A'LEVEL PHYSICS NOTES

P510/1

MECHANICS

BY

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UGANDA ADVANCED CERTIFICATE OF EDUCATION (UACE)

PHYSICS PAPER 1, P510/1

INTRODUCTION

The paper is the physics studied after Uganda Certificate of Education UCE (O' Level physics)

It is one of the papers out of the three papers of physics and the other two papers are coded as **P510/2** and **P510/3**

The paper consists of three sections as A, B and C

Section A; has normally four questions examined from the general mechanics. The student is required to answer not more than two (2) questions in this section of the paper. This section (mechanics) by syllabus requires about **108** periods in the two years of the study

Section B; has normally three questions examined from thermal properties of matter. It sometimes referred to as thermodynamics or simply heat. Students are normally required to answer not more than two (2) questions from this section of the paper. This by syllabus may require about **96** periods

Section C; has normally three questions examined from modern physics. This may by syllabus require about **56** periods.

In this paper candidates (students) are required to answer a total of five (5) questions including at least one but NOT more than two questions from each section. Each question carries an equal marks of twenty (20 marks); making a total of 100% mark. Time allowed for this paper is 2 ½ hours

Paper two, **P510/2** is a theory paper consisting of four sections A, B, C and D covering Geometric optics, Current Electricity, Magnetism and electromagnetism and Electrostatics

Paper **P510/3** is a practical paper and time allowed for this paper is 3 ¼ hours. Normally three questions are set as follows;

- Question 1 is a compulsory and is set from general mechanics
- Question 2 is optional and is set from optics
- Question 3 is optional and is set from electricity

The student is required to answer only two questions out of the three questions in this paper. The total marks is 67 and the details of this paper will be handled under practical lessons.

MECHANICS

This is a branch of physical science which deals with the action and effects of forces on bodies.

The following are the topics to cover under general mechanics in A' level;

1) Basic physical quantities and Dimensions of the physical quantities.
2) Vectors and Scalars quantities
3) Linear motion (kinematics)
4) Force, momentum and impulse
5) Moments and Centre of Gravity, Stability
6) Solid friction
7) Work, Energy and power
8) Projectiles
9) Simple Harmonic Motion (S.H.M)
10) Mechanical property of matter (Elasticity)
11) Circular Motion and Planetary Motion
12) Hydrostatics
13) Hydrodynamics

The student (candidate) is then advised to study the topics listed above a head of the teachers coverage to have a better mastery of the subject content, that is make **learning notes** from the teachers **class notes** and then make **revision notes** (question – answer summary for every topics covered)

CHAPTER ONE: BASIC PHYSICAL QUANTITIES AND DIMENSIONS OF PHYSICAL QUANTITIES

Fundamental quantities

Are those physical quantities which cannot be expressed in terms of any other quantities other than themselves using any mathematical equation.

Examples of fundamental physical quantities are shown with their S.I units in the table below.

Quantity	S.I Unit	Symbol of S.I Unit
Mass	Kilograms	kg
Time	seconds	s
Length	metres	m
Temperature	Kelvin	K
Current	Ampere	A
Amount of substance	Mole	mol.

Derived physical quantities

These are physical quantities which can be expressed in terms of the fundamental physical quantities.

Examples include;

Area – (length) ^{2}

Volume = (length) ^{3}

Density = $\dfrac{mass}{(length)^3}$

Velocity = $\dfrac{length}{time} = ms^{-1}$

Acceleration = $\dfrac{length}{(time)^2}$

Examples of derived physical quantities are shown with their S.I units in the table below.

Derived physical Quantity		Basic Units	
Name	**Symbol**	**Name**	**Symbol**
Area	A	Square meter	m^2
Volume	V	Cubic meter	m^3
Density	ρ	Kilogram per cubic meter	kgm^{-3}
Velocity	v	Meter per second	ms^{-1}
Acceleration	a	Meter per square second	ms^{-2}
Momentum	p	Kilogram meter per second	$kgms^{-1}$

Exercise 1:

Express the following derived quantities in terms of the fundamental quantities;

(a) Force (b) Pressure (c) work (d) momentum

Dimensions of a physical quantity

Is the way the fundamental quantities are related to a derived physical quantity or are the powers to which the fundamental quantities are raised to in a derived physical quantity.

Symbols of dimension, [ ]

e.g. [Mass] -This means that the dimensions of mass.

[Mass]= M

[Time] = T

[Length] = L

[Area] = L^2 -This means that the dimension of area is 2-in length.

[Volume] = L^3 [Force] = MLT^{-2}

[Density] = ML^{-3} [Pressure] = $MT^{-2}L^{-1}$

[Velocity] =LT^{-1} [Work] = ML^2T^{-2}

[Acceleration] =LT^{-2} [Power] = ML^2T^{-3}

Exercise 2:

Find the dimensions of the following derived quantities in terms of M, L, and T;

(a) Density (b) pressure (c) power (d) momentum

Physical derived Quantities without units are called dimensionless physical quantities and examples include;

(i) Relative density
(ii) Refractive index
(iii) Geometrical ratios
(iv) Mechanical advantage
(v) Natural numbers

Exercise 3:

Which of the following quantities are dimensionless quantities?

Weight, velocity ratio, logarithmic numbers, energy, efficiency, coefficient of friction,

Application of dimensions

(i) Checking for the correctness of the equation.

An equation is correct when it is dimensionally consistent i.e. when dimension on the left hand side (L.H.S) are equal to dimensions on the right hand side (R.H.S.)

Example:

Prove that the following equations are dimensionally consistent

(i) $F = \frac{mv^2}{r}$ Where F = force, m =mass, V = velocity, r = radius

[L.H.S] = [F] = MLT^{-2}

[R.H.S] = $\frac{[M].[V]^2}{[r]} = \frac{M\left(LT^{-1}\right)^2}{L} = \frac{ML^2T^{-2}}{L} = MLT^{-2}$

Since [L.H.S] = [R.H.S], then the equation is dimensionally consistent!

(ii) $S = ut+½at^2$

[L.H.S] = [S] = L

[R.H.S] = $[ut+½at^2]$ = [U] [t] +½ [a] $[t^2]$

$\frac{L}{T} xT + ½\text{x}\frac{\text{L}}{\text{T}^2} xT^2$

$\frac{L}{1} + \frac{L}{2}$

$= 3/2\, L$

Since [L.H.S] = [R.H.S], then the equation is dimensionally consistent!

In the above example 3/2 is just a number so it is not a dimension. You have to consider the power on L.

Note. All current equations are dimensionally consistent but not all dimensionally consistent equations are correct.

Example

V=u+2at.

[v] =LT^{-1}

[U+2at] = $\frac{L}{T} \times 2.\frac{L}{T^2} \times T$

$$\frac{L}{T} + \frac{2L}{T}$$
$$= 3LT^{-1}$$

Dimensionally consistent, but it is a wrong equation.

2. Derive the equation

Example 1: Given that the pressure exerted by the liquid in a container depends on:

(i) Depth (h) of the liquid

(ii) Density of the liquid (ρ)

(iii) Acceleration due to gravity (g).

Use the method of dimension to determine the expression for pressure

Where K is a dimensionless constant.

$$p = kh^x p^y g^z$$
$$[p] = [h]^x [p]^y [g]^z$$
$$ML^{-1}T^{-2} = L^X .(ML^{-3})^Y .(LT^{-2})^Z$$
$$ML^{-1}T^{-2} = L^{X-3Y+X} .M^Y .T^{-2Z}$$

Comparing powers

<u>For M:</u>

$$M^1 = M^y$$
$$y = 1$$

<u>For T:</u>

$T^{-2} = T^{-2Z}$

$-2 = -2Z$

$Z = 1$

For L:

$L^{-1} = L^{X-3Y+Z}$

$-1 = x - 3 + 1$

$x = 1$

Since x $=1$, $y = 1$, $z = 1$, then

$P = Kh\rho g$, for K=1, then the equation is written as;

$P = h\rho g$

Example 2:

Given the period of oscillation (Ψ) of a pendulum bob is according to the equation

$\mathbf{\Psi} = k(l^x \times g^y \times m^z)$. Where *l* is the length of a pendulum, *m* is the mass of the pendulum bob and *g* is the acceleration due to gravity. Find the values of *x, y and z*

Solution:

$\Psi = kl^x.g^y.m^z$

$[\Psi] = k\,[L]^x\,[g]^y\,[m]^z$

$T = k\,L^x\,(LT^{-2})^y\,M^{z.}$

$T = k\,L^{x+y}\,T^{-2y}\,M^z$

Comparing powers

For T:

$T = T^{-2y}$, $y= {}^{-}½$

For M:

$M^{0} = M^z$, $Z = 0$

For M:

$L^0 = L^{x+y}$, $x + y = 0$

But $y= {}^{-}½$

Hence $x = ½$

$\Psi = \underline{K\,L^{½} g^{-½} M^0}$

Note: The method of dimensions does not provide the method for finding the constant k in the above two examples!

Exercise 4:

1. Find the values of x, y and z in the equation below:

$F = \rho^x V^y a^z$. Where F is the force, $\boldsymbol{\rho}$ is density, ***V*** is Volume and ***a*** is acceleration due to gravity.

2. Find the values of x, y and z in the equation below:

$F = k\eta^x \nu^y a^z$. Where F is the force, $\boldsymbol{\eta}$ is coefficient of viscosity, ν is velocity and **a** is radius. $[\eta] = ML^{-1}T^{-1}$

3. Assuming the frequency (F) of a uniform stretched wire depends only on the mass per unit length (μ), the length of wire vibrating (L), the tension (T) of the stretching wire,
Find the relationship between these quantities.

CHAPTER TWO: VECTOR & SCALER QUANTITIES

Definition: Vector quantities are physical quantities that have both magnitude and direction.

Examples of Vector quantities

Acceleration, velocity, displacement, weight, magnetic flux density, electric field intensity, moments, force, momentum etc.

Definition: Scalar quantities are physical quantities with only magnitude but no direction.

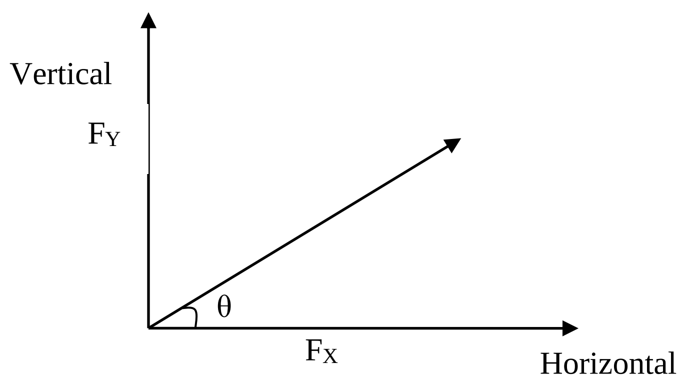
Examples of scalar quantities

Speed, distance, time, mass, volume, pressure, electric potential etc.

COMPOSITION AND RESOLUTION OF VECTORS

Resolution of vectors is the determination of the orthogonal (at an angle) projection of a vector quantity along a specified direction.

Consider a force, F, making an angle, θ with the horizontal as shown in the figure below;



Now we need the projection of F along the horizontal and vertical, these are called the components of the vector along the two directions

$F_X = F\cos\theta$..............(1)

$F_Y = F\sin\theta$...............(2)

Therefore determination of F_Y and F_X is the resolution of F along Y and X directions respectively in the Y-X Cartesian plane.

Generally, a vector $\vec{V}$ is equivalent to a vector $\vec{V}\cos\theta$ along a line making an angle θ, with its own direction and a vector $\vec{V}\sin\theta$ perpendicular to the direction of the first component.

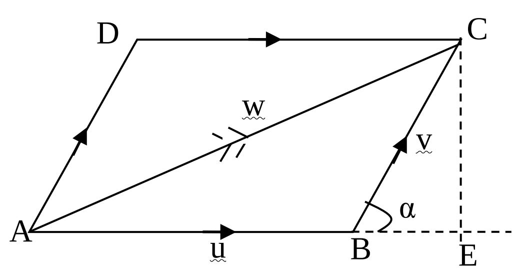
COMPOSITION OF VECTORS

This is done geometrically using the parallelogram rule of addition of vectors unlike scalars which can be added arithmetically. Addition of vectors is thus known as composition of vectors; and it employs the knowledge of mathematics significantly.

<u>Parallelogram law of a vector of vector addition</u>

The parallelogram law states that; *if two vectors at a point are represented in both magnitude and direction by the adjacent sides of a parallelogram drawn from a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point*

Consider the figure below;



$\underset{\sim}{AB} + \underset{\sim}{BC} = \underset{\sim}{AC}$

Suppose a particle is displaced from A to B and then from B to C as shown in the figure. The displacement of the particle from A to B is taken as vector $\overrightarrow{AB}$ and that from B to C as $\overrightarrow{BC}$. The resultant displacement is thus from A to C i.e. $\overrightarrow{AC}$. The vector $\overrightarrow{AC}$ is equivalent to the diagonal of the parallelogram of which vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are the adjacent sides.

Note: vector $\overrightarrow{AC}$ is also equivalent to compounding vectors $\overrightarrow{AD} + \overrightarrow{DC}$

Calculations:

Let $\begin{matrix} \overrightarrow{AB} = \vec{u} \\ \overrightarrow{BC} = \vec{v} \end{matrix}$ in magnitudes

If $\overrightarrow{AD}$ and $\overrightarrow{BC}$ makes angles of α with $\overrightarrow{AB}$, then the resultant $\overrightarrow{AC}$ (whose magnitude is $\underset{\sim}{w}$) is given by;

$w^2 = u^2 + v^2 + 2uv\cos\alpha$..................................(3)

Note: In the above parallelogram figure,

$BE = v\cos\alpha$ and $CE = v\sin\theta$

Equation (3) above which is an expression for the resultant $\overrightarrow{AC}$, derived from Pythagoras theorem as;

$AC^2 = AE^2 + EC^2$

Suppose two vectors $\vec{u}$ and $\vec{v}$ are at right angles (perpendicular) to each other, their resultant w is given by;

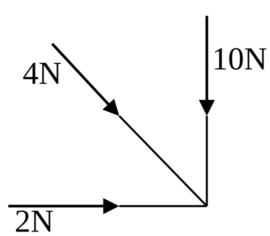
$$w^2 = u^2 + v^2 + 2uv\cos\theta \qquad \textbf{Since } \cos 90^0 = 0$$
$$w^2 = u^2 + v^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

NOTE

- ✓ If $\underset{\rightarrow}{u}$ and $\underset{\rightarrow}{v}$ are forces in equilibrium, then $w^2 = 0$ for two forces that are in equilibrium, their algebraic sum of their resultant components in any two perpendicular direction is zero
- ✓ An alternative of compounding or resolving vectors is by the method of scale drawing.
- ✓ Subtraction of vectors is done similarly by considering the parallelogram law but this time the resultant is given by the short diagonal of the parallelogram.
- ✓ Composition and resolution of vectors are applied to vectors, forces and moments
- ✓ For a given number of vectors, their resultant can be obtained by resolving each of them in two fixed directions **ox** and **oy** at right angles, the components are added in each direction ox and oy to give a single vector along ox and a single vector along oy. Finally these two perpendicular vectors can be compounded in a single vector

Examples

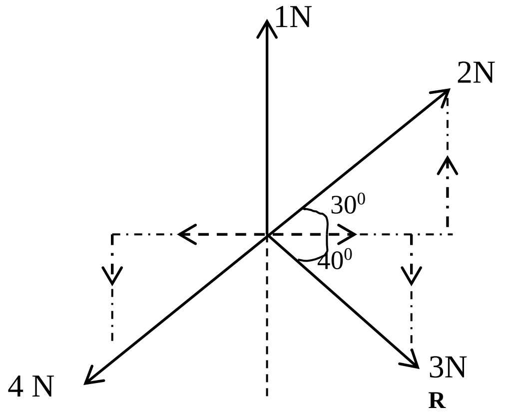
1. Three forces of 2N, 4N and 10N act at a point as shown in the figure. Find their resultant force



Solution

Along x direction;

1. Find the resultant force



$F_x = 2\cos30^0 + 3\cos40^0 - 4\cos60^0$.

$F_x = 2\frac{\sqrt{3}}{2} + 3\cos 40 - 4x\frac{1}{2}$ $= 2.03N$

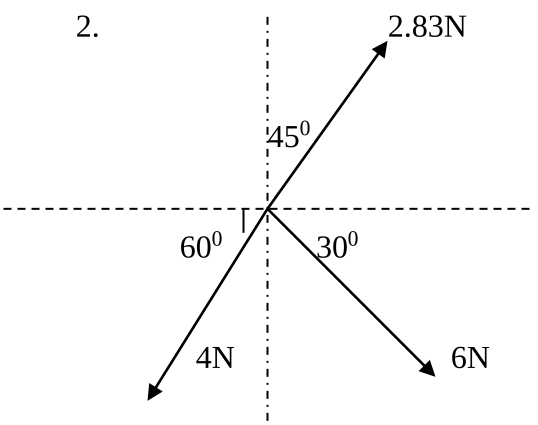
$F_y = 2\sin30^0 – 3\sin40^0 – 4\sin60^0 +1$

$F_y = 2.\frac{1}{2} - 3\sin 40 - 4.\frac{13}{2} + 1$ $= - 3.39N$

Resultant = $\sqrt{F_x{}^2 + F_y{}^2}$ $= \sqrt{2.03^2 + 3.39^2}$

$= 3.95N$

2.



Forces of 2.83N, 4N and 6N act on a particle on Q as shown above. Find the resultant force on the particle

We need the equation in vector matrix for the resultant force acting on the body

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2.38\cos 45 \\ 2.83\sin 45 \end{pmatrix} + \begin{pmatrix} 6\cos 30^0 \\ -6\sin 30^0 \end{pmatrix} + \begin{pmatrix} -4\cos 60^0 \\ -4\sin 60^0 \end{pmatrix}$$

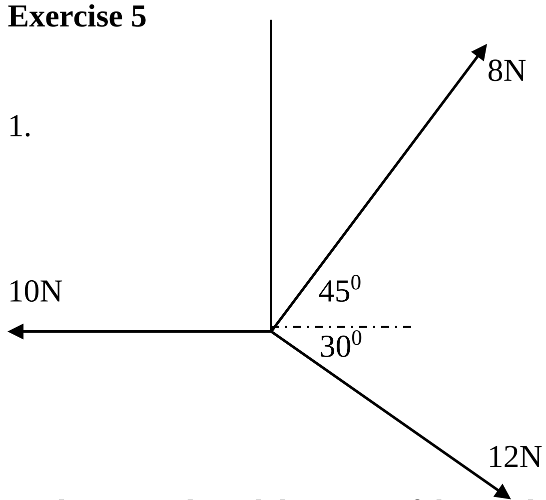
$$= \begin{pmatrix} 2+5.2-2 \\ 2-3-3.46 \end{pmatrix} = \begin{pmatrix} 5.2 \\ -4.46 \end{pmatrix}$$

$$resul\tan t = \sqrt{\ }(5.2)^2 + (4.46)^2$$

$$= 6.85N$$

Exercise 5

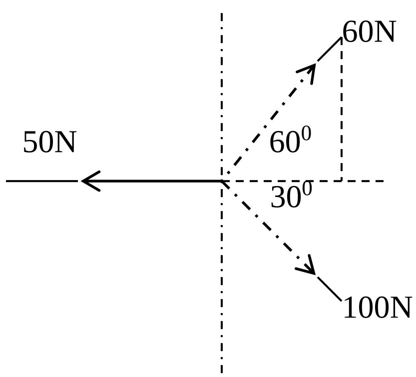
1.



Find magnitude and direction of the resultant.

2) Three forces as shown below act on a body of mass 5.0kg.

Find the acceleration of the body



CHAPTER THREE: UNIFORM MOTION IN A STRAIGHT LINE (KINEMATICS)

Key terms and Concepts

Displacement – Is distance covered in a specified direction.

Speed – Is the rate of change of distance

Velocity – Is the rate of change of displacement

Acceleration – is the rate of change of velocity

Instantaneous Acceleration – Is the acceleration of a moving object at an instant, or is the rate of change of velocity at that instant.

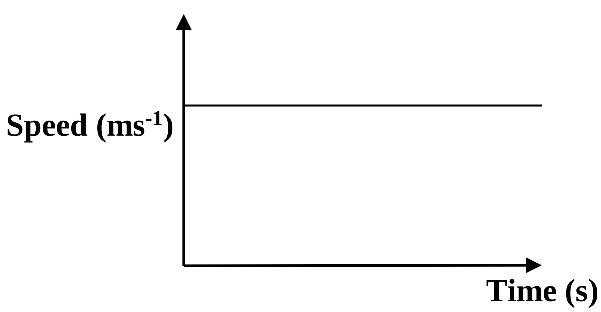
Velocity and acceleration are vector quantities where as speed and distance are scalar quantities

Uniform velocity and Speed motion

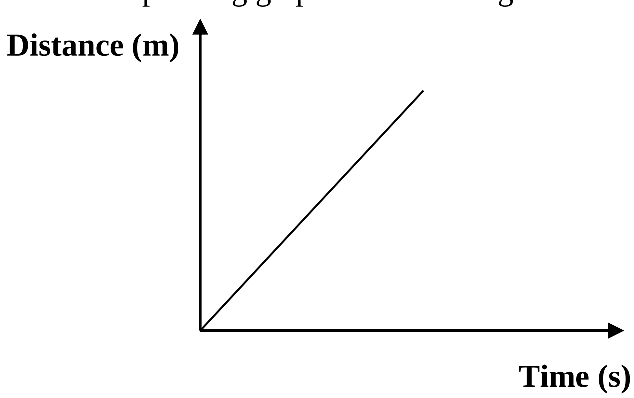
Consider a body moving in a straight line with uniform speed.

During this motion, the body undergoes equal displacement in equal successive time intervals.

The graph of speed against time has the form given below;



The corresponding graph of distance against time is:

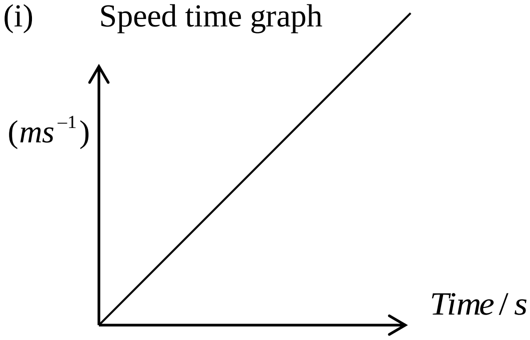


It should be noted that in drawing the graph, it is assumed that the body was at the origin at time t = 0,

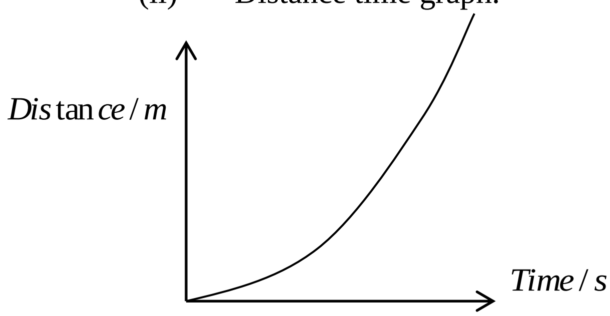
Uniformly accelerated motion

If the velocity changes by equal amounts in equal times, no matter how small the time intervals may be, the acceleration is said to be uniform. The graphs below show uniformly accelerated motion of a body;

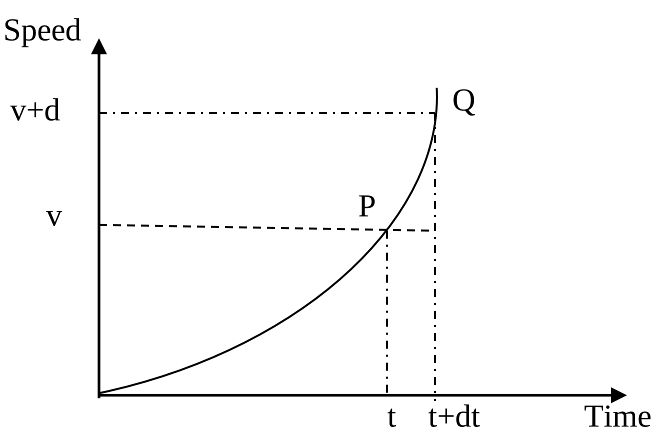
(i) Speed time graph



(ii) Distance time graph.



Suppose a body's speed varies with time. The speed verses time graph might have the form:-



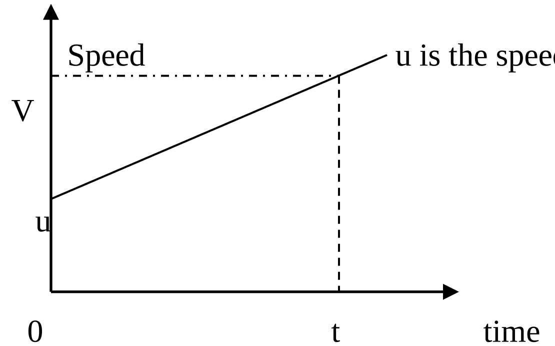
from the graph, we see that the ratio $\frac{\Delta v}{\Delta t}$ is the average acceleration during the time interval Δt and it is equal to the slope of chord PQ

The instantaneous acceleration at time t is therefor given as;

$a = \frac{dv}{dt} = \lim \frac{\Delta v}{\Delta t} = \Delta t \to 0$ Slope of the tangent at the speed-time curve at point P.

The motion of the body is said to be uniform acceleration if acceleration ***a*** is constant.

Thus the speed against time graph for uniformly accelerated motion has the form as shown below;



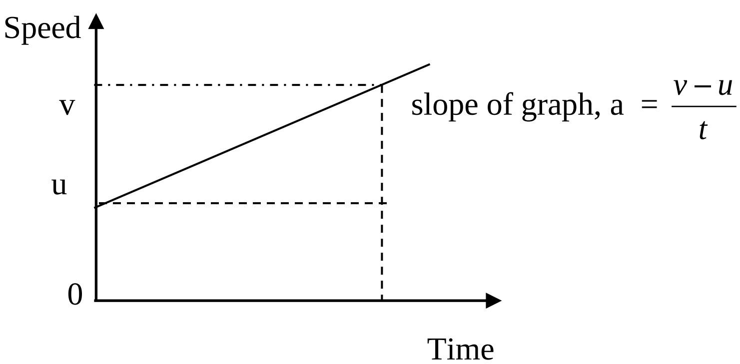
The average acceleration, a $= \frac{v-u}{t}$. In this case, the average acceleration is also the instantaneous acceleration.

From $\frac{v-u}{t} = a$

$$v = u + at$$

Equations of uniformly accelerated motion

The graph of speed against time for uniformly accelerated motion has the form shown in the graph below:-



Thus; $V = u + at \text{.........................}(1)$

The distance travelled, S, in time t is got by finding the area under the speed against time graph.

S = area of the trapezium; $A = \frac{1}{2}(a+b)h$

$$\left(\frac{v+u}{2}\right)t \text{....................................}(i)$$

Replacing v by equation (1) we get

$$S = \left(\frac{u + at + u}{2}\right)t$$

$$S = \left(\frac{2ut + at^2}{2}\right)$$

$$\therefore S = ut + \tfrac{1}{2} at^2$$

$$S = ut + \frac{1}{2}at^2 \text{.........................(2)}$$

Note that this result can be got easily from the relation, distance travelled = average speed x time

$$= \left(\frac{v + u}{2}\right)t$$

Suppose we put t = $\frac{v - u}{a}$ in equation (i), we have;

$$S = \left(\frac{v + u}{2}\right)\left(\frac{v - u}{a}\right) = \frac{v^2 - u^2}{2a}$$

S = $v^2 - u^2 = 2as$

$$\therefore V^2 = u^2 + 2as \text{.........................(3)}$$

Note that this result can be got easily from the relation of distance travelled,

S = average speed x time

S= $\left(\frac{v + u}{2}\right)t$

Suppose we put t = $\frac{v - u}{a}$ in equation (A)

$$S = \left(\frac{v + u}{2}\right)\left(\frac{v - u}{a}\right) = \frac{v^2 - u^2}{2a}$$

$$v^2 - u^2 = 2as$$

$$\therefore v^2 = u^2 + 2as \text{............................(3)}$$

Equations 1, 2, & 3 are the equations of uniformly accelerated motion.

Vertical motion under gravity: Free fall motion

Consider a body falling in a vacuum. Such a body is acted on by the gravitational force alone. The fall is referred to as free fall. In practice, when bodies fall in air, they are acted on by air resistance which will have significant effects on the body's motion if the body's mass is small while the surface area is large, as is the case when a piece of paper is allowed to fall in air.

The acceleration of a freely falling body is constant, and is called the acceleration due to gravity, and is denoted by g. It has a value of $9.81ms^{-2}$ near the poles $9.78ms^{-2}$ at the equator.

Exercise 6

1. Write down the equations of motion for a freely falling body.
2. Suppose a body is projected upwards with initial velocity u. Find the expressions for the time taken to reach the maximum height and also the maximum height attained.

Example:-

1. A ball is thrown vertically upwards with initial speed $20ms^{-1}$. After reaching the maximum height and on the way down it strikes a bird 10m above the ground.

a) How high does the ball rise:-

$V^2 = U^2 + 2as.$ U = $20ms^{-1}$ a = $-9.8ms^{-2}$ V = 0

$0 = (20)^2 + 2(-9.8)s$

$$s = \frac{400}{19.6} = 20.4m$$

b) How fast is the ball moving when it strikes the bird?

$S = (20.4 - 10) = 10.4m$

$V^2 = U^2 + 2as$

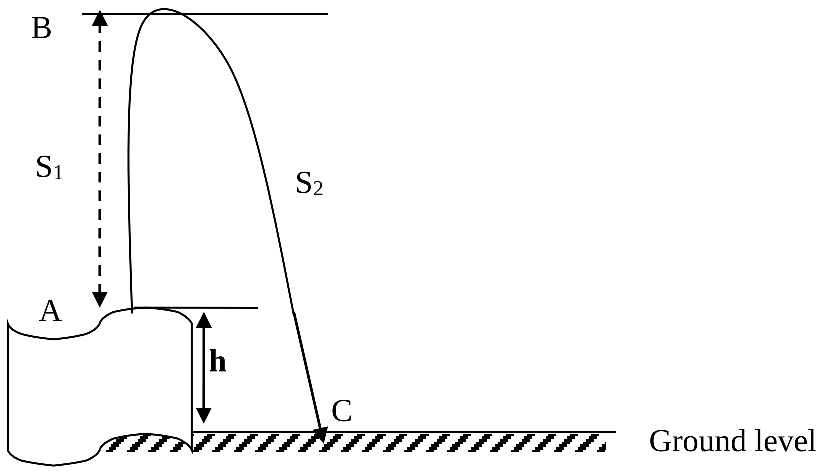
$V^2 = (0)^2 + 2(-9.8)(-10.4)$

$V = \pm 14.28ms^{-1}$

2. A stone is thrown vertically upwards with a speed of $10ms^{-1}$ from a building. If it takes 2.5 seconds to reach the ground, find the height of the building.

Solution

Given initial velocity, u, = $10ms^{-1}$, time for the motion, t, = 2.5 seconds



Between AB, u = 10ms^{-1}, a = -9.81ms^{-2} v = 0, since body is momentarily at rest at point B, which is the maximum height reached, before starting the downward motion

$$v = u + at$$
$$v = 10 - 9.81 \times t$$
$$0 = 10 - 9.81 \times t$$
$$t = \frac{10}{9.81}$$
$$t = 1.02s$$

Distance AB, S, is computed using the Second Equation of motion

$$S = ut + \frac{1}{2}at^2$$
$$S = 10 \times 1.02 - 9.81 \times (1.02)^2$$
$$S = 5.1m$$

Time taken to travel distance BC, $t = 2.5 - 1.02 = 1.48s$

Distance BC, given $u_b = 0$, a = 9.81, t = 1.48s

$$S = ut + \frac{1}{2}at^2$$
$$S = 0 \times 1.48 + 9.81 \times (1.48)^2$$
$$S = 10.7m$$

There the height of the building, **h**, is 10.7 – 5.1 = **5.6m**

Exercise 7

1. A ball is thrown straight upwards with a speed ams^{-1} from a point h m above the ground. Show that time taken to reach the ground is

$$t = \frac{u}{9}\left[1 + \left(1 + \frac{2gh}{u^2}\right)^{1/2}\right]$$

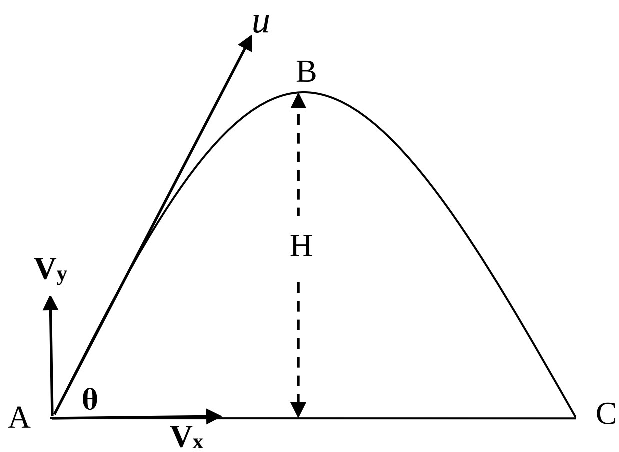
2. A motorist travelling at a constant speed of 50 kmh^{1} passes a motorcyclist just starting off in the same direction. If the motorcyclists maintains a constant acceleration of 2.8ms^{-2}. Calculate; (i) Time taken by motorcyclist to catch up with the motorist (9.9s)

(ii) The speed at which the motorcyclist overtakes the motorist (27.72ms^{-1})

(iii) The distance travelled by the motorcyclists before overtaking (137.2m)

CHAPTER FOUR: PROJECTILES

Consider the motion of an object which is projected with a velocity u at an angle θ to the horizontal as shown in the figure below;



Terms used in projectiles

The angle **θ**, is the angle of projection

The path ABC described by the motion of the particle (body) is called the trajectory

The time taken, T for the trajectory is called the time of flight.

Horizontal motion

Horizontal component of velocity V_X is got by;

$V_X = u_x + a_x t..........(i)$, where V_x, u_x and a_x are the velocity of a body at any time t, initial component of velocity, *u* and horizontal acceleration respectively.

But $u_x = u\cos\theta, a_x = 0.................(ii)$

Combining equations (i) and (ii), we have;

$$V_X = u\cos\theta..........................(1)$$

From the above equation (1) the horizontal velocity is constant throughout motion.

The horizontal distance, *X*, travelled after time, t, is then given as;

$$X = u_x t + \frac{1}{2}a_x t^2.........(i)$$

But $a_x = 0............(ii)$

Combining (i) and (ii), we get;

$$\therefore X = u\cos\theta t..............(2)$$

Vertical motion

$V_y = u_y + a_y t$ where $V_{y,}$ u_y and a_y, are the vertical velocity of a body at any time, *t*, initial velocity component of velocity, *u*, and vertical acceleration respectively.

From the geometry of the trajectory, the vertical component of initial velocity, *u* and acceleration are;

$u_y = u\sin\theta..................(i)$

$a_y = -g.....................(ii)$

Using the first equation of linear motion, $V_y = u_y + a_y t$, we have;

$$V_y = u\sin\theta - gt.........................(3)$$

The vertical displacement, y, is obtained from the second equation of linear motion as below;

$$y = u_y t + \tfrac{1}{2} a_y t^2$$

But $\begin{array}{l} u_y = u\sin\theta..............(i) \\ a_y = -g..................(ii) \end{array}$ and substituting in the second equation of motion, we have;

$$y = u\sin\theta t - \tfrac{1}{2} gt^2 ....(4)$$

Speed, *V*, at any time, *t*, is given by;

$$V = \sqrt{(V_x{}^2 + V_y^2)}.............................(5)$$

The angle, α, the body makes with the horizontal after t is given by

$$\tan\alpha = \frac{Vy}{Vx} = \frac{u\sin\theta - gt}{u\cos\theta}................(6)$$

Maximum height, H

At maximum height, Vy =0

$$Vy^2 = U_y^2 + 2aH$$

$$0 = (U\sin\theta)^2 - 2ghH$$

$$H = \frac{U^2\sin^2\theta}{2g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Time to reach the maximum heights

Using V = u + at

$$0 = Uy + a_y t$$

$$0 = U\sin\theta - gt$$

$$t = U\frac{\sin\theta}{g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Time of flight, T

The time taken by the projectile to move from the point of projection to a point on the plane through the point of projection where the projection lies i.e. time taken to move from A to B.

$$atB,\ \mathrm{y} = 0$$

$$\mathrm{y} = \mathrm{utsin}\theta - \frac{\mathrm{gt}^2}{2}$$

$$0 = 2at\sin\theta - gt^2$$

$$0 = t(2u\sin\theta - gt)$$

$$\text{either } \mathrm{t} = 0 \text{ or } \quad \mathrm{t} = \frac{2\mathrm{u}\sin\theta}{\mathrm{g}}$$

$$\text{Hence } \mathrm{T} = \frac{2\mathrm{u}\sin\theta}{\mathrm{g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Note: Time of flight is twice the time taken to reach height.

Range, R:

It is the distance between the point of projection and a point on the plane through the point of projection where the projectile lands i.e. horizontal distance AB.

$X = ut\cos\theta$..............................(i)

When X = R, t = T=

$T = \dfrac{2u\sin\theta}{g}$.....................$(ii)$

Equation (i) now becomes

$R = uT\cos\theta$....................(iii)

Combining equations (ii) and (iii), we have;

$$R = u \times \frac{2u\sin\theta}{g} \times \cos\theta$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Equation of trajectory

$t = \dfrac{x}{u\cos\theta}$......................$(1)$

$y = ut\sin\theta - \dfrac{gt^2}{2}$..................$(2)$

Substitute equation (1) into equation 2.

$$y = U.\frac{x}{U\cos\theta}.\sin\theta - \frac{g}{2}\frac{x^2}{U^2\cos^2\theta}$$

$$y = \frac{\sin\theta x}{\cos\theta} - \frac{gx^2}{2U^2\cos^2\theta}$$

$$y = (\tan\theta)x - \left(\frac{g}{2u^2Cos^2\theta}\right)x^2$$

$$\mathrm{y} = \mathrm{x}\tan\theta - \tfrac{1}{2}gx^2\,\frac{\sec^2\theta}{u_2}$$

The above equation is in the form y= Ax – Bx^2, where A and B are constants which is an equation of a parabola. Therefore, the trajectory is a parabola.

Note: For any given initial speed, the range is maximum when sin ϴ =1 or ϴ = 45^0

$R_{max} = \frac{U^2}{g}$ *(Prove it !!!!)*

Example

1. Prove that the time of flight T and the horizontal range R, of a projectile are connected by the equation. $gT^2 = 2R\tan\alpha$

Where α is the angle of projection

From equations (9) and (10)

$Tg = 2U\sin\alpha$ …. (a),

$Rg = 2U^2\sin\alpha\cos\alpha$ ……. (b)

Eqn $(a)^2$ ÷ eqn (b)

$$\frac{(Tg)^2}{Rg} = \frac{4U^2\sin^2\alpha}{2U^2\sin\alpha\cos\alpha}$$

$$\frac{T^2 g}{R} = \frac{2\sin\alpha}{\cos\alpha}$$

Hence $T^2 g = 2R\tan\alpha$

2. Two footballers, 120m apart, stand facing each other. One of them kicks a ball from the ground such that the ball takes off at a velocity of $30ms^{-1}$ at 38^0 to the horizontal. Find the speed at which the second footballer must run towards the first footballer in order to trap the ball as it touches the ground, if he starts running at the instant the ball is kicked.

For the first footballer, the time the ball takes to touch the ground is

c) $T = \frac{2u\sin\theta}{g}$

$= \frac{2x30\times\sin 38}{9.8}$

$= 3.78s$

$$R = \frac{u^2\sin 2\alpha}{g}$$

$$R = \frac{30^2\times\sin 76}{9.8}$$

$$R = 89.1$$

The time taken by the second footballer to reach the ball is 3.78s.

The distance travelled by the second footballer is s = 120 – 89.1 = 30.9m

Therefore, the speed of the second footballer distance / time = 30.9/3.78 = 8.2ms^{-1}

3. A projectile is fired from ground level with a velocity of 500ms^{-1}, 30^0 to the horizontal. Find the horizontal range, the greatest height to which it rises and time taken to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range?

$$u = 500ms^{-1} \qquad \alpha = 30^0$$

(i) $$\text{Range} = \frac{u^2 \sin 2\alpha}{g}$$

$$= \frac{500^5}{9.81}\sin(2x30)$$

$$= \underline{\underline{22069.96m}}$$

(ii) $$H = \frac{u^2 \sin^2 \alpha}{2g}$$

$$= \frac{500^2(\sin 30)^2}{2 \times 9.81}$$

= 3185.5m

(iii) Time taken to reach the greatest height.

$$T = \frac{u \sin \alpha}{g}$$

T = (500sin30)/9.81 = 25.5s

(b) $$U_{\min} = (Rg)^{1/2}$$

$$(22069.96 \times 9.81)^{1/2}$$

$$\underline{\underline{465.3ms^{-1}}}$$

Exercise 8:

(1) A body is thrown from the top of a tower 30.4m high with a velocity of $24ms^{-1}$ at an elevation of $3\ 0^0$ above the horizontal. Find the horizontal distance from the roof of the tower of the point where it hits the ground.

(2) A body is projected at such an angle that the horizontal range is three times the greatest height. Given that the range of projection is 400m, find the necessary velocity of projection and angle of projection.

(3) A projectile fired at an angle of 60^0 above the horizontal strikes a building 30m away at a point 15m above the point of projection. (i) Find the speed of projection.
(ii) Find the velocity of the projectile when it strikes the building.

4. An object *P* is projected upwards from a height of 60m above the ground with a velocity of $20ms^{-1}$ at 30^0 to the horizontal. At the same time, an object *Q* is projected from the ground upwards towards *P* at 30^0 to the horizontal. *P* and *Q* collide at a height 60m above the ground while they are both moving downwards. Find,

(i) The speed of projection of Q.
(ii) The horizontal distance between the points of projection.
(iii) The kinetic energy of *P* just before the collision with Q if the as of *P* is 0.5 kg.

CHAPTER FIVE: NEWTON'S LAWS OF MOTION

Law 1. A body continues in its state of rest or of uniform motion unless it is acted on by an external force. The 1st law is sometimes called the law of inertia

Inertia is the reluctance of a body to start moving if it is at rest, or to stop if it is already moving.

Inertia of the body increases with mass. The effect of inertia can be observed by passengers in a bus. There is a forward jerk when the vehicle stops and a backward jerk when the car starts motion.

Linear momentum of the body is the product of its mass and its velocity

$\therefore P = mv \qquad \text{units of P} = \text{kgms}^{-1}$

$[p] = [m].[v]$

$= MLT^{-1}$

Law 2. The rate of change of momentum is directly proportional to the applied resultant force and it takes place in the direction of the force.

$$\frac{dp}{dt} \propto F.$$

F= $k\frac{dp}{dt}$, where K is a constant. But P = mv

$$\therefore \text{F} = \frac{\text{kd}}{\text{dt}}(mv)$$

If m is constant :-

$$\text{F} = \text{km}\,\frac{\text{dv}}{\text{dt}}$$

$$\text{But}\ \frac{\text{dv}}{\text{dt}} = a$$

$$\therefore F = kma$$

A force of 1N acting as a mass of 1kg gives the mass an acceleration of 1ms^{-2}

If F= IN, m= 1kg, $a=1\text{ms}^{-2}$. Substituting the values in the equation we have;

$\therefore 1 = k \times 1 \times 1$

$but k = 1$

$\therefore F = ma = m\dfrac{dv}{dt}$

Law 3. Action and reaction are equal but opposite e.g. when two objects interact with each other the force exerted by the 1st body on the second body is equal but opposite to the force exerted by the 2nd body on the 1st body.

Example

1. A block of mass 2kg is pushed along a table with constant velocity by force of 5N. When the push is increased to 9N, what is the resultant force and acceleration?

$\text{Resultant force} F = 9 - 5 = 4N$

$\text{But } F = ma$

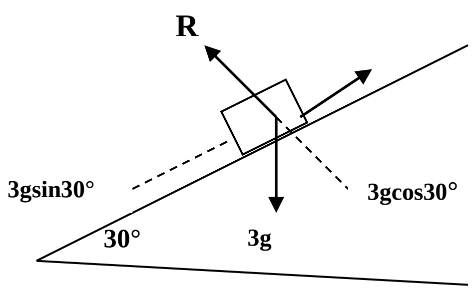
$4 = 2a$

$a = 2ms^{-2}$

2. A body of 3kg slides down a plane which is inclined at 30^0 to the horizontal. Find the acceleration of the body if

(a) The plane is smooth

(b) There is a frictional resistance of 9N.



R is the normal reaction

a)

$F = ma$

$3g\sin 30^0 = 3a.$

$a = 4.9sm^{-1}$

$F = ma$

b) $F = 3g\sin 30^0 - 9 = 3a$

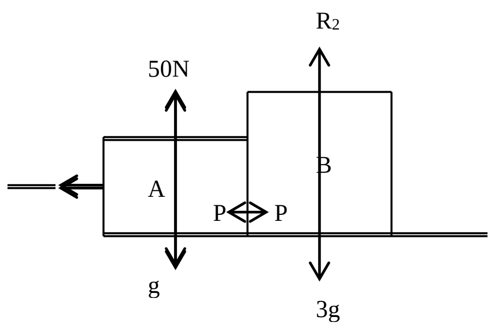
$a = 1.9ms^{-1}$

Note friction force acts in the opposite direction of motion.

3. Two blocks, A of mass 1kg and B of mass 3kg, are side by side and with contact with each other. They are pushed along the smooth flow under the action of a constant force 50N applied to A. Find

i) The acceleration of the blocks

ii) The force exerted on B by A.



$F = ma$

$50 = (1+3)a$

$a = 12.5ms^{-1}$

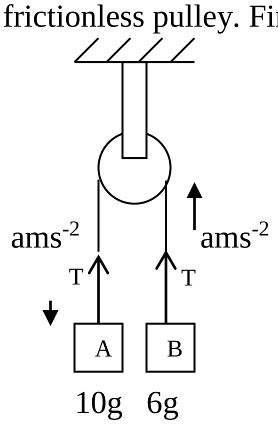
Using A

$50 - p = (1 \times 12.5)$

$p = 50 - 12.5$

$p = 37.5N$

4. A light cord connects 2 objects of masses 10kg and 6kg respectively over a light frictionless pulley. Find the acceleration and tension in the cord



Body A

$10g - T = 10a$ ………… (i)

Body B

$T - 6g = 6a$…………. (ii)

Eqn(i) – eqn (ii)

$4g = 16a$

$a = (1/4)g = 2.45ms^{-2}$

Acceleration, $a = 2.45ms^{-2}$

From eqn (ii) $T=6x2.45 +6x9.81 = 73.6N$

Exercise 8

1. The car of mass 1000kg tows a caravan of mass 600kg up a road which uses 1 metre vertically for every 20 metres of its length. There constant frictional resistance of 200N and 100N to the motion of the car and caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine on constant driving force.

Find

(i) The driving force.

(b) The tension in the tow bar.

2. A rectangular block of mass 10 kg is pulled from rest along a smooth inclined plane by a light inelastic string which passes over a light frictionless pulley and carries a mass of 20kg. The inclined plane makes an angle of 30^0 with the horizontal.

Determine

(i) The acceleration of the block

(ii) The tension in the string

(iii) The K.E of the block when it has moved 2m along the inclined plane.

Impulse

The product of the net force and the time interval during which the force acts is called the impulse

If a steady force F acting on a body of mass in increases the velocity of the body from u to v in the time Δt, the average acceleration

$$\vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}$$

From Newton's second law:

$\vec{F} = m\vec{a}$

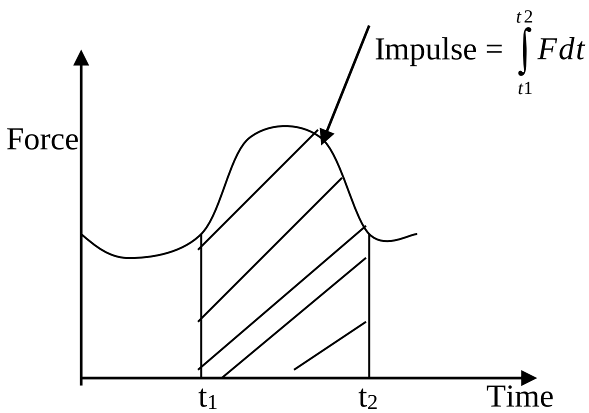
$\vec{F} = m\left(\frac{\vec{v} - \vec{u}}{\Delta t}\right)$

$\vec{F}\,\Delta t = \rightarrow\rightarrow = m(\vec{v} - \vec{u})$

In general, impulse = $\int_{t1}^{t2} F dt$

Where $\vec{v}_1$ and $\vec{v}_2$ the velocities at are times t_1 and t_2

Impulse is the area under the force time curve.



Impulse is a vector quantity.

Units of impulse: NS

[Impulse] = $MLT^{-2}xT$

= MLT^{-1}

Example:

A tennis ball has a mass of 0.07kg. It approaches a racket with a speed of $5ms^{-1}$ and bounces off and returns the way it came from with a speed of $4ms^{-1}$. The ball is in contact with the racket for 0.2 seconds. Calculate:

i) The impulse given to the ball.

ii) The average force exerted on the ball by the racket

i) Impulse = Ft

$$Ft = m(v-u)$$
$$= 0.07(-4-5)$$
$$= 0.07x - 9$$
$$= -0.63Nm$$
$$F = m\left(\frac{v-u}{t}\right)$$
$$= \frac{0.63}{0.2}$$
$$= 3.15N$$

COLLISIONS

Principle of conservation of linear momentum.

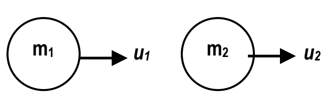
When two or more bodies collide, the total momentum of the system is conserved provided there is no external force acting on the system.

Proof

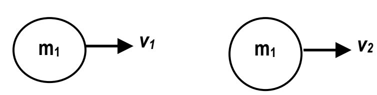
Consider a body of mass m_1 moving with a velocity u_1 to the right. Suppose the body makes a head on collision with another body of mass m_2 moving with velocity u_2 in the same direction.

Let v_1 and v_2 be the velocities of the 2 bodies respectively after collision

Before collision



After collision



Let F_1 be the force exerted on m_2 by m_1 and F_2 the force exerted on m_1 by m_2.

Using Newton's 2nd law.

$$F_1 = M_1\left(\frac{v_1 - u_1}{t}\right), F_2 = m_2\left(\frac{v_2 u_2}{t}\right)$$ t is the time of collision

Using Newton's third law

$$F_1 = -F_2$$

$$m_1\left(\frac{v_1 - u_1}{t}\right) = -m_2\left(\frac{v_2 - u_2}{t}\right)$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

Hence: total momentum before collision = total momentum after collision.

Types of collision

There are three types of collision

- Elastic collision
- Inelastic collision
- Perfectly inelastic collision

Elastic	**Inelastic**	**Perfectly inelastic**
Momentum conserved	Momentum conserved	Momentum conserved
Kinetic energy is conserved	Kinetic energy not conserved	Kinetic energy not conserved
		After collision the particles move together

Elastic collision

Momentum is conserved

$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$

$m_1 (u_1-v_1) = m_2 (v_2-u_2)$ ………………… (i)

Kinetic energy is conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1 + \frac{1}{2}m_2v_2^2$$

$$m_1\left(u_1^2 - v_1^2\right) = m_2\left(v_2^2 - u_2^2\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Diving Equation (i) by (ii), we have;

$$\frac{m_1(u_1 - v_1)}{m_1(u_1^2 - v_1^2)} = \frac{m_2(v_2 - u_2)}{m_2(v_2^2 - u_2^2)}$$

$$\frac{u_1 - v_1}{(u_1 + v_1)(u_1 - v_1)} = \frac{v_2 - u_2}{(v_2 + u_2)(v_2 - u_2)}$$

$$\frac{1}{u_1 + v_1} = \frac{1}{v_2 + u_2}$$

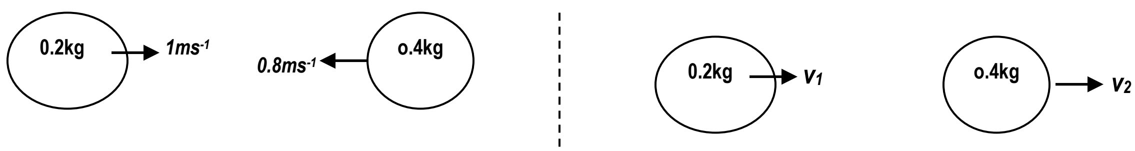
$$u_1 + v_1 = v_2 + u_2$$

$$(u_1 - u_2) = (v_2 - v_2)$$

$$\text{OR } (\mathrm{u}_2 - u_1) = (v_2 - v_1)$$

Example

1. A 200g block moves to the right at a speed of 100cms^{-1} and meets a 400g block moving to the left with a speed of 80cms^{-1}. Find the final velocity of each block if the collision is elastic.



$$(v_2 - v_1) = -(-0.8 - 1)$$

$$v_2 - v_1 = 1.8 \ldots\ldots\ldots\ldots(i)$$

Using law of conservation of momentum.

$$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(0.2 \times 1) + (-0.4 \times 0.0) = (0.2)v_1 + (0.4)v_2$$

$$-0.12 = 0.2v_1 + 0.4v_2$$

$$-0.6 = v_1 + 2v_2 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$v_2 - 1.8 + v_1$$

$$-0.6 = \mathrm{v}_1 + 1.8 + v_1$$

$$\mathrm{v}_1 = -1.2ms^{-1}$$

$$\mathrm{v}_2 = 0.6ms^{-1}$$

2. A neutron of mass m makes a head on elastic collision with a stationary atomic nucleus of mass 12m with a velocity u.

Calculate:

i. the fractional decrease in the kinetic energy of the neutron

ii. The velocity of the nucleus after the collision

$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$

$u_2 = 0$

$m_1u_1 \quad = m_1v_1+12m_2v_2$

$m_1 = m,\ m_2 = 12m$

$u_1 = v_1+12v_2$…………… (i)

From conservation of kinetic energy

$v_2-v_1 = u_1-u_2$

$v_2 - v_1 = u_1$………………… (ii)

From (i) and (ii)

$v_2-v_1 = v_1+12v_2$

$v_2-12v_2 = v_1+v_1$

$-11v_2 = 2v_1$

$$v_2 = \frac{2}{-11}v_1 \text{.................}(iii)$$

Put (iii) in (ii)

$$\frac{-2}{11}v_1\ \frac{v_1}{1} = u_1$$

$$\frac{-2v_1 - 11v_1}{11} = u_1$$

$$\frac{-13v_1}{11} = u_1$$

$$-13v_1 = 11u.$$

$$v_1 = \frac{11u}{-13}$$

$$v_1 = -0.85u.$$

$$\text{Fractional decrease} = \frac{\frac{mu^2}{2} - Kineticenergy\ \text{after collision}}{\frac{\text{mu}^2}{2}}$$

Kinetic energy after collision

$= \frac{1}{2}mv^2$

$= \frac{1}{2}.m.\left(\frac{11v}{13}\right)^2$

$= \frac{m}{2}.\frac{121v^2}{169}$

$\frac{mv^2 121}{338}$

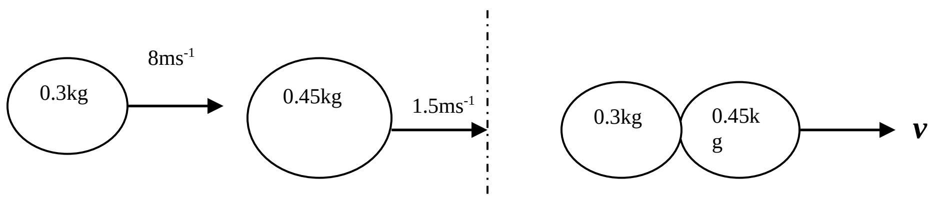
$$\therefore \frac{\frac{mu^2}{2} - \frac{mv^2 121}{338}}{\frac{mu^2}{2}}$$

$$= \frac{169 - 121}{169}$$

$$= 0.28$$

$$= 7/25$$

3. A bullet of mass 300g travelling horizontally at a speed of 8ms^{-1} hits a body of mass 450g moving in the same direction as the bullet at 1.5ms^{-1}. The bullet and the body move together after collision. Find the loss in kinetic energy.



$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.3 \times 8) + (1.5 \times 0.45) = (0.3 + 0.45)v$

$v = 3.075/0.75$

$v = 4.1ms^{-1}$

loss in kinetic energy : $\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$

$\left(\frac{1}{2} \times 0.3 \times 64 + \frac{1}{2} \times 0.45 \times 2.25\right) - \frac{1}{2}(0.75)16.81$

$10.12 - 6.30$

$3.82\, joules$

Exercise

1. An object *A* of mass *m* moving with a velocity of *10ms^{-1}* collides with a stationery object *B* at equal mass *m*. After collision, A moves with a velocity U at an angle of 30^0 to its initial direction and B moves with a velocity V at an angle of 90^0 to the direction U.

   i) Calculate the velocities U and V (U = $5\sqrt{3}$ ms^{-1}, V = 5ms^{-1})

   ii) Determine whether the collision was elastic or not. (Kinetic energy before collision = kinetic energy after collision = 50m, hence collision is elastic)

2. A body of mass 5.0kg is moving with a velocity 2.0ms^{-1} to the right. It collides with a body of mass 3.0 kg moving with a velocity of 2.0ms^{-1} to the left. If the collision is head-on and elastic, determine the velocities of the two bodies after collision. (-1.0 ms^{-1}, 3.0ms^{-1})

3. A car of mass 1000kg travelling at uniform velocity of 20ms-1, collides perfectly inelastically with a stationary car mass 1500kg. Calculate the loss in kinetic energy of the cars as a result of the collision. ($1.2x10^5$J)

CHAPTER SIX: SOLID FRICTION

Friction is a form of energy that opposes motion of bodies in the universe

There are 2 types of friction i.e.

(i) Static friction

(ii) Kinetic friction / sliding friction

Static friction opposes the tendency of one body sliding over the other. Kinetic opposes the sliding of one body over the other.

Limiting friction is the maximum friction between on two surfaces.

Laws of solid friction

(i) Frictional force between 2 surfaces always oppose their relative motion or attempted motion.

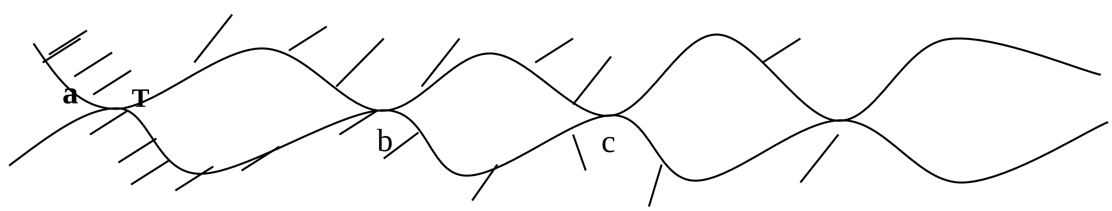
(ii) For given pair of surface in contact, the limiting frictional force is proportional to the normal reaction.

(iii) For two surfaces in contact, the sliding frictional force is proportional to the normal reaction and independent of the relative velocity of these surfaces.

(iv) The frictional force is independent of the area of contact of the given surface provided the normal reaction is constant.

Molecular Theory and the laws of solid friction.

On a microscopic level, even a highly polished surface has bumps and hollow. It follows that when 2 surfaces are put together, the actual area of contact is less than the apparent area of contact



At points of contact like a, b, c, small cold-welded joints are formed by the strong adhesive forces between the molecules in the two surfaces.

These joints have to be broken before one surface can move over the other.

This accounts for law 1.

The actual area of contact is proportional with the normal force (reaction). The frictional force which is determined by the actual area of contact at the joints is expected to be proportional to the normal force.

This accounts for law 1 and 3

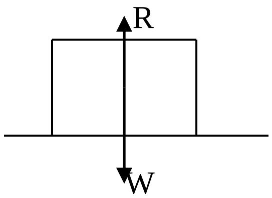
If the apparent area of contact of the body is decreased by turning the body so that it rests on one of the smaller side, the number of contact points is reduced. Since the weight of the body has not altered, there is increased pressure at the contact points and this flattens the bumps so that total contact area and the pressure return to their original values.

Therefore, although the apparent area of contact has been changed, the actual area of contact has not.

This accounts for law 4

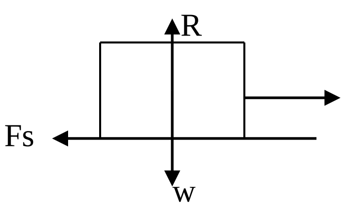
Coefficient of static friction

Consider a block resting on a horizontal surface



The block is in equilibrium under the action of its weight W and normal reaction R.

Suppose a string is fixed to the block and the tension (T) in the string increased gradually, the static frictional force Fs; which posses the tendency of the block to ride over the surface comes in play. In equilibrium Fs=T.



The value F_l of Fs at which the block starts moving is called the limiting frictional force ($0<Fs\leq F_l$)

The ration of the limiting frictional force to the normal reaction is called the co-efficient of static friction μ_s

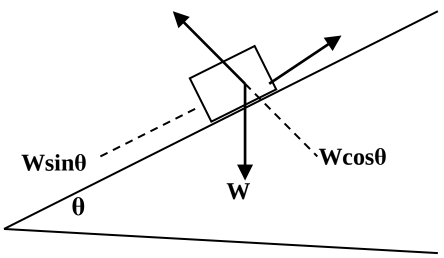
$$\mu_s = \frac{F_l}{R}$$
$$F_s \le F_l = \mu_s R$$
$$F_s \le \mu_s R$$
$$0 \le \mu_s \le 1$$

Measurement of coefficient of static friction, μ_s

Method 1: Using a tilting plane.

A block is placed on a plane and the plane is tilted until when the block begins to slide. The angle of θ of inclination of the plane surface to the horizontal is measured.

The co-efficient of friction is given by $\mu_s = \tan\theta$



When the block is at the point of sliding

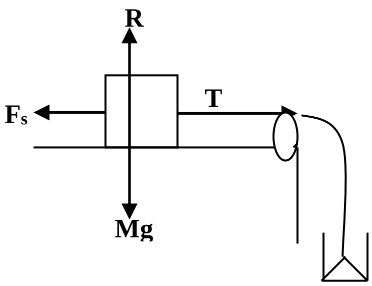
Fs = Wsinθ …………. (i)

R = Wcosθ………….. (ii)

Dividing equation (i) by (ii), we have

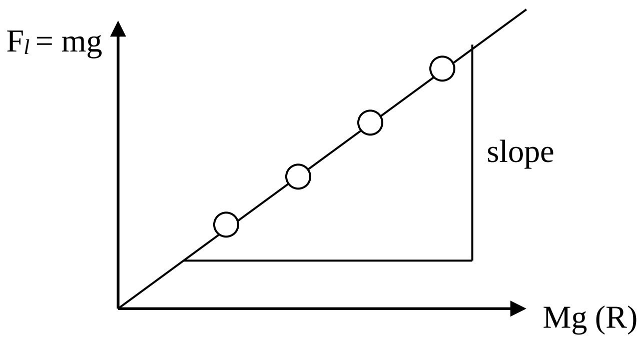
$$\frac{Fr}{R} = \frac{W\sin\theta}{W\cos\theta}$$
$$but \frac{Fr}{R} = \mu_s$$
$$\mu s = \tan\theta$$

Method 2: To determine the co-effienct of static friction.



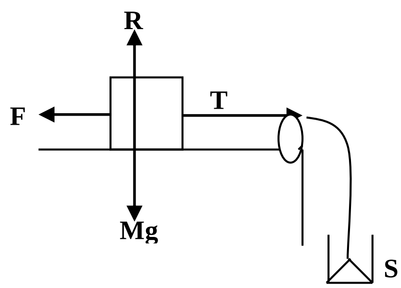
Masses are added to the scale pan until the block just slides. The total mass m of the scale pan and masses added is noted. The prodecures is repeated for different values of R obtained by adding known weights to the block.

A graph of mg against R(Mg) is plotted.



The slope of the graph is μ_s

Co-efficient of kinetic (dynamic friction.



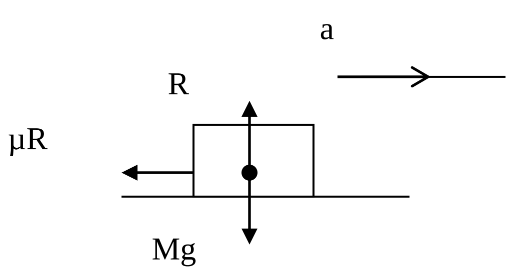
Weights are added to a scale pan S and each time, body is given a slighht push.

At one stage, body continues to move with a constant velocity and kinetic frictional force F is then equal to the weight of the scale pam together with the pan's weight.

On dividing F by the weight of body, the co-efficient of dynamic friction canbe calculated.

Examples

1. A car of mass 200kg moving along a straight road at a speed of 96kmh^{-1} is brough to rest by steady application of the brakes in a distance of 80m. find the co-efiecient of kinetic friction between the tires and the road.



$ma = -\mu R$

$ma = -\mu mg$

$a = -\mu g$

$\mu = \frac{-a}{g}$

$u = 96kmh^{-1} \rightarrow 96 \times \frac{5}{18} ms^{-1} = 26.7ms^{-1}$

$v = 0$

$s = 80m$

$a = ?$

$v^2 = u^2 + 2as$

$0 = (26.7)^2 + (2 \times a \times 80)$

$a = -\frac{26.7^2}{160}$

$a = -4.5ms^{-2}$

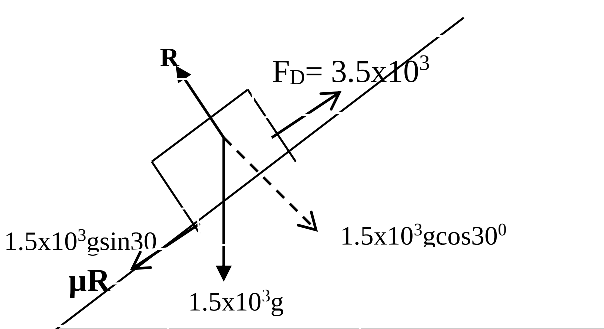
$\therefore \mu = \frac{4.45}{9.8}$

$= 0.45$

2. A car of mass 1.5 x 10^3kg and tractive pull 3.5x10^3N climbs a truck which is inclined at an angle of 30^0 to the horizontaa. The speed of the car at the bottom of the incline is 20ms^{-1} and the coffient of sliding friction is 0.25, calculate

(i) The distance travelled along the incline before the car comes to a halt.

(ii) The time taken ttravelling along the incline before the car comes to a halt.



$F = ma$

$\left(3.5\times10^3\right)-\left(1.5\times10^3 g\sin30+0.25\times1.5\times10^3 g\cos30^0\right)=1.5\times10^3 a$

$\left(3.5\times10^3\right)-\left(750+324.8\right)=1.5\times10^3 a$

$a = -4.69ms^{-2}$

$u = 20ms^{-1}$

$a = -4.69ms^{-2}$

$v = 0$

$s = ?$

$V^2 = u^2 + 2as$

$0 = \left(20^2\right)+\left(2\times-4.69\times s\right).$

$400 = 9.38s.$

$s = 42.6m$

ii) $S = 42.6$

$a = -4.96$

$u = 20$

$v = 0$

$t = ?$

$v = u + at$

$0 = 20 - 4.96t$

$20 = 4.96t.$

$t = 4s$

3. An old car of mass 1500kg and tractive pull 4000N climbs a tract which is incllined at an angle of 30^0 to the horizontal. The velocity of the car at the bottom of the incline is $108kmh^{-1}$ and the co-efiecient of sliding friction is 0.35.

(i) Clculate the distance travelled along the incline before the car comes to a halt.(86.53m)

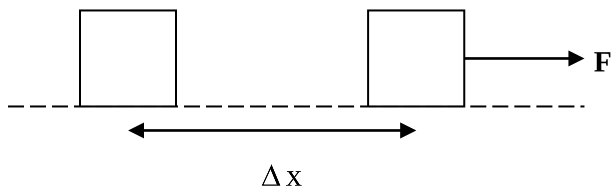
(ii) The time taken to travel along the incline before the car comes to a halt.(5.77s)

CHAPTER SEVEN: WORK, POWER AND ENERGY

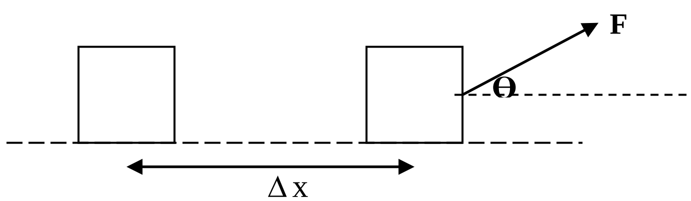
WORK DONE BY A CONSTANT FORCE

Work is defined as the product of the distance moved by the point of aplication of the force and the component of the force in the direction of motion.

Consider a body by mass in resting on a smooth surface.



If a force F moves the object through a distance X, then work done w = F. Δ x If the force puls the block at angle Ө to the horizontal through a horizontal distance Δx



Work done, w = (FcosӨ) Δx

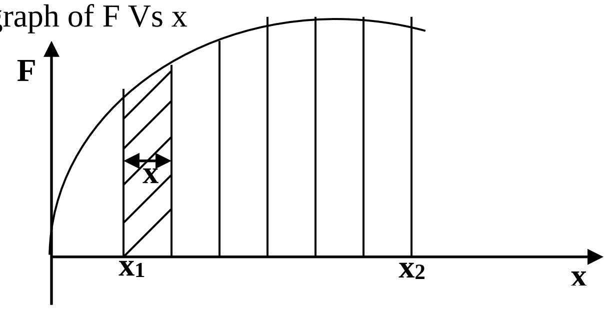
Work is a scalar quantity.

S.I unit J.

<u>Work done by a variable force</u>

Consider a force F = f(x) which val=ries in magnitude

A graph of F Vs x



If it is required to find the work done by the force when its point of application moves from $x = x_1$ and $x = x_2$ then the interval $x_1 \rightarrow x_2$ is subdivided into small displacements, $\Delta x_{1,} \quad \Delta x_2 \ldots\ldots\ldots\ldots \Delta x_n$

The work done by the force during the displacement

$\Delta x_1 is F_1 \Delta x_2$ (since Δx_1 is too small, the F_1 can be considered constant)

For another short interval

Δx_2 work done $= F_2 \Delta x_2$. therefore, work done during the displacement $x_1 \rightarrow x_2 \Delta W = F_1\Delta x_1 + F_2\Delta x_2 + F_3\Delta x_3 + \ldots\ldots\ldots\ldots\ldots\ldots F_n \Delta x_n = \sum_{i=1}^{n} F_1 \Delta x_i$

For $n \rightarrow \infty$

$$W_R = \frac{\lim}{\Delta xi \rightarrow 0} \sum_{i \rightarrow 1}^{n} Fi\Delta xi$$

$= \int_{x_1}^{x_2} F.dx$ = area under the force -distance grgaph

in vector form, work done by a variable force is given by

$$w = \int_{r_1}^{r_2} \vec{f}.\vec{dr}$$

where r is the displacement.

Work – energy theorem

Variable force

Consider an external force F= F(x) which acts as a mass m giving it an acceleration a, by Newton's second law F = ma $= {mdv}/{at}$. The work , ∂x ,done in displacing mass m, throguh a small distance ∂x , under action of a force F.

$$\partial w = F \partial x$$

total work done = $\int_{x_1}^{x_2} Fdx.$

$$\int_{x_1}^{x_2} m\frac{dv}{dt}dx$$

$$\text{but } \frac{\text{dx}}{\text{dt}} = \text{v}$$

$$\text{w} = \int_{\text{v}_1}^{\text{v}_2} \text{mv dv}$$

where v_1 and v_2 are the velocities of the body when at dispacement x_1 and x_2 respectively.

$$W = \left[\frac{mv^2}{2}\right]_{v_1}^{v_2}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2mv_1^2}$$

The above is the expression for the work –energy theorem

It states that, *the work done by the resultant external force is equal to the change in the kinetic energy of the body.*

Constant force

Consider a mass in initially moving of a speed *u* which is subjected to a constant retarding force F. suppose the speed is reduced to v in a distance S

Using
$$v^2 = u^2 + 2as$$
$$as = \frac{v^2 - u^2}{2}$$

Work done by the retarding force

= -FS

= -mas

$$\text{but as} = \frac{\text{v}^2 - u^2}{2}$$

$$W = -m\left(\frac{v^2 - v^2}{2}\right)$$

$$W = \frac{mu^2}{2} - \frac{mv^2}{2}$$

Again work done = change in kinetic energy.

GRAVITATIONAL POTENTIAL ENERGY

Suppose a body of mass m is raised from a height y_1 to a height y_2 above the surface of the earth, the work done by the gravitational force when the body is raised through a small height $\partial y; \partial w = F\partial y$

Where F= gravitational force = -mg

$$\partial w = -mg\partial y$$

Work done to raise the body from height y_1 to height y_2 is

$$\int_{y_1}^{y_2} -mg \quad dy$$

$$= -mg\int_{y_1}^{y_2} dy$$

$$= mg[y]_{y_1}^{y_2}$$

$$\therefore W = -(mgy_2 - mgy_1).$$

From work -energy theorem

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$w = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -mgy_2 + mgy_1$$

$$\frac{1}{2}mv_2^2 + mgy_2 = mgy_1 + \frac{1}{2}mv_1^2$$

The term mgy = gravitational potential energy.

$$\frac{1}{2}mv^2 + mgy = constant......... *$$

Hence $Potential energy + Kinetic energy = \text{mechanical energy}$

Equation $*$ implies that merchanical energy is conserved.

Principal of conservation of mechanical energy

In a given system where the only force acting are conservative forces, the sum of Kinetic energy and Potential energy is constant.

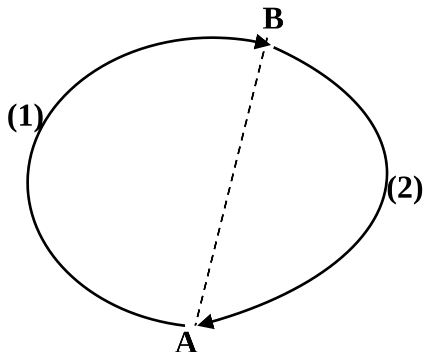
Question

Show that the following obey the law of mechanic energy.

(i) A swinging pendulum

(ii) A falling stone.

Conservative forces.



$W_{AB}^{(1)} = W_{AB}^{(2)}$, then the force being used is a conservative force.

For a conservative force, the work done is independent on the path taken.

Work done when a body moves round a closed path is zero i.e. $W_{AB}^{(1)} + W_{AB}^{(2)} = 0$

Let $W_{AB}^{(1)}$ be the work done to move the mass from A to B via path 1 and $W_{AB}^{(2)}$ be the work done to move the mass from A to B via path (2).

If $W_{AB}^{(1)} = W_{AB}^{(2)}$ then the work done is independent of the path taken in the field of force.

Examples of conservative forces; Gravitational force, Elastic force, Electrostatic force.

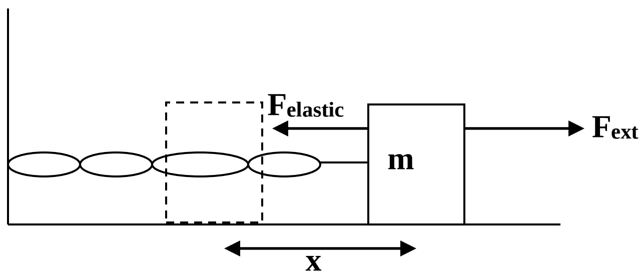
For a conservation force, the work done in moving the body round a close path in the field of force is zero.

In a conservative force field, mechanical energy is conserved

Non- conservative forces: In a conservative force, the work done by a non conservative force round a closed path is not zero and is dependent on the path taken. Example of non-conservative forces: Friction, Air resistance, Viscosity drag.

<u>Elastic potential energy</u>

Consider a mass m resting on a smooth horizontal surface and attached to a spring whose other end is fixed.



Suppose an external force F_{ext} is applied to the mass, so that the spring becomes stretched by a distance x. An equal and opposite force, $F_{elastic}$ i.e elastic force appears in the spring.

$F_{elastic}$ = kx (Hooke's law)

Force is directly proportional to extension provide the elastic limit is not exceeded.

k= force constant, F_{ext} = kx

When a spring is stretched from x = x_1 to x = x_2,

$$W = \int_{x_1}^{x_2} F_{elastic}.\,dx$$

$$-\int_{x_1}^{x_2} kx\ dx$$

$$w = -\left[\frac{kx_2^2}{2} - \frac{kx_1^2}{2}\right]$$

From work – energy theorem

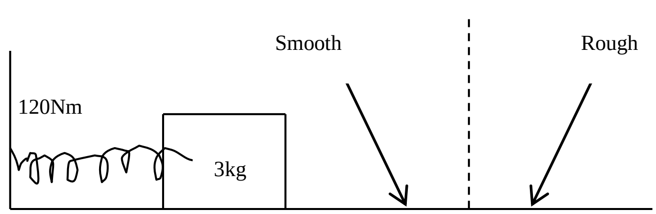
$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\left(\frac{kx_2^2}{2} - \frac{kx_1^2}{2}\right)$$

$$\frac{1}{2}mv_2^2 + \frac{kx_2^2}{2} = \frac{1}{2}mv_1^2 + \frac{kx_1^2}{2}$$

Then term V(x) = $\frac{1}{2}kx^2$ is the elastic potential energy.

Examples

1. A 3.0kg block is held in contact with a compressed spring of a force constant 120Nm^{-1}. The block rests on the smooth portion of a horizontal surface which is partly smooth and partly rough as shown.



When the block is released, it slides without friction until it leaves the spring and then continues to move along the rough portion for 8.0m before it comes to rest. The coefficient of sliding friction between the block and the rough surface is 0.20. Calculate the: (i) maximum kinetic energy the block.

(iii) Compression of the spring before the block was released.

Solution

Kinetic energy = work done against frictional force

$$\frac{1}{2}mv^2 \quad = \quad \mu\text{mg} \times \text{distance}$$

$$\frac{1}{2}v^2 = \mu g \times 8$$

$$v^2 = (2 \times 0.20 \times 9.8 \times 8)$$

$$v = 5.6ms^{-1}$$

Kinetic energy = ½mv^2 = ½x3x $(5.6)^2$ = 47.04J

(ii) elastic energy = ½kx^2

But Kinetic energy = elastic energy

$$47.07 = \frac{1}{2} \times 120 \times x^2$$

$$x^2 = \frac{47.07 \times 2}{120}$$

$$x = 0.89m$$

2. A bullet of mass 10g is fired at short range into a block of wood of mass 990g resting on a smooth horizontal surface and attached to a spring of force constant $100Nm^{-1}$. The bullet remains embedded in the block while the spring is compressed by a distance of 5.0cm. Find the elastic energy of the compressed spring, and the speed of the bullet just before collision with the block.

Elastic energy = ½kx^2 = ½ x 100 x $(0.05)^2$ = 0.125J

Kinetic energy = elastic energy

$$\frac{1}{2}mv^2 = 0.125$$

$$(0.01 + 0.99)v^2 = 0.25$$

$$v = 0.5ms^{-1}$$

Using conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(0.01 \times u) + (0.99 \times 0) = 0.5(0.01 + 0.99)$$

$$u = 50ms^{-1}$$

Exercise

1. A mass of 500g is released from rest so that if falls vertically through a distance of 20cm onto a scale pan, of negligible mass, hung from a spring of force constant $100Nm^{-1}$. Find the position of the scale pan when it first comes to rest. (0.14m)

POWER

It is the rate of doing work.

$power = \frac{dw}{dt} = \frac{d}{dt}(F.S)$; where w is work done, F is force , S is distance travelled.

For constant force

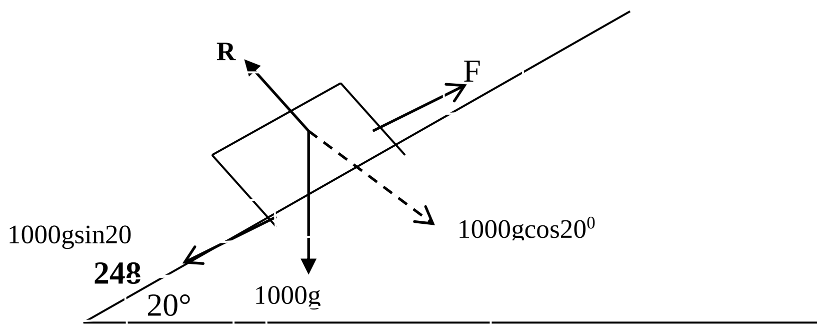
$p = F.\frac{ds}{dt}$ But $\frac{\text{ds}}{\text{dt}} = v$

$\therefore p = FV$

Unit of power is watts

Example

1. A particle of mass 1000kg moves with uniform velocity of 10ms^{-1} up a straight truck inclined at an angle of 20° to the horizontal. The total frictional resistance to motion of the car is 248N. Calculate the power developed in the engine.



$F = 1000g\sin 20 + 248$

$3599.8N$

$P = Fv$

(3599.8×10)

$= 35997.9W$

$= 36kw$

2. Sand is deposited at a uniform rate of 20kgs^{-1} and of negligible kinetic energy onto an empty conveyor belt moving horizontally at a constant speed of 10m / minute.

Find

(i) A force required to maintain a constant velocity.

(ii) The power required to maintain a constant velocity

(iii) The rate of change of K.E of the moving sand

(iv) Why are the latter 2 quantities unequal?

$$v = \frac{10}{60} = \frac{1}{6}\,\text{ms}^{-1}$$

$$\text{F} = \frac{\text{dp}}{\text{dt}} \;\; , \; \text{p} = \text{mv}$$

$$\text{F} = \frac{\text{d(mu)}}{\text{dt}} \qquad v = \frac{dm}{dt}$$

$$\frac{dm}{dt} = 20kgs^{-1}$$

$$F = \frac{1}{6} \times 20$$

$$= 3.33N$$

(ii)

$$P = FV$$

$$= \frac{20}{6} \times \frac{1}{6}$$

$$= \frac{20}{36}\,\text{W}$$

$$\text{W} = 0.56\text{w}$$

Rate of Kinetic Energy

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)v^2$$

$$= \frac{1}{2} \times 20 \times \left(\frac{1}{6}\right)^2$$

$$= 0.28J$$

The two quantities are not equal because there is a frictional force that has to be overcome.

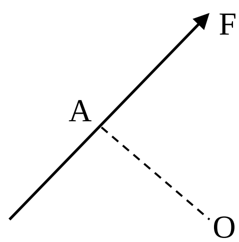
CHAPTER EIGHT: MOMENTS AND COUPLES

MOMENT OF A FORCE

The moment of a force about an axis is the product of the force and the perpendicular distance from the axis to the line of action of the force.

Moment of force is often referred to as the torque of force, and it is a vector quantity

Consider a force F which acts as shown in the figure below;



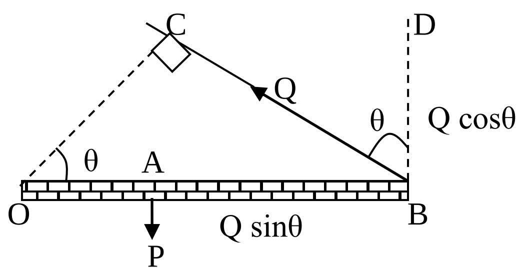
The moment of the force F about the point O, is given by $F \times OA$ whereas the moment of the force about the point A is zero.

AO is the perpendicular distance of the line of action of the force from O.

The S.I unit of moment is the Newton metre (Nm). Moments are either positive (clockwise moments) or negative (anticlockwise moments)

Examples

Consider a uniform beam OB hinged at point O, as shown in the figure below;



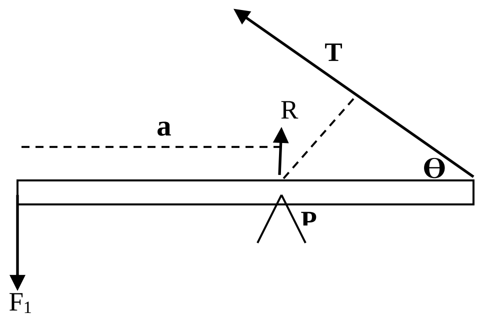
Moment of P about O, is $P \times \overline{OA}$ and moment of Q about O, is $Q \times \overline{OC}$

Considering force Q, its component perpendicular to OB is given by $Q\cos\theta$ and along BD is $Q\sin\theta$

It is important to note that moment of $Q\sin\theta$ about point O is zero while moment of $Q\cos\theta$ about D is $Q\cos\theta \times \overline{OB} = Q\overline{OB}Cos\theta$

But, $\overline{OB}Cos\theta = \overline{OC}$, thus moment of $QCos\theta$ about O = $Q \times \overline{OC}$

Consider a uniform beam pivoted at point P, as shown in the figure below;



Consider taking moments about the point of pivot, P, we have;

Moment of F_1 about P, = $F_1 a$ ……………………… (1)

Moment of R about P = 0 ………………………….. (2)

Moment of T about P = T. a sin Θ…….…………….. (3)

Note that the force, R, is the normal reaction due to support at pivot point P.

COUPLES

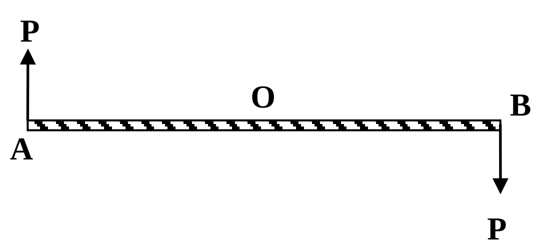
Two equal and opposite parallel forces whose lines of action do not coincide (do not pass through the same point) constitute a couple

The two forces that constitute a couple have a moment of force called a **Torque** and is given by:

$$Torque = \text{one of the forces} \times \text{perpendicular distance between them}$$

Example

Consider two equal and opposite parallel forces, P acting on a beam AB with a centre at O as shown in the figure below.



$$Torque = P \times \overline{AB}$$

A couple tends to change rotation of a system.

PRINCIPAL OF MOMENTS

If a body is in equilibrium, under the action of a number of force, the algebraic sum of the moment of the forces about any axis is zero i.e. total clockwise moments = total anticlockwise moments about the same axis.

Conditions for a body to be in equilibrium

The following are conditions for a rigid body to be in mechanical equilibrium;

- The resultant force on the body must be zero
- The sum of clockwise moments about any point must be equal to the sum of the anticlockwise moments about the same point

(i) Translational equilibrium.

The resultant force must be zero i.e. sum of forces in one direction should be equal to sum of forces in the opposite direction.

(ii) Rotational equilibrium

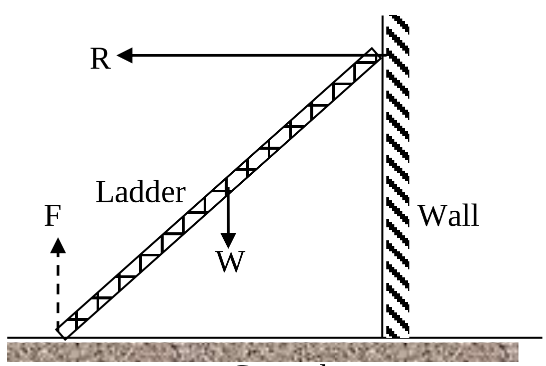
The algebraic sum of moments about any axis must be zero.

Equilibrium of Three Coplanar Forces

Coplanar forces are forces acting in the same plane

If a body is in equilibrium under the action of three coplanar forces, then the resultant of the two of the forces is equal and opposite of the third force

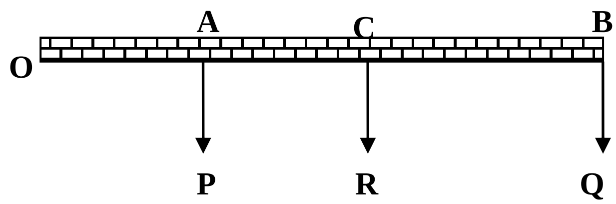
Consider a ladder leaning against a smooth wall as shown in the figure below



- ✓ R is reaction of the wall on the ladder and is perpendicular to the wall if it is smooth.
- ✓ W is the weight of the ladder and acts at its mid-point (centre of gravity)
- ✓ F is the reaction at the foot of the ladder and if the ladder is in equilibrium, then the line of action of force F passes through the point of intersection of the lines of action of W and R

Illustration of the principle of moments of coplanar forces

Consider two parallel forces P and Q acting on a beam as shown in the figure below;



Let O be any point in the plane of the forces

The resultant R = (P+Q) is parallel to forces P and Q and acts through a point C in AB such that;

$P \times \overline{AC} = Q \times \overline{CB}$ (Taking moments about point C)

Therefore sum of moments of P and Q about O is;

$(P \times \overline{OA}) + (Q \times \overline{OB} = P(\overline{OC} - \overline{AC}) + Q(\overline{OC} + \overline{CB}$

$= (P \times OC) - (P \times AC) + (Q \times OC) + (Q \times CB$

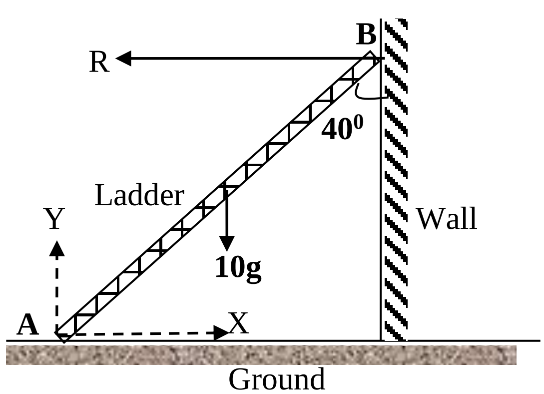
$= (Q + P)OC - (P \times AC) + (Q \times CB)$

$= (R \times OC = (P \times AC) + (Q \times CB)$.........the principle of moments proved

Which states that the moment of the resultant of a number of forces about a point is equal to the algebraic sum of the moments of the individual forces about the same point.

Examples

1. A uniform rod of mass 10kg is smoothly hinged at A and rests in a vertical plane on the end B against a smooth vertical wall. If the rod makes an angle of 40^0 with the wall, find the thrust of the wall and the direction of the reaction at A



Let X and Y represent the components of the reaction in the horizontal and vertical directions respectively.

Resolving forces in the horizontal direction

R = X

Resolving forces in the vertical direction

Y = 10g = 98N

Taking moments about A:

10gx (ABsin40)/2 = R xABcos40

Therefore, R = 41.1N

Hence X = 41.1N

Reaction at A = $\sqrt{(41.1^2 + 98^2)}$ = 106.3N

Direction ϴ = tan^{-1}(98/41.1) = 67.24°

$$\text{Direction} = \text{tcm}^{-1}\left(\frac{98}{41.1}\right)$$

$$= 67.24^0$$

2. (a) Define force and moment

(b) State the conditions for equilibrium of a rigid body under the action of coplanar forces

(c) A 3m long ladder rests at an angle of 60^0 to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5 kg and its centre of gravity is one-third from the bottom of the ladder.

(i) Draw a sketch diagram to show the forces acting on the ladder

(ii) Find the reaction of the ground on the ladder

(d)

Solution

(a) (i) Force is anything that changes or tends to change a body's state of rest or of uniform motion in a straight line

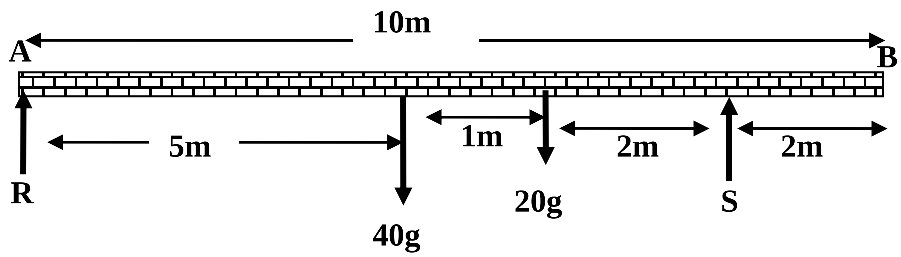
(ii) The moment of a force about an axis is the product of the force and the perpendicular distance from the axis to the line of action of the force

See notes to solve the rest of the questions

3. A uniform beam AB 10 m long and of mass 40 kg rests on two supports, one at A and the other is 2 m from B. if a weight of mass 20 kg is attached to the beam at a point 6m from A. find the pressures on the supports

Solution

Consider the sketch below as the case in the question;



Taking moments about A

$(40g \times 5) + (20g \times 6) = (S \times 8)$

$200g + 120g = 8S$

$8S = 320g$

$S = 40g$

Taking moments about B

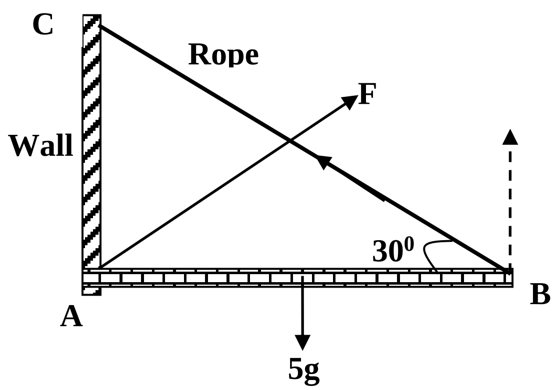
$(40g \times 3) + (20g \times 2) = (R \times 8)$

$120g + 40g = 8R$

$8R = 160g$

$R = 20g$

4. A uniform beam of wood AB is hinged to a wall at A as shown in the figure.



The beam is 4m long and has a mass of 5kg. The beam is maintained horizontal by a rope attached to points B and C, where C is directly above A.

Find;

(i) The tension in the rope

(ii) The reaction of the wall on the beam

Solution;

For beam to be at equilibrium the vertical component of tension, T, is balanced by downward force.

Taking moment about point A

$$\left(5g \times 2\right) = T \sin\theta \times AB$$
$$10g = T \sin 30^0 \times 4$$
$$10g = 2T$$
$$T = \frac{10g}{2} = 5g$$
$$T = 5 \times 9.81 = 49N$$

Resolving forces along the horizontal axis

$$F \cos\theta = T \cos\theta$$
$$F \cos\theta = 49 \cos 30 = 49 \times 0.866 = 42.43................(1)$$

Resolving forces along the vertical axis

$$F \sin\theta + T \sin\theta = 5 \times 9.81$$
$$F \sin\theta = 49 - (49 \sin 30)$$
$$F \sin\theta = 24.5.........................(2)$$

Adding (1) and (2), we have

$$F \cos\theta + F \sin\theta = 66.93.............(3)$$

Squaring both sides of (3), we get

$$F^2 \cos^2\theta + F^2 \sin^2\theta = 4477.88$$
$$F^2 (\cos^2\theta + \sin^2\theta) = 4477.88$$
$$F = \sqrt{\frac{4477.88}{1}} \text{since, trigonometry square terms} = 1$$
$$\mathrm{F} = 66.9\mathrm{N}$$

CHAPTER NINE: CIRCULAR MOTION

Introduction

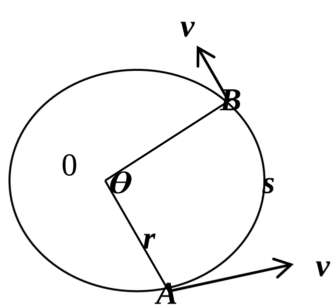
Circular motion is important in physics to describe planetary motion of the Earth, Moon and other planets around the Sun and to launch satellites for space communications

- ✓ If a particle is describing a circular path, its velocity is constantly changing direction- thus the particle must have an acceleration
- ✓ If the magnitude of the velocity is constant, then its speed is uniform and the particle has an acceleration which is directed towards the centre of the circle
- ✓ If the magnitude of the velocity is **NOT** constant, then the acceleration at an instant has a component along the tangent as well as the centre of the circle.

Examples of circular motion

- Rotation of the Earth round the Sun.
- The rim of the balance wheel of a watch moving to and fro about a fixed axis in a circular path.
- The parking of satellites in space orbits to relay information.

Consider a body moving in a circle of radius *r* with uniform speed *v*



Suppose the body moves from point A to point B in time, t, through an angle Θ.

The angle Θ is called the *angular displacement.*

$$\text{Arc length, s} = r\theta..............................(1)$$

Angular velocity,ω, is the rate of change of angular displacement, hence

$$\text{Angular velocity, } \omega = \frac{\theta}{t}......................(2)$$

Speed of the particle if the rate of change of displacement

$$v = \frac{s}{t} = \frac{r\theta}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\text{but } \frac{\theta}{\text{t}} = \omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

combing (3) and (4), we have

$$v = \omega r \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Period (T), time taken to go through one circle.

When Θ =2π, t = T………………… (1)

$$\omega = \frac{\theta}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Combining (1) and (2), we get

$$\omega = \frac{2\pi}{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$But, \text{ v} = \omega\text{r} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

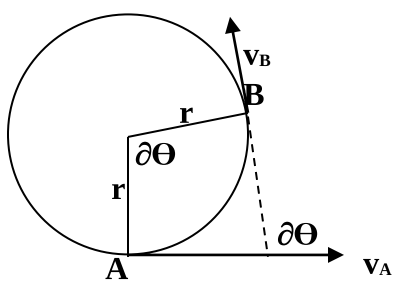
Combing (3) and (4), we get

$$V = \frac{2\pi}{T} r \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

v = ωr

Acceleration of a body moving in a circle

Consider a body moving with constant speed v in a circle of radium r



If it travels from A to B in a short time,∂t,

$$\partial t, \text{ then arc AB} = \text{v}\partial\text{t}$$

$$\text{v} = \frac{\text{AB}}{\partial\text{t}}$$

Also arc AB = r∂Θ

Hence

$r\partial\sigma = v\partial t.$

$$\partial\theta = \frac{v\partial t}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

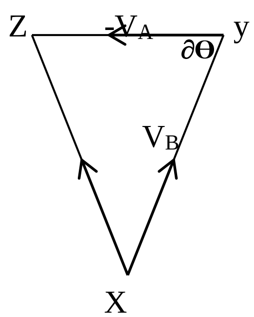
Change of velocity between A and B

$$V_B - V_A$$

$$= V_B + (^{-}V_A)$$

$$XZ = V_B + \left(^{-}V_A\right)$$

But $|V_A| = |V_B| = V$



Arc XZ = $v\partial\Theta$

From equation (1) $\partial\Theta = v\frac{\partial t}{r}$

Hence arc XZ = v. $v\frac{\partial t}{r} = v^2\frac{\partial t}{r}$

The magnitude of the acceleration, a, between A and B is

$$a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{xz}{\partial t}$$

$$a = \frac{v^2\partial t}{r\partial t} = \frac{v^2}{r}$$

But $v = \omega r$

$a = \omega^2 r$

The acceleration of the body moving in a circle is towards the centre of the circle as the case in S.H.M

The force on a body moving in a circle towards the centre of the circular path is called the **centripetal force**

Calculating the centripetal force, F in a circular motion.

From Newtons 2nd law of motion,

$F = ma$..........................(1)

$$But, a = \frac{v^2}{r}..................(2)$$

Combining (1) and (2), we have

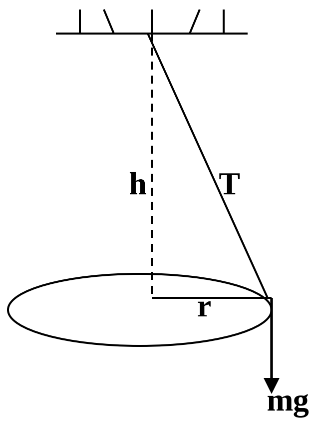
Centripetal force, F = ma = $m\frac{v^2}{r}$...................(3)

Or Centripetal force = $m\omega^2 r$

Example of circular motion

Conical pendulum

Consider a body of mass *m* attached to a string of weight *l*, describing horizontal circle of radius *r* at a uniform speed *v*



$$(\uparrow) T\cos\theta = mg........(1)$$

$$(\rightarrow) T\sin\theta = \frac{mv^2}{r}..................(2)$$

Dividing equation $(2) \div (1)$, we have;

$$\tan\theta = \frac{v^2}{rg}$$

$$r = l\sin\theta$$

$$V^2 = rg\tan\theta.........................(3)$$

$$\text{but } v = \frac{2\pi r}{T}, T = \text{period}$$

$$\frac{4\pi^2 r^2}{T^2} = \text{rg}\tan\theta$$

$$T^2 = \frac{4\pi^2 r}{g\tan\theta}$$

$$T^2 = \frac{4\pi^2 l\sin\theta}{g\tan\theta} \quad = \quad \frac{4\pi^2 l\cos\theta}{g}$$

But $l\cos\Theta = h$

$$T^2 = \frac{4\pi^2 h}{g}\text{.........................}(4)$$

Example

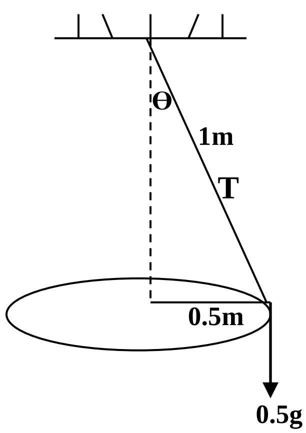
A steel ball of 0.5kg is suspended from a light inelastic string of length 1m. The ball describes a horizontal circle of radius 0.5m

Find

(i) The centripetal speed of the ball

(ii) The angular speed of the ball

(iii) The angle between the string and the radius of the circle if the angular speed is increased to such a values that the tension in the string is 10N



$$\sin\theta = \frac{0.5}{1} = 30°$$

$$(\rightarrow)\frac{mv^2}{r} = T\sin 30\text{..................}(i)$$

$$(\uparrow) T\cos 30^0 = 0.5g$$

$$T = 5.67N$$

From (i)

$$\therefore \frac{0.5v^2}{0.5} = 5.67\sin 30^0$$

$$\therefore \text{certripetal force} = \frac{\text{mv}^2}{\text{r}} = \frac{0.5\times 1.68^2}{0.5}$$

$$= 2.83N$$

(iii) Angular speed ω

$$v = wr$$
$$w = \frac{1.68}{0.5}$$
$$= 3.36 rads^{-1}$$

(iii)

$$(\uparrow) T\cos\theta = 0.5g$$
$$\cos\theta = {0.5g}/{10}$$
$$\theta = 60.66^0$$

Example

1. (a) Define the term angular velocity

(b)A car of mass, m, travels round a circular track of radius, r, with a velocity, v.

(i) Sketch a diagram to show the forces acting on the car

(ii) Show that the car does not overturn if $v^2 \angle \frac{\arg}{2h}$, where ***a*** is the distance between the wheels, ***h*** is the height of the centre of gravity above the ground and ***g*** is the acceleration due to gravity.

(c) A pendulum of mass 0.2kg is attached to one end of an elastic string of length 1.2 m. the bob moves in a horizontal circle with the string inclined at 30^0 to the vertical.

Calculate;

(i) The tension in the string

(ii) The period of the motion

Exercise

1. An object of mass 0.5kg on the end of the string is whirled around in a horizontal circle of radius 2m, with a constant speed of $10ms^{-1}$. Find its angular velocity and the tension in the string. (ω = $5rads^{-1}$, T = 25.5N)

2. A small ball of mass 0.1 kg is suspended by an inextensible string of length 0.5m and is caused to rotate in a horizontal circle of radius 0.4m. Find

(i) The resultant of these forces. (1.3N)

(ii) The period of rotation. (1.1s)

3. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. The bob moves in a horizontal circle with the string inclined at 30° to the vertical. Calculate: (i) the tension in the string

(ii) the period of the motion

4. The period of oscillation of a conical pendulum is 2.0s. If the string makes an angle of 60° to the vertical at the point of suspension, calculate the:

(i) Vertical height of the point of suspension above the circle. (h = 0.994m)

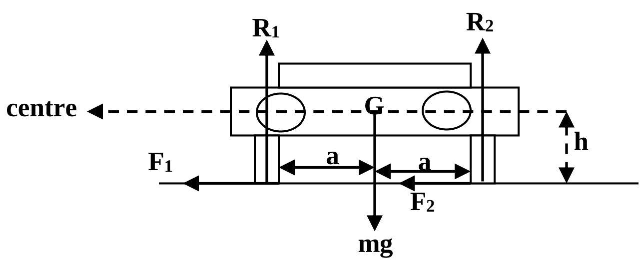
(ii) Length of the string, (l = 1.99m)

(iii) Velocity of the mass attached to the string. (v = 5.41ms^{-1})

APPLICATIONS OF CIRCULAR MOTION

Vehicle on a curved track

(i) Overturning / upsetting / toppling



Consider a vehicle with mass m moving with a speed v in a circle of radius r; let h be the height of the centre of gravity G above the truck and $2a$ the distance between the tyres and that the car is negotiating a bend on a level track

Resolving vertically:

$R_1 + R_2 = mg$...................(1)

Horizontally

$(F_1 + F_2) = \frac{mv^2}{r}$.....................(2)

Taking moments about G:

$$R_1 a + F_1 \times h + F_2 \times h = R_2 .a$$

$$\left(F_1 + F_2\right)\frac{h}{a} = R_2 - R_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substitute equation (2) in equation (3)

$$\frac{mv^2}{r}.\frac{h}{a} = R_2 - R_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Add equation (1) + equation (4)

$$R_1 + R_2 = mg$$

$$R_2 - R_1 = \frac{mv^2 h}{ra}$$

$$\therefore \frac{2\mathrm{R}_2}{\quad} = \frac{m}{\quad}\left(g + \frac{v^2 h}{ra}\right)$$

$$R_2 = \frac{m}{2}\left(g + \frac{v^2 h}{ra}\right)$$

$R_2>0$ implying that the outer tyre never lose contact.

Equation (1) – equation (4)

$$2R_1 = mg - \frac{mv^2 h}{ra}$$

$$R_1 = \frac{m}{2}\left(g - \frac{v^2 h}{ra}\right)$$

When $R_1 = 0$, inner tyre loses contact with the track, then the car is just about to overturn, topple or upset

$$\Rightarrow \frac{m}{2}\left(g - \frac{v^2 h}{ra}\right) = 0$$

$$g - \frac{v^2 h}{ra} = 0$$

$$\mathrm{v}^2 = \frac{rag}{h}$$

$$\mathrm{v} = \sqrt{\frac{\mathrm{rag}}{\mathrm{h}}}$$

If the car is driven at a velocity (speed) $v = \sqrt{\frac{rag}{h}}$, then it's just at a point of toppling/overturning/upsetting.

Therefore, for no toppling/overturning/upsetting, then $v \angle \sqrt{\frac{rag}{h}}$

It can therefore be noted that for upsetting/toppling/overturning:

- ❖ The bend is sharp (r is small)
- ❖ The centre of gravity is high (h is large)
- ❖ The distance between the tires is small (a is small)

Also, if the coefficient of friction between the tyres and the ground is μ, then from equation (1)

$$\mu R_1 + \mu R_2 = \frac{mv^2}{r} \text{..............................}(i)$$

$$\mu(R_1 + R_2) = \frac{mv^2}{r} \text{..............................}(ii)$$

But $(R_1 + R_2) = mg \text{..............................}(iii)$

Combining (ii) and (iii)

$$\mu mg = \frac{mv^2}{r} \text{..............................}(iv)$$

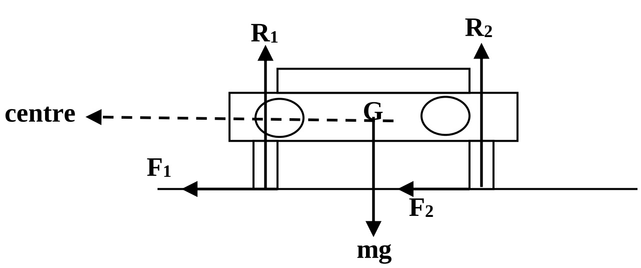
$$\Rightarrow v = \sqrt{\mu rg} \text{..............................}(v)$$

If the is driven at a velocity $v = \sqrt{\mu rg}$, then it's at a point of sliding/ skidding/ slipping.

Therefore, if $v \succ \sqrt{\mu rg}$, the car slids/skids/slips

Skidding

A vehicle will skid when the available centripetal force is not enough to balance the centrifugal force (force away from the centre of the circle), the vehicle fails to negotiate the curve and goes off truck outwards.



For no skidding, the centripetal force must be greater or equal to the centrifugal force i.e.

$$F_1 + F_2 \geq \frac{mv^2}{r}$$

But $F_1 = \mu R_1$ and $F_2 = \mu R_2$

$$\mu(R_1 + R_2) \geq mv^2/r$$

$$\mu mg \geq \frac{mv^2}{r}$$

$$\mu g \geq \frac{v^2}{r}$$

$$v^2 \geq \mu g r$$

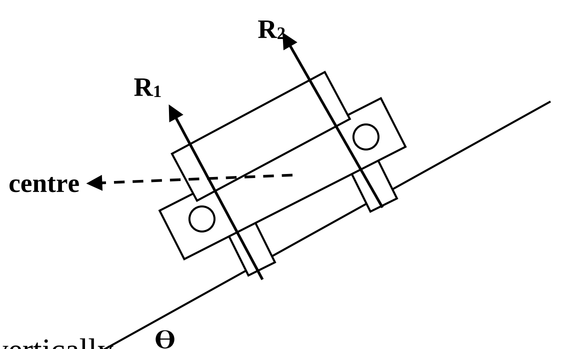
Maximum safe speed, $v_m = \sqrt{\mu r g}$

Skidding will occur if

- The vehicle is moving too fast
- The bend is too sharp (r is small)
- The road is slippery (μ is small)

BANKING OF A TRACK

- This is the building of the track round a corner with the outer edge raised above the inner one. This is done in order to increase the maximum safe speed for no skidding.
- When a road is banked, some extra centripetal force is provided by the horizontal component of the normal reaction
- When determining the angle of banking during the construction of the road, friction is ignored.



Resolving vertically

$$R_1\sin(90-\Theta) + R_2\sin(90-\Theta) = \frac{mv^2}{r}$$

But sin (90- Θ) = cos Θ

(R_1+R_2) Cos Θ = mg ………….. (i)

Horizontally

$$R_1 \cos(90-\theta) + R_2 \cos(90-\theta) = \frac{mv^2}{r}$$

$$(R_1 + R_2)\sin\theta = \frac{mv^2}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

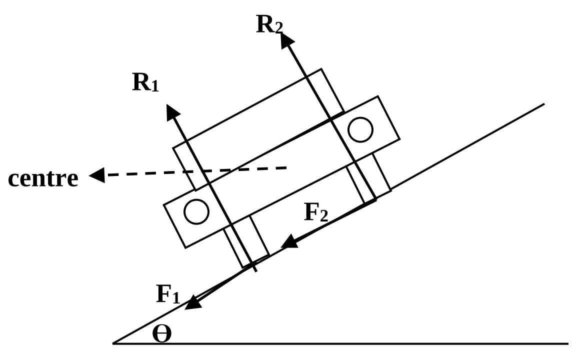
$$eqn\, 2 \div \text{eqn}\, 1$$

$$\tan\theta = \frac{v^2}{rg}$$

Hence Θ <u>is the angle of banking</u>

Case II, When there is friction

Suppose there is friction between the track and the vehicle moving round the bend.



Resolving vertically:

(R_1+R_2) cosΘ = (F_1+F_2) sin Θ + mg

(R_1+R_2) cos Θ – (F_1+F_2) sinΘ = mg

But $F_1 = \mu R_1$, $F_2 = \mu R_2$.

(R_1+R_2) cos Θ – $\mu(R_1+R_2)$ sin Θ = mg

(R_1+R_2) (Sin Θ – μ sin Θ) = mg …………………….. (1)

Horizontally

$$(R1 + R2)\sin\theta + (F1 + F2)\cos\theta = \frac{mv^2}{r}$$

$$(R1 + R2)\sin\theta + \mu(R1 + R2)\cos\theta = \frac{mv^2}{r}$$

$$(R_1 + R_2)(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r} \ldots\ldots\ldots\ldots(2)$$

$$egn\ 2 \div \text{eqn}\,1$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

$$v^2 = rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$\therefore \text{maximum safe speed} = \sqrt{rg\left(\frac{\mu + tan\theta}{1 - \mu\tan\theta}\right)}$$

Question

1.(a) Why a rider has to bend at a certain angle when moving round a bend.

(b) Derive the angle of inclination the rider makes with the horizontal when moving round a bend.

2. A bend of 200m radius on a level road is banked at the correct angle for a speed of $15ms^{-1}$. If a vehicle rounds the bend at $30ms^{-1}$, what is the minimum co-efficient of kinetic friction between the tyres and the road so that the vehicle will not skid.

Solution

Angle of banking

$$= \tan\theta = \frac{v^2}{rg} = \frac{15^2}{(200 \times 9.8)}$$

$$\theta = 6.55^0$$

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$30^2 = 200 \times 9.8 \left(\frac{\mu + \tan\ 6.55}{1 - \mu\tan 6.55}\right)$$

$$900 = 1960 \left(\frac{\mu + 0.1148}{1 - 0.1148\mu}\right)$$

$$900 - 103.32\mu = 1960\mu + 225.008$$

$$2063.32\mu = 674.992$$

$$\mu = 0.327$$

2. A car travels round a bend in road which is a circular arc of radius 62.5m.

The road is banked at angle $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal the coefficient of friction between the tyres of the car and the road surface is 0.4. Find

(i) the greatest speed at which the car can be driven round the bend without slipping.

(ii) The least speed at which this can happen.

(i) Maximum speed

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$v^2 = 62.5 \times 9.8 \left(\frac{0.4 + 5/12}{1 - 0.4 \text{x} 5/12}\right)$$

$$v^2 = 612.5 \left(\frac{\frac{49}{60}}{5/6}\right)$$

$$v^2 = 600.25$$

$$v^2 = 24.5ms^{-1}$$

(ii) Least speed

$$v^2 = rg\tan\theta$$
$$v^2 = 62.5 \times 9.8 \times \frac{5}{12}$$
$$v^2 = 255.208$$
$$v = 15.98ms^{-1}$$

Motion in a vertical circle

This is an example of motion in a circle with non- uniform speed. The body will have a radial component of acceleration as well as a tangential component. Consider a particle of mass is attached to an inextensible string at point O, and projected from the lowest point P with a speed U so that it describes a vertical circle.

Consider a particle at point Q at subsequent time.

The tension T in the string is everywhere normal to the path of the particle and hence to its velocity V. the tension therefore does no work on the particle.

Energy at P, E_P is $E_p = ½mu^2$ …………………….. (1)

P is the reference for zero potential. Energy at Q in E_q is:-

$E_q = ½mv^2 + mgh$.

But $h = r - r\cos\Theta$

$Eq = ½\ mv^2 + mgr\ (1-\cos\Theta)$ ……………………... (2)

Centripetal force of the particle

$$T - mg\cos\Theta = \frac{mv^2}{r}$$

$Mv^2 = r\ (T - mg\cos\Theta)$ …………………………..… (3)

Substitute equation (3) into (2), we have;

$E_q = ½\ r\ (T - mg\cos\Theta) + mgr\ (1-\cos\Theta)$

Using conservation of mechanical energy

$E_q = E_p$.

$\frac{1}{2}r$ (T-mgcosΘ) + mgr (1-cosΘ) = $\frac{1}{2}mu^2$

$\frac{1}{2}r$ (T-mgcosΘ) = $\frac{1}{2}mu^2$- mgr (1-cosΘ)

R (T-mgcosΘ) = mu^2 – 2mgr (1-cosΘ)

(T-mgcosΘ) = $\frac{mu^2}{r} - 2mg(1-\cos\theta)$

$$T = \frac{mu^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$T = \frac{mu^2}{r} + mg(2\cos\theta + \cos\theta - 2).$$

$$T = \frac{mu^2}{r} + mg(3\cos\theta - 2).$$

OR $\mathrm{T} = \frac{\mathrm{mu}^2}{\mathrm{r}} - mg(2 - 3\cos\theta)$

T is greater than zero when $\frac{mu^2}{r} + mg(\cos\theta - 2) > 0$

$$\frac{\mathrm{mu}^2}{\mathrm{r}} > mg(2 - 3\cos\theta)$$

$$u^2 > rg(2 - 3\cos\theta)$$

When $\vartheta = 90$

$\mathrm{u}^2 > rg(2 - 3\cos 90)$

$u^2 > 2rg$

Hence particle overshoots point 0’ when u>$\sqrt{\ }$ 2rg

When Θ = 180^0

u^2 > rg (2-3cos 180)

u^2 > 5rg

Hence particle reaches p' when U> $\sqrt{5rg}$

Therefore particle describes a circle when the initial speed with which you project from P is $u \geq \sqrt{5rg}$

Example

1. A cyclist rounds a curve of 30m radius on a road which is banked at an angle of 20^0 to the horizontal. If the co-efficient of sliding friction between the tires and the road is 0.5; find the greatest speed at which the cyclist can ride without skidding and find into inclination to the horizontal at this speed.

$$v^2 = rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$
$$v^2 = (30 \times 9.8)\left(\frac{0.5 + \tan 20}{1 - 0.5\tan 20}\right)$$
$$v^2 = 294\left(\frac{0.1819}{0.818}\right)$$
$$v = 17.6ms^{-1}$$

$$\tan\theta = \frac{v^2}{rg} = \frac{17.6^2}{30x9.8}$$
$$\theta = 46.5°$$

4(b) A car goes round unbanked curve at 15ms^{-1}the radius of the curve is 60m. Find the least co-efficient of kinetic friction that will allow the car to negotiate the curve without skidding.

$$\mu \geq \frac{v^2}{r}$$
$$\mu \geq \frac{v^2}{rg}$$
$$\mu \geq \frac{15^2}{(60 \times 9.8)} = 0.38$$

Exercise

1. A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1.0m above the ground. The angular speed is gradually increased until the string breaks.

(i) in what position is the string most likely to break?(vertically below point of suspension)

(ii) At what angular speed will the string break? (7.7rads^{-1})

(iii) Find the position where the stone hits the ground when the string breaks.1.22m from point below point of suspension)

2. A car travels round a curved road bend banked at an angle of 22.6°. If the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum speed at which the car negotiates the bend without skidding. (22.4ms^{-1})

CHAPTER TEN: GRAVITATION

Kepler's Laws of Planetary Motion

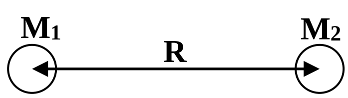
1. Planets revolve in elliptical orbits having the sun at one focus

2. Each planet revolve in such a way that the imaginary line joining it to the sun sweeps out equal areas in equal times

3. The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the sun

Newton's Law of Gravitation

Every particle of matter attracts every other particle with a force which is proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.



$$F \propto \frac{M_1M_2}{R^2}$$

Hence $$F = \frac{G\,M_1M_2}{R^2}$$

Where G is a universal constant known as the Gravitational constant.

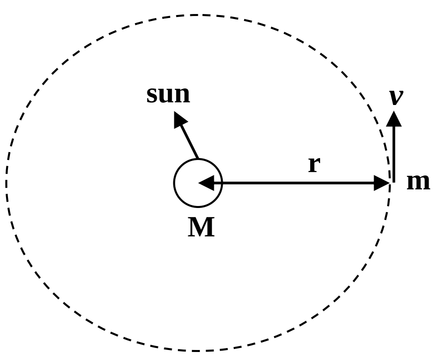
Units of G: Nm^2kg^{-2} or $m^3kg^{-1}s^{-2}$

Numerical value of G= $6.67\text{x}10^{-11}\ Nm^2kg^{-2}$

Question: Show that the dimensions of G are $M^{-1}L^3T^{-2}$

Proof of Kepler's 3rd law

Consider a planet of mass m moves with speed *v* in a circle of radius r round the sun of mass M.



Gravitational attraction of the sun for the planet, $F = \frac{G M m}{r^2}$

If this is centripetal force keeping the planet in orbit, then;

$$\frac{G M m}{r^2} = \frac{mv^2}{r}$$

If T is the time for the planet to make one orbit

$$v = \frac{2\pi r}{T}$$

$$\frac{G M m}{r^2} = \frac{m}{r} x \left(\frac{2\pi r}{T} \right)^2$$

$$GM = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

Since $\frac{4\pi^2}{GM}$ is constant, then $T^2 \propto r^3$ which verifies Kepler's 3rd law.

Parking Orbit

A satellite launched with a speed such that its period equals that of the earth's rotation about its axis and is in the same sense as that of rotation of the earth is called the *Synchronous or Geostationary* satellite. To an observer on the earth's surface, such a satellite appears to be stationary. The orbit of the synchronous satellite is called a *Parking orbit*. Geostationary satellite can be used to relay TV signals and telephone. Messages from one point on the earth surface to other points. In this case a set of 3 synchronous satellites in a triangular array is used.

from

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

When the satellite is in a parking orbit, T = 24 hours = 24 x 3600s

$G = 6.67x10^{-11}\ Nm^2kg^{-2}$

$M = 6x10^{24}kg$

Hence $r = 4.23x10^7m$

Height above the earth for a parking orbit, $h = 4.23x10^7$- Radius of earth

But radius of earth $= 6.4x10^6m$

Therefore, $h = 4.23x10^7 – 6.4x10^6 = 3.59x10^7m$

Variation of acceleration due to gravity

The acceleration due to gravity varies with both altitude and latitude

Variation of acceleration due to gravity with latitude

The acceleration due to gravity increases from $9.78ms^{-2}$ at the equator to $9.83ms^{-2}$ at the poles. The observed variation of g over the earth's surface is due to

(i) the effect of the earth's rotation

(ii) the non- spheroid of their earth

The effect of the earth's rotation: Because the earth rotates about it axis, its gravitational pull on the body on the equator has to provide a centripetal acceleration.

Effect of non- spheroid of the earth: The earth is not a sphere but an oblate spheroid whose equatorial radius exceeds polar radius by about 21.5km i.e. the body at the equator is slightly further away than at the poles. Hence acceleration at the poles is slightly exceeds the acceleration at the equator.

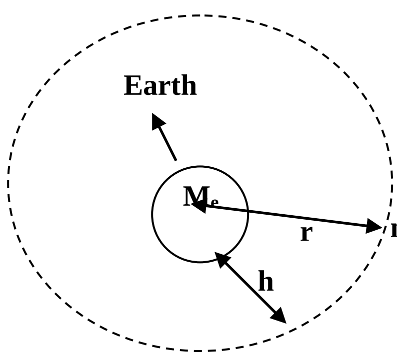
Variation of acceleration due to gravity with altitude

(i) At the earth's surface

$$Mg = \frac{GM_em}{R_e}$$

$$g = \frac{GM_e}{R_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

(ii) Above the earth's surface



If a body is at a point a distance r from the centre of the earth where $r > R_e$

Then $mg' = \frac{GMem}{r^2}$

g' is the acceleration due to gravity at the point a distance r from the centre of earth

$$g' = \frac{GMe}{r^2}$$

Hence $g' \propto \underline{1}$

r^2

but from eqn (i) above, $GM_e = R_e^2 g$

$$g' = \frac{R_e^2 g}{r^2}$$

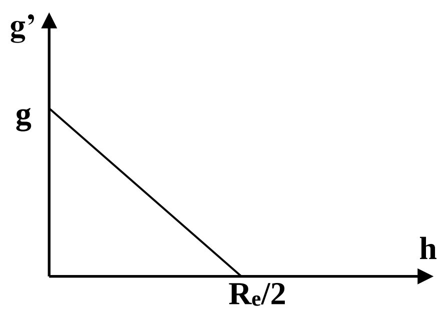
Also $r = h + R_e$

$$g' = \frac{R_e^2 g}{(h + R_e)^2} = g\,(1 + h/Re)^{-2} = g\,(1 - 2h/R_e + 3h^2/R_e^2 + \ldots\ldots\ldots\ldots\ldots\ldots)$$

If h is smaller than R_e, then $(h/R_e)^2$ and higher powers can be ignored as they tend to 0.

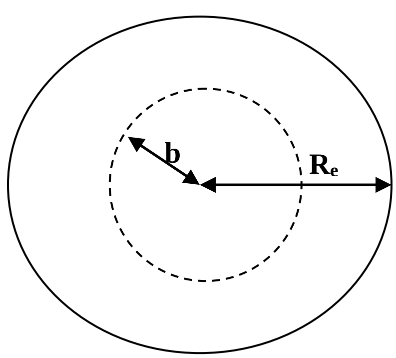
Therefore $g' = g\,(1 - 2h/R_e)$

A graph of g' against h appears as below:



(iii) *Inside the earth's surface*

Consider an object with mass m at a point which is a distance b from the earth's surface, where $b<R_e$. Let g'' be the acceleration due to gravity at this point and M_e' the effective mass of the earth at this point.



Assuming the earth to be a sphere of uniform density, ρ_e

$M_e = 4/3(\pi R_e^3)\rho_e$

$M_e' = 4/3(\pi b^3)\rho_e$

$M_e'/M_e = b^3/R_e^3$

$M_e' = (b^3/R_e^3)M_e$

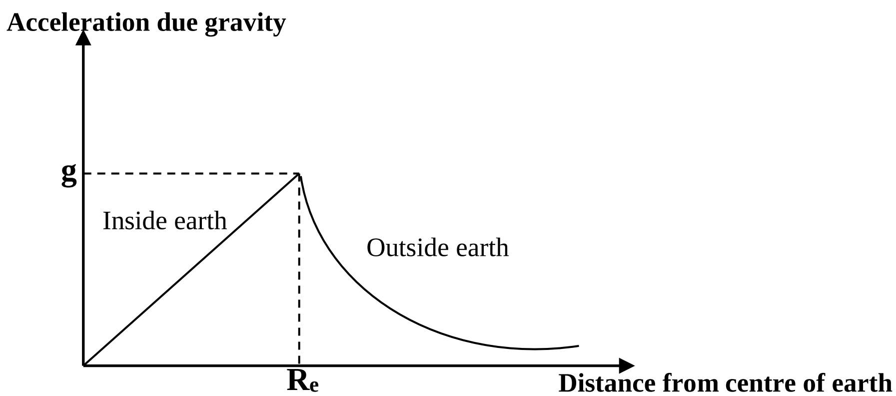
But $mg'' = \frac{G M_e' m}{b^2}$

$$g'' = \frac{G M_e'}{b^2} = \frac{G \times (b^3/R_e^3) M_e}{b^2}$$

$$g'' = \frac{G M_e b}{R_e^2}$$

Hence $g'' \propto b$

Graph showing variation of acceleration due to gravity with distance from centre of the earth.



Mechanical energy of a satellite

(i) Kinetic energy, E_k

Consider a satellite of mass m moving in a circular orbit of radius r. the centripetal force on the satellite is

$$\frac{G M_e m}{r^2} = \frac{mv^2}{r}$$

$$\frac{G M_e m}{r} = mv^2$$

The kinetic energy of the satellite, $E_k = \frac{1}{2}mv^2 = \frac{G M_e m}{2r}$

(ii) **Gravitational Potential energy, E_p**

The force of attraction between the earth and satellite of mass m at a distance x from the centre of the earth is

$$F = \frac{G M_e m}{x^2}$$

If the satellite is to move through ∂x towards the earth, the work done by the gravitational force is

$\partial w = F\partial x = = \frac{G M_e m\partial x}{x^2}$

If the satellite is moved from infinity to a point distance r from the centre of the earth, the work done by the gravitational force is

$$W = \int_{\infty}^{r} \frac{GM_e m}{x^2} dx = GM_e m \left[\frac{-1}{x}\right]_{\infty}^{r} = \frac{-GM_e m}{r}$$

Hence gravitational potential energy is the work done to move a body from infinity to a point in the gravitational field.

Therefore $E_p = \frac{-GM_e m}{r}$

Total mechanical energy $E_T = E_p + E_k = \frac{-GM_e m}{r} + \frac{GM_e m}{2r}$

$$= -\frac{GM_e m}{2r}$$

Note: The satellite has negative total energy hence it is a bound satellite.

Velocity of escape

Velocity of escape is the minimum vertical velocity with which the body must be projected from the earth so that it will never return to the earth.

The work done required for a body to escape = $-\frac{GM_e m}{R_e}$

If the body leaves the earth with speed v_e ad just escapes from its gravitational field

$\frac{1}{2} m v_e^2 = \frac{GM_e m}{R_e}$

Hence, $v_e^2 = \frac{2GM_e}{R_e}$

$v_e = \sqrt{\frac{2GM_e}{R_e}}$

Exercise: Show that velocity of escape can be expressed as

$v_e = \sqrt{2gR_e}$

Effect of friction between a satellite and the atmosphere

The following are the effects of friction between the satellite and the atmosphere

- It reduces the radius of orbit
- The potential energy of the satellite reduces
- The kinetic energy of the satellite increases
- The velocity of the satellite increases
- The mechanical energy of the satellite decreases.

Examples:

1. A satellite of mass 100kg is inn a circular orbit at a height of 3.59×10^7m above the earth's surface. Find the mechanical energy of the satellite. (Mass of earth = 6×10^{24}kg, radius of earth = 6.4×10^6m)

Mechanical energy = $-\dfrac{GM_e m}{2r}$

$r = 3.59 \times 10^7 + R_e = 3.59 \times 10^7 + 6.4 \times 10^6\text{m} = 4.23 \times 10^7\text{m}$

Where $R_e = 6.4 \times 10^6$m, the radius of the earth.

$Me = 6 \times 10^{24}$kg

Mechanical energy = $-\dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 100}{2 \times 4.23 \times 10^7} = -4.71 \times 10^8$ joules

2. A satellite of mass 250kg makes a circular equatorial orbit at a distance 500km above the earth's surface. Find

(i) the radius of the orbit

(ii) the period

(iii) the total energy of the satellite

(i) radius $r = 500 \times 10^3 + 6.4 \times 10^6 = 6.9 \times 10^6$m

(ii) $T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$

Where $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}

$M = 6 \times 10^{24}$kg

$T^2 = \dfrac{4\pi^2 x(6.9x106)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}$

Hence T = 5.69×10^{3}s

(iii) Total energy = $-\frac{GM_e m}{2r}$ = $-\frac{6.67\times10^{-11}\times6\times10^{24}\times250}{2\times6.9\times10^{6}}$ = -7.25×10^{9}J

Exercise

1. A mass is released from a point at a distance of 10R from the centre of the earth, where R is the radius of the earth. Find the speed of the mass at a point a distance of 7R from the centre of the earth. (Assume R = 6.4×10^{6}m)

2. Calculate the ratio of mass of the sun to that of the earth, given that the moon moves round the earth in a circular orbit of radius 4.0×10^{5}km with a period of 27.3days, and the orbital radius of the earth round the sun is 1.5×10^{8}km and its period is 365days. (2.95×10^{5})

3. Calculate the ratio of acceleration due to gravity on the surface of mercury to that on the surface of the earth given that the radius of mercury is 0.38 times that of the earth and the mean density of mercury is 0.68 times that of the earth (0.2584)

CHAPTER ELEVEN: SIMPLE HARMONIC MOTION (S.H.M)

It is a special type of periodic motion in which the acceleration of the body along the path of the body is directed towards a fixed point in the line of motion and is proportional to the displacement of the body from the fixed point.

Characteristic of a body describing Simple harmonic motion

- Motion is periodic
- Acceleration of the body is towards a fixed point
- Acceleration of the body is directly proportional to the distance from the fixed point
- Mechanical energy is conserved.

Equation of simple harmonic motion

Acceleration, $a = -\omega^2 x$

Where ω is angular velocity, x is displacement from fixed point.

Or $$a = \frac{d^2x}{dt^2} = -\omega^2 x$$

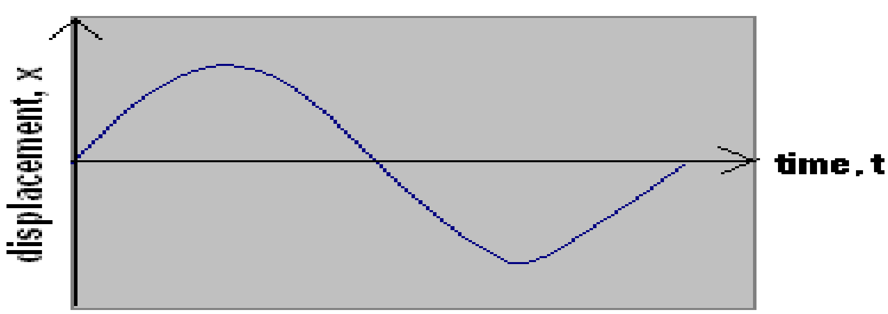
The solution of the above differential equation is

$X = A\cos\omega t$ or $X = A\sin\omega t$

Where A is the maximum displacement of the body called Amplitude.

For $X = A\sin\omega t$ the curve is as below:

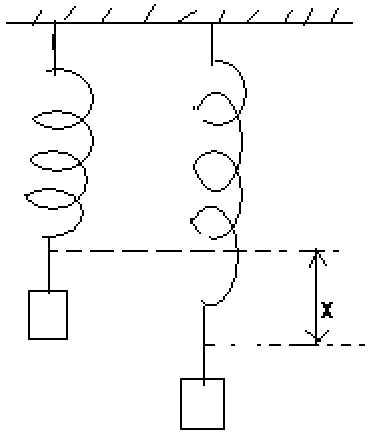
A graph of displacement against time for S.H.M



In general $X = A\sin(\omega t + \Phi)$ Where Φ is the phase angle.

Examples of Simple Harmonic Motion

(i)***Vertical Spiral Spring or Elastic thread***



Consider a body of mass m suspended from a spiral spring of force constant, k, as shown in the diagram. In that case the body will be at equilibrium.

At equilibrium, T = mg

But T = ke (Form Hook's Law)

Where e is the extension in the spring at equilibrium and k is the force constant of the spring.

Hence ke = mg …………………………………(i)

When the mass is pulled through a distance x then released, the resultant upward force on the mass is

F = T' – mg

But T' = k (e + x)

F = k (e + x) – mg

But from (i) ke = mg

F = k (e + x) – ke

F = k x

From Newton's 2nd law, ma = F

ma = - kx

$$a = -\left(\frac{k}{m}\right)x$$

The above equation is in the form $a = -\omega^2 x$

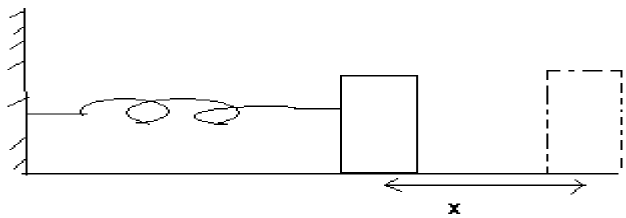
Where $\omega^2 = \left(\frac{k}{m}\right)$

Question: Prove that the period T is given by: T = $2\pi\sqrt{\frac{m}{k}}$

(ii) ***Horizontal*** **Spiral** ***Spring***

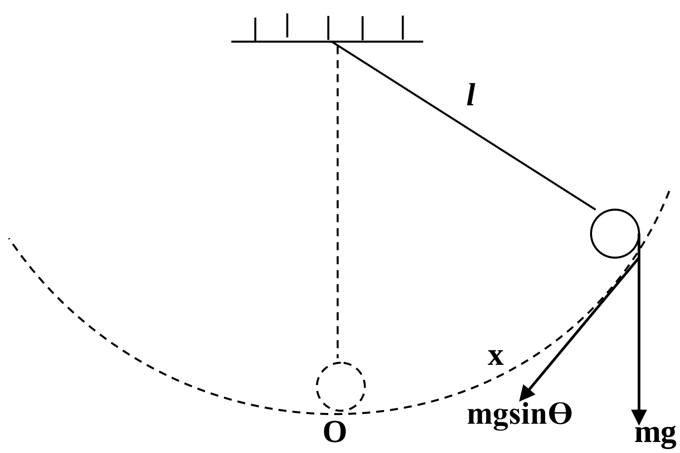
Consider a horizontal spring of force constant k. One end of the spring is fixed and the other end attached to a body of mass m resting on a smooth surface.

If the body is displaced through a distance x



(iii) Simple Pendulum

Suppose a body of mass m attached to a string is displaced through a small angle Θ and then released. The resultant force on the body towards O is mgsinΘ.



By Newton's 2nd law

ma = -mgsinΘ

a = - gsinΘ

If Θ is small and measured in radians $\Theta \cong \sin\Theta = \frac{x}{l}$

$a \cong - g\Theta = g\frac{x}{l}$

Which is in the form $a = -\omega^2 x$

Where $\omega^2 = \frac{g}{l}$

Hence T = $2\pi\sqrt{\frac{l}{g}}$

Example: A simple pendulum has a period of 4.2s. When the length is shortened by 1m, the period is 3.7s. Use these measurements to determine the acceleration due to gravity and the original length of the pendulum.

T = $2\pi\sqrt{\frac{l}{g}}$

$4.2^2\, g = 4\pi^2 x l$……………………… *(1)*

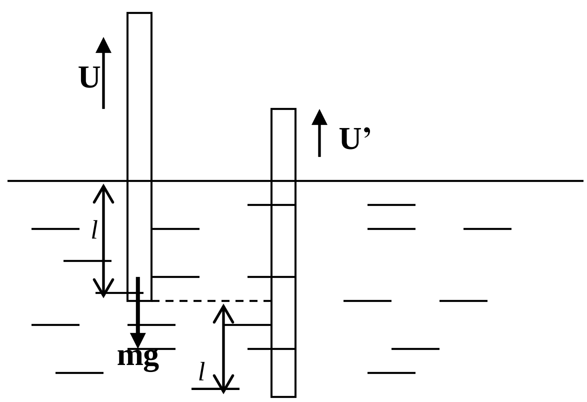
$3.72g = 4\pi 2(l - 1)$………………. (2)

Solving the above two equations, you get

L = , g =

(iv) A Floating cylinder

Consider a cylinder of mass m, floating vertically in a liquid of density ρ to a depth *l*.



In equilibrium, mg = U where U is up thrust

But U = A*l* ρg

mg = A*l*ρg ………………………………(i)

m = A*l*ρ

A is the cross sectional area of the cylinder

Suppose the cylinder is given a small vertical displacement x and released, the net force on the cylinder is U' – mg.

But U' = A (l +x)ρg

Net force = A (l +x)ρg – mg

From Newton's 2nd law; $ma = -A(l + x)\rho g - mg$

From equation (i) $mg = Al\rho g$

Therefore $ma = -Ax\rho g$

$$a = -\frac{(A\rho g)x}{m}$$

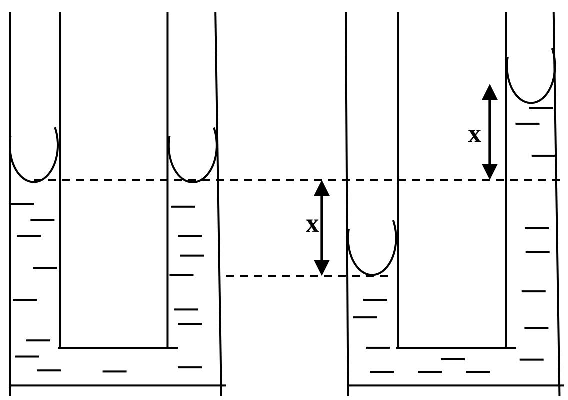
But $a = -\omega^2 x$

Where $\omega^2 = \frac{A\rho g}{m}$

And period $T = 2\pi\sqrt{\frac{A\rho g}{m}} = 2\pi\sqrt{\frac{l}{g}}$

(v) ***Oscillation of a liquid in a U – tube***

Consider a liquid column of length *l* at rest in a U – tube of cross section area A. Suppose the liquid is displaced by a small distance and then released.



Consider the instant when the meniscus a distance x from their equilibrium position. The restoring force of the liquid = 2xAρg, where ρ is the density of the liquid.

Using Newton's 2nd law,

$$ma = -2xA\rho g$$

$$a = -\frac{(2xA\rho g)}{m} = -\frac{(2A\rho g)}{m}x$$

Hence $\omega^2 = -(2A\rho g)$

m

Period $T = 2\pi\sqrt{\frac{l}{2g}}$

Velocity of a body executing Simple Harmonic Motion

Consider the displacement of a body executing Simple harmonic motion to be given by

$X = A\sin(\omega t + \Phi)$

Velocity, $v = dx/dt = A\omega\cos(\omega t + \Phi)$

$\sin(\omega t + \Phi) = X/A$

$\cos(\omega t + \Phi) = \frac{\sqrt{A^2 - x^2}}{A}$

Hence $v = A\omega \frac{\sqrt{A^2 - x^2}}{A} = \omega\sqrt{A^2 - x^2}$

When x = 0, V is maximum

$v_{max} = \omega A$

when X= A, v = 0

Kinetic energy and potential energy of vibrating object

Kinetic enrgy, E_k

Velocity $v = \omega\sqrt{A^2 - x^2}$

Kinetic energy $E_k = ½mv^2 = ½m\omega^2(A^2-x^2)$

$$E_k = ½m\omega^2(A^2-x^2)$$

For a spring of force constant, k ; $\omega^2 = \frac{k}{m}$

$$k = \omega^2 m$$

$$E_k = ½k(A^2-x^2)$$

Potential energy, E_p

Work done against the restoring force is the potential energy

$F = m\omega^2 r$

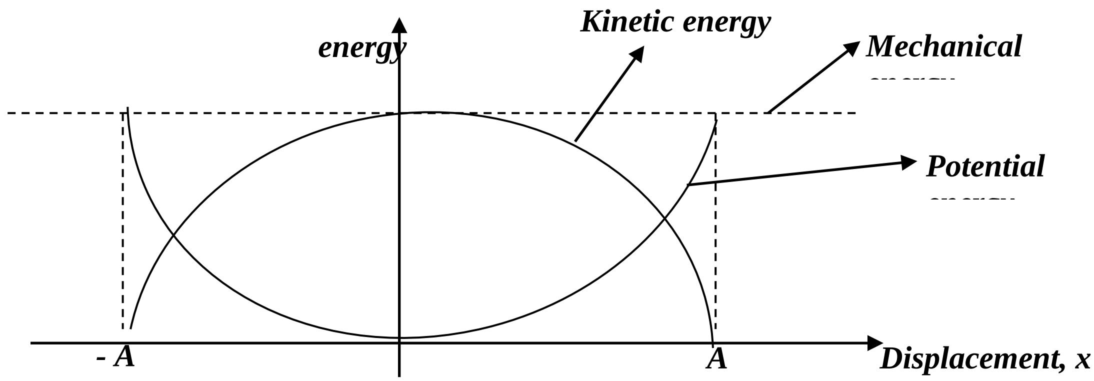
Therefore, $E_p = \int_0^x F dr = \int_0^x m\omega^2 r dr = \frac{1}{2}m\omega^2x^2$

For a vibrating spring, $E_p = \frac{1}{2}kx^2$

Total mechanical energy $E_T = E_k + E_p = \frac{1}{2}m\omega^2(A^2-x^2) + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2$

$$E_T = \frac{1}{2}m\omega^2A^2$$

Note total energy of a vibrating object (a particle undergoing S.H.M) is constant and is directly proportional to the square of the amplitude. Hence mechanical energy is conserved in S.H.M.



Examples:

1. A light spiral spring is loaded with a mass of 50g and it extends by 10cm. Calculate the period of small vertical oscillations

Using $T = 2\pi\sqrt{\frac{m}{k}}$ **,** but mg = ke

$K = mg/e = 0.05 \times 9.81/0.1 = 4.905Nm^{-1}$

Hence $T = 2\pi\sqrt{\frac{0.05}{4.905}} = 0.63s$

2. A body of mass 0.1kg hangs from a long spiral spring. When pulled down 10cm below its equilibrium point A, and released, it vibrates with S.H.M with a period of 2s.

(i) What is the velocity as it passes through A?

(ii) What is its acceleration when it is 5cm above A.

Solution

(i) $v = \omega A$, where $A = 0.1m$, $\omega = 2\pi/T$, but $T = 2s$

$\omega = 2\pi/2 = \pi\ rads^{-1}$

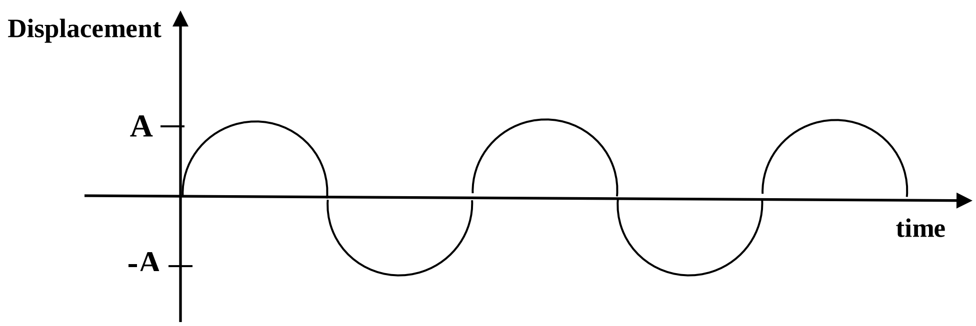
$V = \pi \times 0.1 = 0.314ms^{-1}$

(ii) $a = -\omega^2 x = \pi^2 \times 0.05 = 0.5ms^{-2}$

Types of oscillations

(i) Free oscillations:

Free oscillations occur in the absence of any dissipative forces like air resistance, friction and viscous drag. The amplitude and total mechanical energy remains constant and the system oscillates indefinitely with a period T (the natural period of vibration of the system)



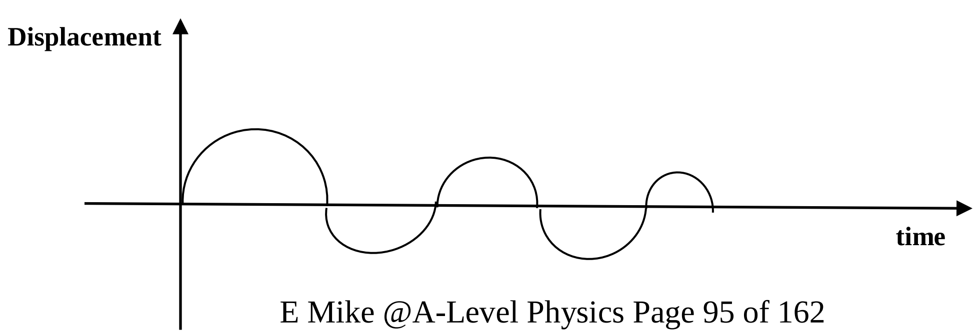
A is amplitude

e.g A simple pendulum will undergo free oscillation in a vacuum.

(ii) Damped oscillations

These are oscillations where the system loses energy to the surrounding due to the dissipative forces. The amplitude reduces with time and oscillations eventually die out. Damped oscillations can be grouped into under damped, critically damped and over damped oscillations.

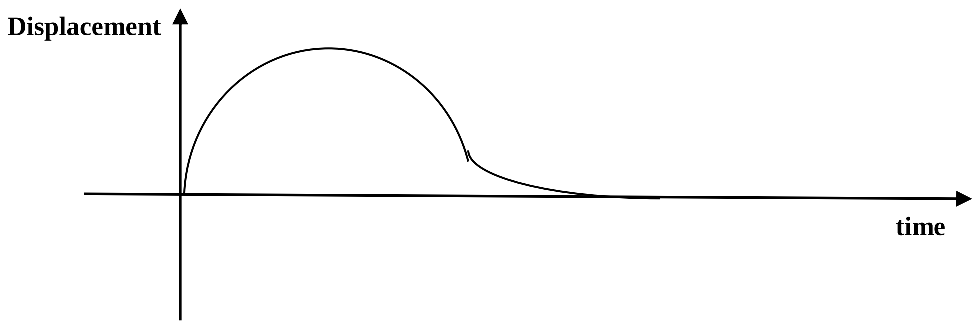
- Under – damped oscillations



The system actually oscillates but gradually dies out due to the dissipative forces. The amplitude of oscillation decreases with time. Examples are a simple pendulum in air, horizontal spring moving over a surface of little roughness.

- Critically damped oscillations

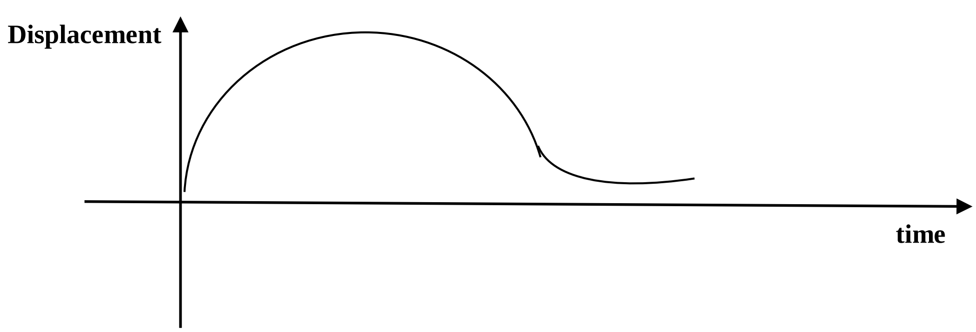
The system does not oscillate when displaced, but returns to the equilibrium position in the minimum possible time



Examples shock absorbers in cars stops the car to oscillate after passing over the hump, toilet doors are critically damped so that they close very quickly.

- Over damped oscillations

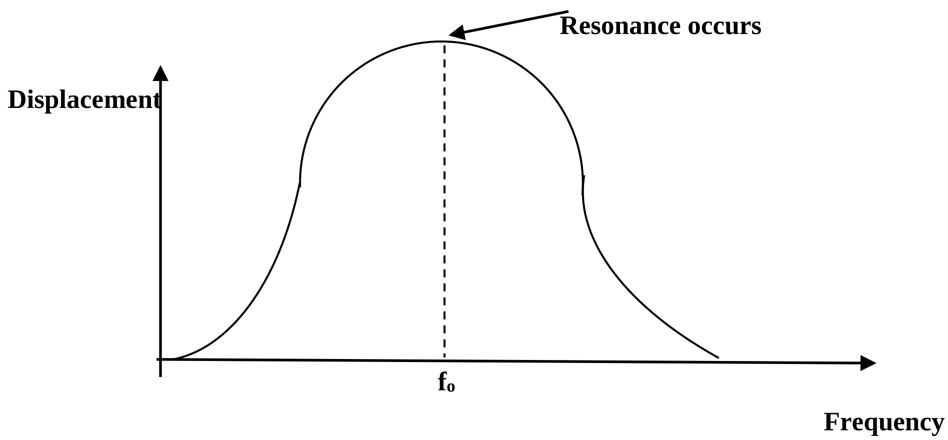
The system does not oscillate but takes a long time to return to the equilibrium position.



Examples: a horizontal spring moving over a very rough surface, a metal cylinder attached to a vertical spring and made to move in a very viscous liquid.

Forced oscillations

These are vibrations where the system is subjected to an external periodic force thus setting the system to oscillate indefinitely. When the periodic force has the same frequency of oscillation as the system, resonance occurs. Examples of forced oscillations are; the oscillation of a diving board, the oscillations of the earth quake and the oscillations of the air column in musical pipe instruments e.t.c



f_o is the fundamental frequency

Exercise:

1. The pendulum of length 130cm has a periodic time T_1. A bob now pulled a side and made to move as a conical pendulum in a horizontal circle of radius 50cm. the period of rotation is T_2. Find the ratio
of T_1: T_2 (1.04)

2. A spring gives a displacement of 5cm for a load of 500g. Find the maximum displacement produced when a mass of 80g is dropped from a height of 10cm onto a light pan attached to the spring.
($5x10^{-2}$m)

3. A small mass rests on a horizontal platform which vibrates vertically in a simple harmonic motion with a period of 0.50s. Find the maximum amplitude of the motion which will allow the mass to remain in contact with the platform throughout the motion.($6.3x10^{-2}$m)
4. A mass of 0.1kg suspended from a spring of force constant $24.5Nm^{-1}$ is pulled vertically downwards through a distance of 5.0cm and released. Find the

(i) period of oscillation (0.4s)

(ii) position of the mass 0.3s after release(0m)

5. A uniform cylindrical rod of length 8cm, cross sectional area $0.02m^2$ and density $900kgm^{-3}$ floats vertically in a liquid of density $1000kgm^{-3}$. The rod is depressed through a distance of 0.5cm and the released.

   i) Show that the rod performs simple harmonic motion

   ii) Find the frequency of the resultant oscillations (1.86Hz)

   iii) Find the velocity of the rod when it is a distance of 0.4cm above the equilibrium position. ($0.035ms^{-1}$)

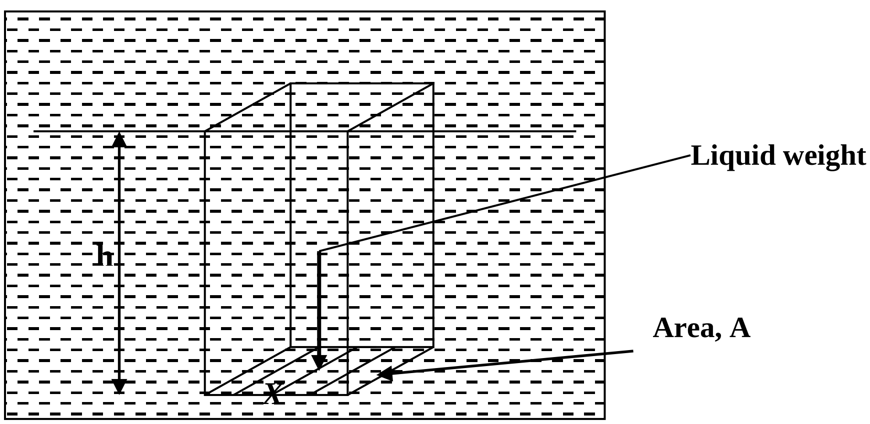
CHAPTER TWELEVE: HYDROSTATICS

Pressure

The pressure at a point in a fluid is the force per unit area acting normal to an infinitesimal area taken about the point. The S.I unit of pressure is Nm^{-2} or Pascal (Pa). The pressure in a column of fluid increase with depth. Pressure in fluids is a scalar quantity since its transmitted equally in all directions and so there is no single direction along which it acts. Therefore at a given point in a liquid, pressure acts in any direction.

Formula for pressure in liquids

Consider a column of liquid of depth, h and density, ρ in a container of crosssectional area, A as shown in the figure below.



Since the volume of this liquid is Ah, the mass of the liquid = Ahρ.

The weight = Ahρg, where g is acceleration due to gravity.

Therefore the pressure, P = $\frac{force}{area} = \frac{Ah\rho g}{A} = h\rho g$

$$\text{Pressurein Liquids} = h\rho g \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Therefore the pressure in liquids is independent of the cross-sectional area and shape of the vessel in which it is poured (proved using communicating tubes in O'level pressure). The pressure acts in all directions, and it depends only on the;

- Depth, h
- Density, ρ
- Atmospheric pressure.

Density

Is the mass per unit volume of a substance. $\Rightarrow density = \frac{mass}{volume}$,

The S.I unit of density is kgm^{-3} and other unit used include gcm^{-3}

Note: 1 gcm^{-3} = 1000 kgm^{-3}

<u>Experiment to determine the density of an irregular solid which floats in water</u>

A thread is tied to the irregular solid and its weight $\boldsymbol{w_1}$ determined. A sinker such as a stone is attached to the irregular solid and the weight w_2 of the two in air is determined. The sinker and the solid are completely immersed in water of known density and their weight w_3 is determined by a spring balance. The sinker is detached from the solid and the weight w_4 of the sinker only when in water is determined.

Weight of irregular solid in water = $(w_3 - w_4)$

From: $\text{Relative density of irregular solid} = \frac{\text{weight in air}}{\text{apparent weight in water}}$

$$= \frac{w_1}{w_1 - (w_3 - w_4)}$$

From: $\text{Density} = \text{Relative density} \times \text{Density of water}$, the density of the irregular solid can be determined.

Relative Density (R.D)

Definition: relative density is the ratio of any mass of the substance to the mass of an equal volume of water

$$R.D = \frac{\text{Mass of substance}}{\text{Mass of an equal volume of water}} = \frac{\text{Density of a substance}}{\text{Density of water}} = \frac{\text{Weight of a substance}}{\text{Wt of an equal vol. of water}}$$

Any of the above formula can be used to find the R.D of both liquids and solids.

To find the R.D of solids only

$$R.D = \frac{\text{mass of substance in air}}{\text{Apparent loss of mass of the substance when in water}}$$

$$= \frac{\text{Weight of substance in air}}{\text{Apparent loss of weight of the substance while in water (upthrust)}}$$

To find the R.D of liquids only

$$R.D = \frac{\text{Apparent loss in mass of substance when in the liquid}}{\text{Apparent loss in mass of the substance when in water}}$$

$$= \frac{\text{Apparent loss in Weight of a substance when in the liquid}}{\text{Apparent loss of weight of the substance while in water (upthrust)}}$$

Example

A block of mass 0.1kg is suspended from a spring balance. When the block is immersed in water of density 1000 kgm^{-3}, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density, the spring balance reads 0.70N. Find the;

(i) Density of the mass

(ii) Density of the liquid

Solution

(i) Mass of solid in air = 0.1 kg

$\Rightarrow$ Weight solid in air $= 0.1 \times 9.81 = 0.981N$

Weight of solid in water = 0.63N

Apparent loss of weight of the solid in water = 0.981 – 0.63 = 0.351N

$$R.D = \frac{\text{Weight of solid in air}}{\text{Apparent loss of weight of solid when immersed in water}}$$

$$= \frac{0.981}{0.51} = 2.795$$

$$\text{But R.D} = \frac{\text{Density of solid}(\rho_s)}{\text{Density of water}}$$

$$\Rightarrow 2.795 = \frac{\rho_s}{1000}$$

$$Density\,of\,solid = 2.795 \times 1000 = 2795 kgm^{-3}$$

(ii) Apparent loss in weight of solid in liquid = 0.981 – 0.7 = 0.281N

$$R.D \text{ of liquid} = \frac{\text{Apparent loss of weight of solid in the liquid}}{\text{Apparent loss of weight of the solid when immersed in water}}$$

$$= \frac{0.281}{0.351} = 0.8$$

$$Density\,of\,liquid = 0.8 \times 1000 = 800\text{kgm}^{-3}$$

Archimedes principles

When an object is immersed in a fluid, it experiences an upward force called up thrust which is equal to the weight of the fluid displaced.

Archimedes principle states that when a body is wholly or partially immersed in a fluid, it experiences an upward force (upthrust) which is equal to the weight of the fluid displaced.

It should be noted that upthrust is also called buoyant force and so the tendency of a fluid to exert an upward force on a body immersed in it is called buoyancy.

Examples

1. A string supports a solid iron object of mass 0.18 kg totally immersed in a liquid of density 800 kgm^{-3}. Calculate the tension in the string if the density of iron is 8000kgm^{-3}.

Solution

Weight of iron in air = mg = 0.18×9.81 = 1.77N

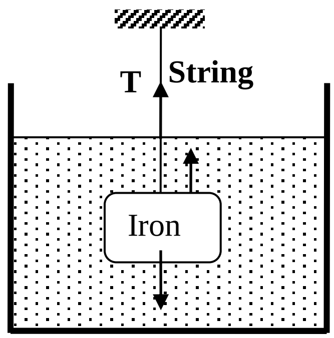
$$\text{Volume of iron} = \frac{\text{mass}}{density} = \frac{0.18}{8000} = 2.25\times10^{-5}m^3$$

$\therefore$ volume of liquid displaced when the iron is immersed $= 2.25\times10^{-5}m^3$

$\Rightarrow$ mass of liquid displaced = volume of liquid × density of liquid $= (2.25\times10^{-5}\times800) = 0.018kg$

$\Rightarrow$ weight of liquid displaced = mg $= 0.018\times9.81 = 0.177$N

Illustration



From the Archimedes principle, upthrust is equal to the weight of the liquid displaced.

Therefore, u = 0.177N

For the iron to attain equilibrium in the liquid, mg = T + u

$\Rightarrow 1.77 = T + 0.17$

$T = 1.593N$

2. A specimen of an alloy of silver and gold whose densities are 10.5 gcm^{-3} and 18.9 gcm^{-3} respectively, weighs 35.2g in air and 33.13 g in water. Find the composition by mass of the alloy, assuming that there has been no volume change in the process of producing the alloy. Assume that the density of water is 1 gcm^{-3}

Solution

Given

$\rho_s = 10.5\,\text{gcm}^{-3}$, $\rho_g = 18.9\,\text{gcm}^{-3}$

Let the volume of silver be v_s, and that of gold be v_g

Also, let the mass of silver be m_s, and that of gold be m_g

Mass of alloy in air, $m_a = 35.2g$

Mass of alloy in water, $m_w = 33.13g$

$\Rightarrow (m_s + m_g) = m_a = 35.2g$..............(i)

Apparent loss in mass when alloy is immersed water $= m_a - m_w = 35.2 - 33.13 = 2.07g$

$$R.D = \frac{\text{mass of alloy in air}}{\text{Apparent loss in mass while in water}} = \frac{35.2}{2.07} = 17$$

Density of alloy $=$ R.D $\times$ density of water $= 17 \times 1 = 17\text{gcm}^{-3}$

Volume of alloy $= \dfrac{m}{\rho} = \dfrac{35.2}{17} = 2.07cm^3$

But volume of alloy $= v_s + v_g = \dfrac{m_s}{\rho_s} + \dfrac{m_g}{\rho_g} = \dfrac{m_s}{10.5} + \dfrac{m_g}{18.9}$

$\therefore 2.07 = \dfrac{m_s}{10.5} + \dfrac{m_g}{18.9}$.................(ii)

From equation (i), $m_g = 35.2 - m_s$...............(iii)

Combing equations (ii) and (iii), we have;

$$2.07 = \frac{m_s}{10.5} + \frac{35.2 - m_s}{18.9}$$

$\therefore 410.79 = 369.6 - 10.5m_s + 18.9m_s \Rightarrow m_s = 4.9g$ and $m_g = 30.3g$

Exercise

1. A piece of metal of mass 2.60g and density 8.4g/cm^3 is attached to a block of wax of mass 1.0g and density 0.92g/cm^3. When the system is placed in a liquid, it floats with wax just submerged. Find the density of the liquid.

2. A cubical block of wood of side 12cm floats at the interface between oil and water with its lower surface 4cm below the interface. The heights of the oil and

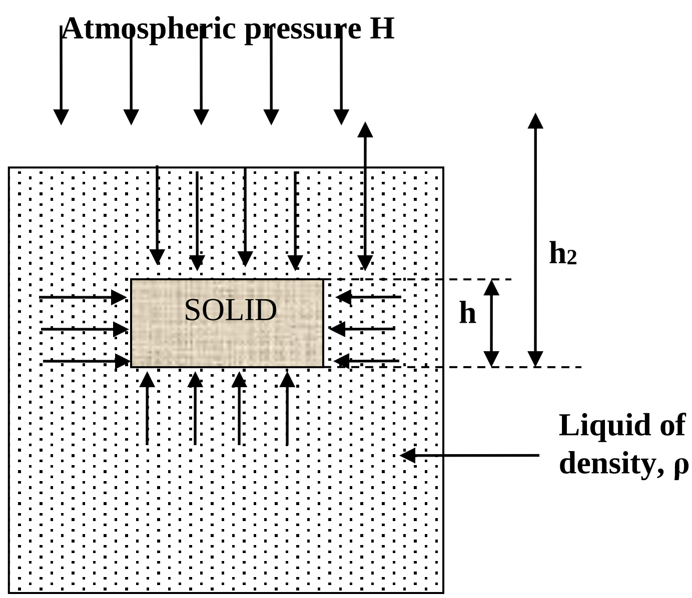
water columns are 10 cm each. If the density of oil is $0.8 gcm^{-3}$ and that of water is $1 gcm^{-3}$, calculate the;

(i) Mass of the block

(ii) Pressure on the lower surface of the block

<u>Proof of Archimedes principle</u>

Consider a uniform solid cylinder of length L, cross sectional area A. Suppose the cylinder is submerged in a liquid of density ρ, so that its face is a depth h, below the surface of the liquid as shown in the figure below



Consider a solid immersed in a liquid, the pressure on the lower surface CD is greater than on the upper surface AB, since the pressure at the greater depth h_2 is more than at h_1 (pressure varies with depth experiment).

It follows that;

- Pressure on the side $= H + h_1\rho g \Rightarrow$ Down ward force on side AB $= A(H + h_1\rho g)$
- Pressure on the side CD $H + h_1\rho g \Rightarrow$ Upward force on side CD $= A(H + h_2\rho g)$

- Since $h_1 \angle h_2$, the downward force is less than upward force.

$\Rightarrow$ Resultant upward force *(upthrust)* $= A(H + h_2\rho g) - A(H + h_1\rho g)$
Therefore, upthrust $= A\rho g(h_2 - h_1)$.........................(i)
Also, volume of solid $= Ah$ $\Rightarrow$ volume of liquid displaced $= Ah$
But $h = (h_2 - h_1)$
$\Rightarrow$ volume of liquid displaced $= A(h_2 - h_1)$
Therefoe, mass of liquid displaced $= A\rho(h_2 - h_1)$
$\Rightarrow$ Weight of liquid displaced $= A\rho g(h_2 - h_1)$................(ii)

It can be seen from equations (i) and (ii) that upthrust is equal to the weight of liquid displaced, therefore up thrust = weight of liquid displaced, which is ***Archimedes principle***

The reader should note that upthrust depends on the density of a body relative to the medium (fluid) in which it is immersed. This explains why a helium filled balloon rises up to a certain height in still air and then stops. Initially, the balloon rises because the upthrust due to air is greater than the weight of the balloon, resulting from the fact that density of air is greater than that of helium. As the balloon rises up, the density of the air decreases, implying that upthrust also decreases. The balloon stops rising when it is at a height when the upthrust is equal to its weight.

Measurement of density or relative density using Archimedes' principle.

For a solid, weigh the mass of solid in air say, m_o. Then weigh its mass when totally immersed in water say, m_1. Then upthrust = $(m_o - m_1)$ g = weight of water displaced.

Therefore relative density of the solid = $\dfrac{m_0}{m_0 - m_1}$

Density of the solid = $\dfrac{m_0}{m_0 - m_1}$ x density of water

For a liquid: Weigh the mass of solid in air say, m_0, then weigh it when totally immersed in the liquid whose density is required say m_1 and finally weigh it when totally immersed in water say m_2.

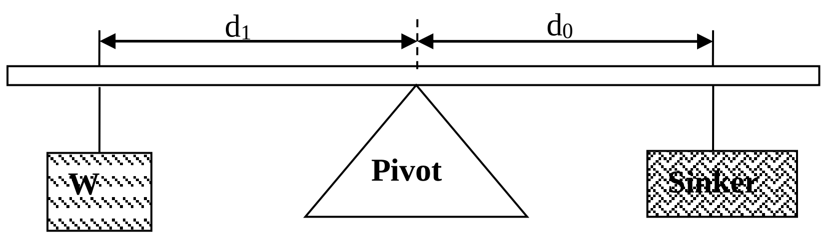
Relative density = $\frac{\text{upthrust in liquid}}{\text{upthrust in water}}$

$$= \frac{m_0 - m_1}{m_0 - m_2}$$

And density $= \frac{m_0 - m_1}{m_0 - m_2}$ x density of water.

An experiment to determine the relative density of a liquid using Archimedes principle and the principle of moments.

Consider the weights of solid and sinker in air as shown below.

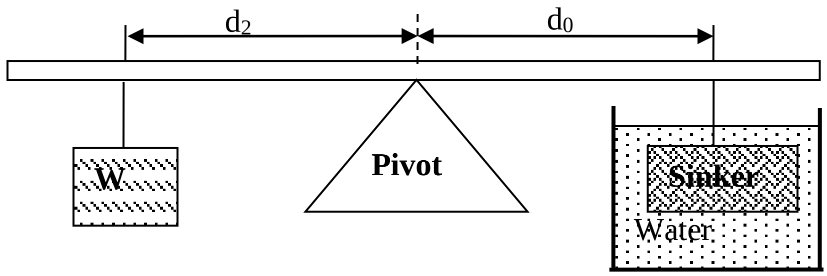


While in air, the sinker (solid) and weight W are attached to the metre rule as shown in the figure above. The weight is adjusted until the metre rule balances horizontally and then distances d_1 and d_0 are measured and recorded.

If the weight of sinker in air is w_1, the taking moments about the pivot gives;

$$w_1 d_0 = W d_1 \Rightarrow w_1 = W \frac{d_1}{d_0} \text{.........................}(i)$$

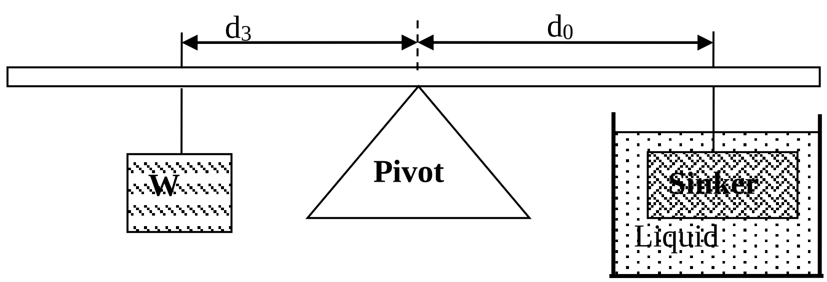
The sinker is then immersed beaker of water while keeping d_0 constant. The position of the weight W is adjusted until balance is restored, then the distance d_2 is measured and recorded as shown in the figure below.



If w_2 is the weight of the sinker in water, then taking moments about the pivot gives;

$$w_2 d_0 = W d_2 \Rightarrow w_2 = W\frac{d_2}{d_0}\text{...........................}(ii)$$

The sinker is then immersed in a liquid in a beaker while keeping d_0 constant. The position of weight W is adjusted until balance is restored. The distance d_3 is measured and recorded as shown in the figure below.



If w_3 is the weight of the sinker in the liquid, then taking moments about the pivot;

$$w_3 d_0 = W d_3 \Rightarrow w_3 = W\frac{d_3}{d_0}\text{...........................}(iii)$$

By definition, Relative density(R.D) = $\dfrac{\text{Apparent loss of weight of the sinker while in liquid}}{\text{Apparent loss in weight of the sinker while in water}}$

$$R.D = \frac{W\frac{d_1}{d_0} - W\frac{d_3}{d_0}}{W\frac{d_1}{d_0} - W\frac{d_2}{d_0}} = \frac{d_1 - d_3}{d_1 - d_2}, \text{ where } d_3 \angle d_2 \angle d_1$$

It should be noted that one of the advantages of such a method is that the relative density can be determined even when the weights are not known.

FLOTATION

A body floats in a fluid if its density is less than that of the fluid in which it is placed

The **law of flotation** states that; A floating body displaces its own weight of the fluid in which it floats.

Proof

For the floating body to be in equilibrium, $\text{upthrust} = \text{Weight of floating body}$

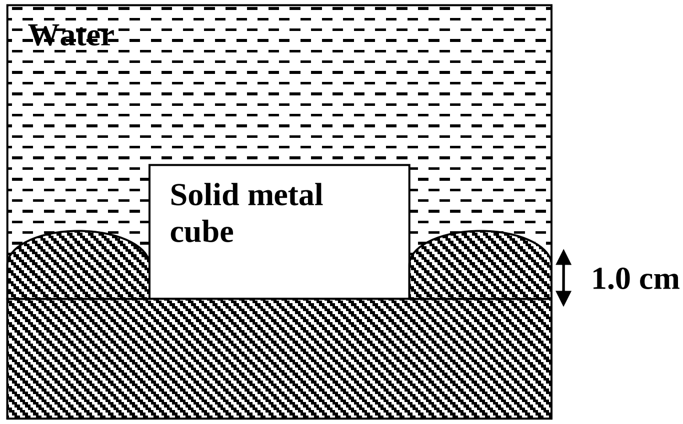
But from Archimedes principle, $\text{upthrust} = \text{weight of fluid displaced}$

$\therefore \text{Weight of floating body} = \text{Weight of fluid displaced}$, Which is the law of flotation

It can therefore be correct to say that; $\text{Mass of floating body} = \text{Mass of fluid displaced}$

Examples

1. A solid metal cube of side 8cm floats vertically at the interface between water and mercury as shown in the figure below. The lower surface of the cube is 1 cm below the interface. Given that the density of mercury is 13600 kgm^{-3}, and that of water is 1000 kgm^{-3}, calculate the density of the metal.



Solution

Volume of solid metal cube = $l \times w \times h = 8 \times 8 \times 8 = 512cm^3 = 5.12 \times 10^{-4} m^3$

Volume of mercury displaced $= 8 \times 8 \times 1 = 64cm^3 = 6.4 \times 10^{-5} m^3$

$\Rightarrow$ Mass of mercury displaced = volume × density = $6.4 \times 10^{-5} \times 13600 = 0.87kg$

Volume of waterv displaced = $8 \times 8 \times 7 = 448\,\text{cm}^3 = 4.48 \times 10^{-4} m^3$

$\Rightarrow$ Mass of water displaced = volume × density = $4.48 \times 10^{-4} \times 1000 = 0.448kg$

$\therefore$ *Total mass of liquid displaced* $= (0.87 + 0.448) = 1.318kg$

But from the law of flotation, mass of floating object is equal to mass of liquid displaced.

$\therefore$ Mass of the metal cube = 1.318kg

From density $= \frac{\text{mass}}{\text{volume}}$, density of solid metal cube $= \frac{1.318}{5.12 \times 10^{-4}} = 2574.2kgm^{-3}$

2. An alloy contains two metals X and Y of densities 3.0×10^3 kgm^{-3} and 5.0×10^3 kgm^{-3} respectively. Calculate the density of the alloy if

(i) The volume of X is twice that of Y

(ii) The mass of X is twice that of Y

Solution

(i) Let the volume of Y be v $\Rightarrow$ volume of X is 2v

Also let the mass of X be x and that of Y be y

From density $= \frac{mass}{volume}$, $3000 = \frac{x}{2v} \Rightarrow x = 6000v..........(i)$ and

$$5000 = \frac{y}{v} \Rightarrow v = \frac{y}{5000}.......(ii)$$

Substituting for v in equation (i), gives

$$x = 6000 \times \frac{y}{5000}, \Rightarrow y = \frac{5}{6}x................(iii)$$

Therefore, total mass of alloy = $x + y = x + \frac{5}{6}x = \left(\frac{11}{6}x\right)$ kg.....(iv)

Also, volume of alloy = $2v + v = 3v$

$$\text{But density} = \frac{\text{mass}}{\text{volume}} = \frac{11/6\, x}{3v} = \frac{11x}{18v}..............(v)$$

Combining equations (i) and (v), we have;

$$Density = \frac{11 \times 6000v}{18v} = 3.7 \times 10^3\, kgm^{-3}$$

(ii) Let the mass of Y be m, $\Rightarrow$ mass of X is 2m

Let the volume of X be v_x and that of Y be v_y

$$3000 = \frac{2m}{v_x} \qquad \Rightarrow v_x = \frac{m}{1500}..................(i)$$

$$5000 = \frac{m}{v_y} \qquad \Rightarrow m = 5000v_y..................(ii)$$

Combining equations (i) and (ii), we have

$$v_x = \frac{5000 v_y}{1500} \quad \Rightarrow v_y = \frac{3}{10} v_x \text{....................}(iii)$$

Total volume $= v_x + v_y = v_x + \frac{3}{10} v_x = \frac{13}{10} v_x \text{ m}^3$

Also, total mass = $2m + m = (3m)$ kg

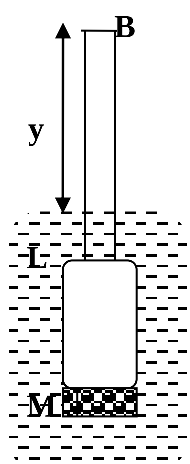
$$\text{But density} = \frac{\text{mass}}{\text{volume}} = Density \text{ of alloy} = \frac{3m}{(\frac{13}{10} v_x)} = \frac{30m}{13 v_x} \text{...............}(iv)$$

Combining equations (i) and (iv), we have;

$$Density\, of\ alloy = \frac{30m}{13 \times \left(\frac{m}{1500}\right)} = 3.5 \times 10^3 kgm^{-3}$$

THE HYDROMETER

This is a device for comparing densities of liquids. It consists of a uniform stem having a loaded bulb at the bottom. The stem is graduated in which it is placed.



Practical hydrometers have a weighted end **M** for stability, a wide bulb to produce sufficient upthrust to counterbalance the weight, and a narrow stem BL for sensitivity. If V is the whole volume of the hydrometer, **a** is the area of the stem and **y** is the length not immersed in a liquid of density,ρ, then upthrust = weight of liquid displaced. Therefore; (V – ay)ρ = w, where w is the weight of the hydrometer.

Examples.

1. A hydrometer consists of a spherical bulb and a cylindrical stem, which has a cross-sectional area of 0.6 cm^3. The total volume of the bulb and the stem is 14.3 cm^3. When immersed in water, the hydrometer floats with 7.6 cm of the stem above the water surface. When in alcohol, it floats with 2.0 cm of the stem above the surface. If the density of water is 1g/cc, calculate the density of alcohol.

Solution

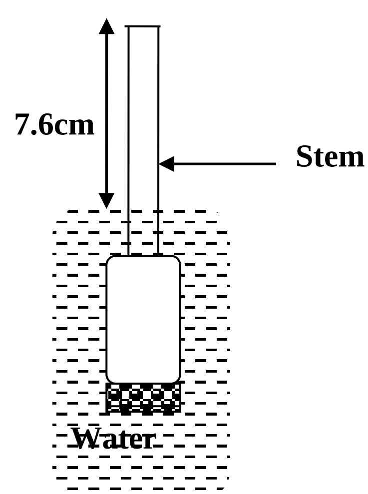
Volume of hydrometer above is equal to volume of its stem above the water

$$= Ah = 0.6 \times 7.6 = 4.56cc$$

Therefore, volume of hydrometer in water $= 14.3 - 4.56 = 9.74cc$

It follows that volume of water displaced = 9.74 cc

Consider the illustration below;



Mass of water displaced = (volume)×(density) = $9.74 \times 1 = 9.74g$

From the law of flotation, a floating body displaces its own weight of the liquid in which it floats

$\Rightarrow mass\,of\ hydrometer = 9.74g$

When the hydrometer is placed in alcohol:

Volume of alcohol displaced = $14.3-(Ah)=14.3-(0.6\times 2)=13.1cc$

Therefore, mass of alcohol displaced = (volume) × (density) = $(13.1\times \rho)g$

But, mass of hydrometer should be equal to the mass of alcohol displaced

$$\Rightarrow 9.74 = 13.1\rho$$
$$\therefore \rho = 0.744g/cc$$
$$Therefore, density\,of\ alcohol = 0.744g/cc = 744kgm^{-3}$$

2. A simple hydrometer consisting of loaded glass blub fixed at the bottom of a glass stem of uniform cross-sectional area sinks in water of density 1.0g/cc, so that a certain mark x on its stem is 4.0 cm below the surface of water. When placed in a liquid of 0.9g/cc, the hydrometer floats with the mark 6.0 cm below the surface of the liquid. How far below the surface will the mark be if the hydrometer is placed in a liquid of density 1.1g/cc.

Solution

Let the cross-sectional area of the stem for the hydrometer be A, also let the weight of the hydrometer in air be w_a

While in water: volume of water displaced = 4A

Mass of water displaced = (volume ×density) = $4A\times 1 = 4A$

Weight of water displaced = $4Ag$

While in the first liquid: volume of liquid displaced = 6A

Mass of liquid displaced = (volume ×density) $6A\times 0.9 = 5.4A$

Weight of first liquid displaced = $5.4Ag$

While in the second liquid: Let the distance from the surface to the mark be x

Volume of second liquid = Ax

Mass of liquid displaced = (volume × density) = $Ax \times 1.1 = 1.1Ax$

Weight of liquid displace = $1.1Axg$

$$\text{Relative density} = \frac{\text{Apparent loss of weight of the hydrometer while in liquid}}{\text{Apparent loss of weight of hydrometer while in water}}$$

$$\textit{considering} \text{ water and first liquid, R.D} = \frac{w_a - 5.4Ag}{w_a - 4Ag}$$

$$\text{But relative density (R.D)} = \frac{\text{Density of substance}}{\text{Density of water}} = \frac{0.9}{1} = 0.9$$

$$\therefore 0.9 = \frac{w_a - 5.4Ag}{w_a - 4Ag} \Rightarrow 0.9w_a - 3.6Ag = w_a - 5.4Ag$$

$$w_a = 18Ag \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$\textit{considering} \text{ water and the second liquid, R.D} = \frac{w_a - 1.1Axg}{w_a - 4Ag}$$

$$\text{But relative density (R.D)} = \frac{\text{Density of substance}}{\text{Density of water}} = \frac{1.1}{1} = 1.1$$

$$\therefore 1.1 = \frac{w_a - 1.1Axg}{w_a - 4Ag} \Rightarrow 1.1w_a - 4.4Ag = w_a - 1.1Axg$$

$$w_a = 44Ag - 11Axg \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Equating equations (i) and (ii), we have

$18\text{Ag} = 44\text{Ag} - 11\text{Axg} \Rightarrow 26 = 11\text{x}$

Therefore x = 2.36 cm

3. A cube of rubber, volume 10^{-3}m^3, floats with half of its volume submerged in a liquid of density 1200kgm^{-3}. Find the depth to which the cube would be submerged in a liquid of density 1000kgm^{-3}.

$L = 10^{-1}m$

Volume = $L^3 = 10^{-3}m^3$

When immersed in liquid of density $1200kgm^{-3}$ volume of liquid displaced = $\frac{1}{2}x10^{-3}$ = $5x10^{-4}m^3$.

$\therefore$ mass of liquid displaced = $5x10^{-4}$ x 1200 = $6x10^{-1}kg$

hence using law of flotation, mass of body = $6x10^{-1}kg$

when immersed in liquid of density 1000 kgm^{-3}

mass of liquid displaced = $6x10^{-1}kg$.

Volume of liquid displaced = $m/d = 6\times10^{-4}m^3$. If h is the depth

$$l^2h = 6\times10^{-4}$$
$$10^{-2}h = 6\times10^{-4}$$
$$h = 6\times10^{-2}m.$$

4. A solid weight 237.5g in air and 12.5g when totally immersed in a liquid of density $0.9gcm^{-3}$. Calculate (a) Density of solid (b) The density of the liquid in which the solid would float with $\frac{1}{5}$ of its volume exposed above the liquid surface.

   When immersed in liquid of density $0.9gcm^{-3}$, Loss in mass = 237.5 – 12-5= 225g.

   Therefore mass of liquid displaced = 225g

Solution

Volume of liquid displaced = $225/0.9$

$= 250cm^3$

Hence volume of the body = $250cm^3$

$\therefore$ Density of solid = $m/v = \frac{237.5}{250} = 0.95gcm^{-3}$

b) Volume of liquid displaced = $\frac{4}{5}\times250 = 200cm^3$

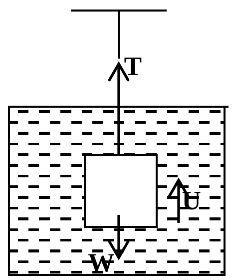
Mass of the liquid displaced = 200ρ

Using law of flotation, Mass of liquid displaced = mass of body =

200 ρ= 237.5

ρ = 1.187gcm^{-3}

3. A string supports a solid iron object of mass 180g, totally immersed in a liquid of density 800kgm^{-3}. Calculate the tension in the string if the density of iron is 8000kgm^{-3}.



Weight of body, W = mg = 0.18x9.81 = 1.764N

Volume of object = $m/d = \frac{0.18}{8000} = \text{volume of liquid displaced}$
$= 2.25\times10^{-5}$

Upthrust, U = Alρg = 2.25x10^{-5}x800x9.8 = 0.176N

Hence tension, T = W –U = 1.764 – 0.176 = 1.5836N

5. A cubical block of wood 10 cm long each side floats at the interface between oil and water with its lower surface 3 cm below the interface. The heights of oil and water columns are 10 cm each. The density of oil is 800 kgm^{-3} and that of water is 1000 kgm^{-3}

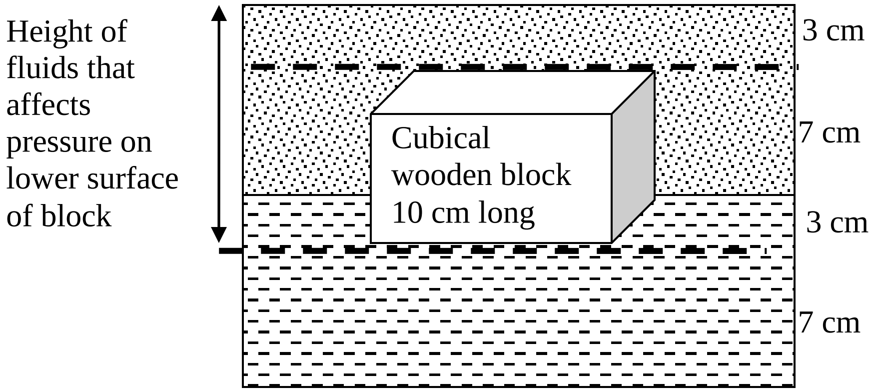
(i) What is the mass and density of the block?

(ii) What is the pressure on the lower surface of the block?

Solution

Consider the illustration below

It is important to note that oil floats on water because it is less dense than water



(i) Volume of oil displaced $= 7\times10\times10 = 700cm^3 = 700\times10^{-6}m^3$

Mass of oil displaced $= 700\times10^{-6}\times800 = 5.6\times10^{-1}kg$

Volume of water displaced $= 3\times10\times10 = 300cm^3 = 300\times10^{-6}m^3$

Mass of water displaced $= 300\times10^{-6}\times1000 = 3\times10^{-1}kg$

Total mass of fluids displaced $= 5.6\times10^{-1} + 3\times10^{-1} = 8.6\times10^{-1}kg = 0.86kg$

But mass of floating body = mass of fluid displaced $\Rightarrow$ mass of block = 0.86 kg

$$\text{Density of block} = \frac{\text{mass}}{\text{volume}} = \frac{0.86}{10\times10\times10\times10^{-6}} = 860kgm^{-3}$$

(ii) Pressure due to oil column $= h\rho g = \frac{10}{100}\times800\times9.81 = 784.8Nm^{-2}$

Pressure due to water column $= h\rho g = \frac{3}{100}\times1000\times9.81 = 294.3Nm^{-2}$

Therefore total pressure on lower surface $= 784.8 + 294.3 = 1079.1Nm^{-2}$

Exercise:

1. A piece of metal of mass $2.60\text{x } 10^{-3}$kg and density $8.4 \text{ x } 10^{3}$ kgm^{-3} is attached to a block of wax of mass $1.0 \text{ x } 10^{-2}$kg and density $9.2 \text{ x } 10^{2}$ kgm^{-3}. When the system is placed in a liquid it floats with wax just submerged. Find the density of the liquid. ($1.13\text{x}10^{3}$ kgm^{-3})

2. A block of mass 0.10kg is suspended from a spring balance. When the block is immersed in water of density $1.0\text{x}10^{3}kgm^{-3}$, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density, the spring balance reads 0.70N. Find
(i) the density of the solid ($2795kgm^{-3}$)
(ii) the density of the liquid (800 kgm^{-3})

3. A string supports a metal block of 2kg which is completely immersed in a liquid of density $8.8\text{x}10^{2}kgm^{-3}$. If the density of the metal is $9\text{x}10^{3}kgm^{-3}$, calculate the tension in the string. (17.7N)

4. A hydrometer floats with 6.0cm of its graduated stem unimmersed, and in oil of relative density 0.8 with 4.0cm of the stem unimmersed. What is the length of the stem unimmersed when the hydrometer is placed in a liquid of relative density 0.9? (5.1cm)

5. A block of volume $1000cm^{3}$mfloats half – immersed in a liquid of relative density 1.2. Calculate the volume of brass, relative density 8.7 which must be attached to the wood in order that the combination just floats in a liquid of relative density 2.2. ($246cm^{3}$)

6. A hydrometer consists of a spherical bulb and a cylindrical stem of cross-sectional area $0.4cm^{2}$. The total volume of the bulb and stem is $13.2cm^{3}$. When immersed in water, the hydrometer floats with 8.0cm of the stem above the water surface in alcohol it floats with 1.0 cm of the stem above the surface. Calculate the density of the alcohol. ($0.78gcm^{-3}$)

7. A solid weighs 237.5 g in air and 212.5g when totally immersed in a liquid of density 0.9g/cc. calculate

(i) Density of the solid

(ii) Density of a liquid in which the solid would float with $\frac{1}{5}$ of its volume exposed above the liquid surface

Ans (i) $\rho = 9500\ kgm^{-3}$ (ii) $\rho = 1190\ kgm^{-3}$

DENSITY ROD

This is a device used to measure the relative density of liquids

Procedure

Let the cross-sectional area of the density rod be A

The rod is submerged in a liquid of density ρ_1 and its length s_1 submerged in liquid is noted

The rod is then submerged in water of density ρ_2 and its length s_2 submerged in water is noted

When the rod is floating in the liquid:
$$\begin{aligned} &volume\,of \text{ liquid displaced} = As_1 \\ &mass\,of \text{ liquid displaced} = A\rho_1 s_1 \\ &weight\,of \text{ liquid displaced} = A\rho_1 s_1 g \end{aligned}$$

But from the law of flotation: weight of liquid displaced is equal to the weight of the floating rod

$$\therefore weight\,of\ \ floating\,rod = A\rho_1 s_1 g.....................(i)$$

When the rod is floating in the water:
$$\begin{aligned} &volume\,of \text{ water displaced} = As_2 \\ &mass\,of \text{ liquid displaced} = A\rho_2 s_2 \\ &weight\,of \text{ liquid displaced} = A\rho_2 s_2 g \end{aligned}$$

$$similarly, \therefore weight\,of\ \ floating\,rod = A\rho_2 s_2 g.....................(ii)$$

But the equations (i) and (ii) are equal $\Rightarrow A\rho_1 s_1 g = A\rho_2 s_2 g$

$$\therefore \frac{\rho_1}{\rho_2} = \frac{s_2}{s_1}.......................(iii)$$

$$\text{But, Relative density} = \frac{\text{Density of a substance } (\rho_1)}{\text{Density of water } (\rho_2)} \Rightarrow \text{Relative density} = \frac{s_2}{s_1}$$

It is important that the reader should note, this method of determining relative density has an advantage of obtaining the R.D if the densities of the substances are not known.

Example

A hydrometer floats in water with 72% of its volume submerged. The hydrometer floats in another liquid with 80% of its volume submerged. Find the relative density of the liquid.

Solution

Let the volume of hydrometer be v, the density of water be ρ_w, and that of the liquid be ρ_l

When the hydrometer is floating in the water:

$$volume\,of \text{ water displaced} = \frac{72}{100}v$$

$$mass\,of \text{ water displaced} = \frac{72}{100}v\rho_w$$

$$weight\,of \text{ water displaced} = \frac{72}{100}v\rho_w g$$

But from the law of flotation: weight of water displaced is equal to the weight of the floating hydrometer

$$\therefore weight\,of\ \ floating\,hydrometr = \frac{72}{100}v\rho_w g \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

When the hydrometer is floating in the liquid:

$$volume\,of \text{ liquid displaced} = \frac{80}{100}v$$

$$mass\,of \text{ liquid displaced} = \frac{80}{100}v\rho_l$$

$$weight\,of \text{ liquid displaced} = \frac{80}{100}v\rho_l g$$

$$\therefore weight\,of\ \ floating\,hydrometr = \frac{80}{100}v\rho_l g \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

But the equations (i) and (ii) are equal $\Rightarrow \frac{72}{100}v\rho_w g = \frac{80}{100}v\rho_l g$

$$\therefore \frac{\rho_l}{\rho_w} = \frac{72}{80} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

But, Relative density $= \frac{\text{Density of a substance } (\rho_l)}{\text{Density of water } (\rho_w)} \Rightarrow$ Relative density $= \frac{72}{80} = 0.9$

Relative density = 0.9

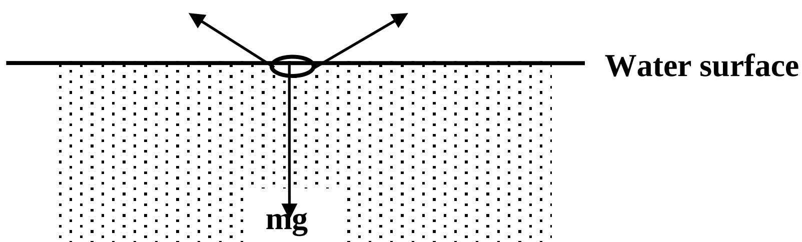
CHAPTER THIRTEEN: SURFACE TENSION

Some observation due to surface tension

1. A drop of water, on closing a tap remained clinging on the tap, as if the water was held in a bag.
2. A thin needle can be made to float on the surface of water though it is denser than water.
3. Mercury gathers in small spherical drops when poured on a smooth surface such as a glass
4. When a capillary tube is dipped in water, water is seen rising up in a tube.
5. Insects can walk on the water surface

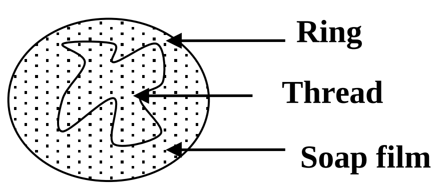
All the above observations show that a liquid surface behaves as if its surface is in a state of tension. The phenomenon is therefore called surface tension.

Consider a floating needle on the surface of water as shown below;

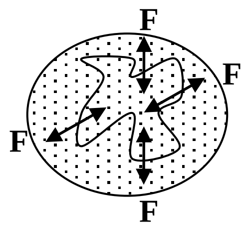


a steel needle can be made to float on water surface despite its greater density than water. The floating needle creates a depression in the water (liquid) surface so that the surface tensional forces F which act in the surface now have upward directed components which are capable of supporting the weight of the needle, thus the needle floats on the water surface as shown in the figure above.

Consider a thread placed on a soap film which is supported by a ring as shown below;

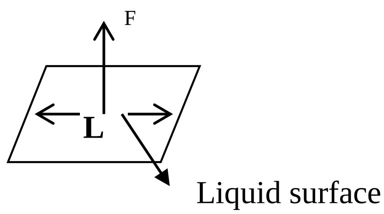


There are equal and opposite forces (surface tensional forces, F, on each side of the thread, and therefore the thread stays where it has been placed as shown below;



Surface Tension or Co-efficient of surface tension (γ)

This is the force per unit length acting in a liquid surface at right angle to an imagining drawn tangentially to the liquid surface.



$\therefore \gamma = F/L$

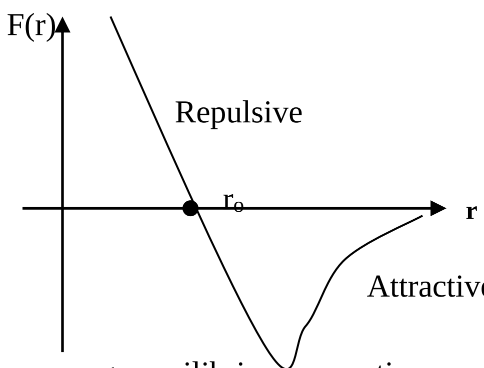
The units of γ are Nm^{-1}

$[\gamma] = MLT^{-2}L^{-1}$

$= MT^{-2}$

Molecular Theory of Surface Tension

The force F(r) between two molecules of a liquid varies with their separation r as shown below

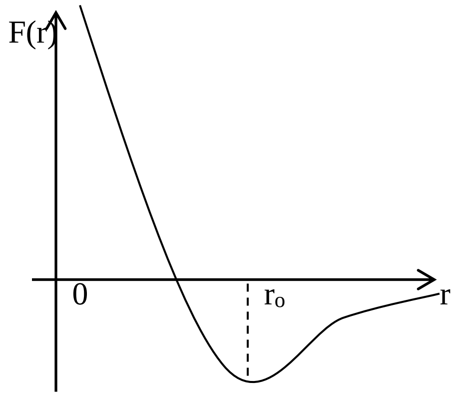


At the average equilibrium separation, r_0, F(r) = 0

For r > r_o = the force is attractive.

For r <r_o = the force is repulsive.

The corresponding potential energy variation with molecular separation is shown below

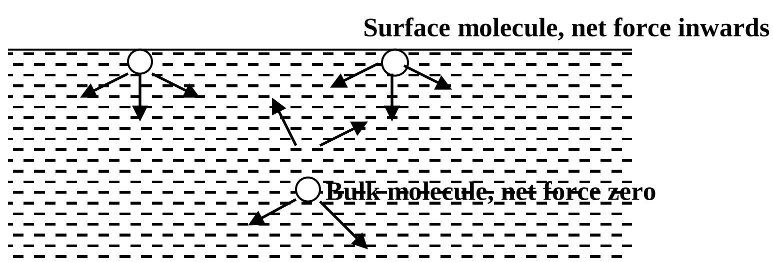


- The molecule within the body of the liquid (built molecule) is attracted equally by neighbours in all directions, hence the force on the bulk molecule is zero, so the intermolecular separation for bulk molecules is r_o.

- For a surface molecule, there is a net inward force because there are no molecules above the surface. Hence to bring a molecule from inside the liquid.

- To the surface, work must be done against the inward attractive force, hence a molecule in the surface of the liquid has a greater potential energy than a molecule in bulk. The potential energy stored in the surface is called free surface energy.

- Molecules at the surface have their separation r > r_o The attractive forces experienced by surface molecules due to their neighbours put them in a state of tension and the liquid surface behaves as a stretched skin.

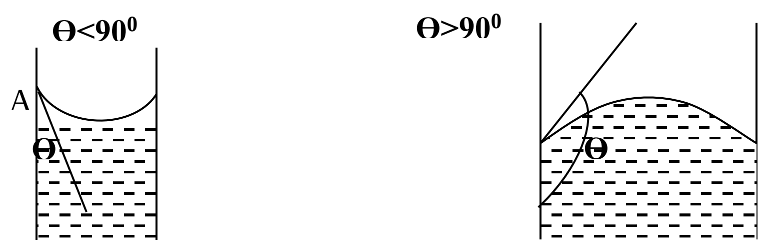


Surface energy and shape of a drop of a liquid

All systems arrange themselves in such a way that they have the minimum possible potential energy. The number of molecules that resides in the surface has to be minimum, and to minimize the number of molecules on the surface, the surface area must be reduced, hence liquid surface contract to the smallest possible area. So free liquid drops are spherical for any given volume because it is the shape which gives the minimum surface area. A large drop flattens out in order to minimize the gravitational potential energy which tends to exceed the surface energy. Due to its large weight, gravitational force distorts the spherical shape of large droplets however a small drop takes on a spherical shape to minimize the surface energy, which to be greater than gravitational potential energy. Therefore the gravitational force can not distort the spherical shape due to very small mass of tiny droplets.

Angle of contact

The angle between the solid surface and the tangent to the liquid surface at the point of intersection with the solid surface as measured through the liquid.



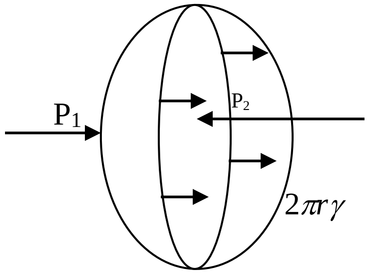
ϴ = angle of contact

A= point of intersection with solid surface

A liquid makes an acute angle of contact with the solid surface if the adhesive forces between the liquid and solid molecules are greater the cohesive forces between the liquid molecules themselves. The angle of contact is zero on a clean glass for pure water. If a liquid makes an acute angle of contact, it is said to wet the solid surface. A liquid makes an obtuse angle of contact with the solid surface if the cohesive forces between the liquid molecules themselves are greater than the adhesive forces between the solid and liquid molecules. Such a liquid is said not to wet the solid surface. The angle of contact of mercury on a glass surface is 140^0. Addition of detergent to a liquid reduces the angle of contact and therefore helps in washing.

Excess pressure inside an air bubble

Consider the equilibrium of one half of an air bubble of radius r, in a liquid of surface tension γ



This half of the bubble is in equilibrium under the action of force F_1 which is due to pressure P_1, F_2 which is due to the pressure p_2 and force F

P_1= pressure outside the bubble

P_2 = pressure inside the bubble

For equilibrium, F_1+F = F_2

$F_1 = P_1.\pi r,$ $\quad$ $F_2 = p_2.\pi r^2,$ $\quad$ $F = 2\pi r\gamma$

Hence $P_1\pi r^2 + 2\pi r\gamma = p_2\pi r^2$

But $(p_2 - p_2)r = 2\gamma$

$p_2 - p_1 = \frac{2\gamma}{r}$ (Excess press for air bubble)

Excess pressure inside a soap bubble

For a soap bubble, it has two surfaces

$F = 2.2\pi r\gamma$

For equilibriuim

$F_1 + F = F_2$

But $F_1 = P_1\pi r$ $\quad$ $F = 4\pi r\gamma$ $\quad$ $F_2 = p_2\pi r^2$

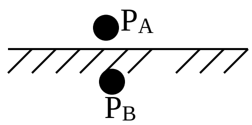
$p_1\pi r^2 + 4\pi r\gamma = p_2\pi r^2$

$(p_2 - p_1)r = 4\gamma$

$(p_2 - p_1) = \frac{4\gamma}{r}$

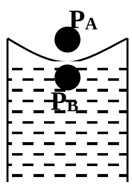
Note. The pressure on the concave side of a liquid surface is always greater than that on a convex side e.g.

Flat surface $P_A = P_B$



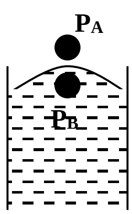
Hence excess is equal o zero on a flat surface.

Concave meniscus



$P_A - P_B = \frac{2\gamma}{r}$ where I is the radius of the meniscus

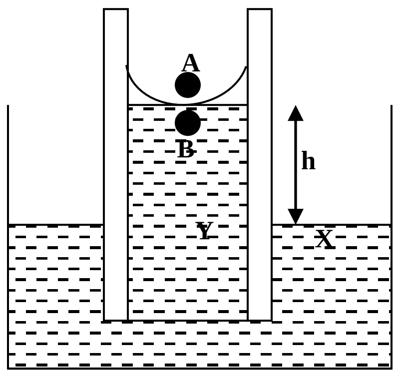
Convex meniscus



$$P_B - P_A = \frac{2\gamma}{r}$$

Capillary Rise

Consider the care of a liquid wets glass.



Pressure at X = pressure at Y = P_o (atmospheric pressure)

But $P_A - P_B = \frac{2\gamma}{r}$ r is radius of meniscus

$P_B = P_A - \frac{2\gamma}{r}$

$$P_y = P_B + h\rho g$$

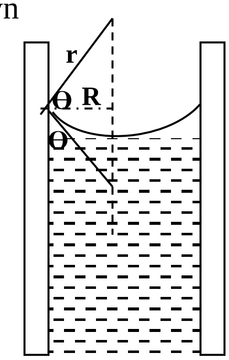
$$p_y = p_A - \frac{2\gamma}{r} + h\rho g$$

But $p_0 = p_0 - \frac{2\gamma}{r} + h\rho g$

$$hpg = \frac{2\gamma}{r}.$$

$\therefore h = \frac{2\gamma}{r\rho g}$ height which liqiud rises

The radius of curvature of the meniscus is related to the radius of the capillary and angle of contact as shown



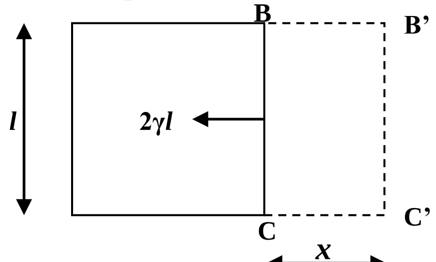
$r = \frac{R}{\cos\theta}$ Hence $h = \frac{2\gamma\cos\theta}{\text{Rpg}}$

Effects of temperature on surface tension

When the temperature of a liquid is raised, the mean kinetic energy of the molecules of the liquid raises on the average of the force of attraction between the molecules decreases since the molecules spend less time in the neighbourhood of the given molecules as a result the intermolecular separation rises hence surface tension of the liquid decreases with rising temperature.

Relationship between surface energy and surface tension

Consider a liquid stretched on a rectangular metal frame



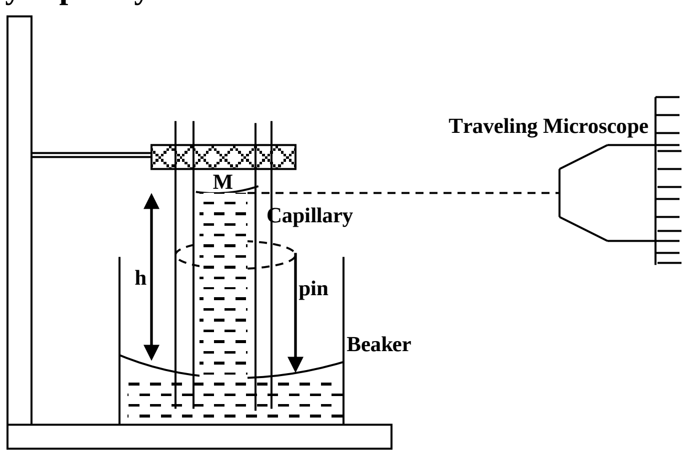
Suppose a film is stretched isothermally (at constant temperature) so that the edge BC moves through a distance ***x*** to B'C'. The work done to stretch the film = $F_0 x$

$$\text{But } F = 2\gamma l \text{ (the film has 2 surfaces)}$$
$$\text{work done} = 2\gamma l x$$
$$\text{increase in area} = 2lx$$
$$\text{Work done to increase a unit area} = \frac{2\gamma lx}{2lx} = \gamma$$

Hence surface tension can also be defined as the work done to increase surface area of a liquid by 1m^2 under isothermal condition.

<u>Measurement of surface Tension</u>

By capillary rise method

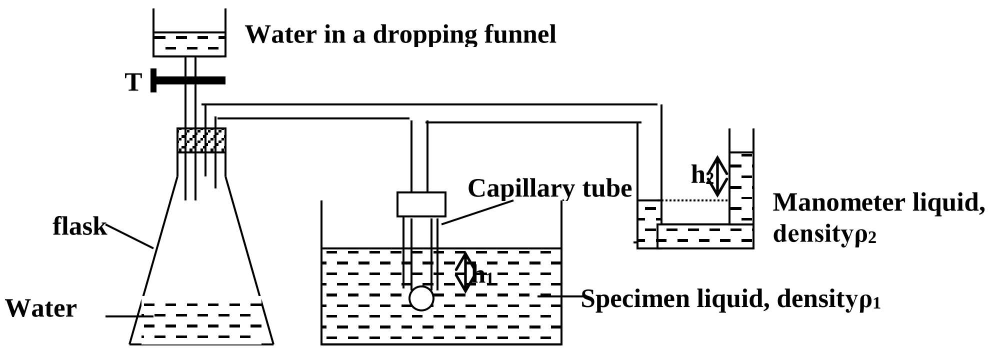


A pin is attached to the capillary tube with its tip just touching the liquid in the beaker. A traveling microscope is focused on the meniscus M. The reading S_1, on the scale is recorded. The beaker is carefully removed and the traveling microscope is focused on the tip of the pin P. The reading S_2 on the scale is recorded.

The capillary rise h = $|S_2 - S_1|$.

The radius, r of the capillary tube is determined measuring its diameter by using a traveling microscope. The angle θ of contact is measured and since the density, ρ of the liquid is known, surface tension can be calculated from; $\gamma = \dfrac{hr\rho g}{2\cos\theta}$

Jaeger's method



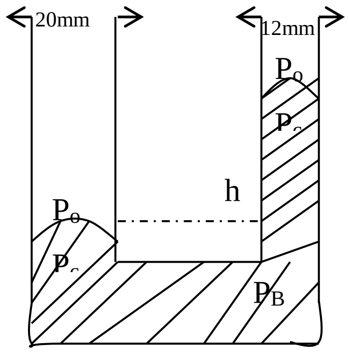
The pressure in the flask is increased gradually by allowing drops to fall down the funnel. Bubbles formed at the tip of the capillary tube dipping in the specimen liquid are observed. When the bubble has grown to a hemispherical shape, the tap T is closed and the reading h2 on the manometer is recorded. The depth, h1 of the end of the capillary tube below the specimen is recorded. Using $\frac{2\gamma\cos\theta}{a} + h_1\rho_1 g = h_2\rho_2 g$

$$\gamma = \frac{(h_2\rho_2 - h_1\rho_1)ga}{2\cos\theta}$$

The radius, a, of the capillary tube is determined measuring its diameter by Z z z traveling microscope. The angle θ of contact is measured and since the density, ρ_1, ρ_2 of the liquids are known, then γ can be calculated.

Examples

1) Mercury is poured into a glass U- tube with vertical limbs of diameters 20mm and 12.00mm respectively. If the angle of contact between mercury and glass is $140^0$and the surface tension of mercury is 0.152 Nm^{-2}. Calculate the difference in the levels of mercury. (Density of mercury = 1.35×10^4 kgm^{-3}).



$$P_A - P_O = \frac{2\gamma\cos\theta}{R_1}$$
$$P_A = P_0 + \frac{2\gamma\cos\theta}{R_1}$$
$$p_B = P_A + hpg$$
$$P_B = P_0 + \frac{2\gamma\cos\theta}{R_1} + hpg.........(i)$$

$$P_C - P_0 = \frac{2\gamma\cos\theta}{R_2}$$
$$P_C = P_0 + \frac{2\gamma\cos\theta}{R_2}..................(ii)$$
$$but P_B = P_c$$
$$hence P_o + \frac{2\gamma\cos\theta}{R_2} = P_o + \frac{2\gamma\cos\theta}{R_1} + h\rho g$$
$$h\rho g = 2\gamma\cos\theta\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$
$$R_1 = 0.006m$$
$$R_2 = 0.01m$$
$$h = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{2\times0.52\cos140}{1.36\times10^4\times9.8}\left(\frac{1}{0.01} - \frac{1}{0.006}\right)$$
$$= 4.9812\times10^{-3}m$$

2. A droplet of mercury of radius 2.0mm falls vertically and on hitting the ground it splits into two droplets each of radius 0.50mm. Calculate the change in surface energy.

Account for the change in (i) above.

1c) Energy of a large droplet

$= 4\pi r_1^2\gamma$

$= 4\pi\left(2\times10^{-3}\right)^2\times0.52$

$= 2.61\times10^{-5}J$

Energy of the split drops

$= 2\left(4\pi r_2^2\gamma\right)$

$= 2.\left(4\times\pi\times0.5\times10^{-3}\right)^2\times0.52)$

$= 3.27\times10^{-6}J$

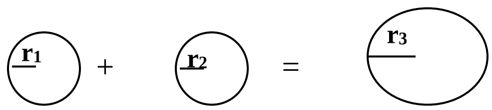
Change in energy

$= 2.61 \times 10^{-5} - 3.27 \times 10^{-6}$

$= 2.283 \times 10^{-5}\, J$

The energy reduces because some of it is lost in overcoming air resistance.

3. Two soap bubbles of radii 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is 2.5 x 10^{-2} Nm^{-1}, calculate the excess pressure inside the resulting soap bubble.



$2 \times 4\pi r_1^2 \gamma + 2 \times 4\pi r_2^2 \gamma = 2 \times 4\pi r_3^2 \gamma$

$r_1^2 + r_2^2 = r_3^2$

$r_1 = 0.02m, r_2 = 0.04m$

$r_3 = \sqrt{\ }(0.0004 + 0.0016)$

$r_3 = 0.045m$

$$\text{excess pressure} = \frac{4\gamma}{r_3} = \frac{4 \times 2.5 \times 10^{-2}}{0.045}$$

$$= 2.22\,\text{Nm}^{-2}.$$

4. In Jaeger method for measuring the surface of a liquid, the lower end of a capillary tube of radius 0.20mm is 25mm below the surface of the liquid whose surface tension is required and whose density is 8.0 x 10^2 kgm^{-3}. The pressure in the hemispherical bubble formed at the end of the tube is measured as 40mm on a water manometer. Calculate the surface tension of the liquid.

r = radius of capillary

h = reading on manometer

ρ = density of water

h_1 = height on tube in liquid

ρ_1 = density of specimen liquid.

$$\gamma = \frac{rg}{2}(hp - h_1 p_1)$$

$$= \frac{0.002 \times 9.8}{2}(0.04 \times 100 - 0.0025 \times 8 \times 10^2)$$

$$= 9.8 \times 10^{-2} \times 20$$

$$= 1.96 \times 10^{-2}\, Nm^{-1}$$

Exercise

1. Calculate the total pressure inside an air bubble of radius 10^{-5}m at a depth of 0.3 m below the surface of the water.

ii) If the bubble is attached to mercury manometer. Calculate the height to which the mercury rises.

2. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipping in water and with 12cm of its tube above the surface.

(i) To what height will water rise in the tube?

(ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface? (Surface tension of water is 7.2×10^{-2}Nm^{-1}, angle of contact =0)

3. An oil drop of radius 5cm falls on the ground and breaks into small drops each of radius 2.5cm. Calculate the work done and the speed of the oil drop when it hits the ground. (Density of oil is 800 kgm^{-3}; coefficient of surface tension of oil $=1.2\times10^{-1}\,Nm^{-1}$

CHAPTER FOURTEEN: HYDRODYNAMICS / FLUIDS IN MOTION

Types of fluid flow

There are two types of fluid flow; streamline/ steady/laminar flow and turbulent flow

Streamline / steady/ Laminar flow

This is the type of orderly fluid flow where the successive particles passing at a certain point in the fluid have the same velocity.

Characteristics of Laminar flow include;

- Lines of liquid flow are parallel to the axis of the tube.
- The particles at the same distance from the axis have the same velocity.
- Laminar flow occurs at low liquid velocities.

It should be noted that the paths that represents the direction of the velocities of the particles of the fluid are called streamlines or lines of flow.

Turbulent flow

This is the type of fluid flow where successive particles passing at a certain point have different velocities and are disorderly

Characteristics of turbulent flow include;

- Lines of liquid flow are not parallel to the axis of the tube.
- The particles at the same distance from the axis have different velocities.
- Turbulent flow occurs at high liquid velocities.

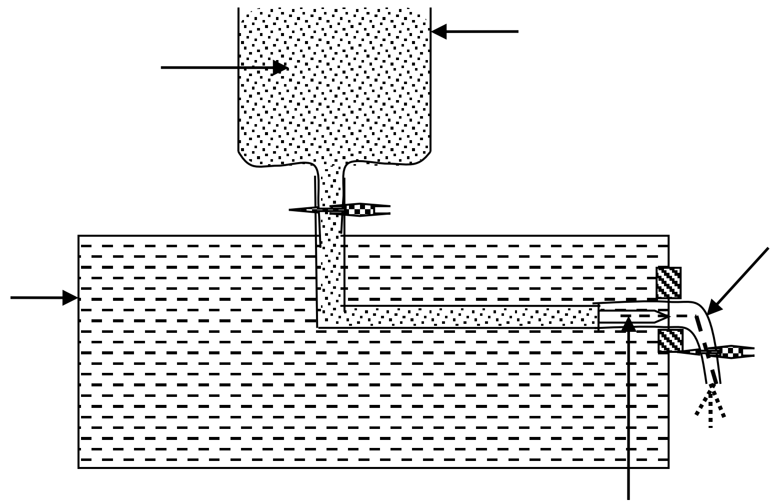
When the fluid flow velocity is increased beyond a critical value (high velocity), wavy currents and sideways movements of the molecules of the fluid occur and turbulence sets in causing the lines of liquid or fluid to move in random direction.

Experiment to demonstrate laminar and turbulent flow

Laminar and turbulent flow can be demonstrated by introducing a small amount of coloured liquid such as Potassium permanganate at the centre of the tube.

Procedure

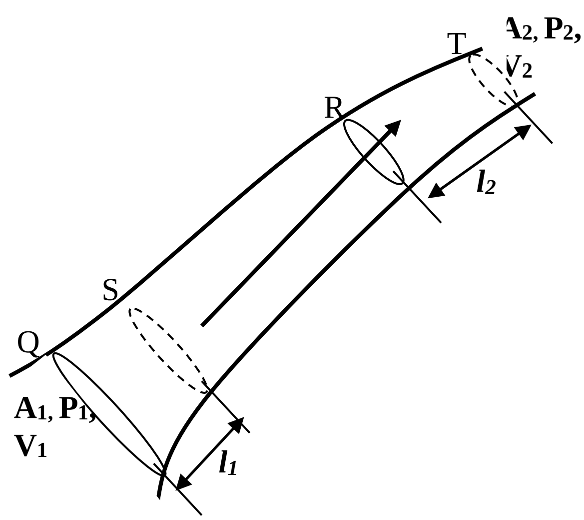
Potassium permanganate solution from a reservoir is fed into the flowing water by a fine jet. The clip is used to control the flow of water along the tube as shown in the figure below.



The clip is opened a little to gradually increase the rate of flow. A fine and thin coloured stream is observed along the centre of the tube for low flow velocities of the fluid, thus showing a streamline flow. As the rate of flow increases, the coloured stream starts to break up (becoming wavy) and the colour rapidly spreads throughout the tube, this demonstrates turbulent fluid flow.

Motion of fluid in a tube/pipe of non-uniform cross-sectional area: (Continuity Equation)

Consider an incompressible liquid (liquid whose density is constant) flowing through a pipe as shown in the figure below;



Consider a fluid such as water of negligible viscosity (non-viscous) flowing steadily in a tube of non-uniform cross-sectional area in the direction shown above.

Also, assume that the velocity at the entrance point Q is V_1, the pressure is P_1 and the cross-sectional area is A_1, while the velocity at point R is V_2, the pressure is P_2 and the cross-sectional area is A_2. Assuming that l_1 and l_2 are so small that the parameters A, P and V remain unchanged between the regions QS and RT.

If the fluid moves from QR to ST in a short time Δt, then the volume between Q and S is equal to the volume between R and T

Volume between Q and S = $A_1 l_1$ and volume between R and T = $A_2 l_2$

$\Rightarrow A_1 l_1 = A_2 l_2$..............................(1)

$\therefore \frac{A_1}{A_2} = \frac{l_2}{l_1}$ and since $A_2 \angle A_1$ then $l_1 \angle l_2$

Therefore, since the distances l_1 and l_2 are covered in the same time interval, l_2 must be covered faster than l_1. This implies that velocity V_2 is greater than V_1.

Therefore, the fluid flows faster at a narrow part than a wider part. It can also be noted that since the volume of fluid entering the tube should be equal to the volume of fluid leaving the tube, then; mass of fluid entering the tube per second is equal to the mass of fluid leaving the tube per second.

From $Mass = volume \times density$, if the density of the fluid is ρ

Similarly, mass of fluid leaving per second = $\frac{A_2 l_2 \rho}{\Delta t}$

$\therefore \frac{A_1 l_1 \rho}{\Delta t} = \frac{A_2 l_2 \rho}{\Delta t} \qquad \Rightarrow A_1 \frac{l_1}{\Delta t} = A_2 \frac{l_2}{\Delta t}$

$But, \frac{\text{distance}}{\text{time}} = velocity \quad \Rightarrow A_1 V_1 = A_2 V_2$..............................(2) Contnuity Equation

It can be seen from equation (2) that; $\frac{A_1}{A_2} = \frac{V_2}{V_1}$ and since $A_2 \angle A_1$, then $V_1 \angle V_2$

Equations (1) and (2) are called equations of continuity

Therefore, if A is the area and V is the velocity, then AV = Constant, and AV is known as the flow rate or volume flux.

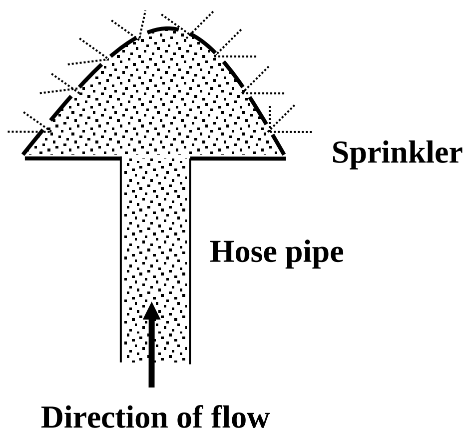
It should be noted that the equations are only true for an incompressible fluid such that its density is constant throughout the tube.

An incompressible fluid is one in which changes in pressure produce no change in the density of the fluid. Therefore liquids and gases are taken to be incompressible.

Example

A lawn sprinkler has 20 holes each of cross-sectional area 2×10^{-2} cm^2. The sprinkler is connected to a hose pipe of cross-sectional area 2.4 cm^2. If the speed of water in the hose pipe is 1.5 ms^{-1}, estimate the speed of the water as it emerges from the holes.

Solution



For sprinkler

$A^n = 2\times10^{-2}cm^2 = 2\times10^{-6}m^2$

since it has 20 holes, then;

Total area of sprinkler, $A_s = 20\times2\times10^{-6} = 40\times10^{-6}m^2$

For hose pipe

$A_h = 2.4cm^2 = 2.4\times10^{-4}m^2$, $\mathrm{V_h} = 1.5ms^{-1}$

From the equation of continuity

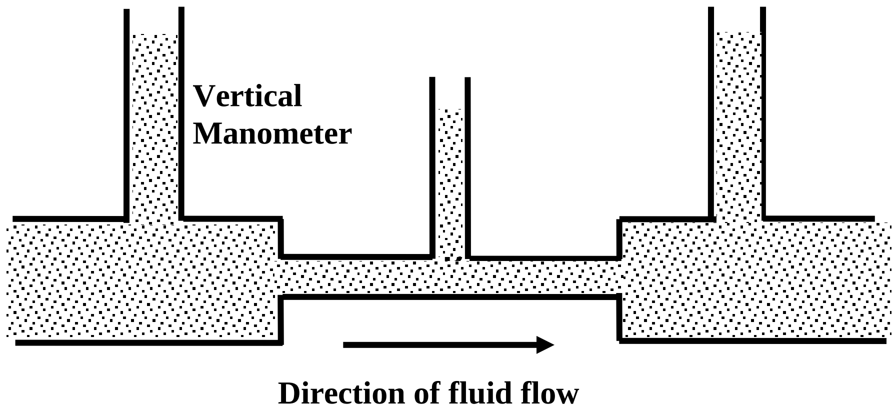
$A_hV_h = A_sV_s$

$\Rightarrow (2.4\times10^{-4})\times1.5 = (40\times10^{-6})\times V_s$

$\therefore V_s = 9ms^{-1}$

Daniel Bernoulli's principle

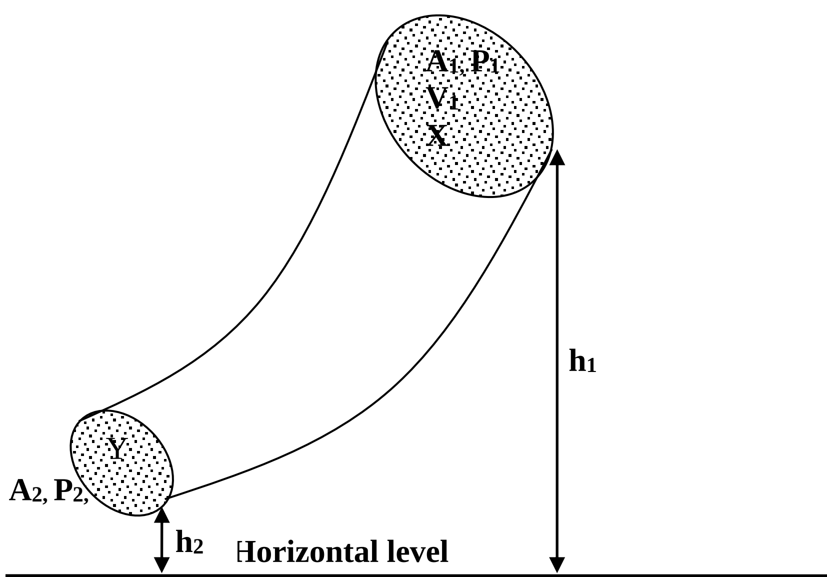
Consider the illustration below for understanding pressure at a point.



When a fluid is at rest, pressure is the same at all point on the same horizontal level. When the fluid is in motion, pressure is not always the same, for example; consider the diagram above, the pressure at the different points is shown by the height of the fluid in the vertical manometers. Pressure is high at parts A and C and falls in part B, but the velocity of fluid is greatest in the narrow part B and least in the wider parts A and C. It therefore follows that a decrease in pressure is accompanied by an increase in velocity of the fluid.

Derivation of Bernoulli's principle

Consider an imaginary tube in a fluid between points X and Y which are at heights h_1 and h_2 from the horizontal



Let for end X, the pressure be P_1, the cross-sectional area be A_1, and the velocity of fluid be V_1 while at Y the pressure be P_2, the cross-sectional area be A_2, and the velocity of fluid be V_2

Also, consider the cross-sectional area of the tube to be constant at a particular time for a small time interval, Δt

A fluid in stream line flow has three types of energy;

(i) Pressure energy

(ii) Potential energy

(iii)Kinetic energy

Pressure energy is the energy possessed by the fluid by virtue of its pressure at a particular point.

Or

It's the work done by the pressure in moving a fluid through a small displacement

For end X:

Work done = $(\text{force}, F)\times(\text{distance}, d)\ldots\ldots\ldots\ldots\ldots\ldots.(i)$

$$\begin{aligned}
&But,\ \text{Force} = (\text{pressure}, P)\times(Area,\ A)\ldots\ldots\ldots\ldots\ldots\ldots(ii)\\
&\text{combining (i) and (ii) we have}\\
&\text{Work done} = (P\times A\times d)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)\\
&\text{But, distance} = (\text{velocity}, V\times \text{time}, \Delta t\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iv)\\
&\text{combining (iii) and (iv) we have;}\\
&\text{Workdone} = (P\times A\times V\times \Delta t).\\
&\therefore Work\ done = \text{pressure energy} = (P_1\times A_1\times V_1\times \Delta t)\ldots\ldots(v)
\end{aligned}$$

Kinetic energy

$$\begin{aligned}
&\text{Kinetic energy} = \frac{1}{2}mV_1^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)\\
&But,\ \text{mass, m} = \text{density}\times\text{volume} = \rho\times(A_1 l_1)\ldots\ldots(ii)\\
&But,\ l_1 = V_1\Delta t\ldots\ldots\ldots\ldots\ldots(iii)\\
&combining\ (ii)\ \text{and}\ (iii)\text{, we have;}\\
&\text{Mass, m} = \rho A_1 V_1 \Delta t \quad \Rightarrow \text{Kinetic energy} = \frac{1}{2}(\rho A_1 V_1 \Delta t)V_1^2\ldots\ldots\ldots.(iv)
\end{aligned}$$

Potential energy

$$Potential\ energy = mgh\ldots\ldots\ldots(i)\ \Rightarrow \text{potential energy} = (\rho A_1 V_1 \Delta t)gh_1\ldots\ldots(ii)$$

Therefore, total energy (mechanical energy) at X is given by:

Pressure energy + Kinetic energy + Potential energy

$$\Rightarrow P_1A_1V_1\Delta t + \frac{1}{2}(\rho A_1V_1\Delta t)V_1^2 + (\rho A_1V_1\Delta t)gh_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly, total energy at end Y is given by:

$$\Rightarrow P_2A_2V_2\Delta t + \frac{1}{2}(\rho A_2V_2\Delta t)V_2^2 + (\rho A_2V_2\Delta t)gh_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If it is assumed that the fluid is;

(i) An incompressible and non – viscous liquid.

(ii) Streamline

(iii) Steady state conditions where velocity is independent of time; then

From the conservation of energy, total energy at X should be equal to total energy at Y

$$\therefore P_1A_1V_1\Delta t + \frac{1}{2}(\rho A_1V_1\Delta t)V_1^2 + (\rho A_1V_1\Delta t)gh_1 = P_2A_2V_2\Delta t + \frac{1}{2}(\rho A_2V_2\Delta t)V_2^2 + (\rho A_2V_2\Delta t)gh_2$$

But volume of fluid entering at X should be equal to the volume leaving at Y

$$\Rightarrow (\rho A_1V_1\Delta t) = (\rho A_2V_2\Delta t)$$

$$\therefore P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2 \ldots\ldots\ldots\ldots\ldots(3)$$

Since P_1, P_2, V_1, V_2, h_1 and h_2 are randomly chosen, then;

$$\Rightarrow \mathrm{P} + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The above equation (4) is called Bernoulli's equation

Therefore, for a pressure P at any part of the tube with a velocity at the same point being V, and the density of the fluid (assumed to be constant) ρ, the Bernoulli's principle can be stated as below;

Bernoulli's principle states that the sum of pressure at any part, the kinetic energy per unit volume and the potential energy per unit volume is always a constant.

OR

The total energy of any incompressible and non-viscous fluid in a streamline flow remains constant throughout the flow.

Note

$$\text{Kinetic energy (K.E)} = \frac{1}{2}mv^2 \Rightarrow \text{K.E per unit mass} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$$

$$\text{Also, K.E per unit volume} = \frac{\frac{1}{2}mv^2}{v} = \frac{1}{2}\frac{m}{v}v^2 = \frac{1}{2}\rho v^2$$

$$\text{Potential energy (P.E)} = mgh \Rightarrow \text{P.E per unit mass} = \text{gh} =$$

$$\text{Also, P.E per unit volume} = \rho gh$$

Examples

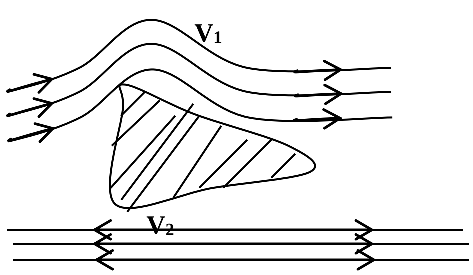
1. A fluid of density 1000 kgm^{-3} flows in a horizontal tube. If the pressure between the ends of the tube (i.e. at entry and exit) is 10^5 Pa and 10^3 Pa respectively and given that the velocity of the fluid at entry is 8ms^{-1}, calculate the velocity of the fluid at exit.

Solution

2. An open tank holds water 1.25m deep. A small hole of cross-section 3cm^3 is made at the bottom of the tank. Assuming that the density of water is 1000 kgm^{-3}, calculate the mass of water per second initially flowing out of the hole.

Solution

<u>Applications of Bernoulli's principle</u>



The orientation of aerofoil relative to the flow direction cause the flow lines to crowd together above the aerofoil corresponding to increased flow velocity. And according to Bernoulli's equation the pressure above reduces. Below the aerofoil, the flow velocity is lower and hence the pressure is higher, hence there is a resultant thrust upwards leading to the lift.

Jets and nozzles

Bernoulli's equation suggests that for fluid flow where potential energy change is very small or zero as in a horizontal pipe, the pressure falls when the velocity rises. The velocity increases at constriction.

The greater the change in cross-sectional area, the greater is the increase of velocity and so the greater is the pressure drop.

$A_1V = A_2V_2$ $\qquad\qquad$ $A_1 > A_2$ $\qquad\qquad$ $V_2 > V_1$

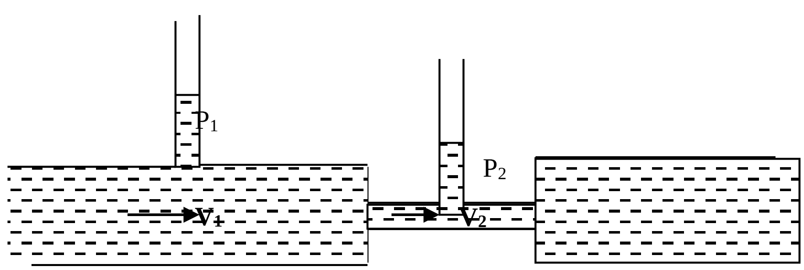
Several devices with jets and nozzles use this effect e.g Bunsen burner, filter pump and paint spray.

iii) Flow meters

These measure the rate of flow of a fluid through a pipe.

a) Ventura flow meter

This consists of a horizontal tube with a constriction and replaces part of a piping of a system.



The two vertical tubes record the pressures in the fluids flowing in the normal part of the tube and in the constriction.

From Bernoulli's equation (ρgy is not considered because pipe and constriction are at the same level)

$$p_1 + \frac{1}{2}\rho v_1{}^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}pv_2^2 - \frac{1}{2}pv_1^2$$

Using the equation of continuity

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2v_1^2}{A_2^2} - v_1^2\right)$$

$$\left(P_1 - P_2\right) = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

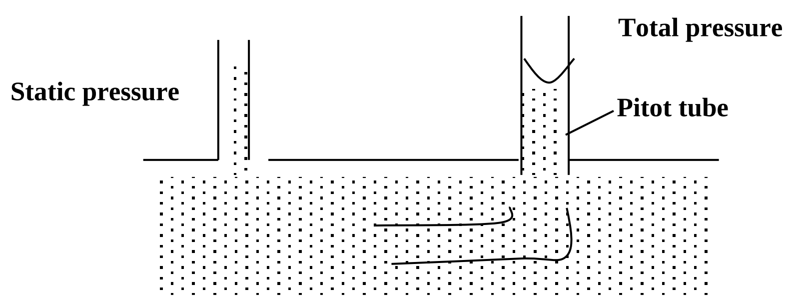
b) Pitot tube

The pressure exerted by a moving fluid called total pressure can be regarded as having two components namely;

i. The static pressure which it would have if it were to rest.

ii. Dynamic pressure which is the pressure equivalent of its velocity $\left(\frac{1}{2}\rho v^2\right)$

A pitot tube measures total pressure.



Total pressure = static pressure + dynamic pressure

Dynamic pressure = total pressure – static pressure

$$\frac{1}{2}\rho v^2 = (\text{Total pressure} - \text{static pressure})$$

$$v^2 = \frac{2}{\rho}(\text{total pressure} - \text{static pressure})$$

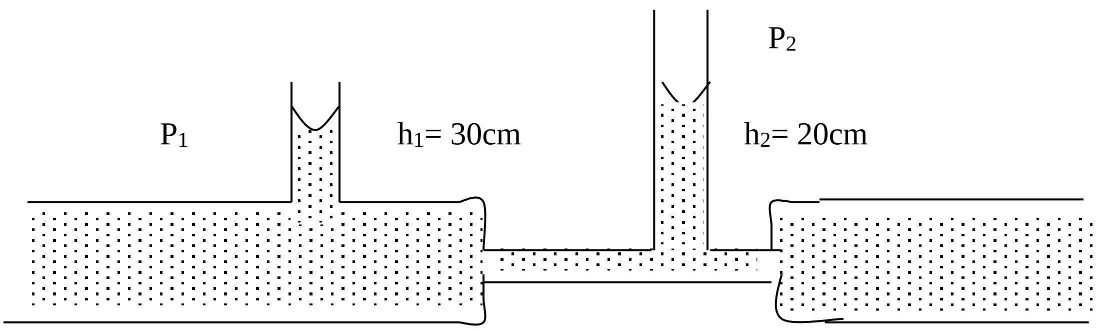
Questions

1. At a certain section of the horizontal water pipe, the static pressure is 1.96 x 10^5Pa, the total pressure is 2.04 x 10^5Pa and area of cross section is 20cm^2, if the density of water is 10^3kgm^{-3}, find the volume flow rate in the pipe.

Solution:

$$v^2 = \frac{2}{\partial}(\text{total pressure - static pressure}).$$

$$v^2 = \frac{2}{10^3}\left(2.04\times10^3 - 1.96\times10^5\right)$$
$$= 0.002((8000)$$
$$v = 4ms^{-1}$$
$$\therefore \text{flow rate} = \text{AV}$$
$$= \frac{20}{10{,}000}\times 4$$
$$= 8\times10^{-4} m^3 s^{-1}$$



2. The above diagram represents a venture-meter, if the cross sectional area of the main pine is 5.81 x 10^{-3}m^2 and that of the constriction is 2.58 x 10^{-3}m^2, find the velocity v

Solution

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$
$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$
$$\text{but } A_1V_1 = A_2V_2$$
$$V_2 = \frac{A_1V_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2 V^2}{A_2^2} - V_1^2\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)V_1^2$$

$$P_1 = P_0 + h_1\rho g \qquad \mathrm{P}_2 = P_0 + h_2\rho g$$

$$(h_1 - h_2)\rho g = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)V^2$$

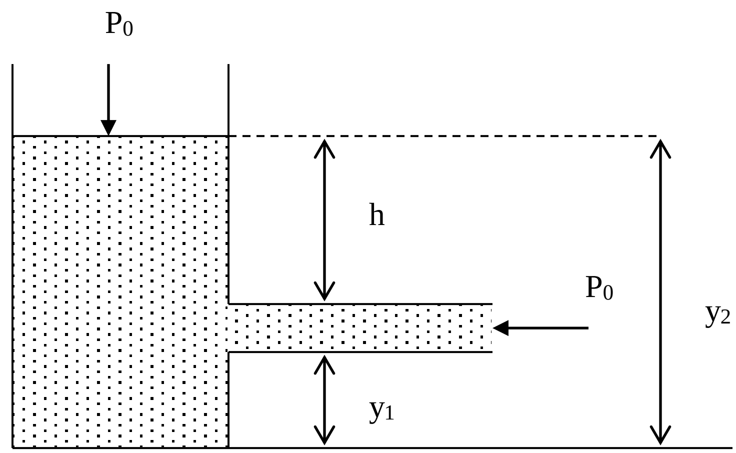
$$V_1^2 = \frac{(h_1 - h_2)}{\frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)}$$

$$V_1^2 = \frac{98}{\frac{1}{2}\times 4.07} \qquad = \frac{98}{2.04}$$

$$V_1^2 = 48.14$$

$$V_1 = 6.9ms^{-1}$$

Flow velocity of a liquid from a tank open to the atmosphere.



By Bernoulli's principal,

$$p_o + \rho g y_2 = P_0 + \rho g y_1 + \frac{1}{2}\rho v^2$$

Where v is the velocity from the orificenear the bottom of the tank

$$\rho g(y_2 - y_1) = \frac{1}{2}\rho v^2$$

but $y_2 - y_1 = h$

$$\rho g y = \frac{1}{2}\rho v^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

VISCOSITY

This is the resistance between fluid layers in contact moving relative to each other.

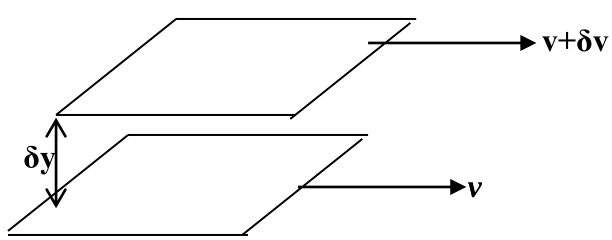
If adjacent layers of a material are displaced laterally over each other, the deformation of the material is called shear.

All liquids and gasses stick to a solid surface so that when they flow, the velocity must gradually decrease to zero as the wall of the pipe is approached, a fluid is therefore sheared when it flows past the solid surface. The opposition set up by the fluid to shear is called the viscosity. So viscosity is a kind of internal friction exhibited to some degree by all fluids.

It arises in liquids because the forced movement of a molecule relative to its neighbours is opposed by the intermolecular forces between them. But viscosity of a liquid is the measure of its resistance to flow. The greater the viscosity, the less easier it is for a liquid to flow and the more sticky it is hence oil is said to be more viscous than water.

Coefficient viscosity, η

Consider two parallel layers of liquid separated by distance δy and having velocities $v + \delta v$



The frictional force F between the layers F shear stress = $\frac{F}{A}$ Where A is the area of the layers.

The rate of change of shear strain is ${dv}/{dy}$, this is also called strain rate or velocity gradient.

For lamina flow

$$\frac{F}{A} \propto \frac{\partial v}{\partial y}$$

$$\frac{F}{A} = \eta \frac{\partial v}{\partial y}$$

$$\eta = \frac{F}{\left(\frac{A\partial v}{\partial y}\right)} = \text{coeffient of viscosity}$$

$$\eta = \frac{\text{shear stress}}{\text{shear strain}}$$

Coefficient of viscosity is the stress which results the motion of one layer of a fluid over another when the velocity gradient is unit or it is the frictional force per unit area when its in a region of unit velocity gradient.

Unit of η is $Nm^{-2}s$ or Pas

Question: Prove that $[\eta] = ML^{-1}T^{-1}$

Poiseulle's equation (For lamina flow only)

The volume rate of flow of a liquid through a pipe depends on;

(i) The radius *r* of the pipe

(ii) The coefficient of viscosity η

(iii) The pressure gradient $\left(\frac{P}{l}\right)$ where P is the pressure head and *l* is the length of the tube.

$$\left(\frac{v}{t}\right) = kr^x\eta^y\left(\frac{p}{l}\right)^z$$

$$[L.H.S] = \frac{[V]}{[t]} = \frac{L^3}{t} = L^3T^{-1}$$

$$[R.H.S] = K[r]^x[\eta]^y\left[\frac{p}{l}\right]^z$$

$$= KL^X(ML^{-1}T^{-1})^y\left(\frac{ML^{-1}T^{-2}}{L}\right)^z$$

$$= KL^XM^yL^{-Y}M^zL^{2Z}T^{-2Z}$$

$= KM^{y+z} L^{x-2z-y} T^{-Y-2Z}$

resolving Left hand side and Right hand side

$M; y + z = 0$.........................(1)

$L; x\text{-}2z\text{-}y = 3$....................(2)

$T; y - 2z = ^{-}1$....................(3)

From equation (3); y + 2z = 1

y = 1 – 2z

Put in equation (1); 1-2z + z = 0

1-z = 0

Z = 1

$\therefore y = -1$

Using equation (2)

$x - 2 + 1 = 3$

$x = 4$

$\therefore$ poiseulle's equation is

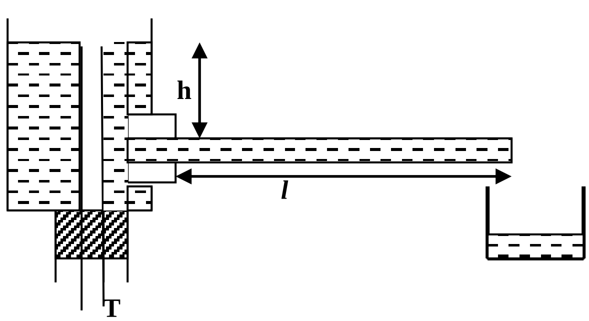
$$\therefore \frac{v}{t} = k\eta^{-1} r^4 \left(\frac{p}{t}\right)$$

$$\frac{v}{t} = \frac{kr^4 p}{\eta l}$$

$$but k = \frac{\pi}{8}$$

$\frac{v}{t} = \frac{\pi}{8}\frac{pr^4}{\eta l} \Rightarrow$ Poiseulle's equation (only for laminar flow)

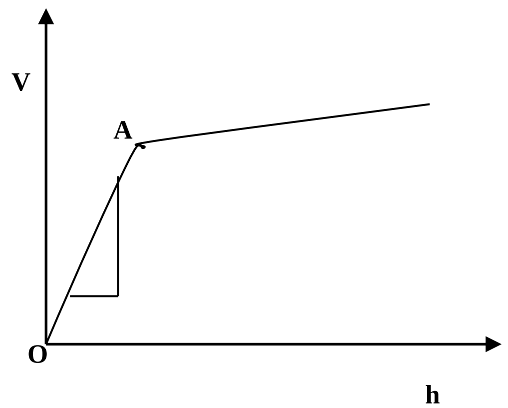
Determining coefficient of viscosity using Poiseulle's equation (Apply only to less viscous liquids e.g. water



The pressure head h is varied by raising or lowering tube T

Liquid flowing through the capillary tube is collected for a measured time. The volume of water, V, flowing per second is calculated.

A graph of V against h is plotted;



The scope, S of the graph an region OA is determined from

$$\frac{V}{t} = \frac{\pi}{8}\frac{pr^4}{\eta l}$$

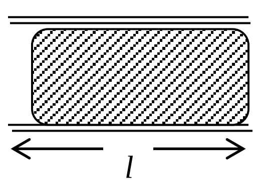
But p = hρg

$$\frac{V}{t} = \frac{\pi}{8}\frac{h\rho g r^4}{\eta l} = \left(\frac{\pi\rho g r^4}{8\eta l}\right)h$$

$$\text{The slope, S} = \left(\frac{\pi\rho \text{g} \text{r}^4}{8\eta \text{l}}\right)$$

$$\eta = \frac{\pi\rho g r^4}{8Sl}$$

In determining the radius of the tube, mercury of known mass is filled in the tube



$$\pi r^2 lpgh = m$$

Stoke's law

A body moving in a fluid experiences a retarding force due to the viscosity of the fluid. This retarding force is called viscous drag.

Note. The difference between viscosity and viscous drag is that viscosity is a frictional force which opposes relative motion between liquid layers whereas viscous drag is a frictional force experienced by a body in a fluid.

The viscous drag F, experienced by a sphere moving in a fluid depend on

(i) The radius r of the sphere

(ii) The velocity v of the sphere.

(iii) The coefficient of viscosity η of the liquid.

For a constant body of similar dimensions moving in a uniform fluid, the force of viscosity depends on the velocity of the body.

Hence $\mathrm{F} = \mathrm{kr}^{\mathrm{x}}\eta^{y}v^{z}$

$MLT^{-2} = [L]^{x}\left(ML^{-1}T^{-1}\right)^{y}\left(LT^{-1}\right)^{z}$

$= L^{x}.M^{y}.L^{-y}.T^{-y}.L^{Z}.t^{-Z}$

$MLT^{-2} = L^{x-y+z}.M^{y}T^{-y-z}$

$M^{1} = M^{y}$, hence $\mathrm{y} = 1$

$\mathrm{L}^{1} = L^{x-y+z}$, hence $\mathrm{x - y + z = 1}$

but $\mathrm{y = 1, x + z = 2}$

$\mathrm{T}^{-2} = T^{-y-z}, hence - y - z = -2$

$but y = 1, z = 1$

$hence x = 1$

$\therefore F = kr\eta v$

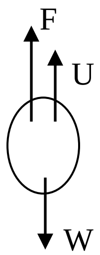
Detailed analysis indicate that

$k = 6\pi$

$\therefore F = 6\pi r\eta v \rightarrow \text{Stoke's law}$

Motion of a metal sphere in a viscous liquid

Consider the forces acting on the sphere as it falls through a liquid



W = weight

U = upthrust

F = viscous drag

The resultant force on the sphere is W- (F+U)

From Newton's second law; $\frac{mdv}{dt} = w - (F + U)$ where m = mass of the sphere

If a is the radius of the sphere, ρ the density of the material of the sphere and σ the density of the liquid then

$W = vpg = \frac{4}{3}\pi a^3 pg$

$U = \frac{4}{3}\pi a^3 \sigma g$

$F = 6\pi a \eta v$

The sphere will accelerate until the net force on it is zero, hence W – (F+U) = 0

When the net force on the sphere is zero, it moves with a constant velocity V_o called ***terminal velocity***.

$W = F + U$

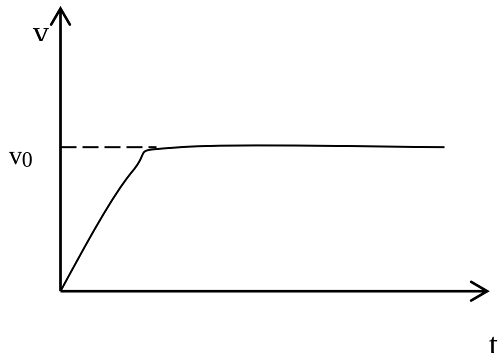
$\frac{4}{3}\pi a^3 pg = \frac{4}{3}\pi a^3 \sigma g + 6\pi a \eta v_0$

$v_0 = \frac{4}{3}\frac{\pi a^3 g}{6\pi a \eta}(p - \sigma)$

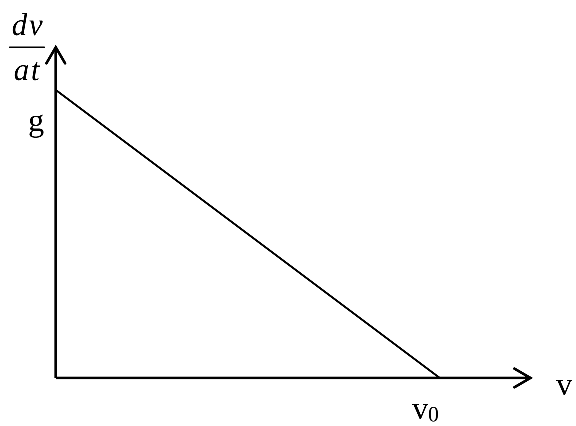
$= \frac{2}{9}\frac{a^2 g}{\eta}(p - \sigma)$

$\therefore v_0 = \frac{2a^2}{9\eta}(p - \sigma)g$

A sketch of velocity against time for a sphere moving in a viscous liquid



A graph of acceleration against velocity.



Measurement of coefficient of viscosity using Stoke's Law

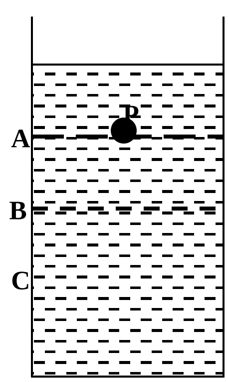
From the equation

$$v_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

$$\eta = \frac{2a^2}{9v_0}(p-\sigma)g$$

The method is suitable for very viscous liquids such as oil.

The densities ρ and σ of the material of the sphere and the specimen liquid respectively are determined.



A tall glass tube T supported vertically in a constant temperature enclosure. Three reference marks ABC are made along the tube T using rubber bands at equal spaces.

A ball bearing is moistened with a specimen liquid and then allowed to fall vertically down a liquid by releasing it. The times t_1 taken by the ball to fall from A to B or t_2 from B to C are measured. The equality of these two times implies that the sphere had attained terminal velocity by the time it reached point A. if t_1 is not equal to t_2 the reference marks are drifted further down the tube and the repeated.

If $t_1 = t_2 = t$, then terminal velocity

$$v_0 = \frac{AB}{t}$$

$$From v_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

The η can be calculated.

When the experiment is repeated with a liquid of coefficient of viscosity η_1 and density σ_1, using the same ball-bearing, then.

$$\eta_1 = \frac{2a^2}{9v_1}(p-\sigma_1)$$

Where v_1 is the new terminal velocity.

$\therefore \frac{\eta}{\eta_1} = \frac{v_1(\rho-\sigma)}{v_0(\rho-\sigma_1)}$ Thus knowing v_1, v, ρ, σ_1, σ, the coefficients of viscosity can be compared.

Effect of temperature on viscosity of fluids

Liquids

The viscosity of a liquid decreases as the temperature rises. When the temperature increases, the molecules of the liquid on the average are further apart and the intermolecular attractive forces decrease.

The resistance to flow decreases hence coefficient of viscosity decreases.

Gases

Viscosity in gases is due to molecules in gases moving from the slower moving layers to the fast moving layers and from the fast moving layers to the slow moving layers. The net result of this is more momentum is carried one way than the other. This is turn means that forces exist on the layers which retard the fast moving layers and accelerate the slower moving layers. The retardation depends on the mass of the molecules and their speeds i.e. the momentum, mv.

When the temperature of the gas is raised, the molecular speeds and hence the momentum increase, leading to an increase in the viscosity of the gas.

Examples

1. A flat plate of area $0.1m^2$ is placed on a flat surface and is separated from the surface by a film of oil 10^{-5}m thick, where η is $1.5Nsm^2$. Calculate the force required to cause the plate to slide on the surface at a constant speed of $1mms^{-1}$.

$$\eta = \frac{\text{shear stress}}{\text{strain rate}} = \frac{F/A}{dv/dy}, 1.5 \Rightarrow \frac{F}{0.1} \times \frac{10^{-5}}{10^{-3}} \quad , \quad \mathrm{F} = \frac{1.5 \times 0.1}{10^{-2}} \quad = 15\mathrm{N}$$

2. The terminal velocity of a spherical oil drop falling in air at 20^0c is 2 x $10^{-7}ms^{-1}$. What is the radius of the drop if its density is $930kgm^{-3}$?

   Assume η of air at 20^0c = 1.8 x 10^{-5}Pas

   Density of air = $1.2kgm^{-3}$

$$V_0 = \frac{2a^2}{9\eta}(p - \sigma)g$$

$$2 \times 10^{-7} = \frac{2a^2}{9 \times 1.81 \times 10^{-5}}(930 - 1.2)9.8$$

$$3.258 \times 10^{-11} = 18204.48a^2$$

$$a = 4.2 \times 10^{-8}m$$

3. A steel ball bearing of diameter 8.0mm is timed as it falls through oil at a steady speed. Over a vertical distance of 0.20m, it takes 0.56s. Assuming the density of steel is 7.8 x 10^{-3} kgm^{-3} and that of oil 9.0 x 10^2 kgm^{-3}. Calculate;
   a) Weight of the ball
   b) Upthrust on the ball
   c) Viscosity of the oil

Using stokes law

$$V = \frac{2a^2}{9\eta}(p-\sigma)g$$

given $a = 4\times10^{-3}$ $\quad V = \frac{0.2}{0.56} \quad = 0.36$

$P = 7.8\times10^3$

$\sigma = 9\times10^2\, kgm^{-3}$

Weight

$= \frac{4}{3}\pi a^3 pg$

$= \frac{4}{3}\pi\left(4\times10^{-3}\right)^2 \times 7.8\times10^3 \times 9.8 = 0.2N$

Upthrust

$= \frac{4}{3}\pi r^3 pg$

$= \frac{4}{3}\pi\left(4\times10^{-3}\right)^3 \times 9\times10^2 \times 9.8$

$= 0.0024N$

Viscosity of oil

$$\eta = \frac{2a^2}{9v_0}(p-\sigma)g$$

$$= \frac{2\times\left(4\times10^{-3}\right)^2}{9\times0.36}\times9.8\left(7.8\times10^3 - 9\times10^2\right)$$

$= 0.6679Pas$

4. A spherical raindrop of radius 2 x 10^{-4} m falls vertically in air at 20^0c. If the densities of air and water are 1.2kgm^{-3} and 1000kgm^{-3} and the viscosity of air 20^0c is 1.8 x 10^{-5} Pas. Calculate the terminal velocity of the drop.

$$V_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

$$= \frac{2\times\left(2\times10^{-4}\right)^2}{9\times1.8\times10^{-5}}(1000-1.2)9.8$$

$= 4.81ms^{-1}$

Exercise

1. Air flows past the upper surface of a horizontal aero plane wing at 250ms^{-1} and past the lower surface of the wing at 200ms^{-1}. The density of air is 1.0kgm^{-3} at the flight altitudes and the area of the wing is 20m^2. Calculate the net lift on the wing. (2.25x10^5N)

2. A pitot – static force fitted on a pressure gauge is used to measure the speed of a boast at sea. Given that the speed of the boat does not exceed $10ms^{-1}$ and the density of sea water is 1050 kgm^{-3}, calculate the maximum pressure on the gauge. ($5.25x10^4$Pa)

CHAPTER FIFTEEN: ELASTICITY

Mechanical properties of materials

The following are used to describe different mechanical characteristics of materials:

Strength: It is the ability of the material to withstand an applied force before the material breaks.

Stiffness: This is the resistance which a material offers to have its shape or size changed.

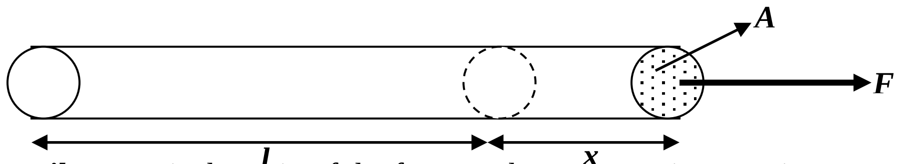
Ductility: This is the ability of a material to be hammered, bent, rolled, or pressed into different shapes. Ductile materials undergo both elastic and plastic deformation.

A material is said to undergo elastic deformation when it can regain its original length after the stretching forces are removed. A material undergoes plastic deformation when it does not regain its original length when the stretching forces are removed.

Brittle material cannot be permanently stretched. It undergoes elastic deformation not plastic deformation.

Tensile Stress, Tensile Strain and Young's Modulus.

Suppose a material of length *l*, cross section, *A*, stretched by an extension *x* when a force F is applied to the material.



Tensile stress is the ratio of the force to the cross section area.i.e.

$$\text{Tensile stress} = \frac{force}{area} = \frac{F}{A}$$

Unit of stress is Nm^{-2} or Pascals (Pa).

Dimensions of stress = [stress] = $ML^{-1}T^{-2}$.

Tensile strain is the ratio of the extension to the original length of the material. i.e. tensile strain $= \frac{extension}{original length} = \frac{x}{l}$

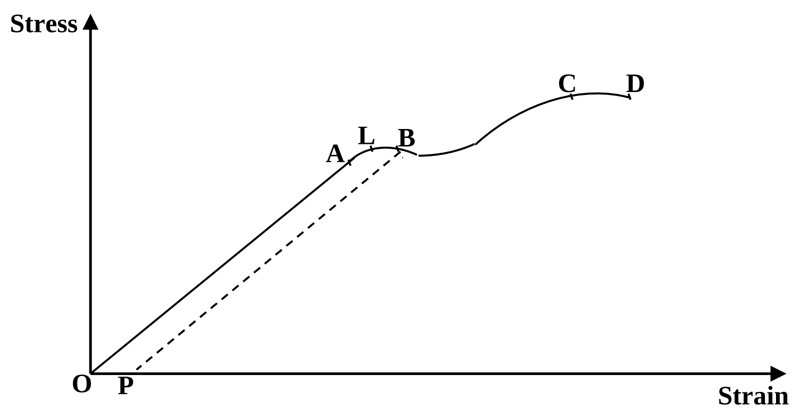
Strain has no units.

Young's Modulus, E.

This is the ratio of tensile stress to tensile strain. Young's Modulus, E $= \frac{Stress}{Strain} = \frac{\frac{F}{A}}{\frac{x}{l}} = \frac{Fl}{Ax}$.

Therefore, stretching force, $F = \frac{EAx}{l}$.

A graph of Stress against strain for a ductile material



OA is a straight line. Up to point A stress is proportional to strain. The portion OA is the Hooke's law region. Region OA is where young's modulus is defined. A is called the *proportional limit*. Along OA and up to L just beyond A, the wire returns to its original length when stress is decreased to zero. L is called *elastic limit*. Beyond L up to B the material becomes plastic. The molecules of the wire begin to slide across each other and some of the energy of the material is dissipated as heat. Point B is the *yield point*. With further increase in stress, work hardening occurs; this is due to the dislocations. When the dislocation density is high slippage of atomic plates became difficult. The dislocations become tangled up with each other. Point C is the *breaking stress or maximum stress*. At this point the material develops kinks. Point D is the where the wire breaks.

Question: Sketch graphs on the same axes of stress against strain for glass, metal wire and rubber, and explain the nature of the graphs.

Force in a metal bar due to contraction or expansion

When a bar is heated, and then prevented from contracting as it cools, a considerable force is exerted at the ends of the bar. Consider a bar of young's modulus, E, a cross sectional A, linear expansivity α, and a decrease in temperature ΔΘ°C. If *l* is the original length of the bar, the decrease in length x if the bar were free to contract = α*l* (ΔΘ). Now

$F = \frac{EAx}{l}$,   but x = α*l* (ΔΘ)

$$F = \frac{EA(\alpha l \Delta\theta)}{l} = EA\alpha(\Delta\theta)$$

Relationship between Young's modulus, E and the force constant, k

From the definition of young's modulus, $E = \frac{Fl}{Ax}, F = \left(\frac{EA}{l}\right)x$ ……… (i)

Using Hooke's law, F = kx ……………………………………………. (ii)

From equations (i) and (ii) $k = \frac{EA}{l}$

Energy stored in a stretching wire

Suppose a wire is stretched by an amount x by applying a force F without exceeding elastic limit. The average force = (0+F)/2 = ½F. Now the work done = force x distance.

Work done = average force x extension= ½F. *x*

.This is the amount of energy stored in the wire.

Further, since $F = \frac{EAx}{l}$,

energy stored $= \frac{EAx^2}{2l}$.

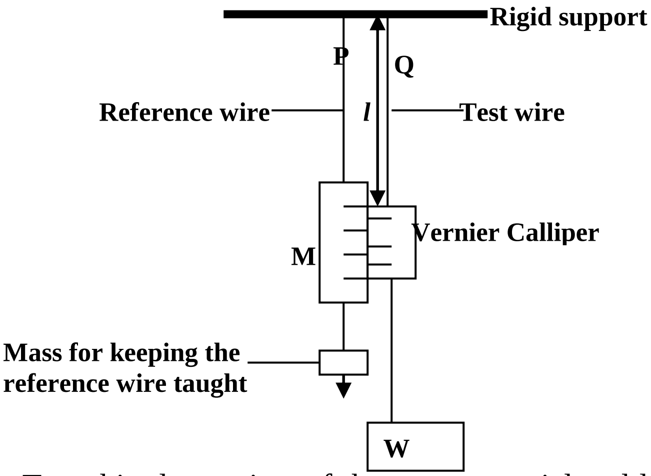
Energy stored per unit volume

energy stored = $\frac{EAx^2}{2l}$ but volume = Al.

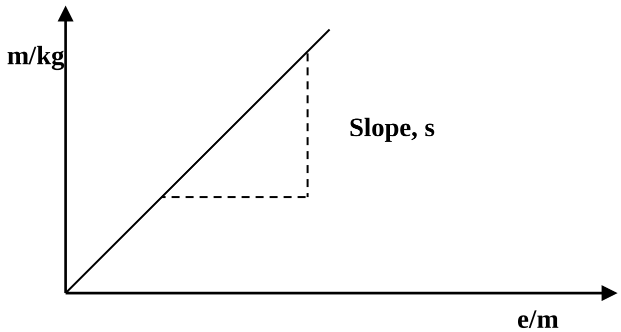
Energy stored per unit volume = $\frac{EAx^2}{2l.Al} = \frac{E}{2}\left(\frac{x}{l}\right)^2 = \frac{Young's\,modulus \times}{2}(strain)^2$

$= \frac{Stress}{2strain} \times (strain)^2$ = ½ x stress x strain

Experiment to determine Young's Modulus for a metal wire



Two thin, long wires of the same material and length P and Q are suspended from a rigid support. P carries a scale M in mm and it's straightened by attaching a weight at its end. Q carries a vernier scale which is along side scale M. Various loads are added to the test wire and the corresponding extensions caused are read off from the vernier scale. After each reading, the load should be removed to check that the wire returns to its original position, showing that elastic limit has not been exceeded. The original length of the wire l is measured from the rigid support up to the vernier scale. Using a micrometer screw gauge, the diameter of the test wire and hence the cross sectional wire A = πr^2 can be obtained. A graph of mass (m) of the load against extension (e) is plotted.



From; $E = \frac{mgl}{eA}$, $m = \frac{EA}{gl}e$

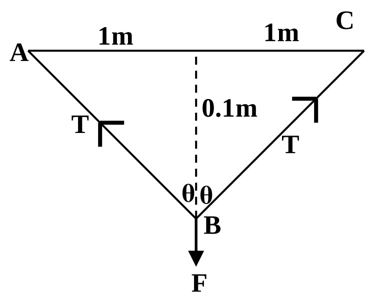
Slope, $s = \frac{EA}{gl}$, Hence $E = \frac{gsl}{A}$

Examples:

1. A metal wire of diameter 2.0x10^{-4}m and length 2m is fixed horizontally between two points 2m apart. Young's modulus for the wire is 2x10^{11}Nm^{-2}.

(i) What force should be applied at the mid point of the wire to depress it by 0.1m.

(ii) Find the work done in (i) above.



1. Solution

$\cos\theta = \frac{0.1}{AB}$ but $AB = \sqrt{(1^2 + 0.1^2)} = 1.005m$

Hence $\cos\theta = \frac{0.1}{1.005}$

ABC = 2xAB = 2x1.005 = 2.01m, Extension, e = 2.01 - 2 =0.01m

$T = \frac{EAe}{l}$ and A = $\pi r^2 = \pi d^2/4$

Resolving vertically, 2Tcosθ = F,

Therefore, $F = \frac{2EAe\cos\theta}{l} = \frac{2E\pi d^2 e\cos\theta}{4l}$

$$F = \frac{2\times 2\times 10^{11}\times \pi \times \left(2\times 10^{-4}\right)^2 \times 0.01\times 0.1}{1\times 4\times 1.005} = 12.5N$$

ii) Work done = ½Fe = ½x12.5x0.01 = 0.0625J

2. A uniform metal bar of length 1.0m and of diameter 2.0cm is fixed between two rigid supports at 25°C. If the temperature of the rod of raised to 75°C. Find (i) the force exerted on the supports. (ii) The energy stored in the rod at 75°C (Young's modulus for the metal = 2.0×10^{11}Pa, coefficient of linear expansion = $1.0 \times 10^{-5}K^{-1}$)

Solution

(i)

$$F = EA\alpha(\Delta\theta)$$
$$F = 2.0 \times 10^{11} \times (\pi \times 0.01^2) \times 1.0 \times 10^{-5}(75 - 25) = 31400N$$

(ii) Energy stored = ½Fe, but e =$\alpha l(\Delta\theta)$

Hence energy stored = ½F$\alpha l(\Delta\theta)$ = ½x31400x1.0×10^{-5}x1x(75-25) =7.85J

Exercise

1. A thin steel wire initially 1.5m long and of diameter 0.5mm is suspended from a rigid support. Calculate (i) the final extension, (3.53×10^{-3}m)
(ii) the energy stored in the wire, when a mass of 3kg is attached to the lower end.
(Young's modulus of steel = $2.0 \times 10^{11}Nm^{-2}$) ($5.19 \times 10^{-2}$J)

2. Two thin wires, one of steel and the other of bronze each 1.5m long and of diameter 0.2cm are joined end to end to form a composite wire of length 3m. What tension in this wire will produce a total extension of 0.064cm? (Young's modulus for steel = 2×10^{11}Pa, Young's modulus for bronze = 1.2×10^{11}Pa) (1009N)

3. A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined end to end to form a composite wire 2.0m long. Find the strain in each wire when the composite stretches by 1.0×10^{-3}m. (Young's moduli for copper and steel are 1.2×10^{11}Pa and 2.0×10^{11}Pa respectively).

4. The ends of a uniform wire of length 2.00m are fixed to points A and B are 2.00m apart in the same horizontal line. When a 5kg mass is attached to the mid-point C of the wire, the equilibrium position of C is 7.5cm below the line AB. Given that young's modulus for the material of the wire is 2.0×10^{11}Pa, find: (i) the strain in the wire,

(ii) the stress in the wire,

(iii) the energy stored in the wire.

Answers

Exercise 1:

a) Force = $\frac{Massxlength}{(time)^2}$

(b) Pressure= $\frac{Mass}{lengthx(time)^2}$

(c) Work = $\frac{forcex(length)^2}{(time)^2}$

(d) Momentum = $\frac{massxlength}{time}$

Exercise 2:

(a) [Density] = ML^{-3}

(b)[Pressure] = $ML^{-1}T^{-2}$

(c) [Power] = $ML^{2}T^{-3}$

(d)[Momentum] = MLT^{-1}

Exercise 3:

Velocity ratio, logarithmic numbers, efficiency, coefficient of friction,

Exercise 4:

1. x = 1, y = 1 and z = 1 2. x = 1, y = 1 and z = 1

Exercise 5:

1. 6.06N 2. 13.3ms^{-2}

Exercise 6:

1. $V = gt$, $S = ½gt^2$, $V^2 = 2gs$ 2. $t = U/g$, $s = U^2/2g$

Exercise 7

2. (i) 9.9s, (ii) 27.7ms^{-1} (iii) 137.2m

Exercise 8:

1. 83.1m, 2. Angle of projection 53.1°, initial speed = 63.9ms^{-1}

Exercise 9:

1. (i) 3004N (ii) 1114.3N

2. (i) 4.905ms^{-2} (ii) 98.1N (iii) 58.87J