - - - - - - - - - - } (ii)$$

$$\Leftrightarrow \frac{d_{400}}{d_0} = \frac{\sin\theta_0}{\sin\theta_{400}} = \frac{\sin 30.4}{\sin 30.0} = 1.012 \text{ - - - - - - - - - - - - - - - } (iii)$$

Recall: *for linear expansivity of a physical property X, we have*

$$X_\theta = X_0(1 + \alpha\theta) \quad \Leftrightarrow \quad d_\theta = d_0(1 + \alpha\theta)$$

$$\Rightarrow d_{400} = d_0(1 + \alpha\theta)\ divide\ through\ by\ d_0$$

$$\frac{d_{400}}{d_0} = (1 + \alpha\theta) combine\ with\ (iii)$$

$$1.012 = (1 + 400\alpha) \quad \therefore \quad \alpha = 3.0\times 10^{-5} K^{-1}$$

5.

Atomic spacing in a crystal

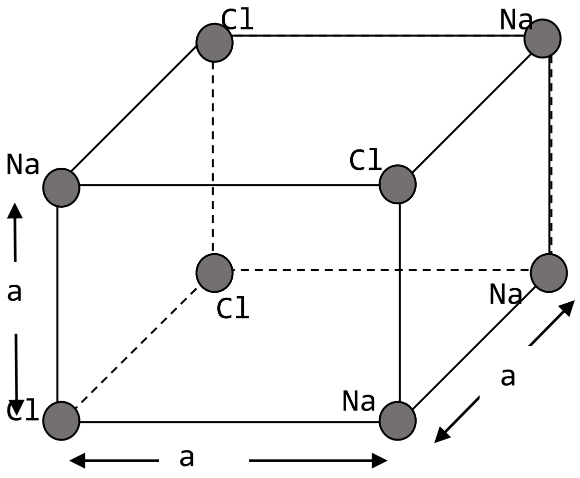
Derivation

Consider a unit cell of sodium chloride.

Let: M = molecular mass

ρ = Density of NaCl crystal

N_A = Avogadro's number



$$The\ mass\ of\ one\ molecule = \frac{M}{N_A}$$

$$\therefore\ the\ volume\ of\ one\ molecule = \frac{M}{\rho N_A}$$

But one molecule of NaCl has two atoms, hence the volume associated with one atom of NaCl is $\frac{1}{2}\left(\frac{M}{\rho N_A}\right)$

Also for a unit cell (cube) the volume = a^3

$$\Rightarrow a^3 = \left(\frac{M}{2\rho N_A}\right) \qquad \Leftrightarrow \qquad a = \left(\frac{M}{2\rho N_A}\right)^{\frac{1}{3}}$$

Example:

Calculate the atomic spacing of sodium chloride if the relative atomic mass of sodium is 23.0g and that of chlorine is 35.5g (assume density of Sodium chloride crystal = 2.2 x $10^3 Kg/m^3$ and N_A = $6.02x10^{23} mol^{-1}$).

If x-rays fall on the crystal causing a first order diffraction at a glancing angle of 5.4^0. Find the wavelength of the X-rays.

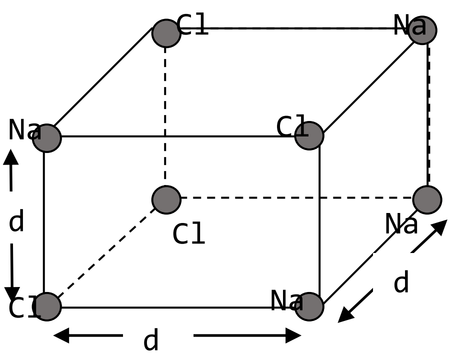
Solution:

(i) Let *d* be the atomic spacing of the crystal

$$The\ mass\ of\ one\ molecule = \frac{M}{N_A},$$

$$\therefore\ the\ volume\ of\ one\ molecule = \frac{M}{\rho N_A}$$

But volume of one atom of NaCl = $\frac{1}{2}\left(\frac{M}{\rho N_A}\right)$



Also volume of the cell = d^3

M = 23.0 + 35.5 = 58.5g = $58.5x10^{-3}$Kg

$$d^3 = \frac{58.5\times10^{-3}}{2\times6.02\times10^{23}\times2.2\times10^3} \qquad \therefore\ d = 2.81\times10^{-10}m$$

$$Using\ Braggs\ law: \quad 2d\sin\theta = n\lambda, \quad and\ n = 1$$

$$\lambda = 2\times2.8\times10^{-10} = 5.6\times10^{-10}m \equiv 5.6\overset{o}{A}$$

Example:

A monochromatic beam of X-rays of wavelength 2.0 x 10-10m is incident on a set of cubic panes in a potassium chloride crystal. First order diffraction maximum are observed at a glancing angle of 18.50. Find the density of potassium chloride if its molecular mass is 74.55g

Example:

The closest spacing between planes of atoms in a crystal of sodium chloride is 0.282nm. First order diffraction of a monochromatic beam of x-rays occurs at an angle of 15030|. How many orders of reflection of these X-rays would be observed from this planes?

Example:

A beam of X-rays of wavelength 1.0x10-10m is incident on a set of cubic planes in a sodium chloride crystal. The first order diffracted beam is obtained for a glancing angle of 10.20. Find

the spacing between the successive planes

the density of the sodium chloride.

X-RAY SPECTRA IN AN X-RAY TUBE

There are two types of X-ray spectra in an X-ray tube, namely;

(i) Continuous spectrum which is independent of the target material.

(ii) Line speectrum which is characteristic of the target material.

GRAPH OF INTENSITY OF X-RAYS AGAINST WAVELENGTH

(A line spectrum is superimposed on the continuous spectrum)

CONTINUOUS SPECTRUM:

This is produced by multiple collisions of the electrons with target atoms and being decelerated. The energy of the emitted X-ray quantum is equal to the energy lost in the deceleration. Different amounts of energy are lost during these collisions. The X-rays given off when the electrons are decelerated will have wavelengths varying from a certain minimum λ_{min} to infinity.

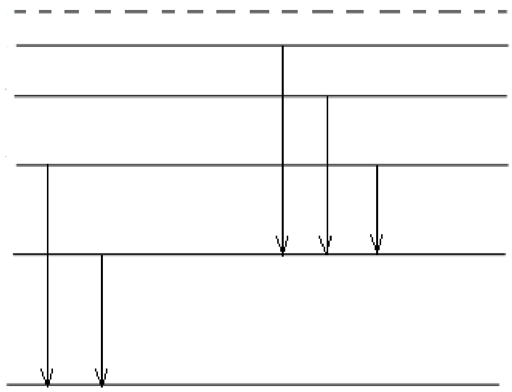
When an electron loses all its energy in a single collision with an atom of the target, a most energetic X-ray photon is given off. The kinetic energy of the electron equal to eV, where V is the accelerating voltage between the filament and the anode, is converted into electromagnetic radiation of energy $hf_{max} = \frac{hc}{\lambda_{min}}$.

Thus $\frac{hc}{\lambda_{min}} = eV$ or $\lambda_{min} = \frac{hc}{eV}$. λ_{min} represents the minimum wavelength of the X-rays produced for a given accelerating voltage V. It is also called the cut-off wavelength.

X-rays with longer wavelengths are the result of electrons losing less than their total energy.

LINE SPECTRUM:

Electrons in an atom belong to shells covering a range of energies. These shells are labelled K, L,M,N,O….. The K-shell has the smallest radius and hence the lowest energy.



When an electron bombards an atom of the target, an electron may be ejected from the K-shell to higher un filled shells. The target atom becomes excited and is unstable. Stability is restored by an electron say from the L-shell making a transition to the vacancy left in the K-shell. The transition is accompanied by emission of an X-ray photon of energy

$hf_k = -E_L - (-E_K) = E_K - E_L$.

The frequency of the X-ray photon is $f_k = \frac{E_K - E_L}{h}$.

The difference in energy, $E_K - E_L$, is very high, hence the corresponding wavelength $\lambda_k = \frac{c}{f_K}$ will be very short.

Electron trasition into the L shell gives rise to the L-lines. These lines are characteristic of the target material.

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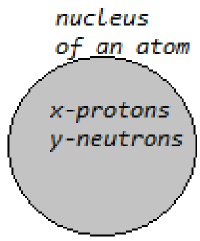
RADIOACTIVITY

This is the spontaneous disintegration of unstable atoms with emission of particles like alpha, beta particles and gamma radiations.

Let m_p and m_n be the masses of a proton and a neutron respectively *(see diagram)*



Consider a nucleus having *x* protons and *y* neutrons, then the total mass of the protons and neutrons $\left(xm_p + ym_n\right)$

If **M** is the mass of the nucleus, it's found that the mass

$$M \neq \left(xm_p + ym_n\right)$$

This difference in mass is called the mass defect and it is responsible for holding the particles (nucleons) together in the nucleus.

From Einstein's mass – energy relation, the energy E associated with this mass defect is such that: $E = \Delta mc^2$ or $E = mc^2$

Where *c* = speed of light $\left(3.0\times10^8 ms^{-1}\right)$ and $m = \Delta m$

Thus mass can be turned into energy and energy can be turned into mass.

This implies that for a reaction that produces an appreciable decrease in mass is a possible source of energy.

When a radioactive substance decays, it may emit three types of radiations in addition to the daughter elements. These are alpha – particles(α), beta – particles (β) and gamma radiations(γ).

i.e. $$\underset{(parent)}{X} \xrightarrow{decay} \underset{(daughter)}{Y} + Radiation$$

The three types of radiations can be identified by:

1) Their different penetrating powers/abilities
2) Their ionizing powers
3) Their behavior in electric and magnetic fields.

Alpha – particles

- They are the least penetrating with a range of a few centimeters in air and can be stopped by paper.
- They produce intense ionization in any gases through which they pass.
- They are deflected by both electric and magnetic fields. Their direction and size of deflection suggests that:
  i) They are positively charged

ii) They are relatively heavier particles.

- Alpha particles are therefore a Helium nuclei containing 2 protons and 2 neutrons.

Beta – particles

- They are more penetrating than the alpha particles with a range of several centimeters in air and a few millimeters in aluminum.
- They are less ionizing than the alpha particles.
- They are more easily deflected than the alpha – particles, and their size and direction of deflection suggest that:
  i) They are negatively charged
  ii) They have a very small mass

Gamma rays

- They are highly penetrating
- They ionize gases to a very small extent
- They are not deflected by both the magnetic and electric fields, indicating that they are uncharged.

Laws of radioactivity

1) When a radioactive substance decays by emission of alpha – particles, its atomic number Z reduces by 2 and mass number A reduces by 4

$$^{A}_{Z}X \xrightarrow{\alpha-decay} {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

2) When a radioactive substance decays by emission of beta – particles, its atomic number Z increase by 1 and mass number A remains constant

$$^{A}_{Z}X \xrightarrow{\beta-decay} {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$

3) When a radioactive substance decays by emission of gamma radiations, both the atomic number Z and mass number remains constant.

$$^{A}_{Z}X \xrightarrow{\gamma-decay} {}^{A}_{Z}Y + \gamma$$ *The nucleus jumps from a higher energy level to a lower energy level*

Alpha decay

This is when the element decays with the emission of a – particles $^{4}_{2}He$

During α - decay the following changes occur;

(a) The atomic number of the daughter nuclide (element) reduces by two.

(b) The mass number of the daughter nuclide reduces by four.

(c) The element shifts two places before in the periodic table

Generally;

$$\underset{Parent\ nuclide}{^{A}_{Z}X} \xrightarrow{\alpha-decay} \underset{daughter}{^{A-4}_{Z-2}Y} + {}^{4}_{2}He$$

Examples; write a balanced equation for the following decay if they occur with emission of an alpha particle.

- Uranium $^{234}_{92}U$ decaying to form thorium (Th).
- Radium $^{226}_{88}Ra$ decaying to form Radon (Rn).

Beta decay

This occurs with the emission of ß – particles,

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. The following changes occur.

 The atomic number of the daughter nuclide increases by one.

e

 The mass number of the daughter nuclide remains unchanged.
 The element shifts one place ahead in the periodic table
Generally;

A
Z
X
ß - decay
A
1Z
Y
+
0
1
Parent nuclide *daughter nuclide*
ß – particle
Examples;

214
83
Bi

214
84
Po
e

+

Problems:
1. A radioactive nuclide
222
86
Rn
238
92
X

222
88
Ra
0
1
+ 2
e
0
1
e

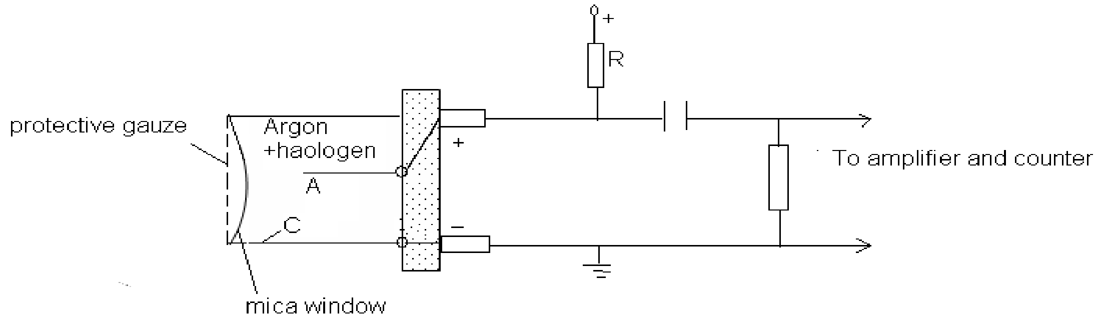
DETECTORS OF NUCLEAR RADIONS

A. Geiger Muller Tube (GMT)

The GMT is used to detect the presence of **X- rays**, **Gamma rays**, **beta particles** and if the window of the tube used is very thin, it detects even **alpha particles**.

Structure

The thin mica window allows the passage and detection of the weak penetrating alpha particles. The GM tube is first evacuated then filled with Neon, Argon plus Halogen gas which is used as a quenching agent.



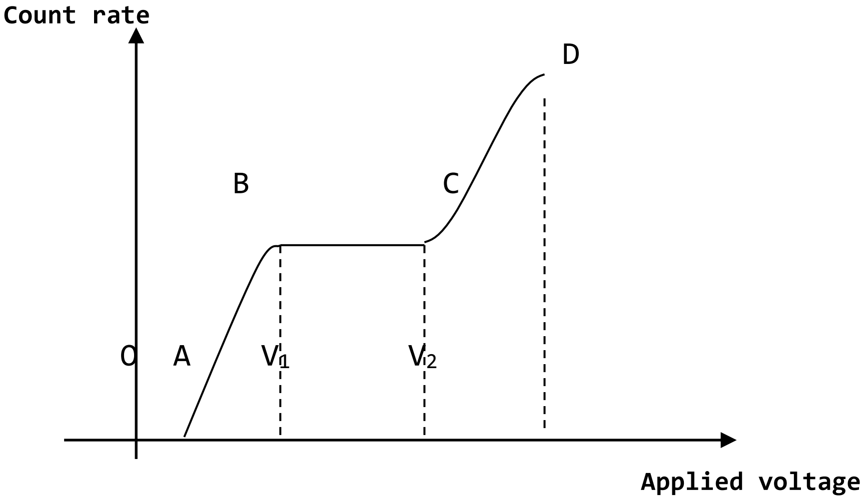
<u>Mode of operation</u>

- When an ionizing particle enters the tube through the window, it ionizes a few gas atoms producing ion pairs. (I.e. creating positively charged ions, and electrons).
- The free electrons are strongly attracted towards the anode. As they are accelerated, they acquire sufficient energy to ionize other gas atoms by collision and produce secondary ions.
- The secondary ions produced are also accelerated by the abode and collide with other atoms to produce more ion pairs.
- This leads to an increase in the number of electrons that move towards the anode.
- An avalanche of charged particles (electrons), which spreads throughout the entire length of the anode, producing a large pulse of current which is then amplified and sent to the detector/counter and an activity registered by the GMT.
- Once the GMT registers an activity then it is an indication that radiations are present.
- The heavy ions move relatively slowly to the cathode reaching after an avalanche has occurred. These ions also have sufficient energy to cause emission of other electrons from the cathode leading to a second avalanche which would cause another pulse.
- To prevent this, a halogen e.g. Bromine is mixed with the gas to act as a quenching agent.

Note: *The anode wire must be very thin so that the charge on it produces an intense electric field close to its surface. This electric field is used to accelerate the electrons towards it from the cathode.*

Characteristics of a GMT

The graph below is obtained when the counter rate is plotted against the operating voltage.



OA – the operating voltage is not enough to attract the ions to the respective electrodes and hence the counter registers no reading. This voltage (i.e. at **A)** is called the *threshold voltage.*

AB – is the proportional region. Increasing the anode and cathode potentials beyond the threshold voltage increases the activity of the GMT and hence an increase in the count rate between **A** and **B.**

- The size of the pulse depends/proportional to the initial ionization

BC – here the count rate is constant. This is called the plateau region.

- Between **BC**, all the negative ions are able to reach the anode because the operating voltage is large enough to attract them.
- Full avalanche is obtained along the entire length of the anode.
- Here the tube is said to be operating normally.

CD: - The count rate increases with an increase with voltage because the quenching agent (Bromine) is unable to absorb the high energy of the accelerated ions.

- In this region, the anode potential is so large that the electrons moving towards the anode collide with other ions or atoms because of their rapid motion. This collision of the ions causes avalanche of electrons thus causing secondary ionization. When secondary ionizations take place, a bigger ionization current flows. The current increases and the detector falsely registers a high count rate.

NOTE:

1) The suitable region for operating the tube is along **AB** because within this region every particle that produces ionization is detected.
2) **Background Count**: When the GM-tube is supplied with a suitable working voltage across and without any radioactive substance nearby, the GM-tube also gives a count rate. This is known as Background radiations.
   The background radiation is due to the presence of the radioactive substances in the air, soil or water near the GM-tube.

Definitions.

Dead time. This is the time taken by the positive ions to move from the anode to the cathode.
During this time the tube is insensitive to the arrival of further ionizing particles.

Recovery time. This is the second period of insensitivity. During this period, pulses are produced but not large enough to be detected. In this time, argon ions are being neutralized by the quenching gas before they reach the anode.

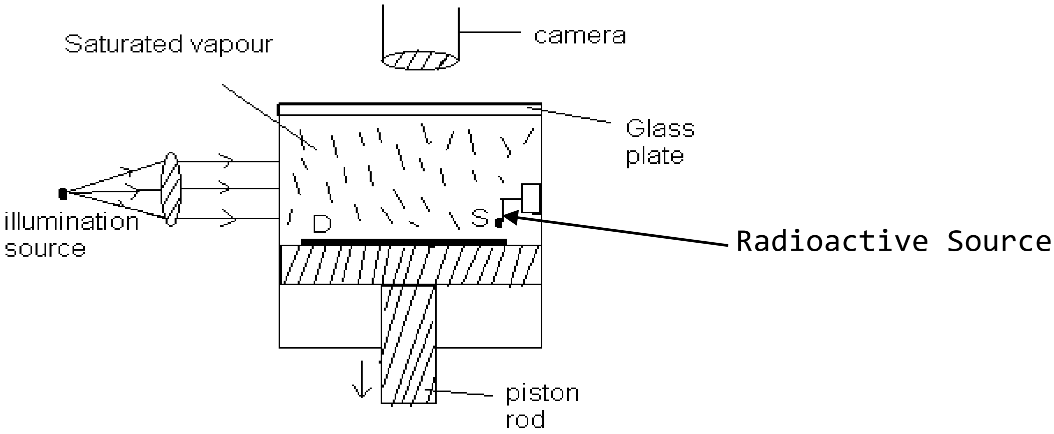
Threshold voltage is the voltage below which there is no sufficient gas amplification to produce pulse high enough to be detected

B) CLOUD CHAMBER

There are two types.

(iv) Wilson cloud chamber (expansion cloud chamber)

- It consists of a chamber containing saturated water or alcohol vapor.
- Photographs of the path of ionizing particles or radiations are taken using the camera.

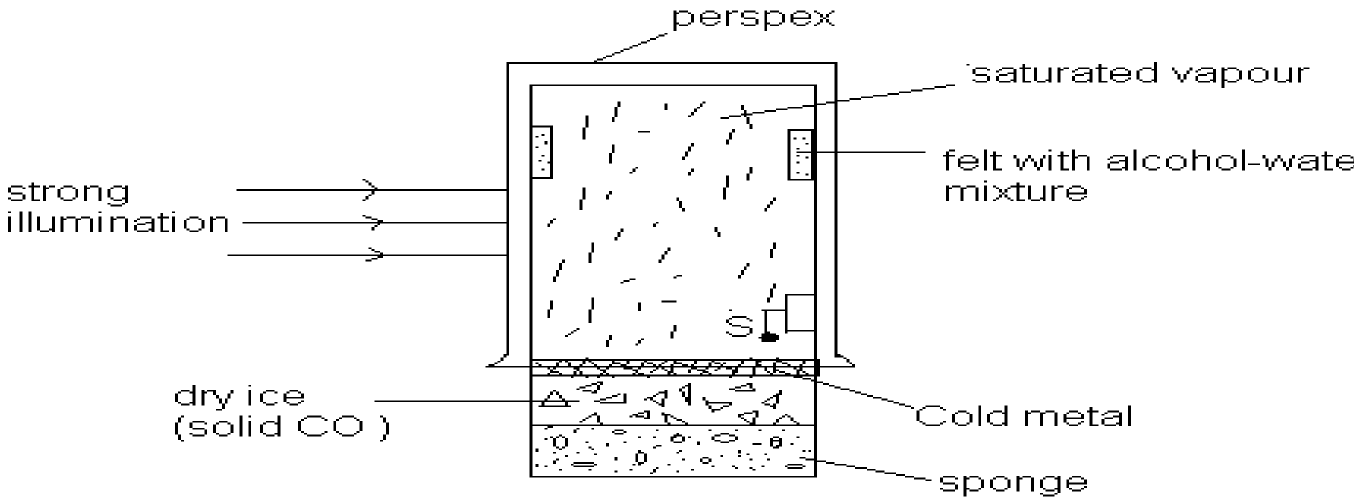


Mode of operation

- Saturated water vapor is introduced into the chamber.
- Air in the chamber suddenly expands adiabatically when the piston is quickly moved downwards. This causes the air to cool.
- The dust nuclei are carried away after a few adiabatic expansions by drops forming on them.
- The dust free air is subjected to a controlled adiabatic expansion, and the air becomes super saturated.
- When super saturated air is exposed to radiations from a radioactive source S, water droplets (tracks) collect around the ions produced.
- The drops are illuminated and photographed.

- Alpha particles produce short, thick, continuous straight tracks
- Beta particles which are less massive produce longer, thin but straggly paths owing to collisions with gas molecules
- Gamma radiations are uncharged and for ionization to take place, it must collide with an atom and eject an electron which then ionizes the vapor.

(v) DIFFUSION CLOUD CHAMBER



- The base of the chamber is maintained at low temperature, about -80^{0}c by the solid carbon dioxide while the top of the chamber is at room temperature, and so there is a temperature gradient between the top and the bottom of the chamber.
- The air at the top of the chamber is saturated with alcohol vapor from the felt ring. This vapor continuously diffuse downwards into the cooler regions so that the air at the chamber is super saturated with alcohol vapor.
- Radiations from the radioactive source **S** cause the ionization of the vapor.
- The ionizations from the radioactive source **S** cause condensation of the vapor on the ions formed, hence the path of the ionizing radiations are traced by series of small droplets of condensation.
- The thickness and length of the path indicate the extent to which ionization has taken place.

- Alpha particles produce short, thick, continuous straight tracks
- Beta particles which are less massive produce longer, thin but straggly paths owing to collisions with gas molecules
- Gamma radiations are uncharged and for ionization to take place, it must collide with an atom and eject an electron which then ionizes the vapor.

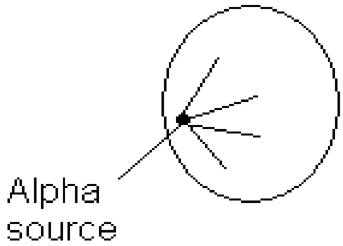
Note:

1) The floor of the chamber is dark to prevent any reflection of light.
2) A small p.d maybe applied between the top and bottom of the chamber to clear the chamber of any ions

NATURE OF TRACKS

ALPHA PARTICLES

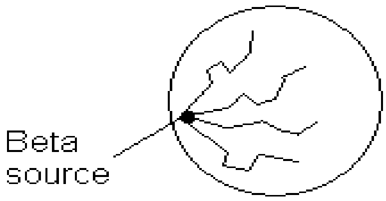
- cause intensive ionization which leads to thick tracks
- they are of short range (short track)
- they are emitted with the same kinetic energy from any given source (tracks of the same length)
- they have a high momentum (*since they are massive*) and therefore are not deflected by collisions with vapor atoms



(straight tracks)

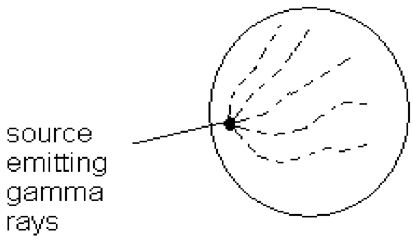
BETA PARTICLES

- more penetrative/longer range than alpha particles (long tracks)
- less ionizing than alpha particles (thin tracks)
- light particles (*i.e. less massive*) moving at high speed (tracks are originally straight, there after become wavy)
- emitted with varying speeds from any given source(tracks of varying lengths)



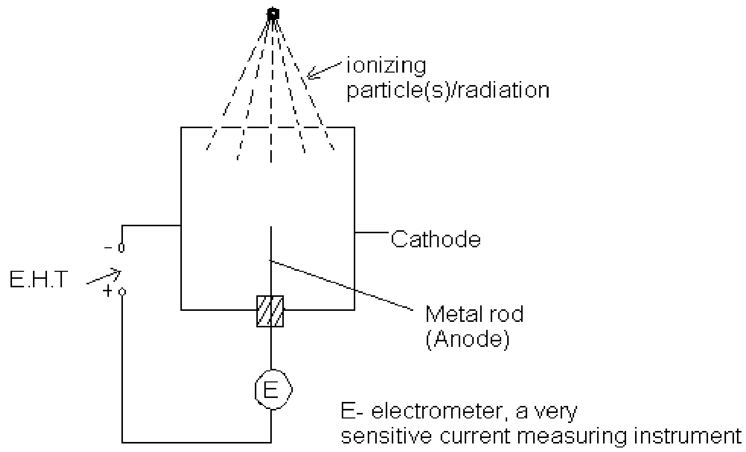
GAMMA RAYS

-they have a very weak ionization potential and very high penetrative



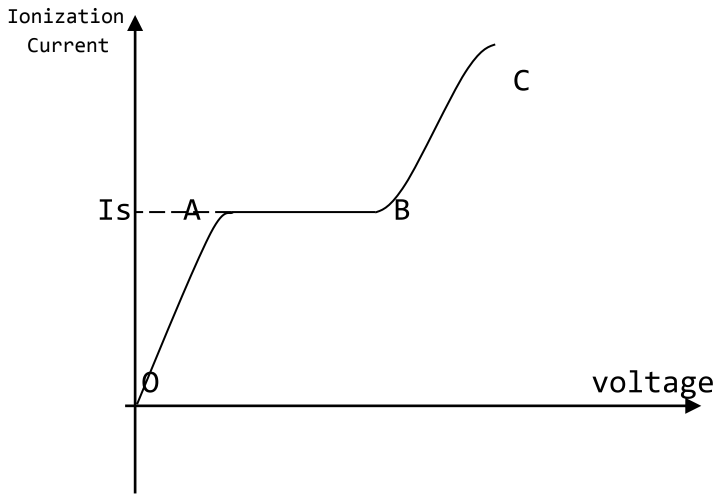
**The tracks are actually those of free electrons which are produced when the gamma rays ionize the air. The features are similar to those of beta particles, except they are much shorter.*

C) IONIZATION CHAMBER



- When ionizing radiation enters the chamber, ion pairs are produced. Positive ions drift for the cathode while the electrons are accelerated towards the anode.
- When the electrons arrive at the anode, an ionizing current pulse flows.
- The current pulse produces a voltage pulse which is amplified and recorded.
- The E.H.T is set to such a value that a constant ionization current I_s flows. In this setting the energy of the incoming radiation is proportional to I_s.

A graph of ionization current (I_s) against voltage.



Region OA:

- The p.d is not large enough to draw all the electrons and positive ions to their respective electrodes.
- Positive atoms and electrons recombine (since the velocities are low), increasing the number of ion pairs.
- The ionization current is proportional to voltage.

Region AB. (saturation region)

- All ion pairs produced per second travel and reach the electrodes. This results in a constant current or *saturation current* I_s.
- The ionization chamber is operated in this region.

Region BC (gas amplification)

- Because of high voltage, the electrons produced by ionization of the neutral gas molecules acquire sufficient energy to cause secondary ionization.
- This results into increased ions in the chamber (gas amplification) and hence a rise in ionization current I_s.

Example: A source of alpha particles has an initial activity of 2.5×10^5Bq. When the alpha particles enter an ionization chamber, a saturation current of 5×10^{-7}A is produced. Find the energy of one alpha particle if the energy required to produce ion pair is 5.0eV.

Approach:

Let the number of the ion pairs produced per second = n

$2ne = 5 \times 10^{-7}$ (*since both positive and negative ions constitute current*)

$n = 1.5625 \times 10^{12}$ ion pairs per second.

But; 1 ion pair produces 5.0ev per second

Therefore 1.5625×10^{12} ion pairs will produce $1.5625 \times 10^{12} \times 5.0\text{ev} = 7.8125 \times 10^{12}$ev

Now: 2.5×10^5 alpha-particles produce 7.8125×10^{12}ev

1 alpha particle will produce = $\dfrac{7.8125 \times 10^{12}}{2.5 \times 10^{6}}$ = 3.125Mev.

Thus the energy of an alpha particle is 3.125Mev.

Unified Atomic Mass Unit (U)

Definition: the Unified Atomic Mass Unit (U) is one – twelfth the mass of one atom of carbon – 12 $\left({}^{12}_{6}\mathrm{C}\right)$

Derivation:

1 mole of a substance contains 6.02x10^{23}atoms

12g of carbon–12 $\left({}^{12}_{6}\mathrm{C}\right)$ contains 6.02x10^{23}atoms

Mass of 1 atom of carbon–12 = $\dfrac{12\times10^{-3}}{6.02\times10^{23}}kg$

$$\therefore\ 1U = \frac{12\times10^{-3}}{6.02\times10^{23}}kg$$
$$= 1.66\times10^{-27}kg$$

From $E = mc^2$

$$= \left(1.66\times10^{-27}\right)\times\left(3.0\times10^{8}\right)$$
$$= 1.494\times10^{-10}J$$
$$\therefore\ 1U = 1.494\times10^{-10}J$$

In electron volts (eV)

$$1U = \frac{1.494\times10^{-10}}{1.6\times10^{-19}}eV = 9.34\times10^{9}eV$$

In MeV: $1U = 934MeV$ *UNEB calculations however it's usually* $1U = 931MeV$

Binding energy (B)

The mass of the nucleus of atom is always less than the total mass of its constituent nucleons (*protons and neutrons*). The difference in mass is called the mass defect.

$$\text{Mass defect} = \left(\textit{mass of the nucleons}\right) - \left(\textit{mass of the nucleus}\right)$$

The reduction in mass is because when the nucleons are combining to form the nucleus, some of the mass is released as energy in the form of gamma rays.

In order to break the nucleus and separate the nucleons, the same amount of energy which was released has to be supplied to the nucleus. This is called the Binding energy.

Definition: Binding Energy is the minimum energy required to break the nucleus into its constituent particles and completely separate them from each other.

Binding energy = (*mass defect in kg*) X (*speed of light*)^2

Examples:

1. The mass of lithium atom ${}_{3}^{7}li$ is 7.01818U. Calculate

   i) the binding energy of lithium atom

   ii) the binding energy per nucleon of lithium atom

Given	mass of proton	= 1.0081U
	mass of neutrons	= 1.0091U
	mass of electron	= 0.00055U
	1U	= 931MeV

Solution:

(i) From ${}_{3}^{7}li$; 3 protons and 4 neutrons (7-3)

Mass of the nucleons: (3X1.0081)U + (4X1.0091)U = 7.0607U

$$\text{Mass defect} = (mass\ of\ the\ nucleons) - (mass\ of\ the\ nucleus)$$

= 7.0607U – 7.01818U

= 0.04252U (*this is the B.E in U*)

But $1U = 931MeV$

B.E = 0.04252 X 931MeV

= 39.58612MeV

(ii) Binding energy per nucleon $\left(\bar{B}\right)$ is defined as the total binding energy of the nucleus divided by the total number of nucleons. i.e. $\left(\bar{B}\right) = \dfrac{Binding\ Energy\ (B)}{mass\ number\ (A)}$

$$\therefore \left(\bar{B}\right) = \frac{39.58612}{7} = 5.65516MeV \approx 5.66MeV$$

N.B: Binding energy per nucleon is the energy that has to be given to the nucleus to remove one nucleon from it. This is a useful measure of the stability of the nucleus. It’s got from the binding energy divided by the mass number.

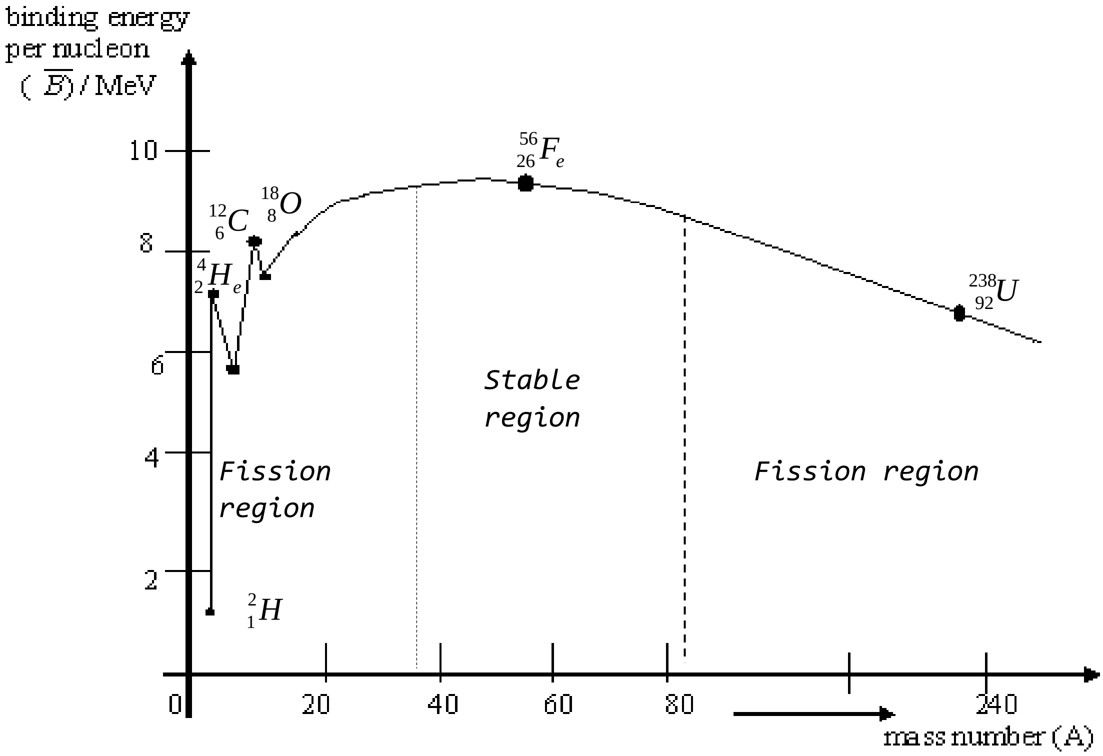
2. Calculate the binding energy and binding energy per nucleon for ${}_{92}^{238}U$. (Atomic mass of ${}_{92}^{238}U$ =238.05076U, mass of: ${}_{1}^{0}n$ = 1.00867U, ${}_{-1}^{0}e$ = 0.00055U, ${}_{1}^{1}p$ = 1.00728U and 1U = 931MeV)

   Ans: 1801.876MeV and 7.58MeV

3. Calculate the binding energy per nucleon of ${}_{26}^{56}F_e$.

   (mass of: ${}_{1}^{0}n$ = 1.008665U, ${}_{1}^{1}p$ = 1.007277U, ${}_{26}^{56}F_e$ = 55.9349U and 1U = 931MeV). Ans: 8.787Mev

Variation of Binding energy per nucleon with mass numbers



Features of the curve above

- Nuclides in the middle part of the graph have the highest binding energy per nucleon and thus are more stable, with iron $^{56}_{26}F_e$ the most stable nucleus in nature.
-

Conclusions from the graph

(1) Energy can be released by assembling light nuclei (A<26) into moderate sized nuclei of greater binding energy per nucleon. A process called nuclear fusion.

(2) Energy can be released by breaking nuclei (A>50) into medium sized nuclei of higher binding energy per nucleon. A process called nuclear fission.

N.B: in all cases the mass defect appears as energy released.

Nuclear Stability

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Radioactive Decay

Radioactive decay follows an exponential law which states that the rate of decay or disintegration of an atomic nuclei in a given element at any time is directly proportional to the number of undecayed atoms present in the sample (element). *This is referred to as the decay law*

Mathematically;

Let the number of radioactive atoms initially present in the sample/element be **N_o** at time t=**t_o**. After some time t=**t**, let the remaining atoms be N, then the rate of disintegration $\frac{dN}{dt}$ is given by

$$\frac{dN}{dt} \propto N$$

$\Rightarrow \frac{dN}{dt} = -\lambda N$, where λ decay constant ----------------------- (1)

The negative sign indicates that N decreases with increase in time.

Separating variables and integrating equation (1)

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\int \frac{dN}{N} = -\lambda \int dt$$

$If\ the initial number of\ nuclei\, N_0\ i.e\ N = N_0\ when t = 0, and\, N\ after t.$

$$Then \quad \int_{N_O}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$(\log_e N)_{N_0}^{N} = -\lambda t$$

$$\log_e N - \log_e N_O = -\lambda t$$

$$\log_e \left(\frac{N}{N_O} \right) = -\lambda t$$

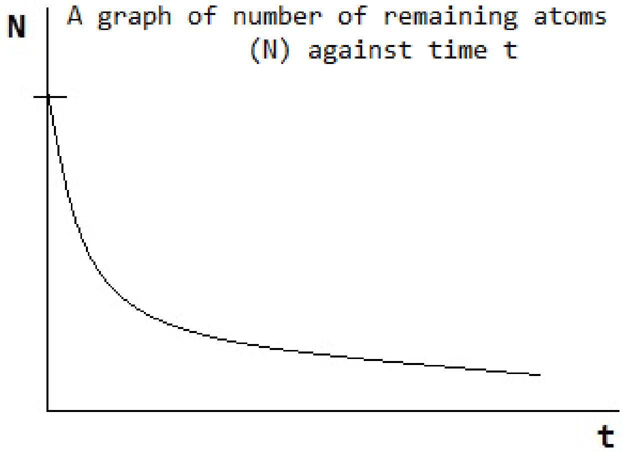
$$\left(\frac{N}{N_O} \right) = e^{-\lambda t}$$

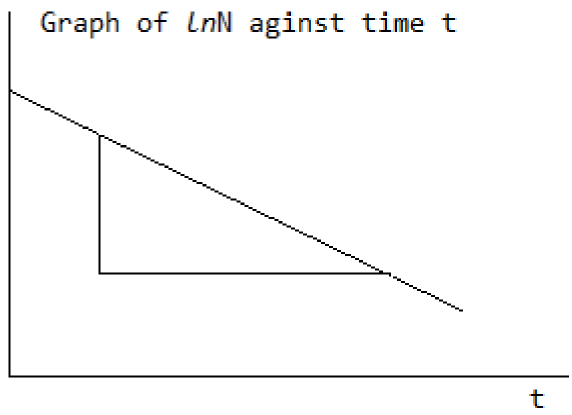
$N = N_0 e^{-\lambda t}$ This is called the decay equation.

Where N_O = *original (parent) nuclei*

N = present nuclei after time t.

Graphs





Activity (A): This is defined as the rate of decay **OR**
Activity is the number of disintegrations per second

Mathematically: $A = \left(\frac{dN}{dt}\right)$ But since the decay process decreases the

$A = -\left(\frac{dN}{dt}\right)$ number of atoms with time, then

Thus from the decay law; $A = -\lambda N$

If we let the initial activity $A_o = -\lambda N_0$ and after time *t*

$A = -\lambda N$

Then $A = A_0 e^{-\lambda t}$ *Units of Activity is the* Becquerel (Bq)

1Bq is one disintegration per second

N.B: The original units of activity was the Curie (Ci), such that

1Ci = 3.71×10^{10}Bq

Decay constant, λ is defined as the fraction of the radioactive nuclei which decays per second. I.e. $\lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right)$.

Decay Constant λ **:** Defined as the probability that a radioactive nucleus of the isotope in the sample would decay in one second. **OR**
The fraction of the radioactive nuclei which decays per second

I.e. $\lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right)$ or $\lambda = \frac{A}{N}$

Half Life $\left(T_{½}\right)$

Definitions:

- The half life of a radioactive source is the time taken for half the number of radioactive nuclei present in the source to disintegrate.
- The half-life of a radioactive substance is the time taken for half the atom in a sample to decay.
- Half-life the time taken for a radioactive substance to reduce to half its initial mass.
- Half life is the time taken for the activity of a radioactive sample to fall to half its original value.

Relationship between λ and $T_{½}$

From the decay equation: $N = N_0e^{-\lambda t}$, when $t = T_{½}$, $N = \frac{1}{2}N_o$

Then $\left(\frac{N}{N_0}\right) = \frac{1}{2}$ $\Rightarrow \frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$. Take natural *log* and simplify

$$\log_e\left(\frac{1}{2}\right) = \log_e e^{-\lambda T_{\frac{1}{2}}}$$

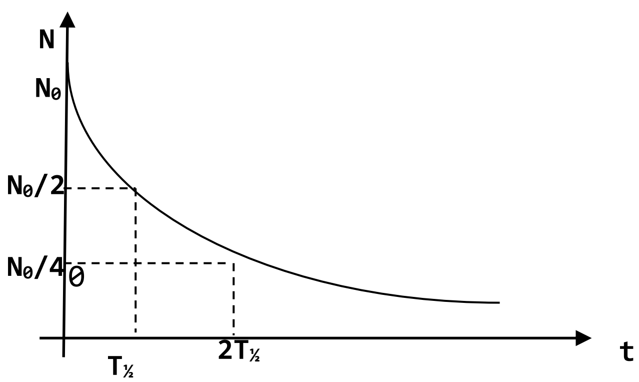
$$\log_e 1 - \log_e 2 = -\lambda T_{\frac{1}{2}}$$

$$\log_e 2 = \lambda T_{\frac{1}{2}}$$

$$T_{\frac{1}{2}} = \frac{\log_e 2}{\lambda}, \quad but \quad \log_e 2 = 0.693$$

$$T_{\frac{1}{2}} = \frac{ln2}{\lambda} \quad \textbf{OR} \quad T_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Consider the decay curve of a radioactive source



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An alternative to half-life

Consider the decay process of a sample containing **N_o** atoms initially

Number of radioactive Nuclei (atoms) present		Number of Half-lives
N_o	----------------------	0
$\frac{N_o}{2}$	----------------------	1
$\frac{N_o}{2^2}$	----------------------	2
$\frac{N_o}{2^3}$	----------------------	3
$\frac{N_o}{2^n}$	----------------------	*n*

E.g. if an atom decays into 12 half-lives, then the number of atoms left will be $\frac{N_o}{2^{12}}$

Nuclear equation

When writing a nuclear equation:

- The total mass number on the left hand must balance with those on the right of the equations
- The atomic number on the left hand must balance with those on the right hand of the equation

Examples:

1. The mass of $1.0\mu g$ of the nuclide $^{32}_{15}X$ decays by emission of a beta particle. Its half - life is 90.0 seconds. Find
   (a) the number of atoms of element *X* initially present
   (b) the initial activity of the sample
   (c) The number of atoms of element *X* present after 12 minutes.
   (d) What is the activity 12 minutes later?
   (e) After how long will the activity fall to $1s^{-1}$

Solution:

a) 32g of element *X* contains $6.02x10^{23}$ atoms

1g of element X will contain $\frac{6.02\times10^{23}}{32}$ atoms

$\therefore$ 1.0x10^{-6}g of element x will contain $\frac{6.02\times10^{23}}{32}\times1.0\times10^{-6}$ atoms

2. The activity of a particular radioactive nuclide falls from 1.0x10^{11} to 2.0x10^{10} in 10.0hours. Calculate the half life of the nuclide.

3. Calculate the activity of a 2.0μg of $^{64}_{26}\mathrm{Cu}$ given that the half life of copper is 13.0 hours and Avogadro's number is 6.02x10^{23} mol^{-I}

Solution:

4. The activity of a mass of $^{14}_{6}\mathrm{C}$ is 5.0x10^{8}Bq and its half life is 5570 years. Estimate the number of $^{14}_{6}\mathrm{C}$ nuclei present in the sample (take *Ln*2 = 0.693).

5. A sample of a radioactive waste has a half life of 80 years. How long will it take for its activity to fall to 20% of its current value?

6. An isotope of Krypton $^{87}_{36}\mathrm{Kr}$ has a half life of 78 minutes. Calculate the activity of 10μg of $^{87}_{36}\mathrm{Kr}$ (N_A = 6.0x10^{23} mol^{-I})

7. A nucleus decays to produce alpha particles of mass 4.00U and daughter of mass 204 releasing 5.21MeVin the process.
   (a) Calculate the K.E of each of the products
   (b) Find the ratio of the K.E of the alpha particle to the K.E of the daughter nucleus

8. $^{210}_{84}\mathrm{P}_o$ decays to Pb – 206 by emission of alpha particles.
   i) Write down the symbolic equation for the reaction
   ii) Calculate the energy in MeV released in each disintegration
   iii) Calculate the K.E of the alpha particle emitted.

$$\begin{aligned}\text{Mass of : } {}^{210}_{84}\mathrm{P}_o &= 209.936730\mathrm{U}\\ {}^{220}_{86}\mathrm{Pb} &= 205.929421\mathrm{U}\\ {}^{4}_{2}\mathrm{He} &= 4.00504\mathrm{U}\\ 1U &= 931\mathrm{MeV}\end{aligned}$$

9. (a) Calculate the energy released during the decay of $^{220}_{86}\mathrm{Rn}$ nucleus into $^{216}_{84}\mathrm{P}_o$ and alpha particle.

(b) Also find the energy of each product

Mass of: $^{216}_{84}P_o = 215.964176U$, $^{220}_{86}Rn = 219.964176U$, $^{4}_{2}He = 4.001566U$, $1U = 931MeV$ *Ans: 6.35MeV*

10. A radioactive source contains 1.0μg of plutonium of mass 239. If the source emits 2300 alpha particles per second, calculate the half life of plutonium.

11. When Uranium $^{238}_{92}U$ decays, the end product is $^{206}_{86}Pb$. The half life is 1.4×10^{17}s. Suppose a rock sample contains $^{206}_{86}Pb$ and $^{238}_{92}U$ in the ratio 1:5 by weight. Calculate;

(a) The number of atoms in the 1.0μg of the rock sample.

(b) The age of the rock (assume the decay law: $N = N_0e^{-\lambda t}$)

$$Ratio \Rightarrow {}^{206}_{86}Pb : {}^{238}_{92}U = 1:5$$

$$But\ mass\ of\ the\ {}^{206}_{86}Pb\ in\ the\ sample = \frac{1}{6}\times1.0g = \frac{1}{6}g$$

$$206g\ of\ {}^{206}_{86}Pb\ contains\ 6.0\times10^{23} atoms$$

$$\therefore \frac{1}{6}g\ of\ {}^{206}_{86}Pb\ will\ contain\ \frac{6.0\times10^{23}}{6\times206} atoms$$

$$= 4.85\times10^{20} atoms$$

(c) First obtain the number of atoms of $^{238}_{92}U$ *present in the sample (2.1 x 10^{21} atoms)*

Initially the total number of atoms present in the sample

$\Rightarrow$ **N₀** = 4.85 x 10^{20} +2.1 x 10^{21} = 2.585x10^{21} atoms

N = 2.1x10^{21} (*i.e. after time t, the Uranium atoms left*)

From the decay law: $N = N_0e^{-\lambda t}$

Time t = 4.24x10^{16} seconds

1. (a) what is meant by the following terms *radioactivity, half-life, and decay constant*

(b) The activity of a sample of dead wood is 10 counts per minute while activity of a living plant is 19counts/min. if the half life of C-14 is 5500yrs, find the age of the sample of wood (assume that $A = A_0e^{-\lambda t}$)

2. *A sample of radioactive material contains 10^8 atoms. The half-life of the material is 2 days. Calculate*

(i) The fraction remaining after 5 days

(ii) The activity of the sample after 5 days.

4. Find the activity of 1g sample of radium $^{226}_{88}Ra$ whose half life is 1620 yrs

5. The nuclide $^{124}_{55}Cs$ decays with a half life is 30.8seconds. If we have $7.5\mu g$ initially,

(a) How many nuclei are present initially? **(ANS** $3.64\times10^{16}\,nuclei$)

(b) How many nuclei are present 2.0 min later? $\left(2.45\times10^{15}\,nuclei\right)$

(c) What is the activity 2.0 min later? $\left(5.51\times10^{13}\,Bq\right)$

(d) After how long will the activity fall to less than one per second? $\left(1526s\right)$

Nuclear fission and nuclear fusion

Nuclear Fission: Is the splitting of the heavy nucleus into light nuclei.

Fission usually occurs in several heavier nuclei when an uncharged particle penetrates deeply into the positively charged nuclei. e.g.

$$^{235}_{92}U \ + \ ^{1}_{0}n \ \rightarrow \ ^{144}_{56}Ba \ + \ ^{92}_{36}Kr \ + \ 2^{1}_{0}n$$

The heavy nucleus splits into lighter nuclei of higher binding energy per nucleon. The mass deficiency (loss in mass) which results is accounted for by the energy released in accordance to Einstein’s mass-energy relation. In most nuclear fission reactions, neutrons are used to induce a reaction because of being neutral, they can penetrate the nucleus.

Nuclear Fusion: is the union of light nuclei to form a heavy nucleus. e.g. $^{2}_{1}H \ + \ ^{2}_{1}H \ \rightarrow \ ^{3}_{2}He \ + \ ^{1}_{0}n$

The sun contains a considerable amount of hydrogen. It is believed that the energy of the sun is due to nuclear fusion of the hydrogen atoms to form a helium isotope as in the equation above.

Note: In both processes;

1) Energy is released
2) They both result in an increase in the Binding energy per nucleon
3) Both take place in the nucleus of an atom.

Examples:

1. Consider the fission reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}\mathrm{Mo} + ^{139}_{57}\mathrm{La} + 2^{1}_{0}n + 7^{0}_{-1}e$$

Calculate the total energy in Joules released by 1kilomole of $^{235}_{92}U$ undergoing fission by the above reaction, neglecting the masses of the electrons.

$massof :^{235}_{92}U = 235.044\mathrm{U}$

$^{1}_{0}n = 1.009U$

$^{144}_{56} =$

$^{92}_{36}Kr$

<<<<<<<<< THERE ARE SOME MISSING INFORMATION>>>>>>>

Calculate the energy released when 10Kg of $^{235}_{92}U$ *undergoes fission according to the equation.*

$$^{235}_{92}U + ^{1}_{0}\cap \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}\cap$$

Given

Mass of $^{235}_{92}U$ ***= 235.04U***

mass of $^{141}_{56}Ba$ = 140.91U

Mass of $^{92}_{36}Kr$ = 91.91U

Mass of $^{1}_{0}\cap$ = 1.00728U

N_A = 6.02 x 10^{23} mol^{-1}

Solution.

Mass defect = total mass of parent nuclei - total mass Of daughter nuclei (reactants) (products)

= (235.04 + 1.00728)U – (140.91 +91.91 + 3 X 1.00728)U

= 0.20U

In 1Kg of $^{235}_{92}U$ ***there are*** $\frac{6.02\ x\ 10^{23}}{235x10^{-3}}\ atoms$

In 10 Kg of $^{235}_{92}U$ ***there*** are $\frac{6.02\ x\ 10^{23}}{235x10^{-3}}\ x\ 10\ \ atoms$

$= 2.56\ x\ 10^{25}\ \ atoms$

Energy of an atom (MeV) = 0.2 x 931 MeV = 186.2 MeV

For 2.56×10^{25} atoms, the energy released in MeV = $2.56 \, x \, 10^{25}$ x 186.2 MeV

= 4.767×10^{27} MeV