

From sine rule,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$ .

From L.H.S ~~b~~-

$$\frac{bc}{a(b+c)} = \frac{k^2 \sin B \sin C}{k \sin A (k \sin B + k \sin C)}$$

$$= \frac{\sin B \sin C}{\sin A \sin B + \sin A \sin C}$$

Dividing by  $\sin B \sin C$  in both numerator and denominator.

$$= \frac{1}{\sin A \left( \frac{1}{\sin C} + \frac{1}{\sin B} \right)}$$

$$= \frac{1}{\sin A (\operatorname{cosec} C + \operatorname{cosec} B)}$$

$$= \frac{1}{\sin A} \cdot \frac{1}{\operatorname{cosec} C + \operatorname{cosec} B}$$

Putting  $A = (\pi - (B + C))$

$$= \frac{1}{\sin(\pi - (B + C))} \cdot \frac{1}{\operatorname{cosec} C + \operatorname{cosec} B}$$

$$= \frac{1}{\sin(\pi - (B + C))} \cdot \frac{1}{\operatorname{cosec} C + \operatorname{cosec} B}$$

$$= \frac{1}{\sin(B+C)} \cdot \frac{1}{\operatorname{cosec} C + \operatorname{cosec} B}.$$

$$= \frac{\operatorname{cosec}(B+C)}{\operatorname{cosec} C + \operatorname{cosec} B} \quad \text{---} \quad \#$$