P425/1

PURE MATHEMATICS

Paper 1

October. 2022

3 hours

INTEGRATION TEST

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

Attempt all the questions in this section.

1. Show that $\int_0^1 \frac{ln(1+x)^2}{1+x}\,dx = (ln2)^2$. *(05 marks)*

2. Find the general solution of $\frac{dy}{dx} = \frac{x^2+y^2}{x(x+y)}$. *(05 marks)*

3. $\int_0^1 \frac{1-3x}{3x^2-2x+3}\,dx$. *(05 marks)*

4. Prove that $\int_0^{\pi/4} \frac{sin^3x}{cos^2x}\,dx = \frac{3}{2}\sqrt{2} - 2$. *(05 marks)*

5. A spherical container of internal radius 12 cm contains water up to a depth of 8 cm. Find the volume of the water. (Leave π in your solution). *(05 marks)*

6. The area of a segment cut off by $y = 5$ from the curve $y = x^2 + 1$ is rotated about the line $y = 5$ is through 360^0. Find the volume of the solid generated. *(05 marks)*

7. Given that $\frac{dy}{dx} + 2xy^2 + 2y^2 = 0$ and $y(0) = -\frac{1}{8}$, show that $y = \frac{1}{(x+4)(x-2)}$. *(05 marks)*

8. Evaluate: $\int_2^5 (x+2)(x-1)^{1/2}\,dx$. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt **only** five questions.*

9. (a) The gradient function of a curve at any point $P(x, y)$ is $3x^2 + ax$. A point (2,1) is one of the stationary points on the curve. Find the value of a and the other stationary point. *(06 marks)*

   (b) Solve the differential equation: $x\frac{dy}{dx} = x^2 - 2y$ given that $y(2) = 4$. *(06 marks)*

10. (a) Evaluate $\int_1^e x^2 log_2 x^4 dx$. *(06 marks)*

    (b) Evaluate: $\int_0^{\pi/4} xtan^2 x dx$. *(06 marks)*

11. (a) Show that $12\sin^2 xcos^2 x = \frac{3}{2}(1 - cos4x)$.

    Hence prove that $\int_{\pi/4}^{\pi/3} 12\sin^2 xcos^2 x dx = \frac{1}{16}\left(2\pi + 3\sqrt{3}\right)$. *(06 marks)*

    (b) Find the volume generated when the area bounded by the curve $y = cos2x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the x-axis correct to 4 significant figures. *(06 marks)*

12. Express $\frac{32x^2+17x+18}{(2-3x)(1+2x)^2}$ into partial fractions. Hence evaluate

    $\int_{-1}^{1} \frac{32x^2+17x+18}{(2-3x)(1+2x)^2} dx$ *(12 marks)*

13. Evaluate the following

    (a) $\int_0^{\pi/4} cos5xsin3x dx$. *(04 marks)*

    (b) $\int_1^4 \frac{1}{\sqrt{x}} . 2^{\sqrt{x}} dx$. *(04 marks)*

    (c) $\int_0^{\pi/4} (1 + tanx)^2 dx$. *(04 marks)*

14. The differential equation $\frac{dp}{dt} = kp(c - p)$ shows the rate at which information flows in a student population c. P represents the number of students who have heard the information in t days and k is a constant.

(a) Show that $\frac{1}{c} ln\left(\frac{p}{c-p}\right) = kt + b$ where b is a constant. *(06 marks)*

(b) A school had a population of 1000 students. Initially 20 students had heard the information. A day later, 100 students the information. How many students had heard the information by the tenth day.

(06 marks)

15. Given the curves $y = 2 + x - x^2$ and $y = 2 - 3x + x^2$

(a) Sketch the two curves on the same axes. *(05 marks)*

(b) Find the area between the two curves. *(03 marks)*

(c) Obtain the volume generated when the area in (b) above is rotated about the x-axis through 360^0. *(04 marks)*

16. (a) Solve the differential equation $e^y \cos^2 x \frac{dy}{dx} = 1$ given $y\left(\frac{\pi}{4}\right) = 0$.

(04 marks)

(b) The population of a certain country increases at a rate directly proportional to the population present at any time. The population doubles after every ten years and the population was 12 million in 2000.

(i) Find the population in 2022. *(06 marks)*

(ii) Find the time taken for the population to reach 50 million.

(02 marks)

GOOD LUCK