1. The function P is given by $P = \frac{a}{t} + bt^2$ and when $t = 1, P = -1$. ${dP}/{dt}$ when $t = -\frac{1}{2}$ is 5. Find the values of a and b.
2. Given that $L = mp^4 + np^2 + 3$. Find $\frac{dL}{dp}$.

   When $p = 1$, $\frac{dL}{dp} = 12$ and when $p = \frac{1}{4}$, $\frac{dL}{dp} = -\frac{3}{4}$. Find the values of m and n.
3. Find the points where the curve $y = 2x^2 + 3x - 5$ cuts the x −axis. Find the equations of the tangents to the curve at those points.

   Prove that the tangents meet together at the point $\left(-\frac{3}{4}, -\frac{49}{4}\right)$.
4. The line $y = 4x - 5$ cuts the curve $y = x^2 - 2x$ at two points.

   (a) Find the gradients of the curve at the two points.

   (b) Show that the tangent at one of the point is horizontal and find the equation of that tangent.
5. A curve has the equation $y = x^3 - px + q$. The tangent to this curve at the point $(2, -8)$ is parallel to the x −axis.

   (a) Find the values of p and q.

   (b) Find also the coordinates of the other point where the tangent is parallel to the x −axis.
6. The curve $y = \frac{a}{x} + bx$ passes through the points $A(1, -1)$ and $B(2, 4)$.

   (a) Find the values of a and b.

   (b) Show that the tangent to the curve at $x = \sqrt{2}$ is parallel to AB.
7. (a) Show that the gradients of the tangents to the curve $y = x^2 - x - 2$ where the curve meets the x −axis are numerically equal.

   (b) Find the equation of these tangents and show that they intersect on the axis of the curve.
8. The normal at the point $A(-1, 2)$ on the curve $y = 3 - x^2$ meets the curve again at B.

   (a) Find the equation of the normal at A and the coordinates of B.

   (b) Find the coordinates of the point C on the curve where the normal at A is parallel to the curve.
9. The tangent to the curve $y = x^2 + \frac{1}{x}$ at $(1, 2)$ meets the curve again at P.

   (a) Find the equation of the tangent.

   (b) Find the coordinates of P.

   (c) Find the equation of the tangent at P.
10. T is the tangent to the curve $y = x^2 + 6x - 4$ at the point $(1, 3)$ and N is the normal to the curve $y = x^2 - 6x + 18$ at $(4, 10)$. Find the coordinates of the point of intersection of T and N.

11. The normal to the curve $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$, where a and b are constants has the equation $4x + y = 22$ at the point where $x = 4$. Find the value of a and b.

12. L is the line $y = 4x - 2$ and C is the curve $y = mx^3 + nx^2 - 1$. The line L is tangent to the curve C at $x = 1$;
   (a) Using the fact that L and C meet at $x = 1$; show that $m + n = 3$.
   (b) Given that L is a tangent to C at $x = 1$; show that $3m + 2n = 4$.
   (c) Hence solve m and n.

13. The straight line $y = -x + 4$ cuts the parabola $y = 16 - x^2$ at the points A and B.
   (a) Find the coordinates of A and B.
   (b) Find the equation of the tangents A and B, and hence determine where the two tangents meet.

14. The velocity $v\ cms^{-1}$ of a particle moving in a straight line is given by $v = 8t - kt^2$ where k is a constant and $t\ s$ is the time from the start. If its acceleration is 0 when $t = 1$, find:
   (a) The value of k.
   (b) The time when the particle comes to instantaneous rest.
   (c) The maximum velocity of the particle.

15. The curve C has equation

$$y = x^3 - 5x + \frac{2}{x}, \quad x \neq 0.$$

The points A and B both lie on C and have coordinates $(1, -2)$ and $(-1, 2)$ respectively.
   (a) Show that the gradient of C at A is equal to the gradient of C at B.
   (b) Show that the equation of the normal to C at A is $4y = x - 9$.
   (c) The normal to C at A meets the x-axis at the point P. The normal to C at B meets the y-axis at the point Q. Find the length of PQ.

16. The curve C has equation $y = 4x^2 + \frac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.
   (a) Show that the value of $\frac{dy}{dx}$ at P is 3.
   (b) Find an equation of the tangent to C at P.
   (c) This tangent meets the x-axis at the point $(k, 0)$. Find the value of k.

17. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$. The point P has coordinates $(3, 0)$.
   (a) Show that P lies on C.
   (b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.
   (c) Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P. Find the coordinates of Q.

18. The curve $y = (x - 1)(x^2 - 4)$ cuts the x-axis at three points; P, $(1, 0)$ and Q.
   (a) Write down the x-coordinate of P and the x-coordinate of Q.
   (b) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$
   (c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.

(d) The tangent to the curve at the point R is parallel to the tangent at the point $(-1,\ 6)$. Find the exact coordinates of R.

19. A curve is defined by the equation;

$$y = \frac{(2x+1)(x+4)}{\sqrt{x}}, \quad x > 0$$

(a) Show that y can be written in the form $Px^{\frac{3}{2}} + Qx^{\frac{1}{2}} + Rx^{-\frac{1}{2}}$, stating the values of the constants P, Q and R.

(b) Find $\frac{dy}{dx}$.

(c) Show that the tangent to the curve at the point where $x = 1$ is parallel to the line with equation $2y = 11x + 3$.

20. A curve has the equation $y = x + \frac{3}{x}$, $x \neq 0$. The point P on the curve has x −coordinate 1.

(a) Show that the gradient of the curve at P is -2.

(b) Find an equation for the normal to the curve at P, giving your answer in the form $y = mx + c$.

(c) Find the coordinates of the point where the normal to the curve at P intersects the curve again.

21. The line $y = 2x + 1$ intersects curve $y = x^2 - 3x + 5$ at points P and Q.

(a) Using algebra, show that P has the coordinates $(1,\ 3)$ and find the coordinates of Q.

(b) Find the equations of the tangents to the curve at points P and Q.

(c) Find the coordinates of the point where the tangent to the curve at P intersects the tangent to the curve at Q.

22. The curve C is given by the equation $y = (x+2)^3$.

(a) Find $\frac{dy}{dx}$.

(b) A straight line l is tangent to C at the point $(-1,\ 1)$. Find the equation of l.

(c) The straight line m is parallel to l and is also a tangent to C. Show that m has the equation $y = 3x + 8$.

23. The curve $y = x^3 + 3x^2 - 4x$ crosses the x −axis at the origin O, and the points A and B.

(a) Find the coordinates of A and B

(b) A line l is tangent to the curve at O. Find the equation of l.

(c) Find the coordinates of the point where l intersects the curve again.

24. Given that;

$$y = \frac{(x-4)^2}{2x^{\frac{1}{2}}}, \quad x > 0.$$

(a) Find the values of the constants A, B and C such that

$$y = Ax^{3/2} + Bx^{1/2} + Cx^{-1/2}.$$

(b) Show that

$$\frac{dy}{dx} = \frac{(3x+4)(x-4)}{4x^{\frac{3}{2}}}.$$

25. A curve has the equation $y = \frac{x}{2} + 3 - \frac{1}{x}$, $x \neq 0$. The point A on the curve has x − coordinate 2.

(a) Find the gradient of the curve at A.

(b) Show that the tangent to the curve at A has equation $3x - 4y + 8 = 0$.

(c) The tangent to the curve at the point B is parallel to the tangent at A. Find the coordinates of B.

26. A curve has the equation $y = x^3 - 5x^2 + 7x$.

(a) Show that the curve only crosses the x −axis at one point.

(b) The point P on the curve has coordinates $(3,\ 3)$. Find an equation for the normal to the curve at P, giving your answer in the form $ax + by = c$, where a, b and c are integers.

(c) The normal to the curve at P meets the coordinate axes at Q and R. Show that the triangle OQR, where O is the origin, has area $28\frac{1}{8}$.

27. A curve has the equation $y = (\sqrt{x} - 3)^2$, $x \geq 0$.

(a) Show that $\frac{dy}{dx} = 1 - \frac{3}{\sqrt{x}}$

(b) The point P on the curve has x −cordinate 4. Find the equation of the normal at P in the form $y = mx + c$.

(c) Show that the normal to the curve at P does not intersect the curve again.

28. A curve is defined by the equation $y = 2 + 3x - x^2$. The line l is the tangent to the curve at the point A where the curve crosses the y −axis.

(a) Find an equation for l.

(b) The line m is the normal to the curve at the point B. Given that l and m are parallel, find the coordinates of B.

Examination questions

1. $\left(a = -\frac{4}{3}, b = \frac{1}{3}\right)$

2. $\frac{dL}{dp} = 4\,m\,p^3 + 2\,n\,p,\ (m = 4, n = -2)$

3. $\left(-\frac{5}{2},\ 0\right)$ *and* $(1,\ 0)$; $y = -7x - 17.5$,
$y = 7x - 7$

4. (*a*) *At* $(1, -1), \frac{dy}{dx} = 0$; *at* $(5,\ 15), \frac{dy}{dx} = 8$
(*b*) $y = -1$

5. (a) $(p = 12, q = 8)$
(*b*) $(-2,\ 24)$

6. (*a*) $(a = -4,\ b = 3)$

7. (*a*) 3 *and* -3
(*b*) *Tangents* $y = -3x - 3$,
$y = 3x - 6$;
intersection at $x = \frac{1}{2}$.

8. (*a*) $y = -\frac{1}{2}x + \frac{3}{2}$; $B\left(\frac{3}{2},\ \frac{3}{4}\right)$
(*b*) $\left(\frac{1}{4},\ \frac{47}{16}\right)$

9. (*a*) $y = x + 1$
(*b*) $P(-1,\ 0)$
(*c*) $y = -3x - 3$

10. $(2,\ 11)$

11. $(a = 2,\ b = 4)$

12. (*c*) $(m = -2,\ n = 5)$

13. (*a*) $A(-3,\ 7)$, $B(4,\ 0)$
(*b*) $y = 6x + 25$ *and* $y = -8x + 32$;
$\left(\frac{1}{2},\ 28\right)$

14. (*a*) $k = 4$
(*b*) $t = 2\ s$
(*c*) $v = 4\ cms^{-1}$

15. (*c*) $\overline{PQ} = \frac{9\sqrt{17}}{4}$

16. (*b*) $y = 3x + 5$
(*c*) $k = -\frac{5}{3}$

17. (*b*) $y = -7x + 21$
(*c*) $\left(5,\ -\frac{46}{3}\right)$

18. (*a*) $P(x = -2)$; $Q(x = 2)$
(*d*) $R\left(\frac{5}{3}, -\frac{22}{27}\right)$

19. (*a*) $P = 2,\ Q = 9,\ R = 4$
(*b*) $\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{9}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$

20. (*b*) $y = \frac{1}{2}x + \frac{7}{2}$
(*c*) $\left(6,\ \frac{13}{2}\right)$

21. (*a*) $Q(4,\ 9)$
(*b*) $y = 4 - x$; $y = 5x - 11$
(*c*) $\left(\frac{5}{2},\ \frac{3}{2}\right)$

22. (*a*) $\frac{dy}{dx} = 3x^2 + 12x + 12$
(*b*) $y = 3x + 4$

23. (*a*) $A(1, 0)$ $B(-4, 0)$
(*b*) $y = -4x$
(*c*) $(-3,\ 12)$

24. (*a*) $A = \frac{1}{2},\ B = -4,\ C = 8$

25. (*a*) $\frac{dy}{dx} = \frac{3}{4}$
(*c*) $B\left(-2,\ \frac{5}{2}\right)$

26. (*a*) *point* $(0,\ 0)$
(*b*) $x + 4y = 15$

27. (*b*) $y = 2x - 7$

28. (*a*) $y = 3x + 2$
(*b*) $B\left(\frac{5}{3},\ \frac{38}{9}\right)$