

# PURE MATHEMATICS JANUARY HOLIDAY ASSESSMENTS

## SECTION A (40 MARKS)

*Attempt all questions in this section*

1. Find  $y$  if  $\log_y 27 - \log_{y^2} 81 = 1$  (05 marks)
2. Integrate with respect to  $x$ ;  $\int e^{2x} \cos x \, dx$  (05 marks)
3. Find the points of intersection of the curve  $x^2 - 3xy + y^2 + 19 = 0$  and the straight line  $x - y - 3 = 0$ .
4. If  $\alpha$  and  $\beta$  are roots of the quadratic equation  $x^2 - x - 3 = 0$ , find the quadratic equation whose roots are  $\frac{1}{(\alpha + 1)}$  and  $\frac{1}{(\beta + 1)}$  (05 marks)
5. Prove that:  $\tan^{-1} \left( \frac{b}{a+b} \right) + \tan^{-1} \left( \frac{a}{a+2b} \right) = \frac{\pi}{3}$ . (05 marks)
6. Given that  $ye^x = \sin x + \cos x$  show that  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$ . (05 marks)
7. Find the equation of the normal to the curve  $x^2 - 2xy - 2y^2 + x = 2$  at the point  $(-4, 1)$ . (05 marks)
8. Find the possible values of  $y$  given that:  

$$\int_0^y (8x^3 - 27x^2 + 26x - 6) dx = 0$$
 (05 marks)

## SECTION B

*Attempt only five questions from this section*

9. (a) Solve the equation  $\cos 7\theta + \cos 5\theta + \cos 3\theta + \cos \theta = 0$  for  $0^\circ \leq \theta \leq 360^\circ$  (06 marks)  
 (b) Prove that:  $\cos 3\beta = 4\cos^3 \beta - 3\cos \beta$ , hence solve the equation  
 $1 + \cos 3\beta = \cos \beta (1 + \cos \beta)$  for  $0^\circ \leq \beta \leq 360^\circ$  (06 marks)
10. Express  $\frac{3-x}{(x+1)(x^2+1)}$  in partial fractions and hence differentiate  $\frac{3-x}{(x+1)(x^2+1)}$  (12 marks)
11. Find the values of  $x$  and  $y$  in the simultaneous equations;  
 (a)  $y \log_2 8 = x$   
 $2^x + 8^y = 8192$  (6mks)  
 (b)  $\log_2 x + 2 \log_4 y = 4$   
 $x + 12y = 52$  (6mks)
12. ) Prove that  $\frac{\sin 3A \sin 6A + \sin A \sin 2A}{\sin 3A \cos 6A + \sin A \cos 2A} = \tan 5A$  6 mks  
 b) Show that  $\tan 3\theta = \frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3 \tan^2 \theta}$  6 mks
13. a) Evaluate i)  $\int_0^{\frac{\pi}{2}} \sin 2\theta \cos \theta \, d\theta$  4 mks  
 ii)  $\int x^2 \sin 2x \, dx$  4 mks  
 iii)  $\int_0^2 \frac{8x}{x^2 - 4x - 12} \, dx$  4 mks

**END**