S.6 MODERN PHYSICS NOTES FOR COPYING DURING MINI HOLIDAY

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Atomic Nucleus

Rutherford's model of the atom

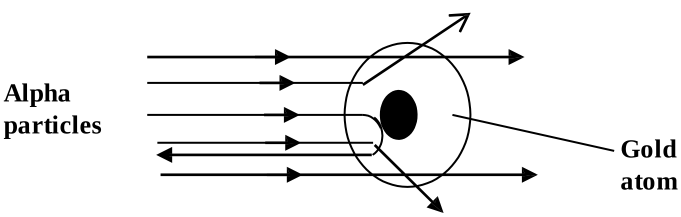
All the positive charge of the atom is concentrated in a small region called the nucleus of diameter less than 10^{-10}m. The negative charge surrounds the positive charge.

This was verified by Rutherford and his team. The experiment involved the scattering of thin Gold foil.

Alpha particles emitted by a radioactive source were directed towards a thin gold foil. The scattered alpha particles were observed on a fluorescent screen on the focal plane of the microscope. Scintillations were observed on the screen whenever the alpha particles struck the ZnS scintillation detector. The microscope was moved to different positions in order to detect the alpha particles.

Observations

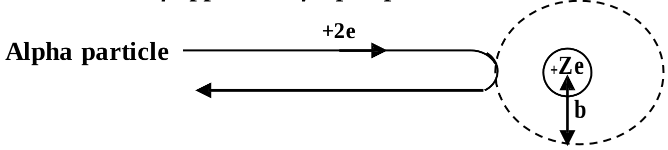
1. The majority of the alpha particles passed through undeflected.
2. A few of the alpha particles were scattered through small angles.
3. Very few alpha particles were deflected through angles greater than 90°.



Conclusion

1. The alpha particles being positively charged, their scattering must be due to the positive charge in the gold atom.
2. Since the majority of the alpha particles passed through un deflected, most of the space inside the atom is empty.
3. Large angle scattering occurred whenever an alpha particle was incident almost head on to the nucleus.
4. Since very few alpha particles were scattered through large angles, it follows that the probability of a head on collision with the nucleus is small and it follows that the nucleus occupies only small proportion of the available space inside an atom.

Closest distance of approach of alpha particles.



Where Z is the proton number or atomic number of the atom.

At closest distance of approach, all alpha particle's kinetic energy is converted into electrostatic potential energy of the alpha particle or nucleus system.

Hence

$$\frac{1}{2}mu^2 = \frac{2Ze^2}{4\pi\varepsilon_0 b}$$

$$\frac{1}{2}mu^2 = \frac{Ze^2}{2\pi\varepsilon_0 b}$$

$$b = \frac{Ze^2}{\pi\varepsilon_0 mu^2}$$

Example

A beam of alpha particles of energy 4.2MeV is incident normal to a gold foil. What is the closest distance of approach by the particles to the nucleus of the gold atom?

(Atomic number of gold = 79)

$$\frac{1}{2}mu^2 = \frac{Ze^2}{2\pi\varepsilon_0 b}$$

$$4.2\times1.6\times10^{-13} = \frac{79\times\left(1.6\times10^{-19}\right)^2}{2\times\pi\times8.85\times10^{-12}\times b}$$

$$b = 5.412\times10^{-14} m$$

Summary, the atom consists of the following main particle: (i) the protons which are positively charged, (ii) the neutrons which carry no charge and the electrons which are found in orbits around the nucleus. The neutrons and protons make up the nucleus of the atom.

The Bohr model of the atom

Bohr postulated that:

(i) Electrons in atoms can exist only in certain discrete orbits and while in these orbits, they don't radiate energy.

(ii) Whenever an electron makes a transition from one orbit to another of lower energy, a quantum of electromagnetic radiation is given off.

The energy of the quantum of radiation emitted is given by $E = hf = E_i - E_f$, where E_i is energy of the electron in the initial orbit, E_f is the energy of the electron in the final orbit, h is Planck's constant and f is the frequency of emitted electron.

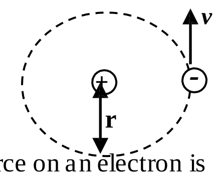
(iii) The angular momentum of an electron in its orbit in an atom is an integral multiple of $\frac{h}{2\pi}$

i.e. $mvr = \frac{nh}{2\pi}$, where n = 1, 2, 3 ……….

The orbit with the lowest energy is called the ground state. All physical systems are in physical equilibrium in the lowest energy state. Other high energy levels are called excited state.

Bohr's theory of the hydrogen atom

Consider an electron in a hydrogen atom to be in a circular orbit of radius, r, about the nucleus.



For circular motion, a centripetal force on an electron is $\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

Hence kinetic energy, $T = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \ldots\ldots\ldots\ldots(i)$

The electric potential energy of the electron, $V(r) = \frac{e}{4\pi\varepsilon_0 r} \times (-e) = \frac{-e^2}{4\pi\varepsilon_0 r} \ldots\ldots\ldots\ldots(ii)$

Total energy, E = T + V(r) = $\frac{e^2}{8\pi\varepsilon_0 r} + \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r} \ldots\ldots\ldots(iii)$

From Bohr's postulates, $mvr = \frac{nh}{2\pi}$

Hence $v^2 = \frac{n^2h^2}{4\pi^2 m^2 r^2} \ldots\ldots\ldots\ldots(iv)$

Substitute equation (iv) in equation (i)

$$\frac{mn^2h^2}{8\pi^2 m^2 r^2} = \frac{e^2}{8\pi\varepsilon_0 r}$$

Hence $r = \frac{n^2h^2\varepsilon_0}{\pi m e^2} \ldots\ldots\ldots\ldots(v)$

Substitute equation (v) in equation (iii)

$$E = \frac{-e^2}{8\pi\varepsilon_0\left(\frac{n^2h^2\varepsilon_0}{\pi m e^2}\right)} = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$$

Hence the allowed electron energies can be obtained from the equation

$E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$, where n is the principal quantum number,; n – 1, 2, 3, ……..

Note: (i) The energy of the electron is always negative. This means that work has to be done to move the electron to infinity where it is considered to have zero energy. The electron is therefore bound to the nucleus.

(ii) Whenever an electron makes a transition from a higher energy level, n_i, to a lower energy level, n_f, the energy of the quantum of radiation emitted is

$$hf = E_i - E_f = \frac{-me^4}{8\varepsilon_0^2 n_i^2 h^2} - \frac{-me^4}{8\varepsilon_0^2 n_f^2 h^2} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Energy of the ground state:- $E_0 = \frac{-me^4}{8\varepsilon_0^2 h^2}$, since n = 1

But $m = 9.11\times10^{-31}, \varepsilon_0 = 8.85\times10^{-12}, h = 6.6\times10^{-34}$

$$E_0 = -2.18\times10^{-18} J$$
$$E_0 = -13.6 eV$$

Hence $E_n = \frac{-13.6}{n^2} eV$

$$E = hf = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$f = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The wave number of the radiation emitted is $\sigma = \frac{f}{c} = \frac{me^4}{8\varepsilon_0^2 h^2 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

The term $\frac{me^4}{8\varepsilon_0^2 h^3 c} = R_H = Rydberg$ constant

$$\sigma = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Spectral lines of hydrogen atoms

Energy levels are grouped into shells. Electrons in one shell have nearly the same energy. The shells are denoted by letters K, L, M, N etc. where K corresponds to n = 1, L to n = 2, M to n = 3 and so on.

Transitions of electron from a high energy level to lower energy level cause electron to lose energy hence producing electromagnetic waves. Transitions from other shells to K- shell emit spectra of wavelength grouped into what is called Lymann series.

Lymann series lie in the Ultra violet region of the spectrometer.

$$\sigma = R_H\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$

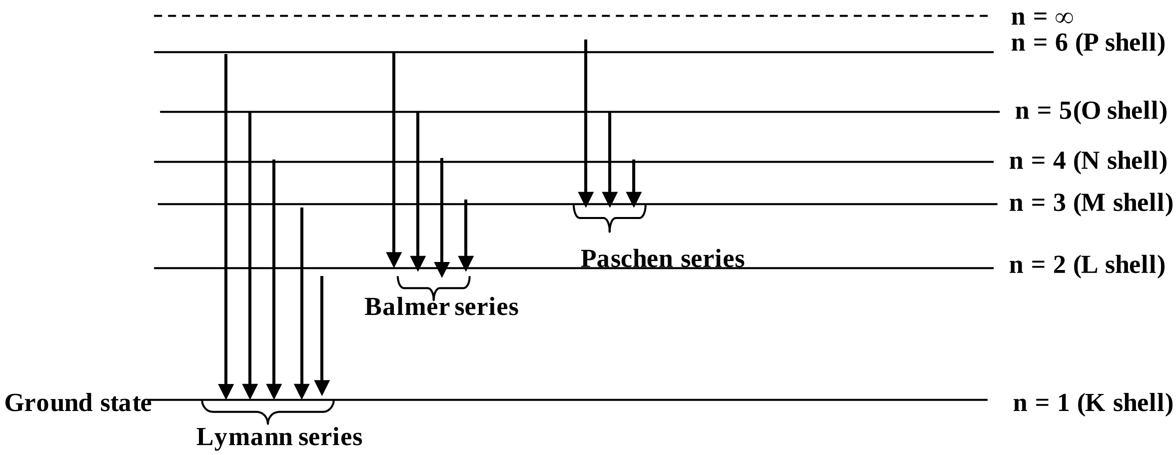
Where n_i = 2, 3, 4…….

Transitions from other high energy levels to the L- shell (n = 2), emits spectra of wavelengths referred to as Balmer series. Balmer series lie in the visible spectrum.

$$\sigma = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right), \qquad n_i = 3, 4, 5\ldots$$

Transition from other high energy levels to the M – shell (n = 3), emits spectra referred to as Paschen series which lie in the infra red region.

$$\sigma = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right), \quad n_i = 4, 5, 6\ldots\ldots.$$



Note Bohr's theory is to simple to explain spectra of more complicated atoms however, the following remain valid:

(i) electrons exist outside the atomic nucleus

(ii) existence of energy levels.

(iii) Emission and absorption of radiation occur in discrete amounts called quanta.

Line emission spectra

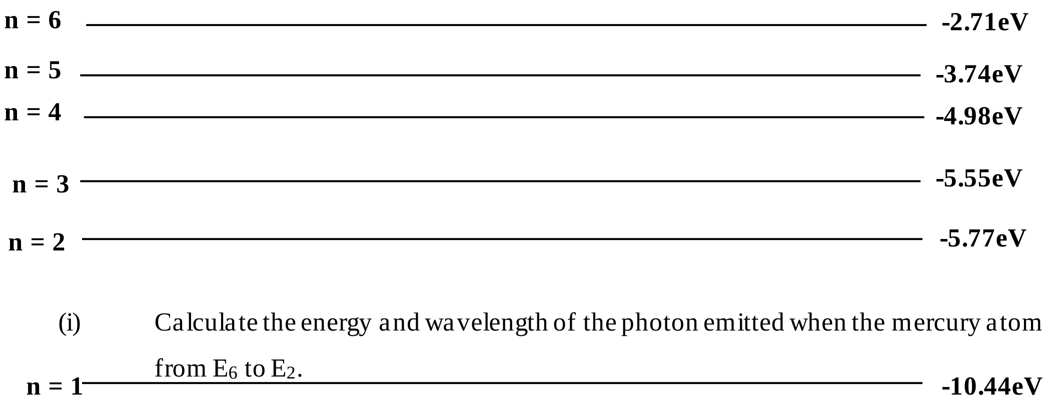
When atoms like H_2, neon etc. are excited due to some form of heat from a frame or electricity, electron transition may occur to higher energy levels. This makes the atom unstable since energy has increased. Electron transition may occur to a vacancy left in the lower energy level and radiation of a definite wavelength or frequency is emitted. A line appear bright against a dark background. The lines are separated which give evidence that energy levels of the atoms are separate.

Line absorption spectra.

An atom's energy can change by only discrete amounts. If a photon of energy, hf, is just enough to excite the atom, such that an electron can jump to one of higher energy levels, the photon will be absorbed. The intensity of the incident radiation is reduced since it has lost a photon. A dark line on a white background is observed, whose wavelength is that of the absorbed photon.

Example

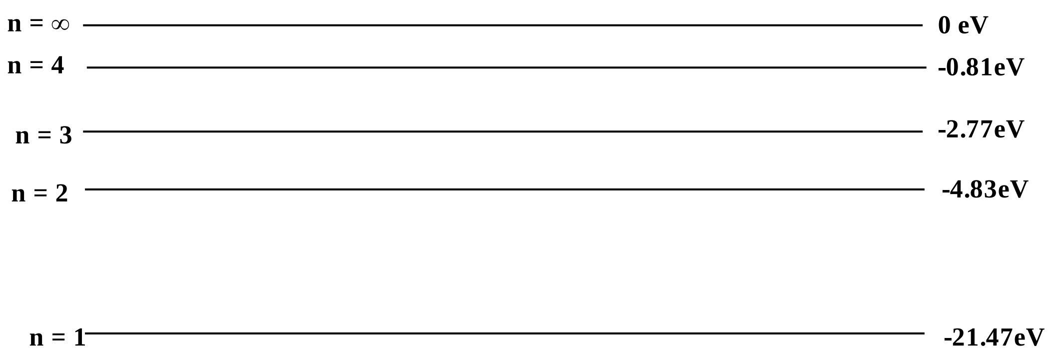
1. The figure below represents the lowest energy levels of mercury.



(i) Calculate the energy and wavelength of the photon emitted when the mercury atom's energy changes from E_6 to E_2.

(ii) Determine which energy levels in the mercury atom are involved in the emission of a line whose wavelength is 546nm.

2. The figure below shows some of the energy levels of a neon atom. In what region of the electromagnetic spectrum does the radiation emit in the transition E_3 to E_2 lie?



The Decay law

The rate of disintegration of a given sample at any time is directly proportional to the number of nuclide N, present at that time, t.

Mathematically

$$\frac{dN}{dt} \propto (-N)$$

The negative sign indicates that N decreases as t increases

$$\frac{dN}{dt} = -\lambda N \quad \text{Where } \lambda \text{ is}$$

Decay constant, λ, is defined as the fraction of the radioactive nuclei which decays per second.

$$\int \frac{dN}{N} = \int -\lambda dt$$
$$In N = -\lambda t + c$$

When t = 0, N = N_0, which is the original number of nuclei.

Hence $l\text{n}N_0 = c$

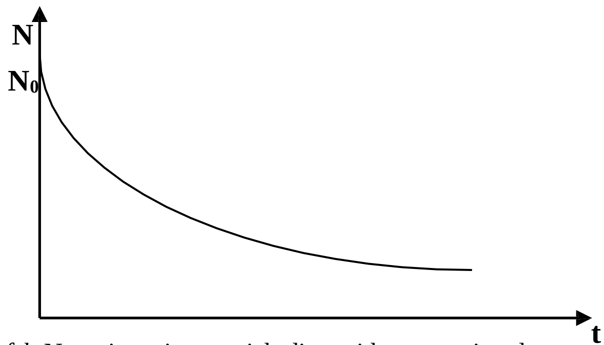
Hence $l\text{n}N = -\lambda t + l\text{n}N_0$

$$\ln\left(\frac{N}{N_O}\right) = -\lambda t$$

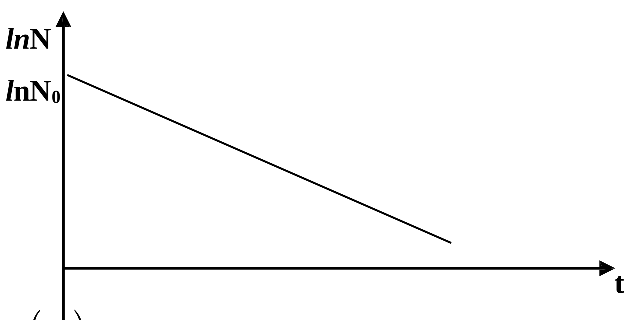
$$or$$

$$N = N_0 e^{-\lambda t}$$

A graph of N against t is called the decay curve



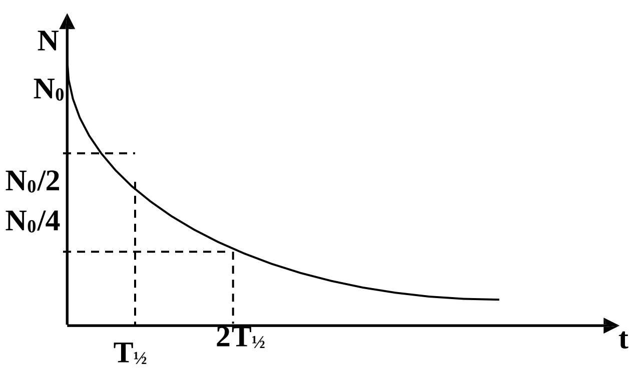
A graph of lnN against t is a straight line with a negative slope.



Half life $\left(T_{1/2}\right)$

The half life of a radioactive source is the time taken for half the number of radioactive nuclei present in the source to disintegrate.

Consider the decay curve of a radioactive source



Relationship between λ and $(T_{1/2})$

When t = $T_{1/2}$, N = $N_0/2$

From

$$N = Noe^{-\lambda t}$$

$$\frac{No}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$In\tfrac{1}{2} = In\, e^{-\lambda T_{1/2}}$$

$$In\tfrac{1}{2} = -\lambda T_{1/2}$$

$$-0.693 = -\lambda T_{1/2}$$

$$\boxed{\lambda = \frac{0.693}{T_{1/2}} = \frac{\ln 2}{T_{1/2}}}$$

Activity of a radioactive source

This is number of disintegration of a radioactive source per second.

$$\text{Activity}\, A = \frac{dN}{dt} = -\lambda N$$

The SI unit of activity is Becquerel (Bq)

IBq = 1disintegration per second

A large unit of activity is curie (Ci)

$$3.70 \times 10^{10} Bq = 1Ci$$

$$\text{Activity}, A = \frac{dN}{dt} = -\lambda N$$

$$N = Noe^{-\lambda t}$$

$$A = -\lambda Noe^{-\lambda t}$$

$$\text{but } A_0 = -\lambda N_0 = \text{initial activity}$$

$$\text{when } t = 0$$

$$\text{hence}$$

$$A = A_0 e^{-\lambda t}$$

Hence Half-life can also be defined as the time taken for the activity of the source to decrease to half the original value.

Example

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1. The half life of a radio isotope is 5.27 years, calculate

i. Its decay constant

ii. The number of years it will take 75% of a given mass of isotope to decay

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27 \times 365 \times 24 \times 3600} = 4 \times 10^{-9} s^{-1}$$

(ii)

$$N = Noe^{-\lambda t}$$
$$N = 0.25No$$
$$0.25No = Noe^{-\lambda t}$$
$$In0.25 = -\lambda t = 4 \times 10^{-9} t$$
$$t = 3.31 \times 10^{8} s$$
$$t = 10.5 years$$

2. The radio isotope ^{60}Co decays by emission of a β particle and a δ ray. Its half life is 5.3 years. Find the activity of the source containing 0.10 gm of ^{60}Co

$$A = \lambda N$$
$$but, \lambda = \frac{In\frac{1}{2}}{5.3 \times 365 \times 24 \times 3600} = 4.15 \times 10^{-9}$$
$$0.10 gm contain \frac{NA}{60} \times 0.10 = \frac{6.02 \times 10 \times 0.10}{60} atoms$$
$$N = 1.003 \times 10^{21} atoms$$
$$A = \lambda N$$
$$A = 115 \times 10^{-9} \times 1.003 \times 10^{21} = 4.16 \times 10^{12} disintegrations^{-1}$$

Exercise:

A silver isotope $^{108}_{47}Ag$ has a half life of 2.4mins. Initially, a sample contain 2.0 x 10^6 nuclei of silver. Find the number of radioactive nuclei left after 1.2 minutes. (ans:1.412x10^6)

Further examples

When Uranium 238, 92 decays, the end product is lead 206,82. The half life is 1.4x10^{17} s. suppose a rock sample contains Pb 206,82 atoms and U238,92 in the ratio 1:5 by weight, calculate the:

(i) Number of Pb 206, 82 atoms in 1.0g of the rock sample.

(ii) Age of the rock. Assume the radioactive decay law $N=N_oe^{-\lambda t}$.

Solution:

(i) Mass of Pb in the sample = 1/6 x 1 = 1/6g

206g of Pb contains 6.02x10^{23} atoms

1/6g of Pb contains (6.02x10^{23}/206)x1/6 atoms = 4.87x10^{20} atoms.

(ii) Mass of Uranium (U) in the sample = 5/6 x1 = 5/6g

238g of U contains 6.02×10^{23} atoms

5/6g Uranium contains $(6.02 \times 10^{23}/238) \times 5/6$g

$= 21.1 \times 10^{20}$ atoms

$N_o = (4.84 + 21.1) \times 10^{20} = 25.97 \times 10^{20}$ atoms.

From $N = N_o e^{-\lambda t}$, $21.1 \times 10^{20} = 25.97 \times 10^{20} e^{-\lambda t}$

$-0.21 = -\lambda t$ and Half life = $0.693/\lambda$

Therefore time, $t = 4.24 \times 10^{16}$s.

Carbon dating

The unstable isotope ^{14}C produced during nuclear reactions in the atmosphere as a result of cosmic ray bombardment give a small portion of ^{14}C in CO_2 in the atmosphere.

Plants take in CO_2 for photosynthesis. When a plant dies it stops taking in CO_2 and its ^{14}Cdecays to ^{14}N by β particle emission.

By measuring the activity of ^{14}C in the remains, the time when the plant died can be estimated.

Example

The activity of a sample of dead wood is 10 counts per minute, while for a living plant is 19 counts per minute. If the half life of ^{14}C is 5500 years, find the age of the wood sample.

$$A = A_0 e^{-\lambda t}$$
$$10 = 19 e^{-\lambda t}$$
$$-\lambda t = \ln\left(\frac{10}{19}\right)$$

but

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln}{5500} yr^{-1}$$

Hence

$$-\frac{\ln 2}{5500} t = \ln\left(\frac{10}{19}\right)$$
$$t = 5093 years$$

Exercise

Wood from a buried ship has a specific activity of 1.2×10^{2}Bqkg^{-1} due to ^{14}C, whereas comparable living wood has an activity of 2×10^{2}Bqjkg^{-1}. What is the age of the ship? (half life of $^{14}C = 5{,}7 \times 10^{3}$years).