

TOPIC: ALGEBRA

Algebra is a part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations or Algebra is the type of mathematics that uses letters to represent numbers.

In algebra, we often use symbols to translate word phrases into algebraic expressions/mathematical expressions.

Note that:

A mathematical expression is made up of a co-efficient, a variable, an operator and a constant as shown in the equation $3k + 4 = 24$

Where 3 → The co-efficient k → A variable

$+$ → An operator 4 and 24 → Constants

A co-efficient is a numerical value that appears alongside a variable or a term in an equation or an expression.

A variable is any letter used to represent an unknown quantity/number in an expression or equation.

An operator is a mathematical symbol used in either an expression or equation. These symbols are $+$, $-$, \times and \div

A constant is a fixed value that doesn't change. A constant has a known value.

WORD PHRASES USED FOR EACH OF THE FOUR OPERATIONS IN ALGEBRA

1. Addition (+)

-Total -More than
-Altogether -Older than
-Increased by -Sum
-Add -Plus

3. Multiplication (x)

-Multiply -Cube of -Square of
-Product -Thrice -Twice
-Square of -Twice

2. Subtraction (-)

-Minus -Range -Take away
-Subtract -Deduct -Less than
-Difference -Decreased by

4. Division (÷)

-Quotient -Divide
-Share -Distribute

Mathematical phrases for the above four operations:

1. Sum of y and 4

$$= (y + 4)$$

6. 5 more than k

$$= (k + 5)$$

2. 8 less than p

$$= (P - 8)$$

3. Add p and q

$$= (P + q) \text{ or } (q + p)$$

4. Square the product of y and 4

$$= (4y)^2$$

5. Add 4 to $\frac{2}{3}$ of a number

Let the number be n.

$$= \frac{2}{3}n + 4$$

7. Square of x

$$= x^2$$

8. Divide r by 6 and add 5 to its results.

$$= \left(\frac{r}{6} + 5\right)$$

9. Subtract 5 from the product of p and 8

$$= (p \times 8) - 5$$

10. Divide the sum of k and 7 by 3

$$= \frac{k+7}{3}$$

General Activity

1. Square the sum of a and 8

2. Add 3 to a number and triple the results.

3. Subtract 8 from x and double the results.

4. Add 5 to p and multiply the results by 7.

5. The product of x and y

6. Add 5 to m and then square the result

7. The sum of twice of x and thrice y.

8. Multiply the difference between b and 6 by 8

9. 28 added to the sum of p and q

10. A half minus the sum of k and 9.

11. Multiply a by 4.

12. Add 9 to q.

13. Subtract h from 2.

14. The quotient of p and q.

15. Subtract k from 16

16. Square root of k

17. A half the sum of x and 4

Meaning of the algebraic expressions:

1. $a + b \rightarrow (a) + (b)$

2. $ab \rightarrow (a) \times (b)$

5. $a(b-c) \rightarrow a \times (b-c)$

6. $py^2 \rightarrow p \times y \times y$

7. $y^4 \rightarrow y \times y \times y \times y$

3. $ab + ac \rightarrow (a \times b) + (a \times c)$

4. $k^2 \rightarrow (k) \times (k)$

8. $7k^2 \rightarrow 7 \times k^2 \text{ or } 7 \times k \times k$

9. $x - y \rightarrow (x) - (y)$

10. $p^3 \rightarrow p \times p \times p$

SUBSTITUTION:

The word **Substitution** simply means to **replace**.

Example 1

1. Given that $a = 2$, $b = 3$ and $c = 4$.

Find the value of $a + b + c$.

$$a + b + c$$

$$2 + 3 + 4$$

$$= 7$$

Example 2

2. If $x = 4$ and $y = 7$. Find the value of:

(i) $3x + 4y$

$$(3 \times x) + (4 \times y)$$

$$(3 \times 4) + (4 \times 7)$$

$$12 + 16$$

$$= 28$$

(ii) $3y - 2x$

$$(3 \times y) + (2 \times x)$$

$$(3 \times 7) + (2 \times 4)$$

$$21 + 8$$

$$= 29$$

(iii) $\sqrt{x} + 3y$

$$\sqrt{x} + (3 \times y)$$

$$\sqrt{4} + (3 \times 7)$$

$$\sqrt{4} + 21$$

$$(2 + 21)$$

$$= 23$$

General activity:

1. Given $p = 3$, $q = 5$ and $r = 2$. Find the value of

(i) $pq + 2r$

(ii) $pq + r$

(iii) $p + q + r$

2. If $a = 5$, $b = 4$ and $c = 0$. Find the value of

(i) abc

(ii) $bc - a$

(iii) $a^2 + bc$

3. Given that $x = 2$ and $y = -3$. Find the value of

(i) $y + x$

(ii) $y^4 + 2x$

(iii) $x^2 + 4y$

4. If $m = 5$, $n = m$ and $p = -2$. Find the value of

(i) mnp

(ii) $\frac{m+n+p}{2}$

(iii) $\frac{pm-3}{2n}$

5. Given that $m = 5$, $n = 6$ and $p = 3m$. Work out the value of

(i) $2m + p^2$

(ii) $mp + n$

(iii) np^2

6. Given that $a = 3$, $b = 7$. Find the value of

(i) $2a + 2b$

(ii) $2b - 3a$

(iii) $a + b$

7. If $x = 4$, $y = -2$ and $z = 3$. Find the value of

(i) $x + z$

(ii) xyz

(iii) $\frac{y \times y \times z}{x}$

8. Given that $n = 4$ and $t = 3$. Find the value of $\frac{3n - t}{t}$

9. Given that $x = \frac{3}{4}$ and $y = \frac{1}{3}$. Find the value of $x + y$

COLLECTING AND SIMPLIFYING ALGEBRAIC TERMS:

Example 1	Example 2	Example 3
a) $5q - 2r + 3q - r$ $(5q + 3q) - 2r - r$ $8q - 3r$	(b) $2 - 3(p - 2)$ $2 - 3p + 6$ $2 + 6 - 3p$ $8 - 3p$	c) $3 + 4(m - 3)$ $3 + 4m - 12$ $3 - 12 + 4m$ $(-9 + 4m)$ or $(4m - 9)$

General Activity:

Simplify the following:

a) $6k + 3k + 6k$

b) $2p + 3p - p$

c) $3p - 6q - p + 2q$

d) Subtract $-5m$ from m

e) $2ab + ab + 5ab$

f) $-2b + 6b - b - 3a$

g) $2p + 3p - p$

h) $3m - p - 4m - 3p$

i) $a + 2b + 3a - b$

j) $3k^2 - k + 6k - k^2$

k) $7xy^2 - 2xy + 3xy^2 + 5x$

l) Subtract $2(x + 3)$ from $3(x + 1)$

m) $2(q - 1) + 3(q - 2)$

n) Subtract $2(x + 3)$ from $3(x + 1)$

o) Subtract $3p - 1$ from $5p - 3$

p) Simplify $\frac{1}{3}(6k - 9p) + \frac{2}{5}(15k + 25p)$

q) Subtract $\frac{2}{3}(6m + 9k)$ from $\frac{3}{4}(8m + 12k)$

r) Simplify: $\frac{1}{2}(4q - 6p) + \frac{1}{2}(6q - 9p)$

s) Subtract $p - 1$ from $2p + 2$

t) Simplify: $\frac{1}{2}(4q - 6p) + \frac{1}{2}(6q - 9p)$

u) Subtract $\frac{2}{3}(6m + 9k)$ from $\frac{3}{4}(8m + 12k)$

FACTORISING EXPRESSIONS COMPLETELY:

1. Factorize $4xy - 12y$ completely.

2	$4xy$	$-12y$
2	$2xy$	$-6y$
y	xy	$-3y$
	x	-3

$$2 \times 2 \times y(x-3)$$

$$4 \times y(x-3)$$

$$\mathbf{4y(x-3)}$$

4. Factorize $3a + 12ab - 18$ completely.

3	$3a$	$+12ab$	-18
	a	$+4ab$	-6

$$\mathbf{3(a+4ab-6)}$$

2. Factorize $2xy^2 + 2xyz$ completely.

2	$2xy^2$	$+2xyz$
x	xy^2	$+xyz$
y	y^2	$+yz$
	y	$+z$

$$2 \times x \times y(y+z)$$

$$\mathbf{2xy(y+z)}$$

3. Factorize $4mn - 18mn^2$

2	$4mn$	$-18mn^2$
m	$2mn$	$-9mn^2$
n	$2n$	$-9n^2$
	2	$-9n$

$$2 \times m \times n(2-9n)$$

$$\mathbf{2mn(2-9n)}$$

EQUATIONS:

An equation is a mathematical sentence with an equal sign to show that two expressions give the same value.

SOLVING EQUATIONS BY SUBTRACTING:

Examples 1

Solve: $k + 4 = 23$

Solution:

$$k + 4 = 23$$

$$k + 4 - 4 = 23 - 4$$

$$k = 19$$

Example 2

Solve: $12 = k + 7$

Solution:

$$12 = k + 7$$

$$12 - 7 = k + 7 - 7$$

$$5 = k$$

$$k = 5$$

Example 3

Solve:

Solution:

$$6 + k = 17$$

$$6 - 6 + k = 17 - 6$$

$$k = 11$$

Solve: $k + 23 = 5$

$$k + (2 \times 2 \times 2) = 5$$

$$k + 8 = 5$$

$$k + 8 - 8 = 5 - 8$$

$$k = -3$$

Activity

1. Solve:

a) $m + 5 = 16$

b) $k + 9 = 25$

c) $y + 3 = 7$

d) $m + 10 = 7$

2. Solve:

a) $4 + k = 8$

b) $6 + t = 9$

c) $12 + y = 7$

d) $52 + e = 35$

e) $2 \times 6 + b = 14$

3. Solve:

a) $5 + g = 9$

b) $6 + k = 19$

c) $32 + y = 15$

d) $22 + m = 10$

4. Solve:

a) $12 = 8 + w$

b) $8 = 5 + d$

c) $7 = 8 + k$

d) $9 = 23 + e$

WORD PROBLEMS INVOLVING SOLVING EQUATIONS BY SUBTRACTING:

Example 1

I think of a number, add 7 to it, the result is 12. What is the number?

Let the number be m

$$m + 7 = 12$$

$$m + 7 - 7 = 12 - 7$$

$$m = 5$$

The number is 5

Example 2:

Derrick had some oranges, his friend gave him 13 more oranges. He had 21 oranges altogether. How many oranges did Derrick have at first?

Let the number of oranges be y

$$y + 13 = 21$$

$$y + 13 - 13 = 21 - 13$$

$$y = 8$$

Derrick had 8 oranges at first

Activity:

1. I think of a number, add 4 to it and the result is 9. Find the number.
2. The sum of two numbers is 20. One of the numbers is 12. Find the other number
3. Mr. Lizard is 12 years old. After how many years will he be 21 years?
4. In primary five class of 36 pupils, 21 are girls and the rest are boys. Find the number of boys in the class.
5. I think of a number, add 6 to it and the answer is 16. What is the number?
6. If the sum of $2y$ and 7 is 31. Find the value of x

SOLVING EQUATIONS INVOLVING ADDITION:

Example 1

Solve: $k - 9 = 7$

$$k - 9 + 9 = 7 + 9$$

$$k = 16$$

Example 3

Solve for m: $4m - 5 = 15$

$$4m - 5 + 5 = 15 + 5$$

$$4m = 20$$

$$\frac{4m}{4} = \frac{20}{4}$$

$$m = 5$$

Example 2

Solve: $6 + y = 21$

$$6 - 6 + y = 21 - 6$$

$$y = 15$$

Example 4

Solve for x: $2x - 4 = 16$

$$2x - 4 + 4 = 16 + 4$$

$$2x = 20$$

$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

Activity:

Solve the following equations

1. $x - 2 = 8$

2. $2m - 1 = 9$

3. $-2k - 24 = 8$

4. $y - 2 = 17$

5. $-3y - 17 = 13$

6. $p - 4 = 11$

7. When 12 is subtracted from a number, the answer is 8. What is the number?

8. Think of a number, take away 7 from it and the result is 9. What is the number?

SOLVING EQUATIONS BY DIVIDING:

Procedures:

- Study the equation.
- Divide both sides by the same value.
- Simplify correctly.
- The quotient is the answer

Example 1:

Solve for k.

$$3k = 21$$

$$3k = 21$$

$$\frac{3k}{3} = \frac{21}{3}$$

$$k = 7$$

Example 2:

Solve for m.

$$5m = 70$$

$$5m = 70$$

$$\frac{5m}{5} = \frac{70}{5}$$

$$m = 14$$

Example 3:

Solve: $-p = 10$

$$-p = 10$$

$$\frac{-p}{-1} = \frac{10}{-1}$$

$$p = -5$$

Example 4:

Solve: $-12m = 6$

$$-12m = 6$$

$$\frac{-12m}{-12} = \frac{6}{-12}$$

$$m = -\left(\frac{1}{6}\right)$$

Activity:

Solve for the unknown letter.

1. $12g = 8$

6. $4p = 24$

11. $5w = 15$

2. $6d = 36$

7. $10p = 100$

12. $3a = 156$

3. $15x = 135$

8. $8m = 16$

13. $13n = 18$

4. $-k = -6$

9. $-15p = -5$

14. $-12m = 18$

5. $-9r = -18$

10. $-14y = -7$

15. $3t = -18$

SOLVING EQUATIONS BY MULTIPLYING:

Procedures.

- Study the equation.
- Multiply either side by the given denominator.

- Divide where need be.
- Simplify correctly.

Example 2

Solve for p: $\frac{p}{2} = 8$

$$\frac{p}{2} = 8$$

$$2 \times \left(\frac{p}{2}\right) = 8 \times 2$$

$$\mathbf{p = 16}$$

Example 2

Solve for y: $\frac{2y}{3} = 10$

$$\frac{2y}{3} = 10$$

$$3 \times \left(\frac{2y}{3}\right) = 10 \times 3$$

$$2y = 30$$

$$\frac{2y}{2} = \frac{30}{2}$$

$$\mathbf{y=15}$$

Example 3

Solve for k: $\frac{2}{5}k = 6$

$$5 \times \left(\frac{2}{5}k\right) = 6 \times 5$$

$$2k = 30$$

$$\frac{2k}{2} = \frac{30}{2}$$

$$\mathbf{k = 15}$$

General Activity.

Solve these equations.

1. $\frac{p}{3} = 9$

2. $\frac{x}{4} + 7 = 19$

3. $\frac{m}{2} + 3 = 15$

4. $\frac{m}{2} = 12$

5. $\frac{r}{5} - 7 = 3$

6. $\frac{2y}{3} - 7 = 3$

5. $\frac{2h}{5} + 6 = 16$

6. $\frac{1}{2}p = 13$

7. $\frac{y}{3} = 9$

SOLVING EQUATIONS INVOLVING SQUARE ROOTS:

Example 1

Solve: $\frac{3}{9}m^2 = 3$

$$9 \times \frac{3}{9}m^2 = 3 \times 9$$

$$3m^2 = 27$$

$$\frac{3}{3}m^2 = \frac{27}{3}$$

$$m^2 = 9$$

$$\sqrt[2]{m^2} = \sqrt[2]{9}$$

$$\mathbf{m = 3}$$

Example 2

Simplify: $x^2 - 3 = 22$

$$x^2 - 3 + 3 = 22 + 3$$

$$x^2 = 25$$

$$\sqrt[2]{x^2} = \sqrt[2]{25}$$

$$\mathbf{x = 5}$$

Example 3

Solve: $g^2 = 16$

$$\frac{\quad}{4}$$

$$4 \times g^2 = 16 \times 4$$

$$\frac{\quad}{4}$$

$$g^2 = 64$$

$$\sqrt[2]{g^2} = \sqrt[2]{64}$$

$$\mathbf{g = 8}$$

General Activity.

Solve the following equations

a) $p^2 + 3 = 11$

b) $4(k^2 - 1) = 21$

c) $m^2 + 4 = 40$

d) $p^2 + 3 = 11$

e) $2(2p^2 - 15) = 9$

f) $\frac{p^2}{3} = 27$

g) $\frac{2g^2 - 4}{3} = 20$

h) $9x^2 = \frac{16}{9}$

i) $y^2 + 4 = 20$

j) $q^3 - 3 = 46$

k) $y^2 + 1 = 101$

l) $3(t^2 - 2) = 42$

m) $\frac{m^2}{2} - 1 = 7$

n) $\frac{3^2}{k} = 2k$

k) $x^2 = 16$

l) $3m^2 = 75$

m) $2p \times 3p = 294$

n) $2k^2 + 2 = 20$

o) $2(g^2 - 20) = 32$

p) $\frac{r^2}{5} + 6 = 26$

q) $\frac{3t^2 + 8}{4} = 20$

SOLVING EQUATIONS INVOLVING CUBE ROOTS:

Example 1

Solve: $\frac{m^3}{2} + 3 = 35$

$\frac{m^3}{2} + 3 = 35$

$\frac{m^3}{2} + 3 - 3 = 35 - 3$

$\frac{m^3}{2} = 32$

$2 \times \frac{m^3}{2} = 32 \times 2$

$m^3 = 64$

$\sqrt[3]{m^3} = \sqrt[3]{64}$

$m = 4$

Example 2

Simplify: $p^3 - 3 = 24$

solution:

$p^3 - 3 = 24$

$p^3 - 3 + 3 = 24 + 3$

$p^3 = 27$

$\sqrt[3]{p^3} = \sqrt[3]{27}$

$p = 3$

Example 3

Solve: $\frac{g^3}{4} = 16$

$4 \times \frac{g^3}{4} = 16 \times 4$

$g^3 = 64$

$\sqrt[3]{g^3} = \sqrt[3]{64}$

$g = 4$

General Activity:

1. $m^3 = 8$

2. $y^3 = 1000$

3. $m^3 + 7 = 15$

4. $r^3 + 3 = 30$

5. $K^3 = 216$

6. $q^3 = 27$

7. $y^3 + 5 = 130$

8. $2w^3 = 128$

9. $p^3 = 64$

10. $x^3 = 125$

11. $\underline{5g^3} - 18 = 14$

12. $3m^3 + 4 = 14$

SOLVING EQUATIONS INVOLVING BRACKETS:**Example 1:**

1. Solve. $3(x - 2) = x + 4$

$3x - 6 = x + 4$

$3x - x = 4 + 6$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Example 2:

Simplify: $3 + 2(m - 1) = 8$

$3 + (2 \times m) + (2 \times -1) = 8$

$3 + 2m - 2 = 8$

$3 - 2 + 2m = 8$

$1 + 2m = 8$

$1 - 1 + 2m = 8 - 1$

$2m = 7$

$\frac{2m}{2} = \frac{7}{2}$

$m = 3\frac{1}{2}$

General Activity.

a) $6(w - 4) = 0$

b) $6(k-2)+3(k+1)=0$

c) $(p - 3) + (p - 4) = 1$

d) $(2p - 5) - (p + q) = 12$

e) $3(y - 1) = 21$

f) $5(k - 4) = 50$

g) $3 - (-p - 1) = 6$

h) $2(k - 2) - 3(k - 4) = -10$

r) $2(m+4) = 20$

i) $5(n + 4) = 30$

j) $5(3 - 4k) - 8(2k + 4) = 19$

k) $7(2r - 5) - (r - + 8) = -17$

l) $5(t - 2) - 3(t - 4) = 14$

m) $5(a - 4) + 3 + 2(a - 3) = 33$

n) $3(x - 3) = 21$

o) $3(3m - 1) - 6(m - 2) = 24$

p) $7(3m - 1) - 11(m + 1) = 12$

q) $3(3x - 1) - 6(x - 2) = 24$

MORE EQUATIONS PART ONE:

Example 1:

Solve: $3k - 4 - k - 3 = 1$

Solution:

$$3k - 4 - k - 3 = 1$$

$$3k - k - 4 - 3 = 1$$

$$2k - 7 = 1$$

$$2k - 7 + 7 = 1 + 7$$

$$2k = 8$$

$$\frac{2K}{2} = \frac{8}{2}$$

$$\mathbf{k = 4}$$

Example 2:

Solve: $2m - 4 + m + 5 + 3m - 8 = 23$

Solution:

$$2m - + m + 5 + 3m - 8 = 23$$

$$2m + m + 3m + 5 - 4 - 8 = 23$$

$$6m - 7 = 23 + 7$$

$$6m = 30$$

$$\frac{6m}{6} = \frac{30}{6}$$

$$\mathbf{m = 5}$$

General Activity:

a) $5m + 5 - 3m + 3 = 14$

b) $4p - 1 - 2p + 2 = 12$

c) $3p - 6 - 2p + 10 = 7$

d) $4w + 8 + 3q - 3 = 12$

e) $3r - 6 - 2r + 10 = 7$

f) $4q - 4 - 3q + 6 = 4$

g) $4n + 2 + 2n + 12 = 44$

h) $2m + 2 + 2m - 16 = 28$

i) $2y + 8 + 2y - 12 = 56$

j) $6k + 6 - k + 2 = 13$

k) $5g + 5 + 2g + 16 = 16$

l) $5t - 5 - t + 21 = 0$

m) $3y - 6 + 2y - 2 = 2$

n) $n - 2 + n - 4 = 0$

o) $5y - 5 - 3y + 9 = 20$

p) $2x - 3 + x + 6 + 2x + 4x - 5 = 43$

q) $4w + 2w + 12 = 24$

r) $3k + 1 + 2k - 8 + k + a = 59$

MORE EQUATIONS PART TWO:

Example 1:

Solve: $2k - 7 = k + 1$

Solution:

$$2k - 7 = k + 1$$

Example 2:

Solve: $2(y + 4) = y + 10$

Solution:

$$2(y + 4) = y + 10$$

$$2k - 7 = k + 1$$

$$2k - k - 7 + 7 = k - k + 1 + 7$$

$$k = 1 + 7$$

$$\mathbf{k = 8}$$

$$2y + 8 = y + 10$$

$$2y - y = 10 - 8$$

$$\mathbf{y = 2}$$

Example 1:

$$\text{Solve: } 3(p - 2) = 2(p - 1)$$

Solution:

$$3(p - 2) = 2(p - 1)$$

$$3p - 6 = 2p - 2$$

$$3p - 2p = -2 + 6$$

$$\mathbf{p = 4}$$

Example 2:

$$\text{Solve: } 4 - 2(m - 1) = 2(m + 5)$$

Solution:

$$4 - 2(m - 1) = 2(m + 5)$$

$$4 - 2m + 2 = 2m + 10$$

$$4 + 2 - 2m = 2m + 10$$

$$6 - 2m = 2m + 10$$

$$-2m - 2m = 10 - 6$$

$$-4m = 4$$

$$\frac{-4m}{-4} = \frac{4}{-4}$$

$$\mathbf{m = -1}$$

General Activity:

a) $5m + 5 - 3m + 3 = 14$

b) $4p - 1 - 2p + 2 = 12$

c) $3p - 6 - 2p + 10 = 7$

d) $4w + 8 + 3q - 3 = 12$

e) $3r - 6 - 2r + 10 = 7$

f) $4q - 4 - 3q + 6 = 4$

g) $4n + 2 + 2n + 12 = 44$

h) $2m + 2 + 2m - 16 = 28$

i) $2y + 8 + 2y - 12 = 56$

j) $6k + 6 - k + 2 = 13$

k) $5g + 5 + 2g + 16 = 16$

l) $5t - 5 - t + 21 = 0$

m) $3y - 6 + 2y - 2 = 2$

n) $n - 2 + n - 4 = 0$

o) $5y - 5 - 3y + 9 = 20$

p) $2x - 3 + x + 6 + 2x + 4x - 5 = 43$

q) $4w + 2w + 12 = 24$

r) $3k + 1 + 2k - 8 + k + a = 59$

SOLVING EQUATIONS INVOLVING FRACTIONS:

Example 1:

Simplify: $\frac{2}{3}(x - 1) = 5$

$$3 \times \frac{2}{3}(x - 1) = 5 \times 3$$

$$2(x - 1) = 5 \times 3$$

$$2x - 2 + 2 = 15 + 2$$

$$2x = 17$$

$$\frac{2x}{2} = \frac{17}{2}$$

$$x = 8\frac{1}{2}$$

Example 2:

Solve: $\frac{4-2y}{3}$

$$3 \times \left(\frac{4-2y}{3}\right) = 3 \times (y + 1)$$

$$4 - 2y = 3(y + 1)$$

$$4 - 2y = 3y + 3$$

$$4 - 3 = 3y + 2y$$

$$1 = 5y$$

$$\frac{5y}{5} = \frac{1}{5}$$

$$y = \frac{1}{5}$$

Example 3

Simplify: $0.2(m-1) = 4$

$$\frac{2}{10}(m - 1) = 4$$

$$10 \times \frac{2}{10}(m - 1) = 4 \times 10$$

$$2(m - 1) = 40$$

$$2m - 2 = 40$$

$$2m - 2 + 2 = 40 + 2$$

$$2m = 42$$

$$\frac{2m}{2} = \frac{42}{2}$$

$$m = 21$$

Example 4:

Solve: $\frac{2m-2}{3} = \frac{m+1}{2}$

$$6 \times \left(\frac{2m-2}{3}\right) = 6 \times \left(\frac{m+1}{2}\right)$$

$$2(2m - 2) = 3(m + 1)$$

$$4m - 4 = 3m + 3$$

$$4m - 3m = 3 + 4$$

$$m = 7$$

3. $\frac{3n-2}{2} - \frac{2n+4}{3} = n + 1$

4. $\frac{2p}{5} + p = 7$

5. $\frac{2k}{3} + 8 = 6$

6. $4x + 7 - \frac{4x}{7} = 31$

9. $\frac{m+1}{3} + \frac{m}{4} = 2$

10. $\frac{p-5}{3} - p = \frac{2p+4}{5}$

11. $3x + 7 - \frac{3x}{4} = 10$

12. $2p - 5 - \frac{3n}{5} = 10$

General Activity:

1. $\frac{2y-1}{3} = y + 1$

2. $\frac{1}{3}(2x - 1) = 3$

7. $3x - 7 + \frac{3}{4} = 23$

8. $\frac{3x+1}{9} = \frac{11x-8}{7}$

13. $\frac{y-4}{y-2} = \frac{2}{3}$

14. $\frac{4x-5}{7} = \frac{5x-4}{11}$

$$15. \frac{1}{3}(x - 2) = \frac{1}{3}$$

$$16. \frac{y - 3}{4} = \frac{x + 2}{9}$$

$$17. \frac{3}{9}n + 6 = 2 + n$$

$$28. \frac{3m + 1}{4} = \frac{m + 3}{2}$$

$$19. \frac{3x + 1}{4} = \frac{x + 2}{2}$$

$$20. \frac{13y}{3} + 2 = 15$$

$$21. \frac{3m}{4} = 9$$

$$22. \frac{x - 3}{3} = \frac{x + 3}{5}$$

$$23. \frac{y - 2}{7} = \frac{4 - 8y}{28}$$

$$24. \frac{k - 1}{2} + \frac{y}{5} = 3$$

$$25. \frac{10}{k} + 4 = 24$$

$$26. m - \frac{2p}{3} = 7$$

$$27. \frac{3x - 1}{4} = \frac{7x + 1}{6}$$

$$28. \frac{2k}{5} + k = 12$$

$$29. \frac{2q}{3} - q = 5$$

$$30. \frac{3y - 8}{4} = \frac{2y - 3}{5}$$

$$31. \frac{x - 11}{3} = \frac{x - (-1)}{5}$$

$$32. \frac{x + 5}{5} + \frac{x}{5} = 5$$

$$33. \frac{x + 3}{3} = \frac{5x + 1}{9}$$

$$34. \frac{4}{p + 1} = \frac{3}{p - 3}$$

$$35. \frac{x + 6}{8} + \frac{x}{4} = 3$$

$$36. \frac{x + 2}{9} = \frac{x + 4}{11}$$

$$37. \frac{x - 5}{2} + \frac{x}{8} = 5$$

$$38. \frac{x + 2}{2} = \frac{4x - 4}{11}$$

APPLICATION OF ALGEBRA USING RATIOS PART ONE:

Example 1:

Joyce is 12 years older than Alice. If the ratio of their age is 5:8 respectively. Find their age.

Let the age of Alice be represented by letter m

Alice Joyce Ratio

m (m + 12) 5 : 8

m:(m+12) = 5:8

$$\frac{m}{m+12} = \frac{5}{8}$$

$$8m - 5m = 60$$

$$3m = 60$$

$$\frac{3m}{3} = \frac{60}{3}$$

$$m = 20$$

Joyce's age = (m+12)

(20+12)

= 32 years

Alice is 20 years old.

Example 2:

Roy is 18 years younger than John and the ratio of their age is 2:3 respectively. How old is each now?

Let the age of John be represented by letter y

John	Samuel	Ratio	$2y = 3y - 54$	Samuel = (y-18)
y	(y - 18)	(3 : 2)	$-y = -54$	(54 - 18)
$y : (y-18) = 3:2$			$\frac{-y}{-1} = \frac{-54}{-1}$	= 36 years
$\frac{y}{y-18} = \frac{3}{2}$			$y = 54$	
$(2 \times y) = 3(y-18)$			John= 54 years	

General Activity:

1. Chekwemoin is 12 years younger than Chelangot. If their age is in the ratio of 4:3 respectively, how old is each?

2. Owino is 6 years older than Okumu. The ratio of their age is 3:4 respectively.

a) How old is Okumu?

b) How old will Owino be in 7 years' time?

3. Tom is 8 years younger than Andrew. How is old each one of them given that their age is in the ratio of 3:5 respectively?

4. A mother is 24 years older than her son. If the ratio of their age is 5:2 respectively, how was each 6 years ago?

5. Ogot is 10 years older Opot. If the ratio of their age is 3:2.

a) How old is Opot?

b) How old was Ogot 12 years ago?

APPLICATION OF ALGEBRA USING RATIOS PART TWO:

Tom is 8 years older than Kamya. Four years ago, the ratio of their age was 4:3 respectively. How old is Kamya now?

Let the age of Kamya be represented by letter k.

$(k-4) : (k+8-4) = 3:4$	$4k-3k = 12+16$
$\frac{k-4}{k+4} = \frac{3}{4}$	$k = 28$
$4(k-4) = 3(k+4)$	$k = 28$
$4k-16 = 3k+12$	Kamya is 28 years old.

Example 2:

The age of Musa and that of Angella are in the ratio of 7:9 respectively. Five years ago, their total age was 54 years. Find Musa's age now.

Let the common term be represented by letter x

$(7x-5) + (9x-5) = 54$	$16x = 64$
$(7x + 9x) - 5 - 5$	$\frac{16x}{16} = \frac{64}{16}$
$16x - 10 = 54$	$x = 4$
$16x - 10 + 10 = 54 + 10$	Musa is 28 years

General Activity:

1. Alice is 12 years older than Joan. Eight years ago, the ratio of their age was 10:7 respectively.

a) How old is Joan now? b) How old will Alice be in 2 years' time?

2. Jan Van is 3 years older than Benjamin. In five years' time, the ratio of their age will 4:5 respectively. How old is Benjamin?

3. Nikki is 4 years younger than Mary. In six years' times, the ratio of their age will be 9:7 respectively. How old is Nikki?

4. The heights of two boys are in the ratio of 2:3. If the height of the shorter boy is 108cm. What is the difference in the heights of the two boys?

5. Walter and Washington shared some money in the ratio of 5:8. Walter got sh.12,000 less than Washington. How much money did Washington get?

SOLVING SIMPLE WORD PROBLEMS INVOLVED IN ALGEBRA (PART ONE):**Example 1:**

Kiplagat had some mangoes and his brother added him more 5 mangoes. If he got 12 mangoes in total, how many mangoes did have at first?

Let the number of mangoes he had at first be represented by letter m.

$m + 5 = 12$	Kiplagat had 7 had 7 mangoes at first.
$m + 5 - 5 = 12 - 5$	
$m = 7$	

Example 2:

Think of a number, multiply it by 3 and the answer is 12. What is the number?

Let the number be represented by letter m

$$m \times 3 = 12$$

$$3m = 12$$

$$\frac{3m}{3} = \frac{12}{3}$$

$$m = 4$$

Example 3:

What number is divided by 3 to give 5?

Let the number be represented by letter n.

$$\frac{n}{3} = 5$$

$$3 \times \frac{n}{3} = 5 \times 3$$

$$n = 15$$

The number is 15

Example 4:

Think of a number, subtract 5 from it and the answer is 2. What is the number?

Let the number be represented by letter y.

$$y - 5 = 2$$

$$y - 5 + 5 = 2 + 5$$

$$y = 7$$

The number is 7

Example 4:

James thought of a number, multiplied it by 4 and the product was 20. What was the number?

Let the number be represented by letter k.

$$k \times 4 = 20$$

$$4k = 20$$

$$\frac{4k}{4} = \frac{20}{4}$$

$$k = 5$$

The number is 5

General Activity:

1. What number is divided by 2 and it gives 7 as the remainder?
2. Find the number which Roy added to 12 to get 25.
3. The sum of a number and 7 is 12. What is the number?
4. What number is divided by 2 and it gives 7 as the remainder?
5. Find the number which Roy added to 12 to get 25.
6. The sum of a number and 7 is 12. What is the number?

7. The product of x and 7 is 21. Find x

8. Ogot has 7 more goats than Joy. Altogether they have 47 cows. How many cows does each one have?

9. A boy is 5 years older than his sister. Their total age is 19 years. Find their ages.

10. Oscar got 6 more books than his brother Mike. Altogether they got 24 books. How many books did Mike get?

11. Reagan is 8 younger than Roy. Their total age is 30 years. How old is Roy?

12. Kato has x pens, Peter has 2x pens and John has 9pens. If the total number of pens which they have is 18. How many pens has Kato?

FORMING AND SOLVING EQUATIONS (PART ONE):

Example 1:

Peter is 5 years older than his sister Apio. Their total is 27 years

a) How old is each now?

Let the age of Apio be represented by letter k.

Apio	Peter	Total age
k	K + 5	27

$$K + k + 5 = 27$$

$$2k + 5 = 27$$

$$2k + 5 - 5 = 27 - 5$$

$$2k = 22$$

$$\frac{2k}{2} = \frac{22}{2}$$

$$k = 11$$

Apio is 11 years old.

Peter is (k+5)

(11 + 5) = 16 years old.

b) How old will Peter be in 7 years' time?

Peter's age now = 16 years

In 7 years' time = (16 + 7) years

= 23 years

Example 2:

A daughter is 18 years younger than her mother. Their total age is 58 years.

a) How old is the mother now?

Let the daughter's age be represented by letter m.

Daughter	Mother	Total age
m - 18	m	58

$$m - 18 + m = 58$$

$$m + m - 18 = 58$$

$$2m - 18 = 58$$

$$2m - 18 + 18 = 58 + 18$$

$$2m = 76$$

$$\frac{2m}{2} = \frac{76}{2}$$

$$m = 38$$

Mother's age = (m)
= 38 years.

b) How old will the daughter be in 10 years to come?

Daughter's age now = (m - 18)

$$(38 - 18)$$

$$= 20 \text{ years}$$

Daughter's age in 10 years' time

$$(20 + 10)$$

$$= 30 \text{ years}$$

Example 3:

A father is 12 years older than his son now. In 5 years' time, their total age will be 76 years. a) How old is each of them now?

Let the son's age now be represented by letter y.

	Son	Father	Total age
Now	y	y + 12	
In 5 years' time	y + 5	y + 12 + 5	76

$$y + 5 + y + 12 + 5 = 76$$

$$y + y + 5 + 12 + 5 = 76$$

$$2y + 22 = 76$$

$$2y + 22 - 22 = 76 - 22$$

$$2y = 54$$

$$\frac{2y}{2} = \frac{54}{2}$$

$$y = 27$$

Son's age now = 27 years

Father's age now = (y + 12) years

$$(27 + 12)$$

$$= 39 \text{ years.}$$

b) How old will the father be then?	Father's age then
Father's age now = $(y + 12)$	$(39 + 5)$
$(27 + 12)$	= 44 years
= 39 years	

Example 4:

7. Cynthia is twice as old as Joseph. Their total age is 24 years.

a) How old is John?

Let the Joseph's age be represented by letter g.

Joseph	Cynthia	Total age
g	2g	24

$$g + 2g = 24$$

$$3g = 24$$

$$\frac{3g}{3} = \frac{24}{3}$$

$$g = 8$$

Joseph is 8

b) How old will Cynthia be in 21 years to come?

Cynthia's age now = $(2g)$	Cynthia's age in 21 years to come.
(2×8)	$(16 + 21)$
= 16 years	= 37 years

Example 5:

Akello is 3 times as old as Awino. The difference in their age is 30 years. How old is each of them now?

Let the age of Awino be represented by letter k.

Awino	Akello	Age difference	$\frac{2k}{2} = \frac{30}{2}$
k	3k	30	k = 15
Age difference = 30			Awino = 15 years
$3k - k = 30$			Akello = (3×15)
$2k = 30$			= 45 years

General Activity:

1. A mother is 18 years older than her son. If their total age is 52 years, how old is the son now?
2. Amoiti is 8 years younger than Mary. Their total age is 34 years.
a) How old is each now? b) How old will Amoiti be in 5 years' time?
3. Lucy is 5 years older than the Dominica. In 7 years' time, their total age will be 33 years.
a) How old is the Dominica now? b) How old will the Lucy be then?
4. John is 4 years older than Mary. In 20 years' time, their total age will be 80 years. How old is John?
b) How old will John be in 20 years' time?
5. A mother is three times as old as her daughter. In seven years' time, their total age will be 74 years.
a) How old is each now? b) Find the difference in their age in seven years' time.
6. Jane is twice as old as Sarah. If their total age is 30 years, how old is Sarah?
7. If Tony is thrice as old as Annet. If their total age is 44 years. How old is Annet?
8. A mother is 18 years older than her son. If their total age is 52 years, how old is the son now?
9. John is 4 years older than Mary. In 20 years' time, their total age will be 80 years. How old is John?
b) How old will John be in 20 years' time?
10. Cynthia is twice as old as Joseph. Their total age is 24 years.
a) How old is John? b) How old will Cynthia be in 21 years to come?
12. Jane is twice as old as Sarah. If their total age is 30 years, how old is Sarah?
13. If Tony is thrice as old as Annet. If their total age is 44 years. How old is Annet?
14. The deserter is 5 years older than the infantry. Their total age will be 33 years in 7 years' time.
a) How old is the Infantry now? b) How old will the Infantry be then?

APPLICATION OF ALGEBRA USING RATIOS:

Example 1:

Joyce is 12 years older than Alice. If the ratio of their age is 5:8 respectively. Find their age.

Let the age of Alice be represented by letter m

Alice	Joyce	Ratio
m	m + 12	5 : 8

$$m : (m + 12) = 5 : 8$$

$$\frac{m}{(m+12)} = \frac{5}{8}$$

$$8(m) = 5(m+12)$$

$$8m = 5m+60$$

$$8m - 5m = 5m - 5m + 60$$

$$3m = 60$$

$$\frac{3m}{3} = \frac{60}{3}$$

$$m = 20$$

$$\text{Alice} = 20 \text{ years}$$

$$\text{Joyce} = (m + 12)$$

$$(20 + 12)$$

$$= 32 \text{ years}$$

Example 2:

Roy is 18 years younger than Joyce and the ratio of their age is 2:3 respectively. How old is each now?

Let the age of John be represented by letter y

Joyce	Roy	Ratio
y	(y - 18)	2 : 3

$$y : (y - 18) = (3 : 2)$$

$$\frac{y}{(y-18)} = \frac{3}{2}$$

$$2(y) = 3(y - 18)$$

$$2y = 3y - 54$$

$$2y - 3y = 3y - 3y - 54$$

$$-y = -54$$

$$\frac{-y}{-1} = \frac{-54}{-1}$$

$$y = 54$$

$$\text{Joyce} = 54 \text{ years}$$

$$\text{Roy} = (y - 18)$$

$$(54 - 18)$$

$$= 36 \text{ years}$$

Example 3:

Tom is 8 years older than Kamya. Four years ago, the ratio of their age was 4:3 respectively. How old is Kamya now?

Let Kamya's age be represented by letter k.

	Kamya	Tom	Ratio
Now	k	K + 8	
4 years ago	K - 4	K + 8 - 4	4 : 3

$$(k-4) : (k+8-4) = 3:4$$

$$\frac{(k-4)}{(k+8-4)} = \frac{3}{4}$$

$$4(k - 4) = 3(k + 8 - 4)$$

$$4(k - 4) = 3(k + 4)$$

$$4k - 16 = 3k + 12$$

$$4k - 3k = 12 + 16$$

$$\mathbf{k = 28}$$

Kamya is 28 years old.

Example 4:

The age of Walter and that of Patrick are in the ratio of 7:9 respectively. Five years ago, their total age was 54 years. Find Musa's age now.

Let the common term be represented by letter x

	Walter	Patrick	Total age
Now	7x	9x	
5 years ago	7x - 5	9x - 5	54

$$(7x - 5) + (9x - 5) = 54$$

$$(7x + 9x - 5 - 5) = 54$$

$$16x - 10 = 54$$

$$16x - 10 + 10 = 54 + 10$$

$$16x = 64$$

$$\frac{16x}{16} = \frac{64}{16}$$

$$\mathbf{x = 4}$$

Walter's age now

$$(7x) \text{ years}$$

$$(7 \times 4)$$

$$\mathbf{= 28 \text{ years.}}$$

Example 5:

The length and width of a rectangular garden are in the ratio of 5 : 3. If the perimeter of the garden is 64cm.

a) Find the actual length and width of the rectangular garden.

Solution:

Let the common term between length and width be represented by letter y.

Length	Width	Perimeter
5y	3y	64cm

$2(L + W) = \text{Perimeter}$	<u>Actual length:</u>	(3×4)
$2(5y + 3y) = 64$	$(5y) \text{ cm}$	$= \mathbf{12cm}$
$10y + 6y = 64$	(5×4)	
$16y = 64$	$= \mathbf{20cm}$	
$\frac{16y}{16} = \frac{64}{16}$	<u>Actual width:</u>	
$y = 4$	$(3y) \text{ cm}$	

b) Calculate how much longer is the length than the width.

Dimension difference = Actual length - Actual width.

$$20\text{cm} - 12\text{cm}$$

$$= \mathbf{8cm}$$

The length is 8cm longer than the width.

General Activity:

1. Alice is 12 years older than Joan. Eight years ago, the ratio of their age was 10:7 respectively.

a) How old is Joan now? b) How old will Alice be in 2 years' time?

2. Jan Van is three years older than Benjamin. In five years' time, the ratio of their age will 4:5 respectively. How old is Benjamin?

3. Nikki is 4 years younger than Mary. In six years' times, the ratio of their age will be 9:7 respectively. How old is Nikki?

4. The heights of two boys are in the ratio of 2:3. If the height of the shorter boy is 108cm. What is the difference in the heights of the two boys?

5. Walter and Washington shared some money in the ratio of 5:8. Walter got sh.12,000 less than Washington. How much money did Washington get?

6. Cheboy is 12 years younger than Mary. If their age is in the ratio of 4:3 respectively, how old is each?

7. Owino is 6 years older than Okumu. The ratio of their age is 3:4 respectively.

a) How old is Okumu?

b) How old will Owino be in 7 years' time?

8. Patrick is 8 years younger than Andrew. How is old each one of them given that their age is in the ratio of 3:5 respectively?

9. A mother is 24 years older than her son. If the ratio of their age is 5:2 respectively, how old was each 6 years ago?

10. Ogot is 10 years older than Opot. If the ratio of their age is 3:2.

a) How old is Opot?

b) How old was Ogot 12 years ago?

11. The length and width of a rectangular field are in the ratio of 1 : 7. Calculate its actual length and width if the perimeter is 84 meters.

b) Find its area.

TO BE CONTINUED BY THE DESERTER@PATRICK