LINEAR MOTION

This chapter deals with the study of motion in a straight line.

TERMS USED IN LINEAR MOTION

❖ **Distance:**

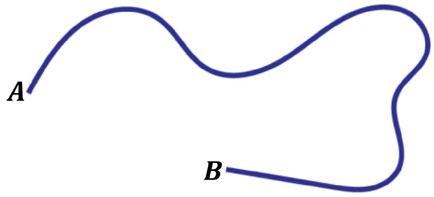
This is the length of path moved by the body

OR

This is the length between two points.

The SI unit of distance is a metre (m).

NOTE: Distance is a scalar quantity.



The person moves from point A to B regardless of any direction he/she takes. The length of path from A to B is called **distance**.

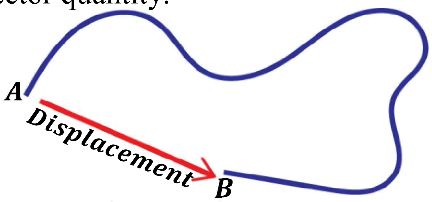
Therefore, distance is described by only magnitude hence a scalar quantity.

❖ **Displacement:**

This is the distance moved in a specified direction.

The SI unit of displacement is a metre (m).

NOTE: Displacement is a vector quantity.



The person moves from point A to B in a specific direction. The distance moved in that direction is called **displacement**.

Therefore, displacement is described by both magnitude and direction hence a vector quantity.

❖ **Speed:**

This is the rate of change of distance with time.

$$\boldsymbol{Speed = \frac{Distance}{Time}}$$

The SI unit of speed is metre per second (ms^{-1} or m/s).

NOTE: Speed is a scalar quantity.

Uniform speed:

This is the constant rate of change of distance with time.

A body is said to move with uniform speed if it covers equal distances in equal time intervals.

If the body moves with varying distances in unit time intervals, then the speed is non-uniform.

❖ **Velocity:**

This is the rate of change of displacement with time.

$$Velocity = \frac{Displacement}{Time}$$

The SI unit of displacement is metre per second (ms^{-1} or m/s).

NOTE: Velocity is a vector quantity.

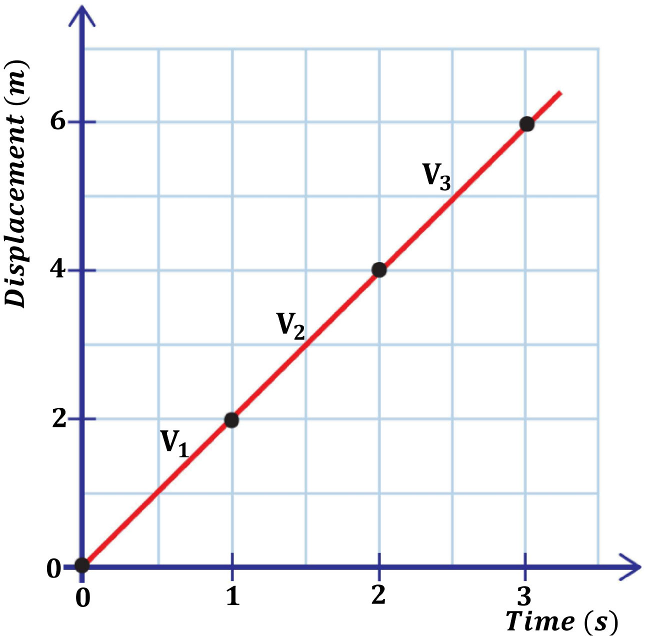
Uniform velocity:

This is the constant rate of change of displacement with time.

A body is said to move with uniform velocity if its displacement changes by equal amounts in equal time intervals.

The figure below shows the displacement-time graph of a student moving around the school compound.

$Displacement\ (m)$	0	2	4	6
$Time\ (s)$	0	1	2	3



A body whose displacement is not constant in given time intervals is said to have non-uniform velocity.

Calculating the corresponding velocities of the student, we get;

$$Velocity = \frac{displacement}{time}$$

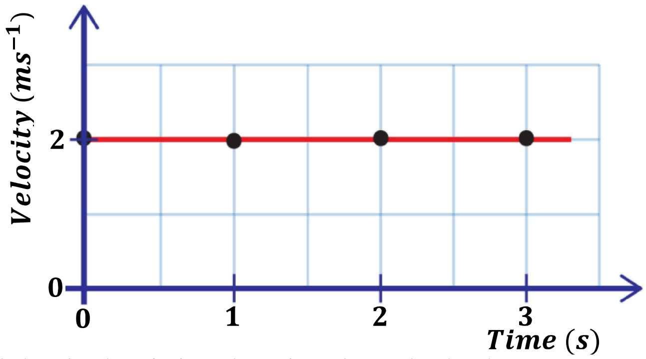
$$V_1 = \frac{2-0}{1-0} = 2ms^{-1}$$

$$V_2 = \frac{4-2}{2-1} = 2ms^{-1}$$

$$V_3 = \frac{6-4}{3-2} = 2ms^{-1}$$

$Displacement\ (m)$	0	2	4	6
$Time\ (s)$	0	1	2	3
$Velocity\ (ms^{-1})$	0	2	2	2

The figure below shows the velocity-time graph for the student's motion.



A straight-line graph is obtained showing that the velocity is constant or uniform.

Differences between Speed and Velocity

SPEED	VELOCITY
• It is the rate of change of distance with time. • It is a scalar quantity.	• It is the rate of change of displacement with time. • It is a vector quantity.

TYPES OF VELOCITIES

Initial velocity, u:

This is the velocity with which the body starts its motion. i.e. it's the starting velocity.

NOTE:

- If a body starts from rest, its initial velocity, $u = 0ms^{-1}$.
- If a stationary body starts its motion, its initial velocity, $u = 0ms^{-1}$.
- If a body starts with a certain velocity, x, then its initial velocity, $u = xms^{-1}$.

Final velocity, v:

This is the velocity with which the body ends its motion. i.e. it's the ending velocity.

NOTE:

- If a body is brought to rest, its final velocity, $v = 0ms^{-1}$.
- If a body stops with a certain velocity, x, then its final velocity, $v = xms^{-1}$.

Average velocity:

This is the average of the initial and final velocity.

$$\boldsymbol{Average\ velocity} = \frac{\boldsymbol{Final\ velocity\ (v) + Initial\ velocity\ (u)}}{\boldsymbol{2}}$$

❖ **Acceleration:**

This is the rate of change of velocity with time.

$$\boldsymbol{Acceleration} = \frac{\boldsymbol{Change\ in\ Velocity}}{\boldsymbol{Time}}$$

$$\boldsymbol{Acceleration} = \frac{\boldsymbol{Final\ velocity\ (v) - Initial\ velocity\ (u)}}{\boldsymbol{Time}}$$

The SI unit of acceleration is metre per second squared (ms^{-2} or m/s^2).

<u>Question</u>:

"A body has an acceleration of $2ms^{-2}$". What do you understand by the statement?

The statement means that the velocity of the body increases by $2ms^{-1}$ every second.

<u>Uniform acceleration</u>:

This is the constant rate of change of velocity with time.

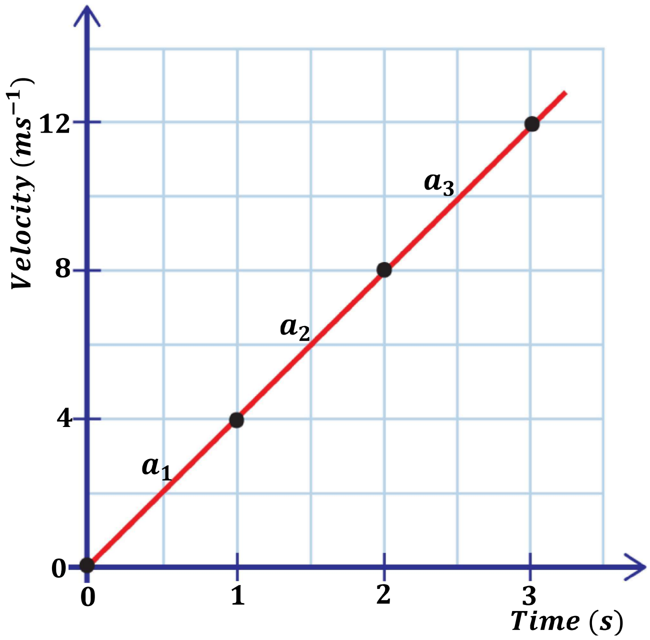
A body is said to move with uniform acceleration if its velocity changes by equal amounts in equal time intervals.

NOTE:

- A body with uniform velocity has **zero acceleration** because there is no change in velocity.
- Acceleration is either positive or negative. If the acceleration is increasing, then it is said to be positive and if it is decreasing (retarding or decelerating), it is said to be negative.

The figure below shows the velocity-time graph of a student moving around the school compound.

$Velocity\ (ms^{-1})$	0	4	8	12
$Time\ (s)$	0	1	2	3



A straight-line graph is obtained showing that the acceleration is uniform.

Calculating the corresponding accelerations of the student, we get;

$$Acceleration = \frac{change\ in\ velocity}{time}$$

$$a_1 = \frac{4-0}{1-0} = 4ms^{-2}$$

$$a_2 = \frac{8-4}{2-1} = 4ms^{-2}$$

$$a_3 = \frac{12-8}{3-2} = 4ms^{-2}$$

❖ **<u>Deceleration (Retardation)</u>:**

When a body is moving with a decreasing velocity, then the body is said to be decelerating or retarding and the acceleration is negative.

<u>Converting units of velocities</u>

Convert the following units.

(a) $108\ kmh^{-1}$ to ms^{-1}

$$\boldsymbol{Distance = 108km}$$
$$\boldsymbol{1km = 1000m}$$
$$\boldsymbol{108km = d}$$
$$\boldsymbol{d = (108 \times 1000)m}$$
$$\boldsymbol{d = 108000m}$$

$$\boldsymbol{time = 1\ hour}$$
$$\boldsymbol{1hr = 3600s}$$

$$\boldsymbol{V = \frac{d}{t}}$$
$$\boldsymbol{V = \frac{108000}{3600}}$$
$$\boldsymbol{V = 30ms^{-1}}$$

(b) $60\ ms^{-1}$ to kmh^{-1}

$$\boldsymbol{Distance = 60m}$$
$$\boldsymbol{1km = 1000m}$$
$$\boldsymbol{d = 60m}$$
$$\boldsymbol{d = \frac{60}{1000}km}$$
$$\boldsymbol{d = 0.06km}$$

$$\boldsymbol{time = 1\ second}$$
$$\boldsymbol{1hr = 3600s}$$
$$\boldsymbol{1s = \frac{1}{3600}hrs}$$

$$\boldsymbol{V = \frac{d}{t}}$$
$$\boldsymbol{V = \frac{0.06}{1/3600}}$$
$$\boldsymbol{V = 216ms^{-1}}$$

<u>Examples</u>:

1. A car starts from rest and acquires a final velocity of $60ms^{-1}$ in $30s$. Find the acceleration of the car.

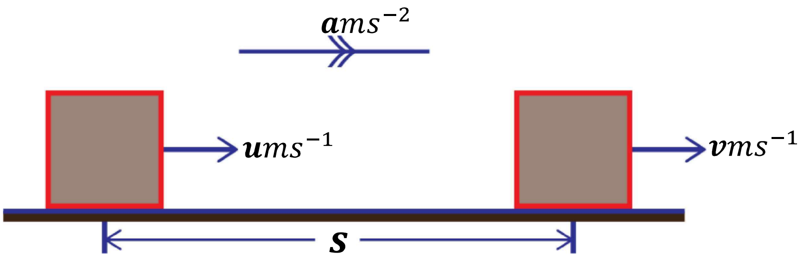
$$\boldsymbol{u = 0ms^{-1}, \quad v = 60ms^{-1}, \quad t = 30s}$$
$$\boldsymbol{a = \frac{v - u}{t}}$$
$$\boldsymbol{a = \frac{60 - 0}{30}}$$
$$\boldsymbol{a = 2ms^{-1}}$$

2. A body moving with a velocity of $80ms^{-1}$ changes to $60ms^{-1}$ in $2s$. Find the deceleration of the body.

$$\boldsymbol{u = 80ms^{-1}, \quad v = 60ms^{-1}, \quad t = 2s}$$
$$\boldsymbol{a = \frac{v - u}{t}}$$
$$\boldsymbol{a = \frac{60 - 80}{2}}$$
$$\boldsymbol{a = -10ms^{-1}}$$

EQUATIONS OF UNIFORMLY ACCELERATED MOTION

Consider a body starting with initial velocity, $\boldsymbol{u}ms^{-1}$ and accelerates uniformly at a rate of, $\boldsymbol{a}ms^{-2}$ to acquire a final velocity, $\boldsymbol{v}ms^{-1}$ in time $\boldsymbol{t}s$ and covers a displacement of $\boldsymbol{s}m$ as shown below



First equation of linear motion:

$$Acceleration = \frac{change\ in\ velocity}{time\ taken}$$

$$Acceleration = \frac{final\ velocity - initial\ velocity}{time\ taken}$$

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$$\boxed{\boldsymbol{v = u + at}} \text{-----------} (\boldsymbol{i})$$

Second equation of linear motion:

From the definition of displacement,

$$Displacement = Average\ velocity \times Time$$

$$s = \left(\frac{u+v}{2}\right) \times t$$

$$2s = (u+v)t$$

From equation (i), $\boldsymbol{v = u + at}$

$$2s = (u + u + at)t$$

$$2s = (2u + at)t$$

$$2s = 2ut + at^2$$

$$s = \frac{2ut}{2} + \frac{at^2}{2}$$

$$\boxed{\boldsymbol{s = ut + \frac{1}{2}at^2}}$$

Third equation of linear motion:

From the definition of displacement,

$$Displacement = Average\ velocity \times Time$$

$$s = \left(\frac{u+v}{2}\right) \times t$$

From equation (i),

$$\boldsymbol{t = \frac{v-u}{a}}$$

$$s = \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right)$$

$$s = \frac{(u+v)(v-u)}{2a}$$

$$2as = uv - u^2 + v^2 - uv$$

$$\boxed{\boldsymbol{v^2 = u^2 + 2as}}$$

In summary, the three equations of uniformly accelerated motion are;

$$\textit{First equation}: \quad \boldsymbol{v = u + at}$$

$$\textit{Second equation}: \quad \boldsymbol{s = ut + \frac{1}{2}at^2}$$

$$\textit{Third equation}: \quad \boldsymbol{v^2 = u^2 + 2as}$$

Examples:

1. A car accelerated from a velocity of $10ms^{-1}$ to $30ms^{-1}$ in $4s$. Calculate
   i) Acceleration of the car.
   ii) Distance moved by the car.

(i)

$$\boldsymbol{u = 10ms^{-1} \quad v = 30ms^{-1} \quad t = 4s}$$

$$\boldsymbol{v = u + at}$$

$$\boldsymbol{a = \frac{v - u}{t}}$$

$$\boldsymbol{a = \frac{30 - 10}{4}}$$

$$\boldsymbol{a = \frac{20}{4}}$$

$$\boldsymbol{a = 5ms^{-2}}$$

(ii)

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

$$\boldsymbol{s = 10 \times 4 + \frac{1}{2} \times 5 \times 4^2}$$

$$\boldsymbol{s = 40 + \frac{1}{2} \times 5 \times 16}$$

$$\boldsymbol{s = 40 + 40}$$

$$\boldsymbol{s = 80m}$$

2. A body starts from rest and accelerates uniformly to a velocity of $15ms^{-1}$ at a rate of $5ms^{-2}$. Calculate the distance moved by the body.

$$\boldsymbol{u = 0ms^{-1} \quad v = 15ms^{-1} \quad a = 5ms^{-2}}$$

$$\boldsymbol{v^2 = u^2 + 2as}$$

$$\boldsymbol{15^2 = 0^2 + 2 \times 5 \times s}$$

$$\boldsymbol{225 = 0 + 10s}$$

$$\boldsymbol{s = \frac{225}{10}}$$

$$\boldsymbol{s = 22.5m}$$

3. A body moving with a velocity of $20ms^{-1}$ accelerates to a velocity of $30ms^{-1}$ in 5 seconds. Calculate;
   i) Acceleration of the body.
   ii) Distance moved by the body.

(i)

$$\boldsymbol{u = 20ms^{-1} \quad v = 30ms^{-1} \quad t = 5s}$$

$$\boldsymbol{v = u + at}$$

$$\boldsymbol{a = \frac{v - u}{t}}$$

$$\boldsymbol{a = \frac{30 - 20}{5}}$$

$$\boldsymbol{a = \frac{10}{5}}$$

$$\boldsymbol{a = 2ms^{-2}}$$

(ii)

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

$$\boldsymbol{s = 20 \times 5 + \frac{1}{2} \times 2 \times 5^2}$$

$$\boldsymbol{s = 100 + \frac{1}{2} \times 2 \times 25}$$

$$\boldsymbol{s = 100 + 25}$$

$$\boldsymbol{s = 125m}$$

4. A car moving with a velocity of $25ms^{-1}$ retards uniformly at a rate of $2.5ms^{-2}$. Calculate;
   i) Velocity of the car after 8s.
   ii) Time it takes to come to rest.
   iii) Distance moved by the car.

$$u = 25ms^{-1} \quad a = -2.5ms^{-2}$$

(i)

$$t = 8s$$
$$v = u + at$$
$$v = 25 + -2.5 \times 8$$
$$v = 25 - 20$$
$$v = 5ms^{-1}$$

(ii)

$$v = 0ms^{-1}$$
$$v = u + at$$
$$0 = 25 + -2.5t$$
$$t = \frac{-25}{-2.5}$$
$$t = 10s$$

(iii)

$$s = ut + \frac{1}{2}at^2$$
$$s = 25 \times 10 + \frac{1}{2} \times (-2.5) \times 10^2$$
$$s = 250 + \frac{1}{2} \times (-2.5) \times 100$$
$$s = 250 - 125$$
$$s = 125m$$

OR

$$v^2 = u^2 + 2as$$
$$0^2 = 25^2 + 2 \times (-2.5) \times s$$
$$0 = 625 - 5s$$
$$s = \frac{625}{5}$$
$$s = 125m$$

5. A car travelling at $90kmh^{-1}$ is uniformly brought to rest in 40 seconds. Calculate its acceleration.

$$u = 90kmh^{-1} \quad v = 0ms^{-1} \quad t = 40s$$

<u>Converting initial velocity to ms^{-1}</u>

$$distance = 90km \quad time = 1hour$$
$$u = \frac{distance\ (m)}{time\ (s)}$$
$$u = \frac{90 \times 1000}{3600}$$
$$u = 25ms^{-1}$$

$$v = u + at$$
$$a = \frac{v - u}{t}$$
$$a = \frac{0 - 25}{40}$$
$$a = \frac{-25}{40}$$
$$a = -0.625ms^{-2}$$

6. A driver of a bus initially travelling at $72kmh^{-1}$ applies the brakes on seeing crossing elephants. The bus comes to rest in 5 seconds. Calculate;
   i) the retardation of the bus.
   ii) Distance travelled in this interval.

$$u = 72kmh^{-1} \quad v = 0ms^{-1} \quad t = 5s$$

<u>Converting initial velocity to ms^{-1}</u>

$$distance = 72km \quad time = 1hour$$
$$u = \frac{distance\ (m)}{time\ (s)}$$
$$u = \frac{72 \times 1000}{3600}$$
$$u = 20ms^{-1}$$

$$v = u + at$$
$$a = \frac{v - u}{t}$$
$$a = \frac{0 - 20}{5}$$
$$a = \frac{-20}{5}$$
$$a = -4ms^{-2}$$

$$s = ut + \frac{1}{2}at^2$$
$$s = 20 \times 5 + \frac{1}{2} \times (-4) \times 5^2$$
$$s = 100 + \frac{1}{2} \times (-4) \times 25$$
$$s = 100 - 50$$
$$s = 50m$$

EXERCISE:

1. A body starts from rest and accelerates uniformly to a velocity of $25ms^{-1}$ at a rate of $2.5ms^{-2}$. Calculate the distance moved by the body.

2. A particle initially moving with a velocity of $5ms^{-1}$ accelerates uniformly at $4ms^{-2}$. Find
   i) the velocity of the particle after 8s.
   ii) the displacement of the particle after 10s.
   iii) displacement by the time its velocity is $25ms^{-1}$.

3. A car moving with a velocity of $25ms^{-1}$ retards uniformly at a rate of $2.5ms^{-2}$. Calculate;
   i) Velocity of the car after 8s.
   ii) Time it takes to come to rest.
   iii) Distance moved by the car.
4. A body traveling at $90kmh^{-1}$is retarded to rest at $20ms^{-2}$. Calculate the distance covered.

5. A car on a straight road accelerates from rest to a speed of $30ms^{-1}$ in 5s. it then travels at the same speed for 5 minutes and then brakes for 10s in order to come to stop. Calculate the;
   i) acceleration of the car during the motion.
   ii) deceleration of the car.
   iii) total distance travelled.

6. Calculate the final (maximum) velocity of a body travelling at $4ms^{-1}$, when it accelerates at $2ms^{-2}$ and covers a distance of 5m.

7. A car travelling at $40ms^{-1}$ is uniformly decelerated to $25ms^{-1}$ for 5s. Calculate the total distance covered.

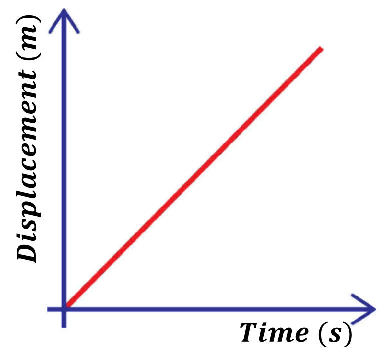
MOTION GRAPHS

These are graphs that represent the motion of the body moving in a straight line.
They include;

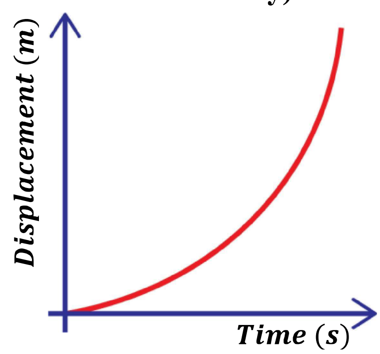
(a) DISPLACEMENT-TIME GRAPHS

These are graphs of motion with displacement of a body along the vertical axis and time along the horizontal axis.
The graphs below show displacement-time graphs for a body;

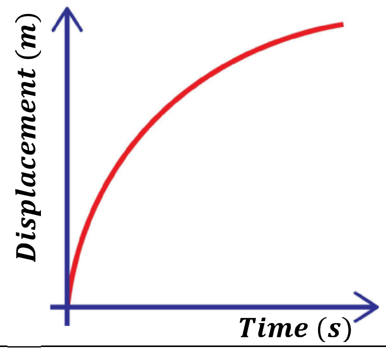
i) <u>Uniform velocity</u>:



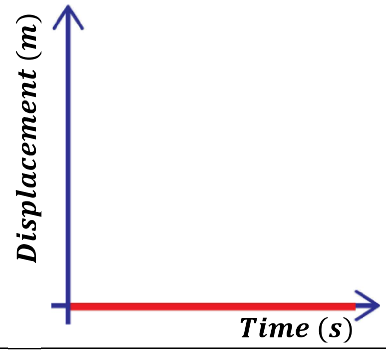
ii) <u>Uniform acceleration</u>: (Non-uniform velocity)



iii) <u>Uniform deceleration</u>:



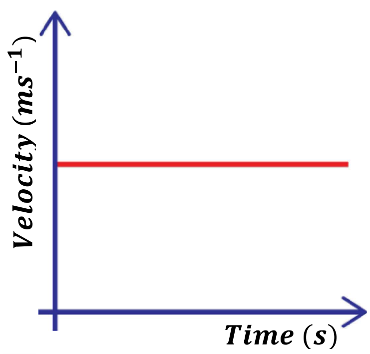
iv) <u>Body at rest</u>:



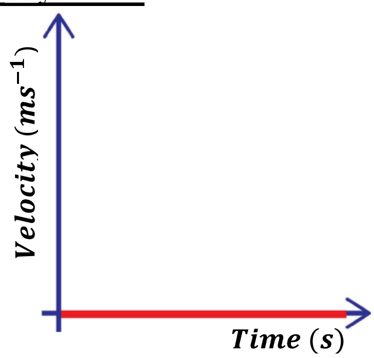
(b) VELOCITY-TIME GRAPHS

These are graphs of motion with velocity of a body along the vertical axis and time along the horizontal axis.
The graphs below show velocity-time graphs for a body;

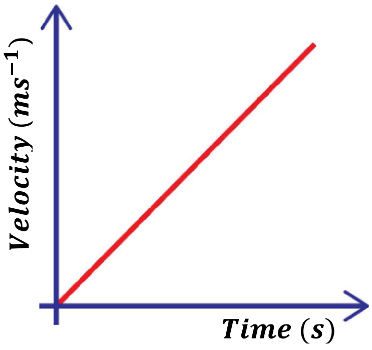
i) <u>Uniform velocity</u>:



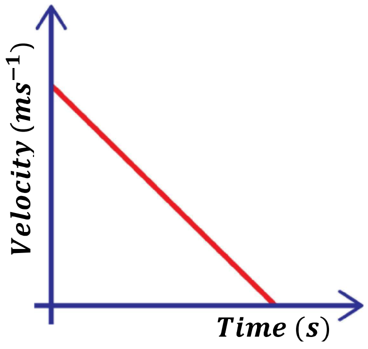
ii) <u>Body at rest</u>:



iii) Uniform acceleration:



iv) Uniform deceleration:

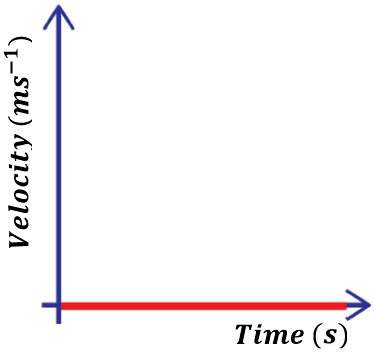


(c) ACCELERATION-TIME GRAPHS

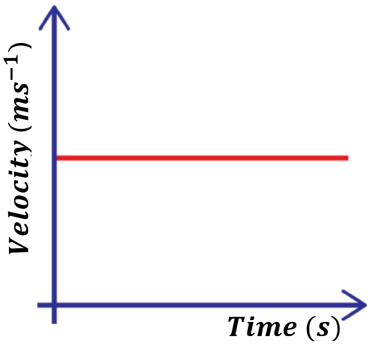
These are graphs of motion with acceleration of a body along the vertical axis and time along the horizontal axis.

The graphs below show acceleration-time graphs for a body;

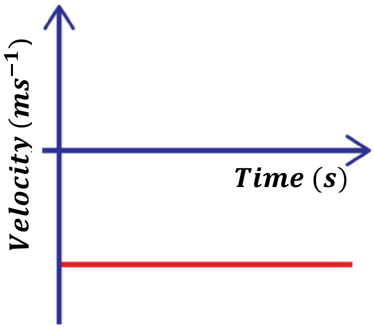
i) Uniform velocity:



ii) Uniform acceleration:

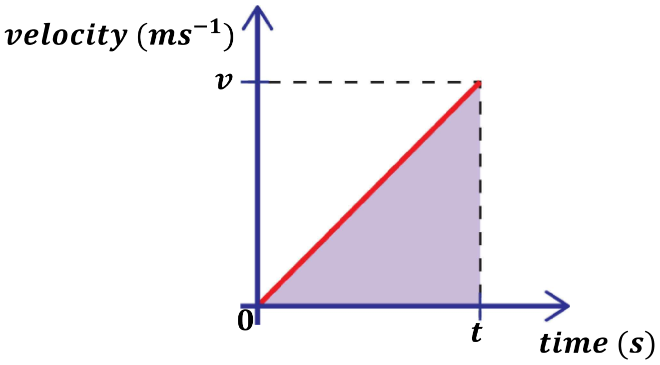


iii) Uniform deceleration:



<u>AREA UNDER A VELOCITY-TIME GRAPH</u>

Consider a body starting from rest and accelerates uniformly to a velocity, $\boldsymbol{v}ms^{-1}$ in time $\boldsymbol{t}s$.



$$Distance\ covered = Average\ velocity \times Time$$

$$Distance\ covered = (\frac{final\ velocity + initial\ velocty}{2}) \times Time$$

$$Distance\ covered = (\frac{v+u}{2}) \times t$$

$$Distance\ covered = (\frac{v+0}{2}) \times t$$

$$\boldsymbol{Distance\ covered = \frac{1}{2}vt = Area\ of\ triangle\ under\ the\ graph}$$

NOTE:

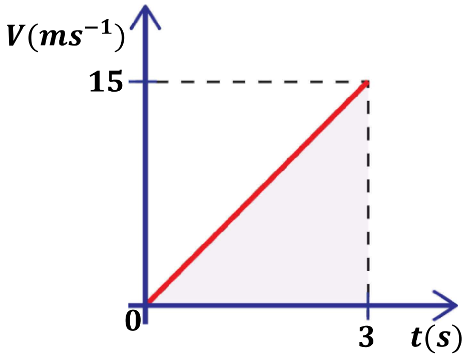
- **Therefore, distance covered by the body is equal to the area under the velocity-time graph.**
- **Average velocity of the body under motion is given by;**

$$\boldsymbol{Average\ velocity = \frac{Total\ distance}{Total\ time}}$$

<u>Examples</u>:

1. Describe the motion of the car for the graphs below.

a)



$$\boldsymbol{From,\quad v = u + at}$$

$$\boldsymbol{Then,\quad a = \frac{v-u}{t}}$$

$$\boldsymbol{u = 0ms^{-1}\quad v = 15ms^{-1}\quad t = 3s}$$

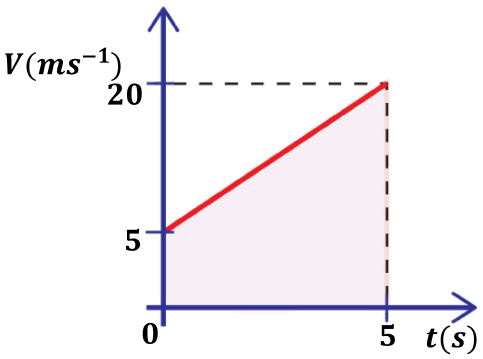
$$\boldsymbol{a = \frac{15-0}{3}}$$

$$\boldsymbol{a = \frac{15}{3}}$$

$$\boldsymbol{a = 5ms^{-2}}$$

A car starts from rest (with velocity of $0ms^{-1}$) and accelerates uniformly at a rate of $5ms^{-2}$ to a velocity of $15ms^{-1}$ in 3 seconds.

b)



$From,\quad v = u + at$

$Then,\quad a = \dfrac{v-u}{t}$

$u = 5ms^{-1}\quad v = 20ms^{-1}\quad t = 5s$

$$a = \frac{20-5}{5}$$

$$a = \frac{15}{5}$$

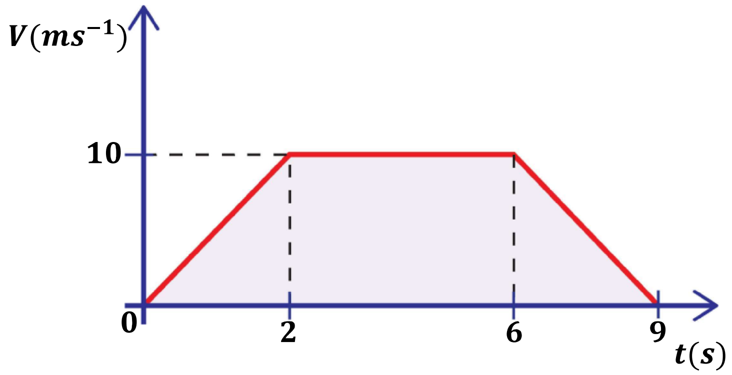
$$a = 3ms^{-2}$$

A car starts from with a velocity of $5ms^{-1}$ and accelerates uniformly at a rate of $3ms^{-2}$ to a velocity of $20ms^{-1}$ in 5 seconds.

OR

A car moving with a velocity of $5ms^{-1}$ accelerates uniformly to a velocity of $20ms^{-1}$ with an acceleration of $3ms^{-2}$.

c)



$\underline{For\ uniform\ acceleration}$

$From,\quad a = \dfrac{v-u}{t}$

$u = 0ms^{-1}\quad v = 10ms^{-1}\quad t = 2s$

$$a = \frac{10-0}{2}$$

$$a = \frac{10}{2}$$

$$a = 5ms^{-2}$$

$\underline{For\ uniform\ deceleration}$

$From,\quad a = \dfrac{v-u}{t}$

$u = 10ms^{-1}\quad v = 0ms^{-1}\quad t = (9-6) = 3s$

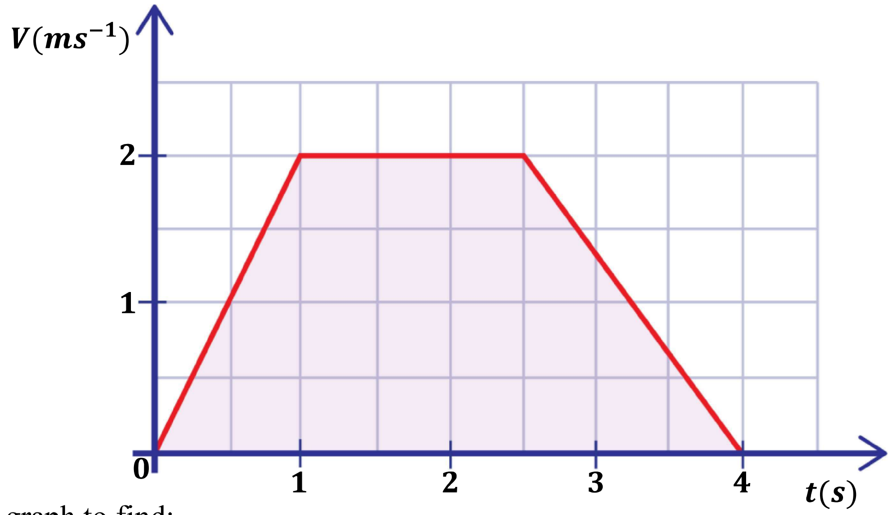
$$a = \frac{0-10}{3}$$

$$a = \frac{-10}{3}$$

$$a = -3.33ms^{-2}$$

A car starts from rest and accelerates uniformly at a rate of $5ms^{-2}$ to velocity of $10ms^{-1}$ in 2s. It maintains this velocity for 4s and it then finally decelerates uniformly at a rate of $3.33ms^{-2}$ to rest in 3 seconds.

2. The figure below is a velocity-time graph of a car.



Use the graph to find;

i) the acceleration of the car.

ii) the deceleration of the car.

iii) the total distance covered by the car.

iv) the average velocity of the car.

i) $\underline{\boldsymbol{For\ uniform\ acceleration}}$

$$\boldsymbol{From,\quad a = \frac{v-u}{t}}$$

$$\boldsymbol{u = 0ms^{-1}\quad v = 2ms^{-1}\quad t = 1s}$$

$$\boldsymbol{a = \frac{2-0}{1}}$$

$$\boldsymbol{a = \frac{2}{1}}$$

$$\boldsymbol{a = 2ms^{-2}}$$

ii) $\underline{\boldsymbol{For\ uniform\ deceleration}}$

$$\boldsymbol{From,\quad a = \frac{v-u}{t}}$$

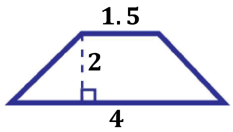
$$\boldsymbol{u = 2ms^{-1}\quad v = 0ms^{-1}\quad t = (4-2.5) = 1.5s}$$

$$\boldsymbol{a = \frac{0-2}{1.5}}$$

$$\boldsymbol{a = \frac{-2}{1.5}}$$

$$\boldsymbol{a = -1.33ms^{-2}}$$

iii) $\underline{\boldsymbol{Total\ distance\ covered}}$



$$\boldsymbol{Distance = Area\ of\ a\ trapezium}$$

$$\boldsymbol{Distance = \frac{1}{2}h(a+b)}$$

$$\boldsymbol{Distance = \frac{1}{2} \times 2(1.5+4)}$$

$$\boldsymbol{Distance = \frac{11}{2}}$$

$$\boldsymbol{Distance = 5.5m}$$

iv) $\underline{\boldsymbol{Average\ velocity}}$

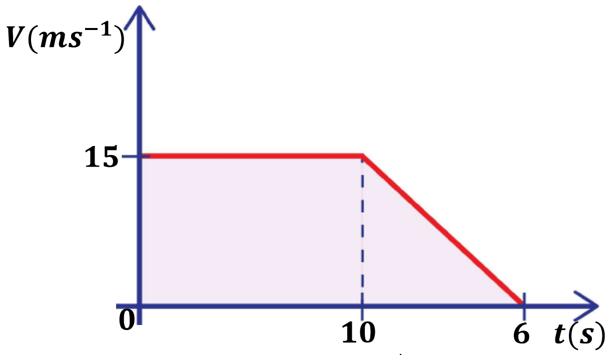
$$\boldsymbol{Average\ velocity = \frac{Total\ distance}{Total\ time}}$$

$$\boldsymbol{Average\ velocity = \frac{5.5}{4}}$$

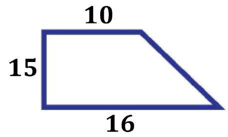
$$\boldsymbol{Average\ velocity = 1.375ms^{-1}}$$

3. Sketch the velocity-time graphs for the information below and calculate the distance covered.

a) A boat is moving with a velocity of $15ms^{-1}$ for $10s$. It is then brought to rest.



<u>Total distance covered</u>



$$Distance = Area\ of\ a\ trapezium$$

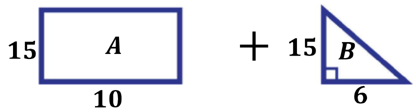
$$Distance = \frac{1}{2}h(a+b)$$

$$Distance = \frac{1}{2} \times 15(10+16)$$

$$Distance = \frac{390}{2}$$

$$Distance = 195m$$

OR



$$Distance = \underset{rectangle\ A}{Area\ of} + \underset{triangle\ B}{Area\ of}$$

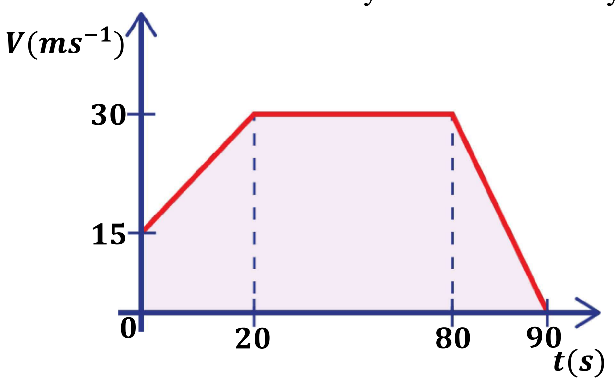
$$Distance = (l \times w) + \frac{1}{2}bh$$

$$Distance = (10 \times 15) + \frac{1}{2} \times 6 \times 15$$

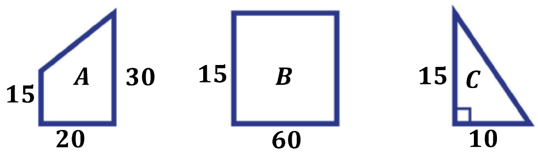
$$Distance = 150 + 45$$

$$Distance = 45m$$

b) A car starts with a velocity of $10ms^{-1}$ and accelerates uniformly for $20s$ to a velocity of $30ms^{-1}$. It then maintains this velocity for $60s$ and finally decelerates uniformly to rest for $10s$.



<u>Total distance covered</u>



$$Distance = \underset{trapezium\ A}{Area\ of} + \underset{rectangle\ B}{Area\ of} + \underset{triangle\ C}{Area\ of}$$

$$Distance = \frac{1}{2}h(a+b) + (l \times w) + \frac{1}{2}bh$$

$$= \frac{1}{2} \times 20(15+30) + (60 \times 15) + \frac{1}{2} \times 10 \times 15$$

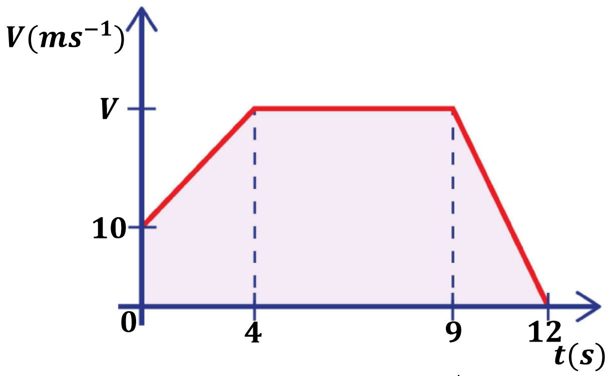
$$Distance = \frac{900}{2} + 900 + \frac{150}{2}$$

$$Distance = 450 + 900 + 75$$

$$Distance = 1425m$$

4. A boat traveling at $10ms^{-1}$ uniformly accelerated for $4s$ at $2ms^{-2}$ to a maximum speed. It then moves with this maximum speed for $5s$ after it is uniformly brought to rest in another $3s$.
   i) Draw a velocity-time graph for the motion of the boat.
   ii) Calculate the maximum speed.
   iii) Calculate the deceleration of the boat.

i)



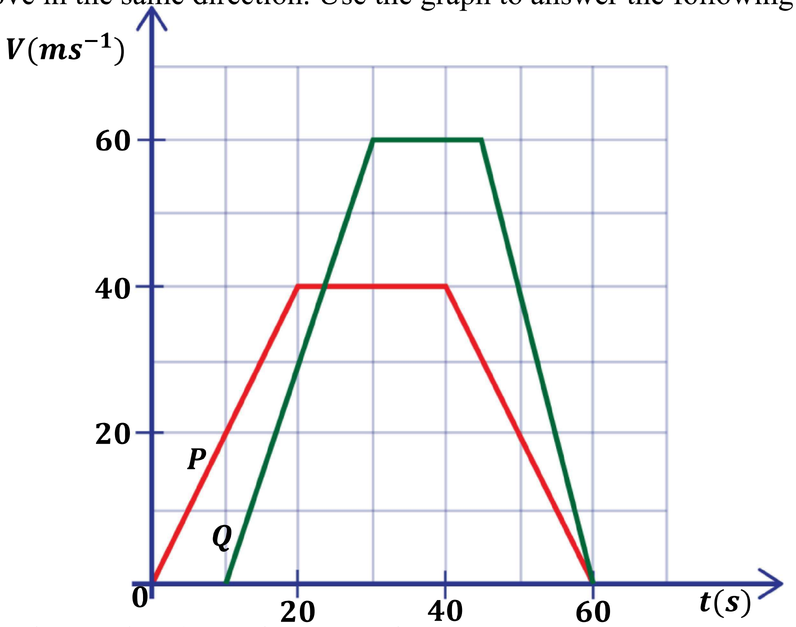
ii) ***Maximum velocity***

$$a = 2ms^{-2} \quad t = 4s \quad u = 10ms^{-1}$$
$$v = u + at$$
$$V = 10 + 2 \times 4$$
$$V = 10 + 8$$
$$V = 18ms^{-1}$$

ii) ***Deceleration of the boat***

$$v = 0ms^{-1} \quad t = 3s \quad u = 18ms^{-1}$$
$$v = u + at$$
$$0 = 18 + a \times 3$$
$$a = \frac{-18}{3}$$
$$a = -6ms^{-2}$$

5. The velocity-time graph below represents the motion of two cars P and Q which start from the same place and move in the same direction. Use the graph to answer the following questions.



   i) Calculate the accelerations of cars P and Q.
   ii) Determine how far the cars are from each other at the end of their accelerations.
   iii) Find the distance covered by car P between the 20^{th} and 40^{th} seconds.
   iv) Describe the motion of car Q.

i) ***For acceleration of car P***

From, $v = u + at$

$u = 0ms^{-1}$ $v = 40ms^{-1}$ $t = 20s$

$$40 = 0 + a \times 20$$

$$a = \frac{40}{20}$$

$$a = 2ms^{-2}$$

For acceleration of car Q

From, $v = u + at$

$u = 0ms^{-1}$ $v = 60ms^{-1}$ $t = 20s$

$$60 = 0 + a \times 20$$

$$a = \frac{60}{20}$$

$$a = 3ms^{-2}$$

ii) ***For distance covered by car P during its acceleration***

From, $S_P = ut + \frac{1}{2}at^2$

$u = 0ms^{-1}$ $a = 2ms^{-2}$ $t = 20s$

$$S_P = 0 \times 20 + \frac{1}{2} \times 2 \times 20^2$$

$$S_P = \frac{800}{2}$$

$$S_P = 400m$$

For distance covered by car P during its acceleration

From, $S_Q = ut + \frac{1}{2}at^2$

$u = 0ms^{-1}$ $a = 3ms^{-2}$ $t = 20s$

$$S_Q = 0 \times 20 + \frac{1}{2} \times 3 \times 20^2$$

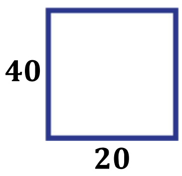
$$S_Q = \frac{1200}{2}$$

$$S_Q = 600m$$

Difference between the distance of the cars at the end of their accelerations.

$$Difference = (600 - 400)m = 200m$$

(iii)



$$Distance = Area\ of\ the\ rectangle$$

$$Distance = l \times w$$

$$Distance = 20 \times 40$$

$$Distance = 800m$$

iv) *deceleration of car Q*

From, $v = u + at$

$u = 60ms^{-1}$ $v = 0ms^{-1}$ $t = (60 - 45) = 15s$

$$0 = 60 + a \times 15$$

$$a = \frac{-60}{15}$$

$$a = -4ms^{-2}$$

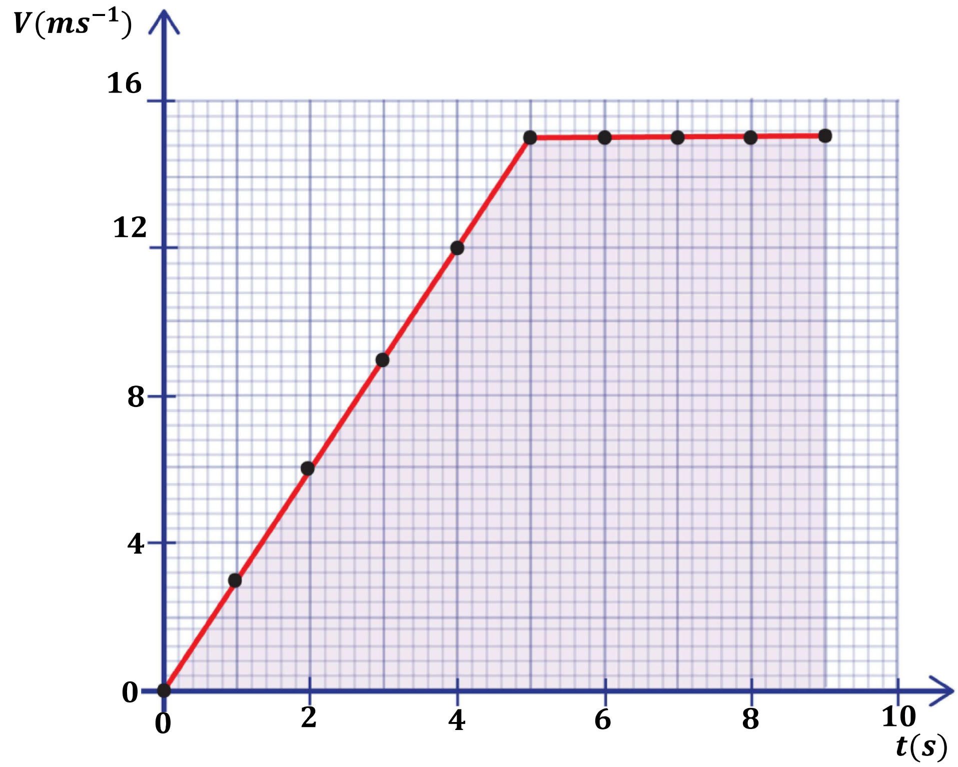
A car P starts from rest and accelerates uniformly at a rate of $3ms^{-2}$ to velocity of $60ms^{-1}$ in 2s. It maintains this velocity for 15s and it then finally decelerates uniformly at a rate of $4ms^{-2}$ to rest in 15 seconds.

6. The table below represents the velocity of a vehicle after a given time.

Velocity (ms^{-1})	0	3	6	9	12	15	15	15	15	15
Time (s)	0	1	2	3	4	5	6	7	8	9

i) Plot a velocity-time graph representing the motion of the vehicle.
ii) Find the slope of the vehicle (slope is equal to acceleration of the vehicle)
iii) Find the total displacement for the whole vehicle.
iv) Use the graph to describe the motion of the vehicle.

i)



ii) ***For uniform acceleration(slope)***

From, $a = \frac{v-u}{t}$

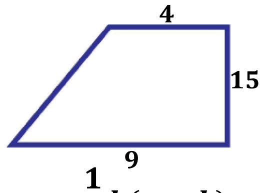
$u = 0ms^{-1}$ $v = 15ms^{-1}$ $t = 5s$

$$a = \frac{15-0}{5}$$

$$a = \frac{15}{5}$$

$$a = 3ms^{-2}$$

iii) ***Total displacement***



$$Distance = \frac{1}{2}h(a+b)$$

$$Distance = \frac{1}{2} \times 15(4+9)$$

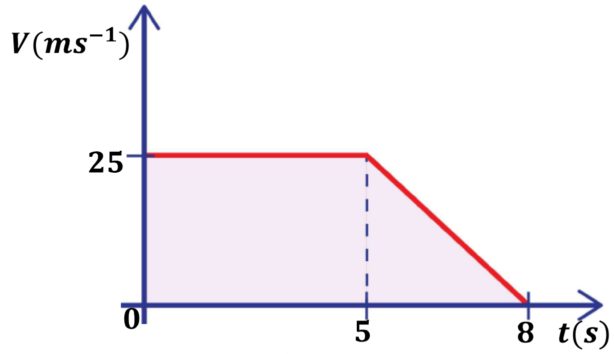
$$Distance = \frac{195}{2}$$

$$Distance = 97.5m$$

iv) **A vehicle starting from rest accelerates uniformly at a rate of $3ms^{-2}$ to a velocity of $15ms^{-1}$ in 5 seconds. It then maintains this new velocity for 4 seconds.**

7. A car of mass 20kg travelling with a uniform velocity of $25ms^{-1}$ for $5s$ brakes and then comes to rest under a uniform deceleration in $8s$.
   i) Sketch a velocity-time graph for the motion.
   ii) Find the retardation.
   iii) Calculate the retarding force of the car.
   iv) Find the total distance travelled.

i)



ii) Retardation

$v = 0ms^{-1}$

$t = 3s \quad u = 25ms^{-1}$

$v = u + at$

$0 = 25 + a \times 3$

$a = \frac{-25}{3}$

$a = -8.33ms^{-2}$

ii) Retarding force

$m = 2kg$

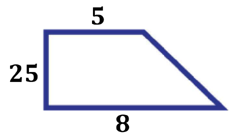
$a = 8.33ms^{-2}$

$F = ma$

$F = 2 \times 8.33$

$F = 16.7N$

iii) Total distance



$Distance = Area\ of\ a\ trapezium$

$Distance = \frac{1}{2}h(a + b)$

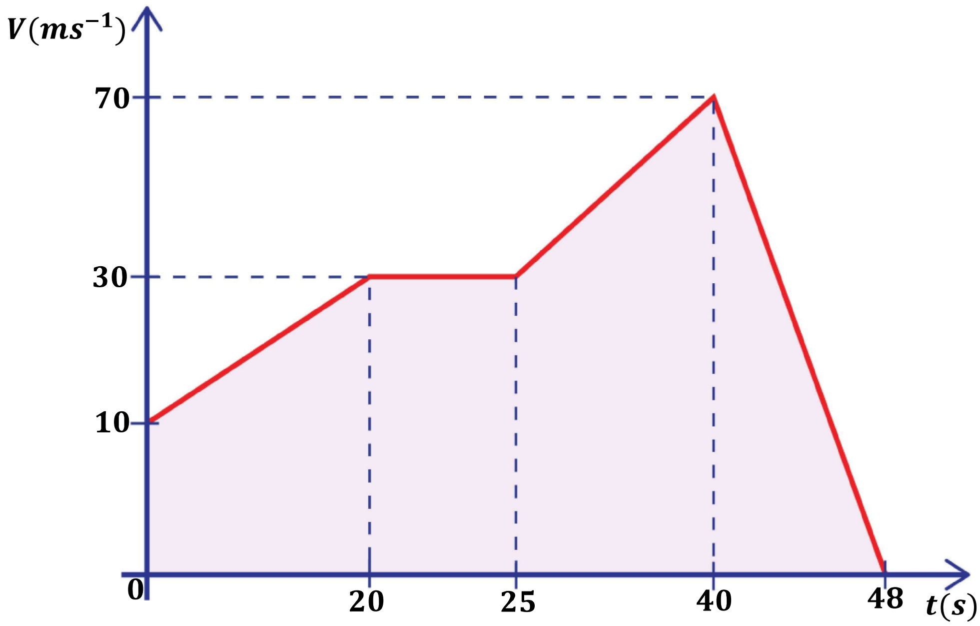
$Distance = \frac{1}{2} \times 25(5 + 8)$

$Distance = \frac{325}{2}$

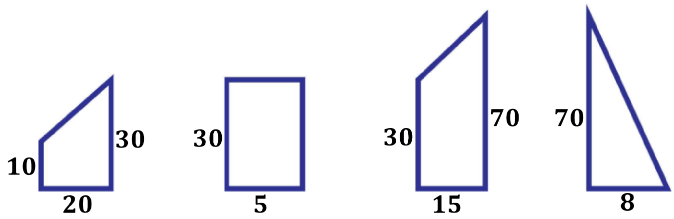
$Distance = 162.5m$

8. A car accelerated uniformly from a velocity of $10ms^{-1}$ to a velocity of $30ms^{-1}$ in $20s$. It then moved with a constant velocity for $5s$. It accelerates uniformly again to a velocity of $70ms^{-1}$ in $15s$. The brakes are then applied and it comes to rest uniformly in a further $8s$.
   i) Draw a velocity-time graph for the motion of the car.
   ii) Calculate the distance covered by the car.
   iii) Calculate the average velocity of the car.

i)



ii)



$$Distance = \text{Area of trapezium } A + \text{Area of rectangle } B + \text{Area of trapezium } C + \text{Area of triangle } D$$

$$Distance = \frac{1}{2}h(a+b) + (l \times w) + \frac{1}{2}h(a+b) + \frac{1}{2}bh$$

$$Distance = \frac{1}{2} \times 20(10+30) + (30 \times 5) + \frac{1}{2} \times 15(30+70) + \frac{1}{2} \times 8 \times 70$$

$$Distance = \frac{800}{2} + 150 + \frac{1500}{2} + \frac{560}{2}$$

$$Distance = 400 + 150 + 750 + 280$$

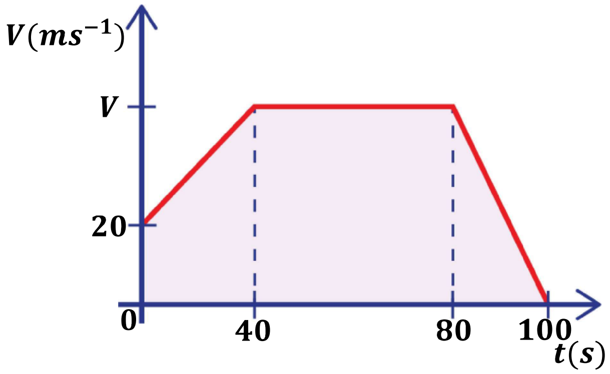
$$Distance = 1580m$$

EXERCISE.

1. A car moves from rest with a uniform acceleration of $1ms^{-2}$ for the first $20s$. It continues at a constant velocity for the next $30s$ and finally takes $10s$ to decelerate uniformly to rest.
   a) Calculate the constant velocity reached after $20s$.
   b) Sketch a velocity-time graph for the whole journey.
   c) Calculate the distance travelled by the car.

2. A car starting from ret at P accelerates uniformly for 10s to a velocity of $25ms^{-1}$. It then moves at this constant velocity for 8s before retarding uniformly for 5s so as to stop at Q.
   Sketch a velocity-time graph for the motion and find;
   i) the distance covered during each of the parts of the journey described.
   ii) the acceleration of the car.
   iii) The retardation of the car.

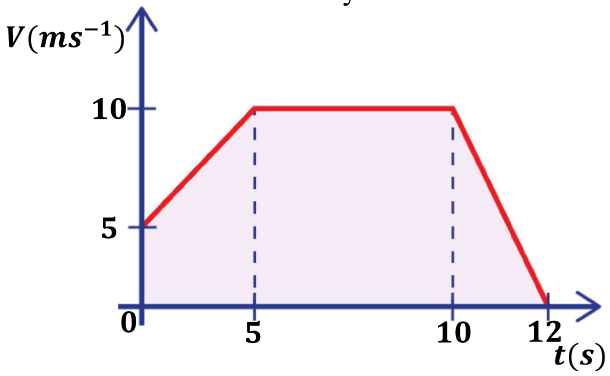
3. The figure shows the motion of a car with an acceleration of $2ms^{-2}$.



   i) Describe the motion of the car.
   ii) Find the distance moved after $50s$.
   iii) Find the total distance travelled by the car.

4. A car travels at a velocity of $20ms^{-1}$ for 6s. It is then uniformly brought to rest in 4s.
   i) Draw a velocity against time graph.
   ii) Calculate the retardation.
   iii) Find the total distance travelled.
   iv) Calculate the average speed of the body.

5. The graph below shows motion of a body. Use it to answer the following questions;



   i) Describe the motion of the body.
   ii) Calculate the acceleration and the retardation of the body.
   iii) Calculate the total distance covered by the body.
   iv) Calculate the average velocity of the body.

6. Plot a velocity-time graph of the body for the information below and use it to answer the questions.

Velocity (ms^{-1})	0	9	18	27	36	45	54
Time (s)	0	5	10	15	20	25	30

   a) Describe the motion of the body.
   b) Calculate the acceleration of the body
   c) Calculate the total displacement of the body.

7. The table below shows the velocity attained by a moving particle in a given time.

Velocity (ms^{-1})	5	13	21	29	39	39	39	27	15	3
Time (s)	0	2	4	6	8	10	12	14	16	18

Draw a velocity-time graph and describe the motion of the particle.
Use it to find;
i) Distance move while accelerating.
ii) Acceleration and retardation of the particle.
iii) The time that would have elapsed when it comes to rest.

MOTION UNDER GRAVITY

When a body is moving under gravity (upwards or downwards), it attains a constant acceleration called acceleration due to gravity (g). This constant acceleration is equal to $\mathbf{9.8}\ ms^{-2} \approx \mathbf{10}\ ms^{-2}$.

Definition:

Acceleration due to gravity is the rate of change of velocity with time for a freely falling body under the force of gravity.

NOTE: Acceleration due to gravity varies from place to place because;

- The earth is not a perfect sphere.
- The earth is always rotating.

Since the force of gravity acts vertically downwards;

- bodies moving vertically downwards are moving in the same direction as the force of gravity so, they have a positive acceleration due to gravity ($+\boldsymbol{g}\ ms^{-2}$).

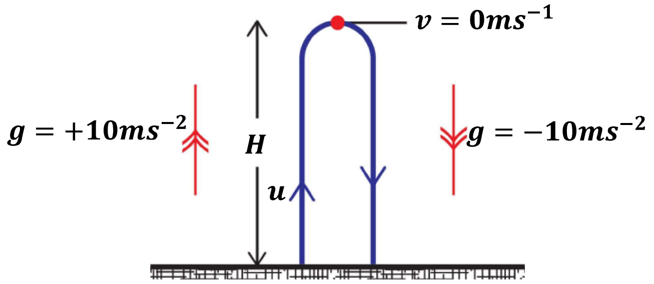
The equations of motion are;

$$First\ equation:\quad \boldsymbol{v = u + gt}$$
$$Second\ equation:\quad \boldsymbol{s = ut + \frac{1}{2}gt^2}$$
$$Third\ equation:\quad \boldsymbol{v^2 = u^2 + 2gs}$$

- bodies moving vertically upwards are moving in the direction opposite to the force of gravity so, they have a negative acceleration due to gravity ($-\boldsymbol{g}\ ms^{-2}$).

The equations of motion are;

$$First\ equation:\quad \boldsymbol{v = u - gt}$$
$$Second\ equation:\quad \boldsymbol{s = ut - \frac{1}{2}gt^2}$$
$$Third\ equation:\quad \boldsymbol{v^2 = u^2 - 2gs}$$



IMPORTANT TERMS:

Maximum height, H:
This is the greatest height reached by the body from the point of projection.
At maximum height, $\boldsymbol{velocity = 0ms^{-1}}$

Time of flight, T:
This is the total time taken by a body from its point of projection until it lands.
Time of flight is twice the time taken by the body to reach maximum height.
$Let\ time\ taken\ to\ reach\ maximum\ height = \boldsymbol{t}$
$\boldsymbol{Then;\quad T = 2t}$

Trajectory:
This is the path followed by a projectile.

<u>Examples</u>:

($\boldsymbol{where\ necessary\ apply}$ $(\boldsymbol{g = 10ms^{-2}})$)

1. A stone is thrown vertically upwards with an initial velocity of $25ms^{-1}$.
   a) Determine the time taken to reach maximum height.
   b) What is the maximum height reached by the stone

a)

$\boldsymbol{At\ maximum\ height, v = 0ms^{-1}}$

$\boldsymbol{u = 25ms^{-1} \quad g = -10ms^{-2}}$

$\boldsymbol{From\ \ v = u + gt}$

$$\boldsymbol{0 = 25 + -10 \times t}$$
$$\boldsymbol{-25 = -10t}$$
$$\boldsymbol{t = \frac{-25}{-10}}$$
$$\boldsymbol{t = 2.5s}$$

b)

$\boldsymbol{From\ \ v^2 = u^2 + 2gs}$

$Let\ maximum\ height = \ H$

$$\boldsymbol{0^2 = 25^2 + 2 \times -10 \times H}$$
$$\boldsymbol{20H = 625}$$
$$\boldsymbol{H = \frac{625}{20}}$$
$$\boldsymbol{H = 31.25m}$$

2. A ball is thrown vertically upwards and reaches a maximum height of 31.25m. Calculate;
   i) Initial velocity of the ball.
   ii) The time taken to return to the hands of the thrower.

i)

$\boldsymbol{At\ maximum\ height\ H, v = 0ms^{-1}}$

$\boldsymbol{u} =? \quad \boldsymbol{g = -10ms^{-2} \quad H = 31.25m}$

$\boldsymbol{From\ \ v^2 = u^2 + 2gs}$

$$\boldsymbol{v^2 = u^2 + 2gH}$$
$$\boldsymbol{0^2 = u^2 + 2 \times -10 \times 31.25}$$
$$\boldsymbol{u^2 = 625}$$
$$\boldsymbol{u = \sqrt{625}}$$
$$\boldsymbol{u = 25ms^{-1}}$$

c)

$\boldsymbol{From\ \ v = u + gt}$

$$\boldsymbol{0 = 25 + -10 \times t}$$
$$\boldsymbol{10t = 25}$$
$$\boldsymbol{t = \frac{25}{10}}$$
$$\boldsymbol{t = 2.5s}$$

$$\boldsymbol{Total\ time, T = 2.5 \times 2}$$
$$\boldsymbol{T = 5s}$$

NOTE:

When a body is thrown vertically upwards the time taken to reach the maximum height is equal to the time taken for the body to fall back from maximum height to the point of projection.

3. A stone is released vertically downwards from the top of a tree and hits the ground after 3s. Find
   i) the height of the tree.
   ii) the velocity with which it hits the ground.

i)

$\boldsymbol{u = 0ms^{-1} \quad g = 10ms^{-2}}\ \ \boldsymbol{h} =? \ \ \boldsymbol{t = 3s}$

$\boldsymbol{From\ \ s = ut + \frac{1}{2}gt^2}$

$$\boldsymbol{h = 0 \times 3 + \frac{1}{2} \times 10 \times 3^2}$$
$$\boldsymbol{h = \frac{90}{2}}$$
$$\boldsymbol{h = 45m}$$

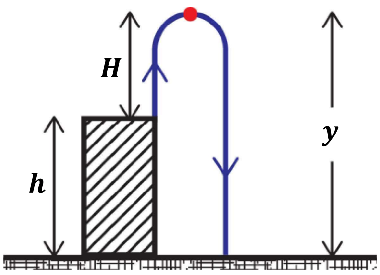
ii)

$\boldsymbol{From\ \ v = u + gt}$

$$\boldsymbol{v = 0 + 10 \times 3}$$
$$\boldsymbol{v = 30ms^{-1}}$$

4. A particle is thrown vertically upwards with a velocity of $20ms^{-1}$ from the edge of a cliff of height 10m. Calculate;
   i) the maximum height reached by the particle.
   ii) time taken to reach maximum height from the cliff top.
   iii) The total time taken by the particle to hit the ground

$u = 20ms^{-1}$ $g = -10ms^{-2}$ $h = 10m$



ii)

$\textbf{From } v = u + gt$

$$0 = 20 + -10 \times t_1$$
$$10t_1 = 20$$
$$t_1 = \frac{20}{10}$$
$$t_1 = 2s$$

i) At maximum height $v = 0ms^{-1}$

$\textbf{From } v^2 = u^2 + 2gs$

$$0^2 = 20^2 + 2 \times -10 \times H$$
$$20H = 400$$
$$H = \frac{400}{20}$$
$$H = 20m$$

iii)

$\textit{Total height}, y = h + H$

$\textit{Total height}, y = 10 + 20 = 30m$

<u>*Time taken to reach the ground*</u>

$u = 0ms^{-1}$ $g = 10ms^{-2}$ $y = 30m = s$

$$\text{From } s = ut + \frac{1}{2}gt^2$$
$$30 = 0 \times t_2 + \frac{1}{2} \times 10 \times {t_2}^2$$
$${t_2}^2 = \frac{30}{5}$$
$$t_2 = \sqrt{6}$$
$$t_2 = 2.45s$$

$\textit{Time taken to reach the ground} = t_1 + t_2$
$= 2 + 2.45$
$= 4.45s$

EXERCISE:

1. A stone is thrown vertically upwards with a velocity of $15ms^{-1}$.
   Calculate
   i) the maximum height reached by the stone.
   ii) the time taken to reach maximum height.

2. A body at a height of 20m above the ground falls freely under gravity to the ground.
   Calculate
   i) the time taken by the body to reach the ground.
   ii) the velocity with which it hits the ground.

3. An object is dropped from a helicopter. If the object hits the ground after 2s, calculate;
   i) the height from which the object was dropped.
   ii) The velocity with which it hits the ground.

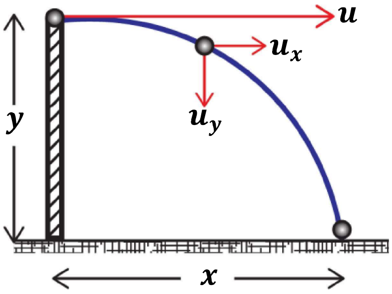
PROJECTILES

A projectile is an object which when given an initial velocity moves under the influence of force of gravity and it is only acted upon by its weight.
The path followed by a projectile is called **trajectory.**
If a projectile motion, a body's velocity consists of two parts i.e. horizontal velocity and vertical velocity.

- The horizontal velocity of the body remains the same throughout the motion since its not affected by the acceleration due to gravity.
- The vertical velocity of the body varies or changes since it is being affected by the acceleration due to gravity.

Therefore, the projectile will have both horizontal and vertical motion.

Consider a ball projected horizontally with an initial velocity $\boldsymbol{u}ms^{-1}$ from the point above the ground.



HORIZONTAL MOTION

$a_x = 0ms^{-2} \quad u_x = u$

$$From\ \ s = u_x t + \frac{1}{2}a_x t^2$$

$$x = ut + \frac{1}{2} \times 0 \times t^2$$

$$\boldsymbol{x = ut}$$

VERTICAL MOTION

$a_y = gms^{-2} \quad u_y = 0$

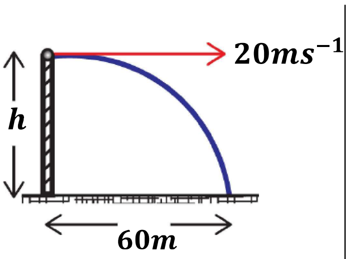
$$From\ \ s = u_y t + \frac{1}{2}a_y t^2$$

$$y = 0 \times t + \frac{1}{2} \times g \times t^2$$

$$\boldsymbol{y = \frac{1}{2}gt^2}$$

Examples:

1. A ball is thrown from the edge of the cliff with a horizontal velocity $20ms^{-1}$ and hits the surface at a distance 60m from the base of the cliff.
   Calculate;
   i) the time it takes to reach the surface.
   ii) the height of the cliff.



i)

$\boldsymbol{x = 60m \ \ y = h \ \ u = 20ms^{-1}}$

$$\boldsymbol{x = ut}$$

$$\boldsymbol{60 = 20 \times t}$$

$$\boldsymbol{t = \frac{60}{20}}$$

$$\boldsymbol{t = 3s}$$

ii)

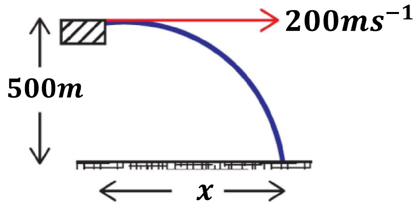
$$\boldsymbol{y = \frac{1}{2}gt^2}$$

$$\boldsymbol{h = \frac{1}{2} \times 10 \times 3^2}$$

$$\boldsymbol{h = \frac{90}{2}}$$

$$\boldsymbol{h = 45m}$$

2. An object is released from an aircraft horizontally with a velocity of $200ms^{-1}$ a height of 500m. Find
   i) how long it takes the object to reach the ground.
   ii) The horizontal distance covered by the object.



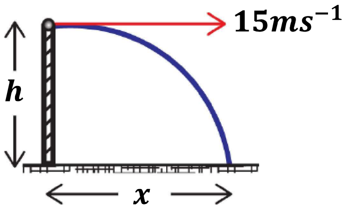
i)

$$y = 500m \quad u = 200ms^{-1}$$
$$y = \frac{1}{2}gt^2$$
$$500 = \frac{1}{2} \times 10 \times t^2$$
$$t^2 = \frac{500}{5}$$
$$t = \sqrt{100}$$
$$t = 10s$$

ii)

$$x = ut$$
$$x = 200 \times 10$$
$$x = 2000m$$

3. A ball of mass 2kg is thrown horizontally with a speed of $15ms^{-1}$ from a top of a building. If it takes 2 seconds to reach the ground, find
   i) the height of the building.
   ii) the vertical velocity with which it hits the ground.
   iii) the kinetic energy of the ball before it hits the ground.



i)

$$m = 2kg \quad y = h \quad u = 20ms^{-1} \quad t = 2s$$
$$y = \frac{1}{2}gt^2$$
$$h = \frac{1}{2} \times 10 \times 2^2$$
$$h = \frac{40}{2}$$
$$h = 20m$$

ii)

$$u_y = 0ms^{-1}$$
$$From \; v_y = u_y + gt$$
$$v_y = 0 + 10 \times 2$$
$$v_y = 20ms^{-1}$$

OR

$$P.E = K.E$$
$$mgh = \frac{1}{2}m\,{v_y}^2$$
$$2 \times 10 \times 20 = \frac{1}{2} \times 2 \times {v_y}^2$$
$${v_y}^2 = 400$$
$$v_y = \sqrt{400}$$
$$v_y = 20ms^{-1}$$

iii)

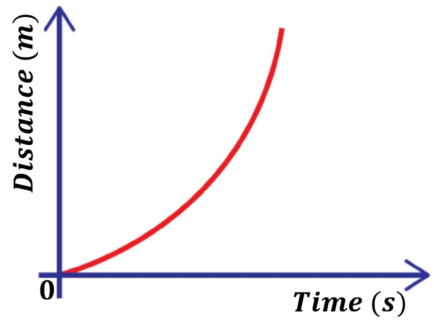
$$K.E = \frac{1}{2}mv^2$$
$$K.E = \frac{1}{2} \times 2 \times 20^2$$
$$K.E = 400\,J$$

MOTION GRAPHS FOR BODIES UNDER GRAVITY

The following graphs describe the motion of the body that has been thrown or projected vertically upwards.

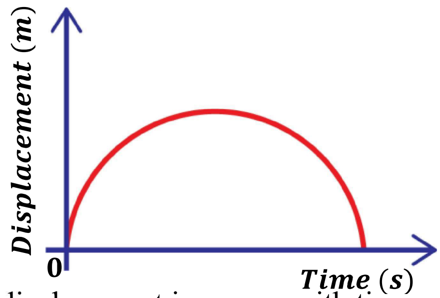
<table>
<tr>
<td>

Distance-time graph



</td>
<td>

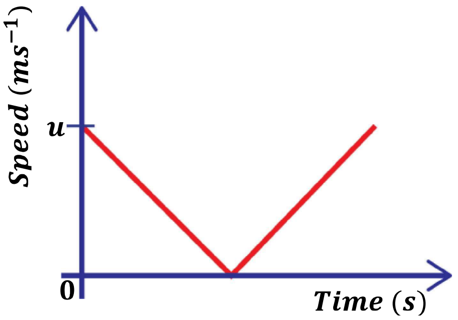
Displacement-time graph



The displacement increases with time and when the body reaches maximum height, it changes direction and start to move downwards

</td>
</tr>
<tr>
<td>

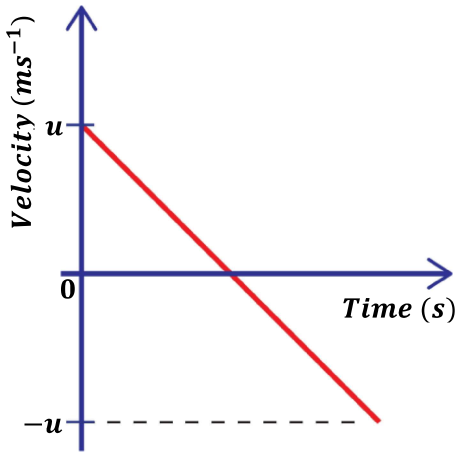
Speed-time graph



The speed of the body decreases as it goes higher. At maximum height the speed becomes $0ms^{-1}$ as the body rests momentarily. The speed increases as the body starts to fall down.

</td>
<td>

Velocity-time graph



The velocity of the body decreases as it goes higher until it reaches maximum height where velocity becomes $0ms^{-1}$.The velocity increases as the body starts to fall down.
The negative velocity means that the body has changed direction.

</td>
</tr>
</table>

EXERCISE:

1. A girl throws a ball horizontally from a window of a room onto the ground. If it takes the ball 4s to hit the ground, find
   i) the vertical height from the point of projection to the ground.
   ii) the velocity with which the ball was projected given that it landed 50m away from the room.

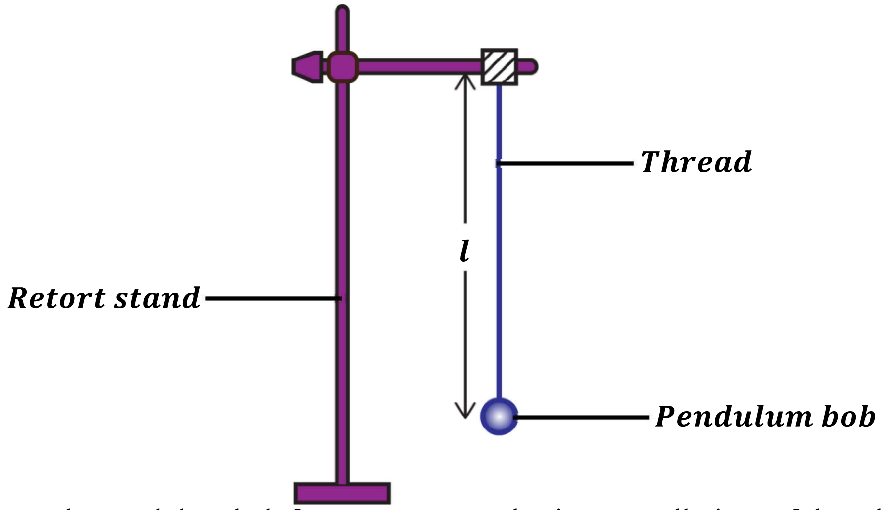
2. A stone is thrown horizontally with a velocity of $6ms^{-1}$ from the edge of the cliff 125m tall. Find how far the stone landed from the bottom of the cliff.

3. A bomb is released from a plane 5000m high with a velocity $30ms^{-1}$. Find the
   i) time it takes to reach the ground.
   ii) horizontal distance it covers by the time it hits the ground.

4. A jet flying at a height of 2km with a horizontal velocity of 40m/s drops a bomb to hit a target. Determine
   a) the time taken before the bomb hits the ground.
   b) the vertical velocity with which it hits the ground.
   c) the horizontal distance covered by the bomb by the time it hits the ground.

5. A bomb is released from a jet fighter plane moving with a velocity of $400ms^{-1}$ to hit a rebel camp in northern Uganda. If the bomb took 10 seconds to hit the target, calculate;
   i) the altitude at which the bomb was released.
   ii) the horizontal distance from the vertical point of the jet fighter plane to the target.
   iii) the velocity with which the bombs hits the target.
   iv) The kinetic energy of the bomb before hitting the target.

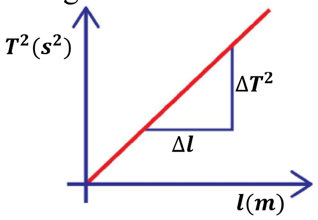
Experiment to determine acceleration due to gravity using a simple pendulum



- Suspend a pendulum bob from a retort stand using a small piece of thread as shown in the diagram above.
- Starting with length, $\boldsymbol{l} = 20cm$, displace the pendulum bob through a small angle and release it to oscillate.
- Measure the time, $\boldsymbol{t}$ for 20 oscillations of the pendulum bob using a stop clock.
- Determine time, $\boldsymbol{T}$ for one oscillation i.e. $\boldsymbol{T} = \frac{\boldsymbol{t}}{\boldsymbol{20}}$
- Repeat the experiment for other increasing values of $\boldsymbol{l}$.
- Record the results in a suitable table including values of $\boldsymbol{T^2}$.

$\boldsymbol{l(m)}$	$\boldsymbol{t(s)}$	$\boldsymbol{T(s)}$	$\boldsymbol{T^2(s^2)}$

- Plot a graph of $\boldsymbol{T^2}$ against $\boldsymbol{l}$ and determine its slope, $\boldsymbol{S}$.



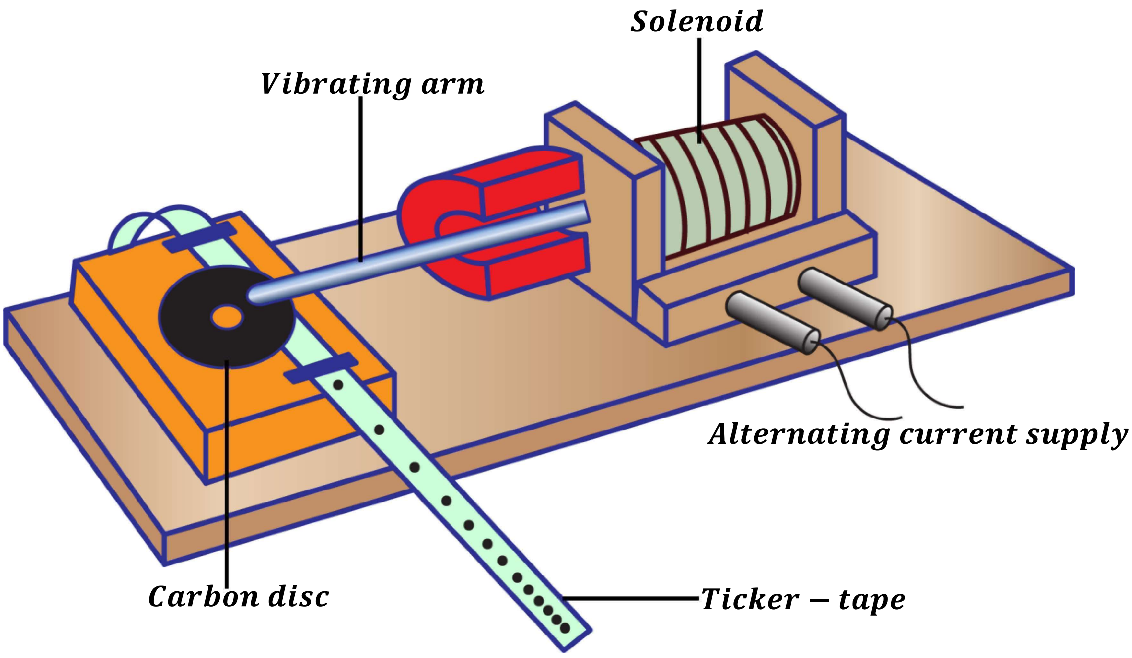
$$S = \frac{\Delta T^2}{\Delta l}$$

- Determine the acceleration due to gravity, $\boldsymbol{g}$ from $\boldsymbol{g} = \frac{\boldsymbol{4\pi^2}}{\boldsymbol{S}}$

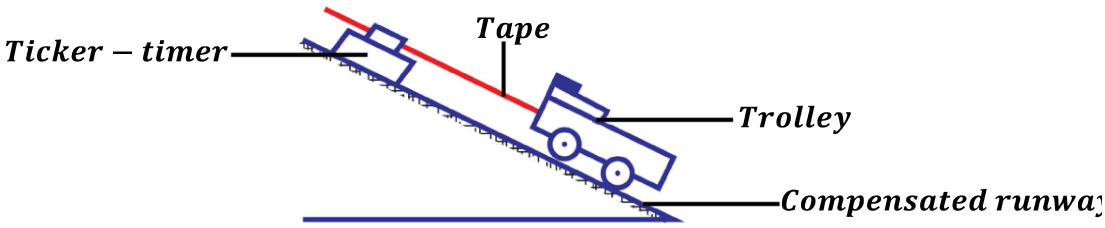
TICKER-TIMER

A ticker-timer is an electric device used in a physics laboratory to study speed, velocity and acceleration of the body.

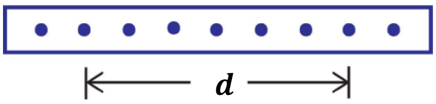
It consists of a vibrating arm which vibrates due to the changing current (alternating current) applied to it. As it vibrates, it prints dots on a ticker-tape which is pulled through it. The dots printed are used to study the motion of the body.



<u>Experiment to determine uniform velocity of a body using a ticker-timer</u>.



- The ticker-tape is tied on a trolley placed on a tilted (inclined) runway.
- The ticker-timer is switched on and a vibrating arm of known frequency, $\boldsymbol{f}$ moves the pin up and down.
- The time taken for one complete vibration (periodic time) is determined from $\boldsymbol{T} = \frac{\mathbf{1}}{\boldsymbol{f}}$
- The trolley is slightly pushed to make it run on the inclined runway.
- The ticker-tape is pulled with uniform velocity such that the dots printed are equally spaced.
- The tape with printed dots is removed from the trolley.
- The distance covered by a certain number of dots is measured and noted.



- Uniform velocity is then calculated from; $\boldsymbol{Uniform\ velocity} = \frac{\boldsymbol{Distance\ covered\ (d)}}{\boldsymbol{Time\ taken}}$

$$where\ Time\ taken = periodic\ time(\boldsymbol{T}) \times number\ of\ spaces$$

NOTE:

Before the experiment, its necessary to compensate for friction. This can be done by tilting the runway until a certain point is reached such that when a trolley is given a slight push, it moves with uniform velocity.

Important definitions:

- **Frequency, f:**
  This is the number of number of dots printed per second.
  OR
  This is the number of vibrations per second.
  The SI unit of frequency is Hertz (**Hz**)

- **Period (Tick), T:**
  This is time taken to print any two successive dots on the tape.
  Its SI unit is a second (**s**).

How to calculate velocity:

- First note the distance between the reference dots.
- Find the period from $\boldsymbol{T = \frac{1}{f}}$
- Find the time taken to print the reference dots from;

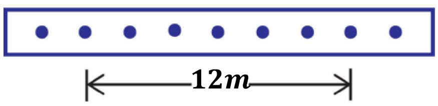
$$Time\ taken = Period\ (\boldsymbol{T}) \times number\ of\ spaces\ between\ the\ reference\ dots$$

$$\boldsymbol{Time\ taken = \frac{1}{f} \times number\ of\ spaces\ between\ the\ reference\ dots}$$

- Then speed or velocity $= \boldsymbol{\frac{Distance\ covered}{Time\ taken}}$

Examples:

1. The figure below is a tape which was pulled through a ticker- timer of frequency 50Hz.



Find

i) the period.
ii) time taken to print the reference dots.
iii) the speed at which a tape is pulled.

i)

$$\boldsymbol{d = 12m, \quad f = 50Hz}$$

$$\boldsymbol{Period, T = \frac{1}{f}}$$

$$\boldsymbol{T = \frac{1}{50}}$$

$$\boldsymbol{T = 0.02s}$$

ii)

$$\boldsymbol{Time\ taken, t = period \times number\ of\ spaces}$$

$$\boldsymbol{t = 0.02 \times 6}$$

$$\boldsymbol{t = 0.12s}$$

iii)

$$\boldsymbol{Speed = \frac{Distance\ (d)}{Time\ taken\ (t)}}$$

$$\boldsymbol{V = \frac{12}{0.12}}$$

$$\boldsymbol{V = 100ms^{-1}}$$

2. A ticker-timer vibrates at a frequency of 50Hz. If the distance between two consecutive dots is 2cm. Find the time that elapses between two consecutive and average speed of the tape.

$$d = 2cm, \quad f = 50Hz$$

<u>Periodic time</u>

$$Period, T = \frac{1}{f}$$
$$T = \frac{1}{50}$$
$$T = 0.02s$$

<u>Average speed</u>

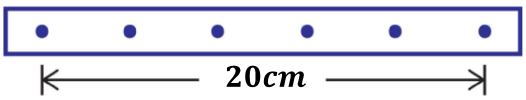
$$Time\ taken, t \ = T \times number\ of\ spaces$$
$$t = \frac{1}{f} \times no.of\ spaces$$
$$t = \frac{1}{50} \times 1$$
$$t = 0.02s$$

$$Speed = \frac{Distance\ (d)}{Time\ taken\ (t)}$$
$$V = \frac{2/100}{0.02}$$
$$V = 1ms^{-1}$$

OR

$$V = \frac{Distance\ (d)}{Time\ taken\ (t)}$$
$$V = \frac{2}{0.02}$$
$$V = 100cms^{-1}$$

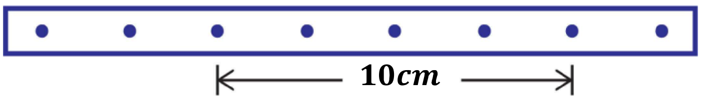
3. The ticker tape below was pulled through a ticker-time of frequency 50Hz. Calculate the speed at which the tape was pulled.



$$f = 50Hz, \qquad d = 20cm = \frac{20}{100} = 0.2m$$
$$Time\ taken, t = period \times number\ of\ spaces$$
$$t = \frac{1}{f} \times number\ of\ spaces$$
$$t = \frac{1}{50} \times 5$$
$$t = 0.1s$$

$$Speed = \frac{Distance\ (d)}{Time\ taken\ (t)}$$
$$V = \frac{0.2}{0.1}$$
$$V = 2ms^{-1}$$

4. The ticker-tape shown below was pulled through a ticker-timer which makes 100 dots every second.

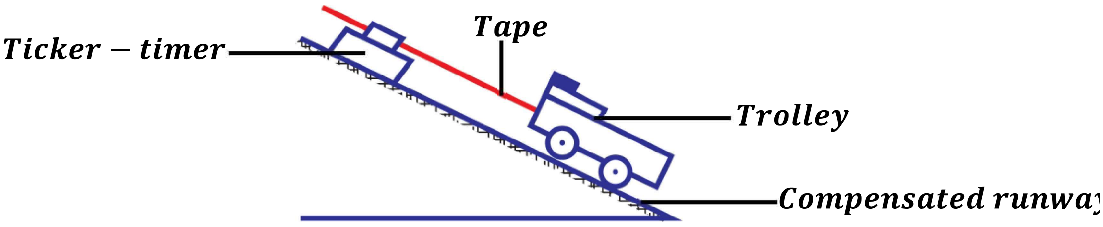


Find the speed at which the tape is pulled.

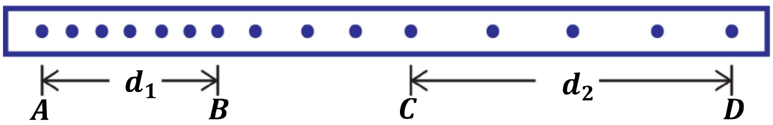
$$f = 100Hz, \qquad d = 10cm = \frac{10}{100} = 0.1m$$
$$Time\ taken, t = period \times number\ of\ spaces$$
$$t = \frac{1}{f} \times number\ of\ spaces$$
$$t = \frac{1}{100} \times 4$$
$$t = 0.04s$$

$$Speed = \frac{Distance\ (d)}{Time\ taken\ (t)}$$
$$V = \frac{0.1}{0.04}$$
$$V = 2.5ms^{-1}$$

Experiment to determine acceleration of a body using a ticker-timer.



- The ticker-tape is tied on a trolley placed on a tilted (inclined) runway.
- The ticker-timer is switched on and a vibrating arm of known frequency, $\boldsymbol{f}$ moves the pin up and down.
- The time taken for one complete vibration (periodic time) is determined from $\boldsymbol{T} = \frac{\boldsymbol{1}}{\boldsymbol{f}}$
- The trolley is slightly pushed to make it run on the inclined runway.
- The ticker-tape is pulled through the ticker-timer such that the dots are printed on it as shown below.



- The distances $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ covered by the selected number of dots is measured and noted.
- If there are $\boldsymbol{n_1}$ spaces between region **AB**, then the time taken to print them is $\boldsymbol{Tn_1}$.
- The initial velocity, $\boldsymbol{u}$ is then calculated from; $\boldsymbol{u} = \frac{\boldsymbol{d_1}}{\boldsymbol{Tn_1}}$
- If there are $\boldsymbol{n_2}$ spaces between region **CD**, then the time taken to print them is $\boldsymbol{Tn_2}$.
- The final velocity, $\boldsymbol{v}$ is then calculated from $\boldsymbol{v} = \frac{\boldsymbol{d_2}}{\boldsymbol{Tn_2}}$
- The acceleration of the trolley can be calculated from $\boldsymbol{a} = \frac{\boldsymbol{v-u}}{\boldsymbol{t}}$

$$where\ \boldsymbol{t} = time\ taken\ from\ the\ mid\ point\ of\ \boldsymbol{AB}\ to\ mid\ point\ of\ \boldsymbol{CD}$$

How to calculate acceleration:

- First note the distances between the reference dots.
- Find the period from $\boldsymbol{T} = \frac{\boldsymbol{1}}{\boldsymbol{f}}$
- Find the times taken $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$ to print the reference dots from;

$$Time\ taken = Period\ (\boldsymbol{T}) \times number\ of\ spaces\ between\ the\ reference\ dots$$

$$\boldsymbol{Time\ taken} = \frac{\boldsymbol{1}}{\boldsymbol{f}} \times \boldsymbol{number\ of\ spaces\ between\ the\ reference\ dots}$$

- Calculate the initial velocity, $\boldsymbol{u}$ and final velocity, $\boldsymbol{v}$ from;

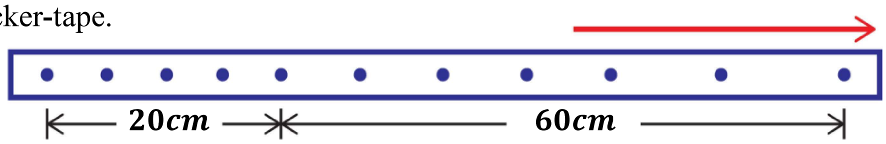
$$\boldsymbol{u} = \frac{\boldsymbol{d_1}}{\boldsymbol{t_1}} \qquad \boldsymbol{v} = \frac{\boldsymbol{d_2}}{\boldsymbol{t_2}}$$

- Calculate the acceleration of the trolley from $\boldsymbol{a} = \frac{\boldsymbol{v-u}}{\boldsymbol{t}}$

NOTE: To calculate time taken, $\boldsymbol{t}$, the spaces are measured from middle of initial distance to the middle of final distance.

Examples:

1. The figure below shows dots printed by a ticker-timer of frequency 100Hz, calculate the acceleration of the ticker-tape.



Initial velocity

$$d_1 = 20cm = \frac{20}{100} = 0.2m$$
$$f = 100Hz,\ \ n_1 = 4spaces$$

$$t_1 = period \times number\ of\ spaces$$
$$t_1 = \frac{1}{f} \times n_1$$
$$t_1 = \frac{1}{100} \times 4$$
$$t_1 = 0.04s$$

$$u = \frac{d_1}{t_1}$$
$$u = \frac{0.2}{0.04}$$
$$u = 5ms^{-1}$$

Final velocity

$$d_2 = 60cm = \frac{60}{100} = 0.6m$$
$$f = 100Hz,\ \ n_2 = 6spaces$$

$$t_2 = period \times number\ of\ spaces$$
$$t_2 = \frac{1}{f} \times n_2$$
$$t_2 = \frac{1}{100} \times 6$$
$$t_2 = 0.06s$$

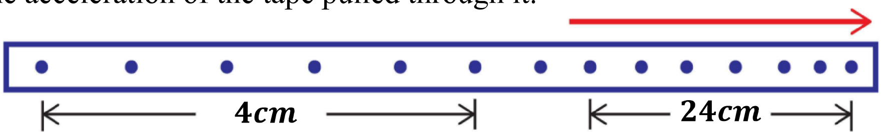
$$v = \frac{d_2}{t_2}$$
$$v = \frac{0.6}{0.06}$$
$$v = 10ms^{-1}$$

Acceleration

$$n = 5spaces$$
$$t = \frac{1}{f} \times n$$
$$t = \frac{1}{100} \times 5$$
$$t = 0.05s$$

$$a = \frac{v-u}{t}$$
$$a = \frac{10-5}{0.05}$$
$$a = \frac{5}{0.05}$$
$$a = 100ms^{-2}$$

2. The figure below shows dots printed by a ticker-timer. If the ticker-timer prints 100 dots per second, calculate the acceleration of the tape pulled through it.



Initial velocity

$$d_1 = 4cm = \frac{40}{100} = 0.4m$$
$$f = 100Hz,\ \ n_1 = 5spaces$$

$$t_1 = period \times number\ of\ spaces$$
$$t_1 = \frac{1}{f} \times n_1$$
$$t_1 = \frac{1}{100} \times 5$$
$$t_1 = 0.05s$$

$$u = \frac{d_1}{t_1}$$
$$u = \frac{0.4}{0.05}$$
$$u = 8ms^{-1}$$

Final velocity

$$d_2 = 24cm = \frac{24}{100} = 0.24m$$
$$f = 100Hz,\ \ n_2 = 6spaces$$

$$t_2 = period \times number\ of\ spaces$$
$$t_2 = \frac{1}{f} \times n_2$$
$$t_2 = \frac{1}{100} \times 6$$
$$t_2 = 0.06s$$

$$v = \frac{d_2}{t_2}$$
$$v = \frac{0.24}{0.06}$$
$$v = 4ms^{-1}$$

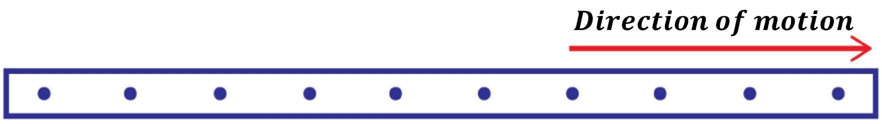
Acceleration

$$n = 7.5spaces$$
$$t = \frac{1}{f} \times n$$
$$t = \frac{1}{100} \times 7.5$$
$$t = 0.075s$$

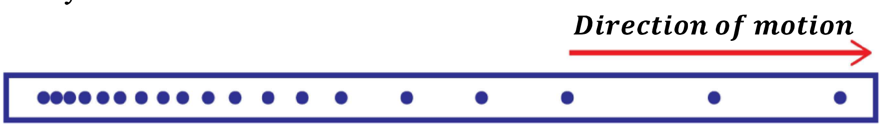
$$a = \frac{v-u}{t}$$
$$a = \frac{4-8}{0.075}$$
$$a = \frac{-4}{0.075}$$
$$a = -53.3ms^{-2}$$

NOTE:
The ticker-tapes below shows the dots printed for bodies in motion.

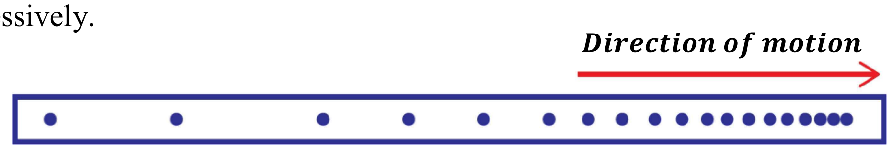
- If the body is moving with **<u>constant or uniform velocity</u>**, the dots are equally spaced along the tape.



- If the body is moving with **<u>uniform acceleration</u>**, the spacing between the dots increase progressively.

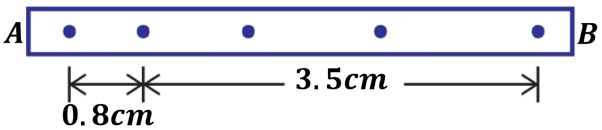


- If the body is moving with **<u>uniform deceleration</u>**, the spacing between the dots decrease progressively.



<u>Further examples</u>:

3. The figure below shows a tape produced by a ticker-timer operating at a frequency of 50Hz.



If the body or trolley was uniformly decelerating,

a) In which direction was it moving?
It was moving from B to A since spacing of the dots reduce progressively from B to A.

b) Calculate the deceleration of the trolley that pulled the tape through the ticker timer.

<u>Initial velocity</u>

$$d_1 = 3.5cm = \frac{3.5}{100} = 0.035m$$

$$f = 50Hz, \quad n_1 = 3spaces$$

$$t_1 = period \times number\ of\ spaces$$

$$t_1 = \frac{1}{f} \times n_1$$

$$t_1 = \frac{1}{50} \times 3$$

$$t_1 = 0.06s$$

$$u = \frac{d_1}{t_1}$$

$$u = \frac{0.035}{0.06} = 0.58ms^{-1}$$

<u>Final velocity</u>

$$d_2 = 0.8cm = \frac{0.8}{100} = 0.008m$$

$$f = 50Hz, \quad n_2 = 1space$$

$$t_2 = period \times number\ of\ spaces$$

$$t_2 = \frac{1}{f} \times n_2$$

$$t_2 = \frac{1}{50} \times 1$$

$$t_2 = 0.02s$$

$$v = \frac{d_2}{t_2}$$

$$v = \frac{0.008}{0.02} = 0.4ms^{-1}$$

<u>Acceleration</u>

$$n = 2spaces$$

$$t = \frac{1}{f} \times n$$

$$t = \frac{1}{50} \times 2$$

$$t = 0.04s$$

$$a = \frac{v-u}{t}$$

$$a = \frac{0.4 - 0.58}{0.04}$$

$$a = \frac{-0.18}{0.04}$$

$$a = -4.5ms^{-2}$$

4. A trolley is pulled from rest with a constant force down an inclined plane. The trolley pulls a tape through a ticker-timer vibrating at 50Hz. The following measurements were made as follows.
$Distance\ between\ 16^{th}\ dot\ and\ 20^{th}\ dot = 20cm$
$Distance\ between\ 40^{th}\ dot\ and\ 50^{th}\ dot = 62cm$
Calculate the acceleration of the trolley.

Initial velocity

$$d_1 = 20cm = \frac{20}{100} = 0.2m$$
$$f = 50Hz,$$
$$n_1 = n^{th} - m^{th}$$
$$n_1 = 20 - 16 = 4spaces$$
$$t_1 = period \times number\ of\ spaces$$
$$t_1 = \frac{1}{f} \times n_1$$
$$t_1 = \frac{1}{50} \times 4$$
$$t_1 = 0.08s$$
$$u = \frac{d_1}{t_1}$$
$$u = \frac{0.2}{0.08} = 2.5ms^{-1}$$

Final velocity

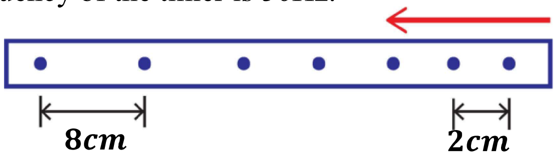
$$d_2 = 62cm = \frac{62}{100} = 0.62m$$
$$f = 50Hz,$$
$$n_2 = n^{th} - m^{th}$$
$$n_2 = 50 - 40 = 10spaces$$
$$t_2 = period \times number\ of\ spaces$$
$$t_2 = \frac{1}{f} \times n_2$$
$$t_2 = \frac{1}{50} \times 10$$
$$t_2 = 0.2s$$
$$v = \frac{d_2}{t_2}$$
$$v = \frac{0.62}{0.2} = 3.1ms^{-1}$$

Acceleration

Spaces in between

$$n_3 = n^{th} - m^{th}$$
$$n_3 = 40 - 20 = 20$$
$$n = 2 + n_3 + 5$$
$$n = 27spaces$$
$$t = \frac{1}{f} \times n$$
$$t = \frac{1}{50} \times 27$$
$$t = 0.54s$$
$$a = \frac{v-u}{t}$$
$$a = \frac{3.1 - 2.5}{0.54}$$
$$a = \frac{0.6}{0.54}$$
$$a = 1.11ms^{-2}$$

5. The figure below shows a tape pulled by a trolley through a ticker-timer. Describe the motion of the trolley if the frequency of the timer is 50Hz.



Initial velocity

$$d_1 = 2cm = \frac{2}{100} = 0.02m$$
$$f = 50Hz,\ n_1 = 1space$$
$$t_1 = period \times number\ of\ spaces$$
$$t_1 = \frac{1}{f} \times n_1$$
$$t_1 = \frac{1}{50} \times 1$$
$$t_1 = 0.02s$$
$$u = \frac{d_1}{t_1}$$
$$u = \frac{0.02}{0.02} = 1ms^{-1}$$

Final velocity

$$d_2 = 8cm = \frac{8}{100} = 0.08m$$
$$f = 50Hz,\ n_2 = 1space$$
$$t_2 = period \times number\ of\ spaces$$
$$t_2 = \frac{1}{f} \times n_2$$
$$t_2 = \frac{1}{50} \times 1$$
$$t_2 = 0.02s$$
$$v = \frac{d_2}{t_2}$$
$$v = \frac{0.08}{0.02} = 4ms^{-1}$$

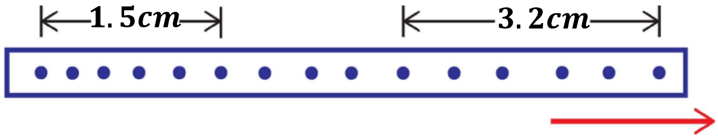
Acceleration

$$n = 5spaces$$
$$t = \frac{1}{f} \times n$$
$$t = \frac{1}{50} \times 5$$
$$t = 0.1s$$
$$a = \frac{v-u}{t}$$
$$a = \frac{4-1}{0.1}$$
$$a = \frac{3}{0.1}$$
$$a = 30ms^{-2}$$

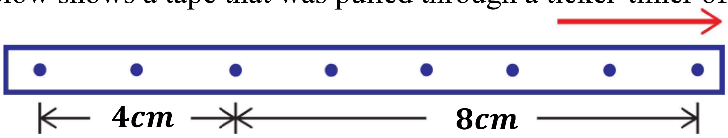
In the first 0.02s, the trolley was moving with a speed of $1ms^{-1}$. It accelerated uniformly at a rate of $30ms^{-2}$ to a final velocity of $4ms^{-1}$ in the last 0.02s.

EXERCISE:

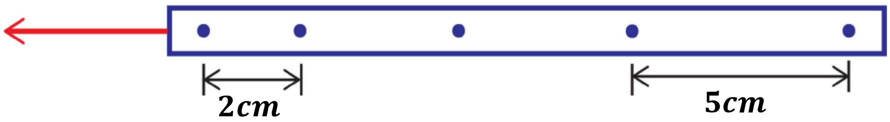
1. A tape was pulled through a ticker-timer which made one dot every second. If it made three dots and the distance between the three dots is 16cm, find the velocity of the tape.
2. A paper tape was attached to a moving trolley and allowed to run through a ticker-timer. The figure below shows a section of the tape.



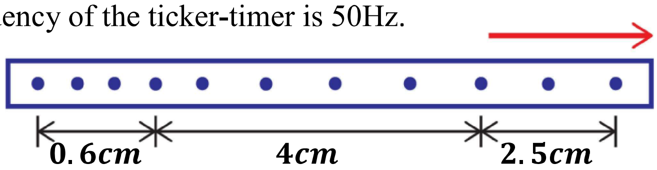
   If the frequency of the tape is 100Hz, determine;
   i) initial and final velocities of the trolley.
   ii) average acceleration of the trolley.
3. In a ticker-timer experiment, the distance occupied by a 6-dot space on the tape is 5.1cm, while the adjacent 6-dot space occupies 6.3cm. Find the acceleration of the body to which the tape is attached, if the ticker frequency is 50Hz.
4. The figure below shows a tape that was pulled through a ticker-timer of frequency 50Hz.



   Describe the motion of the body pulling the tape.
5. The figure below shows dots produced on a tape pulled through a ticker-timer by a moving body.



   a) State the type of motion in the figure above.
   b) Calculate the acceleration of the moving body.
6. A paper tape dragged through a ticker timer by a trolley has the first ten dots covering a distance of 4cm and the next ten dots covering a distance of 7cm. if the frequency of the ticker-timer is 5oHz, calculate the acceleration of the trolley.
7. The distance between the 15th dot and the 18th dot is 10cm. If the ticker-timer is vibrating at 20Hz. Calculate the:
   i) time taken to print the dots.
   ii) average speed of the tape.
8. The figure below shows a piece of tape pulled through a ticker-timer by a trolley down an inclined plane. The frequency of the ticker-timer is 50Hz.



   Calculate the acceleration of the trolley.

CIRCULAR MOTION

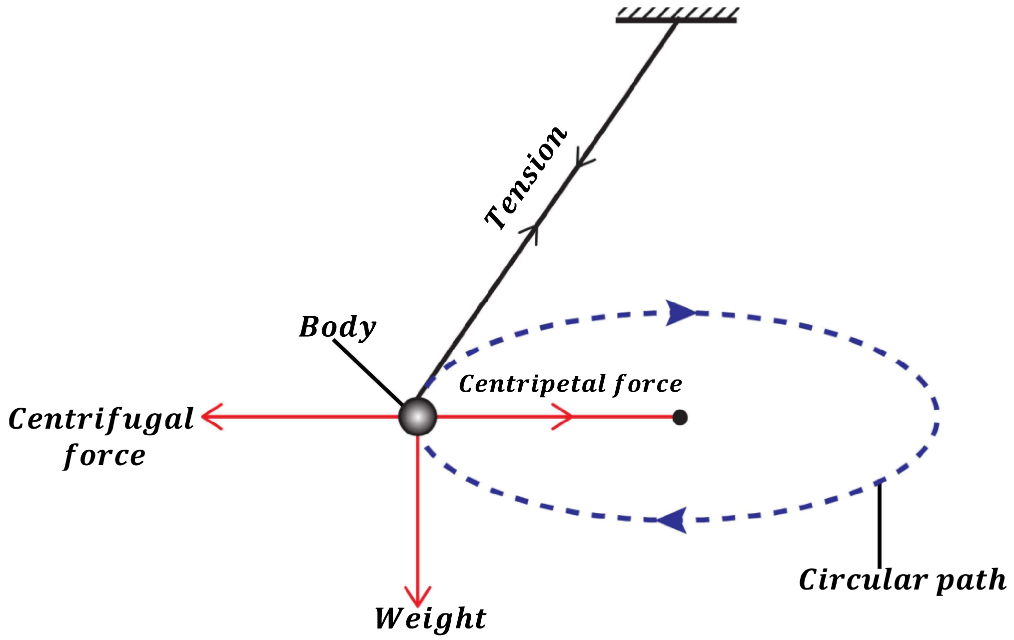
Circular motion is the motion in which a body moves in a circle about a fixed point.

In circular motion;

- The speed of the body is always constant.
- The direction of the body is always changing.
- The velocity of the body keeps on changing due to the changing direction. This is because velocity is a vector quantity which depends on the direction.
- The body has an acceleration due to the changing velocity. This acceleration (centripetal acceleration) acts towards the centre of the circle.

TERMS USED IN CIRCULAR MOTION

The figure below shows a whirled body tied on the string.



Tension:

This is the force exerted by a string on the body moving in circular motion. The tension force provides the centripetal force.

Centripetal force:

This is the force acting on the body towards the centre of the circular path.

Centripetal acceleration:

This is the acceleration that acts on a body towards the centre of the circular path. It is provided by the centripetal force.

Centrifugal force:

This is the force acting on the body away from the centre of the circular path.

Weight:

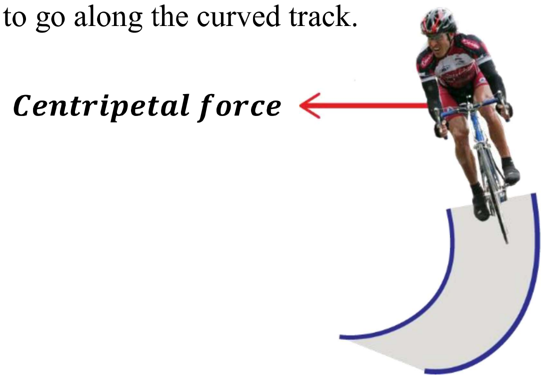
This is the force acting on the body vertically downwards towards the centre of the earth.

NOTE:
Experimental results show that the force required to keep the body moving in a circular path i.e. **centripetal force** increases with;

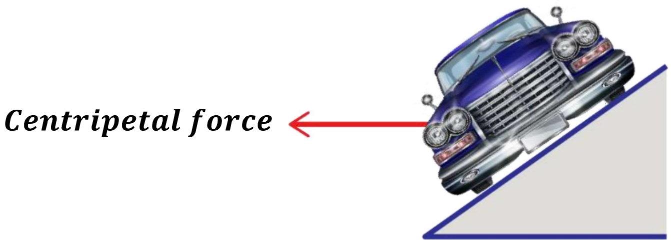
i) an increase in the mass of the body.
ii) an increase in the speed of the body.
iii) a decrease in radius of the circular path.

APPLICATION OF CIRCULAR MOTION IN REAL LIFE SITUATION

Bending of a cyclist round a curve:
A cyclist going round a curve has to lean inwards (bend slightly towards the centre of circular path) in order to take a safe turn without slipping. This slight bending provides the necessary centripetal force so as to be able to go along the curved track.



Banking of tracks:
When a car is going round a circular path, the centripetal force required to keep the car in circular motion is provided by the frictional force between the tyres and the road. This centripetal force prevents the car from sliding even if it is moving fast.
In order for a car not to fully depend on the frictional force, the circular paths are given a small banking i.e. the outer edge of the road is slightly raised above the inner surface. This helps to increase the centripetal force required for a body not to slide.

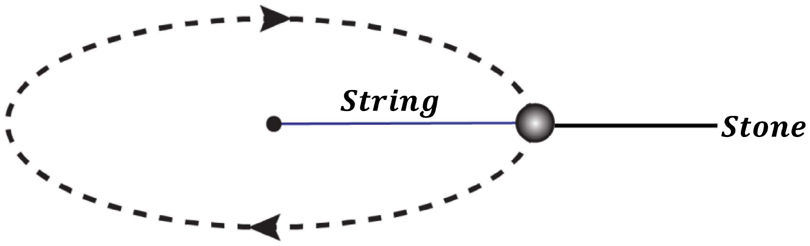


Other examples or applications of circular motion include;

- A stone tied to one end of a string and the other end is rotated about a fixed point.
- An aircraft (plane) making a circular turn.
- Planets or satellites orbiting the earth.
- Electrons orbiting the nucleus.
- Centrifuge used to separate liquids of different densities.
- Washing machines for clothes.

EXERCISE:

1. A stone attached to a string is swung in a vertical circular path in air as shown in the figure below.



   a) Copy the above diagram and on it indicate all the forces acting on the body.
   b) Describe all the forces indicated above.

2. a) Define the term centripetal force.
   b) Explain how a cyclist avoids slipping off the road when moving round a circular path.
   c) State five applications of circular motion in daily life situations.

NEWTON'S LAWS OF MOTION

Sir Isaac Newton carried out very many experiments and through these experiments, he formulated three laws that relate the **forces** acting on the body and the **motion** of the body. These laws are known as *Newton's laws of motion*.

❖ Newton's first law of motion:

It states that every body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force.
The first is also referred to as the **"law of inertia."**

INERTIA:

This is the tendency of a body at rest to remain at rest or to continue moving in a straight if it was already moving.

OR

This is the reluctance of a body to start moving or stop moving if it was already moving.

Factors affecting inertia of a body:

a) **Mass of a body:**
A body with a large mass requires a large force to make it move and it requires a large force to stop it if it was already moving. Hence a body with a large mass has a greater inertia.

b) **Force applied on a body:**
When the force applied on a body is increased, its tendency to remain at rest is also reduced. This would result to movement of the body from its resting state. Thus, a large force applied on the body reduces its inertia.

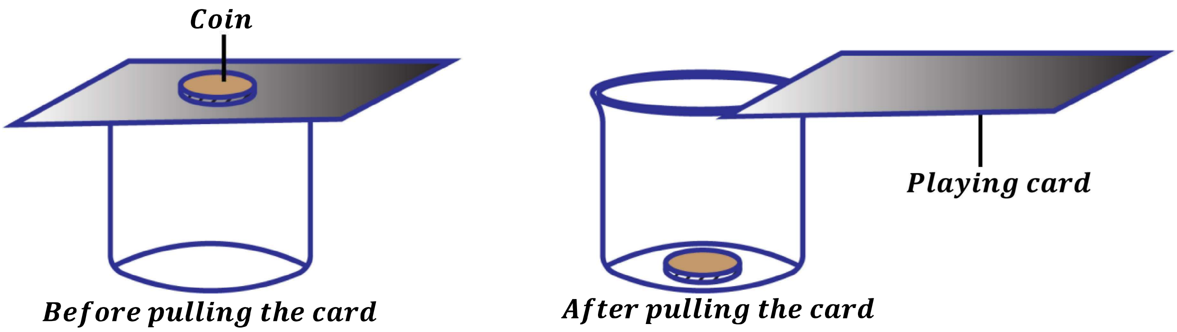
c) **Friction acting on a body:**
The law of inertia states that a body will continue in its state of rest or uniform motion in a straight line unless acted upon by an external force. An example of such a force is the friction force. This force will slow down the motion of the body even though it is moving fast.

Examples or applications of Newton's first law of motion:

- A person riding a bicycle along a level road does not come to rest immediately when he or she stops pedaling i.e. the bicycle continues to move forward for sometime and eventually comes to rest.
Due to inertia, the bicycle continues to move forward for sometime until it is acted upon by an external force such a friction between the tyre and the surface of the road which brings it to rest.

- Passengers in a fast-moving vehicle jerk forward when the vehicle tops suddenly and jerk backward when the vehicle starts moving.
For this reason, passengers are advised to fasten their safety belts when in a vehicle. The seat belts hold passengers onto their seats in case of any sudden stopping. This helps them to avoid getting injuries as a result of jerking forward and hitting the wind screen.

Demonstration of Newton's first law of motion:



- Place a playing card on top of a beaker.
- Put a coin on top of the playing card.
- Pull the card quickly towards you or hit the card out with a sharp flick.

Observation:
It is observed that the coin drops into the beaker.

Explanation:
The coin has inertia meaning that it doesn't change its state of rest when the card is quickly pulled. So, it just drops vertically into the beaker at the same position.
Pulling the card quickly provides a large force to overcome the friction force between the card and the coin.

NOTE: If the card is pulled away at a slow pace, the coin will move together with the card. This is because pulling it slowly provides a less force to overcome the friction force between the card and the coin.

❖ Newton's second law of motion:

It states that the rate of change of momentum of a body is directly proportional to the applied force and it takes place in the direction of force.
Mathematically;

$$Force \propto Rate\ of\ change\ of\ momentum$$

$$Force \propto \frac{Change\ of\ momentum}{Time}$$

$$Force \propto \frac{Final\ momentum - Initial\ momentum}{Time}$$

$$Final\ momentum = mass \times final\ velocity = mv$$

$$Final\ momentum = mass \times initial\ velocity = mu$$

$$F \propto \frac{mv - mu}{t}$$

$$F \propto \frac{m(v-u)}{t}$$

$$But\ \ a = \frac{(v-u)}{t}$$

$$F \propto ma$$

$$\boldsymbol{F = kma} \qquad where\ k\ is\ a\ constant\ of\ proportionality$$

$$If\ \ F = 1N,\ \ m = 1kg,\ \ a = 1ms^{-2}\ \ then\ k = 1$$

$$\boxed{\boldsymbol{F = ma}}$$

NOTE:

The SI unit of force is a Newton (**N**).

A **Newton** is the force which gives a mass of $1kg$ an acceleration of $1ms^{-2}$.

<u>Examples</u>:

1. An object of mass $4kg$ accelerates at a rate of $5ms^{-2}$. Calculate the resultant force acting on it.

$$\boldsymbol{m = 4kg, \quad a = 5ms^{-2}}$$
$$\boldsymbol{F = ma}$$
$$\boldsymbol{F = 4 \times 5}$$
$$\boldsymbol{F = 20N}$$

2. Calculate the acceleration produced by a force of 25N on an object of mass 2tonnes.

$$\boldsymbol{m = 2tonnes = (2 \times 1000) = 2000kg, \quad F = 25N}$$
$$\boldsymbol{F = ma}$$
$$\boldsymbol{25 = 2000 \times a}$$
$$\boldsymbol{a = \frac{25}{2000}}$$
$$\boldsymbol{a = 0.0125ms^{-2}}$$

3. A resultant force of 40N acts on a body of 500g initially at rest for 4s. Calculate
   i) acceleration on the body.
   ii) final velocity of the body.

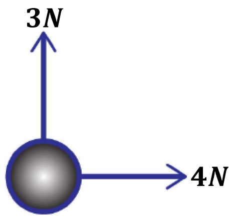
i)

$$\boldsymbol{m = 500g = \frac{500}{1000} = 0.5kg, \quad F = 40N}$$
$$\boldsymbol{F = ma}$$
$$\boldsymbol{40 = 0.5 \times a}$$
$$\boldsymbol{a = \frac{40}{0.5}}$$
$$\boldsymbol{a = 80ms^{-2}}$$

ii)

$$\boldsymbol{u = 0ms^{-1}, \quad t = 4s}$$
$$\boldsymbol{v = u + at}$$
$$\boldsymbol{v = 0 + 80 \times 4}$$
$$\boldsymbol{v = 320ms^{-1}}$$

4. Two forces of 3N and 4N act on the object of mass 2kg as shown below. Find the acceleration of the body.



<u>Resultant force</u>

$$\boldsymbol{F^2 = {F_1}^2 + {F_2}^2}$$
$$\boldsymbol{F^2 = 3^2 + 4^2}$$
$$\boldsymbol{F^2 = 25}$$
$$\boldsymbol{F = \sqrt{25}}$$
$$\boldsymbol{F = 5N}$$

$$\boldsymbol{F = ma}$$
$$\boldsymbol{5 = 2 \times a}$$
$$\boldsymbol{a = \frac{5}{2}}$$
$$\boldsymbol{a = 2.5ms^{-2}}$$

<u>Examples or applications of Newton's second law of motion</u>:

In Newton's second law acceleration depends on two variables i.e. the net force acting on the body and the mass of the body.

$$\boldsymbol{a = \frac{F}{m}}$$

Therefore, acceleration is directly proportional to force acting on the body and inversely proportional to mass of the body. This means an increase in force increases the acceleration of the body and an increase in mass reduces the acceleration of the body.

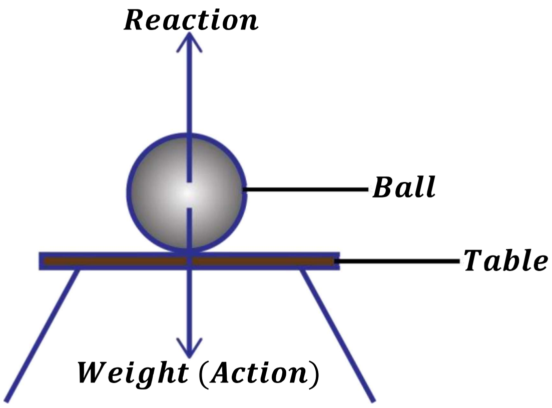
The following are some of daily-life applications of Newton's second law of motion.

- When playing football, the stronger the ball is kicked, the more the increase in velocity it will move with. This is because a stronger kick increases the force acting on the ball thus increasing its acceleration.
- When in a supermarket, its easier to push an empty cart than to push a loaded one. This is because the loaded cart has more mass thus a decrease in its acceleration. So, it will require a large force to accelerate it any further.
- Among two people walking, if one is heavier than the other one, the heavier person will walk slows than the lighter person.
- In formula one racing, the mass of the cars is kept as low as possible. This implies that low mass will increase their acceleration.

❖ <u>Newton's third law of motion</u>:

It states that for every action, there is an equal and opposite reaction.

Consider a ball placed on the table as shown below;



The ball exerts a force equal to its weight onto the table. This force is called an **action**.
At the same time, the table exerts an equal force on the ball but the force acts in an opposite direction. This force is called a **reaction**.

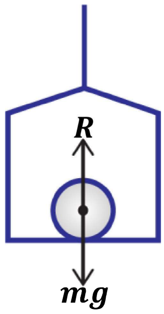
NOTE:

Since $\boldsymbol{weight, W = mg}$
Then also, $\boldsymbol{reaction, R = mg}$

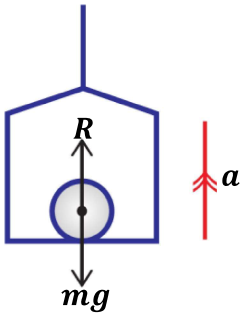
Applications of Newton's third law of motion:

MOTION IN A LIFT (ELEVATOR)

When a body of mass, $\boldsymbol{m}$ is placed on the floor of a lift which is moving with an acceleration, $\boldsymbol{a}ms^{-2}$, it exerts its weight onto the floor of the lift. At the same time the, lift exerts a reaction force in an opposite direction.



a) **If the lift is moving upwards:**



$\boldsymbol{Resultant\ force, F = ma}$

$\boldsymbol{R - mg = ma}$

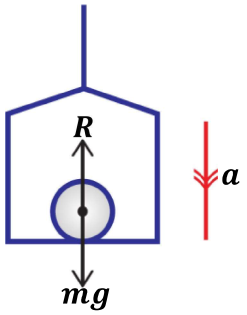
$\boldsymbol{R = mg + ma}$

$\boldsymbol{R = m(g + a)}$

The Reaction (Apparent weight of the body), **R** is greater than the actual weight of the body.

- This explains why a person standing in a lift feels heavier when the lift is moving upwards.

b) **If the lift is moving downwards:**



$\boldsymbol{Resultant\ force, F = ma}$

$\boldsymbol{mg - R = ma}$

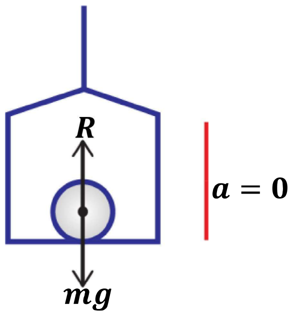
$\boldsymbol{R = mg - ma}$

$\boldsymbol{R = m(g - a)}$

The Reaction (Apparent weight of the body), **R** is less than the actual weight of the body.

- This explains why a person standing in a lift feels lighter when the lift is moving downwards.

c) **If the lift is stationary or moving with uniform velocity:**



$\boldsymbol{Resultant\ force, F = ma}$

Considering upward motion of the lift.

$\boldsymbol{R - mg = ma}$

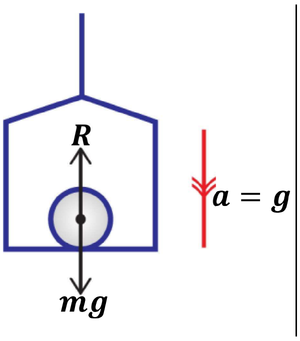
$\boldsymbol{R = mg + m \times 0}$

$\boldsymbol{R = mg}$

The Reaction (Apparent weight of the body), **R** is equal to the actual weight of the body.

NOTE:

If the cable or rope of the lift breaks, the lift will fall freely under the influence of gravity. Therefore, the acceleration of the lift is equal to the acceleration due to gravity, g.



$$\boldsymbol{Resultant\ force, F = ma}$$
$$\boldsymbol{mg - R = ma}$$
$$\boldsymbol{R = mg - ma}$$
$$\boldsymbol{R = mg - mg}$$
$$\boldsymbol{R = 0}$$

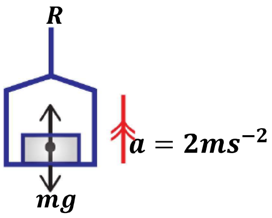
The Reaction (Apparent weight of the body), **R** is 0N.

- This explains why a person standing in a lift feels weightless when the lift is falling freely.

Examples:

1. A girl of mass $50kg$ stands in a stationary lift on earth. Calculate her apparent weight when the lift
   i) accelerates upwards at $2ms^{-2}$.
   ii) accelerates downwards at $2ms^{-2}$.
   iii) falls freely under gravity.

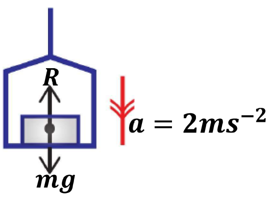
i)



$\boldsymbol{m = 50kg}$

$$\boldsymbol{Resultant\ force, F = ma}$$
$$\boldsymbol{R - mg = ma}$$
$$\boldsymbol{R = mg + ma}$$
$$\boldsymbol{R = 50 \times 10 + 50 \times 2}$$
$$\boldsymbol{R = 600N}$$
$$\boldsymbol{Apparent\ weight = 600N}$$

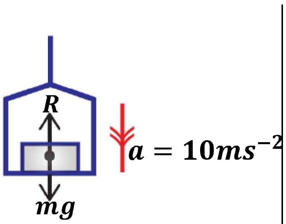
ii)



$\boldsymbol{m = 50kg}$

$$\boldsymbol{Resultant\ force, F = ma}$$
$$\boldsymbol{mg - R = ma}$$
$$\boldsymbol{R = mg - ma}$$
$$\boldsymbol{R = 50 \times 10 - 50 \times 2}$$
$$\boldsymbol{R = 400N}$$
$$\boldsymbol{Apparent\ weight = 400N}$$

iii)

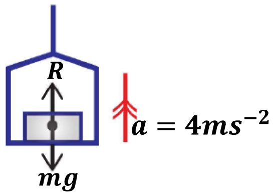


$\boldsymbol{m = 50kg}$

$$\boldsymbol{Resultant\ force, F = ma}$$
$$\boldsymbol{mg - R = ma}$$
$$\boldsymbol{R = mg - ma}$$
$$\boldsymbol{R = 50 \times 10 - 50 \times 10}$$
$$\boldsymbol{R = 0N}$$
$$\boldsymbol{Apparent\ weight = 0N}$$

2. A person of mass $78kg$ is standing inside an electric lift. What is the apparent weight of the person if the;
   i) lift is moving upwards with an acceleration of $4ms^{-2}$.
   ii) lift is descending with an acceleration of $4ms^{-2}$

i)

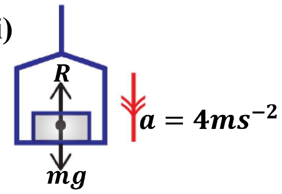


$m = 78kg$

$$Resultant\ force, F = ma$$
$$R - mg = ma$$
$$R = mg + ma$$
$$R = 78 \times 10 + 78 \times 4$$
$$R = 1092N$$

$$Apparent\ weight = 1092N$$

ii)

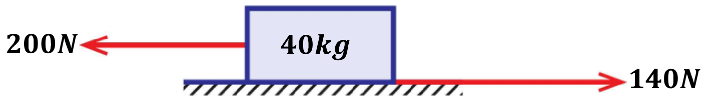


$m = 78kg$

$$Resultant\ force, F = ma$$
$$mg - R = ma$$
$$R = mg - ma$$
$$R = 78 \times 10 - 78 \times 4$$
$$R = 468N$$

$$Apparent\ weight = 468N$$

3. A block of mass $40kg$ is pulled from rest along a horizontal surface by a rope connected to one face of the block as shown below.



Given that the tension in the rope is 200N and that the frictional force between the block and the horizontal surface is 140N, find;

i) the acceleration of the block.

ii) The distance moved in 5s.

i)

$T = 200N, \quad F_R = 140N$

$$Resultant\ force, F = ma$$
$$T - F_R = ma$$
$$200 - 140 = 40a$$
$$a = \frac{60}{40}$$
$$a = 1.5ms^{-2}$$

$$u = 0ms^{-1}, \quad t = 5s$$
$$From, \quad s = ut + \frac{1}{2}at^2$$
$$s = 0 \times 5 + \frac{1}{2} \times 1.5 \times 5^2$$
$$s = \frac{37.5}{2}$$
$$s = 18.75m$$

<u>Other applications of Newton's third law include</u>:

- **Recoiling of a gun when a bullet is fired:**
  When a bullet is fired from a gun, the gun exerts a force on the bullet in the forward direction. This is the action force. The bullet also exerts an equal force on the gun in the backward direction. This is the reaction force. Due to the large mass of the gun, it moves a small distance backward giving a jerk at the shoulder of the gunman. This backward movement of the gun is called recoil of the gun.

- **Rowing of a boat.**
  During rowing of a boat, the boatman pushes the water backwards with the oars (action force). The water also apply an equal and opposite push on the boat which moves the boat forward (reaction)

- **Rocket propulsion:**
  During propulsion of a rockets, fuels are burnt, and the engine produces hot exhaust gases which flow out from the back of the engine (action). In reaction, a thrusting force is produced in an opposite direction.

EXERCISE:

1. A car of mass 5000kg initially moving at a velocity of $50m/s$ accelerates to $100m/s$ in 2 seconds. Calculate the engine force on the car that caused the velocity change.

2. A lift moves up and then down with an acceleration of $3ms^{-2}$. Calculate the reaction by the floor on the passenger of mass 60kg standing in the lift in each case.

3. A block of mass 8kg rests on a rough horizontal surface. It is being pulled from rest by a rope connected to one end of the block. Given that the tension in the rope is 20N and the friction force between the surface and the block is 4N, calculate;
   i) Acceleration of the system.
   ii) Distance moved by the block in 10s.

4. A girl of mass $500g$ stands in a stationary lift on earth. Calculate her apparent weight when the lift
   i) accelerates upwards at $2ms^{-2}$.
   ii) accelerates upwards at $2ms^{-2}$.
   iii) falls freely under gravity.

5. A spring balance carrying a mass of $4.0kg$ on its hook is hanged from the ceiling of a lift. Determine the spring balance reading when the lift is
   i) ascending with an acceleration of $4ms^{-2}$.
   ii) descending with an acceleration of $4ms^{-2}$.
   iii) ascending with a uniform velocity of $4ms^{-2}$.

6. A block of mass $10kg$ accelerates uniformly at a rate $3ms^{-2}$ along a horizontal table when a force of 40N acts on it. Find the frictional force between the block and the table.

7. A trolley of mass 2kg is pulled from rest by a horizontal force of 5N for 1.2 seconds. If there is no frictional force between the horizontal surface and the wheels of the trolley, calculate the
   i) acceleration and velocity of the trolley after 1.2 seconds.
   ii) distance covered by the trolley.
   iii) kinetic energy gained by the trolley.

8. A man with a mass of 85kg steps onto a weighing balance placed on the floor of the lift (Elevator).
   a) What would be the initial reading of the weighing balance?
   b) If the elevator accelerates up at $2.5ms^{-2}$, what is the new weighing balance reading?
   c) If the elevator accelerates downwards at $2.5ms^{-2}$, what is the new weighing balance reading?

LINEAR MOMENTUM

In the game of football, a player with a more momentum is hard to stop. The player has the ability to continue moving because of his mass and velocity. Therefore, any moving body possess momentum.

Definition:

Momentum is the product of mass of a body and its velocity.

$$Momentum = mass \times velocity$$

$$p = m \times v$$

$$\boxed{\boldsymbol{p = mv}}$$

The SI unit of momentum is kilogram metre per second ($\boldsymbol{kgms^{-1}}$).

❖ Momentum is a vector quantity and its direction is the same as that of the velocity.

Factors affecting momentum of a body:

Mass of a body:

The momentum of a body increases with increase in its mass.

This explains why;

- A heavy hammer can drive a nail deeper into a piece of wood than a lighter hammer.
- A heavy footballer is harder to stop once in motion than a small footballer.

Velocity of a body:

The momentum of a body increases with increase in its velocity.

This explains why;

- A fast-moving ball is not easier to stop than a slow-moving ball.
- A fast-moving car causes more damage when it makes an accident than a slow-moving car.

Examples:

1. Find the momentum of a car of mass $600kg$ moving with a constant velocity of $30ms^{-1}$.

$$\boldsymbol{m = 600kg, \quad v = 30ms^{-1}}$$

$$\boldsymbol{Momentum = mass \times velocity}$$

$$\boldsymbol{p = mv}$$

$$\boldsymbol{p = 600 \times 30}$$

$$\boldsymbol{p = 18000kgms^{-1}}$$

2. An object has a mass of 200kg and a momentum of $3800kgms^{-1}$. At what velocity is it moving?

$$\boldsymbol{m = 200kg, \quad p = 3800kgms^{-1}}$$

$$\boldsymbol{Momentum = mass \times velocity}$$

$$\boldsymbol{p = mv}$$

$$\boldsymbol{3800 = 200 \times v}$$

$$\boldsymbol{v = \frac{3800}{200}}$$

$$\boldsymbol{v = 19ms^{-1}}$$

3. A truck of mass 1200kg initially moving with a velocity of $15ms^{-1}$ accelerated uniformly at a rate of $1.5ms^{-2}$ for 10s. Find;
   i) its initial momentum.
   ii) its final velocity after 10s.
   iii) its final momentum.
   iv) the difference in momentum.

$m = 1200kg, \quad u = 15ms^{-1}, \quad t = 10s$
$a = 1.5ms^{-2}$

i) $\underline{initial\ momentum}$
$p_1 = mu$
$p_1 = 1200 \times 15$
$p_1 = 18000kgms^{-1}$

ii) $\underline{final\ velocity}$
$v = u + at$
$v = 15 + 1.5 \times 10$
$v = 30ms^{-1}$

iii) $\underline{final\ momentum}$
$p_2 = mv$
$p_2 = 1200 \times 30$
$p_2 = 36000kgms^{-1}$

iv) $\underline{change\ in\ momentum}$
$p_2 - p_1 = 36000 - 18000$
$p_2 - p_1 = 18000kgms^{-1}$

4. Calculate the kinetic energy possessed by a body of mass $10kg$ moving with a momentum of $200kgms^{-1}$.

$m = 10kg, \quad p = 200kgms^{-1}$
$p = mv$
$200 = 10 \times v$
$$v = \frac{200}{10}$$
$v = 20ms^{-1}$

$$K.E = \frac{1}{2}mv^2$$
$$K.E = \frac{1}{2} \times 10 \times 20^2$$
$K.E = 2000J$

Principle of conservation of linear momentum:

It states that when two or more bodies collide, their total linear momentum remains constant provided no external force is acting.
i.e. $\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$

COLLISIONS:

When bodies come into contact with each other, they are said to have collided. Therefore, they experience a force from each other which changes their momentum.
However, if there are no external forces participating in the collision, the total momentum after collision remains the same as before collision.

Types of collisions:

There are two types of collisions namely;

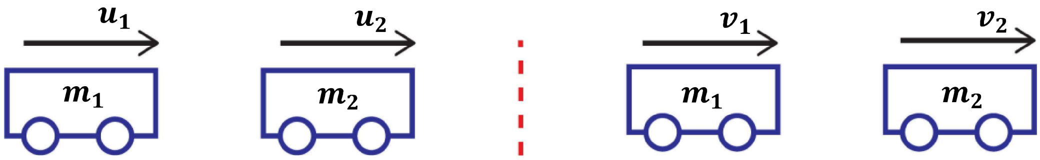
- Elastic collision.
- Inelastic collision.

ELASTIC COLLISION:

This is the type of collision where the colliding bodies separate after collision and move with different velocities.
During elastic collision;

- Momentum is conserved i.e. total momentum before collision is equal to total momentum after collision.
- Kinetic energy is also conserved i.e. total kinetic energy before collision is equal to total kinetic energy after collision.

Consider two bodies of masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ moving with initial velocities, $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ respectively. After their collision, they move with final velocities, $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ respectively.



Before collision ***After collision***

From the principle of linear conservation of momentum;

$$\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$$

$$\boxed{\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}}$$

For kinetic energy;

$$\boldsymbol{Total\ kinetic\ energy\ before\ collision = Total\ kinetic\ energy\ after\ collision}$$

$$\boldsymbol{\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2}$$

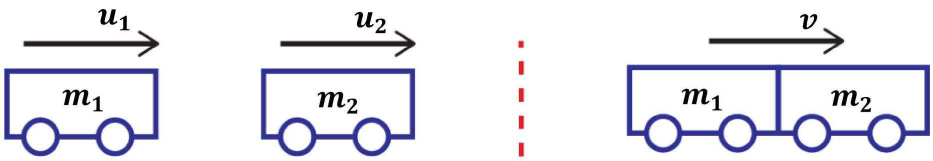
<u>INELASTIC COLLISION</u>:

This is the type of collision where the colliding bodies stay together after collision and move with the same velocity.

During inelastic collision;

- Momentum is conserved i.e. total momentum before collision is equal to total momentum after collision.
- Kinetic energy is not conserved i.e. total kinetic energy before collision is not equal to total kinetic energy after collision.

Consider two bodies of masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ moving with initial velocities, $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ respectively. After their collision, they move with the same final velocity, $\boldsymbol{v}$.



Before collision ***After collision***

From the principle of linear conservation of momentum;

$$\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$$

$$\boldsymbol{m_1u_1 + m_2u_2 = m_1v + m_2v}$$

$$\boxed{\boldsymbol{m_1u_1 + m_2u_2 = (m_1 + m_2)v}}$$

For kinetic energy;

$$\boldsymbol{Total\ kinetic\ energy\ before\ collision \neq Total\ kinetic\ energy\ after\ collision}$$

$$\boldsymbol{\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 \neq \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2}$$

$$\boldsymbol{\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 \neq \frac{1}{2}(m_1 + m_2)v^2}$$

Causes of kinetic energy losses in inelastic collision include;

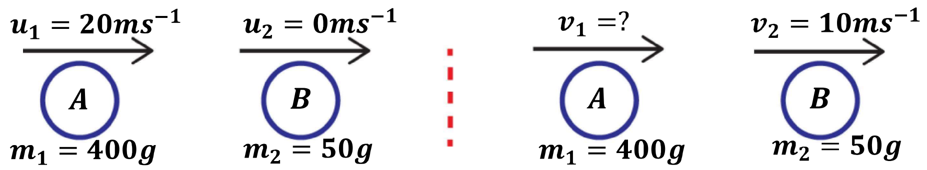
- Some kinetic energy is converted into heat energy leading to increase in temperature of the colliding bodies.
- Some kinetic energy is converted in sound energy as the bodies collide.

<u>DIFFERENCES BETWEEN ELASTIC AND INELASTIC COLLISION</u>

Elastic collision	Inelastic collision
▪ Bodies separate after collision. ▪ Both momentum and kinetic energy are conserved. ▪ Bodies move with different velocities after collision.	▪ Bodies stick together after collision. ▪ Kinetic energy is not conserved but momentum is conserved. ▪ Bodies move with the same velocity after collision.

<u>Examples</u>:
Note: The bodies should have the same units of mass and velocity.

1. Ball A of mass $400g$ moving with a velocity of $20ms^{-1}$ collided with ball B of mass $50g$ at rest. If ball B moves with a velocity of $10ms^{-1}$ after collision in the direction of ball A.
   Find
   i) the velocity of ball A after collision.
   ii) kinetic energy after collision.



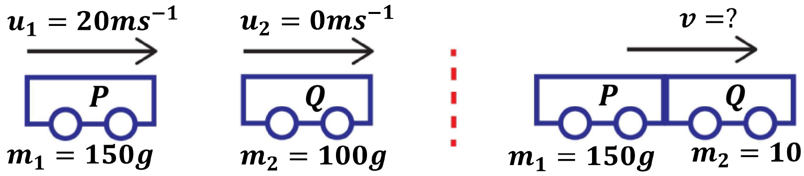
i) $\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$400 \times 20 + 50 \times 0 = 400 \times v_1 + 50 \times 10$$
$$8000 = 400v_1 + 500$$
$$7500 = 400v_1$$
$$v_1 = \frac{7500}{400}$$
$$v_1 = 18.75ms^{-1}$$

ii) **<u>Kinetic energy after collision</u>**

$$K.E = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$$
$$K.E = \frac{1}{2} \times \left(\frac{400}{1000}\right) \times 18.75^2 + \frac{1}{2} \times \left(\frac{50}{1000}\right) \times 10^2$$
$$K.E = \frac{140.625}{2} + \frac{5}{2}$$
$$K.E = 70.3125 + 2.5$$
$$K.E = 72.8125J$$

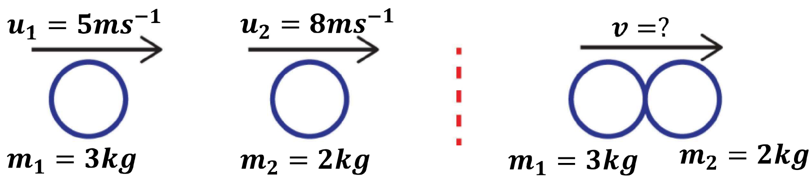
2. A trolley P of mass $150g$ moving with a velocity of $20ms^{-1}$ collides with another stationary trolley Q of mass $100g$. If the two trolleys move together after collision, calculate their common velocity.



$$\textit{Total momentum before collision} = \textit{Total momentum after collision}$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$150 \times 20 + 100 \times 0 = (150 + 100)v$$
$$3000 = 250v$$
$$v = \frac{3000}{250}$$
$$v = 12ms^{-1}$$

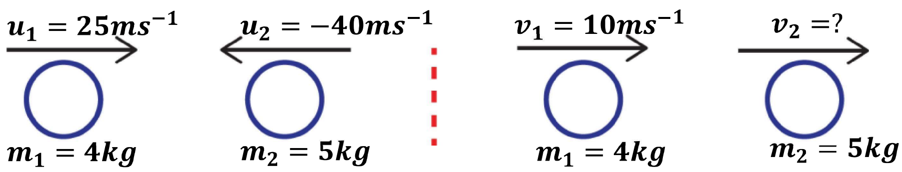
3. A body of mass $3kg$ travelling at $5ms^{-1}$ collides with a $2kg$ body moving at $8ms^{-1}$ in the same direction. If after collision the two bodies moved together, calculate the velocity with which the two bodies move after collision.



$$\textit{Total momentum before collision} = \textit{Total momentum after collision}$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$3 \times 5 + 2 \times 8 = (3 + 2)v$$
$$31 = 5v$$
$$v = \frac{31}{5}$$
$$v = 6.2ms^{-1}$$

4. A body of mass $4kg$ moving with a velocity of $25ms^{-1}$ collided with another body of mass $5kg$ moving with a velocity of $40ms^{-1}$ from the opposite direction. If the $4kg$ mass moves with a velocity of $10ms^{-1}$ after collision, find the velocity of the $5kg$ mass after collision.

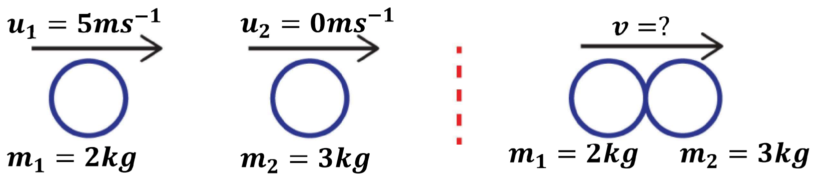


$$\textit{Total momentum before collision} = \textit{Total momentum after collision}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$4 \times 25 + 5 \times -40 = 4 \times 10 + 5 \times v_2$$
$$-100 = 40 + 5v_2$$
$$-140 = 5v_2$$
$$v_2 = \frac{-140}{5}$$
$$v_2 = -28ms^{-1}$$

The negative sign shows a change in direction

5. An object of mass $2kg$ moving at $5ms^{-1}$ collides with another of mass $3kg$ which is at rest. Find
   i) velocity of the two bodies if they stick together after collision.
   ii) loss in kinetic energy.



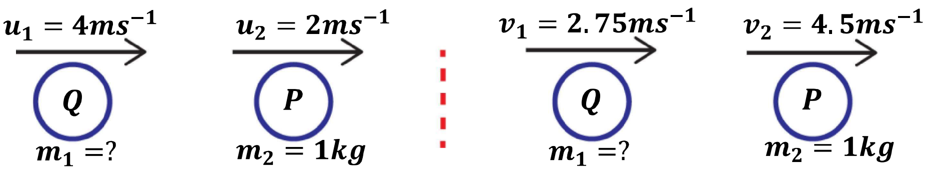
a) $\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$

$$\boldsymbol{m_1u_1 + m_2u_2 = (m_1 + m_2)v}$$
$$\boldsymbol{2 \times 5 + 3 \times 0 = (2 + 3)v}$$
$$\boldsymbol{10 = 5v}$$
$$\boldsymbol{v = \frac{10}{5}}$$
$$\boldsymbol{v = 2ms^{-1}}$$

b) **<u>Loss in kinetic energy</u>**

$$\boldsymbol{Loss = kinetic\ energy\ before\ collision - kinetic\ energy\ after\ collision}$$
$$\boldsymbol{Loss = \left[\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2\right] - \left[\frac{1}{2}(m_1 + m_2)v^2\right]}$$
$$\boldsymbol{Loss = \left[\frac{1}{2} \times 2 \times 5^2 + \frac{1}{2} \times 3 \times 0^2\right] - \left[\frac{1}{2} \times (2 + 3) \times 2^2\right]}$$
$$\boldsymbol{Loss = \left[\frac{25}{2}\right] - \left[\frac{20}{2}\right]}$$
$$\boldsymbol{Loss = 12.5 - 10}$$
$$\boldsymbol{Loss = 2.5\,J}$$

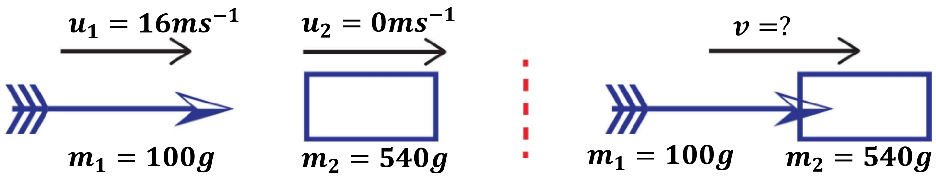
6. A particle P of mass $1kg$ moving with a velocity of $2ms^{-1}$ is knocked directly from behind by another particle Q moving at $4ms^{-1}$. If the velocity of P increases to $4.5ms^{-1}$ and velocity of Q reduces to $2.75ms^{-1}$, find the mass of particle Q.



$\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$

$$\boldsymbol{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$
$$\boldsymbol{m_1 \times 4 + 1 \times 2 = m_1 \times 2.75 + 1 \times 4.5}$$
$$\boldsymbol{4m_1 + 2 = 2.75m_1 + 4.5}$$
$$\boldsymbol{4m_1 - 2.75m_1 = 4.5 - 2}$$
$$\boldsymbol{1.25m_1 = 2.5}$$
$$\boldsymbol{m_1 = \frac{2.5}{1.25}}$$
$$\boldsymbol{m_1 = 2kg}$$

7. An arrow of mass $100g$ moving at a velocity of $16ms^{-1}$ horizontally enters a block of wood of mass $540g$ lying at rest on a smooth surface.
   a) State the type of collision.
   b) Find the common velocity after the impact.
   c) Calculate the loss in kinetic energy.



a) **Inelastic collision**

b) $\boldsymbol{Total\ momentum\ before\ collision = Total\ momentum\ after\ collision}$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$100 \times 16 + 540 \times 0 = (100 + 540)v$$
$$1600 = 640v$$
$$v = \frac{1600}{640}$$
$$v = 2.5ms^{-1}$$

c) **Loss in kinetic energy**

$$m_1 = 100g = \frac{100}{1000} = 0.1kg, \quad m_2 = 540g = \frac{540}{1000} = 0.54kg$$

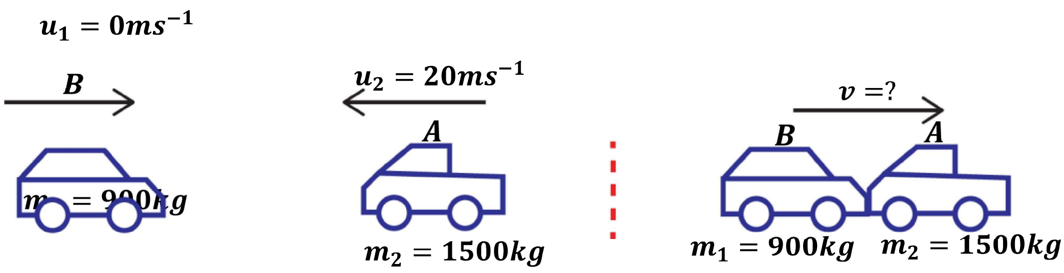
$$Loss = kinetic\ energy\ before\ collision - kinetic\ energy\ after\ collision$$
$$Loss = \left[\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2\right] - \left[\frac{1}{2}(m_1 + m_2)v^2\right]$$
$$Loss = \left[\frac{1}{2} \times 0.1 \times 16^2 + \frac{1}{2} \times 0.54 \times 0^2\right] - \left[\frac{1}{2} \times (0.1 + 0.54) \times 2.5^2\right]$$
$$Loss = \left[\frac{25.6}{2}\right] - \left[\frac{4}{2}\right]$$
$$Loss = 12.8 - 2$$
$$Loss = 10.8J$$

8. The figure below shows a system where vehicle A of mass $1500kg$ travelling at a velocity of $72km/hr$ towards a stationary vehicle B of mass $900kg$.

If A collides with B, the two move together at a constant velocity for 20 seconds, calculate;
   i) the common velocity.
   ii) the distance moved after the impact.

$$u_2 = 72km/hr = \frac{72 \times 1000}{3600} = 20ms^{-1}$$

$$u_1 = 0ms^{-1}$$



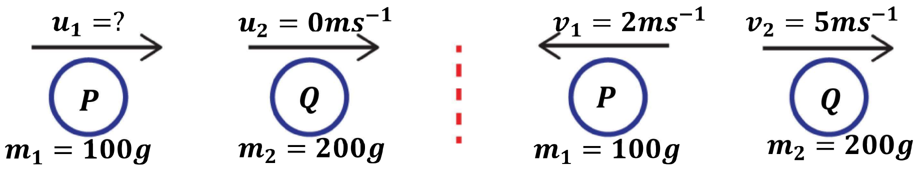
i) $\textit{Total momentum before collision} = \textit{Total momentum after collision}$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$900 \times 0 + 1500 \times 20 = (900 + 1500)v$$
$$30000 = 2400v$$
$$v = \frac{30000}{2400}$$
$$v = 12.5ms^{-1}$$

ii) $t = 20s,\ \ u = 12.5ms^{-1},\ \ a = 0ms^{-2}$ *since velocity is constant*

$$\textit{From},\quad s = ut + \frac{1}{2}at^2$$
$$s = 12.5 \times 20 + \frac{1}{2} \times 0 \times 20^2$$
$$s = 250m$$

9. A moving ball P of mass 100g collides with a stationary ball Q of mass 200g. After collision, P moves backwards with a velocity of $2ms^{-1}$ while Q moves forward with a velocity of $5ms^{-1}$. Calculate;
   i) the initial velocity of P.
   ii) the force exerted by P on Q if the collision took 5s.



i) $\textit{Total momentum before collision} = \textit{Total momentum after collision}$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$100 \times u_1 + 200 \times 0 = 100 \times 2 + 200 \times 5$$
$$100u_1 = 1200$$
$$u_1 = \frac{1200}{100}$$
$$u_1 = 12ms^{-1}$$

ii) <u>**Force exerted by P on Q**</u>

$\textit{For } P,\quad u_1 = 12ms^{-1},\quad v_1 = 2ms^{-1},\quad t = 5s,\quad m_1 = 100g = \frac{100}{1000} = 0.1kg$

$$a = \frac{v_1 - u_1}{t} = \frac{2 - 12}{5} = -2ms^{-2} = 2ms^{-2}(\textit{deceleration})$$
$$F = ma$$
$$F = 0.1 \times 2$$
$$F = 0.2N$$

IMPULSE

When a force is applied on a free object for some amount of time, it changes it velocity thus changing the momentum of the body. This impact created by the force is referred to as **impulse**.

Definition:

Impulse is the product of force and its time of action on the body.

$$Impulse = Force \times Time$$

$$\boxed{\boldsymbol{I = Ft}}$$

The SI unit of impulse is Newton second (**Ns**).

From Newton's second law of motion, $F = ma$

$$I = Ft$$
$$I = mat$$
$$I = m\frac{(v-u)}{t} \times t$$
$$I = m(v-u)$$
$$\boxed{\boldsymbol{I = mv - mu}}$$

Therefore, Impulse can be defined as the change in momentum of the body.

The other unit of Impulse is $\boldsymbol{kgms^{-1}}$.

<u>Examples</u>:

1. A body of mass $2kg$ changes its velocity from $10ms^{-1}$ to $45ms^{-1}$ after a period of time. Calculate the impulse on the body.

$$\boldsymbol{m = 2kg, \quad u = 10ms^{-1}, \quad v = 45ms^{-1}}$$
$$\boldsymbol{I = mv - mu}$$
$$\boldsymbol{I = (2 \times 45) - (2 \times 10)}$$
$$\boldsymbol{I = 90 - 20}$$
$$\boldsymbol{I = 70kgms^{-1}}$$

2. A body of mass 4.5kg accelerates uniformly at $2ms^{-2}$ for 5 seconds. Calculate the impulse on the body.

$$\boldsymbol{m = 4.5kg, \quad a = 2ms^{-2}, \quad t = 5s}$$

$\boldsymbol{F = ma}$	$\boldsymbol{I = Ft}$
$\boldsymbol{F = 4.5 \times 2}$	$\boldsymbol{I = 9 \times 5}$
$\boldsymbol{F = 9N}$	$\boldsymbol{I = 45Ns}$

3. An object is acted upon by a force of 50N for 2 minutes. Calculate the impulse on the object.

$$\boldsymbol{F = 50N, \quad t = 2\ mins = 2 \times 60 = 120s}$$
$$\boldsymbol{I = Ft}$$
$$\boldsymbol{I = 50 \times 120}$$
$$\boldsymbol{I = 6000Ns}$$

4. A footballer kicks a ball of mass 0.25kg initially at rest with a force of 200N that acts on the ball for 0.5s. Find;
   a) the impulse of the force on the ball.
   b) the takeoff velocity of the ball

a) $F = 200N, \quad t = 0.5s, \quad m = 0.25kg$

$$I = Ft$$
$$I = 200 \times 0.5$$
$$I = 100Ns$$

b) $u = 0ms^{-1}, \quad v = ?$

$$I = mv - mu$$
$$100 = 0.25 \times v - 0.25 \times 0$$
$$0.25v = 100$$
$$v = \frac{100}{0.25}$$
$$v = 400ms^{-1}$$

EFFECTS OF IMPULSIVE FORCES ON THE BODY

Though impulsive forces act for a short period of time, they are sometimes disadvantageous to a body on which they act. Some of the negative effects include;

- Impulsive forces tend to change the shape of colliding bodies.
- Impulsive forces tend harm bodies e.g. pain after knocking a stone.

In order to reduce the above negative effects, the time of action of the force on the body is increased or prolonged.

This explains the following applications:

- ❖ A goal keeper draws his hands towards his body when catching a fast-moving ball. This increases the time of action of the force on the ball thus reducing the pain that would be felt by the goal keeper after catching the ball.

- ❖ Goal keepers wear soft gloves that absorb shocks on their hands. The soft gloves reduce the force on the hands by increasing the time of action of the force.

- ❖ The nets at the back of a goal post are made loose to increase the time of action of the impact as the ball hits the net. This prevents the net from getting torn.

- ❖ Shock absorbers are put in vehicles to reduce the force exerted on the vehicles as they move over pot holes. The shock absorbers increase the time of action of the impact of force.

- ❖ High jumpers usually bend their knees on landing. This increases the time of impact hence reducing injuries on the jumpers.

- ❖ High jumpers land in sand or soft cushions that increase the time of action of the impact thus absorbing the shocks on the jumper.

- ❖ In golf, players follow the ball as it is hit. This reduces the reaction force the player feels on hitting the ball by increasing the time of contact.

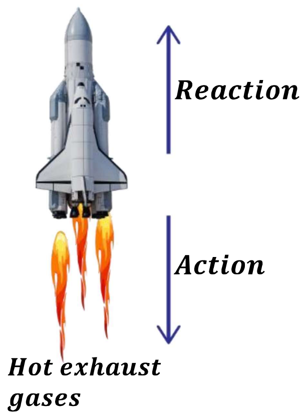
- ❖ Cars are fitted with air bag. During an accident, air bags increase the time of action of the impact thus a less force is exerted on a person over a long period of time. This reduces injuries.

- ❖ Objects that easily break like eggs are packed in soft, shock-absorbing boxes. This reduces the possibility of them cracking on sudden stop or start of motion. The shock-absorbing boxes increase the time of impact on the eggs.

APPLICATIONS OF THE PRINCIPLE OF CONSERVATION OF MOMENTUM

The principle of conservation of linear momentum is applied in:

Rockets:

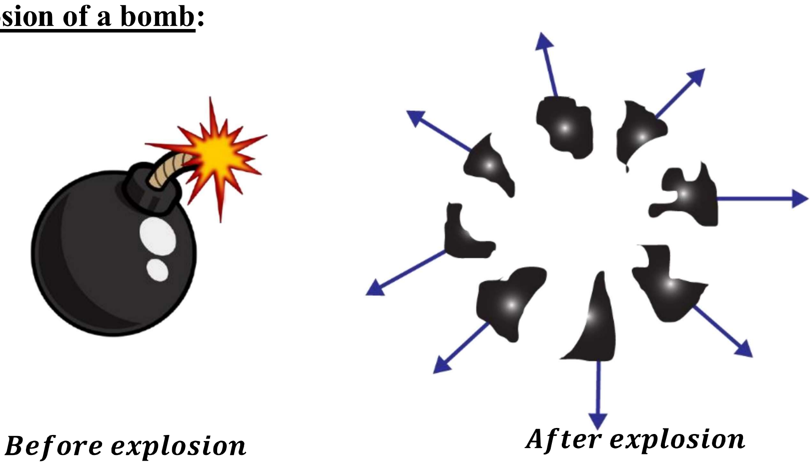


During propulsion of a rockets, fuels are burnt in the rocket engine, and the engine produces hot exhaust gases which escape through the engine nozzle with a large velocity hence a large momentum.
In turn, the escaping gases produce a force which impart an equal but opposite momentum to the rocket. This momentum propels the rocket to move forward with a very high velocity.

Jet planes:

During movement of jet planes, fuels are burnt in the jet engines, and the engine produces hot exhaust gases which escape through the exhaust pipes with a large velocity hence a large momentum.
In turn, the escaping gases produce a force which impart an equal but opposite momentum to the jet plane. This momentum forces the jet plane to move forward with a very high velocity.

Explosion of a bomb:

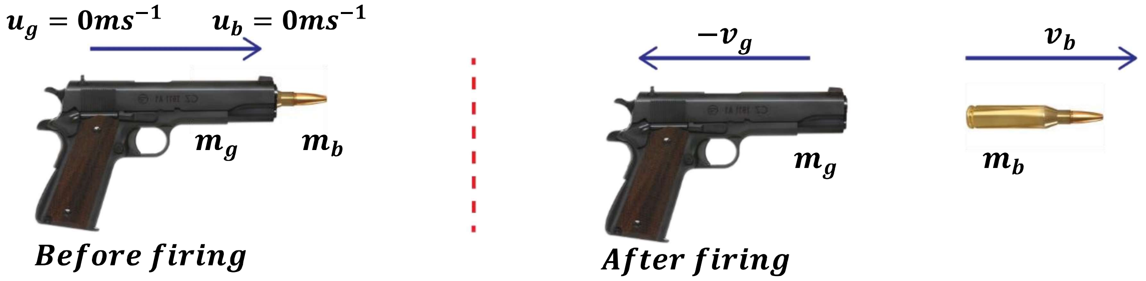


Before a bomb explodes, its total momentum is zero since it is at rest. When it explodes, the bomb breaks into very many fragments (parts) with each fragment having a particular momentum.
A fragment moving in one direction with a particular momentum has another fragment with the same momentum moving in an opposite direction. Therefore, the total momentum of the fragments is also zero thus momentum before explosion is equal to momentum after collision.

Recoil of a gun:

Before firing a bullet from a gun, the gun and the bullet are at rest. After firing, the gun exerts a force a force on the bullet in the forward direction (action) but also the bullet exerts an equal force on the gun in the backward direction (reaction) thus the gun moves backward i.e. recoiling.

Therefore, the bullet receives an equal but opposite momentum to that of the gun.



From the principle of linear conservation of momentum;

$$\boldsymbol{Total\ momentum\ before\ firing = Total\ momentum\ after\ firing}$$

$$\boldsymbol{m_g u_g + m_b u_b = m_g v_g + m_b v_b}$$

$$\boldsymbol{m_g \times 0 + m_b \times 0 = m_g \times -v_g + m_b v_b}$$

$$\boldsymbol{0 = -m_g v_g + m_b v_b}$$

$$\boxed{\boldsymbol{m_g v_g = m_b v_b}}$$

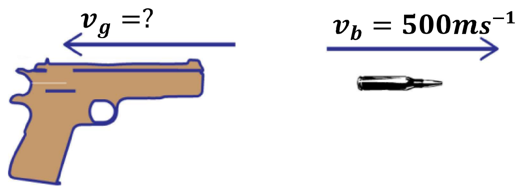
NOTE:

- The velocity of the gun, $\boldsymbol{v_g}$ is called the **Recoil velocity**.
- The velocity of the bullet, $\boldsymbol{v_b}$ is called the **muzzle velocity**.

Examples:

1. A bullet of mass $8g$ is fired from a gun of mass $500g$. If the muzzle velocity of the bullet is $500ms^{-1}$, calculate the recoil velocity of the gun.

$$\boldsymbol{m_g = 500g = \frac{500}{1000} = 0.5kg} \qquad \boldsymbol{m_b = 8g = \frac{8}{1000} = 0.008kg}$$



$$\boldsymbol{momentum\ of\ the\ gun = momentum\ of\ the\ bullet}$$

$$\boldsymbol{m_g v_g = m_b v_b}$$

$$\boldsymbol{0.5 \times v_g = 0.008 \times 500}$$

$$\boldsymbol{0.5 v_g = 4}$$

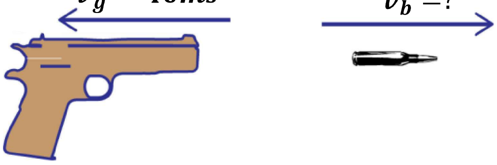
$$\boldsymbol{v_g = \frac{4}{0.5}}$$

$$\boldsymbol{v_g = 8ms^{-1}}$$

2. A bullet of mass $20g$ is fired from a gun of mass $0.4kg$, if the gun recoils with the velocity of $40ms^{-1}$, calculate the velocity of the bullet.

$$m_g = 0.4kg \qquad m_b = 20g = \frac{20}{1000} = 0.02kg$$

$$v_g = 40ms^{-1} \qquad v_b = ?$$



$$momentum\ of\ the\ gun = momentum\ of\ the\ bullet$$
$$m_g v_g = m_b v_b$$
$$0.4 \times 40 = 0.02 \times v_b$$
$$16 = 0.02 v_b$$
$$v_b = \frac{16}{0.02}$$
$$v_b = 800ms^{-1}$$

3. A bullet of mass 20g is fired from the gun of mass $0.4kg$. If the velocity of the bullet is $400ms^{-1}$, calculate;
   i) the recoil velocity of the gun.
   ii) the kinetic energy gained by the gun.

$$m_g = 0.4kg \qquad m_b = 20g = \frac{20}{1000} = 0.02kg \qquad v_b = 400ms^{-1} \qquad v_g = ?$$

i)

$$momentum\ of\ the\ gun = momentum\ of\ the\ bullet$$
$$m_g v_g = m_b v_b$$
$$0.4 \times v_g = 0.02 \times 400$$
$$0.4 v_g = 8$$
$$v_g = \frac{8}{0.4}$$
$$v_g = 20ms^{-1}$$

ii)

$$K.E = \frac{1}{2} m_g {v_g}^2$$
$$K.E = \frac{1}{2} \times 0.4 \times 20^2$$
$$K.E = \frac{160}{2}$$
$$K.E = 80\,J$$

4. A bullet of mass $6g$ is fired from a gun of mass $500g$. If the muzzle velocity of the bullet is $300ms^{-1}$, calculate the recoil velocity of the gun.

$$m_g = 500g = \frac{500}{1000} = 0.5kg \qquad m_b = 6g = \frac{6}{1000} = 0.006kg \qquad v_b = 300ms^{-1} \qquad v_g = ?$$

$$momentum\ of\ the\ gun = momentum\ of\ the\ bullet$$
$$m_g v_g = m_b v_b$$
$$0.5 \times v_g = 0.006 \times 300$$
$$0.5 v_g = 1.8$$
$$v_g = \frac{1.8}{0.5}$$
$$v_g = 3.6ms^{-1}$$

5. A bullet of mass $12g$ travelling at $150ms^{-1}$ penetrates deeply into a fixed soft wood and is brought to rest in 0.015s. calculate how deep the bullet penetrates the wood.

$$m_b = 12g = \frac{12}{1000} = 0.012kg$$
$$u_b = 150ms^{-1}, \quad v_b = 0ms^{-1}, \quad t = 0.015s$$
$$a = \frac{v_b - u_b}{t}$$
$$a = \frac{0 - 150}{0.015}$$
$$a = -10000ms^{-2}$$

$$From \quad s = ut + \frac{1}{2}at^2$$
$$s = 150 \times 0.015 + \frac{1}{2} \times -10000 \times 0.015^2$$
$$s = 2.25 - 1.125$$
$$s = 1.125m$$

EXERCISE:

1. A bus of mass $7500kg$ travelling at $30ms^{-1}$ collides inelastically with a van which is approaching from the opposite side at $32ms^{-1}$. If the van has a mass of $2500kg$, at what velocity do the van and bus travel with after collision?
2. Car A of mass $2000kg$ travelling at $0.5ms^{-1}$ collides with another car B of half the mass of A moving in opposite direction with a velocity of $0.4ms^{-1}$. If the trucks stay together on collision, calculate the common velocity with which they move.
3. A bullet of mass $1.5 \times 10^{-2}kg$ is fired from a riffle of mass $3kg$ with a muzzle velocity of $180kmh^{-1}$. Calculate the recoil velocity of the riffle.
4. A trolley P of mass $150g$ moving with a velocity of $20ms^{-1}$ collides with another stationary trolley X of mass $100g$. If P and X move together after collision, calculate;
   i) momentum of P before collision.
   ii) the velocity of P and X with which they moved after collision.
5. A gun of mass $5kg$ fires a bullet of mass $50g$ at a speed of $500ms^{-1}$. Calculate the recoil velocity of the gun.
6. A car of mass $1500kg$ moving with a velocity of $25ms^{-1}$ collides directly with another car of mass $1400kg$ at rest so that the two stick and move together. Find their common velocity.
7. A bullet of mass $30g$ is fired into a stationary block of wood of mass $480g$ lying on a smooth horizontal surface. If the bullet gets embedded in the block and the two move together at a speed of $15ms^{-1}$. Find;
   i) the speed of the bullet before it hits the block.
   ii) The kinetic energy lost.
8. A moving ball A of mass $200g$ collides directly with a stationary ball B of mass $300g$ so that A bounces with a velocity of $2ms^{-1}$ while B moves forward with a velocity of $3ms^{-1}$. Calculate the initial velocity of A.
9. A particle X of mass $2kg$ originally moving with a velocity of $3ms^{-1}$ collides directly with another particle Y of mass $2kg$ which is moving at a velocity of $2ms^{-1}$ in the opposite direction so that the velocity of X becomes $1ms^{-1}$ after the impact. Find the velocity of Y after the impact.
10. A bullet of mass $40g$ is fired with a velocity of $200ms^{-1}$ from a gun of mass $5kg$. What is the recoil velocity of the gun?

11. A one-tonne car travelling at $20ms^{-1}$ is accelerated at $2ms^{-2}$ for 5 seconds. Calculate the;
    i) change in momentum.
    ii) rate of change of momentum.
    iii) accelerating force acting on the body.
12. A man of mass $6kg$ jumps from a high wall and lands on a hard floor at a velocity of $6ms^{-1}$. Calculate the force exerted on the man's legs if;
    a) he bends his knees on landing o that it takes $1.2s$ for his motion to be stopped.
    b) he does not bend his knees and it takes $0.06s$ to stop his motion.
13. a) Explain why a passenger standing on the floor of a lorry jerks backwards when the lorry starts moving forwards.
    b) A 7-tonne truck initially moving at a velocity of $50ms^{-1}$ accelerates to $80ms^{-1}$ in 3 seconds. Calculate the force on the truck that caused the velocity change.
14. A van of mass 1.5 tonnes travelling at $20ms^{-1}$ hits a wall and is brought to rest as a result in 0.5 seconds. Calculate the;
    i) impulse.
    ii) average force exerted on the wall.
15. A goal keeper is to catch a ball of mass $0.25kg$ travelling at $250ms^{-1}$. Find the impulsive force exerted on the goal keeper's hands
    i) if the impact lasts for $0.2s$.
    ii) if the impact lasts for $1s$ when the goal keeper draws his hands towards his body as he catches the ball.
16. A car of mass $2000kg$ travelling at $5m/s$ collides with a mini-bus of mass $5000kg$ travelling in the opposite direction at $7m/s$. The vehicles stick and move together after collision. If the collision lasts 0.1 seconds.
    a) Determine the velocity of the system after collision to 3 decimal places.
    b) Calculate the impulsive force on the mini-bus.
17. Explain the following observations:
    i) a water jet directed to a spot on the ground digs a hole in the ground after sometime.
    ii) A goal keeper draws hands to his body when catching a fast-moving ball.
    iii) A fast- moving vehicle causes more damage than a slow-moving vehicle when they both a hit an obstacle.
18. A truck of mass $4 \times 10^4 kg$ moving at a velocity $3m/s$ collides with another truck of mass $2 \times 10^4 kg$ which is at rest. The couplings join and the trucks move off together.
    a) State the type of collision.
    b) Calculate the common velocity of the trucks after collision.
    c) Calculate the loss in kinetic energy.
19. A bullet of mass $10g$ is shot from a gun of mass $20kg$ with a muzzle velocity osf $100m/s$. If the barrel (tube of the gun) is 0.2m long, determine;
    i) the acceleration of the bullet.
    ii) recoil velocity of the gun.
20. A car X of mass $1000kg$ travelling at a speed of $20ms^{-1}$ in the direction due east collides heads-on with another car Y of mass $1500kg$ travelling at $15ms^{-1}$ in the direction due west. If the two cars stick together, find their common velocity after collision.