SECTION B: WAVES AND WAVE MOTION

A wave is any disturbance from an equilibrium position that travels with time from one region to another.

Wave motion is a means of transfer of energy from one point to another without the transfer of matter between the points

Waves are classified into two namely;

(i) Electromagnetic waves

(ii) Mechanical waves

Mechanical waves

These are waves that are produced by the disturbances of a material medium and are transmitted by the particles of the medium oscillating to and fro

Mechanical waves require a material medium for their transmission. They can be felt and seen

Examples of mechanical waves

- Sound waves
- Water waves
- Waves on compressed springs
- Waves on stretched string

Electromagnetic waves

These are waves produced by disturbances of varying electric and magnetic fields
They do not require a material medium for their transmission and travel in a vacuum.

Examples of electromagnetic waves

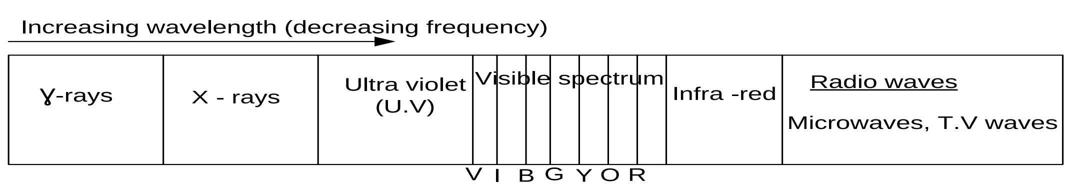
- Light waves
- $\gamma - rays$
- Radio waves
- All other electromagnetic band waves

Note:

All electromagnetic waves travel at a speed of light $ie\ 3x10^8 m/s$

Properties of electromagnetic waves

(i) Electromagnetic waves travel in a vacuum and therefore do not require a material medium for their transmission.
(ii) Electromagnetic waves travel at a speed of light $ie\ 3x10^3 ms^{-1}$
(iii) They are made of varying electric and magnetic vibration.
(iv) They vibrate with a high frequency.
(v) They have no charge.



Differences between mechanical and electromagnetic waves

Mechanical waves	Electromagnetic waves
❖ Need a material medium for their transmission	❖ Can propagate in vacuum
❖ Propagate at relatively low speeds	❖ Propagate at high speeds
❖ Have longer wavelength	❖ Have shorter wavelength
❖ Are due of vibrations of particles in the transmitting medium	❖ Are due of vibrations of particles in the transmitting medium

Types of waves

There are two types of wave motion namely

- Transverse waves
- Longitudinal waves

Transverse waves

These are waves in which displacement of the particles in the medium is perpendicular to the direction of wave travel.

Transverse waves are characterized by crests and troughs

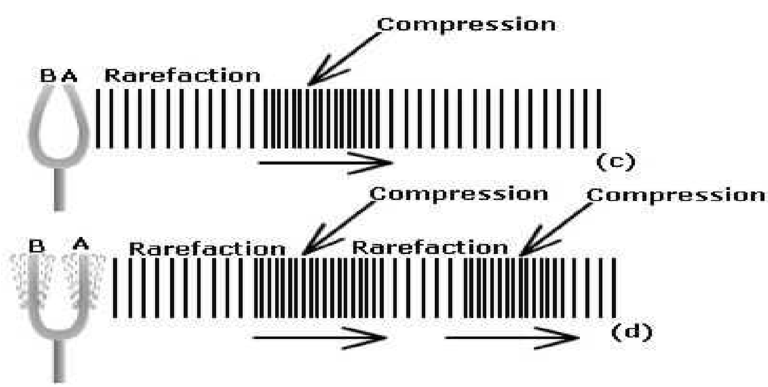
- Crest is the part of the wave above the line of zero disturbance
- Trough is the part of the wave below the line of zero disturbance

Examples

- Water waves
- Waves due plucked strings
- Light waves
- All electromagnetic waves ($eg\ \gamma - rays, X - rays$)

Longitudinal waves

These are waves in which the displacement of the particles is parallel to the direction of travel of the wave.



Longitudinal waves are characterized by compressions and rare factions

- Compressions are regions of high particle density in wave
- Rare factions are regions of low particle density in wave

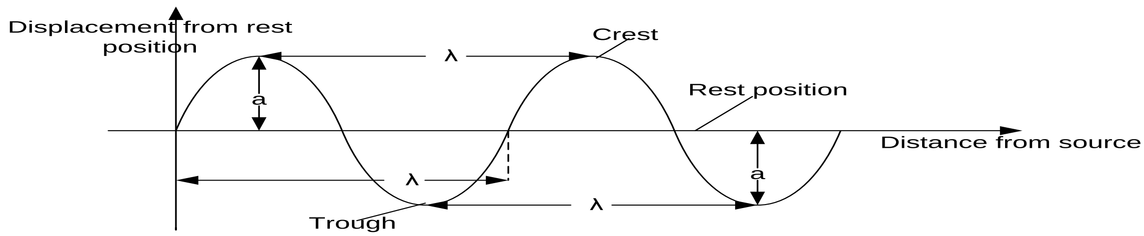
Examples

- Sound waves
- Waves on a compressed spring

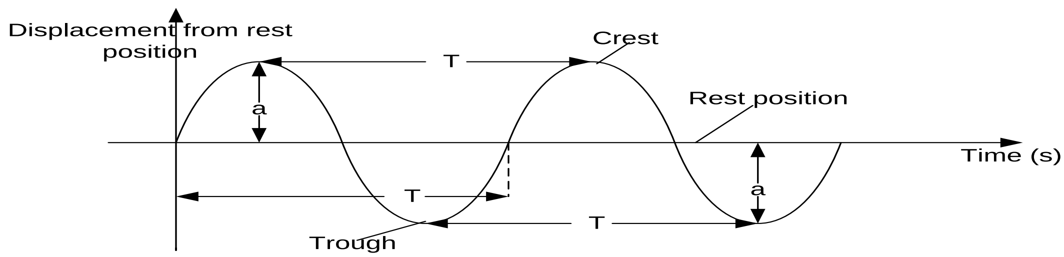
Differences between transverse and longitudinal waves

Transverse waves	Longitudinal waves
Particles vibrate at right angles to the direction of travel of the wave	Particles vibrate along the direction of travel of the wave
Transverse waves are represented by crests and troughs	longitudinal waves are represented by compression and rare faction regions

Representation of a wave



A displacement time graph can also be drawn



Terms used

(1) Amplitude (a)

This is the maximum displacement of a particle of a medium from its equilibrium position.

(2) Wave length (λ)

This is the distance between two successive particles in phase.
Wave length of **a transverse** wave is the distance between two successive crests or successive troughs.

(3) Oscillation or cycle

This is a complete to and fro movement of a wave particle in a medium

(4) Period (T)

This is the time taken for one particle to under one complete oscillation.
Or the time taken for a wave to travel a distance of one wavelength

$$\boxed{T = \frac{1}{f}}$$

Period T is measured in seconds

(5) Frequency (f)

The number of complete oscillations a wave particle makes in one second

$$\boxed{f = \frac{1}{T}}$$

The S.I unit of frequency is Hertz (Hz)

(6) Phase

Particles are in phase when they are exactly at the same point in their paths and are moving in the same direction

(7) Wave front

Is any section through an advancing wave in which all the particles are in the phase.

(8) **A ray.** This is the direction of an advancing wave

(9) Speed (V) of the wave

This is the linear distance travelled by a wave per unit time

$$V = \frac{linear\ distance}{time\ taken}$$

Since one complete wave is produced in time **T** and the length of one complete wave is λ

$$V = \frac{\boldsymbol{\lambda}}{T}$$

$$V = \frac{\mathbf{1}}{T} x\, \boldsymbol{\lambda}$$

But $f = \frac{1}{T}$

$$\boxed{V = f\, \boldsymbol{\lambda}}$$

EXAMPLES

1. Sanyu Fm broadcasts at a frequency of $88.2MHz$. Calculate the wavelength of the radio waves.

Solution

Note: All electromagnetic waves eg radio waves travel at a speed of light $3x10^8m/s$

$f = 88.2MHz = 88.2x10^6Hz,$

$v = 3x10^8m/s$

$v = f\,\lambda$

$3x10^8 = 88.2x10^6\,\lambda$

$$\boldsymbol{\lambda} = \frac{3x10^8}{88.2x10^6}$$

$\lambda = 3.4m$

2. A vibrator produces waves which travel a distance of $12m$ in $4s$. If the frequency of the vibrator is $2Hz$, what is the wavelength of the wave?

Solution

$f = 2Hz, t = 4s, distance = 12m$

$$v = \frac{distance}{time} = \frac{12}{4}$$

$v = 3m/s$

$v = f\,\lambda$

$3 = 2\,\lambda$

$$\boldsymbol{\lambda} = \frac{3}{2}$$

$\lambda = 1.5m$

3. A vibrator has a period of 0.02s and produces circular waves of water in a tank. If the distance between any two consecutive crests is 3cm, what is the speed of the wave?

Solution

$T = 0.02\,s\,,$

But $f = \frac{1}{T}$

$$f = \frac{1}{0.02}$$

$f = 50Hz$

$\lambda = 3cm\,,\ \lambda = 0.03m$

$v = f\,\lambda$

$v = 50x\,0.03$

$v = 1.5m/s$

4. Water waves are produced at a frequency of 50Hz and the distance between 10 successive troughs is 18cm. Calculate the velocity of the waves.

Solution

$f = 50Hz$

$9\,\lambda = 18\,cm, \lambda = \frac{18}{9}cm$

$\lambda = 2cm, \lambda = 0.02\,m$

$v = f\,\lambda$

$v = 50x\,0.02$

$v = 1m/s$

Properties of waves

All waves can be;

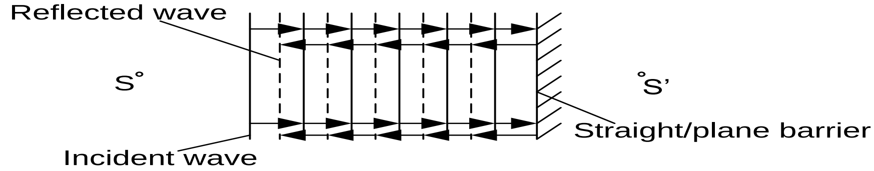
1. Reflected
2. Refracted
3. Diffracted
4. Interfered

1. REFLECTION OF WAVES

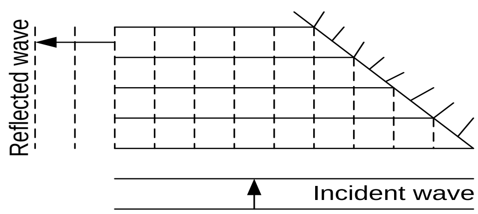
This is the bouncing back of waves when they meet a barrier

a) Plane reflector

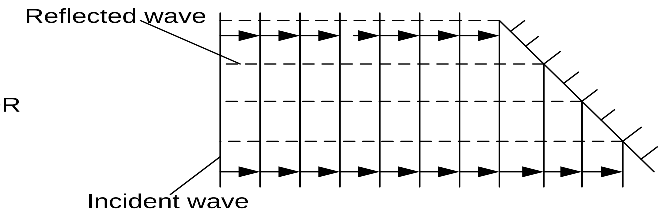
(i) Straight waves incident on a plane reflector



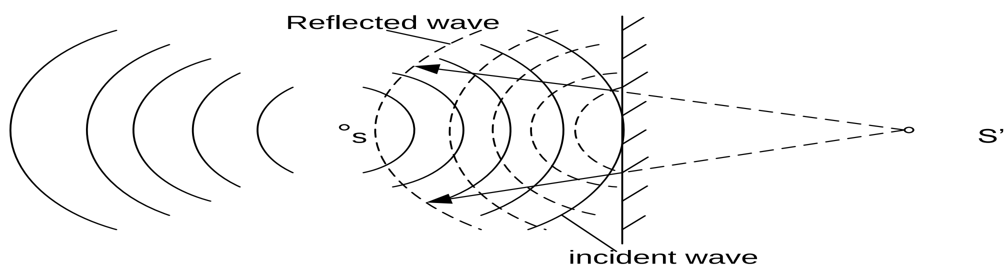
(ii) Straight waves incident on an inclined Plane surface



OR

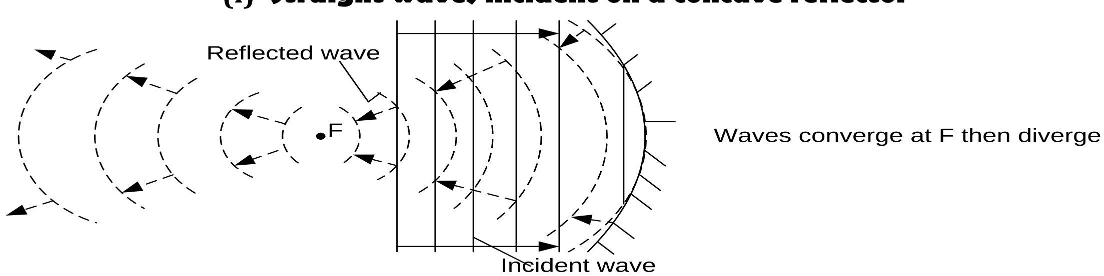


(iii) Circular waves incident on a plane reflector

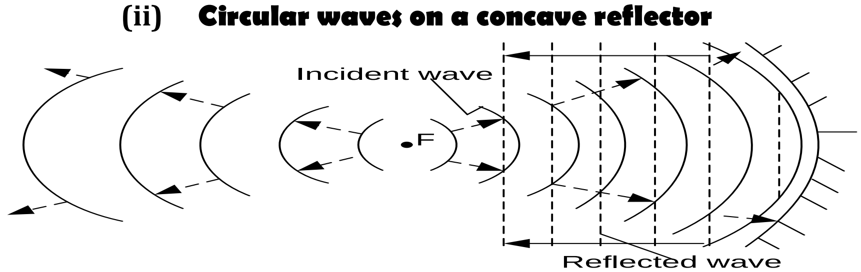


b) Concave reflector

(i) Straight waves incident on a concave reflector

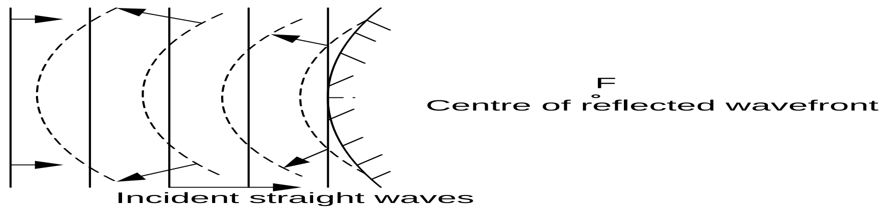


(ii) Circular waves on a concave reflector

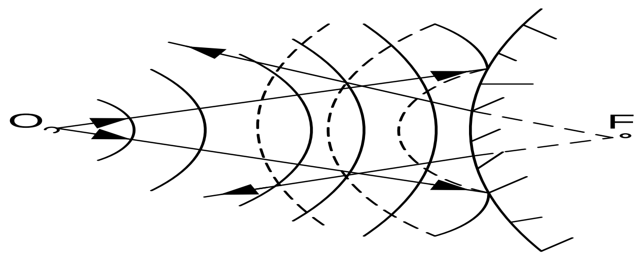


c) Convex reflector

(i) Plane waves incident on a convex reflector



(ii) Circular waves incident on a convex reflector



PROGRESSIVE WAVES

It is a wave in which the disturbance moves from the source to the surrounding places and energy is transferred from one point to another along the wave form

Examples

- Water waves
- All electromagnetic waves

Note: All transverse and longitudinal waves are progressive and the amplitude of a progressive wave is constant

Energy transmitted by a wave

In a progressive wave energy propagates through the medium in the direction in which the wave travels. So every particle in the medium possesses energy due to vibrations. This energy is passed on to the neighboring particles so for any system vibrating in form of simple harmonic motion, the energy of the vibrating particle changes from kinetic to potential energy and back but the total mechanical energy on the wave remains constant.

$$\boldsymbol{k.e = \frac{1}{2}mv^2}$$

But $v = \omega A$

$$\boldsymbol{k.e = \frac{1}{2}m(\omega A)^2}$$

$$\boldsymbol{\omega = 2\pi f}$$

$$\boldsymbol{k.e = \frac{1}{2}m(2\pi f A)^2}$$

$$\boxed{\boldsymbol{K.E = 2\pi^2 f^2 A^2 m}}$$

Where f- frequency

A- Amplitude

M- mass of vibrating particle

$$Also\ energy\ per\ unit\ volume = \frac{E}{V}$$

$$\frac{E}{V} = \frac{\boldsymbol{2\pi^2 f^2 A^2 m}}{\left(\frac{m}{\rho}\right)}$$

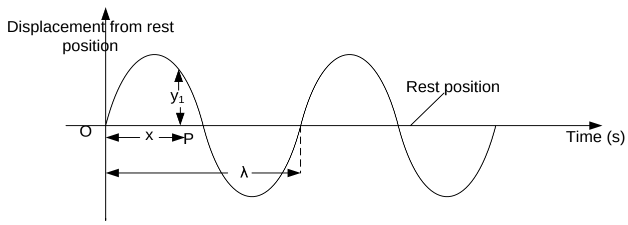
$$\boxed{Energy\ per\ unit\ volume = \boldsymbol{2\pi^2 f^2 A^2 \rho}}$$

Intensity of wave

This is the rate of flow of energy through an area of 1 m^2 perpendicular to the path of travel of wave

Equation of a progressive wave

Consider a wave form below



if the oscillation of the particle at O is simple harmonic with frequency f and angular velocity ω then its displacement y with time is given by

$$\boxed{\boldsymbol{y = asin\omega t}} \ldots\ldots\ldots\ldots\ldots (\mathbf{1})$$

Suppose the wave generated travels towards the right, the particle at P a distance x form O will lag behind by a phase angle φ

$$\boxed{\boldsymbol{y_1 = asin(\omega t - \varphi)}} \ldots\ldots\ldots\ldots\ldots\ldots. (\mathbf{2})$$

From the figure above, the phase angle of $2\pi = \lambda$ and phase angle $\varphi = x$

$2\pi = \lambda \ldots\ldots\ldots\ldots\ldots\ldots (1)$

$\varphi = x \ldots\ldots\ldots\ldots\ldots. (2)$

$$\frac{\varphi}{2\pi} = \frac{x}{\lambda}$$

$$\boxed{\varphi = \frac{2\pi x}{\lambda}}$$

Equation 2 will become

$$\boldsymbol{y_1 = asin}\left(\boldsymbol{\omega t} - \frac{2\pi x}{\lambda}\right)$$

But $\boldsymbol{\omega = 2\pi f = \frac{2\pi}{T}}$

$$\boldsymbol{y_1 = asin}\left(\frac{\boldsymbol{2\pi t}}{\boldsymbol{T}} - \frac{2\pi x}{\lambda}\right)$$

$$\boldsymbol{y_1 = asin\ 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)$$

Generally for a wave travelling to the right the equation of a progressive wave is $\boxed{y = \boldsymbol{asin\ 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)}$

Note;

If the wave is travelling to the left it arrives at P before O. This makes the vibration at P to lead the vibrations at O and its equation is given by $\boxed{y = \boldsymbol{asin\ 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} + \frac{x}{\lambda}\right)}$

Examples

1. A displacement of travelling wave in the direction x- direction is given by $y = \boldsymbol{asin\ 2\pi}\left(\frac{\boldsymbol{t}}{0.5} - \frac{x}{0.2}\right)\boldsymbol{m}$

Find the speed of the wave

Solution

$$y = \boldsymbol{asin\ 2\pi}\left(\frac{\boldsymbol{t}}{\mathbf{0.5}} - \frac{x}{0.2}\right)$$

Compare with $y = \boldsymbol{asin\ 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)$

$\boldsymbol{T = 0.5s}$

$\lambda = 0.2m$

$\boldsymbol{v = f\lambda}$

$$\boldsymbol{v} = \frac{\mathbf{1}}{\mathbf{0.5}} x0.2$$

$\boldsymbol{v = 0.4m/s}$

2. A sound wave propagating in the x- direction is given by $y = \boldsymbol{0.4sin}\ \left[\mathbf{10}\left(\boldsymbol{200t} - \frac{x}{100}\right)\right]\boldsymbol{m}$

Find the speed of the wave

Solution

$y = \mathbf{0.4}\boldsymbol{sin}\left[\mathbf{10}\left(\mathbf{200}\boldsymbol{t} - \frac{x}{100}\right)\right]$ Compare with

$y = \boldsymbol{asin}\ \mathbf{2}\boldsymbol{\pi}\left(\boldsymbol{ft} - \frac{x}{\lambda}\right)$

$\mathbf{2}\boldsymbol{\pi ft} = \mathbf{10}\boldsymbol{x}\mathbf{200}\boldsymbol{t}$

$\boldsymbol{f} = \mathbf{318.5}\boldsymbol{Hz}$

$$\frac{10x}{100} = \frac{2\pi x}{\lambda}$$

$\lambda = 62.8m$

$\boldsymbol{v} = \boldsymbol{f}\lambda$

$\boldsymbol{v} = \mathbf{318.5}x62.8$

$\boldsymbol{v} = \mathbf{2.0}\boldsymbol{x}\mathbf{10^4}\boldsymbol{m/s}$

3. The displacement y in meters of a plain pThe displacement y in meters of a plain progressive wave is given by $y = \boldsymbol{asin}\ \mathbf{2}\boldsymbol{\pi}\left(\mathbf{100}\boldsymbol{t} - \frac{20x}{17}\right)$. Find the wavelength of the wave and the speed of the wave

Solution

$y = \boldsymbol{asin}\ \mathbf{2}\boldsymbol{\pi}\left(\mathbf{100}\boldsymbol{t} - \frac{20x}{17}\right)$.

Compare with $y = \boldsymbol{asin}\ \mathbf{2}\boldsymbol{\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)$

$$\frac{\mathbf{1}}{\boldsymbol{T}} = \mathbf{100}$$

$$\frac{\mathbf{1}}{\lambda} = \frac{\mathbf{10}}{\mathbf{17}}$$

$\lambda = 1.7m$

$\boldsymbol{v} = \boldsymbol{f}\lambda$

$\boldsymbol{v} = \mathbf{100}x1.7$

$\boldsymbol{v} = \mathbf{170}\boldsymbol{m/s}$

Exercise

1. The displacement of a particle in a progressive wave is $y = 2\ sin\ [2\pi(0.25\ x - 100\ t)]$, where x and y are in cm and t is in seconds. Calculate the :
   (i) wave length,
   (ii) velocity of propagation of the wave **An(**$\lambda = \mathbf{4.0}\boldsymbol{cm}, \boldsymbol{V} = \mathbf{4}\boldsymbol{ms^{-1}}$**)**
2. The displacement y given of a wave travelling in the x- direction at time t is:

   $$y = \boldsymbol{asin}\ \mathbf{2}\boldsymbol{\pi}\left(\frac{\boldsymbol{t}}{\mathbf{0.1}} - \frac{x}{2.0}\right) \text{ meter}$$

   Find
   (i) the velocity of the wave
   (ii) the period of the wave **An(**$\boldsymbol{T} = \mathbf{0.1}\ \boldsymbol{s}, \boldsymbol{V} = \mathbf{20}\boldsymbol{ms^{-1}}$**)**
3. A plane progressive wave is given by $y = a\ sin\ (100\ \pi t - \frac{10}{9}\ \pi x\ )$ where x and y are millimetres and t is in seconds

   Calculate the :
   (i) wave length,
   (ii) velocity of propagation
   (iii) period T of the wave
4. The displacement in metres of a plane progressive wave is given by the equation

   $$y = \mathbf{0.5}\boldsymbol{sin}\left[\boldsymbol{\pi}\left(\mathbf{200}\boldsymbol{t} - \frac{20x}{17}\right)\right]$$

   Find
   (i) wavelength and
   (ii) speed, of the wave
5. A progressive wave is represented by the equation $y = \mathbf{0.1}\boldsymbol{sin}\ \left(\mathbf{100}\boldsymbol{\pi t} - \frac{\mathbf{10}\boldsymbol{\pi x}}{9}\right)$mm. find
   (i) Amplitude of the wave. **An(0.1mm)**
   (ii) Frequency **An(50Hz)**
   (iii) Wavelength **An(1.8mm)**
   (iv) Speed of the wave **An(0.09m/s)**
   (v) Speed of particles in the wave motion **An(use** $v = \boldsymbol{\omega a}$**)**
   (vi) Phase difference between a point 0.245m from O and point 1.10m from O. **An(use** $\varphi = \frac{2\pi x}{\lambda}$ **)**

6. A progressive and a stationary wave each has a frequency of 240 Hz and a speed of 80 ms^{-1}. Calculate
   (i) phase difference between two vibrating points in the progressive wave which are 6 cm apart
   (ii) distance between modes in the stationary wave
7. A wave of amplitude 0.2m, wavelength 2.0m and frequency 50Hz. If the initial displacement is zero at point x = 0
   (i) write the expression for the displacement of the wave at any time t.
   (ii) find the speed of the wave
8. Two waves of frequencies 256 Hz and 280 Hz respectively travel with a speed of 340 m s – 1 through a medium. Find the phase difference at a point 2.0m from where they were initially in phase

STATIONARY WAVE / STANDING WAVE

This a wave formed as a result of superposition of two progressive waves of equal amplitude and frequency but travelling at same speed in opposite direction.
Therefore in a stationary wave, energy does not move along with the wave.
Stationary waves are characterized by node (N) and antinodes (A)

Formation of a stationary wave

Stationary waves are formed when two waves of equal frequency and amplitude travelling at same speed in opposite direction are supposed resulting into formation of node and antinode
At antinodes, waves meet in phase and the amplitude is maximum. At nodes, the wave meet antiphase and amplitude is minimum.

Condition for stationary waves to be formed

- Waves must be moving in opposite direction.
- Waves must have the same speed, same frequency and equal amplitude.

Equation of a stationary wave

Consider a progressive wave travelling to the right. The displacement of any particle of the medium is given by

$$y_1 = a\sin(\omega t - \varphi)$$

When this wave is reflected, it travels to the left. The displacement of any particle of medium will be

$$y_1 = a\sin(\omega t + \varphi)$$

When the two waves superpose, the resultant displacement I given by $y = y_1 + y_2$

$$y = a\sin(\omega t - \varphi) + a\sin(\omega t + \varphi)$$

$$\boxed{y = 2a\cos\varphi \sin\omega t}$$

Where amplitude of vibration is $2a\cos\varphi$
Where $\varphi = \frac{2\pi x}{\lambda}$
Note: amplitude of a stationary wave varies with x hence its not constant

Principle of super position of waves

It states that for two wave travelling in the same region, the total displacement at any point is equal to the vector sum of their displacement at that point when the two waves over lap

Examples

A plane progressive wave is given by

$y = a\sin(100\pi t - \frac{10}{9}\pi x)$ where x and y are millimetres and t is in seconds

(i) write the equation of the progressive wave which would give rise to a stationary wave if superimposed on the one above (1 mark)
(ii) find the equation of the stationary wave and hence determine its amplitude of vibration (3 marks)
(iii) determine the frequency and velocity of the stationary wave (4 marks)

solution

(i) $y_2 = a\sin(100\pi t + \frac{10}{9}\pi x)$

(ii) $y = y_1 + y_2$

$$\boldsymbol{y} = a\sin\left(100\pi t - \frac{10}{9}\pi x\right) + a\sin(100\pi t + \frac{10}{9}\pi x)$$

$$\boldsymbol{y} = \boldsymbol{2}a\cos(\frac{10}{9}\pi x)\sin(100\pi t)$$

Amplitude of vibration $is\ \boldsymbol{2a}\cos(\frac{10}{9}\pi x)$

(iii) $2\pi f = 100\pi$

$$f = 50Hz$$
$$v = 0ms^{-1}$$

Differences between stationary waves and progressive waves

Stationary waves	Progressive waves
1. Amplitude of the particles in the medium varies with position along the wave	1. All particles in the transmitting medium oscillate with the same amplitude.
2. Wave energy is not transferred but confined to a particular section of a wave	2. Wave energy is transferred form one point to another along the wave
3. Distance between any two successive nodes or antinodes is equal to $\frac{\lambda}{2}$	3. Distance between any two successive crests or troughs is equal to λ
4. Has nodes and antinodes	4. Doesn't have nodes and antinodes

MECHANICAL OSCILLATION

There are three types of oscillation i.e.

a) Free oscillation b) Damped oscillation c) Forced oscillation

a) Free oscillations

These are oscillations in which the energy of the system remains constant and is not lost to the surrounding. The amplitude of oscillation remains constant with time.

b) Damped oscillations

These are oscillations in which energy of oscillating system loses energy to the surrounding as a result of dissipative forces acting on it. Amplitude of oscillation decreases with time.

C) FORCED OSCILLATIONS

These are oscillations where the system is subjected to a periodic force which sets the system into oscillation. When the periodic force has the same frequency as the natural frequency of the oscillating system then resonance occurs.

SOUND WAVES

Sound is any mechanical vibration whose frequency lies within the audible range.
Sound waves propagate in air by series of compressions and rare factions.

Explain why sound propagates as an adiabatic process

Sound waves propagate in air by series of compressions and rare factions. In compressions the temperature of air rises unless heat is withdrawn. In rare factions, there is a decrease in temperature. The compressions and rare factions occur so fast that heat does not enter or leave the wave. Hence the process is adiabatic.

Characteristic of sound

a) Pitch

This is the characteristic of sound by which the ear assigns a place on a musical scale.
Pitch depends on the frequency of vibration of the sound waves ie it increases as the frequency of sound increases.

b) Loudness

This is the magnitude of the auditory sensation produced by sound.
Or Amount of sound energy entering the ear per second.

Factors that affect Loudness

- Sound intensity
- Amplitude of sound.

ECHOES

An echo is a reflected sound.
The time that elapses between hearing the original sound and hearing the echo depends on;

a) The distance away from the reflecting surface.
b) The speed of sound in the medium.

REVERBERATION

When sound is reflected from a hard surface close to the observer, the echo follows the incident sound so closely that the observer may not be able to distinguish between the two. Instead the observer gets an impression or hears a prolonged original sound. This effect is referred to as reverberation

Briefly explain why reverberation is necessary while making speeches

Too short a reverberation time makes a room sound dead but if it is too long, confusion results. For speeches half a second is acceptable. Reverberation time is made the same irrespective of the size of the audience b lining the walls with a soft material so that there is reduced reflection of sound

Refraction of sound

This is the change in the speed of sound waves as they move from one medium to another of different optical densities.

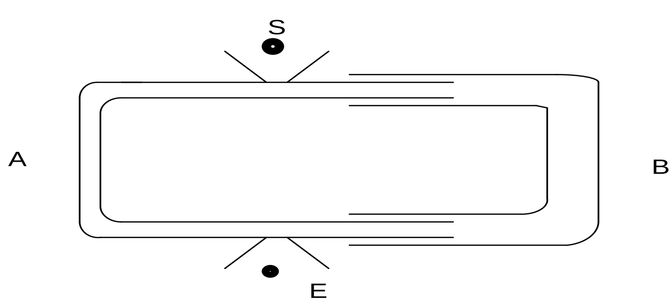
Explain why sound is easily heard at night than during day time

Distant sounds are more audible at night than day because the speed of sound in warm air exceeds that in the cold air and refraction occurs. At night the air is usually colder near the ground than it is higher up and refraction towards the earth occurs. During the day, the air is usually warmer near the ground than it is higher up

Interfernce of sound

Interference of waves is the superposition of waves from different two coherent soruces resulting into alternate regions of maximum and minimium intensity.

Experiment to show interference of longitudinal waves



- Tube A is fixed while B is free to move.
- A note is sounded at S and detected at E.
- Tube B is then **pulled out slowly**. It is noted that the sound detected at E increases to a maximum and reduces to a minimu in intensity at **equal intervals of length** of the tube.
- The alternate maximum and minimum intensity of sound are interference patterns

Differences between sound and light waves

Sound waves	Light waves
- They cant travel through a vacuum	- They can travel through a vacuum
- They travel at a low speed i.e 330m/s	- They travel at a high speed i.e $3x10^8$m/s
- Require a material medium for their transmission	-Do not require a material medium for their transmission
- They cant eject electrons from a metal surface	- They can eject electrons from a metal surface by photo electric emission
- They are longitudinal waves	- They are transverse waves

BEATS

A beat is aperiodic rise and fall in the intensity of sound heard when two notes of nearly equal frequency but similar amplitude are sounded together.

Formation of beats

When two waves of nearly equal frequency and similar amplitude are sounded together they superpose.
When they meet in phase constructive interference takes place and a loud sound is heard. When they meet out of phase destructive interference takes place and a soft sound is heard.
A periodic rise and fall in intensity of sound is heard which is called beats

Beat frequency

Its defined as the number of intense sounds heard per second

Derivation of Beat frequency

Let f_1 and f_2 be frequencies of two sound notes.
Suppose a note of frequency f_1 makes one cycle more than other in time T.
The number of cycles of frequency $f_1 = f_1T$
The number of cycles of frequency $f_2 = f_2T$

$f_1T - f_2T = 1$

$(f_1 - f_2)T = 1$

$\frac{1}{T} = (f_1 - f_2)$

But $\frac{1}{T} = f$

$\boxed{f = f_1 - f_2}$ This is called beat frequency

Uses of frequency

- Used in measurement of frequency of a note
- Determination of frequency of a musical note
- Tuning an instrument to a given note

Measurement of frequency of a note

- A note is sounded together with a tuning fork of known frequency, f_T
- The number of beats, n in t seconds are counted and the beat frequency, $f_b = \frac{n}{t}$ calculated

- One prong of the tuning fork is loaded with plasticine and then the experiment repeated. The new beat frequency $f_b{}^1$ is determined
- If $f_b{}^1 < f_b$ then the frequency of the test note f_n is calculated from $f_n = f_T + f_b$
- If $f_b{}^1 > f_b{}^1$ then the frequency of the test note f_n is calculated from $f_n = f_T - f_b$

Examples

1. Two tuning forks of frequency 256Hz and 250Hz respectively are sounded together in air. Find the number of beats per second produced

   Solution

   $f = f_1 - f_2$ | $f = 256 - 250$ | $\boldsymbol{f = 6\ Hz}$

2. Two sources, one with known freq 224Hz and the other unknown are sounded together .the beat freq recorded is 6Hz. When the unknown source is sounded again together with another known source of 250Hz the beat freq is 20Hz find the unknown freq of the 2nd source

   Solution

   In 1st case possible freq for 2nd source is either 224 – 6 =218 or 224 + 6 = **230**Hz

   In case 2 possible freq for 2nd source is either 250 – 20 = **230** or 250 +20 = 270Hz

   The common answer in both cases is 230Hz which is the freq of the 2nd source

3. Two whistles are sounded simultaneously. The wavelength sound emitted are 5.5m and 6.0m. find; (speed of sound in air 330m/s)
   (a) Beat frequency
   (b) Beat period

   Solution

   $f = f_1 - f_2$

   $f = \frac{v}{\lambda_1} - \frac{v}{\lambda_1}$

   $f = \frac{330}{5.5} - \frac{330}{6}$

   $f = 60 - 55$

   $\boldsymbol{f = 5\ Hz}$

   Beat period $= \frac{1}{f}$

   Beat period $= \frac{1}{5} = 0.2s$

4. Two sources of sound are vibrating simultaneously with frequency of 200Hz and 240Hz. If the speed of sound in air is 340m/s
   (i) How many beats are heard
   (ii) What is the distance between successive locations of maximum intensity

   Solution

   (i) $f = f_1 - f_2$

   $f = 240 - 200$

   $\boldsymbol{f = 40\ Hz}$

   (ii) $\lambda = \frac{v}{f}$

   $\lambda = \frac{v}{f}$

   $\lambda = \frac{340}{40}$

   $\lambda = 8.5m$

5. Two tuning forks x and y are sounded in air to produce beats of frequency 8Hz. Fork x has a known frequency of 512Hz. When y is loaded with a small piece of plasticine, beats of frequency 2Hz are heard when the two forks are sounded together. Find the frequency of y when it is unloaded.

   Solution

   $f = f_x - f_y$

   $8 = 512 - f_y$

   $f_y = \boldsymbol{504\ Hz}$

   or

   $f = f_y - f_x$

   $8 = f_y - 512$

   $f_y = \boldsymbol{520\ Hz}$

   When the fork is unloaded, the frequency of y is 520Hz, since beat frequency reduces

6. Two tuning forks A and B produce three beats per second when sounded together. If fork A has a frequency of 512Hz.
   (a) What is the possible frequency of B
   (b) Explain how you determine the actual frequency of tuning fork B

Solution

(a) $f = f_x - f_y$

$3 = 512 - f_y$

$f_y = \mathbf{509\ Hz}$

or

$f = f_y - f_x$

$3 = f_y - 512$

$f_y = \mathbf{515Hz}$

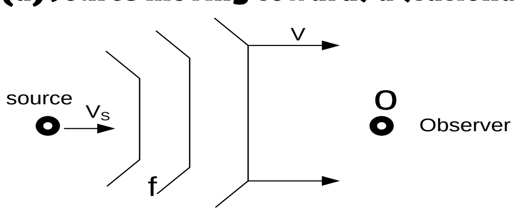
Attach a piece of plasticine on B and determine the new beat frequency by sounding A and B together. If the beat frequency increases, then the actual frequency of B is 509Hz, if the beat frequency decreases then the actual frequency of B is 515Hz

DOPPLER EFFECT

This is the apparent change in frequency and wave length of a wave when there is relative motion between the source of the waves and the observer
Doppler Effect takes place in both sound and light

Doppler Effect in sound
Case 1: source of sound in motion but observer fixed

(a) Source moving towards a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v - v_s$

Apparent change in wavelength,$\lambda_a = \frac{v-v_s}{f}$

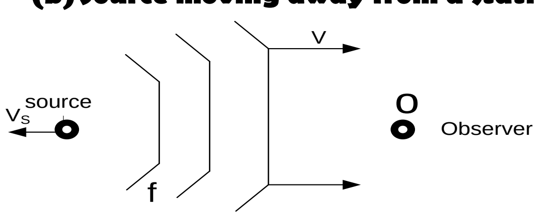
Velocity of wave relative to observer$= v - 0 = v$

Apparent change in frequency,$f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v - v_s}\right) f}$$

Since $v - v_s < v$ then the apparent frequency appears to increase when the source moves towards an observer

(b) Source moving away from a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v + v_s$

Apparent change in wavelength,$\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer$= v - 0 = v$

Apparent change in frequency,$f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v + v_s}\right) f}$$

Since $v + v_s > v$ then the apparent frequency appears to decrease when the source moves away from an observer.

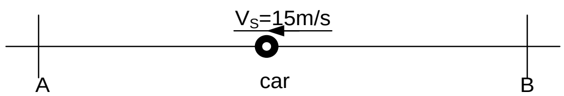
Note:

When the source is in motion, only wavelength and frequency change but the speed of the sound waves is not affected

Examples

1. A car sounds its horn as it travels at a steady speed of 15m/s along a straight road between two stationary observers A and B. Observer A hears a frequency of 538Hz while B hears a lower frequency. Calculate the frequency heard by B if the speed of sound in air is 340m/s

Solution



Since B receives sound of lower frequency, the car is moves away from B

Towards A: $f_A = \left(\frac{v}{v-v_s}\right) f$

$$538 = \left(\frac{340}{340-15}\right) f$$

$f_A = 514.265Hz$

Away from B: $f_B = \left(\frac{v}{v+v_s}\right) f$

$$f_B = \left(\frac{340}{340+15}\right) x514.265$$

$f_B = 492.54Hz$

2. A stationary observer notices that the pitch of the racing car changes in a ratio 4:3. The velocity of sound in air is 320m/s. Calculates the speed of the car.

Solution

Source moving towards:

$$f_1 = \left(\frac{v}{v-v_s}\right) f$$

$$f_1 = \left(\frac{320}{320-v_s}\right) f \ldots\ldots 1$$

Source moving away from:

$$f_2 = \left(\frac{v}{v+v_s}\right) f$$

$$f_2 = \left(\frac{320}{320+v_s}\right) f \ldots\ldots\ldots\ldots 2$$

$$f_1 : f_2 = 4:3$$

$$3f_1 = 4f_2$$

$$3\left(\frac{320}{320-v_s}\right) f = 4\left(\frac{320}{320+v_s}\right) f$$

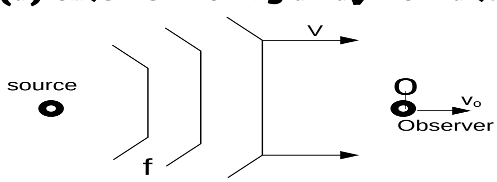
$$v_s = 45.71m/s$$

3. A car sounds its horn as it travels at a steady speed of 20m/s along a straight road between two stationary observers X and Y. Observer X hears a frequency of 560Hz while Y hears a lower frequency. Calculate the frequency heard by Y if the speed of sound in air is 330m/s.

An(495.9Hz)

Case2: observer in motion while the source is stationary

(a) observer moving away from a stationary source



$$v_o - velocity\ of\ the\ observer$$

$$v - velocity\ of\ sound$$

$$f - frequency\ of\ the\ sound\ waves$$

Velocity of wave relative to source= $v - o = v$

Apparent change in wavelength, $\lambda_a = \frac{v}{f}$

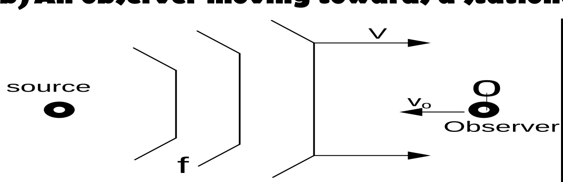
Velocity of wave relative to observer= $v - v_o$

Apparent change in frequency, $f_a = \frac{v-v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v-v_o}{v}\right) f}$$

Since $v - v_o < v$ then the apparent frequency appears to decrease when the observer moves away from the source

(b) An observer moving towards a stationary source



$$v_o - velocity\ of\ the\ observer$$

$$v - velocity\ of\ sound$$

$$f - frequency\ of\ the\ sound\ waves$$

Velocity of wave relative to source= $v - o = v$

Apparent change in wavelength, $\lambda_a = \frac{v}{f}$

Velocity of wave relative to observer= $v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v+v_o}{v}\right)f}$$

Since $v + v_s > v$ then the apparent frequency appears to increase when the observer approaches the source

Note:

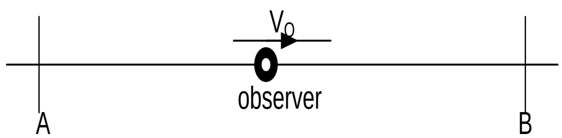
The motion of the observer has no effect on the wavelength of the sound but it affects the relative velocity of sound

Examples

1. An observer moving between two stationary sources of sound along a straight line joining them hears beats at a rate of $4s^{-1}$ at what velocity is the observer moving if the frequency of the sources is 50Hz and speed of sound in air is 340m/s

Solution

let observer move away from A



$4 = f_B - f_A$..........................(1)

Away from A:

$$f_A = \left(\frac{v-v_o}{v}\right)f$$

$$f_A = \left(\frac{340-v_o}{340}\right)x50 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Towards B:

$$f_B = \left(\frac{v+v_o}{v}\right)f$$

$$f_B = \left(\frac{340+v_o}{340}\right)x50 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

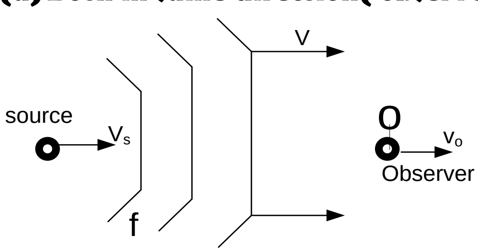
$$\mathbf{4} = \left(\frac{340+v_o}{340}\right)x50 - \left(\frac{340-v_o}{340}\right)x50$$

$$v_o = 13.6m/s$$

2. An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at a rate of $5.0s^{-1}$. At what velocity is the observer moving if the frequency of the sources is 600Hz and speed of sound in air is 330m/s. **An(1.38m/s)**

Case3: observer and source in motion

(a) Both in same direction(observer ahead of source)



$v_o - velocity\ of\ the\ observer$

$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v - v_s$

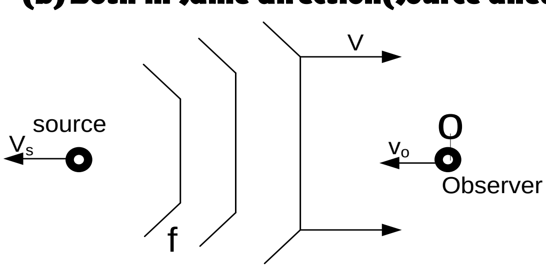
Apparent change in wavelength, $\lambda_a = \frac{v-v_s}{f}$

Velocity of wave relative to observer$= v - v_o$

Apparent change in frequency, $f_a = \frac{v-v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v-v_o}{v-v_s}\right)f}$$

(b) Both in same direction(source ahead of observer)



$v_o - velocity\ of\ the\ observer$

$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v + v_s$

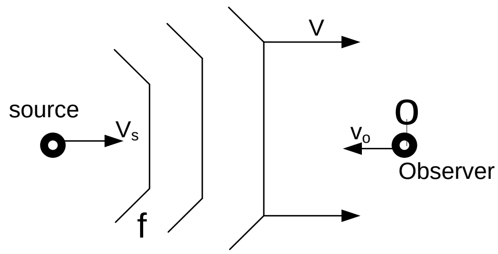
Apparent change in wavelength, $\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer$= v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v+v_o}{v+v_s}\right)f}$$

(c) Both moving towards each other



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source$= v - v_s$

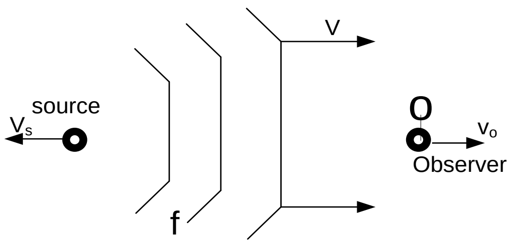
Apparent change in wavelength,$\lambda_a = \frac{v-v_s}{f}$

Velocity of wave relative to observer$= v + v_o$

Apparent change in frequency,$f_a = \frac{v+v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v+v_o}{v-v_s}\right)f}$$

(d) Both moving away from each other



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source$= v + v_s$

Apparent change in wavelength,$\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer$= v - v_o$

Apparent change in frequency,$f_a = \frac{v-v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v-v_o}{v+v_s}\right)f}$$

Generally

$$\boxed{f_a = \left(\frac{v \pm v_o}{v \pm v_s}\right)f}$$

Examples

1. A car A travelling at 36km/h has a horn of frequency 120Hz. A second car B travelling at 54km/h approaching the first car. What is the apparent frequency of the horn to an observer in the second car given that speed of sound in air 320m/s

Solution

$f_a = \left(\frac{v\pm v_o}{v\pm v_s}\right)f$

Since the observer is being approached the apparent frequency increase, therefore the numerator should be maximum and denominator minimum

$$f_a = \left(\frac{v+v_o}{v-v_s}\right)f$$

$$f_a = \left(\frac{320+15}{320-10}\right)x120$$

$$f_a = 129.68Hz$$

2. A cyclist and train approach each other with a speed of 10m/s and 20m/s respectively. A train sounded siren at 480Hz. Calculate the frequency of the note heard by the cyclist.
 (Speed of sound in air is 340m/s)
   (a) Before the train passes him
   (b) After the train has passed him

Solution

(a) Before train passes him

$f_a = \left(\frac{v\pm v_o}{v\pm v_s}\right)f$

Since the train approaches the observer, apparent frequency increases so numerator should be maximum and denominator minimum

$$f_a = \left(\frac{v+v_o}{v-v_s}\right)f$$

$$f_a = \left(\frac{320+10}{320-20}\right)x120$$

$$f_a = 525Hz$$

(b) After passing him

Since the train recedes away from the observer, apparent frequency decreases so numerator should be minimum and denominator maximum

$$f_a = \left(\frac{v - v_o}{v + v_s}\right) f$$
$$f_a = \left(\frac{340 - 10}{340 + 20}\right) x120$$
$$f_a = 440Hz$$

3. A train approaching a hill at 36km/h sounds a whistle of 580Hz. Wind is blowing at 72km/h in the direction of motion of the train. Calculate the frequency of the whistle as heard by an observer on the hill. (Speed of sound in air is 340m/s)

Solution

Apparent frequency $f_a = \left(\frac{v^1+v_o}{v^1-v_s}\right) f$

But resultant speed of sound $v^1 = v + v_w$

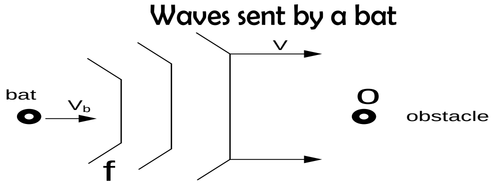
$$f_a = \left(\frac{[v + v_w] + v_o}{[v + v_w] - v_s}\right) f$$

$$f_a = \left(\frac{340 + 20 - 0}{340 + 20 - 10}\right) x580$$
$$f_a = 596.57Hz$$

4. A bat can locate an obstacle by emitting a high frequency sound wave through detecting the reflected waves. A bat flying at a steady speed of 5m/s emits sound waves of frequency 7800Hz and is reflected back.
   (a) Derive the equation of the frequency of sound waves reaching the bat after reflection
   (b) Calculate the frequency of the sound received by the bat given that speed of sound in air is 340m/s. **An(80328.36Hz)**

Solution

Waves sent by a bat



$v_b - velocity\ of\ the\ bat$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source $= v - v_b$

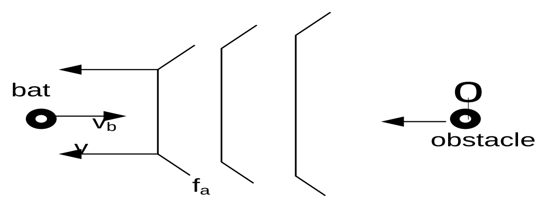
Apparent change in wavelength, $\lambda_a = \frac{v-v_b}{f}$

Velocity of wave relative to observer $= v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v - v_b}\right) f}$$

Waves reflected by the obstacle



$v_b - velocity\ of\ the\ bat$

$v - velocity\ of\ sound$

$f_a - frequency\ of\ the\ reflected\ sound\ waves$

Velocity of wave relative to source $= v$

Apparent change in wavelength, ${\lambda_a}^1 = \frac{v}{f_a}$

Velocity of wave relative to observer $= v + v_b$

Apparent change in frequency, ${f_a}^1 = \frac{v+v_b}{{\lambda_a}^1}$

$$\boxed{{f_a}^1 = \left(\frac{v + v_b}{v}\right) f_a}$$

$${f_a}^1 = \left(\frac{v + v_b}{v}\right)\left(\frac{v}{v - v_b}\right) f$$
$${f_a}^1 = \left(\frac{v + v_b}{v - v_b}\right) f$$

Applications of Doppler Effect

(i) Used in radar speed traps
(ii) Measurement temperature of hot gases
(iii) Used in measurement of speed of the star

Speed traps

- Microwaves of frequency f_o from a stationary radar set are directed towards a motor vehicle moving with speed u
- Microwaves reflected from the moving car are detected at the radar
- The reflected signal mixes with the transmitted signals to obtain beats
- The beat frequency Δf which is equal to the difference between the frequency of the received and transmitted signal is determined
- The speed of the vehicle is $u = \frac{v\Delta f}{2f_o}$

Measurement of plasma temperature

- The broadening $\Delta\lambda$ of a spectral line emitted by the plasma is determined using a diffraction grating
- $\frac{\Delta\lambda}{\lambda_o} = \frac{2u}{c}$
- Assume $u = v_{rms}$
- $\frac{1}{2}\boldsymbol{mu^2} = \frac{3}{2}\boldsymbol{RT}$ where m= $\boldsymbol{molar\ mass}$

  $\boldsymbol{u} = \left(\frac{\boldsymbol{3RT}}{\boldsymbol{m}}\right)^{\frac{1}{2}}$
- $\boldsymbol{T} = \frac{\boldsymbol{m}}{\boldsymbol{12R}}\left(\frac{\Delta\lambda}{\lambda_o}\right)^2 \boldsymbol{C^2}$

Speed of star

- The wavelength , λ of light emitted by the star is measured
- The absorption spectrum of an elemen known to be in the star is examined.
- The wavelenght λ^1 of the missing line is measured
- Doppler shift $= |\lambda^1 - \lambda|$

  $\Rightarrow \left|\frac{\lambda^1-\lambda}{\lambda}\right| = \frac{u_s}{C}$

  $\Rightarrow u_s = \left|\frac{\lambda^1-\lambda}{\lambda}\right| C$

Exercise

1. A car travelling at 72 kmh^{-1} has a siren which produces sound of frequency 500Hz. Calculate the difference between the frequency of sound heard by an observer by the roadside as the car approaches and recedes from the observer . [Speed of sound in air = 320 m s^{-1}]. **An(62.7Hz)**
2. An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at the rate of 4.0. At what velocity is the observer moving if the frequencies of the sources are 500Hz and the velocity of sound when the observer makes the observation is 340 m s^{-1}?
3. a) Explain what is meant by Doppler effect (2)

   b) Deduce an expression for the frequency heard by an observer when:

   (i) He is stationary and a source of sound is moving towards him. (3)

   (ii) He is moving towards a stationary source of sound. (3)

   c) A bat flying at a speed of 30ms^{-1} towards an obstacle emits sound waves of frequency $2.5x10^8$ HZ. The bat hears an echo 0.5 s later. If the speed of the sound in air is 340ms-^{1}, how far is the obstacle from the bat when the bat hears the echo?. Find the apparent freq. Of the echo received by the bat (4)
4. (a) A source of sound moving with velocity u_s approaches an observer moving with velocity u_o, in the same direction. Derive the expression for the frequency of sound heard by the observer. (05 marks)

   (b) Explain what happens to the pitch of the sound heard by the observer in (b) above when the

   (i) observer moves faster than the source (02 marks)

   (ii) observer's velocity is equal to that of the sound (02 marks)

   (c) (i) What is meant by Doppler effect?

   (ii) A car sounds its horn as it travels at a steady speed of 15 ms^{-1} along a straight road between two stationary observers A and B. The observer A hears a frequency of 538 Hz while B hears a lower frequency. Calculate the frequency heard by B, assuming the speed of Sound in air is 340ms $^{-1}$ (4 marks)

(d) (i) Explain how beats are produced

(ii) An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at the rate of 4.0. At what velocity is the observer moving if the frequencies of the sources are 500Hz and the velocity of sound when the observer makes the observation is 340 m s $^{-1}$?

5. (a) (i) A police car sounds a siren of 1000 Hz as it approaches a stationary observer. What is the apparent frequency of the siren as heard by the observer if the speed of sound in air is 340?

(ii) Give any three applications of the Doppler effect

(b) An observer standing by the roadside hers sound of frequency 600 Hz coming from the horn of an approaching car. When the car passes, the frequency appears to change to 560 Hz. Given that the speed of sound in air is 320 ms $^{-1}$, calculate the speed of the car. (5 marks)

6. (a) Describe briefly one application of the Doppler effect (2 marks)

(b) (i) Derive an expression for the frequency of sound observed by a stationary observer infront of a source moving with a velocity U m/s and emitting f pulse each second given that speed of sound on the day is C m/s

(ii) A police car travelling at 108 km/hr is chasing a lorry which is travelling at 72 km/hr. The police car given Emits sound of frequency 400 Hz as it approaches the lorry. Calculate the apparent frequency of the note heard by the lorry driver.

7. (a) (i) Define Doppler effect as applied to sound (1)

(ii) Explain briefly how Doppler effect can be used to measure the star (3)

(iii) A stationery police car by the roadside emits a siren of frequency f_s infront of an approaching taxi moving at a speed of vm/s. Find the expression for frequency received by the taxi driver if the speed of sound on that day was C m/s (3)

(b) A police car operating its siren of frequency 384 Hz travels at 90km/hr. Another car travels at 72 km/h. Given that the speed of sound on the day is 340 m/s, calculate the apparent frequency of the siren as heard by the occupants of the second car if they are travelling away from the police car. (4)

8. (a) Show that if a dog is to chase little Tamale,

(i) the apparent frequency of sound heard by Tamale is $f_T{}^1 = \left(\frac{V-U_T}{V-U_D}\right) f_D$ (5 marks)

(ii) the apparent frequency of sound heard by the dog is $f_D{}^1 = \left(\frac{V+U_D}{V+U_T}\right) f_T$ (3 marks)

Where V – speed of sound in air
U_D – speed of the dog
U_T – speed of Tamale
f_D – frequency of sound from dog
f_T – frequency of sound from Tamale

(b) (i) What is meant by the Doppler effect?

(ii) A police car sounds a siren of 1000 Hz as it approaches a stationary observer. What is the apparent frequency of the siren as heard by the observer if the speed of sound in air is 340ms $^{-1}$ (3 marks)

(iii) State one application of the Doppler effect (1 mark)

9. (a) (i) Explain Doppler effect (2 marks)

(ii) A car travelling at 10ms $^{-1}$ sounds its horn that sends sound waves of frequency 500Hz and this is heard in another car which is travelling behind the first one in the same direction with a velocity of 20 ms $^{-1}$. What frequency will be heard by

(i) the driver of the second car? (3 marks)

(ii) an observer standing some distance ahead of the first car (velocity of sound in air = 330 ms $^{-1}$) (3 marks)

(b) (i) Define Doppler effect as applied to sound (1)

(ii) Explain briefly how Doppler effect can be used to measure the star (3)

(iii) A stationery police car by the roadside emits a siren of frequency f_s infront of an approaching taxi moving at a speed of vm/s. Find the expression for frequency received by the taxi driver if the speed of sound on that day was C m/s

(C) (i) Derive an expression for the frequency of sound observed by an observer moving with a velocity μo m/s towards the stationary source emitting fs pulse each second given that speed of sound on the day is C m/s

(ii) A police car operating its siren of frequency 384 Hz travels at 90km/hr. Another car travels at 72 km/h. Given that the speed of sound on the day is 340 m/s, calculate the apparent frequency of the siren as heard by the occupants of the second car if they are travelling away from the police car. (4)

(d) (i) What is meant by **Doppler effect?** (1 mark)

(ii) A car sounds its horn as it travels at a steady speed of 20ms^{-1} along a straight road between two stationary observers X and Y. Observer X hears a frequency of 560H$_z$ while Y hears a lower frequency.Calculate the frequency heard by Y assuming the speed of sound in air is 330ms^{-1}

(iii) A man moving at 10m/s while blowing a whistle at freq of 1.5kHz towards a vertical tall wall. Calculate the apparent freq. of an echo received by the man

RESONANCE

This is a condition obtained when a system is set to oscillate at its <u>own natural frequency</u> as a result of impulses received form another system vibrating at the <u>same frequency</u>

Other terms

Fundamental frequency

This is the lowest possible frequency that an instrument can produce.

Overtones

These are note of higher frequencies than the fundamental frequency produ ed by an instrument.

Harmonic

These is one of the frequencies that can be produced by a particular instrument

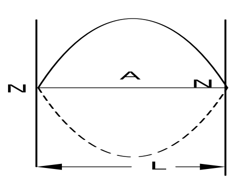
WAVES ON A STRETCHED STRING

When a stretched string is plucked, a progressive wave is formed and it travels to both ends which are fixed and these waves are reflected back to meet the incident wave. The incident and reflected waves both have the same speed, frequency and amplitude and therefore when they superimpose a stationary wave is formed.

Modes of vibration

When a string is plucked in the middle, the wave below is produced

(a) 1st harmonic (fundamental frequency)



$$l = \frac{\lambda}{2}$$
$$\lambda = 2l$$
$$v = f\lambda$$
$$\boxed{v = 2lf_o}$$

A is antinodes;

These are points on a stationary wave where particles have maximum displacement.

N is nodes;
This is a point on a stationary wave in which particles are always at rest (zero displacement)

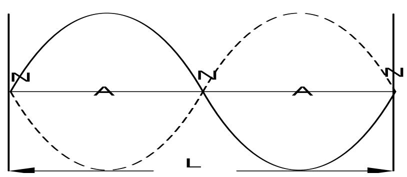
Note:

- The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$ where λ is wavelength.
- When a stationary wave is produced, the distance between the source and reflector is a multiple of $\frac{1}{2}\boldsymbol{\lambda}$.

$$\boxed{distance = n\frac{\boldsymbol{\lambda}}{2}}$$

Where n is the number of loops ie n is $1,2,3 \ldots \ldots \ldots \ldots$

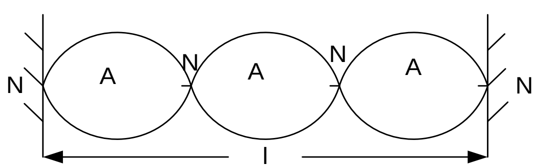
(b) 2nd harmonic (1st overtone)



$l = \lambda$

$v = f\lambda$
$v = lf_1$
$2lf_o = lf_1$
$\boxed{f_1 = 2f_o}$

(c) 3rd harmonic (2nd overtone)



$\lambda = \frac{2}{3}l$
$v = f\lambda$
$2lf_o = \frac{2}{3}lf_2$
$\boxed{f_1 = 3f_o}$

$l = \frac{3}{2}\lambda$

Generally $\boxed{f_n = nf_o}$

$\boxed{f_n = \frac{n\,v}{2\,l}}$ $n = 1,2,3,4,5,6 \quad n^{th} - harmonic$

Velocity of a transverse wave along a stretched string

The velocity of a wave on the sting depends on the following

(i) Tension T
(ii) Mass m
(iii) Length l

V $\propto T^x m^y\, l^z$
V $= \mathrm{k}\, \mathrm{T}^{\mathrm{x}}\mathrm{m}^{\mathrm{y}}\, \mathrm{l}^{\mathrm{z}}$...........(x)
[V] $= [\,K\,][\,T]^x\,[m\,]^y\,[\,l^z]$
K is a dimensionless constant
$LT^{-1} = (MLT^{-2})^{\,x}\,(M)^y\;\;(L)^z$
For powers of T
$-2x = -1$....................(1)

$x = \frac{1}{2}$
For powers of M,
$0 = x + y$...................(2)
$0 = \frac{1}{2} + y$

$$y = -\frac{1}{2}$$

For powers of L,

$1 = x + z$

$$z = \frac{1}{2}$$

$V = k\,T^x m^y l^z$

$V = k\,T^{\frac{1}{2}} m^{\frac{-1}{2}} l^{\frac{1}{2}}$

$$\boxed{V = \sqrt{\frac{Tl}{m}}}$$

Examples

1. A string of length 0.5m has a mass of 5g. The string is stretched between two fixed points and plucked. If the tension is 100N, find the frequency of the second harmonic

Solution

$$v = \sqrt{\frac{Tl}{m}}$$

$$v = \sqrt{\frac{100x0.5}{5x10^{-3}}}$$

$$v = 100m/s$$

$$v = f_0\lambda$$

$$v = 2lf_0$$

$$100 = 2x0.5f_0$$

$$f_0 = 100Hz$$

$$f_2 = 2\,f_0$$

$$f_2 = 2x100$$

$$f_2 = 200Hz$$

Alternatively

$$v = \sqrt{\frac{Tm}{l}}$$

$$v = \sqrt{\frac{100x5x10^{-3}}{0.5}}$$

$$v = 100m/s$$

$$f_n = \frac{n\,v}{2l}$$

$$f_2 = \frac{2x\,100}{2x0.5}$$

$$f_2 = 200Hz$$

2. A wire under a tension of 20N is plucked at the middle to produce a note of frequency 100Hz. Calculate the;
   (i) diameter of the wire if its length is 1m and has a density of $600kgm^{-3}$
   (ii) frequency of the first overtone $\mathbf{An(200Hz)}$

solution

$$v = \sqrt{\frac{Tl}{m}}$$

$$2lf_0 = \sqrt{\frac{Tl}{m}}$$

$$2x1x100 = \sqrt{\frac{20x1}{m}}$$

$$m = 0.0005kg$$

$$\rho = \frac{m}{volume}$$

$$volume = \frac{0.0005}{600} = 8.33x10^{-7}m^3$$

$$volume = \pi r^2 l$$

$$r = \sqrt{\left(\frac{8.33x10^{-7}}{\pi}\right)}$$

$$r = 5.15x10^{-4}m$$

$$d = 2r = 2x5.15x10^{-4}$$

$$= 1.03x10^{-3}m$$

3. A stretched wire of length 0.75m, radius 1.36 mm and density 1380 kg m $^{-3}$ is clamped at both ends and plucked in the middle. The fundamental note produced by the wire has the same frequency as the first overtone in a pipe of length 0.15 m closed at one end.
   (i) Sketch the standing wave pattern in the wire
   (ii) Calculate the tension in the wire

   [The speed of sound along the stretched wire is $\sqrt{\left(\frac{T}{\mu}\right)}$ where T is the tension in the wire and μ is the mass per unit length. Speed of sound in air = 330m s $^{-1}$]

Solution

For the wire at fundamental note

$$V = \sqrt{\left(\frac{T}{\mu}\right)}$$

$$2lf_0 = \sqrt{\frac{Tl}{m}}$$

But $\boldsymbol{m} = \boldsymbol{\rho}\pi r^2 l$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\rho\pi r^2}}$$

$$f_0 = \frac{1}{2x0.75}\sqrt{\frac{T}{\pi(1.36x10^{-3})^2 x1380}}$$ ....**(i)**

For a closed at first overtone

$v = f_1\lambda$

$$330 = f_1\frac{4}{3}l$$

$$f_1 = \frac{330x3}{4x0.15}$$ ....**(ii)**

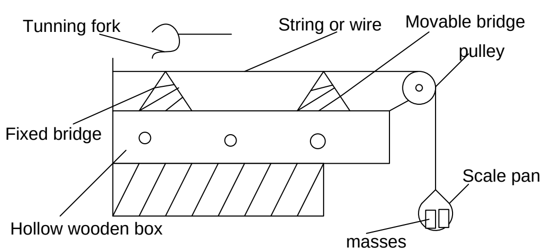
$$\frac{1}{2x0.75}\sqrt{\frac{T}{\pi(1.36x10^{-3})^2 x1380}} = \frac{330x3}{4x0.15}$$

$$T = 4.91x10^4 N$$

4. The wire of a guitar of length 50cm and mass per unit length $1.5x10^{-3}kgm^{-1}$ is under a tension of 173.4N. The wire is plucked at its mid-point. Calculate the;
   (i) frequency **An(340Hz)**
   (ii) wavelength of the tfundamental note **An(1.0m)**
5. A string of length 50cm vibrates in a fundamental mode. Find fundamental frequency of vibration. **An(330Hz)**
6. A wire of length 0.60m and mass $9x10^{-4}kg$ is under a tension of 135N. The wire is plucked such that it vibrates in its third harmonic. Calculate
   (iii) Fundamental frequency **An(250Hz)**
   (iv) Frequency of the third harmonic **An(750Hz)**
7. A wire of length 0.4m and mass $1.2x10^{-3}kg$ is under a tension of 120N. The is plucked in the middle. calculate
   (v) Fundamental frequency **An(250Hz)**
   (vi) Frequency of the third harmonic **An(750Hz)**

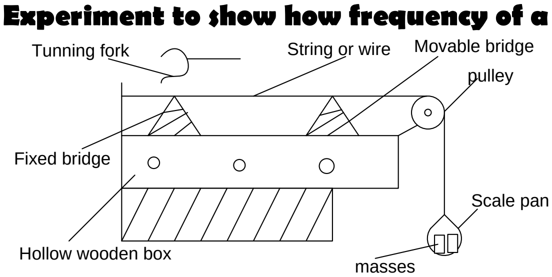
Factors on which frequency of a stretched string depends

Experiment to investigate the variation of frequency of a stretched string with length



- ❖ The experiment is set up as shown above
- ❖ Pluck the string in the middle and place a sounding tuning fork near it
- ❖ Move the bridge B towards A until when a loud sound is heard
- ❖ The distance l between the bridges is measured and recorded together with the frequency of the tuning fork
- ❖ Repeat the above procedures using different tuning forks of different frequency
- ❖ Tabulate your results including values of $\frac{1}{l}$
- ❖ Plot a graph of against $\frac{1}{l}$
- ❖ It's a straight line passing through the origin implying that f ∝ $\frac{1}{l}$

Experiment to show how frequency of a stretched string varies with tension



- ❖ The experiment is set up as shown above
- ❖ The length l between two bridges is kept constant
- ❖ A suitable mass m is attached to the free end of a string (scale pan)
- ❖ Pluck the string in the middle and a tuning fork of known frequency f is sounded near it
- ❖ Vary the mass on the scale pan until when a loud sound is heard
- ❖ Record the mass of the corresponding frequency f in a suitable table including values of f^2

- Repeat the above procedures using different tuning forks of different frequency
- Plot a graph of f^2 against m
- It's a straight line passing through the origin implying that $f^2 \propto m$
- Since $T = mg$, it implies $f \propto \sqrt{T}$ hence frequency increases with increase in frequency

Resonance of air in pipes

When air is blown in a pipe, a longitudinal wave is formed. This wave travels along the pipe and if the pipe is closed the wave will be reflected back. The incident and reflected wave both have the same speed, same frequency and same amplitude. This results into formation of a stationary wave.

These are two type of pipe for air vibrations.

(i) **Open pipes**

This is one that has both ends open eg trumpet, a flute

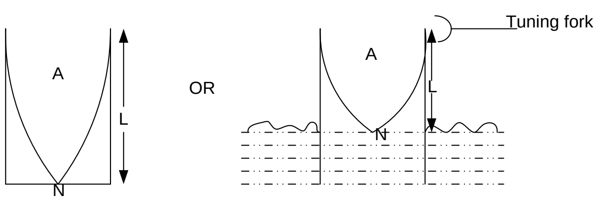
(ii) **Closed pipes**

It is one in which one end is open, while the other is closed eg a long drum.

a) Modes of vibration in closed pipes

For closed pipes, a node is formed at a closed end and an antinode at the open end

First harmonic / fundamental note



The length of the air column is L

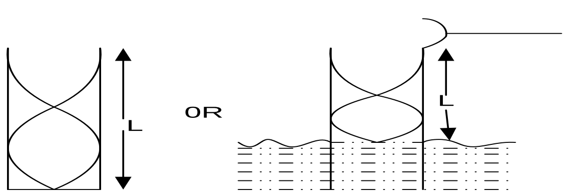
$$L = \frac{1}{4}\lambda$$
$$\lambda = 4\,l$$
$$v = f_0\lambda$$
$$\boxed{v = 4lf_0}$$
$$f_0 = \frac{v}{4l}$$

f_0 is the fundamental frequency

Describe the motion of air I a tube closed at one end and vibrating in its fundamental frequency

Air at end A vibrates with maximum amplitude. The amplitude of vibration decreases as end N is approached. Air at N is stationary. End N is node while end A is antinode

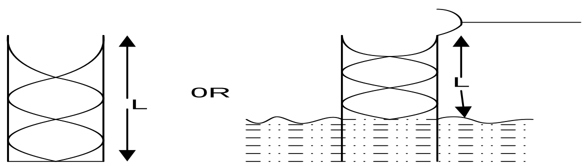
First overtones / third harmonic



The length of the air column is L

$$L = \frac{3}{4}\lambda$$
$$\lambda = \frac{4}{3}l$$
$$v = f_1\lambda$$
$$4lf_0 = f_1\frac{4}{3}l$$
$$f_1 = 3\,f_0$$

Second overtones/ fifth harmonic



The length of the air column is L

$$L = \frac{5}{4}\lambda$$

$$\lambda = \frac{4}{5}\, l$$
$$v = f_2\lambda$$
$$4lf_0 = f_2 x \frac{4\, l}{5}$$
$$f_2 = 5\, f_0$$

Note: In closed pipes, harmonics produced must be with frequencies $f_0, 3f_0, 5f_0, 7f_0 \ldots\ldots\ldots\ldots\ldots$

This implies that only <u>odd harmonics</u> are produced can be produced by closed pipes

$\boxed{f_n = \frac{n\,v}{4\,l}}$ $n = 1,3,5,7,9 \ldots\ldots$ $n^{th} - harmonic$

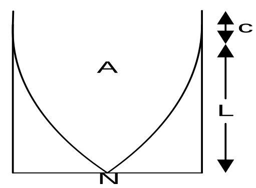
Variation of pressure with displacement of air in a closed pipe

At the mouth of the pipe, the air is <u>free to move</u> and therefore the displacement of air molecules is <u>large</u> and <u>pressure is low</u>. At the closed end the molecules are <u>less free</u> and the displacement is <u>minimal</u> and the <u>pressure is high</u>

END CORRECTIONS

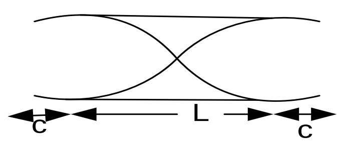
An antinode of stationary wave in a pipe is not formed exactly at the end of the pipe. Instead it is displaced by a distance, c. This distance is called the end correction

The effective length of a wave in the closed pipe of length l is $l + c$



$$l + c = \frac{\lambda}{4}$$

The effective length of a wave in an open pipe of length l is $l + 2c$



$$l + 2c = \frac{\lambda}{2}$$

Note:

C is related to the radius of the pipe by an equation $c = 0.6r$ implying that the end correction is more significant for wide pipes

Examples

1. A cylindrical pipe of length 30cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 825Hz sounded near the open end of the pipe. Determine the mode of vibration of air assuming there is no end correction. Take speed of sound in air as 330m/s.

Solution

$$f_n = \frac{n\,v}{4\,l}$$

$$825 = \frac{n\,x330}{4\,x0.3}$$

$$n = 3$$

But $n = 1,3,5,7,9 \ldots \ldots$

Mode of vibration is third harmonic

2. A cylindrical pipe of length 29cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 860Hz sounded near the open end of the pipe. Determine the mode of vibration of air and end correction. Take speed of sound in air as 330m/s.

Solution

$$f_n = \frac{n\,v}{4\,l}$$

$$860 = \frac{n\,x330}{4\,x0.29}$$

$$n = 3.02$$

$n = 1,3,5,7,9 \ldots \ldots$

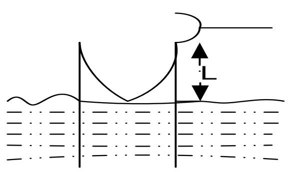
Mode of vibration is fifth harmonic

$$f_n = \frac{n\,v}{4\,(l+c)}$$

$$c = \frac{5x330}{4\,x860} - 029 = 0.1897m$$

3. A long tube is partially immersed in water and a tuning fork of 425Hz is sounded and held above it. If the tube is gradually raised, find the length of the air column when resonance first occurs. [speed of sound in air is 340m/s]

Solution



$f = 425Hz,\ v = 340ms^{-1}$

$v = f\lambda$

$340 = 425x\boldsymbol{\lambda}$

$\lambda = 0.8\ m$

$$L = \frac{1}{4}\boldsymbol{\lambda}$$

$$L = \frac{1}{4}x0.8$$

$$L = 0.2\ m$$

4. A tube 100cm long closed at one end has its lowest frequency at 86.2Hz. With a tube of identical dimensions but open at both ends, the first harmonic occurs at 171Hz. Calculate
   (i) The speed of sound
   (ii) End correction

Solution

(i) For closed pipe: $v = 4(l+c)f_0$

$v = 4(1+c)86.2$……….(1)

For open pipe: $v = 2(l+2c)f_0$

$v = 2(1+2c)171$……….(2)

Equating (1)and (2)

$$4(1+c)86.2 = 2(1+2c)171$$

$$c = 8.25x10^{-3}m$$

(ii) $v = 4(1+c)86.2$

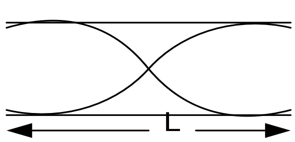
$$v = 4(1+8.25x10^{-3})86.2$$

$$v = 347m/s$$

b) Modes of vibration in open pipes

In open pipes, antinodes are found at the two open ends of the pipe

First harmonic / fundamental note



$$l = \frac{1}{2}\lambda$$

$$\lambda = 2\,l$$

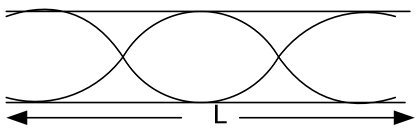
$v = f_0\lambda$

$v = 2lf_0$

$$\boxed{f_0 = \frac{v}{2\,l}}$$

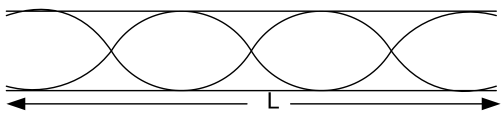
f_0 is the fundamental frequency

Second harmonic / first overtone



$$l = \lambda$$
$$v = f_1 \lambda$$
$$2lf_0 = f_1 l$$
$$f_1 = 2 f_0$$

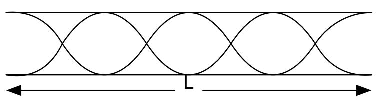
Third harmonic / Second overtone



$$l = \frac{3}{2}\lambda$$

$$\lambda = \frac{2}{3} l$$
$$v = f_2 \lambda$$
$$2lf_0 = f_2 x \frac{2l}{3}$$
$$f_2 = 3 f_0$$

Fourth harmonic / Third overtone



$$l = 2\lambda$$

$$\lambda = \frac{l}{2}$$
$$v = f_3 \lambda$$
$$2lf_0 = f_3 x \frac{l}{2}$$
$$f_3 = 4 f_0$$

Note:

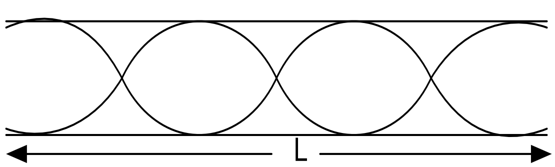
In open pipes, harmonics produced must be with frequencies $f_0, 2f_0, 3f_0, 4f_0, 5f_0 \ldots \ldots \ldots \ldots \ldots$
Open pipes produce both odd and even harmonics and this is why open pipes are preferred as musical instruments.

$\boxed{f_n = \frac{n\,v}{2\,l}}$ $n = 1,2,3,4,5,6$ $\quad n^{th} - harmonic$

Examples

1. The frequency of third harmonic in an open pipe is 660Hz, if the speed of sound in air is 330m/s. Find;
   (i) the length of the air column
   (ii) the fundamental frequency

Solution



i) $f = 660Hz, \quad v = 330ms^{-1}$

$$v = f\lambda$$
$$330 = 660x\boldsymbol{\lambda}$$
$$\lambda = 0.5\ m$$

$$L = \frac{3}{2}\boldsymbol{\lambda}$$
$$L = \frac{3}{2}x0.5$$
$$L = 0.75\ m$$

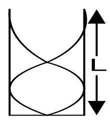
$ii)$ $f_2 = 3 f_0$

$$f_0 = \frac{660}{3}$$
$$f_0 = 220Hz$$

2. If the velocity of sound in air is 330m/s and the fundamental frequency is 110Hz in a closed tube.;
   (i) What is the approximate length of the tube if the tube is resonate at the first overtone
   (ii) What would be the fundamental frequency if the tube was open at both ends

Solution

i) $f_0 = 110Hz,\ v = 330m/s$



$f_1 = 3\,f_0$

$f_1 = 3\,x110 = 330Hz$

$v = \quad f_1\lambda$

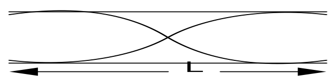
$\lambda = \frac{330}{330} = 1\,m$

$L = \frac{3}{4}\boldsymbol{\lambda}$

$L = \frac{3}{4}x1$

$L = 0.75\,m$

ii)



$L = \frac{1}{2}\boldsymbol{\lambda}$

$\boldsymbol{\lambda} = 2L$

$\boldsymbol{\lambda} = 2x0.75$

$\boldsymbol{\lambda} = \ 1.5m$

$v = \quad f_o\lambda$

$f_0 = \frac{330}{1.5}$

$f_0 = 220Hz$

3. Two organ pipes of length 50cm and 51cm respectively give beats of frequency 7Hz when sounding their fundamental notes together. Neglecting the end corrections calculate the velocity of sound in air.

Solution

$f_1 = \frac{v}{2\,l_1}$ and $f_2 = \frac{v}{2\,l_2}$

$f_b = \frac{v}{2\,l_1} - \frac{v}{2\,l_2}$

$7 = \frac{v}{2}\left(\frac{1}{0.5} - \frac{1}{0.51}\right)$

$\boldsymbol{v = 357m/s}$

4. Two organ open pipes of length 50cm and 51cm respectively give beats of frequency 6Hz when sounding heir fundamental notes together. Neglecting the end corrections calculate the velocity of sound air $\boldsymbol{An(306ms^{-1})}$
5. Two organ pipes of length 92cm and 93cm respectively give beats of frequency 3.0Hz when sounding their fundamental notes together. If the end corrections are 1.5cm and 1.8cm respectively. Calculate the velocity of sound in air.

Solution

$f_1 = \frac{v}{2(l_1+2\,C_1)}$ and $f_2 = \frac{v}{2(l_2+2\,C_2)}$

$f_b = \frac{v}{2(l_1+2\,C_1)} - \frac{v}{2(l_2+2\,C_2)}$

$3 = \frac{v}{2}\left(\frac{1}{0.92+2x0.015} - \frac{1}{0.93+2x0.018}\right)$

$\boldsymbol{v = 344.14m/s}$

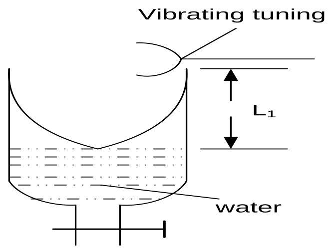
8. A glass tube, open at the top, is held vertically and filled with water. A tuning fork vibrating at 264 Hz is held above the tube and water is allowed to flow out slowly. The first resonance occurs when the water level is 32.5 cm from the top while the second resonance occurs when the level is 96.3 cm from the top. Find:
   (i) the speed of sound in the air column
   (ii) the end correction
9. A uniform tube 50cm long, is held vertically and filled with water. A tuning fork vibrating at 512 Hz is held above the tube and water is allowed to flow out slowly. The first resonance occurs when the water level is 12 cm from the top while the second resonance occurs when the level is 43.3 cm from the top. Find the lowest frequency to which the air could resonate if the tube were empty.

Note:

Different instruments produce <u>different number</u> of overtones. The numbers of overtones produced affect the quality of the note played. Hence the quality of the notes produced by different instruments are different

Qn: Explain why a musical note played on one instrument sounds different from the same note played on another instrument

Experiment: To measure velocity of sound in air by Resonance tube and a tuning fork of a known frequency



- A glass tube which can be drained from the bottom is filled with water.
- A <u>sounding tuning fork</u> of known frequency f is <u>brought to the mouth of tube</u>.
- The water is then slowly drained until <u>a loud sound is heard</u>.
- The tap is closed and the length of the air column l_1 is <u>measured</u>.
- The tuning fork is <u>sounded again</u> at the mouth of the tube and water is drained further until a loud <u>sound is heard</u>.
- The tap is closed and the length of the air column l_2 is <u>measured</u>.
- Velocity of sound in air is obtained from $v = 2f(l_2 - l_1)$

Theory

$l_1 + c = \frac{1}{4}\lambda$ ……………………(1)

$l_2 + c = \frac{3}{4}\lambda$ ……………………(2)

Equation (2) – Equation (1)

$$(l_2 + c) - (l_1 + c) = \frac{3}{4}\lambda - \frac{1}{4}\lambda$$

$$l_2 - l_1 = \frac{1}{2}\lambda$$

$$\lambda = 2(l_2 - l_1)$$

But $v = f\lambda$

$$\boxed{v = 2f(l_2 - l_1)}$$

Example

1. A tuning fork of frequency 256Hz produces resonance in a tube of length 32.5cm and also in one of length 95cm. Calculates the speed of sound in air column of the tube.

Solution

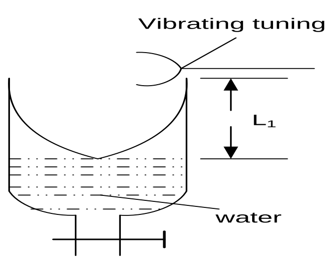
$v = 2f(l_2 - l_1)$

$v = 2\ x\ 256x\left(\frac{95 - 32.5}{100}\right)$

$v = 320ms^{-1}$

2. A uniform tube 50cm long is filled with water and a vibrating tuning fork of frequency 512Hz is sounded and held above the tube. When the level of water is gradually lowered, the air column resonates with the tuning fork when its length is 12cm and again when it is 43.3cm. Calculate
   (i) Speed of sound $An(322.56ms^{-1})$
   (ii) The end corrections $An(3.67cm)$
   (iii) Lowest frequency to which the air can resonate if the tube is empty **An(281.27Hz)**

Experiment: To measure velocity of sound in air using Resonance tube and different tuning forks of a known frequencies



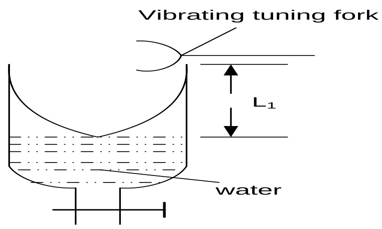
- A glass tube which can be drained from the bottom is filled with water.
- A <u>sounding tuning fork</u> of known frequency f is <u>brought to the mouth of tube</u>.
- The water is then slowly drained until <u>a loud sound is heard</u>.
- The tap is closed and the length of the air column l is <u>measured</u>.
- The experiment is <u>repeated with other</u> tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$
- A graph of l against $\frac{1}{f}$ is plotted and <u>slope s is obtained</u>
- Velocity of sound in air is obtained from $v = 4s$

Theory

$$l + c = \frac{1}{4}\lambda$$
$$l = \frac{1}{4}\lambda - c$$

But $\lambda = \frac{v}{f}$
$$l = \frac{1}{4f}v - c$$

Experiment: To measure end corrections using Resonance tube and different tuning forks of a known frequencies



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column l is measured.
- The experiment is repeated with other tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$
- A graph of l against $\frac{1}{f}$ is plotted and the intercept c of the l axis determined from line graph
- The intercept c is the end corrections

VELOCITY OF SOUND IN GASES

Velocity of sound in gasses depends on the pressure and density of the gas

$$v \propto P\rho$$
$$v = kP\rho$$

Where k- constant

$[V] = [K]\,[P]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y \ldots\ldots\ldots(1)$

For M: $0 = x + y \ldots\ldots\ldots(2)$

For T: $-1 = -2x \ldots\ldots\ldots(3)$

$$x = \frac{1}{2}$$
$$y = -x$$
$$y = -\frac{1}{2}$$

$$v = k\sqrt{\frac{P}{\rho}}$$

But if $k = \sqrt{\gamma}$

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

γ-ratio of molar heat capacity at constant pressure to molar heat capacity at constant volume

Note: The speed of sound in air depends on pressure, density and temperature

Explanation

When temperature of air is increased, the pressure increases. If the air is not restricted in volume it expands leading to a reduction in density. From the above expression a reduction in density leads to increase in velocity. Hence increase in temperature leads to increase in velocity of sound in air

VELOCITY OF SOUND IN SOLIDS

Velocity of sound in solids depends on the young's modulus E and density ρ of the solid

$$v \propto E\rho$$
$$v = kE\rho$$

Where k- constant

$[V] = [K]\,[E]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y \ldots\ldots\ldots(1)$

For M: $0 = x + y \ldots\ldots\ldots(2)$

For T: $-1 = -2x \ldots\ldots\ldots(3)$

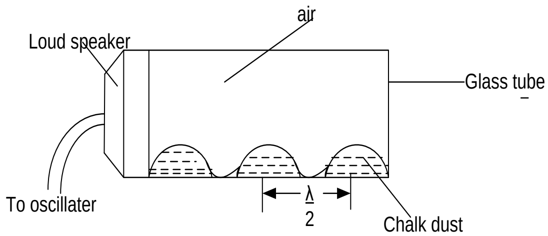
$$x = \frac{1}{2}$$
$$y = -x$$
$$y = -\frac{1}{2}$$

$$v = k\sqrt{\frac{E}{\rho}}$$

But if $k = 1$

$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

Measurement of speed of sound in air using Kundt's dust tube

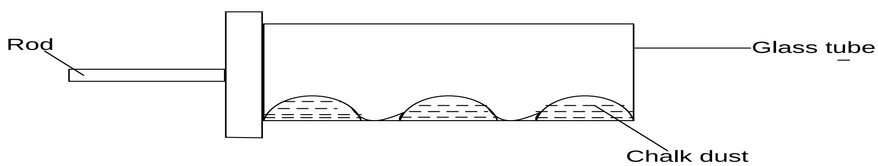


- A long glass tube is placed horizontally with chalk dust inside it
- The open end is fitted with a loud speaker which is connected to an oscillator of known frequency f
- When the oscillator is switched on, sound is produced and a stationary wave is formed in the glass tube which makes the power to settle into well-spaced heaps,
- Measure the distance l between the two consecutive heaps
- Wavelength of the wave generated is given by $\lambda = 2l$
- Speed of sound in air is got from $v = 2lf$

Note: Heaps are found at points where there are no vibrations (nodes)
Measurement of l from outside the tube may not be accurate hence a source of error

Examples

In an experiment to determine the speed of sound in air in a tube, chalk dust settled in heaps as shown in the diagram below;



If the frequency of the vibrating rod is 220Hz and the distance between three consecutive heaps is 1.50m, calculate the speed of sound in air

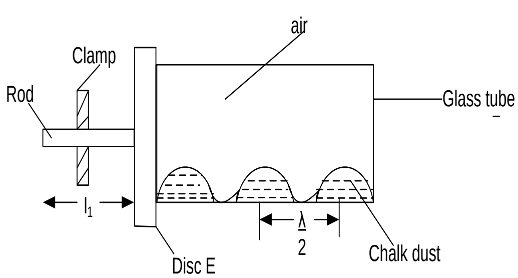
Solution

$\lambda = 1.50m$
$v = \lambda f$

$v = 1.5x220$
$\boldsymbol{v = 330m/s}$

Measurement of speed of sound in a rod using Kundt's dust tube

- Sprinkle some chalk dust along the interior of the tube



- Sprinkle some chalk dust along the interior of the tube
- Clamp the rod at its mid-point with one end projecting into tube
- Connect disc E to the end of the tube such that it just covers the side of the tube
- Strike the rod using a piece of a cloth until when the [powder in the tube settles into heaps
- Measure the distance l_2 between two consecutive heaps and l_1 of the end

- Velocity of sound in the rod is obtained from $v_r = \frac{v_a l_1}{l_2}$

Where v_a is the velocity of sound in air which is in the tube

Theory

$v_r = 2f_r l_1 \ldots\ldots\ldots\ldots\ldots\ldots.(1)$ Where f_r is frequency of the rod

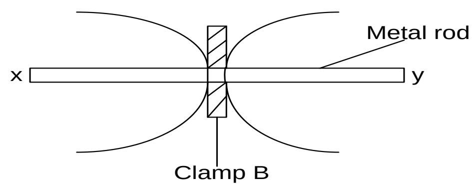
$v_a = 2f_a l_2 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$

But $f_r = f_a$

$$\frac{v_r}{v_a} = \frac{2f_r l_1}{2f_a l_2}$$

$$v_r = \frac{v_a l_1}{l_2}$$

Measurement of speed of sound in a rod



- Rod xy is fixed a clamp B at its a middle point
- The rood is then stroked and a stationary wave is formed due to the vibration of the rod
- A node is formed at the mid-point of the node and antinode ta the free ends x and y
- Measure length l of the rod
- Wavelength of the wave generated by the rod is given by $\lambda = 2l$
- Speed of sound in the rod is got from
$$v = 2lf$$

Uneb 2016

(a) What is meat by the following terms as applied to a waves
   (i) Resonance . (01mark)
   (ii) Frequency. (01mark)

(b) Explain with the aid f suitable diagrams, the terms **fundamental note** and **overtone** as applied to a vibrating air in a closed pipe. (05marks)

(c) Describe how you would determine the speed of sound in air using a resonane tube and several tunning forks. (05marks)

(d) (i) Explain the formation of beats (02marks)
   (ii) Derive the expression for beat frequency (03marks)

(e) Two observers **A** and **B** are provide with sources of sound of frequency 750Hz. If **A** remains stationary while **B** moves away at a velocity of $2.0ms^{-1}$, find the number of beats heard per second by **A**.
(velocity of sound in air = $330ms^{-1}$) (03marks)

Uneb 2015

(a) Distinquish between **progressive waves** and **stationary waves** . (03marks)

(b) (i) What are overtones?. (01mark)
   (ii) Expalin why a musical note played on one instrument sounds different from the same note played on another instrument. (03marks)

(c) A stretched string of length L, is fixed at both ends and then set to vibrate in its allowed mode. The wire is plucked such that it vibrates in its third harmonic. Calculate the frequency of the third harmonic. (04marks)

(d) A wire of length 0.6m and mass $9x10^{-4}kg$ is under tension of 135N. The wire is plucked ssuch that it vibrates in its third harmonic. Calculate the frequency of the third harmonic. (05marks)

(e) Describe the varation of pressure with displacement of air in a closed pipe vibrating with fundamental frequency. (04marks)

Uneb 2013

(a) (i) Distinquish between **free oscillations** and **damped oscillations.** (02marks)
   (ii) What is meant by **resonance** as applied to sound (01mark)

(b) Describe how you would determine the velocity of sound in air using a resonane tube and several tunning forks of different frequencies. (05marks)

(c) A uniform tube 80cm long is filled with water and a small loudspeaker connected to a signal generator is held over the open end of the tube. With the signal generator set at 600Hz, the water level in the tube is lowered until resonance is first obtained when the length of the air column is 69.8cm long, calculate the;

(i) Velocity of sound in air (04marks)

(ii) Fundamental frequency for the tube if it were open at both ends (03marks)

(d) (i) What is meant by **doppler effect** (01marks)

(ii) A motor cyclist and a police car are approaching each other. The motor cyclist is moving at 10m/s and the police car at 20m/s. if the police siren is sounded at 480Hz, calculate the freqeucny of the note heard by the cyclist after the police car passes by. (03marks)

(iii) Give two applications of Doppler effect. (01marks)

Uneb 2012

(a) What is meat by the following terms as applied to sound

(i) Resonance . (01mark)

(ii) Fundamental Frequency. (01mark)

(b) Describe an experiment to determine the end corrections of a resonance tube. (05marks)

(c) A wire of length 50cm, density 8.0gcm^{-3} is stretched between two points. If the wire is set to vibrate at a fundamental frequency of 15Hz, calculate;

(i) The velocity of the wave along the wire. (03marks)

(ii) The tension per unit length of cross-section of the wire. (03marks)

(d) (i) Explain using the principle of superposition of waves the formation of;

(i) beats (04marks)

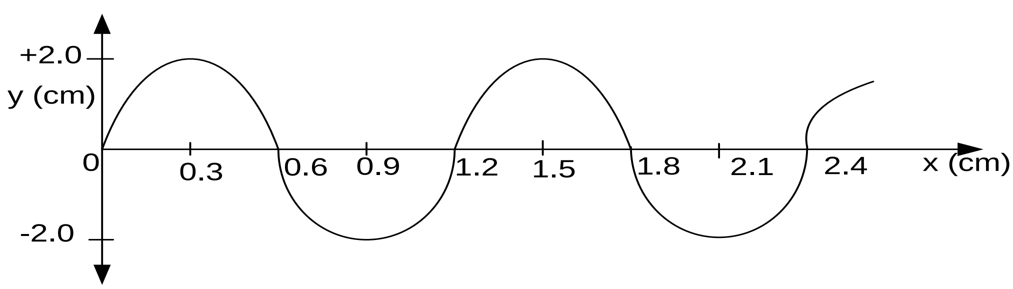
(ii) Stationary wave. (03marks)

Uneb 2011

(a) (i) Define the terms **wave front** and **a ray** in reference to a progressive wave. (02marks)

(ii) Draw a sketch diagram showing reflection of a circular wave by a plane reflector. (02marks)

(b) Figure shows a wave travelling in the positive x-direction away from the origin with a velocity 9ms^{-1}



(i) What is the period of the wave. (03marks)

(ii) Show that the displacement equation for the wave is $y = 2\, sin\frac{5}{3}\pi(9t - x)$. (03marks)

(c) What is meant by **Doppler effect**. (01mark)

(d) One species of bats locates obstacles by emitting high frequency sound waves and detecting the reflected waves. A bat flying at a steady speed of 5ms^{-1} emits sound of frequency 78.0kHz and is reflected back to it.

(i) Derive the equation for the frequency of the sound waves reaching the bat after reflection. (05marks)

(ii) Calculate the frequency of the sound received by the bat given that the speed of sound in air is 340ms^{-1}. (02marks)

(e) (i) What is meant by **intensity** of a sound note. (01marks)

(ii) distinguish between **loudness** and **pitch** of a sound note. (01marks)

Uneb 2010

(a) (i) Define the terms **amplitude** of a wave (01marks)

(ii) State two characteristics of a stationary wave. (02marks)

(iii) A progressive wave $y = asin(\omega t - kx)$ is reflected at a barrier to interfere with the in coming wave. Show that the resultant wave is a stationary one. (04marks)

(b) (i) What is meant by **beats**. (02marks)

(ii) Describe how you can determine the frequency of a musical note using beats. (05marks)

(c) Two open pipes of length 92cm and 93cm are found to give beat frequency of 3.0Hz when each is sounding in its fundamental note. If the end errors are 1.5cm and 1.8cm respectively, calculate the;

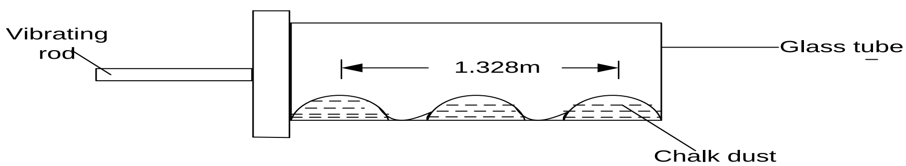
(i) Velocity of sound in air (04marks)

(ii) Frequency of each note. (02marks)

Uneb 2009

(a) (i) A progressive wave is represented by $y = asin2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ is reflected back along the same path. Show how the overlapping of the two waves will give rise to a stationary one. (03marks)

(ii) In an experiment to determine the speed of sound in air, a tube, chalk dust settled in heaps as shown below.



If the frequency of the vibrating rod is 252Hz and the distance between three consecutive heaps is 1.328m, calculate the speed of sound in air. (03marks)

(b) The speed of sound in air is given by $V = \sqrt{\frac{\gamma\, P}{\delta}}$ where p is pressure, δ is density and γ the ratio of the principal heat capacities of air.

Use this expression to explain the effect of temperatureon the speed of air. (03marks)

(c) (i) A train moving with uniform velocity, v_1, sounds its horn as it passes a stationary observer. Derive the expression for apparent frequency of the sound detected by the observer. (03marks)

(ii) If the frequency of thee sound detected by the observer after the train passes is 1.2times lower than the frequency detected in (c) (i), find the speed of the train. [speed of sound in air 340ms^{-1}]. (04marks)

(d) Describe a simple experiment to show interference of longitudinal waves. (04marks)

Uneb 2008

(a) (i) what is **a wave?**. (01mark)

(ii) Explain why an open tube is preferred to a closed tube as a musical instrument. (03marks)

(b) (i) State **two** factors that affect the speed of sound in air. (01mark)

(ii) Explain the term **reverberation**. (02marks)

(iii) What are the implications ooof reverberation in a concert hall? (02marks)

(c) Describe an experiment to determine the velocity of sound in air using a resonane method. (06marks)

(d) (i) What is **a harmonic** in sound (01mark)

(ii) A string of length 0.50m and mass $5.0g$ is stretched between two fixed points. If the tension in the string is 100N. Calculate the frequency of the second harmonic. (04marks)

Uneb 2007

(a) State **three** differences between sound and light waves. (03marks)

(b) Distinquish between **free oscillations** and **damped oscillations**. (02marks)

(c) (i) What is meant by **resonance**? (01mark)

(ii) Describe with aid of a diagram, an experiment to investigate the variation of frequency of a stretched string with length. (06marks)

(d) (i) Calculate the frequency of beats heard by a stationary observer when a source of sound of frequency 80Hz is receeding with a speed of 5.0ms^{-1} towards a vertical wall. [speed of sound in air =340ms^{-1}]. (05marks)

(ii) state **two** uses of beats (02marks)

Uneb 2006

(a) (i) What is meant by **amplitude** and **wavelenght** of a wave (02marks)

(ii) State the differences between a progressive and a stationary wave. (03marks)

(b) The displacement, y of a wave travelling in the x- direction is given at time, t by

$$y = asin2\pi\left(\frac{t}{0.5} - \frac{x}{2.0}\right) \text{ meters}$$

Find the peed of the wave. (04marks)

(c) (i) What is meant bythe terms **overtones** and **beats**. (03marks)

(ii) State **two** uses of beats. (02marks)

(d) A tube 1m long closed at one end has its lowest resonance freqeucny at 86.2Hz. With a tube of identical dimensions but open at both ends, the first resonance occurs at 171Hz. Calculate the speed of sound in air and end corrections. (04marks)

Uneb 2005

(a) Distguish between **progressive** and **stationary** waves. (04marks)

(b) Briefly describe an experiment to show that a wire under tension can vibrate with more than one frequency. (05marks)

(c) A uniform wire of length 1.00m and mass $2.0x10^{-2}kg$ is stretched between two fixed points. The tension of 200N. The wire is plucked in the middle and released. Calculate the ;

(i) Speed of the transverse waves (03marks)

(ii) frequency of the fundamental note. (03marks)

(d) (i) Explain how beats are formed (02marks)

(ii) Derive an expression for beat frequency. (03marks)

Uneb 2004

(a) (i) Distguish between **transverse** and **longitudinal** waves. (02marks)

(ii) Define the wavelenght of a wave. (01marks)

(b) Describe with the aid of a diagram, an experiment to show that how fundamental frequency varies with tension in a given wire. (06marks)

(c) A sound wave propagating in the x- direction is given by an equation

$$y = 2x10^{-7}sin2\pi(8000t - 25x) \text{ meters}$$

Find;

(i) The amplitude. (01marks)

(ii) The speed of the wave (05marks)

(d) Explain why the amplitude of a wave goes on decreasing as the distance from the source increases. (05marks)