

Charge in an electric field

The electric field strength E is defined as the force per unit positive charge. Hence the force on a particle of charge q in an electric field E is given by

$$F = qE \dots\dots\dots (1)$$

The effect of this force is that the particle is being accelerated

From the equation $F = ma \dots\dots\dots (2)$

The acceleration of the particle is $a = \frac{qE}{m} \dots\dots\dots (3) \text{ from (1) and (2)}$

Where m is the mass of the charged particle

The potential difference V between two points in an electric field is defined as the work done per unit positive charge. When a particle of charge q is accelerated through a potential difference V , the energy gained by the particle is

$$\begin{aligned} (\text{The gain in energy}) &= \left(\begin{array}{l} \text{Work done on the charged} \\ \text{particle by the electric field} \end{array} \right) \\ \text{energy} &= qV \dots\dots\dots (4) \end{aligned}$$

If the charged particle is an electron e

$$\text{energy} = eV$$

When the electron is accelerated through a potential difference of one volt

The energy gained

$$\begin{aligned} \text{energy} &= e \times 1 \\ &= 1.6 \times 10^{-19} \text{ J} \end{aligned}$$

This energy is known as **electron volt** ($1eV$).

An electron volt ($1eV$) is the energy gained by an electron accelerated in vacuum through a potential difference of one volt

From the discussion above, $1eV = 1.6 \times 10^{-19} \text{ J}$
This is a unit of energy which is particularly useful in atomic and nuclear physics. *See the conversions to the right*

(a). $3.488 \times 10^{-17} \text{ J} \Rightarrow \frac{3.488 \times 10^{-17}}{1.6 \times 10^{-19}} \equiv 21.8eV$
(b). $1.8eV \Rightarrow 1.8 \times 1.6 \times 10^{-19} \equiv 2.88 \times 10^{-19} \text{ J}$
(c). $4.2MeV \Rightarrow 4.2 \times 10^6 \times 1.6 \times 10^{-19} \equiv 6.72 \times 10^{-13} \text{ J}$

Since Kinetic energy is the energy possessed by a body in motion, then equation (4) can precisely be written as

$$K.E = qV$$

Example: Electrons are emitted with negligible speed from a plane cathode in evacuated tube. The electrons are accelerated towards the plane anode, which is parallel to the cathode and 2.0cm from it, by a p.d of 1000V. Calculate the;

- acceleration of the electrons
- kinetic energy of the electrons as they reach the anode
- speed with which the electrons strike the anode
- time taken by the electrons to move from cathode to anode

Specific charge of an electron $\left(\frac{e}{m}\right) = 1.8 \times 10^{11} \text{ C kg}^{-1}$

Solution:

- (i) the electric field intensity E between cathode and anode (parallel plates) is given by;

$$E = \frac{V}{d} = \frac{1000}{2.0 \times 10^{-2}} = 5.0 \times 10^4 \text{ V m}^{-1}$$

$$a = \frac{eE}{m} = (1.8 \times 10^{11}) \times (5.0 \times 10^4) = 9.0 \times 10^{15} \text{ m s}^{-2}$$

- (ii) $K.E = eV = 1.6 \times 10^{-19} \times 1000 = 1.6 \times 10^{-16} \text{ J}$

- (iii) $K.E = \frac{1}{2}mu^2$ and $K.E = eV$

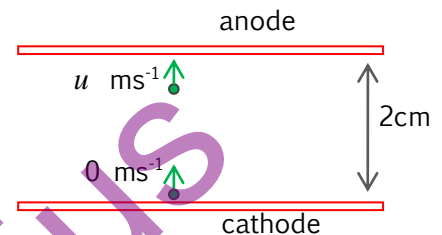
$$\Rightarrow \frac{1}{2}mu^2 = eV$$

$$u = \sqrt{2 \left(\frac{e}{m}\right) V} = \sqrt{2 \times 1.8 \times 10^{11} \times 1000}$$

$$= 1.890 \times 10^7 \text{ m s}^{-1}$$

- (iv) $V = u + at$, $u = 0$, $V = 1.90 \times 10^7 \text{ m s}^{-1}$

$$t = \frac{1.90 \times 10^7}{9.0 \times 10^{15}} = 2.11 \times 10^{-9} \text{ s}$$



Alternatively: using Newtonian mechanics

$$V^2 = u^2 + 2as$$

$$V^2 = 0 + 2 \times 9.0 \times 10^{15} \times 2 \times 10^{-2}$$

$$\Rightarrow V = 1.890 \times 10^7 \text{ m s}^{-1}$$

Question: A uniform electric field is obtained between two parallel plates by using p.d of 10V and a plate separation of 20mm. An electron initially at rest close to the negative plate is moved by the field to the positive plate. Calculate;

- the intensity of the field Ans: $5.0 \times 10^2 \text{ V m}^{-1}$
- the force acting on the electron Ans: $8.0 \times 10^{-17} \text{ N}$
- the speed of the electron as it arrives at the positive plate Ans: $1.9 \times 10^6 \text{ m s}^{-1}$

Mass of an electron = $9.11 \times 10^{-31} \text{ kg}$ and electronic charge = $1.6 \times 10^{-19} \text{ C}$

Electron Dynamics

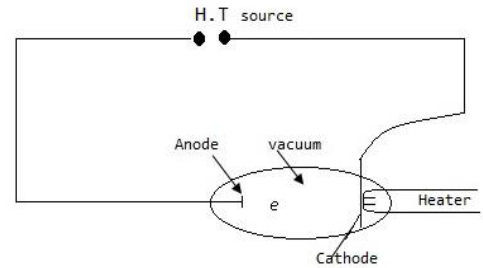
Consider an electron moving from cathode to Anode.

- If the p.d between the cathode and anode is V , then the electron will be accelerated by the electric field and hence it gains K.E.
- If the electron starts from the cathode with zero velocity and reaches the anode with velocity $u \text{ ms}^{-1}$, then the K.E gained by the electron will be

$$\frac{1}{2}mu^2 = eV, \text{ where } e = \text{electronic charge, } 1.6 \times 10^{-19} \text{C}$$

$$u = \sqrt{\left(\frac{2eV}{m}\right)} \quad \text{OR} \quad u = \sqrt{\left\{2\left(\frac{e}{m}\right)V\right\}}.$$

The quantity $\left(\frac{e}{m}\right)$ is called the specific charge of the electron



Example:

Calculate the kinetic energy in (a) joules (b) eV when each of the following charged particles are accelerated from rest through a p.d of 50.0V

- An electron
- A proton
- Ion of iron (Fe^{3+})

Approach:

Theorem

$$\begin{aligned} \left(\text{The gain in K.E} \right) &= \left(\text{Work done on the charged particle by the electric field} \right) \\ \text{K.E} &= QV \end{aligned}$$

(a) For an electron, $Q = e = 1.6 \times 10^{-19} \text{C}$

$$\begin{aligned} \text{K.E} &= 1.6 \times 10^{-19} \times 50 \\ &= 8.0 \times 10^{-19} \text{J} \end{aligned}$$

(b) in eV, $\text{K.E} = 50 \text{eV}$

For a proton, $Q = e = 1.6 \times 10^{-19} \text{C}$

$$\begin{aligned} \text{K.E} &= 1.6 \times 10^{-19} \times 50 \\ &= 8.0 \times 10^{-19} \text{J} \end{aligned}$$

In eV, $\text{K.E} = 50 \text{eV}$

For an ion of iron (Fe^{3+}), $Q = 3e = 4.8 \times 10^{-19} \text{C}$

$$\begin{aligned} \text{K.E} &= 4.8 \times 10^{-19} \times 50 \\ &= 2.4 \times 10^{-17} \text{J} \end{aligned}$$

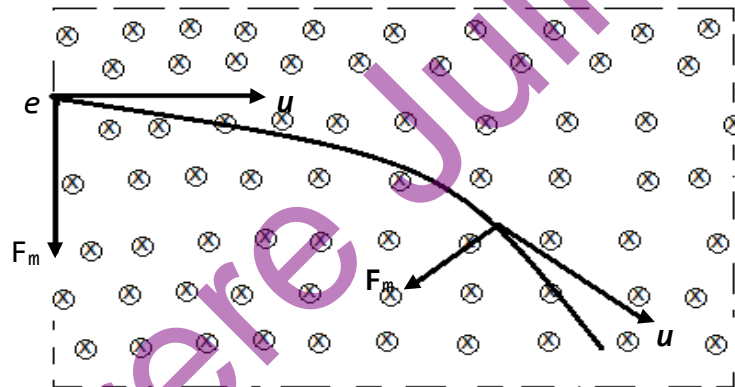
In eV, $\text{K.E} = 150 \text{eV}$

Exercise:

- Calculate the speed of a proton which has been accelerated through a p.d of 400V (mass of a proton = $1.67 \times 10^{-27} \text{kg}$).
Ans: $2.77 \times 10^5 \text{m/s}$
- The K.E of an alpha particle from a radioactive source is 4.0MeV. What is its speed?
(Take $m_\alpha = 6.4 \times 10^{-27} \text{kg}$ $e = 1.6 \times 10^{-19} \text{C}$)
Ans: $1.414 \times 10^7 \text{m/s}$
- An electron is accelerated from rest through a p.d of 1000V. What is: (a). its K.E in eV (b) its speed (Take $m_e = 9.11 \times 10^{-31} \text{kg}$, $1 \text{eV} = 1.6 \times 10^{-19} \text{J}$).
Ans: $1.6 \times 10^{-16} \text{J}$, $1.87 \times 10^7 \text{m/s}$
- What value of the p.d between the cathode and anode that will accelerate an electron from the cathode to a speed of $2.91 \times 10^7 \text{m/s}$.
Ans: 2.4Kv

Deflection of an electron in a magnetic field

Consider an electron entering a uniform magnetic field of flux B, at right angles to its direction of motion with velocity u .



When the electron enters the field, the magnitude of its speed u does not change (*because the magnetic force is perpendicular to the direction of the electron*), But instead its direction changes and the electron moves in a circular arc.

— Let r be the radius of the circular arc (path)

— The centripetal force on the electron $F = \frac{mu^2}{r}$ (1)

— The force due to the magnetic field $F = Beu$ (2)

$$\text{From (1) and (2) } \Rightarrow \frac{mu^2}{r} = Beu \quad \therefore r = \frac{mu}{Be}$$

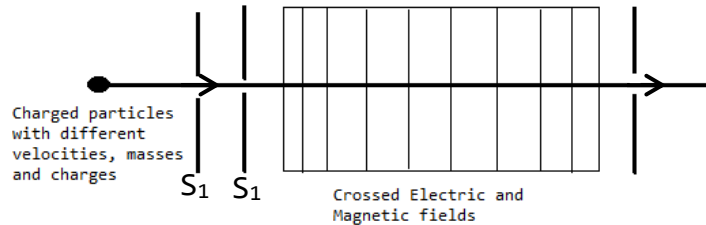
Since the speed of the electron is constant, its K.E is also constant and expressed as Kinetic

energy $\frac{1}{2}mv^2 = \frac{e^2 B^2 r^2}{2m}$ Task: derive this formulae .

Example: An electron moves in a circular path at $3.0 \times 10^6 \text{m/s}$ in a uniform magnetic field of flux $2.0 \times 10^{-4} \text{T}$. Find the radius of the path (mass of an electron $m_e = 9.11 \times 10^{-31} \text{kg}$, $e = 1.6 \times 10^{-19} \text{C}$).
Ans: 8.5cm

CROSSED FIELDS

- Crossed fields are fields in which a uniform magnetic field and a uniform electric field are perpendicular to each other producing deflections opposite to each other.
- If the magnetic force and electric force in the crossed fields are of the same magnitude, there is no deflection on charged particles that enter such fields.



- The slits S_1 and S_2 confine the particles into a narrow beam as they enter the crossed fields.
- The only particles that will emerge at slit S_3 are those which are undeflected by the crossed fields, and therefore they all will emerge with the same velocity U .
- The electric force F_E due to the electric field = eE
- The magnetic force F_m due to the magnetic field = BeU

For crossed fields $F_E = F_m$

$$eE = Beu$$

$$\therefore u = \frac{E}{B} \text{ This is the velocity of the particle emerging at } S_3$$

- Therefore, all particles that emerge at S_3 will have the same velocity $\therefore u = \frac{E}{B}$ regardless of their mass and charge.
- The crossed fields can be used as a velocity selector of particles of a single velocity from a beam of particles of different velocities.

Example: An electron accelerated by a p.d of 1.5KV passes through an electric field crossed with a uniform magnetic field of flux density 0.45T. Calculate the value of the electric field needed for the electron to emerge undeflected.

Solution:

First obtain the speed of the electron *just* before it enters the crossed field

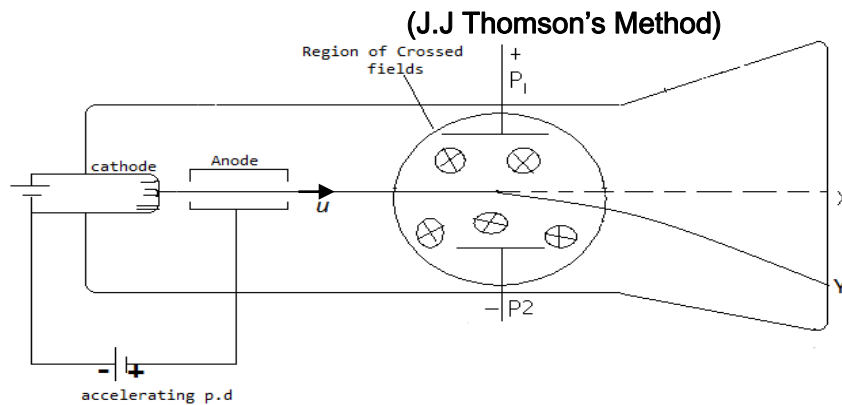
$$\frac{1}{2}mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 2.295 \times 10^7 \text{ ms}^{-1}$$

For an electron to pass through the crossed fields undeflected

$$F_E = F_m \Rightarrow E = Bu$$

$$E = 1.033 \times 10^7 \text{ NC}^{-1}$$

Determination of Specific Charge $\left(\frac{e}{m}\right)$ of an electron



- The electrons are produced thermionically by a hot filament cathode and are accelerated towards a cylindrical anode and pass through it.
- The small hole on the anode confines the electrons to a narrow beam.
- When both the electric field and the magnetic field are off, the electrons reach the screen at X and cause fluorescence.
- If the velocity of the electrons on emerging from the anode is u then

$$eV_a = \frac{1}{2}mu^2 \quad \Rightarrow \left(\frac{e}{m}\right) = \frac{u^2}{2V_a} \quad \dots\dots\dots (1)$$

Where V_a is the accelerating voltage between the cathode and anode

- The magnetic field is switched on and the beam is deflected to position Y.
- In order to bring the beam back to the original position X, the electric field is switched on and adjusted until the beam is at X again. This implies that

The magnetic force = the electric force

$$Beu = eE \quad \text{Hence } u = \frac{E}{B}$$

$$\text{Thus from (1), } \left(\frac{e}{m}\right) = \frac{E^2}{2B^2V_a} \quad \text{but } E = \frac{V}{d},$$

$$\boxed{\left(\frac{e}{m}\right) = \frac{V^2}{2B^2d^2V_a}} \quad \text{where } V \text{ is the P.d between the plates at a separation of } d \text{ apart.}$$

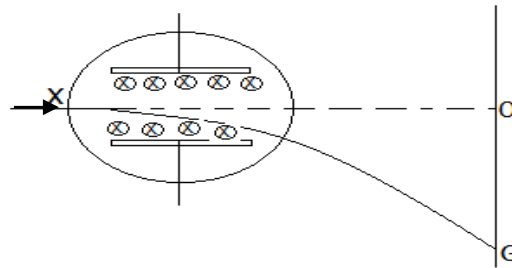
Exercise

- A beam of protons is accelerated from rest through a potential difference of 200V and then enters a uniform magnetic field which is perpendicular to the deflection of the proton beam. If the flux density is 0.2T, calculate the radius of the path which the beam describes. (Proton mass = 1.7×10^{-27} Kg, $e = 1.6 \times 10^{-19}$ C).
- A particle of charge 3.2×10^{-19} C is acceleration from rest through a p.d of 10.0KV. It enters a region of uniform magnetic field of flux density 0.5T. The particle describes a circular path of radius 8.94cm. Find;
 - The K.E of the particle on entering the magnetic field
 - the mass of the particle.

Ans: (i) 3.2×10^{-15} J

(ii). 3.196×10^{-36} kg

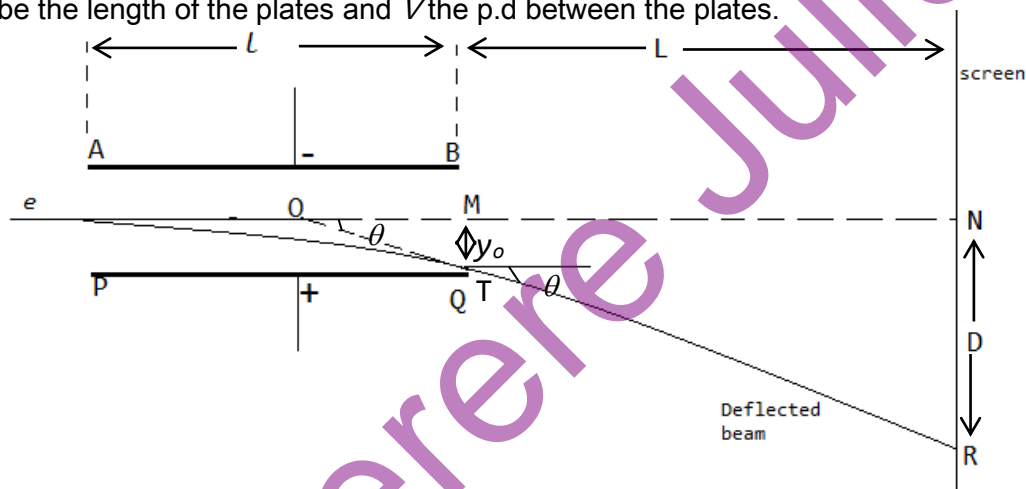
3. In an experiment to determine the specific charge of an ion, the ion is projected horizontally along a region of uniform magnetic field of flux density 0.4 T at X as shown below.



On leaving the magnetic field the ions strike the screen at G . When an electromagnetic field is applied perpendicularly to the magnetic field, the ion returns to O . If the path of the ion in the region of magnetic field is an arc of circle of radius 2.0 cm and the electric field intensity is $1.408 \times 10^5 \text{ Vm}^{-1}$. Calculate the specific charge of the ion. (Ans: $4.4 \times 10^7 \text{ Ckg}^{-1}$)

Motion of an electron in an electric field

Consider two parallel plates AB and PQ such that AB is vertically above PQ and at a distance d apart. Let l be the length of the plates and V the p.d between the plates.



- ✓ The electric force F_E experienced by the electron is given by $F_E = eE$
Where E = the electric field intensity
- ✓ This electric force is directed towards the positive plate causing the deflection of the beam as shown above.
- ✓ But for parallel plates $E = \frac{V}{d}$ Thus $F_E = \frac{eV}{d}$
- ✓ Since the electric field intensity is vertical, there is no horizontal force acting on the electron. Hence, the horizontal component of the velocity of the electron does not change
- ✓ Let u be the horizontal component of the velocity of the electron entering the electric field.

*We analyze the motion of the electron by considering separately its **vertical component of motion** and **horizontal component of motion***

Motion in the X-direction

$$s = ut + \frac{1}{2}at^2 \text{ but } s = x, u_x = u \text{ and } a = 0$$

$$\therefore t = \frac{x}{u} \dots \dots \dots (1)$$

Motion in the y-direction

$$s = ut + \frac{1}{2}at^2 \text{ but } u_y = 0, s = y \text{ and } a_y = \frac{eE}{m} \text{ from } ma = eE$$

$$\text{Thus } y = \frac{1}{2} \left(\frac{eE}{m} \right) \left(\frac{x}{u} \right)^2$$

$$y = \left(\frac{eE}{2mu^2} \right) x^2, \Rightarrow \therefore y \propto x^2 \text{ or } y = kx^2 \text{ which is a parabola}$$

N.B: Thus the motion of the electron while in the electric field is parabolic.

NOTE:

1. If y_0 is the vertical deflection of the electron just at the end of the plate, with E the electric field, then: $y = y_0$ and $x = l$

$$\text{From } s = ut + \frac{1}{2}at^2 \Rightarrow y_0 = \left(\frac{eE}{m} \right) \frac{l^2}{u^2} \dots\dots\dots (2)$$

2. The time t_0 taken for the electron to pass through the electric field (leave the plates)

$$\text{When } t = t_0, x = l \Rightarrow t_0 = \frac{l}{u} \dots\dots\dots (3)$$

3. The velocity V_0 with which electrons leave the plates

$$V_0 = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow v_x = u, \text{ and}$$

$$\Rightarrow v_y = u_y + a_y t, \text{ But } u_y = 0 \text{ and } a_y = \frac{eE}{m}$$

$$\text{Thus } v_y = \left(\frac{eE}{m} \right) t_0 \text{ also } t = t_0 \Rightarrow t_0 = \frac{l}{u}$$

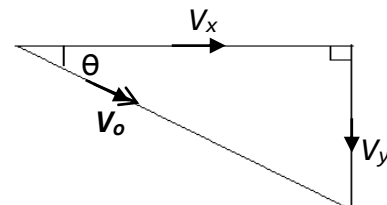
$$v_y = \left(\frac{eE}{m} \right) \frac{l}{u}, \text{ Where } E = \frac{V}{d}$$

$$\text{Substitute for } v_y \text{ and } v_x \text{ in } V_0 = \sqrt{v_x^2 + v_y^2}$$

4. **Direction (angle):** The electron emerges from the region between the plates at an angle θ to the horizontal, given by:

$$\tan \theta = \frac{v_y}{v_x}$$

$$\tan \theta = \frac{v_y}{v_x} = \frac{eEl}{mu^2} \dots\dots\dots (4)$$



5. **The deflection D** of the electron on the screen placed at a distance L from the edge of the plates is obtained from:

$$\tan \theta = \left(\frac{\Delta}{L + \frac{1}{2}l} \right) \text{ but from (4)}$$

$$\left(\frac{eEl}{mu^2} \right) = \left(\frac{\Delta}{L + \frac{1}{2}l} \right), \text{ hence } \Delta = \frac{eEl}{mu^2} \left(L + \frac{1}{2}l \right) \dots\dots\dots (5)$$

Exercise

1. A beam of electrons is accelerated through a p.d of 500V and enters a uniform electric field of strength $3.0 \times 10^3 \text{ V/m}$ created by two parallel plates of length 2.0cm. Calculate:
- the speed of the electrons as they enter the field.
 - the time that each electron spends in the field.
 - the angle from which the electrons have been deflected by the time they emerge from the field.

Solution:

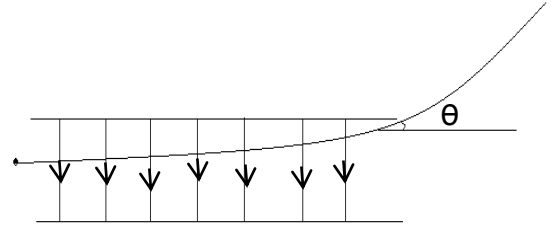
$$(a) \text{ From } \frac{1}{2} mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 1.327 \times 10^7 \text{ ms}^{-1}$$

$$(b) \text{ From } u = \frac{l}{t_0} \quad \therefore t_0 = \frac{2.0 \times 10^{-2}}{1.327 \times 10^7} = 1.51 \times 10^{-9} \text{ s}$$

$$u_x = u = 1.327 \times 10^7 \text{ ms}^{-1}$$

$$(c) u_y = a_y t_0 \text{ But } a_y = \frac{eE}{m} \Leftrightarrow u_y = 7.957 \times 10^5 \text{ ms}^{-1}$$

$$\text{Let } \theta \text{ be the angle; } \tan \theta = \frac{u_y}{u_x} \quad \therefore \theta = 3.43^\circ$$



2. An electron gun operated at $3 \times 10^3 \text{ V}$ is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm. Calculate the deflection of the electrons as they emerge from the region between the charged plates when the p.d is $1.0 \times 10^3 \text{ V}$.

Solution:

$$\text{From } \frac{1}{2} mu^2 = eV \quad \therefore u = \sqrt{\frac{2eV}{m}} = 3.246 \times 10^7 \text{ ms}^{-1}$$

$$\text{Also } s = ut + at^2 \Leftrightarrow y_0 = a_y t_0^2 \text{ but } a_y = \frac{eE}{m} \text{ and } t_0^2 = \frac{l}{u_x}$$

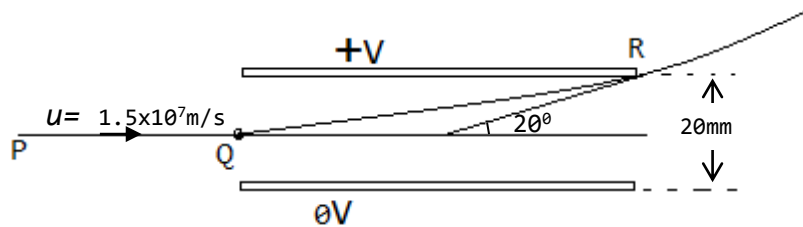
$$\Rightarrow y_0 = \frac{1}{2} \left(\frac{eE}{m} \right) \left(\frac{l}{u_x} \right)^2 \text{ and } E = \frac{V}{d} = 2 \times 10^4 \text{ Vm}^{-1}$$

$$\therefore y_0 = 1.667 \times 10^{-2} \text{ m}$$

3. A beam of electrons is accelerated through a p.d of 2000V and is directed mid-way between two horizontal parallel plates of length 5.0cm and separation of 2.0cm. The p.d across the plates is 80V.
- Calculate the speed of the electrons as they enter the region of between the plates
 - Find the speed of the electrons as they emerge from the region between the plates.
 - Explain the motion of the electrons between the plates.

$$\text{Ans: } 2.652 \times 10^7 \text{ ms}^{-1}, 2.653 \times 10^7 \text{ ms}^{-1}$$

4. A beam of electrons moving at a uniform speed of $1.5 \times 10^7 \text{ m/s}$ in a vacuum enters the space between two plane parallel deflecting plates along the line PQ as shown in the fig below.



PQ is along the middle of the space between the plates. The plates are 40mm long and separated by 20mm. The upper plate is at a positive potential V relative to the lower plate. The electron emerges from the plates at a point R with a speed of $1.60 \times 10^7 \text{ m/s}$ and at an angle of 20° to its initial direction.

- How long does the electron take to move from the Q to R? Hence find the acceleration of the electron during the deflection.
- Find the value of the p.d V which produces the deflection

Solution:

$$\text{From } u = \frac{l}{t_0} \quad \therefore t_0 = \frac{40.0 \times 10^{-3}}{1.5 \times 10^7} = 2.67 \times 10^{-9} \text{ s}$$

$$(a) \text{ Hence: } a_y = \frac{u_y}{t_0} \text{ but } u_y = \sqrt{V^2 - u_x^2} \equiv 5.6 \times 10^6 \text{ ms}^{-1}$$

$$\therefore a_y = 2.1 \times 10^{15} \text{ ms}^{-2}$$

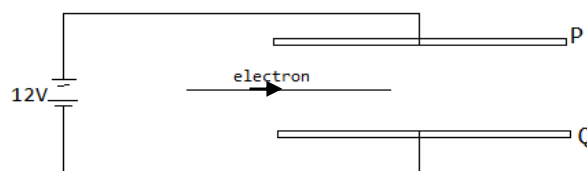
$$(b) a_y = \frac{eE}{m}, \quad E = \frac{V}{d}$$

$$\Rightarrow V \equiv 239 \text{ V}$$

- Two parallel plates 4cm long are held horizontally, 3cm apart in vacuum, one being vertically above the other. The upper plate is at a potential of 300V and the lower is earthed. Electrons having velocity $1.0 \times 10^7 \text{ m/s}$ are ejected horizontally mid-way between the plates. Calculate the deflection of the electron beam as it emerges from the plates. ($\frac{e}{m} = 1.8 \times 10^{11} \text{ C kg}^{-1}$).

Ans: 1.41cm

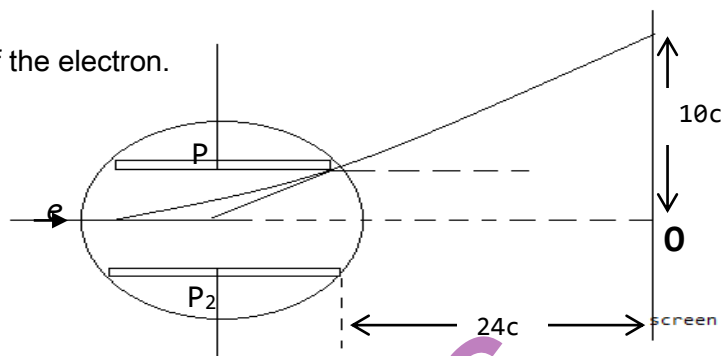
6.



In the diagram above PQ are parallel plates of length 4.0cm and 4.0cm apart. A p.d of 12.0V is applied between the plates P and Q, the space between P and Q is in vacuum. A beam of electrons of speed $1.0 \times 10^6 \text{ m/s}$ is directed mid-way between P and Q. Show that the electron beam emerges from the space between P and Q at an angle of 64.6° to the initial direction of the beam. ($m_e = 9.11 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$)

7. In the fig below, the metal plates P_1 and P_2 are metal plates each of length 2.0cm and are separated by a distance of 5.0mm, in a uniform magnetic field of $4.7 \times 10^{-3} \text{T}$. An electron beam incident mid-way between the plates is deflected by the magnetic field by a distance of 10.0cm on a screen placed a distance of 24cm from the end of the plates. When a p.d of 1.0Kv is applied between P_1 and P_2 , the electron spot on the screen is restored to the undeflected position O.

Calculate the charge to mass ratio of the electron.



Solution:

$$\tan \theta = \frac{\Delta y}{L + \frac{1}{2}l} \equiv \frac{10}{25} \dots\dots\dots (i)$$

$$\text{But also } \tan \theta = \frac{u_y}{u_x} \dots\dots\dots (ii)$$

since the electron spot is restored on the screen

$$F_m = F_E \Rightarrow u = \frac{E}{B} \text{ but } E = \frac{V}{d} = \frac{1 \times 10^3}{5 \times 10^{-3}} = 2 \times 10^5 \text{Vm}^{-1}$$

$$u_x = u = \frac{E}{B} = 4.255 \times 10^7 \text{ms}^{-1}$$

$$u_y = a_y t_0 \text{ But } a_y = \frac{eE}{m}$$

$$\Leftrightarrow u_y = \left(\frac{eE}{m} \right) \frac{l}{u_x} \equiv \left(\frac{e}{m} \right) \frac{El}{u_x}$$

$$\text{from (ii) } \tan \theta = \left(\frac{e}{m} \right) \frac{El}{u_x^2}$$

Combine (i) and (ii)

$$\left(\frac{e}{m} \right) = \left(\frac{10}{25} \right) \frac{u_x^2}{El} \equiv 1.81 \times 10^{11} \text{Ckg}^{-1}$$

8. An electron of energy 10KeV enters mid-way between two horizontal metal plates each of length 5cm and separated by a distance of 2cm. A p.d of 20V is applied across the plates. A fluorescent screen is placed 20cm beyond the plates. Calculate the vertical deflection of the electron on the screen.

(Take $m_e = 9.11 \times 10^{-31} \text{kg}$, $e = 1.6 \times 10^{-19} \text{C}$) Ans: $5.63 \times 10^{-4} \text{m}$

9. A beam of cathode rays is directed mid-way between two parallel metal plates of length 4cm and separation 1cm. The beam is deflected through 10cm on a fluorescent screen placed 20cm beyond the nearest edge of the plates when a p.d of 200V is applied across the plates. If this deflection is annulled (*canceled*) by the magnetic field of flux density $1.14 \times 10^{-3} \text{T}$ applied normal to the electric field between the plates. Find the ratio of mass to the charge of the cathode rays.

Ans: $1.75 \times 10^{11} \text{Ckg}^{-1}$

Solution:

$$\text{since } F_m = F_E \therefore u = u_x = \frac{E}{B} \text{ and } E = \frac{V}{d}$$

$$\text{From } \frac{\Delta y}{L + \frac{1}{2}l} = \frac{u_y}{u_x} \text{ and } u_y = \left(\frac{eE}{m} \right) \left(\frac{l}{u_x} \right)$$

$$\Rightarrow \left(\frac{e}{m} \right) = 1.75 \times 10^{11} \text{Ckg}^{-1}$$

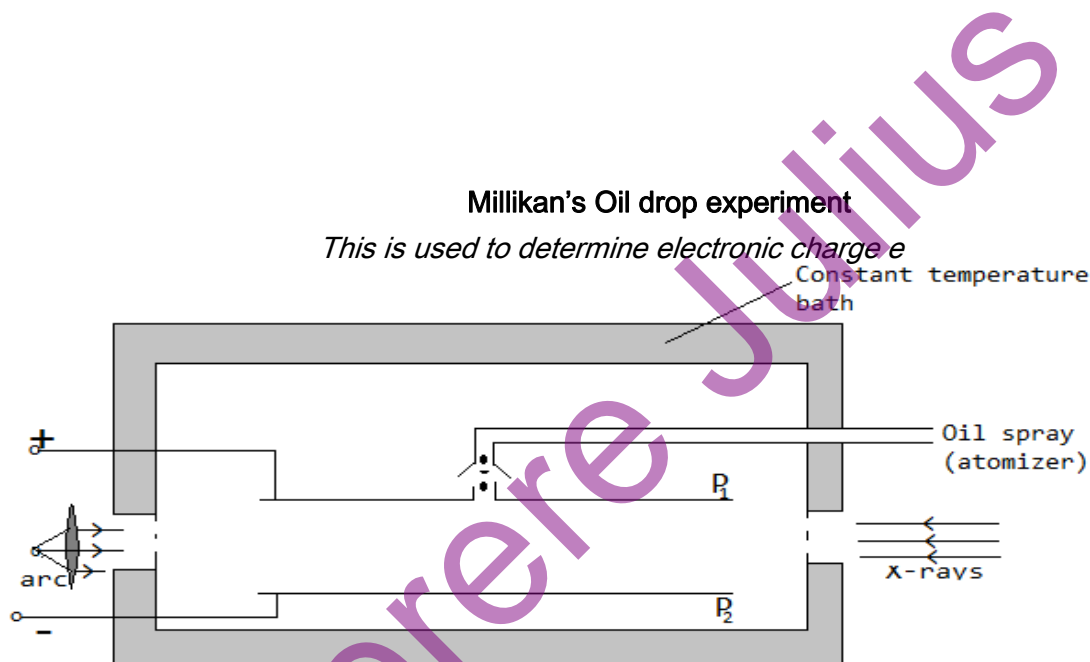
10. In a C.R.O an electron beam passes between the y -deflector plates each 5cm long and 0.5cm apart. The distance between the center of the y -plates and the screen is 20cm and the p.d between the anode and the electron gun is 250V. Determine the deflection in volts per meter of the electron beam on the screen of the C.R.O.

(Take $m_e = 9.11 \times 10^{-31} \text{kg}$, $e = 1.6 \times 10^{-19} \text{C}$) Ans: 1.0

$$\text{From } \frac{1}{2} mu^2 = eV_1 \quad \therefore u = \sqrt{\frac{2eV_1}{m}} = 9.371 \times 10^6 \text{ ms}^{-1}$$

$$u_y = a_y t_0 \quad \text{But } a_y = \frac{eE}{m}, \quad E = \frac{V_2}{d} \quad \text{and } t_0 = \frac{l}{u_x}$$

$$\text{But p.d } V_1 = V_2 \Rightarrow \Delta y \equiv 1.0 \text{m}$$



Procedure

- Set up of the apparatus is as shown above
- Oil drops are introduced between the plates P_1 and P_2 by spraying using the atomizer.
- These oil drops are charged in the process of spraying by friction.
- The oil drops are strongly illuminated by an intense light from the arc lamp so that they appear as bright spots when observed through a low power microscope.
- With no electric field between the plates, record the time t_1 taken for drop to fall from P_1 to P_2 .
- The electric field between the plates is turned on and adjusted so that the drop becomes stationary.

Theory:

Case I

With no electric field:

The oil drop falls with a uniform velocity (called *terminal velocity*) v_1 and has no acceleration.

The terminal velocity $v_1 = \frac{d}{t_1}$, where d is the separation between the plates.

$$\text{Weight} = \text{Upthrust} + \text{Viscous drag} \text{-----} (1)$$

$$\text{weight} = \text{Volume of the drop} \times \text{density of oil} \times g$$

$$\text{weight} = \frac{4}{3}\pi r^3 \rho g$$

$$\text{Upthrust} = \text{weight of the air displaced by the drop}$$

$$\text{weight} = \text{Volume of the drop} \times \text{density of air} \times g$$

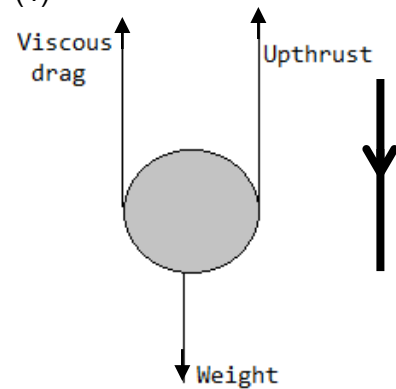
$$\text{Upthrust} = \frac{4}{3}\pi r^3 \sigma g$$

$$\text{Viscous drag} = 6\pi r \eta v_1 \text{ (From Stokes' law).}$$

From (1)

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_1 \text{-----} (2)$$

$$r = \left[\frac{9\eta v_1}{2g(\rho - \sigma)} \right]^{\frac{1}{2}} \text{ This is used to find the radius } r \text{ of the oil drop}$$



Case II

When the electric field is applied so that the drop is stationary, the drop has no velocity and no acceleration.

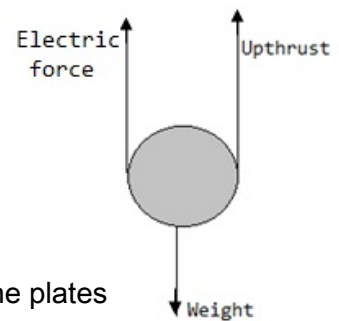
$$\text{Weight} = \text{Upthrust} + \text{Electric force}$$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + qE \text{-----} (3)$$

Combine equations (2) and (3)

$$q = \frac{6\pi r \eta v_1}{E} \text{ Substitute for } r \text{ from above.}$$

$$q = \frac{6\pi \eta}{E} \left(\frac{9\eta v_1}{2g(\rho - \sigma)} \right)^{\frac{1}{2}} v_1, \text{ and } E = \frac{V}{d} \text{ where } V \text{ is the p.d between the plates}$$



Note: the density of air at room temperature is very small compared to that of oil and thus may be assumed negligible (*when it's not given*) in calculations. This implies that the upthrust due to the

displaced air is Zero. Thus $q = \frac{6\pi \eta}{E} \left(\frac{9\eta v_1}{2g\rho} \right)^{\frac{1}{2}} v_1$

Precautions

To improve on the accuracy of the experiment, the following precautions need to be taken into account

- 1) A non-volatile oil or low vapour pressure oil should be used to reduce evaporation. Evaporation would alter the mass of the drop
- 2) The experiment is enclosed in a constant temperature enclosure. This is to eliminate convection currents and changes in the viscosity of air as a result of temperature changes.
- 3) An X-ray tube is used to increase the charge of the oil drop.

Note:

- When Millikan repeated the experiment using different oil drops of different radii and carrying different charges, this set of results were obtained $1.6 \times 10^{-19} \text{ C}$, $3.21 \times 10^{-19} \text{ C}$, $6.46 \times 10^{-19} \text{ C}$, $9.66 \times 10^{-19} \text{ C}$ etc... This results show that the charge q is always an integral multiple of $1.6 \times 10^{-19} \text{ C}$. Hence he concluded that the charge on an electron is $-1.6 \times 10^{-19} \text{ C}$
- This experiment also shows that charge is *quantized*, The fundamental unit of charge e is $1.6 \times 10^{-19} \text{ C}$. All charges are integral multiples of this fundamental charge. It is not possible to have a charge less than the fundamental charge

Example 1:

Oil droplets are introduced into the space between two flat horizontal plates, set 5.0mm apart. The plate voltage is then adjusted to exactly 780V so that one of the droplets is held stationary. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given that the density of the oil used is 900 kg m^{-3} and the viscosity of air is $1.8 \times 10^{-5} \text{ N s m}^{-2}$, find;

- | | |
|---|---|
| The radius of the oil drop | Ans: $1.1 \times 10^{-6} \text{ m}$ |
| i) The charge of the droplets | Ans: $1.56 \times 10^{-5} \text{ V m}^{-1}$ |
| ii) The number of electrons on the drop | Ans: 2 electrons |

Example 2:

- a) Calculate the radius of a drop of oil of density 900 kg m^{-3} which falls with a terminal velocity of $2.9 \times 10^{-4} \text{ m/s}$ through air of viscosity $1.8 \times 10^{-5} \text{ N s m}^{-2}$. Ans: $r = 16.3 \text{ mm}$
- b) If the charge on the drop is $-3e$, what p.d must be applied between the plates 5.0mm apart in order to keep the drop stationary? Ans: 1665.8V

Example 3:

An oil drop carrying a charge of $3e$ falls under gravity in air with a velocity of $4.6 \times 10^{-4} \text{ m/s}$ between two parallel plates 5mm. when a p.d of 4.6Kv is applied between the plates, the drop rises steadily. Assuming the effect of air (buoyancy) on the drop is negligible. Calculate the velocity with which the oil drop rises.

Example 4:

In Millikan's oil drop experiment, an oil drop of density 890 kg m^{-3} and radius $2.35 \times 10^{-4} \text{ m}$ has an excess charge of 10 electrons.

- (i) Find the free fall terminal velocity with the electric field turned off
- (ii) What values of the electric field intensity is needed to produce zero net force on the droplet.

(Assume: density and viscosity of are 1.0 kg m^{-3} and $1.83 \times 10^{-5} \text{ N s m}^{-2}$ respectively)

Example 5:

In a Millikan type apparatus, the horizontal plates are 1.5cm apart. With the electric field is switched off, an oil drop is observed to fall with a steady velocity of $2.5 \times 10^{-2} \text{ cm/s}$. When the field is switched on, the upper plate being positive, the drop just remains stationary when the p.d between the plates is 1500V.

- (a) Calculate the radius of the drop
- (b) How many electronic charges does it carry?

- (c) If the p.d between the plates remains unchanged, with what velocity will the drop move when it has collected two more electrons.

(Take density of oil as 900kgm^{-3} and viscosity of air as

$1.83 \times 10^{-5} \text{Nsm}^{-2}$)

Solution:

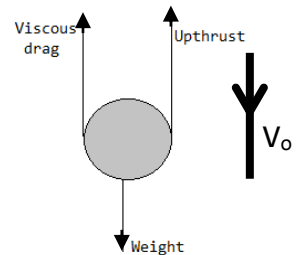
- a) When the drop is falling steadily

Weight = Upthrust + Viscous drag

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_o$$

But the density of air $\sigma = 0$ can be neglected

$$r = \left[\frac{9\eta v_o}{2g\rho} \right]^{\frac{1}{2}} \Rightarrow r = 1.52 \times 10^{-6} \text{m}$$



- b) Let the number of electronic charges (electrons) on the oil drop be n .

$$\therefore \text{Total Charge on the drop } Q = ne \text{----- (1)}$$

When the drop is held stationary, then

Weight = Upthrust + Electric force

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + QE$$

But $\sigma = 0$ can be neglected

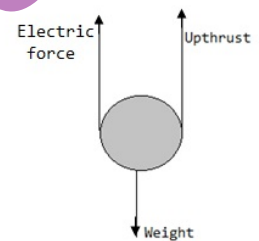
$$Q = \frac{4\pi r^3 \rho g}{3E} \text{ But } E = \frac{V}{d} \Leftrightarrow Q = \frac{4\pi r^3 \rho g d}{3V}$$

$$\therefore Q = \text{This is the total charge on the drop}$$

$$\text{From (1), } n = \frac{Q}{e} \quad n = 8$$

- c) When the drop has collected two more electrons, then

Total Charge $Q = 10e$.



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