S6 MTC REVISION QNS
Paper 1

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DIFFERENTIATION TOPICAL REVISION

Differentiating from first principles

1. x^2 and $\frac{2}{x^2}$
2. x^3
3. $\sqrt{x}$ and $\frac{1}{\sqrt{x}}$
4. $\cos x$ and $\sin x$
5. $\sin 2x$ and $\cos 3x$
6. $x^2 + \cos 3x$
7. $\cos^2 2x$,
8. $2x + \tan x$.
9. $\tan^{-1} x$.

Chain rule / parametric eqn

10. $x = t^2, y = 4t - 1$
11. $y = 3t^2 + 2t, x = 1 - 2t$
12. $x = 2\sqrt{2}$, $y = 5t - 4$
13. $x = \frac{1}{t}$, $y = t^2 + 4t - 3$
14. $x = \frac{2}{3+\sqrt{t}}$; $y = \sqrt{t}$

Product rule

15. $(x^2+1)(x^3+2)$
16. $x^2(x+1)^3$
17. $(1+x)^{3/4}(x-1)^{5/4}$
18. $(x-1)\sqrt{x^2+1}$
19. $\sqrt{(x+1)(x-2)^3}$
20. $(x-1)^2\sqrt[3]{1-2x}$

Quotient rule

21. $\frac{x^2+1}{x^2-1}$
22. $\frac{x}{\sqrt{x^2+1}}$
23. $\sqrt{\frac{(x+2)^3}{x-1}}$
24. $\sqrt{\frac{(x+1)^3}{x+2}}$

Implicit functions

25. $x^2 + 2xy + y^2 = 8$
26. $x^2 - 3xy + y^2 - 2y + 4x = 0$
27. $3x^2 - 4xy = 7$
28. $x^2 + 3xy - y^2 = 0$
29. $x^3 - y^3 - 4x^2 + 3y = 11x + 4$

exponential functions

30. a) $4e^x$ b) e^{-2x}
31. c) e^{ax^2+b}
32. d) $e^{\sqrt{\cos x}}$ e) e^{xe^x}
33. f) $e^{\tan x^2}$
34. g) $e^{\sqrt{x^2+1}}$ h) $e^{-\cot x}$
35. a^x, $2^x, 3^x, 5^x$

logarithmic functions

36. $\ln(2x^3)$
37. $\ln(x^3+1)$
38. $\ln \sec x$
39. $\ln\left(\frac{1+\cos x}{1-\sin x}\right)$
40. $\frac{\ln x}{\sqrt{1+x^2}}$
41. $3x \ln x^2$
42. $\ln \cos x$
43. $\ln(\sec x + \tan x)$
44. $\ln \frac{(x+1)^2}{\sqrt{x-1}}$
45. $\frac{dy}{dx}(\ln x\sqrt{x^2-1})$
46. $\ln \sin^2 x$ (b) $\ln \tan(3x)$
47. $\ln 3\cos^2 x$ (d) $\ln\left(\frac{(x+1)^2}{x-1}\right)$
48. $\ln(x + \sqrt{(x^2-1)})$
49. $\sqrt[3]{\frac{x+1}{x-1}}$
50. $\frac{e^{x^2}\sqrt{\sin x}}{(2x+1)^3}$
51. x^x
52. $(\sin x)^x$
53. 2^x
54. $x 10^{\sin x}$
55. $\ln(x)^x$
56. $x^{\sin x}$

Inverse trigonometric fxns

57. $\cos^{-1} x$ b) $\sin^{-1} x$
58. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
59. $\tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
60. $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

Proofs

61. If $y = e^{2x}\cos 3x$ show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 13y = 0$
62. $y = xe^{-x}$ Show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$
63. Given that $y = \sin\sqrt{x}$, prove that $2\frac{dy}{dx} + y + 4x\frac{d^2y}{dx^2} = 0$
64. If $y = \tan xy$, prove that $\frac{dy}{dx} = \frac{y}{\cos^2 xy - x}$
65. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$. Show that $\frac{dy}{dx} = \frac{1}{1+x^2}$
66. if $e^x = \tan 2y$, then $\frac{d^2y}{dx^2} = \frac{e^x - e^{3x}}{2(1+e^{2x})^2}$.
67. If $x = \sin\theta$ and $y = 1 - \cos\theta$, show that $\left(\frac{d^2y}{dx^2}\right)^2 = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3$
68. If $y = \tan\left[2\tan^{-1}\left(\frac{x}{2}\right)\right]$ show that $\frac{dy}{dx} = \frac{4(1+y^2)}{(4+x^2)}$.
69. If $x = \theta - \sin\theta$, $y = 1 - \cos\theta$, show that $\frac{dy}{dx} = \cot\left(\frac{\theta}{2}\right)$.
70. If $y^x = x^y$, show that $\frac{dy}{dx} = \frac{y(x\ln y - y)}{x(y\ln x - x)}$.
71. If $e^x = \ln(x+y)$, Show that $\frac{d^2y}{dx^2} = (1+e^x)(1+\frac{dy}{dx})$.
72. If $y = \theta - \cos\theta$, $x = \sin\theta$. Show that $\frac{d^2y}{dx^2} = \frac{1+\sin\theta}{\cos^3\theta}$
73. Show that $\frac{d(\tan^{-1}x^x)}{dx} = \frac{(1+\ln x)x^x}{1+x^{2x}}$.

There is still time to change your

Tv.r

RATES AND SMALL CHANGES

Applications of differentiation

1. If $p = 4s^2 - 10s + 7$, find the minimum value of p and the values of s which gives the minimum value of p.
2. Onyango wishes to fence a rectangular farm. He wants the sum of the length and the width of the farm to be 42 cm. Calculate the length and width of the farm for the area of the farm to be as maximum as possible.
3. A cylindrical can is made so that the sum of the height and the circumference of its base is 45π cm. Find the radius of the base of the cylinder if the volume of the can is maximum.
4. The length of a rectangular block is twice its width, and the total surface area is 108 cm². Show that if the width of the block is x cm, the volume is $\frac{4}{3}x(27 - x^2)$. Find the dimensions of the block if the volume is maximum.
5. A cylindrical volume V is to be cut from a solid sphere of radius R. Prove that the maximum volume of the cylindar, V is $V = \frac{4\pi R^3}{3\sqrt{3}}$
6. A rectangular block has a base x cm square. Its surface area is 150 cm². Prove that the volume of the block is $\frac{1}{2}(75x - x^3)$.
   (a) Calculate the dimensions of the block when the volume is maximum.
   (b) The maximum volume.
7. A variable rectangular flower garden has a constant perimeter of 40. Find the length of the side when the area is maximum.
8. ii)A variable rectangle has a constant area of 36 cm². Find the length of the sides when the perimeter is maximum.
9. Mukasa wishes to enclose a rectangular piece of land of area 1250 cm² whose one side is bound by a straight bank of a river. Find the least possible length of barbed wire required.
10. A closed right circular cylinder of base radius r cm and height h cm has volume of 54π cm³. Show that S, the total surface area of the cylinder, is given by $S = \frac{108\pi}{r} + 2\pi r^2$ hence find the radius and height which makes the surface area minimum.
11. A company that manufactures dog food wishes to pack the feed in closed cylindrical tins. What should be the dimensions of each tin if each is to have a volume of 250π cm³ and the minimum possible surface area?
12. A right circular cone of radius r cm has a maximum volume. The sum of its vertical height h and circumference of its base is 15 cm. If the radius varies, show that the maximum volume of the cone is $\frac{125}{3\pi}$ cm³

SMALL CHANGES

13. Given that $y = 3x^2 + 2x - 4$. Use small changes to find the small change in y when x increases from 2 to 2.02.
14. The radius of the circle increases from 5cm to 5.02cm. find the percentage increase in area of the circle
15. A cylinder of radius r and height $8r$. the radius increases from $4cm$ to $4.1cm$. find the approximate increase in volume

Approximating roots

16. use small changes to evaluate
    (i) $\sqrt[3]{28}$, $\sqrt{9.04}$, $\sqrt[3]{1003}$
    (ii) $sin30.5$, $cos47$, $sin56$, cot59.8°.

Percentages in small changes

17. An error of 3% is made in measuring the radius of the sphere. Find the percentage error in the volume
18. The height of a cylinder is 10 cm and the radius is 4 cm. Find the approximate percentage increase in the volume when the radius increases from 4 to 4.02 cm.
19. An error of 2.5% is made in measuring the area of a circle. What is the percentage error in the circumference?
20. The period T of a simple pendulum is calculated from the for $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length of the pendulum and g is the acceleration due to gravity constant. find the percentage change in the period caused by lengthening the pendulum by 2%.

Rates of change

21. A side of a cube is increasing at a rate of 6cm/s. Find the rate of increase in the volume of the cube when the length of the side is 8cm.
22. The volume of a cube is increasing at a rate of 2 cm³/s. Find the rate of change of the side of the base when the length is 3 cm.
23. The area of the circle is increasing at a rate of 3cm²/s. Find the rate of change of the circumference when its radius is 2cm.
24. A spherical balloon is inflated such that the rate at which its radius is increasing is 0.5cm/s. Find the rate at which:
25. the volume is increasing at the instant when $r = 5.0$cm
26. the surface area is increasing when $r = 8.5$ cm
27. The area of the circle is increasing at a rate of 3cm²/s. Find the rate of change of the circumference when its radius is 2cm

Rates in cones

28. A circular cone is held vertex downwards beneath a tap leaking at a rate of 2cm³/s. Find the rise of water level when the level is 6 cm. Given that the height of the cone is 18 cm and its radius is 12 cm.
29. An inverted cone with semi vertical angle of 30° is collecting water leaking from a tap at a rate of 2cm³/s. If the height of water collected is 10cm, find the rate at which the depth is decreasing at that instant.
30. An inverted right circular cone of vertical angle 120° is collecting water from a tap at a steady rate of 18π cm³/min. Find:
    (i)the depth of the water after 12 minutes
    (ii)the rate of increase of the depth at this instant.
31. A rectangular figure with sides $8cm$ by $5cm$, equal sides of $x$cm are removed from each corner and the edge are turned up to make an open box of volume Vcm³. Show that $V = 40x - 26x^2 + 4x^3$ and hence find the maximum possible volume and the value of x.